WBJEE 2012 Question Paper

West Bengal Joint Entrance Examinations

WB JEE

Engineering Entrance Exam

Solved Paper 2012

Physics

1. In a mercury thermometer, the ice point (lower fixed point) is marked as 10° and the steam point (upper fixed point) is marked as 130°. At 40°C temperature, what will this thermometer read?

(a) 78° (b) 66° (c) 62° (d) 58°

2. The magnetic flux linked with a coil satisfies the relation $\phi = 4t^2 + 6t + 9$ Wb, where t is the time in seconds. The emf induced in the coil at $t = 2\,\text{s}$ is

(a) 22 V (b) 18 V (c) 16 V (d) 40 V

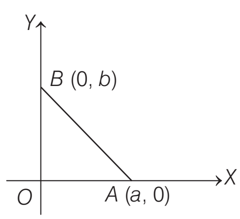
3. Water is flowing through a very narrow tube. The velocity of water below which the flow remains a streamline flow is known as

(a) relative velocity (b) terminal velocity
(c) critical velocity (d) particle velocity

4. If the velocity of light in vacuum is $3 \times 10^8\ \text{ms}^{-1}$, the time taken (in nanosecond) to travel through a glass plate of thickness 10 cm and refractive index 1.5 is

(a) 0.5 (b) 1.0 (c) 2.0 (d) 3.0

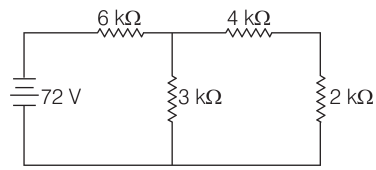
5. A charge $+q$ is placed at the origin O of X-Y axes as shown in the figure. The work done in taking a charge Q from A to B along the straight line AB is



(a) $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{a-b}{ab}\right)$ (b) $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{b-a}{ab}\right)$

(c) $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{b}{a^2}-\dfrac{1}{b}\right)$ (d) $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{a}{b^2}-\dfrac{1}{b}\right)$

6. What current will flow through the 2 kΩ resistor in the circuit shown in the figure?



(a) 3 mA (b) 6 mA (c) 12 mA (d) 36 mA

7. In a region, the intensity of an electric field is given by $\mathbf{E} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ in NC^{-1}. The electric flux through a surface $\mathbf{S} = 10\hat{\mathbf{i}}\ \text{m}^2$ in the region is

(a) $5\text{Nm}^2\text{C}^{-1}$ (b) $10\text{Nm}^2\text{C}^{-1}$
(c) $15\text{Nm}^2\text{C}^{-1}$ (d) $20\text{Nm}^2\text{C}^{-1}$

8. A train approaching a railway platform with a speed of 20 ms^{-1} starts blowing the whistle. Speed of sound in air is 340 ms^{-1}. If the frequency of the emitted sound from the whistle is 640 Hz, the frequency of sound to a person standing on the platform will appear t o be

(a) 600 Hz (b) 640 Hz (c) 680 Hz (d) 720 Hz

9. A straight wire of length 2 m carries a current of 10 A. If this wire is placed in a uniform magnetic field of 0.15 T making an angle of 45° with the magnetic field, the applied force on the wire will be

(a) 1.5 N (b) 3 N (c) $3\sqrt{2}$ N (d) $\dfrac{3}{\sqrt{2}}$ N

10. What is the phase difference between two simple harmonic motions represented by

$x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_2 = A\cos(\omega t)$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

11. Heat is produced at a rate given by H in a resistor, when it is connected across a supply of voltage V. If now the resistance of the resistor is doubled and the supply voltage is made $\frac{V}{3}$, then the rate of production of heat in the resistor will be

(a) $\frac{H}{18}$ (b) $\frac{H}{9}$ (c) $6H$ (d) $18H$

12. Two elements A and B with atomic numbers Z_A and Z_B are used to produce characteristic X-rays with frequencies ν_A and ν_B, respectively. If $Z_A : Z_B = 1:2$, then $\nu_A : \nu_B$ will be

(a) $1:\sqrt{2}$ (b) 1 : 8 (c) 4 : 1 (d) 1 : 4

13. The de-Broglie wavelength of an electron moving with a velocity $\frac{c}{2}$ ($c =$ velocity of light in vacuum) is equal to the wavelength of a photon. The ratio of the kinetic energies of electron and photon is

(a) 1 : 4 (b) 1 : 2 (c) 1 : 1 (d) 2 : 1

14. Two infinite parallel metal planes, contain electric charges with charge densities $+\sigma$ and $-\sigma$ respectively and they are separated by a small distance in air. If the permittivity of air is ε_0, then the magnitude of the field between the two planes with its direction will be

(a) $\frac{\sigma}{\varepsilon_0}$, towards the positively charged plane

(b) $\frac{\sigma}{\varepsilon_0}$, towards the negatively charged plane

(c) $\frac{\sigma}{(2\varepsilon_0)}$, towards the positively charged plane

(d) 0, towards any direction

15. A box of mass 2 kg is placed on the roof of a car. The box would remain stationary untill the car attains a maximum acceleration. Coefficient of static friction between the box and the roof of the car is 0.2 and $g = 10\text{ms}^{-2}$.

This maximum acceleration of the car, for the box to remain stationary, is.

(a) 8 ms^{-2} (b) 6 ms^{-2} (c) 4 ms^{-2} (d) 2 ms^{-2}

16. The dimension of angular momentum is

(a) $[M^0L^1T^{-1}]$ (b) $[M^1L^2T^{-2}]$ (c) $[M^1L^2T^{-1}]$ (d) $[M^2L^1T^{-2}]$

17. If $\mathbf{A} = \mathbf{B} + \mathbf{C}$ have scalar magnitudes of 5, 4, 3 units respectively, then the angle between **A** and **C** is

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\cos^{-1}\left(\frac{4}{5}\right)$

(c) $\frac{\pi}{2}$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$

18. A particle is travelling along a straight line OX. The distance x (in metre) of the particle from O at a time t is given by $x = 37 + 27t - t^3$, where t is time in seconds. The distance of the particle from O when it comes to rest is

(a) 81 m (b) 91 m (c) 101 m (d) 111 m

19. A particle is projected from the ground with a kinetic energy E at an angle of 60° with the horizontal. Its kinetic energy at the highest point of its motion will be

(a) $\frac{E}{\sqrt{2}}$ (b) $\frac{E}{2}$ (c) $\frac{E}{4}$ (d) $\frac{E}{8}$

20. A bullet on penetrating 30 cm into its target loses its velocity by 50%. What additional distance will it penetrate into the target before it comes to rest?

(a) 30 cm (b) 20 cm (c) 10 cm (d) 5 cm

21. When a spring is stretched by 10 cm, the potential energy stored is E. When the spring is stretched by 10 cm more, the potential energy stored in the spring becomes

(a) 2 E (b) 4 E (c) 6 E (d) 10 E

22. Average distance of the Earth from the Sun is L_1. If one year of the Earth $= D$ days, then one year of another planet whose average distance from the Sun is L_2 will be

(a) $D\left(\frac{L_2}{L_1}\right)^{\frac{1}{2}}$ days (b) $D\left(\frac{L_2}{L_1}\right)^{\frac{3}{2}}$ days

(c) $D\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$ days (d) $D\left(\frac{L_2}{L_1}\right)$ days

23. A spherical ball A of mass 4 kg, moving along a straight line strikes another spherical ball B of mass 1 kg at rest. After the collision, A and B move with velocities $v_1 \text{ms}^{-1}$ and $v_2 \text{ms}^{-1}$ respectively, making angles of 30° and 60° with respect to the original direction of motion of A. The ratio $\frac{v_1}{v_2}$ will be

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

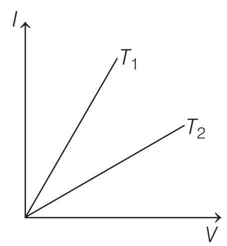
24. The decimal number equivalent to a binary number 1011001 is

(a) 13 (b) 17 (c) 89 (d) 178

25. The frequency of the first overtone of a closed pipe of length l_1 is equal to that of the first overtone of an open pipe of length l_2. The ratio of their lengths $(l_1 : l_2)$ is

(a) 2 : 3 (b) 4 : 5 (c) 3 : 5 (d) 3 : 4

26. The I-V characteristics of a metal wire at two different temperatures (T_1 and T_2) are given in the adjoining figure. Here, we can conclude that



(a) $T_1 > T_2$ (b) $T_1 < T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

27. In a slide caliper, $(m + 1)$ number of vernier divisions is equal to m number of smallest main scale divisions. If d unit is the magnitude of the smallest main scale division, then the magnitude of the vernier constant is

(a) $\frac{d}{(m+1)}$ unit (b) $\frac{d}{m}$ unit

(c) $\frac{md}{(m+1)}$ unit (d) $\frac{(m+1)d}{m}$ unit

28. From the top of a tower, 80 m high from the ground, a stone is thrown in the horizontal direction with a velocity of 8 ms^{-1}. The stone reaches the ground after a time t and falls at a distance of d from the foot of the tower. Assuming, $g = 10 \text{ m/s}^2$, the time t and distance d are given respectively by

(a) 6 s, 64 m (b) 6 s, 48 m

(c) 4 s, 32 m (d) 4 s, 16 m

29. A Wheatstone bridge has the resistance 10Ω, 10Ω, 10Ω and 30Ω in its four arms. What resistance joined in parallel to the 30Ω resistance will bring it to the balanced conditions?

(a) 2Ω (b) 5Ω (c) 10Ω (d) 15Ω

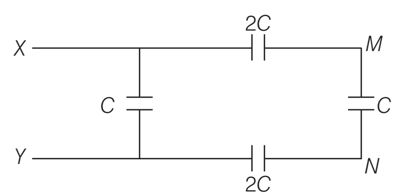
30. An electric bulb marked as 50 W - 200 V is connected across a 100 V supply. The present power of the bulb is

(a) 37.5 W (b) 25 W (c) 12.5 W (d) 10 W

31. A magnetic needle is placed in a uniform magnetic field and is aligned with the field. The needle is now rotated by an angle of 60° and the work done is W. The torque on the magnetic needle at this position is

(a) $2\sqrt{3}W$ (b) $\sqrt{3}W$ (c) $\frac{\sqrt{3}}{2}W$ (d) $\frac{\sqrt{3}}{4}W$

32. In the adjoining figure, the potential difference between X and Y is 60 V. The potential difference between the points M and N will be



(a) 10 V (b) 15 V (c) 20 V (d) 30 V

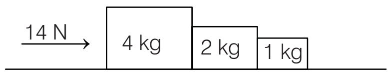
33. A body, when fully immersed in a liquid of specific gravity 1.2 weighs 44 gwt. The same body when fully immersed in water weighs 50 gwt. The mass of the body is

(a) 36 g (b) 48 g (c) 64 g (d) 80 g

34. When a certain metal surface is illuminated with light of frequency ν, the stopping potential for photoelectric current is V_0. When the same surface is illuminated by light of frequency $\frac{\nu}{2}$, the stopping potential is $\frac{V_0}{4}$. The threshold frequency for photoelectric, emission is

(a) $\frac{\nu}{6}$ (b) $\frac{\nu}{3}$ (c) $\frac{2\nu}{3}$ (d) $\frac{4\nu}{3}$

35. Three blocks of masses 4 kg, 2 kg, 1 kg respectively, are in contact on a frictionless table as shown in the figure. If a force of 14 N is applied on the 4 kg block, the contact force between the 4 kg and the 2 kg block will be



(a) 2 N (b) 6 N
(c) 8 N (d) 14 N

36. Let L be the length and d be the diameter of cross-section of a wire. Wires of the same material with different L and d are subjected to the same tension along the length of the wire. In which of the following cases, the extension of wire will be the maximum?

(a) $L = 200$ cm, $d = 0.5$ mm
(b) $L = 300$ cm, $d = 1.0$ mm
(c) $L = 50$ cm, $d = 0.05$ mm
(d) $L = 100$ cm, $d = 0.2$ mm

37. An object placed in front of a concave mirror at a distance of x cm from the pole gives a 3 times magnified real image. If it is moved to a distance of $(x + 5)$ cm, the magnification of the image becomes 2. The focal length of the mirror is

(a) 15 cm (b) 20 cm (c) 25 cm (d) 30 cm

38. 22320 cal of heat is supplied to 100 g of ice at 0° C. If the latent heat of fusion of ice is 80 cal g^{-1} and latent heat of vaporisation of water is 540 cal g^{-1}, the final amount of water thus obtained and its temperature respectively are

(a) 8 g, 100° C (b) 100 g, 90°C
(c) 92 g, 100° C (d) 82 g, 100° C

39. A progressive wave moving along x-axis is represented by $y = A\sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$. The wavelength ($\lambda$) at, which the maximum particle velocity is 3 times the wave velocity is

(a) $\frac{A}{3}$ (b) $\frac{2A}{3\pi}$ (c) $\left(\frac{3}{4}\right)\pi A$ (d) $\left(\frac{2}{3}\right)\pi A$

40. Two radioactive substances A and B have decay constants 5λ and λ, respectively. At $t = 0$, they have the same number of nuclei. The ratio of number of nuclei of A to that of B will be $\left(\frac{1}{e}\right)^2$ after a time interval of

(a) $\frac{1}{\lambda}$ (b) $\frac{1}{2\lambda}$ (c) $\frac{1}{3\lambda}$ (d) $\frac{1}{4\lambda}$

Chemistry

1. Li occupies higher position in the electrochemical series of metals as compared to Cu since

(a) the standard reduction potential of Li^+/Li is lower than that of Cu^{2+}/Cu
(b) the standard reduction potential of Cu^{2+}/Cu is lower, than that of Li/Li
(c) the standard oxidation potential of Li/Li^+ is lower, than that of Cu/Cu^{2+}
(d) Li is smaller in size as compared to Cu

2. $_{11}Na^{24}$ is radioactive and it decays to

(a) $_9F^{20}$ and α-particles (b) $_{13}Al^{24}$ and positron
(c) $_{11}Na^{23}$ and neutron (d) $_{12}Mg^{24}$ and β-particles

3. The paramagnetic behaviour of B_2 is due to the presence of

(a) two unpaired electrons is π_bMO
(b) two unpaired electrons is $\overset{*}{\pi}$ MO
(c) two unpaired electrons is $\overset{*}{\sigma}$ MO
(d) two unpaired electrons is σ_b MO

4. A 100 mL 0.1 = (M) solution of ammonium acetate is diluted by adding 100 mL of water. The pH of the resulting solution will be (pK_a of acetic acid is nearly equal to pK_b of NH_4OH)

(a) 4.9 (b) 5
(c) 7 (d) 10

5. In 2-butene, which one of the following statements is true?

(a) $C_1 — C_2$ bond is a sp^3 - sp^3 - σ-bond
(b) $C_2 — C_3$ bond is a sp^3 - sp^2 - σ-bond
(c) $C_1 — C_2$ bond is a sp^3 - sp^2 - σ-bond
(d) $C_1 — C_2$ bond is a sp^2 - sp^2 - σ-bond

6. The well known compounds, (+)-lactic acid and (−)-lactic acid, have the same molecular formula, $C_3H_6O_3$. The correct relationship between them is

(a) constitutional isomerism
(b) geometrical isomerism
(c) identicalness
(d) optical isomerism

7. The stability of $Me_2C{=}CH_2$ is more than that of $MeCH_2CH = CH_2$ due to

(a) inductive effect of the Me groups
(b) resonance effect of the Me groups
(c) hyperconjugative effect of the Me groups
(d) resonance as well as inductive effect of the Me groups

8. Which one of the following characteristics belongs to an electrophile?

(a) It is any species having electron deficiency which reacts at an electron rich C-centre.
(b) It is any species having electron enrichment, that reacts at an electron deficient C-centre.
(c) It is cationic in nature.
(d) It is anionic in nature.

9. Which one of the following methods is used to prepare Me_3COEt with a good yield?

(a) Mixing EtONa with Me_3CCl
(b) Mixing Me_3CONa with EtCl
(c) Heating a mixture of (1 : 1) EtOH and Me_3COH in the presence of conc. H_2SO_4.
(d) Treatment of Me_3COH with EtMgI

10. 58.4g of NaCl and 180g of glucose were separately dissolved in 1000 mL of water. Identify the correct statement regarding the elevation of boiling point (bp) of the resulting solutions.

(a) NaCl solution will show higher elevation of boiling point.
(b) Glucose solution will show higher elevation of boiling point.
(c) Both the solutions will show equal elevation of boiling point.
(d) The boiling point elevation will be shown by neither of the solutions.

