MET 2019 Question Paper with Solution

Manipal Entrance Test (MET)

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MANIPAL
## Engineering Entrance Exam

Solved Paper 2019

PHYSICS

1. Three blocks of masses 2 kg, 3 kg and 5 kg are connected to each other with light string and are then placed on a frictionless surface as shown in the figure. The system is pulled by a force $F = 10$ N, then tension T_1 is

   (a) 1 N   (b) 5 N
   (c) 8 N   (d) 10 N

2. Two masses m_1 and m_2 are attached to a string which passes over a frictionless smooth pulley. When $m_1 = 10$ kg, $m_2 = 6$ kg, the acceleration of masses is
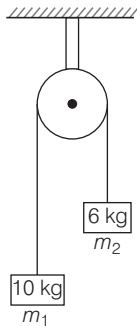
   (a) 20 m/s^2 (b) 5 m/s^2
   (c) 2.5 m/s^2 (d) 10 m/s^2

3. The unit of L/R is (where L = inductance and R = resistance)
   (a) sec   (b) sec^{-1}
   (c) volt   (d) ampere

4. If force is proportional to square of velocity, then the dimensions of proportionality constant is
   (a) $[ML^{-1} T]$ (b) $[ML^{-1} T^0]$
   (c) $[ML\ T^0]$ (d) $[M^0LT^{-1}]$

5. Minimum energy required to take out the only one electron from ground state of He$^+$ is
   (a) 13.6 eV   (b) 54.4 eV
   (c) 27.2 eV   (d) 6.8 eV

6. Photons of 5.5 eV energy fall on the surface of the metal emitting photoelectrons of maximum kinetic energy 4.0 eV. The stopping voltage required for these electrons is
   (a) 5.5 V
   (b) 1.5 V
   (c) 9.5 V
   (d) 4.0 V
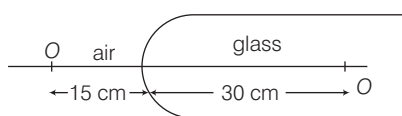
7. Which is different from others in units?
   (a) Phase difference
   (b) Mechanical equivalent
   (c) Loudness of sound
   (d) Poisson's ratio

8. Continuous emission spectrum is produced by
   (a) in candescent electric lamp
   (b) mercury vapour lamp
   (c) sodium vapour lamp
   (d) polyatomic substances

9. A ball is dropped from top of a tower of 100 m height. Simultaneously another ball was thrown upward from bottom of the tower with a speed of 50 m/s. They will cross each other after ($g = 10 \, \text{m/s}^2$)
   (a) 1 s   (b) 2 s   (c) 3 s   (d) 4 s

10. The driver of a car moving towards a rocket launching pad with a speed of 6 m/s observes that the rocket is moving with speed of 10 m/s. The upward speed of the rocket as seen by the stationary observer is
    (a) 4 m/s   (b) 6 m/s   (c) 8 m/s   (d) 11 m/s

11. A satellite revolves very near to the earth surface. Its speed should be around
    (a) 5 km/s   (b) 8 km/s
    (c) 2 km/s   (d) 11 km/s

12. If the density of earth is doubled keeping its radius constant, then acceleration due to gravity g is
    (a) $20 \, \text{m/s}^2$ (b) $10 \, \text{m/s}^2$
    (c) $5 \, \text{m/s}^2$ (d) $2.5 \, \text{m/s}^2$

13. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical ?
    (a) 30°   (b) 45°
    (c) 60°   (d) 90°

14. A body of mass 8 kg is moved by a force F = 3 x N, where x is the distance covered. Initial position is x = 2 m and the final position is x = 10 m. The initial speed is zero. The final speed is
    (a) 6 m/s   (b) 12 m/s
    (c) 18 m/s   (d) 14 m/s

15. Find the equivalent resistance across AB
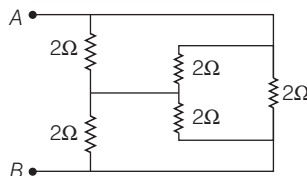
   (a) 1Ω   (b) 2Ω   (c) 3Ω   (d) 4Ω

16. The nuclear reaction $_1\text{H}^1 + {}_1\text{H}^1 \rightarrow {}_2\text{He}^4$ (mass of deuteron = 2.0141 amu and of He = 4.0024 amu) is
    (a) fusion reaction releasing 24 MeV energy
    (b) fusion reaction absorbing 24 MeV energy
    (c) fission reaction releasing 0.0258 MeV energy
    (d) fission reaction absorbing 0.0258 MeV energy

17. A thin metal plate P is inserted half way between the plates of a parallel plate capacitor of capacitance C in such a way that it is parallel to the two plates. The capacitance now becomes
    (a) C   (b) $\dfrac{C}{2}$
    (c) 4C   (d) None of these

18. An inclined plane makes an angle 30° with horizontal. A solid sphere rolling down this inclined plane has a linear acceleration of
    (a) $\dfrac{5g}{14}$ (b) $\dfrac{2g}{3}$
    (c) $\dfrac{g}{3}$ (d) $\dfrac{5g}{7}$

19. A bullet of mass 10 g is fired from a gun of mass 1 kg. If the recoil velocity is 5 m/s, the velocity of the muzzle is
    (a) 0.05 m/s   (b) 5 m/s
    (c) 50 m/s   (d) 500 m/s

20. A particle moves with constant speed v along a circular path of radius r and completes the circle in time T. The acceleration of the particle is
    (a) $\dfrac{2\pi v}{T}$ (b) $\dfrac{2\pi r}{T}$
   (c) $\dfrac{2\pi r^2}{T}$ (d) $\dfrac{2\pi v^2}{T}$

21. The separation between C and O-atoms in CO is 1.2 Å. The distance of carbon atom from the centre of mass is
(a) 0.3 Å (b) 0.7 Å (c) 0.5 Å (d) 0.9 Å

22. A body moves a distance of 10 m under the action of force F = 10 N. If the work done is 25 J, the angle which the force makes with the direction of motion is
(a) 0°   (b) 30°
(c) 60°   (d) None of these

23. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by
(a) 100 J (b) 200 J (c) 300 J (d) 400 J

24. Two wires A and B are of same materials. Their lengths are in the ratio 1 : 2 and diameters are in the ratio 2 : 1. When stretched by force F_A and F_B, respectively, they get equal increase in their lengths. Then the ratio $\dfrac{F_A}{F_B}$ should be

(a) 1 : 2   (b) 1 : 1   (c) 2 : 1   (d) 8 : 1

25. Mixed He^+ and O^{2+} ions (mass of $He^+ = 4$ amu and that of $O^{2+} = 16$ amu) beam passes a region of constant perpendicular magnetic field. If kinetic energy of all the ions is same, then
(a) He^+ ions will be deflected more than those of O^{2+}
(b) He^+ ions will be deflected less than those of O^{2+}
(c) all the ions will be deflected equally
(d) no ions will be deflected

26. In Young's double slit experiment the wavelength of light was changed from 7000 Å to 3500 Å. While doubling the separation between the slits which of the following is not true for this experiment?
(a) The width of the fringes changes
(b) The colour of bright fringes changes
(c) The separation between successive bright fringes changes
(d) The separation between successive dark fringes remains unchanged

27. The coherence of two light sources means that the light waves emitted have
(a) same frequency
(b) same intensity
(c) constant phase difference
(d) same velocity

28. The valence band and conduction band of a solid overlap at low temperature, the solid may be
(a) a metal
(b) a semiconductor
(c) an insulator
(d) None of the above

29. The dominant contribution to current comes from holes in case of
(a) metals
(b) intrinsic semiconductors
(c) p-type extrinsic semiconductors
(d) n-type extrinsic semiconductors

30. A laser device produces amplification in the
(a) microwave region
(b) ultraviolet or visible region
(c) infrared region
(d) None of the above

31. A point object O is placed in front of a glass rod having spherical end of radius of curvature 30 cm. The image would be formed at

(a) 30 cm left
(b) infinity
(c) 1 cm to the right
(d) 18 cm to the left

32. In the formation of a rainbow light from the sun on water droplets undergoes
(a) dispersion only
(b) only total internal reflection
(c) dispersion and total internal reflection
(d) None of the above

33. If no external voltage is applied across p-n junction, there would be
(a) no electric field across the junction
(b) an electric field pointing from n-type to p-type side across the junction
(c) an electric field pointing from p-type to n-type side across the junction
(d) a temporary electric field during formation of p-n junction that would subsequently disappear

34. Light travelling from a transparent medium to air undergoes total internal reflection at an angle of incidence of 45°. Then refractive index of the medium may be
(a) 1.5   (b) 1.3
(c) 1.1   (d) $1/\sqrt{2}$

**35.** A force F acting on a body depends on its displacement S as $F \propto S^{-1/3}$. The power delivered by F will depend on displacement as
(a) $S^{2/3}$ (b) $S^{-5/3}$
(c) $S^{1/2}$ (d) S^0

36. Binding energy per nucleon plot against the mass number for stable nuclei is shown in the figure. Which curve is correct?
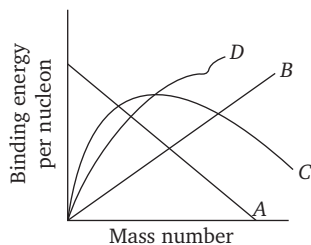
(a) A   (b) B
(c) C   (d) D

37. A light ray from air is incident (as shown in figure) at one end of a glass fiber (refractive index, $\mu = 1.5$) making an incidence angle of 60° on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fiber of length 1 km ?
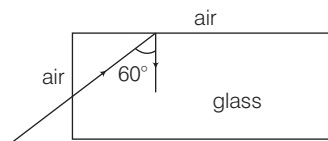
(a) 3.33 µs   (b) 6.67 µs
(c) 5.77 µs   (d) 3.85 µs

38. A thermodynamic system is taken through the cycle PQRSP process. The net work done by the system is
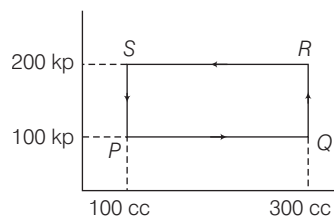
(a) 20 J   (b) – 20 J   (c) 400 J   (d) – 374 J

39. Consider four circuits shown in the figure below. In which circuit power dissipated is greatest?

(Neglect the internal resistance of the power supply).

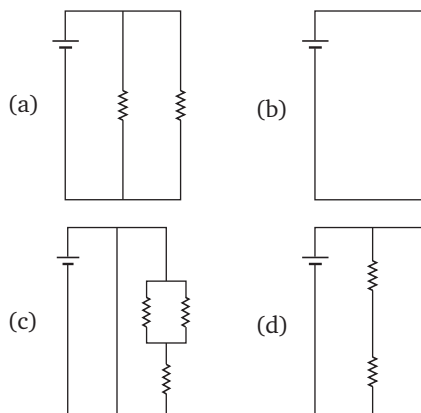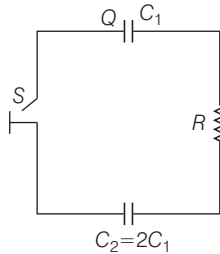40. Two capacitors C_1 and $C_2 = 2C_1$ are connected in a circuit with a switch between them as shown in the figure. Initially the switch is open and C_1 holds charge Q. The switch is closed. At steady state, the charge on each capacitors will be

(a) Q, 2Q

(b) $\dfrac{Q}{3}, \dfrac{2Q}{3}$

(c) $\dfrac{3Q}{2}, 3Q$

(d) $\dfrac{2Q}{3}, \dfrac{4Q}{3}$

41. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest positions) are T_1 and T_2, respectively. Then
(a) $T_1 = T_2$
(b) $T_2 > T_1$
(c) $T_1 > T_2$
(d) tension in the string always remains the same

42. A coil of 100 turns carries a current of 5 mA and creates a magnetic flux of 10^{-5} wb. The inductance is
(a) 0.2 mH   (b) 2.0 mH
(c) 0.02 mH   (d) None of these

43. The starter motor of a car draw a current I= 300 A from the battery of voltage 12 V. If the car starts only after 2 minutes, what is the energy drawn from the battery ?
(a) 3 kJ   (b) 30 kJ
(c) 7.2 kJ   (d) 432 kJ

44. Surface of the lake is at 2°C and depth of the lake is 20 m. Find the temperature of the bottom of the lake
(a) 2°C   (b) 3°C
(c) 4°C   (d) None of these

45. $y_1 = 4\sin(\omega t + kx)$, $y_2 = -4\cos(\omega t + kx)$, the phase difference is
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{3\pi}{2}$ (c) π (d) zero

46. A charged particle of mass 0.003 g is held stationary in space by placing it in a downward direction of electric field of 6×10^4 N/C. Then the magnitude of the charge is
(a) 5×10^{-4} C   (b) 5×10^{-10} C
(c) -18×10^{-6} C   (d) -5×10^{-9} C

47. A parallel plate capacitor has an electric field of 10^5 V / m between the plates. If the charge on the capacitor plate is 1 µC, the force on each capacitor plate is
(a) 0.5 N   (b) 0.05 N
(c) 0.005 N   (d) None of these

48. 1 g of water at atmospheric pressure has volume of 1 cc and when boiled it becomes 1681 cc of steam. The heat of vaporisation of water is 540 cal/g. Then the change in its internal energy in this process is
(a) 540 cal
(b) 500 cal
(c) 1681 cal
(d) None of the above

49. A physicist works in a laboratory where the magnetic field is 2 T. She wears a necklace enclosing area 0.01 m^2 in such a way that the plane of the necklace is normal to the field and is having a resistance, R = 0.01 Ω. Because of power failure, the field decays to 1 T in time 10^{-3} s. Then what is the total heat produced in her necklace?
(a) 10 J   (b) 20 J
(c) 30 J   (d) 40 J

50. The temperature coefficient of resistance of a wire is $0.00125°\,\mathrm{C}^{-1}$. At 300 K, its resistance is 1Ω. At what temperature the resistance of the wire will be 2Ω?
(a) 800 K
(b) 1100 K
(c) 600 K
(d) None of the above

CHEMISTRY

1. The solubility CaF_2 is s moles/litre. Then solubility product is
   (a) s^2 (b) $4s^3$ (c) $3s^2$ (d) s^3

2. If p, T, ρ and R represents pressure, temperature, density and universal gas constant respectively, then the molar mass of the ideal gas is given by
   (a) $\dfrac{\rho RT}{p}$ (b) $\dfrac{\rho T}{pR}$ (c) $\dfrac{p}{\rho RT}$ (d) $\dfrac{RT}{\rho p}$

3. The kinetic energy of a gas molecule is ..... temperature
   (a) independent of
   (b) directly proportional to
   (c) inversely proportional to
   (d) directly proportional to square root of

4. The ratio of rate of diffusion of $SO_2 (M = 64)$ and oxygen (M = 32) is
   (a) $1 : 1$ (b) $2 : 1$
   (c) $1 : 2$ (d) $1 : 1.414$

5. An aqueous solution freezes at $- 0.186°$ C, then elevation in boiling point is
   ($K_b = 0.512, K_f = 1.86$)
   (a) $0.0512°$ C   (b) $100.0512°$ C
   (c) $- 0.0512°$ C   (d) None of these

6. Arrange the following in the increasing order of their bond order
   $$O_2, O_2^+, O_2^- \text{ and } O_2^{2-}$$
   (a) $O_2^{2-}, O_2^-, O_2, O_2^+$ (b) $O_2^{2-}, O_2^-, O_2^+, O_2$
   (c) $O_2^+, O_2, O_2^-, O_2^{2-}$ (d) $O_2, O_2^+, O_2^-, O_2^{2-}$

7. In hydrolysis of a salt of weak acid and strong base, $A^- + H_2O \rightleftharpoons HA + OH^-$, the hydrolysis constant (K_h) is equal to
   (a) $\dfrac{K_w}{K_a}$ (b) $\dfrac{K_w}{K_b}$ (c) $\sqrt{\dfrac{K_a}{C}}$ (d) $\dfrac{K_w}{K_a \times K_b}$

8. For a reaction of the type $aA + bB \longrightarrow$ Products, the $- \dfrac{d[A]}{dt}$ is equal to
   (a) $- \dfrac{d[B]}{dt}$ (b) $- \dfrac{1}{b} \times \dfrac{d[B]}{dt}$
   (c) $- \dfrac{a}{b} \times \dfrac{d[B]}{dt}$ (d) $- \dfrac{b}{a} \times \dfrac{d[B]}{dt}$

9. In a mixture of 1 g H_2 and 8 g O_2 the mole fraction of hydrogen is
   (a) 0.667
   (b) 0.5
   (c) 0.33
   (d) None of the above

10. In acid medium MnO_4^- is reduced to Mn^{2+}, by a reducing agent. Then the equivalent mass of $KMnO_4$ is given by
    (M = molecular mass)
    (a) M/2   (b) M
    (c) M/5   (d) M/3

11. For the reaction
    $$CH_3COOH_{(1)} + 2O_{2(g)} \rightleftharpoons 2CO_{2(g)} + 2H_2O_{(1)}$$
    at 25°C and 1 atm pressure, $\Delta H = - 874$ kJ. Then the change in internal energy (ΔE) is
    (a) $- 874$ kJ   (b) $- 871.53$ kJ
    (c) $- 876.47$ kJ   (d) $+ 874$ kJ

12. The electrophile involved in the sulphonation of benzene is
    (a) SO_3^+ (b) SO_3^{2-}
    (c) H_3^+O (d) SO_3

13. According to the adsorption theory of catalysis, the speed of reaction increases because
    (a) adsorption lowers the activation enegry of the reaction
    (b) the concentration of reactant molecules at the active centres of the catalyst becomes high due to adsorption
    (c) in the process of adsorption, the activation energy of the molecules becomes large
    (d) adsorption produces heat which increases the speed of the reaction

14. The product of reaction between alcoholic silver nitrite with etyle bromide is
    (a) ethene
    (b) ethane
    (c) ethyle nitrile
    (d) nitro ethane

15. The specific conductivity of 0.1 N KCl solution is 0.0129 $ohm^{-1}cm^{-1}$. The resistance of the solution in the cell is 100 ohm. The cell constant of the cell will be
    (a) 1.10   (b) 1.29
    (c) 0.56   (d) 2.80

16. The standard emf of a cell $Zn / Zn^{2+}||$ Fe^{2+}/Fe, if electrode potentials for (Zn/Zn^{2+}) and (Fe^{2+}/Fe) and 0.763 V and -0.44 V respectively is
    (a) $+0.323\,V$ (b) $-1.203\,V$
    (c) $+1.203\,V$ (d) $-0.323\,V$

17. Which of the following has highest second ionisation energy
    (a) Chromium
    (b) Calcium
    (c) Iron
    (d) Cobalt

18. Which of the following plot represents the graph of pH against volume of alkali added in the titration of NaOH and HCl?

