MET 2016 Question Paper with Solution

Manipal Entrance Test (MET)

Manipal

Engineering Entrance Exam

SOLVED PAPER 2016

Physics

1. Two particles of charges $+e$ and $+2e$ are at 16 cm away from each other. Where should another charge q be placed between them, so that the system remains in equilibrium?
(a) 24 cm from $+e$
(b) 12.23 cm from $+e$
(c) 80 cm from $+e$
(d) 6.63 cm from $+e$

2. How does the energy difference between the consecutive energy level vary as the quantum number n increases?
(a) Increases
(b) Decreases
(c) Remains unchanged
(d) First increases and then decreases

3. The ionisation potential of hydrogen atom is 13.6 eV. How much energy need to be supplied to ionise the hydrogen atom in the first excited state?
(a) 13.6 eV (b) 27.2 eV (c) 3.4 eV (d) 6.8 eV

4. If 10% of the main current is to be passed through the moving coil galvanometer of resistance 99 Ω, then the required shunt resistance will be
(a) 9.9 Ω
(b) 10 Ω
(c) 11 Ω
(d) 9 Ω

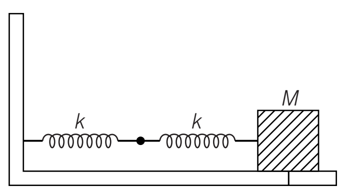
5. A mixture of two gases is contained in a vessel. The gas 1 is monoatomic and gas 2 is diatomic and the ratio of their molecular masses $M_1/M_2 = 1/4$. What is the ratio of root mean square speeds of the molecules of two gases?
(a) 2
(b) 4
(c) 8
(d) 16

6. A current carrying coil is bent sharply so as to convert it into a double loop both carrying current in the same direction. If B be the initial magnetic field at the centre, then what will be the final concentric magnetic field?
(a) 2 B
(b) 4 B
(c) 8 B
(d) Zero

7. White light reflected at normal incidence from a soap film has minima at 6500 Å and maxima at 7500 Å in the visible region with not minimum in between. If μ is $\frac{5}{3}$ for the, then film, the thickness of film is
(a) 7.40×10^{-7} cm
(b) 9.75×10^{-5} mm
(c) 9.70×10^{-7} cm
(d) 9.75×10^{-4} mm

8. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be
(a) $n B$ (b) $n^2 B$ (c) $2nB$ (d) $2n^2 B$

9. Two springs are connected to a block of mass M placed on a frictionless surface as shown below. If both the springs have a spring constant k, then the frequency of oscillation of the block is



(a) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{M}{k}}$

10. Four identical balls, A, B, C and D are placed on a smooth horizontal plane such that B, C and D are in one line and are in contact and A is kept along the same line but a distance away. Now, A is set in motion with speed v along the line and it strikes the ball B. After this collision, which is elastic.

(a) Balls A, B, C will be at rest and D will move with speed v

(b) A will be at rest, where B, C and D will move together with speed $v/3$

(c) A will rebound with speed $v/2$ while B, C and D will move together with speed $v/2$

(d) A, B, C and D will move together with speed $v/4$

11. Two coils X and Y are placed in a circuit such that a current changes by 3 A in coil X and magnetic flux changes of 1.2 Wb occurs in Y. The value of mutual inductance of the coils is

(a) 0.2 H (b) 0.4 H

(c) 0.6 H (d) 3.6 H

12. A projectile has the maximum range 500 m. If the projectile is thrown up an inclined plane of 30° with the same (magnitude) velocity, the distance covered by it along the inclined plane will be

(a) 250 m (b) 500 m

(c) 400 m (d) 1000 m

13. Nuclear reactor in which U-235 is used as fuel. Uses 2 kg of U-235 in 30 days. Then, power output of the reactor will be (given energy released per fission = 185 MeV)

(a) 43.5 MW (b) 58.5 MW

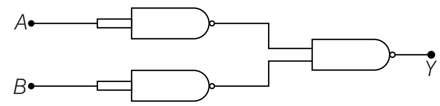
(c) 69.6 MW (d) 73.1 MW

14. Two capacitors of capacitances $3\,\mu F$ and $6\,\mu F$ are charged to a potential of 12 V each. They are now connected to each other, with the positive plate of each joined to the negative plate of the other. The potential difference across each will be

(a) 4 V (b) 12 V

(c) zero (d) 3 V

15. The combination of gates shown in the figure below produces



(a) NOR gate (b) OR gate

(c) AND gate (d) XOR gate

16. A thin dielectric rod of length l lies along X-axis. One end of the rod is placed at the origin and the other end of the rod is placed at $(l, 0)$. A total charge Q is distributed uniformly along the length of the rod. What is the potential at a point $(x, 0)$ when $x > l$?

(a) $\frac{Q}{4\pi\varepsilon_0}\cdot\log_e\left(\frac{x}{x-1}\right)$ (b) $\frac{Q}{4\pi\varepsilon_0 l}\cdot\log_e\left(\frac{x}{l}\right)$

(c) $\frac{Q}{4\pi\varepsilon_0 l}\cdot\log_e\left(\frac{x-l}{x}\right)$ (d) $\frac{Q}{4\pi\varepsilon_0 l}$

17. A ray of light is travelling through a medium of refractive index $\frac{1}{\sqrt{2}}$ with respect to air. When it is incident on the surface making an angle 45° with the surface, which of the following will take place?

(a) Angle of refraction will be 45°

(b) Angle of refraction will be 90°

(c) The ray will be internally reflected

(d) None of the above

18. If a wave travelling in positive x-direction with $A = 0.2$ m/s, velocity $= 360$ m/s and $\lambda = 60$ m, then correct expression for the wave is

(a) $y = 0.2\sin\left[2\pi\left(6t + \frac{x}{60}\right)\right]$

(b) $y = 0.2\sin\left[\pi\left(6t + \frac{x}{60}\right)\right]$

(c) $y = 0.2\sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$

(d) $y = 0.2\sin\left[\pi\left(6t - \frac{x}{60}\right)\right]$

19. In a potentiometer experiment for measuring the emf of a cell, the null point is at 240 cm when we have a 500 Ω resistor in series with the cell and galvanometer. If the series resistance reduced to half, then the null point will be at

(a) 480 cm (b) 240 cm (c) 120 cm (d) 60 cm

20. Two coils each of self-inductance L are connected in parallel. If they are separated by a large distance, then what will be the self-inductance of combination?

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) L (d) $2L$

21. Two bar magnets having same geometry with magnetic moments m and $2m$, are firstly placed in such a way that their similar poles one are on same side. Then, its time period of oscillation is T_1. Now, the polarity of the one of the magnet is reversed. Then, time period of oscillation is T_2, then

(a) $T_1 < T_2$ (b) $T_1 = T_2$ (c) $T_1 > T_2$ (d) $T_2 = \infty$

22. A solenoid of 2.5 m length and 2.0 cm diameter possesses 10 turns per cm. A current of 0.5 A is flowing through it. The magnetic induction at axis inside the solenoid is

(a) $2\pi \times 10^{-4}$ T (b) $2\pi \times 10^{-5}$ T
(c) $2\pi \times 10^{-6}$ T (d) $2\pi \times 10^{-7}$ T

23. In an L-C-R circuit which one of the following statements is correct?

(a) L and R oppose each other
(b) R value increase with frequency
(c) The inductive reactance increases with frequency
(d) The capacitive reactance increases with frequency

24. What will be the kinetic energy of an electron having de-Broglie wavelength 2Å?

(a) 37.5 eV (b) 75 eV
(c) 150 eV (d) 300 eV

25. In an L-C-R series circuit, the values of R, X_L and X_C are 120 Ω, 180Ω and 130Ω, what is the impedance of the circuit?

(a) 120 Ω (b) 130 Ω (c) 180 Ω (d) 330 Ω

26. Particle which can be added to the nucleus of an atom without changing its chemical properties are

(a) neutrons (b) electrons
(c) protons (d) α-particle

27. Two sources give interference pattern which is observed on a screen, placed at a distance D from the sources. The fringe width is given as $2W$. If the distance of sources from the screen is doubled, then the fringe width will

(a) become $W/2$ (b) remains the same
(c) become W (d) become $4W$

28. A particle moves along X-axis from $x = 0$ to $x = 5$ cm under the influence of force given by $F = (7 - 2x + 3x^2)$ N. The work done in the process is

(a) 70 J (b) 270 J (c) 35 J (d) 135 J

29. A sphere of mass m moving with a constant velocity u hits another stationary sphere of the same mass. If e is the coefficient of restitution, then the ratio of velocities of the two spheres after collision $\left(\frac{v_1}{v_2}\right)$ will be

(a) $\frac{1-e}{1+e}$ (b) $\frac{1+e}{1-e}$
(c) $\frac{e+1}{1-e}$ (d) $\frac{e-1}{e+1}$

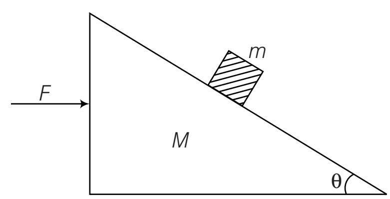
30. The molar specific heat of oxygen at constant pressure $C_p = 7.03$ cal/mol °C and $R = 8.31$ J/mol°C. The amount of heat taken by 5 moles of oxygen when heated at constant volume from 10°C to 20°C will be approximately.

(a) 25 cal (b) 50 cal
(c) 253 cal (d) 500 cal

31. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of this nucleus to decay to $\frac{1}{16}$ th of its initial value is

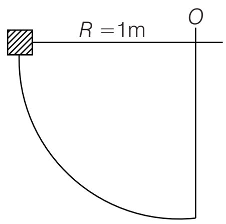
(a) 6.3 μs (b) 40 μs
(c) 300 μs (d) 400 μs

32. A mass m is placed on a wedge (triangular block) of mass M. The wedge moves on a smooth horizontal surface what should be the force F applied on the wedge to the right, so that m remains stationary with respect to wedge. (Ignore any friction)



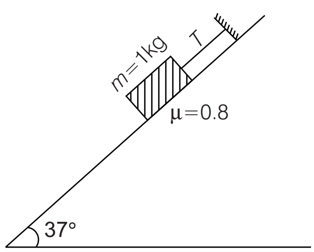
(a) $g \tan\theta$ (b) $mg \cos\theta$
(c) $(M + m) g \tan\theta$ (d) $(M + m)g \operatorname{cosec}\theta$

33. A block of mass 1 kg slides down a curved track that is one quadrant of a circle of radius 1 m. Speed of the block at the bottom is 2 m/s. Work done by the frictional force on the block when it reaches at the bottom is



(a) 8 J (b) –8 J (c) 4 J (d) 0 J

34. In the arrangement, the minimum value of tension in the string to prevent it from sliding down is



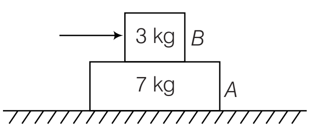
(a) 6 N (b) 8 N (c) 10.8 N (d) zero

35. In a planetary motion, the areal velocity of position vector of a planet depends on angular velocity (ω) and distance (r) of the planet from the sun. The correct relation for areal velocity $\left(\frac{dA}{dt}\right)$ is

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$

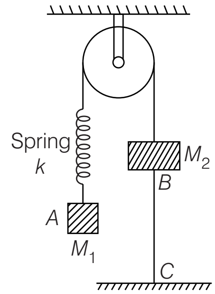
(c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

36. Two blocks A and B are placed one over the other on a smooth horizontal surface. The maximum horizontal force that can be applied on the upper block B, so that A and B move without separation is 49N. The coefficient of friction between A and B is



(a) 0.2 (b) 0.3 (c) 0.5 (d) 0.8

37. In the system shown in figure $M_1 > M_2$ and pulley and threads are ideal. System is held at rest by thread BC. Just after thread BC is burnt.



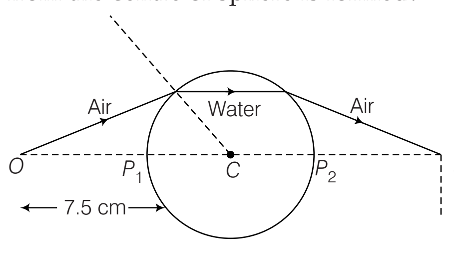
(a) Acceleration M_1 and M_2 will be upward

(b) Magnitude of acceleration of both masses will be $\frac{M_1 - M_2}{M_1 + M_2} g$

(c) Acceleration of M_1 and M_2 will be equal to zero

(d) Acceleration of M_1 will be equal to zero, which that of M_2 will be $\frac{M_1 - M_2}{M_2} g$ upward

38. A thin walled glass sphere of radius 2.5 cm is filled with water. An object (O) is placed at 7.5 cm from the surface of the sphere. Neglecting the effect of glass wall, at what distance the image (I) of the object, measured from the centre of sphere is formed?

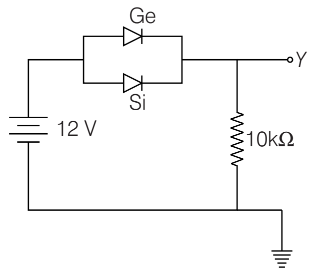


(a) 20 cm (b) 15 cm (c) 10 cm (d) 7.5 cm

39. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0, its potential energy is

(a) $-E_0$ (b) $1.5E_0$ (c) $2E_0$ (d) E_0

40. Two junction diodes one of germanium (Ge) and other of silicon (Si) are connected as shown in figure to a battery of emf 12 V and a load resistance 10 k Ω. The germanium diode conducts at 0.3 V and silicon diode at 0.7 V. When a current flows in the circuit, then the potential of terminal Y will be



(a) 12 V (b) 11 V (c) 11.3 V (d) 11.7 V

41. Fifty six tuning forks are arranged in a series such that each fork gives 6 beats per second with the preceding one. Assuming the frequency of the last fork to be double of the first fork, the frequency of the last fork should be

(a) 220 Hz (b) 330 Hz (c) 440 Hz (d) 660 Hz

42 The wavelength of incident light falling on a photosensitive surface is changed from 2000 Å to 2100 Å. The corresponding change in stopping potential is

(a) 0.03 V (b) 0.3 V (c) 3 V (d) 3.3 V

43. The path difference between two waves

$y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$ is

(a) $\frac{\lambda\phi}{2\pi}$ (b) $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

(c) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (d) $\frac{2\pi\phi}{\lambda}$

44. In photoelectric effect, the photo current

(a) increases with increase of frequency of incident photon
(b) decreases with increase of frequency of incident photon
(c) does not depend on the frequency of photon but depends only on intensity of incident light
(d) depends both on intensity and frequency of incident beam

45. A copper ball of radius r is moving with a uniform velocity v in the mustard oil and the dragging force acting on the ball is F. The dragging force on the copper ball of radius $2r$ with uniform velocity $2v$ in the mustard oil is

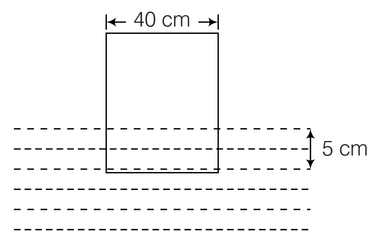
(a) F (b) $2F$ (c) $4F$ (d) $8F$

46. In a photoemissive cell with exciting wave length λ, the fastest electron has a speed v. If the exciting wavelength is changed to $3\lambda/4$, then the speed of the fastest emitted electron will be

(a) $v\left(\frac{3}{4}\right)^{\frac{1}{2}}$ (b) $v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(c) less than $v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (d) greater than $v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

47. A block of wood of side 40 cm floats in water in such a way that its lower face is 5 cm below the free surface of water. What is the weight of the block?



(a) 64 kg
(b) 16 kg
(c) 8 kg
(d) cannot be determined as density of wood is not given

48. A stone is projected from the ground with velocity 50 m/s at an angle 30° with horizontal. It crosses a wall after 3 s. How far beyond the wall the stone will strike the ground? ($g = 10\ \text{m/s}^2$)

(a) 90.2 m (b) 12.5 m (c) 86.5 m (d) $125\sqrt{3}$ m

49. An iron cube floats in a vessel containing mercury at 20°C. If the temperature is increased by 100°C, then the cube will float

(a) lower
(b) higher
(c) at same level
(d) lower or higher depending on mass of cube

50. In Young's double slit experiment, when wavelength used is 600 Å and the screen is 40 cm from the slits, then the fringes are 0.012 cm apart. What is the distance between the slits?

