JEE Main 2022 July Question Paper with Answer

25th, 26th, 27th, 28th & 29th July 2022 (Shift 1 & Shift 2)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If momentum [P], area [A] and time [T] are taken as fundamental quantities, then the dimensional formula for coefficient of viscosity is :

(A) $[P\ A^{-1}\ T^{0}]$ (B) $[P\ A\ T^{-1}]$

(C) $[P\ A^{-1}\ T]$ (D) $[P\ A^{-1}\ T^{-1}]$

Official Ans. by NTA (A)

Sol. Viscosity = pascal.second

$P^x\ A^y\ T^z = [M^1\ L^{-1}\ T^{-1}]$

$[M^1\ L^{+1}\ T^{-1}]^x\ [L^2]^y[T^1]^z = M^1L^{-1}\ T^{-1}$

$M^xL^{+x+2y}\ T^{-x+z} = M^1\ L^{-1}\ T^{-1}$

$x = 1 \quad x + 2y = -1 \quad -x + z = -1$

$y = -1$

$z = 0$

Viscosity $= P^1\ A^{-1}\ T^0$

2. Which of the following physical quantities have the same dimensions ?

(A) Electric displacement ($\vec{D}$) and surface charge density

(B) Displacement current and electric field

(C) Current density and surface charge density

(D) Electric potential and energy

Official Ans. by NTA (A)

Sol. Electric displacement

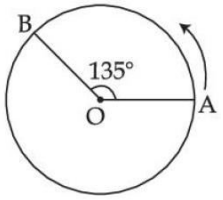
$\vec{D} = \epsilon_0\ \vec{E}$

$$[D] = [\epsilon_0\ E] = \left[\epsilon_0\ \frac{\sigma}{\epsilon_0}\right]$$

$[D] = [\sigma]$

$\rightarrow$ Surface change density = σ.

3. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be : (Given cos135° = - 0.7)



(A) 42 m (B) 47 m

(C) 19 m (D) 40 m

Official Ans. by NTA (B)

Sol. $d = R\theta$

$$60 = R\left(\frac{3\pi}{4}\right)$$

$$R = \frac{60\times 4}{3\pi} = \frac{80}{\pi}\text{m}$$

$$\text{Displacement} = \sqrt{R^2 + R^2 - 2R^2\cos 135}$$

$$\Rightarrow \sqrt{2R^2 - 2R^2(-0.7)}$$

$$\Rightarrow \sqrt{3.4R^2} = \sqrt{3.4\left(\frac{80}{\pi}\right)^2}$$

≈ 47 m

4. A body of mass 0.5 kg travels on straight line path with velocity $v = (3x^2 + 4)$m/s. The net workdone by the force during its displacement from x = 0 to x = 2 m is :

(A) 64 J (B) 60 J

(C) 120 J (D) 128 J

Official Ans. by NTA (B)

Sol. $v_i = 3(0^2) + 4 = 4 \quad \cong \quad x = 0$

$v_F = 3(2)^2 + 4 \quad \cong \quad x = 2$

$= 16$

$$W = \Delta K = \frac{1}{2}m\left(16^2 - 4^2\right)$$

$$= \frac{1}{2}\times\frac{1}{2}(256 - 16)$$

$$= \frac{240}{4} = 60\text{J}$$

5. A solid cylinder and a solid sphere, having same mass M and radius R, roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they reach the ground, will be :

(A) $\sqrt{\frac{5}{3}}$ (B) $\sqrt{\frac{4}{5}}$

(C) $\sqrt{\frac{3}{5}}$ (D) $\sqrt{\frac{14}{15}}$

Official Ans. by NTA (D)

Sol. $V = \sqrt{\frac{2gH}{1+k^2/R^2}}$

$$\frac{V_{cylinder}}{V_{sphere}} = \sqrt{\frac{(1+k^2/R^2)_{sphere}}{(1+k^2/R^2)_{cylinder}}}$$

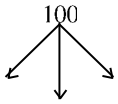
$$= \sqrt{\frac{1+2/5}{1+1/2}} = \sqrt{\frac{7}{5}\times\frac{2}{3}} = \sqrt{\frac{14}{15}}$$

6. Three identical particle A, B and C of mass 100 kg each are placed in a straight line with AB = BC = 13 m. The gravitational force on a fourth particle P of the same mass is F, when placed at a distance 13 m from the particle B on the perpendicular bisector of the line AC. The value of F will be approximately :

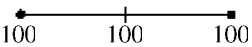
(A) 21 G (B) 100 G

(C) 59 G (D) 42 G

Official Ans. by NTA (B)



Sol.



$$F = \frac{GMM}{r^2} + \sqrt{2}\frac{GMM}{(\sqrt{2}r)^2}$$

$$= \frac{GMM}{r^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$= \frac{G\times10^4}{13^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$F \simeq 100G$$

7. A certain amount of gas of volume V at 27°C temperature and pressure 2×10^7 Nm^{-2} expands isothermally until its volume gets doubled. Later it expands adiabatically until its volume gets redoubled. The final pressure of the gas will be (Use $\gamma = 1.5$)

(A) 3.536×10^5 Pa (B) 3.536×10^6 Pa

(C) 1.25×10^6 Pa (D) 1.25×10^5 Pa

Official Ans. by NTA (B)

Sol. $P_1 = 2\times10^7$ Pa

$P_1V_1 = P_2V_2$

Since $V_2 = 2V_1$ Hence $P_2 = P_1/2$ (isothermal expansion)

$P_2 = 1\times10^7$ Pa

$P_2(V_2)^\gamma = P_3(2V_2)^\gamma$

$$P_3 = \frac{1\times10^7}{2^{1.5}} = 3.536\times10^6$$

8. Following statements are given :

(1) The average kinetic energy of a gas molecule decreases when the temperature is reduced.

(2) The average kinetic energy of a gas molecule increases with increase in pressure at constant temperature.

(3) The average kinetic energy of a gas molecule decreases with increases in volume.

(4) Pressure of a gas increases with increase in temperature at constant pressure.

(5) The volume of gas decreases with increase in temperature.

Choose the correct answer from the options given below :

(A) (1) and (4) only (B) (1), (2) and (4) only

(C) (2) and (4) only (D) (1), (2) and (5) only

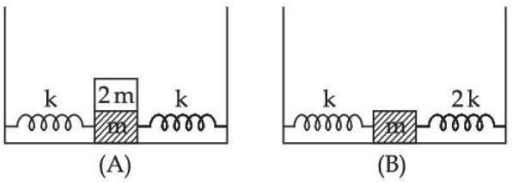
Official Ans. by NTA (A)

Sol. $KE_{avg} = \frac{3}{2}KT$

$$P = \frac{1}{3}\rho V_{rms}^2$$

Note : Statement (4) is correct only if we consider it at constant volume and not constant pressure. Ideally, this question must be bonus but most appropriate answer is option (A)

9. In figure (A), mass '2 m' is fixed on mass 'm' which is attached to two springs of spring constant k. In figure (B), mass 'm' is attached to two spring of spring constant 'k' and '2k'. If mass 'm' in (A) and (B) are displaced by distance 'x' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation.



(A) $\frac{T_1}{T_2}=\frac{3}{\sqrt{2}}$ (B) $\frac{T_1}{T_2}=\sqrt{\frac{3}{2}}$

(C) $\frac{T_1}{T_2}=\sqrt{\frac{2}{3}}$ (D) $\frac{T_1}{T_2}=\frac{\sqrt{2}}{3}$

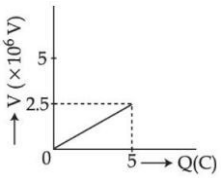
Official Ans. by NTA (A)

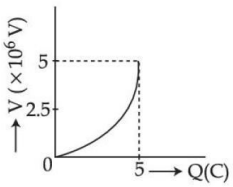
Sol. $T_1=2\pi\sqrt{\frac{3m}{2k}}$

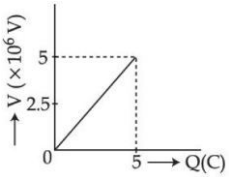
$T_2=2\pi\sqrt{\frac{m}{3k}}$

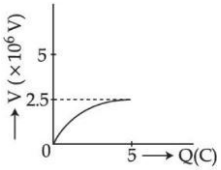
$\frac{T_1}{T_2}=\frac{2\pi\sqrt{\frac{3m}{2k}}}{2\pi\sqrt{\frac{m}{3k}}}=\frac{3}{\sqrt{2}}$

10. A condenser of 2 μF capacitance is charged steadily from 0 to 5C. Which of the following graph represents correctly the variation of potential difference (V) across it's plates with respect to the charge (Q) on the condenser ?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. $Q=CV$

$V=\frac{1}{C}Q$

Straight line with slope $=\frac{1}{C}$

Slope $=\frac{1}{C}=\frac{1}{2\times10^{-6}}=5\times10^5$

11. Two charged particles, having same kinetic energy, are allowed to pass through a uniform magnetic field perpendicular to the direction of motion. If the ratio of radii of their circular paths is 6 : 5 and their respective masses ratio is 9 : 4. Then, the ratio of their charges will be :

(A) 8 : 5 (B) 5 : 4

(C) 5 : 3 (D) 8 : 7

Official Ans. by NTA (B)

Sol. Radius of circular path $R=\frac{\sqrt{2mk}}{qB}$

$q=\frac{\sqrt{2mk}}{RB}$

$\frac{q_1}{q_2}=\sqrt{\frac{m_1}{m_2}}\times\frac{R_2}{R_1}=\sqrt{\frac{9}{4}}\times\frac{5}{6}=\frac{5}{4}$

12. To increase the resonant frequency in series LCR circuit,

(A) Source frequency should be increased

(B) Another resistance should be added in series with the first resistance.

(C) Another capacitor should be added in series with the first capacitor

(D) The source frequency should be decreased

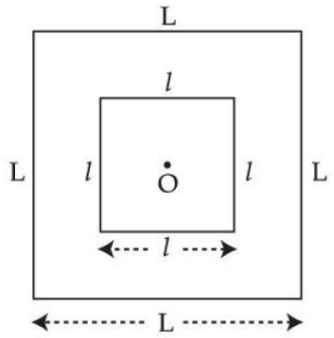
Official Ans. by NTA (C)

Sol. $f=\frac{1}{2\pi\sqrt{LC}}$

To increase the resonating frequency product of L and C should decrease.

By joining capacitor in series, capacitor will decrease

13. A small square loop of wire of side l is placed inside a large square loop of wire L (L >> l). Both loops are coplanar and their centres coincide at point O as shown in figure. The mutual inductance of the system is :

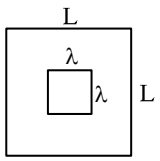


(A) $\frac{2\sqrt{2}\mu_0 L^2}{\pi\ell}$ (B) $\frac{\mu_0\ell^2}{2\sqrt{2}\pi L}$

(C) $\frac{2\sqrt{2}\mu_0\ell^2}{\pi L}$ (D) $\frac{\mu_0 L^2}{2\sqrt{2}\pi\ell}$

Official Ans. by NTA (C)

Sol. Assuming current I in outer loop magnetic field at centre $= 4\times\frac{\mu_0 i}{4\pi\times\frac{L}{2}}\times(2\sin 45°) = \frac{2\sqrt{2}\mu_0 i}{\pi L}$



$$M = \frac{\text{Flux through inner loop}}{i}$$

$$M = \frac{2\sqrt{2}\mu_0\ell^2}{\pi L}$$

14. The rms value of conduction current in a parallel plate capacitor is 6.9 μA. The capacity of this capacitor, if it is connected to 230 V ac supply with an angular frequency of 600 rad/s, will be :

(A) 5 pF (B) 50 pF

(C) 100 pF (D) 200 pF

Official Ans. by NTA (B)

Sol. Current in capacitor $I = \frac{V}{X_C}$

$$I = (V)\times(\omega C)$$

$$C = \frac{I}{V\omega} = \frac{6.9\times10^{-6}}{230\times600} = 50\text{pF}$$

15. Which of the following statement is correct ?

(A) In primary rainbow, observer sees red colour on the top and violet on the bottom

(B) In primary rainbow, observer sees violet colour on the top and red on the bottom

(C) In primary rainbow, light wave suffers total internal reflection twice before coming out of water drops

(D) Primary rainbow is less bright than secondary rainbow.

Official Ans. by NTA (A)

Sol. In primary rainbow, red colour is at top and violet is at bottom.

Intensity of secondary rainbow is less in comparison to primary rainbow.

16. Time taken by light to travel in two different materials A and B of refractive indices μ_A and μ_B of same thickness is t_1 and t_2 respectively. If $t_2 - t_1 = 5\times10^{-10}$ s and the ratio of μ_A to μ_B is 1 : 2. Then the thickness of material, in meter is : (Given v_A and v_B are velocities of light in A and B materials respectively).

(A) $5\times10^{-10}\ v_a$ m (B) 5×10^{-10} m

(C) 1.5×10^{-10} m (D) $5\times10^{-10}\ v_B$ m

Official Ans. by NTA (A)

Sol. $\frac{\mu_A}{\mu_B} = \frac{c/V_A}{c/V_B} = \frac{V_B}{V_A} = \frac{1}{2}$

Let the thickness is d

$$\frac{d}{v_B} - \frac{d}{v_A} = 5\times10^{-10}$$

$$d = \frac{5\times10^{-10}\times v_A v_B}{v_A - v_B}$$

As $v_A = 2v_B \Rightarrow d = 5\times10^{-10}\times 2v_B$

Or $d = 5\times10^{-10}\times v_A$

17. A metal exposed to light of wavelength 800 nm and emits photoelectrons with a certain kinetic energy. The maximum kinetic energy of photo-electron doubles when light of wavelength 500 nm is used. The work function of the metal is (Take hc = 1230 eV-nm).

(A) 1.537 eV (B) 2.46 eV

(C) 0.615 eV (D) 1.23 eV

Official Ans. by NTA (C)

Sol. $k_1 = \frac{1230}{800} - \phi$ ...(1)

$k_2 = 2k_1 = \frac{1230}{500} - \phi$ ...(2)

Eliminating k_1 from (1) and (2) we get

$0 = \frac{1230}{500} - \frac{1230}{400} + \phi$

$\phi = 0.615 eV$

18. The momentum of an electron revolving in n^{th} orbit is given by : (Symbols have their usual meanings)

(A) $\frac{nh}{2\pi r}$ (B) $\frac{nh}{2r}$

(C) $\frac{nh}{2\pi}$ (D) $\frac{2\pi r}{nh}$

Official Ans. by NTA (A)

Sol. Angular momentum is integral multiple of $\frac{h}{2\pi}$

$mvr = \frac{nh}{2\pi}$

So momentum $mv = \frac{nh}{2\pi r}$

19. The magnetic moment of an electron (e) revolving in an orbit around nucleus with an orbital angular momentum is given by :

(A) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$ (B) $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$

(C) $\vec{\mu}_l = -\frac{e\vec{L}}{m}$ (D) $\vec{\mu}_l = \frac{2e\vec{L}}{m}$

Official Ans. by NTA (B)

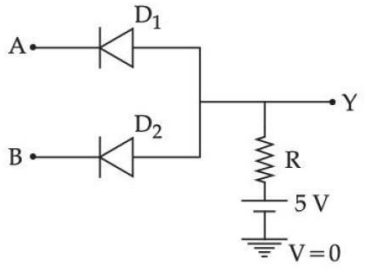
Sol. Ratio of magnetic moment and angular momentum

$\frac{\vec{\mu}}{\vec{L}} = \frac{q}{2m}$

For e^-

$\vec{\mu} = -\frac{e}{2m}\vec{L}$

20. In the circuit, the logical value of A = 1 or B = 1 when potential at A or B is 5V and the logical value of A = 0 or B = 0 when potential at A or B is 0 V.



The truth table of the given circuit will be :

	A	B	Y
	0	0	0
(A)	1	0	0
	0	1	0
	1	1	1

	A	B	Y
	0	0	0
(B)	1	0	1
	0	1	1
	1	1	1

	A	B	Y
	0	0	0
(C)	1	0	0
	0	1	0
	1	1	0

	A	B	Y
	0	0	0
(D)	1	0	1
	0	1	1
	1	1	0

Official Ans. by NTA (A)

Sol. When both A and B have logical value '1' both diode are reverse bias and current will flow in resistor hence output will be 5 volt i.e. logical value '1'.

In all other case conduction will take place, hence output will be zero volt i.e. logical value '0'.

So truth table is

A	B	Y	
0	0	0	
0	1	0	(AND gate)
1	0	0	
1	1	1	

SECTION-B

1. A car is moving with speed of 150 km/h and after applying the brake it will move 27 m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling _____ m distance.

Official Ans. by NTA (3)

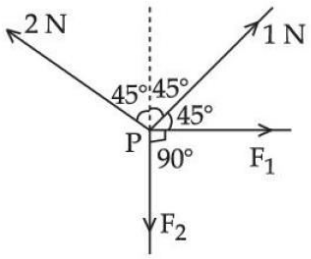
Sol. Stopping distance $=\frac{v^2}{2a}=d$

If speed is made $\frac{1}{3}$rd

$d'=\frac{1}{9}d.\qquad d'=\frac{27}{9}=3.$

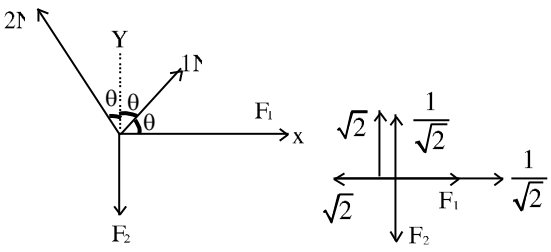
Braking acceleration remains same

2. Four forces are acting at a point P in equilibrium as shown in figure. The ratio of force F_1 to F_2 is 1 : x where x = _____ .



Official Ans. by NTA (3)

Sol.



$\theta=45°$

Taking components along x & y

$F_1=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{2-1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$F_2=\sqrt{2}+\frac{1}{\sqrt{2}}=\frac{2+1}{\sqrt{2}}=\frac{3}{\sqrt{2}}$

$F_1:F_2=1:3$

x = 3

3. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force F, its length increases by 5 cm. Another wire of the same material of length 4L and radius 4r is pulled by a force 4F under same conditions. The increase in length of this wire is _____ cm.

Official Ans. by NTA (5)

Sol. $\Delta\ell_1=\frac{F\ell}{AY}=\frac{F\ell}{\pi r^2Y}=5cm$

$\Delta\ell_2=\frac{4F4\ell}{\pi 16r^2Y}=\frac{F\ell}{\pi r^2Y}=5cm$

4. A unit scale is to be prepared whose length does not change with temperature and remains 20 cm, using a bimetallic strip made of brass and iron each of different length. The length of both components would change in such a way that difference between their lengths remains constant. If length of brass is 40 cm and length of iron will be _____cm.

($\alpha_{iron} = 1.2 \times 10^{-5}\ K^{-1}$ and $\alpha_{brass} = 1.8 \times 10^{-5}\ K^{-1}$).

Official Ans. by NTA (60)

Sol. $\ell_B(1+\alpha_B\Delta T)-\ell_i(1+\alpha_i\Delta T)=\ell_B-\ell_i$

$\alpha_B\ell_B=\ell_i\alpha_i$

$1.8\times10^{-5}\times40=\ell_i\times1.2\times10^{-5}$

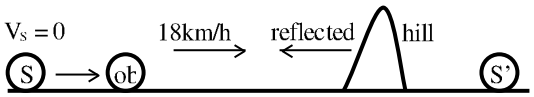
$\ell_i=\frac{1.8\times10^{-5}\times40}{1.2\times10^{-5}}=\frac{3\times40}{2}=60$

$\ell_i=60cm$

5. An observer is riding on a bicycle and moving towards a hill at 18 kmh^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640 Hz and velocity of the sound in air is 320 m/s, the beat frequency between the two sounds heard by observer will be _____ Hz.

Official Ans. by NTA (20)

Sol.



$V_S=0,\ V_{ob}=5m/s$

$f_{direct}=\left(\frac{320-5}{320}\right)640=630Hz$

$f_{reflected}=\left(\frac{320+5}{320}\right)640=650Hz$

$f_{beat}=650-630=20Hz$

6. The volume charge density of a sphere of radius 6 m is 2 μC cm^{-3}. The number of lines of force per unit surface area coming out from the surface of the sphere is _____ × 10^{10} NC^{-1}.

[Given : Permittivity of vacuum

$\in_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1} - m^{-2}$]

Official Ans. by NTA (45)

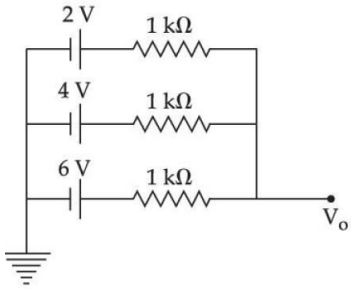
Sol. No. of electric field lines per unit area = electric field.

$$E = \frac{\rho r}{3\in_0}, \text{ for } r = R$$

$$E = \frac{\rho R}{3\in_0} = \frac{2\times 6}{3\times 8.85\times 10^{-12}} = 0.45\times 10^{12}\ NC^{-1}$$

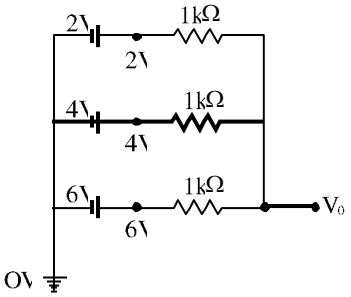
$$= 45\times 10^{10}\ N/C$$

7. In the given figure, the value of V_0 will be ____ V.



Official Ans. by NTA (4)

Sol.



By nodal analysis $\frac{V_0-2}{1k\Omega}+\frac{V_0-4}{1k\Omega}+\frac{V_0-6}{1k\Omega}=0$

$$3V_0 - 12 = 0$$

$$V_0 = 4$$

8. Eight copper wire of length *l* and diameter d are joined in parallel to form a single composite conductor of resistance R. If a single copper wire of length 2*l* have the same resistance (R) then its diameter will be _____ d.

Official Ans. by NTA (4)

Sol. Each wire has resistance $= \rho\frac{4\ell}{\pi d^2} = r$

Eight wire in parallel, then equivalent resistance is

$$\frac{r}{8} = \frac{\rho\ell}{2\pi d^2}$$

Single copper wire of length 2*l* has resistance

$$R = \rho\frac{2\ell\times 4}{\pi d_1^2} = \frac{\rho\ell}{2\pi d^2}$$

$$\Rightarrow d_1 = 4d$$

9. The energy band gap of semiconducting material to produce violet (wavelength = 4000 Å) LED is _____ eV. (Round off to the nearest integer).

Official Ans. by NTA (3)

Sol. $E_g = \frac{hc}{\lambda} = \frac{1242}{\lambda(nm)} = \frac{1242}{400} = 3.105$

Answer rounded to 3 eV

10. The required height of a TV tower which can cover the population of 6.03 lakh is h. If the average population density is 100 per square km and the radius of earth is 6400 km, then the value of h will be _____ m.

Official Ans. by NTA (150)

Sol. $d = \sqrt{2Rh}$

$d = \sqrt{2\times 6400\times h\times 10^{-3}}$ (h in m)

Area = πd^2

= $(\pi \times 2 \times 6400 \times h \times 10^{-3})$ km^2

$6.03 \times 100000 = 100 \times \pi \times 2 \times 6400 \times 10^{-3}$ h

$$h = \frac{6.03\times 10^5}{10\times\pi\times 128}$$

h = 150 m

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. SO_2Cl_2 on reaction with excess of water results into acidic mixture

$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$

16 moles of NaOH is required for the complete neutralisation of the resultant acidic mixture. The number of moles of SO_2Cl_2 used is :

(A) 16 (B) 8

(C) 4 (D) 2

Official Ans. by NTA (C)

Sol. Let $n(SO_2Cl_2) = x$ moles

$\therefore n(H_2SO_4) = x, n(HCl) = 2x$

$\Rightarrow n(H^+) = 4x$

For Neutralisation

$\Rightarrow n(H^+) = n(OH^-)$

$\Rightarrow 4x = 16$

$\Rightarrow \mathbf{x = 4}$

2. Which of the following sets of quantum numbers is not allowed ?

(A) $n = 3, l = 2, m_l = 0, s = +\frac{1}{2}$

(B) $n = 3, l = 2, m_l = -2, s = +\frac{1}{2}$

(C) $n = 3, l = 3, m_l = -3, s = -\frac{1}{2}$

(D) $n = 3, l = 0, m_l = 0, s = -\frac{1}{2}$

Official Ans. by NTA (C)

Sol. $l = 0, 1, 2 \ldots\ldots (n-1)$

$\therefore$ for $n = 3$

$l = 0, 1, 2$

$\Rightarrow l = 3,$

not possible for $n = 3$

3. The depression in freezing point observed for a formic acid solution of concentration 0.5 mL L^{-1} is 0.0405°C. Density of formic acid is 1.05 g mL^{-1}. The Van't Hoff factor of the formic acid solution is nearly : (Given for water $k_f = 1.86$ K kg mol^{-1})

(A) 0.8 (B) 1.1

(C) 1.9 (D) 2.4

Official Ans. by NTA (C)

Sol. $[HCOOH] = 0.5 \text{ ml } l^{-1}$

$\Rightarrow (0.5 \text{ ml} \times 1.05 \text{ g ml}^{-1})$ HCOOH in 1L

$\Rightarrow 0.525$ g HCOOH in 1L

$$m = \frac{(0.525/46)}{1\text{kg}} \text{mol} \quad \text{[Assuming dilute solution]}$$

$$\therefore \Delta T_f = iK_f m \Rightarrow i = \frac{\Delta T_f}{k_f m} = \frac{0.0405 \times 46}{1.86 \times 0.525} = 1.9$$

4. 20 mL of 0.1 M NH_4OH is mixed with 40 mL of 0.05 M HCl. The pH of the mixture is nearest to:

(Given: $K_b(NH_4OH) = 1 \times 10^{-5}$, log 2 = 0.30, log 3 = 0.48, log 5 = 0.69, log 7 = 0.84, log 11 =1.04)

(A) 3.2 (B) 4.2

(C) 5.2 (D) 6.2

Official Ans. by NTA (C)

Sol. $NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

mmole 2 2

\- - 2 mmole

$$\left[NH_4^+\right] = \frac{2\text{mmole}}{60\,\text{ml}} = \frac{1}{30}\text{ M}$$

$$pH = \frac{pK_w - pK_b - \log C}{2} = \frac{14 - 5 + 1.48}{2} = 5.24$$

5.

Match **List - I** with **List - II**

	List - I		List - II
(A)	$N_2(g)+3H_2(g) \rightarrow 2NH_3(g)$	(I)	Cu
(B)	$CO(g)+3H_2(g) \rightarrow CH_4(g)+H_2O(g)$	(II)	$Cu/ZnO-Cr_2O_3$
(C)	$CO(g)+H_2(g) \rightarrow HCHO(g)$	(III)	$Fe_xO_y+K_2O+Al_2O_3$
(D)	$CO(g)+2H_2(g) \rightarrow CH_3OH(g)$	(IV)	Ni

Choose the correct answer from the options given below :

(A) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(B) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(C) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

(D) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

Official Ans. by NTA (C)

Sol. **Factual**

6. The IUPAC nomenclature of an element with electronic configuration $[Rn]5f^{14}6d^{1}7s^{2}$ is :

(A) Unnilbium (B) Unnilunium

(C) Unnilquadium (D) Unniltrium

Official Ans. by NTA (D)

Sol. Atomic Number 103

7. The compound(s) that is(are) removed as slag during the extraction of copper is :

(1) CaO (2) FeO

(3) Al_2O_3 (4) ZnO

(5) NiO

Choose the correct answer from the options given below :

(A) (3) (4) Only (B) (1), (2), (5) Only

(C) (1), (2) Only (D) (2) Only

Official Ans. by NTA (D)

Sol. $FeO + SiO_2 \rightarrow FeSiO_3$

8. The reaction of H_2O_2 with potassium permanganate in acidic medium leads to the formation of mainly:

(A) Mn^{2+} (B) Mn^{4+}

(C) Mn^{3+} (D) Mn^{6+}

Official Ans. by NTA (A)

Sol. $H_2O_2 + MnO_4^- \rightarrow Mn^{2+} + O_2$ (unbalanced)

9. Choose the correct order of density of the alkali metals :

(A) Li < K < Na < Rb < Cs

(B) Li < Na < K < Rb < Cs

(C) Cs < Rb < K < Na < Li

(D) Li < Na < K < Cs < Rb

Official Ans. by NTA (A)

Sol. **Factual**

10. The geometry around boron in the product 'B' formed from the following reaction is

$$BF_3 + NaH \xrightarrow{450K} A + NaF$$

$$A + NMe_3 \rightarrow B$$

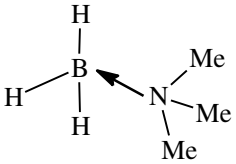
(A) trigonal planar (B) tetrahedral

(C) pyramidal (D) square planar

Official Ans. by NTA (B)

Sol. $BF_3 + NaH \xrightarrow{450K} \underset{\text{(diborane)}}{B_2H_6} + NaF$

$$B_2H_6 + NMe_3 \longrightarrow \underset{\text{symmetrical cleavage}}{2[BH_3 \leftarrow NMe_3]}$$



11. The interhalogen compound formed from the reaction of bromine with excess of fluorine is a :

(A) hypohalite (B) halate

(C) perhalate (D) halite

Official Ans. by NTA (B)

Sol. $Br_2 + \underset{\text{(excess)}}{5\ F_2} \longrightarrow 2\overset{+5}{Br}F_5 \xrightarrow{H_2O} HBrO_3$ (Forms bromate)

12. The photochemical smog does not generally contain :

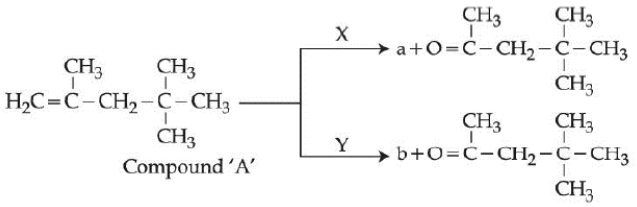
(A) NO (B) NO_2

(C) SO_2 (D) HCHO

Official Ans. by NTA (C)

Sol. **Factual**

13. A compound 'A' on reaction with 'X' and 'Y produces the same major product but different by product 'a' and 'b'. Oxidation of 'a' gives a substance produced by ants.



'X' and 'Y' respectively are :

(A) $KMnO_4/H^+$ and dil. $KMnO_4$, 273 K

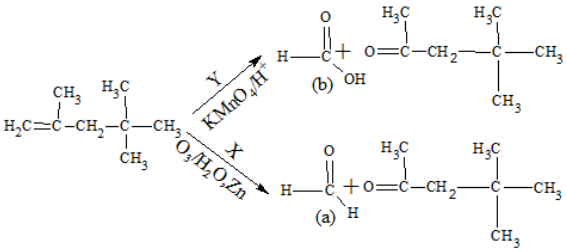
(B) $KMnO_4$,(dilute), 273 K and $KMnO_4/H^+$

(C) $KMnO_4/H^+$ and O_3, H_2O/Zn

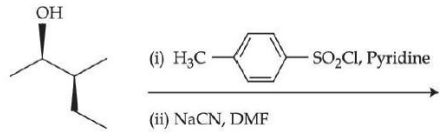
(D) O_3, H_2O/Zn and $KMnO_4/H^+$

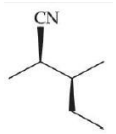
Official Ans. by NTA (D)

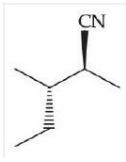
Sol.

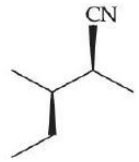


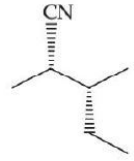
14. Most stable product of the following reaction is:



(A) 

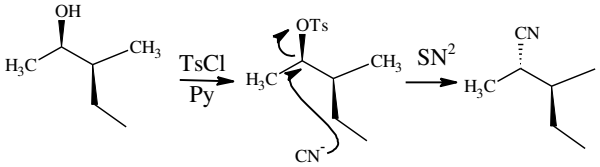
(B) 

(C) 

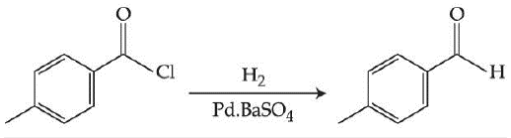
(D) 

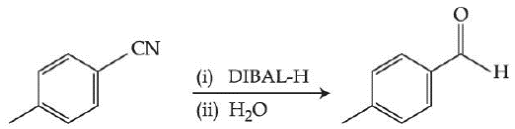
Official Ans. by NTA (B)

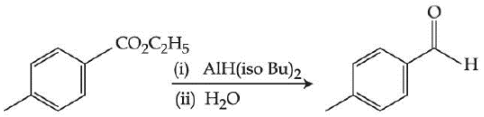
Sol.

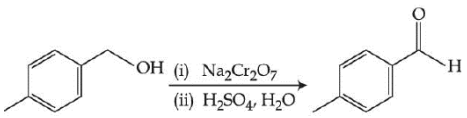


15. Which one of the following reactions does not represent correct combination of substrate and product under the given conditions ?

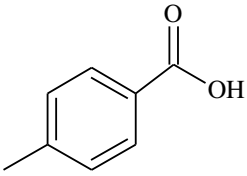
(A) 

(B) 

(C) 

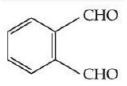
(D) 

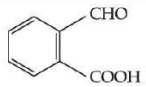
Official Ans. by NTA (D)

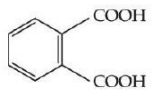
Sol. 

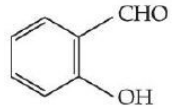
is formed

16. An organic compound 'A' on reaction with NH_3 followed by heating gives compound B. Which on further strong heating gives compound C ($C_8H_5NO_2$). Compound C on sequential reaction with ethanolic KOH, alkyl chloride and hydrolysis with alkali gives a primary amine. The compound A is :

(A) 

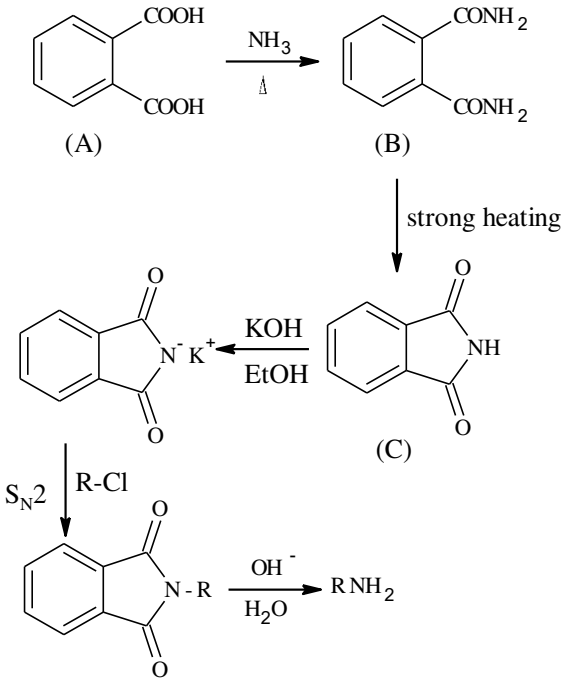
(B) 

(C) 

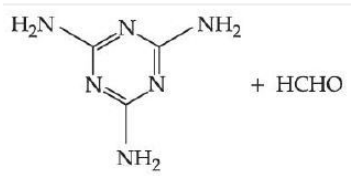
(D) 

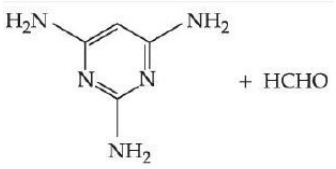
Official Ans. by NTA (C)

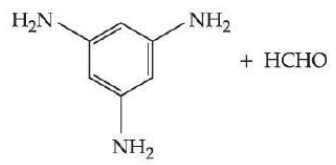
Sol. Gabriel Pthalimide reaction

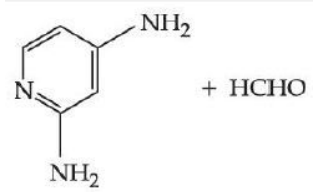


17. Melamine polymer is formed by the condensation of :

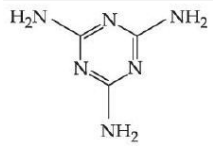
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. Melamine : 

Formaldehyde HCHO

Melamine formaldehyde Resin is melamine polymer

18. During the denaturation of proteins, which of these structures will remain intact ?

(A) Primary

(B) Secondary

(C) Tertiary

(D) Quaternary

Official Ans. by NTA (A)

Sol. Primary structure remains intact during denaturation of proteins

19. Drugs used to bind to receptors, inhibiting its natural function and blocking a message are called :

(A) Agonists

(B) Antagonists

(C) Allosterists

(D) Anti histaminists

Official Ans. by NTA (B)

Sol. Factual

20. Given below are two statements :

Statement I : On heating with $KHSO_4$, glycerol is dehydrated and acrolein is formed.

Statement II : Acrolein has fruity odour and can be used to test glycerol's presence.

Choose the correct option.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (B)

Sol. Acrolein has a pungent, suffocating odour.

Acrolein is used to detect presence of glycerol

SECTION-B

1. Among the following species

$N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$

the number of species showing diamagnetism is

Official Ans. by NTA (2)

Sol. Diamagnetic species are: N_2, O_2^{2-}

2. The enthalpy of combustion of propane, graphite and dihydrogen at 298 K are: –2220.0 kJ mol^{-1}, –393.5 kJ mol^{-1} and –285.8 kJ mol^{-1} respectively. The magnitude enthalpy of formation of propane (C_3H_8) is……kJ mol^{-1}. (Nearest integer)

Official Ans. by NTA (104)

Sol. $3C_{(gr)} + 4H_{2(g)} \rightarrow C_3H_{8(g)}$

$= -103.7$ kJ mol^{-1}

3. The pressure of a moist gas at 27°C is 4 atm. The volume of the container is doubled at the same temperature. The new pressure of the moist gas is …. $\times 10^{-1}$ atm. (Nearest integer)

(Given : The vapour pressure of water at 27°C is 0.4 atm)

Official Ans. by NTA (22)

Sol. $\left[P_{gas}\right]_0 + V.P. = 4$

$\left[P_{gas}\right]_0 = 4 - 0.4 = 3.6$

As volume is doubled, $\left[P_{gas}\right]_{new} = 1.8$ atm

New Total Pressure = 1.8 + 0.4 = 2.2 atm

4. The cell potential for $Zn|Zn^{2+}(aq)||Sn^{x+}|Sn$ is 0.801 V at 298 K. The reaction quotient for the above reaction is 10^{-2}. The number of electrons involved in the given electrochemical cell reaction is. . . . .

(Given $E^0_{Zn^{2+}|Zn} = -0.763V$, $E^0_{Sn^{x+}|Sn} = +0.008V$ and $\frac{2.303RT}{F} = 0.06V$**)**

Official Ans. by

Sol. $E = E^0 - \frac{2.303\,RT}{nF}\log Q$

Here, $E = +0.801V$, $E^0 = 0.008 - (-0.763)$

$= +0.771$ V

$\therefore 0.801 = +0.771 - \frac{0.06}{n}\log 10^{-2}$

$\Rightarrow n = 4$

5. The half life for the decomposition of gaseous compound A is 240 s when the gaseous pressure was 500 Torr initially. When the pressure was 250 Torr, the half life was found to be 4.0 min. The order of the reaction is……. (Nearest integer)

Official Ans. by NTA (1)

Sol. $(t_{1/2})_{500\ torr} = 240$ sec = 4 min.

$(t_{1/2})_{250\ torr} =$ 4 min.

$t_{1/2} \propto a^{1-n}$

As $t_{1/2}$ is independent of initial pressure. Hence, order is 1st order.

6. Consider the following metal complexes :

$[Co(NH_3)]^{3+}$

$[CoCl(NH_3)_5]^{2+}$

$[Co(CN)_6]^{3-}$

$[Co(NH_3)_5(H_2O)]^{3+}$

The spin-only magnetic moment value of the complex that absorbs light with shortest wavelength is B.M. (Nearest integer)

Official Ans. by NTA (0)

Sol. $\Delta_0 \propto \frac{1}{\lambda}$

Here, CN^- being SFL will have maximum CFSE

So, $[Co(CN)_6]^{3-}$ will be d^2sp^3, $\mu = 0$

7. Among Co^{3+}, Ti^{2+}, V^{2+} and Cr^{2+} ions, one if used as a reagent cannot liberate H_2 from dilute mineral acid solution, its spin-only magnetic moment in gaseous state is ……B.M. (Nearest integer)

Official Ans. by NTA (5)

Sol. Co^{3+} can't liberate H_2.

It has d^6 configuration,

Number of unpaired electrons = 4

$\mu = \sqrt{4 \times 6} = 4.92$ B.M.

8. While estimating the nitrogen present in an organic compound by Kjeldahl's method, the ammonia evolved from 0.25 g of the compound neutralized 2.5 mL of 2 M H_2SO_4. The percentage of nitrogen present in organic compound is ..... .

Official Ans. by NTA (56)

Sol. $$\%N = \frac{1.4(N_1V_1)}{\text{mass of organic compound}}$$

$$\%N = \frac{1.4(2.5\times2\times2)}{0.25} = 56$$

9. The number of sp^3 hybridised carbons in an acyclic neutral compound with molecular formula C_4H_5N is :

Official Ans. by NTA (1)

Sol. $$DU = 4+1-\left(\frac{5-1}{2}\right) = 3$$

$H_3C-CH=CH-C\equiv N$

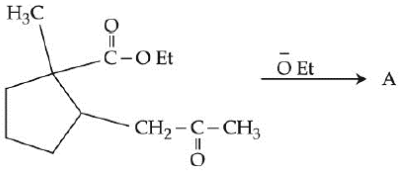
(sp^3)

or

$CH_2=C=CH=CH=NH$

(Zero sp^3 carbon)

10. In the given reaction

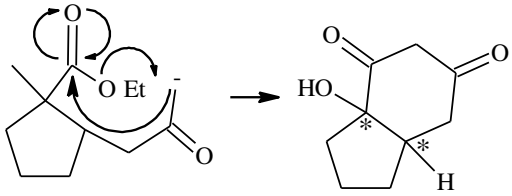


(Where Et is $-C_2H_5$)

The number of chiral carbon/s in product A is

Official Ans. by NTA (2)

Sol.



2 chiral carbons

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The total number of functions,

$f : \{1, 2, 3, 4\} \bullet \{1, 2, 3, 4, 5, 6\}$

such that $f(1) + f(2) = f(3)$, is equal to :

(A) 60 (B) 90

(C) 108 (D) 126

Official Ans. by NTA (B)

Sol. $A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4, 5, 6\}$

Here f(3) can be 2, 3, 4, 5, 6

$f(3) = 2, (f(1), f(2)) \rightarrow (1,1) \rightarrow$ 6 cases

$f(3) = 3, (f(1), f(2)) \rightarrow (1,2), (2,1)$

$\rightarrow 2 \times 6 = 12$ cases

$f(3) = 4, (f(1), f(2)) \rightarrow (1,3), (3,1), (2,2)$

$\rightarrow 3 \times 6 = 18$ cases

$f(3) = 5, (f(1), f(2)) \rightarrow (1,4), (4,1), (2,3), (3,2)$

$\rightarrow 4 \times 6 = 24$ cases

$f(3) = 6, (f(1), f(2)) \rightarrow (1,5),(5,1),(2,4),(4,2),(3,3)$

$\rightarrow 5 \times 6 = 30$ cases

Total number of cases = 6+ 12 + 18 + 24 + 30 = 90

2. If $\alpha,\beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then

$\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to .

