WBJEE 2023 Question Paper

West Bengal Joint Entrance Examinations

WBJEE 2023 Solved Paper

Mathematics

Question 1

$\lim_{x \to \infty} \{x - \sqrt[n]{(x-a_1)(x-a_2)\ldots(x-a_n)}\}$ where $a_1, a_2, \ldots, a_n$ are positive rational numbers. The limit

Options:

A. does not exist

B. is $\frac{a_1 + a_2 + \ldots a_n}{n}$

C. is $\sqrt[n]{a_1 a_2 \ldots a_n}$

D. is $\frac{n}{a_1 + a_2 + \ldots a_n}$

Answer: B

Solution:

Solution:

Question 2

Suppose f : R → R be given by

$$f(x) = \begin{cases} 1 & \text{if } x = 1 \\ e^{(x^{10}-1)} + (x-1)^2 \sin\frac{1}{x-1} & \text{if } x \neq 1 \end{cases}.$$

then

Options:

A. $f'(1)$ does not exist

B. $f'(1)$ exists and is zero

C. $f'(1)$ exist and is 9

D. $f'(1)$ exists and is 10

Answer: D

Solution:

Solution:

Question 3

Let f : [1, 3] → R be continuous and be derivable in (1, 3) and $f'(x) = [f(x)]^2 + 4\ \forall x \in (1, 3)$. Then

Options:

A. $f(3) - f(1) = 5$ holds

B. $f(3) - f(1) = 5$ does not hold

C. $f(3) - f(1) = 3$ holds

D. $f(3) - f(1) = 4$ holds

Answer: B

Solution:

Solution:

Question 4

f (x) is a differentiable function and given $f'(2) = 6$ and $f'(1) = 4$, then $L = \lim_{h \to 0} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$

Options:

A. does not exist

B. equal to -3

C. equal to 3

D. equal to 3/2

Answer: C

Solution:

Solution:

Question 5

Let $\cos^{-1}\left(\frac{y}{b}\right) = \log_e\left(\frac{x}{n}\right)^n$, then $Ay_2 + By_1 + Cy = 0$ is possible for, where $y_2 = \frac{d^2y}{dx^2}$, $y_1 = \frac{dy}{dx}$

Options:

A. A = 2, B = x^2, C = n

B. A = x^2, B = x, C = n^2

C. A = x, B = 2x, C = 3n + 1

D. A = x^2, B = 3x, C = 2n

Answer: B

Solution:

Solution:

Question 6

If I $= \int \frac{x^2\,dx}{(x\sin x + \cos x)^2} = f(x) + \tan x + c$, then f (x) is

Options:

A. $\frac{\sin x}{x\sin x + \cos x}$

B. $\frac{1}{(x\sin x + \cos x)^2}$

C. $\frac{-x}{\cos x(x\sin x + \cos x)}$

D. $\frac{1}{\sin x(x\cos x + \sin x)}$

Answer: C

Solution:

Solution:

Question 7

If $\int \frac{dx}{(x+1)(x-2)(x-3)} = \frac{1}{k}\log_e\left\{\frac{|x-3|^3\,|x+1|}{(x-2)^4}\right\} + c$, then the value of k is

Options:

A. 4

B. 6

C. 8

D. 12

Answer: D

Solution:

Solution:

Question 8

the expression $\frac{\int_0^n [x]\,dx}{\int_0^{n\}x\}} dx}$, where [x] and {x} are respectively integral and fractional part of x and n ∈ N , is equal to

Options:

A. $\frac{1}{n-1}$

B. $\frac{1}{n}$

C. n

D. $n-1$

Answer: D

Solution:

Solution:

Question 9

The value $\int_0^{1/2} \frac{dx}{\sqrt{1-x^{2n}}}$ is (n ∈ N)

Options:

A. less than or equal to $\frac{\pi}{6}$

B. greater than or equal to 1

C. less than $\frac{1}{2}$

D. greater than $\frac{\pi}{6}$

Answer: A

Solution:

Solution:

Question 10

If $I_n = \int_0^{\frac{\pi}{2}} \cos^n x \cos nx \, dx$, then I_1, I_2, I_3... are in

Options:

A. A.P.

B. G.P.

C. H.P.

D. no such relation

Answer: D

Solution:

Solution:

Question 11

If $y = \frac{x}{\log_e |cx|}$ is the solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \varphi\left(\frac{x}{y}\right)$, then $\varphi\left(\frac{x}{y}\right)$ is given by

Options:

A. $\frac{y^2}{x^2}$

B. $-\frac{y^2}{x^2}$

C. $\frac{x^2}{y^2}$

D. $-\frac{x^2}{y^2}$

Answer: B

Solution:

Solution:

Question 12

The function $y = e^{kx}$ satisfies $\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right)\left(\frac{dy}{dx} - y\right) = y\frac{dy}{dx}$. It is valid for

Options:

A. exactly one value of k.

B. two distinct values of k.

C. three distinct values of k.

D. three distinct values of k.

Answer: C

Solution:

Solution:

Question 13

Given $\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y\,\mathrm{cosec}^2 x = 0$. Changing the independent variable x to z by the substitution $z = \log \tan \frac{x}{2}$, the equation is changed to

Options:

A. $\frac{d^2y}{dz^2} + \frac{3}{y} = 0$

B. $2\frac{d^2y}{dz^2} + e^y = 0$

C. $\frac{d^2y}{dz^2} - 4y = 0$

D. $\frac{d^2y}{dz^2} + 4y = 0$

Answer: D

Solution:

Solution:

Question 14

Let f (x) = $\begin{cases} x+1 & -1 \le x \le 0 \\ -x & 0 < x \le 1 \end{cases}$.

Options:

A. f (x) is discontinuous in [−1, 1] and so has no maximum value or minimum value in [−1, 1]

B. f (x) is continuous in [−1, 1] and so has maximum value and minimum value.

C. f (x) is discontinuous in [−1, 1] but still has the maximum and minimum value.

D. f (x) is bounded in [−1, 1] and does not attain maximum or minimum value.

Answer: C

Solution:

Solution:

Question 15

A missile is fired from the ground level rises x meters vertically upwards in sec, where x = 100t − $\frac{25}{2}t^2$. The maximum height reached is

Options:

A. 100m

B. 300m

C. 200m

D. 125m

Answer: C

Solution:

Solution:

Question 16

If a hyperbola passes through the point P($\sqrt{2}$, $\sqrt{3}$) and has foci at (± 2, 0), then the tangent to this hyperbola at P is

Options:

A. y = x$\sqrt{6}$ − $\sqrt{3}$

B. y = x$\sqrt{3}$ − $\sqrt{6}$

C. y = x$\sqrt{6}$ + $\sqrt{3}$

D. y = x$\sqrt{3}$ + $\sqrt{6}$

Answer: A

Solution:

Solution:

Question 17

A, B are fixed points with coordinates (0, a) and (0, b)(a > 0, b > 0). P is variable point (x, 0) referred to rectangular axis. If the angle ∠APB is maximum, then

Options:

A. $x^2 = ab$

B. $x^2 = a + b$

C. $x = \frac{1}{ab}$

D. $x = \frac{a+b}{2}$

Answer: A

Solution:

Solution:

Question 18

The average length of all vertical chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $a \le x \le 2a$, is :

Options:

A. $b\{2\sqrt{3} - \ln(2 + \sqrt{3})\}$

B. $b\{3\sqrt{2} + \ln(3 + \sqrt{2})\}$

C. $a\{2\sqrt{5} - \ln(2 + \sqrt{5})\}$

D. $a\{5\sqrt{2} + \ln(5 + \sqrt{2})\}$

Answer: A

Solution:

Solution:

The equation of the hyperbola is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

For a given value of x in the interval $[a, 2a]$, the corresponding y values can be calculated from the hyperbola equation as

follows :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Solving for y, we get $y = \pm b\sqrt{\frac{x^2}{a^2} - 1}$.

The length of a vertical chord at a given x value will be the difference of the y values, which gives us:

$$\text{Length of chord } = 2b\sqrt{\frac{x^2}{a^2} - 1}$$

To calculate the average length of all vertical chords from x = a to x = 2a, we take the definite integral of this function over that interval, and divide by the length of the interval :

$$\text{Average length of chord } = \frac{1}{2a - a}\int_a^{2a} 2b\sqrt{\frac{x^2}{a^2} - 1}\,dx$$

Simplifying this gives us :

$$\text{Average length of chord } = \frac{2b}{a}\int_a^{2a}\sqrt{\frac{x^2}{a^2} - 1}\,dx$$

$$= \frac{2\int_a^{2a} y\,dx}{\int_a^{2} dx} = \frac{2\int_a^{2a}\frac{b}{a}\sqrt{x^2 - a^2}\,dx}{(x)_a^{2a}}$$

$$= \frac{\frac{2b}{a}\int_a^{2a}\sqrt{x^2 - a^2}\,dx}{a} = \frac{2b}{a^2}\int_0^{2a}\sqrt{x^2 - a^2}\,dx$$

$$= \frac{2b}{a^2}\left(\frac{x}{3}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right|\right)_a^{2a}$$

$$= \frac{2b}{a^2}\left[\frac{(2a)\sqrt{4a^2 - a^2}}{2} - \frac{a^2\ln\left|2a + \sqrt{4a^2 - a^2}\right|}{2} - \frac{a\sqrt{a^2 - a^2}}{2} + \frac{a^2\ln\left|a + \sqrt{a^2 - a^2}\right|}{2}\right]$$

$$= \frac{2b}{a^2}\left[\sqrt{3}a^2 - \frac{a^2\ln|\,2 + \sqrt{3}\,)\,a|}{2} + \frac{a^2\ln|\,a|}{2}\right]$$

$$= 2b\left(\sqrt{3} + \frac{\ln\left|\frac{a}{(2 + \sqrt{3})a}\right|}{2}\right)$$

$$= b(2\sqrt{3} - \ln|\,2 + \sqrt{3}\,|)$$

Question 19

The value of ' a ' for which the scalar triple product formed by the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + widehat\,k$, $\vec{\beta} = \hat{j} + a\,widehat\,k$ and $\vec{\gamma} = a\hat{i} + widehat\,k$ is maximum, is

Options:

A. 3

B. -3

C. $-\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{3}}$

Answer: C

Solution:

Solution:

Question 20

If the vertices of a square are z_1, z_2, z_3 and z_4 taken in the anticlockwise order, then z_3 =

Options:

A. $-iz_1 - (1 + i)z_2$

B. $z_1 - (1 + i)z_2$

C. $z_1 + (1 + i)z_2$

D. $-iz_1 + (1 + i)z_2$

Answer: D

Solution:

Solution:

Question 21

If the n terms a_1, a_2, ……, a_n are in A.P. with increment r, then the difference between the mean of their squares & the square of their mean is

Options:

A. $\frac{r^2\{(n-1)^2-1\}}{12}$

B. $\frac{r^2}{12}$

C. $\frac{r^2(n^2-1)}{12}$

D. $\frac{n^2-1}{12}$

Answer: C

Solution:

Solution:

Question 22

If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P., then x equals

Options:

A. $\log_3 4$

B. $1-\log_3 4$

C. $1-\log_4 3$

D. $\log_4 3$

Answer: B

Solution:

Solution:

Question 23

Reflection of the line $\bar{a}z + a\bar{z} = 0$ in the real axis is given by:

Options:

A. $az + \overline{az} = 0$

B. $\bar{a}z - a\bar{z} = 0$

C. $az - \overline{az} = 0$

D. $\frac{a}{z} + \frac{\bar{a}}{z} = 0$

Answer: A

Solution:

Solution:

Question 24

If one root of $x^2 + px - q^2 = 0$, p and q are real, be less than 2 and other be greater than 2 , then

Options:

A. $4 + 2p + q^2 > 0$

B. $4 + 2p + q^2 < 0$

C. $4 + 2p - q^2 > 0$

D. $4 + 2p - q^2 < 0$

Answer: D

Solution:

Solution:

Question 25

The number of ways in which the letters of the word 'VERTICAL' can be arranged without changing the order of the vowels is

Options:

A. $6! \times 3!$

B. $\frac{8!}{3}$

C. $6! \times 3$

D. $\frac{8!}{3!}$

Answer: D

Solution:

Solution:

Question 26

n objects are distributed at random among n persons. The number of ways in which this can be done so that at least one of them will not get any object is

Options:

A. $n! - n$

B. $n^n - n$

C. $n^n - n^2$

D. $n^n - n!$

Answer: D

Solution:

Solution:

Question 27

Let $P(n) = 3^{2n+1} + 2^{n+2}$ where $n \in N$. Then

Options:

A. P(n) is not divisible by any prime integer.

B. there exists prime integer which divides P(n).

C. P(n) is divisible by 5 for all $n \in N$.

D. P(n) is divisible by 3 for all $n \in N$.

Answer: B

Solution:

Solution:

Question 28

Let A be a set containing n elements. A subset P of A is chosen, and the set A is reconstructed by replacing the elements of P. A subset Q of A is chosen again. The number of ways of choosing P and Q such that Q contains just one element more than P is

Options:

A. ${}^{2n}C_{n-1}$

B. ${}^{2n}C_{n}$

C. ${}^{2n}C_{n+2}$

D. 2^{2n+1}

Answer: A

Solution:

Solution:

Question 29

Let A and B are orthogonal matrices and $\det A + \det B = 0$. Then

Options:

A. A + B is singular

B. A + B is non-singular

C. A + B is orthogonal

D. A + B is skew symmetric

Answer: A

Solution:

Solution:

Question 30

Let A = $\begin{pmatrix} 2 & 0 & 3 \\ 4 & 7 & 11 \\ 5 & 4 & 8 \end{pmatrix}$ **. Then**

Options:

A. det A is divisible by 11

B. det A is not divisible by 11

C. det A = 0

D. A is orthogonal matrix

Answer: A

Solution:

Solution:

Question 31

If the matrix M_r **is given by** $M_r = \begin{pmatrix} r & r-1 \\ r-1 & r \end{pmatrix}$ **for r = 1, 2, 3, ... then** $\det(M_1) + \det(M_2) + \ldots + \det(M_{2008}) =$

Options:

A. 2007

B. 2008

C. $(2008)^2$

D. $(2007)^2$

Answer: C

Solution:

Solution:

Question 32

Let α, β be the roots of the equation $ax^2 + bx + c = 0$, a, b, c real and

$s_n = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix} = k\frac{(a+b+c)^2}{a^4}$ then k =

Options:

A. $b^2 - 4ac$

B. $b^2 + 4ac$

C. $b^2 + 2ac$

D. $4ac - b^2$

Answer: A

Solution:

Solution:

Question 33

Let A, B, C are subsets of set X . Then consider the validity of the following set theoretic statement:

Options:

A. A ∪ (B\C) = (A ∪ B)\(A ∪ C)

B. (A\B)\C = A\(B ∪ C)

C. (A ∪ B)\A = A\B

D. A\C = B\C

Answer: B

Solution:

Solution:

Question 34

Let X be a nonvoid set. If ρ_1 and ρ_2 be the transitive relations on X, then (o denotes the composition of relations)

Options:

A. $\rho_1 \circ \rho_2$ is transitive relation

B. $\rho_1 \circ \rho_2$ is not transitive relation

C. $\rho_1 \circ \rho_2$ is equivalence relation

D. $\rho_1 \circ \rho_2$ is not any relation on X

Answer: C

Solution:

Solution:

Question 35

Let A and B are two independent events. The probability that both A and B happen is $\frac{1}{12}$ and probability that neither A and B happen is $\frac{1}{2}$. Then

Options:

A. P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$

B. P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{6}$

C. P(A) = $\frac{1}{6}$, P(B) = $\frac{1}{2}$

D. P(A) = $\frac{2}{3}$, P(B) = $\frac{1}{8}$

Answer: A

Solution:

Solution:

Question 36

Let S be the sample space of the random experiment of throwing simultaneously two unbiased dice and $E_k = \{(a, b) \in S : ab = k\}$. If $p_k = P(E_k)$, then the correct among the following is :

Options:

A. $p_1 < p_{10} < p_4$

B. $p_1 < p_8 < p_{14}$

C. $p_1 < p_8 < p_{17}$

D. $p_1 < p_{16} < p_5$

Answer: A

Solution:

Solution:

Question 37

If $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in G.P, then the solution set of θ is (Here $n \in N$)

Options:

A. $2n\pi \pm \frac{\pi}{6}$

B. $2n\pi \pm \frac{\pi}{3}$

C. $n\pi + (-1)^n \frac{\pi}{3}$

D. n pi + frac pi 3

Answer: B

Solution:

Solution:

Question 38

The equation $r^2\cos^2\left(\theta - \frac{\pi}{3}\right) = 2$ represents

Options:

A. a parabola

B. a hyperbola

C. a circle

D. a pair of straight lines

Answer: D

Solution:

Solution:

Question 39

Let A be the point (0, 4) in the xy-plane and let B be the point (2t, 0). Let L be the midpoint of AB and let the perpendicular bisector of AB meet the y-axis M . Let N be the midpoint of LM. Then locus of N is

Options:

A. a circle

B. a parabola

C. a straight line

D. a hyperbola

Answer: B

Solution:

Solution:

Question 40

If $4a^2 + 9b^2 - c^2 + 12ab = 0$, then the family of straight lines $ax + by + c = 0$ is concurrent at

Options:

A. $(2, 3)$ or $(-2, -3)$

B. $(-2, 3)$ or $(2, 3)$

C. $(3, 2)$ or $(-3, 2)$

D. $(-3, 2)$ or $(2, 3)$

Answer: A

Solution:

Solution:

Question 41

The straight lines x + 2y − 9 = 0, 3x + 5y − 5 = 0 and ax + by − 1 = 0 are concurrent if the straight line 35x − 22y + 1 = 0 passes through the pointThe straight lines x + 2y − 9 = 0, 3x + 5y − 5 = 0 and ax + by − 1 = 0 are concurrent if the straight line 35x − 22y + 1 = 0 passes through the point

Options:

A. (−a, −b)

B. (a, −b)

C. (−a, b)

D. (a, b)

Answer: D

Solution:

Solution:

Question 42

ABC is an isosceles triangle with an inscribed circle with centre 0 . Let P be the midpoint of BC. If AB = AC = 15 and BC = 10, then OP equals

Options:

A. $\frac{\sqrt{5}}{\sqrt{2}}$ unit

B. $\frac{5}{\sqrt{2}}$ unit

C. $2\sqrt{5}$ unit

D. $5\sqrt{2}$ unit

Answer: B

Solution:

Solution:

Question 43

Let O be the vertex, Q be any point on the parabola $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is:

Options:

A. $x^2 = y$

B. $y^2 = x$

C. $y^2 = 2x$

D. $x^2 = 2y$

Answer: D

Solution:

Solution:

Question 44

The tangent at point $(a\cos\theta, b\sin\theta)$, $0 < \theta < \frac{\pi}{2}$, to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the x-axis at T and y-axis at T_1. Then the value of $\min_{0 < \theta < \frac{\pi}{2}} (OT)(OT_1)$ is

Options:

A. ab

B. 2ab

C. 0

D. 1

Answer: B

Solution:

Solution:

Question 45

Let $A(2\sec\theta, 3\tan\theta)$ and $B(2\sec\varphi, 3\tan\varphi)$ where $\theta + \varphi = \frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$. If (α, β) is the point of intersection of normals to the hyperbola at A and B, then β is equal to

Options:

A. $\frac{12}{3}$

B. $\frac{13}{3}$

C. $-\frac{12}{3}$

D. $-\frac{13}{3}$

Answer: D

Solution:

Solution:

Question 46

If the lines joining the focii of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a > b, and an extremity of its minor axis is inclined at an angle 60˚, then the eccentricity of the ellipse is

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. $\frac{\sqrt{7}}{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:

Question 47

If the distance between the plane $\alpha x - 2y + z = k$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |k| is

Options:

A. 36

B. 12

C. 6

D. $2\sqrt{3}$

Answer: C

Solution:

Solution:

Question 48

The angle between a normal to the plane $2x - y + 2z - 1 = 0$ and the X-axis is

Options:

A. $\cos^{-1}\frac{2}{3}$

B. $\cos^{-1}\frac{1}{5}$

C. $\cos^{-1}\frac{3}{4}$

D. $\cos^{-1}\frac{1}{3}$

Answer: A

Solution:

Solution:

Question 49

Let f (x) = $[x^2]\sin \pi x$, x > 0. Then

Options:

A. f is discontinuous everywhere.

B. f is continuous everywhere.

C. f is continuous at only those points which are perfect squares.

D. f is continuous at only those points which are not perfect squares.

Answer: B

Solution:

Solution:

Question 50

If $y = \log^n x$, where $\log^n$ means $\log_e \log_e \log_e \ldots$ (repeated n times), then $x \log x \log^2 x \log^3 x \ldots\ldots \log^{n-1} x \log^n x \frac{dy}{dx}$ is equal to

Options:

A. $\log x$

B. x

C. 1

D. $\log^n x$

Answer: D

Solution:

Solution:

Question 51

$\int_0^{2\pi} \theta \sin^6 \theta \cos \theta \, d\theta$ is equal to

Options:

A. $\frac{\pi}{16}$

B. $\frac{3\pi}{16}$

C. $\frac{16\pi}{3}$

D. 0

Answer: D

Solution:

Solution:

Question 52

If $x = \sin\theta$ and $y = \sin k\theta$, then $(1 - x^2)y_2 - xy_1 - \alpha y = 0$, for α =

Options:

A. k

B. $-k$

C. $-k^2$

D. k^2

Answer: C

Solution:

Solution:

Question 53

In the interval $(-2\pi, 0)$, the function $f(x) = \sin\left(\frac{1}{x^3}\right)$.

