WBJEE 2013 Question Paper

West Bengal Joint Entrance Examinations

WB JEE

Engineering Entrance Exam

Solved Paper 2013

Physics

Category I

Directions (Q. Nos. 1 to 45) [*Carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of* 1/3 *mark.*]

1. The r.m.s speed of the molecules of a gas at 100°C is v. The temperature at which the r.m.s. speed will be $\sqrt{3}v$ is

(a) 546°C (b) 646°C
(c) 746°C (d) 846°C

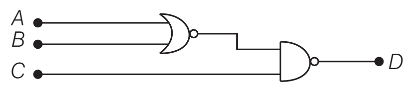
2. The equation of state of a gas is given by $\left(p + \frac{a}{v^3}\right)(V - b^2) = cT$, where p, V, T are pressure, volume and temperature respectively and a, b, c are constants. The dimensions of a and b are respectively

(a) $[ML^8T^{-2}$ and $L^{3/2}]$
(b) $[ML^5T^{-2}$ and $L^3]$
(c) $[ML^5T^{-2}$ and $L^6]$
(d) $[ML^6T^{-2}$ and $L^{3/2}]$

3. A frictionless piston-cylinder based enclosure contains some amount of gas at a pressure of 400 kPa. Then heat is transferred fo the gas at constant pressure in a quasi-static process. The piston moves up slowly through a height of 10 cm. If the piston has a cross-sectional area of 0.3 m^2, the work done by the gas in this process is

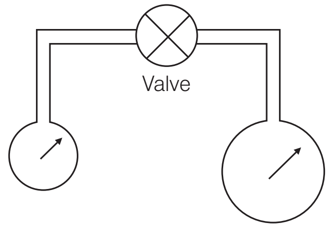
(a) 6 kJ (b) 12 kJ (c) 7.5 kJ (d) 24 kJ

4. A NOR gate and a NAND gate are connected as shown in the figure. Two different sets of inputs are given to this setup. In the first case, the inputs to the gates are $A = 0, B = 0, C = 0$. In the second case, the inputs are $A = 1, B = 0, C = 1$. The output D in the first case and second case respectively are



(a) 0 and 0 (b) 0 and 1
(c) 1 and 0 (d) 1 and 1

5. Two soap bubbles of radii r and $2r$ are connected by a capillary tube-valve arrangement shown in the diagram. The valve is now opened. Then which one of the following will result.



(a) The radii of the bubbles will remain unchanged
(b) The bubbles will have equal radii
(c) The radius of the smaller bubble will decrease and that of the bigger bubble will decrease
(d) The radius of the smaller bubble will decrease and that of the bigger bubble will increase

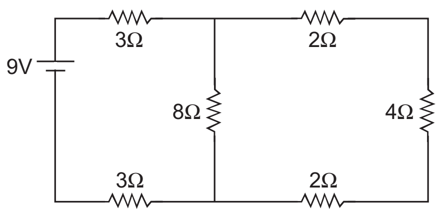
6. The velocity of a car travelling on a straight road is 3.6 kmh^{-1} at an instant of time. Now travelling with uniform acceleration for 10 s, the velocity becomes exactly double. If the wheel radius of the car is 25 cm, then which of the following is the closest to the number of revolutions that the wheel makes during this 10 s?

(a) 84 (b) 95
(c) 126 (d) 135

7. The ionization energy of the hydrogen atom is 13.6 eV. The potential energy of the electron in $n = 2$ state of hydrogen atom is

(a) + 3.4 eV (b) – 3.4 eV
(c) + 6.8 eV (d) – 6.8 eV

8. In the electrical circuit shown in the figure, the current through the 4Ω resistor is



(a) 1A (b) 0.5 A
(c) 0.25 A (d) 0.1 A

9. A current of 1 A is flowing along positive x-axis through a straight wire of length 0.5 m placed in a region of a magnetic field given by $\mathbf{B} = (2\,\mathbf{i} + 4\,\mathbf{j})$T. The magnitude and the direction of the force experienced by the wire respectively are

(a) $\sqrt{18}$ N, along positive z-axis
(b) $\sqrt{20}$ N, along positive x-axis
(c) 2 N, along positive z-axis
(d) 4 N, along positive y-axis

10. S_1 and S_2 are the coherent point sources of light located in the xy-plane at points (0,0) and (0, 3λ) respectively. Here λ is the wavelength of light. At which one of the following points (given as coordinates), the intensity of interference will be maximum?

(a) $(3\lambda, 0)$ (b) $(4\lambda, 0)$
(c) $(5\lambda/4, 0)$ (d) $(2\lambda/3, 0)$

11. Four small objects each of mass m are fixed at the corners of a rectangular wire-frame of negligible mass and of sides a and b ($a > b$). If the wire frame is now roated about an axis passing along the side of length b, then the moment of inertia of the system for this axis of rotation is

(a) $2ma^2$ (b) $4\,ma^2$
(c) $2m\,(a^2 + b^2)$ (d) $2m\,(a^2 - b^2)$

12. The de-Broglie wavelength of an electron (mass $= 1 \times 10^{-30}$ kg, charge $= 1.6 \times 10^{-19}$C) with a kinetic energy of 200 eV is (Planck's constant $= 6.6 \times 10^{-34}$ Js)

(a) 9.60×10^{-11} m (b) 8.25×10^{-11} m
(c) 6.25×10^{-11} m (d) 5.00×10^{-11} m

13. The number of atoms of a radioactive substance of half-life T is N_0 at $t = 0$. The time necessary to decay from $N_0 / 2$ atoms to $N_0 / 10$ atoms will be

(a) $\frac{5}{2}T$ (b) $T \log 5$
(c) $T \log \left[\frac{5}{2}\right]$ (d) $\frac{T}{2} \log 5$

14. A mass M at rest is broken into two pieces having masses m and $(M - m)$. The two masses are then separated by a distance r. The gravitational force between them will be the maximum when the ratio of the masses $[m : (M - m)]$ of the two parts is

(a) 1:1 (b) 1:2 (c) 1:3 (d) 1:4

15. A bullet of mass m travelling with a speed v hits a block of mass M initially at rest and gets embedded in it. The combined system is free to move and there is no other force acting on the system. The heat generated in the process will be

(a) Zero
(b) $\frac{mv^2}{2}$
(c) $\frac{Mmv^2}{2\,(M - m)}$
(d) $\frac{mMv^2}{2\,(M + m)}$

16. A planet moves around the sun in an elliptical orbit with the sun at one of its foci. The physical quantity associated with the motion of the planet that remains constant with time is

(a) velocity (b) centripetal force
(c) linear momentum (d) angular momentum

17. A particle of mass M and charge q is released from rest in a region of uniform electric field magnitude E. After a time t, the distance travelled by the charge is S and the kinetic energy attained by the particle is T. Then, the ratio T/S

(a) remains constant with time t
(b) varies linearly with the mass M of the particle
(c) is independent of the charge q
(d) is independent of the magnitude of the electric field E

18. The specific heat C of a solid at low temperature shows temperature dependence according to the relation $C = DT^3$, where D is a constant and T is the temperature is kelvin. A piece of this solid of mass m kg is taken and its temperature is raised from 20 K to 30 K. The amount of heat required in the process in energy units is

(a) $5\text{x}10^4 Dm$ (b) $(33/4)\text{x}10^4 Dm$
(c) $(65/4)\text{x}10^4 Dm$ (d) $(5/4)\text{x}10^4 Dm$

19. The least distance of vision of a long sighted person is 60 cm. By using a spectacle lens, this distance is reduced to 12 cm. The power of the lens is

(a) + 5.0 D (b) + (20/3) D
(c) −(10/3) D (d) +2.0 D

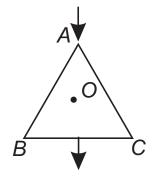
20. A particle of mass M and charge q, initially at rest, is accelerated by a uniform electric field E through a distance D and is then allowed to approach a fixed static charge Q of the same sign. The distance of the closest approach of the charge q will then be

(a) $\frac{qQ}{4\pi\varepsilon_0 D}$ (b) $\frac{Q}{4\pi\varepsilon_0 ED}$
(c) $\frac{qQ}{2\pi\varepsilon_0 D^2}$ (d) $\frac{Q}{4\pi\varepsilon_0 E}$

21. At two different places, the angles of dip are respectively 30° and 45°. At these two places the ratio of horizontal component of earth's magnetic field is

(a) $\sqrt{3}:\sqrt{2}$ (b) $1:\sqrt{2}$ (c) $1:2$ (d) $1:\sqrt{3}$

22. An equilateral triangle is made by uniform wires AB, BC, CA. A current I enters at A and leaves from the mid point of BC. If the lengths of each side of the triangle is L, the magnetic field B at the centroid O of the triangle is

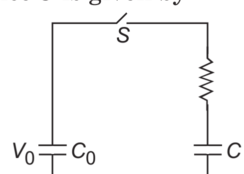


(a) $\frac{\mu_0}{4\pi}\left(\frac{4L}{L}\right)$ (b) $\frac{\mu_0}{2\pi}\left(\frac{4L}{L}\right)$
(c) $\frac{\mu_0}{4\pi}\left(\frac{2L}{L}\right)$ (d) zero

23. A particle is moving with a uniform speed v in a circular path of radius r with the centre at O. When the particle moves from a point P to Q on the circle such that $\angle POQ = \theta$, then the magnitude of the change in velocity is

(a) $2v\sin(2\theta)$ (b) zero
(c) $2v\sin\left(\frac{\theta}{2}\right)$ (d) $2v\cos\left(\frac{\theta}{2}\right)$

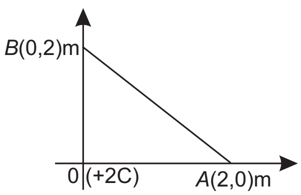
24. A capacitor of capacitance C_0 is charged to a potential V_0 and is connected with another capacitor of capacitance C as shown. After closing the switch S, the common potential across the two capacitors becomes V. The capacitance C is given by



(a) $\frac{C_0(V_0 - V)}{V_0}$ (b) $\frac{C_0(V - V_0)}{V_0}$
(c) $\frac{C_0(V + V_0)}{V}$ (d) $\frac{C_0(V_0 - V)}{V}$

25. As shown in figure below, a charge +2 C is situated at the origin O and another charge +5 C is on the x-axis at the point A. The later charge from the point A is then brought to a point B on the y-axis. The work done is

(given $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F)



(a) 45×10^9 J (b) 90×10^9 J
(c) zero (d) -45×10^9 J

26. An electric cell of emf E is connected across a copper wire of diameter d and length l. The drift velocity of electrons in the wire is v_d. If the length of the wire is changed to $2l$, the new drift velocity of electrons in the copper wire will be

(a) v_d (b) $2v_d$
(c) $v_d/2$ (d) $v_d/4$

27. A bar magnet has a magnetic moment of 200 Am^2. The magnet is suspended in a magnetic field of 0.30 $NA^{-1}m^{-1}$. The torque required to rotate the magnet from its equilibrium position through an angle of 30°, will be

(a) 30 N-m (b) $30\sqrt{3}$ N-m
(c) 60 N-m (d) $60\sqrt{3}$ N-m

28. An ideal mono-atomic gas of given mass is heated at constant pressure. In this process, the fraction of supplied heat energy used for the increase of the internal energy of the gas is

(a) 3/8 (b) 3/5 (c) 3/4 (d) 2/5

29. The glass prisms P_1 and P_2 are to be combined together to produce dispersion without deviation. The angles of the prisms P_1 and P_2 are selected as 4° and 3° respectively. If the refractive index of prism P_1 is 1.54, then that of P_2 will be

(a) 1.48 (b) 1.58 (c) 1.62 (d) 1.72

30. Water is flowing in streamline motion through a horizontal tube. The pressure at a point in the tube is p where the velocity of flow is v. At another point, where the pressure is $p/2$, the velocity of flow is [density of water = ρ]

(a) $\sqrt{v^2 + \dfrac{p}{\rho}}$ (b) $\sqrt{v^2 - \dfrac{p}{\rho}}$

(c) $\sqrt{v^2 + \dfrac{2p}{\rho}}$ (d) $\sqrt{v^2 - \dfrac{2p}{\rho}}$

31. A wire of initial length L and raidus r is stretched by a length l. Another wire of same material but with initial length $2L$ and radius $2r$ is stretched by a length $2l$. The ratio of stored elastic energy per unit volume in the first and second wire is

(a) 1 : 4 (b) 1 : 2 (c) 2 : 1 (d) 1 : 1

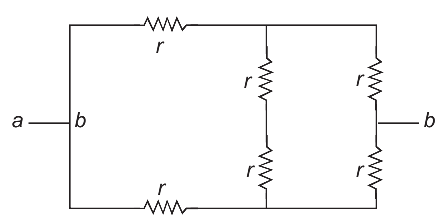
32. Two spheres of the same material, but of radii R and $3R$ are allowed to fall vertically downwards through a liquid of density ρ. The ratio of their terminal velocities is

(a) 1 : 3 (b) 1 : 6
(c) 1 : 9 (d) 1 : 1

33. An alpha particle (^{4}He) has a mass of 4.00300 amu. A proton has a mass of 1.00783 amu and a neutron has a mass of 1.00867 amu respectively. The binding energy of alpha estimated from these data is the closest to

(a) 27.9 MeV (b) 22.3 MeV
(c) 35.0 MeV (d) 20.4 MeV

34. The equivalent resistance between the points a and b of the electrical network shown in the figure is



(a) $6r$ (b) $4r$
(c) $2r$ (d) r

35. An object placed at a distance of 16 cm from a convex lens produces an image of magnification $m\ (m > 1)$. If the object is moved towards the lens by 8 cm, then again an image of magnification m is obtained. The numerical value of the focal length of the lens is

(a) 12 cm (b) 14 cm (c) 18 cm (d) 20 cm

36. A travelling acoustic wave frequency 500 Hz is moving along the positive x-direction with a velocity of 300 ms^{-1}. The phase difference between two points x_1 and x_2 is 60°. Then the minimum separation between the two points is

(a) 1 mm (b) 1 cm (c) 10 cm (d) 10 mm

37. A shell of mass $5M$, acted upon by no external force and initially at rest, bursts into three fragments of masses M, $2M$ and $2M$ respectively. The first two fragments move in opposite directions with velocities of magnitudes $2v$ and v respectively. The third fragment will

(a) move with a velocity v in a direction perpendicular to the other two
(b) move with a velocity $2v$ in the direction of velocity of the first fragment
(c) be at rest
(d) move with velocity v in the direction of velocity of the second fragment

38. A particle moves along x-axis and its displacement at any time is given by $x(t) = 2t^3 - 3t^2 + 4t$ in SI units. The velocity of the particle when its acceleration is zero, is

(a) 2.5 ms^{-1} (b) 3.5 ms^{-1} (c) 4.54 ms^{-1} (d) 8.5 ms^{-1}

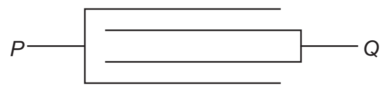
39. The fundamental frequency of a closed pipe is equal to the frequency of the second harmonic of an open pipe. The ratio of their lengths is

(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

40. An alternating current in a circuit is given by $I = 20\sin(100\,\pi\,t + 0.05\,\pi)$ A. The r.m.s. value and the frequency of current respectively are

(a) 10 A and 100 Hz (b) 10 A and 50 Hz
(c) $10\sqrt{2}$ A and 50 Hz (d) $10\sqrt{2}$ A and 100 Hz

41. Four identical plates each of area a are separated by a distance d. The connection is shown below. what is the capacitance between P and Q?



(a) $2a\varepsilon_0/d$ (b) $a\varepsilon_0/(2d)$
(c) $a\varepsilon_0/d$ (d) $4a\varepsilon_0/d$

42. A particle is acted upon by a constant power. Then, which of the following physical quantity remains constant ?

(a) Speed
(b) Rate of change of acceleration
(c) Kinetic energy
(d) Rate of change of kinetic energy

43. In an n-p-n transistor

(a) the emitter has higher degree of doping compared to that of the collector
(b) the collector has higher degree of doping compared to that of the emitter
(c) both the emitter and collector have same degree of doping
(d) the base region is most heavily doped

44. The vectors are given by $\hat{\mathbf{A}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. Another vector $\mathbf{C}$ has the same magnitude as $\mathbf{B}$ but has the same direction as $\mathbf{A}$. Then which of the following vectors represents $\mathbf{C}$?

(a) $\frac{7}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$
(b) $\frac{3}{7}(\hat{i} - 2\hat{j} + 2\hat{k})$
(c) $\frac{7}{9}(\hat{i} - 2\hat{j} + 2\hat{k})$
(d) $\frac{9}{7}(\hat{i} + 2\hat{j} + 2\hat{k})$

45. A car moving at a velocity of 17 ms^{-1} towards an approaching bus that blows a horn at a frequency of 640 Hz on a straight track. The frequency of this horn appears to be 680 Hz to the car driver. If the velocity of sound in air is 340 ms^{-1}, then the velocity of the approaching bus is

(a) 2 ms^{-1} (b) 4 ms^{-1}
(c) 8 ms^{-1} (d) 10 ms^{-1}

Category II

Directions (Q. Nos. 46 to 55) [*Carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of* 2/3 *mark*.]

46. A small mass m, attached to one end of a spring with a negligible mass and an unstretched length L, executes virtual oscillation with angular frequency ω_0. When the mass is rotated with an angular speed ω by holding the other end of the spring at a fixed point, the mass moves uniformly in a circular path in a horizontal plane. Then the increase in length of the spring during the rotation is

(a) $\dfrac{\omega^2 L}{\omega_0^2 - \omega^2}$ (b) $\dfrac{\omega_0^2 L}{\omega^2 - \omega_0^2}$

(c) $\dfrac{\omega^2 L}{\omega_0^2}$ (d) $\dfrac{\omega_0^2 L}{\omega^2}$

47. A sphere of radius R has a volume density of charge $\rho = kr$, where r is the distance from the centre of the sphere and k is constant. The magnitude of the electric field which exists at the surface of the sphere is given by (ε_0 = permittivity of free space)

(a) $\dfrac{4\pi k R^4}{3\varepsilon_0}$ (b) $\dfrac{kR}{3\varepsilon_0}$ (c) $\dfrac{4\pi k R}{\varepsilon_0}$ (d) $\dfrac{kR^2}{4\varepsilon_0}$

48. A body is projected from the ground with a velocity $\mathbf{v} = (3\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\ \mathrm{ms^{-1}}$. The maximum height attained and the range of the body respectively are (given $g = 10\ \mathrm{ms^{-2}}$)

(a) 5 m and 6 m (b) 3 m and 10 m

(c) 6 m and 5 m (d) 3 m and 5 m

49. A cell of emf E is connected to a resistance R_1 for time t and the amount of heat generated in it is H. If the resistance R_1 is replaced by another resistance R_2 and is connected to the cell at the same time t, the amount of heat generated in R_2 is 4H. Then internal resistance of the cell is

(a) $\dfrac{2R_1 + R_2}{2}$ (b) $\sqrt{R_1 R_2}\,\dfrac{2\sqrt{R_2} - \sqrt{R_1}}{\sqrt{R_2} - 2\sqrt{R_1}}$

(c) $\sqrt{R_1 R_2}\,\dfrac{\sqrt{R_2} - 2\sqrt{R_1}}{2\sqrt{R_2} - \sqrt{R_1}}$ (d) $\sqrt{R_1 R_2}\,\dfrac{\sqrt{R_2} - \sqrt{R_1}}{\sqrt{R_2} + \sqrt{R_1}}$

50. A magnetic field $B = 2t + 4t^2$ (where, t = time) is applied perpendicular to the plane of a circular wire of radius r and resistance R. If all the units are in SI the electric charge that flows through the circular wire during $t = 0$ s to $t = 2$ s is

(a) $\dfrac{6\pi r^2}{R}$ (b) $\dfrac{20\pi r^2}{R}$

(c) $\dfrac{32\pi r^2}{R}$ (d) $\dfrac{48\pi r^2}{R}$

51. Two simple harmonic motions are given by

$$x_1 = a\sin\omega t + a\cos\omega t \text{ and}$$
$$x_2 = a\sin\omega t + \frac{a}{\sqrt{3}}\cos\omega t$$

The ratio of the amplitudes of first and second motion and the phase difference between them are respectively

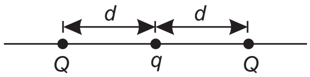
(a) $\sqrt{\dfrac{3}{2}}$ and $\dfrac{\pi}{12}$ (b) $\dfrac{\sqrt{3}}{2}$ and $\dfrac{\pi}{12}$

(c) $\dfrac{2}{\sqrt{3}}$ and $\dfrac{\pi}{12}$ (d) $\sqrt{\dfrac{3}{2}}$ and $\dfrac{\pi}{6}$

52. A cylindrical block floats vertically in a liquid of density ρ_1 kept in a container such that the fraction of volume of the cylinder inside the liquid is x_1. Then some amount of another immiscible liquid of density ρ_2 ($\rho_2 < \rho_1$) is added to the liquid in the container so that the cylinder now floats just fully immersed in the liquids with x_2 fraction of volume of the cylinder inside the liquid of density ρ_1. The ratio ρ_1 / ρ_2 will be

(a) $\dfrac{1 - x_2}{x_1 - x_2}$ (b) $\dfrac{1 - x_1}{x_1 + x_2}$ (c) $\dfrac{x_1 - x_2}{x_1 + x_2}$ (d) $\dfrac{x_2}{x_1} - 1$

53. A particle of mass M and charge q is at rest at the mid point between two other fixed similar charges each of magnitude Q placed a distance $2d$ apart. The system is collinear as shown in the figure. The particle is now displaced by a small amount x ($x << d$) along the joining the two charges and is left to itself. It will now oscillate about the mean position with a time period (ε_0 = permittivity of free space)



(a) $2\sqrt{\dfrac{\pi^3 M\varepsilon_0 d}{Qq}}$
(b) $2\sqrt{\dfrac{\pi^2 M\varepsilon_0 d^3}{Qq}}$
(c) $2\sqrt{\dfrac{\pi^3 M\varepsilon_0 d^3}{Qq}}$
(d) $2\sqrt{\dfrac{\pi^3 M\varepsilon_0}{Qqd^3}}$

54. The stopping potential for photoelectrons from a metal surface is V_1 when monochromatic light of frequency $n\nu$ is incident on it. The stopping potential becomes V_2 when monochromatic light of another frequency is incident on the same metal surface. If h be the Planck's constant and e be the charge of an electron, then the frequency of light in the second case is

(a) $\nu_1 - \dfrac{e}{h}(V_2 + V_1)$
(b) $\nu_1 + \nu\dfrac{e}{h}(V_2 + V_1)$
(c) $\nu_1 - \dfrac{e}{h}(V_2 - V_1)$
(d) $\nu_1 + \dfrac{e}{h}(V_2 - V_1)$

55. 3 moles of mono-atomic gas $(\gamma = 5/3)$ is mixed with 1 mole of a diatomic gas $(\gamma = 7/3)$. The value of γ for the mixture will be

(a) $\dfrac{9}{11}$
(b) $\dfrac{11}{7}$
(c) $\dfrac{12}{7}$
(d) $\dfrac{15}{7}$

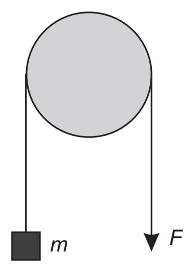
Category III

Directions (Q. Nos. 56 to 60) [*Carry two marks each, for which only one or more than one option may be correct. Marking of correct option will lead to a maximum mark of two on pro data basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked*.]

56. An electron of charge e and mass m is moving in a circular of radius r with a uniform angular speed ω. Then which of the following statements are correct?

(a) The equivalent current flowing in the circular path is proportional to r^2
(b) The magnetic moment due to circular current loop is independent of m
(c) The magnetic moment due to circular current loop equal to $2e/m$ times the angular momentum of the electron
(d) The angular momentum of the particle is proportional to the areal velocity of electron.

57. A block of mass m (= 0.1 kg) is hanging over a frictionless light fixed pulley by an inextensible string of negligible mass. The other end of the string is pulled by a constant force F in the vertically downward direction. The linear momentum of the block increases by 2 kgms^{-1} in 1s after the block starts from rest. Then, (given $g = 10\ \text{ms}^{-2}$)



(a) The tension in the string is F
(b) The tension in the string is 3 N
(c) The work done by the tension on the block is 20 J during this 1s
(d) The work done against the force of gravity is 10 J

58. If E and B are the magnitudes of electric and magnetic fields respectively in some region of space, then the possibilities for which a charged particle may move in that space with a uniform velocity of magnitude v are

(a) $E = vB$
(b) $E \neq 0, B = 0$
(c) $E = 0, B \neq 0$
(d) $E \neq 0, B \neq 0$

59. A bar of length marrying a small mass m at one of its ends rotates with a uniform angular speed ω in a vertical plane about the mid point of the bar. During the rotation, at some instant of time when the bar is horizontal, the mass is detached from the bar but the bar continues to rotate with some ω. The mass moves vertically up, comes back and reaches the bar at the same point. At that place, the acceleration due to gravity is g.

(a) This possible if the quantity $\dfrac{\omega^2 l}{2\pi g}$ is an integer

(b) The total time of flight of the mass is proportional to ω^2

(c) The total distance travelled by the mass in air is proportional to ω^2

(d) The total distance travelled by the mass in air and its total time of flight are both independent on its mass.