11. Equal weights of CH_4 and H_2 are mixed in an empty container at 25°C. The fraction of the total pressure exerted by H_2 is

(a) 1/9 (b) 1/2 (c) 8/9 (d) 16/17

12. Which of the following will show a negative deviation from Raoult's law?

(a) Acetone-benzene
(b) Acetone-ethanol
(c) Benzene-methanol
(d) Acetone-chloroform

13. In a reversible chemical reaction at equilibrium, if the concentration of any one of the reactants is doubled, then the equilibrium constant will

(a) also be doubled
(b) be halved
(c) remains the same
(d) becomes one-fourth

14. Identify the correct statement from the following in a chemical reaction.

(a) The entropy always increases.
(b) The change in entropy alongwith suitable change in enthalpy decides the rate of reaction.
(c) The enthalpy always decreases.
(d) Both the enthalpy and the entropy remain constant.

15. Which one of the following is wrong about molecularity of a reaction?

(a) It may be whole number or fractional.
(b) It is calculated from reaction mechanism
(c) It is the number of molecules of the reactants taking part in a single step chemical reaction.
(d) It is always equal to the order of elementary reaction.

16. Which of the following does not represent the mathematical expression for the Heisenberg uncertainty principle?

(a) $\Delta x \cdot \Delta P \geq h/(4\pi)$ (b) $\Delta x \cdot \Delta v \geq h/(4\pi m)$
(c) $\Delta E \cdot \Delta t \geq h/(4\pi)$ (d) $\Delta E \cdot \Delta x \geq h/(4\pi)$

17. The stable bivalency of Pb and trivalency of Bi is

(a) due to *d* contraction in Pb and Bi
(b) due to relativistic contraction of the 6s-orbitals of Pb and Bi, leading to inert pair effect
(c) due to screening effect
(d) due to attainment of noble liquid configuration

18. The equivalent weight of $K_2Cr_2O_7$ in acidic medium is expressed in terms of its molecular weight (M) as

(a) $M/3$ (b) $M/4$
(c) $M/6$ (d) $M/7$

19. **Which of the following is correct?**

(a) Radius of $Ca^{2+} < Cl^- < S^{2-}$
(b) Radius of $Cl^- < S^{2-} < Ca^{2+}$
(c) Radius of $S^{2-} = Cl^- = Ca^{2+}$
(d) Radius of $S^{2-} < Cl^- < Ca^{2+}$

20. **CO is practically non-polar since**

(a) the σ-electron drift from C to O is almost nullified by the π-electron drift from O to C
(b) the σ-electron drift from O to C is almost nullified by the π-electron drift from C to O
(c) the bond moment is low
(d) there is a triple bond between C and 0

21. **The number of acidic protons in H_3PO_3 are**

(a) 0 (b) 1 (c) 2 (d) 3

22. **When H_2O_2 is shaken with an acidified solution of $K_2Cr_2O_7$ in the presence of ether, the ethereal layer turns blue due to the formation of**

(a) Cr_2O_3 (b) CrO_4^{2-}
(c) $Cr_2(SO_4)_3$ (d) CrO_5

23. **The state of hybridisation of the central atom and the number of lone pairs over the central atom in $POCl_3$ are**

(a) *sp*, 0 (b) sp^2, 0 (c) sp^3, 0 (d) dsp^2, 1

24. **Upon treatment with I_2 and aqueous NaOH, which of the following compounds will from iodoform?**

(a) $CH_3CH_2CH_2CH_2CHO$
(b) $CH_3CH_2COCH_2CH_3$
(c) $CH_3CH_2CH_2CH_2CH_2OH$
(d) $CH_3CH_2CH_2CH(OH)CH_3$

25. **Upon treatment with $Al(OEt)_3$ followed by usual reactions (work up), CH_3CHO will produce**

(a) only $CH_3COOCH_2CH_3$
(b) a mixutre of CH_3COOH and EtOH
(c) only CH_3COOH
(d) only EtOH

26. **Friedel-Craft's reaction using MeCl and anhydrous $AlCl_3$ will take place most efficiently with**

(a) benzene
(b) nitrobenzene
(c) acetophenone
(d) toluene

27. **Which one of the following properties is exhibited by phenol?**

(a) It is soluble in aq. NaOH and evolves CO_2 with aq. $NaHCO_3$.
(b) It is soluble in aq. NaOH and does not evolve CO_2 with aq. $NaHCO_3$.
(c) It is not soluble in aq. NaOH but evolves CO_2 with aq. $NaHCO_3$.
(d) It is insoluble in aq. NaOH and does not evolve CO_2 with aq. $NaHCO_3$.

28. **The basicity of aniline is weaker in comparison to that of methyl amine due to**

(a) hyperconjugative effect of Me-group in $MeNH_2$
(b) resonance effect of phenyl group in aniline
(c) lower molecular weight of methyl amine as compared to that of aniline
(d) resonance effect of $—NH_2$ group in $MeNH_2$

29. **Under identical conditions, the S_N1 reaction will occur most efficiently with**

(a) *tert*-butyl chloride
(b) 1-chlorobutane
(c) 2-methyl-1-chloropropane
(d) 2-chlorobutane

30. **Identify the method by which Me_3CCO_2H can be prepared.**

(a) Treating 1 mole of MeCOMe with 2 moles of MeMgI.
(b) Treating 1 mole of $MeCO_2Me$ with 3 moles of MeMgI.
(c) Treating 1 mole of MeCHO with 3 moles of MeMgI.
(d) Treating 1 mole of dry ice with 1 mol of Me_3CMgI

31. **20 mL 0.1 (N) acetic acid is mixed with 10 mL 0.1 (N) solution of NaOH. The pH of the resulting solution is (pK_a of acetic acid is 4.74)**

(a) 3.74 (b) 4.74
(c) 5.74 (d) 6.74

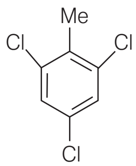
32. **In the brown ring complex $[Fe(H_2O)_5(NO)]SO_4$, nitric oxide behaves as**

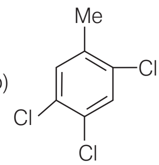
(a) NO^+ (b) netural NO molecule
(c) NO^- (d) NO^{2-}

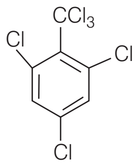
33. **The most contributing tautomeric enol from of $MeCOCH_2CO_2Et$ is**

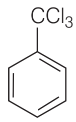
(a) $CH_2 = C(OH)CH_2CO_2Et$
(b) $MeC(OH) = CHCO_2Et$
(c) $MeCOCH = C(OH)OEt$
(d) $CH_2 = C(OH)CH = C(OH)OEt$

34. By passing excess $Cl_2(g)$ in boiling toluene, which one of the following compounds is exclusively formed?

(a) 

(b) 

(c) 

(d) 

35. An equimolar mixture of toluene and chlorobenzene is treated with a mixture of conc. H_2SO_4 and conc. HNO_3. Indicate the correct statement from the following.

(a) *p*-nitrotoluene is formed in excess.
(b) Equimolar amounts of *p*-nitrotoluene and *p*-nitrochlorobenzene are formed.
(c) *p*-nitrochlorobenzene is formed in excess.
(d) *m*-nitrochlorobenzene is formed in excess.

36. Among the following carbocations $Ph_2C^+CH_2Me$ (I), $PhCH_2CH_2CH^+Ph$ (II), Ph_2 $CHCH^+Me$ (III) and $Ph_2C(Me)CH_2$ (IV), the order of stability is

(a) IV > II > I > III (b) I > II > III > IV
(c) II > I > IV > III (d) I > IV > III > II

37. Which of the following is correct?

(a) Evaporation of water causes an increase in disorder of the system.
(b) Melting of ice causes a decrease in randomness of the system.
(c) Condensation of steam causes an increase in disorder of the system.
(d) There is practically no change in the randomness of the system, when water is evaporated.

38. On passing C ampere of current for time t sec through 1 L of 2 (M) $CuSO_4$ solution (Atomic weight of Cu = 63.5), the amount m of Cu (in gram) deposited on cathode will be

(a) $m = Ct/(63.5 \times 96500)$
(b) $m = Ct/(31.25 \times 96500)$
(c) $m = (C \times 96500)/(31.25 \times t)$
(d) $m = (31.25 \times C \times t)/96500$

39. If the first ionisation energy of H atom is 13.6 eV, then the second ionisation energy of He atom is

(a) 27.2 eV (b) 40.8 eV
(c) 54.4 eV (d) 108.8 eV

40. The weight of oxalic acid that will be required to prepare a 1000 mL (N/20) solution is

(a) 126/100 g (b) 63/40 g
(c) 63/20 g (d) 126/20 g

Mathematics

1. If $(\alpha + \sqrt{\beta})$ and $(\alpha - \sqrt{\beta})$ are the roots of the equation $x^2 + px + q = 0$, where α, β, p and q are real, then the roots of the equation $(p^2 - 4q)(p^2x^2 + 4px) - 16q = 0$ are

(a) $\left(\frac{1}{\alpha} + \frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\alpha} - \frac{1}{\sqrt{\beta}}\right)$
(b) $\left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\beta}\right)$ and $\left(\frac{1}{\sqrt{\alpha}} - \frac{1}{\beta}\right)$
(c) $\left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}\right)$ and $\left(\frac{1}{\sqrt{\alpha}} - \frac{1}{\sqrt{\beta}}\right)$
(d) $(\sqrt{\alpha} + \sqrt{\beta})$ and $(\sqrt{\alpha} - \sqrt{\beta})$

2. The number of solutions of the equation $\log_2(x^2 + 2x - 1) = 1$ is

(a) 0 (b) 1
(c) 2 (d) 3

3. The sum of the series

$$1 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \frac{1}{n+1}\,nC_n \text{ is equal to}$$

(a) $\frac{2^{n+1} - 1}{n+1}$ (b) $\frac{3(2^n - 1)}{2n}$
(c) $\frac{2^n + 1}{n+1}$ (d) $\frac{2^n + 1}{2n}$

4. The value of $\sum_{r=2}^{\infty} \frac{1 + 2 + \ldots + (r-1)}{r!}$

(a) e (b) $2e$ (c) $\frac{e}{2}$ (d) $\frac{3e}{2}$

5. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$, $Q = PP^T$, then the value of the determinant of Q is

(a) 2 (b) –2 (c) 1 (d) 0

6. The remainder obtained when $1! + 2! \ldots + 95!$ is divided by 15 is

(a) 14 (b) 3 (c) 1 (d) 0

7. If P, Q and R are angles of ΔPQR, then the value of $\begin{vmatrix} -1 & \cos R & \cos Q \\ \cos R & -1 & \cos P \\ \cos Q & \cos P & -1 \end{vmatrix}$ is equal to

(a) –1 (b) 0 (c) $\frac{1}{2}$ (d) 1

8. The number of real values of α for which the system of equations

$$x + 3y + 5z = \alpha x$$
$$5x + y + 3z = \alpha y$$
$$3x + 5y + z = \alpha z$$

has infinite number of solutions is

(a) 1 (b) 2 (c) 4 (d) 6

9. The total number of injections (one one into mappings) from $\{a_1, a_2, a_3, a_4\}$ to $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ is

(a) 400 (b) 420 (c) 800 (d) 840

10. Let $(1 + x)^{10} = \sum_{r=0}^{10} c_r x^r$ and $(1 + x)^7 = \sum_{r=0}^{7} d_r x^r$. If $P = \sum_{r=0}^{5} c_{2r}$ and $Q = \sum_{r=0}^{3} d_{2r+1}$, then $\frac{P}{Q}$ is equal to

(a) 4 (b) 8 (c) 16 (d) 32

11. Two decks of playing cards are well shuffled and 26 cards are randomly distributed to a player. Then, the probability that the player gets all distinct cards is

(a) ${}^{52}C_{26} / {}^{104}C_{26}$ (b) $2 \times {}^{52}C_{26} / {}^{104}C_{26}$
(c) $2^{13} \times {}^{52}C_{26} / {}^{104}C_{26}$ (d) $2^{26} \times {}^{52}C_{26} / {}^{104}C_{26}$

12. An urn contains 8 red and 5 white balls. Three balls are drawn at random. Then, the probability that balls of both colours are drawn is

(a) $\frac{40}{143}$ (b) $\frac{70}{143}$ (c) $\frac{3}{13}$ (d) $\frac{10}{13}$

13. Two coins are available, one fair and the other two-headed. Choose a coin and toss it once; assume that the unbiased coin is chosen with probability $\frac{3}{4}$. Given that the outcome is head, the probability that the two-headed coin was chosen, is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$
(c) $\frac{1}{5}$ (d) $\frac{2}{7}$

14. Let R be the set of real numbers and the functions $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$. Then, the value of x for which $f(g(x)) = g(f(x))$ is

(a) –1 (b) 0
(c) 1 (d) 2

15. If a, b and c are in arithmetic progression, then the roots of the equation $ax^2 - 2bx + c = 0$ are

(a) 1 and $\frac{c}{a}$ (b) $-\frac{1}{a}$ and $-c$
(c) –1 and $-\frac{c}{a}$ (d) –2 and $-\frac{c}{2a}$

16. The equation $y^2 + 4x + 4y + k = 0$ represents a parabola whose latusrectum is

(a) 1 (b) 2
(c) 3 (d) 4

17. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is equal to

(a) 2 or $-\frac{3}{2}$ (b) –2 or $-\frac{3}{2}$
(c) 2 or $\frac{3}{2}$ (d) –2 or $\frac{3}{2}$

18. If four distinct points $(2k, 3k)$, $(2, 0)$, $(0, 3)$, $(0, 0)$ lie on a circle, then

(a) $k < 0$ (b) $0 < k < 1$
(c) $k = 1$ (d) $k > 1$

19. The line joining $a(b \cos \alpha, b \sin \alpha)$ and $B(a \cos \beta, a \sin \beta)$, where $a \neq b$, is produced to the point $M(x, y)$ so that $AM : MB = b : a$. then, $x \cos \frac{\alpha + \beta}{2} + y \sin \frac{\alpha + \beta}{2}$ is equal to

(a) 0 (b) 1
(c) –1 (d) $a^2 + b^2$

20. Let the foci of the ellipse $\frac{x^2}{9}+y^2=1$ subtend a right angle at a point P. Then, the locus of P is

(a) $x^2+y^2=1$ (b) $x^2+y^2=2$
(c) $x^2+y^2=4$ (d) $x^2+y^2=8$

21. The general solution of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{2x+2y+1}$ is

(a) $\log_e|3x+3y+2|+3x+6y=C$
(b) $\log_e|3x+3y+2|-3x+6y=C$
(c) $\log_e|3x+3y+2|-3x-6y=C$
(d) $\log_e|3x+3y+2|+3x-6y=C$

22. The value of the integral $\int_{\pi/6}^{\pi/2}\left(\frac{1+\sin 2x+\cos 2x}{\sin x+\cos x}\right)dx$ is equal to

(a) 16 (b) 8
(c) 4 (d) 1

23. The value of the integral $\int_0^{\pi/2}\frac{1}{1+(\tan x)^{101}}dx$ is equal to

(a) 1 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

24. The integrating factor of the differential equation $3x\log_e x\frac{dy}{dx}+y=2\log_e x$ is given by

(a) $(\log_e x)^3$ (b) $\log_e(\log_e x)$
(c) $\log_e x$ (d) $(\log_e x)^{1/3}$

25. Number of solutions of the equation $\tan x+\sec x=2\cos x$, $x\in[0,\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

26. The value of the integral $\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx$ is equal to

(a) $\log_e 2$ (b) $\log_e 3$
(c) $\frac{1}{4}\log_e 2$ (d) $\frac{1}{4}\log_e 3$

27. Let $y=\left(\frac{3^x-1}{3^x+1}\right)\sin x+\log_e(1+x)$, $x>-1$. Then, at $x=0$, $\frac{dy}{dx}$ equals

(a) 1 (b) 0
(c) -1 (d) -2

28. Maximum value of the function $f(x)=\frac{x}{8}+\frac{2}{x}$ on the interval $[1, 6]$ is

(a) 1 (b) $\frac{9}{8}$ (c) $\frac{13}{12}$ (d) $\frac{17}{8}$

29. For $-\frac{\pi}{2}<x<\frac{3\pi}{2}$, the value of $\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1+\sin x}\right\}$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) 1 (d) $\frac{\sin x}{(1+\sin x)^2}$

30. The value of the integral $\int_{-2}^{2}(1+2\sin x)e^{|x|}dx$ is equal to

(a) 0 (b) e^2-1 (c) $2(e^2-1)$ (d) 1

31. The maximum value of $|z|$, when the complex number z satisfies the condition $\left|z+\frac{2}{z}\right|=2$ is

(a) $\sqrt{3}$ (b) $\sqrt{3}+\sqrt{2}$
(c) $\sqrt{3}+1$ (d) $\sqrt{3}-1$

32. If $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy)$, where x and y are real, then the ordered pair (x, y) is

(a) $(-3, 0)$ (b) $(0, 3)$
(c) $(0, -3)$ (d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

33. If $\frac{z-1}{z+1}$ is purely imaginary, then

(a) $|z|=\frac{1}{2}$ (b) $|z|=1$ (c) $|z|=2$ (d) $|z|=3$

34. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only one student passed in all the subjects. Then, the number of students failing in all the three subjects

(a) is 12
(b) is 4
(c) is 2
(d) cannot be determined from the given information

35. A vehicle registration number consists of 2 letters of English alphabet followed by 4 digits, where the first digit is not zero. Then, the total number of vehicles with distinct registration numbers is

(a) $26^2 \times 10^4$ (b) ${}^{26}P_2 \times {}^{10}P_4$
(c) ${}^{26}P_2 \times 9 \times {}^{10}P_3$ (d) $26^2 \times 9 \times 10^3$

36. The number of words that can be written using all the letters of the word 'IRRATIONAL' is

(a) $\frac{10!}{(2!)^3}$ (b) $\frac{10!}{(2!)^2}$ (c) $\frac{10!}{2!}$ (d) $10!$

37. Four speakers will address a meeting where speaker Q will always speak P. Then, the number of ways in which the order of speakers can be prepared is

(a) 256 (b) 128 (c) 24 (d) 12

38. The number of diagonals in a regular polygon of 100 sides is

(a) 4950 (b) 4850 (c) 4750 (d) 4650

39. Let the coefficients of powers of x in the 2nd, 3rd and 4th terms in the expansion of $(1+x)^n$, where n is a positive integer, be in arithmetic progression. Then, the sum of the coefficients of odd powers of x in the expansion is

(a) 32 (b) 64 (c) 128 (d) 256

40. Let $f(x) = ax^2 + bx + c$, $g(x) = px^2 + qx + r$ such that $f(1) = g(1)$, $f(2) = g(2)$ and $f(3) - g(3) = 2$. Then $f(4) - g(4)$ is

(a) 4 (b) 5 (c) 6 (d) 7

41. The sum $1 \times 1! + 2 \times 2! + \ldots + 50 \times 50!$ equals

(a) $51!$ (b) $51! - 1$
(c) $51! + 1$ (d) $2 \times 51!$

42. Six numbers are in AP such that their sum is 3. The first term is 4 times the third term. Then, the fifth term is

(a) -15 (b) -3 (c) 9 (d) -4

43. The sum of the infinite series

$$1 + \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12} + \ldots$$

is equal to

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{1}{3}}$

44. The equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$ have a common real root

(a) for no value of a
(b) for exactly one value of a
(c) for exactly two values of a
(d) for exactly three values of a

45. If 64, 27, 36 are the Pth Qth and Rth terms of a GP, then $P + 2Q$ is equal to

(a) R (b) $2R$
(c) $3R$ (d) $4R$

46. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^9 + y^9 + z^9 - \frac{1}{x^9 y^9 z^9}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

47. Let p, q and r be the sides opposite to the angles P, Q and R, respectively in a ΔPQR. If $r^2 \sin P \sin Q = pq$, then the triangle is

(a) equilateral
(b) acute angled but not equilateral
(c) obtuse angled
(d) right angled

48. Let p, q and r be the sides opposite to the angles P, Q and R respectively in a ΔPQR Then, $2pr \sin\left(\frac{P - Q + R}{2}\right)$ equals

(a) $p^2 + q^2 + r^2$ (b) $p^2 + r^2 - q^2$
(c) $q^2 + r^2 - p^2$ (d) $p^2 + q^2 - r^2$

49. Let $P(2, -3)$, $Q(-2, 1)$ be the vertices of the ΔPQR. If the centroid of ΔPQR lies on the line $2x + 3y = 1$, then the locus of R is

(a) $2x + 3y = 9$ (b) $2x - 3y = 7$
(c) $3x + 2y = 5$ (d) $3x - 2y = 5$

50. $\lim_{x \to 0} \frac{\pi^x - 1}{\sqrt{1 + x} - 1}$

(a) does not exist (b) equals $\log_e(\pi^2)$
(c) equals 1 (d) lies between 10 and 11

51. If f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, then

(a) $f(x)$ must be an increasing function
(b) $f(x)$ must be a decreasing function
(c) $|f(x)|$ must be an increasing function
(d) $|f(x)|$ must be a decreasing function

52. Rolle's theorem is applicable in the interval $[-2, 2]$ for the function

(a) $f(x) = x^3$ (b) $f(x) = 4x^4$
(c) $f(x) = 2x^3 + 3$ (d) $f(x) = \pi|x|$

53. The solution of $25\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + y = 0$, $y(0) = 1$, $y(1) = 2e^{1/5}$ is

(a) $y = e^{5x} + e^{-5x}$ (b) $y = (1+x)e^{5x}$
(c) $y = (1+x)e^{x/5}$ (d) $y = (1+x)e^{-x/5}$

54. Let P be the mid point of a chord joining the vertex of the parabola $y^2 = 8x$ to another point on it. Then, the locus of P is

(a) $y^2 = 2x$ (b) $y^2 = 4x$
(c) $\frac{x^2}{4} + y^2 = 1$ (d) $x^2 + \frac{y^2}{4} = 1$

55. The line $x = 2y$ intersects the ellipse $\frac{x^2}{4} + y^2 = 1$ at the points P and Q. The equation of the circle with PQ as diameter is

(a) $x^2 + y^2 = \frac{1}{2}$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = 2$ (d) $x^2 + y^2 = \frac{5}{2}$

56. The eccentric angle in the first quadrant of a point on the ellipse $\frac{x^2}{10} + \frac{y^2}{8} = 1$ at a distance 3 units from the centre of the ellipse is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

57. The transverse axis of a hyperbola is along the X-axis and its length is $2a$. The vertex of the hyperbola bisects the line segment joining the centre and the focus. The equation of the hyperbola is

(a) $6x^2 - y^2 = 3a^2$ (b) $x^2 - 3y^2 = 3a^2$
(c) $x^2 - 6y^2 = 3a^2$ (d) $3x^2 - y^2 = 3a^2$

58. A point moves in such a way that the difference of its distance from two points (8, 0) and (−8, 0) always remains 4. Then, the locus of the point is

(a) a circle (b) a parabola
(c) an ellipse (d) a hyperbola

59. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

(a) 0 (b) 2
(c) 4 (d) 1

60. If a straight line passes through the point (α, β) and the portion of the line intercepted between the axes is divided equally at that point, then $\frac{x}{\alpha} + \frac{y}{\beta}$ is

(a) 0 (b) 1 (c) 2 (d) 4

61. The coefficient of x^{10} in the expansion of $1 + (1+x) + \ldots + (1+x)^{20}$ is

(a) ${}^{19}C_9$ (b) ${}^{20}C_{10}$
(c) ${}^{21}C_{11}$ (d) ${}^{22}C_{12}$

62. The system of linear equations :

$$\lambda x + y + z = 3$$
$$x - y - 2z = 6$$
$$-x + y + z = \mu \text{ has}$$

(a) infinite number of solutions for $\lambda \neq -1$ and all μ
(b) infinite number of solutions for $\lambda = -1$ and $\mu = 3$
(c) no solution for $\lambda \neq -1$
(d) unique solution for $\lambda = -1$ and $\mu = 3$

63. Let A and B be two events with $P(A^C) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^C) = 0.5$. Then, $P(B | A \cup B^C)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

64. Let p, q and r be the altitudes of a triangle with area S and perimeter $2t$. Then, the value of $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$ is

(a) $\frac{S}{t}$ (b) $\frac{t}{S}$ (c) $\frac{S}{2t}$ (d) $\frac{2S}{t}$

65. Let C_1 and C_2 denote the centres of the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$ respectively and let P and Q be their points of intersection.Then, the areas of ΔC_1PQ and ΔC_2PQ are in the ratio

(a) 3 : 1 (b) 5 : 1
(c) 7 : 1 (d) 9 : 1

66. A straight line through the point of intersection of the lines $x+2y=4$ and $2x+y=4$ meets the coordinate axes at A and B. The locus of the mid-point of AB is

(a) $3(x+y)=2xy$ (b) $2(x+y)=3xy$
(c) $2(x+y)=xy$ (d) $x+y=3xy$

67. Let P and Q be the points on the parabola $y^2=4x$, so that the line segment PQ subtends right angle at the vertex. If PQ intersects the axis of the parabola at R, then the distance of the vertex from R is

(a) 1 (b) 2 (c) 4 (d) 6

68. The incentre of an equilateral triangle is (1, 1) and the equation of one side is $3x+4y+3=0$. Then, the equation of the circumcircle of the triangle is

(a) $x^2+y^2-2x-2y-2=0$
(b) $x^2+y^2-2x-2y-14=0$
(c) $x^2+y^2-2x-2y+2=0$
(d) $x^2+y^2-2x-2y+14=0$

69. The value of $\lim_{n\to\infty}\frac{(n!)^{\frac{1}{n}}}{n}$ is

(a) 1 (b) $\frac{1}{e^2}$ (c) $\frac{1}{2e}$ (d) $\frac{1}{e}$

70. The area of the region bounded by the curves $y=x^3$, $y=\frac{1}{x}$, $x=2$ is

(a) $4-\log_e 2$ (b) $\frac{1}{4}+\log_e 2$
(c) $3-\log_e 2$ (d) $\frac{15}{4}-\log_e 2$

71. Let y be the solution of the differential equation $x\frac{dy}{dx}=\frac{y^2}{1-y\log x}$ satisfying $y(1)=1$. Then, y satisfies

(a) $y=x^{y-1}$ (b) $y=x^y$
(c) $y=x^{y+1}$ (d) $y=x^{y+2}$

72. The area of the region, bounded by the curves $y=\sin^{-1}x+x(1-x)$ and $y=\sin^{-1}x-x(1-x)$ in the first quadrant, is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

73. The value of the integral $\int_1^5[|x-3|+|1-x|]\,dx$ is equal to

(a) 4 (b) 8
(c) 12 (d) 16

74. If $f(x)$ and $g(x)$ are twice differentiable functions on (0, 3) satisfying $f''(x)=g''(x)$, $f'(1)=4$, $g'(1)=6$, $f(2)=3$, $g(2)=9$, then $f(1)-g(1)$ is

(a) 4 (b) -4
(c) 0 (d) -2

75. Let $[x]$ denote the greatest integer less than or equal to x, then the value of the integral $\int_{-1}^{1}(|x|-2[x])dx$ is equal to

(a) 3 (b) 2
(c) -2 (d) -3

76. The points representing the complex number z for which $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$ lie on

(a) a circle (b) a straight line
(c) an ellipse (d) a parabola

77. Let a, b, c, p, q and r be positive real numbers such that a, b and c are in GP and $a^p=b^q=c^r$. Then,

(a) p, q, r are in GP (b) p, q, r are in AP
(c) p, q, r are in HP (d) p^2, q^2, r^2 are in AP

78. Let S_k be the sum of an infinite GP series whose first term is k and common ratio is $\frac{k}{k+1}$ $(k>0)$. Then, the value of $\sum_{k=1}^{\infty}\frac{(-1)^k}{S_k}$ is equal to

(a) $\log_e 4$ (b) $\log_e 2-1$
(c) $1-\log_e 2$ (d) $1-\log_e 4$

79. The quadratic equation $2x^2-(a^3+8a-1)x+a^2-4a=0$ possesses roots of opposite sign. Then,

(a) $a\le 0$ (b) $0<a<4$
(c) $4\le a<8$ (d) $a\ge 8$

80. If $\log_e(x^2-16)\le\log_e(4x-11)$, then

(a) $4<x\le 5$ (b) $x<-4$ or $x>4$
(c) $-1\le x\le 5$ (d) $x<-1$ or $x>5$

Answers

Physics

1.	(d)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(a)	**6.**	(a)	**7.**	(d)	**8.**	(c)	**9.**	(d)	**10.**	(b)
11.	(a)	**12.**	(d)	**13.**	(b)	**14.**	(b)	**15.**	(d)	**16.**	(c)	**17.**	(a)	**18.**	(b)	**19.**	(c)	**20.**	(c)
21.	(b)	**22.**	(b)	**23.**	(a)	**24.**	(c)	**25.**	(d)	**26.**	(b)	**27.**	(a)	**28.**	(c)	**29.**	(d)	**30.**	(c)
31.	(b)	**32.**	(d)	**33.**	(d)	**34.**	(b)	**35.**	(b)	**36.**	(c)	**37.**	(d)	**38.**	(a)	**39.**	(d)	**40.**	(b)

Chemistry

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(c)	**8.**	(a)	**9.**	(b)	**10.**	(a)
11.	(c)	**12.**	(d)	**13.**	(c)	**14.**	(b)	**15.**	(a)	**16.**	(d)	**17.**	(b)	**18.**	(c)	**19.**	(a)	**20.**	(a)
21.	(c)	**22.**	(d)	**23.**	(c)	**24.**	(d)	**25.**	(a)	**26.**	(d)	**27.**	(b)	**28.**	(b)	**29.**	(a)	**30.**	(d)
31.	(b)	**32.**	(a)	**33.**	(b)	**34.**	(d)	**35.**	(a)	**36.**	(b)	**37.**	(a)	**38.**	(d)	**39.**	(c)	**40.**	(c)

Mathematics

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(b)	**8.**	(a)	**9.**	(d)	**10.**	(b)
11.	(d)	**12.**	(d)	**13.**	(b)	**14.**	(a)	**15.**	(a)	**16.**	(d)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(d)
21.	(d)	**22.**	(d)	**23.**	(d)	**24.**	(d)	**25.**	(c)	**26.**	(d)	**27.**	(a)	**28.**	(d)	**29.**	(b)	**30.**	(c)
31.	(c)	**32.**	(d)	**33.**	(b)	**34.**	(c)	**35.**	(d)	**36.**	(a)	**37.**	(d)	**38.**	(b)	**39.**	(b)	**40.**	(c)
41.	(b)	**42.**	(d)	**43.**	(b)	**44.**	(b)	**45.**	(c)	**46.**	(c)	**47.**	(d)	**48.**	(b)	**49.**	(a)	**50.**	(b)
51.	(d)	**52.**	(b)	**53.**	(c)	**54.**	(b)	**55.**	(d)	**56.**	(b)	**57.**	(d)	**58.**	(d)	**59.**	(b)	**60.**	(c)
61.	(c)	**62.**	(b)	**63.**	(a)	**64.**	(b)	**65.**	(c)	**66.**	(b)	**67.**	(c)	**68.**	(b)	**69.**	(d)	**70.**	(b)
71.	(b)	**72.**	(c)	**73.**	(c)	**74.**	(b)	**75.**	(a)	**76.**	(a)	**77.**	(c)	**78.**	(d)	**79.**	(b)	**80.**	(c)

Hints & Solutions

Physics

1. $\dfrac{x-10}{130-10} = \dfrac{40}{100}$

$$100x - 1000 = 4800$$

$$100x = 5800 \Rightarrow x = 58°$$

2. Given, $\phi = 4t^2 + 6t + 9\,\text{Wb}$ and $t = 2\,\text{s}$

We know that, $\varepsilon = \dfrac{d\phi}{dt}$

Here, $\dfrac{d\phi}{dt} = 8t + 6 \Rightarrow \left(\dfrac{d\phi}{dt}\right)_{t=2} = 8\times 2 + 6$

$$\varepsilon = 22\,\text{V}$$

3. The velocity of water below, which the flow remains streamline flow is known as critical velocity.

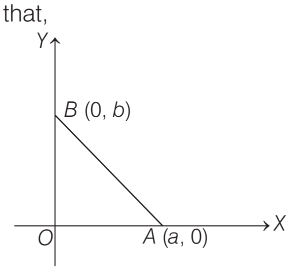
4. Given, $v = 3\times 10^8\,\text{ms}^{-1}$, $d = 10$ m and $\mu = 1.5$

Here, $t = \dfrac{d}{\frac{c}{\mu}} = \dfrac{10\times 10^{-2}}{2\times 10^8}$

$$\left(\therefore \frac{c}{\mu} = \frac{3\times 10^8}{1.5} = 2\times 10^8\,\text{ms}^{-1}\right)$$

$$t = 0.5\times 10^{-9}\,\text{s} = 0.5\,\text{ns}$$

5. We know that,

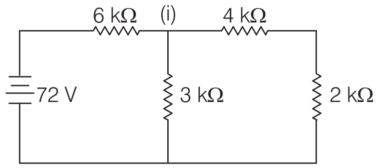


$$W = q\Delta V \text{ and } V = \frac{Q}{4\pi\varepsilon_0 r}$$

Hence, $$W = q\left(\frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 a}\right)$$

$$= \frac{Qq}{4\pi\varepsilon_0}\left(\frac{a-b}{ab}\right)$$

6. From KCL, we have



$$i = \frac{72}{8 \times 10^3} = 9 \times 10^{-3}\text{ A}$$

$$i \times 6 = (9 - i) \times 3$$

$$i = 3\,\text{mA}$$

7. Given, $\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})\text{NC}^{-1}$

and $\mathbf{S} = 10\hat{\mathbf{i}}\ \text{m}^2$

We know that,

$$\phi = \mathbf{E} \cdot \mathbf{S}$$

$$\phi = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (10\hat{\mathbf{i}})$$

$$\phi = 20\text{Nm}^2\text{C}^{-1}$$

8. Given, $v = 340\text{ms}^{-1}$, $u_s = 20\text{ms}^{-1}$ and $v_0 = 640$ Hz

From Doppler's law,

$$\nu = \left(\frac{340}{340 - 20}\right)640$$

$$\nu = 680\text{ Hz}$$

9. Given, $i = 10$ A, $B = 0.15$ T, $\theta = 45°$ and $l = 2$ m

Here,

$$F = ilB\sin\theta = 10 \times 2 \times 0.15 \sin 45°$$

$$= \frac{3}{\sqrt{2}}\text{ N}$$

10. Given,

$$x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right)$$

$$x_2 = A\cos(\omega t) = A\cos t\left(\omega t + \frac{\pi}{2}\right)$$

Phase difference,

$$\Delta\phi = \phi_2 - \phi_1$$

$$\Delta\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi - \pi}{6} = \frac{\pi}{3}$$

11. We know that, $H = \dfrac{V^2}{R}$

Here, in second condition V' becomes $\dfrac{V}{3}$ and R' becomes $2R$.

So, $$H' = \frac{\left(\frac{V}{3}\right)^2}{2R} = \frac{V^2}{18R}$$

$$H' = \frac{H}{18}$$

12. Given, $Z_A : Z_B = 1:2$

We know that,

$$\nu \propto Z^2$$

Hence,

$$\frac{\nu_A}{\nu_B} = \frac{(1)^2}{(2)^2}$$

$$\nu_A : \nu_B = 1:4$$

13. de-Broglie wavelength,

$$\lambda = \frac{h}{mv}$$

Here,

$$\lambda_e = \frac{h}{m_e \frac{c}{2}} \text{ and } \lambda_p = \frac{h}{m_p c}$$

Given, $\lambda_e = \lambda_p$

So, $$\frac{h}{m_e \frac{c}{2}} = \frac{h}{m_p c}$$

$$\frac{m_e}{m_p} = 2$$

Ratio of KE,

$$\frac{K_e}{K_p} = \frac{\frac{1}{2}m_e v_e^2}{\frac{1}{2}m_p v_p^2} = \frac{1}{2}$$

14. $E = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0}$

$E = \dfrac{\sigma}{\varepsilon_0}$, towards the negatively charged plane.

15. Given, $m = 2$ kg, $\mu = 0.2$ and $g = 10\text{ms}^{-2}$

Here, $ma = \mu\, mg$

$\Rightarrow$ $a = \mu\, g$

$\Rightarrow$ $a = 0.2 \times 10$

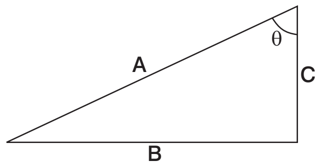
$a = 2\ \text{m/s}^2$

16. Dimension of angular momentum,

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$= [\text{L}][\text{MLT}^{-1}] = [\text{ML}^2\text{T}^{-1}]$$

17. Here, triangle is



$$\theta = \cos^{-1}\left(\frac{|\mathbf{C}|}{|\mathbf{A}|}\right) = \cos^{-1}\left(\frac{3}{5}\right)$$

18. Given, $x = 37 + 27t - t^3$

$$v = \frac{dx}{dt} = 27 - 3t^2$$

According to problem,

$$v = 0 \Rightarrow \quad 27 - 3t^2 = 0$$

Here, $t = 3\text{s}$

$$x = 37 + 27 \times 3 - (3)^2 = 91\text{m}$$

19. At ground, $E = \frac{1}{3}mu^2$

At highest point, $E' = \frac{1}{2}m(u\cos 60°)^2$

$$E' = \frac{E}{4}$$

20. Given, $s = 30$ cm $= 30 \times 10^{-2}$ m and loss in velocity is 50%

Equation of motion

$$v^2 = u^2 + 2as$$

Here, $$\frac{u^2}{4} = u^2 + 2a \times 30 \times 10^{-2} \quad \ldots\text{(i)}$$

$$0 = \frac{u^2}{4} + 3a \times x \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$x = 10\text{ cm}$$

21. For 1st situation,

$$E = \frac{1}{2}k(10 \times 10^{-2})^2 \quad \ldots\text{(i)}$$

For 2nd situation

$$E' = \frac{1}{2}K(20 \times 10^{-2})^2 \quad \ldots\text{(ii)}$$

On comparing Eqs. i and ii, we get

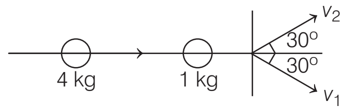
$$E' = 4E$$

22. By Kepler's law,

$$T^2 \propto R^3$$

Hence, $$T = D\left(\frac{L_2}{L_1}\right)^{3/2}$$

23.



Along, Y-axis momentum remains zero.

Here,

$$4v_1 \sin 30° = v_2 \sin 60°$$

$$\frac{v_1}{v_2} = \frac{\sqrt{3}}{4}$$

24. Given, binary number $= 1011001$

Its equivalent decimal number

$$= 2^0 + 0 + 0 + 2^3 + 2^4 + 2^6$$

$$= 1 + 8 + 16 + 64 = 89$$

25. Here, $v_1 = \frac{(2n-1)}{4l_1}v$

$$v_2 = \frac{n}{2l_2}v$$

Hence, $$v_1 = v_2$$

$$\frac{l_1}{l_2} = \frac{3}{4}$$

26. R increases with temperature and slope of V-I graph gives resistance, so $T_1 < T_2$.

27. $(m+1)$ vernier division $= m$ main scale division

One division on vernier scale $= \left(\frac{m}{m+1}\right)$ division on main scale

Vernier constant $= \left(1 - \frac{m}{m+1}\right)d = \frac{d}{m+1}$ unit

28. Given, $h = 80$ m, $v = 8\text{ms}^{-1}$ and $g = 10\text{ms}^{-2}$

Here, $$h = \frac{1}{2}gt^2 \Rightarrow 80 = \frac{1}{2} \times 10 \times t^2$$

$$t = 4\text{ s}$$

$\therefore$ $$x = vt = 8 \times 4 = 32\text{ m}$$

29. In balanced Wheatstone bridge,

$$\frac{1}{10} = \frac{1}{x} + \frac{1}{30}$$

$$x = 15\Omega$$

30. We know that ,

$$R = \frac{V^2}{P}$$

Given, $V = 200\text{V}, P = 50\text{ W}$

and $V' = 100\text{V}$

Hence, $R = \frac{V^2}{P} = \frac{200 \times 200}{50}$

$$P' = \frac{V'^2}{R} = \frac{100 \times 100 \times 50}{200 \times 200} = 12.5\,\text{W}$$

31. Given, work done $= W$ and $\theta = 60°$

We know that,

$$W = MB(1 - \cos\theta)$$
$$W = MB(1 - \cos 60°) = \frac{MB}{2}$$

Hence, $|\tau| = MB \sin 60° = \sqrt{3}W$

32. Capacitors in series,

$$2C_1 \times V_1 = C \times V_2$$
$$2V_1 = V_2 \quad \text{...(i)}$$

and $V_1 + V_2 + V_1 = 60$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$V_2 = 30\,\text{V}$$

33. Balancing equation,

$$W = mg - V\rho g$$

Hence, $44 = m - 1.2V$

$$50 = m - V$$

On solving Eqs. (i) and (ii), we get

$$m = 80\,\text{g}$$

34. In Ist case,

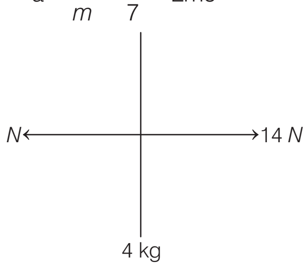
$$h\nu = h\nu_0 + eV_0 \quad \text{...(i)}$$
$$\frac{h\nu}{2} = h\nu_0 + \frac{eV_0}{4} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$V_0 = \frac{\nu}{3}$$

35. We know that, $F = ma$

$$a = \frac{F}{m} = \frac{14}{7} = 2\text{ms}^{-2}$$



Hence, from the figure

$$14 - N = 4a$$
$$14 - N = 8$$
$$N = 6\text{N}$$

36. We know that,

$$\Delta L = \frac{F \times L}{A \times Y} = \frac{4L}{\pi d^2}$$

Here, $\frac{4}{\pi}$ is a constant.

Hence, $\Delta L \propto \frac{L}{d^2}$

37. Given, $v_1 = 3x$ and $v_2 = 2(x + 5)$

In Ist case,

$$\frac{1}{-x} + \frac{1}{-3x} = \frac{1}{f} \quad \text{...(i)}$$
$$\frac{1}{-(x+5)} + \frac{1}{-2(x+5)} = \frac{1}{f} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get $f = 30$ cm

38. Heat required to convert ice to water at 100°C,

$$Q = m \times L + ms\Delta T = 18000\text{ cal}$$

Amount of heat left = 4320 cal

$$m \times L = 4320$$
$$m = 8\,\text{g steam}$$

39. $y = A\sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$

$$v = \frac{2\pi}{\lambda}VA$$
$$(v_p)_{max} = 3v$$
$$\lambda = \frac{2\pi}{3}A$$

40. We know that,

$$N = N_0 e^{-\lambda t}$$

For A,

$$N_A = N_0 e^{-5\lambda t}$$

For B,

$$N_B = N_0 e^{-\lambda t}$$

Given, $\frac{N_A}{N_B} = \frac{1}{e^2}$

$$\frac{N}{N_B} = \frac{1}{e^{4\lambda t}}$$
$$t = \frac{1}{2\lambda}$$

Chemistry

1. In the electrochemical series, metals are arranged in increasing order of their standard reduction potential. The standard reduction potentials of Li^+/Li and Cu^{2+}/Cu are $-3.05\ V$ and $+0.34\ V$ respectively. Therefore, Li occupies higher position in series as compared to Cu.

2. $_{11}Na^{24} \longrightarrow \underset{\text{(Stable)}}{_{12}Mg^{24}} + {}_{-1}\beta^{0}$

3. $B_2 = 5 + 5 = 10e^-$

$$= \sigma 1s^2\, \overset{*}{\sigma} 1s^2, \sigma 2s^2\, \overset{*}{\sigma} 2s^2, \pi 2p_x^1, \pi 2p_y^1$$

Due to the presence of 2 unpaired electrons, in π-bonding orbitals, B_2 shows paramagnetic behaviour.

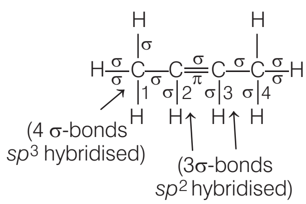
4. Ammonium acetate is a salt of weak acid and weak base. When solutions of such salts diluted resulting pH of solution is calculated from following relationship

$$pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

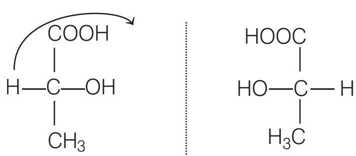
$\therefore \quad pK_a = pK_b$

pH of diluted solution = 7

5. The structure of 2-butene is as

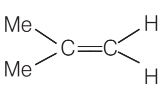


6.

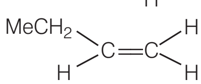


(+) lactic acid (–) lactic acid

Both are optical isomers because they rotate the plane of polarised light in opposite directions.

7. 

has 6-hyper conjugate forms while



has 2-hyperconjugate forms.

Therefore, $Me_2C = CH_2$ is more stable, than $MeCH_2CH = CH_2$.

8. Electrophiles are electron deficient species (neutral or cationic). They attack on electron rich C-atom.

9. $Me_3CONa \xrightarrow[\substack{(1^\circ \text{alkyl halide}) \\ -NaCl}]{EtCl} Me_3COEt$

This reaction is known as Williamson's synthesis.

10. Elevation in boiling point, $\Delta T_b = i \times K_b \times m$

Molality of NaCl solution $= \dfrac{n}{W} \times 1000$

$$= \frac{\frac{58.5}{58.5} \times 1000}{W_{H_2O}} = \frac{1000}{W_{H_2O}}$$

Molality of $C_6H_{12}O_6$ solution

$$= \frac{\frac{180}{180} \times 1000}{W_{H_2O}} = \frac{1000}{W_{H_2O}}$$

Both the solutions have same molarity but the values for NaCl and glucose are 2 and 1 respectively.

$\therefore \Delta T_{b\,(NaCl)} = 2 \times \Delta T_{b(C_6H_{12}O_6)}$

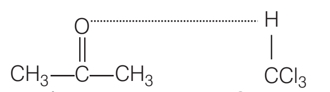
11. Ratio of number of moles of CH_4 and H_2 is

$$CH_4 : H_2 = \frac{x}{16} : \frac{x}{2} = 1 : 8$$

Hence, partial pressure exerted by

$$H_2 = \frac{8}{1+8} \times p_{\text{total}} = \frac{8}{9} \times p_{\text{total}}$$

12.



Acetone Chloroform

Acetone and chloroform will show a negative deviation due to their association after mixing.

13. Equilibrium constant is independent of the concentration of reactants.

14. $\Delta G_{(T,\,p)} = \Delta H - T\Delta S$ determines the existence of a reaction.

15. Molecularity of a reaction can never be fractional. It is always a whole number.

16. From Heisenberg uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \quad \ldots \text{(i)}$$

or $\qquad \Delta x \cdot m\Delta v \geq \dfrac{h}{4\pi}$

or $\qquad \Delta x \cdot \Delta v \geq \dfrac{h}{4\pi m} \qquad \ldots\text{(ii)}$

This principle is also applicable for pairs like energy-time $(\Delta E \cdot \Delta t)$ and angular moment-angle $(\Delta w \cdot \Delta\theta)$ along with position-moment $(\Delta x \cdot \Delta p)$.

Thus, $\qquad \Delta E \cdot \Delta t \geq \dfrac{h}{4\pi} \qquad \ldots\text{(iii)}$

17. ${}_{82}Pb = 6s^2 6p^2$

${}_{83}Bi = 6s^2 6p^3$

Due to inertness of $6s^2$ electrons in Pb and Bi, they show bivalency and trivalency respectively instead of tetra and pentavalency.

18. In acidic medium,

$$K_2\overset{+6}{C}r_2O_7 \longrightarrow Cr^{3+}$$

Equivalent weight of

$$K_2Cr_2O_7 = \frac{\text{Molecular weight}}{\text{Change in oxidation state of Cr}}$$

$$= \frac{M}{2(6-3)} = \frac{M}{6}$$

19.

Species	Atomic Number	Number of electrons
Ca^{2+}	20	$20 - 2 = 18$
Cl^-	17	$17 + 1 = 18$
S^{2-}	16	$16 + 2 = 18$

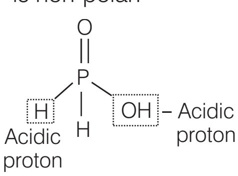
For isoelectronic species, an increase in atomic number, result in decreased size. Thus, order of radius will be

$$Ca^{2+} < Cl^- < S^{2-}$$

20. $C \overset{\overset{\pi}{\longleftarrow}\;\pi}{\underset{\sigma}{\equiv}} O$

In CO molecule, $C—O$ σ moment and $O—C$ π-moment cancels each other. So, that it is non-polar.

21.

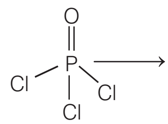


There are two acidic protons in H_3PO_3.

22. $K_2Cr_2O_7 + H_2SO_4 + 4H_2O_2 \longrightarrow$

$$K_2SO_4 + 5H_2O + \underset{\text{Dark blue}}{2CrO_5}$$

23.



$4\,\sigma$-bonds $\Rightarrow sp^3$ hybridisation without lone pair of electrons.

24. $CH_3CH_2CH_2CH(OH)CH_3 \xrightarrow{[O]}$

$$\underset{\text{Keto methyl group}}{CH_3CH_2CH_2COCH_3}$$

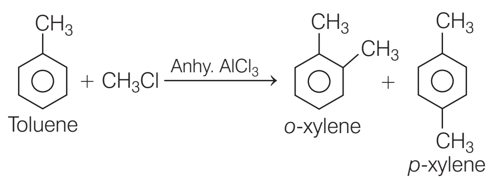
$$CH_3CH_2CH_2\overset{O}{\overset{\|}{C}}—CH_3 + 3I_2 + 4NaOH \longrightarrow$$

$$CH_3CH_2CH_2\overset{O}{\overset{\|}{C}}—ONa + \underset{\text{Iodoform}}{CHI_3}$$

$$+3H_2O + 3NaI$$

25. $\underset{\text{Acetaldehyde}}{2CH_3CHO} \xrightarrow[\text{Tischenko reaction}]{Al(OC_2H_5)_3} \underset{\text{Ethyl acetate}}{CH_3COOCH_2CH_3}$

26.



Friedel-Craft's reaction is an electrophilic substitution reaction and presence of an electron releasing substituent like Me, (makes the benzene nucleus more reactive towards such reactions)

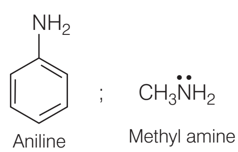
Thus, toluene is most reactive among the given.

27. Phenol is a weak acid. It reacts with NaOH to produce salt.

$$C_6H_5OH + NaOH \longrightarrow \underset{\substack{\text{Sodium}\\\text{phenoxide}}}{C_6H_5ONa} + H_2O$$

But it is not sufficient acidic to evolve CO_2 from $NaHCO_3$ solution.

28.



The lone pair of electrons, present in N-atom in aniline involve in ring resonance. So, it is less basic than methyl amine.

29. S_N1 reaction is most effective in 3° carbon.

$$\underset{tert\text{-butyl chloride}(3°)}{(CH_3)_3—C—Cl} \quad \underset{\text{1-chlorobutane}(1°)}{CH_3CH_2CH_2CH_2Cl}$$

$$\underset{\text{2-methyl-1-chloropropane}(1°)}{Cl—CH_2—CH(CH_3)—CH_3}$$

$$\underset{\text{2-chloro butane }(2°)}{CH_3CH_2CH(Cl)CH_3}$$

30. $\underset{\text{Dry ice}}{O{=}C{=}O} + (CH_3)_3—C—MgI \longrightarrow$

$$O{=}\overset{C(CH_3)_3}{\overset{|}{C}}—OMgI \xrightarrow{HOH}$$

$$(CH_3)_3C—COOH + Mg\begin{matrix} OH \\ I \end{matrix}$$

31.

	CH_3COOH	+	$NaOH$
Initial	$20 \times 0.1 = 2$ mmol		$10 \times 0.1 = 1$ mmol
At time t	$(2-1) = 1$ mmol		$(1-1) = 0$ mmol

$$\longrightarrow \underset{1\text{ m mol}}{CH_3COONa} + H_2O$$

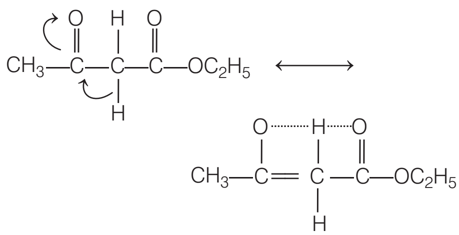
From Henderson's equation

$$pH = pK_a + \log\frac{[CH_3COONa]}{[CH_3COOH]}$$

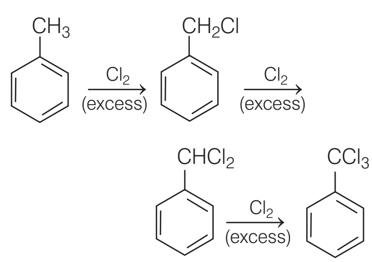
$$= 4.74 + \log\frac{1}{1} = 4.74$$

32. In Fe complex, NO behaves as NO^+

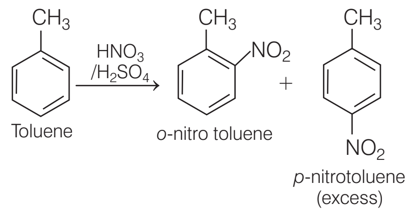
33.

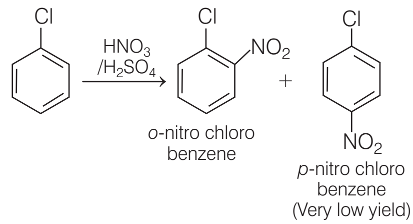


34.



35.





As toluene and chlorobenzene, both can react with mixture of conc. H_2SO_4 and conc. HNO_3, but being more reactive, toluene will yield *p*-nitrotoluene in excess.

36. The order of stability of carbocation is 3° benzylic > 2° benzylic > 2° > 1°.

Thus, the correct order of stability will be

$$\underset{(I)}{Ph—\overset{Ph}{\underset{+}{C}}—CH_2CH_3} > \underset{(II)}{PhCH_2CH_2—\overset{H}{\underset{+}{C}}—Ph}$$

$$> \underset{(III)}{Ph_2CH—\overset{H}{\underset{Me}{C^+}}} > \underset{(IV)}{Ph—\overset{Me}{\underset{Ph}{C}}—CH_2}$$

37. Water evaporates into steam. In this physical state its disorder or randomness increases.

38. $m = \dfrac{ECt}{F} = \dfrac{\frac{63.5}{2} \times C \times t}{96500}$

$$= \frac{31.75 \times C \times t}{96500}$$

39. Second ionisation energy $= 13.6 \times \dfrac{2^2}{1^2} = 54.4$ eV

40. $w = \dfrac{NEV}{1000} = \dfrac{1}{20} \times \dfrac{63 \times 1000}{1000} = \dfrac{63}{20}$ g

Mathematics

1. Since, $(\alpha+\sqrt{\beta})$ and $(\alpha-\sqrt{\beta})$ are the roots of the equation $x^2+px+q=0$

$\therefore$ Sum of roots $=-p$

$\Rightarrow \quad (\alpha+\sqrt{\beta})+(\alpha-\sqrt{\beta})=-p$

$\Rightarrow \quad 2\alpha=-p \Rightarrow \alpha=-\dfrac{p}{2}$

and product of roots $=q$

$(\alpha+\sqrt{\beta})(\alpha-\sqrt{\beta})=q$

$\alpha^2-\beta=q$

$\Rightarrow$

$$\beta=\alpha^2-q=\left(-\frac{p}{2}\right)^2-q=\frac{p^2}{4}-q$$

$\Rightarrow \quad p^2-4q=4\beta$

$\therefore$ The given equation

$(p^2-4q)(p^2x^2+4px)-16q=0$

$4\beta(p^2x^2+4px)-16(\alpha^2-\beta)=0$

$[\alpha^2-\beta=q]$

$\beta(4\alpha^2x^2-8\alpha x)-4(\alpha^2-\beta)=0$

$\alpha^2\beta x^2-2\alpha\beta x+\beta=\alpha^2$

$\Rightarrow \quad (\alpha x\sqrt{\beta}-\sqrt{\beta})^2=\alpha^2$

$\Rightarrow \quad \alpha x\sqrt{\beta}-\sqrt{\beta}=\pm\alpha$

$\therefore \quad x=\dfrac{1}{\alpha}\pm\dfrac{1}{\sqrt{\beta}}$

$\Rightarrow \quad x=\left(\dfrac{1}{\alpha}+\dfrac{1}{\sqrt{\beta}}\right)$ and $\left(\dfrac{1}{\alpha}-\dfrac{1}{\sqrt{\beta}}\right)$

2. Given, $\log_2(x^2+2x-1)=1$

$\Rightarrow \quad \log_2(x^2+2x-1)=\log_2 2$

$\Rightarrow \quad x^2+2x-1=2$

$\Rightarrow \quad x^2+2x-3=0$

$\Rightarrow \quad x^2+3x-x-3=0$

$\Rightarrow \quad x(x+3)-1(x+3)=0$

$\Rightarrow \quad (x+3)(x-1)=0$

$\Rightarrow \quad x=1,-3$

Since, $x=1$ and $x=-3$ are satisfy the given equation therefore the number of solutions of the equation are two.

3. $1+\dfrac{1}{2}\,{}^nC_1+\dfrac{1}{3}\,{}^nC_2+\dots\dfrac{1}{n+1}\,{}^nC_n$

$$=\frac{1}{n+1}\left[(n+1)+\frac{(n+1)n}{2!}+\frac{(n+1)n(n-1)}{3!}+\dots+1\right]$$

$$=\frac{1}{n+1}[{}^{n+1}C_1+{}^{n+1}C_2+\dots{}^{n+1}C_{n+1}]$$

$$=\frac{1}{n+1}[{}^{n+1}C_0+{}^{n+1}C_1+{}^{n+1}C_2+\dots+{}^{n+1}C_{n+1}-1]$$

$[{}^{n+1}C_0=1]$

$$=\frac{1}{n+1}(2^{n+1}-1)$$

4. $\displaystyle\sum_{r=2}^{\infty}\frac{(r-1)r}{2r!}=\sum_{r=2}^{\infty}\frac{1}{2(r-2)!}$

$$=\frac{1}{2}\left[\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\dots\infty\right]=\frac{1}{2}e$$

5. Given, $P=\begin{bmatrix}1&2&1\\1&3&1\end{bmatrix}$

$$\therefore \quad Q=PP^T=\begin{bmatrix}1&2&1\\1&3&1\end{bmatrix}\begin{bmatrix}1&1\\2&3\\1&1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+2\times2+1\times1 & 1\times1+2\times3+1\times1\\1\times1+3\times2+1\times1 & 1\times1+3\times3+1\times1\end{bmatrix}$$

$$=\begin{bmatrix}1+4+1 & 1+6+1\\1+6+1 & 1+9+1\end{bmatrix}=\begin{bmatrix}6&8\\8&11\end{bmatrix}$$

$\therefore$ The determinant of $Q=\begin{bmatrix}6&8\\8&11\end{bmatrix}=66-64=2$

6. Since, $1!=1, 2!=2, 3!=6, 4!=24, 5!=120$ Since, all terms from 5! onwards are divisible by 15 and $1!+2!+3!+4!=33$

$\therefore$ The required remainder after dividing by 15 will be 3.

7. Let $\Delta=\begin{vmatrix}-1 & \cos R & \cos Q\\ \cos R & -1 & \cos P\\ \cos Q & \cos P & -1\end{vmatrix}$

Multiplying C_1 by p and then doing $C_1\to C_1+qC_2+rC_3$, we get

$$\Delta=\frac{1}{p}\begin{vmatrix}-p & \cos R & \cos Q\\ p\cos R & -1 & \cos P\\ p\cos Q & \cos P & -1\end{vmatrix}$$

$$=\frac{1}{p}\begin{vmatrix}-p+q\cos R+r\cos Q & \cos R & \cos Q\\ p\cos R-q+r\cos P & -1 & \cos P\\ p\cos Q+q\cos P-r & \cos P & -1\end{vmatrix}$$

$$=\frac{1}{p}\begin{vmatrix}0 & \cos R & \cos Q\\ 0 & -1 & \cos P\\ 0 & \cos P & -1\end{vmatrix}=0$$

Using the projection formula

$p=q\cos R+r\cos Q$

8. The system of equations are

$$x + 3y + 5z = \alpha x$$
$$5x + y + 3z = \alpha y$$
$$3x + 5y + z = \alpha z$$

$\Rightarrow$ $$(1-\alpha)x + 3y + 5z = 0 \quad \ldots\text{(i)}$$
$$5x + (1-\alpha)y + 3z = 0 \quad \ldots\text{(ii)}$$
$$3x + 5y + (1-\alpha)z = 0 \quad \ldots\text{(iii)}$$

For infinite number of solutions, we must have

$$\begin{vmatrix} 1-\alpha & 3 & 5 \\ 5 & 1-\alpha & 3 \\ 3 & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 9-\alpha & 3 & 5 \\ 9-\alpha & 1-\alpha & 3 \\ 9-\alpha & 5 & 1-\alpha \end{vmatrix} = 0,$$

$C_1 \to C_1 + C_2 + C_3$

Taking $(9-\alpha)$ common from the first column, we get

$$\Rightarrow (9-\alpha)\begin{vmatrix} 1 & 3 & 5 \\ 1 & 1-\alpha & 3 \\ 1 & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow (9-\alpha)\begin{vmatrix} 1 & 3 & 5 \\ 0 & -\alpha-2 & -2 \\ 0 & 2 & -\alpha-4 \end{vmatrix} = 0$$

$$\Rightarrow (9-\alpha)\begin{vmatrix} \alpha+2 & 2 \\ -2 & \alpha+4 \end{vmatrix} = 0$$

$$\Rightarrow (9-\alpha)(\alpha^2 + 6\alpha + 8 + 4) = 0$$

$$\Rightarrow (\alpha - 9)(\alpha^2 + 6\alpha + 12) = 0$$

$\Rightarrow \alpha = 9$ is the only real value of α.

9. Let $A = \{a_1, a_2, a_3, a_4\}$

$B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$

$\therefore n(A) = 4, n(B) = 7$

$\therefore$ The total number of infections $= {}^7P_4$

$$= \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

10. $P = \sum_{r=0}^{5} C_{2r}$

$$= {}^{10}C_0 + {}^{10}C_2 + \ldots\ldots + {}^{10}C_{10} = \frac{2^{10}}{2} = 2^9$$

$$Q = \sum_{r=0}^{3} d_{2r+1} = d_1 + d_3 + d_5 + d_7$$

$$= {}^7C_1 + {}^7C_3 + {}^7C_5 + {}^7C_7 = \frac{2^7}{2} = 2^6$$

$$\therefore \frac{P}{Q} = \frac{2^9}{2^6} = 2^3 = 8$$

11. Since, these are 52 distinct cards in decks and each distinct card is 2 in number. Therefore, 2 decks will also contain only 52 distinct cards two each.

$\therefore$ Probability that the player gets all distinct cards

$$= \frac{{}^{52}C_{26} \times 2^{26}}{{}^{104}C_{26}}$$

12. Total number of selections of three balls in which balls of both colours are drawn

= 2 red balls and 1 white ball

+1 red ball and 2 white balls

$$= {}^8C_2 \times {}^5C_1 + {}^8C_1 \times {}^5C_2 = 140 + 80 = 220$$

$\therefore$ Required probability

$$= \frac{220}{{}^{13}C_3} = \frac{220 \times 6}{13 \times 12 \times 11} = \frac{10}{13}$$

13. Let F denotes fair coin, T denotes two headed, H denotes head occurs

$$\therefore \quad P(F) = \frac{3}{4}, P(T) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore \quad P\left(\frac{T}{H}\right) = \frac{P\left(\frac{H}{T}\right).P(T)}{P\left(\frac{H}{T}\right).P(T) + P\left(\frac{H}{F}\right).P(F)}$$

(By Baye's theorem)

$$= \frac{1 \cdot \frac{1}{4}}{1 \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4}} = \frac{2}{5}$$

14. According to question,

$$f(g(x)) = g(f(x))$$
$$\Rightarrow f(x+1) = g(x^2 + 2x - 3)$$
$$\Rightarrow (x+1)^2 + 2(x+1) - 3 = x^2 + 2x - 3 + 1$$
$$\Rightarrow x^2 + 1 + 2x + 2x + 2 - 3 = x^2 + 2x - 2$$
$$\Rightarrow x^2 + 4x = x^2 + 2x - 2$$
$$\Rightarrow x^2 + 4x - x^2 - 2x + 2 = 0$$
$$\Rightarrow 2x + 2 = 0 \Rightarrow 2x = -2$$
$$\Rightarrow x = -1$$

15. Since, a, b and c are in AP.

$$\therefore \quad 2b = a + c$$

Given, quadratic equation,

$$ax^2 - 2bx + c = 0$$
$$\Rightarrow ax^2 - (a+c)x + c = 0 \quad (2b = a + c)$$
$$\Rightarrow ax^2 - ax - cx + c = 0$$

$\Rightarrow \quad ax(x-1) - c(x-1) = 0$

$\Rightarrow \quad (x-1)(ax-c) = 0$

$\Rightarrow \quad x = 1, \dfrac{c}{a}$

16. $y^2 + 4x + 4y + k = 0$

$y^2 + 2 \times 2y + 4 - 4 + 4x + k = 0$

$$(y+2)^2 = -4x - k + 4$$

$$(y+2)^2 = -4\left(x - \frac{4+k}{4}\right)$$

$\therefore$ Latusrectum $= 4$ units

17. Two circles are orthogonally if and only if

$$2(g_1g_2 + f_1f_2) = c_1 + c_2$$

$\Rightarrow \quad 2[(1\times 0 + k)k] = 6 + k$

$\Rightarrow \quad 2k^2 = 6 + k$

$\Rightarrow \quad 2k^2 - k - 6 = 0$

$\Rightarrow \quad 2k^2 - 4k + 3k - 6 = 0$

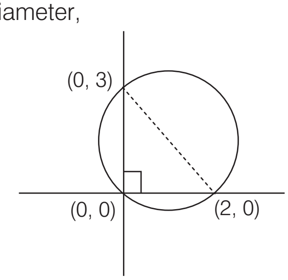
$\Rightarrow \quad 2k(k-2) + 3(k-2) = 0$

$\Rightarrow \quad (k-2)(2k+3) = 0$

$\Rightarrow \quad k = 2, -\dfrac{3}{2}$

18. Since, join of (2, 0) and (0, 3) subtends 90° at (0, 0).

$\Rightarrow$ It is a diameter,



Equation is

$$(x-2)(x-0) + (y-0)(y-3) = 0$$

$$x^2 + y^2 - 2x - 3y = 0$$

$(2k, 3k)$ lies on it

$\Rightarrow \quad 4k^2 + 9k^2 - 4k - 9k = 0$

$\Rightarrow \quad 13k^2 = 13k$

$\Rightarrow \quad k = 1$

Since, $k \neq 0$ otherwise $(2k, 3k)$ will be (0, 0).

19. Since, $AM : BM = b : a$

$\therefore M$ divides AB externally in the ratio $b : a$

$\therefore \quad x = \dfrac{ba\cos\beta - ab\cos\alpha}{b-a} \quad \ldots$(i)

$y = \dfrac{ba\sin\beta - ab\sin\alpha}{b-a} \quad \ldots$(ii)

Divide Eq. (i) by Eq. (ii), we get

$$\frac{x}{y} = \frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha}$$

$$= \frac{2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}}{-2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}}$$

$$\therefore x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0$$

20. $\dfrac{x^2}{9} + \dfrac{y^2}{1} = 1 \Rightarrow e = \sqrt{1 - \dfrac{1}{9}} = \dfrac{2\sqrt{2}}{3}$

Two foci are $(\pm ae, 0)$ i.e., $(\pm 2\sqrt{2}, 0)$

Let $P(h, k)$ be any point on the ellipse

$\therefore \quad \dfrac{k-0}{h-2\sqrt{2}} \times \dfrac{k-0}{h+2\sqrt{2}} = -1$

[From given condition]

$\Rightarrow \quad h^2 - 8 = -k^2$

$\Rightarrow \quad x^2 + y^2 = 8$

21. Given differential equation,

$$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1} \quad \ldots\text{(i)}$$

Put $x + y = v$

$\Rightarrow \quad 1 + \dfrac{dy}{dx} = \dfrac{dv}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{dv}{dx} - 1$

$\therefore$ Eq. (i) becomes,

$$\frac{dy}{dx} = \frac{v+1}{2v+1} \Rightarrow \frac{dv}{dx} - 1 = \frac{v+1}{2v+1}$$

$\Rightarrow \quad \dfrac{dv}{dx} = \dfrac{v+1}{2v+1} + 1$

$\Rightarrow \quad \dfrac{dv}{dx} = \dfrac{v+1+2v+1}{2v+1}$

$\Rightarrow \quad \dfrac{dv}{dx} = \dfrac{3v+2}{2v+1} \Rightarrow \dfrac{2v+1}{3v+2}dv = dx$

$\Rightarrow \quad \dfrac{\left[\dfrac{2}{3}(3v+2) - \dfrac{1}{3}\right]}{3v+2}dv = dx$

$\Rightarrow \quad \left[\dfrac{2}{3} - \dfrac{1}{3} \times \dfrac{1}{(3v+2)}\right]dv = dx$

Integrating both sides, we get

$$\Rightarrow \quad \int\left(\frac{2}{3}-\frac{1}{3}\times\frac{1}{(3v+2)}\right)dv=\int dx+C'$$

Where, c is a constant of integration.

$$\Rightarrow \quad \frac{2}{3}v-\frac{1}{3}\frac{\log(3v+2)}{3}=x+C'$$

$$\Rightarrow \frac{2}{3}(x+y)-\frac{1}{9}\log(3x+3y+2)=x+C'$$

$$\Rightarrow \quad 6x+6y-\log(3x+3y+2)=9x+C$$

$$\Rightarrow \quad 3x-6y+\log(3x+3y+2)=C$$

22. Let $I=\int_{\pi/6}^{\pi/2}\left(\frac{1+\sin 2x+\cos 2x}{\sin x+\cos x}\right)dx$

$$=\int_{\pi/6}^{\pi/2}\left(\frac{1+2\sin x\cos x+2\cos^2 x-1}{(\sin x+\cos x)}\right)dx$$

$$=\int_{\pi/6}^{\pi/2}\frac{2\cos x(\sin x+\cos x)}{(\sin x+\cos x)}dx$$

$$=\int_{\pi/6}^{\pi/2}2\cos x dx=2[\sin x]_{\pi/6}^{\pi/2}$$

$$=2\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{6}\right)=2\left(1-\frac{1}{2}\right)=2\times\frac{1}{2}=1$$

23. Let $I=\int_0^{\frac{\pi}{2}}\frac{1}{1+(\tan x)^{101}}dx$

$$=\int_0^{\frac{\pi}{2}}\frac{dx}{1+\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}^{101}}=\int_0^{\pi/2}\frac{dx}{1+(\cot x)^{101}}$$

$$=\int_0^{\pi/2}\frac{\tan x^{101}}{\tan x^{101}+1}dx$$

$$=\int_0^{\pi/2}\frac{1+\tan x^{101}-1}{1+\tan x^{101}}=[x]_0^{\pi/2}-I \Rightarrow I=\frac{\pi}{2}-I$$

$$\Rightarrow \quad 2I=\frac{\pi}{2} \quad \Rightarrow \quad I=\frac{\pi}{4}$$

24. Given, $3x\log_e x\frac{dy}{dx}+y=2\log_e x$

Dividing both sides by $3x\log_e x$, we get

$$\frac{dy}{dx}+\frac{1}{3x\log_e x}y=\frac{2\log_e x}{3x\log_e x}$$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{1}{3x\log_e x}y=\frac{2}{3x}$$

Which is linear form $\frac{dy}{dx}+Py=Q$, where P and Q are function of x and the integrating factor is given by the following formula $e^{\int Pdx}$

$$\therefore \quad IF=e^{\int\frac{1}{3x\log_e x}dx}$$

Put $\log_e x=t \quad \Rightarrow \quad \frac{1}{x}dx=dt$

$$=e^{1/3\int\frac{dt}{t}}=e^{1/3}\log t$$

$$=e^{\log t^{1/3}}=t^{1/3}=(\log_e x)^{1/3}$$

25. $\tan x+\sec x=2\cos x$

$$\Rightarrow \frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$$

$$\Rightarrow \sin x+1=2\cos^2 x=2(1-\sin^2 x)=2-2\sin^2 x$$

$$\Rightarrow \quad 2\sin^2 x+\sin x-1=0$$

$$\Rightarrow \quad 2\sin^2 x+2\sin x-\sin x-1=0$$

$$\Rightarrow \quad 2\sin x(\sin x+2)-1(\sin x+1)=0$$

$$\Rightarrow \quad (\sin x+2)(2\sin x-1)=0$$

$\Rightarrow$ $\sin x=-2$, which is not possible.

or $\sin x=\frac{1}{2} \quad \Rightarrow \quad x=\frac{\pi}{6},\frac{5\pi}{6}$

$x\in[0,\pi]$

$\therefore$ The number of solution $=2$

26. Let $I=\int_0^{\pi/4}\frac{\sin x+\cos x}{3+\sin 2x}dx$

$$=\int_0^{\pi/4}\frac{\sin x+\cos x}{3+2\sin x\cos x}dx$$

$$=\int_0^{\pi/4}-\frac{\sin x+\cos x}{(\sin x-\cos x)^2-4}dx \quad \ldots(i)$$

Put $\quad \sin x-\cos x=t$

$\Rightarrow \quad (\cos x+\sin x)dx=dt$

When $x=0 \Rightarrow t=-1$

and $x=\frac{\pi}{4}\Rightarrow t=0$

$\therefore$ Eq. (i) becomes,

$$I=-\int_{-1}^{0}\frac{dt}{t^2-4}=-\frac{1}{4}\left[\log\left|\frac{t-2}{t+2}\right|\right]_{-1}^{0}$$

$$=-\frac{1}{4}(\log 1-\log 3)=-\frac{1}{4}(0-\log 3)=\frac{1}{4}\log 3$$

27. Given, $y=\left(\frac{3^x-1}{3^x+1}\right)\sin x+\log_e(1+x)$, $x>-1$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(\frac{3^x-1}{3^x+1}\right)\sin x\right]+\frac{d}{dx}\log_e(1+x)$$

$$=\left(\frac{3^x-1}{3^x+1}\right)\frac{d}{dx}\sin x+\sin x\frac{d}{dx}\left(\frac{3^x-1}{3^x+1}\right)$$

$$+\frac{1}{1+x}\frac{d}{dx}(1+x)$$

$$=\left(\frac{3^x-1}{3^x+1}\right)\cos x+\sin x$$

$$\frac{(3^x+1)\frac{d}{dx}(3^x-1)-(3^x-1)\frac{d}{dx}(3^x+1)}{(3^x+1)^2}+\frac{1}{1+x}$$

$$=\left(\frac{3^x-1}{3^x+1}\right)\cos x+\sin x(3^x+1)(3^x\log_e 3)$$

$$\frac{-(3^x-1)(3^x\log_e 3)}{(3^x+1)^2}+\frac{1}{1+x}$$

$$\therefore\ \left(\frac{dy}{dx}\right)_{at\ x=0}=0+0+\frac{1}{1+0}=1$$

28. $f(x)=\frac{x}{8}+\frac{2}{x}$

$$\therefore\qquad f'(x)=\frac{1}{8}-\frac{2}{x^2}=\frac{x^2-16}{8x^2}$$

For maximum or minimum $f'(x)$ must be vanished

$$\therefore\qquad f'(x)=0\ \Rightarrow\ \frac{x^2-16}{8x^2}=0$$

$$\Rightarrow\qquad x=4,-4\ \text{and}\ x\in[1,6]\neq-4$$

Also, in [1, 4], $f'(x)<0$

$\Rightarrow f(x)$ is decreasing

In [4, 6], $f'(x)>0\Rightarrow f(x)$ is increasing

$$f(1)=\frac{1}{8}+\frac{2}{1}=\frac{17}{8}$$

$$f(8)=\frac{6}{8}+\frac{2}{6}=\frac{3}{4}+\frac{1}{3}=\frac{13}{12}$$

Hence, maximum value of $f(x)$ in [1, 6] is $\frac{17}{8}$.

29. $\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1+\sin x}\right\}=\frac{1}{1+\left(\frac{\cos x}{1+\sin x}\right)^2}\frac{d}{dx}\frac{\cos x}{1+\sin x}$

$$=\frac{(1+`\sin x)^2}{1+\sin^2 x+2\sin x+\cos^2 x}$$

$$\frac{(1+\sin x)(-\sin x)-\cos(\cos x)}{(1+\sin x)^2}$$

$$=\frac{1}{2+2\sin x}\times[-(\sin x+\sin^2 x+\cos^2 x)]$$

$$=\frac{1}{2(1+\sin x)}\times-(1+\sin x)=-\frac{1}{2}$$

30. $\int_{-2}^{2}(1+2\sin x)e^{|x|}dx=\int_{-2}^{2}e^{x}dx+2\int_{-2}^{2}\sin x e^{|x|}dx$

(Since, first function is even and second function is odd)

$$=2\int_0^2 e^{x}dx+2,0=2[e^{x}]_0^2=2(e^2-1)$$

31. We have the identity,

$$|z|=\left|z+\frac{2}{z}-\frac{2}{z}\right|\le\left|z+\frac{2}{z}\right|+\left|\frac{2}{z}\right|$$

$$\Rightarrow\qquad |z|\le 2+\frac{2}{|z|}\ \Rightarrow|z|^2\le 2|z|+2$$

$$\Rightarrow\qquad |z|^2-2|z|+1\le 3\Rightarrow(|z|-1)^2\le 3$$

$$\Rightarrow\qquad -\sqrt{3}\le|z|-1\le\sqrt{3}$$

$$\Rightarrow\qquad -\sqrt{3}+1\le|z|\le\sqrt{3}+1$$

Hence, the maximum value $=\sqrt{3}+1$

32. Let $z=\frac{3}{2}+i\frac{\sqrt{3}}{2}$ and $r=\sqrt{\frac{9}{4}+\frac{3}{4}}=\sqrt{\frac{12}{4}}=\sqrt{3}$

$$\theta=\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$$

$$\therefore\qquad \frac{3}{2}+\frac{i\sqrt{3}}{2}=\sqrt{3}e\,\frac{i\pi}{6}$$

$$\therefore\qquad \left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=\left(\sqrt{3}e\,\frac{i\pi}{6}\right)^{50}$$

$$=(\sqrt{3})^{50}\left(e^{\frac{i\pi}{6}}\right)^{50}=3^{25}e^{\frac{i50\pi}{6}}$$

$$\Rightarrow\qquad \left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}=3^{25}e^{\frac{i25\pi}{3}}$$

$$=3^{25}\left(\cos\frac{25\pi}{3}+i\sin\frac{25\pi}{3}\right)$$

$$=3^{25}(\cos 1500+i\sin 1500)$$

$$=3^{25}[\cos(360\times 4+60)+i\sin(360\times 4+60)]$$

$$=3^{25}(\cos 60+i\sin 60)$$

$$\Rightarrow\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=3^{25}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$$

According to question,

$$\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy)$$

$$3^{25}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=2^{25}(x+iy)$$

Which is true only when $x=\frac{1}{2}$, $y=\frac{\sqrt{3}}{2}$

33. Let $\omega=\frac{z-1}{z+1}$

Then, using componendo and dividendo, we get

$$z=\frac{1+\omega}{1-\omega} \Rightarrow |z|=\left|\frac{\omega+1}{\omega-1}\right|$$

Put $|z|=1$

$\Rightarrow$ $|\omega+1|=|\omega-1|$ ...(i)

Let $\omega=u+iv$

Then, $|\omega+1|=|u+iv+1|=|(u+1)+iv|$

$=\sqrt{(u+1)^2+v^2}$ and $|\omega-1|=\sqrt{(u-1)^2+v^2}$

From Eq. (i), we get

$$\sqrt{(u+1)^2+v^2}=\sqrt{(u-1)^2+v^2}$$

$\Rightarrow$ $(u+1)^2+v^2=(u-1)^2+v^2$

$\Rightarrow$ $u=0$

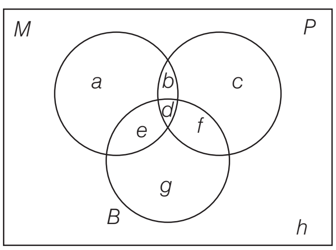
$\therefore \omega=\frac{z-1}{z+1}$ is a pure imaginary number.

34. According to question,

$a+b+c+d+e+f+g+h=100$ ...(i)

$h=1$ ...(ii)

$a+b+d+e=50$ ...(iii)



Venn diagram shows failed students

$b+c+d+f=45$ ...(iv)

$d+e+f+g=40$ ...(v)

$b+e+f=32$ ...(vi)

On solving, the above equations, we get $d=2$

35. The total number of arrangements of 2 letters of English alphabet

$=26\times26$

The total number of arrangements of 4 digit numbers in which first digit is not zero

$=9\times10\times10\times10$

$\therefore$ The total number of vehicles with distinct registration number

$=26\times26\times9\times10\times10\times10=26^2\times9\times10^3$

36. These are 10 letters in the word IRRATIONAL in which these are 2 I, 2 R and 2 A.

$\therefore$ The total number of words $=\frac{10!}{2!2!2!}=\frac{10!}{(2!)^3}$

37. Four speakers will address the meeting in 4! ways $=24$ different ways in which half number of cases will be such that P Speaks before Q and half number of case will be such that P speaks after Q

$\therefore$ Required number of ways $=\frac{24}{2}=12$

38. The number of diagonals in a regular polygon of n sides is given by the following formula ${}^nC_2-n$

$\therefore$ The number of diagonals in a regular polygon of 100 sides

$$={}^{100}C_2-100=\frac{100\cdot99}{2}-100$$

$$=\frac{9900-200}{2}=\frac{9700}{2}=4850$$

39. According to question,

${}^nC_1$, ${}^nC_2$ and ${}^nC_3$ are in AP.

$\Rightarrow$ $\frac{2n(n-1)}{2!}=n+\frac{n(n-1)(n-2)}{3!}$

$\Rightarrow$ $n^2-9n+14=0$

$\Rightarrow$ $(n-7)(n-2)=0$

$\Rightarrow$ $n=7$, since $n\neq2$

$\therefore$ The sum of the coefficients of odd powers of x in the expansion of $(1+`x)^n$ is

$$\frac{2^n}{2}=\frac{2^7}{2}=2^6=64$$

40. Given, $f(x)=ax^2+bx+c$,

$g(x)=px^2+qx+r$

Since, $f(1)=g(1)$

$\Rightarrow$ $a+b+c=p+q+r$ ...(i)

$f(2)=g(2)$

$\Rightarrow$ $4a+2b+c=4p+2q+r$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$3a+b=3p+q$ ...(iii)

$f(3)-g(3)=2$

$\Rightarrow$ $(9a+3b+c)-(9p+3q+r)=2$

$$\Rightarrow \quad 3(3a+b)+c-3(3p+q)-r=2$$
$$\Rightarrow \quad c-r=2 \quad \ldots(\text{iv})$$
$$(3a+b=3p+q)$$

From Eq. (i) we get
$$(a-p)+(b-q)+(c-r)=0$$
$$\Rightarrow \quad (a-p)+(b-q)+2=0 \quad \ldots(\text{v})$$

From Eq. (ii), we get
$$4(a-p)+2)(b-q)+c-r=0$$
$$\Rightarrow \quad 2(a-p)+(b-q)+1=0 \quad \ldots(\text{vi})$$

Subtracting Eq. (v) from Eq. (vi), we get
$$(a-p)-1=0$$
$$a-p=1$$

$\therefore$From Eq. (v), $b-q=-3$

Now,
$$f(4)-g(4)=(16a+4b+c)-(16p+4q+r)$$
$$=16(a-p)+4(b-q)+(c-r)\ldots(\text{vii})$$

Substituting the values of $(a-p),(b-q),(c-r)$ from above in Eq. (vii), we get
$$f(4)-g(4)=16\times 1+4(-3)+2=16-12+2=6$$

41. $1\times 1!+2\times 2!+\ldots+50\times 50!$
$$=(2-1)1!+(3-1)2!+(4-1)3!+\ldots+(51-1)50!$$
$$=(2!-1!)+(3!-2!)+(4!-3!)+\ldots+(51!-50!)$$
$$=51!-1!=51!-1$$

42. Let the numbers be
$a-5d, a-3d, a-d, a+d, a+3d, a+5d$
$$\therefore a-5d+a-3d+a-d+a+d$$
$$+a+3d+a+5d=3 \quad (\therefore \text{Sum}=3)$$
$$\Rightarrow \quad 6a=3 \Rightarrow a=\frac{1}{2}$$

Also, given $T_1=4T_3$, where T_1, T_3 are respectively, first and third terms of AP.
$$\Rightarrow \quad a-5d=4(a-d) \Rightarrow d=-3a=-\frac{3}{2}$$

$\therefore$The fifth term
$$a+3d=\frac{1}{2}+3\left(-\frac{3}{2}\right)=\frac{1}{2}-\frac{9}{2}=-4$$

43. Given series,
$$1+\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9}+\frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12}+\ldots\infty$$
$$=1+\frac{1}{3}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{1\cdot 2}\left(\frac{2}{3}\right)^2+\frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{1\cdot 2\cdot 3}\left(\frac{2}{3}\right)^3\ldots\infty$$
$$=1+\frac{1}{2}\cdot\frac{2}{3}+\frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\left(\frac{2}{3}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3!}$$
$$\left(\frac{2}{3}\right)^3\ldots\infty=\left(1-\frac{2}{3}\right)^{-1/2}=3^{1/2}=\sqrt{3}$$

44. Let α be the common roots
$$\therefore \quad \alpha^2+\alpha+a=0 \quad \ldots(\text{i})$$
$$\text{and} \quad \alpha^2+a\alpha+1=0 \quad \ldots(\text{ii})$$
$$\frac{\alpha^2}{1-a^2}=\frac{\alpha}{a-1}=\frac{1}{a-1} \quad [a\neq 1]$$

Eliminating α, we get
$$(a-1)^2=(1-a^2)(a-1)$$
$$\Rightarrow \quad (a-1)=1-a^2$$
$$\Rightarrow \quad a^2+a-2=0$$
$$\Rightarrow \quad a^2+2a-a-2=0$$
$$\Rightarrow \quad a(a+2)-1(a+2)=0$$
$$\Rightarrow \quad (a+2)(a-1)=0$$
$$\Rightarrow \quad a=-2 \quad [\because a\neq 1]$$

45. Let a be the first term and r be the common ratio of a GP.

$\therefore P$th, Qth and Rth terms of a GP are respectively ar^{P-1}, ar^{Q-1} and ar^{R-1}

According to question,
$$ar^{P-1}=64 \quad \ldots(\text{i})$$
$$ar^{Q-1}=27 \quad \ldots(\text{ii})$$
$$ar^{R-1}=36 \quad \ldots(\text{iii})$$

Dividing Eq, (i) by Eq (ii) we get
$$r^{P-Q}=\left(\frac{4}{3}\right)^3 \quad \ldots(\text{iv})$$

Dividing Eq. (ii) by Eq. (iii), we get
$$r^{Q-R}=\frac{3}{4}$$
$$\Rightarrow \quad r^{3Q-3R}=\left(\frac{3}{4}\right)^3 \quad \ldots(\text{v})$$

Multiplying Eq. (iv) and Eq. (v), we get
$$r^{P-Q}\times r^{3Q-3R}=1$$
$$\Rightarrow \quad r^{P-Q+3Q-3R}=1$$
$$\Rightarrow \quad r^{P+2Q-3R}=r^0$$
$$\Rightarrow \quad P+2Q-3R=0 \Rightarrow P+2Q=3R$$

46. We know that $|\sin^{-1}x|\leq\frac{\pi}{2}$

Hence, from the given relation we observe that each of $\sin^{-1}x, \sin^{-1}y$ and $\sin^{-1}z$ will be $\frac{\pi}{2}$

So that $x = y = z = \sin\frac{\pi}{2} = 1$

$$\therefore \quad x^9 + y^9 + z^9 - \frac{1}{x^9y^9z^9}$$

$$= (1)^9 + (1)^9 + (1)^9 - \frac{1}{(1)^9(1)^9(1)^9}$$

$$= 1 + 1 + 1 - \frac{1}{1\times1\times1} = 3 - 1 = 2$$

47. We know that in ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\therefore \quad r^2 \sin P \sin Q = pq$$

$\Rightarrow r^2 . \frac{p}{2R_1} . \frac{q}{2R_1} = pq$, where R_1 is circumradius of ΔPQR.

$$\Rightarrow \quad r^2 = 4R_1^2 \;\Rightarrow\; r = 2R_1$$

$$\Rightarrow \quad 2R_1 \sin R = 2R_1$$

$$\Rightarrow \quad R_1 = 90°$$

$\therefore \Delta PQR$ is right angled.

48. In ΔPQR, $P + Q + R = 180°$

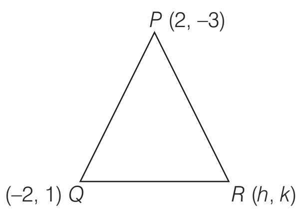
$$\therefore 2pr\sin\left(\frac{P - Q + R}{2}\right) = 2pr\sin\frac{180° - Q - Q}{2}$$

$$= 2pr\sin(90° - Q) = 2pr\cos Q$$

$$\left(\text{In } \Delta PQR \cos Q = \frac{P^2 + r^2 - q^2}{2pr}\right)$$

$$= 2pr\left(\frac{p^2 + r^2 - q^2}{2pr}\right) = p^2 + r^2 - q^2$$

49. Let the vertices of R be (h, k)



$\therefore$Centroid of triangle is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

i.e $\left(\frac{2 + h - 2}{3}, \frac{-3 + 1 + k}{3}\right) \Rightarrow \left(\frac{h}{3}, \frac{-2 + k}{3}\right)$

Since, the point $\left(\frac{h}{3}, \frac{-2 + k}{3}\right)$ lies on the line $2x + 3y = 1$, therefore the point $\left(\frac{h}{3}, \frac{-2 + k}{3}\right)$

Must be satisfy the equation of the given line

i.e, $2\left(\frac{h}{3}\right) + 3\left(\frac{-2 + k}{3}\right) = 1 \Rightarrow \quad \frac{2h - 6 + 3k}{3} = 1$

$$\Rightarrow \quad 2h - 6 + 3k = 3 \;\Rightarrow\; 2h + 3k = 9$$

Replace $h = x$ and $k = y$, we get the required locus of R.

i.e, $\quad 2x + 3y = 9$

50. $\lim_{x\to 0} \frac{\pi^x - 1}{\sqrt{1 + x} - 1} \qquad \left[\frac{0}{0}\text{ from}\right]$

$$= \lim_{x\to 0} \frac{\pi^x \log_e \pi}{\frac{1}{2\sqrt{1+x}}} = \lim_{x\to 0} 2\sqrt{1+x}(\pi^x \log_e \pi)$$

$$= 2\sqrt{1}(\pi^0 \log_e \pi) = 2\log_e \pi = \log_e \pi^2$$

51. Given, $f(x)f'(x) < 0$

$\Rightarrow f(x)$ and $f'(x)$ must be of opposite sign.

(i) Let $f(x) = e^{-x}$

$\therefore \quad f'(x) = -e^{-x}$

$\Rightarrow f(x) > 0$ and $f'(x) < 0, \forall\; x \in R$

(ii) Let $f(x) = -e^{-x}$

$\therefore \quad f'(x) = e^{-x}$

$\Rightarrow f(x) < 0$ and $f'(x) > 0 \;\forall\; x \in R$

But $|f(x)| = |\pm e^{-x}| = e^{-x}$ in both cases

$\therefore \frac{d}{dx}|f(x)| = |-e^{-x}| < 0$ in both case $\forall x \in R$

$\Rightarrow |f(x)|$ must be a decreasing function.

52. If we take $f(x) = 4x^4$, then

(i) $f(x)$ is continuous in $(-2, 2)$

(ii) $f(x)$ is differentiable in $(-2, 2)$

(iii) $f(-2) = f(2)$

So, $f(x) = 4x^4$ satisfies all the conditions of Rolle's theorem therefore $\exists$ a point c such that $f'(c) = 0$

$\Rightarrow 16c^3 = 0 \Rightarrow c = 0 \in (-2, 2)$

53. Let $y = e^{mx}$ be the solution of given differential equation,

$$\Rightarrow \quad \frac{dy}{dx} = me^{mx} \Rightarrow \frac{d^2y}{dx^2} = m^2e^{mx}$$

$\therefore \qquad 25\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + y = 0$

$\Rightarrow \qquad 25m^2e^{mx} - 10me^{mx} + e^{mx} = 0$

$\Rightarrow \qquad e^{mx}(25m^2 - 10m + 1) = 0$

$\Rightarrow$ Auxiliary equation

$\Rightarrow \qquad 25m^2 - 10m + 1 = 0$

$e^{mx} \neq 0$

$\Rightarrow \qquad (5m)^2 - 2(5m) \times 1 + 1 = 0$

$\Rightarrow \qquad (5m - 1)^2 = 0$

$\Rightarrow \qquad m = \frac{1}{5}, \frac{1}{5}$

Since, roots are real and equal, .

$\therefore$General solution is $y = (c_1 + c_2x)e^{x/5}$ ...(i)

$$y(0) = 1 \Rightarrow c_1 = 1$$

$$y(1) = 2e^{1/5} \Rightarrow 2e^{1/5} = (c_1 + c_2)e^{1/5}$$

$\Rightarrow \qquad c_1 + c_2 = 2$

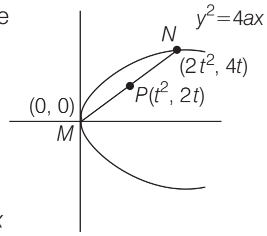
$\Rightarrow \qquad c_1 = 1$

Putting the value of c_1 and c_2 in Eq. (i), we get particular solution

$y = (1 + x)e^{x/5}$

54. Let the chord be MN.

Let other end of chord joining from vertex to it lying on $y^2 = 8x$ be $N(2t^2, 4t)$



$\therefore$Mid-point of $MN = (t^2, 2t)$

$\therefore \quad x = t^2, y = 2t$

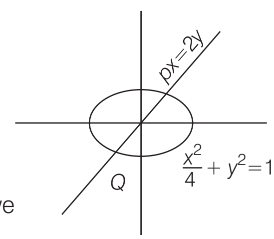
$\Rightarrow x = \left(\frac{y}{2}\right)^2 \Rightarrow y^2 = 4x$

is the required locus of P.

55. Solving $x = 2y$

and

$\frac{x^2}{4} + y^2 = 1$



Put $x = 2y$ in Eq. (ii), we get

$$\frac{(2y)^2}{4} + y^2 = 1 \Rightarrow \frac{4y^2}{4} + y^2 = 1$$

$\Rightarrow \qquad 2y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

$\therefore$From Eq. (i), $x = \pm\sqrt{2}$

$\therefore P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ and $Q\left(-\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

$\therefore$Equation of circle with PQ as diameter is

$$(x - \sqrt{2})(x + \sqrt{2}) + \left(y - \frac{1}{\sqrt{2}}\right)\left(y + \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \qquad x^2 - 2 + y^2 - \frac{1}{2} = 0$

$\Rightarrow \qquad x^2 + y^2 = \frac{5}{2}$

56. Let $P(\sqrt{10}\cos\theta, \sqrt{8}\sin\theta)$ be the required point on $\frac{x^2}{10} + \frac{y^2}{8} = 1$

Whose distance from centre (0, 0) is 3 units.

$\therefore 10\cos^2\theta + 8\sin^2\theta = 9 = 9(\sin^2\theta + \cos^2\theta)$

$\Rightarrow \qquad \tan^2\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

57. Let e be the eccentricity of hyperbola and length of conjugate axis be $2b$. Since, vertex $(a, 0)$ bisects the join of centre (0, 0) and focus $(ae, 0)$

$$a = \frac{ae + 0}{2} \Rightarrow e = 2$$

$$b^2 = a^2(e^2 - 1) = 3a^2$$

$\therefore$Required hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \qquad \frac{x^2}{a^2} - \frac{y^2}{3a^2} = 1 \Rightarrow 3x^2 - y^2 = 3a^2$

58. We know by the definition of hyperbola, "A hyperbola is the locus of a point which moves in such a way that the difference between two fixed points remains constant,

i.e $|PS - PS'| = 2a$

59. Given lines are, $3x + 4y = 9$ ...(i)

$y = mx + 1$ ...(ii)

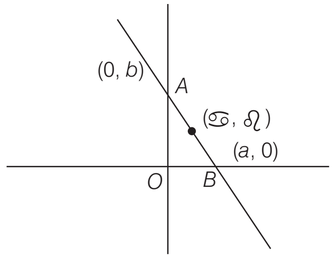
Solving Eqs. (i) and (ii), we get

$$x = \frac{5}{3 + 4m}$$

If $m = -1$ then $x = -5$ and $m = -2$, then $x = -1$ are the only possibilities.