(A) –4 (B) –1

(C) 1 (D) 4

Official Ans. by NTA (B)

Sol. $\alpha,\beta, \gamma, \delta$ root of the equation

$x^4 + x^3 + x^2 + x + 1 = 0$

Which are 5^{th} roots of unity except 1.

then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} =$

$\alpha + \beta + \gamma + \delta = -1$

3. For $n \in N$, let $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ and

$T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}$.

Then the number of elements in the set

$\{n \in N : S_n \cap T_n = \phi\}$ is :

(A) 0 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (D)

Sol. $S_n : |z - (3 - 2i)| = \frac{n}{4}$ is a circle center $C_1(3,-2)$

and radius n/4

$T_n : |z - (2 - 3i)| = \frac{1}{n}$ is a circle center C_2 (2,–3)

and radius 1/n

Here $S_n \cap T_n = \phi$

Both circles do not intersect each other

Case-1 : $C_1C_2 > n/4 + 1/n$

$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$

then n = 1, 2, 3, 4

Case-2 : $C_1C_2 < \left|\frac{n}{4} - \frac{1}{n}\right|$

$\Rightarrow \sqrt{2} < \left|\frac{n^2 - 4}{4n}\right|$

$\Rightarrow$ n has infinite solutions for $n \in \mathbf{N}$

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$3 (\sin 3\theta) x - y + z = 2$

$3 (\cos 2\theta) x + 4y + 3z = 3$

$6x + 7y + 7z = 9$

has no solution is :

(A) 6 (B) 7 (C) 8 (D) 9

Official Ans. by NTA (B)

Sol. The system of equation has no solution.

$$D=\begin{vmatrix}3\sin 3\theta & -1 & 1\\ 3\cos 2\theta & 4 & 3\\ 6 & 7 & 7\end{vmatrix}=0$$

$21\sin 3\theta + 42\cos 2\theta - 42 = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

Number of solution is 7 in $(0,4\pi)$

5. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then $8(\alpha+\beta)$ is equal to :

(A) 4 (B) –8

(C) –4 (D) 8

Official Ans. by NTA (C)

Sol. $\lim\limits_{n\to\infty} n\left(1-\dfrac{n+1}{n^2}\right)^{\frac{1}{2}}+\alpha n+\beta=0$

$$\lim_{n\to\infty} n\left\{1-\frac{1}{2}\left(\frac{n+1}{n^2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{n+1}{n^2}\right)^2+....\right\}+\alpha n+\beta=0$$

$$\lim_{n\to\infty} n-\frac{1}{2}+\frac{1}{n}+....+n\alpha+\beta=0$$

$$\alpha=-1,\beta=\frac{1}{2}$$

$8(\alpha+\beta) = -4$

6. If the absolute maximum value of the function f(x) $= (x^2 - 2x + 7)\, e^{(4x^3-12x^2-180x+31)}$ in the interval [–3, 0] is $f(\alpha)$, then :

(A) $\alpha = 0$ (B) $\alpha = -3$

(C) $\alpha \in (-1,0)$ (D) $\alpha \in (-3,-1)$

Official Ans. by NTA (B)

Sol. $f'(x)=e^{(4x^3-12x^2-180x+31)}\left(12(x^2-2x+7)(x+3)(x-5)+2(x-1)\right)$

for $x \in [-3,0]$

$\Rightarrow f'(x) < 0$

f(x) is decreasing function on [–3,0]

The absolute maximum value of the function f(x) is at $x = -3$

$\Rightarrow \alpha = -3$

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point P(–2, 0) and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of y(x) is :

(A) $\frac{27}{4}$ (B) $\frac{29}{4}$ (C) $\frac{37}{4}$ (D) $\frac{9}{2}$

Official Ans. by NTA (A)

Sol. $y(x) = ax^3 + bx^2 + cx + 5$ is passing through (-2,0) then $8a - 4b + 2c = 5$……(1)

$y'(x) = 3ax^2 + 2bx + c$ touches x-axis at (-2,0)

$12a - 4b + c = 0$ ……(2)

again, for $x = 0$, $y'(x) = 3$

$c = 3$ ……(3)

Solving eq. (1), (2) & (3) $a=-\frac{1}{2},\ b=-\frac{3}{4}$

$$y'(x)=-\frac{3}{2}x^2-\frac{3}{2}x+3$$

y(x) has local maxima at x = 1

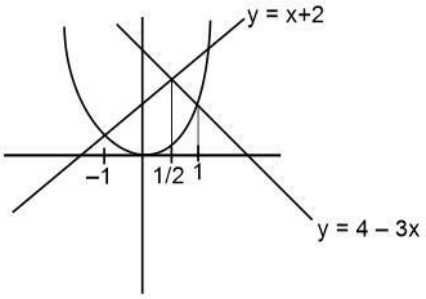
$$y(1)=\frac{27}{4}$$

8. The area of the region given by $A=\{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$ is :

(A) $\frac{31}{8}$ (B) $\frac{17}{6}$ (C) $\frac{19}{6}$ (D) $\frac{27}{8}$

Official Ans. by NTA (B)

Sol.



$$A=\int_{-1}^{\frac{1}{2}}(x+2-x^2)dx+\int_{\frac{1}{2}}^{1}(4-3x-x^2)dx=\frac{17}{6}$$

9. For any real number x, let [x] denote the largest integer less than equal to x. Let f be a real valued function defined on the interval [–10,10] by $f(x)=\begin{cases} x-[x], & \text{if } x \text{ is odd} \\ 1+[x]-x & \text{if } x \text{ is even} \end{cases}$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is :

(A) 4 (B) 2

(C) 1 (D) 0

Official Ans. by NTA (A)

Sol. f(x) is periodic function whose period is 2

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx=\frac{\pi^2}{10}\times 10\int_0^2 f(x)\cos\pi x\,dx$$

$$=\pi^2\left(\int_0^1 (1-x)\cos\pi x\,dx+\int_1^2 (x-1)\cos\pi x\,dx\right)$$

Using by parts

$$=\pi^2\times\frac{4}{\pi^2}=4$$

10. The slope of the tangent to a curve C : y =y(x) at any point [x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$ then e^{α} is equal to :

(A) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$

(B) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

(D) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$

$$\frac{dy}{dx}=e^{2x}-\frac{6e^{x}}{2e^{2x}+9}$$

$$y=\frac{e^{2x}}{2}-\tan^{-1}\left(\frac{\sqrt{2}e^{x}}{3}\right)+c$$

If C passes through the point $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$

$$c=-\frac{\pi}{4}-\tan^{-1}\frac{\sqrt{2}}{3}$$

Again C passes through the point $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$

then $e^{\alpha}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

11. The general solution of the differential equation $(x-y^2)dx+y(5x+y^2)dy=0$ is :

(A) $(y^2+x)^4=C|(y^2+2x)^3|$

(B) $(y^2+2x)^4=C|(y^2+x)^3|$

(C) $|(y^2+x)^3|=C(2y^2+x)^4$

(D) $|(y^2+2x)^3|=C(2y^2+x)^4$

Official Ans. by NTA (A)

Sol. $(x-y^2)dx+y(5x+y^2)dy=0$

$$\frac{dy}{dx}=\frac{y^2-x}{y(5x+y^2)}.\ \text{Let } y^2=v$$

$$\frac{2ydy}{dx}=2\left(\frac{y^2-x}{5x+y^2}\right)$$

$$\frac{dv}{dx}=2\left(\frac{v-x}{5x+v}\right) \qquad v=kx$$

$$k+x\frac{dk}{dx}=2\left(\frac{kx-x}{5x+kx}\right)$$

$$x\frac{dk}{dx}=-\frac{(k^2+3k+2)}{k+5}$$

$$\int\frac{(5+k)}{(k+1)(k+2)}dk=\int-\frac{dx}{x}$$

$$\int\left(\frac{4}{k+1}-\frac{3}{k+2}\right)dk=-\int\frac{dx}{x}$$

$$4\ln(k+1)-3\ln(k+2)=-\ln x+\ln c$$

$$\frac{(k+1)^4}{(k+2)^3}=-\ln x+\ln c$$

$$c(y^2+2x)^3=(y^2+x)^4$$

12. A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

(A) $2x + y = 9$ (B) $3x - 2y = 7$

(C) $x + 2y = 6$ (D) $2x - 3y = 7$

Official Ans. by NTA (C)

Sol. Let $B(x_1, x_1 - 2)$

$$\sqrt{(x_1-4)^2+(x_1-2-3)^2} = \frac{\sqrt{29}}{3}$$

Squaring on both side

$18x_1^2 - 162x_1 + 340 = 0$

$x_1 = \frac{51}{9}$ or $x_1 = \frac{10}{3}$

$y_1 = \frac{33}{9}$ or $y_1 = \frac{4}{3}$

Option (C) will satisfy $\left(\frac{10}{3}, \frac{4}{3}\right)$

13. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is :

(A) $\frac{32\sqrt{2}}{3}$ (B) $\frac{40\sqrt{2}}{3}$ (C) $\frac{64}{3}$ (D) $\frac{32}{3}$

Official Ans. by NTA (C)

Sol. $(\alpha - 0)^2 + (\beta - 1)^2 = (\beta + 1)^2$

$\alpha^2 = 4\beta$

$x^2 = 4y$

$$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right) dx = \frac{64}{3}$$

14. Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point $(2, -5, 11)$, then d^2 is equal to :

(A) $\frac{147}{2}$ (B) 96 (C) $\frac{32}{3}$ (D) 54

Official Ans. by NTA (D)

Sol. $a(x - 3) + b(y + 4) + c(z - 7) = 0$

P : $9a - b - 5c = 0$

$-11a - b + 5c = 0$

After solving DR's $\propto (1, -1, 2)$

Equation of plane

$x - y + 2z = 21$

$d = \frac{8}{\sqrt{6}}$

$d^2 = \frac{32}{3}$

15. Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12$

Consider the statements :

(S1) : $\left|\left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{b}\right)\right| - |\vec{c}| = 6(2\sqrt{2} - 1)$

(S2) : $\angle ABC = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$. Then

(A) both (S1) and (S2) are true

(B) only (S1) is true

(C) only (S2) is true

(D) both (S1) and (S2) are false

Official Ans. by NTA (D)

Sol. $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$

$|\vec{c}|^2 = 36$

$|\vec{c}| = 6$

S1 : $|\vec{a} \times \vec{b} + \vec{c} \times \vec{b}| - |\vec{c}|$

$\left|\left(\vec{a} + \vec{c}\right) \times \vec{b}\right| - |\vec{c}|$

$|-\vec{b} \times \vec{b}| - |\vec{c}|$

$0 - 6 = -6$

S2 : $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\angle ACB) = |\vec{c}|^2$

$\cos(\angle ACB) = \sqrt{\frac{2}{3}}$

16. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

(A) $\frac{33}{2^{32}}$ (B) $\frac{33}{2^{29}}$ (C) $\frac{33}{2^{28}}$ (D) $\frac{33}{2^{27}}$

Official Ans. by NTA (C)

Sol. $np + npq = 24$ …(1)

$np \cdot npq = 128$ …(2)

Solving (1) and (2) :

We get $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 32$.

Now,

$P(X = 1) + P(X = 2)$

$= {}^{32}C_1\, pq^{31} + {}^{32}C_2\, p^2 q^{30}$

$= \frac{33}{2^{28}}$

17. If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is :

(A) $\frac{17}{36}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{19}{36}$

Official Ans. by NTA (A)

Sol. $x^2 + \alpha x + \beta > 0, \forall\, x \in R$

$D = \alpha^2 - 4\beta < 0$

$\alpha^2 < 4\beta$

Total cases $= 6 \times 6 = 36$

Fav. cases $= \beta = 1, \alpha = 1$

$\beta = 2, \alpha = 1, 2$

$\beta = 3, \alpha = 1, 2, 3$

$\beta = 4, \alpha = 1, 2, 3$

$\beta = 5, \alpha = 1, 2, 3, 4$

$\beta = 6, \alpha = 1, 2, 3, 4$

Total favourable cases = 17

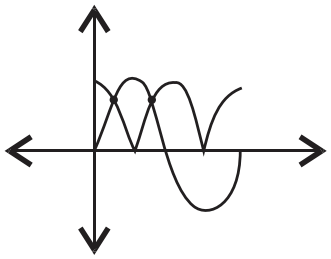
$P(x) = \frac{17}{36}$

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is :

(A) 4 (B) 6 (C) 8 (D) 12

Official Ans. by NTA (C)

Sol.



2 solutions in $(0, 2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

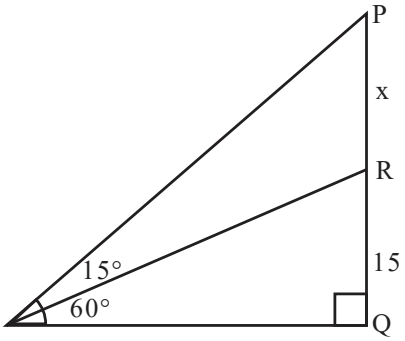
19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15 m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(A) $5(2\sqrt{3}+3)$ m (B) $5(\sqrt{3}+3)$ m

(C) $10(\sqrt{3}+1)$ m (D) $10(2\sqrt{3}+1)$ m

Official Ans. by NTA (A)

Sol.



$\frac{15}{AQ} = \tan 60°$ …(1)

$\frac{15+x}{AQ} = \tan 75°$ …(2)

$\frac{(1)}{(2)} \Rightarrow x = 10\sqrt{3}$

So, $PQ = 5(2\sqrt{3}+3)$ m

20. Which of the following statements is a tautology ?

(A) $((\sim p) \vee q) \Rightarrow p$ (B) $p \Rightarrow ((\sim p) \vee q)$

(C) $((\sim p) \vee q) \Rightarrow q$ (D) $q \Rightarrow ((\sim p) \vee q)$

Official Ans. by NTA (D)

Sol.

p	q	~p	~q	~pvq
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

options	1	2	3	4
	T	T	T	T
	T	F	T	T
	F	T	T	T
	F	T	F	T

SECTION-B

1. Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A^n + (\omega B)^n = A + B\}$ is equal to _____.

Official Ans. by NTA (17)

Sol. $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A^2 = A \Rightarrow A^n = A.$

$\forall\ n \in \{1, 2, \ldots, 100\}$

Now, $B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

$B^2 = -B$

$\Rightarrow B^3 = -B^2 = B$

$\Rightarrow B^5 = B$

$\Rightarrow B^{99} = B$

Also, $\omega^{3k} = 1$

So, n = common of $\{1, 3, 5, \ldots, 99\}$ and $\{3, 6, 9, \ldots, 99\} = 17$

2. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is _____.

Official Ans. by NTA (1492)

Sol.

M	A	N	K	I	N	D

$\left(\frac{4 \times 6!}{2!}\right) + (5! \times 0) + \left(\frac{4! \times 3}{2!}\right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$

3. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8K is equal to _____.

Official Ans. by NTA (6006)

Sol. $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

$T_{r+1} = {}^{15}C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \cdot \frac{(1-x)^{\frac{r}{10}}}{t^r}$

For independent of t,

$30 - 2r - r = 0$

$\Rightarrow r = 10$

So, Maximum value of ${}^{15}C_{10}\ x(1 - x)$ will be at $x = \frac{1}{2}$

i.e. 6006

4. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _____.

Official Ans. by NTA (38)

Sol. $x^2 - 8ax + 2a = 0$ $\quad$ $x^2 + 12bx + 6b = 0$

$p + r = 8a$ $\quad$ $q + s = -12b$

$pr = 2a$ $\quad$ $qs = 6b$

$\frac{1}{p}+\frac{1}{r} = 4$ $\quad$ $\frac{1}{q}+\frac{1}{s} = -2$

$\frac{2}{q} = 4$ $\quad$ $\frac{2}{r} = -2$

$q = \frac{1}{2}$ $\quad$ $r = -1$

$p = \frac{1}{5}$ $\quad$ $s = \frac{-1}{4}$

Now, $\frac{1}{a}-\frac{1}{b}=\frac{2}{pr}-\frac{6}{qs} = 38$

5. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to _____.

Official Ans. by NTA (27560)

Sol. $a_1 = b_1 = 1$

$a_2 = a_1 + 2 = 3$

$a_3 = a_2 + 2 = 5$

$a_4 = a_2 + 2 = 7$

$\Rightarrow a_n = 2n - 1$

$b_2 = a_1 + b_1 = 4$

$b_3 = a_3 + b_2 = 9$

$b_4 = a_4 + b_3 = 16$

$b_n = n^2$

$\sum_{n=1}^{15} a_n b_n$

$\sum_{n=1}^{15} (2n-1)n^2$

$\sum_{n=1}^{15} (2n^3 - n^2)$

$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$

Put n = 15

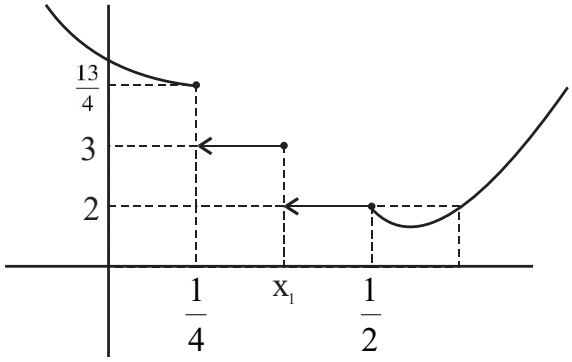
$= \frac{2\times 225\times 16\times 16}{4} - \frac{15\times 16\times 31}{6}$

$= 27560$

6. Let $f(x) = \begin{cases} |4x^2 - 8x + 5|, \text{ if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5], \text{ if } 8x^2 - 6x + 1 < 0 \end{cases}$, where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is _____.

Official Ans. by NTA (3)

Sol.



ND at $\frac{1}{4}, x_1, \frac{1}{2}$

7. If $\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}$ $[(nk + 1) + (nk + 2) + \ldots + (nk + n)] = 33$. $\lim_{n\to\infty} \frac{1}{n^{k+1}} \cdot [1^k + 2^k + 3^k + \ldots + n^k]$, then the integral value of k is equal to _____.

Official Ans. by NTA (5)

Sol. LHS

$\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} [nk \cdot n + 1 + 2 + \ldots + n]$

$= \lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} \cdot \left[n^2 k + \frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty} \frac{(n+1)^{k-1} \cdot n^2 \left(k + \frac{\left(1+\frac{1}{n}\right)}{2}\right)}{n^{k+1}}$

$\Rightarrow \lim_{n\to\infty} \left(1+\frac{1}{n}\right)\left(k + \frac{\left(1+\frac{1}{n}\right)}{2}\right)$

$\Rightarrow \left(k + \frac{1}{2}\right)$

RHS

$\Rightarrow \lim\limits_{n\to\infty}\frac{1}{n^{k+1}}(1^k+2^k+...+n^k)=\frac{1}{k+1}$

LHS = RHS

$\Rightarrow k+\frac{1}{2}=33\cdot\frac{1}{k+1}$

$\Rightarrow (2k+1)(k+1)=66$

$\Rightarrow (k-5)(2k+13)=0$

$\Rightarrow k=5 \text{ or } -\frac{13}{2}$

8. Let the equation of two diameters of a circle $x^2+y^2-2x+2fy+1=0$ be $2px-y=1$ and $2x+py=4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2-y^2=3$ passing through the centre of the circle is equal to _____.

Official Ans. by NTA (2)

Sol. $2p+f-1=0 \quad ...(1)$

$2-pf-4p=0 \quad ...(2)$

$2=p(f+4)$

$p=\frac{2}{f+4}$

$2p=1-f$

$\frac{4}{f+4}=1-f$

$f^2+3f=0$

$f=0 \text{ or } -3$

Hyperbola $3x^2-y^2=3$, $x^2-\frac{y^2}{3}=1$

$y=mx\pm\sqrt{m^2-3}$

It passes (1, 0)

$o=m\pm\sqrt{m^2-3}$

m tends ∞

It passes (1, 3)

$3=m\pm\sqrt{m^2-3}$

$(3-m)^2=m^2-3$

$m=2$

9. The sum of diameters of the circles that touch (i) the parabola $75x^2=64(5y-3)$ at the point $\left(\frac{8}{5},\frac{6}{5}\right)$ and (ii) the y-axis, is equal to _____.

Official Ans. by NTA (10)

Sol. $x^2=\frac{64.5}{75}\left(y-\frac{3}{5}\right)$

equation of tangent at $\left(\frac{8}{5},\frac{6}{5}\right)$

$x\,.\,\frac{8}{5}=\frac{64}{15}\left(\frac{y+\frac{6}{5}}{2}-\frac{3}{5}\right)$

$3x-4y=0$

equation of family of circle is

$\left(x-\frac{8}{5}\right)^2+\left(y-\frac{6}{5}\right)^2+\lambda(3x-4y)=0$

It touches y axis so $f^2=c$

$x^2+y^2+x\left(3\lambda-\frac{16}{5}\right)+y\left(-4\lambda-\frac{12}{5}\right)+4=0$

$\frac{\left(4\lambda+\frac{12}{5}\right)^2}{4}=4$

$\lambda=\frac{2}{5} \text{ or } \lambda=-\frac{8}{5}$

$\lambda=\frac{2}{5}, \quad r=1$

$\lambda=-\frac{8}{5}, \quad r=4$

$d_1+d_2=10$

10. The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane P : $ax-y-z=0$, $(a>0)$. If the image of the point (1, 1, –5) in the plane P is (α, β, γ), then $\alpha+\beta-\gamma$ is equal to _____.

Official Ans. by NTA (3)

Sol. DR's of line of shortest distance

$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&1\\2&2&1\end{vmatrix}=-\hat{i}+2\hat{j}-2\hat{k}$

angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}}=\alpha$

$\cos\alpha=\sqrt{\frac{2}{27}}$, $\sin\alpha=\frac{5}{3\sqrt{3}}$

DR's normal to plane (1, –1, –1)

$\sin\alpha=\left|\frac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right|=\frac{5}{3\sqrt{3}}$

$\sqrt{3}\,|a|=5\sqrt{a^2+2}$

$3a^2=25a^2+50$

No value of (a)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

SECTION-A

1. In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitude are found to be 6V and 2V respectively. The modulation index is

(A) 100% (B) 80%

(C) 60% (D) 50%

Official Ans. by NTA (D)

Sol. modulation index $= \frac{V_{max} - V_{min.}}{V_{max} + V_{min.}} \times 100\%$

$= \frac{6-2}{6+2} \times 100\% = 50\%$

2. The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and is rewound into a circular coil of 5 turns and the same current produces a magnetic induction B_2 at its centre.

The ratio of $\frac{B_2}{B_1}$ is :

(A) $\frac{5}{2}$ (B) $\frac{25}{4}$

(C) $\frac{5}{4}$ (D) $\frac{25}{2}$

Official Ans. by NTA (B)

Sol. $B = \frac{N\mu_0 i}{2R}$

$B_1 = \frac{N_1\mu_0 i}{2R_1}$

For $N_2 = 5$

Radius of coil $= R_2 = \frac{N_1 \times R_1}{N_2}$

$B_2 = \frac{N_2\mu_0 i}{R_2}$

$\frac{B_2}{B_1} = \frac{N_2}{N_1}\frac{R_1}{R_2} = \frac{N_2}{N_1} \times \frac{N_2}{N_1}$; $\frac{B_2}{B_1} = \frac{25}{4}$

TEST PAPER WITH SOLUTION

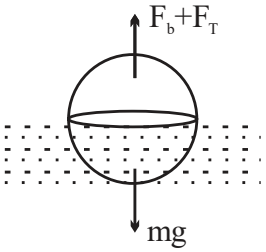
3. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension 7.5×10^{-4} Ncm^{-1}. The radius of drop in cm will be : (Take : g = 10 m/s^2)

(A) $\frac{15}{\sqrt{2\rho - \sigma}}$ (B) $\frac{15}{\sqrt{\rho - \sigma}}$

(C) $\frac{3}{2\sqrt{\rho - \sigma}}$ (D) $\frac{3}{20\sqrt{2\rho - \sigma}}$

Official Ans. by NTA (A)

Sol.



Boyant force + surace tension = mg

$\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$

$2\pi RT = \frac{(2\rho - \sigma)}{2}\cdot\frac{4}{3}\pi R^3 g; \left[V = \frac{4}{3}\pi R^3\right]$

$R^3 = \frac{3T}{(2\rho - \sigma)g} \Rightarrow R = \sqrt{\frac{3 \times 7.5 \times 10^{-2}\,N - m^{-1}}{(2\rho - \sigma) \times 10}}$

$R = \frac{3}{20\sqrt{(2\rho - \sigma)}}\,m = \frac{15}{\sqrt{2\rho - \sigma}}\,cm$

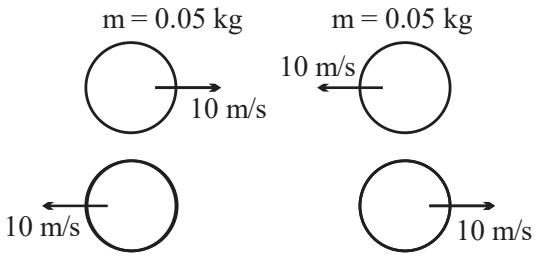
4. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is t = 0.005 s, then what is the force exerted on the ball due to each other?

(A) 100 N (B) 200 N

(C) 300 N (D) 400 N

Official Ans. by NTA (B)

Sol.



Change in momentum of any one ball

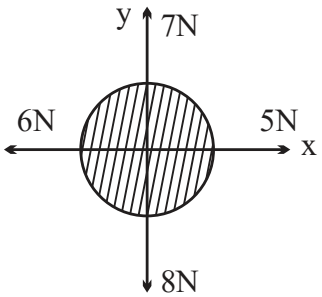
$$|\Delta\vec{P}| = 2\times 0.05\times 10$$

$$|\Delta\vec{P}| = 1$$

$$|\vec{F}_{av}| = \frac{|\Delta\vec{P}|}{\Delta t}$$

$F_{av.}$ = 200 N

5. For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that the net acceleration of body is zero?



(A) $\sqrt{2}$ N, 45° (B) $\sqrt{2}$ N, 135°

(C) $\frac{2}{\sqrt{3}}$ N, 30° (D) 2 N, 45°

Official Ans. by NTA (A)

Sol. Let addition force required is = $\vec{F}$

$$\vec{F} + 5\hat{i} - 6\hat{i} + 7\hat{j} - 8\hat{j} = 0$$

$$\vec{F} = \hat{i} + \hat{j}, |\vec{F}| = \sqrt{2}$$

Angle with x-axis: $\tan\theta = \frac{\text{y component}}{\text{x component}} = \frac{1}{1}$

$\theta = 45°$

6. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is:

(A) $\frac{n}{n-1}$ (B) $\frac{2n}{2n+1}$

(C) $\frac{n+1}{n}$ (D) $\frac{2n+1}{n}$

Official Ans. by NTA (A)

Sol. Capacitance of isolated Conducting sphere = $4\pi\varepsilon_0 R_1$

By enclosing inside another sphere of radius R_2, new capacitance = $\frac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)}$

Given: $\frac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)} = n\times 4\pi\varepsilon_0 R_1$

$$\Rightarrow \frac{R_2}{(R_2 - R_1)} = n \Rightarrow \frac{\frac{R_2}{R_1}}{\left(\frac{R_2}{R_1} - 1\right)} = n$$

$$\Rightarrow \frac{R_2}{R_1} = n\frac{R_2}{R_1} - n \Rightarrow \frac{R_2}{R_1} = \frac{n}{(n-1)}$$

7. The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is $1:\sqrt{2}$. Then, the ratio of V_p to V_d will be

(A) 1 : 1 (B) $\sqrt{2}:1$

(C) 2 : 1 (D) 4 : 1

Official Ans. by NTA (D)

Sol. Kinetic energy gained by a charged particle accelerated by a potential V is qV

KE = qV

$$\Rightarrow \frac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV}$$

$$p = \frac{h}{\lambda}, \text{ thus } \lambda = \frac{h}{\sqrt{2mqV}}$$

$$\text{now } \frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d V_d}{m_p V_p}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sqrt{\frac{2V_d}{V_p}} \Rightarrow \frac{V_p}{V_d} = 4$$

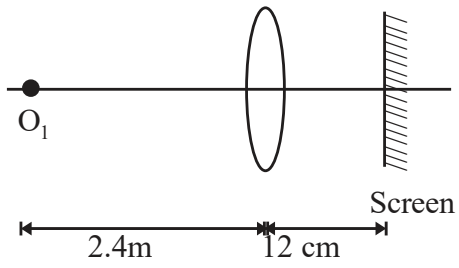
8. For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm from the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen?

(A) 0.8 m (B) 3.2 m

(C) 1.2 m (D) 5.6 m

Official Ans. by NTA (B)

Sol.



Applying lens formula

$$\frac{1}{0.12} + \frac{1}{2.4} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{210}{24}$$

Upon putting the glass slab, shift of image is

$$\Delta x = t\left(1 - \frac{1}{\mu}\right) = \frac{1}{3}\text{cm}$$

$$\text{Now } v = 12 - \frac{1}{3} = \frac{35}{3}\text{cm}$$

Again apply lens formula

$$\frac{1}{0.12} + \frac{1}{u} = \frac{1}{f} = \frac{210}{24}$$

Solving u = – 5.6 m

Thus shift of object is

5.6 – 2.4 = 3.2 m

9. Light wave traveling in air along x-direction is given by $E_y = 540 \sin \pi \times 10^4(x - ct)\ Vm^{-1}$. Then, the peak value of magnetic field of wave will be (Given $c = 3 \times 10^8\ ms^{-1}$)

(A) 18×10^{-7} T (B) 54×10^{-7} T

(C) 54×10^{-8} T (D) 18×10^{-8} T

Official Ans. by NTA (A)

Sol. $E_y = 540 \sin \pi \times 10^4(x - ct)\ Vm^{-1}$

$E_0 = 540\ Vm^{-1}$

$$B_0 = \frac{E_0}{C} = \frac{540}{3\times10^8} = 18\times10^{-7}\text{T}$$

10. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on

(A) Electromagnetic induction

(B) Resonance in ac circuits

(C) Mutual induction in ac circuits

(D) interference of electromagnetic waves

Official Ans. by NTA (B)

)

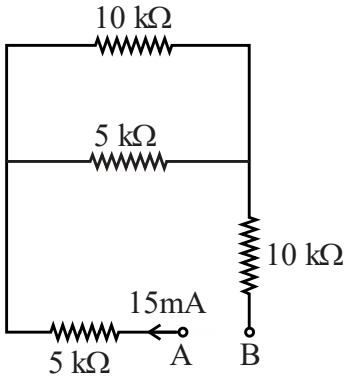
Sol. Metal detector works on the principle of transmitting an electromagnetic signal and analyses a return signal from the target. So it works on the principle of resonance in AC circuit.

11. An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of 1×10^{-4} Wbm^{-2}. The frequency of revolution of the electron will be
(Take mass of electron = 9.0×10^{-31} kg)
(A) 1.6×10^5 Hz (B) 5.6×10^5 Hz
(C) 2.8×10^6 Hz (D) 1.8×10^6 Hz
Official Ans. by NTA (C)

Sol. $f = \frac{1}{T} = \frac{eB}{2\pi m}$

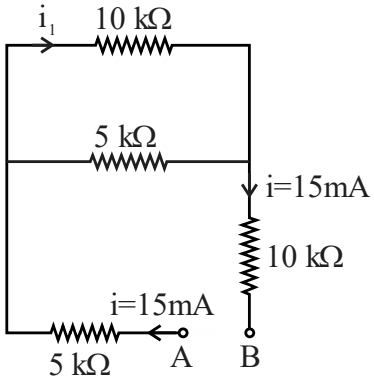
$= \frac{1.6\times10^{-19}\times10^{-4}}{2\pi\times9\times10^{-31}} = 2.8\times10^{6}\,Hz$

12. A current of 15 mA flows in the circuit as shown in figure. The value of potential difference between the points A and B will be



(A) 50V (B) 75V
(C) 150V (D) 275V
Official Ans. by NTA (D)

Sol.



$i_1 = \frac{5}{10+5}\times15mA = 5mA$

$V_A - 5i - 10i_1 - 10i = V_B$

$V_A - V_B = 75 + 50 + 150 = 275$ V

13. The length of a seconds pendulum at a height h = 2R from earth surface will be:
(Given: R=Radius of earth and acceleration due to gravity at the surface of earth $g = \pi^2 m/s^{-2}$)

(A) $\frac{2}{9}$m (B) $\frac{4}{9}$m

(C) $\frac{8}{9}$m (D) $\frac{1}{9}$m

Official Ans. by NTA (D)

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$, $g' = \frac{GM}{9R^2} = \frac{g}{9} = \frac{\pi^2}{9}$

$2 = 2\pi\sqrt{\frac{L}{\pi^2}\times9}$

$\Rightarrow 1 = \pi\sqrt{L}\times\frac{3}{\pi} \Rightarrow L = \frac{1}{9}$m

14. Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be
(A) 1 (B) 2
(C) 3 (D) 4
Official Ans. by NTA (B)

Sol. $v_s = \sqrt{\frac{\gamma RT}{M}}$

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{v_s}{v_{rms}} = \sqrt{\frac{\gamma}{3}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\gamma}{3} = \frac{1}{2} \Rightarrow \gamma = \frac{3}{2}$

$\gamma = 1 + \frac{2}{f_{mix.}}$

$f_{mix.} = \frac{2\times3+n\times5}{n+2} = \frac{6+n\times5}{(n+2)}$

$\gamma = 1 + \frac{2(n+2)}{6+n\times5} = \frac{6+5n+2n+4}{6+5n}$

$\gamma = \frac{7n+10}{6+5n} = \frac{3}{2}$

$14n + 20 = 18 + 15n$

$n = 2$

15. Let η_1 is the efficiency of an engine at $T_1 = 447°C$ and $T_2 = 147°C$ while η_2 is the efficiency at $T_1 = 947°C$ and $T_2 = 47°C$. The ratio $\frac{\eta_1}{\eta_2}$ will be :

(A) 0.41 (B) 0.56
(C) 0.73 (D) 0.70

Official Ans. by NTA (B)

Sol. Efficiency $\eta = 1 - \frac{T_L}{T_H}$

$$\eta_1 = 1 - \frac{147+273}{447+273} = 1 - \frac{420}{720}$$

$$\eta_1 = \frac{300}{720}$$

$$\eta_2 = 1 - \frac{47+273}{947+273} = 1 - \frac{320}{1220}$$

$$\eta_2 = \frac{900}{1220}$$

$$\frac{\eta_1}{\eta_2} = \frac{300}{720} \times \frac{1220}{900} = \frac{122}{72 \times 3}$$

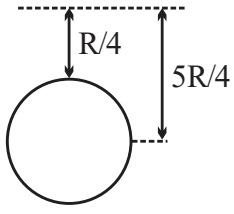
$$\frac{\eta_1}{\eta_2} = 0.56$$

16. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, R = 6400 km. The percentage decrease in the weight of the object will be

(A) 36% (B) 50%
(C) 64% (D) 25%

Official Ans. by NTA (A)

Sol.



$$g_{eff} = \frac{g}{\left(1+\frac{h}{R}\right)^2}; \quad g_{eff} = \frac{g}{\left(1+\frac{1}{4}\right)^2} = \frac{16g}{25}$$

$$\text{change} = \frac{g_{eff} - g}{g} \times 100 = \frac{\frac{16}{25} - 1}{1} \times 100$$

$$= \frac{-9}{25} \times 100 = -36\%$$

Hence % decrease in the weight = 36%

17. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be

(A) 4.9 J (B) 9.8 J
(C) 14.7 (D) 19.6 J

Official Ans. by NTA (B)

Sol. $P_i = P_f$ (no any external force)

$0.2 \times 10 = 10 \times v$

$v = 0.2$ m/sec

$$\text{Loss in K.E.} = \frac{1}{2} \times (0.2) \times 10^2 - \frac{1}{2} \times 10 (0.2)^2$$

$$= \frac{1}{2} \times 10 \times (0.2)[10 - 0.2]$$

$= 9.8$ J

18. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then'tan θ' will be equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 2 (D) 4

Official Ans. by NTA (D)

Sol. R = H

$$\frac{2v_x \times v_y}{g} = \frac{v_y^2}{2g}$$

$$v_x = \frac{v_y}{4}; \quad u\cos\theta = \frac{u\sin\theta}{4}$$

$\tan\theta = 4$

19. The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are 1%, 2% and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be:

(A) 2 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

Sol. $E_H = I^2R \times t$

$$\frac{\Delta E}{E}\times 100 = \frac{2\Delta I}{I}\times 100 + \frac{\Delta R}{R}\times 100 + \frac{\Delta T}{T}\times 100$$

$$= 2 \times 2 + 1 + 3 = 8$$

20. Hydrogen atom from excited state comes to the ground by emitting a photon of wavelength λ. The value of principal quantum number 'n' of the excited state will be :

(R : Rydberg constant)

(A) $\sqrt{\frac{\lambda R}{\lambda - 1}}$ (B) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$

(C) $\sqrt{\frac{\lambda}{\lambda R - 1}}$ (D) $\sqrt{\frac{\lambda R^2}{\lambda R - 1}}$

Official Ans. by NTA (B)

Sol.

$$E_n = \frac{-Rch}{n^2}(1)$$

n=n

$E_{photon} = E_n - E_1$

n=1

$$E_1 = \frac{-Rch}{(1)^2}(1)$$

$$\frac{-Rch}{(n)^2} + \frac{Rch}{1} = \frac{hc}{\lambda}$$

$$\frac{-R}{n^2} + R = \frac{1}{\lambda}$$

$$R - \frac{1}{\lambda} = \frac{R}{n^2}$$

$$\frac{\lambda R - 1}{\lambda} = \frac{R}{n^2}$$

$$n^2 = \frac{\lambda R}{\lambda R - 1} \Rightarrow n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

SECTION-B

1. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is _____ ms^{-2} at a point where its velocity is 20 ms^{-1}.

Official Ans. by NTA (100)

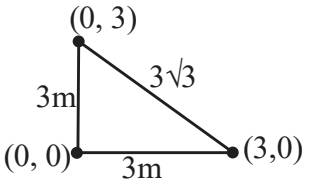
Sol. $\frac{dv}{ds} = 5$

$$a = v\frac{dv}{ds} = 20 \times 5 = 100 \text{ m/sec}^2$$

2. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is

Official Ans. by NTA (2)

Sol.



$$\vec{r}_{com} = \frac{M(0\hat{i} + 0\hat{j}) + M(3\hat{i}) + M(3\hat{j})}{3M}$$

$$\vec{r}_{com} = \hat{i} + \hat{j}$$

$$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x}$$

$$x = 2$$

3. A block of ice of mass 120 g at temperature 0°C is put in 300 gm of water at 25°C. The xg of ice melts as the temperature of the water reaches 0°C. The value of x is

[Use: Specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.5×10^5 Jkg^{-1}]

Official Ans. by NTA (90)

Sol. Energy released by water

$= 0.3 \times 25 \times 4200 = 31500$ J

let m kg ice melts

$m \times 3.5 \times 10^5 = 31500$

$$m = \frac{31500 \times 10^{-5}}{3.5} = 9000 \times 10^{-5}$$

$m = 0.09$ kg $= 90$ gm

$x = 90$

4. $\frac{x}{x+4}$ is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its

(i) third permitted energy level to the second level and

(ii) the highest permitted energy level to the second permitted level.

The value of x will be

Official Ans. by NTA (5)

Sol. $$\frac{13.6\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{13.6\left(\frac{1}{2^2}-0\right)} = \frac{x}{x+4}; \quad \frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{4}} = \frac{x}{x+4}$$

$$\frac{5}{9} = \frac{x}{x+4}$$

$5x + 20 = 9x$

$4x = 20$

$x = 5$

5. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36 cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be _____ cm.

Official Ans. by NTA (18)

Sol. 1.2 = (Potential Gradient) × 36

1.8 = (Potential Gradient) × x

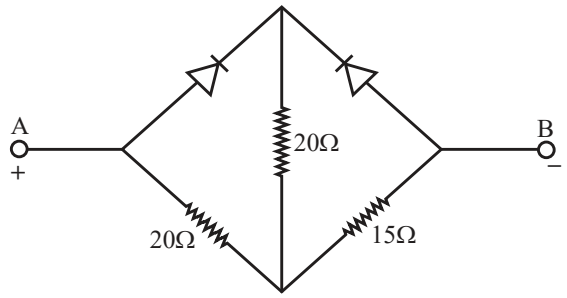
On dividing, we get

$$\frac{2}{3} = \frac{36}{x}$$

$x = 18 \times 3 = 54$ cm

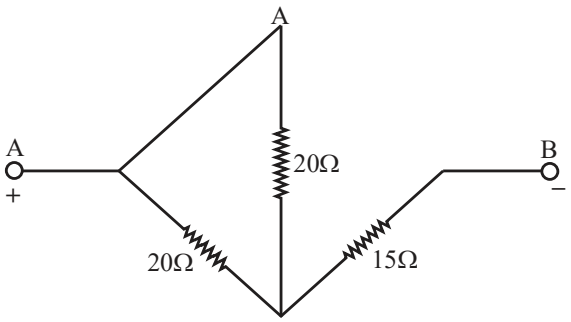
Hence difference = 54 – 36 = 18 cm

6. Two ideal diodes are connected in the network as shown in figure. The equivalent resistance between A and B is _____Ω.

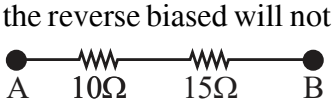


Official Ans. by NTA (25)

Sol.



The forward biased diode will conduct while the reverse biased will not



∴ Equivalent resistance = 10 + 15 = 25Ω

7. Two waves executing simple harmonic motion travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is _____ (degree)

Official Ans. by NTA (60)

Sol. $A_{resultant} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\Rightarrow \sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2\cos\phi}$

$\Rightarrow 3A^2 = 2A^2 + 2A^2\cos\phi$

$\Rightarrow \cos\phi = \frac{1}{2}$

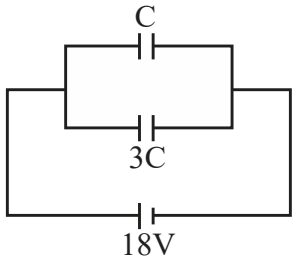
$\therefore \phi = 60°$

$\therefore$ Phase difference = 60 degree

8. Two parallel plate capacitors of capacity C and 3C are connected in parallel combination and charged to a potential difference 18V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be _____ V

Official Ans. by NTA (6)

Sol.



Initial charge on C = 18 CV

initial charge on 3C = 54 CV

Let final common potential difference = V'

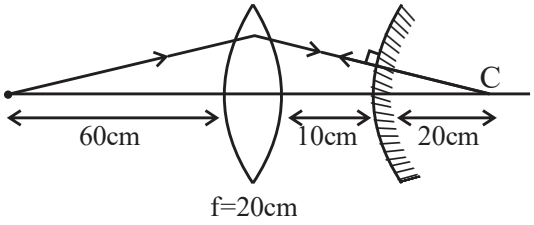
9CV' + 3CV' = 18CV + 54CV

$\Rightarrow$ 12CV' = 72 CV $\Rightarrow$ V' = 6 V

9. A convex lens of focal length 20 cm is placed in front of convex mirror with principal axis coinciding each other. The distance between the lens and mirror is 10 cm. A point object is placed on principal axis at a distance of 60 cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is _____ cm.

