WBJEE 2014 Question Paper

West Bengal Joint Entrance Examinations

WB JEE

Engineering Entrance Exam

Solved Paper 2014

Physics

Category I

Directions (*Q. Nos.* 1-45) *Carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of* 1/3 *mark.*

1. Consider three vectors $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{C} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. A vector $\mathbf{X}$ of the form $\alpha\mathbf{A} + \beta\mathbf{B}$ (α and β are numbers) is perpendicular to $\mathbf{C}$. The ratio of α and β is

(a) 1 : 1 (b) 2 : 1
(c) –1 : 1 (d) 3 : 1

2. Three capacitors $3\,\mu F$, $6\,\mu F$ and $6\,\mu F$ are connected in series to a source of 120 V. The potential difference, in volt, across the $3\,\mu F$ capacitor will be

(a) 24 (b) 30 (c) 40 (d) 60

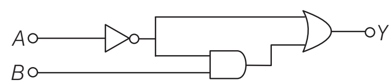
3. A galvanometer having internal resistance $10\,\Omega$ requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full-scale deflection at 120 V, we need to connect a resistance of

(a) $11990\,\Omega$ in series
(b) $11990\,\Omega$ in parallel
(c) $12010\,\Omega$ in series
(d) $12010\,\Omega$ in parallel

4. A whistle whose air column is open at both ends has a fundamental frequency of 5100 Hz. If the speed of sound in air is $340\,ms^{-1}$, the length of the whistle, in cm, is

(a) 5/3 (b) 10/3
(c) 5 (d) 20/3

5. The output Y of the logic circuit given below is



(a) $\overline{A} + B$ (b) $\overline{A}$
(c) $\overline{(\overline{A} + B)}\cdot\overline{A}$ (d) $\overline{(\overline{A} + B)}\cdot A$

6. One mole of an ideal monoatomic gas is heated at a constant pressure from 0°C to 100°C. Then the change in the internal energy of the gas is (Given, $R = 8.32\ J\ mol^{-1}K^{-1}$)

(a) 0.83×10^3 J (b) 4.6×10^3 J
(c) 2.08×10^3 J (d) 1.25×10^3 J

7. The ionization energy of hydrogen is 13.6 eV. The energy of the photon released when an electron jumps from the first excited state ($n = 2$) to the ground state of a hydrogen atom is

(a) 3.4 eV (b) 4.53 eV
(c) 10.2 eV (d) 13.6 eV

8. A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now moved farther apart by pulling at them by means of insulating handles, then

(a) the energy stored in the capacitor decreases
(b) the capacitance of the capacitor increases
(c) the charge on the capacitor decreases
(d) the voltage across the capacitor increases

9. When a particle executing SHM oscillates with a frequency ν, then the kinetic energy of the particle

(a) changes periodically with a frequency of ν
(b) changes periodically with a frequency of 2ν
(c) changes periodically with a frequency of $\nu/2$
(d) remains constant

10. In which of the following pairs, the two physical quantities have different dimensions?

(a) Planck's constant and angular momentum
(b) Impulse and linear momentum
(c) Moment of inertia and moment of a force
(d) Energy and torque

11. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(a) $\dfrac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1}$ (b) $\dfrac{h_1 t_1^2 + h_2 t_2^2}{h_2 t_1 + h_1 t_2}$

(c) $\dfrac{h_1 t_2^2 + h_2 t_1^2}{h_1 t_2 + h_2 t_1}$ (d) $\dfrac{h_1 t_1^2 - h_2 t_2^2}{h_1 t_1 - h_2 t_2}$

12. A small metal sphere of radius a is falling with a velocity v through a vertical column of a viscous liquid. If the coefficient of viscosity of the liquid is η, then the sphere encounters an opposing force of

(a) $6\pi\eta a^2 v$ (b) $\dfrac{6\eta v}{\pi a}$ (c) $6\pi\eta a v$ (d) $\dfrac{\pi\eta v}{6a^3}$

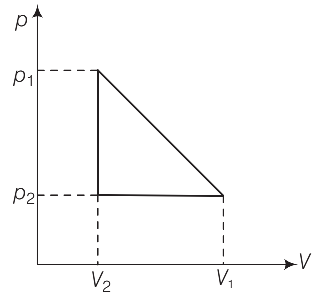
13. A scientist proposes a new temperature scale in which the ice point is 25 X (X is the new unit of temperature) and the steam point is 305 X. The specific heat capacity of water in this new scale is (in J kg^{-1}X^{-1})

(a) 4.2×10^3 (b) 3.0×10^3
(c) 1.2×10^3 (d) 1.5×10^3

14. One mole of a van der Waals' gas obeying the equation

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

undergoes the quasi-static cyclic process which is shown in the p-V diagram. The net heat absorbed by the gas in this process is



(a) $\dfrac{1}{2}(p_1 - p_2)(V_1 - V_2)$

(b) $\dfrac{1}{2}(p_1 + p_2)(V_1 - V_2)$

(c) $\dfrac{1}{2}\left(p_1 + \dfrac{a}{V_1^2} - p_2 - \dfrac{a}{V_2^2}\right)(V_1 - V_2)$

(d) $\dfrac{1}{2}\left(p_1 + \dfrac{a}{V_1^2} + p_2 + \dfrac{a}{V_2^2}\right)(V_1 - V_2)$

15. A wooden block is floating on water kept in a beaker. 40% of the block is above the water surface. Now the beaker is kept inside a lift that starts going upward with acceleration equal to $g/2$. The block will then

(a) sink
(b) float with 10% above the water surface
(c) float with 40% above the water surface
(d) float with 70% above the water surface

16. A smooth massless string passes over a smooth fixed pulley. Two masses m_1 and m_2, $(m_1 > m_2)$ are tied at the two ends of the string. The masses are allowed to move under gravity starting from rest. The total external force acting on the two masses is

(a) $(m_1 + m_2)g$ (b) $\dfrac{(m_1 - m_2)^2}{m_1 + m_2} g$

(c) $(m_1 - m_2)g$ (d) $\dfrac{(m_1 + m_2)^2}{m_1 - m_2} g$

17. To determine the coefficient of friction between a rough surface and a block, the surface is kept inclined at 45° and the block is released from rest. The block takes a time t in moving a distance d. The rough surface is then replaced by a smooth surface and the same experiment is repeated. The block now takes a time $t/2$ in moving down the same distance d. The coefficient of friction is

(a) 3/4 (b) 5/4 (c) 1/2 (d) $1/\sqrt{2}$

18. A metal rod is fixed rigidly at two ends so as to prevent its thermal expansion. If L, α and Y respectively denote the length of the rod, coefficient of linear thermal expansion and Young's modulus of its material, then for an increase in temperature of the rod by ΔT, the longitudinal stress developed in the rod is

(a) inversely proportional to α
(b) inversely proportional to Y
(c) directly proportional to $\Delta T / Y$
(d) independent of L

19. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be

(a) $2r \cos\left(\frac{\theta}{2}\right)$ (b) $2r \cot\left(\frac{\theta}{2}\right)$
(c) $2r \tan\left(\frac{\theta}{2}\right)$ (d) $2r \sin\left(\frac{\theta}{2}\right)$

20. A uniform rod is suspended horizontally from its mid-point. A piece of metal whose weight is w is suspended at a distance l from the mid-point. Another weight W_1 is suspended on the other side at a distance l_1 from the mid-point to bring the rod to a horizontal position. When w is completely immersed in water, w_1 needs to be kept at a distance l_2 from the mid-point to get the rod back into horizontal position. The specific gravity of the metal piece is

(a) $\frac{w}{w_1}$ (b) $\frac{wl_1}{wl - w_1 l_2}$
(c) $\frac{l_1}{l_1 - l_2}$ (d) $\frac{l_1}{l_2}$

21. A drop of some liquid of volume 0.04 cm^3 is placed on the surface of a glass slide. Then another glass slide is placed on it in such a way that the liquid forms a thin layer of area 20 cm^2 between the surfaces of the two slides. To separate the slides a force of 16×10^5 dyne has to be applied normal to the surfaces. The surface tension of the liquid is (in dyne-cm^{-1})

(a) 60 (b) 70 (c) 80 (d) 90

22. An electron in a circular orbit of radius 0.05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in Am^2)

(a) 2.16×10^{-23} (b) 3.21×10^{-22}
(c) 3.21×10^{-24} (d) 1.26×10^{-23}

23. A very small circular loop of radius a is initially (at $t = 0$) coplanar and concentric with a much larger fixed circular loop of radius b. A constant current I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is

(a) $\frac{\pi a^2 \mu_0 I}{2b} \omega \cos(\omega t)$ (b) $\frac{\pi a^2 \mu_0 I}{2b} \omega \sin(\omega^2 t^2)$
(c) $\frac{\pi a^2 \mu_0 I}{2b} \omega \sin(\omega t)$ (d) $\frac{\pi a^2 \mu_0 I}{2b} \omega \sin^2(\omega t)$

24. A luminous object is separated from a screen by distance d. A convex lens is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is

(a) $4d$ (b) $2d$
(c) $\frac{d}{2}$ (d) $\frac{d}{4}$

25. An infinite sheet carrying a uniform surface charge density σ lies on the xy-plane. The work done to carry a charge q from the point $\mathbf{A} = a(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ to the point $\mathbf{B} = a(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ (where a is a constant with the dimension of length and ε_0 is the permittivity of free space) is

(a) $\frac{3\sigma a q}{2\varepsilon_0}$ (b) $\frac{2\sigma a q}{\varepsilon_0}$
(c) $\frac{5\sigma a q}{2\varepsilon_0}$ (d) $\frac{3\sigma a q}{\varepsilon_0}$

26. The intensity of magnetization of a bar magnet is 5.0×10^4 Am^{-1}. The magnetic length and the area of cross-section of the magnet are 12 cm and 1 cm^2 respectively. The magnitude of magnetic moment of this bar magnet is (in SI unit)

(a) 0.6 (b) 1.3 (c) 1.24 (d) 2.4

27. A particle moves with constant acceleration along a straight line starting from rest. The percentage increase in its displacement during the 4th second compared to that in the 3rd second is

(a) 33% (b) 40% (c) 66% (d) 77%

28. A proton of mass m and charge q is moving in a plane with kinetic energy E. If there exists a uniform magnetic field B, perpendicular to the plane of the motion, the proton will move in a circular path of radius

(a) $\frac{2Em}{qB}$ (b) $\frac{\sqrt{2Em}}{qB}$ (c) $\frac{\sqrt{Em}}{2qB}$ (d) $\sqrt{\frac{2Eq}{mB}}$

29. An artificial satellite moves in a circular orbit around the earth. Total energy of the satellite is given by E. The potential energy of the satellite is

(a) $-2E$ (b) $2E$ (c) $\frac{2E}{3}$ (d) $-\frac{2E}{3}$

30. In which of the following phenomena, the heat waves travel along straight lines with the speed of light?

(a) Thermal conduction (b) Forced convection
(c) Natural convection (d) Thermal radiation

31. In the bandgap between valence band and conduction band in a material is 5.0 eV, then the material is

(a) semiconductor (b) good conductor
(c) superconductor (d) insulator

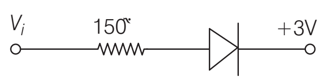
32. A uniform solid spherical ball is rolling down a smooth inclined plane from a height h. The velocity attained by the ball when it reaches the bottom of the inclined plane is v. If the ball is now thrown vertically upwards with the same velocity v, the maximum height to which the ball will rise is

(a) $\frac{5h}{8}$ (b) $\frac{3h}{5}$ (c) $\frac{5h}{7}$ (d) $\frac{7h}{9}$

33. Two coherent monochromatic beams of intensities I and $4I$ respectively are superposed. The maximum and minimum intensities in the resulting pattern are

(a) $5I$ and $3I$ (b) $9I$ and $3I$
(c) $4I$ and I (d) $9I$ and I

34. In the circuit shown assume the diode to be ideal. When V_i increases from 2V to 6 V, the change in the current is (in mA)



(a) zero (b) 20
(c) 80/3 (d) 40

35. If n denotes a positive integer, h the Planck's constant, q the charge and B the magnetic field, then the quantity $\left[\frac{nh}{2\pi qB}\right]$ has the dimension of

(a) area (b) length
(c) speed (d) acceleration

36. In a transistor output characteristics commonly used in common emitter configuration, the base current I_B, the collector current I_C and the collector-emitter voltage V_{CE} have values of the following orders of magnitude in the active region

(a) I_B and I_C both are in μA and V_{CE} in volt
(b) I_B is in μA and I_C is in mA and V_{CE} in volt
(c) I_B is in mA and I_C is in μA and V_{CE} in mV
(d) I_B is in mA and I_C is in mA and V_{CE} in mV

37. The displacement of a particle in a periodic motion is given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$.

This displacement may be considered as the result of superposition of n independent harmonic oscillations. Here n is

(a) 1 (b) 2 (c) 3 (d) 4

38. Consider a black body radiation in a cubical box at absolute temperature T. If the length of each side of the box is doubled and the temperature of the walls of the box and that of the radiation is halved, then the total energy

(a) halves (b) doubles
(c) quadruples (d) remains the same

39. Consider two concentric spherical metal shells of radii r_1 and r_2 ($r_2 > r_1$). If the outer shell has a charge q and the inner one is grounded, the charge on the inner shell is

(a) $\frac{-r_2}{r_1}q$ (b) zero
(c) $\frac{-r_1}{r_2}q$ (d) $-q$

40. Four cells, each of emf E and internal resistance r, are connected in series across an external resistance R. By mistake one of the cells is connected in reverse. Then the current in the external circuit is

(a) $\frac{2E}{4r+R}$ (b) $\frac{3E}{4r+R}$ (c) $\frac{3E}{3r+R}$ (d) $\frac{2E}{3r+R}$

41. Same quantity of ice is filled in each of the two metal containers P and Q having the same size, shape and wall thickness but made of different materials. The containers are kept in identical surroundings. The ice in P melts completely in time t_1 whereas in Q takes a time t_2. The ratio of thermal conductivities of the materials of P and Q is

(a) $t_2 : t_1$

(b) $t_1 : t_2$

(c) $t_1^2 : t_2^2$

(d) $t_2^2 : t_1^2$

42. For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur?

(a) Isobar of original nucleus is produced

(b) Isotope of the original nucleus is produced

(c) Nuclei with higher atomic number than that of the original nucleus is produced

(d) Nuclei with lower atomic number than that of the original nucleus is produced

43. A car is moving with a speed of 72 km-h^{-1} towards a roadside source that emits sound at a frequency of 850 Hz. The car driver listens to the sound while approaching the source and again while moving away from the source after crossing it. If the velocity of sound is 340 ms^{-1}, the difference of the two frequencies, the driver hears is

(a) 50 Hz (b) 85 Hz (c) 100 Hz (d) 150 Hz

44. The energy of gamma (γ) ray photon is E_γ and that of an X-ray photon is E_X. If the visible light photon has an energy of E_v, then we can say that

(a) $E_X > E_\gamma > E_v$ (b) $E_\gamma > E_v > E_x$

(c) $E_\gamma > E_X > E_v$ (d) $E_X > E_v > E_\gamma$

45. The intermediate image formed by the objective of a compound microscope is

(a) real, inverted and magnified

(b) real, erect and magnified

(c) virtual, erect and magnified

(d) virtual, inverted and magnified

Category II

Directions (*Q. Nos.* 46-55) *Carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of* 2/3 *mark.*

46. A solid uniform sphere resting on a rough horizontal plane is given a horizontal impulse directed through its centre so that it starts sliding with an initial velocity v_0. When it finally starts rolling without slipping the speed of its centre is

(a) $\frac{2}{7}v_0$ (b) $\frac{3}{7}v_0$ (c) $\frac{5}{7}v_0$ (d) $\frac{6}{7}v_0$

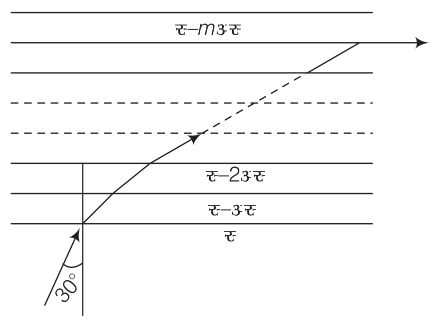
47. The de-Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass is 0.5 MeV)

(a) 1 : 50 (b) 1 : 20

(c) 20 : 1 (d) 50 : 1

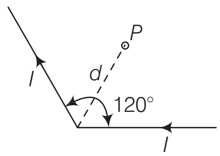
48. A glass slab consists of thin uniform layers of progressively decreasing refractive indices RI (see figure) such that the RI of any layer is $\mu - m\,\Delta\mu$. Here, μ and $\Delta\mu$ denote the RI of 0th layer and the difference in RI between any two consecutive layers, respectively. The integer $m = 0, 1, 2, 3, \ldots$ denotes the numbers of the successive layers.

A ray of light from the 0th layer enters the 1st layer at an angle of incidence of 30°. After undergoing the mth refraction, the ray emerges parallel to the interface. If $\mu = 1.5$ and $\Delta\mu = 0.015$, the value of m is



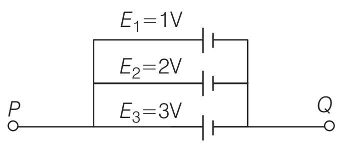
(a) 20 (b) 30 (c) 40 (d) 50

49. A long conducting wire carrying a current I is bent at 120° (see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is (μ_0 is the permeability of free space)



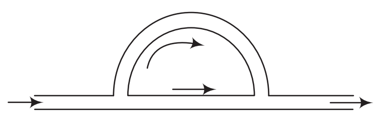
(a) $\frac{3\mu_0 I}{2\pi d}$ (b) $\frac{\mu_0 I}{2\pi d}$ (c) $\frac{\mu_0 I}{\sqrt{3}\,\pi d}$ (d) $\frac{\sqrt{3}\,\mu_0 I}{2\pi d}$

50. A circuit consists of three batteries of emf $E_1 = 1$ V, $E_2 = 2$ V and $E_3 = 3$ V and internal resistances 1 Ω, 2 Ω and 1 Ω respectively which are connected in parallel as shown in the figure. The potential difference between points P and Q is



(a) 1.0 V (b) 2.0 V (c) 2.2 V (d) 3.0 V

51. Sound waves are passing through two routes-one in straight path and the other along a semicircular path of radius r and are again combined into one pipe and superposed as shown in the figure. If the velocity of sound waves in the pipe is v, then frequencies of resultant waves of maximum amplitude will be integral multiples of



(a) $\frac{v}{r(\pi - 2)}$ (b) $\frac{v}{r(\pi - 1)}$

(c) $\frac{2v}{r(\pi - 1)}$ (d) $\frac{v}{r(\pi + 1)}$

52. To determine the composition of a bimetallic alloy, a sample is first weighed in air and then in water. These weights are found to be w_1 and w_2 respectively. If the densities of the two constituents metals are ρ_1 and ρ_2 respectively, then the weight of the first metal in the sample is (where ρ_w is the density of water)

(a) $\frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 - \rho_w) - w_2\rho_2]$

(b) $\frac{\rho_1}{\rho_w(\rho_2 + \rho_1)}[w_1(\rho_2 - \rho_w) + w_2\rho_2]$

(c) $\frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 + \rho_w) - w_2\rho_1]$

(d) $\frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_1 - \rho_w) - w_2\rho_1]$

53. A 10 W electric heater is used to heat a container filled with 0.5 kg of water. It is found that the temperature of water and the container rises by 3°K in 15 min. The container is then emptied, dried and filled with 2 kg of oil. The same heater now raises the temperature of container-oil system by 2°K in 20 min. Assuming that there is no heat loss in the process and the specific heat of water is 4200 J kg^{-1}K^{-1}, the specific heat of oil in the same unit is equal to

(a) 1.50×10^3

(b) 2.55×10^3

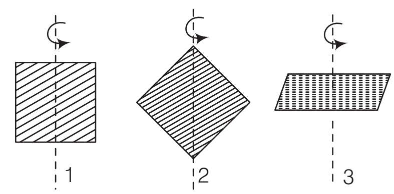
(c) 3.00×10^3

(d) 5.10×10^3

54. An object is placed 30 cm away from a convex lens of focal length 10 cm and a sharp image is formed on a screen. Now a concave lens is placed in contact with the convex lens. The screen now has to be moved by 45 cm to get a sharp image again. The magnitude of focal length of the concave lens is (in cm)

(a) 72 (b) 60 (c) 36 (d) 20

55. Three identical square plates rotate about the axes shown in the figure in such a way that their kinetic energies are equal. Each of the rotation axes passes through the centre of the square. Then the ratio of angular speeds $\omega_1 : \omega_2 : \omega_3$ is



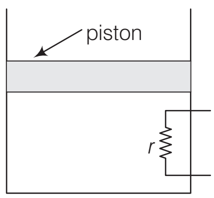
(a) 1 : 1 : 1 (b) $\sqrt{2} : \sqrt{2} : 1$

(c) $1 : \sqrt{2} : 1$ (d) $1 : 2 : \sqrt{2}$

Category III

Directions (*Q. Nos.* 56-60) *Carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective questions - irrespective of the number of correct options marked.*

56. A heating element of resistance r is fitted inside an adiabatic cylinder which carries a frictionless piston of mass m and cross-section A as shown in diagram. The cylinder contains one mole of an ideal diatomic gas. The current flows through the element such that the temperatures rises with time t as $\Delta T = \alpha t + \frac{1}{2}\beta t^2$



(α and β are constants), while pressure remains constant. The atmospheric pressure above the piston is P_0. Then

(a) the rate of increase in internal energy is $\frac{5}{2}R(\alpha + \beta t)$

(b) the current flowing in the element is $\sqrt{\frac{5}{2r}R(\alpha + \beta t)}$

(c) the piston moves upwards with constant acceleration

(d) the piston moves upwards with constant speed

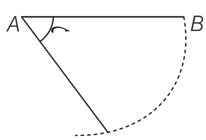
57. A thin rod AB is held horizontally so that it can freely rotate in a vertical plane about the end A as shown in the figure. The potential energy of the rod when it hangs vertically is taken to be zero. The end B of the rod is released from rest from a horizontal position. At the instant the rod makes an angle θ with the horizontal

(a) the speed of end B is proportional to $\sqrt{\sin\theta}$

(b) the potential energy is proportional to $(1 - \cos\theta)$

(c) the angular acceleration is proportional to $\cos\theta$

(d) the torque about A remains the same as its initial value



58. Find the correct statement(s) about photoelectric effect.

(a) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons

(b) Einstein analysis gives a threshold frequency above which no electron can be emitted

(c) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation

(d) The maximum kinetic energy of electrons does not depend on the intensity of radiation

59. Half of the space between the plates of a parallel-plate capacitor is filled with a dielectric material of dielectric constant K. The remaining half contains air as shown in the figure. The capacitor is now given a charge Q. Then

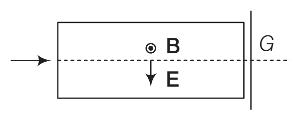
(a) electric field in the dielectric-filled region is higher than that in the air-filled region

(b) on the two halves of the bottom plate the charge densities are unequal

(c) charge on the half of the top plate above the air-filled part is $\frac{Q}{K+1}$

(d) capacitance of the capacitor shown above is $(1 + K)\frac{C_0}{2}$, where C_0 is the capacitance of the same capacitor with the dielectric removed.

