VITEEE 2013 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER VITEEE 2013

PART - I (PHYSICS)

1. The amplitude of an electromagnetic wave in vaccum is doubled with no other changes made to the wave. As a result of this doubling of the amplitude, which of the following statement is correct?
(a) The frequency of the wave changes only
(b) The wave length of the wave changes only
(c) The speed of the wave propagation changes only
(d) Alone of the above is correct

2. An element with atomic number $Z = 11$ emits K_α – X-ray of wavelength λ. The atomic number which emits K_α – X-ray of wavelength 4λ is
(a) 4 (b) 6
(c) 11 (d) 44

3. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are $0.36 m^2 V^{-1}s^{-1}$ and $0.17 m^2V^{-1}s^{-1}$. The electron and hole densities are each equal to 2.5×10^{19} m^3. The electrical conductivity of germanium is
(a) $4.24 \, Sm^{-1}$ (b) $2.12 \, Sm^{-1}$
(c) $1.09 \, Sm^{-1}$ (d) $0.47 \, Sm^{-1}$

4. If a radio-receiever amplifiers all the signal frequencies equally well, it is said to have high
(a) sensitivity (b) selectivity
(c) distortion (d) fidelity

5. If a progressive wave is represented as $y = 2\sin\pi\left(\frac{t}{2} - \frac{x}{4}\right)$ where x is in metre and t is in second, then the distance travelled by the wave in 5 s is
(a) 5m (b) 10m
(c) 25m (d) 32m

6. The gravitational potential at a place varies inversely with x^2(*i.e.*, $V = k/x^2$), the gravitational field at that place is
(a) $2k/x^3$ (b) $-2k/x^3$
(c) k/x (d) $-k/x$

7. A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm are connected end to end. When stretched by a force, the elonation in length 0.50 mm is produced in the copper wire. The stretching force is ($Y_{cu} = 1.1 \times 10^{11}$ N/m^2, $Y_{steel} = 2.0 \times 10^{11}$ N/m^2)
(a) 5.4×10^2 N (b) 3.6×10^2 N
(c) 2.4×10^2 N (d) 1.8×10^2 N

8. If $\bar{v}$, v_{rms} and v_p represent the mean speed, root mean square and most probable speed of the molecules in an ideal monoatomic gas at temperature T and if m is mass of the molecule, then
(a) $v_p < v < v_{rms}$
(b) no molecule can have a speed greater than $\sqrt{2}v_{rms}$
(c) no molecule can have a speed less than $v_p / \sqrt{2}$
(d) None of the above

9. Two balls of equal masses are thrown upwards along the same vertical direction at an interval of 2 s, with the same initial velocity of 39.2 m/s. The two balls will collide at a height of
(a) 39.2m (b) 73.5m
(c) 78.4m (d) 117.6m

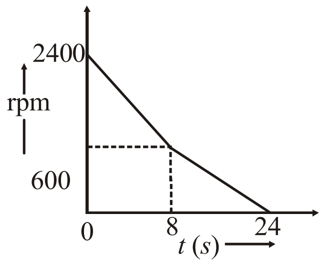
10. The dimensional formula of magnetic flux is
(a) $[M^1L^2T^{-1} A^{-2}]$ (b) $[M^1L^2T^{-2} A^{-1}]$
(c) $[M^1L^2T^{-1} A^{-1}]$ (d) $[M^1L^0T^{-2} A^{-1}]$

11. The time dependence of a physical quantity P is given by $P = P_0 \, e\alpha \, (-\alpha t^2)$, where α is a constant and t is time. The constant α
(a) is a dimensionless
(b) has dimensions of P
(c) has dimensions of T^{-2}
(d) has dimensions of T^2

12. If the potential energy of a gas molecule is $U = \frac{M}{r^6} - \frac{N}{r^{12}}$, M and N being positive constants, then the potential energy at equilibrium must be

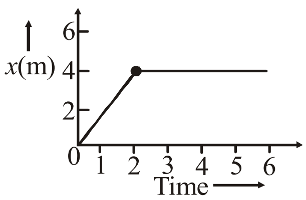
(a) zero (b) $NM^2/4$
(c) $MN^2/4$ (d) $M^2/4N$

13. A table fan rotating at a speed of 2400 rpm is switched off and the resulting variation of revolution/minute with time is shown in figure. The total number of revolutions of the fan before it, comes to rest is



(a) 160 (b) 280
(c) 380 (d) 420

14. In the adjoining figure, the position time graph of a particle of mass 0.1 kg is shown. The impulse at $t = 2$ s is



(a) 0.02 kg m/s (b) 0.1 kg m/s
(c) 0.2 kg m/s (d) 0.4 kg m/s

15. The pressure on a square plate is measured by measuring the force on the plate. If the maximum error in the measurement of force and length are respectively 4% and 2%, then the maximum error in the measurement of pressure is

(a) 1% (b) 2%
(c) 4% (d) 8%

16. The centre of a wheel rolling on a plane surface moves with a speed ν_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed

(a) zero (b) v_0
(c) $2v_0$ (d) $\sqrt{2v_0}$

17. A body of mass 5 m initially at rest explodes into 3 fragments with mass ratio 3:1:1. Two of fragments each of mass 'm' are found to move with a speed of 60 m/s is mutually perpendicular directions. The velocity of third fragment is

(a) $10\sqrt{2}$ (b) $20\sqrt{2}$
(c) $20\sqrt{3}$ (d) $60\sqrt{2}$

18. A body of mass 2 kg moving with velocity of 6 m/s strikes in elastically with another body of same mass mass at rest. The amount of heat evolved during collision is

(a) 18 J (b) 36 J
(c) 9 J (d) 3 J

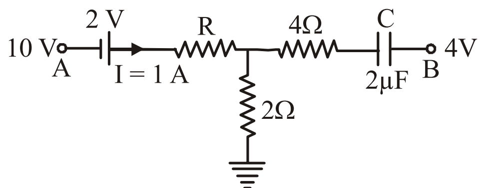
19. Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{4Gm}{R}}$
(c) $\sqrt{\frac{Gm}{2R}}$ (d) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$

20. Four equal charges Q each are placed at four corners of a square of side a each. Work done in carrying a charge $-q$ from its centre to infinity is

(a) zero (b) $\frac{\sqrt{2}q}{\pi\varepsilon_0 a}$
(c) $\frac{q^2}{2\pi\varepsilon_0 a}$ (d) $\frac{\sqrt{2}q^2}{\pi\varepsilon_0 a}$

21. A network of resistances, cell and capacitor C(= 2 μ F) is shown in adjoining figure. In steady state condition, the charge on 2 μ F capacitor is Q, while R is unknown resistance. Values of Q and R are respectively

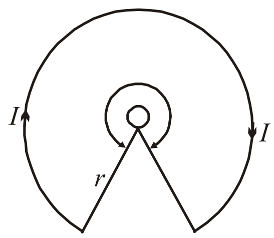


(a) 4 μ C and 10 Ω (b) 4 μ C and 4 Ω
(c) 2 μ C and 2 Ω (d) 8 μ C and 4 Ω

22. As the electron in Bohr's orbit of hydrogen atom passes from state $n = 2$ to, $n = 1$, the KE (K) and the potential energy (U) changes as
(a) K four fold, U also four fold
(b) K two fold, V also two fold
(c) K four fold, U two fold
(d) K two fold, U four fold

23. To get an OR gate from a NAND gate, we need
(a) Only two NAND gates
(b) Two NOT gates obtained from NAND gates and one NAND gate
(c) Four NAND gates and two AND gates obtained from NAND gates
(d) None of the above

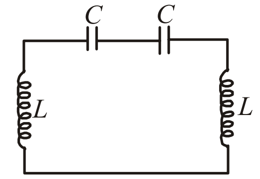
24. If a current I is flowing in a loop of radius r as shown in adjoining figure, then the magnetic field induction at the centre O will be



(a) Zero
(b) $\frac{\mu_0 I\theta}{4\pi r}$
(c) $\frac{\mu_0 I \sin\theta}{4\pi r}$
(d) $\frac{2\mu_0 I \sin\theta}{4\pi r^2}$

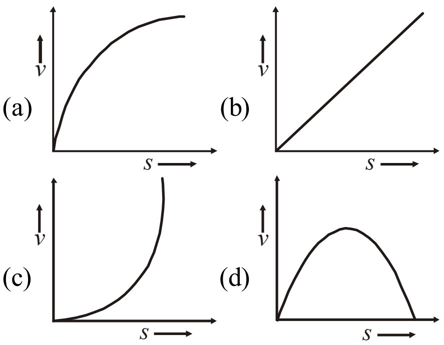
25. Two indentical magnetic dipoles of magnetic moment 1.0 Am^2 each,placed at a separation of 2 m with their axes perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is
(a) $\sqrt{5}\times 10^{-7}\,T$
(b) $5 \times 10^{-7}\,T$
(c) $10^{-7}\,T$
(d) $2 \times 10^{-7}\,T$

26. The natural frequency of the circuit shown in adjoining figure is



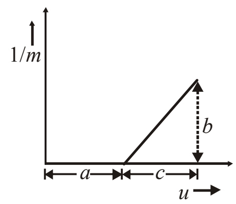
(a) $\frac{1}{2\pi\sqrt{LC}}$
(b) $\frac{1}{2\pi\sqrt{2LC}}$
(c) $\frac{2}{2\pi\sqrt{LC}}$
(d) zero

27. A lead shot of 1 mm diameter falls through a long column of glycerine. The variation of the velocity with distance covered (s) is correctly represented by



28. If ε_0 and μ_0 represent the permittivity and permeability of vaccum and ε and μ represent the permittivity and permeability of medium, then refractive index of the medium is given by
(a) $\sqrt{\frac{\varepsilon_0\mu_0}{\varepsilon\mu}}$
(b) $\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$
(c) $\sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon}}$
(d) $\sqrt{\frac{\mu_0\varepsilon_0}{\mu}}$

29. A students plots a graph between inverse of magnification $1/m$ produced by a convex thin lens and the object distance u as shown in figure. What was the focal length of the lens used?



(a) $\frac{b}{ca}$
(b) $\frac{bc}{a}$
(c) $\frac{c}{b}$
(d) $\frac{b}{c}$

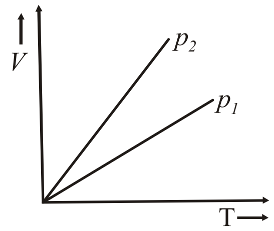
30. Two waves $y_1 = A_1 \sin(\omega t - \beta_1)$ and $y_2 = A_2 \sin(\omega t - \beta_2)$ superimpose to form a resultant wave whose amplitude is
(a) $A_1 + A_2$
(b) $|A_1 + A_2|$
(c) $\sqrt{A_1^2 + A_2^2 - 2A_1A_2 \sin(\beta_1 - \beta_2)}$
(d) $\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\beta_1 - \beta_2)}$

31. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current $3V_0$. When the same surface is illuminated with a light of wave length 2λ, the stopping potential is V_0. The threshold wavelength for this surface to photoelectric effect is

(a) 4λ (b) 6λ

(c) 8λ (d) $\frac{4}{3}\lambda$

32. In the V-T diagram shown in adjoining figure, what is the relation between p_1 and p_2?



(a) $p_2 = p_1$ (b) $p_2 < p_1$

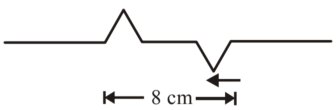
(c) $p_2 > p_1$ (d) insufficient data

33. If a gas mixture contains 2 moles of O_2 and 4 moles of Ar at temperature T, then what will be the total energy of the system (neglecting all vibrational modes)

(a) $11RT$ (b) $15RT$

(c) $8RT$ (d) RT

34. In the adjoining figure, two pulses in a stretched string are shown. If initially their centres are 8 cm apart and they are moving towards each other, with speed of 2cm/s, then total energy of the pulses after 2 s will be



(a) Zero

(b) Purely kinetic

(c) Purely potential

(d) Partly kinetic and partly potential

35. When two waves of almost equal frequency n_1 and n_2 are produced simultaneously, then the time interval between succesive maxima is

(a) $\frac{1}{n_1+n_2}$ (b) $\frac{1}{n_1}+\frac{1}{n_2}$

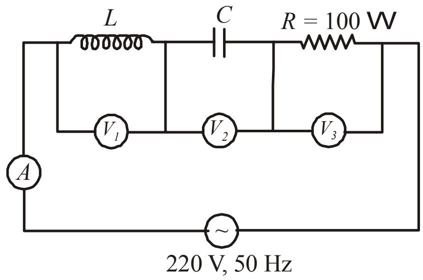
(c) $\frac{1}{n_1}-\frac{1}{n_2}$ (d) $\frac{1}{n_1-n_2}$

36. A long glass capillary tube is dipped in water. It is known that water wets glass. The water level rises by h in the tube. The tube is now pushed down so that only a length $h/2$ is outside the water surface. The angle of contact at the water surface at the upper end of the tube will be

(a) $\tan^{-1}\frac{1}{2}$ (b) 60º

(c) 30º (d) 15º

37. In the adjoining circuit, if the reading of voltmeter V_1 and V_2 are 300 volts each, then the reading voltmeter V_3 and ammeter A are respectively



(a) 220 V, 2.2 A (b) 100 V, 2.0 A

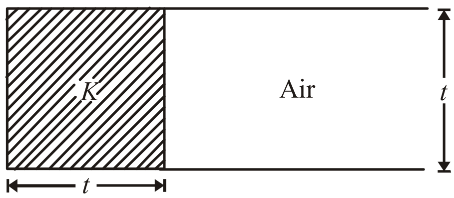
(c) 220 V, 2.0 A (d) 100 V, 2.2 A

38. If the work done in turning a magnet of magnetic moment M by an angle of 90º from the magnetic meridian is n times the corresponding work done to turn it through an angle of 60º, then the value of n is

(a) 1 (b) 2

(b) $\frac{1}{2}$ (d) $\frac{1}{4}$

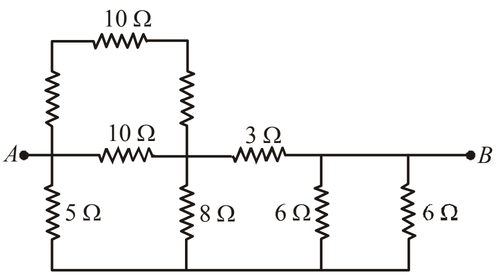
39. The capacitance of a parallel plate capacitor with air as dielectric is C. If a slab of dielectric constant K and of the same thickness as the separation between the plates is introduced so as to fill 1/4th of the capacitor (shown in figure), then the new capacitance is



(a) $(K+2)\frac{C}{4}$ (b) $(K+3)\frac{C}{4}$

(c) $(K+1)\frac{C}{4}$ (d) None of these

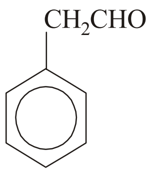
40. Seven resistance are connected between points A and B as shown in adjoining figure. The equivalent resistance between A and B is

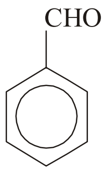


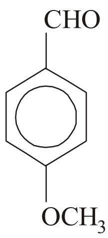
(a) 5 Ω (b) 4.5 Ω
(c) 4 Ω (d) 3 Ω

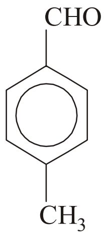
PART - II (CHEMISTRY)

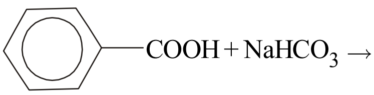
41. Which of the following does not undergo benzoin condensation?