Options:

A. never changes sign.

B. changes sign only once.

C. changes sign more than once but finitely many times.

D. changes sign infinitely many times.

Answer: D

Solution:

Solution:

Question 54

The average ordinate of $y = \sin x$ over $[0, \pi]$ is :

Options:

A. $\frac{2}{\pi}$

B. $\frac{3}{\pi}$

C. $\frac{4}{\pi}$

D. π

Answer: A

Solution:

Solution:
The average value of a function $f(x)$ over an interval $[a, b]$ is given by the formula:

$$\text{Average} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

In this case, we want to find the average value of the function $y = \sin(x)$ over the interval $[0, \pi]$. We can use the formula above, with $f(x) = \sin(x)$, $a = 0$, and $b = \pi$:

$$\text{Average} = \frac{1}{\pi-0}\int_0^{\pi} \sin(x)\,dx$$

The integral of $\sin(x)$ from 0 to π is $-\cos(\pi) + \cos(0) = -(-1) + 1 = 2$.
So, the average value is :

$$\text{Average} = \frac{1}{\pi}\cdot 2 = \frac{2}{\pi}$$

So, the correct answer is Option A, $\frac{2}{\pi}$.

Question 55

The portion of the tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, a > 0 at any point of it, intercepted between the axes

Options:

A. varies as abscissa

B. varies as ordinate

C. is constant

D. varies as the product of abscissa and ordinate

Answer: C

Solution:

Solution:

Question 56

If the volume of the parallelopiped with $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ as conterminous edges is 9 cu. units, then the volume of the parallelopiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})$, $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$, and $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as conterminous edges is

Options:

A. 9 cu. units

B. 729 cu. units

C. 81 cu. units

D. 243 cu. units

Answer: C

Solution:

Solution:

Question 57

Given f (x) = $e^{\sin x} + e^{\cos x}$. The global maximum value of f (x)

Options:

A. does not exist.

B. exists at a point in $\left(0, \frac{\pi}{2}\right)$ and its value is $2e^{\frac{1}{\sqrt{2}}}$.

C. exists at infinitely many points.

D. exists at x = 0 only.

Answer: B

Solution:

Solution:

Question 58

Consider a quadratic equation $ax^2 + 2bx + c = 0$ where a, b, c are positive real numbers. If the equation has no real root, then which of the following is true?

Options:

A. a, b, c cannot be in A.P. or H.P. but can be in G.P.

B. a, b, c cannot be in G.P. or H.P. but can be in A.P.

C. a, b, c cannot be in A.P. or G.P. but can be in H.P.

D. a, b, c cannot be in A.P., G.P. or H.P.

Answer: C

Solution:

Solution:

Question 59

Let $a_1, a_2, a_3, \ldots, a_n$ be positive real numbers. Then the minimum value of $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \ldots + \frac{a_n}{a_1}$ is

Options:

A. 1

B. n

C. ${}^{n}C_{2}$

D. 2

Answer: B

Solution:

Solution:

Question 60

Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $P \begin{pmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{pmatrix}$ be an orthogonal matrix such that $B = PAP^{-1}$ holds. Then

Options:

A. x = 1 = y

B. x = 1, y = 0

C. x = 0, y = 1

D. x = −1, y = 0

Answer: A

Solution:

Solution:

Question 61

Let ρ be a relation defined on set of natural numbers N, as ρ = {(x, y) ∈ N × N : 2x + y = 4}. Then domain A and range B are

Options:

A. A ⊂ {x ∈ N : 1 ≤ x ≤ 20} and
B ⊂ {y ∈ N : 1 ≤ y ≤ 39}

B. A = {x ∈ N : 1 ≤ x ≤ 15} and
B = {y ∈ N : 1 ≤ y ≤ 30}

C. A ≡ N , B = Q

D. A = Q, B = Q

Answer: A

Solution:

Solution:

Question 62

From the focus of the parabola $y^2 = 12x$, a ray of light is directed in a direction making an angle $\tan^{-1}\frac{3}{4}$ with x-axis. Then the equation of the line along which the reflected ray leaves the parabola is

Options:

A. y = 2

B. y = 18

C. y = 9

D. y = 36

Answer: B

Solution:

Solution:

Question 63

The locus of points (x, y) in the plane satisfying $\sin^2 x + \sin^2 y = 1$ consists of

Options:

A. a circle centered at origin

B. infinitely many circles that are all centered at the origin

C. infinitely many lines with slope \pm 1

D. finitely many lines with slope \pm 1

Answer: C

Solution:

Solution:

Question 64

The value of $\lim_{n \to \infty} \left[\left(\frac{1}{2 \cdot 3} + \frac{1}{2^2 \cdot 3} \right) + \left(\frac{1}{2^2 \cdot 3^2} + \frac{1}{2^3 \cdot 3^2} \right) + \ldots + \left(\frac{2}{2^n \cdot 3^n} + \frac{1}{2^{n+1} \cdot 3n} \right) \right.$ is

Options:

A. $\frac{3}{8}$

B. $\frac{3}{10}$

C. $\frac{3}{14}$

D. $\frac{3}{16}$

Answer: B

Solution:

Solution:

Question 65

The family of curves $y = e^{a \sin x}$, where ' a ' is arbitrary constant, is represented by the differential equation

Options:

A. $y \log y = \tan x \frac{dy}{dx}$

B. $y \log x = \cot x \frac{dy}{dx}$

C. $\log y = \tan x \frac{dy}{dx}$

D. $\log y = \cot x \frac{dy}{dx}$

Answer: A

Solution:

Solution:

Question 66

Let f be a non-negative function defined on $\left[0, \frac{\pi}{2}\right]$. If $\int_0^x (f'(t) - \sin 2t)\,dt = \int_x^0 f(t)\tan t\,dt$, $f(0) = 1$ then $\int_0^{\frac{\pi}{2}} f(x)\,dx$ is

Options:

A. 3

B. $3 - \frac{\pi}{2}$

C. $3 + \frac{\pi}{2}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

Solution:

Question 67

A balloon starting from rest is ascending from ground with uniform acceleration of $4\,ft/\,sec^2$. At the end of 5 sec, a stone is dropped from it. If T be the time to reach the stone to the ground and H be the height of the balloon when the stone reaches the ground, then

Options:

A. T = 6 sec

B. $H = 112.5\,ft$

C. T = 5/2 sec

D. 225 ft

Answer:B, C

Solution:

Solution:

Question 68

If $f(x) = 3\sqrt[3]{x^2} - x^2$, then

Options:

A. f has no extrema.

B. f is maximum at two points x = 1 and x = −1.

C. f is minimum at x = 0.

D. f has maximum at x = 1 only.

Answer:B, C

Solution:

Solution:

Question 69

If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1$, then $\frac{z_1}{z_2}$ may be

Options:

A. real positive

B. real negative

C. zero

D. purely imaginary

Answer: C

Solution:

Solution:

Question 70

A letter lock consists of three rings with 15 different letters. If N denotes the number of ways in which it is possible to make unsuccessful attempts to open the lock, then

Options:

A. 482 divides N

B. N is the product of two distinct prime numbers.

C. N is the product of three distinct prime numbers.

D. 16 divides N.

Answer: A, B, C

Solution:

Solution:

Question 71

If R and R^1 are equivalence relations on a set A, then so are the relations

Options:

A. R^{-1}

B. $R \cup R^1$

C. $R \cap R^1$

D. All of these

Answer: A, C

Solution:

Solution:

Question 72

Let f be a strictly decreasing function defined on R such that $f(x) > 0, \forall x \in R$. Let $\frac{x^2}{f(a^2+5a+3)} + \frac{y^2}{f(a+15)} = 1$ be an ellipse with major axis along the y-axis. The value of 'a' can lie in the interval (s)

Options:

A. $(-\infty, -6)$

B. $(-6, 2)$

C. $(2, \infty)$

D. $(-\infty, \infty)$

Answer: A, C

Solution:

Solution:

Question 73

A rectangle ABCD has its side parallel to the line y = 2x and vertices A, B, D are on lines y = 1, x = 1 and x = −1 respectively. The coordinate of C can be

Options:

A. (3, 8)

B. (−3, 8)

C. (−3, −1)

D. (3, −1)

Answer: A, C

Solution:

Solution:

Question 74

Let $f(x) = x^m$, m being a non-negative integer. The value of m so that the equality $f'(a + b) = f'(a) + f'(b)$ is valid for all a, b > 0 is

Options:

A. 0

B. 1

C. 2

D. 3

Answer: A, C

Solution:

Solution:

Question 75

Which of the following statements are true?

Options:

A. If f (x) be continuous and periodic with periodicity T , then $I = \int_a^{a+T} f(x)\,dx$ depend on 'a'.

B. If f (x) be continuous and periodic with periodicity T , then $I = \int_a^{a+T} f(x)\,dx$ does not depend on 'a'.

C. Let f(x) = $\begin{cases} 1 & \text{if x is rational} \\ 0 & \text{if x is irrational} \end{cases}$, then f is periodic of the periodicity T only if T is rational.

D. f defined in (C) is periodic for all T .

Answer: B

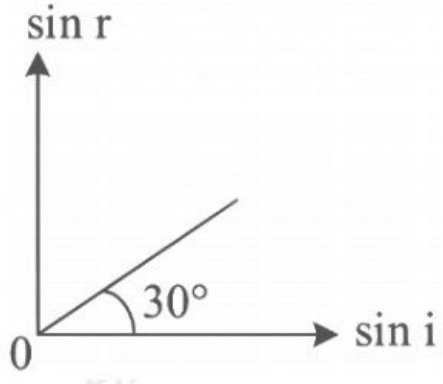
Solution:

Solution:

Physics

Question 41

A ray of monochromatic light is incident on the plane surface of separation between two media X and Y with angle of incidence ' i ' in medium X and angle of refraction ' r ' in medium Y. The given graph shows the relation between sin i and sin r. If V_X and V_Y are the velocities of the ray in media X and Y respectively, then which of the following is true?



