MET 2015 Question Paper with Solution

Manipal Entrance Test (MET)

Solved Paper 2015

Manipal

Engineering Entrance Exam

Physics

1. Two point charges $2q$ and $8q$ are placed at a distance r apart. Where should a third charge $-q$ be placed between them, so that the electrical potential energy of the system is minimum?

(a) At a distance of $r/3$ from $2q$
(b) At a distance of $2r/3$ from $2q$
(c) At a distance of $r/16$ from $2q$
(d) None of the above

2. If levels 1 and 2 are separated by an energy $E_2 - E_1$, such that the corresponding transition frequency falls in the middle of the visible range, calculate the ratio of the populations of two levels in the thermal equilibrium at room temperature.

(a) 1.1577×10^{-38} (b) 2.9×10^{-35}
(c) 2.168×10^{-36} (d) 1.96×10^{-20}

3. The energy of a hydrogen atom in its ground state is -13.6 eV. The energy of the level corresponding to the quantum number $n = 5$ is

(a) -5.40 eV (b) -0.54 eV
(c) -8.5 eV (d) -2.72 eV

4. We have a galvanometer of resistance 25 Ω. It is shunted by 2.5 Ω wire. The part of the total current that flows through the galvanometer is given as

(a) $\frac{i}{i_0} = \frac{4}{11}$ (b) $\frac{i}{i_0} = \frac{3}{11}$ (c) $\frac{i}{i_0} = \frac{2}{10}$ (d) $\frac{i}{i_0} = \frac{1}{11}$

5. Two moles of helium are mixed with n moles of hydrogen. The root mean square (rms) speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. Then, the value of n is

(a) 1 (b) 3 (c) 2 (d) 3/2

6. A coil having n turns and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t second from a magnetic field w_1 weber to w_2 weber. The induced current in the circuit is

(a) $\frac{-(w_2 - w_1)}{5Rt}$
(b) $\frac{-n(w_2 - w_1)}{5Rt}$
(c) $\frac{-(w_2 - w_1)}{Rnt}$
(d) $\frac{-n(w_2 - w_1)}{Rt}$

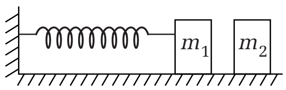
7. A thin film of soap solution ($\mu_s = 1.4$) lies on the top of a glass plate ($\mu_g = 1.5$). When incident light is almost normal to the plate, two adjacent reflection maxima are observed at two wavelengths 420 nm and 630 nm. The minimum thickness of the soap solution is

(a) 420 nm (b) 450 nm
(c) 630 nm (d) 1260 nm

8. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of a permanent magnet such that **B** is in the plane of the coil. If due to current i in the triangle, a torque τ acts on it. The side l of the triangle is

(a) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$
(b) $2\left(\frac{\tau}{\sqrt{3}\,Bi}\right)^{1/2}$
(c) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)^{1/2}$
(d) $\frac{1}{\sqrt{3}}\frac{\tau}{Bi}$

9. The two blocks of masses m_1 and m_2 are kept on a smooth horizontal table as shown in the figure. Block of mass m_1 but not m_2 is fastened to the spring. If now both the blocks are pushed to the left, so that the spring is compressed at a distance d. The amplitude of oscillation of block of mass m_1 after the system released, is



(a) $d\sqrt{\dfrac{m_1}{m_1 + m_2}}$ (b) $d\sqrt{\dfrac{m_2}{m_1 + m_2}}$
(c) $d\sqrt{\dfrac{2m_2}{m_1 + m_2}}$ (d) $d\sqrt{\dfrac{2m_1}{m_1 + m_2}}$

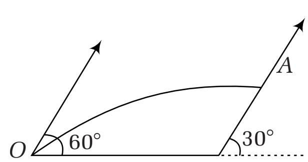
10. A juggler keeps on moving four balls in air throwing the balls after regular intervals. When one ball leaves his hand (speed $= 20 \text{ ms}^{-1}$), the position of other balls (height in metre) will be (take $g = 10 \text{ m s}^{-2}$)

(a) 10, 20, 10
(b) 15, 20, 15
(c) 5, 15, 20
(d) 5, 10, 20

11. Two coils have mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\ \pi$ rad/s. The maximum value of emf in the second coil is

(a) $12\ \pi$ (b) 8π
(c) 5π (d) 2π

12. A ball is projected from the point O with velocity 20 m/s at an angle of 60° with horizontal as shown in the figure. At highest point of its trajectory, it strikes a smooth plane of inclination 30° at point A. The collision is perfectly inelastic. The maximum height from the ground attained by the ball is



(a) 18.75 m (b) 15 m
(c) 22.5 m (d) 20.25 m

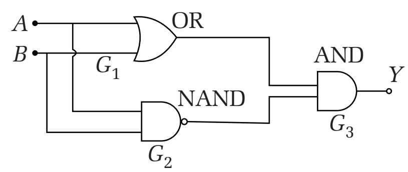
13. In a nuclear reactor, ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 yr, then find the total mass of uranium required.

(a) 36.5×10^3kg (b) 36×10^2kg
(c) 39.5×10^3kg (d) 38.2×10^3kg

14. A capacitor of capacitance 10 μF is charged to potential 50 V with a battery. The battery is now disconnected and an additional charge 200 μC is given to the positive plate of the capacitor. The potential difference across the capacitor will be

(a) 50 V (b) 80 V
(c) 100 V (d) 60 V

15. The following configuration of gate is equivalent to

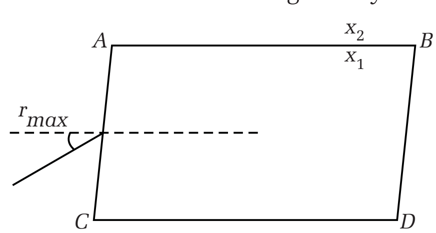


(a) NAND (b) XOR
(c) OR (d) None of these

16. Under what conditions current passing through the resistance R can be increased by short circuiting the battery of emf E_2? The internal resistances of the two batteries are r_1 and r_2 respectively.

(a) $E_2 r_1 > E_1 (R + r_2)$ (b) $E_1 r_2 < E_2(r_1 + R)$
(c) $E_2 r_2 < E (R + r_2)$ (d) $E_1 r_1 > E_2(r_1 + R)$

17. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index n_2 ($n_1 > n_2$). A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max} such that the ray comes out only from the another surface CD is given by



(a) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$

(b) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$

(c) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

(d) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

18. A sinusoidal wave travelling in the positive direction on stretched string has amplitude 20 cm, wavelength 1 m and wave velocity 5 m/s. At $x = 0$ and $t = 0$, it is given that $y = 0$ and $\frac{dy}{dt} < 0$. Find the wave function $y(x,t)$.

(a) $y(x,t) = (0.02\text{ m})\sin[(2\pi\text{m}^{-1})x + (10\pi\text{s}^{-1})t]\text{m}$

(b) $y(x,t) = (0.02\text{ m})\cos[(10\pi\text{s}^{-1})t + (2\pi\text{m}^{-1})x]\text{m}$

(c) $y(x,t) = (0.02\text{ m})\sin[(2\pi\text{m}^{-1})x - (10\pi\text{s}^{-1})t]\text{m}$

(d) $y(x,t) = (0.02\text{ m})\sin[(\pi\text{m}^{-1})x + (5\pi\text{s}^{-1})t]\text{m}$

19. The length of a potentiometer wire is l. A cell of emf E is balanced at length $l/3$ from the positive end of the wire. If the length of the wire is increased by $l/2$. Then, at what distance will the same cell give a balance point?

(a) $\frac{2l}{3}$ (b) $\frac{l}{2}$ (c) $\frac{l}{6}$ (d) $\frac{4l}{3}$

20. Find the inductance of a unit length of two parallel wires, each of radius a, whose centres are at distance d apart and carry equal currents in opposite directions. Neglect the flux within the wire.

(a) $\frac{\mu_0}{2\pi}\ln\left(\frac{d-a}{a}\right)$ (b) $\frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$

(c) $\frac{3\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$ (d) $\frac{\mu_0}{3\pi}\ln\left(\frac{d-a}{a}\right)$

21. Two wires of same lengths are shaped into a square and a circle. If they carry same current, then the ratio of their magnetic moments is

(a) $2 : \pi$ (b) $\pi : 2$ (c) $\pi : 4$ (d) $4 : \pi$

22. A circular coil of 100 turns has an effective radius of 0.05 m and carries a current of 0.1 A. How much work is required to turn it in an external magnetic field of 1.5 Wb/m^2 through 180° about its axis perpendicular to the magnetic field? The plane of the coil is initially perpendicular to the magnetic field.

(a) 0.456 J (b) 2.65 J (c) 0.2355 J (d) 3.95 J

23. The time constant of L-R circuit is

(a) LR (b) $\frac{L}{R}$

(c) $\frac{R}{L}$ (d) $\frac{1}{LR}$

24. In wave mechanics, the angular momentum of an electron is given by

(a) $\sqrt{l\frac{h}{2\pi}}$ (b) $\sqrt{l(l+1)\frac{h}{2\pi}}$

(c) $\sqrt{\frac{h2\pi}{l(l+1)}}$ (d) $\sqrt{\frac{h2\pi}{l}}$

25. The alternating voltage and current in an electric circuit are respectively given by

$E = 100\sin 100\,\pi t,\ I = 5\sin 100\pi t$

The reactance of the circuit will be

(a) 1 Ω (b) 0.05 Ω

(c) 20 Ω (d) zero

26. The biological damage caused by 1 gray α-radiation is compared with 1 gray γ-radiation in the same type of human tissue. The damage caused by the γ-radiation is

(a) more serious as compared to the damage caused by the α-radiation.

(b) less serious as compared to the damage caused by the α-radiation

(c) equally serious as the damage caused by the α-radiation

(d) incomparable with the damage caused by the α-radiation, because γ-radiation are not particles.

27. Two identical coherent sources placed on a diameter of a circle of radius R at separation x ($<< R$) symmetrically about the centre of the circle. The sources emit identical wavelength λ each. The number of points on the circle with maximum intensity is ($x = 5\lambda$)

(a) 20 (b) 22

(c) 24 (d) 26

28. Two point masses 1 and 2 move with uniform velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Their initial position vectors are $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Which of the following should be satisfied for the collision of the point masses?

(a) $\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_2 - \mathbf{r}_1|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 - \mathbf{v}_1|}$ (b) $\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_1 - \mathbf{r}_1|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 - \mathbf{v}_1|}$

(c) $\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 + \mathbf{r}_1|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 + \mathbf{v}_1|}$ (d) $\frac{\mathbf{r}_2 + \mathbf{r}_1}{|\mathbf{r}_2 + \mathbf{r}_1|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 + \mathbf{v}_1|}$

29. A neutron moving with a speed v makes a head on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of neutron for which inelastic collision will take place is

(a) 10.2 eV (b) 20.4 eV
(c) 12.1 eV (d) 16.8 eV

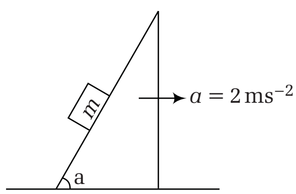
30. The specific heat at constant volume for the monoatomic argon is 0.075 kcal/kg-K, whereas its gram molecular specific heat is $C_V = 2.98$ cal/mol K. The mass of the carbon atom is

(a) 6.60×10^{-23} g (b) 3.30×10^{-23} g
(c) 2.20×10^{-23} g (d) 13.20×10^{-23} g

31. Order of magnitude of density of uranium nuclus is $(m_p = 1.6 \times 10^{-27}\,\text{kg})$

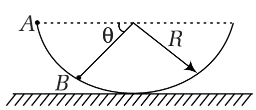
(a) 10^{20} kg/m^3 (b) 10^{17} kg/m^3
(c) 10^{14} kg/m^3 (d) 10^{11} kg/m^3

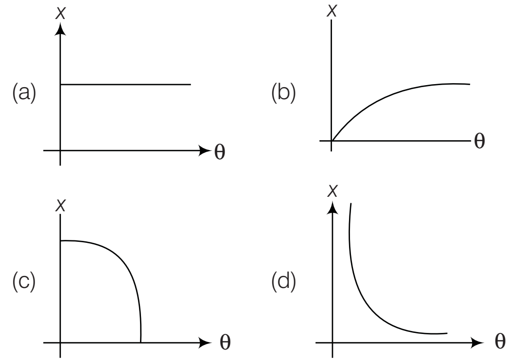
32. A block of mass m is lying on the edge having inclination angle $\alpha = \tan^{-1}\left(\frac{1}{5}\right)$. Wedge is moving with a constant acceleration, $a = 2\text{ms}^{-2}$. The minimum value of coefficient of friction μ, so that m remains stationary with respect to wedge is



(a) $\frac{2}{a}$ (b) $\frac{5}{12}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

33. A small particle of mass m is released from rest from point A inside a smooth hemispherical bowl as shown in the figure. The ratio (x) of magnitude of centripetal force and normal reaction on the particle at any point B varies with θ as





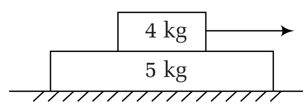
34. A solid cylinder is rolling down on an inclined plane of angle θ. The coefficient of static friction between the plane and the cylinder is μ_s. The condition for the cylinder not to slip is

(a) $\tan\theta \geq 3\,\mu_s$ (b) $\tan\theta > 3\,\mu_s$
(c) $\tan\theta \leq 3\,\mu_s$ (d) $\tan\theta < 3\,\mu_s$

35. For a given density of a planet, the orbital speed of satellite near the surface of the planet of radius R is proportional to

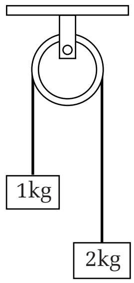
(a) $R^{1/2}$ (b) $R^{3/2}$
(c) $R^{-1/2}$ (d) R^0

36. A large slab of mass 5 kg lies on a smooth horizontal surface, with a block of mass 4 kg lying on the top of it, the coefficient of friction between the block and the slab is 0.25. If the block is pulled horizontally by a force of $F = 6$ N, the work done by the force of friction on the slab between the instants $t = 2$ s and $t = 3$ s is



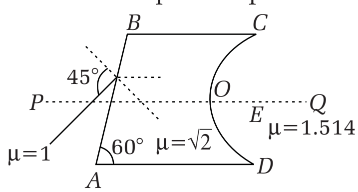
(a) 2.4 J (b) 5.55 J
(c) 4.44 J (d) 10 J

37. Two unequal masses are connected on two sides of a light string passing over a light and smooth pulley as shown in the figure. The system is released from the rest. The larger mass is stoped for a moment, 1s after the system is set into motion. The time elapsed before the string is tight again is



(a) 1/4 s (b) 1/2 s (c) 2/3 s (d) 1/3 s

38. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in figure. After refraction, it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal.

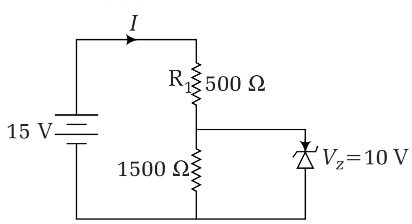


(a) 7 m (b) 7.29 m (c) 6.06 m (d) 8.55 m

39. The ratio of the energy required to raise a satellite upto a height h above the earth to the kinetic energy of the satellite into the orbit is

(a) $h : R$ (b) $R : 2h$ (c) $2h : R$ (d) $R : h$

40. In the circuit diagram, the current through the Zener diode is



(a) 10 mA (b) 3.33 mA (c) 6.67 mA (d) 0 mA

41. The frequency of sonometer wire is f, but when the weights producing the tensions are completely immersed in water, the frequency becomes $f/2$ and on immersing the weights in a certain liquid, the frequency becomes $f/3$. The specific gravity of the liquid is

(a) 4/3 (b) 16/9
(c) 15/12 (d) 32/27

42. Find the frequency of light which ejects electron from a metal surface fully stopped by a retarding potential of 3V. The photoelectric effect begins in this metal at a frequency of 6×10^{15}Hz.

(a) 1.324×10^{15}Hz
(b) 2.295×10^{16}Hz
(c) 3.678×10^{18}Hz
(d) 2.7×10^{14}Hz

43. Equations of a stationary and a travelling waves are as follows, $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin (\omega t - kx)$. The phase difference between two points $x_1 = \dfrac{\pi}{3k}$ and $x_2 = \dfrac{3\pi}{2k}$ are ϕ_1 and ϕ_2 respectively for two waves. The ratio $\dfrac{\phi_1}{\phi_2}$ is

(a) 1 (b) 5/6
(c) 3/4 (d) 6/7

44. Out of a photon and an electron, the equation $E = Pc$, is valid for

(a) both (b) neither
(c) photon only (d) electron only

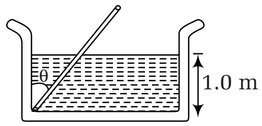
45. A small block of wood of specific gravity 0.5 is submerged at a depth of 1.2 m in a vessel filled with the water. The vessel is accelerated upwards with an acceleration $a_0 = g/2$. Time taken by the block to reach the surface, if it is released with zero initial velocity is

(a) 0.6 s (b) 0.4 s
(c) 1.2 s (d) 1 s

46. An electron beam accelerated from rest through a potential difference of 5000 V in vaccum is allowed to impinge on a surface normally. The incident current is 50 μA and if the electron comes to rest on striking the surface, the force on it is

(a) 1.1924×10^{-8}N (b) 2.1×10^{-8}N
(c) 1.6×10^{-8}N (d) 1.6×10^{-6}N

47. A uniform rod of length 2 m, specific gravity 0.5 and mass 2 kg is hinged at one end to the bottom of a tank of water (specific gravity = 1.0) filled upto a height of 1 m as shown in the figure. Taking the case $\theta \neq 0°$, the force exerted by the hinge on the rod is



(a) 10.2 N upwards
(b) 4.2 N downwards
(c) 8.3 N downwards
(d) 6.2 N upwards

48. A projectile is thrown in upward direction making an angle of 60° with the horizontal direction with a velocity of $150\ \text{ms}^{-1}$. Then, the time after which its inclination with horizontal is 45°, is

(a) $15(\sqrt{3}-1)$ s (b) $15(\sqrt{3}+1)$ s
(c) $7.5(\sqrt{3}-1)$ s (d) $7.5(\sqrt{3}+1)$ s

49. When a copper sphere is heated, percentage change is

(a) maximum in radius
(b) maximum in volume
(c) maximum in density
(d) equal in radius, volume and density

50. In Young's double slit experiment, fringes of width β are produced on the screen kept at a distance of 1m from the slit. When the screen is moved away by 5×10^{-2}m, fringe width changes by 3×10^{-5}m. The separation between the slits is 1×10^{-3}m. The wavelength of light used is……nm.

(a) 500 (b) 600 (c) 700 (d) 400

Chemistry

1. How many moles of $Al_2(SO_4)_3$ would be in 50 g of the substance?

(a) 0.0843 mol (b) 0.952 mol
(c) 0.481 mol (d) 0.140 mol

2. The ionisation energy of hydrogen atom is 13.6 eV. What will be the ionisation energy of He^+?

(a) 13.6 eV (b) 54.4 eV
(c) 122.4 eV (d) Zero

3. In which of the following arrangement the order is not according to the property indicated against it?

(a) $Li < Na < K < Rb$ (increasing metallic radius)
(b) $I < Br < F < Cl$ (increasing electron gain enthalpy, with negative sign)
(c) $B < C < N < O$ (increasing first ionisation enthalpy)
(d) $Al^{3+} < Mg^{2+} < Na^+ < F^-$ (increasing ionic size)

4. The H–O–H bond angle in H_2O is 104.5°. This fact can be explained with the help of

(a) valence shell electron pair repulsion theory (VSEPR)
(b) molecular orbital theory
(c) presence of hydrogen bond
(d) electronegativity difference between hydrogen and oxygen atoms

5. Under which of the following condition, vander waals' gas approaches ideal behaviour?

(a) Extremely lower pressure
(b) Low temperature
(c) High pressure
(d) Low product of pV

6. The enthalpy of combustion of carbon and carbon monoxide are -393.5 and -283 kJ/mol respectively. The enthalpy of formation of carbon monoxide per mole is

(a) 110.5 kJ (b) 676.5 kJ
(c) -676.5 kJ (d) -110.5 kJ

7. Equivalent amounts of H_2 and I_2 are heated in a closed vessel till equilibrium is obtained. If 80% of the hydrogen can be converted to HI, the K_C at this temperature is

(a) 64 (b) 16 (c) 0.25 (d) 4

8. Which of the following is false about H_2O_2?

(a) Act as both oxidising and reducing agent
(b) Two OH bond lie in the same plane
(c) Pale blue liquid
(d) Can be oxidised by ozone

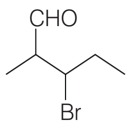
9. KO_2 is used in space and submarines because it

(a) absorb CO_2 and increase O_2 concentration
(b) absorb moisture
(c) absorb CO_2
(d) produce ozone

10. The relative Lewis acid character of boron trihalides is the order

(a) $BI_3 > BBr_3 > BF_3 > BCl_3$
(b) $BI_3 > BBr_3 > BCl_3 > BF_3$
(c) $BF_3 > BCl_3 > BBr_3 > BI_3$
(d) $BCl_3 > BF_3 > BI_3 > BBr_3$

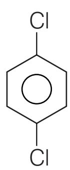
11. The IUPAC name of

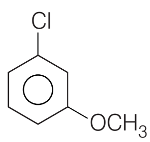


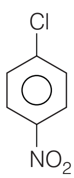
is

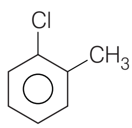
(a) 2-methyl-3-bromo hexanal
(b) 3-bromo-2-methyl butanal
(c) 2-bromo-3-bromo butanal
(d) 3-bromo-2-methyl pentanal

12. Which of the following would react most readily with nucleophiles?

(a) 

(b) 

(c) 

(d) 

13. Ethyl acetoacetate shows, Which type of isomerism?

(a) Chain (b) Optical
(c) Metamerism (d) Tautomerism

14. $C_7H_8 \xrightarrow{3Cl_2,\ heat} A \xrightarrow{Fe/Br_2} B \xrightarrow{Zn/HCl} C$

Here the compound C is

(a) 3-bromo -2,4,5,6-trichloro tolucne
(b) *o*-bromo toluene
(c) *p*-bromo toluene
(d) *m*-bromo toluene

15. An important product in the ozone deplation by chlorofluoro carbons is

(a) Cl_2 (b) OCl
(c) OF_2 (d) O_2F_2

16. The crystalline structure of NaCl is

(a) hexagonal close packing
(b) face centred cubic
(c) square planar
(d) body centred cubic

17. 40% by weight solution will contain how much mass of the solute in 1L solution, density of the solution is 1.2 g/mL?