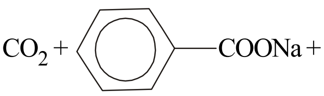
(a) 

(b) 

(c) 

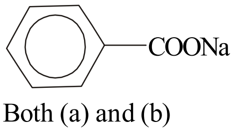
(d) 

42. 



*
C is with the product

(a) CO_2

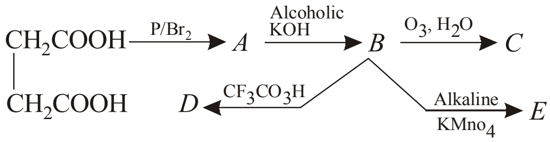
(b) 

(c) Both (a) and (b)
(d) None of the above

43. Benzene diazonium chloride on treatment with hypophosphorous acid and water yield benzene. Which of the following is used as a catalyst in this reaction?

(a) $LiAlH_4$ (b) Red p
(c) Zn (d) Cu+

44. Consider the following reaction sequence,



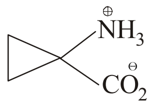
Isomers are

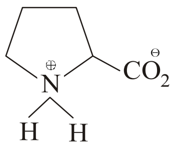
(a) C and E (b) C and D
(c) D and E (d) C,D and E

45. When a monosaccharide forms a cyclic hemiacetal, the carbon atom that contained the carbonyl group is identified as the .... Carbon atom,because

(a) D,the carbonyl group is drawn to the right
(b) L, the carbonyl group is drawn to the left
(c) acetal,it forms bond to an –OR and an –OR'
(d) anomeric, its substituents can assume an β or α position

46. Which of the following is/ are α - amino acid?

(a) 

(b) 

(c) Both (a) and (b) (d) None of these

47. Calculate pH of a buffer prepared by adding 10 mL of 0.10 M acetic acid to 20 mL of 0.1 M sodium acetate [pK_a (CH_3COOH) = 4.74]

(a) 3.00 (b) 4.44
(c) 4.74 (d) 5.04

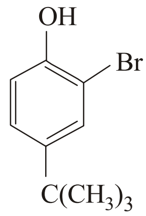
48. The equivalent conductance of silver nitrate solution at 250°C for an infinite dilution was found to be 133.3Ω^{-1} cm^2 $equiv^{-1}$. The transport number of Ag^+ ions in very dilute solution of $AgNO_3$ is 0.464. Equivalent conductances of Ag^+ and NO^-_3 (in Ω^{-1} cm^2 $equiv^{-1}$) at infinite dilution are respectively

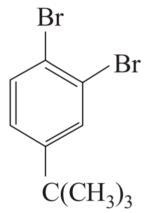
(a) 195.2, 133.3 (b) 61.9, 71.4
(c) 71.4, 61.9 (d) 133.3, 195.2

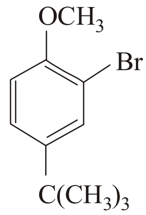
49. Treating anisole with the following reagents, the major product obtained is

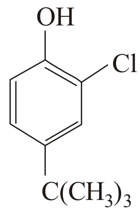
I. $(CH_3)_3\,CCl, AlCl_3$ II. $Cl_2, FeCl_3$

III. HBr, Heat

(a) 

(b) 

(c) 

(d) 

50. Ketones $[R{-}\overset{}{\underset{\underset{O}{\|}}{C}}{-}R']$ where, $R = R'$ = alkyl group can be obtained in one step by

(a) Hydrolysis of esters
(b) Oxidation of primary alcohols
(c) Oxidation of secondary alcohols
(d) Reaction of acid halide with alcohols

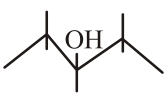
51. An optically active compound 'X' has molecular formula $C_4H_8O_3$. It evolves CO_2 with aqueous $NaHCO_3$. '*X*' reacts with $LiAlH_4$ to give an achiral compound.'*X*' is

(a) $CH_3CH_2\underset{\underset{OH}{|}}{C}HCOOH$

(b) $CH_3\underset{\underset{OMe}{|}}{C}HCOOH$

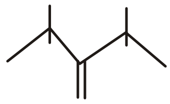
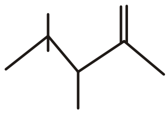
(c) $CH_3\underset{\underset{CH_2OH}{|}}{C}HCOOH$

(d) $CH_3\underset{\underset{OH}{|}}{C}HCH_2COOH$

52. 

$\xrightarrow{\text{conc.}H_2SO_4}$ products.

Product is/are

(a)  (b) 

(c) Both (a) and (b) (d) None is correct

53. Glycerol $\xrightarrow{KHSO_4}$ A $\xrightarrow{HClO}$ B,
A –A and B respectively are

(a) $CH_2{=}CH\overset{\overset{O}{\|}}{C}H$ $\quad \underset{\underset{Cl}{|}}{C}H_2{-}\underset{\underset{OH}{|}}{C}H\overset{\overset{O}{\|}}{C}H$

(b) $CH_2{=}CH\overset{\overset{O}{\|}}{C}H$ $\quad \underset{\underset{OH}{|}}{C}H_2{-}\underset{\underset{Cl}{|}}{C}H\overset{\overset{O}{\|}}{C}H$

(c) CH_3CH_2CHO $\quad CH_3CH_2CH{<}^{OH}_{Cl}$

(d) $CH_2{=}CH\overset{\overset{O}{\|}}{C}H$ $\quad \underset{\underset{ClO}{|}}{C}H_2CH_2CHO$

54. Phenol is heated with phthalic anhydride in the presence of conc. H_2SO_4. The product gives pink colour with alkali. The product is

(a) phenolphthalein (b) bakelite
(c) salicylic acid (d) flurorescein

55. $C_6H_5NH_2 \xrightarrow[0°C]{NaNO_2/HCl} X \xrightarrow{CuCN}$
$Y \xrightarrow{H_2O/H^+} Z$, *Z* is identified as

(a) $C_6H_5{-}NH{-}CH_3$
(b) $C_6H_5{-}CH_2{-}NH_2$
(c) $C_6H_5{-}CH_2{-}COOH$
(d) $C_6H_5{-}COOH$

56. *B* can be obtained from halide by van-Arkel method. This involves reaction

(a) $2BI_3 \xrightarrow[\text{filament}]{\text{Red hot W or Ta}} 2B + 3I_2$

(b) $2BCl_3 + 3H_2 \xrightarrow[\text{filament}]{\text{Red hot W or Ta}} 2B + 6HCl$

(c) Both (a) and (b)
(d) None of the above

57. $NH_4Cl(s)$ is heated in a test tube. Vapours are brought in contact with red litmus paper, which changes it to blue and then to red. It is because of

(a) formation of NH_4OH and HCl
(b) formation of NH_3 and HCl
(c) greater diffusion of NH_3 than HCl
(d) greater diffusion of HCl than NH_3

58. Out of $H_2S_2O_3$, $H_2S_2O_4$, H_2SO_5 and $H_2S_2O_8$ peroxy acids are

(a) $H_2S_2O_3$, $H_2S_2O_8$ (b) H_2SO_5, $H_2S_2O_8$
(c) $H_2S_2O_4$, H_2SO_5 (d) $H_2S_2O_3$, $H_2S_2O_4$

59. The density of solid argon is 1.65 g per cc at -233°C. If the argon atom is assumed to be a sphere of radius 1.54×10^{-8} cm, what per cent of solid argon is apparently empty space? (Ar = 40)

(a) 16.5% (b) 38%
(b) 50% (d) 62%

60. When 1 mole of $CO_2(g)$ occupying volume 10L at 27°C is expanded under adiabatic condition, temperature falls to 150 K. Hence,final volume is

(a) 5 L (b) 20 L
(c) 40 L (d) 80 L

61. Acid hydrolysis of ester is first order reaction and rate constant is given by

$k = \frac{2.303}{t}\log\frac{V_\infty - V_0}{V_\infty - V_t}$ where, V_0, V_t and V_∞ are the volumle of standard NaOH required to neutralise acid present at a given time, if ester is 50% neutralised then

(a) $V_\infty = V_t$ (b) $V_\infty = (V_t - V_0)$
(c) $V_\infty = 2V_t - V_0$ (d) $V_\infty = 2V_t + V_0$

62. A near UV photon of 300 nm is absorbed by a gas and then re-emitted as two photons. One photon is red with wavelength of the second photon is

(a) 1060 nm (b) 496 nm
(c) 300 nm (d) 215 nm

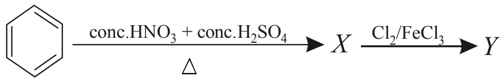
63. Which of these ions is expected to be coloured in aqueous solution?

I. Fe^{3+} II. Ni^{2+} III. Al^{3+}

(a) I and II (b) II and III
(c) I and III (d) I, II and III

64. Select the correct statements(s).

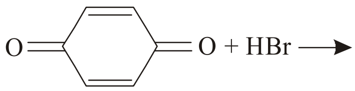
(a) $LiAlH_4$ reduces methyl cyanide to methyl amine
(b) Alkane nitrile has electrophilic as well as nucleophilic centres
(c) saponification is a reversible reaction
(d) Alkaline hydrolysis of methane nitrile forms methanoic acids

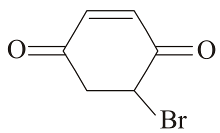
65. 

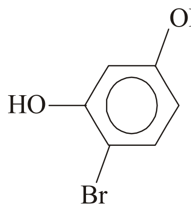
The product Y is

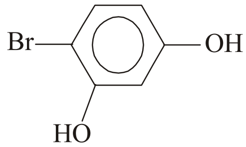
(a) *p*-chloro nitrobenzene
(b) *o*-chloro nitrobenzene
(c) *m*-chloro nitrobenzene
(d) *o, p*-dichloro nitrobenzene

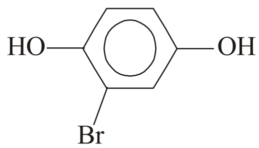
66. End product of the following reaction is



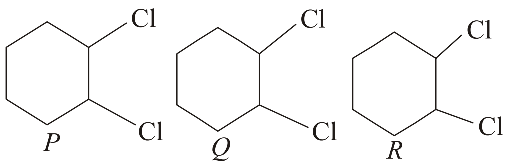
(a) 

(b) 

(c) 

(d) 

67. Following compounds are respectively ... geometrical isomers



	P	Q	R
(a)	cis	cis	trans
(b)	cis	trans	trans
(c)	trans	cis	cis
(d)	cis	trans	cis

68. Which is more basic oxygen in an ester?

$$R-\overset{\overset{O^{\alpha}}{\|}}{C}-\overset{\beta}{O}-R$$

(a) Carbonyl oxygen, α
(b) Carboxyl oxygen, β
(c) Equally basic
(d) Both are acidic oxygen

69. In a Claisen condensation reaction (when an ester is treated with a strong base)
(a) a proton is removed from the α-carbon to form a resonance stabilised carbanion of the ester
(b) carbanion acts as a nucleophile in a nucleophilic acyl substitution reaction with another ester molecule
(c) a new C—C bond is formed
(d) All of the above statements are correct

70. An organic compound *B* is formed by the reaction of ethyl magnesium iodide with a substance *A*, followed by treatment with dilute aqueous acid, Compound B does not react with PCC or PDC in dichloromethane. Which of the following is a possible compound for *A*?

