WBJEE 2015 Question Paper

West Bengal Joint Entrance Examinations

WB JEE

Engineering Entrance Exam

Solved Paper 2015

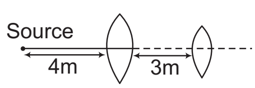
Physics

1. Two particles of mass $\mathbf{m_1}$ and $\mathbf{m_2}$; approach each other due to their mutual gravitational attraction only. Then,
   (a) accelerations of both the particles are equal
   (b) acceleration of the particle of mass m_1 is proportional to m_1
   (c) acceleration of the particle of mass m_1 is proportional to m_2
   (d) acceleration of the particle of mass m_1 is inversely proportional to m_1

2. Three bodies of the same material and having masses $\mathbf{m, m}$ and $\mathbf{3m}$ are at temperatures $\mathbf{40°C}$, $\mathbf{50°C}$ and $\mathbf{60°C}$, respectively. If the bodies are brought in thermal contact, the final temperature will be
   (a) 45°C (b) 54°C
   (c) 52°C (d) 48°C

3. A satellite has kinetic energy **K**, potential energy **V** and total energy **E**. Which of the following statements is true?
   (a) $K = -V/2$ (b) $K = V/2$
   (c) $E = K/2$ (d) $E = -K/2$

4. An object is located 4 m from the first of two thin converging lenses of focal lengths 2 m and 1 m, respectively. The lenses are separated by 3 m. The final image formed by the second lens is located from the source at a distance of

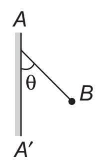


   (a) 8.0 m (b) 5.5 m
   (c) 6.0 m (d) 6.5 m

5. A simple pendulum of length **L** swings in a vertical plane. The tension of the string when it makes an angle θ with the vertical and the bob of mass **m** moves with a speed **v** is (**g** is the gravitational acceleration)
   (a) mv^2/L (b) $mg\cos\theta + mv^2/L$
   (c) $mg\cos\theta - mv^2/L$ (d) $mg\cos\theta$

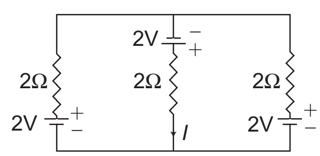
6. The length of a metal wire is $\mathbf{L_1}$ when the tension is $\mathbf{T_1}$ and $\mathbf{L_2}$ when the tension is $\mathbf{T_2}$. The unstretched length of wire is
   (a) $\frac{L_1 + L_2}{2}$ (b) $\sqrt{L_1 L_2}$
   (c) $\frac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$ (d) $\frac{T_2 L_1 + T_1 L_2}{T_2 + T_1}$

7. The line **AA′** is on charged infinite conducting plane which is perpendicular to the plane of the paper. The plane has a surface density of charge σ and **B** is ball of mass **m** with a like charge of magnitude **q**. **B** is connected by string from a point on the line **AA′**. The tangent of angle (θ) formed between the line **AA′** and the string is



   (a) $\frac{q\sigma}{2\epsilon_0 mg}$ (b) $\frac{q\sigma}{4\pi\varepsilon_0 mg}$ (c) $\frac{q\sigma}{2\pi\varepsilon_0 mg}$ (d) $\frac{q\sigma}{\varepsilon_0 mg}$

8. The current **I** is in the circuit shown is



   (a) 1.33 A (b) zero (c) 2.00 A (d) 1.00 A

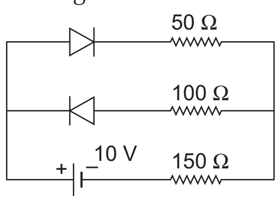
9. A hollow sphere of external radius **R** and thickness **t (<< R)** is made of a metal of density ρ. The sphere will float in water, if

(a) $t \le \frac{R}{\rho}$ (b) $t \le \frac{R}{3\rho}$ (c) $t \le \frac{R}{2\rho}$ (d) $t \ge \frac{R}{3\rho}$

10. A metal wire of circular cross-section has a resistance $\mathbf{R_1}$. The wire is now stretched without breaking, so that its length is doubled and the density is assumed to remain the same. If the resistance of the wire now becomes $\mathbf{R_2}$, then $\mathbf{R_2 : R_1}$ is

(a) 1 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

11. Assume that each diode as shown in the figure has a forward bias resistance of **50 Ω** and an infinite reverse bias resistance. The current through the resistance **150 Ω** is



(a) 0.66 A (b) 0.05 A (c) zero (d) 0.04 A

12. The rms speed of oxygen is **v** at a particular temperature. If the temperature is doubled and oxygen molecules dissociate into oxygen atoms, the rms speed becomes

(a) v (b) $\sqrt{2}v$
(c) $2v$ (d) $4v$

13. Two particles, **A** and **B**, having equal charges, after being accelerated through the same potential difference enter into a region of uniform magnetic field and the particles describe circular paths of radii $\mathbf{R_1}$ and $\mathbf{R_2}$, respectively. The ratio of the masses of **A** and **B** is

(a) $\sqrt{R_1/R_2}$ (b) R_1/R_2
(c) $(R_1/R_2)^2$ (d) $(R_2/R_1)^2$

14. A large number of particles are placed around the origin, each at a distance **R** from the origin. The distance of the center of mass of the system from the origin is

(a) equal to R
(b) less than equal to R
(c) greater then R
(d) greater than equal to R

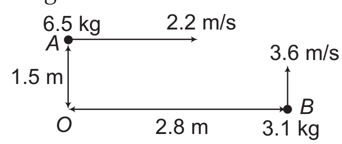
15. A straight conductor 0.1 m long moves in a uniform magnetic field 0.1T. The velocity of the conductor is 15 m/s and is directed perpendicular to the field. The emf induced between the two ends of the conductor is

(a) 0.10 V (b) 0.15 V (c) 1.50 V (d) 15.00 V

16. A ray of light is incident at an angle **i** on a glass slab of refractive index μ. The angle between reflected and refracted light is **90°**. Then, the relationship between **i** and μ is

(a) $i = \tan^{-1}\left(\frac{1}{\mu}\right)$ (b) $\tan i = \mu$
(c) $\sin i = \mu$ (d) $\cos i = \mu$

17. Two particles **A** and **B** are moving as shown in the figure.



Their total angular momentum about the point **O** is

(a) 9.8 kg m^2/ s (b) zero
(c) 52.7 kg m^2/s (d) 37.9 kg m^2/s

18. A 20 cm long capillary tube is dipped vertically in water and the liquid rises upto 10 cm. If the entire system is kept is a freely falling platform, the length of the water column in the tube will be

(a) 5 cm (b) 10 cm (c) 15 cm (d) 20 cm

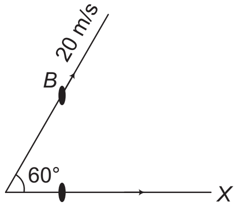
19. A train is moving with a uniform speed of 33 m/s and an observer is approaching the train with the same speed. If the train blows a whistle of frequency 1000 Hz and the velocity of sound is 333 m/s, then the apparent frequency of the sound that the observer hears is

(a) 1220 Hz (b) 1099 Hz (c) 1110 Hz (d) 1200 Hz

20. A photon of wavelength 300 nm interacts with a stationary hydrogen atom in ground state. During the interaction, whole energy of the photon is transferred to the electron of the atom. State which possibility is correct. (Consider, Plank constant $= \mathbf{4 \times 10^{-15}\, eVs}$, velocity of light $= \mathbf{3 \times 10^8\, m/s}$, ionisation energy of hydrogen = **13.6 eV**)

(a) Electron will be knocked out of the atom
(b) Electron will go to any excited state of the atom
(c) Electron will go only to first excited state of the atom
(d) Electron will keep orbiting in the ground state of the atom

21. Particle **A** moves along **X**-axis with a uniform velocity of magnitude 10 m/s. Particle **B** moves with uniform velocity 20 m/s along a direction making an angle of **60°** with the positive direction of **X**-axis as shown in the figure. The relative velocity of **B** with respect to that of **A** is

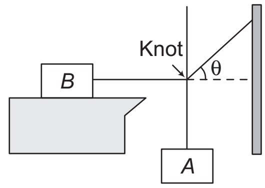


(a) 10 m/s along *X*-axis
(b) $10\sqrt{3}$ m/s along *Y*-axis (perpendicular to *X*-axis)
(c) $10\sqrt{5}$ m/s along the bisection of the velocities of *A* and *B*
(d) 30 m/s along negative *X*-axis

22. When light is refracted from a surface, which of its following physical parameters does not change?
(a) Velocity
(b) Amplitude
(c) Frequency
(d) Wavelength

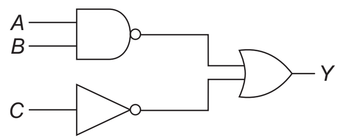
23. A solid maintained at t°_1 **C** is kept in an evacuated chamber at temperature t°_2 **C** ($t_2 >> t_1$). The rate of heat absorbed by the body is proportional to
(a) $t_2^4 - t_1^4$
(b) $(t_2^4 + 273) - (t_1^4 + 273)$
(c) $t_2 - t_1$
(d) $t_2^2 - t_1^2$

24. Block **B** lying on a table weighs **W**. The coefficient of static friction between the block and the table is μ. Assume that the cord between **B** and the knot is horizontal. The maximum weight of the block **A** for which the system will be stationary is



(a) $\frac{W \tan\theta}{\mu}$ (b) $\mu W \tan\theta$
(c) $\mu W\sqrt{1+\tan^2\theta}$ (d) $\mu W \sin\theta$

25. The inputs to the digital circuit are as shown below. The output **Y** is



(a) $A + B + \overline{C}$
(b) $(A + B)\overline{C}$
(c) $\overline{A} + \overline{B} + \overline{C}$
(d) $\overline{A} + \overline{B} + C$

26. Two particles **A** and **B** having different masses are projected from a tower with same speed. **A** is projected vertically upward and **B** vertically downward. On reaching the ground
(a) velocity of *A* is greater than that of *B*
(b) velocity of *B* is greater than that of *A*
(c) both *A* and *B* attain the same velocity
(d) the particle with the larger mass attains higher velocity

27. The work function of metals is in the range of **2 eV** to **5 eV**. Find which of the following wavelength of light cannot be used for photoelectric effect? (Consider, Plank constant $= \mathbf{4 \times 10^{-15} eVs}$, velocity of light $= \mathbf{3 \times 10^8 m/s}$)
(a) 510 nm (b) 650 nm
(c) 400 nm (d) 570 nm

28. A thin plastic sheet of refractive index 1.6 is used to cover one of the slits of a double slit arrangement. The central point on the screen is now occupied by what would have been the 7th bright fringe before the plastic was used. If the wavelength of light is 600 nm, what is the thickness (in **μm**) of the plastic sheet?
(a) 7 (b) 4
(c) 8 (d) 6

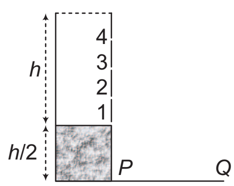
29. The length of an open organ pipe is twice the length of another closed organ pipe. The fundamental frequency of the open pipe is 100 Hz. The frequency of the third harmonic of the closed pipe is

(a) 100 Hz (b) 200 Hz
(c) 300 Hz (d) 150 Hz

30. A **5 μF** capacitor is connected in series with a **10 μF** capacitor. When a 300 V potential difference is applied across this combination, the total energy stored in the capacitors is

(a) 15 J (b) 1.5 J
(c) 0.15 J (d) 0.10 J

31. A cylinder of height **h** is filled with water and is kept on a block of height **h/2**. The level of water in the cylinder is kept constant. Four holes numbered 1, 2, 3 and 4 are at the side of the cylinder and at heights **0, h/4, h/2** and **3h/4**, respectively. When all four holes are opened together, the hole from which water will reach farthest distance on the plane **PQ** is the hole number.



(a) 1 (b) 2
(c) 3 (d) 4

32. The pressure **p**, volume **V** and temperature **T** for a certain gas are related by $\mathbf{p} = \dfrac{\mathbf{AT - BT^2}}{\mathbf{V}}$, where **A** and **B** are constants. The work done by the gas when the temperature changes from $\mathbf{T_1}$ to $\mathbf{T_2}$ while the pressure remains constant, is given by

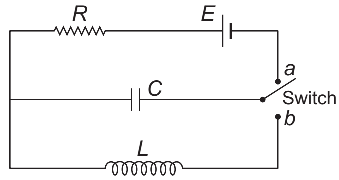
(a) $A(T_2 - T_1) + B(T_2^2 - T_1^2)$

(b) $\dfrac{A(T_2 - T_1)}{V_2 - V_1} - \dfrac{B(T_2^2 - T_1^2)}{V_2 - V_1}$

(c) $A(T_2 - T_1) - \dfrac{B}{2}(T_2^2 - T_1^2)$

(d) $\dfrac{A(T_2 - T_1^2)}{V_2 - V_1}$

33. In the circuit shown below, the switch is kept in position **a** for a long time and is then thrown to position **b**. The amplitude of the resulting oscillating current is given by

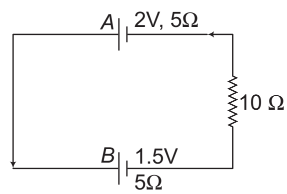


(a) $E\sqrt{L/C}$ (b) E/R (c) infinity (d) $E\sqrt{C/L}$

34. A charge **q** is placed at one corner of a cube. The electric flux through any of the three faces adjacent to the charge is zero. The flux through any one of the other three faces is

(a) $q/3\epsilon_0$ (b) $q/6\epsilon_0$ (c) $q/12\epsilon_0$ (d) $q/24\epsilon_0$

35. Two cells **A** and **B** of emf **2V** and **1.5 V** respectively, are connected as shown in figure through an external resistance **10 Ω**. The internal resistance of each cell is **5Ω**. The potential difference $\mathbf{E_A}$ and $\mathbf{E_B}$ across the terminals of the cells **A** and **B** respectively are



(a) $E_A = 2.0$ V, $E_B = 1.5$ V
(b) $E_A = 2.125$ V, $E_B = 1.375$ V
(c) $E_A = 1.875$ V, $E_B = 1.625$ V
(d) $E_A = 1.875$ V, $E_B = 1.375$ V

36. Two charges **+q** and **−q** are placed at a distance **a** in a uniform electric field. The dipole moment of the combination is $\mathbf{2qa(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})}$, where, θ is the angle between the direction of the field and the line joining the two charges.

Which of the following statement(s) is/are correct?

(a) The torque exerted by the field on the dipole vanishes
(b) The net force on the dipole vanishes
(c) The torque is independent of the choice of coordinates
(d) The net force is independent of *a*

37. Find the right condition(s) for Fraunhoffer diffraction due to a single slit.
(a) Source is at infinite distance and the incident beam has converged at the slit
(b) Source is near to the slit and the incident beam is parallel
(c) Source is at infinity and the incident beam is parallel
(d) Source is near to the slit and the incident beam has converged at the slit

38. The conducting loop in the form of a circle is placed in a uniform magnetic field with its plane perpendicular to the direction of the field. An emf will be induced in the loop, if
(a) it is translated parallel to itself
(b) it is rotated about one of its diameters
(c) it is rotated about its own axis which is parallel to the field
(d) the loop is deformed from the original shape

39. A circular disc rolls on a horizontal floor without slipping and the centre of the disc moves with a uniform velocity **v**. Which of the following values of the velocity at a point on the rim of the disc can have?
(a) v
(b) $-v$
(c) $3v$
(d) Zero

40. Consider two particles of different masses. In which of the following situations the heavier of the two particles will have smaller de-Broglie wavelength?
(a) Both have a free fall through the same height
(b) Both move with the same kinetic energy
(c) Both move with the same linear momentum
(d) Both move with the same speed

Chemistry

41. Match the flame colours of the alkaline earth metal salts in the Bunsen burner.

A. Calcium	p. Brick red
B. Strontium	q. Apple green
C. Barium	r. Crimson

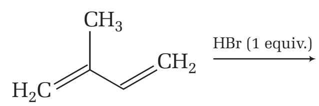
	A	B	C
(a)	p	r	q
(b)	r	p	q
(c)	q	r	p
(d)	p	q	r

42. Extraction of gold (Au) involves the formation of complex ions **X** and **Y**.

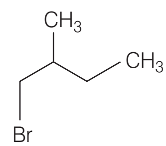
$$\text{Gold ore} \xrightarrow[\text{CN}^-,\ \text{H}_2\text{O},\ \text{O}_2]{\text{Roasting}} \text{HO}^- + \text{X} \xrightarrow{\text{Zn}} \text{Y} + \text{Au}$$

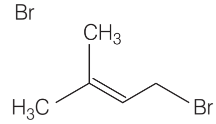
X and **Y** respectively are
(a) $Au(CN)_2^-$ and $Zn(CN)_4^{2-}$
(b) $Au(CN)_4^{3-}$ and $Zn(CN)_4^{2-}$
(c) $Au(CN)_3^-$ and $Zn(CN)_6^{4-}$
(d) $Au(CN)_4^-$ and $Zn(CN)_3^-$

43. The atomic number of cerium (Ce) is 58. The correct electronic configuration of Ce^{3+} ion is
(a) [Xe] $4f^1$
(b) [Kr] $4f^1$
(c) [Xe] $4f^{13}$
(d) [Kr] $4d^1$

44.


The major product of the above reaction is

(a)


(b)


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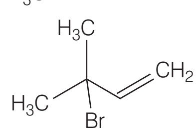

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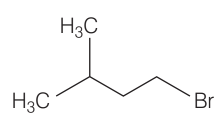

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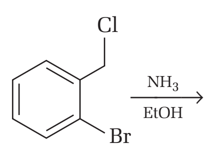
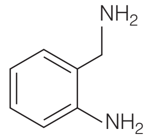
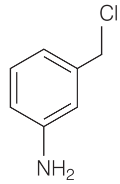
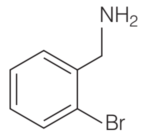
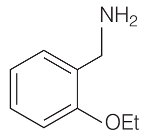


The product of the above reaction is

(a) 

(b) 

(c) 

(d) 

46. Sulphuryl chloride (SO_2Cl_2) reacts with white phosphorus (P_4) to give

(a) PCl_5, SO_2 (b) $OPCl_3$, $SOCl_2$
(c) PCl_5, SO_2, S_2Cl_2 (d) $OPCl_3$, SO_2, S_2Cl_2

47. The number of lone pair of electrons on the central atoms of H_2O, $SnCl_2$, PCl_3 an XeF_2 respectively, are

(a) 2, 1, 1, 3 (b) 2, 2, 1, 3
(c) 3, 1, 1, 2 (d) 2, 1, 2, 3

48. Consider the following salts : NaCl, $HgCl_2$, Hg_2Cl_2, $CuCl_2$, CuCl and AgCl. Identify the correct set of insoluble salts in water.

