VITEEE 2009 Question Paper

Vellore Institute of Technology Engineering Entrance Examination

SOLVED PAPER VITEEE 2009

PART - I (PHYSICS)

1 When a wave traverses a medium the displacement of a particle located at x at a time t is given by $y = a \sin (bt - cx)$. Where a, b and c are constants of the wave. Which of the following is a quantity with dimensions?

(a) $\frac{y}{a}$ (b) bt

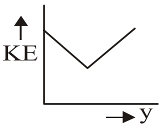
(c) cx (d) $\frac{b}{c}$

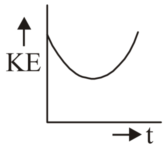
2. A body is projected vertically upwards at time $t = 0$ and it is seen at a height H at time t_1 and t_2 second during its flight. The maximum height attained is (g is acceleration due to gravity)

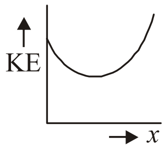
(a) $\frac{g(t_2 - t_1)^2}{8}$ (b) $\frac{g(t_1 + t_2)^2}{4}$

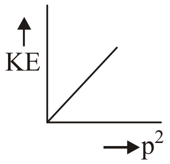
(c) $\frac{g(t_1 + t_2)^2}{8}$ (d) $\frac{g(t_2 - t_1)^2}{4}$

3. A particle is projected up from a point at an angle θ with the horizontal direction. At any time t, if p is the linear momentum, y is the vertical displacement, x is horizontal displacement, the graph among the following which does not represent the variation of kinetic energy KE of the particle is

(A) 

(B) 

(C) 

(D) 

(a) graph (A) (b) graph (B)
(c) graph (C) (d) graph (D)

4. A motor of power P_0 is used to deliver water at a certain rate through a given horizontal pipe. To increase the rate of flow of water through the same pipe n times, the power of the motor is increased to P_1. The ratio of P_1 to P_0 is
(a) $n : 1$ (b) $n^2 : 1$
(c) $n^3 : 1$ (d) $n^4 : 1$

5. A body of mass 5 kg makes an elastic collision with another body at rest and continues to move in the original direction after collision with a velocity equal to $\frac{1}{10}$th of its original velocity. Then the mass of the second body is
(a) 4.09 kg (b) 0.5 kg
(c) 5 kg (d) 5.09 kg

6. A particle of mass 4 m explodes into three pieces of masses m, m and 2m. The equal masses move along X-axis and Y-axis with velocities 4 ms^{-1} and 6 ms^{-1} respectively. The magnitude of the velocity of the heavier mass is

(a) $\sqrt{17}$ ms^{-1} (b) $2\sqrt{13}$ ms^{-1}

(c) $\sqrt{13}$ ms^{-1} (d) $\frac{\sqrt{13}}{2}$ ms^{-1}

7. A body is projected vertically upwards from the surface of the earth with a velocity equal to half the escape velocity. If R is the radius of the earth, maximum height attained by the body from the surface of the earth is

(a) $\frac{R}{6}$ (b) $\frac{R}{3}$

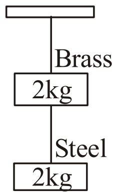
(c) $\frac{2R}{3}$ (d) R

8. The displacement of a particle executing SHM is given by

$$y = 5\sin\left(4t + \frac{\pi}{3}\right)$$

If T is the time period and the mass of the particle is 2g, the kinetic energy of the particle when t = T/4 is given by
(a) 0.4 J (b) 0.5 J
(c) 3 J (d) 0.3 J

9. If the ratio of lengths, radii and Young's modulus of steel and brass wires shown in the figure are a, b and c respectively, the ratio between the increase in lengths of brass and steel wires would be



(a) $\frac{b^2a}{2c}$ (b) $\frac{bc}{2a^2}$

(c) $\frac{ba^2}{2c}$ (d) $\frac{a}{2b^2c}$

10. A soap bubble of radius r is blown up to form a bubble of radius 2r under isothermal conditions. If T is the surface tension of soap solution, the energy spent in the blowing

(a) $3\pi Tr^2$ (b) $6\pi Tr^2$

(c) $12\pi Tr^2$ (d) $24\pi Tr^2$

11. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of 6 cm-s^{-1}. If they coalesce to form one big drop, what will be the terminal speed of bigger drop? (Neglect the buoyancy of the air)

(a) 1.5 cm-s^{-1} (b) 6 cm-s^{-1}

(c) 24 cm-s^{-1} (d) 32 cm-s^{-1}

12. A clock pendulum made of invar has a period of 0.5 s, at 20°C. If the clock is used in a climate where the temperature averages to 30°C, how much time does the clock lose in each oscillation? (For invar, $\alpha = 9 \times 10^{-7}$/°C, g = constan t)

(a) 2.25×10^{-6} s (b) 2.5×10^{-7} s

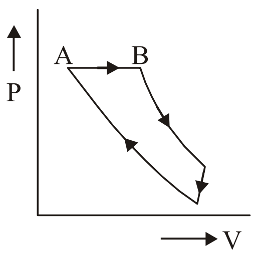
(c) 5×10^{-7} s (d) 1.125×10^{-6}s

13. A piece of metal weighs 45 g in air and 25 g in a liquid of density 1.5×10^3 kg-m^{-3} kept at 30°C. When the temperature of the liquid is raised to 40° C, the metal piece weights 27 g. the density of liquid of 40°C is 1.25×10^3 kg–m^{-3}. the coefficient of linear expansion of metal is

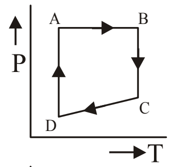
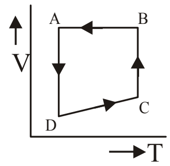
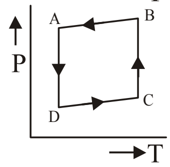
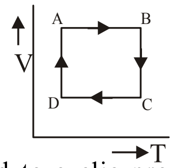
(a) 1.3×10^{-3}/°C (b) 5.2×10^{-3}/°C

(c) 2.6×10^{-3}/°C (d) 0.26×10^{-3}/°C

14. An ideal gas is subjected to a cyclic process ABCD as depicted in the p-V diagram given below:



Which of the following curves represents the equivalent cyclic process?

(a) 
(b) 
(c) 
(d) 

15. An ideal gas is subjected to cyclic process involving four thermodynamic states, the amounts of heat (Q) and work (W) involved in each of these states are

$Q_1 = 6000$ J, $Q_2 = -5500$ J; $Q_3 = -3000$ J; $Q_4 = 3500$ J

$W_1 = 2500$ J; $W_2 = -1000$ J; $W_3 = -1200$ J; $W_4 = x$ J.

The ratio of the net work done by the gas to the total heat absorbed by the gas is η). The values of x and η respectively are

(a) 500 ; 7.5 % (b) 700; 10.5 %

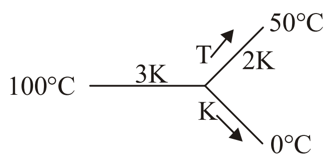
(c) 1000 ; 21 % (d) 1500; 15 %

16. Two cylinders A and B fitted with pistons contain equal number of moles of an ideal monoatomic gas at 400 K. The piston of A is free to move while that of B is held fixed. Same amount of heat energy is given to the gas in each cylinder. If the rise in temperature of the gas in A is 42 K, the rise in temperature of the gas in B is

(a) 21 K (b) 35 K

(c) 42 K (d) 70 K

17. Three rods of same dimensional have thermal conductivity 3 K, 2 K and K. They are arranged as shown in the figure below



Then, the temperature of the junction in steady state is

(a) $\frac{200}{3}$°C (b) $\frac{100}{3}$°C

(c) 75°C (d) $\frac{50}{3}$°C

18. Two sources A and B are sending notes of frequency 680 Hz. A listener moves from A and B with a constant velocity u. If the speed of sound in air is 340 ms^{-1}, what must be the value of u so that he hears 10 beats per second?
(a) 2.0 ms^{-1} (b) 2.5 ms^{-1}
(c) 3.0 ms^{-1} (d) 3.5 ms^{-1}

19. Two identical piano wires have a fundamental frequency of 600 cycle per second when kept under the same tension. What fractional increase in the tension of one wires will lead to the occurrence of 6 beats per second when both wires vibrate simultaneously?
(a) 0.01 (b) 0.02
(c) 0.03 (d) 0.04

20. In the Young's double slit experiment, the intensities at two points P_1 and P_2 on the screen are respectively I_1 and I_2. If P_1 is located at the centre of a bright fringe and P_2 is located at a distance equal to a quarter of fringe width fromP_1, then $\frac{I_1}{I_2}$ is
(a) 2 (b) $\frac{1}{2}$
(c) 4 (d) 16

21. In Young's double slit experiment, the 10^{th} maximum of wavelength λ_1 is at a distance of y_1, from the central maximum. When the wavelength of the source is changed to λ_2, 5^{th} maximum is at a distance of y_2 from its central maximum. The ratio $\left(\frac{y_1}{y_2}\right)$ is
(a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$
(c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$

22. Four light sources produce the following four waves:
(i) $y_1 = a\sin(\omega t + \phi_1)$
(ii) $y_2 = a\sin 2\omega t$
(iii) $y_3 = a\sin(\omega t + \phi_2)$
(iv) $y_4 = a\sin(3\omega t + \phi)$

Superposition of which two waves give rise to interference?
(a) (i) and (ii) (b) (ii) and (iii)
(c) (i) and (iii) (d) (iii) and (iv)

23. The two lenses of an achromatic doublet should have
(a) equal powers
(b) equal dispersive powers
(c) equal ratio of their power and dispersive power
(d) sum of the product of their powers and dispersive power equal to zero

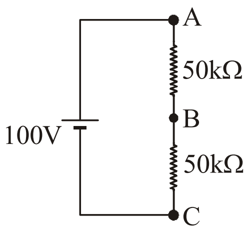
24. Two bar magnets A and B are placed one over the other and are allowed to vibrate in a vibration magnetometer. They make 20 oscillations per minute when the similar poles of A and B are on the same side, while they make 15 oscillations per minute when their opposite poles lie on the same side. If M_A and M_B are the magnetic moments of A and B and if $M_A > M_B$, the ratio of M_A and M_B is
(a) 4 : 3 (b) 25 : 7
(c) 7 : 5 (d) 25 : 16

25. A bar magnet is 10 cm long is kept with its north (N)-pole pointing north. A neutral point is formed at a distance of 15 cm from each pole. Given the horizontal component of earth's field is 0.4 Gauss, the pole strength of the magnet is
(a) 9 A-m (b) 6.75 A-m
(c) 27 A-m (d) 1.35 A-m

26. An infinitely long thin straight wire has uniform linear charge density of $\frac{1}{3}$ cm^{-1}. Then, the magnitude of the electric intensity at a point 18 cm away is (given $\varepsilon_0 = 8.8 \times 10^{-12}\ C^2Nm^{-2}$)
(a) $0.33 \times 10^{11}\ NC^{-1}$ (b) $3 \times 10^{11}\ NC^{-1}$
(c) $0.66 \times 10^{11}\ NC^{-1}$ (d) $1.32 \times 10^{11}\ NC^{-1}$

27. Two point charges –q and +q are located at points (0, 0, – a) and (0, 0, a) respectively. The electric potential at a point (0, 0, z), where z > a is
(a) $\frac{qa}{4\pi\varepsilon_0 z^2}$ (b) $\frac{q}{4\pi\varepsilon_0 a}$
(c) $\frac{2qa}{4\pi\varepsilon_0(z^2 - a^2)}$ (d) $\frac{2qa}{4\pi\varepsilon_0(z^2 + a^2)}$

28. In the adjacent shown circuit, a voltmeter of internal resistance R, when connected across B and C reads $\frac{100}{3}$ V. Neglecting the internal resistance of the battery, the value of R is

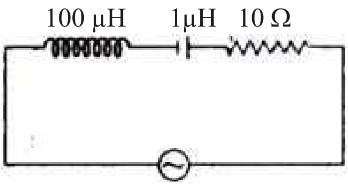


(a) 100 kΩ (b) 75kΩ
(c) 50kΩ (d) 25kΩ

29. A cell in secondary circuit gives null deflection for 2.5 m length of potentiometer having 10 m length of wire. If the length of the potentiometer wire is increased by 1 m without changing the cell in the primary, the position of the null point now is

(a) 3.5m (b) 3m
(c) 2.75m (d) 2.0m

30. The following series L-C-R circuit, when driven by an emf source of angular frequency 70 kilo-radians per second, the circuit effectively behaves like

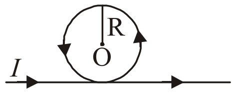


(a) purely resistive circuit
(b) series R-L circuit
(c) series R-C circuit
(d) series L-C circuit with R = 0

31. A wire of length l is bent into a circular loop of radius R and carries a current I. The magnetic field at the centre of the loop is B. The same wire is now bent into a double loop of equal radii. If both loops carry the same current I and it is in the same direction, the magnetic field at the centre of the double loop will be

(a) Zero (b) 2B
(c) 4B (d) 8B

32. An infinitely long straight conductor is bent into the shape as shown below. It carries a current of I ampere and the radius of the circular loop is R metre. Then, the magnitude of magnetic induction at the centre of the circular loop is



(a) $\frac{\mu_0 I}{2\pi R}$ (b) $\frac{\mu_0 nI}{2R}$

(c) $\frac{\mu_0 I}{2\pi R}(\pi+1)$ (d) $\frac{\mu_0 I}{2\pi R}(\pi-1)$

33. The work function of a certain metal is 3.31×10^{-19} J. Then, the maximum kinetic energy of photoelectrons emitted by incident radiation of wavelength 5000 Å is (Given, $h = 6.62 \times 10^{-34}$ J-s, $c = 3 \times 10^8$ ms^{-1}, $e = 1.6 \times 10^{-19}$ C)

(a) 2.48 eV (b) 0.41 eV
(c) 2.07 eV (d) 0.82 eV

34. A photon of energy E ejects a photoelectron from a metal surface whose work function is W_0. If this electron enters into a uniform magnetic field of induction B in a direction perpendicular to the field and describes a circular path of radius r, then the radius r is given by, (in the usual notation)

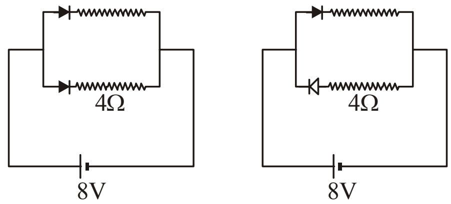
(a) $\frac{\sqrt{2m(E-W_0)}}{eB}$ (b) $\sqrt{2m(E-W_0)eB}$