60. A biconvex lens of focal length f and radii of curvature of both the surfaces R is made of a material of refractive index n_1. This lens is placed in a liquid of refractive index n_2. Now this lens will behave like

(a) either as a convex or as a concave lens depending solely on R

(b) a convex lens depending on n_1 and n_2

(c) a concave lens depending on n_1 and n_2

(d) a convex lens of same focal length irrespective of R, n_1 and n_2

Chemistry

Category I

Directions (Q. Nos. 1 to 45) [*Carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of* 1/3 *mark.*]

1. At 25°C, the solubility product of salt of MX_2 type is 3.2×10^{-8} in water. The solubility (in mol/L) of MX_2 in water at the same temperature will be

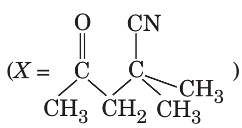
(a) 1.2×10^{-3}

(b) 2×10^{-3}

(c) 3.2×10^{-3}

(d) 1.75×10^{-3}

2. The IUPAC name of the compound X is



(a) 4-cyano-4-methyl-2-oxopentane

(b) 2-cyano-2-methyl-4-oxopentane

(c) 2, 2-dimethyl-4-oxopentanenitrile

(d) 4-cyano-4-methyl-2-pentanone

3. In $SOCl_2$, the Cl—S—Cl and Cl—S—O bond angles are

(a) 130° and 115°

(b) 106° and 96°

(c) 107° and 108°

(d) 96° and 106°

4. (+)-2-chloro-2-phenylethane in toluene racemises slowly in the presence of small amount of $SbCl_2$, due to the formation of

(a) carbanion (b) carbene

(c) free-radical (d) carbocation

5. Acid catalysed hydrolysis of ethyl acetate follows a pseudo-first order kinetics with respect to ester. If the reaction is carried out with large excess of ester, the order with respect to ester will be

(a) 1.5 (b) 0

(c) 0.5 (d) 1

6. The different colours of litmus in acidic, neutral and basic solutions are, respectively.

(a) red, orange and blue

(b) blue, violet and red

(c) red, colourless and blue

(d) red, violet and blue

7. Baeyer's reagent is

(a) alkaline potassium permanganate

(b) acidified potassium permanganate

(c) neutral potassium permanganate

(d) alkaline potassium manganate

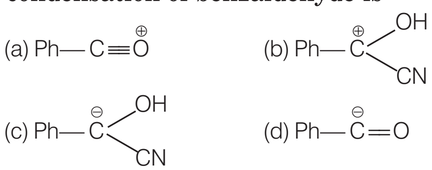
8. The correct order of equivalent conductances at infinite dilution in water at room temperature for H^+, K^+, CH_3COO^- and HO^- ions is

(a) $HO^- > H^+ > K^+ > CH_3COO^-$
(b) $H^+ > HO^- > K^+ > CH_3COO^-$
(c) $H^+ > K^+ > HO^- > CH_3COO^-$
(d) $H^+ > K^+ > CH_3COO^- > HO^-$

9. Nitric acid can be obtained from ammonia *via* the formations of the intermediate compounds

(a) nitric oxide and nitrogen dioxide
(b) nitrogen and nitric oxide
(c) nitric oxide and dinitrogen pentoxide
(d) nitrogen and nitrous oxide

10. In the following species, the one which is likely to be the intermediate during benzoin condensation of benzaldehyde is



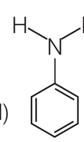
11. In O_2 and H_2O_2, the O—O bond lengths are 1.21 and 1.48 Å respectively. In ozone, the average O—O bond length is

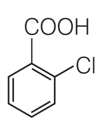
(a) 1.28 Å (b) 1.18 Å (c) 1.44 Å (d) 1.52 Å

12. The change of entropy (dS) is defined as

(a) $dS = \frac{\delta q}{T}$
(b) $dS = \frac{dH}{T}$
(c) $dS = \frac{\delta q_{rev}}{T}$
(d) $dS = \frac{dH - dG}{T}$

13. Correct pair of compounds which gives blue colouration/precipitate and white precipitate, respectively, when their Lassaigne's test is separately done is

(a) NH_2NH_2, HCl and $ClCH_2COOH$
(b) NH_2CSNH_2 and $PhCH_2Cl$
(c) NH_2NH_2, COOH and NH_2CONH_2
(d) 

and 

14. Chlorine gas reacts with red hot calcium oxide to give

(a) bleaching powder and dichlorine monoxide
(b) bleaching powder and water
(c) calcium chloride and chlorine dioxide
(d) calcium chloride and oxygen

15. For a chemical reaction at 27°C, the activation energy is 600 R. The ratio of the rate constants at 327°C to that of at 27°C will be

(a) 2 (b) 40 (c) e (d) e^2

16. 2-methylpropane on monochlorination under photochemical condition give

(a) 2-chloro-2-methylpropane as major product
(b) (1 : 1) mixture of 1-chloro-2-methylpropane and 2-chloro-2-methylpropane
(c) 1-chloro-2-methylpropane as a major product
(d) (1 : 9) mixture of 1-chloro-2-methylpropane and 2-chloro-2-methylpropane

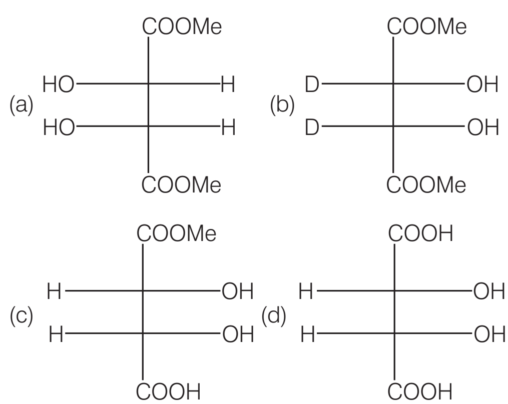
17. The half-life for decay of ^{14}C by β-emission is 5730 yr. The fraction of ^{14}C decays, in a sample that is 22920 yr old, would be

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{7}{8}$ (d) $\frac{15}{16}$

18. A van der Waals' gas may behave ideally when

(a) the volume is very low
(b) the temperature is very high
(c) the pressure is very low
(d) the temperature, pressure and volume all are very high

19. The optically active molecule is



20. In diborane, the number of electrons that accounts for bonding in the bridges is

(a) six (b) two
(c) eight (d) four

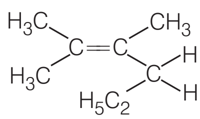
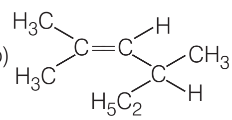
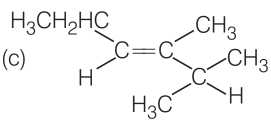
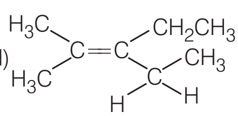
21. The reaction of nitroprusside anion with sulphide ion gives purple colouration due to the formation of

(a) the tetranionic complex of iron (II) coordinating to one NOS^- ion
(b) the dianoinic complex of iron (II) coordinating to one NCS^- ion
(c) the trianoionic complex of iron (III) coordinating to one NOS^- ion
(d) the tetranionic complex of iron (III) coordinating to one NCS^- ion

22. At 25°C, pH of a 10^{-8} M aqueous KOH solution will be

(a) 6.0 (b) 7.02
(c) 8.02 (d) 9.02

23. An optically active compound having molecular formula C_2H_{16} on ozonolysis gives acetone as one of the products. The structure of the compound is

(a) 
(b) 
(c) 
(d) 

24. Mixing of two different ideal gases under isothermal reversible condition will lead to

(a) increase of Gibbs free energy of the system
(b) no change of entropy of the system
(c) increase of entropy of the system
(d) increase of enthalpy of the system

25. The ground state electronic configuration of CO molecule is

(a) $1\sigma^2 2\sigma^2 1\pi^4 3\sigma^2$
(b) $1\sigma^2 2\sigma^2 3\sigma^2 1\pi^2 2\pi^2$
(c) $1\sigma^2 2\sigma^2 1\pi^2 3\sigma^2 2\pi^2$
(d) $1\sigma^2 1\pi^2 2\sigma^2 2\sigma^2$

26. When aniline is nitrated with nitrating mixture in ice cold condition, the major product obtained is

(a) *p*-nitroaniline (b) 2, 4-dinitroaniline
(c) *o*-nitroaniline (d) *m*-nitroaniline

27. The measured freezing point depression for a 0.1 m aqueous CH_3COOH solution is 0.19°C. The acid dissociation constant K_a at this concentration will be (Given, K_f the molal cryoscopic constant $= 1.86$ K kg mol^{-1})

(a) 4.76×10^{-5} (b) 4×10^{-5}
(c) 8×10^{-5} (d) 2×10^{-5}

28. The ore chromite is

(a) $FeCr_2O_4$ (b) $CoCr_2O_3$
(c) $CrFe_2O_4$ (d) $FeCr_2O_3$

29. 'Sulphan' is

(a) a mixture of SO_3 and H_2SO_5
(b) 100% conc. H_2SO_4
(c) a mixture of gypsum and conc. H_2SO_4
(d) 100% oleum (a mixture of 100% SO_3 in 100% H_2SO_4)

30. Pressure-volume (pV) work done by an ideal gaseous system at constant volume is (where E is internal energy of the system)

(a) $-\Delta p / p$ (b) zero (c) $-V\Delta p$ (d) $-\Delta E$

31. Amongst $[Ni(H_2O)_6]^{2+}$, $[Ni(PPh_3)_2Cl_2]$, $[Ni(CO)_4]$ and $[Ni(CN_4)]^{2-}$ the paramagnetic species are

(a) $[NiCl_4]^{2-}$, $[Ni(H_2O)_6]^{2+}$, $[Ni(PPh_3)_2Cl_2]$
(b) $[Ni(CO)_4]$, $[Ni(PPh_3)_2Cl_2]$, $[NiCl_4]^{2-}$
(c) $[Ni(CN)_4]^{2-}$, $[Ni(H_2O)_6]^{2+}$, $[NiCl_4]^{2-}$
(d) $[NI(PPh_3)_2Cl_2]$, $[Ni(CO)_4]$, $[Ni(CN)_4]^{2-}$

32. Ribose and 2-deoxyribose can be differentiated by

(a) Fehling's reagent (b) Tollen's reagent
(c) Barfoed's reagent (d) Osazone formation

33. Number of hydrogen ions present in 10 milionth part of 1.33 cm^3 of pure water at 25°C is

(a) 6.023 million (b) 60 million
(c) 8.01 million (d) 80.23 million

34. The correct order of acid strength of the following substituted phenols in water at 28°C is

(a) *p*-nitrophenol < *p*-fluorophenol < *p*-chlorophenol
(b) *p*-chlorophenol < *p*-fluorophenol < *p*-nitrophenol
(c) *p*-fluorophenol < *p*-chlorophenol < *p*-nitrophenol
(d) *p*-flurophenol < *p*-nitrophenol < *p*-chlorophenol

35. For isothermal expansion of an ideal gas, the correct combination of the thermodynamic parameters will be

(a) $\Delta U = 0, Q = 0, W \neq 0$ and $\Delta H \neq 0$
(b) $\Delta U \neq 0, Q \neq 0, W \neq 0$ and $\Delta H = 0$
(c) $\Delta U = 0, Q \neq 0, W = 0$ and $\Delta H \neq 0$
(d) $\Delta U = 0, Q \neq 0, W \neq 0$ and $\Delta H = 0$

36. Addition of excess potassium iodide solution to a solution of mercuric chloride gives the halide complex

(a) tetrahedral $K_2[HgI_4]$
(b) trigonal $K[HgI_3]$
(c) linear Hg_2I_2
(d) square planar $K_2[HgCl_2I_2]$

37. Amongst the following, the one which can exist in free state as a stable compound is

(a) C_7H_9O (b) $C_8H_{12}O$ (c) $C_6H_{12}O$ (d) $C_{10}H_{17}O$

38. A conductivity cell has been calibrated with a 0.01 M 1 : 1 electrolyte solution (specific conductance, $k = 1.25 \times 10^{-3}$ S cm^{-1}) in the cell and the measured resistance was 800 Ω at 25°C. The cell constant will be

(a) 1.02 cm^{-1} (b) 0.102 cm^{-1}
(c) 1.00 cm^{-1} (d) 0.5 cm^{-1}

39. The orange solid on heating gives a colourless gas and a green solid which can be reduced to metal by aluminium powder. The orange and the green solids are, respectively

(a) $NH_4Cr_2O_7$ and Cr_2O_3 (b) $Na_2Cr_2O_7$ and Cr_2O_3
(c) $K_2Cr_2O_7$ and CrO_3 (d) $(NH_4)_2CrO_4$ and CrO_3

40. The best method for the preparation of 2, 2-dimethylbutane is *via* the reaction of

(a) Me_3CBr and $MeCH_2Br$ in Na/ether
(b) $(Me_3C)_2CuLi$ and $MeCH_2Br$
(c) $(MeCH_2)_2CuLi$ and Me_3CBr
(d) Me_3CMgI and $MeCH_2I$

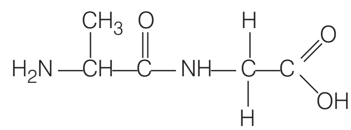
41. The condition of spontaneity of a process is

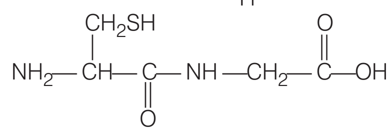
(a) lowering of entropy at constant temperature and pressure
(b) lowering of Gibbs free energy of system at constant temperature and pressure
(c) increase of entropy of system at constant temperature and pressure
(d) increase of Gibbs free energy of the universe at constant temperature and pressure

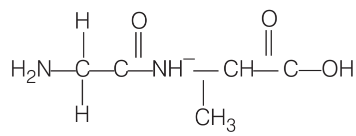
42. The increasing order of O—N—O bond angle in the species NO_2, NO_2^+ and NO_2^- is

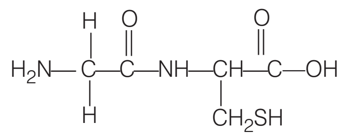
(a) $NO_2^+ < NO_2 < NO_2^-$ (b) $NO_2 < NO_2^- < NO_2^+$
(c) $NO_2^+ < NO_2^- < NO_2$ (d) $NO_2 < NO_2^+ < NO_2^-$

43. The correct structure of the dipeptide gly-ala is

(a) 

(b) 

(c) 

(d) 

44. Equivalent conductivity at infinite dilution for sodium-potassium oxalate $[(COO^-)_2Na^+K^+]$ will be [given molar conductivities of oxalate, K^+ and Na^+ ions at infinite dilution are 148.2, 50.1, 73.5 S cm^2 mol^{-1} respectively]

(a) 271.8 S cm^2 eq^{-1} (b) 67.95 S cm^2 eq^{-1}
(c) 543.6 S cm^2 eq^{-1} (d) 135.9 S cm^2 eq^{-1}

45. For BCl_3, $AlCl_3$ and $GaCl_3$ the increasing order of ionic character is

(a) $BCl_3 < AlCl_3 < GaCl_3$ (b) $GaCl_3 < AlCl_3 < BCl_3$
(c) $BCl_3 < GaCl_3 < AlCl_3$ (d) $AlCl_3 < BCl_3 < GaCl_3$

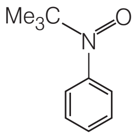
Category II

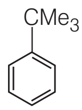
Directions (Q. Nos. 46 to 55) [*Carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of* 2/3 *mark.*]

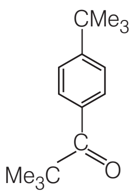
46. In borax the number of B—O—B links and B—OH bonds present are, respectively

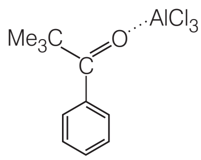
(a) five and four (b) four and five
(c) three and four (d) five and five

47. Reaction of benzene with Me_3CCOCl in the presence of anhydrous $AlCl_3$ gives

(a) 

(b) 

(c) 

(d) 

48. 1×10^{-3} mole of HCl is added to a buffer solution made up of 0.01 M acetic acid and 0.01 M sodium acetate. The final pH of the buffer will be (given, pK_a of acetic acid is 4.75 at 25°C)

(a) 4.60 (b) 4.66 (c) 4.75 (d) 4.8

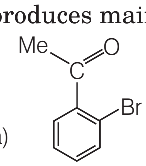
49. On heating, chloric acid decomposes to

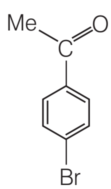
(a) $HClO_4$, Cl_2, O_2 and H_2O
(b) $HClO_2$, Cl_2, O_2 and H_2O
(c) HClO, Cl_2O and H_2O_2
(d) HCl, HClO, Cl_2O, and H_2O

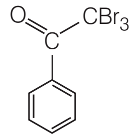
50. The best method for preparation of Me_3CCN is

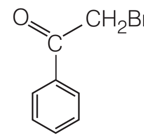
(a) to react Me_3COH with HCN
(b) to react Me_3CBr with NaCN
(c) to react Me_3CMgBr with ClCN
(d) to react Me_3CLi with NH_2CN

51. Bromination of PhCOMe in acetic medium produces mainly

(a) 

(b) 

(c) 

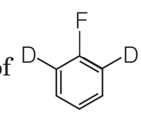
(d) 

52. The standard Gibbs free energy change (ΔG^o) at 25°C for the dissociation of $N_2O_4(g)$ to $NO_2(g)$ is (given, equilibrium constant = 0.15, R = 8.314 JK/mol)

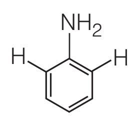
(a) 1.1 KJ
(b) 4.7 KJ
(c) 8.1 KJ
(d) 38.2 KJ

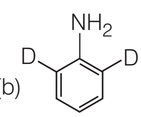
53. Silicon oil is obtained from the hydrolysis and polymerisation of

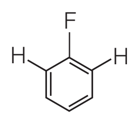
(a) trimetylchlorosilane and dimethyldichlorosilane
(b) trimethylchlorosilane and methyldichlorosilane
(c) methyltrichlorosilane and dimethyldichlorosilane
(d) triethylchlorosilane and dimethyldichlorosilane

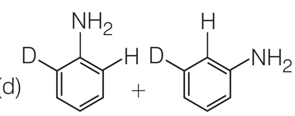
54. Treatment of 

with $NaNH_2$/liq. NH_3 gives

(a) 

(b) 

(c) 

(d) 

55. Identify the correct statement.

(a) Quantum numbers (*n, l, m, s*) are arbitrarily
(b) All the quantum numbers (*n, l, m, s*) for any pair of electron in an atom can be identical under special circumstance
(c) All the quantum numbers (*n, l, m, s*) may not be required to describe an electron of an atom completely
(d) All the quantum numbers (*n, l, m, s*) are required to describe an electron of an atom completely

Category III

Directions [Q. Nos. 56 to 60] [*Carry two marks each, for which only one or more than one option may be correct. Marking of correct option will lead to a maximum mark of two on pro data basis. There will be no negative marking for these questions. However any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.*]

56. In basic medium the amount of Ni^{2+} in a solution can be estimated with the dimethylglyoxime reagent. The correct statement(s) about the reaction and the product is (are)

(a) in a ammoniacal solution Ni^{2+} salts give cherry-red precipitate of nickel(II) dimethylglyoximate

(b) two dimethylglyoximate units are bound to one Ni^{2+}

(c) in the complex two dimethylglyoximate units are hydrogen bonded to each other

(d) each dimethylglyoximate unit forms a six membered chelate ring with Ni^{2+}

57. Correct statement(s) in cases of *n*-butanol and *t*-butanol is (are)

(a) both are having equal solubility in water

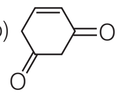
(b) *t*-butanol is more soluble in water than *n*-butanol

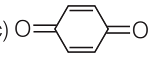
(c) boiling point of *t*-butanol is lower than *n*-butanol

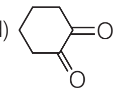
(d) boiling point of *n*-butanol is lower than *t*-butanol

58. Tautomerism is exhibited by

(a) $(Me_3CCO)_3CH$

(b) 

(c) 

(d) 

59. The important advantage(s) of Lintz and Donawitz (L.D.) process for the manufacture of steel is (are)

(a) the process is very quick

(b) operating costs are low

(c) better quality steel is obtained

(d) scrap iron can be used

60. Consider the following reaction for $2NO_2(g) + F_2(g) \longrightarrow 2NO_2F(g)$. The expression for the rate of reaction in terms of the rate of change of partial pressure of reactant and product is/are

(a) rate $= -\frac{1}{2}\left[\frac{dp(NO_2)}{dt}\right]$ (b) rate $= \frac{1}{2}\left[\frac{dp(NO_2)}{dt}\right]$

(c) rate $= -\frac{1}{2}\left[\frac{dp(NO_2F)}{dt}\right]$ (d) rate $= \frac{1}{2}\left[\frac{dp(NO_2F)}{dt}\right]$

Mathematics

Category I

Directions (Q. Nos. 1 to 60) [*Carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of* 1/3 *mark*.]

1. Each of a and b can take values 1 or 2 with equal probability. The probability that the equation $ax^2 + bx + 1 = 0$ has real roots, is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

2. Cards are drawn one-by-one without replacement from a well shuffled pack of 52 cards. Then, the probability that a face card (jack, queen or king) will appear for the first time on the third turn is equal to

(a) $\frac{300}{2197}$ (b) $\frac{36}{85}$ (c) $\frac{12}{85}$ (c) $\frac{4}{51}$

3. There are two coins, one unbiased with probaility $\frac{1}{2}$ or getting heads and the other one is biased with probability $\frac{3}{4}$ of getting heads. A coin is selected at random and tossed. It shows heads up. Then, the probability that the unbiased coin was selected is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$
(c) $\frac{1}{2}$ (d) $\frac{2}{5}$

4. Lines $x + y = 1$ and $3y = x + 3$ intersect the ellipse $x^2 + 9y^2 = 9$ at the points P, Q and R. The area of the ΔPQR is

(a) $\frac{36}{5}$ (b) $\frac{18}{5}$
(c) $\frac{9}{5}$ (d) $\frac{1}{5}$

5. For the variable , the locus of the point of intersection of the lines $3tx - 2y + 6t = 0$ and $3x + 2ty - 6 = 0$ is

(a) the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$
(b) the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$
(c) the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$
(d) the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

6. The locus of the mid-points of the chords of an ellipse $x^2 + 4y^2 = 4$ that are drawn from the positive end of the minor axis, is

(a) a circle with centre $\left(\frac{1}{2}, 0\right)$ and radius 1
(b) a parabola with focus $\left(\frac{1}{2}, 0\right)$ and directrix $x = -1$
(c) an ellipse with centre $\left(0, \frac{1}{2}\right)$, major axis 1 and minor axis $\frac{1}{2}$
(d) a hyperbola with centre $\left(0, \frac{1}{2}\right)$, transverse axis 1 and conjugate axis $\frac{1}{2}$

7. A point P lies on the circle $x^2 + y^2 = 169$. If $Q = (5, 12)$ and $R = (-12, 5)$, then the $\angle QPR$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

8. A point moves, so that the sum of squares of its distance from the points (1, 2) and (−2, 1) is always 6. Then, its locus is

(a) the straight line $y - \frac{3}{2} = -3\left(x + \frac{1}{2}\right)$
(b) a circle with centre $\left(-\frac{1}{2}, \frac{3}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$
(c) a parabola with focus (1, 2) and directrix passing through (−2, 1)
(d) an ellipse with foci (1, 2) and (−2, 1)

9. A circle passing through (0, 0), (2, 6), (6, 2) cut the x-axis at the point $P \neq (0, 0)$. Then, the lenght of OP, where O is the origin, is

(a) $\frac{5}{2}$ (b) $\frac{5}{\sqrt{2}}$ (c) 5 (d) 10

10. For the variable t, the locus of the points of intersection of lines $x - 2y = t$ and $x + 2y = \frac{1}{t}$ is

(a) the straight line $x = y$
(b) the circle with centre at the origin and radius 1
(c) the ellipse with centre at the origin and one focus $\left(\frac{2}{\sqrt{5}}, 0\right)$
(d) the hyperbola with centre at the origin and one focus $\left(\frac{\sqrt{5}}{2}, 0\right)$

11. The number of onto functions from the set {1, 2, …, 11 } to the set {1, 2, …, 10} is

(a) $5 \times 11!$ (b) $10!$ (c) $\frac{11!}{2}$ (d) $10 \times 11!$

12. Let $p(x)$ be a quadratic polynomial with constant term 1. Suppose $p(x)$, when divided by $x - 1$ leaves remainder 2 and when divided by $x + 1$ leaves remainder 4. Then, the sum of the roots of $p(x) = 0$ is

(a) −1 (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

13. The limit of $\left[\frac{1}{x^2}+\frac{(2013)^x}{e^x-1}-\frac{1}{e^x-1}\right]$ as $x \to 0$

(a) approaches $+\infty$ (b) approaches $-\infty$
(c) is equal to $\log_e(2013)$ (d) does not exist

14. Eleven apples are distributed among a girl and a boy, Then, which one of the following statements is true ?

(a) atleast one of them will receive 7 apples
(b) the girl receives atleast 4 apples or the boy receives atleast 9 apples
(c) the girl receives atleast 5 apples or the boy receives atleast 8 apples
(d) the girl receives atleast 4 apples or the boy receives atleast 8 apples

15. If $z_1 = 2+3i$ and $z_2 = 3+4i$ be two points on the complex plane. Then, the set of complex number z satisfying $|z-z_1|^2+|z-z_2|^2 = |z_1-z_2|^2$ represents

(a) a straight line (b) a point
(c) a circle (d) a pair of straight line

16. Five numbers are in HP. The middle term is 1 and the ratio of the second and the fourth terms is 2 : 1. Then, the sum of the first three terms is

(a) $\frac{11}{2}$ (b) 5
(c) 2 (d) $\frac{14}{3}$

17. If $p = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $X = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$. Then, P^3X is equal to

(a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
(c) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

18. If α and β are roots of $x^2 - x + 1 = 0$, then the value of $\alpha^{2013}+\beta^{2013}$ is

(a) 2 (b) –2 (c) –1 (d) 1

19. The number of solutions of the equation $x+y+z=10$ where x, y and z are positive integers

(a) 36 (b) 55
(c) 72 (d) 45

20. The value of the integral

$\int_{-1}^{1}\left\{\frac{x^{2013}}{e^{|x|}(x^2+\cos x)}+\frac{1}{e^{|x|}}\right\}dx$ is equal to

