JEE Main 2022 June Question Paper with Answer

24th, 25th, 26th, 27th, 28th & 29th June 2022 (Shift 1 & Shift 2)

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is :

(A) 3×10^8 Nm^{-2} (B) 9×10^8 Nm^{-2}

(C) 6×10^8 Nm^{-2} (D) 12×10^8 Nm^{-2}

Official Ans. by NTA (C)

Sol. $B = 3 \times 10^{10}$

$$-\frac{\Delta V}{V} = 0.02$$

$$B = \frac{\Delta P}{-\frac{\Delta V}{v}} \Rightarrow \Delta P = -B\left(\frac{\Delta V}{V}\right)$$

$$= (3\times10^{10})(0.02)$$

$$= 6\times10^8 \text{ N/m}^2$$

2. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In an uniform magnetic field, speed and energy remains the same for a moving charged particle.

Reason (R) : Moving charged particle experiences magnetic force perpendicular to its direction of motion.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true.

Official Ans. by NTA (A)

Sol. $\vec{F} = q(\vec{v}\times\vec{B})$

$\boxed{\vec{F}\perp\vec{v}}$

Work done $= \vec{F}\cdot\vec{S}$

Work done = 0

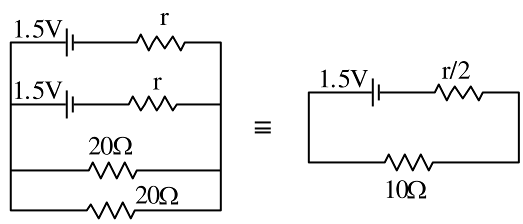
3. Two identical cells each of emf 1.5 V are connected in parallel across a parallel combination of two resistors each of resistance 20Ω. A voltmeter connected in the circuit measures 1.2 V. The internal resistance of each cell is

(A) 2.5Ω (B) 4Ω

(C) 5Ω (D) 10Ω

Official Ans. by NTA (C)

Sol.



$V = E - ir/2$

$$1.2 = 1.5 - i\left(\frac{r}{2}\right)$$

$$i\frac{r}{2} = 0.3$$

$$i = \frac{1.5}{10+\frac{r}{2}} \Rightarrow 10i + \frac{ir}{2} = 1.5$$

$10i = 1.5 - 0.3$

$i = 0.12$ A

$$\Rightarrow \quad r = \frac{0.6}{0.12} = 5\Omega$$

4. Identify the pair of physical quantities which have different dimensions :

(A) Wave number and Rydberg's constant

(B) Stress and Coefficient of elasticity

(C) Coercivity and Magnetisation

(D) Specific heat capacity and Latent heat

Official Ans. by NTA (D)

Sol. $S = \frac{Q}{m\Delta T} = \frac{J}{Kg°C}$

$L = \frac{Q}{m} = \frac{J}{Kg}$

5. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be : [use g = 10 m/s^2]

(A) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(C) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (D) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

Official Ans. by NTA (D)

Sol. $R = \frac{V^2(2\sin\theta\cos\theta)}{g}$

$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$

$\Rightarrow R = \frac{g^2t^2}{\sin^2\theta}.\frac{2\sin\theta\cos\theta}{g}$

$\tan\theta = \frac{2gt^2}{R} = \frac{20t^2}{R}$

$\cot\theta = \frac{R}{20t^2}$

6. A block of mass 10 kg starts sliding on a surface with an initial velocity of 9.8 ms^{-1}. The coefficient of friction between the surface and bock is 0.5. The distance covered by the block before coming to rest is : [use g = 9.8 ms^{-2}]

(A) 4.9 m (B) 9.8 m

(C) 12.5 m (D) 19.6 m

Official Ans. by NTA (B)

Sol. $a = -\mu g = -0.5 \times 9.8 = -4.9 m/s^2$

$d = \frac{v^2}{2a} = \frac{9.8 \times 9.8}{2(4.9)}$

= 9.8 m

7. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev./min. The value of K is : (Assume the string is massless and unstretchable)

(A) 400 (B) 300

(C) 600 (D) 800

Official Ans. by NTA (C)

Sol. $T = M\omega^2 R$

T = 80 N M = 0.1 ω = ? R = 2m

$80 = 0.1\ \omega^2(2)$

$\omega^2 = 400$

$\omega = 20$

$2\pi f = 20$

$f = \frac{10}{\pi}\frac{rev}{s}$

$= \frac{600}{\pi}\frac{rev}{min}$

8. A vertical electric field of magnitude 4.9×10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be : (Given $g = 9.8$ m/s^2)

(A) 1.6×10^{-9} C (B) 2.0×10^{-9} C

(C) 3.2×10^{-9} C (D) 0.5×10^{-9} C

Official Ans. by NTA (B)

Sol. $Mg = qE$

$$(0.1\times10^{-3})(9.8) = 4.9\times10^5 q$$

$$\frac{2\times10^{-4}}{10^5} = q$$

$$\boxed{q = 2\times10^{-9}\,C}$$

9. A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x-y plane, the Kinetic Energy changes by

(A) 50.0 J (B) 12.5 J

(C) 25.0 J (D) 0 J

Official Ans. by NTA (C)

Sol. $F = 4x\hat{i} + 3y^2\hat{j}$

WD = ΔKE

$$W = \int \vec{F}.(dx\hat{i} + dy\hat{j})$$

$$= \int_1^2 4xdx + \int_2^3 3y^2dx$$

$$= (2x^2)_1^2 + (y^3)_2^3$$

$$= (8-2) + (27-8)$$

$$= 6 + 19 = 25J$$

10. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is : [Radius of earth R = 6400 km and $\sqrt{3} = 1.732$]

(A) 3840 km (B) 4685 km

(C) 2133 km (D) 4267 km

Official Ans. by NTA (B)

Sol. $Mg' = \frac{M}{3}g$

$$g' = \frac{g}{3}$$

$$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$$

$$\frac{R}{R+h} = \frac{1}{\sqrt{3}}.$$

$$h = (\sqrt{3}-1)R$$

$$= (1.732-1)6400$$

$$\boxed{h = 4685km}$$

11. A resistance of 40 Ω is connected to a source of alternating current rated 220 V, 50 Hz. Find the time taken by the current to change from its maximum value to rms value :

(A) 2.5 ms (B) 1.25 ms

(C) 2.5 s (D) 0.25 s

Official Ans. by NTA (A)

Sol. Considering sinusoidal AC.

Phase at maximum value $= \frac{\pi}{2}$

Phase at rms value $= \frac{3\pi}{4}$

Thus phase change $= \frac{3\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4}$

Now $\omega = 2\pi f$

$= 2\pi \times 50$

$= 100\,\pi$

time taken $t = \frac{\theta}{\omega} = \frac{\pi/4}{100\pi} = \frac{1}{400}$ s

$t = 2.5 \times 10^{-3} = 2.5$ ms

12. The equations of two waves are given by :

$$y_1 = 5\sin 2\pi(x - vt)cm$$

$$y_2 = 3\sin 2\pi(x - vt + 1.5)cm$$

These waves are simultaneously passing through a string. The amplitude of the resulting wave is

(A) 2 cm (B) 4 cm

(C) 5.8 cm (D) 8 cm

Official Ans. by NTA (A)

Sol. $A_1 = 5 \quad A_2 = 3$

$\Delta\theta = 2\pi(1.5) = 3\pi$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)}$

$= |A_1 - A_2|$

= 2cm

13. A plane electromagnetic wave travels in a medium of relative permeability 1.61 and relative permittivity 6.44. If magnitude of magnetic intensity is 4.5×10^{-2} Am^{-1} at a point, what will be the approximate magnitude of electric field intensity at that point ?

(Given : permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}, speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 16.96 Vm^{-1} (B) 2.25×10^{-2} Vm^{-1}

(C) 8.48 Vm^{-1} (D) 6.75×10^6 Vm^{-1}

Official Ans. by NTA (C)

Sol. $\mu_r = 1.61 \qquad \in_r = 6.44$

$B = 4.5 \times 10^{-2}$

E = ?

$C = \frac{1}{\sqrt{\mu_0 \in_0}} \quad V = \frac{1}{\sqrt{\mu \in}}$

$\frac{C}{V} = \sqrt{\mu_r \in_r} = \sqrt{1.61 \times 6.44}$

$\frac{E}{B} = V = \frac{3 \times 10^8}{\sqrt{1.61 \times 6.44}} = 9.32 \times 10^7 m/s$

$E = 4.5 \times 10^{-2} \times 9.32 \times 10^7$

$= 4.2 \times 10^6$

14. Choose the correct option from the following options given below :

(A) In the ground state of Rutherford's model electrons are in stable equilibrium. While in Thomson's model electrons always experience a net-force.

(B) An atom has a nearly continuous mass distribution in a Rutherford's model but has a highly non-uniform mass distribution in Thomson's model

(C) A classical atom based on Rutherford's model is doomed to collapse.

(D) The positively charged part of the atom possesses most of the mass in Rutherford's model but not in Thomson's model.

Official Ans. by NTA (C)

Sol. According to Rutherford, e^- revolves around nucleus in circular orbit. Thus e^- is always accelerating (centripetal acceleration). An accelerating change emits EM radiation and thus e^- should loose energy and finally should collapse in the nucleus.

15. Nucleus A is having mass number 220 and its binding energy per nucleon is 5.6 MeV. It splits in two fragments 'B' and 'C' of mass numbers 105 and 115. The binding energy of nucleons in 'B' and 'C' is 6.4 MeV per nucleon. The energy Q released per fission will be :

(A) 0.8 MeV (B) 275 MeV

(C) 220 MeV (D) 176 MeV

Official Ans. by NTA (D)

Sol. $Q = (B.E)_P - (B.E)_R$

$= (105 + 115)(6.4) - (220)(5.6)$

= 176 MeV

16. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal ?

(A) 42.8 m (B) 42.8 mm

(C) 21.4 mm (D) 21.4 m

Official Ans. by NTA (C)

Sol. $f_c = 3.5\text{GHz} \quad f_m = 3.5\text{MHz}$

Side band frequencies are $f_c - f_m$ & $f_c + f_m$. which are almost f_c

$$\lambda = \frac{c}{f_c}$$

Minimum length of antenna =

$$\frac{c}{f_c 4} = \frac{\lambda}{4} = \frac{3\times10^8}{3.5\times10^9\times4}$$

= 21.4 mm

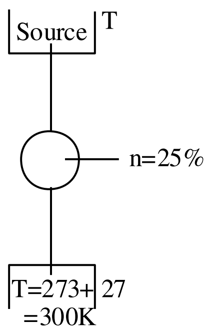
17. A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency ?

(A) Increases by 18°C (B) Increase by 200°C

(C) Increase by 120°C (D) Increase by 73°

Official Ans. by NTA (B)

Sol.



$$1 - \frac{300}{T} = 0.25$$

$$\frac{300}{T} = 0.75$$

T = 400K

If efficiency increased by 100% then new efficiency ⇒ n' = 50%

$$1 - \frac{300}{T'} = 0.5$$

$$T' = 600K$$

Increase in temp = 600 – 400

= 200 K or 200°C

18. A parallel plate capacitor is formed by two plates each of area $30\pi\ \text{cm}^2$ separated by 1 mm. A material of dielectric strength $3.6 \times 10^7\ \text{Vm}^{-1}$ is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6} C, the value of dielectric constant of the material is :

$$\left\{\text{Use}: \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}\right\}$$

(A) 1.66 (B) 1.75

(C) 2.25 (D) 2.33

Official Ans. by NTA (D)

Sol. $$K = \frac{q}{A\in_0 E} = \frac{7\times10^{-6}}{30\pi\times10^{-4}\times\frac{1}{4\pi\times9\times10^9}\times3.6\times10^7}$$

$$K = \frac{36\times7}{30\times3.6} = 2.33$$

19. The magnetic field at the centre of a circular coil of radius r, due to current I flowing through it, is B. The magnetic field at a point along the axis at a distance $\frac{r}{2}$ from the centre is :

(A) B/2 (B) 2B

(C) $\left(\frac{2}{\sqrt{5}}\right)^3 B$ (D) $\left(\frac{2}{\sqrt{3}}\right)^3 B$

Official Ans. by NTA (C)

Sol. $$B_C = \frac{\mu_0 I}{2r}, B_a = \frac{\mu_0 I r^2}{2(x^2+r^2)^{3/2}}$$

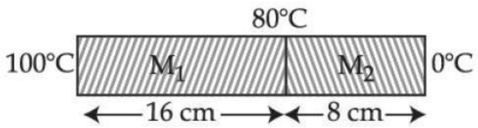
At $x = \frac{r}{2}$

$$B_a = \frac{\mu_0 I r^2}{2\left(\frac{r^2}{4}+r^2\right)^{3/2}}$$

$$= \frac{\mu_0 I r^2}{2\left(\frac{5}{4}r^2\right)^{3/2}} = \frac{\mu_0 I}{2r}\left(\frac{4}{5}\right)^{3/2}$$

$$= \frac{\mu_0 I}{2r}\left(\frac{2}{\sqrt{5}}\right)^3$$

20. Two metallic blocks M_1 and M_2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be : [Assume steady state heat conduction]



(A) 10 K (B) 8 K

(C) 12.5 K (D) 2 K

Official Ans. by NTA (B)

Sol. $\Delta T \ \alpha \ R \ \alpha \ \frac{\ell}{k}$,

$$\frac{\Delta T_1}{\Delta T_2} = \frac{\ell_1}{k_1} \times \frac{k_2}{\ell_2} = \frac{16}{k_1} \times \frac{k}{8}$$

$$\frac{20}{80} = \frac{16}{k_1} \times \frac{k}{8} \rightarrow k_1 = 8k$$

SECTION-B

1. 0.056 kg of Nitrogen is enclosed in a vessel at a temperature of 127°C. The amount of heat required to double the speed of its molecules is _____ k cal. (Take R = 2 cal $mole^{-1}K^{-1}$)

Official Ans. by NTA (12)

Sol. 0.056 kg N_2 = 56 gm of N_2 = 2 mole of N_2

$T_1 = 400$ K, $v \alpha \sqrt{T}$ so $T_2 = 4T_1 = 1600K$

$$Q = \frac{f}{2} nR\Delta T$$

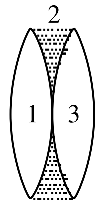
f = 5

Q = 12 k cal

2. Two identical thin biconvex lenses of focal length 15 cm and refractive index 1.5 are in contact with each other. The space between the lenses is filled with a liquid of refractive index 1.25. The focal length of the combination is ______ cm.

Official Ans. by NTA (10)

Sol.



$$\frac{1}{f_1} = \frac{1}{15} = \left(\frac{3}{2} - 1\right)\left[\frac{2}{R}\right]$$

$$\frac{1}{R} = \frac{1}{15}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{15} + \left(\frac{5}{4} - 1\right)\left[\frac{-2}{R}\right] + \frac{1}{15}$$

$$= \frac{1}{15} - \frac{1}{30} + \frac{1}{15}$$

$$= \frac{2 - 1 + 2}{30}$$

$$= \frac{3}{30} = \frac{1}{10}$$

= 10

3. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 10 mV is added to the base-emitter voltage, the base current changes by 10 μA and the collector current changes by 1.5 mA. The load resistance is 5 kΩ. The voltage gain of the transistor will be _____ .

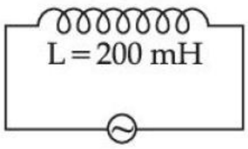
Official Ans. by NTA (750)

Sol. $r_i = \frac{10mV}{10\mu A} = 10^3 \Omega$

$$\beta = \frac{1.5mA}{10\mu A} = 150$$

$$A_V = \left(\frac{R_0}{r_i}\right)\beta = \left(\frac{5000}{1000}\right) \times 150 = 750$$

4. As shown in the figure an inductor of inductance 200 mH is connected to an AC source of emf 220 V and frequency 50 Hz. The instantaneous voltage of the source is 0 V when the peak value of current is $\frac{\sqrt{a}}{\pi}$ A. The value of a is _______.



Official Ans. by NTA (242)

Sol. $f = 50Hz$

$X_L = 2\pi fL$

$= 2\pi(50)(200\times10^{-3})$

$= 20\pi\Omega$

$i_0 = \frac{V_0}{X_L} \Rightarrow \frac{V_{rms}\sqrt{2}}{X_L}$

$= \frac{(220)\sqrt{2}}{20\pi} = \frac{11\sqrt{2}}{\pi}$

$\boxed{i_o = \frac{\sqrt{242}}{\pi}}$

5. Sodium light of wavelengths 650 nm and 655 nm is used to study diffraction at a single slit of aperture 0.5 mm. The distance between the slit and the screen is 2.0 m. The separation between the positions of the first maxima of diffraction pattern obtained in the two cases is _______ $\times 10^{-5}$ m.

Official Ans. by NTA (3)

Sol. $a\sin\theta = \frac{3\lambda}{2}$

$\frac{y}{L} = \theta = \frac{3\lambda}{2a}$ $\qquad L = 2m$

$y_1 = \frac{3\lambda_1 L}{2a}$ $\qquad \lambda_2 = 655$ nm

$y_2 = \frac{3\lambda_2 L}{2a}$ $\qquad \lambda_1 = 650$ nm

$a = 0.5$ nm

$\Delta y = y_2 - y_1 = \frac{3(\lambda_2 - \lambda_1)}{2a}L$

$= \frac{3(655-650)}{2\times0.5\times10^{-3}}\times2\times10^{-9}$

$= \frac{3\times5\times2}{1\times10^{-3}}\times10^{-9}$

$= 3\times10^{-5}$

6. When light of frequency twice the threshold frequency is incident on the metal plate, the maximum velocity of emitted election is v_1. When the frequency of incident radiation is increased to five times the threshold value, the maximum velocity of emitted electron becomes v_2. If $v_2 = x\, v_1$, the value of x will be ______.

Official Ans. by NTA (2)

Sol. $hv = hv_{th} + \frac{1}{2}mv^2$

$v = 2v_{th}$

$2hv_{th} = hv_{th} + \frac{1}{2}mv_1^2$ ….. (1)

$v = 5v_{th}$

$5hv_{th} = hv_{th} + \frac{1}{2}mv_2^2$ ….(2)

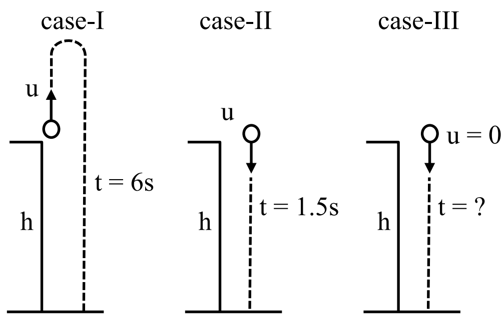
$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hv_{th}}{4hv_{th}}$

$\left(\frac{v_1}{v_2}\right)^2 = \frac{1}{4} \Rightarrow \boxed{v_2 = 2v_1}$

7. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in ________ s.

Official Ans. by NTA (3)

Sol. Let height of tower be h and speed of projection in first two cases be u.



For case-I : 2nd equation $s = ut + \frac{1}{2}at^2$

$h = -u(6) + \frac{1}{2}g(6)^2$

H = – 6u + 18 g …. (i)

For case-II : $h = u(1.5) + \frac{1}{2}g(1.5)^2$

$h = 1.5u + \frac{2.25g}{2}$ … (ii)

Multiplying equation (ii) by 4 we get

4h = 6u + 4.5 g …. (iii)

equation (i) + equation (iii) we get 5h = 22.5g

h = 4.5g … (iv)

For case-III :

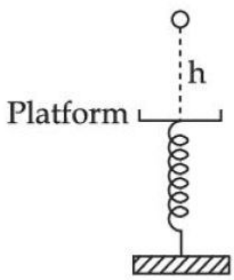
$h = 0 + \frac{1}{2}gt^2$ … (v)

Using equation (4) & equation (5)

$4.5g = \frac{1}{2}gt^2$

$t^2 = 9 \Rightarrow t = 3s$

8. A ball of mass 100 g is dropped from a height h = 10 cm on a platform fixed at the top of vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is ________ Nm^{-1}. (Use $g = 10\ ms^{-2}$)



Official Ans. by NTA (120)

Sol. By energy conservation

PE = KE

$mg\left(H + \frac{H}{2}\right) = \frac{1}{2}kx^2 \quad \left(x = \frac{H}{2}\right)$

$0.100 \times 10 \times \frac{3}{2}(0.10) = \frac{1}{2}k(0.05 \times 0.05)$

$k = \frac{3 \times 0.10}{0.05 \times 0.05}$

$= \frac{3 \times 1000}{25} = 120\ N/m$

9. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be ______ cm.

Official Ans. by NTA (25)

Sol. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$

$\frac{3}{2} = \frac{75cm}{\ell_2}$

$\ell_2 = 50cm$

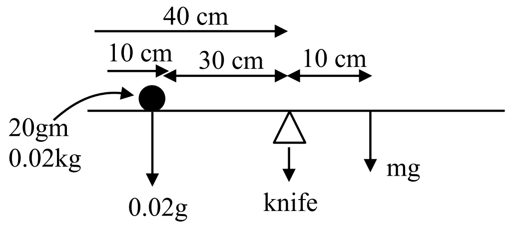
$\ell_1 - \ell_2 = 75 - 50$

= 25 cm

10. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be x × 10^{-2} kg. The value of x is

Official Ans. by NTA (6)

Sol. Let mass of meter scale be m.



Balancing torque about knife edge

(0.02g) × (30 × 10^{-2}) = mg × (10 × 10^{-2})

m = 0.06 kg = 6 × 10^{-2} kg

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. If a rocket runs on a fuel ($C_{15}H_{30}$) and liquid oxygen, the weight of oxygen required and CO_2 released for every litre of fuel respectively are: (Given: density of the fuel is 0.756 g/mL)

(A) 1188 g and 1296 g (B) 2376 g and 2592 g

(C) 2592g and 2376 g (D) 3429 g and 3142 g

Official Ans. by NTA (C)

Sol. $C_{15}H_{30} + \frac{45}{2}O_2 \rightarrow 15CO_2 + 15H_2O$

Mass of fuel = 0.756 × 1000 g

No. of moles of fuel = $\frac{0.756 \times 1000}{210}$

Wt. of oxygen = $\frac{0.756 \times 1000}{210} \times \frac{45}{2} \times 32 = 2592g$

Wt of CO_2 = $\frac{0.756 \times 1000}{210} \times 15 \times 44 = 2376$ g

2. Consider the following pairs of electrons

(A) (a) $n = 3, 1 = 1, m_1 = 1, m_s = +\frac{1}{2}$

(b) $n = 3, 1 = 2, m_1 = 1, m_s = +\frac{1}{2}$

(B) (a) $n = 3, 1 = 2, m_1 = -2, m_s = -\frac{1}{2}$

(b) $n = 3, 1 = 2, m_1 = -1, m_s = -\frac{1}{2}$

(C) (a) $n = 4, 1 = 2, m_1 = 2, m_s = +\frac{1}{2}$

(b) $n = 3, 1 = 2, m_1 = 2, m_s = +\frac{1}{2}$

The pairs of electron present in degenerate orbitals is/are:

(A) Only A

(B) Only B

(C) Only C

(D) (B) and (C)

Official Ans. by NTA (B)

Sol. Based on "n + l" rule only (B) has pair of electron in degenerate orbitals

3. Match List – I with List - II

List – I		List – II	
(A)	$[PtCl_4]^{2-}$	**(I)**	sp^3d
(B)	BrF_5	**(II)**	d^2sp^3
(C)	PCl_5	**(III)**	dsp^2
(D)	$[Co(NH_3)_6]^{3+}$	**(IV)**	sp^3d^2

(A) (A)→(II), (B)→(IV), (C)→(I), (D)→(III)

(B) (A)→(III), (B)→(IV), (C)→(I), (D)→(II)

(C) (A)→(III), (B)→(I), (C)→(IV), (D)→(II)

(D) (A)→(II), (B)→(I), (C)→(IV), (D)→(III)

Official Ans. by NTA (B)

Sol. Answer (B)

List – I		List – II	
(A)	$[PtCl_4]^{2-}$	(III)	dsp^2
(B)	BrF_5	(IV)	sp^3d^2
(C)	PCl_5	(I)	sp^3d
(D)	$[Co(NH_3)_6]^{3+}$	(II)	d^2sp^3

4. For a reaction at equilibrium

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

the relation between dissociation constant (K), degree of dissociation (α) and equilibrium pressure (p) is given by :

(A) $K = \dfrac{\alpha^{\frac{1}{2}}p^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(B) $K = \dfrac{\alpha^{\frac{3}{2}}p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

(C) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(D) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{(1+\alpha)(1-\alpha)^{\frac{1}{2}}}$

Official Ans. by NTA (B)

Sol. $\quad$ **A(g)** $\rightleftharpoons$ $B(g) + \frac{1}{2}C(g)$

Initial : P_i $\quad$ **0** $\quad$ **0**

At eq.: $P_i(1-\alpha)$ $\quad$ $P_i.\alpha$ $\quad$ $P_i\dfrac{\alpha}{2}$

Now, equilibrium pressure (p) ,

$$P = P_i \times \left(1+\frac{\alpha}{2}\right)$$

$$\therefore\ P_A = \left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right)P$$

$$P_B = \left(\frac{\alpha}{1+\frac{\alpha}{2}}\right)P$$

$$P_C = \left(\frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}\right)P$$

$$\therefore K = \frac{P_c^{\frac{1}{2}} \times P_B}{P_A}$$

$$K = \frac{\alpha^{\frac{3}{2}}p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

5. Given below are two statements :

Statement I : Emulsions of oil in water are unstable and sometimes they separate into two layers on standing.

Statement II :For stabilisation of an emulsion, excess of electrolyte is added.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (C)

Sol. Statement I : Fact

Statement II: The principle emulsifying agents for O/W emulsions are proteins, gums natural and synthetic soaps etc…

6. Given below are the oxides:

Na_2O, As_2O_3, N_2O, NO and Cl_2O_7

Number of amphoteric oxides is:

(A) 0 $\quad$ (B) 1

(C) 2 $\quad$ (D) 3

Official Ans. by NTA (B)

Sol. Na_2O = Basic $\quad$ As_2O_3 = Amphoteric

N_2O = Neutral $\quad$ NO = Neutral

Cl_2O_7 = Acidic

7. Match List – I with List – II

	List - I		List - II
(A)	Sphalerite	(I)	$FeCO_3$
(B)	Calamine	(II)	PbS
(C)	Galena	(III)	$ZnCO_3$
(D)	Siderite	(IV)	ZnS

Choose the most appropriate answer from the options given below:

(A) (A) - (IV), (B) – (III), (C) – (II), (D) – (I)

(B) (A) - (IV), (B) – (I), (C) – (II), (D) – (III)

(C) (A) - (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) - (III), (B) – (IV), (C) – (II), (D) – (I)

Official Ans. by NTA (A)

Sol.

	List - I		List - II
(A)	Sphalerite	(IV)	ZnS
(B)	Calamine	(III)	$ZnCO_3$
(C)	Galena	(II)	PbS
(D)	Siderite	(I)	$FeCO_3$

8. The highest industrial consumption of molecular hydrogen is to produce compounds of element:

(A) Carbon (B) Nitrogen

(C) Oxygen (D) Chlorine

Official Ans. by NTA (B)

Sol. Nitrogen . Around 55% of hydrogen around would goes to ammonia production

9. Which of the following statements are correct ?

(A) Both LiCl and $MgCl_2$ are soluble in ethanol.

(B) The oxides Li_2O and MgO combine with excess of oxygen to give superoxide.

(C) LiF is less soluble in water than other alkali metal fluorides.

(D) Li_2O is more soluble in water than other alkali metal oxides.

Choose the most appropriate answer from the options given below:

(A) (A) and (C) only (B) (A), (C) and (D) only

(C) (B) and (C) only (D) (A) and (C) only

Official Ans. by NTA (A)

Sol. (A) Both LiCl and $MgCl_2$ are soluble in ethanol

(B) Li and Mg do not form superoxide

(C) LiF has high lattice energy

(D) Li_2O is least soluble in water than other alkali metal oxides

10. Identify the correct statement for B_2H_6 from those given below.

(A) In B_2H_6, all B-H bonds are equivalent.

(B) In B_2H_6 there are four 3-centre-2-electron bonds.

(C) B_2H_6 is a Lewis acid.

(D) B_2H_6 can be synthesized form both BF_3 and $NaBH_4$.

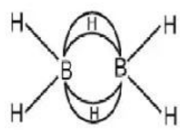
(E) B_2H_6 is a planar molecule.

Choose the most appropriate answer from the options given below :

(A) (A) and (E) only (B) (B), (C) and (E) only

(C) (C) and (D) only (D) (C) and (E) only

Official Ans. by NTA (C)



Sol. (A) (B)

Two 3 centre – 2 – electron bonds

(C) $B_2 H_6$ is e^- deficient species

(E) B_2H_6 is non – Planar molecule

(D) $BF_3 + LiAlH_4 \rightarrow 2B_2H_6 + 3LiF + 3AlF_3$

$NaBH_4 + I_2 \rightarrow B_2H_6 + 2NaI + H_2$

11. The most stable trihalide of nitrogen is:

(A) NF_3 (B) NCl_3

(C) NBr_3 (D) NI_3

Official Ans. by NTA (A)

Sol. Order of stability: -

$NF_3 > NCl_3 > NBr_3 > NI_3$

12. Which one of the following elemental forms is not present in the enamel of the teeth?

(A) Ca^{2+} (B) P^{3+}

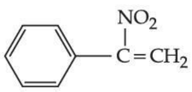
(C) F^- (D) P^{5+}

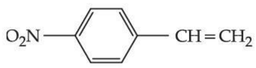
Official Ans. by NTA (B)

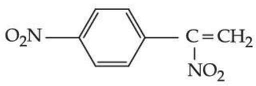
Sol. Calcium and phosphate are the major components of teeth enamel

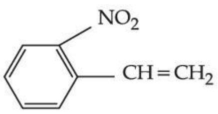
13. In the given reactions sequence, the major product 'C' is :

$C_8H_{10} \xrightarrow[H_2SO_4]{HNO_3} A \xrightarrow[\Delta]{Br_2} B \xrightarrow[KOH]{alcoholic} C$

(A) 

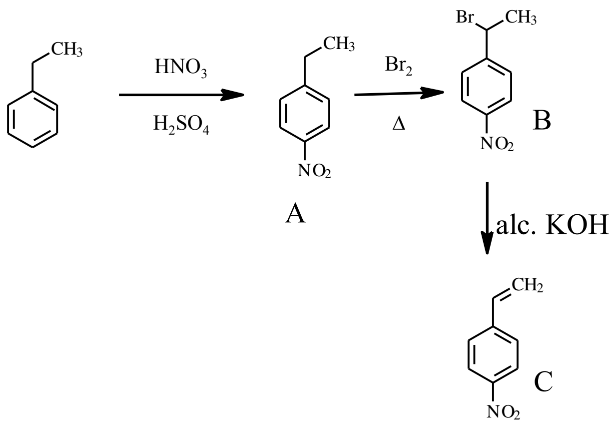
(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. C_8H_{10} DU = 9 –5 = 4



14. Two statements are given below :

Statement I: The melting point of monocarboxylic acid with even number of carbon atoms is higher than that of with odd number of carbon atoms acid immediately below and above it in the series.

Statement II : The solubility of monocarboxylic acids in water decreases with increase in molar mass.

Choose the most appropriate option:

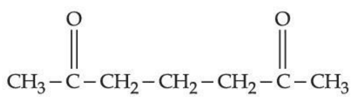
(A) Both Statement I and Statement II are correct.
(B) Both Statement I and Statement II are incorrect.
(C) Statement I is correct but Statement II is incorrect.
(D) Statement I is incorrect but Statement II is correct.

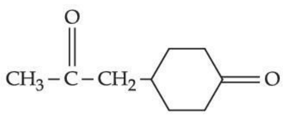
Official Ans. by NTA (A)

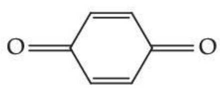
Sol. I . Better packing efficiency of monocarboxylic acids with even number of carbon atoms results in higher M.P

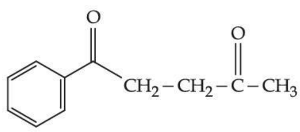
II. As molar mass increases hydrophobic part size increase hence solubility decreases.

15. Which of the following is an example of conjugated diketone?

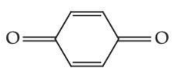
(A) 

(B) 

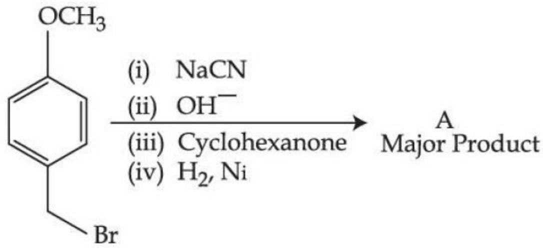
(C) 

(D) 

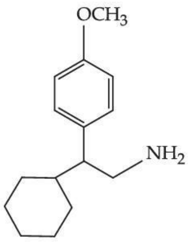
Official Ans. by NTA (C)

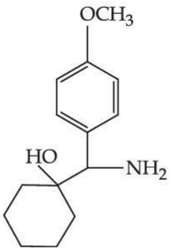
Sol.  **is a conjugated diketone**

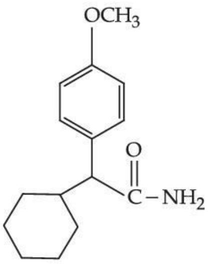
16.

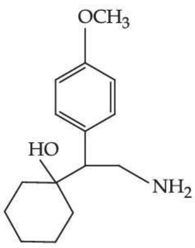


The major product of the above reaction is

(A) 

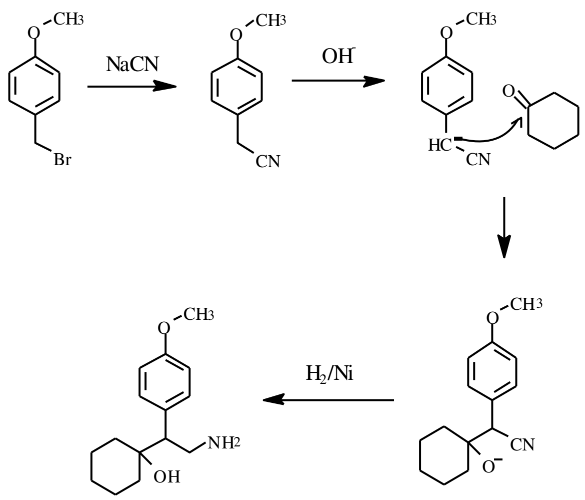
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

Sol.



17. Which of the following is an example of polyester?

(A) Butadiene-styrene copolymer

(B) Melamine polymer

(C) Neoprene

(D) Poly-β-hydroxybutyrate-co-β-hydroxy valerate

Official Ans. by NTA (D)

Sol. **Factual**

18. A polysaccharide 'X' on boiling with dil H_2SO_4 at 393 K under 2-3 atm pressure yields 'Y'.

'Y' on treatment with bromine water gives gluconic acid. 'X' contains β-glycosidic linkages only. Compound 'X' is :

(A) starch (B) cellulose

(C) amylose (D) amylopectin

Official Ans. by NTA (B)

Sol. Cellulose contains β – glycosidic linkages only

19. Which of the following is not a broad spectrum antibiotic?

(A) Vancomycin (B) Ampicillin

(C) Ofloxacin (D) Penicillin G

Official Ans. by NTA (D)

Sol. Penicillin G following is a narrow spectrum antibiotic

20. During the qualitative analysis of salt with cation y^{2+} , addition of a reagent (X) to alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are:

(A) Dimethylglyoxime and Ni^{2+}

(B) Dimethylglyoxime and Co^{2+}

(C) Nessler's reagent and Hg^{2+}

(D) Nessler's reagent and Ni^{2+}

Official Ans. by NTA (A)

Sol. $Ni^{2+} + DMG^{-} \rightarrow [Ni(DMG)_2]\downarrow$

(Bright red precipitate)

SECTION-B

1. Atoms of element X form hcp lattice and those of element Y occupy $\frac{2}{3}$ of its tetrahedral voids. The percentage of element X in the lattice is__________ (Nearest integer)

Official Ans. by NTA (43)

Sol. $X \rightarrow 6 \qquad Y \rightarrow \frac{2}{3} \times 2 \times 6 = 8$

$\% X = \frac{6}{14} \times 100 = 42.8 \simeq 43\%$

2. $2O_3(g) \rightleftharpoons 3O_2(g)$

At 300 K, ozone is fifty percent dissociated. The standard free energy change at this temperature and 1 atm pressure is (–) __J mol $^{-1}$ (Nearest integer)

[Given: ln 1.35 = 0.3 and R = 8.3 J K^{-1} mol^{-1}]

Official Ans. by NTA (747)

Sol. $2O_3 \rightleftharpoons 3O_2(g)$

$\frac{2}{5} \qquad \frac{3}{5}$

$k_p = \frac{P_{O_2}^3}{P_{O_3}^2}$

$k_p = 1.35$

$\Delta G° = -RT \ln k_p$

$= -8.3 \times 300 \times \ln 1.35$

$= -747$ J/mol

3. The osmotic pressure of blood is 7.47 bar at 300 K. To inject glucose to a patient intravenously, it has to be isotonic with blood. The concentration of glucose solution in gL^{-1} is _____(Molar mass of glucose = 180 g mol^{-1}

R = 0.083 L bar K^{-1} mol^{-1}) (Nearest integer)

Official Ans. by NTA (54)

Sol. $\pi = C.R.T$

7.47= C × 0.083 × 300

C = 0.3 M

= 0.3 ×180 gL^{-1}

= 54 gL^{-1}

4. The cell potential for the following cell

$Pt\,|\,H_2(g)\,|\,H^+(aq)\,||\,Cu^{2+}(0.01M)\,|\,Cu(s)$

is 0.576 V at 298 K. The pH of the solution is ___. (Nearest integer)

Official Ans. by NTA (5)

Sol.

Anode : $H_2 \rightarrow 2H^+ + 2e^-$

Cathode : $Cu^{2+} + 2e^- \rightarrow Cu$

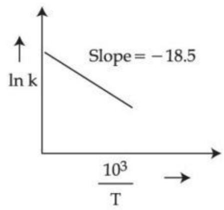
$Cu^{2+} + H_2 \rightarrow 2H^+ + Cu$

$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log \frac{[H^+]^2}{[Cu^{2+}]}$

$0.576 = 0.34 - \frac{0.06}{2} \log \left\{ \frac{[H^+]^2}{(0.01)} \right\}$

$+ 3.93 \; -\log(H^+) + \log 0.1 \Rightarrow pH = 4.93 \simeq 5$

5. The rate constants for decomposition of acetaldehyde have been measured over the temperature range 700 –1000 K. The data has been analysed by plotting In k vs $\frac{10^3}{T}$ graph.The value of activation energy for the reaction is___ kJ mol^{-1}. (Nearest integer) (Given : R = 8.31 J K^{-1} mol^{-1})



Official Ans. by NTA (154)

Sol. $\ln k = \ln A - \frac{Ea}{10^3 RT} \times 10^3 = \ell nA + \frac{10^3}{T}\left[-\frac{Ea}{10^3 RT}\right]$

From the graph

$\frac{-Ea}{10^3 \times R} = -18.5$

Ea = 153.735 kJ/mol

~154

6. The difference in oxidation state of chromium in chromate and dichromate salts is _______

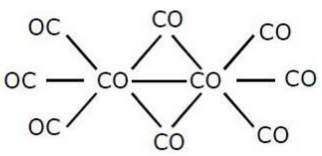
Official Ans. by NTA (0)

Sol. $\overset{+6}{Cr}O_4^{2-}, \overset{+6}{Cr_2}O_7^{2-}$ difference is zero

7. In the cobalt-carbonyl complex: $[Co_2(CO)_8]$, number of Co-Co bonds is "X" and terminal CO ligands is "Y". X + Y =____

Official Ans. by NTA (7)

Sol.



X = 1

Y = 6

8. A 0.166 g sample of an organic compound was digested with conc. H_2SO_4 and then distilled with NaOH. The ammonia gas evolved was passed through 50.0 mL of 0.5 N H_2SO_4. The used acid required 30.0 mL of 0.25 N NaOH for complete neutralization. The mass percentage of nitrogen in the organic compound is____.

Official Ans. by NTA (63)

Sol. m_{eq} of NaOH used = 30 ×0.25

m_{eq} of H_2SO_4 taken = 50 × 0.5

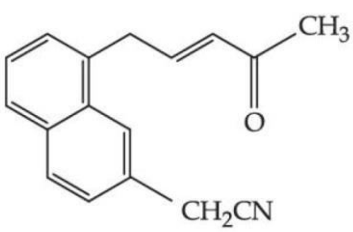
$\therefore$ m_{eq} of H_2SO_4 used

= 50 × 0.25 ×30 ×0.25 = 17.5 m mol of NH_3

$$\therefore \% N = \frac{17.5 \times 10^{-3} \times 14}{0.166} \times 100 = 147.59\%$$

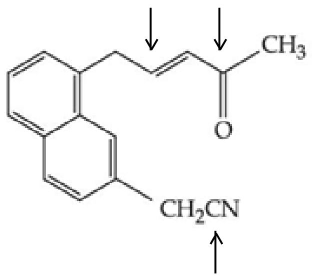
(Not possible)

9. Number of electrophilic centre in the given compound is ____



Official Ans. by NTA (3)

Sol.



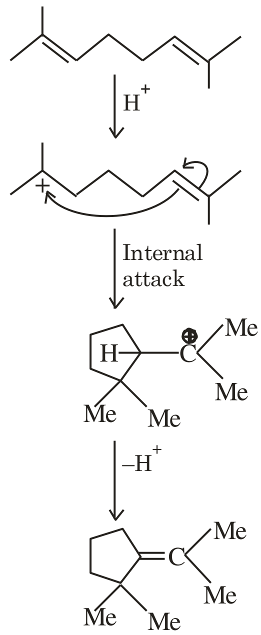
10. The major product 'A' of the following given reaction has _____ sp^2 hybridized carbon atoms.

2,7 – Dimethyl1 – 2, 6 – octadiene

$\xrightarrow{H^+}$ $\underset{\text{Mojor Product}}{A}$

Official Ans. by NTA (2)

Sol. Answer (2)



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A=\{z\in C:1\le|z-(1+i)|\le 2\}$ and $B=\{z\in A:|z-(1-i)|=1\}$. Then, B :

(A) is an empty set

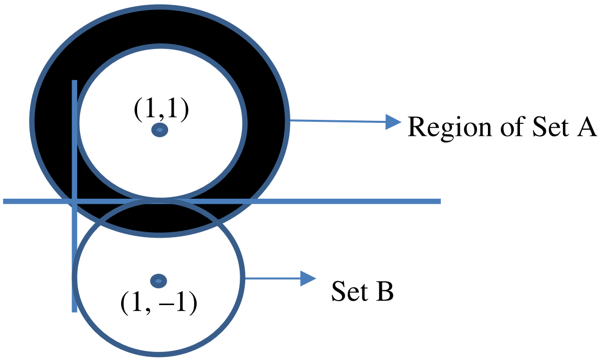
(B) contains exactly two elements

(C) contains exactly three elements

(D) is an infinite set

Official Ans. by NTA (D)

Sol. $A=\{z\in C:1\le|z-(1+i)|\le 2\}$



$B=\{z\in A:|z-(1-i)|=1\}$.

$A\cap B$ has infinite set.

2. The remainder when 3^{2022} is divided by 5 is

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (D)

Sol. $3^{2022} = 9^{1011} = (10-1)^{1011} = 10\,m - 1 = 10\,m - 5 + 4$

$= 5\,(2m-1) + 4$ (m is integer)

Remainder = 4

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds,, it becomes 7 units, then its radius after 9 seconds is :

(A) 9 (B) 10

(C) 11 (D) 12

Official Ans. by NTA (A)

Sol. Let r be the radius of spherical balloon

S = Surface area

$S = 4\pi r^2$

$\frac{dS}{dt}=8\pi r\times\frac{dr}{dt}=k$ (constant)

$4\pi r^2 = kt + C$ (C is constant of integration)

For $t = 0$, $r = 3 \Rightarrow 36\pi = C$

For $t = 5$, $r = 7 \Rightarrow K = 32\pi$

$4\pi r^2 = 32\pi t + 36\pi$

$r^2 = 8t + 9$

for $t = 9$

$r^2 = 81$

$r = 9$

4. Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to _____ .

(A) 13 (B) 6

(C) 4 (D) 3

Official Ans. by NTA (C)

Sol. E_1 = denotes selection for 1st bag

E_2 = denotes selection for 2nd bag

$P(E_1)=\frac{1}{2}$, $P(E_2)=\frac{1}{2}$

A = selected balls are 1 red & 1 black

$P\left(\frac{A}{E_1}\right)=\frac{{}^3C_1\times{}^1C_1}{{}^6C_2}=\frac{1}{5}$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_1 \times {}^2C_1}{(n+5)_{C_2}} = \frac{12}{(n+5)(n+4)}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$\Rightarrow n = 4$

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to______ .

(A) 16 (B) 88/5

(C) 72 (D) –8

Official Ans. by NTA (A)

Sol. $x^2 + y^2 + Ax + By + C = 0$ is passing through (0,6)

$\Rightarrow 6B + C = -36$

The tangent of the parabola $y = x^2$ at (2, 4) is

$4x - y - 4 = 0$ ----(1)

The tangent of circle $x^2 + y^2 + Ax + By + C = 0$ at (2, 4) is

$(4 + A)x + (8 + B)y + 2A + 4B + 2C = 0$ -----(2)

From Equation (1) and (2)

$$\frac{4+A}{4} = \frac{8+B}{-1} = \frac{2A+4B+2C}{-4}$$

$A + 4B = -36$ ---(3)

$3A + 4B + 2C = -4$ ---(4)

From equation (3) and (4)

A+ C = 16

6. The number of values of α for which the system of equations :

$x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

is inconsistent, is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (B)

Sol. $x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

Has inconsistent solution

$$D = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 0$$

$\Rightarrow (\alpha - 1)^2 = 0$

$\alpha = 1$

For $\alpha = 1$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 4 & 3 & 5 \end{vmatrix}$$

$= (10 - 9) - (-5-12) + (-3-8)$

$= 1+17 - 11 \neq 0$

For $\alpha = 1$ the system of equation has Inconsistent solution

7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(A) 18 (B) 24

(C) 36 (D) 96

Official Ans. by NTA (B)

Sol. **Here** α, β roots of equation $3x^2 + \lambda x - 1 = 0$

$$\alpha + \beta = \frac{-\lambda}{3}, \ \alpha\beta = \frac{-1}{3}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$$

$\lambda^2 = 9$

Now $6(\alpha^3 + \beta^3)^2 = 6\left((\alpha+\beta)\left((\alpha+\beta)^2 - 3\alpha\beta\right)\right)^2$

$$= 6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9} + 1\right\}^2 = 24$$

8. The set of all values of k for which $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3, x \in R,$ is the interval :

(A) $\left[\frac{1}{32}, \frac{7}{8}\right)$ (B) $\left(\frac{1}{24}, \frac{13}{16}\right)$

(C) $\left[\frac{1}{48}, \frac{13}{16}\right]$ (D) $\left[\frac{1}{32}, \frac{9}{8}\right)$

Official Ans. by NTA (A)

Sol. **Let S =** $(\tan^{-1}x)^3 + (\cot^{-1}x)^3$

$= \left(\tan^{-1}x + \cot^{-1}x\right) - 3\tan^{-1}x.\cot^{-1}x\left(\tan^{-1}x + \cot^{-1}x\right)$

$= \frac{\pi^3}{8} - \frac{3\pi}{2}\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right)$

$= \frac{3\pi}{2}\left(\tan^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^3}{32}$

$\Rightarrow \frac{\pi^3}{32} \le S < \frac{7}{8}\pi^3$

$= \frac{\pi^3}{32} \le K\pi^3 < \frac{7}{8}\pi^3$

$\frac{1}{32} \le K < \frac{7}{8}$

9. Let $S = \{\sqrt{n} : 1 \le n \le 50 \text{ and n is odd}\}$

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$

If $\sum_{a \in S} \det(\text{adj}A) = 100\lambda$, then λ is equal to

(A) 218 (B) 221

(C) 663 (D) 1717

Official Ans. by NTA (B)

Sol. $S = \{\sqrt{n} : 1 \le n \le 50 \text{ and n is odd}\}$

$= \left\{\sqrt{1}, \sqrt{3}, \sqrt{5}..........\sqrt{49}\right\}$, 25 terms

$|A| = 1 + a^2$

$\sum_{a\in S} \det(\text{adj}A) = \sum_{a\in S} |A|^2 = \sum (1 + a^2)^2$

$= 22100 = 100\lambda$

$\lambda = 221$

10. $f(x) = 4 \log_e(x - 1) - 2x^2 + 4x + 5, x > 1$, which one of the following is NOT correct ?

(A) f is increasing in (1, 2) and decreasing in $(2, \infty)$

(B) $f(x) = -1$ has exactly two solutions

(C) $f'(e) - f''(2) < 0$

(D) $f(x) = 0$ has a root in the interval $(e, e + 1)$

Official Ans. by NTA (C)

Sol. $f(x) = 4 \log_e(x - 1) - 2x^2 + 4x + 5, x > 1$

$f'(x) = \frac{4}{x-1} - 4(x-1)$

For $1 < x < 2 \Rightarrow f'(x) > 0$

For $x > 2 \Rightarrow f'(x) < 0$ (option 1 is correct)

$f(x) = -1$ has two solution (option 2 is correct)

$f(e) > 0$

$f(e+1) < 0$

$f(e).f(e+1) < 0$ (option 4 is correct)

$f'(e) - f''(2) = \frac{4}{e-1} - 4(e-1) + 8 > 0$

(option C is incorrect)

11. the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

(A) $x^2 + \frac{y^2}{81} = 2$ (B) $\frac{y^2}{9} - x^2 = 8$

(C) $y = 4x^2 + 5$ (D) $\frac{x}{3} - y^2 = 2$

Official Ans. by NTA (D)

Sol. **The tangent at (x_1, y_1) to the curve**

$y = x^3 + 3x^2 + 5$

$y - y_1 = \left(3x_1^2 + 6x_1\right)\left(x - x_1\right)$ passing through origin

$-y_1 = \left(3x_1^3 + 6x_1\right)\left(-x_1\right)$

$y_1 = \left(3x_1^3 + 6x_1^2\right)$ --------(1)

And (x_1, y_1) lies on the curve

$y = x^3 + 3x^2 + 5$

$y_1 = x_1^3 + 3x_1^2 + 5$ ----(2)

From equation (1) and (2)

$2y_1 = 3x_1^2 + \frac{15}{2}$

Hence the equation of curve $y = \frac{3}{2}x^2 + \frac{15}{2}$

This curve does not intersect $\frac{x}{3} - y^2 = 2$

12. The sum of absolute maximum and absolute minimum values of the function

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval $[0, 1]$ is :

(A) $3 + \frac{\sin(1)\cos^2(\frac{1}{2})}{2}$ (B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$ (D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

Official Ans. by NTA (B)

Sol. $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$

$f(x) = |(2x - 1)(x + 2)| + \sin x \cos x$

$$f'(x) = \begin{cases} 4x + 3 + \frac{\cos 2x}{4}, & \frac{1}{2} < x < 1 \\ -(4x + 3) + \frac{\cos 2x}{4}, & 0 \le x < \frac{1}{2} \end{cases}$$

For $0 \le x < \frac{1}{2} \Rightarrow f'(x) < 0$

For $\frac{1}{2} < x \le 1 \Rightarrow f'(x) > 0$

f (x) local minima at $x = \frac{1}{2}$ and

local maxima at x = 1

$f\left(\frac{1}{2}\right) + f(1) = 3 + \frac{1}{2}(1 + 2\cos 1)\sin 1$

13. If $\{a_i\}_{i=1}^{n}$ where n is an even integer , is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to:

(A) 48 (B) 96

(C) 92 (D) 104

Official Ans. by NTA (B)

Sol. $\sum_{i=1}^{n} a_i = \frac{n}{2}\{2a_1 + (n+1)\} = 192$

$\Rightarrow 2a_1 + (n-1) = \frac{384}{n}$ ----(1)

$\sum_{i=1}^{n/2} a_{2i} = \frac{n}{4}\left[2a_1 + 2 + \left(\frac{n}{2} - 1\right)2\right] = 120$

$2a_1 + n = \frac{480}{n}$ ----(2)

From equation (2) and (1)

$1 = \frac{480}{n} - \frac{384}{n}$

$n = 480 - 384 = 96$

14. If x = x(y) is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$; then x(e) is equal to :

(A) $e^3(e^e - 1)$ (B) $e^e(e^3 - 1)$

(C) $e^2(e^e + 1)$ (D) $e^e(e^2 - 1)$

Official Ans. by NTA (A)

Sol. $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$

$\frac{dx}{dy} - \frac{2}{y}x = y^2(y+1)e^y$

$I.f = e^{\int \frac{-2}{y}dy} = \frac{1}{y^2}$

$x.\frac{1}{y^2} = \int (y+1)e^y dy$

$\frac{x}{y^2} = (y+1)e^y - e^y + c = y.e^y + c$

$x = y^3e^y + cy^2$

For $x = 0,\ y = 1 \Rightarrow c = -e$

$x = y^3e^y - e.y^2$

$x(e) = e^3(e^e - 1)$

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to:

(A) –2 (B) –4

(C) 2 (D) 4

Official Ans. by NTA (D)

Sol. $\lambda x - 2y = \mu$ is a tangent to the curve

$a^2x^2 - y^2 = b^2$ then

$$a^2x^2 - \left(\frac{\lambda x - \mu}{2}\right)^2 = b^2$$

$$\left(4a^2 - \lambda^2\right)x^2 + 2\lambda\mu x - \mu^2 - 4b^2 = 0$$

Disc. = 0

$$4\lambda^2\mu^2 + 4\left(4a^2 - \lambda^2\right)\left(\mu^2 + 4b^2\right) = 0$$

$$4\lambda^2 b^2 - 4a^2\mu^2 = 16a^2b^2$$

$$\frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$$

16. Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to

(A) $6(3 - \sqrt{3})$ (B) $3 + \sqrt{3}$

(C) $6(3 + \sqrt{3})$ (D) $6(\sqrt{3} + 1)$

Official Ans. by NTA (C)

Sol. $\left|\hat{b}\right|^2 = \left|\vec{c} + 2(\vec{c} \times \hat{a})\right|^2$

$$\left|\hat{b}\right|^2 = |c|^2 + 4|\vec{c} \times \hat{a}|^2 + 4\vec{c}.(\vec{c} \times \hat{a})$$

$$1 = |c|^2 + 4|c|^2 \sin^2\frac{\pi}{12} + 0$$

$$1 = |c|^2 + 4|c|^2\left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)^2$$

$$|c|^2 = \frac{1}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{6}$$

So $6^2|c|^2 = 6(3 + \sqrt{3})$

17. If a random variable X follows the Binomial distribution B (33, p) such that 3P(X = 0) = P(X = 1), then the value of $\frac{P(X = 15)}{P(X = 18)} - \frac{P(X = 16)}{P(X = 17)}$ is equal to

(A) 1320 (B) 1088

(C) $\frac{120}{1331}$ (D) $\frac{1088}{1089}$

Official Ans. by NTA (A)

Sol. n = 33, let probability of success is p and q = 1 – p

$3p(x = 0) = p(x = 1)$

$$3.\,{}^{33}C_0(q)^{33} = {}^{33}C_1 pq^{32}$$

$$p = \frac{1}{12},\ q = \frac{11}{12},\ \frac{q}{p} = 11$$

$$\frac{p(x = 15)}{p(x = 18)} - \frac{p(x = 16)}{p(x = 17)}$$

$$\frac{{}^{33}C_{15}p^{15}q^{18}}{{}^{33}C_{18}p^{18}q^{15}} - \frac{{}^{33}C_{16}p^{16}q^{17}}{{}^{33}C_{17}p^{17}q^{16}} = \left(\frac{q}{p}\right)^3 - \left(\frac{q}{p}\right)$$

$= (11)^3 - 11$

= 1320

18. The domain of the function

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\log_e(x^2 - 3x + 2)}$$ is

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}\right\}$

Official Ans. by NTA (DROP)

Sol. $-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$

$\frac{x^2-5x+6}{x^2-9} - 1 \le 0$

$\frac{1}{x+3} \ge 0$

$x \in (-3, \infty)$ **……(1)**

$\frac{x^2-5x+6}{x^2-9} + 1 \ge 0$

$\frac{2x+1}{x+3} \ge 0$

$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right)$ **……(2)**

after taking intersection

$x \in \left[-\frac{1}{2}, \infty\right)$

$x^2 - 3x + 2 > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

$x^2 - 3x + 2 \ne 1$

$x \ne \frac{3 \pm \sqrt{5}}{2}$

after taking intersection of each solution

$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

19. Let

$S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$.

If $T = \sum_{\theta \in S} \cos 2\theta$, then T + n(S) is equal

(A) $7+\sqrt{3}$ (B) 9

(C) $8+\sqrt{3}$ (D) 10

Official Ans. by NTA (B)

Sol. $\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

$\tan\theta(\sin\theta + 1) = \frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta = 0 \Rightarrow \theta = -\pi, 0, \pi$

$(\sin\theta + 1) = 2.\cos^2\theta = 2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta = -1$ which is not possible

$\sin\theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

n(s) = 5

$T = \cos 0 + \cos 2\pi + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$

$T = 4$

$T + n(s) = 9$

20. The number of choices of $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p\Delta q) \Rightarrow ((p\Delta \sim q) \vee ((\sim p)\Delta q))$ is a tautology, is

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (B)

Sol. For tautology $((p\Delta \sim q) \vee ((\sim p)\Delta q))$ must be true.

This is possible only when $\Delta = \vee \,\&\, \Rightarrow$

SECTION-B

1. The number of one-one function f : {a, b, c, d} → {0, 1, 2, … ..,10} such that 2f(a) – f(b) + 3f(c) + f(d) = 0 is _____ .

Official Ans. by NTA (31)

Sol. 2f(a) + 3f(c) = f(d) – f(b)

Using fundamental principle of counting

Number of one-one function is 31

2. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is_.

Official Ans. by NTA

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 5$

Only one possibilities 3, 3, 3, -2, -2

Number of ways is = $\frac{5!}{3!2!} \times 2 \times 2 = 40$

3. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right) a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If D(3 cos θ, a sin θ) is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to ________ .

Official Ans. by NTA (8)

Sol. $A=\left(\frac{3}{\sqrt{a}},\sqrt{a}\right)$

$B=\left(\frac{-3}{\sqrt{a}},\sqrt{a}\right)$

$C=\left(-\frac{3}{\sqrt{a}},-\sqrt{a}\right)$

Area of ACD

$$\frac{1}{2}\begin{vmatrix}\frac{3}{\sqrt{a}} & \sqrt{a}\\ -\frac{3}{\sqrt{a}} & -\sqrt{a}\\ 3\cos\theta & a\sin\theta\\ \frac{3}{\sqrt{a}} & \sqrt{a}\end{vmatrix}$$

$\frac{1}{2}6\sqrt{a}(\cos\theta-\sin\theta)$

$3\sqrt{a}(\cos\theta-\sin\theta)$

max values of function is $3\sqrt{a}\sqrt{2}$

$3\sqrt{a}\sqrt{2}=12$

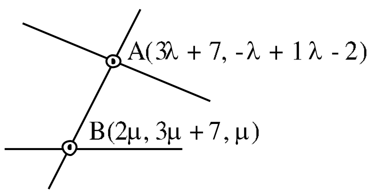
2a = 16

a = 8

4. Let a line having direction ratios 1, -4, 2 intersect the lines $\frac{x-7}{3}=\frac{y-1}{-1}=\frac{z+2}{1}$ and $\frac{x}{2}=\frac{y-7}{3}=\frac{z}{1}$ at the point A and B. Then $(AB)^2$ is equal to ____ .

Official Ans. by NTA (84)

Sol.



DR's of AB

$(3\lambda-2\mu+7,-\lambda-3\mu-6,\lambda-\mu-2)$

$\frac{3\lambda-2\mu+7}{1}=\frac{-\lambda-3\mu-6}{-4}=\frac{\lambda-\mu-2}{2}$

Taking first (2) $-12\lambda+8\mu-28=-\lambda-3\mu-6$

$\lambda-\mu+2=0$

Taking second & third $-2\lambda-6\mu-12=-4\lambda+4\mu+8$

$\lambda-5\mu-10=0$

After solving above two equation $\lambda=-5,\mu=-3$

A = (-8, 6, 7)

B = (-6, -2, -3)

$(AB)^2$ = 4 + 64 + 16 = 84

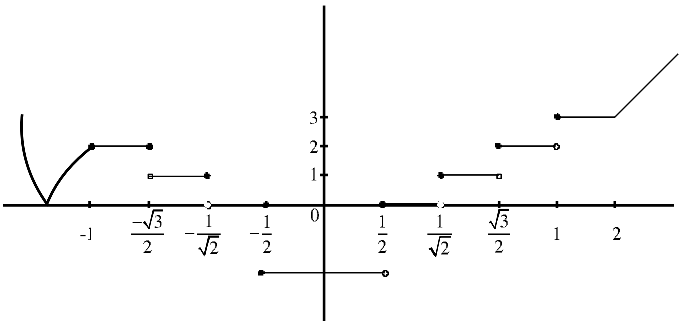
5. The number of points where the function

$$f(x)=\begin{cases}|2x^2-3x-7| & \text{if} \quad x\le -1\\ [4x^2-1] & \text{if} \quad -1<x<1\\ |x+1|+|x-2| & \text{if} \quad x\ge 1\end{cases}$$

[t] denotes the greatest integer $\le t$, is discontinuous is _______ .

Official Ans. by NTA (7)

Sol.



6. Let $f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)f(t)dt$. Then the value of $\left|\int_0^{\pi/2}f(\theta)d\theta\right|$ is _______ .

Official Ans. by NTA (1)

Sol. $f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)\,f(t)\,dt$

$f(\theta)=\sin\theta+\sin\theta\int_{-\pi/2}^{\pi/2}f(t)dt+\cos\theta\int_{-\pi/2}^{\pi/2}tf(t)dt$

Let $A=\int_{-\pi/2}^{\pi/2}f(t)dt,\ B=\int_{-\pi/2}^{\pi/2}tf(t)dt$

$f(\theta)=\sin\theta+A\sin\theta+B\cos\theta$

$f(\theta)=(A+1)\sin\theta+B\cos\theta$

$A=\int_{-\pi/2}^{\pi/2}(A+1)\sin t+B\cos t\ dt$

$A = 2B \qquad ......(1)$

$B=\int_{-\pi/2}^{\pi/2}t((A+1)\sin t+B\cos t)$

$B=\int_{-\pi/2}^{\pi/2}t(A+1)\sin t$

$B=(A+1)2\int_{0}^{\pi/2}t\sin t\ dt$

$B = (A + 1)\ 2.1$

$2A + 2 - B = 0 \ .....(2)$

After solving

$B=-\frac{2}{3},A=-\frac{4}{3}$

$\left|\int_{0}^{\pi/2}f(\theta)d\theta\right|=\left|\int_{0}^{\pi/2}-\frac{1}{3}\sin\theta-\frac{2}{3}\cos\theta\right|$

$= 1$

7. Let $\underset{0\le x\le 2}{\mathrm{Max}}\left\{\frac{9-x^2}{5-x}\right\}=\alpha$ and $\underset{0\le x\le 2}{\mathrm{Min}}\left\{\frac{9-x^2}{5-x}\right\}=\beta$

If $\int_{\beta-\frac{8}{3}}^{2\alpha-1}\mathrm{Max}\left\{\frac{9-x^2}{5-x},x\right\}dx=\alpha_1+\alpha_2\log_e\left(\frac{8}{15}\right)$ then

$\alpha_1+\alpha_2$ is equal to _______

Official Ans. by NTA (34)

Sol. $y=\frac{9-x^2}{5-x}=5+x+\frac{16}{x-5}$

$\frac{dy}{dx}=1-\frac{16}{(x-5)^2}$

So critical point is $x = 1$ in $[0, 2]$

$y(0)=\frac{9}{5},\ y(1)=2,\ y(2)=\frac{5}{3}$

So $\alpha=2$ and $\beta=\frac{5}{3}$

$I=\int_{-1}^{3}\max\left(\frac{9-x^2}{5-x},x\right)$

$I=\int_{-1}^{9/5}\frac{9-x^2}{5-x}dx+\int_{9/5}^{3}x\,dx$

$I=\int_{-1}^{9/5}5+x+\frac{16}{x-5}dx+\int_{9/5}^{3}x\,dx$

After solving

$I=14+\frac{28}{25}+16\ln\left(\frac{8}{15}\right)+\frac{72}{25}$

$\alpha_1=18$ and $\alpha_2=16$

8. If two tangents drawn from a point (α,β) lying on the ellipse $25x^2+4y^2=1$ to the parabola $y^2=4x$ are such that the slope of one tangent is four times the other, then the value of

$(10\alpha+5)^2+(16\beta^2+50)^2$ equals __________

Official Ans. by NTA (2929)

Sol. $\alpha=\frac{1}{5}\cos\theta,\ \beta=\frac{1}{2}\sin\theta$

Equation of tangent to $y^2=4x$

$y=mx+\frac{1}{m}$

It passes through (α,β)

$\frac{1}{2}\sin\theta=m\frac{1}{5}\cos\theta+\frac{1}{m}$

$m^2\left(\frac{\cos\theta}{5}\right)-m\left(\frac{1}{2}\sin\theta\right)+1=0$

It has two roots m_1 and m_2 where $m_1=4m_2$

$m_1+m_2=\frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$

$m_1m_2=\frac{5}{\cos\theta}$

After eliminating m_1 and m_2

$\cos\theta = \frac{-5 \pm \sqrt{29}}{2}$

$\alpha = \frac{-5 \pm \sqrt{29}}{10} \Rightarrow 10\alpha + 5 = \pm\sqrt{29}$

$\beta^2 = \frac{1}{4}\sin^2\theta \Rightarrow 16\beta^2 = -50 \pm 10\sqrt{29}$

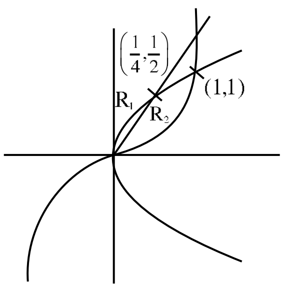
$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 2929$

9. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2.

If max $\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to _____ .

Official Ans. by NTA (19)

Sol.



$S = \int_0^1 \sqrt{x} - x^3$

$= \left[\frac{2x^{3/2}}{3} - \frac{x^4}{4}\right]_1^0$

$= \frac{5}{12}$

$R_1 = \int_0^{1/4} (\sqrt{x} - 2x)dx$

$= \left[\frac{2x^{3/2}}{3} - x^2\right]_0^{1/4} = \frac{1}{48}$

$\therefore \; R_2 = \frac{19}{48}$

So, $\frac{R_2}{R_1} = 19$

10. If the shortest distance between the line

$\vec{r} = (-\hat{i} + 3k) + \lambda(\hat{i} - a\hat{j})$ and

$\vec{r} = (-\hat{j} + 2k) + \mu(\hat{i} - \hat{j} + k)$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to

Official Ans. by NTA (2)

Sol. $a_1 = (-1, 0, 3)$

$a_2 = (0, -1, 2)$

$b_1 = (1, -a, 0)$ dr's of line (1)

$b_2 = (1, -1, 1)$ dr's of line (2)

$\bar{a}_2 - \bar{a}_1 = (1, -1, -1)$

$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$\bar{b}_1 \times \bar{b}_2 = \hat{i}(-a) - \hat{j} + k(a-1)$

$|\bar{b}_1 \times \bar{b}_2| = \sqrt{a^2 + 1 + (a-1)^2}$

$a_2 - a_1 . \bar{b}_1 \times \bar{b}_2 = 2 - 2a$

$\frac{2(1-a)}{\sqrt{a^2 + 1 + (a-1)^2}} = \sqrt{\frac{2}{3}}$

Squaring an both the side

After solving $a = 2, \; \frac{1}{2}$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Identify the pair of physical quantities that have same dimensions :

(A) velocity gradient and decay constant

(B) wien's constant and Stefan constant

(C) angular frequency and angular momentum

(D) wave number and Avogadro number

Official Ans. by NTA (A)

Sol. Velocity gradient $= \frac{dV}{dx} = \frac{1}{S}$

$\lambda = \frac{1}{S}$

2. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be :

(A) $\sqrt{3}$ years (B) 3 years

(C) 9 years (D) $3\sqrt{3}$ years

Official Ans. by NTA (D)

Sol. $T' = T\left(\frac{3R}{R}\right)^{3/2} = 3\sqrt{3}\,T$

3. A stone of mass m, tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is :

(A) the same throughout the motion

(B) minimum at the highest position of the circular path

(C) minimum at the lowest position of the circular path

(D) minimum when the rope is in the horizontal position

Official Ans. by NTA (B)

Sol. Theory

4. Two identical charged particles each having a mass 10 g and charge 2.0×10^{-7} C area placed on a horizontal table with a separation of L between then such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L. [Use g = 10 ms^{-2}]

(A) 12 cm (B) 10 cm

(C) 8 cm (D) 5 cm

Official Ans. by NTA (A)

Sol. $\frac{kq^2}{L^2} = \mu mg \Rightarrow L = \sqrt{\frac{k}{\mu mg}}\,q$

5. A Carnot engine take 5000 kcal of heat from a reservoir at 727°C and gives heat to a sink at 127°C. The work done by the engine is :

(A) 3×10^6 J (B) Zero

(C) 12.6×10^6 J (D) 8.4×10^6 J

Official Ans. by NTA (C)

Sol. $L = \frac{WD}{Q_H}$

$\Rightarrow WD = Q_H\left(1 - \frac{T_L}{T_H}\right)$

$= 5 \times 10^3\left(1 - \frac{400}{1000}\right)$

$= 3000$ kcal

6. Two massless springs with spring constants 2 k and 2 k, carry 50 g and 100 g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitudes will be :

(A) 1 : 2 (B) 3 : 2

(C) 3 : 1 (D) 2 : 3

Official Ans. by NTA (B)

Sol. $V_{max} = \omega A$

$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{9}{2} \times \frac{1}{2}} = \frac{3}{2}$

7. What will be the most suitable combination of three resistors A = 2Ω, B = 4Ω, C = 6 Ω so that $\left(\frac{22}{3}\right)\Omega$ is equivalent resistance of combination?

(A) Parallel combination of A and C connected in series with B.

(B) Parallel combination of A and B connected in series with C.

(C) Series combination of A and C connected in parallel with B.

(D) Series combination of B and C connected in parallel with A.

Official Ans. by NTA (B)

Sol. $\Rightarrow \frac{4}{3}+6=\frac{22}{3}$

8. The soft-iron is a suitable material for making an electromagnet. This is because soft-iron has :

(A) low coercively and high retentively

(B) low coercively and low permeability

(C) high permeability and low retentively

(D) high permeability and high retentively

Official Ans. by NTA (C)

Sol. Theory

9. A proton, a deuteron and an α-particle with same kinetic energy enter into a uniform magnetic field at right angle to magnetic field. The ratio of the radii of their respective circular paths is :

(A) $1:\sqrt{2}:\sqrt{2}$ (B) $1:1:\sqrt{2}$

(C) $\sqrt{2}:1:1$ (D) $1:\sqrt{2}:1$

Official Ans. by NTA (D)

Sol. $R=\frac{\sqrt{2km}}{qB}\propto\frac{\sqrt{m}}{q}$

$\frac{\sqrt{m}}{e}:\frac{\sqrt{2m}}{e}:\frac{\sqrt{4m}}{2e}$

$1:\sqrt{2}:1$

10. Given below are two statements :

Statement-I : The reactance of an ac circuit is zero. It is possible that the circuit contains a capacitor and an inductor.

Statement-II : In ac circuit, the average poser delivered by the source never becomes zero.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II in false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (C)

Sol. if R = 0, P = 0

11. Potential energy as a function of r is given by $U=\frac{A}{r^{10}}-\frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be :

(A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$ (B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$

(C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$ (D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Official Ans. by NTA (C)

Sol. $\frac{-10A}{r^{11}}+\frac{5B}{r^6}=0$

$r^5=\frac{10A}{5B}=\frac{2A}{B}$

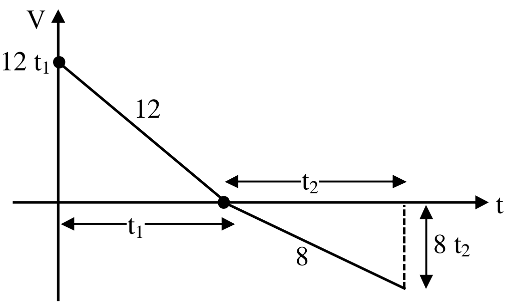
12. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to : [Use g = 10 ms^{-2}]

(A) 1 : 1 (B) $\sqrt{2}:\sqrt{3}$

(C) $\sqrt{3}:\sqrt{2}$ (D) 2 : 3

Official Ans. by NTA (B)

Sol.



$6t_1^2 = 4t_2^2$

13. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be :

(A) 7.5 rad (B) 15 rad

(C) 20 rad (D) 30 rad

Official Ans. by NTA (B)

Sol. $5 = \frac{1}{2}\alpha(1)^2$

$\theta = \frac{1}{2}\alpha(2)^2$

$\theta - 5 = 15$

14. A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms^{-1}. What will be the rise in the temperature of the nail if one fourth of energy of the hammer goes into heating the nail?

[Specific heat capacity of iron = 0.42 $Jg^{-1}\,°C^{-1}$]

(A) 675°C (B) 1600°C

(C) 160.7°C (D) 6.75°C

Official Ans. by NTA (C)

Sol. $\frac{1}{2}\times 1.5\times 60^2 \times \frac{1}{4} = 0.1 \times 420 \times \Delta T$

15. If the charge on a capacitor is increased by 2 C, the energy stored in it increases by 44%. The original charge on the capacitor is (in C) :

(A) 10 (B) 20

(C) 30 (D) 40

Official Ans. by NTA (A)

Sol. $U \propto q^2$

$\Rightarrow q_f = 1.2\,q$

$q_f - q = 2$

$\Rightarrow 1.2\,q - q = 2$

$q = 10$

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) revolves around the cylinder in a circular path. The kinetic of the particle is :

(A) $\frac{\rho q R^2}{4\varepsilon_0}$ (B) $\frac{\rho q R^2}{2\varepsilon_0}$

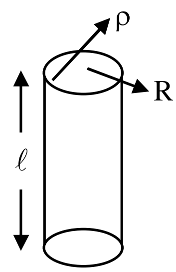
(C) $\frac{q\rho}{4\varepsilon_0 R^2}$ (D) $\frac{4\varepsilon_0 R^2}{q\rho}$

Official Ans. by NTA (A)

Sol. $E = 2\pi r\ell = \frac{\rho\pi r^2 \ell}{\varepsilon_0}$

$qE = \frac{q\rho R^2}{2\varepsilon_0 r} = \frac{mv^2}{r}$

$mv^2 = \frac{q\rho R^2}{2\varepsilon_0}$



17. An electric bulb is rated as 200 W. What will be the peak magnetic field at 4 m distance produced by the radiations coming from this bulb? Consider this bulb as a point source with 3.5% efficiency.

(A) 1.19×10^{-8} T (B) 1.71×10^{-8} T

(C) 0.84×10^{-8} T (D) 3.36×10^{-8} T

Official Ans. by NTA (B)

Sol. $\frac{\eta P}{4\pi r^2} = \frac{cB_0^2}{2\mu_0}$

$B_0 = \sqrt{\frac{\mu_0}{4\pi}\frac{\eta P}{c}}\frac{1}{r}$

$\Rightarrow B_0 = \frac{1}{4}\sqrt{\frac{10^{-7}\times 4\times 3.5}{3\times 10^8}} = 1.71 \times 10^{-8}$ T

18. The light of two different frequencies whose photons have energies 3.8 eV and 1.4 eV respectively, illuminate a metallic surface whose work function is 0.6 eV successively. The ratio of maximum speeds of emitted electrons for the two frequencies respectivly will be :

(A) 1 : 1 (B) 2 : 1

(C) 4 : 1 (D) 1 : 4

Official Ans. by NTA (B)

Sol. $\sqrt{\frac{3.8-0.6}{1.4-0.6}}=\sqrt{\frac{3.2}{0.8}}=2$

19. Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The .ratio of the intensity of maxima and minima will be :

(A) 2 : 3 (B) 16 : 81

(C) 25 : 169 (D) 25 : 1

Official Ans. by NTA (D)

Sol. $\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{9}{4}}=\frac{3}{2}$

$$\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2=5^2=25$$

20. In Bohr's atomic model of hydrogen, let K. P and E are the kinetic energy, potential energy and total energy of the electron respectively. Choose the correct option when the electron undergoes transitions to a higher level :

(A) All K. P and E increase.

(B) K decreases. P and E increase.

(C) P decreases. K and E increase.

(D) K increases. P and E decrease.

Official Ans. by NTA (B)

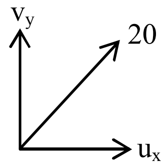
Sol. Based on theory

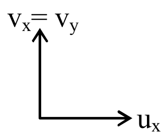
SECTION-B

1. A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2s is 20 ms^{-1}. The maximum height reached by the body during its motion is _______m. (use g = 10ms^{-2})

Official Ans. by NTA (20)

Sol.





$v_y = v_x - 20$

$\sqrt{(u_x-20)^2+u_x^2}=20$

$\Rightarrow 2u_x^2-40u_x=0$

$\therefore u_x = 20$

2. An antenna is placed in a dielectric medium of dielectric constant 6.25. If the maximum size of that antenna is 5.0 mm. it can radiate a signal of minimum frequency of ______GHz.

(Given $\mu_r = 1$ for dielectric medium)

Official Ans. by NTA (6)

Sol. $C'=\frac{C}{\sqrt{\mu_r\varepsilon_r}}=\frac{3\times10^8}{\sqrt{6.25}}=\frac{3\times10^8}{2.5}$

$f\lambda = 1.25 \times 10^8$ s

$\Rightarrow f(5\times10^{-3}\times4)=1.25\times10^8$

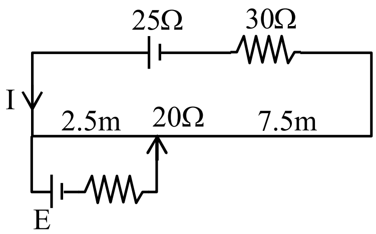
f = 6.25 GHz

So $f\approx6$

3. A potentiometer wire of length 10 m and resistance 20 Ω is connected in series with a 25 V battery and an external resistance 30 Ω. A cell of emf E in secondary circuit is balanced by 250 cm long potentiometer wire. The value of E (in volt) is $\frac{x}{10}$. The value of x is _______.

Official Ans. by NTA (25)

Sol.



$I = \frac{25}{50} = \frac{1}{2} A$

$\therefore \Delta V = 10$ V

10 m $\rightarrow$ 10V

2.5m $\rightarrow$ 2.5V

4. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produce a stationary wave whose equation is given by

$y = (10 \cos \pi x \sin \frac{2\pi t}{T})$ cm

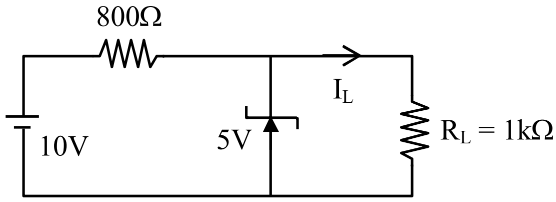
The amplitude of the particle at $x = \frac{4}{3}$ cm will be _______ cm.

Official Ans. by NTA (5)

Sol. $10 \cos\left(\frac{4\pi}{3}\right)$

5. In the given circuit- the value of current I_L will be _______ mA.

(When $R_L = 1k\Omega$)



Official Ans. by NTA (5)

Sol. $I_L = \frac{5}{1000} = 5mA$

6. A sample contains 10^{-2} kg each of two substances A and B with half lives 4 s and 8 s respectively. The ratio of then atomic weights is 1 : 2. The ratio of the amounts of A and B after 16 s is $\frac{x}{100}$. the value of x is _______.

Official Ans. by NTA (25)

Sol. $N_t = N_0 (0.5)^{\frac{t}{t_{1/2}}}$

$= \frac{m}{M} \times N_A (0.5)^{\frac{t}{t_{1/2}}}$

$\frac{N_1}{N_2} = \frac{M_2}{M_1} (0.5)^{t\left[\frac{1}{T_A} - \frac{1}{T_B}\right]}$

$= 2(0.5)^{16 \times \frac{1}{8}} = \frac{2}{4} = \frac{1}{2} = \frac{x}{100}$

7. A ray of ligh is incident at an angle of incidence 60° on the glass slab of refractive index $\sqrt{3}$. After refraction, the light ray emerges out from other parallel faces and lateral shift between incident ray and emergent ray is $4\sqrt{3}$ cm. The thickness of the glass slab is _______ cm.

Official Ans. by NTA (12)

Sol. $\ell = t\sin i\left[1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right]$

$\Rightarrow 4\sqrt{3} = t\sin 60°\left[1-\frac{\cos 60°}{\sqrt{3-\frac{3}{4}}}\right]$

8. A circular coil of 1000 turns each with area $1m^2$ is rotated about its vertical diameter at the rate of one revolution per second in a uniform horizontal magnetic field of 0.07T. The maximum voltage generation will be _______V.

Official Ans. by NTA (440)

Sol. $\in_{max} = BAN\omega$

$= 0.07 \times 1 \times 10^3 \times 2\pi$

$= 140\pi \approx 440$

9. A monoatomic gas performs a work of $\frac{Q}{4}$ where Q is the heat supplied to it. The molar heat capacity of the gas will be ________R during this transformation.

Where R is the gas constant.

Official Ans. by NTA (2)

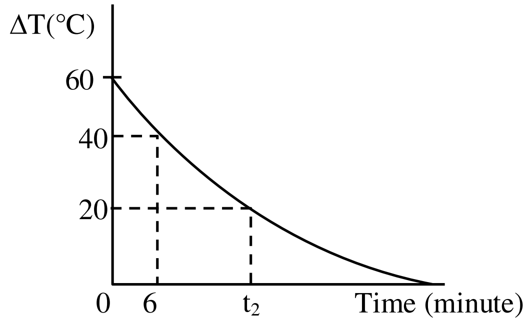
Sol. $\Delta Q = \Delta E + WD \Rightarrow Q = \Delta E + \frac{Q}{4}$

$\Rightarrow n\frac{3R}{2}\Delta T = \Delta E = \frac{3Q}{4}$

$\therefore n\Delta T = \frac{Q}{2R}$

$\therefore C = 2R$

10. In an experment ot verify Newton's law of cooling, a graph is plotted between, the temperature difference (ΔT) of the water and surroundings and time as shown in figure. The initial temperature of water is taken as 80°C. The value of t_2 as mentioned in the graph will be _______.



Official Ans. by NTA (16)

Sol. $T - T_0\ (T_i - T_0)\ e^{-\frac{Bt}{ms}}$

$6\lambda = \ln 1.5$

$40 = 60e^{-\lambda(6)} \Rightarrow 6\lambda = \ln 1.5$

$20 = 60e^{-\lambda t_2} \Rightarrow t_2\lambda = \ln 3$

$\frac{t_2}{6} = \frac{\ln 3}{\ln 1.5}$

$\therefore t_2 = 16.25$ min

So ≈ 16

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. 120 of an organic compound that contains only carbon and hydrogen gives 330g of CO_2 and 270g of water on complete combustion. The percentage of carbon and hydrogen, respectively are.

(A) 25 and 75 (B) 40 and 60

(C) 60 and 40 (D) 75 and 25

Official Ans. by NTA (D)

Sol. Given mass of organic compound = 120

mass of $CO_2(g)$ = 330 g

mass of H_2O (ℓ) = 270 g

mass of carbon = $n_{CO_2} \times 12$

$= \frac{330}{44} \times 12 = 90g$

% of carbon = $\frac{90}{120} \times 100 = 75\%$

mass of hydrogen = $n_{H_2O} \times 2$

$= \frac{270}{18} \times 2 = 30g$

% of hydrogen = $\frac{30}{120} \times 100 = 25\%$

2. The energy of one mole of photons of radiation of wavelength 300 nm is

(Given : h = 6.63 × 10^{-34} Js, N_A = 6.02 × 10^{23}mol^{-1}, c = 3 × 10^8 ms^{-1})

(A) 235 kJ mol^{-1} (B) 325 kJ mol^{-1}

(C) 399 kJ mol^{-1} (D) 435 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. Energy of one mole of photons = $\frac{hc}{\lambda} \times N_A$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} \times 6.02 \times 10^{23}$

= 399.13 × 10^3 Joule/mole

= 399 kJ / mole

3. The correct order of bound orders of C_2^{2-}, N_2^{2-} and O_2^{2-} is, respectively.

(A) $C_2^{2-} < N_2^{2-} < O_2^{2-}$ (B) $O_2^{2-} < N_2^{2-} < C_2^{2-}$

(C) $C_2^{2-} < O_2^{2-} < N_2^{2-}$ (D) $N_2^{2-} < C_2^{2-} < O_2^{2-}$

Official Ans. by NTA (B)

Sol.

Species	Bond order
C_2^{2-}	3
N_2^{2-}	2
O_2^{2-}	1

4. At 25°C and 1 atm pressure, the enthalpies of combustion are as given below:

Substance	H_2	C(graphite)	$C_2H_6(g)$
$\frac{\Delta_C H^\ominus}{kJmol^{-1}}$	–286.0	–394.0	–1560.0

The enthalpy of formation of ethane is

(A) +54.0 kJ mol^{-1} (B) –68.0 kJ mol^{-1}

(C) –86.0 kJ mol^{-1} (D) +97.0 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. $C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)$

$\Delta_C H(C_2H_6) = 2\Delta_f H\ CO_2(g) + 3\Delta_f H(H_2O,\ell)$

$- \Delta_f H(C_2H_6,g)$

$-1560 = 2(-394) + 3\ (-286) - \Delta_f H(C_2H_6,g)$

$\Delta_f H(C_2H_6,g) = -86$ kJ/mole

5. For a first order reaction, the time required for completion of 90% reaction is 'x' times the half life of the reaction. The value of 'x' is

(Given: ln 10 = 2.303 and log 2 = 0.3010)

(A) 1.12 (B) 2.43

(C) 3.32 (D) 33.31

Official Ans. by NTA (C)

Sol. Given $t_{0.90}$ = $t_{0.90} = xt_{1/2}$

First order rate constant

$$K = \frac{\ln 2}{t_{1/2}} = \frac{1}{xt_{1/2}} \ln \frac{A_0}{A_0 - A_0 \times \frac{90}{100}}$$

$$\frac{\ln 2}{t_{1/2}} = \frac{\ln 10}{xt_{1/2}}$$

$$x = \frac{\ln 10}{\ln 2} = \frac{2.303}{2.303 \times 0.3010} = 3.32$$

6. Metals generally melt at very high temperature. Amongst the following, the metal with the highest melting point will be

(A) Hg (B) Ag

(C) Ga (D) Cs

Official Ans. by NTA (B)

Sol. Hg, Ga, Cs are liquid near room temperature But Ag(silver) is solid.

7. Which of the following chemical reactions represents Hall-Heroult Process?

(A) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(B) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(C) $FeO + CO \rightarrow Fe + CO_2$

(D) $2\left[Au(CN)_2\right]^-_{(aq)} + Zn(s) \rightarrow 2Au(s) + \left[Zn(CN_4)\right]^{2-}$

Official Ans. by NTA (B)

Sol. Hall Heroult process is the major industrial process for extraction of aluminium.

8. In the industrial production of which of the following, molecular hydrogen is obtained as a byproduct?

(A) NaOH (B) NaCl

(C)Na metal (D) Na_2CO_3

Official Ans. by NTA (A)

Sol. Sodium hydroxide is generally prepared commercially by electrolysis of sodium chloride in castner Kellner cell.

at cathode : $Na + e^- \xrightarrow{Hg} Na-$ amalgum

$$\text{Anode : } Cl^- \longrightarrow \frac{1}{2}Cl_2 + e^-$$

The Na–amalgam is treated with water to give sodium hydroxide and hydrogen gas :

2Na (amalgam) + $H_2O \rightarrow 2NaOH + H_2 + 2Hg$

9. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?

(A) Baking Soda (B) Soda ash

(C) Washing Soda (D) Caustic Soda

Official Ans. by NTA (A)

Sol. Sodium hydrogencarbonate (Baking soda), $NaHCO_3$ is used in the fire extinguishers.

10. PCl_5 is well known. but NCl_5 is not. Because.

(A) nitrogen is less reactive than phosphorous.

(B) nitrogen doesn't have d-orbitals in its valence shell.

(C) catenation tendency is weaker in nitrogen than phosphorous.

(D) size of phosphorous is larger than nitrogen.

Official Ans. by NTA (B)

Sol. PCl_5 forms five bonds by using the d–orbitals to "expand the octet". But NCl_5 does not exist because there are no d–orbitals in the valence shell (2^{nd} shell). Therefore there is no way to expand the octet.

11. Transition metal complex with highest value of crystal field splitting (Δ_0) will be

(A) $\left[Cr(H_2O)_6\right]^{3+}$ (B) $\left[Mo(H_2O)_6\right]^{3+}$

(C) $\left[Fe(H_2O)_6\right]^{3+}$ (D) $\left[Os(H_2O)_6\right]^{3+}$

Official Ans. by NTA (D)

Sol. CFSE of octahedral complexes with water is greater for 5d series metal centre ion as compared to 3d and 4d series metal centre.

12. Some gases are responsible for heating of atmosphere (green house effect). Identify from the following the gaseous species which does not cause it.

(A) CH_4 (B) O_3

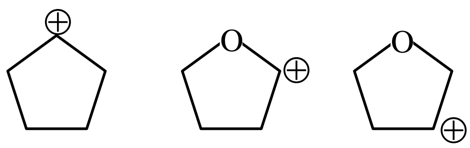
(C) H_2O (D) N_2

Official Ans. by NTA (D)

Sol. CH_4, O_3 and H_2O causes global warming in Tropospheric level.

N_2 does not cause global warming.

13. Arrange the following carbocations in decreasing order of stability.



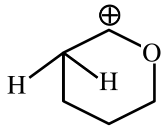
A B C

(A) A > C > B (B) A > B > C

(C) C > B > A (D) C > A > B

Official Ans. by NTA (B)

Sol. Carbocation is stabilised by resonance with lone pairs on oxygen atom and +H effect of 2a hydrogens



B > A > C

14. Given below are two statements.

Statement I : The presence of weaker π- bonds make alkenes less stable than alkanes.

Statement II : The strength of the double bond is greater than that of carbon-carbon single bond.

In the light of the above statements, choose the *correct* answer from the options given below.

(A) Both Statement I and Statement II are correct.

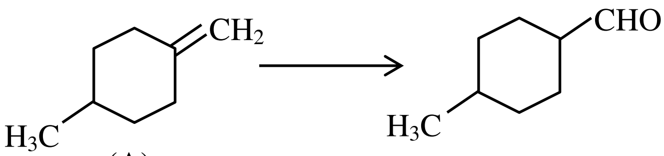
(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (A)

15. Which of the following reagents/ reactions will convert 'A' to 'B'?



(A) (B)

(A) PCC oxidation

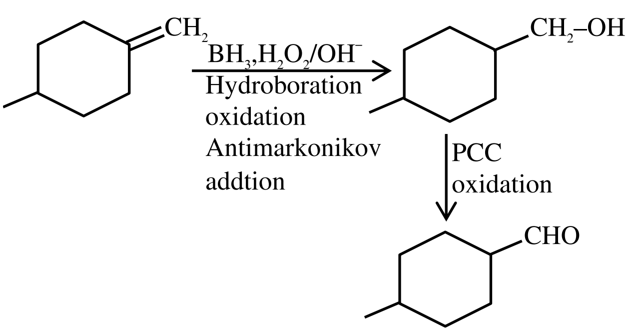
(B) Ozonolysis

(C) $BH_3, H_2O_2 / {}^-OH$ followed by PCC oxidation

(D) HBr, hydrolysis followed by oxidation by $K_2Cr_2O_7$.

Official Ans. by NTA (C)

Sol. BH_3, $H_2O_2/^-OH$ followed by PCC oxidation.



16. Hex-4-ene-2-ol on treatment with PCC gives 'A'. 'A' on reaction with sodium hypoiodite gives 'B', which on further heating with soda lime gives 'C'. The compound 'C' is

(A) 2- pentene (B) proponaldehyde

(C) 2-butene (D) 4-methylpent-2-ene

Official Ans. by NTA (C)

Sol.

$CH_3–CH=CH–CH_2–CH(OH)–CH_3$

↓ PCC

$CH_3–CH=CH–CH_2–C(=O)–CH_3$ (A)

↓ NaOI

$CH_3–CH=CH–CH_2–COOH + CHI_3$ (B)

↓ NaOH+CaO, $–CO_2$

$CH_3–CH=CH–CH_3$ (C)

But-2-ene

17. The conversion of propan-1-ol to n-butylamine involves the sequential addition of reagents. The correct sequential order of reagents is.

(A)(i) $SOCl_2$ (ii) KCN (iii) H_2/Ni,Na(Hg)/C_2H_5OH

(B) (i) HCl (ii) H_2/Ni, Na(Hg)/C_2H_5OH

(C) (i) $SOCl_2$ (ii) KCN (iii) CH_3NH_2

(D) (i) HCl (ii) CH_3NH_2

Official Ans. by NTA (A)

Sol.

$CH_3–CH_2–CH_2–OH$ (Propanol) ⟶ $CH_3–CH_2–CH_2–CH_2NH_2$ (n-Butanamine)

$CH_3–CH_2–CH_2–OH \xrightarrow{SOCl_2} CH_3–CH_2–CH_2–Cl \xrightarrow{KCN} CH_3–CH_2–CH_2–CN \xrightarrow{H_2/Ni,\ Hg/C_2H_5OH} CH_3–CH_2–CH_2–CH_2NH_2$

18. Which of the following is **not** an example of a condensation polymer?

(A) Nylon 6,6 (B) Decron

(C) Buna-N (D) Silicone

Official Ans. by NTA (C)

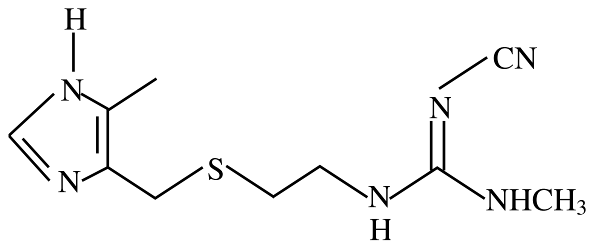
Sol. Buna-N is an addition copolymer of 1,3-butadiene and acrylonitrile.

$CH_2=CH–CH=CH_2 + CH_2=CH(CN)$

$[–CH_3–CH=CH–CH_2–CH_2–CH(CN)–]_n$

Buna-N

19. The structure shown below is of which well-known drug molecule?



(A) Ranitidine (B) Seldane

(C) Cimetidine (D) Codeine

Official Ans. by NTA (C)

20. In the flame test of a mixture of salts, a green flame with blue centre was observed. Which one of the following cations may be present?

(A) Cu^{2+} (B) Sr^{2+}

(C) Ba^{2+} (D) Ca^{2+}

Official Ans. by NTA (A)

Sol. Ion — Colour of the flame

(A) Cu^{+2} — green flame with blue centre

(B) Sr^{2+} — Crimson Red

(C) Ba^{2+} — Apple green

SECTION-B

1. At 300 K, a sample of 3.0 g of gas A occupies the same volume as 0.2 g of hydrogen at 200 K at the same pressure. The molar mass of gas A is____ g mol^{-1} (nearest integer) Assume that the behaviour of gases as ideal. (Given: The molar mass of hydrogen (H_2) gas is 2.0 g mol^{-1})

Official Ans. by NTA (45)

Sol. Given : Ideal gas A and H_2 gas at same pressure and volume.

From ideal gas equation pv = nRT

$n_1T_1 = n_2T_2$

$$\frac{3}{\text{GMM of A}} \times 300 = \frac{0.2}{2} \times 200$$

GMM of A = 45 g/mole

2. A company dissolves 'X' amount of CO_2 at 298 K in 1 litre of water to prepare soda water

X =____ $\times 10^{-3}$g. (nearest integer)

(Given: partial pressure of CO_2 at 298 K= 0.835 bar.

Henry's law constant for CO_2 at 298 K = 1.67 kbar.

Atomic mass of H,C and O is 1, 12 and 6 g mol^{-1}, respectively)

Official Ans. by NTA (1221 OR 1222)

Sol. From Henry law

$$P = K_H X_{CO_2}$$

$$0.835 = 1.67 \times 10^3 \times 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{w_{CO_2}}{44} + \frac{1000}{18}}$$

$$w_{CO_2} = 1.2228\text{g} = 1222.8 \times 10^{-3}\text{ g}$$

Or

$$P = K_H X_{CO_2}$$

$$0.835 = 1.67 \times 10^3 \times \frac{n_{CO_2}}{n_{CO_2} + n_{H_2O}}$$

$$0.835 = 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{1000}{18}}$$

$$w_{CO_2} = 1.2222\text{g} = 1222.2 \times 10^{-3}\text{g}$$

3. PCl_5 dissociates as

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

5 moles of PCl_5 are placed in a 200 litre vessel which contains 2 moles of N_2 and is maintained at 600 K. The equilibrium pressure is 2.46 atm. The equilibrium constant K_p for the dissociation of PCl_5 is_____ $\times 10^{-3}$. (nearest integer)

(Given: R = 0.082 L atm K^{-1} mol^{-1} : Assume ideal gas behaviour)

Official Ans. by NTA (1107)

Sol. Given : 2 mole of N_2 gas was present as inert gas.

Equilibrium pressure = 2.46 atm

$$PCl_5(g) \rightleftharpoons PCl_3(g) + C\ell_2(g)$$

	PCl_5	PCl_3	$C\ell_2$
t = 0	5	0	0
t = Eq^m	5 – x	x	x

from ideal gas equation

PV = nRT

$2.46 \times 200 = (5 - x + x + x + 2) \times 0.082 \times 600$

x = 3

$$K_P = \frac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}} \times \left[\frac{P_{total}}{n_{total}}\right]$$

$$\frac{3 \times 3}{2} \times \frac{2.46}{10} = 1.107 = 1107 \times 10^{-3}$$

4. The resistance of conductivity cell containing 0.01 M KCl solution at 298 K is 1750 Ω. If the conductively of 0.01 M KCl solution at 298 K is 0.152×10^{-3} S cm^{-1}, then the cell constant of the conductivity cell is_______$\times 10^{-3}$ cm^{-1}.

Official Ans. by NT

Sol. $K = \frac{1}{R} \times \text{cell constant}$

$0.152 \times 10^{-3} = \frac{1}{1750}$ cell constant

cell constant = 266×10^{-3}

5. When 200 mL of 0.2 M acetic acid is shaken with 0.6 g of wood charcoal, the final concentration of acetic after adsorption is 0.1 M. The mass of acetic acid adsorbed per garm of carbon is_____ g.

Official Ans. by NTA (2)

Sol. weight of wood charcoal = 0.6 g

Mass of acetic acid adsorbed = $\frac{M_1V_1 - M_2V_2}{1000} \times 60$

$= \frac{0.2 \times 200 - 0.1 \times 200}{1000} \times 60$

= 1.2 g

Mass of acetic acid adsorbed per gram of carbon = $\frac{1.2}{0.6} = 2$

6. (a) Baryte, (b) Galena, (c) Zinc blende and (d) Copper pyrites. How many of these minerals are sulphide based?

Official Ans. by NTA (3)

Sol.

(1) Baryte : $BaSO_4$

(2) Galena : PbS

(3) Zinc blende : ZnS

(4) Copper pyrite : $CuFeS_2$

(2)–(4): sulphide (S^{2-}) ores

7. Manganese (VI) has ability to disproportionate in acidic solution. The difference in oxidation states of two ions it forms in acidic solution is______

Official Ans. by NTA (3)

Sol. MnO_4^{2-} disproportionates in a neutral or acidic solution to give MnO_4^- and Mn^{+4}

$$3MnO_4^{2-} + 3H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

O.S. of Mn in MnO_4^- = +7

O.S. of Mn in MnO_2 = +4

difference = 3

8. 0.2 g of an organic compound was subjected to estimation of nitrogen by Dumas method in which volume of N_2 evolved (at STP) was found to be 22.400 mL. The percentage of nitrogen in the compound is____.[nearest integer]

(Given: Molar mass of N_2 is 28 mol^{-1}. Molar volume of N_2 at STP : 22.4 L)

Official Ans. by NTA (14)

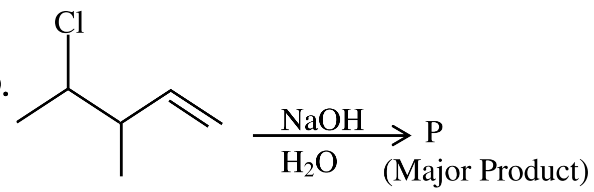
Sol. weight of organic compound = 0.2g

mass of N_2(g) evolved = $\frac{22.4 \times 10^{-3}}{22.4} \times 28$

$= 28 \times 10^{-3}$g

$\% \text{ of } N = \frac{28 \times 10^{-3}}{0.2} \times 100 = 14$

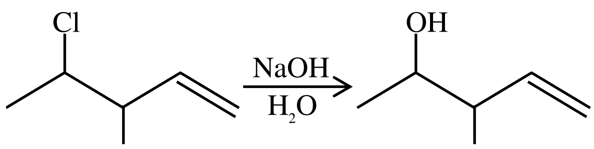
9.



Consider the above reaction. The number of π electrons present in the product 'P' is____.

Official Ans. by NTA (2)

Sol. Number of π electron = 2



10. In alanylglycylleucylalanylvaline, the number of peptide linkages is______.

Official Ans. by NTA (4)

Sol. There are Five amino acids and four peptide linkages.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $x*y = x^2 + y^3$ and $(x*1)*1 = x*(1*1)$.

Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (B)

Sol. $\because (x * 1) * 1 = x * (1 * 1)$

$(x^2 + 1) * 1 = x * (2)$

$(x^2 + 1)^2 + 1 = x^2 + 8$

$x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$

$x^2 = 2$

$\Rightarrow 2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) = 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$

2. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is

(A) $\log_e 3$ (B) $-\log_e 3$

(C) $\log_e 6$ (D) $-\log_e 6$

Official Ans. by NTA (B)

Sol. $(e^{2x} - 4)(6e^{2x} - 3e^x - 2e^x + 1) = 0$

$(e^{2x} - 4)(3e^x - 1)(2e^x - 1) = 0$

$e^{2x} = 4$ or $e^x = \frac{1}{3}$ or $e^x = \frac{1}{2}$

$\Rightarrow$ sum of real roots $= \frac{1}{2}\ell n4 + \ell n\frac{1}{3} + \ell n\frac{1}{2}$

$= -\ell n3$

3. Let the system of linear equations

$x + y + \alpha z = 2$

$3x + y + z = 4$

$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is :

(A) 4 (B) 3

(C) 2 (D) 1

Official Ans. by NTA (C)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$

$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$

$\alpha \neq -3$, $x = 1$, $y = 1$, $z = 0$,

Now points $(\alpha, 1)$, $(1, \alpha)$ & $(1, -1)$ are collinear

$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$

$\alpha^2 + \alpha - 1 - \alpha = 0$

$\alpha = \pm 1$

4. Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30 (B) 32

(C) 36 (D) 40

Official Ans. by NTA (D)

Sol. Using AM ≥ GM

$$\frac{x+x+x+y+y}{5} \geq \left(x^3.y^2\right)^{\frac{1}{5}}$$

$$\frac{3x+2y}{5} \geq \left(2^{15}\right)^{\frac{1}{5}}$$

$(3x + 2y)_{min} = 40$

5. Let $f(x)=\begin{cases} \dfrac{\sin(x-[x])}{x-[x]} & , \quad x\in(-2,-1) \\ \max\{2x,3[|x|]\} & , \quad |x|<1 \\ 1 & , \quad \text{otherwise} \end{cases}$

where [t] denotes greatest integer ≤ t. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is :

(A) (3, 3) (B) (2, 4)

(C) (2, 3) (D) (3, 4)

Official Ans. by NTA (C)

Sol. $f(x)=\begin{cases} \dfrac{\sin(x+2)}{x+2} & , \quad x\in(-2,-1) \\ \max\{2x,0\} & , \quad x\in(-1,1) \\ 1 & , \quad \text{otherwise} \end{cases}$

$f(-2^+) = \lim\limits_{h\to0} f(-2+h) = \lim\limits_{h\to0}\dfrac{\sinh}{h}=1$

f is continuous at x = –2

$f(-1^-) = \lim\limits_{h\to0}\dfrac{\sin(-1-h+2)}{(-1-h+2)}=\sin1$

$f(-1) = f(-1^+) = 0$

$f(1^+) = 1$ & $f(1^-) = 0 \Rightarrow f$ is not continuous at x = 1

f is continuous but not diff. at x = 0

$\left.\begin{array}{l}\Rightarrow \text{f is discontinuous at } x=-1 \ \& \ 1 \\ \ \& \text{ f is not diff. at } x=-1, 0 \ \& \ 1\end{array}\right\} \Rightarrow \begin{array}{l} m=2 \\ n=3\end{array}$

6. The value of the integral $\displaystyle\int_{-\pi/2}^{\pi/2}\frac{dx}{\left(1+e^x\right)\left(\sin^6x+\cos^6x\right)}$ is equal to

(A) 2π (B) 0

(C) π (D) $\dfrac{\pi}{2}$

Official Ans. by NTA (C)

Sol. $\displaystyle I=\int_{-\pi/2}^{0}\frac{dx}{\left(1+e^x\right)\left(\sin^6x+\cos^6x\right)}+\int_{0}^{\pi/2}\frac{dx}{\left(1+e^x\right)\left(\sin^6x+\cos^6x\right)}$

Put x = –t

$$=\int_{\pi/2}^{0}\frac{-dt}{\left(1+e^{-t}\right)\left(\sin^6t+\cos^6t\right)}+\int_{0}^{\pi/2}\frac{dx}{\left(1+e^x\right)\left(\sin^6x+\cos^6x\right)}$$

$$=\int_{0}^{\pi/2}\frac{\left(e^x+1\right)dx}{\left(1+e^x\right)\left(\sin^6x+\cos^6x\right)}$$

$$=\int_{0}^{\pi/2}\frac{dx}{\left(\sin^2x+\cos^2x\right)\left(\sin^4x-\sin^2x\cos^2x+\cos^4x\right)}$$

$$=\int_{0}^{\pi/2}\frac{\left(1+\tan^2x\right)\sec^2x\,dx}{\left(\tan^4x-\tan^2x+1\right)}$$

Put tanx = t

$$=\int_{0}^{\infty}\frac{\left(1+t^2\right)dt}{\left(t^4-t^2+1\right)}$$

$$=\int_{0}^{\infty}\frac{\left(1+\frac{1}{t^2}\right)dt}{t^2-1+\frac{1}{t^2}}=\int_{0}^{\infty}\frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+1}$$

Put $t-\dfrac{1}{t}=z$

$\left(1+\dfrac{1}{t^2}\right)dt=dz$

$$=\int_{-\infty}^{\infty}\frac{dz}{1+z^2}=\left(\tan^{-1}z\right)_{-\infty}^{\infty}$$

$$=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi$$

7. $\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\dots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$

is equal to

(A) $\frac{\pi}{8}+\frac{1}{4}\log_e 2$ (B) $\frac{\pi}{4}+\frac{1}{8}\log_e 2$

(C) $\frac{\pi}{4}-\frac{1}{8}\log_e 2$ (D) $\frac{\pi}{8}+\log_e \sqrt{2}$

Official Ans. by NTA (A)

Sol. $\lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}\right)$

$=\lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{1}{n\left(1+\left(\frac{r}{n}\right)^2\right)\left(1+\left(\frac{r}{n}\right)\right)}\right)$

$=\int_0^1\frac{dx}{(1+x^2)(1+x)}=\frac{1}{2}\int_0^1\frac{1-x}{1+x^2}dx+\frac{1}{2}\int_0^1\frac{1}{1+x}dx$

$=\frac{1}{2}\int\left(\frac{1}{1+x^2}-\frac{x}{1+x^2}\right)dx+\frac{1}{2}\left(\ln(1+x)\right)_0^1$

$=\frac{1}{2}\left[\tan^{-1}x-\frac{1}{2}\ell n\left(1+x^2\right)\right]_0^1+\frac{1}{2}\ell n2$

$=\frac{1}{2}\left[\frac{\pi}{4}-\frac{1}{2}\ell n2\right]+\frac{1}{2}\ell n2$

$=\frac{\pi}{8}+\frac{1}{4}\ell n\ 2$

8. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with

(A) length of latus rectum 3

(B) length of latus rectum 6

(C) focus $\left(\frac{4}{3},0\right)$

(D) focus $\left(0,\frac{3}{4}\right)$

Official Ans. by NTA (A)

Sol. Let Point P(x,y)

$Y-y=y'(X-x)$

$Y=0\Rightarrow X=x-\frac{y}{y'}$

$Q\left(x-\frac{y}{y'},0\right)$

Mid Point of PQ lies on y axis

$x-\frac{y}{y'}+x=0$

$y'=\frac{y}{2.x}\Rightarrow 2\frac{dy}{y}=\frac{dx}{x}$

$2\ell ny=\ell nx+\ell nk$

$y^2=kx$

It passes through (3, 3) $\Rightarrow$ k = 3

curve c $\Rightarrow y^2=3x$

Length of L.R. = 3

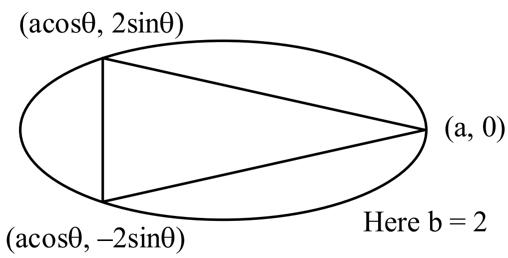
Focus = $\left(\frac{3}{4},0\right)$ **Ans. (A)**

9. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1$, a > 2, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{4}$

Official Ans. by NTA (A)

Sol.