    (a) pH vs Volume
    (b) pH vs Volume
    (c) pH vs Volume
    (d) pH vs Volume
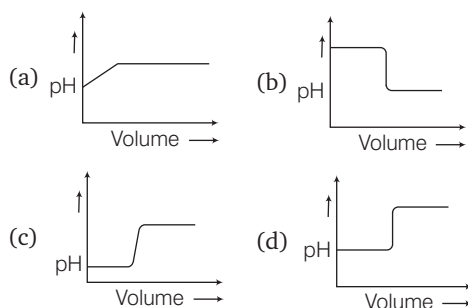
19. CO_2 is a gas, while SiO_2 is a solid, but both are
    (a) acidic
    (b) ionic
    (c) discrete molecules
    (d) covalent containing π-bonds

20. Pure conc. HNO_3 make iron passive, as the surface is covered with protective layer of
    (a) Fe_2O_3
    (b) FeO
    (c) Fe_3O_4
    (d) $Fe(NO_3)_3$

21. Ozone when reacts with potassium iodide solution liberates certain product, which turns starch paper blue. The liberated substance is
    (a) oxygen
    (b) iodine
    (c) hydrogen iodide
    (d) potassium hydroxide

22. Red hot iron absorbs SO_2 giving the product
    (a) $FeS + O_2$ (b) $Fe_2O_3 + FeS$
    (c) $FeO + FeS$ (d) $FeO + S$

23. Ethyl iodide when heated with sodium in dry ether gives pure
    (a) C_4H_{10} (b) C_2H_6
    (c) C_3H_8 (d) C_2H_5OH

24. For the reaction,
    $$CH_3-CH=CH_2 + HOCl \longrightarrow A$$
    The product A is :
    (a) $CH_3-CHCl-CH_2OH$
    (b) $CH_3-\underset{\underset{OH}{|}}{CH}-CH_2-Cl$
    (c) $CH_3-\underset{\underset{Cl}{|}}{CH_2}-CH_2-COCl$
    (d) $CH_3-\underset{\underset{OH}{|}}{\overset{\overset{Cl}{|}}{C}}-CH_3$

25. Which of the following is not correct for ionic crystals?
    (a) They possess high melting point and boiling point
    (b) All are electrolyte
    (c) Exhibit the property of isomorphism
    (d) Exhibit directional properties of the bond

26. Which of the following is not true in Rutherford's nuclear' model of atom?
    (a) Protons and neutrons are present inside nucleus
    (b) Volume of nucleus is very small as compared to volume of atom
    (c) The number of protons and neutrons are always equal
    (d) The number of electrons and protons are always equal

27. All the s-block elements of the periodic table are placed in the groups .....
    (a) IA and IIA
    (b) IIIA and IVA
    (c) B sub groups
    (d) VA to VIIA

28. The magnetic quantum number for d-orbital is given by
    (a) 2
    (b) $0, \pm 1, \pm 2$
    (c) 0, 1, 2
    (d) 5

29. XeF_2 is isostructural with
    (a) ICL_2
    (b) $SbCl_3$
    (c) $BaCl_2$
    (d) TeF_2

30. In the process, $O_2^+ \longrightarrow O_2^{2+} + e^-$ the electron lost is from
    (a) bonding π-orbital
    (b) antibonding π-orbital
    (c) $2p_z$ orbital
    (d) $2p_x$ orbital

31. Bond between A and B can be represented by
    $$A—B, \ A^+B^-, \ A^-B^+$$
    $$\ (I) \quad (II) \quad (III)$$
    If A is more electronegative than B, then least contribution to the actual structure comes from
    (a) I
    (b) II
    (c) III
    (d) All the structures have equal contribution

32. In the following reaction
    $NaOH + S \rightarrow A + Na_2S + H_2O$; A is
    (a) Na_2SO_4
    (b) Na_2SO_3
    (c) Na_2S
    (d) $Na_2S_2O_3$

33. SiF_4 gets hydrolysed giving
    (a) SiO_2
    (b) $Si(OH)_2F_2$
    (c) H_2SiF_6
    (d) $Si(OH)_4$

34. All ores are minerals, while all minerals are not ores, because
    (a) the metal cannot be extracted economically from all the minerals
    (b) minerals are complex compounds
    (c) the minerals are obtained from mines
    (d) All of these are correct

35. In the reaction,
    $P_2O_5 + 3CaO \longrightarrow Ca_3(PO_4)_2$, P_2O_5 acts as
    (a) acidic flux
    (b) basic flux
    (c) basic impurity
    (d) acidic impurity

36. In the given reaction, the oxide of sodium is
    $$\begin{pmatrix} 4Na + O_2 \longrightarrow 2Na_2O \\ Na_2O + H_2O \longrightarrow 2NaOH \end{pmatrix}$$
    (a) acidic
    (b) basic
    (c) amphoteric
    (d) neutral

37. When CO_2 is passed through solution of calcium hydroxide, which one of the following compound is precipitated?
    (a) $Ca(HCO_3)_2$
    (b) CaO
    (c) $CaCO_3$
    (d) $Ca(OH)_2$

38. Ferric alum has the composition
    $(NH_4)_2SO_4 \cdot Fe_2(SO_4)_3 \cdot xH_2O$
    The value of x is
    (a) 7
    (b) 24
    (c) 6
    (d) 15

39. What is the general electronic configuration for 2nd row transition series?
    (a) $[Ne]3d^{1-10}, 4s^2$
    (b) $[Ar]3d^{1-10}, 4s^{1-2}$
    (c) $[Kr]4d^{1-10}, 5s^{1-2}$
    (d) $[Xe]5d^{1-10}, 5s^{1-2}$

40. The existence of two different coloured complexes of $[Co(NH_3)_4Cl_2]^+$ is due to
    (a) ionisation isomerism
    (b) co-ordination isomerism
    (c) linkage isomerism
    (d) geometrical isomerism

41. IUPAC name of the compound
    $$CH_3—\underset{\underset{OH}{|}}{C}H—CH_2—\underset{\underset{CH_3}{|}}{C}H—CH_3 \ \text{is}$$
    (a) 4-methyl pentene-2-ol
    (b) 2-methyl pentanol-4
    (c) 4, 4-dimethyl-butane-2-ol
    (d) 4-methyl pentane-2-ol

42. Alkyl halide on heating with alc.NH_3 in a sealed tube results .....
    (a) $1°$ amine
    (b) $2°$ amine
    (c) $3°$ amine
    (d) all of these

43. Among H—CHO, CH_3CHO and C_6H_5CHO, which will undergo Cannizaro's reaction?
   (a) HCHO and CH_3—CHO
   (b) CH_3—CHO and C_6H_5CHO
   (c) C_6H_5CHO and HCHO
   (d) All of the above

44. The main product of the reaction of CH_3CONH_2 with Br_2 in aqueous potassium hydroxide medium is
   (a) CH_3—CH_2—NH_2 (b) CH_3Br
   (c) $CH_3CONHBr$ (d) CH_3NH_2

45. In the reaction,
   $$HCHO + CH_3MgI \longrightarrow A \xrightarrow{H_2O} B + Mg(OH)I$$
   What are A and B?
   (a) CH_3OMgI and CH_3—OH
   (b) CH_3CH_2OMgI and C_2H_5—O—C_2H_5
   (c) CH_3CH_2OMgI and CH_3—CH_2—OH
   (d) CH_3—CH_2—I and CH_3—CH_2—OH

46. Acetylation of a secondary amine in alkaline medium yields
   (a) N, N-dialkyl acetamide
   (b) N, N-dialkyl amine
   (c) N, N-dialkyl amide
   (d) acetyl dialkyl amine

47. In acid medium nitrobenzene is reduced to aniline as shown in the reaction
   $$C_6H_5—NO_2 + 6[H] \longrightarrow C_6H_5—NH_2 + 2H_2O$$
   The reducing agent used in this reaction is :
   (a) $LiAlH_4$ (b) Sn / HCl
   (c) Na/Alcohol   (d) H_2/Ni

48. Acetyl salicylic acid is used as
   (a) anti oxidant   (b) analgesic drug
   (c) anti biotic drug   (d) anaesthetic

49. C_6H_6 consists of one ring, while naphthlene consists of two rings. Both of them are aromatic and obey the (4n + 2) rule. Thus the number of π-electrons inside rings of C_6H_6 and naphthalene are respectively
   (a) 3, 5
   (b) 5, 10
   (c) 6, 10
   (d) 6, 12

50. In the oxidation of C_6H_5—CH_2—CH_3 by $KMnO_4$ the product formed is
   (a) C_6H_5—CH_2—CHO
   (b) C_6H_5—CH_2—COOH
   (c) C_6H_6—COOH
   (d) C_6H_5—CH_2—OH

MATHEMATICS

1. If the angles between the pair of straight lines represented by the equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ is $\tan^{-1} \frac{1}{3}$.

   Where λ is a non-negative real number, then λ is
   (a) 2   (b) 0
   (c) 3   (d) 1

2. The distance of the line $2x - 3y = 4$ from the point (1, 1) measured parallel to the line $x + y = 1$ is
   (a) $\sqrt{2}$ (b) $5/\sqrt{2}$
   (c) $1/\sqrt{2}$ (d) 6

3. The equation of bisectors of the angles between the lines $|x| = |y|$ are
   (a) $y = \pm x$ and $x = 0$
   (b) $x = \frac{1}{2}$ and $y = \frac{1}{2}$
   (c) $y = 0$ and $x = 0$
   (d) None of the above

4. The base of vertices of an isosceles triangle PQR are Q(1, 3) and R(− 2, 7). The vertex P can be
   (a) (1, 6)   (b) $\left(\frac{1}{2}, 5\right)$
   (c) $\left(\frac{5}{6}, 6\right)$ (d) None of these

5. The normal at the point (3, 4) on a circle cuts the circle at the point (− 1, − 2). Then the equation of the circle is

    (a) $x^2 + y^2 + 2x - 2y - 13 = 0$

    (b) $x^2 + y^2 - 2x - 2y - 11 = 0$

    (c) $x^2 + y^2 - 2x + 2y + 12 = 0$

    (d) $x^2 + y^2 - 2x - 2y + 14 = 0$

6. If $\cos P = \dfrac{1}{7}$ and $\cos Q = \dfrac{13}{14}$, where P and Q both are acute angles. Then the value of P − Q is

    (a) 30°   (b) 60°

    (c) 45°   (d) 75°

7. The equation $3 \cos x + 4 \sin x = 6$ has ..... solution.

    (a) finite   (b) infinite

    (c) one   (d) no

8. If $\sec^{-1} x = \operatorname{cosec}^{-1} y$, then $\cos^{-1} \dfrac{1}{x} + \cos^{-1} \dfrac{1}{y}$ is equal to

    (a) π (b) $\dfrac{\pi}{4}$ (c) $-\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{2}$

9. If n be any integer, then $n(n + 1)(2n + 1)$ is:

    (a) odd number

    (b) integral multiple of 6

    (c) perfect square

    (d) does not necessarily have any of the foregoing proof

10. If $\tan \theta = -\dfrac{4}{3}$, then the value of $\sin \theta$ is

    (a) $-\dfrac{4}{5}$ but $\neq \dfrac{4}{5}$ (b) $-\dfrac{4}{5}$ or $\dfrac{4}{5}$

    (c) $\dfrac{4}{5}$ but $\neq -\dfrac{4}{5}$ (d) $\dfrac{1}{5}$

11. If $c = 2 \cos \theta$, then the value of the determinant $\Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix}$ is

    (a) $\dfrac{\sin 4\theta}{\sin \theta}$ (b) $\dfrac{2 \sin^2 2\theta}{\sin \theta}$

    (c) $4 \cos^2 \theta (2 \cos \theta - 1)$ (d) None of these

12. The equation of the parabola whose vertex is $(-1, -2)$, axis is vertical and which passes through the point (3, 6), is

    (a) $x^2 + 2x - 2y - 3 = 0$

    (b) $2x^2 = 3y$

    (c) $x^2 - 2x + 2y - 3 = 0$

    (d) $x^2 - 2x - 2y - 3 = 0$

13. The length of the axis of the conic $9x^2 + 4y^2 - 6x + 4y + 1 = 0$ are

    (a) $\dfrac{1}{2}, 9$ (b) $3, \dfrac{2}{5}$ (c) $1, \dfrac{2}{3}$ (d) $3, 2$

14. If $f(x) = \cot^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$ and

    $g(x) = \cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$, then

    $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{g(x) - g(a)}, \, 0 < a < \dfrac{1}{2}$, is

    (a) $\dfrac{3}{2(1 + a^2)}$ (b) $\dfrac{3}{2(1 + x^2)}$

    (c) $\dfrac{3}{2}$ (d) $-\dfrac{3}{2}$

15. If $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2x - 1, & 1 < x \end{cases}$, then

    (a) f is discontinuous at $x = 1$

    (b) f is differentiable at $x = 1$

    (c) f is continuous but not differentiable at $x = 1$

    (d) None of the above

16. $\lim\limits_{x \to -2} \dfrac{\sin^{-1}(x + 2)}{x^2 + 2x}$ is equal to

    (a) 0   (b) ∞

    (c) $-\dfrac{1}{2}$ (d) None of these

17. Let $f(x) = x^p \cos\left(\dfrac{1}{x}\right)$, when $x \neq 0$ and $f(x) = 0$, when $x = 0$. Then $f(x)$ will be differentiable at $x = 0$, if

    (a) $p > 0$ (b) $p > 1$

    (c) $0 < p < 1$ (d) $\dfrac{1}{2} < p < 1$

18. The derivative of $f(x) = 3|2 + x|$ at the point $x_0 = -3$ is
    (a) 3
    (b) -3
    (c) 0
    (d) None of the above

19. Derivative of the function
    $f(x) = \log_5(\log_7 x)$, $x > 7$ is
    (a) $\dfrac{1}{x(\log 5)(\log 7)(\log_7 x)}$
    (b) $\dfrac{1}{x(\log 5)(\log 7)}$
    (c) $\dfrac{1}{x(\log x)}$
    (d) None of the above