(c) $\frac{\sqrt{2e(E-W_0)}}{mB}$ (d) $\frac{\sqrt{2m(E-W_0)}}{eB}$

35. Two radioactive materials x_1 and x_2 have decay constants 10λ and λ respectively. if initially they have the same number of nuclei, then the ratio of the number of nuclei of x_1 to that of x_2 will be 1/e after a time

(a) $(1/10\lambda)$ (b) $(1/11\lambda)$
(c) $11/(10\lambda)$ (d) $1/(9\lambda)$

36. Current flowing in each of the following circuit A and B respectively are



(Circuit A) (Circuit B)

(a) 1A, 2A (b) 2 A, 1 A
(c) 4 A, 2 A (d) 2 A, 4 A

37. A bullet of mass 0.02 kg travelling horizontally with velocity 250 ms^{-1} strikes a block of wood of mass 0.23 kg which rests on a rough horizontal surface. After the impact, the block and bullet move together and come to rest after travelling a distance of 40 m. The coefficient of sliding friction of the rough surface is ($g = 9.8\ ms^{-2}$)
(a) 0.75 (b) 0.61
(c) 0.51 (d) 0.30

38. Two persons A and B are located in X-Y plane at the points (0, 0) and (0,10) respectively. (The distances are measured in MKS unit). At a time t = 0, they start moving simultaneously with velocities $\vec{v}_a = 2\hat{j}ms^{-1}$ and $\vec{v}_b = 2\hat{i}ms^{-1}$ respectively. The time after which A and B are at their closest distance is
(a) 2.5s (b) 4s
(c) 1s (d) $\frac{10}{\sqrt{2}}$s

39. A rod of length l is held vertically stationary with its lower end located at a point P, on the horizontal plane. When the rod is released to topple about P, the velocity of the upper end of the rod with which it hits the ground is
(a) $\sqrt{\frac{g}{l}}$ (b) $\sqrt{3gl}$
(c) $3\sqrt{\frac{g}{l}}$ (d) $\sqrt{\frac{3g}{l}}$

40. A wheel of radius 0.4 m can rotate freely about its axis as shown in the figure. A string is wrapped over its rim and a mass of 4 kg is hung. An angular acceleration of 8 rad-s^{-2} is produced in it due to the torque. Then, moment of inertia of the wheel is ($g = 10\ ms^{-2}$)
(a) 2 kg-m^2 (b) 1 kg-m^2
(c) 4 kg-m^2 (d) 8 kg-m^2

PART - II (CHEMISTRY)

41. Given that $\Delta H_f(H) = 218$ kJ/mol, express the H—H bond energy in kcal/mol.
(a) 52.15 (b) 911
(c) 104 (d) 52153

42. Identify the alkyne in the following sequence of reactions,

$$\text{Alkyne} \xrightarrow[\text{Lindlar's catalyst}]{H_2} A \xrightarrow{\text{Ozonolysis}} \underset{\text{only}}{B} \xleftarrow[\text{Process}]{\text{Wacker}} CH_2 = CH_2$$

(a) $H_3C-C-C-CH_3$
(b) $H_3C-CH_2-C\equiv CH$
(c) $H_2C=CH-C\equiv CH$
(d) $HC\equiv C-CH_2-C\equiv CH$

43. Fluorine reacts with dilute NaOH and forms a gaseous product *A*. The bond angle in the molecule of *A* is
(a) 104°40' (b) 103°
(c) 107° (d) 109°28'

44. One mole of alkene X on ozonolysis gave one mole of acetaldehyde and one mole of acetone. The IUPAC name of X is
(a) 2-methyl-2-butene (b) 2-methyl-1-butene
(c) 2-butene (d) 1-butene

45. The number of $p\pi - d\pi$ 'pi' bonds present in XeO_3 and XeO_4 molecules, respectively are
(a) 3, 4 (b) 4, 2
(c) 2, 3 (d) 3, 2

46. The wavelengths of electron waves in two orbits is 3 : 5. The ratio of kinetic energy of electrons will be
(a) 25 : 9 (b) 5 : 3
(c) 9 : 25 (d) 3 : 5

47. Which one of the following sets correctly represents the increase in the paramagnetic property of the ions?
(a) $Cu^{2+} > V^{2+} > Cr^{2+} > Mn^{2+}$
(b) $Cu^{2+} < Cr^{2+} < V^{2+} < Mn^{2+}$
(c) $Cu^{2+} < V^{2+} < Cr^{2+} < Mn^{2+}$
(d) $V^{2+} < Cu^{2+} < Cr^{2+} < Mn^{2+}$

48 . Electrons with a kinetic energy of 6.023×10^4 J/mol are evolved from the surface of a metal, when it is exposed to radiation of wavelength of 600 nm. The minimum amount of energy required to remove an electron from the metal atom is
(a) 2.3125×10^{-19} J (b) 3×10^{-19} J
(c) 6.02×10^{-19} J (d) 6.62×10^{-34} J

49. The chemical entities present in thermosphere of the atmosphere are
(a) O_2^+, O^+, NO^+ (b) O_3
(c) N_2, O_2, CO_2, H_2O (d) O_3, O_2^+, O_2

50. The type of bonds present in sulphuric anhydride are
(a) 3σ and three $p\pi - d\pi$
(b) 3σ, one $p\pi - p\pi$ and two $p\pi - d\pi$
(c) 2σ and three $p\pi - d\pi$
(d) 2σ and two $p\pi - d\pi$

51. In Gattermann reaction, a diazonium group is replaced by X using Y. X and Y are

	X	Y
(a)	$Cl^{\ominus}$	Cu/HCl
(b)	$Cl^{\oplus}$	$CuCl_2$/HCl
(c)	$Cl^{\ominus}$	$CuCl_2$/HCl
(d)	Cl_2	Cu_2O/HCl

52. Which pair of oxyacids of phosphorus contains ' P—H' bonds?
(a) H_3PO_4, H_3PO_3 (b) $H_3PO_5, H_4P_2O_7$
(c) H_3PO_3, H_3PO_2 (d) H_3PO_2, HPO_3

53. Dipole moment of HCl = 1.03 D, HI = 0.38 D. Bond length of HC1 = 1.3 Å and HI =1.6 Å. The ratio of fraction of electric charge, δ, existing on each atom in HCl and HI is
(a) 12 : 1 (b) 2.7 : 1
(c) 3.3 : 1 (d) 1 : 3.3

54. $SiCl_4$ on hydrolysis forms 'X' and HC1. Compound 'X' loses water at 1000°C and gives 'Y'. Compounds 'X' and 'Y' respectively are
(a) H_2SiCl_6, SiO_2 (b) H_4SiO_4, Si
(c) SiO_2, Si (d) H_4SiO_4, SiO_2

55. 1.5 g of $CdCl_2$ was found to contain 0.9 g of Cd. Calculate the atomic weight of Cd.
(a) 118 (b) 112
(c) 106.5 (d) 53.25

56. Aluminium reacts with NaOH and forms compound 'X'. If the coordination number of aluminium in 'X' is 6, the correct formula of X is
(a) $[Al(H_2O)_4(OH)_2]^+$ (b) $[Al(H_2O)_3(OH)_3]$
(c) $[Al(H_2O)_2(OH)_4]^-$ (d) $[Al(H_2O)_6](OH)_3$

57. The average kinetic energy of one molecule of an ideal gas at 27°C and 1 atm pressure is
(a) 900 cal $K^{-1}mol^{-1}$
(b) 6.21×10^{-21} JK^{-1} $molecule^{-1}$
(c) 336.7 JK^{-1} $molecules^{-1}$
(d) 3741.3 JK^{-1} mol^{-1}

58. **Assertion (A) :** K. Rb and Cs form superoxides.
Reason (R) : The stability of the superoxides increases from 'K' to 'Cs' due to decrease in lattice energy.
The correct answer is
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is true but (R) is not true
(d) (A) is not true but (R) is true

59. How many mL of perhydrol is required to produce sufficient oxygen which can be used to completely convert 2 L of SO_2 gas to SO_3 gas?
(a) 10 mL (b) 5 mL
(c) 20 mL (d) 30 mL

60. pH of a buffer solution decreases by 0.02 units when 0.12 g of acetic acid is added to 250 mL of a buffer solution of acetic acid and potassium acetate at 27°C. The buffer capacity of the solution is
(a) 0.1 (b) 10
(c) 1 (d) 0.4

61. Match the following

List I		List II	
(A)	Flespar	(I)	$[Ag_3Sb_3]$
(B)	Asbestos	(II)	$Al_2O_3 . H_2O$
(C)	Pyrargyrite	(III)	$MgSO_4 . H_2O$
(D)	Diaspore	(IV)	$KAlSi_3O_8$
		(V)	$CaMg_3(SiO_3)_4$

The correct answer is

	(A)	(B)	(C)	(D)
(a)	IV	V	II	I
(b)	IV	V	I	II
(c)	IV	I	III	II
(d)	II	V	IV	I

62. Which one of the following order is correct for the first ionisation energies of the elements?
(a) B < Be < N < O (b) Be < B < N < O
(c) B < Be < O < N (d) B < O < Be < N

63. What are X and Y in the following reaction sequence?

$$C_2H_5OH \xrightarrow{Cl_2} X \xrightarrow{Cl_2} Y$$

(a) C_2H_5Cl, CH_3CHO
(b) CH_3CHO, CH_3CO_2H
(c) CH_3CHO, CCl_3CHO
(d) C_2H_5Cl, CCl_3CHO

64. What are A, B, C in the following reactions?
(i) $(CH_3CO_2)_2Ca \xrightarrow{\Delta} A$
(ii) $CH_3CO_2H \xrightarrow[\text{Red P}]{HI} B$
(iii) $2CH_3CO_2H \xrightarrow{P_4O_{10}} C$

	A	B	C
(a)	C_2H_6	CH_3COCH_3	$(CH_3CO)_2O$
(b)	$(CH_3CO)_2O$	C_2H_6	CH_3COCH_3
(c)	CH_3COCH_3	C_2H_6	$(CH_3CO)_2O$
(d)	CH_3COCH_3	$(CH_3CO)_2O$	C_2H_6

65. One per cent composition of an organic compound A is, carbon : 85.71% and hydrogen 14.29%. Its vapour density is 14. Consider the following reaction sequence

$$A \xrightarrow{Cl_2/H_2O} B \xrightarrow[(ii)H_3O^+]{(i)KCN/EtOH} C$$

Identify C.

(a) $CH_3 - \underset{\substack{|\\OH}}{CH} - CO_2H$

(b) $HO - CH_2 - CH_2 - CO_2H$

(c) $HO - CH_2 - CO_2H$

(d) $CH_3 - CH_2 - CO_2H$

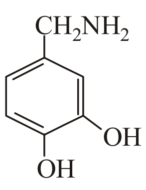
66. How many tripeptides can be prepared by linking the amino acids glycine, alanine and phenyl alanine?

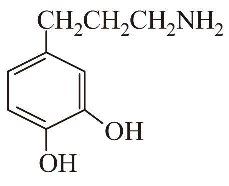
(a) One	(b) Three
(c) Six	(d) Twelve

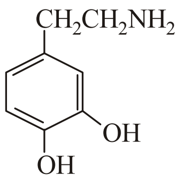
67. A codon has a sequence of A and specifies a particular B that is to be incorporated into a C. What are A, B, C?

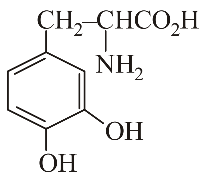
	A	B	C
(a)	3 bases	amino acid	carbohydrate
(b)	3 acids	carbohydrate	protein
(c)	3 bases	protein	amino acid
(d)	3 bases	amino acid	protein

68. Parkinson's disease is linked to abnormalities in the levels of dopamine in the body. The structure of dopamine is

(a)


(b)


(c)


(d)


69. During the depression in freezing point experiment, an equilibrium is established between the molecules of

(a) liquid solvent and solid solvent
(b) liquid solute and solid solvent
(c) liquid solute and solid solute
(d) liquid solvent and solid solute

70. Consider the following reaction,

$$C_2H_5Cl + AgCN \xrightarrow{EtOH/H_2O} X \text{ (major)}$$

Which one of the following statements is true for X?

(I) It gives propionic acid on hydrolysis
(II) It has an ester functional group
(III) It has a nitrogen linked to ethyl carbon
(IV) It has a cyanide group

(a) IV	(b) III
(c) II	(d) I

71. For the following cell reaction,

$$Ag \mid Ag^+ \mid AgCl \mid Cl^{\ominus} \mid Cl_2, Pt$$

$$\Delta G_f^{\circ}(AgCl) = -109 \text{kJ/mol}$$

$$\Delta G_f^{\circ}(Cl^-) = -129 \text{kJ/mol}$$

$$\Delta G_f^{\circ}(Ag^+) = 78 \text{kJ/mol}$$

E° of the cell is

(a) –0.60 V	(b) 0.60 V
(c) 6.0 V	(d) None of these

72. The synthesis of crotonaldehyde from acetaldehyde is an example of....... reaction.

(a) nucleophilic addition
(b) elimination
(c) electrophilic addition
(d) nucleophilic addition-elimination

73. At 25°C, the molar conductances at infinite dilution for the strong electrolytes NaOH, NaCl and $BaCl_2$ are 248×10^{-4}, 126×10^{-4} and 280×10^{-4} $Sm^2 mol^{-1}$ respectively, $\lambda_m^{\circ} Ba(OH)_2$ in $Sm^2 mol^{-1}$ is

(a) 52.4×10^{-4}	(b) 524×10^{-4}
(c) 402×10^{-4}	(d) 262×10^{-4}

74. The cubic unit cell of a metal (molar mass = 63.55 g mol^{-1}) has and edge length of 362 pm. Its density is 8.92g cm -3 . The type of unit cell is

(a) primitive	(b) face centred
(c) body centred	(d) end centred

75. The equilibrium constant for the given reaction is 100.

$$N_2(g) + 2O_2(g) \rightleftharpoons 2NO_2(g)$$

What is the equilibrium constant for the reaction given below?

$$NO_2(g) \rightleftharpoons \frac{1}{2}N_2(g) + O_2(g)$$

(a) 10 (b) 1
(c) 0.1 (d) 0.01

76. For a first order reaction at 27°C, the ratio of time required for 75% completion to 25% completion of reaction is

(a) 3.0 (b) 2.303
(c) 4.8 (d) 0.477

77. The concentration of an organic compound in chloroform is 6.15 g per 100 mL of solution. A portion of this solution in a 5cm polarimeter tube causes an observed rotation of –1.2°. What is the specific rotation of the compound?

(a) +12° (b) –3.9°
(c) –39° (d) +61.5°

78. 20 ml of 0.1 M acetic acid is mixed with 50 mL of potassium acetate. K_a of acetic acid = 1.8×10^{-5} at 27°C. Calculate concentration of potassium acetate if pH of the mixture is 4.8.