(a) 480 g (b) 48 g
(c) 38 g (d) 380 g

18. The standard reduction potential $E°$ for half reactions are,

$$Zn \longrightarrow Zn^{2+} + 2e^- \quad ; E° = +0.76V$$

$$Fe^{2+} + 2e^- \longrightarrow Fe \quad ; E° = +0.41$$

The emf of the cell reaction;

$$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe \text{ is}$$

(a) – 0.35 V (b) + 0. 35 V
(c) + 1.17 V (d) – 1.17 V

19. $N_2 + 3H_2 \rightleftharpoons 2NH_3 ; + 22\ kcal$

The activation energy for the forward reaction is 50 kcal. What is the activation energy for the backword reaction?

(a) – 72 kcal (b) – 28 kcal
(c) + 28 kcal (d) + 72 kcal

20. The coagulating power of an electrolyte for arsenious sulphide decrease in order

(a) $Na^+ > Al^{3+} > Ba^{2+}$ (b) $PO_4^{3-} > SO_4^{2-} > Cl^-$
(c) $Cl^- > SO_4^{2-} > Cl^-$ (d) $Al^{3+} > Ba^{2+} > Na^+$

21. ${}^{228}_{88}X - 3\alpha, -\beta \longrightarrow Y$. The element Y is

(a) ${}^{216}_{82}Pb$ (b) ${}^{217}_{82}Pb$ (c) ${}^{218}_{83}Bi$ (d) ${}^{216}_{83}Bi$

22. Hydrolysis of NCl_3 gives NH_3 and X. Which of the following is X?

(a) $HClO_4$ (b) $HClO_3$
(c) HOCl (d) $HClO_2$

23. In the reaction,

$$4M + 8CN^- + 2H_2O + O_2 \longrightarrow 4[M(CN)_2] + 4OH^-$$

identify the metal M.

(a) Copper (b) Iron
(c) Silver (d) Zinc

24. In which of the following complex ion, the central matal ion is in a state of sp^3d^2 hybridisation?

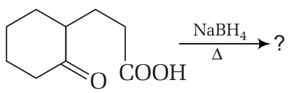
(a) $[CoF_6]^{3-}$ (b) $[Co(NH_3)_6]^{3+}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Cr(NH_3)_6]^{3+}$

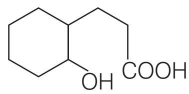
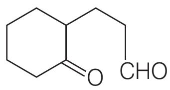
25. In the reaction sequence,

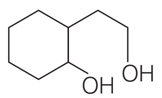
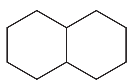
$$C_2H_5Cl + KCN \xrightarrow{C_2H_5OH} X \xrightarrow[\Delta]{H_3O^{\oplus}} Y,$$

what is the molecular formula of Y?

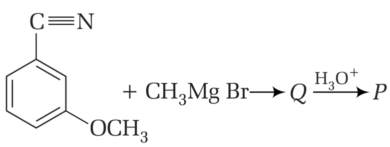
(a) $C_3H_6O_2$ (b) C_3H_5N
(c) $C_2H_4O_2$ (d) C_2H_6O

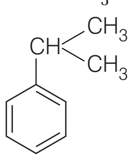
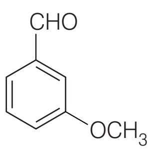
26. 

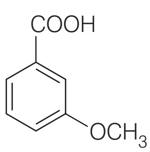
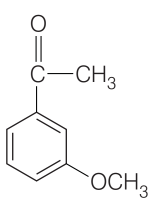
(a)  (b) 

(c)  (d) 

27. The product P in the reaction is



(a)  (b) 

(c)  (d) 

28. An organic compound of molecular formula C_3H_6O did not give a silver mirror with Tollen's reagent, but gave an oxime with hydroxylamine, it may be

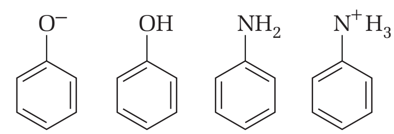
(a) CH_3COCH_3
(b) C_2H_5CHO
(c) $CH_2{=}CH{-}CH_2{-}OH$
(d) $CH_3{-}O{-}CH{=}CH_2$

29. Given the following sequence of reactions,

$$CH_3CH_2I \xrightarrow{NaCN} A \xrightarrow[\text{Partial hydrolysis}]{OH^-} B \xrightarrow{Br_2/NaOH} C.$$ The major product C is

(a) $CH_3CH_2NH_2$ (b) $CH_3CH_2{-}\overset{\overset{\displaystyle O}{\|}}{C}{-}NHBr$
(c) $CH_3CH_2COONH_4$ (d) $CH_3{\cdot}CH_2{-}\overset{\overset{\displaystyle O}{\|}}{C}{-}NBr_2$

30. Coupling of diazonium salts of the following takes place in the order



I II III IV

(a) IV < II < III < I (b) IV < III < II < I
(c) II < IV < I < III (d) I < II < III < IV

31. Which of the following pairs give positive Tollen's test?

(a) Glucose, sucrose
(b) Glucose, fractose
(c) Hexanal, acetophenone
(d) Fractose, sucrose

32. Which one is chain growth polymer?

(a) Teflon (b) Nylon-6 (c) Nylon-66 (d) Bakelite

33. Sodium alkyl benzene sulphonate is used as

(a) soap (b) fertilizer
(c) detergent (d) pesticide

34. Brown ring is made for

(a) NO_3^- (b) Cl^- (c) I^- (d) Br^-

35. The purest zinc is made by

(a) electrolytic refining (b) zone refining
(c) The van-Arkel method (d) Mond process

36. Which of the following is the strongest oxidising agent?

(a) HOCl (b) $HClO_2$ (c) $HClO_3$ (d) $HClO_4$

37. In aerosol, the dispersion medium is

(a) solid (b) liquid
(c) gas (d) Any of these

38. For the two gaseous reactions, following data

$A \longrightarrow B; k_1 = 10^{10} e^{-20,000/T}$

$C \longrightarrow D; k_2 = 10^{12} e^{-24,606/T}$

the temperature at which k_1 becomes equal to k_2 is

(a) 400 K (b) 1000 K
(c) 800 K (d) 1500 K
(e) 500 K

39. How long (in hours) must a current of 5.0 A be maintained to electroplate 60g of calcium from molten $CaCl_2$?

(a) 27 h (b) 8.3 h
(c) 11 h (d) 16 h

40. Calculate the molal depression constant of a solvent which has freezing point 16.6° C and latent heat of fusion 180.75 J/g.

(a) 2.68 (b) 3.86
(c) 4.68 (d) 2.86

41. In CsCl type structure, the coordination number of Cs^+ and Cl^- respectively are

(a) 6, 6 (b) 6,8
(c) 8,8 (d) 8,6

42. 9.2 g N_2O_4 is heated in 1L vessel till equilibrium state is established.

$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$

In equilibrium state, 50% N_2O_4 was dissociated, equilibrium constant will be (molecular wt. of $N_2O_4 = 92$)

(a) 0.1 (b) 0.4
(c) 0.3 (d) 0.2

43. Enthalpy is equal to

(a) $T^2 \left[\frac{\delta (G/T)}{\delta T}\right]_p$

(b) $-T^2 \left[\frac{\delta (G/T)}{\delta T}\right]_p$

(c) $T^2 \left[\frac{\delta (G/T)}{\delta T}\right]_V$

(d) $-T^2 \left[\frac{\delta (G/T)}{\delta T}\right]_V$

44. The isoelectronic pair is

(a) Cl_2O, ICl_2^- (b) Cl_2^-, ClO_2
(c) IF_2^+, I_3^- (d) ClO_2^-, ClF_2^+

45. Which of the following has largest ionic radius?

(a) Cs^+ (b) Li^+
(c) Na^+ (d) K^+

46. The ground state term symbol for an electronic state is governed by

(a) Heisenberg principle (b) Hund's rule
(c) Aufbau principle (d) Pauli exclusion principle

47. 2.76 g of silver carbonate on being strongly heated yeild a residue weighing

(a) 2.16 g (b) 2.48 g
(c) 2.64 g (d) 2.32 g

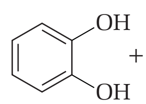
48. The following reaction is an example of ...... reaction.

$$C_2H_4Br_2 \xrightarrow{\text{Alc} \cdot \text{KOH}} C_2H_2$$

(a) addition
(b) dehydrobromination
(c) substitution
(d) bromination

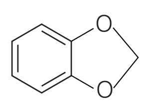
49. I_2 dissolve in KI solution due to formation of

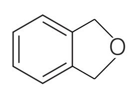
(a) KI_2 and I^-
(b) K^+; I^- and I_2
(c) I_3^-
(d) None of the above

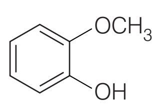
50. 

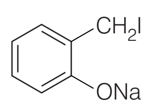
$+ CH_2I_2 + NaOH \longrightarrow$

The product is

(a) 

(b) 

(c) 

(d) 

Mathematics

1. The approximate value of $(1.0002)^{3000}$ is

(a) 1.2 (b) 1.4
(c) 1.6 (d) 1.8

2. $(\mathbf{a} \cdot \hat{\mathbf{i}})(\mathbf{a} \times \hat{\mathbf{i}}) + (\mathbf{a} \cdot \hat{\mathbf{j}})(\mathbf{a} \times \hat{\mathbf{j}}) + (\mathbf{a} \cdot \hat{\mathbf{k}})(\mathbf{a} \times \hat{\mathbf{k}})$ is equal to

(a) 3a (b) a
(c) 0 (d) None of these

3. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + y^2 = 144\}$; then $A \cap B$ contains

(a) one point (b) three points
(c) two points (d) four points

4. The value of c prescribed by Lagrange's mean value theorem, when $f(x) = \sqrt{x^2 - 4}$, $a = 2$ and $b = 3$, is

(a) 2.5 (b) $\sqrt{5}$
(c) $\sqrt{3}$ (d) $\sqrt{3} + 1$

5. The mean deviation from the mean of the series $a, a + d, a + 2d, \dots, a + 2nd$, is

(a) $n(n+1)d$ (b) $\frac{n(n+1)d}{2n+1}$
(c) $\frac{n(n+1)d}{2n}$ (d) $\frac{n(n-1)d}{2n+1}$

6. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is

(a) increasing on $\left[-\frac{1}{2}, 1\right]$ (b) decreasing on R
(c) increasing on R (d) decreasing on $\left[-\frac{1}{2}, 1\right]$

7. If ω is an imaginary cube root of unity, then the value of $(1 + \omega)(1 + \omega^2)(1 + \omega^3)(1 + \omega^4)(1 + \omega^5)\dots(1 + \omega^{3n})$ is

(a) 2^{3n} (b) 2^{2n}
(c) 2^n (d) None of these

8. Let X denotes the number of times heads occur in n tosses of a fair coin. If $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in AP, then the value of n is

(a) 7, 14 (b) 10,14
(c) 12, 7 (d) 14, 12

9. If there is a term containing x^{2r} in $\left(x + \frac{1}{x^2}\right)^{n-3}$, then

(a) $n - 2r$ is a positive integral multiple of 3.
(b) $n - 2r$ is even
(c) $n - 2r$ is odd
(d) None of the above

10. The value of $\sin^{-1}\left\{\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right\}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{2}$

11. If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then $9x^2 - 12xy\cos\theta + 4y^2$ is equal to

(a) 36 (b) $-36\sin^2\theta$
(c) $36\sin^2\theta$ (d) $36\cos^2\theta$

12. If $\tan(\sec^{-1} x) = \sin\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$, then x is equal to

(a) $\pm\frac{3}{\sqrt{5}}$ (b) $\pm\frac{\sqrt{5}}{3}$
(c) $\pm\sqrt{\frac{3}{5}}$ (d) None of these

13. The equation of the tangent to the curve $y = (2x - 1)e^{2(1-x)}$ at the point of its maximum, is

(a) $y - 1 = 0$ (b) $x - 1 = 0$
(c) $x + y - 1 = 0$ (d) $x - y + 1 = 0$

14. The proposition $(p \to \sim p) \wedge (\sim p \to p)$ is

(a) a tautology
(b) a contradiction
(c) neither tautology nor contradiction
(d) both tautology and contradiction

15. The number of ways in which four letters can be selected from the word 'DEGREE', is

(a) 7 (b) 6
(c) $\frac{6!}{3!}$ (d) None of these

16. If $PQRS$ is a convex quadrilateral with 3, 4, 5 and 6 points marked on sides PQ, QR, RS and PS respectively. Then, the number of triangles with vertices on different sides is

(a) 220 (b) 270
(c) 282 (d) 342

17. $\lim_{x \to -1} \left(\frac{x^4 + x^2 + x + 1}{x^2 - x + 1} \right)^{\frac{1 - \cos(x+1)}{(x+1)^2}}$ is equal to

(a) 1 (b) $\sqrt{\frac{2}{3}}$
(c) $\sqrt{\frac{3}{2}}$ (d) $e^{1/2}$

18. The term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3$, is

(a) -3 (b) 0
(c) 1 (d) 3

19. If A is a square matrix such that $A^2 = A$ and $(1 + A)^n = I + \lambda A$, then λ is equal to

(a) $2n - 1$ (b) $2^n - 1$
(c) $2n + 1$ (d) None of these

20. The functions $u = e^x \sin x$ and $v = e^x \cos x$ satisfy the equation

(a) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ (b) $\frac{d^2u}{dx^2} = 2v$
(c) $\frac{d^2v}{dx^2} = -2u$ (d) All of these

21. The derivative of $\tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1 - x^2}}{1 - 2x^2}\right)$ at $x = 0$, is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

22. $\int \frac{1 - x^2}{(1 + x^2)\sqrt{1 + x^4}} dx$ is equal to

(a) $\sqrt{2} \sin^{-1}\left\{\frac{\sqrt{2}x}{x^2 + 1}\right\} + C$ (b) $\frac{1}{\sqrt{2}} \sin^{-1}\left\{\frac{\sqrt{2}x}{x^2 + 1}\right\} + C$
(c) $\frac{1}{2} \sin^{-1}\left\{\frac{\sqrt{2}x}{x^2 + 1}\right\} + C$ (d) None of these

23. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{9}{4}$
(c) $\frac{5}{4}$ (d) 2

24. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive X-axis, is

(a) $\frac{1}{2}$ sq unit (b) 1 sq unit
(c) $\frac{3}{2}$ sq units (d) $\frac{5}{2}$ sq units

25. The value of $\lim_{x \to \infty} \left\{\frac{a_1^{1/x} + a_2^{1/x} + \ldots + a_n^{1/x}}{n}\right\}^{nx}$, is

(a) $a_1 + a_2 + \ldots + a_n$ (b) $e^{a_1 + a_2 + \ldots + a_n}$
(c) $\frac{a_1 + a_2 + \ldots + a_n}{n}$ (d) $a_1 a_2 \ldots a_n$

26. $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals

(a) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(b) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(c) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)\right\} + K$

(d) None of the above

27. The centre of the circle passing through $(0, 0)$ and $(1, 0)$ and touching the circle $x^2 + y^2 = 9$, is

(a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \pm\sqrt{2}\right)$

28. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the value of k is

(a) $\frac{1}{8}$ (b) 8 (c) 4 (d) $\frac{1}{4}$

29. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to the parabola at the point $(2, 3)$ and the X-axis, is

(a) 3 (b) 6
(c) 9 (d) 12

30. Solution of the differential equation

$$x = 1 + xy\frac{dy}{dx} + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{(xy)^3}{3!}\left(\frac{dy}{dx}\right)^3$$
$+\ldots$ is

(a) $y = \log_e(x) + C$ (b) $y = (\log_e x)^2 + C$
(c) $y = \pm\sqrt{(\log_e x)^2 + 2C}$ (d) $xy = x^y + C$

31. Numbers $1, 2, 3, \ldots, 100$ are written down on each of the cards A, B and C. One number is selected at random from each of the cards. The probability that the numbers so selected can be the measures (in cm) of three sides of a right angled triangle, is

(a) $\frac{4}{100^3}$ (b) $\frac{3}{50^3}$
(c) $\frac{3!}{100^3}$ (d) None of these

32. If $f(x) = \begin{cases} \dfrac{\sin(\cos x) - \cos x}{(\pi - 2x)^3}, & x \neq \dfrac{\pi}{2} \\ k, & x = \dfrac{\pi}{2} \end{cases}$

is a continuous at $x = \frac{\pi}{2}$, then k is equal to

(a) 0 (b) $-\frac{1}{6}$ (c) $-\frac{1}{24}$ (d) $-\frac{1}{48}$

33. If [ ] denotes the greatest integer function, then $f(x) = [x] + \left[x + \frac{1}{2}\right]$

(a) is continuous at $x = \frac{1}{2}$
(b) is discontinuous at $x = \frac{1}{2}$
(c) $\lim_{x \to (1/2)^+} f(x) = 2$
(d) $\lim_{x \to (1/2)^-} f(x) = 1$

34. If $f(x) = \min\{1, x^2, x^3\}$, then

(a) $f(x)$ is not everywhere continuous
(b) $f(x)$ is continuous and differentiable everywhere
(c) $f(x)$ is not differentiable at two points
(d) $f(x)$ is not differentiable at one point

35. If $x_n = \cos\frac{\pi}{3^n} + i\sin\frac{\pi}{3^n}$, then $x_1 \cdot x_2 \cdot x_3 \ldots$ is equal to

(a) 1 (b) -1
(c) i (d) $-i$

36. The total number of natural numbers of 6 digits that can be made with digits 1, 2, 3, 4, if all digits are to appear in the same number at least once, is

(a) 1560 (b) 840
(c) 1080 (d) 480

37. The number of solutions of the equation $9x^2 - 18|x| + 5 = 0$ belonging to the domain of definition of $\log_e\{(x+1)(x+2)\}$, is

(a) 1 (b) 2
(c) 3 (d) 4

38. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2^x + 2^y}{2^x - 2^y}$ (b) $\frac{2^x + 2^y}{1 + 2^{x+y}}$
(c) $2^{x-y}\left(\frac{2^y - 1}{1 - 2^x}\right)$ (d) $\frac{2^{x+y} - 2^x}{2^y}$

39. If $3x + y = 0$ is a tangent to the circle with centre at the point $(2, -1)$, then the equation of the other tangent to the circle from the origin, is

(a) $x - 3y = 0$ (b) $x + 3y = 0$
(c) $3x - y = 0$ (d) $2x + y = 0$

40. If $\sin A + \cos A = m$ and $\sin^3 A + \cos^3 A = n$, then

(a) $m^3 - 3m + n = 0$ (b) $n^3 - 3n + 2m = 0$
(c) $m^3 - 3m + 2n = 0$ (d) $m^3 + 3m + 2n = 0$

41. In ΔABC, if $a = 4, b = 3$, $\angle A = 60°$. Then, c is the root of the equation

(a) $c^2 - 3c - 7 = 0$ (b) $c^2 + 3c + 7 = 0$
(c) $c^2 - 3c + 7 = 0$ (d) $c^2 + 3c - 7 = 0$

42. The sum to n terms of the series

$$\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \ldots, \text{ is}$$

(a) $\frac{6n}{n+1}$ (b) $\frac{9n}{n+1}$
(c) $\frac{12n}{n+1}$ (d) $\frac{3n}{n+1}$

43. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \dfrac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(a) $\frac{1}{4}\ln\frac{3}{2}$ (b) $\frac{1}{2}\ln\frac{3}{2}$
(c) $\ln\frac{3}{2}$ (d) $\frac{1}{6}\ln\frac{3}{2}$

44. $\int_1^4 \log_e [x]\, dx$ equals

(a) $\log_e 2$
(b) $\log_e 3$
(c) $\log_e 6$
(d) None of the above

45. The radius of the circle passing through the foci of the ellipse $9x^2 + 16y^2 = 144$ and having its centre at $(0, 3)$, is

(a) 4 (b) 3 (c) $\sqrt{12}$ (d) $\frac{7}{2}$

46. If $f(x) = \dfrac{\sin(e^{x-2} - 1)}{\log(x-1)}$, then $\lim_{x \to 2} f(x)$ is given by

(a) -2 (b) -1
(c) 0 (d) 1

47. If $\tan A$ and $\tan B$ are the roots of the equation $x^2 - ax + b = 0$, then the value of $\sin^2(A + B)$ is

(a) $\dfrac{a^2}{a^2 + (1-b)^2}$ (b) $\dfrac{a^2}{a^2 + b^2}$
(c) $\dfrac{a^2}{(a+b)^2}$ (d) $\dfrac{a^2}{b^2 + (1-a)^2}$

48. In a ΔABC, if $a^2 + b^2 + c^2 = ac + \sqrt{3}ab$, then the triangle is

(a) equilateral
(b) right angled and isosceles
(c) right angled and not isosceles
(d) None of the above

49. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be non-zero vectors such that no two are collinear and $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$. If θ is the acute angle between the vectors $\mathbf{b}$ and $\mathbf{c}$, then $\sin\theta$ equals

(a) $\dfrac{2\sqrt{2}}{3}$ (b) $\dfrac{\sqrt{2}}{3}$
(c) $\dfrac{2}{3}$ (d) $\dfrac{1}{3}$

50. The distance of the point $(1, 1)$ from the line $2x - 3y - 4 = 0$ in the direction of the line $x + y = 1$, is

(a) $\sqrt{2}$ (b) $5\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

51. The two tangents to the curve $ax^2 + 2hxy + by^2 = 1$, $a > 0$ at the points, where it crosses X-axis, are

(a) parallel (b) perpendicular
(c) inclined at an angle $\frac{\pi}{4}$ (d) None of these

52. The greatest value of the function $f(x) = xe^{-x}$ in $[0, \infty)$, is

(a) 0 (b) $\frac{1}{e}$
(c) $-e$ (d) e

53. If an isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Then, area of the triangle is maximum, when θ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

54. The range of the function $f(x) = \log_5(25 - x^2)$ is

(a) $[0, 5]$ (b) $[0, 2)$
(c) $(0, 2)$ (d) None of these

55. The image of the point $P(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$, is

(a) $(3, 5, -2)$ (b) $(-3, 5, 2)$
(c) $(3, -5, 2)$ (d) $(3, 5, 2)$

56. Equation of the plane that contains the lines $\mathbf{r} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$ and $\mathbf{r} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) + \mu(-\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$, is

(a) $r \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = -4$ (b) $r \times (-\hat{i} + \hat{j} + \hat{k}) = 0$
(c) $r \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0$ (d) None of these

57. The distance of the point $(3, 8, 2)$ from the line $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z + 15 = 0$, is

(a) 2 (b) 3
(c) 6 (d) 7

58. Let u_1 and u_2 be two urns such that u_1 contains 3 white, 2 red balls and u_2 contains only 1 white ball. A fair coin is tossed. If head appears, then 1 ball is drawn at random from urn u_1 and put into u_2. However, if tail appears, then 2 balls are drawn at random from u_1 and put into u_2. Now, 1 ball is drawn at random from u_2. Then, probability of the drawn ball from u_2 being white is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

59. Let $D_k = \begin{vmatrix} a & 2^k & 2^{16}-1 \\ b & 3(4^k) & 2(4^{16}-1) \\ c & 7(8^k) & 4(8^{16}-1) \end{vmatrix}$, then the value of $\sum_{k=1}^{16} D_k$ is

(a) 0 (b) $a + b + c$
(c) $ab + bc + ca$ (d) None of these

60. The eccentricity of the conic $x^2 - 4x + 4y^2 = 12$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) None of these

61. The value of $\int \frac{1}{\sin\left(x - \frac{\pi}{3}\right)\cos x} dx$ is

(a) $2\log\left|\sin x + \sin\left(x - \frac{\pi}{3}\right)\right| + C$
(b) $2\log\left|\sin x \cdot \sin\left(x - \frac{\pi}{3}\right)\right| + C$
(c) $2\log\left|\sin x - \sin\left(x - \frac{\pi}{3}\right)\right| + C$
(d) None of the above

62. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$

63. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to X-axis, is

(a) $y - 3z + 6 = 0$
(b) $3y - z + 6 = 0$
(c) $y + 3z + 6 = 0$
(d) $3y - 2z + 6 = 0$

64. Let $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$. If $\mathbf{c}$ is a vector such that $\mathbf{a}\cdot\mathbf{c} = |\mathbf{c}|, |\mathbf{c} - \mathbf{a}| = 2\sqrt{2}$ and the angle between $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}$ is 30°. Then, $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) 2 (d) 3

65. Order and degree of a differential equation

$$\frac{d^2y}{dx^2} = \left\{y + \left(\frac{dy}{dx}\right)^2\right\}^{1/4}$$ are

(a) 4 and 2 (b) 1 and 2
(c) 1 and 4 (d) 2 and 4

66. A cone whose height is always equal to its diameter, is increasing in volume at the rate of $40\ \text{cm}^3/\text{s}$. At what rate is the radius increasing when its circular base area is $1\ \text{m}^2$?

(a) 1 mm/s (b) 0.001 cm/s
(c) 2 mm/s (d) 0.002 cm/s

67. The sum to infinity of the series

$$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$$ is

(a) 2 (b) 3
(c) 4 (d) 6

68. The equations of the straight lines passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is -1, is

(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) None of the above

69. One possible condition for the three points $(a, b), (b, a)$ and $(a^2, -b^2)$ to be collinear, is

(a) $a - b = 2$ (b) $a + b = 2$
(c) $a = 1 + b$ (d) $a = 1 - b$

70. The number of positive integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$, is

(a) 2 (b) 3
(c) 4 (d) 5

General English & Aptitude

1. John M Keynes was advocate of which of the following suggestions?
   (a) Pending money recklessly during recessions is suicidal
   (b) Exorbitant spending during recessions is likely to boost economy
   (c) Aggressive deficit spending is likely to be fatal for economic meltdown
   (d) Government stimulus to economy may not help because of red-tapism
   (e) None of the above

2. Which of the following is true about Keynes philosophy?
   (a) Actual spending money during meltdown is more important than where and on what it is spent
   (b) Government should be selective in approach for spending money during recession
   (c) Filling old bottles with banknotes and burying them is an atrocious proposal
   (d) Creating jobs and prosperity during recessions is almost an impracticable proposal
   (e) None of the above

3. The author of the passage calls Barack Obama and his team as 'Keynesians' because
   (a) Barack Obama has been reluctant to follow Keynes' philosophy
   (b) his team is advising Barack to refrain from Keynes' philosophy
   (c) Barack Obama and his team have decided to fill old bottles with banknotes
   (d) building houses has been under the active consideration of Barack Obama and his team
   (e) None of the above

4. What, according to Keynes, is the 'aggregate demand'?
   (a) Goods and Services Sector
   (b) Stimulation of a short-term activity
   (c) Attempting to rev up the sluggish economy
   (d) Pumping one trillion dollars into economy
   (e) None of the above

5. Highways, bridges, ethanol plants etc., are considered by the author as
   (a) reasonably appropriate propositions to spend money on
   (b) measures that affect the environment adversely
   (c) imprudent proposals to waste money on
   (d) tax saving schemes bestowed on builders
   (e) None of the above

6. Obama's upcoming American Recovery and Reinvestment Plan focuses on which of the following?
   A. Recovery of all debts from the debtors in a phased manner.
   B. Pumping money very liberally in projects that are mandatory.
   C. Investing money recklessly in any project regardless or its utility.
   (a) Only A
   (b) Only B
   (c) Only C
   (d) B and C
   (e) All of the above

Directions (Q. Nos. 7 and 8) *Choose the word which is most opposite in meaning of the word printed in **bold** as used in the passage.*

7. **Moribund**
   (a) Declining
   (b) Waning
   (c) Thriving
   (d) Pessimistic
   (e) Glorifying

8. **Beleaguered**
   (a) Carefree
   (b) Harassed
   (c) Stressful
   (d) Uneventful
   (e) Evaporating

Directions (Q.Nos. 9 and 10) *Choose the word which is most nearly the same in meaning as the word given in **bold** as used in the passage.*

9. **Apocalyptic**
   (a) Unwelcome
   (b) Disastrous
   (c) Risk-free
   (d) Joyous
   (e) Ceremonious

10. **Resuscitate**
   (a) Meltdown
   (b) Devastate
   (c) Mislead
   (d) Save
   (e) Deactivate

Directions (Q.Nos. 11-15) *Rearrange the following seven sentences* (A), (B), (C), (D), (E), (F) *and* (G) *in the proper sequence to form a meaningful paragraph, then answer the questions given below them.*

A. People thoroughly dedicated to social service but not fulfilling the eligibility requirements would not be able to contest elections.

B. Those who fulfil the stipulated criteria of age and formal education may not be necessarily devoted to social service.

C. This system has both advantages and disadvantages.

D. Therefore, imposing such eligibility requirements is likely to be counter-productive.

E. In certain democratic countries, elections can be contested by anybody.

F. People would be deprived of the probable benefit accrued from services of such people.

G. There are no eligibility requirements of formal education and upper age limit stipulated in their Constitution.

11. Which sentence should be the **FOURTH** in the paragraph?

(a) A (b) B
(c) C (d) D
(e) E

12. Which sentence should be the **LAST** in the paragraph?

(a) A (b) B
(c) C (d) D
(e) E

13. Which sentence should be the **FIRST** in the paragraph?

(a) G (b) F
(c) E (d) D
(e) C

14. Which sentence should be the **SECOND** in the paragraph?

(a) G (b) F
(c) E ` (d) D
(e) C

15. Which sentence should be the **THIRD** in the paragraph?

(a) A (b) B
(c) C (d) D
(e) E

Directions (Q. Nos. 16-20) *In each of the following sentences there are two blank spaces. Below each five pairs of words have been denoted by* (a), (b), (c), (d) *and* (e). *Find out which pair of words can be filled up in the blanks in the sentence in the same sequence to make the sentence meaningfully complete.*

16. As public resources are .... there is a need for .... of the private sector in development.

(a) exhaustive, participation
(b) relative, solicitation
(c) necessary, regulation
(d) dwindling, coveting
(e) limited, involvement

17. The Government is likely to .... a new rural infrastructure .... by the end of March.

(a) formulate, development
(b) frame, policy
(c) sanction, funds
(d) inspect, clearance
(e) conceive, inflow

18. State owned banks may lose .... Government business if they .... to fulfil their developmental role.

(a) valuable, compete (b) approved, omit
(c) lucrative, fail (d) significant, neglected
(e) profitable, succeed

19. The company was .... with a .... to improve power distribution in rural areas.

(a) initiated, backing (b) established, aim
(c) promote, priority (d) started, mandate
(e) acquired, belief

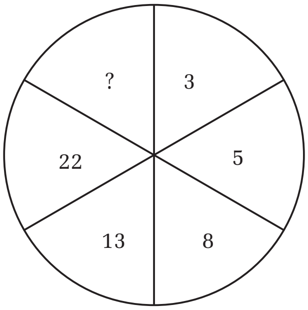
20. The Finance Minister is .... that the country will .... a 9% growth rate.

(a) assured, check (b) convinced, ensure
(c) confident, post (d) proposed, achieve
(e) challenged, sustain

21. In an examination, Raju got more marks than Mukesh but not as many as Priya. Priya got more marks than Gaurav and Kavita. Gaurav got less marks than Mukesh but his marks are not the lowest in the group. Who is second in the descending order of marks?

(a) Priya (b) Kavita
(c) Raju (d) Gaurav

Directions (Q. Nos. 22 and 23) Select the missing number from the given responses.

22. 

(a) 1 (b) 26 (c) 39 (d) 45

23.

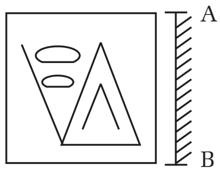
64	36	2
81	25	4
144	16	?

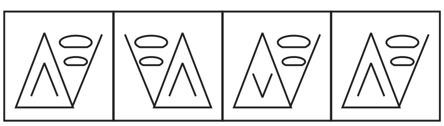
(a) 6 (b) 8 (c) 3 (d) 16

24. Ashish walked 50 m towards East and took a right turn and walked 40 m. He again took a right turn and walked 50 m. How for is he from the starting point?

(a) 10 m (b) 25 m
(c) 30 m (d) 40 m

25. Which of the answer figure is exactly the mirror image of the given figure, when the mirror is held on the line *AB*?





(a) (b) (c) (d)

26. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3 but when divided by 9 leaves no remainder, is

(a) 1677
(b) 1683
(c) 2523
(d) 3363

27. If $\dfrac{x+y}{x-y}=\dfrac{5}{2}$, then value of $\dfrac{x}{y}$ is

(a) $\dfrac{3}{8}$ (b) $\dfrac{8}{3}$
(c) $\dfrac{5}{3}$ (d) $\dfrac{3}{5}$

28. Ram bought a refrigerator with 5% discount on the labelled price. Had he bought it with 20% discount, he would have saved ₹ 1500. At what price did he buy the refrigerator?

(a) ₹ 10000
(b) ₹ 9000
(c) ₹ 8000
(d) ₹ 9500

29. The ratio of the incomes of A and B is $5:4$ and the ratio of their expenditures is $3:2$. If at the end of the year, each saves ₹1600, then the income of A is

(a) ₹ 3400
(b) ₹ 3600
(c) ₹ 4000
(d) ₹ 4400

30. The difference between the compound interest and simple interest on a certain sum of money for 3yr at $6\dfrac{2}{3}$% per annum is ₹ 184, then what is the sum?

(a) ₹ 13500
(b) ₹ 12500
(c) ₹ 11500
(d) ₹ 10500

Answers

Physics

1. (a)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (b)
11. (c)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (b)
21. (c)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (a)
31. (b)	**32.** (b)	**33.** (a)	**34.** (c)	**35.** (d)	**36.** (b)	**37.** (d)	**38.** (c)	**39.** (c)	**40.** (b)
41. (d)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (b)	**46.** (a)	**47.** (c)	**48.** (c)	**49.** (b)	**50.** (b)

Chemistry

1. (d)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (b)
11. (d)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (d)	**20.** (d)
21. (d)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (a)
31. (b)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (d)	**37.** (c)	**38.** (b)	**39.** (d)	**40.** (b)
41. (c)	**42.** (d)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (c)	**50.** (a)

Mathematics

1. (c)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (d)
21. (b)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (c)
31. (d)	**32.** (d)	**33.** (b)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (c)
41. (a)	**42.** (a)	**43.** (a)	**44.** (c)	**45.** (a)	**46.** (d)	**47.** (a)	**48.** (c)	**49.** (a)	**50.** (a)
51. (a)	**52.** (b)	**53.** (a)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (b)	**59.** (a)	**60.** (a)
61. (d)	**62.** (b)	**63.** (a)	**64.** (b)	**65.** (d)	**66.** (d)	**67.** (b)	**68.** (a)	**69.** (c)	**70.** (d)

General English & Aptitude

1. (b)	**2.** (a)	**3.** (e)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (d)
11. (a)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (e)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (c)
21. (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (a)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (a)

Solutions

Physics

1. Let the third charge $-q$ is placed at a distance of x from the charge 2q. Then, the potential energy of the system is

$$U = k\left[\frac{2q\,(8q)}{r} - \frac{(2q)\,(q)}{x} - \frac{(8q)\,(q)}{r-x}\right]$$

Here, $k = \frac{1}{4\pi\varepsilon_0}$

U is minimum, where $\frac{2}{x} + \frac{8}{r-x}$ is maximum

Let $\frac{2}{x} + \frac{8}{r-x} = y$

For y to be maximum, $\frac{dy}{dx} = 0$

$$-\frac{2}{x^2} + \frac{8}{(r-x)^2} = 0$$

$\Rightarrow$ $$\frac{x}{r-x} = \sqrt{\frac{2}{8}} = \frac{1}{2} \Rightarrow x = \frac{r}{3}$$

But at $x = \frac{r}{3}$, $\frac{d^2y}{dx^2}$ = positive i.e. at $x = \frac{r}{3}$, y is minimum or U is maximum. So, there cannot be any point between them where the potential energy is minimum.

2. At thermal equilibrium, the ratio $\frac{N_2}{N_1}$ is given as

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

The middle of the visible range is taken at $\lambda = 550$ nm

$\Rightarrow$ $$E_2 - E_1 = \frac{hc}{\lambda} = 3.16 \times 10^{-19}\,J$$

$\Rightarrow$ $$\frac{N_2}{N_1} = \exp\left(\frac{-3.16 \times 10^{-19}\,J}{(1.38 \times 10^{-23} 1/k)\cdot(300\ k)}\right)$$

$\Rightarrow$ $$\frac{N_2}{N_1} = 1.1577 \times 10^{-38}$$

3. The energy of the hydrogen atom in *n*th state is givne by

$$E_n = \frac{-13.6}{(n^2)} eV$$

Here, $n = 5$

$\therefore$ $$E = \frac{-13.6}{(5)^2} eV = -0.54\ eV$$

4. When a galvanometer is connected to the shunt resistance, then we have
potential drop across the galvanometer = potential drop across the shunt

i.e. $$iG = (i - i_0)\,S \qquad \ldots(i)$$

Here, i = current through the galvanometer
G = resistance of the galvanometer
i_0 = current through the shunt resistance
S = shunt resistance

From Eq. (i), we have the part of the total current that flows through the galvanometer is

$$\frac{i}{i_0} = \frac{S}{G + S}$$

Substituting, S = 2.5 Ω, G = 25 Ω, we get

$$\frac{i}{i_0} = \frac{1}{11}$$

5. $\because$ $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_{sound} = \sqrt{\frac{\gamma RT}{M}}$,

$$v_{rms} = 2\,v_{sound}$$

i.e. $\gamma = \frac{3}{2}$ = ratio of $\frac{C_p}{C_V}$ for the mixture

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

and $$C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

$\therefore$ $$\gamma = \frac{C_P}{C_V} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$\therefore$ $$\frac{3}{2} = \frac{2\left(\frac{5}{2}R\right) + n\left(\frac{7}{2}R\right)}{2\left(\frac{3}{2}R\right) + n\left(\frac{5}{2}R\right)}$$

$\Rightarrow$ $$\frac{3}{2} = \frac{10 + 7n}{6 + 5n}$$

$\Rightarrow$ $n = 2$

6. Current is given by

$$I = \frac{E}{R'} \quad ,$$

Substituting, $E = -\frac{nd\phi}{dt}$, we get

$$I = -\frac{n}{R'}\cdot\frac{d\phi}{dt} \qquad \ldots(i)$$

Given that w_1 and w_2 are the values of the flux associated with one turn of the coil.

Therefore, Eq. (i) becomes,

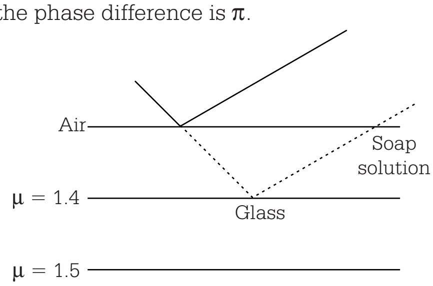
$$I = -\frac{n}{R'}\left[\frac{w_2 - w_1}{t_2 - t_1}\right] \quad \ldots\text{(ii)}$$

Total resistance of the combination is $R' = R + 4R = 5R$, substituting the values of R' and $(t_2 - t_1) = t$ in Eq. (ii), we get

$$I = -\frac{n}{5R}\left(\frac{w_2 - w_1}{t}\right)$$

$$\Rightarrow \qquad I = \frac{-n\,(w_2 - w_1)}{5Rt}$$

7. For the reflection at the air-soap solution interface, the phase difference is π.



For reflection at the interface of soap solution to the glass also there will be a phase difference of π.