(a) 0 (b) $1-e^{-1}$
(c) $2e^{-1}$ (d) $2(1-e^{-1})$

21. For $0 \le P, Q \le \frac{\pi}{2}$, if $\sin P + \cos Q = 2$, then the value of $\tan\left(\frac{P+Q}{2}\right)$ is equal to

(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

22. If $f(x) = 2^{100}x+1$, $g(x) = 3^{100}x+1$, then the set of real numbers x such that $f\{g(x)\} = x$ is

(a) empty
(b) a singleton
(c) a finite set with more than one element
(d) infinite

23. The limit of $\left\{\frac{1}{x}\sqrt{1+x}-\sqrt{1+\frac{1}{x^2}}\right\}$ as $x \to 0$

(a) does not exist (b) is equal to $\frac{1}{2}$
(c) is equal to 0 (d) is equal to 1

24. The value of $\cos^2 75° + \cos^2 45° + \cos^2 15° - \cos^2 30° - \cos^2 60°$ is

(a) 0 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. The maximum and minimum values of $\cos^6\theta + \sin^6\theta$ are respectively

(a) 1 and $\frac{1}{4}$
(b) 1 and 0
(c) 2 and 0
(d) 1 and $\frac{1}{2}$

26. If $z = x + iy$, where x and y are real numbers and $i = \sqrt{-1}$, then the points (x, y) for which $\frac{z-1}{z-i}$ is real, lie on

(a) an ellipse
(b) a circle
(c) a parabola
(d) a straight line

27. If a, b and c are in AP, then the straight line $ax + 2by + c = 0$ will always pass through a fixed point whose coordinates are

(a) (1, – 1)
(b) (–1, 1)
(c) (1, – 2)
(d) (–2, 1)

28. The equation $2x^2 + 5xy - 12y^2 = 0$ represents a

(a) circle
(b) pair of non-perpendicular intersecting straight lines
(c) pair of perpendicular straight lines
(d) hyperbola

29. If one end of a diameter of the circle $3x^2 + 3y^2 - 9x + 6y + 5 = 0$ is (1, 2), then the other end is

(a) (2, 1) (b) (2, 4) (c) (2, – 4) (d) (–4, 2)

30. The line $y = x$ intersects the hyperbola $\frac{x^2}{9} - \frac{y^2}{25} = 1$ at the points P and Q. The eccentricity of ellipse with PQ as major axis and minor axis of length $\frac{5}{\sqrt{2}}$ is

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{5}{\sqrt{3}}$ (c) $\frac{5}{9}$ (d) $\frac{2\sqrt{2}}{9}$

31. The limit of $x \sin\left(e^{\frac{1}{x}}\right)$ as $x \to 0$

(a) is equal to 0
(b) is equal to 1
(c) is equal to $\frac{e}{2}$
(d) does not exist

32. The value of

$$1000\left[\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots + \frac{1}{999\times 1000}\right]$$

is

(a) 1000
(b) 999
(c) 1001
(d) $\frac{1}{999}$

33. If $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$. Then, the matrix $P^3 + 2P^2$ is equal to

(a) P
(b) $I - P$
(c) $2I + P$
(d) $2I - P$

34. The value of determinant

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} \text{ is}$$

(a) 0
(b) $(1 + a^2 + b^2)$
(c) $(1 + a^2 + b^2)^2$
(d) $(1 + a^2 + b^2)^3$

35. If α, β are the roots of the quadratic equation $x^2 + ax + b = 0$, $(b \neq 0)$, then the quadratic equation whose roots are $\alpha - \frac{1}{\beta}, \beta - \frac{1}{\alpha}$, is

(a) $ax^2 + a(b-1)x + (a-1)^2 = 0$
(b) $bx^2 + a(b-1)x + (b-1)^2 = 0$
(c) $x^2 + ax + bv = 0$
(d) $abx^2 + bx + a = 0$

36. If the distance between the foci of an ellipse is equal to the length of the latusrectum, then its eccentricity is

(a) $\frac{1}{4}(\sqrt{5} - 1)$
(b) $\frac{1}{2}(\sqrt{5} + 1)$
(c) $\frac{1}{2}(\sqrt{5} - 1)$
(d) $\frac{1}{4}(\sqrt{5} + 1)$

37. The equation of the circle passing through the point (1, 1) and the points of intersection of $x^2 + y^2 - 6x - 8 = 0$ and $x^2 + y^2 - 6 = 0$ is

(a) $x^2 + y^2 + 3x - 5 = 0$
(b) $x^2 + y^2 - 4x + 2 = 0$
(c) $x^2 + y^2 + 6x - 4 = 0$
(d) $x^2 + y^2 - 4y - 2 = 0$

38. The number of lines which pass through the point (2, – 3) and are at a distance 8 from the point (–1, 2) is

(a) infinite (b) 4 (c) 2 (d) 0

39. Six positive numbers are in GP, such that their product is 1000. If the fourth term is 1, then the last term is

(a) 1000 (b) 100 (c) $\frac{1}{100}$ (d) $\frac{1}{1000}$

40. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$ and $3b^2 = 16ac$, then

(a) $\alpha = 4\beta$ or $\beta = 4\alpha$ (b) $\alpha = -4\beta$ or $\beta = -4\alpha$
(c) $\alpha = 3\beta$ or $\beta = 3\alpha$ (d) $\alpha = -3\beta$ or $\beta = -3\alpha$

41. In the set of all 3×3 real matrices a relation is defined as follows. A matrix A is related to a matrix B, if and only if there is a non-singular 3×3 matrix P, such that $B = P^{-1}AP$. This relation is

(a) reflexive, symmetric but not transitive
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) an equivalence relation

42. For any two real numbers a and b, we define $a\,R\,b$ if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is

(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

43. For the curve $x^2 + 4xy + 8y^2 = 64$ the tangents are parallel to the x-axis only at the points

(a) $(0, 2\sqrt{2})$ and $(0, -2\sqrt{2})$
(b) $(8, -4)$ and $(-8, 4)$
(c) $(8\sqrt{2}, -2\sqrt{2})$ and $(-8\sqrt{2}, 2\sqrt{2})$
(d) $(9, 0)$ and $(-8, 0)$

44. If $f(x) = \begin{cases} x^3 - 3x + 2, & x < 2, \\ x^3 - 6x^2 + 9x + 2, & x \ge 2 \end{cases}$

then

(a) $\lim_{x \to 2} f(x)$ does not exist
(b) f is not continuous at $x = 2$
(c) f is continuous but not differentiable at $x = 2$
(d) f is continuous and differentiable at $x = 2$

45. The value of $I = \int_0^{\frac{\pi}{4}} (\tan^{n+1} x)\, dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} \tan^{n+1}\left(\frac{x}{2}\right) dx$ is

(a) $\frac{1}{n}$ (b) $\frac{n+2}{2n+1}$
(c) $\frac{2n-1}{n}$ (d) $\frac{2n-3}{3n-2}$

46. The limit of $\sum_{n=1}^{1000} (-1)^n x^n$ as $x \to \infty$

(a) does not exist
(b) exists and equals to 0
(c) exists and approaches to $+\infty$
(d) exists and approaches $-\infty$

47. Let $f(\theta) = (1 + \sin^2\theta)(2 - \sin^2\theta)$. Then, for all values of θ

(a) $f(\theta) > \frac{9}{4}$ (b) $f(\theta) < 2$
(c) $f(\theta) > \frac{11}{4}$ (d) $2 \le f(\theta) \le \frac{9}{4}$

48. If $f(x) = e^x (x-2)^2$, then

(a) f is increasing in $(-\infty, 0)$ and $(2, \infty)$ and decreasing in $(0, 2)$.
(b) f is increasing in $(-\infty, 0)$ and decreasing in $(0, \infty)$.
(c) f is increasing in $(2, \infty)$ and decreasing in $(-\infty, 0)$.
(d) f is increasing in $(0, 2)$ and decreasing in $(-\infty, 0)$ and $(2, \infty)$.

49. Let n be a positive even integer. If the ratio of the largest coefficient and the 2nd largest coefficient in the expansion of $(1+x)^n$ is $11:10$. Then, the number of terms in the expansion of $(1+x)^n$ is

(a) 20 (b) 21
(c) 10 (d) 11

50. Five numbers are in AP with common difference $\neq 0$. If the 1st, 3rd and 4th terms are in GP, then

(a) the 5th term is always 0.
(b) the 1st term is always 0.
(c) the middle term is always 0.
(d) the middle term is always -2.

51. Let exp (x) denote the exponential function e^x. If $f(x) = \exp\left(x^{\frac{1}{x}}\right)$, $x > 0$, then the minimum value of f in the interval $[2, 5]$ is

(a) $\exp\left(e^{\frac{1}{e}}\right)$ (b) $\exp\left(2^{\frac{1}{2}}\right)$
(c) $\exp\left(5^{\frac{1}{5}}\right)$ (d) $\exp\left(3^{\frac{1}{3}}\right)$

52. The minimum value of the function $f(x) = 2|x-1| + |x-2|$ is

(a) 0 (b) 1 (c) 2 (d) 3

53. The sum of the series

$$\frac{1}{1\times 2}\,{}^{25}C_0 + \frac{1}{2\times 3}\,{}^{25}C_1 + \frac{1}{3\times 4}\,{}^{25}C_2 + \ldots + \frac{1}{26\times 27}\,{}^{25}C_{25} \text{ is}$$

(a) $\frac{2^{27}-1}{26\times 27}$ (b) $\frac{2^{27}-28}{26\times 27}$

(c) $\frac{1}{2}\left(\frac{2^{26}+1}{26\times 27}\right)$ (d) $\left(\frac{2^{26}-1}{52}\right)$

54. If P, Q and R are angles of an isosceles triangle and $\angle P = \frac{\pi}{2}$, then the value of

$$\frac{\left(\cos\frac{P}{3} - i\sin\frac{P}{3}\right)^3 + (\cos Q + i\sin Q)(\cos R - i\sin R) + (\cos P - i\sin P)}{(\cos Q - i\sin Q)(\cos R - i\sin R)} \text{ is}$$

(a) i (b) $-i$ (c) 1 (d) -1

55. Let $f: R \to R$ be such that f is injective and $f(x)\,f(y) = f(x+y)$ for all $x, y \in R$, if $f(x), f(y)$ and $f(z)$ are in G P, then x, y and z are in

(a) AP always
(b) GP always
(c) AP depending on the values of x, y and z
(d) GP depending on the values of x, y and z

56. The value of the integral

$$\int_1^2 e^x\left(\log_e x + \frac{x+1}{x}\right)dx \text{ is}$$

(a) $e^2(1+\log_e 2)$ (b) $e^2 - e$

(c) $e^2(1+\log_e 2) - e$ (d) $e^2 - e(1+\log_e 2)$

57. The number of solutions of the equation

$$\frac{1}{2}\log_{\sqrt{3}}\left(\frac{x+1}{x+5}\right) + \log_9 (x+5)^2 = 1 \text{ is}$$

(a) 0 (b) 1 (c) 2 (d) infinite

58. If $P = 1 + \frac{1}{2\times 2} + \frac{1}{3\times 2^2} + \ldots$

and $Q = \frac{1}{1\times 2} + \frac{1}{3\times 4} + \frac{1}{5\times 6} + \ldots$, then

(a) $P = Q$ (b) $2P = Q$

(c) $P = 2Q$ (d) $P = 4Q$

59. The area of the region bounded by the parabola $y = x^2 - 4x + 5$ and the straight line $y = x + 1$ is

(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) $\frac{9}{2}$

60. If $f(x) = \sin x + 2\cos^2 x$, $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$. Then, f attains its

(a) minimum at $x = \frac{\pi}{4}$ (b) maximum at $x = \frac{\pi}{2}$

(c) minimum $x = \frac{\pi}{2}$ (d) maximum at $x = \sin^{-1}\left(\frac{1}{4}\right)$

Category II

Directions [(Q. Nos. 61 to 75) *Carry two marks each, for which only one option is correct. Any wrong answer will lead to deduction of* 2/3 *mark.*]

61. An objective type test paper has 5 questions. Out of these 5 questions, 3 questions have four options each (a, b, c, d) with one option being the correct answer. The other 2 questions have two options each, namely true and false. A candidate randomly ticks the options. Then, the probability that he/she will tick the correct option in atleast four questions, is

(a) $\frac{5}{32}$ (b) $\frac{3}{128}$ (c) $\frac{3}{256}$ (d) $\frac{3}{64}$

62. The solution of the differential equation $(y^2 + 2x)\frac{dy}{dx} = y$ satisfies $x = 1, y = 1$. Then, the solution is

(a) $x = y^2(1+\log_e y)$ (b) $y = x^2(1+\log_e x)$

(c) $x = y^2(1-\log_e y)$ (d) $y = x^2(1-\log_e x)$

63. A family of curves is such that the length intercepted on the y-axis between the origin and the tangent at a point is three times the ordinate of the point of contact. The family of curves is

(a) $xy = C$, C is a constant
(b) $xy^2 = C$, C is a constant
(c) $x^2y = C$, C is a constant
(d) $x^2y^2 = C$, C is a constant

64. The solution of the differential equation $y \sin\left(\frac{x}{y}\right) dx = \left\{x \sin\left(\frac{x}{y}\right) - y\right\} dy$ satisfying $y\left(\frac{\pi}{4}\right) = 1$ is

(a) $\cos\frac{x}{y} = -\log_e y + \frac{1}{\sqrt{2}}$
(b) $\sin\frac{x}{y} = \log_e y + \frac{1}{\sqrt{2}}$
(c) $\sin\frac{x}{y} = \log_e x - \frac{1}{\sqrt{2}}$
(d) $\cos\frac{x}{y} = -\log_e x - \frac{1}{\sqrt{2}}$

65. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the x-axis. It intersects the parabola $y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then, $|x_1 - x_2|$ is equal to

(a) $\frac{16}{9}$ (b) $\frac{32}{9}$
(c) $\frac{40}{9}$ (d) $\frac{80}{9}$

66. If $\sin^2\theta + 3\cos\theta = 2$, then $\cos^3\theta + \sec^3\theta$ is equal out to

(a) 1 (b) 4 (c) 9 (d) 18

67. If $[a]$ denote the greatest integer which is less than or equal to a. Then, the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\sin x \cos x]\, dx$ is

(a) $\frac{\pi}{2}$ (b) π
(c) $-\pi$ (d) $-\frac{\pi}{2}$

68. If $x = 1 + \frac{1}{2 \times 1!} + \frac{1}{4 \times 2!} + \frac{1}{8 \times 3!} + \dots$ and $y = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots$

Then, the value of $\log_e y$ is

(a) e (b) e^2 (c) 1 (d) $\frac{1}{e}$

69. If $P = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$, then P^5 is equal to

(a) P (b) $2P$ (c) $-P$ (d) $-2P$

70. The value of the infinite series $\frac{1^2 + 2^2}{3!} + \frac{1^2 + 2^2 + 3^2}{4!} + \frac{1^2 + 2^2 + 3^2 + 4^2}{5!} + \dots$ is

(a) e (b) $5e$ (c) $\frac{5e}{6} - \frac{1}{2}$ (d) $\frac{5e}{6}$

71. The value of integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x - x\cos x)}{x(x + \sin x)} dx$ is

(a) $\log_e \left\{\frac{2(\pi + 3)}{(2\pi + 3\sqrt{3})}\right\}$ (b) $\log_e \left\{\frac{\pi + 3}{2(2\pi + 3\sqrt{3})}\right\}$
(c) $\log_e \left\{\frac{2\pi + 3\sqrt{3}}{2(\pi + 3)}\right\}$ (d) $\log_e \left\{\frac{2(2\pi + 3\sqrt{3})}{\pi + 3}\right\}$

72. If $f(x) = x^{2/3}, x \geq 0$. Then, the area of the region enclosed by the curve $y = f(x)$ and the three lines $y = x$, $x = 1$ and $x = 8$ is

(a) $\frac{63}{2}$ (b) $\frac{93}{5}$ (c) $\frac{105}{7}$ (d) $\frac{129}{10}$

73. If $f(x) = x\left(\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1}\right), x > 1$. Then,

(a) $f(x) \leq 1$ (b) $1 < f(x) \leq 2$
(c) $2 < f(x) \leq 3$ (d) $f(x) > 3$

74. If P be a point on the parabola $y^2 = 4ax$ with focus F. Let Q denote the foot of the perpendicular from P onto the directrix. Then, $\frac{\tan \angle PQF}{\tan \angle PFQ}$ is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

75. If $F(x) = \int_0^x \frac{\cos t}{(1+t^2)}\, dt, 0 \le x \le 2\pi$. Then,

(a) F is increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$.

(b) F is increasing in $(0, \pi)$ and decreasing in $(\pi, 2\pi)$.

(c) F is increasing $(\pi, 2\pi)$ and decreasing in $(0, \pi)$.

(d) F is increasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

Category III

Directions (Q. Nos. 76 to 80) [*Carry two marks each, for which one or more than one option is correct. Marking of correct option will lead to a maximum mark of two on pro data basis. There will be no negative marking for these questions. However any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.*]

76. The equations of the circles, which touch both the axes and the line $4x + 3y = 12$ and have centres in the first quadrant, are

(a) $x^2 + y^2 + x - y + 1 = 0$

(b) $x^2 + y^2 - 2x - 2y + 1 = 0$

(c) $x^2 + y^2 - 12x - 12y + 36 = 0$

(d) $x^2 + y^2 - 6x - 6y + 36 = 0$

77. The area of the region enclosed between parabola $y^2 = x$ and the line $y = mx$ is $\frac{1}{48}$.

Then, the value of m is

(a) –2 (b) –1 (c) 1 (d) 2

78. If $\sin\alpha$, $\cos\alpha$ be the roots of the equation $x^2 - bx + c = 0$. Then, which of the following statements is/are correct ?

(a) $c \le \frac{1}{2}$ (b) $b \le \sqrt{2}$ (c) $c > \frac{1}{2}$ (d) $b > \sqrt{2}$

79. Consider the system of equations

$$x + y + z = 0 \quad \alpha x + \beta y + \gamma z = 0$$

$$\alpha^2 x + \beta^2 y + \gamma^2 z = 0$$

Then, the system of equations has

(a) a unique solution for all values of α, β and γ.

(b) infinite number of solutions, if any two of α, β, γ are equal.

(c) a unique solution, if α, β and γ are distinct.

(d) more than one, but finite number of solutions depending on values of α, β and γ.

80. Which of the following real valued functions is/are not even functions?

(a) $f(x) = x^3 \sin x$

(b) $f(x) = x^2 \cos x$

(c) $f(x) = e^x x^3 \sin x$

(d) $f(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to x.

Answers

Physics

1. (d)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (b)
11. (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (b)
21. (a)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (d)	**30.** (a)
31. (d)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (c)	**38.** (a)	**39.** (b)	**40.** (c)
41. (a)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (a)	**49.** (b)	**50.** (b)
51. (a)	**52.** (a)	**53.** (c)	**54.** (d)	**55.** (b)	**56.** (b, d)	**57.** (a, b, d)	**58.** (a, c, d)	**59.** (a, c, d)	**60.** (b, c)

Chemistry

1. (b)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)
11. (a)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (c)	**20.** (d)
21. (a)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (d)	**30.** (b)
31. (a)	**32.** (d)	**33.** (c)	**34.** (c)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (b)
41. (b)	**42.** (*)	**43.** (c)	**44.** (d)	**45.** (c)	**46.** (a)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (c)
51. (d)	**52.** (b)	**53.** (a)	**54.** (d)	**55.** (d)	**56.** (a, b, c,d)	**57.** (b,c)	**58.** (a, b, d)	**59.** (a, c, d)	**60.** (a, d)

(*) *No option is correct.*

Mathematics

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (d)
11. (d)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (d)
21. (d)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (d)
31. (a)	**32.** (b)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (c)	**40.** (c)
41. (d)	**42.** (d)	**43.** (b)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (a)	**49.** (b)	**50.** (a)
51. (c)	**52.** (b)	**53.** (b)	**54.** (b)	**55.** (a)	**56.** (c)	**57.** (c)	**58.** (c)	**59.** (d)	**60.** (c)
61. (d)	**62.** (a)	**63.** (c)	**64.** (*)	**65.** (b)	**66.** (d)	**67.** (d)	**68.** (a)	**69.** (a)	**70.** (c)
71. (a)	**72.** (d)	**73.** (d)	**74.** (a)	**75.** (d)	**76.** (b, c)	**77.** (a, d)	**78.** (a, b)	**79.** (b, c)	**80.** (c, d)

Hints & Solutions

Physics

1. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

Given, $T = 100°\,C = 373\,K$

$\therefore \quad v = \sqrt{\dfrac{3R \times 373}{M}}$

For $\quad \sqrt{3}\,v = \sqrt{\dfrac{3RT}{M}}$

$\therefore \quad \dfrac{\sqrt{3}\,v}{v} = \dfrac{\sqrt{3RT/M}}{\sqrt{3R \times 373/M}}$

$\Rightarrow \quad \sqrt{3} = \sqrt{\dfrac{T}{373}}$

$\Rightarrow \quad 3 = \dfrac{T}{373}$

$\Rightarrow \quad T = 1119\,K$

or $\quad T = 846°\,C$

2. $[P] = \left[\dfrac{a}{V^3}\right] \Rightarrow [ML^{-1}T^{-2}] = \dfrac{a}{[L^3]^3}$

$\Rightarrow \quad a = [ML^8T^{-2}]$

$[V] = [b^2] \Rightarrow [L^3] = b^2 \Rightarrow b = [L^{3/2}]$

3. $W_{\text{constant pressure}} = p \times \Delta V$

$= 400 \times 10^3 \times 0.3 \times 10 \times 10^{-2}$

$= 400 \times 10 \times 3$

$= 12000 = 12\,kJ$

4. **In first case** $A = 0, B = 0$

$\therefore$ Output of NOR gate, $Y = \overline{A + B} = 1$

This output is the input for NAND gate, *i.e.*, $Y = 1$ and $C = 0$

$\therefore \quad D = \overline{Y \cdot C} = 1$

In second case $\quad A = 1, B = 0$

$\therefore$ Output of NOR gate, $Y = \overline{A + B} = 0$

This output is the input for NAND gate *i.e.*, $Y = 0$ and $C = 1$

$\therefore \quad D = \overline{Y \cdot C} = 1$

5. Pressure difference $= \dfrac{4T}{r}$

For smaller soap, $p_{atm} - p_{i1} = \dfrac{4T}{r}$

For bigger soap, $p_{atm} - p_{i2} = \dfrac{4T}{2r}$

As the pressure inside smaller bubble is greater than pressure inside bigger bubble, so air flows from smaller to bigger and thus radius of the smaller bubble will decrease and that of the bigger bubble will increase.

6. We have, $\quad \theta = 2\pi n = \dfrac{\left(\dfrac{v_f^2}{r^2} - \dfrac{v_i^2}{r^2}\right)}{\left(2\dfrac{a}{r}\right)}$

$\Rightarrow \quad n = \dfrac{v_f^2 - v_i^2}{(2ar)(2\pi)} \approx 95$

7. Given, $E_n = 13.6\,eV$

Energy of an electron in n^{th} state

$$E_n = \frac{-13.6\,z^2 eV}{n^2}$$

$\therefore$ Energy of an electron in $n = 2$ state

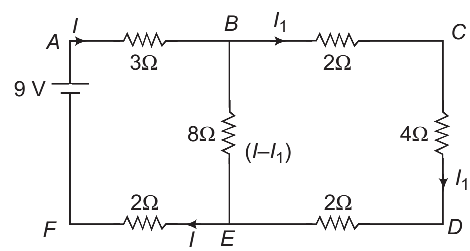
$$E_2 = \frac{18 - 6z^2}{(2)^2} = -3.4\,eV$$

So, $\quad PE = 2E_{n=2}$

$= 2 \times (-3.4)$

$\approx -6.8\,eV$

8. The current through the various branches of the circuit will be shown as



According to Kirchhoff's second law in closed circuit $BCDEB$

$$2I_1 + 4I_1 + 2I_1 - 8(I - I_1) = 0$$

$$\Rightarrow \quad 16I_1 - 8I = 0$$

$$\Rightarrow \quad I_1 = \frac{8I}{16} \Rightarrow I_1 = \frac{1}{2}I \quad \ldots\text{(i)}$$

In closed circuit $ABEFA$

$$-9 + 3I + 8(I - I_1) + 2I = 0$$

$$\Rightarrow \quad 13I - 8I_1 = 9 \Rightarrow 13I - 8\left(\frac{1}{2}I\right) = 9$$

$$\Rightarrow \quad I = \frac{9}{9} = 1\text{A} \quad \ldots\text{(ii)}$$

So current through 4 Ω resistor $I_1 = \frac{1}{2} \times 1 = 0.5\text{A}$

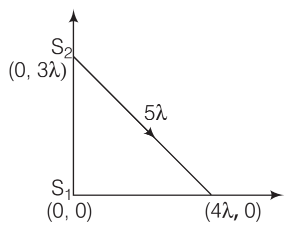
9. The force experienced by the wire placed in magnetic field, $F = Bil$

$$= (2\hat{i} + 4\hat{j})\left(1\hat{i} \times \frac{1}{2}\right)$$

$$= \hat{i} \times \hat{i} + 4 \times \frac{1}{2}(\hat{i} \times \hat{j}) = 2\hat{k}\text{ N}$$

Direction of this force can be found out by Fleming's left hand rule which is along pasitive z-axis.

