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MET 2014 Question Paper with Solution

Manipal Entrance Test (MET)

Solved Paper 2014

Manipal

Engineering Entrance Exam

Physics

1. The equation $\left(p + \frac{a}{V^2}\right)(V - b) = \text{constant}$. The unit of a is

(a) Dyne $\times$ cm^5 (b) Dyne $\times$ cm^4
(c) Dyne/cm^3 (d) Dyne/cm^2

2. If L, C and R denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is

(a) $[ML^{-2}T^{-1}I^0]$ (b) $[M^0L^0T^3I^0]$
(c) $[M^{-1}L^{-2}T^6I^2]$ (d) $[M^0L^0T^2I^0]$

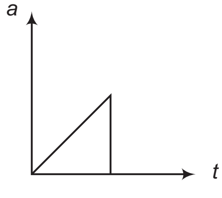
3. Number of particles is given by $n = -D\frac{n_2 - n_1}{x_2 - x_1}$ crossing a unit area perpendicular to X-axis is unit time, where n_1 and n_2 are number of particles per unit volume for the value of x meant to x_2 and x_1. Find dimensions of D called as diffusion constant.

(a) $[M^0LT^2]$ (b) $[M^0L^2T^{-4}]$ (c) $[M^0LT^{-3}]$ (d) $[M^0L^2T^{-1}]$

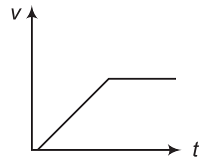
4. A body starts from the origin and moves along the x-axis such that velocity at any instant is given by $(4t^3 - 2t)$, where t is in second and velocity is in m/s. What is the acceleration of the particle, when it is $2m$ from the origin?

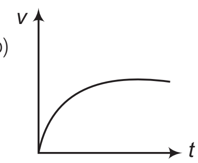
(a) 28 m/s^2 (b) 22 m/s^2
(c) 12 m/s^2 (d) 10 m/s^2

5. The acceleration-time graph of a body is shown below



The most probable velocity-time graph of the body is

(a)


(b)


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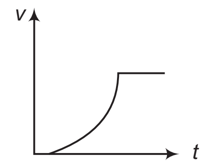

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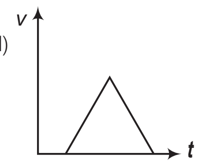


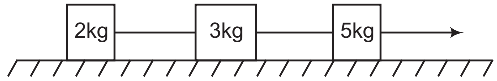
6. Two seconds after projection, a projectile is travelling in a direction inclined at 30° to the horizontal. After one more sec, it is travelling horizontally, the magnitude and direction of its velocity are

(a) $2\sqrt{20}$ m/s, 60° (b) $20\sqrt{3}$ m/s, 60°
(c) $6\sqrt{40}$ m/s, 30° (d) $40\sqrt{6}$ m/s, 30°

7. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered. It is given as KE $= as^2$, where a is a constant. The force acting on the particle is

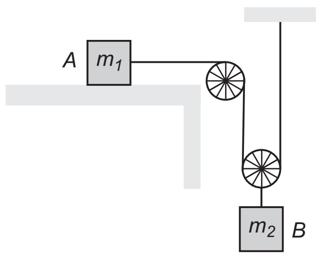
(a) $2a\frac{s^2}{R}$ (b) $2as\left(1 + \frac{s^2}{R^2}\right)^{1/2}$ (c) $2as$ (d) $2a\frac{R^2}{s}$

8. Three blocks of masses 2 kg, 3 kg and 5 kg are connected to each other with light string and are then placed on a frictionless surface as shown in the figure. The system is pulled by a force $F = 10$ N, then tension $T_1 =$



(a) 1 N (b) 5 N (c) 8 N (d) 10 N

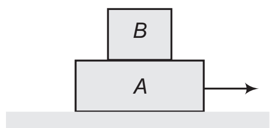
9. The acceleration of block B in the figure will be



(a) $\dfrac{m_2 g}{(4m_1 + m_2)}$ (b) $\dfrac{2m_2 g}{(4m_1 + m_2)}$

(c) $\dfrac{2m_1 g}{(m_1 + 4m_2)}$ (d) $\dfrac{2m_1 g}{(m_1 + m_2)}$

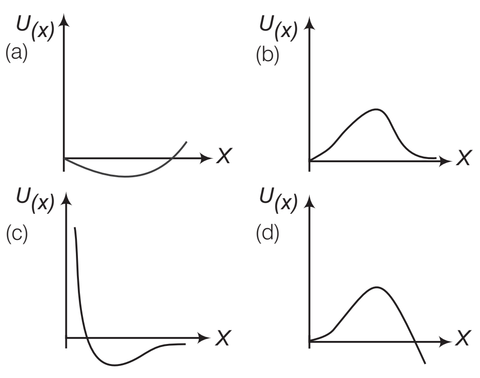
10. A body A of mass 1 kg rests on a smooth surface. Another body B of mass 0.2 kg is placed over A as shown. The coefficient of static friction between A and B is 0.15. B will begin to slide on A, if A is pulled with a force greater than



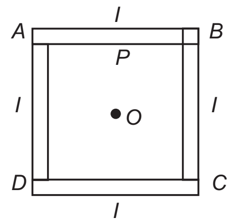
(a) 1.764 N (b) 0.1764 N

(c) 0.3 N (d) It will not slide for any F

11. A particle which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin is $F(x) = -kx + ax^3$. Here, k and a are positive constants. For $x \geq 0$, the functional from of the potential energy $U_{(x)}$ of the particle is



12. Four thin rods of same mass M and same length l, form a square as shown in figure. Moment of inertia of this system about an axis through centre O and perpendicular to its plane is



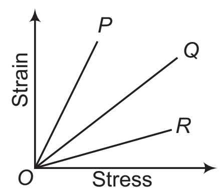
(a) $\dfrac{4}{3}Ml^2$ (b) $\dfrac{Ml^2}{3}$ (c) $\dfrac{Ml^2}{6}$ (d) $\dfrac{2}{3}Ml^2$

13. A wheel is rotating with an angular speed of 20 rad/s. It is stopped to rest by applying a constant torque in 4s. If the moment of inertia of the wheel about its axis is 0.20 kg/m^2, then the work done by the torque in two seconds will be

(a) 10 J (b) 20 J

(c) 30 J (d) 40 J

14. The strain stress curves of three wires of different materials are shown in the figure. P, Q and R are the elastic limits of the wires. The figure shows that



(a) elasticity of wire P is maximum

(b) elasticity of wire Q is maximum

(c) tensile strength of R is maximum

(d) None of the above is true

15. Several spherical drops of a liquid of radius r coalesce to form a single drop to radius R. If T is surface tension and V is volume under consideration, then the release of energy is

(a) $3VT\left(\dfrac{1}{r} + \dfrac{1}{R}\right)$ (b) $3VT\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

(c) $VT\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$ (d) $VT\left(\dfrac{1}{r^2} + \dfrac{1}{R^2}\right)$

16. Two substances of densities ρ_1 and ρ_2 are mixed in equal volume and the relative density of mixture is 4. When they are mixed in equal masses, the relative density of the mixture is 3. The values of ρ_1 and ρ_2 are

(a) $\rho_1 = 6$ and $\rho_2 = 2$ (b) $\rho_1 = 3$ and $\rho_2 = 5$
(c) $\rho_1 = 12$ and $\rho_2 = 4$ (d) None of these

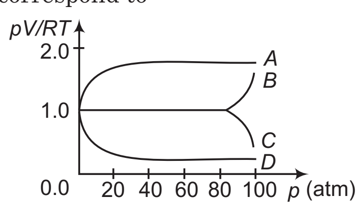
17. The pressure p, volume V and temperature T of a gas in the jar A and the other gas in the jar B at pressure $2p$, volume $V/4$ and temperature $2T$, then the ratio of the number of molecules in the jar A and B will be

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

18. Two identical containers each of volume V_0 are joined by a small pipe. The containers contain identical gases at temperature T_0 and pressure P_0. One container is heated to temperature $2T_0$ while maintaining the other at the same temperature. The common pressure of the gas is P and n is the number of moles of gas in container at temperature $2T_0$

(a) $p = 2p_0$ (b) $p = \frac{4}{3}p_0$
(c) $n = \frac{2}{3}\frac{p_0V_0}{RT_0}$ (d) $n = \frac{3}{2}\frac{p_0V_0}{RT_0}$

19. An experiment is carried on a fixed amount of gas at different temperature and at high pressure such that it deviates from the ideal gas behaviour. The variation of $\frac{pV}{RT}$ with p is shown in the diagram. The correct variation will correspond to

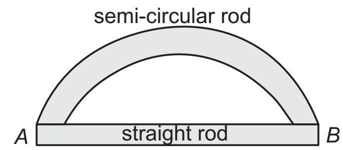


(a) Curve A (b) Curve B
(c) Curve C (d) Curve D

20. Liquid is filled in a flask upto a certain point. When, the flask is heated, the level of the liquid

(a) immediately starts increasing
(b) initially falls and then rises
(c) rises abruptly
(d) falls abruptly

21. Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined as shown in the figure. The points A and B are maintained at different temperature. The ratio of the heat transferred through a cross-section of a semi-circular rod to the heat transferred through a cross-section of the straight rod in a given time is

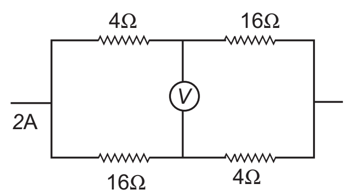


(a) $2 : \pi$ (b) 1 : 2
(c) $\pi : 2$ (d) 3 : 2

22. Two particles executes SHM of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions. Each time their displacement is half of their amplitude. The phase difference between them is

(a) 30° (b) 60°
(c) 90° (d) 120°

23. In the following circuit reading of voltmeter V is



(a) 12 V (b) 8 V
(c) 20 V (d) 16 V

24. The amount of work done in increasing the voltage across the plates of a capacitor from 5V to 10V is W. The work done in increasing it from 10 V to 5 V will be

(a) 0.6 W
(b) 1 W
(c) 1.25 W
(d) 1.67 W

25. The de-Broglie wavelength of a neutron at 27°C is λ. What will be its wavelength at 927°C?

(a) $\lambda/2$ (b) $\lambda/3$
(c) $\lambda/4$ (d) $\lambda/9$

26. The minimum intensity of light to be detected by human eye is 10^{-10} W/m^2. The number of photons of wavelength 5.6×10^{-7} m entering the eye, with pupil area 10^{-6} m^2, per second for vision will be nearly

(a) 100 (b) 200
(c) 300 (d) 400

27. In a photoemissive cell with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed by $3\lambda/4$, the speed of the fastest emitted electron will be

(a) $v(3/4)^{1/2}$
(b) $v(4/3)^{1/2}$
(c) less than $v(4/3)^{1/2}$
(d) greater than $v(4/3)^{1/2}$

28. The ratio of the wavelength for $2 \to 1$ transition in Li^+, He^+ and H is

(a) 1 : 2 : 3 (b) 1 : 4 : 9
(c) 4 : 9 : 36 (d) 3 : 2 : 1

29. The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

30. Nucleus of mass number A, originally at rest, emits the α-particle with speed v. The daughter nucleus recoils with a speed of

(a) $2v/(A+4)$ (b) $4v/(A+4)$
(c) $4v/(A-4)$ (d) $2v/(A-4)$

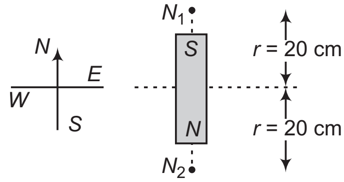
31. When $_{90}Th^{228}$ transforms to $_{83}Bi^{212}$, then the number to the emitted α and β-particles is, respectively

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$
(c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

32. Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 MeV (b) 7.88 MeV
(c) 126.15 MeV (d) 788 MeV

33. A short bar magnet is placed with its south pole towards geographical north. The neutral points are situated at a distance of 20 cm from the centre of the magnet. If $B_H = 0.3 \times 10^{-4}$ Wb/m^2 then the magnetic moment of the magnet is



(a) 9000ab – amp × cm^2 (b) 900ab – amp × cm^2
(c) 1200ab – amp × cm^2 (d) 225ab – amp × cm^2

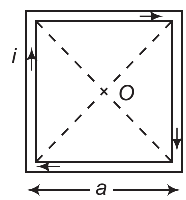
34. A magnet makes 40 oscillations per minute at a place having magnetic field intensity $B_H = 0.1 \times 10^{-5}$. At another place, it takes 2.5 s to complete one vibration. The value of the earth's horizontal field at the place

(a) 0.25×10^{-6} T (b) 0.36×10^{-6} T
(c) 0.66×10^{-8} T (d) 1.2×10^{-6} T

35. The coereivity of a small bar magnet is 4×10^3 A/m. It is inserted inside a solenoid of 500 turns and length 1 m to demagnetise it. The amount of current to be passed through the solenoid will be

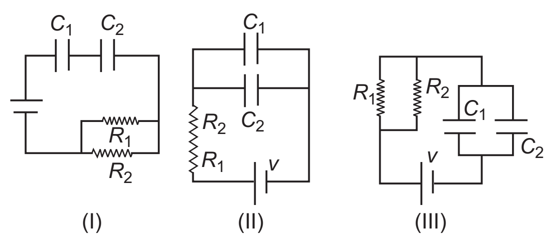
(a) 2.5 A (b) 5 A (c) 8 A (d) 10 A

36. A wire in the form of a square of side a carries a current i. Then, the magnetic induction at the centre of the square wire is (magnetic permeability of free space $= \mu_0$).



(a) $\frac{\mu_0 i}{2\pi a}$ (b) $\frac{\mu_0 i\sqrt{2}}{\pi a}$ (c) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$ (d) $\frac{\mu_0 i}{\sqrt{2}\pi a}$

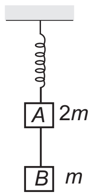
37. Given,
$R_1 = 1\Omega$ $C_1 = 2\mu F$
$R_2 = 2\Omega$ $C_2 = 4\mu F$



The time constant (in μs) for the circuits I, II, III are respectively

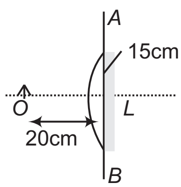
(a) 18, 8/9, 4 (b) 18, 4, 8/9
(c) 4, 8/9, 18 (d) 8/9, 18, 4

38. Two blocks A and B of masses $2m$ and m, respectively, are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the figure. The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively



(a) $g, g/2$ (b) $g/2, g$
(c) g, g (d) $g/2, g/2$

39. A point object is placed at a distance of 20 cm from a thin plano-convex lens of focal length 15 cm, if the plane surface is silvered. The image will form at



(a) 60 cm left of AB
(b) 30 cm left of AB
(c) 12 cm left of AB
(d) 60 cm right of AB

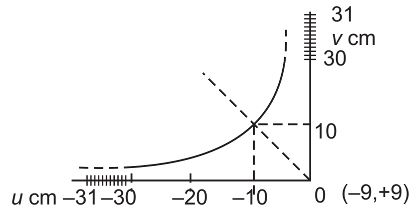
40. A biconvex lens of focal length f forms a circular image of sun of radius r in focal plane. Then,

(a) $\pi r^2 \propto f$
(b) $\pi r^2 \propto f^2$
(c) if lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
(d) if f is doubled, intensity will increase

41. Given a sample of radium-226 having half-life of 4 days. Find, the probability, a nucleus disintegrates after 2 half lives.

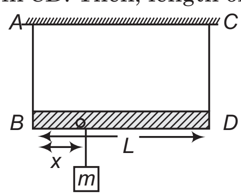
(a) 1 (b) 1/2
(c) 1.5 (d) 3/4

42. Graph of position of image *versus* position of point object from a convex lens is shown. Then, focal length of the lens is



(a) 0.50 ± 0.05 cm (b) 0.50 ± 0.10 cm
(c) 5.00 ± 0.05 cm (d) 5.00 ± 0.10 cm

43. A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further, it is observed that the frequency of 1st harmonic (fundamental frequency) in AB is equal to 2nd harmonic frequency in CD. Then, length of BO is



(a) $L/5$ (b) $4L/5$
(c) $3L/4$ (d) $L/4$

44. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then

(a) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$
(b) $T_A > T_B$ (if $r_A > r_B$)
(c) $T_A > T_B$ (if $m_A > m_B$)
(d) $T_A = T_B$

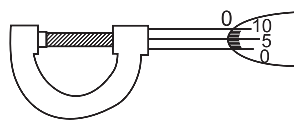
45. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

(a) $\frac{2R}{\sqrt{15}}$ (b) $R\sqrt{\frac{2}{15}}$
(c) $\frac{4R}{\sqrt{15}}$ (d) $\frac{R}{4}$

46. A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$, $l \approx 1$ m, and he commits an error of Δl. For T he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 s. For which of the following data, the measurement of g will be most accurate?

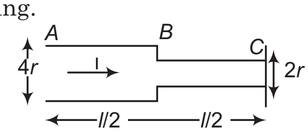
	Δl	ΔT	n	Amplitude of oscillation
(a)	5 mm	0.2 s	10	5 mm
(b)	5 mm	0.2 s	20	5 mm
(c)	5 mm	0.1 s	20	1 mm
(d)	1 mm	0.1 s	50	1 mm

47. The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is



(a) 2.25 mm (b) 2.20 mm
(c) 1.20 mm (d) 1.25 mm

48. Consider the cylindrical element as shown in the figure. Current flowing the through element is I and resistivity of material of the cylinder is ρ. Choose the correct option out the following.



(a) Power loss in first half is four times the power loss in second half
(b) Voltage drop in first half is twice of voltage drop in second half
(c) Current density in both halves are equal
(d) Electric field in both halves is equal

49. Ideal gas is contained in a thermally insulated and rigid container and it is heated through a resistance 100 Ω by passing a current of 1 A for five minutes, then change in internal energy of the gas is

(a) 0 kJ (b) 30 kJ
(c) 10 kJ (d) 20 kJ

50. Two litre of water at initial temperature of 27°C is heated by a heater of power 1 kW. If the lid of kettle is opened, then heat is lost at the constant rate of 160 J/s. Find, the time required to raise the temperature of water to 77°C with the lid open (specific heat of water 4.2 kJ/kg)

(a) 5 min 40 s
(b) 14 min 20 s
(c) 8 min 20 s
(d) 16 min 10 s

51. An open organ pipe resonated with frequency f_1 and 2nd harmonic. Now, one end is closed and the frequency is slowly increased, then it resonates with frequency f_2 and nth harmonic then,

(a) $n = 3, f_2 = \frac{3}{4}f_1$ (b) $n = 5, f_2 = \frac{3}{4}f_1$
(c) $n = 3, f_2 = \frac{5}{4}f_1$ (d) $n = 5, f_2 = \frac{5}{4}f_1$

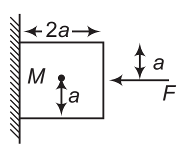
52. Which of the following process does not occur through convection

(a) boiling of water
(b) land breeze and sea breeze
(c) circulation of air around furnace
(d) heating of glass bulb through filament

53. T_1 is the time period of simple pendulum. The point of suspension moves vertically upwards according to $y = kt^2$, where $k = 1\,\text{m/s}^2$. New time period is T_2, then $\frac{T_1^2}{T_2^2} = ?\ (g = 10\,\text{m/s}^2)$

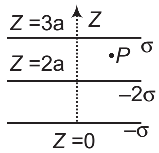
(a) 4/5 (b) 6/5
(c) 5/6 (d) 1

54. In the figure shown, a cubical block is held stationary against a rough wall by applying force 'F', then incorrect statement among the following is



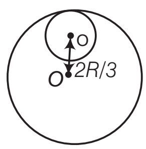
(a) frictional force $f = mg$
(b) $F = N$, N is normal reaction
(c) F does not apply any torque
(d) N does not apply any torque

55. Three infinitely charged sheets are kept parallel to x-y plane having charge densities as shown. Then, the value of electric field at 'P' is



(a) $\frac{-4\sigma}{\varepsilon_0}\hat{k}$ (b) $\frac{4\sigma}{\varepsilon_0}\hat{k}$
(c) $\frac{-2\sigma}{\varepsilon_0}\hat{k}$ (d) $\frac{2\sigma}{\varepsilon_0}\hat{k}$

56. A circular disc of radius $\frac{R}{3}$ is cut from a circular disc of radius R and mass $9M$ as shown. Then, moment of inertia of remaining disc about 'O' perpendicular to the plane of the disc is



(a) $4\,MR^2$ (b) $9\,MR^2$
(c) $\frac{37}{9}MR^2$ (d) $\frac{40}{9}MR^2$

57. A cylindrical conducting rod is kept with its axis along positive z-axis, where a uniform magnetic field exists parallel to z-axis. The current induced in the cylinder is

(a) zero
(b) clockwise as seen from $+z$-axis
(c) anti-clockwise as seen from $+z$-axis
(d) opposite to the direction of magnetic field

58. Ratio of area of hole to beaker is 0.1. Height of liquid in beaker is 3m, and hole is at the height of 52.5 cm from the bottom of beaker, find the square of the velocity of liquid coming out from the hole

(a) $50\ (\text{m/s})^2$ (b) $50.5\ (\text{m/s})^2$
(c) $51\ (\text{m/s})^2$ (d) $42\ (\text{m/s})^2$

59. Temperature of a gas is 20°C and pressure is changed from 1.01×10^5 Pa to 1.165×10^5 Pa. If volume is decreased isothermally by 10%. Bulk modulus of gas is

(a) 1.55×10^5 (b) 0.155×10^5
(c) 1.4×10^5 (d) 1.01×10^5

60. The atomic number (Z) of an element whose k_a wavelength is λ is 11. The atomic number of an element whose k_a, wavelength is 4λ is equal to

(a) 6 (b) 11 (c) 44 (d) 4

Chemistry

1. Which of the following compounds will not react with Mg metal in the presence of ether to give a Grignard reagent?

(a) $CH_2{=\!=}CHBr$
(b) CH_3CH_2I
(c) $CH_2OH\ CH_2Br$
(d) $CH_3OCH_2CH_2I$

2. How many enols (including stereo isomers) exist for 3-hexanone?

(a) Two (b) Three
(c) Four (d) Six

3. Which of the following processes does not yield a primary amine?

(a) $CH_3COCH_3 + NH_3 + H_2 \xrightarrow{Ni}$
(b) $CH_3CH_2CN + H_2 \xrightarrow[140°C]{Ni}$
(c) $CH_3CHO + H_2NOH \xrightarrow{Na,\ C_2H_5OH}$
(d) $CH_3CHO + N_2CH_2 \xrightarrow{NaBH_4}$

4. An aqueous solution of phosphoric acid, H_3PO_4 being titrated has molarity equal to 0.25 M. Which of the following could be normality of this solution?

(a) 0.25 N (b) 0.50 N
(c) 0.75 N (d) All of these

5. For the reaction represented by the equation,

$$CX_4 + 2O_2 \longrightarrow CO_2 + 2X_2O$$

9.0 g of CX_4 completely reacts with 1.74 g of oxygen. The approximate molar mass of X will be

(a) 20 (b) 40
(c) 60 (d) 80

6. Which of the following species contain an element in an oxidation state that is not a whole number?

(a) VO_4^{3-} (b) Mn_2O_3 (c) $S_4O_6^{2-}$ (d) Cl_2O_7

7. Gold is extracted by hydrometallurgical process, based on its property

(a) of being electropositive
(b) of being less reactive
(c) to form complexes which are water soluble
(d) to form salts which are water soluble

8. Due to lanthanide contraction

(a) Fe, Co, Ni have equal size
(b) Zr and Hf have equal size
(c) all *f*-block ions have equal size
(d) all isoelectronic ions have equal size

9. The element which forms ions in dimeric state is

(a) mercury (b) nickel (c) iron (d) cadmium

10. The high oxidising power of fluorine in due to its

(a) high electron affinity
(b) high heat of dissociation and low heat of hydration
(c) low heat of dissociation and high heat of hydration
(d) high heat of dissociation and high heat of hydration

11. Which one of the following is the weakest acid?

(a) HF (b) HCl (c) HBr (d) HI

12. Find the pressure of a sample of CCl_4 if 1.0 mole occupies 35L at 77°C. Assume CCl_4 obeys the van der Waals equation of state. ($a = 20\ L^2\,mol^{-2}$ atm and $b = 0.14\ L\,mol^{-1}$)

(a) 0.56 atm (b) 0.572 atm
(c) 0.82 atm (d) 0.807 atm

13. KBr crystallises in NaCl type of crystal lattice and its density is 2.75 g/cm^3. Number of unit cells of KBr present in a 1.00 mm^3 grain of KBr are

(a) $3 \cdot 6 \times 10^{18}$ (b) $3 \cdot 6 \times 10^{16}$
(c) $2 \cdot 34 \times 10^{23}$ (d) $6 \cdot 02 \times 10^{24}$

14. A body centre cubic lattice is made up of two different types of atoms *A* and *B*. Atom *A* occupies the body centre and *B* occupying the corner positions. One of the corners is left unoccupied per unit cell. Empirical formula of such a solid is

(a) AB (b) A_2B_3 (c) A_5B_7 (d) A_8B_7

15. A substance is carried out through the following transformations

$$\begin{tikzcd} A \arrow[r, "\Delta S_1"] \arrow[d, dash, "\Delta S_4"'] & B \arrow[r, "\Delta S_2"] & C \arrow[d, dash, "\Delta S_3"] \\ D \arrow[rr, dash, "\Delta S_5"'] & & E \end{tikzcd}$$

The equation $\Delta S_1 + \Delta S_2 = \Delta S_3 + \Delta S_4 + \Delta S_5$

(a) is true only if the steps are carried out reversibly
(b) is always true because entropy is a state function
(c) may be true but need more informations on the processes
(d) is incorrect

16. At 298 K the entropy of rhombic sulphur is 32.04 J/mol K and that of monoclinic sulphur is 32.68 J/mol K. The heat of their combustion are respectively–298246 and –297948 J mol^{-1}. ΔG for the reaction; $S_{\text{rhombic}} \longrightarrow S_{\text{monoclinic}}$ will be

(a) 107. 28 J (b) 10. 728 J
(c) 107. 28 kJ (d) 10728 J

17. An atomic orbital has two angular nodes and one radial node. It is a

(a) 2*p* orbital (b) 3*p* orbital
(c) 3*d* orbital (d) 4*d* orbital

18. Which of the following sets of quantum numbers could represent the last electron added to complete the electron configuration for a ground state atom of Br($Z = 35$) according to the Aufbau principle, n ; l ; m_l ; m_s

(a) $4 ; 0 ; 0 ; -\frac{1}{2}$ (b) $4 ; 1 ; 1 ; -\frac{1}{2}$
(c) $3 ; 1 ; 1 ; -\frac{1}{2}$ (d) $4 ; 1 ; 2 ; +\frac{1}{2}$

19. Order the species CF^+, CF and CF^- according to increasing C—F bond length

(a) $CF^+ < CF < CF^-$ (b) $CF^+ < CF^- < CF$
(c) $CF^- < CF < CF^+$ (d) $CF < CF^- < CF^+$

20. When the substances CCl_4, CH_4 and CF_4 are arranged is order of increasing boiling point, what is the correct order?

(a) CCl_4, CF_4, CH_4
(b) CH_4, CF_4, CCl_4
(c) CF_4, CCl_4, CH_4
(d) CF_4, CH_4, CCl_4

21. The nitrite ion in represented by

(I) $[O - N = O]^-$

(II) $[N = O - O]^-$

Which of the structures represents possible resonance forms of this ion?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I and II

22. Equal moles of $NO_2(g)$ and $NO(g)$ are to be placed in a container to produce N_2O according to the reaction,

$$NO_2 + NO \rightleftharpoons N_2O + O_2;\ K_C = 0.914$$

How many moles of NO_2 and NO be placed in the 5.0 L container to have an equilibrium concentration of N_2O to be 0.05 M?

(a) 0.5115 (b) 0.1023 (c) 0.0526 (d) 0.2046

23. The rate of a chemical reaction involving liquids only will not be influenced by the

(a) temperature of the system
(b) number of reacting particles per mL
(c) stability of the bonds
(d) pressure of the system

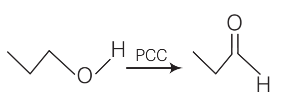
24. Consider the following reaction in aqueous solution.

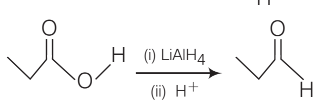
$$5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq) \longrightarrow 3Br_2(aq) + 3H_2O(l)$$

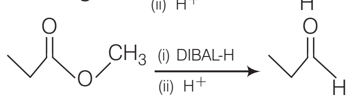
If the rate of appearance of Br_2 at a particular moment during the reaction is 0.025 Ms^{-1}, what is the rate of disappearance (in Ms^{-1}) of Br^- at that moment?

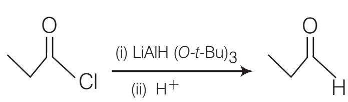
(a) 0.025 ms^{-1} (b) 0.042 ms^{-1}
(c) 0.075 ms^{-1} (d) 0.125 ms^{-1}

25. Shown below are four possible synthesis of propionaldehyde. Which one would not work?

(a) 

(b) 

(c) 

(d) 

26. The compound $C_6H_5CH = N - N = CHC_6H_5$ is produced by the reaction of an axcess of benzaldehyde with which compound?

(a) NH_3 (b) NH_2NH_2
(c) Hydroxyl amine (d) Phenyl hydrazine

27. Which of the following statements is not true for acetaldehyde?

(a) It will undergo an aldol condensation reaction.
(b) It will undergo a Cannizzaro reaction.
(c) It will undergo a haloform reaction
(d) Enolisation is catalysed by acid or base

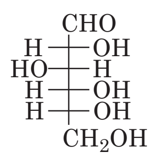
28. Which of the following compounds does not undergo mutarotation?

(a) Glucose (b) Sucrose
(c) Ribose (d) Fructose

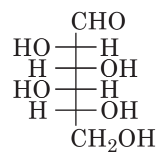
29. If two isomers have been classified correctly as anomers, they may be also called

(a) conformers (b) enantiomers
(c) tautomers (d) diastereomers

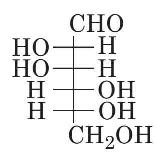
30. Which two of the following aldohexoses give the same osazone derivative?



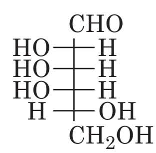
I.



II.



III.



IV.

(a) I and IV (b) I and III
(c) II and III (d) III and IV

31. Which of the following describes the overall three dimensional folding of a polypeptide?

(a) Primary structure (b) Secondary structure
(c) Tertiary structure (d) Quaternary structure

32. What is incorrect regarding natural rubber?

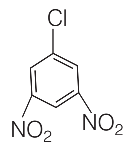
(a) It is *cis*-polyisoprene
(b) It is amorphous in nature
(c) It has syndiotactic stereochemistry
(d) Molecules of rubber have coiled conformation

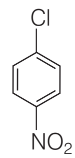
33. Which one of the following is the structure of polyacrylonitrile?

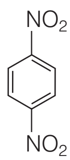
(a) $\left[CH_2 - CH(Cl) \right]_n$ (b) $\left[CH_2 - CH(CN) \right]_n$

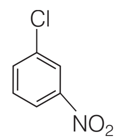
(c) $\left[CH_2 - C(Cl)_2 \right]_n$ (d) $\left[CH_2 - C(CN)(COOCH_3) \right]_n$

34. Which of the aromatic compounds react fastest with methoxide ion?

(a) 

(b) 

(c) 

(d) 

35. Which reagents would you use to carry out the reaction

Ethyl benzene $\longrightarrow$ 2 and 4- chloro 1- ethyl benzene?

(a) Cl_2, light and heat

(b) Cl_2, $FeCl_3$

(c) $SOCl_2$

(d) C_2H_5Cl, $AlCl_3$

36. What represents the best method for producing salicylic acid?

(a) Benzoic acid, strong base, high temperature

(b) Toluene, strong base, water, high temperature

(c) Phenol, base, carbondioxide then acid

(d) Bromobenzene, carbonic acid, high temperature

37. Which pair is usually required for a nucleophilic aromatic substitution reaction?

(a) An — NO_2 substitution and a strong electrophile

(b) A ring bearing a strong activating group and a strong acid

(c) An aryl halide with an — NO_2 and a strong nucleophile

(d) An unsubstituted benzene ring and a strong electrophile

38. What reaction does hydroquinone undergo most readily?

(a) S_N2 substitution (b) Hydrolysis

(c) Oxidation (d) Reduction

39. In physical adsorption, the adsorbent and adsorbate are held together by the

(a) ionic forces (b) covalent forces

(c) van der Waals' forces (d) H-bonding

40. Potassium stearate is obtained by the saponification of an oil or fat. It has formula $CH_3 - (CH_2)_{16} - COOK$. The molecule has a lyophobic terminal CH_3 and a lyophilic terminal — COOK. Potassium stearate is an example of

(a) lyophilic colloid

(b) lyophobic colloid

(c) macromolecular colloid

(d) micelle

41. Suppose that gold is being plated on to another metal in an electrolytic cell. The half-cell reaction producing the Au(s) is $AuCl_4^- \rightarrow Au(s) + 4Cl^- - 3e^-$

If a 0.30 A current runs for 15.00 min, what mass of Au(s) will be plated, assuming all the electrons are used in the reduction of $AuCl_4^-$? The Faraday constant is 96485 C/mol and molar mass of Au is 197.

(a) 0.184 g Au (b) 0.551 g Au

(c) 1.84 g Au (d) 0.613 g Au

42. What is the cell potential (standard emf, E^0) for the reaction below?

$[E^0(Fe^{2+}(aq)Fe) = 0.44 V$ and

$E^0(O_2(g)/H_2O/OH^-) = +0.4 V]$

$$2Fe(s) + O_2(g) + 2H_2O(l) \rightleftharpoons 2Fe^{2+}(aq) + 4OH^-(aq)$$

(a) $E^0_{cell} = -0.48$ V (b) $E^0_{cell} = -0.04$ V

(c) $E^0_{cell} = +0.84$ V (d) $E^0_{cell} = +1.28$ V

43. If in a solvent, n simple molecules of solute combine to form an associated molecule, α is the degree of association, the van't Hoff's factor is equal to

(a) $\dfrac{1}{1 - n\alpha}$ (b) $1 - \alpha + n\alpha$

(c) $1 - \alpha + \dfrac{\alpha}{n}$ (d) $\dfrac{\alpha}{n} - 1 + \alpha$

44. If the osmotic pressure of a 0.010 M aqueous solution of sucrose at 27°C is 0.25 atm, then the osmotic pressure of a 0.010 M aqueous solution of NaCl at 27°C is

(a) 0.062 atm (b) 0.12 atm
(c) 0.25 atm (d) 0.50 atm

45. A certain radioactive isotope ${}^{A}_{Z}X$ (half-life = 10 days) decays to give ${}^{A-4}_{Z-2}X$. If 1.0 g of X is kept in a sealed vessel find the volume of He accumulated at STP in 20 days. Molar mass of ${}^{A}_{Z}X = 253$ u

(a) 22.40 mL (b) 33.40 mL
(c) 55.40 mL (d) 66.40 mL

46. Raw milk sours in about 4 h at 27°C, but in about 48 h in a refrigerator at 17°C. What is the activation energy for souring of milk?

(a) 78.3 kJ mol^{-1}
(b) 46.21 kJ mol^{-1}
(c) 23.5 kJ mol^{-1}
(d) 80.8 kJ mol^{-1}

47. If 200 mL of a 0.031 molar solution of H_2SO_4 are added to 84 mL of a 0.150 M KOH solution, what is the pH of the resulting solution?

(a) 12.4 (b) 1.7
(c) 2.2 (d) 10.9

48. $(SiH_3)_3N$ is

(a) planar (b) pyramidal
(c) octahedral (d) tetrahedral

49. Solid N_2O_5 is

(a) ionic (b) covalent
(c) coordinate covalent (d) metallic

50. Which of the following phosphorus is the most reactive?

(a) Red phosphorus
(b) White phosphorus
(c) Scarlet phosphorus
(d) Violet phosphorus

51. The activity of alkaline earth metals as reducing agent

(a) decreases from Be to Ba
(b) increases from Be to Ba
(c) increases from Be to Ca and decreases from Ca to Ba
(d) decreases from Be to Ca and increases from Ca to Ba

52. Baking powder is more commonly used to make cakes or bread "rise". Filler in baking powder is

(a) $NaHCO_3$ (b) $Ca(H_2PO_4)_2$
(c) starch (d) $NaAl(SO_4)_2$

53. Strongest conjugate base is

(a) ClO^- (b) ClO_2^- (c) ClO_3^- (d) ClO_4^-

54. Out of the following redox reactions

I. $NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$

II. $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

III. $PCl_5 \xrightarrow{\Delta} PCl_3 + Cl_2$

disproportionation is not shown in

(a) I and II (b) II and III
(c) I and III (d) I, II and III

55. The complex $[Fe(H_2O)_5NO]^{2+}$ is formed in the ring test for nitrate ion when freshly prepared $FeSO_4$ solution is added to aqueous solution of NO_3^- followed by addition of Conc. H_2SO_4. This complex is formed by charge transfer in which

(a) Fe^{2+} changes to Fe^{3+} and NO^+ changes to NO
(b) Fe^{2+} changes to Fe^{3+} and NO changes to NO^+
(c) Fe^{2+} changes to Fe^+ and NO changes to NO^+
(d) No charge transfer takes place

56. Which pair is different from the others?

(a) Li — Mg (b) B — Al
(c) Na — K (d) Ca — Mg

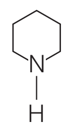
57. The dominant factor in determining the IE of the elements on moving down the groups is its

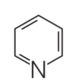
(a) atomic radius (b) effective nuclear charge
(c) Both (a) and (b) (d) None of the above

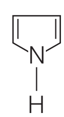
58. Valence electrons in the element A are 3 and that in element B are 6. Most probable compound formed from A and B is

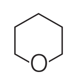
(a) A_2B (b) AB_2
(c) A_6B_3 (d) A_2B_3

59. Which of the following in the strongest base?

(a) 

(b) 

(c) 

(d) 

60. Which of the following best define the term hyperconjugation?

(a) Transfer of H from a σ_{C-H} bond to the π system

(b) Participation of an H-atom on terminal carbon in the electron sharing

(c) Delocalisation of electron density from a properly oriented σ-bond into π-system

(d) Interaction between a properly aligned lone electron pair with the empty π-system

Mathematics

1. If $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$, then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$, is

(a) $x^2 - x + 2 = 0$ (b) $x^2 + 2x + 2 = 0$

(c) $x^2 + x + 2 = 0$ (d) $x^2 + x - 2 = 0$

2. If $1, \omega, \omega^2, \dots \omega^{n-1}$ are n, n^{th} roots of unity, then the value of $(9-\omega)\cdot(9-\omega^2)\cdot(9-\omega^3)\dots(9-\omega^{n-1})$ will be

(a) $\frac{9^n + 1}{8}$ (b) $9^n - 1$

(c) $\frac{9^n - 1}{8}$ (d) $9^n + 1$

3. Middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$ is

(a) $\frac{n!}{(n!)^2}$ (b) $\frac{(2n!)}{(n!)^2}$ (c) $\frac{(2n-1)!}{n!}$ (d) $\frac{(2n)!}{n!}$

4. The common tangent of the parabolas $y^2 = 4x$ and $x^2 = -8y$ is

(a) $y = x + 2$ (b) $y = x - 2$

(c) $y = 2x + 3$ (d) None of these

5. 10 different toys are to be distributed among 10 children. Total number of ways of distributing these toys, so that exactly two children do not get any toy, is

(a) $\frac{10!}{2!\cdot 3!\cdot 7!}$

(b) $\frac{10!}{(2!)^4 \times 6!}$

(c) $(10!)^2\left[\frac{1}{(2!)^4 \times 6!} + \frac{1}{2! \times 3!}\right]$

(d) $\frac{10! \times 10!}{(2!)^2 \times 6!}\left[\frac{25}{84}\right]$

6. The variance of first n natural numbers is

(a) $\frac{n(n+1)}{12}$ (b) $\frac{(n+1)(n+5)}{12}$

(c) $\frac{(n+1)(n-5)}{12}$ (d) $\frac{n(n-5)}{12}$

7. A circle of radius 2 is touching both the axes and a circle with centre (6, 5). The distance between their centres is

(a) 8 units (b) 5 units

(c) 7 units (d) None of these

8. $S = \{1, 2, 3, \dots 20\}$ is to be partitioned into four sets A, B, C and D of equal size. The number of ways it can be done, is

(a) $\frac{20!}{4! \times 5!}$ (b) $\frac{20!}{4^5}$

(c) $\frac{20!}{(5!)^4}$ (d) $\frac{20!}{(4!)^5}$

9. Out of 50 tickets numbered $00, 01, 02, \dots, 49$, one ticket is drawn randomly, the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8, is

(a) $\frac{1}{14}$ (b) $\frac{3}{14}$

(c) $\frac{1}{5}$ (d) None of these

10. If the term free from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then the value of k is

(a) ± 1 (b) ± 3

(c) ± 4 (d) ± 2

11. If $\tan\frac{\alpha}{2}$, and $\tan\frac{\beta}{2}$ are the roots of $8x^2 - 26x + 15 = 0$, then $\cos(\alpha + \beta)$ is equal to

(a) $\frac{627}{725}$ (b) $-\frac{627}{725}$

(c) -1 (d) None of these

12. If $a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ and $a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$, then $(m+n)^{2/3} + (m-n)^{2/3}$ is equal to

(a) $2a^3$ (b) $2a^{1/3}$
(c) $2a^{2/3}$ (d) $2a^3$

13. In a triangle, the lengths of two larger sides are 10 cm and 9 cm. If the angles of the triangle are in AP , then the length of the third side is

(a) $\sqrt{5}-\sqrt{6}$ (b) $\sqrt{5}+\sqrt{6}$
(c) $\sqrt{5}\pm\sqrt{6}$ (d) $5\pm\sqrt{6}$

14. The distance of the point (3, 5) from $2x+3y-14=0$ measured parallel to $x-2y=1$ is

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{7}{\sqrt{13}}$
(c) $\sqrt{5}$ (d) $\sqrt{13}$

15. If the line $\frac{x}{a}+\frac{y}{b}=1$ moves such that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$, then the locus of the foot of the perpendicular from the origin to the line is

(a) straight line (b) circle
(c) parabola (d) ellipse

16. A rhombus is inscribed in the region common to the two circles $x^2+y^2-4x-12=0$ and $x^2+y^2+4x-12=0$ with two of its vertices on the line joining the centres of the circles. The area of rhombus is

(a) $8\sqrt{3}$ (b) $4\sqrt{2}$
(c) $16\sqrt{3}$ (d) $16\sqrt{2}$

17. If the line $y-\sqrt{3}x+3=0$ cuts the parabola $y^2=x+2$ at A and B, then $PA\cdot PB$ is equal to where [where $P=(\sqrt{3},0)$].

(a) $\frac{4(2-\sqrt{3})}{3}$ (b) $\frac{4(\sqrt{3}+2)}{3}$
(c) $\frac{4\sqrt{3}}{3}$ (d) $\frac{2(\sqrt{3}+2)}{3}$

18. AB is a chord of the parabola $y^2=4ax$ with vertex A, BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

(a) a (b) $2a$
(c) $4a$ (d) $8a$

19. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{4}+\frac{4y^2}{7}=1$ and having its centre at $\left(\frac{1}{2},2\right)$ is

(a) $\sqrt{5}$ (b) 3
(c) $\sqrt{12}$ (d) $\frac{7}{2}$

20. The area of the quadrilateral formed by the tangents at the end point of latusrectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is

(a) $\frac{27}{4}$ (b) 9 (c) $\frac{27}{2}$ (d) 27

21. If the chords of the hyperbola $x^2-y^2=a^2$ touch the parabola $y^2=4ax$. Then, the locus of the middle points of these chords is

(a) $y^2=(x-a)x^3$ (b) $y^2(x-a)=x^3$
(c) $x^2(x-a)=x^3$ (d) None of these

22. $\lim_{x\to 0}\frac{e^{\sin x}-1}{x}$ is equal to

(a) 0 (b) e
(c) 1 (d) Does not exist

23. If p and q are two statements, then $(p\wedge q)\vee(q\Leftrightarrow p)$ is

(a) tautology
(b) contradiction
(c) neither tautology nor contradiction
(d) either tautology or contradiction

24. The value of the determinant $\begin{vmatrix}(a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1\\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1\\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1\end{vmatrix}$ is

(a) 0 (b) $2abc$
(c) $a^2b^2c^2$ (d) None of these

25. If $\Delta_r=\begin{vmatrix}1 & n & n\\ 2r & n^2+n+1 & n^2+n\\ 2r-1 & n^2 & n^2+n+1\end{vmatrix}$ and $\sum_{r=1}^{n}\Delta_r=56$, then n is equal to

(a) 4 (b) 6 (c) 7 (d) 8

26. If a, b, c are cube roots of unity, then $\begin{vmatrix} e^a & e^{2a} & e^{3a}-1 \\ e^b & e^{2b} & e^{3b}-1 \\ e^c & e^{2c} & e^{3c}-1 \end{vmatrix}$ is equal to

(a) 0 (b) e (c) e^2 (d) e^3

27. If $fog = |\sin x|$ and $gof = \sin^2 \sqrt{x}$, then $f(x)$ and $g(x)$ are

(a) $f(x) = \sqrt{\sin x}$, $g(x) = x^2$

(b) $f(x) = |x|$, $g(x) = \sin x$

(c) $f(x) = \sqrt{x}$, $g(x) = \sin^2 x$

(d) $f(x) = \sin \sqrt{x}$, $g(x) = x^2$

28. Let $f: R \to R$ be defined by $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, then $(fofof)(x)$ is

(a) $\dfrac{x}{\sqrt{1+x^2}}$ (b) $\dfrac{x}{\sqrt{1+3x^2}}$

(c) x (d) 1

29. If $f(x) = \dfrac{a^x + a^{-x}}{2}$ and $f(x+y) + f(x-y) = kf(x)\, f(y)$, then k is equal to

(a) 2 (b) 4

(c) –2 (d) None of these

30. If $\tan^{-1}\left(\dfrac{a}{x}\right) + \tan^{-1}\left(\dfrac{b}{x}\right) = \dfrac{\pi}{2}$, then x is equal to

(a) $\sqrt{ab}$ (b) $\sqrt{2ab}$ (c) $2ab$ (d) ab

31. $\cos^{-1}\{\cos 2 \cot^{-1}(\sqrt{2}-1)\}$ is equal to

(a) $\sqrt{2}-1$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{3\pi}{4}$ (d) 0

32. $\displaystyle\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

(a) $\tan^{-1}\left(\dfrac{n^2+n}{n^2+n+2}\right)$ (b) $\tan^{-1}\left(\dfrac{n^2-n}{n^2-n+2}\right)$

(c) $\tan^{-1}\left(\dfrac{n^2+n+2}{n^2+n}\right)$ (d) None of these

33. $\displaystyle\lim_{x \to \frac{\pi}{2}} \frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$ is equal to

(a) $\dfrac{1}{8}$ (b) 0 (c) $\dfrac{1}{32}$ (d) ∞

34. $f(x) = \begin{cases} \dfrac{\sin^3(\sqrt{3})\cdot\log(1+3x)}{(\tan^{-1}\sqrt{x})^2\,(e^{5\sqrt{x}}-1)\,x}, & x \neq 0 \\ a, & x = 0 \end{cases}$ is continuous in $[0, 1]$, if 'a' equals to

(a) 0 (b) $\dfrac{3}{5}$

(c) 2 (d) $\dfrac{5}{3}$

35. If $y = |\sin x|^{|x|}$, then the value of $\dfrac{dy}{dx}$ at $x = \dfrac{-\pi}{6}$ is

(a) $\dfrac{2^{-\frac{\pi}{6}}}{6}[6\log 2 - \sqrt{3}\pi]$

(b) $2^{\frac{\pi}{6}}[6\log 2 + \sqrt{3}\pi]$

(c) $\dfrac{2^{-\frac{\pi}{6}}}{6}[6\log 2 + \sqrt{3}\pi]$

(d) None of the above

36. The function x^x is increasing, when

(a) $x > \dfrac{1}{e}$ (b) $x < \dfrac{1}{e}$

(c) $x < 0$ (d) for all x

37. The length of the longest interval, in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing, is

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{3\pi}{2}$ (d) π

38. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes lies in the first quadrant. If its area is 2, then the value of b is

(a) –1 (b) 3

(c) –3 (d) 1

39. The equation of the tangent to the curve $\left(\dfrac{x}{a}\right)^n + \left(\dfrac{y}{b}\right)^n = 2$ at (a, b) is

(a) $\dfrac{x}{a} + \dfrac{y}{b} = 2$ (b) $\dfrac{x}{a} + \dfrac{y}{b} = \dfrac{1}{2}$

(c) $\dfrac{x}{b} - \dfrac{y}{a} = 2$ (d) $ax + by = 2$

40. The minimum radius vector of the curve $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$ is of length

(a) $a - b$ (b) $a + b$

(c) $2a + b$ (d) None of these

41. The largest term in the sequence $a_n = \dfrac{n^2}{n^3 + 200}$ is given by

(a) $\dfrac{529}{49}$ (b) $\dfrac{8}{89}$

(c) $\dfrac{49}{543}$ (d) None of these

42. $f(x) = \begin{cases} |x^3 + x^2 + 3x + \sin x| \cdot \left(3 + \sin \dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

The number of points, where $f(x)$ attains its minimum value, is

(a) 1 (b) 2

(c) 3 (d) infinitely many

43. $\displaystyle\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1)\tan^{-1}\left(x + \frac{1}{x}\right)}\,dx$ is equal to

(a) $\tan^{-1}\left(x + \dfrac{1}{x}\right) + C$

(b) $\cot^{-1}\left(x + \dfrac{1}{x}\right) + C$

(c) $\log\left(x + \dfrac{1}{x}\right) + C$

(d) $\log\left[\tan^{-1}\left(x + \dfrac{1}{x}\right)\right] + C$

44. $\displaystyle\int \frac{\cos x}{\sin^2 x \cdot (\sin x + \cos x)}\,dx$ is equal to

(a) $\log\left|\dfrac{1 + \tan x}{\tan x}\right| - \cot x + C$

(b) $\log\left|\dfrac{\tan x}{1 + \tan x}\right| + C$

(c) $\log\left|\dfrac{\tan x}{1 + \tan x}\right| - \tan x + C$

(d) $\log\left|\dfrac{\tan x}{1 + \tan x}\right| + \cot x + C$

45. $\displaystyle\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ is equal to

(a) $-\dfrac{1}{\sqrt{2}}\tan\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right) + C$

(b) $\dfrac{1}{\sqrt{2}}\tan\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right) + C$

(c) $\dfrac{1}{\sqrt{2}}\cot\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right) + C$

(d) $-\dfrac{1}{\sqrt{2}}\cot\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right) + C$

46. $\displaystyle\int_0^{\frac{3\pi}{2}} \sin\left[\frac{2x}{\pi}\right] \cdot dx$, where $[\cdot]$ denotes the greatest integer function, is equal to

(a) $\dfrac{\pi}{2}(\sin 1 + \cos 1)$ (b) $\dfrac{\pi}{2}(\sin 1 + \sin 2)$

(c) $\dfrac{\pi}{2}(\sin 1 - \cos 1)$ (d) $\dfrac{\pi}{2}(\sin \pi + \sin 2)$

47. The value of $\displaystyle\int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a - x} + \sqrt{x}}\,dx$ is equal to

(a) $\dfrac{a}{2}$ (b) $\dfrac{n \cdot a + 2}{2n}$

(c) $\dfrac{n \cdot a - 2}{2n}$ (d) $\dfrac{n \cdot a}{2}$

48. The area bounded by $y = x|\sin x|$ and X-axis between $x = 0$ and $x = 2\pi$ is

(a) 2π sq units (b) 3π sq units

(c) π sq units (d) 4π sq units

49. The area enclosed by the curves $|y + x| \le 1$, $|y - x| \le 1$ and $2x^2 + 2y^y = 1$ is

(a) $\left(2 + \dfrac{\pi}{2}\right)$sq units (b) $\left(2 - \dfrac{\pi}{2}\right)$sq units

(c) $\left(3 + \dfrac{\pi}{2}\right)$sq units (d) $\left(3 - \dfrac{\pi}{4}\right)$sq units

50. The real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4 \cdot y\dfrac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation, is

(a) $m = 0$ (b) $m = 1$ (c) $m = \dfrac{3}{2}$ (d) $m = \dfrac{2}{3}$

51. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal on the X-axis and passing through $(2, 1)$, is

(a) $x^2 + y^2 - x = 0$

(b) $4x^2 + 2y^2 - 9y = 0$

(c) $2x^2 + 4y^2 - 9x = 0$

(d) $4x^2 + 2y^2 - 9x = 0$

52. If $|\mathbf{a}| = 1, |\mathbf{b}| = 4, \mathbf{a} \cdot \mathbf{b} = 2$ and $\mathbf{c} = 2\mathbf{a} \times \mathbf{b} - 3\mathbf{b}$, then the angle between $\mathbf{b}$ and $\mathbf{c}$ is

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{5\pi}{6}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{2\pi}{3}$

53. If a, b, c are p^{th} and q^{th} and r^{th} terms of a GP, then the vectors $\log a\,\hat{\mathbf{i}} + \log b\,\hat{\mathbf{j}} + \log c\,\hat{\mathbf{k}}$ and $(q-r)\,\hat{\mathbf{i}} + (r-p)\,\hat{\mathbf{j}} + (p-q)\,\hat{\mathbf{k}}$ are

(a) equal (b) parallel
(c) perpendicular (d) None of these

54. The line joining the points $(1, 1, 2)$ and $(3, -2, 1)$ meets the plane $3x + 2y + z = 6$ at the point

(a) (1, 1, 2) (b) (3, −2, 1)
(c) (2, −3, 1) (d) (3, 2, 1)

55. The distance of the point $(1, -5, 9)$, from the plane $\mathbf{r} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ measured along the line $\mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

(a) $3\sqrt{5}$ (b) $10\sqrt{3}$ (c) $5\sqrt{3}$ (d) $3\sqrt{10}$

56. The image of the point with position vector $\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ in the plane $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ is

(a) $\hat{i} + 2\hat{j} + \hat{k}$ (b) $\hat{i} - 2\hat{j} + \hat{k}$
(c) $-\hat{i} - 2\hat{j} + \hat{k}$ (d) $\hat{i} + 2\hat{j} - \hat{k}$

57. In a trial, the probability of success is twice the probability of failure. In six trial, the probability of atleast four successes will be

(a) $\frac{496}{729}$ (b) $\frac{400}{729}$ (c) $\frac{500}{729}$ (d) $\frac{600}{729}$

58. If the integers m and n are chosen at randon between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$

59. If $\sin^{-1} a + \sin^{-1} b + \sin^{-1} c = \pi$, then the value of $a \cdot \sqrt{(1-a^2)} + b \cdot \sqrt{(1-b^2)} + c \cdot \sqrt{(1-c^2)}$ will be

(a) $2abc$ (b) abc (c) $\frac{1}{2}abc$ (d) $\frac{1}{3}abc$

60. The value of λ and μ for which the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have no solution, are

(a) $\lambda = 3, \mu \neq 10$ (b) $\lambda \neq 3, \mu = 10$
(c) $\lambda \neq 3, \mu \neq 10$ (d) None of these

61. The range of $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right), -\infty < x < \infty$ is

(a) $[1, \sqrt{2}]$ (b) $[1, \infty]$
(c) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$ (d) $[-\infty, -1] \cup [1, \infty]$

62. The value of the $\lim_{x \to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{2/x}$, $(a, b, c > 0)$ is

(a) $(abc)^3$ (b) abc
(c) $(abc)^{1/3}$ (d) None of these

63. The function $f(x) = [x] \cos\left[\frac{2x-1}{2}\right]\pi$, where $[\cdot]$ denotes the greatest integer funtion, is discontinuous at

(a) all x
(b) no x
(c) all integral points
(d) x which is not an integer

64. The function $f(x) = (x^2 - 1)\,|x^2 - 3x + 2| + \cos|x|$ is non-differentiable at

(a) −1 (b) 0
(c) 1 (d) 2

65. If $f(x + y) = f(x) \cdot f(y)$ for all x and y and $f(15) = 2$, $f'(0) = 3$, then $f'(5)$ will be

(a) 2 (b) 4
(c) 6 (d) 8

66. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{\sin^2(a+y)}{\sin a}$ (b) $\frac{\cos^2(a+y)}{\cos a}$
(c) $\frac{\sin^2(a+y)}{\cos a}$ (d) $\frac{\cos^2(a+y)}{\sin a}$

67. The approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$, is

(a) −34.995 (b) −33.995
(c) −33.335 (d) −35.993

68. What are the values of c for which Rolle's theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in the interval [0, 2] is verified?

(a) $c = \pm 1$ (b) $c = 1 \pm \frac{1}{\sqrt{3}}$
(c) $c = \pm 2$ (d) None of these

69. The equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$ is

(a) $7x + 5y + 4z + 8 = 0$ (b) $7x + 5y + 4z - 8 = 0$
(c) $7x - 5y + 4z - 8 = 0$ (d) None of these

70. $\lim_{x \to 0} (1^{\text{cosec}^2 x} + 2^{\text{cosec}^2 x} + \ldots + n^{\text{cosec}^2 x})^{\sin^2 x}$ is equal to

(a) 1 (b) $\frac{1}{n}$

(c) n (d) 0

71. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cdot \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

72. The area of the region bounded by $1 - y^2 = |x|$ and $|x| + |y| = 1$ is

(a) 1/3 sq unit

(b) 2/3 sq unit

(c) 4/3 sq units

(d) 1 sq unit

73. The equations of the lines passing through the point (1, 0) and at a distance $\frac{\sqrt{3}}{2}$ from the origin are

(a) $\sqrt{3}x + y - \sqrt{3} = 0, \sqrt{3}x - y - \sqrt{3} = 0$

(b) $\sqrt{3}x + y + \sqrt{3} = 0, \sqrt{3}x - y + \sqrt{3} = 0$

(c) $x + \sqrt{3}y - \sqrt{3} = 0, x - \sqrt{3}y - \sqrt{3} = 0$

(d) None of the above

74. The equation of circle which passes through the origin and cuts off intercepts 5 and 6 from the positive parts of the axes respectively, is $\left(x - \frac{5}{2}\right)^2 + (y - 3)^2 = \lambda$, where λ is

(a) $\frac{61}{4}$ (b) $\frac{6}{4}$

(c) $\frac{1}{4}$ (d) 0

75. A perpendicular is drawn from the point $P(2, 4, -1)$ to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. The equation of the perpendicular from P to the given line is

(a) $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$ (b) $\frac{x+2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$

(c) $\frac{x+2}{-6} = \frac{y-4}{3} = \frac{z+1}{2}$ (d) $\frac{x+2}{6} = \frac{y+4}{3} = \frac{z+1}{2}$

76. If $\cos^2 A + \cos^2 C = \sin^2 B$, then ΔABC is

(a) equilateral

(b) right angled

(c) isosceles

(d) None of the above

77. If $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

$= k(x+y+z)^3$, then the value of k is

(a) 1 (b) 2

(c) 4 (d) 8

78. The minimum value of $\frac{x}{\log x}$ is

(a) e (b) $\frac{1}{e}$

(c) e^2 (d) e^3

79. One mapping (function) is selected at random from all the mappings of the set $A = \{1, 2, 3, \ldots, n\}$ into itself. The probability that the mapping selected is one-one, is

(a) $\frac{n!}{n^{n-1}}$

(b) $\frac{n!}{n^n}$

(c) $\frac{n!}{2n^n}$

(d) None of the above

80. In how many ways, can a student choose a program of 5 courses, if 9 courses are available and 2 specific courses are compulsory for every student?

(a) 34 (b) 36 (c) 35 (d) 37

English

Directions (Q.Nos. 1-5) *Out of the four alternatives, choose the one which best expresses the meaning of the given word.*

1. Remorse
(a) Warmth (b) Delight
(c) Repentance (d) Satisfaction

2. Stigma
(a) Gedit (b) Pride
(c) Shame (d) Clarity

3. Blemish
(a) Adornment (b) Blank
(c) Embellishment (d) Flaw

4. Deteriorate
(a) Boost (b) Improve (c) Build (d) Descend

5. Knave
(a) Fraud (b) Angel
(c) Ugly (d) Perfect

Directions (Q.Nos. 6-10) *Choose the word opposite in meaning to the given word and mark it in the answer sheet.*

6. Enormous
(a) Soft (b) Average
(c) Tiny (d) Weak

7. Commissioned
(a) Started (b) Closed
(c) Finished (d) Terminated

8. Artificial
(a) Red (b) Natural
(c) Truthful (d) Solid

9. Exodus
(a) Influx (b) Home-coming
(c) Return (d) Restoration

10. Mortal
(a) Divine (b) Immortal
(c) Spiritual (d) Eternal

Directions (Q.Nos. 11-15) *Four alternatives are given in each group, one word is correctly spelt. Find the correctly spelt word.*

11. (a) Addvertise (b) Surender
(c) Admiror (d) Pitiable

12. (a) Humilliation (b) Veterinary
(c) Appology (d) Resturant

13. (a) Accute (b) Forstry
(c) Personel (d) Omnious

14. (a) Diference (b) Homogenious
(c) Greatful (d) Possession

15. (a) Committee (b) Commentery
(c) Involuntry (d) Anxiety

Directions (Q.Nos. 16-20) *Out of the four alternatives, choose the one which can be substituted for the given words/sentences.*

16. To examine ones to own thoughts and feelings
(a) Introspection (b) Retrospection
(c) Euthanasia (d) Circumspection

17. The custom of having more than one husband at a time
(a) Polygamy (b) Polyandry
(c) Debauchery (d) Bigamy

18. Murder of a brother
(a) Patricide (b) Regicide
(c) Homicide (d) Fratricide

19. Study of mankind
(a) Pathology (b) Physiology
(c) Philology (d) Anthropology

20. Through which life cannot pass
(a) Dull (b) Dark
(c) Obscure (d) Opaque

Directions (Q.Nos. 21-30) *Some of the words have been left out. First read the passage over and try to understand what it is about. Then, fill in the blanks with the help of the alternatives given.*

Something has happened in the last twenty years that surely must **(21)** anything that has happened before. Some historians are already saying that thrust **(22)** space represents a vital turning point in history. Moon flights are considered **(23)** less than steps in human evolution **(24)** to the time when life on earth emerged from the sea and established itself on land. Of course, not everyone **(25)** enraptured by space. Critics have often said that space flight has been an **(26)** use of resources that should have **(27)** to feeding, clothing and housing people. There is, however, no proof that if we had **(28)** been working on space, we would have done anything of great human value. In fact, research and exploration have a **(29)** spin-offs, quite apart from the fact that they demonstrate that **(30)** is alive and insatiably curious.

21. (a) terminate (b) transcend (c) precede (d) recede

22. (a) on (b) upon (c) in (d) into

23. (a) nothing (b) certainly (c) sufficiently (d) probably

24. (a) exceeding (b) contrasting (c) comparable (d) matching

25. (a) was (b) has been (c) had been (d) being

26. (a) economical (b) extravagant (c) appropriate (d) benevolent

27. (a) devoted (b) allotted (c) reserved (d) gone

28. (a) not (b) occasionally (c) seldom (d) possibly

29. (a) renowned (b) renounced (c) remarkable (d) relevant

30. (a) one (b) man (c) human (d) individual

Directions (Q.Nos. 31-35) *Some of the sentences have errors and some are correct. Find out which part of a sentence has a error and corresponding to the appropriate letter* (a), (b) *and* (c) *If the sentence free from error, then mark* (d).

31. You will not(a)/get well until(b)/you take medicine regularly.(c)/No error(d)

32. You must either(a)/ inform the police else be(b)/ prepared to suffer any loss.(c)/ No error(d)

33. Both the rich along with the poor(a)/ are responsible for a great many vices(b)/with which our society as well as country is in flicted.(c)/ No error(d)

34. Have you ever(a)/ met such a boy(b)/ who has not travelled by train?(c)/ No error(d)

35. The robber had hardly(a)/ put the ornaments in his bag(b)/ than the house wife woke up.(c)/ No error(d)

Directions (Q.Nos. 36-40) *Sentences are given with blanks to be filled with an appropriate word. Four alternatives are suggested for each question. Choose the correct alternative.*

36. I wish my bus .... so late, then I could have reached home before breakfast

(a) had not arrived (b) does not arrive (c) has not arrived (d) arrived

37. .... you examine it carefully you will notice some flaws.

(a) Despots (b) If (c) Unless (d) Until

38. The contractor did not keep his promise that the work .... be finished before the end of the week.

(a) shall (b) could (c) would (d) may

39. If you come across my umbrella anywhere bring it to me, ...?

(a) is not it (b) do not you (c) will you (d) let you

40. Miss Pillai teaches very well ....?

(a) did not she
(b) does not she
(c) was not it
(d) would not she

Answers

Physics

1. (b)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (a)
11. (d)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (b,c)	**19.** (b)	**20.** (b)
21. (a)	**22.** (d)	**23.** (a)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (c)
31. (d)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (d)	**38.** (b)	**39.** (c)	**40.** (b)
41. (b)	**42.** (d)	**43.** (a)	**44.** (d)	**45.** (a)	**46.** (d)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (c)
51. (d)	**52.** (d)	**53.** (b)	**54.** (d)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (a)	**59.** (a)	**60.** (a)

Chemistry

1. (c)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (c)
11. (a)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)
21. (a)	**22.** (a)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (d)	**30.** (b)
31. (c)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (c)	**38.** (c)	**39.** (c)	**40.** (d)
41. (a)	**42.** (c)	**43.** (c)	**44.** (d)	**45.** (d)	**46.** (a)	**47.** (d)	**48.** (a)	**49.** (a)	**50.** (b)
51. (b)	**52.** (c)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (d)	**59.** (a)	**60.** (c)

Mathematics

1. (c)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (b)
11. (b)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (d)
21. (b)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (a)
31. (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (a)	**38.** (c)	**39.** (a)	**40.** (b)
41. (c)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (d)	**46.** (b)	**47.** (c)	**48.** (d)	**49.** (b)	**50.** (c)
51. (d)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (b)	**57.** (a)	**58.** (a)	**59.** (a)	**60.** (a)
61. (a)	**62.** (d)	**63.** (c)	**64.** (d)	**65.** (c)	**66.** (a)	**67.** (a)	**68.** (b)	**69.** (c)	**70.** (c)
71. (a)	**72.** (b)	**73.** (a)	**74.** (a)	**75.** (a)	**76.** (b)	**77.** (b)	**78.** (a)	**79.** (b)	**80.** (c)

English

1. (c)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (b)
11. (d)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (d)	**20.** (d)
21. (b)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (b)
31. (b)	**32.** (b)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (b)

Hints & Solutions

Physics

1. According to the principle of dimensional homogenity $[p] = \left[\frac{a}{V^2}\right]$

$\Rightarrow \quad [a] = [p][V^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$

Hence, the units of $a = \text{gm} \times \text{cm}^5 \times \text{s}^{-2}$

$= \text{Dyne} \times \text{cm}^4$

2. Given, $[C^2LR] = \left[C^2L^2 \frac{R}{L}\right] = \left[(LC)^2\left(\frac{R}{L}\right)\right]$

and we know that frequency of LC circuits $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$. Here the dimension of LC is equal to $[T^2]$.

$\left[\frac{L}{R}\right]$ gives the time constant of L - R circuit, so that the dimension of $\frac{L}{R}$ is equal to $[T]$.

Hence the required dimensions

$\left[(LC)^2\left(\frac{R}{L}\right)\right] = [T^2]^2[T^{-1}] = [T^3]$.

3. Given, number of particle passing from unit area in unit time $= n$

$= \frac{\text{Number of particle}}{A \times t} = \frac{[M^0 L^0 T^0]}{[L^2][T]} = [L^{-2}T^{-1}]$

$[n_1] = [n_2] =$ Number of particle in unit volume $= [L^{-3}]$

Now, from the given formula $[D] = \frac{[n][x_2 - x_1]}{[n_2 - n_1]}$

$= \frac{[L^{-2}T^{-1}][L]}{[L^{-3}]}$

$= [L^2 T^{-1}]$

4. Given that $v = 4t^3 - 2t$

$x = \int v\, dt,\ x = t^4 - t^2 + C$, at $t = 0,\ x = 0$

$\Rightarrow \quad C = 0$

When particle is $2m$ away from the origin, then

$2 = t^4 - t^2 \Rightarrow t^4 - t^2 - 2 = 0$

$\Rightarrow (t^2 - 2)(t^2 + 1) = 0$

$\Rightarrow \quad t = \sqrt{2}$ s

$a = \frac{dv}{dt} = \frac{d}{dt}(4t^3 - 2t)$

$a = 12t^2 - 2$

For $\quad t = \sqrt{2}$ sec

$a = 12 \times (\sqrt{2})^2 - 2 = 22$ m/s^2

5. From given a - t graph acceleration is increasing at constant rate

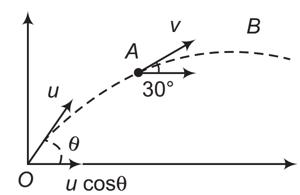
$\therefore \frac{da}{dt} = k$ (constant) $\Rightarrow a = kt$ (by integration)

$\Rightarrow \quad \frac{dv}{dt} = kt \Rightarrow dv = k\,t\,dt$

$\Rightarrow \quad \int dv = k\int t\,dt \Rightarrow v = \frac{kt^2}{2}$

i.e., v is dependent on time parabolically and parabola is symmetric about v-axis and suddenly acceleration becomes zero. *i.e.*, velocity becomes constant. Hence the most stable graph is (c).

6. Let in 2 s body reaches upto point A and after one more sec upto point B.
Total time of ascent for a body is given 3 s



i.e., $\quad t = \frac{u\sin\theta}{g} = 3$

$\therefore \quad u\sin\theta = 10 \times 3$

$u\sin\theta = 30 \quad \ldots$(i)

Horizontal component of velocity remains always constant

$u\cos\theta = v\cos 30° \quad \ldots$(ii)

For vertical upward motion between point O and A

$v\sin 30° = u\sin\theta - g \times 2 \quad$ [Using $v = u - gt$]

$v \sin 30° = 30 - 20$ [As $u \sin\theta = 30$]

$\therefore \quad v = 20$ m/s

Substituting this value v, we get in Eq. (ii)

$u \cos\theta = 20 \cos 30° = 10\sqrt{3}$ ...(iii)

From Eqs. (i) and (iii)

$$u = 20\sqrt{3} \text{ and } \theta = 60°$$

7. In non-uniform circular motion two forces will work on a particle F_c and F_t.

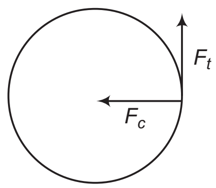
So, the net force $F_{\text{Net}} = \sqrt{F_c^2 + F_t^2}$ ...(i)

Centripetal force $F_c = \dfrac{mv^2}{R} = \dfrac{2as^2}{R}$ ...(ii)

[given $\dfrac{1}{2}mv^2 = as^2$ given]

Again from $\dfrac{1}{2}mv^2 = as^2$

$$\Rightarrow \quad v^2 = \frac{2as^2}{m}$$



$$\Rightarrow \quad v = s\sqrt{\frac{2a}{m}}$$

Tangential acceleration $a_t = \dfrac{dv}{dt} = \dfrac{dv}{ds} \cdot \dfrac{ds}{dt}$

$$\Rightarrow \quad a_t = \frac{d}{ds}\left[s\sqrt{\frac{2a}{m}}\right] \cdot v$$

$$a_t = v\sqrt{\frac{2a}{m}} = s\sqrt{\frac{2a}{m}}\sqrt{\frac{2a}{m}} = \frac{2as}{m}$$

and $\quad F_t = ma_t = 2as$ ...(iii)

Now, on substituting value of F_c and F_t in Eq.(i), we get

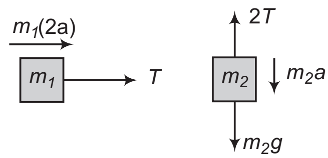
$$\therefore \; F_{\text{Net}} = \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2} = 2as\left[1 + \frac{s^2}{R^2}\right]^{1/2}$$

8. We have $T_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$

Now $T_1 = \dfrac{(3+5)10}{2+3+5} = 8$ N

9. When the block m_2 moves downward with acceleration a, the acceleration of mass m_1 will be $2a$, because it covers double distance in the same time in comparison to m_2.

Let T is the tension in the string.



From the free body diagram of A and B we get

$T = m_1 2a$ ...(i)

$m_2g - 2T = m_2a$ ...(ii)

From Eqs. (i) and (ii), we get

$$a = \frac{m_2g}{(4m_1 + m_2)}$$

10. B will begin to slide on A, if pseudo force is more than limiting friction $F > F_1 \Rightarrow m\left(\dfrac{F}{m+M}\right) > \mu_s R$

$$\Rightarrow \quad m\left(\frac{F}{m+M}\right) > 0.15\,mg$$

$\therefore \quad F > 1.764$ N

11. We know $F = -\dfrac{dU}{dx}$

$\Rightarrow \; dU = -F \cdot dx$

$\Rightarrow \; U = -\int_0^x (-kx + ax^3)\,dx \Rightarrow U = \dfrac{kx^2}{2} - \dfrac{ax^4}{4}$

$\therefore$ We get $U = 0$ at $x = 0$ and $x = \sqrt{\dfrac{2k}{a}}$

Also we get $U =$ negative for $x > \sqrt{\dfrac{2k}{a}}$

From the given function we can see that $F = 0$ at $x = 0$ *i.e.*, slope of U-x graph is zero at $x = 0$.

12. Moment of inertia of rod AB about point $P = \dfrac{1}{12}Ml^2$

MI of rod AB about point $O = \dfrac{Ml^2}{12} + M\left(\dfrac{l}{2}\right)^2 = \dfrac{1}{3}Ml^2$ [from the theorem of parallel axis]

and the system consists of 4 rods of similar type so by the symmetry $I_{\text{system}} = \dfrac{4}{3}Ml^2$

13. $\omega_1 = 20 \text{rad/s}$, $\omega = 0$, $t = 4\text{s}$. So angular retardation $\alpha = \dfrac{\omega_1 - \omega_2}{t} = \dfrac{20}{4} = 5 \text{ rad/s}^2$

Now, angular speed after 2s $\omega_2 = \omega_1 - \alpha t = 20 - 5 \times 2 = 10 \text{ rad/s}$

Work done by torque in 2 s = loss in kinetic energy $= \dfrac{1}{2} I(\omega_1{}^2 - \omega_1{}^2) = \dfrac{1}{2}(0.20)((20)^2 - (10)^2)$

$$= \frac{1}{2}(0.20)(400 - 100)$$

$$= \frac{1}{2}(0.20) \times (300) = \frac{1}{2} \times 60 = 30$$

14. On the graph stress is represented on *x*-axis and strain *y*-axis

So from the graph $y = \cot\theta = \dfrac{1}{\tan\theta} \propto \dfrac{1}{\theta}$

[where, θ is the angle from stress axis]

$\therefore \quad Y_P < Y_Q < Y_R \quad [\text{As } \theta_P < \theta_Q < \theta_R]$

We can say that elasticity of wire *P* is minimum and *R* is maximum.

15. Energy released

$= 4\pi TR^3 \left(\dfrac{1}{r} - \dfrac{1}{R}\right) = 3\left(\dfrac{4}{3}\pi R^3\right) T \left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

$= 3VT\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

16. When substances are mixed in equal volume then density $= \dfrac{\rho_1 + \rho_2}{2} = 4$

$\Rightarrow \quad \dfrac{\rho_1 + \rho_2}{2} = 4 \Rightarrow \rho_1 + \rho_2 = 8 \quad \ldots\text{(i)}$

When substances are mixed in equal masses then density $= \dfrac{2\rho_1 \rho_2}{\rho_1 + \rho_2} = 3$

$\Rightarrow \quad 2\rho_1 \rho_2 = 3(\rho_1 + \rho_2) \quad \ldots\text{(ii)}$

On solving (i) and (ii) we get

$\rho_1 = 6$ and $\rho_2 = 2$.

17. We have, ideal gas equation $pV = \mu RT = \left(\dfrac{N}{N_A}\right) RT$

where, N = number of molecule, N_A

= Avogadro number

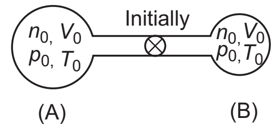
$\therefore \quad \dfrac{N_1}{N_2} = \left(\dfrac{p_1}{p_2}\right)\left(\dfrac{V_1}{V_2}\right)\left(\dfrac{T_2}{T_1}\right) = \left(\dfrac{p}{2p}\right)\left(\dfrac{V}{V/4}\right)\left(\dfrac{2T}{T}\right) = \dfrac{4}{1}$

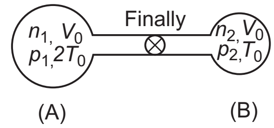
18. Initially for container *A*, $p_0 V_0 = n_0 RT_0$

For container *B*, $p_0 V_0 = n_0 RT_0$

$\therefore \quad n_0 = \dfrac{p_0 V_0}{RT_0}$

Total number of moles $= n_0 + n_0 = 2n_0$





Since, even on heating the total number of moles is conserved

Hence, $\quad n_1 + n_2 = 2n_0 \quad \ldots\text{(i)}$

If p be the common pressure, then

For container *A*, $pV_0 = n_1 R\, 2T_0 \quad \therefore \quad n_1 = \dfrac{pV_0}{2RT_0}$

For container *A*, $pV_0 = n_2 RT_0 \quad \therefore \quad n_2 = \dfrac{pV_0}{RT_0}$

Substituting the value of n_0, n_1 and n_2 in Eq. (i) we get

$$\frac{pV_0}{2RT_0} + \frac{pV_0}{RT_0} = \frac{2 \cdot p_0 V_0}{RT_0} \Rightarrow p = \frac{4}{3} p_0$$

Number of moles in container *A* (at temperature $2T_0$)

$$= n_1 = \frac{PV_0}{2RT_0} = \left(\frac{4}{3} p_0\right) \frac{V_0}{2RT_0}$$

$$= \frac{2}{3} \frac{p_0 V_0}{RT_0} \qquad \left[As\ p = \frac{4}{3} p_0\right]$$

19. At lower pressure, we can assume that given gas behaves as ideal gas so $\dfrac{pV}{RT}$ = constant but when pressure increase, the decrease in volume will not take place in same proportion so $\dfrac{pV}{RT}$ will increases.

20. Since both the liquid and the flask undergoes volume expansion and the flask expands first therefore the level of the liquid initially falls and then rises.

21. $\frac{dQ}{dt} = \frac{KA\Delta\theta}{l}$, for both rods K, A and $\Delta\theta$ are same

$$\therefore \qquad \frac{dQ}{dt} \propto \frac{1}{l}$$

so $$\frac{(dQ/dt)_{\text{semi-circular}}}{(dQ/dt)_{\text{straight}}} = \frac{l_{\text{straight}}}{l_{\text{semi-circular}}} = \frac{2r}{\pi r} = \frac{2}{\pi}$$

22. Let two simple harmonic motions are $y = a\sin\omega t$ and $y = a\sin(\omega t + \phi)$

In the first case, $\frac{a}{2} = a\sin\omega t \Rightarrow \sin\omega t = \frac{1}{2}$

$$\therefore \qquad \cos\omega t = \frac{\sqrt{3}}{2}$$

In the second case, $\frac{a}{2} = a\sin(\omega t + \phi)$

$$\Rightarrow \frac{1}{2} = [\sin\omega t \cdot \cos\phi + \cos\omega t \sin\phi]$$

$$\Rightarrow \frac{1}{2} = \left[\frac{1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi\right]$$

$$\Rightarrow \qquad 1 - \cos\phi = \sqrt{3}\sin\phi$$

$$\Rightarrow \qquad (1-\cos\phi)^2 = 3\sin^2\phi$$

$$\Rightarrow \qquad (1-\cos\phi)^2 = 3(1-\cos^2\phi)$$

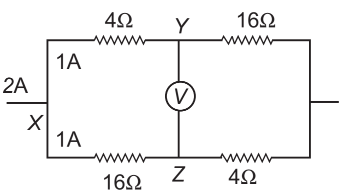
On solving we get

$$\cos\phi = +1 \text{ or } \cos\phi = \frac{-1}{2}$$

i.e., $$\phi = 0 \text{ or } \phi = 120°$$

23. PD between X and Y is $V_{XY} = V_X - V_Y = 1 \times 4$

and PD between X and Z is $V_{XZ} = V_X - V_Z = 1 \times 16 = 16\,\text{V}$



Now, we get potential difference between Y and Z *i.e.*, reading of voltmeter is $V_Y - V_Z = 12\,\text{V}$

24. As we know that work done

$$= U_{\text{final}} - U_{\text{initial}} = \frac{1}{2}C(V_2^2 - V_1^2)$$

When potential difference increases from 5V to 10 V then $W = \frac{1}{2}C(10^2 - 5^2)$ ...(i)

When potential difference increases form 10 V to 15 V, then $W' = \frac{1}{2}C(15^2 - 10^2)$ ...(ii)

On solving Eqs. (i) and (ii) we get

$$W' = 1.67\,W$$

25. $\lambda_{\text{neutron}} \propto \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}}$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{(273+927)}{(273+27)}} = \sqrt{\frac{1200}{300}} = 2 \Rightarrow \lambda_2 = \frac{\lambda}{2}$$

26. On using $I = \frac{P}{A}$; where $P =$ radiation power

$$\Rightarrow \qquad P = I \times A \Rightarrow \frac{nhc}{t\lambda} = IA \Rightarrow \frac{n}{t} = \frac{IA\lambda}{hc}$$

Hence, number of photons entering per sec the eye $\left(\frac{n}{t}\right) = \frac{10^{-10} \times 10^{-6} \times 5.6 \times 10^{-7}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 300$

27. From $E = W_0 + \frac{1}{2}mv^2{}_{\max} \Rightarrow v_{\max} = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}}$

(where $E = \frac{hc}{\lambda}$)

If wavelength of incident light charges from λ to $\frac{3\lambda}{4}$ (decreases).

Let energy of incident light charges from E to E' and speed of fastest electron changes from v to v' then

$$v = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}} \qquad \text{...(i)}$$

and $$v' = \sqrt{\frac{2E'}{m} - \frac{2W_0}{m}} \qquad \text{...(ii)}$$

As $$E \propto \frac{1}{\lambda} \Rightarrow E' = \frac{4}{3}E$$

Hence, $$v' = \sqrt{\frac{2\left(\frac{4}{3}E\right)}{m} - \frac{2W_0}{m}}$$

$$\Rightarrow \qquad v' = \left(\frac{4}{3}\right)^{1/2}\sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}}$$

$$\Rightarrow \qquad v' = \left(\frac{4}{3}\right)^{1/2} X = \sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}} > v$$

So $$v' > \left(\frac{4}{3}\right)^{1/2} v.$$

28. Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \lambda \propto \frac{1}{Z^2} \Rightarrow$

$\lambda_{Li} : \lambda_{He} : \lambda_H = \frac{1}{9} : \frac{1}{4} : \frac{1}{1} = 4 : 9 : 36$

29. By using $N = N_0\left(\frac{1}{2}\right)^{1/T_{1/2}}$

$$\Rightarrow \quad t = \frac{T_{12}\log_e\left(\frac{N_0}{N}\right)}{\log_e(2)}$$

$$\Rightarrow \quad t \propto \log_e \frac{N_0}{N}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{\log_e\left(\frac{N_0}{N}\right)_1}{\log_e\left(\frac{N_0}{N}\right)_2}$$

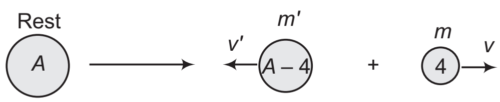
Hence, $\frac{6}{10} = \frac{\log_e(8/1)}{\log_e(N_0/N)}$

$$\Rightarrow \quad \log_e \frac{N_0}{N} = \frac{10}{6}\log_e(8) = \log_e 32$$

$$\Rightarrow \quad \frac{N_0}{N} = 32$$

So, fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$

30. According to conservation of momentum $4v = (A-4)v' \quad \Rightarrow \quad v' = \frac{4v}{A-4}$



31. ${}_{Z=90}\text{Th}^{A=228} \longrightarrow {}_{Z'=83}\text{Bi}^{A'=212}$

Number of α-particles emitted

$$n_\alpha = \frac{A - A'}{4} = \frac{228 - 212}{4} = 4$$

Number of β-particles emitted

$$n_\beta = 2n_\alpha - Z + Z' = 2 \times 4 - 90 + 83 = 1.$$

32. Radius of each nucleus $R = R_0(A)^{1/3} = 1.2(64)^{1/3}$ $= 4.8\,\text{m}$

Distance between two nuclei $(r) = 2R$

So, potential energy $U = \frac{k \cdot q^2}{r}$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\,\text{MeV}$$

33. At neutral point magnetic field due to magnet = Horizontal component of the earth's magnetic field.

$$\Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \Rightarrow \frac{10^{-7} \times 2 \times M \times 1}{(0.2)^3} = 0.3 \times 15^4$$

$$\Rightarrow M = 1.2\,\text{amp} \times \text{m}^2 = 1200\,ab - \text{amp} \times \text{cm}^2.$$

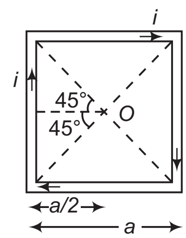
34. By using $T = 2\pi\sqrt{\frac{1}{MB_H}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{(B_H)_2}{(B_H)_1}}$

$$\Rightarrow \quad \frac{60/40}{2.5} = \sqrt{\frac{(B_H)_2}{0.1 \times 10^{-5}}}$$

$$\Rightarrow \quad (B_H)_2 = 0.36 \times 10^{-6}\ \text{T}$$

35. $H = ni \Rightarrow i = \frac{H}{n} = \frac{4 \times 10^3}{500} = 8\ \text{A}$

36. Magnetic field due to one side of the square at centre O.



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i\sin 45°}{a/2}$$

$$\Rightarrow \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\,i}{a}$$

Hence, magnetic field at centre due to all sides

$$B_{\text{net}} = 4B_1 = \frac{\mu_0(2\sqrt{2}\,i)}{\pi a}$$

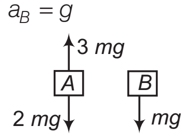
37. $\tau_1 = \frac{8}{9}\mu s$

$$\tau_2 = 18\mu s$$

$$\tau_3 = 4\mu s$$

38. $a_A = \frac{g}{2}$

$$a_B = g$$

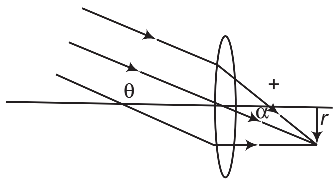


39. $-\frac{1}{F} = \frac{2}{f_l} + \frac{1}{\infty} \Rightarrow F = -\frac{15}{2}$

$$-\frac{2}{15} = \frac{1}{v} - \frac{1}{20}$$

$\Rightarrow v = -12$ cm *i.e.,* 12 cm left of AB.

40. $r = f \tan \alpha$



Hence, $\pi r^2 \propto f^2$

41. Disintegration of each nuclei is independent of any factor. Hence, each nuclei has same change of disintegration.

42. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow f = 5\,\text{cm}$

$$f = \frac{uv}{u+v}$$

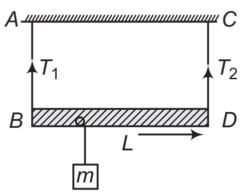
$$\frac{\Delta f}{f} = \left|\frac{\Delta u}{u}\right| + \left|\frac{\Delta v}{v}\right| + \frac{|\Delta u| + |\Delta v|}{|u| + |v|}$$

$$\Delta f = 0.15$$

The most appropriate answer is 5.00 ± 0.10 cm.

43. $\frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{1}{l}\sqrt{\frac{T_2}{\mu}}$

$$T_2 = \frac{T_1}{4}$$

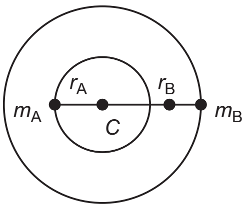


For rotational equilibrium,

$$T_1 x = T_2 (L - x)$$

$$\Rightarrow \quad x = \frac{L}{5}$$

44. $\frac{G m_A m_B}{(r_A + r_B)^2} = \frac{m_A r_A 4\pi^2}{T_A^2} = \frac{m_B r_B 4\pi^2}{T_B^2}$



$$\Rightarrow \quad m_A r_A = m_B r_B$$

$$\therefore \quad T_A = T_B$$

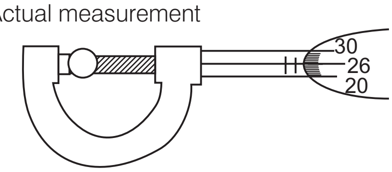
45. $\frac{2}{5} M R^2 = \frac{3}{2} M r^2$

$$r = \frac{2R}{\sqrt{15}}$$

46. Option (d) is correct.

47. Zero error $= 5 \times \frac{0.5}{50} = 0.05\,\text{mm}$

Actual measurement



$$= 2 \times 0.5\,\text{mm} + 25 \times \frac{0.5}{50} - 0.05\,\text{mm}$$

$$= 1\,\text{mm} + 0.25\,\text{mm} - 0.05\,\text{mm} = 1.20\,\text{mm}$$

48. $\frac{R_1}{R_2} = \frac{A_1}{A_2} = \frac{4}{1}$

$$\frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} = \frac{4}{1} \Rightarrow \frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{4}{1}$$

$$\frac{J_1}{J_2} = \frac{1}{4}$$

49. $\Delta W = 0 \quad \therefore \quad \Delta Q = \Delta U$

$\Delta Q = \Delta U = I^2 R \Delta t = 1^2 (100)(5 \times 60) = 30$ kJ

50. Rate of heat gain $= 1000 - 160 = 840$ J/s

$\therefore$ Required time $= \frac{2 \times 4.2 \times 10^3 \times (77 - 27)}{840}$

$$= 500\,\text{s} = 8\,\text{min}\ 20\,\text{s}$$

51. $f_1 = \frac{1}{l}\sqrt{\frac{B}{\rho}}$

$$f_2 = \frac{n}{4l}\sqrt{\frac{B}{\rho}} \Rightarrow \frac{f_1}{f_2} = \frac{4}{n} \Rightarrow f_2 = \frac{n}{4}f_1$$

52. Heating of glass bulb is by radiation.

53. Acceleration of the point of suspension

$$a = \frac{d^2y}{dt^2} = 2k = 2\text{ m/s}^2$$

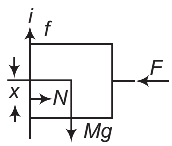
$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} \Rightarrow T_1 = 2\pi\sqrt{\frac{L}{10}} \text{ and } T_2 = 2\pi\sqrt{\frac{L}{12}}$$

$$\frac{T_1^2}{T_2^2} = \frac{6}{5}$$

54. For equilibrium, $f = Mg$

$$F = N$$

For rotational equilibrium normal will shift downward.

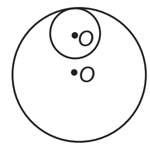


Hence torque due to friction about centre of mass = Torque due to normal reaction about centre of mass.

55. $E_P = \frac{\sigma}{2\varepsilon_0}(-\hat{k}) + \frac{(-2\sigma)}{2\varepsilon_0}(\hat{k}) + \frac{(-\sigma)}{2\varepsilon_0}(\hat{k})$

$$= \frac{-2\sigma}{\varepsilon_0}\hat{k}$$

56. $I_0 = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right] = 4MR^2$

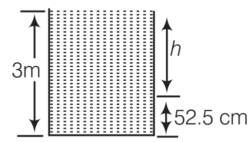


57. Since, B is constant,

$\therefore \quad \frac{d\phi}{dt} = 0$

$\therefore \quad i = 0$

58. $u^2 = \frac{2gh}{\left[1 - \left(\frac{A_0}{A}\right)^2\right]} = 50\ (\text{m/s})^2$



59. $B = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} = -\frac{(1.165 - 1.01) \times 10^5}{0.1}$

$$= 1.55 \times 10^5$$

60. $(Z-1)^2\lambda = \text{constant}$

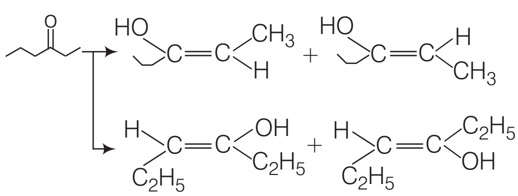
$\therefore \quad (10^2)\lambda = 4\lambda(Z-1)^2$

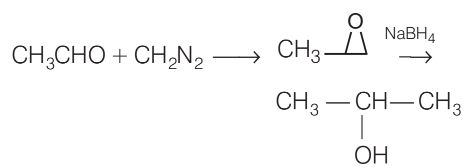
$\Rightarrow \quad Z = 6$

Chemistry

1. Compounds containing acidic hydrogen cannot be used in Grignard's synthesis and the compound CH_2OHCH_2Br contains acidic hydrogen.

2. Four enols, two pairs of geometrical isomers.



3. 

4. As phosphoric acid is a tribasic acid, its n-factor can be 1, 2 or 3. Hence, its normality can be M, $2M$ or $3M$, Where, M is molarity of solution. Therefore, N = 0.25, 0.50 or 0.75.

5. $CX_4 + 2O_2 \longrightarrow CO_2 + 2X_2O$

1 mol 2 mol = 2 × 32 g.

9.0 g 1.74 g.

$\because$ 1.74 g O_2 reacts with CX_4 = 9.0 g

$\therefore$ 64 g O_2 will react with $CX_4 = \frac{9.0 \times 64}{1.74} = 331$

Molecular mass of CX_4 = 331

or $331 = 12 + 4\,XX$

$$X = \frac{331 - 12}{4} = 79.8 \approx 80$$

6. In $S_4O_6^{2-}$, let 'x' be the oxidation number of S.

Thus, the element X may be bromine.

$$4x - 12 = -2$$

$$x = \frac{10}{4} = 2.5$$

7. $4Au(s) + 8CN^- + O_2(g) + 2H_2O \longrightarrow$

$$\underset{\text{soluble complex}}{4[Au(CN)_2]^-} + 4\,OH^-$$

8. The lanthanide contraction cancels almost exactly the normal size increase on descending a group of transition elements. Thus, Zr and Hf have equal size.

9. Mercurous ion is diamagnetic, this indicates that there is no unpaired electron, hence, mercurous ion is not Hg^+ but Hg_2^{2+}.

10. The high oxidising power of fluorine is due to its low heat of dissociation and high heat of hydration. Due to its smallest size, there is considerable repulsion in the non-bonding electrons which weakens the F—F bond. Thus, bond dissociation energy of F_2 is low.

11. HF has the highest bond dissociation enthalpy amongst all the halogen acids. The hydration enthalpy of the fluoride ion is also highest. The former predominates making HF the weakest of the halogen acids.

12. Pressure, $p = \frac{RT}{V - b} - \frac{a}{V^2}$

$$= \frac{0.082 \times 350}{35 - 0.14} - \frac{20}{(35)^2}$$

$$= 0.807 \text{ atm}$$

13. In a unit cell of KBr, there are four formula units of KBr therefore,

Density $(\rho) = \frac{4 \times 119}{6.023 \times 10^{23} \times a^3} = 2.75$

$$a^3 = 2.873 \times 10^{-22} \text{ cm}^3$$

$$= 2.778 \times 10^{-19} \text{mm}^3$$

Number of unit cells per mm^3

$$= \frac{10^{19}}{2.873} = 3.5 \times 10^{18}$$

14. There is one A per unit cell

Number of B per unit cell $= \frac{1}{8} \times 7 = \frac{7}{8}$

Empirical formula $= A_1 B_{7/8}$

$$= A_8 B_7$$

15. State functions do not depend upon path followed.

16. $S_{\text{rhombic}} \longrightarrow S_{\text{monoclinic}}$

$\Delta H = -297948 - (-298246) = 298$ J

$\Delta S = 32.68 - 32.04 = 0.64$

$\Delta G = \Delta H - T\Delta S = 298 - 298 \times 0.64 = 107.28$ J

17. Two angular nodes are found in d-orbitals. Also for a d-orbital to have one radial node it must be $4d$. [$\because$ Number of radial nodes = $n - l - 1$]

18. The valence shell configuration is $4s^2\ 4p^5$ and for the last electron $n = 4, l = 1$

$$m = -1, 0 \text{ or } +1, s = +\frac{1}{2} \text{ or } -\frac{1}{2}$$

Hence, $4, 1, 1, -\frac{1}{2}$

19. The molecular orbital electronic configuration of $CF(15\bar{e})$ is $\sigma 1s^2, \overset{*}{\sigma} 1s^2\ \sigma 2s^2\ \overset{*}{\sigma} 2s^2\ \sigma 2p_z^2$

$\pi 2p_x^2 \approx \pi 2p_y^2, \overset{*}{\pi} 2p_x^1$

Bond order (BO) $= \frac{10 - 5}{2} = 2.5$

In CF^+, there is one electron less in antibonding MO therefore, BO $= \frac{10 - 4}{2} = 3$

In CF^-, there is one electron more in antibonding MO therefore, BO $= \frac{10 - 6}{2} = 2$

Bond length $\propto \frac{1}{\text{Bond order}}$

Hence, the order of bond length in given series is $CF^+ < CF < CF^-$.

20. All are non-polar, molar mass follows the boiling points.

21. In structure II, position of N is changed which is not allowed in resonance.

22. Equilibrium concentration of $N_2O = O_2 = 0.05M$

Let the equilibrium concentration of NO_2 and NO be x M

$$K_c = 0.917 = \frac{(0.05)^2}{x^2} \Rightarrow x = 0.0522\text{ M}$$

Initial concentrations of NO_2 and NO were $0.05 + x = 0.1022$ M each.

Hence, moles of each gas NO_2 and NO taken initially $= 5 \times 0.1022 = 0.511$

23. Pressure of the system will not influenced the rate of a chemical reaction involving liquids.

24. The two rates are related as $-\frac{1}{5}\frac{d[Br^-]}{dt} = \frac{1}{3}\frac{d[Br_2]}{dt}]$

$$-\frac{d[Br^-]}{dt} = \frac{5}{3}\frac{d[Br_2]}{dt} = \frac{5}{3} \times 0.025 = 0.042\ \text{Ms}^{-1}$$

25. $LiAlH_4$ will reduce aldehyde further to alcohol.

26. Condensation of NH_2NH_2 at both nitrogens with C_6H_5CHO gives the desired product.

27. Acetaldehyde contains α-H, so it undergoes aldol condensation rather than Cannizzaro reaction.

28. Sucrose does not have free hemiacetal group, so does not undergo mutarotation.

29. Anomers differs in configuration only at C-1 carbons, they are simultaneously diastereomers.

30. 1 and 3 are C-2 epimers. C-2 epimers give same osazone on treatment with phenyl hydrazine.

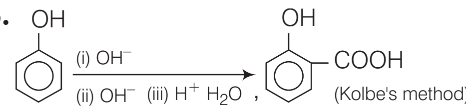
31. The tertiary structure of proteins refers to three dimensional folding of polymer chain.

32. Rubber has isotactic stereochemistry, not syndiotactic.

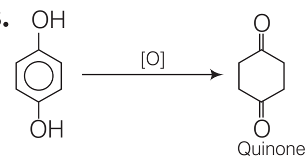
33. $$\underset{\text{Acrylonitrile}}{nCH_2{=}\underset{\substack{|\\CN}}{CH}} \xrightarrow{\text{Polymerisation}} \underset{\text{Polyacrylonitrile}}{\left(CH_2{-}\underset{\substack{|\\CN}}{CH}{-}\right)_n}$$

34. A nitro from *para* positions activate more for S_NAr reaction due to electron withdrawing resonance effect than two nitro from *meta* positions by inductive effect.

35. Ethyl in *o/p* directing group. Direct chlorination of ethyl benzene with $Cl_2/FeCl_3$ would give mixture of 2 and 4-chloroethyl benzene.

36. 

37. An aryl halide with $-NO_2$ substituents are highly activated for aromatic nucleophilic substitution reaction.

38. 

39. In physical adsorption, the adsorbent and adsorbate are held together by the van der Waals' forces.

40. Potassium stearate is an example of micelle.

41. Mass of Au deposited = Number of Faraday passed × Eq. mass

$$= \frac{0.30 \times 15 \times 60}{96485} \times \frac{197}{3} = 0.184\text{ g}$$

42. $E^0_{cell} = E^0_{cathode} - E^0_{anode}$

$$= 0.40 - (-0.44) = 0.84\text{ V}$$

43. $$\underset{(1-\alpha)}{nA} \rightleftharpoons \underset{\frac{\alpha}{n}}{An}$$

$$i = 1 - \alpha + \frac{\alpha}{n}$$

44. Osmotic pressure $= iCRT$, For both the solutions, all other conditions are similar except 'i' which is 1.0 for sucrose while 2.0 for NaCl.

Therefore, osmotic pressure of NaCl

$$= 2 \times 0.25$$
$$= 0.50\text{atm}$$

45. Mass of X decay $= 0.75\,g$

Moles of X decayed $= \dfrac{0.75}{253}$

$= $ moles of He formed

V(mL) of He at STP $= \dfrac{0.75 \times 22400}{253}$

$= 66 \cdot 40$ mL

46. $K \propto \dfrac{1}{t}$

$$\log \frac{k_2}{k_1} = \log \frac{t_1}{t_2} = \frac{E_a}{R}\left(\frac{T_2 - T_1}{T_1 \cdot T_2}\right)$$

$$\log \frac{48}{4} = \frac{E_a}{8.314}\left(\frac{10}{300 \times 290}\right)$$

$$E_a = 78.0 \text{ kJ mol}^{-1}$$

47. mmol of H^+ (initial)$= 200 \times 0.031 \times 2 = 12.4$

mmol of OH^- (initial)$= 84 \times 0.15 = 12.6$

mmol of OH^- (left) after neutralisation $= 0.2$

$[OH^-]_{final} = \dfrac{0.2}{284} = 7 \times 10^{-4}$M

pOH $= 3.15$ and pH $= 10.85$

48. In $(SiH_3)_3N$, lone pair of the nitrogen is transferred to the empty d-orbitals of silicon ($p\pi - d\pi$ overlap) thereby causing planarity of unit.

49. Solid N_2O_5 is ionic. X-ray diffraction shows that solid N_2O_5 exists as $NO_2^+NO_3^-$ (nitronium nitrate).

50. White phosphorus is highly reactive, bursting into flames when exposed to air, and is thus stored under water.

51. The activity of alkaline earth metals as reducing agent increases from Be to Ba.

52. An improved combination of baking powder contains about 40% starch, 30% $NaHCO_3$, 20% $NaAl(SO_4)_2$ and 10% $Ca(H_2PO_4)_2$. Here starch acts as filler.

53. If acid is weak, its conjugate base is strong. Greater the oxidation number of Cl, stronger the acid and thus, weaker the conjugate base.

$$\underset{+1}{HClO}\quad \underset{+3}{HClO_2}\quad \underset{+5}{HClO_3}\quad \underset{+7}{HClO_4}$$
$$\xrightarrow{\hspace{6cm}}$$

Oxidation number of Cl and acid strength increases

$$ClO^-\quad ClO_2^-\quad ClO_3^-\quad ClO_4^-$$
$$\xleftarrow{\hspace{6cm}}$$

Conjugate base strength increases.

Hence, ClO^- is the strongest base.

54. I. $\underset{-3}{NH_4^+}\,\underset{+5}{NO_3^-} \longrightarrow \underset{+1}{N_2O} + 2H_2O$

It is a redox reaction but not a disproportionation reaction. Since, different N-species are involved.

II. $\underset{-3}{NH_4^+}\,\underset{+3}{NO_2^-} \longrightarrow \underset{0}{N_2}$

same as (I)

III. $\underset{+5\;-1}{PCl_5} \longrightarrow \underset{+3\;-1}{PCl_3} + \underset{0}{Cl_2}$

Thus, it is also not a disproportionation reaction.

55. $Fe^{2+} + e^- \longrightarrow Fe^+$

$NO \longrightarrow NO^+ + e^-$

Based on magnetic properties of the complex, it is found that iron has three unpaired electrons thus it exists as Fe^+ formed by reduction of Fe^{2+} by NO which is oxidised to NO^+.

56. Li – Mg $\rightarrow$ diagonal relationship.

B — Al, Na — K and Ca — Mg $\rightarrow$ belongs to same group.

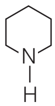
57. Along a group Z (effective nuclear charge) is almost constant. Thus, IE depends mainly on r_n (atomic radius).

58. Valence electron in $A = 3$

Valence electron in $B = 6$

Thus, A is electropositive and B is electronegative. A can lose three electrons and B can gain two electrons to attain stable configuration.

Thus, A exists as A^{3+} and B as B^{2-} thus, compound is A_2B_3.

59. In  lone pair of nitrogen is not involved in resonance as well as it has less s-character than the nitrogen of pyridine, hence the former is better electron donor.

60. Hyperconjugation involve delocalisation of σ-electrons with adjacent pi-system.

Mathematics

1. Given, $a = \cos\dfrac{2\pi}{7} + i.\sin\dfrac{2\pi}{7}$

$\therefore \quad a^7 = \cos 2\pi + i\sin 2\pi \qquad [\because e^{i\theta} = \cos\theta + i\sin\theta]$

$= 1$

Also, $\alpha = a + a^2 + a^4, \beta = a^3 + a^5 + a^6$

then the sum of roots,

$$S = \alpha + \beta = a + a^2 + a^3 + a^4 + a^5 + a^6$$

$$\Rightarrow \quad S = \frac{a(1-a^6)}{1-a} = \frac{a-a^7}{1-a}$$

$$= \frac{a-1}{1-a} = -1 \qquad [\because a^7 = 1]$$

Product of the roots,

$P = \alpha\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$= a^4 + a^5 + 1 + a^6 + 1 + a^2 + 1 + a + a^3 \quad [\because a^7 = 1]$

$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6) = 3 - 1 = 2$

Hence, the required quadratic equation is

$$x^2 + x + 2 = 0$$

2. Let $x = (1)^{1/n}$

$\Rightarrow \quad x^n - 1 = 0$

or $\quad x^n - 1 = (x-1)(x-\omega)(x-\omega^2)\ldots(x-\omega^{n-1})$

$$\Rightarrow \frac{x^n - 1}{x-1} = (x-\omega)(x-\omega^2)(x-\omega^3)\ldots(x-\omega^{n-1})$$

Putting $x = 9$ on both sides, we get

$$(9-\omega)(9-\omega^2)(9-\omega^3)\ldots(9-\omega^{n-1}) = \frac{9^n - 1}{8}$$

3. The given expression is

$$\left(x^2 + \frac{1}{x^2} + 2\right)^n = \left[\left(x + \frac{1}{x}\right)^2\right]^n$$

$$= \left(x + \frac{1}{x}\right)^{2n}$$

The binomial contains $(2n+1)$ terms in its expansion.

$\therefore$ The middle term is T_{n+1}.

$$T_{n+1} = {}^{2n}C_n\,(x)^{2n-n}\left(\frac{1}{x}\right)^n.$$

$$= {}^{2n}C_n$$

$$= \frac{(2n)!}{(2n-n)!\,n!} = \frac{(2n)!}{(n!)^2}$$

4. The equation of the tangent to parabola $y^2 = 4x$ is $y = mx + \dfrac{1}{m}$

This is also the tangent to parabola $x^2 = -8y$

$$\therefore \qquad x^2 = -8\left(mx + \frac{1}{m}\right)$$

$\Rightarrow mx^2 + 8m^2x + 8 = 0$

It has equal roots, if

$$64m^4 = 32m \qquad [\because b^2 = 4ac]$$

$$\Rightarrow \qquad m^3 = \frac{1}{2} \Rightarrow m = \frac{1}{\sqrt[3]{2}}$$

Hence, equation of the tangent is $y = \dfrac{1}{\sqrt[3]{2}}x + \sqrt[3]{2}$.

5. There may be two cases.

***Case* I.** Two children get none and one get three and others get one each.

Then, total number. of ways $= \dfrac{10!}{2!\times 3!\times 7!}\times 10!$

***Case* II.** Two get none and two get 2 each and the others get one each.

Then, total number of ways $= \dfrac{10!}{(2!)^4\times 6!}\times 10!$

Hence, total number of ways

$$= \frac{(10!)^2}{2!\times 3!\times 7!} + \frac{(10!)^2}{(2!)^4\times 6!} = \frac{(10!)^2\times 25}{(2!)^2\times 6!\times 84}$$

6. Variance of first n natural numbers is given by

$$\text{Variance} = \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2$$

$$= \frac{n(n+1)(2n-1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

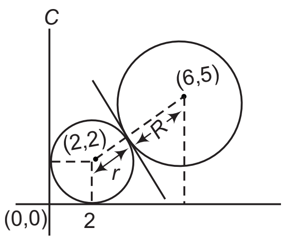
$$= (n+1)\left[\frac{2n-1}{6} - \frac{(n+1)}{4}\right]$$

$$= \frac{(n+1)}{12}[4n - 2 - 3n - 3]$$

$$= \frac{(n+1)}{12} \times (n-5)$$

7. Radius of smaller circle is $r = 2$ cm.

Since, it is touching both the axes, so its centre is (2,2).



The distance between their centres

$$= r + R = \sqrt{(5-2)^2 + (6-2)^2}$$

$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ units}$$

8. Set $S = \{1, 2, 3, \ldots, 20\}$ is to be partitioned into four sets of equal size *i.e.,* having 5-5 numbers.

The number of ways in which it can be done

$$= {}^{20}C_5 \times {}^{15}C_5 \times {}^{10}C_5 \times {}^{5}C_5$$

$$= \frac{20!}{15! \times 5!} \times \frac{15!}{10! \times 5!} \times \frac{10!}{5! \times 5!} \times 1 = \frac{(20!)}{(5!)^4}$$

9. Total number of cases $= {}^{50}C_1 = 50$

Let A be the event of selecting ticket with sum of digits '8'.

Favourable cases to A are $\{08, 17, 26, 35, 44\}$.

Let B be the event of selecting ticket with product of its digits '7'

Favourable cases to B is only $\{17\}$.

$$\text{Now, } P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/50}{5/50} = \frac{1}{5}$$

10. General term in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is

$$T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \cdot \left(\frac{-k}{x^2}\right)^r$$

$$= {}^{10}C_r x^{\frac{10-r}{2}} \cdot (-k)^r \cdot x^{-2r}$$

$$= {}^{10}C_r (-k)^r x^{\left(\frac{10-5r}{2}\right)}$$

$\because$ The term is free from x

$\therefore$ Put $\quad \frac{10-5r}{2} = 0$

$\Rightarrow \quad r = 2$

Now, $\quad {}^{10}C_2(-k)^2 = 405$

$\Rightarrow \quad \frac{10 \times 9}{1 \times 2} \cdot k^2 = 405$

$\Rightarrow \quad k^2 = \frac{405}{45} = 9$

$\Rightarrow \quad k = \pm 3$

11. The given equation is $8x^2 - 26x + 15 = 0$.

$\therefore$ The sum of roots,

$\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{26}{8} = \frac{13}{4}$ and product of roots,

$\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} = \frac{15}{8}$

$$\therefore \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2}}$$

$$= \frac{\frac{13}{4}}{1 - \frac{15}{8}} = -\frac{26}{7}$$

$$\text{Now, } \cos(\alpha+\beta) = \frac{1 - \tan^2\left(\frac{\alpha+\beta}{2}\right)}{1 + \tan^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$\left[\because \cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right]$$

$$= \frac{1 - \left(-\frac{26}{7}\right)^2}{1 + \left(-\frac{26}{7}\right)^2} = \frac{49 - 676}{49 + 676} = -\frac{627}{725}$$

12. Given relations are

$a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ ...(i)

$a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$ ...(ii)

On adding Eq. (i) and (ii) we get

$$(m+n) = a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha + 3a\cos^2\alpha\sin\alpha + a\sin^3\alpha$$

$$= a(\cos\alpha + \sin\alpha)^3$$

Similarly on subtracting Eq. (ii) from Eq. (i), we get

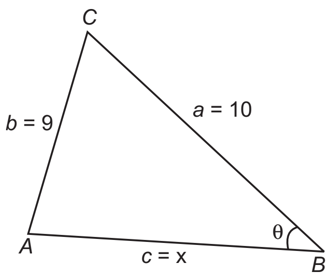
$$m - n = a(\cos\alpha - \sin\alpha)^3$$

Now, $(m+n)^{2/3} + (m-n)^{2/3}$

$$= a^{2/3}\{(\cos\alpha + \sin\alpha)^2 + (\cos\alpha - \sin\alpha)^2\}$$

$$= a^{2/3}\{2(\cos^2\alpha + \sin^2\alpha)\} = 2a^{2/3}$$

13. Let in the figure, $\angle A > \angle B > \angle C$



$\because$ Angles are in A.P.

$\therefore$ $\angle C = \theta - d$, $\angle B = \theta$

and $\angle A = \theta + d$

As $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $\theta + d + \theta + \theta - d = 180°$

$\Rightarrow$ $3\theta = 180°$

$\Rightarrow$ $\theta = 60°$

Now in ΔABC,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos 60° = \frac{(10)^2 + x^2 - 9^2}{2 \times 10 \times x}$$

$$\Rightarrow \frac{1}{2} = \frac{100 + x^2 - 9^2}{20x}$$

$\Rightarrow$ $x^2 - 10x + 19 = 0$

$$\Rightarrow x = \frac{10 \pm \sqrt{100 - 76}}{2}$$

$$= \frac{10 \pm \sqrt{24}}{2} = 5 \pm \sqrt{6}$$

14. Let the equation of the line parallel to $x - 2y = 1$ is $x - 2y + \lambda = 0$

Since, it passes through (3, 5)

$\therefore$ $3 - 10 + \lambda = 0 \Rightarrow \lambda = 7$

$\therefore$ The line is $x - 2y + 7 = 0$.

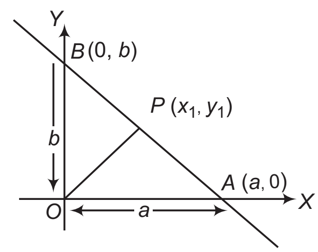
The point of intersection of $x - 2y + 7 = 0$ and $2x + 3y - 14 = 0$ is (1, 4).

$\therefore$ The distance between (3, 5) and (1, 4)

$$= \sqrt{(3-1)^2 + (5-4)^2} = \sqrt{4+1} = \sqrt{5}$$

15. Equation of line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ...(i)

Let the foot of the perpendicular drawn from the origin to the line be $P(x_1, y_1)$.



Since, $OP \perp AB$

$\therefore$ slope of $OP \times$ slope of $AB = -1$

$\Rightarrow$ $\dfrac{y_1}{x_1} \times \dfrac{b}{-a} = -1 \Rightarrow by_1 = ax_1$ ...(ii)

Since, P lies on the line AB, so

$$\frac{x_1}{a} + \frac{y_1}{b} = 1$$

$\Rightarrow$ $bx_1 + ay_1 = ab$ ...(iii)

From (ii) and (iii), we get

$$x_1 = \frac{ab^2}{a^2 + b^2} \quad \text{and} \quad y_1 = \frac{a^2 b}{a^2 + b^2}$$

Now, $x_1^2 + y_1^2 = \left(\dfrac{ab^2}{a^2+b^2}\right)^2 + \left(\dfrac{a^2b}{a^2+b^2}\right)^2$

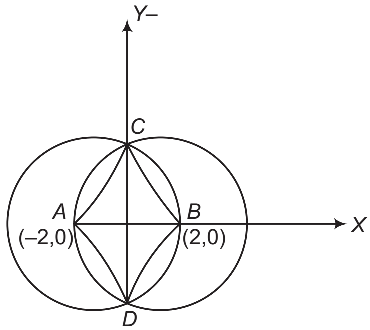
$$= \frac{a^2b^4}{(a^2+b^2)^2} + \frac{a^4b^2}{(a^2+b^2)^2} = \frac{a^2b^2}{(a^2+b^2)^2}(a^2+b^2)$$

$$= \frac{a^2b^2}{a^2+b^2} = \frac{1}{\dfrac{1}{a^2} + \dfrac{1}{b^2}}$$

$$\Rightarrow x_1^2 + y_1^2 = \frac{1}{c^2} \quad \left(\because \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}\right)$$

Thus, the locus of $P(x_1, y_1)$ is $x^2 + y^2 = c^2$, which is a circle.

16. Circles with centres $A(-2, 0)$ and $B(+2, 0)$, each of radius 4 has Y-axis as their common chord.



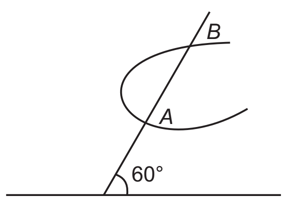
$\Rightarrow \Delta\, ABC$ is an equilateral triangle.
Hence, the area of rhombus $ADBC$ is

$$= 2 \cdot \frac{\sqrt{3}}{4} \times 4^2 = 8\sqrt{3}$$

17. Given $P = (\sqrt{3}, 0)$

$\therefore$ Equation of line AB is

$$\frac{x - \sqrt{3}}{\cos 60°} = \frac{y - 0}{\sin 60°} = r \qquad \text{(say)}$$



$$\Rightarrow \quad x = \sqrt{3} + \frac{r}{2}, \; y = \frac{r\sqrt{3}}{2}$$

$\therefore$ point $\left(\sqrt{3} + \frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ lies on $y^2 = x + 2$

$$\therefore \quad \frac{3r^2}{4} = \sqrt{3} + \frac{r}{2} + 2$$

$$\Rightarrow \quad \frac{3r^2}{4} - \frac{r}{2} - (2 + \sqrt{3}) = 0$$

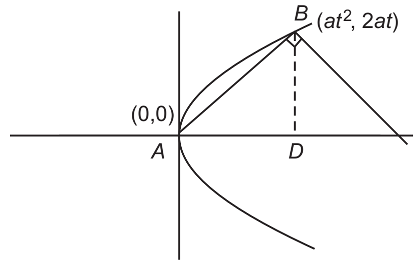
Let the roots be r_1 and r_2, then the product

$$r_1 \times r_2 = PA\,.\,PB = \left|\frac{-(2+\sqrt{3})}{\frac{3}{4}}\right|$$

$$= \frac{4(2 + \sqrt{3})}{3}$$

18. Given equation of parabola is $y^2 = 4ax$.

Let the co-ordinates of B are $(at^2, 2at)$, then the slope of $AB = \frac{2}{t}$



Since, $BC \perp AB$

$\therefore$ Slope of $BC = \frac{-t}{2}$

The equation of BC is

$$y - 2 = -\frac{t}{2}(x - at^2)$$

$\because$ This line meets the X-axis at point C.
So, put $y = 0$, we get

$$\Rightarrow \qquad x = 4a + at^2$$

So, the distance $CD = 4a + at^2 - at^2$

$$= 4a$$

19. Given equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{\frac{7}{4}} = 1$

Here, $\qquad a^2 = 4$

and $\qquad b^2 = \frac{7}{4}$

$\because \qquad b^2 = a^2(1 - e^2)$

$\therefore \qquad \frac{7}{4} = 4(1 - e^2)$

$\Rightarrow \qquad e^2 = 1 - \frac{7}{16} = \frac{9}{16} \Rightarrow e = \frac{3}{4},$

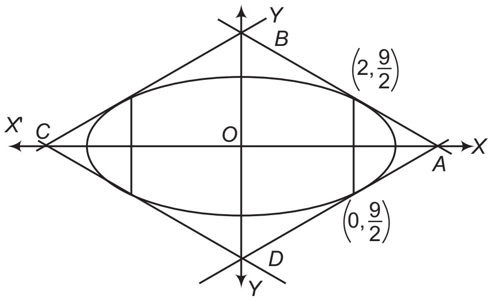
Thus, the foci are $\left(\pm\frac{3}{2}, 0\right)$.

The radius of the required circle

$$= \sqrt{\left(\frac{3}{2} - \frac{1}{2}\right)^2 + (0 - 2)^2}$$

$$= \sqrt{5}$$

20. The quadrilateral formed by the tangents at the end points of the latusrectum is a rhombus. It is symmetrical about the axes.



So, total area is four times the area of the right triangle formed by the tangents and the axes in the first quadrant.

Now, $\quad ae = \sqrt{a^2 - b^2}$

$\Rightarrow \quad ae = 2$

$\therefore$ The co-ordinates of one end point of latusrectum are $\left(2, \frac{5}{3}\right)$.

The equation of tangent at that point is

$\frac{x}{\frac{9}{2}} + \frac{y}{3} = 1$, this meets the co-ordinate *axes* at

$A\left(0, \frac{9}{2}\right)$ and $B\ (3, 0)$.

Area of $\Delta AOB = \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$

Hence, the area of rhombus $ABCD$

$$= 4 \times \frac{27}{4} = 27 \text{ sq units}$$

21. Equation of chord of hyperbola $x^2 - y^2 = a^2$ with mid-point as (h, k) is given by

$$xh - yk = h^2 - k^2$$

$$\Rightarrow \quad y = \frac{h}{k}x - \frac{(h^2 - k^2)}{k}$$

This will touch the parabola $y^2 = 4ax$, if

$$-\left(\frac{h^2 - k^2}{k}\right) = \frac{a}{\frac{h}{k}}$$

$$\Rightarrow \quad ak^2 = h^3 + k^2 h$$

$\therefore$ Locus of the mid-point is $x^3 = y^2\ (x - a)$

22. $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$

$$= \lim_{x \to 0}\left[\frac{(e^{\sin x} - 1)}{\sin x} \times \frac{\sin x}{x}\right]$$

$$= \lim_{\sin x \to 0} \frac{e^{\sin x} - 1}{\sin x} \times \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1 \times 1 = 1$$

23. By truth table

p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$q \Leftrightarrow p$	$\sim(q \Leftrightarrow p)$	$\sim(p \wedge q) \vee \sim(q \Leftrightarrow p)$
T	T	T	F	T	F	F
T	F	F	T	F	T	T
F	T	F	T	F	T	T
F	F	F	T	T	F	T

It is clear that the given statement is neither tautology nor contradiction.

24. Given determinant is

$$\Delta = \begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, we get

$$\Delta = \begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^x - b^{-x})^2 & 1 \\ 4 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 4\begin{vmatrix} 1 & (a^x - a^{-x})^2 & 1 \\ 1 & (b^x - b^{-x})^2 & 1 \\ 1 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$$

$= 0 \quad$ ($\because$ two columns are identical)

25. Putting $r = 1, 2, 3, \ldots, n$ and using the formula

$$\Sigma 1 = n$$

and $\quad \Sigma\, r = \frac{n(n+1)}{2}$

$$\Sigma(2r - 1) = 1 + 3 + 5 + \ldots = n^2$$

$$\therefore \sum_{r=1}^{n} \Delta_r = \begin{vmatrix} n & n & n \\ n(n+1) & n^2 + n + 1 & n^2 + n \\ n^2 & n^2 & n^2 + n + 1 \end{vmatrix} = 56$$

Applying $C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$, we get

$$\begin{vmatrix} 0 & 0 & n \\ 0 & 1 & n^2 + n \\ -n - 1 & -n - 1 & n^2 + n + 1 \end{vmatrix} = 56$$

$$\Rightarrow \quad n(n+1) = 56$$

$$\Rightarrow \quad n^2 + n - 56 = 0$$

$\Rightarrow \quad (n+8)(n-7) = 0$

$\Rightarrow \quad n = 7, (n \neq -8)$

26. $\Delta = \begin{vmatrix} e^a & e^{2a} & e^{3a} \\ e^b & e^{2b} & e^{3b} \\ e^c & e^{2c} & e^{3c} \end{vmatrix} - \begin{vmatrix} e^a & e^{2a} & 1 \\ e^b & e^{2b} & 1 \\ e^c & e^{2c} & 1 \end{vmatrix}$

$= e^a \cdot e^b \cdot e^c \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} + \begin{vmatrix} e^a & 1 & e^{2a} \\ e^b & 1 & e^{2b} \\ e^c & 1 & e^{2c} \end{vmatrix}$

$= e^{(a+b+c)} \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} - \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix}$

$= (e^{a+b+c} - 1) \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix}$

$= 0 \qquad (\because a + b + c = 0)$

27. $fog = f\{g(x)\} = |\sin x| = \sqrt{\sin^2 x}$

Also, $\quad gof = g\{f(x)\} = \sin^2 \sqrt{x}$

Obviously, $\quad \sqrt{\sin^2 x} = \sqrt{g(x)}$

and $\quad \sin^2 \sqrt{x} = \sin^2\{f(x)\}$

i.e., $\quad g(x) = \sin^2 x$

and $\quad f(x) = \sqrt{x}$

28. Given, $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

$(fof)x = \{f(f(x))\}$

$= f\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \dfrac{\dfrac{x}{\sqrt{1+x^2}}}{\sqrt{1+\dfrac{x^2}{1+x^2}}}$

$\Rightarrow \quad (fof)x = \dfrac{x}{\sqrt{1+2x^2}}$

Now, $\quad (fofof)x = f\{(fof)x\}x = f\left(\dfrac{x}{\sqrt{1+2x^2}}\right)$

$= \dfrac{\dfrac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\dfrac{x^2}{1+2x^2}}} = \dfrac{x}{\sqrt{1+3x^2}}$

29. On putting $y = 0$, in the given relation, we get $f(x) + f(x) = k \cdot f(x) \cdot f(0)$

$\Rightarrow \quad 2f(x) = k \cdot f(x) \qquad [\because f(0) = 1]$

$\therefore \quad k = 2$

30. Given, $\tan^{-1}\dfrac{a}{x} + \tan^{-1}\dfrac{b}{x} = \dfrac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\left[\dfrac{\left(\dfrac{a}{x}+\dfrac{b}{x}\right)}{1-\dfrac{ab}{x^2}}\right] = \dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{\dfrac{a+b}{x}}{\dfrac{x^2-ab}{x^2}} = \tan\dfrac{\pi}{2}$

$\Rightarrow \quad x^2 = ab$

$\Rightarrow \quad x = \sqrt{ab}$

31. $\cos^{-1}[\cos\{2 \cdot \cot^{-1}(\sqrt{2}-1)\}]$

$= \cos^{-1}[\cos\{2 \times 67.5\}]$

$= \cos^{-1}[\cos 135°]$

$= 135°$

$= \dfrac{3\pi}{4}$

32. We have,

$\sum_{m=1}^{n} \tan^{-1}\left(\dfrac{2m}{m^4+m^2+2}\right)$

$= \sum_{m=1}^{n} \tan^{-1}\left[\dfrac{2m}{1+(m^2+m+1)(m^2-m+1)}\right]$

$= \sum_{m=1}^{n} \tan^{-1}\left[\dfrac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right]$

$= \sum_{m=1}^{n} \{\tan^{-1}[m^2+m+1] - \tan^{-1}[m^2-m+1]\}$

$= \tan^{-1} 3 - \tan^{-1} 1 + \tan^{-1} 7 - \tan^{-1} 3$

$+ (\tan^{-1} 13 - \tan^{-1} 7) + \ldots + \tan^{-1}(n^2+n+1)$

$- \tan^{-1}(n^2-n+1)$

$= \tan^{-1}\left(\dfrac{n^2+n+1-1}{1+(n^2+n+1)\cdot 1}\right) = \tan^{-1}\left(\dfrac{n^2+n}{2+n^2+n}\right)$

33. Put $x = \frac{\pi}{2} - h$ as $x \to \frac{\pi}{2}, h \to 0$

$\therefore$ Given, limit $= \lim_{h \to 0} \frac{1 - \tan\left(\frac{\pi}{4} + \frac{h}{2}\right)(1 - \cos h)}{1 + \tan\left(\frac{\pi}{4} + \frac{h}{2}\right)(2h)^3}$

$$= \lim_{h \to 0} \tan\left(\frac{h}{2}\right) \lim_{h \to 0} \frac{2 \cdot \sin^2\left(\frac{h}{2}\right)}{8h^3}$$

$$= \lim_{h \to 0} \frac{1}{4}\left(\frac{\tan\frac{h}{2}}{2 \times \frac{h}{2}}\right) \times \lim_{h \to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4}$$

$$= \frac{1}{32}$$

34. We have,

$$f(x) = \begin{cases} \frac{\sin^3(\sqrt{x}) \cdot \log(1 + 3x)}{x \cdot (\tan^{-1}\sqrt{x})^2 \cdot (e^{5\sqrt{x}} - 1)}, & x \neq 0 \\ a, & x = 0 \end{cases}$$

For continuity in $[0, 1]$, $f(0) = f(0^+)$, otherwise it is discontinuous.

$$\therefore \quad \lim_{x \to 0^+} \frac{\sin^3(\sqrt{x}) \cdot \log(1 + 3x)}{x \cdot (\tan^{-1}\sqrt{x})^2 \cdot (e^{5\sqrt{x}} - 1)}$$

$$= \lim_{x \to 0^+}\left[\frac{3}{5} \cdot \frac{\sin^3\sqrt{x}}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^2}{(\tan^{-1}\sqrt{x})^2} \times \frac{\log(1 + 3x)}{3x} \cdot \frac{5\sqrt{x}}{e^{5\sqrt{x}} - 1}\right]$$

$$= \frac{3}{5} \lim_{x \to 0^+} \frac{\sin^3\sqrt{x}}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^2}{\tan^{-1}\sqrt{x}} \times \frac{\log(1 + 3x)}{3x} \cdot \frac{5\sqrt{x}}{e^{5\sqrt{x}} - 1} = a$$

$$\Rightarrow \quad a = \frac{3}{5}$$

35. Given $y = |\sin x|^{|x|}$

In the neighbourhood of $-\frac{\pi}{6}$, $|x|$ and $|\sin x|$ both are negative

i.e., $y = (-\sin x)^{-x}$

Taking log on both sides, we get

$\log y = -x \log(-\sin x)$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (-x)\frac{1}{-\sin x}(-\cos x) + \log(-\sin x)(-1)$$

$$= -x \cdot \cot x - \log(-\sin x)$$

$$= -[x \cot x + \log(-\sin x)]$$

$$\Rightarrow \quad \frac{dy}{dx} = -y[x \cot x + \log(-\sin x)]$$

$$\therefore \left(\frac{dy}{dx}\right)_{x = -\frac{\pi}{6}} = (2)^{-\frac{\pi}{6}} \frac{[6 \log 2 - \sqrt{3}\pi]}{6}$$

36. Let $y = x^x$

$$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$$

For increasing function, $\frac{dy}{dx} > 0$

$\Rightarrow \quad x^x(1 + \log x) > 0$

$\Rightarrow \quad 1 + \log x > 0$

$\Rightarrow \quad \log_e x > \log_e \frac{1}{e}$

$\Rightarrow \quad x > \frac{1}{e}$

$\therefore$ The function is increasing when $x > \frac{1}{e}$

37. Let $f(x) = 3\sin x - 4\sin^3 x = \sin 3x$

Since, $\sin x$ is increasing in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \quad -\frac{\pi}{2} \leq 3x \leq \frac{\pi}{2} \quad \Rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$$

Thus, the length of interval $= \left[\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right] = \frac{\pi}{3}$

38. Given curve is $y = f(x) = x^2 + bx - b$

On differentiating w.r.t.x, we get

$$\frac{dy}{dx} = 2x + b$$

The equation of the tagent at $(1, 1)$ is

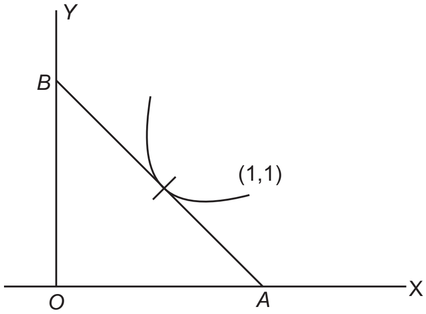
$$y - 1 = \left(\frac{dy}{dx}\right)_{(1, 1)}(x - 1)$$

$\Rightarrow \quad y - 1 = (b + 2)(x - 1)$

$\Rightarrow \quad (2 + b)x - y = 1 + b$

$$\Rightarrow \quad \frac{x}{(1 + b)/(2 + b)} - \frac{y}{(1 + b)} = 1$$

So, $OA = \frac{1 + b}{2 + b}$ and $OB = -(1 + b)$



Now, area of ΔAOB

$$= \frac{1}{2}\frac{(1+b)}{(2+b)}[-(1+b)] = 2 \quad \text{(given)}$$

$\Rightarrow \quad 4(2+b) + (1+b)^2 = 0$

$\Rightarrow \quad 8 + 4b + 1 + b^2 + 2b = 0$

$\Rightarrow \quad b^2 + 6b + 9 = 0$

$\Rightarrow \quad (b+3)^2 = 0 \Rightarrow b = -3$

39. The given curve is $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

$\Rightarrow \quad n\left(\frac{x}{a}\right)^{n-1} \cdot \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \cdot \frac{1}{b}\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{b}{a}\frac{\left(\frac{x}{a}\right)^{n-1}}{\left(\frac{y}{b}\right)^{n-1}}$

At (a, b), $\frac{dy}{dx} = -\frac{b}{a}\frac{(1)^{n-1}}{(1)^{n-1}} = -\frac{b}{a}$

$\therefore$ The tangent at (a, b) is

$\Rightarrow \quad y - b = -\frac{b}{a}(x - a)$

$\Rightarrow \quad ay - ab = -bx + ab$

$\Rightarrow \quad bx + ay = 2ab$

$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 2$

40. Given curve is $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

Let radius vector is 'r'

$\therefore \quad r^2 = x^2 + y^2$

$\Rightarrow \quad r^2 = \frac{a^2y^2}{y^2 - b^2} + y^2 \qquad \left(\because \frac{a^2}{x^2} + \frac{b^2}{y^2} = 1\right)$

For minumum value of r,

$$\frac{d(r^2)}{dy} = 0$$

$\Rightarrow \quad \frac{-2y\, b^2a^2}{(y^2 - b^2)^2} + 2y = 0$

$\Rightarrow \quad y^2 = b(a+b)$

$\therefore \quad x^2 = a(a+b)$

$\Rightarrow \quad r^2 = (a+b)^2$

$\Rightarrow \quad r = a + b$

41. Consider the function $f(x) = \frac{x^2}{x^3 + 200}$

$$f'(x) = x\frac{(400 - x^3)}{(x^3 + 200)^2} = 0$$

$\Rightarrow \quad x^3 = 400$

$\Rightarrow \quad x = (400)^{1/3}$

When $x < 0$, $f'(x) > 0$

$x > 0$, $f'(x) < 0$

$\therefore$ $f(x)$ has maximum at $x = (400)^{1/3}$

Since, $7 < (400)^{1/3} < 8$, either a_7 or a_8 is the greatest term of the sequence.

$\because \quad a_7 = \frac{49}{543}, a_8 = \frac{8}{89}$

and $\quad \frac{49}{543} > \frac{8}{89}$

$a_7 = \frac{49}{543}$ is the greatest.

42. $f(x) = \begin{cases} |x^3 + x^2 + 3x + \sin x|\left(3 + \sin\frac{1}{x}\right) & x \neq 0 \\ 0, & x = 0 \end{cases}$

Let $\quad g(x) = x^3 + x^2 + 3x + \sin x$

$g'(x) = 3x^2 + 2x + 3 + \cos x$

$= 3\left(x^2 + \frac{2x}{3} + 1\right) + \cos x$

$\Rightarrow \quad g'(x) = 3\left\{\left(x + \frac{1}{3}\right)^2 + \frac{8}{9}\right\} + \cos x > 0$

and $2 < 3 + \sin\left(\frac{1}{x}\right) < 4$

Hence, minimum value of $f(x)$ is 0 at $x = 0$

Hence, the number of points = 1

43. Let, $I=\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$

On dividing numerator and denominator by x^2, we get

$$I=\int \frac{1-\frac{1}{x^2}}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$$

Put $x+\frac{1}{x}=t$

$\Rightarrow \left(1-\frac{1}{x^2}\right)dx=dt$ and

$x^2+\frac{1}{x^2}=t^2-2$

$\therefore \quad I=\int \frac{dt}{(t^2-2+3)\tan^{-1}t}$

$=\int \frac{dt}{(1+t^2)\tan^{-1}t}$

$=\log(\tan^{-1}t)+C$

$=\log\left[\tan^{-1}\left(x+\frac{1}{x}\right)\right]+C$

44. Let, $I=\int \frac{\cos x}{\sin^2 x(\sin x+\cos x)}dx$

On dividing numerator and denominator by $\cos^3 x$, we get

$$I=\int \frac{\sec^2 x}{\tan^2 x\,(1+\tan x)}dx$$

Put $\tan x=t \Rightarrow \sec^2 x dx=dt$

$\therefore \quad I=\int \frac{dt}{t^2(1+t)}$

$=\int\left[-\frac{1}{t}+\frac{1}{t^2}+\frac{1}{1+t}\right]dt$

(using partial fraction)

$\Rightarrow \quad I=-\int \frac{1}{t}dt+\int t^{-2}dt+\int \frac{1}{1+t}dt$

$=-\log|t|-\frac{1}{t}+\log|1+t|+C$

$=-\frac{1}{\tan x}+\log\left|\frac{1+\tan x}{\tan x}\right|+C$

$=-\cot x+\log\left|\frac{1+\tan x}{\tan x}\right|+C$

45. Let, $I=\int \frac{dx}{\sin x-\cos x+\sqrt{2}}$

$\Rightarrow \quad I=\int \frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)+\sqrt{2}}$

$=\frac{1}{\sqrt{2}}\int \frac{dx}{1-\cos\left(x+\frac{\pi}{4}\right)}=\frac{1}{\sqrt{2}}\int \frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{8}\right)}$

$=\frac{1}{2\sqrt{2}}\int \frac{dx}{\sin^2\left(\frac{x}{2}+\frac{\pi}{8}\right)}=\frac{1}{2\sqrt{2}}$

$\int \operatorname{cosec}^2\left(\frac{x}{2}+\frac{\pi}{8}\right)dx$

$=\frac{1}{2\sqrt{2}}\left[\frac{-\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\frac{1}{2}}\right]+C$

$=-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+C$

46. $\int_0^{\frac{3\pi}{2}}\sin\left(\frac{2x}{\pi}\right)dx=\int_0^{\frac{\pi}{2}}\sin\left(\frac{2x}{\pi}\right)dx+\int_{\frac{\pi}{2}}^{\pi}\sin\left(\frac{2x}{\pi}\right)dx$
$+\int_{\pi}^{\frac{3\pi}{2}}\sin\left(\frac{2x}{\pi}\right)dx$

$=0+\sin 1\int_{\frac{\pi}{2}}^{\pi}dx+\sin 2\int_{\pi}^{\frac{3\pi}{2}}dx$

$=\frac{\pi}{2}(\sin 1+\sin 2)$

47. Let, $I=\int_{\frac{1}{n}}^{(an-1)/n}\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$

$=\int_{\frac{1}{n}}^{a-\frac{1}{n}}\frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx \quad \ldots(\text{i})$

$=\int_{\frac{1}{n}}^{a-\frac{1}{n}}\frac{\left(\sqrt{\frac{1}{n}+a-\frac{1}{n}-x}\right)}{\sqrt{a-\left(\frac{1}{n}+a-\frac{1}{n}-x\right)}+\sqrt{\frac{1}{n}+a-\frac{1}{n}-x}}dx$

$\Rightarrow \quad I=\int_{\frac{1}{n}}^{a-\frac{1}{n}}\frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx \quad \ldots(\text{ii})$

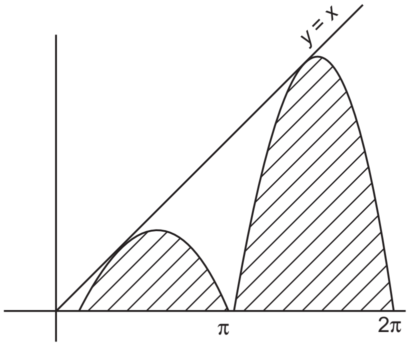
On adding Eqs. (i) and (ii), we get

$2I=\int_{\frac{1}{n}}^{a-\frac{1}{n}}1dx=[x]_{\frac{1}{n}}^{a-\frac{1}{n}}$

$=a-\frac{1}{n}-\frac{1}{n}=\frac{na-2}{n}\Rightarrow I=\frac{na-2}{2n}$

48. $y = x\,|\sin x|$

$$\Rightarrow \quad y = \begin{cases} x\sin x, & 0 \le x \le \pi \\ -x\sin x, & \pi < x \le 2\pi \end{cases}$$



$\therefore$ Required Area $= \int_0^{\pi} x\sin x + \int_{\pi}^{2\pi} (-x\sin x)\,dx$

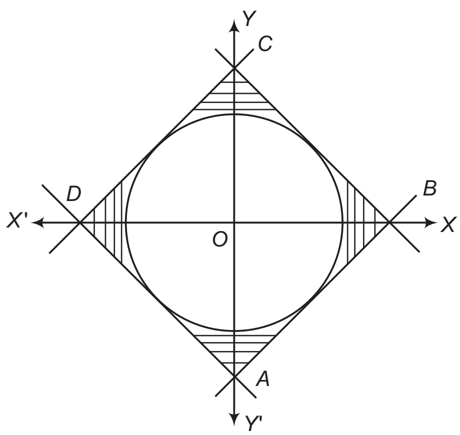
$$= \left[x(-\cos x)\right]_0^{\pi} - \int_0^{\pi} (-\cos x)\,dx$$

$$-\left[\{x(-\cos x)\}\right]_{\pi}^{2\pi} + \int_{\pi}^{2\pi} (-\cos x)\,dx$$

$$= \left[\pi + \sin x\right]_0^{\pi} + \left[(2\pi + \pi) - \sin x\right]_{\pi}^{2\pi}$$

$$= 4\pi \text{ sq units}$$

49. The curves $y + x \le 1$ and $|y - x| \le 1$ form a square $ABCD$ and $2x^2 + 2y^2 = 1$ is a circle of radius $\frac{1}{\sqrt{2}}$ and centre (0, 0).



$\therefore$ Required Area

$$= \text{Area of square} - \text{Area of circle}$$

$$= \left(2 - \frac{\pi}{2}\right) \text{ sq units}$$

50. $y = u^m \Rightarrow \frac{dy}{dx} = m \cdot u^{m-1} \cdot \frac{du}{dx}$

Hence, $2x^4 \cdot mu^{m-1}u^m \cdot \frac{du}{dx} + u^{4m} = 4x^6$

$$\Rightarrow \quad \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4 \cdot u^{2m-1}}$$

$$\therefore \quad 4m = 6 \Rightarrow m = \frac{3}{2}$$

51. Equation of the normal at (x_1, y_1) is

$$y - y_1 = \frac{dx}{dy}(x - x_1)$$

Put $\quad y = 0$, then

$$x = x_1 + y_1\frac{dy}{dx}$$

$$y_1^2 = 2x_1 x$$

$$= 2x_1\left(x_1 + y_1\frac{dy}{dx}\right)$$

$$\Rightarrow \quad y_1^2 = 2x_1^2 + 2x_1 y_1\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y_1^2 - 2x_1^2}{2x_1 y_1} = \frac{\left(\frac{y_1}{x_1}\right)^2 - 2}{2\left(\frac{y_1}{x_1}\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 2}{2y/x}$$

Put $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx},$$

We have, $v + \frac{xdv}{dx} = \frac{v^2 - 2}{2v}$

$$\Rightarrow \quad x \cdot \frac{dv}{dx} = -\frac{(2 + v^2)}{2v}$$

$$\Rightarrow \quad \frac{2v \cdot dv}{2 + v^2} + \frac{dx}{x} = 0$$

On integrating both sides, we get

$$\log(2 + v^2) + \log|x| = \log C$$

$$\Rightarrow \quad \log(2 + v^2)|x| = \log C$$

$$\Rightarrow \quad |x|\left(2 + \frac{y^2}{x^2}\right) = C$$

$\because$ It passes through (2, 1) then, $2\left(2+\frac{1}{4}\right)=C$

$$\Rightarrow \quad C=\frac{9}{2}$$

$$\therefore \quad \frac{|x|(2x^2+y^2)}{x^2}=\frac{9}{2}$$

$$\Rightarrow \quad 2x^2+y^2=\frac{9}{2}|x|$$

$$\Rightarrow \quad 4x^2+2y^2-9|x|=0$$

52. $|a\times b|^2=|a|^2\,|b|^2-2(a\cdot b)^2=16-4=12$ and

$|c|^2=(2\,a\times b-3b)^2=(2\,a\times b-3b)$

$=4|\,a\;\times b|^2+9|\,b|^2=4.12+9.16$

$=192\Rightarrow |\,c|=8\sqrt{3}$

Now, $\quad b\cdot c=b(2\,a\times b-3b)=-3|\,b|^2=-48$

$$\therefore \quad \cos\theta=\frac{b\cdot c}{|\,b|\,|\,c|}=-\frac{48}{4\cdot 8\sqrt{3}}=-\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta=\frac{5\pi}{6}$$

53. Let the first term and common ratio of a GP be α and β, then

$a=\alpha\cdot\beta^{p-1}$, $b=\alpha\cdot\beta^{q-1}$ and $c=\alpha\cdot\beta^{r-1}$

$\therefore \quad \log a=\log\alpha+(p-1)\log\beta,$

$\log b=\log\alpha+(q-1)\log\beta$ and

$\log c=\log\alpha+(r-1)\log\beta$

The dot product of the given two vectors is

$(q-r)\log a+(r-p)\log b+(p-q)\log c$

$\Rightarrow (q-r)[\log\alpha+(p-1)\log\beta]+(r-p)$

$[\log\alpha+(q-1)\log\beta]+(p-q)[\log\alpha+(r-1)\log\beta]$

$\Rightarrow \quad \log\alpha\,[q-r+r-p+p-q]+$

$\log\beta\,[(p-1)(q-r)+(r-p)(q-1)$

$+(r-1)(p-q)]$

$=0+0=0$

$\Rightarrow$ The two vectors are perpedicular.