(a) 0.1 M (b) 0.04 M
(c) 0.4 M (d) 0.02 M

79. Calculate ΔH° for the reaction,

$Na_2O(s) + SO_3(g) \longrightarrow Na_2SO_4(g)$

given the following :

(A) $Na(s) + H_2O(l) \longrightarrow NaOH(s) + \frac{1}{2}H_2(g)$

$\Delta H° = -146$ kJ

(B) $Na_2SO_4(s) + H_2O(l) \longrightarrow 2NaOH(s) + SO_3(g)$

$\Delta H° = +418$ kJ

(C) $2Na_2O(s) + 2H_2(g) \longrightarrow 4Na(s) + 2H_2O(l)$

$\Delta H° = +259$ kJ

(a) +823 kJ (b) –581 kJ
(c) –435 kJ (d) +531 kJ

80. Which one of the following is the most effective in causing the coagulation of an As_2S_3 sol?

(a) KCl (b) $AlCl_3$
(c) $MgSO_4$ (d) $K_3Fe(CN)_6$

PART - III (MATHEMATICS)

81. If $f : [2, 3] \to R$ is defined by $f(x) = x^3 + 3x - 2$, then the range $f(x)$ is contained in the interval

(a) [1, 12] (b) [12, 34]
(c) [35, 50] (d) [–12, 12]

82. The number of subsets of {1, 2, 3, ..., 9} containing at least one odd number is

(a) 324 (b) 396
(c) 496 (d) 512

83. A binary sequence is an array of 0's and l's. The number of n-digit binary sequences which contain even number of 0's is

(a) 2^{n-1} (b) $2^n - 1$
(c) $2^{n-1} - 1$ (d) 2^n

84. If x is numerically so small so that x^2 and higher powers of x can be neglected, then

$$\left(1 + \frac{2x}{3}\right)^{3/2} . (32 + 5x)^{-1/5}$$

is approximately equal to

(a) $\frac{32 + 31x}{64}$ (b) $\frac{31 + 32x}{64}$

(c) $\frac{31 - 32x}{64}$ (d) $\frac{1 - 2x}{64}$

85. The roots of

$$(x - a)(x - a - 1) + (x - a - 1)(x - a - 2) + (x - a)(x - a - 2) = 0$$

$a \in R$ are always

(a) equal (b) imaginary
(c) real and distinct (d) rational and equal

86. Let $f(x) = x^2 + ax + b$, where $a, b \in$ R. If $f(x) = 0$ has all its roots imaginary, then the roots of $f(x) + f'(x) + f''(x) = 0$ are

(a) real and distinct (b) imaginary
(c) equal (d) rational and equal

87. If $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $x^2 - 3x + 2$, then (a, b) is equal to

(a) (–9, – 2) (b) (6, 4)
(c) (9, 2) (d) (2, 9)

88. If x, y, z are all positive and are the p^{th}, q^{th} and r^{th} terms of a geometric progression respectively, then the value of the determinant

$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} \text{ equals}$$

(a) log xyz (b) (p – 1) (q – 1)(r – 1)
(c) pqr (d) 0

89. The locus of z satisfying the inequality $\left|\frac{z+2i}{2z+i}\right|<1$, where $z = x+iy$, is

(a) $x^2+y^2<1$ (b) $x^2-y^2<1$

(c) $x^2+y^2>1$ (d) $2x^2+3y^2<1$

90. If n is an integer which leaves remainder one when divided by three, then $(1+\sqrt{3}i)^n+(1-\sqrt{3}i)^n$ equals

(a) -2^{n+1} (b) 2^{n+1}

(c) $-(-2)^n$ (d) -2^n

91. The period of $\sin^4 x+\cos^4 x$ is

(a) $\frac{\pi^4}{2}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

92. If $3\cos x \neq 2\sin x$, then the general solution of $\sin^2 x-\cos 2x=2-\sin 2x$ is

(a) $n\pi+(-1)^n\frac{\pi}{2}, n\in Z$

(b) $\frac{n\pi}{2}, n\in Z$

(c) $(4n\pm 1)\frac{\pi}{2}, n\in Z$

(d) $(2n-1)\pi, n\in Z$

93. $\cos^{-1}\left(\frac{-1}{2}\right)-2\sin^{-1}\left(\frac{1}{2}\right)+3\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ $-4\tan^{-1}(-1)$ equals

(a) $\frac{19\pi}{12}$ (b) $\frac{35\pi}{12}$ (c) $\frac{47\pi}{12}$ (d) $\frac{43\pi}{12}$

94. In a ΔABC

$$\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$

equals

(a) $\cos^2 A$ (b) $\cos^2 B$

(c) $\sin^2 A$ (d) $\sin^2 B$

95. The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$, $l^2+m^2-n^2=0$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

96. If m_1, m_2, m_3 and m_4 are respectively the magnitudes of the vectors

$\vec{a}_1=2\hat{i}-\hat{j}+\hat{k}$, $\vec{a}_2=3\hat{i}-4\hat{j}-4\hat{k}$,

$\vec{a}_3=\hat{i}+\hat{j}-\hat{k}$, and $\vec{a}_4=-\hat{i}+3\hat{j}+\hat{k}$,

then the correct order of m_1, m_2, m_3 and m_4 is

(a) $m_3<m_1<m_4<m_2$

(b) $m_3<m_1<m_2<m_4$

(c) $m_3<m_4<m_1<m_2$

(d) $m_3<m_4<m_2<m_1$

97. If X is a binomial variate with the range $\{0, 1, 2, 3, 4, 5, 6\}$ and $P(X=2)=4P(X=4)$, then the parameter p of X is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

98. The area (in square unit) of the circle which touches the lines $4x+3y=15$ and $4x+3y=5$ is

(a) 4π (b) 3π

(c) 2π (d) π

99. The area (in square unit) of the triangle formed by $x+y+1=0$ and the pair of straight lines $x^2-3xy+2y^2=0$ is

(a) $\frac{7}{12}$ (b) $\frac{5}{12}$ (c) $\frac{1}{12}$ (d) $\frac{1}{6}$

100. The pairs of straight lines $x^2-3xy+2y^2=0$ and $x^2-3xy+2y^2+x-2=0$ form a

(a) square but not rhombus

(b) rhombus

(c) parallelogram

(d) rectangle but not a square

101. The equations of the circle which pass through the origin and makes intercepts of lengths 4 and 8 on the x and y-axes respectively are

(a) $x^2+y^2\pm 4x\pm 8y=0$

(b) $x^2+y^2\pm 2x\pm 4y=0$

(c) $x^2+y^2\pm 8x\pm 16y=0$

(d) $x^2+y^2\pm x\pm y=0$

102. The point (3, –4) lies on both the circles $x^2+y^2-2x+8y+13=0$ and $x^2+y^2-4x+6y+11=0$

Then, the angle between the circles is

(a) 60° (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{3}{5}\right)$ (d) 135°

103. The equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2+y^2-6x+8=0$ and $x^2+y^2-2x-2y=7$ is

(a) $3x^2+3y^2-8x-13y=0$
(b) $3x^2+3y^2-8x+29y=0$
(c) $3x^2+3y^2+8x+29y=0$
(d) $3x^2+3y^2-8x-29y=0$

104. The number of normals drawn to the parabola $y^2=4x$ from the point (1, 0) is
(a) 0 (b) 1 (c) 2 (d) 3

105. If the circle $x^2+y^2=a^2$ intersects the hyperbola $xy=c^2$ in four points (x_1, y_1) for $i=1, 2, 3$ and 4, then $y_1+y_2+y_3+y_4$ equals
(a) 0 (b) c (c) a (d) c^4

106. The mid point of the chord $4x-3y=5$ of the hyperbola $2x^2-3y^2=12$ is
(a) $\left(0,-\frac{5}{3}\right)$ (b) $(2,1)$
(c) $\left(\frac{5}{4},0\right)$ (d) $\left(\frac{11}{4},2\right)$

107. The perimeter of the triangle with vertices at (1, 0, 0), (0, 1, 0) and (0, 0, 1) is
(a) 3 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{2}$

108. If a line in the space makes angle α, β and γ with the coordinate axes, then

$$\cos 2\alpha+\cos 2\beta+\cos 2\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma \text{ equals}$$

(a) –1 (b) 0 (c) 1 (d) 2

109. The radius of the sphere $x^2+y^2+z^2=12x+4y+3z$ is
(a) 13/2 (b) 13 (c) 26 (d) 52

110. $\lim\limits_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x+3}$ equals
(a) e (b) e^2 (c) e^3 (d) e^5

111. If $f: R\to R$ is defined by

$$f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x}, & if\ x\neq 0\\ a, & if\ x=0\end{cases},$$

then the value of a so that f is continuous at 0 is
(a) 2 (b) 1 (c) –1 (d) 0

112. $x=\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$,

$$y=\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)\Rightarrow\frac{dy}{dx} \text{ is equal to}$$

(a) 0 (b) $\tan t$
(c) 1 (d) sin t cos t

113. $\frac{d}{dx}\left[a\tan^{-1}x+b\log\left(\frac{x-1}{x+1}\right)\right]=\frac{1}{x^4-1}$
$\Rightarrow a-2b$ is equal to
(a) 1 (b) –1 (c) 0 (d) 2

114. $y-e^{a\sin^{-1}x}\Rightarrow(1-x^2)y_{n+2}-(2n+1)xy_{n+1}$ is equal to
(a) $-(n^2+a^2)y_n$ (b) $(n^2-a^2)y_n$
(c) $(n^2+a^2)y_n$ (d) $-(n^2-a^2)y_n$

115. The function $f(x)=x^3+ax^2+bx+c,\ a^2\le 3b$ has
(a) one maximum value
(b) one minimum value
(c) no extreme value
(d) one maximum and one minimum value

116. $\int\left(\frac{2-\sin 2x}{1-\cos 2x}\right)e^x dx$ is equal to
(a) $-e^x\cot x+c$ (b) $e^x\cot x+c$
(c) $2e^x\cot x+c$ (d) $-2e^x\cot x+c$

117. If $I_n=\int\sin^n x\,dx$, then $nI_n-(n-1)I_{n-2}$ equals
(a) $\sin^{n-1}x\cos x$ (b) $\cos^{n-1}x\sin x$
(c) $-\sin^{n-1}x\cos x$ (d) $-\cos^{n-1}x\sin x$

118. The line $x=\frac{\pi}{4}$ divides the area of the region bounded by y = sin x, y = cos x and x-axis $\left(0\le x\le\frac{\pi}{2}\right)$ into two regions of areas A_1 and A_2. Then $A_1 : A_2$ equals
(a) 4 : 1 (b) 3 : 1 (c) 2 : 1 (d) 1 : 1

119. The solution of the differential equation $\frac{dy}{dx}=\sin(x+y)\tan(x+y)-1$ is
(a) cosec (x + y) + tan (x + y) = x + c
(b) x + cosec (x + y) = c
(c) x + tan (x + y) = c
(d) x + sec (x + y) = c

120. If $p\Rightarrow(\sim p\vee q)$ is false, the truth value of p and q are respectively
(a) F, T (b) F, F (c) T, F (d) T, T

SOLUTIONS

PART - I (PHYSICS)

1. (d) Here, $y = a\sin(bt - cx)$

Comparing this equation with geneal wave equation

$$y = a\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

we get $b = \frac{2\pi}{T}$, $c = \frac{2\pi}{\lambda}$

(a) Dimensions of $\frac{y}{a} = \frac{[L]}{[L]} =$ Dimensionless

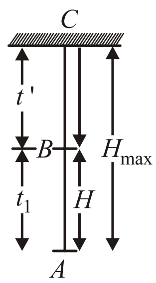
(b) Dimensions of $bt = \frac{2\pi}{T}\cdot t = \frac{[T]}{[T]}$
= Dimensionless

(c) Dimensions of $cx = \frac{2\pi}{\lambda}\cdot x = \frac{[L]}{[L]}$
= Dimensionless

(d) Dimensions of $\frac{b}{c} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = [LT^{-1}]$

2. (b) Let t' be the time taken by the body to fall from point C to B.

Then $t_1 + 2t' = t_2 \Rightarrow t' = \left(\frac{t_2 - t_1}{2}\right)$ ...(i)



Total time taken to reach point C

$$T = t_1 + t' = t_1 + \frac{t_2 - t_1}{2}$$

$$= \frac{2t_1 + t_2 - t_1}{2} = \left(\frac{t_1 + t_2}{2}\right)$$

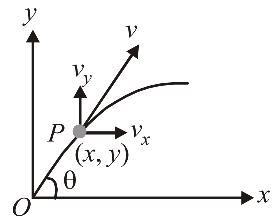
Maximum height attained

$$H_{\max} = \frac{1}{2}g(T)^2 = \frac{1}{2}g\left(\frac{t_1 + t_2}{2}\right)^2$$

$$= \frac{1}{2}g\cdot\frac{(t_1 + t_2)^2}{4}$$

Or, $H_{\max} = \frac{1}{8}g\cdot(t_1 + t_2)^2\, m$

3. (a) Momentum, $p = m\cdot v \Rightarrow v = \left(\frac{p}{m}\right)$



Kinetic energy, KE

$$= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p^2}{m^2}\right) = \frac{1}{2m}p^2$$

or, KE $\propto p^2$ $\left(\because \frac{1}{2m} = \text{constant}\right)$

Hence, the graph between KE and p^2 will be linear.

Now, kinetic energy KE $= \frac{1}{2}mv^2$

The velocity component at point P,

$v_y = (u\sin\theta - gt)$ and $v_x = u\cos\theta$

Resultant velocity at point P,

$$\vec{v} = v_y\hat{j} + v_x\hat{i}$$

$$= (u\sin\theta - gt)\hat{j} + u\cos\theta\hat{i}$$

$$|\vec{v}| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2ugt\sin\theta}$$

$$= \sqrt{u^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2ugt\sin\theta}$$

$\therefore$ KE $= \frac{1}{2}m(u^2 + g^2t^2 - 2ugt \sin\theta)$

i.e., KE $\propto t^2$

Hence, graph will be parabolic intercept on y-axis.

Hence, the graph between KE and t.

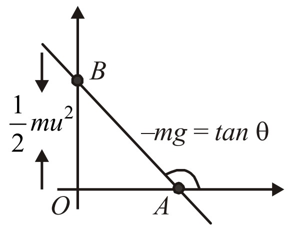
Now, in case of height

$$\text{KE} = \frac{1}{2}m(v^2) \text{ and } v^2 = (u^2 - 2gy)$$

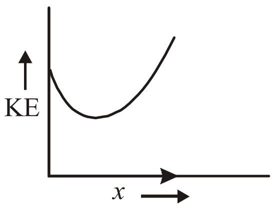
$$\therefore \text{KE} = \frac{1}{2}m(u^2 - 2gy)$$

$$\text{KE} = -mgy + \frac{1}{2}mu^2$$

Intercept on y-axis $= \frac{1}{2}mu^2$



Now, KE $= \frac{1}{2}mv^2$



$$\text{KE} = \frac{1}{2}m\left(\frac{x}{t}\right)^2$$

i.e., KE $\propto x^2$. Thus graph between KE and x will be parabolic.