$\therefore$ The condition for maximum intensity $= 2\mu t = n\lambda$ $\quad\ldots$(i)

For , $\quad n\lambda_1 = (n-1)\,\lambda_2$

$$n\,(420) = (n-1)\,630$$

$$n\,(630 - 420) = 630$$

$$\therefore \qquad n\,(210) = 630$$

$$\Rightarrow \qquad n = 3$$

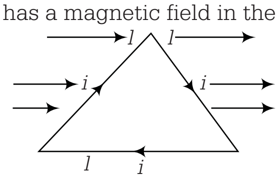
This is the maximum order where they coincide,

From Eq. (i),

$$\therefore \qquad 2 \times 1.4 \times t = 3 \times 420$$

$$\Rightarrow \qquad t = \frac{2 \times 420}{2 \times 1.40} = 450 \text{ nm}$$

8. The current flowing clockwise in an equilateral triangle has a magnetic field in the direction of $\hat{\mathbf{k}}$.



$$\tau = BiNA \sin\theta$$

$$\tau = BiNA \sin 90^\circ$$

$$\tau = Bi \times \frac{\sqrt{3}}{4}\, l^2 \times 1$$

[$\because$ Area of equilateral triangle $= \frac{\sqrt{3}}{4}\, l^2$ and $N = 1$, (assuming]

$$\Rightarrow \qquad l^2 = \frac{4\tau}{\sqrt{3}\; Bi} \Rightarrow l = 2\left[\frac{\tau}{Bi\sqrt{3}}\right]^{1/2}$$

9. Block of mass m_2 shoots off carrying some kinetic energy away from the system. To find its speed, potential energy of spring = maximum kinetic energy of blocks.

$$\frac{kd^2}{2} = (m_1 + m_2)\,\frac{v^2}{2}$$

[k = force constant of spring]

$$v^2 = \frac{kd^2}{m_1 + m_2} \text{ with } m_1 \text{ along on the spring.}$$

Maximum potential energy
= Maximum kinetic energy of m_2

$$\Rightarrow \qquad \frac{1}{2}\, kA^2 = \frac{1}{2} m_1 v^2 \quad \Rightarrow kA^2 = \frac{km_1 d^2}{m_1 + m_2}$$

$$\Rightarrow \qquad A = d\sqrt{\frac{m_1}{m_1 + m_2}}$$

10. Time taken by the small ball to return to the hands of the juggler is $\frac{2v}{g} = \frac{2 \times 20}{10} = 4$ s. So, he is throwing the balls after 1 s each. Let at some instant, he throws the ball number 4. Before 1 s of throwing it, he throws ball 3. So, the height of ball 3 is

$$h = ut - \frac{1}{2}\, gt^2$$

$$h_3 = 20 \times 1 - \frac{1}{2}\,(10)\,(1)^2 = 15 \text{ m}$$

Before 2 s, he throws ball 2. So, the height of ball 2 is

$$h_2 = 20 \times 2 - \frac{1}{2} \times 10\,(2)^2 = 20 \text{ m}$$

Before 3 s, he throws ball 1. So, the height of ball 1 is

$$h_1 = 20 \times 3 - \frac{1}{2} \times 10\,(3)^2$$

$$\Rightarrow \qquad h_1 = 15 \text{ m}$$

11. The instantaneous current in the AC circuit is given by $I = I_0 \sin \omega t$

$$\frac{dI}{dt} = \frac{d}{dt}\,(I_0 \sin \omega t)$$

$$\frac{dI}{dt} = (I_0 \cos\omega t)\,\omega$$

Substituting $I_0 = 10$ A, $\omega = 100\pi$

$\cos(100\pi) = 1$, we get

$$\frac{dI}{dt} = 1000\pi$$

Hence, induced emf is given by

$$E = M\frac{dI}{dt}$$

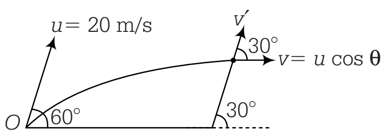
On putting $M = 0.005$ H, $\frac{dI}{dt} = 1000\pi$, we get

$$E = (5\pi)\ V$$

12. Speed of ball before collision is

$$v = u\cos 60° = 20 \times \frac{1}{2} = 10\ m/s$$

Since, collision is perfectly inelastic ($e = 0$), so the ball will not bounce. It will move along the plane with velocity



$$v' = v\cos 30° = \frac{10\sqrt{3}}{2} = 5\sqrt{3}\ m/s$$

∴ Maximum height attained by the ball

$$H = \frac{u^2\sin^2 60°}{2g} + \frac{v'^2}{2g}$$

$$H = \frac{(20)^2\left(\frac{\sqrt{3}}{2}\right)^2}{20} + \frac{(5\sqrt{3})^2}{20}$$

$$= \frac{300}{20} + \frac{75}{20} = 18.75\ m$$

13. Total energy produced by the reactor in time $t = 10$ yr.

$$E = 1000 \times 10^6 \times 10 \times 3.15 \times 10^7\,J$$

$$= 3.15 \times 10^{17}\,J$$

$\because$ Efficiency $= \frac{\text{output energy}}{\text{input energy}}$

Input energy caused by fission $= \frac{\text{output energy}}{\text{efficiency}}$

$$= \frac{3.15 \times 10^{17}}{(10/100)} = 3.15 \times 10^{18}\,J$$

Energy produced by one fission of $^{235}U = 200$ MeV

$$= 200 \times 1.6 \times 10^{-13}\,J$$

$$= 3.2 \times 10^{-11}\,J$$

Therefore, number of fissions required

$$= \frac{\text{total energy}}{\text{energy per fission}}$$

$$= \frac{3.15 \times 10^{18}}{3.2 \times 10^{-11}} = 9.8 \times 10^{28}$$

Hence, mass of uranium required is given by

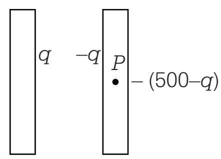
$$m = \frac{N}{N_a} \times 235 = \frac{9.8 \times 10^{28}}{6.02 \times 10^{26}}$$

$$= 38.2 \times 10^3\,kg$$

14. Charge acquired by the plates of the capacitor $q_0 = CV = (10\,\mu F) \times (50\,V) = 500\,\mu C$

Now, let the charge distribution is as follows.

Total charge on positive plate has now become $700\,\mu C$ while that in negative plate is still $-500\,\mu C$,



Here, charges are in μC.

Net electric field at point P is zero.

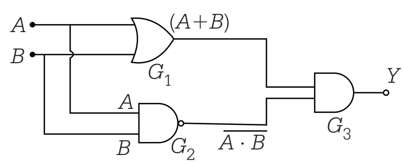
$$\therefore \quad \frac{(700 - q)}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} + \left(\frac{500 - q}{2A\varepsilon_0}\right) = \frac{q}{2A\varepsilon_0}$$

$$\Rightarrow \quad q = 600\,\mu C$$

∴ Potential difference between the plates is

$$\Delta V = \frac{q}{C} = \frac{600\,\mu C}{10\,\mu F} = 60\ V$$

15.



$$Y = (A + B)\cdot\overline{AB}$$

The given output equation can also be written as

$Y = (A + B)\cdot(\overline{A} + \overline{B})$ (Demorgan's theorem)

$$\Rightarrow \quad Y = A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B}$$

$$\Rightarrow \quad Y = 0 + A\overline{B} + \overline{A}B + 0$$

$$\Rightarrow \quad Y = \overline{A}B + A\overline{B}$$

This is the expression for XOR gate.

16. The current through the circuit before the battery of emf E_2 is short circuited, is

$$i_1 = \frac{E_1 + E_2}{r_1 + r_2 + R} \qquad \ldots(i)$$

After short circuiting the battery of emf E_2, current through the resistance R would be

$$i_2 = \frac{E_1}{R + r_1} \qquad \ldots(ii)$$

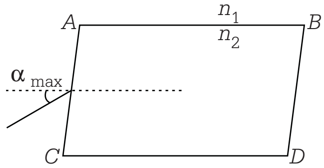
Since, from Eqs. (i) and (ii), we get

$$i_2 > i_1$$

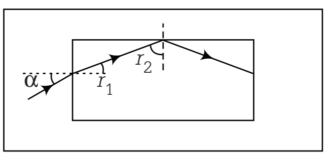
$$\therefore \qquad \frac{E_1}{R + r_1} > \frac{E_1 + E_2}{R + r_1 + r_2}$$

This gives $\qquad E_1 r_2 > E_2(R + r_1)$

17.



Rays comes out only from CD, means rays after refraction from AB get totally internally reflected at AD.



From the figure,

$$r_1 + r_2 = 90$$

$$r_1 = 90 - r_2$$

$$(r_1)_{max} = 90° - (r_2)_{max} \text{ and } (r_2)_{min} = \theta_C$$

(for total internal reflection at AD)

where , $\qquad \sin \theta_C = \frac{n_2}{n_1}$ or $\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$$\therefore \qquad (r_1)_{max} = 90° - \theta_C$$

Now, applying Snell's law at face AB, we wet

$$\frac{n_1}{n_2} = \frac{\sin \alpha_{max}}{\sin (r_1)_{max}} = \frac{\sin \alpha_{max}}{\sin (90 - \theta_C)} = \frac{\sin \alpha_{max}}{\cos \theta_C}$$

$$\sin \alpha_{max} = \frac{n_1}{n_2} \cos \theta_C$$

$$\therefore \qquad \alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2} \cos \theta_C\right]$$

$$= \sin^{-1}\left[\frac{n_1}{n_2} \cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$$

18. We start a general form for a rightward moving wave,

$$y(x, t) = A \sin (kx - \omega t + \phi) \qquad \ldots(i)$$

The given amplitude is $A = 2 \text{ cm} = 0.02 \text{ m}$

The wavelength is given as

$$\lambda = 1 \text{ m}$$

Wave number $= k = 2\pi/\lambda = 2\pi \text{ m}^{-1}$

Angular frequency,

$$\omega = \nu k = 10\,\pi \text{ rad/s}$$

From Eq. (i),

$$y(x, t) = (0.02) \sin [2\pi (x - 5t) + \phi]$$

$\because \qquad$ For $x = 0$, $t = 0$

$$y = 0 \text{ and } \frac{\partial y}{\partial t} < 0$$

i.e. $\qquad 0.02 \sin \phi = 0 \qquad$ (as $y = 0$)

and $\qquad -0.2\pi \cos \phi < 0$

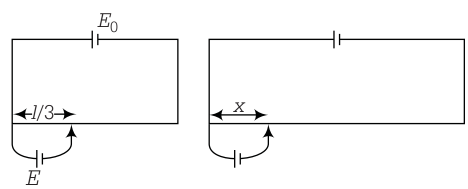
From these conditions, we may conclude that

$$\phi = 2n\pi, \text{ where } n = 0, 2, 4, 6, \ldots$$

Therefore,

$$y(x, t) = (0.02 \text{ m}) \sin [(2\pi \text{m}^{-1})x - (10\pi \text{s}^{-1})\, t] \text{ m}$$

19. Let x be the desired length.



Potential gradient in the first case $= \frac{E_0}{l}$

$$E = \left(\frac{l}{3}\right) \cdot \left(\frac{E_0}{l}\right) = \frac{E_0}{3} \qquad \ldots(i)$$

Potential gradient in the second case.

$$= \frac{E_0}{3l/2} = \frac{2E_0}{3l}$$

$$\therefore \qquad E = (x)\frac{2E_0}{3l} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

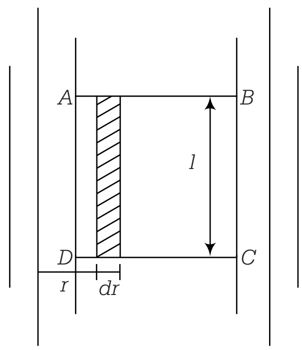
$$\frac{E_0}{3} = \left(\frac{2E_0}{3l}\right) x$$

$$\Rightarrow \qquad x = \frac{l}{2}$$

20. Since, the wires are infinite, so the system of these two wires can be considered as closed rectangle of infinite length and breadth equal to d.

Flux through strip

$$\phi = \int_a^{d-a} \frac{\mu_0 I}{2\pi r}(l dr) = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{d-a}{a}\right)$$



The other wire produces the same result, so the total flux through the rectangle ABCD is

$$\phi_{total} = \frac{\mu_0 I l}{\pi} \ln\left(\frac{d-a}{a}\right)$$

The total inductance of length l

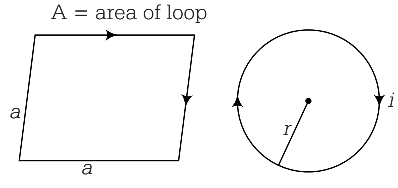
$$L = \frac{l_{total}}{I} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d-a}{a}\right)$$

Inductance per unit length $= \frac{L}{l} = \frac{\mu_0}{\pi} \ln\left(\frac{d-a}{a}\right)$

21. Magnetic moment of a loop is given by $p_m = IA$

where, I = current through the loop

A = area of loop



(i) (ii)

Perimeter of square should be equal to the circumference of circle, since wires have same length.

Therefore, $4a = 2\pi r \Rightarrow a = \frac{\pi r}{2}$ ...(i)

In Fig. (i), magnetic moment of square loop is given by

$$p_{m_1} = I(A_1) \quad \Rightarrow$$

$$\Rightarrow \quad p_{m_1} = \frac{I\pi^2 r^2}{4} \quad \text{[from Eq. (i)]} \ldots\text{(ii)}$$

In Fig. (ii), magnetic moment of circular loop is given by

$$p_{m_2} = I(A_2)$$

$$p_{m_2} = I(\pi r^2) \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$\Rightarrow \quad \frac{p_{m_1}}{p_{m_2}} = \frac{\pi}{4}$$

22. The potential energy of circular loop in the magnetic field $= -MB\cos\theta$, where θ is the angle between normal to plane of coil and magnetic field B.

Initial potential energy,

$$U_i = -NiAB\cos 0° = -NiAB$$

When coil is turned through 180°, therefore final potential energy,

$$U_f = -NiAB\cos 180° = NiAB$$

$\therefore$ Required work, W = gain in potential energy

$$= U_f - U_i = NiAB - (-NiAB) = 2NiAB$$

Here, $N = 100$, $i = 0.1$ A, $B = 1.5$ Wb/m^2 and radius $r = 0.05$ m

$\therefore \quad A = \pi r^2 = 3.14 \times (0.05)^2 = 7.85 \times 10^{-3}\,\text{m}^2$

$\therefore$ Work, $\quad W = 2 \times 100 \times 0.1 \times 7.85 \times 10^{-3} \times 1.5$

$\Rightarrow \quad W = 0.2355$ J

23. The instantaneous current in L-R circuit during growth is given by

$$I = I_0(1 - e^{-\frac{R}{L}t})$$

Since, all the exponential terms are dimensionless, therefore, it follows that factor $\frac{R}{L}t$ should be dimensionless i.e.

$$\frac{R}{L}t = [M^0 L^0 T^0]$$

$$\Rightarrow \quad \frac{R}{L} = \frac{[M^0 L^0 T^0]}{[T]} = [T^{-1}]$$

$$\Rightarrow \quad \left[\frac{L}{R}\right] = [T]$$

Hence, the time constant of L-R circuit is $\left[\frac{L}{R}\right]$.

24. In wave mechanics, the magnitude of angular momentum of an electron is

$$L = \sqrt{l(l+1)}\,\frac{h}{2\pi}$$

where, l can have the values 0, 1, 2......(n + 1)
n is principal quantum number.

25. The general equations of alternating voltage and current in an AC circuit are as follows

$$E = E_0 \sin \omega t \qquad \ldots\text{(i)}$$
$$I = I_0 \sin \omega t \qquad \ldots\text{(ii)}$$

Given that,

$$E = 100 \sin 100\pi t \qquad \ldots\text{(iii)}$$
$$I = 5 \sin 100\ \pi t \qquad \ldots\text{(iv)}$$

Comparing Eqs. (ii) and and (iv), we get,

Peak value of current, $I_0 = 5\,\text{A}$

Comparing Eqs. (i) and (iii), we get

Peak value of voltage, $E_0 = 100\ \text{V}$

Now, r.m.s. value of voltage is given by

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}\ \text{V}$$

Similarly, r.m.s. value of current is given by

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}\ \text{A}$$

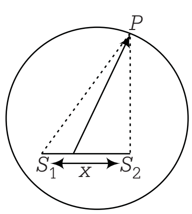
Hence, reactance of the circuit is given by

$$Z = \frac{E_{rms}}{I_{rms}} = \frac{\frac{100}{\sqrt{2}}}{\frac{5}{\sqrt{2}}} = 20\ \Omega$$

26. The damage caused by the γ-radiation is less serious as compared to damage caused by the α-radiation. This is because α-radiation is more dangerous than beta or gamma if ingested because its power to ionize or to disrupt, atoms is 20 times more than β and γ. But if the source is outside the body or at a distance gamma radiation is much dangerous because it could penertrate thick walls of skin.

27. Path difference at P is

$$\Delta x = 2\left(\frac{x}{2}\cos\theta\right) = x\cos\theta$$



For intensity to be maximum

$$\Delta x = n\lambda \qquad (n = 0, 1, \ldots\ldots)$$
$$\Rightarrow \quad x\cos\theta = n\lambda$$
$$\cos\theta = \frac{n\lambda}{x} \text{ or } \cos\theta \not> 1$$

$$\therefore \quad \frac{n\lambda}{x} \not> 1$$
$$\therefore \quad n \not> \frac{x}{\lambda}$$

Substituting $x = 5\lambda$

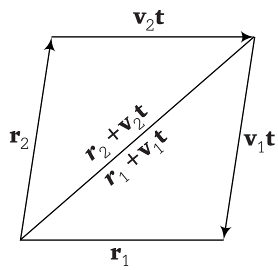
$$n \not> 5 \text{ or } n = 1, 2, 3, 4, 5, \ldots\ldots$$

Therefore, in all four quadrants, there can be 20 maximas. There are more maximas at $\theta = 0°$ and $\theta = 180°$.

But $n = 5$ corresponds to $\theta = 90°$ and $\theta = 270°$, which are coming only twice. While we have multiplied it four times. Therefore, total number of maximas still 20.

28. For collision, $\mathbf{r}_1 + \mathbf{v}_1 t = \mathbf{r}_2 + \mathbf{v}_2 t$

$$\Rightarrow \quad \mathbf{r}_1 - \mathbf{r}_2 = (\mathbf{v}_2 - \mathbf{v}_1)t$$



Equating unit vectors, we get

$$\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_2 - \mathbf{r}_1|} = \frac{\mathbf{v}_1 - \mathbf{v}_2}{|\mathbf{v}_2 - \mathbf{v}_1|}$$

29. Let v = speed of neutron before collision

v_1 = speed of neutron after collision

v_2 = speed of hydrogen atom after collision and ΔE = energy of the excitation

From the conservation of linear momentum,

$$mv = mv_1 + mv_2 \qquad \ldots\text{(i)}$$

From the conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \qquad \ldots\text{(ii)}$$

from Eq. (i),

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2$$

From Eq. (ii),

$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

$$\therefore \quad 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$$
$$= v^2 - \frac{4\Delta E}{m}$$

As, $v_1 - v_2$ must be real

$$\Rightarrow \qquad v^2 - \frac{4\Delta E}{m} \geq 0$$

$$\text{or} \qquad \frac{1}{2}mv^2 \geq 2\Delta E$$

The maximum energy that can be absorbed by hydrogen atom in ground state to go into excited state is 10.2 eV. Therefore, the minimum kinetic energy required is

$$\frac{1}{2}mv^2_{min} = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}$$

30. Molar specific heat

= Molecular weight × gram specific heat.

$$C_V = M \times c_V$$

$$2.98 \text{ cal/mol-K} = M \times 0.075 \text{ kcal/kg-K}$$

$$= \frac{M \times 0.075 \times 10^3}{10^3} \text{ cal/g-K}$$

∴ Molecular weight of argon,

$$M = \frac{2.98}{0.075} = 39.7 \text{ g}$$

i.e. mass of 6.023×10^{23} atom = 39.7 g

$$\text{Therefore, mass of single atom} = \frac{39.7}{6.023 \times 10^{23}} = 6.60 \times 10^{-23} \text{g}$$

31. Radius of a nucleus is given by

$$R = R_0 A^{1/3} \text{ (where, } R_0 = 1.25 \times 10^{-15} \text{ m)}$$

$$= 1.25 \times 10^{-15} A^{1/3} \text{m}$$

Here, A is the mass number and mass of the uranium nucleus will be

$$m \approx Am_p, \; m_p = \text{mass of proton}$$

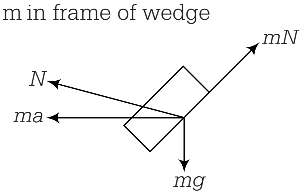
$$= A\,(1.67 \times 10^{-27} \text{ kg})$$

$$\therefore \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{\frac{4}{3}\pi R^3}$$

$$= \frac{A\,(1.67 \times 10^{-27})}{A \times \frac{4}{3} \times \frac{22}{7} \times (1.25 \times 10^{-15})^3}$$

$$\Rightarrow \qquad \rho = 2 \times 10^{17} \text{ kg/m}^3$$

32. FBD of m in frame of wedge



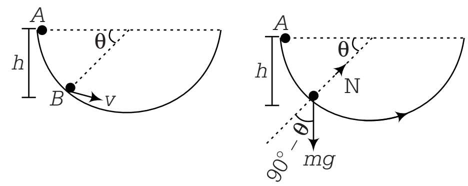
$$N = mg\cos(\alpha) - ma\sin(\alpha)$$

Now, $f = \mu N = ma\cos\alpha + mg\sin\alpha$

$$\Rightarrow \qquad \mu = \frac{a\cos\alpha + g\sin\alpha}{g\cos\alpha - a\sin\alpha}$$

$$\Rightarrow \qquad \mu = \frac{a + g\tan\alpha}{g - a\tan\alpha} = \frac{5}{12} \Rightarrow \mu = \frac{5}{12}$$

33. ∵ $\qquad h = R\sin\theta$



∴ Speed of the particle at point B,

$$v^2 = 2gh = 2Rg\sin\theta$$

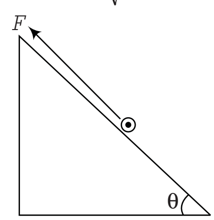
$$\text{Centripetal force, } F = \frac{mv^2}{R} = 2mg\sin\theta \qquad \ldots\text{(i)}$$

Equation of motion gives

$$N - mg\sin\theta = \frac{mv^2}{R} = 2\,mg\sin\theta$$

34. ∵ Linear acceleration for rolling,

$$a = \frac{g\sin\theta}{\sqrt{1 + k^2/R^2}}$$



For cylinder, $\frac{k^2}{R^2} = \frac{1}{2}$

$$\therefore \qquad a_{cylinder} = \frac{2}{3}g\sin\theta$$

For rotation, the torque

$$FR = I\alpha \cdot (MR^2\alpha)/2$$

(where, F = force of friction)

$$\text{But} \qquad R\alpha = a \quad \therefore \quad F = \frac{M}{2}a$$

$$F = \frac{M}{2} \cdot \frac{2}{3}g\sin\theta = \frac{M}{3}g\sin\theta$$

$$\therefore \qquad \mu_s = \frac{F}{N}, \text{ where N is normal reaction,}$$

$$\mu_s = \frac{\frac{M}{3}g\sin\theta}{Mg\cos\theta} = \frac{\tan\theta}{3}$$

∴ For rolling without slipping of a roller down, the inclined plane, $\tan\theta \leq 3\mu_s$.

