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MET 2012 Question Paper with Solution

Manipal Entrance Test (MET)

Manipal

Engineering Entrance Exam

Solved Paper 2012

Physics

1. Oxygen is 16 times heavier than hydrogen. Equal volumes of hydrogen and oxygen are mixed. The ratio of speed of sound in the mixture to that in hydrogen is

(a) $\sqrt{8}$ (b) $\sqrt{2/17}$

(c) $\sqrt{1/8}$ (d) $\sqrt{32/17}$

2. A man, standing between two cliffs, claps his hands and starts hearing a series of echoes at intervals of one second. If the speed of sound in air is 340 ms^{-1}, the distance between the cliffs is

(a) 680 m (b) 1700 m

(c) 340 m (d) 1620 m

3. A beam of light of wavelength 600 nm from a source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fring is

(a) 2.4 cm (b) 2.4 mm

(c) 1.2 mm (d) 1.2 cm

4. A thin plano-convex lens acts like a concave mirror of focal length 0.2 m when silvered from its plane surface. The refractive index of the material of the lens is 1.5. The radius of curvature of the convex surface of the lens will be

(a) 0.1 m (b) 0.75 m

(c) 0.4 m (d) 0.2 m

5. A balloon is rising vertically up with a velocity of 29 ms^{-1}. A stone is dropped from it and it reaches the ground in 10 s. The height of the balloon when the stone was dropped from it is ($g = 9.8\ ms^{-1}$)

(a) 400 m (b) 150 m

(c) 100 m (d) 200 m

6. In a Young's double slit experiment, the separation between the two slits is 0.9 mm and the fringes are observed one metre away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of the monochromatic source of light used is

(a) 450 nm (b) 400 nm

(c) 500 nm (d) 600 nm

7. A count rate meter shows a count of 240 per minute from a given radioactive source. One hour later the meter shows a count rate of 30 per minute. The half-life of the source is

(a) 80 min (b) 120 min

(c) 20 min (d) 30 min

8. Two conductors of the same material have their diameters in the ratio 1 : 2 and their lengths in the ratio 2 : 1. If the temperature difference between their ends is the same, then the ratio of amounts of heat conducted per second through them will be

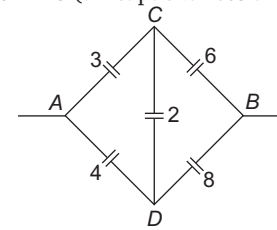
(a) 4 : 1 (b) 1 : 4 (c) 8 : 1 (b) 1 : 8

9. An ideal gas heat engine operates in a Carnot's cycle between 227°C and 127°C. It absorbs 6×10^4 J at high temperature. The amount of heat converted into work is

(a) 1.6×10^4 J (b) 1.2×10^4 J

(c) 4.8×10^4 J (d) 3.5×10^4 J

10. Effective capacitance between *A* and *B* in the figure shown is (all capacitances are in μF)



(a) $\frac{3}{14}\mu F$ (b) $\frac{14}{3}\mu F$
(c) 21 μF (d) 23 μF

11. When the bob of a simple pendulum swings, the work done by tension in the string is
(a) > 0 (b) < 0
(c) zero (d) maximum

12. For a particle executing simple harmonic motion, the kinetic energy K is given by, $K = K_0 \cos^2 \omega t$. The maximum value of potential energy is
(a) K_0 (b) zero
(c) $K_0 / 2$ (d) not obtainable

13. A particle is moving along a circular path of radius 5 m with a uniform speed 5 ms^{-1}. What will be the average acceleration when the particle completes half revolution?
(a) Zero (b) 10 ms^{-2}
(c) 10 π ms^{-2} (d) $\frac{10}{\pi}$ ms^{-2}

14. ECE of copper is 3.3×10^{-7} kg/C. If 100 kWh energy is consumed at 33 V in a copper voltameter, then mass of copper liberated is
(a) 3.6 kg (b) 3.3 kg
(c) 1 kg (d) 1 mg

15. In non-resonant circuit, what will be the nature of the circuit for frequencies higher than the resonant frequency?
(a) Resistive (b) Capacitive
(c) Inductive (d) None of these

16. A spot of light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance of 3 m from S. The velocity of spot P, where $\theta = 45°$, is
(a) 0.5 m/s (b) 0.6 m/s
(c) 0.7 m/s (d) 0.8 m/s

17. A balloon contains 500 m^3 of He at 27°C and 1 atmospheric pressure. The volume of He at –3°C and 0.5 atmospheric pressure will be
(a) 700 m^3 (b) 900 m^3
(c) 1000 m^3 (d) 500 m^3

18. Which of the following is different from others?
(a) Wavelength (b) Velocity
(c) Frequency (d) Amplitude

19. The breaking force for a wire of diameter D of a material is F. The breaking force for a wire of the same material of radius D is
(a) F (b) $2F$
(c) $\frac{F}{4}$ (d) $4F$

20. If pressure of a gas contained in a closed vessel is increased by 0.4% when heated by 1°C, its initial temperature must be
(a) 250 K (b) 250°C
(c) 2500 K (d) 25°C

21. In an electromagnetic wave, the electric and magnetising fields are 100 V/m and 0.265 A/m. The maximum energy flow is
(a) 26.5 W/m^2 (b) 36.5 W/m^2
(c) 46.7 W/m^2 (d) 765 W/m^2

22. A bullet of mass 10 g is fired from a gun of mass 1 kg with recoil velocity of gun is 5 m/s. The muzzle velocity will be
(a) 30 km/min (b) 60 km/min
(c) 30 m/s (d) 500 m/s

23. Work done by air when it expands from 50 L to 150 L at a constant pressure of 2 atmosphere is
(a) 2×10^4 J (b) 2×100 J
(c) 2×10^{11} J (d) 2×10^{15} J

24. In an electron gun, the control grid is given a negative potential, relative to cathode in order to
(a) accelerate the electrons
(b) decrease KE of electrons
(c) repel the electrons
(d) decelerate the electrons

25. In an LCR circuit having L = 8 H, C = 0.5μF and R = 100 Ω in series, the resonance frequency in rad/s is
(a) 600 (b) 200
(c) $250/\pi$ (d) 500

26. Power of a lens is -4 D and for second lens, power is +2D; the total power of the couple is
(a) -2 D (b) 6 D
(c) -6 D (d) -8 D

27. In which one of the following regions of the electromagnetic spectrum will the vibrational motion of molecules give rise to absorption?
(a) Ultraviolet (b) Microwave
(c) Infrared (d) Radio wave

28. During inelastic collision of two particles
(a) $(KE)_{final} = (KE)_{initial}$
(b) $(KE)_{final}$ must be greater than $(KE)_{initial}$
(c) $(KE)_{final}$ must be less than $(KE)_{initial}$
(d) $(KE)_{final}$ must be greater or less than $(KE)_{initial}$

29. Two gases are at absolute temperatures 300 K and 350 K respectively. The ratio of average kinetic energy of their molecules is
(a) 6 : 7 (b) 36 : 49
(c) 7 : 6 (d) 343 : 216

30. The current in a coil changes from 0 to 2 A in 0.05 s. If the induced emf is 80 V, the self-inductance of the coil is,
(a) 1 H (b) 0.5 H
(c) 1.5 H (d) 2 H

31. Equal volumes of monoatomic and diatomic gases at the same temperature are given equal quantities of heat. Then,
(a) the temperature of diatomic gas will be more
(b) the temperature of monoatomic gas will be more
(c) the temperature of both will be zero
(d) nothing can be said

32. A copper wire and a steel wire of the same diameter and length are joined end to end and a force is applied which stretches their combined length by 1 cm. Then, the two wires will have
(a) the same stress and strain
(b) the same stress but different strains
(c) the same strain but different stresses
(d) different stresses and strains

33. The length of the second's pendulum is decreased by 0.3 cm when it is shifted to Chennai from London. If the acceleration due to gravity at London is 981cm/s^2, the acceleration due to gravity at Chennai is
(a) 981 cm/s^2 (b) 978 cm/s^2
(c) 984 cm/s^2 (d) 975 cm/s^2

34. How does, the electric field (E) between the plates of a charged cylindrical capacitor vary with the distance r from the axis of the cylinder?
(a) $E \propto \frac{1}{r^2}$ (b) $E \propto \frac{1}{r}$
(c) $E \propto r^2$ (d) $E \propto r$

35. A thin bar magnet of length $2L$ is bent at the mid-point so that the angle between them is 60°C. The new length of the magnet is
(a) $\sqrt{2}$ L (b) $\sqrt{3}$ L
(c) $2L$ (d) L

36. A capacitor is connected to a cell of emf E and some internal resistance r. The potential difference across the
(a) cell is E (b) cell is < E
(c) capacitor is < E (d) capacitor is > E

37. The radius of the smallest electron orbit in hydrogen-like ion is $(0.51 \times 10^{-10} / 4)$m, then it is
(a) hydrogen atom (b) He^+
(c) Li^{2+} (d) Be^{3+}

38. de-Broglie wavelength of a body of mass 1 kg moving with velocity of 2000 m/s is
(a) 3.32×10^{-27} Å (b) 1.5×10^{7} Å
(c) 0.55×10^{-22} Å (d) None of these

39. A voltmeter of resistance 998 Ω is connected across a cell of emf 2 V and internal resistance 2Ω. The potential difference across the voltmeter is
(a) 1.99 V (b) 3.5 V (c) 5 V (d) 6 V

40. If a person with a spring balance and a body hanging from it goes up and up in an aeroplane, then the reading of the weight of the body as indicated by the spring balance will
(a) goes on increasing
(b) goes on decreasing
(c) first increase and then decrease
(d) remains the same

41. A mass $3m$, initially at rest at the origin explodes into three fragments of equal mass. Two of the fragments have speed v each and move perpendicular to each other. The third fragment will move with a speed
(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{2}$ (c) v (d) $v\sqrt{2}$

42. A nucleus X initially at rest, undergoes alpha decay according to the equation

$$_{92}X^{A} \rightarrow {}_{Z}Y^{228} + \alpha$$

Then, the value of A and Z are
(a) 94, 230 (b) 232,90
(c) 190,32 (d) 230,94

43. The energy gap between the valence band and the conduction band for a material is 6 eV. The material is
(a) an insulator
(b) a metal
(c) an intrinsic semiconductor
(d) a superconductor

44. When the inputs of a two input logic gate are 0 and 0, the output is 1. When the inputs are 1 and 0 the output is 0. The logic gate is of the type
(a) AND (b) NAND
(c) NOR (d) OR

45. If a transistor biased in the common-emitter mode, the emitter current is
(a) much smaller than base curent
(b) much larger than base current
(c) nearly equal to the base current
(d) much smaller than the collector current

46. A sound has an intensity of 2×10^{-8} W / m^2. Its intensity level in decibels is ($\log_{10} 2 = 0.3$)
(a) 23 (b) 3
(c) 43 (d) 4.3

47. A ray reflected successively from two plane mirrors inclined at a certain angle undergoes a deviation of 240°. Then, the number of images observable is
(a) 3 (b) 5
(c) 7 (d) 9

48. If 1 A of current is passed through $CuSO_4$ solution for 10 s, the number of copper atoms deposite at the cathode will be about
(a) 1.6×10^{20} (b) 8×10^{19}
(c) 3.1×10^{19} (d) 6.2×10^{19}

49. Two points *A* and *B* are situated at distance *x* and 2 *x* respectively from the nearer pole of a magnet 2 cm long. The ratio of magnetic field at *A* and *B* is
(a) 4 : 1 exactly (b) 4 : 1 approximately
(c) 8 : 1 exactly (d) 8 : 1 approximately

50. Electromagnetic waves travel only through
(a) material medium
(b) vacuum
(c) oscillating electric and magnetic fields
(d) oscillating electric and magnetic fields whose directions are perpendicular to each other

51. The ratio of magnetic moments of two short magnets which give null deflection in tan *B* position at 12 cm and 18 cm from the centre of a deflection magnetometer is
(a) 2 : 3 (b) 8 : 27
(c) 27 : 8 (d) 4 : 9

52. A 220 V main supply is connected to a resistance of 100 kΩ. The effective current is
(a) 2.2 mA (b) $2.2\sqrt{2}$ mA
(c) $\frac{2.2}{\sqrt{2}}$ mA (d) None of these

53. Two bodies of different masses m_1 and m_2 have equal momenta. Their kinetic energies E_1 and E_2 are in the ratio
(a) $\sqrt{m_1} : \sqrt{m_2}$ (b) $m_1 : m_2$
(c) $m_2 : m_1$ (d) $m_1^2 : m_2^2$

54. When the light enters from air to glass, for which colour the angle of deviation is maximum?
(a) Red (b) Yellow
(c) Blue (d) Violet

55. Two waves are represented by $y_1 = A \sin(kx - \omega t)$ and $y_2 = A \cos(kx - \omega t)$. The amplitude of resultant wave is
(a) 4*A* (b) 2*A*
(c) $\sqrt{2}A$ (d) *A*

56. An atom of mass *M* which is in the state of rest emits a photon of wavelength λ. As a result, the atom will deflect with the kinetic energy equal to (*h* is Planck's constant)
(a) $\frac{h^2}{M\lambda^2}$ (b) $\frac{1}{2}\frac{h^2}{M\lambda^2}$
(c) $\frac{h}{M\lambda}$ (d) $\frac{1}{2}\frac{h}{M\lambda}$

57. If the angular momentum of any rotating body increases by 200%, then the increase in its kinetic energy is
(a) 400% (b) 800%
(c) 200% (d) 100%

58. A cylindrical tank has a hole of 1 cm^2 in its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of 70 cm^2/s, then the maximum height up to which water can rise in the tank is
(a) 2.5 cm (b) 5 cm
(c) 10 cm (d) 0.25 cm

59. X-rays and γ-rays of the same energies may be distinguished by
(a) their velocity
(b) their ionising power
(c) their intensity
(d) method of production

60. Which of the following materials is non-crystalline?
(a) Copper (b) Sodium chloride
(c) Wood (d) Diamond

Chemistry

1. The vapour density of N_2O_4 at a certain temperature is 30. What is the percentage dissociation of N_2O_4 at this temperature?
(a) 46% (b) 53%
(c) 75% (d) 92%

2. Four different colloids have the following gold number, which one has the most effective action?
(a) 5 (b) 10
(c) 15 (d) 13

3. In which of the following case, increase in concentration of ion cause increase in E_{cell} ?
(a) $Pt(H_2)|H^+(aq)$
(b) $Pt|Quinhydrone|H^+(aq)$
(c) $Ag|Ag^+(aq)$
(d) $Ag, AgCl|Cl^-(aq)$

4. The relative lowering of vapour pressure of an aqueous solution containing a non volatile solute is 0.0125. The molality of the solution is
(a) 0.70 (b) 0.30
(c) 0.125 (d) 0.07

5. Which of the following is known as Glauber's salt?
(a) $Na_2S_2O_3 \cdot 5H_2O$ (b) $Na_2SO_4 \cdot 10H_2O$
(c) $Na_2CO_3 \cdot 10H_2O$ (d) $Na(NH_4)HPO_4$

6. $PhCHClBr \xrightarrow{tBuO^-}$ Carbene
In the above reaction which of the following will be most probable?
(a) $:CHBr$ (b) $:CHCl$
(c) $:CPhBr$ (d) $:CPhCl$

7. Which of the following alcohol is unable to turn orange colour of chromic acid to green?
(a) 1° alcohol (b) 2° alcohol
(c) 3° alcohol (d) Allyl alcohol

8. Which of the following test is performed by carbohydrates?
(a) Biuret test (b) Nitroprusside test
(c) Ninhydrin test (d) Molisch's test

9. Which of the following is not used as propellant?
(a) Hydrazine (b) UDMH
(c) MMH (d) Phenyl hydrazine

10. Which of the following species have the same number of electrons in its outermost as well as penultimate shell?
(a) Cl^- (b) O^{2-}
(c) Na^+ (d) Mg^{2+}

11. Which of the following is the formula of Sorel's cement?
(a) $KCl \cdot MgCl_2 \cdot 6H_2O$
(b) $MgCl_2 \cdot 5MgO \cdot xH_2O$
(c) $CaSO_4 \cdot 2H_2O$
(d) $MgCO_3 \cdot CaCO_3$

12. $S_{(rhombic)} + O_2(g) \longrightarrow SO_2(g)$, $\Delta H = -297.5$ kJ
$S_{(monoclinic)} + O_2(g) \longrightarrow SO_2(g)$, $\Delta H = -300$ kJ
The data can predict that
(a) rhombic sulphur is yellow in colour
(b) monoclinic sulphur has metallic lusture
(c) monoclinic sulphur is more stable
(d) $\Delta H_{transition}$ of S_R to S_M is endothermic

13. Which of the following is a measurement of water pollution?
(a) PSC (b) PCB
(c) BOD (d) COD

14. The half-life period of a first order process is 1.6 min. It will be 90% complete in
(a) 5.3 min (b) 10.6 min
(c) 43.3 min (d) 99.7 min

15. Which of the following is tetrabasic acid?
(a) Orthophosphorus acid
(b) Orthophosphoric acid
(c) Metaphosphoric acid
(d) Pyrophosphoric acid

16. Which of the following order for bond angles is correct?
(a) $BF_3 < NF_3 < PF_3 < ClF_3$
(b) $BF_3 < NF_3 < PF_3 > ClF_3$
(c) $ClF_3 < PF_3 < NF_3 < BF_3$
(d) $BF_3 \approx NF_3 < PF_3 < ClF_3$

17. The triad of the nuclei that is isotonic, is
(a) ${}_6C^{14}, {}_7N^{14}, {}_9F^{19}$ (b) ${}_6C^{14}, {}_7N^{15}, {}_9F^{17}$
(c) ${}_6C^{14}, {}_7N^{14}, {}_9F^{17}$ (d) ${}_6C^{12}, {}_7N^{14}, {}_9F^{19}$

18. The oxidation number of cobalt in $K[Co(CO)_4]$ is
(a) –1 (b) –3 (c) +1 (d) +3

19. Out of Cu, Ag, Fe and Zn, the metal which can displace all others from their salt solution, is
(a) Zn (b) Cu
(c) Ag (d) Fe

20. Which of the following has magic number of neutrons?
(a) $_{13}Al^{27}$ (b) $_{26}Fe^{56}$
(c) $_{83}Bi^{209}$ (d) $_{92}U^{238}$

21. If K_a of $HCN = 4 \times 10^{-10}$, then the pH of 2.5×10^{-1} molar HCN(aq) is
(a) 1 (b) 2.5
(c) 4 (d) 5

22. A coffee cup calorimeter initially contains 125 g of water at a temperature of 24.2°C. After adding of 10.5 g of KBr, the temperature becomes 21.1°C. The heat of solution is
(a) 40 J/g (b) 117 J/g
(c) 167.7 J/g (d) 420.05 J/g

23. What product is formed when cyclohexanone is oxidised?
(a) $HOOC \cdot (CH_2)_4 \cdot COOH$
(b) CH_3CH_2COOH
(c) $CH_3(CH_2)_4 \cdot COOH$
(d) None of the above

24. The reaction
$CH_3CN + 4H \xrightarrow{Na/C_2H_5OH} CH_3CH_2NH_2$ is known as
(a) Hofmann's bromamide reaction
(b) Mendius reaction
(c) Perkin's reaction
(d) Sabatier reaction

25. Action of Br_2 on cyclopentene gives
(a) cyclopentyl bromide
(b) 1,2-dibromo cyclopentane
(c) cyclopentyl dibromide
(d) no reaction

26. In a face-centred cubic arrangement of *A* and *B* atoms whose *A* atoms are at the corner of the unit cell and *B* atoms at the face-centre, one of *A* atom is missing from one corner in the unit cell. The simplest formula of the compound is
(a) $A_{24}B_7$ (b) A_7B_{24}
(c) A_7B_4 (d) AB_2

27. Beryllium shows diagonal relationship with
(a) B (b) Mg
(c) Al (d) Na

28. A compound has the molecular formula X_4O_6. If 10 g of X_4O_6 has 5.72 g *X*, then calculate atomic mass of *X*.
(a) 32 amu (b) 64 amu
(c) 67 amu (d) 98 amu

29. Which of the following is Rinmann's green?
(a) ZnO (b) $ZnS + BaSO_4$
(c) $ZnSO_4 \cdot 7H_2O$ (d) $ZnO \cdot CoO$

30. For the valence electron in copper, the four quantum numbers are
(a) $n = 4, l = 0, m = 0, s = \frac{1}{2}$
(b) $n = 4, l = 0, m = +2, s = \frac{1}{2}$
(c) $n = 4, l = 1, m = +2, s = \frac{1}{2}$
(d) $n = 4, l = 1, m = 0, s = \frac{1}{2}$

31. Geometry of BF_3 is
(a) trigonal planar (b) pentagonal
(c) trihedral (d) tetrahedral

32. Which of the following will represent the normality of 10% (*w/V*) acetic acid?
(a) 1.66 N (b) 16.6 N
(c) 3.32 N (d) 33.2 N

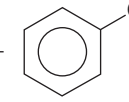
33. The oxidation number of sulphur in $H_2S_2O_8$ is
(a) +2 (b) +4
(c) +6 (d) +7

34. Which of the following reaction requires catalyst?
(a) $S + O_2 \longrightarrow SO_2$
(b) $C + O_2 \longrightarrow CO_2$
(c) $2SO_2 + O_2 \longrightarrow 2SO_3$
(d) All of the above

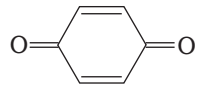
35. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + 22$ kcal
The activation energy for the forward reaction is 50 kcal. What is the activation energy for the backward reaction?
(a) 72 kcal (b) 28 kcal
(c) –72 kcal (d) –28 kcal

36. Malonic acid and succinic acid are distinguished by
(a) heating (b) $NaHCO_3$
(c) both (a) and (b) (d) None of these

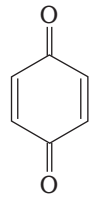
37. When chloroform is treated with chlorine in the presence of sunlight, it yields
(a) urotropine (b) pyrene
(c) chloropicrin (d) chloritone

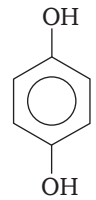
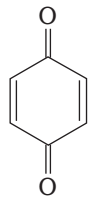
38. $A \xleftarrow[[O]]{CrO_2Cl_2}$  $\xrightarrow[OH^-, (Elb's)]{K_2S_2O_8} [B]$

Which of the following are *A* and *B*?

(a) Both are 

(b) Both are 

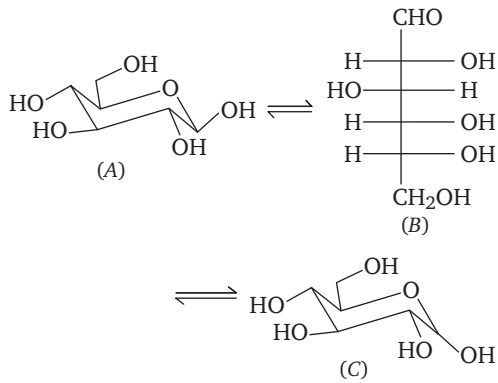
(c)  ;  respectively

(d)  ;  respectively

39. Which of the following will explain synthesis of RNA?
(a) Translation (b) Coagulation
(c) Transcription (d) Replication

40. Which of the following is a narrow spectrum antibiotics?
(a) Penicillin G (b) Dettol
(c) Aspirin (d) Analgin

41. When aniline reacts with oil of bitter almonds, condensation takes place and benzyl derivatives are formed. These are known as
(a) Millon's base (b) Schiff's reagent
(c) Benedict reagent (d) Schiff's base

42. 

Which of the following is correct about (*A*), (*B*) and (*C*)?
(a) *A,B* and *C* are anomers
(b) *A* and *C* are anomers
(c) *A, B* and *C* are metamers
(d) None of the above

43. The IUPAC name of  is
(a) bicyclo (2,1,0) pentane
(b) 1,2-cyclopropyl cyclobutane
(c) 1,2-methylene cyclobutane
(d) cyclopentane (4,3) annulene

44. In the conversion of ethanol into methanol which of the following reagents will be used?
(a) $K_2Cr_2O_7 / H_2SO_4$ (b) $NaOH + CaO$
(c) Cl_2 + aq. KOH (d) All of these

45. PDI for natural polymers is generally close to
(a) zero (b) 1
(c) 10 (d) 100

46. Which of the following is diamagnetic?
(a) H_2 (b) H_2^- (c) H_2^+ (d) He_2^+

47. Which of the following is a property of non-metal?
(a) Electronegativity
(b) Basic nature of oxide
(c) Reducing property
(d) Low ionisation potential

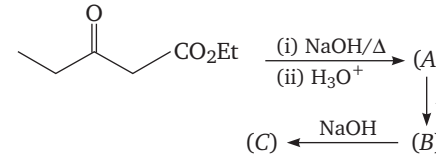
48. One mole of ice is converted into water at 273 K. The entropies of $H_2O(s)$ and $H_2O(l)$ are 38.20 and 60.01 J mol^{-1} K^{-1} respectively. The enthalpy change for the conversion is
(a) 3 kJ mol^{-1} (b) 4 kJ mol^{-1}
(c) 5 kJ mol^{-1} (d) 6 kJ mol^{-1}

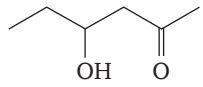
49. Complete the following reaction,
$$P_4 + NaOH + H_2O \longrightarrow ?$$
(a) $PH_3 + Na_2HPO_2$ (b) $PH_3 + NaH_2PO_2$
(c) $H_3PO_4 + NaO$ (d) $PH_3 + Na_2PO_4$

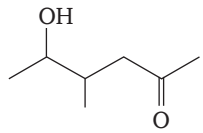
50. The elevation in boiling point would be highest for
(a) 0.08 M $BaCl_2$ (b) 0.15 M KCl
(c) 0.10 M Glucose (d) 0.06 M $Ca(NO_3)_2$

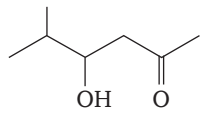
51. What is the magnetic moment of $[FeF_6]^{3-}$?
(a) 2 BM (b) 5.9 BM
(c) 7 BM (d) 35 BM

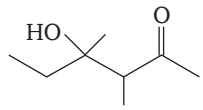
52. In the given reaction, identify compound *C*?



(a) 

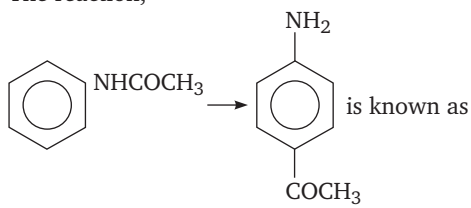
(b) 

(c) 

(d) 

53. The ratio of rate of diffusion of hydrogen and helium gas is
(a) 1 : 1.4 (b) 1 : 1 (c) 1.4 : 1 (d) 1 : 2

54. The reaction,



is known as
(a) substitution reaction
(b) addition reaction
(c) elimination reaction
(d) None of the above

55. Which of the following is responsible for hardness of water?
(a) $CaHCO_3$ (b) $CaCl_2$
(c) Both (a) and (b) (d) None of these

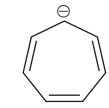
56. Which of the following is differ in nature from others?
(a) Vasopressin (b) Thyroid
(c) Oxytocin (d) Insulin

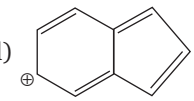
57. Indigo is an example of
(a) vat dye (b) mordant dye
(c) azo dye (d) phthalein dye

58. Solder is an alloy of lead with
(a) Cu (b) Fe
(c) Sn (d) Zn

59. An aqueous solution contains following ions : Hg_2^{2+}, Hg^{2+}, Pb^{2+} and Cd^{2+}. On adding HCl, which of these will precipitate?
(a) Hg_2Cl_2 (b) $PbCl_2$
(c) Hg_2Cl_2 and $PbCl_2$ (d) $HgCl_2$ and $PbCl_2$

60. Which of the following molecules/species are aromatic in nature?

(a)  (b) 

(c)  (d) 

Mathematics

1. The sum of the series $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \ldots$ is
(a) $2e$ (b) $3e$
(c) $2e - 1$ (d) None of these

2. If $f(x) = x^2 + 2bx + 2c^2$
and $g(x) = -x^2 - 2cx + b^2$
are such that min $f(x) >$ max $g(x)$, then relation between *b* and *c* is
(a) no relation
(b) $0 < c < \frac{b}{2}$
(c) $|c| < \sqrt{2}|b|$
(d) $|c| > \sqrt{2}|b|$

3. If $\left|\frac{z_1}{z_2}\right| = 1$ and arg $(z_1 z_2) = 0$, then
(a) $z_1 = z_2$ (b) $|z_2|^2 = z_1 z_2$
(c) $z_1 z_2 = 1$ (d) None of these

4. If cos A = m cos B, and $\cot\frac{A+B}{2} = \lambda \tan\frac{B-A}{2}$, then λ is
(a) $\frac{m}{m-1}$ (b) $\frac{m+1}{m}$
(c) $\frac{m+1}{m-1}$ (d) None of these

5. In a triangle, the angle are in AP and the lengths of the two larger sides are 10 and 9,

respectively, then the length of the third side can be

(a) $5 \pm \sqrt{6}$ (b) 0.7

(c) $15 - \sqrt{6}$ (d) None of these

6. The complex number $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$, lie on

(a) the *x*-axis

(b) the straight line $y = 5$

(c) a circle passing through the origin

(d) None of the above

7. If $x > 1$, $y > 1$, $z > 1$ are in GP, then $\frac{1}{1 + \ln x}$, $\frac{1}{1 + \ln y}, \frac{1}{1 + \ln z}$ are in

(a) AP (b) HP

(c) GP (d) None of these

8. If $\frac{e^x}{1 - x} = B_0 + B_1 x + B_2 x^2 + \ldots + B_n x^n + \ldots$, then $B_n - B_{n-1}$ equals

(a) $\frac{1}{n!}$ (b) $\frac{1}{(n-1)!}$

(c) $\frac{1}{n!} - \frac{1}{(n-1)!}$ (d) 1

9. If $f(x) = \begin{cases} \frac{\sin [x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

where $[x]$ denotes the greatest integer less than or equal to *x*, then $\lim_{x \to 0} f(x)$ equals

(a) 1 (b) 0

(c) –1 (d) None of these

10. If $y(t)$ is a solution of $(1 + t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ equal to

(a) $-\frac{1}{2}$ (b) $e + \frac{1}{2}$

(c) $e - \frac{1}{2}$ (d) $\frac{1}{2}$

11. The locus of the mid-points of the chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin, is

(a) $x + y = 2$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = 2$ (d) $x + y = 1$

12. From a point *A* common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and parabola $y^2 = 4ax$. The area of the quadrilateral formed by the common tangents to the chord of contact of the parabola is

(a) $\frac{15a^3}{4}$ sq units (b) $\frac{15a^2}{4}$ sq units

(c) $\frac{7a^2}{4}$ sq units (d) $\frac{a^2}{4}$ sq unit

13. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then value of the determinant $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 - \omega^2 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$ is

(a) 3ω (b) $3\omega(\omega - 1)$

(c) $3\omega^2$ (d) $3\omega(1 - \omega)$

14. The harmonic mean of the root of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

(a) 2 (b) 4

(c) 6 (d) 8

15. Domain of the function $\log |x^2 - 9|$ is

(a) R (b) $R - [-3, 3]$

(c) $R - \{-3, 3\}$ (d) None of these

16. Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then, $\lim_{x \to 0} \left[\frac{f(1 + x)}{f(1)}\right]^{1/x}$ equals

(a) 1 (b) $e^{1/2}$

(c) e^2 (d) e^3

17. The value of $[\mathbf{a}\ \mathbf{a} + \mathbf{b}\ \mathbf{a} + \mathbf{b} + \mathbf{c}]$ is

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$ (b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

(c) $2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (d) $3[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

18. If $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{c} = -4\mathbf{i} - \mathbf{k}$, then $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

(a) $5\mathbf{i} + 5\mathbf{j} - 15\mathbf{k}$ (b) 0

(c) $12\mathbf{j} + 4\mathbf{k}$ (d) $-3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

19. If $I(m, n) = \int_0^1 t^m (1 + t)^n \, dt$, then the expression for $I(m, n)$ in terms of $I(m + 1, n - 1)$ is

(a) $\frac{2^n}{m + 1} - \frac{n}{m + 1} I(m + 1, n - 1)$

(b) $\frac{n}{m + 1} I(m + 1, n - 1)$

(c) $\frac{2^n}{m + 1} + \frac{n}{m + 1} I(m + 1, n - 1)$

(d) $\frac{m}{m + 1} I(m + 1, n - 1)$

20. The ΔPQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3, 4) and (–4, 3), respectively. Then, $\angle QPR$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

21. The equation of directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(a) $x = -1$ (b) $x = 1$

(c) $x = -\frac{3}{2}$ (d) $x = \frac{3}{2}$

22. If $a > 2b > 0$, then positive value of m for which $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is

(a) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ (b) $\frac{\sqrt{a^2 - 4b^2}}{2b}$

(c) $\frac{2b}{a - 2b}$ (d) $\frac{b}{a - 2b}$

23. If $\alpha \in \left[0, \frac{\pi}{2}\right)$, then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to

(a) $2 \tan \alpha$ (b) 1

(c) 2 (d) $\sec^2 \alpha$

24. If an edge of a cube measure 2 m with a possible error of 0.5 cm. Find the corresponding error in the calculated volume of the cube

(a) 0.6 m^3 (b) 0.06 m^3

(c) 0.006 m^3 (d) 0.0006 m^3

25. Let f be twice differentiable function satisfying $f(1) = 1, f(2) = 4, f(3) = 9$, then

(a) $f''(x) = 2, \forall x \in R$

(b) $f'(x) = 5 = f''(x)$. For some $x \in (1, 3)$

(c) there exists atleast one $x \in (1, 3)$ such that $f''(x) = 2$

(d) None of the above

26. The order of the differential equation whose general solution is given by

$$y = (C_1 + C_2) \cos (x + C_3) - C_4 e^{x + C_5}$$

where, C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is

(a) 5 (b) 4

(c) 3 (d) 2

27. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

(a) $\sqrt{2} \tan^{-1} (\sqrt{\tan x} - \sqrt{\cot x}) + C$

(b) $\frac{2}{\sqrt{2}} \tan^{-1} x + C$

(c) $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(d) None of the above

28. In an election, number of candidates exceeds the number to be elected by 2. A man can vote in 56 ways. Then, the number of candidates is

(a) 5 (b) 6

(c) 7 (d) 8

29. The curve described parametrically by $x = t^2 + t + 1, y = t^2 - t + 1$ represents

(a) a pair of straight lines

(b) an ellipse

(c) a parabola

(d) a hyperbola

30. Tangent is drawn to ellipse $\frac{x^2}{27} + \frac{y^2}{1} = 1$ at $(3\sqrt{3} \cos \theta, \sin \theta)$, where $\left(\theta \in \left(0, \frac{\pi}{2}\right)\right)$. Then, the value of θ such that the sum of intercept on axes made by this tangent is minimum, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

31. Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$

(c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

32. The function $f(x) = \frac{\ln (1 + ax) - \ln (1 - bx)}{x}$ is not defined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$, is

(a) $a - b$ (b) $a + b$

(c) $\ln a + \ln b$ (d) None of these

33. Domain of definition of function $f(x) = \sqrt{\sin^{-1} (2x) + \frac{\pi}{6}}$ for real-valued of x, is

(a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(c) $\left[-\frac{1}{2}, \frac{1}{9}\right]$ (d) $[0, \pi]$

34. If $x\,dy = y(dx + y\,dy)$, $y(1) = 1$ and $y(x) > 0$. Then, $y(-3)$ is equal to

(a) 3 (b) 2

(c) 1 (d) 0

35. Evaluate $\int \frac{(x-1)e^x}{(x+1)^3}\,dx$

(a) 0

(b) $\frac{(x+1)^2}{e^x} + C$

(c) $\frac{e^x}{(x+1)^2} + C$

(d) $\frac{e^x}{(x-1)^2} + (x+1)^2 + C$

36. If $a = \log_2 3$, $b = \log_2 5$, $c = \log_7 2$, then $\log_{140} 63$ in terms of *a, b, c* is

(a) $\frac{2ac+1}{2a+abc+1}$ (b) $\frac{2ac+1}{2a+c+a}$

(c) $\frac{2ac+1}{2c+ab+a}$ (d) None of these

37. Local maximum value of the function $\frac{\log_e x}{x}$ is

(a) 1 (b) *e*

(c) $\frac{1}{e}$ (d) None of these

38. A + iB form of $\frac{(\cos x + i \sin x)(\cos y + i \sin y)}{(\cot u + i)(1 + i \tan v)}$ is

(a) $\sin u \cos v[\cos(x + y - u - v) + i \sin(x + y - u - v)]$

(b) $\sin u \cos v[\cos(x + y + u + v) + i \sin(x + y + u + v)]$

(c) $\sin u \cos v[\cos(x + y + u + v) - i \sin(x + y + u + v)]$

(d) None of the above

39. In the set Q^+ of all positive rational numbers, the operation * is defined by the formula $a * b = \frac{ab}{6}$. Then, the inverse of 9 with respect to * is

(a) 4 (b) 3

(c) $\frac{1}{9}$ (d) $\frac{1}{3}$

40. If $\alpha + \beta + \gamma = \pi$, then the value of $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$ is equal to

(a) $2\sin\alpha$ (b) $2\sin\alpha\cos\beta\sin\gamma$

(c) $2\sin\alpha\sin\beta\cos\gamma$ (d) $2\sin\alpha\sin\beta\sin\gamma$

41. If in a ΔABC

$$\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ac},$$

then $b^2 + c^2$ is equal to

(a) a^2 (b) *ac*

(c) bc (d) None of these

42. In any ΔABC, the least value of $\left(\frac{\sin^2 A + \sin A + 1}{\sin A}\right)$ is

(a) 3 (b) $\sqrt{3}$

(c) 9 (d) None of these

43. Let λ and α be real

$$\lambda x + \sin\alpha \cdot y + \cos\alpha \cdot z = 0,$$
$$x + \cos\alpha \cdot y + \sin\alpha \cdot z = 0,$$
$$-x + \sin\alpha \cdot y - \cos\alpha \cdot z = 0$$

has a non-trivial solution. If $\lambda = 1$, then all the values of α is equals

(a) $n\pi + \frac{\pi}{2}$ (b) $n\pi + \frac{\pi}{4}, n\pi$

(c) $n\pi \pm \frac{\pi}{2}$ (d) $n\pi + \frac{\pi}{4}, \frac{\pi}{4}$

44. If the roots of the equation

$$x^2 - 2ax + a^2 + a - 3 = 0$$

are real and less than 3, then

(a) $a < 2$ (b) $2 \le a \le 3$

(c) $3 < a \le 4$ (d) $a > 4$

45. If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then

(a) $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$ (b) $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$

(c) $\text{Re}\left(\frac{z_1}{z_2}\right) = \text{Im}\left(\frac{z_1}{z_2}\right)$ (d) None of these

46. The number of five digits numbers that can be formed without any restriction is

(a) 990000 (b) 10000

(c) 90000 (d) None of these

47. The middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$ is

(a) $\frac{n}{[(n/2)!]^2}$

(b) $-\frac{(2n!)}{[(n/2)!]^2}$

(c) $\frac{1\cdot 3\cdot 5\ldots(2n+1)}{n!}2^n$

(d) $\frac{(2n)!}{(n!)^2}$

48. The value of $\sin(22°30')$ is

(a) $\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$ (b) $\sqrt{\sqrt{2}-1}$

(c) $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$ (d) None of these

49. If $\frac{\pi}{2}<\theta<\frac{3\pi}{2}$, then the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ is equal to

(a) $-\sec\theta+\tan\theta$ (b) $\sec\theta+\tan\theta$

(c) $\sec\theta-\tan\theta$ (d) None of these

50. If $\int\frac{2^x}{\sqrt{1-4^x}}dx = k\sin^{-1}(2^x)+C$, then k is equal to

(a) log 2 (b) $\frac{1}{2}\log 2$

(c) $\frac{1}{2}$ (d) $\frac{1}{\log 2}$

51. The area of the region bounded by $y=-1, y=2$, $x=y^3$ and $x=0$ is

(a) $\frac{17}{4}$ sq units (b) $\frac{1}{4}$ sq unit

(c) 4 sq units (d) None of these

52. The mid-point of the line joining the common points of the line $2x-3y+8=0$ and $y^2=8x$ is

(a) (3, 2) (b) (5, 6)

(c) (4, −1) (d) (2, −3)

53. The value of $\cos^2 48°-\sin^2 12°$ is

(a) $\frac{\sqrt{5}+1}{8}$ (b) $\frac{\sqrt{5}-1}{8}$

(c) $\frac{\sqrt{5}+1}{16}$ (d) None of these

54. The number of solutions of $\sin^2\theta+3\cos\theta=3$ in $[-\pi,\pi]$, is

(a) 4 (b) 2

(c) 0 (d) None of these

55. The height of the chimney when it is found that on walking towards it 50 m in the horizontal line through its base, the angle of elevation of its top changes from 30° to 60°, is

(a) $25\sqrt{3}$ m (b) 25 m

(c) $25\sqrt{4}$ m (d) $\frac{25}{\sqrt{3}}$ m

56. If $\lim_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right)=2$, then

(a) $a=1, b=1$ (b) $a=1, b=2$

(c) $a=1, b=-2$ (d) None of these

57. If $f(x)=\begin{cases}\frac{|x+2|}{\tan^{-1}(x+2)}, & x\neq -2\\ 2, & x=-2\end{cases}$, then $f(x)$ is

(a) continuous at $x=-2$

(b) not continuous at $x=-2$

(c) differentiable at $x=-2$

(d) continuous but not derivable at $x=-2$.

58. Let P be any point on the curve $x^{2/3}+y^{2/3}=a^{2/3}$. Then, the length of the segment of the tangent between the coordinate axes of length

(a) 3a (b) 4a

(c) 5a (d) a

59. The motion of a particle along a straight line is described by the function $x=(2t-3)^2$, where x is in metre and t is in second. Then, the velocity of the particle at origin is

(a) 0 (b) 1

(c) 2 (d) None of these

60. The value of $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$ is

(a) 1 (b) −1

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

61. The maximum value of $3\cos\theta+4\sin\theta$ is

(a) −5 (b) 5

(c) 25 (d) None of these

62. Which of the following is not a statement?

(a) Every set is a finite set

(b) 8 is less than 6

(c) Where are you going

(d) The sum of interior angles of a triangle is 180°

63. The least positive remainder when $123 \times 125 \times 127$ is divided by 124 is
(a) 4 (b) 120
(c) 121 (d) 130

64. The vertex connectivity of any tree is
(a) one (b) two
(c) three (d) None of these

65. A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done, is
(a) 216 (b) 240
(c) 600 (d) 3125

66. Given positive integers $r > 1$, $n > 2$ and the coefficient of $(3r)$ th and $(r + 2)$ th terms in the binomial expansion of $(1 + x)^{2n}$ are equal, then
(a) $n = 2r$ (b) $n = 2r + 1$
(c) $n = 3r$ (d) None of these

67. The number of solutions of $\log_4 (x - 1) = \log_2 (x - 3)$ is
(a) 3 (b) 1
(c) 2 (d) 0

68. The value of $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \cos^2 t \, dt}{x \sin x}$ is
(a) 2 (b) 1
(c) 0 (d) None of these

69. The value of $\int_2^3 \dfrac{\sqrt{x}}{\sqrt{5 - x} + \sqrt{x}} \, dx$ is
(a) 1 (b) 1/2
(c) 2 (d) None of these

70. The solution of differential equation $(e^x + 1) y \, dy = (y + 1) e^x \, dx$ is
(a) $(e^x + 1)(y + 1) = Ce^y$
(b) $(e^x + 1)|(y + 1)| = Ce^{-y}$
(c) $(e^x + 1)(y + 1) = \pm Ce^y$
(d) None of the above

71. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $11x + \dfrac{1}{2} y + \dfrac{25}{2} = 0$ is
(a) $4x^2 + 4y^2 - 30x - 10y = 25$
(b) $4x^2 + 4y^2 - 30x - 13y - 25 = 0$
(c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$
(d) None of the above

72. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is
(a) $3y = 9x + 2$
(b) $y = 2x + 1$
(c) $2y = x + 8$
(d) $y = x + 2$

73. If k be the perimeter of the ΔABC, then $b \cos^2 \dfrac{C}{2} + c \cos^2 \dfrac{B}{2}$ is equal to
(a) k (b) $2k$
(c) $\dfrac{k}{2}$ (d) None of these

74. Let $f(x)$ be a positive function. Let
$$I_1 = \int_{1-k}^{k} xf\{x(1 - x)\} \, dx,$$
$$I_2 = \int_{1-k}^{k} f\{x(1 - x)\} \, dx,$$
where $2k - 1 > 0$, then $\dfrac{I_1}{I_2}$ is
(a) 2 (b) k
(c) $\dfrac{1}{2}$ (d) 1

75. If P is a variable point on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, with AA' as major axis. Then, the maximum value of the area of the $\Delta APA'$ is
(a) ab sq unit (b) $2ab$ sq unit
(c) $\dfrac{ab}{2}$ sq unit (d) None of these

76. In a ΔABC, if $\dfrac{\cos A}{a} = \dfrac{\cos B}{b} = \dfrac{\cos C}{c}$ and the side $a = 2$, then area of the triangle is
(a) 1 sq unit (b) 2 sq units
(c) $\dfrac{\sqrt{3}}{2}$ sq units (d) $\sqrt{3}$ sq units

77. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is
(a) 1 sq unit (b) 2 sq units
(c) $2\sqrt{2}$ sq units (d) 4 sq units

78. The value of the expression $\sqrt{3}$ cosec $20° - \sec 20°$ is equal to
(a) 2 (b) $\dfrac{2 \sin 20°}{\sin 40°}$
(c) 4 (d) $\dfrac{4 \sin 20°}{\sin 40°}$

79. If in a ΔPQR, sin P, sin Q, sin R are in AP, then
(a) the altitudes are in AP
(b) the altitudes are in HP
(c) the medians are in GP
(d) the medians are in AP

80. A man and a woman appears in an interview for two vacancies in the same post. The probability of man's selection is $\frac{1}{4}$ and that of the woman's selection is $\frac{1}{3}$. What is the probability that none of them will be selected?

(a) $\frac{1}{2}$ (b) $\frac{1}{12}$

(c) $\frac{1}{4}$ (d) None of these

English

Directions (Q. Nos. 1-5) *In the following questions, some of the sentences have errors and some have none. Find out which part of a sentence has an error. The letter of that part is your answer. If there is no error, your answer is (d) i.e., No error.*

1. The whole country (a)/was suffering by (b)/a financial crisis. (c)/No error. (d)

2. I do not know where could he have gone (a)/so early (b)/in the morning. (c)/No error. (d)

3. I suggest that (a)/he goes (b)/to the doctor as soon as he returns from taking the examination. (c)/No error. (d)

4. The introduction of tea and coffee (a)/and such other beverages (b)/have not been without some effect. (c)/No error. (d)

5. Inspite of the roadblock (a)/the guards allowed us (b)/enter the restricted area to search for our friends (c)/No error. (d)

Directions (Q. Nos. 6-10) *In the following questions, out of the four alternatives, choose the one which best expresses the meaning of the given word.*

6. Illicit
(a) immoral (b) illegal
(c) ineligible (d) illegible

7. Flair
(a) talent (b) tendency
(c) bias (d) need

8. Conservation
(a) preservation (b) respiration
(c) correction (d) confusion

9. Abysmal
(a) sickening (b) gloomy
(c) sad (d) bottomless

10. Salient
(a) valient (b) variant
(c) prodent (d) prominent

Directions *(11-15) In the following questions, choose the word(s) opposite in meaning to the given word.*

11. Suppress
(a) stir up (b) rouse
(c) urge (d) incite

12. Loosen
(a) fasten (b) accelerate
(c) delay (d) paste

13. Rebellion
(a) forgiveness (b) retribution
(c) submission (d) domination

14. Idiosyncrasy
(a) insanity (b) sanity
(c) generality (d) singularity

15. Sanguine
(a) diffident (b) hopeless
(c) cynical (d) morose

Directions (Q. Nos. 16-20)

16. (a) prioratise (b) picturise
(c) hesitate (d) perforate

17. (a) mendacous (b) obnoxcious
(c) pernicious (d) ferocious

18. (a) pennetrate (b) irritate
(c) hesitate (d) perforate

19. (a) passagway (b) causeway
(c) subway (d) straightway

20. (a) rapport (b) support
(c) repport (d) purport

Directions (Q. Nos. 21-25) *In the following questions, four alternatives are given for the Idio/Phrase printed in* ***bold*** *in the sentence. Choose the alternative which best expresses the meaning of the Idiom/Phrase.*

21. Don't worry about the silly row. It was just a **storm in a tea cup.**

(a) important matter dealt with ease
(b) hot tea being served
(c) commotion over a trival matter
(d) confusion and chaos

22. The Rajput warriors **set their face against** the invader.
(a) became enemies (b) turned away from
(c) faced difficulty (d) opposed strongly

23. Syria is now **currying favour with** America.
(a) pleasing
(b) favouring
(c) obliging
(d) ingratiating itself with

24. Our principal is not a man **to mince matters.**
(a) to confuse issues
(b) to say something mildly
(c) to mix everything together
(d) to be very modest

25. We tend to **take for granted** the conveniences of modern life.
(a) to consider (b) to admit
(c) to accept readily (d) to care for

Directions (Q. Nos. 26-30) *In the following questions, out of the four alternatives, choose the one which can be substituted for the given words/sentence.*

26. Belief that God is in everything and that everything is God
(a) Atheism (b) Pantheism
(c) Scepticism (d) Animism

27. A picture of a person or a thing drawn in such a highly exaggerated manner as to cause laughter
(a) Cartoon (b) Cacography
(c) Cartography (d) Caricature

28. The state of being miserable be reft of all possessions
(a) Dependant (b) Complacent
(c) Destitute (d) Omnipresent

29. That which cannot be called back
(a) Irresponsible (b) Irrevocable
(c) Irredeemable (d) Incalculable

30. One who journeys from place to place
(a) Quack (b) Cannibal
(c) Itinerant (d) Courier

Directions (Q. Nos. 31-35) *In the following questions, a sentence has been given in Active Voice/Passive Voice. Out of the four alternatives suggested, select the one which best expresses the same sentence in Passive Voice/Active Voice.*

31. We have already done the exercise.
(a) Already, the exercise has been done by us
(b) The exercise has already been done by us
(c) The exercise had been already done by us
(d) The exercise is already done by us

32. The main skills we seek to develop include analysing, interpreting and evaluating ideas.
(a) The main skills sought by us to develop include analysing, interpreting and evaluating ideas
(b) The main skills that we are seeking to be developed include analysing, interpreting and evaluating ideas
(c) The main skills sought to be developed by us include analysing, interpreting and evaluating ideas
(d) The main skills include analysing, interpreting and evaluating kleas which are sought by us to develop

33. Who can question Gandhi's integrity?
(a) By whom Gandhi's integrity can be questioned?
(b) By whom can Gandhi's integrity be questioned?
(c) Gandhi's integrity can be questioned by whom?
(d) Who could have questioned Gandhi's integrity?

34. He presented me a bouquet on my birthday.
(a) A bouquet is presented to me on my birthday by him
(b) I was presented on my birthday a bouquet by him
(c) I was presented a bouquet on my birthday by him
(d) I will be presented a bouquet on my birthday by him

35. This surface feels smooth.
(a) This surface is felt smooth
(b) This surface is smooth when it is felt
(c) This surface when felt is smooth
(d) This surface is smooth as felt

Directions (Q. Nos. 36-40) *In the following questions, a part of the sentence is printed* ***bold.*** *Below are given alternatives to the* ***bold*** *part at (a), (b) and (c), which may improve the sentence. Choose the correct alternative. In case no improvement is needed, your answer is (d).*

36. In the desert, the sun is the master, all else **resigns** before its merciless rays.
(a) collapses (b) falls
(c) retires (d) No improvement

37. I intend **to learn** French next year.
(a) learning (b) learn
(c) have learnt (d) No improvement

38. The police **needed** him for armed robbery.
(a) liked (b) was after
(c) were looking to (d) No improvement

39. There is **no more room** for you in this compartment.
(a) no more seat
(b) no more space
(c) no more accommodation
(d) No improvement

40. It is easy to see why cities grew **on the river banks**
(a) along the river banks
(b) in the river banks
(c) upon the river banks
(d) No improvement

Answers

Physics

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (b)
11. (c)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (a)
21. (a)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (d)
31. (b)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (a)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (b)
41. (d)	**42.** (b)	**43.** (a)	**44.** (c)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (d)	**50.** (d)
51. (b)	**52.** (a)	**53.** (c)	**54.** (d)	**55.** (c)	**56.** (b)	**57.** (b)	**58.** (a)	**59.** (d)	**60.** (c)

Chemistry

1. (b)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (d)	**10.** (a)
11. (b)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (c)
21. (d)	**22.** (c)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (a)
31. (a)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (a)
41. (d)	**42.** (b)	**43.** (a)	**44.** (d)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (b)	**50.** (b)
51. (b)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (c)	**56.** (b)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (c)

Mathematics

1. (a)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (d)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (b)
31. (d)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (c)	**36.** (d)	**37.** (c)	**38.** (a)	**39.** (a)	**40.** (c)
41. (a)	**42.** (a)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (c)	**49.** (a)	**50.** (d)
51. (a)	**52.** (b)	**53.** (a)	**54.** (d)	**55.** (a)	**56.** (c)	**57.** (b)	**58.** (d)	**59.** (a)	**60.** (d)
61. (b)	**62.** (c)	**63.** (c)	**64.** (a)	**65.** (a)	**66.** (a)	**67.** (b)	**68.** (b)	**69.** (b)	**70.** (a)
71. (d)	**72.** (d)	**73.** (c)	**74.** (c)	**75.** (a)	**76.** (d)	**77.** (b)	**78.** (c)	**79.** (b)	**80.** (a)

English

1. (b)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (d)
11. (d)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (c)
21. (c)	**22.** (d)	**23.** (d)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (d)	**37.** (d)	**38.** (c)	**39.** (d)	**40.** (a)

Hints & Solutions

Physics

1. Let one mole of each gas has same volume as V. When they are mixed, then density of mixture is

$$\rho_{mixture} = \frac{\text{mass of } O_2 + \text{mass of } H_2}{\text{volume of } O_2 + \text{volume of } H_2}$$

$$= \frac{32+2}{V+V} = \frac{34}{2V} = \frac{17}{V}$$

Also, $\rho_{H_2} = \frac{2}{V}$

Now, velocity $v = \left(\frac{\gamma P}{\rho}\right)^{1/2}$ or $v \propto \frac{1}{\sqrt{\rho}}$

$$\frac{v_{mixture}}{v_{H_2}} = \sqrt{\left(\frac{\rho_{H_2}}{\rho_{mixture}}\right)}$$

$$= \sqrt{\left(\frac{2/V}{17/V}\right)} = \sqrt{\left(\frac{2}{17}\right)}$$

2. Let the distance between the two cliffs be d. Since, the man is standing midway between the two cliffs, then the distance of man from either end is $d/2$.

The distance travelled by sound (in producing an echo)

$$2 \times \frac{d}{2} = v \times t \quad \Rightarrow \quad d = 340 \times 1 = 340 \text{ m}$$

3. The distance between the first dark (minima) fringes on either side of the central maximum is twice the fringe width of either maximum or minimum.

i.e., $\beta = 2\frac{D\lambda}{d} = \frac{2 \times 2 \times 600 \times 10^{-9}}{1 \times 10^{-3}}$

$$= 2.4 \times 10^{-3} = 2.4 \text{ mm}$$

4. After silvering the plane surface, plano-convex lens behaves as a concave mirror of focal length

$$\frac{1}{F} = \frac{2}{f_{lens}}$$

But $F = 0.2 \text{ m}$

$\therefore$ $f_{lens} = 2F = 2 \times 0.2 = 0.4 \text{ m}$

Now, from lens maker's formula

$$\frac{1}{f_{lens}} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ $\frac{1}{0.4} = (1.5-1) \times \frac{1}{R_1}$ $[\because R_2 = \infty]$

$\Rightarrow$ $R_1 = 0.5 \times 0.4 = 0.2 \text{ m}$

5. For a stone which is thrown downwards from a balloon rising upwards, the equation of motion is

$$h = -ut + \frac{1}{2}gt^2$$

$$= -29 \times 10 + \frac{1}{2} \times 9.8 \times (10)^2$$

$$= -290 + 490$$

$$= 200 \text{ m}$$

6. The distance between two consecutive dark or bright fringes is recognised as β (fringe width) and that between central fringe and first dark fringe on either side is $\frac{\beta}{2}$.

Given, spacing between second dark fringe and central fringe $= \beta + \frac{\beta}{2}$

or $\frac{3\beta}{2} = 1 \text{ mm}$ (Given)

or $\beta = \frac{2}{3} \times 1 \text{ mm}$ or $\frac{\lambda D}{d} = \frac{2}{3} \text{ mm}$

$\therefore$ $\lambda = \frac{2}{3} \times 10^{-3} \times \frac{0.9 \times 10^{-3}}{1}$

$\Rightarrow$ $\lambda = 0.6 \times 10^{-6} \text{ m}$

$\therefore$ Wavelength of the monochromatic source of light

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm}$$

7. The number of counts left after time t

$$N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$\therefore$ $30 = 240\left(\frac{1}{2}\right)^{60/T_{1/2}}$

$[\because t = 1 \text{ h} = 60 \text{ min}]$

or $\frac{30}{240} = \left(\frac{1}{2}\right)^{60/T_{1/2}}$

or $\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{60/T_{1/2}}$

Comparing the powers, we get

$$\therefore \quad \frac{60}{T_{1/2}} = 3$$

$$T_{1/2} = \frac{60}{3}$$

or $$T_{1/2} = 20 \text{ min}$$

8. The resistance of conductor

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$$

or $$R \propto \frac{l}{r^2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2}$$

$$= \frac{2}{1} \times \left(\frac{2}{1}\right)^2 = \frac{8}{1}$$

Thermal potentials between the ends of the rods are same. So, heat conducted per second

$$H = \frac{Q}{t} \propto \frac{1}{R}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{1}{8} = 1 : 8$$

9. Using the relation

$$\frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

or $$\frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

or $$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \left(\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right)$$

or $$W = Q_1\left(1 - \frac{T_2}{T_1}\right)$$

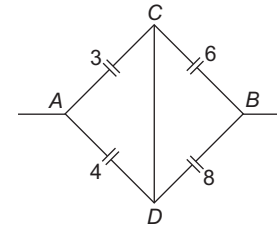
$$\therefore \quad W = 6 \times 10^4\left[1 - \frac{127 + 273}{227 + 273}\right]$$

or $$W = 6 \times 10^4\left(1 - \frac{400}{500}\right)$$

$$= 6 \times 10^4 \times \frac{100}{500}$$

$$= 1.2 \times 10^4 \text{ J}$$

10. The points *C* and *D* will be at same potenitals since, $\frac{3}{6} = \frac{4}{8}$. Therefore, capacitance of 2 μF will be unaffected. So, the equivalent circuit can be shown as the effective capacitance in upper arm in series, is given by



$$C_1 = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2\mu F$$

The effective capacitance in lower arm in series, is given by

$$C_2 = \frac{4 \times 8}{4 + 8} = \frac{32}{12} = \frac{8}{3}\mu F$$

Hence, the resultant capacitance in parallel is given by

$$C = C_1 + C_2 = 2 + \frac{8}{3} = \frac{14}{3}\mu F$$

11. Tension in the string is along the radius of circular path adopted by the bob; while displacement of the bob is along the circumference of the path. Hence, again **F** and **s** are at 90° and so, W = 0.

12. $K_{max} = K_0$ = total energy

As total energy remains conserved in SHM, hence when *U* is maximum in SHM, K = 0, *i.e.*, *E* is also equal to U_{max}, *i.e.*, $U_{max} = E = K_0$.

13. When the particle completes half revolution, change in velocity

$$\Delta v = [5 - (-5)] \text{ m/s} = 10 \text{ m/s}$$

Time taken to complete the half revolution is

$$t = \frac{\pi r}{v} = \frac{\pi \times 5}{5} = \pi \text{ s}$$

Thus, average acceleration $= \frac{\Delta v}{t} = \frac{10}{\pi} \text{ m/s}^2$

14. Mass of copper liberated

$$m = zit = z\left(\frac{P}{V}\right)t$$

$$= (3.3 \times 10^{-7})\left(\frac{100 \times 36 \times 10^5}{33}\right)$$

$$= 3.6 \text{ kg}$$

15. At resonant frequency

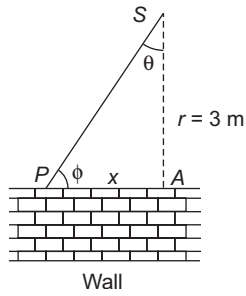
$$X_L = X_C \qquad \left(\omega L = \frac{1}{\omega C}\right)$$

At frequencies higher than resonance frequencies

$$X_L > X_C$$

i.e., behaviour is inductive.

16. The situation is shown in figure.



From figure,

$$x = r \tan\theta$$

$\therefore$ Velocity of *P* is

$$v = \frac{dx}{dt} = r \sec^2\theta \left(\frac{d\theta}{dt}\right)$$

where, $\frac{d\theta}{dt}$ = angular velocity of rotation of spot

$$= \omega$$

$$\therefore \quad v = \omega r \sec^2\theta$$

At $\phi = 45°$, so $\theta = 45°$

Hence, $v = 0.1 \times 3 \times \sec^2 45°$

$$= 0.1 \times 3 \times 2 = 0.6 \text{ m/s}$$

17. Using standard gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad \text{or} \quad V_2 = \frac{p_1V_1T_2}{p_2T_1}$$

$$\therefore \quad V_2 = \frac{1 \times 500 \times (273 - 3)}{0.5 \times (273 + 27)}$$

$$V_2 = \frac{1 \times 500 \times 270}{0.5 \times 300}$$

$$V_2 = 900 \text{ m}^3$$

18. Amplitude is independent of wavelength, velocity and frequency of oscillation.

19. Breaking stress $= \frac{\text{Breaking force}}{\text{Area}} = \text{constant}$

$$\frac{F}{\left(\frac{\pi D^2}{4}\right)} = \frac{F'}{\pi D^2}$$

$$F' = 4F$$

20. $\frac{pV}{T} = \text{constant}$; $V = \text{constant}$

$$\therefore \quad \frac{p}{T} = \text{constant}$$

$$\therefore \quad \frac{p}{T} = \frac{100.4p}{1000} \times \frac{1}{(T + 1)}$$

$$\frac{1}{T} = \frac{100.4}{100} \times \frac{1}{(T + 1)}$$

$$\frac{1}{T} = \frac{100.4}{100T + 100}$$

$$T = 250 \text{ K}$$

21. Maximum rate of energy flow, $S = E_0 \times H_0$

Given, $E_0 = 100 \text{ V/m}$, $H_0 = 0.265 \text{ A/m}$

$$\therefore \quad S = 100 \times 0.265 = 26.5 \text{ W/m}^2$$

22. Conservation of linear momentum gives,

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$v_1 = \frac{-m_2v_2}{m_1}$$

Given, $m_1 = 10 \text{ g} = \left(\frac{10}{1000}\right) \text{kg}$, $m_2 = 1 \text{ kg}$,

$$v_2 = -5 \text{ m/s}$$

$$\therefore \quad v_1 = \frac{+1 \times 5}{10/1000}$$

or $\quad v_1 = 500 \text{ m/s}$

23. Work done

$$W = p(V_2 - V_1)$$

Given, $p = 2 \times 10^5 \text{ N/m}^2$, $V_1 = 50 \times 10^{-3} \text{ m}^3$,

$$V_2 = 150 \times 10^{-3} \text{ m}^3$$

$$\therefore \quad W = 2 \times 10^5 (150 - 50) 10^{-3}$$

$$W = 2 \times 10^5 \times 100 \times 10^{-3}$$

$$W = 2 \times 10^4 \text{ J}$$

24. In an electron gun, the control grid is given a negative potential relative to cathode in order to repel the electrons so that a converging beam of electrons emerges from gun.

25. From the formula, $\omega = \frac{1}{\sqrt{LC}}$

Given, L = 8 H,

$$C = 0.5 \mu\text{F} = 0.5 \times 10^{-6} \text{ F}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{8 \times 0.5 \times 10^{-6}}}$$

$$\Rightarrow \qquad \omega = \frac{1}{2\times 10^{-3}} = 500$$

$$\Rightarrow \qquad \omega = \frac{250}{\pi}\ \text{rad/s}$$

26. Total power

$$P = P_1 + P_2$$
$$P = -4D + 2D$$
$$P = -2D$$

27. Molecular spectra due to vibrational motion lies in the microwave region of electromagnetic spectrum. Due to Kirchhoff's law in spectroscopy the same will be absorbed.

28. During inelastic collision of two particles kinetic energy after collision is not equal to kinetic energy before collision. Here, KE appears in other forms. In some cases, $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation energy) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released.

29. From the relation,

$$\overline{KE} = \frac{3}{2}kT$$

$$\Rightarrow \qquad \frac{(\overline{KE})_1}{(\overline{KE})_2} = \frac{T_1}{T_2} \qquad \text{(become } k \text{ is constant)}$$

Given, $T_1 = 300$ K, $T_2 = 350$ K

$$\therefore \qquad \frac{(KE)_1}{(KE)_2} = \frac{300}{350} = \frac{6}{7}$$

30. From the formula, $\varepsilon = -L\dfrac{dI}{dt}$

Take magnitude on both sides.

$$|\varepsilon| = \left|-L\frac{dI}{dt}\right| \quad \text{or} \quad \varepsilon = L\frac{dI}{dt}$$

$$\therefore \qquad 80 = L\left(\frac{2}{0.05}\right) \quad \Rightarrow \quad 80 = L\times 40$$

$$\Rightarrow \qquad L = \frac{80}{40} \quad \Rightarrow \quad L = 2\,H$$

32. As same force is applied to each wire having equal area of cross-section, so stress is same but extensions will be different for wires of different materials. So, same stress but different strains are there.

33. $L_1 = \dfrac{g_1T^2}{4\pi^2} = \dfrac{g_1}{\pi^2}$; $L_2 = \dfrac{g_2T^2}{4\pi^2} = \dfrac{g_2}{\pi^2}$

Since, length is decreased, g_2 is less than g_1.

$$\therefore \qquad L_1 - L_2 = \frac{g_1 - g_2}{\pi^2}$$

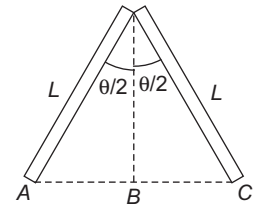
or $\qquad (L_1 - L_2)\pi^2 = g_1 - g_2$

or $\qquad 0.3\times 10 = g_1 - g_2$

$$\therefore \qquad g_2 = 981 - 3 = 978\ \text{cm/s}^2$$

34. Electric field due to outer plate is zero and so the electric field due to inner cylindrical charged conductor varies inversely as the distance from the axis.

35. On bending the magnet, the length of the magnet



$$AC = AB + BC$$
$$= L\sin\left(\frac{\theta}{2}\right) + L\sin\left(\frac{\theta}{2}\right)$$
$$= 2L\sin\left(\frac{\theta}{2}\right) = 2L\sin 30°$$
$$= 2L\times\frac{1}{2} = L$$

36. When a capacitor is fully charged, then no current is drawn by it. When no current flows in the circuit, potential difference across the cell = emf of cell = potential difference across the capacitor.

37. For hydrogen like atom, the radius of nth orbit

$$r_n^Z = \frac{n^2}{Z}a_0$$

Here, $\qquad a_0 = 0.51\times 10^{-10}$ m

$$\therefore \qquad r_n^Z = \frac{0.51\times 10^{-10}}{4}\ \text{m}$$

In the ground state, $n = 1$

$$\therefore \qquad \frac{0.51\times 10^{-10}}{4} = \frac{1^2}{Z}\times 0.51\times 10^{-10}$$

$$\therefore \qquad Z = 4$$

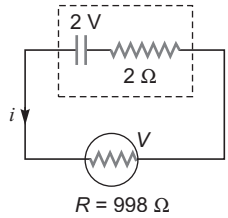
So, the atom is triply ionised beryllium (Be^{3+}).

38. $\lambda = \dfrac{h}{mv} = \dfrac{6.6\times 10^{-34}}{1\times 2000}$

$$= 3.3\times 10^{-37}\ \text{m} = 3.3\times 10^{-27}\ \text{Å}$$

39. From Ohm's law

$$i = \frac{E}{R + r} = \frac{2}{998 + 2} = 2 \times 10^{-3} \text{ A}$$



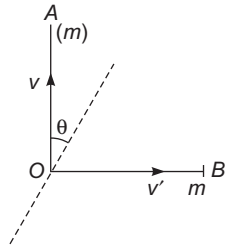
Potential difference across the voltmeter is

$$V = iR = (2 \times 10^{-3}) \times 998$$

$$= 1.996 \text{ V}$$

40. Initially, due to upward acceleration apparent weight of the body increases but then it decreases due to decrease in gravity.

41. From law of conservation of momentum,



$$mv - mv' \cos\theta = 0 \quad \ldots(i)$$

$$mv - mv' \sin\theta = 0 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\theta = 45°$$

$$v' = \frac{v}{\cos\theta} = v\sqrt{2}$$

42. The decay equation is

$${}_{92}X^{A} \longrightarrow {}_{Z}Y^{228} + \alpha$$

α-particle is nucleus of ${}_{2}He^{4}$

$${}_{92}X^{A} \longrightarrow {}_{Z}Y^{228} + {}_{2}He^{4}$$

$$A = 228 + 4 = 232$$

$$Z = 92 - 2 = 90$$

43. Insulators have large energy gap between valence and conduction band (about 6 eV), while semiconductors have a smaller one and conductors, the smallest energy gap.

44. NOR gate is obtained when the output of OR gate is made as the input of NOT gate. Boolean expression for NOR gate is

$$Y = \overline{A + B}$$

Symbol of NOR gate

Truth Table

A	*B*	*C*
0	0	1
1	0	0
0	1	0
1	1	0

45. In common-emitter amplifier, the emitter current is much larger than the base current.

46.

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$$\therefore \quad L = 10 \log_{10}\left(\frac{2 \times 10^{-8}}{10^{-12}}\right)$$

$$(\because \ I_0 = 10^{-12} \text{ W/m}^2)$$

$$L = 10 \log_{10}(2 \times 10^{4})$$

$$L = 10 [\log_{10} 2 + \log_{10} (10)^{4}]$$

$$L = 10 (4.3)$$

$$L = 43 \text{ decibel}$$

47. $D = 360° - 2\theta \Rightarrow 240° = 360° - 2\theta$

$2\theta = 120°, \quad \therefore \quad \theta = 60°$

Then, number of images observable

$$N = \frac{360°}{60°} - 1$$

$$N = 6 - 1$$

$$N = 5$$

48. Number of Cu ions liberated is

$$n = \frac{I \cdot t}{e \text{ (valence electron)}}$$

$$n = \frac{1 \times 10}{1.6 \times 10^{-19} \times 2}$$

$$n = 3.125 \times 10^{19}$$

$$\therefore \quad n \approx 3.1 \times 10^{19}$$

49. For a magnet $B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$ (nearly)

$$\Rightarrow \quad \frac{B_2}{B_1} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \quad \frac{B_2}{B_1} = \left(\frac{x}{2x}\right)^3 \quad \Rightarrow \quad \frac{B_2}{B_1} = \frac{1}{8}$$

Thus, $B_1 : B_2 = 8 : 1$ approximately

50. Electromagnetic waves travel through oscillating electric and magnetic fields whose directions are perpendicular to each other.

51. According to question,

$$\frac{M_1}{d_1^3} = \frac{M_2}{d_2^3}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 = \left(\frac{12}{18}\right)^3$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{8}{27}$$

52. Effective current is the rms value. Here, 220 V is the labelled value of AC which is also the rms value. Hence,

$$I_{rms} = \frac{E_{rms}}{R}$$

$$I_{rms} = \frac{220}{100 \times 10^3}$$

$$I_{rms} = 2.2 \text{ mA}$$

54. The refractive index of glass is the greatest for violet colour and is smaller for red colour and as $\delta \propto (\mu - 1)$ therefore deviation for violet will be maximum.

55. Given, $y_1 = A \sin(kx - \omega t)$

$$y_2 = A \cos(kx - \omega t)$$

or $$y_2 = A \sin\left(kx - \omega t + \frac{\pi}{2}\right)$$

Phase difference of two waves $= \frac{\pi}{2}$

$\because$ Resultant amplitude

$$R = \sqrt{A^2 + A^2 + 2AA\cos\phi}$$

$$= \sqrt{A^2 + A^2 + 2A^2\cos\frac{\pi}{2}}$$

$$= \sqrt{2A^2} \qquad \left(\because \cos\frac{\pi}{2} = 0\right)$$

$$R = \sqrt{2}A$$

56. Momentum of photon, $p = \frac{h}{\lambda}$

Kinetic energy of photon of mass M, $K = \frac{p^2}{2M}$

or $$K = \frac{h^2}{2M\lambda^2}$$

57. $$E = \frac{L^2}{2I}$$

$$\therefore \quad E \propto L^2$$

$$\frac{E_2}{E_1} = \left(\frac{L_2}{L_1}\right)^2$$

$$\frac{E_2}{E_1} = \left[\frac{L_1 + 200\% \text{ of } L_1}{L_1}\right]$$

$$= \left[\frac{L_1 + 2L_1}{L_1}\right]^2 = (3)^2$$

$$E_2 = 9E_1$$

Increment in kinetic energy

$$\Delta E = E_2 - E_1$$

$$= 9E_1 - E_1$$

$$\Delta E = 8E_1$$

$$\frac{\Delta E}{E_1} = 8$$

or percentage increase = 800%

58. The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second $= A\sqrt{2gh}$

Volume of water flowing in per second $= 70 \text{ cm}^3/\text{s}$

$$\therefore \quad A\sqrt{2gh} = 70$$

$$1\sqrt{2 \times 980 \times h} = 70$$

$$h = \frac{4900}{1960} = 2.5 \text{ cm}$$

59. The production of X-rays is an atomic property whereas the production of γ-rays is a nuclear property.

60. Wood is non-crystalline.

Chemistry

1. The reaction is

$$N_2O_4 \rightleftharpoons 2NO_2$$

Molecular weight of $N_2O_4 = 2\times14 + 4\times16$

$= 92$

$\therefore$ Vapour density (D) of N_2O_4

$$= \frac{\text{molecular weight}}{2}$$

$$= \frac{92}{2} = 46$$

From, $\alpha = \dfrac{D-d}{(n-1)d}$

$$\alpha = \frac{46-30}{(2-1)30} = \frac{16}{30}$$

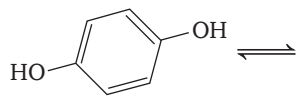
$$= 0.533 = 53.3\%$$

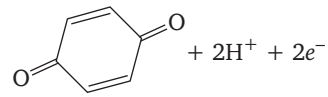
2. Protective power of a lyophilic colloid is measured in terms of gold number. Lower the value of gold number, better is the protective power of a protective colloid. Thus, the order will be

(a) > (b) > (d) > (c)

3. (a) $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log [H^+]$$

(b) 



$+ 2H^+ + 2e^-$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log [H^+]^2$$

(c) $Ag(s) \longrightarrow Ag^+(aq)$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log [Ag^+]$$

(d) $Ag(s) + Cl^-(aq) \longrightarrow AgCl(s) + e^-$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log \frac{1}{[Cl^-]}$$

Thus, in case (d), the E_{cell} increases with increase in the concentration of ions.

4. According to Raoult's law, "Relative lowering of vapour pressure = mole fraction of solute"

$$(X_B) = 0.0125$$

$\because$ $X_B = \dfrac{m\cdot M_A}{10000 + m\cdot M_A}$

$\therefore$ $0.0125 = \dfrac{m\times18}{1000 + m\times18}$

$$12.5 + 0.225m = 18m$$

$$17.775m = 12.5$$

$$m = \frac{12.5}{17.775} = 0.70$$

5.

Formula	Name
(a) $Na_2S_2O_3\cdot5H_2O$	hypo
(b) $Na_2SO_4\cdot10H_2O$	Glauber's salt
(c) $Na_2CO_3\cdot10H_2O$	Washing soda
(d) $Na(NH_4)HPO_4$	Microcosmic salt

6. $PhCHClBr \xrightarrow[-\text{t-BuOH}]{tBuO^-} PhC^-ClBr \longrightarrow Ph\overset{\bullet\bullet}{C}Cl$

7. 3° alcohol offers resistance to oxidation. Hence, Cr^{3+} ions are not formed, and the solution does not turn green.

8. Molisch's test is a general test for carbohydrates, while ninhydrin test is given by all proteins. Nitroprusside test is given by proteins which contains —SH group. Due to the presence of —CONH linkage, alkaline solution of proteins give biuret test.

9. Rocket propellants are a combination of an oxidiser and fuel. Phenyl hydrazine can't act as propellant.

10. $Cl^- = 2, 8, 8$

$O^{2-} = 2, 8$

$Na^+ = 2, 8$

$Mg^{2+} = 2, 8$

Cl^- contains same number of electrons in outermost and penultimate shells.

11. Mixture of $MgCl_2$ and MgO is called Sorel's cement. It is $MgCl_2\cdot5MgO\cdot xH_2O$.

12. Substracting equation (II) from (I), we get

$S_R \longrightarrow S_M$, $\Delta H = +2.5$ kJ

Thus, S_R to S_M transition is an endothermic reaction.

13. (a) PSC (polar stratospheric clouds) are special type of clouds formed over Antarctica in winter that play important role in ozone layer depletion.

(b) PCB (poly chlorinated biphenyls) are toxic chemicals with high stability and resistance to oxidation. These are used for fluids in transformers and capacitors.

(c) BOD (biochemical oxygen demand) is a measure of dissolved oxygen that would be needed by the microorganisms to oxidise organic and inorganic compounds present in polluted water.

(d) COD (chemical oxygen demand) is the amount of oxygen, in ppm, that would be required to oxidise the contaminants.

14. For a first order reaction

$$k = \frac{2.303}{t} \log \frac{a}{a - x}$$

At half-time,

$$x = \frac{a}{2}$$

$$\therefore \quad k = \frac{2.303}{1.6} \times \log \frac{a}{a - \frac{a}{2}}$$

$$= \frac{2.303}{1.6} \times \log 2$$

$$= \frac{2.303}{1.6} \times 0.3010$$

$$= 0.4332$$

For 90% completion,

$$x = 0.9a$$

$$\therefore \quad k = \frac{2.303}{t} \log \frac{a}{a - 0.9a}$$

$$0.4332 = \frac{2.303}{t} \log \frac{a}{0.1\,a}$$

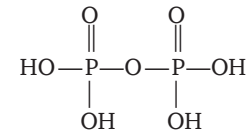
$$= \frac{2.303}{t} \log 10$$

$$= \frac{2.303}{t} \times 1$$

$$t = \frac{2.303}{0.4332}$$

$$= 5.3 \text{ min}$$

15. $H_4P_2O_7$ (pyrophosphoric acid) is tetrabasic because it gives four H^+ ions.



16. Bond angles of ClF_3, PF_3, NF_3 and BF_3 are 80°90′, 101°, 106° and 120° respectively.

17. Isotonic species have the same the number of neutrons.

No. of neutrons in

$$_6C^{14} = 14 - 6 = 8$$

$$_7N^{14} = 14 - 7 = 7$$

$$_9F^{19} = 19 - 9 = 10$$

$$_7N^{15} = 15 - 7 = 8$$

$$_9F^{17} = 15 - 7 = 8$$

$$_6C^{12} = 12 - 6 = 6$$

Hence, $_6C^{14}$, $_7N^{15}$, $_9F^{17}$ are isotonic.

18. $K[Co(CO)_4]$

$$+1 + x + 0 \times 4 = 0$$

$$\therefore \quad x = -1$$

19. The metal placed above in electrochemical series can displace the metals placed below from their salt solution.

The order of given metals in the series is Zn, Fe, Cu, Ag. Thus, Zn metal can displace all other three metals from their salt solutions.

20. Magic numbers are 2, 8, 20, 28, 50 or 82 protons or 2, 8, 20, 28, 50, 82 or 126 neutrons in the nucleus. These numbers impart stability to the nucleus.

(a) $_{13}Al^{27}$, no. of neutrons $= 27 - 13 = 14$

(b) $_{26}Fe^{56}$, no. of neutrons $= 56 - 26 = 30$

(c) $_{83}Bi^{209}$, no. of neutrons $= 209 - 83 = 126$

(d) $_{92}U^{238}$, no. of neutrons $= 238 - 92 = 146$

21. $[H_3O^+] = \sqrt{K_a \times C}$

$$= \sqrt{4 \times 10^{-10} \times 2.5 \times 10^{-1}}$$

$$= \sqrt{10^{-10}} = 10^{-5} \text{ M}$$

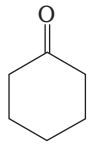
$\because \quad pH = -\log [H_3O^+]$

$\therefore \quad pH = -\log (10^{-5}) = 5$

22. $\Delta H = ms\Delta T = (125 + 10.5) \times 1 \times 3.1$

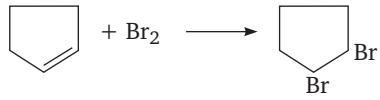
$= 420.05 \text{ cal} = \dfrac{420.05}{10.5} \text{ cal/g}$

$= 40 \text{ cal/g} = 167.7 \text{ J/g}$

23. 

$\longrightarrow$ $HOOC\cdot(CH_2)_4\cdot COOH$
Adipic acid

24. The conversion of —CN to $—CH_2NH_2$ by catalytic reduction is called Mendius reaction.

25. 

26. No. of atoms of *A* from corners of unit cell $= \dfrac{7}{8}$

No. of atoms of *B* from faces of unit cell = 3

$\therefore \quad A : B = \dfrac{7}{8} : 3 = 7 : 24$

Thus, formula of the compound is A_7B_{24}.

27. The elements of II period show similar properties as the elements of III period, which are diagonally placed to them. This is known as diagonal relationship.

II Period	:	Li	Be	B	C	N
III Period	:	Na	Mg	Al	Si	P

28. Molecular weight of $X_4O_6 = (4a + 96)$

$\because$ $(4a + 96)$g X_4O_6 contains X = 4ag

$\therefore$ 10 g X_4O_6 contains $X = \dfrac{4a \times 10}{4a + 96}$

$$5.72 = \frac{4a \times 10}{4a + 96}$$

$$22.88a + 549.12 = 40a$$

$$17.12a = 549.12$$

$$a = 32.07 \text{ amu.}$$

29.

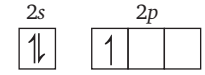
	Formula	Name
(a)	ZnO	Philosopher's wool
(b)	$ZnS + BaSO_4$	Lithopone
(c)	$ZnSO_4 \cdot 7H_2O$	White vitriol
(d)	$ZnO \cdot CoO$	Rinmann's green

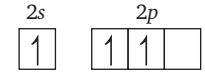
30. $_{29}Cu = 2, 8, 18, 1$

$= 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^1$

$\therefore$ Quantum numbers for $4s^1$ electron are

$$n = 4, \quad l = 0, \quad m = 0, \quad s = +\frac{1}{2}$$

31. $_5B$(G.S.) = 

B(E.S.) = 

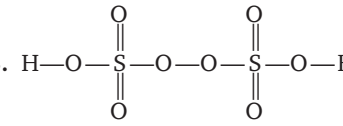
sp^2-hybridisation

Due to sp^2-hybridisation, BF_3 has trigonal planar structure.

32. Normality, $N = \dfrac{w}{E \cdot V}$

$$= \frac{10\,g}{60 \times \frac{100}{1000}\,L}$$

$$= 1.66 \text{ N}$$

33. 

$\because$ 2O-atoms are in peroxide linkage

$\therefore$ Their oxidation state = –1

and, oxidation state of remaining 6O-atoms = – 2

Oxidation state of H-atoms = + 1

Let oxidation state of S atom = x

$S_2 \quad H_2 \quad O_6 \quad O_2$

$\therefore \quad 2x + 2(+1) + 6(-2) + 2(-1) = 0$

$$2x + 2 - 12 - 2 = 0$$

$$x = +6$$

34. Reaction (a) and (b) are exothermic and spontaneous so do not need catalyst. On the other hand, conversion of SO_2 into SO_3 need catalyst. Catalyst used for this reaction may be Pt or V_2O_5 (contact process) or NO (lead chamber process)

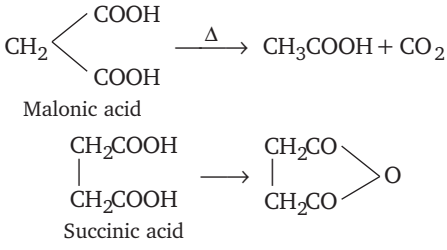
$$2SO_2 + O_2 \xrightarrow{V_2O_5} 2SO_3$$

35. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + 22\,kcal$

$\because$ Activation energy for the forward reaction = 50 kcal

$\therefore$ Activation energy for the backward reaction = 50 + 22 = 72 kcal

36.



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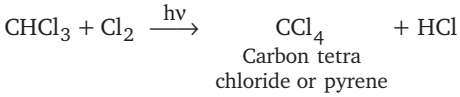

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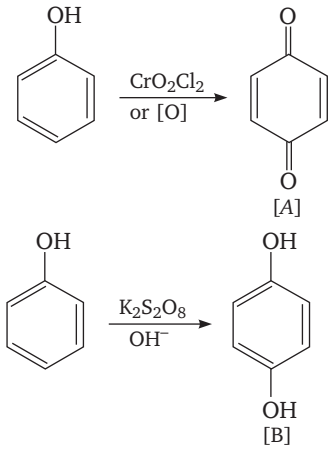
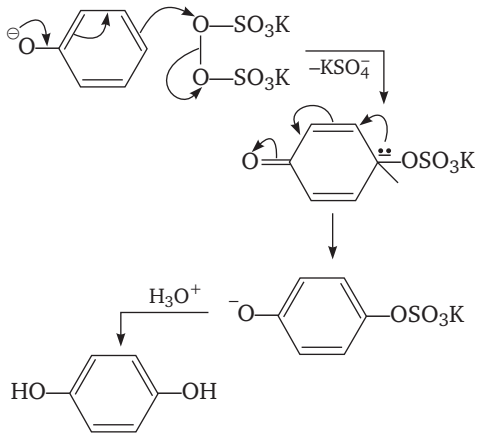


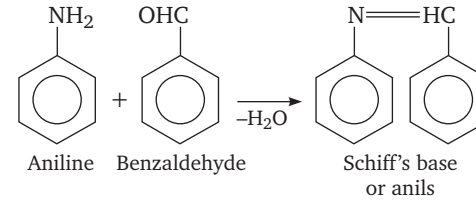
Mechanism



39. In transcription, synthesis of RNA takes place while in translation, synthesis of protein occurs in the cytoplasm of the cell.

40. Penicillin G is a narrow spectrum antibiotics. It is effective against single organism or disease.

41.



42. Only *A* and *C* are anomers.

A and B, B and C } are structural isomers.

43. According to IUPAC nomenclature  is bicyclo (2,1,0) pentane.

44. $CH_3CH_2OH \xrightarrow[K_2Cr_2O_7/H_2SO_4]{[O]} CH_3CHO$

$\xrightarrow[K_2Cr_2O_7/H_2SO_4]{[O]} CH_3COOH \xrightarrow[\Delta]{NaOH + CaO}$

$CH_4 \xrightarrow{Cl_2} CH_3Cl \xrightarrow[-KCl]{aq.\ KOH} CH_3OH$

45. PDI abbreviates as polydispersity index of polymer.

$$PDI = \frac{\overline{M}_w}{M_n}$$

For natural polymers, PDI = 1
and for synthetic polymers, PDI > 1

46. Diamagnetic species have all electrons paired, whereas paramagnetic species have unpaired electrons.

$H_2 = \sigma 1s^2$

No. of unpaired electron = 0 $\Rightarrow$ diamagnetic

$H_2^- = \sigma 1s, \sigma^* 1s^1$

No. of unpaired electron = 1 $\Rightarrow$ paramagnetic

$H_2^- = \sigma 1s^1$

No. of unpaired electron = 1 $\Rightarrow$ paramagnetic

$He_2^+ = \sigma 1s^2, \sigma^* 1s^1$

No. of unpaired electron = 1 $\Rightarrow$ paramagnetic

47. Non-metals are electronegative in nature.

48. $\because \quad \Delta G = \Delta H - T\Delta S$

At equilibrium, $\Delta G = 0$

$\therefore \quad \Delta H = T\Delta S$

$\Delta H = 273 \times (60.01 - 38.20)$

$= 5954.13 \text{ J mol}^{-1} = 5.954 \text{ kJ mol}^{-1}$

$\approx 6 \text{ kJ mol}^{-1}$

49. $\underset{\text{White phosphorus}}{P_4} + 3NaOH + 3H_2O \xrightarrow{\Delta} \underset{\text{Phosphene}}{PH_3} + \underset{\text{Sodium hypophosphate}}{3NaH_2PO_2}$

50. $\because \quad \Delta T_b = \dfrac{1000 \cdot K_b \cdot w}{m \cdot W} \times i$

$\therefore \quad \Delta T_b \propto i \cdot \dfrac{w}{m \cdot \dfrac{W}{1000}} \quad (\because \ K_b = \text{constant})$

or $\quad \Delta T_b \propto i \cdot M$

$\left(\because \text{ Molality} = \dfrac{w}{m \cdot \dfrac{W}{1000}} \text{ and assuming, Molarity, M = molality, } m\right)$

Now, for the given solutions,

(a) For 0.08 M $BaCl_2$, $i \cdot M = 3 \times 0.08 = 0.24$

(b) For 0.15 M KCl, $i \cdot M = 2 \times 0.15 = 0.30$

(c) For 0.10 M glucose, $i \cdot M = 1 \times 0.10 = 0.10$

(d) For 0.06 M $Ca(NO_3)_2$, $i \cdot M = 3 \times 0.06 = 0.18$

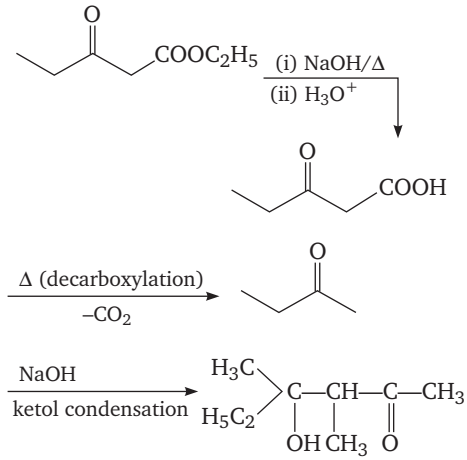
Hence, ΔT_b is maximum for 0.15 M KCl solution.

51. No. of unpaired electrons in Fe = 5

Magnetic moment, $\mu = \sqrt{n(n+2)} = \sqrt{5(5+2)}$

$= \sqrt{35} = 5.92 \text{ BM}$

52.

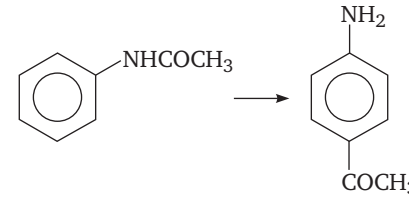


53. $\because \quad \dfrac{r_H}{r_{He}} = \sqrt{\dfrac{M_{He}}{M_H}}$

$= \sqrt{\dfrac{4}{2}} = \sqrt{2} = 1.4$

$\therefore \ r_H : r_{He} = 1.4 : 1$

54.



Above reaction is an example of rearrangement reaction.

55. Temporary hardness of water is due to the presence of bicarbonates of calcium and magnesium while permanent hardness is due to the presence of soluble chlorides and sulphates of calcium and magnesium.

56. Vasopressin, oxytocin, insulin, angiotensin, are peptide hormones whereas thyroid, adrenaline are amine hormones.

57. Indigo is a vat dye. Vat dyes are insoluble in water and cann't be applied directly.

58. Solder is an alloy of tin and lead. Its melting point is quite low, hence, it is very useful in stitching ICs in various electrical instruments.

59. Hg_2^{2+} and Pb^{2+} have been placed in I group of basic radicals as their K_{sp} are less than the II group ions (Hg^{2+}, Cd^{2+}). Hence, only these will precipitate.

60.

$2\pi e^- \ \Rightarrow \ 4 \times 0 + 2 = 4n + 2$

$\therefore$ According to Huckel rule, cyclopropenyl carbocation is aromatic in nature.

Mathematics

1. Let $S = 1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \ldots$

$\therefore \quad T_n = \frac{n^2}{n!} = \frac{n}{(n-1)!}$

$\Rightarrow \quad T_n = \frac{n-1+1}{(n-1)!} = \frac{1}{(n-2)!} + \frac{1}{(n-1)!}$

$\therefore \quad S_n = \Sigma T_n = \Sigma \frac{1}{(n-2)!} + \Sigma \frac{1}{(n-1)!}$

$= e + e$

$= 2e$

2. We have, $f(x) = (x + b)^2 + 2c^2 - b^2$

$\Rightarrow \quad \min f(x) = 2c^2 - b^2$

and $\quad g(x) = b^2 + c^2 - (x + c)^2$

$\Rightarrow \quad \max g(x) = b^2 + c^2$

Thus, $\quad \min f(x) > \max g(x)$

$\Rightarrow \quad 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow \quad c^2 > 2b^2 \quad \Rightarrow \quad |c| > \sqrt{2}|b|$

3. Let $z_1 = \gamma_1 (\cos\theta_1 + i \sin\theta_1)$.

Given that, $\left|\frac{z_1}{z_2}\right| = 1$

$\Rightarrow \quad |z_1| = |z_2| \quad \Rightarrow \quad |z_1| = |z_2| = \gamma_1$

Now, $\arg(z_1 z_2) = 0 \Rightarrow \arg(z_1) + \arg(z_2) = 0$

$\Rightarrow \quad \arg(z_2) = -\theta_1$

Therefore, $z_2 = \gamma_1 (\cos(-\theta_1) + i \sin(-\theta_1)$

$= \gamma_1 (\cos\theta_1 - i \sin\theta_1) = \bar{z}_1$

$\Rightarrow \quad \bar{z}_2 = (\bar{\bar{z}}_1) = z_1 \quad \Rightarrow \quad z_2 \bar{z}_2 = z_1 z_2$

$\Rightarrow \quad |z_2|^2 = z_1 z_2$

4. We have, $\cos A = m \cos B$

$\Rightarrow \quad \frac{\cos A}{\cos B} = \frac{m}{1}$

On applying componendo and dividendo rule, we get

$\Rightarrow \quad \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$

$\Rightarrow \quad \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)} = \frac{m+1}{m-1}$

$\Rightarrow \quad \cot\left(\frac{A+B}{2}\right) = \left(\frac{m+1}{m-1}\right)\tan\left(\frac{B-A}{2}\right)$

But, $\quad \cot\frac{A+B}{2} = \lambda \tan\left(\frac{B-A}{2}\right)$ (given)

$\Rightarrow \quad \lambda = \frac{m+1}{m-1}$

5. Let the angles be $A = x - d$, $B = x$, $C = x + d$, Then, $x - d + x + x + d = 180°$

$\Rightarrow \quad 3x = 180° \quad \Rightarrow \quad x = 60°$

Therefore, two larger angles are *B* and *C*.
Hence, $b = 9$ and $c = 10$.
Now,

$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$

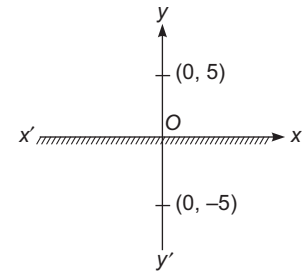
$\Rightarrow \quad \frac{1}{2} = \frac{100 + a^2 - 81}{20a}$

$\Rightarrow \quad a^2 - 10a + 19 = 0$

$\Rightarrow \quad a = 5 \pm \sqrt{6}$

6. $\left|\frac{z - 5i}{z + 5i}\right| = 1 \quad \Rightarrow \quad |z - 5i| = |z + 5i|$

(Using definition $|z - z_1| = |z - z_2|$ gives Perpendicular bisector of z_1 and z_2.)



$\Rightarrow$ Perpendicular bisector of points (0, 5) and (0, – 5).
whch lies on *y*-axis.

7. Let the common ratio of the GP be *r*. Then, $y = xr$ and $z = xr^2$.

$\Rightarrow \quad \ln y = \ln x + \ln r$ and $\ln z = \ln x + 2 \ln r$

Putting $A = 1 + \ln x$, $D = \ln r$

Then, $\quad \frac{1}{1 + \ln x} = \frac{1}{A}$,

$$\frac{1}{1+\ln y}=\frac{1}{1+\ln xr}=\frac{1}{1+\ln x+\ln r}=\frac{1}{A+D}$$

and $\quad \frac{1}{1+\ln z}=\frac{1}{1+\ln x+2\ln r}=\frac{1}{A+2D}$

Here we see that A, $A + D$ and $A + 2D$ are in AP.

$\therefore \quad \frac{1}{A}, \frac{1}{A+D}$ and $\frac{1}{A+2D}$ are in HP.

Therefore, $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$, are in HP.

8. We have,

$$\frac{e^x}{1-x}=B_0+B_1x+B_2x^2+\ldots+B_nx^n+\ldots$$

$$\Rightarrow\ e^x(1-x)^{-1}=B_0+B_1x+B_2x^2+\ldots+B_nx^n+\ldots$$

$$\Rightarrow\ \left(1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots+\frac{x^{n-1}}{(n-1)!}+\frac{x^n}{n!}+\ldots\right)\times$$

$$(1+x+x^2+\ldots+x^{n-1}+x^n+\ldots\infty)$$

$$=B_0+B_1x+B_2x^2+\ldots+B_nx^n+\ldots$$

On comparing the coefficients of x^n and x^{n-1} on both sides, we get

$$\frac{1}{n!}+\frac{1}{(n-1)!}+\ldots+\frac{1}{2!}+\frac{1}{1!}+1=B_n$$

and $\quad \frac{1}{(n-1)!}+\frac{1}{(n-2)!}+\ldots+\frac{1}{2!}+\frac{1}{1!}+1$

$$=B_{n-1}$$

$$B_n-B_{n-1}=\frac{1}{n!}$$

9. As, $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & [x]\neq 0\\ 0, & [x]=0\end{cases}$

$$\Rightarrow\quad f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & x\in R-[0,1)\\ 0, & 0\le x<1\end{cases}$$

$\therefore$ RHL at $x = 0$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}\frac{\sin[0+h]}{[0+h]}=0$$

LHL at $x = 0$

$$\lim_{x\to 0^-} f(x)=\lim_{h\to 0}\frac{\sin[0-h]}{[0-h]}$$

$$=\lim_{h\to 0}\frac{\sin(-1)}{-1}=\sin 1$$

Since, RHL $\neq$ LHL

$\therefore$ Limit does not exist.

10. Here, $\frac{dy}{dt}-\left(\frac{t}{1+t}\right)y=\frac{1}{1+t}$ and $y(0)=-1$

which represents linear differential equation of first order.

$$\text{IF}=e^{-\int\left(\frac{t}{1+t}\right)dt}=e^{-\int\frac{(t+1-1)}{1+t}dt}$$

$$=e^{-t+\log(1+t)}$$

$$=e^{-t}(1+t)$$

$\therefore$ Required solution is

$$y(\text{IF})=[\int Q\cdot(\text{IF})\,dt]+C$$

$$ye^{-t}(1+t)=\int\frac{1}{1+t}e^{-t}(1+t)\,dt+C$$

$$=\int e^{-t}\,dt+C=-e^{-t}+C$$

At $\quad t = 0, y = -1$

$\therefore \quad -1(1+0)=-e^0+C$

$\Rightarrow \quad C = 0$

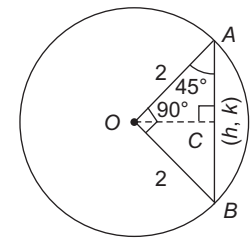
$\therefore \quad ye^{-t}(1+t)=-e^{-t}$

At $\quad t = 1$

$$ye^{-1}(1+1)=-e^{-1}$$

$$\Rightarrow\quad y=-\frac{1}{2}$$

11. As, we have to find the locus of mid-point of chord and we know perpendicular drawn from centre to the chord, it bisects the chord.



Given, $\quad \angle OAB = 90°$

Also, OC bisects the $\angle AOB$

$\therefore \quad \angle COA = \angle OAC = 45°$

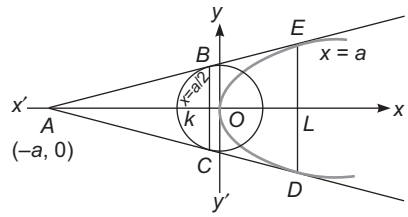
In ΔOAC, $\frac{OC}{OA}=\sin 45° \Rightarrow OC=\frac{2}{\sqrt{2}}=\sqrt{2}$

$\therefore \quad h^2+k^2=OC^2$

$\Rightarrow \quad h^2+k^2=(\sqrt{2})^2$

or $x^2 + y^2 = 2$ is required equation of locus of mid-point of chord subtending right angle at centre.

12. Equation of any tangent of the parabola, $y^2 = 4ax$ is $y = mx + \frac{a}{m}$. This line will touch the circle $x^2 + y^2 = \frac{a^2}{2}$.



$$\therefore \quad BC = 2\sqrt{OB^2 - OK^2}$$

$$= 2\sqrt{\frac{a^2}{2} - \frac{a^2}{4}}$$

$$= 2\sqrt{\frac{a^2}{4}} = a$$

and we know that *DE* is the latusrectum of the parabola, so its length is 4a.

Thus, area of trapezium *BCDE*

$$= \frac{1}{2}(BC + DE)(KL)$$

$$= \frac{1}{2}(a + 4a)\left[\frac{3a}{2}\right]$$

$$= \frac{15a^2}{4} \text{ sq units}$$

13. Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$, the given determinant becomes

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2-\omega^2 & \omega^2 - 1 \\ 0 & \omega^2 - 1 & \omega - 1 \end{vmatrix}$$

$$= 1[(\omega - 1)(-2 - \omega^2) - \{(\omega^2 - 1)(\omega^2 - 1)\}] - 1(0) + 1(0)$$

$$= (-2 - \omega^2)(\omega - 1) - (\omega^2 - 1)^2$$

$$= -2\omega + 2 - \omega^3 + \omega^2 - (\omega^4 + 1 - 2\omega^2)$$

$$= -2\omega + 1 + \omega^2 - (\omega + 1 - 2\omega^2)$$

$$= 3\omega^2 - 3\omega$$

$$= 3\omega(\omega - 1)$$

14. Let α, β be the roots of given quadratic equation. Then,

$$\alpha + \beta = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \quad \text{and} \quad \alpha\beta = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}$$

Again, *H* be the harmonic mean between α and β, then $H = \frac{2\alpha\beta}{\alpha + \beta} = \frac{16 + 4\sqrt{5}}{4 + \sqrt{5}} = 4$

15. For $x = -3, 3; |x^2 - 9| = 0$

Therefore, $\log|x^2 - 9|$ does not exist at

$$x = -3, 3$$

Hence, domain of function is $R - \{-3, 3\}$.

16. Let $y = \left[\frac{f(1 + x)}{f(1)}\right]^{1/x}$

$$\Rightarrow \quad \log y = \frac{1}{x}[\log f(1 + x) - \log f(1)]$$

$$\Rightarrow \lim_{x \to 0} \log y = \lim_{x \to 0} \frac{[\log f(1 + x) - \log f(1)]}{x}$$

$$\Rightarrow \lim_{x \to 0} \log y = \lim_{x \to 0} \left[\frac{1}{f(1 + x)} f'(1 + x)\right]$$

(using L' Hospital's rule)

$$= \frac{f'(1)}{f(1)} = \frac{6}{3} = 2$$

$$\Rightarrow \lim_{x \to 0} y = e^2$$

17. Now, $[\mathbf{a} \quad \mathbf{a} + \mathbf{b} \quad \mathbf{a} + \mathbf{b} + \mathbf{c}]$

$$= [\mathbf{a} \times (\mathbf{a} + \mathbf{b})] \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$$

$$= (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$

$$= [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$$

18. $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{c} = -4\mathbf{i} - \mathbf{k}$

$$\therefore \quad \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b}$$

$$= (4 + 1)(\mathbf{i} + \mathbf{j} - 3\mathbf{k})$$

$$= 5\mathbf{i} + 5\mathbf{j} - 15\mathbf{k}$$

19. Here, $I(m, n) = \int_0^1 t^m (1 + t)^n \, dt$

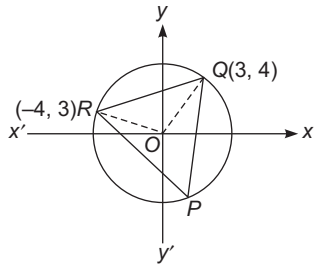
[We apply integration by parts, taking $(1 + t)^n$ as first and t^m as second function]

$$I(m, n) = \left[(1 + t)^n \cdot \frac{t^{m+1}}{m + 1}\right]_0^1 - \int_0^1 n(1 + t)^{n-1} \cdot \frac{t^{m+1}}{m + 1} \, dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1}\int_0^1 (1+t)^{n-1}\cdot t^{m+1}\,dt$$

$$\therefore \quad I(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1}\cdot I(m+1, n-1)$$

20. Let O is the point on centre and P is the point on circumference. Therefore, angle QOR is double the angle QPR. So, it is sufficient to find the angle QOR.



Now, slope of OQ, $m_1 = \frac{4-0}{3-0} = \frac{4}{3}$

Slope of OR, $m_2 = \frac{3-0}{-4-0} = -\frac{3}{4}$

Again, $m_1 m_2 = -1$

Therefore, $\angle QOR = 90°$

Which implies that $\angle QPR = \frac{90°}{2} = 45°$.

21. We have, $y^2 + 4y + 4x + 2 = 0$

$$\Rightarrow \quad (y+2)^2 + 4x - 2 = 0$$

$$\Rightarrow \quad (y+2)^2 = -4\left(x - \frac{1}{2}\right)$$

Replace, $y + 2 = Y, x - \frac{1}{2} = X$

We have, $Y^2 = -4X$

This is a parabola with directrix at $X = 1$

$$\Rightarrow \quad x - \frac{1}{2} = 1$$

$$\Rightarrow \quad x = \frac{3}{2}$$

22. Given, $y = mx - b\sqrt{1+m^2}$ touches both the circles.

So, distance from centre = Radius of both the circles

$$\therefore \quad \frac{|-b\sqrt{1+m^2}|}{\sqrt{m^2+1}} = b$$

and $$\frac{|ma - 0 - b\sqrt{1+m^2}|}{\sqrt{m^2+1}} = b$$

$$\Rightarrow \quad |ma - b\sqrt{1+m^2}| = |-b\sqrt{1+m^2}|$$

$$\Rightarrow \quad m^2a^2 - 2abm\sqrt{1+m^2} + b^2(1+m^2) = b^2(1+m^2)$$

$$\Rightarrow \quad ma - 2b\sqrt{1+m^2} = 0$$

$$\Rightarrow \quad m^2a^2 = 4b^2(1+m^2)$$

$$\Rightarrow \quad m^2(a^2 - 4b^2) = 4b^2$$

$$\Rightarrow \quad m = \frac{2b}{\sqrt{a^2 - 4b^2}}$$

23. Here, $\alpha \in \left[0, \frac{\pi}{2}\right) \Rightarrow \tan\alpha$ is (+ve)

As, we know, if $a, b > 0 \Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$

i.e., AM ≥ GM

$$\therefore \quad \frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \geq \sqrt{\sqrt{x^2+x}\cdot\frac{\tan^2\alpha}{\sqrt{x^2+x}}}$$

(using AM ≥ GM)

$$\Rightarrow \quad \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$$

24. Let x be the length of an edge of a cube and V be the volume of that cube.

$$\therefore \quad V = x^3$$

On differentiating w.r.t. x, we get

$$\frac{dV}{dx} = 3x^2$$

Let δV be error in V and corresponding error δx in x.

$$\therefore \quad \delta V = \frac{dV}{dx}\delta x = 3x^2\,\delta x$$

Given that, $x = 2$ m and $\delta x = 0.5$ cm $= \frac{0.5}{100}$ m

$$\therefore \quad \delta V = 3(2)^2\left(\frac{0.5}{100}\right)$$

$$= \frac{12\times 0.5}{100} = \frac{6}{100} = 0.06 \text{ m}^3$$

25. Let $g(x) = f(x) - x^2$

$\Rightarrow$ g(x) has atleast 3 real roots which are x = 1, 2, 3 (by mean value theorem)

$\Rightarrow$ $g'(x)$ has atleast 2 real roots in $x \in (1, 3)$

$\Rightarrow$ $g''(x)$ has atleast 1 real root in $x \in (1, 3)$

$\Rightarrow$ $f''(x) = 2$ for atleast one $x \in (1, 3)$

26. We have,

$$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5} \quad \ldots(i)$$

$$\Rightarrow y = (C_1 + C_2)\cos(x + C_3) - C_4 e^x \cdot e^{C_5}$$

Now, let $C_1 + C_2 = A, C_3 = B, C_4\, e^{C_5} = C$

$$\Rightarrow \quad y = A\cos(x + B) - Ce^x \quad \ldots(ii)$$

On differentiating w.r.t. *x*, we get

$$\frac{dy}{dx} = -A\sin(x + B) - Ce^x \quad \ldots(iii)$$

Again, differentiating w.r.t. *x*, we get

$$\frac{d^2y}{dx^2} = -A\cos(x + B) - Ce^x \quad \ldots(iv)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -y - 2Ce^x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + y = -2Ce^x \quad \ldots(v)$$

Again, differentiating w.r.t. *x*, we get

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = -2Ce^x \quad \ldots(vi)$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} + \frac{dy}{dx} = \frac{d^2y}{dx^2} + y \text{ [from Eq. (v)]}$$

which is a differential equation of order 3.

27. Let $I = \int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

$$= \int \frac{\tan x + 1}{\sqrt{\tan x}}\, dx$$

Put $\tan x = t^2$

$$\Rightarrow \quad \sec^2 x\, dx = 2t\, dt$$

$$\Rightarrow \quad dx = \frac{2t\, dt}{1 + \tan^2 t} = \frac{2t}{1 + t^4}\, dt$$

$$\therefore \quad I = \int \frac{t^2 + 1}{\sqrt{t^2}} \cdot \frac{2t}{t^4 + 1}\, dt$$

$$= 2\int \frac{t^2 + 1}{t^4 + 1}\, dt$$

$$= 2\int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2} - 2 + 2}\, dt$$

$$= 2\int \frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + (\sqrt{2})^2}\, dt$$

$$= 2\int \frac{du}{u^2 + (\sqrt{2})^2}$$

where, $\quad u = t - \frac{1}{t} \quad \Rightarrow \quad du = \left(1 + \frac{1}{t^2}\right) dt$

$$\Rightarrow \quad I = \frac{2}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= \sqrt{2} \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$$

28. Let number of candidates be *n*. Therefore, n – 2 are to be elected and so one can vote up to (n – 2). Hence, number of ways in which one can vote

$= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_{n-2} = 56$ (given)

$\Rightarrow \quad 2^n - ({}^nC_0 + {}^nC_{n-1} + {}^nC_n) = 56$

$\Rightarrow \quad 2^n - n = 58 \quad \Rightarrow \quad 2^n = 58 + n$

Which is satisfied by n = 6 only.

29. We have, $\quad x = t^2 + t + 1 \quad \ldots(i)$

and $\quad y = t^2 - t + 1 \quad \ldots(ii)$

Now, $\quad x + y = 2(1 + t^2) \quad \ldots(iii)$

and $\quad x - y = 2t \quad \ldots(iv)$

Now, from Eqs. (iii) and (iv), we get

$$x + y = 2\left[1 + \left(\frac{x - y}{2}\right)^2\right]$$

$$\Rightarrow \quad x + y = 2\left[\frac{4 + x^2 + y^2 - 2xy}{4}\right]$$

$$\Rightarrow \quad x^2 + y^2 - 2xy - 2x - 2y + 4 = 0 \quad \ldots(v)$$

$\therefore$ On comparing with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

We get, a = 1, b = 1, c = 4, h = –1, g = –1, f = –1

$\because \quad \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Now, $\Delta = 1\cdot1\cdot4 + 2(-1)(-1)(-1) - 1\times(-1)^2 - 1\times(-1)^2 - 4(-1)^2$

$= 4 - 2 - 1 - 1 - 4$

$= -4$ therefore, $\Delta \neq 0$

and $ab - h^2 = 1\cdot1 - (1)^2 = 1 - 1 = 0$

So, it is equation of a parabola.

30. Equation of tangent is drawn at a $(3\sqrt{3}\cos\theta, \sin\theta)$ to the curve

$$\frac{x^2}{27} + \frac{y^2}{1} = 1$$

is $\frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$

Thus, sum of intercepts

$= (3\sqrt{3}\sec\theta + \operatorname{cosec}\theta) = f(\theta)$ (say)

$\Rightarrow \quad f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$

$$= \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta}$$

Put $f'(\theta) = 0$

$\Rightarrow \quad \sin^3\theta = \frac{1}{3^{3/2}}\cos^3\theta$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = \frac{\pi}{6}$

31. Since, *a, b, c* are in AP.

$\therefore \quad a = A - D, b = A, c = A + D$

Where, *A* is the first term and *D* is the common difference of an AP.

Given, $a + b + c = \frac{3}{2}$

$\Rightarrow \quad (A - D) + A + (A + D) = \frac{3}{2}$

$\Rightarrow \quad 3A = \frac{3}{2}$

$\Rightarrow \quad A = \frac{1}{2}$

$\therefore$ The numbers are $\frac{1}{2} - D, \frac{1}{2}, \frac{1}{2} + D$

Also, $\left(\frac{1}{2} - D\right)^2, \frac{1}{4}, \left(\frac{1}{2} + D\right)^2$ are in GP.

$\Rightarrow \quad \left(\frac{1}{4}\right)^2 = \left(\frac{1}{2} - D\right)^2\left(\frac{1}{2} + D\right)^2$

$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{4} - D^2\right)^2$

$\Rightarrow \quad \frac{1}{4} - D^2 = \pm\frac{1}{4}$

$\Rightarrow \quad D^2 = \left(\because D = 0 \text{ is not possible}\right)$

$\Rightarrow \quad D = \pm\frac{1}{\sqrt{2}}$

$\therefore \quad a = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$

So, out of the given value $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$ is the right choice.

32. For $f(x)$ to be continuous, we must have

$$f(0) = \lim_{x\to 0} f(x)$$

$\therefore \quad \lim_{x\to 0} \frac{\log(1+ax) - \log(1-bx)}{x}$

$= \lim_{x\to 0} \frac{a\log(1+ax)}{ax} + \frac{b\log(1-bx)}{-bx}$

$= a\cdot1 + b\cdot1 \quad \left[\text{using, } \lim_{x\to 0}\frac{\log(1+x)}{x} = 1\right]$

$= a + b$

$\therefore \quad f(0) = a + b$

33. Here, $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$, to find domain, we must have,

$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0 \quad \left(\text{but } -\frac{\pi}{2} \leq \sin^{-1}\theta \leq \frac{\pi}{2}\right)$

$\therefore \quad -\frac{\pi}{6} \leq \sin^{-1}(2x) \leq \sin\frac{\pi}{2}$

$\Rightarrow \quad \sin\left(-\frac{\pi}{6}\right) \leq 2x \leq \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad -\frac{1}{2} \leq 2x \leq 1$

$\Rightarrow \quad -\frac{1}{4} \leq x \leq \frac{1}{2}$

$\Rightarrow \quad x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

34. $x\,dy = y\,(dx + y\,dy), y > 0$

$\Rightarrow \quad x\,dy - y\,dx = y^2\,dy$

$\Rightarrow \quad \frac{x\,dy - y\,dx}{y^2} = dy$

$\Rightarrow \quad -d\left(\frac{x}{y}\right) = dy$

On integrating both sides, we get

$$\frac{x}{y} = -y + C \qquad \ldots(i)$$

As $y(1) = 1 \Rightarrow x = 1, y = 1$

$\therefore \quad C = 2$

$\therefore$ Eq. (i) becomes

$$\frac{x}{y} + y = 2$$

Again, for $x = -3$

$\Rightarrow \quad -3 + y^2 = 2y$

$\Rightarrow \quad y^2 - 2y - 3 = 0$

$\Rightarrow \quad (y + 1)(y - 3) = 0$

As $y > 0$, we take $y = 3$, (neglecting $y = -1$)

35. Let $I = \int \frac{(x-1)e^x}{(x+1)^3}\,dx$

$$\Rightarrow \quad I = \int \left\{\frac{x+1-2}{(x+1)^3}\right\} e^x\,dx$$

$$= \int \left\{\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right\} e^x\,dx$$

$$= \int e^x \cdot \frac{1}{(x+1)^2}\,dx - 2\int e^x \frac{1}{(x+1)^3}\,dx$$

Applying integrating by parts, we get

$$= \left(\frac{1}{(x+1)^2} e^x - \int e^x \frac{(-2)}{(x+1)^3}\,dx\right) - 2\int e^x \frac{1}{(x+1)^3}\,dx$$

$$= \frac{e^x}{(x+1)^2} + C$$

Note We can use the formula

$$\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$$

36. Now, $\log_{140} 63 = \log_{2^2 \times 5 \times 7} (3 \times 3 \times 7)$

$$= \frac{\log_2 (3 \times 3 \times 7)}{\log_2 (2^2 \times 5 \times 7)} = \frac{\log_2 3 + \log_2 3 + \log_2 7}{2\log_2 2 + \log_2 5 + \log_2 7}$$

$$= \frac{2a + \frac{1}{c}}{2 + b + \frac{1}{c}} = \frac{2ac + 1}{2c + bc + 1}$$

37. Let $f(x) = \frac{\log_e x}{x}$

On differentiating, w.r.t. x, we get

$$f'(x) = \frac{1}{x^2} - \frac{\log_e x}{x^2}$$

For maximum or minimum value of $f(x)$,

Put $f'(x) = 0$

$\Rightarrow \quad \frac{1 - \log_e x}{x^2} = 0$

$\Rightarrow \quad \log_e x = 1$

$\Rightarrow \quad x = e$, which lies in $(0, \infty)$.

For $x = e$, $f''(x) = -ve$

Hence, y is maximum at $x = e$ and its maximum value $= \frac{\log_e e}{e} = \frac{1}{e}$

38. LHS

$$= \frac{(\cos x + i \sin x)(\cos y + i \sin y)}{(\cos u + i \sin u)(\cos v + i \sin v)} \sin u \cos v$$

$$= \sin u \cos v\, e^{ix} \cdot e^{iy} \cdot e^{-iu} \cdot e^{-iv}$$

$$= \sin u \cos v[\cos(x + y - u - v) + i \sin(x + y - u - v)]$$

39. Let e be the identity,

$\therefore \quad a * e = a$

$\Rightarrow \quad \frac{ae}{6} = a$

$\Rightarrow \quad e = 6$

$\because \quad a * a^{-1} = e$

$\therefore \quad a * a^{-1} = 6$

$\Rightarrow \quad \frac{aa^{-1}}{6} = 6$

$\Rightarrow \quad a^{-1} = \frac{6 \times 6}{a}$

$\therefore \quad 9^{-1} = \frac{6 \times 6}{9} = 4$

40. $\therefore \quad \sin^2 \alpha + \sin(\beta - \gamma)\sin(\beta + \gamma)$

$$= \sin^2 \alpha + \sin(\pi - \alpha)\sin(\beta - \gamma)$$

$$= \sin \alpha [\sin \alpha + \sin(\beta - \gamma)]$$

$$= \sin \alpha [\sin(\pi - (\beta + \gamma)) + \sin(\beta - \gamma)]$$

$$= \sin \alpha [\sin(\beta - \gamma) + \sin(\beta + \gamma)]$$

$$= 2 \sin \alpha \sin \beta \cos \gamma$$

41. We have, $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ac}$

On multiplying both sides by *abc*

$\Rightarrow 2bc\cos A + ac\cos B + 2ab\cos C = a^2 + b^2$

$\Rightarrow \quad (b^2 + c^2 - a^2) + \frac{(c^2 + a^2 - b^2)}{2}$

$+ (a^2 + b^2 - c^2) = a^2 + b^2$

$\Rightarrow \quad (c^2 + a^2 - b^2) = 2a^2 - 2b^2$

$\Rightarrow \quad b^2 + c^2 = a^2$

42. $\frac{\sin^2 A + \sin A + 1}{\sin A} = \sin A + \frac{1}{\sin A} + 1$

$= \left[\left(\sqrt{\sin A} - \frac{1}{\sqrt{\sin A}}\right)^2 + 3\right] \geq 3$

43. The given system has a non-trivial solution, if

$$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

By expanding the determinant along (C_1), we get

$\lambda(-\cos^2\alpha - \sin^2\alpha) - 1(-\sin\alpha\cos\alpha$

$-\sin\alpha\cos\alpha)$

$-1(\sin^2\alpha - \cos^2\alpha) = 0$

$\lambda = \sin 2\alpha + \cos 2\alpha$

For $\lambda = 1, \sin 2\alpha + \cos 2\alpha = 1$

$\Rightarrow \quad \frac{1}{\sqrt{2}}\sin 2\alpha + \frac{1}{\sqrt{2}}\cos 2\alpha = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \cos\left(2\alpha - \frac{\pi}{4}\right) = \cos\left(2n\pi \pm \frac{\pi}{4}\right)$

$\Rightarrow \quad 2\alpha = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$, n being an integer.

$\Rightarrow \quad \alpha = n\pi + \frac{\pi}{4}, n\pi$

44. Let $f(x) = x^2 - 2ax + a^2 + a - 3 = 0$. Since, $f(x)$ has real roots both less than 3. Therefore, $D > 0$ and $f(3) > 0$

$\Rightarrow \quad 4a^2 - 4(a^2 + a - 3) > 0$

and $\quad a^2 - 5a + 6 > 0$

$\Rightarrow \quad a < 3 \quad \text{and} \quad a < 2 \quad \text{or} \quad a > 3$

$\Rightarrow \quad a < 2$

45. $\because \quad |z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow \quad |z_1| - |z_2| = |z_1 - z_2|$

$\Rightarrow \quad (|z_1| - |z_2|)^2 = |z_1 - z_2|^2$

$\Rightarrow \quad |z_1|^2 + |z_2|^2 - 2|z_1||z_2| = |z_1|^2 + |z_2|^2$

$- 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$\Rightarrow \quad \cos(\theta_1 - \theta_2) = 1$

$\Rightarrow \quad \theta_1 - \theta_2 = 0$

$\Rightarrow \quad \arg(z_1) - \arg(z_2) = 0$

$\Rightarrow \quad \frac{z_1}{z_2}$ is purely real.

$\Rightarrow \quad \text{Im}\left(\frac{z_1}{z_2}\right) = 0$

46. Since, total number of numbers 1 to 5 digits are 99999 and total number of numbers 1 to 4 digits are 9999.

Hence, the total number of numbers of exactly 5 digits = 99999 − 9999 = 90000.

47. $\because \left(x^2 + \frac{1}{x^2} + 2\right)^n = \left[\left(x + \frac{1}{x}\right)^2\right]^n = \left(x + \frac{1}{x}\right)^{2n}$

The number of terms in the expansion of $\left(x + \frac{1}{x}\right)^{2n}$ is $2n + 1$ which is odd. Therefore, $(n + 1)$th term will be middle term.