4. (a) Power of motor initially $= P_0$

Let, rate of flow of motor $= (x)$

Since, power,

$$P_0 = \frac{\text{work}}{\text{time}} = \frac{mgy}{t} = mg\left(\frac{y}{t}\right)$$

$$\frac{y}{t} = x = \text{rate of flow of water} = mgx \quad \text{...(i)}$$

If rate of flow of water is increased by n times, i.e., (nx).

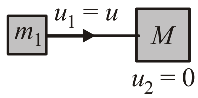
Increased power,

$$P_1 = \frac{mgy'}{t} = mg\left(\frac{y'}{t}\right) = nmgx \quad \text{...(ii)}$$

The ratio of power,

$$\frac{P_1}{P_0} = \frac{nmgx}{mgx} \;;\; P_1 : P_0 = n : 1$$

5. (a) Mass of the first body $m_1 = 5$ kg and for elastic collision coefficient of restitution, $e = 1$.



Let initially body m_1 moves with velocity v after collision velocity becomes $\left(\frac{u}{10}\right)$.

Let after collision velocity of M block becomes (v_2).

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $5u + M \times 0 = 5 \times \frac{u}{10} + Mv_2$

or $5u = \frac{u}{2} + Mv_2$ ...(i)

Since, $v_1 - v_2 = -e(u_1 - u_2)$

or $\frac{u}{10} - v_2 = -1(u)$ or $\frac{u}{10} + u = v_2$

or $\frac{11u}{10} = v_2$ ...(ii)

Substituting value of v_2 in Eq. (i) from Eq. (ii)

$$5u = \frac{u}{2} + M\left(\frac{11u}{10}\right)$$

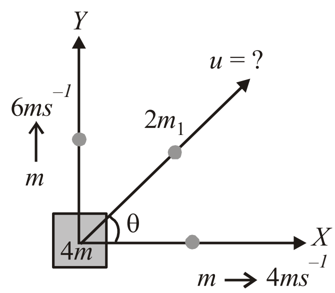
or $5 - \frac{1}{2} = M\left(\frac{11}{10}\right) \Rightarrow M = \frac{9 \times 10}{2 \times 11} = \frac{45}{11}$

$= 4.09$ *kg*

6. (c) Let third mass particle $(2m)$ moves making angle θ with X-axis.

The horizontal component of velocity of $2m$ mass particle $= u \cos\theta$

And vertical component $= u \sin\theta$



From conservation of linear momentum in X-direction

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $\quad 0 = m\times 4 + 2m(u\cos\theta)$

or $\quad -4 = 2u\cos\theta \quad$ or $\quad -2 = u\cos\theta \qquad$ ...(i)

Again, applying law of conservation of linear momentum in Y-direction

$$0 = m\times 6 + 2m(u\sin\theta)$$

$$\Rightarrow -\frac{6}{2} = u\sin\theta \;\text{ or }\; -3 = u\sin\theta \qquad \text{...(ii)}$$

Squaring Eqs. (i) and (ii) and adding,

$$(4)+(9) = u^2\cos^2\theta + u^2\sin^2\theta$$

$$= u^2(\cos^2\theta + \sin^2\theta)$$

or $\quad 13 = u^2$

$\therefore \quad u = \sqrt{13}\ \text{ms}^{-1}$

7. (b) Here, maximum height attained by a projectile

$$h = \frac{v^2R}{2gR - v^2} \qquad \text{...(i)}$$

Velocity of body = half the escape velocity

i.e., $v = \dfrac{v_e}{2}$

or $\quad v = \dfrac{\sqrt{2gR}}{2} \Rightarrow v^2 = \dfrac{2gR}{4} \Rightarrow v^2 = \left(\dfrac{gR}{2}\right)$

Now, putting value of v^2 in Eq. (i), we get

$$\text{Height, } h = \frac{\frac{gR}{2}\cdot R}{2gR - \frac{gR}{2}} = \frac{\frac{gR^2}{2}}{\frac{3gR}{2}} = \frac{R}{3}$$

8. (d) Particle executing SHM.

Displacement $y = 5\sin\left(4t + \dfrac{\pi}{3}\right) \qquad$ ...(i)

Velocity of particle

$$\left(\frac{dy}{dt}\right) = \frac{5d}{dt}\sin\left(4t + \frac{\pi}{3}\right)$$

$$= 5\cos\left(4t + \frac{\pi}{3}\right)\cdot 4 \; = 20\cos\left(4t + \frac{\pi}{3}\right)$$

Velocity at $t = \left(\dfrac{T}{4}\right)$

$$\left(\frac{dy}{dt}\right)_{t=\frac{T}{4}} = 20\cos\left(4\times\frac{T}{4} + \frac{\pi}{3}\right)$$

or $\quad u = 20\cos\left(T + \dfrac{\pi}{3}\right) \qquad$ ...(ii)

Comparing the given equation with standard equation of SHM.

$y = a\sin(\omega t + \phi)$

We get, $\omega = 4$

As $\quad \omega = \dfrac{2\pi}{T} \Rightarrow T = \dfrac{2\pi}{\omega}$

or $\quad T = \dfrac{2\pi}{4} = \left(\dfrac{\pi}{2}\right)$

Now, putting value of T in Eq. (ii), we get

$$u = 20\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = -20\sin\frac{\pi}{3}$$

$$= -10\times\sqrt{3}$$

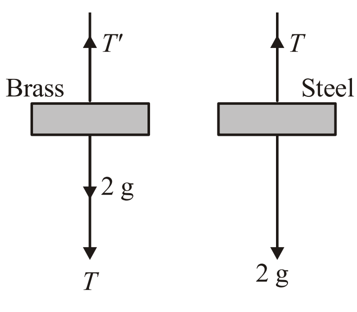
The kinetic energy of particle,

$$\text{KE} = \frac{1}{2}mu^2 \; = \frac{1}{2}\times 2\times 10^{-3}\times(-10\sqrt{3})^2$$

$$= 10^{-3}\times 100\times 3 = 0.3\,\text{J}$$

9. (d) Given, $\dfrac{l_1}{l_2} = a, \; \dfrac{r_1}{r_2} = b, \; \dfrac{Y_1}{Y_2} = c$

Free body diagram of the two blocks brass and steel are



Let Young's modulus of steel is Y_1 and of brass is Y_2.

$$\therefore Y_1 = \frac{F_1 \cdot l_1}{A_1 \cdot \Delta l_1} \quad \text{...(i)}$$

and $$Y_2 = \frac{F_2 \cdot l_2}{A_2 \cdot \Delta l_2} \quad \text{...(ii)}$$

Dividing Eq. (i) by (ii),

$$\frac{Y_1}{Y_2} = \frac{\frac{F_1 \cdot l_1}{A_1 \cdot \Delta l_1}}{\frac{F_2 \cdot l_2}{A_2 \cdot \Delta l_2}}$$

or $$\frac{Y_1}{Y_2} = \frac{F_1 \cdot A_2 \cdot l_1 \cdot \Delta l_2}{F_2 \cdot A_1 \cdot l_2 \cdot \Delta l_1} \quad \text{...(iii)}$$

Force on steel wire from free body diagram
$T = F_1 = (2g)$ newton
Force on brass wire from free body diagram

$F_2 = T' = T + 2g = (4g)$ newton

Now, putting the value of F_1, F_2, in Eq. (iii), we get

$$\frac{Y_1}{Y_2} = \left(\frac{2g}{4g}\right)\cdot\left(\frac{\pi r_2^2}{\pi r_1^2}\right)\cdot\left[\frac{l_1}{l_2}\right]\cdot\left(\frac{\Delta l_2}{\Delta l_1}\right)$$

or $$c = \frac{1}{2}\left(\frac{1}{b^2}\right)\cdot a\left(\frac{\Delta l_2}{\Delta l_1}\right)$$

or $$\frac{\Delta l_1}{\Delta l_2} = \left(\frac{a}{2b^2 c}\right)$$

10. (d) Initially area of soap bubble, $A_1 = 4\pi r^2$
Under isothermal condition radius becomes $2r$.

$\therefore$ Area $A_2 = 4\pi(2r)^2 = 16\pi r^2$

Increase in surface area

$$\Delta A = 2(A_2 - A_1) = 2(16\pi r^2 - 4\pi r^2) = 24\pi r^2$$

Energy spent,

$$W = T \times \Delta A = T \cdot 24\pi r^2 = 24\pi T r^2 \text{ J}$$

11. (c) Let radius of big drop $= R$.

$$\therefore \frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi r^3 \cdot 8$$

$R = 2r$
Here r = radius of small drops.
Now, terminal velocity of drop in liquid

$$v_T = \frac{2}{9} \times \frac{r^2}{\eta}(\rho - \sigma)g$$

where η is coefficient of viscosity and ρ is density of drop σ is density of liquid.
Terminal speed drop is 6 cm s^{-1}

$$\therefore 6 = \frac{2}{9} \times \frac{r^2}{\eta}(\rho - \sigma)g \quad \text{...(i)}$$

Let terminal velocity becomes v' after coalesce, then

$$v' = \frac{2}{9}\frac{R^2}{\eta}(\rho - \sigma)g \quad \text{...(ii)}$$

Dividing Eq. (i) by (ii), we get

$$\frac{6}{v'} = \frac{\frac{2}{9}\frac{r^2}{\eta}(\rho-\sigma)g}{\frac{2}{9}\frac{R^2}{\eta}(\rho-\sigma)g} \Rightarrow \frac{6}{v'} = \frac{r^2}{(2r)^2}$$

or $v' = 24$ cm s^{-1}

12. (a) Time period of oscillation,

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{dT}{T} = \frac{1}{2}\frac{dl}{l}$$

As, $\frac{dl}{l} = \alpha dt$

$$\Rightarrow \frac{dT}{T} = \frac{1}{2}\alpha dt = \frac{1}{2} \times 9 \times 10^{-7} \times (30 - 20)$$
$$= 4.5 \times 10^{-6}$$

$\therefore$ Loss in time $= 4.5 \times 10^{-6} \times 0.5$
$= 2.25 \times 10^{-6}$ s

13. (c) Volume of the metal at 30°C

$$V_{30} = \frac{\text{loss of weight}}{\text{specific gravity} \times g}$$

$$= \frac{(45 - 25)g}{1.5 \times g} = 13.33 \text{ cm}^3$$

Similarly, volume of metal at 40°C

$$V_{40} = \frac{(45 - 27)g}{1.25 \times g} = 14.40 \text{ cm}^3$$

Now, $V_{40} = V_{30}[1 + \gamma(t_2 - t_1)]$

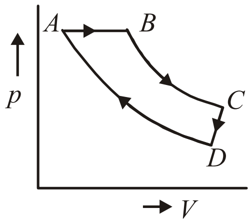
or $$\gamma = \frac{V_{40} - V_{30}}{V_{30}(t_2 - t_1)} = \frac{14.40 - 13.33}{13.33(40 - 30)}$$
$= 8.03 \times 10^{-3}$/°C

$\therefore$ Coefficient of linear expansion of the metal

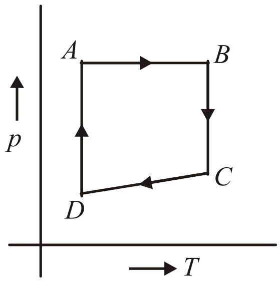
$$\alpha = \frac{\gamma}{3} = \frac{8.03\times10^{-3}}{3} \approx 2.6\times10^{-3}\ /\ °C$$

14. (a) Process $A \to B \to C \to D \to A$ is clockwise.

During $A \to B$, pressure is constant and $B \to C$, process follows $p \propto \frac{1}{V}$ ie., T is constant. During process $C \to D$, both p and V changes and process $D \to A$ follows $p \propto \frac{1}{V}$ i.e., T is constant.



Hence, equivalent cyclic process is has follows.



15. (b) From first law of thermodynamics

$Q = \Delta U + W$ or $\Delta U = Q - W$

$\therefore \Delta U_1 = Q_1 - W_1 = 6000 - 2500 = 3500$ J

$\Delta U_2 = Q_2 - W_2 = -5500 + 1000 = -4500$ J

$\Delta U_3 = Q_3 - W_3 = -3000 + 1200 = -1800$ J

$\Delta U_4 = Q_4 - W_4 = 3500 - x$

For cyclic process $\Delta U = 0$

$\therefore 3500 - 4500 - 1800 + 3500 - x = 0$

or $x = 700$ J

Efficiency, $\eta = \frac{\text{output}}{\text{input}} \times 100$

$$= \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4} \times 100$$

$$= \frac{(2500 - 1000 - 1200 + 700)}{6000 + 3500} \times 100$$

$$= \frac{1000}{9500} \times 100 = 10.5\%$$

16. (c) From first law of thermodynamics

$Q = \Delta U + W$

For cylinder A pressure remains constant.

$\therefore$ Work done by a system

$$W = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$$

For monoatomic gases, $\mu = 1$; $\gamma = \frac{5}{3}$

$$\therefore W = \frac{1\times R}{\frac{5}{3} - 1}(442 - 400) = \frac{3}{2} R \times 42$$

or $W = 63R$

But $\Delta U = 0$, for cylinder A

$\therefore Q = 0 + 63R = 63R$

For cylinder B volume is constant,

$\therefore$ $W = 0$ and $Q = \mu C_V \Delta T$

For monoatomic gas

$$C_V = \frac{3}{2}R \Rightarrow Q = 1\times\frac{3}{2}R\Delta T$$

As heat given on both cylinder is same

$$\therefore 63R = \frac{3}{2}R\Delta T \Rightarrow \Delta T = 42\ \text{K}$$

17. (a) From figure, $H = H_1 + H_2$

$$\Rightarrow \frac{3KA(100-T)}{l} = \frac{2KA(T-50)}{l} + \frac{KA(T-0)}{l}$$

$$\Rightarrow 300 - 3T = 2T - 100 + T \Rightarrow 6T = 400$$

$$\Rightarrow T = \frac{200}{3}°C$$

18. (b) Let listener go from $A \to B$ with velocity (μ).