35. Applying Newton's second law to a circular orbit, we have

$$\frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2} = \frac{GMm}{r^2}$$

where, m is the mass of satellite, and v is the orbital speed

T is the time period

$$\therefore \qquad T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

For $\qquad r \approx R$

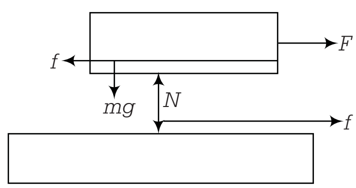
and $\qquad M = \frac{4}{3}\pi R^3 \rho$ (ρ = density of planet)

$$\because \qquad T = \sqrt{\frac{3\pi}{\rho G}}$$

i.e. is independent of R

36. Maximum frictional force between the slab and the block

$$f_{max} = \mu N = \mu mg = \frac{1}{4} \times 4 \times 10 = 10 \text{ N}$$



Evidently, $f \propto f_{max}$

So, the two bodies will move together as a single unit. If be their combined acceleration, then

$$a = \frac{F}{m + M} = \frac{6}{4+5} = \frac{2}{3} \text{ ms}^{-2}$$

Therefore, frictional force acting can be obtained as

$$f = Ma = \frac{2}{3} \times 5 = \frac{10}{3} \text{N}$$

Using $\qquad S = \frac{1}{2}at^2$

$$S(2) = \frac{1}{2} \times \frac{2}{3}(2)^2 = \frac{4}{3}$$

and $\qquad S(3) = \frac{1}{2} \times \frac{2}{3} \times (3)^2 = 3$

Therefore, work done by friction = F [S (3) – S (2)]

$$= \frac{10}{3}\left[3 - \frac{4}{3}\right] = \frac{50}{9} = 5.55 \text{ J}$$

37. Net pulling force = 2g – 1g = 10 N

Mass being pulled = 2 + 1 = 3 kg

$\therefore$ Acceleration of the system is $a = \frac{10}{3}$ m/s^2

$\therefore$ Velocity of both the blocks at t = 1 s will be

$$v_0 = at = \left(\frac{10}{3}\right)(1) = \frac{10}{3} \text{ m/s}$$

Now at this moment, velocity of 2 kg becomes zero, while that of 1 kg block is 10 m/s upwards. Hence, string becomes tight again when displacement of 1 kg block = displacement of 2 kg block.

$$\Rightarrow \qquad v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$\Rightarrow \qquad t = \frac{v_0}{g} = \frac{\frac{10}{3}}{10} = \frac{1}{3} \text{ s}$$

38. *(c)* From Snell's law,

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\Rightarrow \qquad \sin r = \frac{\mu_1}{\mu_2}\sin i = \frac{1}{\sqrt{2}}\sin 45°$$

$$\Rightarrow \qquad \frac{1}{2} = \sin r \Rightarrow r = 30°$$

This means that the ray becomes parallel to side AD inside the slab.

This implies for the second face CD, u = ∞.

Given $\qquad R = 0.4$ m

$$\frac{\mu_3}{v} - \frac{\mu_2}{v} = \frac{\mu_3 - \mu_2}{R}$$

$$\Rightarrow \qquad \frac{1.514}{v} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{R}$$

$$v = \frac{1.514 \times 0.4}{1.514 - 1.414} = \frac{1.514 \times 0.4}{0.1}$$

v = 6.056 = 6.06 m (upto 2 decimal places)

39. Energy required to raise a satellite upto a height h,

$$E_1 = \Delta U = \frac{mgh}{1 + \frac{h}{R}} \qquad \ldots(i)$$

E_2 = energy required to put in orbit

$$= \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}m\left(\frac{GM}{r}\right) \text{ as } v_0 = \text{orbital speed}$$

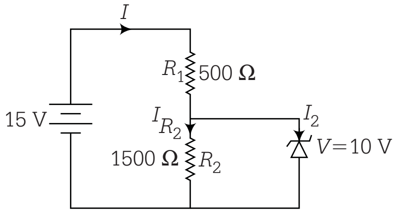
$$= \frac{1}{2}m\left(\frac{GM}{R+h}\right) = \frac{1}{2}m\left(\frac{GM}{R^2}\right)\frac{R}{1+\frac{h}{R}}$$

$$\Rightarrow \qquad E_2 = \frac{mgR}{2\left(1 + \frac{h}{R}\right)} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{E_1}{E_2} = \frac{2h}{R}$$

40.



The voltage drop across R_2 is

$$V_{R_2} = V_z = 10\ V$$

The current through R_2 is

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{10}{1500} = 0.667 \times 10^{-2} A$$
$$= 6.67 \times 10^{-3} A = 6.67\ mA$$

The voltage drop across R_1 is

$$V_{R_1} = 15V - V_{R_2} = 15V - 10\ V = 5\ V$$

The current through R_1 is

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{5V}{500\ \Omega} = 10^{-2} A = 10 \times 10^{-3} = 10\ mA$$

The current through Zener diode is

$$I_z = I_{R_1} - I_{R_2} = (10 - 6.67)\ mA = 3.33\ mA$$

41. $\because f \propto \sqrt{T}$

$$\frac{f_{air}}{f_{water}} = \sqrt{\frac{W_{air}}{W_{water}}} = \sqrt{\frac{V\rho g}{V\rho g - V\rho_w g}}$$
$$= \frac{f}{\frac{f}{2}} = \sqrt{\frac{\rho}{\rho - \rho_w}}$$
$$\Rightarrow \quad 2 = \sqrt{\frac{\rho}{\rho - \rho_w}}$$
$$\Rightarrow \quad 4\rho - 4\rho_w = \rho$$
$$\Rightarrow \quad \rho = \frac{4}{3}\rho_w$$

Similarly in second case,

$$\frac{f}{\frac{f}{3}} = \sqrt{\frac{\rho}{\rho - \rho_L}}$$
$$\Rightarrow \quad 3 = \sqrt{\frac{\frac{4}{3}\rho_w}{4 - \frac{3\rho_L}{\rho_w}}} = \sqrt{\frac{4}{4 - \frac{3\rho_L}{\rho_w}}}$$

Here, $\frac{\rho_L}{\rho_w}$ = specific gravity (say S)

$$\Rightarrow \quad 9 = \frac{4}{4 - 3S}$$
$$\Rightarrow \quad 36 - 27S = 4 \Rightarrow S = \frac{32}{27}$$

42. According to Einstein's photoelectric equation,

$$E = h\nu - w$$

If V_s is retarding or stopping potential and ν_0 is the threshold frequency, then the above equation becomes

$$eV_s = h\nu - h\nu_0$$
$$h\nu = eV_s + h\nu_0$$
$$\nu = \frac{eV_s}{h} + \nu_0$$

Hence, $e = 1.6 \times 10^{-19} C$

$$V_s = 3\ V,\ \nu_0 = 6 \times 10^{-14}\ Hz$$

Therefore, required frequency,

$$\nu = \frac{1.6 \times 10^{-19} \times 3}{6.63 \times 10^{-34}} + 6 \times 10^{14}$$
$$= \frac{4.8}{6.63} \times 10^{15} + 6 \times 10^{14}$$
$$= 0.723 \times 10^{15} + 6 \times 10^{14}$$
$$= 7.23 \times 10^{14} + 6 \times 10^{14}$$
$$= 13.23 \times 10^{14}\ Hz$$
$$= 1.323 \times 10^{15}\ Hz$$

43. At $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$

$\sin kx_1$ or $\sin kx_2$ is not zero.

Therefore, neither of x_1 nor x_2 is a node.

$$\Delta x = x_2 - x_1 = \left(\frac{3}{2} - \frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$$

Since, $\frac{2\pi}{k} > \Delta x > \frac{\pi}{k}$

$$\lambda > \Delta x > \frac{\lambda}{2} \qquad \left\{k = \frac{2\pi}{\lambda}\right\}$$

Therefore, $\phi_1 = \pi$

and $\phi_2 = k$,

$$\Delta x = \frac{7\pi}{6}$$

Therefore, $\frac{\phi_1}{\phi_2} = \frac{6}{7}$

44. Energy is given by

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Rightarrow \quad E^2 = \frac{m_0^2 c^6}{c^2 - v^2}$$

Momentum P is given by

$$P = \frac{m_0 v}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$P^2c^2 = \frac{m_0^2 c^4 v^2}{c^2 - v^2}$$

$\therefore \quad E^2 - P^2c^2 = m_0^2 c^4$

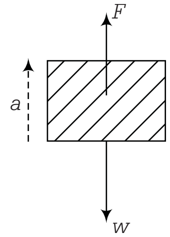
$\Rightarrow \quad E^2 = P^2c^2 + m_0^2 c^4$

For photon, rest mass

$m_0 = 0$, so $E = Pc$

For electron, $m_0 \neq 0$, so $E \neq Pc$

45. Let m be the mass of block and its acceleration in upward direction. Then,



$$a = \frac{\text{upward thrust} - \text{weight}}{m} = \frac{F - w}{m}$$

$$\Rightarrow \quad a = \frac{\left(\frac{m}{0.5 \times 10^3}\right) \times (10^3) \times (g + a_0) - mg}{m}$$

$$\Rightarrow \quad a = 2\left(g + \frac{g}{2}\right) - g = 2g$$

Acceleration of block relative to water

$$a_r = a - a_0 = 2g - \frac{g}{2} = \frac{3g}{2}$$

$$\therefore \quad h = \left(\frac{1}{2}\right) a_r t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{a_r}} = \sqrt{\frac{2h}{\frac{3}{2}g}} = 2\sqrt{\frac{h}{3g}} = 2\sqrt{\frac{1.2}{3 \times 10}} \text{ m}$$

$$\Rightarrow \quad t = 0.4 \text{ s}$$

46. Energy $= \frac{1}{2}mv^2 = 5000$ eV

$$= 5000 \times 1.6 \times 10^{-19} \text{J}$$

$$mv_2 = \sqrt{2 \times 5000 \times (1.6 \times 10^{-19})\ m}$$

$$= 4 \times 10^{-8}\sqrt{m}$$

Number of electrons striking per second is

$$n = \frac{q}{e} = \frac{It}{e} = \frac{50 \times 10^{-6} \times 1}{1.6 \times 10^{-19}}$$

$$\Rightarrow \quad n = 31.25 \times 10^{13}$$

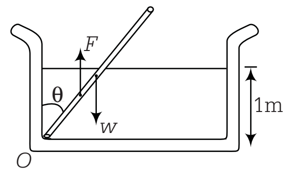
Force = Change of momentum per second

$$= n\,(mv) = 31.25 \times 10^{13} \times 4 \times 10^{-8}\sqrt{m}$$

$$= 125 \times 10^5 \sqrt{9.1 \times 10^{-31}} = 1.1924 \times 10^{-8} \text{N}$$

47. Length of rod inside the water

$$= 1 \sec\theta = \sec\theta$$



Upthrust, $F = \left(\frac{2}{2}\right)(\sec\theta)\left(\frac{1}{500}\right)(1000)(10)$

$\Rightarrow \quad F = 20 \sec\theta$

Weight of rod, $w = 20 \times 10 = 20$ N

For rotational equilibrium of rod, net torque about should be zero.

$$\therefore F\left(\frac{\sec\theta}{2}\right)(\sin\theta) = w = (1 \sin\theta)$$

$$\Rightarrow \quad \frac{20}{2}\sec^2\theta = 20$$

$$\Rightarrow \quad \theta = 45^\circ$$

$$\Rightarrow \quad F = 20 \sec 45^\circ$$

$$F = 20\sqrt{2} \text{ N}$$

For vertical equilibrium of rod, force exerted by the hinge on the rod will be $(20\sqrt{2} - 20)$ N downwards i.e. 8.3 N downwards

48. At the two points of the trajectory during projectile motion, the horizontal component of velocity is same. Then,

$$150 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}} \quad \Rightarrow \quad v = \frac{150}{\sqrt{2}} \text{ ms}^{-1}$$

Initially, $u_y = u \sin 60 = \frac{150\sqrt{3}}{2} \text{ ms}^{-1}$

Finally, $v_y = v \sin 45^\circ = \frac{150}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{150}{2} \text{ ms}^{-1}$

But $v_y = u_y + a_y t$

$$\frac{150}{2} = \frac{150\sqrt{3}}{2} - 10t$$

$$\Rightarrow \quad 10t = \frac{150(\sqrt{3} - 1)}{2}$$

$$\Rightarrow \quad t = 7.5(\sqrt{3} - 1) \text{ s}$$

49. Let R be the radius of sphere, V its volume and ρ its density. Then,

$$\Delta R = R\,\alpha\,\Delta\theta$$

and percentage change in radius

$$\frac{\Delta R}{R} \times 100 = 100\,\alpha\,\Delta\theta \qquad \ldots(i)$$

Now, $\Delta V = \gamma v \Delta\theta = 3\alpha v \Delta\theta$ $\{\because \gamma = 3\alpha\}$

$\therefore$ Percentage increase in volume

$$= \frac{\Delta V}{V} \times 100 = 300\ \alpha\Delta\theta \quad \ldots(ii)$$

Again, $\rho' = \dfrac{\rho}{1+\gamma\Delta\theta} = \dfrac{\rho}{1+3\alpha\Delta\theta}$

$$\therefore \quad \Delta\rho = \rho - \rho'$$

$$= \rho\left[1 - \frac{1}{1+3\,\alpha\Delta\theta}\right] = \frac{(3\alpha\Delta\theta)\,\rho}{1+3\alpha\Delta\theta}$$

Percentage change in density

$$= \frac{\Delta\rho}{\rho} \times 100 = \frac{300\ \alpha\Delta\theta}{1+3\alpha\Delta\theta} \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we see that the percentage change is maximum in volume.

50. Since, we know that in Young's double slit experiment.

$$\beta = \frac{D\lambda}{d} \quad \ldots(i)$$

$$\beta' = \frac{D'\lambda}{d} \quad \ldots(ii)$$

Subtract Eq. (ii) from Eq. (i), we get

$$\beta - \beta' = \frac{\lambda\,(D - D')}{d}$$

Substituting the given values, we get

$$3 \times 10^{-5} = \frac{\lambda \times 5 \times 10^{-2}}{1 \times 10^{-3}}$$

$$\Rightarrow \quad \lambda = \frac{3 \times 10^{-8}}{5 \times 10^{-2}} = 0.6 \times 10^{-6}\,\text{m}$$

$$\Rightarrow \quad \lambda = 600 \times 10^{-9}\,\text{m}$$

$$\Rightarrow \quad \lambda = 600\ \text{nm}$$

Chemistry

1. Number of moles $= \dfrac{\text{Mass}}{\text{Molecular mass}}$

Given, mass of $Al_2(SO_4)_3 = 50$ g

Molecular mass of

$Al_2(SO_4)_3 = 2 \times 27 + 3(32 + 4 \times 16) = 342$ g

$\therefore$ moles of $Al_2(SO_4)_3 = \dfrac{50}{342} = 0.14$ mol

2. Ionisation energy of $He^+ = 13.6 \times Z^2 eV$

$$= 13.6 \times (2)^2 eV$$

$$= 13.6 \times 4\ eV = 54.4\ eV$$

3. (a) Metallic radii increase in a group from top to bottom. Thus, $Li < Na < K < Rb$ is true.

(b) Electron gain enthalpy of $Cl > F$ and afterwards it decreases along a group.

(c) Ionisation enthalpy increase along a period left to right but due to presence of half-filled orbital in N, ionisation enthalpy of $N > O$. Thus $B < C < N < O$ is incorrect.

(d) For isoelectronic species,

ionic size $\propto \dfrac{1}{\text{atomic number}}$

Thus , $Al^{3+} < Mg^{2+} < Na^+ < F^-$ is true.

4. The $H{-}\ddot{\underset{\cdot\cdot}{O}}{-}H$ bond angle in H_2O is 104.5° due to presence of two lone pair of electrons. This fact can be explained with the help of valence shell electron pair repulsion theory (VSEPR).

5. Van der Waals' gas approaches ideal behaviour at extremely low pressure and high temperature.

6. $I : C(s) + O_2(g) \longrightarrow CO_2$; $\Delta H = -393.5$ kJ/mol

$II : CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$; $\Delta H = -283.0$ kJ / mol

On substracting II from I

$III : C(s) + \frac{1}{2}O_2\,(g) \longrightarrow CO\,(g)$; $\Delta H = -110.5$ kJ / mol

This equation III also represents formation of 1 mole of CO and thus enthalpy of formation of CO (g) is – 110.5 kJ / mol

7.

$$H_2 + I_2 \rightleftharpoons 2HI$$

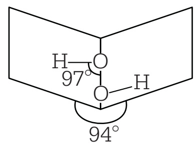
	H_2	I_2	$2HI$
At initial	1	1	0
At equili	1 – 0.8 = 0.2	1 – 0.8 = 0.2	2 × 0.8 = 1.6

$$\because \quad K_C = \frac{[HI]^2}{[H_2]\,[I_2]}$$

$$\therefore \quad K_C = \frac{1.6 \times 1.6}{0.2 \times 0.2}$$

$$\Rightarrow \quad K_C = 64$$

8. H_2O_2 is a pale blue liquid, it can be oxidised by ozone. H_2O_2 acts as both oxidising and reducing agent. The value of dipole moment of H_2O_2 is 2.1 D. Which suggest it cannot be planar infact it has open book like structure.



Structure of H_2O_2

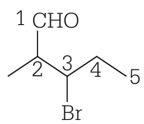
Two O — H lie in different plane

9. KO_2 absorb CO_2 and increase O_2 concentration. So, it is used in space and submarines.

10. $BI_3 > BBr_3 > BCl_3 > BF_3$

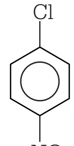
This order can be easily explained on the basis of the tendaney of the halogen atom to back donate its lone pair of electrones to empty p–orbital of the boron atom through $p\pi$-$p\pi$ bonding.

11.



3-bromo-2-methylpentanal

12. Nucleophile always attacks on electron deficient site. Presence of electron withdrawing group such as NO_2 , CHO etc., decreases the electron density on benzene nucleous, hence such group activates the ring towards nucleophilic attack. While presence of electron releasing groups such as *R* or O*R* increase the electron density, thus deactivate the nucleus toward nucleophilie attack. NO_2 group activate the ring more than Cl toward nucleophilic attack hence react readily with nucleophile.

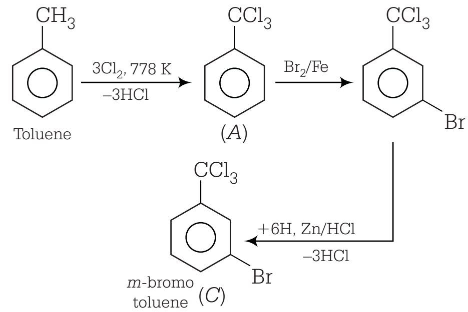


13. Ethyl acetoacetate shows tautomerism.

$$\underset{\text{Keto form}}{CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_2-COOC_2H_5}$$

$$\longleftrightarrow \underset{\text{Enol form}}{CH_3-\overset{\overset{\displaystyle OH}{|}}{C}=CH-COOC_2H_5}.$$

14.



15. We know that,

$$O_3 + CCl_2F_2 \longrightarrow 2OCl + OF_2$$

Thus, in this reaction OCl is produced.

16. Sodium chloride (NaCl) has face centred cubic structure. It contains 4 Na^+ and 4 Cl^- ions. Each Na^+ ino is surrounded by 8 Cl^- ions and each Cl^- ion is also surrounded by 8 Na^+ ions.

17. $$\text{Molarity} = \frac{\text{\% by weight of solute} \times \text{density of solution} \times 10 \text{ (in litre)}}{\text{molecular weight of the solute}}$$

$$\text{Molarity} = \frac{40 \times 1.2 \times 10}{M \times 1000}$$

Again, since

$$\text{Molarity} = \frac{\text{weight of the solute}/M}{\text{volume of the solution (in litre)}}$$

Therefore, weight of solute = 480 g

18. $E_{cell} = E^\circ_{(cathode)} - E^\circ_{(anode)}$

$= 0.41 - 0.76 = -0.35$ V

19. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g); + 22$ kcal

$\because$ The activation energy for the forward reaction $= 50$ kcal

$\therefore$ The activation energy for the backward reaction $= 50 + 72 = +72$ kcal

20. Coagulating power of an electrolyte for arsenious sulphide decreases as $Al^{3+} > Ba^{2+} > Na^+$

21. ${}^{228}_{88}X \xrightarrow{-3\alpha} {}^{216}_{82}Pb \xrightarrow{-\beta} {}^{216}_{83}Bi\ (Y)$

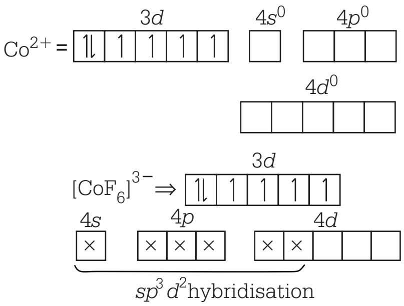
22. Hydrolysis of NCl_3 gives NH_3 or NH_4OH and HClO as

$$NCl_3 + 4H_2O \longrightarrow NH_4OH + 3HOCl$$

23. $4Ag + 8CN^- + 2H_2O + O_2 \longrightarrow 4[Ag(CN)_2]^- + 4OH^-$

This process is called cynide process. It is used in the extraction of silver from argentite (Ag_2S).

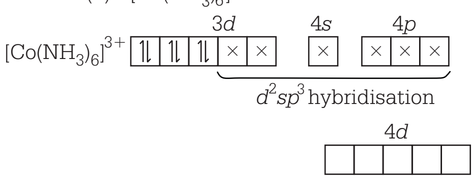
24. (a) $[CoF_6]^{3-}$ ion



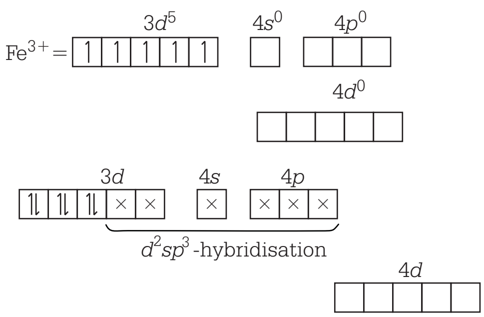
F^- is a weak ligand. It cannot pair up -electrons and forms outer orbital octahedral complex.

NH_3 and CN^- are strong ligands. So, they pair up -electrons and form their inner orbital complex.