(a) 0.024 cm (b) 2.4 cm
(c) 0.24 cm (d) 0.2 cm

Chemistry

1. A gaseous compounds having carbon, hydrogen and oxygen was burnt in the presence of oxygen. After combustion, 1 volume of the gaseous compound produces 2 volumes of CO_2 and 2 volumes of steam. Determines the molecular formula of the compound, it all the volumes were measured under the same conditions of pressure and temperature.
(a) $C_2H_4O_2$ (b) $C_3H_6O_3$
(c) $C_4H_8O_2$ (d) $C_5H_{10}O_3$

2. A silicon chip used in an integrated circuit of computer has a mass of 5.68 mg. The number of Si atoms present in this chip are
(a) 1.44×10^{20} atoms (b) 1.8×10^{20} atoms
(c) 2.1×10^{20} atoms (d) None of these

3. Cost of sugar ($C_{12}H_{22}O_{11}$) is ₹ 50 per kg. Calculate its cost per mol.
(a) ₹ 17.1 per mol
(b) ₹ 27.1 per mol
(c) ₹ 14.1 per mol
(d) None of the above

4. A man writes his biodata with carbon pencil on the plane paper having mass 150 mg. After writing his biodata, he weighs the written paper and find its mass is 152 mg. What is the number of carbon atoms present in the paper?
(a) 1.0036×10^{20} (b) 5.02×10^{20}
(c) 1.0036×10^{23} (d) 0.502×10^{20}

5. The concentration of a solution can be expressed in molarity, normality, formality and molality. Among them, which mode of expression is the most accurate for the all conditions?
(a) Molarity (b) Formality
(c) Normality (d) Molality

6. Calculate the energy of photons having wavelength, 5×10^{-7} m falls on a metal surface of work function, 3.4×10^{-19} J.
(a) 3.97×10^{-19} J
(b) 3.55×10^{-19} J
(c) 2.97×10^{-19} J
(d) 2.57×10^{-19} J

7. Select the correct options according to Hund's rule and Pauli exclusion principle.

I. $\boxed{\uparrow\downarrow}\boxed{}\boxed{}$ II. $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{}$

III. $\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$ IV. $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}$

Codes
(a) Both I and II
(b) Both II and III
(c) Both I and III
(d) Both III and IV

8. The electronic configurations of chromium (at. no. 24) and copper (at. no. 29) are
(a) $[Ar]4s^1 3d^5$ and $[Ar]\ 4s^1 3d^{10}$
(b) $[Ar]\ 4s^2 3d^4$ and $[Ar]\ 4s^1 3d^{10}$
(c) $[Ar]\ 4s^1 3d^5$ and $[Ar]\ 4s^2 3d^9$
(d) $[Ar]\ 4s^2 3d^4$ and $[Ar]\ 4s^2 3d^9$

9. The Pauli exclusion principle applies to the following option.
(a) H (b) H^+
(c) H^- (d) He^+

10. Metals are usually used as catalysts belong to
(a) *s*-block (b) *p*-block
(c) *d*-block (d) *f*-block

11. Which of the following has the highest electron affinity?
(a) K (b) O^-
(c) F^- (d) O

12. Which of the following compounds has lowest boiling point?
(a) KCl (b) CuCl
(c) $CuCl_2$ (d) CsCl

13. Which of the following compounds has zero dipole moment?
(a) CH_2Cl_2 (b) NH_3
(c) CH_4 (d) PH_3

14. Which of the following compounds show hydrogen bonding?
(a) HCl
(b) C_2H_6
(c) RCH_2CHO
(d) RCH_2NHCH_3

15. In van der Waals' equation of gaseous state, the constant 'b' is a measure of
(a) volume occupied by the molecules
(b) intermolecular repulsions
(c) intermolecular attraction
(d) pressure correction

16. Which of the following is incorrect statement for liquid?
(a) Surface tension of a liquid decreases with increase in temperature
(b) Viscosity of a liquid decreases with decrease in temperature
(c) Vapour pressure of a liquid increases with increase in temperature
(d) Droplets of a liquid on flat surface are slightly flattened due to gravity

17. Thermodynamically, in which form of carbon is the most stable?
(a) Coal (b) Fullerenes
(c) Diamond (d) Graphite

18. Enthalpy of formation of HF and HCl are – 161 kJ and – 92 kJ, respectively. Select the incorrect statement on the basis of the above fact.
(a) HF is more stable than HCl
(b) HCl is more stable than HF
(c) Affinity of fluorine to hydrogen is greater than affinity of chlorine to hydrogen
(d) HF and HCl are exothermic compound

19. A reaction is spontaneous at low temperature, but non-spontaneous at high temperature. Which of the following condition is true for the chemical reaction?
(a) $\Delta H < 0$, $\Delta S < 0$
(b) $\Delta H > 0$, $\Delta S = 0$
(c) $\Delta H < 0$, $\Delta S > 0$
(d) $\Delta H > 0$, $\Delta S > 0$

20. The chemical reaction of a gas phase is given below.

$$2A(g) + B(g) \rightleftharpoons C(g) + D(g)$$

Which one of the following changes will affect the value of K_C?
(a) Addition of inert gas
(b) Increasing in temperature
(c) Addition of reactants
(d) Addition of catalyst

21. For the chemical equilibrium,

$$2NO_2(g) \rightleftharpoons N_2O_4(g); + 14.6\,\text{kcal}$$

increase in temperature
(a) favours the formation of N_2O_4
(b) favours the decomposition of N_2O_4
(c) does not affect equilibrium
(d) stop the reaction

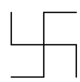
22. The property of the alkaline earth metals which increases with their atomic number?
(a) Ionisation energy
(b) Electronegativity
(c) Solubility of their hydroxides in water
(d) Solubility of their sulphates in water

23. Which of the following element has maximum photoelectric effect?
(a) Li (b) Na
(c) K (d) Cs

24. Among boron halides, which is the strongest Lewis is acid?
(a) BF_3 (b) BI_3
(c) BCl_3 (d) BBr_3

25. The compound gives blood red colouration when its Lassaigne's extract is treated with alkali and ferric chloride?
(a) Benzamide (b) Phenyl hydrozine
(c) Diphenyl sulphide (d) Thiourea

26. The IUPAC name for the hydrocarbon represented by



(a) 3, 3-diethylpentane (b) 2-ethylpentane
(c) tetraethyl carbon (d) *neo*-nonane

27. Keto-enol tautomerism is not observed in
(a) $CH_3COCH_2COCH_3$
(b) $C_6H_5COCH_2COCH_3$
(c) $C_6H_5COCH{=}CH_2$
(d) $C_6H_5COC_6H_5$

28. Which of the following statements are true with respect to electronic displacement in a covalent bond?
I. Inductive effect operates through π-bond.
II. Resonance effect operates through σ-bond.
III. Inductive effect operates through σ-bond.
IV. Resonance effect operates through π-bond.

V. Resonance and inductive effects operate through σ-bond.

(a) III and IV (b) I and II
(c) II and IV (d) I and V

29. The most susceptible to nucleophilic attack at the carbonyl group among the given compound is

(a) CH_3COOCH_3 (b) CH_3CONH_2
(c) CH_3COOCH_3 (d) CH_3COCl

30. Among the following species, the most stable carbonium ion is

(a) $C_6H_5\overset{+}{C}HC_6H_5$
(b) $C_6H_5\overset{+}{C}H_2$
(c) $CH_3\overset{+}{C}H_2$
(d) $C_6H_5CH_2\overset{+}{C}H_2$

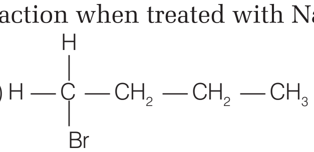
31. For the following,

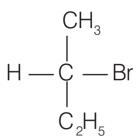
(i) I^- (ii) Cl^- (iii) Br^-

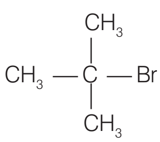
the increasing order of nucleophilicity would be

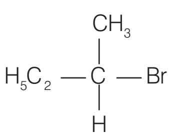
(a) $Br^- < Cl^- < I^-$ (b) $I^- < Cl^- < Br^-$
(c) $Cl^- < Br^- < I^-$ (d) $I^- < Br^- < Cl^-$

32. Which will undergo fastest S_N2 substitution reaction when treated with NaOH?

(a) 

(b) 

(c) 

(d) 

33. Which one of the following is aromatic?

(a) Cycloheptatrienyl cation
(b) Cycloheptrariene
(c) Cyclooctateraene
(d) Cyclopentadienyl cation

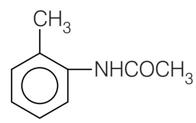
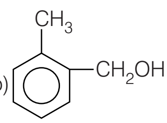
34. The following compound which cannot be prepared by Wurtz reaction?

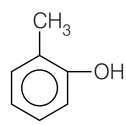
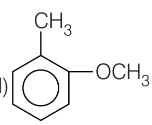
(a) Methane
(b) Ethane
(c) Butane
(d) Propane

35. Anti-Markownikoff's rule addition reaction is not observed in the following compound.

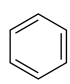
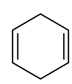
(a) propene
(b) 1-butene
(c) but-2-ene
(d) pent-2-ene

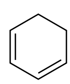
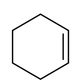
36. The most reactive towards electrophilic reagent is

(a) 
(b) 

(c) 
(d) 

37. The maximum heat of hydrogenation among the following is

(a) 
(b) 

(c) 
(d) 

38. The process of 'eutrophication' is due to

(a) increase in concentration of radioactive substances in water.
(b) the reduction in concentration of the dissolved oxygen in water due to phosphate pollution in water.
(c) increase in concentration of fluoride ion in water.
(d) increase in concentration of insecticide in water.

39. Match the Column I and Column II and pick the correct matching from the codes given below.

Column I	Column II
A. Peroxy acetyl nitrate	1. Waste incineration
B. Polycyclic aromatic	2. Global warming
C. Dioxins	3. Photochemical smog
D. Indigo	4. Carcinogens
E. IR active molecules	5. Vat dye

	A	B	C	D	E
(a)	3	4	1	5	2
(b)	1	2	3	4	5
(c)	3	5	1	2	4
(d)	5	3	1	2	4

40. The flame colours of metal ions are due to
(a) metal excess defect
(b) metal deficiency defect
(c) Schottky defect
(d) Frenkel defect

41. The gas mixture which is used by the divers inside the sea is
(a) $O_2 + N_2$
(b) $O_2 + Ar$
(c) $O_2 + Xe$
(d) $O_2 + He$

42. van't Hoff factor, when benzoic acid is dissolved in benzene, will be
(a) 2 (b) 1
(c) 0.5 (d) 1.5

43. 1 C electricity deposits
(a) half of electrochemical equivalent of Ag
(b) electrochemical equivalent of Ag
(c) 96500 g of Ag
(d) 10.8 g of Ag

44. Which of the following does not influence the rate of reaction?
(a) Molecularity of the reaction
(b) Temperature of the reaction
(c) Concentration of the reactants
(d) Nature of the reactants

45. During the adsorption of krypton on activated charcoal at low temperature.
(a) $\Delta H < 0$ and $\Delta S > 0$
(b) $\Delta H > 0$ and $\Delta S > 0$
(c) $\Delta H < 0$ and $\Delta S < 0$
(d) $\Delta H > 0$ and $\Delta S < 0$

46. In which of the following the pyrophosphate bond of ATP is broken down and linkage takes place between two molecules?
(a) I some rases
(b) Lyases
(c) Transferases
(d) Ligases

47. Which of the following option best describes the 'hydrogen economy?.'
(a) Using dihydrogen in an efficient manner
(b) Dihydrogen is carried and stored in the form of liquid or gas
(c) Addition of dihydrogen to the reactant
(d) Both (a) and (b)

48. Electrolytic reduction of alumina to aluminium by Hall-Heroult process is carried out
(a) in the presence of NaCl
(b) in the presence of Hnorite
(c) in the presence of cryolite which forms a melt with lower melting temperature
(d) in the presence of cryolite which forms a melt with higher melting temperature

49. Which of the following anions is present in the chain structure of silicates?
(a) $(Si_2O_5^{2-})_n$
(b) $(Si_2O_3^{2-})_n$
(c) $Si_2O_4^{4-}$
(d) $Si_2O_7^{6-}$

50. Myosin(s) is/are
(a) fibrous proteins
(b) globular proteins
(c) enzyme
(d) Both (a) and (b)

Mathematics

1. If $2a+3b+6c=0$, then atleast one root of the equation $ax^2+bx+c=0$ lies in the interval
(a) (0, 1) (b) (1, 2)
(c) (2, 3) (d) None of these

2. If $\mathbf{a}$ and $\mathbf{b}$ be two perpendicular unit vectors such that $\mathbf{x}=\mathbf{b}-(\mathbf{a}\times\mathbf{x})$, then $|\mathbf{x}|$ is equal to
(a) 1 (b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$

3. If $a\mathbb{N}=\{an:n\in\mathbb{N}\}$ and $b\mathbb{N}\cap c\mathbb{N}=d\mathbb{N}$, where $a,b,c\in\mathbb{N}$ and b,c are coprime, then
(a) $b=cd$ (b) $c=bd$
(c) $d=bc$ (d) None of these

4. If $f(x)=\int_0^1 e^{|t-x|}dt \quad (0\le x\le 1)$, then the maximum value of $f(x)$ is
(a) $\sqrt{e}-1$ (b) $2(e-1)$ (c) $(e-1)$ (d) $2(\sqrt{e}-1)$

5. The mean deviation from the mean for the set of observations $-1, 0, 4$ is
(a) less than 3 (b) less than 1
(c) greater than 2.5 (d) greater than 4.9

6. If $f(x)$ is a non-negative continuous function for all $x\ge 1$ such that $f'(x)\le pf(x)$, where $p>0$ and $f(1)=0$, then $[f(\sqrt{e})+f(\sqrt{\pi})]$ is equal to
(a) 0 (b) negative
(c) positive (d) None of these

7. If $t_n=\sum_{r=0}^{n}\frac{1}{({}^nC_r)^k}$ and $S_n=\sum_{r=0}^{n}\frac{r}{({}^nc_r)^k}$, where $k\in\mathbb{Z}^+$, then $\cos^{-1}\left(\frac{S_n}{nt_n}\right)$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

8. If X follows a binomial distribution with parameters $n=8$ and $p=\frac{1}{2}$, then $P(|x-4|\le 2)$ is equal to
(a) $\frac{119}{128}$ (b) $\frac{116}{128}$
(c) $\frac{29}{128}$ (d) None of these

9. Sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending power of x is
(a) 2^{59} (b) 2^{58} (c) 2^{30} (d) 2^{29}

10. If $\theta=\sin^{-1}x+\cos^{-1}x-\tan^{-1}x$, $1\le x<\infty$, then the smallest interval in which θ lies is
(a) $\frac{\pi}{2}\le\theta\le\frac{3\pi}{4}$ (b) $0\le\theta\le\frac{\pi}{4}$
(c) $-\frac{\pi}{4}\le\theta\le 0$ (d) $\frac{\pi}{4}\le\theta\le\frac{\pi}{2}$

11. If $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, then the value of x is
(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) None of these

12. If $\cos^{-1}p+\cos^{-1}q+\cos^{-1}r=3\pi$, then $p^2+q^2+r^2+2pqr$ is equal to
(a) 3 (b) 1 (c) 2 (d) -1

13. The points on the curve $xy^2=1$ which are nearest to the origin, are
(a) $\left[\left(\frac{1}{2}\right)^{1/3}, \pm\left(\frac{1}{2}\right)^{-1/6}\right]$ (b) $\left[\left(\frac{1}{2}\right)^{1/3}, 2^{-1/6}\right]$
(c) $\left[2^{1/3}, \pm\left(\frac{1}{2}\right)^{-1/6}\right]$ (d) None of these

14. The statement $p\to(q\to p)$ is equivalent to
(a) $p\to(p\leftrightarrow q)$ (b) $p\to(p\to q)$
(c) $p\to(p\vee q)$ (d) $p\to(p\wedge q)$

15. Total number of polynomials of the form x^3+ax^2+bx+c that are divisible by x^2+1, where $a,b,c\in\{1,2,3,\ldots,10\}$ is equal to
(a) 90 (b) 45 (c) 5 (d) 10

16. Total number of regions in which 'n' coplanar lines can divide the plane, it is known that no two lines are parallel and no three of them are concurrent, is equal to
(a) $\frac{1}{2}(n^2+n+2)$ (b) $\frac{1}{2}(n+3n^2)$
(c) $\frac{1}{2}(3n+n^2)$ (d) (n^2-n+2)

17. The value of $\lim_{n\to\infty}\left[\sqrt[3]{n^2-n^3}+n\right]$ is

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

18. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(a) 210 (b) 105
(c) 70 (d) 112

19. If $\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is to be the square root of two rowed unit matrix, then α, β and γ should satisfy the relation

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$
(c) $1-\alpha^2+\beta\gamma=0$ (d) $\alpha^2+\beta\gamma-1=0$

20. The differential equation of the family of curves $y=Ae^{3x}+Be^{5x}$, where A and B are arbitrary constants, is

(a) $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+15y=0$
(b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}+y=0$
(c) $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$
(d) None of the above

21. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\sin^{-1}(3x-4x^3)$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) 1

22. $\int\frac{(2x^{12}+5x^9)}{(1+x^3+x^5)^3}dx$ equals

(a) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$
(b) $\frac{x^2+2x}{(x^5+x^3+1)}+C$
(c) $\log(x^5+x^3+1+\sqrt{(2x^{12}+5x^9)}+C)]$
(d) None of the above

23. $\begin{vmatrix}\sin\theta & \cos\theta & \sin 2\theta\\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$ equals

(a) $3\sin\theta$ (b) $\sin^3\theta$
(c) 0 (d) None of these

24. Area bounded by the curves $y=x^2$ and $y=2-x^2$ is

(a) $\frac{8}{3}$ sq units (b) $\frac{3}{8}$ sq unit
(c) $\frac{3}{2}$ sq units (d) None of these

25. $\lim_{n\to\infty}\left(\frac{1^2}{1-n^3}+\frac{2^2}{1-n^3}+\dots+\frac{n^2}{1-n^3}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$
(c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

26. The value of $\int\frac{\sin x+\cos x}{3+\sin 2x}dx$ is

(a) $\frac{1}{4}\log\left(\frac{2-\sin x+\cos x}{2+\sin x-\cos x}\right)+C$
(b) $\frac{1}{2}\log\left(\frac{2+\sin x}{2-\sin x}\right)+C$
(c) $\frac{1}{4}\log\left(\frac{1+\sin x}{1-\sin x}\right)+C$
(d) None of the above

27. The equation of the circle passing through the point $(2a, 0)$ and whose radical axis is $x=\frac{a}{2}$ with respect to the circle $x^2+y^2=a^2$, will be

(a) $x^2+y^2-2ax=0$ (b) $x^2+y^2+2ax=0$
(c) $x^2+y^2+2ay=0$ (d) $x^2+y^2-2ay=0$

28. The coordinates of the point on the parabola $y=x^2+7x+2$, which is nearest to the straight line $y=3x-3$, are

(a) $(-2,-8)$ (b) $(1, 10)$
(c) $(2, 20)$ (d) $(-1,-4)$

29. The area bounded by the parabolas $y^2 = 4a(x + a)$ and $y^2 = -4a(x - a)$ is

(a) $\frac{16}{3}a^2$ (b) $\frac{8}{3}a^2$

(c) $\frac{4}{3}a^2$ (d) None of these

30. The solution of the differential equation $\frac{dy}{dx} = \sin(x + y)\tan(x + y) - 1$ is

(a) $\text{cosec}(x + y) + \tan(x + y) = x + C$
(b) $x + \text{cosec}(x + y) = C$
(c) $x + \tan(x + y) = C$
(d) $x + \sec(x + y) = C$

31. Fifteen coupons are numbered 1, 2, ..., 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

(a) $\left(\frac{9}{10}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) None of these

32. Let $h(x) = \min(x, x^2)$, for every real numbers x. Then,

(a) h is not continuous for all x
(b) h is differentiable for all x
(c) $h'(x) \neq 1$, for all $x > 1$
(d) h is not differentiable at two values of x

33. If the function

$$f(x) = \begin{cases} (\cos x)^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$ is continuous

at $x = 0$, then the value of k is

(a) 8 (b) 1
(c) -1 (d) None of these

34. If $f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$, $x \neq 0$ and $f(0) = 0$, then $f(x)$ is

(a) left continuous at 0 (b) right continuous at 0
(c) discontinuous at 0 (d) continuous at 0

35. The value of $\sum_{p=1}^{6} 2\left(\sin\frac{2p\pi}{7} - i\cos\frac{2p\pi}{7}\right)$ is

(a) $2i$ (b) $-2i$
(c) 2 (d) 1

36. A letter lock contains 5 rings each marked with four different letters. The number of all possible unsuccessful attempts to open the lock is

(a) 625 (b) 1024
(c) 624 (d) 1023

37. The domain of definition of the function $f(x) = \frac{1}{\sqrt{|[|x|-1]|-5}}$ is

(a) $(-\infty, -7] \cup [7, \infty)$
(b) $(-\infty, \infty)$
(c) $(7, \infty)$
(d) $(-7, \infty)$

38. If $y^{1/m} + y^{-1/m} = 2x$, then $(x^2 - 1)y_2 + xy_1$ is equal to

(a) m^2y (b) $-m^2y$

(c) $\pm m^2y$ (d) None of these

39. The condition that the straight line $cx - by + b^2 = 0$ may touch the circle $x^2 + y^2 = ax + by$ is

(a) $abc = 1$ (b) $a = c$
(c) $b = ac$ (d) None of these

40. If $0 < \alpha, \beta, \gamma < \frac{\pi}{2}$ such that $\alpha + \beta + \gamma = \frac{\pi}{2}$ and $\cot\alpha, \cot\beta, \cot\gamma$ are in AP, then the value of $\cot\alpha\cot\gamma$ is

(a) 1 (b) 3
(c) $\cot^2\beta$ (d) None of these

41. In a ΔABC, if $b = 2$, $\angle B = 30°$, then the area of the circumcircle of ΔABC (in sq units) is

(a) π (b) 2π (c) 4π (d) 6π

42. The sum of the series

$$\frac{3}{4 \cdot 8} - \frac{3 \cdot 5}{4 \cdot 8 \cdot 12} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12 \cdot 16} - \ldots \text{ is}$$

(a) $\sqrt{\frac{3}{2}} - \frac{3}{4}$ (b) $\sqrt{\frac{2}{3}} - \frac{3}{4}$

(c) $\sqrt{\frac{3}{2}} - \frac{1}{4}$ (d) $\sqrt{\frac{2}{3}} - \frac{1}{4}$

43. The value of $\int_0^{\pi} |\sin^3\theta|\, d\theta$ is

(a) 0 (b) π
(c) $\frac{4}{3}$ (d) $\frac{3}{8}$

44. The value of $\int \frac{\cos x + x \sin x}{x^2 + x \cos x} dx$ is

(a) $\log \left| \frac{\sin x}{1 + \cos x} \right| + C$

(b) $\log \left| \frac{\sin x}{x + \cos x} \right| + C$

(c) $\log \left| \frac{2 \sin x}{x + \cos x} \right| + C$

(d) $\log \left| \frac{x}{x + \cos x} \right| + C$

45. Tangent to the ellipse $\frac{x^2}{32} + \frac{y^2}{18} = 1$ having slope $\frac{-3}{4}$ meet the coordinate axis at A and B. Then, the area of ΔAOB, where O is the origin, is

(a) 12 sq units (b) 8 sq units
(c) 24 sq units (d) 32 sq units

46. The value of $\lim_{x \to \pi/6} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$ is

(a) 3 (b) -3 (c) 6 (d) 0

47. If $\sin A + \cos B = a$ and $\sin B + \cos A = b$, then $\sin (A + B)$ is equal to

(a) $\frac{a^2 + b^2}{2}$ (b) $\frac{a^2 - b^2 + 2}{2}$

(c) $\frac{a^2 + b^2 - 2}{2}$ (d) None of these

48. In a ΔABC, if $\sin A \sin B = \frac{ab}{c^2}$, then the triangle is

(a) equilateral (b) isosceles
(c) right angled (d) obtuse angled

49. The direction cosines l, m, n of two lines are connected by the relations $l + m + n = 0$ $lm = 0$, then the angle between them is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

50. The foci of the conic section

$25x^2 + 16y^2 - 150x = 175$ are

(a) $(0, \pm 3)$ (b) $(0, \pm 2)$
(c) $(3, \pm 3)$ (d) $(0, \pm 1)$

51. If $x - 2y = 4$, then the minimum value of xy is

(a) -2 (b) 0 (c) 1 (d) -3

52. If **b** and **c** are any two non-collinear unit vectors and **a** is any vector, then

$(\mathbf{a} \cdot \mathbf{b})\, \mathbf{b} + (\mathbf{a} \cdot \mathbf{c}) + \frac{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}}{|\mathbf{b} \times \mathbf{c}|} \cdot (\mathbf{b} \times \mathbf{c})$ is equal to

(a) 0 (b) a (c) b (d) **c**

53. If θ is the semi-vertical angle of a cone of maximum volume and given slant height, then $\tan \theta$ is given by

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$

54. The period of the function

$f(x) = \frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$ is

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) None of these

55. The equation of the plane through the points $(2, 3, 1)$ and $(4, -5, 3)$ parallel to X-axis is

(a) $x + 4z = 7$ (b) $y - 4z = 7$
(c) $y + 4z = -7$ (d) $y + 4z = 7$

56. If Q is the image of the point $P(2, 3, 4)$ under the reflection in the plane $x - 2y + 5z = 6$, then the equation of the line PQ is

(a) $\frac{x-2}{-1} = \frac{y-3}{2} = \frac{z-4}{5}$

(b) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$

(c) $\frac{x-2}{-1} = \frac{y-3}{-2} = \frac{z-4}{5}$

(d) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{5}$

57. The equation of the plane passing through the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$ is

(a) $x + 7y + 13z - 96 = 0$
(b) $x + 7y + 13z + 96 = 0$
(c) $x + 7y - 13z - 96 = 0$
(d) $x - 7y + 13z + 96 = 0$

58. $1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \ldots \infty$ equals

(a) $5e$ (b) $4e$ (c) $3e$ (d) $2e$

59. The number of values of x, where $f(x) = \cos x + \cos \sqrt{2}x$ attains its maximum is

(a) 1 (b) 0
(c) 2 (d) infinite

60. A vector perpendicular to the plane containing the points $A(1,-1,2)$, $B(2,0,-1)$, $C(0,2,1)$ is

(a) $4\hat{i} + 8\hat{j} - 4\hat{k}$
(b) $8\hat{i} + 4\hat{j} + 4\hat{k}$
(c) $3\hat{i} + \hat{j} + 2\hat{k}$
(d) $\hat{i} + \hat{j} - \hat{k}$

61. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP'$, then $P'Q^{2005}P$ is

(a) $\begin{bmatrix} 1 & 1 \\ 2005 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

62. $\int \cos^{-3/7} x \sin^{-11/7} x \, dx$ is equal to

(a) $\log|\sin^{4/7} x| + C$
(b) $\frac{4}{7}\tan^{4/7} x + C$
(c) $\frac{-7}{4}\tan^{-4/7} x + C$
(d) $\log|\cos^{3/7} x| + C$

63. All the chords of the hyperbola $3x^2 - y^2 - 2x + 4y = 0$, subtending a right angle at the origin pass through the fixed point

(a) $(1, -2)$
(b) $(-1, 2)$
(c) $(1, 2)$
(d) None of the above

64. The vector equation of the plane through the point $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and perpendicular to the line of intersection of the plane $\mathbf{r}\cdot(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ and $\mathbf{r}\cdot(\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 2$ is

(a) $r\cdot(2\hat{i} + 7\hat{j} - 13\hat{k}) = 1$
(b) $r\cdot(2\hat{i} - 7\hat{j} - 13\hat{k}) = 1$
(c) $r\cdot(2\hat{i} + 7\hat{j} + 13\hat{k}) = 0$
(d) None of the above

65. The degree of the differential equation $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \dots$

(a) 3 (b) 2
(c) 1 (d) Not defined

66. The locus of the mid-points of the chords of the circle $x^2 + y^2 = 16$ which are tangent to the hyperbola $9x^2 - 16y^2 = 144$, is

(a) $(x^2 + y^2)^2 = 16x^2 + 9y^2$
(b) $(x^2 - y^2)^2 = 16x^2 - 9y^2$
(c) $(x^2 + y^2)^2 = 16x^2 - 9y^2$
(d) None of the above

67. The number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 50$ is

(a) 9 (b) 8 (c) 7 (d) 6

68. The intercepts on the straight line $y = mx$ by the line $y = 2$ and $y = 6$ is less than 5, then m belongs to

(a) $\left]-\frac{4}{3}, \frac{4}{3}\right[$
(b) $\left]\frac{4}{3}, \frac{3}{8}\right[$
(c) $\left]-\infty, -\frac{4}{3}\right[\cup \left]\frac{4}{3}, \infty\right[$
(d) $\left]\frac{4}{3}, \infty\right[$

69. A line passing through origin and is perpendicular to two given lines $2x + y + 6 = 0$ and $4x + 2y - 9 = 0$. The ratio in which the origin divides this line, is

(a) 1 : 2 (b) 2 : 1 (c) 4 : 2 (d) 4 : 3

70. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then, the events A and B are

(a) mutually exclusive and independent
(b) independent but not equally likely
(c) equally likely but not independent
(d) equally likely and mutually exclusive

English & General Awareness

Directions (Q. Nos. 1-5) *Rearrange the following five sentences (A), (B), (C), (D) and (E) in proper sequence to form a meaningful paragraph and then answer the questions given below them.*

(A) The history of mankind is full of such fightings between communities, nations and people.

(B) From the primitive weapons of warfare, man has advanced to the modern nuclear weapons.

(C) Ever since the dawn of civilisation, man has been fighting with man.

(D) A modern war is scientific in character, but the effect is the same, wiping human existence out of this Earth.

(E) The only difference now seems to be in the efficiency of the instruments used for killing each other.

1. Which of the following should be the FIRST sentence?
(a) A (b) B
(c) C (d) D
(e) E

2. Which of the following should be the SECOND sentence?
(a) A (b) B
(c) C (d) D
(e) E

3. Which of the following should be the THIRD sentence?
(a) A (b) B
(c) C (d) D
(e) E

4. Which of the following should be the FOURTH sentence?
(a) A (b) B
(c) C (d) D
(e) E

5. Which of the following should be the FIFTH (LAST) sentence?
(a) A (b) B
(c) C (d) D
(e) E

Directions (Q. Nos. 6-10) *In these questions, look at the underlined part of the given sentence. Below the sentence are given three possible substitutions for the underlined part. If any of substitutions; (a), (b) or (c) is better than the underlined part, choose that substitution as your response. If none of the substitution improves the sentence, choose (d) as your response. Thus, a 'No improvement' response will be signified by the letter (d).*

6. <u>It is no good to cry</u> over spilt milk.
(a) It is no good crying
(b) It is of no good to cry
(c) It is of no good crying
(d) No improvement

7. He has been working <u>off and on</u> for several years to compile a dictionary.
(a) on or off
(b) on and off
(c) regularly
(d) No improvement

8. Rohit assured Sunita that he <u>would look at</u> her work while she was on leave.
(a) would overlook
(b) would look after
(c) will look
(d) No improvement

9. Newton wanted to <u>know why did the apple fall</u> to the ground.
(a) know that why did the apple fall
(b) know why the apple fell
(c) know that why the apple fell
(d) No improvement

10. He was extremely unhappy because of <u>the inordinately delay.</u>
(a) the inordinate delaying
(b) the inordinate delay
(c) the inordinately delaying
(d) No improvement

Directions (Q. Nos. 11-12) *Each of the following items consists of a sentence followed by four words or group of words. Select the opposite of the word occurring in the sentence as per the context.*

11. His family <u>severed</u> ties with him for marrying inter-caste.
(a) joined (b) included
(c) detached (d) disrupted

12. Domestic violence is a very <u>inhuman</u> act.
(a) indifferent
(b) compassionate
(c) terrible
(d) ferocious

Fill in the blanks with correct prepositions.

Directions (Q. Nos. 13-14) *Four words are given in each question, out of which only one word is correctly spelt. find the correctly spelt word and mark your answer.*

13. (a) arcitecture
(b) architectere
(c) architecetur
(d) architecture

14. (a) epilogue
(b) eppilogue
(c) epiloge
(d) epiloug

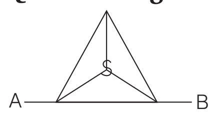
15. The poor have to work .... morning to evening.
(a) in (b) to
(c) from (d) before

16. John, in the morning, started walking towards North and then turn towards opposite side of the Sun. He then turn left again and stops. Which direction is he facing now?
(a) North
(b) West
(c) South
(d) East

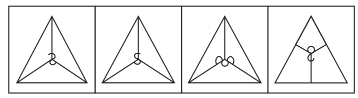
17. If $64 + 7 = 460$ and $25 + 8 = 212$, then $43 + 8 = ?$
(a) 360 (b) 376
(c) 332 (d) 356

18. What is the mirror image of the following figure?

Question Figure



Answer Figures

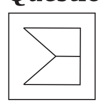


(a) (b) (c) (d)

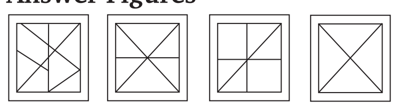
19. If '+' mean '×', '–' means '÷', '×' means '–' and '÷' means '+', then find the value of the following equation. $6 + 64 - 8 \div 45 \times 8$
(a) 85 (b) 76
(c) 87 (d) 75

20. Which of the following has the given figure embedded in it?

Question Figure



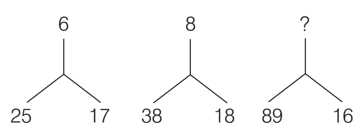
Answer Figures



(a) (b) (c) (d)

21. 1, 4, 2, 3, 2, ?
(a) 2 (b) 5
(c) 3 (d) 4

22.