Options:

A. $V_X = \frac{1}{\sqrt{3}}V_Y$

B. $V_X = \sqrt{3}V_Y$

C. Total internal reflection can happen when the light is incident in medium X .

D. $v_X = \sqrt{3}v_Y$, where v_X and v_Y are frequencies of the light in medium X and Y respectively.

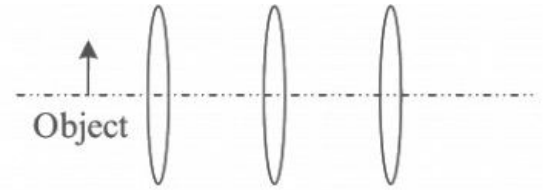
Answer: B

Solution:

Solution:

Question 42

Three identical convex lenses each of focal length f are placed in a straight line separated by a distance f from each other. An object is located at f / 2 in front of the leftmost lens. Then,



Options:

A. Final image will be at f / 2 behind the rightmost lens and its magnification will be -1 .

B. Final image will be at f / 2 behind the rightmost lens and its magnification will be +1

C. Final image will be at f behind the rightmost lens and its magnification will be -1 .

D. Final image will be at f behind the rightmost lens and its magnification will be +1 .

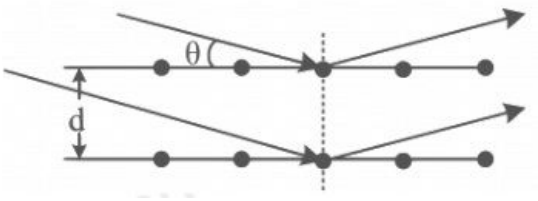
Answer: A

Solution:

Solution:

Question 43

X-rays of wavelength λ gets reflected from parallel planes of atoms in a crystal with spacing d between two planes as shown in the figure. If the two reflected beams interfere constructively, then the condition for maxima will be, (n is the order of interference fringe)



Options:

A. $d\tan\theta = n\lambda$

B. $d\sin\theta = n\lambda$

C. $2d\cos\theta = n\lambda$

D. $2d\sin\theta = n\lambda$

Answer: D

Solution:

Solution:

Question 44

If the potential energy of a hydrogen atom in the first excited state is assumed to be zero, then the total energy of n = ∞ state is,

Options:

A. 3.4 eV

B. 6.8 eV

C. 0

D. ∞

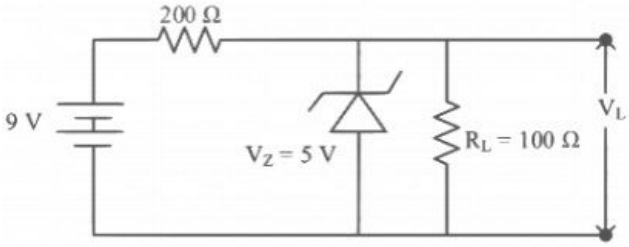
Answer: A

Solution:

Solution:

Question 45

In the given circuit, find the voltage drop V_L in the load resistance R_L.



Options:

A. 5V

B. 3V

C. 9V

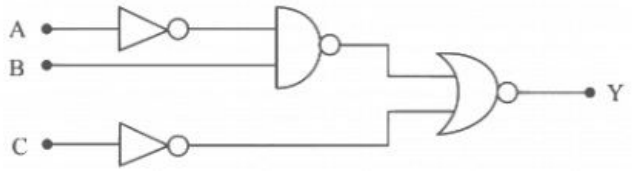
D. 6V

Answer: B

Solution:

Solution:

Question 46



Consider the logic circuit with inputs A, B, C and output Y. How many combinations of A, B and C gives the output Y = 0 ?

Options:

A. 8

B. 5

C. 7

D. 1

Answer: C

Solution:

Solution:

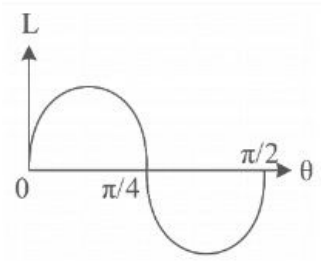
Question 47

A particle of mass m is projected at a velocity u, making an angle θ with

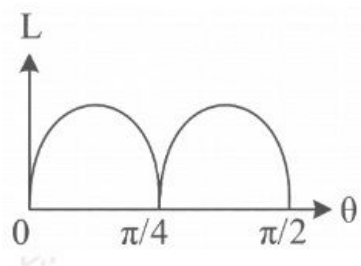
the horizontal (x-axis). If the angle of projection θ is varied keeping all other parameters same, then magnitude of angular momentum (L) at its maximum height about the point of projection varies with θ as,

Options:

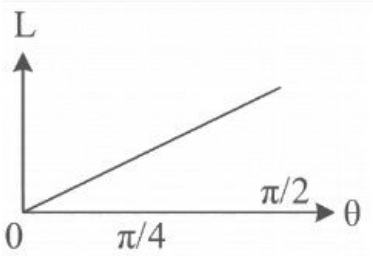
A.



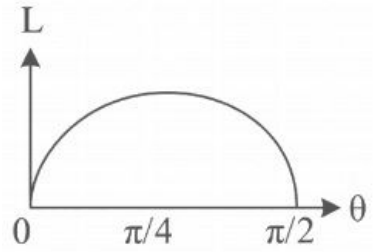
B.



C.



D.



Answer: D

Solution:

Solution:

Question 48

A body of mass 2 kg moves in a horizontal circular path of radius 5m. At

an instant, its speed is $2\sqrt{5}$m / s and is increasing at the rate of 3 m / s^2. The magnitude of force acting on the body at that instant is,

Options:

A. 6N

B. 8N

C. 14N

D. 10N

Answer: D

Solution:

Solution:

Question 49

In an experiment, the length of an object is measured to be 6.50 cm. This measured value can be written as 0.0650m. The number of significant figures on 0.0650m is

Options:

A. 3

B. 4

C. 2

D. 5

Answer: A

Solution:

Solution:

Question 50

A mouse of mass m jumps on the outside edge of a rotating ceiling fan of moment of inertia I and radius R. The fractional loss of angular velocity of the fan as a result is,

Options:

A. $\frac{mR^2}{I + mR^2}$

B. $\frac{I}{I + mR^2}$

C. $\frac{I - mR^2}{I}$

D. $\frac{I - mR^2}{I + mR^2}$

Answer: A

Solution:

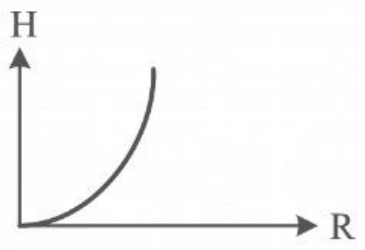
Solution:

Question 51

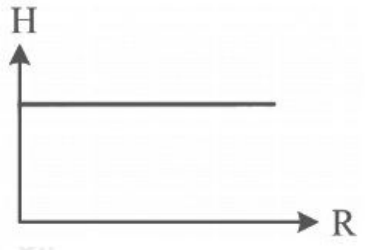
Acceleration due to gravity at a height H from the surface of a planet is the same as that at a depth of H below the surface. If R be the radius of the planet, then H vs. R graph for different planets will be,

Options:

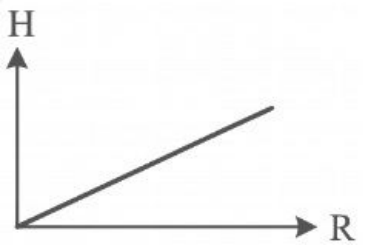
A.



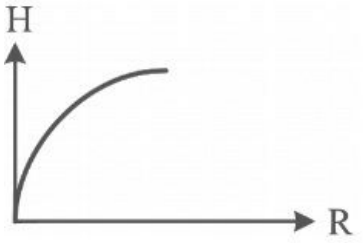
B.



C.



D.



Answer: C

Solution:

Solution:

Question 52

A uniform rope of length 4m and mass 0.4 kg is held on a frictionless table in such a way that 0.6m of the rope is hanging over the edge. The work done to pull the hanging part of the rope on to the table is, (Assume g = 10m / s^2)

Options:

A. 0.36J

B. 0.24J

C. 0.12J

D. 0.18J

Answer: D

Solution:

Solution:

Question 53

The displacement of a plane progressive wave in a medium, travelling towards positive x-axis with velocity 4m / s at t = 0 is given by $y = 3\sin 2\pi\left(-\frac{x}{3}\right)$. Then the expression for the displacement at a later time t = 4 sec will be

Options:

A. $y = 3\sin 2\pi\left(-\frac{x-16}{3}\right)$

B. $y = 3\sin 2\pi\left(\frac{-x-16}{3}\right)$

C. $y = 3\sin 2\pi\left(\frac{-x+1}{3}\right)$

D. $y = 3\sin 2\pi\left(\frac{-x-1}{3}\right)$

Answer: A

Solution:

Solution:
1. The general equation of a wave propagating in one dimension can be expressed as $y = A\sin(\omega t - kx + \varphi)$, where A is the amplitude, ω is the angular frequency, k is the wave number, x is the position, t is the time, and φ is the phase constant.

2. The given wave equation is $y = 3\sin 2\pi\left(-\frac{x}{3}\right)$. This can be rewritten as $y = 3\sin\left(-2\pi\frac{x}{3}\right)$, which matches the form $y = A\sin(\omega t - kx)$ where ωt is zero (i.e., $t = 0$), A is 3 and k is $-\frac{2\pi}{3}$.

3. The negative sign of k indicates that the wave is moving in the negative x-direction, but the problem states that the wave is moving in the positive x-direction. Hence, we should take k as positive, i.e., $k = \frac{2\pi}{3}$.

4. The speed of the wave v is given by $v = \frac{\omega}{k}$. From the problem, we know that $v = 4\,m/s$. By substituting these values, we can solve for ω : $4 = \frac{\omega}{2\pi/3}$ which gives $\omega = \frac{8\pi}{3}$.

5. So, the wave equation at any time t can be written as $y = 3\sin\left[\frac{8\pi}{3}t - \frac{2\pi}{3}x\right]$.

6. Now, we want to find the wave equation at $t = 4$ s. Substituting $t = 4$ into the equation gives $y = 3\sin\left[\frac{8\pi}{3}\times 4 - \frac{2\pi}{3}x\right]$.

7. Simplifying the argument of the sine function, we get $y = 3\sin 2\pi\left(\frac{-x+16}{3}\right)$.

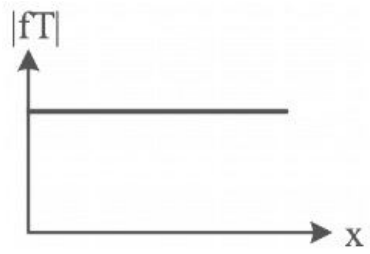
8. Rearranging the argument of the sine function again, we get $y = 3\sin 2\pi\left(-\frac{x-16}{3}\right)$.

Question 54

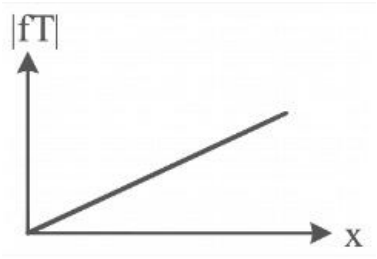
In a simple harmonic motion, let f be the acceleration and t be the time period. If x denotes the displacement, then |fT| vs. x graph will look like,

Options:

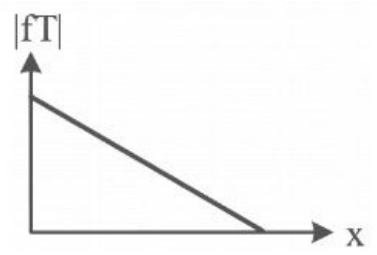
A.



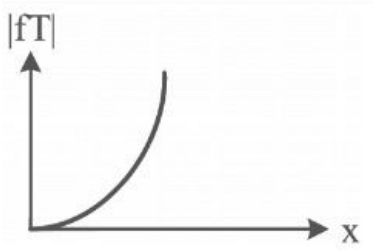
B.



C.



D.