(a) Hg_2Cl_2, CuCl, AgCl (b) $HgCl_2$, CuCl, AgCl
(c) Hg_2Cl_2, $CuCl_2$, AgCl (d) Hg_2Cl_2, CuCl, NaCl

49. In the following compound, the number of **sp**-hybridised carbon is

$$CH_2 = C = CH - \underset{\displaystyle CN}{\underset{|}{CH}} - C \equiv CH$$

(a) 2 (b) 3 (c) 4 (d) 5

50. For the reaction, $A + 2B \longrightarrow C$; the reaction rate is doubled, if the concentration of **A** is doubled. The rate is increased by four times when concentrations of both **A** and **B** are increased by four times. The order of the reaction is

(a) 3 (b) 0 (c) 1 (d) 2

51. At a certain temperature, the value of the slope of the plot of osmotic pressure (π) against concentration (C in mol L^{-1}) of a certain polymer solution is 291R. The temperature at which osmotic pressure is measured is (**R** is gas constant)

(a) 271°C (b) 18°C (c) 564 K (d) 18 K

52. The rms velocity of CO gas molecules at 27°C is approximately 1000 m/s. For N_2 molecules at 600 K, the rms velocity is approximately

(a) 2000 m/s (b) 1414 m/s
(c) 1000 m/s (d) 1500 m/s

53. A gas can be liquefied at temperature **T** and pressure p provided

(a) $T = T_c$ and $p < p_c$ (b) $T < T_c$ and $p > p_c$
(c) $T > T_c$ and $p > p_c$ (d) $T > T_c$ and $p < p_c$

54. The dispersed phase and dispersion medium of fog respectively are

(a) solid, liquid (b) liquid, liquid
(c) liquid, gas (d) gas, liquid

55. The decreasing order of basic character of K_2O, BaO, CaO and MgO is

(a) K_2O > BaO > CaO > MgO
(b) K_2O > CaO > BaO > MgO
(c) MgO > BaO > CaO > K_2O
(d) MgO > CaO > BaO > K_2O

56. In aqueous alkaline solution, two electrons reduction of HO_2^- gives

(a) HO^- (b) H_2O
(c) O_2 (d) O_2^-

57. Cold ferrous sulphate solution on absorption of NO develops brown colour due to the formation of

(a) paramagnetic $[Fe(H_2O)_5(NO)]\ SO_4$
(b) diamagnetic $[Fe(H_2O)_5(N_3)]\ SO_4$
(c) paramagnetic $[Fe(H_2O)_5(NO_3)][SO_4]_2$
(d) diamagnetic $[Fe(H_2O)_4(SO_4)]NO_3$

58. Amongst Be, B, Mg and Al the second ionisation potential is maximum for

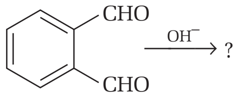
(a) B (b) Be
(c) Mg (d) Al

59. In a mixture, two enantiomers are found to be present in 85% and 15% respectively. The enantiomeric excess **(ee)** is

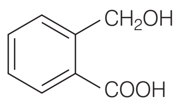
(a) 85% (b) 15%
(c) 70% (d) 60%

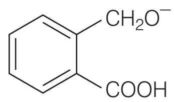
60. 1,4-dimethylbenzene on heating with anhydrous $AlCl_3$ and HCl produces

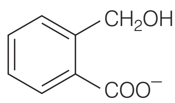
(a) 1, 2-dimethylbenzene
(b) 1, 3-dimethylbenzene
(c) 1, 2, 3-trimethylbenzene
(d) ethylbenzene

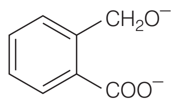
61. 

The product of the above reaction is

(a) 

(b) 

(c) 

(d) 

62. Suppose the mass of a single Ag atom is '**m**'. Ag metal crystallises in fcc lattice with unit cell of length '**a**'. The density of Ag metal in terms of '**a**' and '**m**' is

(a) $\frac{4m}{a^3}$ (b) $\frac{2m}{a^3}$ (c) $\frac{m}{a^3}$ (d) $\frac{m}{4a^3}$

63. For the reactions,
$\mathbf{2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)}$ at 300 K, the value of $\Delta G°$ is $-\mathbf{690.9\,R}$. The equilibrium constant value for the reaction at that temperature is (**R** is gas constant)

(a) 10 atm^{-1} (b) 10 atm
(c) 10 (d) 1

64. At a particular temperature, the ratio of equivalent conductance to specific conductance of a 0.01 N NaCl solution is

(a) $10^5 cm^3$ (b) $10^3 cm^3$
(c) 10 cm^3 (d) $10^5 cm^2$

65. The units of surface tension and viscosity of liquids respectively are

(a) kg $m^{-1}s^{-1}$, Nm^{-1} (b) kg s^{-2}, kg $m^{-1}s^{-1}$
(c) Nm^{-1}, kg $m^{-1}s^{-2}$ (d) kg s^{-1}, kg m^{-2} s^{-1}

66. The ratio of volumes of $\mathbf{CH_3COOH}$ 0.1 (N) to $\mathbf{CH_3COONa}$ 0.1 (N) required to prepare a buffer solution of pH 5.74 is
(given $\mathbf{pK_a}$ of $\mathbf{CH_3COOH}$ is 4.74)

(a) 10 : 1 (b) 5 : 1 (c) 1 : 5 (d) 1 : 10

67. The reaction of methyltrichloroacetate $\mathbf{(Cl_3CCO_2Me)}$ with sodium methoxide (NaOMe) generates

(a) carbocation (b) carbene
(c) carbanion (d) carbon radical

68. Best reagent for nuclear iodination of aromatic compounds is

(a) KI / CH_3COCH_3 (b) I_2 / CH_3CN
(c) KI / CH_3COOH (d) I_2 / HNO_3

69. In the Lassaigne's test for the detection of nitrogen in an organic compound, the appearance of blue coloured compound is due to

(a) ferric ferricyanide
(b) ferrous ferricyanide
(c) ferric ferrocyanide
(d) ferrous ferrocyanide

70. In the following reaction,

$$\mathbf{R\,MgBr + HC(OEt)_3 \xrightarrow{Ether,\ H_3O^+} P}$$

The product **P** is

(a) *R*CHO
(b) R_2CHOEt
(c) R_3CH
(d) *R*CH$(OEt)_2$

71. Addition of sodium thiosulphate solution to a solution of silver nitrate gives '**X**' as white precipitate, insoluble in water but soluble in excess thiosulphate solution to give '**Y**'. On boiling in water, '**Y**' gives '**Z**'. '**X**', '**Y**' and '**Z**' are respectively

(a) $Ag_2S_2O_3$, $Na_3[Ag(S_2O_3)_2]$, Ag_2S
(b) Ag_2SO_4, $Na[Ag(S_2O_3)_2]$, Ag_2S_2
(c) $Ag_2S_2O_3$, $Na_5[Ag(S_2O_3)_3]$, AgS
(d) Ag_2SO_3, $Na_3[Ag(S_2O_3)_2]$, Ag_2O

72. At temperature of 298 K, the emf of the following electrochemical cell

$\mathbf{Ag(s)|Ag^+(0.1\,M)||Zn^{2+}(0.1\,M)|Zn(s)}$

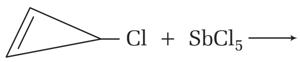
will be (given $\mathbf{E°_{cell} = -1.562\,V}$)

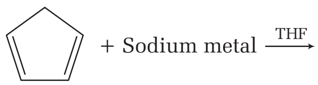
(a) −1.532 V (b) −1.503 V
(c) 1.532 V (d) − 3.06 V

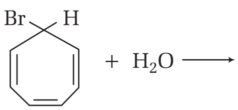
73. For the reaction, $\mathbf{X_2Y_4(l) \longrightarrow 2XY_2(g)}$ at 300 K, the values of Δ**U** and Δ**S** are 2 kcal and 20 cal $\mathbf{K^{-1}}$ respectively. The value of Δ**G** for the reaction is

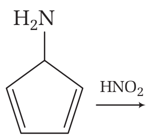
(a) − 3400 cal
(b) 3400 cal
(c) −2800 cal
(d) 2000 cal

74. The total number of aromatic species generated in the following reaction is

(i) 

(ii) 

(iii) 

(iv) 

(a) zero (b) 2
(c) 3 (d) 4

75. Roasted copper pyrite on smelting with sand produces

(a) $FeSiO_3$ as fusible slag and Cu_2S as matte
(b) $CaSiO_3$ as infusible slag and Cu_2O as matte
(c) $Ca_3(PO_4)_2$ as fusible slag and Cu_2S as matte
(d) $Fe_3(PO_4)_2$ as infusible slag and Cu_2S as matte

76. Ionisation potential values of noble gases decrease down the group with increase in atomic size. Xenon forms binary fluorides by the direct reaction of elements. Identify the correct statement(s) from below.

(a) Only the heavier noble gases form such compounds
(b) It happens because the noble gases have higher ionisation energies
(c) It happens because the compounds are formed with electronegative ligands
(d) Octet of electrons provide the stable arrangements

77. Optical isomerism is exhibited by
(**ox** = oxalate anion; **en** = ethylenediamine).

(a) *cis*-$[CrCl_2(ox)_2]^{3-}$
(b) $[Co(en)_3]^{3+}$
(c) *trans*-$[CrCl_2(ox)_2]^{3-}$
(d) $[Co(ox)(en)_2]^{+}$

78. The increase in rate constant of a chemical reaction with increasing temperature is (are) due to the fact(s) that

(a) the number of collisions among the reactant molecules increases with increasing temperature
(b) the activation energy of the reaction decreases with increasing temperature
(c) the concentration of the reactant molecules increases with increasing temperature
(d) the number of reactant molecules acquiring the activation energy increases with increasing temperature

79. Within the list shown below, the correct pair of structures of alanine in pH ranges 2-4 and 9-11 is

I. $\mathbf{H_3N^+—CH(CH_3)CO_2H}$

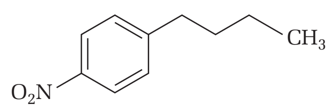
II. $\mathbf{H_2N—CH(CH_3)CO_2^-}$

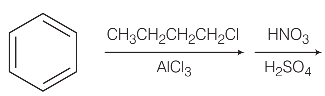
III. $\mathbf{H_3N^+—CH(CH_3)CO_2^-}$

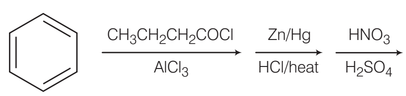
IV. $\mathbf{H_2N—CH(CH_3)CO_2H}$

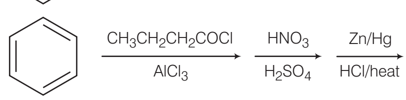
(a) I and II
(b) I and III
(c) II and III
(c) III and IV

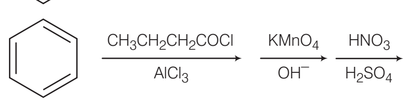
80. Identify the correct method for the synthesis of the compound shown below from the following alternatives.



(a) 

(b) 

(c) 

(d) 

Mathematics

81. Let $\mathbf{a, b, c}$ and $\mathbf{d}$ be any four real numbers. Then, $\mathbf{a^n + b^n = c^n + d^n}$ holds for any natural number $\mathbf{n}$, if

(a) $a + b = c + d$
(b) $a - b = c - d$
(c) $a + b = c + d, a^2 + b^2 = c^2 + d^2$
(d) $a - b = c - d, a^2 - b^2 = c^2 - d^2$

82. If α and β are the roots of $\mathbf{x^2 - px + 1 = 0}$ and γ is a root of $\mathbf{x^2 + px + 1 = 0}$, then $(\alpha + \gamma)(\beta + \gamma)$ is

(a) 0 (b) 1 (c) –1 (d) p

83. The number of irrational terms in the binomial expansion of $\mathbf{(3^{1/5} + 7^{1/3})^{100}}$ is

(a) 90 (b) 88 (c) 93 (d) 95

84. The quadratic expression $\mathbf{(2x + 1)^2 - px + q \neq 0}$ for any real $\mathbf{x}$, if

(a) $p^2 - 16p - 8q < 0$ (b) $p^2 - 8p + 16q < 0$
(c) $p^2 - 8p - 16q < 0$ (d) $p^2 - 16p + 8q < 0$

85. In a certain town, 60% of the families own a car, 30% own a house and 20% own both car and house. If a family is randomly chosen, then what is the probability that this family owns a car or a house but not both?

(a) 0.5 (b) 0.7 (c) 0.1 (d) 0.9

86. The letters of the word 'COCHIN' are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word 'COCHIN', is

(a) 360 (b) 192 (c) 96 (d) 48

87. Let $\mathbf{f : R \to R}$ be a continuous function which satisfies $\mathbf{f(x) = \int_0^x f(t)\,dt}$. Then, the value of $\mathbf{f(\log_e 5)}$ is

(a) 0 (b) 2 (c) 5 (d) 3

88. Let $\mathbf{f : R \to R}$ be defined as $\mathbf{f(x) = \dfrac{x^2 - x + 4}{x^2 + x + 4}}$.

Then, range of the function $\mathbf{f(x)}$ is

(a) $\left[\dfrac{3}{5}, \dfrac{5}{3}\right]$ (b) $\left(\dfrac{3}{5}, \dfrac{5}{3}\right)$
(c) $\left(-\infty, \dfrac{3}{5}\right) \cup \left(\dfrac{5}{3}, \infty\right)$ (d) $\left[-\dfrac{5}{3}, -\dfrac{3}{5}\right]$

89. The least value of $\mathbf{2x^2 + y^2 + 2xy + 2x - 3y + 8}$ for real numbers $\mathbf{x}$ and $\mathbf{y}$, is

(a) 2 (b) 8 (c) 3 (d) $-1/2$

90. Let $\mathbf{f : [-2, 2] \to R}$ be a continuous function such that $\mathbf{f(x)}$ assumes only irrational values. If $\mathbf{f(\sqrt{2}) = \sqrt{2}}$, then

(a) $f(0) = 0$ (b) $f(\sqrt{2} - 1) = \sqrt{2} - 1$
(c) $f(\sqrt{2} - 1) = \sqrt{2} + 1$ (d) $f(\sqrt{2} - 1) = \sqrt{2}$

91. The minimum value of $\mathbf{\cos\theta + \sin\theta + \dfrac{2}{\sin 2\theta}}$ for $\theta \in \mathbf{(0, \pi/2)}$, is

(a) $2 + \sqrt{2}$ (b) 2 (c) $1 + \sqrt{2}$ (d) $2\sqrt{2}$

92. The value of $\mathbf{\lim_{x \to 2} \int_2^x \dfrac{3t^2}{(x - 2)}\,dt}$ is

(a) 10 (b) 12 (c) 8 (d) 16

93. If $\mathbf{\cot\dfrac{2x}{3} + \tan\dfrac{x}{3} = \text{cosec}\,\dfrac{kx}{3}}$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) – 1

94. If $\theta \in \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$, then the value of

$\sqrt{4\cos^4\theta + \sin^2 2\theta} + 4\cot\theta\cos^2\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)$ is

(a) $-2\cot\theta$ (b) $2\cot\theta$
(c) $2\cos\theta$ (d) $2\sin\theta$

95. The number of real solutions of the equation $\mathbf{(\sin x - x)(\cos x - x^2) = 0}$ is

(a) 1 (b) 2 (c) 3 (d) 4

96. The value of $\left(\dfrac{1 + \sqrt{3}i}{1 - \sqrt{3}i}\right)^{64} + \left(\dfrac{1 - \sqrt{3}i}{1 + \sqrt{3}i}\right)^{64}$ is

(a) 0 (b) –1
(c) 1 (d) i

97. Find the maximum value of $\mathbf{|z|}$ when $\left|\mathbf{z - \dfrac{3}{z}}\right| = \mathbf{2}$, where z being a complex number.

(a) $1 + \sqrt{3}$ (b) 3
(c) $1 + \sqrt{2}$ (d) 1

98. Given that, x is a real number satisfying $\frac{5x^2 - 26x + 5}{3x^2 - 10x + 3} < 0$, then

(a) $x < \frac{1}{5}$ (b) $\frac{1}{5} < x < 3$

(c) $x > 5$ (d) $\frac{1}{5} < x < \frac{1}{3}$ or $3 < x < 5$

99. The value of λ such that the following system of equations has no solution, is
$2x - y - 2z = 2$; $x - 2y + z = -4$;
$x + y + \lambda z = 4$

(a) 3 (b) 1 (c) 0 (d) –3

100. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$.