(a) 13 (b) 15
(c) 17 (d) 19

23. Two persons ride towards each other from two places 55 km apart, one riding at 12 km/h and the other at 10 km/h. When will they be 11 km apart?
(a) 2 h and 30 min (b) 1 h and 30 min
(c) 2 h (d) 2 h and 45 min

24. If $x-\frac{1}{x}=2$, then what is the value of $x^2+\frac{1}{x^2}$?

(a) 4 (b) 5 (c) 3 (d) 6

25. The population of a state is 20000. It increases by 20% during the first year and 30% during the second year. The population after two years will be

(a) 32000 (b) 40000
(c) 31200 (d) 30000

26. A reduction of 20% in the price of rice enable a buyer to buy 5 kg more for ₹ 1200. The reduced price per kg of rice will be

(a) 36 (b) 45
(c) 48 (d) 60

27. A man spends 60% of his income on different expenditures. His income is increased by 20% and his expenditure also increased by 10%. Find the percentage decrease in his saving.

(a) 10% (b) 15%
(c) 20% (d) 25%

28. Find the amount which Shyam will get on ₹ 4096, if he gave it for 18 months at $12\frac{1}{2}\%$ per annum, interest being compounded half yearly.

(a) ₹ 5813 (b) ₹ 4515
(c) ₹ 4913 (d) ₹ 5713

29. *A* works twice as fast as *B*. If *B* can complete a piece of work independently in 12 days, then what will be the number of days taken by *A* and *B* together to finish the work?

(a) 4 (b) 6
(c) 8 (d) 18

30. A well of diameter 3 m is dug 14 m deep. The Earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

(a) 4.25 m
(b) 2.250 m
(c) 1.125 m
(d) 1.750 m

Answers

Physics

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (b)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (b)
21. (a)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (d)	**30.** (c)
31. (d)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (c)	**36.** (c)	**37.** (d)	**38.** (c)	**39.** (c)	**40.** (d)
41. (d)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (d)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (d)

Chemistry

1. (a)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (c)
11. (d)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)
21. (b)	**22.** (c)	**23.** (d)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (d)	**30.** (a)
31. (d)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (a)	**40.** (a)
41. (d)	**42.** (c)	**43.** (b)	**44.** (a)	**45.** (c)	**46.** (d)	**47.** (d)	**48.** (c)	**49.** (b)	**50.** (a)

Mathematics

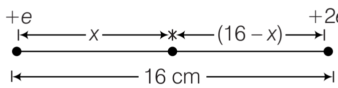
1. (a)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (c)
21. (a)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (a)	**30.** (b)
31. (d)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (d)	**37.** (a)	**38.** (a)	**39.** (b)	**40.** (b)
41. (c)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (c)	**46.** (b)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (c)
51. (a)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (d)	**56.** (b)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (b)
61. (b)	**62.** (c)	**63.** (a)	**64.** (b)	**65.** (c)	**66.** (c)	**67.** (b)	**68.** (c)	**69.** (d)	**70.** (b)

General English & Aptitude

1. (c)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (e)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (b)
11. (a)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (b)
21. (d)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (d)

Explanations

Physics

1. 

For the system to be in equilibrium, we have force on charge q due to + e = force on charge q due to + 2e

i.e. $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e.q}{x^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2e\cdot q}{(16-x)^2}$$

$\Rightarrow \quad 2x^2=(16-x)^2$

$\Rightarrow \quad 2x^2=256+x^2-32x$

$[\because (a-b)^2=a^2+b^2-2ab]$

$\Rightarrow x^2+32x-256=0 \quad \ldots(i)$

On solving Eq. (i), we get

$$x=6.63\text{ cm}$$

2. We know that, $E_n \propto \frac{1}{n^2}$.

As n increases $(E_n - E_{n+1})$ decreases.

3. Ionisation energy of nth state $= -E_n$

For the first excited state, n = 2

and energy in that state is, $\frac{13.6}{2^2}\text{eV}=\frac{13.6}{4}\text{eV}$

$$=3.4\text{ eV}$$

4. Here, $I_g=\frac{I}{10}$, $G=99\ \Omega$

$\because$ The shunt, $S=\frac{I_g}{I-I_g}\cdot G$

$\therefore \quad S=\frac{I/10}{I-\frac{I}{10}}\times 99$

$\Rightarrow \quad S=11\ \Omega$

5. From kinetic theory of gases

$$p=\frac{1}{3}\frac{M}{V}\cdot e^2$$

Since, p and V are constant (same)

$\therefore \quad M_1e_1^2=M_2e_2^2$

$\Rightarrow \quad \frac{e_1^2}{e_2^2}=\frac{M_2}{M_1}$

$\Rightarrow \quad \frac{e_1}{e_2}=\sqrt{\frac{M_2}{M_1}}=\sqrt{4/1}=2 \qquad \left[\text{given}, \frac{M_1}{M_2}=\frac{1}{4}\right]$

6. Radius of the double loop, $r=\frac{R}{2}$

where, R is the radius of single loop B (centre)

$=\frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{R}$.

So, magnetic field due to each coil is double of that due to single coil.

and total magnetic field

$$=2B+2B=4B$$

7. For minima to occur,

$$2\mu t=2n\frac{\lambda_1}{2} \qquad \ldots(i)$$

and for maxima

$$2\mu t=(2n-1)\frac{\lambda_2}{2} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$(2n-1)\lambda_2=2n\lambda_1$

or $\quad (2n-1)7500=(2n)6500$

[given, $\lambda_2=7500$ Å, $\lambda_1=6500$ Å]

or $\quad 1500n-7500=13000n$

or $\quad 2000n=7500$

or $\quad n=3.75$

Substitution the value of (n) in Eq. (i), we get

$$2\times\frac{5}{3}\times t=2\times 3.75\times\frac{6500}{2}\times 10^{-10} \qquad \left[\because \mu=\frac{5}{3}\right]$$

$\Rightarrow \quad t=7.40\times 10^{-7}$ cm

8. Since we know that current, $l=2\pi R=2\pi n\cdot r$

$\left[\because r=\frac{R}{n}\right]$

The magnetic field at the centre of the coil,

$$B=\frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{R} \qquad \ldots(i)$$

and $\quad B'=\left(\frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{r}\right)\cdot n \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$B'=n^2B$$

9. The frequency of oscillation of a vibrating spring-block, system is given by

$$\nu=\frac{1}{2\pi}\sqrt{\frac{k_{eff}}{M}} \qquad \ldots(i)$$

Since, the springs are attached in series, each having spring constant k, so we have

$$\frac{1}{k_{eff}}=\frac{1}{k}+\frac{1}{k}=\frac{2}{k}$$

$\Rightarrow \quad k_{eff}=\frac{k}{2}$

From Eq. (i), we get $\nu=\frac{1}{2\pi}\sqrt{\frac{k_{eff}}{M}}=\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$.

10. It is a case of elastic collision, whole of energy of A is transferred to D.

11. Given, $d\phi = \text{Wb}, M = ?$

$$dI = 3A$$

As, $\phi = MI$

$\therefore$ $d\phi = MdI$

$\therefore$ $M = \dfrac{d\phi}{dI}$

$$= \frac{1.2}{3} = 0.4 \text{ H}$$

12. Since, range is maximum, therefore $\theta = 45°$.

Hence, using $R = u^2 \sin 2\theta / g$.

We find $500 = \dfrac{u^2}{g}$, i.e. $u = [500g]^{\frac{1}{2}}$.

Distance covered along the inclined plane can be obtained using relation

$\Rightarrow$ $v^2 - v_0^2 = 2ax$

$$0 - u^2 = 2 \times (-g \sin 30°) \times x$$

$$[\because a = -g \sin 30°]$$

$\therefore$ $x = \dfrac{u^2}{g} = 500 \text{ m}$

13. Number of atoms in 2 kg of uranium

$$= \frac{6.02 \times 10^{23}}{235} \times 2000$$

$$= 5.12 \times 10^{24}$$

Therefore, energy obtained from these atoms

$$= 5.12 \times 10^{24} \times 185 \text{ MeV}$$

$$= 5.12 \times 10^{24} \times 185 \times 10^6 \text{eV}$$

Energy obtained per second

$$= \frac{5.12 \times 10^{24} \times 185 \times 10^6 \times 1.6 \times 10^{-19}}{30 \times 24 \times 60 \times 60}$$

Solving the above expression

$$= 58.47 \times 10^6 \text{ W}$$

$$\cong 58.5 \text{ MW}$$

14. Given, $C_1 = 3\mu F, C_2 = 6\mu F, V_1 = V_2 = 12 \text{ V}$

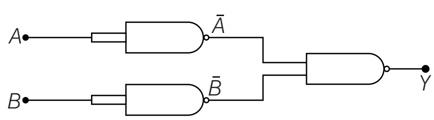
$$V = ?$$

The common potential across capacitors is given by

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V = \frac{3 \times 10^{-6} \times 12 + 6 \times 10^{-6} \times 12}{(3 + 6) \times 10^{-6}} = 12 \text{V}$$

15.



Here, the output of these gate

$$Y = \overline{\bar{A} \cdot \bar{B}}$$

But according to Boolean algebra,

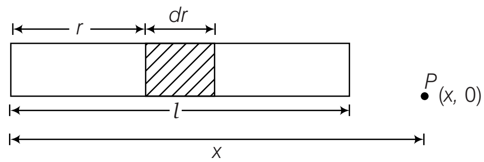
$$\bar{A} \cdot \bar{B} = \overline{A + B}$$

$\therefore$ $Y = \overline{\overline{A + B}} = A + B$

or $Y = A + B$

It means the whole circuit of gates behaves as OR gate.

16.



The potential due to a small element of length dr having charge dQ of the dielectric rod is given by

$$dV = \frac{dQ}{4\pi\varepsilon_0 r}$$

or $dV = \dfrac{\lambda dr}{4\pi\varepsilon_0 r}$ $\left[\because \lambda = \dfrac{dQ}{dr}, \lambda dr = dQ\right]$

or $V = \dfrac{\lambda}{4\pi\varepsilon_0} \displaystyle\int_x^{x-l} \dfrac{dr}{r}$

$$= \frac{\lambda}{4\pi\varepsilon_0} [\log_e r]_x^{x-l}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \log\left(\frac{x - l}{x}\right)$$

$$V = \frac{Q}{4\pi\varepsilon_0 l} \cdot \log e\left(\frac{x - l}{x}\right)$$

When $\lambda = \dfrac{Q}{l}$ (for whole length of rod)

17. Given, $\mu = \dfrac{1}{\sqrt{2}}$ w.r.t. air $i = 45°, r = ?$

Also, $\mu = \dfrac{\sin i}{\sin r}$

$\Rightarrow$ $\dfrac{1}{\sqrt{2}} = \dfrac{\sin 45°}{\sin r} \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{1/\sqrt{2}}{\sin r}$

$\Rightarrow$ $\sin r = 1$

$\Rightarrow$ $r = 90°$

Note that the ray is incident from denser medium and is refracted into a rarer medium air.

18. General equation for a plane progressive wave travelling along positive x-direction is given by

$$y = A \sin(\omega t - kx)$$

$$= A \sin\left[2\pi\left(\nu t - \frac{x}{\lambda}\right)\right] \quad \ldots(i)$$

Here, it is given

$$v = 360 \text{ m/s}, \lambda = 60 \text{ m}$$

We know that, frequency $\nu = \dfrac{v}{\lambda} = \dfrac{360}{60} = 6 \text{ Hz}$

Substituting $A = 0.2 \text{ m/s}, \nu = 6 \text{ Hz}$

$\lambda = 60$ m in Eq. (i), we get

$$y = 0.2 \sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$$

19. The resistance in series with galvanometer does not affect the null point.

So, it will obtained at 240 cm.

20. In parallel combination, the resultant inductance L_P,

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$= \frac{1}{L} + \frac{1}{L} = \frac{2}{L}$$

$$\Rightarrow \quad L_P = \frac{L}{2}$$

21. When similar poles are on same side time period of oscillation T_1 is given by

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 + m_2)\, B_H}} \quad \text{[given]}$$

$$m_1 = 2m,\ m_2 = m$$

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(3m)B_H}} \quad \ldots\text{(i)}$$

When the polarity of magnet is reversed, time period of oscillation T_2 is given by

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 - m_2)B_H}}$$

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{mB_H}} \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$T_2 > T_1$$

or $\quad T_1 < T_2$

22. The magnetic induction along the axis of solenoid

$$B = \mu_0 nI$$

where, n = number of turns per unit length

and $\quad I$ = current

$$\because \quad \frac{\mu_0}{4\pi} = 10^{-7}\ \text{T / A}$$

$$\Rightarrow \quad \mu_0 = 4\pi \times 10^{-7}\ \text{T/A}$$

Given, $\quad n = 10\ \text{cm}^{-1} = 1000\ \text{m}^{-1}$

$$I = 0.5\ \text{A}$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 1000 \times 0.5$$

$$= 2\pi \times 10^{-4}\ \text{T}$$

23. In *L-C-R* circuit, inductive reactance is given by

$$X_L = \omega L$$

$$= 2\pi fL \quad [\because \omega = 2\pi f]$$

$$\therefore \quad X_L \propto f$$

Thus, inductive reactance increases with increase frequency.

24. The de-Broglie wavelength associated with electron

$$\lambda_e = \frac{1.227}{\sqrt{V}}\ \text{nm}$$

$$V = \frac{(12.27)^2}{\lambda_e^2} \times 10^{-2}$$

$$= \frac{150}{\lambda_e^2} \times 10^{-2} \Rightarrow V = \frac{150}{(0.2)^2}$$

$$= \frac{150 \times 10^{-2}}{4 \times 10^{-2}}$$

$$\therefore \quad E = V\ \text{eV} = 37.5\ \text{eV}$$

25. Given, $R = 120\,\Omega$,

$$X_L = 180\ \Omega,\ X_C = 130\ \Omega$$

The impedance of circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(120)^2 + (180 - 130)^2}$$

$$= 130\ \Omega$$

26. Neutrons, as they are neutral particles.

27. Given, $W_1 = 2W$,

Fringe width, $\quad W = \frac{\lambda D}{d}$

$$\Rightarrow \quad 2W = \frac{\lambda D}{d}$$

When, $\quad D' = 2D$

$$W_2 = \frac{\lambda D'}{d} \Rightarrow W_2 = \frac{\lambda 2D}{d}$$

$$= 2 \times 2W = 4\,W$$

28. The given force varies with distance.

Therefore, work done,

$$W = \int_0^{x=5} F \cdot dx$$

$$= \int_0^5 (7 - 2x + 3x^2)dx$$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5$$

$$= [7 \times 5 - (5)^2 + (5)^3 - 0]$$

$$= 135\ \text{J}$$

29. Conservation of momentum (gives)

$$mu + 0 = mv_1 + mv_2$$

$\Rightarrow \quad v_1 + v_2 = u \quad \ldots(i)$

Newton's experimental formula,

$$v_1 - v_2 = -e[u_1 - u_2] \text{ gives}$$

$$v_1 - v_2 = -eu \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$v_1 = \frac{(1-e)u}{2}$$

$$v_2 = \frac{(1+e)u}{2}$$

$\therefore \quad \frac{v_1}{v_2} = \frac{1-e}{1+e}$

30. Given, $C_p = 7.03$ cal/mol °C, $R = 8.315$/mol°C,

$n = 5$ moles, $T_2 = 20°C$, $T_1 = 10°C$,

$$\Delta Q = ?$$

$\therefore$ Heat, $\Delta Q = nC_v \Delta T$

$$= n(C_p - R)\Delta T \quad [\because C_p - C_v = R]$$

$$= 5\left[7.03 - \frac{8.31}{4.2}\right] \times (20 - 10)$$

$$\simeq 253 \text{ cal}$$

31. Activity, $\left(\frac{R}{R_0}\right) = \left(\frac{1}{2}\right)^n$

$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$

or $\quad \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$

$\Rightarrow \quad n = 4$

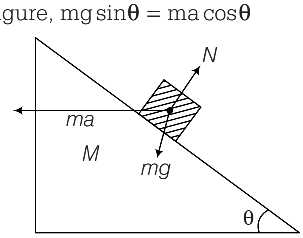
$\Rightarrow \quad \frac{t}{T} = 4 \quad \left[\because n = \frac{t}{T}\right]$

$\Rightarrow \quad t = 4T$

$$= 4 \times 100\ \mu s$$

$$= 400\ \mu s$$

32. From figure, $mg\sin\theta = ma\cos\theta$



$\Rightarrow \quad a = g\tan\theta$

$\therefore$ Force, $\quad F = (M + m)a$

$$= (M + m)g\tan\theta$$

33. Given, $m = 1$ kg, $v = 2$ m/s, $R = 1$ m

$$W = ?$$

From energy conservation,

$$mgh + W = \frac{1}{2}mv^2$$

$$W = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2} \times 1(2)^2 - 1 \times 10 = -8J$$

34. Limiting friction along the plane upward

$$= \mu_s\ mg\cos\theta$$

$$= 0.8 \times 1 \times 10 \times \cos 37°$$

$$= 0.8 \times 10 \times \frac{4}{5}$$

$$= 6.4\ N$$

Force responsible for downward motion

$$= mg\sin\theta$$

$$= 1 \times 10 \times \sin 37°$$

$$= 1 \times 10 \times \frac{3}{5}$$

$$= 6N$$

As, downward force < limiting friction.