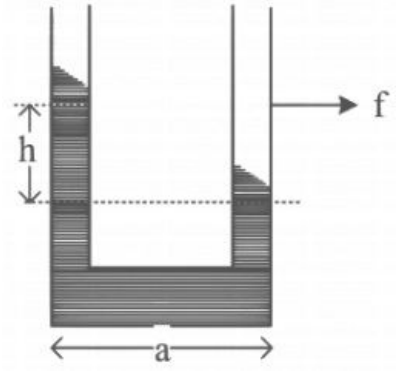


Answer: B

Solution:

Solution:

Question 55



As shown in the figure, a liquid is at same levels in two arms of a U-tube of uniform cross-section when at rest. If the U-tube moves with an acceleration ' f ' towards right, the difference between liquid heights between two arms of the U tube will be, (acceleration due to gravity = g)

Options:

A. $\frac{f}{g}a$

B. $\frac{g}{f}a$

C. a

D. 0

Answer: A

Solution:

Solution:

Question 56

Six molecules of an ideal gas have velocities 1, 3, 5, 5, 6 and 5m / s respectively. At any given temperature, if $\overline{V}$ and V_{rms} represent average and rms speed of the molecules, then

Options:

A. $\overline{V} = 5m / s$

B. $V_{rms} > \overline{V}$

C. $V_{rms}^2 < \overline{V}^2$

D. $V_{rms} = \overline{V}$

Answer: B

Solution:

Solution:

Question 57

As shown in the figure, a pump is designed as horizontal cylinder with a

piston having area A and an outlet orifice having an area ' a '. The piston moves with a constant velocity under the action of force F . If the density of the liquid is ρ, then the speed of the liquid emerging from the orifice is, (assume A > a)

Options:

A. $\sqrt{\frac{F}{\rho A}}$

B. $\frac{a}{A}\sqrt{\frac{F}{\rho A}}$

C. $\sqrt{\frac{2F}{\rho A}}$

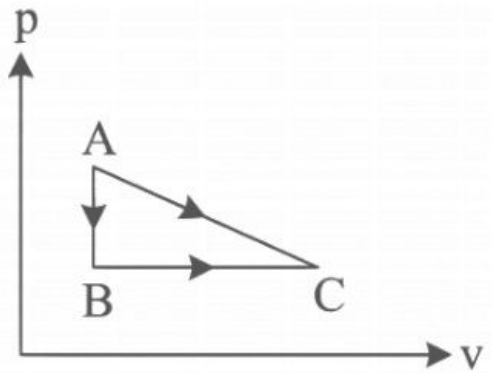
D. $\frac{A}{a}\sqrt{\frac{2F}{\rho A}}$

Answer: C

Solution:

Solution:

Question 58



A given quantity of gas is taken from A to C in two ways; a) directly from A → C along a straight line and b) in two steps, from A → B and then from B → C. Work done and heat absorbed along the direct path A → C is 200J and 280J respectively.

If the work done along A → B → C is 80J, then heat absorbed along this path is,

Options:

A. 80J

B. 0

C. 160J

D. 120J

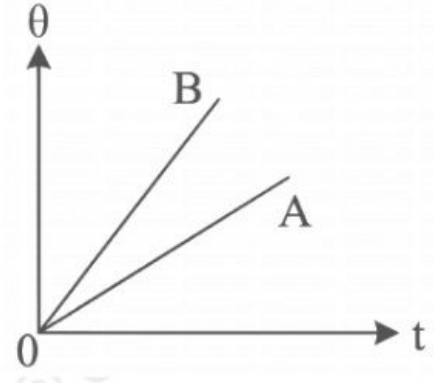
Answer: C

Solution:

Solution:

Question 59

Two substances A and B of same mass are heated at constant rate. The variation of temperature θ of the substances with time t is shown in the figure. Choose the correct statement.



Options:

A. Specific heat of A is greater than that of B.

B. Specific heat of B is greater than that of A.

C. Both have same specific heat.

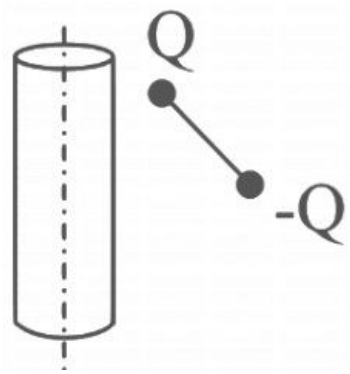
D. None of the above is true.

Answer: A

Solution:

Solution:

Question 60



Consider a positively charged infinite cylinder with uniform volume charge density rho > 0. An electric dipole consisting of +Q and −Q charges attached to opposite ends of a massless rod is oriented as shown in the figure. At the instant as shown in the figure, the dipole will experience,

Options:

A. a force to the left and no torque.

B. a force to the right and a clockwise torque.

C. a force to the right and a counter clockwise torque.

D. no force but only a clockwise torque.

Answer: B

Solution:

Solution:

Question 61

A thin glass rod is bent in a semicircle of radius R. A charge is nonuniformly distributed along the rod with a linear charge density $\lambda = \lambda_0 \sin\theta$ (λ_0 is a positive constant). The electric field at the centre P of the semicircle is,

Options:

A. $-\frac{\lambda_o}{8\pi\varepsilon_o R}\hat{j}$

B. $\frac{\lambda_o}{8\pi\varepsilon_0 R}\hat{j}$

C. $\frac{\lambda_0}{8\pi\varepsilon_0 R}\hat{i}$

D. $-\frac{\lambda_0}{8\pi\varepsilon_0 R}\hat{i}$

Answer: A

Solution:

Solution:

Question 62

12μC and 6μC charges are given to the two conducting plates having same cross-sectional area and placed face to face close to each other as shown in the figure. The resulting charge distribution in μC on surfaces A, B, C and D are respectively,

Options:

A. 9, 3, −3, 9

B. 3, 9, −9, 3

C. 6, 6, −6, 12

D. 6, 6, 3, 3

Answer: A

Solution:

Solution:

Out side face of each plate, charge $= \frac{Q_1 + Q_2}{2} = 9$

Charge on B $= Q_1 - 9 = 3$

Charge on C $= Q_2 - 9 = -3$

Question 63

A wire carrying a steady current I is kept in the x − y plane along the curve y = Asin $\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the z-direction. The magnitude of the magnetic force in the portion of the wire between x = 0 and x = λ is

Options:

A. 0

B. 2lλB

C. lλB

D. lλB/2

Answer: C

Solution:

Solution:

Question 64

The figure represents two equipotential lines in x-y plane for an electric

field. The x-component E_x of the electric field in space between these equipotential lines is,

Options:

A. 100V/m

B. -100V/m

C. 200V/m

D. -200V/m

Answer: B

Solution:

Solution:

Question 65

An electric dipole of dipole moment $\vec{p}$ is placed at the origin of the co-ordinate system along the z-axis. The amount of work required to move a charge ' q ' from the point (a, 0, 0) to the point (0, 0, a) is,

Options:

A. $\frac{pq}{4\pi\varepsilon_0 a}$

B. 0

C. $\frac{-pq}{4\pi\varepsilon_0 a^2}$

D. $\frac{pq}{4\pi\varepsilon_0 a^2}$

Answer: D

Solution:

Solution:

Question 66

The electric field of a plane electromagnetic wave of wave number k and angular frequency ω is given $\vec{E} = E_0\left(\hat{i} + \hat{j}\right)\sin(kz - \omega t)$. Which of the following gives the direction of the associated magnetic field $\vec{B}$?

Options:

A. $\hat{k}$

B. $-\hat{i}+\hat{j}$

C. $-\hat{i}-\hat{j}$

D. $\hat{i}-\hat{k}$

Answer: B

Solution:

Solution:

Question 67

A charged particle in a uniform magnetic field $\vec{B} = B_0\hat{k}$ starts moving from the origin with velocity $v = 3\hat{i} + 4\hat{k}$ m / s. The trajectory of the particle and the time t at which it reaches 2m above x − y plane are,

Options:

A. Circular path 1/2 sec.

B. Helical path, 1/2 sec .

C. Circular path, 2/3 sec.

D. Helidatpath, 2/3 sec.

Answer: B

Solution:

Solution:

Question 68

In an experiment on a circuit as shown in the figure, the voltmeter shows 8V reading. The resistance of the voltmeter is,

Options:

A. 20Ω

B. 320Ω

C. 160Ω

D. 1.44kΩ

Answer: C

Solution:

Solution:

Question 69

An interference pattern is obtained with two coherent sources of intensity ratio n : 1. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be maximum if

Options:

A. n = 1

B. n = 2

C. n = 3

D. n = 4

Answer: A

Solution:

Solution:

Question 70

A circular coil is placed near a current carrying conductor, both lying on the plane of the paper. The current is flowing through the conductor in such a way that the induced current in the loop is clockwise as shown in the figure. The current in the wire is,

Options:

A. time dependent and downward.

B. steady and upward.

C. time dependent and upward.

D. An alternating current.

Answer: C

Solution:

Solution:

Question 71

An amount of charge Q passes through a coil of resistance R. If the current in the coil decreases to zero at a uniform rate during time T , then the amount of heat generated in the coil will be,

Options:

A. $\frac{4Q^2R}{3T}$

B. $\frac{2QR}{3T}$

C. $\frac{Q^2T}{4R}$

D. Q^2RT

Answer: A

Solution:

Solution:

Question 72

A modified gravitational potential is given by V = $-\frac{GM}{r}$ + $\frac{A}{r^2}$. If the constant A is expressed in terms of gravitational constant (G), mass (M) and velocity of light (c), then from dimensional analysis, A is,

Options:

A. $\frac{G^2M^2}{c^2}$

B. $\frac{GM}{c^2}$

C. $\frac{1}{c^2}$

D. Dimensionless

Answer: A

Solution:

Solution:

Question 73

There are n elastic balls placed on a smooth horizontal plane. The masses of the balls are m, $\frac{m}{2}$, $\frac{m}{2^2}$, ... $\frac{m}{2^{n-1}}$ respectively. If the first ball hits the second ball with velocity v_0, then the velocity of the n^{th} ball will be,

Options:

A. $\frac{4}{3}v_0$

B. $\left(\frac{4}{3}\right)^n v_0$

C. $\left(\frac{4}{3}\right)^{n-1} v_0$

D. v_0

Answer: C

Solution:

Solution:

Question 74

An earth's satellite near the surface of the earth takes about 90 min per revolution. A satellite orbiting the moon also takes about 90 min per revolution. Then which of the following is true?
[where ρ_m is density of the moon and ρ_e is density of the earth.]

Options:

A. $\rho_m < \rho_e$

B. $\rho_m > \rho_e$

C. $\rho_m = \rho_e$

D. No conclusion can be made about the densities.

Answer: C

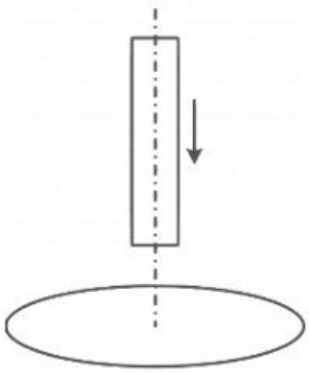
Solution:

Solution:

Question 75

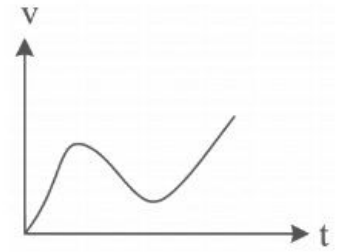
A bar magnet falls from rest under gravity through the centre of a horizontal ring of conducting wire as shown in figure. Which of the

following graph best represents the speed (v) vs. time (t) graph of the bar magnet?