54. The straight line joining the points (1, 1, 2) and

(3, −2, 1) is $\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-2}{-1}=r$ (say)

$\therefore$ point is $(2r+1, 1-3r, 2-r)$ which lies on

$3x+2y+z=6$

$\therefore \quad 3(2r+1)+2(1-3r)+2-r=6$

$\Rightarrow \quad r=1$

So, required point is (3, −2, 1).

55. Let Q be the point where the line through P (1, − 5, 9) and parallel to $r=\hat{i}+\hat{j}+\hat{k}$ meets the plane $r\,(\hat{i}-\hat{j}+\hat{k})=5$.

Equation of PQ, is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$$

$\therefore \quad x=\lambda+1,$

$y=\lambda-5$

and $\quad z=\lambda+9$

lies on the plane $x-y+z=5$

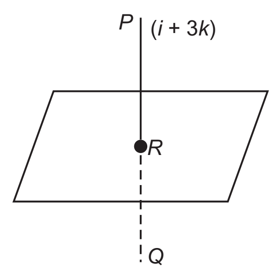
$\Rightarrow \quad \lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow \quad \lambda=-10$

$\therefore$ Co-ordinates of Q are $Q\,(-9,-15,-1)$

$\Rightarrow \quad PQ=\sqrt{10^2+10^2+10^2}=10\sqrt{3}$

56. Let Q be the image of the point $P\,(\hat{i}+3\hat{k})$ in the plane $r\cdot(\hat{i}+\hat{j}+\hat{k})=1$. Then PQ is normal to the plane. Since PQ passes through P and is normal to the given plane, therefore equation of PQ is



$$r=(\hat{i}+3\hat{k})+\lambda\,(\hat{i}+\hat{j}+\hat{k})$$

Since, Q lies on the line PQ, so, let the position vector of Q be $(\hat{i}+3\hat{k})+\lambda\,(\hat{i}+\hat{j}+\hat{k})$

$\Rightarrow (1+\lambda)\,\hat{i}+\lambda\hat{j}+(3+\lambda)\,\hat{k}$

Since R is the mid point of PQ, therefore position vector of R is

$$\frac{(1+\lambda)\,i+\lambda j+(3+\lambda)\hat{k}+\hat{i}+3\hat{k}}{2}$$

or $\left(\frac{\lambda+2}{2}\right)i+\left(\frac{\lambda}{2}\right)\hat{j}+\left(3+\frac{\lambda}{2}\right)\hat{k}$

or $\left(\frac{\lambda}{2}+1\right)\hat{i}+\left(\frac{\lambda}{2}\right)\hat{j}+\left(3+\frac{\lambda}{2}\right)\hat{k}$

Since, R less on the plane $r\cdot(\hat{i}+\hat{j}+\hat{k})=1$

Therefore,

$$\left[\left(\frac{\lambda}{2}+1\right)\hat{i}+\left(\frac{\lambda}{2}\right)\hat{j}+\left(3+\frac{\lambda}{2}\right)\hat{k}\right]\cdot(\hat{i}+\hat{j}+\hat{k})=1$$

$$\Rightarrow \quad \left(\frac{\lambda}{2}+1+\frac{\lambda}{2}+3+\frac{\lambda}{2}\right)=1$$

$$\Rightarrow \quad \lambda=-2$$

So, the postion vector of Q is

$$(\hat{i}+3\hat{k})-2(\hat{i}+\hat{j}+\hat{k})=-\hat{i}-2\hat{j}+\hat{k}$$

57. Let the probability of success and failure be p and q, respectively

Then, $\quad p=2q$

and $\quad p+q=1$

$\Rightarrow \quad 3q-1$

$\Rightarrow \quad q=\frac{1}{3}$

$\therefore \quad p=\frac{2}{3}$

Required probability

$$={}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+{}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)+{}^6C_6\left(\frac{2}{3}\right)^6$$

$$=\frac{496}{729}$$

58. Since, m and n are selected between 1 and 100, hence sample space $=100\times100$

Also, $\quad 7^1=1, 7^2=49, 7^3=343, 7^4=2401$

$7^5=16807$ etc.

Hence, 1, 3, 7 and 9 will be the last digits in the power of 7. Hence, for favourable cases.

nm

↓

1, 2 1, 2 1, 3 … 1, 100

2, 1 2, 2 2, 3 … 2, 100

100, 1 100, 2 100, 3 … 100, 100

for $m=1, \quad n=3, 7, 11, \quad \ldots \quad 97$

$\therefore$ favourable cases $=25$

for $m=2, \quad n=4, 8, 12, 10 \quad \ldots \quad 100$

$\therefore$ favourable cases $=25$

Similarly for every m, favourable n are 25.

$\therefore$ Total favourable cases $=100\times25$

Hence, required probability $=\frac{100\times25}{100\times100}=\frac{1}{4}$

59. Let $\sin^{-1}a=A$

$\sin^{-1}b=B$

$\sin^{-1}C=C$

$\therefore \quad \sin A=a, \quad \sin B=b, \quad \sin C=C$

and $A+B+C=\pi$, then

$\sin 2A+\sin 2B+\sin 2C$

$=4\sin A\sin B\sin C \quad \ldots$(i)

$\Rightarrow \quad \sin A\cos A+\sin B\cos B+\sin C\cos C$

$=2\sin A\sin B\sin C$

$\Rightarrow \quad \sin A\sqrt{1-\sin^2 A}+\sin B\sqrt{1-\sin^2 B}$

$+\sin C\sqrt{1-\sin^2 C}=2\sin A\sin B\sin C \quad \ldots$(ii)

$\Rightarrow \quad a\sqrt{1-a^2}+b\sqrt{1-b^2}+c\sqrt{1-c^2}=2abc$

while $\sin^{-1}a+\sin^{-1}b+\sin^{-1}c=\pi$

60. Given system of equation is

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

If given system of equation has no solution, then $D=0$ and atleast one of the determinants $D_1, D_2, D_3\neq 0$

Here $\quad D=0$

$$\Rightarrow \quad \begin{vmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{vmatrix}=0$$

$\Rightarrow 1(2\lambda-6)-1(\lambda-3)+1(2-2)=0$

$\Rightarrow \quad 2\lambda-6-\lambda+3+0=0$

$\Rightarrow \quad \lambda-3=0$

$\Rightarrow \quad \lambda=3$

Also, $D_1=\begin{vmatrix}6&1&1\\10&2&3\\\mu&2&3\end{vmatrix}\neq 0$

$\Rightarrow 6(6-6)-1(30-3\mu)+1(20-2\mu)\neq 0$

$\Rightarrow \quad 0-30+3\mu+20-2\mu\neq 0$

$\Rightarrow \quad \mu-10\neq 0$

$\Rightarrow \quad \mu\neq 10$

61. Given, $f(x)=sec\left(\frac{\pi}{4}\cos^2 x\right)$

We know that, $\quad 0\leq\cos^2 x\leq 1$

$\therefore \quad f(x)=sec\left(\frac{\pi}{4}\times 1\right)=\sqrt{2}$

$f(x)=sec\left(\frac{\pi}{4}\times 0\right)=1 \quad \therefore$Range $\in[1, \sqrt{2}]$

62. Let $y = \lim_{x \to 10} \left(\dfrac{a^x + b^x + c^x}{3} \right)^{2/x}$

$$\Rightarrow \quad \log y = \lim_{x \to 10} \frac{2}{x} \log \left(\frac{a^x + b^x + c^x}{3} \right)$$

$$= \lim_{x \to 10} \frac{\log (a^x + b^x + c^x) - \log 3}{x}$$

$$\Rightarrow \quad \log y = \log (abc)^{2/3} \quad \text{(by L-Hospital's rule)}$$

$$\Rightarrow \quad y = (abc)^{2/3}$$

63. We have $f(x) = [x] \cos \left[\dfrac{2x - 1}{2} \right] \pi$

Since, $g(x) = [x]$ is always discontinuous at all integral values of points. Hence, $f(x)$ is discontinuous for all integral points.

64. we know that funtion $|x|$ is not differentiable at $x = 0$

$\therefore \quad |x^2 - 3x + 2| = |(x - 1)(x - 2)|$

Hence, it is not differentiable at $x = 1$ and 2

Now, $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos |x|$ is not differentiable at $x = 2$.

For $1 < x < 2$, $f(x) = -(x^2 - 1)(x^2 - 3x + 2) + \cos x$

For $2 < x < 3$, $f(x) = (x^2 - 1)(x^2 - 3x + 2) + \cos x$

$L f'(x) = -(x^2 - 1)(2x - 3) - 2x(x^2 - 3x + 2) - \sin x$

$$Lf'(2) = -3 \sin 2$$

$Rf'(x) = (x^2 - 1)(2x - 3) + 2x(x^2 - 3x + 2) - \sin x$

$Rf'(2) = (4 - 1)(4 - 3) + 0 - \sin 2 = 3 - \sin 2$

Hence, $\quad Lf'(2) \neq Rf'(2)$.

So, $f(x)$ is not differentable at $x = 2$.

65. Let $x = 5, y = 0 \Rightarrow f(5 + 0) = f(5) \cdot f(0)$

$$\Rightarrow \quad f(5) = f(5)\, f(0) \Rightarrow f(0) = 1$$

$$\therefore \quad f'(5) \lim_{h \to 0} = \frac{f(5 + h) - f(5)}{h}$$

$$= \lim_{h \to 0} \frac{f(5)\, f(h) - f(5)}{h}$$

$$= \lim_{h \to 0} 2 \left(\frac{f(h) - 1}{h} \right) \quad [\because f(5) = 2]$$

$$= 2 \lim_{h \to 0} \left(\frac{f(h) - f(0)}{h} \right) = 2 \times f(0)$$

$$= 2 \times 3 = 6$$

66. Here, $x \sin (a + y) + \sin a \cos (a + y) = 0 \quad \ldots$(i)

On differentiating w.r.t. x, we get

$$\frac{d}{dx} [x \sin (a + y)] + \frac{d}{dx} [\sin a \cos (a + y)] = 0$$

$$\Rightarrow \left[x \frac{d}{dx} \{\sin (a + y)\} + \sin (a + y) \frac{d}{dx} (x) \right] + \sin a \frac{d}{dx} \cos (a + y) = 0$$

(using product rule and chain rule)

$$\Rightarrow \quad \left[x \cos (a + y) \frac{d}{dx} (a + y) + \sin (a + y) \right] + \sin a \left[-\sin (a + y) \frac{d}{dx} (a + y) \right] = 0$$

$$\Rightarrow \left[x \cos (a + y) \left(0 + \frac{dy}{dx} \right) + \sin (a + y) \right] - \sin a \sin (a + y) \left(0 + \frac{dy}{dx} \right) = 0$$

$$\Rightarrow \quad x \cos (a + y) \frac{dy}{dx} + \sin (a + y) - \sin a \sin (a + y) \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} [x \cos (a + y) - \sin a \sin (a + y)] = -\sin(a + y)$$

$$\left[\text{from Eq. (i), putting } x = -\sin a \frac{\cos (a + y)}{\sin (a + y)} \right]$$

$$\Rightarrow \quad \frac{dy}{dx} \left[-\sin a \frac{\cos^2 (a + y)}{\sin (a + y)} - \sin a \sin (a + y) \right] = -\sin (a + y)$$

$$\Rightarrow -\frac{dy}{dx} \left[\frac{\sin a \cos^2 (a + y) + \sin a \sin^2 (a + y)}{\sin (a + y)} \right] = -\sin (a + y)$$

$$\Rightarrow \frac{dy}{dx} = \sin (a + y) \left[\frac{\sin (a + y)}{\sin a \{\cos^2 (a + y) + \sin^2 (a + y)\}} \right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin^2 (a + y)}{\sin a} \qquad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

67. Firstly, break the number 5.001 as $x = 5$ and $\Delta x = 0.001$ and use the relation $f(x + \Delta x) \simeq f(x) + \Delta x\, f'(x)$.

Consider, $f(x) = x^3 - 7x^2 + 15$

$\Rightarrow \quad f'(x) = 3x^2 - 14x$

Let $\quad x = 5$

and $\quad \Delta x = 0.001$

Also, $f(x + \Delta x) \simeq f(x) + \Delta x\, f'(x)$

Therefore,

$f(x + \Delta x) \simeq (x^3 - 7x^2 + 15) + \Delta x\,(3x^2 - 14x)$

$$\Rightarrow \quad f(5.001) \simeq (5^3 - 7 \times 5^2 + 15) + (3 \times 5^2 - 14 \times 5)(0.001)$$

(as $x = 5$, $\Delta x = 0.001$)

$$= 125 - 175 + 15 + (75 - 70)(0.001)$$
$$= -35 + (5)(0.001)$$
$$= -35 + 0.005 = -34.995$$

68. Here, we observe that

(a) $f(x)$ is a polynomial, so it is continuous in the interval [0, 2].

(b) $f'(x) = 3x^2 - 6x + 2$ exists for all $x \in (0, 2)$.

So, $f(x)$ is differentiable for all $x \in (0, 2)$ and

(c) $f(0) = 0$, $f(2) = 2^3 - 3(2)^2 + 2(2) = 0$

$\Rightarrow f(0) = f(2)$

Thus, all the three conditions of Rolle's theorem are satisfied.

So, there must exist $c \in [0, 2]$ such that $f'(c) = 0$

$\Rightarrow \quad f'(c) = 3c^2 - 6c + 2 = 0$

$$\Rightarrow \quad c = 1 \pm \frac{1}{\sqrt{3}} \in [0, 2]$$

69. The equation of any plane through the intersection of the planes,

$3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$, is

$(3x - y + 2z - 4) + \lambda\,(x + y + z - 2) = 0 \quad \ldots(i)$

The plane passes through the point (2, 2, 1). Therefore, this point will satisfy Eq. (i).

$\therefore \; (3 \times 2 - 2 + 2 \times 1 - 4) + \lambda\,(2 + 2 + 1 - 2) = 0$

$\Rightarrow \quad (6 - 4) + 3\lambda = 0$

$\Rightarrow \quad 2 + 3\lambda = 0$

$$\Rightarrow \quad \lambda = \frac{-2}{3}$$

On substituting this value of λ in Eq. (i), we get the required plane as

$$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$$

$\Rightarrow \; 9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$

$\Rightarrow \quad 7x - 5y + 4z - 8 = 0$

This is the required equation of the plane.

70. Let $y = \operatorname{cosec}^2 x$,

Required limit $= \lim_{y \to \infty} (1^y + 2^y + \ldots n^y)^{1/y}$

$(0/\infty \text{ form})$

$$= \lim_{y \to \infty} (n^y)^{1/y} \left[\left(\frac{1}{n}\right)^y + \left(\frac{2}{n}\right)^y + \ldots + \left(\frac{n-1}{n}\right)^y + 1\right]^{1/y}$$

$$= \lim_{y \to \infty} n \left[\left(\frac{1}{n}\right)^y + \left(\frac{2}{n}\right)^y + \ldots + \left(\frac{n-1}{n}\right)^y + 1\right]^{1/y}$$

$= n \cdot 1^0 = n \cdot 1 = n$

71. Let $I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad \ldots(i)$

$$\Rightarrow \; I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

$$= \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx \quad \ldots(ii)$$

$$\left[\because \sin\left(\frac{\pi}{2} - x\right) = \cos x \text{ and } \cos\left(\frac{\pi}{2} - x\right) = \sin x\right]$$

On adding Eqs. (i) and (ii), we get

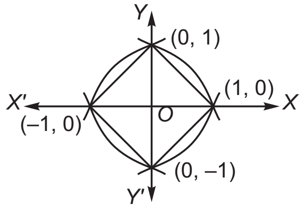
$$2I = \int_0^{\pi/2} \frac{0}{1 + \sin x \cos x} dx = 0 \quad \Rightarrow \quad I = 0$$

72. Since, $|x| + |y| = 1$

$$\Rightarrow \begin{cases} x + y = 1, & x > 0, \quad y > 0 \\ x - y = 1, & x > 0, \quad y < 0 \\ -x + y = 1, & x < 0, \quad y > 0 \\ -x - y = 1, & x < 0, \quad y < 0 \end{cases}$$

and $\quad 1 - y^2 = |x|$

$$\Rightarrow \begin{cases} 1-y^2 = x, & x \geq 0 \\ 1-y^2 = -x, & x < 0 \end{cases}$$



$\therefore$ Required area

$$= \left| 2\int_0^1 \sqrt{(1-x)}\, dx \right| + \left| 2\int_{-1}^0 \sqrt{(x+1)}\, dx \right| - 4\left(\frac{1}{2}\cdot 1\cdot 1\right)$$

$$= \frac{2}{3} \text{ sq unit}$$

73. Let slope of a line be m.

Now, the equation of a line passing through (1, 0) is

$$y - 0 = m(x-1)$$

$$\Rightarrow \; mx - y - m = 0$$

Distance from origin $= \dfrac{\sqrt{3}}{2}$

$$\Rightarrow \quad \frac{|-m|}{\sqrt{1+m^2}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad 4m^2 = 3(1+m^2)$$

$$\Rightarrow \quad m^2 = 3$$

$$\Rightarrow \quad m = \pm\sqrt{3}$$

$\therefore$ Equations of lines are

$$\sqrt{3}x - y - \sqrt{3} = 0$$

and $\quad -\sqrt{3}x - y + \sqrt{3} = 0$

$$\Rightarrow \quad \sqrt{3}x - y - \sqrt{3} = 0$$

and $\quad \sqrt{3}x + y - \sqrt{3} = 0$

74. From figure, we have

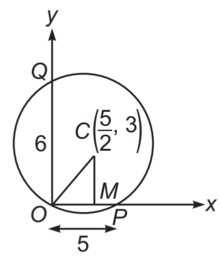
$$OP = 5, \quad OQ = 6$$

and $\quad OM = \dfrac{5}{2}, \quad CM = 3$

$\therefore$ In $\Delta OMC, OC^2 = OM^2 + MC^2$

$$\Rightarrow \quad OC^2 = \left(\frac{5}{2}\right)^2 + (3)^2$$

$$\Rightarrow \quad OC = \frac{\sqrt{61}}{2}$$



Thus, the required circle has its centre $\left(\dfrac{5}{2}, 3\right)$ and radius $\dfrac{\sqrt{61}}{2}$.

So, its equation is $\left(x - \dfrac{5}{2}\right)^2 + (y-3)^2 = \left(\dfrac{61}{4}\right)$.

Hence, $\quad \lambda = \dfrac{61}{4}$

75. Obtain the foot of the perpendicular $N(-4, 1, -3)$.

Find the equations of the line passing through $P(2, 4, -1)$ and $N(-4, 1, -3)$

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$

76. Given, $\cos^2 A + \cos^2 C = \sin^2 B$

Obviously it is not an equilateral triangle because $A = B = C = 60°$ does not satisfy the given condition. But $B = 90°$, then $\sin^2 B = 1$ and

$$\cos^2 A + \cos^2 C = \cos^2 A + \cos^2\left(\frac{\pi}{2} - A\right)$$

$$= \cos^2 A + \sin^2 A = 1$$

Hence, this satisfies the condition, so it is a right angled triangle but not necessarily isosceles triangle.

77. $\Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

(using $C_1 \to C_1 + C_2 + C_3$)

Take out $2(x+y+z)$ common from C_1, we get

$$\Delta = 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$$

(using $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$)

Take out $(x+y+z)$ common from R_2 and R_3, we get

$$\Delta = 2(x+y+z)(x+y+z)(x+y+z) \times \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3, we get

$$\Delta = 2(x+y+z)^3\ [(1)(1-0)]$$

$$= 2(x+y+z)^3 = k(x+y+z)^3 \text{ (given)}$$

$\therefore \quad k = 2$

78. Let $\quad f(x) = \dfrac{x}{\log x}$

$$f'(x) = \frac{\log x - 1}{(\log x)^2}$$

Pur $f'(x) = 0$, for maxima or minima, we get

$$\log x - 1 = 0$$

$\Rightarrow \qquad x = e$

Now, $f''(x) = \dfrac{(\log x)^2 \cdot \dfrac{1}{x} - (\log x - 1) \cdot \dfrac{2\log x}{x}}{(\log x)^4}$

$\Rightarrow \qquad f''(e) = \dfrac{\dfrac{1}{e} - 0}{1} = \dfrac{1}{e} > 0$

$\therefore$ Function is minimum at $x = e$.

Hence, minimum value of $f(x)$ at $x = e$ is $f(e) = e$.

79. Total number of mappings from a set A having elements into itself is n^n.

And the total number of one to one mapping is $n!$.

$\therefore$ Required probability $= \dfrac{n!}{n^n}$

80. Total number of available courses = 9

Out of these 5 courses have to be chosen. But it is given that 2 courses are compulsory for every student *i.e.*, you have to choose only 3 courses instead of 5, out of 7 instead of 9.

It can be done in ${}^7C_3$ ways $= \dfrac{7 \times 6 \times 5}{6} = 35$ ways.