20. If $z = x + iy$, $z^{1/3} = a - ib$, then
    $\dfrac{x}{a} - \dfrac{y}{b} = k(a^2 - b^2)$, where k is equal to :
    (a) 1   (b) 2
    (c) 3   (d) 4

21. The number of real solutions of the equation $1 + |e^x - 1| = e^x(e^x - 2)$ is
    (a) 1   (b) 2
    (c) 4   (d) 8

22. If $u = x^2 + y^2$ and $x = s + 3t$, $y = 2s - t$, then $\dfrac{d^2 u}{ds^2}$ is equal to
    (a) 12   (b) 10
    (c) 32   (d) 36

23. If the equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then $p + q + 1$ is equal to
    (a) 0   (b) 1
    (c) 2   (d) -1

24. The value of a $(a \geq b)$ for which the sum of the cubes of the roots of
    $x^2 - (a - 2)x + (a - 3) = 0$ assumes the least value, is
    (a) 3
    (b) 4
    (c) 5
    (d) None of the above

25. Let z_1, z_2, z_3 be three vertices of an equilateral triangle circumscribing the circle $|z| = \dfrac{1}{2}$. If $z_1 = \dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}$ and z_1, z_2, z_3 were in anticlockwise sense, then z_2 is
    (a) $1 + \sqrt{3}\,i$ (b) $1 - \sqrt{3}\,i$ (c) 1   (d) -1

26. If $z = \dfrac{-2}{1 + \sqrt{3}\,i}$, then the value of arg (z) is
    (a) π (b) $\dfrac{\pi}{3}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{\pi}{4}$

27. Let ω is an imaginary cube roots of unity, then the value of
    $2(1 + \omega)(1 + \omega^2) + 3(2\omega + 1)(2\omega^2 + 1)$
    $\qquad + \ldots + (n + 1)(n\omega + 1)(n\omega^2 + 1)$
    is
    (a) $\left[\dfrac{n(n + 1)}{2}\right]^2 + n$ (b) $\left[\dfrac{n^2(n + 1)^2}{4}\right]$
    (c) $\left[\dfrac{n(n + 1)}{2}\right]^2 - n$ (d) None of these

28. The locus of the point z satisfying
    $\arg\left(\dfrac{z - 1}{z + 1}\right) = k$, (where k is non-zero) is
    (a) a circle with centre on y-axis
    (b) circle with centre on x-axis
    (c) a straight line parallel to x-axis
    (d) a straight line making an angle 60° with the x-axis

29. If $(3, 4, 5)$, $Q(4, 6, 3)$, $R(-1, 2, 4)$, $S(1, 0, 5)$, then the projection of RS on PQ is
    (a) $-\dfrac{2}{3}$ (b) $-\dfrac{4}{3}$ (c) $\dfrac{1}{2}$ (d) 2

30. If a line makes α, β, γ with the positive direction of x, y and z-axes respectively. Then, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$ is equal to
    (a) $\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) -1 (d) 1

31. The projection of a line on a co-ordinate axes are 2, 3, 6. Then the length of the line is
    (a) 7   (b) 5   (c) 1   (d) 11

32. The decimal equivalent of the binary number 10011.1 is
    (a) 19.50   (b) 11001.11
    (c) 5005.55   (d) 19.10

33. The binary represents of 60 is
    (a) 101110   (b) 111100
    (c) 110011   (d) 110000

34. Which of the following statement is not tautology?
    (a) $\sim (p \wedge q) \vee p$
    (b) $(p \wedge q) \Rightarrow p$
    (c) $q \vee \sim (p \wedge q)$
    (d) $(\sim p \wedge q) \cap (\sim p \vee p)$

35. The period of
    $$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right), \ n \in Z, \ n > 2$$
    is
    (a) $2\pi n(n-1)$
    (b) $4n(n-1)$
    (c) $2n(n-1)$
    (d) None of the above

36. For $\theta > \frac{\pi}{3}$, the value of
    $f(\theta) = \sec^2 \theta + \cos^2 \theta$ always lies in the interval
    (a) $(0, 2)$ (b) $[0, 1]$
    (c) $(1, 2)$ (d) $[2, \infty)$

37. The radius of the circle whose arc of length 15 cm makes an angle of $\frac{3}{4}$ radian at the centre, is
    (a) 10 cm   (b) 20 cm
    (c) $11\frac{1}{4}$ cm   (d) $22\frac{1}{2}$ cm

38. If $f_n(x) = e^{f(n-1)(x)}$, for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is equal to
    (a) $f_n(x) f_{n-1}(x)$
    (b) $f_n(x) \frac{d}{dx}\{f_{n+1}(x)\}$
    (c) $f_n(x) \cdot f_{n-1}(x) \dots f_2(x) f_1(x)$
    (d) None of the above

39. If $3^x + 2^{2x} \geq 5^x$, then the solution set for x is
    (a) $(-\infty, 2]$ (b) $[2, \infty)$
    (c) $[0, 2]$ (d) $\{2\}$

40. The number of integral solution of
    $\frac{x+1}{x^2+2} > \frac{1}{4}$ is
    (a) 1   (b) 2
    (c) 5   (d) None of these

41. The triangle PQR of which the angles P, Q, R satisfy $\cos P = \frac{\sin Q}{2 \sin R}$ is
    (a) equilateral   (b) right angled
    (c) any triangle   (d) isosceles

42. If $f(x) = (a - x^n)^{1/n}$, where $a > 0$ and n is a positive integer, then $f[f(x)]$ is equal to
    (a) x^3 (b) x^2
    (c) x (d) None of these

43. A function $f(x) = \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$ is
    (a) maximum at $x = -3$
    (b) maximum at $x = -3$ and maximum at $x = 1$
    (c) maximum at $x = 1$
    (d) function is increasing in its domain

44. The locus of the point $P(x, y)$ satisfying the relation
    $$\sqrt{(x-3)^2 + (y-1)^2}$$
    $$+ \sqrt{(x+3)^2 + (y-1)^2} = 6 \text{ is}$$
    (a) straight line
    (b) pair of straight lines
    (c) circle
    (d) ellipse

45. If z_1, z_2 and z_3 are complex number such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$
    then $|z_1 + z_2 + z_3|$ is
    (a) equal to 1   (b) less than 1
    (c) greater than 3   (d) equal to 3

46. Equation $x * a = b$ has ijn group $(G, \forall)$
    (a) unique solution $b * a^{-1}$
    (b) unique solution $a^{-1} * b$
    (c) unique solution $a^{-1} * b^{-1}$
    (d) many solutions

47. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, then

(a) $f\left(\dfrac{\pi}{4}\right) = 2$ (b) $f(-\pi) = 2$

(c) $f(\pi) = 1$ (d) $f\left(\dfrac{\pi}{2}\right) = -1$

48. The range of
$f(x) = \sec\left(\dfrac{\pi}{4}\cos^2 x\right), -\infty < x < \infty$ is

(a) $[1, \sqrt{2}]$

(b) $[1, \infty)$

(c) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$

(d) $(-\infty, 1] \cup [1, \infty)$

49. The domain of the function
$f(x) = \dfrac{\sin^{-1}(3-x)}{\log(|x|-2)}$ is :

(a) $[2, 4]$

(b) $(3, 4]$

(c) $[2, \infty)$

(d) $(-\infty, 3) \cup [2, \infty)$

50. The remainder obtained when
$1! + 2! + \ldots + 200!$ is divided by 14 is

(a) 3

(b) 4

(c) 5

(d) None of the above

51. $\cos^{-1}\{\cos 2\cot^{-1}(\sqrt{2}-1)\}$ is equal to

(a) $\sqrt{2} - 1$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{3\pi}{4}$ (d) 0

52. The function $f(x) = [x]\cos\left[\dfrac{2x-1}{2}\right]\pi$,

where $[\cdot]$ denotes the greatest integer function, is discontinuous at

(a) all x

(b) no x

(c) all integral points

(d) x which is not an integer

53. A perpendicular is drawn from the point P(2, 4, −1) to the line
$\dfrac{x+5}{1} = \dfrac{y+3}{4} = \dfrac{z-6}{-9}$. The equation of

the perpendicular from P to the given line is

(a) $\dfrac{x-2}{6} = \dfrac{y-4}{3} = \dfrac{z+1}{2}$

(b) $\dfrac{x+2}{6} = \dfrac{y-4}{3} = \dfrac{z+1}{2}$

(c) $\dfrac{x+2}{-6} = \dfrac{y-4}{3} = \dfrac{z+1}{2}$

(d) $\dfrac{x+2}{6} = \dfrac{y+4}{3} = \dfrac{z+1}{2}$

54. One mapping (function) is selected at random from all the mappings of the set A = {1, 2, 3, …, n} into itself. The probability that the mapping selected is one-one, is

(a) $\dfrac{n!}{n^{n-1}}$ (b) $\dfrac{n!}{n^n}$

(c) $\dfrac{n!}{2n^n}$ (d) None of these

55. If $\mathbf{a} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{c} = -4\hat{\mathbf{i}} - \hat{\mathbf{k}}$, then $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

(a) $5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 15\hat{\mathbf{k}}$ (b) 0

(c) $12\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ (d) $-3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

56. The number of five digits numbers that can be formed without any restriction is

(a) 990000 (b) 10000

(c) 90000 (d) None of these

57. The maximum value of $3\cos\theta + 4\sin\theta$ is

(a) − 5 (b) 5

(c) 25 (d) None of these

58. The vertex connectivity of any tree is

(a) one (b) two

(c) three (d) None of these

59. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-collinear vectors such that for some scalars, x, y, z,
$x\mathbf{a} + x\mathbf{b} + x\mathbf{c} = 0$, then

(a) $x = 0, y = 0, z = 0$

(b) $x \neq 0, y \neq 0, z = 0$

(c) $x = 0, y \neq 0, z \neq 0$

(d) $x \neq 0, y \neq 0, z \neq 0$

60. Which is the correct order for a given number α in increasing order?

(a) $\log_2 \alpha, \log_3 \alpha, \log_e \alpha, \log_{10} \alpha$

(b) $\log_{10} \alpha, \log_3 \alpha, \log_e \alpha, \log_2 \alpha$

(c) $\log_{10} \alpha, \log_e \alpha, \log_2 \alpha, \alpha \log_3 \alpha$

(d) None of the above

61. In ΔABC, $\dfrac{b-c}{r_1} + \dfrac{c-a}{r_2} + \dfrac{a-b}{r_3}$ is equal to
    (a) 0   (b) abc
    (c) a + b + c   (d) ab + bc + ca

62. If $n \in N$, then $|\sin nx|$
    (a) $\leq n\,|\sin x|$ (b) $\geq n|\sin x|$
    (c) $= n|\sin x|$ (d) None of these

63. If x follows a binomial distribution with parameters $n = 100$ and $p = \dfrac{1}{3}$, then $p(X = r)$ is maximum when r equals
    (a) 16   (b) 32
    (c) 33   (d) None of these

64. The direction cosines of any normal to the xy-plane are
    (a) 1, 0, 0   (b) 0, 1, 0
    (c) 1, 1, 0   (d) 0, 0, 1

65. If $\int f(x)\,dx = F(x)$, then $\int x^3\,f(x^2)\,dx$ is equal to
    (a) $\dfrac{1}{2}[x^2\,\{F(x)\}^2 - \int \{F(x)\}^2\,dx]$
    (b) $\dfrac{1}{2}[x^2\,F(x^2) - \int F(x^2)\,d(x^2)]$
    (c) $\dfrac{1}{2}[x^2 F(x) - \dfrac{1}{2}\int \{F(x)\}^2\,dx]$
    (d) None of the above

66. The approximate value of $\int_1^5 x^2\,dx$ using trapezoidal rule with n = 4 is
    (a) 41   (b) 41.5   (c) 41.75   (d) 42

67. If the distance between the foci of an ellipse is 6 and the length of the minor axis is 8, then the eccentricity is
    (a) $\dfrac{1}{\sqrt{5}}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{4}{5}$

68. If $I_n = \int \sin^n x\,dx$, then $nI_n - (n-1)I_{n-2}$ equals
    (a) $\sin^{n-1} x \cos x$ (b) $\cos^{n-1} x \sin x$
    (c) $-\sin^{n-1} x \cos x$ (d) $-\cos^{n-1} x \sin x$

69. The adjoining graph
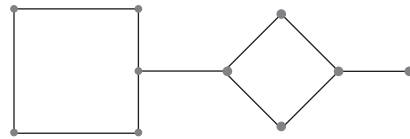
    (a) connected   (b) disconnected
    (c) Neither connected nor disconnected
    (d) None of the above

70. The remainder obtained when 5^{124} is divided by 124 is
    (a) 5   (b) 0   (c) 2   (d) 1

ENGLISH AND GENERAL APTITUDE

Directions (Q. Nos. 1-5) *Read the given passage carefully and answer the questions that follow by selecting the most appropriate option.*

Helicopters are very different from airplanes. They can do three things that airplanes cannot do. First, when airplanes move upward, they must also move forward, but helicopters can move straight up without moving ahead. Second, helicopters can fly backward, which airplanes cannot do. Third, helicopters can use their rotors to hover in the air, which is impossible for airplanes. Helicopters can **perform** actions that airplanes cannot, they are used for different tasks. Since, helicopters can take off without moving forward, they do not need a runway for take off. They are used in **congested** areas where there is no room for airplanes or in isolated areas, which do not have airports. Because they can hover, they are used on firefighting missions to drop water on fires. They are used in logging operations to lift trees out of forests. Helicopters are used as air ambulances to airlift patients out of situations, which are difficult to reach by conventional ambulances.

The police used helicopters to follow suspects on the ground or to search for cars on the ground. Of course, helicopters have military uses because of their design and capabilities.

1. The word 'congested' in the passage means
   (a) the place is roomy
   (b) the place has no place
   (c) the place can be accessed by police vans
   (d) the place has so much of smoke due to fire

2. A helicopter can hover while an airplane cannot, according to the passage. 'Hover' in the passage means
   (a) stay at one place in the air
   (b) move straight up in the air
   (c) go backward in the air
   (d) fly sideways

3. Point out the grammatical category of the word 'perform' in the passage.
   (a) Noun   (b) Verb
   (c) Adverb   (d) Helping verb

4. Why is a helicopter used as an ambulance?
   (a) Its movement can be manoeuvred easily
   (b) It crosses all difficulties of traffic
   (c) It reaches the inaccessible places easily
   (d) It can fire-fight

5. Point out the grammatical category of the word 'very' in the passage.
   (a) Adverb   (b) Adjective
   (c) Noun   (d) Verb

Directions (Q. Nos. 6-8) *Complete the sentence in the following questions with most appropriate choice.*

6. He was very much ashamed ……………… his mischief.
   (a) for   (b) of   (c) with   (d) over

7. …………… he stood first, he would have gone to America.
   (a) If   (b) Had
   (c) Have   (d) Anyways

8. Of the sword and pen, the .... is more powerful.
   (a) last one   (b) later   (c) latter   (d) last

Directions (Q. Nos. 9-11) *Each of the following question consist of a word, followed by four words or group of words. Select the antonym of the words given below.*

9. Adverse
   (a) Hostile   (b) Friendly
   (c) Inimical   (d) Unwilling

10. Expansion
    (a) Depression   (b) Negligent
    (c) Precise   (d) Enlargement

11. Thrift
    (a) Parsimony   (b) Dismantle
    (c) Prodigality   (d) Essential

12. Pick out the correct synonym of the word 'cantankerous'.
    (a) Rash   (b) Quarrelsome
    (c) Noisy   (d) Disrespectful

Directions (Q. Nos. 13-15) *In each of the following questions, choose the alternative which best expresses the meaning of the word given in capital letters.*

13. INTREPID
    (a) Coward   (b) Fearless
    (c) Selfish   (d) Uqly

14. LAMENTABLE
    (a) Deplorable   (b) Contemptible
    (c) Remorseful   (d) Scornful

15. DEFT
    (a) Skilful   (b) Vigorous
    (c) Swift   (d) Store

Directions (Q. Nos. 16-20) *In each of the following questions, there is a certain relationship between two given words on one side of (::) and one word is given on another side (::) while another word is to be found from the given alternatives, having the same relation with this word as the words of the given pair bear. Choose the correct alternative.*

16. King : Throne :: Judge : ?
    (a) Lawyer   (b) Bench
    (c) Court   (d) Trial

17. Lion : Roar :: Ass : ?
    (a) Bark
    (b) Trumpet
    (c) Howl
    (d) Bray

18. Lamb : Frisk :: Mouse : ?
    (a) Trot   (b) Scamper
    (c) Gallop   (d) Flit

19. Circle : Arc :: Square : ?
    (a) Line   (b) Triangle
    (c) Sphere   (d) Rectangle

20. Brinjal : Vegetable :: Orange : ?
    (a) Fruit   (b) Stem
    (c) Leaf   (d) Root

Direction (Q. Nos. 21-23) *In each of the following questions, a group of four words are given. Choose the word which is odd.*

21. Find the odd word.
    (a) Walking
    (b) Running
    (c) Moving
    (d) Reading

22. Find the odd word.
    (a) Ghaziabad   (b) Varanasi
    (c) Gorakhpur   (d) Bhagalpur

23. Find the odd word.
    (a) Orange
    (b) Apple
    (c) Guava
    (d) Grapes

24. In the English alphabet, find the position of S from right.
    (a) 8   (b) 5
    (c) 4   (d) 9

25. In the English alphabet, find the position of L from left.
    (a) 12
    (b) 16
    (c) 11
    (d) 15

26. Which letter is 7th from right in the English alphabet?
    (a) P   (b) C
    (c) T   (d) V

Directions (Q. Nos. 27-28) *In the following series, replace the question mark (?) with the suitable option.*

27. 6, 9, 12, 15, 18, ?
    (a) 21
    (b) 20
    (c) 19
    (d) 22

28. 360, 180, 90, 45, 22.5, ?
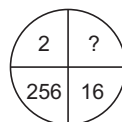   (a) 15
    (b) 11.25
    (c) 20
    (d) 10.5

Directions (Q. Nos. 29-30) *In each of the following questions, a set of figures carrying certain characters is given. Assuming that the characters in each set follow a similar pattern, find the missing character in each case.*

29.
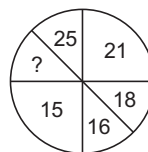
    (a) 28   (b) 30
    (c) 35   (d) 27

30.