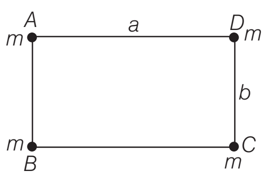
10. The given situation can be show as



The intensity will be maximum at those given points where the path difference between the two interfering waves is an integral multiple of wavelength *i.e.*, $\Delta x = n\lambda$

For the the given points, the intensity will be maximum for $(4\lambda, 0)$.

11. The given situation can be shown as



Moment of inertia of the system about side of length b say CD is

= M.I. of mass at A about CD + M.I. of mass at B about CD + M.I. of mass at C about CD + M.I. of mass ot D about CD

$$= m(a)^2 + m(a)^2 + m(0)^2 + m(0)^2$$

$$= 2ma^2$$

12. The de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mk}}$

Given, $h = 6.6 \times 10^{-34}$ J-s

$m = 1 \times 10^{-30}$ kg

$K = 200\text{ eV} = 200 \times 1.6 \times 10^{-19}$ J

Substituting all these values

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1 \times 10^{-30} \times 200 \times 1.6 \times 10^{-19}}}$$

$$= 0.825 \times 10^{-10} = 8.25 \times 10^{-11}\text{ m}$$

13. We have, $\frac{N(t)}{N_0} = e^{-\lambda t}$ or $N(t) = N_0 e^{-\lambda t}$

$\therefore$ For the given condition,

$$\frac{N_0}{2} = N_0 e^{-\lambda t_1} \text{ and } \frac{N_0}{10} = N_0 e^{-\lambda t_2}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-\lambda t_1} \text{ and } \frac{1}{10} = e^{-\lambda t_2}$$

$$\text{or} \quad e^{\lambda t_1} = 2 \text{ and } e^{\lambda t_2} = 10$$

Taking log on both sides,

$$\lambda t_1 = \log 2 \Rightarrow t_1 = \frac{\log 2}{\lambda}$$

$$\text{or} \quad t_1 = \frac{\log 2 \times \text{T}}{\log 2}$$

$$\text{and} \quad \lambda t_2 = \log 10$$

$$\Rightarrow \quad t_2 = \frac{\log 10}{\lambda}$$

$$\text{or} \quad t_2 = \frac{\log 10 \times \text{T}}{\log 2}$$

$$\therefore \quad (t_2 - t_1) = T\left[\frac{\log 10}{\log 2} - 1\right] = T\left[\frac{\log 10 - \log 2}{\log 2}\right]$$

$$= T\left[\frac{\log 5}{\log 2}\right]$$

$$\Rightarrow (t_2 - t_1) = T \log\left[\frac{5}{2}\right]$$

14. The gravitation force between two masses,

$$F = \frac{Gm_1m_2}{r}$$

here, $m_1 = m$

and $m_2 = (M - m)$

$\therefore$ $$F = \frac{Gm\,(M - m)}{r^2}$$

For maximum gravitational force, $\frac{dF}{dm} = 0$

$\therefore$ $$\frac{d}{dm}[m(M - m)] = 0$$

By solving, we get $m = \frac{M}{2}$

So, $$\frac{m}{(M - m)} = \frac{1}{1}$$

15. Mass of bullet $= m_1 = m$

Initial speed of bullet $= u_1 = v$

Mass of block $= m_2 = M$

Initial speed of block $= u_2 = 0$

Let the common velocity of the bodies after collision $= V$

According to conservation of linear momentum

$$m \times v + M \times 0 = (m + M)V$$

$$V = \frac{mv}{(m + M)}$$

$\therefore$ Heat generated = loss in KE

= Initial KE – Final KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}(m + M)\,v^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}(m + M)\left(\frac{mv}{m + M}\right)^2$$

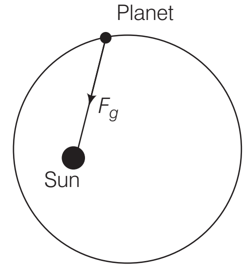
$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{(m + M)}$$

$$= \frac{1}{2}mv^2\left(1 - \frac{m}{m + M}\right)$$

$$= \frac{1}{2}mv^2\left(\frac{m + M - m}{m + M}\right)$$

$$= \frac{1}{2}\frac{mM}{(m + M)}v^2$$

16. A planet revolves around the sun, is an elliptical orbit under the effect of gravitational pull on the planet.



So, torque, $C = r \times F = r\,F \sin 180^\circ\, n = 0$

$$\text{As } C = \frac{dL}{dt};\ \text{so } L = \text{a constant}$$

$\Rightarrow$ Angular momentum is constant.

17. Given, mass of the particle $= M$,

Charge on the particle $= q$

Electric field $= E$

Initial velocity, $u = 0$

$\therefore$ Acceleration, $a = \frac{F}{M} = \frac{qE}{M}$

$\therefore$ Distance travelled in electric field,

$$S = ut + \frac{1}{2}at^2$$

$$S = \frac{1}{2}\left(\frac{qE}{M}\right)t^2$$

Also, kinetic energy $T = \frac{1}{2}M\left(\frac{qEt}{M}\right)^2$

So, $$\frac{T}{S} = \frac{\frac{1}{2}M\left(\frac{qEt}{M}\right)^2}{\frac{1}{2}\left(\frac{qE}{M}\right)t^2} = qE$$

$\Rightarrow$ Ratio of $\frac{T}{S}$ remains constant with time t.

18. Amount of heat required, $Q = \int dQ$

$$= \int_{T_1 = 20}^{T_2 = 30} mcdT \qquad \ldots\text{(i)}$$

Given, $C = DT^3$

$\therefore$ $$Q = \int_{20}^{30} m\,DT^3\,dT = mD\int_{20}^{30} T^3 dT$$

$$= mD\frac{1}{4}[(30)^2 - (20)^2] = \frac{65}{4} \times 10^4\ mD$$

19. Here, $v = -60$ cm, $u = -12$ cm

$\therefore$ By using the relation, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

We have, $\frac{1}{f} = \frac{1}{-60} - \frac{1}{-12}$

$\Rightarrow \quad \frac{1}{f} = \frac{1}{15}$ cm or $\frac{100}{15}$ m

So, the power of lens, $P = \frac{100}{15} = +\frac{20}{3}$ D

20. At the distance of closest approach, the entire K E of a particle is converted into electric potential energy.

i.e., KE = PE

Here, KE of the particle = work done in moving the particle in uniform electric field (E) through a distance (D)

$$= qE \times D$$

PE of the particle $=$ potential $\times$ charge

$$= \frac{q}{4\pi\varepsilon_0 r_0} \times Q$$

$$\therefore \quad qED = \frac{qR}{4\pi\varepsilon_0 r_0} \Rightarrow r_0 = \frac{Q}{4\pi\varepsilon_0 ED}$$

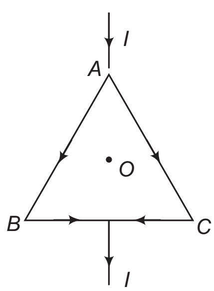
21. The horizontal component of earth's magnetic field is given by, $H = R\cos\delta$

where, δ is the angle of dip so $H_1 = R\cos 30°$

and $\quad H_2 = R\cos 45°$

$$\therefore \quad \frac{H_1}{H_2} = \frac{R\cos 30°}{R\cos 45°} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

22. The given situation can be shown as

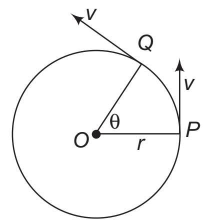


The magnetic field at centroid O = Magnetic field due to left part + Magnetic field due to right part

$$\therefore \quad B = B_1 + B_2 = 0$$

($\because$ direction of B due to both parts is different of O)

23.



$\therefore$ Change in magnitude of velocity

$$|\Delta v| = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$$

$$= 2v\sin\frac{\theta}{2}$$

24. When the switch S is closed, the common potential across the two capacitors becomes V, so charge on isolated plates remains same.

$$\therefore \quad C_0V_0 = (C_0 + C)V$$

$$\Rightarrow \quad CV = C_0V_0 - C_0V$$

$$\Rightarrow \quad C = \frac{C_0\,(V_0 - V)}{V}$$

25. Work done $= U_f - U_i$

$$= \frac{1}{4\pi\varepsilon_0} \times 2 \times 5 \times \left(\frac{1}{2} - \frac{1}{2}\right) = 0$$

26. The drift velocity is given as

$$v_d = \frac{i}{neA} = \frac{E}{R \times neA}$$

$$= \frac{E \times A}{\rho l \times neA} = \frac{E}{lne}$$

When length of wire changed to $2l$, the new drift velocity,

$$v_d{}' = \frac{E}{\rho \times 2l \times ne}$$

$$\therefore \quad \frac{v_d{}'}{v_d} = \frac{E/\rho 2\,lne}{E/\rho\,lne} = \frac{1}{2}$$

$$\Rightarrow \quad v_d{}' = \frac{v_d}{2}$$

27. Given, $M = 200$ A-m^2 B = 0.30 NA^{-1}M^{-1}

and $\quad \theta = 30°$

We know that the Torque,

$$\tau = M \times B$$

$$\Rightarrow \quad |\tau| = MB\sin\theta = 200 \times 0.3 \times \frac{1}{2}$$

$$= 100 \times 0.3 = 30 \text{ N-m}$$

28. Fraction $= \dfrac{\Delta U}{\Delta Q} = \dfrac{C_V}{C_p}$

For monoatomic gas $\dfrac{C_p}{C_V} = \gamma = \dfrac{5}{3}$

So, $\quad \dfrac{C_V}{C_p} = \dfrac{1}{\gamma} = \dfrac{3}{5}$

29. Given for prism P_1

$$\mu = 1.54, A = 4^\circ$$

For prism P_2

$$\mu' = ?, A' = 3^\circ$$

For no deviation $= \delta + \delta' = 0$

$\Rightarrow \quad (\mu - 1)A = (\mu' - 1)A'$

$\Rightarrow \quad (1.54 - 1)4 = (\mu' - 1)3$

On solving we get $\mu' = 1.72$

30. As the water is flowing through the horizontal tube, so in the streamline flow of water, the sun of static pressure and dynamic pressure is constant *i.e.*,

$$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho v_1^2$$

$\Rightarrow \quad v_1 = \sqrt{\dfrac{P}{\rho} + v^2}$

31. Elastic energy per unit volume of wire.

$\mu = \dfrac{1}{2} \times$ Young's modulus $\times (\text{Strain})^2$

$\therefore \quad \dfrac{u_1}{u_2} = \left\{\dfrac{(\text{Strain})_1}{(\text{Strain})_2}\right\}^2 = \dfrac{l^2}{L^2} \times \dfrac{4L^2}{4l^2} = 1:1$

32. The terminal velocity, $v = \dfrac{2r^2(\rho - \sigma)g}{9\eta}$

As, all other parameters are constant, therefore $v \propto r^2$

$\Rightarrow \quad \dfrac{v_R}{v_{3R}} = \dfrac{(R)^2}{(3R)^2} = \dfrac{1}{9}$

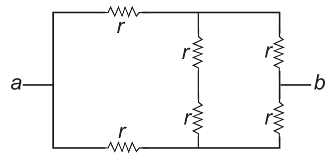
33. The mass defect, $\Delta m = 2(m_p + m_n) - m_{\text{He}}$

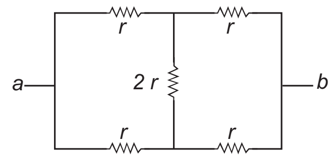
$$= 2(1.00783 + 1.00867) - 4.00300$$
$$= 0.0300 \text{ amu}$$

So, the binding energy, $E = \Delta mc^2$

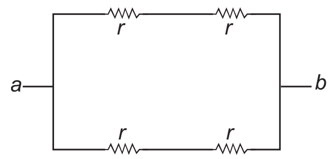
$$= 0.03 \times 931 \text{ MeV} \approx 27.9 \text{ MeV}$$

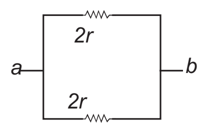
34. The equivalent circuit for the given electrical network is





This is a balanced Wheatstone bridge, so the arm containing $2r$ not in use. So, we have





So, the equivalent resistance between a and b is

$$\frac{1}{r'} = \frac{1}{2r} + \frac{1}{2r} \Rightarrow r' = \frac{2r}{2} = r$$

35. Linear magnification, $m = \dfrac{f}{f + u}$

As given that magnification is same for both cases this is possible if the two different values of u are of opposite sign.

i.e., $\quad \dfrac{f}{f - 16} = \dfrac{-f}{f - 8}$

$\Rightarrow \quad 16 - f = f - 8$

$\Rightarrow \quad 2f = 24$

or $\quad f = 12$ cm

36. By using the relation, $v = \nu\lambda$

We have, $\quad \lambda = \dfrac{v}{\nu} = \dfrac{300}{500} = \dfrac{3}{5}$ m

The phase difference, $\phi = \dfrac{2\pi}{\lambda}(\Delta x)$

$\Rightarrow \quad \dfrac{\pi}{3} = \dfrac{2\pi \times 5}{3}(\Delta x) \Rightarrow \Delta x = \dfrac{1}{10}$ m $= 10$ cm

37. Let the velocity of third fragment is v'. Then by the conservation of linear momentum

$$5(0) = M \times 2v - 2M \times v + 2M \times v'$$

$$\Rightarrow \quad v' = 0$$

i.e., the third fragment will be at rest.

38. Given, $x(t) = 2t^3 - 3t^2 + 4t$

So, velocity, $v = \dfrac{dx}{dt} = (6t^2 - 6t + 4)$

and acceleration $a = \dfrac{dv}{dt} = (12t - 6)$

when acceleration is zero, *i.e.*, $(12t - 6) = 0$

or $\quad t = \dfrac{6}{12} = \dfrac{1}{2}$ s

$\therefore$ Velocity of the particle at zero acceleration is

$$v = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) 4 = 2.5 \text{ m/s}$$

39. Let the length of closed pipe is L_1 and that of pipe is L_2.

Fundamental frequency of closed pipe, $\nu_1 = \dfrac{v}{4L_1}$

and frequency of second harmonic of open pipe,

$$\nu_2 = 2 \times \frac{v}{2L_2}$$

Given, $\quad \nu_1 = \nu_2$

i.e., $\quad \dfrac{v}{4L_1} = \dfrac{2v}{2L_2} \Rightarrow \dfrac{L_1}{L_2} = \dfrac{1}{4}$

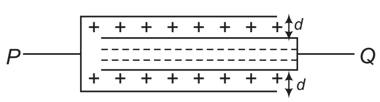
40. Given, $I = 20 \sin(100\pi t + 0.05\pi)$

The root mean square value of alternating current $= I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}} = \dfrac{20}{\sqrt{2}} = 10\sqrt{2}$ A

Also, $\quad \omega = 100\pi$

$\Rightarrow \quad 2\pi f = 100\pi$ or $f = 50$ Hz

41. Suppose the pair of plates is connected to positive terminal of the battery and the pair of plates Q is connected to the negative terminal of the battery.



From figure, it is clear that we have two capacitors C_1 and C_2. Positive plates of C_1 are connected to positive plate of C_2 and negative plate to negative. Therefore C_1 and C_2 are in parallel.

So, $\quad C_p = C_1 + C_2 = 2C = \dfrac{2\varepsilon_0 a}{d}$

42. We have

Power = Rate of doing work $= \dfrac{dW}{dt}$

or, $\quad P = \dfrac{dW}{dt}$ = rate of change in KE

i.e., $\quad P = \dfrac{dW}{dt} = \dfrac{d(\text{KE})}{dt}$ = constant

43. In an *n-p-n* or *p-n-p* transistor, the left hand side thick layer of the transistor is heavily doped known as emitter and right hand side thick layer of the transistor is moderately doped known as collector.

44. (a) Given, $\mathrm{A} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\mathrm{B} = 3\hat{i} + 6\hat{j} + 2\hat{k}$

So, $\quad \mathrm{C} = \dfrac{\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1 + 4 + 4}} \times \sqrt{3^2 + 6^2 + 2^2}$

$$= \frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3} \times \sqrt{49} = \frac{7}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

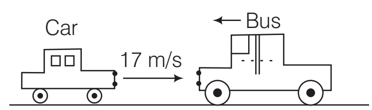
45. Given, velocity of sound, $v = 340$ m/s

Velocity of listner, $v_L = 17$ m/s

Velocity of source $= v_S$

Frequency of horn emitted

$\nu = 640$ Hz



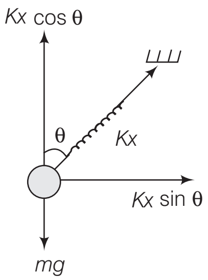
The apparent frequency

$$\nu' = \nu \frac{(v + v_L)}{v - v_s}$$

$$680 = 640\left(\frac{340 + 17}{340 - v_S}\right)$$

On solving we get $v_S = 4$ m/s

46. The given situation can be shown as



From figure, $Kx \sin\theta = m\omega^2 (L + x) \sin\theta$

$$\Rightarrow \qquad Kx = m\omega^2 (L + x) \qquad \ldots(i)$$

Also, $$\sqrt{\frac{K}{m}} = \omega_0$$

$$\Rightarrow \qquad K = m\omega_0^2$$

Substituting the value in Eq. (i)

$$m\omega_0^2 x = m\omega^2 (L + x)$$

$$\Rightarrow \qquad x = \frac{\omega^2 L}{\omega_0^2 - \omega^2}$$

47. Given, $\rho = K \cdot r$

By Gauss's theorem

$$E\,(4\pi r^2) = \frac{\int \rho \times 4\pi r^2\, dr}{\varepsilon_0}$$

$$= \frac{\int Kr \times 4\pi r^2\, dr}{\varepsilon_0}$$

$$\Rightarrow \qquad E = \frac{Kr^2}{4\varepsilon_0}$$

Here $\qquad r = R$

So, $$E = \frac{KR^2}{4\varepsilon_o}$$

48. (a) Given, $v = (3\hat{i} + 10\hat{j})$ m/s

$$\Rightarrow \qquad v_x = 3 \text{ and } v_y = 10$$

$\therefore$ Maximum height attained, $H = \dfrac{v_y^2}{2g}$

$$= \frac{10 \times 10}{2 \times 10} = 5\,\text{m}$$

Range $= v_x \times T = v_x \times \dfrac{2v_y}{g} = \dfrac{3 \times 2 \times 10}{10} = 6\,\text{m}$

49. We have, $H = I^2 R$

According to given condition, $I_1^2 R_1 = H$

and $\qquad I_2^2 R_2 = 4H$

$$\Rightarrow \qquad \frac{E^2}{(R_1 + r)^2} R_1 = H \text{ and } \frac{E^2}{(R_2 + r)^2} R_2 = 4H$$

$$\therefore \qquad \frac{R_2}{(R_2 + r)^2} = \frac{4R_1}{(R_1 + r)^2}$$

$$\Rightarrow \qquad \sqrt{R_2}\,(R_1 + r) = 2\sqrt{R_1}\,(R_2 + r)$$

On solving, $$r = \frac{\sqrt{R_1 R_2}\,[\sqrt{R_1} - 2\sqrt{R_2}]}{2\sqrt{R_1} - \sqrt{R_2}}$$

50. Given, $B = 2t + 4t^2$

at $\qquad t = 0, B_1 = 0$

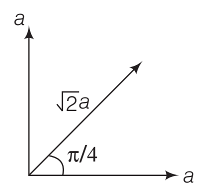
and at $\qquad t = 2, B_2 = 2 \times 2 + 4(2)^2$

$$= 4 + 16 = 20\,\text{Wb/m}^2$$

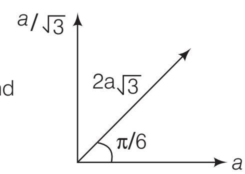
We have, $$\Delta Q = \frac{\Delta\phi}{R} = \frac{\pi r^2 (B_2 - B_1)}{R}$$

$$= \frac{\pi r^2 [20 - 0]}{R} = \frac{20\pi r^2}{R}$$

51. The given situation can be shown as



and

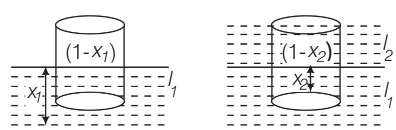


Fon second SHM

Ratio of amplitude $= \dfrac{a_1}{a_2} = \dfrac{\sqrt{3}}{\sqrt{2}}$

and phase difference, $\dfrac{\pi}{4} - \dfrac{\pi}{6} = \dfrac{\pi}{12}$

52. The given situation is shown as



As the Bouyant force in both the cases are same

$$\therefore \qquad \rho_1 x_1 g = \rho_1 x_2 g + \rho_2 (1 - x_2)\, g$$

On solving $$\frac{\rho_1}{\rho_2} = \frac{(1 - x_2)}{(x_1 - x_2)}$$

53. Restoring force on displacement of x

$$F = K\left[\frac{q}{(d-x)^2} - \frac{Qq}{(d+x)^2}\right]$$

$$= KQq\left[\frac{1}{(d-x)^2} - \frac{1}{(d+x)^2}\right]$$

$$= KQq\left[\frac{4dx}{(d^2-x^2)^2}\right]$$

$$= KQq\left[\frac{4dx}{d^4}\right] \quad \text{If } (d >> x)$$

$$\Rightarrow \quad F = KQq\left[\frac{4x}{d^3}\right]$$

Acceleration, $a = \frac{F}{m} = \frac{4KQqx}{Md^3}$

or $\omega^2 = \frac{4KQq}{Md^3}$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{Md^3}{4KQq}}$$

$$= 2\sqrt{\frac{\pi^3 Md^3 \varepsilon_0}{Qq}}$$

54. Here, $h\nu_1 = \phi_0 = eV_1$ ...(i)

and $h\nu_2 = \phi_0 + eV_2$ ...(ii)

From Eqs. (i) and (ii), we have

$$h(\nu_2 - \nu_1) = e(V_2 - V_1)$$

$$\Rightarrow \quad \nu_2 = \frac{e}{h}(V_2 - V_1) + \nu_1$$

55. The number of degrees of freedom for the mixture is

$$f_{\text{mix}} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$$

$$= \frac{3\times 3 + 1\times 5}{4} = \frac{7}{2}$$

$$\therefore \quad \gamma = 1 + \frac{2}{f}$$

$$\Rightarrow \quad \gamma = 1 + \frac{4}{7} = \frac{11}{7}$$

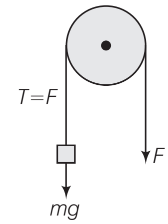
56. Magnetic moment, $m = IA$

$$= \frac{ev}{2\pi r}\times \pi r^2 = \frac{evr}{2}$$

Angular momentum $= 2m\frac{dA}{dt}$

57. The free body diagram

$$F - mg = 2$$



$$\Rightarrow \quad F = 2 + mg = 3\,\text{N}$$

also, $a = \frac{\text{unbalanced force}}{\text{mass}} = \frac{2}{0.1} = 20\ \text{m/s}^2$

$$\therefore \quad S = \frac{1}{2}at^2 = \frac{1}{2}\times 20\times 1 = 10\ \text{m}$$

Hence, work done by tension

$$= F\times 10 = 3\times 10 = 30\ \text{J}$$

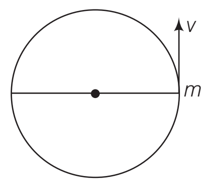
So, the work done against gravity

$$= mg\times S = 1\times 10 = 10\ \text{J}$$

58. The charged particle will move with uniform velocity in that space if $E = 0$ and $B \neq 0$, since a moving charge in a magnetic field experience a force. Also if $E \neq 0$, $B \neq 0$, the charged particle will move with uniform velocity in that space.

If $E = vB$, then particle again moves with uniform velocity.

59. We have, $v = \frac{1}{2}\omega l$



$$T = \frac{2v}{g} = \frac{\omega l}{g} \text{ or } n\frac{2\pi}{\omega} = \frac{\omega l}{g}$$

(as completes n rotations with in T)

$$\therefore \quad n = \frac{l\omega^2}{2\pi g}$$

Distance travelled $= 2h = \frac{2v^2}{2g} = \frac{l^2\omega^2}{4g}$

60. The lens will behave like a convex lens or a concave lens depending upon the value of n_1 and n_2.

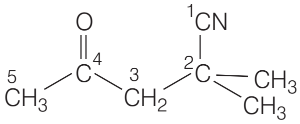
Chemistry

1. For MX_2 type salt,

Solubility product, $K_{sp} = 4s^3$; $3.2 \times 10^{-8} = 4s^3$

or $s = \sqrt[3]{\dfrac{3.2 \times 10^{-8}}{4}} = 2 \times 10^{-3}$ mol/L

2.



2, 2-dimethyl-4-oxopentanenitrile

3. In $SOCl_2$, the Cl—S—Cl and Cl—S—O bond angles are respectively 96° and 106°.

4. A planar carbocation is generated when $SbCl_5$ remove Cl^- from the substrate. This carbocation is subsequently attacked by Cl^- (nucleophile) from both the sides (*i.e.* from top and bottom) to produce a racemic mixture.

5. $CH_3COOC_2H_5 + H_2O \rightleftharpoons CH_3COOH + C_2H_5OH$

If above reaction is carried out with large excess of ester, the rate of reaction does not depend upon cencentration of both the reactants. Due to which, the reaction becomes of zero order.

6.

Medium	Colour of Litmus Paper
Acidic	Red
Neutral	Violet
Basic	Blue

7. Baeyer's reagent is 1% cold dilute alkaline potassium permanganate. It is used to identify unsaturation. All unsaturated compounds lose its purple colour.