$A=\frac{1}{2}a(1-\cos\theta)(4\sin\theta)$

A = 2a(1–cosθ) sinθ

$$\frac{dA}{d\theta} = 2a\left(\sin^2\theta + \cos\theta - \cos^2\theta\right)$$

$$\frac{dA}{d\theta} = 0 \Rightarrow 1 + \cos\theta - 2\cos^2\theta = 0$$

cosθ = 1 (Reject)

OR

$$\cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

$$\frac{d^2A}{d\theta^2} = 2a\left(2\sin^2\theta - \sin\theta\right)$$

$$\frac{d^2A}{d\theta^2} < 0 \text{ for } \theta = \frac{2\pi}{3}$$

$$\text{Now, } A_{max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3}$$

$\boxed{a = 4}$

$$\text{Now, } e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{3}}{2}$$ **Ans. (A)**

10. Let the area of the triangle with vertices A(1, α), B(α, 0) and C(0, α) be 4 sq. units. If the point (α, –α), (–α, α) and (α^2, β) are collinear, then β is equal to

(A) 64 (B) –8

(C) –64 (D) 512

Official Ans. by NTA (C)

Sol. $$\frac{1}{2}\begin{vmatrix} \alpha & 0 & 1 \\ 1 & \alpha & 1 \\ 0 & \alpha & 1 \end{vmatrix} = \pm 4$$

α = ± 8

Now given points (8, –8), (–8, 8), (64, β)

OR (–8, 8), (8, –8), (64, β)

are collinear ⇒ Slope = –1.

$\boxed{\beta = -64}$ **Ans. (C)**

11. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(A) 5 (B) 7 (C) 1 (D) 3

Official Ans. by NTA (D)

Sol. $x^7 - 7x - 2 = 0$

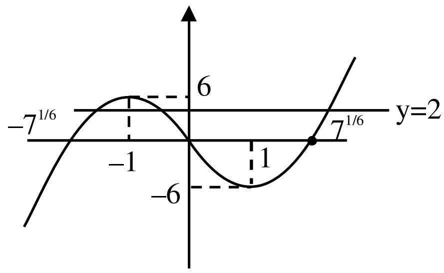
$x^7 - 7x = 2$

f(x) = $x^7 - 7x$ (odd) & y = 2

f(x) = x ($x^2 - 7^{1/3}$) ($x^4 + x^2 \cdot 7^{1/3} + 7^{2/3}$)

f′(x) = 7(x^6 – 1) = 7 (x^2 – 1) ($x^4 + x^2 + 1$)

f′(x) = 0 ⇒ x = ± 1



f(x) = 2 has 3 real distinct solution.

12. A random variable X has the following probability distribution :

X	0	1	2	3	4
P(X)	k	2k	4k	6k	86

The value of P(1 < X < 4 | X ≤ 2) is equal to :

(A) $\frac{4}{7}$ (B) $\frac{2}{3}$

(C) $\frac{3}{7}$ (D) $\frac{4}{5}$

Official Ans. by NTA (A)

Sol. $$P\left(\frac{1<x<4}{x\le 2}\right) = \frac{P(1<x<4 \cap x\le 2)}{P(x\le 2)}$$

$$= \frac{P(1<x\le 2)}{P(x\le 2)} = \frac{P(x=2)}{P(x\le 2)}$$

$$= \frac{4k}{k+2k+4k} = \frac{4}{7}$$

13. The number of solutions of the equation $\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$, $x \in [-3\pi, 3\pi]$ is :

(A) 8 (B) 5

(C) 6 (D) 7

Official Ans. by NTA (D)

Sol. $\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$

$x\in[-3\pi,3\pi]$

$4\left(\cos^2\left(\frac{\pi}{3}\right)-\sin^2 x\right)=\cos^2 2x$

$4\left(\frac{1}{4}-\sin^2 x\right)=\cos^2 2x$

$1-4\sin^2 x=\cos^2 2x$

$1-2(1-\cos 2x)=\cos^2 2x$

let $\cos 2x = t$

$-1+2\cos 2x=\cos^2 2x$

$t^2-2t+1=0$

$(t-1)^2=0$

$\boxed{t=1}$ $\boxed{\cos 2x=1}$

$2x = 2n\pi$

$\boxed{x=n\pi}$

$n = -3, -2, -1, 0, 1, 2, 3$

(D) option is correct.

14. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

(A) 16 (B) 6

(C) 12 (D) 15

Official Ans. by NTA (A)

Sol. SHORTEST distance $\frac{\left|(a_2-a_1)\cdot(b_1\times b_2)\right|}{\left|b_1\times b_2\right|}$

$a_1 = (1, 2, 3)$

$a_2 = (2, 4, 5)$

$\vec{b}_2=2\hat{i}+3\hat{j}+\lambda\hat{k}$

$\vec{b}_2=\hat{i}+4\hat{j}+5\hat{k}$

$$\text{S.D.}=\frac{\left|\left((2-1)\hat{i}+(4-2)\hat{j}+(5-3)\hat{k}\right)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{|b_1\times b_2|}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & \lambda\\ 1 & 4 & 5\end{vmatrix}$$

$=\hat{i}(15-4\lambda)+\hat{j}(\lambda-10)+\hat{k}(5)$

$=(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}$

$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}$

Now

$$\text{S.D.}=\frac{\left|\left(\hat{i}+2\hat{j}+2\hat{k}\right)\cdot\left[(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}\right]\right|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}$$

$$\frac{|15-4\lambda+2\lambda-20+10|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}=\frac{1}{\sqrt{3}}$$

square both side

$3(5-2\lambda)^2=225+16\lambda^2-120\lambda+\lambda^2+100-20\lambda+25$

$12\lambda^2+75-60\lambda=17\lambda^2-140\lambda+350$

$5\lambda^2-80\lambda+275=0$

$\lambda^2-16\lambda+55=0$

$(\lambda-5)(\lambda-11)=0$

$\Rightarrow \lambda = 5, 11$

(A) is correct option.

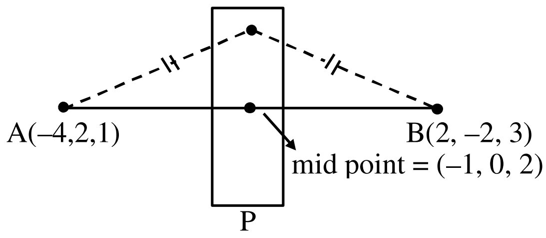
15. Let the points on the plane P be equidistant from the points (–4, 2, 1) and (2, –2, 3). Then the acute angle between the plane P and the plane 2x + y + 3z = 1 is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Official Ans. by NTA (C)

Sol.



Normal vector = $\overrightarrow{AB} = \left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

$= \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right)$

or $2\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$

$P \equiv 3(x + 1) - 2(y) + 1\ (z - 2) = 0$

$P \equiv 3x - 2y + z + 1 = 0$

$P' \equiv 2x + y + 3z - 1 = 0$

angle between P & P' = $\left|\dfrac{\hat{n}_1 \cdot \hat{n}_2}{|n_1||n_2|}\right| = \cos\theta$

$$\theta = \cos^{-1}\left(\frac{6 - 2 + 3}{\sqrt{14} \times \sqrt{14}}\right)$$

$$\theta = \cos^{-1}\left(\frac{7}{14}\right) = = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Option C is correct.

16. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :

(S1) : $2\left|\hat{a} \times \hat{b}\right| = \left|\hat{a} - \hat{b}\right|$

(S2) : The projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$ is $\dfrac{1}{2}$

(A) Only (S1) is true

(B) Only (S2) is true

(C) Both (S1) and (S2) are true

(D) Both (S1) and (S2) are false

Official Ans. by NTA (C)

Sol. $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2,\ \theta \in (0, \pi)$

$\left(\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right).\left(\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right) = 4$

$\left|\hat{a} + \hat{b}\right|^2 + 4\left|\left(\hat{a} \times \hat{b}\right)\right|^2 + 0 = 4$

Let the angle be θ between $\hat{a}$ and $\hat{b}$

$2 + 2\cos\theta + 4\sin^2\theta = 4$

$2 + 2\cos\theta - 4\cos^2\theta = 0$

Let $\cos\theta = t$ then

$2t^2 - t - 1 = 0$

$2t^2 - 2t + t - 1 = 0$

$2t(t - 1) + (t - 1) = 0$

$(2t + 1)(t - 1) = 0$

$t = -\dfrac{1}{2}$ or $t = 1$

$\cos\theta = -\dfrac{1}{2}$ | not possible as $\theta \in (0, \pi)$

$\boxed{\theta = \dfrac{2\pi}{3}}$

Now,

S_1 $2\left|\vec{a} \times \vec{b}\right| = 2\sin\left(\dfrac{2\pi}{3}\right)$

$\left|\hat{a} - \hat{b}\right| = \sqrt{1 + 1 - 2\cos\left(\dfrac{2\pi}{3}\right)}$

$= \sqrt{2 - 2 \times \left(-\dfrac{1}{2}\right)}$

$= \sqrt{3}$

S_1 is correct.

S_2 projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$.

$$\frac{\hat{a}.\left(\hat{a} + \hat{b}\right)}{\left|\hat{a} + \hat{b}\right|} = \frac{1 + \cos\left(\frac{2\pi}{3}\right)}{\sqrt{2 + 2\cos\frac{2\pi}{3}}}$$

$$= \frac{1 - \frac{1}{2}}{\sqrt{1}}$$

$$= \frac{1}{2}$$

C Option is true.

17. If $y = \tan^{-1}(\sec x^3 - \tan x^3)$. $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then

(A) $xy'' + 2y' = 0$

(B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

(C) $x^2y'' - 6y + 3\pi = 0$

(D) $xy'' - 4y' = 0$

Official Ans. by NTA (B)

Sol. $y = \tan^{-1}(\sec x^3 - \tan x^3)$

$= \tan^{-1}\left(\frac{1-\sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x^3\right)}{\sin\left(\frac{\pi}{2}-x^3\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x^3}{2}\right)\right)$

Since $\frac{\pi}{4}-\frac{x^3}{2} \in \left(-\frac{\pi}{2},0\right)$

$y = \left(\frac{\pi}{4}-\frac{x^3}{2}\right)$

$y' = \frac{-3x^2}{2}$, $y'' = -3x$

$4y = \pi - 2x^3$

$4y = \pi - 2x^2\left(\frac{-y''}{3}\right)$

$12y = 3\pi + 2x^2y''$

$x^2y'' - 6y + \frac{3\pi}{2} = 0$

18. Consider the following statements :

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is

(A) $B \to (A \vee C)$

(B) $(\sim B) \wedge (A \wedge C)$

(C) $B \to ((\sim A) \vee (\sim C))$

(D) $B \to (A \wedge C)$

Official Ans. by NTA (B)

Sol. $\sim((A \wedge C) \to B)$

$\sim(\sim(A \wedge C) \vee B)$

Using De-Morgan's law

$(A \wedge C) \wedge (\sim B)$

Option B is correct.

19. The slope of normal at any point (x, y), x > 0, y > 0 on the curve y = y(x) is given by $\frac{x^2}{xy - x^2y^2 - 1}$.

If the curve passes through the point (1, 1), then e.y(e) is equal to

(A) $\frac{1-\tan(1)}{1+\tan(1)}$ (B) $\tan(1)$

(C) 1 (D) $\frac{1+\tan(1)}{1-\tan(1)}$

Official Ans. by NTA (D)

Sol. Slope of normal $= \frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$x^2y^2dx + dx - xydx = x^2dy$

$x^2y^2dx + dx = x^2dy + xydx$

$x^2y^2dx + dx = x(xdy + ydx)$

$x^2y^2dx + dx = xd(xy)$

$\frac{dx}{x} = \frac{d(xy)}{1+x^2y^2}$

$\ln kx = \tan^{-1}(xy)$ … (i)

passes though (1, 1)

$\ln k = \frac{\pi}{4} \Rightarrow k = e^{\frac{\pi}{4}}$

equation (i) be becomes

$\frac{\pi}{4} + \ln x = \tan^{-1}(xy)$

$xy = \tan\left(\frac{\pi}{4} + \ell n\, x\right)$

$xy = \left(\frac{1+\tan(\ell n\, x)}{1-\tan(\ell n\, x)}\right)$ … (ii)

put x = e in (ii)

$\therefore$ ey (e) = $\frac{1+\tan 1}{1-\tan 1}$

20. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in R$. Then $f_{\lambda}*(1) + f_{\lambda}*(-1)$ is equal to :

(A) 36 (B) 48

(C) 64 (D) 72

Official Ans. by NTA (D)

Sol. $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$f_\lambda'(x) = 12\lambda x^2 - 72\lambda x + 36$

$f_\lambda'(x) = 12(\lambda x^2 - 6\lambda x + 3) \geq 0$

$\therefore \lambda > 0$ & $D \leq 0$

$36\lambda^2 - 4 \times \lambda \times 3 \leq 0$

$9\lambda^2 - 3\lambda \leq 0$

$3\lambda(3\lambda - 1) \leq 0$

$\lambda \in \left[0, \frac{1}{3}\right]$

$\therefore \lambda_{largest} = \frac{1}{3}$

$f(x) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$

$\therefore f(1) + f(1) = 72$

SECTION-B

1. Let $S = \{z \in \mathbb{C} : |z-3| \leq 1 \text{ and } z(4+3i) + \bar{z}(4-3i) \leq 24\}$. If $\alpha + i\beta$ is the point in S which is closest to 4i, then $25(\alpha + \beta)$ is equal to _____.

Official Ans. by NTA (80)

Sol. $|z - 3| \leq 1$

represent pt. i/s circle of radius 1 & centred at (3, 0)

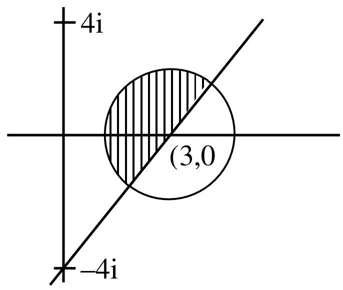
$z(4 + 3i) + \bar{z}(4 - 3i) \leq 24$

$(x + iy)(4 + 3i) + (x - iy)(4 - 3i) \leq 24$

$4x + 3xi + 4iy - 3y + 4x - 3ix - 4iy - 3y \leq 24$

$8x - 6y \leq 24$

$4x - 3y \leq 12$



minimum of (0, 4) from circle = $\sqrt{3^2+4^2} - 1 = 4$

will lie along line joining (0, 4) & (3, 0)

$\therefore$ equation line

$\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12$ … (i)

equation circle $(x - 3)^2 + y^2 = 1$ … (ii)

$\left(\frac{12-3y}{4} - 3\right)^2 + y^2 = 1$

$\left(\frac{-3y}{4}\right)^2 + y^2 = 1$

$\frac{25y^2}{16} = 1 \Rightarrow y = \pm \frac{4}{5}$

for minimum distance $y = \frac{4}{5}$

$\therefore x = \frac{12}{5}$

$\therefore 25(\alpha + \beta) = 25\left(\frac{4}{5} + \frac{12}{5}\right)$

$= 16 \times 5 = 80$

2. Let $S = \left\{\begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, ... 100\}\right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

Official Ans. by NTA (100)

Sol. $A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}\begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} 1 & -a + ab \\ 0 & b^2 \end{bmatrix}$

$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$

$\therefore$ b must be equal to 1

$\therefore$ In this case A^2 will become identity matrix and a can take any value from 1 to 100

$\therefore$ Total number of common element will be 100.

3. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____.

Official Ans. by NTA (576)

Sol. Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 $\to$ Difference of sum at even & odd place is divisible by 11.

Let number of the form abcdefg

$\therefore (a + c + e + g) - (b + d + f) = 11x$

$a + b + c + d + e + f = 31$

$\therefore$ either $a + c + e + g = 21$ or 10

$\therefore b + d + f = 10$ or 21

Case- 1

$a + c + e + g = 21$

$b + d + f = 10$

$(b, d, f) \in \{(1, 2, 7)\ (2, 3, 5)\ (1, 4, 5)\}$

$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$

$\therefore$ Total number in case-1 = $(3! \times 3)(4!) = 432$

Case- 2

$a + c + e + g = 10$

$b + d + f = 21$

$(a, b, e, g) \in \{1, 2, 3, 4)\}$

(b, d, f) & $\{(5, 7, 9)\}$

$\therefore$ Total number in case 2 = $3! \times 4! = 144$

$\therefore$ Total numbers = 144 + 432 = 576

4. The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\} : \text{HCF}(\alpha, 24) = 1\}$ is _____.

Official Ans. by NTA (1633)

Sol. HCF $(\alpha, 24) = 1$

Now, $24 = 2^2.3$

$\to \alpha$ is not the multiple of 2 or 3

Sum of values of α

= S(U) –{S(multiple of 2) + S (multiple of 3) – S(multiple of 6)}

= (1 + 2 + 3 + ... 100) – (2 + 4 + 6 ..... + 100) – (3 + 6 + ... 99) + (6 + 12 + ... + 96)

$= \frac{100 \times 101}{2} - 50 \times 51 - \frac{33}{2} \times (3 + 99) + \frac{16}{2}(6 + 96)$

= 5050 – 2550 – 1683 + 816 = 1633 **Ans.**

5. The remainder on dividing $1 + 3 + 3^2 + 3^3 + ... + 3^{2021}$ by 50 is _____.

Official Ans. by NTA (4)

Sol. $\frac{1.(3^{2022}-1)}{2} = \frac{9^{1011}-1}{2}$

$= \frac{(10-1)^{1011}-1}{2}$

$= \frac{100\lambda + 10110 - 1 - 1}{2}$

$= 50\lambda + \frac{10108}{2}$

$= 50\lambda + 5054$

$= 50\lambda + 50 \times 101 + 4$

Rem (50) = 4.

6. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _____.

Official Ans. by NTA (18)

Sol. $x = 4 - y$

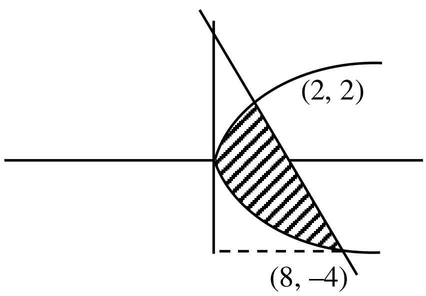
$y^2 = 2(4 - y)$

$y^2 = 8 - 2y$

$y^2 + 2y - 8 = 0$

$y = -4, y = 2$

$x = 8, x = 2$



$\int_{-4}^{2}\left[(4-y)-\frac{y^2}{2}\right]dy$

$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$

$= 8 - 2 - \frac{8}{6} + 16 + \frac{16}{2} - \frac{64}{6}$

$= 22 + 8 - \frac{72}{6}$

$= 30 - 12 = 18$

7. Let a circle C : $(x-h)^2 + (y-k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

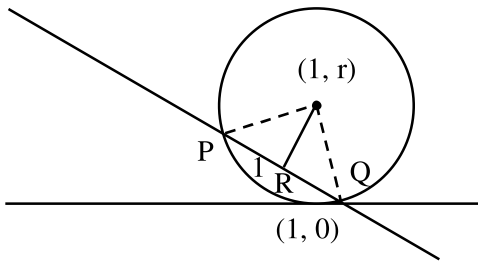
Official Ans. by NTA (7)

Sol. $k = r$

$h = 1$

$OP = r, PR = 1$

$OR = \left|\frac{r+1}{\sqrt{2}}\right|$



$r^2 = 1 + \frac{(r+1)^2}{2}$

$2r^2 = 2 + r^2 + 1 + 2r$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$\boxed{r = 3}, -1$

$h + k + r = 1 + 3 + 3$

$= 7$

8. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to _____.

Official Ans. by NTA (479)

Sol. A = {1, 2, 3, 4} : P(A) = $\frac{3}{4}$ → Correct

B = { 5, 6, 7, 8, 9, 10} ; P(B) = $\frac{1}{4}$ Correct

8 Correct Ans.:

(4, 4): $^4C_4\left(\frac{3}{4}\right)^4 . ^6C_4 . \left(\frac{1}{4}\right)^4 . \left(\frac{3}{4}\right)^2$

(3, 5): $^4C_3\left(\frac{3}{4}\right)^3 . \left(\frac{1}{4}\right)^1 . ^6C_5\left(\frac{1}{4}\right)^5 . \left(\frac{3}{4}\right)$

(2, 6): $^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 . ^6C_6\left(\frac{1}{4}\right)^6$

Total $= \frac{1}{4^{10}}[3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2]$

$= \frac{27}{4^{10}}[2.7 \times 15 + 72 + 2]$

$\Rightarrow K = 479$

9. Let the hyperbola H : $\frac{x^2}{a^2} - y^2 = 1$ and the ellipse E : $3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2 + e_E^2\right)$ is equal to _____.

Official Ans. by NTA (42)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{1} = 1$ $\qquad$ $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$e_H = \sqrt{1 + \frac{1}{a^2}}$ $\qquad$ $e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\ell.R. = \frac{2}{a}$ $\qquad$ $\ell R = \frac{2 \times 3}{2} = 3$

$\frac{2}{a} = 3$

$\boxed{a = \frac{2}{3}}$

$e_H = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

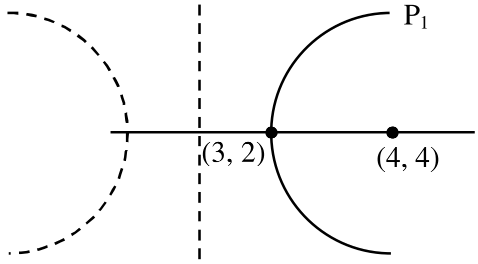
$12\left(e_H^2 + e_E^2\right) = 12\left(\frac{13}{4} + \frac{1}{4}\right)$

$= \frac{12 \times 14}{4} = 42$

10. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line x + 2y = 6. Then the directrix of P_2 is x + 2y = _____.

Official Ans. by NTA (10)

Sol.



P_1: Directorix :

$x + 2y = k$

$x + 2y - k = 0$

$\left|\frac{3 + 4 - K}{\sqrt{5}}\right| = \sqrt{5}$

$|7 - k| = 5$

$7 - K = 5 \qquad 7 - K = -5$

$\boxed{k = 2} \qquad \boxed{k = 12}$

Accepted $\qquad$ Rejected

Passes through focus

$D_1 = x + 2y = 2$] $\Rightarrow d$

$\ell = x + 2y = 6$ $\Rightarrow$ $\boxed{c = 10}$

$D_2 = x + 2y = C$] $\Rightarrow d$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If $Z=\frac{A^2B^3}{C^4}$, then the relative error in Z will be :

(A) $\frac{\Delta A}{A}+\frac{\Delta B}{B}+\frac{\Delta C}{C}$

(B) $\frac{2\Delta A}{A}+\frac{3\Delta B}{B}-\frac{4\Delta C}{C}$

(C) $\frac{2\Delta A}{A}+\frac{3\Delta B}{B}+\frac{4\Delta C}{C}$

(D) $\frac{\Delta A}{A}+\frac{\Delta B}{B}-\frac{\Delta C}{C}$

Official Ans. by NTA (C)

Sol. $Z=\frac{A^2B^3}{C^4}$

In case of error

$$\frac{dZ}{Z}=\frac{2dA}{A}+\frac{3dB}{B}+\frac{4dC}{C}$$

$$\boxed{\frac{\Delta Z}{Z}=\frac{2\Delta A}{A}+\frac{3\Delta B}{B}+\frac{4\Delta C}{C}}$$

2. $\vec{A}$ is a vector quantity such that $|\vec{A}|=$ non-zero constant. Which of the following expressions is true for $\vec{A}$?

(A) $\vec{A}.\vec{A}=0$

(B) $\vec{A}\times\vec{A}<0$

(C) $\vec{A}\times\vec{A}=0$

(D) $\vec{A}\times\vec{A}>0$

Official Ans. by NTA (C)

Sol. $|\vec{A}|\neq 0$

$$\vec{A}\times\vec{A}=|\vec{A}||\vec{A}|\sin 0^\circ\hat{n}=0$$

3. Which of the following relations is true for two unit vectors $\hat{A}$ and $\hat{B}$ making an angle θ to each other?

(A) $|\hat{A}+\hat{B}|=|\hat{A}-\hat{B}|\tan\frac{\theta}{2}$

(B) $|\hat{A}-\hat{B}|=|\hat{A}+\hat{B}|\tan\frac{\theta}{2}$

(C) $|\hat{A}+\hat{B}|=|\hat{A}-\hat{B}|\cos\frac{\theta}{2}$

(D) $|\hat{A}-\hat{B}|=|\hat{A}+\hat{B}|\cos\frac{\theta}{2}$

Official Ans. by NTA (B)

Sol. $|\hat{A}+\hat{B}|=\sqrt{|\hat{A}|^2+|\hat{B}|^2+2|\hat{A}||\hat{B}|\cos\theta}$

$$=\sqrt{1+1+2\cos\theta}$$

$$=\sqrt{2(1+\cos\theta)}$$

$$=\sqrt{2\times 2\cos^2\frac{\theta}{2}}$$

$$=2\cos\frac{\theta}{2}$$

$|\hat{A}-\hat{B}|=\sqrt{|\hat{A}|^2+|\hat{B}|^2-2|\hat{A}||\hat{B}|\cos\theta}$

$$=\sqrt{2-2\cos\theta}$$

$$=2\sin\frac{\theta}{2}$$

$$\frac{|\hat{A}+\hat{B}|}{|\hat{A}-\hat{B}|}=\cot\frac{\theta}{2}$$

4. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be :-

(A) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(B) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(C) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(D) $10\hat{i} + \hat{j} - 5\hat{k}$

Official Ans. by NTA (B)

Sol. $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-8) - \hat{j}(-4-6) + \hat{k}(8-3)$$

$$= -10\hat{i} + 10\hat{j} + 5\hat{k}$$

5. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be : (Given g = acceleration due to gravity at the surface of earth)

(A) g/2

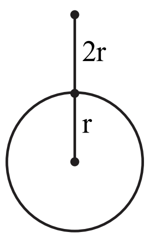
(B) g/4

(C) g/3

(D) g/9

Official Ans. by NTA (D)

Sol. $g = \dfrac{Gm}{r^2}$



$$g' = \frac{Gm}{(3r)^2}$$

$$g' = \frac{Gm}{9r^2}$$

$$g' = \frac{g}{9}$$

6. The terminal velocity (v_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as:-

(A) $r^{1/2}$ (B) r

(C) r^2 (D) r^3

Official Ans. by NTA (C)

Sol. $v_t = \dfrac{2}{9}\dfrac{gr^2(\rho_p - \rho_l)}{\eta}$; $v_t \propto r^2$

7. The relation between root mean square speed (v_{rms}) and most probable speed (v_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be :-

(A) $v_{rms} = \sqrt{\dfrac{2}{3}}v_p$ (B) $v_{rms} = \sqrt{\dfrac{3}{2}}v_p$

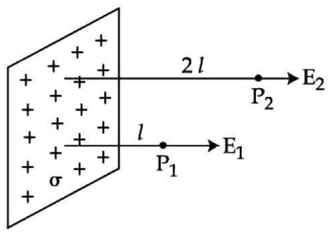
(C) $v_{rms} = v_p$ (D) $v_{rms} = \sqrt{\dfrac{1}{3}}v_p$

Official Ans. by NTA (B)

Sol. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ and $v_{mp} = \sqrt{\dfrac{2RT}{M}}$

Thus $v_{rms} = \sqrt{\dfrac{3}{2}}v_{mp}$

8. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are :



(A) $E_1 = \sigma/\varepsilon_0, E_2 = \sigma/2\varepsilon_0$

(B) $E_1 = 2\sigma/\varepsilon_0, E_2 = \sigma/\varepsilon_0$

(C) $E_1 = E_2 = \sigma/2\varepsilon_0$

(D) $E_1 = E_2 = \sigma/\varepsilon_0$

Official Ans. by NTA (C)

Sol. As the sheet is very large $\vec{E}$ is independent of distance from it.

Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

9. Match List-I with List-II

List-I	List-II
(A) AC generator	(I) Detects the presence of current in the circuit
(B) Galvanometer	(II) Converts mechanical energy into electrical energy
(C) Transformer	(III) Works on the principle of resonance in AC circuit
(D) Metal detector	(IV) Changes an alternating voltage for smaller or greater value

Choose the **correct answer** from the options given below :-

(A) (A)–(II), B–(I), (C)–(IV), (D)–(III)

(B) (A)–(II), B–(I), (C)–(III), (D)–(IV)

(C) (A)–(III), B–(IV), (C)–(II), (D)–(I)

(D) (A)–(III), B–(I), (C)–(II), (D)–(IV)

Official Ans. by NTA (A)

Sol. AC generator converts mechanical energy into electrical energy. Galvanometer shows deflection when current passes through it so it is used to show presence of current in any wire.

Transformer is used to step up or step down the voltage. Metals detectors contain inductor coils and use principle of induction and resonance in AC circuit.

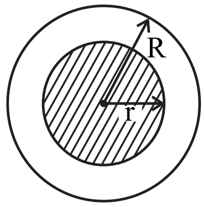
10. A long straight wire with a circular cross-section having radius R, is carrying a steady current I. The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r (r < R) from its centre will be :-

(A) $B \propto r^2$ (B) $B \propto r$

(C) $B \propto \frac{1}{r^2}$ (D) $B \propto \frac{1}{r}$

Official Ans. by NTA (B)

Sol. Use Ampere's law



$$B.2\pi r = \mu_0 . \frac{I}{\pi R^2} . \pi r^2$$

Thus $B \propto r$

11. If wattless current flows in the AC circuit, then the circuit is

(A) Purely Resistive circuit

(B) Purely Inductive circuit

(C) LCR series circuit

(D) RC series circuit only

Official Ans. by NTA (B)

Sol. Purely Inductive circuit

$$\theta = \frac{\pi}{2}$$

$$\cos\frac{\pi}{2} = 0$$

Average power = 0

12. The electric field in an electromagnetic wave is given by $E = 56.5 \sin \omega(t - x/c)\, NC^{-1}$. Find the intensity of the wave if it is propagating along x-axis in the free space. (Given $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$)

(A) 5.65 Wm^{-2} (B) 4.24 Wm^{-2}

(C) 1.9 × 10^{-7} Wm^{-2} (D) 56.5 Wm^{-2}

Official Ans. by NTA (B)

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$$I = \frac{1}{2} \times (8.85 \times 10^{-12})(56.5)^2 \times (3 \times 10^8)$$

= 4.24 Wm^{-2}.

13. The two light beams having intensities I and 9I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be :

(A) 2 I (B) 6 I

(C) 5 I (D) 7 I

Official Ans. by NTA (B)

Sol. $I_P = I + 9I + 2\sqrt{I \times 9I}\cos\frac{\pi}{2}$

$I_P = 10\text{ I}$

$I_Q = I + 9I + 2\sqrt{I \times 9I}\cos\pi$

$= 10\text{ I} - 6\text{I} = 4\text{I}$

$\therefore I_P - I_Q = 10I - 4I = 6I$

14. A light wave travelling linearly in a medium of dielectric constant 4, incident on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be (Given : relative permeability of medium $\mu_r = 1$)

(A) 10° (B) 20°

(C) 30° (D) 60°

Official Ans. by NTA (D)

Sol. For total internal reflection, $i > \theta_C$

$\Rightarrow \sin i > \sin\theta_C$

$\Rightarrow \sin i > \frac{\mu_R}{\mu_D}$ ....(1)

Also $\mu = \sqrt{\mu_r \in_r}$

$\frac{\mu_R}{\mu_D} = \frac{\sqrt{1\times 1}}{\sqrt{4\times 1}} = \frac{1}{2}$

From (1), $\sin i > \frac{1}{2} \Rightarrow i > 30°$, i = 60°

15. Given below are two statements :-

Statement I : Davisson-Germer experiment establishes the wave nature of electrons.

Statement II : If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the **correct answer** from the options given below:-

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (A)

Sol. In Davisson-Germer experiment the electrons exhibit diffraction there by proving that electrons have wave nature. Hence both statement are correct.

Sol. Both the options are correct by concept.

16. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be :-

(A) 1 : 1 (B) 1 : 2

(C) 4 : 1 (D) 2 : 1

Official Ans. by NTA (D)

Sol. $v \propto \frac{Z}{n} \propto Z$ (n = constant)

$\Rightarrow \frac{v_{He^+}}{v_H} = \frac{Z_{He^+}}{Z_H} = \frac{2}{1}$

17. The photodiode is used to detect the optiocal signals. These diodes are preferably operated in reverse biased mode because.

(A) fractional change in majority carriers produce higher forward bias current

(B) fractional change in majority carriers produce higher reverse bias current

(C) fractional change in minority carriers produce higher forward bias current

(D) fractional change in minority carriers produce higher reverse bias current

Official Ans. by NTA (D)

Sol. Very small change in minority charge carriers produces high value of reverse bias current.

18. A signal of 100 THz frequency can be transmitted with maximum efficiency by :

(A) Coaxial cable

(B) Optical fibre

(C) Twisted pair of copper wires

(D) Water

Official Ans. by NTA (B)

Sol. Optical fibre frequency range is 1 THz to 1000 THz.

19. The difference of speed of light in the two media A and B ($v_A - v_B$) is 2.6×10^7 m/s. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 1.303

(B) 1.318

(C) 1.13

(D) 0.12

Official Ans. by NTA (C)

Sol. $v = \frac{c}{\mu}$

$$\Rightarrow v_B = \frac{3\times10^8}{1.47} = 2.04\times10^8 = 20.4\times10^7 \text{ m/s}$$

$$\because v_A - v_B = 2.6\times10^7 \text{ m/s}$$

$$\therefore v_A = (20.4+2.6)\times10^7 = 23\times10^7 \text{ m/s}$$

$$\therefore \frac{\mu_B}{\mu_A} = \frac{v_A}{v_B} = \frac{23\times10^7}{20.4\times10^7} = 1.13$$

20. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) of a galvanometer. Students took the observations for $\frac{1}{3}$ deflection in the galvanometer. Which of the below is **true** for measuring value of G?

(A) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.

(B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of shunt resistance(s).

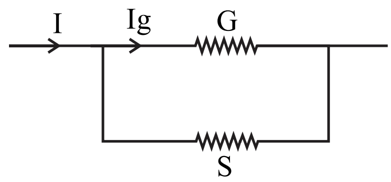
(C) $\frac{1}{3}$ deflection method can be used and in this case, the G equals to three times the value of shunt resistance(s)

(D) $\frac{1}{3}$ deflection method can be used and in this case the G value equals to the shunt resistance(s).

Official Ans. by NTA (B)

Sol. In galvanometer

$$\Rightarrow (I - I_g)S = I_gG$$



$$\frac{I_g}{I} = \frac{S}{S+G}$$

$$\Rightarrow \frac{1}{3} = \frac{S}{S+G} \Rightarrow S+G = 3S \Rightarrow G = 2S$$

SECTION-B

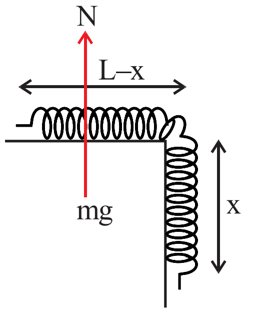
1. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is __________m.

Official Ans. by NTA 2

Sol. Mass per unit length = λ

$N = mg = \lambda(L-x)g$

$fs_{max} = \mu_s N$



$fs_{max} = (0.5)(\lambda)(L-x)g$

And also $fs_{max} = m_x g$

$0.5\lambda(L-x)g = \lambda xg$

$\frac{L-x}{2} = x$

$\frac{L}{2} = \frac{3x}{2} \Rightarrow x = \frac{L}{3} = \frac{6}{3} = 2m$

2. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be ___________ Nm^{-1}.

Official Ans. by NTA 600

Sol. $U_i + K_i = U_f + K_f$

$\Rightarrow 0 + \frac{1}{2}m(12)^2 = \frac{1}{2}K(0.3)^2 + \frac{1}{2}m(6)^2$

$\Rightarrow 0.5(12^2 - 6^2) = K(0.3)^2$

K = 600 N/m

3. The velocity of upper layer of water in a river is 36 kmh^{-1}. Shearing stress between horizontal layers of water is 10^{-3} Nm^{-2}. Depth of the river is __________m. (Co-efficiency of viscosity of water is 10^{-2} Pa.s)

Official Ans. by NTA 100

Sol. $F = \eta A \frac{\Delta v_x}{\Delta y}$

$\frac{F}{A} = \eta \frac{\Delta v_x}{\Delta y}$

$\Rightarrow 10^{-3} = 10^{-2} \times \frac{36 \times 1000}{h \times 3600}$

$\Rightarrow h = 10^{-2} \times \frac{36 \times 1000}{10^{-3} \times 3600} = 100$ m

4. A steam engine intakes 50g of steam at 100°C per minute and cools it down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}, then the heat rejected by the steam engine per minute is ______ × 10^3 cal.

Official Ans. by NTA 31

Sol. Heat rejected = $mL_f + mS\Delta T$

= (50 × 540) + 50 (1) (100 – 20)

= 31000 Cal

= 31 × 10^3 Cal

5. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of the open organ pipe is _____ cm.

Official Ans. by NTA 80

Sol. $f_1 = \frac{2v}{2l_1}$

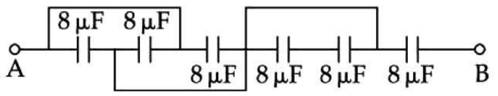
$f_2 = \frac{v}{4l_2}$

$f_1 = f_2$

$= \frac{2v}{2l_1} = \frac{v}{4l_2}$

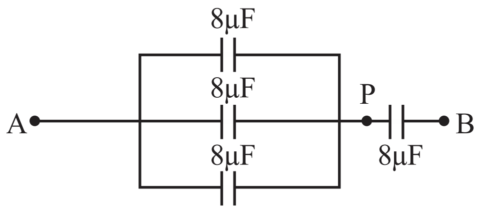
$l_1 = 4l_2 = 80$ cm

6. The equivalent capacitance between points A and B in below shown figure will be _____μF.



Official Ans. by NTA 6

Sol. Two capacitors are short circuited



Finally equivalent capacitance

$$=\frac{24\times8}{24+8}=\frac{24\times8}{32}=6\mu F$$

7. A resistor develops 300 J of thermal energy in 15s, when a current of 2A is passed through it. If the current increases to 3A, the energy developed in 10s is _________ J.

Official Ans. by NTA 450

Sol. $H = i^2Rt$

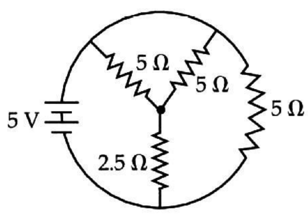
$300 = 2^2 \times R \times 15$

$$\Rightarrow R=\frac{300}{60}=5\Omega$$

Now, for $i = 3A$, $t = 10s$, $R = 5\Omega$

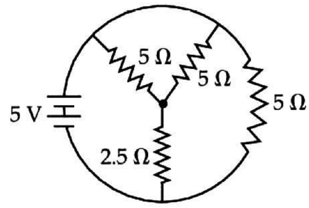
$H = 3^2 \times 5 \times 10 = 450$ J

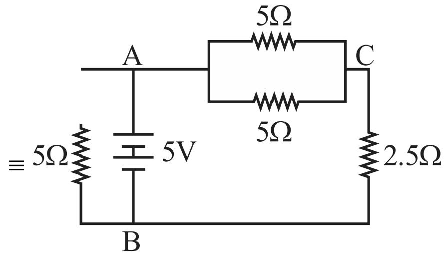
8. The total current supplied to the circuit as shown in figure by the 5V battery is __________A

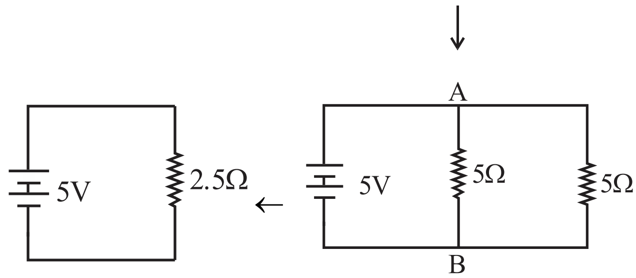


Official Ans. by NTA 2

Sol.







Current supplied by 5V battery

$$=\frac{5V}{2.5\Omega}=2A$$

9. The current in a coil of self inductance 2.0 H is increasing according to $I = 2\sin(t^2)$A. The amount of energy spent during the period when current changes from 0 to 2A is _______ J.

Official Ans. by NTA 4

Sol. $I=2\sin(t^2)\Rightarrow dI=4t\sin(t^2)dt$

If $I=0\Rightarrow t=0$

and $I=2\Rightarrow 2=2\sin t^2$

$$\Rightarrow t=\sqrt{\frac{\pi}{2}}$$

$$E=\int LI\,dI$$

$$=\int 2\times2\sin(t^2)\times4t\cos(t^2)dt$$

$$=8\int_0^{\sqrt{\pi/2}} t\sin(2t^2)dt$$

$$=2\left[-\cos(2t^2)\right]_0^{\sqrt{\pi/2}}$$

$$=2\left[-\cos\pi+\cos0\right]=4$$

10. A force on an object of mass 100g is $(10\hat{i}+5\hat{j})$ N. The position of that object at t = 2s is $(a\hat{i}+b\hat{j})$ m after starting from rest. The value of $\frac{a}{b}$ will be ________

Official Ans. by NTA 2

Sol. $\vec{F}=10\hat{i}+5\hat{j}$

m = 100 g = 0.1 kg

$\vec{a}=\frac{\vec{F}}{m}=100\hat{i}+50\hat{j}$

$\vec{S}=\vec{u}t+\frac{1}{2}\vec{a}t^2=\frac{1}{2}\vec{a}t^2 (\text{as } \vec{u}=0)$

$=\frac{1}{2}(100\hat{i}+50\hat{j})2^2$

$=200\hat{i}+100\hat{j}$

$=a\hat{i}+b\hat{j}$

$a=200, b=100$

$\therefore \frac{a}{b}=2$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Bonding in which of the following diatomic molecule(s) become(s) stronger, on the basis of MO Theory, by removal of an electron ?

(A) NO (B) N_2

(C) O_2 (D) C_2

(E) B_2

Choose the most appropriate answer from the options given below :-

(A) (A), (B), (C) only (B) (B), (C), (E) only

(C) (A), (C) only (D) (D) only

Official Ans. by NTA (C)

Sol. Bond strength $\propto$ Bond order

removal of electron from antibonding MO increases B.O.

NO & O_2 has valence e⁻ in π *orbital.

2. Incorrect statement for Tyndall effect is :-

(A) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

(B) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(C) During projection of movies in the cinemas hall, Tyndall effect is noticed.

(D) It is used to distinguish a true solution from a colloidal solution.

Official Ans. by NTA (B)

Sol. The diameter of dispersed particle should be somewhat below or near the wavelength of light.

3. The pair, in which ions are isoelectronic with Al^{3+} is :-

(A) Br^- and Be^{2+} (B) Cl^- and Li^+

(C) S^{2-} and K^+ (D) O^{2-} and Mg^{2+}

Official Ans. by NTA (D)

Sol. Isoelectronic species have same no. of electrons

Al^{+3}, O^{2-}, Mg^{+2} all have 10 electrons.

4. Leaching of gold with dilute aqueous solution of NaCN in presence of oxygen gives complex [A], which on reaction with zinc forms the elemental gold and another complex [B]. [A] and [B], respectively are :-

(A) $[Au(CN)_4]^-$ and $[Zn(CN)_2(OH)_2]^{2-}$

(B) $[Au(CN)_2]^-$ and $[Zn(OH)_4]^{2-}$

(C) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

(D) $[Au(CN)_4]^{2-}$ and $[Zn(CN)_6]^{4-}$

Official Ans. by NTA (C)

Sol. $Au + NaCN \rightarrow Na[Au(CN)_2]$

$Zn + Na[Au(CN)_2] \rightarrow Na_2[Zn(CN)_4] + Au$

5. Number of electron deficient molecules among the following

PH_3, B_2H_6, CCl_4, NH_3, LiH and BCl_3 is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (C)

Sol. Electron deficient species have less than 8 electrons (or two electrons for H) in their valence (incomplete octet)

B_2H_6, BCl_3 have incomplete octet.

6. Which one of the following alkaline earth metal ions has the highest ionic mobility in its aqueous solution?

(A) Be^{2+} (B) Mg^{2+}

(C) Ca^{2+} (D) Sr^{2+}

Official Ans. by NTA (D)

Sol. Highest ionic mobility corresponds to lowest extent of hydration and highest size of gaseous ion.

Hence Sr^{2+} has the highest ionic mobility in its aqueous solution

7. White precipitate of AgCl dissolves in aqueous ammonia solution due to formation of :

(A) $[Ag(NH_3)_4]Cl_2$ (B) $[Ag(Cl)_2(NH_3)_2]$

(C) $[Ag(NH_3)_2]Cl$ (D) $[Ag(NH_3)Cl]Cl$

Official Ans. by NTA (C)

Sol. $AgCl + 2NH_3 \rightarrow [Ag(NH_3)_2]^+Cl^-$
soluble

8. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it?

(A) It will not prefer to undergo redox reactions.

(B) It will prefer to gain electron and act as an oxidizing agent

(C) It will prefer to give away an electron and behave as reducing agent

(D) It acts as both, oxidizing and reducing agent.

Official Ans. by NTA (B)

Sol. Cerium exists in two different oxidation state +3, +4

$Ce^{+4} + e^- \rightarrow Ce^{3+}$ $E^0 = +1.61$ V

$Ce^{+3} + 3e^- \rightarrow Ce$ $E^0 = -2.336$ V

It shows Ce^{+4} acts as a strong oxidising agent & accepts electron.

9. Among the following, which is the strongest oxidizing agent ?

(A) Mn^{3+} (B) Fe^{3+}

(C) Ti^{3+} (D) Cr^{3+}

Official Ans. by NTA (A)

Sol. Strongest oxidising agent have highest reduction potential value

$E^0_{Mn^{+3}/Mn^{+2}} = 1.51V$ (highest)

10. The eutrophication of water body results in :

(A) loss of Biodiversity

(B) breakdown of organic matter

(C) increase in biodiversity

(D) decrease in BOD.

Official Ans. by NTA (A)

Sol. Eutrophication of water body results in loss of Biodiversity.

11. Phenol on reaction with dilute nitric acid, gives two products. Which method will be most effective for large scale separation ?

(A) Chromatographic separation

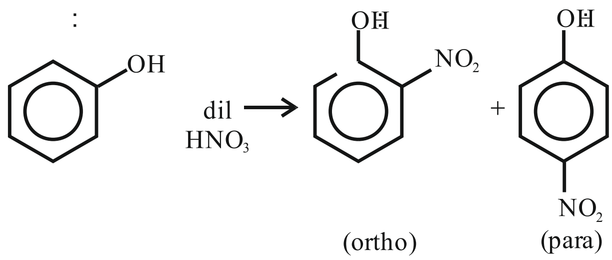
(B) Fractional Crystallisation

(C) Steam distillation

(D) Sublimation

Official Ans. by NTA (C)

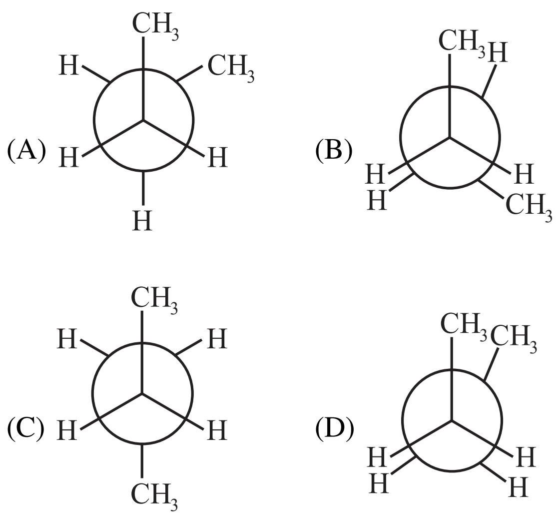
Sol.



Para product has higher boiling point than ortho as intermolecular H-bond is possible in former, where as intramolecular H-bond is possible in ortho product.

Steam distillation can separate them as ortho product is steam volatile.

12. In the following structures, which one is having staggered conformation with maximum dihedral angle?

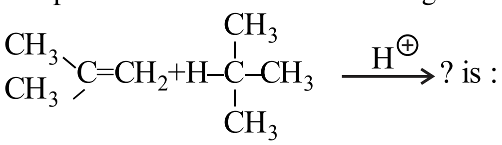


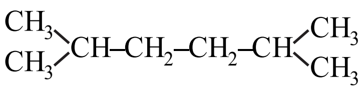
Official Ans. by NTA (C)

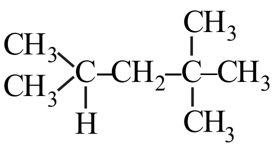
Sol. Dihedral angle : It's the angle b/w 2 specified groups ($-CH_3$ here)

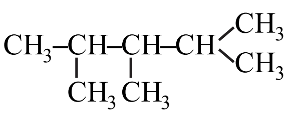
Staggered form is Given in option (C) & the angle is 180^0

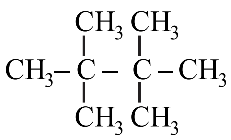
13. The products formed in the following reaction.



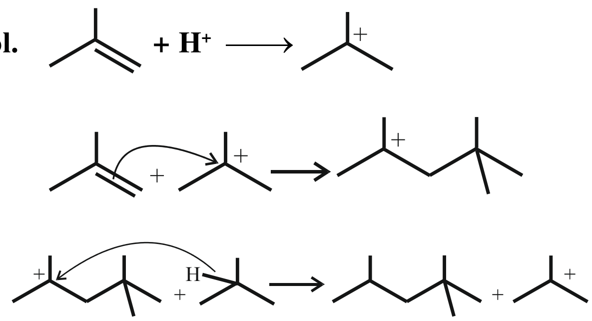
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. 

14. The IUPAC name of ethylidene chloride is :-

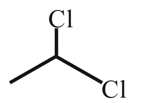
(A) 1-Chloroethene

(B) 1-Chloroethyne

(C) 1,2-Dichloroethane

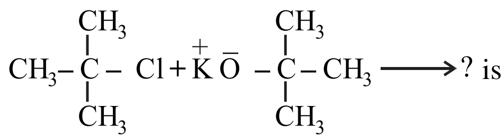
(D) 1,1-Dichloroethane

Official Ans. by NTA (D)

Sol. 

"1, 1-Dichloroethane is Ethylidene chloride"

15. The major product in the reaction



(A) t-Butyl ethyl ether

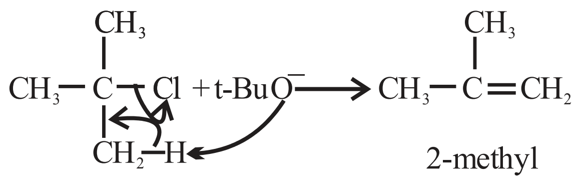
(B) 2,2-Dimethyl butane

(C) 2-Methyl pent-1-ene

(D) 2-Methyl prop-1-ene

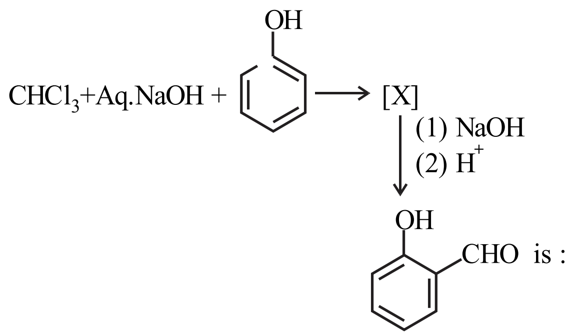
Official Ans. by NTA (D)

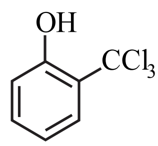
Sol. We have been given a bulky base, hence elimination will take place & not substitution.

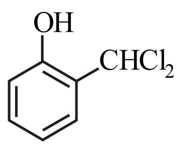


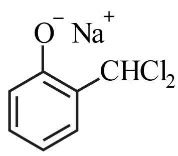
2-methyl prop-1-ene

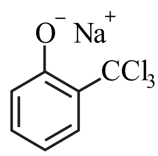
16. The intermediate X, in the reaction



(A) 

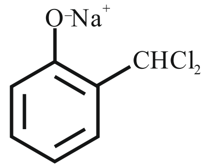
(B) 

(C) 

(D) 

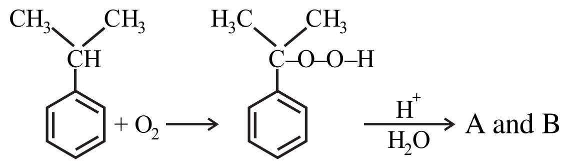
Official Ans. by NTA (C)

Sol. It's a classic Reimer-Tiemann reaction.

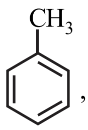


Will be the intermediate formed.

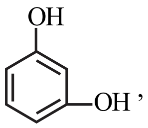
17. In the following reaction :



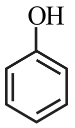
The compounds A and B respectively are :-

(A) 

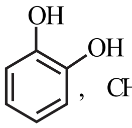
, CH_3COOH

(B) 

, CH_3COOH

(C) 

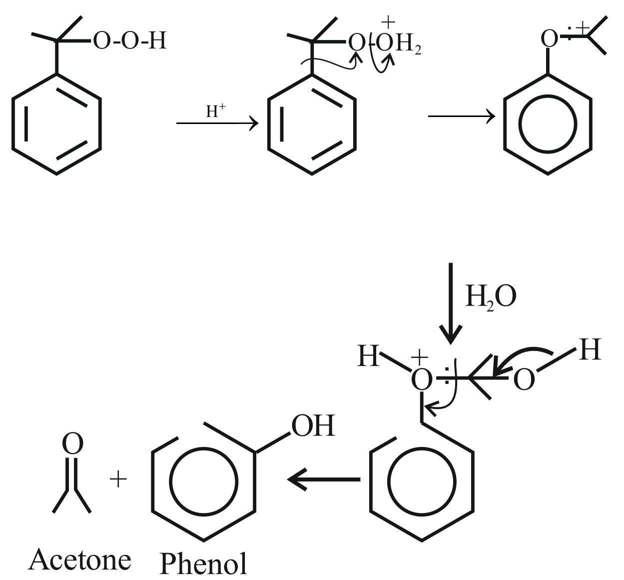
, CH_3COCH_3

(D) 

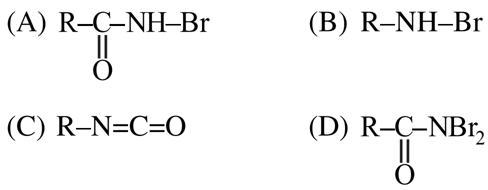
, CH_3COCH_3

Official Ans. by NTA (C)

Sol. Given reaction is cumene-Peroxide method for the preparation of phenol.
In this reaction

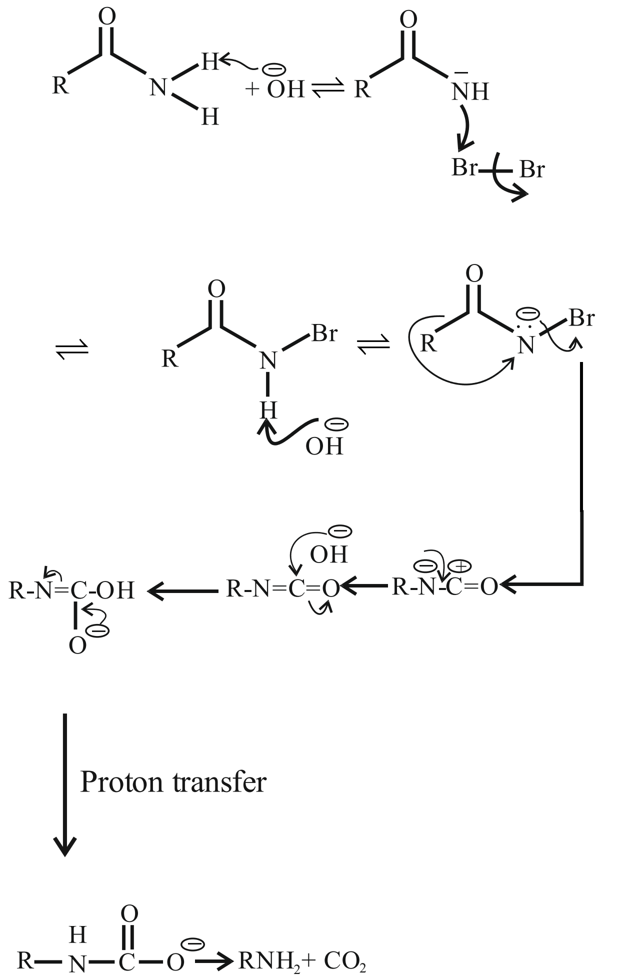


18. The reaction of R–C(=O)–NH_2 with bromine and KOH gives RNH_2 as the end product. Which one of the following is the intermediate product formed in this reaction ?



Official Ans. by NTA (C)

Sol. The given reaction is Hoffmann-Bromide degradation method.



19. Using very little soap while washing clothes, does not serve the purpose of cleaning of clothes because

(A) soap particles remain floating in water as ions

(B) the hydrophobic part of soap is not able to take away grease

(C) the micelles are not formed due to concentration of soap, below its CMC value

(D) colloidal structure of soap in water is completely disturbed.

Official Ans. by NTA (C)

Sol. Micelle formation only takes place above CMC.

20. Which one of the following is an example of artificial sweetner ?

(A) Bithional (B) Alitame

(C) Salvarsan (D) Lactose

Official Ans. by NTA (B)

Sol. Alitame is a second generation dipeptide sweetner that is 200 times sweeter than sucrose.

SECTION-B

1. The number of N atoms is 681 g of $C_7H_5N_3O_6$ is $x \times 10^{21}$. The value of x is _____ ($N_A = 6.02 \times 10^{23}$ mol^{-1}) (Nearest Integer)

Official Ans. by NTA (5418)

Sol. M.M. of $C_7H_5N_3O_6$ is 84 + 5 + 42 + 96 = 227

$$n_{C_7H_5N_3O_6} = \frac{681}{227} = 3$$

$$n_N = \frac{681}{227} \times 3 = 9 \text{ mol}$$

no. of N atoms = 9 × 6.02×10^{23}

$$= 5418 \times 10^{21}$$

∴ The answer is 5418.

2. The distance between Na^+ and Cl^- ions in solid NaCl of density 43.1 g cm^{-3} is ________ × 10^{-10}m. (Nearest Integer)

(Given : $N_A = 6.02 \times 10^{23}$ mol^{-1})

Official Ans. by NTA (1)

Sol. Unit cell formula – Na_4Cl_4

$$\text{Mass per unit cell} = \frac{Z \times M.M.}{N_A} g$$

$$= \frac{4 \times 58.5}{N_A} g$$

$$d_{\text{unit cell}} = \frac{m}{V} = \frac{m}{a^3}$$

$$\Rightarrow \frac{4 \times 58.5}{N_A \cdot a^3} = 43.1$$

$$\Rightarrow a^3 = 9.02 \times 10^{-24} \text{cm}^3$$

$$\Rightarrow a = 2.08 \times 10^{-8} \text{cm}$$

$$\Rightarrow a = 2.08 \times 10^{-10} \text{m}$$

Also $a = 2\left(r_{Na^+} + r_{Cl^-}\right)$

$$\Rightarrow r_{Na^+} + r_{Cl^-} = 1.04 \times 10^{-10} \text{m}$$

∴ The answer is 1

3. The longest wavelength of light that can be used for the ionisation of lithium atom (Li) in its ground state is x × 10^{-8} m. The value of x is ____________. (Nearest Integer)

(Given : Energy of the electron in the first shell of the hydrogen atom is -2.2×10^{-18} J; $h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms^{-1})

Official Ans. by NTA (4)

Sol. We can not calculate I.E. of lithium atom.

4. The standard entropy change for the reaction $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$ is –550 JK^{-1} at 298 K.

[Given : The standard enthalpy change for the reaction is –165 kJ mol^{-1}]. The temperature in K at which the reaction attains equilibrium is _________. (Nearest Integer)

Official Ans. by NTA (300)

Sol. $\Delta G = \Delta H - T\Delta S = 0$ at equilibrium

$\Rightarrow -165\times10^3 - T\times(-505) = 0$

$\Rightarrow T = 300K$

The answer is 300

5. 1 L aqueous solution of H_2SO_4 contains 0.02 m mol H_2SO_4. 50% of this solution is diluted with deionized water to give 1 L solution (A). In solution (A), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is ____________ $\times$ 10^3 m mols.

Official Ans. by NTA (0)

Sol. $n_{H_2SO_4}$ in Sol^n A = 50% of original solution

$= 0.01$ m mol.

$n_{H_2SO_4}$ in Final solution $= 0.01 + 0.01$

$= 0.02$ mmol

$= 0.00002 \times 10^3$ mmol

The answer 0

6. The standard free energy change ($\Delta G°$) for 50% dissociation of N_2O_4 into NO_2 at 27°C and 1 atm pressure is –x J mol^{-1}. The value of x is __________. (Nearest Integer)

[Given : R = 8.31 J K^{-1} mol^{-1}, log 1.33 = 0.1239 ln 10 = 2.3]

Official Ans. by NTA (710)

Sol.

	N_2O_4	$\rightleftharpoons$	$2NO_2$
t = 0	1 mol		
t = t	(1-0.5) mol		0.5×2 mol
	= 0.5 mol		1 mol

$$k_P = \frac{\left(\frac{1}{1.5}\times1\right)^2}{\left(\frac{0.5}{1.5}\times1\right)} = \frac{1}{0.75} = \frac{100}{75}$$

$= 1.33$

$\Delta G^0 = -RT\ell nk_P$

$= -8.31\times300\times\ell n(1.33) = -710.45$ J / mol

$= -710$ J/mol.

7. In a cell, the following reactions take place

$Fe^{2+} \rightarrow Fe^{3+}e^-$ $\quad E^o_{Fe^{3+}/Fe^{2+}} = 0.77$ V

$2I^- \rightarrow I_2 + 2e^-$ $\quad E^o_{I_2/I^-} = 0.54$ V

The standard electrode potential for the spontaneous reaction in the cell is x × 10^{-2}V 298 K. The value of x is __________ (Nearest Integer)

Official Ans. by NTA (23)

Sol. $\underset{\text{Cathode}}{Fe^{+3}} + \underset{\text{anode}}{I^-} \longrightarrow I_2 + Fe^{+2}$

$E^0_{Cell} = E^0_{cathode} - E^0_{anode}$

=0.77–0.54

=0.23

$= 23\times10^{-2}$ V

8. For a given chemical reaction

$$\gamma_1A + \gamma_2B \rightarrow \gamma_3C + \gamma_4D$$

Concentration of C changes from 10 mmol dm^{-3} to 20 mmol dm^{-3} in 10 seconds. Rate of appearance of D is 1.5 times the rate of disappearance of B which is twice the rate of disappearance A. The rate of appearance of D has been experimentally determined to be 9 mmol dm^{-3} s^{-1}. Therefore the rate of reaction is ________mmol dm^{-3} s^{-1}. (Nearest Integer)

Official Ans. by NTA (1)

Sol. $\gamma_1A + \gamma_2B \longrightarrow \gamma_3C + \gamma_4D$

Given : $+\frac{d[D]}{dt} = \frac{-3}{2}\frac{d[B]}{dt}$

$\Rightarrow \frac{-1}{2}\frac{d[B]}{dt} = \frac{+1}{3}\frac{d[D]}{dt}$

$$-\frac{d[B]}{dt}=-2\frac{d[A]}{dt}\Rightarrow-\frac{1}{2}\frac{d[B]}{dt}=\frac{-d(A)}{dt}$$

$$+\frac{d[B]}{dt}=9\ \text{mmol}\,\text{dm}^{-3}\text{s}^{-1}$$

$$\frac{+d[C]}{dt}=\frac{20-10}{10}=1\,\text{mmol}\,\text{dm}^{-3}\text{s}^{-1}$$

$$\frac{+d[C]}{dt}=\frac{1}{9}\times\frac{+d[D]}{dt}$$

$$1A+2B\longrightarrow\frac{1}{3}C+3D$$

$$\Rightarrow 3A+6B\longrightarrow C+9D$$

Rate of reaction = $\frac{+d[C]}{dt}=1\ \text{mmol dm}^{-3}\ \text{s}^{-1}$

9. If $[Cu(H_2O)_4]^{2+}$ absorbs a light of wavelength 600 nm for d–d transition, then the value of octahedral crystal field splitting energy for $[Cu(H_2O)_6]^{2+}$ will be____________ $\times 10^{-21}$ J. (Nearest Integer)

(Given : h = 6.63×10^{-34}Js

and c = 3.08×10^{8} ms^{-1})

Official Ans. by NTA (746)

Sol. $\Delta_t=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3.08\times10^{8}}{600\times10^{-9}}$

$$=\frac{6.63\times3.08\times10^{-17}}{600}$$

$$=0.034034\times10^{-17}$$

$$=340.34\times10^{-21}\,J$$

$$\Delta_0=\frac{9}{4}\Delta_t$$

$$=\frac{9}{4}\times340.34\times10^{-21}$$

$$=765.765\times10^{-21}\,J$$

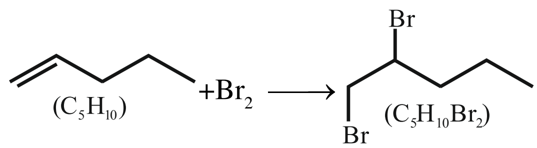
$$\approx766\times10^{-21}\,J$$

Answer = 766

10. Number of grams of bromine that will completely react with 5.0g of pent-1-ene is _______ $\times$ 10^{-2}g. (Atomic mass of Br = 80 g/mol) [Nearest Integer)

Official Ans. by NTA (1143)

Sol.



moles of Br_2 = moles of C_5H_{10}

$$\Rightarrow\frac{w}{160}=\frac{5}{70}$$

$$\Rightarrow w=\frac{5\times160}{70}g$$

$$=11.428\ g$$

$$=1142.8\times10^{-2}\,g\approx1143\times10^{-2}\,g$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (–1, 2) and L_2 at (3, – 6), then the equation of the circle C is

(A) $(x - 1)^2 + (y - 2)^2 = 4$

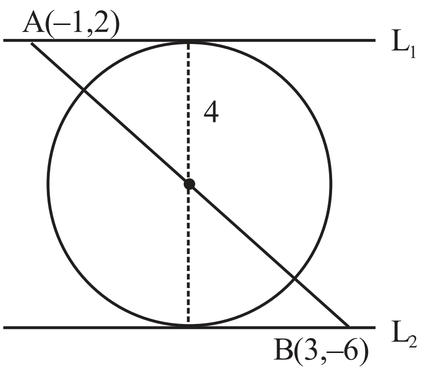
(B) $(x + 1)^2 + (y - 2)^2 = 4$

(C) $(x - 1)^2 + (y + 2)^2 = 16$

(D) $(x - 1)^2 + (y - 2)^2 = 16$

Official Ans. by NTA (C)

Sol.



$L_1 : 4x - 3y + K_1 = 0$

$L_2 : 4x - 3y + K_2 = 0$

now

$-4 - 6 + K_1 = 0 \Rightarrow K_1 = 10$

$12 + 18 + K_2 = 0 \Rightarrow K_2 = -30$

$\Rightarrow$ Tangent to the circle are

$4x - 3y + 10 = 0$

$4x - 3y - 30 = 0$

Length of diameter $2r = \frac{|10+30|}{5} = 8$

$\Rightarrow r = 4$

Now centre is mid point of A & B

$x = 1, y = -2$

Equation of circle

$(x - 1)^2 + (y + 2)^2 = 16$ **Ans.**

2. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Official Ans. by NTA (C)

Sol. $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ ....(1)

Use King's property

$I = \int_0^{\pi} \frac{e^{-\cos x}\sin x}{(1+\cos^2 x)(e^{-\cos x}+e^{\cos x})}dx$ ....(2)

On adding equation (1) and (2), we get

$2I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx = 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$

On putting cos x = t, we get

$I = \int_0^{1} \frac{dt}{1+t^2} = \left(\tan^{-1} t\right)_0^1 = \frac{\pi}{4}$

3. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to

(A) $\frac{5}{2}$ (B) 2

(C) $\frac{3}{2}$ (D) 1

Official Ans. by NTA (A)

Sol. $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=\lambda$

$a + b = 7\lambda,\ b + c = 8\lambda,\ a + c = 9\lambda$

$\Rightarrow\ a + b + c = 12\lambda$

Now $a = 4\lambda,\ b = 3\lambda,\ c = 5\lambda$

$\because\ c^2 = b^2 + a^2$

$\angle C = 90°$

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ab$$

$$\frac{R}{r} = \frac{c}{2\sin C}\times\frac{s}{\Delta} = \frac{c}{2}\times\frac{6\lambda}{\frac{1}{2}ab} = \frac{c}{ab}\times 6\lambda = \frac{5}{2}$$

4. Let $f : N \to R$ be a function such that $f(x+y)=2f(x)f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20}-1)$$

holds, is

(A) 2 (B) 3

(C) 4 (D) 6

Official Ans. by NTA (C)

Sol. $f : N \to R,\ f(x + y) = 2f(x)f(y)$ ....(1)

$f(1) = 2,$

$$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$$

$$= 2f(\alpha)\left(f(1)+f(2)+.....+f(10)\right) \quad ....(2)$$

From (1)

$f(2) = 2f^2(1) = 2^3$

$f(3) = 2f(2)f(1) = 2^5$

$\vdots \qquad \vdots$

$f(10) = 2^9 f^{10}(1) = 2^{19}$

$f(\alpha) = 2^{2\alpha-1}$; $\alpha \in N$

from (2)

$$\sum_{k=1}^{10} f(\alpha+k) = 2\left(2^{2\alpha-1}\right)\left(2+2^3+2^5+....+2^{19}\right)$$

$$\frac{512}{3}(2^{20}-1) = 2^{2\alpha}\left(2\frac{(2^{20}-1)}{3}\right)$$

Hence $\alpha = 4$

5. Let A be a 3 × 3 real matrix such that

$$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}; A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix} \text{ and } A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}.$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has

(A) no solution

(B) infinitely many solutions

(C) unique solution

(D) exactly two solutions

Official Ans. by NTA (B)

Sol. $A = \begin{bmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{bmatrix}$

$$A\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$\Rightarrow\ c_1 = 1,\ c_2 = 1,\ c_3 = 2$

$$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}c_1+a_1\\c_2+a_2\\c_3+a_3\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$\Rightarrow\ a_1 = -2,\ a_2 = -1,\ a_3 = -1$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}a_1+b_1\\a_2+b_2\\a_3+b_3\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$\Rightarrow\ b_1 = 3,\ b_2 = 2,\ b_3 = 1$

$$\Rightarrow\ A = \begin{bmatrix}-2 & 3 & 1\\ -1 & 2 & 1\\ -1 & 1 & 2\end{bmatrix}$$

$$\Rightarrow\ A - 2I = \begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}$$

$|A - 2I| = 0$

Now, $\begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\1\\1\end{bmatrix}$

$-4x_1 + 3x_2 + x_3 = 4$ ....(1)

$-x_1 + x_3 = 1$ .....(2)

$-x_1 + x_2 = 1$ ....(3)

(1) – [(2) + 3(3)]

$0 = 0 \quad \Rightarrow$ infinite solutions

6. Let $f : R \to R$ be defined as $f(x) = x^3 + x - 5$. If $g(x)$ is a function such that $f(g(x)) = x$, $\forall\ x \in R$, then $g'(63)$ is equal to ______.

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$

(C) $\frac{43}{49}$ (D) $\frac{91}{49}$

Official Ans. by NTA (A)

Sol. $f(x) = x^3 + x - 5$

$\Rightarrow$ $f'(x) = 3x^2 + 1$ $\Rightarrow$ increasing function

$\Rightarrow$ invertible

$\Rightarrow$ $g(x)$ is inverse of $f(x)$

$\Rightarrow$ $g(f(x)) = x$

$\Rightarrow$ $g'(f(x))f'(x) = 1$

$f(x) = 63$

$\Rightarrow$ $x^3 + x - 5 = 63$

$\Rightarrow$ $x = 4$

put $x = 4$

$g'(f(4))f'(4) = 1$

$g'(63) \times 49 = 1$ $\{f'(4) = 49\}$

$g'(63) = \frac{1}{49}$

7. Consider the following two propositions:

P1 : $\sim(p \to \sim q)$

P2 : $(p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then:

(A) P1 is TRUE and P2 is FALSE

(B) P1 is FALSE and P2 is TRUE

(C) Both P1 and P2 are FALSE

(D) Both P1 and P2 are TRUE

Official Ans. by NTA (C)

Sol.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \to (\sim p \vee q)$	$p \to \sim q$	$\sim(p \to \sim q)$	$p \wedge \sim q$	p_2
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

$p \to (\sim p \vee q)$ is F when p is true q is false

From table

P1 & P2 both are false

8. If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \dots \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the remainder when K is divided by 6 is

(A) 1 (B) 2

(C) 3 (D) 5

Official Ans. by NTA (D)

Sol. $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \frac{1}{2^3 \cdot 3^8} + \dots\dots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$

$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + \dots\dots + 3^9$

$$= \frac{2^9\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1} = 3^{10} - 2^{10}$$

Now, $3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5)$

$= (211)(275)$

$= (35 \times 6 + 1)(45 \times 6 + 5)$

$= 6\lambda + 5$

Remainder is 5.

9. Let $f(x)$ be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim\limits_{x \to 1} \frac{f(x)}{x - 1}$

(A) – 15 (B) – 60

(C) 60 (D) 15

Official Ans. by NTA (A)

Sol. $\underset{x \to 1}{\text{Lt}} \frac{f(x)}{x - 1} = f'(1)(\text{and } f(1) = 0)$

$f(x) + f'(x) + t''(x) = x^5 + 64$

$f'(x) + f''(x) + f'''(x) = 5x^4$

$f''(x) + f'''(x) + f^{iv}(x) = 20x^3$

$f'''(x) + f^{iv}(x) + f^{v}(x) = 60x^2$

$\therefore$ $f^{v}(x) - f''(x) = 60x^2 - 20x^3$

$\Rightarrow 120 - f''(1) = 40 \Rightarrow f''(1) = 80$

Also $f(1) + f'(1) + f''(1) = 65 \Rightarrow f'(1) = -15$. **Ans.**

10. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1|E_2)=\frac{1}{2}$, $P(E_2|E_1)=\frac{3}{4}$ and $P(E_1\cap E_2)=\frac{1}{8}$. Then:

(A) $P(E_1\cap E_2)=P(E_1)\cdot P(E_2)$

(B) $P(E'_1\cap E'_2)=P(E'_1)\cdot P(E_2)$

(C) $P(E_1\cap E'_2)=P(E_1)\cdot P(E_2)$

(D) $P(E'_1\cap E_2)=P(E_1)\cdot P(E_2)$

Official Ans. by NTA (C)

Sol.

(A) $P(E_1)\cdot P(E_2)=\frac{1}{6}\cdot\frac{1}{4}=\frac{1}{24}\neq P(E_1\cap E_2)$

(B) $P(E'_1\cap E'_2)=1-P(E_1\cup E_2)$

$=1-\left(P(E_1)+P(E_2)-P(E_1\cap E_2)\right)$

$=1-\left(\frac{1}{6}+\frac{1}{4}-\frac{1}{8}\right)=\frac{17}{24}$

$P(E'_1)P(E_2)=\frac{5}{6}\times\frac{1}{4}=\frac{5}{24}$

(C) $P(E_1\cap E'_2)=P(E_1)-P(E_1\cap E_2)=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

(D) $P(E'_1\cap E_2)=P(E_2)-P(E_1\cap E_2)=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$

11. Let $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$. If M and N are two matrices given by $M=\sum_{k=1}^{10}A^{2k}$ and $N=\sum_{k=1}^{10}A^{2k-1}$ then MN^2 is

(A) a non-identity symmetric matrix

(B) a skew-symmetric matrix

(C) neither symmetric nor skew-symmetric matrix

(D) an identify matrix

Official Ans. by NTA (A)

Sol. $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$

$A^2=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}=\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}=-4I$

$A^3=-4A$

$A^4=(-4I)(-4I)=(-4)^2I$

$A^5=(-4)^2A,\quad A^6=(-4)^3I$

$M=\sum_{k=1}^{10}A^{2k}=A^2+A^4+\ldots+A^{20}$

$=[-4+(-4)^2+(-4)^3+\ldots+(-4)^{20}]I$

$=-4\lambda I$

$\Rightarrow$ M is symmetric matrix

$N=\sum_{k=1}^{10}A^{2k-1}=A+A^3+\ldots+A^{19}$

$=A[1+(-4)+(-4)^2+\ldots+(-4)^9]$

$=\lambda A\Rightarrow$ skew symmetric

$\Rightarrow N^2$ is symmetric matrix

$\Rightarrow MN^2$ is non identity symmetric matrix

12. Let $g:(0,\infty)\to R$ be a differentiable function such that

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c,$$

for all $x>0$, where c is an arbitrary constant. Then.

(A) g is decreasing in $\left(0,\frac{\pi}{4}\right)$

(B) g' is increasing in $\left(0,\frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0,\frac{\pi}{2}\right)$

(D) g – g' is increasing in $\left(0,\frac{\pi}{2}\right)$

Official Ans. by NTA (D)

Sol.

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c$$

On differentiating both sides w.r.t. x, we get

$$\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)$$

$$=\frac{(e^x+1)\left(g(x)+xg'(x)\right)-e^x\cdot x\cdot g(x)}{\left(e^x+1\right)^2}$$

$(e^x + 1)x(\cos x - \sin x) + g(x)(e^x + 1 - xe^x)$

$= (e^x+1)\left(g(x)+xg'(x)\right)-e^x\cdot x\cdot g(x)$

$\Rightarrow g'(x) = \cos x - \sin x$

$\Rightarrow g(x) = \sin x + \cos x + C$

$g(x)$ is increasing in $(0, \pi/4)$

$g''(x) = -\sin x - \cos x < 0$

$\Rightarrow$ $g'(x)$ is decreasing function

let $h(x) = g(x) + g'(x) = 2\cos x + C$

$\Rightarrow h'(x) = g'(x) + g''(x) = -2\sin x < 0$

$\Rightarrow$ h is decreasing

let $\phi(x) = g(x) - g'(x) = 2\sin x + C$

$\Rightarrow \phi'(x) = g'(x) - g''(x) = 2\cos x > 0$

$\Rightarrow$ ϕ is increasing

Hence option D is correct.

13. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x)=\frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$ holds?

(A) (2, 3) (B) (–2, – 1)

(C) (1, 2) (D) (–1, 1)

Official Ans. by NTA (A)

Sol. $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$

$\Rightarrow f'(x) = \frac{2x}{x^2+1}+e^{-x} > 0 \quad \forall x \in R$

$\Rightarrow$ f is strictly increasing

$g(x) = \frac{1-2e^{2x}}{e^x} = e^{-x} - 2e^x$

$\Rightarrow g'(x) = -\left(2e^x+e^{-x}\right)<0 \quad \forall x \in R$

$\Rightarrow$ g is decreasing

Now $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$

$\Rightarrow g\left(\frac{(\alpha-1)^2}{3}\right)>g\left(\alpha-\frac{5}{3}\right)$

$\Rightarrow \frac{(\alpha-1)^2}{3}<\alpha-\frac{5}{3}$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0$

$\Rightarrow (\alpha - 2)(\alpha - 3) < 0$

$\Rightarrow \alpha \in (2, 3)$

14. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ $a_i > 0$, $i = 1, 2, 3$ be a vector which makes equal angles with the coordinates axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i}+4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}$, $\vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i}+4\hat{j}$ is equal to

(A) $\sqrt{7}$ (B) $\sqrt{2}$

(C) 2 (D) 7

Official Ans. by NTA (B)

Sol. $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{a}=\lambda\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)=\frac{\lambda}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Now projection of $\vec{a}$ on $\vec{b}=7$

$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 7$

$\frac{\frac{\lambda}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}\right)}{5} = 7$

$\lambda = 5\sqrt{3}$

$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$

now $\vec{b} = 5\alpha(\hat{i}+\hat{j}+\hat{k})+\beta(\hat{i})$

$\vec{a}\cdot\vec{b} = 0$

$\Rightarrow 25\alpha(3) + 5\beta = 0$

$\Rightarrow 15\alpha + \beta = 0 \quad \Rightarrow \quad \beta = -15\alpha$

$\vec{b} = 5\alpha(-2\hat{i}+\hat{j}+\hat{k})$

$\left|\vec{b}\right| = 5\sqrt{3}$

$\Rightarrow \alpha = \pm\dfrac{1}{\sqrt{2}}$

$\vec{b} = \pm\dfrac{5}{\sqrt{2}}(-2\hat{i}+\hat{j}+\hat{k})$

Projection of $\vec{b}$ on $3\hat{i}+4\hat{j}$ is

$$\frac{\vec{b}\cdot(3\hat{i}+4\hat{j})}{5} = \pm\frac{5}{\sqrt{2}}\left(\frac{-6+4}{5}\right) = \pm\sqrt{2}$$

15. Let $y = y(x)$ be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is:

(A) not a critical point

(B) a point of local minima

(C) a point of local maxima

(D) a point of inflection

Official Ans. by NTA (B)

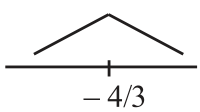
Sol. $(x + 1)dy - y\,dx = e^{3x}(x + 1)^2$

$$\frac{(x+1)dy - ydx}{(x+1)^2} = e^{3x}$$

$$d\left(\frac{y}{x+1}\right) = e^{3x} \quad \Rightarrow \quad \frac{y}{x+1} = \frac{e^{3x}}{3} + C$$

$$\left(0, \frac{1}{3}\right) \Rightarrow C = 0 \quad \Rightarrow \quad y = \frac{(x+1)e^{3x}}{3}$$

$$\frac{dy}{dx} = \frac{1}{3}\left((x+1)3e^{3x} + e^{3x}\right) = \frac{3^{3x}}{3}(3x+4)$$



Clearly, $x = \dfrac{-4}{3}$ is point of local minima

16. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to

(A) $3+4\sqrt{2}$ (B) $-5+6\sqrt{2}$

(C) $-4+3\sqrt{2}$ (D) $7+6\sqrt{2}$

Official Ans. by NTA (C)

Sol. C_1: $x^2 + y^2 = 2$

$C_2 : y^2 = x$

Let tangent to parabola be $y = mx + \dfrac{1}{4m}$.

It is also a tangent of circle so distance from centre of circle (0, 0) will be $\sqrt{2}$.

$$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \quad \Rightarrow \quad 1 = 32m^2 + 32m^4$$

by solving

$m^2 = \dfrac{3\sqrt{2}-4}{8}$, $m^2 = \dfrac{-3\sqrt{2}-4}{8}$ (rejected)

$m = \pm\sqrt{\dfrac{3\sqrt{2}-4}{8}}$

so, $8\,|m_1m_2| = 3\sqrt{2}-4$

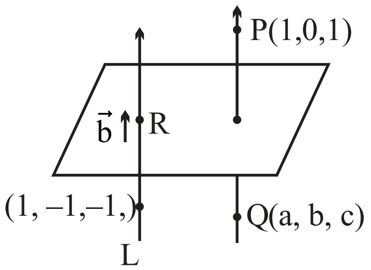
17. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane $S : x + y + z = 5$. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

(A) 2 (B) 5

(C) 7 (D) 11

Official Ans. by NTA (B)

Sol.



Let parallel vector of L = $\vec{b}$

mirror image of Q on given plane x+y+z=5

$$\frac{a-1}{1}=\frac{b-0}{1}=\frac{c-1}{1}=\frac{-2(2-5)}{3}$$

a = 3, b = 2, c = 3

Q≡(3, 2, 3)

$\because \quad \vec{b}\,||\,\overrightarrow{PQ}$

so, $\vec{b}=(1,1,1)$

Equation of line

$$L:\frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}$$

Let point R, $(\lambda+1, \lambda-1, \lambda-1)$

lying on plane x + y + z = 5,

so, 3λ – 1 = 5

$\Rightarrow \quad \lambda = 2$

Point R is (3, 1, 1)

QR^2 = 5 **Ans.**

18. If the solution curve y = y(x) of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, l) and intersects the line y = $\sqrt{3}$ x at the point (α, $\sqrt{3}$ α), then value of $\log_e(\sqrt{3}\,\alpha)$ is equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{12}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Sol. $y^2dx - xy\,dy = -(x^2 + y^2)dy$

$y(y\,dx - x\,dy) = -(x^2 + y^2)\,dy$

$-y(x\,dx - y\,dx) = -(x^2 + y^2)dy$

$$\frac{xdy-ydx}{x^2}=\left(1+\frac{y^2}{x^2}\right)\frac{dy}{y}$$

$$\Rightarrow \frac{d(y/x)}{1+\frac{y^2}{x^2}}=\frac{dy}{y}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \ln y + C$$

$$\left(\alpha, \sqrt{3}\,\alpha\right) \Rightarrow \frac{\pi}{3}=\ln\left(\sqrt{3}\,\alpha\right)+\frac{\pi}{4}$$

$$\therefore \quad \ln\left(\sqrt{3}\,\alpha\right)=\frac{\pi}{12}$$

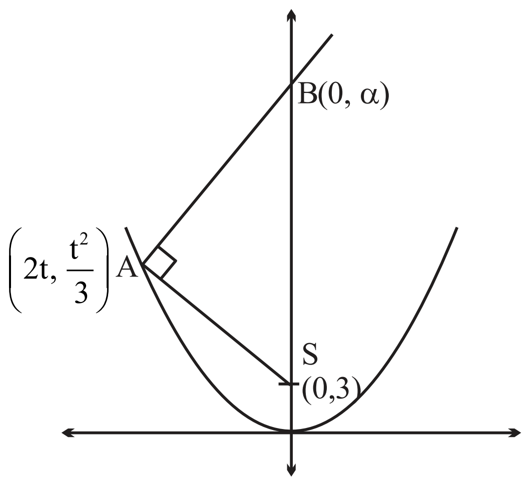
19. Let x = 2t, $y=\frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that SA ⊥ BA, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB, then $\lim\limits_{t\to 1} k$ is equal to

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$

(C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Official Ans. by NTA (D)

Sol.



parabola $x^2 = 12y$

SA ⊥ SB

so, $m_{AS} \cdot m_{AB} = -1$

$$\frac{\left(3-\frac{t^2}{3}\right)}{(0-2t)} \cdot \frac{\left(\alpha-\frac{t^2}{3}\right)}{(0-2t)} = -1$$

by solving

$$3\alpha = \frac{27t^2+t^4}{t^2-9}$$

ordinate of centriod of $\Delta SAB = K = \dfrac{\alpha+\frac{t^2}{3}+3}{3}$

$$k = \frac{9+3\alpha+t^2}{9}$$

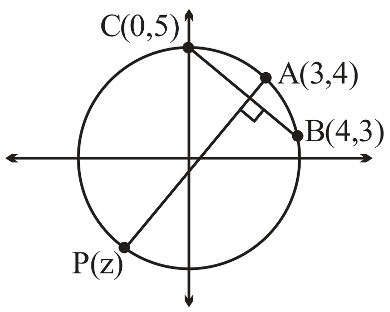
$$\lim_{t\to 1} k = \lim_{t\to 1} \frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{(t^2-9)}\right) = \frac{13}{18}$$

20. Let a circle C in complex plane pass tltrough the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to :

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)-\pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right)-\pi$

(C) $\tan^{-1}(3)-\pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

Sol.



Slope of BC $= \dfrac{3-5}{4-0} = -\dfrac{1}{2}$

Slope of AP = 2

equation of AP : $y - 4 = 2(x - 3)$

$\Rightarrow \quad y = 2(x - 1)$

P lies on circle $x^2 + y^2 = 25$

$\Rightarrow x^2 + (2(x - 1))^2 = 25$

$\Rightarrow x = -\dfrac{7}{5}$ and $y = -\dfrac{24}{5}$

$\Rightarrow \arg(z) = \tan^{-1}\left(\frac{24}{7}\right)-\pi$

SECTION-B

1. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R$. $C_1 + 3\cdot 2C_2 + 5\cdot 3C_3 + ...$ upto 10 terms

$$= \frac{\alpha\times 2^{11}}{2^\beta-1}\left(C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....\text{upto 10 terms}\right)$$

then the value of $\alpha + \beta$ is equal to

Official Ans. by NTA (286)

(BONUS)

Sol. $(1 + x)^{10} = C_0 + C_1x + C_2x^2 + ... + C_{10}x^{10}$

Differentiating

$10(1 + x)^9 = C_1 + 2C_2x + 3C_3x^2 + ... + 10C_{10}x^9$

replace $x \to x^2$

$10(1+x^2)^9 = C_1 + 2C_2x^2 + 3C_3x^4 +....+ 10C_{10}x^{18}$

$10\cdot x(1+x^2)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + ... + 10C_{10}x^{19}$

Differentiating

$$10\left((1+x^2)^9\cdot 1+x\cdot 9(1+x^2)^8 2x\right)$$

$= C_1x + 2\,C_2\cdot 3x^3 + 3\cdot 5\cdot C_3x^4 + ... + 10\cdot 19C_{10}x^{18}$

putting x = 1

$10(2^9+18\cdot 2^8)$

$= C_1 + 3\cdot 2\cdot C_2 + 5\cdot 3\cdot C_3 + ... + 19\cdot 10\cdot C_{10}$

$C_1 + 3\cdot 2\cdot C_2 + ... + 19\cdot 10\cdot C_{10}$

$= 10\cdot 2^9\cdot 10 = 100\cdot 2^9$

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....+\underset{\substack{\uparrow\\ 10^{th}\text{ term}}}{\frac{C_9}{11}}+\underset{\substack{\uparrow\\ 11^{th}\text{ term}}}{\frac{C_{10}}{11}}=\frac{2^{11}-1}{11}$$

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....+\frac{C_9}{11}=\frac{2^{11}-2}{11}$$

Now, $100\cdot 2^9 = \dfrac{\alpha\cdot 2^{11}}{2^\beta-1}\left(\dfrac{2^{11}-2}{11}\right)$

Eqn. of form $y = k\,(2^x - 1)$.

It has infinite solutions even if we take $x, y \in N$.

2. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is ______.

Official Ans. by NTA (63)

Sol. x y z ← odd number

z = 1,3,5,7,9

x+y+ z = 7, 14, 21 [sum of digit multiple of 7]

$\underset{1\text{to}9}{x} + \underset{0\text{to}9}{y}$ = 6, 4, 2, 13, 11, 9, 7, 5, 20, 18, 16, 14, 12

$x + y = 6 \Rightarrow$ (1,5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0)

→ T.N. = 6

$x+y=4 \Rightarrow$ (1,3), (2, 2), (3, 1), (4,0)

→ T.N = 4

$x+y=2 \Rightarrow$ (1,1), (2,0)

→ T.N. = 2

$x+y=13 \Rightarrow$ (4,9), (5,8), (6,7), (7,6), (8,5),(9,4)

→ T.N. = 6

$x+y=11 \Rightarrow$ (2,9), (3,8), (4,7), (5,6), (6,5), (6,5), (7,4), (8,3), (9,2)

→ T.N. = 8

$x+y=9 \Rightarrow (1,8),(2,7),(3,8),(4,5),(5,4),....(8,1),(9,0)$

→ T.N. = 9

$x+y=7 \Rightarrow (1,8),(2,5),(3,4),....$ (8, 1), (7,0)

→ T.N. = 7

$x+y=5 \Rightarrow (1,4),(2,3)$, (3, 2), (4,1),(5,0)

→ T.N. = 5

$x+y=20 \Rightarrow$ Not possible

$x+y=18 \Rightarrow (9,9)$ → T.N. = 1

$x+y=16 \Rightarrow (7,9),(8,8),(9,7)$

→ T.N. = 3

$x+y=14 \Rightarrow (5,9),(6,8),(7,7)$,(8,6),(9,5)

→ T.N. = 5

$x+y=12 \Rightarrow (3,9),(4,8),(5,7),(6,6)$....(9,3)

→ T.N. = 7

3. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}|=4$, $|\vec{b}|=3$ $\theta \in \left(\frac{\pi}{4},\frac{\pi}{3}\right)$. Then

$\left|\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$ is equal to ____

Official Ans. by NTA (576)

Sol. $|\vec{a}|=4, |\vec{b}|=3$ $\theta \in \left(\frac{\pi}{4},\frac{\pi}{3}\right)$

$\left|\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$

$\left|\vec{a}\times\vec{b}-\vec{b}\times\vec{a}\right|^2+4a^2b^2\cos^2\theta$

$2\left|\vec{a}\times\vec{b}\right|^2+4a^2b^2\cos^2\theta$

$4a^2b^2\sin^2\theta+4a^2b^2\cos^2\theta$

$4a^2b^2=4\times16\times9=576$

4. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then 2r + s – 2q + p is equal to

Official Ans. by NTA (7)

Sol. $2x^2 - rx + p = 0 \genfrac{}{}{0pt}{}{\nearrow x_1}{\searrow x_2}$

$y^2 - sy - q = 0 \genfrac{}{}{0pt}{}{\nearrow y_1}{\searrow y_2}$

Equation of the circle with PQ as diameter is

$2(x^2 + y^2) - rx - 2sy + p - 2q = 0$

on comparing with the given equation

r = 11, s = 7

p – 2q = – 22

$\therefore 2r+s-2q+p=22+7-22=7$

5. The number of values of x in the interval $\left(\frac{\pi}{4},\frac{7\pi}{4}\right)$ for which $14\text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$ holds, is ________

Official Ans. by NTA (4)

Sol. $x \in \left(\frac{\pi}{4},\frac{7\pi}{4}\right)$

$14\text{cosec}^2x-2\sin^2x=21-4\cos^2x$

$=21-4\left(1-\sin^2x\right)$

$=17+4\sin^2x$

$14\text{cosec}^2x-6\sin^2x=17$

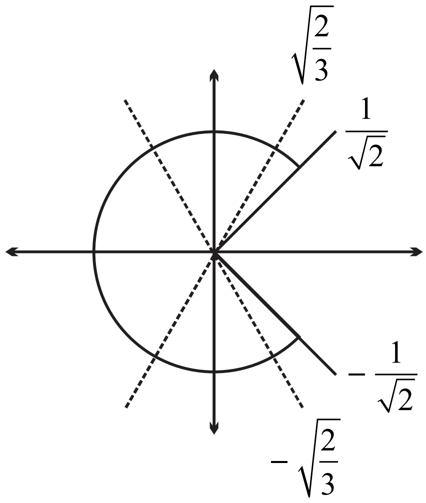
$\text{let}\sin^2x=p$

$\frac{14}{p}-6p=17 \Rightarrow 14-6p^2=17p$

$6p^2+17p-14=0$

$p=-3.5, \frac{2}{3} \Rightarrow \sin^2 x = \frac{2}{3}$

$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$



$\therefore$ Total 4 solutions

6. For a natural number n, let $a_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}$ is

Official Ans. by NTA (4)

Sol. $a_n = 19^n - 12^n$

$$\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}=\frac{31\left(19^9-12^9\right)-\left(19^{10}-12^{10}\right)}{57\alpha_8}$$

$$=\frac{19^9(31-19)-12^9(31-12)}{57\alpha_8}$$

$$=\frac{19^9\cdot 12-12^{19}\cdot 19}{57\alpha_8}$$

$$=\frac{12\cdot 19\left(19^8-12^8\right)}{57\alpha_8}=4$$

7. Let $f : R \to R$ be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x)=f(f(f(x)))+f(f(x))$, the the greatest integer less than or equal to g (1) is ______

Official Ans. by NTA (2)

Sol. $f(x)=\left[2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$

$f(x)=\left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$

$= (4 - x^{50})^{1/50}$

$f(f(x))=\left(4-\left(\left(4-x^{50}\right)^{1/50}\right)^{50}\right)^{1/50} = x$

$g(x) = f(f(f(x)))+f(f(x))$

$= f(x) + x$

$g(1) = f(1) + 1 = 3^{1/50} + 1$

$[g(1)] = [3^{1/50} + 1] = 2$

8. Let the lines

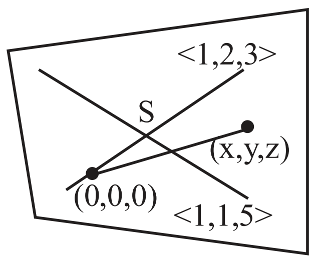
$L_1 : \vec{r}=\lambda(\hat{i}+2\hat{j}+3\hat{k}), \lambda \in R$

$L_2 : \vec{r}=(\hat{i}+3\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j}+5\hat{k}); \mu \in R$

intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2, then the value of $a + b + d$ is equal to _______

Official Ans. by NTA (5)

Sol. Both the lines lie in the same plane



$\therefore$ equation of the plane

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix}=0$$

$\Rightarrow 7x - 2y - z = 0$

$\therefore a + b + d = 5$

9. Let A be a 3 × 3 matrix having entries from. the set {–1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5, is ________

Official Ans. by NTA (414)

Sol. **Case-I:** $1 \rightarrow 7$ times

and $-1 \rightarrow 2$ times

number of possible matrix = $\frac{9!}{7!\,2!} = 36$

Case-II: $1 \rightarrow 6$ times,

$-1 \rightarrow 1$ times

and $0 \rightarrow 2$ times

number of possible matrix = $\frac{9!}{6!\,2!} = 252$

Case-III: $1 \rightarrow 5$ times,

and $0 \rightarrow 4$ times

number of possible matrix = $\frac{9!}{5!\,4!} = 126$

Hence total number of all such matrix A = 414

10. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, ...$ is equal to

Official Ans. by NTA (98)

Sol. $\frac{1}{3}+\frac{5}{9}+\frac{19}{27}+\frac{65}{81}+....$

$$\left(1-\frac{2}{3}\right)+\left(1-\frac{4}{9}\right)+\left(1-\frac{8}{27}\right)+\left(1-\frac{16}{81}\right)....100 \text{ terms}$$

$$100-\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+....\right]$$

$$100-\frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^{100}\right)}{1-\frac{2}{3}}$$

$$100-2\left(1-\left(\frac{2}{3}\right)^{100}\right)$$

$$S=98+2\left(\frac{2}{3}\right)^{100}$$

$$\Rightarrow [S]=98$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3:00 PM to 6:00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1 h_2}$

Reason R: Product of said heights.

$$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right) \cdot \left(\frac{u^2 \cos^2\theta}{2g}\right)$$

Choose the CORRECT answer :

(A) Both A and R are true and R is the correct explanation of A.

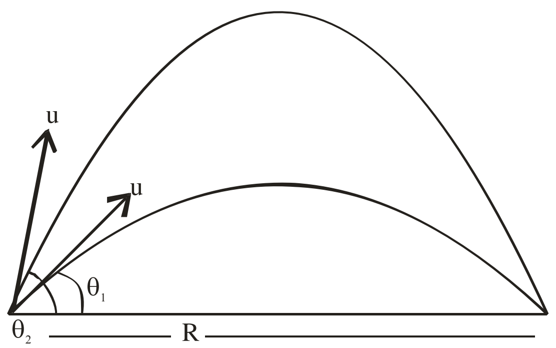
(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol. For same range $\theta_1 + \theta_2 = 90^0$



$$h_1 = \frac{u^2 \sin^2\theta_1}{2g} \qquad h_2 = \frac{u^2 \sin^2\theta_2}{2g}$$

$$h_1 h_2 = \frac{u^2 \sin^2\theta_1}{2g} \times \frac{u^2 \sin^2\theta_2}{2g}$$

$$\theta_2 = 90 - \theta_1$$

$$h_1 h_2 = \frac{u^2 \sin^2\theta_1}{2g} \cdot \frac{u^2 \cos^2\theta_1}{2g}$$

$$= \left[\frac{u^2 \sin\theta_1 \cos\theta_1}{2g}\right]^2$$

$$= \left[\frac{u^2 \sin\theta_1 \cos\theta_1}{2g} \times \frac{2}{2}\right]^2 = \frac{R^2}{16}$$

$$R = 4\sqrt{h_1 h_2}$$

So R is correct explanation of A

2. Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by $X_P(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both the buses have same velocity ?

(A) $\frac{\alpha - f}{1+\beta}$ (B) $\frac{\alpha + f}{2(\beta - 1)}$

(C) $\frac{\alpha + f}{2(1+\beta)}$ (D) $\frac{f - \alpha}{2(1+\beta)}$

Official Ans. by NTA (D)

Sol. $X_P(t) = \alpha t + \beta t^2$ $\qquad X_Q = ft - t^2$

$V_P(t) = \alpha + 2\beta t$ $\qquad V_Q = f - 2t$

$V_P = V_Q$

$\alpha + 2\beta t = f - 2t$

$$t = \frac{f - \alpha}{2\beta + 2}$$

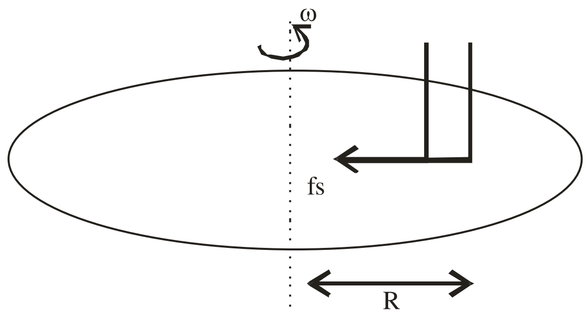
3. A disc with a flat small bottom beaker placed on it at a distance R from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity ω. The coefficient of static friction between the bottom of the beaker and the surface of the disc is μ. The beaker will revolve with the disc if :

(A) $R \le \frac{\mu g}{2\omega^2}$ (B) $R \le \frac{\mu g}{\omega^2}$

(C) $R \ge \frac{\mu g}{2\omega^2}$ (D) $R \ge \frac{\mu g}{\omega^2}$

Official Ans. by NTA (B)

Sol. For beaker to move with disc



$f_s = m\omega^2 R$

We know that $f_s \le f_{s\,max}$

$m\omega^2 R \le \mu mg$

$R \le \frac{\mu g}{\omega^2}$

4. A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube $\left(\text{Given}: \alpha = 5.0\times10^{-4}\,°C^{-1}\right)$

(A) $2.4\times10^6\ cm^3$

(B) $1.2\times10^5\ cm^3$

(C) $6.0\times10^4\ cm^3$

(D) $4.8\times10^5\ cm^3$

Official Ans. by NTA (B)

Sol. Increase in volume $\Delta V = \gamma V_0 \Delta T$

$\gamma = 3\alpha$

So $\Delta V = (3\alpha) V_0 \Delta T$

Total surface area = $6a^2$, where a is side length

$24 = 6a^2$ a= 2m

Volume $V_0 = (2)^3 = 8m^3$

$\Delta V = \left(3\times5\times10^{-4}\right)(8)\times10$

$= 1.2\times10^5 cm^3$

5. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt? [Specific heat of copper: $0.39\,J\,g^{-1}\,°C^{-1}$ and latent heat of fusion of water : 335 J g^{-1}]

(A) 1.5 kg (B) 5.8 kg

(C) 2.9 kg (D) 3.8 kg

Official Ans. by NTA (C)

Sol. Heat given by block to get 0ºC temperature

$\Delta Q_1 = 5\times\left(0.39\times10^3\right)\times(500-0)$

$= 975\times10^3 J$

Heat absorbed by ice to melt m mass

$\Delta Q_2 = m\times\left(335\times10^3\right)J$

$\Delta Q_1 = \Delta Q_2$

$m\times\left(335\times10^3\right) = 975\times10^3$

$m = \frac{975}{335} = 2.910\,kg$

6. The ratio of specific heats $\left(\frac{C_P}{C_V}\right)$ in terms of degree of freedom (f) is given by:

(A) $\left(1+\frac{f}{3}\right)$ (B) $\left(1+\frac{2}{f}\right)$

(C) $\left(1+\frac{f}{2}\right)$ (D) $\left(1+\frac{1}{f}\right)$

Official Ans. by NTA (B)

Sol. Molar heat capacity at constant volume $C_V = \frac{fR}{2}$

where f is degree of freedom.

Molar heat capacity at constant pressure can be written as $C_P = R + C_V = R + \frac{fR}{2} = \left(1+\frac{f}{2}\right)R$

So $\frac{C_P}{C_v} = 1 + \frac{2}{f}$

7. For a particle in uniform circular motion, the acceleration $\vec{a}$ at any point P(R,θ) on the circular path of radius R is (when θ is measured from the positive x –axis and v is uniform speed) :

(A) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

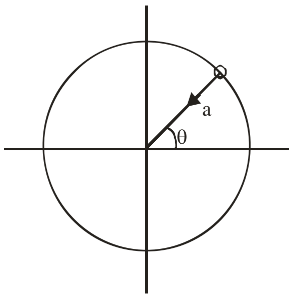
(B) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

(D) $-\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

Official Ans. by NTA (C)

Sol. $a = |\vec{a}| = \frac{V^2}{R}$



$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j}$

$= -\frac{V^2}{R}\cos\theta\hat{i} - \frac{V^2}{R}\sin\theta\hat{j}$

8. Two metallic plates form a parallel plate capacitor. The distance between the plates is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor ?

(A) 2:1 (B) 1:2

(C) 1:4 (D) 4:1

Official Ans. by NTA (A)

Sol. $C_1 = \frac{\in_0 A}{d}$

$C_2 = \frac{\in_0 A}{\frac{d}{2} + \frac{d/2}{\infty}} = \frac{2\in_0 A}{d}$

$\frac{C_2}{C_1} = \frac{2}{1}$

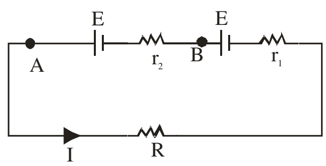
9. Two cells of same emf but different internal resistances r_1 and r_2 are connected in series with a resistance R. The value of resistance R, for which the potential difference across second cell is zero, is

(A) $r_2 - r_1$ (B) $r_1 - r_2$

(C) r_1 (D) r_2

Official Ans. by NTA (A)

Sol. $I = \frac{2E}{R + r_1 + r_2}$ **.....(i)**



But $V_A - V_B = E - Ir_2 = 0$

$\Rightarrow I = \frac{E}{r_2}$ ....(ii)

Comparing values of I from (i) and (ii)

$\frac{E}{r_2} = \frac{2E}{R + r_1 + r_2}$

$\Rightarrow R = r_2 - r_1$

10. Given below are two statements:

Statement – I : Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement – II: Diamagnetism is a result of orbital motions of electrons developing magnetic moments opposite to the applied magnetic field.