    (a) 8
    (b) 4
    (c) 32
    (d) 16

PHYSICS

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(d)	**7.**	(d)	**8.**	(a)	**9.**	(b)	**10.**	(d)
11.	(b)	**12.**	(a)	**13.**	(d)	**14.**	(a)	**15.**	(a)	**16.**	(a)	**17.**	(c)	**18.**	(a)	**19.**	(d)	**20.**	(a)
21.	(b)	**22.**	(d)	**23.**	(c)	**24.**	(d)	**25.**	(c)	**26.**	(a)	**27.**	(c)	**28.**	(a)	**29.**	(c)	**30.**	(b)
31.	(a)	**32.**	(c)	**33.**	(b)	**34.**	(a)	**35.**	(d)	**36.**	(c)	**37.**	(d)	**38.**	(b)	**39.**	(a)	**40.**	(b)
41.	(c)	**42.**	(c)	**43.**	(d)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(b)	**49.**	(a)	**50.**	(b)

CHEMISTRY

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(a)	**6.**	(a)	**7.**	(a)	**8.**	(c)	**9.**	(a)	**10.**	(c)
11.	(a)	**12.**	(d)	**13.**	(a)	**14.**	(d)	**15.**	(b)	**16.**	(a)	**17.**	(a)	**18.**	(a)	**19.**	(a)	**20.**	(c)
21.	(b)	**22.**	(c)	**23.**	(a)	**24.**	(b)	**25.**	(d)	**26.**	(c)	**27.**	(a)	**28.**	(b)	**29.**	(a)	**30.**	(a)
31.	(b)	**32.**	(b)	**33.**	(d)	**34.**	(a)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(b)	**39.**	(c)	**40.**	(d)
41.	(d)	**42.**	(d)	**43.**	(c)	**44.**	(d)	**45.**	(c)	**46.**	(a)	**47.**	(b)	**48.**	(b)	**49.**	(c)	**50.**	(c)

MATHEMATICS

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(b)	**6.**	(b)	**7.**	(d)	**8.**	(d)	**9.**	(b)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(c)	**14.**	(d)	**15.**	(c)	**16.**	(c)	**17.**	(b)	**18.**	(b)	**19.**	(a)	**20.**	(d)
21.	(a)	**22.**	(b)	**23.**	(a)	**24.**	(a)	**25.**	(d)	**26.**	(c)	**27.**	(a)	**28.**	(a)	**29.**	(b)	**30.**	(d)
31.	(a)	**32.**	(a)	**33.**	(b)	**34.**	(d)	**35.**	(c)	**36.**	(d)	**37.**	(b)	**38.**	(c)	**39.**	(a)	**40.**	(c)
41.	(d)	**42.**	(c)	**43.**	(d)	**44.**	(b)	**45.**	(a)	**46.**	(a)	**47.**	(d)	**48.**	(a)	**49.**	(b)	**50.**	(c)
51.	(c)	**52.**	(c)	**53.**	(a)	**54.**	(b)	**55.**	(a)	**56.**	(c)	**57.**	(b)	**58.**	(a)	**59.**	(a)	**60.**	(b)
61.	(a)	**62.**	(d)	**63.**	(c)	**64.**	(d)	**65.**	(b)	**66.**	(d)	**67.**	(c)	**68.**	(c)	**69.**	(a)	**70.**	(a)

ENGLISH AND GENERAL APTITUDE

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(b)	**8.**	(c)	**9.**	(b)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(b)	**14.**	(a)	**15.**	(a)	**16.**	(b)	**17.**	(d)	**18.**	(b)	**19.**	(a)	**20.**	(a)
21.	(d)	**22.**	(d)	**23.**	(d)	**24.**	(a)	**25.**	(a)	**26.**	(c)	**27.**	(a)	**28.**	(b)	**29.**	(b)	**30.**	(b)

Hints & Solutions

PHYSICS

1. (c) Acceleration of the system
$$a = \frac{F}{m} = \frac{10}{2 + 3 + 5}$$
$$= \frac{10}{10} = 1 \ m/s^2$$
$\therefore \qquad T_1 = (3 + 5) \times a = 8 \times 1 = 8 \ N$

2. (c) Acceleration of the system
$$a = \frac{m_1 - m_2}{m_1 + m_2} \ g = \frac{10 - 6}{10 + 6} \times g$$
$$= \frac{4}{16} \times 10 = 2.5 \ m/s^2$$

3. (a) $\because$ L / R is a time constant so the unit of L / R is second.

4. (b) According to the question, $F \propto v^2$
or $\quad F = kv^2$
$$k = \frac{F}{v^2} \Rightarrow \frac{[MLT^{-2}]}{[L^2 T^{-2}]} = [ML^{-1} \ T^0]$$

5. (b) Ionisation energy
$$E = 13.6 \ Z^2$$
For He^+, $Z = 2$
$$E = 13.6 \ (2)^2 = 13.6 \times 4 = 54.4 \ eV$$

6. (d) Max. kinetic energy of photoelectrons
$= 4 \ eV$
If stopping potential is V_0, then
$$eV_0 = 4 \ eV$$
$\therefore \qquad V_0 = 4 \ volt$

7. (d) Out of the given choices, Poisson's ratio has not unit.

8. (a) In candescent electric lamp produces continuous emission spectrum whereas mercury and sodium vapour given line emission spectrum. Polyatomic substance such as H_2, CO_2 and $KMNO_4$ produces band absorption spectrum.

9. (b) Let the two balls cross each other after time t.
Height covered by first ball
$$h_1 = \frac{1}{2} \ gt^2$$
Height covered by second ball
$$h_2 = ut - \frac{1}{2} \ gt^2$$
but $h_1 + h_2 = 100$
$$\frac{1}{2} \ gt^2 + ut - \frac{1}{2} \ gt^2 = 100$$
$$ut = 100$$
$$50 \times t = 100 \Rightarrow t = 2 \ s$$
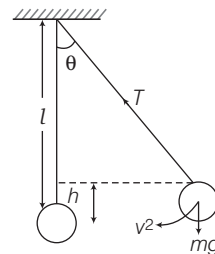
10. (d) Observed speed, $v = \sqrt{(10)^2 + (6)^2}$
$$= \sqrt{100 + 36} = \sqrt{136} \simeq 11 \ m/s$$

11. (b) Orbital speed of satellite near the earth surface is approximately 8 km/s.

12. (a) $g = \dfrac{GM_e}{R_e^2} = G \ \dfrac{\dfrac{4}{3} \pi R_e^3 d}{R_e^2} = \dfrac{4}{3} \pi G R_e d$

$g \propto d$

So, if density is doubled, then acceleration due to gravity will also be doubled.

13. (d) In mean position,

$$T = 3 \ mg$$
Net force, $T - mg = \dfrac{mv^2}{l}$

$$3mg - mg = \frac{mv^2}{l}$$

$$\Rightarrow \qquad v = \sqrt{2\,gl}$$

Let maximum displacement with respect to vertical is θ, then

$$\frac{1}{2}\,mv^2 = mgh$$

$$= mg(1 - l\cos\theta)$$

$$\frac{1}{2}\,m(2\,g\,l) = mgl\,(1 - \cos\theta)$$

$$\cos\theta = 0$$

$$\Rightarrow \qquad \theta = 90°$$

14. (a) Force, $F = m\,a = m\dfrac{dv}{dt}$

$$3x = 8\frac{dv}{dx} \cdot \frac{dx}{dt}$$

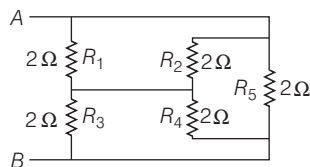$$3x = 8\,v\frac{dv}{dx} \qquad \left[\because \frac{dx}{dt} = v\right]$$

$$3x\,dx = 8v\,dv$$

Integrating both sides

$$3\left[\frac{x^2}{2}\right]_2^{10} = 8\left[\frac{v^2}{2}\right]_0^v$$

$$\frac{3}{2}[10^2 - 2^2] = 4v^2$$

$$\frac{3}{2} \times 96 = 4v^2$$

$$v = 6\,\text{m/s}$$

15. (a) R_1 and R_2 are in parallel order, so their equivalent resistance

$$R' = \frac{2 \times 2}{2 + 2} = 1\,\Omega$$

Similarly, equivalent resistance of R_3 and R_4

$$R'' = 1\,\Omega$$
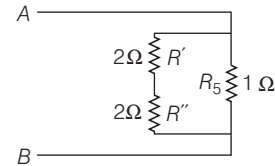
R′ and R″ are in series order, so their equivalent resistance

$$R''' = R' + R''$$

$$= 1 + 1 = 2\,\Omega$$

R′″ and R_5 are in parallel.

So, $\quad R_{AB} = \dfrac{2 \times 2}{2 + 2} = 1\,\Omega$

16. (a) $_1H^1 + {}_1H^1 \longrightarrow {}_2He^4$

Mass defect

$\Delta m = 2 \times$ mass of $_1H^1 -$ mass of $_2He^4$

$$= 2 \times 2.0141 - 4.0024$$

$$= 0.0258$$

Energy released $= \Delta m \times 931$ MeV

$$= 0.0258 \times 931 = 24\,\text{MeV}$$

17. (c) When a thin metal plate is placed between the plates of a parallel plate capacitor, then its capacitance increases so, out of the given choices, 4 C will be the right answer.

18. (a) Linear acceleration, $a = \dfrac{g\,\sin\theta}{1 + \dfrac{k^2}{r^2}}$

For solid sphere $\dfrac{k^2}{r^2} = \dfrac{2}{5}$

$$\therefore \qquad a = \frac{g\,\sin\theta}{1 + \dfrac{2}{5}}$$

$$= \frac{5}{7}\,g\,\sin\theta = \frac{5}{7}\,g\,\sin 30°$$

$$= \frac{5}{7}\,g \times \frac{1}{2} = \frac{5g}{14}$$

19. (d) From law of conservation of momentum

$$MV = mv$$

$$1 \times 5 = 10 \times 10^{-3} \times v$$

$$v = \frac{5}{10 \times 10^{-3}} = 500\,\text{m/s}$$

20. (a) Acceleration in circular motion

$$a = \frac{v^2}{r}$$

But,

$$v = \frac{2\pi r}{T}$$

$$r = \frac{vT}{2\pi}$$

$$\therefore \qquad a = \frac{v^2}{vT / 2\pi}$$

$$= \frac{2\pi v}{T}$$

21. (b) The centre of mass will be slightly farther from C-atom (because it is lighter), so correct choice will be (b).

22. (d) Work done, $W = Fd \cos \theta$

$$25 = 10 \times 10 \cos \theta$$

$$\cos \theta = \frac{1}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{4}\right)$$

23. (c) Ist case,

$$U = \frac{1}{2} kx_1^2$$

$$100 = \frac{1}{2} k (2 \times 10^{-2})^2$$

$$k = \frac{200}{4 \times 10^{-4}}$$

$$= 50 \times 10^4$$

IInd case

$$U = \frac{1}{2} kx_2^2$$

$$= \frac{1}{2} \times 50 \times 10^4 (4 \times 10^{-2})^2$$

$$= 25 \times 10^4 \times 16 \times 10^{-4}$$

$$= 400 \, J$$

So, stored energy increases by

$$400 - 100 = 300 \, J$$

24. (d) Young's modulus, $Y = \frac{F \cdot L}{\pi r^2 \Delta L}$

Both wires are of same material.

So, $\qquad Y_1 = Y_2$

$$\frac{F_A \cdot L_A}{r_A^2 \Delta L_A} = \frac{F_B L_B}{r_B^2 \cdot \Delta L_B}$$

Here, $\qquad \Delta L_A = \Delta L_B$

$$\therefore \qquad \frac{F_A}{F_B} = \frac{L_B}{L_A} \times \left(\frac{r_A}{r_B}\right)^2$$

$$= \frac{2}{1} \times \left(\frac{2}{1}\right)^2 = \frac{8}{1}$$

$$F_A : F_B = 8 : 1$$

25. (c) Radius of path of charged particle

$$r = \frac{\sqrt{2m \, E_k}}{Bq}$$

E_k remains same, so

$$\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \cdot \frac{q_2}{q_1}$$

$$= \sqrt{\frac{4}{16}} \times \frac{2}{1} = \frac{1}{2} \times \frac{2}{1} = \frac{1}{1}$$

So, correct choice is (c).

26. (a) $\beta = \frac{\lambda D}{d}$

$$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \cdot \frac{d_2}{d_1} = \frac{7000}{3500} \times \frac{2d}{d}$$

$$\beta_2 = \frac{\beta_1}{4}$$

So, the width of fringes changes.

27. (c) Coherent sources of light are those sources of light which emit light waves of same wavelength, same frequency and are in same phase or having constant phase difference.

28. (a) In metal, either there is no energy gap between the condition band which is partially filled with electrons and valence band or the condition band and valence band overlap each other.

29. (c) The semiconductors in which majority charge carriers are holes i.e. positinely charged and minority charge are holes, so, in p-type semiconductor, the holes movement results in the formation of the current.

30. (b) A laser device produces amplification in the ultraviolet or visible region.

31. (a) Focal length of glass rod, $f = \dfrac{R}{2}$

$$= \dfrac{30}{2} = 15 \text{ cm}$$

Distance of object, $u = -15$ cm

From formula,

$$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$$

$$\mu_2 = 1.5 \qquad \text{(for glass)}$$
$$\mu_1 = 1 \qquad \text{(for air)}$$

$$\dfrac{1.3}{v} = -\dfrac{1}{-15} = \dfrac{1.5 - 1}{30}$$

$$\dfrac{1.5}{v} + \dfrac{1}{15} = \dfrac{0.5}{30}$$

$$\dfrac{1.5}{v} = \dfrac{1}{60} - \dfrac{1}{15}$$

$$\dfrac{1.5}{v} = \dfrac{-1}{20}$$

$$\Rightarrow \qquad v = -30 \text{ cm}$$

32. (c) Rainbow occurs in the form of concentric coloured circular arcs in the sky, when the sun shines on rain drops during or after the shower. This phenomenon occurs due to combined effect of dispersion, refraction and total internal reflection of sunlight by spherical drop of water.

33. (b) Near the junction positive charge is built on n-side and negative charge on P-side. This sets up potential difference across the junction and internal electric field E_i directed from n-side to P-side. The equilibrium is established when the field E_i becomes strong enough to stop further diffusion of the majority charge carriers.

34. (a) At 45° total internal reflection takes place so, critical angle, $C = 45°$.