Then, $f(100)$ is equal to

(a) 0 (b) 1 (c) 100 (d) 10

101. Let $x_n = \left(1 - \frac{1}{3}\right)^2 \left(1 - \frac{1}{6}\right)^2 \left(1 - \frac{1}{10}\right)^2 \dots \left(1 - \frac{1}{\frac{n(n+1)}{2}}\right)^2$, $n \geq 2$. Then, the value of $\lim_{n \to \infty} x_n$ is

(a) 1/3 (b) 1/9 (c) 1/81 (d) 0

102. The variance of first 20 natural numbers is

(a) 133/4 (b) 279/12

(c) 133/2 (d) 399/4

103. A fair coin is tossed a fixed number of times. If the probability of getting exactly 3 heads equals the probability of getting exactly 5 heads, then the probability of getting exactly one head is

(a) 1/64 (b) 1/32

(c) 1/16 (d) 1/8

104. If the letters of the word 'PROBABILITY' are written down at random in a row, then probability that two B's are together, is

(a) $\frac{2}{11}$ (b) $\frac{10}{11}$ (c) $\frac{3}{11}$ (d) $\frac{6}{11}$

105. Which of the following is not always true?

(a) $|a + b|^2 = |a|^2 + |b|^2$, if **a** and **b** are perpendicular to each other

(b) $|a + \lambda b| \geq |a|$ for all $\lambda \in R$, if **a** and **b** are perpendicular to each other

(c) $|a + b|^2 + |a - b|^2 = 2(|a|^2 + |b|^2)$

(d) $|a + \lambda b| \geq |a|$ for all $\lambda \in R$, if a is parallel to **b**

106. If the four points with position vectors $-2i + j + k$, $i + j + k$, $j - k$ and $\lambda j + k$ are coplanar, then λ is equal to

(a) 1 (b) 2 (c) –1 (d) 0

107. The least positive value of t, so that the lines $x = t + \alpha$, $y + 16 = 0$ and $y = \alpha x$ are concurrent, is

(a) 2 (b) 4 (c) 16 (d) 8

108. If in a ΔABC, $a^2 \cos^2 A - b^2 - c^2 = 0$, then

(a) $\frac{\pi}{4} < A < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < A < \pi$

(c) $A = \frac{\pi}{2}$ (d) $A < \frac{\pi}{4}$

109. $\{x \in R : |\cos x| \geq \sin x\} \cap \left[0, \frac{3\pi}{2}\right]$ is equal to

(a) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{3\pi}{2}\right]$ (b) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

(c) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$ (d) $\left[0, \frac{3\pi}{2}\right]$

110. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \dots\right) = \frac{\pi}{6}$, where $|x| < 2$, then the value of x is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$

111. The area of the region bounded by the curve $y = x^3$, its tangent at (1,1) and X-axis is

(a) $\frac{1}{12}$ sq unit (b) $\frac{1}{6}$ sq unit

(c) $\frac{2}{17}$ sq unit (d) $\frac{2}{15}$ sq unit

112. If $\log_{0.2}(x - 1) > \log_{0.04}(x + 5)$, then

(a) $-1 < x < 4$ (b) $2 < x < 3$

(c) $1 < x < 4$ (d) $1 < x < 3$

113. The number of real roots of equation $\log_e x + ex = 0$ is

(a) 0 (b) 1

(c) 2 (d) 3

114. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) > 2

115. Let $x_1, x_2, \ldots, x_{15}$ be 15 distinct numbers chosen from 1,2,3, …,15.
Then, the value of $(x_1 - 1)(x_2 - 1)(x_3 - 1) \ldots (x_{15} - 1)$ is

(a) always ≤ 0 (b) 0
(c) always even (d) always odd

116. Let $[x]$ denotes the greatest integer less than or equal to x. Then, the value of α for which the function $f(x) = \begin{cases} \frac{\sin[-x^2]}{[-x^2]}, & x \ne 0 \\ \alpha, & x = 0 \end{cases}$ is continuous at $x = 0$, is

(a) $\alpha = 0$ (b) $\alpha = \sin(-1)$
(c) $\alpha = \sin(1)$ (d) $\alpha = 1$

117. For all real values of a_0, a_1, a_2, a_3 satisfying $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$, the equation $a_0 + a_1x + a_2x^2 + a_3x^3 = 0$ has a real root in the interval

(a) [0, 1] (b) [–1, 0] (c) [1, 2] (d) [–2, – 1]

118. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$$

Then, which of the following is true?

(a) *f* is discontinuous for all *x*
(b) *f* is continuous for all *x*
(c) *f* is discontinuous at $x = k\pi$, where *k* is an integer
(d) *f* is continuous at $x = k\pi$, where *k* is an integer

119. A particle starts moving from rest from a fixed point in a fixed direction. The distance s from the fixed point at a time t is given by $s = t^2 + at - b + 17$, where *a* and *b* are real numbers. If the particle comes to rest after 5 s at a distance of $s = 25$ units from the fixed point, then values of **a** and **b** are, respectively

(a) 10, – 33 (b) –10, – 33

(c) –8, 33 (d) –10, 33

120. $\lim_{n \to \infty} \frac{\sqrt{1} + \sqrt{2} + \ldots + \sqrt{n-1}}{n\sqrt{n}}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 0

121. If $\lim_{x \to 0} \frac{axe^x - b\log(1+x)}{x^2} = 3$, then the values of *a* and *b* are, respectively

(a) 2, 2 (b) 1, 2 (c) 2, 1 (d) 2, 0

122. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines $x + y = 3$ and $2x + 2y - 1 = 0$, then

(a) $2 < a < 4$ (b) $-\frac{1}{2} < a < 2$
(c) $0 < a < 2$ (d) $-\frac{1}{2} < a < \frac{3}{2}$

123. Number of intersecting points of the conics $4x^2 + 9y^2 = 1$ and $4x^2 + y^2 = 4$ is

(a) 1 (b) 2
(c) 3 (d) 0

124. The value of λ for which the straight line $\frac{x - \lambda}{3} = \frac{y - 1}{2 + \lambda} = \frac{z - 3}{-1}$ may lie on the plane $x - 2y = 0$, is

(a) 2 (b) 0
(c) $-\frac{1}{2}$ (d) there is no such λ

125. Area of the region bounded by $y = |x|$ and $y = -|x| + 2$ is

(a) 4 sq units
(b) 3 sq units
(c) 2 sq units
(d) 1 sq unit

126. Let $d(n)$ denotes the number of divisors of n including 1 and itself. Then, $d(225)$, $d(1125)$ and $d(640)$ are

(a) in AP
(b) in HP
(c) in GP
(d) consecutive integers

127. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} 2a$ has a real solution, if

(a) $|a| > \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}} < |a| < \frac{1}{\sqrt{2}}$
(c) $|a| > \frac{1}{2\sqrt{2}}$ (d) $|a| \le \frac{1}{2\sqrt{2}}$

128. If $(2+i)$ and $(\sqrt{5}-2i)$ are the roots of the equation $(x^2+ax+b)(x^2+cx+d)=0$, where a, b, c and d are real constants, then product of all the roots of the equation is

(a) 40 (b) $9\sqrt{5}$ (c) 45 (d) 35

129. If $f:[0, \pi/2) \to R$ is defined as

$$f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}.$$ Then, the range of f is

(a) $(2, \infty)$ (b) $(-\infty, -2]$ (c) $[2, \infty)$ (d) $(-\infty, 2]$

130. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ equals

(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

131. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$

(a) -2ω (b) $-3\omega^2$ (c) -1 (d) 0

132. Let $f(x)$ denotes the fractional part of a real number x. Then, the value of $\int_0^{\sqrt{3}} f(x^2)dx$ is

(a) $2\sqrt{3}-\sqrt{2}-1$ (b) 0
(c) $\sqrt{2}-\sqrt{3}+1$ (d) $\sqrt{3}-\sqrt{2}+1$

133. Let $S = \{(a, b, c) \in N \times N \times N : a + b + c = 21, a \le b \le c\}$ and $T = \{(a, b, c) \in N \times N \times N : a, b, c,$ are in AP$\}$, where N is the set of all natural numbers. Then, the number of elements in the set $S \cap T$ is

(a) 6 (b) 7 (c) 13 (d) 14

134. Let $y = e^{x^2}$ and $y = e^{x^2}\sin x$ be two given curves. Then, angle between the tangents to the curves at any point of their intersection is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

135. The value of $2\cot^{-1}\frac{1}{2} - \cot^{-1}\frac{4}{3}$ is

(a) $-\frac{\pi}{8}$ (b) $\frac{3\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

136. If the point $(2\cos\theta, 2\sin\theta)$ for $0 \in (0, 2\pi)$ lies in the region between the lines $x + y = 2$ and $x - y = 2$ containing the origin, then θ lies in

(a) $\left(\theta, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ (b) $[0, \pi]$
(c) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (d) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

137. Number of points having distance $\sqrt{5}$ from the straight line $x - 2y + 1 = 0$ and a distance $\sqrt{13}$ from the line $2x + 3y - 1 = 0$ is

(a) 1 (b) 2
(c) 4 (d) 5

138. Let $P(x)$ be a polynomial, which when divided by $(x-3)$ and $(x-5)$ leaves remainders 10 and 6, respectively. If the polynomial is divided by $(x-3)(x-5)$, then the remainder is

(a) $-2x+16$ (b) 16
(c) $2x-16$ (d) 60

139. The integrating factor of the differential equation $\frac{dy}{dx} + (3x^2\tan^{-1}y - x^3)(1+y^2) = 0$ is

(a) e^{x^2} (b) e^{x^3}
(c) e^{3x^2} (d) e^{3x^3}

140. If $y = e^{-x}\cos 2x$, then which of the following differential equation is satisfied?

(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$ (b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 2y = 0$
(c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 2y = 0$ (d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$

141. Let $f: R \to R$ be differentiable at $x = 0$. If $f(0) = 0$ and $f'(0) = 2$, then the value of $\lim_{x \to 0} \frac{1}{x}[f(x) + f(2x) + f(3x) + \ldots + f(2015x)]$ is

(a) 2015
(b) 0
(c) 2015×2016
(d) 2015×2014

142. If x and y are digits such that $17! = 3556xy428096000$, then $x + y$ equals

(a) 15 (b) 6
(c) 12 (d) 13

143. A person goes to office by car or scooter or bus or train, probability of which are 1/7, 3/7, 2/7 and 1/7, respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is 2/9, 1/9, 4/9, and 1/9, respectively. Given that he reached office in time, the probability that he travelled by a car, is

(a) 1/7 (b) 2/7
(c) 3/7 (d) 4/7

144. The value of $\int \frac{(x-2)\,dx}{\{(x-2)^2 (x+3)^7\}^{1/3}}$ is

(a) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3} + C$ (b) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{3/4} + C$
(c) $\frac{5}{12}\left(\frac{x-2}{x+3}\right)^{4/3} + C$ (d) $\frac{3}{20}\left(\frac{x-2}{x+3}\right)^{5/3} + C$

145. Let $\mathbf{f : N \rightarrow R}$ be such that $\mathbf{f(1) = 1}$ and $\mathbf{f(1) + 2f(2) + 3f(3) + \ldots + nf(n) =}$ $\mathbf{n(n+1)\,f(n)}$, for all $\mathbf{n \in N, n \geq 2}$, where **N** is the set of natural numbers and **R** is the set of real numbers. Then, the value of $\mathbf{f(500)}$ is

(a) 1000 (b) 500
(c) 1/500 (d) 1/1000

146. If 5 distinct balls are placed at random into 5 cells, then the probability that exactly one cell remains empty, is

(a) 48/125 (b) 12/125
(c) 8/125 (d) 1/125

147. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006. What is the probability of death due to lung cancer given that a person is a smoker?

(a) 1/140 (b) 1/70
(c) 3/140 (d) 1/10

148. In a $\Delta\mathbf{ABC}$, if $\angle\mathbf{C} = \mathbf{90°}$, **r** and **R** are the inradius and circumradius of the $\Delta\mathbf{ABC}$ respectively, then $\mathbf{2(r + R)}$ is equal to

(a) $b + c$
(b) $c + a$
(c) $a + b$
(d) $a + b + c$

149. Let α and β be two distinct roots of $\mathbf{a\cos\theta + b\sin\theta = c}$, where a, b, c are three real constants and $\theta \in \mathbf{[0, 2\pi]}$. Then, $\alpha + \beta$ is also a root of the same equation, if

(a) $a + b = c$ (b) $b + c = a$
(c) $c + a = b$ (d) $c = a$

150. For a matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$, if $\mathbf{U_1, U_2}$ and $\mathbf{U_3}$ are $\mathbf{3 \times 1}$ column matrices satisfying

$\mathbf{AU_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{AU_2} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, \mathbf{AU_3} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and U is $\mathbf{3 \times 3}$ matrix whose columns are $\mathbf{U_1, U_2}$ and $\mathbf{U_3}$. Then, sum of the elements of $\mathbf{U^{-1}}$ is

(a) 6 (b) 0 (c) 1 (d) 2/3

151. Let **f** be any continuously differentiable function on **[a, b]** and twice differentiable on **(a, b)** such that $\mathbf{f(a) = f'(a) = 0}$ and $\mathbf{f(b) = 0}$. Then,

(a) $f''(a) = 0$
(b) $f'(x) = 0$ for some $x \in (a, b)$
(c) $f''(x) = 0$ for some $x \in (a, b)$
(d) $f'''(x) = 0$ for some $x \in (a, b)$

152. A relation ρ on the set of real number **R** is defined as follows: $\mathbf{x\rho y}$ if and only if $\mathbf{xy > 0}$. Then, which of the following is/are true?

(a) ρ is reflexive and symmetric
(b) ρ is symmetric but not reflexive
(c) ρ is symmetric and transitive
(d) ρ is an equivalence relation

153. If $\mathbf{\cos x}$ and $\mathbf{\sin x}$ are solutions of the differential equation

$$\mathbf{a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0},$$

where $\mathbf{a_0, a_1}$ and $\mathbf{a_2}$ are real constants, then which of the following is/are always true?

(a) $A\cos x + B\sin x$ is a solution, where A and B are real constants
(b) $A\cos\left(x + \frac{\pi}{4}\right)$ is a solution, where A is a real constant
(c) $A\cos x\sin x$ is a solution, where A is a real constant
(d) $A\cos\left(x + \frac{\pi}{4}\right) + B\sin\left(x - \frac{\pi}{4}\right)$ is a solution, where A and B are real constants

154. Which of the following statements is /are correct for $0 < \theta < \dfrac{\pi}{2}$?

(a) $(\cos\theta)^{1/2} \le \cos\dfrac{\theta}{2}$ (b) $(\cos\theta)^{3/4} \ge \cos\dfrac{3\theta}{4}$

(c) $\cos\dfrac{5\theta}{6} \ge (\cos\theta)^{5/6}$ (d) $\cos\dfrac{7\theta}{8} \le (\cos\theta)^{7/8}$

155. Let $16x^2 - 3y^2 - 32x - 12y = 44$ represents a hyperbola. Then,

(a) length of the transverse axis is $2\sqrt{3}$
(b) length of each latusrectum is $32/\sqrt{3}$
(c) eccentricity is $\sqrt{19/3}$
(d) equation of a directrix is $x = \dfrac{\sqrt{19}}{3}$

156. For the function $f(x) = \left[\dfrac{1}{[x]}\right]$, where $[x]$ denotes the greatest integer less than or equal to x, which of the following statements are true?

(a) The domain is $(-\infty, \infty)$
(b) The range is $\{0\} \cup \{-1\} \cup \{1\}$
(c) The domain is $(-\infty, 0) \cup [1, \infty)$
(d) The range is $\{0\} \cup \{1\}$

157. Which of the following is / are always false?

(a) A quadratic equation with rational coefficients has zero or two irrational roots
(b) A quadratic equation with real coefficients has zero or two non-real roots
(c) A quadratic equation with irrational coefficients has zero or two rational roots
(d) A quadratic equation with integer coefficients has zero or two irrational roots

158. If the straight line $(a-1)x - by + 4 = 0$ is normal to the hyperbola $xy = 1$, then which of the following does not hold?

(a) $a > 1, b > 0$
(b) $a > 1, b < 0$
(c) $a < 1, b < 0$
(d) $a < 1, b > 0$

159. Suppose a machine produces metal parts that contain some defective parts with probability 0.05. How many parts should be produced in order that the probability of atleast one part being defective is 1/2 or more? (Given that, $\log_{10} 95 = 1.977$ and $\log_{10} 2 = 0.3$)

(a) 11
(b) 12
(c) 15
(d) 14

160. Let $f: R \to R$ be such that $f(2x-1) = f(x)$ for all $x \in R$. If f is continuous at $x = 1$ and $f(1) = 1$, then

(a) $f(2) = 1$
(b) $f(2) = 2$
(c) f is continuous only at $x = 1$
(d) f is continuous at all points

Answers

Physics

1. (c)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (d)
21. (b)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (c)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (c)
31. (a)	**32.** (c)	**33.** (c)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (c)	**38.** (b)	**39.** (d)	**40.** (d)

Chemistry

41. (a)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (a)	**49.** (c)	**50.** (c)
51. (b)	**52.** (b)	**53.** (b)	**54.** (c)	**55.** (a)	**56.** (a)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (b)
61. (c)	**62.** (a)	**63.** (a)	**64.** (a)	**65.** (b)	**66.** (d)	**67.** (b)	**68.** (d)	**69.** (c)	**70.** (a)
71. (a)	**72.** (a)	**73.** (c)	**74.** (c)	**75.** (a)	**76.** (a,c)	**77.** (a,b,d)	**78.** (a,d)	**79.** (a)	**80.** (b)

Mathematics

81. (d)	**82.** (a)	**83.** (*)	**84.** (c)	**85.** (a)	**86.** (c)	**87.** (a)	**88.** (a)	**89.** (d)	**90.** (d)
91. (a)	**92.** (b)	**93.** (b)	**94.** (b)	**95.** (c)	**96.** (b)	**97.** (b)	**98.** (d)	**99.** (d)	**100.** (a)
101. (b)	**102.** (a)	**103.** (b)	**104.** (a)	**105.** (d)	**106.** (a)	**107.** (d)	**108.** (b)	**109.** (a)	**110.** (a)
111. (a)	**112.** (c)	**113.** (b)	**114.** (c)	**115.** (b)	**116.** (c)	**117.** (a)	**118.** (d)	**119.** (b)	**120.** (c)
121. (a)	**122.** (b)	**123.** (d)	**124.** (d)	**125.** (c)	**126.** (c)	**127.** (d)	**128.** (c)	**129.** (c)	**130.** (c)
131. (b)	**132.** (c)	**133.** (d)	**134.** (a)	**135.** (d)	**136.** (c)	**137.** (c)	**138.** (a)	**139.** (b)	**140.** (a)
141. (c)	**142.** (a)	**143.** (a)	**144.** (a)	**145.** (d)	**146.** (a)	**147.** (c)	**148.** (c)	**149.** (d)	**150.** (b)
151. (b,c)	**152.** (b,c)	**153.** (a,b,d)	**154.** (a,c)	**155.** (a,b,c)	**156.** (b, c)	**157.** (c)	**158.** (a, c)	**159.** (c,d)	**160.** (c)

Solutions

Physics

1. The gravitational force acting between the two masses m_1 and m_2 is given by

$$F_G = \frac{Gm_1m_2}{r^2}$$

Force on mass m_1,

$$F_1 = \frac{Gm_1m_2}{r^2} = m_1a_1$$

where, a_1 = acceleration

$$\Rightarrow \quad a_1 = \frac{Gm_2}{r^2}$$

$$\Rightarrow \quad a_1 \propto m_2$$

and similarly, $a_2 \propto m_1$

2. Let the final temperature after the masses in thermal contact is θ, then from the principle of calorimetry.

Heat lost = Heat gained

$$\Rightarrow 3ms\,(60-\theta) = ms\,(\theta-50) + ms\,(\theta-40)$$

$$\Rightarrow \quad 3\,(60-\theta) = \theta - 50 + \theta - 40$$

$$\Rightarrow \quad 180 - 3\theta = 2\theta - 90$$

$$\theta = \frac{270}{5} = 54°\text{C}$$

3. As we know that for the earth and satellite system,

Kinetic energy, $K = \dfrac{GMm}{2a}$

Potential energy, $V = -\dfrac{GMm}{a}$

and total energy, $E = -\dfrac{GMm}{2a}$

where, a = radius of the orbit of the satellite and m = mass of the satellite

On the basis of above three expressions for the energies, we have

$$K = \frac{GMm}{2a} = -\left(-\frac{\frac{GMm}{a}}{2}\right) = -\frac{V}{2}$$

4. From the lens formula (for first lens)

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1} \Rightarrow \frac{1}{2} = \frac{1}{v_1} - \frac{1}{(-4)}$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} = \frac{1}{v_1} = \frac{3}{4} \quad \ldots(i)$$

$$\Rightarrow \quad v_1 = \frac{4}{3},\ u_2 = 3 - \frac{4}{3} = 5/3$$

This image will be treated as the source for second lens, then again from lens formula, we have

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

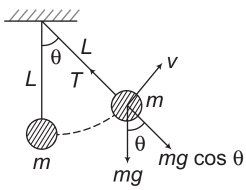
$$\Rightarrow \quad \frac{1}{1} = \frac{1}{v_2} + 5/3 \quad \text{[By equation (i)]}$$

$$\Rightarrow \quad 1 - 5/3 = \frac{1}{v_2} \Rightarrow -3/2$$

This is the final image distance from 2nd lens. So, the overall distance of image from the primary source (or object)

Let $d = 4 + 3 - 1.5 = 5.5$

5. The situation is given below



For motion along a vertical circular track, the required centripetal force is along the radius and towards the centre of the circle is given by

$$T - mg\cos\theta = \frac{mv^2}{L}$$

$$\Rightarrow \quad T = \frac{mv^2}{L} + mg\cos\theta$$

6. Let the initial length of the metal wire is L.

The strain at tension T_1 is $\Delta L_1 = L_1 - L$

The strain at tension T_2 is $\Delta L_2 = L_2 - L$

Suppose, the young's modulus of the wire is Y, then

$$\frac{\frac{T_1}{A}}{\frac{\Delta L_1}{L}} = \frac{\frac{T_2}{A}}{\frac{\Delta L_2}{L}}$$

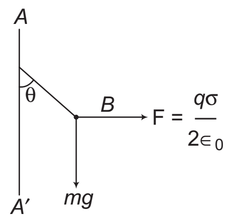
where, A is an cross-section of the wire. assume to be same at all the situations.

$$\Rightarrow \frac{T_1}{A} \times \frac{L}{\Delta L_1} = \frac{T_2}{A} \times \frac{L}{\Delta L_2}$$

$$\Rightarrow \quad \frac{T_1}{(L_1 - L)} = \frac{T_2}{(L_2 - L)}$$

$$T_1(L_2 - L) = T_2(L_1 - L);\ L = \frac{T_2L_1 - T_1L_2}{T_2 - T_1}$$

7. The diagram is as follows



The electric field due to charged infinite conducting sheet is $E = \frac{\sigma}{2\epsilon_0}$.

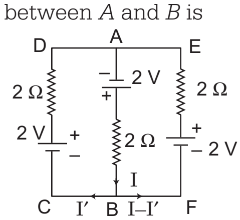
Now, force (electric force) on the charged ball is

$$F = qE = \frac{q\sigma}{2\epsilon_0}$$

The resultant of electric force and mg balance the tension produced in the string.

So, $\tan\theta = \frac{F_e}{mg} = \frac{\frac{q\sigma}{2\epsilon_0}}{mg} = \frac{q\sigma}{2\epsilon_0 mg}$

8. The circuit diagram can be redrawn as The potential between A and B is



For the loop $ABCDA$,

$$+2 - 2I' + 2 - 2I = 0 \quad \ldots(i)$$

For the loop $ABFEA$,

$$2 - 2I + 2 - 2(I - I') = 0$$

$$4 - 2I - 2I + 2I' = 0$$

$$4 - 4I + 2I' = 0$$

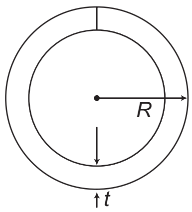
$$2 = 2I - I' \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$2 = 2I - I', 2 = I + I'$$

$$4 = 3I, \ I = 4/3 = 1.33 \text{ A}$$

9. The density of material is ρ



The hollow sphere will float if its weight is less than the weight of the water displaced by the volume of the sphere This implies mass of the sphere is less than that for the same volume of water. Now, mass of spherical cell

$$m_1 = 4\pi R^2 \times t \times \rho$$

While the mass of water having same volume

$$m_2 = \frac{4}{3}\pi R^3 \times \rho_g = \frac{4}{3}\pi R^3$$

where, ρ_g = density of water = $\frac{1 \text{ kg}}{\text{m}^3}$

For the floatation of sphere,

$$m_1 \le m_2$$

$$4\pi R^2 \times t \times \rho \le \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad t\rho \le \frac{R}{3}$$

$$\Rightarrow \quad t \le \frac{R}{3\rho}$$

10. As we know that,

$$R_1 = \rho\frac{l}{a} = \rho\frac{l^2}{V}$$

where, l = length of wire

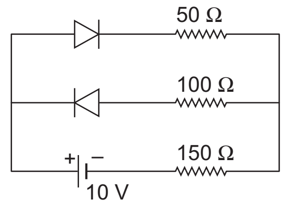
a = area of cross-section of the wire

and V = volume of the wire $R_1 \propto l^2$

$$\Rightarrow \quad \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

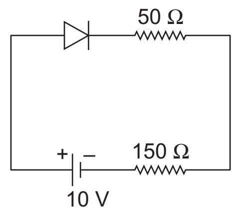
$$\Rightarrow \quad R_2 : R_1 = 4 : 1$$

11. The circuit is



As the lower diode attached to 100 Ω resistance is in reversed biased so, it is non-conducting.

Now, the circuit can be redrawn as,



$\therefore$ Current through the circuit, $I = \frac{V}{R}$

$$= \frac{10}{200} = 0.05 \text{ A}$$

12. As the rms speed is given by

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \quad V_{rms} \propto \sqrt{\frac{T}{M}}$$

When temperature is doubled and molecules dissociates into atoms, then

$$\frac{V_{\text{rms}_1}}{V_{\text{rms}_2}} = \frac{\sqrt{\frac{T}{M}}}{\sqrt{\frac{2T}{M/2}}} = \frac{\sqrt{\frac{T}{M}}}{\sqrt{\frac{4T}{M}}}$$

$$= \frac{\sqrt{T}}{\sqrt{4T}} = \frac{1}{2}$$

If V_{rms_1} is V, then V_{rms_2} will be $2V$

13. Let the masses of two particles are m_1 and m_2.

As the changes are of the same magnitude and being accelerated through same potential, so these charges enter into the magnetic field with the same speeds (let V).

Now, radii of the circular paths followed by two changes is given by

$$R_1 = \frac{m_1 V}{qB}$$

and $$R_2 = \frac{m_2 V}{qB} \Rightarrow \frac{R_1}{R_2} = \frac{m_1}{m_2}$$

14. As large number of particles is situated at a distance R from the origin. If particles are uniformly distributed and make a circular boundary around the origin, then centre of mass will be at the origin.

While if the particles are not uniformly distributed, then centre of mass will lie between particle and origin. This implies the distance between centre of mass and origin is always less than equal to R.

15. Given, length of conductor, $l = 0.1$ m

Mangetic field, $B = 0.1$ T

Velocity of conductor, $V = 15$ m/s

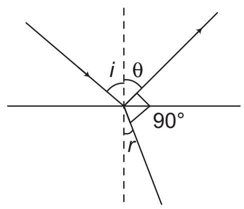
The angle between V and B is 90°

When V and B are mutually perpendicular, then emf (induced) is given by

$$\varepsilon = VBl$$

$$= 15 \times 0.1 \times 0.1 = \frac{15}{100} = 0.15 \text{ V}$$

16. As situation can be diagrammatically as below



From law of reflection,

$$i = \theta$$

Now, $\theta + r + 90° = 180°$

$$\Rightarrow \quad i + r + 90° = 180°$$

$$r = 90° - i$$

Also, from Snell's law

$$\frac{\sin i}{\sin r} = \mu$$

$$\Rightarrow \quad \frac{\sin i}{\sin (90° - i)} = \frac{\sin i}{\cos i} = \mu$$

$$\Rightarrow \quad \tan i = \mu$$

17. Total angular momentum about O is given as,

$$L = L_1 + L_2 = m_1 v_1 r_1 + m_2 v_2 r_2$$

$$= -6.5 \times 2.2 \times 1.5 + 3.1 \times 3.6 \times 2.8$$

$$= -21.45 + 31.248 = 9.8 \text{ kgm}^2/\text{s}$$

18. The height raised by liquid in capillary tube

$$h = \frac{2 l \cos \theta}{\rho g h}$$

As in freely falling platform a body experience weight lessness.

So, the liquid will rise upto to length of the capillary.

i.e. height raised by the liquid will be 20 cm.

19. We have,

Apparent frequency,

$$\nu = \left(\frac{v + u_0}{v - u_s}\right) \nu_0 = \left(\frac{333 + 33}{333 - 33}\right) \times 1000$$

$$= \frac{366}{300} \times 1000 = 1220 \text{ Hz}$$

20. The energy of the photon,

$$E = \frac{hc}{\lambda}$$

$$= \frac{4 \times 10^{-15} \text{eVs} \times 3 \times 10^8 \text{ m/s}}{300 \times 10^{-9} \text{ m}}$$

$$= \frac{4 \times 10^2}{300} \text{ eV}$$

$$= \frac{4}{3} \text{eV} = 1.33 \text{ eV}$$

The ionisation energy is 13.6 eV which is greater than energy of photon, so atom can not come into excited state and will remain in ground state.

21. The component of velocity of B along x-direction

$$V_{Bx} = 20 \cos 60°$$

$$= 20 \times \frac{1}{2} = 10 \text{ m/s}$$

$$V_A = 10\hat{\mathbf{i}}$$

$$V_B = 10\hat{\mathbf{i}} + 10\sqrt{3}\,\hat{\mathbf{j}}$$

$$V_{BA} = V_B - V_A = 10\hat{\mathbf{i}} + 10\sqrt{3}\,\hat{\mathbf{j}} - 10\hat{\mathbf{i}} = 10\sqrt{3}\,\hat{\mathbf{j}}$$

22. When light is refracted its frequency will remain unchanged.

23. As we know that, the rate of cooling is

$$\frac{d\theta}{dt} = bA(\theta - \theta_0)$$

where, bA = constant

$$\Rightarrow \quad \frac{d\theta}{dt} \propto \theta - \theta_0$$

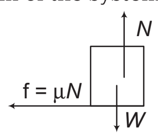
Also $\quad -\frac{dT}{dt} \propto T - T_0$

where, T = temperature

and $\quad t$ = time

So, the rate of heat absorbed by cooled body is proportional to $t_2 - t_1$.

24. Let weight of A is W'. From the free body diagram, For equilibrium of the system,



$$T\cos\theta = \mu N = \mu W \quad \ldots\text{(i)}$$

$$T\sin\theta = W' \quad \ldots\text{(ii)}$$

where, T = tension in the thread lying between knot and the support.

On divindg Eq. (ii) by Eq. (i), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{W'}{\mu W}$$

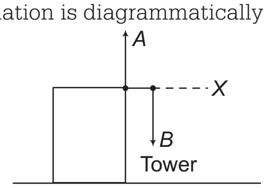
$$\Rightarrow \quad \tan\theta = \frac{W'}{\mu W}$$

$$\Rightarrow \quad W' = \mu W \tan\theta$$

25. The output of digital circuit can be given as, $Y = \overline{AB} + \overline{C}$

It will be same as, $Y = \overline{A} + \overline{B} + \overline{C}$

26. The situation is diagrammatically as below



When A is projected with a vertical speed the after sometime it comes on the same level with same speed as it was projected. Now, the downward speeds of A and B at the level-X is same. So, on reaching the ground, velocity of A and B are same.

27. Given that, $2\text{ eV} \le \phi \le 5\text{ eV}$

Wavelength corresponding to minimum and maximum values of work function are

$$\lambda_{\text{max}} = \frac{hc}{E} = \frac{4\times10^{-15}\times3\times10^{8}}{2}$$

$$= 6\times10^{-7}\text{ m} = 600\text{ nm}$$

and $$\lambda_{\text{min}} = \frac{hc}{E} = \frac{4\times10^{-15}\times3\times10^{8}}{5}$$

$$= 2.4\times10^{-7}\text{ m} = 240\text{ nm}$$

Clearly, wavelength range would be

$$240 \le \lambda \le 600$$

Thus, wavelength 650 nm is not suitable for photoelectric effect.

28. The change in path length $= (\mu - 1)t$. This path length change the position of bright fringe upto seventh bright fringe. The thickness of plastic substance inserted in the path of rays will must be 7 (on assuming fringe width to be units).

29. Let the length of closed organ pipe is l, then,

$$\nu_n = (2n-1)\frac{v}{4l}$$

Its third harmonics (put $n = 2$) in above question,

$$\nu_3 = \frac{3}{4}\frac{v}{l} \quad \ldots\text{(i)}$$

Now, the length of open organ pipe is $2l$, then

$$\nu_n = n\frac{v}{2(2l)} = \frac{nv}{4l}$$

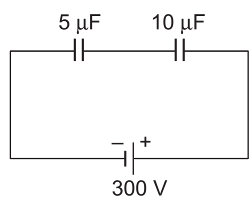
Its fundamental frequency,

$$\nu_0 = \frac{v}{2l} = 100 \quad \ldots\text{(ii)}$$

From Eq. (i) and (ii), we get

$$\nu_3 = \frac{3v}{4l} = 3\times100 = 300\text{ Hz}$$

30. According to question the figure can be drawn as below



The equivalent capacitance,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{10}$$

$$= \frac{2+1}{10} = \frac{3}{10}$$

$$\Rightarrow \quad C_{eq} = \frac{10}{3}\ \mu\text{F}$$

Now, the energy stored in the capacitor is

$$U = \frac{1}{2}CV^2$$
$$= \frac{1}{2} \times \frac{10}{3} \times 10^{-6} \times 300 \times 300$$
$$= \frac{3}{10 \times 2} = \frac{3}{20} = 0.15 \text{ J}$$

31. As we know that the speed of efflux is given by $v = \sqrt{2gh}$, where, h = height above the hole upto which liquid is contained.

$\Rightarrow \qquad v \propto \sqrt{h}$

As the water coming out from hole-1 has maximum speed (horizontal) speed. So, it will reach farthest distance.

32. Given, $p = \frac{AT - BT^2}{V}$

$\Rightarrow \quad pV = AT - BT^2$

$\Rightarrow \quad p\Delta V = A\Delta T - BT\Delta T$

On integrating, we get

$$\text{Work} = \int PdV = A\int_{T_1}^{T_2} dT - B\int_{T_1}^{T_2} TdT$$
$$= A(T_2 - T_1) - \frac{B}{2}[(T_2)^2 - (T_1)^2]$$

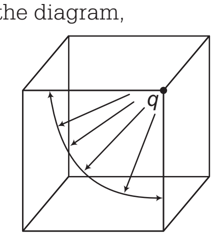
33. When switch is in position *a*, then capacitor will be changed.

When switch is in position *b*, then circuit becomes an LC oscillator with frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$.

In two situation the net reactance or impedance of the circuit is zero.

This implies the current, $i = \frac{E}{\text{zero}}$ i.e. infinite current through the circuit even without any applied emf.

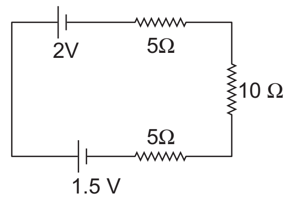
34. Consider the diagram,



In above the flux coverage of three face is shown in figure. Which is like a quadrant in any plane. So, flux will be

$$\phi = \frac{1}{4} \times \frac{q}{6\epsilon_0} = \frac{q}{24\epsilon_0}$$

35. The figure can be redrawn as,



The current through the circuit

$$i = \frac{\text{net emf}}{\text{effective resistance}}$$
$$= \frac{2 - 1.5}{5 + 5 + 10} = \frac{0.5}{20}$$
$$= \frac{1}{40} = 0.025 \text{ A}$$

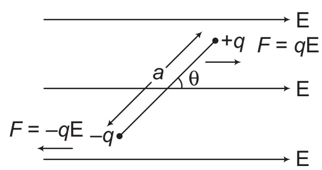
The terminal potential difference of the batteries

$$V_A = \varepsilon_A - ir_A = 2 - 0.025 \times 5$$
$$= 2 - 0.0125 = 1.875 \text{ V}$$

and $V_B = \varepsilon_B - ir_B$

$$= 1.5 - 0.025 \times 5$$
$$= 1.5 - 0.0125 = 1.375 \text{ V}$$

36. The situation can be diagrammatically as,



The dipole moment,

$$\text{P} = 2qa \qquad (\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}})$$

The net force on the dipole is always zero while net torque on the dipole is not zero.

37. The source is at infinity and incident beam is parallel.

38. Whenever, there is a change in the magnetic flux crossing through the loop, an emf induced in it and hence there is an amount of induced current through the loop. If loop is rotated about one of its diameter, the flux through it varies and causing the emf induced in it.

39. In this case (pure rolling) the velocity of point of contact is zero.

40. The de-Broglie equation is

$$\lambda = \frac{h}{mv} \Rightarrow m = \frac{h}{\lambda v}$$

$$\Rightarrow \quad m \propto \frac{1}{\lambda}, \text{ if } \frac{h}{v} = \text{constant}$$

Again, $\frac{h}{v}$ will be constant only when v is constant.

Thus, in present case the de-Broglie wavelength of heavier particle will be smaller. When the velocities of two particles will be same.

Chemistry

41. Flame colours are produced from the movement of the electrons in the metal ions present in the compounds.

Calcium gives brick red colour. Strontium gives crimson colour.

Barium gives apple green colour.

42. Extraction of gold (Au) involves the following sequence of reaction,

$$\text{Gold ore} \xrightarrow[CN^-,\, H_2O,\, O_2]{\text{Roasting}} HO^- + \underset{X}{Au(CN)_2^-}$$

$$\xrightarrow{Zn} \underset{Y}{Zn(CN)_4^{2-}} + Au$$

Hence, $X = Au(CN)_2^-$, $Y = Zn\ (CN)_4^{2-}$

43. Atomic number of cerium = 58

Electronic configuration of

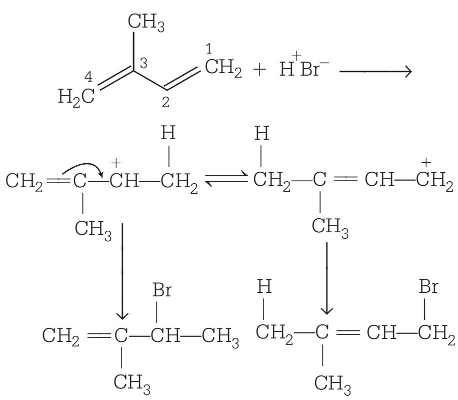
$$Ce = [Xe]\ 4f^2, 5d^0, 6s^2$$

Electronic configuration of

$$Ce^{3+} = [Xe]\ 4f^1$$

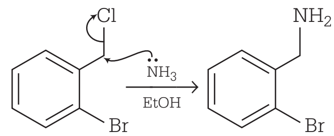
44. $HBr \rightleftharpoons H^+ + Br^-$

For, 1,3-butadiene at room temperature, there is formation of 1,4-addition product.



1,2-addition (Minor product) 1,4-addition (Major product)

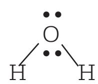
45. It is a substitution reaction, so the product formed is



46. When sulphuryl chloride (SO_2Cl_2) reacts with white phosphorus (P_4), it give phosphorus pentachloride (PCl_5) and sulphur dioxide (SO_2). The reaction is as follows:

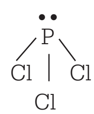
$$P_4 + 10\,SO_2Cl_2 \longrightarrow 4PCl_5 + 10SO_2$$

47. The number of lone pair of electrons on the centra atoms of H_2O, $SnCl_2$, PCl_3 and XeF_2 are 2, 1, 1 and 3 respectively.

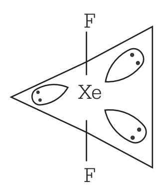


Number of lone pairs = 2

Number of lone pair = 1



Number of lone pair = 1



Number of lone pairs = 3

48. Salts such as NaCl, $HgCl_2$, $CuCl_2$ are soluble in water while salts such as Hg_2Cl_2, CuCl and AgCl are insoluble in water.

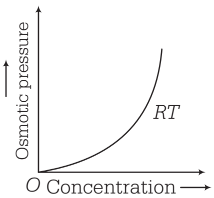
49. $\overset{sp^2}{CH_2} = \overset{sp}{C} = \overset{sp^2}{CH} - \overset{sp^3}{C}H(-\underset{sp}{C}\equiv N) - \overset{sp}{C} \equiv \overset{sp}{CH}$

Thus, number of sp-hybridised C = 4

50. Rate of reaction is doubled when concentration of A is doubled. Again, rate of reaction becomes four times when concentration of A is increased by four times. It is clearly shown that there is no effect on rate of reaction on increasing the concentration of B.

Thus, order with respect to A is 1 and order with respect to B is 0. Total order of reaction = 1

51.



Slope, $RT = 291R$ or $T = 291$ K

$\therefore$ Temperature $= 291 - 273 = 18°C$

52. As from the formula, $v_{rms} = \sqrt{\frac{3RT}{M}}$

Given that $(v_{rms})_{CO} = 1000$ m/s

$(Temp.)_{CO} = 27°C = 300$ K

$(Temp.)_{N_2} = 600$ K

Now putting the values, we get

$$\frac{(v_{rms})_{CO}}{(v_{rms})_{N_2}} = \sqrt{\frac{3R}{3R} \times \frac{T_{CO}}{M_{CO}} \times \frac{M_{N_2}}{T_{N_2}}}$$

$$\frac{1000}{(v_{rms})_{N_2}} = \sqrt{\frac{300}{28} \times \frac{28}{600}} = \frac{1}{\sqrt{2}}$$

or, $(v_{rms})_{N_2} = 1000 \times 1.414$

$= 1414$ m/s

53. A gas can be liquefied at temperature T when it is lower than critical temperature, T_c and pressure p is greater than critical pressure.

Hence, $T < T_c$ and $p > p_c$.

54. The dispersed phase is liquid and dispersion medium is gas in the fog.

55. On the basis of electropositivity of metal, the order of basic character is

$$K_2O > BaO > CaO > MgO$$

56. In aqueous alkaline solution, two electron reduction of HO_2^- gives $\bar{O}H$. The reaction is as follows:

$$H_2O_2 \rightleftharpoons H^+ + HO_2^-$$

$$HO_2^- + H_2O + 2e^- \rightleftharpoons 3\bar{O}H$$

57. Cold $FeSO_4$ solution on absorption of NO develops brown colour due to the formation of $[Fe(H_2O)_5(NO)]SO_4$.

$$FeSO_4 + NO + 5H_2O \longrightarrow \underset{\text{Brown ring complex}}{[Fe(H_2O)_5(NO)]SO_4}$$

This complex has 3.89 BM magnetic moment shows that it has 3 unpaired electrons.

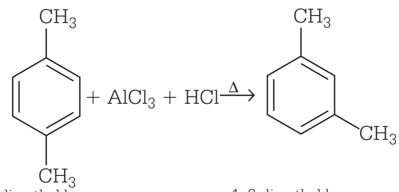
58.

Group II A	Group III A
Be	B
Mg	Al

The electronic configuration of boron is $2s^2 2p^1$. And electronic configuration of B^+ is $2s^2$. Hence, it is difficult to remove second electron from $2s^2$ shell because half-filled and full-filled orbitals are more stable than others.

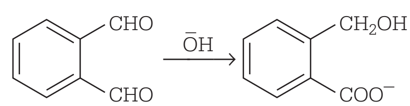
59. 15% will form racemic mixture with another 15%. Hence, the enantiomeric excess is $= (85 - 15) = 70\%$

60.



1, 4-dimethyl benzene 1, 3-dimethyl benzene

61.



It is an example of intramolecular Cannizaro reaction.

62. Given that,

Mass of single Ag atom = m

Unit cell length = a

Total atoms present per unit cell of fcc lattice = 4. Therefore, mass of unit cell becomes $4m$. Volume of unit cell $= a^3$

$$\text{Density}(\rho) = \frac{\text{Mass of unit cell}}{\text{Volume of unit cell}} = \frac{4m}{a^3}$$

63. For the reaction,

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

Given that, $\Delta G° = -690.9R$, $T = 300$ K

From the formula, $\Delta G° = -RT \ln K$

$$-690.9R = -R \times 300 \times \ln K$$

or $\ln k = \frac{690.9}{300} = 2.303$

or $= 2.303 \log K = 2.303$

or $K = 10^1 = 10$

Unit of $K = (\text{atm})^{\Delta n} = (\text{atm})^{2-3} = (\text{atm})^{-1}$

$\therefore \quad K = 10 \text{ atm}^{-1}$

64. As we know that,

Equivalent conductance, (λ)

$$= \frac{\text{specific conductance } (K) \times 1000}{\text{concentration}}$$

or, $\dfrac{\lambda}{K} = \dfrac{1000}{\text{conc.}}$

or, $\dfrac{\lambda}{K} = \dfrac{1000 \ \Omega^{-1} \text{cm}^2 \text{eq}^{-1}}{0.01 \ \Omega^{-1} \text{cm}^{-1}}$

(Given, conc. $= 0.01$ N)

or, $\dfrac{\lambda}{K} = 10^5 \text{ cm}^3 \text{ eq}^{-1}$

65. From the formula,
Surface tension,

$$(\gamma) = \frac{\Delta W}{\Delta A} = \frac{\text{J}}{\text{m}^2} = \frac{\text{kg m}^2 \text{s}^{-2}}{\text{m}^2} = \text{kg s}^{-2}$$

Form the formula, $F = \eta \cdot A \cdot \dfrac{dV}{dx}$

or, η (coefficient of viscosity)

$$= \frac{F}{A \cdot \dfrac{dV}{dx}} = \frac{\text{N}}{\text{m}^2 \cdot \dfrac{\text{ms}^{-1}}{\text{m}}} = \text{Nm}^{-2} \cdot \text{s}$$

or $\text{kg ms}^{-2} \cdot \text{m}^{-2} \cdot \text{s} = \text{kg m}^{-1} \text{ s}^{-1}$

66. Given, that pH $= 5.74$, $pK_a = 4.74$

Suppose that volume of acid solution $= x$ L

Volume of salt solution $= y$ L

From Henderson equation.

$$\text{pH} = pK_a + \log \frac{[\text{Salt}]}{[\text{Acid}]}$$

or, $\text{pH} - pK_a = \log \dfrac{[CH_3COONa]}{[CH_3COOH]}$

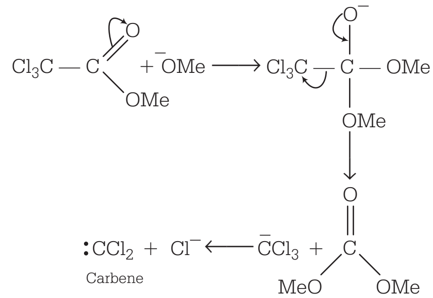
or, $5.74 - 4.74 = 1 = \log \dfrac{[CH_3COONa]}{[CH_3COOH]}$

or $\dfrac{CH_3COONa]}{[CH_3COOH]} = 10$

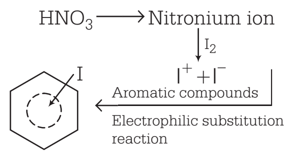
or $\dfrac{[CH_3COOH]}{[CH_3COONa]} = \dfrac{1}{10} = \dfrac{\dfrac{0.1\,x}{x+y}}{\dfrac{0.1\,y}{x+y}}$

Thus, $\dfrac{x}{y} = \dfrac{1}{10}$

67. $\text{NaOMe} \rightleftharpoons \text{Na}^+ + \overset{-}{\text{O}}\text{Me}$



68. Best reagent for nuclear iodination of aromatic compounds is I_2/HNO_3.



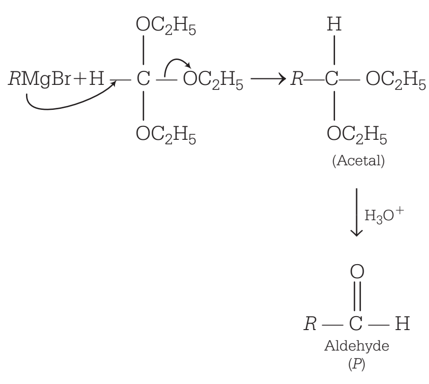
69. To a part of sodium extract, $FeSO_4$ solution is added and the contents are warmed. A few drops of $FeCl_3$ solution are then added and resulting solution is acidified with conc. HCl. The appearance of bluish green or Prussian blue colouration confirms the presence of nitrogen. The reactions that occur during this test are as

$$2NaCN + FeSO_4 \longrightarrow Na_2SO_4 + Fe(CN)_2$$

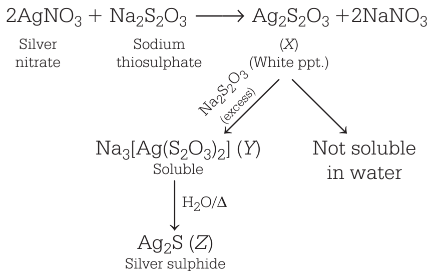
$$Fe(CN)_2 + 4NaCN \longrightarrow \underset{\text{Sodium ferrocyanide}}{Na_4[Fe(CN)_6]}$$

$$3\,Na_4\,[Fe(CN)_6] + 4\,FeCl_3 \longrightarrow \underset{\substack{\text{Ferric ferrocyanide} \\ \text{(Prussian blue)}}}{Fe_4[Fe(CN)_6]_3} + 12NaCl$$

70.



71. The sequence of reactions are



Hence, $X = Ag_2S_2O_3$

$Y = Na_3[Ag(S_2O_3)_2]$ $Z = Ag_2S$

72. From the given cell, the cell reaction is

$$2Ag(s) + Zn^{2+}(0.1\text{ M}) \longrightarrow 2Ag^{+}(0.1\text{ M}) + Zn(s)$$

The Nernst equation is

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{n}\log\frac{[Ag^{+}]^2}{[Zn^{2+}]}$$

$$\text{or, } E_{cell} = (-1.562) - \frac{(0.0591)}{2}\log\frac{(0.1)^2}{(0.1)}$$

(where, $E^{\circ}_{cell} = -1.562$ V)

$$\text{or, } E_{cell} = (-1.562) - \frac{0.0591}{2}\log 10^{-1}$$

$$= -1.562 + \frac{0.0591}{2}$$

$$= -1.562 + 0.02955$$

$$= -1.532\text{ V}$$

73. For the reaction, $X_2Y_4(l) \longrightarrow 2XY_2(g)$

Δn_g = number of gaseous products − number of gaseous reactants

$$\Delta n_g = 2 - 0 = 2$$

Given that, $\Delta U = 2$ kcal

$$\Delta S = 20\text{ cal K}^{-1}$$

From the formula,

$$\Delta H = \Delta U + \Delta n_g RT$$

Putting the values, we get

$$\Delta H = 2 + \left(\frac{2\times 2\times 300}{1000}\right)$$

$$= 3.2\text{ kcal} = 3.2\times 10^3\text{ cal}$$

Also, $\Delta G = \Delta H - T\Delta S$

$$= 3.2\times 10^3 - 300\times 20$$

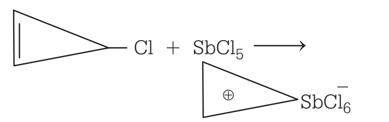
$$= 3.2\times 10^3 - 6\times 10^3$$

$$= -2800\text{ cal}$$

74. Criteria for aromaticity (Huckel rule)

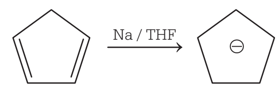
The cyclic π molecular orbital (electron cloud) formed by overlap of *p*-orbitals must contain $(4n+2)\,\pi$-electrons, where n = integer 0, 1, 2, 3, etc. This is known as Huckel rule.

(i)



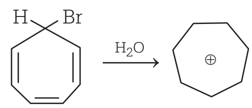
$2\pi e^-$ system (aromatic)

(ii)



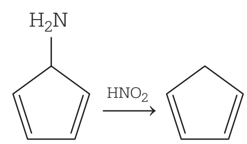
$6\pi e^-$ system (aromatic)

(iii)



Tropylium cation $6\pi e^-$ system (aromatic)

(iv)



$4\pi e^-$ system (non-aromatic)

75. Roasted copper pyrite on smelting with sand produces $FeSiO_3$ as fusible slag and Cu_2S as matte.

For removing the gangue, FeS, silica present in the lining of the Bessemer converter, acts as a flux and forms slag (iron silicate) on reaction with FeO.

$$\underset{\text{Gangue}}{FeS} + 3O_2 \longrightarrow 2FeO + 2SO_2$$

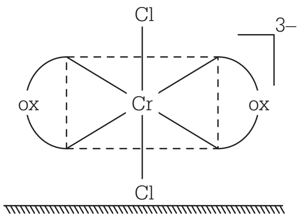
$$\underset{\text{Gangue}}{FeO} + SiO_2 \longrightarrow \underset{\text{Slag}}{FeSiO_3}$$

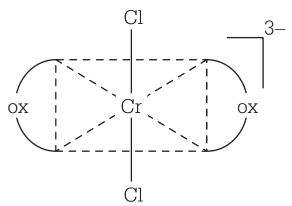
Copper matte mainly contains Cu_2S and FeS.

76. Ionisation potential values of noble gases decrease down the group with increase in atomic size. Xenon forms binary fluorides by the direct reaction of elements.

The reason is that only the heavier noble gases form such compounds. Octet of electrons provide the stable arrangements but this statement is not very much valid in cases of higher element which can be ionised easily due to large distance between nucleus and valence electrons.

77.





trans $[CrCl_2(ox)_2]^{3-}$ isomer- optically inactive (superimposable mirror images and plane of symmetry).

cis-$[CrCl_2(ox)_2]^{3-}$, $[Co(en)_3]^{3+}$ and $[Co(ox)(en)_2]^{+}$ exhibited optical isomerism.

78. The increase in rate constant of a chemical reaction with increasing temperature are due to the following facts.

The number of collisions among the reactant molecules increases with increasing temperature.

The number of reactant molecules acquiring the activation energy increases with increasing temperature.

79. Isoelectric point for neutral amino acids ranges from pH 5.5 to 6.3. Acidic amino acids have isoelectric point around 3 while basic amino acids have ranges from pH 7.6 to 10.8.

At isoelectric point, amino acids exist as Zwitter ions. At low pH, it exists as cation and at high pH, it exists as anion. Hence, alanine (pH range 2-4) will exist as cation while alanine (pH range 9-11) will exist as anion.

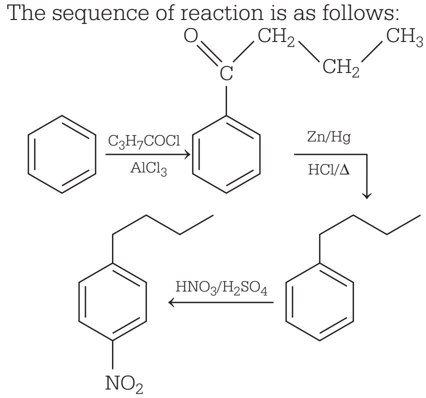
Acidic medium:

$$H_3N^+—CH(CH_3)CO_2H$$

Basic medium:

$$H_2N—CH(CH_3)CO_2^-$$

80. The sequence of reaction is as follows:



(*o*, *p*-director)

Mathematics

81. From option (d), we have

$$a - b = c - d \quad \ldots(i)$$

and $$a^2 - b^2 = c^2 - d^2 \quad \ldots(ii)$$

Consider, $a^2 - b^2 = c^2 - d^2$

$\Rightarrow (a+b)(a-b) = (c-d)(c+d)$

$\Rightarrow \quad a + b = c + d \quad \ldots(iii)$

[using Eq. (i)]

On adding Eqs. (i) and (iii), we get

$2a = 2c \Rightarrow a = c$

$\Rightarrow \quad b = d$ [using Eq. (iii)]

Thus, $a^n + b^n = c^n + d^n$ for all $n \in N$.

82. Since, α and β are the roots of $x^2 - px + 1 = 0$.

$\therefore \quad \alpha + \beta = p$ and $\alpha\beta = 1$

Also, γ is the root of $x^2 + px + 1 = 0$.

$\therefore \quad \gamma^2 + p\gamma + 1 = 0 \Rightarrow \gamma^2 = -p\gamma - 1$

Now, $(\alpha+\gamma)(\beta+\gamma) = \alpha\beta + \alpha\gamma + \beta\gamma + \gamma^2$

$= 1 + \gamma(\alpha+\beta) - p\gamma - 1 = \gamma(\alpha+\beta-p)$

$= \gamma \times 0 = 0 \quad [\because \alpha+\beta = p]$

a

Since, $\gamma = -\alpha$ or $-\beta$

$\therefore \quad (\alpha+\gamma)(\beta+\gamma) = 0$

83. General term of $(3^{1/5} + 7^{1/3})^{100}$ is given by $T_{r+1} = {}^{100}C_r (3^{1/5})^{100-r}(7^{1/3})^r$

$$= {}^{100}C_r \cdot 3^{\frac{100-r}{5}} \cdot 7^{\frac{r}{3}}$$

For a rational term, $\frac{100-r}{5}$ and $\frac{r}{3}$ must be integer.

Hence, $r = 0, 15, 30, 45, 60, 75, 90$

$\therefore$ There are seven rational terms.