Choose the **CORRECT** answer from the options given below : -

(A) Both statement – I and statement -II are true.

(B) Both statement – I and Statement – II are false.

(C) Statement – I is true but statement – II is false.

(D) Statement-I is false but Statement-II is true.

Official Ans. by NTA (A)

Sol. According to curie's law, magnetic susceptibility is inversely proportional to temperature for a fixed value of external magnetic field i.e. $\chi = \frac{C}{T}$.

The same is applicable for ferromagnet & the relation is given as $\chi = \frac{C}{T - T_C}$ (T_C is curie temperature)

Diamagnetism is due to non-cooperative behaviour of orbiting electrons when exposed to external magnetic field.

Hence option (A).

11. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to

(A) B (B) 2 B

(C) 4 B (D) $\frac{B}{2}$

Official Ans. by NTA (A)

Sol. $B_1 = \mu_0 nI$

$$B_2 = \mu_0 \left(\frac{n}{2}\right)(2I)$$

$\Rightarrow B_1 = B_2$

12. A sinusoidal voltage V(t) = 210 sin 3000t volt is applied to a series LCR circuit in which L = 10 mH, C = 25 μF and R = 100Ω. The phase difference (Φ) between the applied voltage and resultant current will be :

(A) $\tan^{-1}(0.17)$ (B) $\tan^{-1}(9.46)$

(C) $\tan^{-1}(0.30)$ (D) $\tan^{-1}(13.33)$

Official Ans. by NTA (A)

Sol. $X_L = 10^{-2} \times 3000 = 30\Omega$

$$X_C = \frac{1}{3000 \times 25 \times 10^{-6}} = \frac{40}{3}\Omega$$

$X = X_L - X_C$

$$= 30 - \frac{40}{3} = \frac{50}{3}$$

$$\tan\delta = \frac{X}{R} = \frac{50}{3 \times 100} = \frac{1}{6}$$

$$\delta = \tan^{-1}\left(\frac{1}{6}\right) = \tan^{-1}(0.17)$$

13. The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s.The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

(A) 2.25 (B) 4.25

(C) 6.25 (D) 8.25

Official Ans. by NTA (A)

Sol. $V = 2 \times 10^8 \, m/s$

$C = 3 \times 10^8 \, m/s$

$$\frac{C}{V} = \sqrt{\mu_r \in_r}$$

$$\frac{9}{4} = 1 \times \in_r$$

$$\in_r = \frac{9}{4} = 2.25$$

14. The interference pattern is obtained with two coherent light sources of intensity ratio 4 :1. And the ratio $\frac{I_{max}+I_{min}}{I_{max}-I_{min}}$ is $\frac{5}{x}$. Then, the value of x will be equal to :

(A) 3 (B) 4

(C) 2 (D) 1

Official Ans. by NTA (B)

Sol. $\frac{I_1}{I_2}=4$

$$\frac{I_{max}}{I_{min}}=\left[\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2$$

$$\frac{I_{max}}{I_{min}}=\left[\frac{2\sqrt{I_2}+\sqrt{I_2}}{2\sqrt{I_2}-\sqrt{I_2}}\right]^2$$

$$\frac{I_{max}}{I_{min}}=9$$

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}}=\frac{10}{8}$$

$$\frac{5}{x}=\frac{10}{8}$$

x = 4

15. A light whose electric field vectors are completely removed by using a good Polaroid, allowed to incident on the surface of the prism at Brewster's angle. Choose the most suitable option for the phenomenon related to the prism.

(A) Reflected and refracted rays will be perpendicular to each other

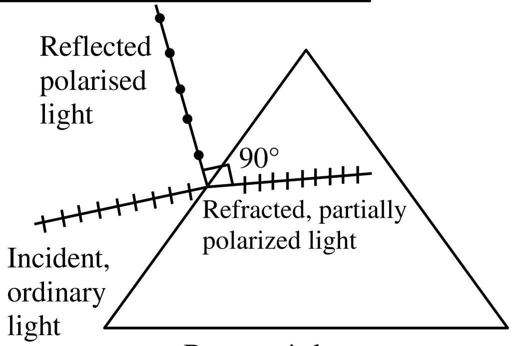
(B) Wave will propagate along the surface of prism

(C) No refraction, and there will be total reflection of light.

(D) No reflection and there will be total transmission of light.

Official Ans. by NTA (D)

Sol.



Brewster's law

But as the incident light electric field vectors are completely removed so there will be no reflection and there will be total transmission of light, explained by an experiment in NCERT.

[Reference NCERT Part-2 Pg-380, (A special case of total transmission)]

Note : Since direction of polarization is not mentioned hence most suitable option (D) corresponding to case in which electric field is absent perpendicular to plane consisting incident and normal.

16. A proton, a neutron, an electron and an α-particle have same energy. If $\lambda_p, \lambda_n, \lambda_e$ and λ_α are the de Broglie's wavelengths of proton, neutron, electron and α particle respectively, then choose the correct relation from the following :

(A) $\lambda_p=\lambda_n>\lambda_e>\lambda_\alpha$

(B) $\lambda_\alpha<\lambda_n<\lambda_p<\lambda_e$

(C) $\lambda_e<\lambda_p=\lambda_n>\lambda_\alpha$

(D) $\lambda_e=\lambda_p=\lambda_n=\lambda_\alpha$

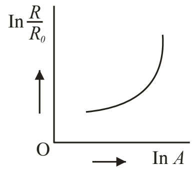
Official Ans. by NTA (B)

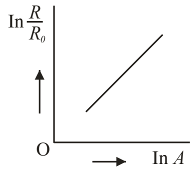
Sol. $\lambda=\frac{h}{\sqrt{2Em}}$

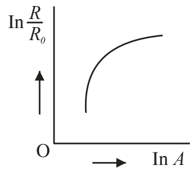
$$\lambda\propto\frac{1}{\sqrt{m}}$$

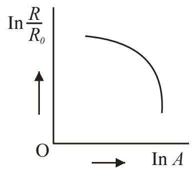
$\therefore$ $\lambda_e>\lambda_p>\lambda_n>\lambda_\alpha$

17. Which of the following figure represents the variation of $\ln\left(\frac{R}{R_0}\right)$ with ln A(If R = radius of a nucleus and A = its mass number)

(A)


(B)


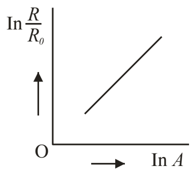
(C)


(D)


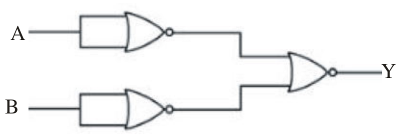
Official Ans. by NTA (B)

Sol. $R = R_0 A^{\frac{1}{3}}$

$\ln\frac{R}{R_0} = \frac{1}{3}\ln A$



18. Identify the logic operation performed by the given circuit :



(A) AND gate (B) OR gate
(C) NOR gate (D) NAND gate

Official Ans. by NTA (A)

Sol. $=\overline{\left[\overline{\left[\overline{A+A}\right]}+\overline{\left[\overline{B+B}\right]}\right]}$

$Y=\overline{\overline{A}+\overline{B}}$ (D' MORGAN LAW)

Y = AB

19. Match List I with List II

List -I		List – II	
A	Facsimile	I.	Static Document Image
B.	Guided media Channel	II.	Local Broadcast Radio
C.	Frequency Modulation	III.	Rectangular wave
D.	Digital Signal	IV.	Optical Fiber

Choose the correct answer from the following options :

(A) A –IV, B-III, C-II, D-I

(B)A-I, B-IV, C-II, D-III

(C) A –IV, B-II, C-III, D-I

(D) A-I, B-II, C-III, D-IV

Official Ans. by NTA (B)

Sol. Question based on the theory given in NCERT.

20. If n represents the actual number of deflections in a converted galvanometer of resistance G and shunt resistance S. Then the total current I when its figure of merit is K will be :

(A) $\frac{KS}{(S+G)}$ (B) $\frac{(G+S)}{nKS}$

(C) $\frac{nKS}{(G+S)}$ (D) $\frac{nK(G+S)}{S}$

Official Ans. by NTA (D)

Sol.

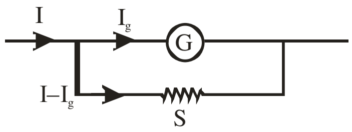


Figure of merit $\frac{I_g}{\theta} = K$

$I_g = Kn$

$I = \frac{I_g}{s}(G+S)$

$I = \frac{nK}{S}(G+S)$

SECTION-B

1. For $z = a^2x^3y^{\frac{1}{2}}$, where 'a' is a constant. If percentage error in measurement of 'x' and 'y' are 4% and 12%, respectively, then the percentage error for 'z' will be %.

Official Ans. by NTA (18)

Sol. $z = a^2x^3y^{1/2}$

$$\frac{\Delta z}{z} = \frac{2\Delta a}{a} + \frac{3\Delta x}{x} + \frac{1}{2}\frac{\Delta y}{y}$$

a is constant

$$\frac{\Delta z}{z} \times 100 = 3(4\%) + \frac{1}{2}(12\%) = 18\%$$

2. A curved in a level road has a radius 75m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48 m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be__ m/s.

Official Ans. by NTA (24)

Sol. $f_{s\max} = \frac{mv^2}{R}$

$\mu mg = \frac{mv^2}{R}$

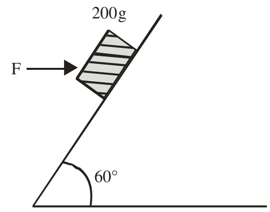
$v = \sqrt{\mu Rg}$

$\frac{v_2}{v_1} = \sqrt{\frac{R_2}{R_1}}$

$\frac{v_2}{30} = \sqrt{\frac{48}{75}}$

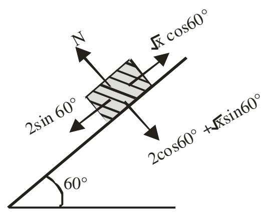
$v_2 = 24\,m/s$

3. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F = \sqrt{x}N$ as shown in figure. The value of x = ________.



Official Ans. by NTA (12)

Sol. mg = 2N



$\sqrt{x}\frac{1}{2} = \frac{2\sqrt{3}}{2}$

x = 12

4. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '2R' are as follows:

I_1= M.I. of solid sphere about its diameter

I_2 = M.I. of solid cylinder about its axis

I_3 = M.I. of solid circular disc about its diameter

I_4 = M.I. of thin circular ring about its diameter

If $2(I_2 + I_3) + I_4 = x.\ I_1$ then the value of x will be ____

Official Ans. by NTA (5)

Sol. $I_1 = \frac{2}{5}M(2R)^2 = \frac{8}{5}MR^2$

$I_1 = \frac{1}{2}M(2R)^2 = 2MR^2$

$I_3 = \frac{M(2R)^2}{4} = MR^2$

$I_4 = \frac{M(2R)^2}{2} = 2MR^2$

$2(I_2 + I_3) + I_4 = x\ I_1$

$8MR^2 = x\frac{8}{5}MR^2$

$x = 5$

5. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius R_1 = 3200 km and R_2 = 800 km respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =

Official Ans. by NTA (2)

Sol. $V = \frac{GM}{r} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$

6. When a gas filled in a closed vessel is heated by raising the temperature by 1°C, its pressure increase by 0.4%. The initial temperature of the gas is______ K.

Official Ans. by NTA (250)

Sol. $pV = nRT$

$\Delta P.V = nR\Delta T$

$\Rightarrow \frac{\Delta P}{P} = \frac{\Delta T}{T} = \frac{0.4}{100}$

$\Rightarrow T = \frac{100\times 1}{0.4} = 250K$

7. 27 identical drops are charged at 22V each. They combine to form a bigger drop. The potential of the bigger drop will be ____ V.

Official Ans. by NTA (198)

Sol. $q \rightarrow nq$

$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r')^3$

$\Rightarrow r' = n^{\frac{1}{3}}r$

$V = \frac{kq}{r} \propto \frac{n}{n^{1/3}} \propto n^{2/3} \propto 27^{2/3} \Rightarrow v' = 9V = 9\times 22 = 198$

8. The length of a given cylindrical wire is increased to double of its original length. The percentage increase in the resistance of the wire will be ____%.

Official Ans. by NTA (300)

Sol. V' = V

$\ell' A = \ell A$

$2\ell A' = \ell A$

$A' = \frac{A}{2}$

$R = \rho\frac{\ell}{A}$ ... (i)

$\ell' = 2\ell$

$A' = \frac{A}{2}$

$R' = \frac{\rho\ell'}{A'} = \frac{\rho 2\ell}{\frac{A}{2}}$

$R' = \frac{4\rho\ell}{A}$

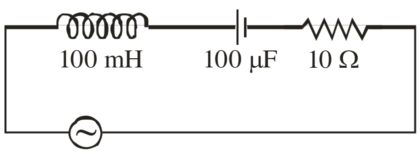
R' = 4R from equation (i)

% increase in resistance

$= \frac{R' - R}{R}\times 100 = \frac{4R - R}{R}\times 100$

= 300 %

9. In a series LCR circuit, the inductance, capacitance and resistance are L = 100mH, C = 100μF and R = 10Ω respectively. They are connected to an AC source of voltage 220V and frequency of 50 Hz. The approximate value of current in the circuit will be____ A.



Official Ans. by NTA (22)

Sol. $X_L = \omega L = 2\pi \times 50 \times 10^{-1} = 10\pi$

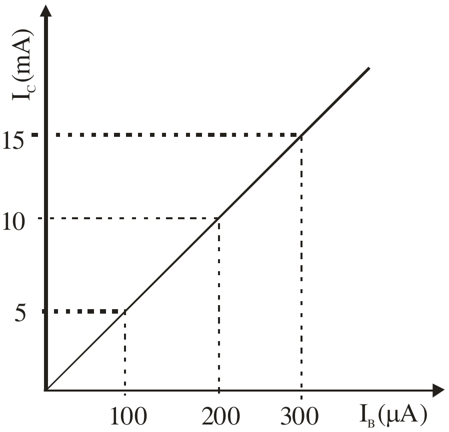
$$X_X = \frac{1}{\omega C} = \frac{1}{2\pi \times 50} \times 10^4 = \frac{100}{\pi}$$

$R = 10\Omega$

$$Z = \sqrt{\left(10\pi - \frac{100}{\pi}\right)^2 + 10^2} \simeq 10\Omega$$

$$i = \frac{E}{2} \simeq \frac{220}{10} \simeq 22\text{Amp}$$

10. In an experiment of CE configuration of n-p-n transistor, the transfer characteristics are observed as given in figure.



If the input resistance is 200Ω and output resistance is 60 Ω the voltage gain in this experiment will be ______

Official Ans. by NTA (15)

Sol. $$\text{Voltage Gain} = \frac{I_C}{I_B} \times \frac{R_0}{R_I} = \frac{10 \times 10^{-3}}{200 \times 10^{-6}} \times \frac{60}{200} = 15$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. The minimum energy that must be possessed by photons in order to produce the photoelectric effect with platinum metal is:

[Given: The threshold frequency of platinum is $1.3 \times 10^{15}\ s^{-1}$ and $h = 6.6 \times 10^{-34}$ J s.]

(A) 3.21×10^{-14} J (B) 6.24×10^{-16} J

(C) 8.58×10^{-19} J (D) 9.76×10^{-20} J

Official Ans. by NTA (C)

Sol. $W = h\nu$

$= 6.6 \times 10^{-34} \times 1.3 \times 10^{15}$

$= 8.58 \times 10^{-19}$ J

2. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (1) and acetylene (g) are –3268 kJ mol^{-1} and –1300 kJ mol^{-1}, respectively. The change in enthalpy for the reaction $3\ C_2H_2(g) \rightarrow C_6H_6(l)$, is

(A) + 324 kJ mol^{-1} (B) +632 kJ mol^{-1}

(C) – 632 kJ mol^{-1} (D) – 732 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. $\Delta H = \Sigma\Delta H_{Combustion}$ (Reactant) - $\Sigma\Delta H_{Combustion}$ (Product)

$= 3 \times (-1300) - [-3268]$

$= -632$ kJ mol^{-1}

3. Solute A associates in water. When 0.7 g of solute A is dissolved in 42.0 g of water, it depresses the freezing point by 0.2°C. The percentage association of solute A in water, is

[Given : Molar mass of A = 93 g mol^{-1}. Molal depression constant of water is 1.86 K kg mol^{-1}]

(A) 50 % (B) 60 %

(C) 70 % (D) 80 %

Official Ans. by NTA (D)

Sol. $\Delta T = i\ k_f \times m$

$$0.2 = i \times 1.86 \times \frac{0.7}{93} \times \frac{1000}{42}$$

$$i = \frac{0.2 \times 93 \times 6}{1.86 \times 100}$$

$i = 0.60$

$2A \rightleftharpoons A_2$

$1-\alpha \qquad \frac{\alpha}{2}$

$$i = 1 - \alpha + \frac{\alpha}{2}$$

$$i = 1 - \frac{\alpha}{2}$$

$$1 - \frac{\alpha}{2} = 0.60$$

$$1 - 0.60 = \frac{\alpha}{2}$$

$\alpha = 0.80$

4. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol L^{-1} at 298 K is

(A) 1.0×10^{-15} (B) 2.7×10^{-12}

(C) 3.2×10^{-10} (D) 4.2×10^{-8}

Official Ans. by NTA (A)

Sol. $Bi_2S_3 \rightleftharpoons 2Bi^{3+} + 3S^{2-}$

$\qquad\qquad 2s \qquad 3s$

$k_{sp} = (2s)^2\ (3s)^3$

$= 4s^2 \times 27\ (s)^3$

$= 108\ (s)^5$

$$(s)^5 = \frac{1.08 \times 10^{-73}}{108}$$

$\Rightarrow s = 10^{-15}$

5. Match List I with List II.

List I	List II
A. Zymase	I. Stomach
B. Diastase	II. Yeast
C. Urease	III. Malt
D. Pepsin	IV. Soyabean

Choose the correct answer from the options given below:

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-II, C-I, D-IV

Official Ans. by NTA (B)

Sol. Zymase naturally occurs in yeast.

Diastase is found in malt.

Urease is found in soyabean

Pepsin is found in stomach

6. The correct order of electron gain enthalpies of Cl, F, Te and Po is

(A) F < Cl < Te < Po (B) Po < Te < F < Cl

(C) Te < Po < Cl < F (D) Cl < F < Te < Po

Official Ans. by NTA (D)

Sol. As Cl has maximum electron affinity among all elements.

Element	**$\Delta_{eg}H$ (kJ/mol)**
F	–328
Cl	–349
Te	–190
Po	–174

7. Given below are two statements.

Statement I: During electrolytic refining, blister copper deposits precious metals

Statement II: In the process of obtaining pure copper by electrolysis method, copper blister is used to make the anode.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (A)

Sol. In the electro-refining, impure metal (here blister copper) is used as an anode while precious metal like Au, Pt get deposited as anode mud.

8. Given below are two statements one is labelled as **Assertion A** and the other is labelled as **Reason R:**

Assertion A : The amphoteric nature of water is explained by using Lewis acid/base concept.

Reason R : Water acts as an acid with NH_3 and as a base with H_2S.

In the light of the above statements choose the correct answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (D)

Sol. $\underset{\text{Acid}}{H_2S} + \underset{\text{Base}}{H_2O} \rightleftharpoons H_3O^+ + HS^-$

$\underset{\text{Acid}}{H_2O} + \underset{\text{Base}}{NH_3} \rightleftharpoons NH_4OH$

9. The correct order of reduction potentials of the following pairs is

A. Cl_2/Cl^-

B. I_2/I^-

C. Ag^+/Ag

D. Na^+/Na

E. Li^+/Li

Choose the correct answer from the options given below.

(A) A > C > B > D > E

(B) A > B > C > D > E

(C) A > C > B > E > D

(D) A > B > C > E > D

Official Ans. by NTA (A)

Sol. $E^\circ_{Cl_2/Cl^-} = +1.36\,V$

$E^\circ_{I_2/I^-} = +0.54\,V$

$E^\circ_{Ag^+/Ag} = +0.80\,V$

$E^\circ_{Na^+/Na} = -2.71\,V$

$E^\circ_{Li^+/Li} = -3.05\,V$

10. The number of bridged oxygen atoms present in compound B formed from the following reactions is

$$Pb(NO_3)_2 \xrightarrow{673\ K} A + PbO + O_2$$

$$A \xrightarrow{\text{Dimerise}} B$$

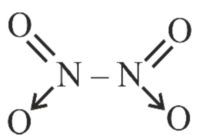
(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (A)

Sol. $$\underset{}{Pb(NO_3)_2} \xrightarrow[673\ K]{\Delta} PbO + \underset{(A)}{NO_2} + O_2$$

$$\underset{A}{2NO_2} \xrightarrow{\text{Dimerise}} \underset{B}{N_2O_4}$$



(no bridged oxygen)

11. The metal ion (in gaseous state) with lowest spin-only magnetic moment value is

(A) V^{2+} (B) Ni^{2+}

(C) Cr^{2+} (D) Fe^{2+}

Official Ans. by NTA (B)

Sol. V^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^3$

[↑][↑][↑][ ][ ] (3d) (unpaired e^- = 3)

Ni^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^8$

[↑↓][↑↓][↑↓][↑][↑] (3d) (unpaired e^- = 2)

Cr^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^4$

[↑][↑][↑][↑][ ] (3d) (unpaired e^- = 4)

Fe^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^6$

[↑↓][↑][↑][↑][↑] (3d) (unpaired e^- = 4)

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Polluted water may have a value of BOD of the order of 17 ppm.

Reason R: BOD is a measure of oxygen required to oxidise both the biodegradable and non-biodegradable organic material in water.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Official Ans. by NTA (C)

Sol. Clean water have BOD less than 5 ppm while highly polluted water has BOD greater or equal to 17 ppm. So, assertion is correct.

BOD is measure of oxygen required to oxidise only bio-degradable organic matter. So, reason is false.

13. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: A mixture contains benzoic acid and napthalene. The pure benzoic acid can be separated out by the use of benzene.

Reason R: Benzoic acid is soluble in hot water.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (D)

Sol. Benzoic acid and Napthalene can be effectively separated by crystallization. Benzoic acid is soluble in hot water whereas Napthalene is insoluble.

Hence assertion is incorrect but reason is correct

14. During halogen test, sodium fusion extract is boiled with concentrated HNO_3 to

(A) remove unreacted sodium

(B) decompose cyanide or sulphide of sodium

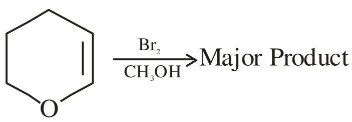
(C) extract halogen from organic compound

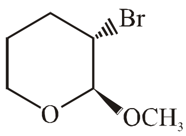
(D) maintain the pH of extract

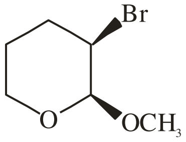
Official Ans. by NTA (B)

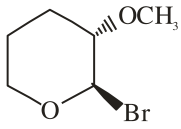
Sol. Sodium fusion extract is boiled with concentrated HNO_3 to remove sodium cyanide and sodium sulphide

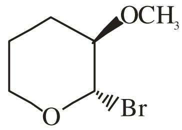
15. Amongst the following, the major product of the given chemical reaction is



(A) 

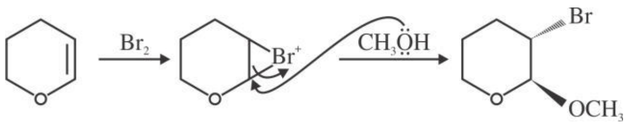
(B) 

(C) 

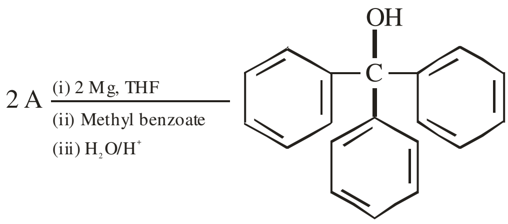
(D) 

Official Ans. by NTA (A)

Sol.



16. In the given reaction



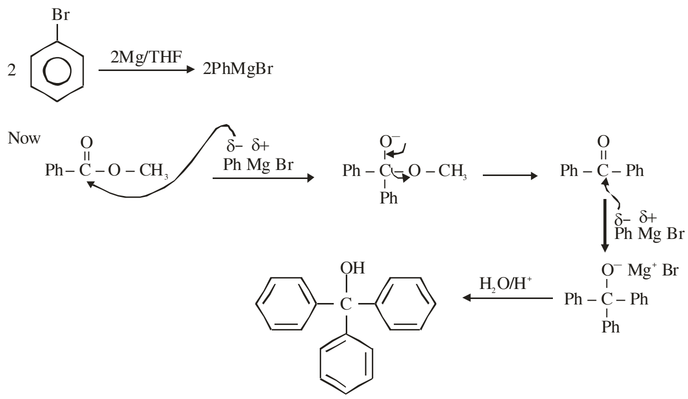
'A' can be

(A) benzyl bromide (B) bromobenzene

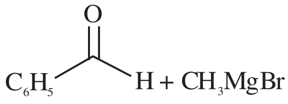
(C) cyclohexyl bromide (D) methyl bromide

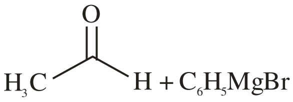
Official Ans. by NTA (B)

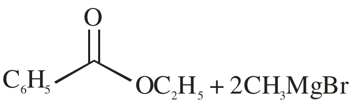
Sol.



17. Which of the following conditions or reaction sequence will NOT give acetophenone as the major product ?

(A) (a)  (b) $Na_2Cr_2O_7$, H^+

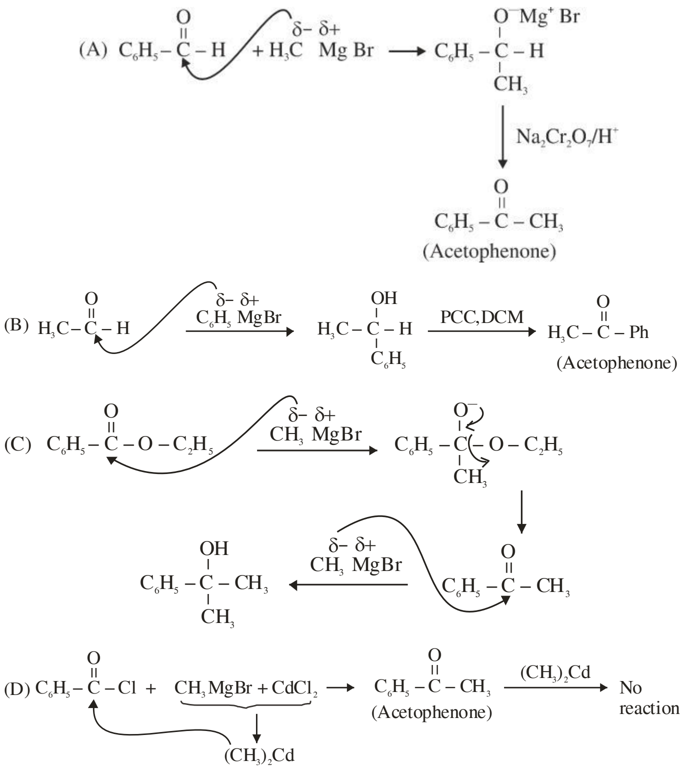
(B) (a)  (b) PCC, DCM

(C) 

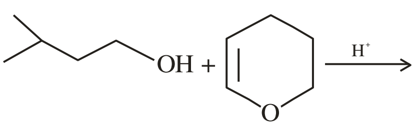
(D) 

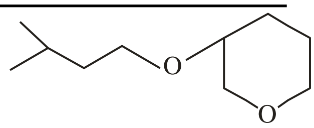
Official Ans. by NTA (C)

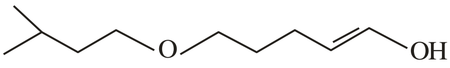
Sol.

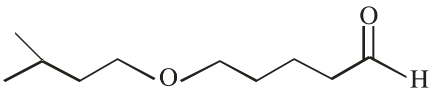


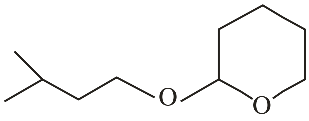
18. The major product formed in the following reaction, is



(A) 

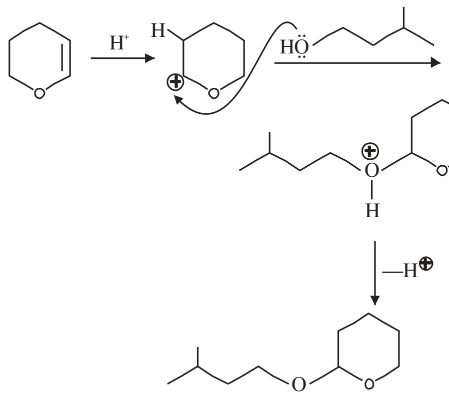
(B) 

(C) 

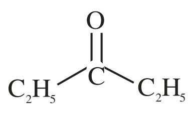
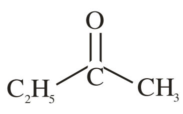
(D) 

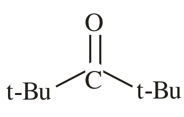
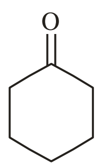
Official Ans. by NTA (D)

Sol.



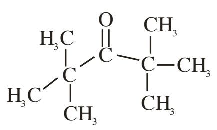
19. Which of the following ketone will NOT give enamine on treatment with secondary amines? [where t-Bu is $-C(CH_3)_3$]

(A)  (B) 

(C)  (D) 

Official Ans. by NTA (C)

Sol. Enamine formation is an example of nucleophilic addition elimination reaction

Since in ketone  Carbonyl

Group is highly sterically hindered hence attack of nucleophile will not be possible.

20. An antiseptic dettol is a mixture of two compounds 'A' and 'B' where A has 6π electrons and B has 2π electrons. What is 'B'?

(A) Bithionol

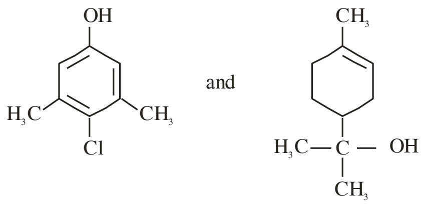
(B) Terpineol

(C) Chloroxylenol

(D) Chloramphenicol

Official Ans. by NTA (B)

Sol. Dettol is mixture of



Chloroxylenol (Compound A) It has $6\pi e^-$ Terpineol (Compound B) It has $2\pi e^-$

Hence compound 'B' is Terpineol.

SECTION-B

1. A protein 'A' contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein 'A' is________ $\times$ 10^3 g mol^{-1} [nearest integer]

Official Ans. by NTA (25)

Sol. 0.30 % glycine is equal to 75

$$1\ \% \longrightarrow \frac{75}{0.30}$$

$$100\ \% \longrightarrow \frac{75}{0.30}\times 100$$

$$= 25000\ g$$

2. A rigid nitrogen tank stored inside a laboratory has a pressure of 30 atm at 06:00 am when the temperature is 27 °C. At 03:00 pm, when the temperature is 45°C, the pressure in the tank will be ____________atm. [nearest integer]

Official Ans. by NTA (32)

Sol. $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{30}{300} = \frac{P_2}{318}$$

$$P_2 = \frac{30}{300}\times 318$$

$$= \frac{1}{10}\times 318$$

$$= 32$$

3. Amongst BeF_2, BF_3, H_2O, NH_3, CCl_4 and HCl, the number of molecules with non-zero net dipole moment is __________.

Official Ans. by NTA (3)

Sol. BeF_2, BF_3 and $CCl_4 \Rightarrow \mu_{net} = 0$

H_2O, NH_3 and $HCl \Rightarrow \mu_{net} \neq 0$

4. At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 27.8 kPa, the half life was fund to be 170 s. The order of the reaction is________. [integer answer]

Official Ans. by NTA (0)

Sol. $t_{1/2} \times \frac{1}{[P_0]^{n-1}}$

$$\frac{t_1}{t_2} = \frac{(P_2)^{n-1}}{(P_1)^{n-1}}$$

$$\frac{340}{170} = \left(\frac{27.8}{55.5}\right)^{n-1}$$

$$\Rightarrow\ 2 = \frac{1}{(2)^{n-1}}$$

$$n = 0$$

5. A solution of $Fe_2(SO_4)_3$ is electrolyzed for 'x' min with a current of 1.5 A to deposit 0.3482 g of Fe. The value of x is______. [nearest integer]

Given : 1 F = 96500 C mol^{-1}

Atomic mass of Fe = 56 g mol^{-1}

Official Ans. by NTA (20)

Sol. $Fe^{3+} + 3e^- \longrightarrow Fe$

$3F \longrightarrow$ 1 mole Fe is deposited

For 56 g $\longrightarrow 3 \times 96500$ (required charge)

For 1g $\longrightarrow \frac{3 \times 96500}{56}$ (required charge)

For 0.3482 g $\longrightarrow \frac{3 \times 96500}{56} \times 0.3482$

$= 1800.06$

$Q = it$

$1800.06 = 1.5\ t$

$t = 20$ min

6. Consider the following reactions :

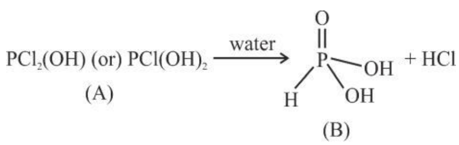
$PCl_3 + H_2O \longrightarrow A + HCl$

$A + H_2O \longrightarrow B + HCl$

number of ionisable protons present in the product B________.

Official Ans. by NTA (2)

Sol. $PCl_3 + H_2O \xrightarrow[\text{hydrolysis}]{\text{Partial}} PCl_2(OH)$ (or) $PCl(OH)_2$ + HCl



no. of ionisable protons in B = 2

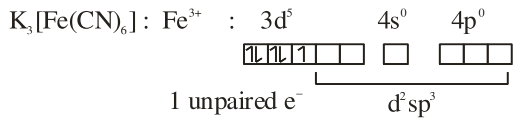
7. Amongst $FeCl_3.3H_2O$, $K_3[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$, the spin-only magnetic moment value of the inner-orbital complex that absorbs light at shortest wavelength is________ B.M. [nearest integer]

Official Ans. by NTA (2)

Sol. $[Fe(H_2O)_3Cl_3]$, $\underbrace{K_3[Fe(CN)_6], [Co(NH_3)_6]Cl_3}_{\text{inner orbital complexes}}$

$K_3[Fe(CN)_6]$ has more value of Δ_0 than that of $[Co(NH_3)_6]Cl_3$; as $\bar{C}N$ is stronger ligand.

More $\Delta_0 \Rightarrow$ smaller value of absorbed λ



Spin only magnetic moment (μ) = $\sqrt{3}$ BM

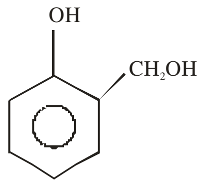
= 1.732 BM

Rounding off ⇒ 2

8. The Novolac polymer has mass of 963 g. The number of monomer units present in it are

Official Ans. by NTA (9)

Sol. Monomer unit of Novolac is



its molecular mass is 124 amu.

Upon considering molecular weight of polymer as 963 amu (In question its given as 963 gram)

Now if during formation of Novolac, (n–1) unit of water are removed then

$n \times 124 = 963 + [18 \times (n-1)]$

n = 9

9. How many of the given compounds will give a positive Biuret test ______ ? Glycine, Glycylalanine, Tripeptide, Biuret

Official Ans. by NTA (2)

Sol. Biuret test is given by all proteins and peptides having atleast two peptide linkages.

Hence positive test must be given by tripeptide and Biuret.

10. The neutralization occurs when 10 mL of 0.1 M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is______. [M is a metal]

Official Ans. by NTA (3)

Sol. Acid + Base $\longrightarrow$ Salt + H_2O

Acid	Base
0.1 M	$M(OH)_2$
10ml	0.05 M
	30 ml

at equivalence point

equivalent of acid = equivalent of base

$0.1 \times 10 \times n = 30 \times 0.05 \times 2$

n = 3

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $A=\{x\in R:|x+1|<2\}$ and $B=\{x\in R:|x-1|\geq 2\}$. Then which one of the following statements is **NOT** true ?

(A) $A-B=(-1,1)$ (B) $B-A=R-(-3,1)$

(C) $A\cap B=(-3,-1]$ (D) $A\cup B=R-[1,3)$

Official Ans. by NTA (B)

Sol. $A:x\in(-3,1)$ $B:x\in(-\infty,-1]\cup[3,\infty)$

$B-A=(-\infty,-3]\cup[3,\infty)=R-(-3,3)$

2. Let $a,b\in R$ be such that the equation $ax^2-2bx+15=0$ has a repeated root α. If α and β are the roots of the equation $x^2-2bx+21=0$, then $\alpha^2+\beta^2$ is equal to:

(A) 37 (B) 58

(C) 68 (D) 92

Official Ans. by NTA (B)

Sol. $ax^2-2bx+15=0$

$2\alpha=\frac{2b}{a},\alpha^2=\frac{15}{a}$

$\frac{\alpha}{2}=\frac{15}{2b}$

$\alpha=\frac{15}{b}$

$x^2-2bx+21=0$

$\left(\frac{15}{b}\right)^2-2b\left(\frac{15}{b}\right)+21=0$

$b^2=25$

$\alpha+\beta=2b,\ \alpha\beta=21$

$\alpha^2+\beta^2=4b^2-42$

$=58$

3. Let z_1 and z_2 be two complex numbers such that $\overline{z}_1=i\overline{z}_2$ and $\arg\left(\frac{z_1}{\overline{z}_2}\right)=\pi$. Then

(A) $\arg z_2=\frac{\pi}{4}$ (B) $\arg z_2=-\frac{3\pi}{4}$

(C) $\arg z_1=\frac{\pi}{4}$ (D) $\arg z_1=-\frac{3\pi}{4}$

Official Ans. by NTA (C)

Sol. $\overline{z}_1=i\overline{z}_2$

$z_1=-iz_2$

$\arg\left(\frac{z_1}{\overline{z}_2}\right)=\pi$

$\arg\left(-i\frac{z_2}{\overline{z}_2}\right)=\pi$ $\arg(z_2)=\theta$

$-\frac{\pi}{2}+\theta+\theta=\pi$

$2\theta=\frac{3\pi}{2}$

$\arg(z_2)=\theta=\frac{3\pi}{4}$, $\arg z_1=\frac{\pi}{4}$

4. The system of equations

$-kx+3y-14z=25$

$-15x+4y-kz=3$

$-4x+y+3z=4$

is consistent for all k in the set

(A) R (B) $R-\{-11,13\}$

(C) $R-\{13\}$ (D) $R-\{-11,11\}$

Official Ans. by NTA (D)

Sol. $\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2$

$\Delta \neq 0 \qquad k \in R - \{11, -11\}$ (Unique sol.)

If k = 11

$\Delta_z = \begin{vmatrix} -11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$

No solution

If k = −11

$\Delta_z = \begin{vmatrix} 11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$

No solution

5. $\lim_{x \to \frac{\pi}{2}} \left(\tan^2 x \left(\left(2\sin^2 x + 3\sin x + 4\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right) \right)$

is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$ (D) $-\frac{1}{6}$

Official Ans. by NTA (A)

Sol.

$\lim_{x \to \frac{\pi}{2}} \tan^2 x \left[\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right] =$

$\lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x\, [\sin^2 x - 3\sin x + 2]}{\sqrt{9} + \sqrt{9}}$

$= \lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x (\sin x - 1)(\sin x - 2)}{6}$

$= \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \tan^2 x (1 - \sin x)$

$= \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \frac{\sin^2 x\, (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{1}{12}$

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

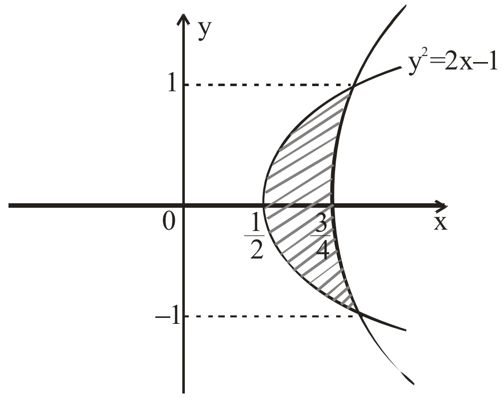
(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Sol. Required area = $2\int_0^1 \left(\frac{y^2 + 3}{4} - \frac{y^2 + 1}{2} \right) dy$

$= 2\int_0^1 \frac{1 - y^2}{4} dy = \frac{1}{2} \left| y - \frac{y^3}{3} \right|_0^1 = \frac{1}{3}$



7. The coefficient of x^{101} in the expression $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ...x^{500}$, $x > 0$, is

(A) ${}^{501}C_{101}(5)^{399}$ (B) ${}^{501}C_{101}(5)^{400}$

(C) ${}^{501}C_{100}(5)^{400}$ (D) ${}^{500}C_{101}(5)^{399}$

Official Ans. by NTA (A)

Sol. $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}$

$= \frac{(5+x)^{501} - x^{501}}{(5+x) - x} = \frac{(5+x)^{501} - x^{501}}{5}$

$\Rightarrow$ coefficient x^{101} in given expression

$= \frac{{}^{501}C_{101} 5^{400}}{5} = {}^{501}C_{101} 5^{399}$

8. The sum $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ is equal to

(A) $\frac{2\cdot3^{12}+10}{4}$ (B) $\frac{19\cdot3^{10}+1}{4}$

(C) $5\cdot3^{10}-2$ (D) $\frac{9\cdot3^{10}+1}{2}$

Official Ans. by NTA (B)

Sol. $S=1\cdot3^0+2.3^1+3\cdot3^2+.....+10.3^9$

$3S=1\cdot3^1+2.3^2.......................+9\times3^9+10\times3^{10}$

$-2S=(1\cdot3^0+3^1+3^2.....3^9)-10.3^{10}$

$S=5\times3^{10}-\left(\frac{3^{10}-1}{4}\right)$

$S=\frac{20.3^{10}-3^{10}+1}{4}=\frac{19.3^{10}+1}{4}$

9. Let P be the plane passing through the intersection of the planes

$\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$, and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P

(B) Y and Y – X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X – Y are on the same side of P

Official Ans. by NTA (C)

Sol. $P_1+\lambda P_2=0$

$\Rightarrow(x+3y-z-5)+\lambda(2x-y+z-3)=0$

$(2,1,-2)$ lies on this plane

$\therefore\lambda=1\Rightarrow$ plane is $3x+2y-8=0$

10. A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is

(A) $y=\sqrt{2}x$ (B) $x=\sqrt{2}y$

(C) $y^2-x^2=2xy$ (D) $x^2-y^2=2xy$

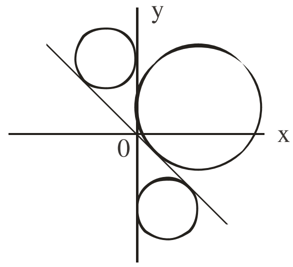
Official Ans. by NTA (D)

Sol. Let (h, k) is centre of circle

$\left|\frac{h-k}{\sqrt{2}}\right|=|h|$

$k^2-h^2+2hk=0$

$\therefore$ Equation of locus is $y^2-x^2+2xy=0$



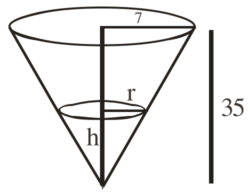
11. Water is being filled at the rate of $1\text{ cm}^3/\text{sec}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is

(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Official Ans. by NTA (C)

Sol. From figure $\frac{r}{h}=\frac{7}{35}\Rightarrow h=5r$



Given $\frac{dV}{dt}=1\Rightarrow\frac{d}{dt}\left(\frac{\pi r^2h}{3}\right)=1$

$\Rightarrow\frac{d}{dt}\left(\frac{5\pi}{3}r^3\right)=1\ \Rightarrow r^2\frac{dr}{dt}=\frac{1}{5\pi}$

Let wet conical surface area = S

$=\pi r\ell=\pi r\sqrt{h^2+r^2}$

$=\sqrt{26}\pi r^2\Rightarrow\frac{dS}{dt}=2\sqrt{26}\pi r\frac{dr}{dt}$

When h = 10 then r = 2 $\Rightarrow\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx, n \in N$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference –2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference –2

Official Ans. by NTA (D)

Sol. $b_n = \int_0^{\pi/2} \frac{1+\cos 2nx}{\sin x} dx$

$b_{n+1} - b_n = \int_0^{\pi/2} \frac{\cos^2(n+1)x - \cos^2 nx}{\sin x} dx$

$= \int_0^{\pi/2} \frac{-\sin(2n+1)x \sin x}{\sin x} dx$

$= \left(\frac{\cos(2n+1)x}{2n+1}\right)_0^{\pi/2} = \frac{-1}{2n+1}$

$\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with c.d.= –2

13. If y = y(x) is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx} - \frac{y}{x} = -\frac{3}{2}\left(\frac{y}{x}\right)^2$ $y = vx$

$\frac{dv}{v^2} = -\frac{3dx}{2x}$

$-\frac{1}{v} = -\frac{3}{2}\ell n|x| + C$

$-\frac{x}{y} = \frac{-3}{2}\ell n|x| + C$

$x = e,\ y = \frac{e}{3}$

$C = -\frac{3}{2}$

When x = 1, y = $\frac{2}{3}$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve x = 12(t + sin t cos t), $y = 12(1+\sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6\left(3+2\sqrt{2}\right)$ (B) $3\left(7+4\sqrt{3}\right)$

(C) 27 (D) 48

Official Ans. by NTA (C)

Sol. $\frac{dy}{dx} = \frac{2(1+\sin t) \times \cos t}{1+\cos 2t}$

$\Rightarrow \frac{2(1+\sin t)\cos t}{2\cos^2 t} = \sqrt{3}$

$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$

15. The value of 2sin(12°) – sin(72°) is :

(A) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (B) $\frac{1-\sqrt{5}}{8}$

(C) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (D) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Official Ans. by NTA (D)

Sol. $\sin 12° + \sin 12° - \sin 72°$

$= \sin 12° - 2\cos 42° \sin 30°$

$= \sin 12° - \sin 48°$

$= -2\cos 30° \sin 18°$

$= -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{3}}{4}(1-\sqrt{5})$

16. A biased die is marked with numbers 2,4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Official Ans. by NTA (D)

Sol. $P(n) = \frac{1}{n}$

$P(2) = \frac{1}{2}$ $P(8) = \frac{1}{8}$

$P(4) = \frac{1}{4}$ $P(16) = \frac{1}{16}$

$P(32) = \frac{2}{32}$

Possible cases

16, 16, 16 and 32, 8, 8

Probability = $\frac{1}{16^3} + \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

(A) $p \Rightarrow q$ (B) $q \Rightarrow p$

(C) $\sim (p \Rightarrow q)$ (D) $\sim (q \Rightarrow p)$

Official Ans. by NTA (C)

Sol. $\sim p \vee q \equiv p \to q$

$\sim q \wedge p \equiv \sim (p \to q)$

Negation of $\sim (p \to q) \to (p \to q)$

is $\sim (p \to q) \wedge (\sim (p \to q))$ i.e. $\sim (p \to q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

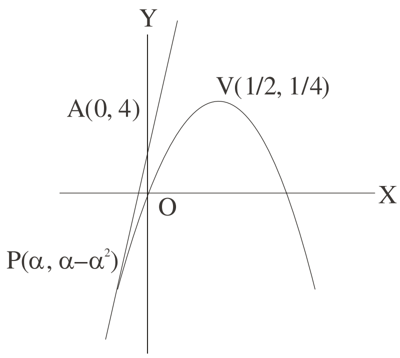
Official Ans. by NTA (C)

Sol. Slope of tangent at P = Slope of line AP

$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$

Solving $\alpha = -2 \Rightarrow P(-2, -6)$

Slope of PV = $\frac{5}{2}$



19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Official Ans. by NTA (B)

Sol. $\tan^{-1}\left[\dfrac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right] \Rightarrow \tan^{-1}\left(\dfrac{\cos\frac{\pi}{4}-1}{\sin\frac{\pi}{4}}\right)$

$\tan^{-1}\left(\dfrac{1-\sqrt{2}}{1}\right)=-\dfrac{\pi}{8}$

20. The line $y=x+1$ meets the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to

(A) 20 (B) 12

(C) 11 (D) 8

Official Ans. by NTA (A)

Sol. Ellipse $x^2+2y^2=4$

Line y = x + 1

Point of intersection

$x^2+2(x+1)^2=4$

$3x^2+4x-2=0$

$|x_1-x_2|=\dfrac{\sqrt{40}}{3}$

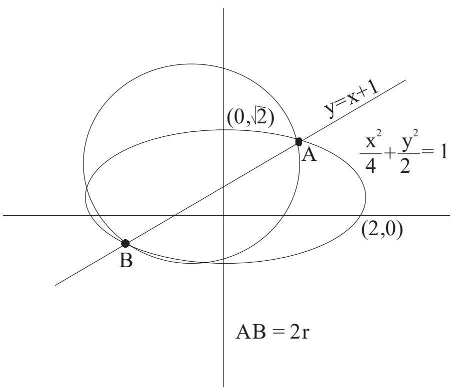
$AB=2r=|x_1-x_2|\sqrt{1+m^2}$,

m is slope of given line

$AB=\dfrac{\sqrt{40}}{3}\sqrt{1+1}$

$2r=\dfrac{\sqrt{80}}{3}\Rightarrow r=\dfrac{\sqrt{80}}{6}$

$(3r)^2=\left(3\times\dfrac{\sqrt{80}}{6}\right)^2=\dfrac{80}{4}=20$



SECTION-B

1. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set

$\{(n,m):n,m\in\{1,2,.....,10\}$ and $nA^n+mB^m=I\}$ is ____

Official Ans. by NTA (1)

Sol. $A^2=A$ and $B^2=B$

Therefore equation $nA^n+mB^m=I$ becomes

nA + mB = I, which gives m = n = 1

Only one set possible

2. Let $f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\ge 0\end{cases}$, where [t] is the greatest integer $\le t$. Then, in the open interval (–1, 1), the number of points where fog is discontinuous is equal to _____

Official Ans. by NTA (62)

Sol. $f(g(x))=\left[2g^2(x)\right]+1$

$=\begin{cases}\left[2(2x-3)^2\right]+1;\ x<0\\ \left[2(2x+3)^2\right]+1; x\ge 0\end{cases}$

$\therefore$ fog is discontinuous whenever $2(2x-3)^2$ or $2(2x+3)^2$ belongs to integer except x = 0.

$\therefore$ 62 points of discontinuity.

3. The value of b > 3 for which

$12\displaystyle\int_3^b\frac{1}{(x^2-1)(x^2-4)}dx=\log_e\left(\frac{49}{40}\right)$, is equal to

Official Ans. by NTA (6)

Sol. $\frac{12}{3}\left[\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx\right]=\log\frac{49}{40}$

$\frac{12}{3}\cdot\left[\frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ell n\left|\frac{x-1}{x+1}\right|\right]_3^b=\log\frac{49}{40}$

$\ell n\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2}=\ell n\frac{49}{50}$

$b = 6$

4. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot3^9$, then β is equal to _____

Official Ans. by NTA (83)

Sol. $T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$= {}^{10}C_r 2^{10-r}3^r x^{30-4r}$

Put r = 0, 1, 2, ... 7 and we get $\beta = 83$

5. If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____

Official Ans. by NTA (21)

Sol. Mean deviation about mean of first n natural numbers is $\frac{n^2-1}{4n}$

$\therefore n=21$

6. Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}\cdot\vec{b}+21=0$, then $\left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)$ is equal to

Official Ans. by NTA (14)

Sol. $\left(\vec{a}\times\vec{b}\right).\vec{b}=0$

$\Rightarrow 13-1-4\lambda=0\Rightarrow\lambda=3$

$\Rightarrow\vec{b}=\hat{i}+\hat{j}+3\hat{k}\ \Rightarrow\ \vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$

$\Rightarrow\ \left(\vec{a}\times\vec{b}\right)\times\vec{b}=\left(13\hat{i}-\hat{j}-4\hat{k}\right)\times\left(\hat{i}+\hat{j}+3\hat{k}\right)$

$\Rightarrow -21\vec{b}-11\vec{a}=\hat{i}-43\hat{j}+14\hat{k}$

$\Rightarrow\ \vec{a}=-2\hat{i}+2\hat{j}-7\hat{k}$

Now $\left(\vec{b}-\vec{a}\right).\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right).\left(\hat{i}-\hat{k}\right)=14$

7. The total number of three-digit numbers, with one digit repeated exactly two times, is

Official Ans. by NTA (243)

Sol. If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1\times2=18$

If 0 not taken then ${}^9C_1\,{}^8C_1.3=216$

Total = 243

8. Let $f(x)=\left|(x-1)(x^2-2x-3)\right|+x-3,\ x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to ____

Official Ans. by NTA (3)

Sol. $f(x)=\begin{cases}\left(x^2-1\right)\left(x-3\right)+\left(x-3\right), x\in(0,1]\cup[3,4)\\ -\left(x^2-1\right)\left(x-3\right)+\left(x-3\right), x\in[1,3]\end{cases}$

$\Rightarrow f'(x)=\begin{cases}3x^2-6x,\ x\in(0,1)\cup(3,4)\\ -3x^2+6x+2,\ x\in(1,3)\end{cases}$

f(x) is non-derivable at x = 1 and x = 3

also $f'(x)=0$ at $x=1+\sqrt{\frac{5}{3}}$ $\Rightarrow$ m + M = 3

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to

Official Ans. by NTA (85)

Sol. $e^2=1+\frac{b^2}{a^2}=\frac{25}{16}\Rightarrow\frac{b^2}{a^2}=\frac{9}{16}$ **……(1)**

$A\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow\frac{64}{5a^2}-\frac{144}{25b^2}=1$ **……(2)**

Solving (1) & (2) $b=\frac{6}{5}$ $a=\frac{8}{5}$

Normal at A is $\frac{\sqrt{5}a^2x}{8}+\frac{5b^2y}{12}=a^2+b^2$

Comparing it $8\sqrt{5}x+\beta y=\lambda$

Gives $\lambda=100,\ \beta=15$

$\lambda-\beta=85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to

Official Ans. by NTA (51)

Sol. $8x+4\sqrt{2}y=1,\ z=0$

$\Rightarrow\frac{x-\frac{1}{8}}{1}=\frac{y-0}{-\sqrt{2}}=\frac{z-0}{0}=\lambda$

$-8x-6\sqrt{3}z=1, y=0$

$\Rightarrow\frac{x+\frac{1}{8}}{3\sqrt{3}}=\frac{y-0}{0}=\frac{z-0}{-4}$

$$\begin{vmatrix}\frac{1}{4} & 0 & 0\\ 1 & -\sqrt{2} & 0\\ 3\sqrt{3} & 0 & -4\end{vmatrix}=\sqrt{2}$$

$d=\frac{1}{\sqrt{51}}$

$\frac{1}{d^2}=51$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

1. An expression for a dimensionless quantity P is given by $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$; where α and β are constants, x is distance ; k is Boltzmann constant and t is the temperature. Then the dimensions of α will be :

(A) $[M^0L^{-1}T^0]$ (B) $[ML^0T^{-2}]$

(C) $[MLT^{-2}]$ (D) $[ML^2T^{-2}]$

Official Ans. by NTA (C)

Sol. $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$

$\frac{kt}{\beta x} = 1 \Rightarrow \beta = \frac{kt}{x} = \frac{ML^2T^{-2}}{L}$

$\left(\because E = \frac{1}{2}kt\right)$

As P is dimensionless

$\Rightarrow [\alpha] = [\beta] = [MLT^{-2}]$

2. A person is standing in an elevator. In which situation, he experiences weight loss ?

(A) When the elevator moves upward with constant acceleration

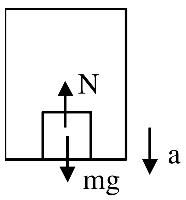
(B) When the elevator moves downward with constant acceleration

(C) When the elevator moves upward with uniform velocity

(D) When the elevator moves downward with uniform velocity

Official Ans. by NTA (B)

Sol.



mg – N = ma

$\Rightarrow N = m(g – a)$

∴ Person experiences weightloss, when acceleration of lift is downward.

3. An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero ?

(A) Momentum (B) Potential energy

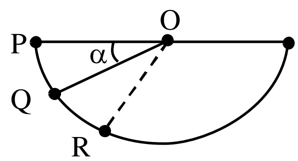
(C) Acceleration (D) Force

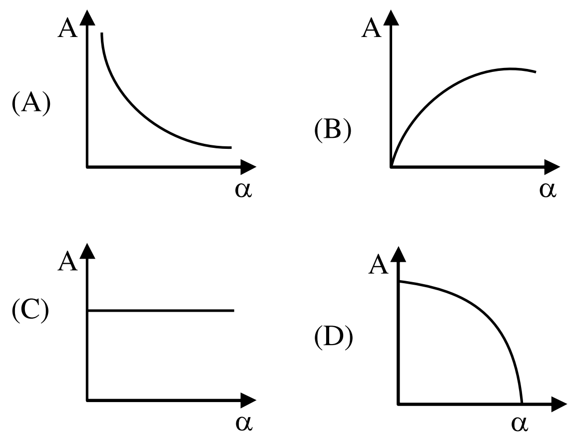
Official Ans. by NTA (A)

Sol. At maximum height, V = 0

∴ Momentum of object is zero.

4. A ball is released from rest from point P of a smooth semi-spherical vessel as shown in figure. The ratio of the centripetal force and normal reaction on the ball at point Q is A while angular position of point Q is α with respect to point P. Which of the following graphs represent the correct relation between A and α when ball goes from Q to R ?

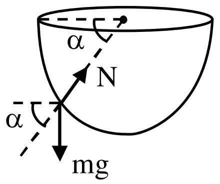




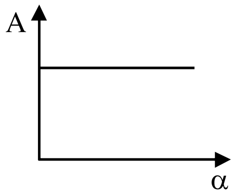
Official Ans. by NTA (C)

Sol. $V = \sqrt{2gR\sin\alpha}$

$N - mg\sin\alpha = \frac{mv^2}{R} = 2mg\sin\alpha$



$\frac{N}{2mg\sin\alpha} = \frac{1}{2} + 1 = \frac{3}{2}$



$\Rightarrow$ A = constant

5. A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 rads^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rads^{-1}).

(A) $\frac{M}{(M+m)}$ (B) $\frac{(M+2m)}{2M}$

(C) $\frac{2M}{(M+2m)}$ (D) $\frac{2(M+2m)}{M}$

Official Ans. by NTA (C)

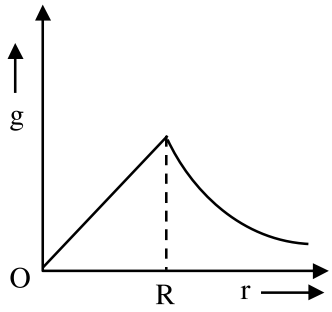
Sol. Applying conservation of angular momentum

$MR^2\omega = (MR^2 + 2mR^2)\omega'$

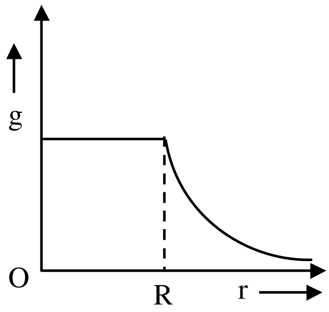
$\omega' = \frac{2M}{M+2m}$

6. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by : (Given R = radius of earth)

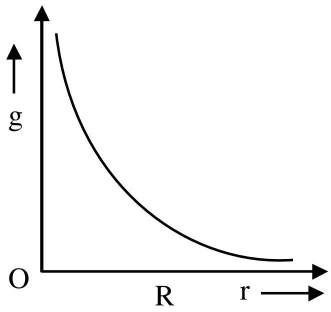
(A)



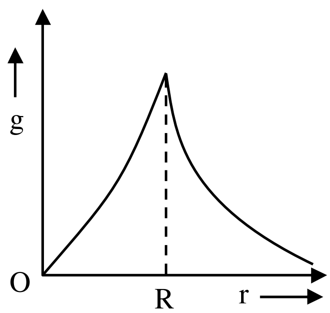
(B)



(C)

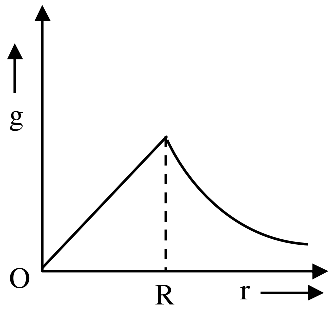


(D)



Official Ans. by NTA (A)

Sol. $g = \begin{cases} \frac{GMr}{R^3}, r \le R \\ \frac{GM}{r^2}, r \ge R \end{cases}$



7. The efficiency of a Carnot's engine, working between steam point and ice point, will be :

(A) 26.81% (B) 37.81%

(C) 47.81% (D) 57.81%

Official Ans. by NTA (A)

Sol. $\eta = \left[1 - \frac{T_L}{T_n}\right] \times 100\%$

$T_L = 0°C = 273K, T_n = 373$ K

$\therefore \eta = 26.809\%$

8. Time period of a simple pendulum in a stationary lift is 'T'. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be :

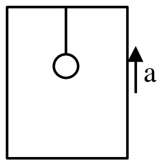
(where g = acceleration due to gravity)

(A) $\sqrt{\frac{6}{5}}T$ (B) $\sqrt{\frac{5}{6}}T$

(C) $\sqrt{\frac{6}{7}}T$ (D) $\sqrt{\frac{7}{6}}T$

Official Ans. by NTA (C)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$



(a) when a = 0, $T = 2\pi\sqrt{\frac{\ell}{g}}$

(b) when $a = \frac{g}{6}$, $T' = 2\pi\sqrt{\frac{\ell}{g + \frac{g}{6}}}$

$\therefore T' = \sqrt{\frac{6}{7}}T$

9. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by : (R = universal gas constant)

(A) $\frac{Mv^2}{7R}$

(B) $\frac{Mv^2}{5R}$

(C) $2\frac{Mv^2}{7R}$

(D) $7\frac{Mv^2}{5R}$

Official Ans. by NTA (B)

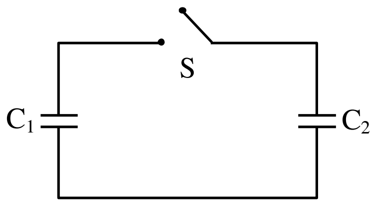
Sol. $\frac{C_P}{C_v} = 1 + \frac{2}{F} = 1.4 \Rightarrow F = 5$

By conservation of energy

$\frac{F}{2}nR\Delta T = \frac{1}{2}[nm]v^2$

$\Delta T = \frac{mv^2}{FR} = \frac{Mv^2}{5R}$

10. Two capacitors having capacitance C_1 and C_2 respectively are connected as shown in figure. Initially, capacitor C_1 is charged to a potential difference V volt by a battery. The battery is then removed and the charged capacitor C_1 is now connected to uncharged capacitor C_2 by closing the switch S. The amount of charge on the capacitor C_2, after equilibrium is :



(A) $\frac{C_1C_2}{(C_1+C_2)}V$ (B) $\frac{(C_1+C_2)}{C_1C_2}V$

(C) $(C_1 + C_2)V$ (D) $(C_1 - C_2)V$

Official Ans. by NTA (A)

Sol. Charge on capacitor C_2

$$= \frac{C_2 \times Q_{total}}{C_{total}} = \frac{C_2[C_1V]}{C_1+C_2} = \frac{C_1C_2V}{C_1+C_2}$$

11. **Assertion (A) :** Non-polar amterials do not have my permanent dipole moment.

Reason (R) : When an non-polar material is placed in a electric field. the centre of the positive charge distribution of it's individual atom or molecule coinsides with the centre of the negative charge distribution.

In the light of above statements, choose the most appropriate answer from the options given below.

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(B) Both (A) and (R) are correct and (R) is not the correct explanation of (A).

(C) (A) is correct but (R) is not correct.

(D) (A) is not correct but (R) is correct.

Official Ans. by NTA (C)

Sol. **S1 :** In nonpolar molecules, centre of +ve charge coincides with centre of –ve charge, hence net dipole moment is comes to zero.

S2 : When non polar material is placed in external field, centre of charges does not coincide, hence give non zero moment in field

12. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = (5t^3 + 4t + 2t - 5)$ Weber. If the resistant of the coil is 5 ohm, then the induced current through the coil at t = 2 sec will be:

(A) 15.6 A (B) 16.6 A

(C) 17.6 A (D) 18.6 A

Official Ans. by NTA (A)

Sol. $\phi = 5t^3 + 4t^2 + 2t - 5$

$$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$$

At t = 2, $|e| = 15 \times 2^2 + 8 \times 2 + 2$

$$\Rightarrow e = 78V \Rightarrow I = \frac{e}{R} = \frac{78}{5} = 15.60$$

13. An aluminium wire is stretched to make its length, 04% larger. Then percentage change in resistance is:

(A) 0.4 % (B) 0.2 %

(C) 0.8 % (D) 0.6 %

Official Ans. by NTA (C)

Sol. $R = \frac{\rho\ell}{A}$

$$\frac{\Delta R}{R} = \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A}$$

$\ell A = k$

$$\frac{\Delta \ell}{\ell} + \frac{\Delta A}{A} = 0$$

$$\frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell}$$

$$\frac{\Delta R}{R} = 2 \times 0.4 = 0.8\%$$

14. A proton and an alpha particle of the same enter in a uniform magnetic field which is acting perpendicular to their direction of motion. The ratio of the circular paths described by the alpha particle and proton is:

(A) 1 : 4 (B) 4 : 1

(C) 2 : 1 (D) 1 : 2

Official Ans. by NTA (C)

Sol. $\frac{R_\alpha}{R_P} = \frac{M_\alpha}{M_P} \times \frac{q_P}{q_\alpha}$

$\frac{R_\alpha}{R_P} = \frac{4}{1} \times \frac{1}{2} = 2$

15. If electric field intensity of a uniform plane electro magnetic wave is given as

E = –301.6 sin(kz – ωt)$\hat{a}_x$ + 452.4 sin(kz – ωt) $\hat{a}_y \frac{V}{m}$

Then, magnetic intensity H of this wave in Am^{-1} will be:'

[Given: Speed of light in vacuum $c = 3 \times 10^8\ ms^{-1}$, permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$]

(A) $+0.8\sin(kz-\omega t)\hat{a}_y + 0.8\sin(kz-\omega t)\hat{a}_x$

(B) $+1.0\times10^{-6}\sin(kz-\omega t)\hat{a}_y + 1.5\times10^{-6}(kz-\omega t)\hat{a}_x$

(C) $-0.8\sin(kz-\omega t)\hat{a}_y - 1.2\sin(kz-\omega t)\hat{a}_x$

(D) $-1.0\times10^{-6}\sin(kz-\omega t)\hat{a}_y - 1.5\times10^{-6}\sin(kz-\omega t)\hat{a}_x$

Official Ans. by NTA (C)

Sol. $\vec{E} = 301.6\sin(kz-\omega t)(-\hat{a}_x) + 452.4\sin(kz-\omega t)\hat{a}_y$

$\vec{B} = \frac{301.6}{C}\sin(kz-\omega t)(-\hat{a}_y)$

$+ \frac{452.4}{C}\sin(kz-\omega t)(-\hat{a}_x)$

$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{301.6}{\mu C}\sin(kz-\omega t)(-\hat{a}_y)$

$+\frac{452.4}{\mu C}\sin(kz-\omega t)(-\hat{a}_x)$

$\vec{H} = -0.8\sin(kz-\omega t)\hat{a}_y - 1.2\sin(kz-\omega t)\hat{a}_x$

For direction

$\vec{E}\times\vec{B}$ is direction of $\vec{C}$

For first part $\hat{E} = -\hat{i}, \hat{B} = ?$

$\hat{E}\times\hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{j}$

Similarly for second

$\hat{E} = \hat{j}, \hat{B} = ?$

$\hat{E}\times\hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{i}$

16. In free space, an electromagnetic wave of 3 GHz of 3 GHz frequency strikes over the edge of an object of size $\frac{\lambda}{100}$, where λ is the wavelength of the wave in free space. The phenomenon, which happens there will be:

(A) Reflection (B) Refraction

(C) Diffraction (D) Scattering

Official Ans. by NTA (D)

Sol. $\frac{a}{\lambda} = \frac{1}{100}$

For reflection size of obstacle must be much larger than wavelength, for diffraction size should be order of wavelength.

Since the object is of size $\frac{\lambda}{100}$, much smaller than wavelength, so scattering will occur.

17. An electron with speed v and a photon with speed c have the same de-Broglie wavelength. If the kinetic energy and momentum of electron are E_e and p_e and that of photon are E_{ph} and p_{ph} respectively. Which of the following is correct?

(A) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$ (B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

(C) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$ (D) $\frac{p_e}{p_{ph}} = \frac{v}{2c}$

Official Ans. by NTA (B)

Sol. $\lambda_e = \lambda_{photon}$

$$\frac{h}{mv} = \frac{h}{P_{photon}} \Rightarrow P_{photon} = mv$$

$$\frac{E_e}{E_{ph}} = \frac{\frac{1}{2}mv^2}{\frac{hc}{\lambda}} = \frac{1}{2}\frac{mv}{P_{Ph}C} \times v = \frac{v}{2C}$$

18. How many alpha and beta particles are emitted when Uranium ${}_{92}U^{238}$ decays to lead ${}_{82}Pb^{206}$?

(A) 3 alpha particles and 5 beta particles

(B) 6 alpha particles and 4 beta particles

(C) 4 alpha particles and 5 beta particles

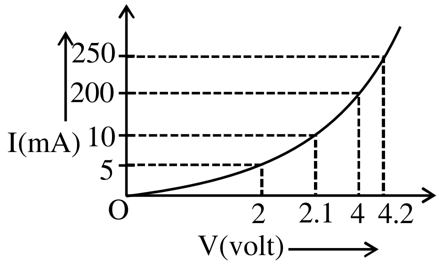
(D) 8 alpha particles and 6 beta particles

Official Ans. by NTA (D)

Sol. ${}^{238}_{92}U \rightarrow \boxed{8}\,{}^{4}_{2}He + \boxed{6}\,{}^{0}_{-1}e + {}^{206}_{82}Pb$

8α particles and 6β particles are emitted.

19. The I-V characteristics of a p-n junction diode in forward bias is shown in the figure. The ratio of dynamic resistance, corresponding to forward bias voltages of 2V and 4V respectively, is :



(A) 1 : 2 (B) 5 : 1

(C) 1 : 40 (D) 20 : 1

Official Ans. by NTA (B)

Sol. $R = \frac{\Delta V}{\Delta i}$

$$\frac{R_1}{R_2} = \frac{\Delta v_1}{\Delta v_2}\frac{\Delta i_2}{\Delta i_1} = \frac{0.1}{0.2} \times \frac{50}{5} = 5$$

20. Choose the correct statement for amplitude modulation:

(A) Amplitude of modulating is varied in accordance with the information signal.

(B) Amplitude of modulated is varied in accordance with the information signal.

(C) Amplitude of carrier signal is varied in accordance with the information signal.

(D) Amplitude of modulated is varied in accordance with the modulating signal.

Official Ans. by NTA (C)

Sol. In amplitude modulation the amplitude of high frequency carrier wave is varied in accordance with message signal

SECTION-B

1. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1}. When it passes directly overhead an anti-aircraft gun, bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be .... °.

Official Ans. by NTA (60)

Sol. Both should have same horizontal component of velocity

$200 = 400 \cos \theta$

$\theta = 60°$

2. A ball of mass 0.5 kg is dropped from the height of 10m. The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is .... m. (Use g = 10 m/s^2).

Official Ans. by NTA (5)

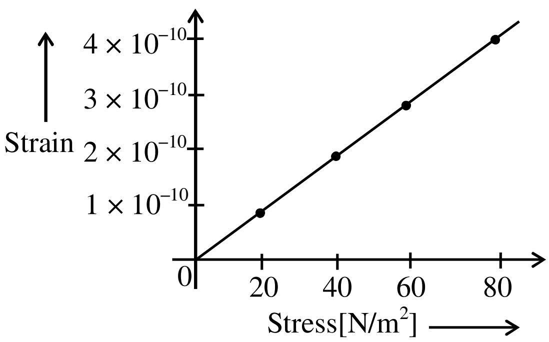
Sol. $v^2 = u^2 + 2as$

$100 = 0 + 2(10)s$

S = 5m

Height from ground = 10 – 5 = 5m

3. The elastic behaviour of material for linear streass and linear strain, is shown in the figure. The energy density for a linear strain of 5×10^{-4} is .... kJ/m^3. Assume that material is elastic upto the linear strain of 5×10^{-4}.



Official Ans. by NTA (25)

Sol. $y = \frac{\text{stress}}{\text{strain}} = 2.0 \times 10^{10}$

Energy density = $\frac{1}{2}\text{stress} \times \text{strain}$

$= \frac{1}{2}(\text{strain})^2 y = \frac{1}{2}(5 \times 10^{-4})^2 \times 20 \times 10^{10}$

$= 25 \times 10^2 \times 10 = 25\frac{kJ}{m^3}$

Ans. 25

4. The elongation of a wire on the surface of the earth is 10^{-4} m. The same wire of same dimensions is elongated by 6×10^{-5}m on another planet. The acceleration due to gravity on the planet will be .... ms^{-2}. (Take acceleration due to gravity on the surface of earth = 10 m/s^{-2})

Official Ans. by NTA (6)

Sol. $\Delta\ell \propto g$

$\frac{\Delta\ell_{earth}}{\Delta\ell_{planet}} = \frac{g_{earth}}{g_{planet}} = \frac{10^{-4}}{6 \times 10^{-5}}$

$g_{planet} = 6\ m/s^2$

Ans. 6.00

5. A 10Ω, 20 mH coil carrying constant current is connected to a battery of 20 V through a switch is opened current becomes zero in 100μs. The average emf induced in the coil is .... V.

Official Ans. by NTA (400)

Sol. $<\varepsilon> = \frac{\int \varepsilon dt}{\int dt} = \frac{\int (Ldi/dt)dt}{\int dt} = \frac{L\int di}{\int dt}$

$<\varepsilon> = \frac{L\Delta i}{\Delta i}$

$i_0 = \frac{V}{R} = \frac{20}{10} = 2A$, if i = 0A

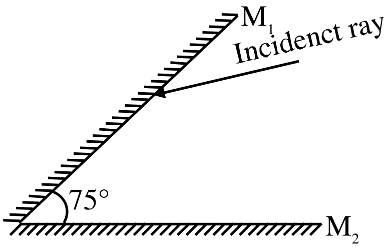
T = 100 μs, L = 20 mH

$<\varepsilon> = \frac{20 \times 10^{-3} \times (2-0)}{100 \times 10^{-6}}$

$= \frac{2 \times 10^3}{5}$

$<\varepsilon> = 400V$

6. A light ray is incident, at an incident angle θ_1, on the system of two plane mirrors M_1 and M_2 having an inclination angle 75° between them (as shown in figure). After reflecting from mirror M_1 it gets reflected back by the mirror M_2 with an angle of reflection 30°. The total deviation of the ray will be .... degree.

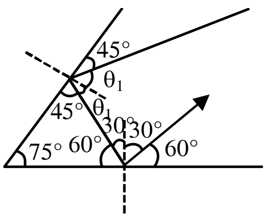


Official Ans. by NTA (210)

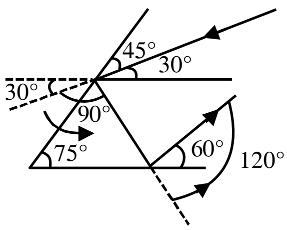
Sol. $\delta_{total} = 360° - 2\theta$

$= 360° - 2 \times 75°$

$\delta_{totakl} = 210°$



$\theta_1 = 45°$



$\delta = 120° + 90° = 210°$

7. In a vernier callipers, each cm on the main scale is divided into 20 equal parts. If tenth vernier scale division coincides with nineth main scale division. Then the value of vernier constant will be .... $\times 10^{-2}$ mm.

Official Ans. by NTA (5)

Sol. 20 MSD = 1cm

$1\text{MSD} = \frac{1}{20}\text{cm}$

10 VSD = 9MSD

$1\text{VSD} = \frac{9}{10}\text{MSD}$

$= \frac{9}{10} \times \frac{1}{20}\text{cm}$

$1\text{VSD} = \frac{9}{200}\text{cm}$

VC = 1 MSD – 1 VSD

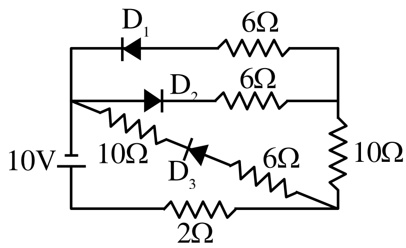
$= \frac{1}{20}\text{cm} - \frac{9}{200}\text{cm}$

$= \frac{1}{200} \times 10\text{mm}$

$\text{VC} = 5 \times 10^{-2}$ mm

Ans. 5

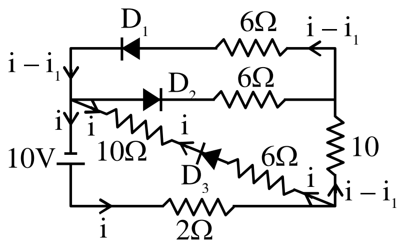
8. As per the given circuit, the value of current through the battery will be .... A.

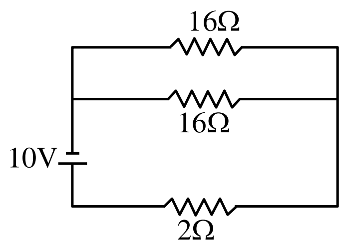


Official Ans. by NTA (1)

)

Sol.





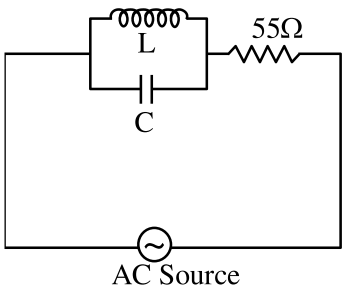
$V = IR_{net}$

$10 = I \times 10$

$I = 1A$

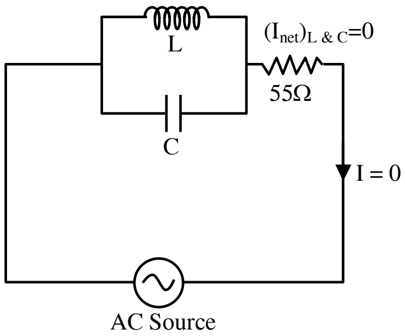
Ans. 1

9. A 110 V , 50 Hz, AC source is connected in the circuit (as shown in figure). The current through the resistance 55 Ω, at resonance in the circuit, will be .... A.



Official Ans. by NTA (0)

Sol. At resonance $I_L = I_C$



Alternatively,

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{X_L}-\frac{1}{X_C}\right)^2}$$

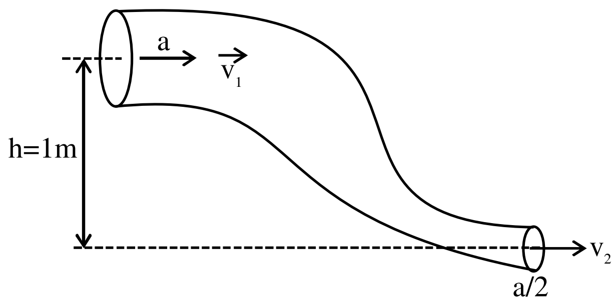
At resonance, $X_L = X_C$ & $Z \to \infty$

$\therefore Z_{\text{total circuit}} \to \infty$ i.e, $I = 0$

Ans. 0

10. An ideal fluid of density 800 kgm^{-3}, flows smoothly through a bent pipe (as shown in figure) that tapers in cross-sectional area from a to $\frac{a}{2}$. The pressure difference between the wide and narrow sections of pipe is 4100 Pa. At wider section, the velocity of fluid is $\frac{\sqrt{x}}{6}ms^{-1}$ for x = ...

(Given g = 10 m^{-2})



Official Ans. by NTA (363)

Sol. From continuity equation

$$av_1=\frac{a}{2}v_2$$

$v_2 = 2v_1$

From Bernoulli's theorem,

$$P_1+\rho gh_1+\frac{1}{2}\rho v_1^2=P_2+\rho gh_2+\frac{1}{2}\rho v_2^2$$

$$P_1-P_2=\rho\left[\left(\frac{v_2^2-v_1^2}{2}\right)+g\left(h_2-h_1\right)\right]$$

$$4100=800\left[\left(\frac{4v_1^2-v_1^2}{2}\right)+10\times(0-1)\right]$$

$$\frac{41}{8}+10=\frac{3v_1^2}{2}$$

$$\frac{121}{8}\times\frac{2}{3}=v_1^2$$

$$v_1=\sqrt{\frac{121}{4\times3}\times\frac{3}{3}}$$

$$v_1=\frac{\sqrt{363}}{6}m/s$$

X = 363.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. A commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/mL, the molarity of this solution is :

(Atomic mass : Cl = 35.5 amu, H = 1 amu)

(A) 10.2 M (B) 12.5 M

(C) 14.0 M (D) 18.2 M

Official Ans. by NTA (C)

Sol. Let total volume = 1000 mL = 1L

total mass of solution = 1460 g

mass of HCl = $\frac{35}{100}\times 1460$

moles of HCl = $\frac{35\times 1460}{100\times 36.5}$

So molarity = $\frac{35\times 1460}{100\times 36.5}$ = 14M

2. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL^{-1} and 40.5 g when filled with an ideal gas at 0.82 atm at 250 K. The molar mass of the gas in g mol^{-1} is :

(Given : R = 0.082 L atm K^{-1} mol^{-1})

(A) 35 (B) 50

(C) 75 (D) 125

Official Ans. by NTA (D)

Sol. Mass of liquid = 135 – 40 = 95 g

Volume of liquid = $\frac{\text{mass}}{\text{density}} = \frac{95}{.95}$ mL

= 100 mL = 0.1 L

mass of ideal gas = 40.5 – 40 g = 0.5 g

PV = nRT

$0.82 \times 0.1 = \left(\frac{0.5}{M}\right) \times 0.082 \times 250$

M = 125

3. If the radius of the 3rd Bohr's orbit of hydrogen atom is r_3 and the radius of 4th Bohr's orbit is r_4. Then :

(A) $r_4 = \frac{9}{16}r_3$ (B) $r_4 = \frac{16}{9}r_3$

(C) $r_4 = \frac{3}{4}r_3$ (D) $r_4 = \frac{4}{3}r_3$

Official Ans. by NTA (B)

Sol. $r = 0.529 \times \frac{n^2}{z}$ Å

$r_3 = 0.529 \times \frac{3^2}{1}$

$r_4 = 0.529 \times \frac{4^2}{1}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9}$

$r_4 = \frac{16r_3}{9}$

4. Consider the ions/molecule

$O_2^+, O_2, O_2^-, O_2^{2-}$

For increasing bond order the correct option is :

(A) $O_2^{2-} < O_2^- < O_2 < O_2^+$

(B) $O_2^- < O_2^{2-} < O_2 < O_2^+$

(C) $O_2^- < O_2^{2-} < O_2^+ < O_2$

(D) $O_2^- < O_2^+ < O_2^{2-} < O_2$

Official Ans. by NTA (A)

Sol.

ion/molecule	Number of e^- in BMO	Number of e^- in ABMO	Bond order
O_2^+	10	5	2.5
O_2	10	6	2
O_2^-	10	7	1.5
O_2^{2-}	10	8	1

Bond order $O_2^{2-} < O_2^- < O_2 < O_2^+$

5. The $\left(\frac{\partial E}{\partial T}\right)_P$ of different types of half cells are as follows :

A	B	C	D
1×10^{-4}	2×10^{-4}	0.1×10^{-4}	0.2×10^{-4}

(Where E is the electromotive force)

Which of the above half cells would be preferred to be used as reference electrode ?

(A) A (B) B

(C) C (D) D

Official Ans. by NTA (C)

Sol. A cell with less variation in EMF with temperature is preferred as reference electrode because it can be used for wider range of temperature without much derivation from standard value so a cell with less $\left(\frac{\partial E}{\partial T}\right)_P$ is preferred.

6. Choose the correct stability order of group 13 elements in their +1 oxidation state.

(A) Al < Ga < In < Tl (B) Tl < In < Ga < Al

(C) Al < Ga < Tl < In (D) Al < Tl < Ga < In

Official Ans. by NTA (A)

Sol. Moving down the group stability of lower oxidation state increases

Al < Ga < In < Tl

7. Given below are two statements :

Statement I : According to the Ellingham diagram, any metal oxide with higher ΔG° is more stable than the one with lower ΔG°.

Statement II : The metal involved in the formation of oxide placed lower in the Ellingham diagram can reduce the oxide of a metal placed higher in the diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (D)

Sol. Metal oxide with lower ΔG° is more stable

Statement II is correct

8. Consider the following reaction :

$$2HSO_4^-(aq) \xrightarrow[\text{(2) Hydrolysis}]{\text{(1) Electrolysis}} 2HSO_4^- + 2H^+ + \mathbf{A}$$

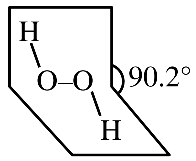
The dihedral angle in product **A** in its solid phase at 110 K is :

(A) 104° (B) 111.5°

(C) 90.2° (D) 111.0°

Official Ans. by NTA (C)

Sol. $2HSO_4^-(aq.) \xrightarrow[\text{(2) Hydrolysis}]{\text{(1) Electrolysis}} 2HSO_4^- + 2H^+ + \underset{(A)}{H_2O_2}$



Solid phase.

9. The correct order of melting point is :

(A) Be > Mg > Ca > Sr (B) Sr > Ca > Mg > Be

(C) Be > Ca > Mg > Sr (D) Be > Ca > Sr > Mg

Official Ans. by NTA (D)

Sol.

	M.P
Be	1560 K
Mg	924 K
Ca	1124 K
Sr	1062 K

10. The correct order of melting points of hydrides of group 16 elements is :

(A) $H_2S < H_2Se < H_2Te < H_2O$

(B) $H_2O < H_2S < H_2Se < H_2Te$

(C) $H_2S < H_2Te < H_2Se < H_2O$

(D) $H_2Se < H_2S < H_2Te < H_2O$

Official Ans. by NTA (A)

Sol.

	M.P
H_2O	273 K
H_2S	188 K
H2Se	208 K
H_2Te	222 K

11. Consider the following reaction :

A + alkali $\rightarrow$ B (Major Product)

If B is an oxoacid of phosphorus with no P–H bond, then A is :

(A) White P_4 (B) Red P_4

(C) P_2O_3 (D) H_3PO_3

Official Ans. by NTA (B)

Red P_4 + Alkali $\rightarrow$ $H_4P_2O_6$ (No P–H bond)

12. Polar stratospheric clouds facilitate the formation of :

(A) $ClONO_2$ (B) HOCl

(C) ClO (D) CH_4

Official Ans. by NTA (B)

Sol. Polar stratospheric clouds provide surface on which hydrolysis of $ClONO_2$ takes place to form HOCl (Hypochlorous acid)

$ClONO_2(g) + H_2O(g) \rightarrow HOCl(g) + HNO_3(g)$

13. Given below are two statements :

Statement I : In 'Lassaigne's Test, when both nitrogen and sulphur are present in an organic compound, sodium thiocyanate is formed.

Statement II : If both nitrogen and sulphur are present in an organic compound, then the excess of sodium used in sodium fusion will decompose the sodium thiocyanate formed to give NaCN and Na_2S.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (A)

Sol. Both statement I & statement II are correct.

14. $(C_7H_5O_2)_2 \xrightarrow{h\nu} [X] + 2\dot{C}_6H_5 + 2CO_2$

Consider the above reaction and identify the intermediate 'X'

(A) $C_6H_5-\overset{\overset{O}{||}}{C}^{\oplus}$

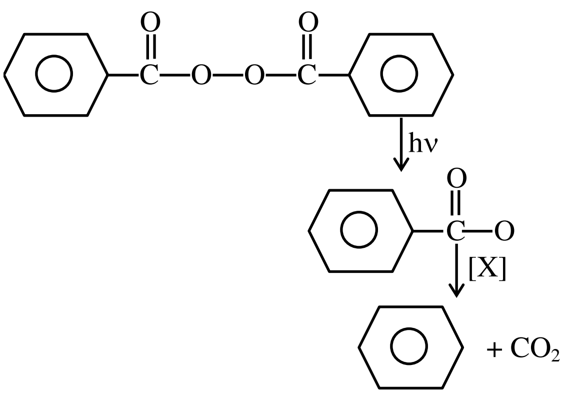
(B) $C_6H_5-\overset{\overset{O}{||}}{C}-O^{\ominus}$

(C) $C_6H_5-\overset{\overset{O}{||}}{C}-\ddot{\underset{\cdot\cdot}{O}}$

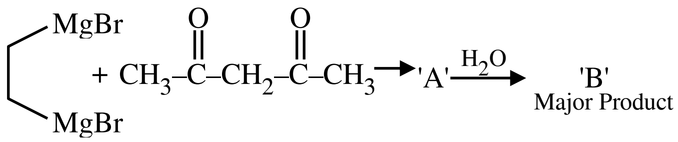
(D) $C_6H_5-\overset{\overset{O}{||}}{C}-\ddot{\underset{\cdot\cdot}{O}}\cdot$

Official Ans. by NTA (D)

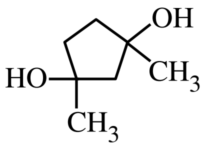
Sol.

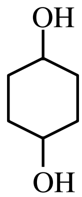


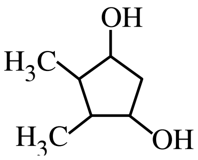
15.

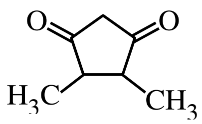


Consider the above reaction sequence and identify the product **B**.

(A) 

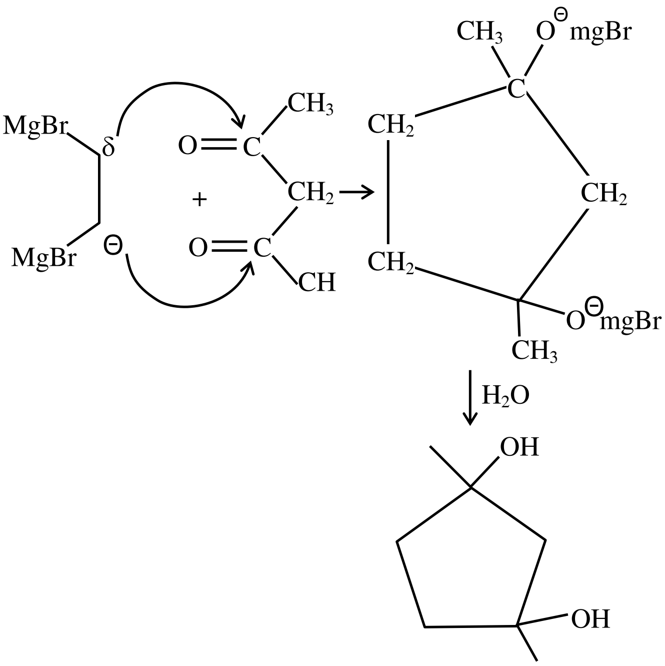
(B) 

(C) 

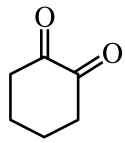
(D) 

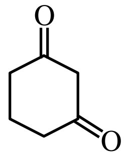
Official Ans. by NTA (A)

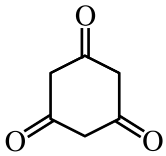
Sol. Although Acetyl Acetone predominantly gives Acid base reaction with G.R due to Active methylene group but according to given option ans should be based on nucleophilic addition reaction (NAR).

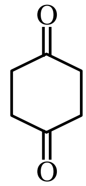


16. Which will have the highest enol content ?

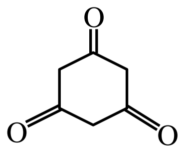
(A) 

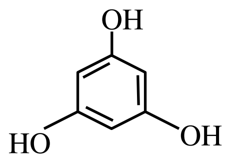
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

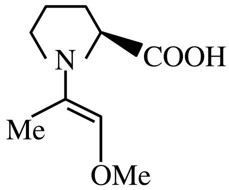
Sol. 

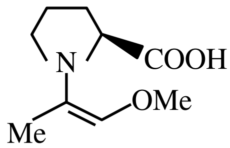
is tautomer of 

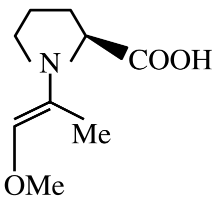
, Which is aromatic in nature.

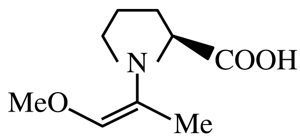
17. Among the following structures, which will show the most stable enamine formation ?

(Where Me is $-CH_3$)

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol. All these enamines are interconvertible through their resonating structures. So most stable form is 'C' due to steric factor.

18. Which of the following sets are **correct** regarding polymer ?

(A) Copolymer : Buna–S

(B) Condensation polymer : Nylon–6,6

(C) Fibre : Nylon–6,6

(D) Thermosetting polymer : Terylene

(E) Homopolymer : Buna–N

Choose the **correct** answer from given options below:

(A) (A), (B) and (C) are correct

(B) (B), (C) and (D) are correct

(C) (A), (C) and (E) are correct

(D) (A), (B) and (D) are correct

Official Ans. by NTA (A)

Sol. Which of the following set are correct regarding polymer.

Bona - 5 is copolymer of butadiene + styrene

Nylon 6.6 is condensation polymer of adipic Acid and hexanediamine.

Nylon 6.6 is fiber

Terylene is fiber not themosetting polymer

Buna-N is copolymer nol Homopolymer

19. A chemical which stimulates the secretion of pepsin is :

(A) Anti histamine (B) Cimetidine

(C) Histamine (D) Zantac

Official Ans. by NTA (C)

Sol. Histamine (It is use for secretion of pepsin & HCl in stomach)

20. Which statement is **not** true with respect to nitrate ion test ?

(A) A dark brown ring is formed at the junction of two solutions.

(B) Ring is formed due to nitroferrous sulphate complex.

(C) The brown complex is $[Fe(H_2O)_5(NO)]SO_4$.

(D) Heating the nitrate salt with conc. H_2SO_4, light brown fumes are evolved.

Official Ans. by NTA (B)

Sol. Ring is formed due to formation of nitrosoferrous sulphate

SECTION–B

1. For complete combustion of methanol

$$CH_3OH(l)+\frac{3}{2}O_2(g)\rightarrow CO_2(g)+2H_2O(l)$$

the amount of heat produced as measured by bomb calorimeter is 726 kJ mol^{-1} at 27°C. The enthalpy of combustion for the reaction is –x kJ mol^{-1}, where x is ______. (Nearest integer)

(Given : R = 8.3 JK^{-1} mol^{-1})

Official Ans. by NTA (727)

Sol. $\Delta U = -726$ KJ/mol

$\Delta ng = 1-3/2 = \frac{-1}{2}$

$\Delta H = \Delta U + \Delta ngRT$

$= -726 - \frac{1}{2}\times\frac{8.3\times 300}{1000}$

$= -727.245$

2. A 0.5 percent solution of potassium chloride was found to freeze at –0.24°C. The percentage dissociation of potassium chloride is ________. (Nearest integer)

(Molal depression constant for water is 1.80 K kg mol^{-1} and molar mass of KCl is 74.6 g mol^{-1})

Official Ans. by NTA (98)

Sol. 0.5% solution of KCl

So $m = \frac{0.5}{74.6} \times \frac{1}{0.1}$

$\Delta T_f = i \times m \times K_f$

$0.24 = i \times \frac{0.5}{74.6} \times \frac{1.80}{0.1}$

$i = \frac{0.24 \times 74.6}{0.5 \times 1.80} \times 0.1$

= 1.989

$1.989 = 1 + \alpha\ (n-1)$

$1.989 = 1 + \alpha$

$\alpha = .989$

$\%\ \alpha = 98.9\%$

Ans 99%

If mass of H_2O = 99.5

$m = \frac{0.5}{74.5} \times \frac{1}{.0995}$

$i = \frac{0.24 \times 74.6 \times .0995}{.5 \times 1.80}$

= 1.979

$1.979 = 1 + \alpha\ (n-1)$

$1.979 = 1 + \alpha$

$\alpha = .979$

$\%\ \alpha = 97.9\ \%$

Ans 98%

3. 50 mL of 0.1 M CH_3COOH is being titrated against 0.1 M NaOH. When 25 mL of NaOH has been added, the pH of the solution will be ________ $\times 10^{-2}$. (Nearest integer)

(Given : pK_a (CH_3COOH) = 4.76)

log 2 = 0.30

log 3 = 0.48

log 5 = 0.69

log 7 = 0.84

log 11 = 1.04

Official Ans. by NTA (476)

Sol. Moles of CH_3COOH = 5 m mole

moles of NaOH = 2.5 m mole

$NaOH + CH_3COOH \longrightarrow CH_3COONa + H_2O$

2.5 m mole   2.5 m mole

0   2.5 m mole   2.5 m mole

so buffer is formed

$pH = pKa + \log\left(\frac{2.5/75}{2.5/75}\right) = pKa$

pH = 4.76

$= 476 \times 10^{-2}$

4. A flask is filled with equal moles of A and B. The half lives of A and B are 100 s and 50 s respectively and are independent of the initial concentration. The time required for the concentration of A to be four times that of B is ________s.

(Given : ln 2 = 0.693)

Official Ans. by NTA (200)

Sol. $k_A = \frac{\ln 2}{100}$; $k_B = \frac{\ln 2}{50}$

$A_t = A_0 \times e^{-k_A t}$

$A_t = A_0 \times e^{\left(\frac{-\ln 2}{100} \times t\right)}$

$B_t = B_0 \times e^{\left(\frac{-\ln 2}{50} \times t\right)}$

$A_0 = B_0$

& $A_t = 4B_t$

$e^{-\frac{\ln 2}{100} \times t} = 4 \times e^{-\frac{\ln 2}{50} \times t}$

$e^{\frac{\ln 2}{100} \times t} = 4$

$e^{\frac{\ln 2}{100} \times t} = 4$

$$\frac{\ln 2}{100} \times t = \ln 4 = 2 \ln 2$$

t = 200 sec

5. 2.0 g of H_2 gas is adsorbed on 2.5 g of platinum powder at 300 K and 1 bar pressure. The volume of the gas adsorbed per gram of the adsorbent is ________ mL.

(Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (9960)

Sol. Volume of $H_2 = \frac{nRT}{p} = \frac{2}{2} \times \frac{0.083 \times 300}{1}$

= 24.92 L

= 24900 mL

So 1 g platinum adsorb $= \frac{24900}{2.5} mLH_2$

= 9960

6. The spin–only magnetic moment value of the most basic oxide of vanadium among V_2O_3, V_2O_4 and V_2O_5 is ________ B.M. (Nearest Integer)

Official Ans. by NTA (3)

Sol. Most basic oxide is V_2O_3

$V^{+3} \rightarrow [A_r]\ 3d^2$

$\mu = \sqrt{2(2+2)} = 2.84$ BM ≈ 3

7. The spin–only magnetic moment value of an octahedral complex among $CoCl_3.4NH_3$, $NiCl_2.6H_2O$ and $PtCl_4.2HCl$, which upon reaction with excess of $AgNO_3$ gives 2 moles of AgCl is ________ B.M. (Nearest Integer)

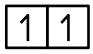
Official Ans. by NTA (3)

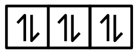
Sol. $CoCl_3.\ 4NH_3 \rightarrow [Co(NH_3)_4Cl_2]Cl$

$NiCl_2.6H_2O \rightarrow [Ni(H_2O)_6]Cl_2$

$PtCl_4\ .\ 2HCl \rightarrow H_2[PtCl_6]$

$[Ni(H_2O)_6]Cl_2 \xrightarrow{2AgNO_3} 2AgCl \downarrow + [Ni(H_2O)_6](NO_3)_2$





$\mu = \sqrt{2(2+2)}$ B.M = 2.84 BM ≈ 3

8. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is ________ (Nearest Integer)

Official Ans. by NTA (18)

Sol. $C_xHyOz + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}\ H_2O$

0.3g 0.2g .1g

$$\frac{n_{CO_2}}{n_{H_2O}} = \frac{x}{y/2} = \frac{0.2/44}{.1/18}$$

$$\frac{2x}{y} = \frac{36}{44} = \frac{9}{11}$$

$$x = \frac{9y}{22}$$

$$\frac{n_{C_xH_yO_z}}{n_{CO_2}} = \frac{1}{x}$$

$$\frac{0.3}{12x + y + 16z} \times \frac{44}{0.2} = \frac{1}{x}$$

66x = 12 x + y + 16 z

54x = y + 16 z

$\frac{54 \times 9y}{22} - y = 16$ z

$\frac{464y}{22} = 16z$

$z = \dfrac{29y}{22}$

$C_xH_yO_z = C_xH_yO_z$

$C_{\frac{9y}{22}}H_yO_{\frac{29y}{22}}$

$C_9H_{22}O_{29}$

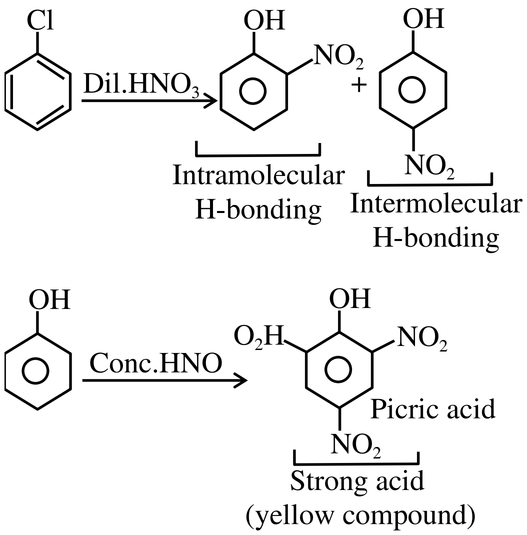
$$\% \text{ of C} = \frac{12\times9}{(12\times9+22+29\times16)}\times 100 = \frac{108}{594}\times100$$

18.18%

9. Compound 'P' on nitration with dil. HNO_3 yields two isomers (A) and (B). These isomers can be separated by steam distillation. Isomers (A) and (B) show the intramolecular and intermolecular hydrogen bonding respectively. Compound (P) on reaction with conc. HNO_3 yields a yellow compound 'C', a strong acid. The number of oxygen atoms is present in compound 'C' ________.

Official Ans. by NTA (7)

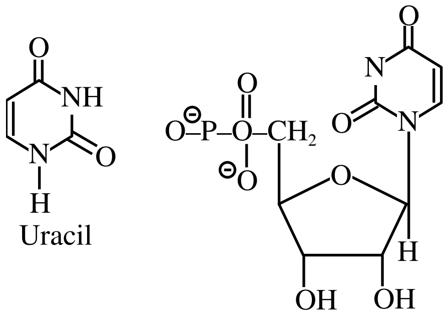
Sol.



10. The number of oxygens present in a nucleotide formed from a base, that is present only in RNA is ________.

Official Ans. by NTA (9)

Sol. Uracil is the base which only present is RNA.



Structure of nucleotides number of 0-9.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $f(x)=\frac{x-1}{x+1}, x \in R-\{0,-1,1\}$. If $f^{n+1}(x)=f(f^n(x))$ for all n $\in$ N, then $f^6(6)+f^7(7)$ is equal to:

(A) $\frac{7}{6}$ (B) $-\frac{3}{2}$ (C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Official Ans. by NTA (B)

Sol. $f(x)=\frac{x-1}{x+1}$

$\Rightarrow f^2(x)=f(f(x))=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{1}{x}$

$f^3(x)=f(f^2(x))=f\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$

$\Rightarrow f^4(x)=f\left(\frac{x+1}{1-x}\right)=-\frac{1}{x}$

$\Rightarrow f^6(x)=-\frac{1}{x}\Rightarrow f^6(6)=-\frac{1}{8}$

$f^7(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$

$\Rightarrow f^7(7)=\frac{8}{-6}=-\frac{4}{3}$

$\therefore -\frac{1}{6}+-\frac{4}{3}=-\frac{3}{2}$

2. Let $A=\left\{z\in C:\left|\frac{z+1}{z-1}\right|<1\right\}$

and $B=\left\{z\in C:\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$.