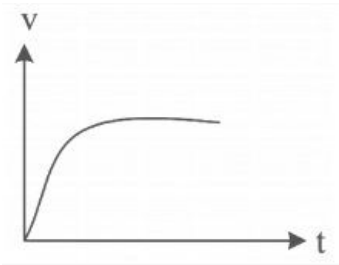


Options:

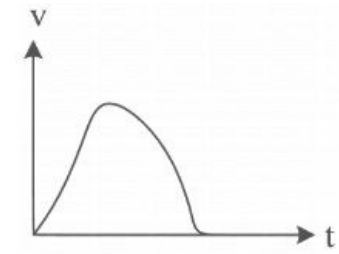
A.



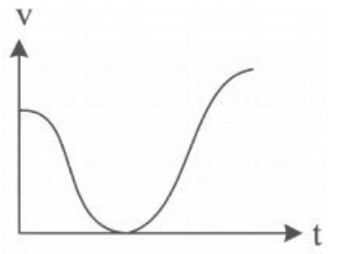
B.



C.



D.



Answer: B

Solution:

Solution:

Question 76

A uniform magnetic field B exists in a region. An electron of charge q and mass m moving with velocity v enters the region in a direction perpendicular to the magnetic field. Considering Bohr angular momentum quantization, which of the following statements is/are true?

Options:

A. The radius of n^{th} orbit $r_n \propto \sqrt{n}$.

B. The minimum velocity of the electron is $\frac{\sqrt{qBh}}{m}$.

C. Energy of the n^{th} level $E_n \propto n$.

D. Transition frequency ω between two successive levels is independent of n.

Answer: D

Solution:

Solution:

Question 77

A train is moving along the tracks at a constant speed u. A girl on the train throws a ball of mass m straight ahead along the direction of motion of the train with speed v with respect to herself. Then

Options:

A. Kinetic energy of the ball as measured by the girl on the train is $mv^2 / 2$.

B. Work done by the girl in throwing the ball is $mv^2 / 2$.

C. Work done by the train is mvu.

D. The gain in kinetic energy of the ball as measured by a person standing by the rail track is $mv^2 / 2$.

Answer: C

Solution:

Solution:
1. Train Frame (the frame of reference of the girl on the train):
- The initial velocity of the ball is 0 (it's at rest before she throws it).
- The final velocity of the ball is v (after she throws it).
- So the kinetic energy (K.E.) of the ball in the train frame, after she throws it, is $\frac{1}{2}mv^2$.

- The work done by the girl is equal to the change in kinetic energy, so it's also $\frac{1}{2}mv^2$.

2. Ground Frame (the frame of reference of a person standing by the track) :
- The initial velocity of the ball is u (equal to the speed of the train, as the ball is at rest relative to the train before she throws it).
- The final velocity of the ball is u + v (the speed of the train plus the speed of the ball relative to the train).
- So, the kinetic energy (K.E.) of the ball in the ground frame, after she throws it, is $\frac{1}{2}m(u+v)^2 = \frac{1}{2}mu^2 + \frac{1}{2}mv^2 + muv$.
- The gain in K.E. of the ball as measured by a person standing on the ground is the final K.E. minus the initial K.E., which is $\frac{1}{2}mu^2 + \frac{1}{2}mv^2 + muv - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + muv$. This corresponds to the work done by the girl plus the work done by the train.

So, in conclusion:
- Option A is correct.
- Option B is correct.
- Option C is correct.
- Option D is incorrect because it's missing the term for the work done by the train (muv).

Question 78

A cyclic process is shown in p-v diagram and T-S diagram. Which of the following statements is/are true?

Options:

A. 1 → 2 : Isobaric, 2 → 3 : Isothermal.

B. 3 → 1 : Isochoric, 2 → 3 : adiabatic.

C. Work done by the system in the complete cyclic process is non-zero.

D. The heat absorbed by the system in the complete cyclic process is non-zero.

Answer: D

Solution:

Solution:

Question 79

The figure shows two identical parallel plate capacitors A and B of capacitances C connected to a battery. The key K is initially closed. The switch is now opened and the free spaces between the plates of the capacitors are filled with a dielectric constant 3 . Then which of the following statements is/are true?

Options:

A. When the switch is closed, total energy stored in the two capacitors is CV^2.

B. When the switch is opened, no charge is stored in the capacitor B.

C. When the switch is opened, energy stored in capacitor B is $\frac{3}{2}CV^2$

D. When the switch is opened, total energy stored in two capacitors is $\frac{5}{3}CV^2$.

Answer: D

Solution:

Solution:

Question 80

A charged particle of charge q and mass m is placed at a distance 2R from the centre of a vertical cylindrical region of radius R where magnetic field varies as $\vec{B} = (4t^2 - 2t + 6)\hat{k}$, where t is time. Then which of the following statements is/are true?

Options:

A. Induced electric field lines form closed loops.

B. Electric field varies linearly with r if r < R, where r is the radial distance from the centerline of the cylinder.

C. The charged particle will move in clockwise direction when viewed from top.

D. Acceleration of the charged particle is $\frac{7q}{2m}$ when t = 2 sec.

Answer: B

Solution:

Solution:

Chemistry

Question 1

Which of the following statements is incorrect?

Options:

A. $[VF_6]^{3-}$ is paramagnetic with 2 unpaired electrons.

B. $[CuCl_4]^{2-}$ is paramagnetic with 1 unpaired electron.

C. $[Co(NH_3)_6]^{3+}$ is diamagnetic

D. $[CoF_6]^{3-}$ is paramagnetic with 2 unpaired electrons.

Answer: D

Solution:

Solution:

Question 2

The calculated spin-only magnetic moment values in BM for $[FeCl_4]^-$ and $[Fe(CN)_6]^{3-}$ are

Options:

A. 5.9 BM, 1.732 BM

B. 4.89 BM, 1.732 BM

C. 3.87 BM, 1.732 BM

D. 1.732 BM, 2.82 BM

Answer: A

Solution:

Solution:

Question 3

BrF_3 self ionises as following

Options:

A. $2BrF_3 \rightleftharpoons BrF^+ + BrF_5^-$

B. $2BrF_3 \rightleftharpoons BrF_2^+ + BrF_4^-$

C. $2BrF_3 \rightleftharpoons BrF_4^+ + BrF_2^-$

D. $2BrF_3 \rightleftharpoons BrF_3^+ + BrF_3^-$

Answer: B

Solution:

Solution:

Question 4

$4f^2$ electronic configuration is found in

Options:

A. Pr

B. Pr^{3+}

C. Nd^{3+}

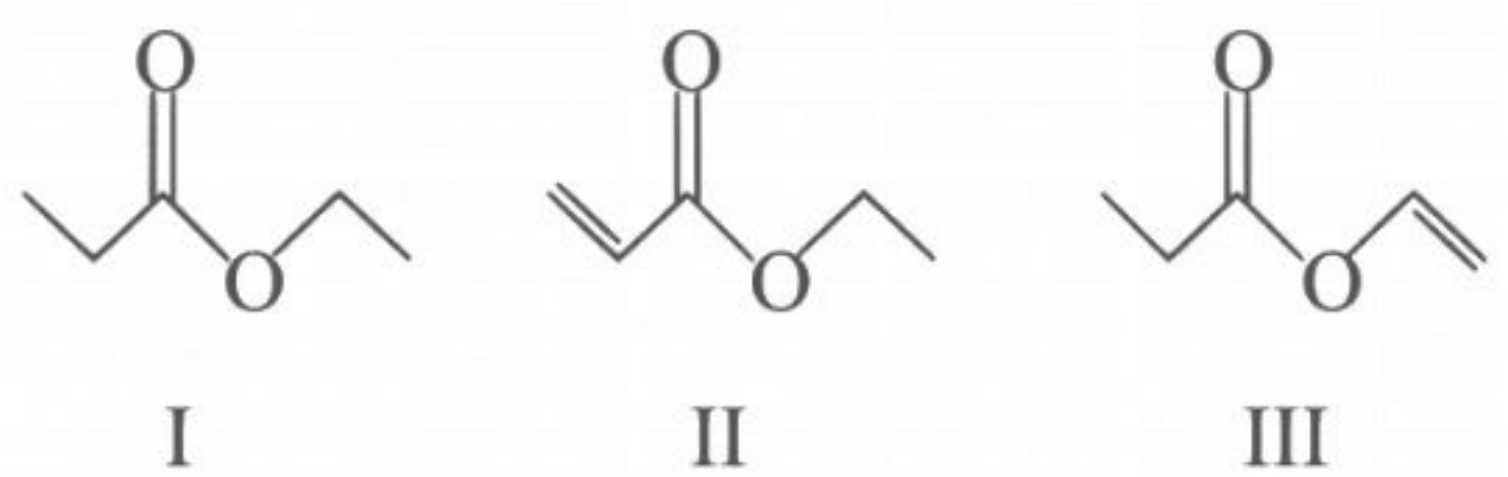
D. Pm^{3+}

Answer: B

Solution:

Solution:
Let's consider the elements:
- Pr (Praseodymium) has an atomic number of 59. Its ground state electron configuration is $[Xe]4f^3 6s^2$.
- Pr^{3+} (Praseodymium ion) will lose 3 electrons. It loses the two 6 s electrons and one from the 4f orbital. So, the configuration is $[Xe]4f^2$.
$-Nd^{3+}$ (Neodymium ion) with an atomic number of 60 , in its ground state has the configuration $[Xe]4f^4 6s^2$. When it loses 3 electrons (two from 6s and one from 4f), its configuration is [Xe] 4f3.
- Pm^{3+} (Promethium ion) with an atomic number of 61 , in its ground state has the configuration $[Xe]4f^5 6s^2$. When it loses 3 electrons (two from 6s and one from 4f), its configuration is $[Xe]4f^4$.
So, the $4f^2$ electronic configuration is found in Pr^{3+}. Hence, Option B is correct.