(a) $CH_2\overset{O}{—}CH_2$ (b) $CH_3CH_2\overset{\overset{O}{\|}}{C}CH_3$

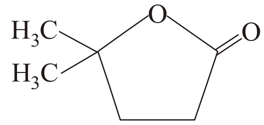
(c) $CH_3\overset{\overset{O}{\|}}{C}H$ (d) $H_2C=O$

71. $CH_3\overset{\overset{O}{\|}}{C}CH_2CH_2\overset{\overset{O}{\|}}{C}OCH_2CH_3$

$\xrightarrow[\text{(ii) } H_2O^+]{\text{(i) } CH_3MgBr\text{(one mole)}}$ A formed in this reaction is

(a) $CH_3\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}CH_2CH_2\overset{\overset{O}{\|}}{C}OCH_2CH_3$

(b) $CH_3\overset{\overset{O}{\|}}{C}CH_2CH_2\overset{\overset{O}{\|}}{C}CH_3$

(c) 

(d) $CH_3\overset{\overset{CH_3}{|}}{\underset{\underset{OH}{|}}{C}}CH_2CH_2\overset{\overset{CH_3}{|}}{\underset{\underset{OH}{|}}{C}}CH_3$

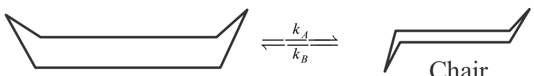
72. For the cell reaction $2Ce^{4+} + Co \rightarrow 2Ce^{3+} + Co^{3+}$; E^o_{cell} cell is 1.89 V. If $E_{Co^{2+}/Co}$ is -0.28 V, what is the value of $E^{\circ}_{Ce^{4+}/Ce^{3+}}$?

(a) 0.28 V (b) 1.61 V
(c) 2.17 V (d) 5.29 V

73. A constant current of 30 A is passed through an aqueous solution of NaCl for a time of 1.00 h. What is the volume of Cl_2 gas at STP produced?

(a) 30.00 L (b) 25.08 L
(c) 12.54 L (d) 1.12 L

74. Consider the following reaction,



The reaction is of first order in each diagram, with an equilibrium constant of 10^4. For the conversion of chair form to boat form $e^{-Ea/RT} = 4.35 \times 10^{-8}$ m at 298 K with pre-exponential factor of 10^{12} s^{-1}. Apparent rate constant (= kA / kB) at 298 K is

(a) 4.35×10^4 s^{-1} (b) 4.35×10^8 s^{-1}
(c) 4.35×10^{-8} s^{-1} (d) 4.35×10^{12} s^{-1}

75. If for the cell reaction, $Zn + Cu^{2+} \rightleftharpoons Cu + Zn^{2+}$ Entropy change ΔS^o is 96.5 J mol^{-1}K^{-1}, then temperature coefficient of the emf of a cell is

(a) 5×10^{-4} VK^{-1} (b) 1×10^{-3} VK^{-1}
(c) 2×10^{-3} VK^{-1} (d) 9.65×10^{-4} VK^{-1}

76. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition, $n = 4$ to $n = 2$ of He^+ spectrum?

(a) $n = 4$ to $n = 2$ (b) $n = 3$ to $n = 2$
(c) $n = 2$ to $n = 1$ (d) $n = 4$ to $n = 3$

77. What is the degeneracy of the level of H-atom that has energy $\left(-\frac{R_H}{9}\right)$?

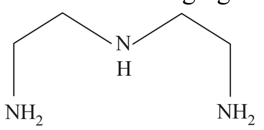
(a) 16 (b) 9
(c) 4 (d) 1

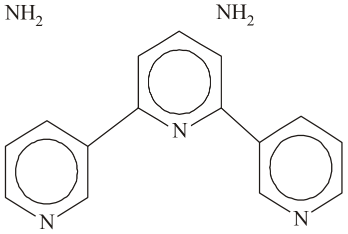
78. Match the following and choose the correct option given below.

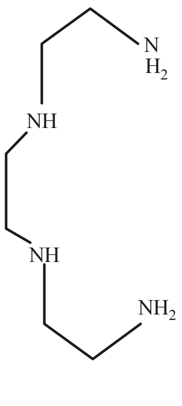
Compound/Type	**Use**
A. Dry ice	I. Anti-knocking compound
B. Semiconductor	II. Electronic diode or triode
C. Solder	III. Joining circuits
D. TEL	IV. Referigerant for preserving food

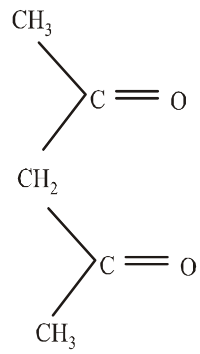
	A	B	C	D
(a)	I	II	IV	III
(b)	II	III	I	IV
(c)	IV	III	II	I
(d)	IV	II	III	I

79. Which of the following ligands is tetradentate?

(a) 

(b) 

(c) 

(d) 

80. What is the EAN of $[Al(C_4O_4)_3]^{3-}$?

(a) 28 (b) 22
(c) 16 (d) 10

PART - III (MATHEMATICS)

81. The relation R defined on set $A = \{x : |x| < 3, x \varepsilon \text{ I}\}$ by $R = \{(x, y) : y = |x|\}$ is

(a) $\{-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$
(b) $\{(-2, -2), (-2, 2), (-1, 1), (0, 0), (1, -2), (1, 2), (2, -1), (2, -2)\}$
(c) $\{0, 0), (1, 1), (2, 2)\}$
(d) None of the above

82. The solution of the differential equation

$$\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)} \text{ is}$$

(a) $f(x) = y+C$ (b) $f(x) = y(x+C)$
(c) $f(x) = x+C$ (d) None of the above

83. If a, b and c are in AP, then determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \text{ is}$$

(a) 0 (b) 1
(c) x (d) $2x$

84. If two events A and B. If odds against A are as 2:1 and those in favour of $A \cup B$ are as 3:1, then

(a) $\frac{1}{2} \le P(B) \le \frac{3}{4}$ (b) $\frac{5}{12} \le P(B) \le \frac{3}{4}$

(c) $\frac{1}{4} \le P(B) \le \frac{3}{5}$ (d) None of these

85. The value of $2 \tan^{-1} (\text{cosec} \tan^{-1} x - \tan \cot^{-1} x)$ is

(a) $\tan^{-1} x$ (b) $\tan x$
(c) $\cot x$ (d) $\text{cosec}^{-1} x$

86. The proposition $\sim(p \Leftrightarrow q)$ is equivalent to

(a) $(p \vee \sim q) \wedge (q \wedge \sim p)$

(b) $(p \wedge \sim q) \vee (q \wedge \sim p)$

(c) $(p \wedge \sim q) \wedge (q \wedge \sim p)$

(d) None of the above

87. If truth values of P be F and q be T. Then, truth value of $\sim(\sim p \vee q)$ is

(a) T (b) F

(c) Either T or F (d) Neither T not F

88. The rate of change of the surface area of a sphere of radius r, when the radius is increasing at the rate of 2 cm/s is proportional to

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) r (d) r^2

89. If N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\,R\,(c, d)$, if $ad\,(b + c) = bc\,(a + d)$, then R is

(a) symmetric only

(b) reflexive only

(c) transitive only

(d) an equivalence relation

90. A complex number z is such that arg $\left(\frac{z-2}{z+2}\right) = \frac{z}{3}$. The points representing this complex number will lie on

(a) an ellipse (b) a parabola

(c) a circle (d) a straight line

91. If a_1, a_2 and a_3 be any positive real numbers, then which of the following statement is true?

(a) $3a_1a_2a_3 \le a_1^3 + a_2^3 + a_3^3$

(b) $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \ge 3$

(c) $(a_1 + a_2 + a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \ge 9$

(d) $(a_1 . a_2 . a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{a_3}{a_3}\right)^3 \ge 27$

92. If $|x^2 - x - 6| = x + 2$, then the values of x are

(a) $-2, 2, -4$ (b) $-2, 2, 4$

(c) $3, 2, -2$ (d) $4, 4, 3$

93. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(a) $4 \le x^2 + y^2 \le 64$

(b) $x^2 + y^2 \le 25$

(c) $x^2 + y^2 \ge 25$

(d) $3 \le x^2 + y^2 \le 9$

94. A tower AB leans towards west making an angle α with the vertical. The angular elevation of B, the top most point of the tower is β as observed from a point C due east of A at a distance 'd' from A. If the angular elevation of B from a point D due east of C at a distance $2d$ from C is r, then $2 \tan \alpha$ can be given as

(a) $3 \cot \beta - 2 \cot \gamma$ (b) $3 \cot \gamma - 2 \cot \beta$

(c) $3 \cot \beta - \cot \gamma$ (d) $\cot \beta - 3 \cot \gamma$

95. If α and β are the roots of $x^2 - ax + b = 0$ and if $\alpha^n + \beta^n = V_n$, then

(a) $V_{n+1} = aV_n + bV_{n-1}$

(b) $V_{n+1} = aV_n + aV_{n-1}$

(c) $V_{n+1} = aV_n - bV_{n-1}$

(d) $V_{n+1} = aV_{n-1} - bV_n$

96. The sum of the series

$$\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \ldots m \text{ terms}\right) \text{is}$$

(a) $\frac{2^{mn} - 1}{2^{mn}(2^n - 1)}$ (b) $\frac{2^{mn} - 1}{2^n - 1}$

(c) $\frac{2^{mn} + 1}{2^n + 1}$ (d) None of these

97. The angle of intersection of the circles $x^2 + y^2 - x + y - 8 = 0$ and $x^2 + y^2 + 2x + 2y - 11 = 0$ is

(a) $\tan^{-1}\left(\frac{19}{9}\right)$ (b) $\tan^{-1}(19)$

(c) $\tan^{-1}\left(\frac{9}{19}\right)$ (d) $\tan^{-1}(9)$

98. The vector $\mathbf{b} = 3\mathbf{j} + 4\mathbf{k}$ is to be written as the sum of a vector $\mathbf{b_1}$ parallel to $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and a vector $\mathbf{b_2}$ perpendicular to $\mathbf{a}$. Then $\mathbf{b_1}$ is equal to

(a) $\frac{3}{2}(\mathbf{i} + \mathbf{j})$ (b) $\frac{2}{3}(\mathbf{i} + \mathbf{j})$

(c) $\frac{1}{2}(\mathbf{i} + \mathbf{j})$ (d) $\frac{1}{3}(\mathbf{i} + \mathbf{j})$

99. If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then the rank of the matrix

$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than

(a) 3 (b) 2
(c) 1 (d) None of these

100. The value of the determinant

$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is

(a) $\alpha^2+\beta^2$ (b) $\alpha^2-\beta^2$
(c) 1 (d) 0

101. The number of integral values of K, for which the equation $7\cos x + 5\sin x = 2K + 1$ has a solution, is

(a) 4 (b) 8
(c) 10 (d) 12

102. The line joining two points $A(2,0)$, $B(3,1)$ is rotated about A in anti-clockwise direction through an angle of 15°. The equation of the line in the now position,is

(a) $\sqrt{3}x-y-2\sqrt{3}=0$

(b) $x-3\sqrt{y}-2=0$

(c) $\sqrt{3}x+y-2\sqrt{3}=0$

(d) $x+\sqrt{3}y-2=0$

103. The line $2x+\sqrt{6}y=2$ is a tangent to the curve $x^2-2y^2=4$. The point of contact is

(a) $(4,-\sqrt{6})$ (b) $(7,-2\sqrt{6})$

(c) $(2,3)$ (d) $(\sqrt{6},1)$

104. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0) is

(a) 133 (b) 190
(c) 233 (d) 105

105. $\int (1+x-x^{-1})\, e^{x+x^{-1}}\, dx$ is equal to

(a) $(x+1)e^{x+x^{-1}}+C$

(b) $(x-1)e^{x+x^{-1}}+C$

(c) $xe^{x+x^{-1}}+C$

(d) $xe^{x+x^{-1}}x+C$

106. If $f(x)=x-[x]$, for every real number x, where $[x]$ is the integral part of x. Then, $\int_{-1}^{1} f(x)\,dx$ is equal to

(a) 1 (b) 2

(c) 0 (d) $\frac{1}{2}$

107. The value of the integral

$\int_{-1/2}^{1/2}\left[\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right]^{1/2} dx$ is

(a) $\log\left(\frac{4}{3}\right)$ (b) $4\log\left(\frac{3}{4}\right)$

(c) $4\log\left(\frac{4}{3}\right)$ (d) $\log\left(\frac{3}{4}\right)$

108. If a tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points A and B respectively, then the area of ΔOAB is equal to (O is the centre of the ellipse)

(a) 12 sq units (b) 48 sq units
(c) 64 sq units (d) 24 sq units

109. The locus of mid points of tangents intercepted between the axes of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will be

(a) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ (b) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=2$

(c) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=3$ (d) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=4$

110. If PQ is a double ordinate of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Such that OPQ is an equilateral triangle, O being the centre of the hyperbola, then the eccentricity 'e' of the hyperbola satisfies

(a) $1 < e < \frac{2}{\sqrt{3}}$ (b) $e = \frac{2}{\sqrt{3}}$

(c) $e = \frac{\sqrt{3}}{2}$ (d) $e > \frac{2}{\sqrt{3}}$

111. The sides *AB, BC* and *CA* of a ΔABC have respectively 3, 4 and 5 points lying on them. The number of triangles that can be constructed using these points as vertices is

(a) 205 (b) 220

(c) 210 (d) None of these

112. In the expansion of $\frac{a+bx}{e^x}$, the coefficient of x^r is

(a) $\frac{a-b}{r!}$ (b) $\frac{a-br}{r!}$

(c) $(-1)^r \frac{a-br}{r!}$ (d) None of these

113. If $n = (1999)!$, then $\sum_{x=1}^{1999} \log_n x$ is equal to

(a) 1 (b) 0

(c) $\sqrt[1999]{1999}$ (d) -1

114. *P* is a fixed point (*a, a, a*) on a line through the origin equally inclined to the axes, then any plane through *P* perpendicular to *OP*, makes intercepts on the axes, the sum of whose reciprocals is equal to

(a) a (b) $\frac{3}{2a}$

(c) $\frac{3a}{2}$ (d) None of these

115. For which of the following values of m, the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$

(a) -4 (b) -2

(c) 2 (d) 4

116. If $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then, $\lim_{x \to 0} \left\{ \frac{f(1+x)}{f(1)} \right\}^{1/x}$ equals to

(a) 1 (b) $e^{1/2}$

(c) e^2 (d) e^3

117. If $f(x) = \begin{cases} (1 + |\sin x|)^{a/|\sin x|} & , -\frac{\pi}{6} < x < 0 \\ b & , x = 0 \\ e^{\tan 2x / \tan 3x} & , 0 < x < -\frac{\pi}{6} \end{cases}$, then the value of a and b, if f is continuous at $x = 0$, are respectively.