Official Ans. by NTA (10)

Sol.



For lens

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} - \frac{1}{(-60)} = \frac{1}{20} \Rightarrow \frac{1}{v} + \frac{1}{60} = \frac{1}{20}$

v = 30 cm

For final image to be formed on the object itself, after refraction from lens the ray should meet the mirror perpendicularly and the image by lens should be on the centre of curvature of mirror

R = 30 – 10 = 20 cm

Focal length of mirror = R/2 = 10 cm

10. Magnetic flux (in weber) in a closed circuit of resistance 20 Ω varies with time t(s) as $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at t = 0.25 s will be _____ mA

Official Ans. by NTA (250)

Sol. $\phi = 8t^2 - 9t + 5$

$\text{emf} = -\frac{d\phi}{dt} = -(16t - 9)$

At t = 0.25 s

Emf = –[(16 × 0.25)–9] = 5V

$\text{Current} = \frac{\text{Emf}}{\text{Resistance}} = \frac{5V}{20\Omega}$

$= \frac{1}{4}A = \frac{1000}{4}mA = 250mA$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Match **List I** with **List II** :

List-I (molecule)	List-II (hybridization; shape)
A. XeO_3	I. sp^3d ; linear
B. XeF_2	II. sp^3; pyramidal
C. $XeOF_4$	III. sp^3d^3; distorted octahedral
D. XeF_6	IV. sp^3d^2 ;square pyramidal

Choose the correct answer from the options given below:

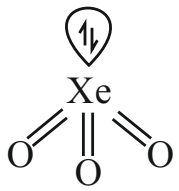
(A) A-II, B-I, C-IV, D-III

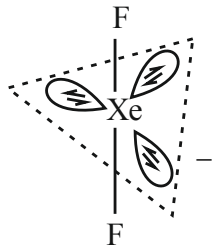
(B) A-II, B-IV, C-III, D-I

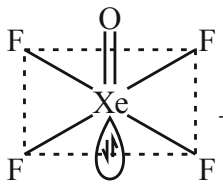
(C) A-IV, B-II, C-III, D-I

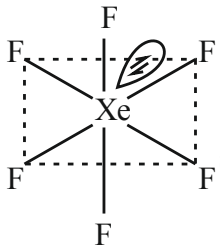
(D) A-IV, B-II, C-I, D-III

Official Ans. by NTA (A)

Sol. (A)  – sp^3 – pyramidal

(B)  – sp^3d – linear

(C)  –sp^3d^2–square pyramidal

(D)  sp^3d^3–distorted octahedral

2. Two solutions A and B are prepared by dissolving 1 g of non-volatile solutes X and Y. respectively in 1 kg of water. The ratio of depression in freezing points for A and B is found to be 1 : 4. The ratio of molar masses of X and Y is :

(A) 1 : 4

(B) 1 : 0.25

(C) 1 : 0.20

(D) 1 : 5

Official Ans. by NTA (B)

Sol. $$\frac{\Delta T_{fx}}{\Delta T_{fy}}=\frac{k_f\cdot m_x}{k_f\cdot m_y}=\frac{\frac{1}{M_x}}{\frac{1}{M_y}}$$

$$\Rightarrow \frac{1}{4}=\frac{M_y}{M_x}$$

$$\Rightarrow M_x : M_y = 1 : 0.25$$

3. Ka_1, Ka_2 and Ka_3 are the respective ionization constants for the following reactions (a),(b), and (c).

(a) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(b) $HC_2O_4^- \rightleftharpoons H^+ + HC_2O_4^{2-}$

(c) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} and K_{a_3} is given as

(A) $K_{a_3}=K_{a_1}+K_{a_2}$ (B) $K_{a_3}=K_{a_1}-K_{a_2}$

(C) $K_{a_3}=K_{a_1}/K_{a_2}$ (D) $K_{a_3}=K_{a_1}\times K_{a_2}$

Official Ans. by NTA (D)

Sol. $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$ K_{a_1}

$H_2C_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$ K_{a_2}

$H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$ $K_{a_3}=K_{a_1}\times K_{a_2}$

4. The molar conductivity of a conductivity cell filled with 10 moles of 20 mL NaCl solution is Λ_{m1} and that of 20 moles another identical cell heaving 80 mL NaCl solution is Λ_{m2}, The conductivities exhibited by these two cells are same.
The relationship between Λ_{m2} and Λ_{m1} is

(A) $\Lambda_{m2} = 2\Lambda_{m1}$ (B) $\Lambda_{m2} = \Lambda_{m1}/2$

(C) $\Lambda_{m2} = \Lambda_{m1}$ (D) $\Lambda_{m2} = 4\Lambda_{m1}$

Official Ans. by NTA (A)

Sol. $\Lambda_m = \kappa \times \frac{1000}{M}$

$\Rightarrow \Lambda_m \propto \frac{1}{M}$

$\frac{\Lambda_{m_1}}{\Lambda_{m_2}} = \frac{M_2}{M_1} = \frac{\frac{20}{80}}{\frac{10}{20}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$

$\Rightarrow \Lambda_{m_2} = 2\Lambda_{m_1}$

5. For micelle formation, which of the following statements are correct?

(A) Micelle formation is an exothermic process.

(B) Micelle formation is an endothermic process.

(C) The entropy change is positive.

(D) The entropy change is negative.

(A) A and D only (B) A and C only

(C) B and C only (D) B and D only

Official Ans. by NTA (A)

Sol. For micelle formation, $\Delta S > 0$ (hydrophobic effect) This is possible because, the decrease in entropy due to clustering is offset by increase in entropy due to desolvation of the surfactant, Also $\Delta H > 0$

6. The first ionization enthalpies of Be, B, N and O follow the order

(A) O < N < B < Be (B) Be < B < N < O

(C) B < Be < N < O (D) B < Be < O < N

Official Ans. by NTA (D)

Sol. **1st I.E.** $\underset{(2p^3)}{N} > \underset{(2p^4)}{O} > \underset{(2s^2)}{Be} > \underset{(2p^1)}{B}$

7. Given below are two statements.

Statement I : Pig iron is obtained by heating cast iron with scrap iron.

Statement II: Pig iron has a relatively lower carbon content than that of cast iron. In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are not correct.

(C) Statement I is correct but Statement II is not correct

(D) Statement I is not correct but Statement II is correct.

Official Ans. by NTA (B)

Sol. Statement –I is incorrect because cast iron is obtained by heating pig iron with scrap iron
Statement–II is also incorrect because pig iron has more carbon content (~4%) than cast iron (~3%)

8. High purity (>99.95%) dihydrogen is obtained by

(A) reaction of zinc with aqueous alkali.

(B) electrolysis of acidified water using platinum electrodes.

(C) electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

(D) reaction of zinc with dilute acid.

Official Ans. by NTA (C)

Sol. High purity (>99.95%) dihydrogen is obtained by electrolysis of warm aqueous $Ba(OH)_2$ solution between Ni-electrodes

9. The correct order of density is

(A) Be > Mg > Ca > Sr

(B) Sr > Ca > Mg > Be

(C) Sr > Be > Mg > Ca

(D) Be > Sr > Mg > Ca

Official Ans. by NTA (C)

Sol. In II'A' group density decreases down the group till Ca and after that it increases.
Correct order of density is
Sr > Be > Mg > Ca

10. The total number of acidic oxides from the following list is: NO, N_2O, B_2O_3, N_2O_5, CO, SO_3, P_4O_{10}

(A) 3 (B) 4

(C) 5 (D) 6

Official Ans. by NTA (B)

Sol. Neutral Oxides — N_2O, NO, CO

Acidic Oxides — B_2O_3, N_2O_5, SO_3, P_4O_{10}

11. The correct order of energy of absorption for the following metal complexes is

A: $[Ni(en)_3]^{2+}$, B: $[Ni(NH_3)_6]^{2+}$, C: $[Ni(H_2O)_6]^{2+}$

(A) C < B < A

(B) B < C < A

(C) C < A < B

(D) A < C < B

Official Ans. by NTA (A)

Sol. Stronger the ligand, larger the splitting & higher the energy of absorption.

$$\underset{(A)}{[Ni(en)_3]^{+2}} > \underset{(B)}{[Ni(NH_3)_6]^{+2}} > \underset{(C)}{[Ni(H_2O)_6]^{+2}}$$

12. **Match List I with List II.**

List–I		List–II	
A.	Sulphate	I.	Pesticide
B.	Fluoride	II.	Bending of bones
C.	Nicotine	III.	Laxative effect
D.	Sodium arsinite	IV.	Herbicide

Choose the correct answer from the options given below:

(A) A-II, B-III. C-IV, D-I

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-II, C-I, D-IV

(D) A-III, B-II, C-IV, D-I

Official Ans. by NTA (C)

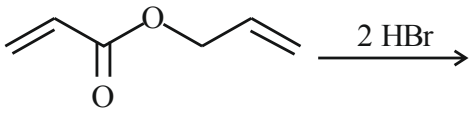
Sol. **A-Sulphate – III (Laxative effect)**

B-Fluoride – II (Bending of bones)

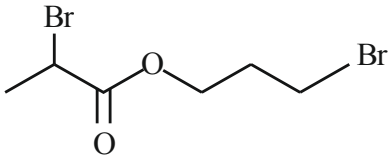
C-Nictoine – I (pesticides)

D-Sodium Arsinite – IV (herbicide)

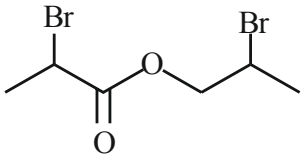
13. Major product of the following reaction is



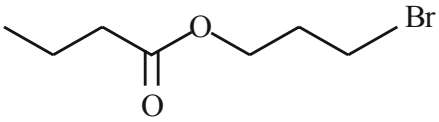
(A)



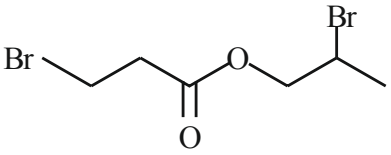
(B)



(C)

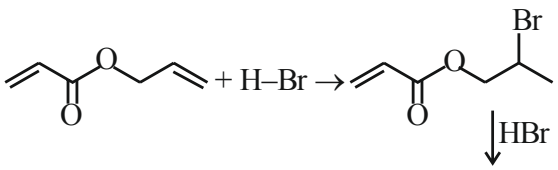


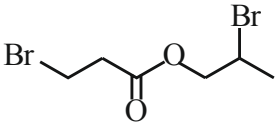
(D)



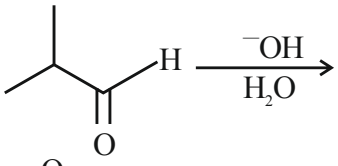
Official Ans. by NTA (D)

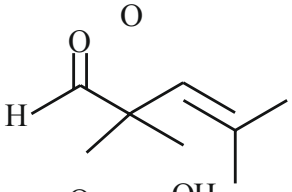
Sol.

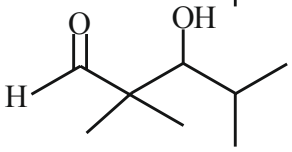


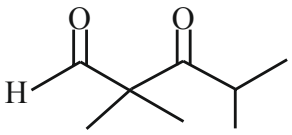


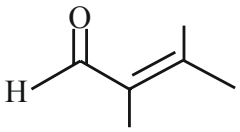
14. What is the major product of the following reaction?



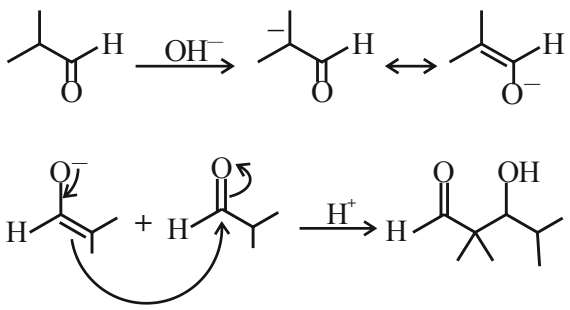
(A) 

(B) 

(C) 

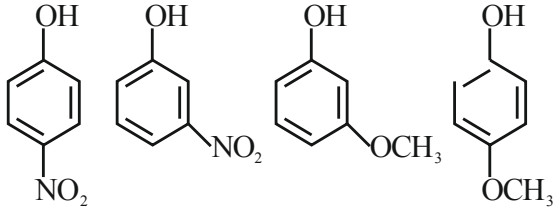
(D) 

Official Ans. by NTA (B)

Sol. 

Aldol formation takes place.

15. Arrange the following in decreasing acidic strength.



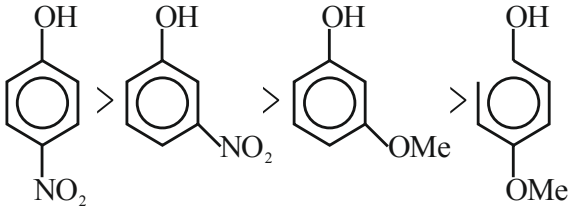
(A) (B) (C) (D)

(A) A > B > C > D (B) B > A > C > D

(C) D > C > A > B (D) D > C > B > A

Official Ans. by NTA (A)

Sol. The correct order of acid strength is



16. $CH_3-CH_2-CN \xrightarrow[Ether]{CH_3MgBr} A \xrightarrow{H_3O^+} B \xrightarrow[HCl]{Zn-Hg} C$

The correct structure of C is

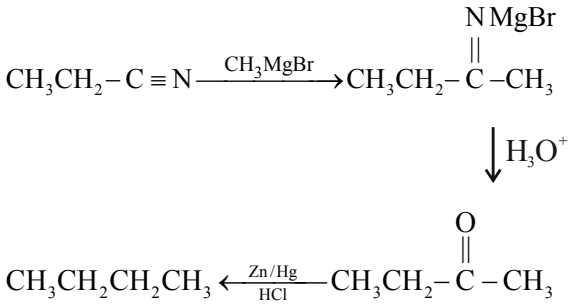
(A) $CH_3—CH_2—CH_2—CH_3$

(B) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

(C) $CH_3-CH_2-\overset{\overset{OH}{|}}{CH}-CH_3$

(D) $CH_3—CH_2—CH=CH_2$

Official Ans. by NTA (A)

Sol. 

(Clemmensen Reduction)

17. **Match List I with List II :**

List-I Polymer	List-II used for items
A. Nylon 6,6	I. Buckets
B. Low density polythene	II. Non-stick utensils
C. High density polythene	III. Bristles of brushes
D. Teflon	IV. Toys

Choose the correct answer from the options given below:

(A) A–III, B-I, C-IV, D-II

(B) A–III, B-IV, C-I, D-II

(C) A–II, B-I, C-IV, D-III

(D) A–II, B-IV, C-I, D-III

Official Ans. by NTA (B)

Sol. **LDPE → Toys**

HDPE → Buckets **(As per NCERT)**

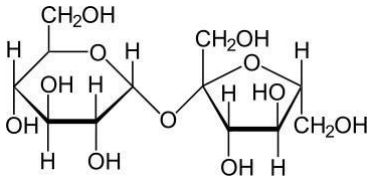
18. Glycosidic linkage between C_1 of α-glucose and C_2 of β-fructose is found in

(A) maltose (B) sucrose

(C) lactose (D) amylose

Official Ans. by NTA (B)

Sol. Theoretical



19. Some drugs bind to a site other than, the active site of an enzyme. This site is known as

(A) non-active site (B) allosteric site

(C) competitive site (D) therapeutic site

Official Ans. by NTA (B)

Sol. Theoretical

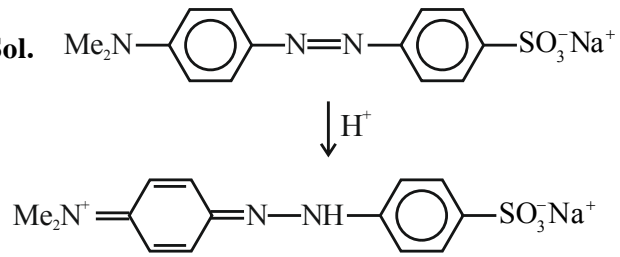
20. In base vs. Acid titration, at the end point methyl orange is present as

(A) quinonoid form (B) heterocyclic form

(C) phenolic form (D) benzenoid form

Official Ans. by NTA (A)

Sol.



(QUINONOID FORM)

SECTION-B

1. 56.0 L of nitrogen gas is mixed with excess of hydrogen gas and it is found that 20 L of ammonia gas is produced. The volume of unused nitrogen gas is found to be____ L.

Official Ans. by NTA (46)

Sol.

N_2	+	$3H_2$	→	$2NH_3$
56L		excess		O
−10L		−30L		+20L
46L				20L

2. A sealed flask with a capacity of 2 dm^3 contains 11 g of propane gas. The flask is so weak that it will burst if the pressure becomes 2 MPa. The minimum temperature at which the flask will burst is _____ °C. [Nearest integer] (Given: R = 8.3 $J\ K^{-1}\ mol^{-1}$. Atomic masses of C and H are 12u and 1u respectively.) (Assume that propane behaves as an ideal gas.)

Official Ans. by NTA (1655)

Sol. Moles of $C_3H_8 = \frac{11}{44} = 0.25$ moles

$PV = nRT$

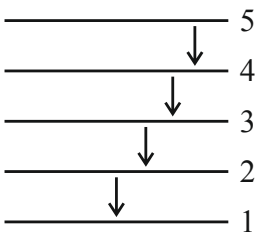
$\Rightarrow 2 \times 10^6 \times 2 \times 10^{-3} = 0.25 \times 8.3 \times T$

$\Rightarrow T = 1927.710\ K = 1654.56°C$

3. When the excited electron of a H atom from n = 5 drops to the ground state, the maximum number of emission lines observed are _____

Official Ans. by NTA (10)

Sol. Since only a single H atom is present, maximum number of spectral lines = 4



4. While performing a thermodynamics experiment, a student made the following observations,

$HCl + NaOH \rightarrow NaCl + H_2O\ \Delta H = -57.3\ kJ\ mol^{-1}$

$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

$\Delta H = -55.3\ kJ\ mol^{-1}$.

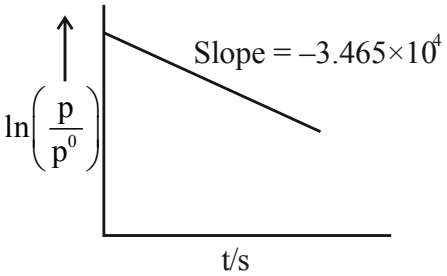
The enthalpy of ionization of CH_3COOH as calculated by the student is _____ $kJ\ mol^{-1}$. (nearest integer)

Official Ans. by NTA (2)

Sol. $\Delta H_{ionisation}$ of $CH_3COOH = |-57.3 - (-55.3)|$

$= 2$ KJ/mol

5. For the decomposition of azomethane.
$CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g)+N_2(g)$ a first order reaction, the variation in partial pressure with time at 600 K is given as



The half life of the reaction is _____ × 10^{-5}s.
[Nearest integer]

Official Ans. by NTA (2)

Sol. For first order reaction

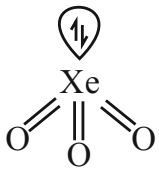
$$k = \frac{1}{t}\ln\left(\frac{P_0}{P}\right)$$

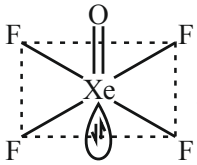
$$\ln\left(\frac{P_0}{P}\right) = kt$$

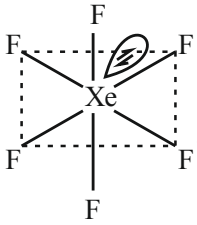
$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{3.465\times10^4} = 2\times10^{-5}$$

6. The sum of number of lone pairs of electrons present on the central atoms of XeO_3, $XeOF_4$ and XeF_6 is _____

Official Ans. by NTA (3)

Sol.  - one lone pair on central atom

 - one lone pair on central atom

 - one lone pair on central atom

7. The spin-only magnetic moment value of M^{3+} ion (in gaseous state) from the pairs Cr^{3+}/Cr^{2+}, Mn^{3+}/Mn^{2}, Fe^{3+}/Fe^{2+} and Co^{3+}/Co^{2+} that has negative standard electrode potential, is B.M.

[Nearest integer]

Official Ans. by NTA (4)

Sol. $E^0_{Cr^{+3}|Cr^{+2}} = -0.41V$

$[Cr^{+3}] = 4s^0 3d^3$

$\mu = \sqrt{n(n+2)}$ B.M

$= \sqrt{15}$ B.M ~ 4 B.M

8. A sample of 4.5 mg of an unknown monohydric alcohol, R–OH was added to methylmagnesium iodide. A gas is evolved and is collected and its volume measured to be 3.1 mL. The molecular weight of the unknown alcohol is ____ g/mol.
[Nearest integer]

Official Ans. by NTA (33)

Sol. $ROH + CH_3MgI \rightarrow ROMgI + CH_4(g)$

moles of CH_4 = moles of ROH

$$\Rightarrow \frac{V}{22400} = \frac{m}{M.M} \text{ (Assuming NTP Condition)}$$

$$\Rightarrow \frac{3.1}{22400} = \frac{4.5\times10^{-3}}{M.M}$$

$\Rightarrow$ MM = 32.51

Nearest Integer = 33

9. The separation of two coloured substances was done by paper chromatography. The distances travelled by solvent front, substance A and substance B from the base line are 3.25 cm. 2.08 cm and 1.05 cm. respectively. The ratio of R_f values of A to B is _____

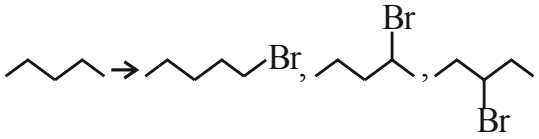
Official Ans. by NTA (2)

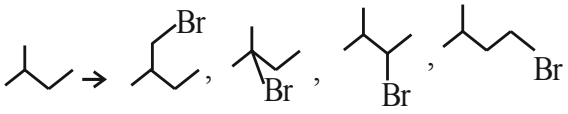
Sol. $$\frac{R_{F_A}}{R_{F_B}} = \frac{\frac{2.08}{3.25}}{\frac{1.05}{3.25}} = \frac{2.08}{1.05} \simeq 2$$

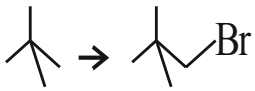
10. The total number of monobromo derivatives formed by the alkanes with molecular formula C_5H_{12} is (excluding stereo isomers)_____

Official Ans. by NTA (8)

Sol. The Alkanes and their monobromodervative are

1. 

2. 

3. 

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. For $z \in \mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is _____

(A) 3 (B) $\frac{7}{2}$

(C) 4 (D) $\frac{9}{2}$

Official Ans. by NTA (C)

2. The number of real values λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solution, is :-

(A) 0 (B) 1

(C) 2 (D) 4

Official Ans. by NTA (C)

3. The number of bijective functions $f : \{1, 3, 5, 7,.....99\} \to \{2, 4, 6, 8,....., 100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$, is _____

(A) ${}^{50}P_{17}$ (B) ${}^{50}P_{33}$

(C) $33! \times 17!$ (D) $\frac{50!}{2}$

Official Ans. by NTA (B)

4. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

5. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to

(A) $\frac{7}{87}$ (B) $\frac{7}{29}$

(C) $\frac{14}{87}$ (D) $\frac{21}{29}$

Official Ans. by NTA (B)

6. $\lim\limits_{x \to \frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2}\sin 2x}$ is equal to

(A) 14 (B) 7

(C) $14\sqrt{2}$ (D) $7\sqrt{2}$

Official Ans. by NTA (A)

7. $\lim\limits_{n \to \infty} \frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}} + \frac{1}{\sqrt{1-\frac{2}{2^n}}} + \frac{1}{\sqrt{1-\frac{3}{2^n}}} + ... + \frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) –2

Official Ans. by NTA (C)

8. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then $P(A \mid B') + P(B \mid A')$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{5}{8}$

(C) $\frac{5}{4}$ (D) $\frac{7}{8}$

Official Ans. by NTA (B)

9. Let [t] denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}([\sin(\pi x)]+e^{[\cos(2\pi x)]})dx$ is equal to

(A) $\frac{52(1-e)}{e}$ (B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$ (D) $\frac{104}{e}$

Official Ans. by NTA (B)

10. Let the point P (α, β) be at a unit distance from each of the two lines L_1 : 3x – 4y + 12 = 0, and L_2 : 8x + 6y + 11 = 0. If P lies below L_1 and above L_2, then 100 (α + β) is equal to

(A) –14 (B) 42

(C) –22 (D) 14

Official Ans. by NTA (D)

11. Let a smooth curve y = f(x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the point (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(A) $2\log_e 2$ (B) 4

(C) 1 (D) $4\log_e 2$

Official Ans. by NTA (B)

12. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ (B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ (D) $\frac{2\sqrt{5}}{7}$

Official Ans. by NTA (A)

13. The tangents at the point A(1, 3) and B(1, –1) on the parabola y^2 – 2x – 2y = 1 meet at the point P. Then the area (in unit^2) of the triangle PAB is :-

(A) 4 (B) 6

(C) 7 (D) 8

Official Ans. by NTA (D)

14. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:-

(A) $\frac{32}{9}$ (B) $\frac{18}{5}$

(C) $\frac{27}{4}$ (D) $\frac{27}{10}$

Official Ans. by NTA (D)

15. A plane E is perpendicular to the two planes 2x – 2y + z = 0 and x – y + 2z = 4, and passes through the point P(1, –1, 1). If the distance of the plane E from the point Q(a, a, 2) is $3\sqrt{2}$, then $(PQ)^2$ is equal to

(A) 9 (B) 12

(C) 21 (D) 33

Official Ans. by NTA (C)

16. The shortest distance between the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$ is

(A) $2\sqrt{29}$ (B) 1

(C) $\sqrt{\frac{37}{29}}$ (D) $\frac{\sqrt{29}}{2}$

Official Ans. by NTA (A)

17. Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}.\vec{b}=3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is :-

(A) $\frac{2}{\sqrt{21}}$ (B) $2\sqrt{\frac{3}{7}}$

(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (D) $\frac{2}{3}$

Official Ans. by NTA (A)

18. If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5 (B) 10.5

(C) 12 (D) 11

Official Ans. by NTA (D)

19. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to

(A) $\frac{3}{16}$ (B) $\frac{1}{16}$

(C) $\frac{1}{32}$ (D) $\frac{9}{32}$

Official Ans. by NTA (B)

20. Consider the following statements :

P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(A) $((P\wedge(\sim R))\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee R))$

(B) $((P\wedge R)\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(C) $((P\wedge R)\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(D) $((P\wedge(\sim R))\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee R))$

Official Ans. by NTA (D)

SECTION-B

1. Let A : {1, 2, 3, 4, 5, 6, 7}. Define B = {T $\subseteq$ A : either $1\notin T$ or $2\in T$} and C = T $\subseteq$ A : T the sum of all the elements of T is a prime number}. Then the number of elements in the set $B\cup C$ is _____

Official Ans. by NTA (107)

2. Let f(x) be a quadratic polynomial with leading coefficient 1 such that f(0) = p, $p\neq 0$ and $f(1)=\frac{1}{3}$. If the equation f(x) = 0 and fofofof(x) = 0 have a common real root, then f(–3) is equal to.....

Official Ans. by NTA (25)

3. Let $A=\begin{bmatrix}1 & a & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}, a,b\in\mathbb{R}$. If for some $n\in N$, $A^n=\begin{bmatrix}1 & 48 & 2160\\ 0 & 1 & 96\\ 0 & 0 & 1\end{bmatrix}$ then n + a + b is equal to _____

Official Ans. by NTA (24)

4. The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + [x^2 + 2x]$ is the interval $\left[\frac{5}{4},2\right]$, where [t] is the greatest integer $\leq t$ is _____

Official Ans. by NTA (15)

5. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=\frac{4y^3+2yx^2}{3xy^2+x^3}, y(1)=1$. If for some $n\in N, y(2)\in[n-1,n)$, then n is equal to _____

Official Ans. by NTA (3)

6. Let f be a twice differentiable function on R. If $f'(0)=4$ and

$$f(x)+\int_0^x (x-t)f'(t)dt=(e^{2x}+e^{-2x})\cos 2x+\frac{2}{a}x,$$

then $(2a+1)^5 a^2$ is equal to _____

Official Ans. by NTA (8)

7. Let $a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+.....+\frac{x^{n-1}}{n}\right)dx$ for $n\in N$. Then the sum of all the elements of the set $\{n\in N: a_n\in(2,30)\}$ is _____

Official Ans. by NTA (5)

8. If the circles $x^2+y^2+6x+8y+16=0$ and $x^2+y^2+2(3-\sqrt{3})x+x+2(4-\sqrt{6})y=k+6\sqrt{3}+8\sqrt{6}, k>0$, touch internally at the point P(α, β), then $(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2$ is equal to _____

Official Ans. by NTA (25)

9. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3-3xy^2+6x^2-5xy-8y^2+9x+14=0$ at the point (–2, 3) be A. Then 8A is equal to _____

Official Ans. by NTA (170)

10. Let $x=\sin(2\tan^{-1}\alpha)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S=\{\alpha\in R: y^2=1-x\}$, then $\sum_{\alpha\in S}16\alpha^3$ is equal to _____

Official Ans. by NTA (130)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

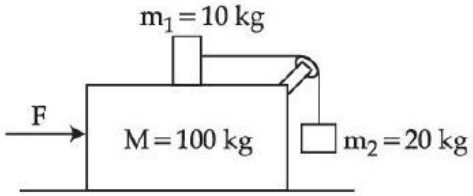
PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Three masses M = 100 kg, m_1 = 10 kg and m_2=20 kg are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is :

(Take g = 10 ms^{-2})



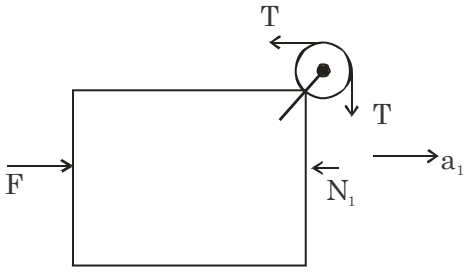
(A) 3360 N (B) 3380 N

(C) 3120 N (D) 3240 N

Official Ans. by NTA (C)

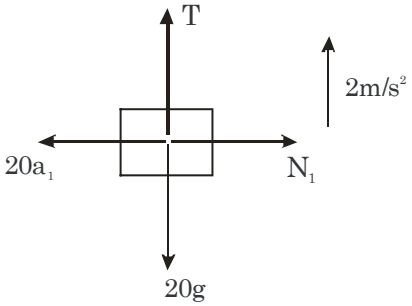
Sol. Let acceleration of 100 kg block = a_1

FBD of 100 kg block w.r.t ground



$F - T - N_1 = 100\ a_1$ ……..(i)

FBD of 20 block wrt 100kg

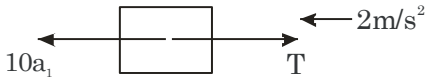


$T - 20g = 20(2)$

$T = 240$ ….. (ii)

$N_1 = 20a_1$ ….. (iii)

FBD of 10 kg block wrt 100 kg



$10a_1 - 240 = 10(2)$

$a_1 = 26\,m/s^2$

$F - 240 - 20(26) = 100 \times 26$

$\Rightarrow F = 3360$ N

2. A radio can tune to any station in 6 MHz to 10 MHz band. The value of corresponding wavelength bandwidth will be :

(A)4 m (B) 20 m

(C) 30 m (D) 50 m

Official Ans. by NTA (B)

Sol. Given: Frequency f_1 = 6MHz

Frequency f_2 = 10MHz

$$\lambda_1 = \frac{c}{f_1}$$

$$\lambda_2 = \frac{c}{f_2}$$

Wavelength bandwidth = λ_2-λ_1 = 20 m

3. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. 10 minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is :

(Take $\log_{10}1.88 = 0.274$)

(A)0.02 min^{-1} (B) 2.7 min^{-1}

(C) 0.063 min^{-1} (D) 6.3 min^{-1}

Official Ans. by NTA (C)

Sol. At t=0 disintegration rate = 4250 dpm

At t=10 disintegration rate = 2250 dpm

$A = A_o e^{-\lambda t}$

$2250 = 4250 e^{-\lambda(10)}$

$\Rightarrow \lambda(10) = \ln\left(\frac{4250}{2250}\right)$

$\Rightarrow \lambda = 0.063 \text{ min}^{-1}$

4. A parallel beam of light of wavelength 900 nm and intensity 100 Wm^{-2} is incident on a surface perpendicular to the beam. Tire number of photons crossing 1 cm^2 area perpendicular to the beam in one second is :

(A) 3×10^{16} (B) 4.5×10^{16}

(C) 4.5×10^{17} (D) 4.5×10^{20}

Official Ans. by NTA (B)

Sol. Wavelength of incident beam $\lambda = 900 \times 10^{-9}$ m

Intensity of incident beam =I= 100 W/m^2

No. of photons crossing per unit sec

$= n = \frac{E_{net}}{E_{single\ photon}} = \frac{IA\lambda}{hc}$

$= \frac{(100)(1\times10^{-4})(900\times10^{-9})}{6.62\times10^{-34}\times3\times10^{8}} = 4.5\times10^{16}$

5. In young's double slit experiment, the fringe width is 12mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm)

(A) 16 (B) 9

(C)48 (D) 12

Official Ans. by NTA (B)

Sol. For a given light wavelength corresponding a medium of refractive index μ

$\lambda_{med} = \frac{\lambda_{vacuum}}{\mu}$

and we know that fringe width $\beta = \frac{\lambda D}{d}$

Therefore, $\beta_{med} = \frac{\beta_{vacuum}}{\mu} = \frac{12}{\frac{4}{3}} = 9$ mm

6. The magnetic field of a plane electromagnetic wave is given by

$\vec{B} = 2\times10^{-8}\sin\left(0.5\times10^{3}x + 1.5\times10^{11}t\right)\hat{j}T$

The amplitude of the electric field would be

(A) $6 Vm^{-1}$ along x-axis

(B) $3 Vm^{-1}$ along z-axis

(C) $6 Vm^{-1}$ along z-axis

(D) $2\times10^{-8} Vm^{-1}$ along z-axis

Official Ans. by NTA (C)

Sol. $c = \frac{E_0}{B_0} \Rightarrow E_0 = cB_0$

$E_0 = \left(3\times10^{8}\right)\left(2\times10^{-8}\right)$

$E_0 = 6 Vm^{-1}$

As, $\vec{B}$ = along y-axis

$\vec{v}$ = along negative x-axis

hence $\vec{E}_0$ = along z-axis

7. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D)2 : 1

Official Ans. by NTA (B)

Sol. In case of L-R circuit

$Z=\sqrt{X_L^2+R^2}$ & power factor

$P_1=\cos\phi=\dfrac{R}{Z}$

As $X_L=R$

$\Rightarrow Z=\sqrt{2}R$

$\Rightarrow P_1=\dfrac{R}{\sqrt{2}R}\Rightarrow P_1=\dfrac{1}{\sqrt{2}}$

In case of L-C-R circuit

$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$

As $X_L=X_C$

$\Rightarrow Z=R$

$\Rightarrow P_2=\cos\phi=\dfrac{R}{R}=1$

$\Rightarrow \dfrac{P_1}{P_2}=\dfrac{1}{\sqrt{2}}$

8. A charge particle is moving in a uniform magnetic field $\left(2\hat{i}+3\hat{j}\right)T$. If it has an acceleration of $\left(\alpha\hat{i}-4\hat{j}\right)m/s^2$, then the value of α will be

(A) 3 (B) 6 (C) 12 (D) 2

Official Ans. by NTA (B)

Sol. As $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$

$\vec{a}=\dfrac{q}{m}\left(\vec{v}\times\vec{B}\right)$

So, $\vec{a}\,\&\,\vec{B}$ are $\perp$ to each other

Hence, $\vec{a}.\vec{B}=0$

$\left(\alpha\hat{i}-4\hat{j}\right).\left(2\hat{i}+3\hat{j}\right)=0$

$\alpha(2)+(-4)(3)=0$

$\alpha=\dfrac{12}{2}\Rightarrow\alpha=6$

9. B_X and B_Y are the magnetic field at the centre of two coils of two coils X and Y respectively, each carrying equal current. If coil X has 200 turns and 20 cm radius and coil Y has 400 turns and 20 cm radius, the ratio of B_X and B_Y is

(A)1 : 1 (B) 1 : 2

(C)2 : 1 (D) 4 : 1

Official Ans. by NTA (B)

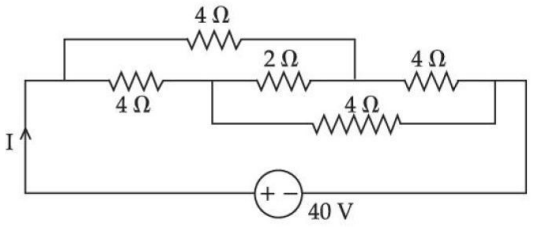
Sol. At centre $B=N\left(\dfrac{\mu_0 i}{2R}\right)$

$B_x=200\left(\dfrac{\mu_0 i}{2\times 20cm}\right)$

$B_y=400\left(\dfrac{\mu_0 i}{2\times 20cm}\right)$

$\dfrac{B_x}{B_y}=\dfrac{1}{2}$

10. The current I in the given circuit will be :

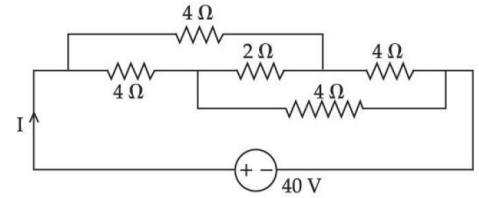


(A) 10A (B) 20 A

(C) 4A (D) 40A

Official Ans. by NTA (A)

Sol.



Given circuit is balanced wheat stone bridge

Hence 2Ω can be neglected

$R_{net}=4\Omega$

$I=\dfrac{40}{4}$

I = 10A

11. The total charge on the system of capacitance $C_1 = 1\mu F,\ C_2 = 2\mu F, C_3 = 4\mu F$ and $C_4 = 3\mu F$ connected in parallel is

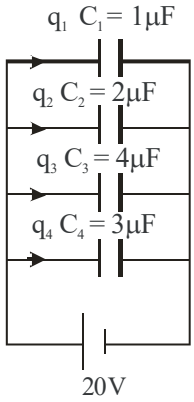
(Assume a battery of 20V is connected to the combination)

(A) $200\mu C$ (B) $200C$

(C) $10\mu C$ (D) $10C$

Official Ans. by NTA (A)

Sol.