So, frictional force = downward force.

Hence, tension in string is zero.

35. Areal velocity, $\frac{dA}{dt} = \frac{L}{2m}$

where, L = angular momentum

$\Rightarrow \quad \frac{dA}{dt} = \frac{mr^2\omega}{2m} \quad [\because L = mr^2\omega]$

$$\propto r^2\omega$$

36. Given, $F = 49$ N, $M_A = 7$ kg, $M_B = 3$ kg, $\mu = ?$

Equation of motion of two blocks-system

$$F_{max} = \mu(M_A + M_B)\cdot g$$

$$\mu = \frac{F}{(M_A + M_B)\cdot g}$$

$$= \frac{49}{(7+3)\cdot 9\cdot 8}$$

$$= \frac{49}{10 \times 9.8}$$

$$= 0.5$$

37. As $M_1 > M_2$, so tension in thread connecting the block B and support C is $(M_1 - M_2)g$. When this thread is burnt, this tension disappears. In this case tension in spring is $M_1 g$ and it is elongated. So, tension in string connecting A and B is $M_1\ g$.

Hence, resultant force on A remains zero because tension in string is balanced by tension in spring. The block B has a net upward force $(M_1 - M_2)g$, so initial acceleration of block B is

$$\frac{(M_1 - M_2)g}{M_2}$$

Thus, initial acceleration of M_1 is zero and that of M_2 is $\frac{(M_1 - M_2)g}{M_2}$ upward.

38. For refraction at first surface,

$$u = -7.5 \text{ cm}, R_1 = 2.5 \text{ cm}$$

$$\mu_1 = 1, \mu_2 = \frac{4}{3}$$

$$\therefore \quad \frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

$$\frac{4/3}{v'} - \frac{1}{(-7.5)} = \frac{4/3 - 1}{2.5}$$

$$\Rightarrow \quad \frac{4}{3v'} = \frac{1}{7.5} - \frac{1}{7.5} = 0$$

$$\Rightarrow \quad v' = \infty$$

It means the ray is parallel within the sphere.
For refraction at second surface,

$$u = -\infty, \mu_1 = \frac{4}{3}$$

$$\mu_2 = 1, R_2 = -2.5 \text{ cm}$$

$$\therefore \quad \frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_2} \text{ gives}$$

$$\Rightarrow \quad \frac{1}{v'} - \frac{4/3}{(-\infty)} = \frac{1 - (4/3)}{(-2.5)}$$

$$\Rightarrow \quad v = +7.5 \text{ cm}$$

i.e. Image is formed at distance 7.5 cm to the right of P_2. Hence, distance of final image P from centre $C = 7.5 + 2.5 = 10$ cm.

39. Total energy, $E_0 = -\frac{GMm}{2r}$ ...(i)

and potential energy, $V = -\frac{GMm}{r}$ ...(ii)

From Eqs. (i) and (ii), we get

$$= 2E_0$$

40. Ge conducts at 0.3 V and Si conducts at 0.7 V. Both Ge and Si are connected in parallel. When current begins to flow, then the potential difference remains at 0.3 V, so no current flows through Si-diode.

$\therefore$ Potential difference across resistive load $= 12 - 0.3$

$$= 11.7 \text{ V}$$

$\therefore$ Potential of Y = 11.7 V

41. From question,

$$n_1 + 55 \times 6 = n_2$$

$$\Rightarrow \quad n + 55 \times 6 = 2n$$

$$\Rightarrow \quad n = 330 \text{ Hz}$$

$$\therefore \quad n_2 = 2n = 2 \times 330 = 660 \text{ Hz}$$

42. Given, $\lambda_1 = 2000 \text{ Å} = 2000 \times 10^{-10}$ m

$$= 2 \times 10^{-7} \text{ m}$$

$$\lambda_2 = 2100 \text{ Å} = 2.1 \times 10^{-7} \text{ m}$$

$$\therefore \quad \frac{hc}{\lambda_1} = W + eV_0 \quad \text{...(i)}$$

$$\text{and} \quad \frac{hc}{\lambda_2} = W + eV_0' \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e(V_0 - V_0')$$

Change in stopping potential

$$\Delta V = V_0 - V_0'$$

$$= \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}}\left(\frac{1}{2 \times 10^{-7}} - \frac{1}{2.1 \times 10^{-7}}\right)$$

$$= \frac{6.6 \times 3}{1.6}\left(\frac{1}{2} - \frac{1}{2.1}\right)$$

$$= \frac{6.6 \times 3 \times 0.1}{1.6 \times 2 \times 2.1} = 0.3 \text{ V}$$

43. Given, $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$$

$$= a_2 \sin\left(\frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda} + \phi\right)$$

$\therefore$ Phase of first wave, $\phi_1 = \left(\omega t - \frac{2\pi x}{\lambda}\right)$

and phase of second wave, $\phi_2 = \frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda} + \phi$

$\therefore$ Phase difference, $\Delta\phi = \phi_2 - \phi_1 = \frac{\pi}{2} + \phi$

$\therefore$ Path difference $= \frac{\lambda}{2\pi} \times$ phase difference

$$= \frac{\lambda}{2\pi}\left(\frac{\pi}{2} + \phi\right)$$

44. When intensity of incident photon increases number of electrons emitted from the surface increases. Thus current increases. It is independent of frequency of photon.

45. From Stokes' law, $F = 6\pi\eta rv$

wehre, η = coefficient of viscosity

and $F' = 6\pi\eta(2r)(2v) = 6\pi\eta rv \times 4$

$\because \quad \frac{F'}{F} = 4$

$F' = 4F$

46. From Einstein's photoelectric equation,

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W \qquad \ldots(i)$$

and $$\frac{1}{2}mv'^2 = \frac{hc}{3\lambda/4} - W \qquad \ldots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\left(\frac{v'}{v}\right)^2 = \frac{\frac{4hc}{3\lambda} - W}{\frac{hc}{\lambda} - W}$$

$\because \quad W > 0$

$\therefore \quad v' > v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

47. Weight of wooden block = Weight of water displaced

$$Mg = V_{in}\sigma_w g$$
$$= (40 \times 40 \times 5) \times 10^{-6} \times 10^3 \text{ g}$$

$\Rightarrow \quad M = 8 \text{ kg}$

48. Time, $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2 \times 50 \times \sin 30°}{10}$$
$$= \frac{2 \times 50 \times \frac{1}{2}}{10} = 5\text{s}$$

Time spent after crossing the wall, $t_2 = 5 - 3 = 2\text{s}$

$\therefore$ Distance, $x_2 = u\cos\theta \times 1_2$

$$= 50 \times \cos 30° \times 2$$
$$= 50 \times \frac{\sqrt{3}}{2} \times 2$$
$$= 50\sqrt{3}$$
$$= 50 \times 1.73 = 86.5 \text{ m}$$

49. Lower (conceptual question).

When temperature is increased, then the density will decrease, so cube will float lower.

50. Given, $D = 40 \text{ cm} = 40 \times 10^{-2} \text{ m}$

$$\lambda = 600 \text{ Å} = 6000 \times 10^{-10} \text{ m}$$
$$W = 0.012 \text{ cm} = 0.012 \times 10^{-2} \text{ m}$$

$\therefore$ Fringe width,

$$W = \frac{D\lambda}{d}$$

$\Rightarrow \quad d = \frac{D\lambda}{W}$

where, d = distance between the slits

$$d = \frac{40 \times 10^{-2} \times 6000 \times 10^{-10}}{0.012 \times 10^{-2}}$$
$$= 2 \times 10^{-3}\text{m} = 2 \text{ mm}$$
$$= 0.2 \text{ cm}$$

Chemistry

1. Suppose the chemical formula of the compound be $C_xH_yO_z$ chemical reaction related to combustion of the compound.

$C_xH_yO_z$	$+ O_2$	$\longrightarrow CO_2$	$+ H_2O$	(Steam)
1 vol.	2 vol.	2 vol.	2 vol.	Given
1. mol	2 mol or 4 atoms	2 mol or 2 atom C and 4 atom O	2 mol. or 4 atom H and 2 atom O	(By Avogadro 15 hypothesis)

Since, 2 molecules of CO_2 have 2 atoms of carbon and these carbon must have come from the compound.

Hence, the value of x is 2. Again, 2 molecules of steam contain 4 atoms of hydrogen and they have come from the compound. Hence, the value of y will be 4. Again, total oxygen atoms in the product side = 6 But 4 oxygen atoms must have come from the oxygen molecules. Hence 2 oxygen atoms definitely come from the compound. Hence, the value of z = 2. So, the molecular formula of the compound is $C_2H_4O_2$.

2. Molar mass of silicon = 28 g = 28000 mg

$\because$ 28000 mg of silicon contain 6.022×10^{23} atoms

$\therefore$ 5.68 mg of silicon contain $\dfrac{6.022 \times 10^{23}}{28000} \times 5.68$

$= 0.00122 \times 10^{23}$

$= 1.22 \times 10^{20}$ atoms

3. Molar mass of sugar $= 12 \times 12 + 1 \times 22 + 11 \times 16$

$= 144 + 22 + 176$

$= 342$ g

$\because$ Cost of 1000 g of sugar is 50 rupees

$\therefore$ Cost of 342 g of sugar $= \dfrac{50}{1000} \times 342$

$=$ ₹ 17.1

4. Mass of C-atoms = 152 − 150 = 2 mg

Molar mass of C-atoms = 12 g = 12000 mg

$\because$ 12000 mg of carbon contains 6.022×10^{23} atoms carbon

$\therefore$ 2 mg of carbon contains $= \dfrac{6.022 \times 10^{23}}{12000} \times 2$

$= 0.0010036 \times 10^{23}$

$= 1.0036 \times 10^{20}$ atoms

5. Since, molarity normality and formality are calculated against the volume of the solution and the volume of the solution changes with change in temperature. Therefore, these qualities do not remain constant with temperature.

$$\text{Molality (m)} = \frac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}}$$

The molality of a solution remains independent of temperature, because it involves only mass, which is independent of temperature.

6. Energy of photons,

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(6.626 \times 10^{-34}\ \text{Js}) \times (3 \times 10^{8}\ \text{ms}^{-1})}{5 \times 10^{-7}}$$

$$= 3.975 \times 10^{-19}\ \text{J}$$

7. According to Pauli exclusion principle, no two electron in an atom can have the same set of four quantum numbers.

Hund's rule states that the pairing of electrons in the orbital of a particular sub-shell (p, d or f) does not take place until all the orbitals of the subshell are singly filled.

Option I and II do not follow Hund's rule and Paulis exclusion principle.

8. Chromium and copper have five and ten electrons in 3d-orbitals rather than four and nine because fully-filled and half-filled orbitals have lower energy and thus have extra stability. Therefore, chromium and copper have d^5 and d^{10}-configurations.

9. Since, He^+ and H have one electron each, while H^+ has no electron, hence Pauli exclusion principle does not apply to them. However, H^- has two electrons, hence this principle applies on it.

10. d-block metals, e.g., Ni, Pt, Fe, etc., are used as catalysts.

11. In general, electron affinity increases in a period from left to right. This is due to decrease in size and increase in nuclear charge as the atomic number increases in a period. Potassium is a metallic element which is electron donor. O^- and F^- ions gain stability by taking one electron.
Only O (oxygen atoms) has tendency to gain electron to attain stability.

12. According to Fajan's rule, as the size of cation decreases, it polarising power increases. Hence, K^+ and Cs^+ having bigger cations contain less polarising power. Cu^{2+} polarise Cl^- ions more than Cu^+ due to its smaller size. Therefore, $CuCl_2$ has more covalent character and hence, its boiling point is less.

13. CH_4 has regular tetrahedral shape, so its dipole moment is zero.

14. RCH_2NHCH_3 shows the hydrogen bonding, because H-atom is directly attached to N-atom.

15. van der Waals' equation for 1 mole of a real gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where, b is volume correction. It arises due to effective size of molecules.

16. Viscosity of a liquid increases with decrease in temperature as kinetic energy of the molecules decreases and hence, intermolecular attraction increases which in turn increases the viscosity of a liquid.

17. Thermodynamically, graphite is the most stable form of carbon.

18. HF is more stable than HCl because more energy is produced in the formation of HF. It means HF has less energy than HCl and hence is more stable. So, we need more energy to break H—F bond. Hence, HF is more stable.

19. $\Delta H < 0$, $\Delta S < 0$

$$\Delta G = \Delta H - T\Delta S$$

When, $\Delta H < 0$ and $\Delta S < 0$, then ΔG will be negative only at low temperature.

20. K_C is a function of temperature

21. The given reaction is exothermic and according to Le-Chatelier principle, high temperature shifts the equilibrium in backward direction, i.e. high temperature favours the decomposition of N_2O_4.

22. Solubility of alkaline earth metal hydroxide in water increases with increase in atomic number.

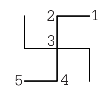
23. The phenomenon of emission of electrons from the surface of metal by visible light is called photoelectric effect.

All alkali metals (except lithium) exhibits photoelectric effect. This ability to exhibit photoelectric effect is due to low value of their ionisation energies. Among alkali metals, Cs having least ionisation energy, exhibits maximum photoelectric effect.

24. Generally, as the electronegativity increases, acidic character increases, but in case of boron halides. BI_3 is the most acidic, i.e. strongest Lewis acid. This is because there is back donation of electrons from F-atom to boron in case of BF_3. This makes BF_3 less electron deficient. The tendency of back donation decreases as the size of halogen atom increases due to large energy difference. Hence, BI_3 is the most electron deficient and thus, the strongest Lewis acid.

25. Thiourea contains both S and N and hence gives blood red colouration when its Lassaigne's extract is treated with alkali and $FeCl_3$.

$$3NaCNS + FeCl_3 \longrightarrow \underset{\substack{\text{Ferric} \\ \text{sulphocyanide} \\ \text{(Blood red)}}}{Fe(CNS)_3} + 3NaCl$$

26. 

3, 3-diethylpentane

27. $C_6H_5COC_6H_5$ does not exhibit tautomerism as it contains no α-H atom.

28. Inductive effect is a permanent effect operates on σ-electrons, while resonance involves delocalisation of π-electrons.

29. Cl^- is the weakest base, so good leaving group.

30. $C_6H_5\overset{+}{C}HC_6H_5$ is the most stable, since the positive charges can be delocalised over both of phenyl rings.

31. Halogens are the most reactive elements. High reactivity is due to their low dissociation energy. Further, among halogen, reactivity decreases as we move down the group. Hence, the correct order of nucleophilicity would be

$$I^- < Br^- < Cl^-$$

32. Primary alkyl halides undergo bimolecular nucleophilic substitution most readily.

33. To be aromatic the following conditions must be ful-filled.

(i) Molecules should have $(4\pi + 2)\pi$-electrons in conjugation.

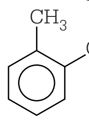
(ii) Molecule should be planar.

Cycloheptatriene has $(4n + 2)\pi$-electrons, but not in conjugation.

34. Compounds containing two or more carbon atoms are prepared by Wurtz reaction. Thus, methane (CH_4) cannot be prepared by Wurtz reaction.

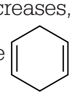
35. Anti-Markownikoff's rule addition reaction is not observed in case of but-2-ene. The reason is that symmetrical alkenes do not exhibit anti-Markownikoff's addition.

36. Due to greater electron releasing effect,



OH is the most reactive towards electrophilic reagent.

37. As, the conjugation increases, heat hydrogenation decreases. Thus, alkene



with two isolated double bonds has the highest heat of hydrogenation.

38. The enrichment of water by plant nutrients is known as eutrophication. It is due to the reduction in concentration of dissolved oxygen in water due to phosphate pollution.

39. Peroxy acetyl nitrate is used in photochemical smog. Polycyclic aromatic hydrocarbons are used as carcinogens. Dioxins are used to burn the waste into ashes. IR active molecules are used in global warming. Indigo is called as vat dye because the dying with indigotin was carried out in wooden vats in the form of water soluble indigotin-white.

40. Flame colours of metal ions are due to metal excess defect.

41. Helium-oxygen mixture is used by deep sea divers because of its very low solubility in blood.

42. When benzoic acid dissolve in benzene, it will be dimerise due to the intermolecular hydrogen bonding.

$$2C_6H_5COOH \xrightarrow{\text{Benzene}} (C_6H_5COOH)_2$$

van't Hoff factor =

$$= \frac{\text{Number of particles after association or dissociation}}{\text{Number of particles before associations or dissociations}}$$

$$= \frac{1}{2} = 0.5$$

43. From Faraday's first law,

Mass deposited , $W = Z.Q$

when $Q = 1\,C$, then $W = Z =$ electrochemical equivalent.

44. Nature and concentration of the reactants its and temperature of the reaction influence the rate of reaction. But molecularity does not affect the rate of reaction as it includes the number of atoms, ions or molecules that must collide with one another to result into a chemical reaction.

45. Adsorption is an exothermic process thus, ΔH is negative (i.e. $\Delta H < 0$). Moreover, adsorption results in more ordered arrangements of molecules, thus entropy decreases (i.e. $\Delta S < 0$).

Hence, low temperature favours the reaction.

46. Ligases catalyse reactions in which the pyrophosphate bond of ATP is broken down and linkage takes place between two molecules.

47. It is a method of using dihydrogen in an efficient manner and carried or stored in the form of liquid or gas. dihydrogen releases more energy than petrol and in more eco-friendly and also can be used in fuel cells.

48. In the metallurgy of alumina, purified Al_2O_3 is mixed with Na_3AlF_6 or CaF_2 which lowers the melting point of the mix and brings conductivity.

$$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$$

49. In $(Si_2O_3^{2-})_n$ two oxygen atoms per SiO_4 tetrahedron are shared giving polymeric anions chain. e.g. spodumene, $LiAl(SiO_3)_2$ and diposide, $CaMg\,(SiO_3)_2$.

50. Myosins are fibrous proteins present in muscles.

Mathematics

1. Consider the function

$$f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

We have, $f(0) = d$

and $f(1) = \frac{a}{3} + \frac{b}{2} + c + d$

$$= \frac{2a + 3b + 6c}{6} + d$$

$$= \frac{0}{6} + d \qquad [\because 2a + 3b + 6c = 0]$$

Therefore, 0 and 1 are roots of the polynomial f(x). Hence, according to Rolle's Theorem, there exists atleast one root of the polynomial $f'(x) = ax^2 + bx + c$ lying between 0 and 1.

2. We have,

$$a \cdot x = 0$$

Also, $\mathbf{x} = \mathbf{b} + (\mathbf{x} \times \mathbf{a})$

$\Rightarrow$ $x \times a = b \times a + (x \times a) \times a$

$\Rightarrow$ $x \times a = b \times a + (x \cdot a)a - (a \cdot a)x$

$\Rightarrow$ $x - b = b \times a - x$

$$|2x| = |b + b \times a|$$

$\Rightarrow$ $|x| = \frac{1}{\sqrt{2}}$

3. We have, $b\mathbb{N} = \{bn : n \in \mathbb{N}\}$, $c\mathbb{N} = \{cn : n \in \mathbb{N}\}$

and $d\mathbb{N} = \{dn : n \in \mathbb{N}\}$

Also, $bc \in b\mathbb{N} \cap c\mathbb{N} = d\mathbb{N}$ [given]

$\therefore$ $bc \in b\mathbb{N} \cap c\mathbb{N}$ or $bc \in d\mathbb{N}$

$\therefore$ $bc = d$

[as b and c are coprime]

4. Given, $f(x) = \int_0^x e^{|t-x|}\,dt + \int_x^1 e^{|t-x|}\,dt$

$$= e^x + e^{1-x} - 2$$

$\therefore$ $f'(x) = e^x - e^{1-x}$

Clearly, $x = \frac{1}{2}$ is the point of minima of f(x).

Also, $f'(0) = f(1) = e - 1$

5. Meen $(\bar{x}) = \frac{-1 + 0 + 4}{3}$

$\therefore$ Mean deviation $= \frac{1}{3}\,[\,|-1-1| + |0-1| + |4-1|\,]$

$$= 2$$

6. $f'(x) - pf(x) \le 0$

$\Rightarrow$ $\frac{d}{dx} e^{-px} f(x) \le 0$

Let $g(x) = e^{-px} f(x)$

$g'(x) \le 0, \forall\ x \ge 1$

$\Rightarrow$ $g(x) \le g(1), \forall\, x \ge 1$

$\Rightarrow$ $g(x) \le 0$

$\Rightarrow$ $f(x) \le 0$

But given, $f(x) \ge 0$

$\therefore$ $f(x) = 0, \ \forall x \ge 1$

7. $S_n = \frac{0}{({}^nC_0)^k} + \frac{1}{({}^nC_1)^k} + \frac{2}{({}^nC_2)^k} + \ldots + \frac{n}{({}^nC_n)^k}$

$\Rightarrow$ $S_n = \frac{n}{({}^nC_0)^k} + \frac{n-1}{({}^nC_1)^k} + \frac{n-2}{({}^nC_2)^k} + \ldots + \frac{0}{({}^nC_0)^k}$

$\Rightarrow$ $2S_n = nt_n$

$\Rightarrow$ $\frac{S_n}{nt_n} = \frac{1}{2}$

Now, $\cos^{-1}\left(\frac{S_n}{nt_n}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

8. We have,

$P(|x - 4| \le 2) = P(-2 \le x - 4 \le 2) = P(2 \le x \le 6)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 7) + P(x = 8)]$

$$= 1 - \left[{}^8C_0\left(\frac{1}{2}\right)^8 + {}^8C_1\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8\right]$$

$$= 1 - \left(\frac{1}{2}\right)^8 (1 + 8 + 8 + 1) = 1 - \frac{18}{2^8} = \frac{119}{128}$$

9. Since, total number of terms $= 59 + 1 = 60$

$\therefore$ Required sum $= \frac{2^{59}}{2} = 2^{58}$

10. $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$\Rightarrow$ $\theta = \cot^{-1}x$

Since, $1 \le x < \infty$, therefore $0 \le \theta \le \frac{\pi}{4}$.

11. Given, $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\therefore$ $\tan^{-1}\left(\frac{2\cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow$ $\frac{2\cos x}{1 - \cos^2 x} = 2\operatorname{cosec} x$

$\Rightarrow$ $\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

12. $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = 3\pi$

We know that, if $y = \cos^{-1}x$, then $-1 \le x \le 1$ and $0 \le y \le \pi$.

Hence, the given equation will hold only when each is π.

$\therefore \quad p = q = r = \cos\pi = -1$

$$\therefore \quad p^2 + q^2 + r^2 + 2pqr$$
$$= (-1)^2 + (-1)^2 + (-1)^2 + 2(-1)(-1)(-1)$$
$$= 1 + 1 + 1 - 2$$
$$= 3 - 2 = 1$$

13. Let $P(x, y)$ be such a point, then

$$OP^2 = x^2 + y^2$$

Let $\quad s = OP^2 = x^2 + \frac{1}{x}$

$$\Rightarrow \quad \frac{ds}{dx} = 2x - \frac{1}{x^2}$$

For maximum or minimum, put $\frac{ds}{dx} = 0$

$$\Rightarrow \quad 2x - \frac{1}{x^2} = 0$$
$$\Rightarrow \quad x^3 = \frac{1}{2}$$
$$\therefore \quad x = \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

Also, $\quad \frac{d^2s}{dx^2} = 2 + \frac{2}{x^3}$

Now, $\left(\frac{d^2s}{dx^2}\right)_{x = \left(\frac{1}{2}\right)^{1/3}} = 2 + \frac{2}{1/2} > 0$

$\therefore$ s is minimum at $x = \left(\frac{1}{2}\right)^{1/3}$.

Thus, for nearest point $x = \left(\frac{1}{2}\right)^{1/3}$,

$$y = \pm\left(\frac{1}{2}\right)^{-1/6}$$

14.

q	p	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

$\therefore$ Statement $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

15. $(x^2 + 1) = (x - 2)(x + 2)$

Let $f(x) = x^3 + ax^2 + bx + c$; $a, b, c \in \{1, 2, \ldots, 10\}$

$f(x)$ is divisible by $x^2 + 1$ iff $f(i) = 0$, $f(-i) = 0$

$\therefore \quad i^3 + ai^2 + bi + c = 0$

$\Rightarrow \quad -i - a + bi + c = 0$

$\Rightarrow \quad a = c, b = 1$

and $\quad -i^3 + ai^2 - bi + c = 0$

$\Rightarrow \quad i - a - bi + c = 0$

$\Rightarrow \quad a = c, b = 1$

$\therefore$ Total number of selection of triplets

$(a, b, c) = (1, 1, 1), (2, 1, 2), \ldots, (10, 1, 10)$

Hence, number of such polynomials = 10.

16. Let number of regions for n lines be $R(n)$.

Clearly, $R(1) = 2$, $R(2) = 4$, …

$R(n) = R(n-1) + n$ i.e. $R(n) - R(n-1) = n$

Putting $n = 2, 3, \ldots, n$ and then adding, we get

$$R(n) - R(1) = 2 + 3 + \ldots + n$$
$$= 2 + 2 + 3 + 4 + \ldots + n = 1 + \frac{n(n+1)}{2}$$
$$= \frac{n^2 + n + 2}{2}$$

17. $\lim_{n\to\infty}\left[\sqrt[3]{n^2 - n^3} + n\right] = \lim_{n\to\infty} n\left[\left(-1 + \frac{1}{n}\right)^{1/3} + 1^{1/3}\right]$

$$= \lim_{n\to\infty} n \cdot \frac{\left(\frac{1}{n} - 1\right) + 1}{\left(\frac{1}{n} - 1\right)^{2/3} + 1 - \left(\frac{1}{n} - 1\right)^{1/3}}$$

$$\left[\because a + b = \frac{a^3 + b^3}{a^2 - ab + b^2}\right]$$

$$= \lim_{n\to\infty} \frac{1}{\left(\frac{1}{n} - 1\right)^{2/3} + 1 - \left(\frac{1}{n} - 1\right)^{1/3}}$$

$$= \frac{1}{1 + 1 + 1} = \frac{1}{3}$$

18. $\frac{x + 1}{x^{2/3} - x^{1/3} + 1} - \frac{x - 1}{x - x^{1/2}}$

$$= \frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{x - 1}{x^{1/2}(x^{1/2} - 1)}$$

$$= \frac{(x^{1/3} + 1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \frac{x^{1/2} + 1}{x^{1/2}}$$

$$= x^{1/3} + 1 - 1 - x^{-1/2} = x^{1/3} - x^{-1/2}$$

$\therefore \left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}=(x^{1/3}-x^{-1/2})^{10}$

T_{r+1} for $(x^{1/3}-x^{-1/2})$ is

${}^{10}C_r\ (x^{1/3})^{10-r}\ (-1)^r\ (x^{-1/2})^r$.

For term independent of x,

$$\frac{10-r}{3}-\frac{r}{2}=0$$

$\Rightarrow \quad 20-2r-3r=0$

$\Rightarrow \quad r=4$

Hence, required coefficient $= {}^{10}C_4\ (-1)^4 = 210$.

19. Since, $\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is a square root of I_2 i.e. two rowed unit matrix.

$\therefore \quad \begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta\\ \gamma\alpha-\gamma\alpha & \gamma\beta+\alpha^2\end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \alpha^2+\beta\gamma\end{bmatrix}$

$\therefore \quad \alpha^2+\beta\gamma=1$

$\Rightarrow 1-\alpha^2-\beta\gamma=0$

20. Given, $y=Ae^{3x}+Be^{5x}$ ...(i)

$\Rightarrow \quad \frac{dy}{dx}=3Ae^{3x}+5Be^{5x}$ ...(ii)

and $\quad \frac{d^2y}{dx^2}=9Ae^{3x}+25Be^{5x}$ ...(iii)

From (i), (ii) and (iii), we have

$$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$$

which is the required differential equation.

21. Let $y=\sin^{-1}(2x\sqrt{1-x^2})$

and $\quad z=\sin^{-1}(3x-4x^3)$

$\Rightarrow \quad y=2\sin^{-1}x$ and

$\quad z=3\sin^{-1}x$

$\Rightarrow \quad y=\frac{2}{3}z$

$\therefore \quad \frac{dy}{dz}=\frac{2}{3}$

22. Let $I=\int\frac{2x^{12}+5x^9}{(1+x^3+x^5)^3}dx=\int\frac{5x^{-6}+2x^{-3}}{(1+x^{-2}+x^{-5})^3}dx$

Put $1+x^{-2}+x^{-5}=t$, $(-2x^{-3}-5x^{-6})\,dx=dt$

$\therefore \quad I=-\int\frac{dt}{t^3}=\frac{1}{2t^2}=\frac{1}{2(1+x^{-2}+x^{-5})^2}$

$$=\frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

23. Let $\Delta=\begin{vmatrix}\sin\theta & \cos\theta & \sin2\theta\\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$

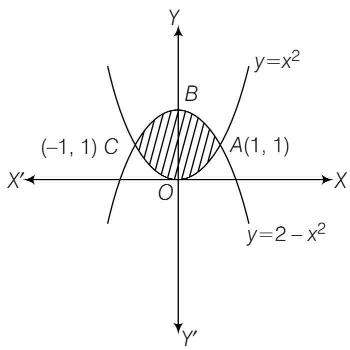
$$=\begin{vmatrix}\sin\theta & \cos\theta & \sin2\theta\\ 2\sin\theta\cos\frac{2\pi}{3} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin2\theta\cos\frac{4\pi}{3}\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

Applying $R_2 \to R_2+R_3$,

$$=-\begin{vmatrix}\sin\theta & \cos\theta & \sin2\theta\\ \sin\theta & \cos\theta & \sin2\theta\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}=0$$

as $\quad R_1=R_2$

24. Required area = 2 Area of curve OABO



$$=2\int_0^1[(2-x^2)-(x^2)]\,dx$$

$$=2\int_0^1(2-2x^2)\,dx$$

$$=4\left[x-\frac{x^3}{3}\right]_0^1=\frac{8}{3}\text{ sq units}$$

25. $\lim\limits_{n \to \infty} \dfrac{1}{1-n^3} \sum\limits_{r=1}^{n} r^2$

$$= \lim_{n \to \infty} \frac{1}{n^3\left(\frac{1}{n^3}-1\right)} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \to \infty} \frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{n^3\left(\frac{1}{n^3}-1\right)6} = -\frac{1}{3}$$

26. Let $I = \displaystyle\int \frac{\sin x + \cos x}{3 + \sin 2x}\, dx$

$$= \int \frac{\sin x + \cos x}{4 + \sin 2x - 1}\, dx$$

$$\Rightarrow \quad I = \int \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2}\, dx$$

Put $\sin x - \cos x = t$

$$\Rightarrow \quad (\cos x + \sin x) dx = dt$$

$$\therefore \quad I = \int \frac{dt}{4 - t^2} = \frac{1}{4} \log \frac{2-t}{2+t} + C$$

$$= \frac{1}{4} \log \left(\frac{2 - \sin x + \cos x}{2 + \sin x - \cos x}\right) + C$$

27. Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Since, it passes through the point $(2a, 0)$, so

$$4a^2 + 4ag + c = 0 \quad \ldots(i)$$

Also, its radical axis with $x^2 + y^2 = a^2$ is

$$2gx + 2fy + c + a^2 = 0$$

But the radical axis is $x = \dfrac{a}{2}$, so we get

$$\frac{2g}{1} = \frac{c+a}{-a/2} \text{ and } f = 0$$

or $\quad ag + c + a = 0 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get g and c

Hence, the equation is $x^2 + y^2 - 2ax = 0$

28. Any point on the parabola is $(x, x^2 + 7x + 2)$.

Its distance from the line $y = 3x - 3$ is given by

$$p = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9+1}}\right|$$

$$= \left|\frac{x^2 + 4x + 5}{\sqrt{10}}\right|$$

$$= \frac{x^2 + 4x + 5}{\sqrt{10}}$$

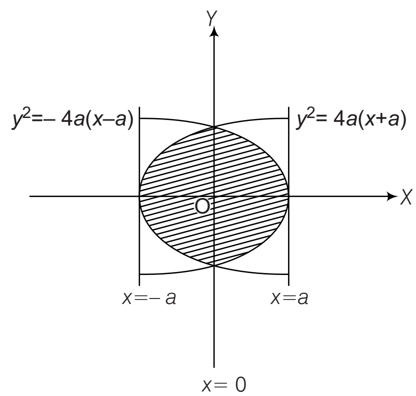
[as $x^2 + 4x + 5 > 0, \forall x \in \mathbf{R}$]

$$\therefore \quad \frac{dp}{dx} = 0$$

$$\Rightarrow \quad x = -2$$

So, the required point is $(-2, -8)$.