(a) $\frac{2}{3}, \frac{3}{2}$ (b) $\frac{2}{3}, e^{2/3}$

(c) $\frac{3}{2}, e^{3/2}$ (d) None of these

118. The domain of the function

$$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2} \text{ is}$$

(a) $]-3, -2.5[\cap]-2.5, -2[$

(b) $[-2, 0[\cup]0, 1[$

(c) $]0, 1[$

(d) None of the above

119. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$, is

(a) $(x-2) = Ke \tan^{-1} y$

(b) $2xe^{\tan^{-1} y} = e^2 \tan^{-1} y + K$

(c) $xe\tan^{-1} y = \tan^{-1} y + K$

(d) $xe2\tan^{-1} y = e\tan^{-1} y + K$

120. If the gradient of the tangent at any point *(x, y)* of a curve which passes through the point $\left(1, \frac{\pi}{4}\right)$ is $\left\{ \frac{y}{x} - \sin^2\left(\frac{y}{x}\right) \right\}$, then equation of the curve is

(a) $y = \cot^{-1}(\log_e x)$

(b) $y = \cot^{-1}\left(\log_e \frac{x}{e}\right)$

(c) $y = x\cot^{-1}(\log_e ex)$

(d) $y = \cot^{-1}\left(\log_e \frac{e}{x}\right)$

SOLUTIONS

PART - I (PHYSICS)

1. (d) As we know, velocity of electromagnetic wave, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

   It is independent of amplitude of electromagnetic wave, frequency and wavelength of electromagnetic wave.

2. (b) According to Moseley's law $\sqrt{v} = a(z-b)$

   or $v = a^2 (z-b)^2$

   or $\frac{c}{\lambda} = a^2 (z-b)^2$

   $\therefore \frac{\lambda_1}{\lambda_2} = \frac{(z_2-1)^2}{(z_1-1)^2}$

   Here $\lambda_1 = \lambda$, $\lambda_2 = 4\lambda$, $z_1 = 11$ and $z_2 = ?$

   $\therefore \frac{1}{4\lambda} = \frac{(z_2-1)^2}{(11-1)^2}$

   or $(z_2 - 1)^2 = 25$ or $z_2 = 6$

3. (b) As we know, conductivity,

   $\sigma = \frac{1}{\rho} = \rho(\mu_e n_e + \mu_n n_n)$

   $= 1.6 \times 10^{-19}$ [0.36 + 0.17] $(2.5 \times 10^{19})]$

   $= 2.12$ Sm^{-1}

4. (d) If a radio receiver amplifies all the signal frequencies equally well, it is said to have high fidelity.

5. (b) Given, $y = 3 \sin \pi\left(\frac{t}{2} - \frac{x}{4}\right)$

   $= 3 \sin\left(\frac{\pi t}{2} - \frac{\pi x}{4}\right)$

   Comparing it with standard equation

   $y = r \sin \frac{2\pi}{\lambda}(vt - x)$

   $= r\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$

   We have, $\frac{2\pi v}{\lambda} = \frac{\pi}{2}$ or $v = \frac{\lambda}{4}$

   and $\frac{2\pi}{\lambda} = \frac{\pi}{4}$ or $\lambda = 8$ m $\therefore$ $v = \frac{8}{4} = 2$ m/s

   So, the distance travelled by wave in t second = vt = 2 × 5 = 10 m

6. (a) Gravitational intensity,

   $I = -\frac{dv}{dx} = -\frac{d}{dx}\left(\frac{k}{x^2}\right) = \frac{2k}{x^3}$

7. (d) For Cu wire, $l_1 = 2.2$ m, $r_1 = 1.5$ mm

   $= 1.5 \times 10^{-3}$n

   $Y_1 = 1.1 \times 10^{11}$ N/m^2

   For steel wire, $l_2 = 1.6$m, $r_2 = 1.5$ mm

   $= 1.5 \times 10^{-3}$ m

   $Y_2 = 2.0 \times 10^{11}$ N/m^2

   Let F be the stretching force in both the wires the

   For Cu wire, $Y_1 = \frac{F}{\pi r_1^2} \times \frac{l_1}{\Delta l_1}$

   $\Rightarrow F = \frac{Y_1 \pi r_1^2 \times \Delta l_1}{l_1}$

   $= \frac{1.1 \times 10^{-11}}{2.2} \times \frac{22}{7} \times (1.5 \times 10^{-3})^2 \times 0.5 \times 10^3$

   $= 1.8 \times 10^2$ N

8. (a) Mean speed, $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 0.92 v_{rms}$

   rms speed, $v_{rms} = \sqrt{\frac{3kT}{m}}$

   Most probable speed v_p

   $= \sqrt{\frac{2kT}{m}} = 0.816 v_{rms}$

   i.e., $v_p < \bar{v} < v_{rms}$

9. (b) Let two balls collide at a height *s* from the ground after *t* second when second ball is thrown upwards.

   $\therefore$ Time taken by first ball to reach the point of collision = $(t + 2)$ s

   $s = 39.2\,(t+2) + \frac{1}{2}(-9.8)(t+2)^2$

$= 39.2\,(t+2) - 4.9\,(t+2)^2$ ...(i)

For second ball

$s = 39.2t + \frac{1}{2}(-9.8)\,t^2$

$= 39.2t - 4.9\,t^2$ ...(ii)

From eqs. (i) and (ii)

$39.2\,(t+2) - 4.9\,(t+2)^2 = (39.2)\,t - 4.9t^2$

On solving we get, t = 3s

From Eq. (ii),

$s = 39.2 \times 3 - 4.9 \times (3)^2 = 117.6 - 44.1 = 73.5$ m

10. (b) Magnetic flux, $\phi = \mathbf{B}.\mathbf{A} = \frac{F}{I\lambda}.\mathbf{A}$

$= \frac{[M^1L^{+1}T^{-2}]L^2}{[A.L]} = [M^1\,L^2\,T^{-2}\,A^{-1}]$

11. (c) Given, $P = P_0 \exp(-\alpha t^2)$

As P and P_0 have the same units, therefore αt^2 must be dimensionless for which

$\alpha = \frac{1}{T^2} = T^{-2}$

12. (d) Given, $U = \frac{M}{r^6} - \frac{N}{r^{12}}$

$\therefore F = \frac{-du}{dr} = \frac{-d}{dr}\left(\frac{M}{r^6} - \frac{N}{r^{12}}\right)$

$= -\left(\frac{-6M}{r^7} + \frac{12N}{r^{13}}\right) = \left(\frac{6M}{r^7} - \frac{12N}{r^{13}}\right)$

For equilibrium position, F = 0

$\therefore \frac{6M}{r^7} = \frac{12N}{r^{13}}$ or $r^6 = \frac{2N}{M}$

Hence, $U = \frac{M}{(2N/M)} - \frac{N}{(2N/M)^2} = \frac{M^2}{4N}$

13. (b) Total number of revolutions = area under n–t graph

$= \frac{1}{2} \times 8 \times \frac{1800}{60} + 8 \times \frac{600}{60} + \frac{1}{2} \times 16 \times \frac{600}{60}$

$= 120 + 80 + 80 = 280$

14. (c) From the graph we can say, upto t = 2.0 s, the body moves with a constant velocity

Slope of position-time graph $= \frac{4}{2} = 2$m/s

After t = 2.0 s, position-time graph is parallel to time axis i.e., body comes to rest.

$\therefore$ Change in velocity = dx = 2 m/s

Impulse = Change in momentum

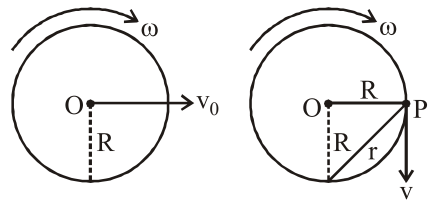
$= mdv = 0.1 \times 2 = 0.2$ kg m/s

15. (d) Pressure $= \frac{\text{force}}{\text{area}} = \frac{F}{L^2}$

$\therefore \frac{\Delta p}{p} = \frac{\Delta F}{F} + \frac{2\Delta L}{L} = 4\% + 2\,(2\%)$

or percentage error, = 8%

16. (d) The situation can be shown as



Here $v_0 = R\omega$

At. P, $v = r\omega$

$= \sqrt{(R^2 + R^2)}\omega = \sqrt{2}R\omega = \sqrt{2}v_0$

17. (b) Using principle of conservation of linear momentum,

$3m \times v = \sqrt{(m \times 60)^2 + (m \times 60)^2}$

$= m \times 60\sqrt{2}$

$\Rightarrow v = 20\sqrt{2}$ m/s

18. (a) Common velocity, $v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$= \frac{2 \times 6 + 2 \times 0}{2 + 2} = 3$ m/s

Initial kinetic energy (E_1)

$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2} \times 2 \times (6)^2 = 36$J

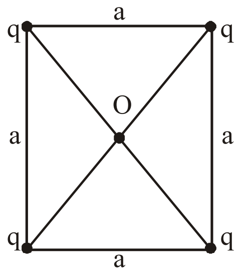
$= \frac{1}{2}(m_1 + m_2)V^2 = \frac{1}{2}(2+2)(3)^2 = 18$J

$\therefore$ Heat evolved = (36 – 18) J = 18 J

19. (a) From given condition

$\frac{Gmm}{(2R)^2} = \frac{mv^2}{R}, v = \frac{Gm}{4R} = \frac{1}{2}\sqrt{\frac{Gm}{R}}$

20. (d) At the centre O of the square due to four equal charges q at the corners, potential

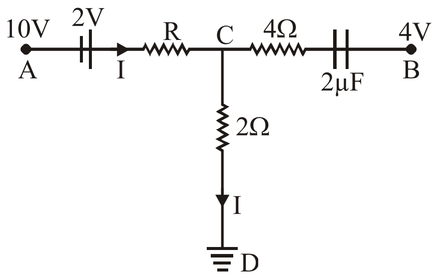


$$V=\frac{4q}{4\pi\varepsilon_0(a\sqrt{2})/2}=\frac{\sqrt{2}q}{\pi\varepsilon_0 a}$$

$\therefore\ W_{0\infty}=-\ W_{\infty 0}$

$$=-(-q)V=\frac{\sqrt{2}q^2}{\pi\varepsilon_0 a}$$

21. (a) In the steady state, current through capacitor



Using Kirchhof's voltage law to the circuit ACD

We have, $10-2+1\times R+1\times 2=0$

or $R=10\Omega$

Potential difference across C and D

$V_C-V_D=2\times 1=2V$

As $V_D=0V$

So, $V_C=2V$

Potential difference across capacitor $=4-2=2V$

$\therefore$ Charge on capacitor $Q=CV=2\mu F\times 2=4\mu C$

22. (a) KE of an electron in nth orbit : $K_n\propto\frac{1}{n^2}$

and PE of an electron in nth orbit :

$U_n\propto\frac{1}{n^2}$

$\therefore$ When an electron passes from state $n=2$ to $n=1$

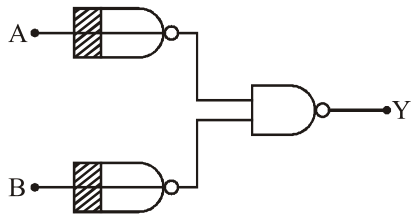
$$\frac{K_2}{K_1}=\frac{1^2}{2^2}=\frac{1}{4}$$

or $K_1=4K_2$

$$\frac{U_2}{U_1}=\frac{1^2}{2^2}=\frac{1}{4}$$

or $U_1=4U_2$

23. (b) To obtain OR gate from NAND gates we need two NOT gates obtained from NAND gates and one NAND gate as figure.



Boolean expression $\gamma=\overline{\overline{A}.\overline{B}}=\overline{\overline{A}}+\overline{\overline{B}}$

$=A+B$ OR gate

24. (b) Magnetic field $B=\frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

Here, $2\pi=\theta$

$$\therefore B=\frac{\mu_0}{4\pi}\frac{\theta I}{r}$$

25. (a) Since axes are perpendicular so mid point lies on axial line of one magnet and on equitorial line of other magnet

$$\therefore B_1=\frac{\mu_0}{4\pi}\frac{2M}{d^3}=\frac{10^{-7}\times 2\times 1}{1^3}=2\times 10^{-7}\,T$$

and $$B_2=\frac{\mu_0}{4\pi}\frac{M}{d^3}=\frac{10^{-7}\times 1}{1^3}=10^{-7}\,T$$

As $B_1\perp B_2$

$\therefore$ Resultant magnetic field

$$=\sqrt{B_1^2+B_2^2}=\sqrt{5}\times 10^{-7}\,T$$

26. (a) In the given circuit, two condensors and the inductor are in series.

$\therefore L_s=L+L=2L$

and $$\frac{1}{C_s}=\frac{1}{C}+\frac{1}{C}=\frac{2}{C}\Rightarrow C_s=\frac{C}{2}$$

$\therefore$ Natural frequency of the circuit

$$v=\frac{1}{2\pi\sqrt{L_sC_s}}=\frac{1}{2\pi\sqrt{2L}\times\sqrt{C/2}}$$

$$=\frac{1}{2\pi\sqrt{LC}}$$

27. (a) In the beginning due to gravity pull, the lead shot will be accelerated and hence will move, with increasing velocity for some time When the viscous force balance the gravity pull, then the shot will move with constant velocity. As in the beginning, the velocity of shot is not fully linear with the effective distance covered by the shot.