Then $A\cap B$ is :

(A) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) an empty set

(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Official Ans. by NTA (B)

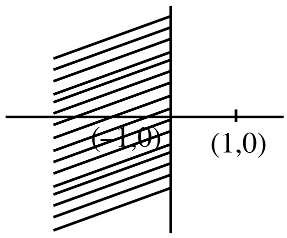
Sol. Set A

$\Rightarrow \left|\frac{z+1}{z-1}\right|<1$

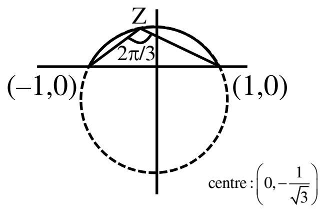
$\Rightarrow |z+1|<|z-1|$

$\Rightarrow (x+1)^2+y^2<(x-1)^2+y^2$

$\Rightarrow x<0$



Set B



$\Rightarrow \arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right)-\tan^{-1}\left(\frac{y}{x+1}\right)=\frac{2\pi}{3}$

$\Rightarrow x^2+y^2+\frac{2y}{\sqrt{3}}-1=0$

$A\cap B$

$\Rightarrow$ Centre $\left(0,-\frac{1}{\sqrt{3}}\right)$

3. Let A be a 3 × 3 invertible matrix. If |adj (24A)| = adj(3adj(2A))|, then $|A|^2$ is equal to :

(A) 6^6 (B) 2^{12} (C) 2^6 (D) 1

Official Ans. by NTA (C)

Sol. |adj (24A)| = |adj 3(adj 2A)|

$\Rightarrow |24a|^2 = (3\ \text{adj}(2A))^2$

$\Rightarrow (24^3\,|A|)^2 = (3^3|\text{adj}\ (2A)|)^2$

$= 3^6(|2A|^2)^2$

$$\Rightarrow 24^6\,|A|^2 = \left(24^3\,|\,A\,|\right)^2 = 3^6 \times 2^{12}\,|A|^4$$

$$\Rightarrow |A|^2 = \frac{24^6}{3^6 \times 2^{12}} = 64$$

4. The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

has no solution, is :

(A) $\left(3, \frac{1}{3}\right)$ (B) $\left(-3, \frac{1}{3}\right)$

(C) $\left(-3, -\frac{1}{3}\right)$ (D) $\left(3, -\frac{1}{3}\right)$

Official Ans. by NTA (C)

Sol. $$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$

$3(-8a - 9) + 2(5a - 18) + 1(21) = 0$

$\Rightarrow a = -3$

$$\text{Also } \Delta_2 = \begin{vmatrix} 3 & -2 & b \\ 5 & 8 & 3 \\ 2 & 1 & -1 \end{vmatrix}^{\frac{1}{3}}$$

If $b = \frac{1}{3}$

$\Delta_2 = 0$

So b must be equal to

$-\frac{1}{3}$

5. The remainder when $(2021)^{2023}$ is divided by 7 is :

(A) 1 (B) 2 (C) 5 (D) 6

Official Ans. by NTA (C)

Sol. $(2021)^{2023} = (7\lambda - 2)^{2023}$

$= {}^{2023}C_0(7A)^{2023} - \ldots\ {}^{2023}C_{2023}2^{2023}$

$= 7t - 2^{2023}$

$\therefore\ -2^{2023} = -2 \times 2^{2022}$

$= -2 \times \left(2^3\right)^{674}$

$= -2\left(1 + 7\mu\right)^{674}$

$= -(7\alpha + 2)$

$\Rightarrow$ remainder $= -2$ or + 5

6. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$ is equal to :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

Official Ans. by NTA (D)

Sol. $$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$$

$$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin\left(\sin^{-1}\sqrt{1 - x^2}\right) - x}{1 - \tan\left(\tan^{-1}\left(\frac{\sqrt{1 - x^2}}{x}\right)\right)}$$

$$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sqrt{1 - x^2} - x}{1 - \left(\frac{\sqrt{1 - x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}(-x) = -\frac{1}{\sqrt{2}}$$

7. Let f, g : R → R be two real valued functions defined as $f(x)=\begin{cases}-|x+3| & , x<0 \\ e^x & , x\ge 0\end{cases}$ and

$g(x)=\begin{cases}x^2+k_1x & , x<0 \\ 4x+k_2 & , x\ge 0\end{cases}$, where k_1 and k_2 are real constants. If (gof) is differentiable at x = 0, then (gof) (–4) + (gof) (4) is equal to :

(A) $4(e^4+1)$ (B) $2(2e^4+1)$

(C) $4e^4$ (D) $2(2e^4-1)$

Official Ans. by NTA (D)

Sol. $f(x)=\begin{cases}x+3 & ; x<-3 \\ -(x+3) & ; -3\le x<0 \\ e^x & ; x\ge 0\end{cases}$

$$g(x)=\begin{cases}x^2+k_1x & ; x<0 \\ 4x+k_2 & ; x\ge 0\end{cases}$$

$$g(f(x))=\begin{cases}f(x)^2+k_1f(x) & ; f(x)<0 \\ 4f(x)+k_2 & ; f(x)\ge 0\end{cases}$$

$$g(f(x))=\begin{cases}(x+3)^2+k_1(x+3) & ; x<-3 \\ (x+3)^2-k_1(x+3) & ; -3\le x<0 \\ 4e^x+k_2 & ; x>0\end{cases}$$

check continuity at x = 0

$$gof(0)=g(f(0^-))=g(f(0^+))$$

$$4+k_2=9-3k_1=4+k_2$$

$$3k_1+k_2=5 \qquad ...(a)$$

differentiate

$$(g(f(x)))'=\begin{cases}2(x+3)+k_1 & ; x<-3 \\ 2(x+3)-k_1 & ; -3\le x<0 \\ 4e^x & ; x\ge 0\end{cases}$$

$$6-k_1=4$$

$$k_1=2 \qquad ...(b)$$

$$\therefore k_1=2, k_2=-1$$

$$gof(x)=\begin{cases}(x+3)^2+2(x+3) & ; x<-3 \\ (x+3)^2-2(x+3) & ; -3\le x<0 \\ 4e^x-1 & ; x\ge 0\end{cases}$$

gof(–4) + gof(4) = $4e^4-2$

$$\Rightarrow 2(2e^4-1)$$

8. The sum of the absolute minimum and the absolute maximum values of the function $f(x)=|3x-x^2+2|-x$ in the interval [–1, 2] is :

(A) $\frac{\sqrt{17}+3}{2}$ (B) $\frac{\sqrt{17}+5}{2}$

(C) 5 (D) $\frac{9-\sqrt{17}}{2}$

Official Ans. by NTA (A)

Sol. $f(x)=\begin{cases}x^2-4x-2, & \forall x\in\left(-1,\frac{3-\sqrt{17}}{2}\right) \\ -x^2+2x+2, & \forall x\in\left(\frac{3-\sqrt{17}}{2},2\right)\end{cases}$

$f'(x)$ when $x\in\left(-1,\frac{3-\sqrt{17}}{2}\right)$

$f'(x) = 2x - 4 = 0 \Rightarrow x = 2$

$f'(x) = 2(x - 2)$ $\Rightarrow f'(x)$ is always ↓

$f(2) = 2$

$f(-1) = 3$

$$f\left(\frac{3-\sqrt{17}}{2}\right)=\frac{\sqrt{17}-3}{2}$$

$f'(x)$ when $x\in\left(\frac{3-\sqrt{17}}{2},2\right)$

$f'(x) = -2x + 2$

$f'(x) = -2(x - 1)$

$f'(x) = 0$ when $x = 1$

$f(1) = 3$

absolute minimum value = $\frac{\sqrt{17}-3}{2}$

absolute maximum value = 3

Sum = $\frac{\sqrt{17}-3}{2}+3=\frac{\sqrt{17}+3}{2}$

9. Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), ab ≠ 0. Then:

(A) $S=\phi$ (B) $n(S)=1$

(C) $S=\{2k : k\in N\}$ (D) $S = N$

Official Ans. by NTA (D)

Sol. $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$

Slope of tangent at (a, b)

$$n.\left(\frac{x}{a}\right)^{n-1}.\frac{1}{a}+n\left(\frac{x}{b}\right)^{n-1}.\frac{1}{b}\frac{dy}{dx}=0$$

$$\left.\frac{dy}{dx}\right|_{(a,b)}=-\frac{b}{a}$$

∴ Equation of tangent

$$y-b=-\frac{b}{a}(x-a)$$

$$\frac{x}{a}+\frac{y}{b}=2\ \forall\ n\in N$$

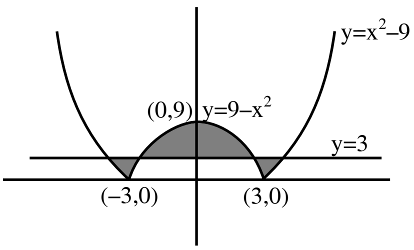
10. The area bounded by the curve $y=|x^2-9|$ and the line $y = 3$ is :

(A) $4\left(2\sqrt{3}+\sqrt{6}-4\right)$ (B) $4\left(4\sqrt{3}+\sqrt{6}-4\right)$

(C) $8\left(4\sqrt{3}+3\sqrt{6}-9\right)$ (D) $8\left(4\sqrt{3}+\sqrt{6}-9\right)$

Official Ans. by NTA (DROP)

Sol.



Area of shaded region

$$=2\int_0^3\left(\sqrt{9+y}-\sqrt{9-y}\right)dy+2\int_3^9\left(\sqrt{9-y}\right)dy$$

$$=2\left[\int_0^3(9+y)^{1/2}dy-\int_0^3(9-y)^{1/2}dy+\int_3^9(9-y)^{1/2}dy\right]$$

$$=2\left[\frac{2}{3}\left[(9+y)^{3/2}\right]_0^3+\frac{2}{3}\left[(9-y)^{3/2}\right]_0^3-\frac{2}{3}\left[(9-y)^{3/2}\right]_3^9\right]$$

$$=\frac{4}{3}\left[12\sqrt{12}-27+6\sqrt{6}-27-\left(0-6\sqrt{6}\right)\right]$$

$$=\frac{4}{3}\left[24\sqrt{3}+12\sqrt{6}-54\right]$$

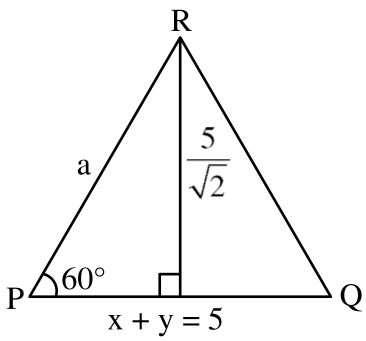
$$=8\left(4\sqrt{3}+2\sqrt{6}-9\right)$$

11. Let R be the point (3, 7) and let P and Q be two points on the line x + y = 5 such that PQR is an equilateral triangle. Then the area of ΔPQR is :

(A) $\frac{25}{4\sqrt{3}}$ (B) $\frac{25\sqrt{3}}{2}$ (C) $\frac{25}{\sqrt{3}}$ (D) $\frac{25}{2\sqrt{3}}$

Official Ans. by NTA (D)

Sol.



$$\sin 60° = \frac{5/\sqrt{2}}{a}$$

$$a=\frac{5\sqrt{2}}{3}$$

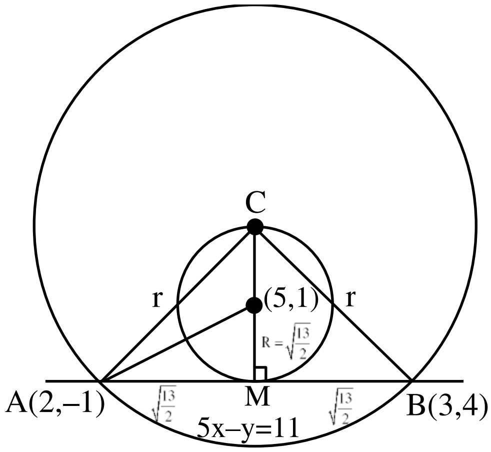
$$\text{Area of } \Delta PQR=\frac{\sqrt{3}}{4}a^2=\frac{25}{2\sqrt{3}}$$

12. Let C be a circle passing through the points A(2, –1) and B(3, 4). The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2+(y-1)^2=\frac{13}{2}$, then r^2 is equal to :

(A) 32 (B) $\frac{65}{2}$ (C) $\frac{61}{2}$ (D) 30

Official Ans. by NTA (B)

Sol.



$AB = \sqrt{26}$

$r^2 = CM^2 + AM^2$

$= \left(2\times\sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2$

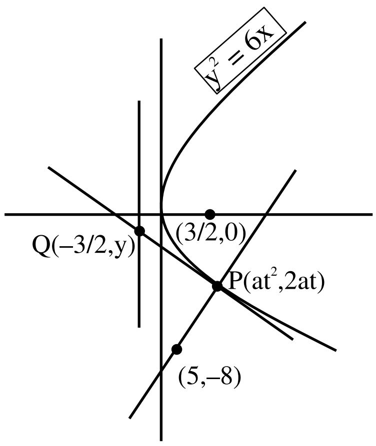
$r^2 = \frac{65}{2}$

13. Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, –8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :

(A) –3 (B) $-\frac{9}{4}$ (C) $-\frac{5}{2}$ (D) –2

Official Ans. by NTA (B)

Sol.



Equation of normal : $y = -tx + 2at + at^3$ $\left(a = \frac{3}{2}\right)$

since passing through (5, –8), we get $t = -2$

Co-ordinate of Q : (6, –6)

Equation of tangent at Q : $x + 2y + 6 = 0$

Put $x = \frac{-3}{2}$ to get $R\left(\frac{-3}{2}, \frac{-9}{4}\right)$

14. If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}$, $z = 2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

(A) $\cos^{-1}\left(\frac{29}{4}\right)$ (B) $\sec^{-1}\left(\frac{29}{4}\right)$

(C) $\cos^{-1}\left(\frac{2}{29}\right)$ (D) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Official Ans. by NTA (B)

Sol. $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$l_2 : \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$

$l_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$

$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$

angle between l_2 & l_3

$\cos\theta = \frac{|1\times(-3)+(-2)(\alpha/2)+2\times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$

$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$

put $\alpha = 3$

$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$

$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$

15. Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane x – 3y + 5z = 8. If the mirror image of the point $\left(2,-\frac{1}{2},2\right)$ in the rotated plane is B(a, b, c), then :

(A) $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$ (B) $\frac{a}{4}=\frac{b}{5}=\frac{c}{-2}$

(C) $\frac{a}{8}=\frac{b}{-5}=\frac{c}{4}$ (D) $\frac{a}{4}=\frac{b}{5}=\frac{c}{2}$

Official Ans. by NTA (A)

Sol. Let equation of rotated plane be :

$(2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$

$(2 + \lambda)x + (3 - 3\lambda)y + (1 + 5\lambda)z + 20 - 8\lambda = 0$

Above plane is perpendicular to $2x + 3y + z + 20 = 0$

So, $(2 + \lambda).2 + (3 - 3\lambda).3 + (1 + 5\lambda).1 = 0 \Rightarrow \lambda = 7$

$\Rightarrow$ Equation of rotated plane : $x - 2y + 4z - 4 = 0$

Mirror image of $A\left(2,\frac{-1}{2},2\right)$ in rotated plane is B(a, b, c)

Equation of AB : $\frac{x-2}{1}=\frac{y+1/2}{-2}=\frac{z-2}{4}=k$

Let coordinate of B be $\left(2+k, \frac{-1}{2}-2k, 2+4k\right)$

midpoint of AB is $\left(2+\frac{k}{2},\frac{-1}{2}-k,2+2k\right)$ which will lie on the plane $x - 2y + 4z - 4 = 0$

Hence $k=\frac{-2}{3}$

Therefore B is $\left(\frac{4}{3},\frac{5}{6},\frac{-2}{3}\right)\equiv\left(\frac{8}{6},\frac{5}{6},\frac{-4}{6}\right)$

So, $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$

16. If $\vec{a}\cdot\vec{b}=1$, $\vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right), \vec{b}\times\left(\vec{c}\times\vec{a}\right),\vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]$ is :

(A) 0 (B) $-6\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

(C) $12\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ (D) $-12\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

Official Ans. by NTA (A)

Sol. $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=3\vec{b}-\vec{c}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\left(\vec{b}.\vec{a}\right)\vec{c}-\left(\vec{b}.\vec{c}\right)\vec{a}=\vec{c}-2\vec{a}$

$\vec{c}\times\left(\vec{b}\times\vec{a}\right)=(\vec{c}.\vec{a})\vec{b}-(\vec{c}.\vec{b})\vec{a}=3\vec{b}-2\vec{a}$

$\left[3\vec{b}-\vec{c},\vec{c}-2\vec{a}, 3\vec{b}-2\vec{a}\right]$

$\left(3\vec{b}-\vec{c}\right).\left[(\vec{c}-2\vec{a})\times\left(3\vec{b}-2\vec{a}\right)\right]$

$\left(3\vec{b}-\vec{c}\right).\left[3\left(\vec{c}\times\vec{b}\right)-2\left(\vec{c}\times\vec{a}\right)-6\left(\vec{a}\times\vec{b}\right)\right]$

$-6\left[\vec{b}\ \vec{c}\ \vec{a}\right]+6\left[\vec{c}\ \vec{a}\ \vec{b}\right]$

17. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is:

(A) $\frac{275}{6^5}$ (B) $\frac{36}{5^4}$ (C) $\frac{181}{5^5}$ (D) $\frac{46}{6^4}$

Official Ans. by NTA (D)

Sol. P(H) = x, P(T) = 1 – x

P(4H, 1T) = P(5H)

${}^5C_1(x)^4\,(1-x)^1 = {}^5C_5\,x^5$

$5(1 - x) = x$

$6x = 5 = 0 \quad x=\frac{5}{6}$

P(atmost 2H)

= P(OH, 5T) + P(1H, 4T) + P(2H, 3T)

$={}^5C_0\left(\frac{1}{6}\right)^5+{}^5C_1\frac{5}{6}.\left(\frac{1}{6}\right)^4+{}^5C_2\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^3$

$=\frac{1}{6^5}(1+25+250)=\frac{276}{6^5}$

$= \frac{46}{6^4}$

18. The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25 M is equal to:

(A) 60 (B) 55 (C) 50 (D) 45

Official Ans. by NTA (A)

Sol. $\sigma^2 = \frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{n}$

Mean = 6

$\frac{a+b+8+5+10}{5} = 6$

a + b = 7

b = 7 – a

$6.8 = \frac{(a-6)^2 + (b-6)^2 + (8-6)^2 + (5-6)^2 + (10-6)^2}{5}$

$34 = (a-6)^2 + (7-a-6)^2 + 4 + 1 + 18$

$a^2 - 7a + 12 = 0 \Rightarrow a = 4$ or $a = 3$

a = 4 a = 3

b = 3 b = 4

$M = \frac{\sum_{i=1}^{5}|x_i - x|}{n}$

$M = \frac{|a-6| + |b-6| + |8-6| + |5-6| + |10-6|}{5}$

when a = 3, b = 4 when a = 4, b = 3

$M = \frac{3+2+2+1+4}{5}$ $M = \frac{2+3+2+1+7}{5}$

$M = \frac{12}{5}$ $M = \frac{12}{5}$

$25M = 25 \times \frac{12}{5} = 60$

19. Let f(x) = $2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If [a, b] is the range of the function then 4a – b is equal to:

(A) 11 (B) 11 – π (C) 11 + π (D) 15 – π

Official Ans. by NTA (B)

Sol. $f'(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2$

$= -2\left[\frac{1}{\sqrt{1-x^2}} + \frac{2}{1+x^2} + 3x + 1\right]$

$f'(x) < 0 \Rightarrow f(x)$ is a dec. function

$f(1) = \pi + 5$

$f(-1) = 5\pi + 5$

Range : $[a, b] \equiv [\pi + 5, 5\pi + 5]$

$a = \pi + 5, b = 5\pi + 5 \Rightarrow 4a - b = 11 - \pi$.

20. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to :

(A) $(p \Delta r) \vee q$ (B) $(p \Delta r) \wedge q$

(C) $(p \wedge r) \Delta q$ (D) $(p \nabla r) \wedge q$

Official Ans. by NTA (A)

Sol. **Case-I** If $\Delta \equiv \nabla \equiv \wedge$

$(p \wedge q) \to ((p \wedge q) \wedge r)$

it can be false if r is false,

so not a tautology

Case-II If $\Delta \equiv \nabla \equiv \vee$

$(p \vee q) \to ((p \vee q) \vee r) \equiv$ tautology

then $(p \vee q) \vee r \equiv (p \Delta r) \vee q$

Case-III if $\Delta = \vee, \nabla = \wedge$

then $(p \wedge q) \to \{(p \vee q) \wedge r\}$

Not a tautology

(Check $p \to T, q \to T, r \to F$)

Case-IV if $\Delta = \wedge, \nabla = \vee$

$(p \wedge q) \to \{(p \wedge q) \vee r\}$

Not a tautology

SECTION-B

1. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$ is ______.

Official Ans. by NTA (36)

Sol. $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$

x = 0 is not the root of this equation so divide it by x^2

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2 + 2 - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$$

$x - \frac{1}{x} = 0$, $\qquad x - \frac{1}{x} = 3$

$x^2 - 1 = 0 \qquad x^2 - 3x - 1 = 0$

$x = \pm 1 \qquad \gamma + \delta = 3$

$\alpha = 1, \beta = -1 \qquad \gamma\delta = -1$

$\alpha^3 + \beta^3 + \gamma^3 + \delta^3$

$1 - 1 + (\gamma + \delta)((\gamma + \delta)^2 - 3\gamma\delta)$

$0 + 3(9 - 3(-1))$

$+ 3(12) = 36$

2. There are ten boys $B_1, B_2, ..., B_{10}$ and five girls $G_1, G_2, ..., G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is__________.

Official Ans. by NTA (1120)

Sol. n(B) = 10

n(a) = 5

The number of ways of forming a group of 3 girls of 3 boys.

$= {}^{10}C_3 \times {}^5C_3$

$$= \frac{10 \times 9 \times 8}{3 \times 2} \times \frac{5 \times 4}{2} = 1200$$

The number of ways when two particular boys B_1 of B_2 be the member of group together

$= {}^8C_1 \times {}^5C_3 = 8 \times 10 = 80$

Number of ways when boys B_1 of B_2 hot in the same group together

$= 1200 \times 80 = 1120$

3. Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to__________.

Official Ans. by NTA (4)

Sol. $x^2 + y^2 = \frac{9}{4} \qquad y = 4x$

Equation tangent in slope form

$$y = mx \pm \frac{3}{2}\sqrt{(1+m^2)} \quad ...(1)$$

$$y = mx + \frac{1}{m} \quad ...(2)$$

compare (1) & (2)

$$\pm\frac{3}{2}\sqrt{(1+m^2)} = \frac{1}{m^2}$$

$9m^2(1 + m^2) = 4$

$9m^4 + 9m^2 - 4 = 0$

$9m^4 + 12m^2 - 3m^2 - 4 = 0$

$3m^2(3m^2 + 4) - (3m^2 + 4) = 0$

$m^2 = -\frac{4}{3}$ (Rejected)

$m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

Equation of common tangent

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$$

on X axis y = 0

OQ = –3

b = |OQ| = 3

a = 6

$b^2 = a^2(1 - e^2) \Rightarrow e^2 = 1 - \frac{9}{36} = \frac{3}{4}$

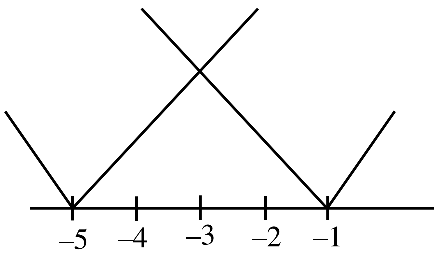
$e = \frac{2b^2}{a} = \frac{2\times 9}{6} = 3$

$\frac{e}{e^2} = \frac{3}{3/4} = 4$

4. Let f(x) = max{|x + 1|, |x + 2|, ..., |x + 5|}. Then $\int_{-6}^{0} f(x)\,dx$ is equal to ______________.

4. Official Ans. by NTA (21)

Sol. f(x) = max{|x+1|, |x+2|, |x + 3|, |x + 4|, |x + 5|}



$$\int_{-6}^{0} f(x)\,dx = \int_{-6}^{-3} |x+1|\,dx + \int_{-3}^{0} |x+5|\,dx$$

$$= -\int_{-6}^{-3} (x+1)\,dx + \int_{-3}^{0} (x+5)\,dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{9}{2} - 3\right) - (18 - 6)\right] + \left[0 - \left(\frac{9}{2} - 15\right)\right]$$

$$= -\left[\frac{3}{2} - 12\right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21$$

5. Let the solution curve y = y(x) of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then y(2) is equal to________.

Official Ans. by NTA (12)

Sol. $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx$

$$(x^2 + 4)\frac{dy}{dx} = 2x^3 + 6xy + 8x$$

$$(x^2 + 4)\frac{dy}{dx} - 6xy = 2x^3 + 8x$$

$$\frac{dy}{dx} - \frac{6x}{x^2+4}y = \frac{2x^3+8x}{x^2+y}$$

L.I. $\frac{dy}{dx} + py = \phi$

$p = \frac{-6x}{x^2+4}$ $\qquad \phi = \frac{2x^3+8x}{x^2+4}$

$$\text{I.F.} = e^{-\int \frac{6x}{x^2+4}dx} = e^{-3\log_e(x^2+4)}$$

$$= e^{\log_e(x^2+4)^{-3}} = \frac{1}{(x^2+4)^3}$$

Sol.

$$y.\frac{1}{(x^2+4)^3} = \int \frac{2x^3+8x}{(x^2+4)^3(x^2+4)}dx$$

$$\frac{y}{(x^2+4)^3} = \int \frac{2x(x^2+4)}{(x^2+4)^3(x^2+4)}dx$$

$x^2 + 4 = t$

2xdx = dt

$$\frac{y}{(x^2+4)^3} = \int \frac{dt}{t^3}$$

$$\frac{y}{(x^2+4)^3} = \frac{-1}{2(x^2+4)^2} + C$$

passes through origin (0, 0)

$$0 = \frac{-1}{2\times 16} + C$$

$$\frac{y}{(x^2+4)^3} = \frac{-1}{2(x^2+4)^2} + \frac{1}{32}$$

$$y = \frac{-(x^2+4)}{2} + \frac{(x^2+4)^3}{32}$$

$$y(2) = -\frac{8}{2} + \frac{8\times 8\times 8}{32} = 12$$

6. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to ___________.

Official Ans. by NTA (80)

Sol. $\sin 10°\left(\frac{1}{2}.2\sin 20°\sin 40°\right).\sin 10°\sin(60°-10°)\sin(60°+10°)$

$\sin 10°\frac{1}{2}(\cos 20°-\cos 60°).\frac{1}{4}\sin 30°$

$\frac{1}{2}.\frac{1}{4}.\frac{1}{2}.\sin 10°\left(\cos 20°-\frac{1}{2}\right)$

$=\frac{1}{32}(2\sin 10°\cos 20°-\sin 10°)$

$=\frac{1}{32}(\sin 30°-\sin 10°-\sin 10°)$

$=\frac{1}{32}\left(\frac{1}{2}-2\sin 10°\right)$

$=\frac{1}{64}(1-4\sin 10°)$

$=\frac{1}{64}-\frac{1}{16}\sin 10°$

Hence $\alpha=\frac{1}{64}$

$16+\alpha^{-1}=80$

7. Let A = {n ∈ N : H.C.F. (n, 45) = 1} and

Let B = {2k : k ∈ {1, 2, ..., 100}}. Then the sum of all the elements of A ∩ B is __________.

Official Ans. by NTA (5264)

Sol. Sum of elements in A ∩ B

$$=\underbrace{(2+4+6+...+200)}_{\text{Multiple of 2}}-\underbrace{(6+12+...+198)}_{\text{Multiple of 2 \& 3 i.e. 6}}$$

$$-\underbrace{(10+20+...+200)}_{\text{Multiple of 5 \& 2 i.e. 10}}+\underbrace{(30+60+...+180)}_{\text{Multiple of 2, 5 \& 3 i.e. 30}}$$

= 5264

8. The value of the integral $\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ is equal to ________.

Official Ans. by NTA (6)

Sol. $I=\frac{48}{\pi^4}\int_0^{\pi}x^2\left(\frac{3\pi}{2}-x\right)\frac{\sin x}{1+\cos^2 x}dx$ ...(1)

Apply king property

$I=\frac{48}{\pi^4}\int_0^{\pi}(\pi-x)^2\left(\frac{\pi}{2}+x\right)\frac{\sin x}{1+\cos^2 x}dx$ ...(2)

(1) + (2)

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+(\pi-2).x.(\pi-2x)\right]dx$ ...(3)

Apply king again

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+(\pi-2)(\pi-x)(2x-\pi)\right]dx$ ...(4)

(3) + (4)

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(\pi-2x)\right]dx$ ...(5)

Apply king

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(2x-\pi)\right]dx$ ...(6)

(5) + (6)

$I=\frac{12}{\pi}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$

Let cosx = t ⇒ sinxdx = –dt

$I=\frac{12}{\pi}\int_1^{-1}\frac{-dt}{1+t^2}=6$

9. Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then A + B is equal to ________.

Official Ans. by NTA (1100)

Sol. $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$

$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$

$A=\sum_{j=1}^{10}\min(i,1)+\min(j,2)+...\min(i,10)$

$$=\underbrace{(1+1+1+...+1)}_{\text{19 times}}+\underbrace{(2+2+2...+2)}_{\text{17 times}}+\underbrace{(3+3+3...+3)}_{\text{15 times}}$$

+ ... (1) 1 times

$$B=\sum_{j=1}^{10}\max(i,1)+\max(j,2)+\ldots\max(i,10)$$

$$=\underbrace{(10+10+\ldots+10)}_{19\text{ times}}+\underbrace{(9+9+\ldots+9)}_{17\text{ times}}+\ldots+1\ 1\text{ times}$$

A + B = 20(1 + 2 + 3 + ... +10)

$$= 20\times\frac{10\times 11}{2} = 10\times 110 = 1100$$

10. Let $S=(0,2\pi)-\left\{\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2},\frac{7\pi}{4}\right\}$. Let y = y(x), x ∈ S, be the solution curve of the differential equation $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$, $y\left(\frac{\pi}{4}\right)=\frac{1}{2}$. if the sum of abscissas of all the points of intersection of the curve y = y(x) with the curve $y=\sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to ________.

Official Ans. by NTA (42)

Sol. $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$

$$\int dy=\int\frac{dx}{(\sin x+\cos x)^2}$$

$$\int dy=\int\frac{\sec^2 x}{(1+\tan x)^2}$$

$$y(x)=-\frac{1}{1+\tan x}+C$$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{2}=-\frac{1}{2}+C$$

C = 1

$$y(x)=\frac{-1}{1+\tan x}+1$$

$$y(x)=\frac{-1+1+\tan x}{1+\tan x}$$

$$y(x)=\frac{\tan x}{1+\tan x}$$

Solving with $y=\sqrt{2}\sin x$

$$\frac{\tan x}{1+\tan x}=\sqrt{2}\sin x$$

sinx = 0 , $\frac{1}{\sqrt{2}}=\sin x+\cos x$

x = π $\quad\frac{1}{2}=\sin\left(x+\frac{\pi}{4}\right)$

$$\sin\frac{\pi}{6}=\sin\left(x+\frac{\pi}{4}\right)$$

$$x+\frac{\pi}{4}=\pi-\frac{\pi}{6},\ 2\pi+\frac{\pi}{6}$$

$$x=\frac{5\pi}{6}-\frac{\pi}{4},\quad x=\frac{13\pi}{6}-\frac{\pi}{4}$$

$$x=\frac{7\pi}{12},\ x=\frac{23\pi}{12}$$

sum of sol.

$$=\pi+\frac{7\pi}{12}+\frac{23\pi}{12}$$

$$=\frac{12\pi+7\pi+23}{12}=\frac{42\pi}{12}=\frac{k\pi}{12}$$

$\Rightarrow k=42$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 3:00 PM to 06:00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The dimension of mutual inductance is :

(A) $[ML^2T^{-2}A^{-1}]$ (B) $[ML^2T^{-3}A^{-1}]$

(C) $[ML^2T^{-2}A^{-2}]$ (D) $[ML^2T^{-3}A^{-2}]$

Official Ans. by NTA (C)

Sol. e_2 : induced emf in secondary coil

i_1 : Current in primary coil

M : Mutual inductance

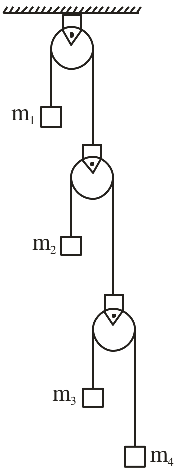
$$e_2 = -M\frac{di_1}{dt}$$

$$M = -\frac{e_2}{\frac{di_1}{dt}}$$

$$[M] = \frac{[e_2]}{\left[\frac{di_1}{dt}\right]} = \frac{\left[\frac{W}{q}\right]}{\left[\frac{di_1}{dt}\right]} = \frac{\frac{[ML^2T^{-2}]}{[AT]}}{[AT^{-1}]}$$

$= [ML^2T^{-2}A^{-2}]$

2. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the accelerations of masses m_1, m_2, m_3 and m_4 respectively. Which of the following relation is true for this arrangement?



(A) $4a_1 + 2a_2 + a_3 + a_4 = 0$

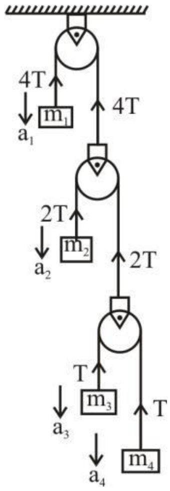
(B) $a_1 + 4a_2 + 3a_3 + a_4 = 0$

(C) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$

(D) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

Official Ans. by NTA (A)

Sol.



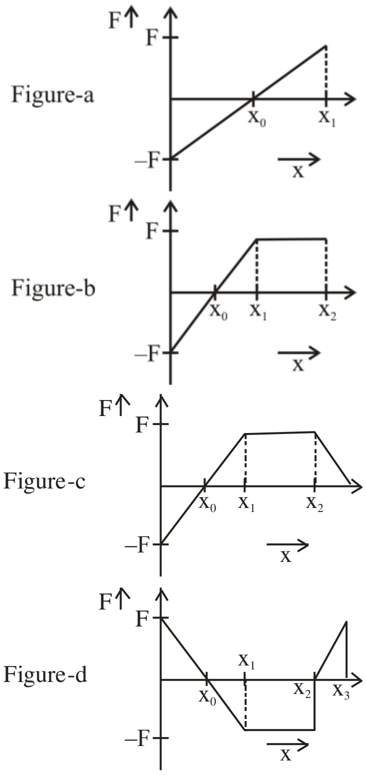
Using costraint

$$\sum \vec{T}.\vec{a} = 0$$

$-4Ta_1 - 2Ta_2 - Ta_3 - Ta_4 = 0$

$4a_1 + 2a_2 + a_3 + a_4 = 0$

3. Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3 and W_4 are the work done corresponding to figure a, b, c and d respectively.



(A) $W_3 > W_2 > W_1 > W_4$

(B) $W_3 > W_2 > W_4 > W_1$

(C) $W_2 > W_3 > W_4 > W_1$

(D) $W_2 > W_3 > W_1 > W_4$

Official Ans. by NTA (A)

Sol. Work done = area under F – x curve. Area below x –axis is negative & area above x-axis is positive.

so

$W_3 > W_2 > W_1 > W_4$

4. Solid spherical ball is rolling on a frictionless horizontal plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is :-

(A) $\frac{2}{5}$ (B) $\frac{2}{7}$ (C) $\frac{1}{5}$ (D) $\frac{7}{10}$

Official Ans. by NTA (B)

Sol. $K_{total} = K_{rotational} + K_{Translational}$

$K_{total} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mV_{cm}^2$

$v_{cm} = R\omega$ for pure rolling

$I_{cm} = \frac{2}{5}mR^2$

$K_{Rot} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\times\frac{2}{5}mR^2\times\frac{v_{cm}^2}{R^2} = \frac{1}{5}mv_{cm}^2$

$K_{Total} = \frac{1}{5}mv_{cm}^2 + \frac{1}{2}mv_{cm}^2 = \frac{7}{10}mv_{cm}^2$

$\frac{K_{Rot}}{K_{Total}} = \frac{\frac{1}{5}mv_{cm}^2}{\frac{7}{10}mv_{cm}^2} = \frac{2}{7}$

5. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R : At equator, the direction of acceleration due to the gravity is towards the center of earth.

In the light of above statements, choose the correct answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A.

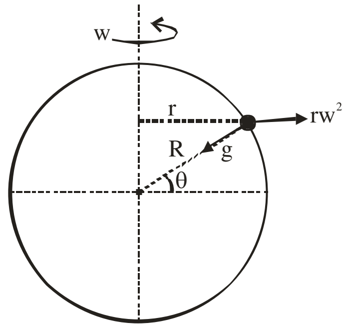
(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (D)

Sol.



Effective acceleration due to gravity is the resultant of g & rw^2 whose direction & magnitude depends upon θ. Hence assertion is false.

When θ = 0° (at equator), effective acceleration is radially inward.

6. If ρ is the density and η is coefficient of viscosity of fluid which flows with a speed v in the pipe of diameter d, the correct formula for Reynolds number R_e is :

(A) $R_e = \frac{\eta d}{\rho v}$ (B) $R_e = \frac{\rho v}{\eta d}$

(C) $R_e = \frac{\rho v d}{\eta}$ (D) $R_e = \frac{\eta}{\rho v d}$

Official Ans. by NTA (C)

Sol. Reynold's number is given by $\frac{\rho v d}{\eta}$

7. A flask contains argon and oxygen in the ratio of 3:2 in mass and the mixture is kept at 27°C. The ratio of their average kinetic energy per molecule respectively will be :

(A) 3 : 2 (B) 9 : 4

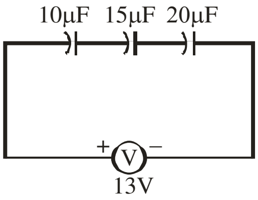
(C) 2 : 3 (D) 1 : 1

Official Ans. by NTA (D)

Sol. Average K.E./molecule $= \frac{f}{2}kT$

So, $\frac{K_{Ar}}{K_{O_2}} = \frac{\frac{3}{2}kT}{\frac{5}{2}kT} = \frac{3}{5}$

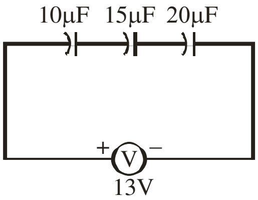
8. The charge on capacitor of capacitance 15μF in the figure given below is :



(A) 60μc (B) 130μc (C) 260 μc (D) 585 μc

Official Ans. by NTA (A)

Sol.



$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20} = \frac{12+8+6}{120} = \frac{26}{120}$$

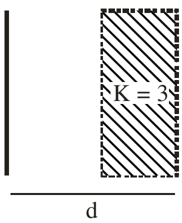
$$C_{eq} = \frac{60}{13}\mu F$$

$$Q = \frac{13 \times 60}{13} = 60\mu C$$

Charge on each capacitor is same

∵ they are in series.

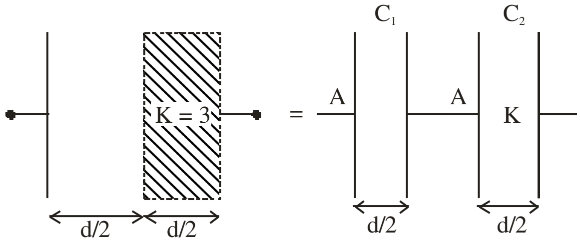
9. A parallel plate capacitor with plate area A and plate separation d=2 m has a capacitance of 4 μF. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant K=3 (as shown in figure) will be :



(A) 2μF (B) 32μF (C) 6μF (D) 8μF

Official Ans. by NTA (C)

Sol. $C_{original} = \frac{A\varepsilon_0}{d}$



$$C_1 = \frac{A\varepsilon_0}{d/2} = \frac{2A\varepsilon_0}{d} = C$$

$$C_2 = \frac{KA\varepsilon_0}{d/2} = \frac{2KA\varepsilon_0}{d} = \frac{6A\varepsilon_0}{d} = 3C$$

C_1 & C_2 are in series

$$C_{new} = \frac{C_1C_2}{C_1+C_2} = \frac{C \times 3C}{C+3C} = \frac{3C}{4}$$

$$= \frac{3}{4} \times \frac{2A\varepsilon_0}{d} = \frac{3}{2} \times \frac{A\varepsilon_0}{d}$$

$$C_{new} = \frac{3}{2}C_{original}$$

$$= \frac{3}{2} \times 4 = 6\mu F$$

10. Sixty four conducting drops each of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface density of bigger drop to the smaller drop will be :

(A) 1 : 4 (B) 4 : 1 (C) 1 : 8 (D) 8 : 1

Official Ans. by NTA (B)

Sol. Let R = radius of combined drop

r = radius of smaller drop

Volume will remain same

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

R = 4r

Q = 64q ;

q : charge of smaller drop

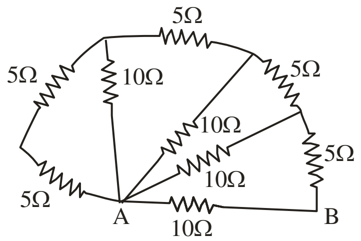
Q : Charge of combined drop

$$\frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{\frac{Q}{4\pi R^2}}{\frac{q}{4\pi r^2}} = \frac{Q}{q} \cdot \frac{r^2}{R^2}$$

$$= 64\frac{r^2}{16r^2} = 4$$

$$\frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{4}{1}$$

11. The equivalent resistance between points A and B in the given network is :

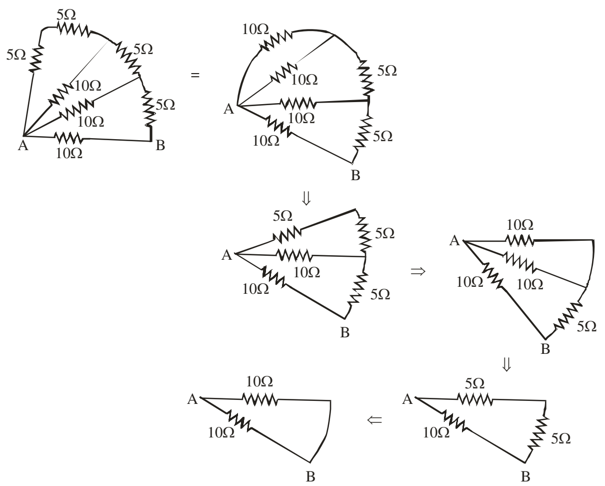


(A) 65Ω (B) 20Ω

(C) 5Ω (D) 2Ω

Official Ans. by NTA (C)

Sol.



$R_{AB} = 5\Omega$

12. A bar magnet having a magnetic moment of 2.0 × 10^5 JT^{-1}, is placed along the direction of uniform magnetic field of magnitude B= 14 × 10^{-5} T. The work done in rotating the magnet slowly through 60° from the direction of field is :

(A) 14 J (B) 8.4 J (C) 4 J (D) 1.4 J

Official Ans. by NTA (A)

Sol. Work done = MB ($\cos\theta_1 - \cos\theta_2$)

$\theta_1 = 0°, \theta_2 = 60°$

$= 2 \times 10^5 \times 14 \times 10^{-5} (1 - 1/2)$

= 14 J

13. Two coils of self inductance L_1 and L_2 are connected in series combination having mutual inductance of the coils as M. The equivalent self inductance of the combination will be :

(A) $\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{M}$ (B) $L_1 + L_2 + M$

(C) $L_1 + L_2 + 2M$ (D) $L_1 + L_2 - 2M$

Official Ans. by NTA (D)

Sol. Current on both the inductor is in opposite direction.

Hence :

$L_{eq} = L_1 + L_2 - 2M$

14. A metallic conductor of length 1m rotates in a vertical plane parallel to east-west direction about one of its end with angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is 0.2 × 10^{-4} T, then emf induced between the two ends of the conductor is :

(A) 5μV (B) 50μV (C) 5mV (D) 50mV

Official Ans. by NTA (B)

Sol. emf induced between the two ends = $\frac{B_H \omega l^2}{2}$

$\frac{0.2\times10^{-4}\times5\times1}{2} = 0.5\times10^{-4} = 50\times10^{-6}\,V = 50\mu V$

15. Which is the correct ascending order of wavelengths?

(A) $\lambda_{visible} < \lambda_{X-ray} < \lambda_{gamma-ray} < \lambda_{microwave}$

(B) $\lambda_{gamma-ray} < \lambda_{X-ray} < \lambda_{visible} < \lambda_{microwave}$

(C) $\lambda_{X-ray} < \lambda_{gamma-ray} < \lambda_{visible} < \lambda_{microwave}$

(D) $\lambda_{microwave} < \lambda_{visible} < \lambda_{gamma-ray} < \lambda_{X-ray}$

Official Ans. by NTA (B)

Sol. From electromagnetic wave spectrum.

λ increases ⟶

γ-ray	x-rays	ultra violet	visible	infrared	microwave	Radio wave

$\lambda_{\text{gamma-ray}} < \lambda_{\text{X-ray}} < \lambda_{\text{visible}} < \lambda_{\text{microwave}}$

16. For a specific wavelength 670 nm of light coming from a galaxy moving with velocity v, the observed wavelength is 670.7 nm.

The value of v is :

(A) 3×10^8 ms^{-1} (B) 3×10^{10} ms^{-1}

(C) 3.13×10^5 ms^{-1} (D) 4.48×10^5 ms^{-1}

Official Ans. by NTA (C)

Sol. $\lambda_{\text{emitted}} = 670$ nm

$\lambda_{\text{obs}} = 670.7$ nm

v = ?

c = 3×10^8 m/s

If v << c

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{v}{c}$$

$$\frac{670.7 - 670}{670} = \frac{v}{c}$$

V = 3.13×10^5 m/s

17. A metal surface is illuminated by a radiation of wavelength 4500 Å. The ejected photo-electron enters a constant magnetic field of 2 mT making an angle of 90° with the magnetic field. If it starts revolving in a circular path of radius 2 mm, the work function of the metal is approximately :

(A) 1.36 eV (B) 1.69 eV (C) 2.78 eV (D) 2.23 eV

Official Ans. by NTA (A)

Sol. $\lambda = 4500$ Å

B = 2mT, R = 2mm

$$R = \frac{\sqrt{2Km}}{qB}$$

$$\frac{(qBR)^2}{2m} = K$$

$$\frac{\left(1.6 \times 10^{-19} \times 2 \times 10^{-3} \times 2 \times 10^{-3}\right)^2}{2 \times 9.1 \times 10^{-31}} = K$$

$$\frac{(6.4)^2}{2 \times 9.1} \times \frac{10^{-50}}{10^{-31}} = K$$

K = 2.25×10^{-19} J

$$= \frac{2.25 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV} = 1.40 \text{eV}$$

$$E = \frac{12400}{4500} = 2.76 \text{eV}$$

$\phi = E - K = (2.76 - 1.40)$ eV = 1.36 eV

18. A radioactive nucleus can decay by two different processes. Half-life for the first process is 3.0 hours while it is 4.5 hours for the second process. The effective half- life of the nucleus will be :

(A) 3.75 hours (B) 0.56 hours

(C) 0.26 hours (D) 1.80 hours

Official Ans. by NTA (D)

Sol. $\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{\ln 2}{(t_{1/2})_{eq}} = \frac{\ln 2}{(t_{1/2})_1} + \frac{\ln 2}{(t_{1/2})_2}$$

$$(t_{1/2})_{eq} = \frac{(t_{1/2})_1 \times (t_{1/2})_2}{(t_{1/2})_1 + (t_{1/2})_2}$$

$$= \frac{3 \times 4.5}{3 + 4.5} = \frac{3 \times 4.5}{7.5} = \frac{3 \times 3}{5} = 1.8 \text{ hr}$$

19. The positive feedback is required by an amplifier to act an oscillator. The feedback here means :

(A) External input is necessary to sustain ac signal in output.

(B) A portion of the output power is returned back to the input.

(C) Feedback can be achieved by LR network.

(D) The base-collector junction must be forward biased.

Official Ans. by NTA (B)

Sol. When the amplifier connects with positive feedback, it acts as the oscillator the feedback here is positive feedback which means some amount of voltage is given to the input.

20. A sinusoidal wave $y(t) = 40\sin(10 \times 10^6 \pi t)$ is amplitude modulated by another sinusoidal wave $x(t) = 20\sin(1000\pi t)$. The amplitude of minimum frequency component of modulated signal is :

(A) 0.5 (B) 0.25 (C) 20 (D) 10

Official Ans. by NTA (D)

Sol. $y(t) = 40 \sin(10 \times 10^6 \pi t)$

$x(t) = 20\sin(1000\pi t)$

$\Rightarrow \omega_c = 10^7 \pi$

$\omega_m = 10^3 \pi$

$A_C = 40$

$A_m = 20$

Equation of modulated wave = $(A_C + A_m \sin \omega_m t) \sin \omega_c t$

$$= A_c\left(1 + \frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$$

$$= A_c(1 + \mu \sin \omega_m t) \sin \omega_c t, \qquad \mu = \frac{A_m}{A_c}$$

$$= A_c \sin\omega_c t + \frac{\mu A_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

Amplitude of minimum frequency = $\frac{\mu A_c}{2} = \frac{A_m}{A_c} \times \frac{A_c}{2} = \frac{A_m}{2} = 10$

SECTION-B

1. A ball is projected vertically upward with an initial velocity of 50 ms^{-1} at t = 0s. At t = 2s. another ball is projected vertically upward with same velocity. At t =________s, second ball will meet the first ball (g =10 ms^{-2}).

Official Ans. by NTA (6)

Sol. Let they meet at t = t

So first ball gets t sec.

& 2^{nd} gets (t – 2) sec. & they will meet at same height

$$h_1 = 50t - \frac{1}{2}gt^2$$

$$h_2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$h_1 = h_2$

$$50t - \frac{1}{2}gt^2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$$100 = \frac{1}{2}g\left[t^2 - (t-2)^2\right]$$

$$100 = \frac{10}{2}[4t - 4]$$

$5 = t - 1$

t = 6 sec.

2. A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is ______________Ns.

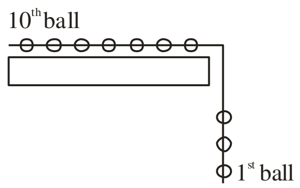
Official Ans. by NTA (12)

Sol. Impulse = change in momentum

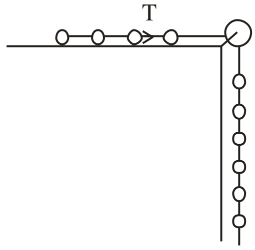
$= m[v - (-v)] = 2\,mv$

$= 2 \times 0.4 \times 15 = 12$ Ns

3. A system to 10 balls each of mass 2 kg are connected via massless and unstretchable string. The system is allowed to slip over the edge of a smooth table as shown in figure. Tension on the string between the 7^{th} and 8^{th} ball is____________N when 6^{th} ball just leaves the table.



Official Ans. by NTA (36)

Sol. 

$$a = \frac{6mg}{10m} = \frac{6g}{10} = \frac{3g}{5}$$

taking 8,9,10 together 

T = 3 ma

$$= 3m \times \frac{3g}{5}$$

= 36 N

4. A geyser heats water flowing at a rate of 2.0 kg per minute from 30°C to 70°C. If geyser operates on a gas burner, the rate of combustion of fuel will be ____________g min^{-1}

[Heat of combustion = 8×10^3 Jg^{-1}

Specific heat of water = 4.2 Jg^{-1} $°C^{-1}$]

Official Ans. by NTA (42)

Sol. m = 2000 gm/min

Heat required by water/min = mSΔT

= (2000) × 4.2 × 40 J/min

= 336000 J/min

The rate of combustion = $\left(\frac{dm}{dt}L\right) = 336000 J/min$

$$\frac{dm}{dt} = \frac{336000}{8 \times 10^3} g/min$$

= 42 gm/min

5. A heat engine operates with the cold reservoir at temperature 324 K.

The minimum temperature of the hot reservoir, if the heat engine takes 300 J heat from the hot reservoir and delivers 180 J heat to the cold reservoir per cycle, is ___________K.

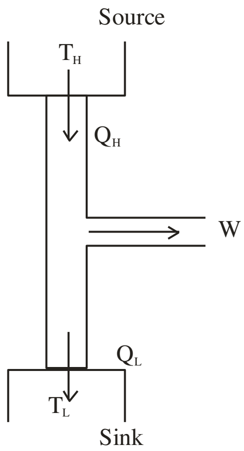
Official Ans. by NTA (540)

Sol. T_c = 324 k

T_H = ?

Q_H = 300 J

Q_L = 180 J



$$1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

$$T_H = \frac{Q_H}{Q_L} \times T_L = \frac{300}{180} \times 324 = 540 K$$

6. A set of 20 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is_____Hz.

Official Ans. by NTA (152)

)

Sol. $f_1 = f$

$f_2 = f + 4$

$f_3 = f + 2 \times 4$

$f_4 = f + 3 \times 4$

$f_{20} = f + 19 \times 4$

$f + (19 \times 4) = 2 \times f$

f = 76 Hz.

Frequency of last tuning forks = 2f

= 152 Hz

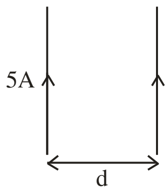
7. Two 10 cm long, straight wires, each carrying a current of 5A are kept parallel to each other. If each wire experienced a force of 10^{-5} N, then separation between the wires is _________cm.

Official Ans. by NTA (5)

Sol. It should be mentioned, 10 cm wire is part of long wire.

Force experienced by unit length of wire

$$=\frac{\mu_0 I_1 I_2}{2\pi d},\ I_1 = I_2 = 5A$$



Force experienced by wires of length 10 cm

$$=\frac{\mu_0 I_1 I_2}{2\pi d}\times 10\times 10^{-2}$$

$$10^{-5}=\frac{2\times 10^{-7}\times 5\times 5}{d}\times 10\times 10^{-2}$$

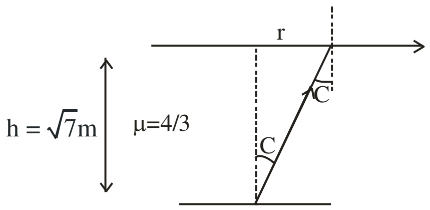
$d = 50 \times 10^{-3}$ m

$d = 50 \times 10^{-1}$ cm = 5 cm.

8. A small bulb is placed at the bottom of a tank containing water to a depth of $\sqrt{7}$ m. The refractive index of water is $\frac{4}{3}$. The area of the surface of water through which light from the bulb can emerge out is $x\pi$ m^2. The value of x is_______.

Official Ans. by NTA (9)

Sol. C : Criticle angle



$$\tan C=\frac{r}{h}$$

r = h tan C

$$\sin C=\frac{1}{\mu}=\frac{3}{4}$$

$$\tan C=\frac{3}{\sqrt{7}}$$

$$r=\sqrt{7}\times\frac{3}{\sqrt{7}}=3$$

Area of surface = $\pi r^2 = 9\pi m^2$

9. A travelling microscope is used to determine the refractive index of a glass slab. If 40 divisions are there in 1 cm on main scale and 50 Vernier scale divisions are equal to 49 main scale divisions, then least count of the travelling microscope is ______$\times 10^{-6}$ m.

Official Ans. by NTA (5)

Sol. 50 VSD = 49 MSD

$$1VSD=\frac{49}{50}MSD$$

Least count = 1 MSD – 1 VSD

$$=\left(1-\frac{49}{50}\right)MSD=\frac{1}{50}MSD$$

$$1MSD=\frac{1}{40}cm$$

$$\text{Least count}=\frac{1}{50\times 40}\ cm$$

$$=\frac{1}{2000}\ cm=\frac{1}{2}\times 10^{-5}m$$

$= 0.5 \times 10^{-5}$ m

$= 5 \times 10^{-6}$ m

10. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 6630 Å is 0.42 V. If the threshold frequency is x × 10^{13}/s, where x is ______ (nearest integer).

(Given, speed light = 3 × 10^8 m/s, Planck's constant = 6.63 × 10^{-34} Js)

Official Ans. by NTA (35)

Sol. Stopping potential V_0 = 0.42 V

λ = 6630 Å

$E = \phi + eV_0$

E : energy of incident photon

V_0 : Stopping potential

$\phi = E - eV_0$

$$E=\frac{12400}{6630}eV=1.87\ eV$$

ϕ = (1.87 – 0.42) = 1.45 eV

$\phi = h\nu_0$; ν_0 : threshold frequency

$1.45 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times \nu_0$

$\nu_0 = 0.35 \times 10^{15}$

$= 35 \times 10^{13}$ sec^{-1}

= 35

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. The number of radial and angular nodes in 4d orbital are. respectively

(A) 1 and 2 (B) 3 and 2

(C) 1 and 0 (D) 2 and 1

Official Ans. by NTA (A)

Sol. Radial node = $n - l - 1$

$= 4 - 2 - 1$

$= 1$

Angular node (l) = 2

2. Match List I with List II.

List I Enzyme	List II Conversion of
A. Invertase	I. Starch into maltose
B. Zymase	II. Maltose into glucose
C. Diastase	III. Glucose into ethanol
D. Maltase	IV. Cane sugar into glucose

Choose the most appropriate answer from the options given below :

(A) A-III, B-IV. C-II. D-I

(B) A-III. B-II. C-I. D-IV

(C) A-IV, B-IIL C-I. D-II

(D) A-IV, B-II. C-III. D-I

Official Ans. by NTA (C)

Sol. Invertase : Cane sugar $\rightarrow$ Glucose and fructose

Zymase : Glucose $\rightarrow$ Ethanol and CO_2

Diastase : Starch $\rightarrow$ Maltose

Maltase : Maltose $\rightarrow$ Glucose

3. Which of the following elements in considered as a metalloid?

(A) Sc (B) Pb (C) Bi (D) Te

Official Ans. by NTA (D)

Sol. Sc, Pb, Bi are metals

Te is a metalloid

4. The role of depressants in Froth Flotation method* is to

(A) selectively prevent one component of the ore from coming to the froth.

(B) reduce the consumption of oil for froth formation.

(C) stabilize the froth.

(D) enhance non-wettability of the mineral particles.

Official Ans. by NTA (A)

Sol. Depressant prevent one component from coming to the froth.

For eg., in Galena ore, the depressant (NaCN) prevents impurity (ZnS) from coming to the froth.

5. Boiling of hard water is helpful in removing the temporary hardness by converting calcium hydrogen carbonate and magnesium hydrogen carbonate to

(A) $CaCO_3$ and $Mg(OH)_2$

(B) $CaCO_3$ and M_2CO_3

(C) $Ca(OH)_2$ and $MgCO_3$

(D) $Ca(OH)_2$ and $Mg(OH)_2$

Official Ans. by NTA (A)

Sol. $Mg(HCO_3)_2 \xrightarrow{\text{Boil}} Mg(OH)_2 + 2CO_2\uparrow$

$Ca(HCO_3)_2 \xrightarrow{\text{Boil}} CaCO_3 + H_2O + CO_2\uparrow$

6. s-block element which cannot be qualitatively confirmed by the flame test is

(A) Li (B) Na (C) Rb (D) Be

Official Ans. by NTA (D)

Sol.

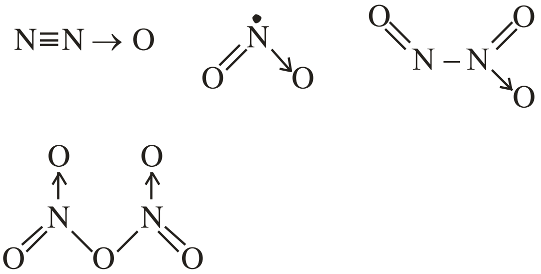
	Flame color
Li	Crimson Red
Na	Yellow
Rb	Red violet
Be	No color

7. The oxide which contains an odd electron at the nitrogen atom is

(A) N_2O (B) NO_2 (C) N_2O_3 (D) N_2O_5

Official Ans. by NTA (B)

Sol.



8. Which one of the following is an example of disproportionation reaction?

(A) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(B) $MnO_4^{2-} + 4H^+ + 4e^- \rightarrow MnO_2 + 2H_2O$

(C) $10I^- + 2MnO_4^- + 16H^+ \rightarrow 2Mn^{2+} + 8H_2O + 5I_2$

(D) $8MnO_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8MnO_2 + 6SO_4^{2-} + 2OH^-$

Official Ans. by NTA (A)

Sol. Reduction

$$\overset{+6}{3\,MnO_4^{2-}} + 4H^+ \rightarrow 2\,\overset{+7}{MnO_4^-} + \overset{+4}{MnO_2} + 2H_2O$$

Oxidation

9. The most common oxidation state of Lanthanoid elements is +3. Which of the following is likely to deviate easily from +3 oxidation state?

(A) Ce (At. No. 58) (B) La (At. No. 57)

(C) Lu (At. No. 71) (D) Gd (At. No. 64)

Official Ans. by NTA (A)

Sol. $Ce = [Xe]\,4f^1\,5d^1\,6s^2$

$Ce^{3+} = [Xe]\,4f^1\,5d^0$

$Ce^{+4} = [Xe]\,4f^0\,5d^0$ (Noble gas configuration)

10. The measured BOD values for four different water samples (A-D) are as follows:

A = 3 ppm: B=18 ppm: C-21 ppm: D=4 ppm. The water samples which can be called as highly polluted with organic wastes, are

(A) A and B (B) A and D

(C) B and C (D) B and D

Official Ans. by NTA (C)

Sol. Clean water $\longrightarrow$ B.O.D. < 5 ppm

Highly polluted water $\longrightarrow$ B.O.D. > 17 ppm

11. The correct order of nucleophilicity is

(A) $F^- > OH^-$ (B) $H_2\ddot{O} > OH^-$

(C) $R\ddot{O}H > RO^-$ (D) $NH_2^- > NH_3$

Official Ans. by NTA (D)

Sol. Nucleophilicity $\propto$ electro density on donor atom

$\propto$ size of donor atom (in gas)

$\propto \dfrac{1}{\text{EN of atom}}$ (for period)

12. Oxidation of toluene to Benzaldehyde can be easily carried out with which of the following reagents?

(A) CrO_3/acetic acid, H_3O^+

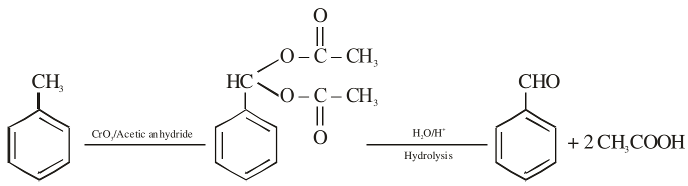
(B) CrO_3/acetic anhydride, H_3O^+

(C) $KMnO_4$/HCl, H_3O^+

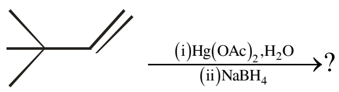
(D) CO/HCl, anhydrous $AlCl_3$

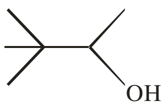
Official Ans. by NTA (B)

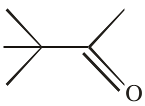
Sol.

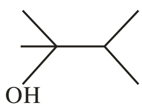


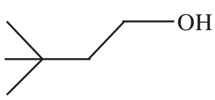
13. The major product in the following reaction



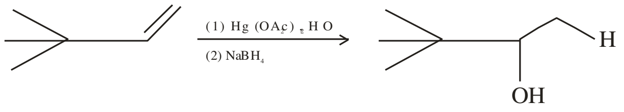
(A) 

(B) 

(C) 

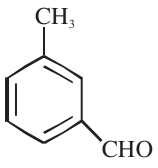
(D) 

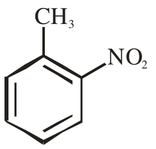
Official Ans. by NTA (A)

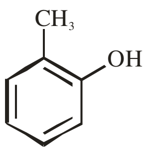
Sol. 

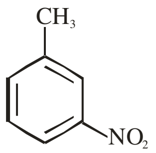
Oxymercuration – Demercuration
Addition of H_2O
Markovnikov's addition without rearrangement

14. Halogenation of which one of the following will yield m-substituted product with respect to methyl group as a major product?

(A) 

(B) 

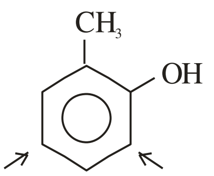
(C) 

(D) 

Official Ans. by NTA (C)

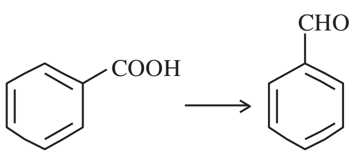
Sol. Electrophile will attack at ortho and para position with respect to better electron releasing group (ERG)

ERG : – OH > – CH_3



Para position with respect to – OH (+R) group and it will be meta position with respect to – CH_3 group.

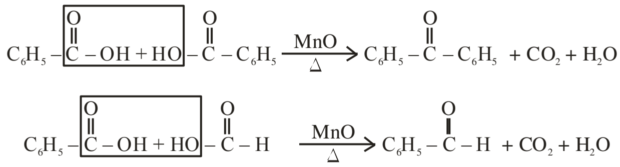
15. The reagent, from the following, which converts benzoic acid to benzaldehyde in one step is



(A) $LiAlH_4$ (B) $KMnO_4$
(C) MnO (D) $NaBH_4$

Official Ans. by NTA (C)

Sol.



16. The final product 'A' in the following reaction sequence

$$CH_3CH_2-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} ? \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} A$$

(A) $CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$

(B) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CN$

(C) $CH_3-CH-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-COOH$

(D) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CONH_2$

Official Ans. by NTA (A)

Sol.

$$CH_3CH_2-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} CH_3CH_2-\overset{\overset{CN}{|}}{\underset{\underset{OH}{|}}{C}}-CH_3 \xrightarrow[\text{Heat}]{95\%\ H_2SO_4}$$

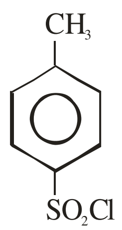
$$CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$$

17. Which statement is NOT correct for p-toluenesulphonyl chloride?

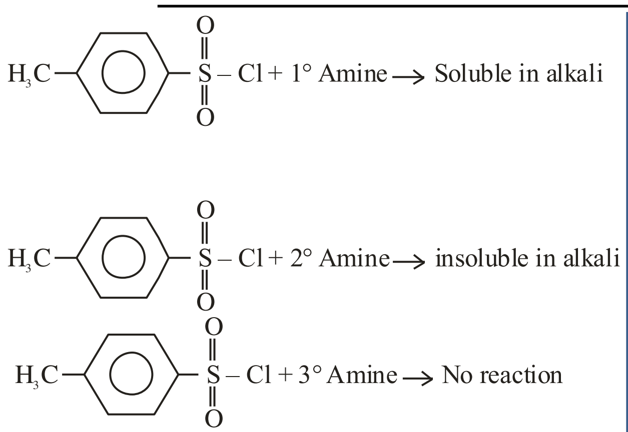
(A) It is known as Hinsberg's reagent.
(B) It is used to distinguish primary and secondary amines.
(C) On treatment with secondary amine, it leads to a product, that is soluble in alkali.
(D) It doesn't react with tertiary amines.

Official Ans. by NTA (C)

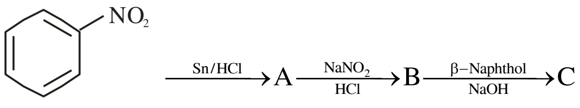
Sol.



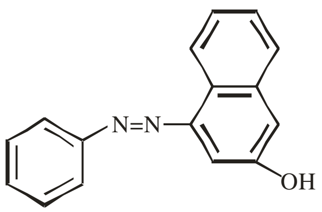
Hinsberg's reagent

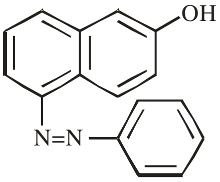


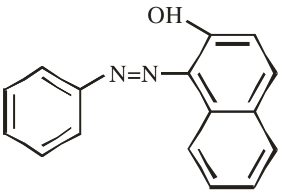
18. The final product 'C' is the following series series of reactions

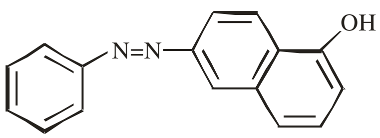


is

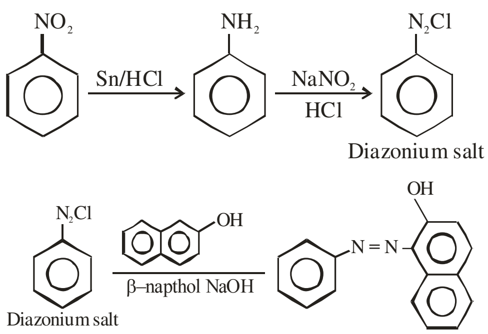
(A) 

(B) 

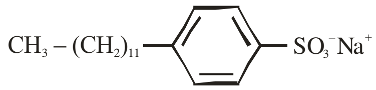
(C) 

(D) 

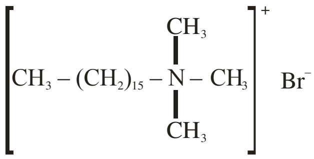
Official Ans. by NTA (C)

Sol. 

19. Which of the following is NOT an example of synthetic detergent?

(A) 

(B) $CH_3 – (CH_2)_{16} –COO^- Na^+$

(C) 

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$

Official Ans. by NTA (B)

Sol. Refer NCERT (Page No. 452)

20. Which one of the following is a water soluble vitamin, that is not excreted easily?

(A) Vitamin B_2 (B) Vitamin B_1

(C) Vitamin B_6 (D) Vitamin B_{12}

Official Ans. by NTA (D)

Sol. Refer NCERT (Page No. 426)

SECTION-B

1. CNG is an important transportation fuel. When 100 g CNG is mixed with 208 oxygen in vehicles, it leads to the formation of CO_2 and H_2O and produces large quantity of heat during this combustion, then the amount of carbon dioxide, produced in grams is________. [nearest integer]

[Assume CNG to be methane]

Official Ans. by NTA (143)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Mole $\frac{100}{16}$ $\frac{208}{32}$

$= 6.25$ $= 6.5$

$\frac{\text{Mole}}{\text{Stoi. Coeff.}}$ $\frac{6.25}{1}$ $\frac{6.5}{2} = 3.25$

So, O_2 is limiting reagent

<u>Mole – Mole analysis</u>

$\frac{n_{O_2}}{2} = \frac{n_{co_2}}{1}$

$\frac{6.5}{2} = n_{co_2}$

Mass of $CO_2 = \frac{6.5}{2} \times 44 = 143$ gm

2. In a solid AB. A atoms are in ccp arrangement and B atoms occupy all the octahedral sites. If two atoms from the opposite faces are removed, then the resultant stoichiometry of the compound is A_xB_y. The value of x is_______. [nearest integer]

Official Ans. by NTA (3)

Sol. $A \rightarrow 4 - \left(2 \times \frac{1}{2}\right) = 3$

$B \rightarrow 12 \times \frac{1}{4} + 1 \times 1 = 4$

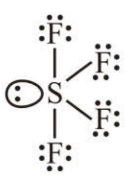
So, Compound is A_3B_4

The value of x is 3.

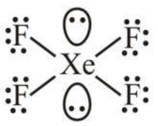
3. Amongst SF_4, XeF_4, CF_4 and H_2O, the number of species with two lone pairs of electrons ______ .

Official Ans. by NTA (2)

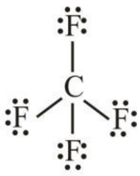
Sol.



Total lone pairs = 13

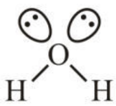


Total lone pairs = 14



Total lone pairs = 12

Total lone pairs = 2



4. A fish swimming in water body when taken out from the water body is covered with a film of water of weight 36 g. When it is subjected to cooking at 100°C, then the internal energy for vaporization in kJ mol^{-1} is ________.

[nearest integer]

[Assume steam to be an ideal gas. Given $A_{vap}H^{\ominus}$ for water at 373 K and 1 bar is 41.1 kJ mol^{-1} ; R = 8.31 $JK^{-1}mol^{-1}$]

Official Ans. by NTA (38)

Sol. $H_2O\ (l) \rightarrow H_2O(g)$

$n = \frac{36}{18} = 2$ mol

$\Delta U = \Delta H - \Delta n_g RT$

$= 41.1 - \frac{1 \times 8.31 \times 373}{1000}$ kJ / mol

= 38 kJ/mol

5. The osmotic pressure exerted by a solution prepared by dissolving 2.0 g of protein of molar mass 60 kg mol^{-1} in 200 mL of water at 27°C is _______ Pa. [integer value]

(use R = 0.083 L bar mol^{-1} K^{-1})

Official Ans. by NTA (415)

Sol. $\pi = iCRT$

$= \frac{1 \times 2}{60000 \times 0.2} \times 0.083 \times 300$

= 0.00415 bar ($\because$ 1 bar = 10^5 Pa)

So, 0.00415 × 10^5 Pa = 415 Pa

6. 40° of HI undergoes decomposition to H_2 and I_2 at 300 K. $\Delta G^{\ominus}$ for this decompostion reaction at one atmosphere pressure is________ J mol^{-1}. [nearest integer]

(Use R = 8.31 J K^{-1} mol^{-1}; log 2 = 0.3010. In 10 = 2.3, log 3 = 0.477)

Official Ans. by NTA (2735)

Sol. $HI \rightleftharpoons \frac{1}{2}H_2 + \frac{1}{2}I_2$

t_i	1		
teq	1 – 0.4	$\frac{0.4}{2}$	$\frac{0.4}{2}$

$K_p = \frac{(0.2)^{\frac{1}{2}}(0.2)^{\frac{1}{2}}}{1 - 0.4} = \frac{0.2}{0.6} = \frac{1}{3}$

$\Delta G = \Delta G^\circ + RT\ \ln K = 0$

$\Delta G^\circ = -RT\ \ln K \Rightarrow -8.31 \times 300 \times 2.3 \times \log\left(\frac{1}{3}\right)$

= 2735 J/mol

7. $Cu(s) + Sn^{2+}$ (0.001M) $\rightarrow Cu^{2+}$ (0.01M) + Sn(s)

The Gibbs free energy change for the above reaction at 298 K is x × 10^{-1} kJ mol^{-1};

The value of x is_______. [nearest integer]

$\left[\text{Given}: E^{\ominus}_{Cu^{2+}/Cu} = 0.34V; E^{\ominus}_{Sn^{2+}/Sn} = -0.14V; F = 96500C\ mol^{-1}\right]$

Official Ans. by NTA (983)

Sol. $Cu_{(s)} + Sn^{2+}$ (0.001 M) $\rightarrow Cu^{2+}$ (0.01 M) + $Sn_{(s)}$

$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$

$= -0.14 - (0.34)$

$= -0.48$ V

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[Cu^{2+}]}{[Sn^{2+}]}$$

$$= -0.48 - \frac{0.059}{2}\log\frac{0.01}{0.001}$$

$= -0.509$

$\Delta G = -nF E_{cell}$

$= -2 \times 96500 \times (-0.5095)$

= 98333.5 J/mol

= 98.335 kJ/mol

= 983.35 × 10^{-1} kJ/mol

Nearest Integer : 983

8. Catalyst A reduces the activation energy for a reaction by 10 kJ mol^{-1} at 300 K. The ratio of rate constants, $\frac{k_{T,Catalysed}}{k_{T,Uncatalysed}}$ is e^x. The value of x is _______. [nearest integer]

[Assume theat the pre-exponential factor is same in both the cases.

Given R = 8.31 J K^{-1} mol^{-1}]

Official Ans. by NTA (4)

Sol.

$$K = Ae^{\frac{-Ea}{RT}}$$

$$K_{cat} = Ae^{-\frac{E_a^1}{RT}}, \quad K_{uncat.} = Ae^{-\frac{Ea}{RT}}$$

$$\frac{K_{cat}}{K_{uncat.}} = e^{\frac{E_a - E_a^1}{RT}} = e^{\frac{10\times1000}{8.31\times300}} = e^{4.009} = e^x$$

$\therefore\ x = 4$

9. Reaction of $[Co(H_2O)_6]^{2+}$ with excess ammonia and in the presence of oxygen results into a diamagnetic product. Number of electrons present in t_{2g}–orbitals of the product is _______ .

Official Ans. by NTA (6)

Sol. $[Co(H_2O)_6]^{2+}$ +NH_3(excess) $\rightarrow$ $[Co(NH_3)_6]^{3+}$ + $6H_2O$

Diamagnetic

↓

Low spin complex

$Co^{3+} \Rightarrow 3d^6\ 4s^0$

$\Rightarrow t_{2g}^{\ 6}\ e_g^{\ 0}$

Total number electrons = 6

10. The moles of methane required to produce 81 g of water after complete combustion is _______ × 10^{-2} mol. [nearest integer]

Official Ans. by NTA (225)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

POAC on H atom

$n_{CH4} \times 4 = n_{H2O} \times 2$

$$n_{CH_4} = \frac{81}{18} \times 2 \times \frac{1}{4} = \frac{81}{36}$$

$n_{CH_4} = 2.25$

$= 225 \times 10^{-2}$

Nearest Integers = 225

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 03 : 00 PM to 06 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let f : $\mathbb{R} \to \mathbb{R}$ be defined as f(x) = x-1 and $g : \mathbb{R} - \{1, -1\} \to \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$.

Then the function fog is :

(A) one-one but not onto function

(B) onto but not one-one function

(C) both one-one and onto function

(D) neither one-one nor onto function

Official Ans. by NTA (D)

Sol. $f(x) = x - 1$; $g(x) = \frac{x^2}{x^2-1}$

$f(g(x)) = g(x) - 1$

$= \frac{x^2}{x^2-1} - 1 = \frac{x^2 - x^2 + 1}{x^2 - 1}$

$f(g(x)) = \frac{1}{x^2-1}$; $x \neq \pm 1$, even function

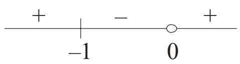
$\to$ Hence f(g(x)) is many one function

$y = \frac{1}{x^2 - 1}$

$y \cdot x^2 - y = 1$

$x^2 = \left(\frac{1+y}{y}\right)$

$\left(\frac{1+y}{y}\right) \geq 0$



Range:- $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range $\neq$ Co-domain $\Rightarrow$ f(g(x)) is into function

2. If the system of equations $\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$,

has infinitely many solutions, then the ordered pair (α, β) is equal to :

(A) (1,–3) (B) (–1, 3)

(C) (1, 3) (D) (–1, –3)

Official Ans. by NTA (C)

For infinitely many solutions,

$\Delta = 0 = \Delta_x = \Delta_y = \Delta_z$

Sol. $\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \alpha(10 - 9) - 1(5 - 3) + 1(3 - 2) = 0$

$\Rightarrow \alpha - 2 + 1 = 0$

$\Rightarrow \alpha = 1$

$\Delta_x = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow 5(10 - 9) - 1(20 - 3\beta) + 1(12 - 2\beta)$

$\Rightarrow 5 - 20 + 3\beta + 12 - 2\beta$

$\Rightarrow -3 + \beta = 0$

$\Rightarrow \beta = 3$

3. If $A = \sum_{n=1}^{\infty} \frac{1}{\left(3 + (-1)^n\right)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(3 + (-1)^n\right)^n}$, then $\frac{A}{B}$ is equal to :

(A) $\frac{11}{9}$ (B) 1 (C) $-\frac{11}{9}$ (D) $-\frac{11}{3}$

Official Ans. by NTA (C)

Sol. $A = \left(\frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$A = \left(\frac{1}{2} + \frac{1}{2^3} + \ldots..\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$A = \left(\frac{\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}\right)$

$\Rightarrow A = \frac{1}{2} \times \frac{4}{3} + \frac{1}{16} \times \frac{16}{15} \Rightarrow A = \frac{11}{15}$

$B = \frac{-1}{2} + \frac{1}{4^2} + \frac{-1}{2^3} + \frac{1}{4^4} + \ldots..\infty$

$B = \left(\frac{-1}{2} + \frac{-1}{2^3} + \ldots.\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \ldots..\infty\right)$

$$B=\frac{-\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$\Rightarrow B=-\frac{1}{2}\times\frac{4}{3}+\frac{1}{16}\times\frac{16}{15}$$

$$B=-\frac{9}{15}$$

$$\frac{A}{B}=\frac{11}{15}\times\frac{15}{(-9)}$$

$$\frac{A}{B}=-\frac{11}{9}$$

4. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to :

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Official Ans. by NTA (C)

Sol. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4};\left(\frac{0}{0}\right)$

$$\lim_{x\to 0}\left(\frac{2\cdot\sin\left(\frac{x+\sin x}{2}\right)\cdot\sin\left(\frac{x-\sin x}{2}\right)}{x^4}\right)$$

$$\lim_{x\to 0}2\left(\frac{\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)}\right)\left(\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right)\left(\frac{\frac{x+\sin x}{2}}{x^4}\right)\left(\frac{x-\sin x}{2}\right)$$

$$\lim_{x\to 0}\left(\frac{x^2-\sin^2 x}{2x^4}\right):\left(\frac{0}{0}\right)$$

Apply L-Hopital Rule :

$$\lim_{x\to 0}\frac{2x-2\sin x\cos x}{2.4.x^3}$$

$\lim\limits_{x\to 0}\frac{2x-\sin 2x}{8x^3};\frac{0}{0}$: Again apply L-Hopital rule

$$\lim_{x\to 0}\frac{2-2\cos(2x)}{8(3)x^2}$$

$$\lim_{x\to 0}\frac{2(1-\cos(2x))}{24(4x^2)}\times 4\Rightarrow\frac{2}{24}\times\frac{1}{2}\times 4\Rightarrow\frac{1}{6}$$

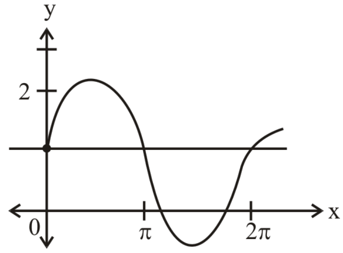
5. Let f (x) = min {1, 1 + x sin x}, $0\le x\le 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to

(A) (2, 0) (B) (1, 0)

(C) (1, 1) (D) (2, 1)

Official Ans. by NTA (B)

Sol.



No. of non-differentiable points = 1 (m)

No. of not continuous points = 0 (n)

(m, n) = (1, 0)

6. Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum, is :

(A) 2 : 5 (B) 19:45 (C) 3 : 8 (D) 19 : 15

Official Ans. by NTA (B)

Sol. Surface area = $76x^2+3\pi r^2$ = constant (K)

$$V=40x^3+\frac{2}{3}\pi r^3$$

$[76x^2+3\pi r^2=K]$

$$r^2=\frac{K-76x^2}{3\pi}$$

$$r=\left(\frac{K-76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$V=40x^3+\frac{2}{3}\pi\left(\frac{K-76x^2}{3\pi}\right)^{\frac{3}{2}}$$

$$\frac{dV}{dx}=120x^2+\frac{2}{3}\pi.\frac{3}{2}\left(\frac{K-76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right)$$

Put

$$\frac{dV}{dx}=0\Rightarrow 120x^2+\frac{2}{3}\pi.\frac{3}{2}\left(\frac{K-76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right)=0$$

$$\Rightarrow 120x^2=\frac{152x}{3}\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$\Rightarrow\frac{45}{19}x^2=x\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\quad ;x\ne 0$$

$$\Rightarrow\frac{45}{19}x=\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\Rightarrow\left(\frac{45}{19}\right)^2x^2=\frac{k-76x^2}{3\pi}$$

$$\Rightarrow\left(\frac{45}{19}\right)^2x^2=r^2\Rightarrow\frac{x^2}{r^2}=\left(\frac{19}{45}\right)^2$$

$$\Rightarrow\frac{x}{r}=\frac{19}{45}$$

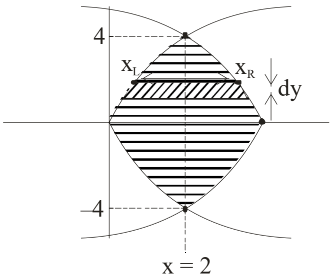
7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to :-

(A) $\frac{32}{3}$ (B) $\frac{40}{3}$ (C) 16 (D) 19

Official Ans. by NTA (C)

Sol. $y^2 = 8x$; $y^2 = 16(3 - x)$

$$y^2 = -16(x - 3)$$



finding their intersection pts.

$y^2 = 8x$ & $y^2 = -16(x - 3)$

$8x = -16x + 48$

$24x = 48$

$x = 2$; $y = \pm 4$

Required Area

$$A = 2.\int_0^4 (x_R - x_L)dy$$

$$= 2.\int_0^4 \left(\underbrace{3 - \frac{y^2}{16}}_{(x_R} - \underbrace{\frac{y^2}{8}}_{x_L)}\right)dy$$

$$= 2\left(3y - \frac{y^3}{3\times16} - \frac{y^3}{3\times8}\right)_0^4$$

$$= 2\left(3\times4 - \frac{4\times4\times4}{3\times16} - \frac{4\times4\times4\times2}{3\times8\times2}\right)$$

$$= 2\left(12 - \frac{4}{3} - \frac{8}{3}\right) = 2\times12\left(1 - \frac{1}{3}\right) = 2\times12\times\frac{2}{3} = 16$$

8. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c, g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to :

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$ (B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$ (D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

Official Ans. by NTA (A)

Sol. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c$

Put $x = \cos 2\theta$

$dx = -2\sin 2\theta \cdot d\theta$

$$= \int \frac{1}{\cos 2\theta}\tan\theta(-4\sin\theta\cdot\cos\theta)d\theta$$

$$= \int \frac{1}{\cos 2\theta}(-4\sin^2\theta)d\theta$$

$$= -2\int \frac{1-\cos 2\theta}{\cos 2\theta}d\theta$$

$$= -\frac{2}{2}\ln|\sec 2\theta + \tan 2\theta| + 2\theta + c$$

$$= \ln|\sec 2\theta - \tan 2\theta| + 2\theta + c$$

$$= \ln\left|\frac{1 - \sin 2\theta}{\cos 2\theta}\right| + \cos^{-1}x + c$$

$$= \underbrace{\ln\left|\frac{1 - \sqrt{1-x^2}}{x}\right| + \cos^{-1}x}_{g(x)} + c$$

$\therefore g(1) = 0$

$$g(x) = \ln\left|\frac{1 - \sqrt{1-x^2}}{x}\right| + \cos^{-1}x$$

$$g\left(\frac{1}{2}\right) = \ln\left|2 - \sqrt{3}\right| + \frac{\pi}{3}$$

$$g\left(\frac{1}{2}\right) = \ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| + \frac{\pi}{3}$$

9. If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx} + 2y = xe^x, y(1) = 0$, then the local maximum value of the function $z(x) = x^2y(x) - e^x$, $x \in R$ is :

(A) $1 - e$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{4}{e} - e$

Official Ans. by NTA (D)

Sol. $x\frac{dy}{dx} + 2y = xe^x$

$$\frac{dy}{dx} + \frac{2y}{x} = e^x$$

I.F. $= x^2$

$$y.x^2 = \int x^2e^x dx$$

$= \int e^x \left(x^2 + 2x - 2x - 2 + 2\right) dx$

$yx^2 = e^x (x^2 - 2x + 2) + c$

$y(1) = 0$

$0 = e(1 + 0) + c$

$c = -e$

$z(x) = x^2 y(x) - e^x$

$= e^x(x^2 - 2x + 2) - e - e^x$

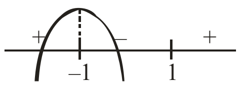
$= e^x (x - 1)^2 - e$

$\frac{dz}{dx} = e^x.2\left(x-1\right) + e^x\left(x-1\right)^2 = 0$

$x^x (x - 1) (2 + x - 1) = 0$

$e^x (x - 1) (x + 1) = 0$

$x = -1, 1$



$x = -1$ local maxima. Then maximum value is

$z(-1) = \frac{4}{e} - e$

10. If the solution of the differential equation

$\frac{dy}{dx} + e^x\left(x^2 - 2\right)y = \left(x^2 - 2x\right)\left(x^2 - 2\right)e^{2x}$ satisfies

$y(0) = 0$, then the value of $y(2)$ is ________ .

(A) –1 (B) 1 (C) 0 (D) e

Official Ans. by NTA (C)

Sol. $I.F. = e^{\int e^x(x^2-2)dx} = e^{\int e^x(x^2-2x+2x-2)dx}$

$= e^{e^x(x^2-2x)}$

$y.e^{e^x(x^2-2x)} = \int e^{e^x(x^2-2x)} e^x \left(x^2 - 2x\right)\left(x^2 - 2\right) e^x dx$

Let $e^x(x^2 - 2x) = t$

So, $y.e^{e^x(x^2-2x)} = \int e^t.t\ dt$

At $x = 0,\ t = 0$

$x = 2,\ t = 0$

$= t \cdot e^t - e^t + c$

$x = 0;\ 0 \cdot 1 = 0 - 1 + c \Rightarrow c = 1$

for $x = 2;\ y \cdot 1 = 0 - 1 + 1 = 0$

$y(2) = 0$

11. If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to :

(A) 6 (B) 9

(C) 10 (D) 12

Official Ans. by NTA (B)

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

equation of tangent to the ellipse is

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = mx \pm \sqrt{16m^2 + 9}$ ....(i)

$x^2 + y^2 = 12$

equation of tangent to the circle is

$y = mx \pm \sqrt{12}\sqrt{1 + m^2}$ ....(ii)

for common tangent equate eq. (i) and (ii)

$\Rightarrow 16m^2 + 9 = 12(1 + m^2)$

$16\ m^2 - 12\ m^2 = 3$

$4\ m^2 = 3$

$12\ m^2 = 9$

12. The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Official Ans. by NTA (C)

Sol. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

P(4,3) •———•———• Q $\left(2\cos\theta, \sqrt{2}\sin\theta\right)$

D

Coordinate of D is

$\left(\frac{2\cos\theta + 4}{2}, \frac{\sqrt{2}\sin\theta + 3}{2}\right) \equiv (h, k)$

$\frac{2h - 4}{2} = \cos\theta$ ....(i)

$\frac{2k - 3}{\sqrt{2}} = \sin\theta$ ....(ii)

$(i)^2 + (ii)^2$, then we get

$$\left(\frac{2h-4}{2}\right)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1 \Rightarrow \frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{1}{2}\right)}=1$$

∴ Required eccentricity is

$$e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

13. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $(8,3\sqrt{3})$ on it passes through the point :

(A) $(15,-2\sqrt{3})$ (B) $(9,2\sqrt{3})$

(C) $(-1,9\sqrt{3})$ (D) $(-1,6\sqrt{3})$

Official Ans. by NTA (C)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{9}=1$: $(8,3\sqrt{3})$ lie on Hyperbola then

$\frac{64}{a^2}-\frac{27}{9}=1 \Rightarrow a^2=\frac{64}{4}=16$

equation of normal at $(8,3\sqrt{3})$:

$\frac{16x}{8}+\frac{9y}{3\sqrt{3}}=16+9$

$2x+\sqrt{3}y=25$

Check options.

14. If the plane 2x + y – 5z = 0 is rotated about its line of intersection with the plane 3x – y + 4z – 7 = 0 by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :

(A) (2, –2, 0) (B) (–2, 2, 0)

(C) (1, 0, 2) (D) (–1, 0, –2)

Official Ans. by NTA (C)

Sol. (2x + y – 5z) + λ(3x – y + 4z – 7) = 0

Rotated by π/2

(2 + 3 λ)x + (1 – λ)y + (–5 + 4λ)z – 7λ = 0

2x + y – 5z = 0

2(2 + 3 λ) + (1 – λ) – 5(–5 + 4 λ) = 0

⇒ 4 + 6 λ + 1 – λ + 25 – 20 λ = 0

30 = 15 λ

λ = 2

Required plane :- 8x – y + 3z – 14 = 0

Check options

15. If the lines $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{j}-\hat{k})$ and $\vec{r}=(\alpha\hat{i}-\hat{j})+\mu(2\hat{i}-3\hat{k})$ are co-planar, then distance of the plane containing these two lines from the point (•, 0, 0) is :

(A) $\frac{2}{9}$ (B) $\frac{2}{11}$

(C) $\frac{4}{11}$ (D) 2

Official Ans. by NTA (B)

Sol. $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{j}-\hat{k})$ ….. L1

$\vec{r}=(\alpha\hat{i}-\hat{j})+\mu(2\hat{i}-3\hat{k})$ ….. L2

• L1 and L2 are coplanar

$$\therefore \begin{vmatrix} 0 & 3 & -1 \\ 2 & 0 & -3 \\ (1-\alpha) & 0 & 1 \end{vmatrix}=0$$

–3 (2 + 3 (1 – •)) = 0

2 + 3 – 3• = 0

• 3• = 5

$\Rightarrow \alpha=\frac{5}{3}$

Now,

$$\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix}=\hat{i}(-9)-\hat{j}(2)+k(-6)$$

= (9, 2, 6)

Equation of plane :

9 (x – 1) + 2(y + 1) + 6(z – 1) = 0

9x + 2y + 6z – 13 = 0

Perpendicular distance from (•, 0, 0)

$$=\left|\frac{\left(9\cdot\frac{5}{3}+0+0-13\right)}{\sqrt{81+36+4}}\right|=\frac{2}{\sqrt{121}}=\frac{2}{11}$$

16. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k},\vec{b}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}.\hat{j}=7$, then $\vec{v}.(\hat{i}+\hat{k})$ is equal to :

(A) 6 (B) 7 (C) 8 (D) 9

Official Ans. by NTA (D)

Sol. $\vec{v}=\lambda\vec{a}+\mu\vec{b}$

$\vec{v}=\lambda(1,1,2)+\mu(2,-3,1)$

$\vec{v}=(\lambda+2\mu,\lambda-3\mu,2\lambda+\mu)$

$\vec{v}.\hat{j}=7$ $\quad$ $\vec{v}.\frac{\vec{c}}{|\vec{c}|}=\frac{2}{\sqrt{3}}$

$\lambda-3\mu=7$ $\quad$ $\vec{v}.\vec{c}=2$

$\lambda+2\mu-\lambda+3\mu+2\lambda+\mu=2$

$2\lambda+6\mu=2$

$\lambda+3\mu=1$

$\lambda-3\mu=7$

$2\lambda=8$

$\lambda=4$

$\mu=-1$

We get $\vec{v}=(2,7,7)$

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to :

(A) 10 (B) 36 (C) 43 (D) 60

Official Ans. by NTA (C)

Sol. No. of observations: - 50

$\text{mean}(\bar{x})=15$

Standard deviation $(\sigma)=2$

Let incorrect observation is x_1 & correct observation is (x'_1)

Given $x_1+x'_1=70$

$\bar{x}=\frac{x_1+x_2+....+x_{50}}{50}=15(\text{given})$

$\Rightarrow x_1+x_2+.....x_{50}=750$ ....(i)

Now

Mean of correct observation is 16

$\frac{x'_1+x_2+....+x_{50}}{50}=16$

$x'_1+x_2+x_3+....x_{50}=16\times50$ ....(ii)

eq. (ii) – eq. (i)

$\Rightarrow x'_1-x_1=16\times50-15\times50$

$x'_1-x_1=50$ & $x_1+x'_1=70$

$x'_1=60$

$x_1=10$

$\Rightarrow 4=\frac{x_1^2+x_2^2+....+x_{50}^2}{50}-15^2$ ....(iii)

$\Rightarrow \sigma^2=\frac{x'^2_1+x_2^2+....x_{50}^2}{50}-16^2$ ....(iv)

from (iii)

$\Rightarrow 4=\frac{(10)^2}{50}+\frac{x_2^2+x_3^2+....+x_{50}^2}{50}-225$

$\Rightarrow 4=2-225+\frac{(x_2^2+x_3^2+....+x_{50}^2)}{50}$

$\Rightarrow 227=\frac{(x_2^2+x_3^2+....x_{50}^2)}{50}$

From (iv)

$\sigma^2=\frac{(60)^2}{50}+\left(\frac{x_2^2+x_3^2+....+x_{50}^2}{50}\right)-(16)^2$

$\sigma^2=\frac{60\times60}{50}+227-256$

$\sigma^2=72+227-256$

$\sigma^2=43$

18. $16\sin(20°)\sin(40°)\sin(80°)$ is equal to :

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 3 (D) $4\sqrt{3}$

Official Ans. by NTA (B)

Sol. $16\sin20°\sin40°\sin80°$

$=16\sin40°\sin20°\sin80°$

$=4(4\sin(60-20)\sin(20)\sin(60+20))$

$=4\times\sin(3\times20°)$

$[\because \sin3\theta=4\sin(60-\theta)\times\sin\theta\times\sin(60+\theta)]$

$=4\times\sin60°$

$=4\times\frac{\sqrt{3}}{2}=2\sqrt{3}$

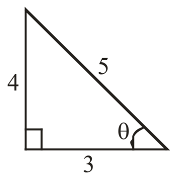
19. If the inverse trigonometric functions take principal values, then $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Sol. Let

$\tan^{-1}\frac{4}{3}=\theta \Rightarrow \tan\theta=\frac{4}{3}$



$E=\cos^{-1}\left(\frac{3}{10}\cos\theta+\frac{2}{5}\sin\theta\right)$

$=\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\cdot\frac{4}{5}\right)$

$=\cos^{-1}\left(\frac{9}{50}+\frac{8}{25}\right)=\cos^{-1}\left(\frac{25}{50}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

20. Let $r \in \{p, q, \sim p, \sim q\}$ be such that the logical statement $r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$ is a tautology. Then 'r' is equal to :

(A) p (B) q (C) ~p (D) ~q

Official Ans. by NTA (C)

Sol. By options

(1)

p=r	q	~p	$r\vee(\sim p)$	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
T	F	F	T	F	T	T
F	T	T	T	F	F	F
T	T	F	T	T	T	T
F	F	T	T	F	F	F

(2)

p	~p	$r\vee(\sim p)$	q=r	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(3)

p	q	r = ~p	$r\vee(\sim p)$	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(4)

~p	p	q	$r\vee(\sim p)$	r=~q	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

Now final answer is option no. 3.

SECTION-B

1. Let $f:\mathbb{R}\to\mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x), \forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14.\frac{f'(4)}{f'(2)}$ is equal to ______.

Official Ans. by NTA (248)

Sol. Put y = 2

$f(x + y) = 2^x.f(y) + 4^y.f(x).$

$f(x + 2) = 2^x.3 + 16f(x)$

$f'(x + 2) = 16f'(x) + 3.2^x \ln 2$

$f'(4) = 16f'(2) + 12\ln 2$ ....(i)

$f(y + 2) = 4f(y) + 3\cdot 4^y$

$f'(y+2) = 4f'(y) + 3.4^y \ln 4$

$f'(4) = 4f'(2) + 96 \ln 2$ ....(ii)

solving eq. (i) and (ii), we get

$f'(2) = 7 \ln 2$

from equation (i), we get

$f'(4) = 124 \ln 2$

Now, $\Rightarrow 14.\frac{f'(4)}{f'(2)}$

$14 \times \frac{124 \ln 2}{7 \ln 2}$

$= 248.$

2. Let p and q be two real numbers such that p + q = 3 and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is equal to ________.

Official Ans. by NTA (4)

Sol. $p + q = 3 \quad p^4 + q^4 = 369$

$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$

$(p+q)^2 = 9$

$p^2 + q^2 = 9 - 2pq$

$$\frac{1}{\left(\frac{1}{p}+\frac{1}{q}\right)^2} = \frac{(qp)^2}{(q+p)^2} = \frac{(qp)^2}{9}$$

$p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$

$369 = (9 - 2pq)^2 - 2(pq)^2$

$369 = 81 + 4p^2q^2 - 36pq - 2p^2q^2$

$288 = 2p^2q^2 - 36pq$

$144 = p^2q^2 - 18pq$

$(pq)^2 - 2 \times 9 \times pq + 9^2 = 144 + 9^2$

$(pq - 9)^2 = 225$

$pq - 9 = \pm 15$

$pq = \pm 15 + 9$

$pq = 24, -6$

(24 is rejected because $p^2 + q^2 = 9 - 2pq$ is negative)

$$\frac{(qp)^2}{9} = \frac{1(-6)^2}{9} = 4$$

3. If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then $\left|\sum_{n=1}^{15}\left(Z^n + (-1)^n \frac{1}{Z^n}\right)^2\right|$ is equal to _______ .

Official Ans. by NTA (2)

Sol. $z^2 + z + 1 = 0 \Rightarrow z = w, w^2$

$$\left|\sum_{n=1}^{15}\left(z^n + (-1)\frac{1}{z^n}\right)^2\right| = \left|\sum_{n=1}^{15}\left(z^{2n} + \frac{1}{z^{2n}} + 2(-1)^n\right)\right|$$

$$= \left|\sum_{n=1}^{15} w^{2n} + \frac{1}{w^{2n}} + 2(-1)^n\right|$$

$$= \left|\frac{w^2(1-w^{30})}{1-w^2} + \frac{\frac{1}{w^2}\left(1-\frac{1}{w^{30}}\right)}{1-\frac{1}{w^2}} + 2(-1)\right|$$

$$= \left|\frac{w^2(1-1)}{1-w^2} + \frac{\frac{1}{w^2}(1-1)}{1-\frac{1}{w^2}} - 2\right|$$

$= |0 + 0 - 2| = 2$

4. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, Y = \alpha I + \beta X + \gamma X^2$ and $Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2, \alpha, \beta, \gamma \in \mathbb{R}$. If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to _____ .

Official Ans. by NTA (100)

Sol. $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$Y = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}, Z = \begin{bmatrix} \alpha^2 & -\alpha\beta & \beta^2 - \alpha\gamma \\ 0 & \alpha^2 & -\alpha\beta \\ 0 & 0 & \alpha^2 \end{bmatrix}$

$Y \cdot Y^{-1} = I$

$$\begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}\begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{\alpha}{5} = 1 \Rightarrow \alpha = 5$

$-\frac{2}{5}\alpha + \frac{\beta}{5} = 0 \Rightarrow \beta = 10$

$\frac{\alpha}{5} - \frac{2\beta}{5} + \frac{\gamma}{5} = 0 \quad \Rightarrow \gamma = 15$

$\Rightarrow (\alpha - \beta + \gamma)^2 = (5 - 10 + 15)^2 = 100$

5. The total number of 3–digit numbers, whose greatest common divisor with 36 is 2, is _______ .

Official Ans. by NTA (150)

Sol. $36 = 2 \times 2 \times 3 \times 3$

Number should be odd multiple of 2 and does not having factor 3 and 9

Odd multiple of 2 are

102, 106, 110, 114 ……998 (225 no.)

No. of multiples of 3 are

102, 114, 126 ……. 990 (75 no.)

Which are also included multiple of 9

Hence,

Required = 225 – 75 = 150

6. If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+...+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$, m and n are coprime, then m + n is equal to _____ .

Official Ans. by NTA (102)

Sol. ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + ...{}^{59}C_{19} + {}^{60}C_{20}$

$\left(\frac{1}{41}+1\right){}^{41}C_1 + {}^{42}C_2 + ...$

$\left[\frac{42}{41}\left(\frac{2}{42}\right)+1\right]{}^{42}C_2 + {}^{43}C_3 + ...$

$\left(\frac{2}{41}+1\right){}^{42}C_2 + {}^{43}C_3 + ...$

$\left(\frac{43}{41}\times\frac{3}{43}+1\right){}^{43}C_3 + {}^{44}C_4 + ...$

$\frac{3+41}{41}.{}^{43}C_3 + ...$

Similarly :

$\frac{20+41}{41}$

$\Rightarrow m = 61 ; n = 41$

m + n = 102

7. If $a_1(> 0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _____.

Official Ans. by NTA (40)

Sol. $a_1 > 0, a_2, a_3, a_4, a_5 \to$ G.P.

$3a_2 + a_3 = 2a_4$

$3ar + ar^2 = 2ar^3$

$3 + r = 2r^2$

$2r^2 – r – 3 = 0$

$r = – 1$ & $r = \frac{3}{2}$

$a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$

$a (r + r^3 – 2r^2) = 1$

$a\left(\frac{3}{2}+\frac{27}{8}-\frac{18}{4}\right) = 1$

$a = \frac{8}{3}$

When $r = –1$, $a = -\frac{1}{4}$ (rejected, $a_1 > 0$)

$r = \frac{2}{3}, a = \frac{8}{3}$ (selected)

Now

$a_2 + a_4 + 2a_5$

$= \frac{8}{3}\times\frac{3}{2}+\frac{8}{3}\times\frac{27}{8}+2\times\frac{8}{3}\times\frac{81}{16}$

$= 4 + 9 + 27 = 40$

8. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to ____.

Official Ans. by NTA (3)

Sol. $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{(2-x^2)}{(x^2+2)\sqrt{4+x^4}}dx$

$\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{x^2\left(\frac{2}{x^2}-1\right)dx}{x\left(x+\frac{2}{x}\right)\times x\sqrt{\frac{4}{x^2}+x^2}}$

$$\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{\left(\frac{2}{x^2}-1\right)dx}{\left(x+\frac{2}{x}\right)\sqrt{\left(x+\frac{2}{x}\right)^2-4}}$$

$$x+\frac{2}{x}=t$$

$$dt=\left(1-\frac{2}{x^2}\right)dx$$

$$I=-\frac{24}{\pi}\int\frac{dt}{t\sqrt{t^2-4}}$$

$$= -\frac{24}{\pi}\times\frac{1}{2}\sec^{-1}\left(\frac{x+\frac{2}{x}}{2}\right)\Bigg|_0^{\sqrt{2}}$$

$$= -\frac{12}{\pi}\left[\sec^{-1}\left(\frac{2\sqrt{2}}{2}\right)-\sec^{-1}(\infty)\right]$$

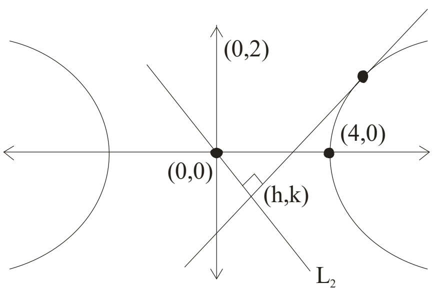
$$= -\frac{12}{\pi}\left[\frac{\pi}{4}-\frac{2\pi}{2\times 2}\right]=-\frac{12}{\pi}\left[-\frac{\pi}{4}\right]$$

$= 3$

9. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to ______ .

Official Ans. by NTA (12)

Sol.



$$\frac{x\sec\theta}{4}-\frac{y\tan\theta}{2}=1$$

$$m_1=\frac{\sec\theta\times 2}{4(\tan\theta)}=\frac{\sec\theta}{2\tan\theta}$$

$$m_2=\frac{k}{h}$$

$$m_1m_2=-1$$

$$\frac{k}{h}\cdot\frac{\sec\theta}{2\tan\theta}=-1$$

$$\frac{k}{2h\sin\theta}=-1$$

$$\sin\theta=\frac{-k}{2h}\qquad \cos\theta=\frac{\sqrt{4h^2-k^2}}{2h}$$

also

$$\frac{h\sec\theta}{4}-\frac{k\tan\theta}{2}=1$$

$$\frac{h}{4}\frac{2h}{\sqrt{4h^2-k^2}}-\frac{k}{2}\left(\frac{-k}{\sqrt{4h^2-k^2}}\right)=1$$

$$h^2+k^2=2\sqrt{4h^2-k^2}$$

$$(x^2+y^2)^2 = 4(4x^2-y^2)$$

$$(x^2+y^2)^2 = 16x^2-4y^2$$

$\alpha = 16, \beta = -4$

$\alpha + \beta = 16 - 4 = 12$

10. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to _____ .

Official Ans. by NTA (33)

Sol. $2\times2\times2\times2\times2\times2=64$

Divisible by 21 when divided by 3.

Case – I : All 1 $\rightarrow$ (1)

Case – II : All 8 $\rightarrow$ (1)

Case – III : 3 ones & 3 eights

$$\frac{6!}{3!\times 3!}=20$$

Required probability $\therefore p=\frac{22}{64}$

$$96p=96\times\frac{22}{64}=33$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 9:00 AM to 12:00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

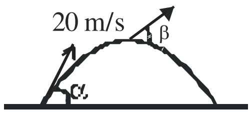
1. A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1} . After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be : ($g = 10\ ms^{-2}$)

(A) $\tan\alpha + 5\sec\alpha$ (B) $\tan\alpha - 5\sec\alpha$

(C) $2\tan\alpha - 5\sec\alpha$ (D) $2\tan\alpha + 5\sec\alpha$

Official Ans. by NTA (B)

Sol.



$v_x = u_x = 20\cos\alpha$

$v_y = 20\sin\alpha - 10\times10$

$\tan\beta = \frac{v_y}{v_x} = \frac{20\sin\alpha - 100}{20\cos\alpha}$

$= \tan\alpha - 5\sec\alpha$

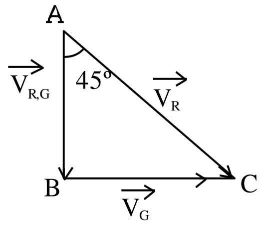
2. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}\,kmh^{-1}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is :

(A) $30\,kmh^{-1}$ (B) $\frac{25}{\sqrt{2}}kmh^{-1}$

(C) $\frac{30}{\sqrt{2}}kmh^{-1}$ (D) $25\,kmh^{-1}$

Official Ans. by NTA (C)

Sol.



$V = \tan\theta = \frac{V_G}{V_{RG}}$

$1 = \frac{V_G}{V_{RG}} \Rightarrow 15\sqrt{2} = V_{RG}$

3. A sliver wire has mass (0.6 ± 0.006) g, radius (0.5 ± 0.005) mm and length (4 ± 0.04) cm. The maximum percentage error in the measurement of its density will be :

(A) 4% (B) 3%

(C) 6% (D) 7%

Official Ans. by NTA (A)

Sol. $M = (0.6 \pm 0.006)g$

$r = (0.5 \pm 0.005)mm$

$l = (4 \pm 0.04)cm$

$\rho = \frac{m}{V}$

$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}$

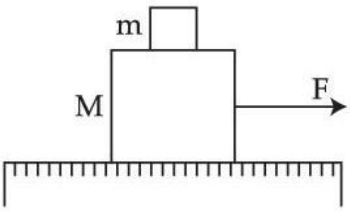
$(Volume\ of\ cylinder = \pi r^2 l)$

$= \frac{0.006}{0.6} + \frac{2\times0.005}{0.5} + \frac{0.04}{4}$

$100 \times \frac{\Delta\rho}{\rho} = 4\times10^{-2}\times100$

$\frac{\Delta\rho}{\rho}\times100 = 4\%$

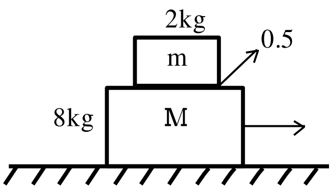
4. A system of two blocks of masses m = 2 kg and M = 8 kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be :



(A) 9.8 N (B) 39.2 N
(C) 49 N (D) 78.4 N

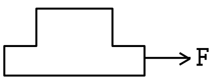
Official Ans. by NTA (C)

Sol.