29. Required area $= 4\displaystyle\int_0^a \sqrt{4a(a-x)}\, dx$



$$= 4 \cdot 2\sqrt{a} \cdot \left[\frac{-2(a-x)^{3/2}}{3}\right]_0^a$$

$$= \frac{16}{3} a^2$$

30. We have,

$$\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1 \quad \ldots(i)$$

Put $x + y = v$, we get

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dv}{dx} - 1$$

Eq. (i) reducing to

$$\frac{dv}{dx} = \sin v \tan v$$

$$\Rightarrow \quad \frac{dv}{\sin v \tan v} = dx$$

$$\Rightarrow \quad \int \operatorname{cosec} v \cot v \, dv = \int dx$$

$$\Rightarrow \quad -\operatorname{cosec} v \cot v = x + C$$

$$\Rightarrow \quad -\operatorname{cosec}(x+y) = x + C$$

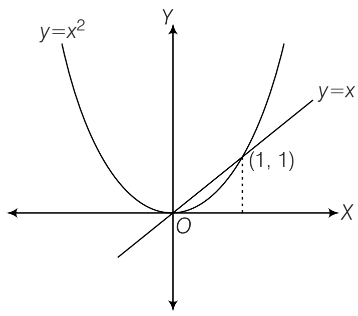
$$\Rightarrow \quad x + \operatorname{cosec}(x+y) = C$$

31. Every time the selected coupon must be from 1 to 9. The probability of such a selection at any time is $\frac{9}{15} = \frac{3}{5}$ and this can happen 7 times. i.e. $\left(\frac{9}{15}\right)^7$.

Put of these $\left(\frac{9}{15}\right)^7, \left(\frac{8}{15}\right)^7$ cases do not contain the number 9.

Thus, the required probability $= \left(\frac{9}{15}\right)^7 - \left(\frac{8}{15}\right)^7$

32. $h(x) = \min(x, x^2) = \begin{cases} x, & x < 0 \\ x^2, & 0 \le x < 1 \\ x, & x \ge 1 \end{cases}$



It is evident from the above graph that the given function is continuous for all x.

Since, there are sharp edges at $x = 0$ and $x = 1$, the function is not differentiable at these points.

Also, at $x \ge 1$, the function represents a straight line having slope 1, therefore $h'(x) = 1, \forall x \ge 1$.

33. For $f(x)$ to be continuous at $x = 0$, we must have

$$\lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \to 0} (\cos x)^{1/x} = k$$

$$\Rightarrow \lim_{x \to 0} \{1 + (\cos x - 1)\}^{1/x} = k$$

$$\Rightarrow e^{\lim_{x \to 0} \frac{\cos x - 1}{x}} = k$$

$$\Rightarrow e^{-\lim_{x \to 0} \frac{2\sin^2 x/2}{x}} = k$$

$$\Rightarrow e^0 = k$$

$$\therefore k = 1$$

34. $\because f'(0^-) = \lim_{h \to 0} f(0 - h)$

$$\Rightarrow f'(0^-) = \lim_{h \to 0} \frac{e^{-1/h} - 1}{e^{1/-h} + 1}$$

$$\Rightarrow f'(0^-) = \frac{0 - 1}{0 + 1}$$

$$\therefore f'(0^-) = -1$$

and $f'(0^+) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{e^{1/h} - 1}{e^{1/h} + 1}$

$$\Rightarrow f'(0^+) = \frac{1 - 0}{1 + 0} = 1$$

35. Let $E = -2i \sum_{p=1}^{6} \left(\cos \frac{2p\pi}{7} + i \sin \frac{2p\pi}{7}\right)$

$$\Rightarrow E = -2i \sum_{p=1}^{6} e^{i\left(\frac{2p\pi}{7}\right)}$$

Let $\alpha = e^{i\left(\frac{2\pi}{7}\right)}$ ...(i)

$$\therefore E = -2i(\alpha + \alpha^2 + \ldots + \alpha^6)$$

$$\Rightarrow E = -2i[(1 + \alpha + \alpha^2 + \ldots + \alpha^6) - 1]$$

$$\Rightarrow E = -2i\left[\left(\frac{1 - \alpha^7}{1 - \alpha}\right) - 1\right]$$

Since, $\alpha^7 = \left(e^{i\left(\frac{2\pi}{7}\right)}\right)^7 = e^{i2\pi} = 1$

$$\Rightarrow E = -2i\left\{\frac{1 - \left(e^{i\frac{2\pi}{7}}\right)^7}{1 - e^{i\left(\frac{2\pi}{7}\right)}} - 1\right\}$$

$$\Rightarrow E = -2i\left\{\frac{1 - 1}{1 - e^{i\left(\frac{2\pi}{7}\right)}} - 1\right\}$$

$$\Rightarrow E = -2i(0 - 1)$$

$$\therefore E = 2i$$

36. Number of options on 1st ring = 4

Number of options on 2nd ring = 4

Number of options on 3rd ring = 4

Number of options on 4rd ring = 4

Number of options on 5rd ring = 4

$\therefore$ Total number of ways of all the options

$= 4 \times 4 \times 4 \times 4 \times 4 = 1024$

$\therefore$ Number of all possible unsuccesful attempts to open the lock is $= 1024 - 1 = 1023$

37. Domain of f is defined, if

$$|[|x| - 1]| - 5 > 0$$

$$\Rightarrow |[|x| - 1]| > 5$$

$\Rightarrow$ either $[|x| - 1] < -5$ or $[|x| - 1] > 5$

I

$$[|x| - 1] < -5$$
$$\Rightarrow |x| - 1 < -5$$
$\Rightarrow$ $|x| < -4$, which is not possible.

II

$$[|x| - 1] > 5$$
$$\Rightarrow |x| - 1 \geq 6$$
$$\Rightarrow |x| \geq 7$$
$$\therefore x \in (-\infty, -7] \cup [7, \infty)$$

38. We have, $y^{1/m} + y^{-1/m} = 2x$

or $$y^{1/m} + \frac{1}{y^{1/m}} = 2x$$

or $$(y^{1/m})^2 - 2xy^{1/m} + 1 = 0$$

$$\therefore y^{1/m} = x \pm \sqrt{(x^2 - 1)}$$

or $$y = \left[x \pm \sqrt{x^2 - 1}\right]^m$$

$$\therefore \quad y_1 = m[x \pm \sqrt{(x^2 - 1)}]^{m-1}\left[1 \pm \frac{2x}{2\sqrt{x^2 - 1}}\right]$$

$$\Rightarrow \quad y_1 = \frac{\pm m [x \pm \sqrt{(x^2 - 1)}]^m}{\sqrt{(x^2 - 1)}} = \frac{\pm my}{\sqrt{(x^2 - 1)}}$$

$$\Rightarrow (x^2 - 1)y_1^2 = m^2y^2$$
$$\Rightarrow (x^2 - 1)2y_1y_2 + 2xy_1^2 = 2m^2yy_1$$
$$\Rightarrow (x^2 - 1)y_2 + xy_1 = m^2y$$

39. We have, $cx - by + b^2 = 0$ is a tangent of circle $x^2 + y^2 - ax - by = 0$.

$$\therefore \text{ Centre} = \left(\frac{a}{2}, \frac{b}{2}\right), \text{Radius}(r) = \sqrt{\frac{a^2 + b^2}{4}}$$

Distance from centre to the line $cx - by + b^2 = 0$,

$$\Rightarrow \quad r = \left|\frac{\frac{ac}{2} - \frac{b^2}{2} + b^2}{\sqrt{b^2 + c^2}}\right|$$

$$\Rightarrow \quad \sqrt{\frac{a^2 + b^2}{4}} = \frac{\frac{ac + b^2}{2}}{\sqrt{b^2 + c^2}}$$

$$\Rightarrow (b^2 + c^2)(a^2 + b^2) = (ac + b^2)^2$$
$$\Rightarrow b^4 + b^2(a^2 + c^2) + a^2c^2 = b^4 + 2b^2ac + a^2c^2$$
$$\Rightarrow a^2 + c^2 = 2ac$$
$$\Rightarrow (a - c)^2 = 0$$
$$\Rightarrow a = c$$

40. Given, $\alpha + \beta + \gamma = \frac{\pi}{2}$

$\Rightarrow \cot\alpha + \cot\beta + \cot\gamma = \cot\alpha\cot\beta\cot\gamma$

$\therefore$ $\cot\alpha, \cot\beta, \cot\gamma$ are in AP.

$$\Rightarrow 3\cot\beta = \cot\alpha\cot\beta\cot\gamma$$
$$\Rightarrow \cot\alpha\ \cot\gamma = 3$$

41. Since, $R = \frac{b}{2\sin B} = \frac{2}{2\sin 30°} = \frac{2}{1}$

$\therefore$ Area of circumcircle $= \pi R^2 = \pi(2)^2$

$= 4\pi$ sq units

42. $$\frac{3}{4\cdot 8} - \frac{3\cdot 5}{4\cdot 8\cdot 12} + \frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12\cdot 16} - \ldots + \frac{3}{4} - \frac{3}{4}$$

$$= 1 - \frac{1}{4} + \frac{1\cdot 3}{2\cdot 4\cdot 4} - \frac{1\cdot 3\cdot 5}{4\cdot 4\cdot 2\cdot 4\cdot 3} + \ldots - \frac{3}{4}$$

$$= \left[1 + \frac{1}{1!}\left(-\frac{1}{4}\right) + \frac{1(1+2)}{2!}\left(-\frac{1}{4}\right)^2 + \frac{1(1+2)(1+4)}{3!}\left(-\frac{1}{4}\right)^3 + \ldots\right] - \frac{3}{4}$$

$$= \left(1 - \frac{1}{4}\right)^{-1/2} - \frac{3}{4} = \sqrt{\frac{2}{3}} - \frac{3}{4}$$

43. Let $I = \int_0^\pi \sin^3\theta\, d\theta$

$[\because \sin\theta > 0 \text{ for } 0 < \theta < \pi]$

$$= \int_0^\pi \sin\theta(1 - \cos^2\theta)\, d\theta$$

Put $\cos\theta = t$

$$\Rightarrow -\sin\theta\, d\theta = dt$$

$$\therefore \quad I = \int_{-1}^1 (1 - t^2)dt = \left[t - \frac{t^3}{3}\right]_{-1}^1$$

$$= 2 - \frac{2}{3} = \frac{4}{3}$$

44. $$\int \frac{\cos x + x\sin x}{x^2 + x\cos x}\, dx$$

$$= \int \frac{x + \cos x - x + x\sin x}{x^2 + x\cos x}\, dx$$

$$= \int \frac{dx}{x} - \int \frac{x(1 - \sin x)}{x(x + \cos x)}\, dx$$

$$= \log x - \log(x + \cos x) + C$$

$$= \log\left|\frac{x}{x + \cos x}\right| + C$$

45. Equation of tangent with the slope $\frac{-3}{4}$ is

$$y = \frac{-3}{4}x + C.$$

According to condition of tangency,

$$C = \sqrt{32 \times \left(\frac{-3}{4}\right)^2 + 18} = \sqrt{18 + 18} = 6$$

$\therefore \qquad y = \frac{-3}{4}x + 6$

$\Rightarrow 4y + 3x = 24$

Since, it meet the coordinate axes in A and B.

$\therefore$ $A \equiv (8, 0)$ and $B \equiv (0, 6)$

$\therefore$ Required area $= \frac{1}{2} \times 8 \times 6 = 24$ sq units

46. $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

$$= \lim_{x \to \frac{\pi}{6}} \frac{4\sin x \cos x + \cos x}{4\sin x \cos x - 3\cos x}$$

[by L′ Hospital's rule]

$$= \lim_{x \to \frac{\pi}{6}} \frac{\cos x(4\sin x + 1)}{\cos x(4\sin x - 3)}$$

$$= \frac{4\sin\frac{\pi}{6} + 1}{4\sin\frac{\pi}{6} - 3} = -3$$

47. On squaring and adding the given equations, we get

$$\sin^2 A + \cos^2 B + 2\sin A \cos B + \sin^2 B + \cos^2 A + 2\sin B \cos A = a^2 + b^2$$

$\Rightarrow \ 2\sin(A + B) + 2 = a^2 + b^2$

$\Rightarrow \qquad \sin(A + B) = \dfrac{a^2 + b^2 - 2}{2}$

48. Given, $\sin A \sin B = \dfrac{ab}{c^2}$

$\Rightarrow \ c^2 = \dfrac{ab}{\sin A \sin B} = \left(\dfrac{a}{\sin A}\right)\left(\dfrac{b}{\sin B}\right)$

$\Rightarrow \ c^2 = \left(\dfrac{c}{\sin C}\right)^2 \qquad \left[\because \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}\right]$

$\Rightarrow \qquad \sin^2 C = 1$

$\Rightarrow \qquad C = 90°$

Hence, ΔABC is a right angled triangle.

49. Given, $l + m + n = 0 \qquad \ldots(i)$

and $\qquad lm = 0$

Either $\qquad m = 0$ or $l = 0$

If $l = 0$, then put in Eq. (i), we get $\ m = -n$

$\therefore$ Direction ratios are $0, -n, n$ i.e. $0, -1, 1$

If $m = 0$, then put in Eq. (i), we get $l = -n$

$\therefore$ Direction ratios are $-n, 0, n$ i.e. $-1, 0, 1$.

$$\therefore \ \cos\theta = \frac{0 \times (-1) + (-1) \times 0 + 1 \times 1}{\sqrt{(0)^2 + (-1)^2 + (1)^2}\ \sqrt{(-1)^2 + (0)^2 + (1)^2}} = \frac{1}{2}$$

$\Rightarrow \qquad \theta = \dfrac{\pi}{3}$

50. Given equation can be written as

$$\frac{(x-3)^2}{16} + \frac{y^2}{25} = 1$$

Here, $a^2 = 16$ and $b^2 = 25$

$\therefore \qquad e = \sqrt{1 - \dfrac{16}{25}} = \dfrac{3}{5}$

Hence, the foci of conic section are $(0, \pm\ be)$ i.e. $(3, \pm 3)$.

51. Given, $x - 2y = 4$

Let $\qquad A = xy$

$\Rightarrow \qquad A = 2y^2 + 4y$

$\Rightarrow \qquad \dfrac{dA}{dy} = 4 + 4y$

For extremum value, $\dfrac{dA}{dy} = 0 \Rightarrow y = -1$

Now, $\quad \dfrac{d^2A}{dy^2} = 4 > 0, \qquad$ (minimum)

At $\quad y = -1, x = 4 + 2(-1) = 2$

$\therefore \ A = xy = 2(-1) = -2$

$\therefore$ Hence, the minimum value of xy is -2.

52. Let $b = \hat{j}$ and $c = \hat{j}$

$\therefore \qquad |b \times c| = |\hat{k}| = 1$

Again, let $\qquad a = a_1\hat{i} + a_2\hat{j} + a_3\hat{j}$

Now, $\qquad a \cdot b = a \cdot i = a_1, a \cdot \bar{c} = a \cdot \hat{j} = a_2$

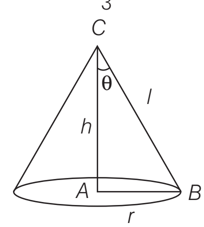
and $\quad a \cdot \dfrac{b \cdot c}{|b \times c|} = a \cdot k = a_3$

$\therefore \qquad (a \cdot b)\, b + (a \cdot c)\, c + \dfrac{a \cdot (b \times c)}{\bar{b} \times c|} \cdot (b \times c)$

$= a_1 b + a_2 c + a_3 (b \times c) = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = a$

53. Volume of cone, $V = \dfrac{\pi}{3} r^2 h$

$\Rightarrow \qquad V = \dfrac{\pi}{3} r^2 \sqrt{l^2 - r^2}$



On differentiating w.r.t. r, we get

$$\frac{dV}{dr} = \frac{\pi}{3}\left[2r\sqrt{l^2 - r^2} + \frac{r^2}{2\sqrt{l^2 - r^2}}(-2r)\right]$$

Put $\frac{dV}{dr} = 0$, we get

$$r = \pm l\sqrt{\frac{2}{3}}$$

At $r = l\sqrt{\frac{2}{3}}, \frac{d^2V}{dr^2} < 0,$ (maximum)

$\therefore$ $h = \sqrt{l^2 - \frac{2}{3}l^2} = \frac{l}{\sqrt{3}}$

In Δ ABC, $\tan\theta = \frac{r}{h} = \frac{l\sqrt{\frac{2}{3}}}{l/\sqrt{3}} = \sqrt{2}$

54. $f(x) = \frac{2\sin 8x\cos x - 2\sin 6x\cos 3x}{2\cos 2x\cos x - 2\sin 3x\sin 4x}$

$= \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)}$

$= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} = \frac{2\cos 5x\sin 2x}{2\cos 2x\cos 5x}$

$= \tan 2x$

$\therefore$ Period of $f(x) = \frac{\pi}{2}$

55. We know that the equation of a plane parallel to X-axis is

$$by + cz + d = 0.$$

Since, it passes through the points (2, 3, 1) and (4, −5, 3).