Total charge = $q_1 + q_2 + q_2 + q_4$

$= 1\times 20 + 2\times 20 + 4\times 20 + 3\times 20 = 200\mu C$

12. When a particle executes simple Harmonic motion, the nature of graph of velocity as function of displacement will be :

(A)Circular (B)Ellipitical

(C)Sinusoidal (D) Straight line

Official Ans. by NTA (B)

Sol. For a particle in SHM, its speed depends on position as

$v = \omega\sqrt{A^2 - x^2}$

Where ω is angular frequency and A is amplitude

Now $v^2 = \omega^2 A^2 - \omega^2 x^2$

So, $\dfrac{v^2}{(\omega A)^2} + \dfrac{x^2}{(A)^2} = 1$

So graph between v and x is elliptical

13. 7 mole of certain monoatomic ideal gas undergoes a temperature increase of 40K at constant pressure. The increase in the internal energy of the gas in this process is

(Given $R = 8.3\ JK^{-1}mol^{-1}$)

(A) 5810 J (B) 3486 J

(C)11620J (D) 6972 J

Official Ans. by NTA (B)

Sol. For a quasi-static process the change in internal energy of an ideal gas is

$\Delta U = nC_V\ \Delta T$

$= n\times\dfrac{3R}{2}\times\Delta T$

[molar heat capacity at constant volume for mono atomic gas = $\dfrac{3R}{2}$]

$\Delta U = 7\times\dfrac{3}{2}\times 8.3\times 40 = 3486J$

14. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be:

(A) P (B) 8P (C) 32P (D) 64 P

Official Ans. by NTA (C)

Sol. Constant entropy means process is adiabatic

$PV^{\gamma} = \text{constant}$

$V_2 = \dfrac{V_1}{8}$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$P_1V_1^{\gamma} = P_2\left(\dfrac{V_1}{8}\right)^{5/3}$

$P_1V_1^{5/3} = \dfrac{P_2V_1^{5/3}}{32}$

$P_2 = 32P_1$

15. A water drop of radius 1cm is broken into 729 equal droplets. If surface tension of water is 75 dyne/cm, then the gain in surface energy upto first decimal place will be :

[Given $\pi = 3.14$]

(A) 8.5×10^{-4} J (B) 8.2×10^{-4} J

(C) 7.5×10^{-4} J (D) 5.3×10^{-4} J

Official Ans. by NTA (C)

Sol. Initial surface energy = TA

Where T is surface tension and A is surface area

$$U_i = \left(\frac{75\times10^{-5}}{10^{-2}}\frac{N}{m}\right)\times\left[4\pi\left(1\times10^{-2}\right)^2\right]$$

$$= 75\times10^{-3}\times4\pi\times10^{-4} = 942\times10^{-7}\ J$$

To get final radius of drops by volume conservation

$$\frac{4}{3}\pi R^3 = 729\left(\frac{4}{3}\pi r^3\right)$$

R = Initial radius

r = final radius

$$r = \frac{R}{(729)^{1/3}} = \frac{R}{9} = \frac{1}{9}\text{cm}$$

Final surface energy

$$U_f = 729[TA]$$

$$= 729\left[\frac{75\times10^{-5}}{10^{-2}}\frac{N}{m}\right]\times\left[4\pi\left(\frac{1}{9}\times10^{-2}\right)^2\right]$$

$$= 729\left[75\times10^{-3}\times\frac{4\pi\times10^{-4}}{81}\right]$$

$$= 9\left[942\times10^{-7}\ J\right]$$

Gain in surface energy

$$\Delta U = 9\times942\times10^{-7} - 942\times10^{-7}$$

$$= 8\times942\times10^{-7}\ J = 7536\times10^{-7}\ J$$

$$= 7.5\times10^{-4}\ J$$

16. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be :

(Radius of earth = 6400km)

(A) 1 % (B) 3%

(C) 4% (D) 0.5%

Official Ans. by NTA (A)

Sol. Acceleration due to gravity at a height h<< R is

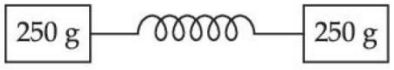
$$g' = g\left(1-\frac{2h}{R}\right)$$

$$\therefore \frac{\Delta g}{g} = \frac{2h}{R}$$

$$\Rightarrow \frac{\Delta g}{g}\times100 = \frac{2h}{R}\times100$$

$$= 2\times\frac{32}{6400}\times100 = 1\%$$

17. As per the given figure, two blocks each of mass 250g are connected to a spring of spring constant $2Nm^{-1}$. If both are given velocity v in opposite directions, then maximum elongation of the spring is :

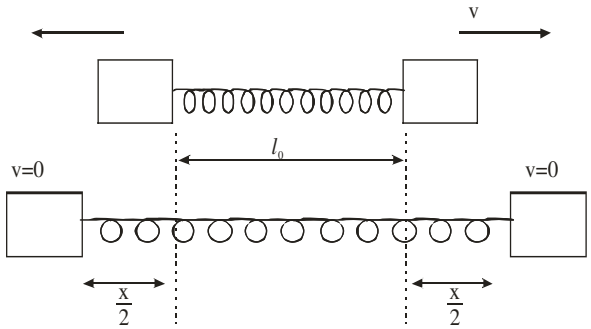


(A) $\frac{v}{2\sqrt{2}}$ (B) $\frac{v}{2}$

(C) $\frac{v}{4}$ (D) $\frac{v}{\sqrt{2}}$

Official Ans. by NTA (B)

Sol.



using energy conservation

$$\frac{1}{2}mv^2\times2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{4}v^2 = \frac{1}{2}\times2\times x^2$$

$$\therefore x = \frac{v}{2}$$

18. A monkey of mass 50kg climbs on a rope which can withstand the tension (T) of 350N. If monkey initially climbs down with an acceleration of $4m/s^2$ and then climbs up with an acceleration of $5m/s^2$. Choose the correct option $\left(g = 10m/s^2\right)$

(A) T = 700N while climbing upward

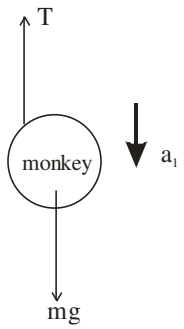
(B) T = 350 N while going downward

(C) Rope will break while climbing upward

(D) Rope will break while going downward

Official Ans. by NTA (C)

Sol. F.B.D of monkey while moving downward

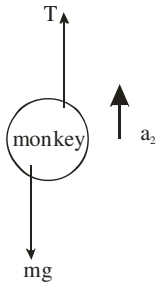


Using Newton's second law

$mg - T = ma_1$

$\therefore$ $500 - T = 50 \times 4 \Rightarrow T = 300$ N

F.B.D of monkey while moving up



Using Newton's second law of motion

$T - mg = ma_2$

$\Rightarrow$ $T - 500 = 50 \times 5$

$\Rightarrow$ $T = 750$ N

Breaking strength of string = 350 N

$\therefore$ String will break while monkey is moving upward

19. Two projectile thrown at 30^0 and 45^0 with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is

(A) $1:\sqrt{2}$ (B) $2:1$

(C) $\sqrt{2}:1$ (D) $1:2$

Official Ans. by NTA (C)

Sol. **Time taken to reach maximum height**

$$t = \frac{u\sin\theta}{g}$$

$$\therefore \frac{u_1\sin\theta_1}{g} = \frac{u_2\sin\theta_2}{g}$$

$$\Rightarrow u_1\sin 30 = u_2\sin 45$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{1/\sqrt{2}}{1/2} = \frac{\sqrt{2}}{1}$$

20. A screw gauge of pitch 0.5mm is used to measure the diameter of uniform wire of length 6.8cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is :

[Screw gauge has 50 divisions on the circular scale]

(A) $6.8cm^2$ (B) $3.4cm^2$

(C) $3.9cm^2$ (D) $2.4cm^2$

Official Ans. by NTA (B)

Sol. $L.C. = \frac{P}{N} = \frac{0.5mm}{50} = 0.01$ mm

Length of wire = 6.8 cm

Diameter of wire = 1.5 mm + 7 × L.C

= 1.5 mm + 7 × .01 = 1.57 mm

Curved surface area = $\pi D\ell$

$= 3.14 \times 6.8 \times 1.57 \times 10^{-1}\ cm^2$

$= 3.352\ cm^2 = 3.4\ cm^2$

SECTION-B

1. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1-x)$. The y component vector of the initial velocity is _____ $\hat{j}$

(Take $g = 10 m/s^2$)

Official Ans. by NTA (5)

Sol. $u_x = 1$

$y = 5x(1-x)$

$$\frac{dy}{dt} = 5\frac{dx}{dt} - 10x\frac{dx}{dt}$$

For initial y-component of velocity

$$u_y = \left(\frac{dy}{dt}\right)_{x=0} \Rightarrow 5(1) = 5$$

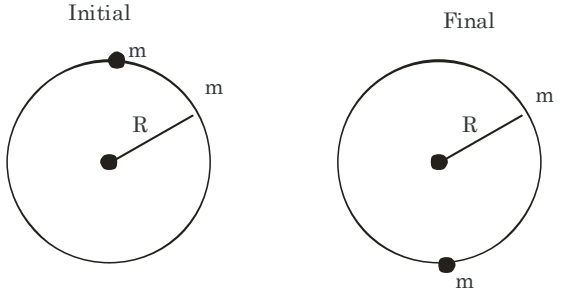
$$\vec{u}_y = 5\hat{j}$$

2. A disc of mass 1 kg and radius R is free of rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}$ rad s^{-1} where x = _____

$\left(g = 10 ms^{-2}\right)$

Official Ans. by NTA (5)

Sol.



using conservation of mechanical energy

$$mg2R = \frac{1}{2}I_{disc}\ \omega^2 + \frac{1}{2}I_{particle}\omega^2$$

$$mg2R = \frac{\omega^2}{2}\left[\frac{mR^2}{2} + mR^2\right]$$

$$mg2R = \frac{\omega^2}{2}\frac{3}{2}mR^2$$

$$\frac{3}{4}\omega^2 = \frac{2g}{R}$$

$$\omega^2 = \frac{8g}{3R}$$

$$\omega = \sqrt{\frac{80}{3R}}$$

Given $\omega = 4\sqrt{\frac{x}{3R}}$

$$16\frac{x}{3R} = \frac{80}{3R}$$

$x = 5$

3. In an experiment of determine the Young's modulus of wire of a length exactly 1m, the extension in the length of the wire is measured as 0.4mm with an uncertainty of ± 0.02 mm when a load of 1kg is applied. The diameter of the wire is measured as 0.4mm with an uncertainty of ± 0.01 mm. The error in the measurement of Young's modulus (ΔY) is found to be $x \times 10^{10}\ Nm^{-2}$. The value of x is _____

[Take $g = 10 m/s^2$]

Official Ans. by NTA (2)

Sol. $L = 1m$

$\Delta L = 0.4 \times 10^{-3}$ m

$m = 1kg$

$d = 0.4 \times 10^{-3} m$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$Y = \frac{FL}{A\Delta L} = \frac{(mg).(1)}{\left(\frac{\pi d^2}{4}\right)0.4\times10^{-3}}$$

$$\Rightarrow \frac{10\times4}{\pi\left(0.4\times10^{-3}\right)^2\times0.4\times10^{-3}}$$

$$Y = \frac{40}{\pi\left(0.4\times10^{-3}\right)^3}$$

$$Y = \frac{40\times7}{22\times64\times10^{-3}\times10^{-9}}$$

$Y = 0.199 \times 10^{-12}$ N/m^2

$$\frac{\Delta Y}{Y} = \frac{\Delta F}{F} + \frac{\Delta L}{L} + \frac{\Delta A}{A} + \frac{\Delta(\Delta L)}{(\Delta L)}$$

$$= \frac{0.02}{0.4} + 2\frac{\Delta d}{d} = \frac{0.2}{4} + 2\times\frac{0.01}{0.4}$$

$$= \frac{0.1}{2} + \frac{0.1}{2} = 0.1$$

$$\Rightarrow \Delta Y = 0.1\times Y$$

$= 0.199 \times 10^{11} = 1.99 \times 10^{10}$

4. When a car is approaching the observer, the frequency of horn is 100Hz. After passing the observer, it is 50Hz. If the observer moves with the car, the frequency will be $\frac{x}{3}$ Hz where x = ___

Official Ans. by NTA (200)

Sol. $$f_1 = 100 = f_0\left(\frac{C}{C - V_s}\right)$$

C = speed of sound

V_S = speed of source

$$f_2 = 50 = f_0\left(\frac{C}{C + V_s}\right)$$

$$\frac{f_1}{f_2} = 2 = \frac{C + V_s}{C - V_s}$$

$2C - 2V_s = C + V_s$

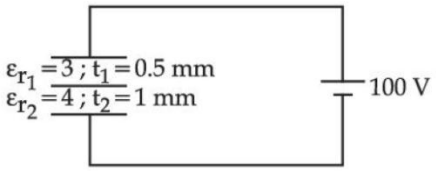
$3V_s = C$

$$V_S = \frac{C}{3}$$

$$100 = f_0\frac{C}{\frac{2C}{3}} = \frac{3}{2}f_0$$

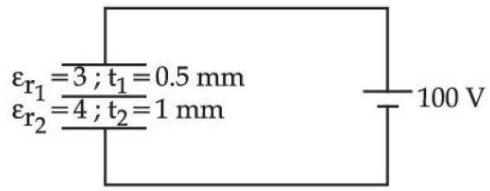
$$f_0 = \frac{200}{3}$$

5. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness $(t_1 \text{ and } t_2)$ as shown in figure. The two different dielectric material are separated by a conducting foil F. The voltage of the conducting foil is ____V.



Official Ans. by NTA (60)

Sol.



Capacitance of each capacitor

$$C_1 = \frac{A3\in_0}{\frac{1}{2}} = 6A\in_0$$

$C_2 = A4\in_0 = 4A\in_0$

Equivalent capacitance

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} \Rightarrow \frac{24}{10}A\in_0$$

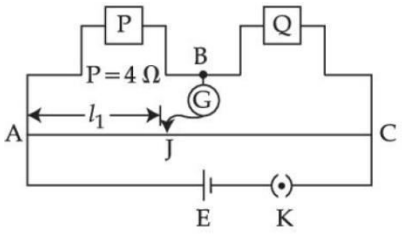
$$q_{net} = C_{eq}(\Delta V) \Rightarrow 240\,A\in_0$$

$$\Delta V_2 = \frac{240\ A\in_0}{4A\in_0} = 60V$$

$(\Delta V_2 = \text{Potential drop across } C_2)$

$V_{foil} = 60V$

6. Resistance are connected in a meter bridge circuit as shown in the figure. The balancing length l_1 is 40cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80cm measured from the same end. Then the value of x will be _____Ω



Official Ans. by NTA (20)

Sol. Initially, $\frac{P}{Q}=\frac{40cm}{60cm}=\frac{2}{3}$ ...(1)

Finally, $\frac{P+x}{Q}=\frac{80cm}{20cm}=\frac{4}{1}$ ...(2)

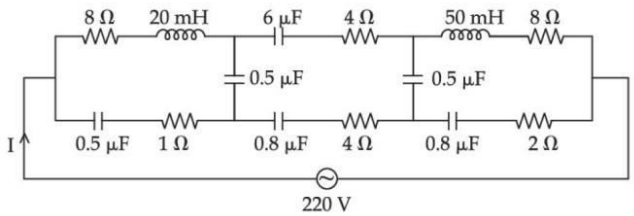
Divide (2) by (1)

$$\frac{P+x}{P}=4\times\frac{3}{2}=6$$

$$\Rightarrow \quad 1+\frac{x}{P}=6\Rightarrow\frac{x}{P}=5$$

$\therefore$ $x = 5P = 5 \times 4 = 20\Omega$

7. The effective current I in the given circuit at very high frequencies will be ____A



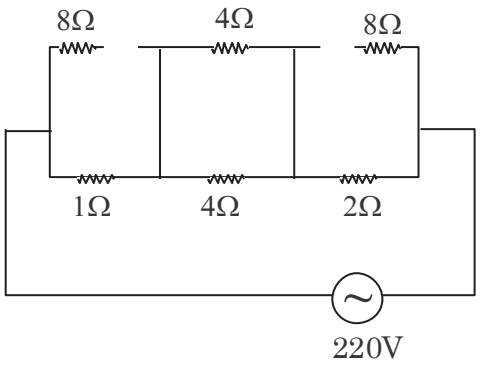
Official Ans. by NTA (44)

Sol. At very high frequencies,

$$X_C=\frac{1}{\omega C}\approx 0$$

Also $X_L=\omega L\approx\infty$

Thus, equivalent circuit can be redrawn as

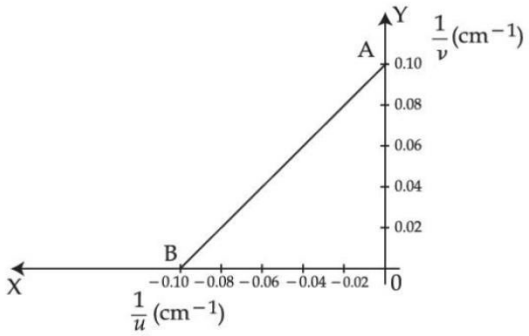


$Z = 1 + 2 + 2 = 5\Omega$

$$I=\frac{220V}{5\Omega}=44A$$

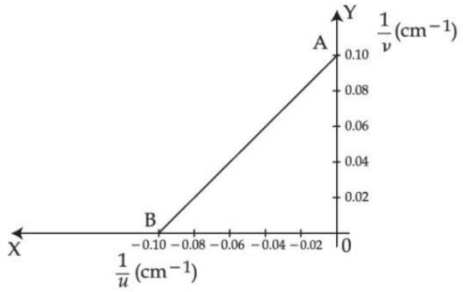
8. The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of length is 1.5 and its both the surfaces have same radius of curvatures R. The value of R will be ____cm.

(Where u = object distance , v = image distance)



Official Ans. by NTA (10)

Sol.



For point B, $\frac{1}{u} = -0.10\,cm^{-1}, \frac{1}{v} = 0$

$\therefore$ Thus, u = –10cm, v = ∞

i.e. f = 10 cm

$$\Rightarrow \quad \frac{1}{10cm} = (1.5-1)\left(\frac{2}{R}\right) = \frac{1}{R} \Rightarrow R = 10cm$$

9. In a hydrogen spectrum ,λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "aλ" between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer Series where a = _____

Official Ans. by NTA (5)

Sol. **For first line of Lyman**

$$\frac{1}{\lambda} = R\left(1-\frac{1}{4}\right) = R\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda = \frac{4}{3R} \quad \ldots(1)$$

3rd line(Paschen)

$$\frac{1}{\lambda_3} = R\left(\frac{1}{3^2}-\frac{1}{6^2}\right) = \frac{R}{9}\times\frac{3}{4}$$

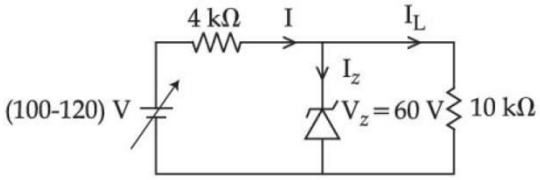
2nd line(Balmer)

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{R}{4}\times\frac{3}{4}$$

Thus $a\lambda = \lambda_3 - \lambda_2 = \frac{12}{R} - \frac{16}{3R} = \frac{20}{3R}$ putting (1)

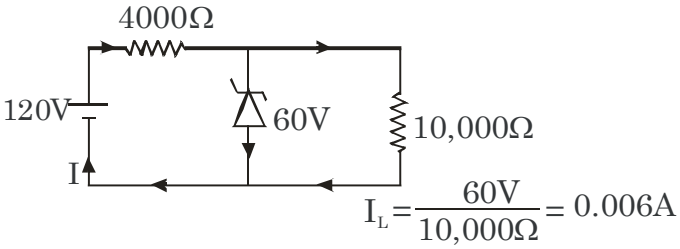
$$a\left(\frac{4}{3R}\right) = \frac{20}{3R} \Rightarrow a = 5$$

10. In the circuit shown below, maximum zener diode current will be ____mA



Official Ans. by NTA (9)

Sol. Consider input 120V



$$I = \frac{(120-60)\,V}{4000\Omega} = 0.015A$$

Thus $I_2 = I - I_L$

= 0.015 – 0.006 = 0.009 A = 9 mA

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Match **List - I** with **List - II**.

List - I (Compound)	**List – II (Shape)**
(A) BrF_5	(I) bent
(B) $[CrF_6]^{3-}$	(II) square pyramidal
(C) O_3	(III) trigonal bipyramidal
(D) PCl_5	(IV) octahedral

Choose the **correct** answer from the options given below :

(A) (A) – (I), (B) - (II), (C) - (III), (D) - (IV)

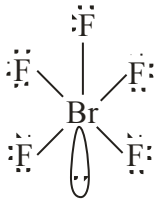
(B) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(C) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(D) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

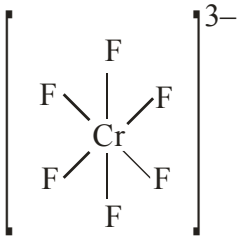
Official Ans. by NTA (C)

Sol.



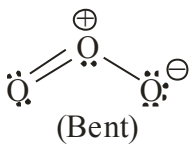
(Square pyramidal)

$[CrF_6]^{3-}$:



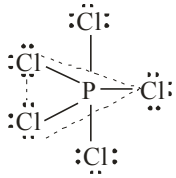
(Octahedral)

O_3 :



(Bent)

PCl_5 :



(Trigonal bipyramidal)

2. Match **List - I** with **List - II.**

List –I (Processes/Reactions)	**List – II (Catalyst)**
(A) $2SO_2(g)+O_2(g)\rightarrow 2SO_3(g)$	(I) Fe(s)
(B) $4NH_3(g)+5O_2(g)\rightarrow 4NO(g)+6H_2O(g)$	(II) Pt(s)-Rh(s)
(C) $N_2(g)+3H_2(g)\rightarrow 2NH_3(g)$	(III) V_2O_5
(D) Vegetable oil(l)+$H_2\rightarrow$Vegetable ghee(s)	(IV) Ni(s)

Choose the correct answer from the options given below :

(A) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

(B) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

(C) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(D) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)

Official Ans. by NTA (B)

Sol. $2SO_2(g)+O_2(g)\xrightarrow{V_2O_5}2SO_3(g)$:

contact process

$4NH_3(g)+5O_2(g)\xrightarrow{Pt(s)-Rh(s)}4NO(g)+6H_2O(g)$:

Ostwald's process

$N_2(g)+3H_2(g)\xrightarrow{Fe(s)}2NH_3(g)$; Haber's process

Vegetable oil $(l)+H_2(g)\xrightarrow{Ni(s)}$ vegetable ghee

: Hydrogenation

3. Given two statements below :

Statement I : In Cl_2 molecule the covalent radius is double of the atomic radius of chlorine.

Statement II : Radius of anionic species is always greater than their parent atomic radius.

Choose the **most appropriate** answer from options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (D)

Sol. In Cl_2 molecule, the covalent radius is half of the internuclear distance, so statement(I) is false.

For the same element, anion has lower effective nuclear charge than atom $\Rightarrow$ so anion is larger than atom. $\Rightarrow$ statement (II) is correct.

4. Refining using liquation method is the most suitable for metals with :

(A) Low melting point

(B) High boiling point

(C) High electrical conductivity

(D) Less tendency to be soluble in melts than impurities

Official Ans. by NTA (A)

Sol. Liquation is used to purify metals having lower melting point than impurities present in them.

5. Which of the following can be used to prevent the decomposition of H_2O_2?

(A) Urea

(B) Formaldehyde

(C) Formic acid

(D) Ethanol

Official Ans. by NTA (A)

Sol. Urea acts as stabiliser for H_2O_2.

6. Reaction of $BeCl_2$ with $LiAlH_4$ gives :

(A) $AlCl_3$

(B) BeH_2

(C) LiH

(D) LiCl

(E) $BeAlH_4$

Choose the **correct** answer from options given below :

(A) (A), (D) and (E)

(B) (A) , (B) and (D)

(C) (D) and (E)

(D) (B) , (C) and (D)

Official Ans. by NTA (B)

Sol. $2BeCl_2 + LiAlH_4 \rightarrow 2BeH_2 + LiCl + AlCl_3$

7. Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of "X" with 6-equivalents of "Y". "X" and "Y", respectively are :

(A) $B(OH)_3$ and NH_3 (B) B_2H_6 and NH_3

(C) B_2H_6 and HN_3 (D) NH_3 and B_2O_3

Official Ans. by NTA (B)

Sol. $3B_2H_6 + 6NH_3 \xrightarrow{\Delta} 2B_3N_3H_6 + 12\ H_2$

8. Which of the given reactions is not an example of disproportionation reaction ?

(A) $2H_2O_2 \rightarrow 2H_2O + O_2$

(B) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

(D) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Official Ans. by NTA (C)

Sol. $\overset{-1}{2H_2O_2} \longrightarrow \overset{2-}{2H_2O} + \overset{0}{O_2}$: Disproportionation

$\overset{+4}{2NO_2} + H_2O \rightarrow \overset{+5}{HNO_3} + \overset{+3}{HNO_2}$: Disproportionation

$MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$: reduction

$\overset{+6}{3MnO_4^{2-}} + 4H^+ \rightarrow \overset{+7}{2MnO_4^-} + \overset{+4}{MnO_2} + 2H_2O$: Disproportionation

9. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium.

The overall change in the oxidation number of manganese in the reaction is :

(A) 5 (B) 1

(C) 7 (D) 2

Official Ans. by NTA (A)

Sol. In acidic medium,

$\overset{+7}{MnO_4^-} \rightarrow Mn^{+2}$

change in ox. no. = 5

10. $\dot{C}l + CH_4 \rightarrow A + B$

A and B in the above atmospheric reaction step are

(A) C_2H_6 and Cl_2 (B) $\dot{C}HCl_2$ and H_2

(C) $\dot{C}H_3$ and HCl (D) C_2H_6 and HCl

Official Ans. by NTA (C)

Sol. $\dot{C}l + CH_4 \longrightarrow \dot{C}H_3 + HCl$

11. Which technique among the following, is most appropriate in separation of a mixture of 100 mg of p-nitrophenol and picric acid ?

(A) Steam distillation

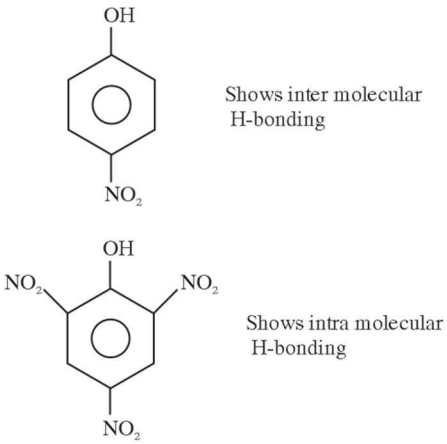
(B) 2-5 ft long column of silica gel

(C) Sublimation

(D) Preparative TLC (Thin Layer Chromatography)

Official Ans. by NTA (D)

Sol.



Solvent polarity has been related to R_f value of nitrocompounds.

100 mg p-nitrophenol and picric acid have different R_f value on silica gel plate

$\therefore$ Preparative TLC is best to separate 100 mg of para nitrophenol and picric acid

12. The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to :

(A) Hyperconjugation in substrate

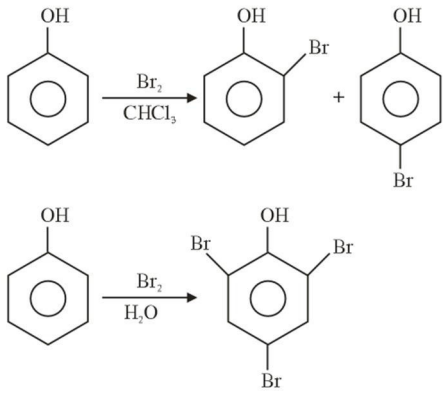
(B) Polarity of solvent

(C) Free radical formation

(D) Electromeric effect of the substrate

Official Ans. by NTA (B)

Sol.

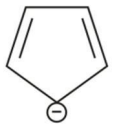


Difference in reactions is observed due to solvent polarity, which

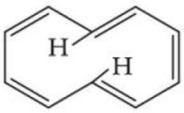
(i) Ionizes phenol to make more reactive phenoxide ion

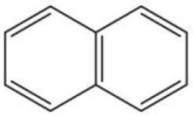
(ii) Increases electrophilicity of bromine.

13. Which of the following compounds is **not** aromatic?

(A) 

(B) 

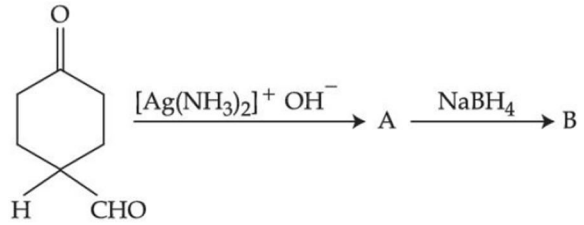
(C) 

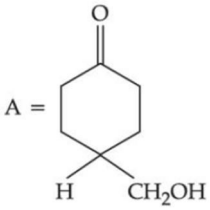
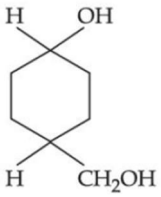
(D) 

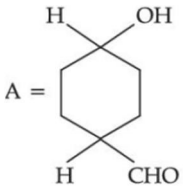
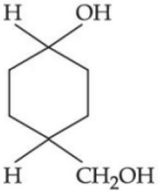
Official Ans. by NTA (C)

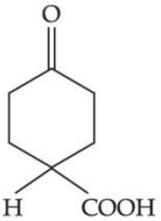
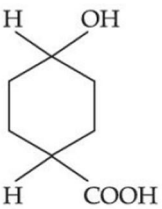
Sol. [10] Annulene, although follow $(4n + 2)\pi$ electron rule, but it is non-aromatic due to its non planar nature. It is nonplanar due to repulsion of C – H bonds present inside the ring.

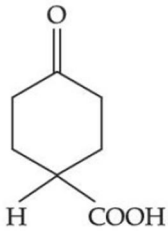
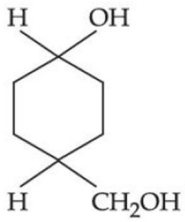
14. The products formed in the following reaction, A and B are



(A) A =  , B = 

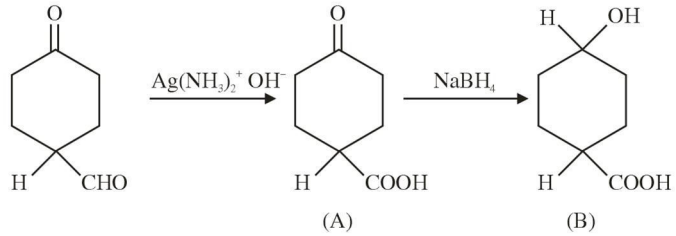
(B) A =  , B = 

(C) A =  , B = 

(D) A =  , B = 

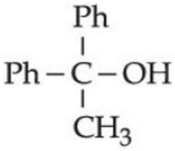
Official Ans. by NTA (C)

Sol.



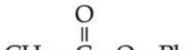
$NaBH_4$ does not reduce carboxylic acid.

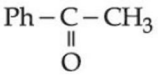
15. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide (PhMgBr) followed by acidic hydrolysis ?



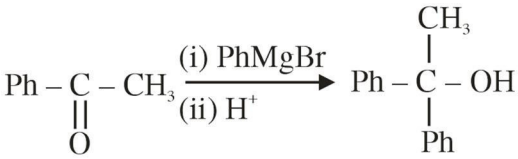
(A) $CH_3 - C \equiv N$

(B) $Ph - C \equiv N$

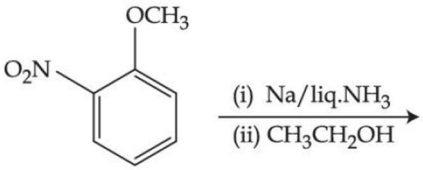
(C) 

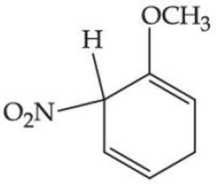
(D) 

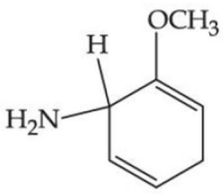
Official Ans. by NTA (D)

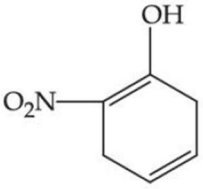
Sol. 

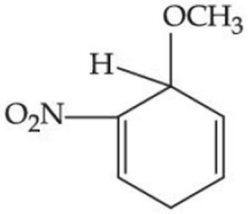
16. The major product of the following reaction is



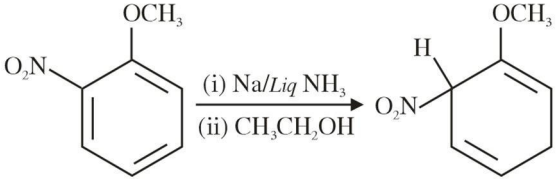
(A) 

(B) 

(C) 

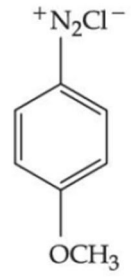
(D) 

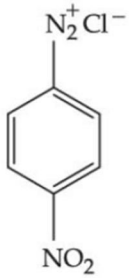
Official Ans. by NTA (A)

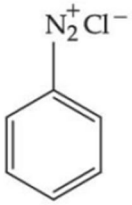
Sol. 

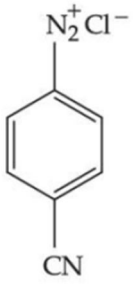
Given reaction is an example of birch reduction.

17. The correct stability order of the following diazonium salt is

(A) 

(B) 

(C) 

(D) 

(A) (A) > (B) > (C) > (D)

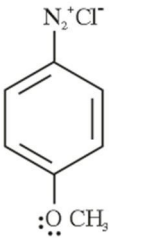
(B) (A) > (C) > (D) > (B)

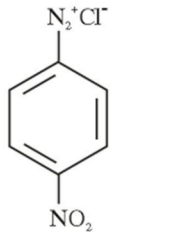
(C) (C) > (A) > (D) > (B)

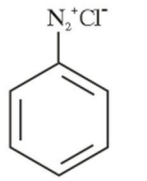
(D) (C) > (D) > (B) > (A)

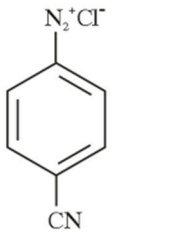
Official Ans. by NTA (B)

Sol.

(A) 
(+ M group)

(B) 
(– M & – I group)

(C) 

(D) 
(– M & – I group)

Since diazonium ion is a cation hence it is stabilized by electron donating groups and destabilized by electron withdrawing group.

Hence Stability order should be A > C > D > B.

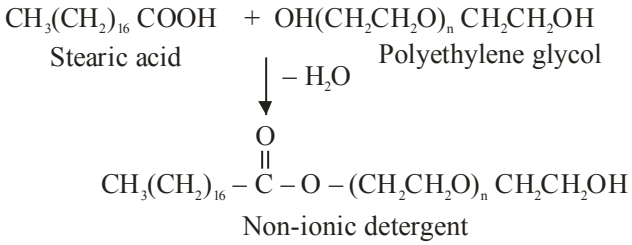
18. Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents ?

(A) Cationic detergent (B) Soap

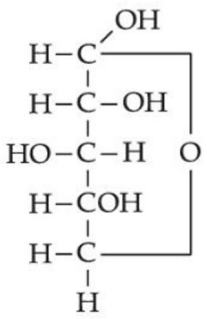
(C) Anionic detergent (D) Non-ionic detergent

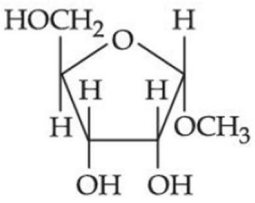
Official Ans. by NTA (D)

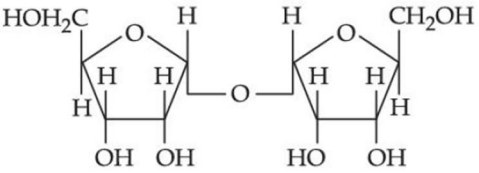
Sol.

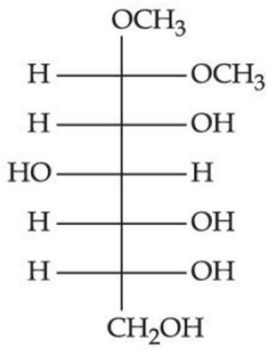


19. Which of the following is reducing sugar?

(A) 

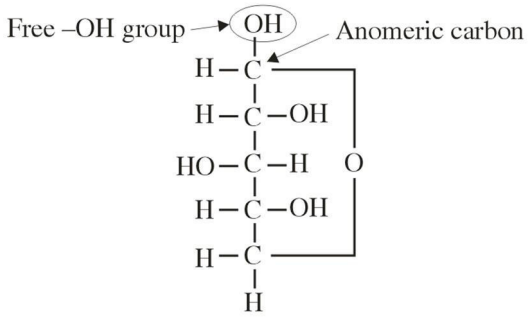
(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. If any sugar is having free –OH group at anomeric carbon then it will be a reducing sugar



20. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does **not** give o and p-methylaniline.

Reason (R) : The — NH_2 group of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields *m*-methyl aniline as the product.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

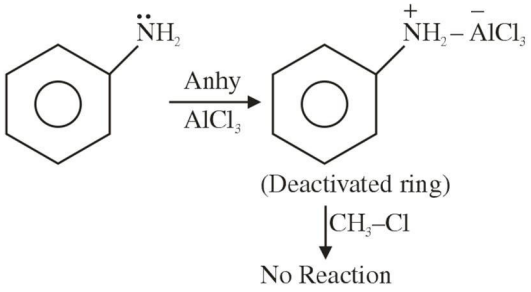
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

Official Ans. by NTA (C)

Sol.



Friedel Craft Alkylation does not occur on this deactivated ring.

SECTION-B

1. Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is_____. (Nearest Integer)

(Given : Atomic mass of Mg is 24 g mol^{-1}, $N_A = 6.02 \times 10^{23}$ mol^{-1})

Official Ans. by NTA (24)

Sol. $$ppm = \frac{W_{Mg}}{V_{soln}} \times 10^6 = 48$$

$$\Rightarrow W_{Mg} = \frac{48 \times 2 \times 1000}{10^6}$$

$$= 48 \times 2 \times 10^{-3}\ g$$

$$n_{Mg} = \frac{W_{Mg}}{24} = \frac{48 \times 2 \times 10^{-3}}{24}$$

$$= 4 \times 10^{-3}$$

Number of Mg atoms $= 4 \times 10^{-3} \times 6.02 \times 10^{23}$

$= 4 \times 6.02 \times 10^{20}$

$= 24.08 \times 10^{20}$

$\therefore \quad x = 24.08$

2. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is bar.

(Nearest Integer)

Official Ans. by NTA (2)

Sol. Let $W_{H_2} = 40\ g \Rightarrow n_{H_2} = \frac{40}{2} = 20$

$$W_{O_2} = 60g \Rightarrow n_{O_2} = \frac{60}{32} = \frac{15}{8}$$

$$P_{H_2} = \left(\frac{20}{20 + \frac{15}{8}}\right) \times 2.2$$

$$= \frac{20}{20 + 1.875} \times 2.2$$

$$= \frac{20}{21.875} \times 2.2$$

$$= 2.0114$$

$$\simeq 2.01 \text{ bar}$$

3. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is_____. (Nearest Integer)

(Mass of electron is 9.1×10^{-31} kg and mass of neutron is 1.6×10^{-27} kg)

Official Ans. by NTA (1758)

Sol. $v_e = x\, v_N$

$\lambda_e = \lambda_N$

$$\Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_N v_N}$$

$$v_e = \frac{m_N}{m_e}.v_N$$

$$= \frac{1.6\times10^{-27}}{9.1\times10^{-31}} v_N$$

$v_e = 1758.24 \times v_N$

$\therefore$ x = 1758.24

4. 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure.

The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is – x kJ mol^{-1}. The value of x is_____. (Nearest Integer)

(Given : Heat capacity of bomb calorimeter 20.0 kJ K^{-1} . Assume coal to be pure carbon)

Official Ans. by NTA (200)

Sol. $C(s) + O_2(g) \rightarrow CO_2(g)$; ΔH = -x kJ/mole

$Q = C\Delta T = 20$ kJ × 2

40 kJ heat is released for 2.4 g of C

For 1 mole 'C' :

$$Q = \frac{40}{2.4}\times12$$

$$= \frac{400}{24}\times12 = 200 \text{ kJ/mole}$$

$Q = \Delta E = \Delta H = 200$ kJ ($\because \Delta n_g = 0$)

x = 200

5. When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is x × 10^{-2} M. (Nearest Integer)

(Molar mass of nitric acid is 63 g mol^{-1})

Official Ans. by NTA (54)

Sol. $n_{HNO_3} = 0.5 \times 0.8$

= 0.4 mole

$$(n_{HNO_3})_{remains} = 0.4 - \frac{11.5}{63}$$

= 0.4 – 0.1825

= 0.2175

$$\text{Molarity} = \frac{0.2175}{400}\times1000$$

$$= \frac{0.2175}{0.4}$$

= 0.5437 mole/lit.

$\simeq$ 0.54 mole/lit.

= 54 × 10^{-2} mol/lit.

6. At 298 K, the equilibrium constant is 2 × 10^{15} for the reaction :

$Cu(s) + 2Ag^+(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$

The equilibrium constant for the reaction

$$\frac{1}{2}Cu^{2+}(aq) + Ag(s) \rightleftharpoons \frac{1}{2}Cu(s) + Ag^+(aq)$$

is x × 10^{-8}. The value of x is_____.

(Nearest Integer)

Official Ans. by NTA (2)

Sol. $$K'_{eq} = \frac{1}{\sqrt{K_{eq}}} = \frac{1}{\sqrt{2\times10^{15}}} = x\times10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}}\times\frac{1}{10^7} = x\times10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}}\times10^{-7} = x\times10^{-8}$$

$$\frac{10}{\sqrt{20}} = x$$

$$\Rightarrow x = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5} = 2.236$$

$\simeq$ 2.24

7. The amount of charge in F (Faraday) required to obtain one mole of iron from Fe_3O_4 is _____. (Nearest Integer)

Official Ans. by NTA (8)

Sol. $Fe_3O_4 \xrightarrow{+8e^-} 3Fe$

Charge for 1 mole Fe = 8/3 F

= 2.67 F

8. For a reaction A $\rightarrow$ 2B + C the half lives are 100 s and 50 s when the concentration of reactant A is 0.5 and 1.0 mol L^{-1} respectively. The order of the reaction is___ _____. (Nearest Integer)

Official Ans. by NTA (2)

Sol. $t_{\frac{1}{2}} \propto \frac{1}{[A_0]^{n-1}}$

$[100] \propto \frac{1}{(0.5)^{n-1}}$

$(50) \propto \frac{1}{(1)^{n-1}}$

$[2]^1 = \left[\frac{1}{0.5}\right]^{n-1}$

$[2]^1 = [2]^{n-1}$

$n - 1 = 1$

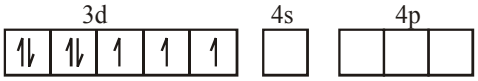
$n = 2$

order = 2

9. The difference between spin only magnetic moment values of $[Co(H_2O)_6]Cl_2$ and $[Cr(H_2O)_6]Cl_3$ is_____.

Official Ans. by NTA (0)

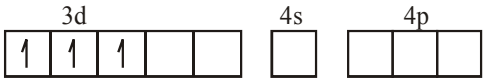
Sol. $[Co(H_2O)_6]^{2+}$

Co^{+2} : 

number of unpaired e^- = 3

$\mu = \sqrt{15}BM$

$[Cr(H_2O)_6]^{3+}$

Cr^{+3} : 

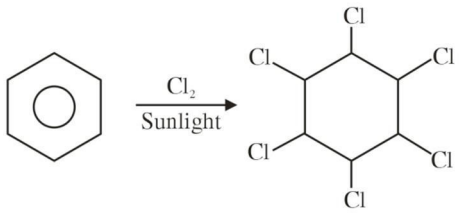
number of unpaired e^- = 3

$\mu = \sqrt{15}BM$

Difference in spin only magnetic moment = 0

10. In the presence of sunlight, benzene reacts with Cl_2 to give product, X. The number of hydrogens in X is_____.

Official Ans. by NTA (6)

Sol. 

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let f: R $\to$ R be a continuous function such that f (3x) – f (x) = x. If f (8) = 7, then f (14) is equal to :

(A) 4 (B) 10

(C) 11 (D) 16

Official Ans. by NTA (B)

Sol. $f(x) - f(x/3) = x/3$

$f(x/3) - f(x/3^2) = x/3^2$

.... on adding

$$f(x) - \lim_{n\to\infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} \ldots \infty\right)$$

$$f(x) - f(0) = \frac{x}{2}$$

$f(8) = 7$; $f(0) = 3$

$f(x) = x/2 + 3$

$f(14) = 10$

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ?

(A) $\arg z_2 = \pi - \tan^{-1} 3$

(B) $\arg (z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Official Ans. by NTA (D)

Sol. $AB = AO.\, z^{-i\pi/2} = -2 + i$

So $OB = (-2 + i) + (1 + 2i)$

$z_2 = -1 + 3i$

$\therefore |2z_1 - z_2| = \sqrt{10}$

3. If the system of linear equations.

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is :

(A) $3\sqrt{5}$ (B) 4

(C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Official Ans. by NTA (D)

Sol. $$D = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0 \Rightarrow \lambda = 4$$

Also $D_1 = D_2 = D_3 = 0$

So $\mu = -2$

Point $\left(4, -2, -\frac{1}{2}\right)$

Distance from plane = $\frac{10}{3}$

4. Let A be a 2 × 2 matrix with det (A) = –1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be :

(A) –1 (B) 2

(C) 1 (D) $-\sqrt{2}$

Official Ans. by NTA (B)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc = -1$

$|A + I||adj\ A + I| = 4$

$\Rightarrow ad - bc + a + d + 1 = 2$ or -2

$a + d = 2$ or -2

5. The odd natural number a, such that the area of the region bounded by y= 1, y = 3, x = 0, x = y^a is $\frac{364}{3}$, equal to :

(A) 3 (B) 5

(C) 7 (D) 9

Official Ans. by NTA (B)

Sol. $A=\int_1^3 y^a.dy=\left.\frac{y^{a+1}}{a+1}\right|_1^3=\frac{364}{3}$

$\Rightarrow a=5$

6. Consider two G.Ps. 2, 2^2, 2^3, ... and 4, 4^2, 4^3,.... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

(A) 560 (B) 1540

(C) 1330 (D) 2600

Official Ans. by NTA (C)

Sol. $\left(\left(2^1 2^2.....2^{60}\right)\left(4^1.4^2.....4^n\right)\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$\left(2^{30\times 61}\,4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$2^{1830+n^2+n}=2^{\frac{(225)(60+n)}{8}}$

$= 8n^2-217n+1140=0$

$n=20,\ \frac{57}{8}$

$\sum_{k=1}^{n} nk-k^2=\frac{n^2(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}$

$= 1330$

7. If the function

$$f(x)=\begin{cases}\frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)-\{0\}\\ k, & x=0\end{cases}$$

is continuous at x = 0, then k is equal to :

(A) 1 (B) –1

(C) e (D) 0

Official Ans. by NTA (A)

Sol. $\lim_{x\to 0}\frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$

$\lim_{x\to 0}\frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2}=1$

$\therefore k=1$

8. If f(x) = $\begin{cases}x+a, & x\le 0\\ |x-4|, & x>0\end{cases}$ and

g(x) = $\begin{cases}x+1, & x<0\\ (x-4)^2+b, & x\ge 0\end{cases}$

are continuous on R, then (gof) (2) + (fog) (–2) is equal to :

(A) –10 (B) 10

(C) 8 (D) –8

Official Ans. by NTA (D)

Sol. $f(x)=\begin{cases}x+a & ;x\le 0\\ |x-4| & ;x>0\end{cases}$; $g(x)=\begin{cases}x+1 & ;x<0\\ (x-4)^2+b & ;x\ge 0\end{cases}$

For continuity a = 4 and b = –15

$g(f(2))+f(g(-2))$

$= g(2)+f(-1)=-8$

9. Let $f(x)=\begin{cases} x^3-x^2+10x-7\ , & x\le 1 \\ -2x+\log_2(b^2-4), & x>1 \end{cases}$

Then the set of all values of b, for which f(x) has maximum value at x = 1, is :

(A) (–6, –2)

(B) (2, 6)

(C) $[-6,-2)\cup(2,6]$

(D) $[-\sqrt{6}\ ,-2)\cup(2,\sqrt{6}\ ]$

Official Ans. by NTA (C)

Sol. $f(1)=3$

For $x<1,\ f'(x)=3x^2-2x+10>0$

$\Rightarrow f(x)$ is increasing

For $x>1, f'(x)<0$

$\Rightarrow$ function is decreasing.