Hence, number of irrational terms

$$= 101 - 7 = 94$$

84. Given equation is $(2x+1)^2 - px + q \neq 0$

$$\Rightarrow \quad 4x^2 + 4x + 1 - px + q \neq 0$$

$$\Rightarrow \quad 4x^2 + (4-p)x + (1+q) \neq 0$$

Now, $\quad D < 0$

$$\Rightarrow \quad (4-p)^2 - 4(4)(1+q) < 0$$

$$\Rightarrow \quad 16 - 8p + p^2 - 16 - 16q < 0$$

$$\Rightarrow \quad p^2 - 8p - 16q < 0$$

85. Let A be the set of families who own a car and B be the set of families who own a house.

Then, $\quad P(A) = 60\%,\ P(B) = 30\%$

and $\ P(A \cap B) = 20\%$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 60 + 30 - 20 = 70\%$$

Now, families who own a car or a house but not both $= A \Delta B$

$$\Rightarrow \quad P(A \Delta B) = P(A \cup B) - P(A \cap B)$$

$$= 70 - 20 = 50\% = 0.5$$

86. The number of words starting with CC = 4!

The number of words starting with CH = 4!

The number of words starting with CI = 4!

The number of words starting with CN = 4!

COCHIN is the first word in the list of words beginning with CO.

$\therefore$ Number of words that appear before the word COCHIN = 96

87. Given, $\quad f(x) = \int_0^x f(t)\,dt \quad \ldots\text{(i)}$

Using Leibnitz theorem, we get

$$f'(x) = f(x) \quad \Rightarrow \quad f(x) = ke^x$$

On putting $x = 0$ in Eq. (i), we get

$$f(0) = \int_0^0 f(t)\,dt$$

$$\Rightarrow \quad ke^0 = 0 \qquad \left[\because \int_a^a f(x)\,dx = 0\right]$$

$$\Rightarrow \quad k = 0 \qquad [\because e^o = 1]$$

$$\therefore \quad f(x) = 0 \quad \Rightarrow \quad f(\log_e 5) = 0$$

88. Let $y = \dfrac{x^2 - x + 4}{x^2 + x + 4}$

$$\Rightarrow \quad x^2 y + xy + 4y = x^2 - x + 4$$

$$\Rightarrow (y-1)x^2 + (y+1)x + 4y - 4 = 0$$

For x to be real, discriminant of the above quadratic equation should be greater than or equal to 0.

$$\Rightarrow \quad (y+1)^2 - 4(y-1)(4y-4) \geq 0$$

$$\Rightarrow \quad (y+1)^2 - 16(y-1)^2 \geq 0$$

$$\Rightarrow (y + 1 + 4y - 4)(y + 1 - 4y + 4) \geq 0$$

$$\Rightarrow \quad (5y-3)(5-3y) \geq 0$$

$$\therefore \quad y \in \left[\frac{3}{5}, \frac{5}{3}\right]$$

89. $2x^2 + y^2 + 2xy + 2x - 3y + 8$

$$= \frac{1}{2}[4x^2 + 2y^2 + 4xy + 4x - 6y + 16]$$

$$= \frac{1}{2}[(y^2 - 8y) + (4x^2 + y^2 + 4xy + 4x + 2y) + 16]$$

$$= \frac{1}{2}[(y-4)^2 + (2x + y + 1)^2 - 1]$$

So, least value will be $-\dfrac{1}{2}$ at

$$y = 4 \quad \text{and} \quad x = \frac{-5}{2}.$$

90. If a function $f(x)$ assumes only irrational values and which is also continuous, then $f(x)$ must be a constant function.

$$\Rightarrow \quad f(x) = \sqrt{2} \text{ as } f(\sqrt{2}) = \sqrt{2}$$

$$\therefore \quad f(\sqrt{2} - 1) = \sqrt{2}$$

91. Here, $\cos\theta + \sin\theta + \dfrac{2}{\sin 2\theta},\ \theta \in \left(0, \dfrac{\pi}{2}\right)$

For minimum value, $\sin 2\theta$ must be maximum.

$$\therefore \quad 2\theta = \frac{\pi}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{4}$$

Hence, $\cos\dfrac{\pi}{4} + \sin\dfrac{\pi}{4} + \dfrac{2}{\sin\dfrac{\pi}{2}} = \sqrt{2} + 2$

92. $\displaystyle\lim_{x \to 2} \int_2^x \frac{3t^2}{(x-2)}\,dt = \frac{\displaystyle\lim_{x \to 2} \int_2^x 3t^2\,dt}{\displaystyle\lim_{x \to 2}(x-2)}$

$$= \frac{\displaystyle\lim_{x \to 2} 3x^2}{1} \text{ [using L' Hospital's rule]}$$

$$= 3 \times (2)^2 = 12$$

93. Given, $\cot\dfrac{2x}{3} + \tan\dfrac{x}{3} = \operatorname{cosec}\dfrac{kx}{3}$

Let $\theta = \dfrac{x}{3}$, then

$\therefore \quad \cot 2\theta + \tan\theta = \operatorname{cosec} k\theta$

$\Rightarrow \quad \dfrac{\cos 2\theta}{\sin 2\theta} + \dfrac{\sin\theta}{\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \dfrac{2\cos^2\theta - 1}{2\sin\theta\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \dfrac{2\cos^2\theta - 1 + 2\sin^2\theta}{2\sin\theta\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \quad \dfrac{1}{2\sin\theta\cos\theta} = \operatorname{cosec} k\theta$

$\Rightarrow \quad \operatorname{cosec} 2\theta = \operatorname{cosec} k\theta$

$\therefore \quad k = 2$

94. $\sqrt{4\cos^4\theta + \sin^2 2\theta} + 4\cot\theta\cos^2\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)$

$= \sqrt{4\cos^4\theta + (2\sin\theta\cos\theta)^2} + 2\cot\theta\left[2\cos^2\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)\right]$

$= \sqrt{4\cos^4\theta + 4\sin^2\theta\cos^2\theta} + 2\cot\theta\left[1 + \cos\left(\dfrac{\pi}{2} - \theta\right)\right]$

$[\because 2\cos^2\theta = 1 + \cos 2\theta]$

$= \sqrt{4\cos^2\theta(\cos^2\theta + \sin^2\theta)} + 2\cot\theta(1 + \sin\theta)$

$= |2\cos\theta| + 2\cot\theta + 2\cos\theta$

$= -2\cos\theta + 2\cot\theta + 2\cos\theta$

$\left[\because \text{ for } \theta \in \left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right), |\cos\theta| = -\cos\theta\right]$

$= 2\cot\theta$

95. Given, $(\sin x - x)(\cos x - x^2) = 0$

$\Rightarrow \quad \sin x = x$ or $\cos x = x^2$

Now, if $\sin x = x$, then only one solution i.e. $x = 0$ is possible. Also, if $\cos x = x^2$, then two solutions are possible. Hence, there are total 3 solutions.

96. We know that, $\omega = \dfrac{-1 + \sqrt{3}i}{2}$

$\Rightarrow \; 1 - \sqrt{3}i = -2\omega \quad$ and $\quad \omega^2 = \dfrac{-1 - \sqrt{3}i}{2}$

$\Rightarrow \; 1 + \sqrt{3}i = -2\omega^2$

Now, $\left(\dfrac{1 + \sqrt{3}i}{1 - \sqrt{3}i}\right)^{64} + \left(\dfrac{1 - \sqrt{3}i}{1 + \sqrt{3}i}\right)^{64}$

$= \left(\dfrac{-2\omega^2}{-2\omega}\right)^{64} + \left(\dfrac{-2\omega}{-2\omega^2}\right)^{64}$

$= \omega^{64} + \dfrac{1}{\omega^{64}} = \omega + \omega^2 \qquad [\because \omega^3 = 1]$

$= -1 \qquad [\because 1 + \omega + \omega^2 = 0]$

97. We have,

$|z| = \left|z - \dfrac{3}{z} + \dfrac{3}{z}\right|$

$\Rightarrow \quad |z| \le \left|z - \dfrac{3}{z}\right| + \left|\dfrac{3}{z}\right|$

[using triangle inequality]

$\Rightarrow \quad |z| \le 2 + \left|\dfrac{3}{z}\right| \qquad \left[\because \left|z - \dfrac{3}{z}\right| = 2\right]$

$\Rightarrow \quad |z| \le 2 + \dfrac{3}{|z|}$

$\Rightarrow \quad |z|^2 - 2|z| - 3 \le 0$

$\Rightarrow \quad (|z| - 3)(|z| + 1) \le 0$

$\Rightarrow \quad -1 \le |z| \le 3$

Thus, the maximum value of $|z|$ is 3.

98. We have, $\dfrac{5x^2 - 26x + 5}{3x^2 - 10x + 3} < 0$

$\Rightarrow \quad \dfrac{5x^2 - 25x - x + 5}{3x^2 - 9x - x + 3} < 0$

$\Rightarrow \quad \dfrac{5x(x-5) - 1(x-5)}{3x(x-3) - 1(x-3)} < 0$

$\Rightarrow \quad \dfrac{(x-5)(5x-1)}{(x-3)(3x-1)} < 0$

$\therefore \quad x \in \left(\dfrac{1}{5}, \dfrac{1}{3}\right) \cup (3, 5)$

99. If the given system of equations has no solution,

then $\begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 2(-2\lambda - 1) + 1(\lambda - 1) - 2(1 + 2) = 0$

$\Rightarrow \quad -4\lambda - 2 + \lambda - 1 - 6 = 0$

$\Rightarrow \quad -3\lambda - 9 = 0$

$\Rightarrow \quad \lambda = -3$

100. We have, $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Taking common x, $(x-1)$ from R_2 and R_3 respectively, we get

$$f(x)=x(x-1)\begin{vmatrix}1 & x & x+1\\ 2 & x-1 & x+1\\ 3x & x(x-2) & (x+1)x\end{vmatrix}$$

Taking common $(x+1)$ from C_3, we get

$$f(x)=x(x-1)(x+1)\begin{vmatrix}1 & x & 1\\ 2 & x-1 & 1\\ 3x & x(x-2) & x\end{vmatrix}$$

Taking common x from R_3, we get

$$f(x)=x^2(x-1)(x+1)\begin{vmatrix}1 & x & 1\\ 2 & x-1 & 1\\ 3 & x-2 & 1\end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, we get

$$f(x)=x^2(x-1)(x+1)\begin{vmatrix}-1 & 1 & 0\\ -1 & 1 & 0\\ 3 & x-2 & 1\end{vmatrix}=0$$

[$\because$ R_1 is identical with R_2]

$\Rightarrow \quad f(100)=0$

101. We have, $x_n=\left[\left(1-\frac{1}{3}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{10}\right)\ldots\left(1-\frac{2}{n(n+1)}\right)\right]^2$

$$\Rightarrow \quad x_n=\left[\prod_{n=2}^{n}\left(\frac{n^2+n-2}{n(n+1)}\right)\right]^2$$

$$=\left[\prod_{n=2}^{n}\left(\frac{(n+2)(n-1)}{n(n+1)}\right)\right]^2$$

$$=\left[\prod_{n=2}^{n}\left(\frac{n+2}{n+1}\right)\cdot\prod_{n=2}^{n}\left(\frac{n-1}{n}\right)\right]^2$$

$$=\left[\prod_{n=2}^{n}\left(\frac{n+2}{n+1}\right)\right]^2\left[\prod_{n=2}^{n}\left(\frac{n-1}{n}\right)\right]^2$$

$$\Rightarrow \quad x_n=\left(\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\ldots\cdot\frac{n+2}{n+1}\right)^2\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{n-1}{n}\right)^2$$

$$\Rightarrow \quad x_n=\left(\frac{n+2}{3}\right)^2\left(\frac{1}{n}\right)^2$$

$$\Rightarrow \quad x_n=\frac{1}{9}\left(\frac{n+2}{n}\right)^2$$

$$\Rightarrow \quad x_n=\frac{1}{9}\left(1+\frac{2}{n}\right)^2$$

$$\Rightarrow \quad \lim_{n\to\infty} x_n=\frac{1}{9}(1+0)^2=\frac{1}{9}$$

102. Variance of n natural numbers

$$=\frac{n^2-1}{12}=\frac{(20)^2-1}{12} \qquad [\because n=20]$$

$$=\frac{400-1}{12}=\frac{399}{12}=\frac{133}{4}$$

103. Let the coin be tossed n times.

Let getting head is consider to be success.

$\therefore \quad p=\frac{1}{2}$, $q=1-p=1-\frac{1}{2}=\frac{1}{2}$

It is given that,

$$P(X=3)=P(X=5)$$

$$\Rightarrow \quad {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3}={}^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5}$$

$$\Rightarrow \quad {}^nC_3={}^nC_5$$

$$\Rightarrow \quad n=3+5 \qquad [\because {}^nC_x={}^nC_y \Rightarrow x+y=n]$$

$$\Rightarrow \quad n=8$$

Now, $P(X=1)={}^8C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{8-1}$

$$={}^8C_1\times\left(\frac{1}{2}\right)^8=\frac{1}{32}$$

104. Total number of ways in which the letters of the word 'PROBABILITY' can be arranged in a row $=\frac{11!}{2!\,2!}$

Number of ways in which two B's are together $=\frac{10!}{2!}$

$\therefore$ Required probability

$$=\frac{\frac{10!}{2!}}{\frac{11!}{2!\,2!}}=\frac{10!\times 2!}{11!}=\frac{2}{11}$$

105. Option (a) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other, then $\mathbf{a}\cdot\mathbf{b}=0$

Now consider,

$$|\mathbf{a}+\mathbf{b}|^2=(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})$$

$$=|\mathbf{a}|^2+|\mathbf{b}|^2$$

So, option (a) is always true.

Option (b) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other, then $\mathbf{a}\cdot\mathbf{b}=0$

Now consider,

$$|\mathbf{a}+\lambda\mathbf{b}|^2=(\mathbf{a}+\lambda\mathbf{b})\cdot(\mathbf{a}+\lambda\mathbf{b})$$
$$=|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2$$
$$\Rightarrow |\mathbf{a}+\lambda\mathbf{b}|=\sqrt{|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2}$$
$$\geq|\mathbf{a}| \text{ for all } \lambda\in R$$

So, option (b) is always true.

Option (c) consider,

$$|\mathbf{a}+\mathbf{b}|^2+|\mathbf{a}-\mathbf{b}|^2=(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})+(\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$$
$$=|\mathbf{a}|^2+\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{a}+|\mathbf{b}|^2+|\mathbf{a}|^2-\mathbf{a}\cdot\mathbf{b}-\mathbf{b}\cdot\mathbf{a}+|\mathbf{b}|^2$$
$$=2(|\mathbf{a}|^2+|\mathbf{b}|^2)$$

So, option (c) is always true.

Option (d) consider,

$$\mathbf{a}=-\mathbf{b} \text{ and } \mathbf{b}\neq 0$$

Then, $|\mathbf{a}+\lambda\mathbf{b}|\geq|\mathbf{a}|$

$\Rightarrow\quad |-\mathbf{b}+\lambda\mathbf{b}|\geq|-\mathbf{b}|$

$\Rightarrow\quad |\mathbf{b}||\lambda-1|\geq|\mathbf{b}| \Rightarrow |\lambda-1|\geq 1$

which is not true for all λ, as we consider $\lambda=\frac{1}{2}$, then it is not true.

Hence, option (d) is not always true.

106. If four points (x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3) and (x_4,y_4,z_4) are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix}=0$$

Now, $$\begin{vmatrix} 3 & 0 & 0 \\ 2 & 0 & -2 \\ 2 & \lambda-1 & 0 \end{vmatrix}=0$$

$$\Rightarrow\quad 3(0+2\lambda-2)=0$$
$$\Rightarrow\quad \lambda=1$$

107. Consider the given equation of lines,

$$x-(t+\alpha)=0 \quad \ldots\text{(i)}$$
$$y+16=0 \quad \ldots\text{(ii)}$$
and $$-\alpha x+y=0 \quad \ldots\text{(iii)}$$

Since, these lines are concurrent, therefore the system of equation is consistent.

Now, $$\begin{vmatrix} 1 & 0 & -(t+\alpha) \\ 0 & 1 & 16 \\ -\alpha & 1 & 0 \end{vmatrix}=0$$

$$\Rightarrow\quad 1(0-16)-(t+\alpha)(0+\alpha)=0$$
$$\Rightarrow\quad -16-\alpha(t+\alpha)=0$$
$$\Rightarrow\quad \alpha(t+\alpha)+16=0$$
$$\Rightarrow\quad \alpha^2+t\alpha+16=0$$

Clearly, α should be real.

$\therefore\ t^2-4\times 16\geq 0 \Rightarrow t^2-64\geq 0$

Hence, least positive value of t is 8.