Refractive index, $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 45°}$

$$= \sqrt{2} = 1.414$$

$$\approx 1.5$$

35. (d) $\because F \propto S^{-1/3}$ or $a \propto S^{-1/3}$

or $Vdv \propto S^{-1/3}ds \left(a = V \cdot \dfrac{dV}{ds} \right)$

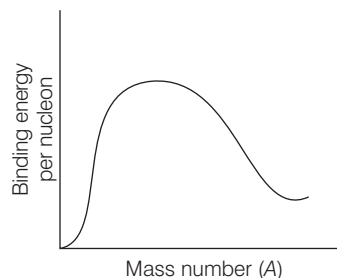Integrating we have, $V^2 \propto S^{2/3}$

or $\qquad\qquad V \propto S^{1/3}$

Now, power $P \propto F \cdot V \propto S^{-1/3} \cdot S^{1/3}$

or $\qquad\qquad P \propto S^0$

36. (c) Binding Energy per nuclear

$$= \dfrac{\text{Total binding energy}}{\text{Number of nucleon (A)}}$$

37. (d) Velocity of light in fiber

$$v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

Straight length is 1 km,

Distance covered by light,

$$s = \dfrac{1 \text{ km}}{\sin 60°} = \dfrac{1000}{\sqrt{3}/2} = \dfrac{2000}{\sqrt{3}}$$

So, time taken $= \dfrac{s}{v} = \dfrac{2000/\sqrt{3}}{2 \times 10^8} = 3.85 \,\mu s$

38. (b) $W_{PQ} = P\Delta V$

$$= 100 \times 10^3 (300 - 100) \times 10^{-6}$$

$$= 100 \times 10^{-3} \times 200 = 20 \text{ J}$$

$W_{QR} = 0$ ($\because$ V is constant)

$W_{RS} = P\Delta V$

$$= 200 \times 10^3 (100 - 300) \times 10^{-6}$$

$$= -200 \times 10^{-3} \times 200 = -40 \text{ J}$$

$W_{SP} = 0$

Net work done

$$= 20 + (-40) = -20 \text{ J}$$

39. (a) Let each resistance is R

$$P = \frac{E^2}{R_{eq}}$$

In choice (a) $R_{eq} = \frac{R \times R}{R + R} = \frac{R}{2}$

$$\Rightarrow \qquad P_1 = \frac{E^2}{\dfrac{R}{2}} = \frac{2E^2}{R}$$

In choice (b)

$$R_{eq} = R + R = 2R$$

$$P_2 = \frac{E^2}{2R} = 0.5\frac{E^2}{R}$$

In choice (c) $R_{eq} = \frac{R}{2} + R = \frac{3R}{2}$

$$P_3 = \frac{2E^2}{3R} = 0.6\frac{E^2}{R}$$

In choice (d) $\quad R = \frac{2R \times R}{2R + R} = \frac{2R}{3}$

$$P_4 = \frac{3E^2}{2R} = 1.5\frac{E^2}{R}$$

So, P_1 is maximum.

40. (b) Charge redistributes on the capacitors in the ratio of their capacitance.

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{C_1}{2C_1} = \frac{1}{2}$$

$$Q_1 : Q_2 = 1 : 2$$

$$\therefore \qquad Q_1 = \frac{1}{1+2} \times Q = \frac{Q}{3}$$

$$Q_2 = \frac{2}{1+2}Q = \frac{2}{3}Q$$

41. (c) Tension in the string
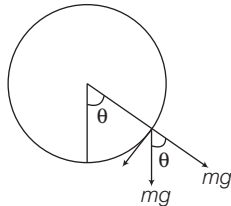
$$T = \frac{mv^2}{l} + mg\cos\theta$$

So, $\qquad T \propto \cos\theta$

$$\frac{T_1}{T_2} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 30°}{\cos 60°}$$

$$= \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{1}$$

$$\therefore \qquad T_1 > T_2$$

42. (c) $L = \dfrac{\phi}{nI} = \dfrac{10^{-5}}{100 \times 5 \times 10^{-3}} = \dfrac{10^{-2}}{100 \times 5}$

$$= \frac{10^{-4}}{5} H = \frac{10^{-1}}{5} mH$$

$$= 0.02\, mH$$

43. (d) Energy drawn from the battery

$$E = V\,I\,t$$

$$= 12 \times 300 \times 2 \times 60$$

$$= 432 \times 10^3\, J$$

$$= 432\, kJ$$

44. (d) From $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$

$$\frac{1 \times 10^5}{2 + 273} = \frac{h\,d\,g + 1 \times 10^5}{T_2}$$

$$[\because P_1 = \text{atmospheric pressure}]$$

$h = 20\, m$, d = density of water

$$= 1 \times 10^3\, kg/m^3$$

$$\therefore \quad \frac{1 \times 10^5}{275} = \frac{20 \times 10^3 \times 10 + 1 \times 10^5}{T_2}$$

$$\frac{1 \times 10^5}{275} = \frac{3 \times 10^5}{T_2}$$

$$T_2 = 275 \times 3 = 825\, K$$

$$= 825 - 273$$

$$= 552°\, C$$

45. (b) $y_1 = 4\sin(\omega t + kx)$ \qquad ...(i)

$$y_2 = -4\cos(\omega t + kx)$$

$$= -4\sin(\omega t + kx + \pi/2)$$

$$= 4\sin\left(\omega t + kx + \frac{3\pi}{2}\right) \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

The phase difference, $\Delta\phi = \dfrac{3\pi}{2}$

46. (b) $m = 0.003$ g $= 0.003 \times 10^{-3}$ kg

$E = 6 \times 10^4$ N/C

The particle is stationary, so

Electric force = Weight of the particle

$$qE = mg$$

$$q = \frac{mg}{E} = \frac{0.003 \times 10^{-3} \times 10}{6 \times 10^4}$$

$$= \frac{3 \times 10^{-10} \times 10}{6} = 5 \times 10^{-10} C$$

47. (b) Force on each plate, $F = \dfrac{qE}{2}$

$$= \frac{1 \times 10^{-6} \times 10^5}{2} = \frac{10^{-1}}{2}$$

$$= 0.05 \, N$$

48. (b) Given heat, $Q = mL = 1 \times 540$ cal

$$= 540 \times 4.18 \times 10^7 \, J$$

Work done, $W = P\Delta V$

$P = 1 \, atm = 1 \times 10^5 \, N/m^2 = 10^6 \, dyne/cm^2$

$W = 10^6 \times 1680 \, J$

Change in internal energy

$$\Delta U = Q - W$$

$$= 540 \times 4.18 \times 10^7 - 10^6 \times 1680$$

$$= \frac{540 \times 4.18 \times 10^7 - 10^6 \times 1680}{4.18 \times 10^7} \, cal$$

$$= 500 \, cal$$

49. (a) Induced emf

$$E = -\frac{\Delta\phi}{\Delta t} = -A\frac{\Delta B}{\Delta t}$$

$$= -0.01 \times \frac{1-2}{10^{-3}} = \frac{0.01}{10^{-3}}$$

$$= 10 \, V$$

$\therefore$ Heat produced, $H = \dfrac{E^2}{R} t$

$$= \frac{(10)^2}{0.01} \times 10^{-3}$$

$$= \frac{100 \times 10^{-3}}{0.01}$$

$$= 10 \, J$$

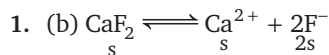**50.** (b) $R_t = R_0(1 + \alpha \, \Delta T)$

$$2 = 1(1 + 0.00125 \times \Delta T)$$

$$\Delta T = \frac{1}{0.00125} \quad \text{(neglecting } -\text{ve sign)}$$

$$T = 800 + 300 = 1100 \, K$$

CHEMISTRY

1. (b) $CaF_2 \underset{s}{\rightleftharpoons} \underset{s}{Ca^{2+}} + \underset{2s}{2F^-}$

$$[Ca^{2+}] = s$$

$$[F^-] = 2s$$

$$K_{sp} = [Ca^{2+}] \cdot [F^-]^2 = s \cdot (2s)^2$$

$$K_{sp} = 4s^3$$

2. (a) If p, T, ρ and R represents pressure, temperature, density and universal gas constant then ideal gas equation

$pV = nRT$ for n mole of gas

$pV = RT$ for 1 mole of gas

We know that

$$M = V \times \rho$$

$$V = \frac{M}{\rho}$$

Put the value of V in gas equation

$$p \times \frac{M}{\rho} = RT$$

then molar mass of gas

$$M = \frac{\rho RT}{p}$$

3. (b) The kinetic energy of a gas molecule is directly proportional to absolute temperature. According to kinetic theory of gases, the pressure P exerted by a gas is given by

$$p = \frac{1}{3} \frac{Mu^2}{V}$$

or $\qquad pV = \dfrac{1}{3} Mu^2$

According to gas equation $pV = RT$

$\therefore \qquad \dfrac{1}{3} Mu^2 = RT$

or $\qquad \dfrac{1}{3} \dfrac{Mu^2}{N} = \dfrac{RT}{N}$

We know that

$$\dfrac{R}{N} = K$$

K = Boltzmann constant

or $\qquad \dfrac{1}{2} Mu^2 = \dfrac{3}{2} KT$

But $\dfrac{1}{2} Mu^2$ is average kinetic energy per molecule of the gas.

$\therefore \qquad$ Average K.E. $= \dfrac{3}{2} KT$

$$K.E. \propto T$$

Thus average kinetic energy of one mole of any gas is directly proportional to its absolute temperature.

4. (d) Molecular weight of $SO_2(M_1) = 64$

Molecular weight of $O_2(M_2) = 32$

According to Graham's law of diffusion

$$\dfrac{r_1}{r_2} = \sqrt{\dfrac{M_2}{M_1}}$$

$$\dfrac{r_1}{r_2} = \sqrt{\dfrac{32}{64}}$$

$$= \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$$

or $\qquad = \dfrac{1}{1.414}$

$$r_1 : r_2 = 1 : 1.414$$

5. (a) Elevation in boiling point $\Delta T_b = K_b \times m$

Dipression in freezing point

$$\Delta T_f = K_f \times m$$

$$\dfrac{\Delta T_b}{\Delta T_f} = \dfrac{K_b}{K_f}$$

$$\dfrac{\Delta T_b}{0.186} = \dfrac{0.512}{1.86}$$

$$\Delta T_b = 0.0512°C$$

6. (a) Species : $O_2^{2-}, O_2^-, O_2, O_2^+$

Bond Order : 1   1.5   2   2.5

7. (a) In hydrolysis of a salt of weak acid and strong base.

$$A^- + H_2O \rightleftharpoons HA + OH^-$$

the hydrolysis constant K_h

$$K_h = \dfrac{[HA][OH^-]}{[A^-]}$$

(According to law of mass action)

($\because$ conc. of water remains constant)

For the ionisation of weak acid HA,

we has $HA \rightleftharpoons H^+ + A^-$

$$K_a = \dfrac{[H^+][A^-]}{[HA]}$$

Also we know that

$$\dfrac{K_w}{K_a} = \dfrac{[H^+][OH^-][HA]}{[H^+][A^-]}$$

$$\dfrac{K_w}{K_a} = \dfrac{[OH^-][HA]}{[A^-]}$$

or $\qquad \dfrac{K_w}{K_a} = K_h$

8. (c) $aA + bB \longrightarrow$ Products

Rate of reactant for A $= -\dfrac{1}{a} \dfrac{d[A]}{dt}$

Rate of reactant for B $= -\dfrac{1}{b} \dfrac{d[B]}{dt}$

the $-\dfrac{d[A]}{dt}$ is equal to $= -\dfrac{a}{b} \dfrac{d[B]}{dt}$

9. (a) Number of moles of hydrogen

$$(H_2) = \dfrac{\text{Weight in gram of } H_2}{\text{Molecular weight of } H_2}$$

Weight of H_2 in gram = 1g

molecular weight of $H_2 = 2$

Number of moles of $H_2 = \dfrac{1}{2} = 0.5$ moles

Number of moles of

$$O_2 = \frac{\text{Weight in g of } O_2}{\text{Molecular weight of } O_2} = \frac{8}{32}$$

$$= 0.25 \text{ moles}$$

Mole fraction of

$$H_2 = \frac{\text{Number of moles of } H_2}{\text{Total number of moles}}$$

$$= \frac{0.5}{0.25 + 0.5} = 0.667$$

10. (c) $\overset{+7}{MnO_4^-} \to Mn^{+2}$ (Acid medium). Change in oxidation number = + 5 Equivalent weight of $KMnO_4 = \dfrac{\text{Molecular weight of } KMnO_4}{\text{Change in oxidation number}}$

$$= \frac{M}{5}$$

11. (a) For the reaction

$$CH_3COOH \text{ (l)} + 2O_2(g) \rightleftharpoons 2CO_2 \text{ (g)} + 2H_2O \text{ (l)}$$

$$\Delta H = -874 \text{ kJ}$$

We know that $\Delta H = \Delta E + P\Delta V$

From ideal gas equation $P\Delta V = \Delta nRT$
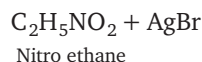
Then $\Delta H = \Delta E + \Delta nRT$

Number of moles of reactant

$$= \text{Number of moles of products}$$

$\therefore \qquad\qquad \Delta n = 0$

Then $\qquad\qquad \Delta H = \Delta E$

$$\Delta E = -874 \text{ kJ}$$

12. (d) The electrophile involved in the sulphonation of benzene is SO_3

$$2H_2SO_4 \to SO_3 + H_3O^+ + HSO_4^-$$

13. (a) In heterogeneous catalysis, adsorption lowers the activation energy of the reaction.

14. (d) $C_2H_5Br + AgNO_2(\text{alc.}) \to$

$$C_2H_5NO_2 + AgBr$$
$$\text{Nitro ethane}$$

15. (b) Specific conductivity

$$(K) = \frac{1}{R} \times \text{Cell constant}$$

Cell constant = $K \times R$

$$= 0.0129 \times 100 = 1.29$$

16. (a) Cell $Zn \,|\, Zn^{2+} \,||\, Fe^{2+} \,|\, Fe$

Electrode potential for

$$Zn \,/\, Zn^{2+} = 0.763 \text{ V}$$

Electrode potential for

$$Fe^{2+}/Fe = -0.44 \text{ V}$$

EMF of cell = $E^\circ_{\text{oxidation (Fe)}} + E^\circ_{\text{reduction (Zn)}}$

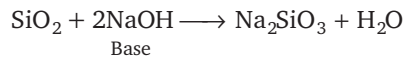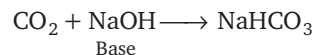$$= -0.44 + 0.763$$

$$= 0.323 \text{ V}$$

17. (a) The electronic configuration of $_{24}C_r$ is $[Ar]_{18} \, 3d_1^5 4s^1$. Chromium after loss of one electron gain stable configuration due to presence of half filled d-orbitals, therefore its second ionisation enthalphy is highest.

18. (a) If pH is less then 7, the solution is acidic but if the pH is more than 7 the solution becomes basic.

$$0 \underset{\text{Acid}}{\rule{1.5cm}{0.4pt}} 7 \underset{\text{Base}}{\rule{1.5cm}{0.4pt}} 14$$
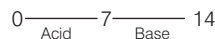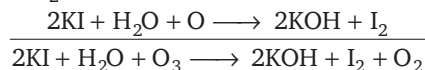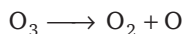
If alkali is added in acid solution the pH increases.

19. (a) CO_2 is a gas, while SiO_2 is a solid but both are acidic in nature, both reacts with base and forms carbonate and silicates.

$$CO_2 + \underset{\text{Base}}{NaOH} \longrightarrow NaHCO_3$$

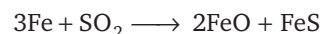$$SiO_2 + 2\underset{\text{Base}}{NaOH} \longrightarrow Na_2SiO_3 + H_2O$$

20. (c) HNO_3 is a strong oxidising agent. So it oxidise iron and form a layer of Fe_3O_4 that works as protective layer.

21. (b) O_3 (Ozone) works as oxidising agent. It reacts with potassium iodide (KI) solution and liberate iodine vapours that turns starch paper blue.
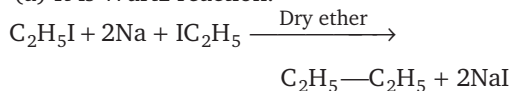
$$O_3 \longrightarrow O_2 + O$$

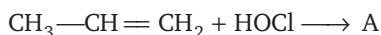$$\frac{2KI + H_2O + O \longrightarrow 2KOH + I_2}{2KI + H_2O + O_3 \longrightarrow 2KOH + I_2 + O_2}$$

22. (c) It is the oxidation process.

$$3Fe + SO_2 \longrightarrow 2FeO + FeS$$

Fe is oxidised to form FeO and FeS.