28. (b) Refractive index of a medium $\mu = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

29. (c) Lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $\frac{u}{v} - 1 = \frac{u}{f}$

$\Rightarrow \frac{1}{m} - 1 = \frac{u}{f}$

or $\frac{1}{m} = \left(\frac{1}{f}\right)u + 1$

this is the equation of a straight line whose slope

$\frac{1}{f} = \frac{b}{c} \therefore f = \frac{c}{b}$

30. (d) Amplitudes A_1 and A_2 are added as vectors. Angle between the two vectors is the phase difference $(\beta_1 - \beta_2)$ between them.
$\therefore$ Resultant wave,

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1 - \beta_2)}$

31. (a) Here, case (i) $e(3V_0) = \frac{hc}{\lambda} - \phi_0$ ...(i)

Case (ii) $eV_0 = \frac{hc}{2\lambda} - \phi_0$ ...(ii)

From eqs. (i) and (ii),

$\frac{3hc}{2\lambda} - 3\phi_0 = \frac{hc}{\lambda} - \phi_0$

or $\frac{3hc}{2\lambda} - \frac{hc}{\lambda} = 3\phi_0 - \phi_0 = \frac{hc}{2\lambda} = 2\phi_0$

or, $\phi_0 = \frac{hc}{4\lambda}$

$\therefore$ Threshold wavelength

$\lambda_0 = \frac{hc}{\phi_0} = \frac{hc}{hc} \times 4\lambda = 4\lambda$

32. (b) In an isobaric process, p = constant
Hence, $V \propto T$

i.e., $V = \left(\frac{nR}{P}\right)T$

$\therefore$ V–T graph is a straight line with slope $\propto \frac{1}{p}$

$(\text{slope})_2 > (\text{slope})_1$
$\therefore p_2 < p_1$

33. (a) Total energy

$U = 2\left(\frac{n_1}{2}RT\right) + 4\left(\frac{n_2}{2}RT\right)$

For O_2, n = 5 and for Ar, $n_2 = 3$

$\therefore U = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11RT$

34. (b) Given, speed of each pulse = 2 cm/s Therefore distance travelled by both pulses in 2s = 4 cm toward each other. On their superposition, the resultant displacement at every point will be zero.
Hence, total energy will be purely kinetic.

35. (d) Time interval between two successive maxima = time interval between two successive beats $= \frac{1}{n_1 - n_2}$

36. (b) Here, $h = \frac{2s\cos 0°}{r\rho g} = \frac{2s}{r\rho g}$ ...(i)

According to question,

$\frac{h}{2} = \frac{2s\cos\theta'}{r\rho g}$ ...(ii)

Dividing eq. (ii) by (i) we get,

$\frac{1}{2} = \cos\theta'$

or $\theta' = 60°$

37. (a) Given, $V_1 = V_2 = 300V; V_3 = ?, i = ?$

As, $V = \sqrt{V_3^2 + (V_1 - V_2)^2}$

$\therefore 220 = \sqrt{V_3^2} = V_3 \Rightarrow V_3 = 220V$

$\therefore I = \frac{V_3}{R} = \frac{220}{100} = 2.2A$

38. (b) We have, $W = -MB(\cos\theta_2 - \cos\theta_1)$

So, $W_1 = -MB(\cos 90° - \cos 0°) = MB$

and $W_2 = -MB(\cos 60° - \cos 0°) = \frac{1}{2}MB$

As $W_1 = nW_2$

$\therefore n = \frac{W_1}{W_2} = \frac{MB}{\frac{1}{2}MB} = 2$

39. (b) Capacitance, $C = \frac{\varepsilon_0 A}{d}$

As one–fourth of capacitor is filled with dielectric of constant K, then,

$C_1 = \frac{K\varepsilon_0 A/4}{d}$

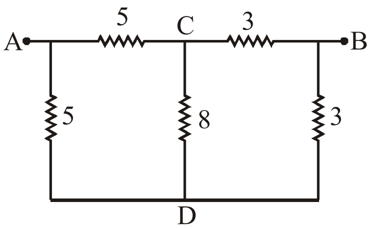
and $C_2 = \frac{\varepsilon 3A/4}{d}$

Both C_1 and C_2 are in parallel.

$\therefore C_P = C_1 + C_2 = \frac{K\varepsilon_0 A}{4d} + \frac{3\varepsilon_0 A}{4d}$

$= (K+3)\frac{\varepsilon_0 A}{4d} = (K+3)\frac{C}{4}$

40. (c) The equivalent circuit of the given circuit is



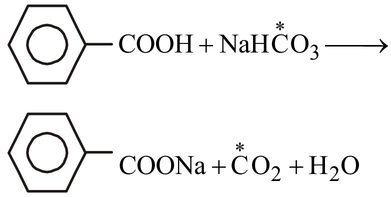
This is a balanced Wheatstone bridge. Therefore, the arm CD becomes in effective. Hence 5Ω and 3Ω are in series and they together are in parallel with $(5+3)\,\Omega$

$\therefore$ Net resistance $= \frac{(5+3)+(5+3)}{(5+3)\times(5+3)} = 4\Omega$

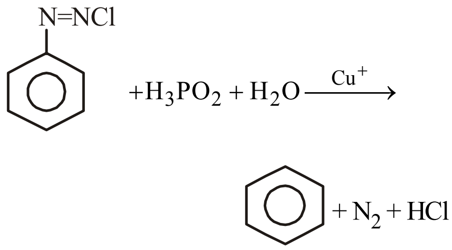
PART - II (CHEMISTRY)

41. (a) Benzoin condensation is performed by aromatic aldehydes (i.e., compounds in which –CHO group is directly attached with benzene ring).

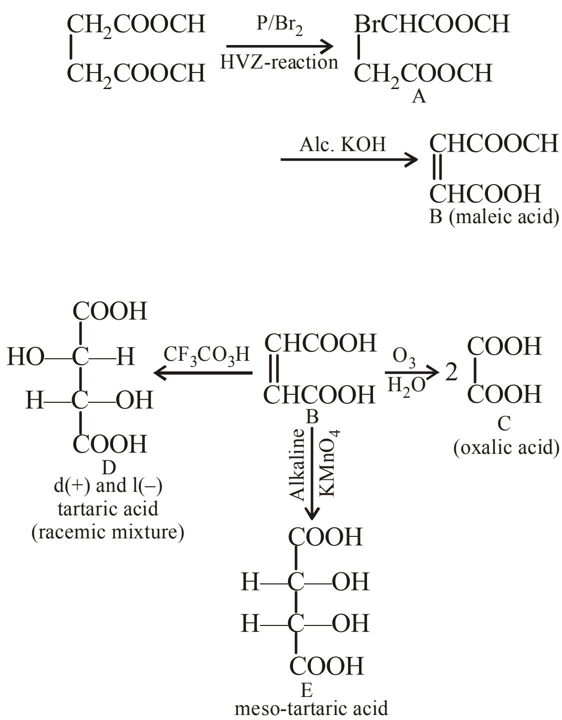
42. (a)



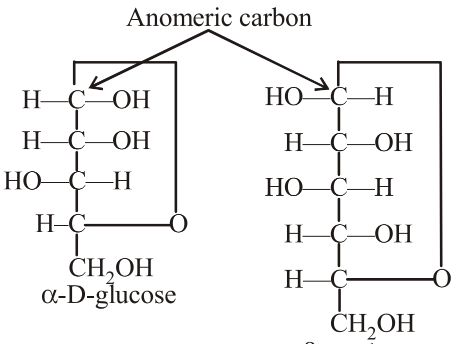
43. (d)



44. (c)



45. (d) When two cyclic forms of a carbohydrate differ in configuration only at hemiacetal carbons, they are said to be anomers. Thus, anomers are cyclic forms of carbohydrates that are epimeric at hemiacetal carbon and this carbon (C–1 of aldose) is called anomeric carbon, e.g.,



α-D-glucose β-D-glucose

46. (c) α– amino acids are bifunctional organic compounds. They contain a basic amino group ($-NH_2$) on the α–carbon and one acidic carboxyl group (–COOH).

47. (d) $[CH_3COOH] = \text{millimoles of } CH_3COOH$
$= 0.1 \times 10 = 1.0$
$[CH_3COONa] = \text{millimoles of } CH_3COONa$
$= 0.1 \times 20 = 2.0$
From, Henderson Hasselbalch equation,

$$pH = pK_a + \log \frac{[\text{conjugate base}]}{[\text{acid}]}$$

$$= 4.74 + \log \frac{2}{1} = 4.74 + 0.30 = 5.04$$

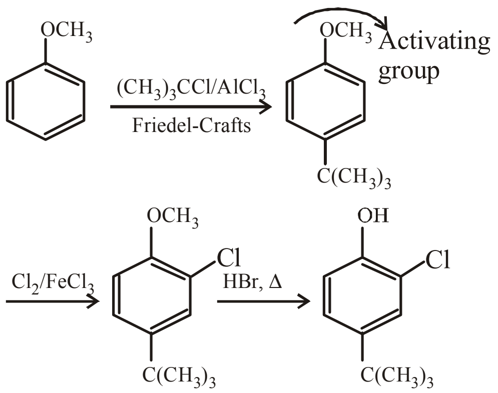
48. (b) $\lambda^{\infty}\ (Ag^+)$ = transport number of $Ag^+ \times \Lambda^{\infty}_{(AgNO_3)}$
$= 0.464 \times 133.3 = 61.9\ \Omega^{-1}\ cm^2\ equiv^{-1}$
By Kohlrausch's law

$$\Lambda^{\infty}_{(AgNO_3)} = \lambda^{\infty}_{(Ag^+)} + \lambda^{\infty}_{(NO_3^-)}$$

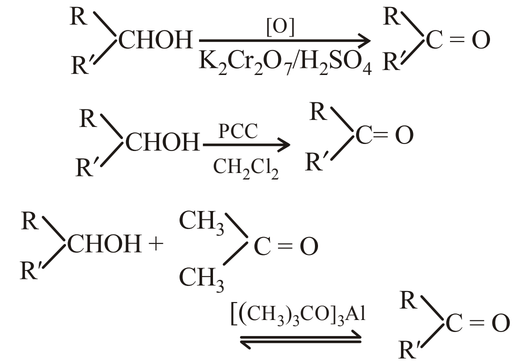
$$\therefore\ \Lambda^{\infty}_{(NO_3^-)} = \Lambda^{\infty}_{(AgNO_3)} - \lambda^{\infty}_{(Ag^+)}$$

$= 133.3 - 61.9 = 71.4\ \Omega^{-1}\ cm^2\ equiv^{-1}$

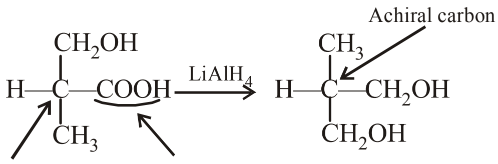
49. (d)



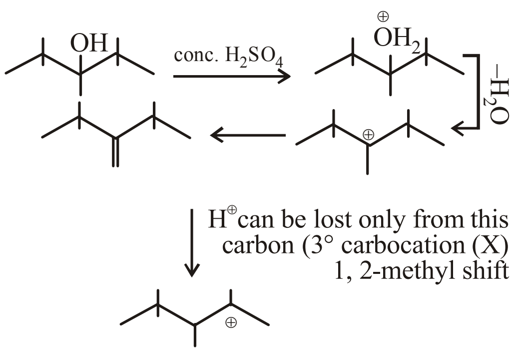
50. (c) Oxidation of Ketones, yield secondary alcohol



51. (c) Since, $X \xrightarrow{NaHCO_3} CO_2$
Hence, it must contain —COOH group.



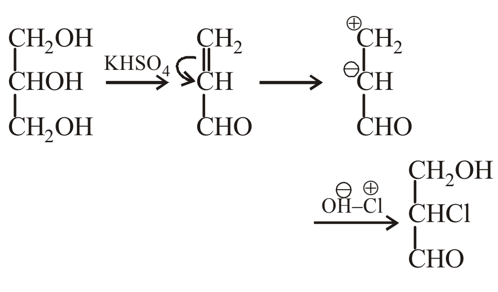
52. (a)



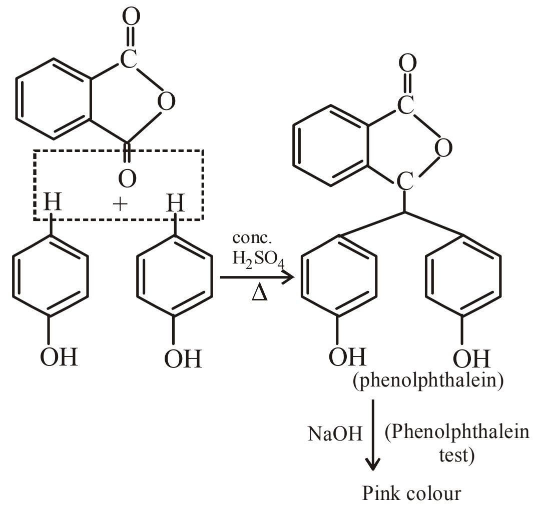
(3° carbocation (Y))

Y is less soluble than (X) due to lack of symmetry Chiral carbon. This is reduced to $-CH_2OH$

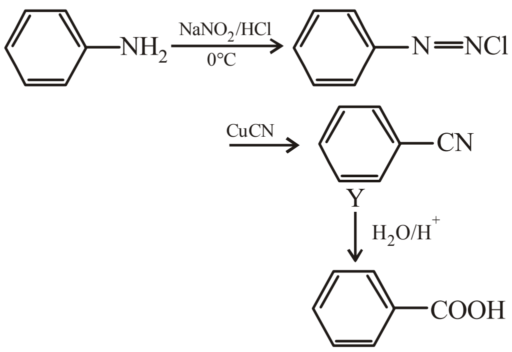
53. (b)



54. (a)



55. (d)



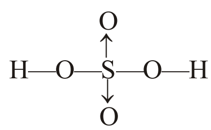
56. (a) According to Van–Arkel method, pyrolysis of BI_3 is carried out in the presence of red hot W or Ta filament.