$\lim\limits_{x\to 1^+} f(x)=-2+\log_2(b^2-4)$

For maximum value at x = 1

$3\ge -2+\log_2(b^2-4)$

$32\ge b^2-4>0$

$b\in[-6,-2)\cup(2,6]$

10. If $a=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{2n}{n^2+k^2}$ and $f(x)=\sqrt{\dfrac{1-\cos x}{1+\cos x}}, x\in(0,1)$, then :

(A) $2\sqrt{2}f\left(\dfrac{a}{2}\right)=f'\left(\dfrac{a}{2}\right)$

(B) $f\left(\dfrac{a}{2}\right)f'\left(\dfrac{a}{2}\right)=\sqrt{2}$

(C) $\sqrt{2}f\left(\dfrac{a}{2}\right)=f'\left(\dfrac{a}{2}\right)$

(D) $f\left(\dfrac{a}{2}\right)=\sqrt{2}f'\left(\dfrac{a}{2}\right)$

Official Ans. by NTA (C)

Sol. $a=\dfrac{1}{n}\sum\limits_{k=1}^{n}\dfrac{2}{1+\left(\dfrac{k}{n}\right)^2}=\int_0^1\dfrac{2}{1+x^2}dx=\dfrac{\pi}{2}$

$f(x)=\tan\left(\dfrac{x}{2}\right);\ x\in(0,1)$

$f\left(\dfrac{\pi}{4}\right)=\sqrt{2}-1$

$f'\left(\dfrac{\pi}{4}\right)=\dfrac{1}{2}\sec^2\left(\dfrac{\pi}{8}\right)=\dfrac{\sqrt{2}}{\sqrt{2}+1}$

$f'\left(\dfrac{\pi}{4}\right)=\sqrt{2}\,f\left(\dfrac{\pi}{4}\right)$

11. If $\dfrac{dy}{dx}+2y\tan x=\sin x,\ 0<x<\dfrac{\pi}{2}$ and $y\left(\dfrac{\pi}{3}\right)=0$, then the maximum value of y(x) is

(A) $\dfrac{1}{8}$ (B) $\dfrac{3}{4}$

(C) $\dfrac{1}{4}$ (D) $\dfrac{3}{8}$

Official Ans. by NTA (A)

Sol. $\dfrac{dy}{dx}+2y\tan x=\sin x$

$I.F=e^{\int 2\tan x\,dx}=e^{\ln(\sec x)^2}=\sec^2 x$

$y(\sec^2 x)=\int \sin x\sec^2 x\,dx+C$

$y.\sec^2 x=\sec x+C$

Put $x=\dfrac{\pi}{3},\ y=0$

$y=\cos x-2\cos^2 x$

$=\dfrac{1}{8}-2\left(\cos x-\dfrac{1}{4}\right)^2$

$\therefore y_{max}=\dfrac{1}{8}$

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let f(x, y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the point C, D . Then the area of the quadrilateral ACBD is equal to

(A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$ (D)9

Official Ans. by NTA (B)

Sol. $(x-1)^2+(y-2)^2+(x+2)^2+(y-1)^2=14$

$\Rightarrow x^2+y^2+x-3y-2=0$

Put x = 0

$\Rightarrow y^2-3y-2=0$

$\Rightarrow y=\frac{3\pm\sqrt{17}}{2}$

Put y = 0

$\Rightarrow x^2+x-2=0$

$(x+2)(x-1)=0$

$\therefore A(-2,0), B(1,0), C\left(0,\frac{3+\sqrt{17}}{2}\right), D\left(0,\frac{3-\sqrt{17}}{2}\right)$

Area = $\frac{1}{2}.3.\sqrt{17}=\frac{3\sqrt{17}}{2}$

13. Let the tangent drawn to the parabola $y^2=24x$ at the point (α,β) is perpendicular to the line 2x +2y = 5. Then the normal to the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at the point $(\alpha+4,\beta+4)$ does NOT pass through the point :

(A) (25, 10) (B) (20, 12)

(C) (30, 8) (D) (15, 13)

Official Ans. by NTA (D)

Sol. Tangent at (α,β) has slope 1

$\beta^2=24\alpha$

Equation of tangent $y\beta=12(x+\alpha), \frac{12}{\beta}=1$

$\Rightarrow \alpha=6, \beta=12$

$\therefore(\alpha+4, \beta+4)=(10,16)$

Normal at (10, 16) to $\frac{x^2}{36}-\frac{y^2}{144}=1$ is

2x + 5y = 100

14. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line x + y – z = 0 = x – 2y + 3z – 5 is :

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$ (D)1

Official Ans. by NTA (A)

Sol. d.r's of the line = $\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-1\\1&-2&3\end{vmatrix}=\hat{i}-4\hat{j}-3\hat{k}$

$\therefore$ equation of line is

$\vec{r}=\hat{i}+2\hat{j}+4\hat{k}+\lambda(\hat{i}-4\hat{j}-3\hat{k})$

Let A(1, 2, 4) and P be $(1+\lambda,2-4\lambda,4-3\lambda)$

$\therefore \overrightarrow{PA}.(\hat{i}-4\hat{j}-3\hat{k})=0$

$\lambda=\frac{1}{2}$

$\Rightarrow P\left(\frac{1}{2},2,\frac{-5}{2}\right)$

$|AP|=\sqrt{\frac{21}{2}}$

15. Let $\vec{a}=\alpha\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}, \alpha>0$. If the projection of $\vec{a}\times\vec{b}$ on the vector $-\hat{i}+2\hat{j}-2\hat{k}$ is 30, then α is equal to

(A) $\frac{15}{2}$ (B) 8

(C) $\frac{13}{2}$ (D)7

Official Ans. by NTA (D)

Sol. $\vec{a}\times\vec{b}=(1-\alpha)\hat{i}+(\alpha^2-2)\hat{j}+(\alpha-2)\hat{k}$

Projection of $\vec{a}\times\vec{b}$ on $-\hat{i}+2\hat{j}-2\hat{k}$

$$=\frac{(\vec{a}\times\vec{b}).(-\hat{i}+2\hat{j}-2\hat{k})}{3}=30$$

$$\Rightarrow 2\alpha^2-\alpha-91=0$$

$$\Rightarrow \alpha=7,-\frac{13}{2}$$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X=1)=\frac{4}{243}$, then P(X = 4 or 5) is equal to :

(A) $\frac{5}{9}$ (B) $\frac{64}{81}$

(C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Official Ans. by NTA (C)

Sol. $np = \alpha$ .....(1)

$npq = \alpha/3$ .....(2)

From (1) & (2)

$q = 1/3$ & $p = 2/3$

$${}^nC_1\, q^{n-1}p^1=\frac{4}{243}$$

$$\frac{n}{3^n}=\frac{2}{243}$$

$n = 6$

$$P(4 \text{ or } 5) = {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+{}^6C_5\left(\frac{2}{3}\right)^5\cdot\left(\frac{1}{3}\right)^0$$

$$=\frac{16}{27}$$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1)=\frac{2+3p}{6}$, $P(E_2)=\frac{2-p}{8}$ and $P(E_3)=\frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to :

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) 1

Official Ans. by NTA (D)

Sol. $0 \le P(E_i) \le 1$ for i = 1, 2, 3

$\Rightarrow -2/3 \le p \le 1$

E_1 & E_2 & E_3 are mutually exclusive

$P(E_1) + P(E_2) + P(E_3) \le 1$

$\Rightarrow 2/3 \le p \le 1$

$p_1 = 1$, $p_2 = 2/3$

$p_1 + p_2 = 5/3$

18. Let

$S = \{\theta\in[0,2\pi]: 8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\}$. Then

$n(S) + \sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to :

(A) 0 (B) –2

(C) –4 (D) 12

Official Ans. by NTA (C)

Sol. $8^{2\sin^2\theta}+8^{2-2\sin^2\theta}=16$

$$y+\frac{64}{y}=16$$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = ½$

$$n(S)+\sum_{\theta\in S}\frac{1}{\cos(\pi/4+2\theta)\sin(\pi/4+2\theta)}$$

$= 4 + (–2) \times 4 = –4$

19. $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(A) 1 (B) 2

(C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Official Ans. by NTA (B)

Sol. $\tan\left(2(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8})+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2$

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

(A) a tautology
(B) a contradiction

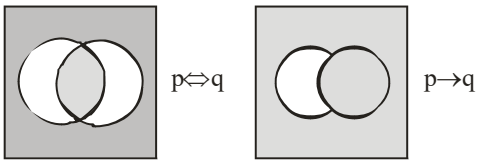
(C) equivalent to $(p \Rightarrow q) \wedge q$

(D) equivalent to $(p \Rightarrow q) \wedge p$

Official Ans. by NTA (D)

Sol. $(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



SECTION-B

1. If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to-

Official Ans. by NTA (272)

Sol. $(px - q)^2 + (qx - r)^2 = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$

$\Rightarrow \frac{q^2+r^2}{p^2} = 272$

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is

Official Ans. by NTA (180)

Sol. $3\times\frac{5!}{2!2!}+\frac{5!}{3!\times 2!}+\frac{5!}{2!}+\frac{5!}{3!}=180$

3. The series of positive multiples of 3 is divided into sets : {3}, {6, 9,12}, {15, 18, 21, 24, 27},... Then the sum of the elements in the 11^{th} set is equal to_____,

Official Ans. by NTA (6993)

Sol. $S_{11} = 3[101 + 102 + ... + 121]$

$= \frac{3}{2}(222)\times 21 = 6993$

4. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$\Rightarrow (x-1)^2 (x + 1)(x^5 + 3x - 1) = 0$

Let $f(x) = x^5 + 3x - 1$

$f'(x) > 0 \ \forall \ x \in R$

Hence 3 real distinct roots.

5. If the coefficients of x and x^2 in the expansion of $(1 + x)^p(1 - x)^q$, p, q $\le$ 15, are –3 and–5 respectively, then the coefficient of x^3 is equal to_____.

Official Ans. by NTA (23)

Sol. Since coefficient of x is –3

$\Rightarrow {}^pC_1 - {}^qC_1 = -3$

$\Rightarrow p - q = -3$ ....(1)

Comparing coefficients of x^2

$-{}^pC_1\,{}^qC_1 + {}^pC_2 + {}^qC_2 = -5$

$-pq + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5$ ....(2)

Solving (1) and (2)

p = 8, q = 11

Coefficient of x^3 is

$-{}^{q}C_3 + {}^{p}C_3 + {}^{p}C_1\,{}^{q}C_2 - {}^{p}C_2\,{}^{q}C_1$

$= -{}^{11}C_3 + {}^{8}C_3 + {}^{8}C_1\,{}^{11}C_2 - {}^{8}C_2\,{}^{11}C_1$

$= 23$

6. If

$n(2n+1)\int_0^1 (1-x^n)^{2n}\,dx = 1177\int_0^1 (1-x^n)^{2n+1}dx$, then

$n \in N$ is equal to _____

Official Ans. by NTA (24)

Sol. Let $I_1 = \int_0^1 (1-x^n)^{2n}\,dx$, $I_2 = \int_0^1 (1-x^n)^{2n+1}\,dx$

$I_2 = \int_0^1 (1-x^n)^{2n+1}\,.\,1\,dx$

$= (1-x^n)^{2n+1}.x\Big|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n}(-nx^{n-1})\,x\,dx$

$I_2 = -n(2n+1)\{I_2 - I_1\}$

$(2n^2+n+1)I_2 = n(2n+1)I_1$

$\frac{I_1}{I_2} = \frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$

$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$

7. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and x(>3) be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $\left(\alpha, 6\sqrt{10}\right)$ in the first quadrant, then α is equal to _____

Official Ans. by NTA (6)

Sol. $x^4 = 3yx.y' - 3y^2$

$\Rightarrow 3xy\frac{dy}{dx} = 3y^2 + x^4$

Put $y^2 = t, y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$

$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$

$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

Put (3, 3) , C = –2

$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

$3y^2 = x^4 - 6x^2$

$x^4 - 6x^2 = 1080$

$\therefore x = 6$

8. The equations of the sides AB, BC and CA of a triangle ABC are 2x + y = 0, x + py = 15a and x – y = 3 respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to

Official Ans. by NTA (3)

Sol. Coordinates of A(1, –2), $B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and orthocentre H(2, a)

Slope of AH = p

a + 2 = p .....(1)

Slope of BH = –1

31a – 2ab = 15a + 4p–2 .....(2)

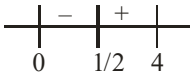
From (1) and (2)

a = 1 & p = 3

9. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a – 1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha}+\frac{y}{\beta}=1$, then $\alpha + \beta$ is equal to-

Official Ans. by NTA (45)

Sol. $f'(x) = 4x - \frac{1}{x}$



$a = \frac{1}{2}$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$\frac{1}{y_1}=\frac{3-y_1}{4-x_1} \Rightarrow y_1^2 - 6y_1 + 8 = 0$

$y_1 = 2, 4$

$\Rightarrow$ P(8, 4) as P(2, 2) rejected

Equation of normal at P.

$y - 4 = -4(x - 8)$

$\frac{x}{9}+\frac{y}{36}=1$

$\alpha = 9, \beta = 36$

$\alpha + \beta = 45$

10. Let Q and R be two points on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is_____.

Official Ans. by NTA (153)

Sol. Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$

$\lambda = 1, 3$

Q (1, 1, 3) ; R (5, 7, 7) ; P (4, 2, 7)

Area of triangle PQR = ½ $\left|\vec{PQ}\times\vec{PR}\right|$

$= \sqrt{153}$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Two projectiles are thrown with same initial velocity making an angle of 45° and 30° with the horizontal respectively. The ratio of their respective ranges will be

(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$

(C) $2:\sqrt{3}$ (D) $\sqrt{3}:2$

Official Ans. by NTA (C)

Sol. Let projection speed is u

$$R_1=\frac{u^2\text{Sin}(90^\circ)}{g};R_2=\frac{u^2\sin(60^\circ)}{g}$$

$$\frac{R_1}{R_2}=\frac{2}{\sqrt{3}}$$

2. In a Vernier Calipers. 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4th Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to 1 mm. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6th Vernier scale division exactly. coincides with the main scale reading. The diameter of the spherical body will be :

(A) 3.02 cm (B) 3.06 cm

(C) 3.10 cm (D) 3.20 cm

Official Ans. by NTA (C)

Sol. 1 M.S.D = 1mm

9 M.S.D = 10 V.S.D

1 V.S.D = 0.9 M.S.D = 0.9 mm

L.C of vernier caliper = 1–0.9 = 0.1 mm = 0.01 cm

zero error =–(10–4)×0.1mm = – 0.6 mm

Reading = M.S.R + V.S.R – Zero error

= 3cm + 6 × 0.01 – [–0.06]

= 3 + 0.06 + 0.06

= 3.12 cm

Nearest given answer in the options is 3.10

3. A ball of mass 0.15 kg hits the wall with its initial speed of 12 ms^{-1} and bounces back without changing its initial speed. If the force applied by the wall on the ball during the contact is 100 N. calculate the time duration of the contact of ball with the wall.

(A) 0.018 s (B) 0.036 s

(C) 0.009 s (D) 0.072 s

Official Ans. by NTA (B)

Sol. $\vec{P}_i=0.15\times12\left(\hat{i}\right)$

$\vec{P}_f=0.15\times12\left(-\hat{i}\right)$

$\left|\overrightarrow{\Delta P}\right|=3.6\text{ kg}-\text{m/s}$

$3.6=F\Delta t$

$3.6=100\ \Delta t$

$\Delta t=0.036$ sec

4. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be :

(A) 1:1 (B) 2:1 (C) 1:4 (D) 4:1

Official Ans. by NTA (B)

Sol. $K.E=\frac{P^2}{2m}$

$K_1=\frac{P_1^2}{2(8)}\ ;\ K_2=\frac{P_2^2}{2(2)}$

$K_1=K_2$

So,

$4P_2^2=P_1^2$

$\frac{P_1}{P_2}=2$

5. Two uniformly charged spherical conductors A and B of radii 5 mm and 10 mm are separated by a distance of 2 cm. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitudes of the electric fields at the surface of the sphere A and B will be :

(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 1 : 4

Official Ans. by NTA (B)

Sol. $V_A = V_B$

$$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$$

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}$$

$$E_A = \frac{KQ_A}{R_A^2}; E_B = \frac{KQ_B}{R_B^2}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{R_B^2}{R_A^2} = \frac{R_B}{R_A} = \frac{2}{1}$$

6. The oscillating magnetic field in a plane electromagnetic wave is given by $B_y = 5 \times 10^{-6}$ sin 1000π $(5x - 4 \times 10^8 t)$T. The amplitude of electric field will be :

(A) 15×10^2 Vm^{-1} (B) 5×10^{-6} Vm^{-1}

(C) 16×10^{12} Vm^{-1} (D) 4×10^2 Vm^{-1}

Official Ans. by NTA (D)

Sol. $B_0 = 5 \times 10^{-6}$

$$v = \text{Speed of wave} = \frac{4 \times 10^8}{5} = 8 \times 10^7 \quad \left[\therefore v = \frac{w}{k}\right]$$

$$E_0 = vB_0 = 40 \times 10^1$$

$$= 4 \times 10^2 \text{ V/m}$$

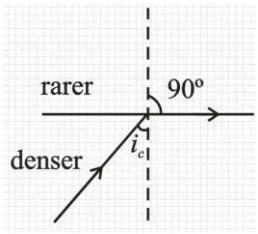
7. Light travels in two media M_1 and M_2 with speeds 1.5×10^8 ms^{-1} and 2.0×10^8 ms^{-1} respectively. The critical angle between them is:

(A) $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{4}\right)$ (D) $\sin^{-1}\left(\frac{2}{3}\right)$

Official Ans. by NTA (A)

Sol.



$$v = \frac{c}{n}$$

$$n_d \sin i_c = n_r \sin 90^\circ$$

$$\sin i_c = \frac{n_r}{n_d} = \frac{v_d}{v_r}$$

$$\sin i_c = \frac{1.5 \times 10^8}{2 \times 10^8} = \frac{1.5}{2}$$

$$\sin i_c = \frac{3}{4}$$

$$\tan i_c = \frac{3}{\sqrt{4^2 - 3^2}} \Rightarrow \frac{3}{\sqrt{7}}$$

$$i_c = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$$

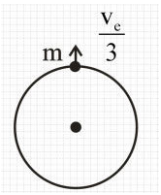
8. A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:
(Take radius of earth = 6400 km and g=10 ms^{-2})

(A) 800 km (B) 1600 km

(C) 2133 km (D) 4800 km

Official Ans. by NTA (A)

Sol.



$$v_e = \sqrt{\frac{2Gm}{R}}$$

$$\frac{-GMm}{R} + \frac{1}{2}m\frac{V_e^2}{9} = -\frac{GMm}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{V_e^2}{18}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$

$$\frac{GM}{R+h} = \frac{8GM}{9R}$$

$$\frac{1}{R+h} = \frac{8}{9R}$$

$$9R = 8R + 8h$$

$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800\text{km}$$

9. The maximum and minimum voltage of an amplitude modulated signal are 60 V and 20 V respectively. The percentage modulation index will be :

(A) 0.5% (B) 50% (C) 2% (D) 30%

Official Ans. by NTA (B)

Sol. $V_{max} = 60$

$V_{min} = 20$

% modulation =

$$\left(\frac{V_{max}-V_{min}}{V_{max}+V_{min}}\right)100 \Rightarrow \left(\frac{60-20}{60+20}\right)100 \Rightarrow \left(\frac{40}{80}\right)100$$

$\Rightarrow$ 50%

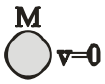
10. A nucleus of mass M at rest splits into two parts having masses $\frac{M'}{3}$ and $\frac{2M'}{3}\left(M' < M\right)$. The ratio of de Broglie wavelength of two parts will be :

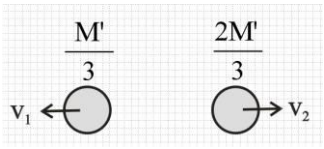
(A) 1 : 2 (B) 2 : 1

(C) 1 : 1 (D) 2 : 3

Official Ans. by NTA (C)

Sol.





$\left|\vec{P}_1\right| = \left|\vec{P}_2\right|$

Here $\vec{P}$ is momentum

So $\lambda = \frac{h}{P}$

Hence both will have same de broglie wavelength.

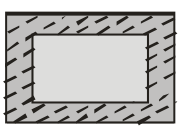
11. An ice cube of dimensions 60 cm × 50 cm × 20 cm is placed in an insulation box of wall thickness 1 cm. The box keeping the ice cube at 0°C of temperature is brought to a room of temperature 40°C. The rate of melting of ice is approximately: (Latent heat of fusion of ice is 3.4×10^5 J kg^{-1} and thermal conducting of insulation wall is 0.05 $Wm^{-1}{}^{o}C^{-1}$)

(A) 61×10^{-1} kg s^{-1} (B) 61×10^{-5} kg s^{-1}

(C) 208 kg s^{-1} (D) 30×10^{-5} kg s^{-1}

Official Ans. by NTA (B)

Sol.



$$\frac{dQ}{dt} = \frac{KA\Delta T}{\ell}$$

A = 2 (0.6 × 0.5 + 0.5 × 0.2 + 0.2 × 0.6)

= 2(0.3 + 0.1 + 0.12)

= 2(0.4 + 0.12)

= 2(0.52)

= 1.04 m^2

$$R_{th} = \frac{\ell}{KA} \Rightarrow \frac{1\times10^{-2}}{0.05\times1.04} \Rightarrow \frac{10^{-2}}{0.052}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{R_{th}} \Rightarrow \frac{40\times0.052}{10^{-2}} \Rightarrow 2.08\times10^2 \text{ J/s}$$

$2.08 \times 10^2 = m \times 3.4 \times 10^5$

$$m = \frac{2.08}{3.4\times10^3} \Rightarrow 0.61\times10^{-3} \text{ kg/s}$$

$= 61 \times 10^{-5}$ Kg/s

12. A gas has n degrees of freedom. The ratio of specific heat of gas at constant volume to the specific heat of gas at constant pressure will be :

(A) $\frac{n}{n+2}$ (B) $\frac{n+2}{n}$

(C) $\frac{n}{2n+2}$ (D) $\frac{n}{n-2}$

Official Ans. by NTA (A)

Sol. $C_v = \frac{nR}{2}$ $C_p = \frac{(n+2)R}{2}$

$$\frac{C_v}{C_p} = \frac{n}{n+2}$$

13. A transverse wave is represented by y = 2sin $(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be ;

(A) 4π (B) 2π

(C) π (D) 2

Official Ans. by NTA (A)

Sol. $y = 2 \sin(\omega t - kx)$

Maximum particle velocity = A ω

Wave velocity = $\frac{\omega}{k}$

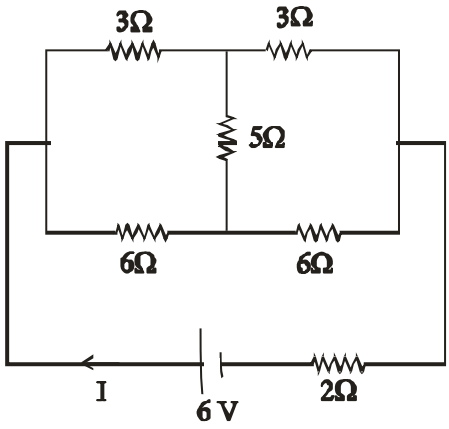
$\frac{\omega}{k} = A\,\omega$

$k = \frac{1}{A} = \frac{2\pi}{\lambda}$

$\lambda = 2\pi A$

$= 4\,\pi$ cm

14. A battery of 6 V is connected to the circuit as shown below. The current I drawn from the battery is :

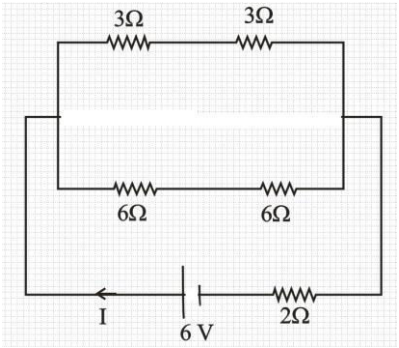


(A) 1A (B) 2A

(C) $\frac{6}{11}$A (D) $\frac{4}{3}$A

Official Ans. by NTA (A)

Sol. Balanced wheat stone bridge in circuit so there is no current in 5 Ω resistor so it can be removed from the circuit.



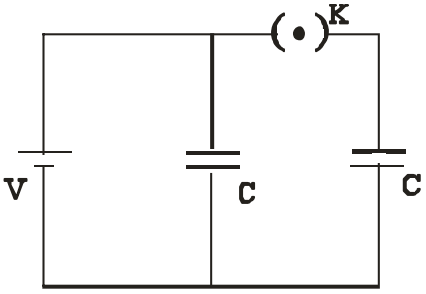
$$R_{eq} = \frac{6\times12}{6+12} + 2$$

$$= \frac{6\times12}{18} + 2$$

$$R_{eq} = 6\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6}{6} = 1\ \text{Amp.}$$

15. A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be :

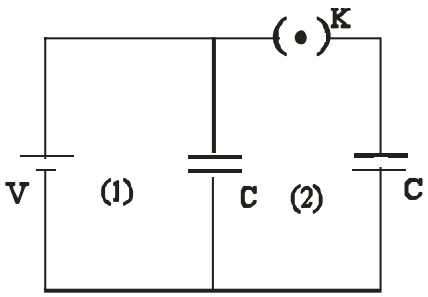


(A) $\frac{1}{10}$ (B) $\frac{2}{5}$

(C) $\frac{5}{13}$ (D) $\frac{5}{26}$

Official Ans. by NTA (C)

Sol.



(1) Switch is closed

$C_{eq} = 2C$

Energy $E_1 = \frac{1}{2}C_{eq}V^2$

$= \frac{1}{2}2C\times V^2$

$E_1 = CV^2$

(ii) When switch is opened charge on right capacitor remain CV while potential on left capacitor remain same

Dielectric K = 5

$C' = KC$

$C' = 5C$

$$E_2 = \frac{1}{2}(5C)V^2 + \frac{(CV)^2}{2(5C)}$$

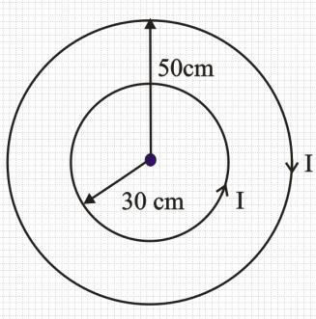
$$E_2 = \frac{5CV^2}{2} + \frac{CV^2}{10}$$

$$E_2 = \frac{13CV^2}{5}$$

$$\frac{E_1}{E_2} = \frac{CV^2}{\frac{13CV^2}{5}} = \frac{5}{13}$$

$$\frac{E_1}{E_2} = \frac{5}{13}$$

16. Two concentric circular loops of radii r_1=30 cm and r_2=50 cm are placed in X-Y plane as shown in the figure. A current I = 7A is flowing through them in the direction as shown in figure. The net magnetic moment of this system of two circular loops is approximately :

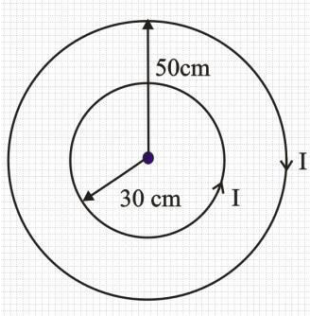


(A) $\frac{7}{2}\hat{k}$ Am^2 (B) $-\frac{7}{2}\hat{k}$ Am^2

(C) $7\hat{k}$ Am^2 (D) $-7\hat{k}$ Am^2

Official Ans. by NTA (B)

Sol.



Magnetic moment

$$\vec{M} = -i\pi(0.5)^2\hat{k} + i\pi(0.3)^2\hat{k}$$

$$\vec{M} = -7\times\frac{22}{7}\left(\frac{25}{100} - \frac{9}{100}\right)\hat{k}$$

$$= -22\left(\frac{16}{100}\right)\hat{k}$$

$$\vec{M} = -3.52\hat{k}\ \text{Am}^2$$

$$= -\frac{7}{2}\hat{k}\ \text{Am}^2$$

17. A velocity selector consists of electric field $\vec{E} = E\hat{k}$ and magnetic field $\vec{B} = B\hat{j}$ with B=12 mT. The value E required for an electron of energy 728 eV moving along the positive x-axis to pass undeflected is :
(Given, mass of electron = 9.1×10^{-31}kg)

(A) 192 kVm^{-1} (B) 192 m Vm^{-1}

(C) 9600 kVm^{-1} (D) 16 kVm^{-1}

Official Ans. by NTA (A)

Sol. $\vec{E} = E\hat{k}$ B = 12 mT

$\vec{B} = B\hat{j}$ Energy = 728 eV

$$\text{Energy} = \frac{1}{2}mv^2$$

$$728\text{eV} = \frac{1}{2}\times9.1\times10^{-31}\times v^2$$

$$728\times1.6\times10^{-19} = \frac{1}{2}\times9.1\times10^{-31}\times v^2$$

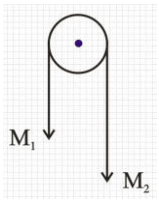
$v = 16 \times 10^6$ m/s

$E = vB$

$E = 16\times10^6 \times 12 \times 10^{-3}$

$E = 192 \times 10^3$ V/m

18. Two masses M_1 and .M_2 are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass M_2 is twice that of M_1. the acceleration of the system is a_1. When the mass M_2 is thrice that of M_1. The acceleration of The system is a_2. The ratio $\frac{a_1}{a_2}$ will be:



(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{1}{2}$

Official Ans. by NTA (B)

Sol. $$a = \frac{m_2g - m_1g}{m_1 + m_2}$$

Case 1 $M_2 = 2m_1$

$$a_1 = \frac{2m_1g - m_1g}{3m_1}$$

$$a_1 = g/3$$

Case -2

$M_2 = 3m_1$

$a_2 = \frac{3m_1g - m_1g}{4m_1}$

$a_2 = \frac{g}{2}$

$\frac{a_1}{a_2} = \frac{\frac{g}{3}}{\frac{g}{2}} = \frac{2}{3}$

19. Mass numbers of two nuclei are in the ratio of 4:3. Their nuclear densities will be in the ratio of

(A) 4:3 (B) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$

(C) 1 : 1 (D) $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

Official Ans. by NTA (C)

Sol. Radius of nucleus $R = R_0A^{\frac{1}{3}}$

Density of nucleus = $\frac{\text{Mass of nucleus}}{\text{volume of nucleus}}$

$\rho = \frac{m \times A}{\frac{4}{3}\pi R^3}$ Where m : mass of proton or neutron

$\rho = \frac{m \times A}{\frac{4}{3}\pi R_0{}^3 A}$

$\rho \propto A^0$

Hence density of nucleus is independent of mass number

20. The area of cross section of the rope used to lift a load by a crane is $2.5 \times 10^{-4}m^2$. The maximum lifting capacity of the crane is 10 metric tons. To increase the lifting capacity of the crane to 25 metric tons, the required area of cross section of the rope should be :
(take g =10 ms^{-2})
(A) $6.25 \times 10^{-4}m^2$
(B) $10 \times 10^{-4}m^2$
(C) $1 \times 10^{-4}m^2$
(D) $1.67 \times 10^{-4}m^2$

Official Ans. by NTA (A)

Sol. Since breaking stress (Maximum lifting capacity) is the property of material so it will remain same.

breaking stress = $\frac{\text{Maximum lifting capacity}}{\text{Area of cross section of rope}}$

$\frac{10}{2.5\times10^{-4}} = \frac{25}{A}$

$A = 625 \times 10^{-6}$

$= 6.25 \times 10^{-4}\ m^2$

SECTION-B

1. If $\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ m and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ m. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be_____ m.

Official Ans. by NTA (2)

Sol. $\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$m and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$m

Component of $\vec{A}$ along $\vec{B} = \vec{A}\cdot\hat{B}$

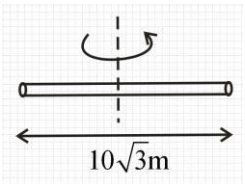
$= \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{2+6-2}{\sqrt{1^2+2^2+2^2}}$

$= \frac{6}{3} = 2$

2. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be_____ m. Given, the length of the rod is $10\sqrt{3}$ m.

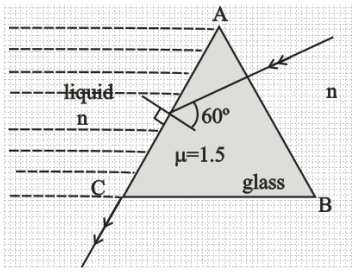
Official Ans. by NTA (5)

Sol.



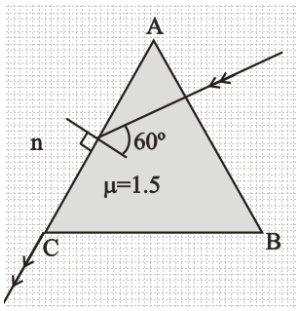
$I = \frac{m\ell^2}{12} = mk^2 \Rightarrow k^2 = \frac{\ell^2}{12} \Rightarrow k = \frac{\ell}{\sqrt{12}} = \frac{\ell}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5$

3. In the given figure, the face AC of the equilateral prism is immersed in a liquid of refractive index 'n'. For incident angle 60° at the side AC, the refracted light beam just grazes along face AC. The refractive index of the liquid $n=\frac{\sqrt{x}}{4}$. The value of x is_____ .
(Given refractive index of glass = 1.5)



Official Ans. by NTA (27)

Sol.



Using snell's law at face AC

$1.5 \sin 60^\circ = n \times \sin 90^\circ$

$1.5\times \frac{\sqrt{3}}{2} = n = \frac{\sqrt{x}}{4}$

$3\sqrt{3} = \sqrt{x}$

$x = 27$

4. Two lighter nuclei combine to form a comparatively heavier nucleus by the relation given below:
${}^{2}_{1}X + {}^{2}_{1}X = {}^{4}_{2}Y$
The binding energies per nucleon ${}^{2}_{1}X$ and ${}^{4}_{2}Y$ are 1.1 MeV and 7.6 MeV respectively. The energy released in this process is_____ . MeV.

Official Ans. by NTA (26)

Sol. Energy released in the given process = Binding energy of product – Binding energy of reactants

$= 7.6 \times 4 - (1.1 \times 2) \times 2$

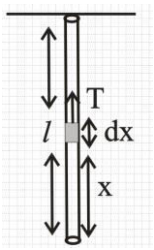
$= 30.4 - 4.4$

$=26$ MeV

5. A uniform heavy rod of mass 20 kg. Cross sectional area 0.4 m^2 and length 20 m is hanging from a fixed support. Neglecting the lateral contraction, the elongation in the rod due to its own weight is $x \times 10^{-9}$ m. The value of x is_____.
:(Given. Young's modulus $Y=2 \times 10^{11}$ Nm^{-2} and $g=10$ ms^{-2})

Official Ans. by NTA (25)

Sol.



$Y = \frac{T}{A}\frac{dx}{dy}$

$m = 20$ kg

$\mathbf{A = 0.}4\ m^2$

$l = 20m$

let extension is dy in length dx

$Y = \frac{\text{stress}}{\text{strain}}$

$Y = \frac{\frac{T}{A}}{\frac{dy}{dx}} = \frac{T}{A}.\frac{dx}{dy}$

$dy = \frac{Tdx}{AY}$

Tension at a distance x from lower end = $\frac{mg}{\ell}x$

So. $\int_0^{\Delta l} dy = \int_0^{\ell} \frac{mg}{\ell} x \frac{dx}{AY}$

$\Delta \ell = \frac{mg}{\ell AY}\left[\frac{x^2}{2}\right]_0^{\ell}$

$\Delta \ell = \frac{mg\ell}{2AY}$

$\Delta \ell = \frac{20\times 10\times 20}{2\times 0.4\times 2\times 10^{11}}$

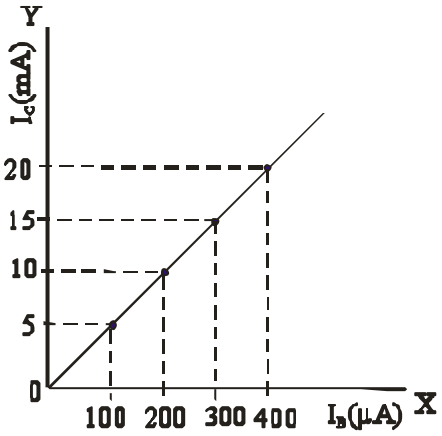
2500×10^{-11}

$\Delta \ell = 25\times 10^{-9}$

$= x \times 10^{-9}$

$x = 25$

6. The typical transfer characteristic of a transistor in CE configuration is shown in figure. A load resistor of 2 kΩ is connected in the collector branch of the circuit used. The input resistance of the transistor is 0.50 kΩ. The voltage gain of the transistor is



Official Ans. by NTA (200)

Sol. Current gain in C-E configuration

$$\Rightarrow \beta = \frac{\Delta I_C}{\Delta I_B}$$

$R_C = 2k\Omega$, R_B=0.50 kΩ

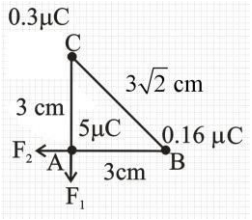
$$\text{Voltage gain} = \frac{\Delta I_C R_C}{\Delta I_B R_B} = \frac{5\times10^{-3}}{100\times10^{-6}} \times \frac{2}{0.5}$$

$$= \frac{10^{-2}}{5\times10^{-5}} = \frac{1000}{5} = 200$$

7. Three point charges of magnitude 5μC, 0.16μC and 0.3μC are located at the vertices A, B, C of a right angled triangle whose sides are AB = 3cm, BC = $3\sqrt{2}$ cm and CA=3 cm and point A is the right angle corner. Charge at point A experiences _____ N of electrostatic force due to the other two charges.

Official Ans. by NTA (17)

Sol.



$$F_1 = \frac{k\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$

$$= \frac{9\times10^9\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$

$$= 1.5\times10 = 15 \text{ N}$$

$$F_2 = \frac{9\times10^9\times5\times0.16\times10^{-12}}{9\times10^{-4}} = 8\text{N}$$

force experienced by charge at $A = \sqrt{F_1^2 + F_2^2}$

$$= \sqrt{15^2 + 8^2}$$

$$= \sqrt{289} = 17 \text{ N}$$

8. In a coil of resistance 8Ω, the magnetic flux due to an external magnetic field varies with time as $\phi = \frac{2}{3}\left(9 - t^2\right)$. The value of total heat produced in the coil, till the flux becomes zero, will be_____J.

Official Ans. by NTA (2)

Sol. $\phi = \frac{2}{3}(9 - t^2) = 0$

$t = 3$ sec

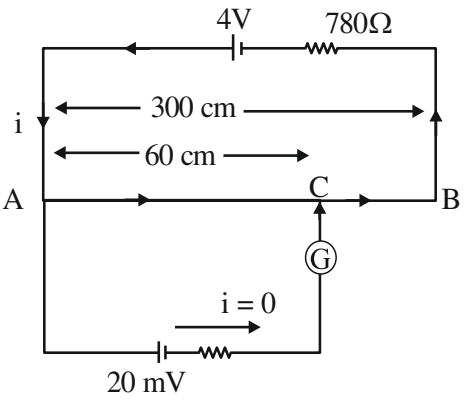
$$e = \frac{-d\phi}{dt} = -\frac{2}{3}(0 - 2t) = \frac{4t}{3}$$

$$\text{Heat produced in 3 sec} = \int \frac{e^2}{r} dt = \int_0^3 \frac{16t^2}{9\times8} dt = 2\text{J}$$

9. A potentiometer wire of length 300 cm is connected in series with a resistance 780 Ω and a standard cell of emf 4V. A constant current flows through potentiometer wire. The length of the null point for cell of emf 20 mV is found to be 60 cm. The resistance of the potentiometer wire is____Ω.

Official Ans. by NTA (20)

Sol.



Let resistance of potentiometers wire is R

$$i = \frac{4}{R+780}$$

Potential difference across AB

$$= \frac{4R}{R+780}$$

Potential difference across AC

$$= \frac{4R \times 60}{(R+780)\times 300} = \frac{4R}{5(R+780)}$$

This should be equal to 20 mV

$$\frac{4R}{5(R+780)} = 20\times10^{-3} = 2\times10^{-2}$$

$4R = 10^{-1}(R + 780)$

$$4R = \frac{R}{10} + 78$$

$$4R - \frac{R}{10} = 78$$

$$\frac{39R}{10} = 78$$

$$\boxed{R = 20\Omega}$$

10. As per given figures, two springs of spring constants K and 2K are connected to mass m. If the period of oscillation in figure (a) is 3s, then the period of oscillation in figure (b) will be $\sqrt{x}$ s. The value of x is_____.

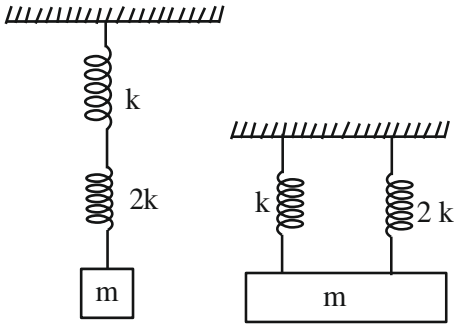


figure (a) figure (b)

Official Ans. by NTA (2)

Sol.

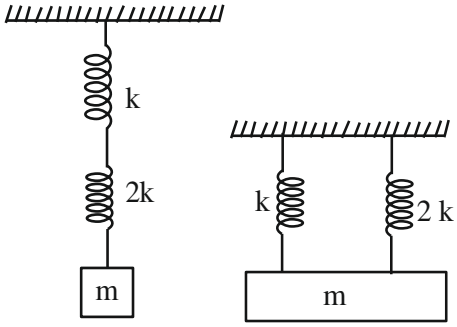


figure (a) figure (b)

For figure (a) :

$$K_{eq} = \frac{K\times 2K}{K+2K} = \frac{2K}{3}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{2K/3}} = 2\pi\sqrt{\frac{3m}{2K}}$$

For figure (b):

$$K_{eq} = 3K,\ T' = 2\pi\sqrt{\frac{m}{3K}}$$

$$\frac{T'}{T} = \sqrt{\frac{m\times 2K}{3K\times 3m}} = \frac{\sqrt{2}}{3}$$

$T' = \sqrt{2}$

$x = 2$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Hemoglobin contains 0.34% of iron by mass. The number of Fe atoms in 3.3 g of hemoglobin is : (Given : Atomic mass of Fe is 56 u, N_A in 6.022×10^{23} mol^{-1})

(A) 1.21×10^5 (B) 12.0×10^{16}

(C) 1.21×10^{20} (D) 3.4×10^{22}

Official Ans. by NTA (C)

Sol. No. of Fe atoms $= \frac{0.34}{100} \times \frac{3.3}{56} \times 6.022 \times 10^{23}$

$= 1.206 \times 10^{20}$

2. Arrange the following in increasing order of their covalent character.

(A) CaF_2 (B) $CaCl_2$

(C) $CaBr_2$ (D) CaI_2

Choose the correct answer from the options given below.

