VITEEE 2015 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER VITEEE 2015

PART - I (PHYSICS)

1. When a hydrogen atom is raised from ground energy level to excited energy level, then

(a) potential energy increases and kinetic energy decreases
(b) kinetic energy increases and potential energy decreases
(c) Both KE and PE increase
(d) Both KE and PE decrease

2. The half life for α-decay of uranium $_{92}U^{228}$ is 4.47×10^8 yr. If a rock contains 60% of original $_{92}U^{228}$ atoms, then its age is
[take log 6 = 0.778, log 2 = 0.3]

(a) 1.2×10^7 yr (b) 3.3×10^8 yr
(c) 4.2×10^9 yr (d) 6.5×10^9 yr

3. A nuclear transformation is given by $Y\,(n, \alpha) \rightarrow {}_3Li^7$. The nucleus of element Y is

(a) $_5Be^{11}$ (b) $_5B^{10}$
(c) $_5B^9$ (d) $_6C^{12}$

4. The angular momentum of an electron in Bohr's hydrogen atom whose energy is –3.4 eV, is

(a) $\frac{5h}{2\pi}$ (b) $\frac{h}{2\pi}$
(c) $\frac{h}{\pi}$ (d) $\frac{2h}{3\pi}$

5. When the momentum of a photon is changed by an amount p' then the corresponding change in the de-Broglie wavelength is found to be 0.20%. Then, the original momentum of the photon was

(a) $300p'$ (b) $500p'$
(c) $400p'$ (d) $100p'$

6. Suppose a beam of electrons with each electron having energy E_0 incident on a metal surface kept in an evacuated chamber. Then ,

(a) electrons can be emitted with any energy with a maximum of E_0
(b) no electrons will be emitted as only photons can emit electrons
(c) electrons can be emitted but all with an energy E_0
(d) electrons can be emitted with any energy with a maximum of $E_0 - \phi$, where ϕ being work function

7. An n-type semiconductor is

(a) neutral
(b) positively charged
(c) negatively charged
(d) negatively or positively charged depending on the amount of impurity added

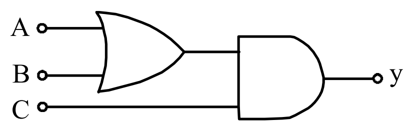
8. In the half wave rectifier circuit operating with 50 Hz mains frequency. The fundamental frequency in the ripple will be

(a) 100 Hz (b) 20 Hz
(c) 50 Hz (d) 25 Hz

9. The input resistance of a common emitter amplifier is 330 Ω and the load resistance is 5 kΩ. A change of base current is 15 μA results in the change of collector current by 1mA. The voltage gain of amplifier is

(a) 1000 (b) 10001
(c) 1010 (d) 1100

10. To get an output $y = 0$ from the circuit shown in the figure, the input C must be



(a) 0 (b) 1
(c) either 0 or 1 (d) None of these

11. Equal charges q each are placed at the vertices of an equilateral triangle of side r. The magnitude of electric field intensity at any vertex is

(a) $\frac{2q}{4\pi\varepsilon_0 r^2}$ (b) $\frac{q}{4\pi\varepsilon_0 r^2}$

(c) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 r^2}$ (d) $\frac{\sqrt{2}q}{4\pi\varepsilon_0 r^2}$

12. Two points masses, m each carrying charges $-q$ and $+q$ are attached to the ends of a massless rigid non-conducting wire of length *'L'*. When this arrangement is placed in a uniform electric field, then it deflects through an angle i. The minimum time needed by rod to align itself along the field is

(a) $2\pi\sqrt{\frac{mL}{qE}}$ (b) $\frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$

(c) $\pi\sqrt{\frac{2mL}{qE}}$ (d) $2\pi\sqrt{\frac{3mL}{qE}}$

13. A condenser of capacitance C is fully charged by a 200V supply. It is then discharged through a small coil of resistance wire embedded in thermally insulated block of specific heat 250 J/kg-K and of mass 100 g. If the temperature of the block rises by 0.4 K, then the value of C is

(a) 300μF (b) 200μF

(c) 400μF (d) 500μF

14. The capacitance of a parallel plate capacitor with air as medium is 3 μF. As a dielectric is introduced between the plates, the capacitance becomes 15 μF. The permittivity of the medium in $C^2N^{-1}m^{-2}$ is

(a) 8.15×10^{-11} (b) 0.44×10^{-10}

(c) 15.2×10^{12} (d) 1.6×10^{-14}

15. The masses of three copper wires are in the ratio 2 : 3 : 5 and their lengths are in the ratio 5 : 3 : 2.Then, the ratio of their electrical resistances is

(a) 1 : 9 : 15 (c) 2 : 3 : 5

(b) 5 : 3 : 2 (d) 125 : 30 : 8

16. A 30V-90W lamp is operated on a 120 V DC line. A resistor is connected in series with the lamp in order to glow it properly. The value of resistance is

(a) 10 Ω (b) 30 Ω

(c) 20 Ω (d) 40 Ω

17. In a potentiometer experiment, the balancing length of a cell is 560 cm. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60 cm.The internal resistance of a cell is

(a) 1.4 Ω (b) 1.6 Ω

(c) 0.12 Ω (d) 1.2 Ω

18. Two sources of equal emf are connected to a resistance R. The internal resistance of these sources are r_1 and r_2 $(r_1 > r_2)$. If the potential difference across the source having internal resistance r_2 is zero, then

(a) $R = \frac{r_1 r_2}{r_2 - r_1}$ (b) $R = r_2\left(\frac{r_1 + r_2}{r_2 - r_1}\right)$

(c) $R = \left(\frac{r_1 r_2}{r_2 + r_1}\right)$ (d) $R = r_2 - r_1$

19. An electron of mass 9.0×10^{-31}kg under the action of a magnetic field moves in a circle of radius 2 cm at a speed of 3×10^6 m/s. If a proton of mass 1.8×10^{27} kg was to move in a circle of same radius in the same magnetic field, then its speed will become

(a) 1.5×10^3 m/s (b) 3×10^6 m/s

(c) 6×10^4 m/s (d) 2×10^6 m/s

20. A horizontal rod of mass 0.01kg and length 10 cm is placed on a frictionless plane inclined at an angle 60° with the horizontal and with the length of rod parallel to the edge of the inclined plane. A uniform magnetic field is applied 'Vertically downwards. If the current through the rod is 1.73 A, then the value of magnetic field induction B for which the rod remains stationary on the inclined plane is

(a) 1 T (c) 3 T

(b) 2.5 T (d) 4 T

21. A current of 2 A is flowing in the sides of an equilateral triangle of side 9 cm. The magnetic field at the centroid of the triangle is

(a) 1.66×10^{-5} T (b) 1.22×10^{-4} T

(c) 1.33×10^{-5} T (d) 1.44×10^{-4} T

22. The direction of magnetic field **dB** due to current element **dl** at a distance **r** is the direction of

(a) $r \times dl$ (b) $dl \times r$

(c) (rdl)r (d) dl

23. A galvanometer with a scale divided into 100 equal divisions has a current sensitivity of 10 divisions per milliampere and a voltage sensitivity of 2 divisions per millivolt. The galvanometer resistance will be

(a) 4 Ω (b) 5 Ω

(c) 3 Ω (d) 7 Ω

24. The earth is considered as a short magnet with its centre coinciding with the geometric centre of earth. The angle of dip ϕ related to the magnetic latitude $\alpha\lambda$as

(a) $\tan\phi = \dfrac{1}{2\tan\alpha}$ (b) $\tan\lambda = 2\tan\phi$

(c) $\tan\lambda = 2\tan\phi$ (d) $\tan\phi = 2\tan\lambda$

25. Which of the following statement related to hysteresis loop is incorrect?

(a) The curve of B against H for a ferromagnetic material is called hysteresis loop

(b) The area of *B-H* curve is a measure of power dissipated per cycle per unit area of the specimen

(c) Coercitivity is a measure of the magnetic field required to destroy the residual magnetism of ferromagnetic material

(d) The retentivity of a specimen is the measure of magnetic field remaining in the specimen when the magnetising field is removed

26. A magnetic needle lying parallel to the magnetic field requires W units of work to turn it through an angle 45°. The torque required to maintain the needle in this position will be

(a) $\sqrt{2}W$ (b) $\dfrac{1}{\sqrt{3}W}$

(c) $(\sqrt{2}-1)W$ (d) $\dfrac{W}{(\sqrt{2}-1)}$

27. An induced emf has

(a) a direction same as field direction

(b) a direction opposite to the field direction

(c) no direction of its own

(d) None of the above

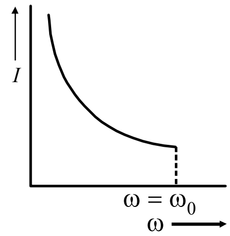
28. A coil of area 5 cm^2 having 20 turns is placed in a uniform magnetic field of 10^3 gauss. The normal to the plane of coil makes an angle 30° with the magnetic field. The flux through the coil is

(a) 6.67×10^{-4} wb (b) 3.2×10^{-5} Wb

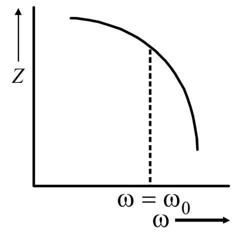
(c) 5.9×10^{-4} wb (d) 8.65×10^{-4} wb

29. The current graph for resonance in LC circuit is

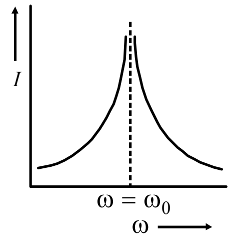
(a)



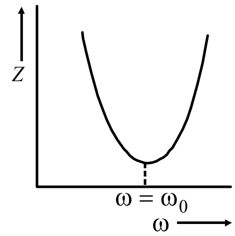
(b)



(c)



(d)



30. The value of inductance L for which the current is maximum in series LCR circuit with C=10 μF and ω =1000 rad/s

(a) 10 mH (b) 50 mH

(c) 200 mH (d) 100 mH

31. A ray of light is incident on a plane mirror at an angle of 30°. At what angle with the horizontal must a plane mirror be placed so that the reflected ray becomes vertically upwards?

(a) 40° (b) 20°

(c) 30° (d) 60°

32. A compound microscope having magnifying power 35 with its eye-piece of focal length 10 cm. Assume that the final image is at least distance of distinct vision then the magnification produced by the objective is

(a) –4 (b) 5

(c) 10 (d) –10

33. The refractive index for a prism is given as $\mu = \cot\frac{A}{2}$. Then, angle of minimum deviation in terms of angle of prism is

(a) 90°–A (b) *2A*

(c) 180°–A (d) 180°–2A

34. Two convex lenses of power 2D and 5D are separated by a distance $\frac{1}{3}$ m. The power of optical system formed is

(a) +2 D (b) –2 D

(c) –3 D (d) +3 D

35. Two light rays having the same wavelength in vacuum are in phase initially. Then, the first ray travels a path L_1 through a medium of refractive index μ_1 while the second ray travels a path L_2 through a medium of refractive index μ_2. The two waves are then combined to observe interference. The phase difference between the two waves is

(a) $\frac{2\pi}{\lambda}\left(\frac{L_1}{\mu_1} - \frac{L_2}{\mu_2}\right)$ (b) $\frac{2\pi}{\lambda}(L_2 - L_1)$

(c) $\frac{2\pi}{\lambda}(\mu_2 L_1 - \mu_1 L_2)$ (d) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$

36. Two polaroids are kept crossed to each other. If one of them is rotated an angle 60°, the percentage of incident light now transmitted through the system is

(a) 10% (b) 20%

(c) 25% (d) 12.5%

37. An electromagnetic wave propagating along north lies its electric field vertically upward. The magnetic field vector points towards

(a) downward (b) east

(c) north (d) south

38. Pick out the wrong statement.

(a) Gauss's law of magnetism is given by $\alpha \oint \mathbf{B} \cdot \mathbf{ds} = 0$

(b) An EM wave is a wave radiated by a charge at rest and propagates through electric field only

(c) A time varying electric field is a source of changing magnetic field

(d) Faraday's law of EM induction is $\oint \mathbf{E.dl} = -\frac{d\phi_B}{dt}$

39. When sunlight is scattered by atmospheric atoms and molecules the amount of scattering of light of wavelength 880nm is *A*. Then, the amount of scattering· of light of wavelength 330 nm is approximately

(a) 10 A (b) 20 A

(c) 40 A (d) 50.5 A

40. The ratio of volume occupied by an atom to the volume of the nucleus is

(a) 10^5:1 (b) 10^{20}:1

(c) 10^{15}:1 (d) 1:10^{15}

PART - II (CHEMISTRY)

41. When copper is treated with a certain concentration of nitric acid, nitric oxide and nitrogen dioxide are liberated in equal volumes according to the equation,

$$x\text{Cu} + y\text{HNO}_3 \longrightarrow \text{Cu(NO}_3)_2 + \text{NO} + \text{NO}_2 + \text{H}_2\text{O}$$

The coefficients of *x* and *y* are respectively

(a) 2 and 3 (b) 2 and 6

(c) 1 and 3 (d) 3 and 8

42. A saturated solution of H_2S in 0.1 M HCl at 25°C contains S^{2-} ion concentration of 10^{-23} mol L^{-1}. The solubility product of some sulphides are CuS=10^{-44}, FeS = 10^{-14}, MnS = 10^{-15}, CdS = 10^{-25}. If 0.01 M solution of these salts in 1M HCl are saturated with H_2S, which of these will be precipitated?