60. A stream of electrons and protons are directed towards a narrow slit in a screen (see figure). The intervening region has a uniform electric field **E** (vertically downwards) and a uniform magnetic field **B** (out of the plane of the figure) as shown. Then



(a) electrons and protons with speed $\frac{|E|}{|B|}$ will pass through the slit

(b) protons with speed $|E|/|B|$ will pass through the slit, electrons of the same speed will not

(c) neither electrons nor protons will go through the slit irrespective of their speed

(d) electrons will always be deflected upwards irrespective of their speed

Chemistry

Category I

Directions (*Q. Nos.* 1-45) *Carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of* 1/3 *mark.*

1. The emission spectrum of hydrogen discovered first and the region of the electromagnetic spectrum in which it belongs, respectively are

(a) Lyman, ultraviolet (b) Lyman, visible
(c) Balmer, ultraviolet (d) Balmer, visible

2. The electronic configuration of Cu is

(a) [Ne] $3s^2, 3p^6, 3d^9, 4s^2$
(b) [Ne] $3s^2, 3p^6, 3d^{10}, 4s^1$
(c) [Ne] $3s^2, 3p^6, 3d^3, 4s^2, 4p^6$
(d) [Ne] $3s^2, 3p^6, 3d^5, 4s^2, 4p^4$

3. As per de-Broglie's formula a macroscopic particle of mass 100 g and moving at a velocity of 100 cm s^{-1} will have a wavelength of

(a) 6.6×10^{-29} cm (b) 6.6×10^{-30} cm
(c) 6.6×10^{-31} cm (d) 6.6×10^{-32} cm

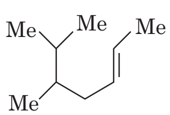
4. For one mole of an ideal gas, the slope of V *vs.* T curve at constant pressure of 2 atm is X L mol^{-1} K^{-1}. The value of the ideal universal gas constant 'R' in terms of X is

(a) X L atm $mol^{-1}K^{-1}$ (b) $\frac{X}{2}$ L atm $mol^{-1}K^{-1}$
(c) $2X$ L atm $mol^{-1}K^{-1}$ (d) $2X$ atm $L^{-1}mol^{-1}K^{-1}$

5. At a certain temperature the time required for the complete diffusion of 200 mL of H_2 gas is 30 min. The time required for the complete diffusion of 50 mL of O_2 gas at the same temperature will be

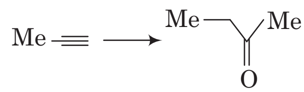
(a) 60 min (b) 30 min
(c) 45 min (d) 15 min

6. The IUPAC name of the following molecule is



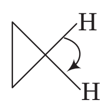
(a) 5,6-dimethylhept-2-ene
(b) 2,3-dimethylhept-5-ene
(c) 5,6-dimethylhept-3-ene
(d) 5-*iso*-propylhex-2-ene

7. The reagents to carry out the following conversion are

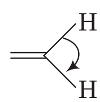


(a) $HgSO_4$/dil $\cdot H_2SO_4$
(b) BH_3; H_2O_2/NaOH
(c) OsO_4; HIO_4
(d) $NaNH_2$/ CH_3I; $HgSO_4$/ dil $\cdot H_2SO_4$

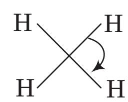
8. The correct order of decreasing H—C—H angle in the following molecule is



I



II



III

(a) I > II > III
(b) II > I > III
(c) III > II > I
(d) I > III > II

9. During the emission of a positron from a nucleus, the mass number of the daughter element remains the same but the atomic number

(a) is decreased by 1 unit
(b) is decreased by 2 units
(c) is increased by 1 unit
(d) remains unchanged

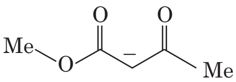
10. β-emission is always accompanied by

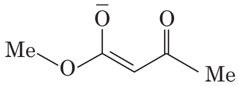
(a) formation of antineutrino and α-particle
(b) emission of α-particle and γ-ray
(c) formation of antineutrino and γ-ray
(d) formation of antineutrino and positron

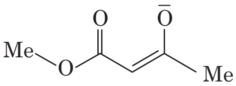
11. Four gases P, Q, R and S have almost same values of 'b' but their 'a' values (a, b are van der Waals' constants) are in the order $Q < R < S < P$. At a particular temperature, among the four gases, the most easily liquefiable one is

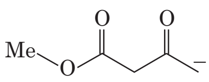
(a) P (b) Q (c) R (d) S

12. Among the following structures the one which is not a resonating structure of others is


I

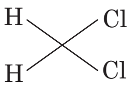

II

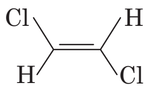

III

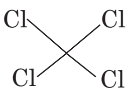

IV

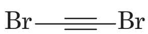
(a) I (b) II
(c) III (d) IV

13. The compound that will have a permanent dipole moment among the following is


I

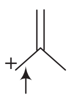

II

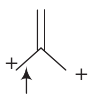

III

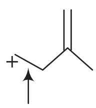

IV

(a) I (b) II
(c) III (d) IV

14. The correct order of decreasing length of the bond as indicated by the arrow in the following structures is


I


II


III

(a) I > II > III (b) II > I > III
(c) III > II > I (d) I > III > II

15. An atomic nucleus having low n/p ratio tries to find stability by

(a) the emission of an α-particle
(b) the emission of a positron
(c) capturing an orbital electron (K-electron capture)
(d) emission of a β-particle

16. $({}_{32}Ge^{76}, {}_{34}Se^{76})$ and $({}_{14}Si^{30}, {}_{16}S^{32})$ are examples of

(a) isotopes and isobars
(b) isobars and isotones
(c) isotones and isotopes
(d) isobars and isotopes

17. ${}_{98}Cf^{246}$ was formed along with a neutron when an unknown radioactive substance was bombarded with ${}_{6}C^{12}$. The unknown substance was

(a) ${}_{91}Pa^{234}$ (b) ${}_{90}Th^{234}$ (c) ${}_{92}U^{235}$ (d) ${}_{92}U^{238}$

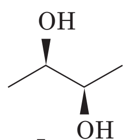
18. The rate of a certain reaction is given by, rate = $k\,[H^+]^n$.
The rate increases 100 times when the pH changes from 3 to 1. The order (n) of the reaction is

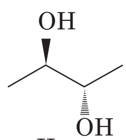
(a) 2 (b) 0 (c) 1 (d) 1.5

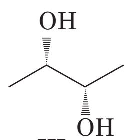
19. The values of ΔH and ΔS of a certain reaction are $-400\ \text{kJ mol}^{-1}$ and $-20\ \text{kJ mol}^{-1}\text{K}^{-1}$ respectively. The temperature below which the reaction is spontaneous, is

(a) 100 K (b) 20°C (c) 20 K (d) 120°C

20. The correct statement regarding the following compounds is

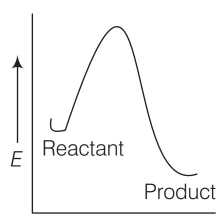

I


II

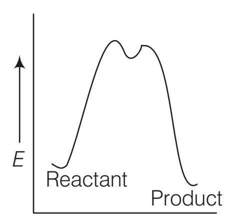

III

(a) all three compounds are chiral
(b) only I and II are chiral
(c) I and III are diastereomers
(d) only I and III are chiral

21. The correct statement regarding the following energy diagrams is



Reaction M



Reaction N

(a) Reaction M is faster and less exothermic than reaction N
(b) Reaction M is slower and less exothermic than reaction N
(c) Reaction M is faster and more exothermic than reaction N
(d) Reaction M is slower and more exothermic than reaction N

22. In the following reaction, the product E is

$$\begin{array}{c}CHO\\ |\\ CHO\end{array} \xrightarrow[\text{(ii) } H^+]{\text{(i) NaOH}} E$$

(a) $\begin{array}{l}CH_2OH\\ |\\ CHO\end{array}$ (b) $\begin{array}{l}CHO\\ |\\ CO_2H\end{array}$

(c) $\begin{array}{l}CH_2OH\\ |\\ CO_2H\end{array}$ (d) $\begin{array}{l}CO_2H\\ |\\ CO_2H\end{array}$

23. If Cl_2 is passed through hot aqueous NaOH, the products formed have Cl in different oxidation states. These are indicated as

(a) −1 and +1 (b) −1 and +5
(c) +1 and +5 (d) −1 and +3

24. Commercial sample of H_2O_2 is labeled as 10 V. Its % strength is nearly

(a) 3 (b) 6 (c) 9 (d) 12

25. The enthalpy of vaporisation of a certain liquid at its boiling point of 35°C is 24.64 kJ mol^{-1}. The value of change in entropy for the process is

(a) 704 $JK^{-1}mol^{-1}$ (b) 80 $JK^{-1}mol^{-1}$
(c) 24.64 $JK^{-1}mol^{-1}$ (d) 7.04 $JK^{-1}mol^{-1}$

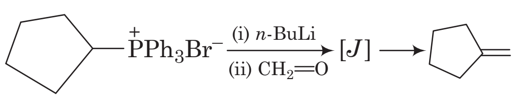
26. Given that

$$C + O_2 \longrightarrow CO_2; \Delta H^\circ = -x \text{ kJ}$$
$$2CO + O_2 \longrightarrow 2CO_2; \Delta H^\circ = -y \text{ kJ}$$

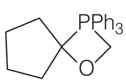
The heat of formation of carbon monoxide will be

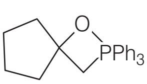
(a) $\frac{y-2x}{2}$ (b) $y+2x$ (c) $2x-y$ (d) $\frac{2x-y}{2}$

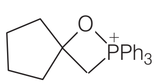
27. The intermediate J in the following Wittig reaction is



(a) 

(b) 

(c) 

(b) 

28. Among the following compounds, the one(s) that gives (give) effervescence with aqueous $NaHCO_3$ solution is (are)

$(CH_3CO)_2O$	CH_3COOH	PhOH	CH_3COCHO
I	II	III	IV

(a) I and II (b) I and III
(c) Only II (d) I and IV

29. The 4th higher homologue of ethane is

(a) butane (b) pentane (c) hexane (d) heptane

30. In case of heteronuclear diatomics of the type AB, where A is more electronegative than B, bonding molecular orbital resembles the character of A more than that of B. The statement

(a) is false
(b) is true
(c) cannot be evaluated since data is not sufficient
(d) is true only for certain systems

31. The hydrides of the first elements in groups 15-17, namely NH_3, H_2O and HF respectively show abnormally high values for melting and boiling points. This is due to

(a) small size of N, O and F
(b) the ability to form extensive intermolecular H-bonding
(c) the ability to form extensive intramolecular H-bonding
(d) effective van der Waals' interaction

32. The quantity of electricity needed to separately electrolyze 1 M solution of $ZnSO_4$, $AlCl_3$ and $AgNO_3$ completely is in the ratio of

(a) 2 : 3 : 1 (b) 2 : 1 : 1
(c) 2 : 1 : 3 (d) 2 : 2 : 1

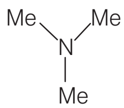
33. The amount of electrolytes required to coagulate a given amount of AgI colloidal solution (–ve charge) will be in the order

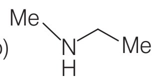
(a) $NaNO_3 > Al_2(NO_3)_3 > Ba(NO_3)_2$
(b) $Al_2(NO_3)_3 > Ba(NO_3)_2 > NaNO_3$
(c) $Al_2(NO_3)_3 > NaNO_3 > Ba(NO_3)_2$
(d) $NaNO_3 > Ba(NO_3)_2 > Al_2(NO_3)_3$

34. The value of ΔH for cooling 2 mole of an ideal monoatomic gas from 225°C to 125°C at constant pressure will be [given $C_p = \frac{5}{2}R$]

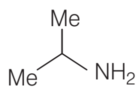
(a) 250 R (b) −500 R
(c) 500 R (d) −250 R

35. An amine C_3H_9N reacts with benzene sulphonyl chloride to form a white precipitate which is insoluble in *aq*. NaOH. The amine is

(a) 

(b) 

(c) 

(b) 

36. The number of amino acids and number of peptide bonds in a linear tetrapeptide (made of different amino acids) are respectively

(a) 4 and 4 (b) 5 and 5
(c) 5 and 4 (d) 4 and 3

37. Among the following, the one which is not a "green house gas", is

(a) N_2O (b) CO_2 (c) CH_4 (d) O_2

38. The pH of 10^{-4} M KOH solution will be

(a) 4 (a) 11 (c) 10.5 (d) 10

39. The system that contains the maximum number of atoms is

(a) 4.25 g of NH_3 (b) 8 g of O_2
(c) 2 g of H_2 (d) 4 g of He

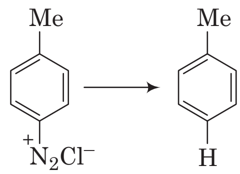
40. Among the following observations, the correct one that differentiates between SO_3^{2-} and SO_4^{2-} is

(a) both form precipitate with $BaCl_2$, SO_3^{2-} dissolves in HCl but SO_4^{2-} does not
(b) SO_3^{2-} forms precipitate with $BaCl_2$, SO_4^{2-} does not
(c) SO_4^{2-} forms precipitate with $BaCl_2$, SO_3^{2-} does not
(d) both form precipitate with $BaCl_2$, SO_4^{2-} dissolves in HCl but SO_3^{2-} does not

41. Metal ion responsible for the Minimata disease is

(a) Co^{2+} (b) Hg^{2+}
(c) Cu^{2+} (d) Zn^{2+}

42. The reagent with which the following reaction is best accomplished is



(a) H_3PO_2 (b) H_3PO_3
(c) H_3PO_4 (d) $NaHSO_3$

43. In DNA, the consecutive deoxynucleotides are connected *via*

(a) phosphodiester linkage
(b) phosphomonoester linkage
(c) phosphotriester linkage
(d) amide linkage

44. The reaction of aniline with chloroform under alkaline conditions leads to the formation of

(a) phenylcyanide
(b) phenylisonitrile
(c) phenylcyanate
(d) phenylisocyanate

45. The two half-cell reactions of an electrochemical cell is given as

$$Ag^+ + e^- \longrightarrow Ag; \quad E^\circ_{Ag^+/Ag} = -0.3995\text{ V}$$
$$Fe^{2+} \longrightarrow Fe^{3+} + e^-; E^\circ_{Fe^{3+}/Fe^{2+}} = -0.7120\text{ V}$$

The value of cell EMF will be

(a) −0.3125 V (b) 0.3125 V
(c) 1.114 V (d) −1.114 V

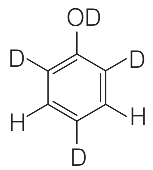
Category II

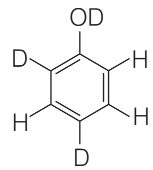
Directions (*Q. Nos.* 46-55) *Carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of* 2/3 *mark.*

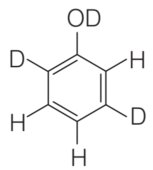
46. The compressibility factor (Z) of one mole of a van der Waals' gas of negligible 'a' value is

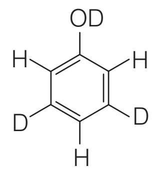
(a) 1 (b) $\frac{bp}{RT}$ (c) $1 + \frac{bp}{RT}$ (d) $1 - \frac{bp}{RT}$

47. When phenol is treated with D_2SO_4 / D_2O, some of the hydrogens get exchanged. The final product in this exchange reaction is

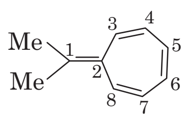
(a) 

(b) 

(c) 

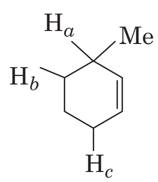
(d) 

48. The most likely protonation site in the following molecule is



(a) C-1 (b) C-2
(c) C-3 (d) C-6

49. The order of decreasing ease of abstraction of hydrogen atoms in the following molecule



(a) $H_a > H_b > H_c$
(b) $H_a > H_c > H_b$
(c) $H_b > H_a > H_c$
(d) $H_c > H_b > H_a$

50. At 25°C, the molar conductance of 0.007 M hydrofluoric acid is 150 mho $cm^2 mol^{-1}$ and its $\Lambda^\circ_m = 500$ mho $cm^2 mol^{-1}$. The value of the dissociation constant of the acid at the given concentration at 25°C is

(a) 7×10^{-4} M (b) 7×10^{-5} M
(c) 9×10^{-3} M (d) 9×10^{-4} M

51. To observe an elevation of boiling point of 0.05°C, the amount of a solute (mol. wt. = 100) to be added to 100 g of water ($K_b = 0.5$) is

(a) 2 g (b) 0.5 g (c) 1 g (d) 0.75 g

52. The volume of ethyl alcohol (density 1.15 g/cc) that has to be added to prepare 100 cc of 0.5 M ethyl alcohol solution in water is

(a) 1.15 cc (b) 2 cc (c) 2.15 cc (d) 2.30 cc

53. The bond angle in NF_3 (102.3°) is smaller than NH_3 (107.2°). This is because of

(a) large size of F compared to H
(b) large size of N compared to F
(c) opposite polarity of N in the two molecules
(d) small size of H compared to N

54. A piece of wood from an archaeological sample has 5.0 counts min^{-1} per gram of C-14, while a fresh sample of wood has a count of 15.0 $min^{-1} g^{-1}$. If half-life of C-14 is 5770 yr, the age of the archaeological sample is

(a) 8,500 yr (b) 9,200 yr
(c) 10,000 yr (d) 11,000 yr

55. The structure of XeF_6 is experimentally determined to be distorted octahedron. Its structure according to VSEPR theory is

(a) octahedron
(b) trigonal bipyramid
(c) pentagonal bipyramid
(d) tetragonal bipyramid

Category III

Directions (*Q. Nos.* 56-60) *Carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rate basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question - irrespective of the number of correct options marked.*

56. Two gases X (molecular weight M_X) and Y (molecular weight M_Y; $M_Y > M_X$) are at the same temperature T in two different containers. Their root mean square velocities are C_X and C_Y respectively. If the average kinetic energies per molecule of two gases X and Y are E_X and E_Y respectively then which of the following relation(s) is(are) true?

(a) $E_X > E_Y$
(b) $C_X > C_Y$
(c) $E_X = E_Y = (3/2)RT$
(d) $E_X = E_Y = (3/2)k_BT$

57. For a spontaneous process, the correct statement(s) is (are)

(a) $(\Delta G_{\text{system}})_{T,p} > 0$
(b) $(\Delta S_{\text{system}}) + (\Delta S_{\text{surroundings}}) > 0$
(c) $(\Delta G_{\text{system}})_{T,p} < 0$
(d) $(\Delta U_{\text{system}})_{T,V} > 0$

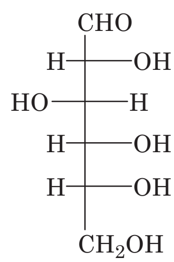
58. The formal potential of Fe^{3+}/Fe^{2+} in a sulphuric acid and phosphoric acid mixture ($E° = +0.61$ V) is much lower than the standard potential ($E° = +0.77$ V). This is due to

(a) formation of the species $[FeHPO_4]^+$
(b) lowering of potential upon complexation
(c) formation of the species $[FeSO_4]^+$
(d) high acidity of the medium

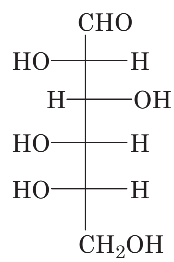
59. Cupric compounds are more stable than their cuprous counterparts in solid state. This is because

(a) the endothermic character of the 2nd IP of Cu is not so high
(b) size of Cu^{2+} is less than Cu^+
(c) Cu^{2+} has stabler electronic configuration as compared to Cu^+
(d) the lattice energy released for cupric compounds is much higher than Cu^+

60. Among the following statements about the molecules X and Y, the one(s) which correct is (are)



X



Y

(a) X and Y are diastereomers
(b) X and Y are enantiomers
(c) X and Y are both aldohexoses
(d) X is a D-sugar and Y is an L-sugar

Mathematics

Category I

Directions (*Q. Nos.* 1-60) *Carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of* 1/3 *mark.*

1. The number of solution(s) of the equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ is/are

(a) 2 (b) 0
(c) 3 (d) 1

2. The value of $|z|^2 + |z-3|^2 + |z-i|^2$ is minimum when z equals

(a) $2 - \frac{2}{3}i$ (b) $45 + 3i$ (c) $1 + \frac{i}{3}$ (d) $1 - \frac{i}{3}$

3. If $f(x)=\begin{cases}2x^2+1, & x\le 1\\ 4x^3-1, & x>1\end{cases}$, then $\int_0^2 f(x)\,dx$ is

(a) 47/3 (b) 50/3 (c) 1/3 (d) 47/2

4. If $\lim_{x\to 0}\dfrac{2a\sin x-\sin 2x}{\tan^3 x}$ exists and is equal to 1, then the value of a is

(a) 2 (b) 1 (c) 0 (d) –1

5. The solution of the equation

$$\log_{101}\log_7(\sqrt{x+7}+\sqrt{x})=0 \text{ is}$$

(a) 3 (b) 7 (c) 9 (d) 49

6. The integrating factor of the differential equation $(1+x^2)\dfrac{dy}{dx}+y=e^{\tan^{-1}x}$ is

(a) $\tan^{-1}x$ (b) $1+x^2$

(c) $e^{\tan^{-1}x}$ (d) $\log_e(1+x^2)$

7. If $\sqrt{y}=\cos^{-1}x$, then it satisfies the differential equation $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=c$, where c is equal to

(a) 0 (b) 3 (c) 1 (d) 2

8. The number of digits in 20^{301} (given, $\log_{10}2=0.3010$) is

(a) 602 (b) 301 (c) 392 (d) 391

9. The area of the region bounded by the curves $y=x^2$ and $x=y^2$ is

(a) 1/3 (b) 1/2 (c) 1/4 (d) 3

10. If R be the set of all real numbers and $f:R\to R$ is given by $f(x)=3x^2+1$. Then, the set $f^{-1}([1,6])$ is

(a) $\left\{-\sqrt{\frac{5}{3}}, 0, \sqrt{\frac{5}{3}}\right\}$ (b) $\left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right]$

(c) $\left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$ (d) $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

11. The value of $\tan\dfrac{\pi}{5}+2\tan\dfrac{2\pi}{5}+4\cot\dfrac{4\pi}{5}$ is

(a) $\cot\dfrac{\pi}{5}$ (b) $\cot\dfrac{2\pi}{5}$ (c) $\cot\dfrac{4\pi}{5}$ (d) $\cot\dfrac{3\pi}{5}$

12. Let $f(x)$ be a differentiable function in [2, 7]. If $f(2)=3$ and $f'(x)\le 5$ for all x in (2, 7), then the maximum possible value of $f(x)$ at $x=7$ is

(a) 7 (b) 15

(c) 28 (d) 14

13. Let the number of elements of the sets A and B be p and q, respectively. Then, the number of relations from the set A to the set B is

(a) 2^{p+q} (b) 2^{pq} (c) $p+q$ (d) pq

14. In a ΔABC, $\tan A$ and $\tan B$ are the roots of $pq(x^2+1)=r^2x$. Then, ΔABC is

(a) a right angled triangle

(b) an acute angled triangle

(c) an obtuse angled triangle

(d) an equilateral triangle

15. If $y=4x+3$ is parallel to a tangent to the parabola $y^2=12x$, then its distance from the normal parallel to the given line is

(a) $\dfrac{213}{\sqrt{17}}$ (b) $\dfrac{219}{\sqrt{17}}$

(c) $\dfrac{211}{\sqrt{17}}$ (d) $\dfrac{210}{\sqrt{17}}$

16. Let the equation of an ellipse be $\dfrac{x^2}{144}+\dfrac{y^2}{25}=1$. Then, the radius of the circle with centre $(0,\sqrt{2})$ and passing through the foci of the ellipse is

(a) 9 (b) 7 (c) 11 (d) 5

17. The straight lines $x+y=0$, $5x+y=4$ and $x+5y=4$ form

(a) an isosceles triangle

(b) an equilateral triangle

(c) a scalene triangle

(d) a right angled triangle

18. If $\sin^{-1}\left(\dfrac{x}{13}\right)+\operatorname{cosec}^{-1}\left(\dfrac{13}{12}\right)=\dfrac{\pi}{2}$, then the value of x is

(a) 5 (b) 4

(c) 12 (d) 11

19. The value of λ for which the curve $(7x+5)^2+(7y+3)^2=\lambda^2(4x+3y-24)^2$ represents a parabola is

(a) $\pm\dfrac{6}{5}$ (b) $\pm\dfrac{7}{5}$

(c) $\pm\dfrac{1}{5}$ (d) $\pm\dfrac{2}{5}$

20. Let $f(x)=x+1/2$. Then, the number of real values of x for which the three unequal terms $f(x)$, $f(2x)$, $f(4x)$ are in HP is

(a) 1 (b) 0

(c) 3 (d) 2

21. Let $f(x) = 2x^2 + 5x + 1$. If we write $f(x)$ as
$f(x) = a(x+1)(x-2) + b(x-2)(x-1) + c(x-1)(x+1)$
for real numbers a, b, c, then
(a) there are infinite number of choices for a, b, c
(b) only one choice for a but infinite number of choices for b and c
(c) exactly one choice for each of a, b, c
(d) more than one but finite number of choices for a, b, c

22. If α, β are the roots of $ax^2 + bx + c = 0$ $(a \neq 0)$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$ $(p \neq 0)$, then the ratio of the squares of their discriminants is
(a) $a^2 : p^2$ (b) $a : p^2$
(c) $a^2 : p$ (d) $a : 2p$

23. Let p, q be real numbers. If α is the root of $x^2 + 3p^2x + 5q^2 = 0$, β is a root of $x^2 + 9p^2x + 15q^2 = 0$ and $0 < \alpha < \beta$, then the equation $x^2 + 6p^2x + 10q^2 = 0$ has a root γ that always satisfies
(a) $\gamma = \frac{\alpha}{4} + \beta$ (b) $\beta < \gamma$
(c) $\gamma = \frac{\alpha}{2} + \beta$ (d) $\alpha < \gamma < \beta$

24. The equation of the common tangent with positive slope to the parabola $y^2 = 8\sqrt{3}\,x$ and the hyperbola $4x^2 - y^2 = 4$ is
(a) $y = \sqrt{6}\,x + \sqrt{2}$ (b) $y = \sqrt{6}\,x - \sqrt{2}$
(c) $y = \sqrt{3}\,x + \sqrt{2}$ (d) $y = \sqrt{3}\,x - \sqrt{2}$

25. The point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 35 = 0$ has coordinates
(a) (9, –24) (b) (1, 81)
(c) (4, –16) (d) (–9, –24)

26. Let z_1, z_2 be two fixed complex numbers in the argand plane and z be an arbitrary point satisfying $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$. Then, the locus of z will be
(a) an ellipse
(b) a straight line joining z_1 and z_2
(c) a parabola
(d) a bisector of the line segment joining z_1 and z_2

27. The function $f(x) = \dfrac{\tan\left\{\pi\left[x - \frac{\pi}{2}\right]\right\}}{2 + [x]^2}$, where $[x]$ denotes the greatest integer $\leq x$, is
(a) continuous for all values of x
(b) discontinuous at $x = \frac{\pi}{2}$
(c) not differentiable for some values of x
(d) discontinuous at $x = -2$

28. The function $f(x) = a \sin|x| + be^{|x|}$ is differentiable at $x = 0$ when
(a) $3a + b = 0$ (b) $3a - b = 0$
(c) $a + b = 0$ (d) $a - b = 0$

29. If the coefficient of x^8 in $\left(ax^2 + \frac{1}{bx}\right)^{13}$ is equal to the coefficient of x^{-8} in $\left(ax - \frac{1}{bx^2}\right)^{13}$, then a and b will satisfy the relation
(a) $ab + 1 = 0$ (b) $ab = 1$
(c) $a = 1 - b$ (d) $a + b = -1$

30. If $I = \int_0^2 e^{x^4}(x - \alpha)\,dx = 0$, then α lies in the interval
(a) (0, 2) (b) (–1, 0)
(c) (2, 3) (d) (–2, –1)

31. The solution of the differential equation $y\dfrac{dy}{dx} = x\left[\dfrac{y^2}{x^2} + \dfrac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$ is (where, c is a constant)
(a) $\phi\left(\frac{y^2}{x^2}\right) = cx$ (b) $x\phi\left(\frac{y^2}{x^2}\right) = c$
(c) $\phi\left(\frac{y^2}{x^2}\right) = cx^2$ (d) $x^2\phi\left(\frac{y^2}{x^2}\right) = c$

32. Suppose that the equation $f(x) = x^2 + bx + c = 0$ has two distinct real roots α and β. The angle between the tangent to the curve $y = f(x)$ at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ and the positive direction of the x-axis is
(a) 0° (b) 30° (c) 60° (d) 90°

33. The function $f(x) = x^2 + bx + c$, where b and c real constants, describes

(a) one-to-one mapping
(b) onto mapping
(c) not one-to-one but onto mapping
(d) neither one-to-one nor onto mapping

34. Let $n \geq 2$ be an integer,

$A = \begin{bmatrix} \cos(2\pi/n) & \sin(2\pi/n) & 0 \\ -\sin(2\pi/n) & \cos(2\pi/n) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and I is the identity matrix of order 3. Then,

(a) $A^n = I$ and $A^{n-1} \neq I$
(b) $A^m \neq I$ for any positive integer m
(c) A is not invertible
(d) $A^m = O$ for a positive integer m