8. Equivalent conductance (Λ_{eq}) is defined as the conducting power of all the ions produced by one gram equivalent of an electrolyte in a given solution.In case of weak electrolytes (as CH_3COOH, NH_4OH, AgCl etc.), ionisation is very small compared to strong electrolytes (as KCl, NaOH etc.), hence Λ_{eq} of weak electrolyte is low. Moreover, ionisation of H^+ is maximum among given ions. Thus, correct of order of Λ_{eq} is

$$H^+ > OH^- > K^+ > CH_3COO^-$$

9. Ostwald process of for manufacture of nitric acid,

$$\underset{\text{ammonia}}{4NH_3} + SO_2 \xrightarrow[800-900°C]{\text{Pt gauge}} \underset{\text{nitric oxide}}{4NO} + 6H_2O$$

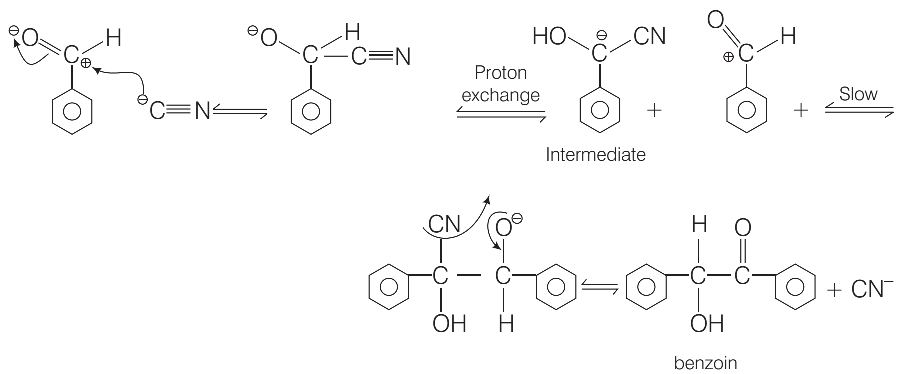
$$\Delta H = -21.5\,\text{kcal}$$

$$2NO + O_2 \xrightarrow{50°C} \underset{\text{nitrogen dioxide}}{2NO_2}$$

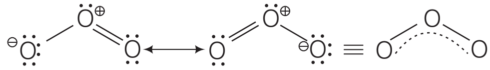
$$4NO_2 + 2H_2O + O_2 \longrightarrow \underset{\text{nitric acid}}{4HNO_3}$$

10. When benzaldehyde is heated with aqueous ethanolic NaCN or KCN, it dimerises to form an α-hydroxy ketone called benzoin, and this reaction is formed as benzoin condensation.

It involves self condensation of an aromatic aldehyde in the presence of CN^- as catalyst.



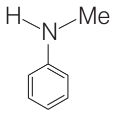
11. Ozone shows following resonance structure



It contains bond angle 116.8° and O—O bond length 1.28 Å.

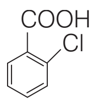
12. The entropy is the measure of disorder or randomness in a system. When a system changes from one state to another, the change of entropy, dS is given by

$$dS = \frac{\delta q_{rev}}{T}$$

13. 

$+ \text{Na} \xrightarrow{\Delta}$ sodium extract (NaCN)

$\xrightarrow[\text{FeCl}_3]{\text{FeSO}_4} \underset{\text{Prussion blue}}{Fe_4[Fe(CN)_6]_3}$



$+ \text{Na} \xrightarrow{\Delta}$ sodium extract (NaCl)

$\xrightarrow{\text{AgNO}_3} \underset{\text{White Precipetate}}{AgCl \downarrow} + NaNO_3$

Hydrozine, $NH_2 \cdot NH_2$ does not respond Lassaigne's test because it does not contain any carbon and hence, NaCN is not formed. Compound, $NH_2 — \underset{\underset{S}{||}}{C} — NH_2$ contains both N and S, hence, it will give red colour in Lassaigne test.

14. $2CaO + Cl_2 \xrightarrow{\text{Red hot}} CaCl_2 + O_2 \uparrow$

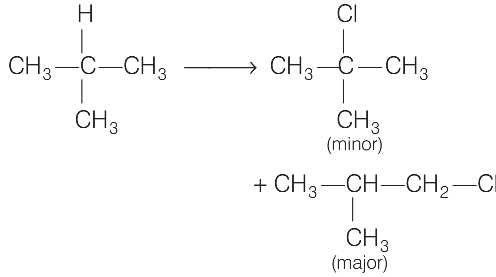
15. From Arrhenius equation

$$\ln \frac{k_2}{k_1} = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

or $$\ln \frac{k_2}{k_1} = \frac{600R}{R}\left(\frac{1}{300} - \frac{1}{600}\right)$$

or $$\ln \frac{k_2}{k_1} = 600\left(\frac{2-1}{600}\right)$$

or $$\ln \frac{k_2}{k_1} = 1 \text{ or } \frac{k_2}{k_1} = e$$

16. 

There are nine primary hydrogens and one tertiary hydrogen in 2-methyl propane. Tertiary hydrogen atoms react with Cl about 5.5 times as fast as primary. Even though the primary hydrogens are less reactive, there are so many of them that the primary product is the major product.

17. $n = \frac{\text{total time } (t)}{\text{half-life } (t/2)} = \frac{22920}{5730} = 4$

Left amount, $N = N_0 \left(\frac{1}{2}\right)^n$

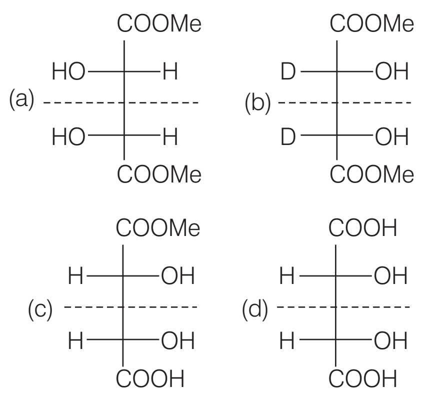
$$= N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$\therefore$ Decayed fraction $= N_0 - \frac{N_0}{16}$

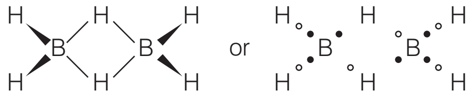
$$= \frac{16\,N_0 - N_0}{16} = \frac{15\,N_0}{16}$$

18. At very low pressure, compressibility factor, Z is approximatedly one.

$\therefore$ van der Waals' gas may behave ideally.

19. 

Only molecule (c) is optically active. Other moelcules are *meso*, due to presence of plane of symmetry.

20. 

Structure of B_2H_6

• Boron electron
∘ Hydrogen electron

In diborone, odd-electron bonds are found in the bridges. From figure, it is clear that 4 electrons are present for bonding in the bridges.

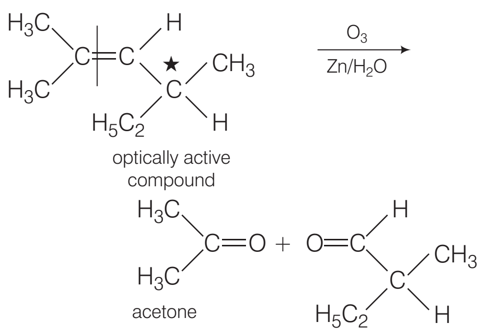
21. $Na_2S + \underset{\text{sodium nitropruside'}}{Na_2[Fe(CN)_5NO]} \longrightarrow \underset{\text{purple colour}}{Na_4[Fe(CN)_5[NOS]}$

$\underset{\text{tetra anionic complex of iron (II) co-ordinating}}{Na_4[Fe(CN)_5NOS]} \rightleftharpoons 4Na^+ + \underset{\text{to one } NOS^- \text{ ions.}}{[Fe(CN)_5NOS]^{4-}}$

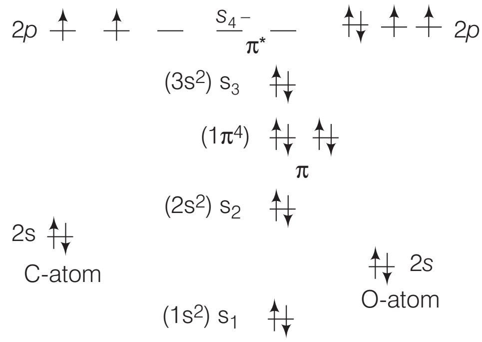
22. $NaOH \rightleftharpoons Na^+ + OH^-$ $\qquad [OH^-] = 10^{-8}M$

$H_2O \rightleftharpoons H^+ + OH^-$ $\qquad [OH^-] = 10^{-7}M$

$$\therefore [OH^-]_{total} = (10^{-8} + 10^{-7})M$$
$$= 10^{-7}(1.1)$$
$$= 1.1 \times 10^{-7}$$
$$\therefore \quad pOH = \log 1.1 \times 10^{-7} \simeq 6.98$$
$$\therefore \quad pH = 14 - 6.98$$
$$= 7.02$$

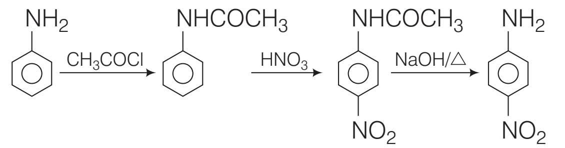
23. 

24. On mixing of two different ideal gases under isothermal reversible conditions, ΔS_{mix} is always positive *i.e.*, increasing.

25. 

CO
(Carbon Monoxide)

26. Direct nitration of aniline with nitric acid gives a complex mixture of mono, di- and tri-nitro compounds and oxidation products. If —NH_2 group is protected by acetylation and then nitrated with nitrating mixture, *p*-isomer is the main product.



p-nitroaniline

27. $\because \qquad \Delta T_f = i \times K_f \times m$

$$\therefore \qquad i = \frac{\Delta T_f}{K_f \times m} = \frac{0.19}{1.86 \times 0.1} = 1.02$$

Again from, $\alpha = \dfrac{i-1}{n-1} = \dfrac{1.02-1}{2-1}$

$$= 0.02 = 2.0 \times 10^{-2}$$

for $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

$$Ka = C\alpha^2$$
$$= 0.1 \times (2 \times 10^{-3})^2$$
$$= 4 \times 10^{-5}$$

28. Ore chomite is $FeCr_2O_4$.

29. H_2SO_4 saturated with SO_3 is called oleum or sulphan.

$$H_2SO_4 + SO_3 \longrightarrow H_2S_2O_7$$

30. From first law of thermodynamics

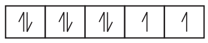
$$\Delta E = q + W$$

where, work do net (W) = $p\,\Delta V$

$$= p \times V \qquad (\because \Delta V = 0)$$
$$= 0$$

31. Species having unpaired electrons are paramagnetic

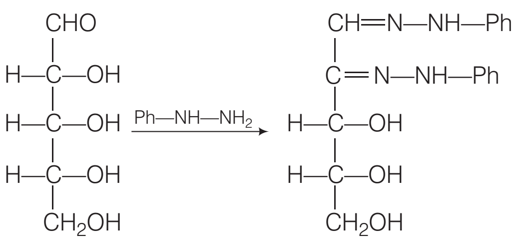
$$Ni^{2+} = [Ar]\,3d^8$$



(i) In $[NiCl_4]^{2-}$ and $[Ni(H_2O)_6]^{2+}$ ligands Cl^- and H_2O are weak ligands, therefore no pairing will be possible. Thus, there are two unpained electrons.

(ii) In $[Ni(Ph_3)_2Cl_2]$, although PPh_3 has d-acceptance nature but presence of Cl, makes electrons unpaired.

32. Ribose forms osazone with Ph—NH—NH_2 whereas in deoxyribose, one —OH group is missing, due to which, it cannot form osazone.



ribose osazone

33. 10 million $= 10^{-7}$

$\therefore$ 10 millionth part of $1.33\,cm^3 = 1.33 \times 10^{-7}\,cm^3$

$$= 1.33 \times 10^{-7}\,mL$$

For pure water, $[H^+] = 10^{-7}$ mol/L

$\Rightarrow$ 1 L water contains $[H^+] = 10^{-7}$ mol

or 1 mL water contains $[H^+] = \dfrac{10^{-7}}{1000} = 10^{-10}$ mol

or 10 millionth part of $1{\cdot}33\,cm^3$ water contains

$[H^+]$ $= 1{\cdot}33 \times 10^{-7} \times 10^{-10}$ mol

$$= 1{\cdot}33 \times 10^{-17}\text{ mol}$$

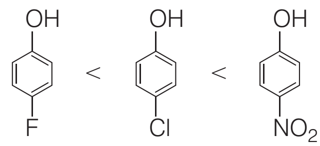
$\therefore$ Number of H^+ ions $= 1.33 \times 10^{-17} \times N_A$

$$= 1.33 \times 10^{-17} \times 6.022 \times 10^{23}$$
$$= 8.009 \times 10^6 = 8.01\text{ million}$$

34. The order of electron withdrawing tendency from benzene ring i.e.,

$$—F < —Cl < —NO_2$$

$\therefore$ Correct order of acidic strength of substituted phenols will be



35. For isothermal expansion of an ideal gas

$$\Delta T = 0$$

$\therefore$ From $\Delta U = nC_v\,\Delta T$,

$$\Delta U = 0 \text{ and, from}$$
$$\Delta H = nC_p\Delta t = 0$$

From first law of thermodynamics,

$$\Delta U = Q + W$$

as $\Delta U = 0 \Rightarrow Q \neq 0$

and $W \neq 0$

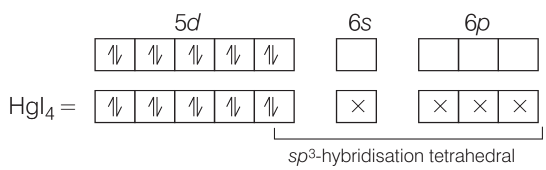
$\therefore$ Parameters are

$$\Delta U = 0, Q \neq 0\,;\ W \neq 0$$

and $\Delta H = 0$

36. $HgCl_2 + 4KI \longrightarrow K_2[HgI_4] + 2KCl$

$$Hg = [Xe]\,4f^{14}, 5d^{10}6s^2$$
$$Hg^{2+} = [Xe]\,4f^{14}, 5d^{10}, 6s^\circ$$



37. Molecules having whole number for degree of unsaturation can exist in free state as stable compounds.

Degree of unsaturation $= \dfrac{\Sigma n\,(v-2)}{2} + 1$

where, n = number of atoms of a particular type

v = valency of the atom.

(a) For C_7H_9O,

$$DU = \frac{7\,(4-2) + 9\,(1-2) + 1\,(2-2)}{2} + 1 = 3{\cdot}5$$

(b) For $C_8H_{12}O$,

$$DU = \frac{8(4-2)+9(1-2)+1(2-2)}{2}+1 = 3\cdot0$$

(c) For $C_6H_{11}O$,

$$DU = \frac{6(4-2)+11(1-2)+1(2-2)}{2}+1 = 1\cdot5$$

(d) For $C_{10}H_{17}O_2$,

$$DU = \frac{10(4-2)+17(1-2)+2(2-2)}{2}+1 = 2\cdot5$$

$\therefore$ $C_8H_{12}O$ will exist in free state or a stable compound.

38. Given, $\kappa = 1\cdot25\times10^{-3}\text{S cm}^{-1}$

$$\rho = \frac{1}{\kappa} = \frac{1}{1.25\times10^{-3}}\text{S}^{-1}\text{ cm}$$

From $$R = \rho\frac{l}{A}$$

$$800 = \frac{1}{1.25\times10^{-3}}\times\frac{l}{A}$$

(where, $\frac{l}{A}$ = cell constant)

$\therefore$ $$\frac{l}{A} = 800\times1.25\times10^{-3}$$

$$= 1\text{cm}^{-1}$$

39. Orange solid is ammonium dichromate.

$$\underset{\text{orange solid}}{(NH_4)_2Cr_2O_7} \xrightarrow{\Delta} \underset{\text{colourless gas}}{N_2\uparrow} + \underset{\text{green solid}}{Cr_2O_3} + 4H_2O$$

$$Cr_2O_3 + 2Al \longrightarrow 2Cr + Al_2O_3$$

40. 2, 2 – dimethyl butane is prepared from Corey - House alkane synthesis.

$$(Me_3C)_2CuLi + MeCH_2Br \xrightarrow{S_N2}$$

$$Me_3C\text{—}CH_2CH_3 + \overset{Br}{Me_3C}\ CuLi$$

41. From Gibbs Helmholtz equation,

$$\Delta G = \Delta H - T\Delta S$$

A process will be spontaneous, if

$$\Delta G = -\text{ve}$$

42. No option is correct.

As the number of lone pair of electrons increases, bond angle decreases.

NO_2^+ ion is isoelectronic with CO_2 molecule. It is a linear ion and its central atom (N^+) undergoes *sp*-hybridisation. Hence, its bond angle is 180°.

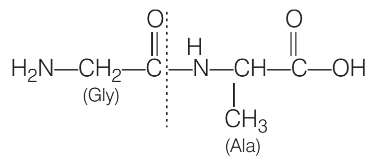
In NO_2^- ion, N-atom undergoes sp^2 hybridisation. The angle between hybrid orbital should be 120° but one lone pair of electrons is lying on N-atom, hence bond angle decreases to 115°.

In NO_2 molecule, N-atom has one unpaired electron in sp^2-hybrid orbital. The bond angle should be 120° but actually, it is 132°. It may be due to one unpaired electron in sp^2-hybrid orbital.

Therefore, the increasing order of bond angle is

$$\underset{115^\circ}{NO_2^-} < \underset{132^\circ}{NO_2} < \underset{180^\circ}{NO_2^+}$$

43. The structure of dipeptide gly-ala is



44. $\lambda_m^\infty = \lambda_m^\infty\text{ (oxalate)} + \lambda_m^\infty(Na^+) + \lambda_m^\infty(K^+)$

$$= (148\cdot2 + 73\cdot5 + 50\cdot1)$$

$$= 271.85\text{ cm}^2\text{ mol}^{-1}$$

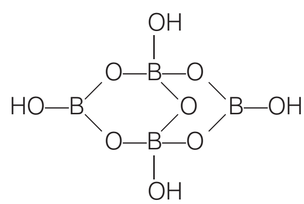
$\because$ $$\lambda_{eq}^\infty = \frac{\lambda_m^\infty}{n\cdot\text{factor}} = \frac{271.8}{2}$$

$$= 135.9\text{ Scm}^2\text{eq}^{-1}$$

45. Since, ionic character is inversely proportional to polarising power of cation therefore correct order is

$$AlCl_3 > GaCl_3 > BCl_3$$

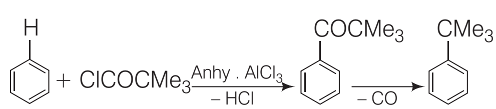
46.



5 B — O — B bonds

4 B — OH bonds

47.



48.

	CH_3COO^-	+ H^+	$\longrightarrow$ CH_3COOH
Initially	0.01	0.001	0.01
at equi.	0.01 − 0.001 = 0.009		0.01 + 0.001 = 0.011

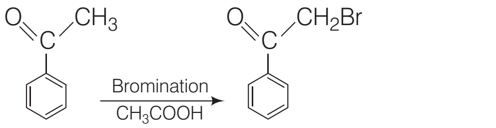
$$pH = pK_a + \log \frac{(\text{salt})}{(\text{acid})} = 4.75 + \log \frac{0.009}{0.011}$$

$$= 4.66$$

49. $3HClO_3 \xrightarrow{\Delta} HClO_4 + Cl_2 + 2O_2 + H_2O$

50. $\begin{matrix} Me_3C \\ Br \end{matrix}\!\!>\! Mg + Cl—CN \longrightarrow Me_3C—CN + Mg\!<\!\!\begin{matrix} Cl \\ Br \end{matrix}$

51.



52. $\Delta G° = -2.303\, RT \log K$

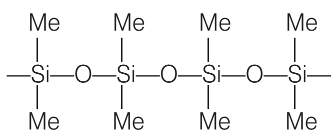
$= -2.303 \times 8.314 \times 298 \times \log 0.15$

$= -2.303 \times 8.314 \times 298 \times (-0.82)$

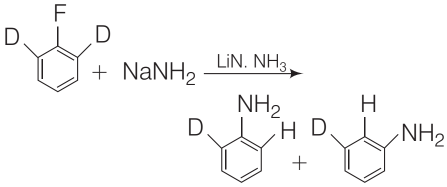
$= 4678.7$ J $= 4.67$ kJ

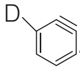
53. Silicon oil in a polymer of trimethylchloro silane and dimethyldichloro silane. These are useful as broad spectrum antifoaming agents.

$Si(MeI_3)Cl + Si(Me)_2Cl_2 + H_2O$ □ $\xrightarrow{\text{Polymerisation}}$



54.



This reaction proceeds *via* benzyne mechanism with intermediate as .

55. All the quantum numbers (n, l, m, s) are required to describe an electron of an atom completely.

e.g., n describes the position and energy of the electron in an orbit or shell. *l* is used to describe subshell and the shape of the orbital occupied by the electron.

m describes the preferred orientation of orbitals in space.

s describes the spining of an electron on its axis.

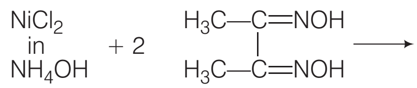
56. Estimation of Ni^{2+} is carried out as.

Filtrate of group III + $NH_4OH + NH_4Cl \xrightarrow{\Delta}$ passing H_2S gas $\rightarrow$ black ppt of NiS.

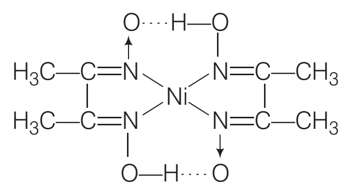
This black ppt of NiS is soluble in conc. HCl in presence of oxidising agent like $KClO_3$

$NiS + \underset{\text{Conc}}{2HCl} + O \longrightarrow NiCl_2 + H_2.O + S$

Now this $NiCl_2$, in basic medium, treated with dimethyl glyoxime, cherry red ppt of nickel (II) dimethyl glyoximate is obtained, in which two dimethyl glyoximate units are hydrogen bonded to each other and each unit forms a six-membered chelate ring with Ni^{2+}.

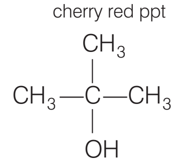


dimethyl glyoxime



nickel (II) dimethyl glyoximate
cherry red ppt

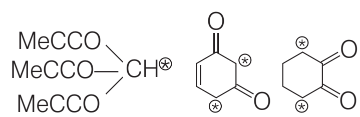
57. $CH_3CH_2CH_2CH_2OH$
n-butanol



t-butanol

More branching results high solubility and low boiling point.

58. Availability of acidic α – H- atoms at (*) marked positions, enable the compounds to show keto-enol tautomerism.



(a) (b) (d)

59. Lintz and Donawitz (L.D.) process is an important process for manufacturing of steel. In India, Rourkela steel plant is based upon this process.

This process is carried out in a converter similar to bessemer converter having lining (usually refractory made of MgO and (CaO).

In this method scrap iron can be used along with flux in converter : On passing oxygen gas at 4 – 12 atm pressure into converter, temperature of 2000 – 2500° C range produced which separates C, Si, Mn etc., impurities as slag from iron quickly remaining pure iron is mixed with spiegeleisen (alloy Fe, Mn, C) to produce better quality steel.

60. $2NO_2(g) + F_2(g) \longrightarrow 2NO_2F(g)$

$$\text{Rate of reaction} = -\frac{1}{2}\left[\frac{dp(NO_2)}{dt}\right]$$
$$= -\left[\frac{dp(F_2)}{dt}\right]$$
$$= +\frac{1}{2}\left[\frac{dp(NO_2F)}{dt}\right]$$

Mathematics

1. The given equation

$ax^2 + bx + 1 = 0$ ...(i)

has real roots.

$\therefore$ Discriminant $(D) \geq 0$

$\Rightarrow \quad b^2 - 4a \geq 0$ ...(ii)

From Eq. (ii), we observe that

a has to be 1 and b has to be 2.