A_s ———— B_s

680 Hz   680 Hz

And the apparent frequency of sound from source A by listener using Doppler's effect,

$$n' = n\left(\frac{v - v_o}{v + v_s}\right) \Rightarrow n' = 680\left(\frac{340 - u}{340 + 0}\right)$$

The apparent frequency of sound from source B by listener

$$n'' = n\left(\frac{v + v_o}{v - v_s}\right) = 680\left(\frac{340 + u}{340 - 0}\right)$$

Listener hear 10 beats per second.
Hence, $n'' - n' = 10$

$$\Rightarrow 680\left(\frac{340 + u}{340}\right) - 680\left(\frac{340 - u}{340}\right) = 10$$

$$\Rightarrow 2(340 + u - 340 + u) = 10$$

$$\Rightarrow u = 2.5 \text{ ms}^{-1}$$

19. (b) When both the wires vibrate simultaneously, beats per second,

$$n_1 \pm n_2 = 6$$

or $$\frac{1}{2l}\sqrt{\frac{T}{m}} \pm \frac{1}{2l}\sqrt{\frac{T'}{m}} = 6$$

$$\Rightarrow \frac{1}{2l}\sqrt{\frac{T'}{m}} - \frac{1}{2l}\sqrt{\frac{T}{m}} = 6$$

$$\Rightarrow \frac{1}{2l}\sqrt{\frac{T'}{m}} - 600 = 6 \Rightarrow \frac{1}{2l}\sqrt{\frac{T'}{m}} = 606 \quad \text{...(i)}$$

Given that fundamental frequency

$$\frac{1}{2l}\sqrt{\frac{T}{m}} = 600 \quad \text{...(ii)}$$

Dividing Eq. (i) by (ii),

$$\frac{\frac{1}{2l}\sqrt{\frac{T'}{m}}}{\frac{1}{2l}\sqrt{\frac{T}{m}}} = \frac{606}{600}$$

$$\Rightarrow \sqrt{\frac{T'}{T}} = (1.01) \Rightarrow \frac{T'}{T} = (1.02)\%$$

$$\Rightarrow T' = T(1.02)$$

Increase in tension

$$\Delta T' = T \times 1.02 - T = (0.02T)$$

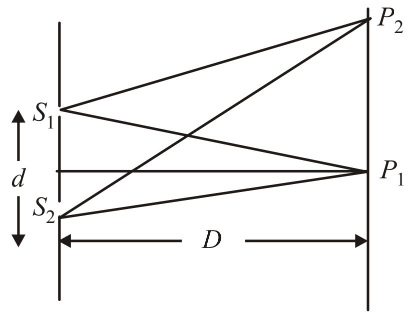
$$\therefore \Delta T = 0.02$$

20. (d) Fringe width $\beta = \dfrac{\lambda D}{d}$

Let be the amplitude of the place where constructive inference takes place.
The position of fringe at p_2.

$$\Rightarrow x = \frac{n\lambda D}{d}$$

Given, $\beta' = \left(\dfrac{\beta}{4}\right)$



$$\therefore \frac{\lambda D}{4d} = \frac{n\lambda D}{d} \quad \text{or} \quad n = \frac{1}{4}$$

$$\therefore \frac{I_1}{I_2} = \frac{a^2}{\left(\frac{a}{4}\right)^2} = 16:1$$

21. (a) Position fringe from central maxima

$$y_1 = \frac{n\lambda_1 D}{d}$$

Given, $n = 10$

$$\therefore y_1 = \frac{10\lambda_1 D}{d} \quad \text{...(i)}$$

For second source

$$y_2 = \frac{5\lambda_2 D}{d} \quad \text{...(ii)}$$

$$\therefore \frac{y_1}{y_2} = \frac{\frac{10\lambda_1 D}{d}}{\frac{5\lambda_2 D}{d}} = \frac{2\lambda_1}{\lambda_2}$$

22. (c) Inteference takes place between two waves having equal frequency and propagate in same direction.

Hence, $y_1 = a\sin(\omega t + \phi_1)$

$y_3 = a'\sin(\omega t + \phi_2)$

will give interference as the two waves have same frequency ω.

23. (d) The two lenses of an achromatic doublet should have, sum of the product of their powers and dispersive power = zero.

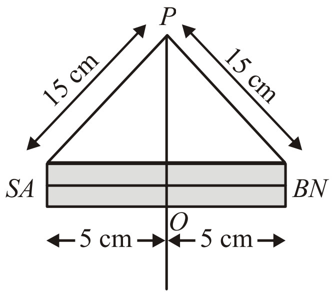
24. (b) Ratio of magnetic moments

$$\frac{M_A}{M_B} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{v_s^2 + v_d^2}{v_s^2 - v_d^2}$$

$$= \frac{\left(\frac{1}{20}\right)^2 + \left(\frac{1}{15}\right)^2}{\left(\frac{1}{15}\right)^2 - \left(\frac{1}{20}\right)^2} = \frac{400 + 225}{400 - 225}$$

$$M_A : M_B = 25 : 7$$

25. (d) Here, length of magnet = 10 cm = 10 × 10^{-2} m, r = 15 × 10^{-2} m



$$OP = \sqrt{225 - 25} = \sqrt{200} \text{ cm}$$

Since, at the neutral point, magnetic field due to the magnet is equal to B_H,

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{M}{(OP^2 + AO^2)^{3/2}}$$

$$0.4 \times 10^{-4} = 10^{-7} \times \frac{M}{(200 \times 10^{-4} + 25 \times 10^{-4})^{3/2}}$$

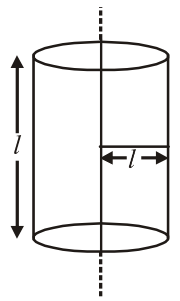
$$\Rightarrow \frac{0.4 \times 10^{-4}}{10^{-7}} \times (225 \times 10^{-4})^{3/2} = M$$

$$\Rightarrow 0.4 \times 10^3 \times 10^{-6} (225)^{3/2} = M$$

$$\Rightarrow M = 1.35 \text{ A-m}$$

26. (a) Charge density or charge per unit length of long wire

$\lambda = \frac{1}{3}$ Cm^{-1} and r = 18 × 10^{-2} m



According to Gauss theorem

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E \oint dS = \frac{q}{\varepsilon_0} \quad \text{or} \quad E \times 2\pi r l = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{2\pi\varepsilon_0 r l} = \frac{q/l}{2\pi\varepsilon_0 r}$$

$$= \frac{\lambda \times 2}{2\pi\varepsilon_0 r \times 2} = \frac{\lambda \times 2}{4\pi\varepsilon_0 r}$$

$$= 9 \times 10^9 \times \frac{1}{3} \times 2 \times \frac{1}{18 \times 10^{-2}}$$

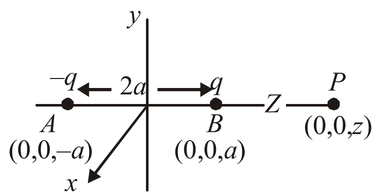
$$= 0.33 \times 10^{11} \text{ NC}^{-1}$$

27. (c) Potential at P due to $(+q)$ charge of dipole

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(z - a)}$$

Potential at P due to $(-q)$ charge of dipole

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{(z + a)}$$



Total potential at P due to electric dipole

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(z - a)} - \frac{1}{4\pi\varepsilon_0} \frac{q}{(z + a)}$$

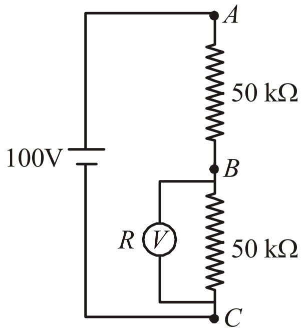
$$= \frac{q}{4\pi\varepsilon_0} \frac{(z + a - z + a)}{(z - a)(z + a)}$$

$$\text{or} \quad V = \frac{2qa}{4\pi\varepsilon_0 (z^2 - a^2)}$$

28. (c) Internal resistance of voltmeter = R. Effective resistance across B and C

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{50} = \frac{50 + R}{50R}$$

$$\text{or} \quad R' = \left(\frac{50R}{50 + R}\right)$$



According to Ohm's law, $V' = IR'$

or $\frac{100}{3} = I \cdot \left(\frac{50R}{50+R}\right)$

or $\frac{100}{3}\left(\frac{50+R}{50R}\right) = I$ ...(i)

Now, total resistance of circuit

$R'' = 50 + \frac{50R}{50+R}$

or $R'' = \frac{(2500+100R)}{(50+R)}$

Now, $V'' = IR''$

$\Rightarrow 100 = \frac{100}{3}\left(\frac{50+R}{50R}\right)\frac{2500+100R}{(50+R)}$

$\Rightarrow 150R = 2500 + 100R$ or $R = 50\text{k}\Omega$

29. (c) Here length of potentiometer wire, $l = 10$ m

Resistance of potentiometer wire

$R = \rho \times \frac{l}{A}$ or $R = \left(\rho \times \frac{10}{A}\right)$

The value of 2.5 m length wire

$R' = \frac{\rho \times 10}{A \times 10} \times 2.5 \Rightarrow R' = \left(\frac{2.5\rho}{A \times 10}\right)$

Potential, $V' = I \times R' = I\left(\frac{2.5\rho}{A \times 10}\right)$

Again the length of potentiometer wire is increased by 1 m.

Resistance of null position

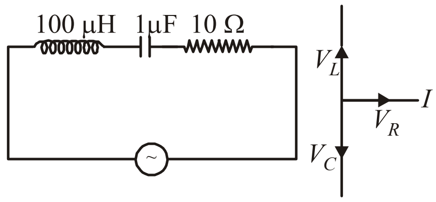
$R'' = \left(\frac{\rho \times l}{11 \times A}\right)$

$\therefore V'' = IR''$ and $V = V'$

$\Rightarrow \frac{I \times 2.5\rho}{A \times 10} = \frac{\rho \times l}{11 \times A} \times I$

or $\frac{2.5 \times 11}{10} = l = 2.75$ m

30. (c)



Impedance, $Z = \sqrt{(X_L \sim X_C)^2 + R^2}$

or $Z = \sqrt{\left(\omega L \sim \frac{1}{\omega C}\right)^2 + R^2}$

Inductive reactance

$X_L = \omega L = 70 \times 10^3 \times 100 \times 10^{-6} = 7\Omega$

Capacitance reactance

$X_C = \frac{1}{\omega C} = \frac{1}{70 \times 10^3 \times 1 \times 10^{-6}}$

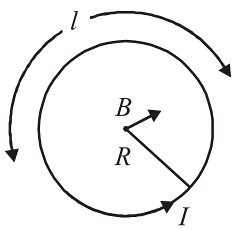
$= \frac{1}{7 \times 10^{-2}} = \frac{10^2}{7} = \frac{100}{7}$

As $X_C > X_L$

So, circuit behaves like R-C circuit.

31. (c) At the centre of the loop, magnetic field

$B = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot 2\pi R}{R^2}$ ...(i)



For the wire which is looped double let radius becomes r

Then, $\frac{l}{2} = 2\pi r$ or $\frac{l}{4\pi} = (r)$

$\therefore B' = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot 2\pi r \times 2}{r^2}$

or $B' = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot \frac{l}{2} \cdot 2}{\left(\frac{1}{4\pi}\right)^2}$

or $B' = \frac{\mu_0}{4\pi} \cdot \frac{Il \times 16\pi^2}{l^2}$ ...(ii)

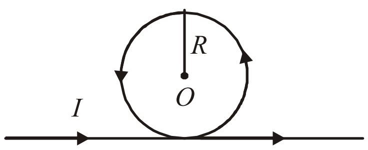
Now, $B = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot l}{\left(\frac{l}{2\pi}\right)^2} \left[R = \frac{1}{2\pi}\right]$ ...(iii)

Dividing Eq. (ii) by (iii),

$$\frac{B'}{B} = \frac{\frac{\mu_0}{4\pi} \cdot \frac{I \cdot l \cdot 16\pi^2}{l^2}}{\frac{\mu_0}{4\pi} \cdot \frac{Il \cdot 4\pi^2}{l^2}} = 4$$

or $B' = 4B$

32. (c) Magnetic field due to long wire at O point



$B_1 = \frac{\mu_0}{2\pi}\left(\frac{I}{R}\right)$ (upward)

Magnetic field due to loop at O point

$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot 2\pi R}{R^2}$

$\Rightarrow B_2 = \frac{\mu_0}{2} \cdot \frac{I}{R}$ (in upward direction)

Resultant magnetic field at centre O

$B = B_1 + B_2$

$\Rightarrow B = \frac{\mu_0 I}{2\pi \cdot R}(\pi + 1)T$

33. (b) Work function $W_0 = 3.31 \times 10^{-19}$ J

Wavelength of incident radiation

$\lambda = 5000 \times 10^{-10}$ m

According to Einstein's photoelectric equation $E = W_0 + KE$

$\Rightarrow \frac{hc}{\lambda} = 3.31 \times 10^{-19} + \text{KE}$

$$\text{KE} = -3.31 \times 10^{-19} + \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}$$

$$= -3.31 \times 10^{-19} + \frac{6.62 \times 3}{5} \times 10^{-19}$$

$= (-3.31 \times 1.324 \times 3) \times 10^{-19}$

$= (3.972 - 3.31) \times 10^{-19}$

$= 0.662 \times 10^{-19}$ J

or $E = \frac{0.662 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.41$ eV

34. (d) From Einstein's photoelectric equation

$E = W_0 + \frac{1}{2}mv^2$ or $\sqrt{\frac{2(E - W_0)}{m}} = v$

A charged particle placed in uniform magnetic field experience a force

$F = evB = \frac{mv^2}{r}$

or $r = \frac{mv}{eB}$

or $r = \frac{m\sqrt{\frac{2(E - W_0)}{m}}}{eB} \Rightarrow r = \frac{\sqrt{2m(E - W_0)}}{eB}$

35. (d) Here, $N_1 = N_0 e^{-10\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$\Rightarrow \frac{N_1}{N_2} = \frac{1}{e} = e^{-1} = e^{(-10\lambda + \lambda)t} = e^{-9\lambda t}$

$\Rightarrow t = \frac{1}{9\lambda}$

36. (c) Here current flows in circuit A as both (p-n) junction diode act as forward biasing.

Total resistnace R.

$\frac{1}{R} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ or $R = 2\Omega$

According to Ohm's law

$V = I_A R$

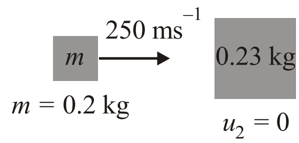
or $8 = I_A \times 2$ or $I_A = 4$A

In circuit B, lower p-n-junction diode is reverse biased. Hence, no current will flow but upper diode is forward biased so current cna flow through it

$V = I_B R$

or $8 = I_B \times 4$ or $I_B = 2$ A

37. (c) After collision the bullet and block move together and comes to rest after covering a distance of 40 m.



By conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow 0.02\times 250 + 0.23\times 0 = 0.02v + 0.23v$

$5 + 0 = v(0.25)$ or $v = 20\ \text{ms}^{-1}$

Now, by conservation of energy

or $\frac{1}{2}Mv^2 = \mu R\cdot d$

or $\frac{1}{2}\times 0.25\times 400 = \mu\times 0.25\times 9.8\times 40$

$\Rightarrow \mu = \frac{200}{9.8\times 40} = 0.51$

38. (a) Let after the time (t) the position of A is $(0, v_At)$ and position of $B = (v_Bt, 10)$. Distance between them

$y = \sqrt{(0 - v_Bt)^2 + (v_At - 10)^2}$

or $y^2 = (2t)^2 + (2t - 10)^2$

or $y^2 = l = 4t^2 + 4t^2 + 100 - 40t$

$\Rightarrow l = 8t^2 + 100 - 40t$

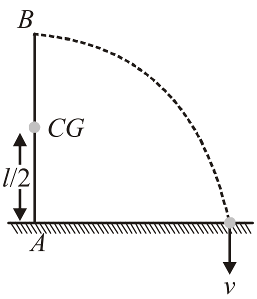
Now, $\frac{dl}{dt} = (16t - 40) = 0$

or

$t = \frac{40}{16} = 2.5\text{ s}$ $\quad\because \frac{d^2l}{dt^2} = 16 = (+\text{ve})$

So, l will be minimum.

39. (b) Here, potential energy of the metre stick will be converted into rotational kinetic energy.



Because centre of gravity of stick lies at the middle of the rod,

PE of metre stick $= \frac{mgl}{2}$

Rotational kinetic energy $E = \frac{1}{2}I\omega^2$

I about point $A = \frac{ml^2}{3}$.

By law of conservation of energy

$mg\left(\frac{l}{2}\right) = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{ml^2}{3}\left(\frac{v_B}{l}\right)^2$

By solving, we get $v_B = \sqrt{3gl}$

40. (a) Given, $r = 0.4$ m,
$\alpha = 8\text{ rad s}^{-1}$,
$m = 4$ kg, $I = ?$
Torque, $\tau = I\alpha$

$mgr = I\cdot\alpha$

$4\times 10\times 0.4 = I\times 8$ or $I = \frac{16}{8} = 2\text{ kg.m}^2$

PART - II (CHEMISTRY)

41. (c) Given : $\Delta H_f(\text{H}) = 218$ kJ/mol

i.e., $\frac{1}{2}H_2 \longrightarrow H;\quad \Delta H = 218$ kJ/mol

or $H_2 \longrightarrow 2H;\quad \Delta H = 436$ kJ/mol

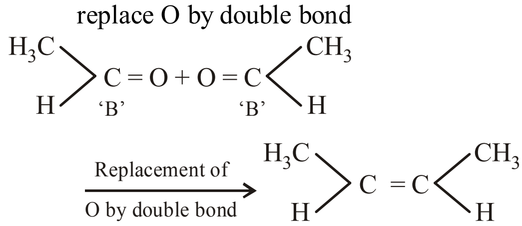
$= \frac{436}{4.18} = 104.3$ kcal/mol

Hence, H–H bond energy is 104.3 kcal/mol.

42. (a) In Wacker process, alkene is oxidised into aldehyde.

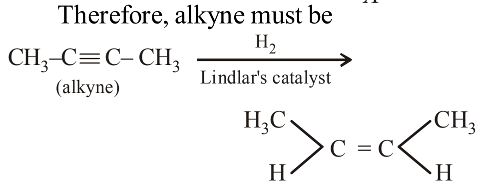
$$CH_2 = CH_2 + \frac{1}{2}O_2 \xrightarrow[H_2O]{PdCl_2\cdot CuCl_2} \underset{(B)}{CH_3CHO}$$

Since only alkenes produce aldehydes, on ozonolysis hence 'A' must be an alkene. Now to find the structure of alkene we should add two molecules of aldehyde and replace O by double bond



'A'

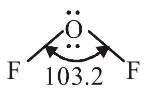
Therefore, alkyne must be



'A'

43. (b) $2F_2 + \underset{\text{dilute}}{2NaOH} \longrightarrow 2NaF + \underset{(A)}{OF_2} \uparrow + H_2O$

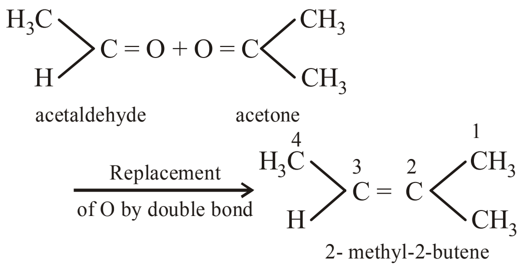
The structure of 'A' (OF_2) is as



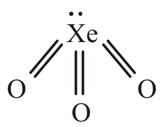
σ bonds made by O = 2

Due to repulsion between two lone pairs of electrons, its shape gets distorted. Therefore, the bond angle in the molecule is 103°.

44. (a) To decide the structure of alkene that undergoes ozonolysis, add the products and replace O by double (=) bond. Thus,

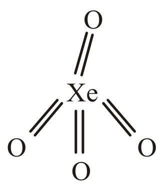


45. (a) **Structure of XeO_3**



$\Rightarrow$ $3p\pi$-$d\pi$ pi bonds.

Structure of XeO_4



$\Rightarrow$ $4p\pi$-$d\pi$ bonds.

46. (a) According to **de-Broglie's** equation.

$$\lambda = \frac{h}{mv} \Rightarrow \lambda^2 = \frac{h^2}{m^2v^2}$$

$$\text{or} \quad mv^2 = \frac{h^2}{m\lambda^2}$$

$$\because \text{KE}(K) = \frac{1}{2}mv^2$$

$$\therefore \text{KE } (K) = \frac{1}{2}\frac{h^2}{m\lambda^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\therefore K_1 : K_2 = 25 : 9$$

47. (c) Paramagnetic nature depends upon the number of unpaired electrons. Higher the number of unpaired electrons, higher the paramagnetic property will be.

Cu^{2+} = [Ar] $3d^9$, no. of unpaired electrons = 1
V^{2+} = [Ar] $3d^3$, no. of unpaired electrons = 3
Cr^{2+} = [Ar] $3d^4$, no. of unpaired electrons = 4
Mn^{2+} = [Ar] $3d^5$, no. of unpaired electrons = 5

Hence, correct order is

$$Cu^{2+} < V^{2+} < Cr^{2+} < Mn^{2+}$$

48. (a) $\because$ 1 mol = 6.023×10^{23} atoms

KE of 1 mol = 6.023×10^4 J

or KE of 6.023×10^{23} atoms = 6.023×10^4 J

$$\therefore \text{KE of 1 atom} = \frac{6.023 \times 10^4}{6.023 \times 10^{23}}$$
$$= 1.0 \times 10^{-19} \text{ J}$$

$$h\nu_{\text{energy}} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}}$$
$$= 3.313 \times 10^{-19} \text{ J}$$

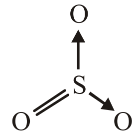
Now since Threshold energy

$$= h\nu - \text{KE}$$
$$= 3.313 \times 10^{-19} - 1.0 \times 10^{-19}$$
$$= 2.313 \times 10^{-19} \text{ J}$$

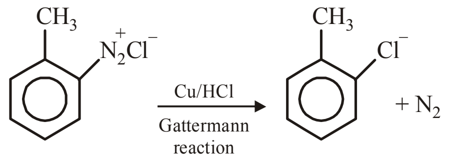
Hence minimum amount of energy required to remove an electron from the metal ion will be 2.313×10^{-19} J.

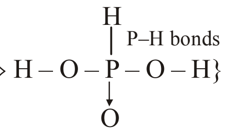
49. (a) The earth's thermosphere also includes the region of the atmosphere, called the *ionosphere*. The ionosphere is the region of the atmosphere that is filled with charged particles such as O_2^+, O^+, NO^+. The high temperature in the thermosphere can cause molecules to ionize.

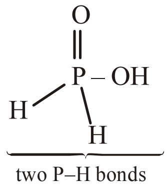
50. (b) The formula of sulphuric anhydride is SO_3 and its structure is as follows :



$\Rightarrow$ 3σ, $1p\pi$-$p\pi$, $2\ p\pi$-$d\pi$ bonds are present.

51. (a) 

52. (c) $\mathbf{H_3PO_3} \Rightarrow$ 

$\mathbf{H_3PO_2} \Rightarrow$ 

53. (c) Dipole moment,

$(\mu) = \delta \times d$

where, δ = magnitude of electric charge

d = distance between particles (or bond length)

$\therefore \delta = \frac{\mu}{d}$

or $\frac{\delta_{HCl}}{\delta_{HI}} = \frac{\mu_{HCl}}{d_{HCl}} \times \frac{d_{HI}}{\mu_{HI}}$

$= \frac{1.03 \times 1.6}{1.3 \times 0.38} = 3.3 : 1$

54. (d) $SiCl_4 + 4H_2O \longrightarrow H_4SiO_4 + 4HCl$

$H_4SiO_4 \xrightarrow[1000°C]{\Delta} SiO_2 + 2H_2O$

55. (c) Mass percent of Cd in $CdCl_2$

$= \frac{0.9}{1.5} \times 100 = 60\%$

$\therefore$ Mass percent of Cl_2 in $CdCl_2$

$= 100 - 60 = 40\%$

$\because$ 40% part (Cl_2) has atomic weight

$= 2 \times 35.5 = 71.0$

$\therefore$ 60% part (Cd) has atomic weight

$= \frac{71.0 \times 60}{40} = 106.5$

56. (c) $2Al + 2NaOH + 2H_2O \longrightarrow \underset{\text{sodium meta aluminate}}{2NaAlO_2} + 3H_2$

Sodiummetaaluminate, thus formed, is soluble in water and changes into the complex $[Al(H_2O)_2(OH)_4]^-$, in which coordination number of Al is 6.

57. (b) Average kinetic energy per molecule

$= \frac{3}{2}kT$

or $\frac{3}{2}\frac{R}{N_0}T = \frac{3}{2} \times \frac{8.314}{6.023 \times 10^{23}} \times 300$

$= 6.21 \times 10^{-21}\ JK^{-1}$ molecule^{-1}

58. (c) The species having an O–O bond and O in an oxidation state of $-\frac{1}{2}$ are super oxides and is represented as O_2^-. Usually these are formed by active metals such as KO_2, RbO_2 and CsO_2. For the salts of larger anions (like O_2^-), lattice energy increases in a group. Since, lattice energy is the driving force for the formation of an ionic compound and its stability, the stability of the superoxides from 'K' to 'Cs' also increases.

59. (a) 30% solution of H_2O_2 is known as perhydrol .

H_2O_2 decomposes as

$2H_2O_2 \longrightarrow 2H_2O + O_2$

Volume strength of 30% H_2O_2 solution is 100 i.e., 1 mL of this solution on decomposition gives 100 mL oxygen.

$$\begin{array}{lll} SO_2 + & \frac{1}{2}O_2 \longrightarrow & SO_3 \\ 1L & \frac{1}{2}L & 1L \\ 2L & 1L & 2L \end{array}$$

Since, 100 mL of oxygen is obtained by

= 1 mL of H_2O_2

$\therefore$ 1000 mL of oxygen will be obtained by

$= \frac{1}{100} \times 1000$ mL of H_2O_2 = 10 mL of H_2O_2

60. (d) Buffer capacity, $\beta = \frac{dC_{HA}}{d_{pH}}$,

where, dC_{HA} = no. of moles of acid added per litre

d_{pH} = change in pH

$dC_{HA} = \frac{\text{moles of acetic acid}}{\text{volume}}$

$= \frac{0.12/60}{250/1000} = \frac{1}{125}$

$\therefore \beta = \frac{1/125}{0.02} = \frac{1}{2.5} = 0.4$

61. (b) (A) Felspar (orthoclase) ($KAlSi_3O_8$)
(B) Asbestos $\{CaMg_3(SiO_3)_4\}$
(C) Pyrargyrite (Ruby silver) (Ag_3SbS_3)
(D) Diaspore ($Al_2O_3 . H_2O$)

62. (c) Along a period first ionisation energy increases. Thus, the first IE of the elements of the second period should follow the order $Be < B < N < O$
But in practice. The first IE of these elements follows the order $B < Be < O < N$
The lower IE of B than that of Be is because in B ($1s^2, 2s^2 2p^1$), electron is to be removed from $2p$ which is easy while in Be ($1s^2, 2s^2$), electron is to be removed from $2s$ which is difficult. The low IE of O than that of N is because of the half-filled $2p$ orbitals in N ($1s^2, 2s^2 2p^3$).

63. (c) $CH_3CH_2OH + Cl_2 \xrightarrow[-2HCl]{} \underset{\substack{X \\ \text{Acetaldehyde}}}{CH_3CHO}$

$$\xrightarrow[-3HCl]{3Cl_2} \underset{\substack{Y \\ \text{chloral}}}{CCl_3CHO}$$

64. (c) I. $\boxed{\begin{matrix} CH_3COO \\ CH_3COO \end{matrix} > Ca} \xrightarrow[-CaCO_3]{\Delta} CH_3COCH_3$

II. $CH_3COOH \longrightarrow 6HI$

$$\xrightarrow{\text{Red P}} CH_3CH_3 + 3I_2 + 2H_2O$$

III. $\begin{matrix} CH_3CO\boxed{OH} \\ CH_3COO\boxed{H} \end{matrix} \xrightarrow{\Delta,\ P_4O_{10}} \begin{matrix} CH_3CO \\ CH_3CO \end{matrix} > O + H_2O$

65. (b) $C = 85.71\% = \dfrac{85.71}{12} = 7.14;\ \dfrac{7.14}{7.14} = 1$

$H = 14.29\% = \dfrac{14.29}{1} = 14.29;\ \dfrac{14.29}{7.14} = 2$

$\therefore$ Empirical formula $= CH_2$
and, empirical formula weight $= 12 + 2 = 14$
Now since molecular formula weight
$= 2 \times$ vapour density
$= 2 \times 14 = 28$

$\therefore n = \dfrac{28}{14} = 2$

$\therefore$ Molecular formula $= (CH_2)_2 = C_2H_4$

$$\underset{(A)}{CH_2 = CH_2} + HOCl \longrightarrow \underset{(B)}{\underset{\substack{| \\ OH}}{CH_2} - CH_2 - Cl}$$

$$+ KCN \xrightarrow{EtOH} \underset{\substack{| \\ OH}}{CH_2} - CH_2 - CN$$

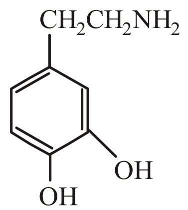
$$\xrightarrow{H_3O^+} \underset{(C)}{\underset{\substack{| \\ OH}}{CH_2} - CH_2 - COOH}$$

66. (c) Tripeptides are amino acids polymers in which three individual amino acid units, called residues, are linked together by amide bonds.
glycine ($NH_2–CH_2–COOH$), alanine ($CH_3 - \underset{\substack{| \\ NH_2}}{CH} - COOH$) and phenyl alanine

$C_6H_5 - CH_2 - \underset{\substack{| \\ NH_2}}{C}HCOOH$ can be linked in six different ways.