35. Ram is visiting a friend. Ram knows that his friend has 2 children and 1 of them is a boy. Assuming that a child is equally likely to be a boy or a girl, then the probability that the other child is a girl, is

(a) 1/2 (b) 1/3 (c) 2/3 (d) 7/10

36. The value of the sum

$({}^nC_1)^2 + ({}^nC_2)^2 + ({}^nC_3)^2 + \ldots + ({}^nC_n)^2$ is

(a) $({}^{2n}C_n)^2$ (b) ${}^{2n}C_n$
(c) ${}^{2n}C_n + 1$ (d) ${}^{2n}C_n - 1$

37. The remainder obtained when $1! + 2! + 3! + \ldots + 11!$ is divided by 12 is

(a) 9 (b) 8 (c) 7 (d) 6

38. Out of 7 consonants and 4 vowels, the number of words (not necessarily meaningful) that can be made, each consisting of 3 consonants and 2 vowels, is

(a) 24800 (b) 25100
(c) 25200 (d) 25400

39. Let $S = \frac{2}{1}\,{}^nC_0 + \frac{2^2}{2}\,{}^nC_1 + \frac{2^3}{3}\,{}^nC_2 + \ldots + \frac{2^{n+1}}{n+1}\,{}^nC_n$. Then, S equals

(a) $\frac{2^{n+1}-1}{n+1}$ (b) $\frac{3^{n+1}-1}{n+1}$
(c) $\frac{3^n-1}{n}$ (d) $\frac{2^n-1}{n}$

40. Let R be the set of all real numbers and $f : [-1, 1] \to R$ be defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then,

(a) f satisfies the conditions of Rolle's theorem on $[-1, 1]$
(b) f satisfies the conditions of Lagrange's mean value theorem on $[-1, 1]$
(c) f satisfies the conditions of Rolle's theorem on $[0, 1]$
(d) f satisfies the conditions of Lagrange's mean value theorem on $[0, 1]$

41. If a, b and c are positive numbers in a GP, then the roots of the quadratic equation $(\log_e a)\,x^2 - (2\log_e b)\,x + (\log_e c) = 0$ are

(a) -1 and $\frac{\log_e c}{\log_e a}$
(b) 1 and $-\frac{\log_e c}{\log_e a}$
(c) 1 and $\log_a c$
(d) -1 and $\log_c a$

42. There is a group of 265 persons who like either singing or dancing or painting. In this group 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of persons who like only dancing and painting is

(a) 10 (b) 20 (c) 30 (d) 40

43. The range of the function $y = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ is

(a) $[0, \sqrt{3/2}]$ (b) $[0, 1]$
(c) $[0, 3/\sqrt{2}]$ (d) $[0, \infty)$

44. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \cos(t^2)\,dt}{x \sin x}$ is

(a) 1 (b) -1
(c) 2 (d) $\log_e 2$

45. Let $f(x)$ be a differentiable function and $f'(4) = 5$. Then, $\lim_{x \to 2} \frac{f(4) - f(x^2)}{x - 2}$ equals

(a) 0 (b) 5
(c) 20 (d) -20

46. The sum of the series $\sum_{n=1}^{\infty} \sin\left(\frac{n!\,\pi}{720}\right)$ is

(a) $\sin\left(\frac{\pi}{180}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{540}\right)$
(b) $\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)$
(c) $\sin\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{30}\right)+\sin\left(\frac{\pi}{120}\right)+\sin\left(\frac{\pi}{360}\right)+\sin\left(\frac{\pi}{720}\right)$
(d) $\sin\left(\frac{\pi}{180}\right)+\sin\left(\frac{\pi}{360}\right)$

47. Let I denote the 3×3 identity matrix and P be a matrix obtained by rearranging the columns of I. Then,

(a) there are six distinct choices for P and $\det(P)=1$
(b) there are six distinct choices for P and $\det(P)=\pm 1$
(c) there are more than one choices for P and some of them are not invertible
(d) there are more than one choices for P and $P^{-1}=I$ in each choice

48. The coefficient of x^3 in the infinite series expansion of $\frac{2}{(1-x)(2-x)}$, for $|x|<1$, is

(a) $-\frac{1}{16}$ (b) $\frac{15}{8}$ (c) $-\frac{1}{8}$ (d) $\frac{15}{16}$

49. For every real number x,

let $f(x)=\frac{x}{1!}+\frac{3}{2!}x^2+\frac{7}{3!}x^3+\frac{15}{4!}x^4+\dots$. Then, the equation $f(x)=0$ has

(a) no real solution
(b) exactly one real solution
(c) exactly two real solutions
(d) infinite number of real solutions

50. Let S denote the sum of the infinite series $1+\frac{8}{2!}+\frac{21}{3!}+\frac{40}{4!}+\frac{65}{5!}+\dots$. Then,

(a) $S<8$ (b) $S>12$
(c) $8<S<12$ (d) $S=8$

51. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. Then, $\lim_{n\to\infty}\frac{[n\sqrt{2}]}{n}$ is equal to

(a) 0 (b) 2
(c) $\sqrt{2}$ (d) 1

52. Suppose that $f(x)$ is a differentiable function such that $f'(x)$ is continuous, $f'(0)=1$ and $f''(0)$ does not exist. Let $g(x)=xf'(x)$. Then,

(a) $g'(0)$ does not exist (b) $g'(0)=0$
(c) $g'(0)=1$ (d) $g'(0)=2$

53. Let z_1 be a fixed point on the circle of radius 1 centred at the origin in the argand plane and $z_1\neq\pm 1$. Consider an equilateral triangle inscribed in the circle with z_1, z_2, z_3 as the vertices taken in the counter clockwise direction. Then, $z_1z_2z_3$ is equal to

(a) z_1^2 (b) z_1^3
(c) z_1^4 (d) z_1

54. Suppose that z_1, z_2, z_3 are three vertices of an equilateral triangle in the argand plane. Let $\alpha=\frac{1}{2}(\sqrt{3}+i)$ and β be a non-zero complex number. The points $\alpha z_1+\beta, \alpha z_2+\beta, \alpha z_3+\beta$ will be

(a) the vertices of an equilateral triangle
(b) the vertices of an isosceles triangle
(c) collinear
(d) the vertices of a scalene triangle

55. The curve $y=(\cos x+y)^{1/2}$ satisfies the differential equation

(a) $(2y-1)\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2+\cos x=0$
(b) $\frac{d^2y}{dx^2}-2y\left(\frac{dy}{dx}\right)^2+\cos x=0$
(c) $(2y-1)\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)^2+\cos x=0$
(d) $(2y-1)\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2+\cos x=0$

56. In the argand plane, the distinct roots of $1+z+z^3+z^4=0$ (z is a complex number) represent vertices of

(a) a square (b) an equilateral triangle
(c) a rhombus (d) a rectangle

57. In a ΔABC, a, b, c are the sides of the triangle opposite to the angles A, B, C, respectively. Then, the value of $a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)$ is equal to

(a) 0 (b) 1
(c) 3 (d) 2

58. Let α, β be the roots of $x^2 - x - 1 = 0$ and $S_n = \alpha^n + \beta^n$, for all integers $n \geq 1$. Then, for every integer $n \geq 2$.

(a) $S_n + S_{n-1} = S_{n+1}$
(b) $S_n - S_{n-1} = S_{n+1}$
(c) $S_{n-1} = S_{n+1}$
(d) $S_n + S_{n-1} = 2S_{n+1}$

59. A fair six-faced die is rolled 12 times. The probability that each face turns up twice is equal to

(a) $\frac{12!}{6!\,6!\,6^{12}}$ (b) $\frac{2^{12}}{2^6 6^{12}}$ (c) $\frac{12!}{2^6 6^{12}}$ (d) $\frac{12!}{6^2 6^{12}}$

60. If α, β are the roots of the quadratic equation $x^2 + px + q = 0$, then the values of $\alpha^3 + \beta^3$ and $\alpha^4 + \alpha^2\beta^2 + \beta^4$ are respectively

(a) $3pq - p^3$ and $p^4 - 3p^2q + 3q^2$
(b) $-p(3q - p^2)$ and $(p^2 - q)(p^2 + 3q)$
(c) $pq - 4$ and $p^4 - q^4$
(d) $3pq - p^3$ and $(p^2 - q)(p^2 - 3q)$

Category II

Directions (*Q. Nos.* 61-75) *Carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of* 2/3 *mark.*

61. The solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x \log_e x} = \frac{1}{x}$$

under the condition $y = 1$ when $x = e$ is

(a) $2y = \log_e x + \frac{1}{\log_e x}$
(b) $y = \log_e x + \frac{2}{\log_e x}$
(c) $y \log_e x = \log_e x + 1$
(d) $y = \log_e x + e$

62. Let $f(x) = \max\{x + |x|, x - [x]\}$, where $[x]$ denotes the greatest integer $\leq x$. Then, the value of $\int_{-3}^{3} f(x)\, dx$ is

(a) 0 (b) 51/2
(c) 21/2 (d) 1

63. Let $X_n = \left\{z = x + iy : |z|^2 \leq \frac{1}{n}\right\}$ for all integers $n \geq 1$. Then, $\bigcap_{n=1}^{\infty} X_n$ is

(a) a singleton set
(b) not a finite set
(c) an empty set
(d) a finite set with more than one element

64. Applying Lagrange's Mean Value Theorem for a suitable function $f(x)$ in $[0, h]$, we have $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$. Then, for $f(x) = \cos x$, the value of $\lim_{h \to 0^+} \theta$ is

(a) 1 (b) 0 (c) 1/2 (d) 1/3

65. The equation of hyperbola whose coordinates of the foci are $(\pm 8, 0)$ and the length of latusrectum is 24 units, is

(a) $3x^2 - y^2 = 48$ (b) $4x^2 - y^2 = 48$
(c) $x^2 - 3y^2 = 48$ (d) $x^2 - 4y^2 = 48$

66. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

(a) $\frac{3p}{4p + 3}$ (b) $\frac{5p}{3p + 2}$
(c) $\frac{5p}{4p + 1}$ (d) $\frac{4p}{3p + 1}$

67. $\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}$

(a) is equal to zero (b) lies between 0 and 3
(c) is a negative number (d) lies between 3 and 6

68. Suppose $M = \int_0^{\pi/2} \frac{\cos x}{x + 2}\, dx$, $N = \int_0^{\pi/4} \frac{\sin x \cos x}{(x + 1)^2}\, dx$. Then, the value of $(M - N)$ equals

(a) $\frac{3}{\pi + 2}$ (b) $\frac{2}{\pi - 4}$
(c) $\frac{4}{\pi - 2}$ (d) $\frac{2}{\pi + 4}$

69. For any two real numbers θ and ϕ, we define $\theta R \phi$, if and only if $\sec^2\theta - \tan^2\phi = 1$. The relation R is

(a) reflexive but not transitive
(b) symmetric but not reflexive
(c) both reflexive and symmetric but not transitive
(d) an equivalence relation

70. The minimum value of $2^{\sin x} + 2^{\cos x}$ is

(a) $2^{1-1/\sqrt{2}}$ (b) $2^{1+1/\sqrt{2}}$
(c) $2^{\sqrt{2}}$ (d) 2

71. We define a binary relation ~ on the set of all 3×3 real matrices as $A \sim B$, if and only if there exist invertible matrices P and Q such that $B = PAQ^{-1}$. The binary relation ~ is

(a) neither reflexive nor symmetric
(b) reflexive and symmetric but not transitive
(c) symmetric and transitive but not reflexive
(d) an equivalence relation

72. Let α, β denote the cube roots of unity other than 1 and $\alpha \neq \beta$. Let $S = \sum_{n=0}^{302} (-1)^n \left(\frac{\alpha}{\beta}\right)^n$. Then, the value of S is

(a) either -2ω or $-2\omega^2$ (b) either -2ω or $2\omega^2$
(c) either 2ω or $-2\omega^2$ (d) either 2ω or $2\omega^2$

73. Let t_n denotes the nth term of the infinite series $\frac{1}{1!} + \frac{10}{2!} + \frac{21}{3!} + \frac{34}{4!} + \frac{49}{5!} + \dots$. Then, $\lim_{n \to \infty} t_n$ is

(a) e (b) 0 (c) e^2 (d) 1

74. A particle starting from a point A and moving with a positive constant acceleration along a straight line reaches another point B in time T. Suppose that the initial velocity of the particle is $u > 0$ and P is the mid-point of the line AB. If the velocity of the particle at point P is v_1 and if the velocity at time $\frac{T}{2}$ is v_2, then

(a) $v_1 = v_2$ (b) $v_1 > v_2$ (c) $v_1 < v_2$ (d) $v_1 = \frac{1}{2} v_2$

75. A poker hand consists of 5 cards drawn at random from a well-shuffled pack of 52 cards. Then, the probability that a poker hand consists of a pair and a triple of equal face values (for example, 2 sevens and 3 kings or 2 aces and 3 queens, etc.) is

(a) $\frac{6}{4165}$ (b) $\frac{23}{4165}$
(c) $\frac{1797}{4165}$ (d) $\frac{1}{4165}$

Category III

Directions (*Q. Nos.* 76-80) *Carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question-irrespective of the number of correct options marked.*

76. If $u(x)$ and $v(x)$ are two independent solutions of the differential equation

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0,$$

then additional solution(s) of the given differential equation is(are)

(a) $y = 5u(x) + 8v(x)$
(b) $y = c_1\{u(x) - v(x)\} + c_2 v(x)$, c_1 and c_2 are arbitrary constants
(c) $y = c_1 u(x)\, v(x) + c_2 u(x) / v(x)$, c_1 and c_2 are arbitrary constants
(d) $y = u(x)\, v(x)$

77. The angle of intersection between the curves $y = [\,|\sin x| + |\cos x|\,]$ and $x^2 + y^2 = 10$, where $[x]$ denotes the greatest integer $\leq x$, is

(a) $\tan^{-1} 3$ (b) $\tan^{-1}(-3)$
(c) $\tan^{-1}\sqrt{3}$ (d) $\tan^{-1}(1/\sqrt{3})$

78. Let $f(x) = \begin{cases} \int_0^x |1 - t|\, dt, & x > 0 \\ x - \frac{1}{2}, & x \leq 1 \end{cases}$. Then,

(a) $f(x)$ is continuous at $x = 1$
(b) $f(x)$ is not continuous at $x = 1$
(c) $f(x)$ is differentiable at $x = 1$
(d) $f(x)$ is not differentiable at $x = 1$

79. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the three circles $x^2 + y^2 - 5 = 0$, $x^2 + y^2 - 8x - 6y + 10 = 0$ and $x^2 + y^2 - 4x + 2y - 2 = 0$ at the extremities of their diameters, then

(a) $c = -5$ (b) $fg = 147/25$
(c) $g + 2f = c + 2$ (d) $4f = 3g$

80. For two events A and B, let $P(A) = 0.7$ and $P(B) = 0.6$. The necessarily false statement(s) is/are

(a) $P(A \cap B) = 0.35$
(b) $P(A \cap B) = 0.45$
(c) $P(A \cap B) = 0.65$
(d) $P(A \cap B) = 0.28$

Answers

Physics

1. (a)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (c)
11. (a)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (c)
21. (c)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (d)
31. (d)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (a)	**36.** (b)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (a)
41. (a)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (d)	**49.** (d)	**50.** (b)
51. (a)	**52.** (a)	**53.** (b)	**54.** (d)	**55.** (b)	**56.** (a)	**57.** (a,c)	**58.** (a,d)	**59.** (b,c,d)	**60.** (c,d)

Chemistry

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (c)
11. (a)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (d)
21. (c)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (c)	**30.** (b)
31. (b)	**32.** (a)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (d)	**37.** (d)	**38.** (d)	**39.** (c)	**40.** (a)
41. (b)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (b)	**50.** (d)
51. (c)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (c)	**56.** (b,d)	**57.** (b,c)	**58.** (a,b,d)	**59.** (a,b,d)	**60.** (b,c,d)

Mathematics

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (a)
21. (c)	**22.** (a)	**23.** (d)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (a)
31. (c)	**32.** (a)	**33.** (d)	**34.** (a)	**35.** (c)	**36.** (d)	**37.** (a)	**38.** (c)	**39.** (b)	**40.** (d)
41. (c)	**42.** (a)	**43.** (c)	**44.** (a)	**45.** (d)	**46.** (c)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (c)
51. (c)	**52.** (c)	**53.** (b)	**54.** (a)	**55.** (a)	**56.** (b)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (d)
61. (a)	**62.** (c)	**63.** (a)	**64.** (c)	**65.** (a)	**66.** (c)	**67.** (c)	**68.** (d)	**69.** (d)	**70.** (a)
71. (d)	**72.** (a)	**73.** (b)	**74.** (b)	**75.** (a)	**76.** (a,b)	**77.** (a)	**78.** (a,d)	**79.** (d)	**80.** (c,d)

Hints & Solutions

Physics

1. Vector $\mathbf{X}$ of the form $\alpha\mathbf{A} + \beta\mathbf{B}$

$$\mathbf{X} = \alpha\mathbf{A} + \beta\mathbf{B}$$
$$= \alpha(\hat{\imath} + \hat{\jmath} - 2\hat{k}) + \beta(\hat{\imath} - \hat{\jmath} + \hat{k})$$
$$\mathbf{X} = \hat{\imath}(\alpha + \beta) + \hat{\jmath}(\alpha - \beta) + \hat{k}(-2\alpha + \beta)$$

A vector $\mathbf{X}$ is perpendicular to $\mathbf{C}$, *i.e.*, $\mathbf{X}\cdot\mathbf{C} = 0$

$$[\hat{\imath}(\alpha + \beta) + \hat{\jmath}(\alpha - \beta) + \hat{k}(-2\alpha + \beta)]\cdot[2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) = 0$$

or $\quad 2(\alpha + \beta) - 3(\alpha - \beta) + 4(-2\alpha + \beta) = 0$

or $\quad 2\alpha + 2\beta - 3\alpha + 3\beta - 8\alpha + 4\beta = 0$

or $\quad -9\alpha + 9\beta = 0$ or $\alpha = \beta$

or $\quad \dfrac{\alpha}{\beta} = \dfrac{1}{1}$ or $\alpha : \beta = 1 : 1$

2. The combination of three charges in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} \Rightarrow C = \frac{6}{4} = 1.5\ \mu\text{F}$$

The charge of this circuit

$$q = CV = 1.5 \times 120$$
$$q = 180\ \mu\text{C}$$

The potential difference across the $3\ \mu\text{F}$

$$q = CV$$
$$V = \frac{q}{C} = \frac{180}{3} = 60\ \text{V}$$

3. Resistance $R = \dfrac{V}{i_g} - G = \dfrac{120}{0.01} - 10$

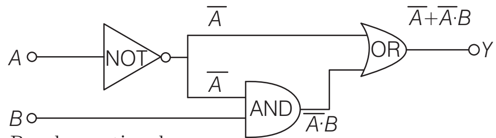
$$= 12000 - 10$$
$$R = 11990\ \Omega$$

To convert a galvanometer into a voltmeter, high resistance should connect in series.

4. For an open pipe, the frequency

$$f = \frac{v}{2l} \Rightarrow 5100 = \frac{340}{2 \times l}$$
$$l = \frac{340}{5100 \times 2} \Rightarrow l = \frac{2}{30 \times 2} = \frac{1}{30}\ \text{m}$$
$$l = \frac{100}{30} = \frac{10}{3}\ \text{cm}$$

5.



By absorption laws

$$Y = \overline{A} + \overline{A}\cdot B$$

or $\quad Y = (\overline{A}\cdot B) + \overline{A} = \overline{A}\cdot(B + 1) = \overline{A}\cdot 1 = \overline{A}$

6. We know that $\Delta U = nC_v\Delta T$

where $\quad \Delta T = (T_2 - T_1)$

$$T_1 = 0°\ \text{C} = 273\ \text{K}$$
$$T_2 = 100°\ \text{C} = 373\ \text{K}$$
$$n = 1\ \text{(monoatomic gas)}$$

$$R = 8.32\ \text{J mol}^{-1}\text{K}^{-1} = 1 \times \frac{3}{2} R \times (373 - 273)$$
$$= 1 \times \frac{3}{2} \times 8.32 \times 100 = 3 \times 8.32 \times 50$$

$\Rightarrow 1248$ kJ

or $\quad = 1.25 \times 10^3$ J

7. Energy required to removed electron in the ($n = 2$) stage

The energy of the photon

$$= R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow 13.6\ \text{eV}\left(\frac{1}{1} - \frac{1}{4}\right)$$
$$= 13.6\ \text{eV}\left(\frac{4 - 1}{4}\right) = 13.6\ \text{eV} \times \frac{3}{4} = \frac{40.8}{4}\ \text{eV}$$
$$= 10.2\ \text{eV}$$

8. We know that

The parallel plate capacitor $C = \dfrac{\varepsilon_0 \cdot A}{d}$

When d (distance between two parallel plate) increase, then C will be decrease

$\because \quad Q = CV \quad$ where Q = constant

$\therefore$ The voltage across the capacitor increases.

9. We know that

Kinetic energy $K = \dfrac{1}{2} mu^2$

where $u = \dfrac{dy}{dt} = \omega a \cos \omega t$

So, $K = \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t$

Hence, kinetic energy varies periodically with double the frequency of SHM. So, when a particle executing SHM oscillates with a frequency ν, then the kinetic energy of particle changes periodically with a frequency of 2ν.

10. (a) Planck's constant $= \frac{\text{Energy}}{\text{Frequency}} = \frac{[ML^2T^{-2}]}{[T^{-1}]}$

$$= [ML^2T^{-1}]$$

Angular momentum = Moment of inertia × Angular velocity

$$= [ML^2] \times [T^{-1}] = [ML^2T^{-1}]$$

(b) Impulse = Force × Time

$$= [MLT^{-2}]\,[T] = [MLT^{-1}]$$

and linear momentum = Mass × Velocity

$$= [M]\,[LT^{-1}] = [MLT^{-1}]$$

(c) Moment of inertia = Mass × (Distance)^2

$$= [ML^2]$$

and moment of force = Force × Distance

$$= [MLT^{-2}]\,[L] = [ML^2T^{-2}]$$

(d) Energy $= [ML^2T^{-2}]$ and torque $= [ML^2T^{-2}]$

So, (c) option has different dimensions.

11. For vertically moment (for h_1)

$$h_1 = u \sin \theta t_1 - \frac{1}{2} g t_1^2$$

or
$$t_1 = \frac{h_1 + \frac{1}{2} g t_1^2}{u \sin \theta} \quad \text{...(i)}$$

$$h_2 = u \sin \theta t_2 - \frac{1}{2} g t_2^2 \quad (\text{for } h_2)$$

or
$$t_2 = \frac{h_2 + \frac{1}{2} g t_2^2}{u \sin \theta} \quad \text{...(ii)}$$

Divide Eq. (i) by Eq. (ii)

$$\frac{t_1}{t_2} = \frac{h_1 + \frac{1}{2} g t_1^2 / u \sin \theta}{h_2 + \frac{1}{2} g t_2^2 / u \sin \theta}$$

$$\Rightarrow \quad h_1 t_2 - h_2 t_1 = \frac{g}{2} (t_1 t_2^2 - t_1^2 t_2)$$

The time of flight of the ball

$$T = \frac{2u \sin \theta}{g} = \frac{2}{g} (u \sin \theta) \quad \text{[from Eq. (i)]}$$

$$= \frac{2}{g} \left[\frac{h_1 + \frac{1}{2} g t_1^2}{t_1} \right] = \frac{2}{t_1} \left[\frac{h_1}{g} + \frac{t_1^2}{2} \right]$$

$$= \frac{h_1}{t_1} \times \frac{2}{g} + t_1 = \frac{h_1}{t_1} \times \left(\frac{t_1 t_2^2 - t_1^2 t_2}{h_1 t_2 - h_2 t_1} \right) + t_1$$

$$= \frac{h_1 t_1 t_2^2 - h_1 t_1^2 t_2 + h_1 t_1^2 t_2 - h_2 t_1^3}{t_1 (h_1 t_2 - h_2 t_1)}$$

$$= \left(\frac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1} \right)$$

12. Stokes established that if a sphere of radius a moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is $F = 6\pi\eta a v$

13. Given, 305 X − 25 X = 100° C

(∵ X is the new unit of temperature)

$$(305 - 25)\text{ X} = 100°\text{ C} \Rightarrow 280\text{ X} = 100°\text{ C}$$

$$\therefore \quad 1°\text{ C} = 2.8\text{ X}$$

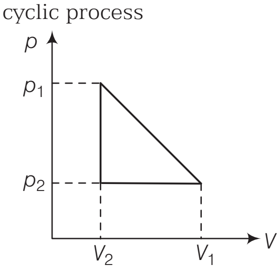
The specific heat capacity of water

$$= 4200 \frac{\text{joule}}{\text{kg °C}} = 4200 \times \frac{\text{joule}}{\text{kg} \times 2.8\text{ X}}$$

$$= 1500\text{ J/kg-X}$$

$$= 1.5 \times 10^3\text{ J kg}^{-1}\text{X}^{-1}$$

14. For the cyclic process



Heat absorbed = Work done

$$= \text{Area} = \frac{1}{2} (\Delta p) \times \Delta V$$

$$= \frac{1}{2} (p_1 - p_2) \times (V_1 - V_2)$$

$$= \frac{1}{2} (p_1 - p_2)(V_1 - V_2)$$

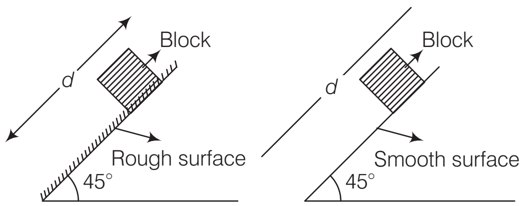
16. We know that

Acceleration, $a_{CM} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)^2 \times g \quad (\because m_1 > m_2)$

So, resultant external force,

$$F = (m_1 + m_2)\, a_{CM}$$
$$= (m_1 + m_2) \times \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)^2 \times g$$
$$= \dfrac{(m_1 - m_2)^2}{(m_1 + m_2)} \times g$$

17. If the same wedge is made rough then time taken by it to come down becomes n times more. The coefficient of friction



$$\mu = \left[1 - \dfrac{1}{n^2}\right] \tan\theta \Rightarrow \mu = \left[1 - \dfrac{1}{2^2}\right] \tan 45°$$
$$\mu = \dfrac{4-1}{4} \Rightarrow \mu = \dfrac{3}{4}$$

18. Strain $= \alpha \cdot \Delta T$

Stress $Y \propto \Delta T$

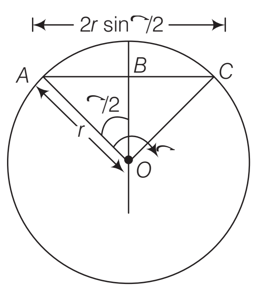
where L = length of the rod

α = coefficient of linear thermal expansion

Y = Young's modulus of its material

So, the longitudinal stress developed in the rod is directly proportional to $\dfrac{\Delta T}{Y}$.

19.

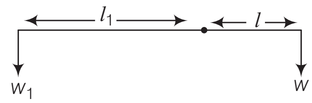


In ΔAOB $\sin\dfrac{\theta}{2} = \dfrac{AB}{AO} \quad (\because AO = r)$

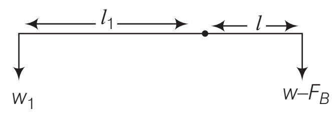
$$AB = AO \sin\dfrac{\theta}{2} \Rightarrow AB = r\sin\dfrac{\theta}{2}$$
$$AC = AB + BC \qquad (\because AB = BC)$$
$$= r\sin\dfrac{\theta}{2} + r\sin\dfrac{\theta}{2}$$
$$AC = 2r\sin\dfrac{\theta}{2}$$

So, the magnitude of the corresponding linear displacement will be $2r\sin\dfrac{\theta}{2}$.

20.



$$Wl = W_1 l_1 \qquad \text{...(i)}$$



$$W - F_B = Wl\left(1 - \dfrac{1}{\rho}\right)$$

where ρ = specific gravity

$$Wl\left(1 - \dfrac{1}{\rho}\right) = W_1 l_2$$
$$\left(1 - \dfrac{1}{\rho}\right) = \dfrac{W_1 l_2}{Wl} \qquad \text{[from Eq.(i)]}$$
$$\left(1 - \dfrac{1}{\rho}\right) = \dfrac{W_1 l_2}{W_1 l_1}$$
$$1 - \dfrac{1}{\rho} = \dfrac{l_2}{l_1} \Rightarrow \dfrac{1}{\rho} = 1 - \dfrac{l_2}{l_1}$$
$$\dfrac{1}{\rho} = \dfrac{l_1 - l_2}{l_1} \quad \text{or } \rho = \dfrac{l_1}{l_1 - l_2}$$

21. Let, thickness of layer be x

So, volume V = Area $\times\, x$

$$V = A \times x \qquad (\because x = 2r)$$
$$x = V / A$$
$$\therefore \quad 2r = \dfrac{V}{A} \Rightarrow r = \dfrac{V}{2A} \qquad \text{...(i)}$$
$$\text{and} \quad \Delta P = \dfrac{T}{r}$$

We know that $F = \Delta P \times A$

$$= \dfrac{T}{r} \times A$$

$$F = \frac{T}{\left(\frac{V}{2A}\right)} \times A \qquad \text{[from Eq. (i)]}$$

$$T = \frac{F \times V}{2A^2}$$

where $F = 16 \times 10^5$ dyne , $V = 0.04$ cm^3,

$$A = 20 \text{ cm}^2 = \frac{16 \times 10^5 \times 0.04}{2 \times 20^2}$$

$$= \frac{8 \times 10^5 \times 4}{20^2 \times 100} = \frac{8 \times 10^5 \times 4}{400 \times 100}$$

$$= 8 \times 10^5 \times 10^{-4} = 80 \text{ dyne/cm}$$

$$= 80 \text{ dyne cm}^{-1}$$

22. Given, $r = 0.05$ nm $= 0.05 \times 10^{-9}$ m

$(\because 1 \text{ nm} = 10^{-9} \text{ m})$

$$n = 10^{16} \text{ revolutions}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

We know that

The magnetic moment $M = Ai$

$$M = \pi r^2 \times ne$$

$$M = 3.14 \times (0.05 \times 10^{-9})^2 \times 10^{16} \times 1.6 \times 10^{-19}$$

$$M = 0.1256 \times 10^{-18} \times 10^{16} \times 10^{-19}$$

or $M = 1.26 \times 10^{-23}$ Am^2

23. We know that $\tau = NBA\omega \sin \omega t$

where N = number of loops = 1

$$B = \frac{\mu_0 I}{2b} \text{ newton/amp-m}$$

$$A = \pi a^2 \text{ metre}^2$$

$$\therefore \qquad \tau = \frac{\mu_0 I}{2b} (\pi a^2)\, \omega \sin \omega t$$

$$= \frac{\pi a^2 \mu_0 I}{2b} \cdot \omega \sin \omega t$$

24. Focal length of a convex lens by displacement method

Focal length of the convex lens

$$f = \frac{a^2 - b^2}{4a}$$

where a = distance between the image and object

and b = distance between two positions of lens

Fact *The distance between the two points should be greater than four time the focal length of the convex lens, i.e., $d > 4f$.*

So, the maximum possible focal length of this convex lens is $\frac{d}{4}$.

25. The given

$$\mathbf{A} = a(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$\mathbf{B} = a(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$= a(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) - a(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$\mathbf{AB} = a(-4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$\text{Work done} = q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathbf{k}} \cdot \mathbf{AB} \qquad \text{(along to } Z\text{-axis)}$$

$$= q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathbf{k}} \cdot a(-4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \frac{3q\sigma a}{2\varepsilon_0}$$

$(\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1 \text{ and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0)$

26. We know that

Intensity of magnetisation

$$I = \frac{M}{V}$$

[where M = magnetic moment, V = volume]

So $\qquad M = IV = 5.0 \times 10^4 \times \frac{12}{100} \times \frac{1}{(100)^2}$

$$= 60 \times 10^4 \times 10^{-6} = 0.6 \text{ Am}^2$$

27. We know that

$$S_{n\text{th}} = u + \frac{1}{2} a(2n - 1)$$

$$S_{3\text{rd}} = 0 + \frac{1}{2} a(2 \times 3 - 1) = \frac{5}{2} a \quad (\text{for } n = 3 \text{ s})$$

$$S_{4\text{th}} = 0 + \frac{1}{2} a(2 \times 4 - 1) = \frac{7}{2} a \quad (\text{for } n = 4 \text{ s})$$

So, the percentage increase

$$= \frac{S_{4\text{th}} - S_{3\text{rd}}}{S_{3\text{rd}}} \times 100$$

$$= \frac{\frac{7}{2} a - \frac{5}{2} a}{\frac{5}{2} a} \times 100$$

$$= \frac{\frac{2a}{2}}{\frac{5}{2} a} \times 100 = 2 \times 20 = 40\%$$

28. Given, Kinetic energy $= E$

$$\text{Mass} = m$$
$$\text{Magnetic field} = B$$
$$\text{Charge} = q$$

We know that

$$F = qvB \sin \theta$$

(motion of a charged particle in a uniform magnetic field)

If $\quad \theta = 90°$

Then $\quad F = qvB \quad \ldots\text{(i)}$

We know that also (centripetal force)

$$F = \frac{mv^2}{r} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$qvB = \frac{mv^2}{r},\ r = \frac{mv}{qB} \quad \left[\because E = \frac{1}{2} mv^2,\ v = \sqrt{\frac{2E}{m}}\right]$$

$$\therefore \quad r = \frac{m\sqrt{\frac{2E}{m}}}{qB} \Rightarrow r = \frac{\sqrt{2Em}}{qB}$$

29. We know that

The potential energy of the satellite

$$U = -\frac{GM_e m}{R_e} \quad \ldots\text{(i)}$$

The kinetic energy of the satellite

$$K = \frac{1}{2} \frac{GM_e m}{R_e} \quad \ldots\text{(ii)}$$

The total energy

$$E = U + K = -\frac{GM_e m}{R_e} + \frac{1}{2} \frac{GM_e m}{R_e}$$

$$E = -\frac{1}{2} \frac{GM_e m}{R_e}$$

$$2E = -\frac{GM_e m}{R_e} \Rightarrow -2E = U$$

So, $\quad \text{PE} = -2\,(\text{TE})$

$$\text{PE} = -2E$$

30. Thermal radiation is the heat waves travel along straight lines with the speed of light.

31. The band gap of 5 eV corresponds to that of an insulator.

32. We know that total kinetic energy of a body rolling without slipping

$$K_{\text{total}} = K_{\text{rot}} + K_{\text{trans}}$$

For solid spherical ball,

$$I = \frac{2}{5} mR^2 \text{ (along to diameter)}$$

and $v = R\omega$, where R is radius of spherical ball

So, $\quad K_{\text{total}} = \frac{1}{2}\left(\frac{2}{5} mR^2\right)\omega^2 + \frac{1}{2} mR^2\omega^2$

$$= \frac{7}{10} mR^2\omega^2$$

$$K = \frac{7}{10} mv^2$$

Potential energy $=$ Kinetic energy

$$mgh = \frac{7}{10} mv^2$$

$$v^2 = \frac{10}{7} gh \quad \ldots\text{(i)}$$

For vertical projection,

$$v^2 = u^2 + 2gh'$$

$$\frac{10}{7} gh = 0 + 2gh' \Rightarrow h' = \frac{5}{7} h$$

33. We know that

The maximum intensities

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \quad \ldots\text{(i)}$$

The minimum intensities

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 \quad \ldots\text{(ii)}$$

So, the ratio of the maximum and minimum of intensities is

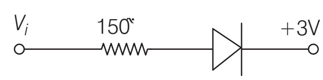
$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{4I} + \sqrt{I})^2}{(\sqrt{4I} - \sqrt{I})^2}$$

$$= \left(\frac{3\sqrt{I}}{\sqrt{I}}\right)^2$$

$$= \frac{9}{1} = 9:1$$

34. $I_{\text{initial}} = 0,$



$$I_{\text{final}} = \frac{3}{150} = 0.02 \text{ A}$$

So, change in $I = 0.02 \text{ A} = 20 \text{ mA}$

35. $\left[\dfrac{nh}{2\pi qB}\right]$

where n and 2π dimensionless quantity

So, $\quad [h] = [mvr]$

$$[qB] = \left[\frac{qvB}{v}\right] = \left[\frac{F}{v}\right]$$

$$\therefore \quad \left[\frac{mvr}{F/v}\right] = \frac{[mvr]\,[v]}{[F]}$$

$$= \frac{[mv^2 r]}{\left[\dfrac{mv^2}{r}\right]} = [r^2] = \text{Area}$$

36. I_B is in μ amp, I_C is in m amp and V_{CE} in volt.

37. Given, $Y = 4\cos^2\left(\dfrac{t}{2}\right)\cdot\sin(1000t)$

$$= 2 \times 2\cos^2\left(\frac{t}{2}\right)\cdot\sin(1000t)$$

$$\left[\because 2\cos^2\frac{t}{2} = (1+\cos t)\right]$$

$$= 2(1+\cos t)\sin(1000t)$$

$$= 2\sin(1000t) + 2\cos t\cdot\sin(1000t)$$

$$= 2\sin(1000t) + 2\sin(1000t)\cdot\cos t$$

$$= 2\sin(1000t) + \sin(1000t+t) + \sin(1000t-t)$$

$$[\because 2\sin A\cdot\cos B = \sin(A+B) + \sin(A-B)]$$

$$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$$

So, $n = 3$

38. Assuming temperature of the body and cubica box is same initially, *i.e.,* T and finally it becomes $T/2$.

Because, temperature of body and surrounding remains same.

Hence, no net loss of radiation occur through the body. This total energy remains constant.

39. Let, the charge on the inner shell be q'

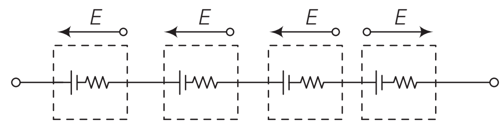
The total charge will be zero

So, $\quad \dfrac{kq'}{r_1} + \dfrac{kq}{r_2} = 0 \qquad (\because r_2 > r_1)$

$$\Rightarrow \quad q' = -\frac{Kq/r_2}{K/r_1}$$

$$q' = -\left(\frac{r_1}{r_2}\right)q$$

40. Total emf of the cell $= 3E - E = 2E$



Total internal resistance $= 4r$

$\therefore$ Total resistance of the circuit $= 4r + R$

So, the current in the external circuit $\left(\because i = \dfrac{V}{R}\right)$

$$\therefore \quad i = \frac{2E}{4r+R}$$

41. We know that

Relation between temperature gradient (TG) and thermal conductivity (K)

So, $\quad \dfrac{d\theta}{dt} = -KA\dfrac{d\theta}{dx} = -KA \times (\text{TG})$

i.e., $\quad \text{TG} \propto \dfrac{1}{K} \left(\dfrac{d\theta}{dt} = \text{constant}\right)$

or $\quad K \propto \dfrac{1}{T}$

$$K_1 \propto \frac{1}{t_1} \qquad \text{...(i)}$$

$$K_2 \propto \frac{1}{t_2} \qquad \text{...(ii)}$$

From the Eqs. (i) and (ii), we get

$$\frac{K_1}{K_2} = \frac{1/t_1}{1/t_2}$$

$$K_1 : K_2 = t_2 : t_1$$

42. Isotope of the original nucleus is produced.

43. By Doppler's Effect

When observer is moving with velocity v_0, towards a source at rest then approach frequency

$$N_{\text{Approach}} = N\left(\frac{v+v_0}{v}\right)$$

where $N = 850$ Hz, $v = 340\ \text{ms}^{-1}$, $v_0 = 72\ \text{kmh}^{-1}$

$$= 20\ \text{ms}^{-1} = 850\left(\frac{340+20}{340}\right)$$

Similarly, when observer is moving away from source, then

$$N_{\text{Separation}} = N\left(\frac{v-v_0}{v}\right) = 850\left(\frac{340-20}{340}\right)$$

The different of the two frequency,

$N_{\text{Approach}} - N_{\text{Separation}}$

$$= 850\left(\frac{360}{340}\right) - 850\left(\frac{320}{340}\right)$$

$$= \frac{850}{340} \times 40 = \frac{850}{8.5} = 100 \text{ Hz}$$

44. $E_\gamma \geq 100$ keV

$$E_X = 100 \text{ eV to } 100 \text{ keV}$$

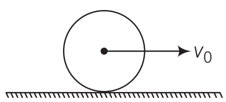
$$E_v = 2.48 \text{ eV}$$

So, we can say that

$$E_\gamma > E_X > E_v$$

45. The intermediate image formed by the objective of a compound microscope is real, inverted and magnified.

46. Let, the final velocity be v



So, angular momentum will remain conserved along point of contact

By conservation of angular momentum

Angular momentum will remain conserved along point of contact

$$I\omega = \text{constant}$$

$$mv_0 r = mvr + \frac{2}{5} mr^2 \times \omega \qquad \left(\because \omega = \frac{v}{r}\right)$$

$$mv_0 r = mvr + \frac{2}{5} mr^2 \left(\frac{v}{r}\right)$$

$$v_0 = v + \frac{2}{5} v$$

$$v_0 = \frac{7}{5} v \Rightarrow v = \frac{5}{7} v_0$$

47. de-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mK}}$

The kinetic energy of the electron

$$K_{\text{electron}} = \frac{1}{2m} \cdot \frac{h^2}{\lambda^2} \qquad \ldots\text{(i)}$$

where h = Planck constant

λ = wavelength

The photon energy

$$E_{\text{photon}} = \frac{hc}{\lambda} \qquad \ldots\text{(ii)}$$

From Eqs. (ii) and (i), we get

$$\frac{E_{\text{photon}}}{K_{\text{electron}}} = \frac{hc/\lambda}{h^2/2m\cdot\lambda^2} = \frac{hc\cdot\lambda^2 \times 2m}{h^2\cdot\lambda}$$

where $m = 0.5$ MeV $= 5 \times 10^5$ eV

$$\frac{h}{\lambda c} = 50 \times 10^3 \text{ eV} = \frac{2m\lambda c}{h}$$

$$= \frac{2m}{h/\lambda c} = \frac{2 \times 5 \times 10^5}{50 \times 10^3} \qquad \left(\because m = \frac{h}{c\lambda}\right)$$

$$= 20 : 1$$

48. By Snell's law,

$$\mu \sin i = \text{constant}$$

$$\mu \sin i = (\mu - m\,\Delta\mu) \sin r$$

where $\mu = 1.5$, $i = 30°$, $r = 90°$, $\Delta\mu = 0.015$

$$1.5 \sin 30° = (1.5 - m \times 0.15) \sin 90°$$

$$\frac{3}{2} \times \frac{1}{2} = (1.5 - m \times 0.015) \times 1$$

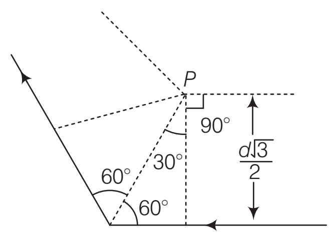
$$\frac{3}{4} = \frac{3}{2} - \frac{15m}{1000}$$

$$15 \text{ m} = \left(\frac{3}{2} - \frac{3}{4}\right) \times 1000$$

$$15m = \frac{6-3}{4} \times 1000 \Rightarrow m = \frac{3}{4} \times \frac{1000}{15}$$

$$m = \frac{3000}{60} \Rightarrow m = 50$$

49. We know that



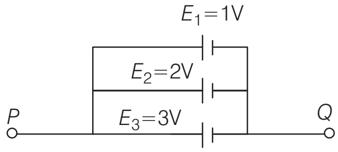
$$B_{\text{net}} = 2\left[\frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{i}{\frac{d\sqrt{3}}{2}} \times (\sin 90° + \sin 30°)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2i}{d\sqrt{3}} \times \left(1 + \frac{1}{2}\right)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2i}{d\sqrt{3}} \times \frac{3}{2}\right] = \frac{\sqrt{3}\,\mu_0 i}{2\pi d}$$

50. 1 Ω, 2 Ω and 1 Ω are in parallel



So, the required internal resistance

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{1}{r} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1}$$

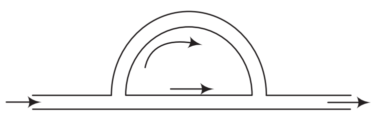
$$\frac{1}{r} = \frac{2+1+2}{2} \Rightarrow r = \frac{2}{5}\ \Omega$$

The potential difference between points P and Q

$$E_{\text{diff}} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{1/r} = \frac{\frac{1}{1} + \frac{2}{2} + \frac{3}{1}}{5/2}$$

$$= \frac{\frac{2+2+6}{2}}{5/2} = \frac{10/2}{5/2} = \frac{5}{5} \times 2 = 2\ \text{V}$$

51. Path difference $= (\pi r - 2r)$



$$= (\pi - 2)\ r = n\lambda \quad (\text{where} f = \text{frequency})$$

$$v = f \times \lambda$$

$$\Rightarrow \quad \frac{v}{\lambda} = f \Rightarrow f = \left[\frac{v}{(\pi - 2)\ r}\right] n$$

So, multiples integral $= \left[\frac{v}{(\pi - 2)\ r}\right]$

52. By Archimedes' Principle

$$F = v\rho_w g \Rightarrow (w_1 - w_2)\ g = v\rho_w g$$

Let, the total volume be v and first metal weight be x

$$w_1 - w_2 = (v_1 + v_2)\ \rho_w$$

$$w_1 - w_2 = v_1\rho_w + v_2\rho_w \qquad \left(\because v = \frac{m}{\rho}\right)$$

$$w_1 - w_2 = \left(\frac{x}{\rho_1}\rho_w + \frac{w_1 - x}{\rho_2}\rho_w\right)$$

$$w_1 - w_2 = \frac{x\rho_2\rho_w + (w_1 - x)\ \rho_w\rho_1}{\rho_1\rho_2}$$

$$w_1\rho_1\rho_2 - w_2\rho_1\rho_2 = x\rho_2\rho_w + w_1\rho_w\rho_1 - x\rho_w\rho_1$$

$$x(\rho_2 - \rho_1)\ \rho_w = \rho_1[w_1(\rho_2 - \rho_w) - w_2\rho_2]$$

$$x = \frac{\rho_1}{\rho_w(\rho_2 - \rho_1)}[w_1(\rho_2 - \rho_w) - w_2\rho_2]$$

53. $m_1 s_1 \Delta t + m_2 s_2 \Delta t = \text{Work done}$

$m_1 s_1 \Delta t + m_2 s_2 \Delta t = P_1 t_1$

where $m_1 = 0.5$ kg

Specific heat $s_1 = 4200$ J/kg-K

$$\Delta t = \Delta t_1 = \Delta t_2 = 3\ \text{K}$$

$$P_1 = P_2 = 10\ \text{W}$$

$$t_1 = 15 \times 60 = 900\ \text{s}$$

$s_2 =$ Specific heat capacity of container

$$0.5 \times 4200 \times (3-0) + m_2 s_2 \times (3-0)$$

$$= 10 \times 15 \times 60$$

$$2100 \times 3 + m_2 s_2 \times 3 = 9000$$

$$m_2 s_2 = \frac{9000 - 6300}{3}$$

$$m_2 s_2 = 900$$

Similarly, in case of oil

$$m_1 s_0 \Delta t + m_2 s_2 \Delta t = P_2 t_2$$

where $s_0 =$ specific heat capacity of oil

$$P_1 = P_2 = 10\ \text{W}$$

$$2 \times s_0 \times 2 + 900 \times 2 = 10 \times 20 \times 60$$

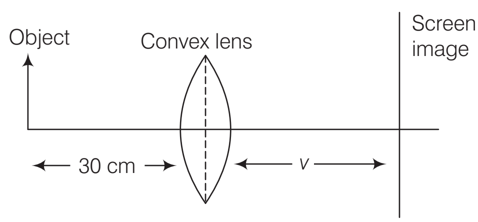
$$4s_0 + 1800 = 12000$$

$$4s_0 = 12000 - 1800$$

$$s_0 = \frac{10200}{4} = 2550$$

$$= 2.55 \times 10^3\ \text{J kg}^{-1}\text{K}^{-1}$$

54. For the first condition



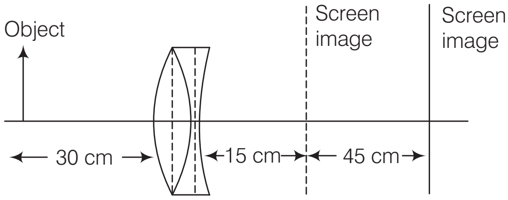
$$\frac{1}{f_1} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{v} + \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30}$$

$$\Rightarrow \quad v = \frac{30}{2} = 15\ \text{cm}$$

where $f_1 = 10$ cm, $u = -30$ cm

$v = ?$



For the second condition when concave lens is placed

$$v' = (15 + 45) \text{ cm} = 60 \text{ cm}$$

$$\frac{1}{F} = \frac{1}{v'} - \frac{1}{u}$$

(where F = focal length of combination)

$$\therefore \quad \frac{1}{F} = \frac{1}{60} + \frac{1}{30}$$

$$F = \frac{60}{3} \text{ cm} = 20 \text{ cm}$$

The magnitude of focal length of concave lens

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{20} = \frac{1}{10} + \frac{1}{f_2} \Rightarrow f_2 = -20 \text{ cm}$$

(Negative sign for concave lens)

55. We know that

$$K = \frac{1}{2} I\omega^2$$

where K = kinetic energy

I = moment of inertia

ω = angular speed

So, $\quad \omega \propto \frac{1}{\sqrt{I}}$

$$\omega_1 : \omega_2 : \omega_3 = \frac{1}{\sqrt{I_1}} : \frac{1}{\sqrt{I_2}} : \frac{1}{\sqrt{I_3}}$$

$$= \frac{1}{\sqrt{1}} : \frac{1}{\sqrt{1}} : \frac{1}{\sqrt{2}} = \sqrt{2} : \sqrt{2} : 1$$

56. We know that

Internal energy, $U = \frac{nfRT}{2} = \frac{5R}{2}\left(\alpha t + \frac{1}{2}\beta t^2\right)$

Differentiate with respect to t

$$\frac{dU}{dt} = \frac{5R}{2}[\alpha + \beta t]$$

But, $\quad dQ = nC_p dT$

$$\therefore \quad \frac{dQ}{dt} = nC_p \times \frac{dT}{dt} \Rightarrow i^2 r = \frac{7}{2} R \times [\alpha + \beta t]$$

$$i = \sqrt{\frac{7R}{2r}(\alpha + \beta t)}$$

$$pV = nRT$$

or $\quad V = \frac{nRT}{p} = \frac{nR}{p}\left(\alpha t + \frac{1}{2}\beta t^2\right)$

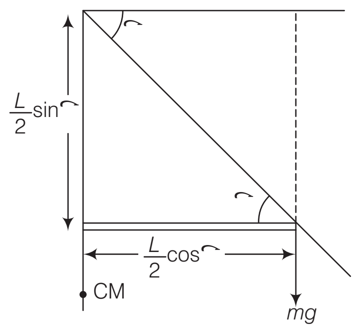
$$X = \frac{nR}{pA}\left(\alpha t + \frac{1}{2}\beta t^2\right)$$

$$v = \frac{nR}{pA}(\alpha + \beta t)$$

and acceleration $= \frac{nR}{pA} \times \beta$

So, the rate of increase in internal energy is $\frac{5}{2} R(\alpha + \beta t)$ and the piston moves upwards with constant acceleration.

57. Loss in potential energy = gain in kinetic energy



$$\Rightarrow \quad mg \frac{L}{2} \sin\theta = \frac{1}{2} I\omega^2$$

So, $\quad \omega \propto \sqrt{\sin\theta}$

and $\quad v \propto \sqrt{\sin\theta} \qquad \left(\because K = \frac{1}{2} I\omega^2\right)$

The speed of end B is proportional to $\sqrt{\sin\theta}$

and

$$U = mgh = mg\frac{L}{2}(1 - \sin\theta) \qquad (\because \tau = I\alpha)$$

$$\Rightarrow \quad mg\frac{L}{2}\cos\theta = \frac{mI^2}{3} \times \alpha$$

$$\alpha \propto \cos\theta$$

The angular acceleration is proportional to $\cos\theta$.

58. There is no significant time delay between the absorption of a suitable radiation and the emission of electrons and the maximum kinetic energy of electrons does not depend on the intensity of radiation.

So, (a) and (d) are correct options.

59. We know that

$$C_1 = \frac{K\varepsilon_0 A}{2d}, C_2 = \frac{\varepsilon_0 A}{2d}$$

and $$C_{eq} = \frac{\varepsilon \cdot A}{2d}(K+1)$$

$$C_{eq} = \frac{C_0}{2}(K+1)$$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{1}$$

$$\Rightarrow \quad \frac{\sigma_1}{\sigma_2} = \frac{K}{1}$$

$$Q_1 = \frac{KQ}{K+1} \quad \text{and} \quad Q_2 = \frac{Q}{K+1}$$

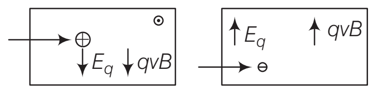
So, $$E = \frac{\sigma}{\varepsilon_0 K} \Rightarrow \frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} \times \frac{K_1}{1}$$

$$= \frac{Q_1}{Q_2} \times \frac{K_2}{K_1} = \frac{K}{1} \times \frac{1}{K}$$

$$= 1:1$$

So, (b), (c) and (d) are correct options.

60. Neither electrons nor protons will go through the slit irrespective of their speed.



And electrons will always be deflected upwards irrespective of their speed.

So, (c) and (d) are correct options.

Chemistry

1. In 1885, Balmer for the first time showed that the wave numbers of spectral lines present in the visible region in hydrogen spectrum are given by

$$\bar{\nu}\,(\text{cm}^{-1}) = 109677\left(\frac{1}{(2)^2} - \frac{1}{n^2}\right)$$

Here, $n = 3, 4, 5, \ldots$

Thus, Balmer spectrum of hydrogen was discovered first and it lies in the visible region.

2. Atomic number of Cu is 29.

Thus, its electronic configuration can be written as

$$\text{Cu}_{29} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^1$$

(Since, fully filled orbitals are more stable, so one electron from $4s$ orbital transfers into $3d$ orbital.)

The electronic configuration of neon is

$$\text{Ne}_{10} = 1s^2, 2s^2, 2p^6$$

Thus, using inert gas, the configuration of Cu can also be represented as

$$\text{Cu}_{29} = [\text{Ne}]\, 3s^2, 3p^6, 3d^{10}, 4s^1$$

3. According to de-Broglie,

wavelength, $$\lambda = \frac{h}{mv}$$

Given, $m = 100$ g, $v = 100$ cm s^{-1}

and $$h = 6.6 \times 10^{-34} \text{ J-s}$$

$$= 6.6 \times 10^{-27} \text{ erg-s}$$

On substituting values, we get

$$\lambda = \frac{6.6 \times 10^{-27}}{100 \times 100} = 6.6 \times 10^{-31} \text{ cm}$$

4. Ideal gas equation is

$$pV = nRT \quad \text{or} \quad V = \frac{nRT}{p} + C$$

or $$V = \frac{RT}{p} + 0 \qquad \text{(for 1 mol)}$$

When V *vs* T is plotted, a straight line is obtained, slope of which is given by R/p. Thus,

$$\text{Slope} = \frac{R}{p} \Rightarrow X = \frac{R}{2}$$

or $$R = 2X \text{ L atm mol}^{-1}\text{K}^{-1}$$

5. According to Graham's law of diffusion,

Rate of diffusion, $$r \propto \frac{1}{\sqrt{M}} = \frac{V}{t}$$

[Here, M = molecular mass,
V = volume and t = time]

Thus, for H_2 gas,

$$\frac{200}{30} = \frac{1}{\sqrt{2}} \qquad \ldots\text{(i)}$$

For O_2 gas,

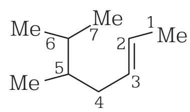
$$\frac{50}{t} = \frac{1}{\sqrt{32}} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

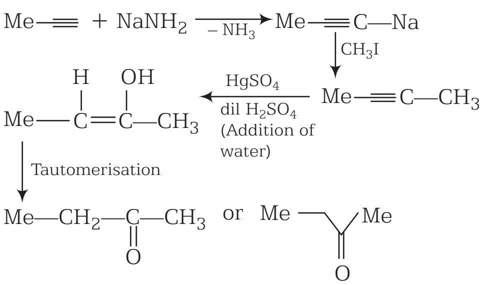
$$\frac{200 \times t}{30 \times 50} = \frac{\sqrt{32}}{\sqrt{2}}$$

$$t = \frac{\sqrt{16} \times 30 \times 50}{200}$$

$$= \frac{4 \times 30}{4} = 30 \text{ min}$$

6. 

The longest chain contains 7C-atom, so the root word is hept and suffix -ene is added for double bond along with its position. Since, two methyl groups are also present at 5 and 6 positions, so the correct IUPAC name of the compound is 5,6-dimethylhept-2-ene.

7. 

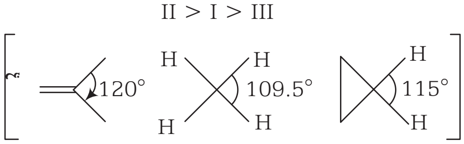
(Here, Me means methyl group)
Thus, the correct set of reagents is
$NaNH_2 / CH_3I; HgSO_4 / \text{dil } H_2SO_4$.

8. H—C—H bond angle is about 120°, when C is bonded through a double bond but in case of CH_4, the H—C—H bond angle is only 109° 28′.

Cyclic compounds have a H—C—H bond angle greater than 109° 28′ due to hindrance.

Thus, the correct order of H—C—H bond angle is

II > I > III



9. When a positron is emitted, a proton is converted into neutron and neutrino. Thus, the number of protons and hence the atomic number decreases by one unit.

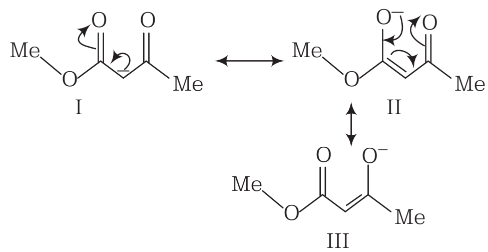
$$\underset{\text{Proton}}{{}^{1}_{1}H} \longrightarrow \underset{\text{Neutron}}{{}^{1}_{0}n} + \underset{\text{Positron}}{{}_{+1}e^{0}} + \underset{\text{Neutrino}}{\nu}$$

10. During the β-emission, a neutron gets converted into a proton along with the formation of antineutrino and electron.

$${}_{0}n^{1} \longrightarrow \underset{\text{Proton}}{{}_{1}H^{1}} + \underset{\beta\text{ particle}}{{}_{-1}e^{0}} + \underset{\text{Anti neutrino}}{\bar{\nu}} + \gamma\text{- ray}$$

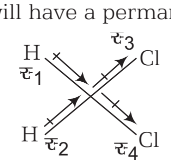
11. Higher the value of *a*, more will be the tendency to get liquefy. Since, value of *a* is highest for P_1 thus, *P* is the most liquefiable gas among the given.

12. Resonating structures involve movement of electrons, but they do not involve movement of atom or group of atoms.

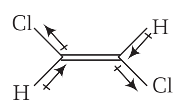


Thus, structure IV is not a resonating structure of others.

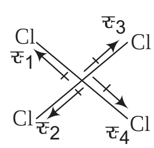
13. Symmetrical molecules and symmetrical *trans*-alkene have a net dipole moment zero. CH_2Cl_2 is not a symmetrical molecule, thus it will have a permanent dipole.



$\mu_1 + \mu_2 - \mu_3 - \mu_4 = 0$
μ_3 and $\mu_4 > \mu_1$ and μ_2
Thus, $\mu_{net} \neq 0$



Symmetrical
trans-alkene
$\mu_{net} = 0$



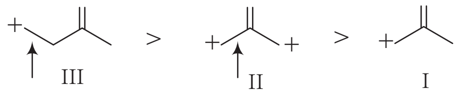
$\mu_1 + \mu_2 = \mu_3 + \mu_4$
∴ $\mu_{net} = 0$

$\mu_{net} = 0$

14. The length of carbon-carbon single bond is always greater than that of the carbon-carbon double bond.

In III, positive charge is not in conjugation of double bond, so the bond does not acquire a double bond character. However, chances of acquiring double bond character (of the indicated bond) is much more in I than that in case of II.

Thus, the correct order of decreasing bond length is



III (No double bond character) > II (Both the positive charge have equal tendency to impart double bond character) > I (Readily acquire double bond character)

15. Emission of a positron results in the conversion of a proton into neutron, along with the release of neutrino. Thus, the number of protons decreases and of neutron increases, *i.e., n/p* ratio increases and the nuclei becomes stable.

$$\underset{\text{Proton}}{{}^1_1\text{H}} \longrightarrow \underset{\text{Neutron}}{{}^1_0n} + \underset{\text{Positron}}{{}_{+1}e^0} + \nu$$

16. ${}_{32}\text{Ge}^{76}$ and ${}_{34}\text{Se}^{76}$ have the same mass number but different atomic number, so they are isobars.

Number of neutrons,

in $\quad {}_{14}\text{Si}^{30} = 30 - 14 = 16$

in $\quad {}_{16}\text{S}^{32} = 32 - 16 = 16$

Because of the presence of same number of neutrons, ${}_{14}\text{Si}^{30}$ and ${}_{16}\text{S}^{32}$ are called isotones.

17. Let, the unknown radioactive substance be ${}_xA^y$.

$${}_xA^y + {}_6\text{C}^{12} \longrightarrow {}_{98}\text{Cf}^{246} + {}_0n^1$$

On comparing atomic numbers, we get

$$x + 6 = 98 + 0$$
$$x = 98 - 6 = 92$$

On comparing mass numbers, we get

$$y + 12 = 246 + 1$$
$$y = 247 - 12$$
$$= 235$$

Since, uranium has the atomic number 92, thus, the given unknown substance was ${}_{92}\text{U}^{235}$.

18. Given, rate $= k\,[\text{H}^+]^n$

Initial pH = 3 so $[\text{H}^+] = 1 \times 10^{-3}$, initial rate $= r_1$

Final pH = 1 so $[\text{H}^+] = 1 \times 10^{-1}$,

Final rate $= r_2 = 100r_1$

On substituting values, we get

$$r_1 = k\,[1 \times 10^{-3}]^n \quad \text{...(i)}$$

$$r_2 = 100r_1 = k\,[10^{-1}]^n \quad \text{...(ii)}$$

Dividing eqs (i) by (ii),

$$\frac{r_1}{100r_1} = \left[\frac{1 \times 10^{-3}}{1 \times 10^{-1}}\right]^n$$

$$\frac{1}{100} = \left[\frac{10^{-2}}{1}\right]^n \Rightarrow \left[\frac{1}{100}\right]^1 = \left[\frac{1}{100}\right]^n$$

$\therefore \quad n = 1$

Thus, the reaction is of first order.

19. Given, $\quad \Delta H = -400 \text{ kJ mol}^{-1}$

$$\Delta S = -20 \text{ kJ mol}^{-1}\text{K}^{-1}$$

Gibbs Helmholtz equation is

$$\Delta G = \Delta H - T\Delta S$$

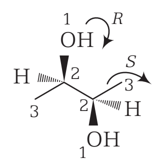
For a reaction to be spontaneous, ΔG must be less than 0, *i.e.,* negative.

$\therefore \quad 0 \geq \Delta H - T\Delta S$

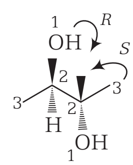
$$T\,\Delta S \leq \Delta H$$

$$T \leq \frac{\Delta H}{\Delta S} \leq \frac{-400}{-20} \leq 20 \text{ K}$$

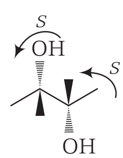
20.



2-*R*, 3-*R* configuration, so it is a chiral compound.



2-*R*, 3-*S* configuration, *i.e.,* it has plane of symmetry or in other words, rotation of half part is cancelled by the other half, so it is not a chiral compound.



2-*S*, 3-*S* configuration, so it is also a chiral compound.

21. In reaction *M*, there is only an intermediate step, so it is a fast reaction. Further, in reaction *M* energy of product is much lesser than that of the reactant, as compared to as shown in reaction *N*.

Thus, reaction *M* is faster and more exothermic than reaction *N*.

22. Aldehydes lacking α-H atom, undergo Cannizzaro reaction if treated with concentrated alkali (NaOH), *i.e.,* one mole of the aldehyde is reduced to alcohol and other mole is oxidised to salt of acid. Thus, the given aldehyde (glyoxal) undergoes intramolecular Cannizzaro reaction, *i.e.,* its half part is oxidised and half is reduced.

Reduced part

$$\begin{array}{c} CHO \\ | \\ CHO \end{array} \xrightarrow{1.NaOH} \begin{array}{c} CH_2OH \\ | \\ COONa \end{array} \xrightarrow{H^+} \begin{array}{c} CH_2OH \\ | \\ CO_2H \end{array}$$

Oxidised part

23. If Cl_2 is passed through hot aqueous NaOH, it results in the formation of sodium chloride and sodium chlorate as

$$\underset{\text{Hot}}{6NaOH} + 3Cl_2 \longrightarrow 5NaCl + NaClO_3 + 3H_2O$$

In NaCl, let oxidation state of Cl be x.

$$+1 + x = 0 \quad \text{or} \qquad x = -1$$

In $NaClO_3$, let oxidation state of Cl be y.

$$+1 + y + (-2)\,3 = 0$$
$$y - 5 = 0$$
$$\therefore \qquad y = +5$$

Thus, the obtained compounds have Cl in -1 and $+5$ oxidation states.

24. '10 V H_2O_2' means 1 L of this solution will produce 10 L O_2 at STP.

$$\underset{68\text{ g}}{2H_2O_2} \longrightarrow 2H_2O + \underset{22.4\text{ L at STP}}{O_2}$$

$\because$ 22.4 L of O_2 is obtained from $H_2O_2 = 68$ g

$\therefore$ 10 L of O_2 will be obtained from

$$H_2O_2 = \frac{68}{22.4} \times 10 = 30.36 \text{ g}$$

$\therefore$ 1000 mL of the given solution contains 30.36 g H_2O_2 and 100 mL of the given solution contains

$$\frac{30.36 \times 100}{1000} = 3.03 \text{ g } H_2O_2$$

Thus, % strength of H_2O_2 is 3.03 (≈ 3).

25. Entropy of vaporisation (ΔS)

$$= \frac{\text{enthalpy of vaporisation } (\Delta H)}{\text{boiling point (in K)}}$$

Given, $\Delta H = 24.64$ kJ mol^{-1} and

boiling point $= 35 + 273 = 308$ K

$$\therefore \qquad \Delta S = \frac{24.64 \times 10^3 \text{ J mol}^{-1}}{308 \text{ K}}$$
$$= 80 \text{ JK}^{-1}\text{mol}^{-1}$$

26. Given,

$$C + O_2 \longrightarrow CO_2;\ \Delta H^\circ = -x \text{ kJ} \qquad \text{...(i)}$$
$$2CO + O_2 \longrightarrow 2CO_2;\ \Delta H^\circ = -y \text{ kJ} \qquad \text{...(ii)}$$

Required equation is

$$C + \frac{1}{2}O_2 \longrightarrow CO, \Delta H_1^\circ = ?$$

On reversing eq. (ii), we get

$$2CO_2 \longrightarrow 2CO + O_2;\ \Delta H_2^\circ = +y \text{ kJ} \qquad \text{...(iii)}$$

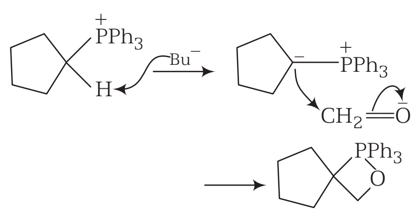
Dividing eq. (iii) by 2 gives

$$CO_2 \longrightarrow CO + \frac{1}{2}O_2;\ \Delta H_3^\circ = +\frac{y}{2} \text{ kJ} \qquad \text{...(iv)}$$

On adding eq. (i) and (iv), we get (required equation)

$$C + \frac{1}{2}O_2 \longrightarrow CO;\ \ \Delta H_1^\circ = \Delta H^\circ + \Delta H_3^\circ$$
$$= \left(-x + \frac{y}{2}\right) \text{kJ}$$
$$= \frac{y - 2x}{2} \text{ kJ}$$

27.



28. Only carboxylic acids and their derivatives like anhydride are capable to give effervescence with aqueous $NaHCO_3$ solution, because they are more acidic as compared to H_2CO_3 (carbonic acid).

II $CH_3COOH + NaHCO_3$
$$\longrightarrow CH_3COONa + H_2O + CO_2$$

I $(CH_3CO)_2O + 2NaHCO_3$
$$\longrightarrow 2\,CH_3COONa + H_2O + CO_2$$

29. The molecular formula of ethane is C_2H_6. To get its fourth homologue, we have to add $4 \times CH_2 = C_4H_8$ to it.

$$C_2H_6 + C_4H_8 = C_6H_{14}$$

C_6H_{14} is the molecular formula of hexane.

Thus, hexane is the fourth homologue of ethane.

30. The given statement is true as electrons are shifted more towards the more electronegative atom.

31. Hydrides of N, O and F because of the small size and high electronegativity of elements, have ability to form extensive intermolecular (*i.e.,* between two molecules) hydrogen bonding. Thus, a large amount of energy is required to break these bonds, *i.e.,* the melting and boiling points of hydrides of these elements are abnormally high.

32. From the Faraday's IInd law, we know that

$$W \propto Zit, \quad W \propto \frac{E}{96500}\, it$$

or $$it \propto \frac{1}{E} \text{ or electricity} \propto \frac{1}{E}$$

Thus, the amount of electricity required for $ZnSO_4$, $AlCl_3$ and $AgNO_3$ is in the ratio of 2 : 3 : 1. (Here, 2, 3 and 1 are the valencies of metal atom in the given compounds).

Alternatively

In $ZnSO_4$,

$$\underset{1\text{ mol}}{Zn^{2+}} + \underset{\substack{2\text{ mol}\\ \text{or 2 Faraday}}}{2e^-} \longrightarrow Zn$$

In $AlCl_3$,

$$\underset{1\text{ mol}}{Al^{3+}} + \underset{3\text{ F}}{3e^-} \longrightarrow Al$$

In $AgNO_3$,

$$\underset{1\text{ mol}}{Ag^+} + \underset{1\text{ F}}{e^-} \longrightarrow Ag$$

Thus, the ratio of amounts of electricity required is 2 : 3 : 1

33. Since, AgI is a negatively charged sol, so according to Hardy Schulze law, higher the positive charge on the metal atoms, more is the tendency of metal atom to cause coagulation of negatively charged colloid.

The order of positive charge is

$$Na^+ < Ba^{2+} < Al^{3+}$$

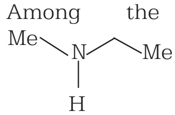
Thus, the order of amount of electrolytes required is in the order

$$NaNO_3 > Ba(NO_3)_2 > Al_2(NO_3)_3$$

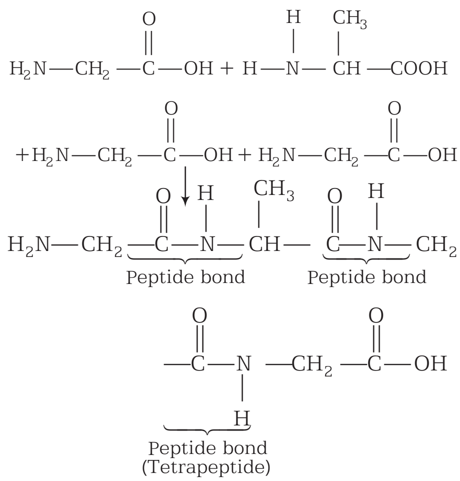
Note *In paper, the formula of aluminium nitrate is incorrect. Correct formula of aluminium nitrate is* $Al_2(NO_3)_3$.

34. $$\Delta H = -nC_p\Delta T = -2 \times \frac{5}{2} R(225 - 125)$$
$$= -5R(100) = -500R$$

35. Since, the amine gives white precipitate with benzene sulphonyl chloride which is insoluble in *aq.* NaOH, so it must be a secondary amine (*i.e.,* amine having $RNHR$ group).

Among the given compound, only  is a secondary amine, so it represents the structure of given amine.

36. Combination of four amino acid units gives a linear tetrapeptide, in which three peptide bonds are present.



37. Among the given gases, N_2O, CO_2 and CH_4 have the ability to trap thermal radiation but O_2 does not. That's why it is not a green house gas.

Note O_2 *is the gas, presence of which is essential for life and in absence of which life is not possible.*

38. 10^{-4} M KOH $= 10^{-4}$ M $[OH^-]$

We know that

$$[H^+][OH^-] = 1 \times 10^{-14}$$

$$[H^+] \times 10^{-4} = 1 \times 10^{-14}$$

$$[H^+] = \frac{1 \times 10^{-14}}{1 \times 10^{-4}} = 1 \times 10^{-10} \text{ M}$$

$$\therefore \quad pH = -\log [H^+] = -\log (1 \times 10^{-10})$$
$$= 10$$

39. Number of atoms $= \dfrac{\text{mass}}{\text{molar mass}} \times N_A$

$\times$ Number of atoms in 1 mole

$\therefore$ Number of atoms in 4.25 g NH_3

$$= \frac{4.25}{17} \times N_A \times 4$$
$$= N_A$$

Number of atoms in 8 g

$$O_2 = \frac{8}{32} \times N_A \times 2 = \frac{N_A}{2}$$

Number of atoms in 2 g

$$H_2 = \frac{2}{2} \times N_A \times 2 = 2N_A$$

Number of atoms in 4 g

$$He = \frac{4}{4} \times N_A \times 1 = N_A$$

Thus, 2 g H_2 contains the maximum number of atoms among the given.

40. SO_3^{2-} and SO_4^{2-} when treated with $BaCl_2$, give white ppt of $BaSO_3$ and $BaSO_4$ respectively.

$$BaCl_2 + SO_3^{2-} \longrightarrow \underset{\text{Barium sulphide}}{BaSO_3} + 2Cl^-$$

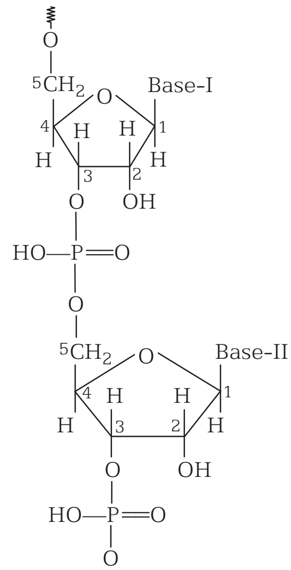
$$BaCl_2 + SO_4^{2-} \longrightarrow \underset{\text{Barium sulphate}}{BaSO_4} + 2Cl^-$$

Out of these two, SO_3^{2-} is soluble in HCl but SO_4^{2-} does not.

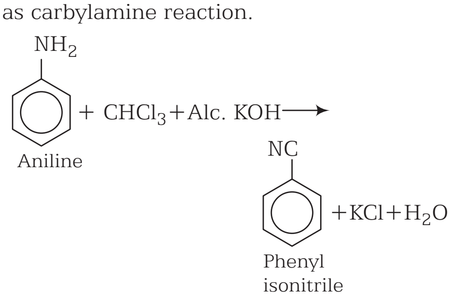
41. Eating animal poisoned with mercury (Hg^{2+}) causes deformity, known as minamata (minimata) disease which is characterised by diarrhoea, hemolysis, impairment of various senses, meningitis and death.

42. H_3PO_2 is the reagent that converts $-N_2^+Cl^-$ group into H. During this process N_2 and HCl gases are released.

43. In DNA, the consecutive deoxynucleotides are linked through phosphodiester linkage.



44. Aniline is a primary amine (contains $-NH_2$ group). When it is treated with chloroform under alkaline conditions, it results in the formation of a bad smelling compound, known by the name isonitrile or carbylamine. This reaction is known as carbylamine reaction.



45. Species with more negative $E°$ (standard reduction potential) generally acts as reducing agent while with less negative $E°$ acts as oxidising agent. Thus, the overall reaction is

$$Ag^+ + Fe^{2+} \longrightarrow Fe^{3+} + Ag$$

$$\Delta E° = E°_{OA} - E°_{RA}$$
$$= -0.3995 - (-0.7120) \text{ V}$$
$$= -0.3995 + 0.7120 \text{ V}$$
$$= +0.3125 \text{ V}$$

46. van der Waals' equation is

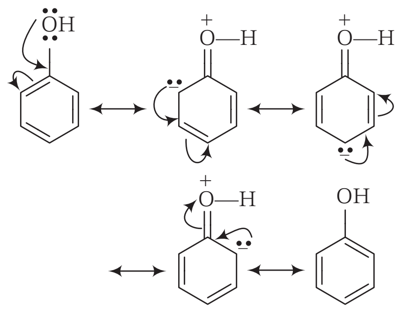
$$\left(p + \frac{a}{V^2}\right)(V - b) = RT \qquad \text{(for 1 mole)}$$

If a is negligible, $p + \frac{a}{V^2} \approx p$

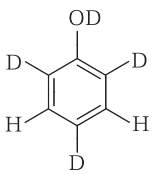
$$p(V - b) = RT \Rightarrow pV - pb = RT$$

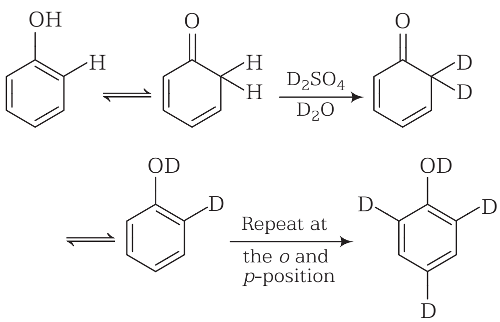
or $$\frac{pV}{RT} - \frac{pb}{RT} = 1 \Rightarrow \frac{pV}{RT} = 1 + \frac{pb}{RT}$$

47. In acidic medium, phenol exists in following resonating structures.



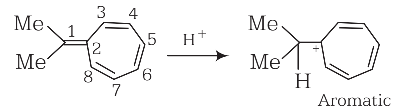
Since, *o/p* positions are electron rich sites, so electrophile will attack on these site, *i.e.*, hydrogen of these sites get exchanged by D (deuterium). Hence, the final product of the

reaction is  which is formed as



48. Addition of a proton at C^1 results in the formation of tropylium carbocation (an aromatic species with more stability due to delocalisation of π electrons), thus, it is the most reactive site towards protonation.

Addition of proton at any other carbon atom, interupt in the delocalisation of π electrons by disturbing planarity of molecules and hence, makes it less stable. Thus, these all are less reactive towards protonation.