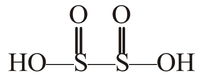
$$2BI_3 \xrightarrow{\text{Red hot w or Ta filament}} 2B + 3I_2$$

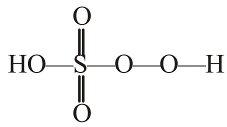
57. (c) $NH_4Cl(s) \xrightarrow{\Delta} NH_3 \uparrow + HCl \uparrow$

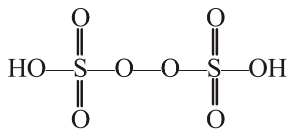
Graham's law of diffusion says, lighter gas will diffuse most rapidly. Therefore, NH_3 will be (mol. wt. = 17) diffused rapidly than HCl. (mol. wt. = 36.5).

58. (b) Peroxy acids contain —O—O— linkage.

$H_2S_2O_3$ 

$H_2S_2O_4$ 

H_2SO_5 

$H_2S_2O_8$ 

59. (d) Volume of one molecule

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\ (1.54 \times 10^{-8})^3\ cm^3$$

$= 1.53 \times 10^{-23}\ cm^3$

Volume of molecules in 1.65 g Ar

$$= \frac{1.65}{40} \times N_0 \times 1.53 \times 10^{-23} = 0.380\ cm^3$$

Volume of solid containing 1.65 g Ar = 1 cm^3

$\therefore$ Empty space = 1 – 0.380 = 0.620

$\therefore$ Per cent of empty space = 62%

60. (d) In adiabatic expansion

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

for CO_2 (triatomic gas) is,

$\gamma = 1.33$

$$\therefore \left(\frac{150}{300}\right) = \left(\frac{10}{V_2}\right)^{0.33}$$

$$\left(\frac{1}{2}\right) = \left(\frac{10}{V_2}\right)^{0.33}$$

$$\left(\frac{1}{2}\right)^3 = \frac{10}{V_2}$$

$$\frac{1}{8} = \frac{10}{V_2}$$

$V_2 = 80\ L$

61. (c) $RCOOR' + H_2O \xrightarrow{H^+} RCOOH + R'OH$

At t = 0,	a	0	0
At time t,	a – x	x	x
At time ∞,	a – a	a	a

At t = 0, V_0 = volume of NaOH due to H^+ (catalyst)

$$V_t = x + V_0$$

$$V_\infty = a + V_0$$

If ester is 50% hydrolysed then, $x = \frac{a}{2}$

or $V_t = \frac{a}{2} + V_0$

or $a = 2V_t - 2V_0$

$\therefore\ V_\infty = 2V_t - 2V_0 + V_0$

$= 2V_t - V_0$

62. (b) Energy values are additives.

$E = E_1 + E_2$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{1}{300} = \frac{1}{760} + \frac{1}{\lambda_2}$$

$$\frac{1}{\lambda_2} = \frac{1}{300} - \frac{1}{760}$$

$\lambda = 495.6\,\text{nm} = 496\,\text{nm}$

63. (a) I. $Fe^{2+} = Ar$ [⇅ | ↑ | ↑ | ↑ | ↑] $3d^6$

4 unpaired electrons,

$\therefore$ Coloured ion,

II. $Ni^{2+} = Ar$ [⇅ | ⇅ | ⇅ | ↑ | ↑] $3d^8$

2 unpaired electrons,

$\therefore$ Coloured ion

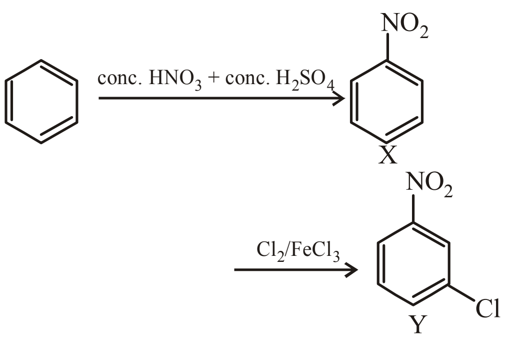
III. $Al^{3+} = [Ne]$

No unpaired electron in 3d, colourless ion.

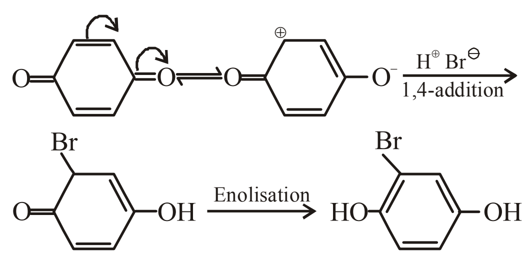
64. (b) $CH_3C \equiv \ddot{N}: \rightarrow CH_3\overset{\oplus}{C} = \overset{\ominus}{\ddot{N}}:$

electrophilic nucleophilic

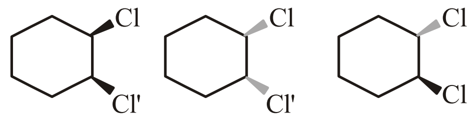
65. (c)



66. (d)



67. (a)

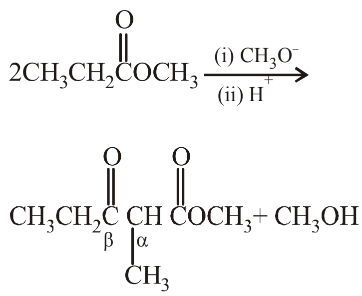


cis *p.* *cis* *Q.* *trans* *R*

68. (a) $R-\overset{\overset{O^\alpha}{\|}}{C}-\overset{\beta}{O}-R'$

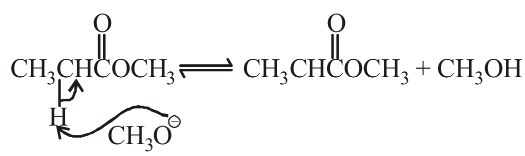
α–O oxygen atom can donate lone pair of electron more easily, therefore, it is more basic than β–oxygen.

69. (d) When two molecules of an ethylacetate undergo condensation reaction, in presence of sodium ethoxide involving the reaction is called as Claisen condensation and product is a β–keto ester.

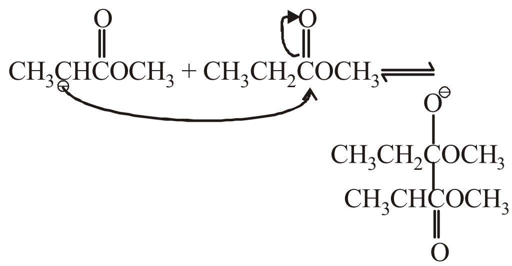


Mechanism

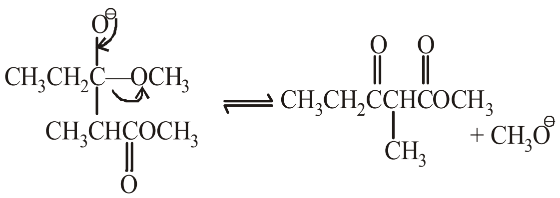
Step I



Step II



Step III

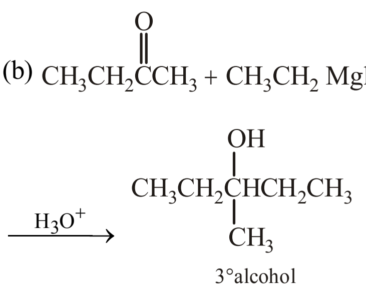


70. (b) B is a tertiary alcohol based on given properties.

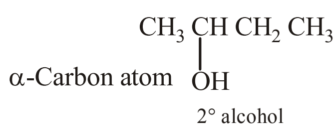
(a) $CH_2-CH_2+CH_3CH_2MgI \xrightarrow{H_3O^+}$

$CH_3CH_2CH_2CH_2OH$
1° alocohol

(b)



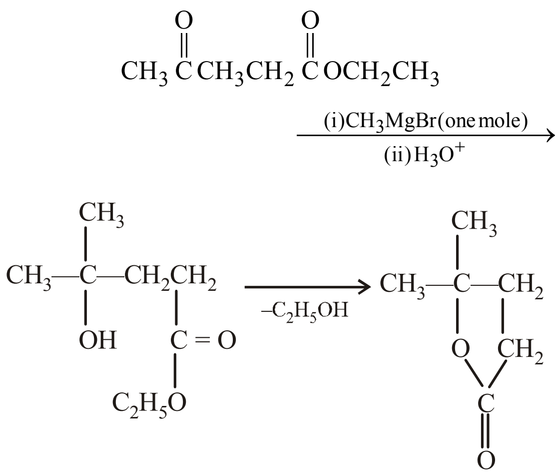
(c) $CH_3CHO + CH_3CH_2MgI \xrightarrow{H_3O^+}$



(d) $HCHO + CH_3CH_2MgI \xrightarrow{H_3O^+}$

$CH_3CH_2CH_2OH$
1° alcohol

71. (c) Keto group is more reactive for addition of Grignard reagent.



72. (b) $E^\circ_{cell} = E^\circ_{ox} + E^\circ_{red}$

$= -E^\circ_{Co/Co^{2+}} + E^\circ_{Ce^{4+}/Ce^{3+}}$

$1.89 = -(-0.28) + E^\circ_{Ce^{4+}/Ce^{3+}}$

$\therefore\ E^\circ_{Ce^{4+}/Ce^{3+}} = -1.879 - 0.28 = 1.61\ V$

73. (c) Due to electrolysis

$$2H_2O(l) + 2e^- \rightarrow H_2(g) + 2OH^-(aq)$$

$$2Cl^-(aq) \rightarrow Cl_2(g) + 2e^-$$

$$2H_2O(l) + 2Cl^-(aq) \rightarrow H_2(g) + Cl_2(g) + 2OH^-(aq)$$

OH^- formed = NaOH formed = Z i t

$= \dfrac{E}{96500} \times i \times t$

$= \dfrac{40}{96500} \times 30 \times 1 \times 60 \times 60 = 44.77\ g$

$= \dfrac{44.77}{40} = 1.12\ mol$

Cl_2 formed $= \dfrac{1}{2}$ mol of NaOH

$= \dfrac{1.12}{2} = 0.56\ mol$

$= 0.56 \times 22.4\ L$ at STP $= 12.54\ L$

74. (b) $K_B = Ae^{-E_a/RT}$

$= 10^{12} \times 4.35 \times 10^{-8}$

$= 4.35 \times 10^{4}\ s^{-1}$

Also equilibrium constant, $k = \frac{k_A}{k_B} = 10^4$

$\therefore k_A = k_B \times 10^4 = 4.35 \times 10^8\ s^{-1}$

75. (a) $\Delta G = \Delta H - nFT\left(\frac{dE}{dT}\right)_P$

and $\Delta G = \Delta T - T\Delta S$

$\therefore \frac{\Delta S}{nF} = \left(\frac{dE}{dT}\right)_P$

or $\frac{96.5}{2 \times 96500} = \left(\frac{dE}{dT}\right)_P$

$\therefore \left(\frac{dE_{cell}}{dT}\right)_P = \frac{1 \times 10^{-3}}{2} = 5 \times 10^{-4}\ VK^{-1}$

76. (c) We have to compare wavelength of transition in the H–spectrum with the Balmer transition n = 4 to n = 2 of He^+ spectrum.

$\because \lambda_H = \lambda_{He^+}$

$\therefore R_H Z_H^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = R_H Z_{He^+}^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$

$1 \times \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 4 \times \left(\frac{1}{4} - \frac{1}{16}\right)$

$\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 4 \times \frac{4-1}{16}$

$\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{3}{4}$

If $n_1 = 1$, then $n_2 = 2, 3, ...$

For first line $n_2 = 2$, $n_1 = 1$

$$\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{1}{1} - \frac{1}{4} = \frac{3}{4}$$

Hence, trnasition $n_2 = 2$ to $n_1 = 1$ will give spectrum of the same wavelength as that of Balmer transition, $n_2 = 4$ to $n_1 = 2$ in He^+.

77. (b) Energy of the electron in the nth orbit in terms of R_H is

$E_n = -\frac{R_H Z^2}{n^2}$

where, Z = atomic number, n^2 = degeneracy

For H–atom, $E_n = -\frac{R_H (1)^2}{n^2}$

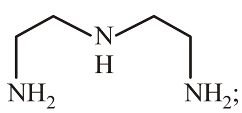
$-\frac{R_H}{9} = -\frac{R_H}{n^2}$

$\therefore n^2 = 9$

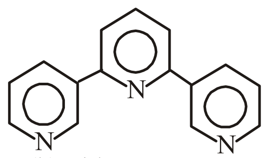
78. (d)

	Compound	Symbol/ formla	Uses
A.	Dry ice	CO_2	Referigerant for preserving food
B.	Semicon ductor	Ge	Electronic diode and triode in computer
C.	Solder	Sn/Pb	Joining circuits
D.	TEL	$(C_2H_5)_4$ Pb	Antiknocking compound for petroleum products

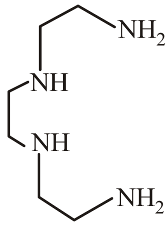
79. (c)



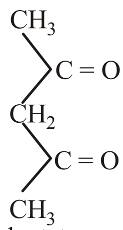
(a) tridentate



(b) tridentate



(c) tetradentate



(d) bidentate

80. (b) Effective atomic number EAN = Atomic number – oxidation number + 2 × coordination number

For $[Al(C_2O_4)_3]^{3-}$

Z = 13

ON = 3

CN = 6

$\therefore$ EAN = 13 – 3 + 2 × 6 = 22

PART - III (MATHEMATICS)

81. (a) $A = \{x : |x| < 3, x \in I\}$

$A = \{x : -3 < x < 3, x \in I\} = \{-2, -1, 0, 1\}$

Also, $R = \{(x, y) : y = |x|\}$

$\therefore R = \{(-2, 2), (-1, 1), (1, 1), (0, 0), (2, 2)\}$

82. (b) $\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$

$\Rightarrow yf'(x)\,dx - f(x)\,dy = y^2\,dx$

$\Rightarrow \frac{yf'(x)dx - f(x)dy}{y^2} = dx$

$\Rightarrow d\left\{\frac{f(x)}{y}\right\} = dx$

On integration, we get

$\frac{f(x)}{y} = x + C$

$\Rightarrow f(x) = y(x + C)$

83. (a) Let $\Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 2(2b-a-c) \\ x+4 & x+5 & x+2c \end{vmatrix}$

(using $R_2 \to 2R_2 - R_1 - R_3$)

But a, b and c are in AP using $2b = a + c$, we get

$\Delta = \frac{1}{2}\begin{vmatrix} x+2 & x+3 & x+2a \\ 0 & 0 & 0 \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$

Since, all elements of R_2 are zero.