Question 5



The correct order of C = O bond length in ethyl propanoate (I), ethyl propanoate (II) and ethenyl propanoate (III) is

Options:

A. I > II > III

B. III > II > I

C. I > III > II

D. II > I > III

Answer: D

Solution:

Solution:

Question 6

Select the molecule in which all the atoms may lie on a single plane is

Options:

A. 4-Nitrobenzaldehyde

B. 4-Methoxybenzaldehyde

C. 4-Methylnitrobenzene

D. 4-Nitroacetophenone

Answer: A

Solution:

Solution:

Question 7

The IUPAC name of $CH_3CH = \underset{\underset{CHO}{|}}{C} - CH_2 - CH_3$

Options:

A. 3-Formyl-2-pentene

B. 2-Ethylbut-2-enal

C. 3-Ethylbut-3-enal

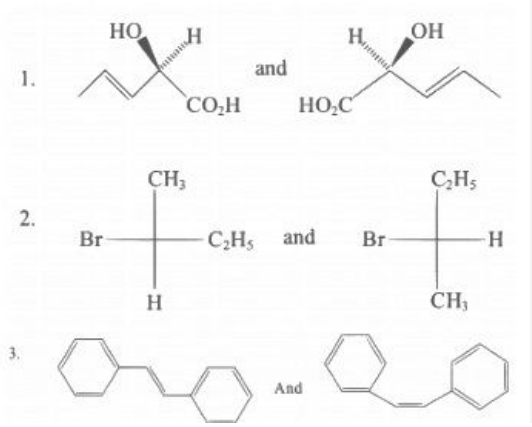
D. 2-Ethylcrotonaldehyde

Answer: B

Solution:

Solution:

Question 8



The relationship between the pair of compounds shown above are respectively,

Options:

A. enantiomer, diastereomer, diastereomer

B. enantiomer, enantiomer, diastereomer

C. enantiomer, homomer (identical), diastereomer

D. homomer (identical), diastereomer, geometrical isomer

Answer: C

Solution:

Solution:

Question 9

The correct stability order of the following carbocations is

(I) $H_2\overset{\oplus}{C} - CH = CH - CH_3$

(II) $\overset{\oplus}{C}H_2 - CH = CH - BMe_2$

(III) $H_2\overset{\oplus}{C} - CH = CH - NMe$

(IV) $H_2\overset{\oplus}{C} - CH = CH - OMe$

Options:

A. II > I > III > IV

B. III > I > II > IV

C. III > IV > I > II

D. IV > III > II > I

Answer: C

Solution:

Solution:

Question 10

The correct order of boiling points of N-ethylethanamine (I), ethoxyethane (II) and butan-2-ol (III) is

Options:

A. III < II < I

B. II < III < I

C. II < I < III

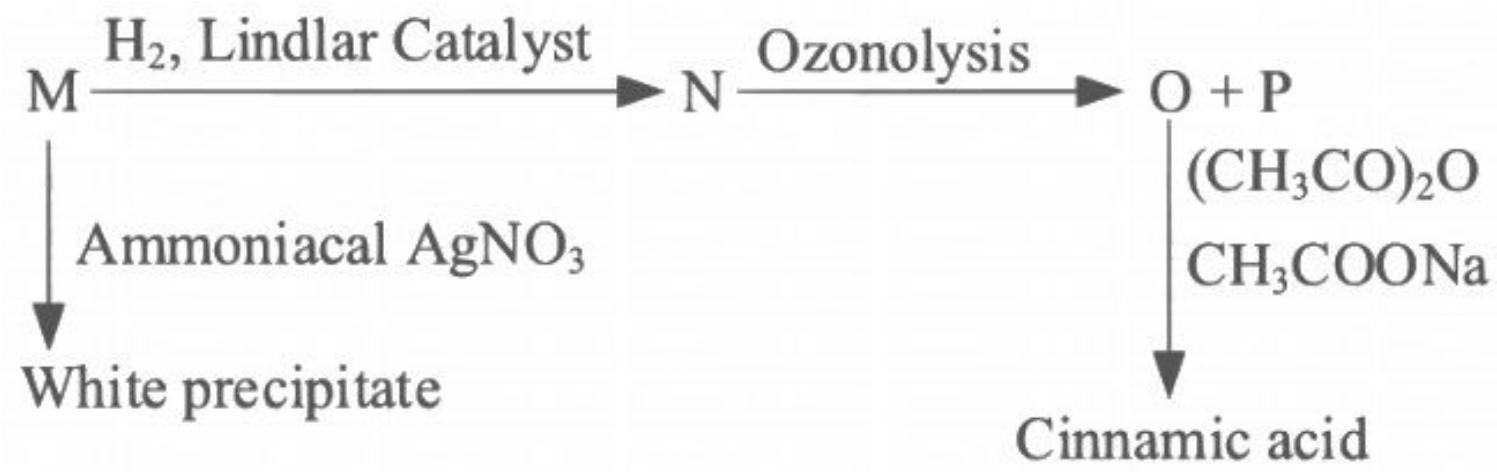
D. III < I < II

Answer: C

Solution:

Solution:

Question 11



Structure of M is,

Options:

A. $Ph - C \equiv CH$

B. $Ph - C \equiv C - CH_3$

C. $H_3C - C \equiv CH$

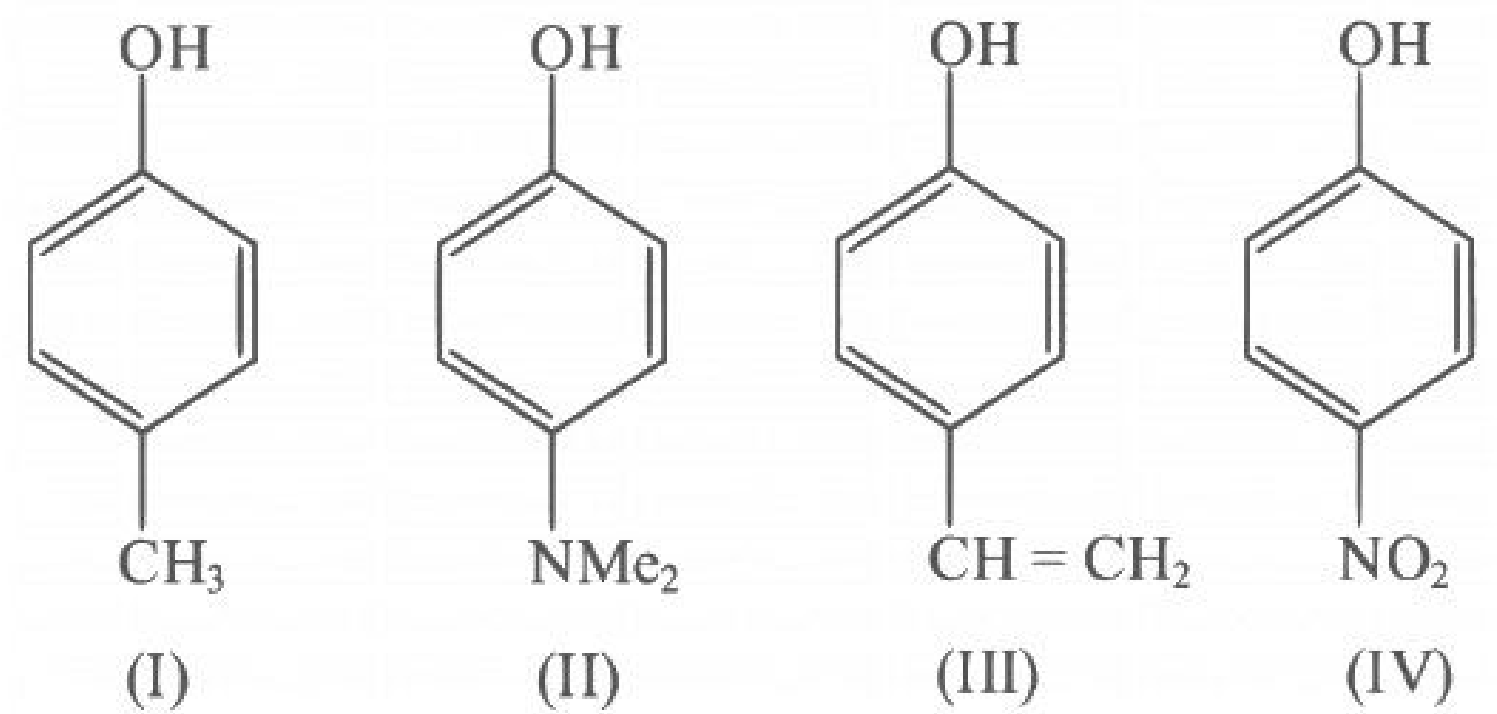
D. $H_3C - C \equiv C - CH_3$

Answer: A

Solution:

Solution:

Question 12



The correct order of acidity of above compounds is

Options:

A. II > IV > I > III

B. III > IV > II > I

C. IV > II > III > I

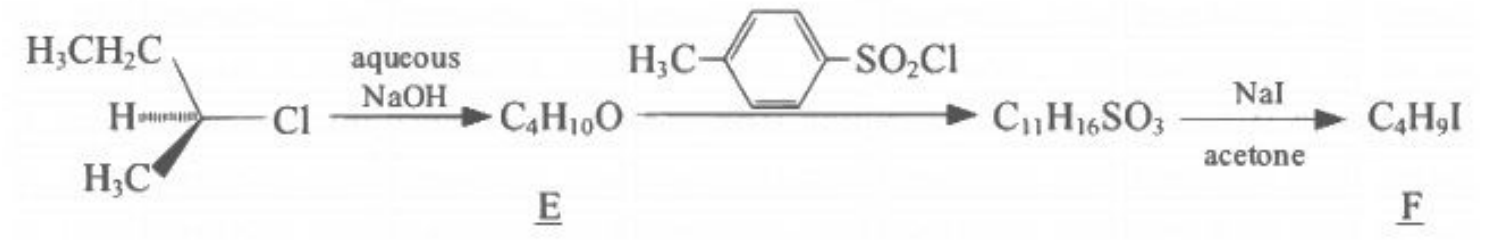
D. IV > III > I > II

Answer: D

Solution:

Solution:

Question 13

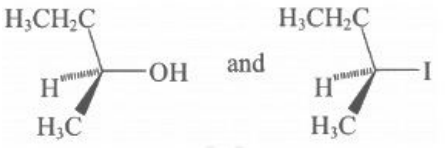


If all the nucleophilic substitution reactions at saturated carbon atoms in the above sequence of reactions follow S_N2 mechanism, then xE and

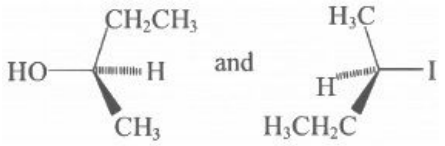
xF will be respectively,

Options:

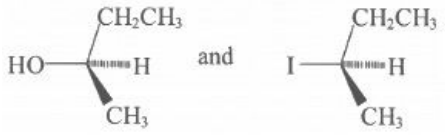
A.



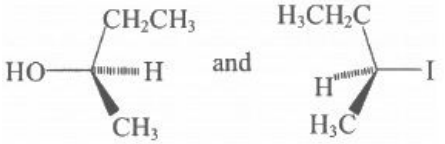
B.



C.



D.

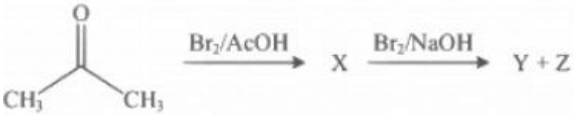


Answer: D

Solution:

Solution:

Question 14



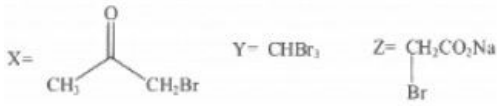
The correct option for the above reaction is

Options:

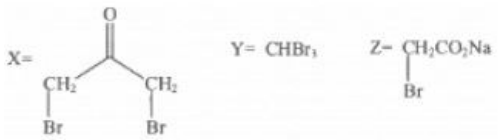
A.