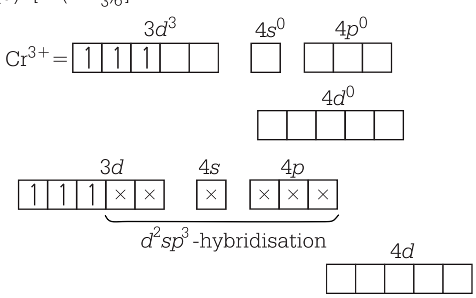
(b) $[Co(NH_3)_6]^{3+}$ ion



(c) $[Fe(CN)_6]^{3-}$ ion



(d) $[Cr(NH_3)_6]^{3+}$ ion



25. $C_2H_5Cl + KCN \xrightarrow{C_2H_5OH} \underset{(X)}{C_2H_5CN} + KCl$

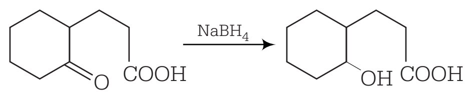
$C_2H_5CN \xrightarrow[\Delta]{H_3O^+,\ 2H_2O} C_2H_5COOH + NH_3$

or

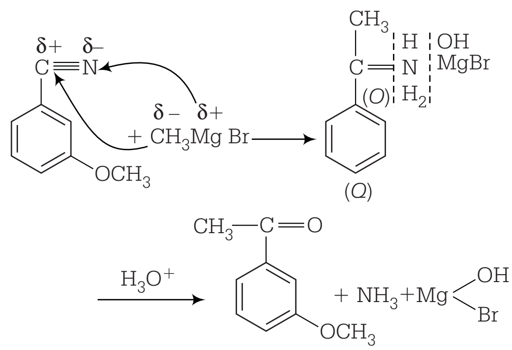
$(C_3H_6O_2)$

(y)

26. $NaBH_4$ reduces aldehyde into 1° alcohol,



27.



3-methoxy acetophenone

(*P*)

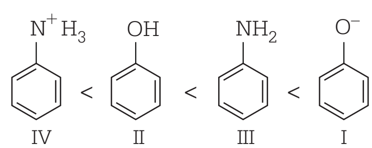
28. (i) Organic compound give an oxime with hydroxyl amine, therefore, it must be an aldehyde or ketone.

(ii) Organic compound did not give silver mirror with Tollen's reagent, therefore it cannot be an aldehyde. Therefore, compound is ketone and its molecular formula with be CH_3COCH_3.

29. $CH_3CH_2I \xrightarrow{NaCN} \underset{(A)}{CH_3CH_2CN} \xrightarrow{OH^-}$

$\underset{(B)}{CH_3CH_2CONH_2} \xrightarrow{Br_2/NaOH} \underset{(C)}{CH_3CH_2NH_2}$

30. Coupling of diazonium salts takes place in the following order as



31. Aldehyde and α-hydroxy ketones give positive Tollen's test. Glucose has an aldehyde group and fructose is an α-hydroxy ketone.

32. Chain growth polymers are formed by the chain growth polymerisation or chain polymerisation. This polymerisation process involves in a series of reaction each of which consumes a relative particles and produces another similar particle resulting a chain reaction. Teflon is a chain growth polymer. It is the polymer of tetraflouro ethylene.

$$\underset{\text{Tetrofluoro ethylene}}{n(CF_2 = CF_2)} \xrightarrow[\text{Pressure}]{\text{High}} \underset{\text{Teflon}}{-\!\!\left(CF_2 - CF_2\right)\!\!_n-}$$

33. Sodium alkyl benzene sulphonate is used as detergent.

34. Ring test for nitrate ions is

$$FeSO_4 + NO + 5H_2O \longrightarrow \underset{\text{Dark brown ring}}{[Fe(H_2O)_5NO]\,SO_4}$$

35. Zinc is purified by zone-refining method.

36. Perchloric acid ($HClO_4$) is the strongest acid among these acidic because the acidic character of oxoacid increases with increasing the oxidation number of a particular halogen atom.

$$HO\overset{+1}{Cl} < H\overset{+2}{Cl}O_2 < H\overset{+5}{Cl}O_3 < H\overset{+7}{Cl}O_4$$

37. Aerosol is colloidal system of solid in gas, e.g, smoke. So, dispersion medium in aerosol is gas.

38. Given, $k_1 = 10^{10}\, e^{-20,000/T}$

$$k_2 = 10^{12}\, e^{-24,606/T}$$

$$k_1 = k_2 \Rightarrow \; 10^{10}\, e^{-20,000/T} = 10^{12}\, e^{-24,606/T}$$

$$e^{-\frac{20,000}{T} + \frac{24606}{T}} = 10^2$$

$$e^{\frac{4,606}{T}} = 10^2$$

On taking log both sides,

$$\frac{4.606}{2.303\ T} = \log 10^2$$

$$2 \log 10 \times T = \frac{4606}{2.303}$$

$$T = \frac{4606}{2.303 \times 2} = \frac{4606}{4.606} = 1000\ K$$

39. $w = 60g,\ i = 5A$

$$\text{Equivalent of Ca} = \frac{\text{atomic weight}}{\text{Valency}} = \frac{40}{2} = 20$$

According to first law of faraday electrolysis

$$w = Zit = \frac{\text{Equivalent weight}}{96500} \times i \times T$$

$$60 = \frac{20}{96500} \times 5 \times T$$

$$T = \frac{96500 \times 60}{20 \times 5}\ s$$

$$T = \frac{96500 \times 60}{20 \times 5 \times 60 \times 60}\ h$$

$$= 16.08\ h. \approx 16\ h.$$

40. Given,

$$T_f = 273 + 16.6 = 289.6\ K$$

$$L_f = 180.75\ J/g$$

$$R = 8.314\ J/K/mol$$

$$K_f = ?,$$

$$\therefore \quad K_f = \frac{RT_f^2}{1000 \times L_f} = \frac{8.314 \times (289.6)^2}{1000 \times 180.75}$$

$$\therefore \quad K_f = 3.86$$

41. The coordination number is 8:8 in $Cs^+ : Cl^-$.

The coordination number is 6:6 in $Na^+ : Cl^-$.

42. $N_2O_4\ (g) \rightleftharpoons 2NO_2\ (g)$

Molar concentration of

$$[N_2O_4] = \frac{9.2}{92} = 0.1\ mol/L$$

In equilibrium state,
when it 50% dissociates

$$[N_2O_4] = 0.05\ M$$

$$[NO_2] = 0.1\ M$$

Now, from $\quad K_C = \dfrac{[NO_2]^2}{[N_2O_4]}$

$$K_C = \frac{0.1 \times 0.1}{0.05} = 0.2$$

43. The Gibbs-Helmholtz equation is as

$$G = H + T\left[\frac{\delta G}{\delta T}\right]_p$$

Dividing above equation by T^2

$$\frac{G}{T^2} = \frac{H}{T^2} + \frac{1}{T}\left[\frac{\delta G}{\delta T}\right]_p$$

This on rearrangement becomes.

$$\left[\frac{\delta (G/T)}{\delta T}\right] = -\frac{H}{T^2}$$

Thus, enthalpy, $H = -T^2 \left[\dfrac{\delta G/T}{\delta T}\right]_p$

Where, $\quad H = \text{enthalpy}$

44. $Cl_2O = 2 \times 17 + 16 = 42$ electrons

$ICl_2^- = 53 + 2 \times 17 + 1 = 88$ electrons

$Cl_2^- = 2 \times 17 + 1 = 35$ electrons

$IF_2^+ = 53 + 2 \times 9 - 1 = 70$ electrons

$ClO_2 = 17 + 2 \times 8 = 33$ electrons

$ClO_2 = 17 + 2 \times 8 + 1 = 33$ electrons

$ClO_2^- = 17 + 2 \times 8 + 1 = 34$ electrons

$ClF_2^+ = 17 + 2 \times 9 - 1 = 34$ electrons

ClO_2^- and ClF_2^+ contain 34 electrons each.

Hence they are isoelectronic.

45. Atomic and ionic radii increase from top to bottom in a group due to the inclusion of another shell at every step. Hence, Cs^+ ion will be the largest among given IA group ions, i.e.

Na^+, Li^+ and K^+.

46. In the ground state of an atom, the number of states is limited by Hund's rule. These are $\frac{\text{Ln}}{\text{Lr.}(n-r)!}$ ways in which electron in orbital may be arranged which do not violate Paulis exclusion principle.

Where, n = number of maximum electrons that can be filled in an orbital

and r = number of electrons present in orbital.

But the valid ground state term is calculated by Hund's rule of maximum multiplicity. As Hund's rule gives the most stable electronic configuration of electrons.

47. $\underset{2 \times 276\text{ g}}{2\,Ag_2CO_3} \xrightarrow[\Delta]{} \underset{4 \times 108\text{g}}{4Ag} + 2CO_2 + O_2$

$\therefore$ 2×276 g of Ag_2CO_3 gives $= 4 \times 108$ g Ag

$\therefore$ 1 g of Ag_2CO_3 will give $= \frac{4 \times 108}{2 \times 276}$

$\therefore$ 2.76 g of Ag_2CO_3 will give $= \frac{4 \times 108 \times 2.76}{2 \times 276}$

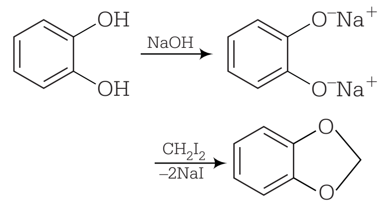
$= 2.16$ g

48. $\begin{array}{c} CH_2Br \\ | \\ CH_2Br \end{array} + 2\underset{\text{Alc.}}{KOH} \xrightarrow{\Delta} \underset{\text{Acetylene}}{\begin{array}{c} CH \\ ||| \\ CH \end{array}} + 2KBr + 2H_2O$

This is dehydrohalogenation reaction.

49. I_2 form complex ion I_3^- in KI solution due to which it dissolve in it.

50. Catechol is most acidic out of all dihydric phenols.



The reaction is Williamson's synthesis type reaction.

Mathematics

1. Let $y = x^{3000}$, $x = 1$ and $x + \Delta x = 1.0002$

Then, $\Delta x = 1.0002 - 1 = 0.0002$

Also, $y = 1$, when $x = 1$

Now, $y = x^{3000}$

$\Rightarrow \quad \frac{dy}{dx} = 3000\, x^{2999}$

$\Rightarrow \quad \left[\frac{dy}{dx}\right]_{x=1} = 3000$

$\because \quad \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \quad \Delta y = 3000 \times 0.0002 = \frac{6}{10} = 0.6$

Hence, $(1.0002)^{3000} = y + \Delta y = 1 + 0.6 = 1.6$

2. Let $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$.

Then, $\mathbf{a} \cdot \hat{\mathbf{i}} = a_1, \mathbf{a} \cdot \hat{\mathbf{j}} = a_2, \mathbf{a} \cdot \hat{\mathbf{k}} = a_3$

$\mathbf{a} \times \hat{\mathbf{i}} = -a_2\hat{\mathbf{k}} + a_3\hat{\mathbf{j}}, \mathbf{a} \times \hat{\mathbf{j}} = a_1\hat{\mathbf{k}} - a_3\hat{\mathbf{i}}$

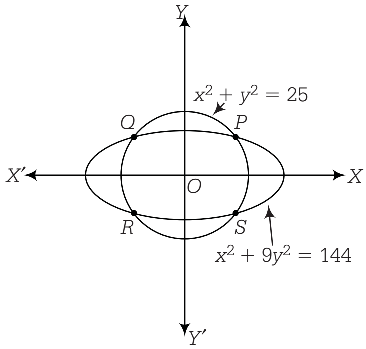
and $\mathbf{a} \times \hat{\mathbf{k}} = -a_1\hat{\mathbf{j}} + a_2\hat{\mathbf{i}}$

$\therefore$ $(\mathbf{a} \cdot \hat{\mathbf{i}})(\mathbf{a} \times \hat{\mathbf{i}}) + (\mathbf{a} \cdot \hat{\mathbf{j}})(\mathbf{a} \times \hat{\mathbf{j}}) + (\mathbf{a} \cdot \hat{\mathbf{k}})(\mathbf{a} \times \hat{\mathbf{k}})$

$= a_1(-a_2\hat{\mathbf{k}} + a_3\hat{\mathbf{j}}) + a_2(a_1\hat{\mathbf{k}} - a_3\hat{\mathbf{i}}) + a_3(-a_1\hat{\mathbf{j}} + a_2\hat{\mathbf{i}})$

$= \mathbf{0}$

3. Clearly, A is the set of all points on the circle $x^2 + y^2 = 25$ and B is the set of all points on the ellipse $x^2 + 9y^2 = 144$. These two intersect at four points P, Q, R and S.



Hence, $A \cap B$ contains four points.

4. Clearly, $f(x) = \sqrt{x^2 - 4}$ is continuous on [2, 3] and differentiable on (2, 3).

So, by Lagrange's mean value theorem, there exists $c \in (2, 3)$ such that

$$f'(c) = \frac{f(3) - f(2)}{3 - 2}$$

$$\Rightarrow \quad \frac{c}{\sqrt{c^2 - 4}} = \sqrt{5} - 0$$

$$\left[\because f(x) = \sqrt{x^2 - 4} \Rightarrow f'(x) = \frac{x}{\sqrt{x^2 - 4}}\right]$$

$$\Rightarrow \quad c^2 = 5(c^2 - 4)$$

$$\Rightarrow \quad 4c^2 = 20$$

$$\Rightarrow \quad c = \sqrt{5}$$

5. The mean of the series $a, a + d, a + 2d, \ldots a \; + 2nd$ is $\bar{x} = \frac{1}{2n + 1}[a + a + d + a + 2d + \ldots + a + 2nd]$

$$\Rightarrow \bar{x} = \frac{1}{2n + 1}\left\{\frac{2n + 1}{2}(a + a + 2nd)\right\} = a + nd$$

$\therefore$ Mean deviation from mean,

$$MD = \frac{1}{2n + 1}\sum_{r=0}^{2n}|(a + rd) - (a + nd)|$$

$$\Rightarrow \quad MD = \frac{1}{2n + 1}\sum_{r=0}^{2n}|r - n|\, d$$

$$\Rightarrow \quad MD = \frac{1}{2n + 1}\{2d(1 + 2 + \ldots + n)\} = \frac{n(n + 1)d}{2n + 1}$$

6. We have,

$$f(x) = xe^{x(1 - x)}$$

On differentiating both sides w.r.t. , we get

$$f'(x) = e^{x(1 - x)} + x(1 - 2x)e^{x(1 - x)}$$

$$\Rightarrow \quad f'(x) = (1 + x - 2x^2)e^{x(1 - x)}$$

$$\Rightarrow \quad f'(x) = -(x - 1)(2x + 1)e^{x(1 - x)}$$

Since, $e^{x(1 - x)} > 0$ for all x. Therefore, signs of $f'(x)$ for different values of x are as shown below:

$$-\infty \xleftarrow{\quad - \quad} -1/2 \xrightarrow{\quad + \quad} 1 \xrightarrow{\quad - \quad} \infty$$

Clearly, f(x) is increasing on $\left[-\frac{1}{2}, 1\right]$ and decreasing on $\left(-\infty, -\frac{1}{2}\right] \cup [1, \infty)$.

7. We have,

$$(1 + \omega)(1 + \omega^2)(1 + \omega^3)(1 + \omega^4)\ldots(1 + \omega^{3n})$$

$$= [\{(1 + \omega)(1 + \omega^2)\}\{(1 + \omega^4)(1 + \omega^5)\}\ldots \{(1 + \omega^{3n-2})(1 + \omega^{3n-1})\} \times (1 + \omega^3)(1 + \omega^6)(1 + \omega^9)\ldots(1 + \omega^{3n})]$$

$$= 2^n[\{(-\omega^2)(-\omega)\}\{(-\omega^2)(-\omega)]\ldots\{(-\omega^2)(-\omega)\}]$$

$$\Rightarrow [(1 + w)(1 + w^2)\{(1 + w)(1 + w^2)\}\{(1 + w)(1 + w^2)\} \ldots \{(1 + w)(1 + w^2)\}] \times (1 + 1)(1 + 1)\ldots(1 + 1)$$

$$= 2^n$$

8. Clearly, X is a binomial variate with parametres n and $p = \frac{1}{2}$ such that

$$P(X = r) = {}^nC_r p^r q^{n-r} = {}^nC_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{n-r}$$

$$= {}^nC_r\left(\frac{1}{2}\right)^n$$

Now, $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in AP.

$$\Rightarrow \quad 2P(X = 5) = P(X = 4) + P(X = 6)$$

$$\Rightarrow \quad 2\cdot{}^nC_5\left(\frac{1}{2}\right)^n = {}^nC_4\left(\frac{1}{2}\right)^n + {}^nC_6\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad 2\cdot{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \quad 2\cdot\frac{n!}{(n-5)!\,5!} = \frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!}$$

$$\Rightarrow \quad \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}$$

$$\Rightarrow \quad n^2 - 21n + 98 = 0$$

$$\Rightarrow \quad (n - 7)(n - 14) = 0$$

$$\therefore \quad n = 7 \text{ or } 14$$

9. Suppose $(s + 1)$th term contains x^{2r}.

Then, we have

$$T_{s+1} = {}^{n-3}C_s x^{n-3-s}\left(\frac{1}{x^2}\right)^s$$

$$= {}^{n-3}C_s x^{n-3-3s}$$

This will contain x^{2r}, if

$$n-3-3s=2r$$

$$\Rightarrow \quad s=\frac{n-3-2r}{3}$$

$$\Rightarrow \quad s=\frac{n-2r}{3}-1$$

$$\Rightarrow \quad s+1=\frac{n-2r}{3}$$

$$\Rightarrow \quad n-2r=3(s+1)$$

Hence, $(n-2r)$ is a positive integral multiple of 3.

10. We have, $\sin^{-1}\left\{\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right\}$

$$=\sin^{-1}\left\{\cot\left(\sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}}+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}\right)\right\}$$

$$=\sin^{-1}\left\{\cot\left(\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi}{4}\right)\right\}$$

$$=\sin^{-1}\left(\cot\frac{\pi}{2}\right)$$

$$=\sin^{-1}0=0$$

11. We have,

$$\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$$

$$\Rightarrow \quad \cos^{-1}\left\{\frac{xy}{6}-\sqrt{1-\frac{x^2}{4}}\cdot\sqrt{1-\frac{y^2}{9}}\right\}=\theta$$

$$\Rightarrow \quad xy-\sqrt{4-x^2}\cdot\sqrt{9-y^2}=6\cos\theta$$

$$\Rightarrow \quad (xy-6\cos\theta)^2=(4-x^2)(9-y^2)$$

$$\Rightarrow \quad -12xy\cos\theta+36\cos^2\theta=36-4y^2-9x^2$$

$$\Rightarrow \quad 9x^2+4y^2-12xy\cos\theta=36\sin^2\theta$$

12. We have,

$$\tan(\sec^{-1}x)=\sin\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$$

$$\Rightarrow \quad \sqrt{\sec^2(\sec^{-1}x)-1}=\sqrt{1-\cos^2\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)}$$

$$\Rightarrow \quad \sqrt{x^2-1}=\sqrt{1-\frac{1}{5}}$$

$$\Rightarrow \quad x^2-1=\frac{4}{5}$$

$$\Rightarrow \quad x^2=\frac{9}{5}$$

$$\Rightarrow \quad x=\pm\frac{3}{\sqrt{5}}$$

13. We have,

$$y=(2x-1)e^{2(1-x)}$$

On differentiating both sides w.r.t. , we get

$$\frac{dy}{dx}=2e^{2(1-x)}-2(2x-1)e^{2(1-x)}$$

$$\Rightarrow \quad \frac{dy}{dx}=2e^{2(1-x)}(2-2x)=4e^{2(1-x)}(1-x)$$

At points of maximum, we must have

$$\frac{dy}{dx}=0\Rightarrow x=1$$

Now, $$\frac{d^2y}{dx^2}=-8e^{2(1-x)}(1-x)-4e^{2(1-x)}$$

$$\Rightarrow \quad \left[\frac{d^2y}{dx^2}\right]_{x=1}=-4<0$$

So, y is maximum at x = 1. Clearly, y = 1 for x = 1.

Thus, the point of maximum is (1, 1).

The equation of the tangent at (1, 1) is

$$y-1=0\,(x-1)$$

$$\Rightarrow \quad y=1$$

14. The truth table of $(p\to\sim p)\wedge(\sim p\to p)$ is as follows:

p	$\sim p$	$p\to\sim p$	$\sim p\to p$	$(P\to\sim P)\wedge(\sim p\to p)$
T	F	F	T	F
F	T	T	F	F

Clearly, last column of the above truth table contains F only. So, the given statement is a contradiction.

15. There are 6 letters in the word 'DEGREE', namely 3E's and D, G, R. Four letters out of six can be selected in the following ways:

(i) 3 like letters and 1 different, viz. EEE + D, G, R

(ii) 2 like letters and 2 different, viz. EEE + any two of D, G, R

(iii) All different letters, viz. EDGR

So, the total number of ways

$$={}^3C_1+{}^3C_2+1=3+3+1=7$$

16. Total number of triangles = Number of triangles with vertices on sides (PQ, QR, RS + QR, RS, PS + RS, PS, PQ + PS, PQ, QR)

$$= {}^3C_1 \times {}^4C_1 \times {}^5C_1 + {}^4C_1 \times {}^5C_1 \times {}^6C_1 + {}^5C_1 \times {}^6C_1 \times {}^3C_1 + {}^6C_1 \times {}^3C_1 \times {}^4C_1$$

$$= 60 + 120 + 90 + 72 = 342$$

17. We observe that

$$\frac{x^4 + x^2 + x + 1}{x^2 - x + 1} \to \frac{2}{3} \text{ as } x \to -1$$

But $\quad \dfrac{1 - \cos(x+1)}{(x+1)^2} \to \dfrac{0}{0}$ as $x \to -1$

$$\therefore \quad \lim_{x \to -1} \left(\frac{x^4 + x^2 + x + 1}{x^2 - x + 1}\right)^{\frac{1-\cos(x+1)}{(x+1)^2}}$$

$$= \left(\frac{2}{3}\right)^{\lim_{x \to -1} \frac{1-\cos(x+1)}{(x+1)^2}} = \left(\frac{2}{3}\right)^{\lim_{x \to -1} \frac{2\sin^2\left(\frac{x+1}{2}\right)}{4\left(\frac{x+1}{2}\right)^2}} = \left(\frac{2}{3}\right)^{1/2}$$

18. We have,

$$\left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3$$

$$= \left({}^4C_0\, x^4 - {}^4C_1 x^2 + {}^4C_2 - {}^4C_3\, \frac{1}{x^2} + {}^4C_4\, \frac{1}{x^4}\right) \times \left({}^3C_0 x^3 + {}^3C_1 x + {}^3C_2\, \frac{1}{x} + {}^3C_3\, \frac{1}{x^3}\right)$$

Clearly, there is no term free from x on RHS. Therefore, the term independent of x on LHS is zero.

19. We have,

$$A^2 = A$$

$\therefore \quad (I + A)^2 = (I + A)(I + A) = I + 2A + A^2 = I + 3A$

and $\quad (I + A)^3 = (I + A)^2 (I + A)$

$= (I + 3A)(I + A) \quad [\because (I + A)^2 = I + 3A]$

$= I + 4A + 3A^2$

$= I + 7A \quad [\because A^2 = A]$

Thus, we have

$$(I + A)^2 = I + 3A \text{ and } (I + A)^3 = I + 7A$$

$\Rightarrow \quad (I + A)^2 = I + (2^2 - 1)\, A$

and $\quad (I + A)^3 = I + (2^3 - 1)\, A$

Hence, $\quad (I + A)^n = I + (2^n - 1)\, A$

$\therefore \quad (I + A)^n = I + \lambda A$

$\Rightarrow \quad \lambda = 2^n - 1$

20. We have, $u = e^x \sin x$ and $v = e^x \cos x$

On differentiating both sides w.r.t. , we get

$$\frac{du}{dx} = e^x \sin x + e^x \cos x = u + v$$

and $\dfrac{dv}{dx} = e^x \cos x - e^x \sin x = v - u$

$\therefore\ v\dfrac{du}{dx} - u\dfrac{dv}{dx} = v(u + v) - u(v - u) = u^2 + v^2$

Now, $\quad \dfrac{d^2u}{dx^2} = \dfrac{du}{dx} + \dfrac{dv}{dx} = u + v + v - u = 2v$

and $\quad \dfrac{d^2v}{dx^2} = \dfrac{dv}{dx} - \dfrac{du}{dx}$

$= (v - u) - (v + u) = -2u$

21. Let $y = \tan^{-1}\left(\dfrac{\sqrt{1 + x^2} - 1}{x}\right)$

Putting $x = \tan\theta$, we get

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{1}{2}\tan^{-1} x$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2(1 + x^2)}$$

and $z = \tan^{-1}\left(\dfrac{2x\sqrt{1 - x^2}}{1 - 2x^2}\right)$

Putting $x = \sin\theta$, we get

$$z = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right) = \tan^{-1}(\tan 2\theta)$$

$\Rightarrow \quad z = 2\theta = 2\sin^{-1} x$

On differentiating both sides w.r.t. x, we get

$$\frac{dz}{dx} = \frac{2}{\sqrt{1 - x^2}}$$

Thus, $\dfrac{dy}{dz} = \dfrac{dy/dx}{dz/dx} = \dfrac{1}{4(1 + x^2)}\sqrt{1 - x^2}$

$$\therefore \quad \left[\frac{dy}{dz}\right]_{x=0} = \frac{1}{4}$$

22. Let $\quad I = \displaystyle\int \frac{1 - x^2}{(1 + x^2)\sqrt{1 + x^4}}\, dx$

$$= \int \frac{\frac{1}{x^2} - 1}{\left(x + \frac{1}{x}\right)\sqrt{x^2 + \frac{1}{x^2}}}\, dx$$

$$= -\int \frac{1}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-(\sqrt{2})^2}}\, d\left(x+\frac{1}{x}\right)$$

$$= \frac{1}{\sqrt{2}}\operatorname{cosec}^{-1}\left(\frac{x+\frac{1}{x}}{\sqrt{2}}\right)+C$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{\sqrt{2}x}{x^2+1}\right)+C$$

23. We have,

$$\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & a & b \\ 0 & c-a & a-b \\ 0 & b-a & c-b \end{vmatrix} = 0$$

[applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$\Rightarrow \quad (c-a)(c-b)+(a-b)^2 = 0$

$\Rightarrow \quad a^2+b^2+c^2-ab-bc-ca = 0$

$\Rightarrow 2a^2+2b^2+2c^2-2ab-2bc-2ca = 0$

$\Rightarrow \quad (a-b)^2+(b-c)^2+(c-a)^2 = 0$

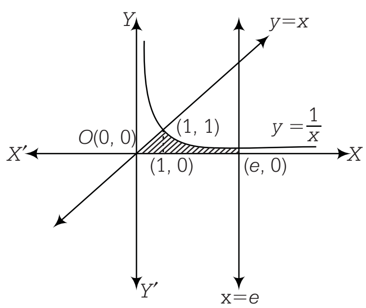
$\Rightarrow \quad a = b = c$

$\therefore$ ΔABC is an equilateral triangle.

$\Rightarrow \quad A = B = C = \dfrac{\pi}{3}$

$\therefore \quad \sin^2 A + \sin^2 B + \sin^2 C = 3\sin^2\dfrac{\pi}{3} = \dfrac{9}{4}$

24. Required area $= \int_0^1 x\,dx + \int_1^e \frac{1}{x}dx$



$$= \left[\frac{x^2}{2}\right]_0^1 + [\log x]_1^e = \frac{1}{2}+1 = \frac{3}{2}$$

25. Let $x = \dfrac{1}{y}$. Then,

$$\lim_{x\to\infty}\left\{\frac{a_1^{1/x}+a_2^{1/x}+\ldots+a_n^{1/x}}{n}\right\}^{nx}$$

$$= \lim_{y\to 0}\left\{\frac{a_1^y+a_2^y+\ldots+a_n^y}{n}\right\}^{n/y}$$

$$= \lim_{y\to 0}\left\{1+\frac{a_1^y+a_2^y+\ldots+a_n^y-n}{n}\right\}^{n/y}$$

$$= e^{\lim_{y\to 0}\left\{\frac{a_1^y-1}{y}+\frac{a_2^y-1}{y}+\ldots+\frac{a_n^y-1}{y}\right\}}$$

$$= e^{\log a_1+\log a_2+\ldots+\log a_n}$$

$$= e^{\log (a_1 a_2 \ldots a_n)}$$

$$= a_1 a_2 a_3 \ldots a_n$$

26. Let $I = \displaystyle\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$

Put $\sec x + \tan x = t$

Then, $\quad \sec x - \tan x = \dfrac{1}{t}$

and $\quad \sec x(\sec x+\tan x)\,dx = dt$

$\Rightarrow \quad \sec x\,dx = \dfrac{1}{t}dt$

Also, $\quad \sec x = \dfrac{1}{2}\left(t+\dfrac{1}{t}\right)$

$$\therefore \quad I = \frac{1}{2}\int \frac{\frac{1}{t}\left(t+\frac{1}{t}\right)}{t^{9/2}}dt$$

$$\Rightarrow \quad I = \frac{1}{2}\int \frac{1}{t^{9/2}}+\frac{1}{t^{13/2}}\,dt$$

$$\Rightarrow \quad I = -\frac{1}{7t^{7/2}}-\frac{1}{11t^{11/2}}+K$$

$$\Rightarrow \quad I = -\frac{1}{t^{11/2}}\left\{\frac{t^2}{7}+\frac{1}{11}\right\}+K$$

$$\Rightarrow \quad I = -\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$$

27. Let the equation of the circle be

$$x^2+y^2+2gx+2fy+c=0$$

which passes through (0, 0) and (1, 0).

$\therefore \quad c = 0$ and $1+2g+c = 0$

$\Rightarrow \quad c = 0, g = -\dfrac{1}{2}$

The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches the circle $x^2 + y^2 = 9$.

$\therefore \quad \sqrt{g^2 + f^2} = 3 \pm \sqrt{g^2 + f^2 - c}$

$[\because \ C_1C_2 = r_1 \pm r_2]$

$\Rightarrow \quad \sqrt{\frac{1}{4} + f^2} = 3 \pm \sqrt{\frac{1}{4} + f^2}$

$\Rightarrow \quad 3 = 2\sqrt{\frac{1}{4} + f^2}$

$\Rightarrow \quad f^2 = \frac{9}{4} - \frac{1}{4} = 2$

$\Rightarrow \quad f = \pm \sqrt{2}$

Hence, the coordinates of the centre are

$\left(\frac{1}{2}, \pm \sqrt{2}\right)$.

28. Given equation of parabola is

$$y^2 - kx + 8 = 0$$

$\Rightarrow \quad y^2 = k\left(x - \frac{8}{k}\right)$

$\Rightarrow \quad (y - 0)^2 = k\left(x - \frac{8}{k}\right)$

The equation of the directrix of this parabola is

$$x - \frac{8}{k} = -\frac{k}{4} \qquad [\because x = -a]$$

$\Rightarrow \quad x = \frac{8}{k} - \frac{k}{4}$

But the equation of the directrix is given as $x - 1 = 0$.

$\therefore \quad \frac{8}{k} - \frac{k}{4} = 1$

$\Rightarrow \quad k^2 + 4k - 32 = 0$

$\Rightarrow \quad (k - 4)(k + 8) = 0$

$\therefore \quad k = -8, 4$

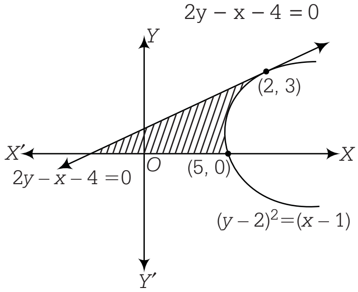
29. Given equation of parabola is

$$y^2 - 4y - x + 5 = 0$$

Then, the equation of tangent at $(2, 3)$ is

$$3y - 2(y + 3) - \frac{(x + 2)}{2} + 5 = 0$$

$\Rightarrow \quad 2y - x - 4 = 0$



$\therefore$ Required area is given by

$$A = \int_0^3 (x_2 - x_1)\,dy$$

$$= \int_0^3 [\{(y-2)^2 + 1\} - \{2y - 4\}]\,dy$$

$$= \int_0^3 (y^2 - 6y + 9)\,dy = \int_0^3 (3 - y)^2\,dy$$

$$= -\left[\frac{(3-y)^3}{3}\right]_0^3$$

$\therefore \quad A = 9$

30. We have,

$$x = e^{xy\frac{dy}{dx}}$$

$\Rightarrow \quad \log x = xy\frac{dy}{dx}$

$\Rightarrow \quad ydy = \frac{\log x}{x}dx$

$\Rightarrow \quad ydy = \log x \, d(\log x)$

On integrating both sides, we get

$$\frac{y^2}{2} = \frac{(\log_e x)^2}{2} + C$$

$\Rightarrow \quad y^2 = (\log_e x)^2 + 2C$

$\Rightarrow \quad y = \pm\sqrt{(\log_e x)^2 + 2C}$

31. Number of ways of selecting three numbers one on each card $= 100 \times 100 \times 100 = 100^3$

We know that $(2n + 1)$, $(2n^2 + 2n)$ and $(2n^2 + 2n + 1)$ are Pythagorean triplets.

$\therefore$ For $n = 1, 2, 3, 4, 5, 6$, we get the lengths of three sides of a right angled triangle such that its hypotenuse is less than or equal to 100 cm.

Now, one Pythagorean triplet (e.g. 3, 4, 5; 5, 12, 13; etc.) can be chosen in 3! ways.

$\therefore$ Number of ways of selecting 6 Pythagorean triplets $= 6 \times 3!$

Hence, required probability $= \frac{6 \times 3!}{100^3}$

32. It is given that $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$\therefore \quad k = \lim_{x \to \pi/2} f(x)$

$\Rightarrow \quad k = \lim_{x \to \pi/2} \frac{\sin(\cos x) - \cos x}{(\pi - 2x)^3}$

$\Rightarrow \quad k = \lim_{x \to \pi/2} \frac{\sin(\cos x) - \cos x}{\cos^3 x} \times \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{8\left(\frac{\pi}{2} - x\right)^3}$

$\therefore \quad k = -\frac{1}{6} \times \frac{1}{8} = -\frac{1}{48} \quad \left[\because \lim_{x \to 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}\right]$

33. We have,

$$f(x) = [x] + \left[x + \frac{1}{2}\right] = \begin{cases} 0, & \text{if } 0 < x < \frac{1}{2} \\ 1, & \text{if } x = \frac{1}{2} \\ 1, & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

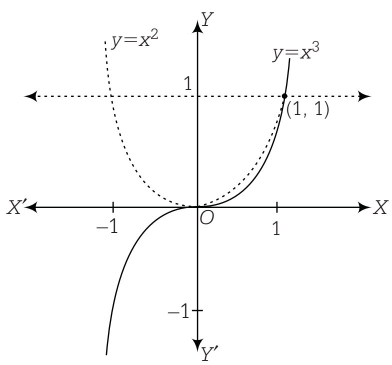
Clearly, $f(x)$ is discontinuous at $x = \frac{1}{2}$.

Also, $\lim\limits_{x \to 1/2^-} f(x) = 0$ and $\lim\limits_{x \to 1/2^+} f(x) = 1$

34. It is evident from the graph of $f(x)$ that

$$f(x) = \begin{cases} 1, & x \geq 1 \\ x^3, & x < 1 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous but it is not differentiable at $x = 1$.



35. We have,

$$x_n = \cos\left(\frac{\pi}{3^n}\right) + i \sin\left(\frac{\pi}{3^n}\right)$$

$\therefore x_1 \cdot x_2 \cdot x_3 \ldots$

$$= \cos\left(\frac{\pi}{3} + \frac{\pi}{3^2} + \ldots\right) + i \sin\left(\frac{\pi}{3} + \frac{\pi}{3^2} + \ldots\right)$$

$$= \cos\left(\frac{\pi/3}{1 - \frac{1}{3}}\right) + i \sin\left(\frac{\pi/3}{1 - \frac{1}{3}}\right)$$

$$= \cos\frac{\pi}{2} + i \sin\frac{\pi}{2} = 0 + i = i$$

36. There can be 2 types of numbers.

(i) Any one of the digits 1, 2, 3, 4 repeats thrice and the remaining digits only once i.e., of the type 1, 2, 3, 4, 4, 4.

(ii) Any two of the digits 1, 2, 3, 4 repeat twice and the remaining two only once i.e., of the type 1, 2, 3, 3, 4, 4.

Number of numbers of the type 1 2 3 4 4 4

$$= \frac{6!}{3!} \times {}^4C_1 = 480$$

Number of numbers of the type 1 2 3 3 4 4

$$= \frac{6!}{2!2!} \times {}^4C_2 = 1080$$

So, the required number $= 480 + 1080 = 1560$

37. We have,

$$9x^2 - 18|x| + 5 = 0$$

$$\Rightarrow \quad 9|x|^2 - 18|x| + 5 = 0$$

$$\Rightarrow \quad (3|x| - 1)(3|x| - 5) = 0$$

$$\Rightarrow \quad |x| = \frac{1}{3} \text{ or } |x| = \frac{5}{3}$$

$$\Rightarrow \quad x = \pm\frac{1}{3} \text{ or } x = \pm\frac{5}{3}$$

Now, $\log_e\{(x + 1)(x + 2)\}$ is defined, if

$$(x + 1)(x + 2) > 0$$

$$\Rightarrow \quad x < -2 \text{ or } x > -1$$

Clearly, $x = -\frac{5}{3}$ does not satisfy this condition. But all other values of x satisfy the above conditions. Hence, the number of solutions lying in the domain of definition of the given function is 3.

38. We have,

$$2^x + 2^y = 2^{x + y}$$

On differentiating both sides w.r.t. , we get

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x + y} \log 2\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \quad (2^y - 2^{x + y})\frac{dy}{dx} = 2^{x + y} - 2^x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2^x(2^y - 1)}{2^y(1 - 2^x)} = 2^{x - y}\left(\frac{2^y - 1}{1 - 2^x}\right)$$

39. Since, $3x + y = 0$ is a tangent to the circle with centre at $(2, -1)$.

$\therefore$ Radius = Length of the perpendicular from $(2, -1)$ on $3x + y = 0$

$$\Rightarrow \quad \text{Radius} = \frac{6 - 1}{\sqrt{9 + 1}} = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}$$

So, the equation of the circle is

$$(x-2)^2+(y+1)^2=\frac{5}{2}$$

$$\Rightarrow \quad x^2+y^2-4x+2y+\frac{5}{2}=0$$

The combined equation of the tangents drawn from the origin to this circle is

$$SS_1=T^2$$

where, $S=x^2+y^2-4x+2y+\frac{5}{2}$,

$$S_1=0^2+0^2-4\times 0+2\times 0+\frac{5}{2}$$

$$=\frac{5}{2}$$

and $T=x(0)+y(0)-2(x+0)+(y+0)+\frac{5}{2}$

$$=-2x+y+\frac{5}{2}$$

$$\therefore \left(x^2+y^2-4x+2y+\frac{5}{2}\right)\left(\frac{5}{2}\right)=\left(-2x+y+\frac{5}{2}\right)^2$$

$$\Rightarrow \quad 3x^2-8xy-3y^2=0$$

$$\Rightarrow \quad 3x+y=0 \text{ and } x-3y=0$$

40. We have,

$\sin A+\cos A=m$ and $\sin^3 A+\cos^3 A=n$

Now, $\quad \sin A+\cos A=m$

$$\Rightarrow \quad (\sin A+\cos A)^3=m^3$$

$$\Rightarrow \quad \sin^3 A+\cos^3 A+3\sin A\cos A\,(\sin A+\cos A)=m^3$$

$$\Rightarrow \quad n+3\sin A\cos A\, m=m^3 \quad \ldots(i)$$

Again, $\quad \sin A+\cos A=m$

$$\Rightarrow \quad \sin^2 A+\cos^2 A+2\sin A\cos A=m^2$$

$$\Rightarrow \quad \sin A\cos A=\frac{m^2-1}{2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$n+3m\frac{(m^2-1)}{2}=m^3$$

$$\Rightarrow \quad 2n+3m^3-3m=2m^3$$

$$\Rightarrow \quad m^3-3m+2n=0$$

41. We have, $a=4$, $b=3$ and $\angle A=60°$

$$\because \quad \cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\therefore \quad \cos 60°=\frac{9+c^2-16}{6c}$$

$$\Rightarrow \quad \frac{1}{2}\times 6c=c^2-7$$

$$\Rightarrow \quad c^2-3c-7=0$$

42. Let T_r be the th term of the given series. Then,

$$T_r=\frac{2r+1}{1^2+2^2+\ldots+r^2}=\frac{2r+1}{\frac{r(r+1)(2r+1)}{6}}$$

$$=\frac{6}{r(r+1)}$$

$$\Rightarrow \quad T_r=6\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

So, the required sum is given by

$$\sum_{r=1}^{n}T_r=6\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$=6\left[\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{n+1}\right]$$

$$=6\left[1-\frac{1}{n+1}\right]=\frac{6n}{n+1}$$

43. Let $\quad I=\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}\frac{x\sin x^2}{\sin x^2+\sin(\ln 6-x^2)}dx$

Putting $x^2=t$

$$\Rightarrow \quad 2x\,dx=dt$$

$$\therefore \quad I=\frac{1}{2}\int_{\ln 2}^{\ln 3}\frac{\sin t}{\sin t+\sin(\ln 6-t)}dt \quad \ldots(i)$$

Using $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$, we get

$$I=\frac{1}{2}\int_{\ln 2}^{\ln 3}\frac{\sin(\ln 6-t)}{\sin(\ln 6-t)+\sin t}dt \quad \ldots(ii)$$

On adding Eqs.(i) and (ii), we get

$$2I=\int_{\ln 2}^{\ln 3}1\,dt=[t]_{\ln 2}^{\ln 3}$$

$$\Rightarrow \quad 2I=\frac{1}{2}(\ln 3-\ln 2)=\frac{1}{2}\ln\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad I=\frac{1}{4}\ln\left(\frac{3}{2}\right)$$

44. Let $I=\int_1^4 \log_e[x]\,dx$

$$=\int_1^2 \log_e 1\,dx+\int_2^3 \log_e 2\,dx+\int_3^4 \log_e 3\,dx$$

$$=\log_e 2+\log_e 3=\log_e 6$$

45. We have,

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

The eccentricity e of the ellipse is given by

$$e=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$$

So, the coordinates of the foci are $(\pm\sqrt{7},0)$.