So, the required probability $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. **Ist turn** Total number of face card = 12

Total number of elements in sample space, $n(s) = 52$

$\therefore P_1$ (no face card in first turn) $= \frac{52-12}{52} = \frac{40}{52}$

IInd turn P_2 (no face card in second turn)

$$= \frac{(52-13)}{(52-1)} = \frac{39}{51}$$

IIIrd turn P_3 (face card in third turn)

$$= \frac{(52-40)}{(51-1)} = \frac{12}{50}$$

$\therefore$ Required P (face card on third turn)

$$= P_1 \times P_2 \times P_3$$
$$= \frac{40}{52} \times \frac{39}{51} \times \frac{12}{50} = \frac{12}{85}$$

3. Let $E \rightarrow$ Event of head showing up

$E_1 \rightarrow$ Event of biased coin chosen

$E_2 \rightarrow$ Event of unbiased coin chosen

Now, $P(E_2) = \frac{1}{2}$ and $P(E_1) = \frac{1}{2}$

Also, $P\left(\frac{E}{E_2}\right) = \frac{1}{2}$ and $P\left(\frac{E}{E_1}\right) = \frac{3}{4}$

(by conditional probability)

By Baye's theorem

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_1)\cdot P\left(\frac{E}{E_1}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{3}{4}} = \frac{2}{5}$$

4. Given equation of ellipse

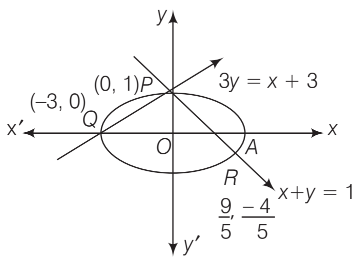
$$x^2 + 9y^2 = 9$$

$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{1} = 1$ ...(i)

and equation of lines

$$x + y = 1 \text{ and } 3y = x + 3$$

or $\quad \frac{x}{(-3)} + \frac{y}{1} = 1$



$\therefore$ Area of $\Delta PQR = \left|\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ -3 & 0 & 1 \\ 9/5 & -4/5 & 1 \end{vmatrix}\right|$

$$= \frac{1}{2}\left[\frac{24}{5} + \frac{12}{5}\right]$$

$$= \frac{1}{2} \times \frac{36}{5} = \frac{18}{5}$$

5. Given equation of lines are

$$3t\,x - 2\,y + 6t = 0 \qquad \ldots\text{(i)}$$

and $\quad 3\,x + 2\,ty - 6 = 0 \qquad \ldots\text{(ii)}$

On multiplying Eq. (i) by t and then adding in Eq. (ii), we get

$$(3t^2 + 3)x + 6t^2 - 6 = 0$$

$$\Rightarrow \quad x = \frac{2(1-t^2)}{(1+t^2)}$$

$$\Rightarrow \quad x + xt^2 = 2 - 2t^2$$

$$\Rightarrow \quad (x + 2)\,t^2 = (2 - x)$$

$$\Rightarrow \quad t^2 = \frac{2 - x}{2 + x} \qquad \ldots\text{(iii)}$$

On multiplying Eq. (ii) by t and then subtract from Eq. (i), we get

$$(-2 - 2t^2)y + 6t + 6t = 0$$

$$12t = 2(1 + t^2)y$$

On squaring both sides, we get

$$144t^2 = 4y^2(1 + t^2)^2$$

$$\Rightarrow 144\left(\frac{2-x}{2+x}\right) = 4y^2\left(1 + \frac{2-x}{2+x}\right)^2 \text{[form Eq. (iii)]}$$

$$\Rightarrow \quad 36\left(\frac{2-x}{2+x}\right) = y^2\left(\frac{4}{2+x}\right)^2$$

$$\Rightarrow \quad 36\,\frac{(2-x)}{(2+x)} = \frac{16y^2}{(2+x)^2}$$

$$\Rightarrow \quad 36\,(4 - x^2) = 16y^2$$

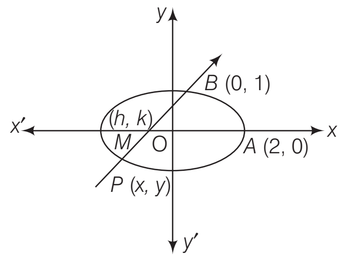
$$\Rightarrow \quad 9(4 - x^2) = 4y^2$$

$$\Rightarrow \quad 36 - 9x^2 = 4y^2$$

$$\Rightarrow \quad 9x^2 + 4y^2 = 36$$

$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{9} = 1$ which represents an ellipse.

6.



Given equation of an ellipse is

$$x^2 + 4y^2 = 4$$

$$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{1} = 1 \qquad \ldots\text{(i)}$$

$\therefore$ Coordinate of positive end of minor axis is $B(0, 1)$. Let mid-point of the chord BP is $M(h, k)$

Then, $\quad (h, k) = \left(\frac{0 + x}{2}, \frac{1 + y}{2}\right)$

$$\Rightarrow \quad h = \frac{x}{2} \quad \Rightarrow x = 2h$$

and $\quad k = \frac{1 + y}{2} \Rightarrow y = 2k - 1$

$$\therefore \quad P(x, y) \equiv \{2h, (2k - 1)\}$$

Since, the point 'P' lies cllipes so form Eq. (i), we get

$$(2h)_1^2 + 4(2k - 1)^2 = 4$$

$$\Rightarrow \quad 4h^2 + 4(2k - 1)^2 = 4$$

$$\Rightarrow \quad h^2 + 4k^2 + 1 - 4k = 1$$

$$\Rightarrow \quad h^2 + 4k^2 - 4k = 0$$

Thus, required locus is an ellipse whose equation is

$$x^2 + 4y^2 - 4y = 0$$

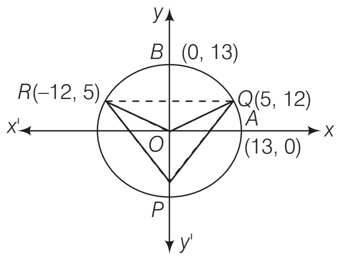
$$\Rightarrow \quad \frac{(x-0)^2}{1} + \frac{\left(y - \frac{1}{2}\right)^2}{\left(\frac{1}{4}\right)} = 1$$

whose centre $\left(0, \frac{1}{2}\right)$ and major and minor axis $\frac{1}{2}$

7. Given equation of circle is

$$x^2 + y^2 = 169$$

Its centre $= (0, 0)$ and radius $= 13$



Now, slope of $OR = \dfrac{-5}{12} = m_1 \left(\because \text{slope} = \dfrac{y_2 - y}{x_2 - x}\right)$

and slope of $OQ = \dfrac{12}{5} = m_2$

$\because \quad m_1 \cdot m_2 = -1 \Rightarrow \angle ROQ = \dfrac{\pi}{2}$

We know that, angle made by the chord of circle at circumference is equal to the half of the angle made by the same chord at the centre of circle.

$\therefore \quad \angle QPR = \dfrac{1}{2} \cdot \angle ROQ = \dfrac{1}{2} \times \dfrac{\pi}{2} = \dfrac{\pi}{4}$

8. Let P be any point, whose coordinate is (h, k). Given,

P moves, so that the sum of squares of its distances from the points $A(1, 2)$ and $B(-2, 1)$ is 6.

i.e., $(PA)^2 + (PB)^2 = 6$

$\Rightarrow \quad (h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 6$

$\Rightarrow \quad h^2 + 1 - 2h + k^2 + 4 - 4k + h^2 + 4 + 4h + k^2 + 1 - 2k = 6$

$\Rightarrow \quad 2h^2 + 2k^2 + 2h - 6k + 4 = 0$

$\Rightarrow \quad h^2 + k^2 + h - 3k + 2 = 0$

$\therefore$ Required locus is

$$x^2 + y^2 + x - 3y + 2 = 0$$

Which represent a circle.

Whose centre is $\left(\dfrac{-1}{2}, \dfrac{3}{2}\right)$

and radius $= \sqrt{\dfrac{1}{4} + \dfrac{9}{4} - 2} = \sqrt{\dfrac{5}{2} - 2} = \dfrac{1}{\sqrt{2}}$

9. Let the equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots\text{(i)}$$

When, circle (i) passes through the origin

Then, $\quad c = 0 \quad \ldots$(ii)

When, circle (i) passes through the point (2, 6).

Then, $4 + 36 + 4g + 12f + 0 = 0$

$\Rightarrow \quad 4g + 12f + 40 = 0$

$\Rightarrow \quad g + 3f = -10 \quad \ldots$(iii)

When, circle (i) passes through the point (6, 2)

Then, $36 + 4 + 12g + 4f + 0 = 0$ [from Eq. (i)]

$\Rightarrow \quad 12g + 4f + 40 = 0$

$\Rightarrow \quad 3g + f + 10 = 0 \quad \ldots$(iv)

On solving Eqs. (iii) and (iv), we get

$$g = \frac{-5}{2} \text{ and } f = \frac{-5}{2}$$

$\therefore$ Equation of circle becomes

$$x^2 + y^2 - 5x - 5y = 0 \quad \ldots\text{(v)}$$

Circle cut the x-axis.

So, put $y = 0$ in Eq. (v), we get

$$x^2 - 5x = 0 \Rightarrow x(x-5) = 0$$

$\Rightarrow \quad x = 5$

So, the circle cut the x-axis at point $P(5, 0)$.

$\therefore$ The length of $OP = 5$

10. Given equation of lines are

$$x - 2y = t \quad \ldots\text{(i)}$$

and $\quad x + 2y = \dfrac{1}{t} \quad \ldots$(ii)

On multiplying Eqs. (i) and (ii), we get

$$(x - 2y)(x + 2y) = t \times \frac{1}{t}$$

$\Rightarrow \quad x^2 - 4y^2 = 1$

$\Rightarrow \quad \dfrac{(x-0)^2}{1} - \dfrac{(y-0)^2}{(1/4)} = 1$

which represent a hyperbola.

Here, $a^2 = 1$ and $b^2 = \dfrac{1}{4}$

$\therefore$ Eccentricity $\quad (e) = \sqrt{\dfrac{a^2 + b^2}{a^2}} = \sqrt{\dfrac{1 + \frac{1}{4}}{1}} = \dfrac{\sqrt{5}}{2}$

and focus

$$= (\pm ae, 0) = \left(\pm 1 \times \frac{\sqrt{5}}{2}, 0\right) = \left(\pm \frac{\sqrt{5}}{2}, 0\right)$$

and centre $= (0, 0)$

11. Let $\quad A = \{1, 2, \ldots, 11\}$

$\therefore \quad n(A) = 11$ and $B = \{1, 2, \ldots, 10\}$

$\therefore \quad n(B) = 10$

$\therefore$Hence number of onto function

$$= {}^{n(A)}C_{n(B)} \times n(B)! \times n(B)$$

$$= {}^{11}C_{10} \times 10! \times 10$$

$$= (11 \times 10!) \times 10 = 11! \times 10$$

12. Let $\quad p(x) = ax^2 + bx + c \quad \ldots$(i)

Given, constant term 'c' $= 1$

$\therefore \quad p(x) = ax^2 + bx + 1 \quad \ldots$(ii)

Now, by given condition, $p(1) = 2$ (remainder)

$\Rightarrow \quad a + b + 1 = 2$

$\Rightarrow \quad a + b = 1 \quad \ldots$(iii)

and $\quad p(-1) = 4$ (remainder)

$\Rightarrow \quad a - b + 1 = 4$

$\Rightarrow \quad a - b = 3 \quad \ldots$(iv)

On adding Eqs. (iii) and (iv), we get

$2a = 4 \Rightarrow a = 2$ form eys (iii) $b = -1$

On putting the values of a and b in Eq. (ii), we get

$$p(x) = 2x^2 - x + 1 = 0$$

$$\therefore \text{Sum of the roots} = -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)} = \frac{-(-1)}{2}$$

$$= \frac{1}{2}$$

13. $\lim_{x \to 0} \left\{\frac{1}{x^2} + \frac{(2013)^x}{e^x - 1} - \frac{1}{e^x - 1}\right\}$

$$= \lim_{x \to 0} \left\{\frac{1}{x^2} + \frac{(2013)^x - 1}{e^x - 1}\right\}$$

$$= \lim_{x \to 0} \left\{\frac{1}{x^2} + \frac{(2013)^x - 1}{x} \cdot \frac{x}{e^x - 1}\right\}$$

$$= \lim_{x \to 0} \frac{1}{x^2} + \lim_{x \to 0} \frac{(2013)^x - 1}{x} \cdot \lim_{x \to 0} \frac{x}{e^x - 1}$$

$$= +\infty + \log(2013) \cdot 1$$

$$= +\infty$$

14. When eleven apples are distributed among a girl and a boy, then the girl receives atleast 14 apples or the boys receives atleast 8 apples. (by hypothesis)

15. Given, $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$

Now, we have

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2 \quad$ (let $z = x + iy$)

$\Rightarrow \quad |(x + iy) - (2 + 3i)|^2 + |(x + iy) - (3 + 4i)|^2$

$= |(2 + 3i) - (3 + 4i)|^2$

$\Rightarrow |(x - 2) + i(y - 3)|^2 + |(x - 3) + i(y - 4)|^2$

$= |-1 - i|^2$

$\Rightarrow (x - 2)^2 + (y - 3)^2 + (x - 3)^2 + (y - 4)^2 = 1 + 1$

$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 - 6y$

$+ x^2 + 9 - 6x + y^2 + 16 - 8y = 2$

$\Rightarrow \quad 2x^2 + 2y^2 - 10x - 14y + 36 = 0$

$\Rightarrow \quad x^2 + y^2 - 5x - 7y + 18 = 0$

which represent a circle with centre $\left(\frac{5}{2}, \frac{7}{2}\right)$ and

radius $\sqrt{\frac{25}{4} + \frac{49}{4} - 18} = \frac{1}{\sqrt{2}}$

16. Let $a - 2d, a - d, a, a + d, a + 2d$ are in AP,

then $\frac{1}{a - 2d}, \frac{1}{a - d}, \frac{1}{a}, \frac{1}{a, + d}, \frac{1}{a + 2d}$ are in HP.

Given, middle term $= 1$

$\Rightarrow \quad \frac{1}{a} = 1 \Rightarrow a = 1$

and $\quad \frac{\text{Second term}}{\text{Fourth term}} = \frac{2}{1}$

$\Rightarrow \quad \frac{1}{a - d} \times a + d = \frac{2}{1}$

$\Rightarrow \quad a = 3d \Rightarrow d = \frac{1}{3} \quad (\because a = 1)$

$\therefore$Sum of first three terms

$$= \frac{1}{1 - \frac{2}{3}} + \frac{1}{1 - \frac{1}{3}} + 1 = 3 + \frac{3}{2} + 1 = \frac{11}{2}$$

17. Given, $P = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$\Rightarrow \quad P = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

Now, $P^2 = P \cdot P = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix} 1-1 & -1-1 \\ 1+1 & -1+1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$P^3 = P \cdot P^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0-1 & -1-0 \\ 0+1 & -1+0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

Also, given $X = \begin{pmatrix} 1/\sqrt{2} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\therefore \quad P^3 X = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -1-1 \\ 1-1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

18. Given equation is $x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1-4}}{2} \qquad \left(\because x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{1 \pm i\sqrt{3}}{2} = \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$$

$$\Rightarrow -x = \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$$

$$\Rightarrow +x = \omega, -\omega^2$$

Since, (α, β) are the roots of given equation.

Then, $\alpha = -\omega$ and $\beta = -\omega^2$

$$\therefore \alpha^{2013} + \beta^{2013} = -(\omega)^{2013} + (-\omega^2)^{2013}$$

$$= -\omega^{2013} - \omega^{4026}$$

$$= -(\omega^3)^{671} - (\omega^3)^{1342}$$

$$= -(1)^{671} - (1)^{1342} \qquad (\because \omega^3 = 1)$$

$$= -1 - 1 = -2$$

19. Given equation, is $x + y + z = 10$

where, x, y and z are positive integers.

$\therefore$Required number of solutions $= {}^{(10-1)}C_{(3-1)}$

$$= {}^9C_2 = \frac{9 \times 8}{2} = 36$$

20. Let $I = \int_{-1}^{1} \left\{ \frac{x^{2013}}{e^{|x|}(x^2 + \cos x)} + \frac{1}{e^{|x|}} \right\} dx$

$$\Rightarrow \quad I = \int_{-1}^{1} \frac{x^{2013}}{e^{|x|}(x^2 + \cos x)} dx + \int_{-1}^{1} \frac{1}{e^{|x|}} dx$$

Here, $\frac{x^{2013}}{e^{|x|}(x^2 + \cos x)}$ is an odd function

and $\frac{1}{e^{|x|}}$ is an even function.

$$\left\{ \because \int_{-a}^{a} f(x) dx = \begin{cases} 2\int_0^a f(x)\, dx; & f(x) \text{ is even} \\ 0, & f(x) \text{ is odd} \end{cases} \right\}$$

$$\therefore \quad I = 0 + 2\int_0^1 e^{-x} dx = -2(e^{-x})_0^1 = -2(e^{-1} 1)$$

$$= 2(1 - e^{-1})$$

21. Given, $0 \le P, Q \le \frac{\pi}{2}$

and $\qquad \sin P + \cos Q = 2 \qquad \ldots$(i)

This equation hold only

when, $\qquad P = \frac{\pi}{2}$

and $\qquad Q = 0$

LHS $= \sin P + \cos Q = \sin \frac{\pi}{2} + \cos 0$

$$= 1 + 1 = 2 = \text{RHS}$$

$$\therefore \quad \tan\left(\frac{P + Q}{2}\right) = \tan\left(\frac{\frac{\pi}{2} + 0}{2}\right) = \tan\frac{\pi}{4} = 1$$

22. Given, $\quad f(x) = 2^{100} \cdot x + 1$

and $\quad g(x) = 3^{100} \cdot x + 1$

Now, $\quad f\{g(x)\} = x$

$$\Rightarrow \quad f(3^{100}. x + 1) = x$$

$$\Rightarrow \quad 2^{100}\{3^{100} \cdot x + 1\} + 1 = x$$

$$\Rightarrow \quad 6^{100} \cdot x + 2^{100} + 1 = x$$

$$\Rightarrow \quad x(1 - 6^{100}) = (1 + 2^{100})$$

$$\Rightarrow \quad x = \frac{1 + 2^{100}}{1 - 6^{100}}$$

Hence, $fog(x) = x$ represent a singleton set.

23. $\lim_{x\to 0}\left\{\frac{\sqrt{1+x}}{x}-\sqrt{1+\frac{1}{x^2}}\right\}$

$= \lim_{x\to 0}\left\{\frac{\sqrt{1+x}-\sqrt{1+x^2}}{x}\right\}\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}}-\frac{x}{\sqrt{1+x^2}}}{1}$ (Use L-Hospital rule)

$= \frac{1}{2\sqrt{1+0}}-0=\frac{1}{2}$

24. Given

$\cos^2 75° + \cos^2 45° + \cos^2 15° - \cos^2 30° - \cos^2 60°$

$= \left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$= \left(\frac{3+1-2\sqrt{3}}{8}\right)+\frac{1}{2}+\left(\frac{3+1+2\sqrt{3}}{8}\right)-\frac{3}{4}-\frac{1}{4}$

$= \frac{1}{2}-\frac{\sqrt{3}}{4}+\frac{1}{2}+\frac{1}{2}+\frac{\sqrt{3}}{4}-\frac{3}{4}-\frac{1}{4}=\frac{3}{2}-1=\frac{1}{2}$

25. Let $f(\theta)=\sin^6\theta+\cos^6\theta$

$\Rightarrow \quad f(\theta)=(\sin^2\theta)^3+(\cos^2\theta)^3$

$= (\sin^2\theta+\cos^2\theta)$

$(\sin^4\theta+\cos^4\theta-\sin^2\theta\cdot\cos^2\theta)$

$[\because a^3+b^3=(a+b)(a^2+b^2-ab)]$

$= 1\cdot\{(\sin^2\theta+\cos^2\theta)^2-3\sin^2\theta\cdot\cos^2\theta\}$

$= 1\cdot\left(1-\frac{3}{4}\cdot 4\sin^2\theta\cdot\cos^2\theta\right)$

$= 1-\frac{3}{4}(\sin 2\theta)^2 \quad (\because \sin 2A = 2\sin A\cos A)$

$= 1-\frac{3}{8}(1-\cos 4\theta)$

$= 1-\frac{3}{8}+\frac{3}{8}\cos 4\theta$

$f(\theta)=\frac{5}{8}+\frac{3}{8}\cdot\cos 4\theta \quad \ldots(\text{i})$

$\because \quad -1\le\cos 4\theta\le 1$

$\Rightarrow \quad \frac{-3}{8}\le\frac{3}{8}\cos 4\theta\le\frac{3}{8}$

$\Rightarrow \quad \frac{5}{8}-\frac{3}{8}\le\frac{5}{8}+\frac{3}{8}\cos 4\theta\le\frac{5}{8}+\frac{3}{8}$

$\Rightarrow \quad \frac{1}{4}\le f(\theta)\le 1$ [from Eq. (i)]

So, the maximum value is 1 and minimum value is $\frac{1}{4}$.

26. Given, $z=x+iy$

Now, $\frac{z-1}{z-i}=\frac{(x+iy)-1}{(x+iy)-i}$

$= \frac{(x-1)+iy}{x+i(y-1)}\times\frac{x-i(y-1)}{x-i(y-1)}$

$= \frac{x(x-1)+ixy-i(x-1)(y-1)+y(y-1)}{x^2+(y-1)^2}$

$= \frac{(x^2+y^2-x-y)+i(xy-xy+y+x-1)}{(x^2+y^2+1^{-2y})}$

$= \left(\frac{x^2+y^2-x-y}{x^2+y^2-2y+1}\right)+i\left(\frac{x+y-1}{x^2+y^2-2y+1}\right) \ldots(\text{i})$

Given, $\frac{z-1}{z-i}$ is real.

So, its imaginary part should be zero.

i.e., $\quad \frac{x+y-1}{x^2+y^2-2y+1}=0$

$\Rightarrow \quad x+y=1$

$(\because x^2+y^2-2y+1\ne 0)$

which represent a straight line.

27. Given that, a, b and c are in AP

$\therefore \quad 2b=a+c$

$\Rightarrow \quad c=2b-a \quad \ldots(\text{i})$

and equation of straight line is

$ax+2by+c=0$

$\Rightarrow \quad ax+2by+(2b-a)=0$

$\Rightarrow \quad a(x-1)+b(2y+2)=a\cdot 0+b\cdot 0$

On comparing, we get

$x-1=0\Rightarrow x=1$

and $\quad 2y+2=0$

$\Rightarrow \quad y=-1$

Hence, the required fixed point is $(1,-1)$.

28. Given equation is

$$2x^2 + 5xy - 12y^2 = 0 \qquad \dots\text{(i)}$$

$$\Rightarrow \quad 2x^2 + 8xy - 3xy - 12y^2 = 0$$

$$\Rightarrow \quad 2x(x + 4y) - 3y(x + 4y) = 0$$

$$\Rightarrow \quad (x + 4y)(2x - 3y) = 0$$

$$\therefore \quad x + 4y = 0 \text{ and } 2x - 3y = 0$$

which represent a pair of straight lines.

Compair Eq. (i) with $ax^2 + 2hxy + by^2 = 0$

we get $a = 2$ and $b = -12$

$$\because \quad a + b \neq 0$$

So, lines are not perpendicular to each other.

Hence, it is a pair of non-perpendicular intersecting straight lines.

29. Given equaitson of circle is

$$3x^2 + 3y^2 - 9x + 6y + 5 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 3x + 2y + \frac{5}{3} = 0$$

Centre $= \left(\frac{3}{2}, -1\right)$

and radius $= \sqrt{\frac{9}{4} + 1 - \frac{5}{3}}$

$$= \sqrt{\frac{19}{12}} = \frac{1}{2}\sqrt{\frac{19}{3}}$$

We know that, centre of the circle is the mid-point of the diameter.

Lives one and of point of dianetev in (1, 2) Let the other end point of diameter is (h, k).

Then, $\left(\frac{3}{2}, -1\right) = \left(\frac{1+h}{2}, \frac{2+k}{2}\right)$

$$\Rightarrow \quad \frac{1+h}{2} = \frac{3}{2}$$

$$\Rightarrow \quad 1 + h = 3$$

$$\Rightarrow \quad h = 2$$

and $\frac{2+k}{2} = -1$

$$\Rightarrow \quad 2 + k = -2$$

$$\Rightarrow \quad k = -4$$

So, the other end point is $(2, -4)$.

30. Given equation of hyperbola and line are

$\frac{x^2}{9} - \frac{y^2}{25} = 1$ and $y = x$ respectively.

For intersection point of both curve put $y = x$, we get

$$\frac{x^2}{9} - \frac{x^2}{25} = 1$$

$$\Rightarrow \quad x^2 = \frac{9 \times 25}{16} = \left(\frac{15}{4}\right)^2$$

$$\Rightarrow \quad x = \pm\frac{15}{4} \text{ and } y = \pm\frac{15}{4}$$

$\therefore$ Intersetion points $P\left(\frac{15}{4}, \frac{15}{4}\right)$

and $Q\left(\frac{-15}{4}, \frac{-15}{4}\right)$

Since, PQ is major axis, then its length

$$= 2\sqrt{2} \cdot \frac{15}{4} = \frac{15}{\sqrt{2}}$$

and length of minor axis is $\frac{5}{\sqrt{2}}$ (given)

i.e., Major axis, $2a = \frac{15}{\sqrt{2}} \Rightarrow a = \frac{15}{2\sqrt{2}}$

and minor axis, $2b = \frac{5}{\sqrt{2}} \Rightarrow b = \frac{5}{2\sqrt{2}}$

$\therefore$ Eccentricity of an ellipse

$$= \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$= \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{9}$$

31. $\lim_{x \to 0} x \sin e^{(1/x)}$

$$\text{LHL} = f(0 - 0) = \lim_{h \to 0} (-h)\sin e^{(-1/h)}$$

$$= -0 \times \sin(e^{-\infty}) = -0 \times \sin(0) = 0$$

$$= 0 \times (\text{a finite number between } -1 \text{ to } +1)$$

$$= 0 \qquad (\because -1 \leq \sin x \leq 1)$$

$\because$ LHL = RHL

$\therefore \lim_{x \to 0} x \sin(e^{1/x})$ exist and equal to 0

32. $1000\left[\frac{1}{1 \times 2} \times \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{999 \times 1000}\right]$

$$= 1000\left\{\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{999} - \frac{1}{1000}\right)\right]$$

$$= 1000\left(1-\frac{1}{1000}\right)$$

$$= 1000 \times \frac{999}{1000}$$

$$= 999$$

33. Given, $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

The characteristic equation of P is

$$|P - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & -1-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\{(1+\lambda)(2+\lambda)\} = 0$$

$$\Rightarrow (1-\lambda^2)(2+\lambda) = 0$$

$$\Rightarrow 2 - 2\lambda^2 + \lambda - \lambda^3 = 0$$

$$\Rightarrow \lambda^3 + 2\lambda^2 - \lambda - 2 = 0$$

We know that, Caylay Hamilton theorem states that 'Every square matrix satisfy its characteristic equation'.

$$\therefore \quad P^3 + 2P^2 - P - 2I = 0$$

$$\Rightarrow \quad P^3 + 2P^2 = P + 2I$$

34. Let

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Apply $C_1 \to C_1 - bC_3$ and $C_2 \to aC_3 + C_2$

$$\Delta = \begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a \\ 2b-b+a^2b+b^3 & -2a+a-`a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} (1+a^2+b^2) & 0 & -2b \\ 0 & (1+a^2+b^2) & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & (1-a^2-b^2) \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & (1-a^2-b^2) \end{vmatrix}$$

$$= (1+a^2+b^2)^2 \{(1-a^2-b^2+2a^2)+2b^2\}$$

$$= (1+a^2+b^2)^2 (1+a^2+b^2) = (1+a^2+b^2)^3$$

35. Given equation is, $x^2 + ax + b = 0, (b \neq 0)$

its roots are α and β.