B.

C.



D.



Answer: A

Solution:

Solution:

Question 15

Arrange the following in order of increasing mass
I. 1 mole of N_2
II. 0.5 mole of O_3
III. 3.011×10^{23} molecules of O_2
IV. 0.5 gram atom of O_2

Options:

A. IV < III < II < I

B. IV < I < III < II

C. III < II < IV < I

D. I < III < II < IV

Answer: A

Solution:

Solution:

Question 16

Two base balls (masses : m_1 = 100g, and m_2 = 50g) are thrown. Both of

them move with uniform velocity, but the velocity of m_2 is 1.5 times that of m_1. The ratio of de Broglie wavelengths $\lambda(m_1) : \lambda(m\ 2)$ is given by

Options:

A. 4 : 3

B. 3 : 4

C. 2 : 1

D. 1 : 2

Answer: B

Solution:

Solution:

Question 17

What is the edge length of the unit cell of a body centred cubic crystal of an element whose atomic radius is 75 pm ?

Options:

A. 170 pm

B. 175 pm

C. 178 pm

D. 173.2 pm

Answer: D

Solution:

Solution:

Question 18

The root mean square (rms) speed of X_2 gas is xm / s at a given temperature. When the temperature is doubled, the X_2 molecules dissociated completely into atoms. The root mean square speed of the sample of gas then becomes (in m / s)

Options:

A. x/2

B. x

C. 2x

D. 4x

Answer: C

Solution:

Solution:

Question 19

Which of the following would give a linear plot? (k is the rate constant of an elementary reaction and T is temp. in absolute scale)

Options:

A. k vs T

B. k vs 1/T

C. In k vs T

D. In k vs 1/T

Answer: D

Solution:

Solution:

Question 20

The equivalent conductance of NaCl, HCl and CH_3COONa at infinite dilution are 126.45, 426.16 and 91 ohm $^{-1}cm^2eq^{-1}$ respectively at 25°C. The equivalent conductance of acetic acid (at infinite dilution) would be

Options:

A. $461.61 ohm^{-1}cm^2eq^{-1}$

B. $390.71 ohm^{-1}cm^2eq^{-1}$

C. cannot be determined from the given data.

D. $208.71 ohm^{-1}cm^2eq^{-1}$

Answer: B

Solution:

Solution:

Question 21

For the reaction A + B → C, we have the following data:

Initial concentration of A (in molarity)	Initial concentration of B (in molarity)	Rate (initial) (Relevant unit)
1	10	100
1	1	1
10	1	10

The order of the reaction with respect to A and B are

Options:

A. Not possible to tell with the given data.

B. First order with respect to both A and B.

C. First order with respect to A and second order with respect to B.

D. Second order with respect to A and first order with respect to B.

Answer: C

Solution:

Solution:

Question 22

If in case of a radio isotope the value of half-life ($T_{1/2}$) and decay constant (λ) are identical in magnitude, then their value should be

Options:

A. 0.693/2

B. $(0.693)^{\frac{1}{2}}$

C. $(0.693)^2$

D. 0.693

Answer: B

Solution:

Solution:

Question 23

Suppose a gaseous mixture of He, Ne, Ar and Kr is treated with photons of the frequency appropriate to ionize Ar. What ion(s) will be present in the mixture?

Options:

A. Ar^+

B. $Ar^+ + Kr^+$

C. $Ar^+ + He^+ + Ne^+$

D. $He^+ + Ar^+ + Kr^+$

Answer: B

Solution:

Solution:

Question 24

A solution containing 4g of polymer in 4.0 litre solution at 27˚C shows an osmotic pressure of 3.0×10^{-4} atm. The molar mass of the polymer in g / mol is

Options:

A. 820000

B. 82000

C. 8200

D. 820

Answer: B

Solution:

Solution:

Question 25

The equivalent weight of KIO_3 in the given reaction is (M = molecular mass):
$2\,Cr(OH)_3 + 4OH^- + KIO_3 \rightarrow 2CrO_4^{2-} + 5H_2O + KI$

Options:

A. M

B. M/2

C. M /6

D. M/8

Answer: C

Solution:

Solution:

Question 26

At STP, the dissociation reaction of water is $H_2O \rightleftharpoons H^+(aq.) + OH^-(aq.)$, and the pH of water is 7.0. The change of standard free energy (ΔG°) for the above dissociation process is given by

Options:

A. $20301\,cal/mol$

B. $19091\,cal/mol$

C. $20096\,cal/mol$

D. $21301\,cal/mol$

Answer: D

Solution:

Solution:

Question 27

Na_2CO_3 is prepared by Solvay process but K_2CO_3 cannot be prepared by the same because

Options:

A. K_2CO_3 is highly soluble in H_2O

B. $KHCO_3$ is sparingly soluble

C. $KHCO_3$ is appreciably soluble

D. $KHCO_3$ decomposes

Answer: C

Solution:

Solution:

Question 28

The molecular shapes of SF_4, CF_4 and XeF_4 are

Options:

A. the same with 2, 0 and 1 lone pairs of electrons on the central atoms, respectively.

B. the same with 1,1 and 1 lone pairs of electrons on the central atoms, respectively.

C. different with 0,1 and 2 lone pairs of electrons on the central atoms, respectively.

D. different with 1, 0 and 2 lone pairs of electrons on the central atoms, respectively.

Answer: D

Solution:

Solution:

Question 29

The species in which nitrogen atom is in a state of sp hybridisation is

Options:

A. NO_3^-

B. NO_2

C. NO_2^+

D. NO_2^-

Answer: C

Solution:

Solution:

Question 30

The correct statement about the magnetic properties of $[Fe(CN)_6]^{3-}$ and $[FeF]^{3-}$ is

Options:

A. Both are paramagnetic

B. Both are diamagnetic

C. $[Fe(CN)_6]^{3-}$ is diamagnetic, $[FeF_6]^{3-}$ is paramagnetic

D. $[Fe(CN)_6]^{3-}$ is paramagnetic, $[FeF]^{3-}$ is diamagnetic

Answer: A

Solution:

Solution:

Question 31

Nickel combines with a uninegative monodentate ligand (X^-)to form a paramagnetic complex $[NiX_4]^{2-}$. The hybridisation involved and number of unpaired electrons present in the complex are respectively

Options:

A. sp^3, two

B. dsp^2, zero

C. dsp^2, one

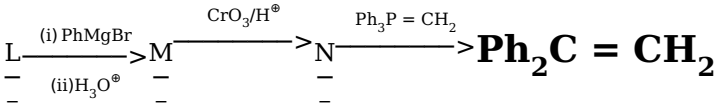
D. sp^3, one

Answer: A

Solution:

Solution:

Question 32



'L' in the above sequence of reaction is/are (where L ≠ M ≠ N)

Options:

A. Benzaldehyde

B. Methyl benzoate

C. Benzoyl chloride

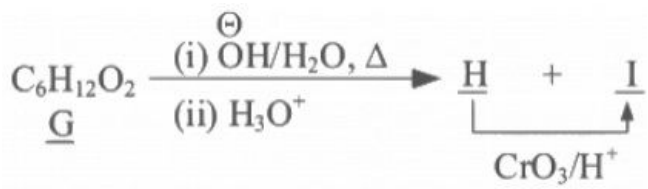
D. Benzonitrile

Answer: A

Solution:

Solution:

Question 33



' G ' in the above sequence of reactions is

Options:

A. $(CH_3)_2CHCOOCH_2CH_3$

B. $CH_3CH_2CH_2COOCH_2CH_3$

C. $CH_3CH_2COOCH_2CH_2CH_3$

D. $CH_3CH_2COOCH(CH_3)_2$

Answer: C

Solution:

Solution:

Question 34

Case - 1 : An ideal gas of molecular weight M at temperature T.
Case - 2 : Another ideal gas of molecular weight 2M at temperature T/2.
Identify the correct statement in context of above two cases.

Options:

A. Average kinetic energy and average speed will be the same in the two cases.

B. Both the averages are halved.

C. Both the averages are doubled.

D. Only average speed is halved in the second case.

Answer: B

Solution:

Solution:

Question 35

63g of a compound (Mol. Wt. = 126) was dissolved in 500g distilled water. The density of the resultant solution as 1.126g / ml. The molarity of the solution is

Options:

A. 1.25M

B. 1.0M

C. 0.75M

D. 1.1M

Answer: B

Solution:

Solution:

Question 36

An electron in the 5d orbital can be represented by the following (n, I , m) values

Options:

A. (5, 2, 1)

B. (5, 1, −1)

C. (5, 0, 1)

D. (5, 2, −1)

Answer: A, D

Solution:

Solution:

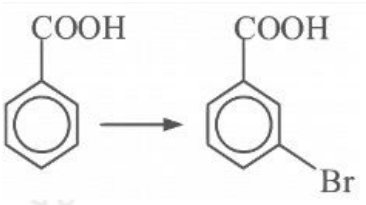
Question 37

The conversion(s) that can be carried out by bromine in carbon tetrachloride solvent is/are

Options:

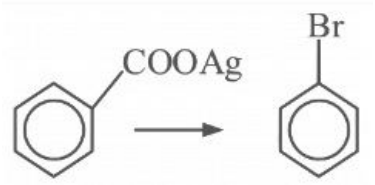
A. $PhCH = CHCH_3 \rightarrow PhCHBrCHBrCH_3$

B.



C. $CH_3CH_2COOH \rightarrow CH_3CHBrCOOH$

D.



Answer: A, D

Solution:

Solution:

Question 38

The correct set(s) of reactions to synthesize benzoic acid starting from benzene is/are

Options:

A. (i) Br_2/Fe, (ii) Mg / dry ether, (iii) CO_2, (iv) $H_3O^{\oplus}$

B. (i) Br_2/Fe, (ii) NH_3, 25˚, (iil) $NaNO_2$, dil. HCl, 0˚ to 5˚, (iv) CuCN/KCN, (v) dil. HCl, Δ

C. (i) CH_3Cl, Anhydrous $AlCl_3$, (ii) $KMnO_4|^{\oplus}OH$, Δ, (iii) $H_3^{\oplus}O$

D. (i) CH_3COCl, Anhydrous $AlCl_3$,
(ii) Br_2, NaOH,
(iii) $H_3O^{\oplus}$

Answer:A, C, D

Solution:

Solution:

Question 39

Which statement(s) is/are applicable above critical temperature?

Options:

A. A gas cannot be liquified.

B. Surface tension of a liquid is very high.

C. A liq. phase cannot be distinguished from a gas phase.

D. Density changes continuously with P or V.

Answer: A, D

Solution:

Solution:

Question 40

Which of the following mixtures act(s) as buffer solution?

Options:

A. NaOH + CH_3COOH (1: 1 mole ratio)

B. NH_4OH + HCl (2: 1 mole ratio)

C. CH_3COOH + NaOH (2: 1 mole ratio)

D. CH_3COOH + NaOH (1:2 mole ratio)

Answer: B, C

Solution: