WBJEE 2020 Question Paper

West Bengal Joint Entrance Examinations

WB JEE

Engineering Entrance Exam

Solved Paper 2020

Physics

Category I (Q. Nos. 1 to 30)

Carry 1 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

1. The intensity of light emerging from one of the slits in a Young's double slit experiment is found to be 1.5 times the intensity of light emerging from the other slit. What will be the approximate ratio of intensity of an interference maximum to that of an interference minimum?

(a) 2.25 (b) 98 (c) 5 (d) 9.9

2. In a Fraunhofer diffraction experiment, a single slit of width 0.5 mm is illuminated by a monochromatic light of wavelength 600 nm. The diffraction pattern is observed on a screen at a distance of 50 cm from the slit. What will be the linear separation of the first order minima?

(a) 1.0 mm (b) 1.1 mm
(c) 0.6 mm (d) 1.2 mm

3. If R is the Rydberg constant in cm^{-1}, then hydrogen atom does not emit any radiation of wavelength in the range of

(a) $\frac{1}{R}$ to $\frac{4}{3R}$ cm (b) $\frac{7}{5R}$ to $\frac{19}{5R}$ cm
(c) $\frac{4}{R}$ to $\frac{36}{5R}$ cm (d) $\frac{9}{R}$ to $\frac{144}{7R}$ cm

4. A nucleus X emits a β-particle to produce a nucleus Y. If their atomic masses are M_x and M_y respectively, then the maximum energy of the β-particle emitted is (where, m_e is the mass of an electron and c is the velocity of light)

(a) $(M_x - M_y - m_e)c^2$ (b) $(M_x - M_y + m_e)c^2$
(c) $(M_x - M_y)c^2$ (d) $(M_x - M_y - 2m_e)c^2$

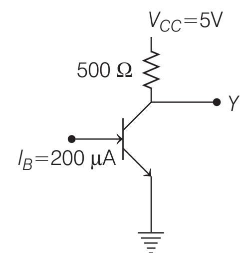
5. For nuclei with mass number close to 119 and 238, the binding energies per nucleon are approximately 7.6 MeV and 8.6 MeV, respectively. If a nucleus of mass number 238 breaks into two nuclei of nearly equal masses, what will be the approximate amount of energy released in the process of fission?

(a) 214 MeV (b) 119 MeV
(c) 2047 MeV (d) 1142 MeV

6. A common emitter transistor amplifier is connected with a load resistance of 6 kΩ. When a small AC signal of 15 mV is added to the base-emitter voltage, the alternating base current is 20 μA and the alternating collector current is 1.8 mA. What is the voltage gain of the amplifier?

(a) 90 (b) 640 (c) 900 (d) 720

7.



In the circuit shown, the value of β of the transistor is 48. If the supplied base current is 200 μA, what is the voltage at the terminal Y?

(a) 0.2 V (b) 0.5 V (c) 4 V (d) 4.8 V

8. The frequency ν of the radiation emitted by an atom when an electron jumps from one orbit to another is given by $\nu = k\delta E$, where k is a constant and δE is the change in energy level due to the transition. Then, dimension of k is

(a) $[ML^2T^{-2}]$
(b) the same dimension of angular momentum
(c) $[ML^2T^{-1}]$
(d) $[M^{-1}L^{-2}T]$

9. Consider the vectors $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{C} = \frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$. What is the value of $\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$?

(a) 1 (b) 0
(c) $3\sqrt{2}$ (d) $18\sqrt{5}$

10. A fighter plane, flying horizontally with a speed 360 km/h at an altitude of 500 m drops a bomb for a target straight ahead of it on the ground. The bomb should be dropped at what approximate distance ahead of the target? Assume that acceleration due to gravity (g) is 10 ms^{-2}. Also, neglect air drag.

(a) 1000 m (b) $50\sqrt{5}$ m
(c) $500\sqrt{5}$ m (d) 866 m

11. A block of mass m rests on a horizontal table with a coefficient of static friction μ. What minimum force must be applied on the block to drag it on the table?

(a) $\frac{\mu}{\sqrt{1+\mu^2}}mg$ (b) $\frac{\mu-1}{\mu+1}mg$
(c) $\frac{\mu}{\sqrt{1-\mu^2}}mg$ (d) $\mu\, mg$

12. A tennis ball hits the floor with a speed v at an angle θ with the normal to the floor. If the collision is inelastic and the coefficient of restitution is ε, what will be the angle of reflection?

(a) $\tan^{-1}\left(\frac{\tan\theta}{\varepsilon}\right)$ (b) $\sin^{-1}\left(\frac{\sin\theta}{\varepsilon}\right)$
(c) $\theta\varepsilon$ (d) $\theta\frac{2\varepsilon}{\varepsilon+1}$

13. The bob of a swinging second pendulum (one whose time period is 2 s) has a small speed v_0 at its lowest point. Its height from this lowest point 2.25 s after passing through it, is given by

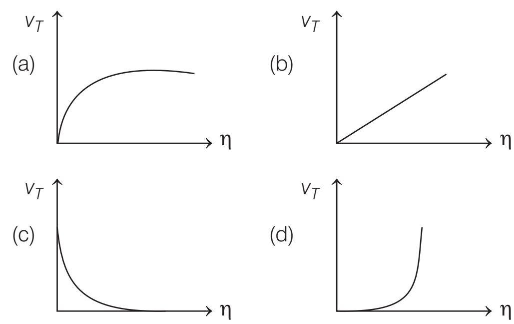
(a) $\frac{v_0^2}{2g}$ (b) $\frac{v_0^2}{g}$
(c) $\frac{v_0^2}{4g}$ (d) $\frac{9v_0^2}{4g}$

14. A steel and a brass wire, each of length 50 cm and cross-sectional area 0.005 cm^2 hang from a ceiling and are 15 cm apart. Lower ends of the wires are attached to a light horizontal bar. A suitable downward load is applied to the bar, so that each of the wires extends in length by 0.1 cm. At what distance from the steel wire, the load must be applied?

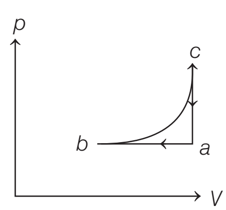
[Young's modulus of steel $= 2 \times 10^{12}$ dyne/cm^2 and that of brass $= 1 \times 10^{12}$ dyne/cm^2]

(a) 7.5 cm (b) 5 cm
(c) 10 cm (d) 3 cm

15. Which of the following diagrams correctly shows the relation between the terminal velocity v_T of a spherical body falling in a liquid and viscosity η of the liquid?



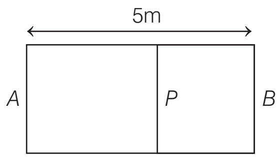
16. An ideal gas undergoes the cyclic process $abca$ as shown in the given p - V diagram.



It rejects 50 J of heat during *ab* and absorbs 80 J of heat during *ca*. During *bc*, there is no transfer of heat and 40 J of work is done by the gas. What should be the area of the closed curve *abca*?

(a) 30 J (b) 40 J (c) 10 J (d) 90 J

17. A container *AB* in the shape of a rectangular parallelopiped of length 5 m is divided internally by a movable partition *P* as shown in the figure.



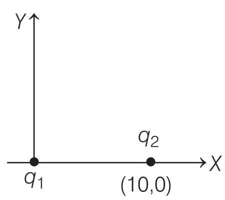
The left compartment is filled with a given mass of an ideal gas of molar mass 32 while the right compartment is filled with an equal mass of another ideal gas of molar mass 18 at same temperature. What will be the distance of *P* from the left wall *A* when equilibrium is established?

(a) 2.5 m (b) 1.8 m (c) 3.2 m (d) 2.1 m

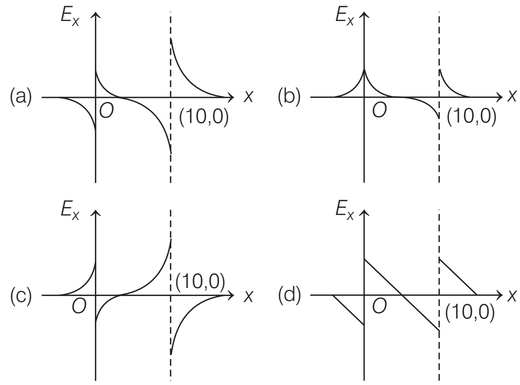
18. When 100 g of boiling water at 100°C is added into a calorimeter containing 300 g of cold water at 10°C, temperature of the mixture becomes 20°C. Then, a metallic block of mass 1 kg at 10°C is dipped into the mixture in the calorimeter. After reaching thermal equilibrium, the final temperature becomes 19°C. What is the specific heat of the metal in CGS unit?

(a) 0.01 (b) 0.3 (c) 0.09 (d) 0.1

19. As shown in the figure, a point charge $q_1 = +1 \times 10^{-6}$ C is placed at the origin in *xy*-plane and another point charge $q_2 = +3 \times 10^{-6}$ C is placed at the coordinate (10, 0).



In that case, which of the following graph(s) shows most correctly the electric field vector in E_x in *x*-direction?



20. Four identical point masses, each of mass *m* and carrying charge $+q$ are placed at the corners of a square of sides *a* on a frictionless plane surface. If the particles are released simultaneously, the kinetic energy of the system when they are infinitely far apart is

(a) $\frac{q^2}{a}(2\sqrt{2}+1)$ (b) $\frac{q^2}{a}(\sqrt{2}+2)$

(c) $\frac{q^2}{a}(\sqrt{2}+4)$ (d) $\frac{q^2}{a}(\sqrt{2}+1)$

21. A very long charged solid cylinder of radius *a* contains a uniform charge density ρ. Dielectric constant of the material of the cylinder is *K*. What will be the magnitude of electric field at a radial distance *x* ($x < a$) from the axis of the cylinder?

(a) $\rho\frac{x}{\varepsilon_0}$ (b) $\rho\frac{x}{2k\varepsilon_0}$

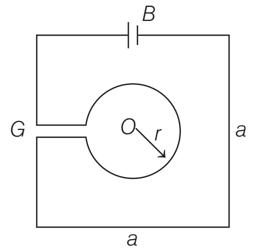
(c) $\rho\frac{x^2}{2a\varepsilon_0}$ (d) $\rho\frac{x}{2k}$

22. A galvanometer can be converted to a voltmeter of full scale deflection V_0 by connecting a series resistance R_1 and can be converted to an ammeter of full scale deflection I_0 by connecting a shunt resistance R_2. What is the current flowing through the galvanometer at its full scale deflection?

(a) $\frac{V_0 - I_0R_2}{R_1 - R_2}$ (b) $\frac{V_0 + I_0R_2}{R_1 + R_2}$

(c) $\frac{V_0 - I_0R_1}{R_2 - R_1}$ (d) $\frac{V_0 + I_0R_1}{R_1 + R_2}$

23.

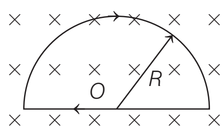


As shown in the figure, a single conducting wire is bent to form a loop in the form of a circle of radius r concentrically inside a square of side a, where $a : r = 8 : \pi$. A battery B drives a current through the wire. If the battery B and the gap G are of negligible sizes, determine the strength of magnetic field at the common centre O.

(a) $\frac{\mu_0 I}{2\pi a}\sqrt{2}(\sqrt{2}-1)$ (b) $\frac{\mu_0 I}{2\pi a}(\sqrt{2}+1)$

(c) $\frac{\mu_0 I}{\pi a}2\sqrt{2}(\sqrt{2}+1)$ (d) $\frac{\mu_0 I}{\pi a}2\sqrt{2}(\sqrt{2}-1)$

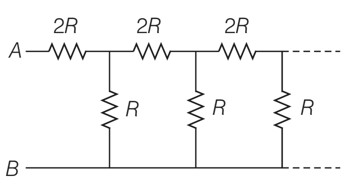
24.



As shown in the figure, a wire is bent to form a D-shaped closed loop, carrying current I, where the curved part is a semi-circle of radius R. The loop is placed in a uniform magnetic field $\mathbf{B}$, which is directed into the plane of the paper. The magnetic force felt by the closed loop is

(a) zero (b) IRB (c) $2IRB$ (d) $\frac{1}{2}IRB$

25.

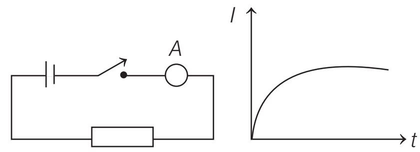


What will be the equivalent resistance between the terminals A and B of the infinite resistive network shown in the figure?

(a) $\frac{(\sqrt{3}+1)R}{2}$ (b) $\frac{(\sqrt{3}-1)R}{2}$

(c) $3\frac{R}{2}$ (d) $(\sqrt{3}+1)R$

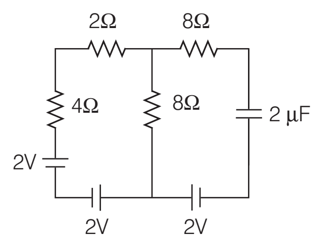
26.



When a DC voltage is applied at the two ends of a circuit kept in a closed box, it is observed that the current gradually increases from zero to a certain value and then remains constant. What do you think that the circuit contains?

(a) A resistor alone
(b) A capacitor alone
(c) A resistor and an inductor in series
(d) A resistor and a capacitor in series

27. Consider the circuit shown.



If all the cells have negligible internal resistance, what will be the current through the $2\,\Omega$ resistor when steady state is reached?

(a) 0.66 A (b) 0.29 A (c) 0 A (d) 0.14 A

28. Consider a conducting wire of length L bent in the form of a circle of radius R and another conductor of length a ($a << R$) is bent in the form of a square. The two loops are then placed in same plane such that the square loop is exactly at the centre of the circular loop. What will be the mutual inductance between the two loops?

(a) $\mu_0\frac{\pi a^2}{L}$ (b) $\mu_0\frac{\pi a^2}{16L}$

(c) $\mu_0\frac{\pi a^2}{4L}$ (d) $\mu_0\frac{a^2}{4\pi L}$

29. An object, is placed 60 cm in front of a convex mirror of focal length 30 cm. A plane mirror is now placed facing the object in between the object and the convex mirror such that it covers lower half of the convex mirror. What should be the distance of the plane mirror from the object, so that there will be no parallax between the images formed by the two mirrors?

(a) 40 cm (b) 30 cm (c) 20 cm (d) 15 cm

30. A thin convex lens is placed just above an empty vessel of depth 80 cm. The image of a coin kept at the bottom of the vessel is thus formed 20 cm above the lens. If now water is poured in the vessel upto a height of 64 cm, what will be the approximate new position of the image? Assume that refractive index of water is 4/3.

(a) 21.33 cm, above the lens
(b) 6.67 cm, below the lens
(c) 33.67 cm, above the lens
(d) 24 cm, above the lens

Category II (Q. Nos. 31 to 35)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

31. A conducting circular loop of resistance $20\,\Omega$ and cross-sectional area 20×10^{-2} m^2 is placed perpendicular to a spatially uniform magnetic field **B**, which varies with time t as $\mathbf{B} = 2\sin(50\ \pi t)$ T. Find the net charge flowing through the loop in 20 ms starting from $t = 0$.

(a) 0.5 C (b) 0.2 C (c) 0 C (d) 0.14 C

32. A pair of parallel metal plates are kept with a separation d. One plate is at a potential $+V$ and the other is at ground potential. A narrow beam of electrons enters the space between the plates with a velocity v_0 and in a direction parallel to the plates. What will be the angle of the beam with the plates after it travels an axial distance L?

(a) $\tan^{-1}\left(\frac{eVL}{mdv_0}\right)$ (b) $\tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

(c) $\sin^{-1}\left(\frac{eVL}{mdv_0}\right)$ (d) $\cos^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

33. A metallic block of mass 20 kg is dragged with a uniform velocity of 0.5 ms^{-1} on a horizontal table for 2.1 s. The coefficient of static friction between the block and the table is 0.10. What will be the maximum possible rise in temperature of the metal block, if the specific heat of the block is 0.1 CGS unit? Assume g =10 ms^{-2} and uniform rise in temperature throughout the whole block. [Ignore absorption of heat by the table]

(a) 0.0025°C (b) 0.025°C
(c) 0.001°C (d) 0.05°C

34. Consider an engine that absorbs 130 cal of heat from a hot reservoir and delivers 30 cal heat to a cold reservoir in each cycle. The engine also consumes 2 J energy in each cycle to overcome friction. If the engine works at 90 cycles per minute, what will be the maximum power delivered to the load? [Assume the thermal equivalent of heat is 4.2 J/cal]

(a) 816 W (b) 819 W
(c) 627 W (d) 630 W

35. Two pith balls, each carrying charge $+q$ are hung from a hook by two strings. It is found that when each charge is tripled, angle between the strings double. What was the initial angle between the strings?

(a) 30° (b) 60° (c) 45° (d) 90°

Category III (Q. Nos. 36 to 40)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero marks will be awarded.

36. A point source of light is used in an experiment of photoelectric effects. If the distance between the source and the photoelectric surface is doubled, which of the following may result?

(a) Stopping potential will be halved.
(b) Photoelectric current will decrease.
(c) Maximum kinetic energy of photoelectrons will decrease.
(d) Stopping potential will increase slightly.

37. Two metallic spheres of equal outer radii are found to have same moment of inertia about their respective diameters. Then, which of the following statement(s) is/are true?

(a) The two spheres have equal masses.
(b) The ratio of their masses is nearly 1.67 : 1.
(c) The spheres are made of different materials.
(d) Their rotational kinetic energies will be equal when rotated with equal uniform angular speed about their respective diameters.

38. A simple pendulum of length l is displaced, so that its taught string is horizontal and then released. A uniform bar pivoted at one end is simultaneously released from its horizontal position. If their motions are synchronous, what is the length of the bar?

(a) $\frac{3l}{2}$ (b) l (c) $2l$ (d) $\frac{2l}{3}$

39. A 400 Ω resistor, a 250 mH inductor and a 2.5 μF capacitor are connected in series with an AC source of peak voltage 5 V and angular frequency 2kHz. What is the peak value of the electrostatic energy of the capacitor?

(a) 2 μJ (b) 2.5 μJ
(c) 3.33 μJ (d) 5 μJ

40. A charged particle moves with constant velocity in a region, where no effect of gravity is felt but an electrostatic field **E** together with a magnetic field B may be present. Then, which of the following cases are possible?

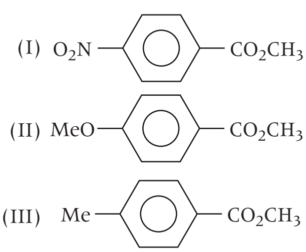
(a) $E \neq 0, B \neq 0$
(b) $E \neq 0, B = 0$
(c) $E = 0, B = 0$
(d) $E = 0, B \neq 0$

Chemistry

Category-I (Q. Nos. 41 to 70)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

41.



For the above three esters, the order of rates of alkaline hydrolysis is

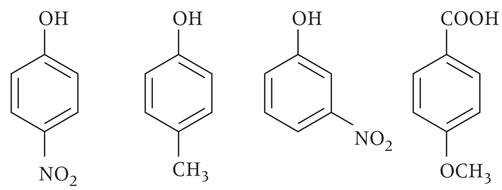
(a) I > II > III (b) II > III > I
(c) I > III > II (d) III > I > II

42. $\mathrm{Ph{-}CDO} \xrightarrow[\text{Warm}]{50\%\ aq.\mathrm{NaOH}} \mathrm{Ph{-}COO\overset{\ominus}{H}}$ + an alcohol.

This alcohol is

(a) Ph—CHD—OH (b) Ph—CHD—OD
(c) Ph—CD_2—OH (d) Ph—CD_2—OD

43. The correct order of acidity for the following compounds is :



(a) II < IV < III < I (b) II < III < I < IV
(c) II < III < IV < I (d) III < II < I < IV

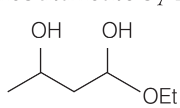
44. For the following carbocations, the correct order of stability is

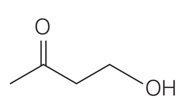
I. $\overset{\oplus}{C}H_2—COCH_3$ II. $\overset{\oplus}{C}H_2—OCH_3$

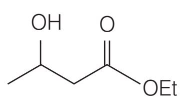
III. $\overset{\oplus}{C}H_2—CH_3$

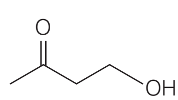
(a) III < II < I (b) II < I < III
(c) I < II < III (d) I < III < II

45. The reduction product of ethyl 3-oxobutanoate by $NaBH_4$ in methanol is

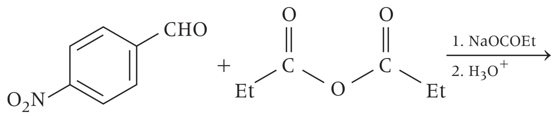
(a) 

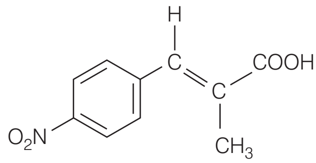
(b) 

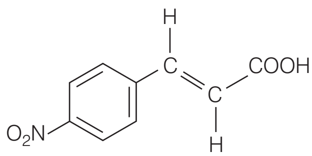
(c) 

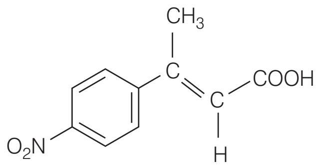
(d) 

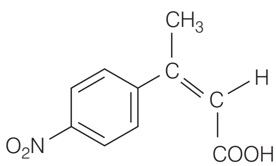
46. What is the major product of the following reaction?



(a) 

(b) 

(c) 

(d) 

47. The maximum number of electrons in an atom in which the last electron filled has the quantum numbers $n = 3$, $l = 2$ and $m = -1$ is

(a) 17 (b) 27
(c) 28 (d) 30

48. In the face centred cubic lattice structure of gold the closest distance between gold atoms is ('a' being the edge length of the cubic unit cell)

(a) $a\sqrt{2}$ (b) $\frac{a}{\sqrt{2}}$ (c) $\frac{a}{2\sqrt{2}}$ (d) $2\sqrt{2}a$

49. The equilibrium constant for the following reactions are given at 25°C

$$2A \rightleftharpoons B + C, \quad K_1 = 1.0$$
$$2B \rightleftharpoons C + D, \quad K_2 = 16$$
$$2C + D \rightleftharpoons 2P, \quad K_3 = 25$$

The equilibrium constant for the reaction

$$P \rightleftharpoons A + \frac{1}{2} B \text{ at } 25°\text{C is}$$

(a) $\frac{1}{20}$ (b) 20 (c) $\frac{1}{42}$ (d) 21

50. Among the following, the ion which will be more effective for flocculation of $Fe(OH)_3$ solution is

(a) PO_4^{3-} (b) SO_4^{2-}
(c) SO_3^{2-} (d) NO_3^-

51. The mole fraction of ethanol in water is 0.08. Its molality is

(a) 6.32 mol kg^{-1} (b) 4.83 mol kg^{-1}
(c) 3.82 mol kg^{-1} (d) 2.84 mol kg^{-1}

52. 5 mL of 0.1 M $Pb(NO_3)_2$ is mixed with 10 mL of 0.02 M KI. The amount of PbI_2 precipitated will be about

(a) 10^{-2}mol (b) 10^{-4}mol
(c) 2×10^{-4}mol (d) 10^{-3}mol

53. At 273 K temperature and 76 cm Hg pressure, the density of a gas is 1.964 g L^{-1}. The gas is

(a) CH_4 (b) CO (c) He (d) CO_2

54. Equal masses of ethane and hydrogen are mixed in an empty container at 298 K. The fraction of total pressure exerted by hydrogen is

(a) 15:16 (b) 1:1 (c) 1:4 (d) 1:6

55. An ideal gas expands adiabatically against vacuum. Which of the following is correct for the given process?

(a) $\Delta S = 0$ (b) $\Delta T = -$ ve
(c) $\Delta U = 0$ (d) $\Delta P = 0$

56. K_f (water) = 1.86 K kg mol^{-1}. The temperature at which ice begins to separate from a mixture of 10 mass % ethylene glycol is

(a) – 1.86 °C (b) – 3.72°C
(c) – 3.3°C (d) – 3°C

57. The radius of the first Bohr orbit of a hydrogen atom is 0.53×10^{-8}cm. The velocity of the electron in the first Bohr orbit is

(a) 2.188×10^8cm s^{-1} (b) 4.376×10^8 cm s^{-1}
(c) 1.094×10^8 cm s^{-1} (d) 2.188×10^9 cm s^{-1}

58. Which of the following statements is not true for the reaction, $2F_2 + 2H_2O \rightarrow 4HF + O_2$?

(a) F_2 is more strongly oxidising than O_2
(b) F—F bond is weaker than O = O bond
(c) H—F bond is stronger than H—O bond
(d) F is less electronegative than O

59. The number of unpaired electrons in the uranium ($_{92}U$) atom is

(a) 4 (b) 6 (c) 3 (d) 1

60. How and why does the density of liquid water change on prolonged electrolysis?

(a) Decreases, as the proportion of H_2O increases
(b) Remains unchanged
(c) Increases, as the proportion of D_2O increases
(d) Increases, as the volume decreases

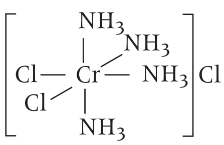
61. The difference between orbital angular momentum of an electron in a $4f$-orbital and another electron in a $4s$-orbital is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

62. Which of the following has the largest number of atoms?

(a) 1 g of Ag (b) 1 g of Fe
(c) 1 g of Cl_2 (d) 1 g of Mg

63. Indicate the correct IUPAC name of the coordination compound shown in the figrue.



(a) *Cis*-dichlorotetraminochromium (III) chloride
(b) *Trans*-dichlorotetraminochromium (III) chloride
(c) *Trans*-tetraminedichlorochromium (III) chloride
(d) *Cis*-tetraamminedichlorochromium (III) chloride

64. What will be the mass of one atom of ^{12}C?

(a) 1 amu (b) 1.9923×10^{-23} g
(c) 1.6603×10^{-22} g (d) 6 amu

65. Bond order of He_2, He_2^+ and He_2^{2+} are respectively

(a) $1, \frac{1}{2}, 0$ (b) $0, \frac{1}{2}, 1$
(c) $\frac{1}{2}, 1, 0$ (d) $1, 0, \frac{1}{2}$

66. To a solution of a colourless efflorescent sodium salt, when dilute acid is added, a colourless gas is evolved along with formation of a white precipitate. Acidified dichromate solution turns green, when the colourless gas is passed through it. The sodium salt is

(a) Na_2SO_3 (b) Na_2S
(c) $Na_2S_2O_3$ (d) $Na_2S_4O_6$

67. The reaction for obtaining the metal (M) from its oxide (M_2O_3) ore is given by

$$M_2O_3(s) + 2Al(l) \xrightarrow{\text{Heat}} Al_2O_3(l) + 2M(s),$$

(s = solid, l = liquid) in that case, M is

(a) copper (b) calcium
(c) iron (d) zinc

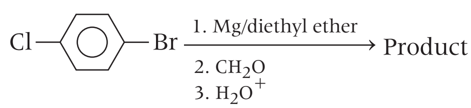
68. In the extraction of Ca by electro reduction of molten $CaCl_2$ some CaF_2 is added to the electrolyte for the following reason :

(a) To keep the electrolyte in liquid state at temperature lower than the m.p. of $CaCl_2$
(b) To effect precipitation of Ca
(c) To effect the electrolysis at lower voltage
(d) To increase the current efficiency

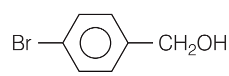
69. The total number of alkyl bromides (including stereoisomers) formed in the reaction $Me_3C—CH = CH_2 + HBr \rightarrow$ will be

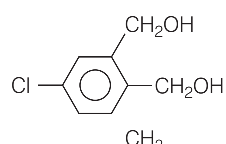
(a) 1 (b) 2
(c) 3 (d) No bromide forms

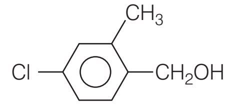
70.

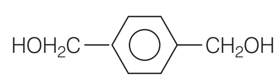


The product in the above reaction is

(a)


(b)


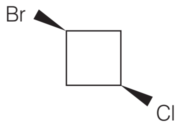
(c)


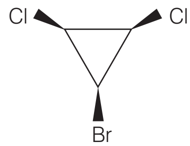
(d)


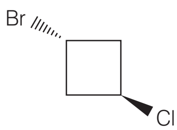
Category-II (Q. Nos. 71 to 75)

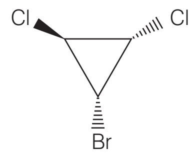
Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

71. Which of the following compounds is asymmetric?

(a) 

(b) 

(c) 

(d) 

72. For a reaction $2A + B \rightarrow P$, when concentration of B alone is doubled, $t_{1/2}$ does not change and when concentrations of both A and B is doubled, rate increases by a factor of 4. The unit of rate constant is,

(a) s^{-1} (b) $L\ mol^{-1}s^{-1}$
(c) $mol\ L^{-1}s^{-1}$ (d) $L^2mol^{-2}s^{-1}$

73. A solution is saturated with $SrCO_3$ and SrF_2. The $[CO_3^{2-}]$ is found to be 1.2×10^{-3}M. The concentration of F^- in the solution would be

(a) 3.7×10^{-6}M
(b) 3.2×10^{-3}M
(c) 5.1×10^{-7}M
(d) 3.7×10^{-2}M

Given: $K_{sp}(SrCO_3) = 7.0 \times 10^{-10}$,
$K_{sp}(SrF_2) = 7.9 \times 10^{-10}$

74. A homonuclear diatomic gas molecule shows 2-electron magnetic moment. The one-electron and two-electron reduced species obtained from above gas molecule can act as both oxidising and reducing agents. When the gas molecule is one-electron oxidised the bond length decreases compared to the neutral molecule. The gas molecule is

(a) N_2 (b) Cl_2
(c) O_2 (d) B_2

75. $$CH_3—O—CH_2—Cl \xrightarrow[\Delta]{aq.^{\ominus}OH} CH_3—O—CH_2—OH$$

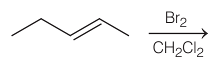
Which information below regarding this reaction is applicable?

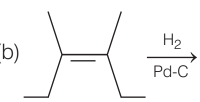
(a) It follows S_N2 pathway, because it is a primary alkyl chloride
(b) It follows S_N1 pathway, because the intermediate carbocation is resonance stabilised
(c) S_N1 pathway is not followed, because the intermediate carbocation is destabilised by $-I$-effect of oxygen
(d) A mixed S_N1 and S_N2 pathway is followed

Category-III (Q. Nos. 76 to 80)

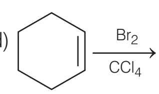
Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and no incorrect answer is marked, then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero mark will be awarded.

76. Which of the following reactions give(s) a *meso*-compound as the main product?

(a) 

(b) 

(c) 

(d) 

77. For spontaneous polymerisation, which of the following is (are) correct?

(a) ΔG is negative (b) ΔH is negative
(c) ΔS is positive (d) ΔS is negative

78. Which of the following statement(s) is/are incorrect?

(a) A sink of SO_2 pollutant is O_3 in the atmosphere
(b) FGD is a process of removing NO_2 from atmosphere
(c) NO_x in fuel gases can be removed by alkaline scrubbing
(d) The catalyst used to convert CCl_4 to CF_4 by HF is SbF_5

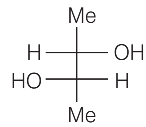
79. SiO_2 is attacked by which one/ones of the following?

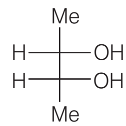
(a) HF (b) Conc.HCl
(c) Hot NaOH (d) Fluorine

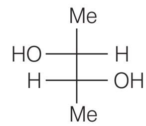
80. $\text{Me—C} \equiv \text{C—Me} \xrightarrow[\text{EtOH, } -33°\text{C}]{\text{Na/NH}_3\text{(liq.)}} \underline{\underline{X}}$

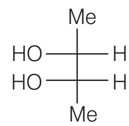
$\xrightarrow{\text{dil.alkaline KMnO}_4} \text{Product(s)}$

The product(s) from the above reaction will be

(a) 

(b) 

(c) 

(d) 

Mathematics

Category-I (Q. Nos. 1 to 50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

1. Let $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$. Then

(a) $x^2y_2 + xy_1 + n^2y = 0$
(b) $xy_2 - xy_1 + 2n^2y = 0$
(c) $x^2y_2 + 3xy_1 - n^2y = 0$
(d) $xy_2 + 5xy_1 - 3y = 0$
$\left(\text{Here, } y_2 = \frac{d^2y}{dx^2}, y_1 = \frac{dy}{dx}\right)$

2. Let $\phi(x) = f(x) + f(1 - x)$ and $f''(x) < 0$ in $[0, 1]$, then

(a) ϕ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2}, 1\right]$
(b) ϕ is monotonic increasing in $\left[\frac{1}{2}, 1\right]$ and monotonic decreasing in $\left[0, \frac{1}{2}\right]$
(c) ϕ is neither increasing nor decreasing in any sub-interval of [0, 1]
(d) ϕ is increasing in [0, 1]

3. $\displaystyle\int \frac{f(x)\,\phi'(x) + \phi(x)\,f'(x)}{(f(x)\,\phi(x) + 1)\sqrt{f(x)\,\phi(x) - 1}}\,dx =$

(a) $\sin^{-1}\sqrt{\frac{f(x)}{\phi(x)}} + c$
(b) $\cos^{-1}\sqrt{(f(x))^2 - (\phi(x))^2} + c$
(c) $\sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\,\phi(x) - 1}{2}} + c$
(d) $\sqrt{2}\tan^{-1}\sqrt{\frac{f(x)\,\phi(x) + 1}{2}} + c$

4. The value of
$\displaystyle\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx + \sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$ is equal to

(a) 27 (b) 54
(c) -54 (d) 0

5. $\displaystyle\int_0^2 [x^2]dx$ is equal to

(a) 1 (b) $5 - \sqrt{2} - \sqrt{3}$
(c) $3 - \sqrt{2}$ (d) 8/3

6. If the tangent to the curve $y^2 = x^3$ at (m^2, m^3) is also a normal to the curve at (M^2, M^3), then the value of mM is

(a) $-\frac{1}{9}$ (b) $-\frac{2}{9}$
(c) $-\frac{1}{3}$ (d) $-\frac{4}{9}$

7. If $x^2 + y^2 = a^2$, then $\displaystyle\int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx =$

(a) $2\pi a$ (b) πa
(c) $\frac{1}{2}\pi a$ (d) $\frac{1}{4}\pi a$

8. Let f, be a continuous function in [0, 1], then $\lim_{n \to \infty} \sum_{j=0}^{n} \frac{1}{n} f\left(\frac{j}{n}\right)$ is

(a) $\frac{1}{2} \int_0^{\frac{1}{2}} f(x)\,dx$
(b) $\int_{\frac{1}{2}}^{1} f(x)\,dx$
(c) $\int_0^1 f(x)\,dx$
(d) $\int_0^{\frac{1}{2}} f(x)\,dx$

9. Let f be a differentiable function with $\lim_{x \to \infty} f(x) = 0$. If $y' + yf'(x) - f(x)f'(x) = 0$, $\lim_{x \to \infty} y(x) = 0$, then (where $y' \equiv \frac{dy}{dx}$)

(a) $y + 1 = e^{f(x)} + f(x)$
(b) $y - 1 = e^{f(x)} + f(x)$
(c) $y + 1 = e^{-f(x)} + f(x)$
(d) $y - 1 = e^{-f(x)} + f(x)$

10. If $x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x\right] dx$, $x > 0$ and $y(1) = \frac{\pi}{2}$, then the value of $\cos\left(\frac{y}{x}\right)$ is

(a) 1 (b) $\log x$ (c) e (d) 0

11. Let $f(x) = 1 - \sqrt{(x^2)}$, where the square root is to be taken positive, then

(a) f has no extrema at $x = 0$
(b) f has minima at $x = 0$
(c) f has maxima at $x = 0$
(d) f' exists at 0

12. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ $[a > 0]$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a is equal to

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 3

13. If a and b are arbitrary positive real numbers, then the least possible value of $\frac{6a}{5b} + \frac{10b}{3a}$ is

(a) 4
(b) $\frac{6}{5}$
(c) $\frac{10}{3}$
(d) $\frac{68}{15}$

14. If $2\log(x+1) - \log(x^2 - 1) = \log 2$, then $x =$

(a) only 3
(b) – 1 and 3
(c) only – 1
(d) 1 and 3

15. The number of complex numbers p such that $|p| = 1$ and imaginary part of p^4 is 0, is

(a) 4
(b) 2
(c) 8
(d) infinitely many

16. The equation $z\bar{z} + (2 - 3i)z + (2 + 3i)\bar{z} + 4 = 0$ represents a circle of radius

(a) 2 unit
(b) 3 unit
(c) 4 unit
(d) 6 unit

17. The expression $ax^2 + bx + c$ (a, b and c are real) has the same sign as that of a for all x if

(a) $b^2 - 4ac > 0$
(b) $b^2 - 4ac \neq 0$
(c) $b^2 - 4ac \leq 0$
(d) b and c have the same sign as that of a

18. In a 12 storied building, 3 persons enter a lift cabin. It is known that they will leave the lift at different floors. In how many ways can they do so if the lift does not stop at the second floor?

(a) 36
(b) 120
(c) 240
(d) 720

19. If the total number of m-element subsets of the set $A = \{a_1, a_2, \ldots, a_n\}$ is k times the number of m element subsets containing a_4, then n is

(a) $(m-1)k$
(b) mk
(c) $(m+1)k$
(d) $(m+2)k$

20. Let $I(n) = n^n$, $J(n) = 1.3.5 \ldots\ldots (2n-1)$ for all $(n > 1)$, $n \in N$, then

(a) $I(n) > J(n)$
(b) $I(n) < J(n)$
(c) $I(n) = J(n)$
(d) $I(n) = \frac{1}{2}J(n)$

21. If $c_0, c_1, c_2, \ldots, c_{15}$ are the binomial coefficients in the expansion of $(1+x)^{15}$, then the value of $\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \ldots + 15\frac{c_{15}}{c_{14}}$ is

(a) 1240
(b) 120
(c) 124
(d) 140

22. Let $A = \begin{pmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{pmatrix}$ and $\det A = 5$, then

(a) $t = 1$ (b) $t = 2$ (c) $t = -1$ (d) $t = -2$

23. Let $A = \begin{pmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{pmatrix}$. The value of x for which the matrix A is not invertible is

(a) 6 (b) 12 (c) 3 (d) 2

24. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 real matrix with $\det A = 1$. If the equation $\det(A - \lambda I_2) = 0$ has imaginary roots (I_2 be the identity matrix of order 2), then

(a) $(a+d)^2 < 4$ (b) $(a+d)^2 = 4$
(c) $(a+d)^2 > 4$ (d) $(a+d)^2 = 16$

25. If $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = ka^2b^2c^2$, then $k =$

(a) 2 (b) -2 (c) -4 (d) 4

26. If $f : S \to R$, where S is the set of all non-singular matrices of order 2 over R and $f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$, then

(a) f is bijective mapping
(b) f is one-one but not onto
(c) f is onto but not one-one
(d) f is neither one-one nor onto

27. Let the relation ρ be defined on R by $a \rho b$ holds if and only if $a - b$ is zero or irrational, then

(a) ρ is equivalence relation
(b) ρ is reflexive and symmetric but is not transitive
(c) ρ is reflexive and transitive but is not symmetric
(d) ρ is reflexive only

28. The unit vector in ZOX plane, making angles 45° and 60° respectively with $\alpha = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\beta = \hat{j} - \hat{k}$ is

(a) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ (b) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$
(c) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$ (d) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$

29. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game. What is the probability that A wins if A begins?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{7}{12}$ (d) $\frac{8}{15}$

30. A rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds he must fire to have more than 50% chance of hitting it at least once, is

(a) 5 (b) 7 (c) 9 (d) 11

31. $\cos(2x + 7) = a(2 - \sin x)$ can have a real solution for

(a) all real values of a (b) $a \in [2, 6]$
(c) $a \in (-\infty, 2) \setminus \{0\}$ (d) $a \in (0, \infty)$

32. The differential equation of the family of curves $y = e^x (A\cos x + B\sin x)$ where A, B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} - 9x = 13$ (b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
(c) $\frac{d^2y}{dx^2} + 3y = 4$ (d) $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - xy = 0$

33. The equation $r\cos\left(\theta - \frac{\pi}{3}\right) = 2$ represents

(a) a circle (b) a parabola
(c) an ellipse (d) a straight line

34. The locus of the centre of the circles which touch both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4ax$ externally is

(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

35. Let each of the equations $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ represent two straight lines passing through the origin. If they have a common line, then the other two lines are given by

(a) $x - y = 0, x - 3y = 0$ (b) $x + 3y = 0, 3x + y = 0$
(c) $3x + y = 0, 3x - y = 0$ (d) $(3x - 2y) = 0, x + y = 0$

36. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at P and Q respectively. The point O divides the segment PQ in the ratio

(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

37. Area in the first quadrant between the ellipses $x^2 + 2y^2 = a^2$ and $2x^2 + y^2 = a^2$ is

(a) $\frac{a^2}{\sqrt{2}} \tan^{-1} \frac{1}{\sqrt{2}}$ (b) $\frac{3a^2}{4} \tan^{-1} \frac{1}{2}$

(c) $\frac{5a^2}{2} \sin^{-1} \frac{1}{2}$ (d) $\frac{9\pi a^2}{2}$

38. The equation of circle of radius $\sqrt{17}$ unit, with centre on the positive side of X-axis and through the point $(0, 1)$ is

(a) $x^2 + y^2 - 8x - 1 = 0$

(b) $x^2 + y^2 + 8x - 1 = 0$

(c) $x^2 + y^2 - 9y + 1 = 0$

(d) $2x^2 + 2y^2 - 3x + 2y = 4$

39. The length of the chord of the parabola $y^2 = 4ax (a > 0)$ which passes through the vertex and makes an acute angle α with the axis of the parabola is

(a) $\pm\ 4a \cot\alpha \operatorname{cosec} \alpha$ (b) $4a \cot\alpha \operatorname{cosec} \alpha$

(c) $-\ 4a \cot\alpha \operatorname{cosec} \alpha$ (d) $4a \operatorname{cosec}^2 \alpha$

40. A double ordinate PQ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is such that ΔOPQ is equilateral, O being the centre of the hyperbola. Then the eccentricity e satisfies the relation

(a) $1 < e < \frac{2}{\sqrt{3}}$ (b) $e = \frac{2}{\sqrt{3}}$

(c) $e = \frac{\sqrt{3}}{2}$ (d) $e > \frac{2}{\sqrt{3}}$

41. If B and B' are the ends of minor axis and S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then the area of the rhombus $SBS'B'$ will be

(a) 12 sq units (b) 48 sq units

(c) 24 sq units (d) 36 sq units

42. The equation of the latusrectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$. Then, the length of the latusrectum is

(a) $4\sqrt{2}$ units (b) $2\sqrt{2}$ units

(c) 8 units (d) $8\sqrt{2}$ units

43. The equation of the plane through the point $(2, -1, -3)$ and parallel to the lines $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-4}$ and $\frac{x}{2} = \frac{y-1}{-3} = \frac{z-2}{2}$ is

(a) $8x + 14y + 13z + 37 = 0$

(b) $8x - 14y - 13z - 37 = 0$

(c) $8x - 14y - 13z + 37 = 0$

(d) $8x - 14y + 13z + 37 = 0$

44. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{2\sqrt{3}}{5}$ (b) $\frac{\sqrt{2}}{10}$

(c) $\frac{4}{5\sqrt{2}}$ (d) $\frac{\sqrt{5}}{6}$

45. Let $f(x) = \sin x + \cos ax$ be periodic function. Then,

(a) a is any real number

(b) a is any irrational number

(c) a is rational number

(d) $a = 0$

46. The domain of $f(x) = \sqrt{\left(\frac{1}{\sqrt{x}} - \sqrt{(x+1)}\right)}$ is

(a) $x > -1$ (b) $(-1, \infty) \setminus \{0\}$

(c) $\left(0, \frac{\sqrt{5}-1}{2}\right]$ (d) $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

47. Let $y = f(x) = 2x^2 - 3x + 2$. The differential of y when x changes from 2 to 1.99 is

(a) 0.01 (b) 0.18 (c) $-$ 0.05 (d) 0.07

48. If $\lim_{x \to 0} \left(\frac{1+cx}{1-cx}\right)^{1/x} = 4$, then $\lim_{x \to 0} \left(\frac{1+2cx}{1-2cx}\right)^{1/x}$ is

(a) 2 (b) 4 (c) 16 (d) 64

49. Let $f : R \to R$ be twice continuously differentiable (or f'' exists and is continuous) such that $f(0) = f(1) = f'(0) = 0$. Then

(a) $f''(c) = 0$ for some $c \in R$

(b) there is no point for which $f''(x) = 0$

(c) at all points $f''(x) > 0$

(d) at all points $f''(x) < 0$

50. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12$. Then

(a) $f(x)$ has 13 non-zero real roots

(b) $f(x)$ has exactly one real root

(c) $f(x)$ has exactly one pair of imaginary roots

(d) $f(x)$ has no real root

Category-II (Q. Nos. 51-65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

51. The area of the region $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$ is

(a) $\frac{\pi^2}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{4} - \frac{1}{2}$ (d) $\frac{\pi^2}{3}$

52. In open interval $\left(0, \frac{\pi}{2}\right)$,

(a) $\cos x + x\sin x < 1$
(b) $\cos x + x\sin x > 1$
(c) no specific order relation can be ascertained between $\cos x + x\sin x$ and 1
(d) $\cos x + x\sin x < \frac{1}{2}$

53. If the line $y = x$ is a tangent to the parabola $y = ax^2 + bx + c$ at the point $(1, 1)$ and the curve passes through $(-1, 0)$, then

(a) $a = b = -1, c = 3$ (b) $a = b = \frac{1}{2}, c = 0$
(c) $a = c = \frac{1}{4}, b = \frac{1}{2}$ (d) $a = 0, b = c = \frac{1}{2}$

54. If the vectors $\alpha = \hat{i} + a\hat{j} + a^2\hat{k}$, $\beta = \hat{i} + b\hat{j} + b^2\hat{k}$ and $\gamma = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ c & c^2 & 1 + c^3 \end{vmatrix} = 0$, then the value of abc is

(a) 1 (b) 0 (c) -1 (d) 2

55. Let z_1 and z_2 be two imaginary roots of $z^2 + pz + q = 0$, where p and q are real. The points z_1, z_2 and origin form an equilateral triangle if

(a) $p^2 > 3q$ (b) $p^2 < 3q$
(c) $p^2 = 3q$ (d) $p^2 = q$

56. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \ne 0$ [a, b, c, d are all real], then $P(x) \cdot Q(x) = 0$ has

(a) atleast two real roots (b) two real roots
(c) four real roots (d) no real root

57. Let $A = \{x \in R: -1 \le x \le 1\}$ and $f: A \to A$ be a mapping defined by $f(x) = x|x|$. Then f is

(a) injective but not surjective
(b) surjective but not injective
(c) neither injective nor surjective
(d) bijective

58. Let $f(x) = \sqrt{x^2 - 3x + 2}$ and $g(x) = \sqrt{x}$ be two given functions. If S be the domain of fog and T be the domain of gof, then

(a) $S = T$ (b) $S \cap T = \phi$
(c) $S \cap T$ is a singleton (d) $S \cap T$ is an interval

59. Let ρ_1 and ρ_2 be two equivalence relations defined on a non-void set S. Then

(a) both $\rho_1 \cap \rho_2$ and $\rho_1 \cup \rho_2$ are equivalence relations
(b) $\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so
(c) $\rho_1 \cup \rho_2$ is equivalence relation but $\rho_1 \cap \rho_2$ is not so
(d) neither $\rho_1 \cap \rho_2$ nor $\rho_1 \cup \rho_2$ is equivalence relation.

60. Consider the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The portion of the tangent at any point of the curve intercepted between the point of contact and the directrix subtends at the corresponding focus an angle of

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

61. A line cuts the X-axis at $A(7, 0)$ and the Y-axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the X-axis at $P(a, 0)$ and the Y-axis at $Q(0, b)$. If AQ and BP intersect at R, the locus of R is

(a) $x^2 + y^2 + 7x + 5y = 0$
(b) $x^2 + y^2 + 7x - 5y = 0$
(c) $x^2 + y^2 - 7x + 5y = 0$
(d) $x^2 + y^2 - 7x - 5y = 0$

62. Let $0 < \alpha < \beta < 1$. Then, $\lim_{n \to \infty} \int_{1/(k+\beta)}^{1/(k+\alpha)} \frac{dx}{1 + x}$ is

(a) $\log_e \frac{\beta}{\alpha}$ (b) $\log_e \frac{1 + \beta}{1 + \alpha}$
(c) $\log_e \frac{1 + \alpha}{1 + \beta}$ (d) ∞

63. $\lim_{x \to 1}\left(\frac{1}{\ln x} - \frac{1}{(x-1)}\right)$

(a) Does not exist (b) 1

(c) $\frac{1}{2}$ (d) 0

64. Let $y = \frac{1}{1 + x + \ln x}$, then

(a) $x\frac{dy}{dx} + y = x$ (b) $x\frac{dy}{dx} = y(y\ln x - 1)$

(c) $x^2\frac{dy}{dx} = y^2 + 1 - x^2$ (d) $x\left(\frac{dy}{dx}\right)^2 = y - x$

65. Consider the curve $y = be^{-x/a}$, where a and b are non-zero real numbers. Then

(a) $\frac{x}{a} + \frac{y}{b} = 1$ is tangent to the curve at (0, 0)

(b) $\frac{x}{a} + \frac{y}{b} = 1$ is tangent to the curve, where the curve crosses the axis of y

(c) $\frac{x}{a} + \frac{y}{b} = 1$ is tangent to the curve at $(a, 0)$

(d) $\frac{x}{a} + \frac{y}{b} = 1$ is tangent to the curve at $(2a, 0)$

Category-III (Q. Nos. 66 to 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and no incorrect answer is marked, then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. The area of the figure bounded by the parabola $x = -2y^2$, $x = 1 - 3y^2$ is

(a) $\frac{1}{3}$ sq unit (b) $\frac{4}{3}$ sq unit

(c) 1 sq unit (d) 2 sq units

67. A particle is projected vertically upwards. If it has to stay above the ground for 12 sec, then

(a) velocity of projection is 192 ft/sec

(b) greatest height attained is 600 ft

(c) velocity of projection is 196 ft/sec

(d) greatest height attained is 576 ft

68. The equation $x^{(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5} = 3\sqrt{3}$ has

(a) at least one real root

(b) exactly one real root

(c) exactly one irrational root

(d) complex roots

69. In a certain test, there are n questions. In this test 2^{n-i} students gave wrong answers to at least i questions, where $i = 1, 2, \ldots, n$. If the total number of wrong answers given is 2047, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

70. A and B are independent events. The probability that both A and B occur is $\frac{1}{20}$ and the probability that neither of them occurs is $\frac{3}{5}$. The probability of occurrence of A is

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

71. The equation of the straight line passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is -1 is

(a) $\frac{x}{2} - \frac{y}{3} = 1$ (b) $\frac{x}{-2} + \frac{y}{1} = 1$

(c) $-\frac{x}{3} + \frac{y}{2} = 1$ (d) $\frac{x}{1} - \frac{y}{2} = 1$

72. Consider a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at any point. The locus of the mid-point of the portion intercepted between the axes is

(a) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (b) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

(c) $\frac{1}{3x^2} + \frac{1}{4y^2} = 1$ (d) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

73. Let $y = \frac{x^2}{(x+1)^2(x+2)}$. Then, $\frac{d^2y}{dx^2}$ is

(a) $2\left[\frac{3}{(x+1)^4} - \frac{3}{(x+1)^3} + \frac{4}{(x+2)^3}\right]$

(b) $3\left[\frac{2}{(x+1)^3} + \frac{4}{(x+1)^2} - \frac{5}{(x+2)^3}\right]$

(c) $\frac{6}{(x+1)^3} - \frac{4}{(x+1)^2} + \frac{3}{(x+1)^3}$

(d) $\frac{7}{(x+1)^3} - \frac{3}{(x+1)^2} + \frac{2}{(x+1)^3}$

74. Let $f(x) = \frac{1}{3} x \sin x - (1 - \cos x)$. The smallest positive integer k such that $\lim_{x \to 0} \frac{f(x)}{x^k} \neq 0$ is

(a) 4 (b) 3 (c) 2 (d) 1

75. Tangent is drawn at any point $P(x, y)$ on a curve, which passes through $(1, 1)$. The tangent cuts X-axis and Y-axis at A and B respectively. If $AP : BP = 3 : 1$, then

(a) the differential equation of the curve is $3x\frac{dy}{dx} + y = 0$

(b) the differential equation of the curve is $3x\frac{dy}{dx} - y = 0$

(c) the curve passes through $\left(\frac{1}{8}, 2\right)$

(d) the normal at $(1, 1)$ is $x + 3y = 4$

Answers

Physics

1. (b)	2. (d)	3. (b)	4. (a)	5. (a)	6. (d)	7. (a)	8. (d)	9. (b)	10. (a)
11. (a)	12. (a)	13. (c)	14. (b)	15. (c)	16. (a)	17. (b)	18. (d)	19. (a)	20. (c)
21. (b)	22. (a)	23. (d)	24. (a)	25. (d)	26. (c)	27. (c)	28. (b)	29. (a)	30. (a)
31. (c)	32. (b)	33. (a)	34. (c)	35. (b)	36. (b)	37. (d)	38. (a)	39. (d)	40. (a,c,d)

Chemistry

41. (c)	42. (c)	43. (b)	44. (d)	45. (c)	46. (a)	47. (d)	48. (b)	49. (a)	50. (a)
51. (b)	52. (b)	53. (d)	54. (a)	55. (c)	56. (c)	57. (a)	58. (d)	59. (a)	60. (c)
61. (a)	62. (d)	63. (d)	64. (b)	65. (b)	66. (c)	67. (c)	68. (a)	69. (c)	70. (d)
71. (d)	72. (b)	73. (d)	74. (c)	75. (d)	76. (a, b)	77. (a, b,d)	78. (a, b, d)	79. (a, c, d)	80. (a, c)

Mathematics

1. (a)	2. (a)	3. (c)	4. (d)	5. (b)	6. (d)	7. (c)	8. (c)	9. (c)	10. (b)
11. (c)	12. (a)	13. (a)	14. (a)	15. (c)	16. (b)	17. (c)	18. (d)	19. (b)	20. (a)
21. (b)	22. (d)	23. (c)	24. (a)	25. (d)	26. (d)	27. (b)	28. (b)	29. (d)	30. (b)
31. (c)	32. (b)	33. (d)	34. (d)	35. (b)	36. (b)	37. (a)	38. (a)	39. (b)	40. (d)
41. (c)	42. (d)	43. (*)	44. (b)	45. (c)	46. (c)	47. (c)	48. (c)	49. (a)	50. (b)
51. (c)	52. (b)	53. (c)	54. (c)	55. (c)	56. (a)	57. (d)	58. (d)	59. (b)	60. (c)
61. (c)	62. (b)	63. (c)	64. (b)	65. (b)	66. (b)	67. (a, d)	68. (a, c)	69. (b)	70. (c, d)
71. (a, b)	72. (d)	73. (a)	74. (c)	75. (a, c)					

Note (*) None of the option is correct.

Answer *with* Explanations

Physics

1. *(b)* Let intensity from one slit = I_1

Intensity from other slit = I_2

According to question,

$$I_1 = 1.5 I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = 1.5 = \frac{3}{2}$$

$\therefore$ Ratio of intensity, $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$$= \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2$$

$$= \left(\frac{1.732+1.414}{1.732-1.414}\right)^2$$

$$= \left(\frac{3.146}{0.318}\right)^2 = (9.89)^2 \simeq 98$$

2. *(d)* Given, slit width, $a = 0.5\,\text{mm} = 0.5 \times 10^{-3}$ m

Wavelength, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m

Distance of screen, $D = 50$ cm $= 50 \times 10^{-2}$ m

$\therefore$ Linear separation $= \frac{2D\lambda}{a}$

$$= \frac{2 \times 50 \times 10^{-2} \times 600 \times 10^{-9}}{0.5 \times 10^{-3}}$$

$$= 12 \times 10^{-4}\,\text{m}$$

$$= 1.2 \times 10^{-3}\,\text{m} = 1.2\,\text{mm}$$

3. *(b)* Wavelength of emitted radiation is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For Lyman series,

$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, where $n = 2, 3, 4, \ldots.$

The range of wavelength in this series is given by

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}\ \text{cm}^{-1}$$

and $\frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R\ \text{cm}^{-1}$

Thus for Lyman : $\left[\frac{1}{R}\ \text{to}\ \frac{4}{3R}\ \text{cm}\right]$

For Balmer series,

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, where $n = 3, 4, 5, \ldots$

The range of wavelength in this series is given by

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}\ \text{cm}^{-1}$$

and $\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}\ \text{cm}^{-1}$

Thus, for Balmer: $\left[\frac{4}{R}\ \text{to}\ \frac{36}{5R}\ \text{cm}\right]$

For Paschen series,

$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, where $n = 4, 5, 6, \ldots$

The range of wavelength in this series is given by

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}\ \text{cm}^{-1}$$

and $\lambda_{min} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}\ \text{cm}^{-1}$

Thus, for Paschen : $\left[\frac{9}{R}\ \text{to}\ \frac{144}{7R}\ \text{cm}\right]$

For Brackett series,

$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, where $n = 5, 6, 7, \ldots$

The range of wavelength in this series is given by

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = \frac{9R}{400}\ \text{cm}^{-1}$$

and $\frac{1}{\lambda_{min}} = R\left(\frac{1}{4^2} - \frac{1}{\infty^2}\right) = \frac{R}{16}\ \text{cm}^{-1}$

Thus, for Brackett : $\left[\frac{16}{R}\ \text{to}\ \frac{400}{9R}\ \text{cm}\right]$

For Pfund series,

$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, where $n = 6, 7, 8, \ldots.$

The range of wavelength in this series is given by

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{5^2} - \frac{1}{6^2}\right) = \frac{11R}{900}\ \text{cm}^{-1}$$

and $\frac{1}{\lambda_{min}} = R\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{R}{25}\ \text{cm}^{-1}$

Thus, for Pfund: $\left[\frac{25}{R}\ \text{to}\ \frac{900}{11R}\ \text{cm}\right]$

Hence, wavelength range of $\left(\frac{7}{5R}\ \text{to}\ \frac{19}{5R}\ \text{cm}\right)$ does not emit.

4. *(a)* The process involved is as

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + \nu + Q$$

$\therefore$ Mass defect, $m_x = M_x - Zm_e$

and $m_y = M_y - (Z+1)m_e$

where, M_x is atomic mass of X, M_y is atomic mass of Y and m_e is mass of an electron.

Total mass defect, $\Delta m = m_x - m_y = (M_x - M_y - m_e)$

$\therefore$ Binding energy, $E = \Delta mc^2$

So, maximum energy of β-particle emitted

$$= (M_x - M_y - m_e)\, c^2$$

5. *(a)* Given, binding energy per nucleon of nuclei with mass number 119 = 7.6 MeV

Binding energy per nucleon of nuclei with mass number 238 = 8.6 MeV

According to question,

fission of nucleus 238 = 119 + 119

Hence, total energy of nucleus before fission,

$$E_i = 238 \times 8.6 \text{ MeV}$$

Total energy of nucleus after fission,

$$E_f = 238 \times 7.6 \text{ MeV}$$

$\therefore$ Energy released in the process of fission

$$= E_i - E_f$$
$$= 238 \times 8.6 - 238 \times 7.6$$
$$= 238 \text{ MeV}$$

Hence, released energy is closest to 214 MeV.

6. *(d)* Given, load resistance, $R_L = 6\text{ k}\Omega = 6000\ \Omega$

Base voltage, $V_B = 15\text{ mV} = 15 \times 10^{-3}$ V

Base current, $I_B = 20\mu\text{A} = 20 \times 10^{-6}$ A

Collector current, $I_C = 1.8\text{ mA} = 1.8 \times 10^{-3}$ A

$\therefore$ Voltage gain = β (Current gain) $\times$ Resistance gain

$$= \frac{I_C}{I_B} \times \frac{R_L}{R_B}$$

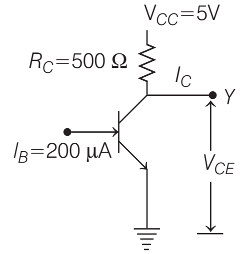
$$= \frac{I_C R_L}{I_B R_B} = \frac{I_C R_L}{V_B} \qquad [\because V_B = I_B R_B]$$

$$= \frac{1.8 \times 10^{-3} \times 6000}{15 \times 10^{-3}}$$

$$= 720$$

7. *(a)* Given, β of transistor = 48

Base current, $I_B = 200\mu\text{A} = 200 \times 10^{-6}$ A



We know that, current gain, $\beta = \dfrac{I_C}{I_B}$

$$\Rightarrow \quad 48 = \frac{I_C}{200 \times 10^{-6}}$$

$$\Rightarrow \quad I_C = 48 \times 200 \times 10^{-6}$$
$$= 96 \times 10^{-4} \text{ A}$$

Now, $V_{CC} = I_C R_C + V_{CE}$

$\Rightarrow \quad V_{CE} = V_{CC} - I_C R_C$

From figure, $V_{CC} = 5$ V and $R_C = 500\ \Omega$

$$\Rightarrow \quad V_{CE} = 5 - 96 \times 10^{-4} \times 500$$
$$= 5 - 4.8 = 0.2 \text{ V}$$

8. *(d)* Given, frequency, $\nu = k\delta E$

where, k is constant and δE is change in energy.

$$\Rightarrow \quad k = \frac{\nu}{\delta E} = \frac{\nu}{h\nu} \qquad [\because \delta E = h\nu]$$
$$= \frac{1}{h}$$

$\because$ We know that, energy, $E = h\nu$

$$\Rightarrow \quad h = \frac{E}{\nu} = \frac{\text{ML}^2\text{T}^{-2}}{\text{T}^{-1}}$$

$$h = [\text{ML}^2\text{T}^{-1}]$$

$\therefore$ Dimension of $k = \dfrac{1}{h} = \dfrac{1}{[\text{ML}^2\text{T}^{-1}]} = [\text{M}^{-1}\text{L}^{-2}\text{T}]$

9. *(b)* Given, $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{B} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$,

$$\mathbf{C} = \frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

$$\because \quad \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

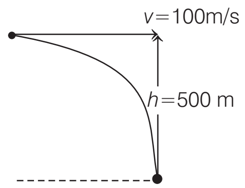
$$\therefore \quad \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = \frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\,(-3\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

$$= \frac{6}{\sqrt{5}} - \frac{6}{\sqrt{5}} = 0$$

10. *(a)* Given, speed of fighter plane, $v = 360$ km/h

$$= 360 \times \frac{5}{18} \text{ m/s} = 100 \text{ m/s}$$

Altitude, $h = 500$ m



Now, from equation of motion,

$$h = ut + \frac{1}{2}gt^2$$

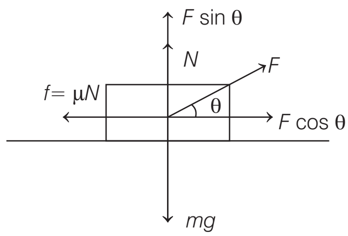
$$\Rightarrow \quad 500 = 0(t) + \frac{1}{2} \times 10 \times t^2 \qquad \begin{bmatrix} \because u = 0 \text{ and} \\ g = 10 \text{ m/s}^2 \end{bmatrix}$$

$$\Rightarrow \quad t^2 = 100 \quad \Rightarrow \quad t = 10 \text{ s}$$

$\therefore$ The bomb should be dropped at the distance,

$$x = vt = 100 \times 10 = 1000 \text{ m}$$

11. *(a)* The block diagram is as shown below,



According to above diagram,

$$mg = N + F\sin\theta$$

$$\Rightarrow \quad N = mg - F\sin\theta \qquad \ldots \text{(i)}$$

and $\quad F\cos\theta = \mu N$

$$\Rightarrow \quad F\cos\theta = \mu(mg - F\sin\theta) \qquad [\text{using Eq. (i)}]$$

$$\Rightarrow \quad F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

For $F_{\min}$,

$$\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$$

$$\Rightarrow \quad -\sin\theta + \mu\cos\theta = 0$$

$$\Rightarrow \quad \tan\theta = \mu$$

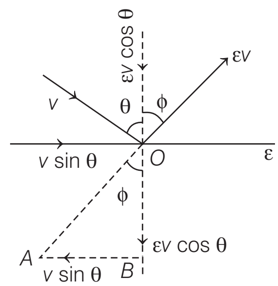
$$\because \quad \sin\theta = \frac{\mu}{\sqrt{1+\mu^2}} \text{ and } \cos\theta = \frac{1}{\sqrt{1+\mu^2}}$$

$$\therefore \quad F_{\min} = \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu \cdot \mu}{\sqrt{1+\mu^2}}}$$

$$= \frac{\mu mg}{\frac{1+\mu^2}{\sqrt{1+u^2}}} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\Rightarrow \quad F_{\min} = \frac{\mu \cdot mg}{\sqrt{1+\mu^2}}$$

12. *(a)* Given, a tennis ball hits the floor with a speed v making an angle θ with the normal as shown in the figure below.



The coefficient of restitution = ε

Now, from the above diagram,

In ΔAOB, $\tan\phi = \frac{v\sin\theta}{\varepsilon v\cos\theta} = \frac{\tan\theta}{\varepsilon}$

$\therefore$ Angle of reflection, $\phi = \tan^{-1}\left(\frac{\tan\theta}{\varepsilon}\right)$

13. *(c)* Time period of second pendulum, $T = 2$ s

Displacement equation of simple pendulum,

$$x = A\sin\omega t \qquad \ldots \text{(i)}$$

Given, $\quad t = 2.25 \text{ s} = 2 + 0.25$

$$= 2 + \frac{1}{4}$$

$$\therefore \quad t = T + \frac{T}{8}$$

Velocity of second pendulum,

$$v = \frac{dx}{dt}$$

$$= \frac{d}{dt} \cdot (A\sin\omega t) \qquad [\text{from Eq. (i)}]$$

$$v = A\omega\cos\omega t$$

Velocity at $\quad t = T$,

$$v = A\,\omega\cos\left(\frac{2\pi}{T} \cdot T\right) = A\omega = v_0 \qquad \ldots \text{(ii)}$$

Velocity at $t = T + \frac{T}{8}$,

$$v_1 = A\omega\cos\omega\left(T + \frac{T}{8}\right)$$

$$= A\omega\cos\frac{2\pi}{T}\left(T + \frac{T}{8}\right)$$

$$= A\omega \cos 2\pi \cdot \frac{9}{8}$$

$$= A\omega \cos\left(2\pi + \frac{\pi}{4}\right)$$

$$= A\omega \cos \frac{\pi}{4}$$

$$= \frac{A\omega}{\sqrt{2}}$$

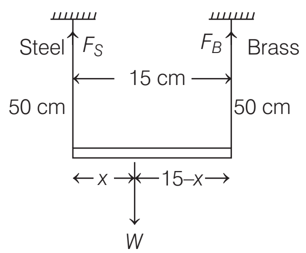
$$v_1 = \frac{v_0}{\sqrt{2}} \qquad \text{[from Eq. (ii)]}$$

By equation of motion,

$$v_1^2 = v_0^2 - 2gh$$

$$\Rightarrow \quad \left(\frac{v_0}{\sqrt{2}}\right)^2 = v_0^2 - 2gh \Rightarrow h = \frac{v_0^2}{4g}$$

14. *(b)* Let x be the distance from steel wire, where load is applied as shown in the figure below.



Given, Young's modulus of steel,

$$Y_{\text{steel}} = 2 \times 10^{12} \text{ dyne/cm}^2$$

Young's modulus of brass, $Y_{\text{brass}} = 1 \times 10^{12}$ dyne/cm^2

Cross-sectional area of wires, $A = 0.005\,\text{cm}^2$

Extension in wires, $\Delta l = 0.1$ cm

We know that,

Young's modulus, $Y_{\text{steel}} = \frac{\text{stress}}{\text{strain}} = \frac{F_S / A}{\frac{\Delta l}{l}} = \frac{F_S \cdot l}{\Delta l\, A}$

$$\Rightarrow \quad F_S = Y_{\text{steel}} \times \frac{\Delta l}{l} \times A$$

$$= 2 \times 10^{12} \times \frac{0.1}{50} \times 0.005$$

$$\Rightarrow \quad F_S = 2 \times 10^7 \text{ dyne}$$

Similarly,

$$F_B = Y_{\text{brass}} \times \frac{\Delta l}{l} \times A$$

$$= 1 \times 10^{12} \times \frac{0.1}{50} \times 0.005$$

$$= 10^7 \text{ dyne}$$

From given figure,

$$F_S \times x = F_B \times (15 - x)$$

$$\Rightarrow \quad 2 \times 10^7 \times x = 10^7 \times (15 - x)$$

$$\Rightarrow \quad 2x = 15 - x$$

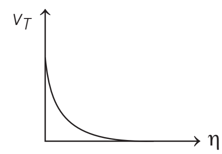
$$\Rightarrow \quad 3x = 15 \Rightarrow x = 5\,\text{cm}$$

15. *(c)* We know that,

terminal velocity, $v_T = \frac{2r^2}{9\eta}(\rho_S - \rho_L)\, g$

$\Rightarrow v_T \propto \frac{1}{\eta}$, where η is viscosity of the liquid.

Hence, graph between terminal velocity v_T and viscosity of liquid η is as shown below.



16. *(a)* Given, during ab, heat rejected, $Q_{ab} = -50$ J

During ca, heat absorbed, $Q_{ca} = 80$ J

During bc, heat transferred, $Q_{bc} = 0$ J

Work done by the gas, $W = 40$ J

$\therefore$ Area of closed curve $abca = Q_{ab} + Q_{bc} + Q_{ca}$

Area $= -50 + 0 + 80$

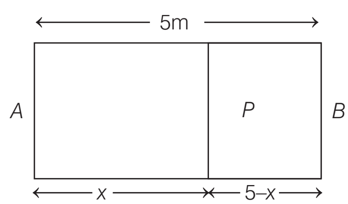
Area $= 30$ J

17. *(b)* Given, length of rectangular container, $l = 5$ m

The number of moles of two gases are

$$\mu_1 = \frac{M}{32} \text{ and } \mu_2 = \frac{M}{18}$$

The given situation is shown in the figure given below.



where, x is the distance of movable partition P from the left wall A.

At equilibrium, $p_1 = p_2 = p$

and $T_1 = T_2 = T$

By ideal gas equation,

$$\frac{p_1 V_1}{\mu_1} = \frac{p_2 V_2}{\mu_2} = RT$$

$$\Rightarrow \quad \frac{V_1}{\mu_1} = \frac{V_2}{\mu_2} \quad [\because p_1 = p_2]$$

$$\frac{a \cdot x}{\mu_1} = \frac{a \cdot (5-x)}{\mu_2} \quad [\because a = \text{area of each wall } A \text{ and } B]$$

$$\frac{x}{\mu_1} = \frac{5-x}{\mu_2}$$

$$\Rightarrow \quad \frac{x}{M/32} = \frac{5-x}{M/18}$$

$$\Rightarrow \quad 32x = 18(5-x)$$

$$\Rightarrow \quad 32x = 90 - 18x$$

$$\Rightarrow \quad 50x = 90$$

$$\Rightarrow \quad x = \frac{90}{50} = 1.8 \text{ m}$$

18. *(d)* Given, mass of boiling water = 100 g

Mass of cold water = 300 g

Fall in temperature = 100 − 20 = 80°C

Let heat capacity of calorimeter = ms

Specific heat of water = 1 cal g^{-1} $°C^{-1}$

$\therefore$ Heat lost by boiling water = $ms\Delta T = 100 \times 1 \times 80$

and heat gained by cold water

$$= 300 \times 1 \times 10 + ms \times 10$$

Now, according to the principle of calorimetry,

heat lost = heat gained

$$100 \times 1 \times 80 = 300 \times 1 \times 10 + ms \times 10$$

$$\Rightarrow \quad ms = 500 \text{ cal/°C}$$

Let specific heat of metal is s_b.

Mass of block = 1 kg = 1000 g

Rise in temperature of metallic block, $\Delta t = 19 - 10 = 9°C$

Then, again from principle of calorimetry,

$$m_{\text{mixture}} \times s\Delta T + ms\Delta T = m_{\text{block}} \times s_b \times \Delta t$$

$$(100 + 300) \times 1 \times (20-19) + 500 \times (20-19) = 1000 \times s_b \times 9$$

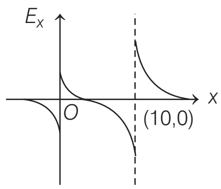
$$\Rightarrow \quad s_b = 0.1 \text{ cal/g°C}$$

19. *(a)* Given, $q_1 = 1 \times 10^{-6}$ C, $q_2 = 3 \times 10^{-6}$ C

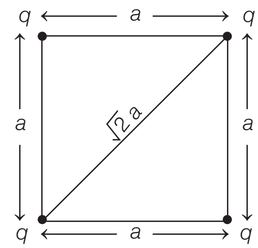
$\therefore \quad q_2 = 3q_1;\ q_2 > q_1$

As we know that, electric field, $E \propto \frac{1}{x^2}$

So, the graph showing variation of E_x in x-direction is as given below.



20. *(c)* The given system of charges is as shown below,



$\because$ Potential energy, $\text{PE} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{k q_1 q_2}{r}$

Here, $\quad q_1 = q_2 = q$

and $\quad r = a$

$$\Rightarrow \quad \text{PE} = \frac{kqq}{a} = \frac{kq^2}{a}$$

Due to four identical point masses,

$$\text{PE}_i = \frac{4kq^2}{a} + \frac{2kq^2}{\sqrt{2}a}$$

According to law of conservation of energy,

$$\text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f$$

Initially, $\text{KE}_i = 0$

and at infinity, $\text{PE}_f = 0$

$$\therefore \quad \text{KE}_f = \text{PE}_i = \frac{4kq^2}{a} + \frac{2kq^2}{\sqrt{2}a}$$

$$= \frac{kq^2}{a}(4 + \sqrt{2})$$

$\because$ In CGS unit, $k = 1$

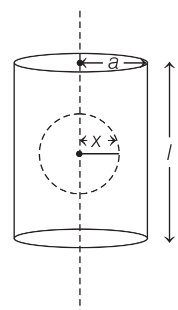
$$\therefore \quad \text{KE} = \frac{q^2}{a}(4 + \sqrt{2})$$

21. *(b)* Given, radius of solid cylinder = a

Charge density of cylinder = ρ

Dielectric constant of material of cylinder = k

The electric field at a radial distance $x (x < a)$ as shown is calculated as



From Gauss' law,

$\int \mathbf{E} \cdot d\mathrm{A} = \frac{Q_{\text{in}}}{k\varepsilon_0}$, where k is dielectric constant of material.

$$\Rightarrow E(2\pi x l) = \frac{\rho(\pi x^2 l)}{k\varepsilon_0} \qquad [\because \text{charge, } Q_{\text{in}} = \rho(\pi x^2 l)]$$

$$\Rightarrow \quad E = \frac{\rho x}{2k\varepsilon_0}$$

22. *(a)* A galvanometer can be converted to a voltmeter of full scale deflection V_0 by connecting a series resistance R_1, then

$$R_1 = \frac{V_0}{I_g} - G$$

$$\Rightarrow \quad V_0 = I_g(G + R_1) \quad \dots \text{(i)}$$

where, I_g = current through the galvanometer and G = resistance of galvanometer.

Similarly, a galvanometer can be converted to an ammeter by connecting a shunt resistance R_2, then

$$R_2 = \left(\frac{I_g}{I_0 - I_g}\right)G$$

$$\Rightarrow \quad I_g = I_0\left(\frac{R_2}{G + R_2}\right) \quad \dots \text{(ii)}$$

where, I_0 = total current.

Now, from Eq. (i), we get

$$\frac{V_0}{I_g} - R_1 = G \quad \dots \text{(iii)}$$

From Eq. (ii), we get

$$G + R_2 = \frac{I_0}{I_g} \times R_2$$

$$\Rightarrow \quad G = \frac{I_0}{I_g} \times R_2 - R_2 \quad \dots \text{(iv)}$$

From Eqs. (iii) and (iv), we get

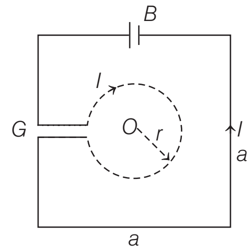
$$\frac{V_0}{I_g} - R_1 = \frac{I_0}{I_g} \times R_2 - R_2$$

$$\Rightarrow \quad \frac{V_0}{I_g} - \frac{I_0}{I_g} \times R_2 = R_1 - R_2$$

$$\Rightarrow \quad \frac{1}{I_g}(V_0 - I_0 R_2) = R_1 - R_2$$

$$\Rightarrow \quad I_g = \frac{V_0 - I_0 R_2}{R_1 - R_2}$$

23. *(d)* According to given figure,



Given, $a : r = 8 : \pi$

$$\frac{a}{r} = \frac{8}{\pi} \Rightarrow r = \frac{\pi a}{8} \quad \dots \text{(i)}$$

Magnetic field due to square, $\mathbf{B}_1 = \frac{\mu_0 I}{\pi \frac{a}{2}} \times \sqrt{2}$, outward

Magnetic field due to circular loop, $\mathbf{B}_2 = \frac{\mu_0 I}{2r}$, inward

Strength of magnetic field at the common centre O,

$$B = B_1 + B_2 = \frac{\mu_0 I}{2r} - \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

Substituting value of r from Eq (i), we get

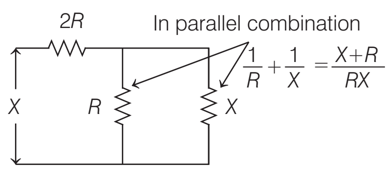
$$B = \frac{\mu_0 I \times 8}{2\pi a} - \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

$$= \frac{\mu_0 I}{\pi a}(4 - 2\sqrt{2})$$

$$= \frac{\mu_0 I}{\pi a} 2\sqrt{2}(\sqrt{2} - 1)$$

24. *(a)* A wire is bent to form a D-shaped loop carrying current I, where the curved part is semi-circle of radius R. The loop is placed in a uniform magnetic field **B** which is directed into the plane of the paper. As single current by flowing in the loop, so net magnetic force on a closed current loop in a uniform magnetic field **B** is zero.

25. *(d)* For equivalent resistance, between the terminals A and B of the infinite resistance network, we redraw the given circuit as



Now the $2R$ and $\frac{RX}{X + R}$ is in series, so

$$X = 2R + \frac{RX}{R + X}$$

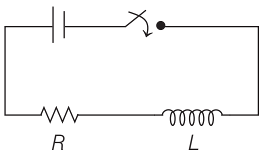
$$X^2 - 2RX - 2R^2 = 0$$

$$\Rightarrow \quad X = \frac{2R \pm \sqrt{4R^2 - 4(1)(-2R^2)}}{2}$$

$$\Rightarrow \quad X = \frac{2R \pm \sqrt{12R^2}}{2} = \frac{2R \pm 2\sqrt{3}R}{2} = (1 \pm \sqrt{3})R$$

$$\Rightarrow \quad X = (1 + \sqrt{3})\, R$$

26. *(c)* When a DC voltage is applied at the two ends of a circuit kept in a closed box, it is observed that the current gradually increases from zero to a certain value and then remains constant. As capacitor blocks DC. So, the circuit must contains a resistor and an inductor in series as shown below.



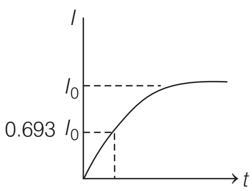
From Graph

Growth of current, $I = I_0(1 - e^{-t/\tau_L})$

At $\quad t = 0, I = 0$

At $\quad t = \tau_L$,

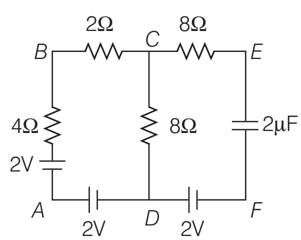
$$I = I_0\left(1 - \frac{1}{e}\right) = 0.693\, I_0$$



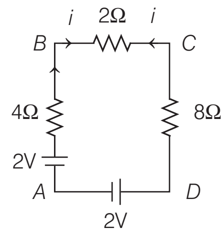
where, $\tau_L = \frac{L}{R}$ = time constant and

$I_0 = \frac{E}{R}$ = maximum current.

27. *(c)* The given circuit is as shown below.

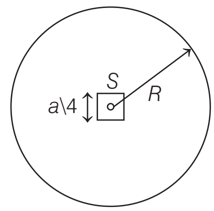


∵ In steady state, the combination of resistance and capacitance in loop *CEFD* will not work. So, we can eliminate this combination and hence the equivalent circuit is as shown below.



∵ In the above circuit, the current i will flow from B to C and same current i will flow from C to B. Hence, total current through the 2Ω resistor is zero.

28. *(b)* A conducting wire of length L bent in the form of a circle of radius R and another conductor of length a is bent in the form of a square. The two loops are then placed in same plane such that the square loop is exactly at the centre of circular loop as shown in the figure



For circular loop,

$$2\pi R = L \qquad \ldots \text{(i)}$$

For square of side S,

$$4S = a \qquad \ldots \text{(ii)}$$

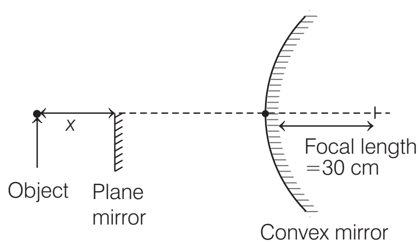
Flux linked, $\phi = \frac{\mu_0 I S^2}{2R}$

Mutual inductance, $M = \frac{\phi}{I} = \frac{\mu_0 S^2}{2R}$

Substituting R from Eq. (i) and S from Eq. (ii), we get

$$M = \frac{\mu_0 a^2}{2(16)} \frac{2\pi}{L} = \frac{\mu_0 a^2 \pi}{16L}$$

29. *(a)* The given situation is as shown in figure below.



Object distance from convex mirror, $u = -60$ cm

Focal length of convex mirror, $f = 30$ cm

Using mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{60}=\frac{1}{30}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{30}+\frac{1}{60}=\frac{2+1}{60}=\frac{3}{60}$$

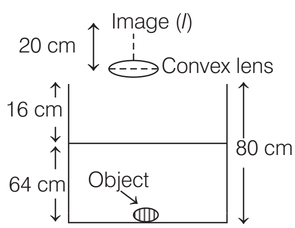
$\Rightarrow \quad v = 20\text{ cm}$

Distance between the object and image = 60 + 20 = 80 cm

As there is no parallex between the image formed by the two mirrors, i.e. the images are formed at the same place.

So, the plane mirror is at a distance of 40 cm from the object.

30. *(a)* A thin convex lens is placed just above an empty vessel of depth 80 cm. The image (I) of a coin kept at the bottom of the vessel is thus formed 20 cm above the lens. If now water is poured in the vessel upto a height of 64 cm as shown in the figure below.



Given, object distance, $u = -80\text{ cm}$

Image distance, $v = +20\text{ cm}$

Using lens formula, we get

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\Rightarrow \frac{1}{f}=\frac{1}{20}+\frac{1}{80}=\frac{5}{80}$$

$$\Rightarrow f = 16\text{ cm}$$

Now, water is poured in the vessel upto height of 64 cm, then object distance,

$$u = 16+\frac{64}{\mu}=16+\frac{64\times 3}{4}=64\text{ cm}$$

(given, refractive index, $\mu = 4/3$)

Again, using lens formula,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\Rightarrow \frac{1}{16}=\frac{1}{v}+\frac{1}{64} \qquad (\because u=-64\text{ cm})$$

$$\Rightarrow \frac{1}{v}=\frac{3}{64}$$

$\therefore$ $v = 21.33\text{ cm}$, above the lens.

31. *(c)* Given, resistance of the circular loop = 20 Ω

Electric charge flowing due to the induced current, $q = I\cdot dt = \frac{\Delta\phi}{R}=\frac{\phi_2-\phi_1}{R}$, where ϕ_1, ϕ_2 are magnetic flux at $t_1 = 0$ and $t_2 = 20$ ms.

As the loop is perpendicular to the magnetic field **B**, the flux passing through the loop is,

$$\phi = \mathbf{B}\cdot\mathbf{A} = BA\cos 90^\circ = 0$$

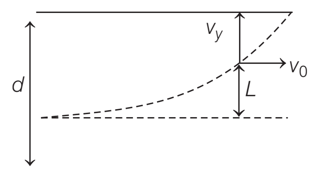
Hence, $q = \frac{\Delta\phi}{R}=\frac{\phi_2-\phi_1}{R}=0$

$\therefore \quad q = 0\text{ C}$

32. *(b)* Distance between the parallel plates = d

The axial distance of beam from the centre of parallel plates = L

As, time, $t = \frac{\text{distance}}{\text{velocity}}=\frac{L}{v_0}$



y-component of velocity, $v_y = \left(\frac{qE}{m}\right)t = \frac{e}{m}\cdot\frac{V}{d}\times\frac{L}{v_0}$

$$\Rightarrow \quad v_y = \frac{eVL}{mdv_0}$$

$$\therefore \tan\theta = \frac{v_y}{v_z}=\frac{eLV}{mdv_0.v_0}=\frac{eVL}{mdv_0^2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$$

33. *(a)* Due to dragging movement of the block, work (W_f) will be done against force of friction (f).

Now, force of friction, $f = \mu mg$

where, μ = coefficient of static friction = 0.1,

mass, $m = 20$ kg and $g = 10\text{ ms}^{-2}$.

$\Rightarrow \quad f = 0.1\times 20\times 10 = 20\text{ N}$

This work done (W_f) will produce heat energy, $H = mc\Delta T$

Hence, $W_f = H$

$\Rightarrow$ Force × Displacement = $mc\Delta T$

$\Rightarrow \quad f\times vt = mc\Delta T \qquad [\because s = v\cdot t]$

Here, $v = 0.5\text{ ms}^{-1}$, $t = 2.1$ s

and $c = 0.1$ CGS unit $= 0.1\times 4.2\times 10^3$ SI unit

$$\Rightarrow \quad 20\times 0.5\times 2.1 = 20\times 0.1\times 4.2\times 10^3\times\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{20\times 0.5\times 2.1}{20\times 0.1\times 4.2\times 10^3}=0.0025^\circ\text{C}$$

34. *(c)* We have, power $= \dfrac{\text{work done}}{\text{time}}$

Given, time = 1 min = 60 s

Now, work done per cycle = (130 − 30)

$= 100 \text{ cal} = 100 \times 4.2 \text{ J}$

As 2J of energy is consumed due to friction by the engine, hence work done per cycle

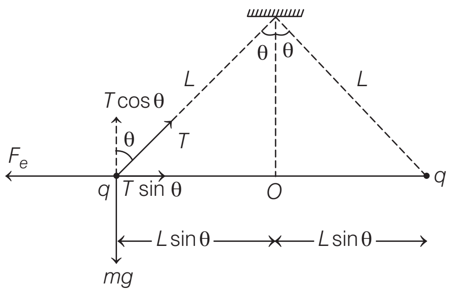
$= 100 \times 4.2 - 2$

$= 420 - 2 = 418 \text{ J}$

Total work done for 90 cycles = 418×90 J

$\therefore$ Power $= \dfrac{418 \times 90}{60} = 627$ W

35. *(b)* Let force acting due to electrostatic attraction of charges be F_e as shown in the figure below.



Now, $$T \sin\theta = F_e \quad \dots\text{(i)}$$
$$T\cos\theta = mg \quad \dots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get
$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_e}{mg} = k\frac{q_1 \times q_2}{r^2}$$

Here, $q_1 = q_2 = q$

Distance, $r = 2L\sin\theta$

$$\Rightarrow \quad \tan\theta = \frac{kq^2}{(2L\sin\theta)^2 \cdot mg} \quad \dots\text{(iii)}$$

Now, in the second case, when the charges are tripled, new charge $q' = 3q$

As the angle between the strings are doubled, θ becomes 2θ, hence from Eq. (iii), we get
$$\tan 2\theta = \frac{k(3q)(3q)}{(2L\sin 2\theta)^2 \cdot mg}$$
$$\tan 2\theta = \frac{9kq^2}{(2L\sin 2\theta)^2 mg} \quad \dots\text{(iv)}$$

On dividing Eq. (iv) by Eq. (iii), we get
$$\frac{\tan 2\theta}{\tan\theta} = \frac{9kq^2}{4L^2\sin^2 2\theta \cdot mg} \times \frac{4L^2\sin^2\theta \cdot mg}{kq^2}$$
$$\frac{\tan 2\theta}{\tan\theta} = \frac{9\sin^2\theta}{\sin^2 2\theta}$$

As, $$\tan 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

and $$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\Rightarrow \quad \frac{2}{1-\tan^2\theta} = \frac{9\sin^2\theta}{4\sin^2\theta\cos^2\theta} = \frac{9}{4\cos^2\theta}$$
$$\Rightarrow \quad \frac{2}{1-\tan^2\theta} = \frac{9}{4}\sec^2\theta$$
$$\Rightarrow \quad \frac{2}{1-\tan^2\theta} = \frac{9}{4}(1+\tan^2\theta)$$
$$[\because 1+\tan^2\theta = \sec^2\theta]$$

Let $\tan^2\theta = x$
$$\frac{2}{1-x} = \frac{9}{4}(1+x)$$
$$\Rightarrow \quad 8 = 9 - 9x^2$$
$$\Rightarrow \quad 9x^2 = 1$$
$$\Rightarrow \quad x^2 = \frac{1}{9} \Rightarrow x = \frac{1}{3}$$
$$\Rightarrow \quad \tan^2\theta = \frac{1}{3}$$
$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$
$$\Rightarrow \quad \theta = 30^\circ$$

Hence, the angle between the strings

$= 2\theta = 2 \times 30^\circ = 60^\circ$

36. *(b)* The photoelectric current depends on the intensity of incident radiation by relation,
$$i \propto I$$

But the intensity of radiation is inversely proportional to the square of distance between the source and the photoelectric surface,

i.e. $$I \propto \frac{1}{d^2}$$

$\therefore$ $$i \propto \frac{1}{d^2}$$

So, when the distance(d) is doubled, the photoelectric current will decrease. The stopping potential and maximum kinetic energy are independent of distance, so they remain same.

37. *(d)* As, inner radii are not given, so we cannot calculate their masses. Also, it is not given about the nature of material of spheres.

The rotational kinetic energy of sphere,
$$(\text{KE})_{\text{rot}} = \frac{1}{2}I\omega^2$$

where, I = moment of inertia

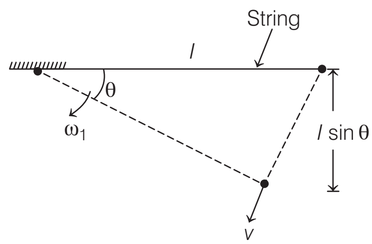
and ω = angular speed.

It is given that, the two metallic spheres have same moment of inertia (I) about their respective diameters. So, their rotational kinetic energies will be equal when rotated with equal uniform angular speed (ω) about their respective diameters.

38. *(a)* The velocity of the particle at extreme position,

$$v = \sqrt{2gh} \qquad [\text{from } v^2 - u^2 = 2gh]$$

Let when released from extreme position, the pendulum traverse an angular distance of θ as shown below.



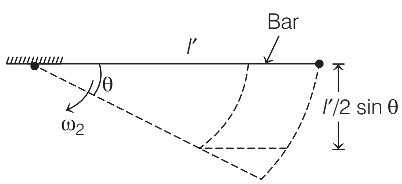
Here, $h = l \sin\theta$

Then, linear velocity, $v = \sqrt{2gl\sin\theta}$

and angular velocity, $\omega_1 = \dfrac{v}{l} = \dfrac{\sqrt{2gl\sin\theta}}{l}$

$$= \sqrt{\frac{2g\sin\theta}{l}}$$

For a bar moved through the same angular distance θ as shown below.



Here, distance travelled, $h = \dfrac{l'}{2}\sin\theta$, where l' is the length of bar.

Potential energy = Rotational kinetic energy

$$mgh = \frac{1}{2}I\omega_2^2$$

$$\Rightarrow \quad mg\frac{l'}{2}\sin\theta = \frac{1}{2}\frac{m(l')^2}{3} \times \omega_2^2 \quad \left[\because \text{for rod,} \quad I = \frac{ml^2}{3}\right]$$

$$\Rightarrow \quad gl'\sin\theta = \frac{(l')^2}{3}\omega_2^2$$

$$\Rightarrow \quad \omega_2 = \sqrt{\frac{3gl'\sin\theta}{(l')^2}} = \sqrt{\frac{3g\sin\theta}{l'}}$$

As the motions of pendulum and bar are synchronous, so their angular velocities will be equal, i.e.

$$\omega_1 = \omega_2$$

$$\Rightarrow \quad \sqrt{\frac{2g\sin\theta}{l}} = \sqrt{\frac{3g\sin\theta}{l'}}$$

$$\Rightarrow \quad \frac{2}{l} = \frac{3}{l'} \quad \Rightarrow \quad l' = \frac{3}{2}l$$

$\therefore$ The length of the bar is $\dfrac{3}{2}l$.

39. *(d)* Given, $R = 400\ \Omega$, $L = 250\text{ mH} = 250 \times 10^{-3}$ H,

$$C = 2.5\mu\text{F} = 2.5 \times 10^{-6}\text{ F}, V_S = 5\text{ V}$$

and $\omega = 2\text{ kHz} = 2000\text{ rad/s} = 2 \times 10^3$ rad/s

$\therefore$ Inductive reactance,

$$X_L = \omega L = 2 \times 10^3 \times 250 \times 10^{-3} = 500\ \Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2 \times 10^3 \times 2.5 \times 10^{-6}} = 200\ \Omega$$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(400)^2 + (500 - 200)^2}$$

$$= \sqrt{250000} = 500\ \Omega$$

The peak voltage across capacitor,

$$(V_P)_C = I_P X_C = \frac{V_S}{Z}X_C = \frac{5}{500} \times 200 = 2\text{V}$$

$\therefore$ Peak value of electrostatic energy of capacitor,

$$(U_P)_C = \frac{1}{2}C(V_P)_C^2$$

$$= \frac{1}{2} \times 2.5 \times 10^{-6} \times (2)^2$$

$$= 5 \times 10^{-6}\text{ J}$$

or $\qquad (U_P)_C = 5\,\mu\text{J}$

40. *(a, c, d)*

It is given that the charged particle is moving with constant velocity in a region of no gravity, so following cases are possible

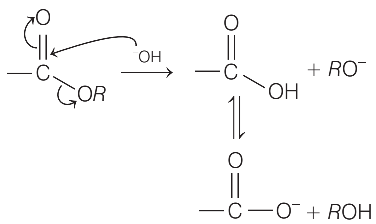
(i) The particle may move in a straight line in any direction, when $\mathbf{E} = 0$ and $\mathbf{B} = 0$.

(ii) The particle may move in a circle with constant velocity. The centripetal force for circular motion is provided by magnetic field.
So, $\mathbf{E} = 0$ and $\mathbf{B} \neq 0$.

(iii) The particle may move in a helical path with constant velocity. For helical motion, the condition is that both fields must be present, i.e. $\mathbf{E} \neq 0$ and $\mathbf{B} \neq 0$.

Chemistry

41. *(c)* Alkaline hydrolysis of an ester (carboxylic acid derivative) follows acyl S_N2 mechanism. It is as follows:



Rate of S_N2 mechanism depends on the polar nature of $\rangle C=O$ group of —COOR group.

Electron withdrawing group ($-R > -I$) increases the rate of S_N2 as it promotes the nucleophilic attack whereas electron donating group ($+R > +I$) decreases the rate of S_N2 reaction as it demotes the nucleophilic attack.

Here, the nature and order of functional groups attached *para* to benzene ring are:

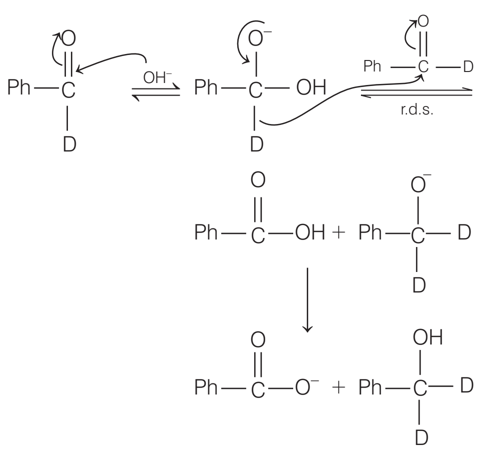
$$\underset{(-R)}{—NO_2(I)} > \underset{(+I)}{—Me(III)} > \underset{(+R)}{—OMe(II)}$$

So, the order of rate of alkaline hydrolysis is

I > III > II

42. *(c)* Given reaction proceed *via* shift of D^-.

It is shown as follows :



The alcohol formed in is $Ph—CD_2—OH$.

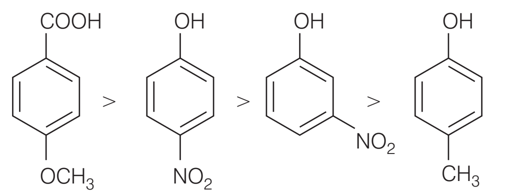
So, correct option is (c).

43. *(b)* Among the given compounds, compound IV is a carboxylic acid whereas rest are phenols. Carboxylic acids are more acidic than phenol due to more stability of carboxylate ion than phenoxide ion.

So, compound IV is the most acidic among the given compounds.

Further, acidicity of phenols increases by the presence of electron withdrawing groups (EWG) such as $—NO_2$. The effect of EWG is more pronounced at *ortho-* and *para*-positions than at *m*-position due to combined effect of $-I$ and $-R$ effect of $—NO_2$ group at *p*-position. On the other hand, electron donating group (EDG) such as $—CH_3$ decreases the acidity of phenols.

Therefore, the correct order of acidity is as follows :



(IV) > (I) > (III) > (II)

44. *(d)* Carbocations are stabilised by the presence of electron donating group ($+R, +I, +H$) and destabilised by presence of electron withdrawing group($-R, -H, -I$).

I. $\overset{+}{C}H_2—\underset{(-I)}{\overset{O}{\overset{\|}{C}}}—CH_3$

$-I$-effect of $—\overset{O}{\overset{\|}{C}}—CH_3$ destabilises the carbocation. So, it is least stable carbocation.

II. $[\overset{+}{C}H_2—\ddot{\underset{..}{O}}\,CH_3 \longleftrightarrow CH_2=\overset{+}{O}CH_3]$

It is stabilised by $+R$-effect. So, it is the most stable carbocation.

III. $\overset{+}{C}H_2—CH_3$

Hyperconjugation of $—CH_3$ group stabilises the ion to some extent but not as effective as $—OCH_3$.

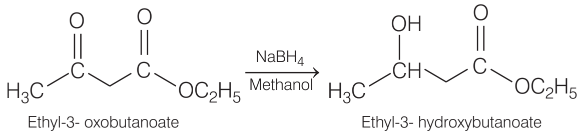
Therefore, its stability is intermediate between I and II.

Hence, the correct order of stability of carbocations is II > III > I.

So, the correct option is (d).

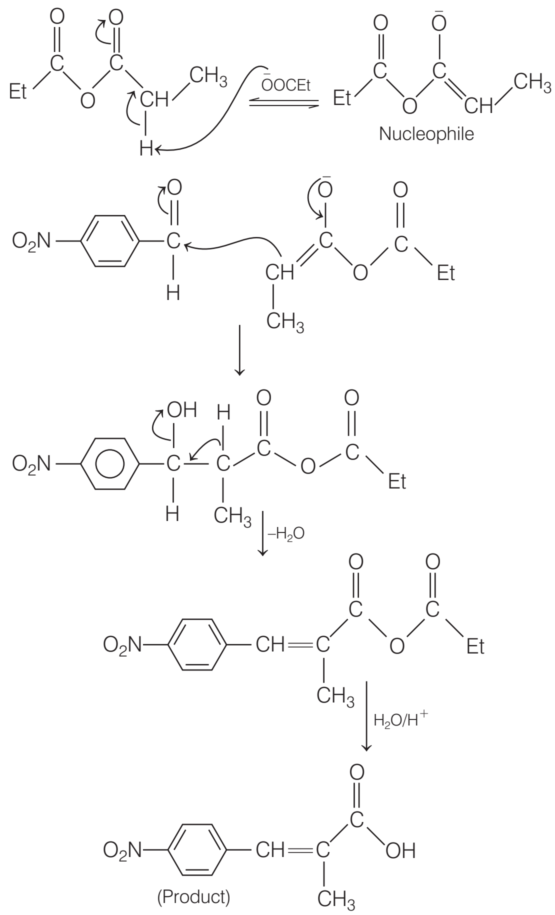
45. *(c)* $NaBH_4$ reduces aldehydes, ketones and acid chlorides into alcohols. It cannot reduce ester and amides. Therefore, in the given reaction, $NaBH_4$ reduces only the *keto* group without affecting the ester group.

The reaction takes place as:



Thus, correct option is (c).

46. *(a)* The given reaction proceeds as follows:



So, the correct option is (a).

47. *(d)* Given,

$$n = 3$$
$$l = 2$$

so, subshell = $3d$

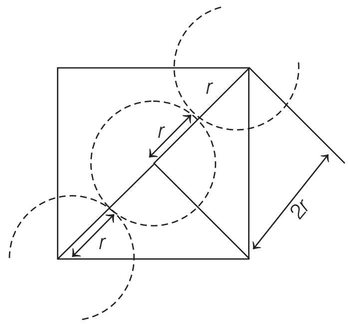
Since, five orientations of d-orbitals are degenerated, therefore $m = -1$ can be assigned to any one of them which means all the $3d$-orbitals are filled. Therefore, the expected maximum number of electrons in an atom of the given quantum numbers are 30.

So, the correct option is (d).

48. *(b)* The given crystal lattice is face centred cubic lattice, which has $Z_{\text{eff}} = 4$.

Also, for fcc, atomic radius $(r) = \frac{a}{2\sqrt{2}}$.

The arrangement of Au-atom in a fcc lattice is shown below:



From the above figure, it is clear that the closest distance between two Au-atoms = $2r$

$$= 2 \times \frac{a}{2\sqrt{2}} = \frac{a}{\sqrt{2}}$$

Hence, option (b) is correct.

49. *(a)* For $2A \rightleftharpoons B + C$, $K_1 = 1.0$

$$K_1 = \frac{[B][C]}{[A]^2} \quad \text{... (i)}$$

For $2B \rightleftharpoons C + D$, $K_2 = 16$

$$K_2 = \frac{[C][D]}{[B]^2} \quad \text{... (ii)}$$

For $2C + D \rightleftharpoons 2P$, $K_3 = 25$

$$K_3 = \frac{[P]^2}{[C]^2[D]} \quad \text{... (iii)}$$

Add equations (i), (ii) and (iii), we get

$$K_1 \cdot K_2 \cdot K_3 = \frac{[P]^2}{[B][A]^2}$$

$$25 \times 16 \times 1 = \frac{[P]^2}{[B][A]^2} \quad \text{... (iv)}$$

On reversing and taking the square root of equation (iv), we get the resultant K for the reaction, $P \rightleftharpoons A + \frac{1}{2} B$ as follows:

$$K = \sqrt{\frac{1}{K_1 \cdot K_2 \cdot K_3}}$$

$$= \frac{[B]^{1/2}[A]}{[P]}$$

$$\Rightarrow \quad = \sqrt{\frac{1}{25 \times 16}}$$

$$\therefore \quad K = \frac{1}{20}$$

So, the option (a) is correct answer.

50. *(a)* According to Hardy-Schulze rule, "greater the valency of coagulating ion or flocculating ion (oppositively charged ion) added, the greater is its power to cause coagulation. To coagulate a positively charged sol, i.e. $Fe(OH)_3$ the maximum power of coagulating of negative ion is PO_4^{3-}.

So, the correct option is (a).

51. *(b)* Let, the number of moles of ethanol = n

The number of moles of water = N

Now, mole fraction of ethanol

$$(\chi_B) = \left[\frac{n}{n+N}\right] = 0.08$$

Mole fraction of water (χ_A) = 1-mole fraction of ethanol = 0.92

$$\text{Molality} = \frac{\chi_B \times 1000}{(1-\chi_B) \times M_A}$$

$$\therefore m = \frac{1000 \times \text{mole fraction of ethanol}}{(1 - \text{mole fraction of ethanol}) \times \text{molecular mass of } H_2O}$$

$$= \frac{1000 \times 0.08}{(1 - .08) \times 18} = 4.83 \text{ mol / kg}$$

So, the correct option is (b).

52. *(b)* The balanced chemical reaction is

$$Pb(NO_3)_2(aq) + 2KI(aq) \longrightarrow PbI_2 \downarrow + 2KNO_3(aq)$$

Initial moles of $Pb(NO_3)_2 = 5 \times 0.1 \times 10^{-3}$

$$= 5 \times 10^{-4}$$

Initial moles of $KI = 10 \times 0.02 \times 10^{-3}$

$$= 2 \times 10^{-4}$$

Therefore,

$$\begin{array}{lcccc} & Pb(NO_3)_2(aq) + & 2KI(aq) \longrightarrow & PbI_2 \downarrow + & 2KNO_3(aq) \\ \text{Initially} & 5 \times 10^{-4} & 2 \times 2 \times 10^{-4} & 0 & 0 \end{array}$$

It is clear that for the formation of 1 mole of PbI_2, we require 4×10^{-4} moles of KI. But the given moles of KI are 2×10^{-4}. Hence, it is a limiting reagent and the amount of PbI_2 formed will depend upon the moles of KI.

Thus, amount of PbI_2 precipitated will be about $= 2 \times 10^{-4}$ mol

So, correct option is (c).

53. *(d)* Given,

Density = 1.964 g / L

Pressure = 76 cm Hg = 1 atm

We know that density,

$$d = \frac{RT}{RM}$$

$$\therefore \quad M = \frac{0.0821 \text{ L atm / K mol} \times 273 \text{ K}}{1 \text{ atm} \times 1.964 \text{ g / L}}$$

$$= 44 \text{ g / mol}$$

Among the given gases, CO_2 gas has molecular mass of 44 g/mol.

So, option(d) is correct.

54. *(a)* Equal masses, i.e. w kg of ethane and hydrogen are mixed in an empty container.

$$\therefore \text{Moles of } C_2H_6 = \frac{w}{30}$$

$$\text{Moles of } H_2 = \frac{w}{2}$$

From Raoult's law, we have

$$p_{H_2} = p_{\text{total}} \times \text{mole fraction of } H_2$$

$$\text{or} \quad p_{H_2} = p_{\text{total}} \times \frac{\frac{w}{2}}{\frac{w}{2} + \frac{w}{30}}$$

$$\therefore \quad p_{H_2} = p_{\text{total}} \times \frac{15}{16}$$

$$\frac{p_{H_2}}{p_{\text{total}}} = \frac{15}{16}$$

Thus, fraction of total pressure exerted by hydrogen is 15:16.

So, the correct option is (a).

55. *(c)* For the work against vacuum, $W = 0$

For an adiabatic process, $q = 0$

From first law of thermodynamics, we have

$$\Delta U = q + W$$

$$\therefore \quad \Delta U = 0$$

Hence, the correct option for the process is (c).

56. *(c)* Given,

mass of ethylene glycol = 10 g

molecular mass = 62 g / mol

K_f (water) = 1.86 K kg mol^{-1}

As we know that,

$$\Delta T_f = K_f \times \text{molality}$$
$$= 1.86 \times \left(\frac{10}{62} \times \frac{1000}{90}\right)$$
$$= 3.3^\circ C$$

$\therefore$ Temperature at which the ice begins to separate $= -3.3^\circ C$.

So, the correct option is (c).

57. *(a)* Given,

Radius of orbit of H-atom $= 0.53 \times 10^{-8}$ cm

For first Bohr orbit, $n = 1$

As we know,

$$mvr = \frac{nh}{2\pi}$$
$$v = \frac{nh}{2\pi mr}$$
$$v = \frac{1 \times 6.626 \times 10^{-27}}{2 \times 3.14 \times 9.1 \times 10^{-28} \times 0.53 \times 10^{-8}}$$
$$v = 2.188 \times 10^{8} \text{ cm/s}$$

So, the option (a) is correct.

58. *(d)* Among the given statements, option (d) is incorrect. It can be corrected as, F is more electronegative than O because it is the most electronegative element in periodic table.

Rest all the given statements are true.

So, correct option is (d).

59. *(a)* The electronic configuration of uranium is $_{92}U : [Rn]^{86} 5f^3 6d^1 7s^2$.

It consist of your electrons, i.e. three from f-orbital and one from d-orbital.

So, the correct answer is option (a).

60. *(c)* On prolonged electrolysis, the density of liquid water increases as the proportion of D_2O increases. This is because on prolonged electrolysis, the concentration of heavy water (D_2O) in liquid water increases which has higher density than ordinary water (H_2O). If electrolysis continued, then almost pure D_2O is obtained.

61. *(a)* Orbital angular momentum $= \sqrt{l(l+1)}\,\dfrac{h}{2\pi}$.

For $4f$-orbital, $l = 3$,

$\therefore$ Orbital angular momentum $= \sqrt{3(3+1)}\,\dfrac{h}{2\pi}$

$$= 2\sqrt{3}\,\frac{h}{2\pi}$$

For $4s$-orbital, $l = 0$

$\therefore$ Orbital angular momentum = 0.

So, the difference between orbital angular momentum $= 2\sqrt{3}\,\dfrac{h}{2\pi} - 0 = 2\sqrt{3}\,\dfrac{h}{2\pi}$

Therefore, the difference between orbital angular momentum of an electron in a $4f$-orbital and another electron in $4s$-orbital is $2\sqrt{3}$.

Thus, the correct option is (a).

62. *(d)* Number of atoms $= \dfrac{\text{mass}}{\text{molar mass}} \times N_A \times$ number of atoms in 1 mole.

The calculation of number of atoms in given options are as follow :

(a) Number of atoms in 1 g of Ag

$$= \frac{1}{107.87} \times N_A \times 1 = 0.0092\, N_A$$

(b) Number of atoms in 1g of Fe

$$= \frac{1}{55.85} \times N_A \times 1 = 0.0179\, N_A$$

(c) Number of atoms in 1g of Cl_2

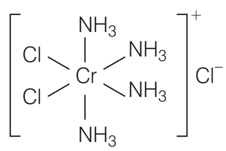
$$= \frac{1}{70} \times N_A \times 2 = 0.0285\, N_A$$

(d) Number of atoms in 1g of Mg

$$= \frac{1}{24.30} \times N_A \times 1 = 0.0411\, N_A$$

Thus, 1g of Mg contains the maximum number of atoms among the given options.

63. *(d)*



Let the oxidation state of central atom (Cr) be x.

Then,

$$(x \times 1) + (4 \times 0) + (2 \times -1) = +1$$
$$x - 2 = +1$$
$$x = +3$$

Thus, the correct IUPAC name of complex is *cis*-tetraamminedichlorochromium (III) chloride.

So, the correct answer is (d).

64. *(b)* We know,

mass of N_A atoms $= 12$ g of C

or mass of 6.023×10^{23} atoms $= 12$ g

$\therefore$ Mass of 1 atom $= \dfrac{12}{6.023 \times 10^{23}}$ g $= 1.9923 \times 10^{-23}$ g

So, the correct option is (b).

65. *(b)* We know that,

Bond order

$= \frac{1}{2}$ [number of electrons in bonding orbitals – number of electrons in antibonding orbitals]

The electronic configuration and bond order (B.O.) of given options is as follows:

(i) $He_2(4e^-) = \sigma(1s)^2\sigma^*(1s)^2$

$$B.O = \frac{2-2}{2} = 0$$

(ii) $He_2^+(3e^-) = \sigma(1s)^2\sigma^*(1s)^1$

$$B.O = \frac{2-1}{2} = \frac{1}{2}$$

(iii) $He_2^{2+}(2e^-) = \sigma(1s)^2$

$$B.O = \frac{2-0}{2} = 1$$

Therefore, the bond order of He_2, He_2^+ and He_2^{2+} are respectively 0, $\frac{1}{2}$, 1.

66. *(c)* Colourless, efflorescent sodium salt is $Na_2S_2O_3$. The colourless gas evolved in SO_2. Complete reaction is as follows :

$$\underset{\text{(Colourless, efflorescent)}}{Na_2S_2O_3(s)} + 2H^+(\text{dil.}) \longrightarrow \underset{\text{White ppt.}}{S(s)} + 2Na^+(aq) + SO_2\uparrow + H_2O(l)$$

Now, when the colourless gas is passed through it acidified dichromate solution turns green.

The reaction taking place is

$$K_2Cr_2O_7(aq) + 3SO_2(aq) + 2H^+(aq) \longrightarrow 2K^+(aq) + \underset{\text{(Green)}}{2Cr^{3+}(aq)} + H_2O(l) + 3SO_4^{-2}(aq)$$

So, the correct option is (c).

67. *(c)* Given reaction is an example of thermite process. In thermite reaction, aluminium metal is oxidised by an oxide of another metal most commonly iron oxide.

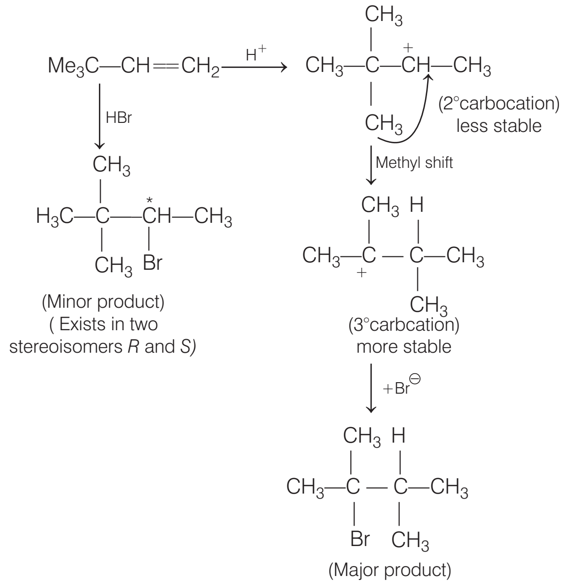
So, Fe is the metal (M).

Hence, option (c) is correct.

68. *(a)* For the extraction of Ca by electro-reduction of molten $CaCl_2$ some CaF_2 is added to the electrolyte because CaF_2 decreases the melting point of $CaCl_2$. It keep the electrolyte in liquid state of temperature than the m.p. of $CaCl_2$

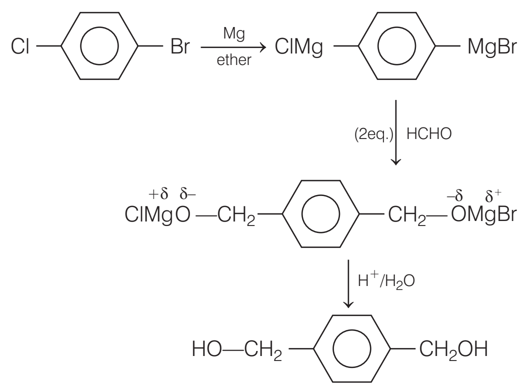
So, option (a) is correct.

69. *(c)* The given reaction proceeds as follows:



Therefore, the total number of alkyl bromides (including stereoisomers) formed in the reaction are 3. Thus, the correct option is (c).

70. *(d)* The given reaction can be completed as follows:



(Product)

So, the correct option is (d).

71. *(d)* Compounds which do not possess any type of symmetry are referred as asymmetric.

Among the given options, only compound (d) do not possess any type of symmetry, i.e. absence of plane of symmetry and centre of symmetry, so compound (d) is asymmetric. Options (a), (b) and (c) have plane of symmetry.

Hence, correct option is (d).

72. *(b)* Rate law : $r = K[A]^{\alpha}[B]^{\beta}$

As the $t_{1/2}$ does not depend on the concentration of B. Therefore, order w.r.t. B follows first order kinetics.

$\therefore \quad B = 1$

Now, according to the question,

$$\frac{r_2}{r_1} = \frac{[2A]^{\alpha}\,[2B]^{\beta}}{[A]^{\alpha}[B]^{\beta}}$$

$$\Rightarrow \quad 4 = 2^{\alpha} \cdot 2^{\beta}$$

$$2^2 = 2^{\alpha + \beta}$$

$$\alpha + \beta = 2$$

As, $\beta = 1$ because B follows first order kinetic.

$$\alpha = 1$$

$\therefore$ Overall order = 2

Hence, unit of rate constant for second order $= \text{L mol}^{-1}\text{s}^{-1}$.

So, the correct option is (b).

73. *(d)* Given,

$$K_{sp}(SrCO_3) = 7.0 \times 10^{-10}$$

$$K_{sp}(SrF_2) = 7.9 \times 10^{-10}$$

$$[CO_3^{2-}] = 1.2 \times 10^{-3}\text{M}$$

Now, for the reaction,

$$SrCO_3 \rightleftharpoons Sr^{2+} + CO_3^{2-}$$

$$K_{sp} = [Sr^{2+}][CO_3^{2-}]$$

$$7 \times 10^{-10} = [Sr^{2+}] \times 1.2 \times 10^{-3}$$

$$\therefore \quad [Sr^{2+}] = \frac{7 \times 10^{-10}}{1.2 \times 10^{-3}}$$

Also, $SrF_2 \rightleftharpoons Sr^{2+} + 2F^-$

$$K_{sp} = [Sr^{2+}]\,[F^-]^2$$

$$[F^-]^2 = \frac{K_{sp}}{[Sr^{2+}]}$$

$$[F^-] = \sqrt{\frac{K_{sp}}{[Sr^{2+}]}}$$

$$= \sqrt{\frac{7.9 \times 10^{-10}}{\frac{7 \times 10^{-10}}{1.2 \times 10^{-3}}}}$$

$$\therefore \quad [F^-] = 3.7 \times 10^{-2}\text{M}$$

So, correct option is (d).

74. *(c)* As the gas molecule shows 2-electron magnetic moment which means it contains 2 unpaired electrons.

Among the given gases, O_2 contains 2 unpaired electrons.

$$O_2 = 8 + 8 = 16$$

Electronic configuration of O_2

$$= \sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2$$

$$< \sigma 2p_z^2 < \pi 2p_x^2 = \pi 2p_y^2 < \pi^* 2p_x^1 = \pi p_y^1 < \sigma^* 2p_z$$

$$\therefore \quad \text{Bond order of } O_2 = \frac{1}{2}(10 - 5) = \frac{5}{2} = 2.5$$

Now, in oxidised form, O_2 exists as O_2^+,

$$O_2 \xrightarrow{-e^-} O_2^+ \; (15\,e^-)$$

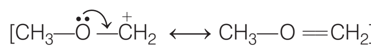
So, bond order of $O_2^+ = 2$

$$\text{As bond ordder} \propto \frac{1}{\text{Bond length}}$$

The bond length, in the oxidised form, of molecule gets decreased. Thus, the gas molecule is O_2.

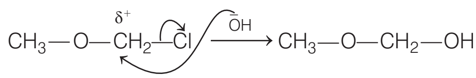
75. *(d)* For the given reaction statement (d) is correct and applicable. For this reaction, a mixed S_N1 and S_N2 pathway is followed. The given reaction can proceed *via* S_N1 pathway, So as,

$$CH_3—O—CH_2—Cl \longrightarrow$$



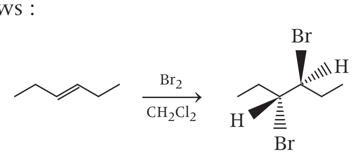
Here, the carbocation is stabilised by resonance.

The reaction can also be proceeded *via* S_N2 path, since the transition state is resonance stabilised.

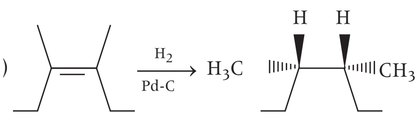


So, the correct option is (d).

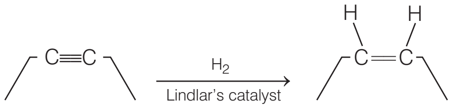
76. *(a, b)* Both reactions (a) and (b) give a *meso*-compound as the main product the reactions are as follows :

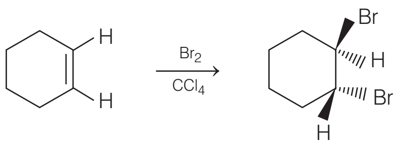
(a) 

Meso-compound

(b) 

Meso-compound

(c) 

(d) 

So, options (a) and (b) are correct.

77. ***(a, b, d)*** For spontaneous polymerisation,

ΔG = negative

Polymerisation process involves the following reaction:

$$nA \longrightarrow -\!\!\left(A - A - A - A\right)\!\!-_n$$

So, ΔS = negative due to association of atoms.

Also, we know that,

$$\Delta G = \Delta H - T\Delta S \quad \text{or} \quad \Delta H = \Delta G + T\Delta S$$

Hence, ΔH = negative.

So, correct options are a, b and d.

78. ***(a, b, d)*** Statements (a, b and d) are incorrect whereas statement (c) is correct. The corrected form are as follows :

- A sink of SO_2 pollutant is ocean.
- FGD (Flue-gas desulfurisation) is a process of removing SO_2 from atmosphere.
- The catalyst used to convert CCl_4 to CF_4 by HF is AgF.

So, the correct options are (a), (b) and (d).

79. ***(a, c, d)*** SiO_2 is attacked by HF, hot NaOH and fluorine. Reactions involved are as follows :

(a) $SiO_2(s) + 6HF(aq) \longrightarrow H_2SiF_6(aq) + 2H_2O(l)$

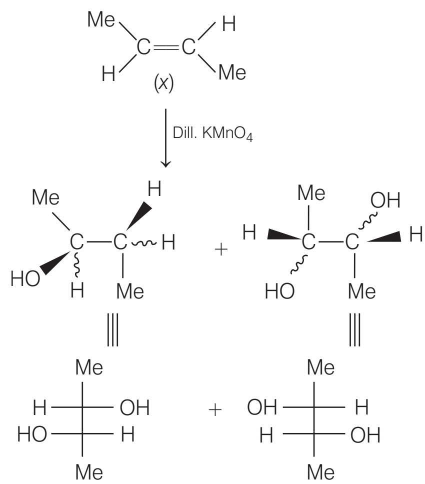
(c) $SiO_2(s) + \text{(hot) } 2NaOH(aq) \longrightarrow Na_2SiO_3(s) + H_2O(l)$

(d) $SiO_2(s) + F_2(g) \longrightarrow SiF_4(aq) + O_2(g)$

So, the correct options are (a) (c) and (d).

80. ***(a, c)*** Complete reaction is as follows :

$$\text{Me—C} \equiv \text{C—Me} \xrightarrow[\text{EtOH, } -33^\circ\text{C}]{\text{Na / NH}_3\text{(liq.)}}$$



Racemic mixture

So, the correct options are (a) and (c).

Mathematics

1. ***(a)*** We have,

$$\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$$

$\Rightarrow \quad y = b\cos(n\log x - n\log n)$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = -b\sin(n\log x - n\log n) \times \frac{n}{x}$$

$\Rightarrow \quad x\dfrac{dy}{dx} = -nb\sin(n\log x - n\log n)$

Again differentiating both sides w.r.t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{n^2 b\cos(n\log x - n\log n)}{x}$$

$\Rightarrow \quad x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = -n^2y$

$\Rightarrow \quad x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + n^2y = 0$

2. ***(a)*** We have,

$\phi(x) = f(x) + f(1-x)$ and $f''(x) < 0$ in $[0, 1]$

$\phi'(x) = f'(x) - f'(1-x)$

For monotonic increasing, $f'(x) - f'(1-x) \geq 0$

$\Rightarrow f'(x) \geq f'(1-x)$

Since, $f'(x)$ is decreasing.

$\therefore x \leq 1 - x \Rightarrow x \leq \dfrac{1}{2}$

$\Rightarrow$ $\phi(x)$ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2}, 1\right]$

3. *(c)* Let

$$I = \int \frac{f(x)\phi'(x) + \phi(x)\, f'(x)}{(f(x)\phi(x) + 1)\sqrt{f(x)\phi(x) - 1}}\, dx$$

Put $f(x)\phi(x) - 1 = t^2$

$\Rightarrow (f(x)\phi'(x) + \phi(x) f'(x))dx = 2t\, dt$

$$\therefore \quad I = \int \frac{2tdt}{(t^2 + 2)t}$$

$$\Rightarrow \quad I = 2\int \frac{dt}{t^2 + 2}$$

$$= \frac{2}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$$

$$\Rightarrow \quad I = \sqrt{2} \tan^{-1} \sqrt{\frac{f(x)\phi(x) - 1}{2}} + c$$

4. *(d)* We have,

$$I = \sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x\, dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x\, dx$$

$$I = \sum_{n=1}^{10} \left[\int_{-2n-1}^{-2n} \sin^{27} x\, dx + \int_{2n}^{2n+1} \sin^{27} x\, dx\right]$$

Let $\quad I_1 = \int_{-2n-1}^{-2n} \sin^{27} x\, dx$

put $\quad x = -t \Rightarrow dx = -dt$

When $x = -2n - 1$

$\Rightarrow \quad t = 2n + 1$ and $x = -2n$

$\Rightarrow \quad t = 2n$

$$\therefore \quad I_1 = \int_{2n+1}^{2n} \sin^{27}(-t)\,(-dt)$$

$$= \int_{2n+1}^{2n} \sin^{27} t\, dt$$

$$\text{Hence, } I = \sum_{n=1}^{10} \left[\int_{2n+1}^{2n} \sin^{27}x\, dx + \int_{2n}^{2n+1} \sin^{27}x\, dx\right]$$

$$I = \sum_{n=1}^{10} \left[-\int_{2n}^{2n+1} \sin^{27} x\, dx + \int_{2n}^{2n+1} \sin^{27} x\, dx\right] = 0$$

5. *(b)* Let $I = \int_0^2 [x^2]\, dx$

$$\Rightarrow \quad I = \int_0^1 0.dx + \int_1^{\sqrt{2}} dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3\, dx$$

$$\Rightarrow \quad I = 0 + \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$\Rightarrow \quad I = 5 - \sqrt{2} - \sqrt{3}$$

6. *(d)* Given, curve

$$y^2 = x^3$$

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(m^2,\, m^3)} = \frac{3(m^4)}{2m^3} = \frac{3}{2}m$$

$\therefore$ Slope of tangent at (m^2, m^3) is $\frac{3}{2}m$.

Now slope of normal at (M^2, M^3) to the curve $y^2 = x^3$ is

$$-\frac{dx}{dy} = \frac{-2y}{3x^2}$$

$$\Rightarrow \quad \left(-\frac{dx}{dy}\right)_{(M^2,\, M^3)} = \frac{-2M^3}{3M^4} = \frac{-2}{3M}$$

Given, slope of tangent = slope of normal

$$\therefore \quad \frac{3m}{2} = \frac{-2}{3M}$$

$$\Rightarrow \quad mM = \frac{-4}{9}$$

7. *(c)* Given, $x^2 + y^2 = a^2$

On differentiating both sides w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{y^2}$$

$$\text{Let} \quad I = \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$I = \int_0^a \sqrt{1 + \frac{x^2}{y^2}}\, dx$$

$$\Rightarrow \quad I = \int_0^a \sqrt{\frac{x^2 + y^2}{y^2}}\, dx$$

$$\Rightarrow \quad I = \int_0^a \frac{a}{\sqrt{a^2 - x^2}}\, dx \qquad [\because x^2 + y^2 = a^2]$$

$$\Rightarrow \quad I = a\left[\sin^{-1} \frac{x}{a}\right]_0^a$$

$$\Rightarrow \quad I = a\left[\sin^{-1}\frac{a}{a} - \sin^{-1} 0\right]$$

$$\Rightarrow \quad I = a[\sin^{-1}1 - \sin^{-1} 0]$$

$$= a\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2}a$$

8. *(c)* We have,

$$\lim_{n \to \infty} \sum_{j=0}^{n} \frac{1}{n} f\left(\frac{j}{n}\right)$$

By standard formula, we write

$$\lim_{n \to \infty} \sum_{j=0}^{n} \frac{1}{n} f\left(\frac{j}{n}\right) = \int_0^1 f(x)\, dx$$

Where, $\lim_{n \to \infty} \left(\frac{j}{n}\right)$(as $j = 0) = 0$

and $\lim_{n \to \infty} \left(\frac{j}{n}\right)$(as $j = n) = 1$

9. *(c)* We have,

$$y' + yf'(x) - f(x)\, f'(x) = 0$$

$$\Rightarrow \quad y' + yf'(x) = f(x)\, f'(x)$$

Which is a linear differentiable equation of the form $\frac{dy}{dx} + Py = Q$

$$\therefore \quad \text{I.F} = e^{\int f'(x)\, dx} = e^{f(x)}$$

Solution of given differential equation is

$$y \cdot e^{f(x)} = \int f(x)\, f'(x) \cdot e^{f(x)} dx$$

$$y \cdot e^{f(x)} = \int te^t dt \quad [\because \text{put } f(x) = t \Rightarrow f'(x)\, dx = dt]$$

$$\Rightarrow \quad y \cdot e^{f(x)} = te^t - e^t + c$$

$$\Rightarrow \quad y \cdot e^{f(x)} = f(x)e^{f(x)} - e^{f(x)} + c$$

$$\Rightarrow \quad y = f(x) - 1 + ce^{-f(x)}$$

$$\Rightarrow \quad y + 1 = f(x) + ce^{-f(x)}$$

When $x \to \infty$, $f(x) = 0$ and $y = 0$

$$\therefore \quad 0 + 1 = 0 + c$$

$$\Rightarrow \quad c = 1$$

Hence, $y + 1 = f(x) + e^{-f(x)}$

10. *(b)* We have,

$$x \sin\left(\frac{y}{x}\right) dy = \left(y \sin\left(\frac{y}{x}\right) - x\right) dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = v - \operatorname{cosec} v$$

$$\Rightarrow \quad -\frac{dv}{\operatorname{cosec} v} = \frac{dx}{x}$$

$$\Rightarrow \quad \int -\sin v\, dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \cos v = \log xc$$

$$\Rightarrow \quad \cos\frac{y}{x} = \log cx \quad \ldots\text{(i)}$$

Put $x = 1$, $y = \frac{\pi}{2}$ in Eq. (i), we get

$$\cos\frac{\pi}{2} = \log c$$

$$\Rightarrow \quad \log c = 0$$

$$\Rightarrow \quad c = 1$$

$$\cos\frac{y}{x} = \log x$$

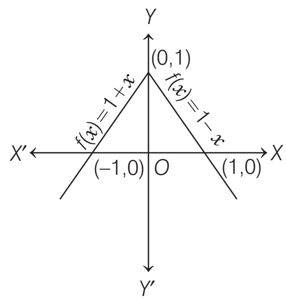
11. *(c)* We have,

$$f(x) = 1 - \sqrt{x^2}$$

$$f(x) = 1 - |x|$$

$$f(x) = \begin{cases} 1 + x, & x < 0 \\ 1 - x, & x \geq 0 \end{cases}$$

Graph of $f(x)$



Clearly, $f(x)$ has maxima at $x = 0$.

12. *(a)* Given,

$$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$$

$$f'(x) = 6x^2 - 18ax + 12a^2$$

For maxima or minima $f'(x) = 0$

$$\therefore \quad 6x^2 - 18ax + 12a^2 = 0$$

$$x^2 - 3ax + 2a^2 = 0$$

$$(x - 2a)(x - a) = 0$$

$$\Rightarrow \quad x = 2a, a$$

$$f''(x) = 12x - 18a$$

$$\therefore \quad f''(2a) = 24a - 18a > 0$$

and $f''(a) = 12a - 18a < 0$

$\therefore f(x)$ is maximum at $x = a$

and minimum at $x = 2a$

$\therefore \quad p = a, q = 2a$

Given, $\quad p^2 = q$

$\Rightarrow \quad a^2 = 2a$

$\Rightarrow \quad a = 2$

13. *(a)* $\frac{6a}{5b} + \frac{10b}{3a}$

We know that, AM $\geq$ GM

$$\therefore \quad \frac{\frac{6a}{5b} + \frac{10b}{3a}}{2} \geq \sqrt{\frac{6a}{5b} \times \frac{10b}{3a}}$$

$$\Rightarrow \quad \frac{6a}{5b} + \frac{10b}{3a} \geq 2 \times 2 = 4$$

Hence, least possible value of $\frac{6a}{5b} + \frac{10b}{3a}$ is 4.

14. *(a)* We have,

$2\log (x+1) - \log(x^2 - 1) = \log 2$

$$\Rightarrow \quad \log \frac{(x+1)^2}{x^2 - 1} = \log 2$$

$\Rightarrow \quad (x+1)^2 = 2(x^2 - 1)$

$\Rightarrow \quad x^2 + 2x + 1 = 2x^2 - 2$

$\Rightarrow \quad x^2 - 2x - 3 = 0$

$\Rightarrow \quad (x-3)(x+1) = 0$

$\Rightarrow \quad x = 3, x \neq -1$

15. *(c)* Let complex number

$P = \cos\theta + i \sin\theta$

$\therefore \quad |P| = 1$ and $\theta \in [0, 2\pi)$

Now, $P^4 = (\cos\theta + i\sin\theta)^4$

$\Rightarrow \quad P^4 = \cos 4\theta + i \sin 4\theta$

Imaginary part of $P^4 = 0$

$\therefore \quad \sin 4\theta = 0$

$$\Rightarrow \quad 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$$

$0 \leq \theta \leq 2\pi$

$$\therefore \quad 0 \leq \frac{n\pi}{4} < 2\pi$$

$= 0 \leq x < 8$

$\Rightarrow \quad n = 8$

Hence, 8 such complex number.

16. *(b)* Given,

$z\bar{z} + (2-3i)z + (2+3i)\bar{z} + 4 = 0$

This is the form of

$z\bar{z} + \bar{a}z + a\bar{z} + b$

Here, $a = 2 + 3i$, $b = 4$

$\therefore$ Radius of circle $= \sqrt{|a|^2 - b}$

$= \sqrt{(4+9) - 4}$

$= \sqrt{13 - 4} = 3$

17. *(c)* The expression $ax^2 + bx + c$ has the same sign as that of a for all x if $D \leq 0$

$\therefore \quad b^2 - 4ac \leq 0$

18. *(d)* In 12 storied building 3 person leave the lift any 10 floors.

$\therefore$ Total number of ways

$$= {}^{10}P_3 = \frac{10!}{7!} = 10 \times 9 \times 8 = 720$$

19. *(b)* ${}^nC_m = k \cdot {}^{n-1}C_{m-1}$

$$\Rightarrow \quad \frac{n!}{m!\,(n-m)!} = k \cdot \frac{(n-1)!}{(m-1)!\,(n-m)!}$$

$$\Rightarrow \quad \frac{n(n-1)!}{m(m-1)!} = \frac{k \cdot (n-1)!}{(m-1)!}$$

$\Rightarrow \quad n = mk$

20. *(a)* Since, AM $\geq$ GM

$$\therefore \quad \frac{1 + 3 + 5 + 7 + \dots + (2n-1)}{n} > (J(n))^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{n^2}{n} > (J(n))^{\frac{1}{n}}$$

$\Rightarrow \quad n^n > J(n)$

$\Rightarrow \quad I(n) > J(n)$

21. *(b)* $(1+x)^{15} = c_0 + c_1 x + c_2 x^2 + \dots + c_{15} x^{15}$

Now, $\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \dots + 15\frac{c_{15}}{c_{14}}$

$$\frac{15+1-1}{1} + \frac{2(15+1-2)}{2} + \frac{3(15+1-3)}{3} + \dots + \frac{15(15+1-15)}{15}$$

$$\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n+1-r}{r} \text{ Here, } n = 15\right]$$

$\therefore \quad 15 + 14 + 13 + \dots + 1$

$$= \frac{15 \times 16}{2} = 120$$

22. *(d)* Given, $A = \begin{bmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$|A| = (3-t)(0+1) - 1(0) + 0$

$\Rightarrow \quad |A| = 3 - t$

Given, $|A| = 5$

$\therefore \quad 3 - t = 5$

$\Rightarrow \quad t = -2$

23. *(c)* Given, $A = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix}$

Since, A is not invertible.

$\therefore \quad |A| = 0$

$|A| = 12(18 + 4) - 24(3x + 2) + 5(-2x + 6) = 0$

$\Rightarrow 264 - 72x - 48 - 10x + 30 = 0$

$\Rightarrow 82x = 246 \quad \Rightarrow x = 3$

24. *(a)* Given, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $|A| = 1$ and $|A - \lambda I| = 0$ has imaginary roots.

$$|A| = ad - bc = 1$$

$$A - \lambda I = \begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix}$$

$$|A - \lambda I| = (a - \lambda)(d - \lambda) - bc$$

$$|A - \lambda I| = ad - (a + d)\lambda + \lambda^2 - bc$$

$$= \lambda^2 - (a + d)\lambda + 1 \quad (\because ad - bc = 1)$$

$|A - \lambda I|$ has imaginary roots.

$\therefore \quad (a + d)^2 - 4 < 0 \quad \Rightarrow \quad (a + d)^2 < 4$

25. *(d)* Let $A = \begin{vmatrix} a^2 & bc & c^2 + ab \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix}$

Taking common a, b, c from C_1, C_2 and C_3 respectively.

$$\therefore \quad A = abc \begin{vmatrix} a & c & c + a \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

Apply $R_1 \to R_1 + R_2 + R_3$

$$A = abc \begin{vmatrix} 2(a + b) & 2(b + c) & 2(c + a) \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

$$A = 2abc \begin{vmatrix} a + b & b + c & c + a \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

Apply $R_1 \to R_1 - R_2$

$$A = 2abc \begin{vmatrix} 0 & c & c \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

$$A = 2abc^2 \begin{vmatrix} 0 & 1 & 1 \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

Apply $C_2 \to C_2 - C_3$

$$A = 2abc^2 \begin{vmatrix} 0 & 0 & 1 \\ a + b & b - a & a \\ b & b & c \end{vmatrix}$$

$$A = 2abc^2 \,|(a + b)b - (b - a)\, b|$$

$$= 2abc^2 \,|ab + b^2 - b^2 + ab|$$

$$A = 4a^2b^2c^2$$

$\therefore \quad k = 4$

26. *(d)* We have,

$$f : S \to R$$

$$f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$$

$$f\left[\begin{pmatrix} 4 & 3 \\ 2 & 3 \end{pmatrix}\right] = 12 - 6 = 6$$

$$f\left[\begin{pmatrix} 2 & 6 \\ 1 & 6 \end{pmatrix}\right] = 12 - 6 = 6$$

$\therefore$ Hence, f is not one-one.

Since, $O \in R$ but S does not contain any singular matrix.

$\therefore f$ is not onto.

Hence, f is neither one-one nor onto.

27. *(b)* Given,

$a\rho b = a - b$ is zero or irrational.

For reflexive,

$(a, a) = a - a = 0$. Hence, ρ is reflexive.

For symmetric,

$(a, b) \in \rho \Rightarrow (b, a) \in \rho$

$\Rightarrow a - b$ is zero or irrational then $b - a$ is zero or irrational.

$\therefore \ \rho$ is symmetric.

For transitive,

If $a = 3$, $b = \sqrt{3}$ and $c = 5$

$(a, b) \in \rho$, $(b, c) \in \rho \Rightarrow (a, c) \notin \rho$

$\therefore \rho$ is not transitive.

28. *(b)* Let the unit vector in ZOX plane be

$$\mathbf{a} = x\hat{\mathbf{i}} + z\hat{\mathbf{k}}, |\mathbf{a}| = 1$$

$$\mathbf{a} \cdot \alpha = |\mathbf{a}||\alpha| \cos 45^\circ$$

$$\Rightarrow \quad (x\hat{\mathbf{i}} + z\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 1 \times 3 \times \frac{1}{\sqrt{2}}$$

$[\because \alpha = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}]$

$$2x - z = \frac{3}{\sqrt{2}}$$

and $\mathbf{a} \cdot \beta = |\mathbf{a}||\beta| \cos 60°$

$\Rightarrow (x\hat{i} + z\hat{k}) \cdot (\hat{j} - \hat{k}) = 1 \times \sqrt{2} \times \frac{1}{2}$ $[\because \beta = \hat{j} - \hat{k}]$

$$-z = \frac{1}{\sqrt{2}}$$

$$z = -\frac{1}{\sqrt{2}} \quad \text{and} \quad x = \frac{1}{\sqrt{2}}$$

$\therefore$ $$\mathbf{a} = \frac{1}{\sqrt{2}}\hat{\mathbf{i}} - \frac{1}{\sqrt{2}}\hat{\mathbf{k}}$$

29. *(d)* Four person A, B, C, D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game.

Probability of an even number $= \frac{1}{2}$

$\therefore$ $P(A) = P(B) = P(C) = P(D) = \frac{1}{2}$

$$P(\overline{A}) = P(\overline{B}) = P(\overline{C}) = P(\overline{D}) = \frac{1}{2}$$

Required probability

$= P(A) + P(\overline{A})\, P(\overline{B})\, P(\overline{C})\, P(\overline{D})\, P(A)$
$+ P(\overline{A})\, P(\overline{B})\, P(\overline{C})\, P(\overline{D})\, P(\overline{A})\, P(\overline{B})\, P(\overline{C})\, P(\overline{D})\, P(A) + \dots$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^9 + \dots$$

$$= \frac{1/2}{1 - (1/2)^4} = \frac{1/2}{15/16} = \frac{8}{15}$$

30. *(b)* Given, $P = \frac{10}{100} = \frac{1}{10}$

$$q = 1 - P = 1 - \frac{1}{10} = \frac{9}{10}$$

$$P(X \geq 1) \geq \frac{50}{100} = \frac{1}{2}$$

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X \geq 1) = 1 - {}^nC_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^n$$

$$1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{2}$$

$$\left(\frac{9}{10}\right)^n \leq \frac{1}{2}$$

For $n = 7$ the least number of round to must so fire.

31. *(c)* By using Sandwich theorem.

32. *(b)* We have,

$$y = e^x (A \cos x + B \sin x)$$

$$\frac{dy}{dx} = e^x (A \cos x + B \sin x) + e^x (-A \sin x + B \cos x)$$

$\Rightarrow$ $$\frac{dy}{dx} = y + e^x (-A \sin x + B \cos x)$$

$\Rightarrow$ $$\frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x (-A \sin x + B \cos x) + e^x (-A \cos x - B \sin x)$$

$\Rightarrow$ $$\frac{d^2y}{dx} = \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - y$$

$\Rightarrow$ $$\frac{d^2y}{dx^2} = \frac{2dy}{dx} - 2y$$

$\Rightarrow$ $$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0$$

33. *(d)* We have,

$$r \cos\left(\theta - \frac{\pi}{3}\right) = 2$$

$$r\left[\cos\theta \cos\frac{\pi}{3} + \sin\theta \sin\frac{\pi}{3}\right] = 2$$

$$\frac{r\cos\theta}{2} + \frac{r\sin\theta \cdot \sqrt{3}}{2} = 2,$$

$$x + \sqrt{3}y = 4 \quad [\because x = r\cos\theta, y = r\sin\theta]$$

$\therefore$ It represents a straight line.

34. *(d)* Let (h, k) be the centre and r be the radius of the variable circle.

Since, variable circle touches the circle $x^2 + y^2 = a^2$

$\therefore$ $$\sqrt{h^2 + k^2} = a \quad \dots\text{(i)}$$

Also variable circle touches the circle $x^2 + y^2 = 4ax$

$\therefore$ $$\sqrt{(h - 2a)^2 + k^2} = 2a + r \quad \dots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\sqrt{(h - 2a)^2 + k^2} - \sqrt{h^2 + k^2} = a$$

$\therefore$ Locus of centre of variable circle is

$$\sqrt{(x - 2a)^2 + y^2} - \sqrt{x^2 + y^2} = a$$

which represents the hyperbola.

35. *(b)* We have, $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have common line.

$y = x$ satisfies both lines.

$\therefore$ $x^2 + 2x^2 + ax^2 = 0$

$3x^2 + ax^2 = 0 \Rightarrow a = -3$

Equation of lines are

$x^2 + 2xy - 3y^2 = 0$ and $-3x^2 + 2xy + y^2 = 0$

$(x + 3y)(x - y) = 0$ and $(3x + y)(x - y) = 0$

The other lines are $x + 3y = 0$ and $3x + y = 0$.

36. *(b)* Given lines,

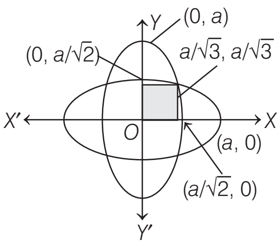
$4x+2y=9$ and $2x+y+6=0$

Lines having distance from origin are $\dfrac{9}{2\sqrt{5}}$ and $\dfrac{6}{\sqrt{5}}$ respectively.

The point O divide the segment PQ in the ratio $=\dfrac{9}{2\sqrt{5}}\div\dfrac{6}{\sqrt{5}}=3:4$

37. *(a)* Given curve

$x^2+2y^2=a^2$ and $2x^2+y^2=a^2$



$$\therefore \text{ Required area} = \left(\frac{a^2}{6}+\int_{a/\sqrt{3}}^{a/\sqrt{2}}\sqrt{a^2-2x^2}\,dx\right)\times 2$$

$$=\frac{a^2}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

38. *(a)* Given, the radius of circle $=\sqrt{17}$

With centre on positive sides of X-axis

$\therefore$ Equation of circle is $(x-a)^2+y^2=17$

Since, it passes through $(0,1)$

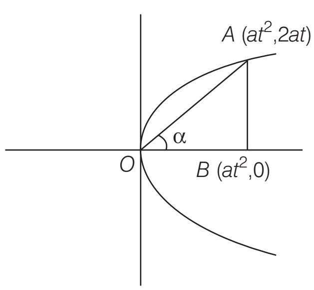
$\therefore \quad a^2+1=17$

$\Rightarrow \quad a=4$

$\therefore$ Equation of circle is

$$x^2+y^2-8x-1=0$$

39. *(b)* Given, parabola $y^2=4ax$



Let the point A on parabola $(at^2, 2at)$ in ΔOAB

$$\cot\alpha=\frac{at^2}{2at}=\frac{t}{2}$$

$$t=2\cot\alpha$$

In ΔOAB

$$\sin\alpha=\frac{AB}{OA}$$

$\Rightarrow \quad OA=AB\operatorname{cosec}\alpha=2at\operatorname{cosec}\alpha$

$\therefore \quad OA=2a(2\cot\alpha)\operatorname{cosec}\alpha$

$OA=4a\cot\alpha\operatorname{cosec}\alpha$

$\therefore$ Length of chord $=4a\cot\alpha\operatorname{cosec}\alpha$

40. *(d)* A double PQ of the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

Such that ΔOPQ is an equilateral where O is origin.

Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\theta, -b\tan\theta)$

ΔOPQ is an equilateral

$\therefore \quad OP=PQ=OQ$

$a^2\sec^2\theta+b^2\tan^2\theta=4b^2\tan^2\theta$

$$a^2\sec^2\theta=3b^2\tan^2\theta$$

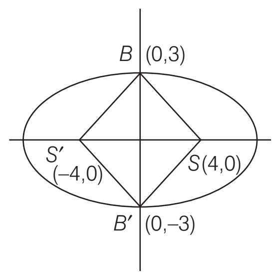
$$\frac{b^2}{a^2}=\frac{\sec^2\theta}{3\tan^2\theta}=\frac{1}{3}\operatorname{cosec}^2\theta$$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{\operatorname{cosec}^2\theta}{3}}=\frac{1}{\sqrt{3}}\sqrt{3+\operatorname{cosec}^2\theta}$$

$e>\dfrac{2}{\sqrt{3}} \qquad [\because \operatorname{cosec}^2\theta\ge 1]$

41. *(c)* Equation of ellipse

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$



Here, $B=(0,3)$, $B'=(0,-3)$

$S=(4,0)$, $S'=(0,-4)$

Area of rhombus $SBS'B'=\dfrac{1}{2}SS'\times BB'$

$$=\frac{1}{2}\times 8\times 6=24 \text{ sq units}$$

42. *(d)* Given, equation of latusrectum of parabola is

$$x+y=8$$

and equation of tangent at vertex is $x+y=12$

Since both are parallel.