(a) All

(b) All except MnS

(c) AU except MnS and FeS

(d) Only-CuS

43. Consider the water gas equilibrium reaction,

$C(s) + H_2O\,(g) \rightleftharpoons CO\,(g) + H_2(g)$

Which of the following statements is true at equilibrium?

(a) If the amount of C(s) is increased, less water would be formed

(b) If the amount of C(s) is increased, more CO and H_2 would be formed

(c) If the pressure on the system is increased by halving the volume, more water would be formed

(d) If the pressure on the system is increased by halving the volume, more CO and H_2 would be formed

44. The chemical composition of slag formed during the smelting process in the extraction of copper is

(a) $Cu_2O + FeS$ (b) $FeSiO_3$

(c) $CuFeS_2$ (d) $Cu_2S + FeO$

45. $X Cl_2\text{ (excess)} + Y Cl_2 \longrightarrow X Cl_4 + Y \downarrow$

$YO \xrightarrow[>400°C]{\Delta} \frac{1}{2}O_2 + Y$

Ore of Y would be,

(a) siderite (b) malachite

(c) hornsilver (d) cinnabar

46. For the given reaction,

$H_2\,(g) + Cl_2\,(g) \longrightarrow 2H^+\,(aq) + 2Cl^-\,(aq)$;

$\Delta G° = -262.4\text{kJ}$

The value of free energy of formation ($\Delta G°_f$) for the ion Cl^{-1} (aq), therefore will be

(a) $-131.2\text{ kJ mol}^{-1}$ (b) $+131.2\text{ kJ mol}^{-1}$

(c) $-262.4\text{ kJ mol}^{-1}$ (d) $+262.4\text{ kJ mol}^{-1}$

47. The molarity of NO_3^- in the solution after 2L of 3M $AgNO_3$ is mixed with 3L of 1M $BaCl_2$ is

(a) 1.2 M (b) 1.8 M

(c) 0.5 M (d) 0.4 M

48. Amongest

$NO_3^-, AsO_3^{3-}, CO_3^{2-}, ClO_3^-, SO_3^{2-}$ and BO_3^{3-} ,

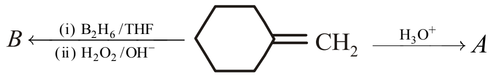
the non-planar species are

(a) CO_3^{2-}, SO_3^{2-} and BO_3^{2-}

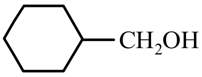
(b) AsO_3^{3-}, ClO_3^- and SO_3^{2-}

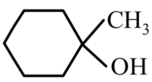
(c) NO_3^-, CO_3^{2-} and BO_3^{3-}

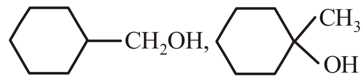
(d) SO_3^{2-}, NO_3^- and BO_3^{3-}

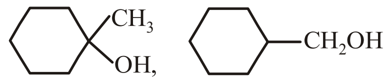
49. 

A and B respectively are

(a) Both 

(b) Both 

(c) 

(d) 

50. A certain metal when irradiated by light ($r = 3.2 \times 10^{16}$Hz) emits photoelectrons with twice kinetic energy as did photoelectrons when the same metal is irradiated by light ($r = 2.0 \times 10^{16}$Hz). The v_0 of metal is

(a) 1.2×10^{14} Hz (b) 8×10^{15} Hz

(c) 1.2×10^{16} Hz (d) 4×10^{12} Hz

51. Gaseous benzene reacts with hydrogen gas in presence of a nickel catalyst to form gaseous cyclohexane according to the reaction,

$C_6H_6(g) + 3H_2(g) \longrightarrow C_6H_{12}(g)$

A mixture of C_6H_6 and excess H_2 has a pressure of 60 mm of Hg in an unknown volume. After the gas had been passed over a nickel catalyst and all the benzene converted to cyclohexane, the pressure of the gas was 30 mm of Hg in the same volume at the same temperature. The fraction of C_6H_6 (by volume) present in the original volume is

(a) 1/3 (b) 1/4

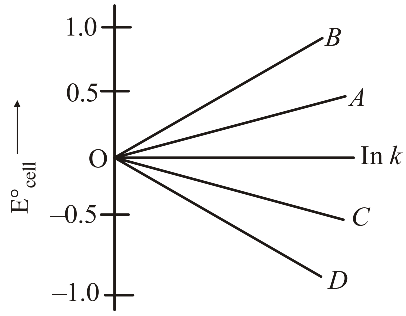
(c) 1/5 (d) 1/6

52. An alloy of copper, silver and gold is found to have copper atom constituting the ccp lattice. If silver atom occupy the edge centres and gold atom is present at body centred, the alloy has a formula

(a) Cu_4Ag_2Au (b) Cu_4Ag_4Au

(c) Cu_4Ag_3Au (d) $CuAgAu$

53. Given, $\Delta G° = -nFE°_{cell}$ and $\Delta G° = -RT\ln k$. The value of $n = 2$ will be given by the slope of which line in the figure



(a) *OA* (b) *OB*
(c) *OC* (d) *OD*

54. The false statements among the following are

I. A primary carbocation is less stable than a tertiary carbocation.

II. A secondary propyl carbocation is less stable than allyl carbocation.

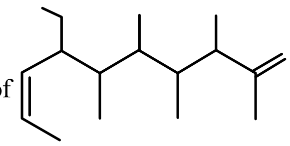
III. A tertiary free radical is more stable than a primary free radical.

IV. Isopropyl carbanion is more stable than ethyl carbanion.

(a) I and II (b) II and Ill
(c) I and IV (d) II and IV

55. A colourless water soluble solid *A* on heating gives equimolar quantities of *B* and C. *B* gives dense white fumes with HCl and *C* does so with NH_3. *B* gives brown precipitate with Nessler's reagent and *C* gives white precipitate with nitrates of Ag^+, Pb^+ and Hg^+. *A* is

(a) NH_4Cl (b) NH_4CO_3
(c) NH_4NO_2 (d) $FeSO_4$

56. The IUPAC name of  is

(a) 4-ethyl-5,6,7,9-tetramethyldeca-2, 9-diene
(b) 7-ethyl-2,4,5,6-tetramethyldeca-1, 8-diene
(c) 7-ethyl-2,4,5,6-tetramethyldeca-1, 7-diene
(d) 7-(1-propenyl)-2,3,4,5-tetramethyl non-1-ene

57. Caffeine has a molecular weight of 194 u. If it contains 28.9% by mass of nitrogen, number of atom of nitrogen in one molecule of caffeine is

(a) 4 (b) 6
(c) 2 (d) 3

58. A compound *X* on heating gives a colourless gas. The residue is dissolved in water to obtain *Y*. Excess CO_2 is passed through aqueous solution of *Y* when *Z* is formed. *Z* on gentle heating gives back *X*. The compound *X* is

(a) $Ca(HCO_3)_2$ (b) $CaCO_3$
(c) $NaHCO_3$ (d) $Na_2\,CO_3$

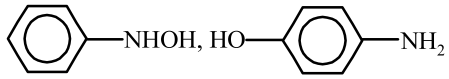
59. Which two sets of reactants best represents the amphoteric character of $Zn(OH)_2$?

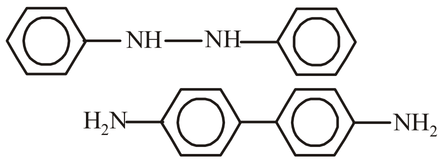
Set I $Zn(OH)_2$(s) and OH (*aq*)
Set II $Zn(OH)_2$(s) and H_2O (*l*)
Set III $Zn(OH)_2$(s) and H^+ (*aq*)
Set IV $Zn(OH)_2$(s) and NH_3 (*aq*)

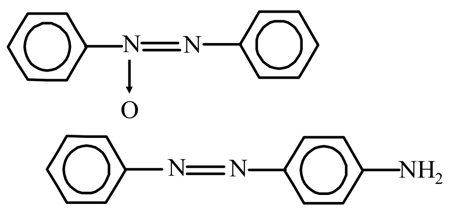
(a) III and II (b) I and III
(c) IV and I (d) II and IV

60. $C_6H_5—NO_2 \xrightarrow[NH_4Cl]{\text{Zn dust}} A \xrightarrow[\text{conc. HCl}]{\text{cold}} B.$

A and *B* respectively are

(a) 

(b) 

(c) 

(d) None of the above

61. Point out incorrect stability order

(a) $[Cu(NH_3)_4\,]^{2+} < [Cu(en)_2]^{2+} < [Cu(trien)]^{2+}$
(b) $[Fe(H_2O)_6]^{3+} < [Fe(NO_2)_6]^{3-} < [Fe\,(NH_3)_6]^{3+}$
(c) $[Co(H_2O)_6]^{3+} < [Rh(H_2O)_6\,]^{3+} < [Ir\,(H_2O)_6]^{3+}$
(d) $[Cr(NH_3)_6\,]^{+} < [Cr(NH_3)_6\,]^{2+} < [Cr\,(NH_3)_6]^{3+}$

62. Consider the following changes

$M(s) \longrightarrow M(g)$ ...(1)
$M(g) \longrightarrow M^{2+}(g) + 2e^-$ ...(2)
$M(g) \longrightarrow M^+(g) + e^-$ ...(3)
$M^+(g) \longrightarrow M^{2+}(g) + e^-$ ...(4)
$M(g) \longrightarrow M^{2+}(g) + 2e^-$ ...(5)

The second ionisation energy of M could be determined from the energy values associated with

(a) $1+2+4$ (c) $1+5-3$
(b) $2+3-4$ (d) 5–3

63. In benzene, the triple bond consists of
(a) one *sp-sp* sigma bond and two *p-p* pi bonds
(b) two *sp-sp* sigma bonds and one *p-p* pi bond
(c) one sp^2-sp^2 sigma bond, one *p-p* pi bond
(d) one sp^2-sp^2 sigma bond, one sp^2-sp^2 pi bond and one *p-p* pi bond

64. In keto-enol tautomerism of dicarbonyl compounds; the enol-form is preferred in contrast to the keto-form, this is due to
(a) presence of carbonyl group on each side of —CH_2— group
(b) resonance stabilisation of enol form
(c) presence of methylene group
(d) rapid chemical exchange

65. An organic compound having carbon, hydrogen and sulphur contains 4% of sulphur. The minimum molecular weight of the compound is
(a) 200 (b) 400
(c) 600 (d) 800

66. Which one of the following is a case of negative adsorption?
(a) Acetic acid solution in contact with animal charcoal.
(b) Dilute KCl solution in contact with blood charcoal.
(c) Concentration KCl solution in contact with blood charcoal.
(d) H_2 gas in contact with charcoal at 300 K.

67. The concentrations of the reactant A in the reacntion $A \rightarrow B$ at different times are given below

Concentration (*M*)	**Time (Minutes)**
0.069	0
0.052	17
0.035	34
0.018	51

The rate constant of the reaction according to the correct order of reaction is
(a) 0.001 M/min (b) 0.001 min^{-1}
(c) 0.001 min/M (d) 0.001 M^{-1} min^{-1}

68. The ratio of slopes of K_{max} vs V and V_0 vs v curves in the photoelectric effects gives (v = frequency, K_{max} = maximum kinetic energy, v_0 = stopping potential)
(a) the ratio of Planck's constant of electronic charge
(b) work function
(c) Planck's constant
(d) charge of electron

69. With excess of water, both P_2O_5 and PCl_5 give
(a) H_3PO_3 (b) H_3PO_2
(c) H_3PO_4 (d) $H_4P_2O_7$

70. The dissolution of $AI(OH)_3$ by a solution of NaOH results in the formation of
(a) $[AI(H_2O)_4(OH)_2]^+$
(b) $[AI(H_2O)_3(OH)_3]$
(c) $[AI(H_2O)_2(OH)_4]^-$
(d) $[AI(H_2O)_6(OH)_3]$

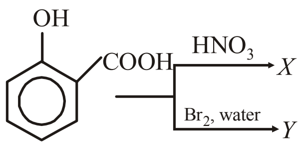
71. Which of the following does not exist?
(a) $KI + I_2 \longrightarrow KI_3$
(b) $KF + F_2 \longrightarrow KF_3$
(c) $KBr + ICl_2 \longrightarrow K[BrICl]$
(d) $KF + BrF_3 \longrightarrow K[BrF_4]$

72. If the ionisation energy and electron affinity of an element are 275 and 86 kcal mol^{-1} respectively, then the electronegativity of the element on the Mulliken scale is
(a) 2.8 (b) 0.0
(c) 4.0 (d) 2.6

73. Which of the following sets of reactants is used for preparation of paracetamol from phenol?
(a) HNO_3, H_2/Pd, $(CH_3CO)_2O$
(b) H_2SO_4, H_2/Pd, $(CH_3CO)_2O$
(c) $C_6H_5N_2Cl$, $SnCl_2$/HCl, $(CH_3CO)_2O$
(d) Br_2/H_2O, Zn / HCl, $(CH_3CO)_2O$

74. A certain compound gives negative test with ninhydrin and positive test with Benedict's solution. The compound is
(a) a protein (b) a monosaccharide
(c) a lipid (d) an amino acid

75. Super glue or crazy glue is
(a) poly (methyl methacrylate)
(b) poly (ethyl acrylate)
(c) poly (methyl α-cyanoacrylate)
(d) poly (ethyl methacrylate)

76. 

X and Y respectively are

(a) picric acid, 2, 4, 6-tribromophenol
(b) 5-nitrophenol acid, 5-bromosalicylic acid
(c) o-nitrophenol, O-bromophenol
(d) 3, 5-dinitrosalicylic acid, 3, 5-dibromosalicylic acid

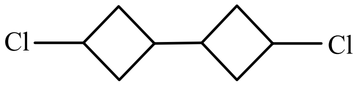
77. In the cannizzaro reaction given below

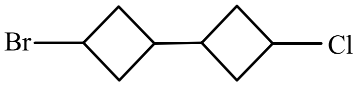
$$2Ph—CHO \xrightarrow{OH^-} Ph—CH_2—OH + PhCO_2^-$$

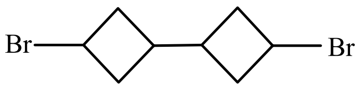
the slowest step is

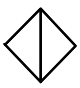
(a) the attack of $\bar{O}H$ at the carbonyl group
(b) the transfer of hydride ion to the carbonyl group
(c) the abstraction of a proton from the carboxylic acid
(d) the deprotonation of $Ph\text{-}CH_2OH$

78. The reaction of 1-bromo-3-chlorocyclobutane with metallic sodium in dioxane under reflux conditions gives

(a) 

(b) 

(c) 

(d) 

79. Identify Z in the following reaction sequence

$$CH_3CH_2CH_2OH \xrightarrow[160-180°C]{Conc.\,H_2SO_4} X$$
$$X \downarrow Y \xrightarrow[(ii)\ NaNH_2]{(i)\ Alc.\ KOH} Z$$

(a) $CH_3—CH(NH_2)—CN_2\,NH_2$
(b) $CH_3—CHOH—CH_2OH$
(c) $CH_3—C(OH)=CH_2$
(d) $CH_3—C\equiv CH$

80 Which of the following reactions is used to prepare isobutane?

(a) Wurtz reaction of C_2H_5Br
(b) Hydrolysis of n-butylmagnesium iodide
(c) Reduction of propanol with red phosphorus and HI
(d) Decarboxylation of 3-methylbutanoic acid

PART - III (MATHEMATICS)

81. The differential equation
$(3x + 4y + 1)dx + (4x + 5y + 1)dy = 0$
represents a family of

(a) circles (b) parabolas
(c) ellipses (d) hyperbolas

82. If $\Delta(r) = \begin{vmatrix} r & r^3 \\ 1 & n(n+1) \end{vmatrix}$, then $\sum_{r=1}^{n} \Delta(r)$ is equal to

(a) $\sum_{r=1}^{n} r^2$ (b) $\sum_{r=1}^{n} r^3$
(c) $\sum_{r=1}^{n} r$ (d) $\sum_{r=1}^{n} r^4$

83. If A, B, C are three events associated with a random experiment, then

$P(A)\,P\left(\frac{B}{A}\right)P\left(\frac{C}{A} \cap B\right)$ is

(a) $P(A \cup B \cap C)$ (b) $P(A \cap B \cap C)$
(c) $P\left(\frac{C}{A} \cap B\right)$ (d) $P\left(\frac{B}{A}\right)$

84. If $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix}$, then rank (A) is equal to

(a) 4 (b) 1
(c) 2 (d) 3

85. The probability of atleast one double six being thrown in n throws with two ordinary dice is greater than 99%.
Then, the least numerical value of n is

(a) 100 (b) 164
(c) 170 (d) 184

86. Find the value of k for which the simultaneous equations $x + y + z = 3$; $x + 2y + 3z = 4$ and $x + 4y + kz = 6$ will not have a unique solution.

(a) 0 (b) 5
(c) 6 (d) 7

87. If the complex number z lies on a circle with centre at the origin and radius $\frac{1}{4}$, then the complex number $-1+8z$ lies on a circle with radius

(a) 4 (b) 1
(c) 3 (d) 2

88. If line $y = 2x + c$ is a normal to the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$, then

(a) $c=\frac{2}{3}$ (b) $c=\sqrt{\frac{73}{5}}$

(c) $c=\frac{14}{\sqrt{73}}$ (d) $c=\sqrt{\frac{5}{7}}$

89. If $x^2+x+1=0$, then the value of $\sum_{n=1}^{6}\left(x^n+\frac{1}{x^n}\right)^2$ is

(a) 13 (b) 12
(c) 9 (d) 14

90. If p : It rains today, q : I go to school, r : I shall meet my friends and s : I shall go for a movie, then which of the following is the proportion? If it does not rain or if I do not go to school, then I shall meet my friend and go for a movie.

(a) $(\sim p \wedge \sim q) \Rightarrow (r \wedge s)$

(b) $\sim (p \wedge q) \Rightarrow (r \wedge s)$

(c) $\sim (p \vee q) \Rightarrow (r \vee s)$

(d) None of these

91. If the matrix $A=\begin{bmatrix}1 & 3 & 1\\ -1 & 2 & -3\\ 0 & 1 & 2\end{bmatrix}$ then adj (adj A) is equal to

(a) $\begin{bmatrix}12 & 36 & 12\\ -12 & 24 & -36\\ 0 & 12 & 24\end{bmatrix}$ (b) $\begin{bmatrix}12 & 26 & -12\\ 24 & 36 & -36\\ 0 & 12 & -24\end{bmatrix}$

(c) $\begin{bmatrix}12 & -12 & 36\\ 24 & -24 & -36\\ 0 & 12 & 24\end{bmatrix}$ (d) None of there

92. Which of the following options is not the asymptote of the curve $3x^3+2x^2y-7xy^2+2y^3-14xy+7y^2+4x+5y=0$?

(a) $y=\frac{-1}{2}x-\frac{5}{6}$ (b) $y=x-\frac{7}{6}$

(c) $y=2x+\frac{3}{7}$ (d) $y=3x-\frac{3}{2}$

93. If N is a set of natural numbers, then under binary operation $a\cdot b=a+b$, $(N,\cdot)$ is

(a) quasi-group (b) semi-group
(c) monoid (d) group

94. $\int\frac{dx}{\cos x+\sqrt{3}\sin x}$ equals

(a) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\frac{1}{3}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\log\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)+C$

95. If (2, 7, 3) is one end of a diameter of the sphere $x^2+y^2+z^2-6x-12y-2z+20=0$, then the coordinates of the other end of the diameter are

(a) (–2, 5, –1) (b) (4, 5, 1)
(c) (2, –5, 1) (d) (4, 5, –1)

96. The two lines $x=my+n$, $z=py+q$ and $x=m'y+n'$, $z=p'y+q'$ are perpendicular to each other, if

(a) $mm'+pp'=1$ (b) $\frac{m}{m'}+\frac{p}{p'}=-1$

(c) $\frac{m}{m'}+\frac{p}{p'}=1$ (d) $mm'+pp'=-1$

97. A tetrahedron has vertices at O(0, 0, 0), $A(1,-2,1)$, $B(-2,1,1)$ and $C(1,-1,2)$. Then, the angle between the faces OAB and ABC will be

(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{-1}{6}\right)$

(c) $\cos^{-1}\left(\frac{-1}{3}\right)$ (d) $\cos^{-1}\left(\frac{1}{4}\right)$

98. If a line segment OP makes angles of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with X-axis and Y-axis, respectively. Then, the direction cosines are

(a) $\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2},\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}},\frac{1}{2},\frac{1}{\sqrt{2}}$

(c) $1,\sqrt{3},1$ (d) $1,\frac{1}{\sqrt{3}},1$

99. If p, q, r are simple propositions with truth values T, F, T, then the truth value of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is

(a) true (b) false

(c) true, if r is false (d) true, if q is true

100. On the interval [0, 1], the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

101. If $|z| \geq 3$, then the least value of $\left|z+\frac{1}{4}\right|$ is

(a) $\frac{11}{2}$ (b) $\frac{11}{4}$

(c) 3 (d) $\frac{1}{4}$

102. The normal at the point $(at_1^2, 2at_1)$ on the parabola meets the parabola again in the point $(at_2^2, 2at_2)$, then

(a) $t_2 = -t_1 + \frac{2}{t_1}$ (b) $t_2 = -t_1 - \frac{2}{t_1}$

(c) $t_2 = t_1 - \frac{2}{t_1}$ (d) $t_2 = t_1 + \frac{2}{t_1}$

103. If $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then the angle θ between **a** and **b** is given by

(a) $\tan^{-1}(1)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\sec^{-1}(1)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

104. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(a) $(4\sqrt{2}-2)$ sq units

(b) $(4\sqrt{2}+2)$ sq units

(c) $(4\sqrt{2}-1)$ sq units

(d) $(4\sqrt{2}+1)$ sq units

105. If a, b and c are three non-coplanar vectors, then $(\mathbf{a}+\mathbf{b}-\mathbf{c}) \cdot [(\mathbf{a}-\mathbf{b}) \times (\mathbf{b}-\mathbf{c})$ equals

(a) 0 (b) $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$

(c) $\mathbf{a} \cdot \mathbf{c} \times \mathbf{b}$ (d) $3\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$

106. If there is an error of m% in measuring the edge of cube, then the percent error in estimating its surface area is

(a) $2m$ (b) $3m$

(c) $1m$ (d) $4m$

107. If the rectangular hyperbola is $x^2 - y^2 = 64$. Then, which of the following is not correct?

(a) The length of latus rectum is 16

(b) The eccentricity is $\sqrt{2}$

(c) The asymptotes are parallel to each other

(d) The directrices are $x = \pm 4\sqrt{2}$

108. The equation of tangents to the hyperbola $3x^2 - 2y^2 = 6$, which is perpendicular to the line $x - 3y = 3$, are

(a) $y = -3x \pm \sqrt{15}$ (b) $y = 3x \pm \sqrt{6}$

(c) $y = -3x \pm \sqrt{6}$ (d) $y = 2x \pm \sqrt{15}$

109. $\lim_{x \to \pi/4} \frac{\tan x - 1}{\cos 2x}$ is equal to

(a) 1 (b) 0

(c) −2 (d) −1

110. The area of the region bounded by the curves $x^2 + y^2 = 9$ and $x + y = 3$ is

(a) $\frac{9\pi}{4} + \frac{1}{2}$ (b) $\frac{9\pi}{4} - \frac{1}{2}$

(c) $9\left(\frac{\pi}{4} - \frac{1}{2}\right)$ (d) $9\left(\frac{\pi}{4} + \frac{1}{2}\right)$

111. For any three vectors **a**, **b** and **c**, [a + b, b + c, c + a] is

(a) [a b c] (b) 3 [a b c]

(c) 2 [a b c] (d) 0

112. $\int_0^{\pi/2} \sin 2x . \log \tan x \, dx$ is equal to

(a) 0 (b) 2

(c) 4 (d) 7

113. If the mean and variance of a binomial distribution are 4 and 2, respectively. Then, the probability of atleast 7 successes is

(a) $\frac{3}{214}$ (b) $\frac{4}{173}$

(c) $\frac{9}{256}$ (d) $\frac{7}{231}$

114. The shortest distance between the lines

$$\frac{x-7}{3}=\frac{y+4}{-16}=\frac{z-6}{7}$$

and $\frac{x-10}{3}=\frac{y-30}{8}=\frac{4-z}{5}$ is

(a) $\frac{234}{7}$ units (b) $\frac{288}{21}$ units

(c) $\frac{221}{3}$ units (d) $\frac{234}{21}$ units

115. If a plane passing through the point (2, 2, 1) and is perpendicular to the planes $3x+2y+4z+1=0$ and $2x+y+3z+2=0$. Then, the equation of the plane is

(a) $2x-y-z-1=0$ (b) $2x+3y+z-1=0$

(c) $2x+y+z+3=0$ (d) $x-y+z-1=0$

116. From a city population, the probability of selecting a male or smoker is $\frac{7}{10}$, a male smoker is $\frac{2}{5}$ and a male, if a smoker is already selected, is $\frac{2}{3}$. Then, the probability of

(a) selecting a male is $\frac{3}{2}$

(b) selecting a smoker is $\frac{1}{5}$

(c) selecting a non-smoker is $\frac{2}{5}$

(d) selecting a smoker, if a male is first selected, is given by $\frac{8}{5}$

117. At $t = 0$, the function $f(t) = \frac{\sin t}{t}$ has

(a) a minimum

(b) a discontinuity

(c) a point of inflexion

(d) a maximum

118. Using Rolle's theorem, the equation

$a_0x^n + a_1x^{n-1} + \ldots + a_n = 0$

has atleast one root between 0 and 1, if

(a) $\frac{a_0}{n}+\frac{a_1}{n-1}+\ldots.+a_{n-1}=0$

(b) $\frac{a_0}{n-1}+\frac{a_1}{n-2}+\ldots.+a_{n-2}=0$

(c) $na_0+(n-1)a_1+\ldots.+a_{n-1}=0$

(d) $\frac{a_0}{n+1}+\frac{a_1}{n}+\ldots.+a_n=0$

119. Which of the following inequality is true for $x>0$?

(a) $\log(1+x)<\frac{x}{1+x}<x$

(b) $\frac{x}{1+x}<x<\log(1+x)$

(c) $x<\log(1+x)<\frac{x}{1+x}$

(d) $\frac{x}{1+x}<\log(1+x)<x$

120. The solution of $\frac{d^2x}{dy^2}-x=k$, where k is a non-zero constant, vanishes when $y = 0$ and tends of finite limit as y tends to infinity, is

(a) $x = k(1 + e^{-y})$ (b) $x = k(e^y + e^{-y} - 2)$

(c) $x = k(e^{-y} - 1)$ (d) $x = k(e^y - 1)$

SOLUTIONS

PART - I (PHYSICS)

1. (a) As r increase, the potential energy increases. Thus, it decreases kinetic energy of hydrogen atom. So, when an atom jumps from one energy level to the higher level, its potential energy increases and kinetic energy decreases.

2. (b) Given: $T_{1/2} = 4.47 \times 10^8$ yr

$$\frac{N}{N_0} = \frac{60}{100} = \left(\frac{1}{2}\right)^n \Rightarrow 2^n = \frac{10}{6}$$

Apply logarithm on both sides

n log2 = log 10 – log 6

$\Rightarrow$ n × 0.3 = 1 – 0.778 = 0.22

$$\Rightarrow \quad n = \frac{0.222}{0.3} = 0.74$$

So, $t = nT_{1/2} = 0.74 \times 4.47 \times 10^8$

or, $t = 3.3 \times 10^8$ yr

3. (b) Y(n, α) the nucleus splits into α-particle and neutrons

i.e. $_ZY^A + {}_0n^1 \longrightarrow {}_3L_i{}^7 + {}_2He^4$

So A + 1 = 7 + 4 ⇒ A = 10

and Z + 0 = 3 + 2 or Z = 5

Hence, the nucleus of element Y is boron $_5Y^{10} = {}_5B^{10}$.

4. (c) Energy of electron in nth orbit of hydrogen atom

$$E_n = -\frac{13.6}{n^2} eV$$

$$\Rightarrow \quad 3.4 = -\frac{13.6}{n^2} \Rightarrow n^2 = 4$$

or, n = 2

Angular momentum of electron

$$L = \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi}$$

5. (b) As, we know de-Broglic wavelength,

$$\lambda = \frac{h}{p}$$

$$\therefore \quad \lambda \propto \frac{1}{p}$$

$$\Rightarrow \quad \frac{\Delta p}{p} = -\frac{\Delta\lambda}{\lambda} \therefore \left|\frac{\Delta p}{p}\right| = \left|\frac{\Delta\lambda}{\lambda}\right|$$

$$\Rightarrow \quad \frac{p'}{p} = \frac{0.20}{100} = \frac{1}{500}$$

or, p = 500 p′

6. (a) The emitted electrons may lie near the surface and can have a maximum amount of energy E_0.

If they are from deep inside, then energy is less than E_0.

7. (a) The n-type semiconductor has excess of free electrons for conduction. The total number of electrons in an atom is equal to the total number of protons in the nucleus. So, n-type semiconductor is neutral.

8. (c) The output is obtained for half cycle only in half wave rectifier. Therefore, frequency of the ripple is same as that of the input i.e. 50 Hz.

9. (c) Given: $\Delta I_C = 1mA = 10^{-3}$ A

$\Delta I_b = 15\ \mu A = 15 \times 10^{-6}$ A

$R_L = 5k\Omega = 5 \times 10^3\ \Omega$.

Ri = 330 Ω

The voltage gain of an amplifier

$$A_r = \frac{\Delta I_C \times R_L}{\Delta I_b \times R_i}$$

$$= \frac{10^{-3} \times 5 \times 10^3}{15 \times 10^{-6} \times 330} \approx 1010$$

10. (a) As we know, output of OR gate

Y = A + B

Output of AND gate

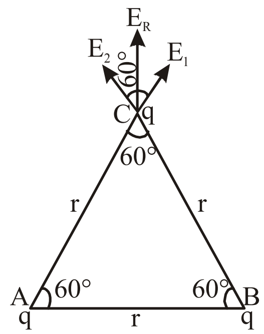
Y′ = Y.C

$\Rightarrow$ Y′ = (A + B).C

If C = 0 irrespective of A and B, then output Y must be zero.

11. (c) Due to charge at A and B magnitude of intensity of electric field at point C

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



Net intensity at point C is

$$E_R = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 60°}$$

$$= \sqrt{E_1^2 + E_1^2 + 2E_1^2 \times \frac{1}{2}} = \sqrt{3}E_1 = \frac{\sqrt{3}q}{4\pi\varepsilon_0 r^2}$$

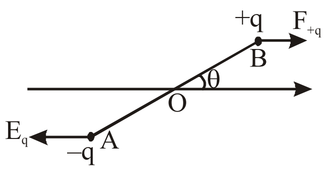
12. (b) Torque when the wire is brought in a uniform field E.

$\tau = qEL \sin\theta$

$= qEL\theta$ [$\because \theta$ is very small]

Moment of inertia of rod AB about O

$$I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$



$\tau = I\alpha$.

$$\therefore \quad \alpha = \frac{\tau}{I} = \frac{qEL\theta}{\frac{mL^2}{2}}$$

$$\Rightarrow \quad \omega^2\theta = \frac{2qEL\theta}{mL^2} \quad [\because \theta = \omega^2\theta]$$

$$\Rightarrow \quad \omega^2 = \frac{2qE}{mL}$$

Time period of the wire

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mL}{2qE}}$$

The rod will become parallel to the field in time $\frac{T}{4}$.

$$\therefore \quad t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$$

13. (d) The energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C \times (200)^2 = 2C \times 10^4 \text{ J}$$

This energy is used to heat up the block. Let $\Delta\theta$ be the rise in temperature, then heat energy

$Q = ms\Delta\theta = 0.1 \times 250 \times 0.4 = 10J$

Now,

$2C \times 10^4 = 10$

$$\Rightarrow \quad C = \frac{10}{2 \times 10^4} = 5 \times 10^{-4} = 500\ \mu F$$

14. (b) Capacitance of air capacitor

$$C_0 = \frac{\varepsilon_0 A}{d} = 3\mu F \quad ....(i)$$

When a dielectric of permittivity ε_r and dielectric constant K is introduced between the plates, then

$$\text{Capacitance, } C = \frac{K\varepsilon_0 A}{d} = 15\mu F \quad ....(ii)$$

Dividing eq. (ii) by (i), we get

$$\frac{C}{C_0} = \frac{d}{\frac{\varepsilon_0 A}{d}} = \frac{15}{3}$$

$\Rightarrow$ K = 5

$\therefore$ permittivity of the medium

$\varepsilon_r = \varepsilon_0 K$

$= 8.85 \times 10^{-12} \times 5 = 0.44 \times 10^{-10}$

15. (d) using, $R = \rho\frac{l}{A}$

$$R_1 : R_2 : R_3 = \frac{l_1}{A_1} : \frac{l_2}{A_2} : \frac{l_3}{A_3}$$

$$= \frac{l_1^2}{V_1} : \frac{l_2^2}{V_2} : \frac{l_3^2}{V_3}$$

$$= \frac{l_1^2}{(m_1 d)} : \frac{l_2^2}{(m_2 d)} : \frac{l_3^2}{(m_3 d)}$$

$= \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3}$

$= \frac{5^2}{2} : \frac{3^2}{3} : \frac{2^2}{5} = 125 : 30 : 8$

16. (b) Resistance of lamp

$$R_0 = \frac{V^2}{P} = \frac{(30)^2}{90} = 10\Omega$$

Current in the lamp

$$I = \frac{V}{R_0} = \frac{30}{10} = 3A$$

As the lamp is operated on 120V DC, then resistance becomes

$$R' = \frac{V'}{i} = \frac{120}{3} = 40\Omega$$

For proper glow, a resistance R is joined in series with the bulb

$R' = R + R_0$

$\Rightarrow$ $R^{\alpha} = R' - R_0 = 40 - 10 = 30\Omega$

17. (d) Let us Consider a cell of emf E and balancing length l_1

$E = kl_1$

potential difference is balanced by length l_2.

$V = kl_2$

Internal resistance of the cell

$$r = \left(\frac{E-V}{V}\right)R = \left(\frac{E}{V} - 1\right)R = \left(\frac{l_1}{l_2} - 1\right)R$$

$$= \left(\frac{560}{560-60} - 1\right)10 = \left(\frac{56}{50} - 1\right)10$$

$$= \frac{6}{5} = 1.2\Omega$$

18. (d) Let the emf of each source be E. When they are connected in series, the current in the circuit

$$I = \frac{E_{tot}}{R_{tot}} = \frac{E+E}{r_1 + r_2 + R} = \frac{2E}{r_1 + r_2 + R}$$

$\therefore$ potential drop across the cell of internal resistance $r_2, \left(\frac{2E}{r_1 + r_2 + R}\right) r_2$

Hence, $E - \frac{2E}{(r_1 + r_2 + R)} r_2 = 0$

$r_1 + r_2 + R = 2r_2$

$\Rightarrow$ $R = r_2 - r_1$

19. (a) Here, the magnetic force (Bqv) will provide the necessary centripetal force $\left(\frac{mv^2}{r}\right)$

$\therefore$ $Bqv = \frac{mv^2}{r}$

$\Rightarrow$ $Bqr = mv$

For electron and proton, the magnetic field B, charge q and radius r, all same.

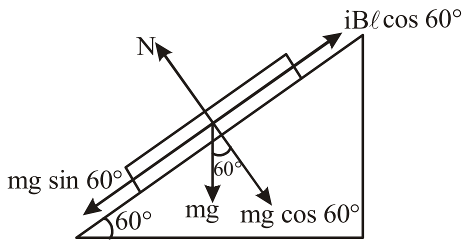
$\therefore$ mv = constant

i.e. $m_e v_e = m_p v_p$

$$v_P = \left(\frac{m_e}{m_p}\right) v_e = \left(\frac{9 \times 10^{-31}}{1.8 \times 10^{-27}}\right) 3 \times 10^6$$

$= 1.5 \times 10^3$ m/s

20. (a) Here two forces acting on the rod simultaneously.



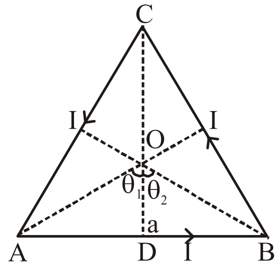
From FBD, mg sin 60 = Bil cos 60°

$$B = \frac{mg}{il} \tan 60°$$

$$= \frac{0.01 \times 10}{173 \times 0.1} \times \sqrt{3} = 1T$$

21. (c) Due to current through side AB Magnetic field at the centre O

$$B_1 = \frac{\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$$



As the magnetic field due to each of the three sides is the same in magnitude and direction.

$\therefore$ Total magnetic field at O is sum of all the fields.

i.e. $B = 3B_1 = \frac{3\mu_0 I}{4\pi a}[\sin\theta_1 + \sin\theta_2]$

Here, $\tan\theta_1 = \frac{AD}{OD} \Rightarrow \tan 60° = \frac{\frac{\ell}{2}}{a}$

$\Rightarrow \quad a = \frac{\ell}{2\sqrt{3}} = \frac{9\times10^{-2}}{2\sqrt{3}}$

Now B

$= 3\times\frac{4\pi\times10^{-7}\times2}{4\pi\times\frac{9\times10^{-2}}{2\sqrt{3}}}[\sin60°+\sin60°]$

$= \frac{4\sqrt{3}}{9}\times10^{-5}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right]$

$= 1.33\times10^{-5}$ T

22. (b) The direction of dB is the direction of vector dl × r. From right hand screw rule, if we place a right handed screw at the point where the magnetic field is needed to be determined and turn its handle from dl to r, then the direction in which the screw advances gives the direction of field dB.

23. (b) Given: current sensitivity = 10 div/mA and there are 100 division on the scale.

$\therefore$ Current required for full scale deflection.

$I_g = \frac{1}{10}\times100\text{mA} = 10\text{mA} = 0.01$ A

Also voltage sensitivity = 2 div/mV

$\therefore$ voltage required for full scale deflection

$V_g = \frac{1}{2}\times100\text{mV} = 0.05\text{V}$

Galvanometer resistance is given by

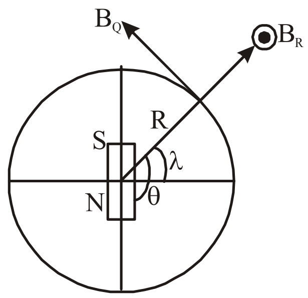
$G = \frac{V_g}{I_g} = \frac{0.05}{0.01} = 5\Omega$

24. For a dipole at position (R, Q)

$B_R = \frac{\mu_0}{4\pi}.\frac{2M\cos\theta}{R^3}$ ....(i)

and $B_Q = \frac{\mu_0}{4\pi}.\frac{M\sin\theta}{R^3}$ ....(ii)

Also $\tan\phi = \frac{B_V}{B_H} = -\frac{B_R}{B_Q}$ ....(iii)



Dividing eq. (i) by (ii)

$\frac{B_R}{B_Q} = \frac{2\cos\theta}{\sin\theta} = 2\cot\theta$ ....(iv)

From eq. (iii) and (iv)

$\tan\phi = -2\cot\theta$

From figure, $\theta = 90° + \lambda$

$\therefore$ $\tan\phi = -2\cot(90+\lambda)$

$\tan\phi = 2\tan\lambda$

25. (b) The hysteresis loop i.e. area of B-H curve is a measure of energy dissipated per cycle per unit volume of the specimen. It depends on the nature of magnetic material.

26. (d) Work done by magnet to turn from angle θ_1 to θ_2

$W = MB(\cos\theta_1 - \cos\theta_2)$

$= MB(\cos0° - \cos45°)$

$= MB\left(1-\frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)MB$

Also torque acting on the magnet

$\tau = MB\sin45° = \frac{MB}{\sqrt{2}}$

$W = \left(\sqrt{2}-1\right).\tau \Rightarrow \tau = \frac{W}{\left(\sqrt{2}-1\right)}$

27. (c) From Lentz's law, the direction of induced emf in a circuit is such that it opposes the magnetic flux that produces it.
So, if the magnetic flux linked with a closed circuit increases the induced current flows in a direction so as to develop a magnetic flux in the opposite direction of original flux.
If the magnetic flux linked with a closed circuit decreases then the induced current flows in the same direction of original flux. So, the induced emf has not direction of its own.

28. (a) Given : N = 20
$B = 10^3$ gauss
$= 10^3 \times 10^{-4}$ T = 0.1T
$A = 5\ cm^2 = 5 \times 10^{-4}\ m^2$
$\theta = 80°$
∴ Flux through the coil
$\phi = NBA \cos\theta$
$= 20 \times 0.1 \times 5 \times 10^{-4} \times \cos 30°$

$$= 10 \times 10^{-4} \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \times 10^{-4} = 865 \times 10^{-4}\,\text{wb}$$

29. (c) In LC circuit, if $X_L = X_C$ then $\omega = \frac{1}{\sqrt{LC}}$

$I_0\ \infty$, so $Z = \frac{E_0}{I_0} = 0.$

As $\frac{1}{\sqrt{LC}}$ is the natural frequency of LC circuit, therefore for an LC circuit if the frequency of applied AC becomes equal to the natural frequency of an AC circuit then the amplitude of current becomes infinite due to zero impedance.

30. (d) Maximum current flows in the circuit in resonance condition Current in the LCR circuit

$$i = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

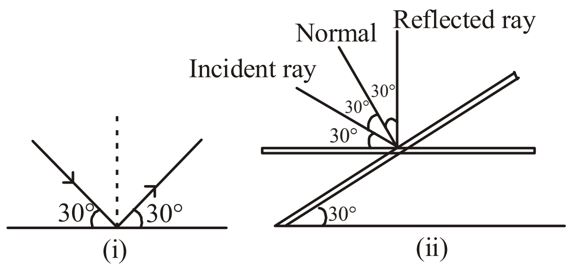
For current to be maximum denominator should be minimum
$(X_L - X_C)^2 = 0$

$$\Rightarrow X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\therefore L = \frac{1}{\omega^2 C} = \frac{1}{(100)^2 \times 10 \times 10^{-6}}$$

$$L = \frac{1}{10}\text{H} = 0.1\text{ H} = 100\text{ mH}$$

31. (c) When a light ray falls on a mirror at an angle 30°, then the reflected ray will make the same angle with the plane as shown in Fig. (i)



In order to make the reflected ray vertical, the mirror should be rotated at an angle of 60°.
So, the mirror should be tilted by $\frac{60°}{2} = 30°$ Fig. (ii)

32. (d) For a compound microscope, magnifying power
$MP = m_e \times m_0$
When the final image is at least distance of distance vision then

$$M_e = 1 + \frac{D}{f_e}$$

$$\therefore MP = m_0\left[1 + \frac{D}{f_e}\right]$$

$$\Rightarrow -35 = m_0\left[1 + \frac{25}{10}\right]$$

$\Rightarrow -35 = m_0 \times 35$
$\Rightarrow m_0 = -10$
The negative sign shows that the image formed by objective is inverted.

33. (d) Using prism formula,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad ....(i)$$

where, A = angle of prism
δ_m = angle of minimum deviation

Given, $\mu = \cot\left(\frac{A}{2}\right) = \frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

So, from Eq. (i)

$$\frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\Rightarrow \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A}{2} + \frac{\delta_m}{2}\right)$

$\Rightarrow \delta_m = \pi - 2A = 180° - 2A$

34. (d) Given:
$P_1 = 2D$; $P_2 = 3D$

$d = \frac{1}{3} m$

We know that $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1.f_2}$

Equivatent power,
$\therefore$ $P = P_1 + P_2 - dP_1.P_2$

$= 2 + 3 - \frac{1}{3} \times 2 \times 3 = 3D$

35. (d) First ray optical path = $\mu_1 L_1$
second ray optical path = $\mu_2 L_2$
So, phase difference

$\Delta\phi = \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda} \times \Delta x$

$\therefore$ $\Delta\phi = \frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$

36. (d) Let the intensity of unpolarised light be I_0, so the intensity of first polaroid is $\frac{I_0}{2}$.
On rotating through 60°, the intensity of light from second polaroid

$I = \left(\frac{I_0}{2}\right)(\cos 60)^2 = \frac{I_0}{2}\frac{1}{4} = \frac{I_0}{8} = 0.125 I_0$

$\therefore$ percentage of incident light transmitted through the system = 12.5%.

37. (b) As the electromagnetic wave is the crossed field of electric and magnetic waves, so the direction of propagation of EM wave is the direction of vector E × B. Here E is upward and (E × B) is towards north.
So, from right hand thumb rule B will be along east.

38. (c) An electromagnetic wave is the wave radiated by an accelerated charge and propagates through space as coupled electric and magnetic field. These fields are oscillating perpendicular to each other.

39. (d) From Rayleigh's law of scattering, intensity

$I \propto \frac{1}{\lambda^4}$

$\therefore$ $\frac{I_1}{I_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$

$\Rightarrow \frac{I_1}{I_2} = \left(\frac{330}{880}\right)^4 = \left(\frac{3}{8}\right)^4 = \frac{81}{4096}$

$I_2 = \frac{4096}{81}Å = (50.557)Å$

40. (c) As we knows, radius of an atom,
$\gamma_A \approx 10^{-10}$ m
radius of nucleus,. $\gamma_B \approx 10^{-15}$ m
So, ratio of their volumes

$$\frac{V_A}{V_N} = \frac{\frac{4}{3}\pi r_A^3}{\frac{4}{3}\pi r_N^3} = \left(\frac{r_A}{I_N}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3$$

$\therefore$ $V_A : V_N = 10^{15} : 1$

PART - II (CHEMISTRY)

41. (b) Balanced equations are

$$3Cu + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O \quad ...(i)$$

$$Cu + 4HNO_3 \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O \quad ...(ii)$$

Here NO and NO_2 are evolved in equal volumes, hence, on adding Eqs. (i) and (ii)

$$4Cu + 12\ HNO_3 \longrightarrow 4Cu(NO_3)_2 + 2NO + 2NO_2 + 6H_2O$$

or $2Cu + 6HNO_3 \longrightarrow 2Cu(NO_3)_2 + NO + NO_2 + 3H_2O$

Hence, coefficients x and y of Cu and HNO_3 are 2 and 6 respectively.

42. (c) $[S^{2-}] = 10^{-23}$ mol L^{-1}.
$[M^{2+}] = 10^{-2}M$
Ionic product, $K_{1P} = [M^{2+}][S^{2-}] = 10^{-25}$,
∴ ionic product is greater than K_{sp} of CuS and CdS.
Therefore, all except MnS and FeS are precipitated.

43. (c) $K = \dfrac{[CO(g)][H_2(g)]}{[H_2O(g)]}$

Concentration will increase, on halving the volume. There are two terms in numerator. So to keep K constant, concentration of $[H_2O]$ should increase much more.

44. (d) During smelting process of copper from copper pyrites reactions are

$Cu_2O + FeS \longrightarrow Cu_2S + FeO$

$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$

45. (d) $\underset{(xCl_2)}{SnCl_2} + \underset{(yCl_2)}{HgCl_2} \longrightarrow \underset{(xCl_2)}{SnCl_4} + \underset{(y)}{Hg}$

$$HgO \xrightarrow[>400°C]{\Delta} \underset{(y)}{Hg} + \frac{1}{2}O_2$$

So ore of γ is HgS i.e., Cinnabar.

46. (a) $(\Delta G°)\text{reaction} = \Delta G_f^0 \text{ (Products)} - \Delta G_f^0 \text{ (reactants)}$

∴ $264.4 = [2\,\Delta G_f^0\ (H^+) + 2\,\Delta G_f^0\ (Cl^{-})]$

or $264.4 = -\left[\Delta G_f^0(H_2) + \Delta G_f^0(Cl_2)\right]$

$= \left[0 + 2\Delta G_f^0\left(Cl^-\right)\right] + [0 + 0]$

or, $-262.4 = 2\Delta\, G_f^0\left(Cl^-\right)$

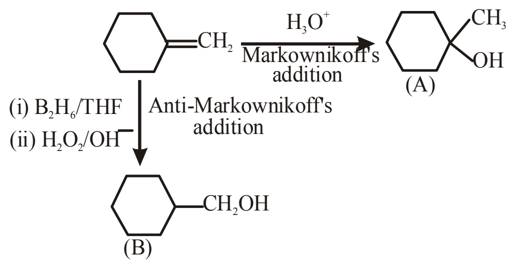
or, $\Delta\, G_f^0\left(Cl^-\right) = -131.\ 2 kJmol^{-1}$.

47. (a) 2L of 3M $AgNO_3$ will contains 6 moles of $AgNO_3$.
3L of 1 M $BaCl_2$ will contain 3 moles of $BaCl_2$.
$2AgNO_3 + BaCl_2 \longrightarrow 2AgCl + Ba(NO_3)_2$.
So, 6 moles of $AgNO_3$ will react with 3 moles on $BaCl_2$ it means, two solution will react completely to form 3 moles of $Ba(NO_3)_2 \equiv 6$ moles of NO_3^- ions in 2 + 3 = 5L solution

Hence, molarity of $NO_3^- = \dfrac{6}{5} = 1.2$ M

48. (b) AsO_3^{3-}, ClO_3^{3-}, and SO_3^{2-} have sp^2 hybridisation and hence are non-planar species, while NO_3^-, CO_3^{2-} and BO_3^{3-} have sp^2 hybridisation and hence are planar species

49. (d)



50. (b) $(KE)_1 = hv_1 - hv_0$
$(KE)_2 = hv_2 - hv_0$
As, $(KE)_1 = 2 \times (KE)_2$
∴ $hv_1 - hv_0 = 2(hv_2 - hv_0)$
or, $hv_0 = 2hv_2 - hv_1$
or, $v_0 = 2v_2 - v_1$
$= 2\times (2 \times 10^{16}) - (3.2 \times 10^{16})$
$= 0.8 \times 10^{16}$ Hz $= 8 \times 10^{15}$ Hz

51. (d) Let initially, pressure of C_6H_6 (g) is p_1 mm and for $H_2(g)$ is p_2 mm

∴ $p_1 + p_2 = 60$ mm ....(i)

After the reaction pressure of

$C_6H_6(g) = 0$ (as all C_6H_6 has reacted)

$H_2(g) = p_2 - 3p_1$

So, total pressure = $p_2 - 3p_1 + p_1 = 30$ mm

$p_2 - 2p_1 = 30$ mm ....(ii)

On solving equation (i) and (ii)

$p_1 = 10$ mm, $p_2 = 50$ mm

Fraction of C_6H_6 by volume = moles fraction fraction of pressure $= \frac{10}{60} = \frac{1}{6}$

52. (c) In the unit cell number of Cu atoms (fcc/ccp)

$= 8 \times \frac{1}{6} + 6 \times \frac{1}{2} = 4$

As Ag atoms occupying edge centred

$= 12 \times \frac{1}{4} = 3$

and Au atoms are presents at the body centred = 1

∴ formula, Cu_4Ag_3Au.

53. (b) As we know that $-nFE°_{cell} = -RT \ln k$ or

$E°_{cell} = \frac{RT}{nF} \ln k.$

Plot of ln k or $E°_{cell}$ will have slope

$= \frac{1}{2}\frac{RT}{F}$

54. (d) Since 2° propyl carbocation is little more stable than allyl carbocation and ethyl carbanion is more stable than isopropyl carbanion.

55. (a) $\underset{\substack{A \\ \text{colourless}}}{NH_4Cl} \xrightarrow{\Delta} \underbrace{\underset{B}{NH_3} + \underset{C}{HCl}}_{\text{equimolar quantities}}$

$\underset{B}{NH_3} + HCl \longrightarrow \underset{\text{dense white fumes}}{NH_4Cl}$

$NH_3 + 2\underset{\text{Nessler's reagent}}{K_2[HgI_4]} + 3KOH \longrightarrow$

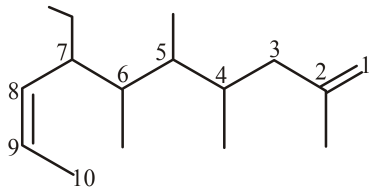
$\underset{\text{brown ppt.}}{H_2 NHgO.HgI} + 7KI + H_2O$

iodine of Million's base

$\underset{C}{HCl} + MNO_3 \longrightarrow \underset{\text{white ppt.}}{MCl} + HNO_3$

$(M = Ag^+, Pb^+, Hg^+)$

56. (b)



The correct IUPAC name is 7-ethyl- 2, 4, 5, 6-tetramethyldeca-1, 8-diene

57. (a) M.wt. of caffeine = 194 u

% of N present in one molecular of caffeine is 28.9% of

$194u = \frac{28.9}{100} \times 194 = 56$ u

Mass of one N atom = 14 m

= 1N atom (∵ 14 m = 14 u)

∴ $56u = \frac{56}{14}$ N atom = 4N atom

58. (b) $\underset{X}{CaCO_3} \xrightarrow{\Delta} CaO + \underset{\text{Colourless gas}}{CO_2 \uparrow}$

$\underset{\text{Residue}}{CaO} + H_2O \longrightarrow \underset{Y}{Ca(OH)_2}$

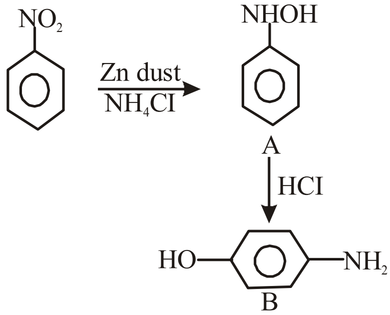
$\underset{Y}{Ca(OH)_2} + \underset{\text{Excess}}{2CO_2} \longrightarrow \underset{Z}{Ca(HCO_3)_2}$

$\underset{Z}{Ca(HCO_3)_2} \xrightarrow{\Delta} \underset{x}{CaCO_3} + CO_2 \uparrow + H_2O$

59. (b) $\underset{\text{Acid}}{Zn(OH)_2} + \underset{\text{Base}}{2O^-H} \longrightarrow \underset{\text{Salt}}{ZnO_2^{2-}} + \underset{\text{Water}}{2H_2O}$

$\underset{\text{Base}}{Zn(OH)_2} + \underset{\text{Acid}}{2H^+} \longrightarrow \underset{\text{Salt}}{Zn^{2+}} + \underset{\text{Water}}{2H_2O}$

The amphoteric character of $Zn(OH)_2$ is represented by I and III

60. (a) 

61. (b) Increasing order of stability

(a) $[Cu(NH_3)_4]^{2+} < [Cu(en)_2]^{2+} < [Cu\ (trein)^{2+}$.

Their formation of entropy increases in the same order. Ligand denticity is increased.

(b) $[Fe(H_2O)_6]^{3+} < [Fe(NO_2)_6]^{3-} < [Fe(NH_3)_6]^{3+}$.

∵ NH_3 is weaker ligand than NO_2^-.

∴ The correct stability order is

$[Fe(H_2O)^{3+} < [Fe(NH_3)_6]^{3+} < [Fe(NO_2)_6]^{3-}$

(c) $[Co(H_2O)_6]^{3+} < [Rh(H_2O)_6]^{3+} < [Ir(H_2O)_6]^{3+}$

Z_{eff} value increases from Co^{3+} to Ir^{3+}.

(d) $[Cr(NH_3)_6]^{1+} < [Cr(NH_3)_6]^{2+} < [Cr(NH_3)_6]^{3+}$

O.S of Cr atom increases from +1 to +3.

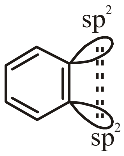
62. (d) The amount of energy required to take out an electron from the monopositive cation is called second ionisation energy

$M(g) \longrightarrow M^{2+}(g) + 2e^-$ (v)

$M(g) \longrightarrow M^+(g) + e^-$ (iii)

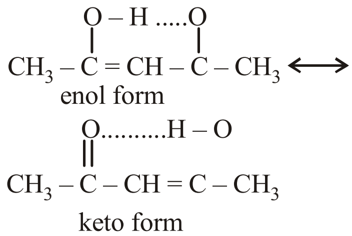
On subtracting eq(iii) form eq. (v) we get,

$M \longrightarrow M^{2+} + e^-$.

63. (d) 

In benzene, the triple bond consists of one sp^2–sp^2 σ-bond, one sp^2–sp^2, π-bond and one p–p π-bond.

64. (b) Resonance stabilisation of enol form is



65. (d) The minimum m. wt. must contain at least one S atom.

$$\therefore \quad \%\ S = \frac{\text{weight of one S} - \text{atom}}{\text{minimum m.wt.}} \times 100$$

$$4 = \frac{32}{\text{minimum m.wt.}} \times 100$$

$$\text{minimum m. wt.} = \frac{32}{4} \times 100 = 800$$

66. (b) When the concentration of the adsorbate is less on the surface as compare to its concentration in the bulk is called negative adsorption. Add from left in this adsorption, concentration of dilute KCl solution is less on the surface of blood charcoal as compare to its concentration in solution.

67. (a)

Interval	Conc.change	Rate
0 – 17 min	0.069 – 0.052 = 0.017 M	$\frac{0.017}{17} = 0.001$
17 – 34 min	0.052 – 0.035 = 0.017M	$\frac{0.017}{17} = 0.001$
34 – 51 min	0.035 – 0.018 = 0.017 M	$\frac{0.017}{17} = 0.001$

∵ Rate remains constant. So, it is independent of concentration, the reaction is of zero order.

According to rate law

Rate = $K(conc.)^0 = 0.001M/min$

68. (d) $hv = hv_0 + ev_0$.

$$v_0 = \frac{h}{e}v - \frac{h}{e}v_0$$

On comparing this equation with the straight line equation, i.e y = mx + c

The slope of v_0 vs v is

(v_0 is stopping potential)

$$(\text{slope})_1 = \frac{h}{e}$$

Likewise,

$h\nu = h\nu_0 + K_{max}$.

or $K_{max} = h\nu - h\nu_0$

Thus, slope of K_{max} vs v is

$$(\text{slope})_2 = h \therefore \frac{(\text{slope})_2}{(\text{slope})_1} = \frac{h}{h/e} = e$$

69. (c) Both P_2O_5 and PCl_5 give H_3PO_4 With excess of water.

$P_2O_5 + 3H_2O \longrightarrow 2H_3PO_4$

$PCl_5 + 4H_2O \longrightarrow H_3PO_4 + 5HCl$

70. (c) $Al(OH)_3$ dissolves in NaOH solution to give $Al(OH)_4^-$ ion which is supposed to have the octahedral complex species $[Al(OH)_4(H_2O)_2]^-$ in aqueous solution.

$$Al(OH)_3 + NaOH(aq) \longrightarrow [Al(OH)_4(H_2O)_2]^-(aq) + Na^+(aq)$$

71. (b) In the absence of d-orbitals F_2 does not combine with F^- to form F_3^-. ion.

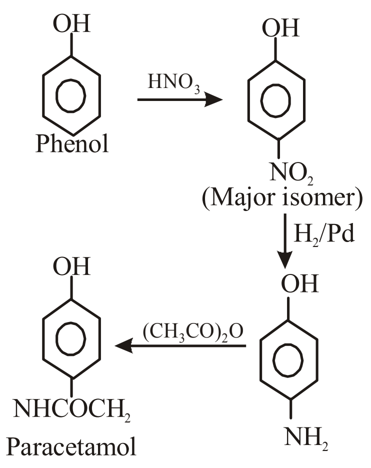
72. (a) According to Mulliken, electronegativity of an atom is average of ionization energy and electron affinity (in eV).

$$n_m = \frac{IE + EA}{2}$$

If ionization energy and electron affinity are in $kcalmol^{-1}$.

$$n = \frac{IE + EA}{125} = \frac{275 + 86}{125} = 288$$

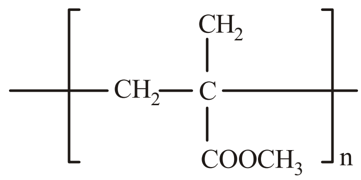
73. (a) For the preparation of paracetamol



74. (b) A compound which gives a negative test with ninhydrin, it cannot be a protein or an amino acid. As, it gives a positive test with Benedict's solution. So, it must be a monosaccharide but not a lipid.

75. (c) 

Methyl-α-cyanoacrylate



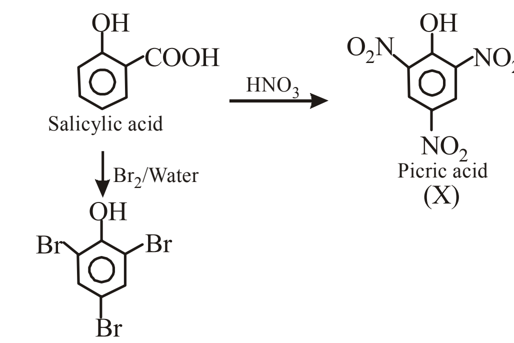
Poly (methyl α-cyanoacrylate)

Super glue of crazy glue

76. (a) Nitration and bromination of salicylic acid, give picric acid (X) and 2, 4, 6–tribromophenol (Y) respectively.

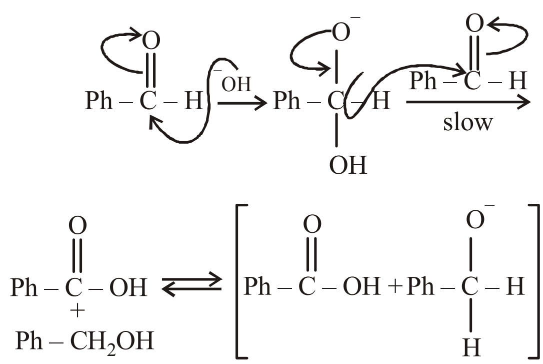
1. Decarboxylation

Bromination



2, 4, 6- tribromophenol(Y)

77. (b) Hydride ion transfer to the carbonyl group is the slowest or the rate determining step.



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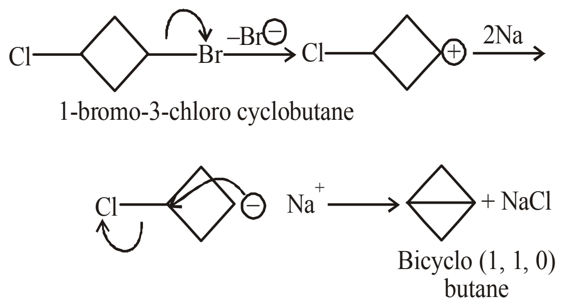
78. (d) This is Wurtz reaction. Bromides have high reactivity than chlorides in Wurtz reaction therefore, reaction occurs from Br atoms,



79. (d)

$$CH_3CH_2CH_2OH \xrightarrow[160°-180°C]{Conc\ H_2SO_4} CH_3CH = CH_2$$

$$\xrightarrow{Br_2} \xrightarrow{alc.\ KOH} \underset{(Y)}{CH_3CHBr - CH_2Br}$$

$$[\underset{(A)}{CH_3C(Br) = CH_2} + \underset{(B)}{CH_3CH = CHBr}]$$

$$\xrightarrow[-HBr]{NaNH_2} \underset{(Z)}{CH_3C \equiv CH}$$

Alcoholic KOH brings about dehydrobromination of Y and give a mixture of vinyl bromide (A and B) while $NaNH_2$ being a strong base than alc. KOH readily brings about dehydrobromination of less reactive vinyl bromide to give propyne $CH_3\ C \equiv CH$ i.e. (Z).

80. (d) 3-methylbutanoic acid gives isobutane on decarboxylation i.e.,

$$CH_3 - \underset{\underset{CH_3}{|}}{CH} - CH_2COOH \xrightarrow[\Delta]{NaOH/CaO}$$

3-methylbutanoic acid

$$CH_3 - \underset{\underset{CH_3}{|}}{CH} - CH_3 + CO_2 \uparrow$$

Isobutane

While Wurtz reaction of C_2H_5Br gives. n-butane and hydrolysis of n-butyl magnesium bromide gives n-butane but reduction of propanol with HI/P gives propane.

PART - III (MATHEMATICS)

81. (d) The given differential equation is

$(3x + 4y + 1)\,dx + (4x + 5y + 1)\,dy = 0$ ....(i)

Comparing eq. (i) with Mdx + Ndy = 0, we get

$M = 3x + 4y + 1$

and $N = 4x + 5y + 1$

Here, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 4$

Hence, eq. (i) is exact and solution is given by

$$\int (3x + 4y + 1)\,dx + \int (5y + 1)\,dy = C$$

$$\Rightarrow \frac{3x^2}{2} + 4xy + x + \frac{5y^2}{2} + y - C = 0$$

$$\Rightarrow 3x^2 + 8xy + 2x + 5y^2 + 2y - 2C = 0$$

$$\Rightarrow 3x^2 + 2.4xy + 2x + 5y^2 + 2y + C' = 0 \quad ...(ii)$$

where, $C' = -2C$

On comparing eq. (ii) with standard form of conic section

$ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$

We get,

$a = 3,\ h = 4,\ b = 5$

Here, $h^2 - ab = 16 - 15 = 1 > 0$

Hence, the solution of differential equation represents family of hyperbolas.

82. (b) $\Delta(r) = \begin{vmatrix} r & r^3 \\ 1 & n(n+1) \end{vmatrix}$

$$\Rightarrow \sum_{r=1}^{n} \Delta(r) = \begin{vmatrix} \sum_{r=1}^{n} r & \sum_{r=1}^{n} r^3 \\ 1 & n(n+1) \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & \frac{[n(n+1)]^2}{2} \\ 1 & n(n+1) \end{vmatrix}$$

$$= \frac{[n(n+1)]^2}{2} - \frac{[n(n+1)]^2}{4}$$

$$= \frac{[n(n+1)]^2}{2} = \sum_{r=1}^{n} r^3$$

83. (b) $P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A}\cap B\right)$

$= P(A\cap B)P\left(\frac{C}{A}\cap B\right)$

$= \frac{P(A\cap B)P[C\cap(A\cap B)]}{P(A\cap B)}$

$= P(A\cap B\cap C)$

84. (d) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & -9 & -2 \end{bmatrix}$

[Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$]

$\approx \begin{bmatrix} 1 & 3 & 1 \\ 0 & -5 & -3 \\ 0 & 0 & \frac{17}{5} \end{bmatrix}$ $\left[R_3 \to R_3 - \frac{9}{5}R_2\right]$

$\therefore$ rank (A) = 3

85. (b) The probability of getting a double six in one throw of two dice

$= \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

$\therefore$ $p = \frac{1}{36}$,

$q = 1 - p$

$= 1 - \frac{1}{36} = \frac{35}{36}$

Now, $(p + q)^m$

$= q^n + {}^nC_1q^{n-1}p + {}^nC_2q^{n-2}p^2 + \ldots + {}^nC_rq^{n-r}p^r + \ldots + p^n$

The probability of getting atleast one double six in n throws with two dice.