49. Abstraction of (H_a) hydrogen atom results in the formation of 3° allylic carbocation whereas abstraction of H_c gives 2° allylic carbocation (which is less stable as compared to former one). However, abstraction of H_b gives only a 2° carbocation which is less stable as compared to former two. More the stability of generated carbocation, higher is the tendency of abstraction of hydrogen.

Thus, the order of decreasing ease of abstraction of hydrogen atoms from the given molecule is

$$H_a > H_c > H_b$$

50. Degree of dissociation, $\alpha = \frac{\lambda^\circ_c}{\lambda^\circ_m} = \frac{150}{500} = 0.3$

Given, $\quad C = 0.007$ M

Hydrofluoric acid dissociates in the following manner

$$\begin{array}{llll} HF & \longrightarrow H^+ & + F^- & \\ C & 0 & 0 & \text{Initially} \\ C - C\alpha & C\alpha & C\alpha & \text{At time } t \\ = C(1-\alpha) & & & \end{array}$$

Dissociation constant,

$$K_a = \frac{[H^+][F^-]}{[HF]} = \frac{C\alpha \cdot C\alpha}{C(1-\alpha)} = \frac{C\alpha^2}{(1-\alpha)}$$

On substituting values, we get

$$K_a = \frac{0.007 \times (0.3)^2}{(1 - 0.3)} = \frac{63 \times 10^{-3} \times 10^{-2}}{0.7}$$

$$= 9 \times 10^{-4} \text{ M}$$

51. Elevation of boiling point,

$$\Delta T_b = \frac{w \times K_b \times 1000}{M \times W}$$

(Here, w and W = weights of solute and solvent respectively,

M = molecular weight of solute and

K_b = constant)

On substituting values, we get

$$0.05 = \frac{w \times 0.5 \times 1000}{100 \times 100}$$

or $$w = \frac{0.05 \times 100 \times 100}{0.5 \times 1000} = 1\text{ g}$$

52. Molarity

$$= \frac{\text{mass} \times 1000}{\text{molecular weight} \times \text{volume of solution}}$$

Molecular weight of ethyl alcohol,

$$C_2H_5OH = 12 \times 2 + 5 + 16 + 1$$
$$= 24 + 22 = 46\text{ g mol}^{-1}$$

On substituting the values, we get

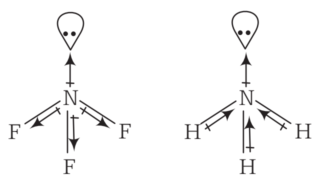
$$0.5\text{ M} = \frac{\text{mass} \times 1000}{46 \times 100}$$

$$\text{Mass} = \frac{0.5 \times 46}{10} = 2.3\text{ g}$$

$\therefore$ Volume of ethyl alcohol required,

$$V = \frac{\text{mass of ethyl alcohol}}{\text{density of ethyl alcohol}} = \frac{2.3\text{ g}}{1.15\text{ g / cc}} = 2\text{ cc}$$

53. In NF_3, N is less electronegative as compared to F but in NH_3, it is more electronegative than H. And in case of same central atom, as the electronegativity of other atoms increases, bond angle decreases. Thus, bond angle in NF_3 is smaller than that in NH_3 because of the difference in the polarity of N in these molecules.



54. Age of archaeological sample,

$$t = \frac{2.303}{k} \log \left(\frac{N_0}{N} \right)$$

(Here, N_0 = initial activity,

N = remaining activity)

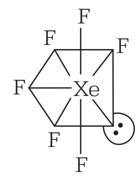
and $$k = \frac{0.693}{t_{1/2}}$$

(where $t_{1/2}$ = half-life)

$$= \frac{0.693}{5770}\text{ yr}^{-1}$$

$\therefore$ $$t = \frac{2.303 \times 5770}{0.693} \log \left(\frac{15}{5} \right)$$
$$= 19175.0 \log 3\text{ yr}$$
$$= 9148\text{ yr} \approx 9200\text{ yr}$$

55. In XeF_6, Xe, the central atom contains, 8 valence electrons. Out of which 6 are utilised with fluorine in bonding (*i.e.*, it contains six bond pairs of electrons) while one pair remains as lone pair.



Thus, the total pairs = 6 + 1 = 7

Hence, its shape is pentagonal bipyramid according to VSEPR theory.

56. Given, molecular weight of $X = M_X$

Mol. wt. of $Y = M_Y$ and $M_Y > M_X$

Root mean square velocities $= C_X$ and C_Y respectively

Average KE/molecule = E_X and E_Y respectively

We know that,

Root mean square velocity,

$$C = \sqrt{\frac{3RT}{M}} \text{ or } C \propto \sqrt{\frac{1}{M}}$$

$\therefore$ $$\frac{C_X}{C_Y} = \frac{M_Y}{M_X}$$

Since $M_Y > M_X$

$\therefore$ $C_X > C_Y$

KE/molecules = $\frac{3}{2} nRT$

$\Rightarrow$ $$E_X \neq E_Y \neq \frac{3}{2} RT$$

Further, $$E_X = E_Y = \frac{3}{2} k_B T$$

57. For a process to be spontaneous,

$$\Delta S_{\text{system}} + \Delta S_{\text{surrounding}} > 0$$
$$(\Delta G_{\text{system}})_{T,\,p} < 0$$
$$(\Delta U_{\text{system}})_{T,\,V} < 0$$

58. Since, H_2SO_4 and H_3PO_4 both are strong acids so their presence makes the medium more acidic. The Fe^{3+} ions form complex, $[FeHPO_4]^+$ with these acid and hence, the concentration of Fe^{3+} ion reduces which results in lowering in the formal reduction potential.

59. Electronic configurations of cuprous (Cu^+) and cupric (Cu^{2+}) ions are as follows

$$Cu^+ = [Ar]\, 3d^{10} 4s^0$$

$$Cu^{2+} = [Ar]\, 3d^9 4s^0$$

Thus, electronic configuration of Cu^+ is more stable but it is less stable because in Cu^{2+} due to its small size the nuclear charge is sufficient to hold 27 electrons but in Cu^+ such a condition is not true.

Further, the IInd IP of Cu is not very high as compared to its Ist IP. Consequently a large amount of lattice energy is released for cupric compounds as compared to Cu^+ compounds.

60. Both the given molecules contain 6 carbon atoms with an aldehyde group, so these are called aldohexoses. In X, the —OH attached to second last carbon in on right hand side while in Y it is on left hand side, so X is D-sugar and Y is L-sugar. X and Y are the mirror images of each other, *i.e.,* these are enantiomers.

Mathematics

1. Given equation is

$$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1} \qquad \text{...(i)}$$

On squaring both sides, we get

$$(x+1) + (x-1) - 2\sqrt{x^2-1} = 4x - 1$$

$$\Rightarrow \quad 2x - 2\sqrt{x^2-1} = 4x - 1$$

$$\Rightarrow \quad -2\sqrt{x^2-1} = 2x - 1$$

Again, squaring both sides, we get

$$4(x^2-1) = 4x^2 + 1 - 4x$$

$$\Rightarrow \quad -4 = +1 - 4x$$

$$\Rightarrow \quad 4x = 5$$

$$\Rightarrow \quad x = \frac{5}{4}$$

But, when we put $x = \frac{5}{4}$ in Eq. (i), we get

$$\sqrt{\frac{5}{4}+1} - \sqrt{\frac{5}{4}-1} = \sqrt{4 \times \frac{5}{4} - 1}$$

$$\Rightarrow \quad \sqrt{\frac{9}{4}} - \sqrt{\frac{1}{4}} = \sqrt{5-1} \quad \Rightarrow \frac{3}{2} - \frac{1}{2} = 2$$

$\Rightarrow 1 = 2$, which is not true.

Hence, no value of x satisfy the given equation.

2. Let $z = x + iy$

$$\therefore \quad |z|^2 + |z-3|^2 + |z-i|^2$$

$$= |x+iy|^2 + |(x-3)+iy|^2 + |x+i(y-1)|^2$$

$$= x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-1)^2$$

$$= x^2 + y^2 + x^2 - 6x + 9 + y^2 + x^2 + y^2 + 1 - 2y$$

$$= 3x^2 + 3y^2 - 6x - 2y + 10$$

$$= 3(x^2 - 2x + 1) + 3\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) + 10 - 3 - \frac{1}{3}$$

$$= 3(x-1)^2 + 3\left(y - \frac{1}{3}\right)^2 + \frac{20}{3}$$

It is minimum, when $x - 1 = 0$ and $y - \frac{1}{3} = 0$

$$\therefore \quad x = 1 \quad \text{and} \quad y = \frac{1}{3}$$

$$\therefore \quad z = 1 + \frac{1}{3}i$$

3. Given, $f(x) = \begin{cases} 2x^2 + 1, & x \le 1 \\ 4x^3 - 1, & x > 1 \end{cases}$

$$\therefore \int_0^2 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx$$

$$= \int_0^1 (2x^2+1)\,dx + \int_1^2 (4x^3-1)\,dx$$

$$= \left[\frac{2x^3}{3} + x\right]_0^1 + \left[\frac{4x^4}{4} - x\right]_1^2$$

$$= \frac{2}{3}(1)^3 + 1 - (0+0) + [(2)^4 - 2 - \{(1)^4 - 1\}]$$

$$= \frac{2}{3} + 1 + [16 - 2 - 0]$$
$$= \frac{2}{3} + 15 = \frac{2 + 45}{3}$$
$$= \frac{47}{3} \text{ sq units}$$

4. $\lim_{x \to 0} \frac{2a \sin x - \sin 2x}{\tan^3 x}$

$$= \lim_{x \to 0} \frac{2a\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) - \left(2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \dots\right)}{\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots\right)^3}$$

$$= \lim_{x \to 0} \frac{(2a - 2)x + \left(-\frac{2a}{3!} + \frac{2^3}{3!}\right)x^3 + \left(\frac{2a}{5!} - \frac{2^5}{5!}\right)x^5 + \dots}{x^3\left(1 + \frac{x^2}{3} + \frac{2}{15}x^4 + \dots\right)^3}$$

Since, it is given that given limit is exist, then

$$2a - 2 = 0 \Rightarrow a = 1$$

5. Given, $\log_{101} \log_7 (\sqrt{x + 7} + \sqrt{x}) = 0$

$\therefore \quad \log_7 (\sqrt{x + 7} + \sqrt{x}) = (101)^0$

$\Rightarrow \quad \log_7 (\sqrt{x + 7} + \sqrt{x}) = 1$

$\Rightarrow \quad (\sqrt{x + 7} + \sqrt{x}) = 7^1$

$\Rightarrow \quad \sqrt{x + 7} + \sqrt{x} = 7$

On squaring both sides, we get

$$(x + 7) + x + 2\sqrt{x^2 + 7x} = 49$$

$\Rightarrow \quad 2x - 42 = -2\sqrt{x^2 + 7x}$

$\Rightarrow \quad x - 21 = -\sqrt{x^2 + 7x}$

Again, squaring both sides, we get

$$x^2 + 441 - 42x = x^2 + 7x$$

$\Rightarrow \quad 49x = 441$

$\Rightarrow \quad x = \frac{441}{49} \Rightarrow x = 9$

6. Given equation is

$$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$$

or it can be rewritten as

$$\frac{dy}{dx} + \frac{1}{(1 + x^2)}y = \frac{e^{\tan^{-1} x}}{1 + x^2}$$

It is a linear differential equation of the form of

$$\frac{dy}{dx} + Py = Q$$

where, $P = \frac{1}{1 + x^2}$ and $Q = \frac{e^{\tan^{-1} x}}{1 + x^2}$

$\therefore$ Integrating factor, $\text{IF} = e^{\int P\,dx}$

$$= e^{\int \frac{1}{1 + x^2} dx} = e^{\tan^{-1} x}$$

7. Given, $\sqrt{y} = \cos^{-1} x \Rightarrow y = (\cos^{-1} x)^2$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2(\cos^{-1} x) \times \frac{-1}{\sqrt{1 - x^2}}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -2\left[\frac{\sqrt{1 - x^2} \times \frac{-1}{\sqrt{1 - x^2}} - \cos^{-1} x \times \left(\frac{1}{2}\right)\frac{(-2x)}{(1 - x^2)^{1/2}}}{(\sqrt{1 - x^2})^2}\right]$$

$$= -2\left[\frac{-1 + \frac{x \cos^{-1} x}{(1 - x^2)^{1/2}}}{(1 - x^2)}\right]$$

$$\frac{d^2y}{dx^2} = \left[\frac{2 - \frac{2x \cos^{-1} x}{(1 - x^2)^{1/2}}}{(1 - x^2)}\right]$$

$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} = 2 + x\frac{dy}{dx}$

$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

But, it is given

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = c$$

$\therefore \quad c = 2$

8. Let $y = 20^{301}$

Number of digits = Integral part of $(301 \log_{10} 20) + 1$

$= \text{Integral part of } [301\,(\log_{10} 10 + \log_{10} 2)] + 1$

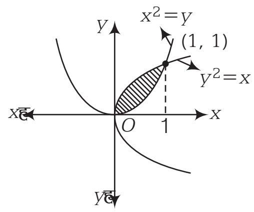
$= \text{Integral part of } [301(1 + 0.3010)] + 1$

$= \text{Integral part of } [301 \times 1.3010] + 1$

$= \text{Integral part of } [391.601] + 1$

$= 391 + 1 = 392$

9. Given curves are $y = x^2$ and $x = y^2$, which is the form of parabola.



The point of intersection, $x = (x^2)^2$

$\Rightarrow \quad x = x^4 \Rightarrow x(1 - x^3) = 0$

$\Rightarrow \quad x = 0 \text{ and } 1 = x^3$

$\Rightarrow \quad x = 0 \text{ and } x = 1$

When $x = 0$, then $y = 0$

When $x = 1$, then $y = 1^2 = 1$

$\therefore$ The point of intersection is (0, 0) and (1, 1).

$\therefore$ Area of shaded region

$$= \int_0^1 (y_2 - y_1)\, dx$$

$$= \int_0^1 [\sqrt{x} - x^2]\, dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3}(1)^{3/2} - \frac{(1)^3}{3} - 0 - 0$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq units}$$

10. Given, $f(x) = 3x^2 + 1$

Let $\quad y = 3x^2 + 1$

$\Rightarrow \quad 3x^2 = y - 1 \Rightarrow x^2 = \dfrac{y-1}{3}$

$\Rightarrow \quad x = \pm\sqrt{\dfrac{y-1}{3}}$

$\therefore \quad f^{-1}(x) = \pm\sqrt{\dfrac{x-1}{3}}$

When $x \in [1, 6]$

Then, $\quad f^{-1}(x) \in \left[-\sqrt{\dfrac{5}{3}}, \sqrt{\dfrac{5}{3}}\right]$

11. $\tan\dfrac{\pi}{5} + 2\tan\dfrac{2\pi}{5} + 4\cot\dfrac{4\pi}{5}$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\,\frac{\sin\frac{2\pi}{5}}{\cos\frac{2\pi}{5}} + 4\,\frac{\cos\frac{4\pi}{5}}{\sin\frac{4\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\,\frac{\sin\frac{2\pi}{5}}{\cos\frac{2\pi}{5}} + \frac{4\left(\cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}\right)}{2\sin\frac{2\pi}{5}\cos\frac{2\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\,\frac{\sin\frac{2\pi}{5}}{\cos\frac{2\pi}{5}} + \frac{2\left(\cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}\right)}{\sin\frac{2\pi}{5}\cos\frac{2\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + 2\left[\frac{\sin^2\frac{2\pi}{5} + \cos^2\frac{2\pi}{5} - \sin^2\frac{2\pi}{5}}{\cos\frac{2\pi}{5}\sin\frac{2\pi}{5}}\right]$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + \frac{2\cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}}$$

$$= \frac{\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} + \frac{2\left(\cos^2\frac{\pi}{5} - \sin^2\frac{\pi}{5}\right)}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$$

$$= \frac{\sin^2\frac{\pi}{5} + \cos^2\frac{\pi}{5} - \frac{\sin^2\pi}{5}}{\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$$

$$= \frac{\cos^2\frac{\pi}{5}}{\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$$

$$= \cot\frac{\pi}{5}$$

12. Given, $\quad f'(x) \le 5$

$\therefore \quad \dfrac{f(7) - f(2)}{x - 2} \le 5$

[by Lagrange mean value theorem]

$\Rightarrow \quad \dfrac{f(7) - 3}{7 - 2} \le 5$

$\Rightarrow \quad f(7) \le 5(7 - 2) + 3$

$\therefore \quad f(7) \le 28$

13. Given, number of elements of A and B are p and q, respectively.

$\therefore$ The number of relations from the set A to the set B is 2^{pq}.

14. Given equation can be rewritten as

$$x^2 - \frac{r^2}{pq}x + 1 = 0$$

$\therefore \quad \tan A + \tan B = \dfrac{r^2}{pq}$

and $\quad \tan A \tan B = 1$

We know, $\quad A + B + C = 180°$

$\Rightarrow \quad (A + B) = 180° - C$

$\Rightarrow \quad \tan(A + B) = \tan(180° - C)$

$\Rightarrow \quad \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\Rightarrow \quad \dfrac{r^2/pq}{1-1} = -\tan C$

$\Rightarrow \quad \infty = -\tan C \Rightarrow C = 90°$

Hence, ΔABC is a right angled triangle.

15. Given equation of parabola is

$$y^2 = 12x \qquad \text{...(i)}$$

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx} = 12 \Rightarrow \frac{dy}{dx} = \frac{6}{y}$$

Since, the normal to the curve is parallel to the line $y = 4x + 3$.

$\therefore$ Slope of normal curve = Slope of line

$\Rightarrow \quad -\dfrac{y}{6} = 4$

$\Rightarrow \quad y = -24$

From Eq. (i), we get

$$(-24)^2 = 12x$$

$\Rightarrow \quad 24 \times 24 = 12x$

$\Rightarrow \quad x = 48$

$\therefore$ Normal point on a curve is $(48, -24)$.

$\therefore$ Distance from $(48, -24)$ to the line $4x - y + 3 = 0$ is

$$\frac{4 \times 48 + 24 + 3}{\sqrt{4^2 + 1^2}} = \frac{219}{\sqrt{17}}$$

16. Given equation of ellipse is

$$\frac{x^2}{144} + \frac{y^2}{25} = 1$$

Here, $\quad a^2 = 144$ and $b^2 = 25$

Now, $\quad e = \sqrt{1 - \dfrac{b^2}{a^2}} = \sqrt{1 - \dfrac{25}{144}}$

$\quad = \sqrt{\dfrac{119}{144}} = \dfrac{\sqrt{119}}{12}$

$\therefore$ Foci of an ellipse $= (\pm ae, 0)$

$\quad = \left(\pm 12 \times \dfrac{\sqrt{119}}{12}, 0\right)$

$\quad = (\pm\sqrt{119}, 0)$

Since, the circle with centre $(0, \sqrt{2})$ and passing through foci $(\pm\sqrt{119}, 0)$ of the ellipse.

$\therefore$ Radius of circle $= \sqrt{(\sqrt{119} - 0)^2 + (0 - \sqrt{2})^2}$

$\quad = \sqrt{119 + 2} = \sqrt{121} = 11$

17. Given lines are

$$x + y = 0 \qquad \text{...(i)}$$

$$5x + y = 4 \qquad \text{...(ii)}$$

and $\quad x + 5y = 4 \qquad \text{...(iii)}$

On solving above equations pairwise, we get

$$A(1, -1),\ B\left(\frac{2}{3}, \frac{2}{3}\right),\ C(-1, 1)$$

Now, $\quad AB = \sqrt{\left(\dfrac{2}{3} - 1\right)^2 + \left(\dfrac{2}{3} + 1\right)^2}$

$\quad = \sqrt{\dfrac{1}{9} + \dfrac{25}{9}} = \sqrt{\dfrac{26}{9}} = \dfrac{\sqrt{26}}{3}$

$\quad BC = \sqrt{\left(-1 - \dfrac{2}{3}\right)^2 + \left(1 - \dfrac{2}{3}\right)^2}$

$\quad = \sqrt{\dfrac{25}{9} + \dfrac{1}{9}} = \sqrt{\dfrac{26}{9}} = \dfrac{\sqrt{26}}{3}$

and $\quad CA = \sqrt{(1 + 1)^2 + (-1 - 1)^2}$

$\quad = \sqrt{4 + 4} = 2\sqrt{2}$

Here, we see that $AB = BC$

Hence, given lines form an isosceles triangle.

18. Given,

$$\sin^{-1}\left(\frac{x}{13}\right) + \operatorname{cosec}^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2} \qquad \text{...(i)}$$

Let $\quad \text{cosec}^{-1}\dfrac{13}{12} = y$

Then, $\quad \text{cosec}\, y = \dfrac{13}{12} \Rightarrow \sin y = \dfrac{12}{13}$

$\therefore \quad \cos y = \sqrt{1-\sin^2 y}$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{25}{169}} = \frac{5}{13} \Rightarrow y = \cos^{-1}\frac{5}{13}$$

Eq. (i) becomes,

$$\sin^{-1}\left(\frac{x}{13}\right) + \cos^{-1}\left(\frac{5}{13}\right) = \frac{\pi}{2} \quad \text{...(i)}$$

We know that

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

$\therefore$ Both angles of Eq. (i) should be same.

$\therefore \quad x = 5$

19. Given curve is

$$(7x+5)^2 + (7y+3)^2 = \lambda^2(4x+3y-24)^2$$

$\Rightarrow \quad 49x^2 + 25 + 70x + 49y^2 + 9 + 42y$

$= \lambda^2(16x^2 + 9y^2 + 576 + 24xy - 144y - 192x)$

$\Rightarrow$

$(49-16\lambda^2)\, x^2 + (49-9\lambda^2)\, y^2 + (70+192\lambda^2)\, x$

$\quad + (42+144\lambda^2)\, y - 24\lambda^2 xy + (25-576\lambda^2) = 0$

On comparing with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$$a = 49-16\lambda^2,\ b = (49-9\lambda^2),\ h = -12\lambda^2$$

Condition for parabola is $ab = h^2$.

$\therefore \quad (49-16\lambda^2)(49-9\lambda^2) = (-12\lambda^2)^2$

$\Rightarrow \quad 2401 - 441\lambda^2 - 784\lambda^2 + 144\lambda^4 = 144\lambda^4$

$\Rightarrow \quad 2401 - 1225\lambda^2 = 0$

$\Rightarrow \quad \lambda^2 = \dfrac{2401}{1225}$

$\Rightarrow \quad \lambda^2 = \dfrac{(49)^2}{(35)^2}$

$\Rightarrow \quad \lambda = \pm\dfrac{49}{35}$

$\Rightarrow \quad \lambda = \pm\dfrac{7}{5}$

20. Given, $\quad f(x) = x + \dfrac{1}{2} = \dfrac{2x+1}{2}$

$\therefore \quad f(2x) = 2x + \dfrac{1}{2}$

$\Rightarrow \quad f(2x) = \dfrac{4x+1}{2}$

and $\quad f(4x) = 4x + \dfrac{1}{2} \Rightarrow f(4x) = \dfrac{8x+1}{2}$

Since, $f(x)$, $f(2x)$ and $f(4x)$ are in HP.

$\therefore \dfrac{1}{f(x)}, \dfrac{1}{f(2x)}$ and $\dfrac{1}{f(4x)}$ are in AP.

$$\Rightarrow \quad \frac{1}{f(2x)} = \frac{\dfrac{1}{f(x)} + \dfrac{1}{f(4x)}}{2}$$

$$\Rightarrow \quad \frac{2}{4x+1} = \frac{\dfrac{2}{2x+1} + \dfrac{2}{8x+1}}{2}$$

$$\Rightarrow \quad \frac{2}{4x+1} = \frac{10x+2}{(2x+1)(8x+1)}$$

$\Rightarrow \quad (2x+1)(8x+1) = (5x+1)(4x+1)$

$\Rightarrow \quad 16x^2 + 10x + 1 = 20x^2 + 9x + 1$

$\Rightarrow \quad 4x^2 - x = 0 \Rightarrow x(4x-1) = 0$

$\Rightarrow \quad x = \dfrac{1}{4} \quad [\because x \neq 0]$

Hence, one real value of x for which the three unequal terms are in HP.

21. Given, $\quad f(x) = 2x^2 + 5x + 1 \quad \text{...(i)}$

Also, $f(x) = a(x+1)(x-2) + b(x-2)(x-1)$

$\quad + c(x-1)(x+1)$

$= a(x^2 - x - 2) + b(x^2 - 3x + 2)$

$\quad + c(x^2 - 1)$

$\Rightarrow \quad f(x) = (a+b+c)\, x^2 + (-a-3b)\, x$

$\quad + (-2a+2b-c) \quad \text{...(ii)}$

On equating the coefficients of x^2, x and constant term in Eqs. (i) and (ii), we get

$$a+b+c = 2,\ -a-3b = 5$$

and $\quad -2a+2b-c = 1$

On solving above equations, we get

$$a = -\frac{35}{4},\ b = \frac{5}{4} \text{ and } c = \frac{38}{4}$$

Hence, exactly one choice for each of b and c.

22. Given, α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$

$$\therefore \quad \alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}$$

and $\alpha + h + \beta + h = -\dfrac{q}{p}, (\alpha + h)(\beta + h) = \dfrac{r}{p}$

Now, $(\alpha + h) - (\beta + h) = \alpha - \beta$

$$\Rightarrow \quad [(\alpha + h) - (\beta + h)]^2 = (\alpha - \beta)^2$$

$$\Rightarrow \quad [(\alpha + h) + (\beta + h)]^2 - 4(\alpha + h)(\beta + h) = (\alpha + \beta)^2 - 4\alpha\beta$$

$$\Rightarrow \quad \frac{q^2}{p^2} - \frac{4r}{p} = \frac{b^2}{a^2} - \frac{4c}{a}$$

$$\Rightarrow \quad \frac{q^2 - 4pr}{p^2} = \frac{b^2 - 4ac}{a^2}$$

$$\therefore \quad \frac{b^2 - 4ac}{q^2 - 4pr} = \frac{a^2}{p^2}$$

Hence, the ratio of the square of their discriminants is $a^2 : p^2$.