(A) B < A < C < D (B) A < B < C < D

(C) A < B < D < C (D) A < C < B < D

Official Ans. by NTA (B)

Sol. According to Fajan's rule,

Covalent character $\propto$ size of Anion

3. Class XII students were asked to prepare one litre of buffer solution of pH 8.26 by their chemistry teacher. The amount of ammonium chloride to be dissolved by the student in 0.2 M ammonia solution to make one litre of the buffer is (Given pK_b (NH_3) = 4.74; Molar mass of NH_3 = 17 g mol^{-1}; Molar mass of NH_4Cl = 53.5 g mol^{-1})

(A) 53.5 g (B) 72.3 g

(C) 107.0 g (D) 126.0 g

Official Ans. by NTA (C)

Sol. POH = 14 – 8.26

$$= pK_b + \log\frac{[NH_4^+]}{[NH_3]}$$

$$= 5.74 = 4.74 + \log\frac{[NH_4^+]}{0.2} \Rightarrow [NH_4^+] = 2$$

Hence

$NH_4Cl = 2 \times 53.5 = 107$ g

4. At 30°C, the half life for the decomposition of AB_2 is 200 s and is independent of the initial concentration of AB_2. The time required for 80% of the AB_2 to decompose is (Given: log 2 = 0.30; log 3 = 0.48)

(A) 200 s (B) 323 s

(C) 467 s (D) 532 s

Official Ans. by NTA (C)

Sol. $T_{1/2}$ = 200 s and 1st order reaction

$$K = \frac{2.303\log 2}{200} = \frac{2.303}{t}\log\frac{A_0}{0.2A_0}$$

$$\frac{\log 2}{200} = \frac{1}{t}\log 5$$

$$t = \frac{7}{3} \times 200 = 466.67s = 467 \text{ s}$$

5. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Finest gold is red in colour, as the size of the particles increases, it appears purple then blue and finally gold.

Assertion R : The colour of the colloidal solution depends on the wavelength of light scattered by the dispersed particles.

In the light of the above statements, choose the most appropriate answer from the options given below;

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

6. The metal that has very low melting point and its periodic position is closer to a metalloid is :

(A) Al (B) Ga

(C) Se (D) In

Official Ans. by NTA (B)

Sol.

	Melting point
Al →	933 K
Ga →	303 K
In →	430 K
Se →	490 K

7. The metal that is not extracted from its sulphide ore is :

(A) Aluminium (B) Iron

(C) Lead (D) Zinc

Official Ans. by NTA (A)

Sol. Al is extracted from $Al_2O_3 \cdot 2H_2O$ i.e., Bauxite ore

8. The products obtained from a reaction of hydrogen peroxide and acidified potassium permanganate are

(A) Mn^{4+}, H_2O only (B) Mn^{2+}, H_2O only

(C) Mn^{4+}, H_2O, O_2 only (D) Mn^{2+}, H_2O, O_2 only

Official Ans. by NTA (D)

Sol. $6H^+ + 2MnO_4^- + 5H_2O_2 \longrightarrow 2Mn^{+2} + 8H_2O + 5O_2$

9. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : LiF is sparingly soluble in water.

Reason R : The ionic radius of Li^+ ion is smallest among its group members, hence has least hydration enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below .

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (C)

Sol. Due to high lattice energy LiF is sparingly soluble in water. Li^+ has high hydration energy among its group members due to smallest size.

10. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Boric acid is a weak acid

Reason R : Boric acid is not able to release H^+ ion on its own. It receives OH^- ion from water and releases H^+ ion.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A

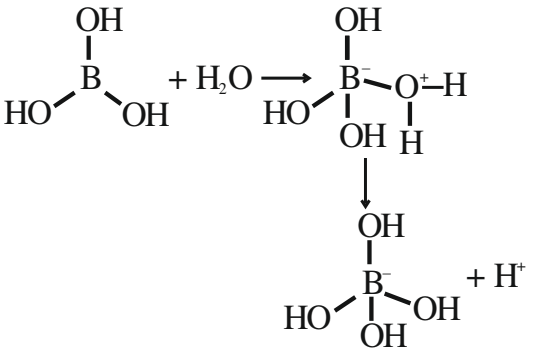
(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol.



11. The metal complex that is diamagnetic is (Atomic number : Fe, 26; Cu, 29)

(A) $K_3[Cu(CN)_4]$

(B) $K_2[Cu(CN)_4]$

(C) $K_3[Fe(CN)_4]$

(D) $K_4[FeCl_6]$

Official Ans. by NTA (A)

Sol. $K_3[Cu(CN)_4]$

O.N. of copper is Cu^{+1}

$Cu^{+1} = [Ar]3d^{10} \Rightarrow$ Diamagnetic

12. Match List I with List II

List I Pollutant	List II Source
A. Microorganisms	I. Strip mining
B. Plant nutrients	II. Domestic sewage
C. Toxic heavy metals	III. Chemical fertilizer
D. Sediment	IV. Chemical factory

Choose the correct answer from the options given below :

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-I, C-IV, D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-IV, C-III, D-II

Official Ans. by NTA (A)

Sol.

List I Pollutant	List II Source
A. Microorganisms	Domestic sewage
B. Plant nutrients	Chemical fertilizer
C. Toxic heavy metals	Chemical factory
D. Sediment	Strip mining

13. The correct decreasing order of priority of functional groups in naming an organic compound as per IUPAC system of nomenclature is :

(A) —COOH > —$CONH_2$ > —COCl > —CHO

(B) —SO_3H > —COCl > —$CONH_2$ > —CN

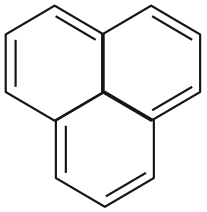
(C) —COOR > —COCl > —NH_2 > $\rangle C = o$

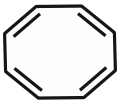
(D) —COOH > —COOR > —$CONH_2$ > —COCl

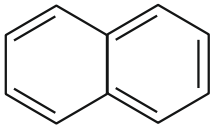
Official Ans. by NTA (B)

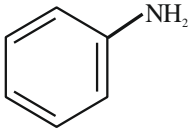
Sol. —SO_3H > —COCl > —$CONH_2$ > —CN

14. Which of the following is not an example of benzenoid compound ?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

15. Hydrolysis of which compound will give carbolic acid ?

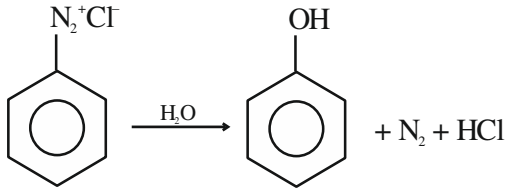
(A) Cumene

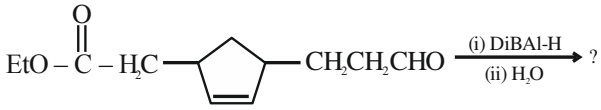
(B) Benzenediazonium chloride

(C) Benzal chloride

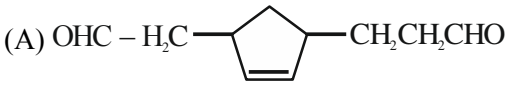
(D) Ethylene glycol ketal

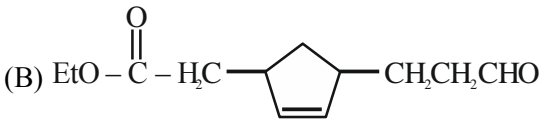
Official Ans. by NTA (B)

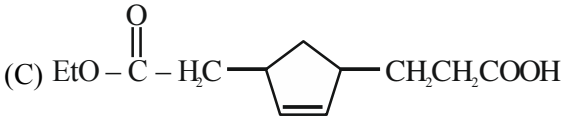
Sol. 

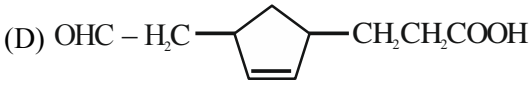
16. 

Consider the above reaction and predict the major product.

(A) 

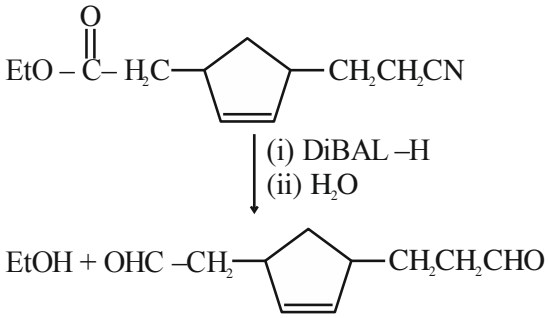
(B) 

(C) 

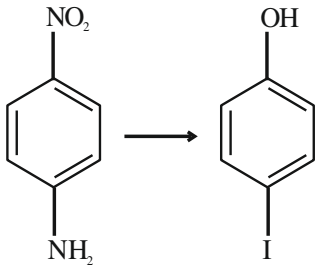
(D) 

Official Ans. by NTA (A)

Sol.



17. The correct sequential order of the reagents for the given reaction is :



(A) HNO_2, Fe/H^+, HNO_2, KI, H_2O/H^+

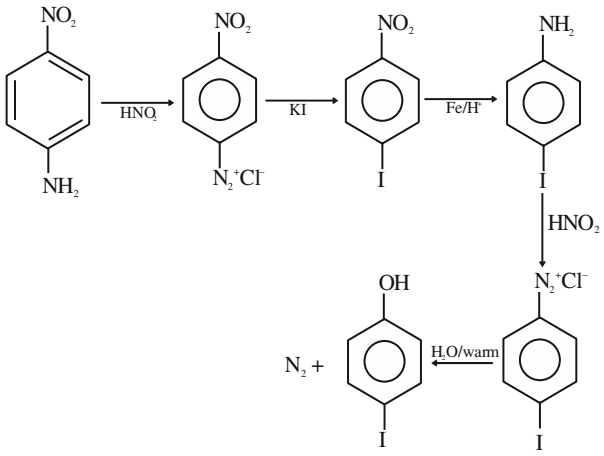
(B) HNO_2, KI, Fe/H^+, HNO_2, H_2O/warm

(C) HNO_2, KI, HNO_2, Fe/H^+, H_2O/H^+

(D) HNO_2, Fe/H^+, KI, HNO_2, H_2O/warm

Official Ans. by NTA (B)

Sol.



18. Vulcanization of rubber is carried out by heating a mixture of :

(A) isoprene and styrene

(B) neoprene and sulphur

(C) isoprene and sulphur

(D) neoprene and styrene

Official Ans. by NTA (C)

Sol. Vulcanization of rubber is carried out by heating a mixture of isoprene & sulphur

19. Animal starch is the other name of :

(A) amylose (B) maltose

(C) glycogen (D) amylopectin

Official Ans. by NTA (C)

Sol. Glycogen

20. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Phenolphthalein is a pH dependent indicator, remains colourless in acidic solution and gives pink colour in basic medium

Reason R : Phenolphthalein is a weak acid. It doesn't dissociate in basic medium.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (C)

Sol. Phenolphthalein dissociate in basic medium

$HPh(aq) \rightleftharpoons H^+ + Ph^-$

(colourless) (Pink)

SECTION-B

1. A 10 g mixture of hydrogen and helium is contained in a vessel of capacity 0.0125 m^3 at 6 bar and 27°C. The mass of helium in the mixture is _____ g. (nearest integer)

Given : R = 8.3 $JK^{-1}mol^{-1}$ (Atomic masses of H and He are 1u and 4u, respectively)

Official Ans. by NTA (8)

Sol. $PV = n_{mix}RT$

$$n_{mix} = \frac{6 \times 12.5}{0.083 \times 300} \approx 3$$

Let mole of He = x

Mole of $H_2 = 3 - x$

$4x + 2(3 - x) = 10$

$\boxed{x = 2mol}$

Mass of He = 8g

2. Consider an imaginary ion $^{48}_{22}X^{3-}$. The nucleus contains 'a'% more neutrons than the number of electrons in the ion. The value of 'a' is ____. [nearest integer]

Official Ans. by NTA (4)

Sol. $^{48}_{22}X^{3-}$

No. of neutrons = 26

No. of electrons = 25

% of extra neutrons

than electrons = $\frac{26-25}{25}\times 100 = 4$

3. For the reaction

$H_2F_2(g) \rightarrow H_2(g) + F_2(g)$

$\Delta U = -59.6$ kJ mol^{-1} at 27°C.

The enthalpy change for the above reaction is (–) ___ kJ mol^{-1} [nearest integer] Given : R = 8.314 JK^{-1} mol^{-1}.

Official Ans. by NTA (57)

Sol. $\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = -59.6 + 1 \times 8.314 \times 300 \times 10^{-3} = -57.10$

4. The elevation in boiling point for 1 molal solution of non-volatile solute A is 3K. The depression in freezing point for 2 molal solution of A in the same solvent is 6 K. The ratio of K_b and K_f i.e., K_b/K_f is 1 : X. The value of X is [nearest integer]

Official Ans. by NTA (1)

Sol. $\Delta T_b = iK_bm_1$ $\Delta T_f = iK_fm_2$

$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b \times 1}{K_f \times 2} \Rightarrow \frac{3}{6} = \frac{1}{2} = \frac{K_b}{K_f} \times \frac{1}{2}$

$\frac{K_b}{K_f} = \frac{1}{1} \Rightarrow x = 1$

5. 20 mL of 0.02 M hypo solution is used for the titration of 10 mL of copper sulphate solution, in the presence of excess of KI using starch as an indicator. The molarity of Cu^{2+} is found to be _____ $\times 10^{-2}$ M [nearest integer]

Given : $2Cu^{2+} + 4I^- \rightarrow Cu_2I_2 + I_2$

$I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$

Official Ans. by NTA (4)

Sol. $n_{eq.}$ of I_2 = n_{eq} of $Na_2S_2O_3 = 20 \times 0.002 \times 1$

$2 \times n_{mol}$ of $I_2 = 0.4$

n_{mol} of $I_2 = 0.2$ m mol

n_{mol} of $Cu^{+2} = 0.2 \times 2 \times 10^{-3}$

$[Cu^{+2}] = \frac{0.4\times10^{-3}}{10\times10^{-3}} = 0.04 = 4\times10^{-2}$

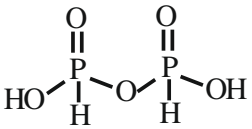
6. The number of non-ionisable protons present in the product B obtained from the following reaction is _____. $C_2H_5OH + PCl_3 \rightarrow C_2H_5Cl + A$

$A + PCl_3 \rightarrow B$

Official Ans. by NTA (2)

Sol. $C_2H_5OH + PCl_3 \longrightarrow C_2H_5Cl + H_3PO_3$

$H_3PO_3 + PCl_3 \longrightarrow H_4P_2O_5 + HCl$



7. The spin-only magnetic moment value of the compound with strongest oxidizing ability among MnF_4, MnF_3 and MnF_2 is _____ B.M. [nearest integer]

Official Ans. by NTA (5)

Sol.

MnF_4	MnF_3	MnF_2
+4	+3	+2
E.C = [Ar]3d^3	[Ar]3d^4	[Ar]3d^5

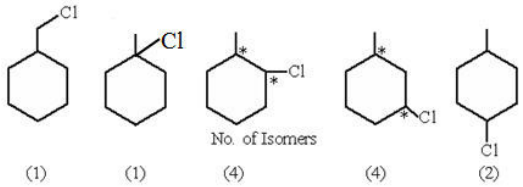
Hence $MnF_3 \Rightarrow$ strongest O.A

$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89 = 5$

8. Total number of isomers (including stereoisomers) obtain on monochlorination of methylcyclohexane is _____.

Official Ans. by NTA (12)

Sol.



9. A 100 mL solution of CH_3CH_2MgBr on treatment with methanol produces 2.24 mL of a gas at STP. The weight of gas produced is _____ mg. [nearest integer]

Official Ans. by NTA (3)

Sol. $CH_3–CH_2–MgBr + CH_3OH \longrightarrow$

$$CH_3–CH_3 + Mg\begin{matrix}OCH_3\\ \diagdown\\ \diagup\\ Br\end{matrix}$$

$$n = \frac{2.24\times10^{-3}}{22.4} = 10^{-4}$$

$W = n \times M$

$= 10^{-4} \times 30 = 3$ mg

10. How many of the following drugs is/are example(s) of broad spectrum antibiotic ? Ofloxacin, Penicillin G, Terpineol, Salvarsan

Official Ans. by NTA (1)

Sol. Ofloxacin

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The minimum value of the sum of the squares of the roots of $x^2+(3-a)x+1=2a$ is:

(A) 4 (B) 5

(C) 6 (D) 8

Official Ans. by NTA (C)

Sol. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

let $f(a) = (3 - a)^2 - 2(1 - 2a)$

$f(a) = a^2 - 2a + 7$

$f(a) = (a - 1)^2 + 6$

$f(a)\big)_{min.} = 6$

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z-i|-|z+5i|=0$, then

(A) $x + 2y - 4 = 0$ (B) $x^2 + y - 4 = 0$

(C) $x + 2y + 4 = 0$ (D) $x^2 - y + 3 = 0$

Official Ans. by NTA (C)

Sol. $|z-i|-|z+5i|=0$

$\Rightarrow |x + (y - 1)i| = |x + (y + 5)i|$

$x^2 + (y - 1)^2 = x^2 + (y + 5)^2$

$(y - 1)^2 - (y + 5)^2 = 0$

$(2y + 4)(-6) = 0$

$y = -2$

$\therefore x^2 + (-2)^2 = 4$

$x = 0$

$Z \equiv (0, -2)$, check options

3. Let $A = \begin{bmatrix}1\\1\\1\end{bmatrix}$ and $B = \begin{bmatrix}9^2 & -10^2 & 11^2\\12^2 & 13^2 & -14^2\\-15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of $A'BA$ is:

(A) 1224 (B) 1042 (C) 540 (D) 539

Official Ans. by NTA (D)

Sol. $A'BA = [1\ \ 1\ \ 1]\begin{bmatrix}9^2 & -10^2 & 11^2\\12^2 & 13^2 & -14^2\\-15^2 & 16^2 & 17^2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$

$=[9^2+12^2-15^2 \quad -10^2+13^2+16^2 \quad 11^2-14^2+17^2]\begin{bmatrix}1\\1\\1\end{bmatrix}$

$= [9^2+12^2-15^2 -10^2+13^2+16^2 + 11^2-14^2+17^2]$

$= [539]$

4. $\sum\limits_{\substack{i,j=0\\i\neq j}}^{n} {}^nC_i\ {}^nC_j$ is equal to

(A) $2^{2n} - {}^{2n}C_n$ (B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}{}^{2n}C_n$ (D) $2^{n-1} + {}^{2n-1}C_n$

Official Ans. by NTA (B)

Sol. $\sum\limits_{\substack{i,j=0\\i\neq j}}^{n} {}^nC_i\ {}^nC_j$

$= \sum\limits_{i=0}^{n} {}^nC_i \cdot \sum\limits_{j=0}^{n} {}^nC_j - \sum\limits_{i=j=0}^{n} \left({}^nC_i\right)^2$

$= (2^n)(2^n) - {}^{2n}C_n$

$= 2^{2n} - {}^{2n}C_n$

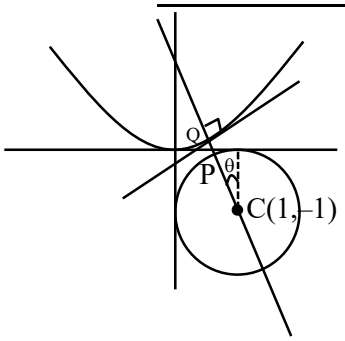
5. Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{4}\right)$

(C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{4}, 1\right)$

Official Ans. by NTA (C)

Sol.



$Q=\left(t,t^2\right)$

$m_{CQ}=m_{normal}$

$\frac{t^2+1}{t-1}=-\frac{1}{2t}$

Let $f\left(t\right)=2t^3+3t-1$

$f\left(\frac{1}{4}\right)f\left(\frac{1}{3}\right)<0\Rightarrow t\in\left(\frac{1}{4},\frac{1}{3}\right)$

$P\equiv\left(1+\cos\left(90+\theta\right),-1+\sin\left(90+\theta\right)\right)$

$P=\left(1-\sin\theta,-1+\cos\theta\right)$

$m_{normal}=m_{CP}\Rightarrow-\frac{1}{2t}=\frac{\cos\theta}{-\sin\theta}\Rightarrow\tan\theta=2t$

$x=1-\sin\theta=1-\frac{2t}{\sqrt{1+4t^2}}=g\left(t\right)\ \ \left(let\right)$

$\Rightarrow g'\left(t\right)<0$

$g\left(t\right)\downarrow function$

$t\in\left(\frac{1}{4},\frac{1}{3}\right)$

$\Rightarrow g\left(t\right)\in\left(0.44,0.485\right)\in\left(\frac{1}{4},\frac{1}{2}\right)$

6. If the maximum value of a, for which the function $f_a(x)=\tan^{-1}2x-3ax+7$ is non-decreasing in $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

(A) $8-\frac{9\pi}{4\left(9+\pi^2\right)}$ (B) $8-\frac{4\pi}{9\left(4+\pi^2\right)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ (D) $8-\frac{\pi}{4}$

Official Ans. by NTA (A)

Sol. $f_a(x)=\tan^{-1}2x-3ax+7$

$f_a'(x)=\frac{2}{1+4x^2}-3a\geq0$

$a\leq\left(\frac{2}{3(1+4x^2)}\right)_{min.}$ at $x=\pm\frac{\pi}{6}$

$a_{max}=\bar{a}=\frac{6}{9+\pi^2}$

$f_{\bar{a}}(\frac{\pi}{8})=\tan^{-1}\frac{\pi}{4}-3\frac{6}{9+\pi^2}\frac{\pi}{8}+7=\tan^{-1}\frac{\pi}{4}-\frac{9\pi}{4\left(\pi^2+9\right)}+7$

7. Let $\beta=\lim_{x\to0}\frac{\alpha x-\left(e^{3x}-1\right)}{\alpha x\left(e^{3x}-1\right)}$ for some $\alpha\in\mathbb{R}$. Then the value of $\alpha+\beta$ is :

(A) $\frac{14}{5}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (C)

Sol. $\beta=\lim_{x\to0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$

$\beta=\lim_{x\to0}\frac{1+\alpha x-\left[1+3x+\frac{9x^2}{2!}+.....\right]}{(\alpha x)\frac{(e^{3x}-1)}{3x}3x}$

$\beta=\lim_{x\to0}\frac{(\alpha x-3x)-\frac{9x^2}{2!}-.....}{3\alpha x^2}$

For existence of limit $\alpha-3=0$

$\alpha=3$

Limit $\beta=\frac{-3}{2\alpha}$

$\beta=-\frac{1}{2}$

Now,

$\alpha+\beta=\frac{5}{2}$

8. The value of $\log_e 2\frac{d}{dx}(\log_{\cos x}\operatorname{cosec}x)$ at $x=\frac{\pi}{4}$ is

(A) $-2\sqrt{2}$ (B) $2\sqrt{2}$ (C) -4 (D) 4

Official Ans. by NTA (D)

Sol. $\log_e 2\dfrac{d}{dx}(\log_{\cos x}\operatorname{cosec}x)$

Let,

$y=\log_{\cos x}\operatorname{cosec}x$

$y=-\dfrac{\ln(\sin x)}{\ln(\cos x)}$

$\dfrac{dy}{dx}=-\dfrac{[\cot x\cdot\ln(\cos x)+\tan x\cdot\ln(\sin x)]}{(\ln(\cos x))^2}$

$\left.\dfrac{dy}{dx}\right)_{x=\frac{\pi}{4}}=\dfrac{4}{\ln 2}$

Now,

$\Rightarrow \log_e 2\cdot\dfrac{4}{\ln 2}=4$

9. $\int_0^{20\pi}(|\sin x|+|\cos x|)^2\,dx$ is equal to :-

(A) $10(\pi+4)$ (B) $10(\pi+2)$

(C) $20(\pi-2)$ (D) $20(\pi+2)$

Official Ans. by NTA (D)

Sol. $I=\int_0^{20\pi}(|\sin x|+|\cos x|)^2\,dx$; (Jack property)

$I=40\int_0^{\pi/2}(\sin x+\cos x)^2\,dx$

$I=40\int_0^{\pi/2}(1+\sin 2x)\,dx$

$I=20[\pi+2]$

10. Let the solution curve y = f(x) of the differential equation $\dfrac{dy}{dx}+\dfrac{xy}{x^2-1}=\dfrac{x^4+2x}{\sqrt{1-x^2}}, x\in(-1,1)$ pass through the origin. Then $\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx$ is equal to

(A) $\dfrac{\pi}{3}-\dfrac{1}{4}$ (B) $\dfrac{\pi}{3}-\dfrac{\sqrt3}{4}$

(C) $\dfrac{\pi}{6}-\dfrac{\sqrt3}{4}$ (D) $\dfrac{\pi}{6}-\dfrac{\sqrt3}{2}$

Official Ans. by NTA (B)

Sol. $\dfrac{dy}{dx}+\dfrac{xy}{x^2-1}=\dfrac{x^4+2x}{\sqrt{1-x^2}}$

I.F = $e^{\int\frac{x}{x^2-1}dx}$

I.F = $\sqrt{1-x^2}$

Solution of D.E.

$y\cdot\sqrt{1-x^2}=\int\dfrac{x^4+2x}{\sqrt{1-x^2}}\cdot\sqrt{1-x^2}\,dx$

$y\cdot\sqrt{1-x^2}=\int(x^4+2x)\,dx$

$y\cdot\sqrt{1-x^2}=\dfrac{x^5}{5}+x^2+C$

At x = 0, y = 0, get C = 0

$y=\dfrac{x^5}{5\sqrt{1-x^2}}+\dfrac{x^2}{\sqrt{1-x^2}}$

Now,

$\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx=\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\dfrac{x^5}{5\sqrt{1-x^2}}dx+\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\dfrac{x^2}{\sqrt{1-x^2}}dx$

$\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx=0+2\int_0^{\frac{\sqrt3}{2}}\dfrac{x^2}{\sqrt{1-x^2}}dx$

$\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx=\dfrac{\pi}{3}-\dfrac{\sqrt3}{4}$

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2+3y^2=5$ from the point (1,3) is

(A) $\tan^{-1}\left(\dfrac{16}{7\sqrt5}\right)$ (B) $\tan^{-1}\left(\dfrac{24}{7\sqrt5}\right)$

(C) $\tan^{-1}\left(\dfrac{32}{7\sqrt5}\right)$ (D) $\tan^{-1}\left(\dfrac{3+8\sqrt5}{35}\right)$

Official Ans. by NTA (B)

Sol. Equation of tangent to the ellipse $2x^2+3y^2=5$ is

$y=mx\pm\sqrt{\dfrac{5}{2}m^2+\dfrac{5}{3}}$

It pass through (1, 3)

$$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

$$3m^2 + 12m - \frac{44}{3} = 0$$

Let θ be the angle between the tangents

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{3\sqrt{320}}{-35}\right|$$

$$\theta = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$$

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

(A) $y = 4(x-2)$ (B) $y = 4(x-1)$

(C) $y = 4(x+1)$ (D) $y = 4(x+2)$

Official Ans. by NTA (B)

Sol. Equation of tangent of $y = x^2$ be

$tx = y + at^2$ .....(1)

$$y = tx - \frac{t^2}{4}$$

Solve with $y = -(x-2)^2$

$$tx - \frac{t^2}{4} = -(x-2)^2$$

$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

$D = 0$

$$(t-4)^2 - 4\cdot\left(4 - \frac{t^2}{4}\right) = 0$$

$t^2 - 4t = 0$

$t = 0$ or $t = 4$

From eq. (1), required common tangent is

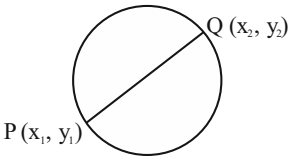
$y = 4(x-1)$

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of (a+b-c) is

(A) 12 (B) 13 (C) 14 (D) 16

Official Ans. by NTA (A)

Sol.



Equation of circle diameter form

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(where x_1 , x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)

$x^2 + y^2 - 4x + 2y - 13 = 0$

Now,

Compare it with the given equation, we get

$a = -2, b = 1, c = -13$

Now

$a + b - c = 12$

14. If the line $x-1 = 0$, is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

(A) $\left(-2\sqrt{5}, 6\right)$ (B) $\left(-\sqrt{5}, 3\right)$

(C) $\left(\sqrt{5}, -2\right)$ (D) $\left(2\sqrt{5}, 3\sqrt{6}\right)$

Official Ans. by NTA (C)

Sol. $$\frac{x^2}{6/k} - \frac{y^2}{6} = 1 \quad \text{.....(1)}$$

$$e^2 = 1 + \frac{6}{6/k}$$

$$e = \sqrt{1+k}$$

$$a = \sqrt{\frac{6}{k}}$$

Eq. of directrix $x = \frac{a}{e} \Rightarrow x = \sqrt{\frac{6}{k(k+1)}}$

$$\frac{6}{k(k+1)} = 1$$

$k = 2$

From eq. (1) ,we get $2x^2 - y^2 = 6$

Check options

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{6}$

Official Ans. by NTA (A)

Sol. $\vec{n}_1=\hat{i}\times(\hat{i}+\hat{j})=\hat{k}$

$\vec{n}_2=(\hat{i}+\hat{k})\times(\hat{i}-\hat{j})$

$=\hat{i}+\hat{j}-\hat{k}$

Line of intersection along $\vec{n}_1\times\vec{n}_2$

$=\hat{k}\times(\hat{i}+\hat{j}-\hat{k})=-\hat{i}+\hat{j}$

D.R of $\vec{a}=-\hat{i}+\hat{j}$

D.R of $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$

$\vec{a}\cdot\vec{b}=-3$ and $(\vec{a}\wedge\vec{b})=\theta$

$\cos\theta=\frac{-3}{\sqrt{2}\times3}$

$\theta=\frac{3\pi}{4}$

16. If $0<x<\frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(A) $4\sqrt{(1-x^2)}(1-2x^2)$

(B) $4x\sqrt{(1-x^2)}(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}(1-4x^2)$

(D) $4\sqrt{(1-x^2)}(1-4x^2)$

Official Ans. by NTA (B)

Sol. $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}=k$

$\sin^{-1}x=k\alpha$

$\cos^{-1}x=k\beta$

$k=\frac{\pi}{2(\alpha+\beta)}$ ....(i)

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)=\sin(4\sin^{-1}x)$

$=2\sin(2\sin^{-1}x)\cos(2\sin^{-1}x)$

$=4x\sqrt{1-x^2}(1-2x^2)$

17. Negation of the Boolean expression $p\Leftrightarrow(q\Rightarrow p)$ is

(A) $(\sim p)\wedge q$ (B) $p\wedge(\sim q)$

(C) $(\sim p)\vee(\sim q)$ (D) $(\sim p)\wedge(\sim q)$

Official Ans. by NTA (D)

Sol. $\sim(p\leftrightarrow(q\rightarrow p))$

$\sim(p\leftrightarrow q)=(p\wedge\sim q)\vee(q\wedge\sim p)$

$\sim(p\leftrightarrow(q\rightarrow p))=(p\wedge\sim(q\rightarrow p))\vee((q\rightarrow p)\wedge\sim p)$

$(p\wedge\sim(q\rightarrow p))=p\wedge(q\wedge\sim p)=(p\wedge\sim p)\wedge q=c$

$(q\rightarrow p)\wedge\sim p=(\sim q\vee p)\wedge\sim p=\sim p\wedge(\sim q\vee p)$

$=(\sim p\wedge\sim q)\vee(\sim p\wedge p)=\sim p\wedge\sim q$

$\sim(p\leftrightarrow(q\rightarrow p))=c\vee(\sim p\wedge\sim q)=\sim p\wedge\sim q$

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then $54\,P(X\le 2)$ is equal to

(A) $\frac{73}{27}$ (B) $\frac{146}{27}$

(C) $\frac{146}{81}$ (D) $\frac{126}{81}$

Official Ans. by NTA (B)

Sol. $np=4$

$npq=4/3$

$n=6,\ p=2/3,\ q=1/3$

$54(P(X=2)+P(X=1)+P(X=0))$

$54\left({}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4+{}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5+{}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6\right)$

$=\frac{146}{27}$

19. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Official Ans. by NTA (A)

Sol. $I=\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$

$\frac{\sqrt{3}}{2}\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(\frac{\sqrt{3}}{2}+\sin 2x\right)}dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x-\sin x)}{\sin 60^\circ+\sin 2x}dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\sin x\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$

$\int \frac{\left(\cos\left(x-\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$

$\frac{1}{2}\left(\int \frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}-\int \frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$

$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$

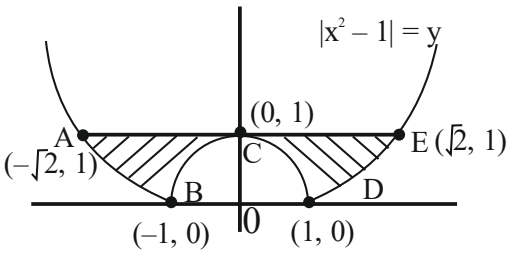
20. The area bounded by the curves $y = |x^2-1|$ and $y = 1$ is

(A) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (B) $\frac{4}{3}\left(\sqrt{2}-1\right)$

(C) $2\left(\sqrt{2}-1\right)$ (D) $\frac{8}{3}\left(\sqrt{2}-1\right)$

Official Ans. by NTA (D)

Sol. $y = |x^2 - 1|$



Area = ABCDEA

$=2\left(\int_0^1\left(1-(1-x^2)\right)dx+\int_1^{\sqrt{2}}\left(1-(x^2-1)\right)dx\right)$

$=\frac{8}{3}(\sqrt{2}-1)$

SECTION-B

1. Let A = {1,2,3,4,5,6,7} and B = {3,6,7,9}. Then the number of elements in the set $\{C\subseteq A : C\cap B\neq\phi\}$ is_____

Official Ans. by NTA (112)

Sol. A = {1,2,3,4,5,6,7} and

B = {3,6,7,9}

Total subset of A = 2^7 = 128

$C\cap B=\phi$ when set C contains the element 1, 2, 4, 5

$\therefore S = \{C \subseteq A; C \cap B \neq \phi\}$

= Total – $(C \cap B = \phi)$

= $128 - 2^4 = 112$

2. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+a\hat{j}-\hat{k})$ and $\vec{r} = (\hat{i}+\hat{j}) + \mu(-\hat{i}+\hat{j}-a\hat{k})$ from the point (2,1,4) is $\sqrt{3}$, is_____.

Official Ans. by NTA (20)

Sol. $\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+a\hat{j}-\hat{k})$

$\vec{r} = (\hat{i}+\hat{j}) + \mu(-\hat{i}+\hat{j}-a\hat{k})$

D.R's of plane containing these lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix} = \hat{i}(1-a^2) - \hat{j}(-a-1) + \hat{k}(1+a)$$

$\vec{n} = (1-a)\hat{i} + \hat{j} + \hat{k}$

One point in plane : (1, 1, 0)

$\therefore$ equation of plane is

$(1 - a)(x - 1) + (y - 1) + (z - 0) = 0$

$(1 - a)x + y + z + a - 2 = 0$

$$\therefore D = \frac{|(1-a)2+1+4+a-2|}{\sqrt{(1-a)^2+1+1}}$$

$\Rightarrow |5 - a| = \sqrt{3}.\sqrt{a^2 - 2a + 3}$

$\Rightarrow a^2 + 2a - 8 = 0$

$\Rightarrow a = 2, -4$

$\therefore$ largest value of a = 2

3. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1,2,3,4,5 and 6 without repetition of digits. Then the total number of such numbers is _____.

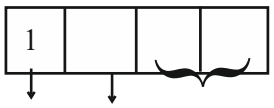
Official Ans. by NTA (30)

Sol. Here 1st digit is 1 or 2 only

Case-I

If first digit is 1

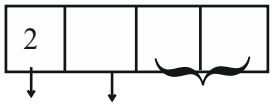
Then last two digits can be 24, 32, 36, 52, 56, 64



$1 \times 3 \times 6$ = 18 ways

Case – II

If first digit is 2 then last two digit can be 16, 36, 56, 64



$1 \times 3 \times 4$ = 12 ways

Total ways = 12 + 18 = 30 ways

4. If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to

Official Ans. by NTA (166)

Sol. $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$

$$\Rightarrow \frac{1}{2}\sum_{k=1}^{10} \frac{(k^2+k+1)-(k^2-k+1)}{(k^2+k+1)(k^2-k+1)}$$

$$\Rightarrow \frac{1}{2}\left(\sum_{k=1}^{10}\left(\frac{1}{(k^2-k+1)} - \frac{1}{k^2+k+1}\right)\right)$$

$$\Rightarrow \frac{55}{111} = \frac{m}{n}$$

m + n = 166

5. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____.

Official Ans. by NTA (3)

Sol. $2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

So, the common solution is

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

Sum = $\frac{7\pi + 11\pi}{6} = 3\pi = k\pi$

K = 3

6. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to_____.

Official Ans. by NTA (238)

Sol. Wrong mean = $\mu_1 = 30$

Wrong S.D = $\sigma_1 = 5$

$\frac{\sum x_i}{40} = 30$

$\Rightarrow \sum x_i = 1200$

$\sigma_1^2 = 25$

$\Rightarrow \frac{\sum x_i^2}{40} - 30^2 = 25$

$\Rightarrow \sum x_i^2 = 925 \times 40 = 37000$

New sum = $\sum x_i' = 1200 - 10 - 12 = 1178$

New mean = $\mu_1' = \frac{1178}{38} = 31$

New $\sum x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$

New S.D, $\sigma_1' = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$

$36756 - (31)^2 \times 38 = 38\sigma^2$

$\Rightarrow 38\sigma^2 = 238$

7. The plane passing through the line L: ℓ x-y+3(1- ℓ) z = 1, x+2y - z = 2 and perpendicular to the plane 3x+2y+z = 6 is 3x-8y+7z=4. If θ is the acute angle between the line L and the y-axis, then 415 $\cos^2\theta$ is equal to_____.

Official Ans. by NTA (125)

Sol. $\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1-\ell)\hat{k}$

$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$

Direction ratio of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1-\ell) \\ 1 & 2 & -1 \end{vmatrix}$

$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$

3x – 8y + 7z = 4 will contain the line $(6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$

Normal of 3x – 8y + 7z = 4 will be perpendicular to the line

$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$

$\Rightarrow \ell = \frac{2}{3}$

$\therefore$ direction ratio of line $\left(-1, \frac{5}{3}, \frac{7}{3}\right)$

Angle with y axis

$\cos\theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$

$\cos\theta = \frac{5}{\sqrt{83}}$

$\therefore 415\cos^2\theta = \frac{25}{83} \times 415 = 125$

8. Suppose y = y(x) be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at x=0, then the value of a-4b is equal to _____.

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - y = 2 - e^{-x}$

I.F. $= e^{-\int dx} = e^{-x}$

$\therefore$ solution of D.E

$y \cdot e^{-x} = \int \left(2e^{-x} - e^{-2x}\right) dx$

$\Rightarrow y = -2 + \frac{e^{-x}}{2} + C.e^{x}$

$\because \lim_{x\to\infty} y$ is finite

$\therefore \lim_{x\to\infty} \left(-2 + \frac{e^{-x}}{2} + C.e^{x}\right) \to$ finite

This is possible only when C = 0

$\therefore y = y(x) = -2 + \frac{e^{-x}}{2}$

$\frac{dy}{dx} = -\frac{1}{2}e^{-x}$

$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{1}{2} = m$, $y(0) = -2 + \frac{1}{2} = \frac{-3}{2}$

$\therefore$ equation of tangent

$y + \frac{3}{2} = -\frac{1}{2}(x - 0)$

$\Rightarrow x + 2y = -3$

$a = -3$, $b = \frac{-3}{2}$

$a - 4b = -3 + 6 = 3$

9. Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

Official Ans. by NTA (53)

Sol. 1st term = 100 = a

Last term = 199 = ℓ

If 3 term

a, a + d, a + 2d

$a_n = \ell = a + (n - 1)d$

$d_i = \frac{\ell - a}{n - 1}$

$n \to$ number of terms

n=3, $d_1 = \frac{199 - 100}{2}$

$= \frac{99}{2} \notin I$

n = 4, $d_2 = \frac{99}{3} = 33 \in I$

n = 10, $d_3 = \frac{99}{9} = 11 \in I$

n = 12, $d_4 = \frac{99}{11} = 9 \in I$

$\therefore \sum d_i = 33 + 11 + 9 = 53$

10. The number of matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a,b,c,d $\in$ {-1,0,1,2,3,……,10}, such that $A = A^{-1}$, is_____.

Official Ans. by NTA (50)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given $A = A^{-1}$

$\therefore A^2 = A \cdot A^{-1} = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore a^2 + bc = 1$ ….(1)

$ab + bd = 0$ …..(2)

$ac + cd = 0$ …(3)

$bc + d^2 = 1$ …….(4)

(1) – (4) gives

$a^2 - d^2 = 0$

$\Rightarrow (a + d) = 0$ or $a - d = 0$

Case – I

$a + d = 0 \Rightarrow (a, d) = (-1, 1), (0, 0), (1, -1)$

(a) $(a, d) = (-1, 1)$

$\therefore$ from equation (1)

$1 + bc = 1 \Rightarrow bc = 0$

$b = 0$ C = 12 possibilities

$c = 0$ b = 12 possibilities

but (0, 0) is repeated

$\therefore 2 \times 12 = 24$

24 – 1 (repeated) = 23 pairs

(b) $(a, d) = (1, -1) \Rightarrow bc = 0 \rightarrow$ 23 pairs

(c) $(a, d) = (0, 0) \Rightarrow bc = 1$

$\Rightarrow$ (b, c) = (1, 1) & (–1, –1), 2 pairs

Case – II

$a = d$

from (2) and (3)

$a \neq 0$ then $b = c = 0$

$a^2 = 1$

$a = \pm 1 = d$

$(a, d) = (1, 1), (-1, -1) \rightarrow$ 2 pairs

$\therefore$ Total = 23 + 23 + 2 + 2

= 50 pairs

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of :

(A) 15% (B) 25% (C) 75% (D) 5%

Official Ans. by NTA (B)

Sol. Dimensional formula for Torque

$[\tau] = [ML^2T^{-2}]$

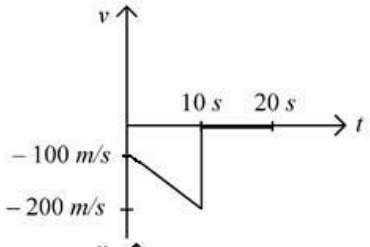
Now

Percentage error in torque = % τ = % M + 2 % L 2 % T

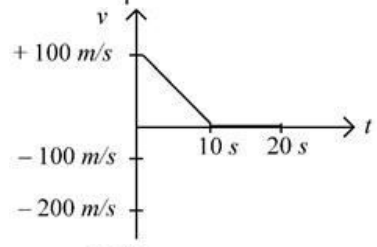
% τ = 25%

2. A bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10 s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time t = 20 s will be : (Take g = 10 m/s^2)

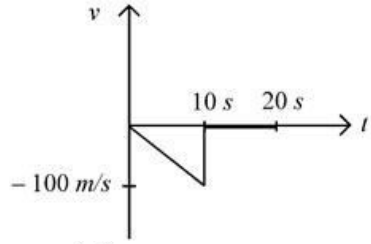
(A)



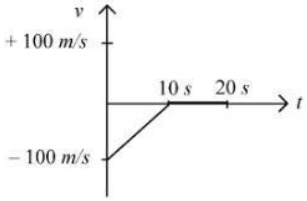
(B)



(C)



(D)



Official Ans. by NTA (A)

Sol. V = –100 –10t

3. Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep belt moving with the same velocity will be :

(A) 1.25 W (B) 2.5 W

(C) 6.25 W (D) 12.5 W

Official Ans. by NTA (D)

Sol. Thrust = λV_{rel}

= 2.5 N

Now, Power =F × V = 12.5 W

4. A bag is gently dropped on a conveyor belt moving at a speed of 2 m/s. The coefficient of friction between the conveyor belt and bag is 0.4 Initially, the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion is : [Take g = 10 m/s^{-2}]

(A) 2 m (B) 0.5 m

(C) 3.2 m (D) 0.8 ms

Official Ans. by NTA (B)

Sol. In frame of belt

$a = \mu g = 4$ m/s^2 , v = 2m/s , u = 0

$v^2 = u^2 + 2as$

$\Rightarrow$ s = 0.5 m

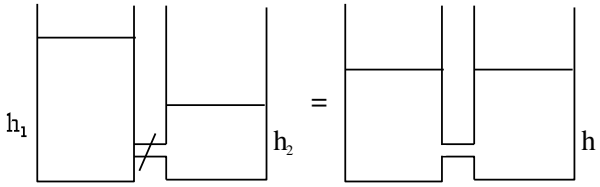
5. Two cylindrical vessels of equal cross-sectional area 16 cm^2 contain water upto heights 100 cm and 150 cm respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take density of water = 10^3 kg/m^3 and g = 10 ms^{-2}]

(A) 0.25 J (B) 1 J

(C) 8 J (D) 12 J

Official Ans. by NTA (B)

Sol.



$$h = \frac{h_1 + h_2}{2}$$

Now,

$W = U_i - U_f$

$$W = (\rho A h_1)\ g\ \frac{h_1}{2} + (\rho\ A h_2) g \frac{h_2}{2} - \rho\ A(h_1+h_2)g\left(\frac{h_1+h_2}{4}\right)$$

$$W = \frac{\rho A g}{2}\left[h_1^2 + h_2^2 - \frac{(h_1+h_2)^2}{2}\right]$$

$W = 1$ J

6. Two satellites A and B having masses in the ratio 4:3 are revolving in circular orbits of radii 3r and 4 r respectively around the earth. The ratio of total mechanical energy of A to B is :

(A) 9 : 16 (B) 16 : 9

(C) 1 : 1 (D) 4 : 3

Official Ans. by NTA (B)

Sol. Given that $\frac{m_1}{m_2} = \frac{4}{3}$, $\frac{r_1}{r_2} = \frac{3}{4}$

Now $TE = \frac{1}{2}\ mv^2 + \left(\frac{-GMm}{r}\right)$

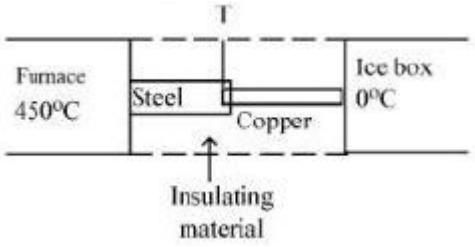
but $\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow mv^2 = \frac{GMm}{r}$

$$\Rightarrow TE = -\frac{GMm}{2r} \propto \frac{m}{r}$$

$$\frac{TE_1}{TE_2} = \frac{m_1}{m_2}\cdot\frac{r_2}{r_1} = \frac{4}{3}\times\frac{4}{3} = \frac{16}{9}$$

7. If K_1 and K_2 are the thermal conductivities L_1 and L_2 are the lengths and A_1 and A_2 are the cross sectional areas of steel and copper rods respectively such that $\frac{K_2}{K_1} = 9, \frac{A_1}{A_2} = 2, \frac{L_1}{L_2} = 2$.

Then, for the arrangement as shown in the figure. The value of temperature T of the steel – copper junction in the steady state will be :

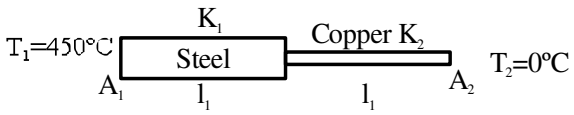


(A) 18°C (B) 14 °C

(C) 45 °C (D) 150 °C

Official Ans. by NTA (C)

Sol.



$$\frac{d\theta}{dt} = \frac{K_1 A_1}{l_1}(T_1 - T) = \frac{K_2 A_2}{l_2}(T - T_2)$$

$$\Rightarrow \frac{450 - T}{T - 0} = \frac{K_2 A_2 l_1}{K_1 A_1 l_2} = 9 \times \frac{1}{2} \times 2$$

$\Rightarrow 450 - T = 9T \Rightarrow T = 45°C$

8. Read the following statements :

A. When small temperature difference between a liquid and its surrounding is doubled the rate of loss of heat of the liquid becomes twice.

B. Two bodies P and Q having equal surface areas are maintained at temperature 10°C and 20 °C. The thermal radiation emitted in a given time by P and Q are in the ratio 1 : 1.15

C. A carnot Engine working between 100 K and 400 K has an efficiency of 75%

D. When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below :

(A) A, B, C only (B) A, B only

(C) A, C only (D) B, C, D only

Official Ans. by NTA (A)

Sol. Heat Transfer

A. by Newton's low of colling $\frac{d\theta}{dt} = \propto \Delta T$

B. $H = \frac{d\theta}{dt} = \sigma e A T^4 \Rightarrow \frac{H_P}{H_Q} = \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{283}{293}\right)^4$

$H_P : H_Q = 1\ (1.03)^4 = 1 : (1.03)^4 = 1 : 1.15$

$\Rightarrow$ B is correct

C. $\eta = 1 - \frac{100}{400} = \frac{3}{4} = 75\%$

D. is wrong as $\frac{d\theta}{dt} \propto \Delta T$

9. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is 1:4, then

A. The r.m.s. velocity of gas molecules in two vessels will be the same.

B. The ratio of pressure in these vessels will be 1 : 4

C. The ratio of pressure will be 1 : 1

D. The r.m.s. velocity of gas molecules in two vessels will be in the ratio of 1 : 4

(A) A and C only (B) B and D only

(C) A and B only (D) C and D only

Official Ans. by NTA (C)

Sol. **KTG**

A. $V_{Rms} = \sqrt{\frac{3RT}{M_w}} \Rightarrow V_{Rms}$ is same

B. $\frac{P_1}{P_2} = \frac{N_1}{N_2} \Rightarrow$ B is correct

Ans [A & B only are correct]

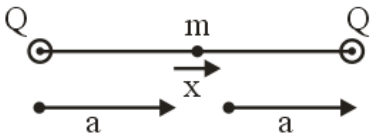
10. Two identical positive charges Q each are fixed at a distance of '2a' apart from each other. Another point charge q_0 with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be :

(A) $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0 Q}}$ (B) $\sqrt{\frac{q_0 Q}{4\pi^3\varepsilon_0 ma^3}}$

(C) $\sqrt{\frac{2\pi^2\varepsilon_0 ma^3}{q_0 Q}}$ (D) $\sqrt{\frac{8\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

Official Ans. by NTA (A)

Sol. **Electrostatics**



$F = m\ acc^n = \frac{KQq_0}{(a-x)^2} - \frac{KQq_0}{(a+x)^2}$

$m\ acc^n = \frac{KQq_0[2a][2x]}{(a^2-x^2)^2}$

$\Rightarrow acc^n \approx \left(\frac{4kQq_0}{ma^3}\right)x$

$T = 2\pi\sqrt{\frac{\pi\varepsilon_0 ma^3}{Qq_0}}$

$T = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{Qq_0}}$

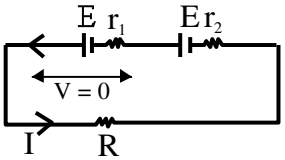
11. Two sources of equal emfs are connected in series. This combination is connected to an external resistance R. The internal resistances of the two sources are r_1 and r_2 ($r_1 > r_2$). If the potential difference across the source of internal resistance r_1 is zero then the value of R will be

(A) $r_1 - r_2$ (B) $\frac{r_1 r_2}{r_1 + r_2}$

(C) $\frac{r_1 + r_2}{2}$ (D) $r_2 - r_1$

Official Ans. by NTA (A)

Sol.



$I = \frac{2E}{r_1 + r_2 + R}$

$IR = E - Ir_2$

$I(R + r_2) = E$

$I = \frac{E}{R + r_2}$

$\frac{2E}{r_1 + r_2 + R} = \frac{E}{R + r_2}$

$2R + 2r_2 = r_1 + r_2 + R$

$R = r_1 - r_2$

12. Two bar magnets oscillate in a horizontal plane in earth's magnetic field with time periods of 3 s and 4 s respectively. If their moments of inertia are in the ratio of 3 : 2 then the ratio of their magnetic moments will e :

(A) 2 : 1 (B) 8 : 3

(C) 1 : 3 (D) 27 : 16

Official Ans. by NTA (B)

Sol. $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{I_1}{M_1B_H}}}{2\pi\sqrt{\frac{I_2}{M_2B_H}}} = \frac{3}{4}$$

$$\sqrt{\frac{I_1}{I_2}\times\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\sqrt{\frac{I_1}{I_2}}\times\sqrt{\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\sqrt{\frac{3}{2}}\times\sqrt{\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\frac{3}{2}\times\frac{M_2}{M_1} = \frac{9}{16}$$

$$\frac{M_1}{M_2} = \frac{8}{3}$$

13. A magnet hung at 45º with magnetic meridian makes an angle of 60º with the horizontal. The actual value of the angle of dip is

(A) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ (B) $\tan^{-1}\left(\sqrt{6}\right)$

(C) $\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (D) $\tan^{-1}\left(\sqrt{\frac{1}{2}}\right)$

Official Ans. by NTA (A)

Sol. $\tan\theta' = \frac{\tan\theta}{\cos\alpha}$

$\theta' = 60^\circ$

$\alpha = 45^\circ$

$$\sqrt{3} = \frac{\tan\theta}{\frac{1}{\sqrt{2}}}$$

$$\tan\theta = \sqrt{\frac{3}{2}}$$

$$\theta = \tan^{-1}\sqrt{\frac{3}{2}}$$

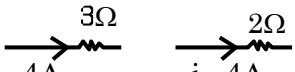
14. A direct current of 4 A and an alternating current of peak value 4 A flow through resistance of 3 Ω and 2 Ω respectively. The ratio of heat produced in the two resistances in same interval of time will be :

(A) 3 : 2 (B) 3 : 1

(C) 3 : 4 (D) 4 : 3

Official Ans. by NTA (B)

Sol.



DC AC

$H_1 = i^2R_1t$ $H_2 = i_{rms}^2R_2t \left\{i_{rms} = \frac{i_0}{\sqrt{2}}\right\}$

$H_1 = 16(3)t$ $H_2 = \frac{i_0^2}{2}R_2t$

$H_2 = 16t$

$H_1 : H_2 = 3 : 1$

15. A beam of light travelling along X-axis is described by the electric field $E_y = 900 \sin \omega(t - x/c)$. The ratio of electric force to magnetic force on a charge q moving along Y-axis with a speed of 3×10^7 ms^{-1} will be :

[Given speed of light = 3×10^8 ms^{-1}]

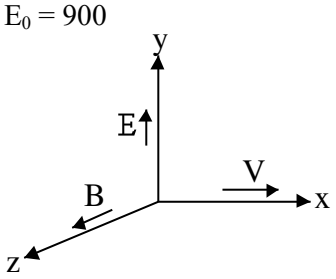
(A) 1 : 1 (B) 1 : 10

(C) 10 : 1 (D) 1 : 2

Official Ans. by NTA (C)

Sol. $E_y = 900\sin\left(\omega t - \frac{\omega x}{c}\right)$

$E_0 = 900$



$F_E = qE_0$

$F_B = qvB_0$

$\frac{F_E}{F_B} = \frac{E_0}{vB_0} = \frac{c}{v} = \frac{3\times10^8}{3\times10^7} = 10:1$

16. A microscope was initially placed in air (refractive index 1). It is then immersed in oil (refractive index 2). For a light whose wavelength in air is λ, calculate the change of microscope's resolving power due to oil and choose the correct option

(A) Resolving power will be $\frac{1}{4}$ in the oil than it was in the air

(B) Resolving power will be twice in the oil than it was in the air.

(C) Resolving power will be four times in the oil than it was in the air.

(D) Resolving power will be $\frac{1}{2}$ in the oil than it was in the air.

Official Ans. by NTA (C)

Sol. $(R.P)_{air} = \frac{2\sin\theta}{1.22\lambda}$

$(R.P)_{oil} = \frac{2\sin\theta}{1.22\lambda_{oil}} = \frac{2\sin\theta\times\mu}{1.22\lambda}$

$(R.P)_{oil} = (R.P)_{air} \times 2$

17. An electron (mass m) with an initial velocity $\vec{v} = v_0\hat{i}(v_0 > 0)$ is moving in an electric field $\vec{E} = -E_0\hat{i}(E_0 > 0)$ where E_0 is constant. If at t = 0 de Broglie wavelength is $\lambda_0 = \frac{h}{mv_0}$, then its de Broglie wavelength after time t is given by

(A) λ_0 (B) $\lambda_0\left(1+\frac{eE_0t}{mv_0}\right)$

(C) $\lambda_0 t$ (D) $\frac{\lambda_0}{\left(1+\frac{eE_0t}{mv_0}\right)}$

Official Ans. by NTA (D)

Sol. $e \longrightarrow V_0$

$E = -E_0\hat{i}$

$\lambda_0 = \frac{h}{mv_0}$

$v = v_0 + \frac{eE_0t}{m}$

$\lambda = \frac{h}{mv} = \frac{h}{m\left(v_0 + \frac{eE_0}{m}t\right)}$

$\lambda' = \frac{h}{mv_0\left(1+\frac{eE_0}{mv_0}t\right)}$

$\lambda' = \frac{\lambda_0}{\left(1+\frac{eE_0}{mv_0}t\right)}$

18. What is the half-life period of a radioactive material if its activity drops to $1/16^{th}$ of its initial value of 30 years ?

(A) 9.5 years (B) 8.5 years

(C) 7.5 years (D) 10.5 years

Official Ans. by NTA (C)

Sol. $A = A_0e^{-\lambda t}$

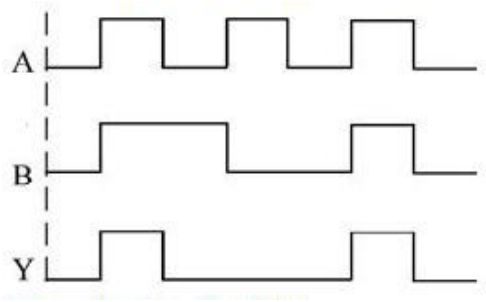
$\Rightarrow -\lambda t = \ell n\left(\frac{A}{A_0}\right)$

$\Rightarrow -\frac{\ell n2}{t_{1/2}}\times30 = \ell n\left(\frac{1}{16}\right)$

$\Rightarrow -\frac{\ell n2}{t_{1/2}}\times30 = -4\ell n2$

$\Rightarrow t_{1/2} = \frac{30}{4} = 7.5$ yrs

19. A logic gate circuit has two inputs A and B and output Y. The voltage waveforms of A, B and Y are shown below



The logic gate circuit is

(A) AND gate (B) OR gate

(C) NOR gate (D) NAND gate

Official Ans. by NTA (A)

Sol. By making Truth table

A	B	Output
0	0	0
1	1	1
0	1	0
1	0	0

Comparing with output of AND gate

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

$\Rightarrow$ logic gate present is AND gate

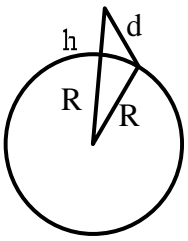
20. At a particular station, the TV transmission tower has a height of 100 m. To triple its coverage range, height of the tower should be increased to

(A) 200 m (B) 300 m

(C) 600 m (D) 900 m

Official Ans. by NTA (D)

Sol.



Let d be range

$d^2 = (h+R)^2 - R^2$

$= h^2+R^2+2RH-R^2$

$d^2 = h^2+2Rh$

as R >>>> h then

$d \approx \sqrt{2Rh}$ .... (1)

Now, if coverage is to be increased 3 times

$3d = \sqrt{2Rh'}$ .... (2)

Divide 2 and 1 $\frac{3d}{d} = \sqrt{\frac{2Rh'}{2Rh}}$

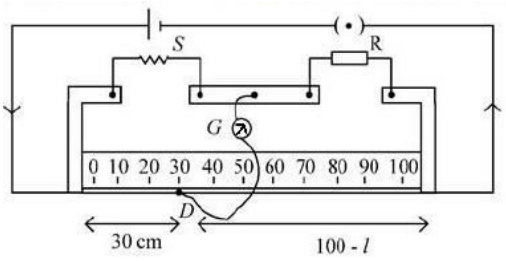
$9 = \frac{h'}{h}$

$9h = h'$

If h = 100 m then tower of height 900 m is required

SECTION-B

1. In meter bridge experiment for measuring unknown resistance 'S', the null point is obtained at a distance 30 cm from the left side as shown at point D. If R is 5.6 kΩ, then the value of unknown resistance 'S' will be _____ Ω.



Official Ans. by NTA (2400)

Sol. $\frac{S}{30} = \frac{5.6\times10^3}{70}$

$S = \frac{3}{7}\times5.6\times10^3 = 2400$

2. The one division of main scale of vernier callipers reads 1 mm and 10 divisions of Vernier scale is equal to the 9 divisions on main scale. When the two jaws of the instrument touch each other the zero of the Vernier lies to the right of zero of the main scale and its fourth division coincides with a main scale division. When a spherical bob is tightly placed between the two jaws, the zero of the Vernier scale lies in between 4.1 cm and 4.2 cm and 6^{th} Vernier division coincides with a main scale division. The diameter of the bob will be_____ 10^{-2} cm

Official Ans. by NTA (412)

Sol. 10 VSD = 9 MSD

1 VST = .9 MSD

L.C. = .1 mm = .01 cm

+ve zero error = .4mm

= 0.04 cm

Negative zero error = 4.1 cm + 6 × .01

= 4.12 cm

= 412 × 10^{-2} cm

3. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the two beams are $\pi/2$ and $\pi/3$ at points A and B respectively. The difference between the resultant intensities at the two points is xI. The value of x will be _____ .

Official Ans. by NTA (2)

Sol. $\phi_A = \frac{\pi}{2}$

$\phi_B = \frac{\pi}{3}$

$I_A = I + 4I + 2\sqrt{I}\ \sqrt{4I}\ \cos\left(\frac{\pi}{2}\right)$

= 5I + 4I (0) = 5I

$I_B = I + 4I + 2\sqrt{I}\ \sqrt{4I}\ \cos(60^o)$

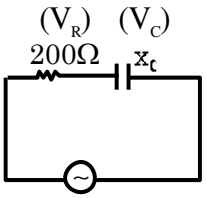
$= 5I + 4I \times \frac{1}{2} = 7I$

$I_B - I_A = 7I - 5I = 2I$, (x = 2)

4. To light, a 50 W, 100 V lamp is connected, in series with a capacitor of capacitance $\frac{50}{\pi\sqrt{x}}\mu F$, with 200 V, 50Hz AC source. The value of x will be ____ .

Official Ans. by NTA (3)

Sol. $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$



100 V

$R = \frac{100\times10^2}{50} = R = 200\Omega$

$V_R^2 + V_C^2 = V^2$

$(100)^2 + V_C^2 = (200)^2$

$i = \frac{100}{200} = \frac{1}{2}$; $V^2 = 40000$

$V = I \times X_C$; $V_C^2 = 30000$

$V_C = 100\sqrt{3}$

$X_C = 200\sqrt{3}$

$200\sqrt{3} = \frac{1}{\omega C}$

$C = \frac{1}{20\times50\times20\sqrt{3}} = \frac{50\times10^{-6}}{\sqrt{x}}$

$\sqrt{x} = 50\times10^{-6}\times100\times200\sqrt{3}$

X = 3

5. A 1 m long copper wire carries a current of 1 A. If the cross section of the wire is 2.0 mm^2 and the resistivity of copper is 1.7×10^{-8} Ω m. the force experienced by moving electron in the wire is _____ $\times 10^{-23}$ N. (charge on electron $= 1.6 \times 10^{-19}$ C)

Official Ans. by NTA (136)

Sol. $l = 1m$

$i = 1A$

Area $= 2 \times 10^{-6}$

$\rho = 1.7 \times 10^{-8}$

$$R = \frac{\rho \ell}{A} = \frac{1.7\times10^{-8}\times1}{2\times10^{-5}} = \frac{1.7}{2}\times10^{-2}$$

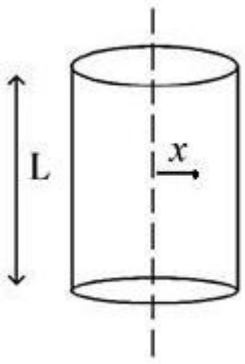
$$v = \frac{1.7}{2}\times10^{-2}$$

$$F = 1.6 \times 10^{-19} \times \frac{1.7}{2} \times 10^{-2}$$

$$= 1.36 \times 10^{-21}$$

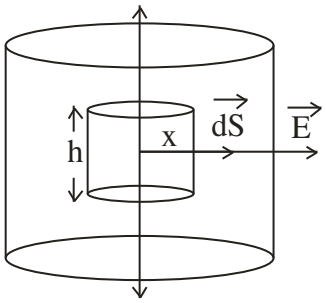
$$= 136 \times 10^{-23}$$

6. A long cylindrical volume contains a uniformly distributed charge of density ρ Cm^{-3}. The electric field inside the cylindrical volume at a distance $x = \frac{2\varepsilon_0}{\rho}$ m from its axis is _____ Vm^{-1}



Official Ans. by NTA (1)

Sol.



$$\int E ds \cos 0 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E.2\pi xh = \frac{\rho \times \pi x^2 h}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho x}{2\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho}{2\varepsilon_0} \times \frac{2\varepsilon_0}{\rho} = 1$$

7. A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha - 1}$, then the value of α will be ____.

Official Ans. by NTA (16)

Sol. $$\frac{1}{2}kA^2 = \frac{p^2}{2m}$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = \frac{m_2}{m_1} = \frac{1024}{900}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{32}{30} = \frac{16}{15} = \frac{16}{16-1}$$

$$\therefore \alpha = 16$$

8. A square aluminium (shear modulus is $25 \times 10^9\ Nm^{-2}$) slab of side 60 cm and thickness 15 cm is subjected to a shearing force (on its narrow face) of 18.0×10^4 N. The lower edge is riveted to the floor. The displacement of the upper edge is ____ μ m.

Official Ans. by NTA (48)

Sol. $\frac{F}{A} = \eta\frac{x}{\ell} \Rightarrow \frac{F\ell}{A\eta} = x$

$\Rightarrow x = \frac{18\times10^4\times60\times10^{-2}}{60\times10^{-2}\times15\times10^{-2}\times25\times10^9}$

$= 48 \times 10^{-6}\ m = 48\ \mu m$

9. A pulley of radius 1.5 m is rotated about its axis by a force $F = (12t - 3t^2)$ N applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is 4.5 kg m^2, the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is ____ .

Official Ans. by NTA (18)

Sol. $\tau = I\alpha \Rightarrow (12t - 3t^2)1.5 = 4.5\,\alpha$

$\Rightarrow \alpha = 4t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 4t - t^2 \Rightarrow \omega = \int_0^t (4t - t^2)dt$

$\Rightarrow \omega = 2t^2 - t^3/3$

For $\omega = 0 = 2t^2 - \frac{t^3}{3} \Rightarrow t^2\left(2 - \frac{t}{3}\right) = 0$

$\Rightarrow t = 0, 6.$

$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \Rightarrow \theta = \int_0^6 (2t^2 - \frac{t^3}{3})dt$

$= \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6$

$= 6^3\left(\frac{2}{3} - \frac{6}{12}\right) = 6^3\left(\frac{8-6}{12}\right)$

$= \frac{6^3}{6} = 36$

No. of revolutions $= \frac{36}{2\pi} = \frac{18}{\pi}$

$\therefore$ K = 18

10. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is _____ .

Official Ans. by NTA (1)

Sol. Time of flight is same

$\Rightarrow$ vertical component of velocity is same

$\Rightarrow H_{max}$ is same

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. 250 g solution of D-glucose in water contains 10.8% of carbon by weight. The molality of the solution is nearest to

(Given: Atomic Weights are H, 1u ; C, 12u ; O, 16u)

(A) 1.03 (B) 2.06

(C) 3.09 (D) 5.40

Official Ans. by NTA (B)

Sol. $C_6H_{12}O_6 \rightarrow$ Glucose

We know: $\frac{\text{mass of C}}{\text{mass of glucose}} = \frac{72}{180}$

Given: $\%C = 10.8 = \frac{\text{mass of C}}{\text{mass of solution}} \times 100$

$\frac{10.8 \times 250}{100} = \text{mass of C} \Rightarrow \text{Mass of C} = 27 \text{ gm}$

$\therefore$ mass of glucose = 67.5 gm

$\therefore$ moles of glucose = 0.375 moles

Mass of solvent = 250 – 67.5 gm = 182.5 gm

$\therefore$ Molality $= \frac{0.375}{0.1825} = 2.055 \approx 2.06$

2. Given below are two statements.

Statement I : O_2 , Cu^{2+} and Fe^{3+} are weakly attracted by magnetic field and are magnetized in the same direction as magnetic field.

Statement II : NaCl and H_2O are weakly magnetized in opposite direction to magnetic field.

In the light of the above statements, choose the ***most appropriate*** answer form the options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (A)

Sol. O_2, Cu^{2+} and Fe^{3+} are paramagnetic,

$\therefore$ Weakly attracted by magnetic field.

NaCl and H_2O are diamagnetic,

$\therefore$ Weakly repelled by magnetic field.

3. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Energy of 2s orbital of hydrogen atom is greater than that of 2s orbital of lithium.

Reason R : Energies of the orbitals in the same subshell decrease with increase in the atomic number.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A.**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A.**

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Official Ans. by NTA (A)

Sol. Energy of orbitals decreases on increasing the atomic number.

4. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Activated charcoal adsorbs SO_2 more efficiently than CH_4.

Reason R : Gases with lower critical temperatures are readily adsorbed by activated charcoal.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A.**

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A.**

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (C)

Sol. SO_2 is absorbed to a greater extent than CH_4 on activated charcoal under same conditions.

Gases with **higher critical temperature** are readily absorbed by activated charcoal.

5. Boiling point of a 2% aqueous solution of a non-volatile solute A is equal to the boiling point of 8% aqueous solution of a non-volatile solute B. The relation between molecular weights of A and B is.

(A) $M_A = 4M_B$ (B) $M_B = 4M_A$

(C) $M_A = 8M_B$ (D) $M_B = 8M_A$

Official Ans. by NTA (B)

Sol. **For A :** 100 gm solution $\rightarrow$ 2 gm solute A

$$\therefore \text{Molality} = \frac{2/M_A}{0.098}$$

For B : 100 gm solution $\rightarrow$ 8 gm solute B

$$\therefore \text{Molality} = \frac{8/M_B}{0.092}$$

$$\because (\Delta T_B)_A = (\Delta T_B)_B$$

$\therefore$ Molality of A = Molality of B

$$\therefore \frac{2}{0.098M_A} = \frac{8}{0.092M_B}$$

$$\frac{2}{98} \times \frac{92}{8} = \frac{M_A}{M_B}$$

$$\frac{1}{4.261} = \frac{M_A}{M_B}$$

$$\therefore M_B = 4.261 \times M_A$$

6. The **incorrect** statement is

(A) The first ionization enthalpy of K is less than that of Na and Li

(B) Xe does not have the lowest first ionization enthalpy in its group

(C) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.

(D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

Official Ans. by NTA (D)

Sol. Ionization enthalpy order :

Li > Na > K

He > Ne > Ar > Kr > Xe > Rn

Sr > Rb

Zn > Ga

7. Which of the following methods are not used to refine any metal?

(A) Liquation (B) Calcination

(C) Electrolysis (D) Leaching

(E) Distillation

Choose the **correct** answer from the options given below:

(A) B and D only (B) A, B, D and E only

(C) B, D and E only (D) A, C and E only

Official Ans. by NTA (A)

Sol. Calcination and leaching are the methods of concentration of ore and not that of refining.

8. Given below are two statements:

Statement I : Hydrogen peroxide can act as an oxidizing agent in both acidic and basic conditions.

Statement II: Density of hydrogen peroxide at 298 K is lower than that of D_2O.

In the light of the above statements. Choose the ***correct*** answer from the options.

(A) Both statement I and Statement II are ture

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (C)

Sol. Depending on the nature of reducing agent H_2O_2 can act as an oxidising agent in both acidic as well as basic medium.

Density of D_2O = 1.1 g/cc

Density of H_2O_2 = 1.45 g/cc

9. Given below are two statements:

Statement I : The chlorides of Be and Al have Cl-bridged structure. Both are soluble in organic solvents and act as Lewis bases.

Statement II: Hydroxides of Be and Al dissolve in excess alkali to give beryllate and aluminate ions.

In the light of the above statements. Choose the correct answer from the options given below.

(A) Both statement I and Statement II are true

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (D)

Sol. Be_2Cl_4 is lewis acid and Al_2Cl_6 has complete octet. Be and Al are amphoteric metals therefore dissolve in acid as well as alkaline solution and form beryllate and aluminate ions in excess alkali.

10. Which oxoacid of phosphorous has the highest number of oxygen atoms present in its chemical formula?

(A) Pyrophosphorous acid

(B) Hypophosphoric acid

(C) Phosphoric acid

(D) Pyrophosphoric acid

Official Ans. by NTA (D)

Sol. Pyrophosphorous acid $\rightarrow H_4P_2O_5$.

Hypophosphoric acid $\rightarrow H_4P_2O_6$.

Phosphoric acid $\rightarrow H_3PO_4$.

Pyrophosphoric acid $\rightarrow H_4P_2O_7$.

11. Given below are two statements:

Statement I : Iron (III) catalyst, acidified $K_2Cr_2O_7$ and neutral $KMnO_4$ have the ability to oxidise I^- to I_2 independently.

Statement II: Manganate ion is paramagnetic in nature and involves $p\pi - p\pi$ bonding.

In the light of the above statements, choose the ***correct*** answer from the options.

(A) Both statement I and Statement II are ture

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (B)

Sol. Neutral $KMnO_4$ oxidises I^- to IO_3^-

Manganate ion has $d\pi - p\pi$ bonding.

12. The total number of Mn = O bonds in Mn_2O_7 is

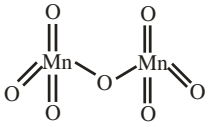
_____

(A) 4 (B) 5

(C) 6 (D) 3

Official Ans. by NTA (C)

Sol.



13. Match List I with List II

List I Pollutant	List II Disease /sickness
A. Sulphate (>500 ppm)	I. Methemoglobinemia
B. Nitrate (>50 ppm)	II. Brown mottling of teeth
C. Lead (> 50 ppb)	III. Laxative effect
D. Fluoride (>2 ppm)	IV. Kidney damage

Choose the correct answer from the options given below:

(A) A-IV, B –I, C-II, D-III

(B) A-III, B –I, C-IV, D-II

(C) A-II, B –IV, C-I, D-III

(D) A-II, B –IV, C-III, D-I

Official Ans. by NTA (B)

Sol. A. Sulphate (>500 ppm) - Causes Laxative effect that leads to dehydration

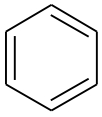
B. Nitrate (>50 ppm) - Causes Methemoglobinemia, skin appears blue

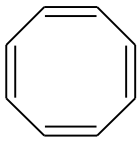
C. Lead (> 50 ppb) – It damage kidney and RBC

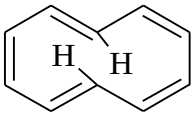
D. Fluoride (>2 ppm) – It Causes Brown mottling of teeth

14. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : [6] Annulene. [8] Annulene and cis –[10] Annulene, are respectively aromatic, not-aromatic and aromatic.

[6] Annulene 

[8] Annulene 

Cis-[10] Annulene 

Reason R : Planarity is one of the requirements of aromatic systems.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A.**

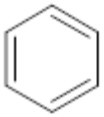
(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A.**

(C) **A** is correct but **R** is not correct.

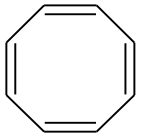
(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (A)

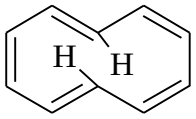
Sol. Assertion A : Not correct , Reason R : correct


Arom atic

[6] – annulene


- Not aromatic

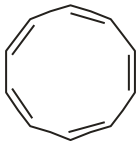
[8] – annulene



- Not aromatic

[10] – annulene

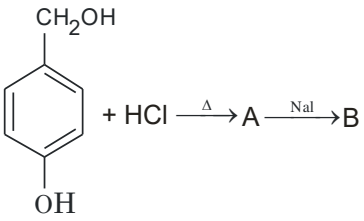
In [10] –Annulene – the hydrogen atoms in the 1 and 6 position interfere with each other and force the molecule out of planarity



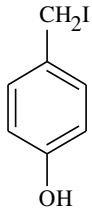
all -cis(10)annulene

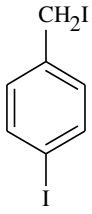
If this annulene with five cis double bonds were planar, each internal angle would be 144°. Since a normal double bond has bond angle of 120°, this would be from ideal. This compound can be made but it does not adopt a planar conformation and therefore is not aromatic even though it has ten π electrons.

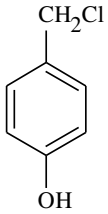
15.

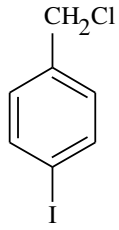


In the above reaction product B is:

(A) 

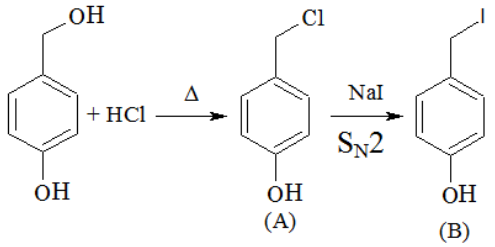
(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol.



16. Match **List I** with **List II**

List I **Polymers**	**List II** **Commenrcial names**
A. Phenol-formaldehyde resin	I. Glyptal
B. Copolymer of 1,3-butadiene and styrene	II. Novolac
C. Polyester of glycol and phthalic acid	III. Buna-S
D. Polyester of glycol and terephthalic acid	IV. Dacron

Choose the correct answer from the options given below:

(A) A-II, B –III, C-IV, D-I

(B) A-II, B –III, C-I, D-IV

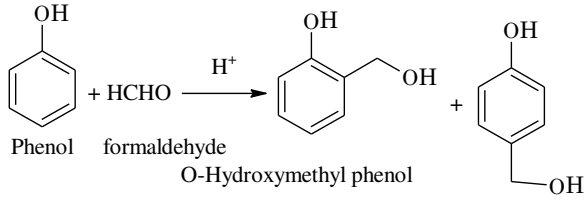
(C) A-II, B –I, C-III, D-IV

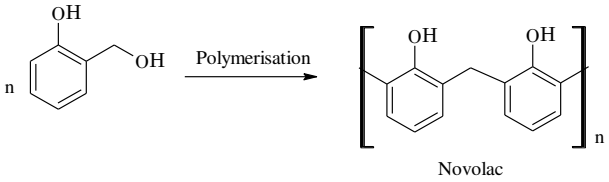
(D) A-III, B –II, C-IV, D-I

Official Ans. by NTA (B)

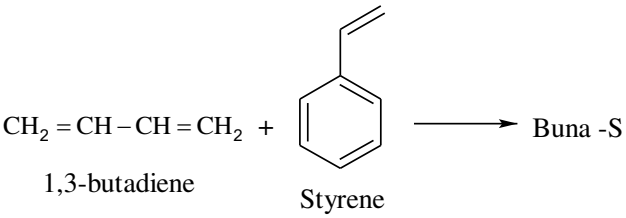
Sol.

(A)

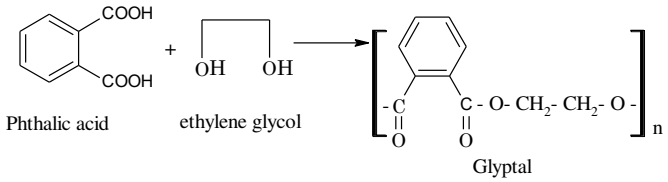




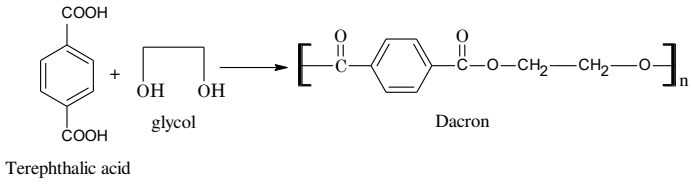
(B)



(C)



(D)



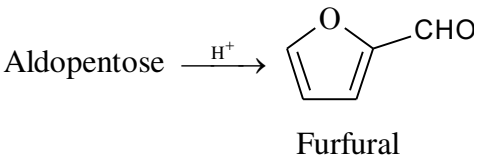
17. A sugar 'X' dehydrates very slowly under acidic condition to give furfural which on further reaction with resorcinol gives the coloured product after sometime. Sugar 'X' is

(A) Aldopentose (B) Aldotetrose

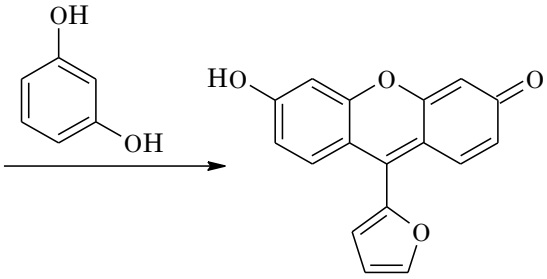
(C) Oxalic acid (D) Ketotetrose

Official Ans. by NTA (A)

Sol.

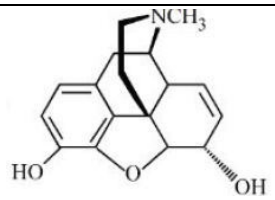
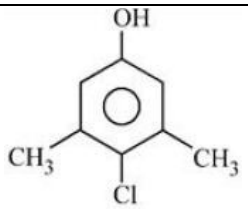
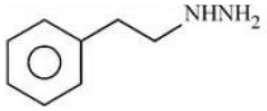
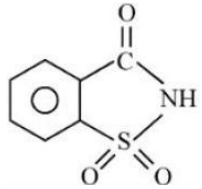


Furfural



Cherry red product
(seliwanoff's test)

18. Match **List I** with **List II**

List I	**List II**
A. 	I. Anti-depressant
B. 	II. 550 times sweeter than cane sugar
C. 	III. Narcotic analgesic
D. 	IV. Antiseptic

Choose the correct answer from the options given below:

(A) A-IV, B –III, C-II, D-I

(B) A-III, B –I, C-II, D-IV

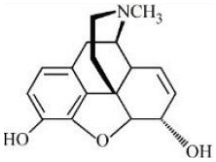
(C) A-III, B –IV, C-I, D-II

(D) A-III, B –I, C-IV, D-II

Official Ans. by NTA (C)

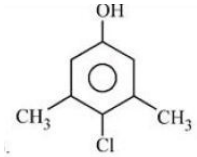
Sol.

(A)



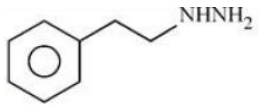
It is morphine use for relief for pain, known for narcotic analgesic

(B)



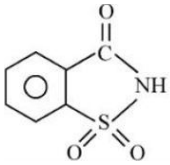
Chloroxylenol used as an antiseptic

(C)



Phenelzine (Nardil) use as Antidepressant

(D)



Saccharin 550 times sweeter than cane sugar

19. In Carius method of estimation of halogen. 0.45 g of an organic compound gave 0.36 g of AgBr. Find out the percentage of bromine in the compound.

(Molar masses : AgBr = 188 g mol^{-1}: Br = 80 g mol^{-1})

(A) 34.04% (B) 40.04%

(C) 36.03% (D) 38.04%

Official Ans. by NTA (A)

Sol. Mass of organic compound = 0.45 gm

Mass of AgBr obtained = 0.36 gm

$$\therefore \text{ Moles of AgBr} = \frac{0.36}{188}$$

$$\therefore \text{ Mass of Bromine} = \frac{0.36}{188} \times 80 = 0.1532\text{gm}$$

$$\therefore \text{ \% Br in compound} = \frac{0.1532}{0.45} \times 100 = 34.04\%$$

20. Match **List I** with **List II**

List I	List II
A. Benzenesulphonyl chloride	I. Test for primary amines
B. Hoffmann bromamide reaction	II. Anti Saytzeff
C. Carbylamine reaction	III. Hinsberg reagent
D. Hoffmann orientation	IV. Known reaction of Isocyanates.

Choose the correct answer from the options given below:

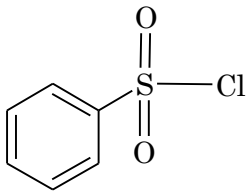
(A) A-IV, B –III, C-II, D-I

(B) A-IV, B –II, C-I, D-III

(C) A-III, B –IV, C-I, D-II

(D) A-IV, B –III, C-I, D-II

Official Ans. by NTA (C)

Sol. **(A)**  → Hinsberg reagent

Benzen sulphonyl chloride

(B) Hoffmann bromamide reaction → known reaction of isocynates

$R-CO-NH_2 + X_2 + 4\ NaOH \rightarrow R\text{-}NH_2 + 2NaX + Na_2CO_3 + 2H_2O$

Intermediate : R- N = C = O (isocyanate)

(C) Carbylamine reaction → Test for primary amine

$R\text{-}NH_2$ or $Ar - NH_2 + CHCl_3 + 3KOH \rightarrow RNC$ or $Ar\text{ - }NC + 3KCl + 3H_2O$

(D) Hoffmann orientation → Anti saytzeff (Formation of less substituted alkene as major product)

SECTION-B

1. 20 mL of 0.02 M $K_2Cr_2O_7$ solution is used for the titration of 10 mL of Fe^{2+} solution in the acidic medium.

The molarity of Fe^{2+} solution is _____ $\times 10^{-2}$ M. (Nearest Integer)

Official Ans. by NTA (24)

Sol. Eq. of $K_2Cr_2O_7$ = Eq. of Fe^{2+}

$\Rightarrow$ (Molarity × volume × n.f) of $K_2Cr_2O_7$ = (molarity × volume × n.f) of Fe^{2+}

$\Rightarrow 0.02 \times 20 \times 6 = M \times 10 \times 1$

$\Rightarrow M = 0.24$

$\Rightarrow$ Molarity $= 24 \times 10^{-2}$

2. $2NO + 2H_2 \rightarrow N_2 + 2H_2O$

The above reaction has been studied at 800°C. The related data are given in the table below

Reaction serial number	Initial pressure of H_2 / kPa	Initial Pressure of NO/ kPa	Initial rate $\left(\frac{-dp}{dt}\right)/(kPa/s)$
1	65.6	40.0	0.135
2	65.6	20.1	0.033
3	38.6	65.6	0.214
4	19.2	65.6	0.106

The order of the reaction with respect to NO is____

Official Ans. by NTA (2)

Sol. On decreasing pressure of NO by a factor of '2' the rate of reaction decreases by a factor of '4'.