$= (q + p)^n - q^n$

$= 1 - q^n = 1 - \left(\frac{35}{36}\right)^n$

$\therefore$ $1 - \left(\frac{35}{36}\right)^n > 0.99$

$\Rightarrow$ $\left(\frac{35}{36}\right)^n < 0.01$

$\Rightarrow$ n(log 35 – log 36) < log 0.01

$\Rightarrow$ n[15441 – 15563] < –2

$\Rightarrow$ – 0.0122n < – 2

$\Rightarrow$ 0.0122n > 2 $\Rightarrow$ n > 163.9

So, the least value of n is 164.

86. (d) The given system of equations will be consistent with unique solution, when

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & k \end{vmatrix} \neq 0$

$\Rightarrow$ $1(2k - 12) + 1(3 - k) + 1(4 - 2) \neq 0$

$\Rightarrow$ $k - 12 + 3 + 2 \neq 0 \Rightarrow k - 7 \neq 0 \Rightarrow k \neq 7$

87. (d) Given: $|z| = \frac{1}{4}$

Let $z' = -1 + 8z$

$\Rightarrow$ $z = \frac{z'+1}{8} \Rightarrow |z| = \frac{|z'+1|}{8}$

$\Rightarrow$ $\frac{1}{4} = \frac{|z'+1|}{8} \Rightarrow |z'+1| = 2$

$\therefore$ z' lies on a circle with centre (–1, 0) and radius 2.

88. (c) If the line $y = mx + c$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

$c^2 = \frac{m^2(a^2 - b^2)^2}{a^2 + b^2m^2}$

[Here, m = 2, $a^2 = 9$ and $b^2 = 16$]

$= \frac{(2)^2(9-16)^2}{9+16\times(2)^2}$

$= \frac{4\times 49}{9+64} = \frac{4\times 49}{73} = \frac{196}{73}$

$\therefore$ $c = \frac{14}{\sqrt{73}}$

89. (b) Given equation is $x^2 + x + 1 = 0$

$\Rightarrow$ $x = \omega$ and $x = \omega^2$

Case I : When $x = \omega$

Then

$$\sum_{n=1}^{6}\left[x^n + \frac{1}{x^n}\right]^2 = \sum_{n=1}^{6}\left[\omega^n + \omega^{2n}\right]^2 \quad \left[\because \frac{1}{\omega} = \omega^2\right]$$

$= (\omega + \omega^2)^2 + (\omega^2 + \omega^4)^2 + (\omega^3 + \omega^6)^2 + (\omega^4 + \omega^8)^2 + (\omega^5 + \omega^{10})^2 + (\omega^6 + \omega^{12})^2$

$= (-1)^2 + (-1)^2 + (2)^2 + (-1)^2$
$+ (-1)^2 + (2)^2 = 12$

Case II: When $x = \omega^2$

Then

$$\sum_{n=1}^{6}\left[x^n + \frac{1}{x^n}\right]^2 = \sum_{n=1}^{6}\left[\omega^{2n} + \omega^n\right]^2 \quad \left[\because \frac{1}{\omega^2} = \omega\right]$$

$= 12$

90. (b) Correct result is as follows:
$(\sim p \vee \sim q) \Rightarrow (r \wedge s)$
or $\sim(p \wedge q) \Rightarrow r \wedge s$

91. (a) $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & -3 \\ 0 & 1 & 2 \end{bmatrix}$

$\Rightarrow$ $|A| = 1.(4 + 3) - 3(-2 + 0) + 1(-1 - 0)$
$= 7 + 6 - 1 = 12$

So, adj (adj A) $= |A|^{n-2} = A$
$= (12)^{3-2} A = 12A$

$$= 12\begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & -3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 36 & 12 \\ -12 & 24 & -36 \\ 0 & 12 & 24 \end{bmatrix}$$

92. (c) $\phi_3(m) = 3 + 2m - 7m^2 + 2m^3$
$\phi_2(m) = -14m + 7m^2$
$\phi'_3(m) = 2 - 14m + 6m^2$

Now, putting $\phi_3(m) = 0$, we have
$3 + 2m - 7m^2 + 2m^3 = 0$

$\Rightarrow$ $(1 - m)(1 + 2m)(3 - m) = 0$

$\Rightarrow$ $m = -\frac{1}{2}, 1, 3$

We know that
$c\phi'_n(m) + \phi_{n-1}(m) = 0$, which in the given case becomes
$c(2 - 14m + 6m^2) + (-14m + 7m^2) = 0$

$$\Rightarrow \quad c = \frac{14m - 7m^2}{2 - 14m + 6m^2}$$

So, when $m = -\frac{1}{2}, c = -\frac{5}{6}$

When $m = 1$, $c = -\frac{7}{6}$

When $m = 3$, $c = -\frac{3}{2}$

$\therefore$ Asymptotes are $y = -\frac{1}{2}x - \frac{5}{6}$,

$y = x - \frac{7}{6}$ and $y = 3x - \frac{3}{2}$.

93. (b) The structure (N,.) satisfies the closure property, associativity and commutativity but the identity element 0 does not belong to N.
So, N is a semi-group.

94. (a) $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$$= \frac{1}{2}\int \frac{dx}{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x}$$

$$= \frac{1}{2}\int \frac{dx}{\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x}$$

$$= \frac{1}{2}\int \frac{dx}{\cos\left(x - \frac{\pi}{3}\right)}$$

$$= \frac{1}{2}\int \sec\left(x - \frac{\pi}{3}\right)dx$$

$$= \frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + C$$

$$= \frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$$

95. (d) Given sphere is
$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$
Centre $\equiv (3, 6, 1)$
Here, one end of diameter is (2, 7, 3).
Let the other end of the diameter be (x, y, z)

Centre of the sphere will be the mid-point of the ends of diameter.

So, $(3,6,1) = \left(\frac{2+x}{2}, \frac{7+y}{2}, \frac{3+z}{2}\right)$

$\Rightarrow$ $2 + x = 6 \Rightarrow x = 4$

$\Rightarrow$ $7 + y = 12 \Rightarrow y = 5$

and $3 + z = 2 \Rightarrow z = -1$

Therefore, $(x, y, z) \equiv (4, 5, -1)$

96. (d) Given lines are

$x = my + n,\ z = py + q$

and $x = m'y + n',\ z = p'y + q'$

Above equations can be rewritten as

$$\frac{x-n}{m} = \frac{y-0}{1} = \frac{z-q}{p}$$

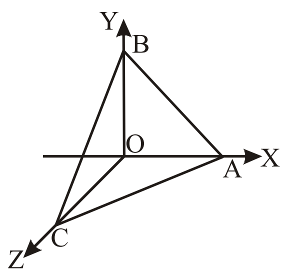
and $\frac{x-n'}{m'} = \frac{y-0}{1} = \frac{z-q'}{p'}$

Lines will be perpendicular, if

$mm' + 1 + pp' = 0$

$\Rightarrow$ $mm' + pp' = -1$

97. (c)



Vector perpendicular to face OAB = $\vec{n}_1$

$= \overrightarrow{OA} \times \overrightarrow{OB}$

$= (\hat{i} - 2\hat{j} + \hat{k}) \times (-2\hat{i} + \hat{j} + \hat{k})$

$= (-2-1)\hat{i} + (-2-1)\hat{j} + (1-4)\hat{k}$

$= -3\hat{i} - 3\hat{j} - 3\hat{k}$

Vector perpendicular to face ABC = $\vec{n}_2$.

$= \overrightarrow{AB} \times \overrightarrow{AC}$

$= (-3\hat{i} + 3\hat{j}) \times (\hat{j} + \hat{k})$

$= -3\hat{i} + 3\hat{j} - 3\hat{k}$

Since, angle between faces is equal to angle between their normals.

$\therefore$ $\cos\theta = \frac{\vec{n}_1 . \vec{n}_2}{|n_1||n_2|}$

$= \frac{(-3)(3) + (-3)(3) + (-3)(-3)}{\sqrt{9+9+9}\sqrt{9+9+9}}$

$= \frac{-9-9+9}{\sqrt{27}\sqrt{27}} = -\frac{1}{3}$

$\Rightarrow$ $\theta = \cos^{-1}\left(\frac{-1}{3}\right)$

98. Let α, β and γ be the angles made by the line segment OP with X-axis, Y-axis and Z-axis, respectively.

Given: $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{3}$

We know that, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore$ $\cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{3} + \cos^2\gamma = 1$

$\Rightarrow$ $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\Rightarrow$ $\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow$ $\cos^2\gamma = \frac{1}{4}$

$\Rightarrow$ $\cos\gamma = \frac{1}{\sqrt{2}}$

$\therefore$ $\gamma = \frac{\pi}{4}$

Hence, direction cosines are $\cos\alpha$, $\cos\beta$, $\cos\gamma$

i.e. $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}$.

99. (a) $\sim p \vee q$ means $F \vee F = F$, $\sim r$ means F

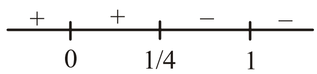
$\therefore$ $[(\sim p \vee q) \wedge \sim r] \Rightarrow p$ means T

100. (b) Let $f(x) = x^{25}(1-x)^{75},\ x \in [0, 1]$

$\Rightarrow$ $f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74}$

$= 25x^{24}(1-x)^{74}\{(1-x) - 3x\}$

$= 25x^{24}(1-x)^{74}(1-4x)$



We can see that $f'(x)$ is positive for $x < \frac{1}{4}$

and $f'(x)$ is negative for $x > \frac{1}{4}$.

Hence, $f(x)$ attains maximum at $x = \frac{1}{4}$.

101. (b) $\left|z+\frac{1}{4}\right|$

$= \left|z-\left(\frac{-1}{4}\right)\right| \geq |z| - \left|\frac{-1}{4}\right|$

$= \left|(-z)-\frac{1}{4}\right| \geq \left|3-\frac{1}{4}\right| = \frac{11}{4}$

$\therefore \quad \left|z+\frac{1}{4}\right| \geq \frac{11}{4}$

102. (b) Equation of the normal at point $\left(at_1^2, 2at_1\right)$ on parabola is

$y = -t_1 x + 2at_1 + at_1^3$

It also passes through $\left(at_2^2, 2at_2\right)$

So, $2at_2 = -t_1\left(at_2^2\right) + 2at_1 + at_1^3$

$\Rightarrow \quad 2t_2 - 2t_1 = -t_1\left(t_2^2 - t_1^2\right)$

$\Rightarrow \quad t_1 + t_2 = \frac{-2}{t_1}$

$\Rightarrow \quad t_2 = -t_1 - \frac{2}{t_1}$

103. (c) $\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

$= \dfrac{1\times 2 + (-1)\times(-1) + 2\times(1)}{\sqrt{1+1+4}\sqrt{4+1+1}}$

$= \dfrac{2+2+2}{6} = \dfrac{6}{6} = 1$

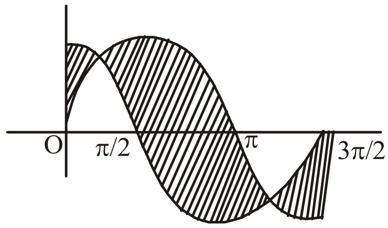
So, $\theta = 0°$ or $\theta = 2\pi$

$\because$ $\sec 2\pi = 1$

$\therefore$ $2\pi = \sec^{-1}(1)$

$\Rightarrow$ $\theta = \sec^{-1}(1)$

104. (a)



Required area

$= \int_0^{\pi/4}(\cos x - \sin x)\,dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)\,dx$

$\int_{5\pi/4}^{3\pi/2}(\cos x - \sin x)\,dx$

$= \left[\sin x + \cos x\right]_0^{\pi/4} + \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$

$+ \left[\sin x + \cos x\right]_{5x/4}^{3\pi/4}$

$= \left(4\sqrt{2} - 2\right)$ sq units

105. (b) [a + b – c] . [(a – b) × (b – c)
= (a + b – c). [a × b – a × c – b × b + b × c]
= a. (a × b) – a. (a × c) + a. (b × c) + b. (a × b) – b (a × c) + b. (b × c) – c. (a × b) + c. (a × c) – c.(b × c)
= a. (b × c) – b. (a × c) – c. (a × b)
= [a b c] – [b a c] – [c a b]
=[a b c] + [a b c] – [a b c]
= [a b c] = a. (b × c)

106. (a) Surface area A of a cube of side x is given by $A = 6x^2$.
On differentiating w.r.t. x, we get

$\frac{dA}{dx} = 12x$

Let the change in x be Δx = m% of x

$= \frac{mx}{100}$

Change in surface area,

$\Delta A = \left(\frac{dA}{dx}\right)\Delta x = (12x)\Delta x$

$= 12x\left(\frac{mx}{100}\right) = \frac{12x^2 m}{100}$

$\therefore$ The approximate change in surface area

$= \frac{2m}{100} \times 6x^2$

= 2m% of original surface area

107. (d) Given equation of rectangular hyperbola is $x^2 - y^2 = 8^2$

Length of latusrectum = 2 × (8) = 16

and eccentricity = $\sqrt{2}$

The asymptotes are perpendicular lines.