108. We have, $a^2\cos^2 A-b^2-c^2=0$

$$\Rightarrow\quad a^2\cos^2 A=b^2+c^2$$

Also in ΔABC, we have

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{a^2\cos^2 A-a^2}{2bc}$$
$$=\frac{-a^2(1-\cos^2 A)}{2bc}=\frac{-a^2\sin^2 A}{2bc}<0$$
$$[\because 0<A<\pi; a,b,c>0]$$

$\Rightarrow \cos A<0 \Rightarrow A$ lies in IInd quadrant

$$\Rightarrow\quad \frac{\pi}{2}<A<\pi$$

109. Given, $\{x\in R: |\cos x|\geq\sin x\}\cap\left[0,\frac{3\pi}{2}\right]$

If we draw, the graph of $|\cos x|$ and $\sin x$, clearly $|\cos x|\geq\sin x$ when

$$x\in\left[0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\frac{3\pi}{2}\right]$$
$$\therefore\quad \left[0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\frac{3\pi}{2}\right]\cap\left[0,\frac{3\pi}{2}\right]$$
$$=\left[0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\frac{3\pi}{2}\right]$$

110. We have, $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}-\frac{x^4}{8}+\ldots\right)=\frac{\pi}{6}$

$$\Rightarrow\quad \sin^{-1}\left(\frac{x}{1-\left(\frac{-x}{2}\right)}\right)=\frac{\pi}{6} \qquad \left[\because S_\infty=\frac{a}{1-r}\right]$$
$$\Rightarrow\quad \sin^{-1}\left(\frac{2x}{2+x}\right)=\frac{\pi}{6}$$
$$\Rightarrow\quad \frac{2x}{2+x}=\sin\frac{\pi}{6} \Rightarrow \frac{2x}{2+x}=\frac{1}{2}$$
$$\Rightarrow\quad 4x=2+x \Rightarrow 3x=2 \Rightarrow x=\frac{2}{3}$$

111. We have, $y = x^3$ and $A(1,1)$

$\therefore \quad \dfrac{dy}{dx} = 3x^2 \qquad \text{...(i)}$

On putting $x=1$ in Eq. (i), we get

$$\frac{dy}{dx} = 3(1)^2 = 3$$

$\therefore$ Equation of tangent at $A(1,1)$ is

$$y-1 = 3(x-1) \Rightarrow y = 3x-2$$

$\therefore$ Required area

$$= \int_0^1 x^3 dx - \int_{2/3}^1 (3x-2)\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 - \left[\frac{3x^2}{2} - 2x\right]_{2/3}^1$$

$$= \frac{1}{4} - \left[\left(\frac{3}{2}-2\right) - \left(\frac{2}{3}-\frac{4}{3}\right)\right]$$

$$= \frac{1}{12} \text{ sq unit}$$

112. We have, $\log_{0.2}(x-1) > \log_{0.04}(x+5)$

$\Rightarrow \quad \log_{0.2}(x-1) > \log_{(0.2)^2}(x+5)$

$\Rightarrow \quad \log_{0.2}(x-1) > \dfrac{1}{2}\log_{0.2}(x+5)$

$\Rightarrow \quad 2\log_{0.2}(x-1) > \log_{0.2}(x+5)$

$\Rightarrow \quad \log_{0.2}(x-1)^2 > \log_{0.2}(x+5)$

$\Rightarrow \quad (x-1)^2 < x+5$

$[\because \log_a x > \log_a y \Rightarrow x<y, \text{ if } 0<a<1]$

$\Rightarrow \quad x^2 - 2x + 1 < x+5$

$\Rightarrow \quad x^2 - 3x - 4 < 0$

$\Rightarrow \quad x^2 - 4x + x - 4 < 0$

$\Rightarrow x(x-4) + 1(x-4) < 0$

$\Rightarrow \quad (x-4)(x+1) < 0 \Rightarrow x \in (-1,4)$

But $\quad x > 1 \Rightarrow x \in (1,4)$

113. We have, $\log_e x + ex = 0$

$\Rightarrow \quad \log_e x = -ex$

Since, both the graphs intersect at only one point. Hence, the number of real roots of equation is one.

114. Given, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow \sin x(\sin^2 x - \cos^2 x)$

$-\cos x(\sin x\cos x - \cos^2 x)$

$+\cos x(\cos^2 x - \sin x\cos x) = 0$

$\Rightarrow \sin x(\sin x - \cos x)(\sin x + \cos x)$

$-\cos^2 x(\sin x - \cos x) - \cos^2 x$

$(\sin x - \cos x) = 0$

$\Rightarrow \quad (\sin x - \cos x)$

$(\sin^2 x + \sin x\cos x - 2\cos^2 x) = 0$

$\Rightarrow (\sin x - \cos x)(\sin^2 x + 2\sin x$

$\cos x - \sin x\cos x - 2\cos^2 x) = 0$

$\Rightarrow \quad (\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$

$\Rightarrow \quad \sin x - \cos x = 0$ or $\sin x + 2\cos x = 0$

$\Rightarrow \quad \tan x = 1$ or $\tan x = -2$

But $\quad x \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

$\Rightarrow$ Only one solution i.e. $x = \dfrac{\pi}{4}$

115. Since, $x_1, x_2, \ldots, x_{15}$ are among 1, 2, 3, …, 15. So, x_1 can take any value from 1, 2, 3, …, 15. Among these values one of the number must be 1, hence product will be 0.

116. We have, $f(x) = \begin{cases} \dfrac{\sin[-x^2]}{[-x^2]}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

Now, $\displaystyle\lim_{x\to 0} \frac{\sin[-x^2]}{[-x^2]} = \frac{\sin(-1)}{(-1)} = \sin(1)$

Since, $f(x)$ is continuous at $x=0$.

$\Rightarrow \quad \displaystyle\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \quad \sin(1) = \alpha$

117. Let $f(x) = \dfrac{a_3x^4}{4} + \dfrac{a_2x^3}{3} + \dfrac{a_1x^2}{2} + a_0x$

$\therefore \quad f(0) = 0,\ f(1) = \dfrac{a_3}{4} + \dfrac{a_2}{3} + \dfrac{a_1}{2} + a_0 = 0$

$\Rightarrow \quad f(0) = f(1)$

$\Rightarrow \quad f'(x) = 0$ has atleast one real root in [0, 1].

[according to Rolle's theorem]

$\therefore \quad f'(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

Hence, $a_3x^3 + a_2x^2 + a_1x + a_0$ must has a real root in the interval [0, 1].

118. We have, $f(x) = \begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$

If $f(x)$ is continuous, then $\sin|x| = 0$

$\Rightarrow \quad x = k\pi$, where k is an integer.

119. Given, $s = t^2 + at - b + 17$

After 5 sec, $\left[\frac{ds}{dt}\right]_{t=5} = [2t+a]_{t=5} = 0$

$\Rightarrow \quad a = -10$

At $t=5$ sec, $s = 25$ units

$\therefore \quad 25 = 5^2 + 5a - b + 17$

$[\because s = 25 \text{ and } t = 5]$

$\Rightarrow \quad 25 = 25 + 5(-10) - b + 17$

$\Rightarrow \quad 50 - 17 = -b$

$\Rightarrow \quad b = -33$

120. $\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\ldots+\sqrt{n-1}}{n\sqrt{n}}$

$= \lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\ldots+\sqrt{n-1}+\sqrt{n}}{n\sqrt{n}} - \frac{n}{n\times n}\right)$

$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/2} - \lim_{n\to\infty}\frac{1}{n}\times\frac{n}{n}$

$= \int_0^1 \sqrt{x}\, dx + 0 = \frac{2}{3}$

121. We have,

$\lim_{x\to 0}\frac{axe^x - b\log(1+x)}{x^2} = 3 \quad \left[\frac{0}{0}\text{ form}\right]$

Using L' Hospital's rule, we get

$\lim_{x\to 0}\frac{ae^x + axe^x - \frac{b}{1+x}}{2x} = 3 \quad \left[\frac{0}{0}\text{ form}\right]$

$\Rightarrow a - b = 0 \Rightarrow a = b$

Using L' Hospital's rule, we get

$\lim_{x\to 0}\frac{ae^x + ae^x + axe^x + \frac{b}{(1+x)^2}}{2} = 3$

$\Rightarrow \quad \lim_{x\to 0} 2ae^x + axe^x + \frac{b}{(1+x)^2} = 6$

$\Rightarrow \quad 2a + b = 6 \quad \ldots\text{(i)}$

$\Rightarrow \quad 3a = 6 \Rightarrow a = 2$

On putting the value of a in Eq. (i), we get $b = 2$

122. We have, $y^2 - 4y = 4x - 4a$

$\Rightarrow \quad (y-2)^2 - 4 = 4x - 4a$

$\Rightarrow \quad (y-2)^2 = 4x - 4a + 4$

$\Rightarrow \quad (y-2)^2 = 4[x - (a-1)]$

Hence, vertex is $(a-1, 2)$.

Since, vertex lies between the lines $x + y = 3$ and $2x + 2y - 1 = 0$, then

$(a - 1 + 2 - 3)(2a - 2 + 4 - 1) < 0$

$\Rightarrow \quad (a-2)(2a+1) < 0 \Rightarrow a \in \left(-\frac{1}{2}, 2\right)$

123. For curve I,

$4x^2 + 9y^2 = 1 \Rightarrow \frac{x^2}{\left(\frac{1}{2}\right)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$

which is an equation of ellipse with $a = \frac{1}{2}$ and $b = \frac{1}{3}$.

For curve II, $4x^2 + y^2 = 4 \Rightarrow \frac{x^2}{1} + \frac{y^2}{4} = 1$

$\Rightarrow \quad \frac{x^2}{(1)^2} + \frac{y^2}{(2)^2} = 1$

which is also an equation of ellipse with $a = 1$ and $b = 2$.

Hence, there is no intersecting point.

124. If a line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ lies on a plane $a_2 x + b_2 y + c_2 z = d$, then

(i) $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

(ii) $a_2 x_1 + b_2 y_1 + c_2 z_1 = d$

According to the question,

$3(1) + (2+\lambda)(-2) + (-1)(0) = 0$

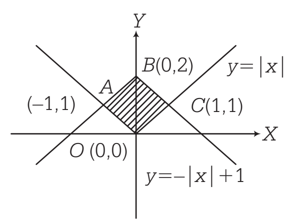
$\Rightarrow \quad 3 - 4 - 2\lambda = 0$

$\Rightarrow \quad \lambda = -1/2$

Also, $\quad \lambda - 2 = 0 \Rightarrow \lambda = 2$

So, there is no such value of λ.

125.



In figure, $B \equiv (0,2) \Rightarrow OB = 2$

and $OABC$ is a square.

So, side of length $= \sqrt{2}$ units

$\therefore$ Area of $OABC = 2$ sq units

126. Here, $225 = 3^2 \times 5^2$

$\Rightarrow d(225) = 3 \times 3 = 9$

and $1125 = 3^2 \times 5^3$

$\Rightarrow \quad d(1125) = 3 \times 4 = 12$

$640 = 2^7 \times 5$

$\Rightarrow \quad d(640) = 8 \times 2 = 16$

Hence, 9, 12 and 16 are in GP.

127. We know that, $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \frac{-\pi}{2} \le 2\sin^{-1} 2a \le \frac{\pi}{2}$

$\Rightarrow \frac{-\pi}{4} \le \sin^{-1} 2a \le \frac{\pi}{4}$

$\Rightarrow \sin\left(\frac{-\pi}{4}\right) \le 2a \le \sin\left(\frac{\pi}{4}\right)$

$\Rightarrow \frac{-1}{\sqrt{2}} \le 2a \le \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{-1}{2\sqrt{2}} \le a \le \frac{1}{2\sqrt{2}} \Rightarrow |a| \le \frac{1}{2\sqrt{2}}$

128. If one root of a quadratic equation is of the form $a + ib$, then other root will be $a - ib$.

So, all the roots are $2 \pm i$, $\sqrt{5} \pm 2i$.

$\therefore$ Product of all the roots

$= (2 + i)(2 - i)(\sqrt{5} + 2i)(\sqrt{5} - 2i)$

$= (4 + 1)(5 + 4) = 5 \times 9 = 45$

129. We have, $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$= 1(1 + \tan^2\theta) - \tan\theta$

$(-\tan\theta + \tan\theta) + 1(\tan^2\theta + 1)$

$= 2(1 + \tan^2\theta) - 0 = 2\sec^2\theta$

$\therefore$ Range of $f = (2, \infty)$

130. Given, $AB = B$ and $BA = A$

Now, $A^2 + B^2 = A \cdot A + B \cdot B$

$= A(BA) + B(AB)$

$= (AB)A + (BA)B$

$= BA + AB = A + B$

131. $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega^2 & -\omega \\ 0 & \omega & -\omega^2 \\ -1+\omega & \omega & -\omega^2 \end{vmatrix} \quad [\because C_1 \to C_1 + C_2]$

$= (-1 + \omega)(-\omega^4 + \omega^2) = (\omega - 1)(\omega^2 - \omega)$

$= \omega^3 - \omega^2 - \omega^2 + \omega = -3\omega^2$

132. Let $I = \int_0^{\sqrt{3}} f(x^2)\,dx = \int_0^{\sqrt{3}} \{x^2\}\,dx$

$= \int_0^{\sqrt{3}} (x^2 - [x^2])\,dx$

$= \int_0^{\sqrt{3}} x^2\,dx - \int_0^{\sqrt{3}} [x^2]\,dx$

$= \left[\frac{x^3}{3}\right]_0^{\sqrt{3}} - [\int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]\,dx]$

$= \sqrt{3} - \left[\int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx\right]$

$= \sqrt{3} - [0 + (x)_1^{\sqrt{2}} + (2x)_{\sqrt{2}}^{\sqrt{3}}]$

$= \sqrt{3} - \sqrt{2} + 1 - 2\sqrt{3} + 2\sqrt{2}$

$= \sqrt{2} - \sqrt{3} + 1$

133. We have, $a + b + c = 21$ and $\frac{a + c}{2} = b$

$\Rightarrow \quad a + c + \frac{a + c}{2} = 21$

$\Rightarrow \quad a + c = 14 \Rightarrow \frac{a + c}{2} = 7$

$\Rightarrow \quad b = 7$

So, a can take values from 1 to 6,

c can have values from 8 to 13

or $\quad a = b = c = 7 \quad [\because a \le b \le c]$

So, there are 7 such triplets.

134. For intersecting points, $e^{x^2} = e^{x^2} \sin x$

$\Rightarrow \quad e^{x^2}(\sin x - 1) = 0$

$\Rightarrow \quad e^{x^2} = 0$ or $\sin x = 1$

But $\quad e^{x^2} \ne 0 \Rightarrow \sin x = 1$

$\Rightarrow \quad x = \frac{\pi}{2}$

Now, $\quad y = e^{x^2}$

$\therefore \quad \frac{dy}{dx} = e^{x^2} \cdot 2x = 2xe^{x^2}$

Also, $y = e^{x^2} \sin x$

$\frac{dy}{dx} = e^{x^2} \cdot 2x\sin x + e^{x^2} \cos x = 2xe^{x^2}$

$[\because \sin x = 1, \cos x = 0]$

Since, both the curves has equal slope.

Hence, angle between the tangents at intersecting point is 0.

135. $2\cot^{-1}\frac{1}{2}-\cot^{-1}\frac{4}{3}$

$$=2\tan^{-1}2-\tan^{-1}\frac{3}{4}$$

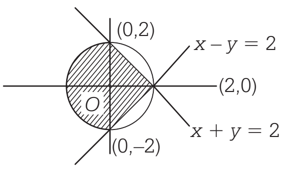
$$=\pi+\tan^{-1}\frac{4}{-3}-\tan^{-1}\frac{3}{4}$$

$$=\pi-\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{3}{4}$$

$$=\pi-\left\{\tan^{-1}\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\cdot\frac{3}{4}}\right\}$$

$$=\pi-\tan^{-1}\infty=\pi-\frac{\pi}{2}=\frac{\pi}{2}$$

136.



$(2\cos\theta, 2\sin\theta)$ will lie on the circle $x^2+y^2=4$ (from the above figure). Since, point lies on the region containing origin.

So, point will be on the shaded region.

$\therefore \quad \theta\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

137. Let the point be (h, k).

Then, $\left|\frac{h-2k+1}{\sqrt{1^2+(-2)^2}}\right|=\sqrt{5}$

$\Rightarrow \quad h-2k+1=\pm 5 \quad \ldots(i)$

Also, $\left|\frac{2h+3k-1}{\sqrt{2^2+3^2}}\right|=\sqrt{13}$

$\Rightarrow \quad 2h+3k-1=\pm 13 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get four points. So, there are such 4 points.

138. $\because \; P(x)=(x-3)(x-5)$

$$=Q(x)+(ax+b)$$

Given, $\quad P(3)=10$ and $P(5)=6$

$\Rightarrow \quad 3a+b=10 \quad \ldots(i)$

and $\quad 5a+b=6 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$a=-2$ and $b=16$

$\therefore$ Remainder $=-2x+16$

139. Given, $\frac{dy}{dx}=-(3x^2\tan^{-1}y-x^3)(1+y^2)$

$\Rightarrow \quad \frac{dy}{dx}=+x^3(1+y^2)-3x^2(\tan^{-1}y)(1+y^2)$

$\Rightarrow \frac{1}{(1+y^2)}\cdot\frac{dy}{dx}=x^3-3x^2\tan^{-1}y$

$\Rightarrow \frac{1}{1+y^2}\cdot\frac{dy}{dx}+3x^2\tan^{-1}y=x^3$

Put $\tan^{-1}y=t \Rightarrow \frac{1}{1+y^2}\cdot\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore \quad \frac{dt}{dx}+3tx^2=x^3$

which is linear differential equation in t.

Now, $\quad \text{IF}=e^{\int 3x^2\,dx}=e^{x^3}$

140. Given, $\quad y=e^{-x}\cos 2x \quad \ldots(i)$

$\therefore \quad \frac{dy}{dx}=e^{-x}(-\sin 2x)2+\cos 2x\cdot e^{-x}(-1)$

$\Rightarrow \quad \frac{dy}{dx}=-2\sin 2x\cdot e^{-x}-y$

$\Rightarrow \quad \frac{dy}{dx}+y=-2\sin 2x\cdot e^{-x}$

$\Rightarrow \frac{d^2y}{dx^2}+\frac{dy}{dx}=-2[\sin 2x\cdot e^{-x}(-1)+e^{-x}2\cos 2x]$

$=2\sin 2x\cdot e^{-x}-4y$ [from Eq. (i)]

$\Rightarrow \quad \frac{d^2y}{dx^2}+\frac{dy}{dx}=-\frac{dy}{dx}-y-4y$

$\Rightarrow \quad \frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$

141. Given, $f(0)=0$ and $f(0)=2$

$\therefore \lim_{x\to 0}\frac{1}{x}[f(x)+f(2x)+f(3x)+\ldots+f(2015x)]$

$$=\lim_{x\to 0}\frac{f'(x)+2f'(2x)+3f'(3x)+\ldots+2015f'(2015x)}{1}$$

[applying L' Hospital's rule]

$$=\frac{2+2\times 2+3\times 2+\ldots+2015\times 2}{1}$$

$=2[1+2+3+\ldots+2015]$

$$= \frac{2(2015)(2015+1)}{2} = \frac{2 \times 2015 \times 2016}{2}$$

$$= 2015 \times 2016$$

142. Given, $17! = 3556\,xy428096000$

Since, 17! is divisible by 9, so sum of the digits $(48 + x + y)$ must be divisible by 9.

So, $x + y$ can be 15 or 6.

Also, 17! is divisible by 11, so $|10 + x - y|$ must be multiple of 11 or 0. The only possibility is $|x - y| = 1$.