23. (a) It is Wurtz reaction.

$$C_2H_5I + 2Na + IC_2H_5 \xrightarrow{\text{Dry ether}}$$
$$C_2H_5\text{—}C_2H_5 + 2NaI$$

So, C_4H_{10} is formed.

24. (b) For the reaction

$$CH_3\text{—}CH = CH_2 + HOCl \longrightarrow A$$

This is addition reaction, this reaction involves in unsaturated compound or that compound which has multiple bonds ($=$ or $\equiv$ bonds)

$$CH_3\text{—}CH = CH_2 + HO\overset{+}{C}l \longrightarrow$$

$$\begin{array}{c} CH_3\text{—}CH\text{—}CH_2 \\ \quad\quad | \quad\quad | \\ \quad\quad OH \quad Cl \end{array}$$
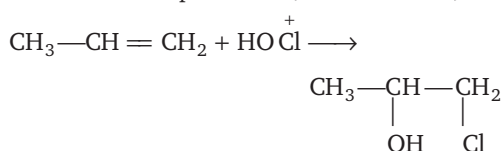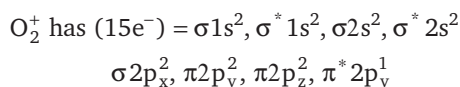
According to Markownfikoff's rule, the –ve part of reagent is added to that carbon atom of the double bond which bears the least number of hydrogen atom.

25. (d) Ionic crystals have no directional bonds as the oppositely charged ions are attracted by electrostatic forces and arranged in a definite ratio.

26. (c) The sum of protons and neutrons give the mass number. The number of neutrons and protons are not equal.

27. (a) According to the modern periodic table. If the last electron of element is placed in s-orbital, then elements are known as s-block element. Group IA has elements with ns^1 configuration and IIA has elements with ns^2 configuration.

28. (b) Azimuthal quantum number l for d orbitals is 2. so the value of magnetic quantum number is $\quad\quad -l$ to $+l$

then $\quad\quad -2$ to $+2 -2, -1, 0, +1, +2$

29. (a) $\quad$ F—$\overset{..}{\underset{..}{Xe}}$—F sp^3d and linear

$\quad$ Cl—$\overset{..}{\underset{..}{I}}$—F $\quad sp^3d$ and linear

$\quad$ $\overset{..}{Sb}$ $\quad sp^3$ and pyramidal
$\quad$ $\diagup | \diagdown$
$\quad$ Cl Cl Cl

$\quad$ Cl—$\overset{..}{\underset{..}{Ba}}$—Cl $\quad sp$ and linear

$\quad$ $\overset{..}{Te}{\bf :}$
$\quad$ $\diagup \quad \diagdown$
$\quad$ F $\quad\quad$ F $\quad sp^3$ and Y-shape

30. (a) $O_2^+ \longrightarrow O_2^{2+} + e^-$

O_2^+ has $(15e^-) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$
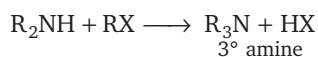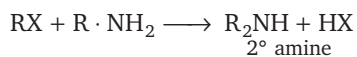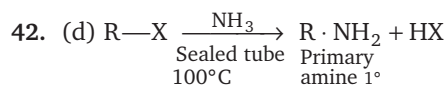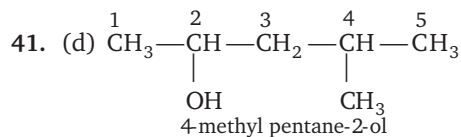$\quad\quad \sigma 2p_x^2, \pi 2p_y^2, \pi 2p_z^2, \pi^* 2p_y^1$

So O_2^+ lost a electron to form O_2^{2+}, the e^- is from antibonding π orbital.

31. (b) Structure II shows A to be electropositive. Hence, It will have least contribution.

32. (b) $NaOH + 2S \longrightarrow Na_2SO_3 + Na_2S + H_2O$
$\quad\quad\quad\quad\quad\quad\quad$ Sodium sulphite

33. (d) $SiF_4 + 4H_2O \longrightarrow Si(OH)_4 + 4HCl$
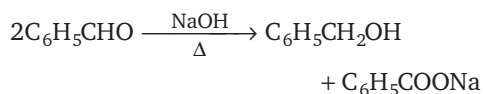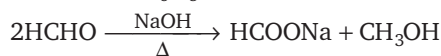$\quad\quad\quad\quad\quad\quad\quad\quad$ Silicic acid

The trend towards hydrolysis decreases down the group. Silicon have vacant orbital to which water molecules can co-ordinate and hence their halides are hydrolysed by water.

34. (a) For extraction of metal only those minerals are used (ores) for which the process is economic.

35. (a) P_2O_5 is an acidic oxide, acting as an acidic flux.

36. (b) Basic Oxide of metals are basic in nature and form basic hydroxide with water.

37. (c) CO_2 gas is passed in $Ca(OH)_2$ solution the solution becomes milky white to form $CaCO_3$ and if the CO_2 is passed in excess the milky white solution become colourless due to the formation of $Ca(HCO_3)_2$.

$$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 \downarrow + H_2O$$

38. (b) $(NH_4)_2SO_4 \cdot Fe_2(SO_4)_3 \cdot 24H_2O$

39. (c) Transition elements have three series 3d, 4d and 5d.

3d series has elements 21 (Sc) to 30 (Zn) atomic number. 4d series has elements 39 (Y) to 48 (Cd) atomic number.

$[Kr]4d^{1-10}, 5s^{1-2}$ has IInd row series.

40. (d) This complex is of the type MA_4B_2 with co-ordination number 6. It shows geometrical isomerism. Cis form is violet coloured and trans form is green.

41. (d)

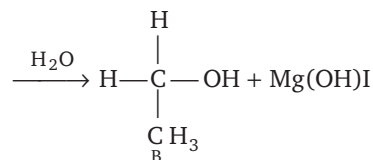$$\overset{1}{C}H_3 - \overset{2}{C}H - \overset{3}{C}H_2 - \overset{4}{C}H - \overset{5}{C}H_3$$

with OH on C2 and CH_3 on C4

OH CH_3

4-methyl pentane-2-ol

42. (d) $R{-}X \xrightarrow[\substack{\text{Sealed tube} \\ 100°C}]{NH_3} \underset{\substack{\text{Primary} \\ \text{amine } 1°}}{R \cdot NH_2} + HX$

$RX + R \cdot NH_2 \longrightarrow \underset{2° \text{ amine}}{R_2NH} + HX$

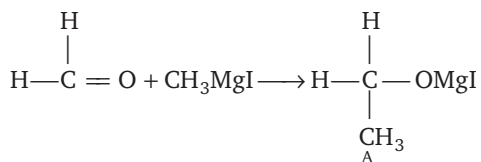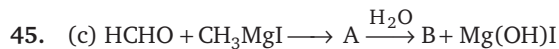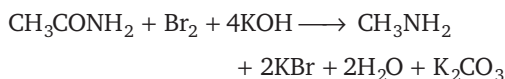$R_2NH + RX \longrightarrow \underset{3° \text{ amine}}{R_3N} + HX$

43. (c) Cannizaro reaction is possible in which aldehyde where the α-hydrogen atoms are absent.

e.g. $HCHO, C_6H_5CHO$

$2HCHO \xrightarrow[\Delta]{NaOH} HCOONa + CH_3OH$

$2C_6H_5CHO \xrightarrow[\Delta]{NaOH} C_6H_5CH_2OH$

$+ C_6H_5COONa$

44. (d) This is Hoffmann's bromide reaction. In this reaction, product compound has one less 'C' atom than reactant.

So it causes shortening of 'C' chain.

$-CONH_2 \longrightarrow -NH_2$

$CH_3CONH_2 + Br_2 + 4KOH \longrightarrow CH_3NH_2$

$+ 2KBr + 2H_2O + K_2CO_3$

45. (c) $HCHO + CH_3MgI \longrightarrow A \xrightarrow{H_2O} B + Mg(OH)I$

$$H{-}\underset{}{\overset{}{C}} = O + CH_3MgI \longrightarrow \underset{A}{H - \overset{\displaystyle H}{\underset{\displaystyle CH_3}{C}} - OMgI}$$

$$\xrightarrow{H_2O} \underset{B}{H - \overset{\displaystyle H}{\underset{\displaystyle CH_3}{C}} - OH} + Mg(OH)I$$

46. (a) $\underset{\substack{\text{Secondary} \\ \text{amine}}}{R_2NH} + \underset{\substack{\text{Acetyl} \\ \text{chloride}}}{CH_3COCl} \longrightarrow \underset{\substack{N, N, \text{dialkyl} \\ \text{acetamide}}}{R_2NCO \cdot CH_3}$
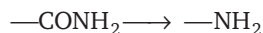
47. (b) Reducing agent Sn/HCl is used.
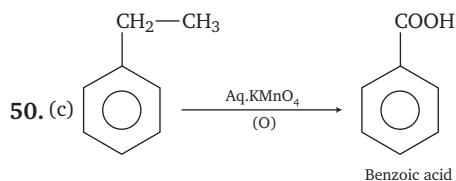
It changes $-NO_2$ group into $-NH_2$.

48. (b) Acetyl salicylic acid is used as analgesic drug. It is also known as aspirin.

49. (c) Benzene, $(4n + 2)\pi$ electrons

$n = 1$ (6π electrons)

Naphthalene, $(4n + 2)\pi$; $n = 2$, 10π electrons

Both obey Huckel's rule.

50. (c)

Benzoic acid

MATHEMATICS

1. (a) Let θ be the angle between the given pair of straight lines

$$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0 \qquad \text{...(i)}$$

∴ $\qquad \theta = \tan^{-1}\dfrac{1}{3} \qquad$ (given)

or $\qquad \tan\theta = \dfrac{1}{3}$

We know that,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} \qquad \text{...(ii)}$$

where, $\qquad h = -\dfrac{3}{2},\ a = 1,\ b = \lambda$

put these value in Eq. (ii), we get

$$\frac{1}{3} = \frac{2\sqrt{\dfrac{9}{4} - \lambda \cdot 1}}{1 + \lambda}$$

⇒ $\qquad (1 + \lambda) = 3\sqrt{9 - 4\lambda}$

On squaring both sides, we get

$$(1 + \lambda)^2 = (3\sqrt{9 - 4\lambda}\,)^2$$

$$1 + \lambda^2 + 2\lambda = 81 - 36\lambda$$

⇒ $\qquad \lambda^2 + 38\lambda - 80 = 0$

⇒ $\qquad \lambda^2 + 40\lambda - 2\lambda - 80 = 0$

⇒ $\ \lambda(\lambda + 40) - 2(\lambda + 40) = 0$

⇒ $\qquad (\lambda - 2)\,(\lambda + 40) = 0$

⇒ $\qquad \lambda = 2, -40 \ \Rightarrow\ \lambda = 2$

$\qquad$ (∵ 'λ' is non-negative real number)

2. (a) Given that equation of lines are

$$2x - 3y = 4 \qquad\qquad \text{...(i)}$$

and $\qquad x + y = 1$

or $\qquad x + y - 1 = 0 \qquad\qquad \text{...(ii)}$

Now, let any line parallel to $x + y - 1 = 0$ is given by $x + y + \lambda = 0$ which passes through (1, 1).

∴ $\qquad 1 + 1 + \lambda = 0 \ \Rightarrow \lambda = -2$

∴ Required line is $x + y - 2 = 0$

or $\qquad x + y = 2 \qquad\qquad \text{...(iii)}$

∴ Point of intersection of lines (i) and (iii) is (2, 0).

∴ Required distance

$$= \sqrt{(2 - 1)^2 + (0 - 1)^2} = \sqrt{1 + 1} = \sqrt{2}$$

3. (c) The equation of lines are $x + y = 0$ and $x - y = 0$

∴ The equation of bisectors of the angles between these lines are

$$\frac{x + y}{\sqrt{1 + 1}} = \pm\,\frac{x - y}{\sqrt{1 + 1}}$$

⇒ $\qquad x + y = \pm\,(x - y) \qquad \text{...(ii)}$

Taking + ve sign, we get $y = 0$.

Taking – ve sign, we get $x = 0$.

Hence the equation of bisectors are

$$x = 0,\ y = 0\,.$$

4. (c) Given that, the base of an isosceles triangle PQR are Q(1, 3) and R(– 2, 7). Let the co-ordinates of point P are (x, y).
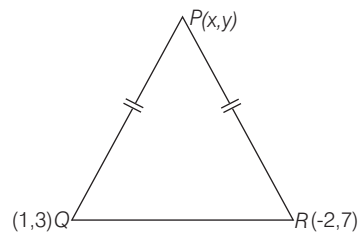
∵ In an isosceles triangle PQ = PR

i.e. $\quad \sqrt{(x - 1)^2 + (y - 3)^2}$

$$= \sqrt{(x + 2)^2 + (y - 7)^2}$$

⇒ $(x - 1)^2 + (y - 3)^2 = (x + 2)^2 + (y - 7)^2$

⇒ $x^2 - 2x + 1 + y^2 - 6y + 9$

$$= x^2 + 4x + 4 + y^2 - 14y + 49$$

⇒ $- 2x - 6y + 10 = 4x - 14y + 49 + 4$

⇒ $\qquad - 6x + 8y = 43$

Here, option 'c' (5/6, 6) satisfies the equation.

∴ Co-ordinate of P are (5/6, 6).

5. (b) Given that normal cuts the circle at points $A(3, 4)$ and $B(-1, -2)$. As we know, normal line is always passing through the centre of the circle and AB is the diameter of the circle. Therefore, A and B are the end points of a diameter.

∴ Required equation of circle is

$(x - 3)(x + 1) + (y - 4)(y + 2) = 0$

$\Rightarrow x^2 - 2x - 3 + y^2 - 2y - 8 = 0$

$\Rightarrow \quad x^2 + y^2 - 2x - 2y - 11 = 0$

6. (b) Given that, $\cos P = \dfrac{1}{7}, \cos Q = \dfrac{13}{14}$

∵ $\sin P = \sqrt{1 - \cos^2 P} = \sqrt{1 - \dfrac{1}{49}} = \dfrac{1}{7}\sqrt{48}$

Now, $\sin Q = \sqrt{1 - \cos^2 Q} = \sqrt{1 - \dfrac{169}{196}} = \dfrac{\sqrt{27}}{14}$

Now,

$\cos(P - Q) = \cos P \cos Q + \sin P \sin Q$

$\cos(P - Q) = \dfrac{1}{7} \times \dfrac{13}{14} + \dfrac{1}{7}\sqrt{48} \times \dfrac{1}{14}\sqrt{27}$

$\qquad = \dfrac{13}{98} + \dfrac{36}{98} = \dfrac{49}{98} = \dfrac{1}{2}$

$\Rightarrow \quad \cos(P - Q) = \dfrac{1}{2}$

$\Rightarrow \quad \cos(P - Q) = \cos 60°$

$\therefore \qquad P - Q = 60°$

7. (d) Given that $3\cos x + 4\sin x = 6$

Since the maximum value of

$(3\cos x + 4\sin x) = +\sqrt{3^2 + 4^2} = 5$. Hence there is no 'x' satisfying $3\cos x + 4\sin x = 6$.

∴ No solution, as $|\sin x| \leq 1, |\cos x| \leq 1$ and both of them do not attain their maximum value for the same angle.

Alternate Method, $3\cos x + 4\sin x = 6$

Let $\qquad r\cos\theta = 3$

and $\qquad r\sin\theta = 4$

$\Rightarrow \qquad\qquad \theta = \tan^{-1}\left(\dfrac{4}{3}\right)$

$\Rightarrow \quad 5(\cos\theta \cos x + \sin\theta \sin x) = 6$

$\Rightarrow \cos(\theta - x) = \dfrac{6}{5}$ or $\cos(x - \theta) = \dfrac{6}{5}$

or $\cos(x - \theta) > 1$ but $-1 \leq \cos x \leq 1$.

∴ There exist no solution in $[-1, 1]$.