So, $x \pm y = 0$

Now, directrices are

$$x = \pm\frac{8}{\sqrt{2}} = \pm 4\sqrt{2}$$

108. (a) Equation of hyperbola is

$3x^2 - 2y^2 = 6$

$$\Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 1$$

So, $a^2 = 2$ and $b^2 = 3$

Given, equation of line is x – 3y = 3.

∴ Slope of given line = $\frac{1}{3}$

∴ Slope of line perpendicular to given line, m = –3

The equation of tangents are

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$= -3x \pm \sqrt{2 \times 9 - 3}$$

$$= -3x \pm \sqrt{18 - 3}$$

$$= -3x \pm \sqrt{15}$$

109. (d) $$\lim_{x\to\pi/4} \frac{\tan x - 1}{\cos 2x} = \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{\cos 2\left(\frac{\pi}{4} + h\right)}$$

$$\left[\because x = \frac{\pi}{4} + h\right]$$

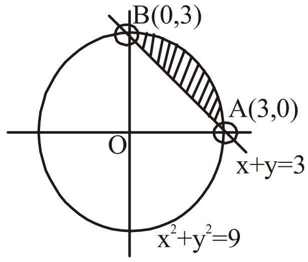
$$= \lim_{h\to 0} \frac{\left(\frac{1 + \tan h}{1 - \tan h}\right) - 1}{\cos\left(\frac{\pi}{2} + 2h\right)}$$

$$= \lim_{h\to 0} \frac{1 + \tan h - 1 + \tan h}{-\sin 2h(1 - \tanh)}$$

$$= \lim_{h\to 0} \frac{-2\tan h}{2\sin h\cos h(1 - \tan h)}$$

$$= \lim_{h\to 0} \frac{-1}{\cos^2 h(1 - \tan h)} = -1$$

110. (c)



Area of required region

$$= \frac{1}{4} \times \text{Area of circle} - \text{Area of } \Delta OAB$$

$$= \frac{1}{4} \times \pi \times (3)^2 - \frac{1}{2} \times 3 \times 3$$

$$= 9\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

111. (c) [a + b, b + c, c + a]

= (a + b) . [(b + c) × (c + a)]

= (a + b) . [b × c + b × a + c × c + c × a]

= (a + b). (b × c + b × a + c × a)

[∵ c × c = 0]

= a. (b × c) + a. (b × a) + a.(c × a) + b. (b × c) + b. (b × a) + b . (c × a)

= a.(b × c) + b.(c × a)

= [a b c] + [a b c]

= [a b c] + [a b c] = 2 [a b c]

112. (a) Let $I = \int_0^{\pi/2} (\log \tan x) . \sin 2x dx$ ...(i)

$$I = \int_0^{\pi/2} \log \tan\left(\frac{\pi}{2} - x\right) \sin 2\left(\frac{\pi}{2} - x\right) dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a - x)dx\right]$$

$$\Rightarrow I = \int_0^{\pi/2} \log\cot x. \sin 2x\, dx \quad ....(ii)$$

$[\because \sin(\pi - 2x) = \sin 2x]$

On adding eqs (i) and (ii), we get

$$2I \int_0^{\pi/2} \log\tan x.\sin 2x\,dx + \int_0^{\pi/2} \log\cot x \sin 2x\,dx$$

$$= \int_0^{\pi/2} \sin 2x \log(\tan x.\cot x) dx$$

$[\because \log m + \log n = \log(m . n)]$

$= \int_0^{\pi/2} \sin 2x \log 1 dx$

$\Rightarrow$ $I = 0 [\because \log 1 = 0]$

$\therefore$ $\int_0^{\pi/2} \sin 2x \log(\tan x)\, dx = 0$

113. (c) Here, mean = 4 and variance = 2

$\Rightarrow$ np = 4 and npq = 2

So, $\frac{npq}{np} = \frac{2}{4} \Rightarrow q = \frac{1}{2}$

Then, $p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

Mean = np = 4

$\Rightarrow$ $n \times \frac{1}{2} = 4 \Rightarrow n = 8$

$\therefore$ $P(X = r) = {}^nC_r p^r q^{n-r}$

$= {}^8C_r \left(\frac{1}{2}\right)^8 \quad \left[\because p = q = \frac{1}{2}\right]$

The required probability of atleast 7 successes is

$P(X \geq 7) = P(X = 7) + P(X = 8)$

$= \left({}^8C_7 + {}^8C_8\right)\left(\frac{1}{2}\right)^8$

$= \left(\frac{8!}{7!1!} + \frac{8!}{8!0!}\right)\left(\frac{1}{2}\right)^8$

$= (8+1)\left(\frac{1}{2}\right)^8 = \frac{9}{256}$

114. (b) Given, lines are $\frac{x-7}{3} = \frac{y+4}{-16} = \frac{z-6}{7}$

and $\frac{x-10}{3} = \frac{y-30}{8} = \frac{4-z}{5}$

The vector form of given lines are

$r = 7\hat{i} - 4\hat{j} + 6\hat{k} + \lambda\left(3\hat{i} - 16\hat{j} + 7\hat{k}\right)$

and $r = 10\hat{i} + 30\hat{j} + 4\hat{k} + \mu\left(3\hat{i} + 8\hat{j} - 5\hat{k}\right)$

On comparing these equations with $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$, we get

$\vec{a}_1 = 7\hat{i} - 4\hat{j} + 6\hat{k}$

$\vec{a}_2 = 10\hat{i} + 30\hat{j} + 4\hat{k}$

$b_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$

and $\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$

Shortest distance = $\left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|b_1 \times b_2|}\right|$

$= \left|\frac{\left(3\hat{i} + 34\hat{j} - 2\hat{k}\right).\left(24\hat{i} + 36\hat{j} + 72\hat{k}\right)}{84}\right|$

$= \left|\frac{72 + 1224 - 144}{84}\right| = \left|\frac{1152}{84}\right| = \frac{288}{21}$ units

115. (a) Equation of plane passing through (2, 2, 1) is

$a(x - 2) + b(y - 2) + c(z - 1) = 0$ ....(i)

Since, above plane is perpendicular to

$3x + 2y + 4z + 1 = 0$

and $2x + y + 3z + 2 = 0$

$\therefore$ $3a + 2b + 4c = 0$ ....(ii)

and $2a + b + 3c = 0$ ....(iii)

$[\because$ for perpendicular, $a_1a_2 + b_1b_2 + c_1c_2 = 0]$

On multiplying eq. (iii) by 2, we get

$4a + 2b + 6c = 0$ ....(iv)

On subtracting eq. (iv) from eq. (ii), we get

$\Rightarrow$ $c = \frac{-a}{2}$

On putting $c = \frac{-a}{2}$ in eq. (iii), we get $b = \frac{-a}{2}$

On putting $b = \frac{-a}{2}$ and $c = \frac{-a}{2}$ in eq. (i),

we get $a(x-2) - \frac{a}{2}(y-2) - \frac{a}{2}(z-1) = 0$

$\Rightarrow$ $\frac{a}{2}[2(x - 2) - (y - 2) - (z - 1)] = 0$

$\Rightarrow$ $2x - 4 - y + 2 - z + 1 = 0$

$\Rightarrow$ $2x - y - z - 1 = 0$

116. (c) Suppose, A : a male is selected

B: a smoker is selected

Given:

$$P(A\cup B)=\frac{7}{10}, P(A\cap B)=\frac{2}{5} \text{ and } P\left(\frac{A}{B}\right)=\frac{2}{3}$$

The probability of selecting a smoker..

$$P(B)=\frac{P(A\cap B)}{P\left(\frac{A}{B}\right)}$$

$$=\frac{2\times 3}{5\times 2}=\frac{3}{5}$$

The probability of selecting a non-smoker

So, P(B) = 1 – P(B)

$$=1-\frac{3}{5}=\frac{2}{5}$$

The probability of selecting a male

$$P(A)=P(A\cup B)+P(A\cap B)-P(B)$$

$$=\frac{7}{10}+\frac{2}{5}-\frac{3}{5}$$

$$=\frac{7+4-6}{10}=\frac{1}{2}$$

Probability of selecting a smoker, if a male is first selected, is given by

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{2}{5}\times\frac{2}{1}=\frac{4}{5}$$

117. (d) Given: $f(t)=\frac{\sin t}{t}$

At t = 0, we will check continuity of the function.

LHL = f(0 – h)

$$=\lim_{h\to 0}\frac{\sin(0-h)}{(0-h)}=\lim_{h\to 0}\frac{-\sin h}{-h}=1$$

RHL = f(0 + h)

$$\lim_{h\to 0}\frac{\sin(0+h)}{(0+h)}$$

$$=\lim_{h\to 0}\frac{\sin h}{h}=1$$

and f(0) = 1

LHL = RHL = f(0)

So, the function is continuous at t = 0

Now, we check the function is maximum or minimum.

$$f'(t)=\frac{1}{t}\cos t-\frac{1}{t^2}\sin t$$

and $f''(t)=\frac{-1}{t}\sin t-\frac{1}{t^2}\cos t-\frac{1}{t^2}\cos t+\frac{2}{t^3}\sin t$

$$=\frac{-\sin t}{t}-\frac{2\cos t}{t^2}+\frac{2\sin t}{t^3}$$

For maximum or minimum value of f(x), put

f′(x) = 0

$$\Rightarrow \frac{\cos t}{t}-\frac{\sin t}{t^2}=0\Rightarrow\frac{\tan t}{t}=1$$

Now $\lim_{t\to 0} f''(t)$

$$=-\lim_{t\to 0}\left(\frac{\sin t}{t}\right)-2\lim_{t\to 0}\left(\frac{t\cos t-\sin t}{t^3}\right)$$

$$\left[\frac{0}{0}\text{ form}\right]$$

$$=-1-2\lim_{t\to 0}\left(\frac{\cos t-t\sin t-\cos t}{3t^2}\right)$$

[using L' Hospital rule]

$$=-1+\frac{2}{3}\lim_{t\to 0}\frac{\sin t}{t}$$

$$=-1+\frac{2}{3}\times 1=\frac{-1}{3}<0$$

So, function f(t) is maximum at t = 0

118. (d) Consider the function f defined by

$$f(x)=a_0\frac{x^{n+1}}{n+1}+a_n\frac{x^n}{n}+...+a_{n-1}\frac{x^2}{2}+a_nx$$

Since, f(x) is a polynomial, so it is continuous and differentiable for all x.

f(x) is continuous in the closed interval [0, 1] and differentiable in the open interval (0, 1).

Also, f(0) = 0

and

$$f(1)=\frac{a_0}{n+1}+\frac{a_1}{n}+....+\frac{a_{n-1}}{2}+a_n=0\,[\text{say}]$$

i.e. f(0) = f(1)

Thus, all the three conditions of Rolle's theorem are satisfied. Hence, there is atleast one value of x in the open interval (0, 1) where $f'(x) = 0$

i.e. $a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$

119. (d) Let $f(x) = \log(1+x) - \frac{x}{1+x}$

$\therefore \quad f'(x) = \frac{1}{1+x} - \frac{(1+x).1 - x.1}{(1+x)^2}$

$= \frac{1}{1+x} - \frac{1}{(1+x)^2} = \frac{x}{(1+x)^2}$

which is positive. $[\because x > 0]$

$\therefore$ f(x) is monotonic increasing, when x > 0.

$\Rightarrow$ f(x) > f(0)

Now, f(0) = log 1 – 0 = 0

$\therefore$ f(x) > 0

$\Rightarrow \log(1+x) - \frac{x}{1+x} > 0$

$\Rightarrow \frac{x}{1+x} < \log(1+x)$ ....(i)

Also, for x > 0,

$x^2 > 0 \Rightarrow x^2 + x > x$

$\Rightarrow x(x+1) > x$

$\Rightarrow x > \frac{x}{x+1}$ ....(ii)

From eqs. (i) and (ii), we get

$\frac{x}{x+1} < \log(1+x) < x$

$[\because \log(1+x) < x \text{ for } x > 0]$

120. (c) We can write given differential equation as,

$(D^2 - 1)x = k$ ....(i)

where, $D \equiv \frac{d}{dy}$

Its auxiliary equation is $m^2 - 1 = 0$, so that m = 1, –1

Hence, $CF = C_1 e^y + C_2 e^{-y}$.

where C_1, C_2 are arbitrary constants

Now, also $PI = \frac{1}{D^2 - 1} k$

$= k.\frac{1}{D^2 - 1} e^{0.y}$

$= K.\frac{1}{0^2 - 1} e^{0.y} = -K$

So, solution of eq. (i) is

$x = C_1 e^y + C_2 e^{-y} - k$ ....(ii)

Given that x = 0, when y = 0

So, $0 = C_1 + C_2 - k$ (From (ii))

$\Rightarrow C_1 + C_2 = k$ ....(iii)

Multiplying both sides of eq. (ii) by e^{-y}, we get

$x.\, e^{-y} = C_1 + C_2 e^{-2y} - ke^{-y}$ ....(iv)

Given that $x \to m$ when $y \to \infty$, m being a finite quantity.

So, eq (iv) becomes

$x \times 0 = C_1 + C_2 \times 0 - (k \times 0)$

$\Rightarrow C_1 = 0$ ....(v)

From eqs. (iv) and (v), we get

$C_1 = 0$ and $C_2 = k$

Hence, eq. (ii) becomes

$x = ke^{-y} - k = k(e^{-y} - 1)$