$\therefore \quad T_{n+1} = {}^{2n}C_n\,(x)^{2n-n}\left(\frac{1}{x}\right)^n$

$= {}^{2n}C_n\,x^n \cdot \frac{1}{x^n}$

$= {}^{2n}C_n = \frac{(2n)!}{n!\,n!} = \frac{(2n)!}{(n!)^2}$

48. We know,

$\sin^2 A = \left(\frac{1 - \cos 2A}{2}\right)$

$\Rightarrow \quad \sin^2(22°30') = \frac{1 - \cos 45°}{2}$

$= \frac{\left(1 - \frac{1}{\sqrt{2}}\right)}{2}$

$= \frac{\sqrt{2} - 1}{2\sqrt{2}}$

$\Rightarrow \quad \sin(22°30') = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$

49. $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \frac{1-\sin\theta}{\sqrt{\cos^2\theta}} = \frac{1-\sin\theta}{|\cos\theta|}$

$= \frac{1-\sin\theta}{-\cos\theta} \qquad \left(\because \frac{\pi}{2} < \theta < \frac{3\pi}{2}\right)$

$= -\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= -\sec\theta + \tan\theta$

50. Let $I = \int \frac{2^x}{\sqrt{1-4^x}}\,dx$

Let $2^x = t$

$\Rightarrow \quad 2^x \log 2\,dx = dt$

$\therefore \quad I = \frac{1}{\log 2}\int \frac{1}{\sqrt{1-t^2}}\,dt$

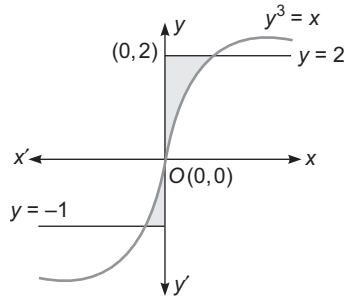
$= \frac{1}{\log 2}\sin^{-1}(t) + C$

$= \frac{1}{\log 2}\sin^{-1}(2^x) + C$

But $I = k\sin^{-1}(2^x) + C,$

$\Rightarrow \quad k = \frac{1}{\log 2}$

51. Required area $= \int_{-1}^{2} |x_1|\,dy$



$= \int_{-1}^{0}(-x_1)\,dy + \int_{0}^{2} x_1\,dy$

$= -\int_{-1}^{0} y^3\,dy + \int_{0}^{2} y^3\,dy$

$= -\left[\frac{y^4}{4}\right]_{-1}^{0} + \left[\frac{y^4}{4}\right]_{0}^{2} = \frac{1}{4} + 4$

$= \frac{17}{4}$ sq units.

52. Let (x_1, y_1) be the mid-point of the line joining the common points of the given line and the given parabola. Then, the equation of line is

$yy_1 - 4(x + x_1) = y_1^2 - 8x_1$

$\Rightarrow \quad 4x - yy_1 + y_1^2 - 4x_1 = 0$

This line and $2x - 3y + 8 = 0$ represents the same line.

$\therefore \quad \frac{4}{2} = \frac{-y_1}{-3} = \frac{y_1^2 - 4x_1}{8}$

$\Rightarrow \quad y_1 = 6,\ y_1^2 - 4x_1 = 16$

$\Rightarrow \quad y_1 = 6,\ x_1 = \frac{36-16}{4}$

$\Rightarrow \quad y_1 = 6,\ x_1 = 5$

$\therefore$ The required point is (5, 6).

53. $\cos^2 48° - \sin^2 12° = \frac{1}{2}(2\cos^2 48° - 2\sin^2 12°)$

$= \frac{1}{2}[(1 + \cos 96°) - (1 - \cos 24°)]$

$= \frac{1}{2}(\cos 96° + \cos 24°)$

$= \frac{1}{2}\left[2\cos\left(\frac{96°+24°}{2}\right)\cos\left(\frac{96°-24°}{2}\right)\right]$

$= \cos 60°\cos 36°$

$= \frac{1}{2} \times \left(\frac{\sqrt{5}+1}{4}\right)$

$= \frac{\sqrt{5}+1}{8}$

54. $\sin^2\theta + 3\cos\theta = 3$

$\Rightarrow \quad 1 - \cos^2\theta + 3\cos\theta = 3$

$\Rightarrow \quad \cos^2\theta - 3\cos\theta + 2 = 0$

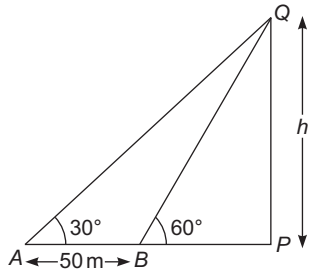
$\Rightarrow \quad (\cos\theta - 1)(\cos\theta - 2) = 0$

$\therefore \quad \cos\theta = 1 \qquad (\because \cos\theta \neq 2)$

$\Rightarrow \quad \theta = 0° \text{ as } \theta \in [-\pi, \pi]$

$\therefore$ The number of solution is 1.

55. Let $PQ = h$ be the height of chimney. A and B are the two points 50 m apart.



In ΔAPQ,

$$\tan 30° = \frac{h}{AP}$$

$$\Rightarrow \quad AP = h \cot 30° \qquad \ldots(i)$$

and in ΔQBP,

$$\tan 60° = \frac{h}{BP}$$

$$\Rightarrow \quad BP = h \cot 60° \qquad \ldots(ii)$$

$$\because \quad AP - BP = 50$$

$$\therefore \quad h(\cot 30° - \cot 60°) = 50$$

$$\Rightarrow \quad h = \frac{50}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{50\sqrt{3}}{3-1}$$

$$= \frac{50\sqrt{3}}{2} = 25\sqrt{3} \text{ m}$$

56. $$\lim_{x \to \infty} \left(\frac{x^3 + 1}{x^2 + 1} - (ax + b) \right) = 2$$

$$\Rightarrow \lim_{x \to \infty} \frac{x^3(1-a) - bx^2 - ax + (1-b)}{x^2 + 1} = 2$$

$$\Rightarrow \lim_{x \to \infty} \frac{x(1-a) - b - \frac{a}{x} + \frac{1-b}{x^2}}{1 + \frac{1}{x^2}} = 2$$

$$\Rightarrow \quad 1 - a = 0 \quad \text{and} \quad -b = 2$$

$$\Rightarrow \quad a = 1 \quad \text{and} \quad b = -2$$

57. $$f(x) = \begin{cases} \dfrac{|x+2|}{\tan^{-1}(x+2)}, & x \neq -2 \\ 2, & x = -2 \end{cases}$$

$$\therefore \quad \lim_{x \to -2^-} f(x) = \lim_{h \to 0} f(-2-h)$$

$$= \lim_{h \to 0} \frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$= \lim_{h \to 0} \frac{-h}{\tan^{-1} h} = -1$$

and $$\lim_{x \to -2^+} f(x) = \lim_{h \to 0} f(-2+h)$$

$$= \lim_{h \to 0} \left[\frac{|-2+h+2|}{\tan^{-1}(-2+h+2)} \right]$$

$$= \lim_{h \to 0} \frac{h}{\tan^{-1} h} = 1$$

$$\because \quad \lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$$

So, $f(x)$ is not continuous as well as not differentiable at $x = -2$.

58. Let the coordinates of the point P be (x_1, y_1). This point lies on the curve

$$x^{2/3} + y^{2/3} = a^{2/3} \qquad \ldots(i)$$

$$\therefore \quad x_1^{2/3} + y_1^{2/3} = a^{2/3} \qquad \ldots(ii)$$

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = -\left(\frac{y_1}{x_1}\right)^{1/3}$$

The equation of the tangent at (x_1, y_1) to the given curve is

$$y - y_1 = -\frac{y_1^{1/3}}{x_1^{1/3}} (x - x_1)$$

$$\Rightarrow \quad x x_1^{-1/3} + y y_1^{-1/3} = x_1^{2/3} + y_1^{2/3}$$

$$\Rightarrow \quad x x_1^{-1/3} + y y_1^{-1/3} = a^{2/3} \quad \text{[using Eq. (ii)]}$$

This tangent meets the coordinate axes at $A(a^{2/3} x_1^{1/3}, 0)$ and $B(0, a^{2/3} y_1^{1/3})$.

$$\therefore \quad AB = \sqrt{(0 - a^{2/3} x_1^{1/3})^2 + (a^{2/3} y_1^{1/3} - 0)^2}$$

$$= \sqrt{a^{4/3}(x_1^{2/3} + y_1^{2/3})} \quad \text{[from Eq. (ii)]}$$

$$= \sqrt{a^{4/3} \cdot a^{2/3}}$$

$$= \sqrt{a^2} = a$$

59. Given that, $x = (2t - 3)^2$

On differentiating w.r.t. x, we get

$$\frac{dx}{dt} = 2(2t - 3)(2)$$

At origin, $x = 0$

$$\therefore \quad 2t - 3 = 0 \quad \Rightarrow \quad t = \frac{3}{2}$$

$$\therefore \quad \text{Velocity} = \frac{dx}{dt} = 4\left(2 \times \frac{3}{2} - 3\right) = 0$$

60. $\therefore \quad \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{1}{2\sin\frac{\pi}{7}} \times$$

$$\left[2\sin\frac{\pi}{7}\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{4\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}\right]$$

$$= \frac{1}{2\sin\frac{\pi}{7}} \times$$

$$\left[\sin\frac{3\pi}{7} - \sin\frac{\pi}{7} + \sin\frac{5\pi}{7} - \sin\frac{3\pi}{7} + \sin\pi - \sin\frac{5\pi}{7}\right]$$

$$= -\frac{1}{2}$$

61. The maximum value of $3\cos\theta + 4\sin\theta$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{25} = 5$$

62. 'Where are you going' is not a statement.

63. Since, $123 = -1 \pmod{124}$

$125 = 1 \pmod{124}$

$127 = 3 \pmod{124}$

$\therefore \quad 123 \times 125 \times 127 = (-1)(1)(3) \bmod (124)$

$= -3 \bmod (124)$

$= 121 \bmod (125)$

64. The vertex connectivity of any tree is one.

65. Since, a five digit number is formed by using digits 0, 1, 2, 3, 4 and 5, divisible by 3 *i.e.,* only possible when sum of digits is multiple of three which gives two cases.

Case I Using digits 0, 1, 2, 4, 5 the number of ways $= 4 \times 4 \times 3 \times 2 \times 1 = 96$.

Case II Using digits 1, 2, 3, 4, 5 the number of ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\therefore$ Total numbers formed $= 120 + 96 = 216$

66. 3r th term in the expansion of $(1 + x)^{2n}$

$$= {}^{2n}C_{3r-1}\, x^{3r-1}$$

and $(r + 2)$ th term in the expansion of $(1 + x)^{2n}$

$$= {}^{2n}C_{r+1}\, x^{r+1}$$

Given that the binomial coefficients of $(3r)$ th and $(r + 2)$ th terms are equal.

$$\therefore \quad {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$$

$$\Rightarrow \quad 3r - 1 = r + 1$$

or $\quad 2n = (3r - 1) + (r + 1)$

$$\Rightarrow \quad 2r = 2 \quad \text{or} \quad 2n = 4r$$

$$\Rightarrow \quad r = 1 \quad \text{or} \quad n = 2r$$

But $\quad r > 1$

$$\therefore \quad n = 2r$$

67. $\log_4 (x - 1) = \log_2 (x - 3) = \log_{4^{1/2}} (x - 3)$

$$\Rightarrow \quad \log_4 (x - 1) = 2\log_4 (x - 3)$$

$$\Rightarrow \quad \log_4 (x - 1) = \log_4 (x - 3)^2$$

$$\Rightarrow \quad (x - 1) = (x - 3)^2$$

$$\Rightarrow \quad x^2 + 9 - 6x = x - 1$$

$$\Rightarrow \quad x^2 - 7x + 10 = 0$$

$$\Rightarrow \quad (x - 2)(x - 5) = 0$$

$$\Rightarrow \quad x = 2 \quad \text{or} \quad x = 5$$

Hence, $x = 5$

$[\because x = 2$ makes $\log (x - 3)$ undefined.$]$

68. $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \cos^2 t \, dt}{x \sin x}$

Using L' Hospital's rule, we get

$$= \lim_{x \to 0} \frac{\cos^2 (x^2) \cdot 2x - 0}{x\cos x + \sin x}$$

$$= \lim_{x \to 0} \frac{2\cos^2 (x^2)}{\cos x + \frac{\sin x}{x}}$$

$$= \frac{2}{2} = 1$$

69. Let $\quad I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5 - x} + \sqrt{x}}\, dx \qquad \ldots(i)$

$$\therefore \quad I = \int_2^3 \frac{\sqrt{5 - x}}{\sqrt{x} + \sqrt{5 - x}}\, dx \qquad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_2^3 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}}\, dx$$

$$= \int_2^3 1\, dx = 3 - 2 = 1$$

$$\Rightarrow \quad I = \frac{1}{2}$$

70. The given differential equation is

$$(e^x + 1)\cdot y\, dy = (y+1)e^x\, dx$$

$$\Rightarrow \quad \frac{y\, dy}{(y+1)} = \frac{e^x}{(e^x+1)}\, dx$$

$$\Rightarrow \quad \left(\frac{y+1-1}{y+1}\right) dy = \frac{e^x}{e^x+1}\, dx$$

$$\Rightarrow \quad dy - \frac{1}{y+1}\, dy = \frac{e^x}{e^x+1}\, dx$$

On integrating, we get

$$\Rightarrow \quad y - \log|y+1| = \log(e^x+1) + \log k$$

$$\Rightarrow \quad y = \log|(y+1)(e^x+1)|\, k$$

$$\Rightarrow \quad (y+1)(e^x+1) = e^y C$$

71. The required equation of circle is

$$(x^2 + y^2 + 13x - 3y) + \lambda\left(11x + \frac{1}{2}y + \frac{25}{2}\right) = 0 \quad \ldots(i)$$

This circle passes through (1, 1).

$$(1^2 + 1^2 + 13 - 3) + \lambda\left(11 + \frac{1}{2} + \frac{25}{2}\right) = 0$$

$$\therefore \quad 12 + \lambda(24) = 0$$

$$\Rightarrow \quad \lambda = -\frac{1}{2}$$

On putting this value of λ in Eq. (i), we get

$$x^2 + y^2 + 13x - 3y - \frac{11}{2}x - \frac{1}{4}y - \frac{25}{4} = 0$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 52x - 12y - 22x - y - 25 = 0$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 30x - 13y - 25 = 0$$

72. Tangent to the curve $y^2 = 8x$ is $y = mx + \frac{2}{m}$.

So, it must satisfy $xy = -1$

$$\Rightarrow x\left(mx + \frac{2}{m}\right) = -1 \quad \Rightarrow \quad mx^2 + \frac{2}{m}x + 1 = 0$$

Since, it has equal roots.

$$\therefore \quad D = 0$$

$$\Rightarrow \quad \frac{4}{m^2} - 4m = 0$$

$$\Rightarrow \quad m^3 = 1 \quad \Rightarrow \quad m = 1$$

So, equation of common tangent is $y = x + 2$.

73. $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$

$$= \frac{b}{2}(1 + \cos C) + \frac{c}{2}(1 + \cos B)$$

$$= \frac{b}{2} + \frac{c}{2} + \frac{1}{2}(b\cos C + c\cos B)$$

$$= \frac{b}{2} + \frac{c}{2} + \frac{a}{2} \qquad (\because\ a = b\cos C + c\cos B)$$

$$= \frac{a+b+c}{2} = \frac{k}{2}$$

74. $I_1 = \int_{1-k}^{k} xf\{x(1-x)\}\, dx$

$$= \int_{1-k}^{k} (1-x)f\{(1-x)(1-(1-x))\}\, dx$$

$$\left(\because\ \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx\right)$$

$$= \int_{1-k}^{k} (1-x)f\{(1-x)x\}\, dx$$

$$= \int_{1-k}^{k} f\{x(1-x)\}\, dx$$

$$- \int_{1-k}^{k} xf\{x(1-x)\}\, dx$$

$$\Rightarrow \quad I_1 = I_2 - I_1 \quad \Rightarrow \quad 2I_1 = I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{2}$$

75. The maximum area corresponds to when P is at either end of the minor axis.

$\therefore$ Required area $= \frac{1}{2}(2a)b = ab$ sq unit

76. We have, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \quad \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$$

$$\Rightarrow \quad \cot A = \cot B = \cot C$$

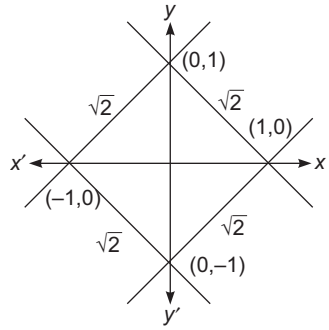
$$\Rightarrow \quad A = B = C = 60°$$

$\therefore$ ΔABC is an equilateral triangle.

$\therefore$ Area of triangle $= \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4} \times 2^2 = \sqrt{3} \text{ sq units}$$

77. The region is clearly square with vertices at the points (1, 0), (0, 1), (–1, 0) and (0, –1). So, its area = $\sqrt{2} \times \sqrt{2} = 2$ sq units.



78. $\sqrt{3}$ cosec 20° – sec 20°

$$= \tan 60° \operatorname{cosec} 20° - \sec 20°$$

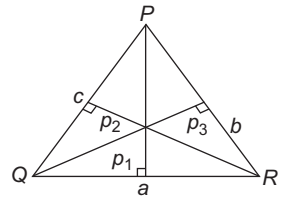
$$= \frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\cos 60° \sin 20° \cos 20°}$$

$$= \frac{\sin (60° - 20°)}{\cos 60° \sin 20° \cos 20°}$$

$$= \frac{\sin 40°}{\frac{1}{2} \sin 20° \cos 20°}$$

$$= \frac{2 \sin 20° \cos 20°}{\frac{1}{2} \sin 20° \cos 20°} = 4$$

79. By sine rule



$$\frac{a}{\sin P} = \frac{b}{\sin Q} = \frac{c}{\sin R} = k \quad \text{(say)}$$

Also, $\frac{1}{2} a p_1 = \Delta$

$\Rightarrow \frac{2\Delta}{a} = p_1$

$\Rightarrow p_1 = \frac{2\Delta}{k \sin P}$

Similarly, $p_1 = \frac{2\Delta}{k \sin Q}$ and $p_3 = \frac{2\Delta}{k \sin R}$

Since, sin P, sin Q, sin R are in AP.

$\therefore \frac{1}{\sin P} \cdot \frac{1}{\sin Q}, \frac{1}{\sin R}$ are in HP

$\Rightarrow \frac{2\Delta}{k \sin P}, \frac{2\Delta}{k \sin Q}, \frac{2\Delta}{k \sin R}$ are in HP

$\Rightarrow p_1, p_2, p_3$ are in HP.

80. Let E_1 be the events that man will be selected and E_2 the events that woman will be selected.

Then, $P(E_1) = \frac{1}{4}$

So, $P(\bar{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$

and $P(E_2) = \frac{1}{3}$

$\Rightarrow P(\bar{E}_2) = \frac{2}{3}$

$\therefore$ E_1 and E_2 are independent events.

$\therefore P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \times P(\bar{E}_2)$

$$= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$