8. (d) Given that, $\sec^{-1} x = \text{cosec}^{-1} y$

or $\qquad \cos^{-1}\left(\dfrac{1}{x}\right) = \sin^{-1}\left(\dfrac{1}{y}\right)$

On adding $\cos^{-1}\left(\dfrac{1}{y}\right)$ on both sides, we get

$\cos^{-1}\left(\dfrac{1}{x}\right) + \cos^{-1}\left(\dfrac{1}{y}\right) = \sin^{-1}\left(\dfrac{1}{y}\right) + \cos^{-1}\left(\dfrac{1}{y}\right)$

$\Rightarrow \cos^{-1}\left(\dfrac{1}{x}\right) + \cos^{-1}\left(\dfrac{1}{y}\right) = \dfrac{\pi}{2}$

$\qquad\qquad \left[\because \sin^{-1}\theta + \cos^{-1}\theta = \dfrac{\pi}{2}\right]$

9. (b) Given that n be any integer, then $n(n + 1)(2n + 1)$ always an integral multiple of even numbers (i.e. 6, 30, 84, ...)
when

$n = 1, n(n + 1)(2n + 1) = 1(2)(3) = 6$

$n = 2, n(n + 1)(2n + 1) = 2(3)(5) = 30$

and so on.

10. (b) We have, $\tan\theta = -\dfrac{4}{3}$
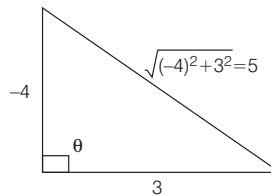
∵ $\tan\theta$ is less than 0 and $0 < \theta < \dfrac{\pi}{2}$

or $\qquad \dfrac{3\pi}{2} < \theta < 2\pi$.

∴ $\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = -\dfrac{4}{5}$ (i.e. $-$ve)

But, θ lies also between $\dfrac{3\pi}{2}$ and 2π

∴ $\qquad\qquad \sin\theta = \dfrac{4}{5}$ also.

11. (d) Given that $c = 2\cos\theta$

$$\therefore \Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix} = \begin{vmatrix} 2\cos\theta & 1 & 0 \\ 1 & 2\cos\theta & 1 \\ 6 & 1 & 2\cos\theta \end{vmatrix}$$

On expanding, we get

$$\Delta = 2\cos\theta\,[(4\cos^2\theta - 1)]$$
$$\qquad - 1[(2\cos\theta - 6)] + 0[(1 - 12\cos\theta)]$$
$$= 8\cos^3\theta - 2\cos\theta - 2\cos\theta + 6$$
$$= 8\cos^3\theta - 4\cos\theta + 6$$

12. (a) The equation of parabola whose vertex is $(-1, -2)$ is

$$(x + 1)^2 = 4a(y + 2)$$

which passes through $(3, 6)$

i.e. $16 = 4a \times 8 \implies a = \dfrac{1}{2}$

$\therefore$ Required equation is

$(x + 1)^2 = 2(y + 2)$ or $x^2 + 2x - 2y - 3 = 0$

13. (c) Given that equation of conic is

$$9x^2 + 4y^2 - 6x + 4y + 1 = 0$$
$$\implies \quad 9x^2 - 6x + 4y^2 + 4y + 1 = 0$$
$$\implies \quad 9\left(x - \frac{1}{3}\right)^2 + 4\left(y + \frac{1}{2}\right)^2 = 1$$

or, $\quad \dfrac{\left(x - \frac{1}{3}\right)^2}{\frac{1}{9}} + \dfrac{\left(y + \frac{1}{2}\right)^2}{\frac{1}{4}} = 1$

Comparing it with standard equation of ellipse $\dfrac{X^2}{a^2} + \dfrac{Y^2}{b^2} = 1$, we get $a = \dfrac{1}{3}$, $b = \dfrac{1}{2}$

Length of major axis $= 2b = 2 \times \dfrac{1}{2} = 1$

and length of minor axis $= 2a = 2 \times \dfrac{1}{3} = \dfrac{2}{3}$

14. (d) Given that, $f(x) = \cot^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$

and $\quad g(x) = \cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

Put $x = \tan\theta$ in $f(x)$, we get

$$f(x) = \cot^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$
$$\implies \quad f(x) = \dfrac{\pi}{2} - 3\tan^{-1}x$$

Again put $x = \tan\theta$ in $g(x)$, we get

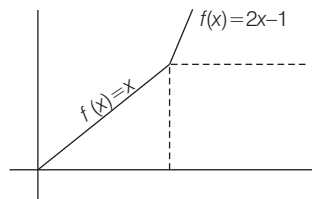$$g(x) = \cos^{-1}\left(\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$$
$$g(x) = 2\tan^{-1}x$$

$\therefore \displaystyle\lim_{x \to a} \dfrac{f(x) - f(a)}{g(x) - g(a)}$

$$= \lim_{x \to a} \dfrac{\left(\dfrac{\pi}{2} - 3\tan^{-1}x\right) - \dfrac{\pi}{2} + 3\tan^{-1}a}{2\tan^{-1}x - 2\tan^{-1}a}$$
$$= \lim_{x \to a} \dfrac{-3(\tan^{-1}a - \tan^{-1}x)}{2(\tan^{-1}a - \tan^{-1}x)} = -\dfrac{3}{2}$$

15. (c) Given that, $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2x - 1, & 1 < x \end{cases}$

From graph $f(x)$ is continuous, $\forall \, x \in R$.

i.e. $f'(x) = 1$ when $0 \le x \le 1$

and $f'(x) = 2 - 0$ when $x > 1$.

$\therefore$ Left hand limit $\ne$ Right hand limit

$\therefore$ At $x = 1$, $f(x)$ is continuous but not differentiable.

16. (c) We have, $y = \displaystyle\lim_{x \to -2} \dfrac{\sin^{-1}(x + 2)}{x^2 + 2x}$

$$\implies \quad y = \lim_{x \to -2} \dfrac{\sin^{-1}(x + 2)}{(x + 2)} \times \dfrac{1}{x}$$
$$\implies \quad y = \lim_{x \to -2} (1) \times \dfrac{1}{x} \quad \left[\because \lim_{x \to \theta} \dfrac{\sin\theta}{\theta} = 1\right]$$
$$= -\dfrac{1}{2}$$

17. (b) Given that, $f(x) = \begin{cases} x^p \cos\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$f(x)$ will be differentiable at $x = 0$, if

$\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$ exists finitely.

$\Rightarrow \quad \lim\limits_{x \to 0} \dfrac{x^p \cos\left(\dfrac{1}{x}\right)}{x}$ exists finitely

$\Rightarrow \quad \lim\limits_{x \to 0} x^{p-1} \cos\left(\dfrac{1}{x}\right)$ exists finitely

$\Rightarrow \qquad p - 1 > 0 \quad \Rightarrow \quad p > 1$

18. (b) We have, $f(x) = 3\,|\,2 + x\,|$

as $f(x) = |\,x\,| \Rightarrow f'(x) = \dfrac{x}{|\,x\,|}$

Now, $f(x) = 3\,|\,x + 2\,|$

$\Rightarrow \quad f(x) = 3\,\dfrac{x + 2}{|\,x + 2\,|} = -3 \qquad$ (as $x_0 = -3$)

19. (a) Given that, $f(x) = \log_5 (\log_7 x)$

or it can be written as

$f(x) = \dfrac{\log_e (\log_7 x)}{\log_e 5} \quad \left[\because \log_a b = \dfrac{\log b}{\log a}\right]$

On differentiating with respect to x, we get

Now, $f'(x) = \dfrac{1}{\log_e 5}\left[\dfrac{1}{\log_7 x} \times \dfrac{d}{dx}(\log_7 x)\right]$

$= \dfrac{1}{\log_e 5} \times \dfrac{1}{\log_7 x} \times \dfrac{1}{\log_e 7} \cdot \dfrac{1}{x}$

$= \dfrac{1}{x(\log 5)(\log 7)(\log_7 x)}$

20. (d) Here, $z^{1/3} = a - ib$

$\Rightarrow z = (a - ib)^3$

$\Rightarrow z = a^3 - 3a^2(ib) + 3a(ib)^2 - (ib)^3$

$\Rightarrow x + iy = (a^3 - 3b^2 a) + i(-3a^2 b + b^3)$

Equating real and imaginary parts, we get

$x = a(a^2 - 3b^2)$ and $y = b(b^2 - 3a^2)$

Now, $\dfrac{a(a^2 - 3b^2)}{a} - \dfrac{b(b^2 - 3a^2)}{b}$

$= k(a^2 - b^2)$

$\Rightarrow 4a^2 - 4b^2 = k(a^2 - b^2) \Rightarrow k = 4$

21. (a) We have, $1 + |\,e^x - 1\,| = e^x(e^x - 2)$

$1 + 1 + |\,e^x - 1\,| = e^{2x} - 2e^x + 1$

$\qquad\qquad\qquad = (e^x - 1)^2$

$1 + 1 + |\,e^x - 1\,| = |\,e^x - 1\,|^2$

Let $(e^x - 1) = y$, then

$y^2 - y - 2 = 0 \quad \Rightarrow \quad y = 2, -1$

$\Rightarrow \quad |\,e^x - 1\,| = 2 \quad \Rightarrow \quad e^x - 1 = \pm 2$

$\Rightarrow \qquad e^x = 1 \pm 2 \Rightarrow e^x = 3, -1$

or $\qquad e^x = 3 \qquad \Rightarrow x = \log_e 3$

$\therefore$ There is one real solution of the equation.

22. (b) Given that,

$\qquad\qquad u = x^2 + y^2$

and $\qquad x = s + 3t,\ y = 2s - t$

Now, $\qquad u = x^2 + y^2$

$\qquad\qquad = (s + 3t)^2 + (2s - t)^2$

$\qquad\qquad = s^2 + 9t^2 + 6st + 4s^2 + t^2 - 4st$

$\Rightarrow \qquad u = 5s^2 + 2st + 10t^2$

On differentiating with respect to s, we get

$\dfrac{du}{ds} = 10s + 2t$

Again differentiating $\dfrac{d^2 u}{ds^2} = 10$

23. (a) Given equations are

$\qquad\qquad x^2 + px + q = 0 \qquad \ldots(i)$

$\qquad\qquad x^2 + qx + p = 0 \qquad \ldots(ii)$

Let α be the common root, then

$\alpha^2 + p\alpha + q = 0$ and $\alpha^2 + q\alpha + p = 0$

$\Rightarrow \qquad \dfrac{\alpha^2}{p^2 - q^2} = \dfrac{\alpha}{q - p} = \dfrac{1}{q - p}$

$\Rightarrow \quad \alpha^2 = \dfrac{(p - q)(p + q)}{-(p - q)} = -(p + q) \ \ldots(iii)$

and $\quad \alpha = \dfrac{q - p}{q - p} = 1 \qquad\qquad \ldots(iv)$

$\therefore$ From Eq. (iii)

$$\alpha^2 = -p + q$$

$$\Rightarrow \quad (1)^2 = -(p + q) \quad \text{[from Eq. (iv)]}$$

$$\Rightarrow \quad p + q = -1 \Rightarrow p + q + 1 = 0$$

24. (a) Let α and β be the roots of the equation $x^2 - (a - 2)x + (a - 3) = 0$

$$\therefore \quad \alpha + \beta = (a - 2) \text{ and } \alpha\beta = a - 3$$

Now, $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$$= (a - 2)^3 - 3(a - 3)(a - 2)$$

$$= a^3 - 9a^2 + 27a - 26 = (a - 3)^3 + 1$$

It assumes the least value, if $(a - 3)^3 = 0$

$$\therefore \quad a = 3$$

25. (d) Given that, $|z| = \dfrac{1}{2}$ and $z_1 = \dfrac{1}{2} + \dfrac{\sqrt{3}\,i}{2}$

Here, one of the number must be a conjugate of $z_1 = \dfrac{1}{2} + \dfrac{\sqrt{3}\,i}{2}$ i.e. $z_2 = \dfrac{1}{2} - \dfrac{\sqrt{3}\,i}{2}$

and $z_2 = z_1 e^{-i\,2\pi/3}$

$$\Rightarrow \quad z_2 = \left(\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)\left[\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right]$$

$$= -2 \times \dfrac{1}{2} = -1$$

$$\left[\because (1 + i\sqrt{3})\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right) = -2\right]$$

26. (c) Given that, $z = \dfrac{-2}{1 + \sqrt{3}i} \times \dfrac{1 - \sqrt{3}i}{1 - \sqrt{3}i}$

$$= -\left(\dfrac{1 - i\sqrt{3}}{2}\right) = -\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}$$

$$\therefore \quad \theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{\sqrt{3}}{2} \times 2\right)$$

$$= \tan^{-1}(\sqrt{3}) = \dfrac{\pi}{3}$$

$$\therefore \quad \arg(z) = \pi - \theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}.$$

27. (a) Given that,

$$1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1$$

$$\therefore \quad 2(1 + \omega)(1 + \omega^2) + 3(2\omega + 1)(2\omega^2 + 1)$$

$$+ \ldots + (n + 1)(n\omega + 1)(n\omega^2 + 1)$$

$$= 2(1) + 3(3) + 4(7) + \ldots + (n^3 + 1)$$

Let $\quad T_n = n^3 + 1 \Rightarrow \Sigma T_n = \left[\dfrac{n(n + 1)}{2}\right]^2 + n$

28. (a) We have, $\arg\left(\dfrac{z - 1}{z + 1}\right) = k, (k \neq 0)$

Put $z = x + iy$, we get $\arg\left(\dfrac{x - 1 + iy}{x + 1 + iy}\right) = k$

$$\Rightarrow \arg\left[\dfrac{(x - 1 + iy)(x + 1 - iy)}{(x + 1 + iy)(x + 1 - iy)}\right] = k$$

$$\Rightarrow \arg\left[\dfrac{(x^2 - 1) + iy(2) + y^2}{(x + 1)^2 + y^2}\right] = k$$

$$\Rightarrow \arg\left[\dfrac{x^2 + y^2 - 1 + 2iy}{(x + 1)^2 + y^2}\right] = k$$

$$\Rightarrow \dfrac{2y}{x^2 + y^2 - 1} = k$$

$$\left[\because \arg(z) = \tan^{-1}\dfrac{y}{x}\right]$$

$$\Rightarrow (x^2 + y^2)k - k - 2y = 0$$

$$\Rightarrow kx^2 + ky^2 - 2y - k = 0$$

$$\Rightarrow x^2 + y^2 - \dfrac{2y}{k} - 1 = 0$$

Which represent a circle of centre $(0, 1/k)$ i.e. on y-axis.

29. (b) We have, $P(3, 4, 5), Q(4, 6, 3), R(-1, 2, 4)$ and $S(1, 0, 5)$.

$\therefore$ Direction ratios of PQ are $(1, 2, -2)$ and direction cosines of PQ are

$$\dfrac{1}{\sqrt{1^2 + 2^2 + (-2)^2}}, \dfrac{2}{\sqrt{1^2 + 2^2 + (-2)^2}},$$

$$\dfrac{-2}{\sqrt{1^2 + 2^2 + (-2)^2}}$$

$$= \dfrac{1}{\sqrt{9}}, \dfrac{2}{\sqrt{9}}, -\dfrac{2}{\sqrt{9}} = \dfrac{1}{3}, \dfrac{2}{3}, -\dfrac{2}{3}$$

$\therefore$ Required projection of RS on PQ is

$$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$$

$$= \dfrac{1}{3}(2) + \left(\dfrac{2}{3}\right)(-2) + \left(-\dfrac{2}{3}\right)(1)$$

$$= \dfrac{2}{3} - \dfrac{4}{3} - \dfrac{2}{3} = -\dfrac{4}{3}$$

30. (d) Let α, β, γ be the angles which the line makes with the positive directions at the axes of x, y and z respectively, then $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are called the direction cosines of the line and these are usually denoted by

$$l = \cos \alpha, \ m = \cos \beta, \ n = \cos \gamma$$

$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = l^2 + m^2 + n^2 = 1$

31. (a) Let $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

Here, $(a_1\hat{\mathbf{i}}) \cdot \hat{\mathbf{i}} = a_1 = 2$; $(a_2\hat{\mathbf{j}}) \cdot \hat{\mathbf{j}} = a_2 = 3$

and $(a_3\hat{\mathbf{k}}) \cdot \hat{\mathbf{k}} = a_3 = 6$

$\therefore \ \mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

Now, required length $= |\mathbf{a}| = \sqrt{2^2 + 3^2 + 6^2}$
$$= \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

32. (a) $(10011.1)_2 = 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1$
$$\times 2^1 + 1 \times 2^0 + 1 \times 2^{-1}$$
$$= 16 + 2 + 1 + 0.5 = (19.5)_{10}$$

33. (b)

2	60	
2	30	0
2	15	0
2	7	1
2	3	1
	1	1

$\therefore \qquad (60)_{10} = (111100)_2$

34. (d) $(\sim p \wedge q) \cap (\sim p \vee p)$ is true.