$\therefore$ The distance between latusrectum and equation of tangent at vertex is 'a'

$\therefore \quad a = \dfrac{4}{\sqrt{1^2 + 1^2}} = \dfrac{4}{\sqrt{2}} = 2\sqrt{2}$

$\therefore$ Length of latusrectum $= 4a = 8\sqrt{2}$ unit

43. ***(*)*** The equation of the plane through the points $(2, -1, -3)$ and parallel to the line,

$\dfrac{x-1}{2} = \dfrac{y+2}{3} = \dfrac{z}{-4}$ and $\dfrac{x}{2} = \dfrac{y-1}{-3} = \dfrac{z-2}{2}$

$$\begin{vmatrix} x-2 & y+1 & z+3 \\ 2 & 3 & -4 \\ 2 & -3 & 2 \end{vmatrix} = 0$$

$\Rightarrow \; (x-2)(6-12) - (y+1)(4+8) + (z+3)(-6-6) = 0$

$\Rightarrow \; -6x + 12 - 12y - 12 - 12z - 36 = 0$

$\Rightarrow \; -6x - 12y - 12z - 36 = 0$

$\Rightarrow \; x + 2y + 2z + 6 = 0$

44. ***(b)*** Given line,

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

and plane $2x - 2y + z = 5$

Angle between line and plane is

$\sin\theta = \dfrac{(3)(2) + 4(-2) + 5(1)}{\sqrt{3^2 + 4^2 + 5^2}\sqrt{2^2 + (-2)^2 + (1)^2}}$

$\sin\theta = \dfrac{6 - 8 + 5}{\sqrt{50} \times 3}$

$\sin\theta = \dfrac{1}{5\sqrt{2}} = \dfrac{\sqrt{2}}{10}$

45. ***(c)*** Given, $f(x) = \sin x + \cos ax$

$\sin x$ is periodic with 2π.

$f(x)$ is periodic only on $\sin x$ and $\cos x$ both are periodic.

$\therefore$ $\cos ax$ is periodic at $\dfrac{2\pi}{a}$.

LCM of 2π and $\dfrac{2\pi}{a}$ is possible only a is rational number.

46. ***(c)*** Given, $f(x) = \sqrt{\left(\dfrac{1}{\sqrt{x}} - \sqrt{x+1}\right)}$

$f(x)$ is defined $\dfrac{1}{\sqrt{x}} - \sqrt{x+1} \geq 0$ and $x > 0$

$$x^2 + x - 1 \leq 0, \; x > 0$$

$$x = \frac{-1 \pm \sqrt{5}}{2}, \; x > 0$$

$\therefore$ Domain of $f(x)$ is $\left(0, \dfrac{\sqrt{5}-1}{2}\right]$.

47. ***(c)*** Given, $y = 2x^2 - 3x + 2$

$$\frac{dy}{dx} = 4x - 3$$

x changes from 2 to 1.99

$\therefore \quad \Delta x = 1.99 - 2 = -0.01$

The differential of y when x charges 2 to 1.99 is

$$dy = (4x - 3)\,\Delta x$$

$$dy = (8 - 3)[-(0.01)] \qquad [\because x = 2]$$

$$dy = -0.05$$

48. ***(c)*** We have,

$$\lim_{x \to 0}\left(\frac{1 + cx}{1 - cx}\right)^{\frac{1}{x}} = 4$$

$\Rightarrow \quad e^{\lim\limits_{x \to 0}\left(\frac{1+cx}{1-cx} - 1\right) \times \frac{1}{x}} = 4$

$\Rightarrow \quad e^{\lim\limits_{x \to 0}\left(\frac{2cx}{1-cx}\right) \times \frac{1}{x}} = 4$

$\Rightarrow \quad e^{2c} = 4$

Now, $\lim\limits_{x \to 0}\left(\dfrac{1 + 2cx}{1 - 2cx}\right)^{\frac{1}{x}} = e^{4c} = (e^{2c})^2 = (4)^2 = 16$

49. ***(a)*** Given,

f is twice continuously and differentiable and $f(0) = f(1) = f'(0) = 0$

$\therefore \quad f(x) = Kx^2(x-1)$

$$f(x) = K(x^3 - x^2)$$

$$f'(x) = K(3x^2 - 2x)$$

$$f'(x) = 0 \Rightarrow x = 0 \text{ and } \frac{2}{3}$$

Hence, Rolle's theorem is applicable in $f'(x)$.

$\therefore f''(c) = 0$ for some $c \in R$

50. ***(b)*** We have,

$f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12$

$f'(x) = 13x^{12} + 11x^{10} + 9x^8 + 7x^6 + 5x^4 + 3x^2 + 1$

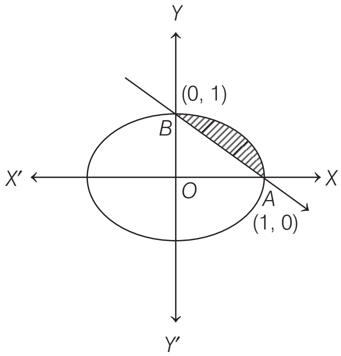
$f'(x) > 0$ for all value of $x \in R$.

Hence, $f(x)$ has exactly are real roots.

51. ***(c)*** Area of region

$$\{(x, y) = x^2 + y^2 \leq 1 < x + y\}$$

$\therefore \quad x^2 + y^2 = 1$ and $x + y = 1$



Area of shaded region = Area of quadrant − Area of triangle

$$= \frac{\pi}{4}(1)^2 - \frac{1}{2} \times 1 \times 1$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

52. *(b)* Let $f(x) = \cos x + x \sin x - 1$

$$f'(x) = -\sin x + x \cos x + \sin x$$

$$f'(x) = x \cos x > 0; x \in \left(0, \frac{\pi}{2}\right)$$

$f(x)$ is increasing on $x \in \left(0, \frac{\pi}{2}\right)$.

$\therefore \quad \cos x + x \sin x - 1 > 0$

$\Rightarrow \quad \cos x + x \sin x > 1$

53. *(c)* Given parabola

$$y = ax^2 + bx + C$$

put $\quad x = 1,\ y = 1$, we get

$$1 = a + b + c \quad \text{...(i)}$$

$y = x$ is tangent of the parabola at $(1, 1)$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(1,1)} = 2ax + b$$

$$\Rightarrow \quad 1 = 2a + b \quad \text{...(ii)}$$

Also curve passes through $(-1, 0)$

$$\therefore \quad 0 = a - b + c \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$a = c = \frac{1}{4}$$

and $\quad b = \frac{1}{2}$

54. *(c)* Given $\alpha = \hat{i} + a\hat{j} + a^2\hat{k}$, $\beta = \hat{i} + b\hat{j} + b^2\hat{k}$ and $\gamma = \hat{i} + c\hat{j} + c^2\hat{k}$

$$\therefore \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$$\text{Now,} \begin{vmatrix} a & a^2 & 1 + a^3 \\ b & b^2 & 1 + b^3 \\ c & c^2 & 1 + c^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (1 + abc) = 0$$

$$\Rightarrow \quad abc + 1 = 0 \left[\because \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0\right]$$

$$\Rightarrow \quad abc = -1$$

55. *(c)* Given, z_1 and z_2 be two imaginary roots of $z^2 + pz + q = 0$

$\therefore \quad z_1 + z_2 = -p,\ z_1 z_2 = q$

z_1, z_2 and origin form an equilateral triangle.

$\therefore \quad z_1^2 + z_2^2 = z_1 z_2$

$\Rightarrow \quad (z_1 + z_2)^2 - 2z_1 z_2 = z_1 z_2$

$\Rightarrow \quad p^2 - 2q = q \Rightarrow p^2 = 3q$

56. *(a)* Given,

$P(x) = ax^2 + bx + c$, $Q(x) = -ax^2 + dx + c$

Discriminant for $P(x) = b^2 - 4ac$

and Discriminant for $Q(x) = d^2 + 4ac$

If $ac > 0$, $Q(x)$ has real roots and $ac < 0$, $P(x)$ has real roots.

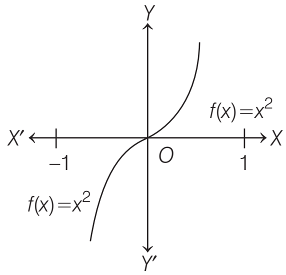
$\therefore P(x) \cdot Q(x) = 0$ has atleast two real roots.

57. *(d)* We have,

$$f(x) = x|x|,\ x \in [-1, 1]$$

$$f(x) = \begin{cases} -x^2, & -1 \le x < 0 \\ x^2, & 0 \le x < 1 \end{cases}$$

Graph of $f(x)$ is



Clearly $f(x)$ is bijective function.

58. *(d)* We have,

$$f(x) = \sqrt{x^2 - 3x + 2}$$

$$g(x) = \sqrt{x}$$

$$fog = f(g(x)) = \sqrt{x - 3\sqrt{x} + 2}$$

fog is defined $x - 3\sqrt{x} + 2 \ge 0$ and $x \ge 0$

$$(\sqrt{x} - 1)(\sqrt{x} - 2) \ge 0$$

$$x \in (-\infty, 1] \cup [4, \infty)$$

$$gof = g(f(x)) = \sqrt{\sqrt{x^2 - 3x + 2}}$$

gof is defined $x^2 - 3x + 2 \ge 0$

$$(x-1)(x-2) \ge 0$$

$$x \in (-\infty, 1] \cup [2, \infty)$$

$\therefore$ Domain of fog is $(-\infty, 1] \cup [4, \infty)$

Domain of gof is $(-\infty, 1] \cup [2, \infty)$

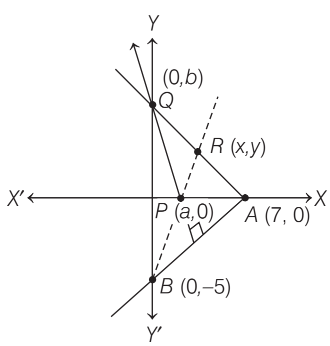
$\therefore$ $S \cap T$ is an interval.

59. *(b)* Given, ρ_1 and ρ_2 be two equivalent relation by theorem,

$\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so.

60. *(c)* Since, the length of tangent between the point of contact and the point where it meets the directrix subtends right angle at the corresponding focus.

61. *(c)*



Here, P is the orthocentre of ΔABQ.

$\therefore$ Slope of $BR \times$ Slope of $AR = -1$

$$\Rightarrow \quad \left(\frac{k+5}{h}\right) \times \left(\frac{k}{h-7}\right) = -1$$

$$\Rightarrow \quad x^2 + y^2 - 7x + 5y = 0$$

62. *(b)* Let

$$I = \lim_{n \to \infty} \sum_{k=1}^{n} \int_{(1/k+\beta)}^{1/(k+\alpha)} \frac{dx}{1+x}$$

$$I = \lim_{n \to \infty} \sum_{k=1}^{n} [\log(1+x)]_{\frac{1}{k+\beta}}^{\frac{1}{k+\alpha}}$$

$$I = \lim_{n \to \infty} \sum_{k=1}^{n} \left[\log\left(1 + \frac{1}{k+\alpha}\right) - \log\left(1 + \frac{1}{k+\beta}\right)\right]$$

$$I = \lim_{n \to \infty} \sum_{k=1}^{n} \log\left(\frac{k+\alpha+1}{k+\alpha}\right)\left(\frac{k+\beta}{k+\beta+1}\right)$$

$$I = \lim_{n \to \infty} \log\left(\frac{\alpha+2}{\alpha+1}\right)\left(\frac{\beta+1}{\beta+2}\right)\left(\frac{\alpha+3}{\alpha+2}\right)\left(\frac{\beta+2}{\beta+3}\right) \ldots \left(\frac{\alpha+n+1}{\alpha+n}\right)\left(\frac{\beta+n}{\beta+n+1}\right)$$

$$I = \lim_{n \to \infty} \left(\log\left(\frac{\beta+1}{\alpha+1}\right)\left(\frac{\alpha+n+1}{\beta+n+1}\right)\right)$$

$$I = \log_e\left(\frac{\beta+1}{\alpha+1}\right)$$

63. *(c)* Let $P = \lim_{x \to 1}\left(\frac{1}{\log x} - \frac{1}{x-1}\right)$

$$P = \lim_{x \to 1}\left(\frac{x - 1 - \log x}{(x-1)\log x}\right)$$

Apply L-Hospital rules,

$$P = \lim_{x \to 1}\left(\frac{1 - \frac{1}{x}}{\frac{x-1}{x} + \log x}\right)$$

$$= \frac{x-1}{(x-1) + x\log x}$$

Again apply L-Hospital rule,

$$P = \lim_{x \to 1} \frac{1}{1 + \frac{x}{x} + \log x}$$

$$= \lim_{x \to 1} \frac{1}{2 + \log x}$$

$$P = \frac{1}{2+0} = \frac{1}{2}$$

64. *(b)* We have,

$$y = \frac{1}{1 + x + \log x}$$

$(1 + x + \log x) y = 1$

On differentiating both sides, we get

$$(1 + x + \log x)\frac{dy}{dx} + \left(1 + \frac{1}{x}\right) y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(x+1)y}{x(1 + x + \log x)}$$

$$\Rightarrow \quad x\frac{dy}{dx} = -(x+1)(y^2) \qquad \left[\because y = \frac{1}{1 + x + \log x}\right]$$

$$\Rightarrow \quad x\frac{dy}{dx} = -y(x+1)y$$

$$\Rightarrow \quad x\frac{dy}{dx} = y(y\log x - 1) \quad [\because (1 + x)y = 1 - y\log x]$$

65. *(b)* Given, curve $y = be^{-\frac{x}{a}}$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{b}{a}e^{-\frac{x}{a}}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)(0, b) = -\frac{b}{a}$$

$\therefore$ Equation of tangent

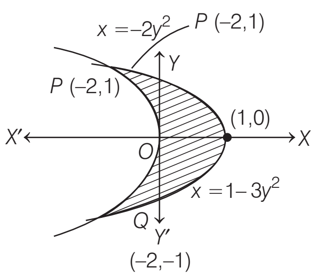
$$y - b = -\frac{b}{a}(x - 0)$$

$$\Rightarrow \quad y - b = -\frac{bx}{a}$$

$$\Rightarrow \quad \frac{y}{b} - 1 = \frac{-x}{a}$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 1$$

66. *(b)* Intersecting points of both curves are $P(-2, 1)$ and $Q(-2, -1)$.



$\therefore$ Area of the bounded region

$$= \int_{-1}^{1} ((1 - 3y^2) - (-2y^2))\, dy$$

$$= 2\int_0^1 (1 - y^2)\, dy$$

$$= 2\left(y - \frac{y^3}{3}\right)_0^1$$

$$= 2\left(1 - \frac{1}{3}\right) = \frac{4}{3} \text{ sq unit}$$

67. *(a, d)* We have, $v = u - gt$

at $t = 6$, $v = 0$

$= u - gt = 0$

$$\Rightarrow \quad u = 6g = 192\,\text{ft/sec} \qquad (\because g = 32\ ft/\text{sec}^2)$$

Now, $h = ut - \frac{1}{2}gt^2$

$$= 192.6 - \frac{1}{2} \times 32 \times 6 \times 6$$

$= 576$ ft

68. *(a, c)* Given, equation

$$x^{(\log_3 x)^2} - \frac{9}{2}\log_3 (x + 5) = 3\sqrt{3}$$

$$\Rightarrow \left((\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right)\log_3 x = \frac{3}{2}\log_3 3$$

put $\log_3 x = t$

$$\therefore \quad \left(t^2 - \frac{9}{2}t + 5\right)t = \frac{3}{2}$$

$$\Rightarrow \quad 2t^3 - 9t^2 + 10t - 3 = 0$$

$$\Rightarrow \quad (t - 3)(t - 1)\left(t - \frac{1}{2}\right) = 0$$

$$\Rightarrow \quad t = 3, 1, \frac{1}{2}$$

$$\Rightarrow \quad \log_3 x = 3, 1, \frac{1}{2}$$

$$\Rightarrow \quad x = 3^3, 3^1, 3^{\frac{1}{2}}$$

69. *(b)* Given, total number of questions are n and total wrong answers are 2047

Also, total wrong answer given

$$= \Sigma k \times (2^{n-k} - 2^{n-(k+1)})$$

$$\Rightarrow \quad 2^{n-1} + \ldots + 1 = 2047$$

$$\Rightarrow \quad 2^n = 2048 = 2^{11}$$

$$\Rightarrow \quad n = 11$$

70. *(c, d)* Given, $P(A' \cap B') = \frac{3}{5}$

$$\Rightarrow \quad 1 - P(A \cup B) = \frac{3}{5}$$

$$\Rightarrow \quad P(A \cup B) = \frac{2}{5}$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A) + P(B) - P(A)\ P(B) = \frac{2}{5} \quad ...(i)$$

(since A and B are independent events)

Also, given $P(A \cap B) = \frac{1}{20}$

$$\Rightarrow \quad P(A)\ P(B) = \frac{1}{20} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$P(A) = \frac{1}{4} \text{ or } \frac{1}{5}$$

71. *(a, b)* Equation of straight line having intercepts on the coordinate axes are a and b, are

$$\frac{x}{a} + \frac{y}{b} = 1 \quad ...(i)$$

Given, $a + b = -1 \Rightarrow b = -(a+1)$

Line (i) passing through the point (4, 3).

$$\therefore \quad \frac{4}{a} + \frac{3}{b} = 1$$

$$\Rightarrow \quad \frac{4}{a} - \frac{3}{a+1} = 1$$

$$\Rightarrow \quad 4(a+1) - 3a = a(a+1)$$

$$\Rightarrow \quad a^2 + a - a = 4$$

$$\Rightarrow \quad a = \pm 2$$

When $a = 2$, then $b = -3$

and when $a = -2$, then $b = 1$

$\therefore$ Equation of straight line are $\frac{x}{2} - \frac{y}{3} = 1, \frac{x}{-2} + \frac{y}{1} = 1$

72. *(d)* Equation of tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at point (x_1, y_1) is

$$\frac{xx_1}{2} + \frac{yy_1}{1} = 1 \quad ...(i)$$

Let mid-point of intercept be $P(h, k)$.

$$\therefore \quad h = \frac{1}{x_1}$$

$$\Rightarrow \quad x_1 = \frac{1}{h}$$

$$\text{and} \quad k = \frac{1}{2y_1}$$

$$\Rightarrow \quad y_1 = \frac{1}{2k}$$

$\therefore$ Required locus of the mid-point

$$= \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

73. *(a)* Given, $y = \frac{x^2}{(x+1)^2 (x-2)}$

By using partial fraction,

$$\frac{x^2}{(x+1)^2 (x+2)} = \frac{4}{(x+2)} - \frac{3}{x+1} + \frac{1}{(x+1)^2}$$

$$\therefore \quad y = \frac{4}{x+2} - \frac{3}{x+1} + \frac{1}{(x+1)^2}$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-4}{(x+2)^2} + \frac{3}{(x+1)^2} - \frac{2}{(x+1)^3}$$

Again differentiating w.r.t. x

$$\frac{d^2y}{dx^2} = \frac{8}{(x+2)^3} - \frac{6}{(x+1)^3} + \frac{6}{(x+1)^4}$$

$$= 2\left[\frac{4}{(x+2)^3} - \frac{3}{(x+1)^3} + \frac{3}{(x+1)^4}\right]$$

74. *(c)* Given, $f(x) = \frac{1}{3} x \sin x - (1 - \cos x)$

$$\therefore \lim_{x \to 0} \frac{f(x)}{x^k} = \lim_{x \to 0} \frac{x \sin x - 3(1 - \cos x)}{3x^k}$$

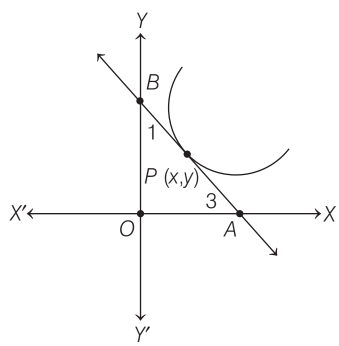
$$= \lim_{x \to 0} \frac{2x \sin \frac{x}{2} \cos \frac{x}{2} - 6 \sin^2 \frac{x}{2}}{3x^k}$$

$$= \frac{1}{3} \lim_{x \to 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right) \cdot \lim_{x \to 0} \left(\frac{2x \cos \frac{x}{2} - 6 \sin \frac{x}{2}}{2x^{k-1}}\right)$$

$$= \frac{1}{3} \lim_{x \to 0} \left(\frac{x \cos \frac{x}{2} - 3 \sin \frac{x}{2}}{x^{k-1}}\right)$$

For, $\lim_{x \to 0} \frac{f(x)}{x^k} \neq 0 \Rightarrow k - 1 = 1 \Rightarrow k = 2$

75. *(a, c)* Given, $AP : BP = 3 : 1$



Equation of tangent AB is $Y - y = \frac{dy}{dx}(X - x)$

$\therefore$ Coordinates of A are $\left(\frac{x\frac{dy}{dx} - y}{y'}, 0\right)$.

and coordinates of B are $\left(0, y - x\frac{dy}{dx}\right)$.

Now, $y = \frac{1 \times 0 + 3 \times \left(y - x\frac{dy}{dx}\right)}{4}$

$\Rightarrow \quad 4y = 3y - 3x\frac{dy}{dx}$

$\Rightarrow \quad 3x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \frac{3dy}{y} + \frac{dx}{x} = 0$

$\Rightarrow \quad 3\log y + \log x = c$

$\Rightarrow \quad xy^3 = c$

at $x = 1$, $y = 1$, we get $c = 1$

$\therefore \quad xy^3 = 1$

$\Rightarrow \quad x(3y^2)\frac{dy}{dx} + y^3 = 0$

$\Rightarrow \quad 3x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{3x}$

$\therefore \quad \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{3}$

$\therefore$ Slope of normal $= 3$

$\therefore$ Equation of normal $= y - 1 = 3(x - 1)$

$\Rightarrow \quad y - 3x + 2 = 0$