67. (d) A codon is a specific sequence of three adjacent bases on a strand of DNA or RNA that provides genetic code information for a particular amino acid.

68. (c) The IUPAC name of dopamine is 2-(3,4-dihydroxyphenyl) ethylamine and its structure is as follows :



69. (a) Freezing point of a substance is the temperature at which the solid and the liquid forms of the substance are in equilibrium.

70. (b) $C_2H_5 - Cl + AgCN \xrightarrow[H_2O]{EtOH/}$

$$C_2H_5 - N\underset{\Downarrow}{C} + AgCl$$

(N-linked to ethyl carbon)

71. (a) For the given cell,
$Ag \mid Ag^+ \mid AgCl \mid Cl^- \mid Cl_2, Pt$
The cell reactions are as follows

At anode : $Ag \longrightarrow Ag^+ + e^-$

At cathode : $AgCl + e^- \longrightarrow Ag(s) + Cl^-$

Net cell reaction : $AgCl \longrightarrow Ag^+ + Cl^-$

$\therefore\ \Delta G^{o}_{reaction} = \Sigma\Delta G^{o}_{p} - \Sigma\Delta G^{o}_{R}$

$= (78 - 129) - (-109)$

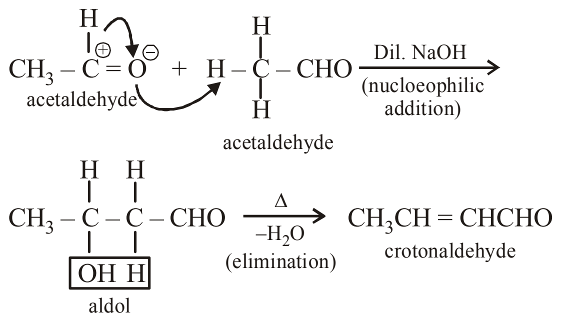
$= +58$ kJ/mol

$\Delta G^{o} = -nFE^{o}$

$58 \times 10^{3}\ J = -1 \times 96500 \times E^{o}_{cell}$

$\Rightarrow E^{o}_{cell} = \dfrac{-58 \times 1000}{96500} = -0.6\,V$

72. (d) Crotonaldehyde is produced by the aldol condensation of acetaldehyde.



73. (b) $BaCl_2 + 2NaOH \longrightarrow Ba(OH)_2 + 2NaCl$

$\lambda^{\infty}_{m\,Ba(OH)_2} = \lambda^{\infty}_{m\ BaCl_2} + 2\lambda^{\infty}_{m\ NaOH} - 2\lambda^{\infty}_{m\ NaCl}$

$= 280 \times 10^{-4} + 2 \times 248 \times 10^{-4} - 2 \times 126 \times 10^{-4}$

$= (280 + 496 - 252) \times 10^{-4}$

$= 524 \times 10^{-4}\ Sm^{2}\ mol^{-1}$

74. (b) Density, $d = \dfrac{MZ}{N_0 a^3}$

where, Z = number of atoms in unit cell

$\therefore Z = \dfrac{dN_0 a^3}{M}$

$= \dfrac{8.92 \times 6.023 \times 10^{23} \times (362 \times 10^{-10})^3}{63.55}$

$= 4.0$

Thus, metal has face centred unit cell.

75. (c) $N_2 + 2O_2 \rightleftharpoons 2NO_2$

$K_1 = \dfrac{[NO_2]^2}{[N_2][O_2]^2}$

or $100 = \dfrac{[NO_2]^2}{[N_2][O_2]^2}$ ..(i)

Again, $[NO_2] \rightleftharpoons \dfrac{1}{2}N_2 + O_2$

$K_2 = \dfrac{[N_2]^{1/2}[O_2]}{[NO_2]}$

or $K_2^2 = \dfrac{[N_2][O_2]^2}{[NO_2]^2}$ ...(ii)

Eqs. (i) × (ii), we get $100 \times K_2^2 = 1$

$\Rightarrow K_2^2 = \dfrac{1}{100}$ or $K_2 = \dfrac{1}{10} = 0.1$

76. (c) For a first order reaction,

$t = \dfrac{2.303}{\lambda}\log_{10}\dfrac{a}{a-x}$

Let initial amount of reactant is 100.

$\dfrac{t_1}{t_2} = \dfrac{\log\dfrac{100}{100-75}}{\log\dfrac{100}{100-25}}$ [$\because \lambda$ remains constant]

$= \dfrac{\log\dfrac{100}{25}}{\log\dfrac{100}{75}} = \dfrac{\log 4}{\log 4/3}$

$= \dfrac{\log 4}{\log 4 - \log 3}$

$= \dfrac{2 \times 0.3010}{2 \times 0.3010 - 0.4771}$

$= \dfrac{0.6020}{0.1249} = 4.81$

77. (c) $[\alpha] = \dfrac{[\alpha]_{observed}}{l \times C} = \dfrac{-1.2}{5 \times \dfrac{6.15}{1000}} = -39°$

78. (a) Let the concentration of potassium acetate is x. According to Henderson's equation,

$pH = pK_a + \log\dfrac{[salt]}{[acid]}$

$4.8 = -\log\,(1.8 \times 10^{-5}) + \log\dfrac{x \times 50}{20 \times 0.1M}$

$4.8 = 4.74 + \log 25x$

or $\log 25x = 0.06$

$25x = 1.148$

$\therefore x = 0.045\text{M}$

79. (b) By $'2A + \frac{C}{2} - B'$, we get

$Na_2O + SO_3 \longrightarrow Na_2SO_4$;

$\Delta H = -2 \times 146 + \frac{259}{2} - 418$

$\Rightarrow \Delta H = -580.5 \approx 581$ kJ

80. (a) According to Hardy Schulze rule, greater the valency of the coagulating ion, greater is its coagulating power. Thus, out of the given, $AlCl_3(Al^{3+})$ is most effective for causing coagulation of As_2S_3 sol.

PART - III (MATHEMATICS)

81. (b) Given, $f(x) = x^3 + 3x - 2$

$\Rightarrow f'(x) = 3x^2 + 3$

Put $f'(x) = 0 \Rightarrow 3x^2 + 3 = 0$

$\Rightarrow x^2 = -1$

$\therefore f(x)$ is either increasing or decreasing.

At $x = 2$, $f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3$, $f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12, 34]$.

82. (c) The total number of subsets of given set is $2^9 = 512$

Even numbers are $\{2, 4, 6, 8\}$.

Case I : When selecting only one even number $= {}^4C_1 = 4$

Case II : When selecting only two even numbers $= {}^4C_2 = 6$

Case III : When selecting only three even numbers $= {}^4C_3 = 4$

Case IV : When selecting only four even numbers $= {}^4C_4 = 1$

$\therefore$ Required number of ways

$= 512 - (4 + 6 + 4 + 1) - 1 = 496$

[Here, we subtract 1 for due to the null set]

83. (a) The required number of ways = The even number of 0's i.e., $\{0, 2, 4, 6, \ldots\}$

$$= \frac{n!}{n!} + \frac{n!}{2!(n-2)!} + \frac{n!}{4!(n-4)!}$$

$$= {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = 2^{n-1}$$

84. (a) $\left(1 + \frac{2x}{3}\right)^{3/2} (32 + 5x)^{-1/5}$

$$= \left[1 + \frac{3}{2}\left(\frac{2x}{3}\right)\right](32)^{-1/5}\left(1 + \frac{5}{32}x\right)^{-1/5}$$

(neglecting higher powers of x)

$$= [1 + x]2^{-1}\left[1 - \frac{1}{5}\left(\frac{5}{32}\right)x\right]$$

(neglecting higher powers of x)

$$= \frac{1}{2}(1 + x)\left(1 - \frac{x}{32}\right)$$

$$= \frac{(1 + x)(32 - x)}{64} = \frac{32 + 31x}{64}$$

(neglecting x^2 term)

85. (c) $(x - a)(x - a - 1) + (x - a - 1)(x - a - 2)$
$+ (x - a)(x - a - 2) = 0$

Let $x - a = t$, then

$t(t - 1) + (t - 1)(t - 2) + t(t - 2) = 0$

$\Rightarrow t^2 - t + t^2 - 3t + 2 + t^3 - 2t = 0$

$\Rightarrow 3t^2 - 6t + 2 = 0$

$$\Rightarrow t = \frac{6 \pm \sqrt{36 - 24}}{2(3)} = \frac{6 \pm 2\sqrt{3}}{2(3)}$$

$$\Rightarrow x - a = \frac{3 \pm \sqrt{3}}{3}$$

$$\Rightarrow x = a + \frac{3 \pm \sqrt{3}}{3}$$

Hence, x is real and distinct.

86. (b) $f(x) = x^2 + ax + b$ has imaginary roots.

$\therefore$ Discriminant, $D < 0 \Rightarrow a^2 - 4b < 0$

$f'(x) = 2x + a$

$\Rightarrow f''(x) = 2$

Also, $f(x) + f'(x) + f''(x) = 0$ ...(i)

$\Rightarrow x^2 + ax + b + 2x + a + 2 = 0$

$\Rightarrow x^2 + (a + 2)x + b + a + 2 = 0$

$$\therefore x = \frac{-(a + 2) \pm \sqrt{(a + 2)^2 - 4(a + b + 2)}}{2}$$

$$= \frac{-(a + 2) \pm \sqrt{a^2 - 4b - 4}}{2}$$

Since, $a^2-4b<0$

$\therefore a^2-4b-4<0$

Hence, Eq. (i) has imaginary roots.

87. (c) $f(x)=2x^4-13x^2+ax+b$ is divisible by $(x-2)(x-1)$.

$\therefore f(2)=2(2)^4-13(2)^2+a(2)+b=0$

$\Rightarrow 2a+b=20$ ...(i)

and $f(1)=2(1)^4-13(1)^2+a+b=0$

$\Rightarrow a+b=11$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a=9, b=2$

88. (d) Let a and R be the first term and common ratio of a GP respectively.

So, $T_p=aR^{p-1}=x$

$T_q=aR^{q-1}=y$ and $T_r=aR^{r-1}=z$

$\Rightarrow \log x=\log a+(p-1)\log R$

$\log y=\log a+(q-1)\log R$

and $\log z=\log a+(r-1)\log R$

$$\therefore \begin{vmatrix}\log x & p & 1\\ \log y & q & 1\\ \log z & r & 1\end{vmatrix}=\begin{vmatrix}\log a+(p-1)\log R & p & 1\\ \log a+(q-1)\log R & q & 1\\ \log a+(r-1)\log R & r & 1\end{vmatrix}$$

$$=\begin{vmatrix}\log a & p & 1\\ \log a & q & 1\\ \log a & r & 1\end{vmatrix}+\begin{vmatrix}(p-1)\log R & p & 1\\ (q-1)\log R & q & 1\\ (r-1)\log R & r & 1\end{vmatrix}$$

$$=\log a\begin{vmatrix}1 & p & 1\\ 1 & q & 1\\ 1 & r & 1\end{vmatrix}+\log R\begin{vmatrix}p-1 & p-1 & 1\\ q-1 & q-1 & 1\\ r-1 & r-1 & 1\end{vmatrix}$$

$(C_2\to C_2-C_3)$

$=0+0=0$ ($\because$ two columns are identical)

89. (c) Let $z=x+iy$

Given : $\left|\dfrac{z+2i}{2z+i}\right|<1$

$$\Rightarrow \frac{\sqrt{(x)^2+(y+2)^2}}{\sqrt{(2x)^2+(2y+1)^2}}<1$$

$\Rightarrow x^2+y^2+4+4y<4x^2+4y^2+1+4y$

$\Rightarrow 3x^2+3y^2>3\Rightarrow x^2+y^2>1$

90. (c) $(1+\sqrt{3}i)^n+(1-\sqrt{3}i)^n$

$$=\left[2\left(\frac{1+\sqrt{3}i}{2}\right)\right]^n+\left[2\left(\frac{1-\sqrt{3}i}{2}\right)\right]^n$$

$=(-2\omega^2)^n+(-2\omega)^n$

$=(-2)^n[(\omega^2)^{3r+1}+(\omega)^{3r+1}]$

($\because$ $n=3r+1$, where r is an integer)

$=(-2)^n(\omega^2+\omega)=-(-2)^n$

91. (d) Let $f(x)=\sin^4 x+\cos^4 x$

$=(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$=1-\dfrac{1}{4}\cdot 2(\sin 2x)^2$

$=1-\dfrac{1}{4}(1-\cos 4x)$ $=\dfrac{3}{4}+\dfrac{\cos 4x}{4}$

$\therefore$ Period of $f(x)=\dfrac{2\pi}{4}=\dfrac{\pi}{2}$

92. (c) $\sin^2 x-\cos 2x=2-\sin 2x$

$\Rightarrow 1-\cos^2 x-(2\cos^2-1)=2-2\sin x\cos x$

$\Rightarrow -3\cos^2 x+2\sin x\cos x=0$

$\Rightarrow \cos x(2\sin x-3\cos x)=0$

$\Rightarrow \cos x=0,\quad (\because 2\sin x-3\cos x\neq 0)$

$\Rightarrow x=2n\pi\pm\dfrac{\pi}{2}$

$\Rightarrow x=(4n\pm 1)\dfrac{\pi}{2},\ n\in Z$

93. (d) $\cos^{-1}\left(-\dfrac{1}{2}\right)-2\sin^{-1}\left(\dfrac{1}{2}\right)+3\cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$

$-4\tan^{-1}(-1)$

$=\pi-\cos^{-1}\left(\dfrac{1}{2}\right)-2\left(\dfrac{\pi}{6}\right)+3\left(\pi-\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)\right)$

$+4\tan^{-1}(1)$

$=\pi-\dfrac{\pi}{3}-\dfrac{\pi}{3}+3\left(\pi-\dfrac{\pi}{4}\right)+4\cdot\dfrac{\pi}{4}$