23. Since, α is a root of

$$x^2 + 3p^2x + 5q^2 = 0$$

$$\therefore \quad \alpha^2 + 3p^2\alpha + 5q^2 = 0 \quad \text{...(i)}$$

and β is a root of

$$x^2 + 9p^2x + 15q^2 = 0$$

$$\therefore \quad \beta^2 + 9p^2\beta + 15q^2 = 0 \quad \text{...(ii)}$$

Let $f(x) = x^2 + 6p^2x + 10q^2$

Then, $f(\alpha) = \alpha^2 + 6p^2\alpha + 10q^2$

$$= (\alpha^2 + 3p^2\alpha + 5q^2) + 3p^2\alpha + 5q^2$$

$$= 0 + 3p^2\alpha + 5q^2 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad f(\alpha) > 0$$

and $f(\beta) = \beta^2 + 6p^2\beta + 10q^2$

$$= (\beta^2 + 9p^2\beta + 15q^2) - (3p^2\beta + 5q^2)$$

$$= 0 - (3p^2\beta + 5q^2) \quad \text{[from Eq. (ii)]}$$

$$\Rightarrow \quad f(\beta) < 0$$

Thus, $f(x)$ is a polynomial such that $f(\alpha) > 0$ and $f(\beta) < 0$.

Therefore, there exists γ satisfying $\alpha < \gamma < \beta$ such that $f(\gamma) = 0$.

24. Equation of tangent in slope form of parabola $y^2 = 8\sqrt{3}\, x$ is

$$y = mx + c \quad \text{...(i)}$$

where, $c = \dfrac{a}{m}$

$$\therefore \quad c = \frac{2\sqrt{3}}{m} \quad \text{...(ii)}$$

Also, tangent to the hyperbola

$$4x^2 - y^2 = 4$$

or $\dfrac{x^2}{1} - \dfrac{y^2}{4} = 1$ is

$$c^2 = a^2m^2 - b^2$$

$$c^2 = 1m^2 - 4$$

$$\Rightarrow \quad \left(\frac{2\sqrt{3}}{m}\right)^2 = m^2 - 4 \quad \text{[from Eq. (ii)]}$$

$$\Rightarrow \quad \frac{12}{m^2} = m^2 - 4$$

$$\Rightarrow \quad m^4 - 4m^2 - 12 = 0$$

$$\Rightarrow \quad m^4 - 6m^2 + 2m^2 - 12 = 0$$

$$\Rightarrow \quad m^2(m^2 - 6) + 2(m^2 - 6) = 0$$

$$\Rightarrow \quad (m^2 + 2)(m^2 - 6) = 0$$

$$\Rightarrow \quad m^2 - 6 = 0 \text{ and } m^2 + 2 \neq 0$$

$$\Rightarrow \quad m^2 = 6$$

$$\Rightarrow \quad m = \pm\sqrt{6}$$

i.e., $m = \sqrt{6}$ as m is positive slope.

$\therefore$ From Eq. (i),

$$y = \sqrt{6}\, x + \frac{2\sqrt{3}}{\sqrt{6}}$$

$$\Rightarrow \quad y = \sqrt{6}\, x + \sqrt{2}$$

25. Given equation of parabola is

$$y^2 = 64x \quad \text{...(i)}$$

The point at which the tangent to the curve is parallel to the line is the nearest point on the curve.

On differentiating both sides of Eq. (i), we get

$$2y\frac{dy}{dx} = 64$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{32}{y}$$

Also, slope of the given line is $-\frac{4}{3}$.

$\therefore \qquad -\frac{4}{3} = \frac{32}{y} \Rightarrow y = -24$

From Eq. (i), $(-24)^2 = 64x \Rightarrow x = 9$

$\therefore$ Hence, the required point is (9, −24).

26. We know that

$|z - z_1| + |z - z_2| = k$ will represent an ellipse, if $|z_1 - z_2| < k$.

Hence, the equation $|z - z_1| + |z - z_2| = 2|z_1 - z_2|$ represent an ellipse.

27. Given, $f(x) = \frac{\tan\left\{\pi\left[x - \frac{\pi}{2}\right]\right\}}{2 + [x]^2}$

Since, $\left[x - \frac{\pi}{2}\right]$ is an integer for all x, therefore $\pi\left[x - \frac{\pi}{2}\right]$ is an integral multiple of π for all x.

Hence, $\tan\left\{\pi\left[x - \frac{\pi}{2}\right]\right\} = 0$ for all x

Also, $2 + [x]^2 \neq 0$ for all x

Hence, $f(x) = 0$ for all x.

Hence, $f(x)$ is continuous and derivable for all x.

28. Given, $f(x) = a \sin|x| + be^{|x|}$

We know that $\sin|x|$ and $e^{|x|}$ is not differentiable at $x = 0$.

Therefore, for $f(x)$ to differentiable at $x = 0$, we must have $a = b = 0$.

$\therefore \qquad a + b = 0$

29. The general term in $\left(ax^2 + \frac{1}{bx}\right)^{13}$ is

$$T_{r+1} = {}^{13}C_r (ax^2)^{13-r} \left(\frac{1}{bx}\right)^r$$
$$= {}^{13}C_r a^{13-r} \times b^{-r} (x)^{26-3r}$$

For coefficient of x^8, put $26 - 3r = 8$

$\Rightarrow \qquad 3r = 18 \Rightarrow r = 6$

$\therefore \qquad T_7 = {}^{13}C_6 a^{13-6} b^{-6} x^8$
$= {}^{13}C_6 a^7 b^{-6} x^8$

Now, the general term in $\left(ax - \frac{1}{bx^2}\right)^{13}$ is

$$T'_{r+1} = {}^{13}C_r (ax)^{13-r} \left(-\frac{1}{bx^2}\right)^r$$
$$= {}^{13}C_r a^{13-r} \times b^{-r} \times (x)^{13-3r} (-1)^r$$

For coefficient of x^{-8}, put $13 - 3r = -8$

$\Rightarrow \qquad 3r = 21 \Rightarrow r = 7$

$\therefore \qquad T'_8 = (-1)^7\, {}^{13}C_7 a^{13-7} b^{-7} x^{-8}$
$= (-1)^7\, {}^{13}C_7 a^6 b^{-7} x^{-8}$

According to the given condition,

Coefficient of x^8 in $\left(ax^2 + \frac{1}{bx}\right)^{13}$

$= $ Coefficient of x^{-8} in $\left(ax - \frac{1}{bx^2}\right)^{13}$

$\therefore\ {}^{13}C_6 a^7 b^{-6} = -{}^{13}C_7 a^6 b^{-7}$

$\Rightarrow \quad {}^{13}C_7 \frac{a^7}{b^6} = -{}^{13}C_7 \frac{a^6}{b^7}$

$\Rightarrow \qquad \frac{a^7}{a^6} = -\frac{b^6}{b^7} \Rightarrow \quad a = -\frac{1}{b} \quad \Rightarrow \quad ab + 1 = 0$

30. Given, $I = \int_0^2 e^{x^4}(x - \alpha)\, dx = 0$

$\Rightarrow \qquad \int_0^2 e^{x^4} x\, dx = \int_0^2 e^{x^4} \alpha\, dx$

Here, we see that $\int_0^2 e^{x^4} x\, dx$ gives us an area between two curves e^{x^4} and x from $x = 0$ to $x = 2$.

Similarly, $\int_0^2 e^{x^4} \alpha\, dx$ gives us a same area, so α should lies between 0 to 2, *i.e.*, $\alpha \in (0, 2)$.

31. Given differential equation can be rewritten as

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x\,\phi\left(\frac{y^2}{x^2}\right)}{y\,\phi'\left(\frac{y^2}{x^2}\right)}$$

Put $y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Given equation becomes,

$$v + x\frac{dv}{dx} = \frac{vx}{x} + \frac{x\,\phi\left(\frac{v^2x^2}{x^2}\right)}{vx\,\phi'\left(\frac{v^2x^2}{x^2}\right)}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{\phi(v^2)}{v\,\phi'(v^2)}$$

$$\Rightarrow \quad \frac{v\,\phi'(v^2)}{\phi(v^2)}\,dv=\frac{dx}{x}$$

On integrating both sides, we get

$$\frac{1}{2}\log\phi(v^2)=\log x+\log c_1$$

$$\Rightarrow \quad \log\phi(v^2)=2\log xc_1$$

$$\Rightarrow \quad \phi(v^2)=(xc_1)^2 \Rightarrow \phi\left(\frac{y^2}{x^2}\right)=x^2c_1^2$$

$$\Rightarrow \quad \phi\left(\frac{y^2}{x^2}\right)=x^2c \qquad [\text{put } c_1^2=c]$$

32. Since, α and β are the roots of

$$f(x)=x^2+bx+c$$

$$\therefore \quad \alpha+\beta=-b \text{ and } \alpha\beta=c$$

$$\therefore \quad f\left(\frac{\alpha+\beta}{2}\right)=\left(\frac{\alpha+\beta}{2}\right)^2+b\left(\frac{\alpha+\beta}{2}\right)+c$$

$$=\left(-\frac{b}{2}\right)^2+b\left(-\frac{b}{2}\right)+c$$

$$=\frac{b^2}{4}-\frac{b^2}{2}+c=-\frac{b^2}{4}+c$$

Now, $\dfrac{dy}{dx}=f'(x)=2x+b$

At point $\left(\dfrac{\alpha+\beta}{2}, f\left(\dfrac{\alpha+\beta}{2}\right)\right)$,

i.e., $\left(-\dfrac{b}{2}, -\dfrac{b^2}{4}+c\right)$,

$$\frac{dy}{dx}=2\left(-\frac{b}{2}\right)+b=0$$

Hence, the slope of the tangent to the curve and the positive direction of x-axis is $0°$.

33. Given function is

$$f(x)=x^2+bx+c$$

It is a quadratic equation in x.

So, we will get a parabola either downward or upward.

Hence, it is a many one mapping and not onto mapping.

Hence, it is neither one-to-one nor onto mapping.

34. Given,

$$A=\begin{bmatrix}\cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\text{Now, } A\times A=\begin{bmatrix}\cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\cos^2\left(\frac{2\pi}{n}\right)-\sin^2\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)+\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & 0\\ -\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right)-\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & -\sin^2\left(\frac{2\pi}{n}\right)+\cos^2\left(\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\left(2\times\frac{2\pi}{n}\right) & 2\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & 0\\ -2\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & \cos\left(2\times\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos\left(2\times\frac{2\pi}{n}\right) & \sin\left(2\times\frac{2\pi}{n}\right) & 0\\ -\sin\left(2\times\frac{2\pi}{n}\right) & \cos\left(2\times\frac{2\pi}{n}\right) & 0\\ 0 & 0 & 1\end{bmatrix}$$

Similarly,

$$A^n = \begin{bmatrix} \cos\left(2^{n-1} \times \frac{2\pi}{n}\right) & \sin\left(2^{n-1} \times \frac{2\pi}{n}\right) & 0 \\ -\sin\left(2^{n-1} \times \frac{2\pi}{n}\right) & \cos\left(2^{n-1} \times \frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

and

$$A^{n-1} = \begin{bmatrix} \cos\left(2^{n-2} \times \frac{2\pi}{n}\right) & \sin\left(2^{n-2} \times \frac{2\pi}{n}\right) & 0 \\ -\sin\left(2^{n-2} \times \frac{2\pi}{n}\right) & \cos\left(2^{n-2} \times \frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\neq I$$

$\therefore$ $A^m \neq I$ for any positive integer m.

35. Total sample space $= \{B_1B_2,\ B_1G_1,\ G_1B_2\}$

Favourable ways $= \{B_1G_1,\ G_1B_2\}$

$\therefore$ Required probability $= \frac{2}{3}$

36. We know that

$$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n \quad \dots\text{(i)}$$

and $$(x + 1)^n = {}^nC_0x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \dots + {}^nC_n \quad \dots\text{(ii)}$$

On multiplying Eqs. (i) and (ii), we get

$$(1 + x)^{2n} = ({}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n) \times ({}^nC_0x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \dots + {}^nC_n)$$

Coefficient of x^n in RHS

$$= ({}^nC_0)^2 + ({}^nC_1)^2 + \dots + ({}^nC_n)^2$$

and coefficient of x^n in LHS $= {}^{2n}C_n$

$$\therefore ({}^nC_0)^2 + ({}^nC_1)^2 + \dots + ({}^nC_n)^2 = \frac{2n!}{n!\,n!}$$

$$\Rightarrow ({}^nC_1)^2 + \dots + ({}^nC_n)^2 = \frac{(2n)!}{n!\,n!} - 1$$

$$= {}^{2n}C_n - 1$$

37. Let $S = 1! + 2! + 3! + 4! + \dots + 11!$

Here, we see that from 4! to 11!, we always get a 12 factor, so it is always divisible by 12.

Now, $\quad 1! + 2! + 3! = 1 + 2 + 6 = 9$

Hence, when S is divided by 12, the remainder is 9.

38. 3 consonants can be selected from 7 consonants

$$= {}^7C_3 \text{ ways}$$

2 vowels can be selected from 4 vowels

$$= {}^4C_2 \text{ ways}$$

$\therefore$ Required number of words

$$= {}^7C_3 \times {}^4C_2 \times 5!$$

[selected 5 letters can be arrange in 5!, so get, a different words]

$$= 35 \times 6 \times 120 = 25200$$

39. We know that

$$(1 + x)^n = {}^nC_0 + x\ {}^nC_1 + x^2\ {}^nC_2 + \dots + x^n\ {}^nC_n$$

On integrating both sides from 0 to 2, we get

$$\left[\frac{(1 + x)^{n+1}}{n + 1}\right]_0^2$$

$$= \left[x\ {}^nC_0 + \frac{x^2}{2}\ {}^nC_1 + \frac{x^3}{3}\ {}^nC_2 + \dots + \frac{x^{n+1}}{n+1}\ {}^nC_n\right]_0^2$$

$$\Rightarrow \frac{(3)^{n+1}}{n+1} - \frac{1}{n+1} = 2\ {}^nC_0 + \frac{2^2}{2}\ {}^nC_1 + \frac{2^3}{3}\ {}^nC_2 + \dots + \frac{2^{n+1}}{n+1}\ {}^nC_n - 0$$

$$\Rightarrow \frac{2}{1}\ {}^nC_0 + \frac{2^2}{2}\ {}^nC_1 + \frac{2^3}{3}\ {}^nC_2 + \dots + \frac{2^{n+1}}{n+1}\ {}^nC_n$$

$$= \frac{3^{n+1} - 1}{n + 1}$$

40. Given, $$f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Continuity at $x = 0$,

$$\text{LHL} = \lim_{x \to 0^-} x \sin\frac{1}{x} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} x \sin\frac{1}{x} = 0$$

and $\quad f(0) = 0$

$\therefore \quad \text{LHL} = \text{RHL} = f(0)$

Hence, $f(x)$ is continuous for all values of x.

Differentiability at $x = 0$

$$\text{LHD} = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{h \sin \frac{1}{h}}{h}$$

$$= -\lim_{h \to 0} \sin \frac{1}{h}$$

$$= -\sin\left(\frac{1}{0}\right) = \text{some definite value}$$

Thus, $f(x)$ is not differentiable at $x = 0$.

Now, $\quad f(0) = 0$ and $f(1) = 1 \sin 1$

$\therefore$ $f(0) \neq f(1)$, Rolle's theorem is not satisfy.

Now, $\quad f'(c) = \dfrac{f(1) - f(0)}{b - 0}$

$$= \frac{1 \sin(1) - 0}{1 - 0} = \sin 1 = \sin 1$$

which lies between 0 to 1.

Hence, their exist a real number $c \in (0, 1)$.

Hence, Lagrange's mean value theorem is satisfy on [0, 1].

41. Since, a, b and c are in GP.

$\therefore \quad b^2 = ac$

Given equation is

$$(\log_e a)\, x^2 - (2 \log_e b)\, x + (\log_e c) = 0$$

Put $x = 1$, we get

$$\log_e a - 2 \log_e b + \log_e c = 0$$

$\Rightarrow \quad 2 \log_e b = \log_e a + \log_e c$

$\Rightarrow \quad \log_e b^2 = \log_e ac$

$\Rightarrow \quad b^2 = ac$, which is true.

Hence, one of the root of given equation is 1.

Let another root be α.

$\therefore$ Sum of roots, $1 + \alpha = \dfrac{2 \log_e b}{\log_e a} = \dfrac{\log_e b^2}{\log_e a}$

$\Rightarrow \quad \alpha = \dfrac{\log_e ac}{\log_e a} - 1$

$$= \frac{(\log_e a + \log_e c)}{\log_e a} - 1$$

$$= \frac{\log_e c}{\log_e a} = \log_a c$$

Hence, roots are 1 and $\log_a c$.

42. Let D denotes dancing, P denotes painting and S denotes singing.

$\therefore \quad n(D \cup P \cup S) = 265,$

$n(S) = 200,\ n(D) = 110,\ n(P) = 55,$

$n(S \cap D) = 60,\ n(S \cap P) = 30$

and $\quad n(D \cap P \cap S) = 10$

$\because n(D \cup P \cup S) = n(D) + n(P) + n(S) - n(D \cap P)$

$\quad - n(P \cap S) - n(S \cap D) + n(D \cap P \cap S)$

$\therefore \quad 265 = 110 + 55 + 200 - n(D \cap P)$

$\quad - 30 - 60 + 10$

$\Rightarrow \quad 265 = 285 - n(D \cap P)$

$\Rightarrow \quad n(D \cap P) = 20$

$\therefore$ Only person who like dancing and painting

$= n(D \cap P) - n(D \cap P \cap S)$

$= 20 - 10 = 10$

43. Given function is $y = 3 \sin\left(\sqrt{\dfrac{\pi^2}{16} - x^2}\right)$

Here, domain is $\left[0, \dfrac{\pi}{4}\right]$

When, $\quad x = 0$

$$y = 3 \sin\left(\sqrt{\frac{\pi^2}{16} - 0}\right)$$

$$= 3 \sin\left(\frac{\pi}{4}\right) = 3 \times \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

When $x = \dfrac{\pi}{4}$, $\ y = 3 \sin\left(\sqrt{\dfrac{\pi^2}{16} - \dfrac{\pi^2}{16}}\right)$

$$= 3 \sin(0) = 0$$

Hence, range of given function is $\left[0, \dfrac{3}{\sqrt{2}}\right]$.

44. $\displaystyle \lim_{x \to 0} \frac{\int_0^{x^2} \cos(t^2)\, dt}{x \sin x}$ $\quad \left[\dfrac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 0} \frac{\cos(x^4) \times 2x}{\sin x + x \cos x} \quad \text{[L' Hospital's rule]}$$

$$= \lim_{x \to 0} \frac{2\,[\cos x^4 - x \sin(x^4) \times 4x^3}{\cos x + \cos x - x \sin x}$$

[L' Hospital's rule]

$$= \frac{2\,[\cos 0 - 0]}{\cos 0 + \cos 0 - 0} = \frac{2}{1 + 1} = 1$$

45. Given, $f'(4) = 5$

Now, $\lim_{x \to 2} \dfrac{f(4) - f(x^2)}{x - 2}$ $\left[\dfrac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 2} \frac{0 - f'(x^2) \cdot 2x}{1}$$

$$= \frac{-f'(4) \cdot 2 \times 2}{1}$$

$$= -(5) \times 4 = -20$$

46. We know that

$$\sin n\pi = 0, \forall\ n \in N.$$

$$\therefore\ E = \sum_{n=1}^{\infty} \sin\left(\frac{n!\,\pi}{720}\right) = \sin\left(\frac{\pi}{720}\right) + \sin\left(\frac{2!\,\pi}{720}\right) + \sin\left(\frac{3!\,\pi}{720}\right) + \sin\left(\frac{4!\,\pi}{720}\right) + \sin\left(\frac{5!\,\pi}{720}\right) + \sin\left(\frac{6!\,\pi}{720}\right) + \ldots + \sin\left(\frac{720!\,\pi}{720}\right)$$

$$= \sin\left(\frac{\pi}{720}\right) + \sin\left(\frac{\pi}{320}\right) + \sin\left(\frac{\pi}{120}\right) + \sin\left(\frac{\pi}{30}\right) + \sin\left(\frac{\pi}{6}\right) + \sin \pi + \ldots + \sin\left(\frac{720!\,\pi}{720}\right)$$

Here, we see that after five terms of the above series all angles are multiple of π, *i.e.,* $\sin n\pi$, so all further values are zero.

$$\therefore\quad E = \sin\left(\frac{\pi}{720}\right) + \sin\left(\frac{\pi}{320}\right) + \sin\left(\frac{\pi}{120}\right) + \sin\left(\frac{\pi}{30}\right) + \sin\left(\frac{\pi}{6}\right)$$

47. Given, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Then, $\det(I) = 1$

If we take I as

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, $\det(I_1) = -1$

Similarly, there are four other possibilities,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

who will give a determinant either -1 or 1.

Hence, there are six distinct choices for P and $\det(P) = \pm 1$.

48. Given expression can be rewritten as

$$2(1-x)^{-1} \times 2^{-1}\left(1 - \frac{x}{2}\right)^{-1}$$

$$= [1 + x + x^2 + x^3 + \ldots]\left[1 + \left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \ldots\right]$$

$\therefore$ Coefficient of x^3 in the above expression is

$$= \left[\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + 1\right]$$

$$= \left[\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1\right]$$

$$= \left[\frac{1 + 2 + 4 + 8}{8}\right] = \frac{15}{8}$$

49. Given,

$$f(x) = \frac{x}{1!} + \frac{3}{2!}x^2 + \frac{7}{3!}x^3 + \frac{15}{4!}x^4 + \ldots$$

$$= \frac{(2^1 - 1)x}{1!} + \frac{(2^2 - 1)x^2}{2!} + \frac{(2^3 - 1)x^3}{3!} + \frac{(2^4 - 1)}{4!}x^4 + \ldots$$

$$= \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \ldots - \left(\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\right)$$

$$= 1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \ldots - \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\right)$$

$$\Rightarrow \quad f(x) = e^{2x} - e^x$$

When we put $x = 0$, we get

$$f(0) = e^0 - e^0 = 1 - 1 = 0$$

Hence, exactly one real solution exists.

50. Let $S = 1 + \frac{8}{2!} + \frac{21}{3!} + \frac{40}{4!} + \frac{65}{5!} + \ldots$

Again, let $S_1 = 1 + 8 + 21 + 40 + 65 + \ldots + T_n$

and $S_1 = \quad + 1 + 8 + 21 + 40 + \ldots + T_n$

$$0 = 1 + 7 + 13 + 19 + 25 + \ldots - T_n$$

$$T_n = 1 + 7 + 13 + 19 + 25 + \ldots + n \text{ terms}$$

$$= \frac{n}{2}[2(1) + (n-1)6]$$

$$= n[1 + 3(n-1)] = n(3n-2)$$

$\therefore \quad S = \sum \frac{n(3n-2)}{n!}$

$$= \sum \frac{3n-2}{(n-1)!}$$

$$= \sum \frac{3n-3+1}{(n-1)!}$$

$$S = \sum \frac{3}{(n-2)!} + \sum \frac{1}{(n-1)!}$$

$$= 3e + e \qquad \left[\because e = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\right]$$

$$= 4e$$

We know $\quad 2 < e < 3$

$\therefore \quad 8 < 4e < 12$

$\Rightarrow \quad 8 < S < 12$

51. We have,

$$n\sqrt{2} - 1 < [n\sqrt{2}] \le n\sqrt{2} \qquad [\because x - 1 \le [x] \le x]$$

$\Rightarrow \quad \sqrt{2} - \frac{1}{n} < [n\sqrt{2}] \le 1$

$\therefore$ By Sandwich theorem,

$$\lim_{n \to \infty}\left(\sqrt{2} - \frac{1}{n}\right) = \sqrt{2}$$

52. Given, $f'(0) = 1$ and $f''(0)$ does not exist.

Also, given $g(x) = xf'(x)$

$\therefore \quad g'(x) = xf''(x) + f'(x)$

Put $x = 0$, we get

$$g'(0) = 0\, f''(0) + f'(0)$$

$$= 0 + 1 = 1$$

53. Given, $z_1 \ne \pm 1$

Since, z_2 and z_3 can be obtained by rotating vector representing through $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$, respectively

$\therefore \quad z_2 = z_1\omega$

and $\quad z_3 = z_1\omega^2$

$\therefore \quad z_1z_2z_3 = z_1 \times z_1\omega \times z_1\omega^2$

$$= z_1^3\omega^3$$

$$= z_1^3 \qquad [\because \omega^3 = 1]$$

54. Since, z_1, z_2 and z_3 are the vertices of an equilateral triangle, therefore

$$|z_1 - z_2| = |z_2 - z_3|$$

$$= |z_3 - z_1| = k \qquad \text{(say)}$$

Also, $\quad \alpha = \frac{1}{2}(\sqrt{3} + i)$

$\Rightarrow \quad |\alpha| = \frac{1}{2}\sqrt{3+1} = \frac{1}{2} \times 2 = 1$

Let $\quad A = \alpha z_1 + \beta,\ B = \alpha z_2 + \beta$

and $\quad C = \alpha z_3 + \beta$

Now, $\quad |AB| = |\alpha z_2 + \beta - (\alpha z_1 + \beta)|$

$$= |\alpha(z_2 - z_1)|$$

$$= |\alpha||z_2 - z_1|$$

$$= |1||z_2 - z_1|$$

$$= 1|z_2 - z_1|$$

$$= |z_2 - z_1| = k$$

Similarly, $\quad BC = CA = k$

Hence, the points $\alpha z_1 + \beta$, $\alpha z_2 + \beta$ and $\alpha z_3 + \beta$ are the vertices of an equilateral triangle.

55. Given curve is

$$y = (\cos x + y)^{1/2}$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2(\cos x + y)^{1/2}} \times \left(-\sin x + \frac{dy}{dx}\right)$$

$$= \frac{\left(-\sin x + \frac{dy}{dx}\right)}{2y}$$

$\Rightarrow \quad 2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$

$\Rightarrow \quad (2y - 1)\frac{dy}{dx} = -\sin x$

Again, differentiating both sides w.r.t. x, we get

$$(2y-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(2\frac{dy}{dx}\right) = -\cos x$$

$\Rightarrow \quad (2y-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(2\frac{dy}{dx}\right) + \cos x = 0$

56. Given equation is

$$1 + z + z^3 + z^4 = 0$$

$$\Rightarrow \quad (1 + z) + z^3(1 + z) = 0$$

$$\Rightarrow \quad (1 + z)(1 + z^3) = 1$$

$$\Rightarrow \quad z = -1, z^3 = -1$$

$$\Rightarrow \quad z = -1, z = -1, -\omega, -\omega^2$$

Hence, roots are in cubic roots of unity.