84. (b) $P(A) = \frac{1}{3}, P(A \cup B) = \frac{3}{4}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\leq P(A) + P(B)$

$\Rightarrow \frac{3}{4} \leq \frac{1}{3} + P(B)$

$\Rightarrow P(B) \geq \frac{5}{12}$

Also, $B \leq A \cup B$

$\Rightarrow P(B) \leq P(A \cup B) = \frac{3}{4}$

$\therefore \frac{5}{12} \leq P(B) \leq \frac{3}{4}$

85. (a) $2\tan^{-1}(\text{cosec}\,\tan^{-1}x - \tan\cot^{-1}x)$

$= 2\tan^{-1}\left[\text{cosec}\left\{\text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}\right\} - \tan\left\{\tan^{-1}\frac{1}{x}\right\}\right]$

$= 2\tan^{-1}\left[\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right]$

$= 2\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$

$= 2\tan^{-1}\left\{\frac{\sec\theta - 1}{\tan\theta}\right\}$ (put $x = \tan\theta$)

$= 2\tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$

$= 2\tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right\}$

$= 2\tan^{-1}\tan\frac{\theta}{2}$

$= 2.\frac{\theta}{2} = \theta = \tan^{-1}x$

86. (b) $\sim(p \Leftrightarrow q) \equiv \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)]$

$\equiv \sim(p \Rightarrow q) \vee (\sim(q \Rightarrow p))$

($\because$ De–Morgan's law)

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

87. (b)

p	**q**	**$\sim$p**	**$\sim$p $\vee$ q**	**$\sim$($\sim$p $\vee$ q)**
F	T	T	T	F

$\therefore$ Truth value of $\sim(\sim p \vee q)$ is F.

88. (c) Surface area of sphere,
$S = 4\pi r^2$

and $\frac{dr}{dt} = 2$

$\therefore \frac{ds}{dt} = 4\pi \times 2r\frac{dr}{dt} = 8\pi r \times 2 = 16\pi r$

$\Rightarrow \frac{ds}{dt} \propto r$

89. (d) For $(a, b), (c, d) \in N \times N$
$(a, b) R (c, d)$
$\Rightarrow ad(b + c) = bc(a + d)$
Reflexive : $ab(b + a) = ba(a + b), \forall ab \in N$
$\therefore (a, b) R (a, b)$
So, R is reflexive,
Symmetric : $ad(b + c) = bc(a + d)$
$\Rightarrow bc(a + d) = ad(b + c)$
$\Rightarrow cd(d + a) = da(c + b)$
$\Rightarrow (c, d) R (a, b)$
So, R is symmetric.
Transitive : For $(a, b), (c, d), (e, f) \in N \times N$
Let $(a, b) R (c, d), (c, d) R (e, f)$
$\therefore ad(b + c) = bc(a + d), cf(d + e) = de(c + f)$
$\Rightarrow adb + adc = bca + bcd$ ...(i)
and $cfd + cfe = dec + def$ ...(ii)
On multiplying eq. (i) by ef and eq. (ii) by ab and then adding, we have
$adbef + adcef + cfdab + cfeab$
$= bcaef + bcdef + decab + defab$
$\Rightarrow adcf(b + e) = bcde(a + f)$
$\Rightarrow af(b + e) = be(a + f)$
$\Rightarrow (a, b) R (e, f)$
So, R is transitive.
Hence R is an equivalence relation.

90. (c) $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$

$\Rightarrow \arg\left(\frac{x-2+iy}{x+2+iy}\right) = \frac{\pi}{3}$

$\Rightarrow \arg(x - 2 + iy) - \arg(x + 2 + iy) = \frac{\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{3}$

$\Rightarrow \frac{4y}{x^2 + y^2 - 4} = \sqrt{3}$

$\Rightarrow \sqrt{3}(x^2 + y^2) - 4y - 4\sqrt{3} = 0$
which is an equation of a circle.

91. (d) We know that, $GM \ge HM$

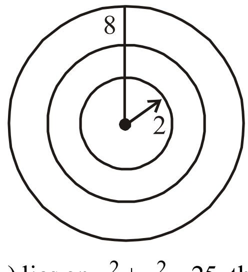
$\Rightarrow (a_1.a_2.a_3)^{1/3} \ge \frac{3}{\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)}$

$\Rightarrow (a_1.a_2.a_3) \ge \frac{27}{\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^3}$

$\Rightarrow (a_1.a_2.a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^3 \ge 27$

92. (b) $|x^2 - x - 6| = x + 2$, then
Case I : $x^2 - x - 6 < 0$
$\Rightarrow (x - 3)(x + 2) < 0$
$\Rightarrow -2 < x < 3$
In this case, the equation becomes
$x^2 - x - 6 = -x - 2$
or $x^2 - 4 = 0$
$\therefore x = \pm 2$
Clearly, x = 2 satisfies the domain of the equation in this case. So, x = 2 is a solution.
Case II : $x^2 - x - 6 \ge 0$
So, $x \le -2$ or $x \ge 3$
In this case, the equation becomes
$x^2 - x - 6 = 0 = x + 2$
i.e., $x^2 - 2x - 8 = 0$ or $x = -2, 4$
Both these values lie in the domain of the equation in this case, so x = –2, 4 are the roots.
Hence, roots are x = –2, 2, 4.

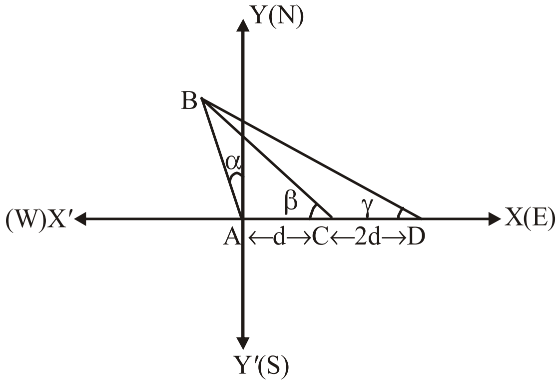
93. (a) Let (h, k) be any point in the set, then equation of circle is
$(x - h)^2 + (y - k)^2 = 9$



(h, k) lies on $x^2 + y^2 = 25$, then $h^2 + k^2 = 25$
$\Rightarrow 2 \le$ Distance between the two circles ≤ 8
$\Rightarrow 2 \le \sqrt{h^2 + k^2} \le 8$
$\Rightarrow 4 \le h^2 + k^2 \le 64$
$\therefore$ Locus of (h, k) is $4 \le (x^2 + y^2) \le 64$

94. (c) By m – n theorem at C

$(d + 2d) \cot \beta = d \cot \gamma - 2d \cot (90° + \alpha)$



$3d \cot \beta = d \cot \gamma + 2d \tan \alpha$

$\Rightarrow 3 \cot \beta = \cot \gamma + 2 \tan \alpha$

$\therefore 2 \tan \alpha = 3 \cot \beta - \cot \gamma$

95. (c) Multiplying $x^2 - ax + b = 0$ by x^{n-1}, we get

$x^{n+1} - ax^n + bx^{n-1} = 0$ ...(i)

α, β are roots of $x^2 - ax + b = 0$, therefore they will satisfy (i).

Also, $\alpha^{n+1} - a\alpha^n + b\alpha^{n-1} = 0$ ...(ii)

and $\beta^{n+1} - a\beta^n + b\beta^{n-1} = 0$ ...(iii)

On adding eqs. (ii) and (iii), we get

$(\alpha^{n+1} + \beta^{n+1}) - a(\alpha^n + \beta^n) + b(\alpha^{n-1} + \beta^{(n-1)}) = 0$

$\Rightarrow V_{n+1} - aV_n + bV_{n-1} = 0 \ (\because \alpha^n + \beta^n = V_n)$

$\Rightarrow V_{n+1} = aV_n - bV_{n-1}$

96. (a) $\sum_{r=0}^{n} (-1)^r . {}^nC_r$

$$\left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \text{...upto m terms}\right)$$

$$= \sum_{r=0}^{n} (-1)^r . {}^nC_r \frac{1}{2^r} + \sum_{r=0}^{n} (-1)^r . {}^nC_r . \frac{3^r}{2^{2r}} + \sum_{r=0}^{n} (-1)^r . {}^nC_r . \frac{7^r}{2^{3r}} + \ldots$$

$$= \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n + \ldots \text{ upto m terms}$$

$$= \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \ldots \text{ upto m terms}$$

$$= \frac{\frac{1}{2^n}\left(1 - \frac{1}{2^{mn}}\right)}{\left(1 - \frac{1}{2^n}\right)} = \frac{2^{mn} - 1}{2^{mn}(2^n - 1)}$$

97. (c) Angle of intersection between two circles is given by

$$\cos \theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2} = \frac{\frac{17}{2} + 13 - \frac{10}{4}}{2\sqrt{\frac{17}{2}}.\sqrt{13}}$$

$$\left\{\begin{array}{l} \text{here, } r_1 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + 8} \\ \quad = \sqrt{\frac{17}{2}}, \\ r_2 = \sqrt{13} \\ \text{and } \quad d = c_1c_2 = \sqrt{\frac{10}{2}} \end{array}\right.$$

$$\Rightarrow \cos \theta = \left(\frac{19}{\sqrt{442}}\right)$$

$$\text{or } \tan \theta = \left(\frac{9}{19}\right)$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{9}{19}\right)$$

98. (a) $\mathbf{b}_1 \parallel \mathbf{a} \Rightarrow \mathbf{b}_1 = a(\mathbf{i} + \mathbf{j})$

$\mathbf{b}_2 = \mathbf{b} - \mathbf{b}_1 = (3 - a)\,\mathbf{i} - a\mathbf{j} + 4\mathbf{k}$

Also, $\mathbf{b}_2 . \mathbf{a} = 0$

$$\Rightarrow (3 - a) - a \Rightarrow a = \frac{3}{2}$$

$$\therefore \mathbf{b}_1 = \frac{3}{2}(\mathbf{i} + \mathbf{j})$$

99. (b) The given matrix is $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$,

using $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Delta = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & 0 \end{bmatrix} = 0$$

($\because$ points are collinear i.e., area of triangle = 0)

$$\Rightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} = 0$$

So, the rank of matrix is always less than 2.

100. (d) On solving the determinant, we have

$1(1 - \cos^2\beta) - \cos(\alpha - \beta)[\cos(\alpha - \beta) - \cos\alpha . \cos\beta] + \cos\alpha[\cos\beta . \cos(\alpha - \beta) - \cos\alpha]$

$= 1 - \cos^2\beta - \cos^2\alpha - \cos^2(\alpha - \beta) + 2\cos\alpha . \cos\beta . \cos(\alpha - \beta)$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos(\alpha - \beta)[2\cos\alpha . \cos\beta - \cos(\alpha - \beta)]$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos(\alpha - \beta)\cos(\alpha + \beta)$
$[\cos(\alpha + \beta) + \cos(\alpha - \beta) - \cos(\alpha - \beta)]$

$= 1 - \cos^2\beta - \cos^2\alpha + \cos^2\alpha . \cos^2\beta - \sin^2\alpha . \sin^2\beta$

$= 1 - \cos^2\beta - \cos^2\alpha(1 - \cos^2\beta) - \sin^2\alpha . \sin^2\beta$

$= 1 - \cos^2\beta - \cos^2\alpha . \sin^2\beta - \sin^2\alpha . \sin^2\beta$

$= (1 - \cos^2\beta) - \sin^2\beta(\sin^2\alpha + \cos^2\alpha)$

$= \sin^2\beta - \sin^2\beta\, 1) = 0$

101. (b) $-\sqrt{7^2 + 5^2} \le (7\cos x + 5\sin x) \le \sqrt{7^2 + 5^2}$

$\Rightarrow -\sqrt{74} \le (2K + 1) \le \sqrt{74}$

$\Rightarrow -8.6 \le (2K + 1) \le 8.6$

$\Rightarrow -9.6 \le 2K \le 7.6$

$\Rightarrow -4.8 \le K \le 3.8$

So, integral values of K are
$-4, -3, -2, -1, 0, 1, 2, 3$ (eight values)

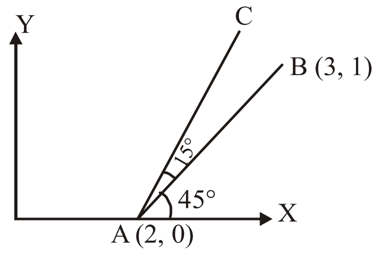
102. (a) Slope of AB $= \frac{1}{1}$

$\Rightarrow \tan\theta = m_1 = 1$ or $\theta = 45°$

Thus, slope of new line is $\tan(45° + 15°)$

$= \tan 60° = \sqrt{3}$

($\because$ it is rotated anti-clockwise, so the angle will be $45° + 15° = 60°$)



Hence, the equation is $y = \sqrt{3}x + c$

But it passes through (2, 0),

So, $c = -2\sqrt{3}$

Thus, required equation is $y = \sqrt{3}x - 2\sqrt{3}$

103. (a) Solving the equation of line and curve, we get

$$x^2 - 2\left\{\frac{2 - 2x}{\sqrt{6}}\right\}^2 = 4$$

$\Rightarrow x^2 - \frac{1}{3} \times 4(1 + x^2 - 2x) = 4$

$\Rightarrow 3x^2 - 4 - 4x^2 + 8x = 12$

$\Rightarrow x^2 - 8x + 16 = 0$

$\Rightarrow (x - 4)^2 = 0 \Rightarrow x = 4$

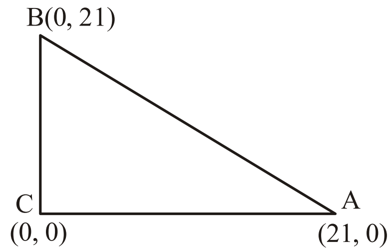
and $\sqrt{6} . y = 2 - 2(4) = -6$

$\Rightarrow y = -\sqrt{6}$

$\therefore$ Point of contact is $(4, -\sqrt{6})$.