35. (c) Given that, $f(x) = \sin\left(\dfrac{\pi x}{n-1}\right) + \cos\left(\dfrac{\pi x}{n}\right)$

Let $f(x) = g(x) + p(x), \ \forall \ n > 2$

$\therefore$ Period of $g(x) = \dfrac{2\pi \times (n-1)}{\pi} = 2(n-1)$

and period of $p(x) = \dfrac{2\pi n}{\pi} = 2n$

Period of $f(x) = $ LCM of $p(x)$ and $q(x)$
$$= 2n(n-1)$$

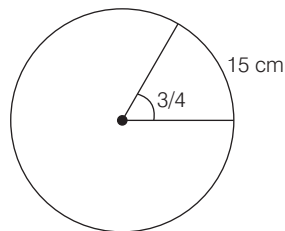**36.** (d) We have, $f(\theta) = \sec^2 \theta + \cos^2 \theta$

Since $\quad AM \geq GM$
$$\frac{\sec^2 \theta + \cos^2 \theta}{2} \geq (\sec^2 \theta \cdot \cos^2 \theta)^{1/2}$$

$\Rightarrow \qquad \dfrac{\sec^2 \theta + \cos^2 \theta}{2} \geq \sqrt{\sec^2 \theta \cdot \cos^2 \theta}$

$\Rightarrow \qquad \dfrac{\sec^2 \theta + \cos^2 \theta}{2} \geq 1 \left[\because \sec \theta = \dfrac{1}{\cos \theta}\right]$

$\Rightarrow \quad \sec^2 \theta + \cos^2 \theta \geq 2 \quad \Rightarrow \quad f(\theta) \geq 2$

$\therefore$ The value of $f(\theta)$ always lies in the interval $[2, \infty)$.

37. (b) Given that, arc = 15 cm.

$$\theta = \frac{3}{4} \text{ radian}$$

We know that, $\text{Angle} = \dfrac{\text{Arc}}{\text{Radius}}$

$\Rightarrow \qquad \text{Radius} = \dfrac{\text{Arc}}{\text{Angle}} = \dfrac{15 \times 4}{3} = 20$ cm

38. (c) $\dfrac{d}{dx}\{f_n(x)\} = \dfrac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= e^{f_{n-1}(x)} \dfrac{d}{dx}\{f_{n-1}(x)\} = f_n(x) \cdot \dfrac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= f_n(x) \cdot e^{f_{n-2}(x)} \cdot \dfrac{d}{dx}\{f_{n-2}(x)\}$

$= f_n(x) \cdot f_{n-1}(x) \dots f_2(x) \dfrac{d}{dx}\{f_1(x)\}$

$= f_n(x) \cdot f_{n-1}(x) \dots f_2(x) \cdot f_1(x) \cdot \dfrac{d}{dx} e^{f_0(x)}$

$= f_n(x) \cdot f_{n-1}(x) \dots f_2(x) \cdot f_1(x) \cdot \dfrac{d}{dx}(f_0(x))$

$= f_n(x) \cdot f_{n-1}(x) \dots f_2(x) \cdot f_1(x) \cdot \dfrac{d}{dx}(x)$

$$[\because f_0(x) = x \text{ (given)}]$$

$= f_n(x) \cdot f_{n-1}(x) \dots f_2(x) \cdot f_1(x).$

39. (a) We have, $3^x + 2^{2x} \geq 5^x$

$3^x + (2^2)^x \geq 5^x; \ 3^x + 4^x \geq 5^x$

$\Rightarrow \qquad \left(\dfrac{3}{5}\right)^x + \left(\dfrac{4}{5}\right)^x \geq 1$

$\Rightarrow \quad (\sin\theta)^x + (\cos\theta)^x \geq 1$

[by triangle inequality]

$\Rightarrow \qquad\qquad x \leq 2$

∴ Solution set is $(-\infty, 2]$

40. (c) We have, $\dfrac{x+1}{x^2+2} > \dfrac{1}{4}$

$\Rightarrow \quad x^2 + 2 - 4x - 4 < 0 \Rightarrow x^2 - 4x - 2 < 0$

$\Rightarrow \qquad\qquad (x-2)^2 - (\sqrt{6})^2 < 0$

$\Rightarrow \{(x-2) - \sqrt{6}\}(x - 2 + \sqrt{6}) < 0$

$\Rightarrow \qquad 2 - \sqrt{6} < x < 2 + \sqrt{6}$

$\Rightarrow \quad 2 - 2.449 < x < 2 + 2.449$

$\Rightarrow \qquad -0.449 < x < 4.449$

∴ In between − 0.445 and 4.449 five values of x lies, i.e. 0, 1, 2, 3, 4.

∴ There are 5 integral solution of inequality.

41. (d) Let sides of a triangle are p, q, r.

Applying sine rule $\dfrac{p}{\sin P} = \dfrac{q}{\sin Q} = \dfrac{r}{\sin R} = \lambda$

∵ $\qquad\qquad \cos P = \dfrac{\sin Q}{2\sin R}$

$\Rightarrow \quad \dfrac{q^2 + r^2 - p^2}{2qr} = \dfrac{q}{2r}$

$\Rightarrow \quad q^2 + r^2 - p^2 = q^2 \Rightarrow r^2 = p^2 \Rightarrow r = p$

∴ Triangle is isosceles.

42. (c) Given that, $f(x) = (a - x^n)^{1/n}$

∴ $\qquad f[f(x)] = [a - \{f(x)\}^n]^{1/n}$

$\qquad\qquad = [a - (a - x^n)]^{1/n} = x$

43. (d) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + 2x - 3}$

On differentiating with respect to x, we get

$f'(x) = \dfrac{(x^2 + 2x - 3)(2x - 3) - (x^2 - 3x + 2)(2x+2)}{(x^2 + 2x - 3)^2}$

$= \dfrac{\begin{array}{c}(2x^3 + 4x^2 - 6x - 3x^2 - 6x + 9) \\ -(2x^3 - 6x^2 + 4x + 2x^2 - 6x + 4)\end{array}}{(x^2 + 2x - 3)^2}$

$= \dfrac{5x^2 - 10x + 5}{(x^2 + 2x - 3)^2} > 0$

$= \dfrac{5(x-1)^2}{(x+3)^2(x-1)^2} > 0 = \dfrac{5}{(x+3)^2} > 0$

∴ f(x) is increasing in its domain.

44. (b) Given that,

$\sqrt{(x-3)^2 + (y-1)^2} + \sqrt{(x+3)^2 + (y-1)^2} = 6$

$\Rightarrow \sqrt{(x-3)^2 + (y-1)^2} = 6$

$\qquad\qquad\qquad - \sqrt{(x+3)^2 + (y-1)^2}$

On squaring both sides, we get

$(x-3)^2 + (y-1)^2 = 36 + (x+3)^2 + (y-1)^2$

$\qquad\qquad\qquad - 12\sqrt{(x+3)^2 + (y-1)^2}$

$\Rightarrow x^2 - 6x + 9 + (y^2 - 2y + 1)$

$\qquad\qquad = 36 + x^2 + 6x + 9 + y^2 - 2y + 1$

$\qquad\qquad\qquad - 12\sqrt{(x+3)^2 + (y-1)^2}$

$\Rightarrow (x+3) = \sqrt{(x+3)^2 + (y-1)^2}$

On squaring both sides of above equation

$\Rightarrow (x+3)^2 = [(x+3)^2 + (y-1)^2]$

$\Rightarrow (y-1)^2 = 0$

$\Rightarrow$ Which represents a pair of straight lines.

45. (a) Since, $|z_1| = |z_2| = |z_3| = 1$

we get $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

Now, $\quad 1 = \left| \dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} \right|$

$\qquad\qquad = |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = |\overline{z_1 + z_2 + z_3}|$

$\qquad\qquad = |z_1 + z_2 + z_3|$

46. (a) Unique solution of $x * a = b$ is taken by $x = b * a^{-1}$.

47. (d) We have, $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$ ...(i)

Put the value of $\pi = 3.141$

$\Rightarrow \quad \pi^2 = $ app. 9.6 in (i)

$\Rightarrow f(x) = \cos 9x + \cos 10x \qquad [\because \pi = 3.141]$

Put $\qquad\qquad\qquad x = \dfrac{\pi}{4}$,

$f\left(\dfrac{\pi}{4}\right) = \cos\dfrac{9\pi}{4} + \cos\dfrac{10\pi}{4} = \dfrac{1}{\sqrt{2}} + 0 = \dfrac{1}{\sqrt{2}}$

Now, $f(-\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

and $f(\pi) = \cos 9\pi + \cos 10\pi = 0$

$$\therefore \quad f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos\frac{10\pi}{2}$$

$$= \cos\frac{9\pi}{2} + \cos 5\pi = 0 + (-1) = -1$$

48. (a) Given that, $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$

Since, $\cos^2 x$ lies between 0 to 1.

i.e. $0 \le \cos^2 x \le 1 \,\forall\, x \in (-\infty, \infty)$

$\Rightarrow \qquad 0 \le \frac{\pi}{4}\cos^2 x \le \frac{\pi}{4}$

$\Rightarrow \qquad \sec 0 \le \sec\left(\frac{\pi}{4}\cos^2 x\right) \le \sec\frac{\pi}{4}$

$\Rightarrow \qquad 1 \le \sec\left(\frac{\pi}{4}\cos^2 x\right) \le \sqrt{2}$

49. (b) Given that, $f(x) = \dfrac{\sin^{-1}(3-x)}{\log(|x|-2)}$

From numerator of the function, we have

$\sin^{-1}(3-x)$, here $-1 \le 3-x \le 1$

$\Rightarrow \qquad -4 \le -x \le -2$

$\Rightarrow \qquad 2 \le x \le 4 \qquad \qquad \text{...(i)}$

From denominator of the function, we have

$\qquad |x|-2 > 1 \text{ or } |x| > 3$

$\Rightarrow \qquad x < -3 \cup x > 3 \qquad \text{...(ii)}$

From both cases, we get the domain of the given function.

i.e. $\quad 3 < x \le 4$ or $x \in (3, 4]$.

50. (c) $(1! + 2! + 3! + 4! + 5! + \ldots + 200!) \div 14$

$= 1 + 2 + 6 + 24 + 120 + 720 + (14M)$

$= 873 + 14M = 5 + 868 + 14M$

$= 5 + 14 \times 62 + 14M = 5 + 14(62 + M)$

Hence, when $1! + 2! + \ldots + 200!$ is divided by 14, remainder is 5.

51. (c) $\cos^{-1}[\cos\{2 \cdot \cot^{-1}(\sqrt{2} - 1\}]$

$= \cos^{-1}[\cos\{2 \times 67.5\}]]$

$= \cos^{-1}[\cos 135°] = 135° = \dfrac{3\pi}{4}$

52. (c) We have $f(x) = [x]\cos\left[\dfrac{2x-1}{2}\right]\pi$

Since, $g(x) = [x]$ is always discontinuous at all integral values of points. Hence, $f(x)$ is discontinuous for all integral points.

53. (a) Obtain the foot of the perpendicular $N(-4, 1, -3)$.

Find the equations of the line passing thorugh $P(2, 4, -1)$ and $N(-4, 1, -3)$.

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$

54. (b) Total number of mappings from a set A having elements into itself is n^n. And the total number of one to one mapping is $n!$.

$\therefore$ Required probability $= \dfrac{n!}{n^n}$.

55. (a) $\mathbf{a} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$,

$\mathbf{c} = -4\hat{\mathbf{i}} - \hat{\mathbf{k}}$

$\therefore \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b}$

$= (4+1)(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 15\hat{\mathbf{k}}$

56. (c) Since, total number of numbers 1 to 5 digits are 99999 and total number of numbers 1 to 4 digits are 9999.

Hence, the total number of numbers of exactly 5 digits $= 99999 - 9999 = 90000$.

57. (b) The maximum value of $3\cos\theta + 4\sin\theta$

$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

58. (a) The vertex connectivity of any tree is one.

59. (a) Since, $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = 0$

Also, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-collinear vectors, then $x = y = z = 0$.

60. (b) Since, 10, 3, e, 2 are in decreasing order.

$\therefore \log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$ are in increasing order.

61. (a) Now, $\dfrac{b-c}{r_1} + \dfrac{c-a}{r_2} + \dfrac{a-b}{r_3}$

$$= \frac{[(b-c)(s-a) + (c-a)(s-b) + (a-b)(s-c)]}{\Delta}$$

$= 0$

62. (d) Let $T = |\sin nx|$

Put $\quad n = 2$, then

$\qquad T = |\sin 2x| = |2\sin x\cos x| \le 2|\sin x|$

63. (c) Since, $(n + 1) p = \dfrac{101}{3}$ is not an integer.

Therefore, $p(X = r)$ is maximum when
$$r = \left[\dfrac{101}{3}\right] = 33.$$

64. (d) The equation of xy-plane is $z = 0$

∴ Direction cosines of its normal are 0, 0, 1.

65. (b) We have, $\int f(x)\, dx = F(x)$

∴ $\int x^3 f(x^2) dx = \dfrac{1}{2} \int x^2 f(x^2)\, d(x^2)$

66. (d) $\int_1^5 x^2\, dx$

$= \dfrac{4}{4}\left[\dfrac{1}{2} f(1) + f(2) + f(3) + f(4) + \dfrac{1}{2} f(5)\right]$

$= \dfrac{1}{2}[x^2 F(x^2) - \int F(x^2)\, d(x^2)]$

$= \left[\dfrac{1}{2} + 4 + 9 + 16 + \dfrac{25}{2}\right] = 42$ [using $f(x) = x^2$]

67. (c) Here, $2ae = 6$ and $2b = 8 \Rightarrow b = 4$

$\Rightarrow b^2 = 16 \Rightarrow a^2(1 - e^2) = 16$

$\Rightarrow \dfrac{9}{e^2}(1 - e^2) = 16 \Rightarrow 9 - 9e^2 = 16e^2$

$\Rightarrow 25e^2 = 9 \Rightarrow e = \dfrac{3}{5}$

68. (c) We know that, if $I_n = \int \sin^n x\, dx$, then

$I_n = \dfrac{\sin^{n-1} x \cos x}{n} + \dfrac{n-1}{n} I_{n-2}$

where n is a positive integer.

$\Rightarrow n I_n - (n-1) I_{n-2} = -\sin^{n-1} x \cos x$

69. (a) The given graph is a connected graph.

70. (a) We have, $5^{124} = (5^3)^{41} \cdot 5$

Now, $\quad 5^3 = 1 \pmod{124}$

∴ $\quad (5^3)^{41} = 1 \pmod{124}$

$\quad (5^3)^{41} \cdot 5 = 1 \pmod{124}$

$\Rightarrow \quad 5^{124} = 5 \pmod{124}$

GENERAL APTITUDE

16. (a) As 'kind' is related to 'judge', in the same way 'Throne' is related to 'Bench'.

17. (b) 'Roar' is the sound produced by 'Lion', similarly 'Bray' is the sound produced by 'Ass'.

18. (a) 'Frisk' is the name given to the movement of 'Lamb' similarly 'Scamper' is the name given to the movement of 'Mouse'.

19. (d) As 'Arc' is a part of 'Circle', in the same way 'Line' is a part of 'Square'.

20. (d) 'Brinjal' is a 'Vegetable', in the same way 'Orange' is a 'Fruit'.

21. (d) All except 'Reading' are related to motion.

22. (d) All except 'Bhagalpur' are in the state of Uttar Pradesh.

23. (d) Only Grapes grow in bunches and so it is odd one among others.

24. (a) Position of S from left = 19

∴ Position of S from right = $27 - 19 = 8$

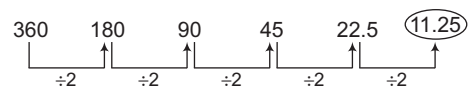**25.** (a) From CFILORUX, Position of L from left = 12

26. (c) 7th letter from right = $(27 - 7)$ th = 20th letter from left = T

27. (a) The pattern is as follows

∴ ? = 20

28. (b) The pattern is as follows

∴ ? = 11. 25

29. (b) Moving anti-clockwise, the pattern is as follows $15 + 1 = 16; 16 + 2 = 18$

$18 + 3 = 21$ and $\quad 21 + 4 = 25$

So, missing number $25 + 5 = 30$

30. (b) The pattern in clockwise direction

$2 \xrightarrow{\times 2} \boxed{4} \xrightarrow{\times 4} 16 \xrightarrow{\times 16} 256$ ∴ ? = 4