60. Let the line be $AB = \frac{x}{a} + \frac{y}{b} = 1$ ...(i)

$\therefore$ The coordinates of A and B are respectively $(a, 0)$ and $(0, b)$



Since (α, β) is the mid-point of AB

$\therefore \quad \alpha = \frac{a}{2}, \beta = \frac{b}{2}$

$\therefore \quad a = 2\alpha$ and $b = 2\beta$

From Eq. (i), $\frac{x}{2\alpha} + \frac{y}{2\beta} = 1$

$\Rightarrow \quad \frac{x}{\alpha} + \frac{y}{\beta} = 2$

61. The given series is in GP. Hence, its sum

$$S = \frac{1\{(1+x)^{20+1} - 1\}}{(1+x) - 1} = \frac{(1+x)^{21} - 1}{x}$$

Therefore, the required coefficient of x^{10} in the expansion of $\frac{(1+x)^{21} - 1}{x}$

= Coefficient of x^{11} in the expansion of $(1+x)^{21} - 1 = {}^{21}C_{11}$

62. Augmented matrix $[A : B] = \begin{bmatrix} 1 & -1 & -2 & 6 \\ -1 & 1 & 1 & \mu \\ \lambda & 1 & 1 & 3 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -1 & -2 & 6 \\ 0 & 0 & -1 & \mu + 6 \\ \lambda + 1 & 0 & -1 & 9 \end{bmatrix}$$

Applying $R_2 \to R_2 + R_1, R_3 \to R_3 + R_1$

$$\sim \begin{bmatrix} 1 & -1 & -2 & 6 \\ 0 & 0 & -1 & \mu + 6 \\ \lambda + 1 & 0 & 0 & 3 - \mu \end{bmatrix}$$

Infinite number of solutions for $\lambda = -1$ and $\mu = 3$

63. $P\left(\frac{B}{A \cup B^c}\right) = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^C)}$

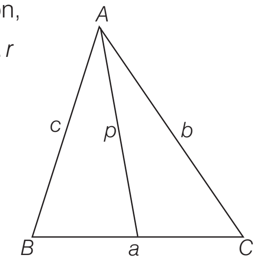
$$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{P(A) - P(A \cap B^C)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$

64. According to question,

$S = \frac{1}{2} a.\, p = \frac{1}{2} b.q = \frac{1}{2} c.r$



$\therefore \quad \frac{1}{p} = \frac{a}{2S}, \frac{1}{q} = \frac{b}{2S},$

$\frac{1}{r} = \frac{c}{2S}$

$\therefore \quad \frac{1}{p} + \frac{1}{q} + \frac{1}{r}$

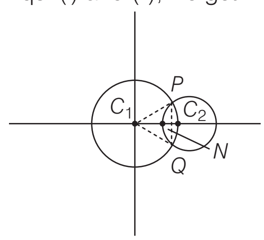
$$= \frac{1}{2S}(a + b + c) = \frac{2t}{2S} = \frac{t}{S}$$

65. $x^2 + y^2 = 4 \quad \ldots$(i)

$(x - 2)^2 + y^2 = 1 \quad \ldots$(ii)

$C_1 = (0, 0), C_2 = (2, 0)$

On solving Eqs. (i) and (ii), we get



$$x = \frac{7}{4}, y = \pm \frac{\sqrt{15}}{4}$$

$$C_1N = \frac{7}{4}, PQ = \frac{2\sqrt{15}}{4} = \frac{\sqrt{15}}{2}$$

$$\frac{\text{Area}(\Delta C_1PQ)}{\text{Area}(\Delta C_2PQ)} = \frac{\frac{1}{2}.PQ.C_1N}{\frac{1}{2}.PQ.C_2N}$$

$$= \frac{C_1N}{C_2N} = \frac{\frac{7}{4}}{2 - \frac{7}{4}} = \frac{\frac{7}{4}}{\frac{1}{4}} = 7 : 1$$

Hence, the required option is (c).

66. Given lines are, $x + 2y = 4 \quad \ldots$(i)

and $\quad 2x + y = 4 \quad \ldots$(ii)

Solving Eqs. (i) and (ii), we get

$$x = \frac{4}{3}, y = \frac{4}{3}$$

Let $\quad AB : \frac{x}{a} + \frac{y}{b} = 1 \quad \ldots$(iii)

Meets X-axis at A and Y - axis at B.

If the straight line (iii) pases through the

Point $\left(\frac{4}{3}, \frac{4}{3}\right)$, then $\frac{4}{3a} + \frac{4}{3b} = 1$

$$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} = \frac{3}{4} \quad \ldots \text{(iv)}$$

Let the mid-point of AB be (h, k)

$\therefore \quad h = \frac{a+0}{2}, k = \frac{0+b}{2} \Rightarrow a = 2h, b = 2k$

$\therefore$ From Eq. (iv), $\frac{1}{2h} + \frac{1}{2k} = \frac{3}{4} \Rightarrow 2h + 2k = 3hk$

Replace x, y from h, k respectively, we get required locus

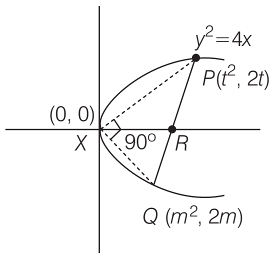
i.e $\quad 2(x+y) = 3xy$

67. According to question, it is given that slope of PX and QX are perpendicular to each other, i.e,

Slope of $PX \times$ slope of $XQ = -1$

$$\Rightarrow \quad \frac{2t}{t^2} \times \frac{2m}{m^2} = -1 \Rightarrow t_m = -4$$

$$PQ : \frac{x - t^2}{t^2 - m^2} = \frac{y - 2t}{2t - 2m}$$



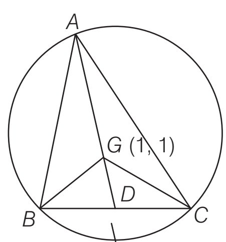
Let PQ meets axis of parabola i.e. X-axis at $R(\alpha, 0)$, then

$$\frac{\alpha - t^2}{(t+m)(t-m)} = \frac{0 - 2t}{2(t-m)} \Rightarrow \alpha - t^2 = t^2 - tm$$

$$= -t^2 + 4 \Rightarrow \alpha = 4 \Rightarrow AR = 4$$

68. Since, triangle is equilateral therefore incentre (1, 1) lies on the centroid of the ΔABC.

$\therefore GD =$ Length of perpendicular from the point $G(1, 1)$ to the line $3x + 4y + 3 = 0$



$$= \frac{3(1) + 4(1) + 3}{\sqrt{3^2 + 4^2}} = 2$$

$$AG = 2GD = 4$$

$\therefore$ Equation of circumcircle with centre at (1, 1) and radius = 4 units

$$(x-1)^2 + (y-1)^2 = 4^2$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 2y - 14 = 0$$

69. $\lim_{x \to \infty} \frac{(n!)^{1/n}}{n} = \lim_{n \to \infty} \left(\frac{n!}{n^n}\right)^{1/n}$

We have, $\frac{n!}{n^n} = \frac{1 \cdot 2 \cdot 3 \ldots . n}{n \cdot n \cdot n \ldots . n}$

$$\therefore \left\{\frac{n!}{n^n}\right\}^{1/n} = \left\{\frac{1}{n} \cdot \frac{2}{n} \cdot \frac{3}{n} \ldots . \frac{r}{n} \ldots \frac{n}{n}\right\}^{1/n}$$

$$\Rightarrow \lim_{n \to \infty} \left\{\frac{n!}{n^n}\right\}^{1/n} = \lim_{n \to \infty} \left(\frac{1}{n} \cdot \frac{2}{n} \cdot \frac{3}{n} \ldots \frac{r}{n} \ldots \frac{n}{n}\right)^{1/n}$$

Let $\quad A = \lim_{n \to \infty} \left\{\frac{n!}{n^n}\right\}^{1/n}$

Then, $\quad A = \lim_{n \to \infty} \left\{\frac{1}{n} \cdot \frac{2}{n} \cdot \frac{3}{n} \ldots \frac{r}{n} \ldots \frac{n}{n}\right\}^{1/n}$

$$\Rightarrow \quad \log A = \lim_{n \to \infty} \frac{1}{n} \Sigma \log\left(\frac{r}{n}\right) = \int_0^1 \log x \, dx$$

$$= \left[x \log x - \int \frac{1}{x} . x \, dx\right]_0^1$$

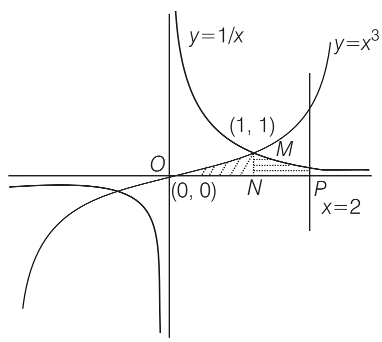
Integrating by parts

$$= [x \log x - x]_0^1 = -1$$

$$\Rightarrow \quad A = e^{-1} = \frac{1}{e}$$

70. First of all we draw the graph,

$$y = x^3, y = \frac{1}{x}, x = 2$$



$\therefore$ Required area i.e $(OMPNO)$

$$=\int_0^1 x^3\,dx+\int_1^2 \frac{1}{x}dx$$

$$=\left[\frac{x^4}{4}\right]_0^1+[\log_e x]_1^2=\frac{1}{4}+\log_e 2$$

71. Let $y=x^y$

$\Rightarrow$ $\log y = y\log x$

Differentiating both sides w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx}=y\times\frac{1}{x}+\log x\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}-\log x\frac{dy}{dx}=\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}-\log x\right)=\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1-y\log x}{y}\right)=\frac{y}{x}$$

$$\Rightarrow \frac{xdy}{dx}=\frac{y^2}{(1-y\log x)}$$

72. $\sin^{-1}x$ is defined, if $-1\le x\le 1$

In first quadrant $0\le x\le 1$ and $x(1-x)\ge 0$

$\therefore$ $y=\sin^{-1}x+x(1-x)$ ...(i)

Lies above $y=\sin^{-1}x-x(1-x)$ ...(ii)

On solving, we get

$$2x(1-x)=0$$

$$\Rightarrow x=0,1$$

$\therefore$ Required area $=\int_0^1 (y_1-y_2)dx$

$$=\int_0^1\{\sin^{-1}x+x(1-x)\}-\{\sin^{-1}x-x(1-x)\}]dx$$

$$=2\int_0^1 (x-x^2)dx$$

$$=2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=2\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{1}{3}$$

73. $\int_1^5 [|x-3|+|1-x|]dx$

$$=\int_1^5|x-3|\,dx+\int_1^5|x-3|\,dx+\int_1^5|1-x|\,dx$$

$$=\int_1^3|x-3|\,dx+\int_3^5|x-3|\,dx+\int_1^5|1-x|\,dx$$

$$=\int_1^3 -(x-3)dx+\int_3^5(x-3)dx+\int_1^5 -(1-x)dx$$

$$=\int_1^3(3-x)dx+\int_3^5(x-3)dx+\int_1^5(x-1)dx$$

$$=\left[3x-\frac{x^2}{2}\right]_1^3+\left[\frac{x^2}{2}-3x\right]_3^5+\left[\frac{x^2}{2}-x\right]_1^5$$

$$=\left(3\times3-\frac{9}{2}\right)-\left(3\times1-\frac{1}{2}\right)+\left(\frac{5\times5}{2}-3\times5\right)$$
$$-\left(\frac{3\times3}{2}-3\times3\right)+\left(\frac{5\times5}{2}-5\right)-\left(\frac{1}{2}-1\right)$$

$$=\left(9-\frac{9}{2}\right)-\left(3-\frac{1}{2}\right)+\left(\frac{25}{2}-15\right)-\left(\frac{9}{2}-9\right)$$
$$+\left(\frac{25}{2}-5\right)-\left(-\frac{1}{2}\right)$$

$$=\frac{9}{2}-\frac{5}{2}-\frac{5}{2}+\frac{9}{2}+\frac{15}{2}+\frac{1}{2}$$

$$=\frac{9-5-5+9+15+1}{2}=\frac{24}{2}=12$$

74. According to question

$$f''(x)=g''(x)$$

Integrating w.r.t. x, we get

$$f'(x)=g'(x)+C_1$$

Put $x=1\Rightarrow f'(1)=g'(1)+C_1$

$\Rightarrow$ $4=6+C_1$

$\therefore$ $C_1=-2$

$\therefore$ $f'(x)=g'(x)-2$

Again, integrating w.r.t. x, we get

$$f(x)=g(x)-2x+C_2$$

$\Rightarrow$ $3=9-4+C_2$

$\Rightarrow$ $C_2=-2$

$\therefore$ $f(x)=g(x)-2x-2$

Put $x=1$, we get

$$f(1)-g(1)=-2(1)-2=-4$$

75. Let $I=\int_{-1}^1(|x|-2[x])dx$

$$=\int_{-1}^0(|x|-2[x])dx+\int_0^1(|x|-2[x])dx$$

$$=\int_{-1}^0(-x-2(-1))dx+\int_0^1(x-2(0))dx$$

$$=\int_{-1}^0(-x+2)dx+\int_0^1 x\,dx$$

$$=\left(-\frac{x^2}{2}+2x\right)_{-1}^0+\left[\frac{x^2}{2}\right]_0^1$$

$$=-\left(-\frac{1}{2}+2(-1)\right)+\frac{1}{2}=\frac{1}{2}+2+\frac{1}{2}=1+2=3$$

76. Let $z = x + iy$

$$\therefore \quad \frac{z-2}{z+2} = \frac{x+iy-2}{x+iy+2}$$

$$= \frac{(x-2)+iy}{(x+2)+iy} \times \frac{(x+2)-iy}{(x+2)-iy}$$

$$= \frac{(x-2)(x+2)+iy(x+2)-iy(x-2)-i^2y^2}{(x+2)^2-(iy)^2}$$

$$= \frac{x^2-4+ixy+2iy-ixy+2iy+y^2}{(x+2)^2+y^2}$$

$$= \frac{(x^2+y^2-4)+4iy}{(x+2)^2+y^2}$$

$$= \frac{x^2+y^2-4}{(x^2+4+4x+y^2)} + \frac{4iy}{x^2+4+4x+y^2}$$

$$\therefore \ \arg\left(\frac{z-2}{z+2}\right) = \tan^{-1}\frac{4y}{x^2+4+4x+y^2}$$

$$\times \frac{x^2+4+4x+y^2}{x^2+y^2-4} = \frac{\pi}{3} \qquad \text{(given)}$$

$$\therefore \ \tan\frac{\pi}{3} = \frac{4y}{x^2+y^2-4}$$

$$\sqrt{3} = \frac{4y}{x^2+y^2-4}$$

$x^2 + y^2 - 4 - \frac{4}{\sqrt{3}}y = 0$, which represents a circle.

77. Let $a^p = b^q = c^r = k$

$$\therefore \qquad a = k^{1/p}, b = k^{1/q}, c = k^{1/r}$$

Since, a, b, c are in GP.

$$\frac{b}{a} = \frac{c}{b}$$

$$\frac{k^{1/q}}{k^{1/p}} = \frac{k^{1/r}}{k^{1/q}}$$

$$k^{1/q-1/p} = k^{1/r-1/q}$$

$$\frac{1}{q} - \frac{1}{p} = \frac{1}{r} - \frac{1}{q}$$

$\therefore \ \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in GP.

$\Rightarrow p, q, r$ are in HP.

78. $S_1 = 1 + \frac{1}{2} + \frac{1}{2^2} + \dots \infty = \frac{1}{1-\frac{1}{2}} = 2$

$$S_2 = 2 + 2.\frac{2}{3} + 2\left(\frac{2}{3}\right)^2 + \dots\infty = \frac{2}{1-\frac{2}{3}} = 6$$

$$S_3 = 3 + 3\left(\frac{3}{4}\right) + 3\left(\frac{3}{4}\right)^2 + \dots\infty = \frac{3}{1-\frac{3}{4}} = 12$$

$$S_4 = 4 + 4\left(\frac{4}{5}\right) + 4\left(\frac{4}{5}\right)^2 + \dots\infty = \frac{4}{1-\frac{4}{5}} = 20$$

$$\therefore \quad \sum_{k=1}^{\infty}\frac{(-1)^k}{S_k} = -\frac{1}{S_1} + \frac{1}{S_2} - \frac{1}{S_3} + \frac{1}{S_4} - \dots\infty$$

$$= -\frac{1}{2} + \frac{1}{6} - \frac{1}{12} + \frac{1}{20} - \dots\infty$$

$$= -\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} - \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} - \dots\infty$$

$$= -\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) - \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) - \dots\infty$$

$$= -1 + 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots\infty\right)$$

$$= -1 - 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots\infty\right) + 2\infty$$

$$= -1 - 2\log_e 2 + 2 = 1 - \log_e 4$$

79. Since, roots are opposite sign.

$\therefore$ Product of the roots < 0

$\Rightarrow \qquad a^2 - 4a < 0$

$\Rightarrow \qquad a(a-4) < 0$

$\Rightarrow \qquad 0 < a < 4$

80. Given, $\log_e(x^2 - 16) \le \log_e(4x - 11)$ if and only if

$$x^2 - 16 \le 4x - 11$$

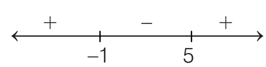
$\Rightarrow \qquad x^2 - 4x - 5 \le 0$

$\Rightarrow \qquad x^2 - 5x + x - 5 \le 0$

$\Rightarrow \qquad x(x-5) + 1(x-5) \le 0$

$\Rightarrow \qquad (x-5)(x+1) \le 0$

Sign scheme of $x^2 - 4x - 5 \le 0$



$\Rightarrow -1 \le x \le 5$