Hence, these roots are the vertices of an equilateral triangle.

57. $\sum a^3 \sin(B - C)$

$$= \sum k^3 \sin^3 A \sin(B - C)$$

$$\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\right]$$

$$= \sum k^3 [\sin^2 A \sin(B + C) \sin(B - C)]$$

$$= k^3 \left[\left\{\sin^2 A \times \frac{1}{2}(\cos 2C - \cos 2B)\right\} + \sin^2 B \times \frac{1}{2}(\cos 2A - \cos 2C) + \sin^2 C \times \frac{1}{2}(\cos 2B - \cos 2A)\right]$$

$$= \frac{k^3}{2}[\sin^2 A(1 - 2\sin^2 C - 1 + 2\sin^2 B) + \sin^2 B(1 - 2\sin^2 A - 1 + 2\sin^2 C) + \sin^2 C(1 - 2\sin^2 B - 1 + 2\sin^2 A)]$$

$$= \frac{k^3}{2}[-2\sin^2 A \sin^2 C + 2\sin^2 A \sin^2 B - 2\sin^2 B \sin^2 A + 2\sin^2 B \sin^2 C - 2\sin^2 C \sin^2 B + 2\sin^2 C \sin^2 A]$$

$$= \frac{k^3}{2}[0] = 0$$

58. Given, α and β are the roots of $x^2 - x - 1 = 0$

$$\therefore \quad \alpha + \beta = 1$$

and $\quad \alpha\beta = -1$

Now, consider option (a),

$$S_n + S_{n-1} = S_{n+1}$$

Put $n = 2$, we get, $\quad S_2 + S_1 = S_3$

$$\therefore \quad (\alpha^2 + \beta^2) + (\alpha + \beta) = \alpha^3 + \beta^3$$

$$[\because S_n = \alpha^n + \beta^n, \text{ given}]$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta + (\alpha + \beta) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$\Rightarrow \quad 1^2 - 2(-1) + 1 = 1^3 - 3(-1)(1)$$

$$\Rightarrow \quad 1 + 2 + 1 = 1 + 3$$

$$\Rightarrow \quad 4 = 4, \text{ which is true.}$$

Hence, option (a) is correct.

59. Required probability

$$= {}^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2 \times {}^{4}C_2 \times {}^{2}C_2 \times \left(\frac{1}{6}\right)^{12}$$

$$= \frac{12!}{10! \times 2!} \times \frac{10!}{8! \times 2!} \times \frac{8!}{6! \times 2!} \times \frac{6!}{4! \times 2!} \times \frac{4!}{2! \times 2!} \times \frac{2!}{2! \times 1!} \times \left(\frac{1}{6}\right)^{12}$$

$$= \frac{12!}{2^6 \times 6^{12}}$$

60. $\because$ Sum of roots, $\alpha + \beta = -p$ and $\alpha\beta = q$

$$\therefore \quad (\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= (-p)^3 - 3q(-p)$$

$$= -p^3 + 3pq$$

and $\alpha^4 + \alpha^2\beta^2 + \beta^4 = (\alpha^4 + \beta^4) + (\alpha\beta)^2$

$$= (\alpha^2 + \beta^2)^2 - (\alpha\beta)^2$$

$$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - (\alpha\beta)^2$$

$$= [(-p)^2 - 2q]^2 - 3(q)^2$$

$$= [p^2 - 2q]^2 - 3q^2$$

$$= p^4 - 4p^2q + 4q^2 - q^2$$

$$= p^4 - 4p^2q + 3q^2$$

$$= p^4 - 3p^2q - p^2q + 3q^2$$

$$= p^2(p^2 - 3q) - q(p^2 - 3q)$$

$$= (p^2 - q)(p^2 - 3q)$$

61. Given, differential equation is

$$\frac{dy}{dx} + \frac{y}{x \log_e x} = \frac{1}{x}$$

It is a linear equation of the form

$$\frac{dy}{dx} + Py = Q$$

where $\quad P = \dfrac{1}{x \log_e x}$ and $Q = \dfrac{1}{x}$

$\therefore$ Integrating factor, $IF = e^{\int \frac{1}{x \log_e x} dx}$

$$= e^{\log(\log_e x)}$$
$$= \log_e x$$

$\therefore$ Solution of differential equation is

$$y \times \log_e x = \int \frac{1}{x} \log_e x \, dx$$

$$\Rightarrow \quad y \times \log_e x = \frac{(\log_e x)^2}{2} + C \quad \ldots(i)$$

When $y = 1$ and $x = e$, then

$$1 \times \log_e e = \frac{(\log_e e)^2}{2} + C$$

$$\Rightarrow \quad 1 = \frac{1}{2} + C$$

$$\Rightarrow \quad C = \frac{1}{2}$$

On putting $C = \frac{1}{2}$ in Eq. (i), we get

$$y \times \log_e x = \frac{(\log_e x)^2}{2} + \frac{1}{2}$$

$$\Rightarrow \quad 2y = \log_e x + \frac{1}{\log_e x}$$

62. Given, $f(x) = \max\{x + |x|, x - [x]\}$

$$= \begin{cases} 2x, & x \geq 0 \\ x - [x], & x \leq 0 \end{cases}$$

$$\therefore \quad \int_{-3}^{3} f(x)\,dx = \int_{-3}^{0} x - [x]\,dx + \int_{0}^{3} 2x\,dx$$

$$= 3\int_{-1}^{0} (1 + x)\,dx + 2\int_{0}^{3} x\,dx$$

[$\because$ $x - [x]$ is a periodic function at $x = 1$]

$$= 3\left[x + \frac{x^2}{2}\right]_{-1}^{0} + 2\left[\frac{x^2}{2}\right]_{0}^{3}$$

$$= 3\left[0 - 0 - \left(-1 + \frac{1}{2}\right)\right] + 3^2 - 0$$

$$= 3\left[\frac{1}{2}\right] + 9 = \frac{3}{2} + 9$$

$$= \frac{21}{2}$$

63. Given, $X_n = \left\{z = x + iy : |z|^2 \leq \frac{1}{n}\right\}$

$$= \left\{x^2 + y^2 \leq \frac{1}{n}\right\}$$

$$\therefore \quad X_1 = \{x^2 + y^2 \leq 1\},$$

$$X_2 = \left\{x^2 + y^2 \leq \frac{1}{2}\right\}$$

$$X_3 = \left\{x^2 + y^2 \leq \frac{1}{3}\right\}$$

....

....

....

$$X_\infty = \{x^2 + y \leq 0\}$$

$$\therefore \quad \bigcap_{n=1}^{\infty} X_n = X_1 \cap X_2 \cap X_3 \cap \ldots \cap X_\infty$$

$$= \{x^2 + y^2 = 0\}$$

Hence, $\bigcap_{n=1}^{\infty} X_n$ is a singleton set.

64. We know that in a Lagrange mean value theorem there exist $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\therefore \quad f'(\theta h) = \frac{f(h) - \cos 0}{h - 0}$$

$$\Rightarrow \quad -\sin(\theta h) = \frac{\cos h - \cos 0}{h - 0}$$

[$\because f(x) = \cos x$]

$$= \frac{\cos h - 1}{h}$$

$$= \frac{\left(1 - \frac{h^2}{2!}\right) - 1}{h}$$

[neglecting higher power of h]

$$\Rightarrow \quad -\sin(\theta h) = \frac{-\frac{h^2}{2}}{h}$$

$$\Rightarrow \quad \sin(\theta h) = \frac{\frac{h}{2}}{1}$$

$$\Rightarrow \quad \theta h = \sin^{-1}\left(\frac{h}{2}\right)$$

$$\Rightarrow \quad \theta = \frac{\sin^{-1}\frac{h}{2} \times \frac{1}{2}}{h \times \frac{1}{2}}$$

$$\therefore \quad \lim_{h\to\theta^+} \theta = \frac{1}{2} \lim_{h\to 0^+} \frac{\sin^{-1}\frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

65. Let equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{...(i)}$$

Given, foci, $(\pm 8, 0) = (\pm ae, 0)$

$$\Rightarrow \quad ae = 8 \qquad \text{...(ii)}$$

and length of latusrectum $= \dfrac{2b^2}{a}$

$$\therefore \quad 24 = \frac{2b^2}{a}$$

$$\Rightarrow \quad b^2 = 12a \qquad \text{...(iii)}$$

$\therefore$ From Eq. (ii),

$$a^2e^2 = 64$$

$$\Rightarrow \quad a^2\left(\frac{a^2+b^2}{a^2}\right) = 64$$

$$\Rightarrow \quad a^2 + b^2 = 64$$

$$\Rightarrow \quad a^2 + 12a = 64$$

$$\Rightarrow \quad a^2 + 12a - 64 = 0$$

$$\Rightarrow \quad a^2 + 16a - 4a - 64 = 0$$

$$\Rightarrow \quad a(a+16) - 4(a+16) = 0$$

$$\Rightarrow \quad (a+16)(a-4) = 0$$

$$\Rightarrow \quad a = 4$$

[$\because$ a cannot be negative]

On putting $a = 4$ in Eq. (iii), we get

$$b^2 = 12 \times 4 \Rightarrow b^2 = 48$$

$\therefore$ From Eq. (i),

$$\frac{x^2}{16} - \frac{y^2}{48} = 1$$

$$\Rightarrow \quad 3x^2 - y^2 = 48$$

66. Let, E_1 = Student does not know the answer

E_2 = Student knows the answer

and E = Student answer correctly.

$$\therefore \quad P(E_1) = 1 - p,$$

$$P(E_2) = p$$

$$P\left(\frac{E}{E_2}\right) = 1 \text{ and } P\left(\frac{E}{E_1}\right) = \frac{1}{5}$$

$\therefore$ The probability that student did not tick the answer randomly

= The probability that student tick the answer correctly

$$= \frac{P(E_2)\,P\left(\frac{E}{E_2}\right)}{P(E_1)\,P\left(\frac{E}{E_1}\right) + P(E_2)\,P\left(\frac{E}{E_2}\right)}$$

$$= \frac{p(1)}{(1-p)\frac{1}{5} + p(1)}$$

$$= \frac{p}{\frac{1-p+5p}{5}} = \frac{5p}{1+4p}$$

67. Let $\dfrac{2\pi r}{7} = \theta$

$$\Rightarrow \quad 2\pi r = 3\theta + 4\theta$$

$$\Rightarrow \quad 4\theta = 2\pi r - 3\theta$$

$$\Rightarrow \quad \sin 4\theta = \sin(2\pi r - 3\theta)$$

$$\Rightarrow \quad \sin 4\theta = -\sin 3\theta$$

$$\Rightarrow \quad 2\sin 2\theta \cos 2\theta = -[3\sin\theta - 4\sin^3\theta]$$

$$\Rightarrow \quad 2 \times 2\sin\theta\cos\theta(2\cos^2\theta - 1)$$

$$= -3\sin\theta + 4\sin^3\theta$$

$$\Rightarrow \sin\theta[8\cos^3\theta - 4\cos\theta + 3 - 4(1-\cos^2\theta)] = 0$$

$$\Rightarrow \quad 8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1 = 0 \qquad \text{...(i)}$$

Thus, $\cos\dfrac{2\pi}{7}$, $\cos\dfrac{4\pi}{7}$ and $\cos\dfrac{6\pi}{7}$ are the roots of the above equation.

$$\therefore \quad \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$$

68. Given, $M = \displaystyle\int_0^{\pi/2} \frac{\cos x}{(x+2)}\,dx$

and $\quad N = \displaystyle\int_0^{\pi/4} \frac{\sin x\cos x}{(x+1)^2}\,dx$

$$\Rightarrow \quad N = \int_0^{\pi/4} \frac{1}{2}\frac{\sin 2x}{(x+1)^2}\,dx$$

Put $2x = t \Rightarrow dx = \dfrac{dt}{2}$

$\therefore \quad N = \int_0^{\pi/2} \frac{\sin t}{4\left(\frac{t}{2}+1\right)^2} dt$

$= \int_0^{\pi/2} \frac{\sin t}{(t+2)^2} dt$

$= \int_0^{\pi/2} \frac{\sin x}{(x+2)^2} dx$

$\therefore \quad M - N = \int_0^{\pi/2} \underset{\text{II}}{\cos x} \times \underset{\text{I}}{\frac{1}{(x+2)}} dx$

$- \int_0^{\pi/2} \frac{\sin x}{(x+2)^2} dx$

$= \left[\frac{\sin x}{x+2}\right]_0^{\pi/2} - \int_0^{\pi/2} - \frac{\sin x}{(x+2)^2} dx$

$- \int_0^{\pi/2} \frac{\sin x}{(x+2)^2} dx$

$= \frac{\sin \frac{\pi}{2}}{\frac{\pi}{2}+2} = \frac{1}{\frac{\pi+4}{2}} = \frac{2}{\pi+4}$

69. Given relation is defined as

$\theta R \phi$ such that $\sec^2\theta - \tan^2\phi = 1$

For Reflexive

When $\theta R \theta$

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow$ $1 = 1$, which is true.

Thus, it is reflexive.

For Symmetric

When $\theta R \phi$

$$\sec^2\theta - \tan^2\phi = 1$$

$\Rightarrow (1 + \tan^2\theta) - (\sec^2\phi - 1) = 1$

$\Rightarrow \quad 2 + \tan^2\theta - \sec^2\phi = 1$

$\Rightarrow \quad \sec^2\phi - \tan^2\theta = 1$

$\Rightarrow \phi R \theta$

Thus, it is symmetric.

For Transitive

When $\theta R \phi$ and $\phi R \psi$, then

$$\sec^2\theta - \tan^2\phi = 1$$

and $\quad \sec^2\phi - \tan^2\psi = 1$

Now, $\theta R \psi$

Then, $\quad \sec^2\theta - \tan^2\psi = 1$

$\Rightarrow \quad \sec^2\theta - \tan^2\psi + 1 = 1 + 1$

$\Rightarrow \sec^2\theta - \tan^2\psi + \sec^2\phi - \tan^2\phi = 1 + 1$

$\Rightarrow \quad \theta R \phi$ and $\phi R \psi$

Thus, it is transitive.

Hence, it is an equivalence relation.

70. We know that

$$\text{AM} \geq \text{GM}$$

$\therefore \quad \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x}\, 2^{\cos x}}$

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2\sqrt{2^{\sin x + \cos x}}$

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2 \times 2^{\frac{\sin x + \cos x}{2}}$

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2^{1 + \frac{\sin x + \cos x}{2}}$

But $\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) \geq -\sqrt{2}$

$\therefore \quad 2^{\sin x} + 2^{\cos x} \geq 2^{1 - \frac{\sqrt{2}}{2}}$

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2^{1 - \frac{1}{\sqrt{2}}}, \forall x \in R$

Hence, minimum value is $2^{1-\frac{1}{2}}$.

71. Let the relation defined as

$$R = \{(A, B) : B = PAQ^{-1}\}$$

For reflexive

$$A = IAI^{-1}$$

$\Rightarrow \quad (A, A) \in R$

$\Rightarrow R$ is reflexive

For symmetric

Let $(A, B) \in R$

$\therefore \quad B = PAQ^{-1}$

$\Rightarrow \quad BQ = PA(Q^{-1}Q)$

$\Rightarrow \quad BQ = PA$

$\Rightarrow \quad P^{-1}(BQ) = P^{-1}P(A)$

$\Rightarrow \quad P^{-1}BQ = A$

$\Rightarrow (B, A) \in R$

$\Rightarrow R$ is symmetric.

For transitive

Let $(A, B) \in R, (B, C) \in R$

Then, $\quad A = PBQ^{-1}$ and $B = RCS^{-1}$

$\Rightarrow \quad A = PRCS^{-1}Q^{-1}$

$\Rightarrow \quad A = (PR)\, C\, (QS)^{-1}$

$\Rightarrow \quad (A, C) \in R$

$\Rightarrow R$ is transitive.

Hence, R is an equivalence relation.

72. ***Case* I** Let $\alpha = \omega$ and $\beta = \omega^2$

$$\therefore \quad S = \sum_{n=0}^{302} (-1)^n \left(\frac{\omega}{\omega^2}\right)^n$$

$$= \sum_{n=0}^{302} (-1)^n (\omega^2)^n$$

$$= 1 - \omega^2 + \omega^4 - \omega^6 + \omega^8 - \omega^{10} + \omega^{12} + \ldots + \omega^{600} - \omega^{602} + \omega^{604}$$

$$= 1 - \omega^2 + \omega - 1 + \omega^2 - \omega + 1 + \ldots + 1 - \omega^2 + \omega$$

$$= 0 + \ldots + 1 - \omega^2 + \omega$$

$$= -\omega^2 - \omega^2 = -2\omega^2 \qquad [\because 1 + \omega + \omega^2 = 0]$$

***Case* II** Let $\alpha = \omega^2$ and $\beta = \omega$

$$\therefore \quad S = \sum_{n=0}^{302} (-1)^n \left(\frac{\omega^2}{\omega}\right)^n$$

$$= \sum_{n=0}^{302} (-1)^n \left(\frac{\omega^4}{\omega^3}\right)^n$$

$$= \sum_{n=0}^{302} (-1)^n (\omega)$$

$$= 1 - \omega + \omega^2 - \omega^3 + \omega^4 - \omega^5 + \omega^6 - \ldots + \omega^{300} - \omega^{301} + \omega^{302}$$

$$= 1 - \omega + \omega^2 - 1 + \omega - \omega^2 + 1 - \ldots + 1 - \omega + \omega^2$$

$$= 0 + \ldots + 1 + \omega^2 - \omega$$

$$= -\omega - \omega = -2\omega$$

73. Let $\quad S = 1 + 10 + 21 + 34 + 49 + \ldots + t'_n$

and $\quad S = 1 + 10 + 21 + 34 + \ldots + t'_n$

$\quad\;\; 0 = 1 + 9 + 11 + 13 + 15 + \ldots - t'_n$

$$\Rightarrow t'_n = 1 + [9 + 11 + 13 + 15 + \ldots (n-1) \text{ term}]$$

$$= 1 + \left[\frac{n-1}{2}\{2 \times 9 + (n-2)\, 2\}\right]$$

$$= 1 + (n-1)\, [9 + n - 2]$$

$$= 1 + (n-1)(n+7)$$

$$\therefore \quad t_n = \frac{t'_n}{n!} = \frac{1 + (n-1)(n+7)}{n!}$$

$$= \frac{1 + (n-1)(n+7)}{n!}$$

$$= \frac{1 + n^2 + 6n - 7}{n!} = \frac{n^2 + 6n - 6}{n!}$$

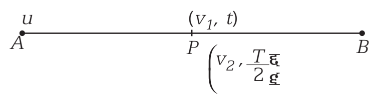
$$= \frac{n}{(n-1)!} + \frac{6}{(n-1)!} - \frac{1}{n!}$$

$$= \frac{n - 1 + 1}{(n-1)!} + \frac{6}{(n-1)!} - \frac{1}{n!}$$

$$= \frac{1}{(n-2)!} + \frac{7}{(n-1)!} - \frac{1}{n!}$$

$$\therefore \quad \lim_{n \to \infty} t_n = \lim_{n \to \infty}\left[\frac{1}{(n-2)!} + \frac{7}{(n-1)!} - \frac{1}{n!}\right] = 0$$

74 . Let time taken from point A to P is t.



When a particle moving from point A to P, using first equation of motion

$$v = u + at$$

When final velocity is v_1 and time t_2, then

$$v_1 = u + at$$

$$\Rightarrow \quad v_1 - u = at \qquad \ldots\text{(i)}$$

When final velocity is v_2 and time $T/2$, then

$$v_2 = u + \frac{aT}{2} \qquad \ldots\text{(ii)}$$

When a particle moving from point P to B using first equation of motion

$$v = u + at$$

When intial velocity v_1 and time $T - t$, then

$$0 = v_1 - a\,(T - t)$$

$$\Rightarrow \quad v_1 = aT - at \qquad \ldots\text{(iii)}$$

When initial velocity v_2 and time $\frac{T}{2}$, then

$$0 = v_2 - \frac{aT}{2}$$

$$\Rightarrow \quad v_2 = \frac{aT}{2} \Rightarrow aT = 2v_2 \qquad \ldots\text{(iv)}$$

$\therefore$ From Eq. (iii),

$$v_1 = 2v_2 - (v_1 - u)$$

[from Eqs. (i) and (iv)]

$$\Rightarrow \quad 2v_1 = 2v_2 + u$$

$$\Rightarrow \quad v_1 = v_2 + \frac{u}{2}$$

$$\therefore \quad v_1 > v_2$$

75. $\therefore$ Required probability

$$= \frac{{}^{13}C_1 \times {}^{4}C_2 \times {}^{12}C_1 \times {}^{4}C_3}{{}^{52}C_5}$$

$$= \frac{13 \times 6 \times 12 \times 4}{\dfrac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}}$$

$$= \frac{13 \times 6 \times 12 \times 4}{52 \times 51 \times 10 \times 49 \times 2}$$

$$= \frac{6}{4165}$$

76. We know that $u(x)$ and $v(x)$ are two independent solutions of the given differential equation, then their linear combination is also the solution of the given equation.

Here, we see that $y = 5u(x) + 8v(x)$ is a linear combination and $y = c_1\{u(x) - v(x)\} + c_2 v(x)$ is also a linear combination of two independent solutions.

77. Given, $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 10$

We know that $(|\sin x| + |\cos x|) \in [1, \sqrt{2}]$

$$\therefore \quad y = 1$$

The point of intersection of given curve is

$$x^2 + 1^2 = 10$$

$$\Rightarrow \quad x^2 = 9$$

$$\Rightarrow \quad x = \pm 3$$

$\therefore$ Point of intersection is $(\pm 3, 1)$

Now, $\quad x^2 + y^2 = 10$

$$\Rightarrow \quad 2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$$

At point $(-3, 1)$

$$\frac{dy}{dx} = \frac{3}{1} = 3 \Rightarrow m_1 = 3$$

Slope of line $y = 1$ is $m_2 = 0$

$\therefore$ Angle between two curves is

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = 3$$

$$\Rightarrow \quad \theta = \tan^{-1}(3)$$

78. Given,

$$f(x) = \begin{cases} \int_0^x |1-t|\,dt, & x > 1 \\ x - \frac{1}{2}, & x \le 1 \end{cases} \quad \ldots\text{(i)}$$

Now, for $x > 1$, $\int_0^x |1-t|\,dt$

$$= \int_0^1 (1-t)\,dt + \int_1^x (t-1)\,dt$$

$$= \left[t - \frac{t^2}{2}\right]_0^1 + \left[\frac{t^2}{2} - t\right]_1^x$$

$$= \left[1 - \frac{1}{2} - 0\right] + \left[\frac{x^2}{2} - x - \frac{1}{2} + 1\right]$$

$$= \frac{1}{2} + \frac{x^2}{2} - x - \frac{1}{2} + 1$$

$$\Rightarrow \quad \frac{x^2}{2} - x + 1$$

$\therefore$ Eq. (i) becomes,

$$f(x) = \begin{cases} \frac{x^2}{2} - x + 1, & x > 1 \\ x - \frac{1}{2}, & x \le 1 \end{cases}$$

Continuity at $x = 1$,

$$\text{LHL} = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1}\left(x - \frac{1}{2}\right) = 1 - \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1}\left(\frac{x^2}{2} - x + 1\right)$$

$$= \frac{1}{2} - 1 + 1 = \frac{1}{2}$$

and $\quad f(1) = 1 - \frac{1}{2} = \frac{1}{2}$

$$\therefore \quad f(1) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

Hence, $f(x)$ is continuous at $x = 1$.

Differentiability at $x = 1$,

$$\text{LHD} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{(1-h) - \frac{1}{2} - \left(1 - \frac{1}{2}\right)}{-h} = \lim_{h \to 0} \frac{-h}{-h} = 1$$

$$\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(1+h)^2}{2} - (1+h) + 1 - \left(1 - \frac{1}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1 + h^2 + 2h}{2} - 1 - h + 1 - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2} + \frac{h^2}{2} + h - h - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{h^2}{2h} = 0$$

$\therefore$ $\text{LHD} \neq \text{RHD}$

Hence, $f(x)$ is not differentiable at $x = 1$.

79. Centres and constant terms in the circles $x^2 + y^2 - 5 = 0$, $x^2 + y^2 - 8x - 6y + 10 = 0$ and $x^2 + y^2 - 4x + 2y - 2 = 0$ are $C_1'(0, 0)$, $c_1 = -5$, $C_2'(4, 3)$, $c_2 = 10$ and $C_3'(2, -1)$, $c_3 = -2$

Also, centre and constant of circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

is $C_4'(-g, -f)$ and $c_4 = c$

Since, the first circle intersect all the three at the extremities of diameter, therefore they are orthogonal to each other.

$\therefore$ $2(g_1 g_2 + f_1 f_2) = c_1 + c$

$\therefore$ $2[-g(0) + (-f)(0)] = c - 5$

$\Rightarrow$ $c = 5$

$2[-g(4) + (-f)(3)] = c + 10$

$\Rightarrow$ $-2(4g + 3f) = 5 + 10$

$\Rightarrow$ $4g + 3f = -\frac{15}{2}$ ...(i)

and $2[-g(2) + (-f)(-1)] = c - 2$

$\Rightarrow$ $2[-2g + f] = 5 - 2$

$\Rightarrow$ $-2g + f = \frac{3}{2}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$f = -\frac{9}{10} \text{ and } g = -\frac{12}{10}$$

$\therefore$ $fg = \frac{-9}{10} \times -\frac{12}{10} = \frac{27}{25}$

and $4f = 4 \times \frac{-9}{10}$

$\Rightarrow$ $4f = \frac{-36}{10}$

$\Rightarrow$ $4f = 3g$

80. Given, $P(A) = 0.7$ and $P(B) = 0.6$

We know that

$$P(A \cap B) \leq P(B)$$

$\therefore$ $P(A \cap B) \leq 0.6$ ...(i)

From Booley's inequality

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

$\therefore$ $P(A \cap B) \geq [(P(A) + P(B) - (2-1)]$

$\Rightarrow$ $P(A \cap B) \geq [0.7 + 0.6 - 1]$

$\Rightarrow$ $P(A \cap B) \geq 0.3$ ...(ii)

From Eqs. (i) and (ii), we get

$$0.3 \leq P(A \cap B) \leq 0.6$$

Hence, options (c) and are (d) always incorrect.