Then, sum of roots $= \alpha + \beta = -a$ ...(i)

Product of roots $= \alpha \cdot \beta = b$ ...(ii)

Now,

$$\left(\alpha - \frac{1}{\beta}\right) + \left(\beta - \frac{1}{\alpha}\right) = (\alpha+\beta) - \left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= -a - \frac{(-a)}{b} \text{ [from Eqs.(i) and (ii)]}$$

$$= -a + \frac{a}{b} = \frac{a}{b}(1-b) \quad \text{...(iii)}$$

and $\left(\alpha - \frac{1}{\beta}\right)\left(\beta - \frac{1}{\alpha}\right) = \alpha\beta - 1 - 1 + \frac{1}{\alpha\beta}$

$$= b + \frac{1}{b} - 2 \text{ [from Eq. (ii)]} \quad \text{...(iv)}$$

$$= \frac{1}{b}(b^2 - 2b + 1) = \frac{1}{b}(b-1)^2$$

$\therefore$ Required of quadratic equation whose roots are $\left(\alpha - \frac{1}{\beta}\right)$ and $\left(\beta - \frac{1}{\alpha}\right)$ is

$$x^2 - \left\{\left(\alpha - \frac{1}{\beta}\right) + \left(\beta - \frac{1}{\alpha}\right)\right\}x + \left\{\left(\alpha - \frac{1}{\beta}\right)\left(\beta - \frac{1}{\alpha}\right)\right\} = 0$$

On putting the values from Eqs. (i) and (ii), we get

$$x^2 - \frac{a}{b}(1-b)x + \frac{1}{b}(b-1)^2 = 0$$

$$\Rightarrow bx^2 + a(b-1)x + (b-1)^2 = 0, b \neq 0$$

36. Given,

In ellipse the distance between the foci

= Length of the latusrectum

$$\Rightarrow 2ae = \frac{2b^2}{a}$$

$$\Rightarrow a^2e = b^2 \Rightarrow e = \frac{b^2}{a^2} \quad \text{...(i)}$$

$$\because \quad e^2 = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow e^2 = 1 - e \quad \text{[from Eq. (i)]}$$

$$\Rightarrow e^2 + e - 1 = 0$$

$$\Rightarrow e = \frac{-1 \pm \sqrt{1+4}}{2}$$

(by quadratic formula)

$$\Rightarrow \quad e = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow \quad e = \frac{\sqrt{5}-1}{2} \qquad (\because 1 > e > 0)$$

37. Let $\quad S_1 \equiv x^2 + y^2 - 6x - 8 = 0$

and $\quad S_2 \equiv x^2 + y^2 - 6 = 0$

Now, the equation of the circle passing through the point (1, 1) and the point of intersection of S_1 and S_2 is

$$S_1 + \lambda S_2 = 0$$

$$\Rightarrow \quad (x^2 + y^2 - 6x - 8) + \lambda(x^2 + y^2 - 6) = 0$$

$$\Rightarrow \quad (1+\lambda)x^2 + (1+\lambda)y^2 - 6x$$

$$+ (-8 - 6\lambda) = 0 \quad \ldots\text{(i)}$$

Since, Eq. (i) passes through the point (1, 1).

$\because$ Put $x = 1$, $y = 1$ in Eq. (ii), we get

$$(1+\lambda) + (1+\lambda) - 6 + (-8 - 6\lambda) = 0$$

$$\Rightarrow \quad -4\lambda - 12 = 0$$

$$\Rightarrow \quad \lambda = -3$$

On putting the value of 'λ' in Eq. (i), we get

$$-2x^2 - 2y^2 - 6x + 10 = 0$$

$$\Rightarrow \quad x^2 + y^2 + 3x - 5 = 0$$

38. The distance from the point (–1, 2) of the line which passes through (2, – 3)

$$= \sqrt{(2+1)^2 + (-3-2)^2}$$

$$= \sqrt{9+25} = \sqrt{34} < 8$$

Given that, the distance between the point (– 1, 2) and the line is 8.

But the maximum distance of the line passing through (2, – 3) from (– 1, 2) is $\sqrt{34}$. So, there is no such line possible.

39. Let the six terms of GP are

$$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$$

Now, according to the question

$$\frac{a}{r^5} \cdot \frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 \cdot ar^5 = 1000$$

$$\Rightarrow \quad a^6 = (10)^3 \Rightarrow a^2 = 10 \qquad \ldots\text{(i)}$$

Also, given fourth term $= ar = 1$

$$\Rightarrow \quad a^2 r^2 = 1$$

$$\Rightarrow \quad r^2 = \frac{1}{10} \qquad \ldots\text{(ii)} \qquad \text{[from Eq. (i)]}$$

$\therefore$ Last term $= ar^5 = [a^2(r^2)^5]^{1/2}$

$$= \left[10 \times \frac{1}{10^5}\right]^{1/2} \text{[from Eqs. (i) and (ii)]}$$

$$= \left(\frac{1}{10^4}\right)^{1/2} = \frac{1}{100}$$

40. Given that, (α, β) are roots of the quadratic equation

$$ax^2 + bx + c = 0$$

and $\quad 3b^2 = 16ac \qquad \ldots\text{(i)}$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a} \qquad \ldots\text{(ii)}$$

From Eq. (i), we get

$$3b \cdot b = 16 \cdot a \cdot c$$

$$\Rightarrow \quad 3\left(\frac{b}{a}\right)^2 = 16\left(\frac{c}{a}\right) \text{ (divide by } a^2\text{)}$$

$$\Rightarrow \quad 3(\alpha + \beta)^2 = 16\alpha\beta \text{ [from Eq. (ii)]}$$

$$\Rightarrow 3\alpha^2 + 3\beta^2 + 6\alpha\beta = 16\alpha\beta$$

$$\Rightarrow \quad 3\alpha^2 + 3\beta^2 = 10\alpha\beta$$

$$\Rightarrow \quad 3\left(\frac{\alpha}{\beta}\right) + 3\left(\frac{\beta}{\alpha}\right) = 10$$

Let $\quad \dfrac{\alpha}{\beta} = x$

Then, $\quad 3x + \dfrac{3}{x} = 10$

$$\Rightarrow \quad 3x^2 - 10x + 3 = 0$$

$$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$$

$$\Rightarrow \quad 3x(x-3) - 1(x-3) = 0$$

$$\Rightarrow \quad (x-3)(3x-1) = 0$$

$$\Rightarrow \quad x = \frac{1}{3}, 3 \Rightarrow \frac{\alpha}{\beta} = \frac{1}{3}, 3$$

$$\therefore \quad \beta = 3\alpha \text{ or } \alpha = 3\beta$$

41. Let the relation defined as

$$R = \{(A, B) \mid B = P^{-1}AP\} \qquad \ldots\text{(i)}$$

For reflexive, $\quad A = I^{-1}AI$

$$\Rightarrow \quad (A, A) \in R$$

$\Rightarrow \quad R$ is reflexive

For symmetric

Let $(A, B) \in R$

$\because \quad B = P^{-1}AP$

$\Rightarrow \quad PB = AP \Rightarrow PBP^{-1} = A$

$\Rightarrow \quad A = (P^{-1})^{-1}B(P^{-1})$

$\Rightarrow \quad (B, A) \in R \Rightarrow R$ is symmetric.

For transitive

Let $\quad (A, B) \in R, (B, C) \in R$

$\because \quad A = P^{-1}BP$ and $B = Q^{-1}CQ$

$\Rightarrow \quad A = P^{-1}Q^{-1}CQP = (QP)^{-1}C(QP)$

$\Rightarrow \quad (A, C) \in R$

$\Rightarrow \quad R$ is transitive.

Since, R is reflexive, symmetric and transitive.

So, R is an equivalence relation.

42. Let the given relation defined as

$$R = \{(a, b) \mid \sin^2 a + \cos^2 b = 1\}$$

For reflexive, $\sin^2 a + \cos^2 a = 1$

$$(\because \sin^2 \theta + \cos^2 \theta = 1, \forall \theta \in R)$$

$\Rightarrow \quad {}_aR_a \Rightarrow (a, a) \in R$

$\Rightarrow \quad R$ is reflexive.

For symmetric, $\sin^2 a + \cos^2 b = 1$

$\Rightarrow \quad 1 - \cos^2 a + 1 - \sin^2 b = 1$

$\Rightarrow \quad \sin^2 b + \cos^2 a = 1$

$\Rightarrow \quad bRa$

Hence, R is symmetric.

For transitive

Let aRb, bRc

$\Rightarrow \quad \sin^2 a + \cos^2 b = 1 \quad \ldots(i)$

and $\quad \sin^2 b + \cos^2 c = 1 \quad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$\sin^2 a + (\sin^2 b + \cos^2 b) + \cos^2 c = 2$

$\Rightarrow \quad \sin^2 a + \cos^2 c + 1 = 2$

$\Rightarrow \quad \sin^2 a + \cos^2 c = 1$

Hence, R is transitive also.

Therefore, relation R is an equivalence relation.

43. Given curve is, $x^2 + 4xy + 8y^2 = 64 \quad \ldots(i)$

On differentiating w.r.t x, we get

$$2x + 4\left(y + x\frac{dy}{dx}\right) + 16y\frac{dy}{dx} = 0$$

$\Rightarrow \quad 2x + 4y + (4x + 16y)\dfrac{dy}{dx} = 0$

$\Rightarrow \quad \dfrac{dy}{dx} = -\dfrac{(x + 2y)}{2(x + 4y)}$

Since, tangent are parallel to x-axis only.

i.e., $\quad \dfrac{dy}{dx} = 0$

$\Rightarrow \quad -\dfrac{(x + 2y)}{2(x + 4y)} = 0 \quad \Rightarrow \quad x + 2y = 0$

$\ldots(ii)$

Now, on putting the valus of x from Eqs. (i) in (ii), we get

$$4y^2 - 8y^2 + 8y^2 = 64$$

$\Rightarrow \quad y^2 = 16$

$\Rightarrow \quad y = \pm 4$

From Eq. (ii)

When $\quad y = 4, x = -8$

and when $\quad y = -4, x = 8$

Hence required points are $(-8, 4)$ and $(8, -4)$

44. Given,

$$f(x) = \begin{cases} x^3 - 3x + 2, & x < 2 \\ x^3 - 6x^2 + 9x + 2, & x \geq 2 \end{cases}$$

LHL

$= f(2 - 0) = \lim\limits_{h \to 0} (2 - h)^3 - 3(2 - h) + 2$

$= (2)^3 - 6 + 2 = 8 - 6 + 2 = 4$

RHL $= f(2 + 0) = \lim\limits_{h \to 0} (2 + h)^3 - 6(2 + h)^2$

$+ 9(2 + h) + 2$

$= (2)^3 - 6(2)^2 + 9(2) + 2$

$= 8 - 24 + 18 + 2 = 4$

$\because \quad$ LHL = RHL

$\therefore \quad \lim\limits_{x \to 2} f(x)$ exist

and $\quad f(2) = (2)^3 - 6(2)^2 + 9(2) + 2$

$= 8 - 24 + 18 + 2 = 4$

$\therefore \quad \text{LHL} = \text{RHL} = f(2)$

So, $f(x)$ is continuous at $x = 2$

Now, $\quad f'(x) = \begin{cases} 3x^2 - 3, & x < 2 \\ 3x^2 - 12x + 9, & x \ge 2 \end{cases}$

$\therefore \quad Lf'(2) = 3(2)^2 - 3 = 12 - 3 = 9$

and $\quad Rf'(2) = 3(2)^2 - 12(2) + 9$

$= 12 - 24 + 9 = -3$

$\because \quad Lf'(2) \ne Rf'(2)$

$\therefore f(x)$ is not differentiable at $x = 2$

Hence, f is continuous but not differentiable at $x = 2$.

45. Given,

$$I = \int_0^{\pi/4} (\tan^{n+1} x)\, dx + \frac{1}{2}\int_0^{\pi/2} \tan^{n-1}\left(\frac{x}{2}\right) dx$$

In second integral, put $t = \dfrac{x}{2} \Rightarrow dx = 2\,dt$

$\Rightarrow$ Also, when $x = 0$ then $t = 0$,

When $x = \pi/2$, then $t = \pi/4$

Then, $I = \int_0^{\pi/4} (\tan^{n+1} x)\, dx + \int_0^{\pi/4} \tan^{n-1} t\, dt$

$$I = \int_0^{\pi/4} \tan^{n+1} x \cdot dx + \int_0^{\pi/4} \tan^{n-1} x\, dx$$

$$\left\{\because \int_a^b f(x)\, dx = \int_a^b f(y)\, dy\right\}$$

$\Rightarrow \quad I = \int_0^{\pi/4} (\tan^{n+1} x + \tan^{n-1} x)\, dx$

$\Rightarrow \quad I = \int_0^{\pi/4} \tan^{n-1} x \cdot (\tan^2 x + 1)\, dx$

$\Rightarrow \quad I = \int_0^{\pi/4} \tan^{n-1} x\, (\sec^2 x)\, dx$

Put $\quad t = \tan x$

$\Rightarrow \quad dt = \sec^2 x\, dx$

Also, when $x = 0$, then $t = 0$

when $\quad x = \pi/4$, then $t = 1$

$$I = \int_0^1 t^{n-1} dt = \left[\frac{t^n}{n}\right]_0^1 = \frac{1}{n}$$

46. $\lim\limits_{x \to \infty} \sum\limits_{n=1}^{1000} (-1)^n x^n$

$$= \lim_{x \to \infty} \{-x + x^2 - x^3 + x^4 + \ldots + x^{1000}\}$$

$$= \lim_{x \to \infty} (-x) \cdot \left\{\frac{(-x)^{1000} - 1}{(-x-1)}\right\} = \lim_{x \to \infty} \frac{x^{1001} - x}{x + 1}$$

$$= \lim_{x \to \infty} \frac{x^{1000} - 1}{1 + \left(\frac{1}{x}\right)} = +\infty$$

47. Given

$$f(\theta) = (1 + \sin^2\theta)(2 - \sin^2\theta)$$

$$= 2 + 2\sin^2\theta - \sin^2\theta - \sin^4\theta$$

$$= -\sin^4\theta + \sin^2\theta + 2$$

$$= -(\sin^4\theta - \sin^2\theta - 2)$$

$$= -\left\{\sin^4\theta - \sin^2\theta + \frac{1}{4} - \frac{9}{4}\right\}$$

$$= +\frac{9}{4} - \left(\sin^2\theta - \frac{1}{2}\right)^2 \qquad \ldots(\text{i})$$

$\because \quad -1 \le \sin\theta \le 1 \Rightarrow 0 \le \sin^2\theta \le 1$

$\Rightarrow \quad -\dfrac{1}{2} \le \sin^2\theta - \dfrac{1}{2} \le \dfrac{1}{2}$

$\Rightarrow \quad 0 \le \left(\sin^2\theta - \dfrac{1}{2}\right)^2 \le \dfrac{1}{4}$

$\Rightarrow \quad 0 \ge -\left(\sin^2\theta - \dfrac{1}{2}\right)^2 \ge -\dfrac{1}{4}$

$\Rightarrow \quad \dfrac{9}{4} \ge \dfrac{9}{4} - \left(\sin^2\theta - \dfrac{1}{2}\right)^2 \ge \dfrac{9}{4} - \dfrac{1}{4}$

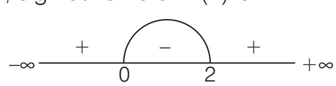
$\Rightarrow \quad 2 \le f(\theta) \le \dfrac{9}{4}$ [from Eq. (i)].

48. Given function is, $f(x) = e^x(x-2)^2$

$\Rightarrow \quad f'(x) = e^x(x-2)^2 + 2(x-2)e^x$

$= e^x(x-2)(x-2+2) = x(x-2)e^x$

Now, sign scheme of $f'(x)$ is



So, f is increasing in $(-\infty, 0)$ and $(2, \infty)$ and decreasing in $(0, 2)$.

49. Let the number of terms, $n = 2m$

Now, by condition

$$\frac{\text{Largest coefficient in } (1+x)^n}{\text{Second largest coefficient in } (1+x)^n} = \frac{11}{10}$$

(given)

$\Rightarrow \quad \dfrac{^{2m}C_m}{^{2m}C_{m-1}} = \dfrac{11}{10}$

$\Rightarrow \quad \dfrac{(m-1)!\,(m+1)!}{m!\,m!} = \dfrac{11}{10}$

$\Rightarrow \quad \dfrac{1}{m}\,\dfrac{m(m-1)!\,(m+1)\,m!}{m!\,m!} = \dfrac{11}{10}$

$\Rightarrow \quad \dfrac{m+1}{m}\,\dfrac{m!\,m!}{m!\,m!} = \dfrac{11}{10}$

$\Rightarrow \quad 10m + 10 = 11\text{m} \Rightarrow m = 10$

$\therefore \quad n = 20$

Hence, total number of term

$$= n + 1 = 20 + 1 = 21$$

50. Let the tine number is in AP are

$$(a-2d),\ (a-d),\ a,\ (a+d),\ (a+2d)$$

where, $d \neq 0$

Given, 1st, 3rd and 4th terms are in GP.

$\Rightarrow \quad a^2 = (a-2d)(a+d)$

$\Rightarrow \quad a^2 = a^2 - 2ad + ad - 2d^2$

$\Rightarrow \quad 2d^2 + ad = 0 \Rightarrow d(2d + a) = 0$

$\because \quad d \neq 0$

$\therefore \quad a + 2d = 0 \Rightarrow a = -2d$

Hence, terms are

$$-4d, -3d, -2d, -d, 0$$

$\therefore$ The fifth term is always 0.

51. Given that,

$$f(x) = e^{(x)^{\frac{1}{x}},\ x>0}$$

Taking log on both sides, we get

$$\log + (x) = (x)^{\frac{1}{x}} = g(x) \text{ (say)} \qquad \ldots\text{(i)}$$

Here, $\quad g(x) = x^{\frac{1}{x}}$

$\Rightarrow \quad \log g(x) = \dfrac{1}{x}\log x$

On differentiating w.r.t. x, we get

$$\frac{1}{g(x)}\cdot g'(x) = \frac{x\cdot\frac{1}{x} - \log x}{x^2}$$

$$= \left(\frac{1-\log x}{x^2}\right)$$

$\Rightarrow \quad g'(x) = x^{\left(\frac{1}{x}-2\right)}(1-\log x)$

For maximum or minimum of $g(x)$ put

$$g'(x) = 0$$

$\Rightarrow \quad x^{\left(\frac{1}{x}-2\right)}(1-\log x) = 0$

$\Rightarrow \quad \log x = 1 = \log e$

$\Rightarrow \quad x = e$

and $\quad g''(x)|_{x=e} > 0$

So, $g(x)$ is minimum at $x = e$

$\therefore g(x)$ increases in $(0, e)$ and decreases in (e, ∞), it will be minimum at either 2 or 5

$\therefore 2^{\frac{1}{2}} > 5^{\frac{1}{5}} \Rightarrow$ Minimum value of $f(x) = e^{(5)^{\frac{1}{5}}}$

52. Given,

$$f(x) = 2|x-1| + |x-2|$$

$$\Rightarrow \quad f(x) = \begin{cases} -2(x-1) - (x-2), & x < 1 \\ 2(x-1) - (x-2), & 1 \le x < 2 \\ 2(x-1) + (x-2), & x \ge 2 \end{cases}$$

$$\Rightarrow \quad f(x) = \begin{cases} -3x + 4, & x < 1 \\ x, & 1 \le x < 2 \\ 3x - 4, & x \ge 2 \end{cases}$$

$$f'(x) = \begin{cases} -3, & x < 1 \\ 1, & 1 \le x < 2 \\ 3, & x \ge 2 \end{cases}$$

So, $f(x)$ will be minimum at $x = 1$

and the minimum value is 1.

53. Given series is,

$$\frac{1}{1\times 2}\,{}^{25}C_0 + \frac{1}{2\times 3}\,{}^{25}C_1 + \frac{1}{3\times 4}\,{}^{25}C_2 + \ldots + \frac{1}{26\times 27}\,{}^{25}C_{25}$$

$$\because \quad \int_0^x (1+x)^{25}dx = \int_0^x [{}^{25}C_0 + {}^{25}C_1 x + {}^{25}C_2 x^2 + \ldots + {}^{25}C_{25}x^{25}]dx$$

On integrating w.r.t. x, taking limits 0 to x, we get

$$\left[\frac{(1+x)^{26}}{26}\right]_0^x$$

$$= \left[{}^{25}C_0 x + {}^{25}C_1\cdot\frac{x^2}{2} + {}^{25}C_2\,\frac{x^3}{3} + \ldots + {}^{25}C_{25}\cdot\frac{x^{26}}{26}\right]_0^x$$

$$\Rightarrow \quad \left\{\frac{1}{26}(1+x)^{26}-\frac{1}{26}\right\}$$

$$= {}^{25}C_0 x + {}^{25}C_1 \cdot \frac{x^2}{2} + \ldots + {}^{25}C_{25} \cdot \frac{x^{26}}{26}$$

Again, integrating w.r.t. x, taking limits 0 to 1, we get

$$\frac{1}{26}\int_0^1 [(1+x)^{26}-1]dx$$

$$= \int_0^1 \left[{}^{25}C_0 x + {}^{25}C_1 \cdot \frac{x^2}{2} + \ldots + {}^{25}C_{25}\frac{x^{26}}{26}\right]dx$$

$$\Rightarrow \quad \frac{1}{26}\left[\frac{(1+x)^{27}}{27}-x\right]_0^1$$

$$= \left[{}^{25}C_0 \frac{x^2}{2} + {}^{25}C_1 \cdot \frac{x^3}{2\times 3} + \ldots + {}^{25}C_{25}\frac{x^{27}}{26\times 27}\right]_0^1$$

$$\Rightarrow \quad \frac{1}{26}\left\{\frac{2^{27}}{27}-1-\frac{1}{27}\right\} = \frac{1}{2}\,{}^{25}C_0 + \frac{1}{2\times 3}\cdot{}^{25}C_1 + \ldots + \frac{1}{26\times 27}\cdot{}^{25}C_{25}$$

$$\therefore \quad \frac{1}{1\times 2}\cdot{}^{25}C_0 + \frac{1}{2\times 3}\,{}^{25}C_1 + \frac{1}{3\times 4}\,{}^{25}C_2 + \ldots + \frac{1}{26\times 27}\cdot{}^{25}C_{25} = \frac{2^{27}-28}{26\times 27}$$

54. Given that, P, Q and R are angles of an isosceles triangle and $\angle P = \frac{\pi}{2}$

$$\therefore \quad Q = R = \frac{\pi}{4} \; (\because P + Q + R = 180^\circ)$$

Now, $\left(\cos\frac{P}{3} - i\sin\frac{P}{3}\right)^3 + (\cos Q + i\sin Q)$
$(\cos R - i\sin R)$
$+ (\cos P - i\sin P)(\cos Q - i\sin Q)$
$(\cos R - i\sin R)$

$$= \left(e^{-i\frac{P}{3}}\right)^3 + e^{iQ}\cdot e^{-iR} + e^{-ip}e^{-iQ}\cdot e^{-iR}$$

$$(\because \cos\theta + i\sin\theta = e^{i\theta})$$

$$= e^{-iP} + e^{i(Q-R)} + e^{-i(P+Q+R)}$$

$$= e^{-i\pi/2} + e^{i(0)} + e^{-i\pi} \qquad (Q = R = \frac{\pi}{2}\; P = \frac{\pi}{2})$$

$$= -i + 1 + (-1) = -i$$

55. Let the function, $f(x) = a^{kx}$

Which define in $f: R \to R$ and injective also.

Now, we have

$$f(x)f(y) = f(x+y)$$

$$\Rightarrow \quad a^{kx}\cdot a^{ky} = a^{k(x+y)}$$

$$\Rightarrow \quad a^{k(x+y)} = a^{k(x+y)}$$

$\because \quad f(x), f(y)$ and $f(z)$ are in GP

$$\therefore \quad f(y)^2 = f(x)\cdot f(z)$$

$$\Rightarrow \quad a^{2ky} = a^{kx}\cdot a^{kz}$$

$$\Rightarrow \quad e^{2ky} = e^{k(x+z)}$$

On comparing, we get

$$2ky = k(x+z) \quad \Rightarrow 2y = x + z$$

$\Rightarrow$ x, y and z are in AP.

56. Let $I = \int_1^2 e^x\left(\log_e x + \frac{x+1}{x}\right)dx$

$$\Rightarrow \quad I = \int_1^2 \left(e^x\cdot\log_e x + e^x + \frac{e^x}{x}\right)dx$$

$$\Rightarrow \quad I = \int_1^2 e^x \log_e x\,dx + \int_1^2 e^x dx + \int_1^2 \frac{e^x}{x}dx$$

$$\Rightarrow \quad I = \int_1^2 e^x \log_e x\,dx + [e^x]_1^2 + [e^x \log_e x]_1^2 - \int_1^2 e^x \log_e x\,dx$$

$$\Rightarrow \quad I = (e^2 - e^1) + (e^2\log_e 2 - 0)$$

$$= e^2(1+\log_e 2) - e$$

57. Given equation is

$$\frac{1}{2}\log_{\sqrt{3}}\left(\frac{x+1}{x+5}\right) + \log_9 (x+5)^2 = 1$$

$$\Rightarrow \quad \frac{1}{2}\cdot\left(\frac{1}{1/2}\right)\log_3\left(\frac{x+1}{x+5}\right) + \frac{1}{2}\log_3(x+5)^2 = 1$$

$$\left(\because \log_{a^n} b = \frac{1}{n}\log_a b\right)$$

$$\Rightarrow \quad \frac{2}{2}\log_3\left(\frac{x+1}{x+5}\right) + \frac{1}{2}\cdot 2\log_3(x+5) = \log_3 3$$

$$\Rightarrow \quad \log_3\left\{\left(\frac{x+1}{x+5}\cdot(x+5)\right)\right\} = \log_3 3$$

$$(\because \log m + \log n = \log mn \text{ and } \log_n n = 1)$$

$$\Rightarrow \quad (x+1) = 3 \quad \Rightarrow x = 2$$

So, only one solution is possible.