$=\dfrac{\pi}{3}+3\cdot\dfrac{3\pi}{4}+\pi=\dfrac{43\pi}{12}$

94. (c) We know that, $2s = a + b + c$

$$\therefore \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$

$$= \frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$$

$$= 4\frac{s(s-a)}{bc} \times \frac{(s-b)(s-c)}{bc}$$

$$= 4\cos^2\frac{A}{2} \times \sin^2\frac{A}{2} = \sin^2 A$$

95. (c) $l + m + n = 0, \Rightarrow l = -m - n$

and $l^2 + m^2 - n^2 = 0$

$$\therefore (-m-n)^2 + m^2 - n^2 = 0$$

$$\Rightarrow 2m^2 + 2mn = 0$$

$$\Rightarrow 2m(m+n) = 0$$

$$\Rightarrow m = 0 \text{ or } m + n = 0$$

If $m = 0$, then $l = -n$

$$\therefore \frac{l_1}{-1} = \frac{m_1}{0} = \frac{n}{1}$$

and if $m + n = 0 \Rightarrow m = -n$, then $l = 0$

$$\therefore \frac{l_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$$

ie., $(l_1, m_1, n_1) = (-1, 0, 1)$

and $(l_2, m_2, n_2) = (0, -1, 1)$

$$\therefore \cos\theta = \frac{0+0+1}{\sqrt{1+0+1}\sqrt{0+1+1}} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

96. (a) $m_1 = |\vec{a}_1| = \sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$$m_2 = |\vec{a}_2| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41}$$

$$m_3 = |\vec{a}_3| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

and $m_4 = |\vec{a}_4| = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11}$

$$\therefore m_3 < m_1 < m_4 < m_2$$

97. (a) Here, $n = 6$

According to the question

$$^6C_2 p^2 q^4 = 4 \cdot {}^6C_4 p^4 q^2$$

$$\Rightarrow q^2 = 4p^2 \Rightarrow (1-p)^2 = 4p^2$$

$$\Rightarrow 3p^2 + 2p - 1 = 0$$

$$\Rightarrow (p+1)(3p-1) = 0$$

$$\Rightarrow p = \frac{1}{3} \quad (\because p \text{ cannot be negative})$$

98. (d) Given lines are parallel.

$$d = \frac{15-5}{\sqrt{4^2+3^2}} = \frac{10}{5}$$

$\Rightarrow d = 2 =$ diameter of the circle

$\therefore$ Radius of circle = 1

$\therefore$ Area of circle = $\pi r^2 = \pi$ sq unit

99. (c) $x^2 - 2xy - xy + 2y^2 = 0$

$$\Rightarrow (x-2y)(x-y) = 0$$

$\Rightarrow x = 2y, x = y$ ...(i)

Also, $x + y + 1 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$A\left(-\frac{2}{3}, -\frac{1}{3}\right), B\left(-\frac{1}{2}, -\frac{1}{2}\right), C(0,0)$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} -\frac{2}{3} & -\frac{1}{3} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\right] = \frac{1}{2}\left[\frac{1}{6}\right] = \frac{1}{12}$$

100. (c) Given pair of lines are

$$x^2 - 3xy + 2y^2 = 0$$

and $x^2 - 3xy + 2y^2 + x - 2 = 0$

$$\therefore (x-2y)(x-y) = 0$$

and $(x - 2y + 2)(x - y - 1) = 0$

$$\Rightarrow x - 2y = 0, x - y = 0$$

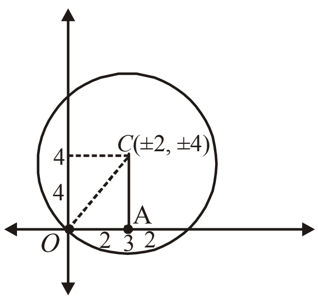
and $x - 2y + 2 = 0, x - y - 1 = 0$

The lines $x - 2y = 0, x - 2y + 2 = 0$ and $x - y = 0, x - y - 1 = 0$ are parallel.

Also, angle between $x - 2y = 0$ and $x - y = 0$ is not 90°.

$\therefore$ It is a parallelogram.

101. (a) In ΔOAC, $OC^2 = 2^2 + 4^2 = 20$



$\therefore$ Required equation of circle is

$(x \pm 2)^2 + (y \pm 4)^2 = 20$

$\Rightarrow x^2 + y^2 \pm 4x \pm 8y = 0$

102. (d) Given circles are

$x^2 + y^2 - 2x + 8y + 13 = 0$

and $x^2 + y^2 - 4x + 6y + 11 = 0$

Here $C_1 = (1, -4)$, $C_2 = (2, -3)$

$\Rightarrow r_1 = \sqrt{1 + 16 - 13} = 2$

and $r_2 = \sqrt{4 + 9 - 11} = \sqrt{2}$

So, $d = C_1C_2 = \sqrt{(2-1)^2 + (-3+4)^2} = \sqrt{2}$

$\therefore \cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2} = \frac{2-4-2}{2 \times 2 \times \sqrt{2}} = -\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 135°$

103. (b) Let the required equation of circle be

$x^2 + y^2 + 2gx + 2fy = 0$

The above circle cuts the given circles orthogonally.

$\therefore 2(-3g) + 2f(0) = 8 \Rightarrow 2g = -\frac{8}{3}$

and $-2g - 2f = -7$

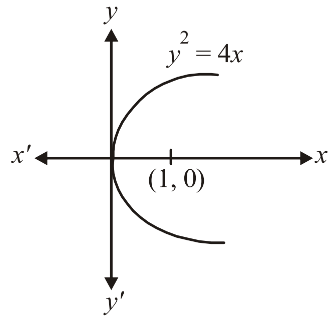
$\Rightarrow 2f = +7 + \frac{8}{3} = \frac{29}{3}$

$\therefore$ Required equation of circle is

$x^2 + y^2 - \frac{8}{3}x + \frac{29}{3}y = 0$

or $3x^2 + 3y^2 - 8x + 29y = 0$

104. (b) Given curve is $y^2 = 4x$.



Also, point (1, 0) is the focus of the parabola.
It is clear from the graph that only one normal is possible.

105. (a) $x^2y^2 = c^4$

$\Rightarrow y^2(a^2 - y^2) = c^4$

$\Rightarrow y^4 - a^2y^2 + c^4 = 0$

Let y_1, y_2, y_3 and y_4 are the roots.

$\therefore y_1 + y_2 + y_3 + y_4 = 0$

106. (b) Solving $4x - 3y = 5$ and $2x^2 - 3y^2 = 12$

$\therefore 2\left(\frac{5+3y}{4}\right)^2 - 3y^2 = 12$

$\Rightarrow \frac{(25 + 9y^2 + 30y)}{8} - 3y^2 = 12$

$\Rightarrow 15y^2 - 30y + 71 = 0$

$\Rightarrow y = \frac{30 \pm \sqrt{900 - 4260}}{30} = 1 \pm \frac{\sqrt{-3360}}{30}$

Also, $2x^2 - 3\left(\frac{4x-5}{3}\right)^2 = 12$

$\Rightarrow 10x^2 - 40x + 61 = 0$

$\Rightarrow x = \frac{40 \pm \sqrt{1600 - 4 \times 10 \times 61}}{2 \times 10}$

$= \frac{40 \pm \sqrt{-840}}{20} = 2 \pm \frac{\sqrt{-840}}{20}$

$\therefore$ Points are $A\left(2 + \frac{\sqrt{-840}}{20}, 1 + \frac{\sqrt{-3360}}{30}\right)$

and $B\left(2 - \frac{\sqrt{-840}}{20}, 1 - \frac{\sqrt{-3360}}{30}\right)$.

$\therefore$ Mid point of AB is (2, 1).

107. (d) Let $A=(1,0,0), B=(0,1,0)$ and $C=(0,0,1)$

So, $AB=\sqrt{(0-1)^2+(1-0)^2+0^2}=\sqrt{2}$

$BC=\sqrt{0^2+(0-1)^2+(1-0)^2}=\sqrt{2}$

and $CA=\sqrt{(1-0)^2+0^2+(0-1)^2}=\sqrt{2}$

$\therefore$ Perimeter of triangle

$=AB+BC+CA=\sqrt{2}+\sqrt{2}+\sqrt{2}=3\sqrt{2}$

108. (c) $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$

$+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=(\cos^2\alpha-\sin^2\alpha)+(\cos^2\beta-\sin^2\beta)$

$(\cos^2\gamma-\sin^2\gamma)+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

109. (a) Given equation of sphere is

$x^2+y^2+z^2-12x-4y-3z=0$

Centre of sphere is $\left(6,\ 2,\ \frac{3}{2}\right)$.

$\therefore$ Radius of sphere $=\sqrt{(6)^2+(2)^2+\left(\frac{3}{2}\right)^2}$

$=\sqrt{36+4+\frac{9}{4}}=\sqrt{\frac{169}{4}}=\frac{13}{2}$

110. (c) $\lim\limits_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x+3}=\lim\limits_{x\to\infty}\left(1+\frac{3}{x+2}\right)^{x+3}$

$=\lim\limits_{x\to\infty}\left[\left(1+\frac{3}{x+2}\right)^{\frac{x+2}{3}}\right]^{\frac{3(x+3)}{x+2}}$

$=e^{\lim\limits_{x\to\infty}3\left(\frac{1+\frac{3}{x}}{1+\frac{2}{x}}\right)}=e^3$

111. (d) $f(x)=\begin{cases}\dfrac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0\\ a, & \text{if } x=0\end{cases}$

$\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\frac{2\sin x-\sin 2x}{2x\cos x}\left(\frac{0}{0}\text{ form}\right)$

$=\lim\limits_{x\to 0}\frac{2\cos x-2\cos 2x}{2(\cos x-x\sin x)}$

$=\lim\limits_{x\to 0}\frac{2-2}{2(1-0)}=0$

Since, $f(x)$ is contineous at $x=0$

$\therefore f(0)=\lim\limits_{x\to 0}f(x)\Rightarrow a=0$

112. (c) We have, $x=\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and $y=\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow x=\tan^{-1}t$ and $y=\tan^{-1}t$

$\therefore y=x\Rightarrow\frac{dy}{dx}=1$

113. (b) $\frac{d}{dx}\left[a\tan^{-1}x+b\log\left(\frac{x-1}{x+1}\right)\right]=\frac{1}{x^4-1}$

On integrating both sides, we get

$a\tan^{-1}x+b\log\left(\frac{x-1}{x+1}\right)$

$=\frac{1}{2}\int\left[\frac{1}{x^2-1}-\frac{1}{x^2+1}\right]dx$

$\Rightarrow a\tan^{-1}x+b\log\left(\frac{x-1}{x+1}\right)$

$=\frac{1}{4}\log\left(\frac{x-1}{x+1}\right)-\frac{1}{2}\tan^{-1}x$

$\Rightarrow a=-\frac{1}{2},\ b=\frac{1}{4}$

$\therefore a-2b=-\frac{1}{2}-2\left(\frac{1}{4}\right)=-1$

114. (c) $y=e^{a\sin^{-1}x}$

On differentiating w.r.t. x, we get

$y_1=e^{a\sin^{-1}x}a\cdot\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow y_1\sqrt{1-x^2}=ay$

$\Rightarrow(1-x^2)y_1^2=a^2y^2$

Again differentiating w.r.t. x, we get

$(1-x^2)2y_1y_2-2xy_1^2=a^2 2yy_1$

$\Rightarrow(1-x^2)y_2-xy_1-a^2y=0$

Using Leibnitz's rule,

$$(1-x^2)y_{n+2}+{}^nC_1y_{n+1}(-2x)+{}^nC_2y_n(-2)$$
$$-xy_{n+1}-{}^nC_1y_n-a^2y_n=0$$
$$\Rightarrow (1-x^2)y_{n+2}+xy_{n+1}(-2n-1)$$
$$+y_n[-n(n-1)-n-a^2]=0$$
$$\Rightarrow (1-x^2)y_{n+2}-(2n+1)xy_{n+1}=(n^2+a^2)y_n$$

115. (c) Given, $f(x)=x^3+ax^2+bx+c,\ a^2\le 3b$

$$\Rightarrow f'(x)=3x^2+2ax+b$$

Put $f'(x)=0$

$$\Rightarrow 3x^2+2ax+b=0$$
$$\Rightarrow x=\frac{-2a\pm\sqrt{4a^2-12b}}{2\times 3}$$
$$=\frac{-2a\pm 2\sqrt{a^2-3b}}{3}$$

Since, $a^2\le 3b$,
$\therefore$ x has an imaginary value.
Hence, no extreme value of x exists.

116. (a) Let $I=\int\left(\frac{2-\sin 2x}{1-\cos 2x}\right)e^x\,dx$

$$=\int\left(\frac{2-2\sin x\cos x}{2\sin^2 x}\right)e^x\,dx$$
$$=\int \operatorname{cosec}^2 xe^x\,dx-\int \cot x\,e^x dx$$
$$=-\cot xe^x-\int(-\cot x)e^x\,dx$$
$$-\int \cot x\,e^x\,dx+c$$
$$=-\cot xe^x+c$$

117. (c) We know that, if

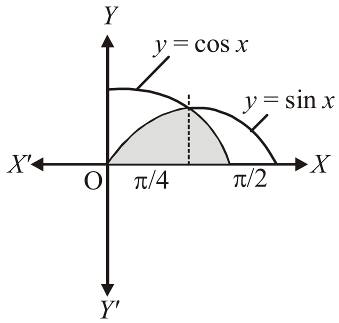
$I_n=\int \sin^n x\,dx$, then

$$I_n=-\frac{\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}I_{n-2}$$

where n is a positive integer.

$$\Rightarrow nI_n-(n-1)I_{n-2}=-\sin^{n-1}x\cos x$$

118. (d)



Area, $A_1=\int_0^{\pi/4}\sin x\,dx$

$$=-[\cos x]_0^{\pi/4}=1-\frac{1}{\sqrt{2}}$$
$$=\frac{\sqrt{2}-1}{\sqrt{2}}$$

and area, $A_2=\int_{\pi/4}^{\pi/2}\cos x\,dx$

$$=[\sin x]_{\pi/4}^{\pi/2}=\left[1-\frac{1}{\sqrt{2}}\right]=\frac{\sqrt{2}-1}{\sqrt{2}}$$
$$\therefore A_1:A_2=\frac{\sqrt{2}-1}{\sqrt{2}}:\frac{\sqrt{2}-1}{\sqrt{2}}=1:1$$

119. (b) $\frac{dy}{dx}=\sin(x+y)\tan(x+y)-1$

Put $x+y=z$

$$\Rightarrow 1+\frac{dy}{dx}=\frac{dz}{dx}$$
$$\Rightarrow \frac{dz}{dx}-1=\sin z\tan z-1$$
$$\Rightarrow \int\frac{\cos z}{\sin^2 z}dz=\int dx$$

Putting $\sin z=t\Rightarrow \cos z\,dz=dt$, we have

$$\int\frac{1}{t^2}dt=x-c\quad\Rightarrow -\frac{1}{t}=x-c$$
$$\Rightarrow -\operatorname{cosec} z=x-c\Rightarrow x+\operatorname{cosec}(x+y)=c$$

120. (c) $p\Rightarrow(\sim p\vee q)$ is false means p is true and $\sim p\vee q$ is false.

$\Rightarrow$ p is true and both $\sim p$ and q are false.

$\Rightarrow$ p is true and q is false.