$(a_A)_{\max} = 0.5g = 4.9\, m/s^2$

For moving together



$F_{\max} = m_T a_A$

$= 10 \times 4.9$

$= 49N$

5. Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) cm and (x, 0) cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is :

(A) 4 cm towards the 10 kg block
(B) 2 cm away from the 10 kg block
(C) 2 cm towards the 10 kg block
(D) 4 cm away from the 10 kg block

Official Ans. by NTA (C)

Sol. $\Delta x_G = \dfrac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$

$0 = \dfrac{10 \times 6 + 30(\Delta x_2)}{40}$

$\Delta x_2 = -2cm$

Block of mass 30 kg will to move towards 10 kg.

6. A 72 Ω galvanometer is shunted by a resistance of 8 Ω. The percentage of the total current which passes through the galvanometer is :

(A) 0.1% (B) 10 %
(C) 25% (D) 0.25%

Official Ans. by NTA (B)

Sol. $S = \dfrac{R_G}{\dfrac{I}{I_g} - 1}$

$8 = \dfrac{72}{\dfrac{I}{I_g} - 1}$

$\dfrac{I}{I_g} - 1 = 9$

$\dfrac{I}{I_g} = 10 \Rightarrow \dfrac{I_g}{I} = \dfrac{1}{10}$ $\quad \% I = \dfrac{I_g}{I} \times 100 = 10\%$

7. Given below are two statements :

Statement I : The law of gravitation holds good for any pair of bodies in the universe.

Statement II : The weight of any person becomes zero when the person is at the centre of the earth.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both statement I and Statement II are true
(B) Both statement I and Statement II are false
(C) Statement I is true but Statement II are false
(D) Statement I is false but Statement II is true

Official Ans. by NTA (A)

Sol. Since it is universal law so it hold good for any pair of bodies.

The value of **g** at centre is zero.

So statement I and Statement II are true.

8. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision)

(A) 50.0% (B) 66.6%

(C) 55.5% (D) 33.3%

Official Ans. by NTA (C)

Sol. Velocity after collision

$$V_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2}$$

$$V_2 = \frac{(5m - m)0 + 2m.u_0}{m + 5m} = \frac{u_0}{3}$$

$$\%\Delta KE = \frac{\frac{1}{2}5m\left(\frac{u_0}{3}\right)^2 - 0}{\frac{1}{2}mu_0^2} \times 100$$

$$= \frac{5u_0^2}{9u_0^2} \times 100 = \frac{500}{9} = 55.6\%$$

9. The velocity of a small ball of mass 'm' and density d_1, when dropped in a container filled with glycerine, becomes constant after some time. If the density of glycerine is d_2, then the viscous force acting on the ball, will be :

(A) $mg\left(1 - \frac{d_1}{d_2}\right)$ (B) $mg\left(1 - \frac{d_2}{d_1}\right)$

(C) $mg\left(\frac{d_1}{d_2} - 1\right)$ (D) $mg\left(\frac{d_2}{d_1} - 1\right)$

Official Ans. by NTA (B)

Sol.
$$F_V = mg - F_B$$
$$= mg - \left(\frac{m}{d_1} \times d_2\right)g$$
$$= mg\left(1 - \frac{d_2}{d_1}\right)$$

10. The susceptibility of a paramagnetic material is 99. The permeability of the material in Wb/A-m is :
[Permeability of free space $\mu_0 = 4\pi \times 10^{-7} Wb/A-m$]

(A) $4\pi \times 10^{-7}$ (B) $4\pi \times 10^{-4}$

(C) $4\pi \times 10^{-5}$ (D) $4\pi \times 10^{-6}$

Official Ans. by NTA (C)

Sol. Susceptibility $\chi = 99$

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$
$$\mu = \mu_0(1 + \chi)$$
$$= 4\pi \times 10^{-7}[1 + 99]$$
$$= 4\pi \times 10^{-5}$$

11. The current flowing through an ac circuit is given by

$I = 5\sin(120\pi t)A$

How long will the current take to reach the peak value starting from zero?

(A) $\frac{1}{60}s$ (B) 60s

(C) $\frac{1}{120}s$ (D) $\frac{1}{240}s$

Official Ans. by NTA (D)

Sol. $\omega = 120\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{60}$ sec

time taken to reach peak value $= \frac{T}{4} = \frac{1}{240}s$

12. Mach List-I with List – II :

List – I List – II

	List-I		List-Ii
(a)	Ultraviolet rays	(i)	Study crystal structure
(b)	Microwaves	(ii)	Greenhouse effect
(c)	Infrared waves	(iii)	Sterilizing surgical instrument
(d)	X-rays	(iv)	Radar system

Choose the correct answer from the options given below :

(A) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)

(B) (a) – (iii), (b) – (i), (c) – (ii), (d) – (iv)

(C) (a) – (iv), (b) – (iii), (c) – (ii), (d) – (i)

(D) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)

Official Ans. by NTA (A)

Sol. (Fact)

13. An α particle and a carbon 12 atom has same kinetic energy K. The ratio of their de-Broglie wavelength $\left(\lambda_{\alpha} : \lambda_{C12}\right)$ is :

(A) $1:\sqrt{3}$ (B) $\sqrt{3}:1$

(C) 3 : 1 (D) $2:\sqrt{3}$

Official Ans. by NTA (B)

Sol. $k=\dfrac{P^2}{2m} \Rightarrow P\alpha\sqrt{m}$

Now $\lambda=\dfrac{h}{p}$

So, $\lambda\alpha\dfrac{1}{p} \Rightarrow \lambda\ \alpha\ \dfrac{1}{\sqrt{m}}$

$\dfrac{\lambda_{\alpha}}{\lambda_{C12}}=\dfrac{\sqrt{3}}{1}$

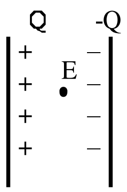
14. A force of 10N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be :

(A) 5 N (B) 10 N

(C) 20 N (D) Zero

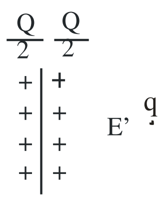
Official Ans. by NTA (A)

Sol.



$$F=qE=q\left(\frac{Q}{A\in_0}\right)=\frac{qQ}{A\in_0}=10N$$

Now, when one plate is removed.



$$E'=\frac{Q}{2A\in_0}$$

$$F=qE'=\frac{Qq}{2A\in_0}=5N$$

15. The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is :

(A) 6 s (B) 8 s

(C) 12s (D) 36 s

Official Ans. by NTA (D)

Sol. $X=A\sin\omega t\left(t=3,\ X=\dfrac{A}{2}\right)$

$\Rightarrow \dfrac{A}{2}=A\sin 3\omega$

$\Rightarrow \sin 3\omega=\dfrac{1}{2}$

$\Rightarrow 3\omega=\dfrac{\pi}{6}$

$\Rightarrow \omega=\dfrac{\pi}{18}=\dfrac{2\pi}{T}$

$\Rightarrow T=36s$

16. An observer moves towards a stationary source of sound with a velocity equal to one-fifth of the velocity of sound. The percentage change in the frequency will be :

(A) 20% (B) 10%

(C) 5% (D) 0%

Official Ans. by NTA (A)

Sol. $f_0=\left(\dfrac{v+v_0}{v}\right)f_s$

$f_0=\left(\dfrac{v+\dfrac{v}{5}}{v}\right)f_s$

$f_0=\dfrac{6}{5}f_s$

% change $=\dfrac{f_0-f_s}{f_s}\times 100$

$=\dfrac{1}{5}\times 100=20\%$

17. Consider a light ray travelling in air is incident into a medium of refractive index $\sqrt{2n}$. The incident angle is twice that of refracting angle. Then, the angle of incidence will be :

(A) $\sin^{-1}\left(\sqrt{n}\right)$ (B) $\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

(C) $\sin^{-1}\left(\sqrt{2n}\right)$ (D) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

Official Ans. by NTA (D)

Sol.

$$i = 2r$$

$$\sin i \times n_1 = \sin r \times n_2$$

$$\sin i \times 1 = \sin\frac{i}{2} \times \sqrt{2n}$$

$$\frac{\sin i}{\sin\frac{i}{2}} = \sqrt{2n}$$

$$\frac{2\sin\frac{i}{2}\cos\frac{i}{2}}{\sin\frac{i}{2}} = \sqrt{2n}$$

$$\cos\frac{i}{2} = \sqrt{\frac{n}{2}}$$

$$\frac{i}{2} = \cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$$

$$i = 2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$$

18. A hydrogen atom in is ground state absorbs 10.2 eV of energy. The angular momentum of electron of the hydrogen atom will increase by the value of : (Given, Plank's constant = 6.6×10^{-34} Js)

(A) $2.10\times10^{-34}\,Js$ (B) $1.05\times10^{-34}\,Js$

(C) $3.15\times10^{-34}\,Js$ (D) $4.2\times10^{-34}\,Js$

Official Ans. by NTA (B)

Sol. $13.6\left(\frac{1}{1^2}-\frac{1}{n^2}\right)=10.2$

$$n = 2$$

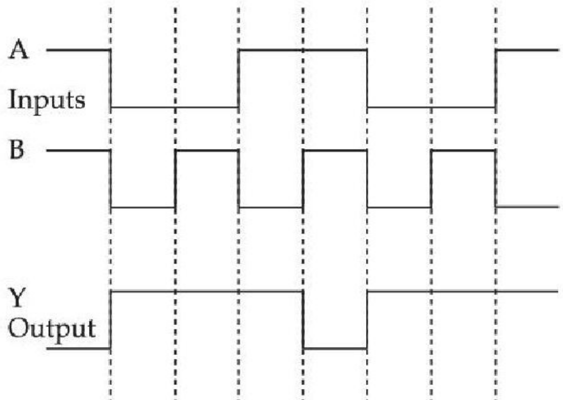
$$L_i = \frac{h}{2\pi}\times 1$$

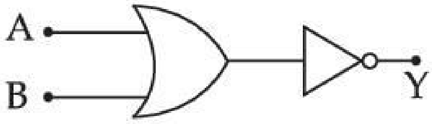
$$L_F = \frac{2h}{2\pi}$$

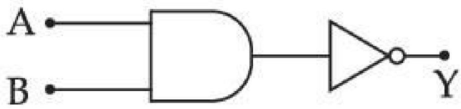
$$\Delta L = L_F - L_i = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\times\frac{22}{7}}$$

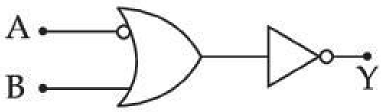
$$=1.05\times10^{-34}\,J-s$$

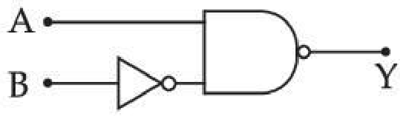
19. Identify the correct Logic Gate for the following output (Y) of two inputs A and B.



(A)


(B)


(C)


(D)


Official Ans. by NTA (B)

Sol.

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1
0	1	1
1	0	1
NAND Gate		

$Y = \overline{A.B}$

20. A mixture of hydrogen and oxygen has volume 2000 cm^3, temperature 300 K, pressure 100 kPa and mass 0.76 g The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be :

(A) $\frac{1}{3}$ (B) $\frac{3}{1}$

(C) $\frac{1}{16}$ (D) $\frac{16}{1}$

Official Ans. by NTA (B)

Sol. PV = nRT

$$n = \frac{100\times10^3\times2000\times10^{-6}}{\frac{25}{3}\times300}$$

$n = 80\times10^{-3}$

$n_1 + n_2 = 0.08$

$n_1\times2 + n_2\times32 = 0.76$

$(0.08 - n_2)2 + n_2(32) = 0.76$

$n_2 = 0.02$

$n_1 = 0.06$

$\frac{n_1}{n_2} = \frac{3}{1}$

SECTION-B

1. In a carnot engine, the temperature of reservoir is 527°C and that of sink is 200 K. If the workdone by the engine when it transfers heat from reservoir to sink is 12000 kJ, the quantity of heat absorbed by the engine from reservoir is ________ $\times 10^6$ J.

Official Ans. by NTA (16)

Sol. $(T_2)T_{sink} = 200K$

$(T_1)T_{Reservoir} = 527 + 273 = 800K$

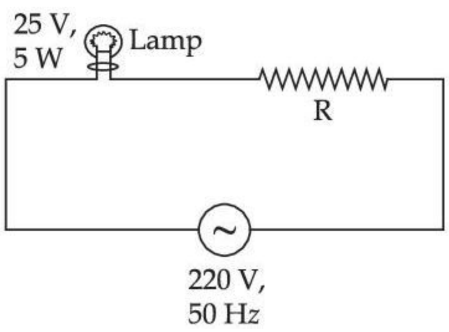
$W = 12000KJ = 12\times10^6 J$

$Q_1 = ?$

$$\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} = 1 - \frac{200}{800} = \frac{12\times10^6}{Q_1}$$

$$\frac{3}{4} = \frac{12\times10^6}{Q_1} = Q_1 = 16\times10^6 J$$

2. A 220 V, 50 Hz AC source is connected to a 25 V, 5 W lamp and an additional resistance R in series (as shown in figure) to run the lamp at its peak brightness, then the value of R (in ohm) will be ______ .



Official Ans. by NTA (975)

Sol. P = Vi

5 = 25i

$i = \frac{1}{5}$

$V_R = iR$

$(220 - 25) = \frac{1}{5}R$

$R = 195\times5 = 975\Omega$

3. In Young's double slit experiment the two slits are 0.6 mm distance apart. Interference pattern is observed on a screen at a distance 80 cm from the slits. The first dark fringe is observed on the screen directly opposite to one of the slits. The wavelength of light will be _____ nm.

Official Ans. by NTA (450)

Sol. $d = 0.6 \times 10^{-3}$

$D = 80 \times 10^{-2}$

1st Dark fringe $= \frac{D\lambda}{2d} = \frac{d}{2}, \qquad \lambda = \frac{d^2}{D}$

$= 450 \times 10^{-9} m$

4. A beam of monochromatic light is used to excite the electron in Li^{++} from the first orbit to the third orbit. The wavelength of monochromatic light is found to be $x \times 10^{-10} m$. The value of x is _______ .

[Given hc = 1242 eV nm]

Official Ans. by NTA (114)

Sol. Z = 3

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$n_1 = 1, \quad n_2 = 3,$

$$\frac{1}{\lambda} = R(9)\left(\frac{1}{1} - \frac{1}{9}\right) = 8R$$

$$\lambda = \frac{1}{8R} = 114 \times 10^{-10} m$$

5. A cell, shunted by a 8 Ω resistance, is balanced across a potentiometer wire of length 3m. The balancing length is 2 m when the cell is shunted by 4 Ω resistance. The value of internal resistance of the cell will be _____ Ω.

Official Ans. by NTA (8)

Sol. $\frac{V_1}{V_2} = \frac{3}{2} = \frac{E - i_1 r}{E - i_2 r}$

$$= \frac{E - \frac{E}{8+r} \times r}{E - \frac{E}{4+r} \times r}$$

$$\frac{3}{2} = \frac{8(4+r)}{4(8+r)}$$

24+ 3r = 16 + 4r

$r = 8\Omega$

6. The current density in a cylindrical wire of radius 4 mm is $4 \times 10^6 Am^{-2}$. The current through the outer portion of the wire between radial distance $\frac{R}{2}$ and R is ____ π A.

Official Ans. by NTA (48)

Sol. $J = \frac{I}{A}$

$I = JA$

$$= 4 \times 10^6 \times \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right]$$

$$= 4 \times 10^6 \times \pi R^2 \times \frac{3}{4}$$

$$= 4 \times 10^6 \times \pi \times \left(4 \times 10^{-3}\right)^2 \times \frac{3}{4} = 48\pi A.$$

7. A capacitor of capacitance 50 pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is ____ nJ.

Official Ans. by NTA (125)

Sol. Energy loss $= \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$

$$= \frac{1}{2}\frac{50 \times 50 \times 10^{-12} \times 10^{-12}}{(50+50)10^{-12}}(100-0)^2 = 125 nJ$$

8. The height of a transmitting antenna at the top of a tower is 25 m and that of receiving antenna is, 49 m. The maximum distance between them, for satisfactory communication in LOS (Line-Of-Sight) is $K\sqrt{5}\times10^2 m$. The value of K is ______ .

[Assume radius of Earth is $64\times10^{+5} m$] (Calculate upto nearest integer value)

Official Ans. by NTA (192)

Sol. $LOS=\sqrt{2Rh_T}+\sqrt{2Rh_R}$

$=\sqrt{2R}\left(\sqrt{h_T}+\sqrt{h_R}\right)$

$=\sqrt{2\times64\times10^5}\left(\sqrt{25}+\sqrt{49}\right)$

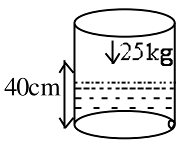
$=192\sqrt{5}\times10^2\ m.$

$K=192$

9. The area of cross-section of a large tank is 0.5 m^2. It has a narrow opening near the bottom having area of cross-section 1 cm^2. A load of 25 kg is applied on the water at the top in the tank. Neglecting the speed of water in the tank, the velocity of the water, coming out of the opening at the time when the height of water level in the tank is 40 cm above the bottom, will be _______ cms^{-1}. [Take g = 10 ms^{-2}]

Official Ans. by NTA (300)

Sol.



$$P_0+\frac{250}{0.5}+\rho g\left(40\times10^{-2}\right)=P_0+\frac{1}{2}\rho v^2$$

$$500+\frac{1000\times10\times40}{100}=\frac{1}{2}\times1000\times v^2$$

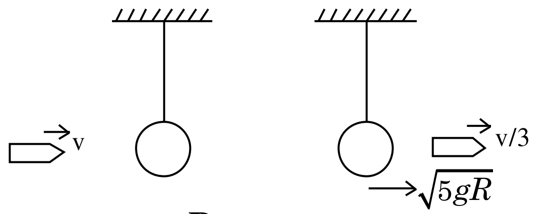
V = 3 m/s

V = 300 cm/s

10. A pendulum of length 2 m consists of a wooden bob of mass 50 g. A bullet of mass 75 g is fired towards the stationary bob with a speed v. The bullet emerges out of the bob with a speed $\frac{v}{3}$ and the bob just completes the vertical circle. The value of v is ________ ms^{-1}. (if g = 10 m/s^2)

Official Ans. by NTA (10)

Sol. Considering Only Horizontal direction



$P_i = P_f$

$$(75v)+0=50\left(\sqrt{5gR}\right)+75\frac{v}{3}$$

$$75\left(v-\frac{v}{3}\right)=50\sqrt{100}$$

$$v=10\,m/s$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : At 10°C, the density of a 5M solution of KCl [atomic masses of K and Cl are 39 & 35.5 g mol^{-1}]. The solution is cooled to -21°C. The molality of the solution will remain unchanged.

Reason (R) : The molality of a solution does not change with temperature as mass remains unaffected with temperature.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(B) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(C) **(A)** is true but **(R)** is false

(D) **(A)** is false but **(R)** is true

Official Ans. by NTA (A)

Sol. Molality is independent of temperature and hence both assertion and reason are true.

2. Based upon VSEPR theory, match the shape (geometry) of the molecules in List-I with the molecules in List-II and select the most appropriate option

List-I (Shape)	**List-II (Molecules)**
(A) T-shaped	(I) XeF_4
(B) Trigonal planar	(II) SF_4
(C) Square planar	(III) ClF_3
(D) See-saw	(IV) BF_3

(A) (A) – I, (B) – (II), (C) – (III), (D) – (IV)

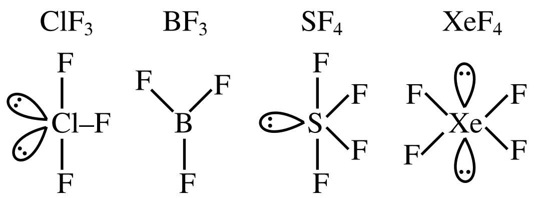
(B) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

(C) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

(D) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

Official Ans. by NTA (B)

Sol.



3. Match List-I with List-II

	List-I	**List-II**
(A)	Spontaneous process	(I) $\Delta H < 0$
(B)	Process with $\Delta P = 0$, $\Delta T = 0$	(II) $\Delta G_{T,P} < 0$
(C)	$\Delta H_{reaction}$	(III) Isothermal and isobaric process
(D)	Exothermic process	(IV) [Bond energies of molecules in reactants] - [Bond energies of product molecules

Choose the correct answer from the options given below:

(A) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)

(B) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(C) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

Official Ans. by NTA (B)

Sol. (A) For a spontaneous process $\Delta G_{T,P} < 0$

(B) $\Delta P = 0 \rightarrow$ Isobaric process

$\Delta T = 0 \rightarrow$ Isothermal process

(C) $\Delta H_{reaction}$ = (Σ Bond energies of reactants) - (Σ bond energies of products)

(D) $\Delta H < 0$ is for exothermic reaction

4. Match List-I with List-II

List-I	**List-II**
(A) Lyophilic colloid	(I) Liquid-liquid colloid
(B) Emulsion	(II) protective colloid
(C) Positively charged	(III) $FeCl_3$ + NaOH
(D) Negatively charged colloid	(IV) $FeCl_3$ + hot water

Choose the correct answer from the options given below:

(A) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(B) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

(C) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

(D) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

Official Ans. by NTA (A)

Sol. (A) Protective colloids are lyophilic colloids

(B) Emulsions are liquid in liquid colloidal solutions

(C) $FeCl_3$ + hot water forms positively charged colloidal solution of hydrated ferric oxide.

(D) $FeCl_3$ + NaOH forms negatively charged colloidal solution due to preferential adsorption of OH^- ions

5. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason(R)

Assertion (A): The ionic radii of O^{2-} and Mg^{2+} are same.

Reason (R) : Both O^{2-} and Mg^{2+} are isoelectronic species

In the light of the above statements, choose the correct answer from the options given below

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Official Ans. by NTA (D)

Sol. Ionic radius of O^{2-} is more than that of Mg^{2+}

Both O^{2-} and Mg^{2+} are isoelectronic with 10 electrons

6. Match List-I with List-II

List-I	**List-II**
(A) Concentration of gold ore	(I) Aniline
(B) Leaching of alumina	(II) NaOH
(C) Froth stabiliser	(III) SO_2
(D) Blister copper	(IV) NaCN

Choose the correct answer from the options given below.

(A) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(B) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(C) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

(D) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

Official Ans. by NTA (B)

Sol. Gold is concentrated by cyanidation

Leaching of alumina is done by NaOH

Froth stabiliser is aniline

Blister copper has condensed SO_2 on the surface

7. Addition of H_2SO_4 to BaO_2 produces:

(A) BaO, SO_2 and H_2O (B)$BaHSO_4$ and O_2

(C) $BaSO_4$, H_2 and O_2 (D) $BaSO_4$ and H_2O_2

Official Ans. by NTA (D)

Sol. $BaO_2 + H_2SO_4 \rightarrow BaSO_4 + H_2O_2$

This is a common method to prepare hydrogen peroxide

8. $BeCl_2$ reacts with $LiAlH_4$ to give

(A) Be + Li[$AlCl_4$] + H_2

(B) Be + AlH_3 + LiCl + HCl

(C) $BeH_2 + LiCl + AlCl_3$

(D) $BeH_2 + Li[AlCl_4]$

Official Ans. by NTA (C)

Sol. $2BeCl_2 + LiAlH_4 \rightarrow 2\ BeH_2 + LiCl + AlCl_3$

This is the method to prepare BeH_2

9. Match List-I with List-II

List-I	List-II
(Si-Compounds)	**(Si-Polymeric/other products)**
(A) $(CH_3)_4Si$	(I) Chain silicone
(B) $(CH_3)Si(OH)_3$	(II) Dimeric silicone
(C) $(CH_3)_2Si(OH)_2$	(III) Silane
(D) $(CH_3)_3Si(OH)$	(IV) 2D – Silicone

Choose the correct answer from the options given below:

(A) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

(B) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)

(C) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(D) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Official Ans. by NTA (D)

Sol. $(CH_3)_4Si$ is a silane

$(CH_3)Si(OH)_3$ polymerise to form 2D silicone

$(CH_3)_2Si(OH)_2$ polymerise to form chain silicone

$(CH_3)_3Si(OH)$ form dimer $(CH_3)_3Si\text{-}O\text{-}Si(CH_3)_3$

10. Heating white phosphorus with conc. NaOH solution gives mainly

(A) Na_3P and H_2O (B) H_3PO and NaH

(C) $P(OH)_3$ and NaH_2PO_4 (D) PH_3 and NaH_2PO_2

Official Ans. by NTA (D)

Sol. $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$

11. Which of the following will have maximum stabilization due to crystal field?

(A) $[Ti(H_2O)_6]^{3+}$ (B) $[Co(H_2O)_6]^{2+}$

(C) $[Co(CN)_6]^{3-}$ (D) $[Cu(NH_3)_4]^{2+}$

Official Ans. by NTA (C)

Sol. Co^{3+} has maximum effective nuclear charge and CN^- is the strongest ligand in the given options

12. Given below are two statements:

Statement I: Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide

Statement II: Photochemical smog has components, ozone, nitric oxide, acrolein, formaldehyde, PAN etc.

In the light of above statements, choose the **most appropriate** answer from the options give below

(A) Both **Statement I** and **Statement II** are correct

(B) Both **Statement I** and **Statement II** are incorrect

(C) **Statement I** is correct but **statement II** is incorrect

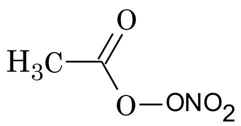
(D) **Statement I** is incorrect but **Statement II** is correct

Official Ans. by NTA (A)

Sol. Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide

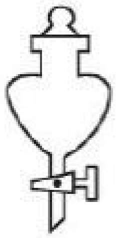
Photochemical smog has components, ozone, nitric oxide, acrolein, formaldehyde, PAN etc.

$CH_4 + O_3 \rightarrow HCHO + H_2O + CH_2 = CH - CHO +$

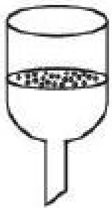


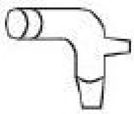
(PAN - peroxyacetyl nitrate)

13. Which of the following is structure of a separating funnel?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. It is used to separate liquid-liquid mixture which is immiscible with different densities

14. **'A'** and **'B'** respectively are:

$$\mathbf{A} \xrightarrow[(2)\,Zn-H_2O]{(1)\,O_3} \text{Ethane-1,2-dicarbaldehyde} + \text{Glyoxal/Oxaldehyde}$$

$$\mathbf{B} \xrightarrow[(2)\,Zn-H_2O]{(1)\,O_3} \text{5-oxohexanal}$$

(A) 1-methylcyclohex-1, 3-diene & cyclopentene

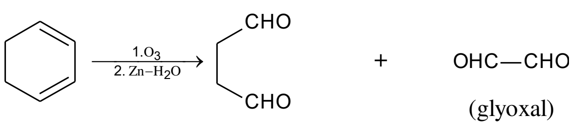
(B) Cyclohex-1, 3-diene & cyclopentene

(C) 1-methylcyclohex-1,4-diene & 1-methylcyclopent-1-ene

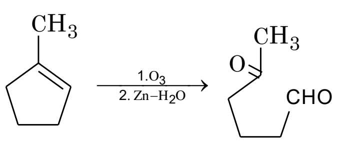
(D) Cyclohex-1,3-diene & 1-methylcyclopent-1-ene

Official Ans. by NTA (D)

Sol.

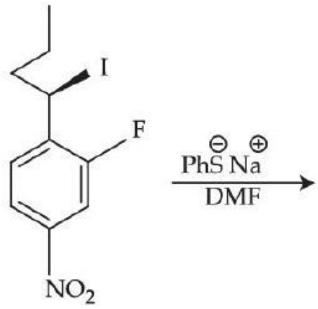


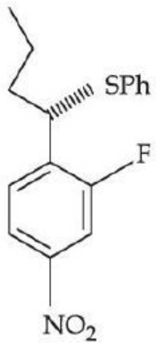
ethane-1, 2-dicarbaldehyde

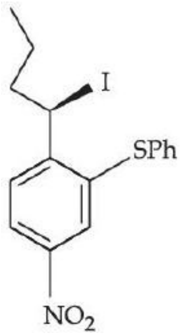


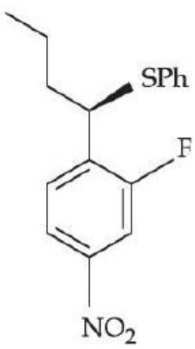
5-oxohexanal

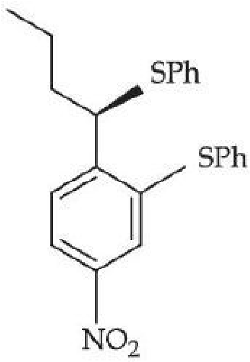
15. The major product of the following reaction is:



(A) 

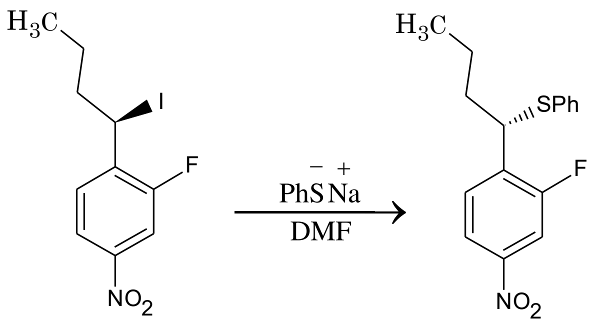
(B) 

(C) 

(D) 

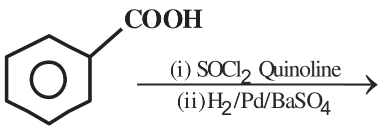
Official Ans. by NTA (A)

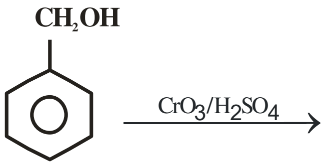
Sol.

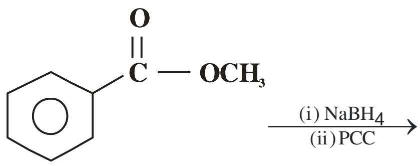


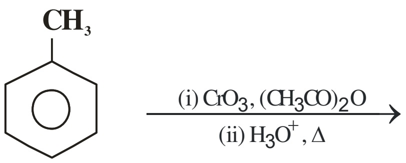
It is bimolecular nucleophilic substitution (SN^2) which occur at benzylic carbon by inversion in contiguration. This reaction cannot undergo substitution at benzene ring

16. Which of the following reactions will yield benzaldehyde as a product?

(A) 

(B) 

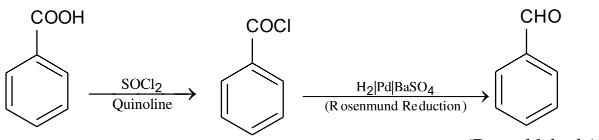
(C) 

(D) 

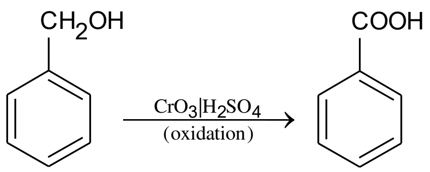
(A) (B) and (C)
(B) (C) and (D)
(C) (A) and (D)
(D) (A) and (C)

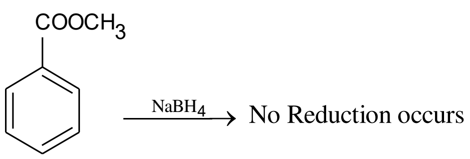
Official Ans. by NTA (C)

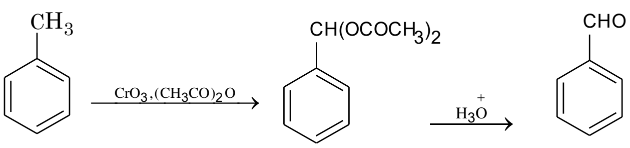
Sol.



(Benzaldehyde)







(Benzaldehyde)

17. Given below are two statements:

Statements-I : In Hofmann degradation reaction, the migration of only an alkyl group takes place from carbonyl carbon of the amide to the nitrogen atom.

Statement-II : The group is migrated in Hofmann degradation reaction to electron deficient atom.

In the light of the above statement, choose the **most appropriate** answer from the options given below:

(A) Both **Statement-I** and **Statement-II** are correct

(B) Both **Statement-I** and **Statement-II** are incorrect

(C) **Statement-I** is correct but **Statement-II** is incorrect

(D) **Statement-I** is incorrect but **Statement-II** is correct

Official Ans. by NTA (D)

Sol. $R-CO-NH_2 + Br_2 + NaOH \rightarrow$

$$R-NH_2 + Na_2CO_3 + NaBr + H_2O$$

$$R-CO-NH_2 + \bar{O}H \rightarrow R-CO-\bar{N}H \xrightarrow{Br_2}$$

$$R-CO-NH-Br \xrightarrow{OH^-} R-CO-\bar{N}-Br$$

$$\xrightarrow{\text{migration of } \bar{R}} R-NCO \xrightarrow{2\bar{O}H} RNH_2 + CO_3^{2-}$$

In this reaction of alkyl as well as aryl group can migrate to electron deficient nitrogen atom.

18. Match List-I with List-II

List-I	List-II
(Polymer)	**(Used in)**
(A) Bakelite	(I) Radio and television Cabinets
(B) Glyptal	(II) Electrical switches
(C) PVC	(III) Paints and Lacquers
(D) Polystyrene	(IV) Water pipes

Choose the correct answer from the options given below:

(A) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(B) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(C) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(D) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

Official Ans. by NTA (A)

Sol. Bakelite- It is thermosetting polymer used for making electrical switches.

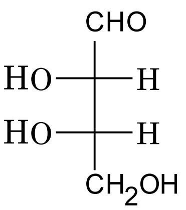
Glyptal – manufacture of paints and lacquers

PVC – manufacture of water pipes, rain coats, hand bags

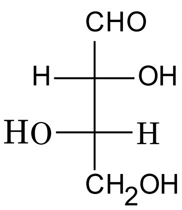
Polystyrene – manufacture of radio and television cabinets

19. L-isomer of a compound 'A' ($C_4H_8O_4$) gives a positive test with $[Ag(NH_3)_2]^+$. Treatment of 'A' with acetic anhydride yield triacetate derivative. Compound 'A' produces an optically active compound (B) and an optically inactive compound (C) on treatment with bromine water and HNO_3 respectively, compound (A) is:

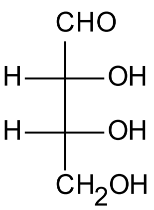
(A)



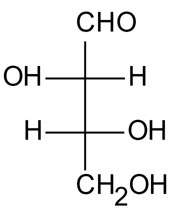
(B)



(C)

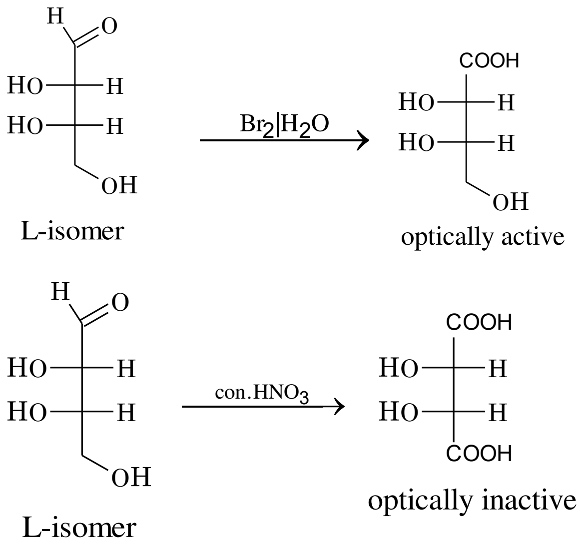


(D)



Official Ans. by NTA (A)

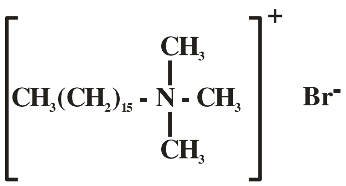
Sol.



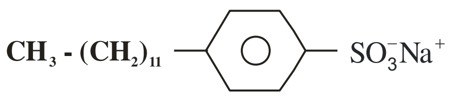
20. Match List-I with List-II

List-I

(A)



(B)



(C) $C_{17}H_{35}COO^-Na^+ + Na_2CO_3 + \text{Rosinate}$

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$

List-II

(I) Dishwashing powder

(II) Toothpaste

(III) Laundry soap

(IV) Hair conditioner

(A) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)

(B) (A) – (IV), (B) – (II), (C) – (III), (D) – (I)

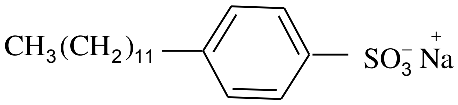
(C) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(D) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Official Ans. by NTA (B)

Sol. (A) $\left[CH_3(CH_2)_{15}-N(CH_3)_3\right]^+ Br^-$

is cationic detergents used in hair conditioner



(B)

Is anionic detergent used in tooth pastes

(C) $C_{17}H_{35}COO^-Na^+ + Na_2CO_3 + \text{Rosin ate}$ is used as laundary soap

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)_NCH_2CH_2OH$ is non-ionic detergents formed from stearic acid and poly ethylene glycol used as liquid dishwashing detergents

SECTION-B

1. Metal deficiency defect is shown by $Fe_{0.93}O$. In the crystal, some Fe^{2+} cations are missing and loss of positive charge is compensated by the presence of Fe^{3+} ions. The percentage of Fe^{2+} ions in the $Fe_{0.93}O$ crystals is ________ . (Nearest integer)

Official Ans. by NTA (85)

Sol. In $Fe_{0.93}O$ for every 93 Fe ions 14 are Fe^{+3} and (93 – 14) = 79 are Fe^{+2} ions

$\therefore$ % $Fe^{+2} = \frac{79}{93}\times 100 = 84.9\%$

$\therefore$ nearest integer = 85%

2. If the uncertainty in velocity and position of a minute particle in space are, 2.4×10^{-26} (ms^{-1}) and 10^{-7}(m) respectively. The mass of the particle in g is ________ (Nearest integer)

(Given : $h = 6.626 \times 10^{-34}$ Js)

Official Ans. by NTA (22)

Sol. $\Delta V = 2.4 \times 10^{-26}\ ms^{-1}$

$\Delta x = 10^{-7}$ m

$\because \Delta p.\Delta x = \frac{h}{4\pi}$

$\therefore m\Delta V.\Delta x = \frac{h}{4\pi}$

$\Rightarrow m\times 2.4\times 10^{-26}\times 10^{-7} = \frac{6.626\times 10^{-34}}{4\times \pi}$

$m = \frac{6.626}{9.6\times \pi}\times 10^{-1}$

m = 0.02198 kg

m = 21.98 gm

nearest integer = 22

3. 2g of a non-volatile non-electrolyte solute is dissolved in 200 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 8. The elevation in boiling points of A and B are in the ratio $\frac{x}{y}$ (x : y). The value of y is________ (Nearest integer)

Official Ans. by NTA (8)

Sol. Given : $\frac{(K_b)_A}{(K_b)_B} = \frac{1}{8}$

$\therefore \frac{(\Delta T_B)_A}{(\Delta T_B)_B} = \frac{(K_b)_A.m}{(K_b)_B.m} = \frac{1}{8} = \frac{x}{y}$

$\because \frac{x}{y} = \frac{1}{8}$

$\therefore y = 8$ (nearest integer)

4. $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

In an experiment, 2.0 moles of NOCl was placed in a one-litre flask and the concentration of NO after equilibrium established, was found to be 0.4 mol/L. The equilibrium constant at 30°C is ______ $\times 10^{-4}$.

Official Ans. by NTA (125)

Sol. $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

t=0	2M	-	-
$t=t_{eq}$	(2-x)M	x M	$\frac{x}{2}$ M

$\because x = 0.4 M$

$\therefore [NOCl]_{eq} = 1.6$ M

$[NO]_{eq} = 0.4$ M

$[Cl_2]_{eq} = 0.2$ M

$$\Rightarrow K_c = \frac{[NO]^2[Cl_2]}{[NOCl]^2} = \frac{[0.4]^2[0.2]}{[1.6]^2}$$

$$K_c = \frac{32}{2.56} \times 10^{-3}$$

$K_c = 12.5 \times 10^{-3}$

$K_c = 125 \times 10^{-4}$

Integer answer is 125

5. The limiting molar conductivities of NaI, $NaNO_3$ and $AgNO_3$ are 12.7, 12.0 and 13.3 mS m^2 mol^{-1}, respectively (all at 25°C). The limiting molar conductivity of AgI at this temperature is _____ mS m^2 mol^{-1}

Official Ans. by NTA (14)

Sol. Given

(1) λ_m^∞ (NaI) = 12.7 mS m^2 mol^{-1}

(2) λ_m^∞ ($NaNO_3$) = 12.0 mS m^2 mol^{-1}

(3) λ_m^∞ ($AgNO_3$) = 13.3 mS m^2 mol^{-1}

λ_m^∞ (Ag I) = (1) + (3) - (2)

= 12.7 + 13.3 – 12.0

= 26.0 – 12.0

λ_m^∞ (Ag I) = 14.0

6. The rate constant for a first order reaction is given by the following equation:

$$\ln k = 33.24 - \frac{2.0 \times 10^4 K}{T}$$

The Activation energy for the reaction is given by ______ kJ mol^{-1}. (In Nearest integer)

(Given: R = 8.3 J K^{-1} mol^{-1})

Official Ans. by NTA (166)

Sol. $\ln k = \ln A - \frac{E_A}{RT}$

Given: $\ln k = 33.24 - \frac{2.0 \times 10^4}{T}$

$\therefore$ on comparing $\frac{E_A}{R} = 2.0 \times 10^4$

$\therefore E_A = 2.0 \times 10^4 \times R$

$\Rightarrow E_A = 2.0 \times 10^4 \times 8.3$ J

$\Rightarrow E_A = 16.6 \times 10^4 J = 166$ kJ

7. The number of statement(s) correct from the following for copper (at no. 29) is/are ______

(A) Cu(II) complexes are always paramagnetic

(B) Cu(I) complexes are generally colourless

(C) Cu(I) is easily oxidized

(D) In Fehling solution, the active reagent has Cu(I)

Official Ans. by NTA (3)

Sol. A,B,C are correct and D is incorrect because Fehling solution has Cu(II)

8. Acidified potassium permanganate solution oxidises oxalic acid. The spin-only magnetic moment of the manganese product formed from the above reaction is ______ B.M. (Nearest Integer)

Official Ans. by NTA (6)

Sol. $2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 \rightarrow K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$

Mn^{2+} has 5 unpaired electrons therefore the magnetic moment is $\sqrt{35}$ BM

9. Two elements A and B which form 0.15 moles of A_2B and AB_3 type compounds. If both A_2B and AB_3 weigh equally, then the atomic weight of A is _____ times of atomic weight of B.

Official Ans. by NTA (2)

Sol. Given : Molar mass of $A_2B = AB_3$

$$\therefore (2A + B) = (A + 3B) \begin{bmatrix} A \rightarrow \text{Atomic wt. of A} \\ B \rightarrow \text{Atomic wt. of B} \end{bmatrix}$$

$\Rightarrow A = 2B$

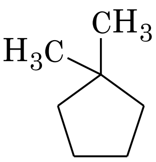
$\therefore$ atomic wt. of A is $\underline{\ 2\ }$ times of atomic wt. of B

Integer answer is 2

10. Total number of possible stereoisomers of dimethyl cyclopentane is ___________

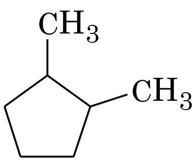
Official Ans. by NTA (6)

Sol. Dimethyl cyclopentane



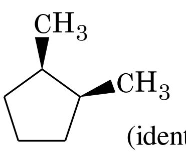
1,1-dimethylcyclopentane

no stereoisomer



1,2-dimethylcyclopentane

will show stereo isomerism, Its stereo isomers are



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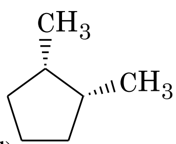
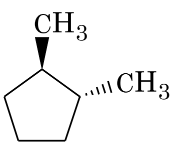
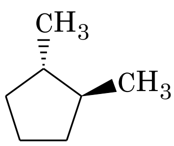
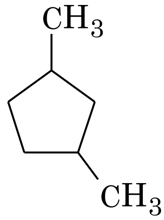
(identical)





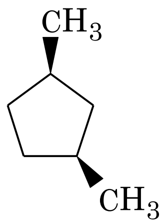


(enantiomers)



1,3-dimethylcyclopentane

will show stereo isomerism, Its stereo isomers are



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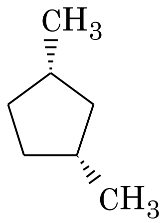
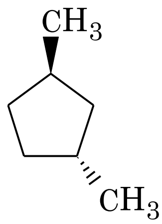
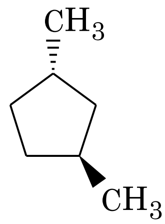
(identical)





(enantiomers)



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Sol. $\Rightarrow$ Let z = x + iy, x, y ∈ R

Now $\bar{z} = iz^2$

then $x - iy = i(x^2 - y^2 + 2xyi)$

$x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow x = -2xy \;\&\; -y = x^2 - y^2$

$\Rightarrow x(1 + 2y) = 0$

$x = 0$ or $y = -\frac{1}{2}$

Put $x = 0$ in $-y = x^2 - y^2$

We get $y = y^2$

$\Rightarrow y = 0, 1$

Similarly

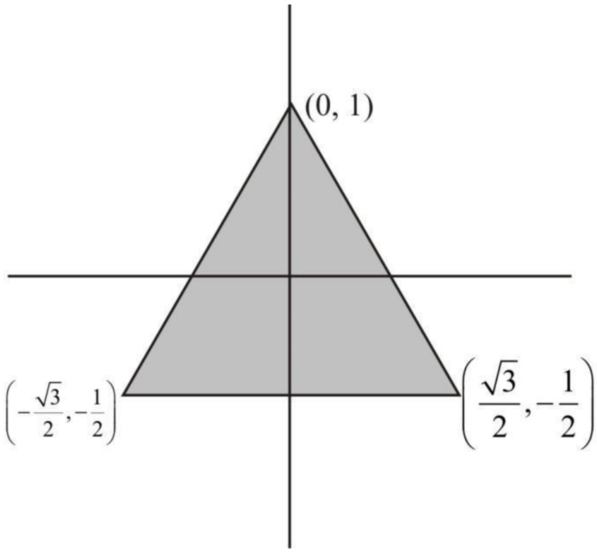
Put $y = -\frac{1}{2}$ in $-y = x^2 - y^2$

$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$

$\Rightarrow x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

$z = \left(0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$



$Area = \frac{1}{2}.\left(\sqrt{3}\right)\left(\frac{3}{2}\right)$

$= \frac{3\sqrt{3}}{4}$

2. Let the system of linear equations x + 2y + z = 2, αx + 3y – z = α, -αx + y + 2z = -α be inconsistent. Then α is equal to :

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$

(C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Official Ans. by NTA (D)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$

$= (6 + y) - 2\left(\left(2\alpha - \alpha\right) + 1\left(\alpha + 3\alpha\right)\right.$

$= 7 - 2\alpha + 4\alpha$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$= 14 + 2\alpha$

$\alpha = -x_2 = 7$

$\Delta_1 \neq 0$

3. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and |a| < 1, |b| < 1, |c| < 1, abc ≠ 0, then

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Official Ans. by NTA (C)

Sol. $x = 1 + a + a^2 = \dots\dots\dots$

$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$

$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$

$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$

a, b, c are in A.P.

$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are in A.P.

$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

4. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is :

(A) 10 (B) 8

(C) 7 (D) 5

Official Ans. by NTA (B)

Sol. Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-(2x + 2g)}{(2y + 2f)}$

Comparing with $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$

$\Rightarrow$ b = 0, a = - 2, c = 2

$\Rightarrow$ -2g = -2 $\Rightarrow$ g = 1 2f = -2

f = -1

Now circle will be

$x^2 + y^2 + 2x - 2y + c = 0$

its passes through (2, 5)

which will give c = -23

so circle will be $x^2 + y^2 + 2x - 2y - 23 = 0$

centre C = (-1, 1)

and radius 5

Now P is (11, 6)

So minimum distance of P from circle will be

$= \sqrt{(11+1)^2 + (6-1)^2} - 5$

= 13 – 5

= 8

5. Let a be an integer such that $\lim_{x \to 7} \frac{18 - [1 - x]}{[x - 3a]}$ exists, where [t] is greatest integer ≤ t. Then a is equal to :

(A) -6 (B) -2

(C) 2 (D) 6

Official Ans. by NTA (A)

Sol. $\lim_{x\to 7}\frac{18-[1-x]}{[x]-3a}$

L.H.L. $\lim_{x\to 7-}\frac{18-[1-x]}{[x]-3a}$

$=\frac{18-(-6)}{6-3a}$

$=\frac{24}{6-3a}$

R.H.L. $\lim_{x\to 7+}\frac{18-[1-x]}{[x]-3a}$

$=\frac{18-(-7)}{7-3a}$

$=\frac{25}{7-3a}$

Now L.H.L. = R.H.L.

$\frac{24}{6-3a}=\frac{25}{7-3a}$

$\Rightarrow 168-72a=150-75a$

$\Rightarrow 18=-3a$

$\Rightarrow$ a = -6

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

(A) 4 (B) 2

(C) 1 (D) 0

Official Ans. by NTA (B)

Sol. Let $f(x)=x^4-4x+1$

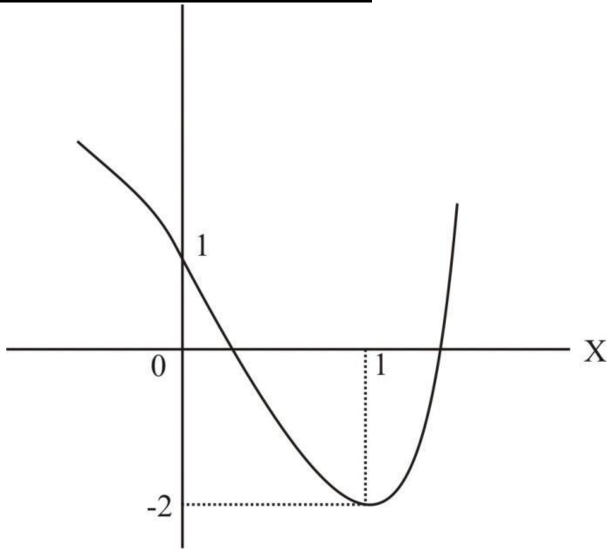
$f'(x) = 4x^3 - 4$

$f'(x) = 0 \Rightarrow x = 1$

x = 1 is point of minima.

f(1) = - 2

f(0) = 1



Hence 2 solutions.

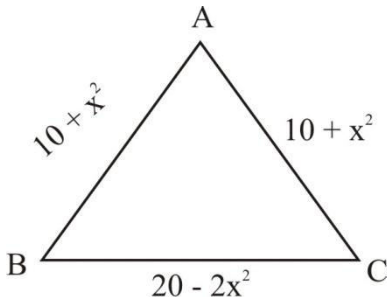
7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for x = k, the area of the triangle is maximum, then $3k^2$ is equal to :

(A) 5 (B) 8

(C) 10 (D) 12

Official Ans. by NTA (C)

Sol.



$a = 20 - 2x^2$, $b = 10 + x^2$, $c = 10 + x^2$

$=\frac{a+b+c}{2}$

= 20

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{20(2x^2)(10-x^2)(10-x^2)}$

$=2\sqrt{10}\sqrt{x^2(10-x^2)^2}$

$=2\sqrt{10}\left|x(10-x^2)\right|$

$= 2\sqrt{10}\,|10x - x^3|$

S = 10x – x^3

$\frac{ds}{dx} = 10 - 3x^2$

$\frac{ds}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$

3x^2 = 10

8. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5, |y| < 2$, then :

(A) $x^2y'' + xy' - 25y = 0$

(B) $x^2y'' - xy' - 25y = 0$

(C) $x^2y'' - xy' + 25y = 0$

(D) $x^2y'' + xy' + 25y = 0$

Official Ans. by NTA (D)

Sol. $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$\cos^{-1}\left(\frac{y}{2}\right) = 5\log_e\left(\frac{x}{5}\right)$

$\frac{-1}{\sqrt{1-\frac{y^2}{4}}}.\frac{y'}{2} = 5.\frac{1}{\frac{x}{5}}\times\frac{1}{5}$

$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$

$-xy' = 5\sqrt{4-y^2}$

$-xy'' - y' = 5.\frac{1}{2\sqrt{4-y^2}}(-2y\ y')$

$\Rightarrow xy'' + y' = \frac{5y'.y}{\sqrt{4-y^2}}$

$xy'' + y' = 5.\left(\frac{-5}{x}\right)y$

$x^2y'' + xy' = -25y$

9. $\int\frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x + C$, Where C is a constant, then $\frac{d^3f}{dx^3}$ at x = 1 is equal to :

(A) $-\frac{3}{4}$

(B) $\frac{3}{4}$

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

Official Ans. by NTA (B)

Sol. $\int\left(\frac{x^2+1}{(x+1)^2}\right)e^x.dx$

$= \int\left(\frac{x^2-1+2}{(x+1)^2}\right)e^x\,dx$

$= \int\left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right)e^x\,dx$

$= \int(f(x) + f'(x))e^x\,dx$

$= f(x)\,e^x + c$

Where $f(x) = \frac{x-1}{x+1}$

$f'(x) = \frac{2}{(x+1)^2}$

$f''(x) = \frac{-4}{(x+1)^3}$

$= \frac{12}{(x+1)^4}$

$f''(1) = \frac{12}{16}$

$= \frac{3}{4}$

10. The value of the integral $\int_{-2}^{2} \frac{|x^3+x|}{(e^{x|x|}+1)}dx$ is equal to :

(A) $5e^2$ (B) $3e^{-2}$

(C) 4 (D) 6

Official Ans. by NTA (D)

Sol. $f(x) = \frac{|x^3+x|}{(e^{x|x|}+1)} dx$

$$\int_{-2}^{2} f(x)dx = \int_{0}^{2}(f(x)+f(-x))dx$$

$$= \int_{0}^{2}\left(\frac{|x^3+x|}{(e^{x|x|}+1)} + \frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx$$

$$= \int_{0}^{2}\left(\frac{|x^3+x|}{(e^{x|x|}+1)} + \frac{|x^3+x|}{(e^{-x|x|}+1)}\right)dx$$

$$= \int_{0}^{2}\left(\frac{x^3+x}{(e^{x^2}+1)} + \frac{x^3+x}{(e^{-x^2}+1)}\right)dx$$

$$I = \int_{0}^{2}\left(\frac{x^3+x}{1+e^{x^2}} + \frac{e^{x^2}(x^3+x)}{1+e^{x^2}}\right)dx$$

$$= \int_{0}^{2}(x^3+x)dx$$

$$= \left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2$$

= 4 + 2 = 6

11. If $\frac{dy}{dx} + \frac{2^{x-y}(2^y-1)}{2^x-1} = 0, x, y > 0, y(1) = 1$, then y(2) is equal to :

(A) $2 + \log_2 3$ (B) $2 + \log_2 2$

(C) $2 - \log_2 3$ (D) $2 - \log_2 3$

Official Ans. by NTA (D)

Sol. $\frac{dy}{dx} + \frac{2^{x-y}(2^y-1)}{2^x-1} = 0$,

x, y > 0, y(1) = 1, y (2) = ?

$$\frac{dy}{dx} = -\frac{2^x(2^y-1)}{2^y(2^x-1)}$$

$$\int \frac{2^y}{2^y-1}dy = -\int \frac{2^x}{2^x-1}dx$$

$$\frac{1}{\ln 2}\int \frac{2^y \ln 2}{2^y-1}dy = -\frac{1}{\ln^2}\int \frac{2^x \ln 2}{2^x-1}dx$$

$$\frac{1}{\ln 2}\ln|2^y-1| = \frac{-1}{\ln 2}\ln|2^x-1| + C$$

At x= 1, y = 1

Putting this values in above relation we get C = 0

$$\ln|2^y-1| + \ln|2^x-1| = 0$$

$$(2^x-1)(2^y-1) = 1$$

$$2^y - 1 = \frac{1}{2^x+1}$$

At x = 2

$$2^y = \frac{1}{3}+1 = \frac{4}{3}$$

$$y = \log_2\frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$$

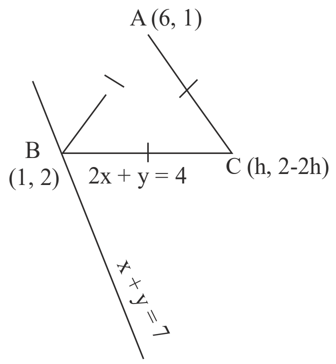
12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is 2x + y = 4. Let the point B lie on the line x + 3y = 7. If (α, β) is the centroid ΔABC, then $15(\alpha+\beta)$ is equal to :

(A) 39 (B) 41

(C) 51 (D) 63

Official Ans. by NTA (C)

Sol.



Point B (1, 2)

Now let C be (h, 4 - 2h)

(As C lies on 2x + y = 4)

$\because \Delta$ is isosceles with base BC

$\therefore AB = AC$

$$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$$

$$\sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$$

$$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$$

$$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$$

$h = \frac{19}{5}$ or h = 1

Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid $\left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right)$

$$\left(\frac{35+19}{15}, \frac{15-18}{15}\right)$$

$$\left(\frac{54}{15}, \frac{-3}{15}\right)$$

$$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$$

$$15(\alpha + \beta) = 51$$

13. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b,$ be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal to :

(A) 29 (B) 31

(C) 32 (D) 34

Official Ans. by NTA (B)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{16 \times \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$16 = a^2$

$b^2 = 15$

14. If two straight lines whose direction cosines are given by the relations l + m – n = 0, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :

(A) 6 (B) 4

(C) 3 (D) 2

Official Ans. by NTA (A)

Sol. l + m – n = 0

$3l^2 + m^2 + cl(l + m) = 0$

n = l + m

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$(3+c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0$ .....(1)

$\because$ lies are parallel.

Roots of (1) must be equal

$\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

c = 6 or c = - 2

+ve value of c = 6

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \ldots, 10\}$ is :

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (A)

Sol. $\vec{a} = i + j - k$

$\vec{c} = 2i - 3j + 2k$

$\vec{b} \times \vec{c} = \vec{a}$

$|\vec{b}| \in \{1, 2 \ldots\ldots 10\}$

$\because \vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{a}$ is perpendicular to $\vec{b}$ as well as $\vec{a}$ is perpendicular to $\vec{c}$

Now $\vec{a}.\vec{c} = 2 - 3 - 2 = -3 \neq 0$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

No. of vectors $\vec{b} = 0$

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3,……, 18 and are arranged in the increasing order $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Official Ans. by NTA (C)

Sol. No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3……………….18

$n(s) = {}^{18}C_5$

$x_1 \quad (x_2) \quad x_3 \quad (x_4) \quad x_5$

$\quad\;\; 7 \qquad\quad 11$

$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$

$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$

$\frac{1}{17 \times 4} = \frac{1}{68}$

17. Let X be a random variable having binomial distribution B(7, p). If P(X = 3) = 5P(X = 4), then the sum of the mean and the variance of X is :

(A) $\frac{105}{16}$ (B) $\frac{7}{16}$

(C) $\frac{77}{36}$ (D) $\frac{49}{16}$

Official Ans. by NTA (C)

Sol. B (7, p)

n = 7 p = p

given

$P(x = 3) = 5P(x = 4)$

$${}^7C_3 \times p^3(1-p)^4 = 5.{}^7C_4 p^4(1-p)^3$$

$$\frac{{}^7C_3}{5\times {}^7C_4} = \frac{p}{1-p}$$

1 – p = 5p

6p = 1

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

n = 7

Mean = np $= 7\times\frac{1}{6} = \frac{7}{6}$

Var = npq $= 7\times\frac{1}{6}\times\frac{5}{6} = \frac{35}{36}$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42+35}{36}$$

$$= \frac{77}{36}$$

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to :

(A) -1 (B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$ (D) $-\frac{1}{4}$

Official Ans. by NTA (B)

Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right)$$

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= -\frac{1}{2}$$

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$ (D) $-\frac{3\pi}{4}$

Official Ans. by NTA (A)

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$

20. The Boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :

(A) $q \rightarrow (p \wedge q)$ (B) $p \rightarrow q$

(C) $p \rightarrow (p \rightarrow q)$ (D) $p \rightarrow (p \vee q)$

Official Ans. by NTA (D)

Sol. (~(p^q)) ∨ q

= (~p ∨ ~q) ∨ q

= ~p ∨ ~ q ∨ q

= ~ p ∨ t

= this statement is a tautology option D

p ⇒ (p ∨ q) is also a tautology.

OR

p	q	P^q	~(p^q)	~(p^q) ∨ q	p ∨ q	$p \to (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

SECTION-B

1. Let $f : R \rightarrow R$ be a function defined $f(x) = \frac{2e^{2x}}{e^{2x}+e}$.

Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+.....+f\left(\frac{99}{100}\right)$ is equal to____________.

Official Ans. by NTA (99)

Sol.

$$f(x)+f(1-x)=\frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2-2x}}{e^{2-ex}+e}=\left[\frac{e^{2x}}{e^{2x}+e}+\frac{e^{2}}{e^{2}+e^{2x+1}}\right]$$

$$=2\left[\frac{e^{2x-1}}{e^{2x-1}+1}+\frac{1}{1+e^{2x-1}}\right]=2$$

$$f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+.....+f\left(\frac{99}{100}\right)$$

$$=\left\{f\left(\frac{1}{100}\right)+f\left(\frac{99}{100}\right)\right\}+\left\{f\left(\frac{2}{100}\right)+f\left(\frac{98}{100}\right)\right\}+.....+f\left\{\left(\frac{49}{100}\right)+f\left(\frac{51}{100}\right)\right\}+f\left(\frac{1}{2}\right)$$

$$=(2+2+2+----49\,times)+\frac{2e}{e+e}$$

$$=98+1=99$$

2. If the sum of all the roots of the equation $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$ is $\log_e$ P, then p is equal to _______.

Official Ans. by NTA (45)

Sol. $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$]

$(e^{x})^{3}-11(e^{x})^{2}-45+\frac{81e^{x}}{2}=0$]

$e^{x}=t$

$2t^{3}-22t^{2}+81t-90=0$

$t_1t_2t_3=45$

$e^{x_1}.e^{x_2}.e^{x_3}=45$

$e^{x_1+x_2+x_3}=45$

$\log_e e^{x_1+x_2+x_3}=\log_e 45$

$x_1+x_2+x_3=\log_e 45$

$\log_e P=\log_e 45$

P = 45

3. The positive value of the determinant of the matrix A, whose $Adj(Adj(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$, is ___________.

Official Ans. by NTA (14)

Sol. $Adj(AdjA) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

$|Adj(AdjA)| = \begin{bmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

$= (14)^3[3 - 2(-5) - 1(-1)] = (14)^3[14] = (14)^4$

$|A|^4 = (14)^4 \Rightarrow |A| = 14$

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _______.

Official Ans. by NTA (56)

Sol. 16 cubes < 11 Blue, 5 Red

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

$x_1, x_6 \geq 0, \qquad x_2, x_3, x_4, x_5 \geq 2$

$x_2 = t_1 + 2$

$x_3 = t_3 + 2$

$x_4 = t_4 + 2$

$x_5 = t_5 + 2$

$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$

No. of solutions $= {}^{6+3-1}C_3 = {}^8C_3 = 56$

5. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k l$, where l, k ∈ N and l is co-prime to 5, then k is equal to _____.

Official Ans. by NTA (5)

Sol. $\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}}\right)^{60}$

$T_{r+1} = {}^{60}C_r \left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r} \left(\frac{5^{1/2}}{x^{1/3}}\right) r$

$= {}^{60}C_r 5 \frac{3r - 60}{4} . x \frac{180 - 5r}{6}$

$\frac{180 - 5r}{6} = 10 \Rightarrow r = 24$

Coeff. of $x^{10} = {}^{60}C_{24} 5^3 = \frac{\lfloor 60}{\lfloor 24 \lfloor 36} 5^3$

Powers of 5 in $= {}^{60}C_{24}.5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$

6. Let

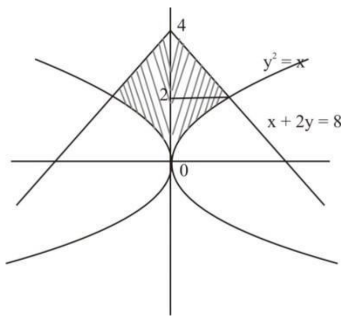
$A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\}$ and

$A_2 = \{(x, y) : |x| + |y| \leq k\}$. If 27 (Area A_1) = 5 (Area A_2), then k is equal to :

Official Ans. by NTA (6)

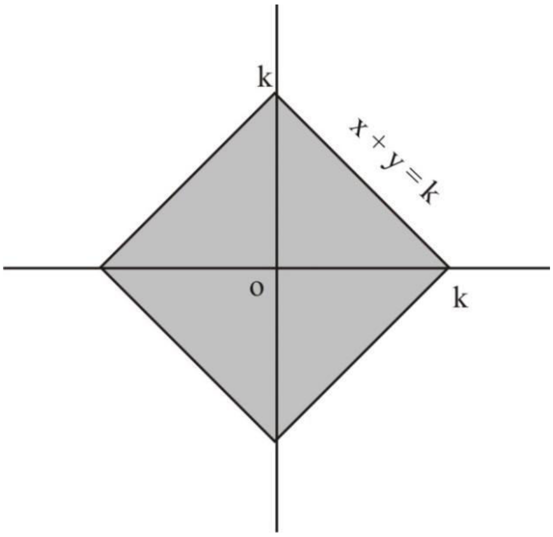
Sol. $A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\}$ and

$A_2 = \{(x, y) : |x| + |y| \leq k\}$.



$$area(A_1) = 2\left[\int_0^2 y^2 dy + \int_2^4 (8-2y)dy\right]$$

$$= 2\left[\left(\frac{y^3}{3}\right)_0^2 + \left(8y - y^2\right)_2^4\right]$$



$$area(A_1) = 2\times\frac{20}{3} = \frac{40}{3}$$

Area (A_2) = $4\times\frac{1}{2}k^2$

Area (A_2) = $2k^2$

Now

27 (Area A_1) = 5 (Area A_2)

$9\times 4 = k^2$

k = 6

7. If the sum of the first ten terms of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+....$ is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to ___________.

Official Ans. by NTA (276)

Sol. $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+.......$

$$T_n = \frac{n}{4n^4+1}$$

$$= \frac{n}{(2n^2+1)^2-(2n)^2} = \frac{n}{(2n^2+2n+1)(2n^2-2n+1)}$$

$$= \frac{1}{4}\left[\frac{1}{2n^2-2n+1} - \frac{1}{2n^2+2n+1}\right]$$

$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4}\left[\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+........\frac{1}{200+20+1}\right]$$

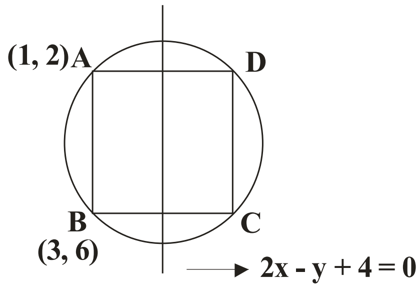
$$= \frac{1}{4}\left[1-\frac{1}{221}\right] = \frac{1}{4}\times\frac{220}{221} - \frac{55}{221} = \frac{m}{n}$$

m + n = 55 + 221 = 276

8. A rectangle R with end points of the one of its dies as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is 2x – y + 4 = 0, then the area of R is ________.

Official Ans. by NTA (16)

Sol.



Eq. of line AB

y = 2x

Slope of AB = 2

Slope of given diameter = 2

So the diameter is parallel to AB

Distance between diameter and line AB

$$= \left(\frac{4}{\sqrt{2^2+12}}\right) = \frac{4}{\sqrt{5}}$$

Thus BC = $2\times\frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

$$AB = \sqrt{(1-3)^2+(2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

Area = AB × BC = $\frac{8}{\sqrt{5}}\times 2\sqrt{5} = 16$ Ans.

9. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to ________.

Official Ans. by NTA (63)

Sol. Vertex and focus of parabola $y^2 = 2x$

are V (0, 0) and $S\left(\frac{1}{2}, 0\right)$ resp.

Let equation of circle be

$(x - h)^2 + (y - k)^2 = 4$

$\because$ Circle passes through (0, 0)

$\Rightarrow h^2 + k^2 = 4$ ……(1)

$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \text{……(2)}$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = +\frac{\sqrt{63}}{4}$$

$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre $\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$$

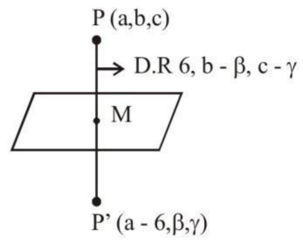
$$4\alpha - 8 = \sqrt{63}$$

$$(4\alpha - 8)^2 = 63$$

10. Let the mirror image of the point (a, b, c) with respect to the plane 3x – 4y + 12z + 19 = 0 be (a- 6, β, γ). If a + b + c = 5, then 7 β - 9 γ is equal to ____________.

Official Ans. by NTA (137)

Sol.



$$M = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \text{ ....(1)}$$

Since PP' is parallel to normal of the plane then

$$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$$

$$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$$

$$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$$

$$\Rightarrow a = -\beta - \gamma - 11$$

Now putting these values in (1) we get

$$6(-\beta - \gamma - 11) - 4(\beta - 8) + 12(\gamma + 24) - 4\beta + 12\gamma + 20 = 0$$

$$\Rightarrow 7\beta - 9\gamma = 170 - 33 = 137$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The SI unit of a physical quantity is pascal-second. The dimensional formula of this quantity will be

(A) $[ML^{-1}T^{-1}]$ (B) $[ML^{-1}T^{-2}]$

(C) $[ML^{2}T^{-1}]$ (D) $[M^{-1}L^{3}T^{0}]$

Official Ans. by NTA (A)

Sol. Pascal second

$$\frac{F}{A}t = \frac{MLT^{-2}}{L^2}T = ML^{-1}T^{-1}$$

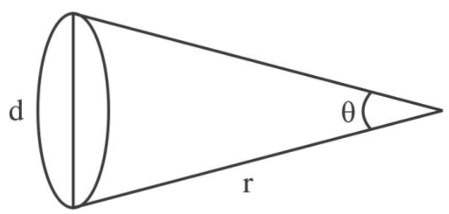
2. The distance of the Sun from earth is 1.5×10^{11} m and its angular diameter is (2000) s when observed from the earth. The diameter of the Sun will be :

(A) 2.45×10^{10} m (B) 1.45×10^{10} m

(C) 1.45×10^{9} m (D) 0.14×10^{9} m

Official Ans. by NTA (C)

Sol.



$$\theta = \frac{d}{r}$$

$$\frac{2000}{60\times60}\times\frac{\pi}{180} = \frac{d}{1.5\times10''}$$

$$\Rightarrow d = \frac{2000}{60\times60}\times\frac{\pi}{180}\times1.5\times10''$$

$$= \frac{\pi\times1.5}{3\times6\times18}\times10'' = 1.45\times10^{9}$$

3. When a ball is dropped into a lake from a height 4.9 m above the water level, it hits the water with a velocity v and then sinks to the bottom with the constant velocity v. It reaches the bottom of the lake 4.0 s after it is dropped. The approximate depth of the lake is :

(A) 19.6 m (B) 29.4 m

(C) 39.2 m (D) 73.5 m

Official Ans. by NTA (B)

Sol. $V^2 = 2 \times 9.8 \times 4.9$

$V = 9.8$ m/s

Depth = distance travelled in 3 seconds

$= 9.8 \times 3 = 29.4$ m

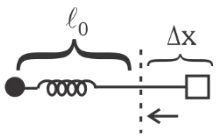
4. One end of a massless spring of spring constant k and natural length l_0 is fixed while the other end is connected to a small object of mass m lying on a frictionless table. The spring remains horizontal on the table. If the object is made to rotate at an angular velocity ω about an axis passing through fixed end, then the elongation of the spring will be:

(A) $\frac{k - m\omega^2 l_0}{m\omega^2}$ (B) $\frac{m\omega \; l_0}{k + m\omega^2}$

(C) $\frac{m\omega^2 l_0}{k - m\omega^2}$ (D) $\frac{k + m\omega^2 l_0}{m\omega^2}$

Official Ans. by NTA (C)

Sol.



$K\,\Delta x = m(\ell_0 + \Delta x)w^2$

$K\,\Delta x = m\,\ell_0\,w^2 + mw^2\,\Delta x$

$$\Delta x = \frac{m\ell_0 w^2}{k - mw^2}$$

5. A stone tide to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as it reaches a position where the string is horizontal, is $\sqrt{x(u^2 - gL)}$. The value of x is

(A) 3 (B) 2

(C) 1 (D) 5

Official Ans. by NTA (B)

Sol. $v = \sqrt{u^2 - 2gL}$

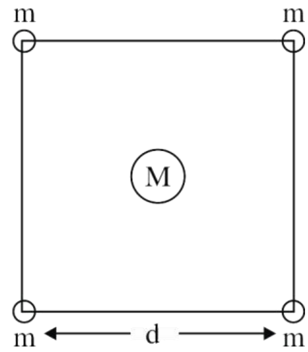
$\Delta v = \sqrt{u^2 + v^2}$

$\Delta v = \sqrt{u^2 + v^2 - 2gL}$

$\Delta v = \sqrt{2u^2 - 2gL}$

$\Delta v = \sqrt{2\left(u^2 - gL\right)}$ x = 2

6. Four spheres each of mass m form a square of side d (as shown in figure). A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is :



(A) $-\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right]$

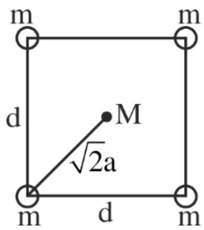
(B) $-\frac{Gm}{d}\left[(4+\sqrt{2})M + 4\sqrt{2}m\right]$

(C) $-\frac{Gm}{d}\left[3m^2 + 4\sqrt{2}M\right]$

(D) $-\frac{Gm}{d}\left[6m^2 + 4\sqrt{2}M\right]$

Official Ans. by NTA (A)

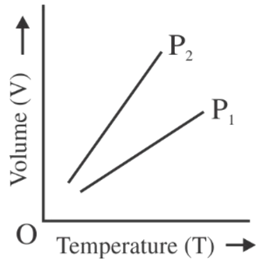
Sol.



$$-\frac{Gm^2}{d} \times 4 - \frac{Gm^2}{\sqrt{2}d} \times 2 - \frac{GMm}{d} \times 4\sqrt{2}$$

$$-\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right]$$

7. For a perfect gas, two pressures P_1 and P_2 are shown in figure. The graph shows:



(A) $P_1 > P_2$

(B) $P_1 < P_2$

(C) $P_1 = P_2$

(D) Insufficient data to draw any conclusion

Official Ans. by NTA (A)

Sol. $PV = nRT$

$$\frac{V}{T} = \frac{nR}{P}$$

$$\frac{nR}{P_1} < \frac{nR}{P_2}$$

$$P_2 < P_1$$

8. According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0ºC

B. The mean free path of gas molecules decreases if the density of molecules is increased.

C. The mean free path of gas molecules increases if temperature is increased keeping pressure constant.

D. Average kinetic energy per molecule per degree of freedom is $\frac{3}{2}k_BT$ (for monoatomic gases)

Choose the most appropriate answer from the options given below:

(A) A and C only (B) B and C only

(C) A and B only (D) C and D only

Official Ans. by NTA (B)

Sol. $\lambda = \dfrac{kT}{\sqrt{2}\pi d^2 P}$

9. A lead bullet penetrates into a solid object and melts. Assuming that 40% of its kinetic energy is used to heat it, the initial speed of bullet is:

(Given, initial temperature of the bullet = 127ºC, Melting point of the bullet = 327ºC,

Latent heat of fusion of lead = 2.5×10^4 J Kg^{-1},

Specific heat capacity of lead = 125J/kg K)

(A) 125 ms^{-1} (B) 500 ms^{-1}

(C) 250 ms^{-1} (D) 600 ms^{-1}

Official Ans. by NTA (B)

Sol. $m \times 125 \times 200 + m \times 2.5 \times 10^4 = \frac{1}{2}mv^2 \times \frac{40}{100}$

V = 500 m/s

10. The equation of a particle executing simple harmonic motion is given by $x = \sin\pi\left(t + \frac{1}{3}\right)$ m. At t = 1s, the speed of particle will be

(Given : $\pi = 3.14$)

(A) 0 cm s^{-1} (B) 157 cm s^{-1}

(C) 272 cm s^{-1} (D) 314 cm s^{-1}

Official Ans. by NTA (B)

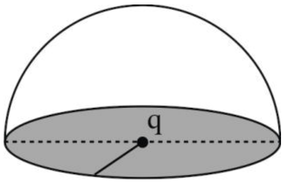
Sol. $x = \sin\pi\left(t + \frac{1}{3}\right)$

$$x = \sin\left(\pi t + \frac{\pi}{3}\right)$$

$$V = \frac{dx}{dt} = \cos\left(\pi t + \frac{\pi}{3}\right)\pi$$

$$= -\pi \times \frac{1}{2} = 157 \text{ cm/s}$$

11. If a charge q is placed at the centre of a closed hemispherical non-conducting surface, the total flux passing through the flat surface would be :

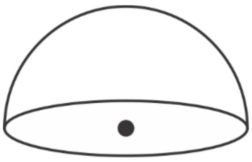


(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{2\varepsilon_0}$

(C) $\frac{q}{4\varepsilon_0}$ (D) $\frac{q}{2\pi\varepsilon_0}$

Official Ans. by NTA (B)

Sol.



Total flux through complete spherical surface is $\frac{q}{\varepsilon_0}$.

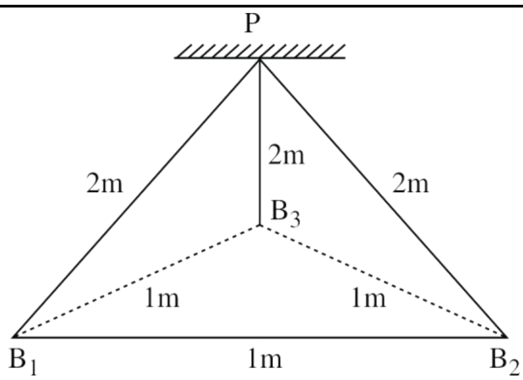
So the flux through curved surface will be $\frac{q}{2\varepsilon_0}$.

The flux through flat surface will be zero.

Remark : Electric flux through flat surface is zero but no option is given, option is available for electric flux passing through curved surface.

12. Three identical charged balls each of charge 2C are suspended from a common point P by silk threads of 2m each (as shown in figure). They form an equilateral triangle of side 1m.

The ratio of net force on a charged ball to the force between any two charged balls will be :

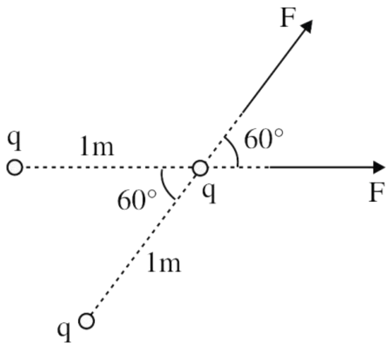


(A) 1 : 1 (B) 1 : 4

(C) $\sqrt{3}:2$ (D) $\sqrt{3}:1$

Official Ans. by NTA (D)

Sol.



$$F = \frac{k(2)(2)}{(1)^2}$$

(F = Force between two charges).

F = 4k

$$F_{net} = 2F\cos 30^\circ = 2\cdot F\cdot\frac{\sqrt{3}}{2} = F\sqrt{3}$$

(F_{net} = Net electrostatic force on one charged ball)

$$\frac{F_{net}}{F} = \frac{\sqrt{3}\,F}{F} = \left(\sqrt{3}\right)$$

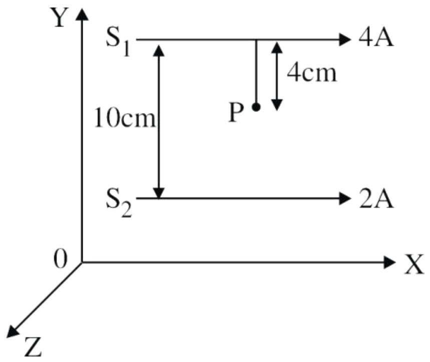
Remark: Net force on any one of the ball is zero. But no option given in options.

13. Two long parallel conductors S_1 and S_2 are separated by a distance 10 cm and carrying currents of 4A and 2A respectively. The conductors are placed along x-axis in X-Y plane. There is a point P located between the conductors (as shown in figure).

A charge particle of 3π coulomb is passing through the point P with velocity

$\vec{v} = (2\hat{i} + 3\hat{j})m/s$; where $\hat{i}$ & $\hat{j}$ represents unit vector along x & y axis respectively.

The force acting on the charge particle is $4\pi \times 10^{-5}(-x\hat{i} + 2\hat{j})N$. The value of x is :

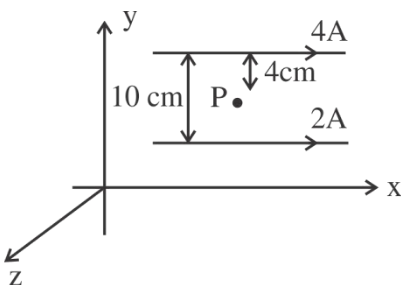


(A) 2 (B) 1

(C) 3 (D) –3

Official Ans. by NTA (C)

Sol.



$B_{net} = B_1 - B_2 = \frac{\mu_0 \times 4}{2\pi[.04]} - \frac{\mu_0 \times 2}{2\pi[.06]}$

$\vec{B}_{net} = \frac{\mu_0}{2\pi}\left[\frac{200}{3}\right](-\hat{k})$

$\vec{F} = q\left[\vec{v} \times \vec{B}\right]$

$= [3\pi]\left[(2\hat{i} + 3\hat{j}) \times \left(\frac{\mu_0}{2\pi}\right)\left(\frac{200}{3}\right) - \hat{k}\right]$

$= 3\pi \times \frac{\mu_0}{2\pi}\left(\frac{200}{3}\right)\left[2 \times \hat{j} - 3\left(\hat{i}\right)\right]$

$= \left(4\pi \times 10^{-7}\right)(100)\left(-3\hat{i} + 2\hat{j}\right)$

$= 4\pi \times 10^{-5} \times \left[-3\hat{i} + 2\hat{j}\right]$

14. If L, C and R are the self inductance, capacitance and resistance respectively, which of the following does not have the dimension of time ?

(A) RC (B) $\frac{L}{R}$

(C) $\sqrt{LC}$ (D) $\frac{L}{C}$

Official Ans. by NTA (D)

Sol. $\left(\frac{L}{C}\right)$ does not have dimension of time.

RC, $\frac{L}{R}$ are time constant while $\sqrt{LC}$ is reciprocal of angular frequency or having dimension of time.

15. Given below are two statements:

Statement I : A time varying electric field is a source of changing magnetic field and vice-versa. Thus a disturbance in electric or magnetic field creates EM waves.

Statement II : In a material medium. The EM wave travels with speed $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both statement I and statement II are true.

(B) Both statement I and statement II are false.

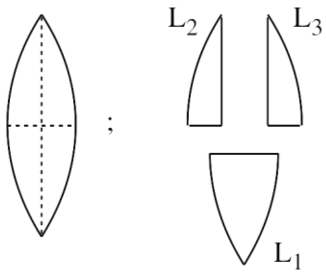
(C) Statement I is correct but statement II is false.

(D) Statement I is incorrect but statement II is true.

Official Ans. by NTA (C)

Sol. The statement II is wrong as the velocity of em wave in a medium is $\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$.

16. A convex lens has power P. It is cut into two halves along its principal axis. Further one piece (out of the two halves) is cut into two halves perpendicular to the principal axis (as shown in figure). Choose the incorrect option for the reported pieces.



(A) Power of $L_1 = \frac{P}{2}$

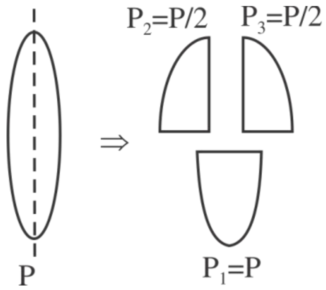
(B) Power of $L_2 = \frac{P}{2}$

(C) Power of $L_3 = \frac{P}{2}$

(D) Power of $L_1 = P$

Official Ans. by NTA (A)

Sol.



17. If a wave gets refracted into a denser medium, then which of the following is true?

(A) wavelength speed and frequency decreases.

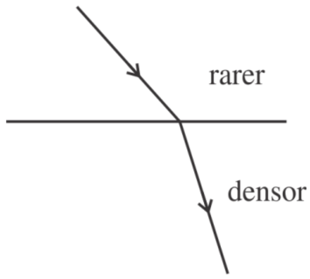
(B) wavelength increases, speed decreases and frequency remains constant.

(C) wavelength and speed decreases but frequency remains constant.

(D) wavelength, speed and frequency increases.

Official Ans. by NTA (C)

Sol.



No change in frequency but speed and wave-length decreases.

18. Given below are two statements:

Statement I : In hydrogen atom, the frequency of radiation emitted when an electron jumps from lower energy orbit (E_1) to higher energy orbit (E_2), is given as $hf = E_1 - E_2$.

Statement-II : The jumping of electron from higher energy orbit (E_2) to lower energy orbit (E_1) is associated with frequency of radiation given as $f = (E_2 - E_1)/h$

This condition is Bohr's frequency condition.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both statement I and statement II are true.

(B) Both statement I and statement II are false

(C) Statement I is correct but statement II is false

(D) Statement I is incorrect but statement II is true.

Official Ans. by NTA (D)

Sol. When electron jump from lower to higher energy level, energy absorbed so statement-I incorrect.

When electron jump from higher to lower energy level, energy of emitted photon

$E = E_2 - E_1$

$hf = E_2 - E_1 \Rightarrow f = \frac{E_2 - E_1}{h}$

so statement-II is correct.

19. For a transistor to act as a switch, it must be operated in

(A) Active region

(B) Saturation state only

(C) Cut-off state only

(D) Saturation and cut-off state

Official Ans. by NTA (D)

Sol. Transistor act as a switch in saturation and cut of region.

20. We do not transmit low frequency signal to long distances because

(a) The size of the antenna should be comparable to signal wavelength which is unreal solution for a signal of longer wavelength.

(b) Effective power radiated by a long wavelength baseband signal would be high.

(c) We want to avoid mixing up signals transmitted by different transmitter simultaneously.

(d) Low frequency signal can be sent to long distances by superimposing with a high frequency wave as well.

Therefore, the most suitable options will be :

(A) All statements are true

(B) (a), (b) and (c) are true only

(C) (a), (c) and (d) are true only

(D) (b), (c) and (d) are true only

Official Ans. by NTA (C)

Sol. (a) For low frequency or high wavelength size of antenna required is high.

(b) E P R is low for longer wavelength.

(c) yes we want to avoid mixing up signals transmitted by different transmitter simultaneously.

(d) Low frequency signals sent to long distance by superimposing with high frequency.

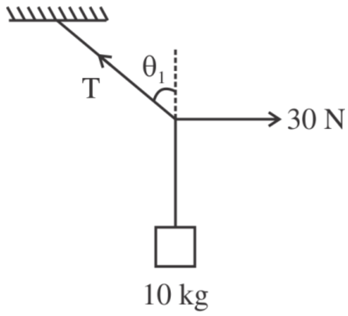
SECTION-B

1. A mass of 10 kg is suspended vertically by a rope of length 5m from the roof. A force of 30 N is applied at the middle point of rope in horizontal direction. The angle made by upper half of the rope with vertical is $\theta = \tan^{-1}(x \times 10^{-1})$. The value of x is ______ .

(Given $g = 10\ m/s^2$)

Official Ans. by NTA (3)

Sol.



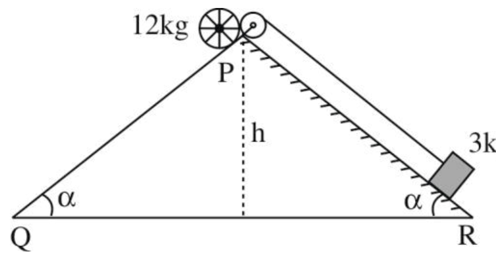
$T \sin\theta = 30$

$T \cos\theta = 100$

$\Rightarrow \tan\theta = 0.3$

2. A rolling wheel of 12 kg is on an inclined plane at position P and connected to a mass of 3 kg through a string of fixed length and pulley as shown in figure. Consider PR as friction free surface.

The velocity of centre of mass of the wheel when it reaches at the bottom Q of the inclined plane PQ will be $\frac{1}{2}\sqrt{xgh}$ m/s. The value of x is ______ .



Official Ans. by NTA (3)

Sol. Net loss in PE = Gain in KE

$$12\,gh - 3gh = \frac{1}{2}3v^2 + \frac{1}{2}12v^2 + \frac{1}{2}\left[12r^2\right]\left(\frac{v}{r}\right)^2$$

$$9gh = \frac{1}{2}[3+12+12]v^2$$

$$v^2 = \frac{2gh}{3} \Rightarrow v = \frac{1}{2}\sqrt{\frac{8}{3}gh}$$

$$x = \frac{8}{3} \simeq 3$$

3. A diatomic gas ($\gamma = 1.4$) does 400 J of work when it is expanded isobarically. The heat given to the gas in the process is _______ J.

Official Ans. by NTA (1400)

Sol. $Q = nC_p\Delta T = \frac{n\nu}{\nu-1}R\Delta T$

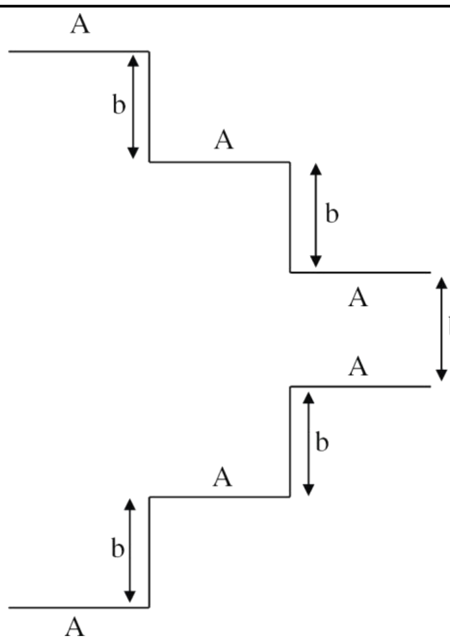
$$Q = \frac{\nu}{\nu-1}\omega = \frac{1.4}{0.4}\times 400 = 1400\text{ J}$$

4. A particle executes simple harmonic motion. Its amplitude is 8 cm and time period is 6s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is ________ s.

Official Ans. by NTA (1)

Sol. $t = \frac{\Delta\phi}{\omega} = \frac{\pi/2 - \pi/6}{2\pi/6} = \frac{\pi/3}{\pi/3} = 1\text{ sec}$

5. A paralle plate capacitor is made up of stair like structure with a palte area A of each stair and that is connected with a wire of length b, as shown in the figure. The capacitance of the arrangement is $\frac{x}{15}\frac{\varepsilon_0 A}{b}$. The value of x is _______ .



Official Ans. by NTA (23)

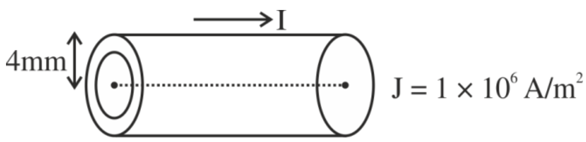
Sol. Parallel combination

$$c_{eq} = \varepsilon_0 A\left[\frac{1}{5b}+\frac{1}{3b}+\frac{1}{b}\right] = \frac{23}{15}\frac{\varepsilon_0 A}{b}$$

6. The current density in a cylindrical wire of radius r = 4.0 mm is 1.0×10^6 A/m^2. The current through the outer portion of the wire between radial distances r/2 and r is $x\pi$ A; where x is _______ .

Official Ans. by NTA (12)

Sol.



$$I = \int JdA$$

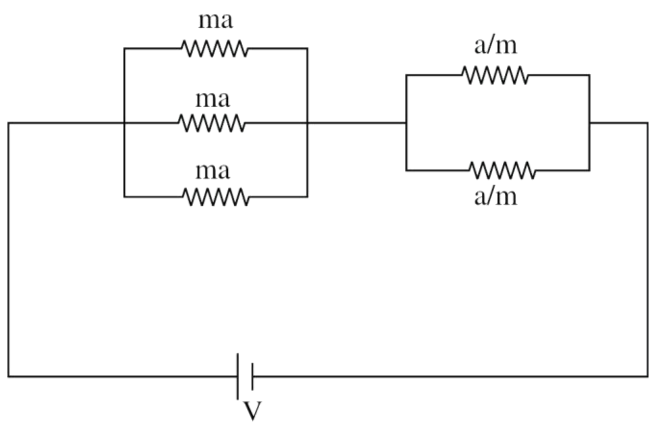
$$= \int 10^6 \times 2\pi x dx$$

$$= 10^6 \times 2\pi . \frac{x^2}{2}\Big]_{\frac{r}{2}}^{r}$$

$$= \pi \times 10^6\left[r^2 - \frac{r^2}{4}\right] = 12\pi$$

x = 12

7. In the given circuit 'a' is an arbitrary constant. The value of m for which the equivalent circuit resistance is minimum, will be $\sqrt{\frac{x}{2}}$. The value of x is _____ .



Official Ans. by NTA (3)

Sol. $R = \left(\frac{ma}{3}\right) + \left(\frac{a}{2m}\right)$

$$\frac{dR}{dm} = \frac{a}{3} - \frac{a}{2m^2} = 0$$

$$\frac{a}{3} = \frac{a}{2m^2}$$

$$m^2 = \frac{3}{2}$$

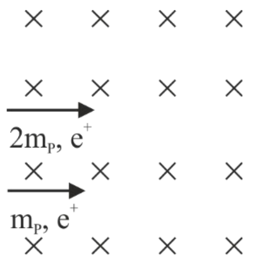
$$m = \sqrt{\frac{3}{2}}$$

$x = 3$

8. A deuteron and a proton moving with equal kinetic energy enter into to a uniform magnetic field at right angle to the field. If r_d and r_p are the radii of their circular paths respectively, then the ratio $\frac{r_d}{r_p}$ will be $\sqrt{x}:1$ where x is ______ .

Official Ans. by NTA (2)

Sol.



$$R = \frac{mv}{q_B}$$

$$R_D = \frac{(2m_P)\,v_D}{e\,B}$$

$$R_P = \frac{(m_P)\,v_P}{e\,B}$$

$$\frac{R_D}{R_P} = \frac{2v_D}{v_P} = \frac{2v_D}{\sqrt{2}v_D} = \frac{\sqrt{2}}{1}$$

$$\frac{1}{2}(2mp)v_D^2 = \frac{1}{2}m_P.v_P^2$$

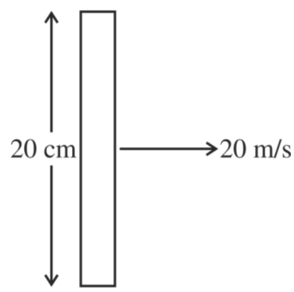
$$\sqrt{2}v_D = v_P$$

$x = 2$

9. A metallic rod of length 20 cm is palced in North-South direction and is moved at a constant speed of 20 m/s towards East. The horizontal component of the Earth's magnetic field at that place is 4×10^{-3} T and the angle of dip is 45°. The emf induced in the rod is _______ mV.

Official Ans. by NTA (16)

Sol.



$B_H = 4\times10^{-3}\,T$

$\theta \rightarrow 45°$

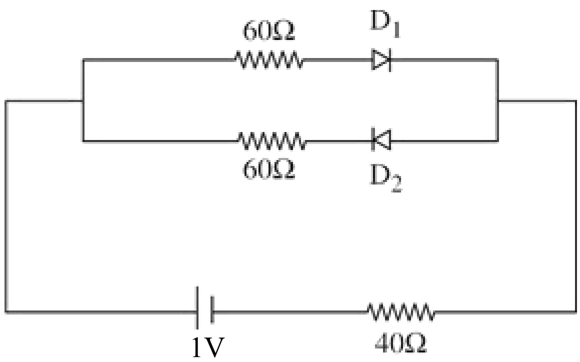
$B_V = B_H$

$\in = (\vec{V}\times\vec{B})\cdot\vec{\ell}$

$= ((4\times10^{-3})(20))\frac{20}{100}$

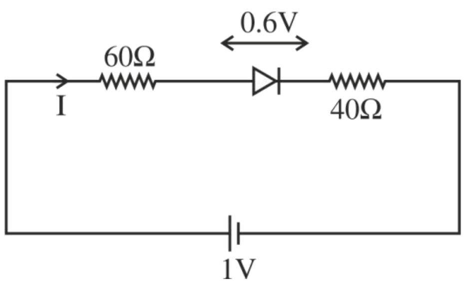
$= 16 \times 10^{-3}$ V $= 16$ mV

10. The cut-off voltage of the diodes (shown in figure) in forward bias is 0.6 V. The current through the resister of 40 Ω is ________ mA.



Official Ans. by NTA (4)

Sol.



$1 - I(60) - 0.6 - I(40) = 0$

$\frac{0.4}{100} = I$

$I = 4$ mA

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

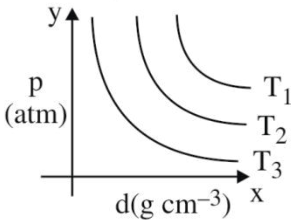
(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | **TEST PAPER WITH SOLUTIONS**

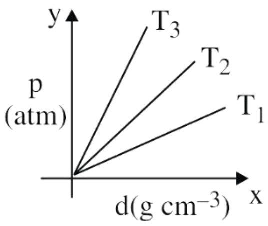
SECTION-A

1. Which amongst the given plots is the correct plot for pressure (p) vs density (d) for an ideal gas ?

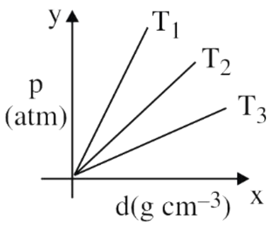
(A) $T_3 > T_2 > T_1$



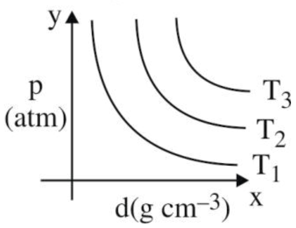
(B) $T_3 > T_2 > T_1$



(C) $T_3 > T_2 > T_1$



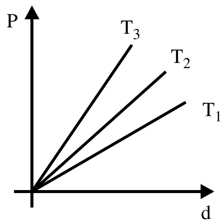
(D) $T_3 > T_2 > T_1$



Official Ans. by NTA (B)

Sol. P vs d :

$$P = \left(\frac{RT}{M}\right) d$$



$T_3 > T_2 > T_1$

2. Identify the **incorrect** statement for PCl_5 from the following.

(A) In this molecule, orbitals of phosphorous are assumed to undergo sp^3d hybridization.

(B) The geometry of PCl_5 is trigonal bipyramidal.

(C) PCl_5 has two axial bonds stronger than three equatorial bonds.

(D) The three equatorial bonds of PCl_5 lie in a plane.

Official Ans. by NTA (C)

Sol. In PCl_5, axial bonds are weaker than equatorial.

3. **Statement I : Leaching of gold** with cyanide ion in absence of air / O_2 leads to cyano complex of Au(III).

Statement II : Zinc is oxidized during the displacement reaction carried out for gold extraction.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (D)

Sol. Statement-1 : wrong, Au^+ is correct, not Au^{+3}

Statement-2 : correct

4. The correct order of increasing intermolecular hydrogen bond strength is

(A) $HCN < H_2O < NH_3$

(B) $HCN < CH_4 < NH_3$

(C) $CH_4 < HCN < NH_3$

(D) $CH_4 < NH_3 < HCN$

Official Ans. by NTA (C)

Sol. Order of H-Bonding

$CH_4 < HCN < NH_3$

NCH . . . NCH

H_2NH . . . NH_3

5. The correct order of increasing ionic radii is

(A) $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

(B) $N^{3-} < O^{2-} < F^{-} < Na^{+} < Mg^{2+}$

(C) $F^{-} < Na^{+} < O^{2-} < Mg^{2+} < N^{3-}$

(D) $Na^{+} < F^{-} < Mg^{2+} < O^{2-} < N^{3-}$

Official Ans. by NTA (A)

Sol. $N^{-3} > O^{-2} > F^{-} > Na^{+} > Mg^{+2}$ (Radii)

(Isoelectronic species)

6. The gas produced by treating an aqueous solution of ammonium chloride with sodium nitrite is

(A) NH_3 (B) N_2

(C) N_2O (D) Cl_2

Official Ans. by NTA (B)

Sol. $NH_4Cl + NaNO_2 \rightarrow NH_4NO_2 + NaCl$

$$\downarrow$$

$$N_2 + 2H_2O$$

7. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Flourine forms one oxoacid.

Reason R : Flourine has smallest size amongst all halogens and is highly electronegative

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol. Both A and R are correct and R is the correct explanation of A.

8. In 3d series, the metal having the highest M^{2+}/M standard electrode potential is

(A) Cr (B) Fe

(C) Cu (D) Zn

Official Ans. by NTA (C)

Sol. $Cr^{+2}/Cr \rightarrow -0.90$ V

$Fe^{+2}/Fe \rightarrow -0.44$ V

$Cu^{+2}/Cu \rightarrow +0.34$ V

$Zn^{+2}/Zn \rightarrow -0.76$ V

So Ans. Cu^{+2}/Cu

9. The 'f' orbitals are half and completely filled, respectively in lanthanide ions

(Given: Atomic no. Eu, 63; Sm, 62; Tm, 69; Tb, 65; Yb, 70; Dy, 66]

(A) Eu^{2+} and Tm^{2+} (B) Sm^{2+} and Tm^{3+}

(C) Tb^{4+} and Yb^{2+} (D) Dy^{3+} and Yb^{3+}

Official Ans. by NTA (C)

Sol. $Tb \rightarrow 4f^9 6s^2$

$Tb^{+4} \rightarrow 4f^7$

$Yb \rightarrow 4f^{14} 6s^2$

$Yb^{+2} \rightarrow 4f^{14}$

10. Arrange the following coordination compounds in the increasing order of magnetic moments. (Atomic numbers: Mn = 25; Fe = 26)

(A) $[FeF_6]^{3-}$

(B) $[Fe(CN)_6]^{3-}$

(C) $[MnCl_6]^{3-}$ (high spin)

(D) $[Mn(CN)_6]^{3-}$

(A) A < B < D < C (B) B < D < C < A

(C) A < C < D < B (D) B < D < A < C

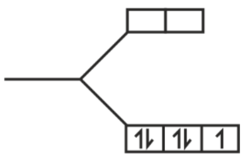
Official Ans. by NTA (B)

Sol. (A) $[FeF_6]^{3-}$

$Fe^{+3} \rightarrow 3d^5\ 4s^0$

n = 5

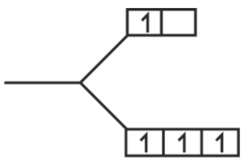
(B) $[Fe(CN)_6]^{3-}$



$Fe^{+3} \rightarrow 3d^5\ 4s^0$

n = 1

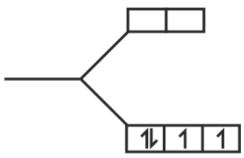
(C) $[MnCl_6]^{3-}$



$Mn^{+3} \rightarrow 3d^4\ 4s^0$

n = 4

(D) $[Mn(CN)_6]^{3-}$



$Mn^{+3} \rightarrow 3d^4\ 4s^0$

n = 2

$\mu \Rightarrow A > C > D > B$

11. On the surface of polar stratospheric clouds, hydrolysis of chlorine nitrate gives A and B while its reaction with HCl produces B and C. A, B and C are, respectively

(A) HOCl, HNO_3, Cl_2

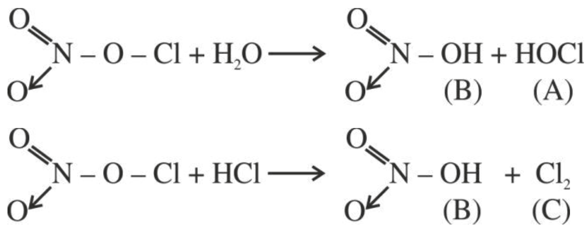
(B) Cl_2, HNO_3, HOCl

(C) $HClO_2$, HNO_2, HOCl

(D) HOCl, HNO_2, Cl_2O

Official Ans. by NTA (A)

Sol.

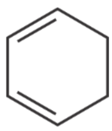


12. Which of the following is most stable?

(A) 

(B) 

(C) 

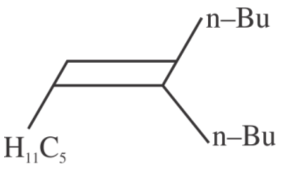
(D) 

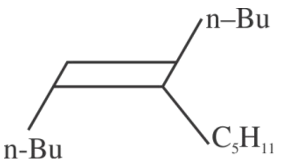
Official Ans. by NTA (A)

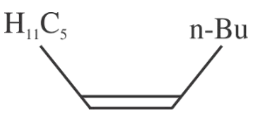
Sol.  is most stable as it is aromatic.

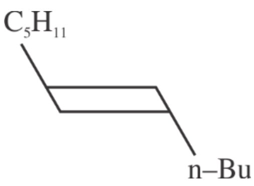
13. What will be the major product of following sequence of reactions?

$$n-Bu-\equiv \xrightarrow[\text{(ii) Lindlar cat, } H_2]{\text{(i) n-BuLi, } n-C_5H_{11}Cl}$$

(A) 

(B) 

(C) 

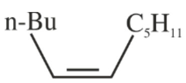
(D) 

Official Ans. by NTA (C)

Sol. $n-Bu-C\equiv CH \xrightarrow[\text{(acid base reaction)}]{\text{nBuLi}}$

$n-Bu-C\equiv C^{-}Li^{+}$

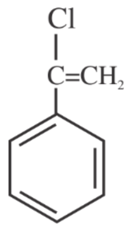
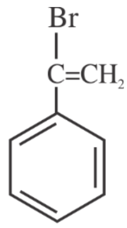
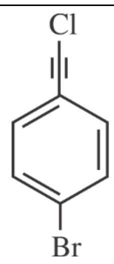
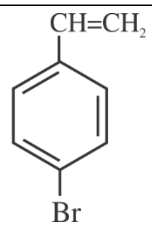
$n-C_5H_{11}Cl \downarrow$ (SN reaction)



$\xleftarrow[\text{Lindlar's Catalyst}]{H_2}$ $n-Bu-C\equiv C-C_5H_{11}$

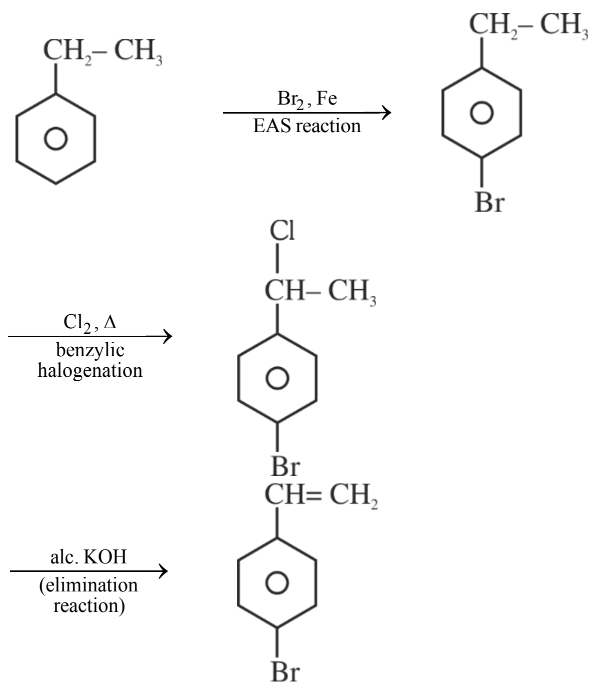
14. Product 'A' of following sequence of reactions is

$$\text{Ethylbenzene} \xrightarrow[\text{(b) } Cl_2, \Delta \text{ (c) alc. KOH}]{\text{(a) } Br_2.Fe} \text{'A'(Major product)}$$

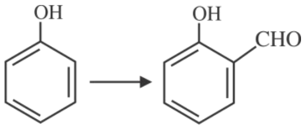
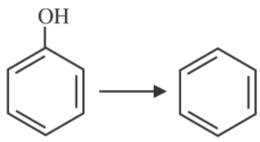
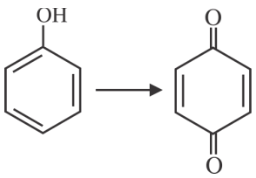
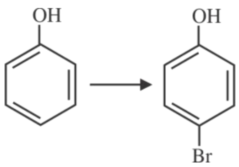
(A)	
(B)	
(C)	
(D)	

Official Ans. by NTA (D)

Sol.