58. Given,

$$P = 1 + \frac{1}{2\times 2} + \frac{1}{3\times 2^2} + \dots$$

and $$Q = \frac{1}{1\times 2} + \frac{1}{3\times 4} + \frac{1}{5\times 6} + \dots$$

Now, $$\frac{P}{2} = \frac{(1/2)^1}{1} + \frac{(1/2)^2}{2} + \frac{(1/2)^3}{3} + \dots$$

$$\Rightarrow \quad -p/2 = -\left(\frac{1/2}{1}\right)^1 - \frac{(1/2)^2}{} - \frac{(1/2)^3}{3} \dots$$

$$\Rightarrow \quad -P/2 = \log_e\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad -\frac{P}{2} = \log_e \frac{1}{2}$$

$$\Rightarrow \quad P = 2\log_e 2 \quad \dots(i)$$

Now, $Q = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \dots$

$$\Rightarrow \quad Q = \log_e 2 \quad \dots(ii)$$

Now, from Eqs. (i) and (ii), we get

$$P = 2Q$$

59. Given equation of parabola is

$$y = x^2 - 4x + 5$$

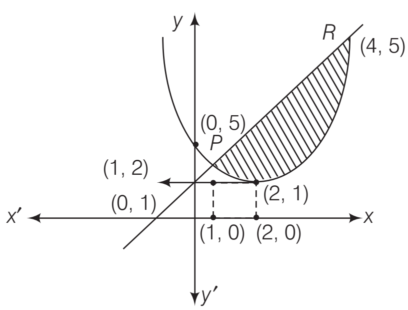
$$\Rightarrow \quad y = (x-2)^2 + 1$$

$$\Rightarrow \quad (x-2)^2 = (y-1) \quad \dots(i)$$

and equation of line is

$$y = x + 1$$

$$\Rightarrow \quad x - y = -1 \quad \dots(ii)$$



On putting the value of $(y-1)$ from Eqs. (ii) in (i), we get

$$(x-2)^2 = x \quad \Rightarrow \quad x^2 + 4 - 4x = x$$

$$\Rightarrow \quad x^2 - 5x + 4 = 0$$

$$\Rightarrow \quad x^2 - 4x - x + 4 = 0$$

$$\Rightarrow \quad x(x-4) - 1(x-4) = 0$$

$$\Rightarrow \quad (x-1)(x-4) = 0 \text{ or } x = 1, 4$$

then from is (ii) $\quad y = 2, 5$

$\therefore$ Required area $= \int_1^4 \{(x+1) - (x^2 - 4x + 5)\}\,dx$

$$= \int_1^4 (-x^2 + 5x - 4)dx = \left[\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right]_1^4$$

$$= \left[-\frac{64}{3} + 40 - 16 + \frac{1}{3} - \frac{5}{2} + 4\right]$$

$$= (-21 - \frac{5}{2} + 28) = \frac{9}{2}$$

60. Given,

$$f(x) = \sin x + 2\cos^2 x, x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

$\therefore \quad f'(x) = \cos x - 4\cos x \cdot \sin x$

and $\quad f''(x) = -\sin x - 4\cos 2x$

For maximum or minimum of $f(x)$

Put $\quad f'(x) = 0$

$$\Rightarrow \quad \cos x - 4\cos x \cdot \sin x = 0$$

$$\Rightarrow \quad \cos x(1 - 4\sin x) = 0$$

$$\Rightarrow \quad \cos x = 0 = \cos\frac{\pi}{2} \text{ and } \sin x \neq \frac{1}{4}$$

$$\because \quad x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \Rightarrow x = \frac{\pi}{2}$$

Now, $$f''\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} - 4\cos\pi$$

$$= -1 + 4 = 3 > 0 \text{ (min)}$$

So, $f(x)$ is minimum at $x = \frac{\pi}{2}$

and its minimum value is

$$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + 2\cos^2\frac{\pi}{2} = 1 - 2\times 0 = 1$$

61. Total sample space, $n(S) = 4^3 \cdot 2^2$ and total number of favourable cases

$$n(E) = ({}^3C_1 \cdot 3 + {}^2C_1 \cdot 1) + 1$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)}$

$$= \frac{{}^3C_1 \cdot 3 + {}^2C_1 \cdot 1 + 1}{4^3 \cdot 2^2} = \frac{3\cdot 3 + 2 + 1}{4^3 \cdot 4}$$

$$= \frac{12}{64\cdot 4} = \frac{3}{64}$$

62. Given differential equation is

$$(y^2+2x)\frac{dy}{dx}=y \Rightarrow \frac{dx}{dy}=\left(\frac{y^2+2x}{y}\right)$$

$$\Rightarrow \quad \frac{dx}{dy}=y+\frac{2x}{y}$$

$$\Rightarrow \quad \frac{dx}{dy}-\frac{2}{y}\cdot x=y$$

$$\text{IF} \;=e^{\int -\frac{2}{y}dy}=e^{-2\log y}=y^{-2}=\frac{1}{y^2}$$

$\therefore$ Complete solution is

$$x\cdot\frac{1}{y^2}=\int y\cdot\frac{1}{y^2}dy+C$$

$$\Rightarrow \quad \frac{x}{y^2}=\int\frac{dy}{y}+C=\log_e y+C$$

$$\Rightarrow \quad x=y^2\log_e y+Cy^2 \qquad \ldots(i)$$

$At + x=1, y=1$ then from Eq. (i), we get

$$1=0+C\Rightarrow C=1$$

$\therefore$ From Eq. (i), we get

$$x=y^2(\log_e y+1)$$

Which is the required solution.

63. Let the general equation of tangent.

Which passes through the point (x, y) is

$$(Y-y)=\frac{dy}{dx}(X-x)$$

$$\Rightarrow \quad Y-y=X\frac{dy}{dx}-x\frac{dy}{dx} \qquad \ldots(i)$$

for length of Y-intercept, put $X=0$ in Eq. (i), we get

$$Y-y=-x\frac{dy}{dx}$$

$$\Rightarrow \quad Y=y-x\frac{dy}{dx} \qquad \ldots(ii)$$

Now,according to the guestion

Y-intercept $=3\times$ ordinate of the point of contact

$$\Rightarrow \quad y-x\frac{dy}{dx}=3y$$

$$\Rightarrow \quad -x\frac{dy}{dx}=2y$$

$$\Rightarrow \quad \int\frac{dy}{y}=-\int\frac{2dx}{x} \quad \text{(on integrating)}$$

$$\Rightarrow \quad \log y=-2\log x+\log C$$

$$\Rightarrow \quad \log y+\log x^2=\log C$$

$\Rightarrow$ $yx^2=C$, where C is a constant.

64. Given differential equation is

$$y\sin\left(\frac{x}{y}\right)dx=\left\{x\sin\left(\frac{x}{y}\right)-y\right\}dy$$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{x\sin\left(\frac{x}{y}\right)-y}{y\sin\left(\frac{x}{y}\right)}=\frac{x}{y}-\frac{1}{\sin\left(\frac{x}{y}\right)} \qquad \ldots(i)$$

On putting $\quad v=\dfrac{x}{y} \quad\Rightarrow\quad x=vy$

$\Rightarrow$ $\dfrac{dx}{dy}=v\cdot 1+y\dfrac{dv}{dy}$ in Eq. (i), we get

$$v+y\frac{dv}{dy}=v-\frac{1}{\sin v}$$

$$\Rightarrow \quad y\frac{dv}{dy}=-\frac{1}{\sin v}$$

$$\Rightarrow \quad -\int\sin v\,dv=\int\frac{dy}{y} \qquad \text{(on integrating)}$$

$$\Rightarrow \quad \cos v=\log y+C$$

$$\Rightarrow \quad \cos\left(\frac{x}{y}\right)=\log y+C \qquad \ldots(i)$$

Given at $x=\dfrac{\pi}{4}$, $y=1$ then from Eq. (i)

$$\Rightarrow \quad \cos\left(\frac{\pi}{4}\right)=\log(1)+C$$

$$\Rightarrow \quad \left(C=\frac{1}{\sqrt{2}}\right)$$

On putting the value of C in Eq. (i), we get

$$\cos\left(\frac{x}{y}\right)=\log_e y+\frac{1}{\sqrt{2}}$$

Which is the required solution. So no option is correct.

65. Given lines are

$$x+y=4 \text{ and } x-y=2$$

On solving these lines, we get

$$x=3 \text{ and } y=1$$

Now, the equation of line which passes through the intersection point $(3, 1)$ having slope

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$ is $(y-1) = \frac{3}{4}(x-3)$

$$\Rightarrow \quad 4y - 4 = 3x - 9$$

$$\Rightarrow \quad 3x - 4y = 5 \quad \ldots(\text{i})$$

Now for the intersection point of the line (i) with parabola $y^2 = 4(x-3)$. Put $y = \left(\frac{3x-5}{4}\right)$,

then we get $\frac{(3x-5)^2}{16} = 4(x-3)$

$$\Rightarrow \quad 9x^2 + 25 - 30x = 64x - 192$$

$$\Rightarrow \quad 9x^2 - 94x + 217 = 0$$

$$\Rightarrow \quad x = \frac{94 \pm \sqrt{8836 - 7812}}{18} = \frac{94 \pm \sqrt{1024}}{18}$$

$$\Rightarrow \quad x = \frac{94 \pm 32}{18} = \frac{126}{18} \text{ or } \frac{62}{18}$$

$$\Rightarrow \quad x_1 = \frac{21}{3} = 7$$

and $\quad x_2 = \frac{31}{9}$

$$\therefore \quad |x_1 - x_2| = \left|7 - \frac{31}{9}\right| = \left|\frac{32}{9}\right| = \frac{32}{9}$$

66. Given, $\sin^2\theta + 3\cos\theta = 2$

$$\Rightarrow \quad 1 - \cos^2\theta + 3\cos\theta = 2$$

$$\Rightarrow \quad \cos^2\theta - 3\cos\theta + 1 = 0$$

$$\Rightarrow \quad \cos\theta = \frac{3 \pm \sqrt{9-4}}{2} \text{ (by quadratic formula)}$$

$$\Rightarrow \quad \cos\theta = \frac{3 \pm \sqrt{5}}{2}$$

Here, $\cos\theta = \frac{3-\sqrt{5}}{2}$

$$\left(\because \cos\theta \neq \frac{3+\sqrt{5}}{2} \quad -1 \le \cos\theta \le 1\right)$$

and $\quad \sec\theta = \frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3-\sqrt{5}}$

$$= \frac{2(3+\sqrt{5})}{4} = \frac{3+\sqrt{5}}{2}$$

Now, $\cos^3\theta + \sec^3\theta = (\cos\theta + \sec\theta)$

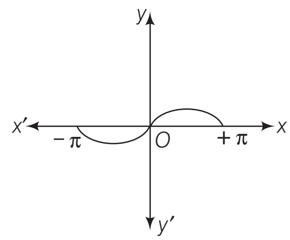
$$(\cos^2\theta + \sec^2\theta - \cos\theta \cdot \sec\theta)$$

$$= \left(\frac{3-\sqrt{5}+3+\sqrt{5}}{2}\right)$$

$$\{(\cos\theta + \sec\theta)^2 - 3\cos\theta \cdot \sec\theta\}$$

$$= 3 \cdot \{(3)^2 - 3\} = 3(9-3) = 3 \times 6 = 18$$

67. Let $I = \int_{-\pi/2}^{\pi/2} [\sin x \cdot \cos x]\,dx = \int_{-\pi/2}^{\pi/2} \left[\frac{1}{2}\sin 2x\right] dx$



Put $\quad \theta = 2x \Rightarrow d\theta = 2dx$

Also, when $\quad x = -\pi/2$, then $\theta = -\pi$

when $\quad x = \frac{\pi}{2}$, then $\theta = \pi$

Then, $\quad I = \frac{1}{2}\int_{-\pi}^{\pi} \left[\frac{1}{2}\sin\theta\right] d\theta$

$$= \frac{1}{2}\left[\int_{-\pi}^{0} (-1)\,dx + \int_{0}^{\pi} (0)\,dx\right]$$

$$= \frac{1}{2}[-x]_{-\pi}^{0} + 0 = -\frac{\pi}{2}$$

68. Given,

$$x = 1 + \frac{1}{2 \times 1!} + \frac{1}{4 \times 2!} + \frac{1}{8 \times 3!} + \ldots$$

$$\Rightarrow \quad x = 1 + \frac{(1/2)^1}{1!} + \frac{(1/2)^2}{2!} + \frac{(1/2)^3}{3!} + \ldots$$

$$\Rightarrow \quad x = e^{1/2} \Rightarrow x^2 = e \quad \ldots(\text{i})$$

and $\quad y = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \ldots$

$$\Rightarrow \quad y = 1 + \frac{(x^2)^1}{1!} + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \ldots$$

$$\Rightarrow \quad y = e^{x^2} = e^e \quad [\text{from Eq. (i)}]$$

Taking log on both sides, we get

$$\log_e y = e$$

69. Given, $P = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$

$$P^2 = P \cdot P = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = P$$

$\therefore \quad P^4 = P^2 = P$

$\Rightarrow P^5 = P^2 = P$

70. Given infinite series is

$$\frac{1^2 + 2^2}{3!} + \frac{1^2 + 2^2 + 3^2}{4!} + \frac{1^2 + 2^2 + 3^2 + 4^2}{5!} + \dots$$

nth term, $t_n = \dfrac{1^2 + 2^2 + 3^2 + 4^2 + \dots + (r+1)^2}{(r+2)!}$

Now , $S_n = \Sigma t_n = \displaystyle\sum_{r=1}^{n} \frac{1^2 + 2^2 + 3^2 + \dots + (r+1)^2}{(r+2)!}$

$$= \sum_{r=1}^{n} \frac{(r+1)(r+2)(2n+3)}{6(r+2)!}$$

$$\left[\because \sum n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \sum_{r=1}^{n} \frac{(2r+3)}{6 \cdot r!} = \frac{1}{6}\sum_{r=1}^{n}\left\{\frac{2}{(r-1)!} + \frac{3}{r!}\right\}$$

$$= \frac{1}{6}\left\{\left(\frac{2}{1} + \frac{3}{1!}\right) + \left(\frac{2}{1!} + \frac{3}{2!}\right) + \left(\frac{2}{2!} + \frac{3}{3!}\right) + \dots\right\}$$

$$= \frac{1}{6}\left\{\left(\frac{2}{1!} + \frac{2}{1!} + \frac{2}{2!} + \dots\right) + \left(\frac{3}{1!} + \frac{3}{2!} + \frac{3}{3!} + \dots\right)\right\}$$

$$= \frac{1}{6}\left\{2\left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots\right) + 3\left(\frac{1}{1!} + \frac{1}{2!} + \dots\right)\right\}$$

$$= \frac{1}{6}\{2e + 3(e-1)\}$$

$$= \frac{1}{6}\{2e + 3e - 3\}$$

$$= \frac{1}{6}(5e - 3) = \frac{5e}{6} - \frac{1}{2}$$

71. Let $I = \displaystyle\int_{\pi/6}^{\pi/3} \frac{(\sin x - x\cos x)}{x(x + \sin x)}dx$

$$\Rightarrow \quad I = \int_{\pi/6}^{\pi/3} \frac{(x + \sin x) - x(1 + \cos x)}{x(x + \sin x)}dx$$

$$\Rightarrow \quad I = \int_{\pi/6}^{\pi/3}\left(\frac{1}{x} - \frac{1 + \cos x}{x + \sin x}\right)dx$$

$$\Rightarrow \quad I = [\log x]_{\pi/6}^{\pi/3} - \int_{\pi/6}^{\pi/3}\frac{1 + \cos x}{x + \sin x}\cdot dx$$

$$\left\{\begin{array}{l}\text{put } t = x + \sin x \\ dt = (1 + \cos x)dx\end{array}\right. \text{in IInd term}$$

$$\Rightarrow \quad I = \left(\log\frac{\pi}{3} - \log\frac{\pi}{6}\right) - \int_{\left(\frac{\pi}{6} + \frac{1}{2}\right)}^{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)}\frac{dt}{t}$$

$$\Rightarrow \quad I = \log 2 - [\log t]_{\left(\frac{\pi}{6} + \frac{1}{2}\right)}^{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow \quad I = \log 2 - \left[\log\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) - \log\left(\frac{\pi}{6} + \frac{1}{2}\right)\right]$$

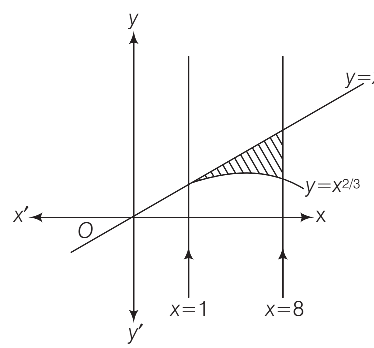
$$\Rightarrow \quad I = \log 2 - \log\left(\frac{2\pi + 3\sqrt{3}}{\pi + 3}\right)$$

$$\left(\because \log m - \log n = \log\frac{m}{n}\right)$$

$$I = \log\left(\frac{2(\pi + 3)}{2\pi + 3\sqrt{3}}\right)$$

$$= \log\left(\frac{2\pi + 6}{2\pi + 3\sqrt{3}}\right)$$

72. Given, $f(x) = x^{2/3}$, $x \geq 0$ and line $y = x$



$\therefore$ Required area $A = \displaystyle\int_{x=1}^{8}(x - x^{2/3})\,dx$

$$= \left[\frac{x^2}{2} - \frac{3}{5}x^{5/3}\right]_1^8 = \left(32 - \frac{3}{5}\times 32\right) - \left(\frac{1}{2} - \frac{3}{5}\right)$$

$$= 32 \times \frac{2}{5} - \frac{(5-6)}{10} = \frac{64}{5} + \frac{1}{10}$$

$$= \frac{128+1}{10} = \frac{129}{10}$$

73. Given function is

$$f(x) = x\left\{\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1}\right\}, x > 1$$

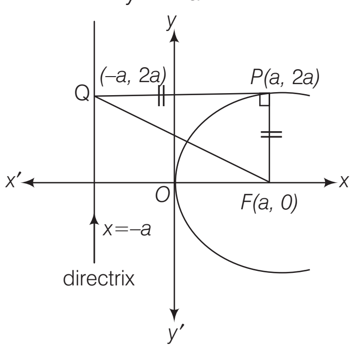
$$= x\left\{\frac{2x}{x^2-1} + \frac{1}{x}\right\} = \frac{2x^2}{x^2-1} + 1$$

$$= \left\{\frac{2}{\left(1 - \frac{1}{x^2}\right)} + 1\right\} > 3 \qquad (\because x > 1)$$

$\therefore \quad f(x) > 3$

74. Equation of parabola is

$$y^2 = 4ax \qquad \ldots(i)$$



Let the parametric coordinate of point P on the parabola is $(a, 2a)$.

Now, $\qquad QF = 2\sqrt{2}a,$

$\qquad PQ = 2a$ and $PF = 2a$

we observe that, $QF^2 = PQ^2 + PF^2$

$\Rightarrow \qquad 8a^2 = 4a^2 + 4a^2 = 8a^2$

So, ΔQPF form a right angle isoceles triangle.

In which, $\angle PQF = \angle PFQ$

$\Rightarrow \qquad \tan\angle PQF = \tan\angle PFQ$

$\Rightarrow \qquad \dfrac{\tan\angle PQF}{\tan\angle PFQ} = 1$

75. Given function is

$$F(x) = \int_0^x \frac{\cos t}{(1+t^2)}dt, 0 \le x \le 2\pi$$

On differentiation w.r.t. x., (apply Leibnitz rule)

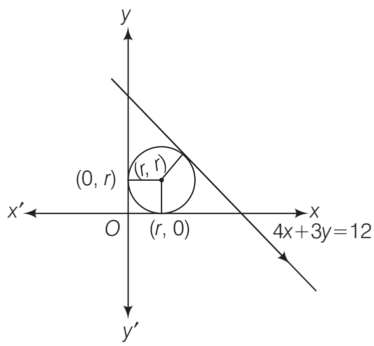
$$F'(x) = \frac{\cos x}{1+x^2} \times 1 = \frac{\cos x}{1+x^2}, \text{ where } (1+x^2) > 0$$

Here, $\cos x > 0 \Rightarrow x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

and $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

So, F is increasing in $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

76. Radius (r) = perpendicular distance on line $4x + 3y = 12$ from centre



$\Rightarrow \qquad r = \dfrac{|4r + 3r - 12|}{\sqrt{16+9}}$

$\Rightarrow \qquad |7r - 12| = 5r$

$\Rightarrow \qquad 7r - 12 = \pm 5r$

$\therefore \qquad 2r = 12 \Rightarrow r = 6$

and $\qquad 12r = 12 \Rightarrow r = 1$

(i) When centre is $(1, 1)$ and radius is 1, then equation of circle is

$$(x-1)^2 + (y-1)^2 = 1$$

$\Rightarrow \qquad x^2 + y^2 - 2x - 2y + 1 = 0$

(ii) When centre is $(6, 6)$ and radius is 2, then equation of circle is

$$(x-6)^2 + (y-6)^2 = 36$$

$\Rightarrow \qquad x^2 + y^2 - 12x - 12y + 36 = 0$

77. Equation of parabola is $y^2 = x$ and line $y = mx$

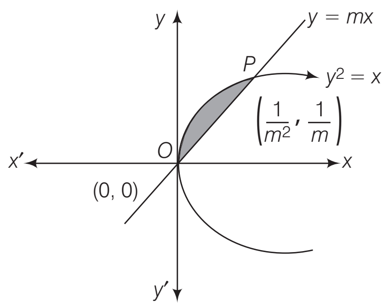
For intersection point of both curves put $x = y^2$, we get

$$y = my^2 \Rightarrow y(my - 1) = 0$$

$$\Rightarrow \quad y = 0 \text{ or } y = \frac{1}{m}$$

Then, $\quad x = 0$ or $x = \dfrac{1}{m^2}$

$\therefore$ Intersection points are (0, 0) and $P\left(\dfrac{1}{m^2}, \dfrac{1}{m}\right)$



$\therefore$ Required area

$$= \int_0^{1/m} \left|\left(\frac{y}{m} - y^2\right)\right| dy = \left|\left[\frac{y^2}{2m} - \frac{y^3}{3}\right]_0^{1/m}\right|$$

$$= \left|\frac{1}{2m^3} - \frac{1}{3m^3}\right| = \left|\frac{1}{6m^3}\right| = \frac{1}{48} \qquad \text{(given)}$$

$$\Rightarrow \quad \frac{1}{6m^3} = \pm \frac{1}{48} \Rightarrow m^3 = \pm 8$$

Now, if $m^3 = 8$

$\Rightarrow \quad m^3 = (2)^3 \Rightarrow m = 2$

If $\quad m^3 = -8$

$\Rightarrow \quad m^3 = (-2)^3 \Rightarrow m = -2$

78. Given equation is

$$x^2 - bx + c = 0 \qquad \ldots\text{(i)}$$

and roots are $\sin\alpha$ and $\cos\alpha$

Also $\quad \sin\alpha + \cos\alpha = b \qquad \ldots$(ii)

and $\quad \sin\alpha \cdot \cos\alpha = c \qquad \ldots$(iii)

$$\because \quad \sin^2\alpha + \cos^2\alpha = (\sin\alpha + \cos\alpha)^2 - 2\sin\alpha \cdot \cos\alpha$$

$\Rightarrow \quad 1 = b^2 - 2c, \qquad$ [from Eqs. (ii) and (iii)]

$$\because \quad -\sqrt{1+1} \le (\sin\alpha + \cos\alpha) \le \sqrt{1+1}$$

$\Rightarrow \quad -\sqrt{2} \le b \le \sqrt{2} \qquad$ [from Eq. (ii)]

and $2\sin\alpha \cdot \cos\alpha = x2c \qquad$ [from Eq. (iii)]

$\Rightarrow \quad \sin 2\alpha = 2c \; (\because -1 \le \sin 2\alpha \le 1)$

$\Rightarrow \quad 2c \le 1 \;\Rightarrow\; c \le \dfrac{1}{2}$

79. Given system of equations is

$$x + y + z = 0$$
$$\alpha x + \beta y + \gamma z = 0$$
$$\alpha^2 x + \beta^2 y + \gamma^2 z = 0$$

The coefficient matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

(i) The system of equations has a unique solution, if α, β and γ are distinct

i.e., $\quad |A| \ne 0$

(ii) The system of equations has infinite number of solutions, if any two of α, β and γ are equal.

i.e., $\quad |A| = 0$

80. we know that,

if $f(-x) = f(x)$, then function is even and if $f(-x) = -f(x)$, then function is odd.

(a) $f(x) = x^3 \sin x$

$$f(-x) = (-x)^3 \sin(-x)$$
$$= -x^3(-\sin x)$$
$$= x^3 \sin x = f(x)$$

So, $f(x)$ is even.

(b) $f(x) = x^2 \cos x$

$$f(-x) = (-x)^2 \cos(-x) = x^2 \cos x = f(x)$$

So, $f(x)$ is even.

(c) $f(x) = e^x x^3 \sin x$

$$f(-x) = e^{-x}(-x)^3 \sin(-x)$$
$$= e^{-x} x^3 \sin x \ne f(x)$$

$\therefore \quad f(x)$ is not even.

(d) $f(x) = x - [x]$

$$f(-x) = (-x) - [-x] \ne f(x)$$

$\therefore f(x)$ is not even.