104. (b) $x + y = 21$

The number of integral solutions to the equations are $x + y < 21$, i.e., $x < 21 - y$



$\therefore$ Number of integral coordinates
$= 19 + 18 + ... + 1$

$$= \frac{19(19+1)}{2} = \frac{19 \times 20}{2} = 190$$

105. (c) $\int (1 + x - x^{-1}) e^{x+x^{-1}} dx$

$$= \int [x.e^{x+x^{-1}}\left(1 - \frac{1}{x^2}\right) + e^{x+x^{-1}}]dx$$

$[\because \int xf'(x) + f(x)dx = xf(x) + C]$

$$\therefore \int (1 + x - x^{-1}) e^{x+x^{-1}} dx = xe^{x+x^{-1}} + C$$

106. (a) $f(x) = x - [x], -1 \le x < 0$
$\Rightarrow f(x) = x + 1$
When $0 \le x < 1$
$\Rightarrow f(x) = x$

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx$$

$$= \int_{-1}^{0} (x+1)dx + \int_{0}^{1} x\,dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= 0 - \left[\frac{(-1)^2}{2} - 1\right] + \frac{1}{2} = 1$$

107. (c) $\int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right]^{1/2} dx$

$$= \int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2\right]^{1/2} dx$$

$$= \int_{-1/2}^{1/2} \left|\frac{4x}{x^2-1}\right| dx$$

$$= \int_{-1/2}^{0} \left|\frac{4x}{1-x^2}\right| dx + \int_{0}^{1/2} \left|\frac{4x}{1-x^2}\right| dx$$

$$= -4\int_{-1/2}^{0} \frac{x}{1-x^2}dx + 4\int_{0}^{1/2} \frac{x}{1-x^2}dx$$

$$= 2\{\log(1-x^2)\}_{-1/2}^{0} - 2\{\log(1-x^2)\}_{0}^{1/2}$$

$$= -2\log\left(1 - \frac{1}{4}\right) - 2\log\left(1 - \frac{1}{4}\right)$$

$$= -4\log\frac{3}{4} = 4\log\frac{4}{3}$$

108. (d) Let P (x_1, y_1) be a point on the ellipse.

$$\frac{x^2}{18} + \frac{y^2}{32} = 1$$

$$\Rightarrow \frac{x_1^2}{18} + \frac{y_1^2}{32} = 1 \ ... \text{(i)}$$

The equation of the tangent at (x_1, y_1) is $\frac{xx_1}{18} + \frac{yy_1}{32} = 1$. This meets the axes at A $\left(\frac{18}{x_1}, 0\right)$ and B $\left(0, \frac{32}{y_1}\right)$. It is given that slope of the tangent at (x_1, y_1) is $-\frac{4}{3}$

So, $-\frac{x_1}{18} \cdot \frac{32}{y_1} = -\frac{4}{3}$

$$\Rightarrow \frac{x_1}{y_1} = \frac{3}{4}$$

$$\Rightarrow \frac{x_1}{3} = \frac{y_1}{4} = K \quad \text{(say)}$$

$\therefore x_1 = 3K$ and $y_1 = 4K$
Putting x_1, y_1 in (i), we get
$K^2 = 1$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}$ OA.OB

$$= \frac{1}{2} \cdot \frac{18}{x_1} \cdot \frac{32}{y_1} = \frac{1}{2} \frac{(18)(32)}{(3K)(4K)} = \frac{24}{K^2}$$

$= 24$ sq units $\quad (\because K^2 = 1)$

109. (d) Let mid–point of part PQ which is in between the axis is R (x_1, y_1), then coordinates of P and Q will be $(2x_1, 0)$ and $(0, 2y_1)$, respectively.

$\therefore$ Equation of line PQ is $\frac{x}{2x_1} + \frac{y}{2y_1} = 1$

$\Rightarrow y = -\left(\frac{y_1}{x_1}\right) x + 2y_1$

If this line touches the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

then it will satisfy the condition,
$c^2 = a^2m^2 + b^2$

So, $(2y_1)^2 = a^2\left(\frac{-y_1}{x_1}\right)^2 + b^2$

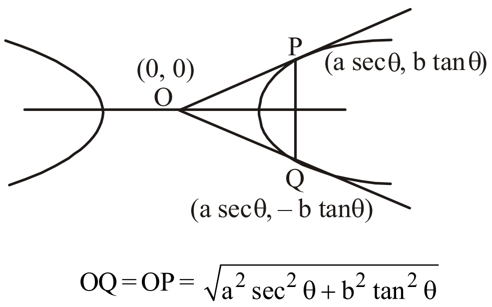
$\Rightarrow 4y_1^2 = \left\{\frac{a^2y_1^2}{x_1^2}\right\} + b^2$

$\Rightarrow 4 = \left(\frac{a^2}{x_1^2}\right) + \left(\frac{b^2}{y_1^2}\right) \Rightarrow \left(\frac{a^2}{x_1^2}\right) + \left(\frac{b^2}{y_1^2}\right) = 4$

$\therefore$ Required locus of (x_1, y_1) is

$\left(\frac{a^2}{x^2} + \frac{b^2}{y^2}\right) = 4$

110. (d) Let P $(a \sec\theta, b\tan\theta)$, Q $(a\sec\theta, -b\tan\theta)$ be end points of double ordinates and (0, 0) is the centre of the hyperbola.
So, $PQ = 2b\tan\theta$



$OQ = OP = \sqrt{a^2\sec^2\theta + b^2\tan^2\theta}$
Since, $OQ = OP = PQ$

$\therefore 4b^2\tan^2\theta = a^2\sec^2\theta + b^2\tan^2\theta$
$\Rightarrow 3b^2\tan^2\theta = a^2\sec^2\theta$
$\Rightarrow 3b^2\sin^2\theta = a^2$
$\Rightarrow 3a^2(e^2 - 1)\sin^2\theta = a^2$
$\Rightarrow 3(e^2 - 1)\sin^2\theta = 1$

$\Rightarrow \frac{1}{3(e^2-1)} = \sin^2\theta < 1, (\because \sin^2\theta < 1)$

$\Rightarrow \frac{1}{e^2-1} < 3 \Rightarrow e^2 - 1 > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3}$

$\therefore e > \frac{2}{\sqrt{3}}$

111. (a) There are 3 + 4 + 5 = 12 points in a plane.
The number of required triangles
= (The number of triangles formed by these 12 points) – (The number of triangles formed by the collinear points)
$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3)$
$= 220 - (1 + 4 + 10) = 205$

112. (c) $(a + bx)e^{-x}$
$= (a + bx)$

$\left\{1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + (-1)^n\frac{x^n}{n!} + ...\right\}$

$\therefore$ The coefficient of x^r = a.

$\frac{(-1)^r}{r!} + b\frac{(-1)^{r-1}}{(r-1)!} = \frac{(-1)^r}{r!}(a - br)$

113. (a) $\sum_{x=1}^{1999} \log_n x$
$= \log_{(1999)!} 1 + \log_{(1999)!} 2 + ... + \log_{(1999)!} 1999$
$= \log_{(1999)!}(1.2.3.... 1999)$
$= \log_{(1999)!}(1999)! = 1$

114. (d) Since, the line is equally inclined to the axes and passes through the origin, its direction ratios are 1, 1,1.

So, its equation is $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$.

A point P on it is given by (a, a, a). So, equation of the plane through P (a, a, a) and perpendicular to OP is
$1(x - a) + 1(y - a) + 1(z - a) = 0$
($\because$ OP is normal to the plane)

i.e., $x+y+z=3a$

$\Rightarrow \frac{x}{3a}+\frac{y}{3a}+\frac{z}{3a}=1$

Intercepts on axes are 3a, 3a and 3a, therefore sum of reciprocals of these intercepts.

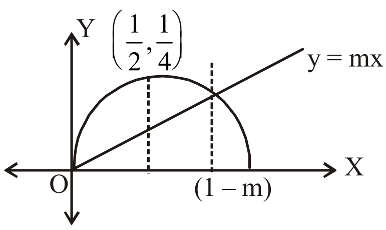
$= \frac{1}{3a}+\frac{1}{3a}+\frac{1}{3a}=\frac{1}{a}$

115. (b) The equation of curve is $y = x - x^2$

$\Rightarrow x^2 - x = y$

$\Rightarrow \left(x-\frac{1}{2}\right)^2 = -\left(y-\frac{1}{4}\right)$

which is a parabola whose vertex is $\left(\frac{1}{2},\frac{1}{4}\right)$



Hence, finding the point of intersection of the curve and the line,

$x - x^2 = mx \Rightarrow x(1-x-m)=0$

i.e., $x = 0$ or $x = 1 - m$

$\therefore \frac{9}{2}=\int_0^{1-m}(x-x^2-mx)dx$

$=\left\{\frac{x^2}{2}-\frac{x^3}{3}-m\frac{x^2}{2}\right\}_0^{1-m}$

$=(1-m)\frac{(1-m)^2}{2}-\frac{(1-m)^3}{3}=\frac{(1-m)^3}{6}$

$\therefore (1-m)^3=\frac{6\times 9}{2}=27$

$\Rightarrow 1-m=(27)^{1/3}=3$

$\Rightarrow m=-2$

Also, $(1-m)^3-(3)^3=0$

$\therefore (1-m)^3=3^3 \Rightarrow 1-m=3$

or $m=-2$

116. (c) $e^{\lim_{x\to 0}\frac{1}{x}[\log f(1+x)-\log f(1)]}$

$= e^{\lim_{x\to 0}\frac{f'(1+x)/f(1+x)}{1}}$

$= e^{f'(1)/f(1)} = e^{613} = e^2$

117. (b) $f(x)=\begin{cases}(1+|\sin x|)^{a/|\sin x|}, & -\frac{\pi}{6}<x<0\\ b &, \quad x=0\\ e^{\tan 2x/\tan 3x} & ,0<x<\frac{\pi}{6}\end{cases}$

For f (x) to be continuous at x = 0

$\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^-}(1+|\sin x|)^{a/|\sin x|}$

$= e^{\lim_{x\to 0}\left\{|\sin x|\frac{a}{|\sin x|}\right\}} = e^a$

Now, $\lim_{x\to 0^+} e^{\tan 2x/\tan 3x}$

$= \lim_{x\to 0^+} e^{\left(\frac{\tan 2x}{2x}\times 2x\right)\Big/\left(\frac{\tan 3x}{3x}\times 3x\right)}$

$= \lim_{x\to 0^+} e^{2/3} = e^{2/3}$

Since, f(x) is continuous at x = 0.

$\therefore e^a = e^{2/3} \Rightarrow a = \frac{2}{3}$

and $b = e^{2/3}$

118. (b) $x+2\geq 0$, i.e., $x\geq -2$ or $-2\leq x$

$\log_{10}(1-x) \neq 0$

$\Rightarrow 1-x\neq 1 \Rightarrow x\neq 0$

Again, $1-x>0$

$\Rightarrow 1>x \Rightarrow x<1$

Combining all the results for values of x, we get

$-2\leq x<0$ and $0<x<1$

119. (b) $(1+y^2)+(x-e^{\tan^{-1}y})\dfrac{dy}{dx}=0$

$(1+y^2)\dfrac{dx}{dy}+x=e^{\tan^{-1}y}$

$\dfrac{dx}{dy}+\dfrac{x}{1+y^2}=\dfrac{e^{\tan^{-1}y}}{(1+y^2)}$

$\text{IF}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

$\Rightarrow x.\ e^{\tan^{-1}y}=\int\dfrac{e^{\tan^{-1}y}}{1+y^2}.e^{\tan^{-1}y}.dy$

$\Rightarrow\ x(e^{\tan^{-1}y})=\dfrac{e^{2\tan^{-1}y}}{2}+c$

$\therefore\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+K$

120. (c) $\dfrac{dy}{dx}=\dfrac{y}{x}-\sin^2\left(\dfrac{y}{x}\right)$

Put $y=vx\Rightarrow\dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

$v+x\dfrac{dv}{dx}=v-\sin^2 v$

$\Rightarrow -\text{cosec}^2 v\ dv=\dfrac{dx}{x}$

Integrating both sides, we get

$-\int\text{cosec}^2 v dv=\int\dfrac{dx}{x}$

$\Rightarrow \cot v=\log x+C$

$\cot\dfrac{y}{x}=\log x+C$

Curve passes through the point $\left(1,\dfrac{\pi}{4}\right)$

$\therefore C=1$

$\Rightarrow \cot\dfrac{y}{x}=\log x+\log_e e$

$\Rightarrow\ \cot\dfrac{y}{x}=\log xe$

$\Rightarrow y=x\cot^{-1}(\log xe)$