15. Match List I with List II

List I	List II
A. 	I. Br_2 in CS_2
B. 	II. $Na_2Cr_2O_7/H_2SO_4$
C. 	III. Zn
D. 	IV. $CHCl_3/NaOH$

Choose the correct answer from the options given below:

(A) A-IV, B-III, C-II, D-I

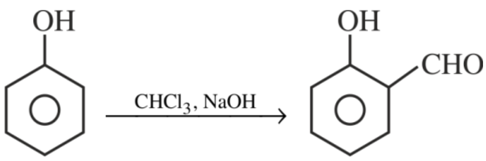
(B) A-IV, B-III, C-I, D-II

(C) A-II, B-III, C-I, D-IV

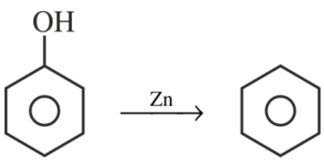
(D) A-IV, B-II, C-III, D-I

Official Ans. by NTA (A)

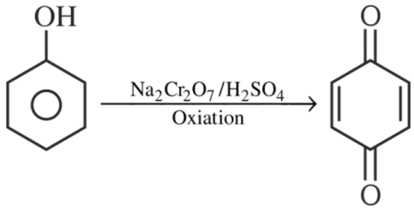
Sol. (A)



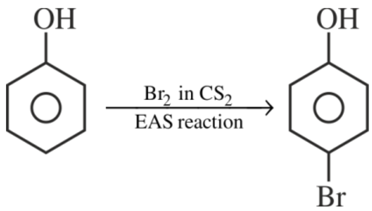
(B)



(C)



(D)



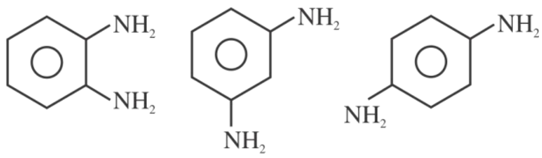
16. Decarboxylation of all six possible forms of diaminobenzoic acids $C_6H_3(NH_2)_2COOH$ yields three products A, B and C. Three acids give a product 'A', two acids gives a product 'B' and one acid give a product 'C'. The melting point of product 'C' is

(A) 63ºC (B) 90ºC

(C) 104ºC (D) 142ºC

Official Ans. by NTA (D)

Sol.



B A C

M.P. 142ºC

17. Which is true about Buna-N?

(A) It is a linear polymer of 1, 3-butadiene.

(B) It is obtained by copolymerization of 1, 3-butadiene and styrene.

(C) It is obtained by copolymerization of 1, 3-butadiene and acrylonitrile.

(D) The suffix N in Buna-N stands for its natural occurrence

Official Ans. by NTA (C)

Sol. It is copolymerization of 1, 3-butadiene and acrylonitrile.

18. Given below are two statements.

Statments I: Maltose has two α-D-glucose units linked at C_1 and C_4 and is a reducing sugar.

Statement II: Maltose has two monosaccharides: α-D-glucose and β-D-glucose linked at C_1 and C_6 and it is a non-reducing sugar.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true

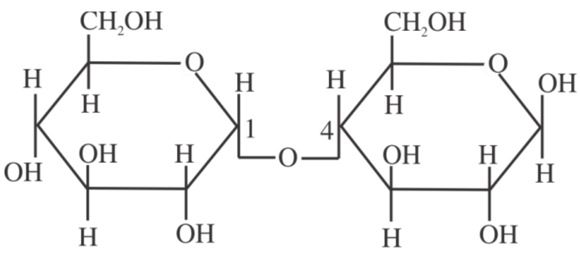
(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (C)

Sol.



Maltose

19. Match List I with List Ii

List I	List II
A. Antipyretic	I. Reduces pain
B. Analgesic	II. Reduces stress
C. Tranquilizer	III. Reduces fever
D. Antacid	IV. Reduces acidity (Stomach)

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-I, C-IV, D-II

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-III, C-II, D-IV

Official Ans. by NTA (A)

Sol.

A. Antipyretic	Reduces fever
B. Analgesic	Reduces pain
C. Tranquilizer	Reduces stress
D. Antacid	Reduces acidity (Stomach)

20. Match List I with List II

List I (Anion)	List II (Gas evolved on reaction with dil. H_2SO_4)
A. CO_3^{2-}	I. Colourless gas which turns lead acetate paper black
B. S^{2-}	II. Colourless gas which turns acidified potassium dichromate solution green.
C. SO_3^{2-}	III. Brown fumes which turns acidified KI solution containing starch blue.
D. NO_2^-	IV. Colourless gas evolved with brisk effervescence, which turns lime water milky.

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV

(B) A-II, B-I, C-IV, D-III

(C) A-IV, B-I, C-III, D-II

(D) A-IV, B-I, C-II, D-III

Official Ans. by NTA (D)

Sol. CO_3^{2-} will give $CO_2(g)$ which will turns lime water milky.

S^{2-} will give H_2S (g), will turns lead acetate paper black

SO_3^{2-} will give SO_2 (g), which will turns acidified potassium dichromate solution green.

NO_2^- will give brown $NO_2(g)$ will turn KI solution blue.

SECTION-B

1. 116 g of a substance upon dissociation reaction, yields 7.5 g of hydrogen, 60g of oxygen and 48.5 g of carbon. Given that the atomic masses of H, O and C are 1, 16 and 12 respectively. The data agrees with how many formulae of the following?

(A) CH_3COOH (B) HCHO

(C) CH_3OOCH_3 (D) CH_3CHO

Official Ans. by NTA (2)

Sol. $\%H = \frac{7.5}{116} \times 100 = 6.5$

$\%O = \frac{60}{116} \times 100 = 51.7$

$\%C = \frac{48.5}{116} \times 100 = 41.8$

Relative atomicities = $H \Rightarrow 6.5$

$O \Rightarrow \frac{51.7}{16} = 3.25$

$C \Rightarrow \frac{41.8}{12} = 3.5$

Emperically formula is approx.. CH_2O

(A) $C_2H_4O_2$ (B) CH_2O relate to this formula.

2. Consider the following set of quantum numbers

	n	l	m_l
A.	3	3	-3
B.	3	2	-2
C.	2	1	+1
D.	2	2	+2

The number of correct sets of quantum numbers is ________

Official Ans. by NTA (2)

Sol. Quantum no. of set (B) and (C) can be correct.

(A) and (D) are wrong as $n = \ell$ is not possible.

3. BeO reacts with HF in presence of ammonia to give [A] which on thermal decomposition produces [B] and ammonium fluoride. Oxidation state of Be in [A] is_______

Official Ans. by NTA (2)

Sol.

$$BeO + HF + NH_3 \rightarrow \underset{[A]}{(NH_4)_2[BeF_4]} \xrightarrow{\Delta} NH_4F + \underset{[B]}{BeF_2}$$

Oxidation state of Be in A is (+2)

4. When 5 moles of He gas expand isothermally and reversibly at 300 K from 10 litre to 20 litre, the magnitude of the maximum work obtained is _____ J. [nearest integer] (Given: R = 8.3 J $K^{-1}mol^{-1}$ and log 2 = 0.3010)

Official Ans. by NTA (8630)

Sol. n = 5 mol

T = 300 K

$V_1 = 10$ L

$V_2 = 20$ L

$w = -nRT\,\ell n \frac{V_2}{V_1}$

$= -5 \times 8.3 \times 300 \times \ell n \frac{20}{10}$

$= -8630.38$ J

5. A solution containing 2.5×10^{-3} kg of a solute dissolved in 75×10^{-3} kg of water boils at 373.535 K. The molar mass of the solute is ______ g mol^{-1}. [nearest integer] (Given: K_b (H_2O) = 0.52 K Kg mol^{-1}, boiling point of water = 373.15K)

Official Ans. by NTA (45)

Sol. $w = 2.5$ g $K_b = 0.52$

$w_{solvent} = 75$ g   M = Mol. Wt. of solute

$T'_B = 373.535\,K$

$T_B^o = 373.15\,K$

$\Delta T_B = 0.385 = K_b$ molality

$$0.385 = 0.52 \times \left(\frac{2.5}{M} \times \frac{1000}{75}\right)$$

$M = 45$ g mol^{-1}

6. pH value of 0.001 M NaOH solution is______.

Official Ans. by NTA (11)

Sol. 0.001 M NaOH

$[OH^-] = 10^{-3}$

pOH = 3

pH = 11

7. For the reaction taking place in the cell:

$Pt(s) \mid H_2(g) \mid H^+(aq) \parallel Ag^+(aq) \mid Ag(s)$

$E^o_{Cell} = +0.5332$ V.

The value of $\Delta_f G^0$ is ________ kJ mol^{-1}. (in nearest integer)

Official Ans. by NTA (51)

Sol. $\frac{1}{2}H_2 + Ag^+ \rightarrow H^+ + Ag$

$\Delta G° = -\,nE°\,F$

$= -1 \times 0.5332 \times 96500$ J

$= -51.35$ kJ

$(n = 2 \text{ for } H_2 + 2Ag^+ \rightarrow 2H^+ + 2Ag)$

8. It has been found that for a chemical reaction with rise in temperature by 9K the rate constant gets doubled. Assuming a reaction to be occurring at 300 K, the value of activation energy is found to be ______ kJ mol^{-1}. [nearest integer]

(Given ln 10 = 2.3, R = 8.3 $JK^{-1}mol^{-1}$, log2 = 0.30)

Official Ans. by NTA (59)

Sol. $$\log_{10}\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{300} - \frac{1}{309}\right)$$

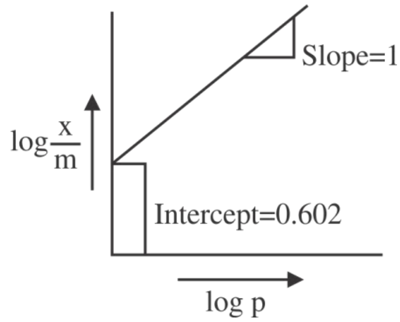
$$0.3 = \frac{E_a}{2.303 \times 8.3}\left(\frac{9}{300 \times 309}\right)$$

$$E_a = \frac{0.3 \times 2.303 \times 8.3 \times 300 \times 309}{9}$$

= 59065.04 J

$E_a = 59.06$ kJ

9.



If the initial pressure of a gas is 0.03 atm, the mass of the gas adsorbed per gram of the adsorbent is ______ $\times 10^{-2}$g.

Official Ans. by NTA (12)

Sol. $$\frac{x}{m} = kP^{\frac{1}{n}}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

From graph

$$\text{Slope} = \frac{1}{n} = 1 \Rightarrow n = 1$$

Intercept = log k = 0.602

k = 4

$$\frac{x}{m} = 4 \times (0.03)^{\frac{1}{1}}$$

$$\frac{x}{m} = 12 \times 10^{-2}$$

10. 0.25 g of an organic compound containing chlorine gave 0.40 g of silver chloride in Carius estimation. The percentage of chlorine present in the compound is ______. [in nearest integer]

(Given: Molar mass of Ag is 108 g mol^{-1} and that of Cl is 35.5 g mol^{-1})

Official Ans. by NTA (40)

Sol. wt. of organic compound = 0.25 g

$$\text{mass of Cl} = \frac{35.5}{143.5} \times 0.4g$$

mass % of Cl in the organic compound

$$= \frac{35.5 \times 0.4}{143.5 \times 0.25} \times 100$$

= 39.58%

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The number of points of intersection of $|z-(4+3i)|=2$ and $|z|+|z-4|=6$, $z \in C$ is :

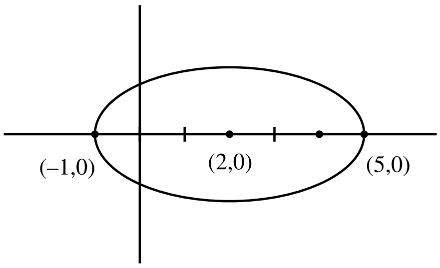
(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (C)

Sol. C : $(x-4)^2+(y-3)^2=4$

$E:\dfrac{(x-2)^2}{9}+\dfrac{y^2}{5}=1$



Lower Extremity of vertical diameter of circle $\rightarrow (4,1)$

Put in ellipse $\Rightarrow \dfrac{(4-2)^2}{9}+\dfrac{1}{5}-1$

$=\dfrac{4}{9}+\dfrac{1}{5}-1$

$=\dfrac{29}{45}-1<0$

Two Solutions

Answer (C)

2. Let $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of which the squares of all the values of a for $2f'(10)-f'(5)+100=0$ is :

(A) 117 (B) 106

(C) 125 (D) 136

Official Ans. by NTA (C)

Sol. $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

$f(x)=a\begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

$= a\,[1\,(a^2+ax)+1\,(ax+x^2)]$

$\Rightarrow f(x)=a\,(x+a)^2$

so, $f'(x)=2a\,(x+a)$

as, $2f'(10)-f'(5)+100=0$

$\Rightarrow 2\times 2a\,(10+a)-2a\,(5+a)+100=0$

$\Rightarrow 40a+4a^2-10a-2a^2+100=0$

$2a^2+30a+100=0$

$\Rightarrow a^2+15a+50=0$

$(a+10)\,(a+5)=0$

$a=-10$ or $a=-5$

Required $=(-10)^2+(-5)^2=125$

3. Let for some real numbers α and β, $a=\alpha-i\beta$. If the system of equations $4ix+(1+i)y=0$ and $8\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution then $\dfrac{\alpha}{\beta}$ is equal to :

(A) $-2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $-2-\sqrt{3}$

Official Ans. by NTA (B)

Sol. $a = \alpha - i\beta$; $\alpha \in R$; $\beta \in R$

$4ix + (1 + i)\, y = 0$ and

$8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$

$\begin{vmatrix} 4i & 1+i \\ 8e^{i2\pi/3} & \bar{a} \end{vmatrix} = 0$

$\Rightarrow 4i\bar{a} - (1 + i)\, 8e^{i2\pi/3} = 0$

$\Rightarrow 4i(\alpha + i\beta) - 8(1 + i)\left(\frac{-1+i\sqrt{3}}{2}\right) = 0$

$\Rightarrow i\alpha - \beta + 1 + \sqrt{3} + i(1 - \sqrt{3}) = 0$

$\Rightarrow \beta = \sqrt{3} + 1$

$\alpha = \sqrt{3} - 1$

So, $\frac{\alpha}{\beta} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

4. Let A and B be two 3 × 3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$ then $|adj(B\,adj(2A))|$ is equal to

(A) 16 (B) 32

(C) 64 (D) 128

Official Ans. by NTA (C)

Sol. $AB = i$

$|adj(B\,adj(2A)| = |B\,adj(2A)|^2$

$= |B|^2\,|adj(2A)|^2$

$= |B|^2\,(|2A|^2)^2 = |B|^2\,(2^6\,|A|^2)^2$

$|A| = \frac{1}{8}$ and $|AB| = 1 \Rightarrow |A|\,|B| = 1$

$\Rightarrow \frac{1}{8}|B| = 1$

$\Rightarrow |B| = 8$

required value = 64

5. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$ then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$ (B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$ (D) $\frac{7^2}{3^3}$

Official Ans. by NTA (C)

Sol. $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

Considering infinite sequence,

$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + ...$

$\Rightarrow \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + ...$

$\Rightarrow \frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + ...$

$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + ...$

$\Rightarrow \frac{6^2 S}{7^2} = \frac{2}{1 - \frac{1}{7}} = \frac{2}{6} \times 7$

$\Rightarrow S = \frac{2 \times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$

6. If $a_1, a_2, a_3 ....$ and $b_1, b_2, b_3 ....$ are A.P. and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10} b_{10}$ then $a_4 b_4$ is equal to

(A) $\frac{35}{27}$ (B) 1

(C) $\frac{27}{28}$ (D) $\frac{28}{27}$

Official Ans. by NTA (D)

Sol. $a_1, a_2, a_3 ....A.P.$; $a_1 = 2$; $a_{10} = 3; d_1 = \frac{1}{9}$

$b_1, b_2, b_3, ...$ A.P. ; $b_1 = \frac{1}{2}$; $b_{10} = \frac{1}{3}$; $d_2 = \frac{-1}{54}$

[Using $a_1 b_1 = 1 = a_{10} b_{10}$; d_1 & d_2 are common differences respectively]

$a_4.b_4 = (2 + 3d_1)\left(\frac{1}{2} + 3d_2\right)$

$= \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right)$

$= \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \frac{28}{27}$

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}dt$, then the ordered pair (m, n) is equal to

(A) (3, 2) (B) (2, 3)

(C) (2, 2) (D) (3, 4)

Official Ans. by NTA (B)

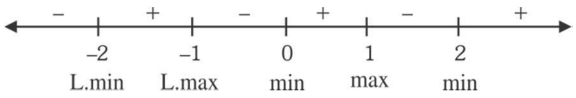
Sol. m = L · max

N = L · min

$$f(x)=\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$$

$$f'(x)=\frac{(x^4-5x^2+4)2x}{2+e^{x^2}}=\frac{2x(x^2-1)(x^2-4)}{2+e^{x^2}}$$

$$=\frac{2x(x-1)(x+1)(x-2)(x+2)}{2+e^{x^2}}$$



so, m = 2 and n = 3

8. Let f be a differentiable function in $\left(0,\frac{\pi}{2}\right)$.

If $\int_{\cos x}^{1} t^2 f(t)dt=\sin^3 x+\cos x$ then $\frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(A) $6-9\sqrt{2}$ (B) $6-\frac{9}{\sqrt{2}}$

(C) $\frac{9}{2}-6\sqrt{2}$ (D) $\frac{9}{\sqrt{2}}-6$

Official Ans. by NTA (B)

Sol. At right hand vicinity of x = 0 given equation does not satisfy

$\because \text{LHS}=\int_{1^-}^{1} t^2 f(t)dt=0,\ \text{RHS}=\lim_{x\to 0^+}\left(\sin^3 x+\cos x\right)=1$

LHS ≠ RHS hence data given in question is wrong hence BONUS

Correct data should have been

$$\int_{\cos x}^{1} t^2 f(t)dt=\sin^3 x+\cos x-1$$

Calculation for option

differentiating both sides

$-\cos^2 x\, f(\cos x).(-\sin x) = 3\sin^2 x.\cos x-\sin x$

$\Rightarrow f(\cos x) = 3\tan x - \sec^2 x$

$\Rightarrow f'(\cos x)(-\sin x) = 3\sec^2 x - 2\sec^2 x\tan x$

$\Rightarrow f'(\cos x)\cos x = \frac{2}{\cos^2 x}-\frac{3}{\sin x.\cos x}$

When $\cos x = \frac{1}{\sqrt{3}}$; $\sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$\therefore f'\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{3}}=6-\frac{9}{\sqrt{2}}$.

9. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where [.] denotes the greatest integer function is equal to

(A) $1+6\log_e\left(\frac{6}{7}\right)$ (B) $1-6\log_e\left(\frac{6}{7}\right)$

(C) $\log_e\left(\frac{7}{6}\right)$ (D) $1-7\log_e\left(\frac{6}{7}\right)$

Official Ans. by NTA (A)

Sol. $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx=-\int_1^0 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$

$=(-1)\left[\int_1^{1/2}\frac{1}{7}dx+\int_{1/2}^{1/3}\frac{1}{7^2}dx+\int_{1/3}^{1/4}\frac{1}{7^3}dx+......\infty\right]$

$=\left(\frac{1}{7}+\frac{1}{2.7^2}+\frac{1}{3.7^3}+...\infty\right)-\left(\frac{1}{7.2}+\frac{1}{7^2.3}+\frac{1}{7^2.4}...\infty\right)$

$=-\ln\left(1-\frac{1}{7}\right)-7\left(\frac{1}{7^2.2}+\frac{1}{7^3.3}+\frac{1}{7^4.4}+.....\infty\right)$

$\left[\text{as } \ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}....\infty\right]$

$\left[\text{as } \ln(1-x)=-\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}....\infty\right)\right]$

$=-\ln\frac{6}{7}-7\left(-\ln\left(1-\frac{1}{7}\right)-\frac{1}{7}\right)$

$=6\ln\frac{6}{7}+1$

10. If the solution curve of the differential equation $\left((\tan^{-1} y)-x\right)dy=(1+y^2)dx$ passes through the point (1, 0) then the abscissa of the point on the curve whose ordinate is tan (1) is :

(A) 2e (B) $\frac{2}{e}$

(C) 2 (D) $\frac{1}{e}$

Official Ans. by NTA (B)

Sol. $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$

$\text{I.f}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

$xe^{\tan^{-1}y}=\int\frac{\tan^{-1}y}{1+y^2}e^{\tan^{-1}y}dy$

$x\cdot e^{\tan^{-1}y} = (\tan^{-1}y - 1)e^{\tan^{-1}y} + c$

$\because$ (1, 0) lies exit c = 2.

For y = tan1 $\Rightarrow x = \frac{2}{e}$

11. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x+y-29=0$, is $x^2+ay^2+bxy+cx+dy+k=0$ then a + b + c +d + k is equal to

(A) 575 (B) –575

(C) 576 (D) –576

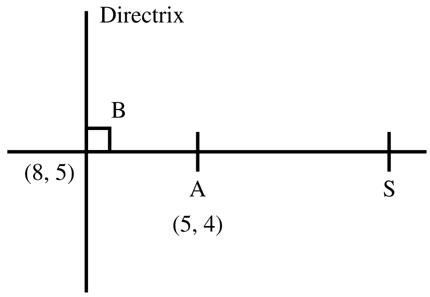
Official Ans. by NTA (D)

Sol. Vertex (5,4)

Directrix : 3x + y – 29 = 0

Co-ordinates of B (foot of directrix)

$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$



x = 8, y = 5

S = (2, 3) (focus)

Equation of parabola

PS = PM

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

a + b + c + d + k = 9 – 6 + 134 – 2 – 711 = – 576

12. The set of values of k for which the circle C : $4x^2+4y^2-12x+8y+k=0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C is :

(A) An empty set (B) $\left(6, \frac{95}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$ (D) $\left(9, \frac{92}{9}\right]$

Official Ans. by NTA (D)

Sol. C : $4x^2 + 4y^2 - 12x + 8y + k = 0$

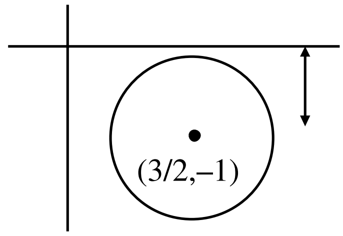
$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}}$ $\Rightarrow k \le 13$ ... (1)

(i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \le 0 \Rightarrow k \le \frac{92}{9}$ ... (2)

(ii) C lies in 4th quadrant



r < 1

$\Rightarrow \frac{\sqrt{13-k}}{2} < 1$

$\Rightarrow k < 9$ .... (3)

Hence (1) ∩ (2) ∩ (3) $\Rightarrow k \in \left(9, \frac{92}{9}\right]$

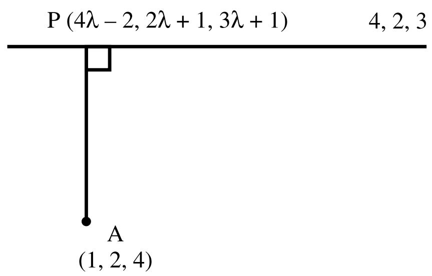
13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$

(A) 5 (B) $\frac{50}{13}$

(C) 4 (D) $\frac{63}{13}$

Official Ans. by NTA (A)

Sol.



$$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda$$

$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$

$\overrightarrow{AP} = (4\lambda - 3)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 5)\hat{k}$

$\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AP} \cdot \vec{b} = 0$

$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$

$29\lambda = 12 + 2 + 15 = 29$

$\lambda = 1$

$P = (2, 3, 2)$

$3x + 4y + 12z + 23 = 0$

$$d = \left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|$$

$$d = \left|\frac{65}{13}\right| = 5$$

14. The shortest distance between the lines $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and $\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$ is :

(A) $\frac{18}{\sqrt{5}}$ (B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$ (D) $6\sqrt{3}$

Official Ans. by NTA (A)

Sol. $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

$A = (3, 2, 1)$ $B = (-3, 6, 5)$

$\vec{n_1} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{n_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{BA} = 6\hat{i} - 4\hat{j} - 4\hat{k}$

$$\text{SHORTEST DISTANCE} = \frac{\left[\overrightarrow{BA}\ \vec{n_1}\ \vec{n_2}\right]}{\left|\vec{n_1} \times \vec{n_2}\right|}$$

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$$

$= 10\hat{i} - 8\hat{j} - 4\hat{k}$

$[\overrightarrow{BA}\ \vec{n_1}\ \vec{n_2}] = 60 + 32 + 16 = 108$

$|\vec{n_1} \times \vec{n_2}| = \sqrt{100+64+16} = \sqrt{180}$

$$\text{S.D} = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$$

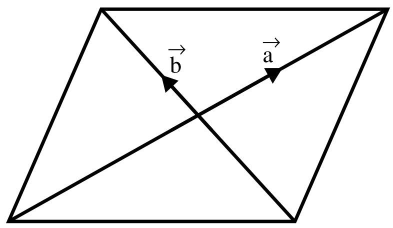
15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute. $|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}\left(\vec{a} \times \vec{b}\right) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Official Ans. by NTA (D)

Sol.



$$\text{Area} = \frac{1}{2}\left|\vec{a} \times \vec{b}\right| = 2\sqrt{2} \Rightarrow \left|\vec{a} \times \vec{b}\right| = 4\sqrt{2}$$

$|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$\therefore \left|\vec{a} \times \vec{b}\right| = 4\sqrt{2} \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{4} = 4\sqrt{2}$

$\Rightarrow |\vec{b}| = 8$

Now, $\vec{c} = 2\sqrt{2}\left(\vec{a} \times \vec{b}\right) - 2\vec{b}$

$$|\vec{c}| = \sqrt{(2\sqrt{2})^2\left|\vec{a} \times \vec{b}\right|^2 + \left(2|\vec{b}|\right)^2} = 16\sqrt{2}$$

Now, $\vec{b}.\vec{c} = -2|\vec{b}|^2$

$\Rightarrow 8 \times 16\sqrt{2} \times \cos\alpha = -2.64$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$

16. The mean and variance of the data4, 5, 6, 6, 7, 8, x, y where x < y are 6, and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to

(A) 162 (B) 320
(C) 674 (D) 420

Official Ans. by NTA (B)

Sol. mean $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 48 - 36 = 12$

Variance

$= \frac{1}{8}(16+25+36+36+49+64+x^2+y^2) - 36 = \frac{9}{4}$

$\Rightarrow x^2 + y^2 = 80$

$\therefore x = 4; y = 8$

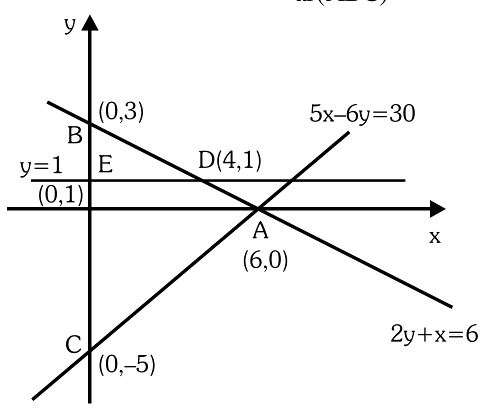
$x^4 + y^2 = 256 + 64 = 320$

17. If a point A(x, y) lies in the region bounded by the y-axis, straight lines 2y + x = 6 and 5x – 6y = 30, then the probability that y < 1 is :

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$
(C) $\frac{2}{3}$ (D) $\frac{6}{7}$

Official Ans. by NTA (B)

Sol. Required probability $= \frac{ar(ADEC)}{ar(ABC)}$



$= 1 - \frac{ar(BDE)}{ar(ABC)}$

$= 1 - \frac{\frac{1}{2}\times 2\times 4}{\frac{1}{2}\times 8\times 6} = 1 - \frac{1}{6} = \frac{5}{6}$

18. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$ (B) $\frac{25}{26}$
(C) $\frac{50}{51}$ (D) $\frac{52}{51}$

Official Ans. by NTA (A)

Sol. $\tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1} n$

so, $\sum_{n=1}^{50}(\tan^{-1}(n+1) - \tan^{-1} n)$

$= \tan^{-1} 51 - \tan^{-1} 1$

$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right) = \cot(\tan^{-1} 51 + \tan^{-1} 1)$

$= \frac{1}{\tan(\tan^{-1}51 - \tan^{-1}1)} = \frac{1+51\times 1}{51-1} = \frac{52}{50} = \frac{26}{25}$

19. $\alpha = \sin 36°$ is a root of which of the following equation

(A) $10x^4 - 10x^2 - 5 = 0$ (B) $16x^4 + 20x^2 - 5 = 0$

(C) $16x^4 - 20x^2 + 5 = 0$ (D) $16x^4 - 10x^2 + 5 = 0$

Official Ans. by NTA (C)

Sol. $\cos 72° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 1 - 2\sin^2 36° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$

$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$

$\Rightarrow (5 - 8\alpha^2)^2 = 5$

$\Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$

$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0$

$\Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$

20. Which of the following statement is a tautology?

(A) $((\sim q)\wedge p)\wedge q$
(B) $((\sim q)\wedge p)\wedge(p\wedge(\sim p))$
(C) $((\sim q)\wedge p)\vee(p\vee(\sim p))$
(D) $(p\wedge q)\wedge(\sim(p\wedge q))$

Official Ans. by NTA (C)

Sol. (A) $(\sim q\wedge p)\wedge q = (\sim q\wedge q)\wedge p = f$

(B) $(\sim q\wedge p)\wedge(p\wedge\sim p) = \sim q\wedge(p\wedge\sim p) = f$

(C) $(\sim q\wedge p)\vee(p\vee\sim p) = (\sim q\wedge p)\vee(t) = t$

(D) $(p\wedge q)\wedge(\sim(p\wedge q)) = f$

SECTION-B

1. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define $f : S \to S$ as $f(n)=\begin{cases} 2n, & \text{if } n=1,2,3,4,5 \\ 2n-11 & \text{if } n=6,7,8,9,10 \end{cases}$.

Let $g : S \to S$ be a function such that $fog(n)=\begin{cases} n+1 & ,\text{if } n \text{ is odd} \\ n-1 & ,\text{if } n \text{ is even} \end{cases}$, then

g(10) ((g(1) + g(2) + g(3) + g(4) + g(5)) is equal to:

Official Ans. by NTA (190)

Sol. $f^{-1}(n)=\begin{cases} \frac{n}{2} & ; \quad n=2,4,6,8,10 \\ \frac{n+11}{2} & ; \quad n=1,3,5,7,9 \end{cases}$

$f(g(n))=\begin{cases} n+1 & ; \quad n \in \text{odd} \\ n-1 & ; \quad n \in \text{even} \end{cases}$

$\Rightarrow \quad g(n)=\begin{cases} f^{-1}(n+1) & ; \quad n \in \text{odd} \\ f^{-1}(n-1) & ; \quad n \in \text{even} \end{cases}$

$\therefore \quad g(n)=\begin{cases} \frac{n+1}{2} & ; \quad n \in \text{odd} \\ \frac{n+10}{2} & ; \quad n \in \text{even} \end{cases}$

$g(10)\cdot[g(1)+g(2)+g(3)+g(4)+g(5)]$

$= 10\cdot[1+6+2+7+3] = 190$

2. Let α, β be the roots of the equation $x^2-4\lambda x+5=0$ and α, γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$.

If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to :

Official Ans. by NTA (98)

Sol. $x^2-4\lambda x+5=0 \langle^{\alpha}_{\beta}$

$x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+\left(7+3\lambda\sqrt{3}\right)=0 \langle^{\alpha}_{\gamma}$

$\alpha+\beta=4\lambda$

$\alpha+\gamma=3\sqrt{2}+2\sqrt{3}$

$\beta+\lambda=3\sqrt{2} \qquad \alpha\gamma=7+3\lambda\sqrt{3}$

$\therefore \quad \alpha=2\lambda+\sqrt{3} \qquad \alpha\beta=5$

$\beta=2\lambda-\sqrt{3} \qquad 4\lambda^2=8 \Rightarrow \lambda=\sqrt{2}$

$\therefore \quad (\alpha+2\beta+\lambda)^2=\left(4\alpha+3\sqrt{2}\right)^2=\left(7\sqrt{2}\right)^2=98$

3. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is :

Official Ans. by NTA (180)

Sol. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

$a+b+c+d=p,\ p \in \{3, 5, 7\}$

Case-(i)

$a+b+c+d=3$; a, b, c, d ∈ {0, 1, 2, 3}

No. of ways $= {}^{3+4-1}C_{4-1} = {}^{6}C_3 = 56$ .... (1)

Case-(ii)

$a+b+c+d=5$; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

No. of ways $= {}^{5+4-1}C_{4-1} = {}^{8}C_3 = 56$ .... (2)

Case-(iii)

$a+b+c+d=7$

No. of ways = total ways when a, b, c, d ∈ {0, 1, 2, 3, 4, 5, 6, 7}– total ways when a, b, c, d ∉ {6, 7}

No of ways $= {}^{7+4-1}C_{4-1} = \left(\frac{\lfloor 4}{\lfloor 3}+\frac{\lfloor 4}{\lfloor 2}\right)$

$= {}^{10}C_3 - 16 = 104$ .... (3)

Hence total no. of ways = 180

4. If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is :

Official Ans. by NTA (57)

Sol. coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n} \to {}^7C_0$	1
$x^{6n-5} \to 2 \cdot {}^7C_1$	$1+14$
$x^{5n-10} \to 2^2 \cdot {}^7C_2$	$1+14+84$
$x^{4n-15} \to 2^3 \cdot {}^7C_3$	$1+14+84+280$
$x^{3n-20} \to 2^4 \cdot {}^7C_4$	$1+4+84+280+560=939$
$x^{2n-25} \to 2^5 \cdot {}^7C_5$	

$3n-20 \ge 0 \cap 2n-25<0 \cap n \in I$

$\therefore \quad 7 \le n \le 12$

Sum = 7 + 8 + 9 + 10 + 11 + 12 = 57

5. Let [t] denote the greatest integer ≤ t and {t} denote the fractional part of t. Then integral value of α for which the left hand limit of the function

$$f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$$ at x = 0 is equal to

$\alpha-\frac{4}{3}$ is ____

Official Ans. by NTA (3)

Sol. $f(x)=[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

$$\lim_{x\to 0^-} f(x)=\alpha-\frac{4}{3} \Rightarrow 0+\frac{\alpha^{-1}-2}{-1}=\alpha-\frac{4}{3}$$

$$\Rightarrow 2-\frac{1}{\alpha}=\alpha-\frac{4}{3}$$

$$\Rightarrow \alpha+\frac{1}{\alpha}=\frac{10}{3}$$

$\Rightarrow \alpha = 3;\ \alpha \in I$

6. If $y(x)=(x^{x^x}),\ x>0$ then $\dfrac{d^2x}{dy^2}+20$ at x = 1 is equal to:

Official Ans. by NTA (16)

Sol. $y = (x) = (x^x)^x$

$\ln y(x) = x^2 \cdot \ln x$

$\dfrac{1}{y(x)} \cdot y'(x) = \dfrac{x^2}{x} + 2x \cdot \ln x$

$y'(x) = y(x)\,[x + 2x \ln x]$

$y(1) = 1;\ y'(1) = 1$

$y''(x) = y'(x)\,[x + 2x \cdot \ln(x)]$

$+ y(x)\,[1 + 2(1 + \ln x)]$

$y''(1) = 1\,[1 + 0] + 1\,(1 + 2) = 4$

$$\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^3 \cdot \frac{d^2x}{dy^2}$$

$$\Rightarrow 4 = -\frac{d^2x}{dy^2}$$

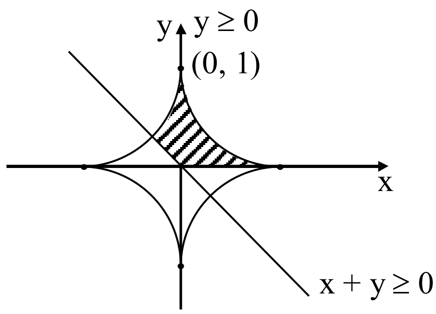
$$\frac{d^2x}{dy^2} = -4$$

Ans. –4 + 20 = 16

7. If the area of the region $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}} \le 1\ x+y \ge 0,\ y \ge 0\right\}$ is A, then $\dfrac{256A}{\pi}$ is

Official Ans. by NTA (36)

Sol.



$$A=\frac{3}{2}\int_0^1 (1-x^{2/3})^{3/2}dx$$

Let $x = \sin^3\theta$

$$A=\frac{3}{2}\int_0^{\pi/2}\left(1-\sin^2\theta\right)^{3/2}.3\sin^2\theta\cos\theta\, d\theta$$

$$=\frac{3}{2}\int_0^{\pi/2} 3\sin^2\theta\cos^4\theta\, d\theta$$

$$=\frac{9}{2}\int_0^{\pi/2} \sin^2\theta\cos^4\theta\, d\theta$$

$$A=\frac{9}{2}\times\frac{1.3.1}{(2+4)(4)(2)}\cdot\frac{\pi}{2}$$

$$\Rightarrow A=\frac{9\pi}{64} \Rightarrow \frac{64A}{\pi}=9$$

$$\Rightarrow \frac{256A}{\pi}=36 \textbf{ Ans.}$$

8. Let v be the solution of the differential equation $(1-x^2)dy=\left(xy+(x^3+2)\sqrt{1-x^2}\right)dx,\ -1<x<1$ and y (0) = 0 if $\int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{1-x^2}\,y(x)dx=k$ then k^{-1} is equal to :

Official Ans. by NTA (320)

Sol. $(1-x^2)\dfrac{dy}{dx} = xy + (x^3+2)\sqrt{1-x^2}$

$\Rightarrow \dfrac{dy}{dx} + \left(\dfrac{-x}{1-x^2}\right)y = \dfrac{x^3+2}{\sqrt{1-x^2}}$

$\text{IF} = e^{\int \frac{-x}{1-x^2}dx} = \sqrt{1-x^2}$

$y(x)\cdot\sqrt{1-x^2} = \dfrac{x^4}{4} + 2x + c$

$y(0) = 0 \Rightarrow c = 0$

$\sqrt{1-x^2}\,y(x) = \dfrac{x^4}{4} + 2x$

$\text{required value} = \int_{-1/2}^{1/2}\left(\dfrac{x^4}{4}+2x\right)dx - \dfrac{1}{4}\cdot 2\int_0^{1/2} x^4 dx$

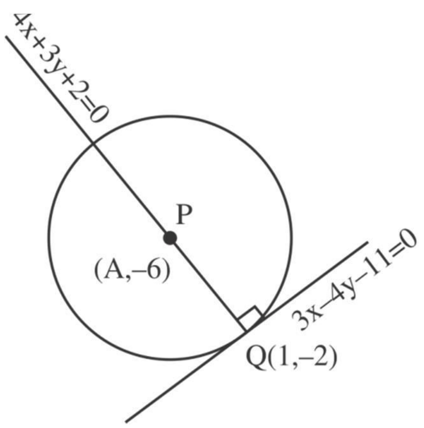
$= \dfrac{1}{10}\left(x^5\right)_0^{1/2} = \dfrac{1}{320}$

$k^{-1} = 320$

9. Let a circle C of radius 5 lie below the x-axis. The line $L_1 = 4x+3y-2$ passes through the centre P of the circle C and intersects the line $L_2: 3x-4y-11=0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x-12y+51=0$ is

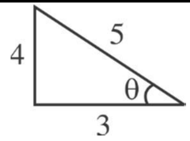
Official Ans. by NTA (11)

Sol.



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$\dfrac{x}{-25} = \dfrac{y}{50} = \dfrac{1}{-25}$

$\dfrac{x-1}{\cos\theta} = \dfrac{y+2}{\sin\theta} = \pm 5$

$y = -2 + 5\left(-\dfrac{4}{5}\right) = -6$

$x = 1 + 5\left(\dfrac{3}{5}\right) = 4$

Req. distance

$\left|\dfrac{5(4)-12(-6)+51}{13}\right|$

$= \left|\dfrac{20+72+51}{13}\right|$

$= \dfrac{143}{13} = 11$

10. Let $S = \{E, E_2 .... E_8\}$ be a sample space of random experiment such that $P(E_n) = \dfrac{n}{36}$ for every n = 1, 2….8. Then the number of elements in the set $\left\{A \subset S : P(A) \geq \dfrac{4}{5}\right\}$ is ______

Official Ans. by NTA (19)

Sol. $P(A') < \dfrac{1}{5} = \dfrac{36}{180}$

5 times the sum of missing number should be less than 36.

If 1 digit is missing = 7

If 2 digit is missing = 9

If 3 digit is missing = 2

If 0 digit is missing = 1

Alternate

A is subset of S hence

A can have elements:

type 1 : { }

type 2: $\{E_1\}, \{E_2\}, \ldots\ldots \{E_8\}$

type 3: $\{E_1, E_2\}, \{E_1, E_3\}\ldots\ldots \{E_1, E_8\}$

⋮

⋮

type 6: $\{E_1, E_2, \ldots\ldots. E_5\}, \ldots\ldots.\{E_4, E_5, E_6, E_7, E_8\}$

type 7: $\{E_1, E_2, \ldots\ldots. E_6\}, \ldots\ldots.\{E_3, E_4, \ldots\ldots\ldots.. E_8\}$

type 8: $\{E_1, E_2, \ldots\ldots. E_7\}\{E_2, E_3, \ldots\ldots\ldots E_8\}$

type 9: $\{E_1, E_2, \ldots\ldots. E_8\}$

As $P(A) \geq \frac{4}{5}$;

Note : Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements.

$\{E_5, E_6, E_7, E_8\}$ is $\frac{5}{36}+\frac{6}{36}+\frac{7}{36}+\frac{8}{36}=\frac{13}{18}<\frac{4}{5}$

Now for Type 5 acceptable elements let's call probability as P_5

$$P_5 = \frac{n_1+n_2+n_3+n_4+n_5}{36} \leq \frac{4}{5}$$

$\Rightarrow n_1 + n_2 + n_3 + n_4 + n_5 \geq 28.8$

Hence, 2 possible ways $\{E_5, E_6, E_7, E_8, \mathbf{E_3} \textbf{ or } \mathbf{E_4}\}$

$P_6 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 \geq 28.8$

$\Rightarrow$ 9 possible ways

$P_7 \Rightarrow n_1 + n_2 + \ldots\ldots\ldots. + n_7 \geq 288$

$\Rightarrow$ 7 possible ways

$P_8 \Rightarrow n_1 + n_2 + \ldots\ldots\ldots. + n_8 \geq 28.8$

$\Rightarrow$ 1 possible way

Total = 19

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Product of Pressure (P) and time (t) has the same dimension as that of coefficient of viscosity.

Reason R:

Coefficient of viscosity = $\dfrac{\text{Force}}{\text{Velocity gradient}}$

Question: Choose the correct answer from the options given below :

(A) Both A and R true, and R is correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (C)

Sol. Pressure and time

$P : \dfrac{N}{m^2}, \text{Time} : \text{Sec}$

$Pt = \dfrac{N\,\text{sec}}{m^2}$

$\eta = \dfrac{F}{6\pi rv} : \dfrac{N}{m.m/\text{sec}} : \dfrac{N\,\text{sec}}{m^2}$

2. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$. where k is a constant. The power delivered to the particle by the force acting on it is given as

(A) zero

(B) $mk^2r^2t^2$

(C) mk^2r^2t

(D) mk^2rt

Official Ans. by NTA (C)

Sol. $a = k^2rt^2 = \dfrac{V^2}{r}$

$V = krt$

$a_t = \dfrac{dv}{dt} = kr$

$F_t = ma_t = mkr$

$P = \vec{F}.\vec{V}$

$= F\cos\theta V = F_t V = mkr(krt)$

$P = mk^2r^2t$

3. Motion of a particle in x-y plane is described by a set of following equations $x = 4\sin\left(\dfrac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$m. The path of particle will be –

(A) circular

(B) helical

(C) parabolic

(D) elliptical

Official Ans. by NTA (A)

Sol. $x = 4\sin\left(\dfrac{\pi}{2} - \omega t\right)$ $y = 4\cos(\omega t)$

$x = 4\cos(\omega t)$ $y = 4\sin(\omega t)$

Eliminate 't' to find relation between x and y

$x^2 + y^2 = y^2\cos^2\omega t + y^2\sin^2\omega t = 4^2$

$x^2 + y^2 = 4^2$

4. Match List-I with List-II

	List-I		List-II
A	Moment of inertia of solid sphere of radius R about any tangent	I	$\frac{5}{3}MR^2$
B	Moment of inertia of hollow sphere of radius (R) about any tangent	II	$\frac{7}{5}MR^2$
C	Moment of inertia of circular ring of radius (R) about its diameter.	III	$\frac{1}{4}MR^2$
D	Moment of inertia of circular disc of radius (R) about any diameter.	IV	$\frac{1}{2}MR^2$

Question: Choose the correct answer from the options given below

(A) A-II, B-II, C-IV, D-III

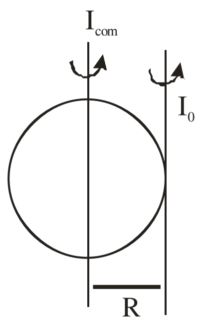
(B) A-I, B-II, C-IV, D-III

(C) A-II, B-I, C-III, D-IV

(D) A-I, B-II, C-III, D-IV

Official Ans. by NTA (A)

Sol. Solid sphere

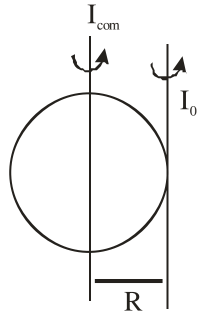


$I_0 = I_{com} + MR^2$ (Parallel Axis theorem)

$$I_0 = \frac{2}{5}MR^2 + MR^2$$

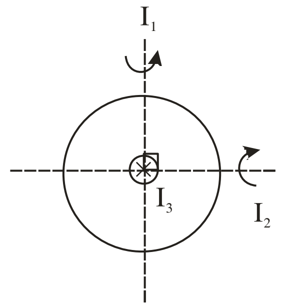
$$I_0 = \frac{7}{5}MR^2$$

Hollow sphere



$$I_0 = I_{com} + MR^2$$

$$= \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$



$I_1 + I_2 + I_3$ (Perpendicular axis theorem)

By symmetry MOI

About 1" and 2" Axis are same i.e.

$$I_1 = I_2$$

$$\therefore\ 2I_1 = I_3 = MR^2 \ \left(I_{com} = MR^2\right)$$

$$I_1 = \frac{MR^2}{2}$$

Similarly in disc

$$2I_1 = \frac{MR^2}{2}\left\{I_{com} = \frac{MR^2}{2}\right\}$$

$$I_1 = \frac{MR^2}{4}$$

5. Two planets A and B of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits?

(A) $2r_A^2 = r_B^2$

(B) $r_A^3 = 2r_B^3$

(C) $r_A^3 = 3r_B^3$

(D) $T_A^2 - T_B^2 = \frac{\pi^2}{GM}\left(r_B^3 - 4r_A^3\right)$

Official Ans. by NTA (C)

Sol. $T = \frac{2\pi}{\sqrt{Gm_a}} r^{\frac{3}{2}}$

$T^2 \propto r^3$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\Rightarrow \left(\frac{2}{1}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow r_A^3 = 4r_B^3$

6. A water drop of diameter cm is broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In this process the gain in surface energy will be :

(A) 2.8×10^{-4} J (B) 1.5×10^{-3} J

(C) 1.9×10^{-4} J (D) 9.4×10^{-5} J

Official Ans. by NTA (A)

Sol. d = 2cm; r = 1 cm; T = 0.075

$\Delta SE = T\,\Delta A$

$= 0.075\left(A_f - A_1\right)$

$A_i = 4\pi r^2$

$A_f = 4\pi r_0^2 \times 64$

By volume conservation

$\frac{4}{3}\pi r^3 = 64\cdot\frac{4}{3}\pi r_0^3$

$r_0 = \frac{r}{4}$

$A_f = 4\pi\left(\frac{r}{4}\right)^2\cdot 64 = 16\pi r^2$

$\Delta SE = 0.075\left(16\pi r^2 - 4\pi r^2\right)$

$= 0.075\left(12\pi(0.01)^2\right)$

$= 2.8\times10^{-4}$ J

7. Given below are two statement :

Statement – I : What μ amount of an ideal gas undergoes adiabatic change from state (P_1, V_1, T_1) to state (P_2, V_2, T_2), the work done is $W = \frac{1R(T_2 - T_1)}{1-\gamma}$, where $\gamma = \frac{C_P}{C_V}$ and R = universal gas constant,

Statement — II: In the above case. when work is done on the gas. the temperature of the gas would rise.

Choose the correct answer from the options given below:

(A) Both statement—I and statement-II are true.

(B) Both statement—I and statement-II are false.

(C) Statement-I is true but statement-II is false.

(D) Statement-I is false but statement-II is true.

Official Ans. by NTA (A)

Sol. $W_{adiabatic} = \dfrac{NR(T_f - T_i)}{1-\gamma} \rightarrow$ statment 1

$Q = W + \Delta U$

$0 = W + \Delta U$

$\Delta U = -W$

If work is done on the gas, i.e. work is negative $\therefore \Delta U$ is positive.

$\therefore$ Temperature will increase.

8. Given below are two statements :

Statement-I : A point charge is brought in an electric field. The value of electric field at a point near to the charge may increase if the charge is positive.

Statement-II : An electric dipole is placed in a non-uniform electric field. The net electric force on the dipole will not be zero.

Choose the correct answer from the options given below :

(A) Both statement-I and statement-II are true.

(B) Both statement-I and statement-I are false.

(C) Statement-I is true but statement-II is false.

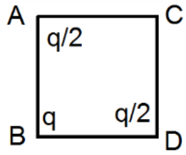
(D) Statement-I is false but statement-II is true.

Official Ans. by NTA (A)

Sol. If the electric field is in the positive direction and the positive charge is to the left of that point then the electric field will increase. But to the left of the positive charge the electric field would decrease.

If the dipole is kept at the point where the electric field is maximum then the force on it will be zero.

9. The three charges q/2, q and q/2 are placed at the corners A, B and C of a square of side 'a' as shown in figure. The magnitude of electric field (E) at the comer D of the square, is :



(A) $\dfrac{q}{4\pi \in_0 a^2}\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}\right)$

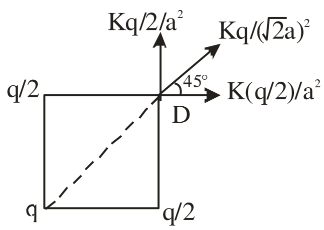
(B) $\dfrac{q}{4\pi \in_0 a^2}\left(1+\dfrac{1}{\sqrt{2}}\right)$

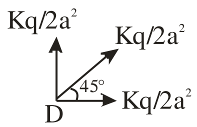
(C) $\dfrac{q}{4\pi \in_0 a^2}\left(1-\dfrac{1}{\sqrt{2}}\right)$

(D) $\dfrac{q}{4\pi \in_0 a^2}\left(\dfrac{1}{\sqrt{2}}-\dfrac{1}{2}\right)$

Official Ans. by NTA (A)

Sol.



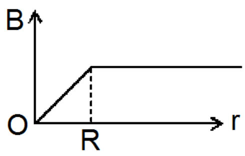


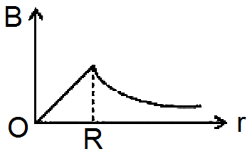
$$(E_{net})_D = \frac{kq}{2a^2} + \frac{\sqrt{2}kq}{2a^2}$$

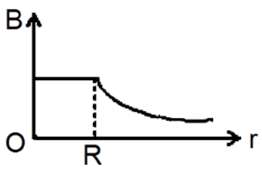
$$(E_{net})_D = \frac{kq}{a^2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$$

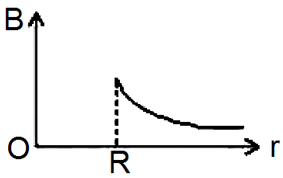
$$(E_{net})_D = \frac{q}{4\pi \in_0 a^2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$$

10. An infinitely long hollow conducting cylinder with radius R carries a uniform current along its surface. Choose the correct representation of magnetic field (B) as a function of radial distance (r) from the axis of cylinder.

(A) 

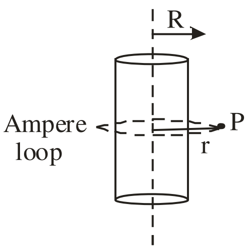
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

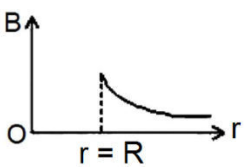
Sol.



1) $r < R, B_p = 0$

2) $r \geq R, B_p = \dfrac{\mu_0 I}{2\pi r}$

$B_p \propto \dfrac{1}{r}$



11. A radar sends an electromagnetic signal of electric field (E_0) = 2.25 V/m and magnetic field (B_0) = 1.5 x 10^{-8} T which strikes a target on line of sight at a distance of 3 km in a medium. After that, a pail of signal (echo) reflects back towards the radar vit1i same velocity and by same path. If the signal was transmitted at time t_0 from radar. then after how much time echo will reach to the radar?

(A) 2.0×10^{-5} s

(B) 4.0×10^{-5} s

(C) 1.0×10^{-5} s

(D) 8.0×10^{-5} s

Official Ans. by NTA (B)

Sol. $C = \dfrac{E_0}{B_0} = \dfrac{2.25}{1.5 \times 10^{-8}} = 1.5 \times 10^{8}\,ms^{-1}$

$t = \dfrac{6 \times 10^3}{1.5 \times 10^8} = 4 \times 10^{-5}\,s$

12. The refracting angle of a prism is A and refractive index of the material of the prism is cot (A/2). Then the angle of minimum deviation will be -

(A) 180 – 2A (B) 90 – A

(C) 180 + 2A (D) 180 – 3A

Official Ans. by NTA (A)

Sol. $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

$\mu = \cot\dfrac{A}{2}$

$\Rightarrow \quad \sin\left(\dfrac{A+\delta_m}{2}\right) = \cos\dfrac{A}{2}$

$\delta_m = 180 - 2A$

13. The aperture of the objective is 24.4 cm. The resolving power of this telescope. If a light of wavelength 2440 Å is used to see the object will be

(A) 8.1×10^{6} (B) 10.0×10^{7}

(C) 8.2×10^{5} (D) 1.0×10^{-8}

Official Ans. by NTA (C)

Sol. $R.P = \frac{d}{1.22\lambda} = \frac{24.4\times10^{-2}}{1.22\times2440\times10^{-10}} = 8.2\times10^{5}$

14. The de Brogue wavelengths for an electron and a photon are λ_e and λ_p respectively. For the same kinetic energy of electron and photon. which of the following presents the correct relation between the de Brogue wavelengths of two ?

(A) $\lambda_p \propto \lambda_e^2$ (B) $\lambda_p \propto \lambda_e$

(C) $\lambda_p \propto \sqrt{\lambda_e}$ (D) $\lambda_p \propto \sqrt{\frac{1}{\lambda_e}}$

Official Ans. by NTA (A)

Sol. $\lambda_e = \frac{h}{\sqrt{2mk}}$

Also for photon $k = \frac{hc}{\lambda_p}$

$\lambda_e = \frac{h\sqrt{\lambda_p}}{\sqrt{2m\,hc}}$

$\lambda_p \propto \lambda e^2$

15. The Q-value of a nuclear reaction and kinetic energy of the projectile particle, K_p are related as :

(A) $Q = K_p$ (B) $(K_p + Q) < O$

(C) $Q < K_p$ (D) $(K_p + Q) > 0$

Official Ans. by NTA (D)

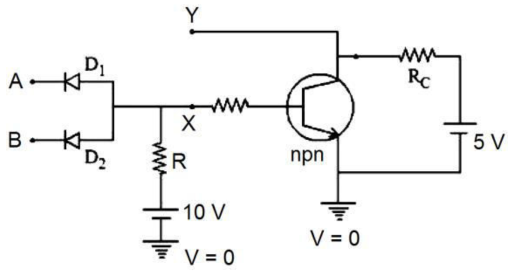
Sol. $x + p \rightarrow \gamma + b$

$Q = k_\gamma + k_b - k_p$

$Q + k_p = k_\gamma + k_b$

$Q + k_p > 0$

16. In the following circuit, the correct relation between output (Y) and inputs A and B will be :



(A) $Y = AB$ (B) $Y = A + B$

(C) $Y = \overline{AB}$ (D) $Y = \overline{A+B}$

Official Ans. by NTA (C)

Sol. This is NAND gate

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

17. For using a multimeter to identify diode from electrical components. choose the correct statement out of the following about the diode :

(A) It is two terminal device which conducts current in both directions.

(B) It is two terminal device which conducts current in one direction only

(C) It does not conduct current gives an initial deflection which decays to zero.

(D) It is three terminal device which conducts current in ne direction only between central terminal and either of the remaining two terminals

Official Ans. by NTA (B)

Sol. In forward bias diode conducts

In revers bias it does not conducts.

18. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : n-p-n transistor permits more current than a p-n-p transistor.

Reason R : Electrons have greater mobility as a charge carrier.

Choose the correct answer from the options given below :

(A) Both A and R true. and R is correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (A)

Sol. Theory

19. Match List-I with List-II

	List-I		List-II
A	Television signal	I	03 KHz
B	Radio signal	II	20 KHz
C	High Quality Music	III	02 MHz
D	Human speech	IV	06 MHz

Choose the correct answer from the options given below :

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-III, C-I, D-II

(C) A-IV, B-III, C-II, D-I

(D) A-I, B-II, C-IV, D-III

Official Ans. by NTA (C)

Sol. Theory

20. The velocity of sound in a gas. in which two wavelengths 4.08m and 4.16m produce 40 beats in 12s, will be :

(A)2.82.8 ms^{-1} (B) 175.5 ms^{-1}

(C) 353.6 ms^{-1} (D) 707.2 ms^{-1}

Official Ans. by NTA (D)

Sol. $f_b = f_1 - f_2$

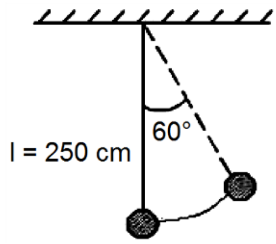
$$\frac{v}{4.08} - \frac{v}{4.16} = \frac{40}{12}$$

$$\Rightarrow v = 707.2$$

SECTION - B

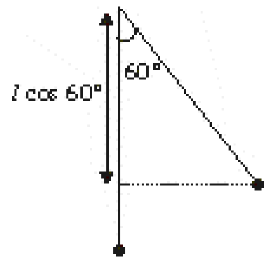
1. A pendulum is suspended by a string of length 250 cm. The mass of the bob of the pendulum is 200 g. The bob is pulled aside until the string is at 60° with vertical as shown in the figure. After releasing the bob. the maximum velocity attained by the bob will be ____________ ms^{-1}.

(if $g = 10\ m/s^2$)



Official Ans. by NTA (5)

Sol. $V_{max} = \sqrt{2gh}$

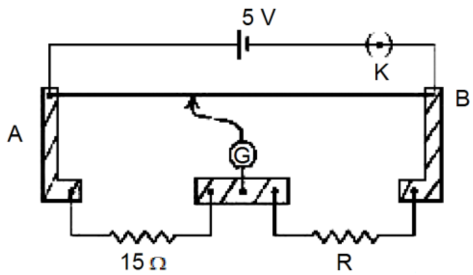


The speed will be highest at the lowest position.

$$h = (\ell - \ell\cos 60°) = \frac{\ell}{2}$$

$$V_{max} = \sqrt{2\times g\times\frac{\ell}{2}} = \sqrt{10\times 2.5} = 5\,m/s$$

2. A meter bridge setup is shown in the figure. It is used to determine an unknown resistance R using a given resistor of 1 5 Ω. The galvanometer (G) shows null deflection when tapping key is at 43 cm mark from end A. If the end correction for end A is 2 cm. then the determined value of R will be _______ Ω.



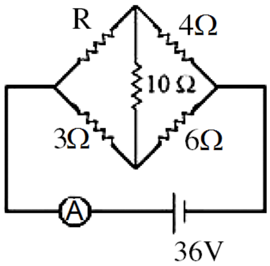
Official Ans. by NTA (19)

Sol. Using the conditions of a balanced wheat stone bridge and adding the end correction.

$$\frac{15}{(43+2)}=\frac{R}{(102-45)}\Rightarrow R=\frac{57}{45}\times 15$$

$R=19\Omega$

3. Current measured by the ammeter Ⓐ in the reported circuit when no current flows through 10 Ω resistance. will be _________ A.



Official Ans. by NTA (10)

Sol. Using the condition of a balanced wheat stone bridge,

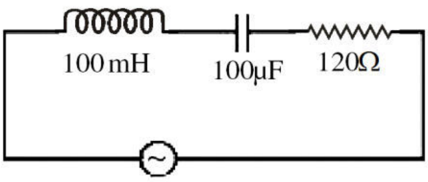
$$\Rightarrow \frac{R}{3}=\frac{4}{6}\Rightarrow R=2\Omega$$

So the effective resistance of the circuit is

$$R_{eq}=\frac{6\times 9}{6+9}=\frac{18}{5}\Omega$$

$$i=\frac{36}{R_{eq}}=10A$$

4. An AC source is connected to an inductance of 100 mH, a capacitance of 100 μ F and a resistance of 120 Ω as shown in figure. The time in which the resistance having a thermal capacity 2 J/°C will get heated by 16°C is _______s.



Official Ans. by NTA (15)

Sol. $|(X_L-X_C)|=|10-10^2|=90\Omega$

Z = Impendance

$$=\sqrt{(X_L-X_C)^2+R^2}=\sqrt{(90)^2+(20)^2}=150\Omega$$

$$i_{rms}=\frac{V_{rms}}{z}=\left(\frac{2}{15}\right)A$$

Now $i_{rms}^2 R\,\Delta t=ms(\Delta T)$

$\Rightarrow\ \Delta t=15\,\text{sec}$

5. The position vector of 1 kg object is $\vec{r}=(3\hat{i}-\hat{j})m$ and its velocity $\vec{v}=(3\hat{j}+k)ms^{-1}$. The magnitude of its angular momentum is $\sqrt{x}$ Nm where x is ______ .

Official Ans. by NTA (91)

Sol. Using $\vec{L}=\vec{r}\times\vec{p}=\vec{r}\times m\vec{v}, m=1kg$

$$\vec{L}=(3\hat{i}-\hat{j})\times(3\hat{j}+\hat{k})=(9\hat{k}-3\hat{j}-\hat{i})N-s$$

$$\Rightarrow|\vec{L}|=\sqrt{91}\,N-s$$

6. A man of 60 kg is running on the road and suddenly jumps into a stationary trolly car of mass 120 kg. Then. the trolly car starts moving with velocity 2 ms^{-1}. The velocity of the running man was _______________ ms^{-1}. when he jumps into the car.

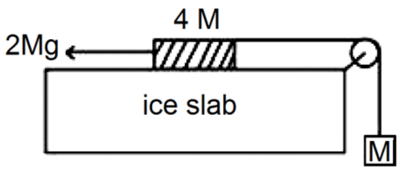
Official Ans. by NTA (6)

Sol. Taking the system as man and trolley and using conservation of linear momentum.

60 × v = (60 + 120) × 2

$\Rightarrow$ v = 6 m/s

7. A hanging mass M is connected to a four times bigger mass by using a string-pulley arrangement. as shown in the figure. The bigger mass is placed on a horizontal ice-slab and being pulled by 2 Mg force. In this situation. tension in the string is $\frac{x}{5}$ Mg for x = _________. Neglect mass of the string and friction of the block (bigger mass) with ice slab. (Given g = acceleration due to gravity)



Official Ans. by NTA (6)

Sol. Using $\vec{F}_{net} = \mu\vec{a}$,

$$2Mg - T = 4Ma$$
$$T - Mg = Ma$$
$$\Rightarrow a = \frac{g}{5}$$

$$T = Mg + Ma = Mg + \frac{Mg}{5} = \frac{6}{5}Mg$$

8. The total internal energy of two mole monoatomic ideal gas at temperature T = 300 K will be J. (Given R = 8.31 J/mol.K)

Official Ans. by NTA (7479)

Sol. $U = nC_vT$

$$= 2 \times \frac{3}{2}R \times 300$$

$$= 900R = 900 \times 8 \cdot 31 = 7479 \text{ J}$$

9. A sing1y ionized magnesium atom (A24) ion is accelerated to kinetic energy 5 keV and is projected perpendicularly into a magnetic field B of the magnitude 0.5 T. The radius of path formed will be _____________ cm.

Official Ans. by NTA (10)

Sol. $R = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

10. A telegraph line of length loo km has a capacity of 0.01 μF/km and it carries an alternating current at 0.5 kilo cycle per second. If minimum impedance is required, then the value of the inductance that needs to be introduced in series is ________ mH. $\left(\text{if } \pi = \sqrt{10}\right)$

Official Ans. by NTA (100)

Sol. For minimum impedance

$$X_L = X_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C} = 10^{-1}\text{H} = 100\,\text{mH}$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. The incorrect statement about the imperfections in solids is :

(A) Schottky defect decreases the density of the substance.

(B) Interstitial defect increases the density of the substance.

(C) Frenkel defect does not alter the density of the substance.

(D) Vacancy defect increases the density of the substance.

Official Ans. by NTA (D)

Sol. Due to vacancy defect density of the substance will decrease.

.

2. The Zeta potential is related to which property of colloids"

(A) Colour

(B) Tyndall effect

(C) Charge on the surface of colloidal particles

(D) Brownian movement

Official Ans. by NTA (C)

Sol. The potential difference between the fixed and diffused layer of charges in a colloidal particle is called zeta potential

3. Element "E" belongs to the period 4 and group 16 of the periodic table. The valence shell electron configuration of the element, which is just above 'E' in the group is

(A) $3s^2$. $3p^4$ (B) $3d^{10}$. $4s^2$, $4p^4$

(C) $4d^{10}$. $5s^2$, $5p^4$ (D) $2s^2$, p4

Official Ans. by NTA (A)

Sol. $E \Rightarrow [Ar]\ 3d^{10}\ 4s^2\ 4p^4$

Element above $E \Rightarrow [Ne]\ 3s^2\ 3p^4$

4. Given are two statements one is labelled as Assertion A and other is labelled as Reason R.
Assertion A :Magnesium can reduce Al_2O_3 at a temperature below 1350°C, while above 1350°C aluminium can reduce MgO.
Reason R : The melting and boiling points of magnesium are lower than those of aluminium.
In light of the above statements. choose most appropriate answer from the options given below:
(A) Both A and R are correct. and R is correct explanation of A.
(B) Both A and R are correct. but R is NOT the correct explanation of A.
(C) A is correct R is not correct.
(D) A is not correct. R is correct.
Official Ans. by NTA (B)

Sol. From Ellingham diagram given in NCERT, it can be seen that Mg, MgO line crosses Al, Al_2O_3 line after 1350ºC hence assertion is true.

Yes, Mg have lower MP and BP than aluminium but it does not explain the above fact.

5. Dihydrogen reacts with CuO to give
(A) CuH_2
(B) Cu
(C) Cu_2O
(D) $Cu(OH)_2$
Official Ans. by NTA (B)

Sol. $CuO + H_2 \rightarrow Cu + H_2O$ (under hot conditions)

6. Nitrogen gas is obtained by thermal decomposition of
(A) $Ba(NO_3)_2$ (B) $Ba(N_3)_2$
(C) $NaNO_2$ (D) $NaNO_3$
Official Ans. by NTA (B)

Sol. $Ba(N_3)_2 \rightarrow Ba + 3N_2$

7. Given below are two statements :
Statement -I :The pentavalent oxide of group- 15 element. E_2O_5. is less acidic than trivalent oxide. E_2O_3. of the same element.
Statement -II :The acidic character of trivalent oxide of group 15 elements. E_2O_3. decreases down the group.
In light of the above statements. choose most appropriate answer from the options given below:
(A) Both Statement I and Statement II are true.
(B) Both Statement I and Statement II are false.
(C) Statement I true. but statement II is false.
(D) Statement I is false but statement II is true.
Official Ans. by NTA (D)

Sol. As +ve oxidation state increases, EN of element increases hence acidic character increases. Down the group, non-metallic character decreases, acidic character decreases.

Acidic character : $E_2O_5 > E_2O_3$

Down the group, acidic character of E_2O_3 decreases

8. Which one of the lanthanoids given below is the most stable in divalent form?
(A) Ce (Atomic Number 58)
(B) Sm (Atomic Number 62)
(C) Eu (Atomic Number 63)
(D) Yb (Atomic Number 70)
Official Ans. by NTA (C)

Sol. $E^o_{M^{3+}/M^{2+}} \Rightarrow \begin{matrix} \text{Eu} & \text{Yb} \\ -0.35 & -1.05 \end{matrix}$

Hence, due to more reduction potential in Eu as compared to Yb, it can concluded that Eu^{2+} is more stable than Yb^{2+}.

9. Given below are two statements :
Statement I : $[Ni(CN)4]^{2-}$ is square planar and diamagnetic complex. with dsp^2 hybridization for Ni but $[Ni(CO)_4]$ is tetrahedral. paramagnetic and with sp^3-hybridication for Ni.
Statement II: $[NiCl_4]^{2-}$ and $[Ni(CO)_4]$ both have same d-electron configuration have same geometry and are paramagnetic.
In light the above statements. choose the correct answer form the options given below:
(A) Both Statement I and Statement II are true.
(B) Both Statement I and Statement II are false.
(C) Statement I is correct but statement II is false.
(D) Statement I is incorrect but statement II is true.
Official Ans. by NTA (B)

Sol. $[Ni(CN)_4]^{2-}$: d^8 configuration, SFL, sq. planar splitting (dsp^2), diamagnetic.

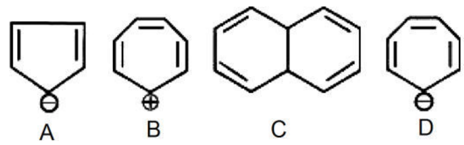
$[Ni(CO)_4]$: d^{10} config (after excitation), SFL, tetrahedral splitting (sp^3), diamagnetic.

$[NiCl_4]^{2-}$: d^8 config, WFL, tetrahedral splitting (sp^3), paramagnetic(2 unpaired e^-).

10. Which amongst the following is not a pesticide ?
(A) DDT
(B) Organophosphates
(C) Dieldrin
(D) Sodium arsenite
Official Ans. by NTA (D)

11. Which one of the following techniques is not used to spot components of a mixture separated on thin layer chromatographic plate?
(A) I_2 (Solid)
(B) U.V. Light
(C) Visualisation agent as a component of mobile phase
(D) Spraying of an appropriate reagent
Official Ans. by NTA (C)

12. Which of the following structures are aromatic in nature?

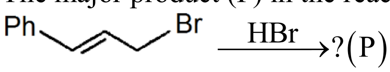


(A) A,B,C and D
(B) Only A and B
(C) Only A and C
(D) Only B, C and D
Official Ans. by NTA (B)

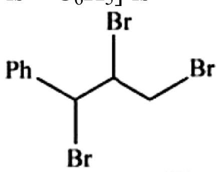
Sol. A, B aromatic

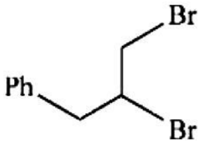
C,D is nonaromatic

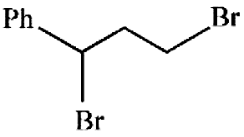
13. The major product (P) in the reaction

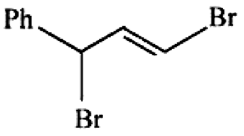


[Ph is – C_6H_5] is

(A) 

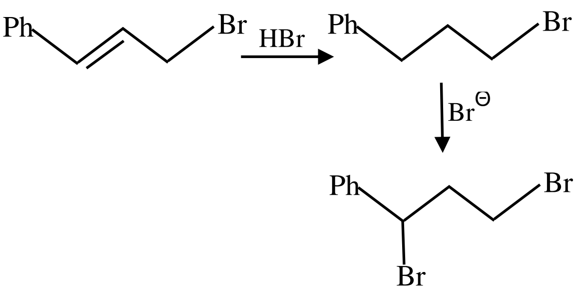
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

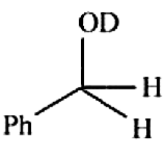
Sol.

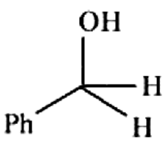


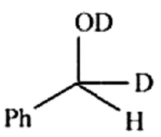
14. The correct structure of product 'A' formed in the following reaction.

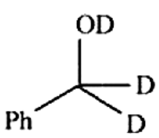
$$PhCHO + Ph\cdot CHO \xrightarrow[\text{in } D_2O]{\text{NaOD}} A + Ph-\overset{O}{\overset{\|}{C}}-O^-$$

$(Ph \text{ is} - C_6H_5)$

(A) 

(B) 

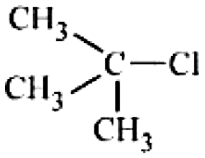
(C) 

(D) 

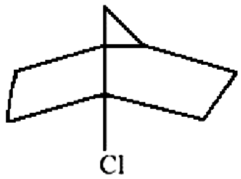
Official Ans. by NTA (A)

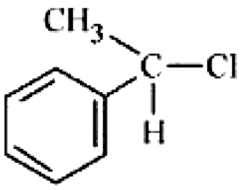
Sol. $PhCH = O + PhCH = O \xrightarrow{OD/D_2O} PhCH_2OD + PhCO_2^-$

15. Which one of the following compounds is inactive towards S_N1 reaction?

(A) 

(B) 

(C) 

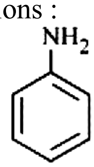
(D) 

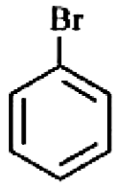
Official Ans. by NTA (C)

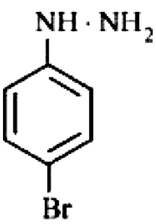
Sol. **Sol.** The carbocation fromed is very unstable.

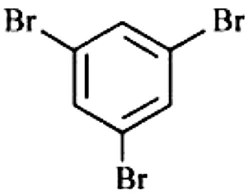
So it is inactive towards S_N1

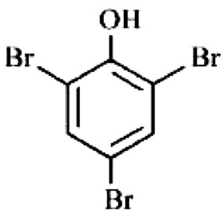
16. Identify the major product formed in the following sequence of reactions :



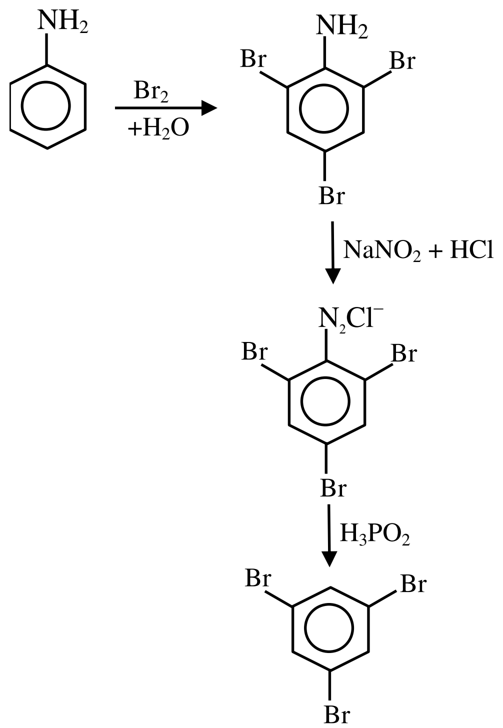
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (C)



Sol.

17. A primary aliphatic amine on reaction with nitrous acid in cold (273 K) and there after raising temperature of reaction mixture to room temperature (298 K). Gives a/an

(A) nitrile (B) alcohol

(C) diazonium salt (D) secondary amine

Official Ans. by NTA (B)

Sol. $R-NH_2 \xrightarrow[+HCl]{NaNO_2} R-N_2^+ \rightarrow R^+ \xrightarrow[H_2O]{} R-OH$

18. Which one of the following is **NOT** a copolymer ?

(A) Buna-S (B) Neoprene

(C) PHBV (D) Butadiene-styrene

Official Ans. by NTA (B)

Sol. Buna-S, PHBr and Butadiene-styrene are copolymer. Only neoprene is namopolymer.

19. Stability of α - Helix structure of proteins depends upon

(A) dipolar interaction

(B) H-bonding interaction

(C) van der Waals forces

(D) π -stacking interaction

Official Ans. by NTA (B)

20. The formula of the purple colour formed in Laissaigne's test for sulphur using sodium nitroprusside is

(A) $NaFe[Fe(CN)_6]$ (B) $Na[Cr(NH_3)_2(NCS)_4]$

(C) $Na_2[Fe(CN)_5(NO)]$ (D) $Na_4[Fe(CN)_5(NOS)]$

Official Ans. by NTA (D)

Sol. $Na_2S + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5NO_5]$

SECTION-B

1. A 2.0 g sample containing MnO_2 is treated with HCl liberating Cl_2. The Cl_2 gas is passed into a solution of KI and 60.0 mL of 0.1 M $Na_2S_2O_3$ is required to titrate the liberated iodine. The percentage of MnO_2 in the sample is ________. (Nearest integer)

[Atomic masses (in u) Mn = 55; Cl = 35.5; O = 16, I = 127, Na = 23, K = 39, S = 32]

Official Ans. by NTA (13)

Sol.

$MnO_2 + HCl \longrightarrow Cl_2 + Mn^{+2}$

6 meq 6 meq

$= 3$ m mol

$Cl_2 + KI \longrightarrow Cl^- + I_2$

6 meq 6 meq

$I_2 + Na_2S_2O_3 \longrightarrow I^- + Na_2S_4O_6$

6 meq 6 m mol

$= 6$ meq

$$\%MnO_2 = \frac{3\times10^{-3}\times87}{2}\times100$$

= 13.05%

Ans. 13

2. If the work function of a metal is 6.63×10^{-19} J, the maximum wavelength of the photon required to remove a photoelectron from the metal is ________ nm. (Nearest integer)

[Given : $h = 6.63 \times 10^{-34}$ J s, and $c = 3 \times 10^8$ m s^{-1}]

Official Ans. by NTA (300)

Sol. $\phi = 6.63 \times 10^{-19} J = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{\lambda}$

$\Rightarrow \lambda = 3 \times 10^{-7} m = 300$ nm

3. The hybridization of P exhibited in PF_5 is sp^xd^y. The value of y is ________.

Official Ans. by NTA (1)

Sol. $PF_5 \Rightarrow sp^3d$ hybridisation

(5 sigma bonds, zero lone pair on central atom)

Value of y = 1

4. 4.0 L of an ideal gas is allowed to expand isothermally into vacuum until the total volume is 20 L. The amount of heat absorbed in this expansion is ________ L atm.

Official Ans. by NTA (0)

Sol. For free expansion: $P_{ext} = 0$, $w = 0$

$q = 0$, $\Delta U = 0$

Ans. 0

5. The vapour pressures of two volatile liquids A and B at 25°C are 50 Torr and 100 Torr, respectively. If the liquid mixture contains 0.3 mole fraction of A, then the mole fraction of liquid B in the vapour phase is $\frac{x}{17}$. The value of x is ________.

Official Ans. by NTA (14)

Sol. $\frac{y_B}{1-y_B} = \frac{P_B^o}{P_A^o}\left[\frac{X_B}{1-X_B}\right]$

$\Rightarrow \frac{y_B}{1-y_B} = \frac{100}{50}\left[\frac{0.7}{0.3}\right] = \frac{14}{3}$

$\Rightarrow y_B = \frac{14}{17}$

Ans. 14

6. The solubility product of a sparingly soluble salt A_2X_3 is 1.1×10^{-23}. If specific conductance of the solution is 3×10^{-5} S m^{-1}, the limiting molar conductivity of the solution is $x \times 10^{-3}$ S m^2 mol^{-1}. The value of x is ________.

Official Ans. by NTA (3)

$$A_2X_{3(s)} \rightleftharpoons 2A^{+3}_{(aq)} + 3X^{-2}_{(aq)}$$

solubility = sM   2s   3s

$(2s)^2(3s)^3 = 1.1 \times 10^{-23}$

$108\ s^5 = 1.1\times10^{-23}$

$s \simeq 10^{-5} M = 10^{-5}\frac{mol}{L} = 0.01\frac{mol}{m^3}$

Now $\wedge_m \simeq \wedge_m^{\infty} = \frac{k}{m} = \frac{k}{s}$

$\Rightarrow \wedge_m^{\infty} = \frac{3\times10^{-5}}{0.01} = 3\times10^{-3}$ S-m^2/mol

Ans. 3

7. The quantity of electricity in Faraday needed to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} is ________.

Official Ans. by NTA (6)

Sol. $Cr_2O_7^{-2} + 6e^- \longrightarrow 2Cr^{+3}$

1mol   6mol

$\Rightarrow$ number of faradays = moles of electrons = 6

8. For a first order reaction $A \rightarrow B$, the rate constant, $k = 5.5\times10^{-14} s^{-1}$. The time required for 67% completion of reaction is $x \times 10^{-1}$ times the half life of reaction. The value of x is ________ (Nearest integer)

(Given : log 3 = 0.4771)

Official Ans. by NTA (16)

Sol. $t_{67\%} = \frac{1}{k}\ln\left(\frac{1}{1-0.67}\right) = \frac{t_{1/2}}{\ln 2}\times\ln\left(\frac{1}{1-\frac{2}{3}}\right)$

$t_{67\%} = \frac{t_{1/2}}{\log 2}\times\log 3 = \frac{t_{1/2}\times0.4771}{0.301}$

$\Rightarrow t_{67\%} = 1.585 \times t_{1/2}$

$X \times 10^{-1} = 1.585$

$\Rightarrow X = 15.85$

Ans.16

9. Number of complexes which will exhibit synergic bonding amongst, $[Cr(CO)_6]$, $[Mn(CO)_5]$ and $[Mn_2(CO)_{10}]$ is ________.

Official Ans. by NTA (3)

Sol. Carbonyl complex compounds have tendency to show synergic bonding.

10. In the estimation of bromine, 0.5 g of an organic compound gave 0.40 g of silver bromide. The percentage of bromine in the given compound is _________% (nearest integer)

(Relative atomic masses of Ag and Br are 108u and 80u, respectively).

Official Ans. by NTA (34)

Sol

$$\underset{0.5\,g}{O.C} \longrightarrow \underset{0.4\,g}{AgBr}$$

$$\text{mol of Br} = \text{mol of AgBr} = \frac{0.4}{188}$$

$$\%\,Br = \%Br = \frac{\frac{0.4}{188}\times 80}{0.5}\times 100$$

$= 34.04\%$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If

$$\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)},$$

Where $\alpha \in R$, then the value of 16α is equal to

(A) 1411 (B) 1320

(C) 1615 (D) 1855

Official Ans. by NTA (A)

Sol. $\sum_{R=1}^{31} {}^{31}C_R \cdot {}^{31}C_{R-1}$

$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + ... + {}^{31}C_{31} \cdot {}^{31}C_{30}$

$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + ... + {}^{31}C_{30} \cdot {}^{31}C_0$

$= {}^{62}C_{30}$.

Similarly

$$\sum_{R=1}^{30}\left({}^{30}C_R \cdot {}^{30}C_{R-1}\right)={}^{60}C_{29}$$

$$ {}^{62}C_{30}-{}^{60}C_{29}=\frac{62!}{30!32!}-\frac{60!}{29!31!}$$

$$=\frac{60!}{29!\,31!}\left\{\frac{62\cdot 61}{30\cdot 32}-1\right\}$$

$$=\frac{60!}{30!31!}\left(\frac{2822}{32}\right)$$

$$\therefore\ 16\alpha=16\times\frac{2822}{32}=1411$$

2. Let a function $f:\mathbb{N}\to\mathbb{N}$ be defined by

$$f(n)=\begin{cases}2n, & n=2,4,6,8,.....\\ n-1, & n=3,7,11,15,.....\\ \frac{n+1}{2}, & n=1,5,9,13,.....\end{cases}$$

then, f is

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) one-one and onto

Official Ans. by NTA (D)

Sol. $f(x)=\begin{cases}4R & ; \quad n=2R\\ 4R-2 & ; \quad n=4R-1\\ 2R-1 & ; \quad n=4R-3\end{cases}$

$(R\in N)$

Note that for any element, it will fall into exactly. one of these sets.

$\{y : y = 4R; y \in N\}$

$\{y : y = 4R-2; y \in N\}$

$\{y : y = 2R-1; y \in N\}$

Corresponding to that y, we will get exactly one value of n.

Thus, f is one – one & onto.

3. If the system of linear equations

$2x + 3y – z = – 2$

$x + y + z = 4$

$x-y+|\lambda|z=4\lambda-4$

where $\lambda\in\mathbb{R}$, has no solution, then

(A) $\lambda=7$ (B) $\lambda=-7$

(C) $\lambda=8$ (D) $\lambda^2=1$

Official Ans. by NTA (B)

Sol. $\begin{vmatrix}2 & 3 & -1\\ 1 & 1 & 1\\ 1 & -1 & |\lambda|\end{vmatrix}=0$

$\Rightarrow |\lambda|=7\Rightarrow \lambda=\pm 7$ ...(1)

System :

$2x+3y-z=-2$ ...(2)

$x+y+z=4$ ...(3)

$x-y+|\lambda|z=4\lambda-4$ ...(4)

Eliminating y from equal (2) & (3) we get

$x+4z=14$ ...(5)

$(3)+(4)\Rightarrow x+\left(\frac{|\lambda|+1}{2}\right)z=2\lambda$ ...(6)

Clearly for $\lambda=-7$, system is inconsistent.

4. Let A be a matrix of order 3 × 3 and det (A) = 2. Then det (det (A) adj (5 adj (A^3))) is equal to ____.

(A) 512×10^6 (B) 256×10^6

(C) 1024×10^6 (D) 256×10^{11}

Official Ans. by NTA (A)

Sol. |(det (A)) adj (5 adj (A))|

$= |2\text{adj}(5\text{adj}(A^3))|$

$= 2^3\, |\text{adj}(5\text{ adj}(A^3))|$

$= 2^3 .\, |5\text{adj}(A^3)|^2$

$= 2^3\, (5^3.|\text{adj}(A^3)|)^2$

$= 2^3.\, 5^6.\, |\text{adj}A^3|^2$

$= 2^3\, .\, 5^6\, ((|A|^3)^2)^2$

$= 2^3.5^6.2^{12} = 2^{15} \times 5^6$

$= 2^9 \times 10^6$

$= 512 \times 10^6.$

5. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

(A) 36 (B) 48

(C) 60 (D) 72

Official Ans. by NTA (D)

Sol. To make a no. divisible by 3 we can use the digits 1,2,5,6,7 or 1,2,3,5,7.

Using 1,2,5,6,7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 2 = 48$

Using 1,2,3,5,7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 1 = 24$

Required answer is 72.

6. Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

(A) 33 (B) 37

(C) 43 (D) 47

Official Ans. by NTA (C)

Sol. $A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$

$(A_4)^4 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$ …(1)

$A_2 + A_4 = \frac{7}{36}$

$A_2 = \frac{1}{36}$ …(2)

$A_6 = 1$

$A_8 = 6$

$A_{10} = 36$

$A_6 + A_8 + A_{10} = 43$

7. Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [-8x^2 + 6x - 1]dx$ is equal to

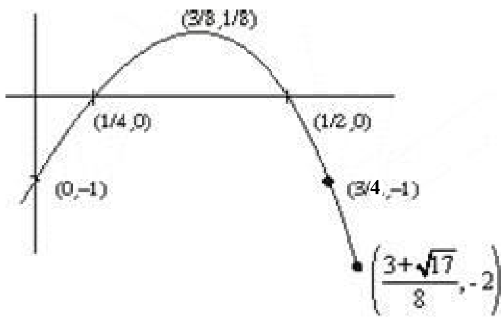
(A) -1 (B) $-\frac{5}{4}$

(C) $\frac{\sqrt{17}-13}{8}$ (D) $\frac{\sqrt{17}-16}{8}$

Official Ans. by NTA (C)

Sol. $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$

$= \int_0^{1/4} -1\,dx + \int_{1/4}^{1/2} 0\,dx + \int_{1/2}^{3/4} -1\,dx$



$+ \int_{3/4}^{\frac{3+\sqrt{17}}{8}} -2\,dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3\,dx$

$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$

$$= -\left(\frac{1}{4}-0\right)-\left(\frac{3}{4}-\frac{1}{2}\right)-2\left(\frac{3+\sqrt{17}}{8}-\frac{3}{4}\right)-3\left(1-\frac{3+\sqrt{17}}{8}\right)$$

$$= -\frac{1}{4}-\frac{1}{4}+\frac{-6-2\sqrt{17}}{8}+\frac{3}{2}-3+\frac{9+3\sqrt{17}}{8}$$

$$= \frac{\sqrt{17}-13}{8}$$

8. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as

$$f(x)=\begin{cases}[e^x], & x<0\\ ae^x+[x-1], & 0\le x<1\\ b+[\sin(\pi x)], & 1\le x<2\\ [e^{-x}]-c, & x\ge 2\end{cases}$$

where $a,b,c\in\mathbb{R}$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true ?

(A) There exists $a,b,c\in\mathbb{R}$ such that f is continuous of $\mathbb{R}$.

(B) If f is discontinuous at exactly one point, then $a + b + c = 1$.

(C) If f is discontinuous at exactly one point, then $a+b+c\neq 1$.

(D) f is discontinuous at atleast two points, for any values of a, b and c.

Official Ans. by NTA (C)

Sol. $f(x)$ is discontinuous at $x = 1$

For continuous at $x = 0$; $a = 1$

For continuous at $x = 2$; $b + c = 1$

$a + b + c = 2$

9. The area of the region

$S=\{(x,y):y^2\le 8x, y\ge\sqrt{2}x, x\ge 1\}$ is

(A) $\frac{13\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

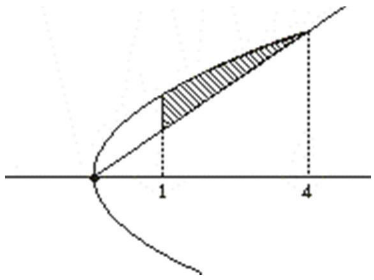
(C) $\frac{5\sqrt{2}}{6}$ (D) $\frac{19\sqrt{2}}{6}$

Official Ans. by NTA (B)

Sol. $y^2=8x$ ...(1)

$y=\sqrt{2}x$ ...(2)

$y^2=2x^2$



$\Rightarrow 8x = 2x^2$

$\Rightarrow x = 0 \ \& \ 4$

$$\text{Area}: = \int_1^4 2\sqrt{2}\sqrt{x}-\sqrt{2}x\,dx$$

$$= 2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{3/2}\right)_1^4-\sqrt{2}\left(\frac{x^2}{2}\right)_1^4$$

$$= \frac{4\sqrt{2}}{3}(8-1)-\frac{\sqrt{2}}{3}(16-1)$$

$$= \frac{28\sqrt{2}}{3}-\frac{15\sqrt{2}}{2}=\frac{11\sqrt{2}}{6}$$

10. Let the solution curve $y = y(x)$ of the differential equation,

$$\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]x\frac{dy}{dx}=x+\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]y$$

pass through the points (1, 0) and $(2\alpha,\alpha),\alpha>0$. Then α is equal to

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6}+\sqrt{e}-1\right)$ (B) $\frac{1}{2}\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

(C) $\exp\left(\frac{\pi}{6}+\sqrt{e}+1\right)$ (D) $2\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

Official Ans. by NTA (A)

Sol. $$\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)x\frac{dy}{dx}=x+\left(\frac{x}{\sqrt{x^2y^2}}+e^{\frac{y}{x}}\right)y$$

$$\Rightarrow \quad e^{\frac{y}{x}}(x\,dy-y\,dx)+\frac{x}{\sqrt{x^2-y^2}}(xdy-ydx)=xdx$$

Dividing both side by x^2

$$\Rightarrow \quad e^{\frac{y}{x}}\left(\frac{xdy-y\,dx}{x^2}\right)+\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}\left(\frac{xdy-ydx}{x^2}\right)=\frac{dx}{x}$$

$$\Rightarrow \quad e^{\frac{y}{x}}\mid d\left(\frac{t}{x}\right)+\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}d\left(\frac{y}{x}\right)=\frac{dy}{x}$$

Integrate both side.

$$\int e^{\frac{y}{x}}d\left(\frac{y}{x}\right)+\int\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}d\left(\frac{y}{x}\right)=\int\frac{dx}{x}$$

$$\Rightarrow e^{\frac{y}{x}}+\sin^{-1}\left(\frac{y}{x}\right)=\ln x+c$$

It passes through (1, 0)

$1 + 0 = 0 + c \Rightarrow c = 1$

It passes through $(2\alpha,\alpha)$

$$e^{\frac{1}{2}}+\sin^{-1}\frac{1}{2}=\ln 2\alpha+1$$

$$\Rightarrow \ln 2\alpha=\sqrt{e}+\frac{\pi}{6}-1$$

$$\Rightarrow 2\alpha=e^{\left(\sqrt{e}+\frac{\pi}{6}-1\right)}$$

$$\Rightarrow \alpha=\frac{1}{2}e^{\left(\frac{\pi}{6}+\sqrt{e}-1\right)}$$

11. Let y = y(x) be the solution of the differential equation $x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2y-y-4x^3\right)=0, x>1,$ with y(2) = –2. Then y(3) is equal to

(A) –18 (B) –12
(C) –6 (D) –3

Official Ans. by NTA (A)

Sol. $x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2y-y-4x^3\right)=0$

$$x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2-1\right)y=4x^3$$

$$\frac{dy}{dx}+\frac{\left(3x^2-1\right)}{\left(x-x^3\right)}y=\frac{4x^3}{\left(x-x^3\right)}$$

$$\frac{dy}{dx}+Py=Q$$

$$IF=e^{\int P\,dx}=e^{\int\frac{3x^2-1}{x-x^3}dx}$$

$$x-x^3=t \Rightarrow IF=e^{\int\frac{-dt}{t}}$$

$$=e^{-\ell nt}=\frac{1}{t}$$

$$\therefore IF=\frac{1}{x-x^3}$$

$$y\times IF=\int Q\times IF\,dx$$

$$y\left(\frac{1}{x-x^3}\right)=\int\frac{4x^3}{x-x^3}\times\frac{1}{\left(x-x^3\right)}dx$$

$$=\int\frac{4x^3}{\left(x-x^3\right)^2}dx$$

$$=\int\frac{4x}{\left(1-x^2\right)^2}dx \qquad 1-x^2=K$$

$$=2\int\frac{-dK}{K^2} \qquad -2xdx=dK$$

$$=-2\left(-\frac{1}{K}\right)+c$$

$$\frac{y}{x-x^3}=\frac{2}{K}+c$$

$$\frac{y}{x-x^3}=\frac{2}{1-x^2}+c$$

At x = 2 , y = – 2

$$\frac{-2}{2-8}=\frac{2}{1-4}+c$$

$$\frac{1}{3}=\frac{-2}{3}+c$$

$\therefore$ C = 1

$$\frac{y}{x-x^3}=\frac{2}{1-x^2}+1$$

Put x = 3

$$\frac{y}{3-27}=\frac{2}{1-9}+1$$

$$\frac{y}{-24}=-\frac{1}{4}+1$$

$$\frac{y}{-24} = \frac{3}{4}$$

$$y = \frac{3}{4}(-24) = -18$$

12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ________.

(A) 0 (B) 1

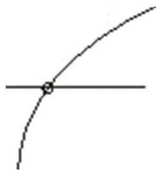
(C) 3 (D) 5

Official Ans. by NTA (B)

Sol. $f(x) = x^7 + 5x^3 + 3x + 1$

$f'(x) = 7x^6 + 15x^2 + 3 > 0$

$\therefore$ $f(x)$ is strictly increasing function



$x \to -\infty$, $y \to -\infty$

$x \to \infty$, $y \to \infty$

$\therefore$ no. of real solution = 1

13. Let the eccentricity of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If y = 2x + c is a tangent to the hyperbola H, then the value of c^2 is equal to

(A) 18 (B) 20

(C) 24 (D) 32

Official Ans. by NTA (B)

Sol. $y = mx \pm \sqrt{a^2m^2 - b^2}$

$m = 2,\ c^2 = a^2m^2 - b^2$

$c^2 = 4a^2 - b^2$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$\frac{5}{2} = 1 + \frac{b^2}{a^2}$$

$$\frac{3}{2} = \frac{b^2}{a^2} \Rightarrow b^2 = \frac{3a^2}{2}$$

$$\frac{2b^2}{a} = 6\sqrt{2}$$

$$\frac{2}{a} \times \frac{3a^2}{2} = 6\sqrt{2}$$

$3a = 6\sqrt{2}$

$\boxed{a = 2\sqrt{2}}$

$$b^2 = \frac{3}{2} \times 8 = 12$$

$b = 2\sqrt{3}$

$\therefore\ c^2 = 4 \times 8 - 12$

$c^2 = 20$

14. If the tangents drawn at the point O(0, 0) and $P(1+\sqrt{5}, 2)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$ (D) $\frac{7+3\sqrt{5}}{2}$

Official Ans. by NTA (C)

Sol. Tangent at O

$-(x + 0) - 2(y + 0) = 0$

$\Rightarrow \boxed{x + 2y = 0}$

Tangent at P

$x(1 + \sqrt{5}) + y.2 - (x + 1 + \sqrt{5}) - 2(y + 2 = 0)$

Put $x = -2y$

$-2y(1 + \sqrt{5}) + 2y + 2y - 1 - \sqrt{5} - 2y - 4 = 0$

$$-2\sqrt{5}y = 5 + \sqrt{5} \Rightarrow y = \left(\frac{\sqrt{5}+1}{2}\right)$$

$Q\left(\sqrt{5}+1, -\frac{\sqrt{5}+1}{2}\right)$

Length of tangent OQ = $\frac{5+\sqrt{5}}{2}$

Area = $\frac{RL^3}{R^2+L^2}$

$R=\sqrt{5}$

$$=\frac{\sqrt{5}\times\left(\frac{5+\sqrt{5}}{2}\right)^3}{5+\left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$$=\frac{\sqrt{5}}{2}\times\frac{4\times\left(125+75+75\sqrt{5}+5\sqrt{5}\right)}{\left(20+25+10\sqrt{5}+5\right)}$$

$$=\frac{5+3\sqrt{5}}{2}$$

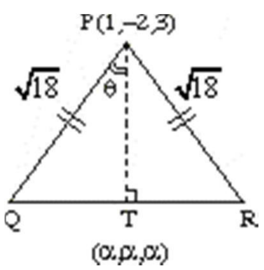
15. If two distinct point Q, R lie on the line of intersection of the planes –x + 2y – z = 0 and $3x-5y+2z=0$ and $PQ=PR=\sqrt{18}$ where the point P is (1, –2, 3), then the area of the triangle PQR is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

Official Ans. by NTA (B)

Sol.



$-x+2y-z=0$

$3x-5y+2z=0$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & -1\\ 3 & -5 & 2\end{vmatrix}$$

$$=\hat{i}(-1)-\hat{j}(1)+\hat{k}(-1)$$

$$\vec{n}=-\hat{i}-\hat{j}-\hat{k}$$

Equation of LOI is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

DR: of PT $\rightarrow \alpha-1, \alpha+2, \alpha-3$

DR: of QR $\rightarrow$ 1, 1, 1

$\Rightarrow (\alpha-1)\times1+(\alpha+2)\times1+(\alpha-3)\times1=0$

$3\alpha=2$

$$\boxed{\alpha=\frac{2}{3}}$$

$$PT^2=\frac{1}{9}+\frac{64}{9}+\frac{49}{9}$$

$$PT^2=\frac{114}{9}$$

$$\boxed{PT=\frac{\sqrt{114}}{3}}$$

$$\cos\theta=\frac{\sqrt{114}}{3}\times\frac{1}{3\sqrt{2}}=\frac{\sqrt{57}}{9}=\frac{\sqrt{19\times3}}{3\times3}$$

$$=\frac{\sqrt{19}}{3\sqrt{3}}$$

$$\cos2\theta=\frac{2\times19}{27}-1=\frac{11}{27}$$

$$\sin2\theta=\sqrt{1-\left(\frac{11}{27}\right)^2}=\frac{\sqrt{38}\sqrt{16}}{27}$$

$$=\frac{4}{27}\sqrt{38}$$

$$\text{Area }=\frac{1}{2}\times\sqrt{18}\sqrt{18}\times\frac{4}{27}\sqrt{38}$$

$$=\frac{18}{2}\times\frac{4}{27}\sqrt{38}=\frac{36}{27}\sqrt{38}\quad=\frac{4}{3}\sqrt{38}$$

16. The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes 5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the points (2, 1, 3) and (0, 1, 2), respectively, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Official Ans. by NTA (A)

Sol. Equation of plane passing through the intersection of planes 5x + 8y + 13z –29 = 0 and 8x – 7y + z – 20 = 0 is

$5x+8y+3z-29+\lambda(8x-7y+z-20)=0$ and

if it is passing through (2,1,3) then $\lambda=\frac{7}{2}$

P_1: Equation of plane through intersection of $5x+8y+13z-29=0$ and $8x-7y+z-20=0$ and the point (2, 1, 3) is

$$5x+8y+3z-29+\frac{7}{2}(8x-7y+z-20)=0$$

$\Rightarrow 2x-y+z=6$

Similarly P_2 : Equation of plane through intersection of

5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the point (0,1,2) is

$\Rightarrow x+y+2z=5$

Angle between planes $=\theta=\cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right)=\frac{\pi}{3}$

17. Let the plane $P:\vec{r}\cdot\vec{a}=d$ contain the line of intersection of two planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=6$ and $\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})=7$. If the plane P passes through the point $\left(2,3,\frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

(A) 90 (B) 93

(C) 95 (D) 97

Official Ans. by NTA (B)

Sol. Equation of plane passing through line of intersection of planes $P_1:\vec{r}\left((\hat{i}+3\hat{j}-\hat{k}\right)=6$ and $P_2:\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})=7$ is

$P_1+\lambda P_2=0$

$$\left(\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})-6\right)+\lambda\left(\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})-7\right)=0$$

and it passes through point $\left(2,3,\frac{1}{2}\right)$

$$\Rightarrow \left(2+9-\frac{1}{2}-6\right)+\lambda\left(-12+15-\frac{1}{2}-7\right)=0$$

$\Rightarrow \lambda=1$

Equation of plane is $\vec{r}\cdot(-5\hat{i}+8\hat{j}-2\hat{k})=13$

$|\vec{a}|^2=25+64+4=93$; d = 13

Value of $\frac{|13\vec{a}|^2}{d^2}=93$

18. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

(A) $\frac{19}{36}$ (B) $\frac{15}{36}$

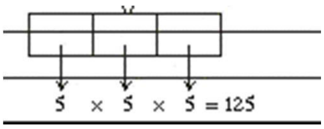
(C) $\frac{13}{36}$ (D) $\frac{23}{36}$

Official Ans. by NTA (A)

Sol. Atleast two digits are odd

= exactly two digits are odd + exactly there 3 digits are odd

For exactly three digits are odd



For exactly two digits odd :

If 0 is used then : $2 \times 5 \times 5 = 50$

If 0 is not used then : ${}^3C_1 \times 4 \times 5 \times 5 = 300$

Required Probability $= \frac{475}{900} = \frac{19}{36}$

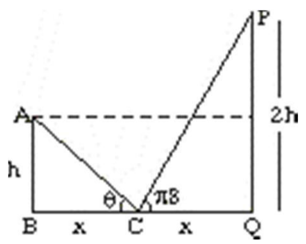
19. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to

(A) $\frac{3-2\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$

(C) $\frac{3-2\sqrt{2}}{4}$ (D) $\frac{3-\sqrt{2}}{4}$

Official Ans. by NTA (C)

Sol.



Let BC = CQ = x & AB = h and PQ = 2 h

$\tan\theta = \frac{h}{x}, \tan\frac{\pi}{8} = \frac{2h}{x}$

$\frac{\tan\theta}{\tan\left(\frac{\pi}{8}\right)} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}\tan\left(\frac{\pi}{8}\right) = \frac{1}{2}\left(\sqrt{2}-1\right)$

$\tan^2\theta = \frac{1}{4}\left(3-2\sqrt{2}\right)$

20. Let p, q, r be three logical statements. Consider the compound statements

$S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2 : p \rightarrow (q \vee r)$

Then, which of the following is **NOT** true ?

(A) If S_2 is True, then S_1 is True

(B) If S_2 is False, then S_1 is False

(C) If S_2 is False, then S_1 is True

(D) If S_1 is False, then S_2 is False

Official Ans. by NTA (C)

Sol. $s_1 : (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv \sim p \vee (q \vee r)$

$s_2 : p \rightarrow (q \vee r)$

$\equiv \sim p \vee (q \vee r) \rightarrow$ By conditional law

$s_1 \equiv s_2$

SECTION-B

1. Let R_1 and R_2 be relations on the set {1, 2, …, 50} such that

$R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$

and $R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$.

Then, the number of elements in $R_1 - R_2$ is ______.

Official Ans. by NTA (8)

Sol. Here, $p, p^n \in \{1, 2, \ldots 50\}$

Now p can take values 2,3,5,7,11,13,17,23,29,31,37,41,43 and 47.

∴ we can calculate no. of elements in R_1 as

$(2, 2^0), (2, 2^1) .. (2, 2^5)$

$(3, 3^0), \ldots (3, 3^3)$

$(5, 5^0), \ldots (5, 5^2)$

$(7, 7^0), \ldots (7, 7^2)$

$(11, 11^0), \ldots (11, 11^1)$

And rest for all other two elements each

$\therefore \quad n(R_1) = 6+4+3+3+(2\times 10) = 36$

Similarly for R_2

(2, 2°), (2,2')

(47, 47°), (47, 47')

$\therefore \quad n(R_2) = 2\times 14 = 28$

$\therefore \quad n(R_1) - n(R_2) = 36 - 28 = 8$

2. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$ is ______.

Official Ans. by NTA (2)

Sol. $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$

Let f(x) = $e^{2x}\left(e^{2x} + \frac{1}{e^{2x}} + 4\left(e^{x} + \frac{1}{e^{x}}\right) - 58\right)$

$e^{x} + \frac{1}{e^{x}}$

Let h(t) = $t^2 + 4t - 58 = 0$

$t = \frac{-4 \pm \sqrt{16 + 4.58}}{2}$

$\frac{-4 \pm 2\sqrt{62}}{2}$

$t_1 = -2 + 2\sqrt{62}$

$t_2 = -2 - 2\sqrt{62}$ (not possible)

$t \geq 2$

$e^{x} + \frac{1}{e^{x}} = -2 + 2\sqrt{62}$

$e^{2x} - (-2 + 2\sqrt{62})\, e^{x} + 1 = 0$

$(-2 + 2\sqrt{62}) - 4$

$4 + 4.62 - 8\sqrt{62} - 4$

$248 - 8\sqrt{62} > 0$

$\frac{-b}{2a} > 0$

both roots are positive

2 real roots

3. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to ______.

Official Ans. by NTA (17)

Sol. We have

$$\text{Variance} = \frac{\sum_{r=1}^{15} x_r^2}{15} - \left(\frac{\sum_{r=1}^{15} x_r}{15}\right)^2$$

Now, as per information given in equation

$\frac{\sum x_r^2}{15} - 8^2 = 3^2 \Rightarrow \sum x_r^2 = \log 5$

Now, the new $\sum x_r^2 = \log 5 - 5^2 + 20^2 = 1470$

And, new $\sum x_r = (15\times 8) - 5 + (20) = 135$

$\therefore \quad \text{Variance} = \frac{1470}{15} - \left(\frac{135}{15}\right)^2 = 98 - 81 = 17$

4. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a}\cdot\vec{c} = 5$, $\vec{b} \perp \vec{c}$, then 122 ($c_1 + c_2 + c_3$) is equal to ______.