$\therefore$ $3b + c + d = 0$

and $-5b + 3c + d = 0$

$\Rightarrow$ $\frac{b}{1-3} = \frac{c}{-8} = \frac{d}{14}$

$\Rightarrow$ $\frac{b}{-2} = \frac{c}{-8} = \frac{d}{14}$

$\therefore$ Equation of plane is $-2y - 8z + 14 = 0$

i.e. $y + 4z = 7$.

56. Since, the point Q is image of P, therefore PQ is perpendicular to the plane $x - 2y + 5z = 6$

$\therefore$ Required equation of the line is

$$\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}.$$

57. The intersection of two planes is

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z$

$+ (-6 + 5\lambda) = 0$ ...(i)

Since, this plane is perpendicular to the plane $4x + 5y - 3z - 8 = 0$.

$\therefore$ $(1 + 2\lambda)4 + (1 + 3\lambda)5 + (1 + 4\lambda)(-3) = 0$

$\Rightarrow$ $\lambda = \frac{-6}{11}$

On putting the value of λ in Eq. (i), we get

$\left(-\frac{1}{11}\right)x + \left(-\frac{7}{11}\right)y + \left(-\frac{13}{11}\right)z + \left(-\frac{96}{11}\right) = 0$

$\Rightarrow$ $x + 7y + 13z + 96 = 0$

58. Here, $T_n = \frac{n^3}{n!} = \frac{n^2 - 1}{(n-1)!} + \frac{1}{(n-1)!}$

$= \frac{n+1}{(n-2)!} + \frac{1}{(n-1)!}$

$= \frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}$

$\therefore \Sigma T_n = \Sigma\left(\frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}\right)$

$= e + 3e + e = 5e$

59. Given, $f(x) = \cos x + \cos\sqrt{2}x$

$\Rightarrow$ $f'(x) = -\sin x - \sqrt{2}\sin\sqrt{2}x$

$\Rightarrow$ $f''(x) = -\cos x - 2\cos\sqrt{2}x$

For extremum value, put $f'(x) = 0$

$\Rightarrow$ $-\sin x - \sqrt{2}\sin\sqrt{2}x = 0$

$\Rightarrow$ $x = 0$

At $x = 0$, $f''(x) < 0$, (maximum)

Hence, $f(x)$ is maximum only once.

60. We know that a vector perpendicular to the plane containing the point A, B and C is given by $A \times B + B \times C + C \times A$.

Given, $\mathbf{A} = \hat{i} - \hat{j} + 2\hat{k}$, $\mathbf{B} = 2\hat{i} + 0\hat{j} - \hat{k}$

and $\mathbf{C} = 0\hat{i} + 2\hat{j} + \hat{k}$

Now, $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{i} + 5\hat{j} + 2\hat{k}$

$\mathbf{B} \times \mathbf{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\mathbf{C} \times \mathbf{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{i} + \hat{j} - 2\hat{k}$

Thus, $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{C} + \mathbf{C} \times \mathbf{A}$

$= (\hat{i} + 5\hat{j} + 2\hat{k}) + (2\hat{i} - 2\hat{j} + 4\hat{k}) + (5\hat{i} + \hat{j} - 2\hat{k})$

$= 8\hat{i} + 4\hat{j} + 4\hat{k}$

61. We have,

$$P'P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad P'P = I$ or $P' = P^{-1}$

As, $\quad Q = PAP'$

$\therefore P'Q^{2005}P = P'[(PAP')(PAP') \dots 2005 \text{ times}]\ P$

$$= \underbrace{(P'P)A\,(P'P)A\,(P'P)\dots(P'P)A\,(P'P)}_{2005 \text{ times}}$$

$$= IA^{2005} = A^{2005}$$

Now, $\quad A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \dots A^{2005}$$

$$= \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad P'Q^{2005}P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

62. $m + n = -\frac{3}{7} + \left(\frac{-11}{7}\right) = -2$ (–ve integer)

Let $\quad I = \int \cos^{-3/7}x\,(\sin^{(-2+\frac{3}{7})}x)\,dx$

$$= \int \cos^{-3/7}x \sin^{-2} x \sin^{3/7} x dx$$

$$= \int \frac{\operatorname{cosec}^2 x}{\left(\frac{\cos^{3/7} x}{\sin^{3/7} x}\right)} dx = \int \frac{\operatorname{cosec}^2 x\, dx}{\cot^{3/7} x}$$

Put $\cos x = t \Rightarrow -\operatorname{cosec}^2 x dx = dt$

$$\therefore \quad I = -\int \frac{dt}{t^{3/7}} = -\frac{t^{\frac{3}{7}+1}}{-\frac{3}{7}+1} + C$$

$$= -\frac{7}{4}t^{4/7} + c = -\frac{7}{4}\cot^{4/7} x + C$$

$$= -\frac{7}{4}\tan^{-4/7} x + C$$

63. Let $ax + by = 1$

be the chord $\quad \dots$(i)

Making the equation of hyperbola homogeneous using Eq. (i), we get

$$3x^2 - y^2 + (-2x + 4y)(ax + by) = 0$$

or $\quad (3 - 2a)x^2 + (-1 + 4b)y^2 + (-2b + 4a)xy = 0$

Since, the angle subtended at the origin is a right angle.

$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$\Rightarrow \quad (3 - 2a) + (-1 + 4b) = 0$

$\Rightarrow \quad a = 2b + 1$

$\therefore$ Chords are $(2b + 1)x + by - 1 = 0$

or $\quad b(2 + y) + (x - 1) = 0,$

which clearly pass through the fixed point $(1, -2)$.

64. The line of intersection of the planes

$\mathbf{r}\cdot(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ and $\mathbf{r}\cdot(\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 2$

is common to both the planes.

Therefore, it is perpendicular to normals to the two planes i.e. $\mathbf{n_1} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{n_2} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. Hence, it is parallel to the vector $\mathbf{n_1} \times \mathbf{n_2} = -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 13\hat{\mathbf{k}}$.

Thus, we have to find the equation of the plane passing through $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and normal to the vector $\mathbf{n} = \mathbf{n_1} \times \mathbf{n_2}$.

The equation of the required plane is

$$(\mathbf{r} - \mathbf{a})\cdot\mathbf{n} \text{ or } \mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$$

or $\quad \mathbf{r}\cdot(-2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 13\hat{\mathbf{k}})$

$$= (\hat{\mathbf{i}} + 2\hat{\mathbf{i}} + \hat{\mathbf{k}})\cdot(-2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 13\hat{\mathbf{k}})$$

or $\quad \mathbf{r}\cdot(2\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 13\hat{\mathbf{k}}) = 1$

65. Given, $x = 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \dots$

or $\quad x = e^{\frac{dy}{dx}}$

$\Rightarrow \quad \frac{dy}{dx} = \log_e x$

Hence, the degree of the above differential equation is 1.

66. Let $P(x_1, y_1)$ be the mid-point of a chord of the circle.

The equation of the chord is $T = S_1$

i.e. $\quad xx_1 + yy_1 - 16 = x_1^2 + y_1^2 - 16$

or $\quad y = -\frac{x_1}{y_1}x + \frac{x_1^2 + y_1^2}{y_1}$

Since, it touches the hyperbola $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$.

$\therefore \left(\frac{x_1^2 + y_1^2}{y_1}\right) = 4^2 \cdot \frac{x_1^2}{y_1^2} - 3^2 \quad [\because c^2 = a^2m^2 - b^2]$

$\therefore$ Locus of (x_1, y_1) is

$$\frac{(x^2 + y^2)^2}{y^2} = \frac{16x^2 - 9y^2}{y^2}$$

or $\quad (x^2 + y^2)^2 = 16x^2 - 9y^2$

67. Given, $^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 50$

$$\Rightarrow \quad \frac{(n-1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-2)!} \le 50$$

$$\Rightarrow \quad \frac{(n+1)!}{3!}\left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!}\right] \le 50$$

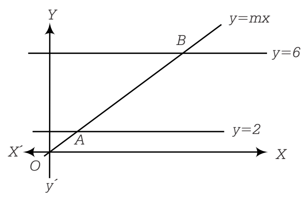
$$\Rightarrow \quad (n+1)!\left(\frac{n-1-3}{(n-1)!}\right) \le 300$$

$$\Rightarrow \quad (n+1)n\,(n-4) \le 300$$

For $n = 8$, it satisfy to the above inequality.

But $n = 1$ it does not satisfy the above inequality.

68. Given lines are $y = mx, y = 2$ and $y = 6$.



Here, coordinates of points A and B are $\left(\frac{2}{m}, 2\right)$, $\left(\frac{6}{m}, 6\right)$, respectively.

$$\therefore \quad AB = \sqrt{\left(\frac{2}{m} - \frac{6}{m}\right)^2 + (2-6)^2} < 5 \qquad \text{[given]}$$

$$\Rightarrow \quad \left(\frac{2}{m} - \frac{6}{m}\right)^2 + (4)^2 < 25$$

$$\Rightarrow \quad \left(\frac{2}{m} - \frac{6}{m}\right)^2 < 9$$

$$\Rightarrow \quad -3 < \frac{2}{m} - \frac{6}{m} < 3$$

$$\Rightarrow \quad -\frac{4}{3} > m > \frac{4}{3}$$

$$\therefore \quad m \in \left]-\infty, -\frac{4}{3}\right[\cup \left]\frac{4}{3}, \infty\right[$$

69. Equation of line perpendicular to $2x + y + 6 = 0$ and passes through origin is $x - 2y = 0$

Now, the point of intersection of $2x + y + 6 = 0$ and $x - 2y = 0$ is $\left(-\frac{12}{5}, -\frac{6}{5}\right)$.

Similarly, the point of intersection of $x - 2y = 0$ and $4x + 2y - 9 = 0$ is $\left(\frac{9}{5}, \frac{9}{10}\right)$.

Let the origin divide the line $x - 2y = 0$ in the ratio $\lambda : 1$

$$\therefore \quad x = \frac{\frac{9}{5}\lambda - \frac{12}{5}}{\lambda + 1} = 0$$

$$\Rightarrow \quad \frac{9}{5}\lambda = \frac{12}{5}$$

$$\Rightarrow \quad \lambda = \frac{12}{9} = \frac{4}{3}$$

70. Given, $P(\overline{A \cup B}) = \frac{1}{6}$

$$\Rightarrow \quad 1 - P(A \cup B) = \frac{1}{6}$$

$$\Rightarrow \quad 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{6}$$

$$\Rightarrow \quad P(\overline{A}) - P(B) + P(A \cap B) = \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{4} - P(B) + \frac{1}{4} = \frac{1}{6}$$

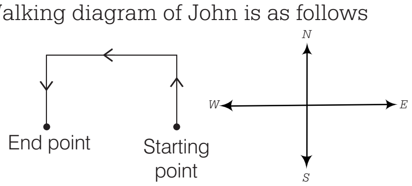
$$\Rightarrow \quad P(B) = \frac{1}{3}$$

and $\quad P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

Since, $P(A \cap B) = P(A) \cdot P(B)$, so the events A and B are independent events but not equally likely.

General Aptitude

16. Walking diagram of John is as follows



It is clear from the above diagram that John is in facing South direction now.

17. As, $64 + 7 = 460$

$\Rightarrow \quad 64 \times 7 + 12 = 460$

and $\quad 25 + 8 = 212$

$\Rightarrow \quad 25 \times 8 + 12 = 212$

Same as, $\quad 43 + 8 = ?$

$\Rightarrow \quad 43 \times 8 + 12 = \boxed{356}$

$\therefore \quad ? = 356$

19. Given equation

$$6 + 64 - 8 \div 45 \times 8$$

Now, changing the sign as per the question,

$$6 \times 64 \div 8 + 45 - 8$$
$$= \frac{6 \times 64}{8} + 45 - 8$$
$$= 6 \times 8 + 45 - 8$$
$$= 48 + 45 - 8$$
$$= 93 - 8 = 85$$

21.

$$1 \quad 4 \quad 2 \quad 3 \quad 2 \quad \boxed{4}$$

+2

−2

22. As,

$$25 + 17 = 42 \Rightarrow \frac{42}{7} = 6$$

and $\quad 38 + 18 = 56 \Rightarrow \dfrac{56}{7} = 8$

Similarly, $\quad 89 + 16 = 105$

$$\Rightarrow \quad \frac{105}{7} = \boxed{15}$$

23. Relative speed of both the persons

$$= 12 + 10 = 22 \text{ km/h}$$

Now, distance between both of them

$$= 55 - 11 = 44 \text{ km}$$

$\therefore$ Required time, when distance is 11 km between both of them $= \dfrac{\text{Total distance}}{\text{Relative speed}}$

$$= \frac{44}{22} = 2 \text{ h}$$

Hence, they will be apart 11 km after 2 h.

24. Given, $x - \dfrac{1}{x} = 2$

On, squaring both the sides, we get

$$x^2 + \frac{1}{x^2} - 2 = 4$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 4 + 2 \Rightarrow x^2 + \frac{1}{x^2} = 6$$

25. Population after 2 yr

$$= 20000 \left(1 + \frac{20}{100}\right)\left(1 + \frac{30}{100}\right)$$
$$= 20000 \times \frac{120}{100} \times \frac{130}{100}$$
$$= 2 \times 120 \times 130 = 31200$$

26. Let initial price of rice = ₹ x per kg

Now, a reduction of 20% in price,

Reduced price of rice

$$= x \times \frac{80}{100} = ₹\ \frac{4x}{5} \text{ per kg}$$

Now, according to the question,

$$\frac{1200}{\frac{4x}{5}} - \frac{1200}{x} = 5$$

$$\Rightarrow \quad \frac{6000}{4x} - \frac{1200}{x} = 5$$

$$\Rightarrow \frac{1500}{x} - \frac{1200}{x} = 5 \Rightarrow \frac{300}{x} = 5$$

$$\Rightarrow \quad x = \frac{300}{5} = 60$$

$\therefore$ Reduced price of rice

$$= \frac{4x}{5} = \frac{4 \times 60}{5} = ₹\ 48 \text{ per kg}$$

27. Let total income of a person = ₹ 100

$\therefore$ Then, spend on different expenses = ₹ 60

$\therefore$ His savings = 100 − 60 = ₹ 40

Now, income after increment of 20%

$$= \frac{120}{100} \times 100 = ₹\ 120$$

and expences after increment of 10%

$$= 120 \times (60 + 10)\%$$
$$= 120 \times \frac{70}{100} = ₹\ 84$$

$\therefore$ New savings = 120 − 84 = ₹ 36

Now, percentage decrease

$$= \left(\frac{40 - 36}{40}\right) \times 100\%$$
$$= \frac{4}{40} \times 100\% = \frac{100}{10}\% = 10\%$$

28. Here, $\quad$ = ₹ 4096, $\quad = 12\frac{1}{2}\% = \frac{25}{2}\%$,

and $\quad = 18 \text{ months} = \dfrac{18}{12} \text{ yr} = \dfrac{3}{2} \text{ yr}$

Now, money received to Shyam

$$= 4096 \left(1 + \frac{25}{2 \times 100} \times \frac{1}{2}\right)^{\frac{3}{2} \times 2}$$
$$= 4096 \left(1 + \frac{1}{16}\right)^3 = 4096 \left(\frac{17}{16}\right)^3$$
$$= 4096 \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16}$$
$$= 17 \times 17 \times 17 = ₹\ 4913$$

29. Time taken to complete the work by = 12 days

Since, work twice as fast as .

$\therefore$ Time taken to complete the work by = 6 days

Now, one day work of $= \frac{1}{12}$

and one day work of $= \frac{1}{6}$

$\therefore$ One day work of $(A + B) = \frac{1}{6} + \frac{1}{12}$

$$= \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$$

Hence, time taken by and together to finish the work = 4 days.

30. Let the height of embankment = h m

Given, radius of well $= \frac{3}{2} = 1.5$ m

$\therefore \quad \frac{4}{3} \times \frac{22}{7} \times [(4)^2 - (1.5)^2] \times h = \frac{22}{7} \times (1.5)^2 \times 14$

$\Rightarrow \quad \frac{4}{3} \times (16 - 2.25) \times h = 2.25 \times 14$

$\Rightarrow \quad \frac{4}{3} \times 13.75 \times h = 2.25 \times 14$

$\Rightarrow \quad h = \frac{2.25 \times 14 \times 3}{4 \times 13.75} = \frac{225 \times 7 \times 3}{2 \times 1375} = \frac{189}{110}$

$= 1.718 \approx 1.750$ m

Hence, the height of embankment is 1.750 m.