$\therefore$ Radius of the circle

$$=\sqrt{(\sqrt{7}-0)^2+(0-3)^2}$$
$$=\sqrt{7+9.}=\sqrt{16}=4$$

46. We have,

$$\lim_{x\to 2} f(x)=\lim_{x\to 2}\frac{\sin(e^{x-2}-1)}{\log(x-1)}$$

$$=\lim_{x\to 2}\left\{\frac{\sin(e^{x-2}-1)}{e^{x-2}-1}\cdot\frac{e^{x-2}-1}{x-2}\cdot\frac{x-2}{\log[1+(x-2)]}\right\}$$

$$=1\times 1\times 1=1$$

47. Since, tan A and tan B are the roots of the equation $x^2-ax+b=0$. Therefore,

$$\tan A+\tan B=a \text{ and } \tan A\tan B=b$$

Now, $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}=\dfrac{a}{1-b}$

Now, $\sin^2(A+B)=\dfrac{1}{2}[1-\cos 2(A+B)]$

$$\Rightarrow \quad \sin^2(A+B)=\frac{1}{2}\left\{1-\frac{1-\tan^2(A+B)}{1+\tan^2(A+B)}\right\}$$

$$\Rightarrow \quad \sin^2(A+B)=\frac{1}{2}\left\{1-\frac{1-\dfrac{a^2}{(1-b)^2}}{1+\dfrac{a^2}{(1-b)^2}}\right\}$$

$$\Rightarrow \quad \sin^2(A+B)=\frac{1}{2}\left\{\frac{2a^2}{a^2+(1-b)^2}\right\}$$

$$\Rightarrow \quad \sin^2(A+B)=\frac{a^2}{a^2+(1-b)^2}$$

48. We have, $a^2+b^2+c^2=ac+\sqrt{3}\,ab$

$$\Rightarrow \quad \frac{a^2}{4}+c^2-ac+\frac{3a^2}{4}+b^2-\sqrt{3}ab=0$$

$$\Rightarrow \quad \left(\frac{a}{2}-c\right)^2+\left(\frac{\sqrt{3}a}{2}-b\right)^2=0$$

$$\Rightarrow \quad \frac{a}{2}-c=0 \text{ and } \frac{\sqrt{3}}{2}a-b=0$$

$$\Rightarrow \quad a=2c \text{ and } \sqrt{3}a=2b$$

$$\Rightarrow \quad a=\frac{2b}{\sqrt{3}}=2c=\lambda \qquad \text{[say]}$$

$$\Rightarrow \quad a=\lambda,\ b=\frac{\sqrt{3}}{2}\lambda \text{ and } c=\frac{\lambda}{2}$$

Now, $b^2+c^2=\left(\dfrac{\sqrt{3}\lambda}{2}\right)^2+\left(\dfrac{\lambda}{2}\right)^2$

$$=\frac{3\lambda^2}{a}+\frac{\lambda^2}{4}=\lambda^2$$

$\therefore \quad b^2+c^2=a^2$

Hence, the triangle is right angled but not isosceles.

49. We have,

$$(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\,\mathbf{a}$$

$$\Rightarrow \quad (\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{b}\cdot\mathbf{c})\,\mathbf{a}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\,\mathbf{a}$$

$$\Rightarrow \quad (\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-\left\{(\mathbf{b}\cdot\mathbf{c})+\frac{1}{3}|\mathbf{b}||\mathbf{c}|\right\}\mathbf{a}=\mathbf{0}$$

$$\Rightarrow \quad \mathbf{a}\cdot\mathbf{c}=0 \text{ and } \mathbf{b}\cdot\mathbf{c}+\frac{1}{3}|\mathbf{b}||\mathbf{c}|=0$$

[$\because$ **a**, **b** are non-collinear]

$$\Rightarrow \quad |\mathbf{b}||\mathbf{c}|\cos\theta+\frac{1}{3}|\mathbf{b}||\mathbf{c}|=0$$

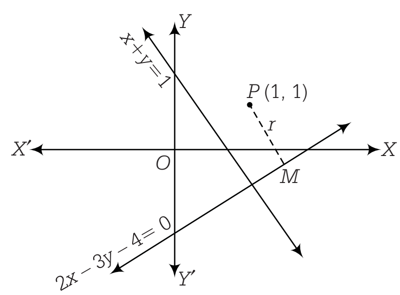
$$\Rightarrow \quad \cos\theta=-\frac{1}{3}$$

$$\Rightarrow \quad \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\left(-\frac{1}{3}\right)^2}$$

$$=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$$

50. The equation of a line through P (1, 1) and parallel to x + y = 1, is

$$\frac{x-1}{\cos\frac{3\pi}{4}}=\frac{y-1}{\sin\frac{3\pi}{4}}$$



Let PM = r.
Then, the coordinates of M given by

$$\frac{x-1}{\cos\frac{3\pi}{4}} = \frac{y-1}{\sin\frac{3\pi}{4}} = r \text{ is } \left(1-\frac{r}{\sqrt{2}}, 1+\frac{r}{\sqrt{2}}\right).$$

$\because$ M lies on $2x - 3y - 4 = 0$.

$\therefore \quad 2\left(1-\frac{r}{\sqrt{2}}\right) - 3\left(1+\frac{r}{\sqrt{2}}\right) - 4 = 0$

$\Rightarrow \quad 2 - \frac{2r}{\sqrt{2}} - 3 - \frac{3r}{\sqrt{2}} - 4 = 0$

$\Rightarrow \quad -5 - \frac{5r}{\sqrt{2}} = 0$

$\Rightarrow \quad r = -\sqrt{2}$

Hence, $PM = \sqrt{2}$

51. At the point where the given curve crosses X-axis, we have

$$y = 0$$

$\Rightarrow \quad ax^2 = 1$

[putting $y = 0$ in $ax^2 + 2hyx + by^2 = 1$]

$\Rightarrow \quad x = \pm\frac{1}{\sqrt{a}}$

Thus, the given curve cuts X-axis at $P\left(\frac{1}{\sqrt{a}}, 0\right)$ and $Q\left(-\frac{1}{\sqrt{a}}, 0\right)$.

Now, $ax^2 + 2hxy + by^2 = 1$

On differentiating both sides w.r.t. , we get

$2ax + 2h\left(x\frac{dy}{dx} + y\right) + 2by\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{ax + hy}{hx + by}$

$\Rightarrow \quad \left[\frac{dy}{dx}\right]_P = -\frac{a}{h}$ and $\left[\frac{dy}{dx}\right]_Q = -\frac{a}{h}$

$\Rightarrow \quad \left[\frac{dy}{dx}\right]_P = \left[\frac{dy}{dx}\right]_Q$

Hence, the tangents are parallel.

52. We have,

$$f(x) = xe^{-x}$$

On differentiating both sides w.r.t. , we get

$$f'(x) = e^{-x}(1-x)$$

Put $\quad f'(x) = 0$

$\Rightarrow \quad x = 1$

Now, $\quad f(0) = 0$

and $\quad \lim_{x\to\infty} f(x) = \lim_{x\to\infty} xe^{-x}$

$$= \lim_{x\to\infty}\frac{x}{e^x} = \lim_{x\to\infty}\frac{1}{e^x} = 0$$

Hence, the greatest value of $f(x)$ is $\frac{1}{e}$.

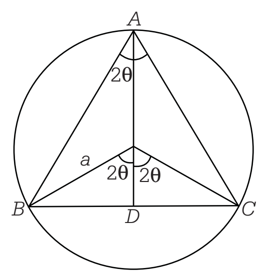
53. We have,

$AD = a + a\cos 2\theta$ and $BC = 2BD = 2a\sin 2\theta$

$\therefore$ Area of ΔABC is given by

$$\Delta = \frac{1}{2}BC \times AD = \frac{2}{2}a^2(\sin 2\theta + \sin 2\theta\cos 2\theta)$$

$\Rightarrow \quad \Delta = a^2\sin 2\theta + \frac{1}{2}a^2\sin 4\theta$



On differentiating both sides w.r.t. θ, we get

$$\frac{d\Delta}{d\theta} = 2a^2\cos 2\theta + 2a^2\cos 4\theta$$

For maximum or minimum, put $\frac{d\Delta}{d\theta} = 0$.

$\Rightarrow \quad \cos 2\theta = -\cos 4\theta$

$\Rightarrow \quad 2\theta = \pi - 4\theta$

$\Rightarrow \quad \theta = \frac{\pi}{6}$

Now, $\frac{d^2\Delta}{d\theta^2} = -2a^2\sin 2\theta \times 2 - 2a^2\sin 4\theta \times 4$

$$= -4a^2\sin 2\theta - 8a^2\sin 4\theta$$

At $\quad \theta = \frac{\pi}{6}, \frac{d^2\Delta}{d\theta^2} = -4a^2\sin\frac{\pi}{3} - 8a^2\sin\frac{2\pi}{3}$

$$= -4a^2 \times \frac{\sqrt{3}}{2} - 8a^2 \times \frac{\sqrt{3}}{2}$$

$$= -6a^2\sqrt{3} < 0$$

Hence, Δ is maximum for $\theta = \frac{\pi}{6}$.

54. Clearly, $f(x)$ is defined, if

$$25 - x^2 > 0$$

$$\Rightarrow \quad -5 < x < 5$$

Let $y = \log_5(25 - x^2)$, then

$$5^y = 25 - x^2$$

$$\Rightarrow \quad x^2 = 25 - 5^y$$

$$\Rightarrow \quad x = \pm\sqrt{25 - 5^y}$$

For x to be real, we must have

$$25 - 5^y \geq 0$$

$$\Rightarrow \quad 5^y \leq 25$$

$$\Rightarrow \quad y \leq 2$$

Also, $y = f(x) \to -\infty$ as $x \to \pm 5$.

Hence, range $(f) = (-\infty, 2]$

55. We know that, the image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$= \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

So, the image of the point $P(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$ is given by

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \frac{-2(2-3+4+3)}{4+1+1}$$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = -2$$

$$\Rightarrow \quad x = -3, y = 5, z = 2$$

Hence, the required point is $(-3, 5, 2)$.

56. The lines are parallel to the vectors $\mathbf{b}_1 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{b}_2 = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$.

$\therefore$ The plane normal to the vector $\mathbf{n}$ is

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

The required plane passes through $(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ and is normal to the vector $\mathbf{n}$.

$\therefore$ Its equation is

$$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$$

$$\Rightarrow \quad \mathbf{r} \cdot (-3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (-3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

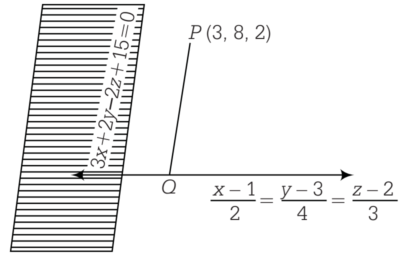
$$\Rightarrow \quad \mathbf{r} \cdot (-3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = -3 + 3$$

$$\Rightarrow \quad \mathbf{r} \cdot (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$$

57. Let $P(3, 8, 2)$ be the given point.

Let the equation of a line through $P(3, 8, 2)$ and parallel to the plane $3x + 2y - 2z + 15 = 0$ be

$$\frac{x-3}{a} = \frac{y-8}{b} = \frac{z-2}{c} \quad \ldots\text{(i)}$$



Then, normal to the plane is perpendicular to the parallel line.

Then, $\quad 3a + 2b - 2c = 0 \quad \ldots\text{(ii)}$

Line (i) intersects the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ at .

$$\therefore \quad \begin{vmatrix} 3-1 & 8-3 & 2-2 \\ a & b & c \\ 2 & 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 2 & 5 & 0 \\ a & b & c \\ 2 & 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2(3b - 4c) - 5(3a - 2c) = 0$$

$$\Rightarrow \quad 15a - 6b - 2c = 0 \quad \ldots\text{(iii)}$$

On solving Eqs. (ii) and (iii) by cross-multiplication, we get

$$\frac{a}{-4-12} = \frac{b}{-30+6} = \frac{c}{-18-30}$$

$$\Rightarrow \quad \frac{a}{-16} = \frac{b}{-24} = \frac{c}{-48} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$

Substituting a, b and c in Eq. (i), we get

$$\frac{x-3}{2} = \frac{y-8}{3} = \frac{z-2}{6} = \lambda \quad \text{[say]}$$

Let the coordinates of Q be $(2\lambda + 3, 3\lambda + 8, 6\lambda + 2)$.

It lies on the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$

$$\therefore \quad \lambda + 1 = \frac{3\lambda + 5}{4} = 2\lambda$$

$$\Rightarrow \quad \lambda = 1$$

$\therefore$ Coordinate of $Q = (2 + 3, 3 + 8, 6 + 2)$ i.e. $Q = (5, 11, 8)$.

Hence, $PQ = \sqrt{(5-3)^2 + (11-8)^2 + (8-2)^2}$

$$= \sqrt{4 + 9 + 36} = 7$$

58. By total probability theorem,

$$\text{Required probability} = \frac{1}{2}\times\left(\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right)$$
$$+\frac{1}{2}\left(\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right)$$
$$=\frac{1}{2}\left(\frac{3}{5}+\frac{1}{5}\right)+\frac{1}{2}\left(\frac{3}{10}+\frac{1}{30}+\frac{2}{5}\right)$$
$$=\frac{2}{5}+\frac{1}{2}\left(\frac{9+1+12}{30}\right)$$
$$=\frac{2}{3}+\frac{22}{60}=\frac{24+22}{60}$$
$$=\frac{46}{60}=\frac{23}{30}$$

59. We have,

$$\sum_{k=1}^{16} D_k = \begin{vmatrix} a & \left(\sum_{k=1}^{16} 2^k\right) & 2^{16}-1 \\ b & 3\left(\sum_{k=1}^{16} 4^4\right) & 2(4^{16}-1) \\ c & 7\left(\sum_{k=1}^{16} 8^k\right) & 4(8^{16}-1) \end{vmatrix}$$

$$\Rightarrow \sum_{k=1}^{16} D_k = \begin{vmatrix} a & 2\left(\frac{2^{16}-1}{2-1}\right) & 2^{16}-1 \\ b & 3\times 4\left(\frac{4^{16}-1}{4-1}\right) & 2(4^{16}-1) \\ c & 7\times 8\left(\frac{8^{16}-1}{8-1}\right) & 4(8^{16}-1) \end{vmatrix}$$

$$\Rightarrow \sum_{k=1}^{16} D_k = \begin{vmatrix} a & 2(2^{16}-1) & (2^{16}-1) \\ b & 4(4^{16}-1) & 2(4^{16}-1) \\ c & 8(8^{16}-1) & 4(8^{16}-1) \end{vmatrix}$$

$$\Rightarrow \sum_{k=1}^{16} D_k = 2\begin{vmatrix} a & (2^{16}-1) & (2^{16}-1) \\ b & 2(4^{16}-1) & 2(4^{16}-1) \\ c & 4(8^{16}-1) & 4(8^{16}-1) \end{vmatrix}$$

[taking 2 common from C_2]

$$\Rightarrow \sum_{k=1}^{16} D_k = 2\times 0 = 0$$

60. We have,

$$x^2-4x+4y^2=12$$
$$\Rightarrow \quad (x-2)^2+4(y-0)^2=8$$
$$\Rightarrow \quad \frac{(x-2)^2}{(2\sqrt{2})^2}+\frac{(y-0)^2}{(\sqrt{2})}=1$$

which is an ellipse whose major and minor axes are $2a = 2\sqrt{2}$ and $2b=\sqrt{2}$ repectively.

$\therefore$ Its eccentricity e is given by

$$e=\sqrt{1-\frac{b^2}{a^2}}$$
$$=\sqrt{1-\frac{2}{8}}=\frac{\sqrt{3}}{2}$$

61. Let $I=\int \frac{1}{\sin\left(x-\frac{\pi}{3}\right)\cos x}dx$

$$=\frac{1}{\cos\frac{\pi}{3}}\int\frac{\cos\left\{x-\left(x-\frac{\pi}{3}\right)\right\}}{\sin\left(x-\frac{\pi}{3}\right)\cos x}dx$$
$$=2\int\frac{\cos x\cos\left(x-\frac{\pi}{3}\right)+\sin x\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x-\frac{\pi}{3}\right)\cos x}dx$$
$$=2\int\left\{\cot\left(x-\frac{\pi}{3}\right)+\tan x\right\}dx$$
$$=2\left\{\log\left|\sin\left(x-\frac{\pi}{3}\right)\right|+\log|\sec x|\right\}+C$$
$$=2\log\left|\sin\left(x-\frac{\pi}{3}\right)\sec x\right|+C$$

62. The equation of any tangent to $y^2=8ax$ is

$$y=mx+\frac{2a}{m} \quad \ldots(i)$$

If it touches $x^2+y^2=2a^2$, then

$$\left(\frac{2a}{m}\right)^2=2a^2(1+m^2)$$

$[\because c^2=a^2(1+m^2)]$

$$\Rightarrow \quad 2=m^2(m^2+1)$$
$$\Rightarrow \quad m^4+m^2-2=0$$
$$\Rightarrow \quad (m^2+2)(m^2-1)=0$$
$$\Rightarrow \quad m^2-1=0$$
$$\Rightarrow \quad m=\pm 1$$

Putting the values of m in Eq. (i), we get $y=\pm(x+2a)$ as the equations of common tangents.

63. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

or $(2\lambda + 1)x + (3\lambda + 1)y + (1 - \lambda)z + 4\lambda - 1 = 0$ ...(i)

It is parallel to X-axis.

i.e. $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

$\therefore\ 1(2\lambda + 1) + 0 \times (3\lambda + 1) + 0(1 - \lambda) = 0$

$\Rightarrow \quad \lambda = -\frac{1}{2}$

On substituting $\lambda = -\frac{1}{2}$ in Eq. (i), we get

$$\left(2 \times -\frac{1}{2} + 1\right)x + \left(3 \times \frac{-1}{2} + 1\right)y + \left(1 + \frac{1}{2}\right) z + 4 \times \left(-\frac{1}{2}\right) - 1 = 0$$

$\Rightarrow \quad \frac{-1}{2}y + \frac{3}{2}z - 3 = 0$

$\Rightarrow \quad y - 3z + 6 = 0$

which is the equation of the required plane.

64. We have, $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $|\mathbf{a} \times \mathbf{b}| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4 + 4 + 1} = 3$

$\therefore \quad |(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| = |\mathbf{a} \times \mathbf{b}|\,|\mathbf{c}| \sin 30^\circ$

$= \frac{3}{2}|\mathbf{c}|$

Given, $|\mathbf{c} - \mathbf{a}| = 2\sqrt{2}$

$\Rightarrow \quad |\mathbf{c} - \mathbf{a}|^2 = 8$

$\Rightarrow \quad |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2(\mathbf{a} \cdot \mathbf{c}) = 8$

$\Rightarrow \quad |\mathbf{c}|^2 + 9 - 2|\mathbf{c}| = 8$

$\Rightarrow \quad |\mathbf{c}|^2 - 2|\mathbf{c}| + 1 = 0$

$\Rightarrow \quad |\mathbf{c}| = 1$

Hence, $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| = \frac{3}{2}$

65. The given differential equation can be written as

$$\left(\frac{d^2y}{dx^2}\right)^4 = \left\{y + \left(\frac{dy}{dx}\right)^2\right\}$$

Clearly, its order and degree are 2 and 4 respectively.

66. Let h be the height, r be the radius of the base and be the volume of the cone at time t. Then,

$$V = \frac{1}{3}\pi r^2 h$$

$\Rightarrow \quad V = \frac{2}{3}\pi r^3 \qquad [\because h = 2r, \text{ given}]$

On differentiating both sides w.r.t. , we get

$$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$$

$\Rightarrow \quad 40 = 2(10)^4 \frac{dr}{dt}$

$\left[\because \pi r^2 = 1\ m^2 = 10^4\ cm^2 \text{ and } \frac{dV}{dt} = 40\ cm^3/s, \text{ given}\right]$

$\Rightarrow \quad \frac{dr}{dt} = \frac{2}{1000}\ cm/s = 0.002\ cm/s$

67. Let S be the sum of the given series.

i.e. $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots$

$\Rightarrow \quad (S - 1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots$ ...(i)

$\Rightarrow \quad (S - 1) \times \frac{1}{3} = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$\frac{2}{3}(S - 1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots$$

$\Rightarrow \quad \frac{2}{3}(S - 1) = \frac{2}{3} + \frac{\frac{4}{3^2}}{1 - \frac{1}{3}}$

$\Rightarrow \quad \frac{2}{3}(S - 1) = \frac{2}{3} + \frac{2}{3}$

$\Rightarrow \quad S - 1 = 2$

$\Rightarrow \quad S = 3$

68. Let the equation of the line be

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad \dots(i)$$

which passes through (4, 3).

$\therefore \quad \frac{4}{a} + \frac{3}{b} = 1$ ...(ii)

It is given that $a + b = -1$ ...(iii)

On solving Eqs. (i) and (ii), we get

$$a = -2, b = 1 \text{ or } a = 2, b = -3$$

On substituting the values of a and b in Eq. (i), we get

$$\frac{x}{-2} + \frac{y}{1} = 1 \text{ and } \frac{x}{2} + \frac{y}{-3} = 1$$

69. Given points will be collinear, if

$$\begin{vmatrix} a & b & 1 \\ b & a & 1 \\ a^2 & -b^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} a & b & 1 \\ b-a & a-b & 0 \\ a^2-a & -b^2-b & 0 \end{vmatrix} = 0$$

[applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$\Rightarrow \quad (a-b)\begin{vmatrix} a & b & 1 \\ -1 & 1 & 0 \\ a^2-a & -b^2-b & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-b)(b^2+b-a^2+a) = 0$$

$$\Rightarrow \quad (a-b)\{(a+b)-(a^2-b^2)\} = 0$$

$$\Rightarrow \quad (a-b)(a+b)(1-a+b) = 0$$

$$\Rightarrow \quad a = b \text{ or } a+b = 0 \text{ or } a = 1+b$$

70. We have,

$$x^2 + 9 < (x+3)^2 < 8x + 25$$

$$\Rightarrow \quad x^2 + 9 < x^2 + 6x + 9 < 8x + 25$$

$$\Rightarrow \quad x^2 + 9 < x^2 + 6x + 9$$

and $\quad x^2 + 6x + 9 < 8x + 25$

$$\Rightarrow \quad 6x > 0 \text{ and } x^2 - 2x - 16 < 0$$

$$\Rightarrow \quad x > 0 \text{ and } 1 - \sqrt{17} < x < 1 + \sqrt{17}$$

$$\Rightarrow \quad 0 < x < 1 + \sqrt{17}$$

$$\therefore \quad x = 1, 2, 3, 4, 5$$