Official Ans. by NTA (150)

Sol. $\vec{a}\cdot\vec{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5$ **...(1)**

$\vec{b}\cdot\vec{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0$ **...(2)**

And $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0 \Rightarrow \begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0$ ...(3)

By solving (1), (2), (3) we get

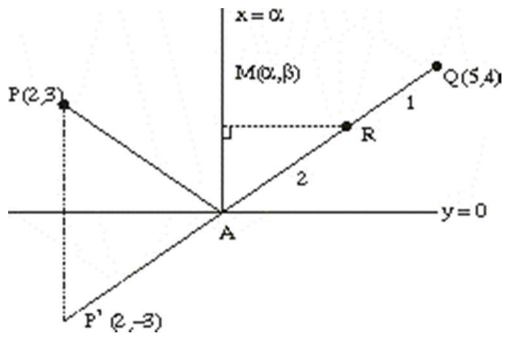
$c_1 = \frac{10}{122}, c_2 = \frac{-85}{122}, c_3 = \frac{225}{122}$

$\therefore 122(c_1 + c_2 + c_3) = 150$

5. A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to ______.

Official Ans. by NTA (31)

Sol.



By observation we see that $A(\alpha,0)$.

And $\beta =$ y -cordinate of R

$$=\frac{2\times4+1\times0}{2+1}=\frac{8}{3} \quad \ldots(1)$$

Now P' is image of P in y = 0 which will be $P'(2,-3)$

$\therefore$ Equation of P'Q is $(y+3)=\frac{4+3}{5-2}(x-2)$

i.e. $3y+9=7x-14$

$A\equiv\left(\frac{23}{7},0\right)$ by solving with y = 0

$\therefore\ \alpha=\frac{23}{7} \quad \ldots(2)$

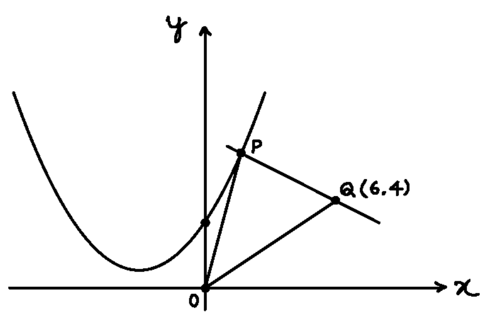
By (1), (2)

$7\alpha+3\beta=23+8=31$

6. Let ℓ be a line which is normal to the curve $y=2x^2+x+2$ at a point P on the curve. If the point Q(6, 4) lies on the line ℓ and O is origin, then the area of the triangle OPQ is equal to ______.

Official Ans. by NTA (13)

Sol. $y=2x^2+x+2$



$\frac{dy}{dx}=4x+1$

Let P be (h, k), then normal at P is

$$y-k=-\frac{1}{4h+1}(x-h)$$

This passes through Q (6,4)

$$\therefore 4-k=-\frac{1}{4h+1}(6-h)$$

$$\Rightarrow (4h+1)(4-k)+6-h=0$$

Also $k=2h^2+h+2$

$$\therefore (4h+1)(4-2h^2-h-2)+6+h=0$$

$$\Rightarrow 4h^3-3h^2+3h-8=0$$

$$\Rightarrow h=1, k=5$$

Now area of ΔOPQ will be $=\frac{1}{2}\begin{vmatrix}1 & 0 & 0\\ 1 & 1 & 5\\ 1 & 6 & 4\end{vmatrix}=13$

7. Let A = {1, a_1, a_2...... a_{18}, 77} be a set of integers with 1 < a_1 < a_2 < ..... < a_{18} < 77. Let the set $A+A=\{x+y : x, y\in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots + a_{18}$ is equal to ________.

Official Ans. by NTA (702)

Sol. $a_1, a_2, a_3, \ldots, a_{18}$, 77

are in AP i.e. 1, 5, 9, 13, ..., 77.

Hence $a_1 + a_2 + a_3 + \ldots + a_{18}$ = 5+9+13+...18 terms

=702

8. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}$, $x\neq0$ is $2^8\cdot\ell$, where ℓ is an odd integer, is ________.

Official Ans. by NTA (2)

Sol. $\left(2x^3+\frac{3}{x^k}\right)^{12}$

$t_{r+1} = {}^{12}C_r \left(2x^3\right)^r \left(\frac{3}{x^k}\right)^{12-r}$

$x^{3r-(12-r)k} \rightarrow$ constant

$\therefore 3r - 12k + rk = 0$

$\Rightarrow k = \frac{3r}{12-r}$

$\therefore$ possible values of r are 3,6,8,9,10 and corresponding values of k are 1,3,6,9,15

Now ${}^{12}C_r = 220, 924, 495, 220, 66$

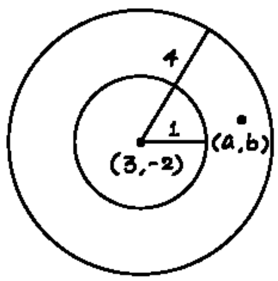
$\therefore$ possible values of k for which we will get 2^8 are 3, 6

9. The number of elements in the set

$\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is _______.

Official Ans. by NTA (40)

Sol. $1 < |Z - 3 + 2i| < 4$



$1 < (a-3)^2 + (b+2)^2 < 16$

$(0,\pm 2), (\pm 2, 0), (\pm 1, \pm 2), (\pm 2, \pm 1)$

$(\pm 2, \pm 3), (3\pm, \pm 2), (\pm 1, \pm 1), (2\pm, \pm 2)$

$(\pm 3, 0), (0, \pm 3), (\pm 3 \pm 1), (\pm 1, \pm 3)$

Total 40 points

10. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x-h)^2 + (y-k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ____.

Official Ans. by NTA (816)

Sol. Normal are

$y + 2x = \sqrt{11} + 7\sqrt{7},$

$2y + x = 2\sqrt{11} + 6\sqrt{7}$

Center of the circle is point of intersection of normals i.e.

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$

Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from center i.e. $4\sqrt{\frac{7}{5}}$

Now $(5h - 8k)^2 + 5r^2 = 816$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Velocity (v) and acceleration (a) in two systems of units 1 and 2 are related as $v_2 = \frac{n}{m^2} v_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constants. The relations for distance and time in two systems respectively are:

(A) $\frac{n^3}{m^3} L_1 = L_2$ and $\frac{n^2}{m} T_1 = T_2$

(B) $L_1 = \frac{n^4}{m^2} L_2$ and $T_1 = \frac{n^2}{m} T_2$

(C) $L_1 = \frac{n^2}{m} L_2$ and $T_1 = \frac{n^4}{m^2} T_2$

(D) $\frac{n^2}{m} L_1 = L_2$ and $\frac{n^4}{m^2} T_1 = T_2$

Official Ans. by NTA (A)

Sol. $\frac{L_2}{T_2} = \frac{n}{m^2} \frac{L_1}{T_1}$

$\frac{L_2}{T_2^2} = \frac{L_1}{T_1^2 \times mn}$

$\frac{n}{m^2} \times \frac{T_2}{T_1} = \frac{T_2^2}{T_1^2 \times mn}$

$\frac{n^2}{m} = \frac{T_2}{T_1}$

$\frac{L_2}{L_1} = \frac{n^4}{m^2} \times \frac{1}{mn}$

$\frac{L_2}{L_1} = \frac{n^3}{m^3}$

2. A ball is spun with angular acceleration $\alpha = 6t^2 - 2t$ where t is in second and α is in rads^{-2}. At t = 0, the ball has angular velocity of 10 rads^{-1} and angular position of 4 rad. The most appropriate expression for the angular position of the ball is:

(A) $\frac{3}{2}t^4 - t^2 + 10t$

(B) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(C) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(D) $2t^4 - \frac{t^3}{2} + 5t + 4$

Official Ans. by NTA (B)

Sol. $\frac{dw}{dt} = 6t^2 - 2t$

$\int_{10}^{w} dw = 2t^3 - t^2$

$w = 10 + 2t^3 - t^2$

$\frac{d\theta}{dt} = 10 + 2t^3 - t^2$

$\int_4^{\theta} d\theta = 10 + 2t^3 - t^2$

$\int_4^{\theta} d\theta = 10t + \frac{t^4}{2} - \frac{t^3}{3}$

$\theta = 4 + 10t + \frac{t^4}{2} - \frac{t^3}{3}$

3. A block of mass 2 kg moving on a horizontal surface with speed of 4 ms^{-1} enters a rough surface ranging from x = 0.5 m to x = 1.5 m. The retarding force in this range of rough surface is related to distance by $F = -kx$ where k = 12 Nm^{-1}. The speed of the block as it just crosses the rough surface will be:

(A) Zero (B) 1.5 ms^{-1}

(C) 2.0 ms^{-1} (D) 2.5 ms^{-1}

Official Ans. by NTA (C)

Sol. $a = \frac{-kx}{2} = \frac{-12x}{2} = -6x$

$$\frac{vdv}{dx} = -6x$$

$$\int_4^v vdv = -\int_{\frac{1}{2}}^{3/2} 6xdx$$

$$\frac{v^2 - 4^2}{2} = -\frac{6}{2}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

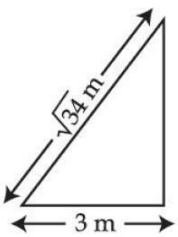
$$v^2 - 16 = -6\left(\frac{9}{4} - \frac{1}{4}\right)$$

$$v^2 = 16 - 6 \times 2 = 4$$

V = 2 m/s

4. A $\sqrt{34}$ m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor 3 m away from the wall as shown in the figure. If F_f and F_w are the reaction forces of the floor and the wall, then ratio of F_w/F_f will be:

(Use g = 10 m/s^2)

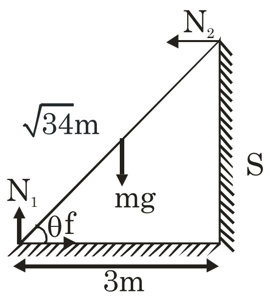


(A) $\frac{6}{\sqrt{110}}$ (B) $\frac{3}{\sqrt{113}}$

(C) $\frac{3}{\sqrt{109}}$ (D) $\frac{2}{\sqrt{109}}$

Official Ans. by NTA (C)

Sol.



$f = N_2$

$N_1 = mg$

$$N_2 \times \ell \sin\theta = mg\frac{\ell}{2}\cos\theta$$

$$N_2 = \frac{mg}{2}\cot\theta$$

$$\frac{F_w}{F_f} = \frac{\frac{mg}{2}\cot\theta}{\sqrt{(mg)^2 + \left(\frac{mg}{2}\cot\theta\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{4}{\cot^2\theta}}}$$

$$= \frac{3}{\sqrt{109}}$$

5. Water fall from a 40 m high dam at the rate of 9×10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydroelectric energy number of 100W lamps, that can be lit, is:

(Take g = 10 ms^{-2})

(A) 25 (B) 50

(C) 100 (D) 18

Official Ans. by NTA (B)

Sol. $\frac{9 \times 10^4 \times g \times 40}{3600} \times 0.5 = n \times 100$

$$\frac{10^4 \times 0.5}{100} = n$$

$100 \times 0.5 = n$

n = 50

6. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be :

(A) $\frac{2}{9}F$ (B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$ (D) F

Official Ans. by NTA (C)

Sol. $F = \frac{Gm^2}{r^2}$

$$F' = \frac{G\left(\frac{4m}{3}\right)\times\left(\frac{2m}{3}\right)}{r^2}$$

$$F' = \frac{8}{9}F$$

7. A water drop of radius $1\mu m$ falls in a situation where the effect of buoyant force is negligible. Co-efficient of viscosity of air is $1.8\times10^{-5}\,Nsm^{-2}$ and its density is negligible as compared to that of water $10^6\,gm^{-3}$. Terminal velocity of the water drop is:

(Take acceleration due to gravity = 10 ms^{-2})

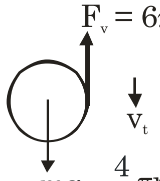
(A) $145.4\times10^{-6}\,ms^{-1}$ (B) $118.0\times10^{-6}\,ms^{-1}$

(C) $132.6\times10^{-6}\,ms^{-1}$ (D) $123.4\times10^{-6}\,ms^{-1}$

Official Ans. by NTA (D)

Sol.

$F_v = 6\pi\eta r v_t$



$mg = \frac{4}{3}\pi r^3 \rho g$

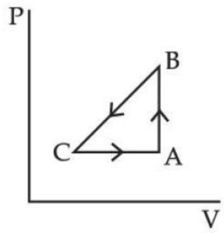
$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g$$

$$v_t = \frac{4}{3}\times\frac{\pi r^3 \rho g}{6\pi\eta r}$$

$$v_t = \frac{4}{3}\times\frac{\pi r^3 \rho g}{6\pi\eta r} = \frac{2\times10^{-12}\times10^3\times10}{9\times1.8\times10^{-5}}$$

$= 123.4 \times 10^{-6}$ m/s

8. A sample of an ideal gas is taken through the cyclic process ABCA as shown in figure. It absorbs, 40 J of heat during the part AB, no heat during BC and rejects 60J of heat during CA. A work 50J is done on the gas during the part BC. The internal energy of the gas at A is 1560J. The work done by the gas during the part CA is:

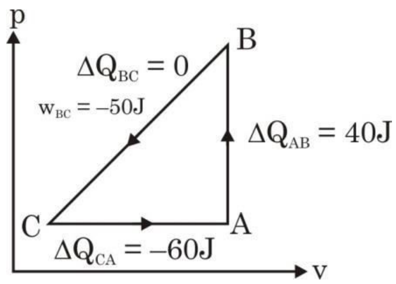


(A) 20 J (B) 30 J

(C) –30J (D) –60 J

Official Ans. by NTA (B)

Sol.



$\Delta Q_{cycle} = 40 - 60 = \Delta W$

$\Rightarrow \Delta W = -20J = W_{BC} + W_{CA}$

$\Rightarrow W_{CA} = -20J - W_{BC}$

$= -20 - (-50)$

$= 30$ J

9. What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxygen?

(A) The velocity of atomic oxygen remains same

(B) The velocity of atomic oxygen doubles

(C) The velocity of atomic oxygen becomes half

(D) The velocity of atomic oxygen becomes four times

Official Ans. by NTA (B)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$T \to 2T$

$M \to \frac{M}{2}$

$V_{rms} \propto \sqrt{\frac{T}{M}}$

$\Rightarrow (V_{rms})_{atomic} = (V_{rms})_{molecular} \times \sqrt{\frac{2}{1/2}} = 2(V_{rms})_{molecular}$

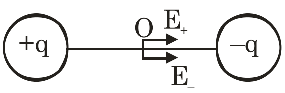
10. Two point charges A and B of magnitude $+8\times10^{-6}C$ and $-8\times10^{-6}C$ respectively are placed at a distance d apart. The electric field at the middle point O between the charges is $6.4\times10^{4}\,NC^{-1}$. The distance 'd' between the point charges A and B is:

(A) 2.0 m (B) 3.0 m

(C) 1.0 m (D) 4.0 m

Official Ans. by NTA (B)

Sol.



$E_0 = 2\times\frac{Kq}{(d/2)^2}$

$\Rightarrow E_0 = 8\frac{Kq}{d^2}$

$\Rightarrow d^2 = \frac{8\times9\times10^{9}\times8\times10^{-6}}{6.4\times10^{4}}$

d = 3 m

11. Resistance of the wire is measured as 2Ω and 3Ω at 10°C and 30°C respectively. Temperature co–coefficient of resistance of the material of the wire is :

(A) $0.033°C^{-1}$ (B) $-0.033°C^{-1}$

(C) $0.011°C^{-1}$ (D) $0.055°C^{-1}$

Official Ans. by NTA (A)

Sol. $R = R_0(1+\alpha\Delta T)$

$3 = R_0(1+\alpha(30-0))$

$2 = R_0(1+\alpha(10-0))$

$\frac{3}{2} = \frac{1+30\alpha}{1+10\alpha}$

$\alpha = \frac{1}{30} = 0.033$

12. The space inside a straight current carrying solenoid is filled with a magnetic material having magnetic susceptibility equal to 1.2×10^{-5}. What is fractional increase in the magnetic field inside solenoid with respect to air as medium inside the solenoid?

(A) 1.2×10^{-5} (B) 1.2×10^{-3}

(C) 1.8×10^{-3} (D) 2.4×10^{-5}

Official Ans. by NTA (A)

Sol. $\chi = 1.2\times10^{-5}$

$\mu_r = 1+\chi = 1+1.2\times10^{-5}$

Fractional Change

$= \frac{\Delta B}{B} = \frac{\mu_0\mu_r ni - \mu_0 ni}{\mu_0 ni} = (\mu_r - 1)$

$= 1.2\times10^{-5}$

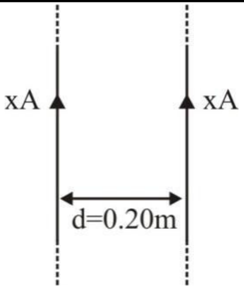
13. Two parallel, long wires are kept 0.20 m apart in vacuum, each carrying current of x A in the same direction. If the force of attraction per meter of each wire is $2\times10^{-6}N$, then the value of x is approximately:

(A) 1 (B) 2.4

(C) 1.4 (D) 2

Official Ans. by NTA (C)

Sol.



Force per unit length = $\frac{\mu_0 i_1 i_2}{2\pi d}$

$= \frac{\mu_0 \cdot x^2}{2\pi \times 0.2}$

$F = 2 \times 10^{-6} = \frac{4\pi \times 10^{-7} \times x^2}{2\pi \times 0.2}$

$\Rightarrow \; 10^{-6} = 10^{-7} \frac{x^2}{0.2}$

$\Rightarrow \; x^2 = 10 \times 0.2$

$= 2$

$\Rightarrow \; x = \sqrt{2} \approx 1.4$ Amp.

14. A coil is placed in a time varying magnetic field. If the number of turns in the coil were to be halved and the radius of wire doubled, the electrical power dissipated due to the current induced in the coil would be:

(Assume the coil to be short circuited.)

(A) Halved

(B) Quadrupled

(C) The same

(D) Doubled

Official Ans. by NTA (D)

Sol. $P = \frac{\varepsilon^2}{R} = \frac{\left(NA\frac{dB}{dt}\right)^2 \times A_C}{\rho\ell}$

$P' = \frac{\left(\frac{NA}{2}\frac{dB}{dt}\right)^2 \times 4A_C}{\rho\ell/2}$

$\Rightarrow P' = 2P$

15. An EM wave propagating in x-direction has a wavelength of 8 mm. The electric field vibrating y-direction has maximum magnitude of 60 Vm^{-1}. Choose the correct equations for electric and magnetic fields if the EM wave is propagating in vacuum :

(A) $E_y = 60\sin\left[\frac{\pi}{4}\times 10^3\left(x - 3\times 10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 2\sin\left[\frac{\pi}{4}\times 10^3\left(x - 3\times 10^8 t\right)\right]\hat{k}T$

(B) $E_y = 60\sin\left[\frac{\pi}{4}\times 10^3\left(x - 3\times 10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^3\left(x - 3\times 10^8 t\right)\right]\hat{k}T$

(C) $E_y = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^3\left(x - 3\times 10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 60\sin\left[\frac{\pi}{4}\times 10^3\left(x - 3\times 10^8 t\right)\right]\hat{k}T$

(D) $E_y = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^4\left(x - 4\times 10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 60\sin\left[\frac{\pi}{4}\times 10^4\left(x - 4\times 10^8 t\right)\right]\hat{k}T$

Official Ans. by NTA (B)

Sol. $B_0 = \frac{E_0}{c} = \frac{60}{3\times 10^8} = 2\times 10^{-7}T$

$E \times B$ must be direction of propagation.

So, $B \rightarrow$ z-axis

$k = \frac{2\pi}{\lambda} = \frac{\pi}{4}\times 10^3 m^{-1}$

$E_y = 60\sin\left[\frac{\pi}{4}\times 10^3(x - 3\times 10^8 t)\right]\hat{j}\,Vm^{-1}$

$B_z = 2\times 10^{-7}\sin\left[\frac{\pi}{4}\times 10^3(x - 3\times 10^8 t)\right]k\,T$

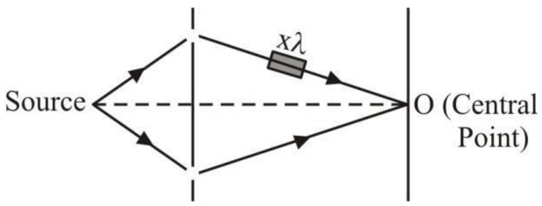
16. In young's double slit experiment performed using a monochromatic light of wavelength λ, when a glass plate $(\mu = 1.5)$ of thickness $x\lambda$ is introduced in the path of the one of the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:

(A) 3 (B) 2

(C) 1.5 (D) 0.5

Official Ans. by NTA (B)

Sol.



Path difference at O = $(\mu - 1)t$.

If the intensity at O remains (maximum) unchanged, path difference must be n λ.

$\Rightarrow (\mu - 1)t = n\lambda$

$(1.5 - 1)x\lambda = n\lambda$

$\Rightarrow x = 2n$

For n = 1, x = 2

17. Let K_1 and K_2 be the maximum kinetic energies of photo–electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2, respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then:

(A) $K_1 > \frac{K_2}{3}$ (B) $K_1 < \frac{K_2}{3}$

(C) $K_1 = \frac{K_2}{3}$ (D) $K_2 = \frac{K_1}{3}$

Official Ans. by NTA (B)

Sol. $\frac{hc}{\lambda_1} - \phi = K_1$

$\frac{hc}{\lambda_2} - \phi = K_2$

$\lambda_1 = 3\lambda_2$

$3K_1 = \frac{3hc}{\lambda_1} - 3\phi$

$3K_1 = \frac{hc}{\lambda_2} - 3\phi$

$3K_1 = K_2 - 2\phi$

$3K_1 < K_2$

$K_1 < \frac{K_2}{3}$

18. Following statements related to radioactivity are given below:

(A) Radioactivity is a random and spontaneous process and is dependent on physical and chemical conditions.

(B) The number of un–decayed nuclei in the radioactive sample decays exponentially with time.

(C) Slope of the graph of $\log_e$(no. of undecayed nuclei) Vs. time represents the reciprocal of mean life time (τ).

(D) Product of decay constant (λ) and half–life time ($T_{1/2}$) is not constant.

Choose the most appropriate answer from the options given below:

(A) (A) and (B) only

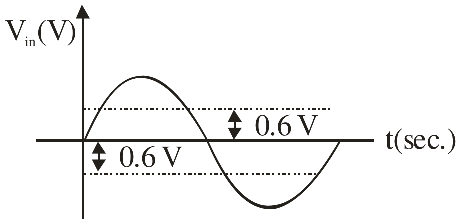
(B) (B) and (D) only

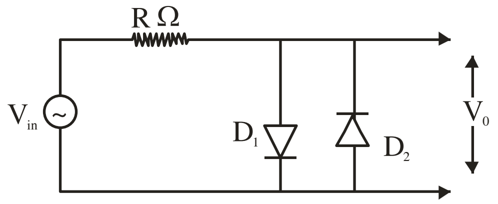
(C) (B) and (C) only

(D) (C) and (D) only

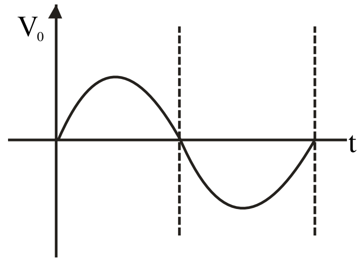
Official Ans. by NTA (C)

19. In the given circuit the input voltage V_{in} is shown in figure. The cut–in voltage of p–n junction diode (D_1 or D_2) is 0.6 V. Which of the following output voltage (V_0) waveform across the diode is correct?

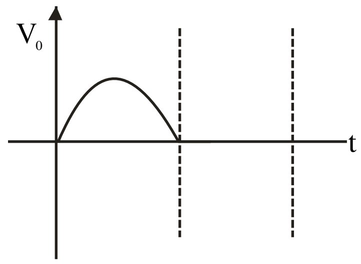




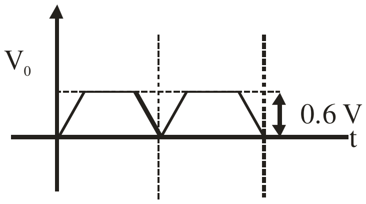
(A)



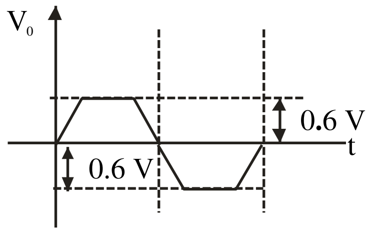
(B)



(C)



(D)



Official Ans. by NTA (D)

Sol. In +ve half cycle

$D_1 \rightarrow$ F.B.; $D_2 \rightarrow$ R.B.

0 – 0.6 V

V_{out} same as V_{in}

In –ve half cycle

$D_2 \rightarrow$ F.B.; $D_1 \rightarrow$ R.B.

20. Amplitude modulated wave is represented by $V_{AM} = 10\left[1 + 0.4\cos\left(2\pi \times 10^4 t\right)\right]\cos\left(2\pi \times 10^7 t\right)$.

The total bandwidth of the amplitude modulated wave is :

(A) 10 kHz (B) 20 MHz

(C) 20 kHz (D) 10 MHz

Official Ans. by NTA (C)

Sol. Bandwidth = 2 f_m

= 2 × 10^4 Hz = 20 × 10^3 Hz

= 20 kHz

SECTION-B

1. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm and 1.20 mm. The percentage error is $\frac{x}{121}\%$. The value of x is ___

Official Ans. by NTA (150)

)

Sol. $X = \frac{1.22mm + 1.23mm + 1.19mm + 1.20mm}{4}$

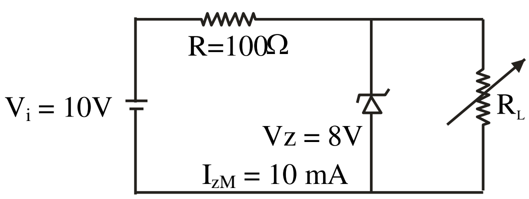
X = 1.21 mm

$\Delta x = \frac{0.01 + 0.02 + 0.02 + 0.01}{4} = \frac{0.06}{4} = 0.015$

Percentage error = $\frac{0.015}{1.21} \times 100$

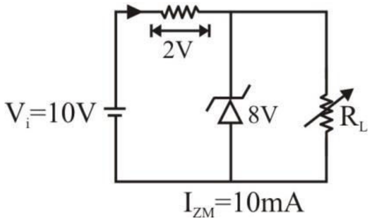
X = 150

2. A Zener of breakdown voltage V_Z = 8V and maximum zener current, I_{ZM} = 10 mA is subjected to an input voltage V_i = 10V with series resistance $R = 100\Omega$. In the given circuit R_L represents the variable load resistance. The ratio of maximum and minimum value of R_L is ______



Official Ans. by NTA (2)

Sol.



$I = \frac{2}{100} = 20mA$ $V_L = I_L R_L$ $8 = 10 \times 10^{-3} \times R_{L_{max}}$ $\frac{4}{5} \times 10^3 = R_{L_{max}}$ $\boxed{800 = R_{L_{max}}}$	$I = I_Z + I_L$ $I_L = 10$ mA If $\boxed{I_Z = 0}$ $I_{L_{max}} = 20mA$ $V_L = I_{L_{max}} \times R_{L_{min}}$ $\frac{8}{20} \times 10^3 = R_{L_{min}}$ $\boxed{400 = R_{L_{min}}}$

$$\frac{R_{L_{max}}}{R_{L_{min}}} = \frac{800}{400} = 2$$

3. In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index 7/5, is $\frac{1}{\alpha}$. The value of α is ________

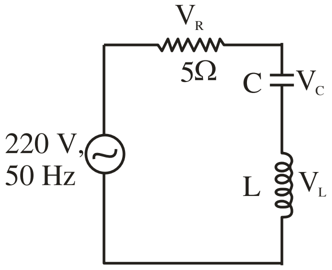
Official Ans. by NTA (4)

Sol. $\beta = \frac{0.35 \times 5}{7} = 0.25$

$\frac{1}{\alpha} = \frac{25}{100}$

$\boxed{\alpha = 4}$

4. In the given circuit, the magnitude of V_L and V_C are twice that of V_R. Given that f = 50 Hz, the inductance of the coil is $\frac{1}{K\pi}$ mH. The value of K is _______



Official Ans. by NTA (0)

Sol. $V_L = V_C = 2V_R$

$X_L = X_C = 2R$

$X_L = 10\Omega$

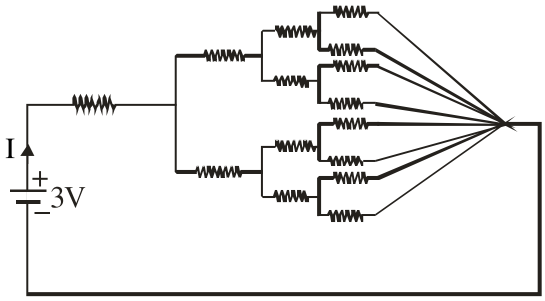
$\omega L = 10$

$2\pi f L = 10$

$L = \frac{10}{2\pi f} = \frac{1}{10\pi} H = \frac{1000}{10\pi} mH$

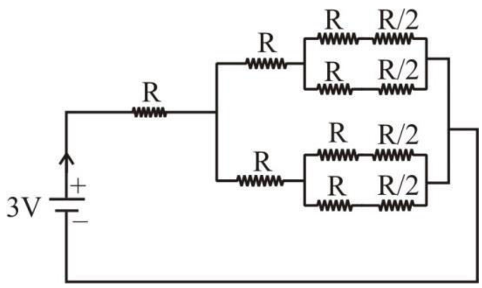
$L = \frac{1}{\frac{1}{100}\pi}$; $\quad K = \frac{1}{100} = 0.01 \approx 0$

5. All resistances in figure are 1Ω each. The value of current 'I' is $\frac{a}{5}$A. The value of a is _______



Official Ans. by NTA (8)

Sol.

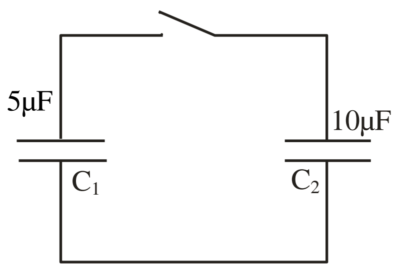


$R_{eq} = \frac{15R}{8} = \frac{15}{8}\Omega$

$I = \frac{3}{\frac{15}{8}} = \frac{8}{5}A$

$\therefore \ a = 8$

6. A capacitor C_1 of capacitance $5\mu F$ is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance $10\mu F$ as shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is ________ μC.



Official Ans. by NTA (100)

Sol. Before closing the switch

$Q = C_1V_0 = 5 \times 30 = 150\mu C$

After closing the switch

$$V = \frac{Q}{C_1 + C_2} = \frac{150}{10+5} = 10\ V$$

$$Q_2 = C_2V = 10 \times 10 = 100\mu C$$

7. A tuning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is ______cm.

(Velocity of sound in air is 340 ms^{-1})

Official Ans. by NTA (50)

Sol. Assumption : Ignore word "**fundamental mode**" in question.

$$\lambda = \frac{V}{f} = \frac{340}{340} = 1\text{m}$$

First resonating length = $\frac{\lambda}{4} = 25$ cm

Second resonating length = $\frac{3\lambda}{4} = 75$ cm

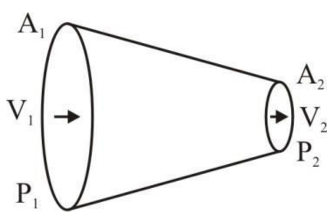
Third resonating length = $\frac{5\lambda}{4} = 125$ cm

Height of water required = 125 – 75 = 50 cm

8. A liquid of density 750 kgm^{-3} flows smoothly through a horizontal pipe that tapers in cross–sectional area from $A_1 = 1.2 \times 10^{-2} m^2$ to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow sections of the pipe is 4500 Pa. The rate of flow of liquid is _______ $\times 10^{-3} m^3s^{-1}$.

Official Ans. by NTA (24)

Sol.



$$A_2 = \frac{A_1}{2}$$

$$P_1 - P_2 = 4500\text{Pa}$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) \quad \ldots(1)$$

And $A_1V_1 = A_2V_2$

$$\Rightarrow \quad V_2 = 2V_1 \quad \ldots(2)$$

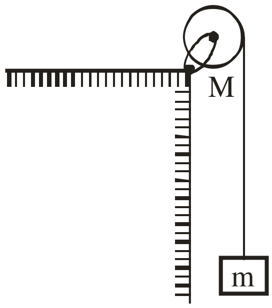
$$4500 = \frac{1}{2} \times 750 \times 3V_1^2$$

$V_1 = 2$ m/s

Volume flow rate = $A_1V_1 = 24 \times 10^{-3}\ m^3s^{-1}$

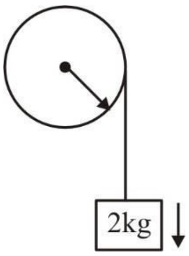
9. A uniform disc with mass M = 4 kg and radius R = 10 cm is mounted on a fixed horizontal axle as shown in figure. A block with mass m = 2 kg hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is ________N.

(Take $g = 10\ ms^{-2}$)



Official Ans. by NTA (10)

Sol.



$2g - T = 2a$ ...(1)

$TR = \frac{MR^2}{2}\alpha$ ...(2)

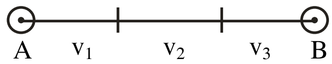
$\alpha = \frac{a}{R}$ ...(3)

$T = 2a$

$2g - T = 2a$

$T = g = 10N$

10. A car covers AB distance with first one–third at velocity $v_1\ ms^{-1}$, second one–third at $v_2\ ms^{-1}$ and last one–third at $v_3\ ms^{-1}$. If $v_3 = 3v_1$, $v_2 = 2v_1$ and $v_1 = 11\ ms^{-1}$ then the average velocity of the car is ________ ms^{-1}.



Official Ans. by NTA (18)

Sol. $<\vec{v}> = \frac{\text{Displacement}}{\text{time}}$

(Let displacement be l)

$$= \frac{\ell}{\left(\frac{\ell}{V_3} + \frac{\ell}{V_2} + \frac{\ell}{V_1}\right)\frac{1}{3}}$$

$$= \frac{3}{\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}} = \frac{3}{\frac{1}{11} + \frac{1}{22} + \frac{1}{33}}$$

= 18 m/s

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The molecular formula of the compound is,

Given : Atomic masses of C, H and N are 12, 1 and 14 amu respectively.

The molar mass of the compound A is 162 g mol^{-1}.

(A) $C_4H_6N_2$ (B) C_2H_3N

(C) C_5H_7N (D) $C_{10}H_{14}N_2$

Official Ans. by NTA (D)

Sol.

C	74%	$\frac{74}{12}=6.16$	$\frac{6.16}{1.23}=5$
N	17.3%	$\frac{17.3}{14}=1.23$	$\frac{1.23}{1.23}=1$
H	8.7%	$\frac{8.7}{1}=8.7$	$\frac{8.7}{1.23}=7$

Emperical formula = C_5NH_7

Emperical weight = 81

Multiplying factor = $\frac{162}{81}=2$

Molecular formula = $C_{10}N_2H_{14}$

2. Consider the following statements :

(A) The principal quantum number 'n' is a positive integer with values of 'n' = 1, 2, 3, ….

(B) The azimuthal quantum number 'l' for a given 'n' (principal quantum number) can have values as 'l' = 0, 1, 2, …. n

(C) Magnetic orbital quantum number 'm_l' for a particular 'l' (azimuthal quantum number) has $(2l + 1)$ values.

(D) ±1/2 are the two possible orientations of electron spin.

(E) For l = 5, there will be a total of 9 orbital.

Which of the above statements are **correct**?

(A) (A), (B) and (C)

(B) (A), (C), (D) and (E)

(C) (A), (C) and (D)

(D) (A), (B), (C) and (D)

Official Ans. by NTA (C)

Sol. (A) Number of values of n = 1, 2, 3 … ∞

(B) Number of values of ℓ = 0 to (n – 1)

(C.) Number of values of $m = -\ell$ to $+\ell$

Total values = $2\ell + 1$

(D) Values of spin = $\pm\frac{1}{2}$

(E) For ℓ = 5 number of orbitals = $2\ell + 1 = 11$

3. In the structure of SF_4, the lone pair of electrons on S is in.

(A) equatorial position and there are two lone pair-bond pair repulsions at 90°

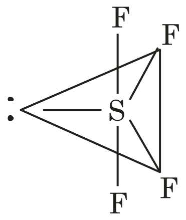
(B) equatorial position and there are three lone pair-bond pair repulsions at 90°

(C) axial position and there are three lone pair – bond pair repulsion at 90°.

(D) axial position and there are two lone pair – bond pair repulsion at 90°.

Official Ans. by NTA (A)

Sol.



sp^3d, See-Saw

4. A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH 4. The ratio of $\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$ required to make buffer is ……….

Given : $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$

(A) 0.03 (B) 0.13

(C) 0.23 (D) 0.33

Official Ans. by NTA (B)

Sol. $pH = pK_a + \log\frac{[Salt]}{[Acid]}$

$$4 = 5 - \log 1.3 + \log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$\log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = \log 1.3 - 1 = \log\frac{1.3}{10}$$

$$\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = 0.13$$

5. Match List-I with List-II.

List-I		List-II	
(A)	Negatively charged sol	(I)	$Fe_2O_3 \cdot xH_2O$
(B)	Macromolecular colloid	(II)	CdS sol
(C)	Positively charged sol	(III)	Starch
(D)	Cheese	(IV)	a gel

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(B) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

(C) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) – (I), (B) – (III), (C) – (II), (D) – (IV)

Official Ans. by NTA (C)

Sol. Negative charged sol = CdS (II)

Macromolecular colloid = starch (III)

Positively charged sol = $Fe_2O_3.xH_2O$ (I)

Cheese = gel (IV)

6. Match List-I with List-II.

List-I (Oxide)		List-II (Nature)	
(A)	Cl_2O_7	(I)	Amphoteric
(B)	Na_2O	(II)	Basic
(C)	Al_2O_3	(III)	Neutral
(D)	N_2O	(IV)	Acidic

Choose the **correct** answer from the options given below :

(A) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(B) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(C) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

(D) (A) – (I), (B) – (II), (C) – (IIII), (D) – (IV)

Official Ans. by NTA (B)

Sol. Cl_2O_7 Acidic

Na_2O Basic

Al_2O_3 Amphoteric

N_2O Neutral

7. In the metallurgical extraction of copper, following reaction is used :

$FeO + SiO_2 \rightarrow FeSiO_3$

FeO and $FeSiO_3$ respectively are.

(A) gangue and flux (B) flux and slag

(C) slag and flux (D) gangue and slag

Official Ans. by NTA (D)

Sol. FeO = Gangue

$FeSiO_3$ = Slag

8. Hydrogen has three isotopes : protium (^1H), deuterium (^2H or D) and tritium (^3H or T). They have nearly same chemical properties but different physical properties. They differ in

(A) number of protons

(B) atomic number

(C) electronic configuration

(D) atomic mass

Official Ans. by NTA (D)

Sol. They have different neutrons and mass number

9. Among the following basic oxide is :

(A) SO_3 (B) SiO_2

(C) CaO (D) Al_2O_3

Official Ans. by NTA (C)

Sol. SO_3, SiO_2 = Acidic

CaO = Basic

Al_2O_3 = Amphoteric

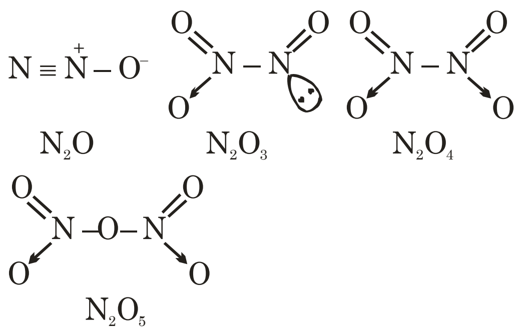
10. Among the given oxides of nitrogen; N_2O, N_2O_3, N_2O_4 and N_2O_5, the number of compound/(s) having N–N bond is :

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (C)

Sol.



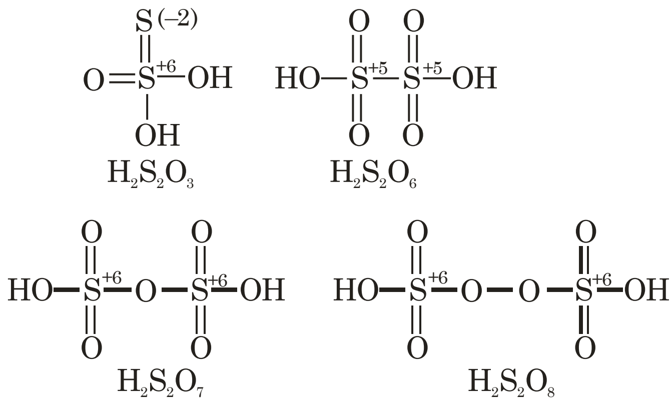
11. Which of the following oxoacids of sulphur contains "S" in two different oxidation states?

(A) $H_2S_2O_3$ (B) $H_2S_2O_6$

(C) $H_2S_2O_7$ (D) $H_2S_2O_8$

Official Ans. by NTA (A)

Sol.



12. Correct statement about photo-chemical smog is :

(A) It occurs in humid climate.

(B) It is a mixture of smoke, fog and SO_2

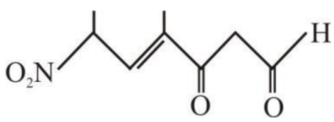
(C) It is reducing smog.

(D) It results from reaction of unsaturated hydrocarbons.

Official Ans. by NTA (D)

Sol. Photo chemical smog results from the action of sunlight on unsaturated hydro carbons and nitrogen oxide

13. The correct IUPAC name of the following compound is :



(A) 4-methyl-2-nitro-5-oxohept-3-enal

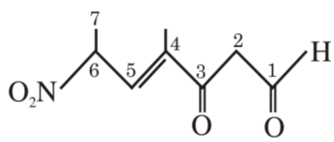
(B) 4-methyl-5-oxo-2-nitrohept-3-enal

(C) 4-methyl-6-nitro-3-oxohept-4-enal

(D) 6-formyl-4-methyl-2-nitrohex-3-enal

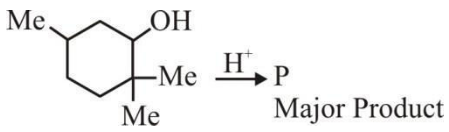
Official Ans. by NTA (C)

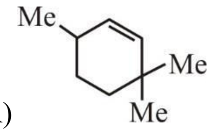
Sol.

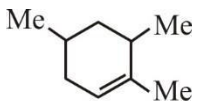


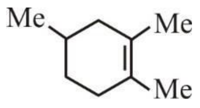
4-Methyl-6-nitro-3-oxohept-4-enal

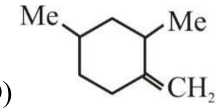
14. The major product (P) of the given reaction is (where, Me is $-CH_3$)



(A) 

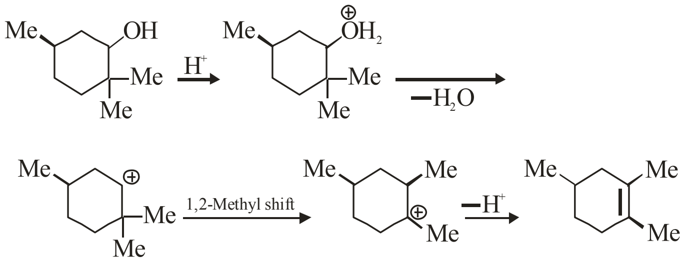
(B) 

(C) 

(D) 

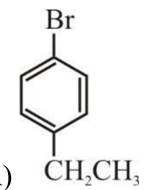
Official Ans. by NTA (C)

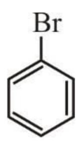
Sol.

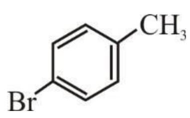


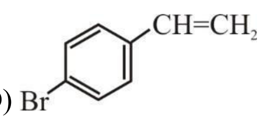
15. $A \xrightarrow[\text{(ii) } CN^- \text{ (iii) } H_2O/H^+]{\text{(i) } Cl_2, \Delta}$ 4-Bromophenyl acetic acid.

In the above reaction 'A' is

(A) 

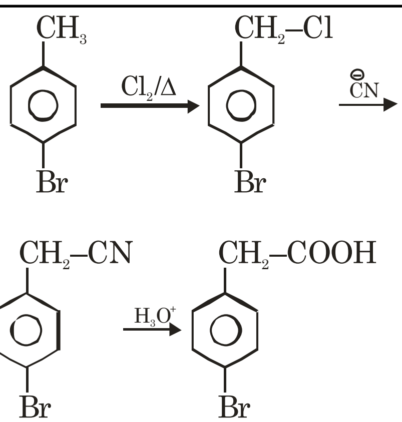
(B) 

(C) 

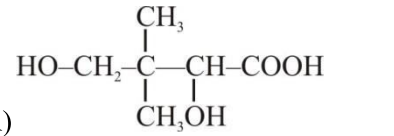
(D) 

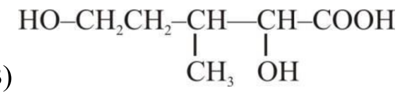
Official Ans. by NTA (C)

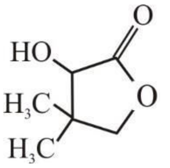
Sol.

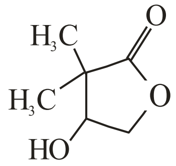


16. Isobutyraldehyde on reaction with formaldehyde and K_2CO_3 gives compound 'A'. Compound 'A' reacts with KCN and yields compound 'B', which on hydrolysis gives a stable compound 'C'. The compound 'C' is :

(A) 

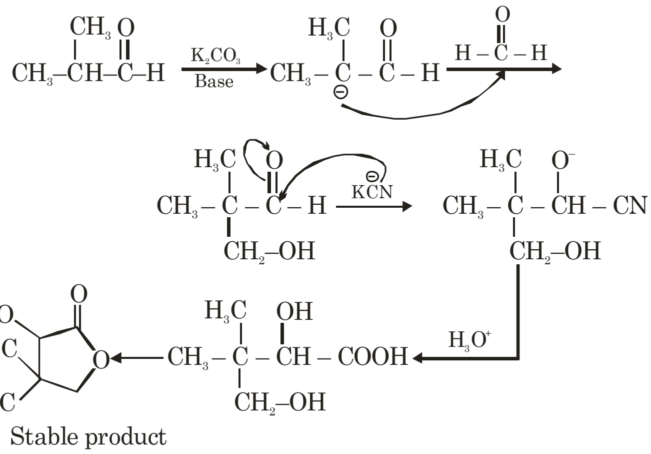
(B) 

(C) 

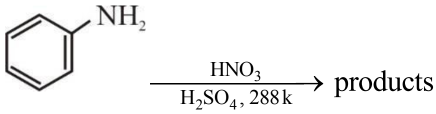
(D) 

Official Ans. by NTA (C)

Sol.



17. With respect to the following reaction, consider the given statements :



(A) o-Nitroaniline and p-nitroaniline are the predominant products

(B) p-Nitroaniline and m-nitroaniline are the predominant products

(C) HNO_3 acts as an acid

(D) H_2SO_4 acts as an acid

(A) (A) and (C) are correct statements.

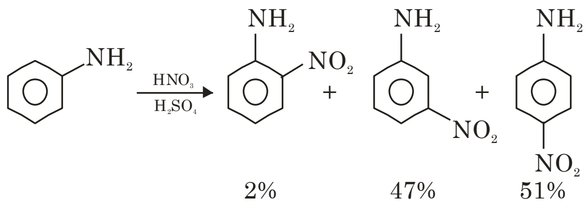
(B) (A) and (D) are correct statements.

(C) (B) and (D) are correct statements.

(D) (B) and (C) are correct statements.

Official Ans. by NTA (C)

Sol.



$\underset{\text{Base}}{HNO_3} + \underset{\text{Acid}}{H_2SO_4} \rightarrow NO_2^+$

18. Given below are two statements, one is Assertion (A) and other is Reason (R).

Assertion (A) : Natural rubber is a linear polymer of isoprene called cis-polyisoprene with elastic properties.

Reason (R) : The cis-polyisoprene molecules consist of various chains held together by strong polar interactions with coiled structure.

In the light of the above statements, choose the **correct** one from the options given below :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Official Ans. by NTA (C)

Sol. Natural rubber is linear polymer of isoprene (2-methyl-1,3-butadiene) and is also called cis-1,4-polyisoprene. The cis-polyisoprene molecules consists of various chains held together by weak Vander Waal's interactions and has a coiled structure

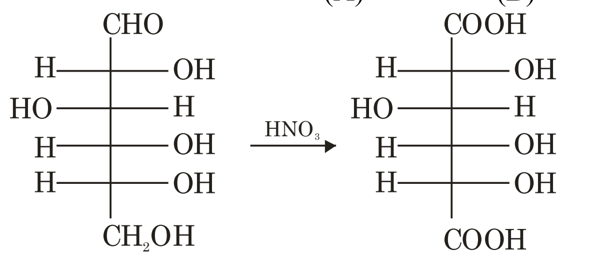
19. When sugar 'X' is boiled with dilute H_2SO_4 in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with HNO_3 yields saccharic acid where as 'B' is laevorotatory. The compound 'X' is :

(A) Maltose (B) Sucrose

(C) Lactose (D) Strach

Official Ans. by NTA (B)

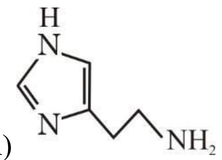
Sol. $\underset{[\alpha] = 66.6°}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H^+} \underset{\substack{\text{D-Glucose} \\ [\alpha] = +52.7° \\ (A)}}{C_6H_{12}O_6} + \underset{\substack{\text{D-Fructose} \\ [\alpha] = -92.2° \\ (B)}}{C_6H_{12}O_6}$



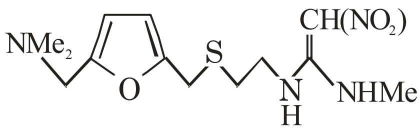
Sachharic acid

20. The drug tegamet is :

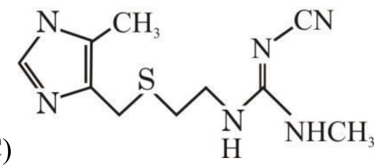
(A)



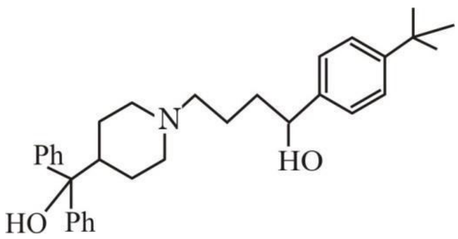
(B)



(C)

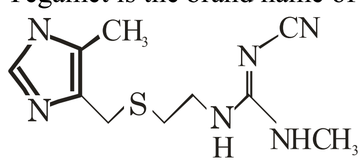


(D)



Official Ans. by NTA (C)

Sol. Tegamet is the brand name of Cimetidine



SECTION-B

1. 100 g of an ideal gas is kept in a cylinder of 416 L volume at 27°C under 1.5 bar pressure. The molar mass of the gas is ______ g mol^{-1}. (Nearest integer) (Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (4)

Sol. $1.5 \times 416 = \frac{100}{M} \times 0.083 \times 300$

M = 3.99

Ans. 4

2. For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_C H^{\ominus}$ = –601.70 kJ mol^{-1}, the magnitude of change in internal energy for the reaction is _____ kJ. (Nearest integer)

(Given : R = 8.3 J K^{-1} mol^{-1})

Official Ans. by NTA (600)

Sol. $Mg(s) + \frac{1}{2}O_2(g) \rightarrow MgO(s)$

$\Delta H = \Delta U + \Delta n_g RT$

$-601.70 \times 10^3 = \Delta U - \frac{1}{2} \times 8.3 \times 300$

$-601.70\,kJ = \Delta U - 1.245\,kJ$

$\Delta U = -600.455\,kJ$

Ans. 600

3. 2.5 g of protein containing only glycine ($C_2H_5NO_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be 5.03 × 10^{-3} bar. The total number of glycine units present in the protein is ____

(Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (330)

Sol. $\pi = CRT$

$5.03 \times 10^{-3} = C \times 0.083 \times 300$

$C = 0.202 \times 10^{-3} M$

Moles of protein = 0.202 × 10^{-3} × 0.5

= 10^{-4} × 1.01

$1.01 \times 10^{-4} = \frac{2.5}{M}$

M(molar mass of protein) = 24752

$\therefore$ No. of glycine units = $\frac{24752}{75} = 330.03$

4. For the given reactions

$Sn^{2+} + 2e^- \rightarrow Sn$

$Sn^{4+} + 4e^- \rightarrow Sn$

The electrode potentials are; $E^o_{Sn^{2+}/Sn}$ = –0.140 V and $E^o_{Sn^{4+}/Sn}$ = 0.010 V. The magnitude of standard electrode potential for Sn^{4+}/Sn^{2+} i.e. $E^o_{Sn^{4+}/Sn^{2+}}$ is ______ × 10^{-2} V. (Nearest integer)

Official Ans. by NTA (16)

Sol. $Sn^{2+} + 2e^- \rightarrow Sn \qquad \Delta G^0_1 = +2 \times 0.140 \times F$

$Sn^{+4} + 4e^- \rightarrow Sn \qquad \Delta G^0_2 = -4 \times 0.01 \times F$

$Sn^{+4} + 2e^- \rightarrow Sn^{+2} \qquad \Delta G^0_3 = -2 \times E^0_{Sn^{+4}/Sn^{+2}} \times F$

$\Delta G^0_3 = \Delta G^0_2 - \Delta G^0_1$

–2 × E^0 × F = –(0.04 + 0.28) × F

E^0 = 0.16 volt = 16 × 10^{-2} V

Ans 16

5. A radioactive element has a half life of 200 days. The percentage of original activity remaining after 83 days is ______. (Nearest integer)

(Given : antilog 0.125 = 1.333, antilog 0.693 = 4.93)

Official Ans. by NTA (75)

Sol. $t = \frac{t_{1/2}}{0.3} \log \frac{[A]_0}{[A]_t}$

$83 = \frac{200}{0.3} \log \frac{[A]_0}{[A]_t}$

$0.125 = \log \frac{[A]_0}{[A]_t}$

$\frac{[A]_0}{[A]_t} = 1.333 \simeq \frac{4}{3}$

$\therefore \frac{[A]_t}{[A]_0} \times 100 = \frac{3}{4} \times 100 = 75\%$

Ans. 75

6. $[Fe(CN)_6]^{4-}$

$[Fe(CN)_6]^{3-}$

$[Ti(CN)_6]^{3-}$

$[Ni(CN)_4]^{2-}$

$[Co(CN)_6]^{3-}$

Among the given complexes, number of paramagnetic complexes is _____.

Official Ans. by NTA (2)

Sol. $[Fe(CN)_6]^{4-}$ Diamagnetic

$[Fe(CN)_6]^{3-}$ Paramagnetic (1 unpaired electron)

$[Ti(CN)_6]^{3-}$ Paramagnetic (1 unpaired electron)

$[Ni(CN)_4]^{2-}$ Diamagnetic

$[Co(CN)_6]^{3-}$ Diamagnetic

Ans. 2

7. (a) $CoCl_3 \cdot 4\ NH_3$

(b) $CoCl_3 \cdot 5NH_3$

(c) $CoCl_3 \cdot .6NH_3$ and

(d) $CoCl(NO_3)_2 \cdot 5NH_3$

Number of complex(es) which will exist in cis-trans is/are

Official Ans. by NTA (1)

Sol. (a) $CoCl_3 \cdot 4\ NH_3 = [Co(NH_3)_4\ Cl_2]Cl$

Can exhibit G.I.

(b) $CoCl_3 \cdot 5NH_3 = [Co(NH_3)_5\ Cl]Cl_2$

Can't exhibit G.I.

(c) $CoCl_3 \cdot .6NH_3 = [Co(NH_3)_6]Cl_3$

Can't exhibit G.I.

(d) $CoCl(NO_3)_2 \cdot 5NH_3 = [Co(NH_3)_5\ Cl](NO_3)_2$

OR

$= [Co(NH_3)_5\ (NO_3)]Cl(NO_3)$

Both can't exhibit G.I.

8. The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793g of CO_2 and 0.442 g of H_2O. The percentage of oxygen composition in the organic compound is ___________. (nearest integer)

Official Ans. by NTA (46)

Sol. Mole of CO_2 = Moles of C = $\frac{0.793}{44}$

Weight of 'C' = $\frac{0.793}{44} \times 12 = 0.216$ gm

Moles of 'H' = $\frac{0.442}{18} \times 2$

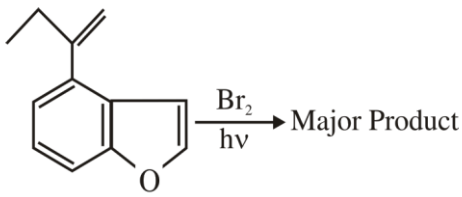
Weight of 'H' = $\frac{0.442}{18} \times 2 \times 1 = 0.049$ gm

$\therefore$ Weight of 'O'=0.492–0.216–0.049= 0.227 gm

$\%\ of\ 'O' = \frac{0.227}{0.492} \times 100 = 46.13\%$

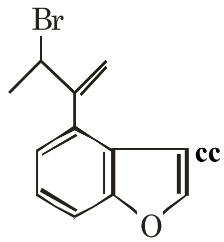
Ans. 46

9. The major product of the following reaction contains _______ bromine atom(s).



Official Ans. by NTA (1)

Sol.



No. of Br atoms = 1

10. 0.01 M $KMnO_4$ solution was added to 20.0 mL of 0.05 M Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of $KMnO_4$ solution left in the burette after the end point is _______ mL. (nearest integer)

Official Ans. by NTA (30)

Sol. $N_1V_1 = N_2V_2$

$0.01 \times 5 \times V_1 = 0.05 \times 1 \times 20$

$V_1 = 20$ ml used

$\therefore$ Volume left = 50 – 20 = 30 ml

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let $R_1 = \{(a,b) \in N \times N : |a-b| \le 13\}$ and $R_2 = \{(a,b) \in N \times N : |a-b| \ne 13\}$. Then on N:

(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_2 is an equivalence relation but R_1 is not

Official Ans. by NTA (B)

Sol. $R_1 = \{(a,b) \in N \times N : |a-b| \le 13\}$

$R_2 = \{(a,b) \in N \times N : |a-b| \ne 13\}$.

For R_1 :

i) Reflexive relation

$(a,a) \in N \times N : |a-a| \le 13$

ii) Symmetric relation

$(a,b) \in R_1, (b,a) \in R_1 : |b-a| \le 13$

iii) Transitive relation

$(a,b) \in R_1, (b,c) \in R_1, (a,c) \in R_1 :$

$(1, 3) \in R_1$, $(3, 16) \in R_1$ but $(1, 16) \notin R_1$

For R_2 :

i) Reflexive relation

$(a,a) \in N \times N : |a-a| \ne 13$

ii) Symmetric relation

$(b,a) \in N \times N : |b-a| \ne 13$

iii) Transitive relation

$(a,b) \in R_2, (b,c) \in R_2, (a,c) \in R_2$

$(1, 3) \in R_2$, $(3, 14) \in R_2$ but $(1, 14) \notin R_2$

2. Let f(x) be a quadratic polynomial such that f(–2) + f(3) = 0. If one of the roots of f(x) = 0 is –1, then the sum of the roots of f(x) = 0 is equal to :

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$

(C) $\frac{13}{3}$ (D) $\frac{14}{3}$

Official Ans. by NTA (A)

Sol. $f(-2) + f(3) = 0$

$f(x) = (x + 1)(ax + b)$

$f(-2) + f(3) = -1(-2a + b) + 4(3a + b) = 0$

$2a - b + 12a + 4b = 0$

$14a + 3b = 0$

$\frac{-b}{a} = \frac{14}{3}$

Sum of roots $= \left(-1 + \frac{-b}{a}\right) = -1 + \frac{14}{3} = \frac{11}{3}$

3. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

(A) 205 (B) 615

(C) 510 (D) 430

Official Ans. by NTA (D)

Sol. $t_1 + t_2 + t_3 + t_4 = 30$

Coefficient of x^{30} in $(1 + x + x^2 + \ldots + x^{30})^2$

$(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$

$x^6\left(\frac{1-x^{31}}{1-x}\right)^2 (1+x+x^2+x^3)(1+x+x^2+x^3+x^4)$

$x^6(1-x^{31})^2(1-x^4)(1-x^5)(1-x)^{-4}$

$x^6(1-x^4-x^5+x^9)(1+x^{62}-2x^{31}(1-x)^{-4})$

$x^6(1-x^4-x^5+x^9)(1-x)^{-4}$

Coefficient of x^n in $(1-x)^{-r}$ is ${}^{n+r-1}C_{r-1}$

$\Rightarrow {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

$2925 - 1771 - 1540 + 816$

$= 430$

OR

$x_2 \in [4,7], x_3 \in [2,6]$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = 24$

total ways $=$

${}^{24+4-1}C_{4-1} - {}^{20+4-1}C_{4-1} - {}^{19+4-1}C_{4-1} + {}^{15+4-1}C_{4-1}$

$= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3 = 430$

4. The term independent of x in the expression of $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}, x \neq 0$ is

(A) $\frac{7}{40}$ (B) $\frac{33}{200}$

(C) $\frac{39}{200}$ (D) $\frac{11}{50}$

Official Ans. by NTA (B)

Sol. $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is

$$^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r$$

General term is $^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-5r}$

Now, term independent of x

$1 \times$ coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$-1 \times$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ +

$3 \times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

for coefficient of x^0 $33-5r=0$ not possible

for coefficient of x^{-2} $33-5r=-2$

$35=5r \Rightarrow r=7$

for coefficient of x^{-3} $33-5r=-3$

$36=5r$ not possible

So term independent of x is

$$(-1)\ ^{11}C_7\left(\frac{5}{2}\right)^4\left(-\frac{1}{5}\right)^7=\frac{33}{200}$$

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is

(A) 21 (B) 22

(C) 23 (D) 24

Official Ans. by NTA (C)

Sol. $d=\frac{100-a}{n+1}$

$A_1=a+d$

$A_n=100-d$

$\Rightarrow \frac{A_1}{A_n}=\frac{1}{7} \Rightarrow \frac{a+d}{100-d}=\frac{1}{7}$

$\Rightarrow 7a+8d=100$

$\Rightarrow 7a+8\left(\frac{100-a}{n+1}\right)=100$ ....(1)

$\because a+n=33$ ...(2)

Now, by Eq. (1) and (2)

$7n^2-132n-667=0$

$\boxed{n=23}$ and $n=\frac{-29}{7}$ reject.

6. Let $f, g: \mathbf{R} \to \mathbf{R}$ be functions defined by

$$f(x)=\begin{cases}[x] & , x<0 \\ |1-x| & , x\geq 0\end{cases} \text{ and}$$

$$g(x)=\begin{cases}e^x-x & , x<0 \\ (x-1)^2-1 & , x\geq 0\end{cases}$$

where [x] denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly :

(A) one point (B) two points

(C) three points (D) four points

Official Ans. by NTA (B)

Sol. Check continuity at x = 0 and also check continuity at those x where g(x) = 0

g(x) = 0 at x = 0, 2

$fog(0^+)=-1$

$fog(0)=0$

Hence, discontinuous at x = 0

$fog(2^+)=1$

$fog(2^-)=-1$

Hence, discontinuous at x = 2

7. Let $f:\mathbf{R}\to\mathbf{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right)=\sqrt{2}, f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$ and let $g(x)=\int_x^{\pi/4}\left(f'(t)\sec t+\tan t\sec t\, f(t)\right)dt$ for $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right)$. Then $\lim_{x\to\left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

(A) 2 (B) 3

(C) 4 (D) –3

Official Ans. by NTA (B)

Sol. $g(x)=\int_x^{\pi/4}\left(f'(t)\sec t+\tan t\sec t f(t)\right)dt$

$g(x)=\int_x^{\pi/4} d\left(f(t)\cdot\sec t\right)=f(t)\sec t\Big|_x^{\pi/4}$

$g(x)=f\left(\frac{\pi}{4}\right)\sec\frac{\pi}{4}-f(x)\cdot\sec x$

$g(x)=2-f(x)\sec x=2-\left(\frac{f(x)}{\cos x}\right)$

$\lim_{x\to\left(\frac{\pi}{2}\right)^-} g(x)=2-\lim_{x\to\left(\frac{\pi}{2}\right)^-}\left(\frac{f(x)}{\cos x}\right)$

Using L'Hopital Rule

$=2-\lim_{x\to\left(\frac{\pi}{2}\right)^-}\frac{f'(x)}{(-\sin x)}$

$=2+\frac{f'\left(\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}}=2+\frac{1}{1}=3$

8. Let $f:\mathbf{R}\to\mathbf{R}$ be continuous function satisfying $f(x) + f(x + k) = n$, for all $x\in\mathbf{R}$ where $k > 0$ and n is a positive integer. If $I_1=\int_0^{4nk} f(x)dx$ and $I_2=\int_{-k}^{3k} f(x)dx$, then

(A) $I_1+2I_2=4nk$ (B) $I_1+2I_2=2nk$

(C) $I_1+nI_2=4n^2k$ (D) $I_1+nI_2=6n^2k$

Official Ans. by NTA (C)

Sol. $f(x)+f(x+k)=n$

$\Rightarrow f(x)=f(x+2k)$

f(x) is periodic with period 2k

$I_1=\int_0^{4nk} f(x)dx=2n\int_0^{2k} f(x)dx$

$I_2=\int_{-k}^{3k} f(x)dx=2\int_0^{2k} f(x)dx$

Now,

$f(x)+f(x+k)=n$

$\Rightarrow\int_0^k f(x)dx+\int_0^k f(x+k)dx=nk$

$\Rightarrow\int_0^k f(x)dx+\int_k^{2k} f(x)dx=nk$

$\Rightarrow\int_0^{2k} f(x)dx=nk$

$\Rightarrow I_1=2n^2k,\ I_2=2nk$

$\Rightarrow I_1+nI_2=4n^2k$

9. The area of the bounded region enclosed by the curve $y=3-\left|x-\frac{1}{2}\right|-|x+1|$ and the x-axis is

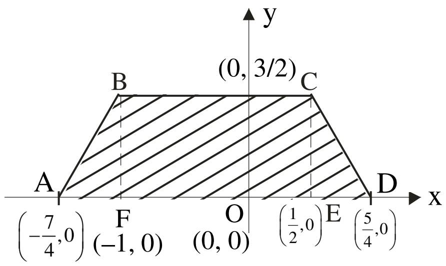
(A) $\frac{9}{4}$ (B) $\frac{45}{16}$

(C) $\frac{27}{8}$ (D) $\frac{63}{16}$

Official Ans. by NTA (C)

Sol. $y=\begin{cases}3+(x+1)+\left(x-\frac{1}{2}\right), & x<-1\\ 3-(x+1)+\left(x-\frac{1}{2}\right), & -1\le x<\frac{1}{2}\\ 3-(x+1)-\left(x-\frac{1}{2}\right), & \frac{1}{2}\le x\end{cases}$

$$y=\begin{cases}\frac{7}{2}+2x, & x<-1\\ \frac{3}{2}, & -1\le x<\frac{1}{2}\\ \frac{5}{2}-2x, & \frac{1}{2}\le x\end{cases}$$



Area bounded = ar ABF + ar BCEF + ar CDE

$$=\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)$$

$=\frac{27}{8}$ sq. units.

10. Let x = x(y) be the solution of the differential equation $2ye^{x/y^2}dx+\left(y^2-4xe^{x/y^2}\right)dy=0$ such that x(1) = 0. Then, x(e) is equal to

(A) $e\log_e(2)$ (B) $-e\log_e(2)$

(C) $e^2\log_e(2)$ (D) $-e^2\log_e(2)$

Official Ans. by NTA (D)

Sol. $2ye^{x/y^2}dx+\left(y^2-4xe^{x/y^2}\right)dy=0$

$$2e^{x/y^2}[ydx-2xdy]+y^2dy=0$$

$$2e^{x/y^2}\left[\frac{y^2dx-x\cdot(2y)dy}{y}\right]+y^2dy=0$$

Divide by y^3

$$2e^{x/y^2}\left[\frac{y^2dx-x\cdot(2y)dy}{y^4}\right]+\frac{1}{y}dy=0$$

$$2e^{x/y^2}d\left(\frac{x}{y^2}\right)+\frac{1}{y}dy=0$$

Integrating

$$\int 2e^{x/y^2}d\left(\frac{x}{y^2}\right)+\int\frac{1}{y}dy=0$$

$$2e^{x/y^2}+\ell ny+c=0$$

(0, 1) lies on it.

$$2e^0+\ell n1+c=0\Rightarrow c=-2$$

Required curve : $\boxed{2e^{x/y^2}+\ell ny-2=0}$

For x (e)

$$2e^{x/e^2}+\ell ne-2=0\Rightarrow x=-e^2\log_e 2$$

11. Let the slope of the tangent to a curve y = f(x) at (x, y) be given by 2 tanx (cosx – y). if the curve passes through the point $\left(\pi/4,0\right)$, then the value of $\int_0^{\pi/2}ydx$ is equal to

(A) $\left(2-\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ (B) $2-\frac{\pi}{\sqrt{2}}$

(C) $\left(2+\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ (D) $2+\frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}=2\tan x\cos x-2\tan x\cdot y$

$$\frac{dy}{dx}+(2\tan x)y=2\sin x$$

Integrating factor $=e^{\int 2\tan x dx}=\frac{1}{\cos^2x}$

$$y\left(\frac{1}{\cos^2x}\right)=\int\frac{2\sin x}{\cos^2x}dx$$

$$y\sec^2x=\frac{2}{\cos x}+C$$

$y = 2\cos x + C\cos^2 x$

Passes through $\left(\frac{\pi}{4}, 0\right)$

$0 = \sqrt{2} + \frac{C}{2} \Rightarrow C = -2\sqrt{2}$

$f(x) = 2\cos x - 2\sqrt{2}\cos^2 x$: Required curve

$\int_0^{\pi/2} y dx = 2\int_0^{\pi/2} \cos x dx - 2\sqrt{2}\int_0^{\pi/2} \cos^2 x dx$

$= [2\sin x]_0^{\pi/2} - 2\sqrt{2}\left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2}$

$= 2 - \frac{\pi}{\sqrt{2}}$

12. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P(2, 3), intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to

(A) $\frac{110}{13}$ (B) $\frac{132}{13}$

(C) $\frac{142}{13}$ (D) $\frac{151}{13}$

Official Ans. by NTA (B)

Sol. Points A lies on L_2

$A\left(\alpha, 4 + \frac{4}{3}\alpha\right)$

Points B lies on L_1

$B\left(\beta, 2 - \frac{2}{5}\beta\right)$

Points P divides AB internally in the ratio 1 : 3

$\Rightarrow P(2,3) = P\left(\frac{3\alpha + \beta}{4}, \frac{3\left(4 + \frac{4}{3}\alpha\right) + 1\left(2 - \frac{2}{5}\beta\right)}{4}\right)$

$\Rightarrow \alpha = \frac{3}{13}, \beta = \frac{95}{13}$

Point $A\left(\frac{3}{13}, \frac{56}{13}\right), B\left(\frac{95}{13}, -\frac{12}{13}\right)$

Vertex C of triangle is the point of intersection of L_1 & L_2

$\Rightarrow C\left(-\frac{15}{13}, \frac{32}{13}\right)$

$$\text{area } \Delta ABC = \frac{1}{2}\left\|\begin{matrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & -\frac{12}{13} & 1 \\ -\frac{15}{13} & \frac{32}{13} & 1 \end{matrix}\right\|$$

$$= \frac{1}{2\times 13^3}\left\|\begin{matrix} 3 & 56 & 13 \\ 95 & -12 & 13 \\ -15 & 32 & 13 \end{matrix}\right\|$$

area $\Delta ABC = \frac{132}{13}$ sq. units.

13. Let a > 0, b > 0. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of 77a+ 44b is equal to

(A) 100 (B) 110

(C) 120 (D) 130

Official Ans. by NTA (D)

Sol. $e = \sqrt{1 + \frac{b^2}{a^2}}, \ \ell = \frac{2b^2}{a}$

Given $e^2 = \frac{11}{14}\ell$

$1 + \frac{b^2}{a^2} = \frac{11}{14}\cdot\frac{2b^2}{a}$

$\frac{a^2 + b^2}{a^2} = \frac{11}{7}\cdot\frac{b^2}{a}$ ......(1)

Also $e' = \sqrt{1+\frac{a^2}{b^2}}, \ \ell' = \frac{2a^2}{b}$

Given $(e')^2 = \frac{11}{8}\ell'$

$$1+\frac{a^2}{b^2} = \frac{11}{8}.\frac{2a^2}{b}$$

$$\frac{a^2+b^2}{b^2} = \frac{11}{4}.\frac{a^2}{b} \qquad ....(2)$$

New (1) ÷ (2)

$$\frac{b^2}{a^2} = \frac{4}{7}.\frac{b^3}{a^3}$$

$\therefore 7a = 4b$ …… (3)

From (2)

$$\frac{\frac{16b^2}{49}+b^2}{b^2} = \frac{11}{4}.\frac{16b^2}{49b}$$

$$\frac{65}{49} = \frac{11}{4}.\frac{16}{49}.b$$

$$\therefore b = \frac{4\times 65}{11\times 16} \qquad ..... (4)$$

We have to find value of

77a + 44b

11(7a + 4b) = 11(4b + 4b) = 11×8b

$\therefore$ Value of 11×8b $= 11\times 8\times\frac{4\times 65}{16\times 11} = 130$

14. Let $\vec{a} = \alpha\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b} = -2\hat{i}+\alpha\hat{j}+\hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2+4)}$, then the value of $2|\vec{a}|^2+(\vec{a}\cdot\vec{b})|\vec{b}|^2$ is equal to

(A) 10 (B) 7

(C) 9 (D) 14

Official Ans. by NTA (D)

Sol. $\vec{a} = \alpha\hat{i}+2\hat{j}-\hat{k}$, $\vec{b} = -2\hat{i}+\alpha\hat{j}+\hat{k}$,

area of parallelogram $= |\hat{a}\times\hat{b}|$

$$|\hat{a}\times\hat{b}| = \sqrt{(\alpha+2)^2+(\alpha-2)^2+(\alpha^2+4)^2}$$

Given $|\hat{a}\times\hat{b}| = \sqrt{15(\alpha^2+4)}$

$$2(\alpha^2+4)+(\alpha^2+4)^2 = 15(\alpha^2+4)$$

$$(\alpha^2+4)^2 = 13(\alpha^2+4)$$

$$\Rightarrow \alpha^2+4 = 13 \ \therefore \alpha^2 = 9$$

$$2|\vec{a}|^2+(\vec{a}.\vec{b})|\vec{b}|^2$$

$$|\vec{a}|^2 = \alpha^2+4+1 = \alpha^2+5$$

$$|\vec{b}|^2 = 4+\alpha^2+1 = \alpha^2+5$$

$$\vec{a}.\vec{b} = -2\alpha+2\alpha-1 = -1$$

$$\therefore 2|\vec{a}|^2+(\vec{a}.\vec{b})|\vec{b}|^2$$

$$2(\alpha^2+5)-1(\alpha^2+5) = \alpha^2+5 = 14$$

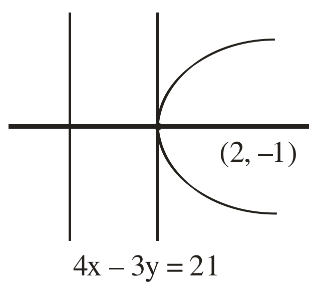
15. If vertex of a parabola is (2, – 1) and the equation of its directrix is 4x – 3y = 21, then the length of its latus rectum is

(A) 2 (B) 8

(C) 12 (D) 16

Official Ans. by NTA (B)

Sol.



$$a = \frac{|8+3-21|}{5} = \frac{10}{5} = 2$$

$\therefore$ latus rectum = 4a = 8

16. Let the plane ax + by + cz = d pass through (2, 3, –5) and is perpendicular to the planes 2x + y – 5z = 10 and 3x + 5y – 7z = 12.

If a, b, c, d are integers d > 0 and gcd (|a|, |b|, |c|, d) = 1, then the value of a + 7b + c + 20d is equal to

(A) 18 (B) 20

(C) 24 (D) 22

Official Ans. by NTA (D)

Sol. DR'S normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

$\therefore$ eq^n of plane

$18x - y + 7z = d$

It passes through (2, 3, –5)

$36 - 3 - 35 = d \qquad \therefore d = -2$

$\therefore$ Eq^n of plane

$18x - y + 7z = -2$

$-18x + y - 7z = 2$

$\therefore$ a= –18, b= 1, c = –7, d= 2

$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$

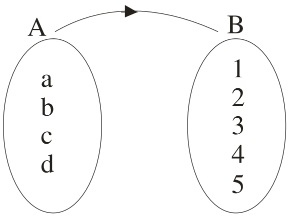
17. The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfies f(a) + 2f(b) – f(c) = f(d) is :

(A) $\frac{1}{24}$ (B) $\frac{1}{40}$

(C) $\frac{1}{30}$ (D) $\frac{1}{20}$

Official Ans. by NTA (D)

Sol.



$n(s) = {}^5C_4 \times 4! = 120$

f(a) +	2f(b) =	f(c) +	f(d)
5	2×1	3	4
4	2×2	3	5
1	2×3	2	5

$n(A) = 2! \times 3 = 6$

$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$

18. The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to

(A) 1 (B) 2

(C) 3 (D) 6

Official Ans. by NTA (C)

Sol. $T_r = \tan^{-1}\left[\frac{(r+2)-(r+1)}{1+(r+2)(r+1)}\right]$

$= \tan^{-1}(r+2) - \tan^{-1}(r+1)$

$T_1 = \tan^{-1}3 - \tan^{-1}2$

$T_2 = \tan^{-1}4 - \tan^{-1}3$

$T_n = \tan^{-1}(n+2) - \tan^{-1}(n+1)$

$S_n = \tan^{-1}(n+2) - \tan^{-1}2 = \tan^{-1}\left(\frac{n+2-2}{1+2(n+2)}\right)$

$= \tan^{-1}\left(\frac{n}{2n+5}\right)$

$\lim_{n\to\infty} 6\tan\left(\tan^{-1}\left(\frac{n}{2n+5}\right)\right)$

$= \lim_{n\to\infty} \frac{6n}{2n+5} = \frac{6}{2} = 3$

19. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Official Ans. by NTA (C)

Sol. $\left(\vec{a} \times (2\hat{i} + \hat{k})\right) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}\right)$

$= \left(2\hat{i} - 13\hat{j} - 4\hat{k}\right) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}\right)$

$$-(6+2)\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

Projection of $\vec{a}$ on vector $2\hat{i}+2\hat{j}+\hat{k}$ is

$$\vec{a}\cdot\frac{\left(2\hat{i}+2\hat{j}+\hat{k}\right)}{3} = \frac{5}{3}$$

20. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) – 7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Official Ans. by NTA (A)

Sol. $\cot\alpha = 1, \sec\beta = \frac{-5}{3}, \cos\beta = \frac{-3}{5}, \tan\beta = \frac{-4}{3}$

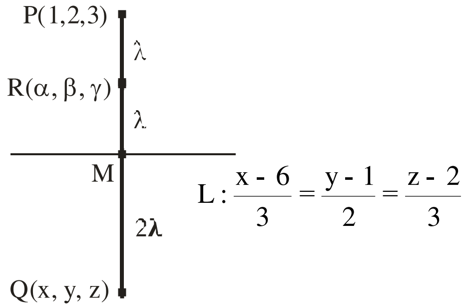
$$\tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$$

SECTION-B

1. Let the image of the point P(1, 2, 3) in the line $L: \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. let $R(\alpha,\beta,\gamma)$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha+\beta+\gamma)$ is equal to

Official Ans. by NTA (125)

Sol.



Let M be the mid-point of PQ

$\therefore M = (3\lambda+6,\ 2\lambda+1,\ 3\lambda+2)$

Now, $\overrightarrow{PM} = (3\lambda+5)\hat{i} + (2\lambda-1)\hat{j} + (3\lambda-1)\hat{k}$

$\because \overrightarrow{PM} \perp \left(3\hat{i}+2\hat{j}+3\hat{k}\right)$

$\therefore 3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1) = 0$

$\lambda = \frac{-5}{11}$

$\therefore M\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$

Since R is mid-point of PM

$22(\alpha+\beta+\gamma) = 125$

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Official Ans. by NTA (0)

)

Sol. $20 = \frac{\sum_{i=1}^{7}|x_i - 62|^2}{7}$

$\Rightarrow |x_1-62|^2 + |x_2-62|^2 + ... + |x_7-62|^2 = 140$

If $x_1 = 49$

$|49-62|^2 = 169$

then,

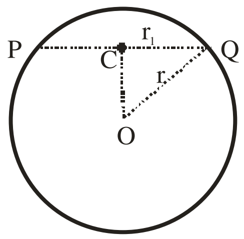
$|x_2-62|^2 + ... + |x_7-62|^2 =$ Negative Number

which is not possible, therefore, no student can fail.

3. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $\left(x-2\sqrt{2}\right)^2 + \left(y-2\sqrt{2}\right)^2 = r^2$, then the value of r^2 is equal to

Official Ans. by NTA (10)

Sol.



PQ is diameter of circle

$S: x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$

$C\left(\sqrt{2}, 3\sqrt{2}\right), O\left(2\sqrt{2}, 2\sqrt{2}\right)$

$r_1 = \sqrt{6}$

$S_1: \left(x-2\sqrt{2}\right)^2 + \left(y-2\sqrt{2}\right)^2 = r^2$

Now in ΔOCQ

$|OC|^2 + |CQ|^2 = |OQ|^2$

$4 + 6 = r^2$

$r^2 = 10$

4. If $\lim\limits_{x\to 1} \dfrac{\sin\left(3x^2 - 4x + 1\right) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of (a – b) is equal to

Official Ans. by NTA (11)

Sol. $\lim\limits_{x\to 1} \dfrac{\sin\left(3x^2 - 4x + 1\right) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$

For finite limit

$a + b - 5 = 0$ …(1)

Apply L'H rule

$\lim\limits_{x\to 1} \dfrac{\cos(3x^2 - 4x + 1)(6x - 4) - 2x}{(6x^2 - 14x + a)} = -2$

For finite limit

$6 - 14 + a = 0$

$\boxed{a = 8}$

From (1) $\boxed{b = -3}$

Now (a – b) = 11

5. Let for n = 1, 2, ….., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\dfrac{1}{(n+1)^2}$. Then the value of $\dfrac{1}{26} + \sum\limits_{n=1}^{50}\left(S_n + \dfrac{2}{n+1} - n - 1\right)$ is equal to

Official Ans. by NTA (41651)

$S_n = \dfrac{n^2}{1 - \dfrac{1}{(n+1)^2}} = \dfrac{n(n+1)^2}{(n+2)}$

$S_n = \dfrac{n(n^2 + 2n + 1)}{(n+2)}$

$S_n = \dfrac{n[n(n+2)+1]}{(n+2)}$

$S_n = n\left[n + \dfrac{1}{n+2}\right]$

$S_n = n^2 + \dfrac{n+2-2}{(n+2)}$

$S_n = n^2 + 1 - \dfrac{2}{(n+2)}$

Now $\dfrac{1}{26} + \sum\limits_{n=1}^{50}\left[(n^2 - n) - 2\left(\dfrac{1}{n+2} - \dfrac{1}{n+1}\right)\right]$

$= \dfrac{1}{26} + \left[\dfrac{50\times 51\times 101}{6} - \dfrac{50\times 51}{2} - 2\left(\dfrac{1}{52} - \dfrac{1}{2}\right)\right]$

$= 41651$

6. If the system of linear equations

$2x - 3y = \gamma + 5,$

$\alpha x + 5y = \beta + 1,$ where $\alpha, \beta, \gamma \in \mathbf{R}$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to

Official Ans. by NTA (58)

Sol. $2x - 3y = \gamma + 5$

$\alpha x + 5y = \beta + 1$

Infinite many solution

$\frac{\alpha}{2}=\frac{5}{-3}=\frac{\beta+1}{\gamma+5}$

$\alpha=\frac{-10}{3}, \quad 5\gamma+25=-3\beta-3$

$9\alpha=-30, \quad 3\beta+5\gamma=-28$

Now, $9\alpha+3\beta+5\gamma=-58$

$|9\alpha+3\beta+5\gamma|=58$

7. Let $A=\begin{pmatrix}1+i & 1\\ -i & 0\end{pmatrix}$ where $i=\sqrt{-1}$.

Then, the number of elements in the set

$\{n\in\{1,2,.....,100\}: A^n=A\}$ is

Official Ans. by NTA (25)

Sol. $A=\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}$

$A^2=\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}$

$A^2=\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}$

$A^4=\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}$

$A^4=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

$A^{4n+1}=A$

$n=1,5,9,.......,97$

$\Rightarrow$ total elements in the set is 25.

8. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z}=iz^2+z^2-z$ is equal to

Official Ans. by NTA (2)

Sol. $z+\bar{z}=iz^2+z^2$

Consider z = x + iy

$2x=(i+1)(x^2-y^2+2xyi)$

$\Rightarrow 2x=x^2-y^2-2xy$ and $x^2-y^2+2xy=0$

$\Rightarrow 2x=-4xy$

$\Rightarrow x=0$ or $y=\frac{-1}{2}$

Case 1 : $x=0\Rightarrow y=0$ here $z=0$

Case 2 : $y=\frac{-1}{2}$

$\Rightarrow 4x^2-4x-1=0$

$(2x-1)^2=2$

$2x-1=\pm\sqrt{2}$

$x=\frac{1\pm\sqrt{2}}{2}$

Here $z=\frac{1+\sqrt{2}}{2}-\frac{i}{2}$ or $z=\frac{1-\sqrt{2}}{2}-\frac{i}{2}$

Sum of squares of modulus of z

$=0+\frac{(1+\sqrt{2})^2+1}{4}+\frac{(1-\sqrt{2})^2+1}{4}=\frac{8}{4}=2$

9. Let S = {1, 2, 3, 4}. Then the number of elements in the set $\{f: S\times S\to S : f$ is onto and $f(a, b) = f(b, a) \geq a\ \forall (a,b)\in S\times S\}$ is

Official Ans. by NTA (37)

Sol. (1, 1), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4) – all have one choice for image.

(2, 1), (1, 2), (2, 2) – all have three choices for image

(3, 2), (2, 3), (3, 1), (1, 3), (3, 3) – all have two choices for image.

So the total functions = 3 × 3 × 2 × 2 × 2 = 72

Case 1 : None of the pre-images have 3 as image

Total functions = 2 × 2 × 1 × 1 × 1 = 4

Case 2 : None of the pre-images have 2 as image

Total functions = 2 × 2 × 2 × 2 × 2 = 32

Case 3 : None of the pre-images have either 3 or 2 as image

Total functions = 1 × 1 × 1 × 1 × 1 = 1

$\therefore$ Total onto functions = 72 – 4 – 32 + 1 = 37

10. The maximum number of compound propositions, out of

$p \vee r \vee s,\ p \vee r \vee \sim s,\ p \vee \sim q \vee s,$

$\sim p \vee \sim r \vee s,\ \sim p \vee \sim r \vee \sim s,\ \sim p \vee q \vee \sim s,$

$q \vee r \vee \sim s,\ q \vee \sim r \vee \sim s,\ \sim p \vee \sim q \vee \sim s$

that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

Official Ans. by NTA (9)

Sol. If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Two balls A and B are placed at the top of 180 m tall tower. Ball A is released from the top at t = 0 s. Ball B is thrown vertically down with an initial velocity 'u' at t = 2 s. After a certain time, both balls meet 100 m above the ground. Find the value of 'u' in ms^{-1}. [use g = 10 ms^{-2}] :

(A) 10 (B) 15
(C) 20 (D) 30

Official Ans. by NTA (D)

Sol. Let they meet at time t.

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 80}{10}}$

= 4 sec

Time taken by ball B to meet A = 2 sec

using $S = ut + \frac{1}{2}at^2$

$-80 = -u \times 2 + \frac{1}{2}(-10)(2)^2$

u = 30

2. A body of mass M at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be :

(A) 15 ms^{-1} (B) 25 ms^{-1}
(C) 35 ms^{-1} (D) 50 ms^{-1}

Official Ans. by NTA (B)

Sol. Mass of pieces by $\frac{M}{4}, \frac{M}{4}, \frac{M}{2}$

conserving momentum

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$

$\vec{P}_3 = -\left(\vec{P}_1 + \vec{P}_2\right)$

As $\vec{P}_1$ & $\vec{P}_2$ are perpendicular

so $P_3 = \sqrt{P_1^2 + P_2^2}$

$P_3 = (50)\frac{M}{4}$

& $P_3 = \frac{M}{2}v$

so v = 25

3. The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci?

(A) 30 days (B) 35 days
(C) 40 days (D) 25 days

Official Ans. by NTA (B)

Sol. $\frac{A}{A_0} = \frac{N}{N_0}$

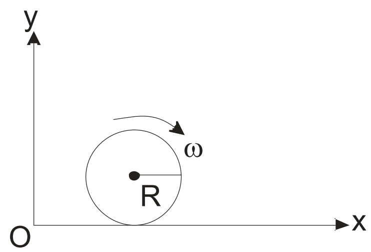
$\frac{2 \times 10^{-5}}{2.56 \times 10^{-3}} = \frac{N}{N_0}$

$\frac{N}{N_0} = \frac{1}{128} \Rightarrow N = \frac{N_0}{128}$

After 7 half life activity comes down to given value $T = 7 \times 5$

= 35 days

4. A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3}R^2\omega$. The value of a will be :



(A) 2 (B) 3
(C) 5 (D) 4

Official Ans. by NTA (C)

Sol. L_0 = angular momentum of shell about O.

As shell is rolling

so $V_{cm} = \omega R$

$L_0 = mV_{cm}R + I\omega$

$= 1 \times \omega R \times R + \frac{2}{3}R^2\omega$

$= \frac{5}{3}R^2\omega$

so a = 5

5. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be :

(Given gas constant R = 8.3 JK^{-1}-mol^{-1})

(A) 249 J (B) 415 J

(C) 498 J (D) 830 J

Official Ans. by NTA (C)

Sol. No of moles $= \frac{44.8}{22.4} = 2$

Gas is mono atomic so $C_V = \frac{3}{2}R$

$\Delta Q = nC_V\Delta T$

$= 2 \times \frac{3}{2}R(20)$

= 60 R

= 60 × 8.3

= 498 J

6. A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to :

(A) $\sqrt{L_1L_2}$ (B) $\frac{L_1 + L_2}{2}$

(C) $2L_1 - L_2$ (D) $3L_1 - 2L_2$

Official Ans. by NTA (C)

Sol. By Hooke's Law

so F α ΔL

$\frac{F_1}{F_2} = \frac{\Delta L_1}{\Delta L_2}$

$\frac{10}{20} = \frac{(L_1 - L)}{(L_2 - L)}$

$L = 2L_1 - L_2$

7. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The photoelectric effect does not take place, if the energy of the incident radiation is less than the work function of a metal.

Reason R : Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct

Official Ans. by NTA (B)

Sol. To free the electron from metal surface minimum energy required, is equal to the work function of that metal.

So Assertion A, is correct.

$h\nu = w_0 + K.E._{max}$

if $h\nu = w_0$

$\Rightarrow K.E._{max} = 0$

Hence reason R, is correct, But R is not the correct explanation of A.

8. A particle of mass 500 gm is moving in a straight line with velocity $\upsilon = b\,x^{5/2}$. The work done by the net force during its displacement from x = 0 to x = 4 m is : (Take b = 0.25 $m^{-3/2}\,s^{-1}$).

(A) 2 J (B) 4 J

(C) 8 J (D) 16 J

Official Ans. by NTA (D)

Sol. By work energy theorem

work done by net force = ΔK.E.

$\Rightarrow w = \frac{1}{2}m\upsilon_f^2 - \frac{1}{2}m\upsilon_i^2$

$w = \frac{1}{2} \times 0.5 \times (0.25)^2 \times (4)^5$

$\boxed{w = 16J}$ (D)

9. A charged particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charged particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charged particle :

(A) 1 : 1 (B) 1 : 2

(C) 2 : 1 (D) 1 : 4

Official Ans. by NTA (C)

Sol. radius of paerticle in cyclotron

$$r = \frac{\sqrt{2mK.E.}}{qB}$$

So ratio of new radius to original

$$\frac{r_n}{r_0} = \sqrt{\frac{(K.E.)_n}{(K.E)_0}} = \sqrt{4} \Rightarrow 2:1 \ \ (C)$$

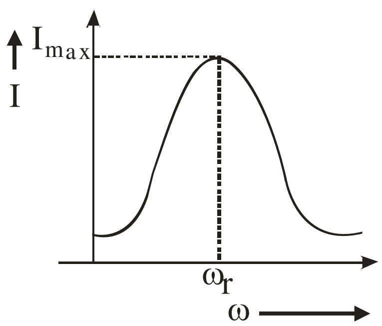
10. For a series LCR circuit, I vs ω curve is shown :

(a) To the left of ω_r, the circuit is mainly capacitive.

(b) To the left of ω_r, the circuit is mainly inductive.

(c) At ω_r, impedance of the circuit is equal to the resistance of the circuit.

(d) At ω_r, impedance of the circuit is 0.



Choose the **most appropriate** answer from the options given below :

(A) (a) and (d) only (B) (b) and (d) only

(C) (a) and (c) only (D) (b) and (c) only

Official Ans. by NTA (C)

Sol. at ω_r, $X_C = X_L$

$$\Rightarrow \frac{1}{\omega_r C} = \omega_r L$$

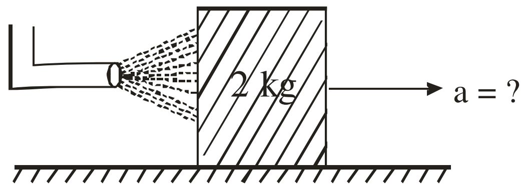
So if $\omega < \omega_r$ then x_C will increase and X_L will decrease.

Hence to left of ω_r circuit is capacitive

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

at ω_r, $Z = \sqrt{R^2 + O^2} = R$ (C)

11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be :



Plane

(A) 3 (B) 6

(C) 5 (D) 4

Official Ans. by NTA (C)

Sol. $F = \frac{dp}{dt} = \upsilon \frac{dm}{dt}$

$\Rightarrow Ma = 10 \times 1$

$\Rightarrow 2a = 10$

$a = 5m/sec^2$

12. In Vander Waals equation $\left[P + \frac{a}{V^2}\right][V - b] = RT$; P is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to :

(A) $\frac{P}{V}$ (B) $\frac{V}{P}$

(C) PV (D) PV^3

Official Ans. by NTA (C)

Sol. By principle of homogenity

$[P] = \left[\frac{a}{v^2}\right]$ and [b] = [v]

$$\Rightarrow \left[\frac{a}{b}\right] = [PV] \ \ (C)$$

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A}+\vec{B}$ is equal to two times the magnitude of $\vec{A}-\vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be :

(A) $\sin^{-1}\left(\frac{3}{5}\right)$ (B) $\sin^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{5}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

Official Ans. by NTA (C)

Sol. $(a^2 + b^2 + 2ab\cos\theta) = 4(a^2 + b^2 - 2ab\cos\theta)$

put a = b we get

$2a^2 + 2a^2\cos\theta = 8a^2 - 8a^2\cos\theta$

$\cos\theta = \frac{3}{5}$

14. The escape velocity of a body on a planet 'A' is 12 kms^{-1}. The escape velocity of the body on another planet 'B', whose density is four times and radius is half of the planet 'A', is :

(A) 12 kms^{-1} (B) 24 kms^{-1}

(C) 36 kms^{-1} (D) 6 kms^{-1}

Official Ans. by NTA (A)

Sol. $V_{escape} = \sqrt{\frac{2Gm}{R}} \Rightarrow \sqrt{\frac{2G\rho \times \frac{4}{3}\pi R^3}{R}}$

$V_{escape} \propto \sqrt{\rho R^2}$

$\therefore$ if ρ is 4 times and Radius is halved.

$\Rightarrow V_{escape}$ will remain same $\therefore$ Ans (A)

15. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is :

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$

(C) $\sqrt{3}$ (D) 1

Official Ans. by NTA (A)

Sol. $B_H = B\cos\theta$

$\therefore B = \frac{B_H}{\cos\theta} = \frac{0.5G}{\cos 30°} \Rightarrow \frac{G}{\sqrt{3}}$

16. A longitudinal wave is represented by $x = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to :

(A) 2π (B) 5π

(C) π (D) $\frac{5\pi}{2}$

Official Ans. by NTA (B)

Sol. $V_P \max = 4V_{wave}$

$\omega A = 4\left(\frac{\omega}{k}\right) \Rightarrow A = \frac{4\lambda}{2\pi}$

$\lambda = \frac{2\pi A}{4} \Rightarrow \frac{20\pi}{4} \Rightarrow 5\pi$

17. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will :

(A) increase by 50% (B) decrease by 15%

(C) increase by 25% (D) increase by 33%

Official Ans. by NTA (A)

Sol. $E \Rightarrow \frac{1}{2}(KC)v^2$

$\therefore$ % change

$$\Rightarrow \frac{\frac{1}{2}K_2CV^2 - \frac{1}{2}K_1CV^2}{\frac{1}{2}K_1CV^2} = \frac{K_2 - K_1}{K_1} \times 100$$

$$\Rightarrow \frac{15-10}{10} \times 100 = 50\%$$

18. A positive charge particle of 100 mg is thrown in opposite direction to a uniform electric field of strength 1×10^5 NC^{-1}. If the charge on the particle is 40 μC and the initial velocity is 200 ms^{-1}, how much distance it will travel before coming to the rest momentarily :

(A) 1 m (B) 5 m

(C) 10 m (D) 0.5 m

Official Ans. by NTA (D)

Sol. Distance travelled by particle before stopping

$$\frac{V^2}{2a} = S \Rightarrow \frac{v^2 m}{2qE} \Rightarrow \frac{(200)^2 \times 100 \times 10^{-6}}{2 \times 40 \times 10^{-6} \times 10^5} = 0.5m$$

19. Using Young's double slit experiment, a monochromatic light of wavelength 5000 Å produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000Å is used and the separation between the slits is doubled, then the new fringe width will be :

(A) 0.5 mm (B) 1.0 mm

(C) 0.6 mm (D) 0.3 mm

Official Ans. by NTA (D)

Sol. Fringe width $\beta = \frac{D\lambda}{d}$

$\lambda_1 = 5000$ Å

$$\beta_1 = \frac{D}{d}(5000\times10^{-10}) = 5\times10^{-4}\text{m} \quad \ldots.(I)$$

$$\beta_2 = \frac{D}{(2d)}(6000\times10^{-10}) = x \ \ (\text{let}) \ldots.(II)$$

Divide (II) & (I)

$$\frac{\beta_2}{\beta_1} = \frac{3000\times10^{-10}}{5000\times10^{-10}} = \frac{x}{5\times10^{-4}}$$

$x = 3 \times 10^{-4}$ m or 0.3 mm

20. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission :

(A) 375×10^7 (B) 75×10^7

(C) 375×10^8 (D) 75×10^9

Official Ans. by NTA (B)

Sol. Frequency at 1000 nm = $\frac{3\times10}{1000\times10^{-9}} \Rightarrow 3\times10^{14}\text{Hz}$

available for channel band width

$$= \frac{2}{100}\times3\times10^{14} \Rightarrow 6\times10^{12}\text{Hz}$$

Bandwidth for 1 channel = 8000 Hz

∴ No. of channel

$$= \frac{6\times10^{12}}{8\times10^3} \Rightarrow \frac{600}{8}\times10^7 = 75\times10^7$$

SECTION-B

1. Two coils require 20 minutes and 60 minutes respectively to produce same amount of heat energy when connected separately to the same source. If they are connected in parallel arrangement to the same source; the time required to produce same amount of heat by the combination of coils, will be__________min.

Official Ans. by NTA (15)

Sol. $\frac{dQ}{dt} = i^2R = \frac{V^2}{R}$ (we know)

$\Rightarrow$ In 't' time , $\Delta Q = \left(\frac{V^2}{R}\right)t$

Given that, (for same source, v = same)

$$Q_0 = \frac{v^2}{R_1}\times20 = \frac{V^2}{R_2}\times60 \quad \ldots.(1)$$

$$\Rightarrow \boxed{R_2 = 3R_1} \quad \ldots..(ii)$$

If they are connected in parallel then

$$\text{Req} = \frac{R_2R_1}{R_1+R_2} = \frac{3R_1.R_1}{3R_1+R_1} = \left(\frac{3R_1}{4}\right)$$

To produce same heat, using equation …(1)

$$Q_0 = \frac{V^2}{R_1}\times20 = \frac{v^2}{\left(\frac{3R_1}{4}\right)}\times t$$

$$t = \frac{3\times20}{4} = 15\,\text{min}$$

2. The intensity of the light from a bulb incident on a surface is 0.22 W/m^2. The amplitude of the magnetic field in this light-wave is______$\times10^{-9}$ T.

(Given : Permittivity of vacuum $\epsilon_0 = 8.85\times10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}$, speed of light in vacuum c = 3×10^8 ms^{-1})

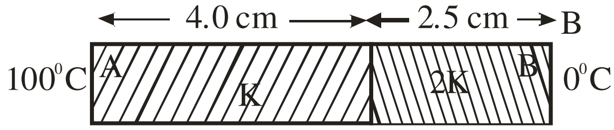
Official Ans. by NTA (43)

Sol. $I = \left(\frac{1}{2}\varepsilon_0E_0^2\right)C$

$$\Rightarrow E_0 \Rightarrow \sqrt{\frac{2I}{\varepsilon_0C}} \Rightarrow \sqrt{\frac{2\times0.22}{8.85\times10^{-12}\times3\times10^8}} = 12.873$$

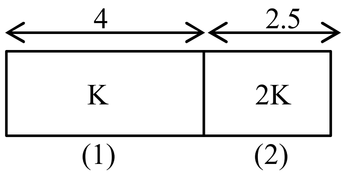
$$B \Rightarrow \frac{E_0}{C} \Rightarrow \frac{12.873}{3\times10^8} = 4.291\times10^{-8} = 43\times10^{-9}$$

3. As per the given figure, two plates A and B of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $\left(1+\frac{5}{\alpha}\right)$K, then the value of α will be ________.



Official Ans. by NTA (21)

Sol.



$$\frac{\Delta Q}{\Delta t}=\left(\frac{1}{R}\right)\Delta T$$

R : Thermal resistivity

$$\therefore R_1=\frac{L_1}{K_1A}=\frac{L_1}{K(120)}$$

$L_1 = 4$ cm

$A = 120\ cm^2$

$$R_2=\frac{2.5}{(2K)(120)}$$

Now, R_{eq} of this series combination

$R_{eq} = R_1 + R_2$

where $L_{eq} = 4 + 2.5 = 6.5$

$$\frac{L_{eq}}{K_{eq}(A)}=\frac{4}{K(120)}+\frac{\frac{5}{2}}{2K(120)}$$

$$\frac{6.5}{K_{eq}(120)}=\frac{4}{K(120)}+\frac{5}{4K(120)}$$

$$\frac{6.5}{K_{eq}}=\frac{21}{4K}$$

$$K_{eq}=\frac{26}{21}K=\left(1+\frac{5}{21}\right)K$$

$\therefore\ a = 21$

4. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is__________.

Official Ans. by NTA (700)

Sol. A = 10 cm

$\therefore$ Total Energy $=\frac{1}{2}KA^2$

By energy conservation we can final v at x = 5

$$\frac{1}{2}K(10)^2=\frac{1}{2}K(5)^2+\frac{1}{2}mv^2$$

$$V=\sqrt{\frac{75K}{m}}$$

Now, velocity is tripled through external mean so the amplitude of SHM will charge and so the total energy, (but potential) energy at this moment will remain same)

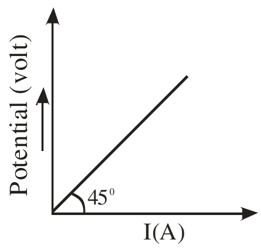
$$\therefore\frac{1}{2}K(5)^2+\frac{1}{2}m\left(3\sqrt{\frac{75K}{m}}\right)^2=\frac{1}{2}KA^2$$

$\Rightarrow 25\ K + 675\ K = KA^2$

$\therefore A=\sqrt{700}$

$\therefore\ x = 700$

5. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as $x \times 10^{-3}\ \Omega$ cm. The value of x is __________. [Take $\pi = 3.14$]



Official Ans. by NTA (144)

Sol. $1=\rho\frac{\ell}{A}$

$1=\frac{\rho\times 31.4}{\frac{\pi(2.4)^2}{4}}$

$\frac{\pi(2.4)^2}{4}=\rho\times 314$

$\frac{2.4\times 2.4}{4}=\rho\times 10$

$\frac{0.6\times 2.4}{10}=\rho$

$\frac{1.44}{10}=\rho$

$0.144 = \rho$

$144 \times 10^{-3} = \rho$

6. 300 cal. of heat is given to a heat engine and it rejects 225 cal. of heat. If source temperature is 227°C, then the temperature of sink will be __ ^{0}C.

Official Ans. by NTA (102)

Sol. $1-\frac{Q_2}{Q_1}=1-\frac{T_2}{T_1}$

$\frac{Q_2}{Q_1}=\frac{T_2}{T_1}$

$\frac{225}{300}=\frac{T_2}{500}$

$\frac{500\times 225}{300}=T_2$

$375 = T_2$

$102°C = T_2$

7. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles 60° and 90° respectively, when an α particle is approaching a gold nucleus. For $d_1 = x\, d_2$, the value of x will be_______.

Official Ans. by NTA (3)

Sol. $\sqrt{d}\propto \cot\frac{\theta}{2}$

$\cot^2 30° = x \cot^2 45°$

$3 = x$

8. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by 100 μA, it brings a change of 10 mA in collector current. If the load resistance is 2 kΩ and input resistance is 1 kΩ, the value of power gain is $x \times 10^4$. The value of x is _______.

Official Ans. by NTA (2)

Sol. $\Delta i_B = 100\ \mu A$ $\qquad \beta=\frac{\Delta i_C}{\Delta i_B}$

$\Delta i_C = 10$ mA

$\text{power} = \beta^2\times\frac{R_0}{R_{in}}$

$\text{Power}=\left(\frac{10}{0.1}\right)^2\times\frac{2}{1}$

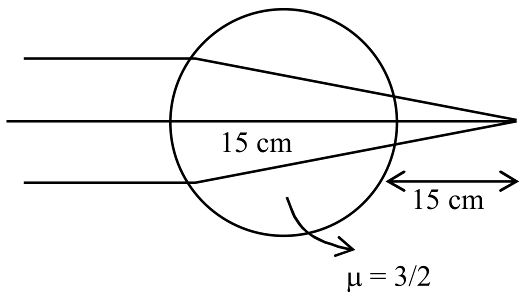
Power = 100 × 100 × 2

Gain = 2×10^4

9. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30 cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is _________ mm.

Official Ans. by NTA (225)

Sol.