$\therefore \quad x + y = 15$

143. Let the event of person goes to office by a car, scooter, bus or train be A, B, C and D, respectively.

We have, $P(A) = \frac{1}{7}$, $P(B) = \frac{3}{7}$, $P(C) = \frac{2}{7}$

and $P(D) = \frac{1}{7}$

Let E = He reached office in time

We have,

$$P\left(\frac{\overline{E}}{A}\right) = \frac{2}{9}, P\left(\frac{\overline{E}}{B}\right) = \frac{1}{9}, P\left(\frac{\overline{E}}{C}\right) = \frac{4}{9}$$

and $P\left(\frac{\overline{E}}{D}\right) = \frac{1}{9}$

$$P\left(\frac{A}{E}\right) = \frac{P(A)\cdot P\left(\frac{E}{A}\right)}{P(A)\cdot P\left(\frac{E}{A}\right) + P(B)\cdot P\left(\frac{E}{B}\right) + P(C)\cdot P\left(\frac{E}{C}\right) + P(D)\cdot P\left(\frac{E}{D}\right)}$$

$$= \frac{\frac{1}{7}\cdot\frac{7}{9}}{\frac{1}{7}\cdot\frac{7}{9} + \frac{3}{7}\cdot\frac{8}{9} + \frac{2}{7}\cdot\frac{5}{9} + \frac{1}{7}\cdot\frac{8}{9}}$$

$$= \frac{7}{7 + 24 + 10 + 8}$$

$$= \frac{7}{49} = \frac{1}{7}$$

144. Let $I = \int \frac{(x-2)\,dx}{\{(x-2)^2(x+3)^7\}^{1/3}}$

$$= \int \frac{dx}{(x-2)^{-1/3}(x+3)^{7/3}}$$

$$= \int \frac{dx}{(x-2)^2\left(\frac{x+3}{x-2}\right)^{7/3}}$$

Put $\frac{x+3}{x-2} = t \Rightarrow \frac{-5\,dx}{(x-2)^2} = dt$

$$\therefore \quad I = -\frac{1}{5}\int \frac{dt}{t^{7/3}} = -\frac{1}{5}\int t^{-7/3}\,dt$$

$$= -\frac{1}{5}\left[\frac{t^{-7/3+1}}{-\frac{7}{3}+1}\right] + C = -\frac{1}{5}\left[\frac{t^{-4/3}}{-\frac{4}{3}}\right] + C$$

$$= \frac{3}{20t^{4/3}} + C = \frac{3}{20}\left(\frac{x-2}{x+3}\right)^{4/3} + C$$

145. We have, $f : N \to R$ such that $f(1) = 1$

and $f(1) + 2f(2) + 3f(3) + \ldots + nf(n)$

$$= n(n+1)\,f(n), \forall\, n \geq 2$$

Clearly, $f(1) + 2f(2) = 2(2+1)\,f(2)$

$\Rightarrow \quad f(1) = 6f(2) - 2f(2)$

$\Rightarrow \quad f(1) = 4f(2)$

$\Rightarrow \quad f(2) = \frac{f(1)}{4} = \frac{1}{4}$

Similarly,

$$f(1) + 2f(2) + 3f(3) = 3(3+1)\,f(3)$$

$\Rightarrow \quad 1 + \frac{1}{2} + 3f(3) = 12f(3)$

$\Rightarrow \quad 9f(3) = \frac{3}{2} \Rightarrow f(3) = \frac{1}{6}$

and $f(1) + 2f(2) + 3f(3) + 4f(4) = 4(5)f(4)$

$\Rightarrow \quad 1 + \frac{1}{2} + \frac{1}{2} = 16f(4)$

$\Rightarrow \quad f(4) = \frac{2}{16} = \frac{1}{8}$

In general, $f(n) = \frac{1}{2n}$

$\therefore \quad f(500) = \frac{1}{1000}$

146. Clearly, 5 distinct balls can be placed into 5 cells in 5^5 ways.

Now, the number of ways of selecting one cell is ${}^5C_1$.

Let the selected cell be empty. Now, we are left with 5 balls and 4 cells.

Now, number of ways of placing 5 distinct balls into 4 cells such that each cell have atleast one ball

$$= 4^5 - {}^4C_1(4-1)^5 + {}^4C_2(4-2)^5 - {}^4C_3(4-3)^5$$

$$= 1024 - 4 \times 3^5 + 6 \times 2^5 - 4$$

$= 1024 - 972 + 192 - 4 = 1216 - 976$
$= 240$

Thus, the number of ways of placing 5 distinct balls such that exactly one cell remains empty $= {}^5C_1 \times 240$

$\therefore$ Required probability

$$= \frac{{}^5C_1 \times 240}{5^5} = \frac{240}{625} = \frac{48}{125}$$

147. Let

S = Event that person is smoker

NS = Event that person is non-smoker

D = Event that death is due to lung cancer. Now, probability of death due to cancer that a person is a smoker,

$$P(D) = P(S)\cdot P\left(\frac{D}{S}\right) + P(NS)\cdot P\left(\frac{D}{NS}\right)$$

$$\Rightarrow 0.006 = \frac{20}{100} \times P\left(\frac{D}{S}\right) + \frac{80}{100} \times \frac{1}{10} \times P\left(\frac{D}{S}\right)$$

$$\Rightarrow \frac{6}{1000} = \frac{2}{10} P\left(\frac{D}{S}\right) + \frac{8}{100} P\left(\frac{D}{S}\right)$$

$$\Rightarrow \quad P\left(\frac{D}{S}\right)\left[\frac{2}{10} + \frac{8}{100}\right] = \frac{6}{1000}$$

$$\Rightarrow \quad P\left(\frac{D}{S}\right)\left[\frac{20+8}{100}\right] = \frac{6}{1000}$$

$$\therefore \quad P\left(\frac{D}{S}\right) = \frac{6}{1000} \times \frac{100}{28}$$

$$= \frac{3}{140}$$

148. Tangents drawn from external points are of equal length.

$\Rightarrow \quad BD = BE = a - r \quad [\because CD = r]$

and $\quad AF = AE = b - r \quad [\because CF = r]$

$\because \quad AE + BE = AB$

$\Rightarrow \quad b - r + a - r = 2R$

[$\because AB$ is a diameter of circumcircle]

$\Rightarrow \quad b + a = 2R + 2r$

$\Rightarrow \quad 2(r + R) = a + b$

149. Given equation is $a\cos\theta + b\sin\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) + \frac{2b\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = c$$

$$\left[\because \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \text{ and } \sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right]$$

$$\Rightarrow a\left(1-\tan^2\frac{\theta}{2}\right) + 2b\tan\frac{\theta}{2} = c\left(1+\tan^2\frac{\theta}{2}\right)$$

$$\Rightarrow a - a\tan^2\frac{\theta}{2} + 2b\tan\frac{\theta}{2} - c - c\tan^2\frac{\theta}{2} = 0$$

$$\Rightarrow (c+a)\tan^2\frac{\theta}{2} - 2b\tan\frac{\theta}{2} + (c-a) = 0$$

Let α and β be the roots of the equation.

$$\therefore \alpha + \beta = \frac{2b}{c+a} \quad \text{and} \quad \alpha\beta = \frac{c-a}{c+a}$$

$$\text{Now, } \tan\frac{\alpha+\beta}{2} = \frac{\frac{2b}{c+a}}{1 - \frac{c-a}{c+a}}$$

$$= \frac{\frac{2b}{c+a}}{\frac{c+a-c+a}{c+a}} = \frac{b}{a}$$

Since, $\frac{b}{a}$ is a root of the equation.

$$\therefore \quad (c+a)\frac{b^2}{a^2} - 2b\left(\frac{b}{a}\right) + c - a = 0$$

$$\Rightarrow b^2c + b^2a - 2b^2a + ca^2 - a^3 = 0$$

$$\Rightarrow \quad -b^2a + b^2c + ca^2 - a^3 = 0$$

$$\Rightarrow \quad b^2c - b^2a + ca^2 - a^3 = 0$$

$$\Rightarrow \quad b^2(c-a) + a^2(c-a) = 0$$

$$\Rightarrow \quad (c-a)(b^2 + a^2) = 0$$

$$\Rightarrow \quad c - a = 0 \quad \text{or} \quad b^2 + a^2 = 0$$

$$\Rightarrow \quad c = a \quad \text{or} \quad b^2 + a^2 = 0$$

150. Let $U_i = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix}$, where $i = 1, 2, 3$

$$\therefore \quad AU_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a_1 \\ 2a_1 + b_1 \\ 3a_1 + 2b_1 + c_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \left[\because AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]$$

$\therefore \quad a_1 = 1,\ 2a_1 + b_1 = 0$

$\Rightarrow \quad 2 + b_1 = 0$

$\Rightarrow \quad b_1 = -2$

and $3a_1 + 2b_1 + c_1 = 0$

$\Rightarrow \quad 3 + 2(-2) + c_1 = 0$

$\Rightarrow \quad 3 - 4 + c_1 = 0$

$\Rightarrow \quad c_1 = 1$

Similarly, $AU_2 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a_2 \\ 2a_2 + b_2 \\ 3a_2 + 2b_2 + c_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \qquad \left[\because AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}\right]$$

$\therefore \quad a_2 = 2$ and $2a_2 + b_2 = 3$

$\Rightarrow \quad 2 \times 2 + b_2 = 3$

$\Rightarrow \quad 4 + b_2 = 3$

$\Rightarrow \quad b_2 = -1$

and $\quad 3a_2 + 2b_2 + c_2 = 0$

$\Rightarrow \quad 3 \times 2 + 2(-1) + c_2 = 0$

$\Rightarrow \quad 6 - 2 + c_2 = 0$

$\Rightarrow \quad c_2 = -4$

Now, $AU_3 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} a_3 \\ b_3 \\ c_3 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a_3 \\ 2a_3 + b_3 \\ 3a_3 + 2b_3 + c_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \left[\because AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right]$$

$\therefore \quad a_3 = 2$ and $2a_3 + b_3 = 3$

$\Rightarrow \quad 2(2) + b_3 = 3$

$\Rightarrow \quad 4 + b_3 = 3$

$\Rightarrow \quad b_3 = -1$

and $\quad 3a_3 + 2b_3 + c_3 = 1$

$\Rightarrow \quad 3 \times 2 + 2(-1) + c_3 = 1$

$\Rightarrow \quad 6 - 2 + c_3 = 1$

$\Rightarrow \quad c_3 = -3$

$$\therefore\ U = \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{pmatrix}$$

$|U| = 1\,(3 - 4) - 2\,(6 + 1) + 2\,(8 + 1)$

$= -1 - 14 + 18 = 3$

$$\text{adj}\, U = \begin{bmatrix} -1 & -7 & 9 \\ -2 & -5 & 6 \\ 0 & -3 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$$

$$\Rightarrow U^{-1} = \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} & 0 \\ -\frac{7}{3} & -\frac{5}{3} & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

Hence, the sum of all elements of U^{-1} is 0.

151. We have, f is continuous and differentiable function on $[a, b]$.

Also, $\quad f(a) = f(b) = 0.$

By Rolle's theorem $\exists\, c \in (a, b)$

such that $\quad f'(c) = 0$

Thus, $\exists\, x \in (a, b)$ such that $f'(x) = 0$

Let $\quad$ at $x = c \in (a,b)$, $f'(c) = 0$

Now, f is continuously differentiable on $[a, b]$.

$\Rightarrow \quad f'$ is continuous on $[a, b]$

Also, f is twice differentiable on (a, b)

$\therefore\ f'$ is differentiable on (a, b)

and $\quad f'(a) = 0 = f'(c)$

By Rolle's theorem $\exists\, k \in (a, c)$ such that $f''(k) = 0$

Thus, $\exists\, x \in (a, c)$ such that $f''(x) = 0$

So, $\exists\, x \in (a, b)$ such that $f''(x) = 0$

Let us consider, $f(x) = (x - a)^2\,(x - b)$,

where $\quad f(a) = f(b) = f'(a) = 0$ but

$f''(a) \neq 0$ and $f'''(x) \neq 0$ for any $x \in (a, b)$

152. We have, $x \rho y : xy > 0$

(i) Reflexive suppose $x \rho x \in R \quad$ (Relation)

$\Rightarrow \quad x^2 > 0$

which is not true when $x = 0$.

Hence, relation is not reflexive.

(ii) Symmetric $\ x \rho y \in R$.

$\Rightarrow \quad xy > 0$

$\Rightarrow \quad yx > 0$

$\Rightarrow \quad y \rho x \in R$

If $(x, y) \in R$, then hence, relation is symmetric.

(iii) Transitive $\ x \rho y \in R$

$\Rightarrow \quad xy > 0$

and $\quad y \rho z \in R$

$\Rightarrow \quad yz > 0$

$\Rightarrow \quad xy^2 z > 0$

$\Rightarrow \quad xz > 0$

$\Rightarrow \quad (x, z) \in R$

If $(x, y) \in R$, then $(y, z) \in R$

$\Rightarrow \quad (x, z) \in R$

Hence, relation is transitive.

153. Option (a) Let $f(x) = \cos x$ and $g(x) = \sin x$

Consider the Wronskian of $f(x)$ and $g(x)$,

$$W = \begin{vmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x = 1 \neq 0$$

Thus, the functions are linearly independent. So, the general solution of given differential equation is given by $y = A\cos x + B\sin x$, where A and B are real constants.

[$\because$ if y_1 and y_2 are linearly independent solutions of the differential equation $ay'' + by' + c = 0$, then the general solution is $y = c_1y_1 + c_2y_2$, where c_1 and c_2 are constants].

Hence, option (a) is true.

Option (b) Let $y = A\cos\left(x + \frac{\pi}{4}\right)$

$$= A\left(\cos x \cdot \cos\frac{\pi}{4} - \sin x \cdot \sin\frac{\pi}{4}\right)$$

$$= \frac{A}{\sqrt{2}}(\cos x - \sin x)$$

$$= \frac{A}{\sqrt{2}}\cos x + \left(-\frac{A}{\sqrt{2}}\right)\sin x$$

which is in the form of general solution.

Hence, option (b) is true.

Option (c) Let $y = A\cos x \sin x$, which can not be expressed in the form of general solution.

Option (d) Let $y = A\cos\left(x + \frac{\pi}{4}\right) + B\sin\left(x - \frac{\pi}{4}\right)$

$$= A\left(\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}}\right) + B\left(\sin x \cdot \frac{1}{\sqrt{2}} - \cos x \cdot \frac{1}{\sqrt{2}}\right)$$

$= \cos x\left(\frac{A}{\sqrt{2}} - \frac{B}{\sqrt{2}}\right) + \sin x\left(\frac{B}{\sqrt{2}} - \frac{A}{\sqrt{2}}\right)$ which is in the form of general solution.

Hence, option (d) is true.

154. $0 < \theta < \frac{\pi}{2}$; $\cos\theta$ is strictly decreasing function.

Option (a) When $\theta > \frac{\theta}{2}$, then

$$\cos\theta \leq \cos\frac{\theta}{2}$$

$\therefore \quad \sqrt{\cos\theta} \leq \cos\frac{\theta}{2}$ [correct]

Option (b) When $\theta > \frac{3\theta}{4}$, then

$$\cos\theta \leq \cos\frac{3\theta}{4}$$

$\therefore \quad (\cos\theta)^{3/4} \leq \cos\frac{3\theta}{4}$ [incorrect]

Option (c) When $\theta > \frac{5\theta}{6}$, then

$$\cos\theta \leq \cos\frac{5\theta}{6}$$

$\therefore \quad (\cos\theta)^{5/6} \leq \cos\frac{5\theta}{6}$ [correct]

Option (d) When $\frac{7\theta}{8} < \theta$, then

$$\cos\theta \leq \cos\frac{7\theta}{8}$$

But $\quad (\cos\theta)^{7/8} \leq \cos\frac{7\theta}{8}$ [incorrect]

155. Given equation of hyperbola is

$$16x^2 - 3y^2 - 32x - 12y = 44$$

$$\Rightarrow 16x^2 + 16 - 32x - 3y^2 - 12 - 12y = 44 + 4$$

$$\Rightarrow \quad 16(x-1)^2 - 3(y+2)^2 = 48$$

$$\Rightarrow \quad \frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$$

On comparing the equation with standard equation of hyperbola,

we get $\quad a = \sqrt{3}, b = 4$

Now, length of transverse axis

$$= 2a = 2\sqrt{3}$$

and length of latusrectum

$$= \frac{2b^2}{a} = \frac{2 \times 16}{\sqrt{3}} = \frac{32}{\sqrt{3}}$$

$\therefore$ Eccentricity $(e) = \sqrt{1 + \frac{b^2}{a^2}}$

$$= \sqrt{1 + \frac{16}{3}} = \sqrt{\frac{19}{3}}$$

Equation of directrix is $x = \pm \frac{a}{e}$

$\Rightarrow x - 1 = \pm \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{19}}$

$\Rightarrow \quad x = 1 \pm \frac{3}{\sqrt{19}}$

156. We have, $f(x) = \left[\frac{1}{[x]}\right]$

Domain $= R - \{f(x) = 0\}$

Now, $f(x)$ will be zero, when $[x] = 0$.

$\Rightarrow \quad x \in [0, 1)$

$\therefore$ Domain of $f(x) = R - [0, 1)$

and range of $f(x) = \{-1, 0, 1\}$

157. If a quadratic equation has irrational coefficients, then roots are also irrational.

So, option (c) is always false.

158. Given equation straight line is

$$(a-1)x - by + 4 = 0$$

Slope $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$

$$= \frac{-(a-1)}{-b} = \frac{a-1}{b}$$

Equation of hyperbola is

$$xy = 1$$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{x^2}$

Slope of normal $= x^2 > 0$

$\therefore \quad \frac{a-1}{b} > 0$

$\Rightarrow \quad a - 1 > 0$ and $b > 0$

or $\quad a - 1 < 0$ and $b < 0$

$\Rightarrow \quad a > 1, b > 0$ or $a < 1, b < 0$

159. Given probability of defective part

$$= 0.05 = \frac{1}{20}$$

Probability of non-defective part

$$= 1 - 0.05 = 0.95 = \frac{19}{20}$$

We know that, $P(X = r) = {}^nC_r\, p^r q^{n-r}$

where, $\quad p = \frac{1}{20}, q = \frac{19}{20}$

$r \geq 1$ and $n = ?$

Also, $\quad P(X \geq 1) \geq \frac{1}{2}$

$\Rightarrow 1 - P(X = 0) \geq \frac{1}{2}$

$\Rightarrow 1 - {}^hC_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{n-0} \geq \frac{1}{2}$

$\Rightarrow \quad 1 - \frac{1}{2} \geq \left(\frac{19}{20}\right)^n$

$\Rightarrow \quad \frac{1}{2} \geq \left(\frac{19}{20}\right)^n$

$\Rightarrow \quad \frac{1}{2} \geq \left(\frac{95}{100}\right)^n$

$\Rightarrow \quad \log 2^{-1} \geq n\,[\log 95 - \log 100]$

$\Rightarrow \quad -\log 2 \geq n\,[\log 95 - 2]$

$\Rightarrow \quad -(0.3) \geq n\,[1.977 - 2]$

$\Rightarrow \quad n \geq \frac{0.3}{0.023}$

$\Rightarrow \quad n \geq \frac{300}{23}$

$\Rightarrow \quad n \geq 13.04$

$\therefore \quad n = 14, 15$

160. Given, $f : R \to R$

and $\quad f(2x - 1) = f(x), x \in R$

Hence, f is continuous at $x = 1$ and $f(1) = 1$.