$\frac{\frac{3}{2}}{V}-\frac{1}{\infty}=\frac{\frac{3}{2}-1}{15}$

$\frac{3}{2V}=\frac{1}{30}$

V = 45 cm

$$\frac{1}{V}-\frac{\frac{3}{2}}{15}=\frac{1-\frac{3}{2}}{\frac{2}{-15}}$$

$$\frac{1}{V}-\frac{1}{10}=\frac{1}{30}$$

$$\frac{1}{V}=\frac{1}{10}+\frac{1}{30}=\frac{4}{30}$$

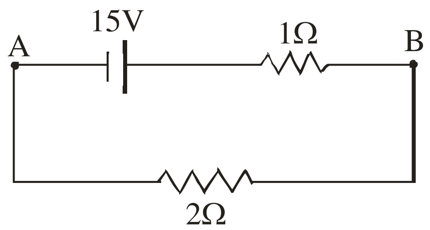
$$\boxed{V=7.5}$$

$$\boxed{V=22.5}$$

v = 225 mm

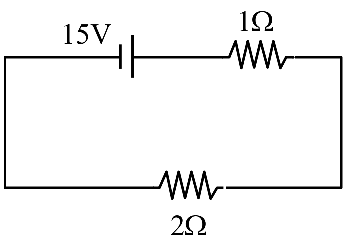
10. For the network shown below, the value $V_B - V_A$ is

________V.



Official Ans. by NTA (10)

Sol.



$$i=\frac{15}{3}=5A$$

15 – 5 (1) = 10 Volt

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. Production of iron in blast furnace follows the following equation

$Fe_3O_4(s) + 4CO(g) \rightarrow 3Fe(l) + 4CO_2(g)$

when 4.640 kg of Fe_3O_4 and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is :

[Given : Molar Atomic mass (g mol $^{-1}$): Fe = 56

Molar Atomic mass (g mol^{-1}) : 0 = 16

Molar Atomic mass (g mol^{-1}): = C = 12

(A) 1400 (B) 2200

(C) 3360 (D) 4200

Official Ans. by NTA (C)

Sol. Moles of $Fe_3O_4 = \frac{4.640 \times 10^3}{232} = 20$

Moles of $CO = \frac{2.52 \times 10^3}{28} = 90$

So limiting Reagent = Fe_3O_4

So moles of Fe formed = 60

Weight of Fe = 60 × 56 = 3360 gms

2. Which of the following statements are **correct** ?

(A) The electronic configuration of Cr is [Ar] $3d^5$ $4s^1$.

(B) The magnetic quantum number may have a negative value.

(C) In the ground state of an atom, the orbitals are filled in order of their increasing energies.

(D) The total number of nodes are given by n – 2 .

Choose the **most appropriate** answer from the options given below :

(A) (A), (C) and (D) only

(B) (A) and (B) only

(C) (A) and (C) only

(D) (A), (B) and (C) only

Official Ans. by NTA (D)

Sol. (A) Cr = $[Ar]3d^5 4s^1$

(B) m = $-\ell$ to $+\ell$

(C) According to Aufbau principle, orbitals are filled in order of their increasing energies.

(D) Total nodes = n – 1

3. Arrange the following in the decreasing order of their covalent character :

(A) LiCl

(B) NaCl

(C) KCl

(D) CsCl

Question: Choose the **most appropriate** answer from the options given below :

(A) (A)> (C) > (B)> (D)

(B) (B) > (A) > (C) > (D)

(C) (A) > (B) > (C) > (D)

(D) (A) > (B) > (D) > (C)

Official Ans. by NTA (C)

Sol. LiCl > NaCl > KCl > CsCl (Covalent character)

4. The solubility of AgCl will be maximum in which of the following ?

(A) 0.01 M KCl

(B) 0.01 M HC1

(C) 0.01 M $AgNO_3$

(D) Deionised water

Official Ans. by NTA (D)

Sol. In deionized water no common ion effect will take place so maximum solubility

5. Which of the following is a **correct** statement ?

(A) Brownian motion destabilises sols.

(B) Any amount of dispersed phase can be added to emulsion without destabilising it.

(C) Mixing two oppositely charged sols in equal amount neutralises charges and stabilises colloids.

(D) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

Official Ans. by NTA (D)

Sol. As equal & similar charge particle will repel each other, hence will never precipitate.

6. The electronic configuration of Pt (atomic number 78) is:

(A) [Xe] $4f^{14}\,5d^{9}\,6s^{1}$

(B) [Kr] $4f^{14}\,5d^{10}$

(C) [Xe] $4f^{14}\,5d^{10}$

(D) [Xe] $4f^{14}\,5d^{8}\,6s^{2}$

Official Ans. by NTA (A)

Sol. $_{78}Pt$ = [Xe] $4f^{14}$ $5d^{9}$ $6s^{1}$ (Exceptional electronic configuration)

7. In isolation of which one of the following metals from their ores, the use of cyanide salt is not commonly involved ?

(A) Zinc

(B) Gold

(C) Silver

(D) Copper

Official Ans. by NTA (D)

Sol. For ZnS, KCN is used as depressant.

For Gold and silver $\Rightarrow$ leaching [Cyanide process]

8. Which one of the following reactions indicates the reducing ability of hydrogen peroxide in basic medium ?

(A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(C) $2MnO_4^- + 3H_2O_2 \rightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$

(D) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

Official Ans. by NTA (C)

Sol. In option (A) and (C) reducing action of hydrogen peroxide is shown.

In option (A) it is in acidic medium, in option (B) it is in basic medium.

or

For reducing ability H_2O_2 changes to O_2 , i.e. oxidize , so in option 'A' & 'C' O_2 is formed but 'A' is in acidic medium so option - C correct.

9. Match the **List-I** with **List- II.**

List-I (Metal)	**List-II (Emitted light wavelength (nm))**
(A) Li	(I) 670.8
(B) Na	(II) 589.2
(C) Rb	(III) 780.0
(D) Cs	(IV) 455.5

Choose the **most appropriate** answer from the options given below:

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(C) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Official Ans. by NTA (A)

Sol. NCERT Table 10.1.5

Metal	**Li**	**Na**	**K**	**Rb**	**Cs**
Colour	Crimson red	Yellow	Violet	Red Violet	Blue
λ/nm	670.8	589.2	766.5	780.0	455.5

10. Match the **List-I** with **List- II.**

List-I (Metal)	List-II Application
(A) Cs	(I) High temperature thermometer
(B) Ga	(II) Water repellent sprays
(C) B	(III) Photoelectric cells
(D) Si	(IV) Bullet proof vest

Choose the most appropriate answer from the option given below:

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Official Ans. by NTA (A)

Sol. Caesium is used in devising photoelectric cells.

Boron fibres are used in making bullet–proof vest.

Silicones being surrounded by non–polar alkyl groups are water repelling in nature.

Gallium is less toxic and has a very high boiling point, so it is used in high temperature thermometers.

11. The oxoacid of phosphorus that is easily obtained from a reaction of alkali and white phosphorus and has two P-H bonds, is :

(A) Phosphonic acid

(B) Phosphinic acid

(C) Pyrophosphorus acid

(D) Hypophosphoric acid

Official Ans. by NTA (B)

Sol. $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

oxoacid = H_3PO_2 (hypo phosphorus acid) or (phosphinic acid)

12. The acid that is believed to be mainly responsible for the damage of Taj Mahal is

(A) Sulfuric acid (B) Hydrofluoric acid

(C) Phosphoric acid (D) Hydrochloric acid

Official Ans. by NTA (A)

Sol. $CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + H_2O + CO_2$

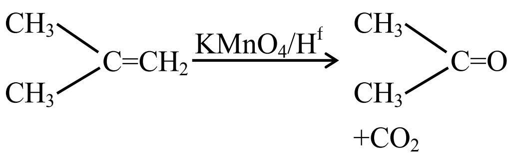
13. Two isomers 'A' and 'B' with molecular formula C_4H_8 give different products on oxidation with $KMnO_4$ in acidic medium. Isomer 'A' on reaction with $KMnO_4/H^+$ results in effervescence of a gas and gives ketone. The compound 'A' is

(A) But-1-ene (B) cis-But-2-ene

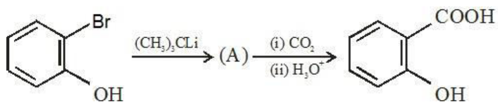
(C) trans-But-2ene (D) 2-methyl propene

Official Ans. by NTA (D)

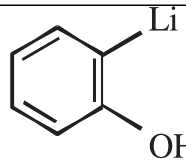
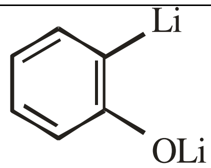
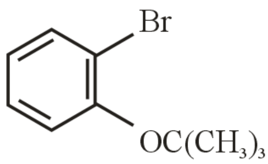
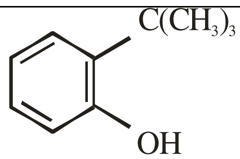
Sol.



14.

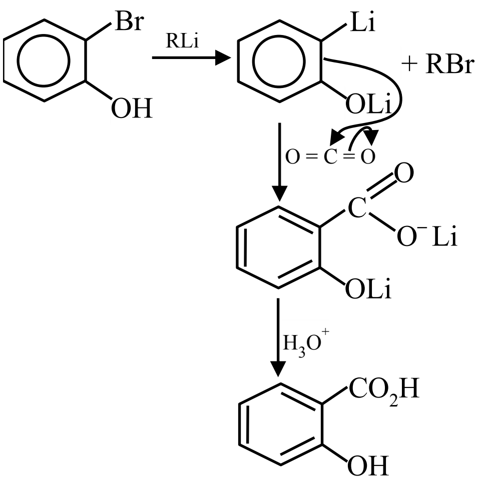


In the given conversion the compound A is:

(A)	
(B)	
(C)	
(D)	

Official Ans. by NTA (B)

Sol.



15. Given below are two statements :

Statement I : The esterification of carboxylic acid with an alcohol is a nucleophilic acyl substitution.

Statement II : Electron withdrawing groups in the carboxylic acid will increase the rate of esterification reaction.

Choose the **most appropriate** option :

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

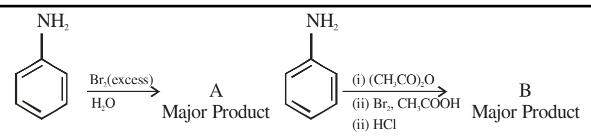
Official Ans. by NTA (A)

Sol. $R{-}OH + R{-}\overset{O}{\overset{\|}{C}}{-}OH \longrightarrow R{-}O{-}\overset{O}{\overset{\|}{C}}{-}R$

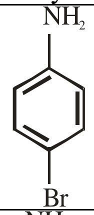
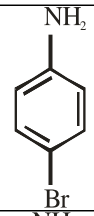
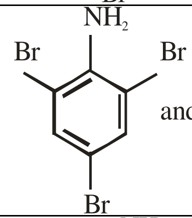
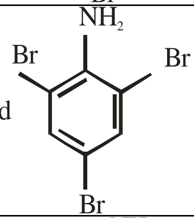
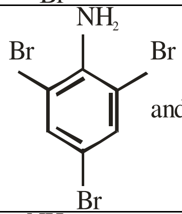
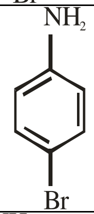
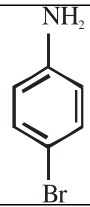
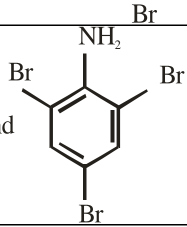
nucleophilic acyl substitution

electron with drawing group on carboxylic acid will increase the rate of esterification

16.

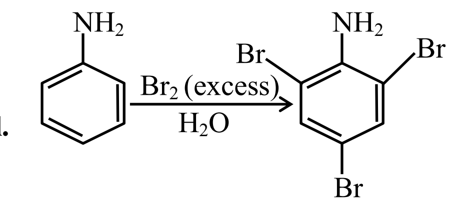


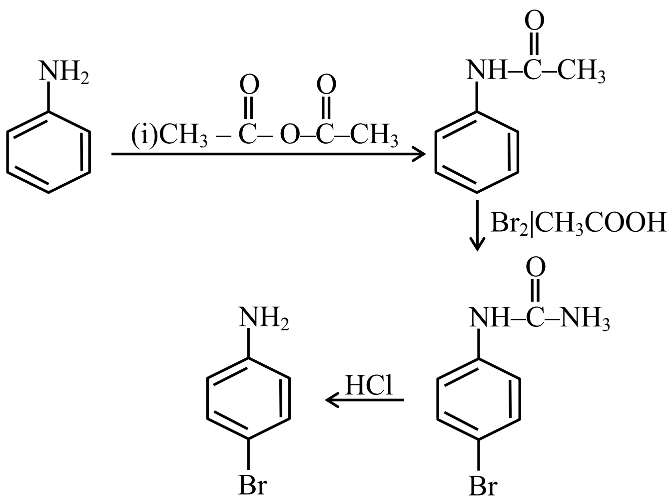
Consider the above reaction, the product A and product B respectively are

(A)	 and 
(B)	 and 
(C)	 and 
(D)	 and 

Official Ans. by NTA (C)

Sol.





17. The polymer, which can be stretched and retains its original status on releasing the force is

(A) Bakelite (B) Nylon 6,6

(C) Buna-N (D) Terylene

Official Ans. by NTA (C)

Buna – N is synthetic rubber which can be stretched and retains its original status on releasing the force.

18. Sugar moiety in DNA and RNA molecules respectively are

(A) β-D-2-deoxyribose, β -D-deoxyribose

(B) β-D-2-deoxyribose, β -D-ribose

(C) β-D-ribose, β -D-2-deoxyribose

(D) β-D-deoxyribose, β -D-2-deoxyribose

Official Ans. by NTA (B)

Sol. DNA contains $\Rightarrow \beta - D - 2 -$ deoxyribose

RNA contains $\Rightarrow \beta - D -$ ribose

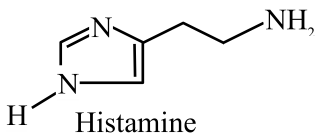
19. Which of the following compound **does not** contain sulphur atom ?

(A) Cimetidine (B) Ranitidine

(C) Histamine (D) Saccharin

Official Ans. by NTA (C)

Sol.



Histamine

Histamine is nitrogenous compound it does not contain sulpher.

20. Given below are two statements.

Statement I : Phenols are weakly acidic.

Statement II : Therefore they are freely soluble in NaOH solution and are weaker acids than alcohols and water.

Choose the **most appropriate** option:

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (C)

Sol. Phenol are weakly acidic. Phenol is more acidic than alcohol & H_2O statement (I) is correct. (II) is incorrect.

SECTION-B

1. Geraniol, a volatile organic compound, is a component of rose oil. The density of the vapour is 0.46 gL^{-1} at 257°C and 100 mm Hg. The molar mass of geraniol is ___________ (Nearest Integer)

[Given R = 0.082 L atm K^{-1} mol^{-1}]

Official Ans. by NTA (152)

Sol. Assuming ideal behaviour $P = \frac{dRT}{M}$

$P = \frac{100}{760}$ atm, $T = 257 + 273 = 530$ K

$d = 0.46$ gm/L

So $M = \frac{0.46 \times 0.082 \times 530}{100} \times 760$

$= 151.93 \approx 152$

2. 17.0 g of NH_3 completely vapourises at – 33.42°C and 1 bar pressure and the enthalpy change in the process is 23.4 kJ mol^{-1}. The enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is ________ kJ.

Official Ans. by NTA (117)

Sol. Given data is for 1 moles and asked for 5 moles so value is 23.4 × 5 = 117 kJ

3. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C. The percentage of dissociation of the acid is ________ . (Nearest integer)

[Given : Density of acetic acid is 1.02 g mL^{-1}
Molar mass of acetic acid is 60 g mol^{-1}
K_f (H_2O) =1.85 K kg mol^{-1}]

Official Ans. by NTA (5)

Sol. M = d × V = 1.02 × 1.2 = 1.224 gm

Moles of acetic acid = 0.0204 moles in 2L

So molality = 0.0102 mol/kg

Now $\Delta T_f = i \times K_f \times M$

$i = 1 + \alpha$ for acetic acid

$0.0198 = (1 + \alpha) \times 1.85 \times 0.0102$

$\alpha = 0.04928$

≅ 5%

4. A dilute solution of sulphuric acid is electrolysed using a current of 0.10 A for 2 hours to produce hydrogen and oxygen gas. The total volume of gases produced at STP is ________ cm^3. (Nearest integer) [Given : Faraday constant F = 96500 C mol^{-1} at STP, molar volume of an ideal gas is 22.7 L mol^{-1}]

Official Ans. by NTA (127)

Sol. At anode

$2H_2O \rightarrow O_2(g) + 4H^+ + 4e^-$

At cathode

$2H^+ + 2e^- \rightarrow H_2(g)$

Now number of gm eq. $= \dfrac{i \times t}{96500}$

$$= \frac{0.1 \times 2 \times 60 \times 60}{96500}$$

$= 0.00746$

$$V_{O_2} = \frac{0.00746}{4} \times 22.7 = 0.0423$$

$$V_{H_2} = \frac{0.00746}{2} \times 22.7 = 0.0846$$

$V_{Total} \approx$ 127 ml or cc

5. The activation energy of one of the reactions in a biochemical process is 532611 J mol^{-1}. When the temperature falls from 310 K to 300 K, the change in rate constant observed is $k_{300} = x \times 10^{-3}\ k_{310}$. The value of x is ________.

[Given: ln10 = 2.3
R=8.3 J K^{-1} mol^{-1}]

Official Ans. by NTA (1)

Sol. $$\ell n\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ell n\left(\frac{K_2}{K_1}\right) = \frac{532611}{8.3} \times \left(\frac{10}{310 \times 300}\right)$$

where K_2 is at 310 K & K_1 is at 300 K

$$\ell n\left(\frac{K_2}{K_1}\right) = 6.9$$

$= 3 \times \ell n10$

$$\ell n\frac{K_2}{K_1} = \ell n10^3$$

$K_2 = K_1 \times 10^3$

$K_1 = K_2 \times 10^3$

So K = 1

6. The number of terminal oxygen atoms present in the product B obtained from the following reaction is ________.

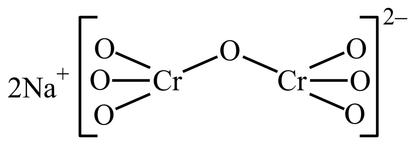
$FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow A + Fe_2O_3 + CO_2$

$A + H^+ \rightarrow B + H_2O + Na^+$

Official Ans. by NTA (6)

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

$2Na_2CrO_4 + 2H^+ \rightarrow \underbrace{Na_2Cr_2O_7}_{B} + 2Na^+ + H_2O$



7. An acidified manganate solution undergoes disproportionation reaction. The spin-only magnetic moment value of the product having manganese in higher oxidation state is _________ B.M. (Nearest integer)

Official Ans. by NTA (0)

Sol. $3MnO_4^{2-} + 4H^+ \longrightarrow 2\overset{+7}{Mn}O_4^- + \overset{+4}{Mn}O_2 + 2H_2O$

$\overset{+7}{Mn}$ = no. of unpaired electrons is '0'

$\mu = 0$ B.M.

8. Kjeldahl's method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of 1 M H_2SO_4 solution. The percentage of nitrogen in the compound is _________. (Nearest integer)

Official Ans. by NTA (64)

Sol. Meq of H_2SO_4 used by NH_3 = $12.5 \times 1 \times 2 = 25$

% of N in the compound = $\dfrac{25\times10^{-3}\times14\times100}{0.55} = 63.6$

or

Meq. of H_2SO_4 = Meq. of NH_3

$12.5 \times 1 \times 2 = 25$ meq. of NH_3

= 25 millimoles of NH_3

So Millimoles of 'N' = 25

Moles of 'N' = 25×10^{-3}

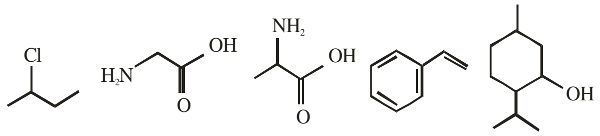
wt. of N = $14 \times 25\times10^{-3}$

$\%N = \dfrac{14\times25\times10^{-3}}{0.55}\times100$

= 63.66

$\approx 64\%$

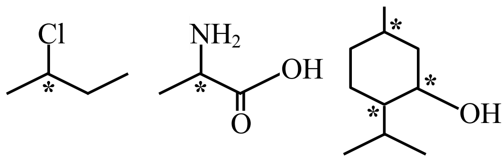
9. Observe structures of the following compounds



The total number of structures/compounds which possess asymmetric carbon atoms is __________.

Official Ans. by NTA (3)

Sol.



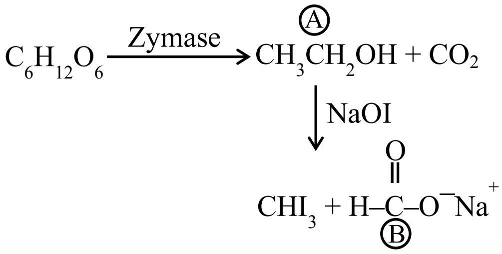
Number of compounds containing asymmetric carbons are three.

10. $C_6H_{12}O_6 \xrightarrow{\text{Zymase}} A \xrightarrow[\Delta]{\text{NaOI}} B + CHI_3$

The number of carbon atoms present in the product B is __________.

Official Ans. by NTA (1)

Sol.



no. of carbon atoms present in B is 1

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Question ID: 101761

The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :

(A) $\frac{133}{10^4}$ (B) $\frac{18}{10^3}$

(C) $\frac{19}{10^3}$ (D) $\frac{271}{10^4}$

Official Ans. by NTA (C)

Sol. Let matrix A is singular then $|A| = 0$

Number of singular matrix = All entries are same + only two prime number are used in matrix

$= 10 + 10 \times 9 \times 2$

$= 190$

Required probability $= \frac{190}{10^4} = \frac{19}{10^3}$

2. Question ID: 101762

Let the solution curve of the differential equation

$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$.

Then y(2) is equal to :

(A) 15 (B) 11

(C) 13 (D) 17

Official Ans. by NTA (A)

Sol. $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow x\frac{dv}{dx} = \sqrt{v^2 + 16}$

$\Rightarrow \int \frac{dv}{\sqrt{v^2+16}} = \int \frac{dx}{x}$

$\Rightarrow \ell n\,|v + \sqrt{v^2+16}| = \ln x + \ln C$

$\Rightarrow y + \sqrt{y^2 + 16x^2} = Cx^2$

As $y(1) = 3 \Rightarrow C = 8$

$\Rightarrow y(2) = 15$

3. Question ID: 101763

If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), the $2a + b + 2c$ is equal to :

(A) 54 (B) 50

(C) −6 (D) −42

Official Ans. by NTA (C)

Sol. $\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-2(6-4+28-2)}{3^2+1^2+4^2}$

$\Rightarrow a = \frac{-84}{13} + 2, b = \frac{28}{13} + 4, C = \frac{-112}{13} + 7$

$\Rightarrow 2a + b + 2c = -6$

4. Question ID: 101764

Let $f : R \to R$ be a function defined by :

$$f(x) = \begin{cases} \max\limits_{t \le x}\{t^3 - 3t\} ; & x \le 2 \\ x^2 + 2x - 6 ; & 2 < x < 3 \\ [x-3] + 9 ; & 3 \le x \le 5 \\ 2x + 1 ; & x > 5 \end{cases}$$

Where [t] is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)dx$. Then the ordered pair (m, I) is equal to :

(A) $\left(3, \frac{27}{4}\right)$ (B) $\left(3, \frac{23}{4}\right)$

(C) $\left(4, \frac{27}{4}\right)$ (D) $\left(4, \frac{23}{4}\right)$

Official Ans. by NTA (C)

Sol. $f(x)=\begin{cases} x^3-3x, x\le -1 \\ 2, -1<x<2 \\ x^2+2x-6, 2<x<3 \\ 9, 3\le x<4 \\ 10, 4\le x<5 \\ 11, x=5 \\ 2x+1, x>5 \end{cases}$

Clearly f (x) is not differentiable at

x = 2, 3, 4, 5 $\Rightarrow$ m = 4

$I=\int_{-2}^{-1}(x^3-3x)dx+\int_{-1}^{2}2\cdot dx=\frac{27}{4}$

5. Question ID: 101765

Let $\vec{a}=\alpha\hat{i}+3\hat{j}-\hat{k}, \vec{b}=3\hat{i}-\beta\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-2\hat{k}$ where α, β ∈ R, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b}\times\vec{c}=-6\hat{i}+10\hat{j}+7\hat{k}$, then the value of α + β equal to :

(A) 3 (B) 4

(C) 5 (D) 6

Official Ans. by NTA (A)

Sol. $\frac{a\cdot c}{|\vec{c}|}=\frac{10}{3}$

$\Rightarrow \frac{\alpha+6+2}{\sqrt{1+4+4}}=\frac{10}{3}\Rightarrow \alpha=2$

and $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix}=-6\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow 2\beta-8=-6\Rightarrow \beta=1$

$\Rightarrow \alpha+\beta=3$

6. Question ID : 101766

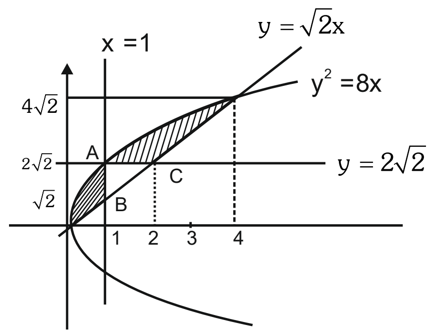
The area enclosed by y^2 = 8x and $y=\sqrt{2}\,x$ that lies outside the triangle formed by $y=\sqrt{2}x, x=1, y=2\sqrt{2}$, is equal to :

(A) $\frac{16\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{13\sqrt{2}}{6}$ (D) $\frac{5\sqrt{2}}{6}$

Official Ans. by NTA (C)

Sol.



Area of ΔABC $=\frac{1}{2}(\sqrt{2})\cdot 1=\frac{\sqrt{2}}{2}$

So required Area $=\int_0^4\left(\sqrt{8x}-\sqrt{2}x\right)dx-\frac{\sqrt{2}}{2}$

$=\frac{32\sqrt{2}}{3}-8\sqrt{2}-\frac{\sqrt{2}}{2}=\frac{13\sqrt{2}}{6}$

7. Question ID: 101767

If the system of linear equations

2x + y – z = 7

x – 3y + 2z = 1

x + 4y + δz = k, where δ, k ∈ R

has infinitely many solutions, then δ + k is equal to:

(A)–3 (B) 3 (C) 6 (D) 9

Official Ans. by NTA (B)

Sol. $\begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix}=0$

$\Rightarrow \delta=-3$

And $\begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ K & 4 & -3 \end{vmatrix}=0\Rightarrow K=6$

$\Rightarrow$ δ + K = 3

Alternate

2x + y – z = 7 ....(1)

x – 3y + 2z = 1 ....(2)

x + 4y + δz = k ....(3)

Equation (2) + (3)

We get 2x + y + (2 + δ) z = 1 + K ......(4)

For infinitely solution

Form equation (1) and (4)

$2+\delta=-1\Rightarrow \boxed{\delta=-3}$

$1+k=7\Rightarrow \boxed{k=6}$

δ + k = 3

8. Question ID: 101768

Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to :

(A) 50 (B) 250

(C) 1250 (D) 1500

Official Ans. by NTA (A)

Sol. $X^2 = 1 - 2i \Rightarrow \alpha^2 = 1 - 2i,\ \beta^2 = 1 - 2i$

Hence $\alpha^8 = \beta^8$

$|\alpha^8 + \beta^8| = |2\alpha^8| = 2|\alpha^2|^4$

$= 2\sqrt{5}^4 = 50$

9. Question ID: 101769

Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q)\Delta((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

(A) $\wedge$ (B) $\vee$

(C) $\Rightarrow$ (D) $\Leftrightarrow$

Official Ans. by NTA (C)

Sol. $p \vee q \Rightarrow q$

$\Rightarrow \sim (p \vee q) \vee q$

$\Rightarrow (\sim p \wedge \sim q) \vee q$

$\Rightarrow (\sim p \vee q) \wedge (\sim q \vee q)$

$\Rightarrow (\sim p \vee q) \wedge t = \sim p \vee q$

Now by taking option C

$(p \wedge q) \Rightarrow \sim p \vee q$

$\Rightarrow \sim p \vee \sim q \vee \sim p \vee q$

$\Rightarrow t$

Hence C

10. Question ID: 101770

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + \ldots + A^{10}$ is equal to :

(A) $\left(\frac{3^{10}-3}{2}\right)A$ (B) $\left(\frac{3^{10}-1}{2}\right)A$

(C) $\left(\frac{3^{10}+1}{2}\right)A$ (D) $\left(\frac{3^{10}+3}{2}\right)A$

Official Ans. by NTA (A)

Sol. $A = \begin{pmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{pmatrix}$

$A^2 = 3A$

$A^3 = 3^2A$

$A^2 + A^3 + \ldots A^{10}$

$= 3A + 3^2A + \ldots + 3^9A = \frac{3(3^9 - 1)}{3 - 1}A$

$= \frac{3^{10} - 3}{2}A$

11. Question ID: 101771

Let a set $A = A_1 \cup A_2 \cup \ldots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$ $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is :

(A) reflexive, symmetric but not transitive

(B) reflexive, transitive but not symmetric

(C) reflexive but not symmetric and transitive

(D) an equivalence relation

Official Ans. by NTA (D)

Sol. A = {1, 2, 3}

R = {(1, 1), (1, 2), (1, 3) (2, 1), (2, 2), (2, 3) (3,1), (3, 2) (3, 3)}

12. Question ID: 101772

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to

(A) $\frac{6}{343}$ (B) $\frac{7}{216}$

(C) $\frac{8}{343}$ (D) $\frac{49}{216}$

Official Ans. by NTA (B)

Sol. $a_2 = 1, a_3 = 3\ a_4 = 6$

$$a_n = \frac{n(n-1)}{2}$$

$$S = \sum_{n=2}^{\infty} \frac{n(n-1)}{2(7^n)}$$

$$S = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \frac{15}{7^5} + \ldots\ldots$$

$$\frac{S}{7} = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \frac{10}{7^6} + \ldots$$

$$6\frac{S}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \frac{4}{7^5} + \ldots$$

$$6\frac{S}{7^2} = \frac{1}{7^3} + \frac{2}{7^4} + \frac{3}{7^5} + \ldots$$

$$6\frac{S}{7}\cdot\frac{6}{7} = \frac{1}{7^2} + \frac{1}{7^3} + \ldots = \frac{1/7^2}{1-1/7}$$

$$6\times 6\frac{S}{7^2} = \cdot\frac{1}{7\times 6}$$

$$S = \frac{7}{6^3} = \frac{7}{216}$$

Alternate

$a_{n+2} = 2a_{n+1} - a_n + 1$

$$\Rightarrow \frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\frac{a_n}{7^n} + \frac{1}{7^{n+2}}$$

$$\Rightarrow \sum_{n=2}^{\infty}\frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\sum_{n=2}^{\infty}\frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\sum_{n=2}^{\infty}\frac{a_n}{7^n} + \sum_{n=2}^{\infty}\frac{1}{7^{n+2}}$$

Let $\sum_{n=2}^{\infty}\frac{a_n}{7^n} = p$

$$\Rightarrow \left(p - \frac{a_2}{7^2} - \frac{a_3}{7^3}\right) = \frac{2}{7}\left(p - \frac{a_2}{7^2}\right) - \frac{1}{49}p + \frac{1/7^4}{1-\frac{1}{7}}$$

$\because a_2 = 1, a_3 = 3$

$$\Rightarrow p - \frac{1}{49} - \frac{3}{343} = \frac{2}{7}p - \frac{2}{7^3} - \frac{p}{49} + \frac{1}{6.7^3}$$

$$\Rightarrow p = \frac{7}{216}$$

13. Question ID: 101773

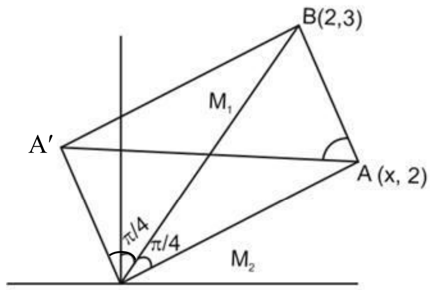
The distance between the two points A and A' which lie on y = 2 such that both the line segments AB and A' B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to :

(A) 10 (B) $\frac{48}{5}$

(C) $\frac{52}{5}$ (D) 3

Official Ans. by NTA (C)

Sol.



$M_1 = 3/2$ $M_2 = 2/x$

$$\tan\pi/4 = \left|\frac{3/2 - 2/x}{1 + 6/2x}\right| = 1$$

$\Rightarrow x_1 = 10,\quad x_2 = -2/5$

$\Rightarrow AA^1 = 52/5$

14. Question ID: 101774

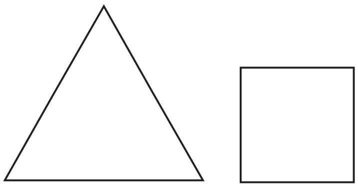
A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

(A) $\frac{22}{9+4\sqrt{3}}$ (B) $\frac{66}{9+4\sqrt{3}}$

(C) $\frac{22}{4+9\sqrt{3}}$ (D) $\frac{66}{4+9\sqrt{3}}$

Official Ans. by NTA (B)

Sol.



$3a = x$ $\quad$ $4b = 22 - x$

$a = 2/13$

$$A_T = \frac{\sqrt{3}}{4}a^2 + b^2$$

$$= \frac{\sqrt{3}}{4}x^2/9 + \frac{(22-x)^2}{16}$$

$$\frac{dA}{dx} = 0 \Rightarrow x\left(\frac{\sqrt{3}}{2\times 9} + \frac{1}{8}\right) - \frac{22}{8} = 0$$

$$\Rightarrow x\left(\frac{4\sqrt{3}+9}{36}\right) = \frac{11}{2}$$

$a = x/3$

$$a = \left(\frac{11/2}{\frac{4\sqrt{3}+9}{36}}\right)\left(\frac{1}{3}\right) = \frac{66}{4\sqrt{3}+9}$$

15. Question ID: 101775

The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is :

(A) $R - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

(B) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Official Ans. by NTA (D)

Sol. $$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$$

$$-\pi/2 \le \sin^{-1}\frac{1}{4x^2-1} \le \pi/2$$

Always $-1 \le \frac{1}{4x^2-1} \le 1$

$$x \in \left(\infty, \frac{1}{\sqrt{2}}\right) \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$$

16. Question ID: 101776

If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to :

(A) 6 $\quad$ (B) 7

(C) 8 $\quad$ (D) 9

Official Ans. by NTA (D)

Sol. General term

$$T_{r+1} = \frac{|\underline{10}}{|\underline{r_1}\,|\underline{r_2}\,|\underline{r_3}}(3)^{r_1}(-2)^{r_2}(5)^{r_3}(x)^{3r_1+2r_2-5r_3}$$

$3r_1 + 2r_2 - 5r_3 = 0$ ...(1)

$r_1 + r_2 + r_3 = 10$ ...(2)

from equation (1) and (2)

$r_1 + 2(10 - r_3) - 5r_3 = 0$

$r_1 + 20 = 7r_3$

$(r_1, r_2, r_3) = (1, 6, 3)$

constant term = $\frac{|\underline{10}}{|\underline{1}\,|\underline{6}\,|\underline{3}}(3)^1(-2)^6(5)^3$

$= 2^9.3^2.5^4.7^1$

$l = 9$

17. Question ID: 101777

$\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right)dx$,

Where [t] denotes greatest integer less than or equal to t, is equal to :

(A) -3 $\quad$ (B) -2

(C) 2 $\quad$ (D) 0

Official Ans. by NTA (D)

Sol. $$I = \int_0^5 \cos\left(\pi x - \pi\left[\frac{x}{2}\right]\right)dx$$

$$\Rightarrow I = \int_0^2 \cos(\pi x)dx + \int_2^4 \cos(\pi x - \pi)dx + \int_4^5 \cos(\pi x - 2\pi)dx$$

$$\Rightarrow I = \left[\frac{\sin \pi x}{\pi}\right]_0^2 + \left[\frac{\sin(\pi x - \pi)}{\pi}\right]_2^4 + \left[\frac{\sin(\pi x - 2\pi)}{\pi}\right]_4^5$$

$\Rightarrow I = 0$

18. Question ID: 101778

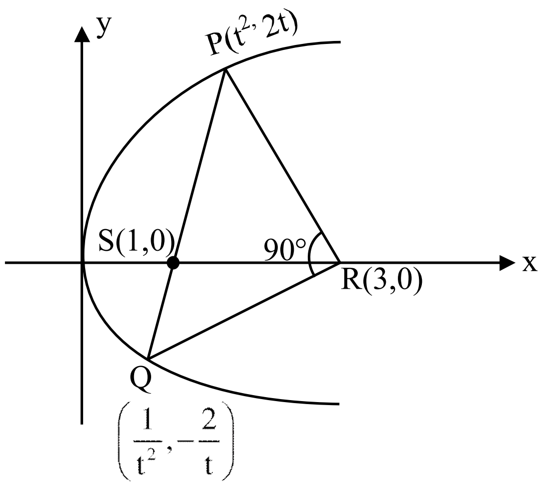
Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to :

(A) $1+\sqrt{2}$ (B) $3+2\sqrt{2}$

(C) $1+2\sqrt{3}$ (D) $4+5\sqrt{3}$

Official Ans. by NTA (B)

Sol. PQ is focal chord



$m_{PR} \cdot m_{PQ} = -1$

$$\frac{2t}{t^2-3}\times\frac{-2/t}{\frac{1}{t^2}-3} = -1$$

$(t^2-1)^2 = 0$

$\Rightarrow t = 1$

$\Rightarrow$ P & Q must be end point of latus rectum: P(1, 2) & Q(1, –2)

$\therefore \frac{2b^2}{a} = 4$ & $ae = 1$

$\because$ We know that $b^2 = a^2(1-e^2)$

$\therefore a = 1+\sqrt{2}$

$\because e^2 = 1-\frac{b^2}{a^2}$

$\therefore e^2 = 3-2\sqrt{2}$

$\frac{1}{e^2} = 3+2\sqrt{2}$

19. Question ID: 101779

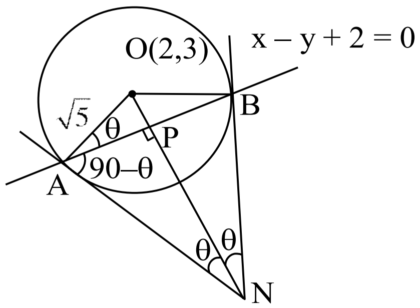
Let the tangent to the circle $C_1 : x^2+y^2=2$ at the point $M(-1,1)$ intersect the circle $C_2 : (x-3)^2+(y-2)^2=5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{6}$ (D) $\frac{5}{3}$

Official Ans. by NTA (C)

Sol. $OP = \left|\frac{2-3+2}{\sqrt{2}}\right|$



$OP = \frac{3}{\sqrt{2}}$

$AP = \sqrt{OA^2 - OP^2}$

$= \frac{1}{\sqrt{2}}$

$\tan\theta = 3$

$\therefore \sin\theta = \frac{3}{\sqrt{10}} = \frac{AP}{AN}$

$\Rightarrow AN = \frac{\sqrt{5}}{3} = BN$

Area of $\Delta ANB = \frac{1}{2}.(AN^2)\sin 2\theta = \frac{1}{6}$

20. Question ID: 101780

Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively.

If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to:

(A) 13 (B) 15

(C) 17 (D) 18

Official Ans. by NTA (B)

Sol. $\bar{x} = \frac{\sum x_i}{5} = \frac{24}{5} \Rightarrow \sum x_i = 24$

$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$

$\Rightarrow \sum x_i^2 = 154$

$x_1 + x_2 + x_3 + x_4 = 14$

$\Rightarrow x_5 = 10$

$\sigma^2 = \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \frac{49}{4} = a$

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 4a + 49$

$x_5^2 = 154 - 4a - 49$

$\Rightarrow 100 = 105 - 4a \Rightarrow 4a = 5$

$4a + x_5 = 15$

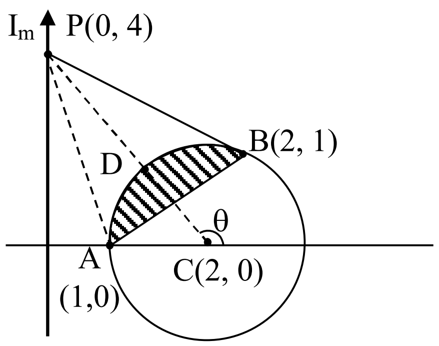
SECTION-B

1. Question ID: 101781

Let $S = \{z \in \mathbb{C} : |z - 2| \le 1, z(1 + i) + \bar{z}(1 - i) \le 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5(|z_1|^2 + |z_2|^2) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of α + β is equal to ______.

Official Ans. by NTA (26)

Sol. $|z - 2| \le 1$



$(x - 2)^2 + y^2 \le 1$ ….. (1)

&

$z(1 + i) + \bar{z}(1 - i) \le 2$

Put $z = x + iy$

$\therefore x - y \le 1$ ….. (2)

$PA = \sqrt{17}$, $PB = \sqrt{13}$

Maximum is PA & Minimum is PD

Let D $(2 + \cos\theta, 0 + \sin\theta)$

$\therefore m_{cp} = \tan\theta = -2$

$\cos\theta = -\frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore D\left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\Rightarrow z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2i}{\sqrt{5}}$

$|z_1| = \frac{25 - 4\sqrt{5}}{5}$ & $z_2 = 1$

$\therefore |z_2|^2 = 1$

$\therefore 5\left(|z_1|^2 + |z_2|^2\right) = 30 - 4\sqrt{5}$

$\therefore \alpha = 30$

$\beta = -4$

$\therefore \alpha + \beta = 26$

2. Question ID: 101782

Let $y = y(x)$ be the solution of the differential equation

$\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}, 0 < x < \pi/2$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$.

If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to ______.

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + \frac{\sqrt{2}}{2\cos^4 x - \cos 2x} y = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$

$\int \frac{dx}{2\cos^4 x - \cos 2x}$

$= \int \frac{dx}{\cos^4 x + \sin^4 x} = \int \frac{\operatorname{cosec}^4 x \, dx}{1 + \cot^4 x}$

$= -\int \frac{t^2 + 1}{t^4 + 1} dt = -\int \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} dt = \frac{-1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right)$

$\cot x = t$

$= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2}\cot 2x\right)$

$\therefore$ IF $= e^{-\tan^{-1}(\sqrt{2}\cot 2x)}$

$$ye^{-\tan^{-1}(\sqrt{2}\cot 2x)} = \int x\,dx$$

$$ye^{-\tan^{-1}\left(\sqrt{2}\cot 2x\right)} = \frac{x^2}{2} + c$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32} + c \Rightarrow c = 0$$

$$y = \frac{x^2}{2}e^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{\tan^{-1}\left(\sqrt{2}\cot\frac{2\pi}{3}\right)}$$

$$= \frac{\pi^2}{18}e^{-\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$$

$$\alpha = \sqrt{\frac{2}{3}} \Rightarrow 3\alpha^2 = 2$$

3. Question ID: 101783

Let d be the distance between the foot of perpendiculars of the points P(1, 2 – 1) and Q(2, – 1, 3) on the plane – x + y + z = 1. Then d^2 is equal to ________.

Official Ans. by NTA (26)

Sol. Points P(1, 2, –1) and Q (2, –1, 3) lie on same side of the plane.

Perpendicular distance of point P from plane is

$$\left|\frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

Perpendicular distance of point Q from plane is

$$= \left|\frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \overrightarrow{PQ}$ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

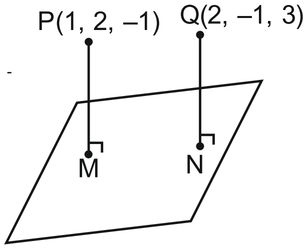
$$\Rightarrow \left|\overrightarrow{PQ}\right| = \sqrt{(1-2)^2 + (2+1)^2 + (-1-3)^2}$$

$= \sqrt{26}$

$$\left|\overrightarrow{PQ}\right|^2 = 26 = d^2$$

Alternate

$$-x + y + z - 1 = 0$$



$M(x_1, y_1, z_1)$

$$\frac{x_1-1}{-1} = \frac{y_1-2}{1} = \frac{z_1+1}{1} = \frac{1}{3}$$

$$x_1 = \frac{2}{3}, y_1 = \frac{7}{3}, z_1 = \frac{-2}{3}$$

$$M\left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$$

$N(x_2, y_2, z_2)$

$$\frac{x_2-2}{-1} = \frac{y_2+1}{1} = \frac{z_2-3}{1} = \frac{1}{3}$$

$$x_2 = \frac{5}{3}, y_2 = \frac{-2}{3},\ z_2 = \frac{10}{3}$$

$$N = \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$$

$d^2 = 1^2 + 3^2 + 4^2 = 26$

4. Question ID: 101784

The number of elements in the set $S = \{\theta \epsilon [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is ________.

Official Ans. by NTA (32)

Sol. $3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta = 0$ OR $\cos 2\theta = -1/3$

$\theta \in [-4\pi, 4\pi]$

$$2\theta = (2n+1).\frac{\pi}{2}$$

$\therefore\ \theta = \pm\pi/4. \pm 3\pi/4 .... \pm 15\pi/4$

Similarly $\cos 2\theta = -1/3$ gives 16 solution

5. Question ID: 101785

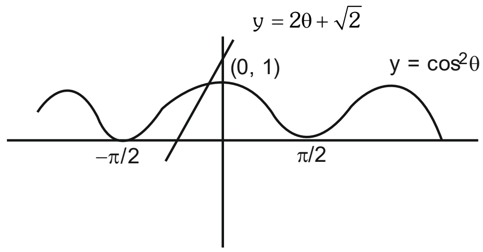
The number of solutions of the equation $2\theta - cos^2\theta + \sqrt{2} = 0$ is R is equal to ______.

Official Ans. by NTA (1)

Sol. $2\theta - \cos^2\theta + \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$



Both graphs intersect at one point.

6. Question ID: 101786

$50\,tan\left(3tan^{-1}\left(\frac{1}{2}\right) + 2cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) + 4\sqrt{2}\tan\left(\frac{1}{2}tan^{-1}(2\sqrt{2})\right)$ is equal to _____.

Official Ans. by NTA (29)

Sol. $50\tan\left(3\tan^{-1}\frac{1}{2} + 2\cos^{-1}\frac{1}{\sqrt{5}}\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2(\tan^{-1}\frac{1}{2} + \tan^{-1}2)\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2.\frac{\pi}{2}\right) + 4\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50\left(\tan\tan^{-1}\frac{1}{2}\right) + 4$

$= 25 + 4 = 29$

7. Question ID: 101787

Let c, k ∈ R. If $f(x) = (c + 1) x^2 + (1 - c^2) x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all x, y ∈ R, then the value of $|2(f(1) + f(2) + f(3) + ...+f(20))|$ is equal to _______.

Official Ans. by NTA (3395)

Sol. $f(x) = (c + 1) x^2 + (1 - c^2) x + 2k$ ....(1)

& $f(x + y) = f(x) + f(y) - xy \quad \forall\, xy \in R$

$\lim_{y\to0}\frac{f(x+y)-f(x)}{y} = \lim_{y\to0}\frac{f(y)-xy}{y} \Rightarrow f'(x) = f'(0) - x$

$f(x) = -\frac{1}{2}x^2 + f'(0).x + \lambda$ but $f(0) = 0 \Rightarrow \lambda = 0$

$f(x) = -\frac{1}{2}x^2 + (1-c^2).x$ ....(2)

∴ as $f'(0) = 1 - c^2$

Comparing equation (1) and (2)

We obtain, $c = -\frac{3}{2}$

∴ $f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$

Now $|2\sum_{x=1}^{20} f(x)| = \sum_{x=1}^{20} x^2 + \frac{5}{2}.\sum_{x=1}^{20} x$

$= 2870 + 525$

$= 3395$

8. Question ID: 101788

Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a > 0, b > 0, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then value of $a^2 + b^2$ is equal to ___________.

Official Ans. by NTA (88)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Given $e^2=1+\frac{b^2}{a^2}\Rightarrow\frac{11}{4}=1+\frac{b^2}{a^2}\Rightarrow b^2=\frac{7}{4}a^2$

$\therefore\ \frac{x^2}{(a)^2}-\frac{y^2}{\left(\frac{\sqrt{7}}{2}a\right)^2}=1$ Now given

$2a+2.\frac{\sqrt{7}a}{2}=4\left(2\sqrt{2}+\sqrt{14}\right)$

$a\left(2+\sqrt{7}\right)=4\sqrt{2}(2+\sqrt{7})$

$a=4\sqrt{2}\Rightarrow a^2=32$

$b^2=\frac{7}{4}\times16\times2=56$

9. Question ID: 101789

Let $P_1:\vec{r}.(2\hat{i}+\hat{j}-3\hat{k})=4$ be a plane. Let P_2 be another plane which passes through the points (2, -3, 2) (2, − 2, − 3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to _____.

Official Ans. by NTA (28)

Sol. $P_1:\vec{r}.(2\hat{i}+\hat{j}-3\hat{k})=4$

P_1**:** $2x + y - 3z = 4$

$P_2\ \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix}=0$

$\Rightarrow -5x+5y+z+23=0$

Let a, b, c be the d'rs of line of intersection

Then $a=\frac{16\lambda}{15};\ b=\frac{13\lambda}{15};\ c=\frac{15\lambda}{15}$

$\therefore\ \alpha = 13 : \beta = 15$

10. Question ID: 101790

Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in$ {1, 2, 3, ………, 100} for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers.

Then the number of such permutations $b_1b_2b_3b_4$ is equal to _________.

Official Ans. by NTA (18915)

Sol. $b_i\in\{1,2,3.........100\}$

Let A = set when b_1 b_2 b_3 are consecutive

$n(A)=\frac{97+97+.......+97}{98\text{times}}=97\times98$

Similarly when b_2b_3 b_4 are consecutive

N(A) = 97 × 98

$n(A\cap B)=\frac{97+97+-------97}{98\text{times}}=97\times98$

Similarly when b_2 b_3 b_4 are consecutive

n(B) = 97 × 98

n(A∩B) = 97

n(AUB) = n(A)+n(B) − n(A∩B)

Number of permutation = 18915

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10m in t s,. the distance travelled by the toy in the next t s will be :

(A) 10m (B) 20m

(C) 30m (D) 40m

Official Ans. by NTA (C)

Sol. u = 0, Say acceleration is a

For t s $10 = \frac{1}{2}at^2$

For 2t s $10 + x = \frac{1}{2}a(2t)^2$

$\frac{10+x}{10} = \frac{4}{1}$

x = 30 m

2. At what temperature a gold ring of diameter 6.230 cm be heated so that it can be fitted on a wooden bangle of diameter 6.241 cm? Both the diameters have been measured at room temperature (27ºC). (Given: coefficient of linear thermal expansion of gold $\alpha_L = 1.4 \times 10^{-5}\ K^{-1}$)

(A) 125.7ºC (B) 91.7ºC

(C) 425.7º (D) 152.7ºC

Official Ans. by NTA (D)

Sol. $\Delta \ell = 6.241 - 6.230 = 0.011$ cm

$\Delta \ell = \ell\ \alpha\ \Delta\theta$

$0.011 = 6.230 \times 1.4 \times 10^{-5}(\theta - 27)$

$\theta - 27 = \frac{0.011 \times 10^5}{6.230 \times 1.4}$

$\theta \approx 153.11$ nearest is 152.7°C.

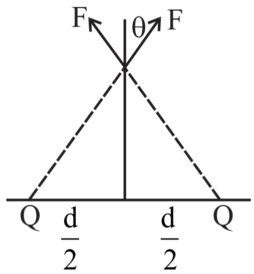
3. Two point charges Q each are placed at a distance d apart. A third point charge q is placed at a distance x from mid-point on the perpendicular bisector. The value of x at which charge q will experience the maximum Coulomb's force is :

(A) x = d (B) $x = \frac{d}{2}$

(C) $x = \frac{d}{\sqrt{2}}$ (D) $x = \frac{d}{2\sqrt{2}}$

Official Ans. by NTA (D)

Sol.



$F = \frac{KQq}{\left(x^2 + \frac{d^2}{4}\right)}$

Net force on g = 2 F cosθ

$F_{net} = \frac{2KQqx}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$

For maximum F_{net}

$\frac{dF_{net}}{dx} = 0$

we get $x = \frac{d}{2\sqrt{2}}$

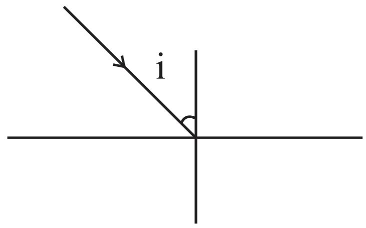
4. The speed of light in media 'A' and 'B' are 2.0 × 10^{10} cm/s and 1.5 × 10^{10} chm/s respectively. A ray of light enters from the medium B to A at an incident angle 'θ'. If the ray suffers total internal reflection, then

(A) $\theta = \sin^{-1}\left(\frac{3}{4}\right)$ (B) $\theta > \sin^{-1}\left(\frac{2}{3}\right)$

(C) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$ (D) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

Official Ans. by NTA (D)

Sol. $\sin i_c = \frac{n_r}{n_d} = \frac{C_d}{C_r} = \frac{1.5\times10^{10}}{2\times10^{10}}$



$\sin i_c = \frac{3}{4}$

$i_c = \sin^{-1}\left(\frac{3}{4}\right)$

for T I R $\quad \theta > i_c$

$\theta > \sin^{-1}\left(\frac{3}{4}\right)$

5. In the following nuclear rection,

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\gamma} D_4$$

Mass number of D is 182 and atomic number is 74. Mass number and atomic number of D_4 respectively will be___.

(A) 174 and 71 (B) 174 and 69

(C) 172 and 69 (D) 172 and 71

Official Ans. by NTA (A)

Sol. Say for D_4 Atomic No = Z

Mass Number = A

A = 182 – 4 – 4 = 174

Z = 74 – 2 + 1 – 2 = 71

6. The electric field at the point associated with a light wave is given by

$E = 200\ [\sin(6 \times 10^{15})\ t + \sin(9 \times 10^{15})\ t]\ Vm^{-1}$

Given : $h = 4.14 \times 10^{-15}$ eVs

If this light falls on a metal surface having a work function of 2.50 eV, the maximum kinetic energy of the photoelectrons will be :

(A) 1.90 eV (B) 3.27 eV

(C) 3.60 eV (D) 3.42 eV

Official Ans. by NTA (D)

Sol. For maximum KE we will take

higher frequency $\left(f = \frac{9\times10^{15}}{2\pi} Hz\right)$

$K_{max} = hf - \phi$

$= \frac{9\times10^{15}\times4.14\times10^{-15}}{2\pi} - 2.50$

3.43 eV nearest is 3.42 eV

7. A capacitor is discharging through a resistor R. Consider in time t_1, the energy stored in the capacitor reduces to half of its initial value and in time t_2, the charge stored reduces to one eighth of its initial value. The ratio t_1/t_2 will be :

(A) 1/2 (B) 1/3

(C) 1/4 (D) 1/6

Official Ans. by NTA (D)

Sol. In t_1 time energy becomes half so charge will become $\frac{1}{\sqrt{2}}$ time

$q = Q_0\ e^{-\frac{t_1}{RC}} = \frac{Q_0}{\sqrt{2}}$

and $q = Q_0\ e^{-\frac{t_1}{RC}} = \frac{Q_0}{8} = \left(\frac{Q_0}{\sqrt{2}}\right)^6$

$t_2 = 6t_1$

$\frac{t_1}{t_2} = \frac{1}{6}$

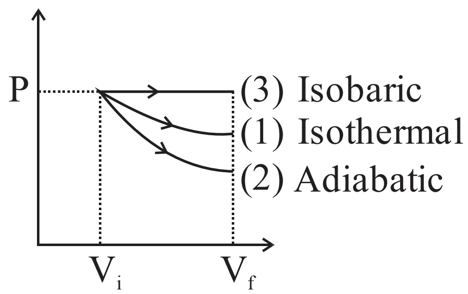
8. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal. W_2. if the process is purely adiabatic and W_3 if the process is purely isobaric. Then, choose the coned option

(A) $W_1 < W_2 < W_3$ (B) $W_2 < W_3 < W_1$

(C) $W_3 < W_1 < W_2$ (D) $W_2 < W_1 < W_3$

Official Ans. by NTA (D)

Sol.



Area under curve is work

$W_2 < W_1 < W_3$

9. Two long current carrying conductors are placed parallel to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid-point between the two conductors due to current flowing in them is 300 μT. The equal current flowing in the two conductors is :

(A) 30A in the same direction.

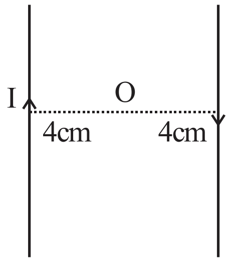
(B) 30A in the opposite direction.

(C) 60A in the opposite direction.

(D) 300A in the opposite direction.

Official Ans. by NTA (B)

Sol.



$B \text{ at } O = 2\,\dfrac{\mu_0 I}{2\pi r}$

$\dfrac{2\times 4\pi\times 10^{-7} I}{2\pi\ 4\times 10^{-2}} = 3\times 10^{-4}T$

I = 30A in opp. direction

10. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be :

(A) 40 hours (B) 36 hours

(C) 30 hours (D) 25 hours

Official Ans. by NTA (B)

Sol. $T = \dfrac{2\pi}{\sqrt{GM}} r^{3/2}$

$\dfrac{T_1}{T_2} = \left(\dfrac{r_1}{r_2}\right)^{3/2} = \left(\dfrac{1}{3}\right)^{3/2}$

$T_2 = T_1\ 3\sqrt{3} = 21\ \sqrt{3}$ hours

≈ 36 hours

11. The TV transmission tower at a particular station has a height of 125 m. For dubling the coverage of its range, the height of the tower should be increased by :

(A)125 m (B) 250 m

(C) 375 (D) 500 m

Official Ans. by NTA (C)

Sol. Range $d = \sqrt{2Rh}$

$d_2 = 2d_1$

$\sqrt{2Rh_2} = 2\sqrt{2Rh_1}$

$h_2 = 4h_1 = 500$ m

$\Delta h = 500 \text{ m} - 125 \text{ m} = 375$ m

12. The motion of a simple pendulum excuting S.H.M. is represented by following equation.

Y = A sin (πt + ϕ), where time is measured in second.

The length of pendulum is :

(A) 97.23 cm (B) 25.3 cm

(C) 99.4 cm (D) 406.1 cm

Official Ans. by NTA (C)

Sol. $\omega = \sqrt{\dfrac{g}{\ell}} = \pi$

$\dfrac{g}{\ell} = \pi^2 \Rightarrow \ell = \dfrac{g}{\pi^2}$

$\ell = \dfrac{980}{\pi^2} \approx 99.4\,\text{cm}$

13. A vessel contains 16g of hydrogen and 128 g of oxygen at standard temperature and pressure. The volume of the vessel in cm^3 is :

(A) 72×10^5 (B) 32×10^5

(C) 27×10^4 (D) 54×10^4

Official Ans. by NTA (C)

Sol. No of moles of H_2 = 8 moles

No of moles of O_2 = 4 moles

Total moles = 12 moles

At STP 1 mole occupy = $22.4\ell = 22.4 \times 10^3\ cm^3$

12 moles will occupy = $12 \times 22.4 \times 10^3\ cm^3$

$\approx 26.8 \times 10^4\ cm^3$

14. Given below are two statements :

Statement I: The electric force changes the speed of the charged particle and hence changes its kinetic energy: whereas the magnetic force does not change the kinetic energy of the charged particle.

Statement II: The electric force accelerates the positively charged particle perpendicular to the direction of electric field. The magnetic force accelerates the moving charged particle along the direction of magnetic field. In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

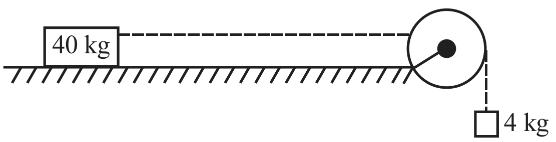
(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (C)

Sol. Electric field can change speed and kinetic energy but magnetic field can not change speed Δ KE. Because magnetic force is always ⊥ to velocity.

15. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below. The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is. (Given g = 10 ms^{-2}.)



(A) 1 ms^{-2} (B) 1/5 ms^{-2}

(C) 4/5 ms^{-2} (D) 8/11 ms^{-2}

Official Ans. by NTA (D)

Sol. For 4 kg block

$4g - T = 4a$

For 40 kg block

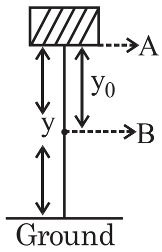
$T - 40g \times 0.02 = 40\,a$

Adding both eq.

$40 - 8 = 44a$

$a = \frac{32}{44} = \frac{8}{11} m/s^2$

16. In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is :



(A) $\frac{1}{2} mg\,y_0^2$ (B) $\frac{1}{2} mg\,y^2$

(C) $mg(y - y_0)$ (D) mgy_0

Official Ans. by NTA (D)

Sol. Work done by gravity = $K_B - K_A$

$mgy_0 = K_B - 0$

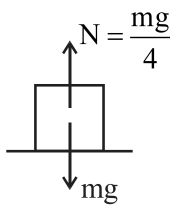
$K_B = mgy_0$

17. A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of 'a' will be :

(A) $\frac{g}{4}$ (B) $\frac{g}{2}$ (C) $\frac{3g}{4}$ (D) g

Official Ans. by NTA (C)

Sol.



$mg - N = ma$

$a = g - \frac{g}{4}$

$a = \frac{3g}{4}$

18. If the electric potential at any point (x, y, z)m in space is given by $V = 3x^2$ volt. The electric field at the point (1, 0, 3) m will be :

(A) 3 Vm^{-1}, directed along positive x-axis.

(B) 3 Vm^{-1}, directed along negative x-axis.

(C) 6 Vm^{-1}, directed along positive x-axis.

(D) 6 Vm^{-1}, directed along negative x-axis.

Official Ans. by NTA (D)

Sol. $E_x = -\frac{\partial V}{\partial x} = -6x$

At (1, 0, 3)

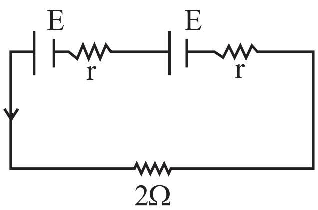
$\vec{E} = -6V/m\ \hat{i}$

19. The combination of two identical cells, whether connected in series or parallel combination provides the same current through an external resistance of 2Ω. The value of internal resistance of each cell is :

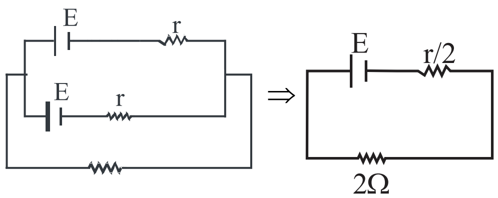
(A) 2Ω (B) 4Ω (C) 6Ω (D) 8Ω

Official Ans. by NTA (A)

Sol.



$I_1 = \frac{2E}{2r+2}$



$I_2 = \frac{E}{\frac{r}{2}+2} = \frac{2E}{r+4}$

$I_1 = I_2$

$2r + 2 = r + 4$

$2r - r = 2\Omega \Rightarrow r = 2\Omega$

20. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can throw the same ball?

(A) 25 m (B) 50 m

(C) 100 m (D) 200 m

Official Ans. by NTA (B)

Sol. **R =** $\frac{u^2 \sin 2\theta}{g}$ $R_{max} = \frac{u^2}{g} = 100$

$H_{max} = \frac{u^2}{2g} = \frac{100}{2} = 50m$

SECTION-B

1. The vernier constant of Vernier callipers is 0.1 mm and it has zero error of (–0.05) cm. While measuring diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be_____ $\times 10^{-2}$ cm.

Official Ans. by NTA (180)

Sol. Measured diameter = MSR + VSR × VC

$= 1.7 + 0.01 \times 5$

$= 1.75$

Corrected = Measured – Error

$= 1.75 - (-0.05)$

$= 1.80$ cm

$= 180 \times 10^{-2}$ cm

180

2. A small spherical ball of radius 0.1 mm and density 10^4 kg m^{-3} falls freely under gravity through a a distance h before entering a tank of water. If after entering the water the velocity of ball does not change and it continue to fall with same constant velocity inside water, then the value of h wil be_____m.

(Given g = 10 ms^{-2}, viscosity of water = 1.0×10^{-5} N-sm^{-2}).

Official Ans. by NTA (20)

Sol. Speed after falling through height h

Should be equal to terminal velocity

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(d-\rho)g}{\eta}$$

$$\sqrt{2gh} = \frac{2}{9}\frac{10^{-8}(10000-1000)\times 10}{10^{-5}}$$

$$= \frac{2}{9}\times 10^{-8}\frac{9\times 10^4}{10^{-5}} = 20$$

2 × 10 × h = 400

h = 20 m

3. In an experiment to determine the velocity of sound in air at room temperature using a resonance is observed when the air column has a length of 20.0 cm for a tuning fork of frequency 400 Hz is used. The velocity of the sound at room temperature is 336 ms^{-1}. The third resonance is observed when the air column has a length of ____cm.

Official Ans. by NTA (104)

Sol. For first resonance

$$\ell_1 + e = \frac{\lambda}{4}$$

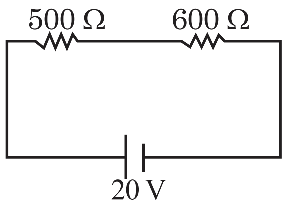
$$\lambda = \frac{336}{400}\times 100 \text{ cm} = 84 \text{ cm} \Rightarrow \frac{\lambda}{4} = 21 \text{ cm}$$

e = 21 – 20 = 1 cm

For third resonance

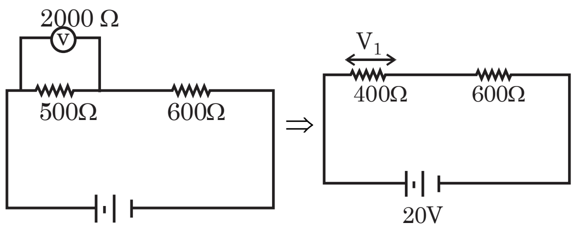
$$\ell_3 + e = \frac{5\lambda}{4} = 105 \text{ cm} \Rightarrow \ell_3 = 104 \text{ cm}$$

4. Two resistors are connected in series across a battery as shown in figure. If a voltmeter of resistance 2000 Ω is used to measure the potential difference across 500 Ω resister, the reading of the voltmeter will be____V.



Official Ans. by NTA (8)

Sol.



$$I = \frac{20}{1000} A$$

$$V_1 = I \times 400 = \frac{20}{1000}\times 400$$

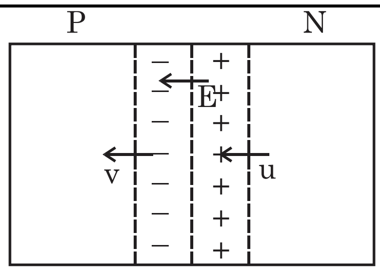
= 8 V

5. A potential barrier of 0.4 V exists across a p-n junction. An electron enters the junction from the n-side with a speed of 6.0×10^5 ms^{-1}. The speed with which electron enters the p side will be $\frac{x}{3}\times 10^5 \text{ ms}^{-1}$ the value of x is _______.

(Given mass of electron = 9×10^{-31} kg, charge on electron = 1.6×10^{-19}C.)

Official Ans. by NTA (14)

Sol.



Work done by Electric field = $K_f - K_i$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -1.6 - 10^{-19} \times 0.4$$

$$\frac{1}{2}9\times10^{-31}(v^2 - u^2) = -0.64 \times 10^{-19}$$

$$u^2 - v^2 = \frac{2\times0.64\times10^{12}}{9}$$

$$v^2 = \left(36 - \frac{128}{9}\right)\times10^{10}$$

$$v = \frac{14}{3}\times10^5 \text{ m/s}$$

x = 14

6. The displacement current of 4.425 μA is developed in the space between the plates of parallel plate capacitor when voltage is changing at a rate of 10^6 Vs^{-1}. The area of each plate of the capacitor is 40 cm^2. The distance between each plate of the capacitor is x × 10^{-3}m. The value of x is,

(Permittivity of free space, $E_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$)

Official Ans. by NTA (8)

Sol. Displacement Current = Conduction Current

$$= \frac{dq}{dt}$$

$$I_d = \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

$$d = \frac{8.85\times10^{-12}\times4\times10^{-3}\times10^6}{4.425\times10^{-6}}$$

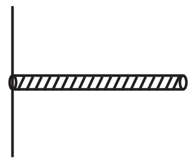
= 8 mm

X = 8

7. The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be_______.

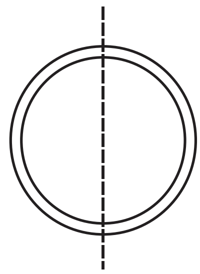
Official Ans. by NTA (8)

Sol.



$$I_1 = \frac{ml^2}{3}$$

$$\ell = 2\pi r \Rightarrow \frac{\ell}{r} = 2\pi$$



$$I_1 = \frac{mr^2}{2}$$

$$\frac{I_1}{I_2} = \frac{2}{3}\left(\frac{\ell}{r}\right)^2$$

$$= \frac{2}{3}\times4\pi^2 = \frac{8\pi^2}{3}$$

x = 8

8. The half life of a radioactive substance is 5 years. After x years a given sample of the radioactive substance gest reduced to 6.25% of its initial value of x is _______.

Official Ans. by NTA (20)

Sol. **T1/2 = 5 year**

$$N = N_0\left(\frac{1}{2}\right)^{\text{No of half lives}}$$

$$\frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

Time = 4 half lives = 20 years

9. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-3} cm. If the distance between the slits is 1 mm, then the wavelength of the light will be ______nm.

Official Ans. by NTA (600)

Sol. $\beta = \frac{\lambda D}{d}$

$\Delta\beta = \frac{\lambda}{d}\Delta D$

$\lambda = \frac{\Delta\beta . d}{\Delta D}$

$= \frac{3\times10^{-5}\times1\times10^{-3}}{5\times10^{-2}}$

$= 60\times10^{-8} = 600\times10^{-9}$m

= 600nm

10. An inductor of 0.5 mH, a capacitor of 200 μF and a resistor of 2 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the frequency of ac source will be___$\times 10^{2}$ Hz.

Official Ans. by NTA (5)

Sol. If Current is in phase with emf then the frequency of source $= \frac{1}{2\pi\sqrt{LC}}$ (Resonant frequency)

$$\frac{1}{2\pi\sqrt{\frac{1}{2}\times10^{-3}\times2\times10^{-4}}}$$

$= \frac{1}{2\pi}\times\sqrt{10}\times1000 = 500$ Hz

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

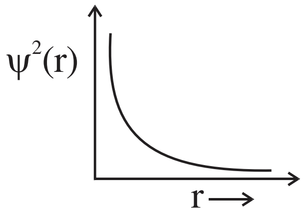
1. Using the rules for significant figures, the correct answer for the expression $\frac{0.02858 \times 0.112}{0.5702}$ will be:

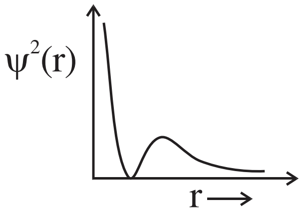
(A) 0.005613 (B) 0.00561
(C) 0.0056 (D) 0.006

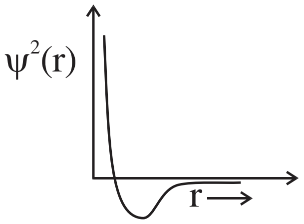
Official Ans. by NTA (B)

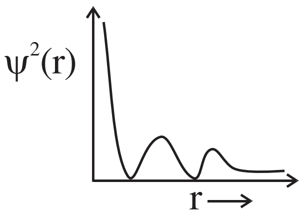
Sol. Reported answer should not be more precise than least precise term in calculations, so there should be three significant figures in reported answer.

2. Which of the following is the correct plot for the probability density $\psi^2(r)$ as a function of distance 'r' of the electron form the nucleus for 2s orbital?

(A)


(B)


(C)


(D)


Official Ans. by NTA (B)

Sol. For 2s, number of radial nodes = 2 – 0 – 1 = 1 and value of ψ^2 is always positive.

3. Consider the species CH_4, NH_4^+ and BH_4^-. Choose the correct option with respect to the there species:

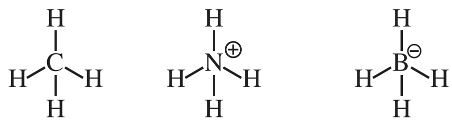
(A) They are isoelectronic and only two have tetrahedral structures

(B) They are isoelectronic and all have tetrahedral structures

(C) Only two are isoelectronic and all have tetrahedral structures

(D) Only two are isoelectronic and only two have tetrahedral structures

Official Ans. by NTA (B)

Sol.


All are tetrahedral and each have 10 electrons.

4. 4.0 moles of argon and 5.0 moles of PCl_5 are introduced into an evacuated flask of 100 litre capacity at 610 K. The system is allowed to equilibrate. At equilibrium, the total pressure of mixture was found to be 6.0 atm. The K_p for the reaction is [Given : $R = 0.082 L\ atm\ K^{-1}\ mol^{-1}$]

(A) 2.25 (B) 6.24
(C) 12.13 (D) 15.24

Official Ans. by NTA (A)

Sol. PCl_5 = 5 mole

Ar = 4 mole

$$P_{Total} = \frac{9 \times 0.82 \times 610}{100} = 4.5\,atm$$

$$P_{PCl_5} = \frac{5 \times 4.5}{9} = 2.5\ ;\ P_{Ar} = \frac{4 \times 4.5}{9} = 2$$

$$PCl_5 \rightleftharpoons PCl_3 + Cl_2$$

2.5 – P P P

$P_{total} = 2.5 - P + P + P + P_{Ar} = 6$

$P = 1.5$

$$K_p = \frac{1.5 \times 1.5}{1} = 2.25$$

5. A 42.12% (w/v) solution of NaCl causes precipitation of a certain sol in 10 hours. The coagulating value of NaCl for the sol is

[Given : Molar mass : Na = 23.0 g mol^{-1}; Cl = 35.5 g mol^{-1}]

(A) 36 mmol L^{-1}

(B) 36 mol L^{-1}

(C) 1440 mol L^{-1}

(D) 1440 mmol L^{-1}

Official Ans. by NTA (D)

Sol. Data insufficient.

6. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The first ionization enthalpy for oxygen is lower than that of nitrogen.

Reason R : The four electrons in 2p orbitals of oxygen experience more electron-electron repulsion.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol. Ionisation energy = N > O.

In oxygen atom, 2 of the 4 2p electrons must occupy the same 2p orbital resulting in an increased electron electron-repulsion.

7. Match List I with List II.

List I Ore	List II Composition
A. Siderite	I. Fe CO_3
B. Malachite	II. $CuCO_3.Cu(OH)_2$
C. Sphalerite	III. ZnS
D. Calamine	IV. $ZnCO_3$

Choose the correct answer from the options given below:

(A) A-I, B-II, C-III, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-III, C-I, D-II

(D) A-I, B-II, C-IV, D-III

Official Ans. by NTA (A)

Sol. Siderite – $FeCO_3$

Malachite – $CuCO_3.Cu(OH)_2$

Calamine – $ZnCO_3$

Sphalerite – ZnS

8. Given below are two statements .

Statement I : In $CuSO_4.5H_2O$, Cu–O bonds are present.

Statement II : In $CuSO_4.5H_2O$, ligands coordinating with Cu(II) ion are O-and S-based ligands.

In the light of the above statements, choose the correct answer from the options given below

(A) Both Statement I and Statement II are correct

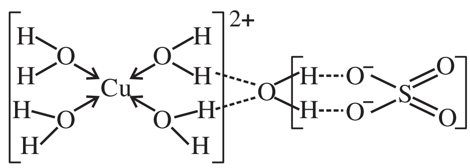
(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (C)

Sol.



9. Amongst baking soda, caustic soda and washing soda carbonate anion is present in :

(A) washing soda only.

(B) washing soda and caustic soda only.

(C) washing soda and baking soda only.

(D) baking soda, caustic soda and washing soda.

Official Ans. by NTA (A)

Sol. Baking soda $\rightarrow NaHCO_3$

Washing soda $\rightarrow Na_2CO_3.10H_2O$

Caustic soda $\rightarrow NaOH$

10. Number of lone pair (s) of electrons on central atom and the shape of BrF_3 molecule respectively, are :

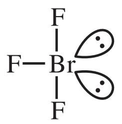
(A) 0, triangular planar.

(B) 1, pyramidal.

(C) 2, bent T-shape.

(D) 1, bent T-shape

Official Ans. by NTA (C)

Sol. 

Steric no. = 5 (sp^3d), lone pair = 2

Bent T shape.

11. Aqueous solution of which of the following boron compounds will be strongly basic in nature?

(A) $NaBH_4$ (B) $LiBH_4$

(C) B_2H_6 (D) $Na_2B_4O_7$

Official Ans. by NTA (D)

Sol. $Na_2B_4O_7$ gives H_3BO_3 and NaOH (strong base) in water.

12. Sulphur dioxide is one of the components of polluted air. SO_2 is also a major contributor to acid rain. The correct and complete reaction to represent acid rain caused by SO_2 is :

(A) $2\ SO_2 + O_2 \rightarrow 2\ SO_3$

(B) $SO_2 + O_3 \rightarrow SO_3 + O_2$

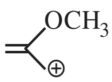
(C) $SO_2 + H_2O_2 \rightarrow H_2SO_4$

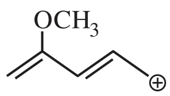
(D) $2\ SO_2 + O_2 + 2\ H_2O \rightarrow 2\ H_2SO_4$

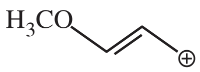
Official Ans. by NTA (D)

Sol. $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$ (Acid rain)

13. Which of the following carbocations is most stable :

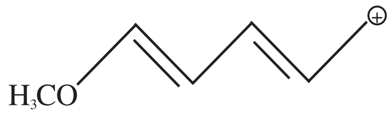
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (D)

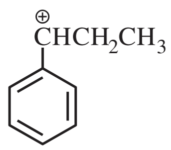
Sol. 

Is most stable carbocation

14.  $+ CH_3CH_2CH_2Cl \xrightarrow{\text{Anhydrous, } AlCl_3}$ 'A' Major Product

The stable carbocation formed in the above reaction is :

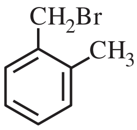
(A) $CH_3CH_2\overset{\oplus}{C}H_2$ (B) $CH_3\overset{\oplus}{C}H_2$

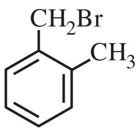
(C) $CH_3-\overset{\oplus}{C}H-CH_3$ (D) 

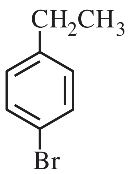
Official Ans. by NTA (C)

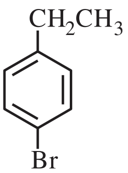
Sol. $CH_3-\overset{\oplus}{C}H-CH_3$ is formed in the above reaction

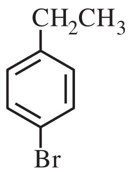
15. Two isomers (A) and (B) with Molar mass 184 g/mol and elemental composition C, 52.2%; H, 4.9% and Br 42.9% gave benzoic acid and p-bromobenzoic acid, respectively on oxidation with $KMnO_4$. Isomer 'A' is optically active and gives a pale yellow precipitate when warmed with alcoholic $AgNO_3$. Isomer 'A' and 'B' are, respectively :

(A) $H_3C-CHBr-C_6H_5$ and 

(B) 

and 

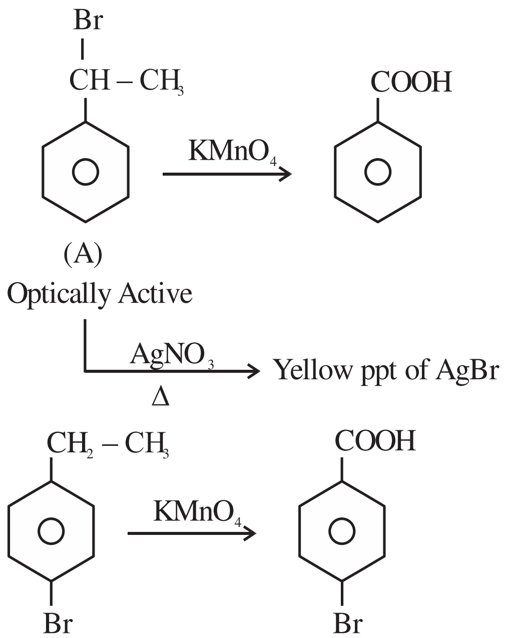
(C) $H_3C-CHBr-C_6H_5$ and 

(D) 

and $H_3C-CHBr-C_6H_5$

Official Ans. by NTA (C)

Sol.



16. In Friedel-Crafts alkylation of aniline, one gets :

(A) alkylated product with ortho and para substitution.

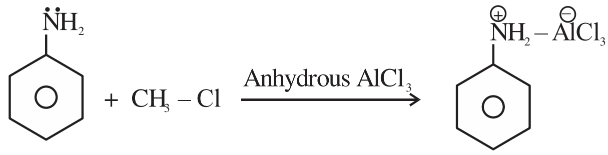
(B) secondary amine after acidic treatment.

(C) an amide product.

(D) positively charged nitrogen at benzene ring.

Official Ans. by NTA (D)

Sol.



17. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Dacron is an example of polyester polymer.

Reason R: Dacron is made up of ethylene glycol and terephthalic acid monomers.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

(A) Both **A** and **B** are correct and **R** is the correct explanation of **A**.

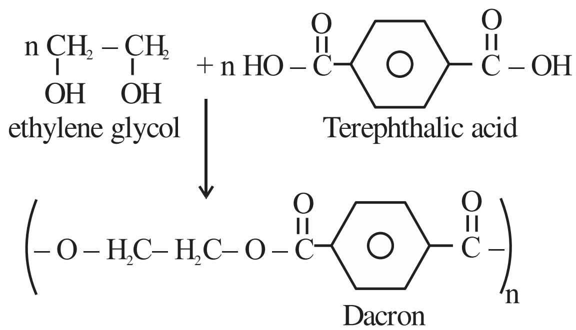
(B) Both **A** and **B** are correct but **R** is NOT the correct explanation of **A**.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (A)

Sol.



18. The structure of protein that is unaffected by heating is :

(A) secondary structure (B) tertiary structure

(C) primary structure (D) quaternary structure

Official Ans. by NTA (C)

Sol. Primary structure of protein is unaffected by physical 'or' chemical changes.

19. The mixture of chloroxylenol and terpineol is an example of :

(A) antiseptic (B) pesticide

(C) disinfectant (D) narcotic analgesic

Official Ans. by NTA (A)

Sol. Antiseptic Dettol is mixture of chloroxylenol and terpineol.

20. A white precipitate was formed when $BaCl_2$ was added to water extract of an inorganic salt. Further, a gas 'X' with characteristic odour was released when the formed white precipitate was dissolved in dilute HCl. The anion present in the inorganic salt is :

(A) I^- (B) SO_3^{2-}

(C) S^{2-} (D) NO_2^-

Official Ans. by NTA (B)

Sol. $BaCl_2 + SO_3^{2-} \rightarrow \underset{\text{white}}{BaSO_3\downarrow} \xrightarrow{\text{dil.HCl}} \underset{\text{burning sulphur like smell}}{SO_2\uparrow}$

SECTION-B

1. A box contains 0.90 g of liquid water in equilibrium with water vapour at 27°C. The equilibrium vapour pressure of water at 27°C 32.0 Torr. When the volume of the box is increased, some of the liquid water evaporates to maintain the equilibrium pressure. If all the liquid water evaporates, then the volume of the box must be____ litre. [nearest integer]

(Given: R = 0.082 L atm K^{-1} mol^{-1})

(Ignore the volume of the liquid water and assume water vapours behave as an ideal gas.)

Official Ans. by NTA (29)

)

Sol. $V = \frac{nRT}{P} = \frac{0.90 \times 0.82 \times 300 \times 760}{18 \times 32} = 29.21$

2. 2.2 g of nitrous oxide (N_2O) gas is cooled at a constant pressure of 1 atm from 310 K to 270 K causing the compression of the gas from 217.1 mL to 167.75 mL. The change in internal energy of the process, ΔU is '–x' J. The value of 'x' is __.

[nearest integer]

(Given: atomic mass of N = 14 g mol^{-1} and of O = 16 g mol^{-1}.

Molar heat capacity of N_2O is 100 JK^{-1} mol^{-1})

Official Ans. by NTA (195)

Sol. $N_2O\,\text{moles} = \frac{2.2}{44} = \frac{1}{20}$

$\Delta H = nC_p\Delta T = \frac{1}{20} \times 100(-40) = -200J$

$\Delta U = q_p + w$

$w = -P_{ext.}\,\Delta V$

$W = -1\frac{(167.75 - 217.1)}{1000} \times 101.3J$

$w = +5J$

$\Delta U = -200 + 5 = -195J$

3. Elevation in boiling point for 1.5 molal solution of glucose in water is 4K. The depression in freezing point for 4.5 molal solution of glucose in water is 4K. The ratio of molal elevation constant to molal depression constant (K_b/K_f) is____.

Official Ans. by NTA (3)

Sol. $\Delta T_b = iK_b m$

$\Delta T_f = iK_f m$

$$\frac{4}{4} = \frac{K_b 1.5}{K_f 4.5}$$

$$\frac{K_b}{K_f} = 3$$

4. The cell potential for the given cell at 298 K

$Pt \mid H_2(g, 1\ bar) \mid H^+(aq) \parallel Cu^{2+}(aq) \mid Cu(s)$

is 0.31V. The pH of the acidic solution is found to be 3, whereas the concentration of Cu^{2+} is 10^{-x} M. The value of x is _____.

(Given: $E^{\ominus}_{Cu^{2+}/Cu} = 0.34$ V and $\frac{2.303RT}{F} = 0.06V$)

Official Ans. by NTA (7)

Sol. $H_2(g) + Cu^{2+}(aq.) \rightarrow 2H^+(aq.) + Cu(s)$

$$0.31 = 0.34 - \frac{0.06}{2}\log\frac{[H^+]^2}{[Cu^{2+}]}$$

$[Cu^{2+}] = 10^{-7}$ M

x = 7

5. The equation

$k = (6.5 \times 10^{12}\ s^{-1})\ e^{-26000K/T}$

is followed for the decomposition of compound A. The activation energy for the reaction is ____ kJ mol^{-1}. [nearest integer]

(Given: R = 8.314 J K^{-1} mol^{-1})

Official Ans. by NTA (216)

)

Sol. $K = Ae^{-Ea/RT} = (6.5 \times 10^{12}\ s^{-1})\ e^{-26000K/T}$

$$\frac{Ea}{8.314} = 26000$$

Ea = 216.164 kJ/mol.

6. Spin only magnetic moment of $[MnBr_6]^{4-}$ is____ B.M. (round off to the closest integer)

Official Ans. by NTA (6)

Sol. $Mn^{2+} \rightarrow t_{2g^{111}} e_{g^{11}}$

$\mu_s = \sqrt{35}$

= 5.91

= 6

7. For the reaction given below:

$CoCl_3 . xNH_3 + AgNO_3\ (aq) \rightarrow$

If two equivalents of AgCl precipitate out, then the value of x will be _____.

Official Ans. by NTA (5)

Sol. $CoCl_3.xNH_3 + AgNO_3 \rightarrow AgCl\downarrow$

2 mol

$[Co(NH_3)_5Cl]Cl_2 + AgNO_3 \rightarrow AgCl\downarrow$

2 mol

x = 5

8. The number of chiral alcohol(s) with molecular formula $C_4H_{10}O$ is______.

Official Ans. by NTA (1)

Sol. $CH_3 - CH_2 - CH_2 - CH_2 - OH$

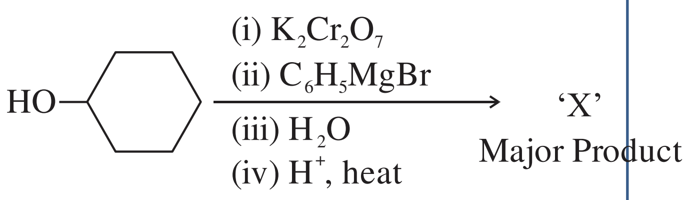
$CH_3 - \overset{\star}{C}H(OH) - CH_2 - CH_3$ (±)

$CH_3 - CH(CH_3) - CH_2 - OH$

$CH_3 - C(CH_3)(OH) - CH_3$

Out of which only two are chiral

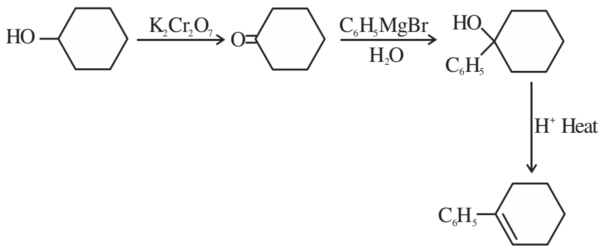
9. In the given reaction



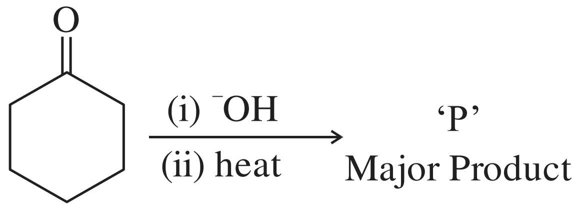
the number of sp^2 hybridised carbon (s) in compound 'X' is_____.

Official Ans. by NTA (8)

Sol.



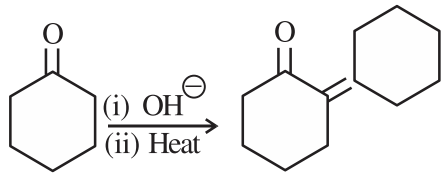
10. In the given reaction,



The number of π electrons present in the product 'P' is _____.

Official Ans. by NTA (4s)

Sol.



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

(A) 1 (B) α

(C) 1 + α (D) 1 + 2 α

Official Ans. by NTA (A)

Sol. $x^4 + x^2 + 1 = 0$

$\Rightarrow (x^2 + x + 1)(x^2 - x + 1) = 0$

$\Rightarrow x = \pm\omega, \pm\omega^2$ where $\omega = 1^{1/3}$ and imaginary.

So $\alpha^{1011} + \alpha^{2022} - \alpha^{3033} = 1 + 1 - 1 = 1$

2. Let arg (z) represent the principal argument of the complex number z. The, |z| = 3 and arg (z – 1) – arg (z + 1) = $\frac{\pi}{4}$ intersect:

(A) Exactly at one point

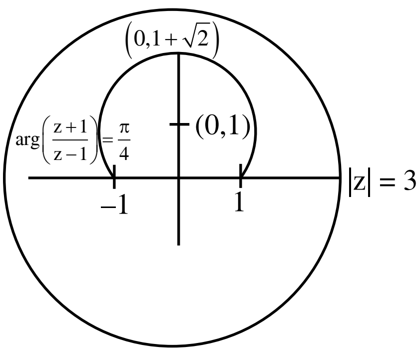
(B) Exactly at two points

(C) Nowhere

(D) At infinitely many points.

Official Ans. by NTA (C)

Sol.



3. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1\,(\text{adj}A) + {}^5C_2\,(\text{adj}A)^2 - \ldots - {}^5C_5\,(\text{adj}A)^5$, then the sum of all elements of the matrix B is:

(A) –5 (B) –6

(C) –7 (D) –8

Official Ans. by NTA (C)

Sol. $B = (I - \text{adj}A)^5 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}^5 = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

Sum of its all elements = –7.

4. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots$ is equal to:

(A) $\frac{425}{216}$ (B) $\frac{429}{216}$

(C) $\frac{288}{125}$ (D) $\frac{280}{125}$

Official Ans. by NTA (C)

Sol. $S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \ldots$

$\frac{S}{6} = \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} + \frac{22}{6^4} + \ldots$

on subtraction

$\frac{5}{6}S = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \frac{13}{6^4} + \ldots$

$\frac{5}{36}S = 1 + \frac{4}{6^2} + \frac{7}{6^3} + \frac{10}{6^4} + \frac{13}{6^5} + \ldots$

on subtraction

$\frac{25}{36}S = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + \ldots = \frac{8}{5}$

$S=\frac{288}{125}$

5. The value of $\lim\limits_{x\to 1}\frac{\left(x^2-1\right)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to:

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{3}$

(C) $\frac{\pi^2}{2}$ (D) π^2

Official Ans. by NTA (D)

Sol. $\lim\limits_{x\to 1}\frac{(x^2-1)\sin^2\pi x}{(x^2-1)(x-1)^2}=\lim\limits_{x\to 1}\left(\frac{\sin((1-x)\pi))}{\pi(1-x)}\right)^2\pi^2=\pi^2$

6. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x) = (x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in N$. The, which of the following is NOT true?

(A) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(B) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local manima.

(C) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(D) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

Official Ans. by NTA (C)

Sol. $f'(x)=(x-3)^{n_1-1}(x-5)^{n_2-1}(n_1+n_2)\left(x-\frac{5n_1+3n_2}{n_1+n_2}\right)$

Option (3) is incorrect since

for $n_1 = 3$, $n_2 = 5$

$f'(x) = 8(x-3)^2(x-5)^4\left(x-\frac{30}{8}\right)$

minima at $x=\frac{30}{8}$

7. Let f be a real valued continuous function on [0,1] and $f(x)=x+\int_0^1(x-t)f(t)dt$. Then which of the following points (x,y) lies on the curve $y = f(x)$?

(A) (2, 4) (B) (1, 2)

(C) (4, 17) (D) (6, 8)

Official Ans. by NTA (D)

Sol. $f(x)=\left(1+\int_0^1 f(t)dt\right)x-\int_0^1 tf(t)dt$

$f(x) = Ax - B$ ...(i)

$A=1+\int_0^1 f(t)dt=1+\int_0^1(At-B)dt$

$\Rightarrow A=2(1-B)$ ...(ii)

Also $B=\int_0^1 tf(t)dt=\int_0^1(At^2-Bt)dt$

$A=\frac{9}{2}B$ ...(iii)

From (2), (3)

$A=\frac{18}{13},\ B=\frac{4}{13}$

so $f(6) = 8$

8. If $\int_0^2\left(\sqrt{2x}-\sqrt{2x-x^2}\right)dx=$

$\int_0^1\left(1-\sqrt{1-y^2}-\frac{y^2}{2}\right)dy+\int_1^2\left(2-\frac{y^2}{2}\right)dy+I$

(A) $\int_0^1\left(1+\sqrt{1-y^2}\right)dy$

(B) $\int_0^1\left(\frac{y^2}{2}-\sqrt{1-y^2}+1\right)dy$

(C) $\int_0^1\left(1-\sqrt{1-y^2}\right)dy$

(D) $\int_0^1\left(\frac{y^2}{2}+\sqrt{1-y^2}+1\right)dy$

Official Ans. by NTA (C)

Sol. $LHS=\int_0^2\left(\sqrt{2x}-\sqrt{2x-x^2}\right)dx=\frac{8}{3}-\frac{\pi}{2}$

$RHS=\int_0^1\left(1-\sqrt{1-y^2}-\frac{y^2}{2}\right)dy+\int_1^2\left(2-\frac{y^2}{2}\right)dy+I$

$I+\frac{5}{3}-\frac{\pi}{4}$

So, $I = 1-\frac{\pi}{4} = \int_0^1\left(1-\sqrt{1-y^2}\right)dy$

9. If y = y (x) is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$ and y(0)= 0, then $6\left(y'(0)+\left(y\left(\log_e\sqrt{3}\right)\right)^2\right)$ is equal to:

(A) 2 (B) –2

(C) –4 (D) –1

Official Ans. by NTA (C)

Sol. $\frac{dy}{1+y^2}+\frac{2e^x}{1+e^{2x}}dx=0$ (i)

on integration

$\tan^{-1}y+2\tan^{-1}e^x=c$

$\because y(0)=0$

so, $C=\frac{\pi}{2}\Rightarrow \tan^{-1}y+2\tan^{-1}e^x=\frac{\pi}{4}$

from eq.(i), $\left(\frac{dy}{dx}\right)_{x=0}=-1$

$\arg y(\ln\sqrt{3})=-\frac{1}{\sqrt{3}}$

$6\left[y'(0)+(y(\ln\sqrt{3})^2\right]=6\left[-1+\frac{1}{3}\right]=-4$

10. Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S.Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line y = 3x + 5 touch the parabola P at A and B. Then the value of a for which A,B and S are collinear is:

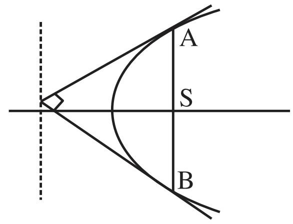
(A) 8 only (B) 2 only

(C) $\frac{1}{4}$ only (D) any a > 0

Official Ans. by NTA (D)

Sol. Lines making angle $\frac{\pi}{4}$ with y = 3x + 5 have slope –2 & 1/2.

Which are perpendicular to each-other so, A, S, B are collinear for all a > 0.



11. Let a triangle *ABC* be inscribed in the circle $x^2 - \sqrt{2}(x+y)+y^2=0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to:

(A) $\left(\sqrt{2}+\sqrt{6}\right)/3$ (B) $\left(\sqrt{6}+\sqrt{3}\right)/2$

(C) $\left(3+\sqrt{3}\right)/4$ (D) $\left(\sqrt{6}+2\sqrt{3}\right)/4$

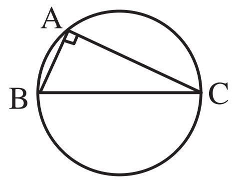
Official Ans. by NTA (Dropped)

Sol. Radius of given circle is 1.

BC = diameter = 2, $AB=\sqrt{2}$

$AC=\sqrt{BC^2-AB^2}=\sqrt{2}$

$\Delta ABC=\frac{1}{2}AB.AC=1$



12. Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane px – qy + z = 5, for some p, q$\in\mathbb{R}$. The shortest distance of the plane from the origin is:

(A) $\sqrt{\frac{3}{109}}$ (B) $\sqrt{\frac{5}{142}}$

(C) $\sqrt{\frac{5}{71}}$ (D) $\sqrt{\frac{1}{142}}$

Official Ans. by NTA (B)

Sol. (2, –1, –3) satisfy the given plane.

So 2p + q = 8 .... (i)

Also given line is perpendicular to normal plane so

3p + 2q – 1 = 0 .... (ii)

$\Rightarrow p = 15, q = -22$

Eq. of plane $15x - 22y + z - 5 = 0$

its distance from origin $= \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$

13. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is:

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 4

Official Ans. by NTA (C)

Sol. $AB \equiv x - 2y + 1 = 0$

$AC \equiv 2x - y - 1 = 0$

So A(1, 1)

Altitude from B is $BH = x + 2y - 7 = 0 \Rightarrow B(3, 2)$

Altitude from C is $CH = 2x + y - 7 = 0 \Rightarrow C(2, 3)$

Centroid of $\Delta ABC = E(2, 2)$ $OE = 2\sqrt{2}$

14. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$. Then, which of the following points lies on T?

(A) (2, 1, 0) (B) (1, 2, 1)

(C) (1, 2, 2) (D) (1, 3, 2)

Official Ans. by NTA (B)

Sol. Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$ is given by Q(4, 8, 7)

Eq. of plane $T = \begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2x - z = 1$ so B(1, 2, 1) lies on it.

15. Let A, B, C be three points whose position vectors respectively are:

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non-collinear, then the length of the median, in ΔABC, through A is:

(A) $\frac{\sqrt{82}}{2}$ (B) $\frac{\sqrt{62}}{2}$

(C) $\frac{\sqrt{69}}{2}$ (D) $\frac{\sqrt{66}}{2}$

Official Ans. by NTA (A)

Sol. $\overrightarrow{AB} \| \overrightarrow{AC}$ if $\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Mid-point of BC $= M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$

16. The probability that a relation R from {x,y} to {x,y} is both symmetric and transitive, is equal to:

(A) $\frac{5}{16}$ (B) $\frac{9}{16}$

(C) $\frac{11}{16}$ (D) $\frac{13}{16}$

Official Ans. by NTA (A)

Sol. Total no. of relations = $2^{2\times 2}$ = 16

Fav. relation = ϕ, {(x, x)}, {(y, y)}, {(x, x)(y, y)}

{(x, x), (y, y), (x, y)(y, x)}

Prob. = $\frac{5}{16}$

17. The number of values of $a \in \mathbb{N}$ such that the variance of 3, 7, 12 a, 43 – a is a natural number is:

(A) 0 (B) 2

(C) 5 (D) infinite

Official Ans. by NTA (A)

Sol. Mean =13

Variance $= \frac{9+49+144+a^2+(43-a)^2}{5} - 13^2 \in N$

$\Rightarrow \frac{2a^2 - a + 1}{5} \in N$

$\Rightarrow$ $2a^2 - a + 1 - 5n = 0$ must have solution as natural numbers

its D = 40n – 7 always has 3 at unit place

$\Rightarrow$D can't be perfect square

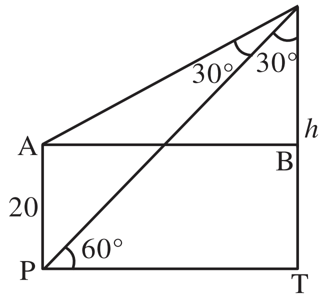
So, a can't be integer.

18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is:

(A) $15\sqrt{3}$ (B) $20\sqrt{3}$

(C) $20+10\sqrt{3}$ (D) 30

Official Ans. by NTA (4)



Sol. $PT = \frac{h}{\sqrt{3}} = AB$

$\frac{AB}{h-20} = \sqrt{3}$

h = 3(h – 20)

h = 30

19. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to:

(A) $p \wedge (\sim q) \wedge r$ (B) $(\sim p) \wedge (\sim q) \wedge r$

(C) $(\sim p) \wedge q \wedge r$ (D) $p \wedge q \wedge (\sim r)$

Official Ans. by NTA (C)

Sol. $P \vee q \Rightarrow (\sim r \vee p)$

$\equiv \sim (p \vee q) \vee (\sim r \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (p \vee \sim r)$

$\equiv [\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim r$

$\equiv [\sim q \vee p) \vee \sim r$

Its negation is $\sim p \wedge q \wedge r$

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\,\alpha$ and $6^n - 5n - 1 = 25\,\beta$, then $\alpha - \beta$ is equal to:

(A) $1 + {}^nC_2\,(8-5) + {}^nC_3\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-1} - 5^{n-1})$

(B) $1 + {}^nC_3\,(8-5) + {}^nC_4\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-2} - 5^{n-2})$

(C) ${}^nC_3\,(8-5) + {}^nC_4\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-2} - 5^{n-2})$

(D) ${}^nC_4\,(8-5) + {}^nC_5\,(8^2 - 5^2) + ... + {}^nC_n\,(8^{n-3} - 5^{n-3})$

Official Ans. by NTA (C)

Sol. $\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + ...$

$\beta = {}^nC_2 + {}^nC_3 5 + {}^nC_4 5^2 + ...$

option (3) will be the answer.

SECTION-B

1. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + \left(\vec{b} \times \vec{c}\right) = \vec{0}$ and $\vec{b} . \vec{c} = 5$. Then, the value of $3\left(\vec{c} . \vec{a}\right)$ is equal to____.

Official Ans. by NTA (DROP)

Sol. $\vec{a} + \vec{b} \times \vec{c} = 0$

$\vec{a} \times \vec{b} + |\vec{b}|^2 \vec{c} - 5\vec{b} = 0$

It gives $\vec{c} = \frac{1}{3}(10\hat{i} + 3\hat{j} + 2\hat{k})$

so $3\vec{a}.\vec{c} = 10$

But it does not satisfy $\vec{a} + \vec{b} \times \vec{c} = 0$.

This question has data error.

Alternate (Explanation) :

According to given $\vec{a}$ & $\vec{b}$

$\vec{a} \cdot \vec{b} = 1 - 2 + 3 = 2$ .... (i)

but given equation

$\vec{a} = -(\vec{b} \times \vec{c})$

$\Rightarrow \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

which contradicts.

2. Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^{\alpha}+1}{\beta e^{\alpha}}$. then the value of $\alpha + \beta$ is equal to_____.

Official Ans. by NTA (14)

Sol. $\frac{dy}{dx} + \frac{2x}{x-1}.y = \frac{1}{(x-1)^2}$

$y = \frac{1}{(x-1)^2}\left[\frac{e^{2x}+1}{2e^{2x}}\right]$

$y(3) = \frac{e^6+1}{8e^6}$

$\alpha + \beta = 14$

3. Let 3, 6, 9, 12,... upto 78 terms and 5, 9, 13, 17,... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to___.

Official Ans. by NTA (2223)

Sol. For series of common terms

a=9, d=12, n=19

$S_{19} = \frac{19}{2}[2(9) + 18(12)] = 2223$

4. The number of solutions of the equation sin x = $\cos^2$ x in the interval (0,10) is__.

Official Ans. by NTA (4)

Sol. $\sin^2x + \sin x - 1 = 0$

$\sin x = \frac{-1+\sqrt{5}}{2} = +ve$

Only 4 roots

5. For real numbers a,b (a > b > 0), let

$$Area\left\{(x,y): x^2 + y^2 \le a^2 \text{and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\right\} = 30\pi$$

and

$$Area\left\{(x,y): x^2 + y^2 \ge b^2 \text{and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$$

Then the value of $(a-b)^2$ is equal to__.

Official Ans. by NTA (12)

Sol. given $\pi a^2 - \pi ab = 30\pi$ and $\pi ab - \pi b^2 = 18\pi$

on subtracting, we get $(a-b)^2 = a^2 - 2ab + b^2 = 12$

6. Let f and g be twice differentiable even functions on (–2, 2) such that $f\left(\frac{1}{4}\right) = 0, f\left(\frac{1}{2}\right) = 0, f(1) = 1$ and $g\left(\frac{3}{4}\right) = 0, g(1) = 2$ Then, the minimum number of solutions of $f(x)\,g''(x) + f'(x)g'(x) = 0$ in $(-2,2)$ is equal to__.

Official Ans. by NTA (4)

Sol. Let $h(x) = f(x)\,g'(x) \to 5$ roots

$\because$ f (x) is even $\Rightarrow$

$$f\left(\frac{1}{4}\right)=f\left(\frac{1}{2}\right)=f\left(-\frac{1}{2}\right)=f\left(\frac{1}{4}\right)=0$$

g(x) is even $\Rightarrow g\left(\frac{3}{4}\right)=g\left(-\frac{3}{4}\right)=0$

$g'(x) = 0$ has minimum one root

$h'(x)$ has at last 4 roots

7. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and n respectively. If r is a positive integer such $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to__.

Official Ans. by NTA (5)

Sol. $T_{r+1} = (-1)^r \cdot {}^{15}C_r \cdot 2^{15-r}\, x^{\frac{15-2r}{5}}$

$m = {}^{15}C_{10}2^5$

$n = -1$

so $mn^2 = {}^{15}C_5 2^5$

8. The total number of four digit numbers such that each of the first three digits is divisible by the last digit, is equal to______.

Official Ans. by NTA (1086)

Sol. Let the number is abcd, where a,b,c are divisible by d.

	No. of such numbers
d = 1,	$9 \times 10 \times 10 = 900$
d = 2	$4 \times 5 \times 5 = 100$
d = 3	$3 \times 4 \times 4 = 48$
d = 4	$2 \times 3 \times 3 = 18$
d = 5	$1 \times 2 \times 2 = 4$
d = 6, 7, 8, 9	$4 \times 4 = 16$
	1086

9. Let $M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$, where α is a non-zero real number an $N=\sum_{k=1}^{49}M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ____.

Official Ans. by NTA (1)

Sol. $M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$; $M^2=\begin{bmatrix}-\alpha^2 & 0\\ 0 & -\alpha^2\end{bmatrix}=-\alpha^2 I$

$N = M^2 + M^4 + \ldots\ldots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \ldots.]I$

$= -\alpha^2\dfrac{(1-(-\alpha^2)^{49})}{1+\alpha^2}.I$

$I - M^2 = (1 + \alpha^2)\, I$

$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$

$\alpha = 1$

10. Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is_____.

Official Ans. by NTA (18)

Sol. $f(g(x) = 8x^2 - 2x$

$g(f(x) = 4x^2 + 6x + 1$

So, $g(x) = 2x - 1$ $\qquad$ $g(2) = 3$

& $f(x) = 2x^2 + 3x + 1$

$f(2) = 8 + 6 + 1 = 15$

Ans. 18