$\therefore$ Order of reaction w.r.t. 'NO' = 2

3. Amongst the following the number of oxide(s) which are paramagnetic in nature is

Na_2O, KO_2, NO_2, N_2O, ClO_2, NO, SO_2, Cl_2O

Official Ans. by NTA (4)

Sol. KO_2, NO_2, ClO_2, NO are paramagnetic.

4. The molar heat capacity for an ideal gas at constant pressure is 20.785 J $K^{-1}mol^{-1}$. The change in internal energy is 5000 J upon heating it from 300K to 500K. The number of moles of the gas at constant volume is ___ [Nearest integer]

(Given: R = 8.314 J K^{-1} mol^{-1})

Official Ans. by NTA (2)

Sol. $C_{p,m} = C_{v,m} + R$

$\Rightarrow C_{v,m} = 20.785 - 8.314 = 12.471$ J k^{-1} ml^{-1}

$\Delta U = nC_{v,m}\Delta T$

$$\Rightarrow n = \frac{5000}{12.471\times 200} = \frac{25}{12.471} \approx 2$$

5. According to MO theory, number of species/ions from the following having identical bond order is_____:

$CN^-, NO^+, O_2, O_2^+, O_2^{2+}$

Official Ans. by NTA (3)

Sol. CN^-, NO^+, O_2^{2+} have bond order = 3

6. At 310 K, the solubility of CaF_2 in water is 2.34×10^{-3}g /100 mL. The solubility product of CaF_2 is ___ $\times 10^{-8}$ $(mol/L)^3$. (Given molar mass : CaF_2 = 78 g mol^{-1})

Official Ans. by NTA (0)

Sol. Solubility of CaF_2 = S mole/L

$$S = \frac{2.34\times 10^{-3}}{0.1\times 78} = \frac{2.34}{78}\times 10^{-2} = 3 \times 10^{-4} \text{ mol/L}$$

$K_{sp}(CaF_2) = 4S^3 = 4(3 \times 10^{-4})^3$

$= 108 \times 10^{-12}$

$= 0.0108 \times 10^{-8}$ $(mol/L)^3$

7. The conductivity of a solution of complex with formula $CoCl_3(NH_3)_4$ corresponds to 1 : 1 electrolyte, then the primary valency of central metal ion is_____

Official Ans. by NTA (1)

Sol. $[Co(NH_3)_4Cl_2]Cl$

Primary valency = oxidation no. = +3

8. In the titration of $KMnO_4$ and oxalic acid in acidic medium, the change in oxidation number of carbon at the end point is_____

Official Ans. by NTA (1)

Sol. Oxidation state of carbon changes from +3 to +4.

$$2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4(dil.) \rightarrow$$

$$K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$$

9. Optical activity of an enantiomeric mixture is +12.6° and the specific rotation of (+) isomer is +30°. The optical purity is____%

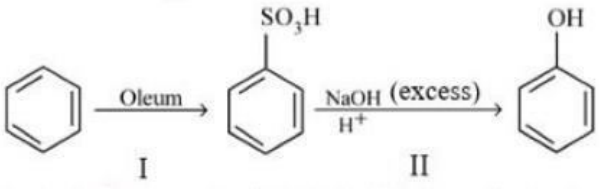
Official Ans. by NTA (42)

Sol.

$$\%\text{ optical purity} = \frac{\text{observed rotation of mixture}\times 100}{\text{rotation of pure enantiomer}}$$

$$= \frac{+12.6^\circ}{+30^\circ}\times 100 = 42$$

10. In the following reaction

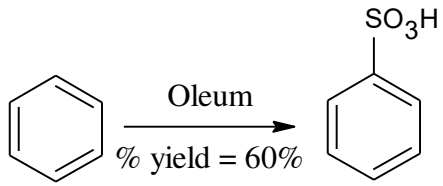


The % yield for reaction I is 60% and that of reaction II is 50%. The overall yield of the complete reaction is ___% [nearest integer]

Official Ans. by NTA (30)

Sol.

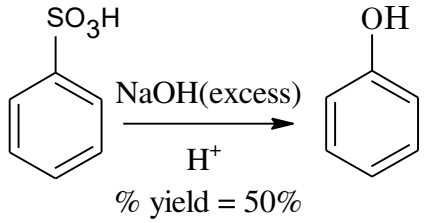
(I)



Let initial moles of reactant taken = n

Total moles obtained for benzene sulphonic acid (with % yield = 60%) = 0.6n

(II)



Moles of benzene sulphonic acid before reaction II = 0.6n

Moles obtained for phenol (with % yield = 50%) = 0.6×0.5n =0.3n

So over all % yield of complete reaction = $\frac{0.3n}{n} \times 100 = 30$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $a\ R_1\ b \Leftrightarrow ab \geq 0$ and $a\ R_2 b \Leftrightarrow a \geq b$, then

(A) R_1 is an equivalence relation but not R_2

(B) R_2 is an equivalence relation but not R_1

(C) both R_1 and R_2 are equivalence relations

(D) neither R_1 nor R_2 is an equivalence relation

Official Ans. by NTA (D)

Sol. $R_1 = \{xy \geq 0, x, y \in R\}$

For reflexive $x \times x \geq 0$ which is true.

For symmetric

If $xy \geq 0 \Rightarrow yx \geq 0$

If x = 2, y = 0 and z = -2

Then $x.y \geq 0$ & $y.z \geq 0$ but x.z ≥ 0 is not true

$\Rightarrow$ not transitive relation.

$\Rightarrow R_1$ is not equivalence

R_2 if $a \geq b$ it does not implies $b \geq a$

$\Rightarrow R_2$ is not equivalence relation

$\Rightarrow D$

2. Let $f, g: \mathbb{N} - \{1\} \to \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^{α} divides a, and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Official Ans. by NTA (D)

Sol. $f : N - \{1\} \to N$ $\quad f(a) = \alpha$

Where α is max of powers of prime P such that p^{α} divides a. Also g(a) = a + 1

$\therefore$ f(2) = 1 $\quad$ g(2) = 3

f (3) = 1 $\quad$ g(3) = 4

f(4) = 2 $\quad$ g(4) = 5

f(5) = 1 $\quad$ g(5) = 6

$\Rightarrow$ f (2) + g (2) = 4

$(f(3) + g(3)) = 5$

f(4) + g (4) = 7

f (5) + g (5) = 7

$\therefore$ Many one f(x) + g(x) does not cotain 1

$\Rightarrow$ into function

$\therefore$ Ans. (D) [neither one-one nor onto]

3. Let the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2$, $z \in \mathbb{C}$ is attained at z = z_0. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to

(A) 1000 $\quad$ (B) 1024

(C) 1105 $\quad$ (D) 1196

Official Ans. by NTA (A)

Sol. $z_0 = \left(\frac{0+3+0}{3}, \frac{0+6+0}{3}\right) = (1, 2)$

$v_0 = |1+2i|^2 + |1+2i-3|^2 + |1+2i-6i|^2 = 30$

Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$

$= \left|2(1+2i)^2 - (1-2i)^3 + 3\right|^2 + 900$

$= \left|2(1-4+4i) - (1-4-4i)(1-2i) + 3\right|^2 + 900$

$= |8+6i|^2 + 900 = 100 + 900 = 1000$

4. Let $A=\begin{pmatrix}1 & 2\\ -2 & -5\end{pmatrix}$. Let $\alpha,\beta\in\mathbb{R}$ be such that $\alpha A^2+\beta A=2I$. Then $\alpha+\beta$ is equal to -

(A) -10 (B) -6

(C) 6 (D) 10

Official Ans. by NTA (D)

Sol. Characteristic equation of matric A

$|A-\lambda I|=0$

$\begin{vmatrix}1-\lambda & 2\\ -2 & -5-\lambda\end{vmatrix}=0$

$\Rightarrow \lambda^2+4\lambda=1$

$\Rightarrow A^2+4A=I$

$\Rightarrow 2A^2+8A=2I$ …….. (1)

Given that $\alpha A^2+\beta A=2I$ …… (2)

Comparing equation (1) & (2) we get

$\alpha=2,\quad \beta=8$

$\therefore \alpha+\beta=10$

Ans. (D) (10)

5. The remainder when $(2021)^{2022}+(2022)^{2021}$ is divided by 7 is

(A) 0 (B) 1

(C) 2 (D) 6

Official Ans. by NTA (A)

Sol. $(2021)^{2022}+(2022)^{2021}$

$=(2023-2)^{2022}+(2023-1)^{2021}$

$=7n_1+2^{2022}+7n_2-1$

$=7(n_1+n_2)+8^{674}-1$

$=7(n_1+n_2)+(7-1)^{674}-1$

$=7(n_1+n_2)+7n_3+1-1$

$=7(n_1+n_2+n_3)$

$\therefore$ Given number is divisible by 7 hence remainder is zero

6. Suppose $a_1,a_2,\ldots,a_n,\ldots$ be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms of the sum of first nine terms of the progression is 5 : 17 and $110<a_{15}<120$, then the sum of the first ten terms of the progression is equal to -

(A) 290 (B) 380

(C) 460 (D) 510

Official Ans. by NTA (B)

Sol. $\frac{S_5}{S_9}=\frac{5}{17}\Rightarrow\frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)}=\frac{5}{17}$

$\Rightarrow d=4a$

$a_{15}=a+14d=57a$

Now, $110<a_{15}<120$

$\Rightarrow 110<57a<120$

$\Rightarrow a=2\ \therefore\ d=8$

$S_{10}=\frac{10}{2}(2\times2+9\times8)=380$

7. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined as

$f(x)=a\sin\left(\frac{\pi[x]}{2}\right)+[2-x],\ a\in\mathbb{R}$, where [t] is the greatest integer less than or equal to t. If $\lim\limits_{x\to-1}f(x)$ exists, then the value of $\int_0^4 f(x)dx$ is equal to :

(A) -1 (B) -2

(C) 1 (D) 2

Official Ans. by NTA (B)

Sol. $\lim\limits_{x\to -1^+} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x]=-a+2$

$\lim\limits_{x\to -1^-} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x]=0+3=3$

$\lim\limits_{x\to -1} f(x)$ exist when $a=-1$

Now,

$\int_0^4 f(x)dx=\int_0^1 f(x)dx+\int_1^2 f(x)dx+\int_2^3 f(x)dx+\int_3^4 f(x)dx$

$=\int_0^1(0+1)dx+\int_1^2(-1+0)dx+\int_2^3(0-1)dx+\int_3^4(1-2)dx$

$=1-1-1-1=-2$

8. $I=\int_{\pi/4}^{\pi/3}\left(\frac{8\sin x-\sin 2x}{x}\right)dx$. Then

(A) $\frac{\pi}{2}<I<\frac{3\pi}{4}$

(B) $\frac{\pi}{5}<I<\frac{5\pi}{12}$

(C) $\frac{5\pi}{12}<I<\frac{\sqrt{2}}{3}\pi$

(D) $\frac{3\pi}{4}<I<\pi$

Official Ans. by NTA (C)

Sol. Consider

$f(x) = 8\sin x - \sin 2x$

$f'(x) = 8\sin x - 2\cos 2x$

$f''(x) = -8\sin x + 4\sin 2x$

$= -8\sin x(1-\cos x)$

$\therefore f''(x) < 0 \quad x\in\left(\frac{\pi}{4},\frac{\pi}{3}\right)$

$\therefore f'(x)$ is $\downarrow$ function

$f'\left(\frac{\pi}{3}\right)<f'(x)<f'\left(\frac{\pi}{4}\right)$

$5<f'(x)<\frac{8}{\sqrt{2}}$

$5<f'(x)<4\sqrt{2}$

$5x<f(x)<4\sqrt{2}x$

$5<\frac{f(x)}{x}<4\sqrt{2}$

$\int_{\pi/4}^{\pi/3}5<\int\frac{f(x)}{x}<\int_{\pi/4}^{\pi/3}4\sqrt{2}$

$\int_{\pi/4}^{\pi/3}5<\int\frac{8\sin x-\sin 2x}{x}<\int_{\pi/4}^{\pi/3}4\sqrt{2}$

$\frac{5\pi}{12}<I<\frac{\sqrt{2}\pi}{3}$

9. The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2+y^2+4\sqrt{3}x-4=0$ is equal to

(A) $\frac{1}{3}\left(2-12\sqrt{3}+8\pi\right)$

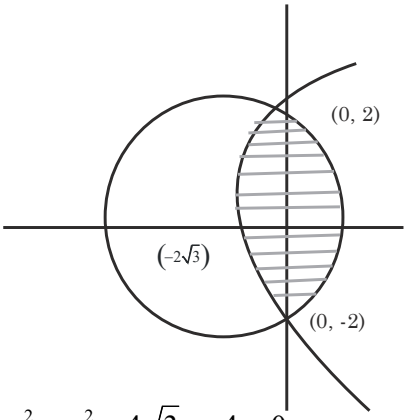
(B) $\frac{1}{3}\left(2-12\sqrt{3}+6\pi\right)$

(C) $\frac{1}{3}\left(4-12\sqrt{3}+8\pi\right)$

(D) $\frac{1}{3}\left(4-12\sqrt{3}+6\pi\right)$

Official Ans. by NTA (C)

Sol.



$x^2+y^2+4\sqrt{3}x-4=0$

$y^2 = 8x + 4$

Point of intersections are (0, 2) & (0, -2)

Both are symmetric about x-axis

Area $=2\int_0^2\left(\sqrt{16-y^2}-2\sqrt{3}\right)-\left(\frac{y^2-4}{8}\right)dy$

On solving $Area=\frac{1}{3}\left[8\pi+4-12\sqrt{3}\right]$

10. Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (A)

Sol. $\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$

1f = e^{-x}

$\therefore$ solution is $ye^{-x} = \int xe^{-x}dx$

$\Rightarrow ye^{-x} = -xe^{-x} - e^{-x} + c$

$\Rightarrow y = -x - 1 + ce^{x}$

$y_1(0) = 0 \Rightarrow c = 1$

$\therefore y_1 = -x - 1 + e^{x}$ ....(1)

$y_2(0) = 1 \Rightarrow c = 2$

$\therefore y_2 = -x - 1 + 2e^{x}$ .....(2)

Now $y_2 - y_1 = e^{x} > 0 \therefore y_2 \neq y_1$

$\therefore$ Number of points of intersection of y_1 & y_2 is zero.

11. Let P (*a*, b) be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of *a* and *B* be the product of all possible values of *b*. Then the value of A + B is equal to :

(A) 0 (B) 25

(C) 40 (D) 65

Official Ans. by NTA (D)

Sol. P(a, b) is point on $y^2 = 8x$, such that tangent at P pass through centre of $x^2 + y^2 - 10x - 14y + 65 = 0$ i.e. (5, 7)

Tangent at P(at^2, 2at) is $ty = x + at^2$

A = 2 & it pass through (5, 7)

$7t = 5 + 2t^2$

$\Rightarrow t = 1, t = \frac{5}{2}$

$\therefore P\left(at^2, 2at\right) \Rightarrow (2,4)$ when t = 1

& $\left(\frac{25}{2}, 10\right)$ when $t = \frac{5}{2}$

$\therefore A = 2 \times \frac{25}{2} = 25$

B = 4 × 10 = 40 $\therefore A + B = 65$

12. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{i} + 12k$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

(A) 2 (B) $\frac{39}{5}$

(C) 9 (D) $\frac{46}{5}$

Official Ans. by NTA (D)

Sol. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix}$$

$\Rightarrow (4 + 5\beta)\hat{i} + (3\beta - 4\alpha)\hat{j} + (-5\alpha - 3)\hat{k}$

$= -\hat{i} + 9\hat{j} + 12\hat{k}$

$\therefore 4 + 5\beta = -1, 3\beta - 4\alpha = 9, -5\alpha - 3 = 12$

$\beta = -1, \quad \alpha = -3$

$\therefore \vec{a} = -3\hat{i} + \hat{j} - \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\therefore \vec{a} + \vec{b} = -4\hat{j} + 3\hat{k}$

$|\vec{a}|^2 = 11, |\vec{b}|^2 = 50$

$\vec{a}.\vec{b} = -9 + (-5) - 4 = -18$

$\therefore$ Projectile of $\left(\vec{b}-2\vec{a}\right)$ on $\vec{a}+\vec{b}$ is

$$\frac{\left(\vec{b}-2\vec{a}\right).\left(\vec{a}+\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|}$$

$$=\frac{\left|\vec{b}\right|^2-2\left|\vec{a}\right|^2-\left(\vec{a}.\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|}=\frac{50-22-(-18)}{5}=\frac{46}{5}$$

Ans. $\left(\frac{46}{5}\right)$

13. Let $\vec{a}=2\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=\alpha\hat{i}+\beta\hat{j}+2\hat{k}$. If $\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right).\hat{k}=\frac{23}{2}$, then $\left|\vec{b}\times 2\hat{j}\right|$ is equal to

(A) 4 (B) 5

(C) $\sqrt{21}$ (D) $\sqrt{17}$

Official Ans. by NTA (B)

Sol. $\vec{a}=2\hat{i}-\hat{j}+5\hat{k},\ \vec{b}=\alpha\hat{i}+\beta\hat{j}+2\hat{k}$

$\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right).\hat{k}=\frac{23}{2}$, then $\left|\vec{b}\times 2\hat{j}\right|$ is

$$\left(\left(\vec{a}.\hat{i}\right)\vec{b}-\left(\vec{b}.\hat{i}\right)\vec{a}\right).\hat{k}=\frac{23}{2}$$

$$\left(\vec{a}.\hat{i}\right)\left(\vec{b}.\hat{i}\right)-\left(\vec{b}.\hat{i}\right)\left(\vec{a}.\hat{k}\right)=\frac{23}{2}$$

$$2\times 2-\alpha\times 5=\frac{23}{2}\Rightarrow 5\alpha=4-\frac{23}{2}\Rightarrow\alpha=\frac{-3}{2}$$

$$\vec{b}\times 2\hat{j}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\ \alpha&\beta&2\\ 0&2&0\end{vmatrix}=-4\hat{i}+2\alpha\hat{k}$$

$$\therefore\left|\vec{b}\times 2\hat{j}\right|=\sqrt{16+4\alpha^2}=\sqrt{16+4\times\frac{9}{4}}=5$$

14. Let S be the sample space of all five digit numbers. If p is the probability that a randomly selected number from S, is a multiple of 7 but not divisible by 5, then $9p$ is equal to

(A) 1.0146 (B) 1.2085

(C) 1.0285 (D) 1.1521

Official Ans. by NTA (C)

Sol. n(S) = all 5 digit nos $= 9\times 10^4$

A : no is multiple of 7 but not divisible by 5

Smallest 5 digit divisible by 7 is 10003

Largest 5 digit divisible by 7 is 99995

$\therefore$ 99995 = 10003 + (n – 1) 7 n = 12857

Numbers divisible by 35

99995 = 10010 + (P-1) 35 $\Rightarrow$ P = 2572

$\therefore$ Numbers divisible by 7 but not by 35 are

12857 – 2572 = 10285

$\therefore P=\frac{10285}{90000}$ $\therefore$ 9P = 1.0285

Ans. (C) [1.0285]

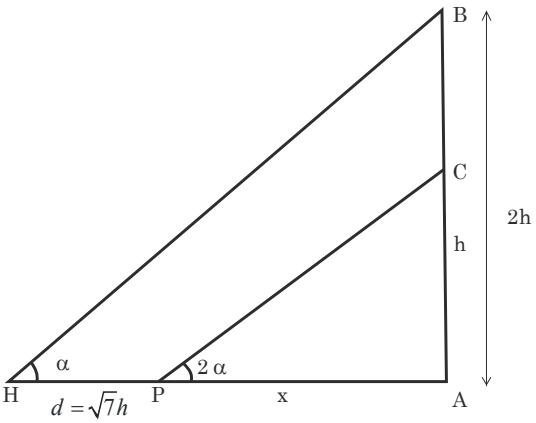
15. Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α. If $d=\sqrt{7}h$, then tan α is equal to

(A) $\sqrt{5}-2$ (B) $\sqrt{3}-1$

(C) $\sqrt{7}-2$ (D) $\sqrt{7}-\sqrt{3}$

Official Ans. by NTA (C)

Sol.



$$\tan 2\alpha=\frac{h}{x}$$

and $\tan\alpha = \dfrac{2h}{x+\sqrt{7}h}$

$$\tan\alpha = \frac{2h}{h\cot 2\alpha + \sqrt{7}h}$$

$$\tan\alpha = \frac{2}{\frac{\left(1-\tan^2\alpha\right)}{2\tan\alpha} + \sqrt{7}}$$

Put $\tan\alpha = t$ & simplify

$\Rightarrow \tan\alpha = \sqrt{7} - 2$

16. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is

(A) p (B) $\sim p$

(C) q (D) $\sim q$

Official Ans. by NTA (C)

Sol. Given $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$

Taking r = q

p	q	~p	~q	p^q	P^~q	$(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

So, clear $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$

17. If the plane P passes through the intersection of two mutually perpendicular planes 2x + ky – 5z = 1 and 3kx – ky + z = 5, k < 3 and intercepts a unit length on positive x-axis, then the intercept made by the plane P on the y-axis is

(A) $\frac{1}{11}$ (B) $\frac{5}{11}$

(C) 6 (D) 7

Official Ans. by NTA (D)

Sol. Two given planes mutually perpendicular

2(3k) + k (-k) + (-5) 1 = 0

k = 1, 5

but k < 3 So k = 1

Plane passing through these planes is

2x + y – 5z – 1 + λ (3x – y + z – 5) = 0

$$\frac{x}{\frac{5\lambda+1}{2+3\lambda}} + \frac{y}{\frac{5\lambda+1}{1-\lambda}} + \frac{z}{\frac{5\lambda+1}{\lambda-5}} = 1$$

Given $\dfrac{5\lambda+1}{2+3\lambda} = 1 \Rightarrow \lambda = \dfrac{1}{2}$

So intercept on y - axis = $\dfrac{5\lambda+1}{1-\lambda} = 7$

18. Let A(1, 1), B(-4, 3) C(-2, -5) be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangle APB and ABC. Respectively.

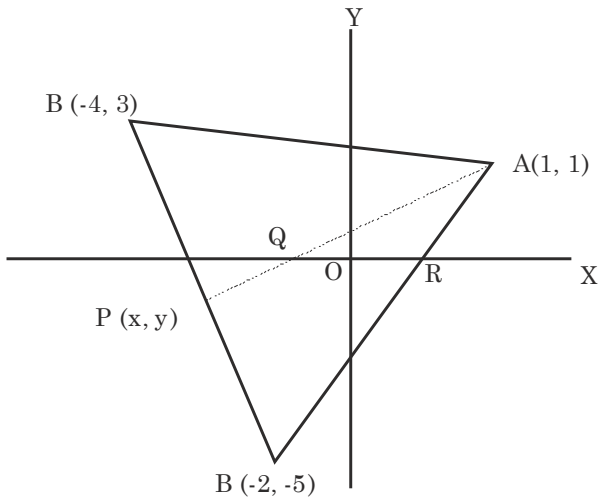
If $\Delta_1 : \Delta_2 = 4:7$, then the area enclosed by the lines AP, AC and the x-axis is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{1}{2}$ (D) 1

Official Ans. by NTA (C)

Sol.



Given $\Delta_1 = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$

& $\Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -4 & 3 & 1 \\ -2 & -5 & 1 \end{vmatrix}$

Given $\frac{\Delta_1}{\Delta_2} = \frac{4}{7} \Rightarrow \frac{-2x-5y+7}{36} = \frac{4}{7}$

$\Rightarrow 14x + 35y = -95$ ....(1)

Equation of BC is 4x + y = -13 ....(2)

Solve equation (1) & (2)

Point $P\left(\frac{-20}{7}, \frac{-11}{7}\right)$

Here point $Q\left(\frac{-1}{2}, 0\right)$ & $R\left(\frac{1}{2}, 0\right)$

So Area of triangle AQR $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

19. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, $g, c \in \mathbb{R}$ passes through the point (6, 1) and its centre lies on the line x – 2cy = 8, then the length of intercept made by the circle on x-axis is

(A) $\sqrt{11}$ (B) 4

(C) 3 (D) $2\sqrt{23}$

Official Ans. by NTA (D)

Sol. Given circle $x^2 + y^2 - 2gx + 6y - 19c = 0$

Passes through (6, 1)

12g + 19 c = 43 ....(1)

Centre (g, -3) lies on given line

So, g + 6c = 8 ....(2)

Solve equation (1) & (2)

c = 1 & g = 2

equation of circle $x^2 + y^2 - 4x + 6y - 19 = 0$

Length of intercept on x-axis

$= 2\sqrt{g^2 - c} = 2\sqrt{23}$

20. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} \int_0^x (5 - |t-3|)\,dt, & x > 4 \\ x^2 + bx \quad , & x \le 4 \end{cases}$$

where $b \in \mathbb{R}$. If f is continuous at x = 4, then which of the following statements is NOT true ?

(A) f is not differentiable at x = 4

(B) $f'(3) + f'(5) = \frac{35}{4}$

(C) f is increasing in $\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$

(D) f has a local minima at $x = \frac{1}{8}$

Official Ans. by NTA (C)

Sol. Given $f(x)\begin{cases} \int_0^x (5 - |t-3|)\,dt, & x > 4 \\ x^2 + bx \quad , & x \le 4 \end{cases}$

f(x) is continuous at x = 4

So $\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x) = f(4)$

So $16 + 4b = \int_0^3 (2 - t)\,dt + \int_3^4 (8 - t)\,dt$

$\Rightarrow 16 + 4b = 15$

So $b = \frac{-1}{4}$

At x = 4

LHD = 2x + b = $\frac{31}{4}$

RHD = 5 - |x – 3| = 4

LHD ≠ RHD

Option (A) is true

and f'(3) + f'(5) = $\frac{23}{4} + 3 = \frac{35}{4}$

Option (B) is true

$\because f(x) = x^2 - \frac{x}{4}$ at $x \le 4$

$f'(x) = 2x - \frac{1}{4}$

This function is not increasing.

In the interval in $x \in \left(-\infty, \frac{1}{8}\right)$

Option (C) is NOT TRUE.

This function f(x) is also local minima at $x = \frac{1}{8}$

SECTION-B

1. For $k \in \mathbb{R}$, let the solutions of the equation $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right) = k, 0 < |x| < \frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to _____.

Official Ans. by NTA (12)

Sol. $\cos\left(\sin^{-1}x\right) = \cos\left(\cos^{-1}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

$\cot\left(\tan^{-1}\sqrt{1-x^2}\right) = \cot\ \cot^{-1}\left(\sqrt{\frac{1}{\sqrt{1-x^2}}}\right) = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right) = \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$

$\Rightarrow \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = k$

$\Rightarrow 1-2x^2 = k^2\left(1-x^2\right)$

$\Rightarrow \left(k^2-2\right)x^2 = k^2 - 1$

$x^2 = \frac{k^2-1}{k^2-2}$

$\alpha = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \alpha^2 = \frac{k^2-1}{k^2-2}$

$\beta = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \beta^2 = \frac{k^2-1}{k^2-2}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2\left(\frac{k^2-2}{k^2-1}\right)$ & $\frac{\alpha}{\beta} = -1$

Sum of roots = $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\alpha}{\beta} = b$

$\Rightarrow \frac{2\left(k^2-2\right)}{k^2-1} - 1 = b$ .....(1)

Product of roots $= \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\frac{\alpha}{\beta} = -5$

$\Rightarrow \frac{2\left(k^2-2\right)}{k^2-1}(-1) = -5$

$\Rightarrow 2k^2 - 4 = 5k^2 - 5$

$\Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3}$ .... Put in (1)

$\Rightarrow b = \frac{2\left(k^2-2\right)}{k^2-1} - 1 = 5 - 1 = 4$

$\frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$

2. The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is _____.

Official Ans. by NTA (2)

Sol. n = 10, $\bar{x} = \frac{\sum x_i}{10} = 15$

$6^2 = \frac{\sum x_i^2}{10} - \left(\bar{x}\right)^2 = 15$

$\Rightarrow \sum_{i=1}^{10} x_i = 150$

$$\Rightarrow \sum_{i=1}^{9} x_i + 25 = 150$$

$$\Rightarrow \sum_{i=1}^{9} x_i = 125$$

$$\Rightarrow \sum_{i=1}^{9} x_i + 15 = 140$$

Actual mean $= \frac{140}{10} = 14 = \bar{x}_{new}$

$$\sum_{i=1}^{9} \frac{x_i^2 + 25^2 - 15^2}{10} = 15$$

$$\Rightarrow \sum_{i=1}^{9} x_i^2 + 625 = 2400$$

$$\sum_{i=1}^{9} x_i^2 = 1775$$

$$\sum_{i=1}^{9} x_i^2 + 15^2 = 2000 = \left(\sum x_i^2\right)_{actual}$$

$$6^2_{actual} = \frac{\left(\sum x_i^2\right)_{actual} - \left(\bar{x}_{new}\right)^2}{10}$$

$$= \frac{2000}{10} - 14^2$$

= 200-196 = 4

$(S.D)_{actual} = 6 = 2$

3. Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $4ax - y + 5z - 7a = 0$ = 2x – 5y – z – 3, $a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha + \beta + \gamma$ equals _____.

Official Ans. by NTA (12)

Sol. Equation of plane

$4ax - y + 5z - 7a + \lambda$ (2x – 5y – z – 3)= 0

this satisfy (4, -1, 0)

$16a + 1 - 7a + \lambda(8 + 5 - 3) = 0$

$9a + 1 + 10\lambda = 0$ .....(1)

Normal vector of the plane A is $(4a + 2\lambda, -1 - 5\lambda, 5 - \lambda)$ vector along the line which contained the plane A is

i – 2j + k

$\therefore 4a + 2\lambda + 2 + 10\lambda + 5 - \lambda = 0$

$11\lambda + 4a + 7 = 0$ .....(2)

Solve (1) and (2) to get a = 1, $\lambda = -1$

Now equation of plane

$x + 2y + 3z - 2 = 0$

Let the point in the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4} = t$

is $(7t + 3, -t + 2, -4t + 3)$ satisfy the equation of plane A

7t + 3 – 2t + 4 + 9 -12t – 2 = 0

t = 2

So $\alpha + \beta + \gamma = 2t + 8 = 12$

4. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E, then the value of 113l *is* equal to _____.

Official Ans. by NTA (1552)

Sol. Hyp : $\frac{y^2}{64} - \frac{x^2}{49} = 1$

An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$.

So $b^2 = 64$

$$e_H = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{1+\frac{49}{64}}$$

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$e_E = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{a^2}{64}}$$

$$b = 8, \sqrt{\frac{1-a^2}{64}} \times \frac{\sqrt{113}}{8} = \frac{1}{2} \Rightarrow \sqrt{64-a^2} \times \sqrt{113} = 32$$

$$\left(64-a^2\right) = \frac{32^2}{113}$$

$$\Rightarrow a^2 = 64 - \frac{32^2}{113}$$

$$l = \frac{2a^2}{b} = \frac{2}{8}\left(64 - \frac{32^2}{113}\right) = \frac{1552}{113}$$

$$113l = 1552$$

5. Let y = y(x) be the solution curve of the differential equation

$$\sin\left(2x^2\right)\log_e\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx = 0,$$

$0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to _____.

Official Ans. by NTA (1)

Sol. $$\sin\left(2x^2\right)\ln\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx = 0$$

$$\ln\left(\tan x^2\right)dy + \frac{4xydx}{\sin\left(2x^2\right)} - \frac{4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)}{\sin\left(2x^2\right)}dx = 0$$

$$d\left(y.\ln\left(\tan x^2\right)\right) - 4\sqrt{2}x\frac{\left(\sin x^2 - \cos x^2\right)}{\sqrt{2}-2\sin x^2\cos x^2}dx = 0$$

$$d\left(y\ln\left(\tan x^2\right)\right) - \frac{4x\left(\sin x^2 - \cos x^2\right)}{\left(\sin x^2 + \cos^2\right)-1}dx = 0$$

$$\Rightarrow \int d\left(y\ln\left(\tan x^2\right)\right) + 2\int\frac{dt}{t^2-1} = \int 0$$

$$\Rightarrow y\ln\left(\tan x^2\right) + 2.\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| = c$$

$$y\ln\left(\tan x^2\right) + \ln\left(\frac{\sin x^2 + \cos x^2 - 1}{\sin x^2 + \cos x^2 + 1}\right) = c$$

Put y = 1 and $x = \sqrt{\frac{\pi}{6}}$

$$1\ln\left(\frac{1}{\sqrt{3}}\right) + \ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)} = c$$

Now $x = \sqrt{\frac{\pi}{3}} \Rightarrow y\left(\ln\sqrt{3}\right) + \ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)} = \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{\sqrt{3}-1}{\sqrt{3}+3}\right)$

$$y\left(\ln\sqrt{3}\right) = \ln\left(\frac{1}{\sqrt{3}}\right)$$

$\Rightarrow$ y = -1

$$|y| = 1$$

6. Let *M* and *N* be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of M + N equals _____.

Official Ans. by NTA (2)

Sol. $y^5 - 9xy + 2x = 0$

$$5y^4\frac{dy}{x} - 9x\frac{dy}{dx} - 9y + 2 = 0$$

$$\frac{dy}{dx}\left(5y^4 - 9x\right) = 9y - 2$$

$$\frac{dy}{dx} = \frac{9y-2}{5y^4-9x} = 0 \text{ (for horizontal tangent)}$$

$y = \frac{2}{9} \Rightarrow$ Which does not satisfy the original equation $\Rightarrow$ M = 0.

Now $5y^4 - 9x = 0$ (for vertical tangent)

$5y^4 (9y - 2) - 9y^5 = 0$

$y^4[45y - 10 - 9y] = 0$

$y = 0$ (Or) $36y = 10$

$$y = \frac{5}{18}$$

$$y = 0 \Rightarrow x = 0 \;\&\; y = \frac{5}{18} \Rightarrow x =$$

$(0,0)$ $\left(x, \frac{5}{18}\right)$

N = 2

M + N = 0 + 2 = 2

7. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \le 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _____.

Official Ans. by NTA (10620)

Sol. $f(x) = 2x^2 - x - 1$

$|f(x)| \le 800$

$2n^2 - n - 801 \le 0$

$$n^2 - \frac{1}{2}n - \frac{801}{2} \le 0$$

$$\left(n - \frac{1}{4}\right)^2 - \frac{801}{2} - \frac{1}{16} \le 0$$

$$\left(n - \frac{1}{4}\right)^2 - \frac{6409}{16} \le 0$$

$$\left(n - \frac{1}{4} - \frac{\sqrt{6409}}{4}\right)\left(n - \frac{1}{4} + \frac{\sqrt{6409}}{16}\right) \le 0$$

$$\frac{1 - \sqrt{6409}}{4} \le n \le \frac{1 + \sqrt{6409}}{4}$$

$$n = \{-19, -18 - 17, \ldots\ldots 0, 1, 2, \ldots\ldots, 20\}$$

$$\sum_{n \in S} f(x) = \sum \left(2x^2 - x - 1\right)$$

$$= 2\left[19^2 + 18^2 + \ldots\ldots + 1^2 + 1^2 + 2^2 + \ldots\ldots + 19^2 + 20^2\right]$$

$$= 4\left[1^2 + 2^2 + \ldots\ldots + 19^2\right] + 2\left[20^2\right] - 20 - 40$$

$$= \frac{4 \times 19 \times 20 \times (2 \times 19 + 1)}{6} + 2 \times 400 - 60$$

$$= \frac{4 \times 19 \times 20 \times 39}{6} + 800 - 60 - 9880 + 800 - 60$$

= 10620

8. Let S be the set containing all 3 × 3 matrices with entries from {-1, 0, 1}. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is _____.

Official Ans. by NTA (5376)

Sol. $Tr\left(AA^T\right) = 6$

$$AA^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Now given $a^2 + d^2 + g^2 + b^2 + e^2 + h^2 + c^2 + f^2 + i^2 = 6$

$= {}^9C_3 \times 2^6$

= 5376

9. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to _____.

Official Ans. by NTA (75)

Sol. $\lambda + \ell = 75$

$$x^2 + 4y^2 + 2x + 8y - \lambda = 0$$

$$\frac{(x+1)^2}{\lambda+5} + \frac{(y+1)^2}{\frac{\lambda+5}{4}} = 1$$

$$\because \frac{2b^2}{a} = 4$$

$$\frac{2(\lambda+5)}{4} = 4\left(\sqrt{\lambda+5}\right)$$

$$\Rightarrow \lambda = 59$$

$$\lambda \neq -5$$

$$l = 2a = 2\sqrt{\lambda+5} = 2\sqrt{65} = 16$$

$$\Rightarrow \lambda + \ell = 59 + 16 = 75$$

10. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\mathrm{Re}(z) + \mathrm{Im}(z))$ is equal to _____.

Official Ans. by NTA (0)

Sol. $S = \{z \in C : z^2 + \bar{z} = 0\}$

Let z = x + iy

$z^2 = x^2 - y^2 + 2ixy$

$\bar{z} = x - iy$

$$z^2 + \bar{z} = x^2 - y^2 + x + i(2xy - y) = 0$$

$$\Rightarrow x^2 + x - y^2 = 0 \ \& \ 2xy - y = 0$$

$$y = 0 \text{ or } x = \frac{1}{2}$$

If y = 0; x = 0, - 1

If $x = \frac{1}{2}; y = \frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

$$\sum_{z \in S} \left(\mathrm{Re}(z) + \mathrm{Im}(z) = \left(0 - 1 + \frac{1}{2} + \frac{1}{2}\right) + 0 + 0 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right)$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

1. An expression of energy density is given by $u = \frac{\alpha}{\beta}\sin\left(\frac{\alpha x}{kt}\right)$, where α, β are constants, x is displacement, k is Boltzmann constant and t is the temperature. The dimensions of β will be :

(A) $[ML^2T^{-2}\theta^{-1}]$ (B) $[M^0L^2T^{-2}]$

(C) $[M^0L^0T^0]$ (D) $[M^0L^2T^0]$

Official Ans. by NTA (D)

Sol. $\frac{\alpha[L]}{[ML^2T^{-2}]} = [M^0L^0T^0]$

$\alpha = [ML^1T^{-2}]$

$\frac{\alpha}{\beta} = \frac{[ML^2T^{-2}]}{[L^3]} \Rightarrow \beta = \frac{[ML^1T^{-2}][L^3]}{ML^2T^{-2}}$

2. A body of mass 10 kg is projected at an angle of 45° with the horizontal. The trajectory of the body is observed to pass through a point (20, 10). If T is the time of flight, then its momentum vector, at time $t = \frac{T}{\sqrt{2}}$, is _____

[Take g = 10 m/s^2]

(A) $100\hat{i} + (100\sqrt{2} - 200)\hat{j}$

(B) $100\sqrt{2}\,\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(C) $100\,\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(D) $100\sqrt{2}\,\hat{i} + (100\sqrt{2} - 200)\hat{j}$

Official Ans. by NTA (D)

Sol. $y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)} \Rightarrow 10 = 20 - \frac{(10)(100)}{u^2}$

$u = 20$

$T = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$

$\vec{v} = 10\sqrt{2}\,\hat{i} + (10\sqrt{2} - 10(2))\hat{j}$

Momentum $\vec{p} = M\vec{v} = 100\sqrt{2}\,\hat{i} + (100\sqrt{2} - 200)\hat{j}$

3. A block of mass M slides down on a rough inclined plane with constant velocity. The angle made by the incline plane with horizontal is θ. The magnitude of the contact force will be :

(A) Mg

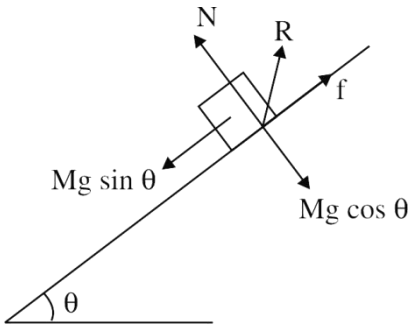
(B) Mg cos θ

(C) $\sqrt{Mg\sin\theta + Mg\cos\theta}$

(D) $Mg\sin\theta\sqrt{1+\mu}$

Official Ans. by NTA (A)

Sol.



$N = Mg\cos\theta$

$f = Mg\sin\theta$

$R = \sqrt{N^2 + f^2}$

$R = Mg$

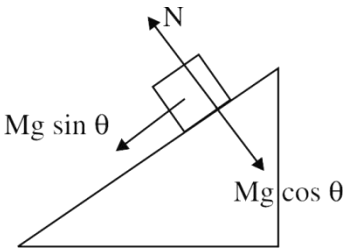
4. A block 'A' takes 2 s to slide down a frictionless incline of 30° and length '*l*', kept inside a lift going up with uniform velocity '*v*'. If the incline is changed to 45°, the time taken by the block, to slide down the incline, will be approximately:

(A) 2.66 s (B) 0.83 s

(C) 1.68 s (D) 0.70 s

Official Ans. by NTA (C)

Sol.



$a = g \sin\theta$

$$\ell = \frac{1}{2} g \sin 30° (2)^2 \quad \ldots.(1)$$

$$\ell = \frac{1}{2} g \sin 45° \, t^2 \quad \ldots.(2)$$

$$\left(\frac{1}{2}\right)(4) = \frac{1}{\sqrt{2}} t^2 \Rightarrow t = \sqrt{2\sqrt{2}} \simeq 1.68$$

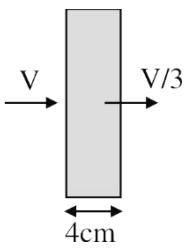
5. The velocity of the bullet becomes one third after it penetrates 4 cm in a wooden block. Assuming that bullet is facing a constant resistance during its motion in the block. The bullet stops completely after travelling at (4 + x) cm inside the block. The value of x is:

(A) 2.0 (B) 1.0

(C) 0. 5 (D) 1.5

Official Ans. by NTA (C)

Sol.



$$\left(\frac{V}{3}\right)^2 = V^2 - 2a(4) \Rightarrow a = \frac{8V^2}{9(8)} = \frac{V^2}{9}$$

$$0 = V^2 - 2a(4 + x)$$

$$\Rightarrow \quad V^2 = 2\left(\frac{V^2}{9}\right)(4 + x)$$

$$4.5 = 4 + x$$

$$x = 0.5$$

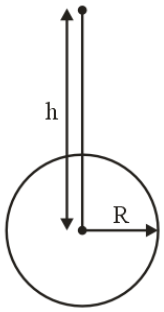
6. A body of mass m is projected with velocity λv_e in vertically upward direction from the surface of the earth into space. It is given that v_e is escape velocity and $\lambda < 1$. If air resistance is considered to the negligible, then the maximum height from the centre of earth, to which the body can go, will be (R : radius of earth)

(A) $\frac{R}{1+\lambda^2}$ (B) $\frac{R}{1-\lambda^2}$

(C) $\frac{R}{1-\lambda}$ (D) $\frac{\lambda^2 R}{1-\lambda^2}$

Official Ans. by NTA (B)

Sol.



$$-\frac{GMm}{R} + \frac{1}{2} m\lambda^2 V_e^2 = -\frac{GMm}{h}$$

$$-\frac{GMm}{R} + \frac{1}{2}\lambda^2 \frac{2GMm}{R} = -\frac{GMm}{h}$$

$$\frac{\lambda^2}{R} - \frac{1}{R} = \frac{-1}{h}$$

$$\frac{1}{h} = \frac{1-\lambda^2}{R}$$

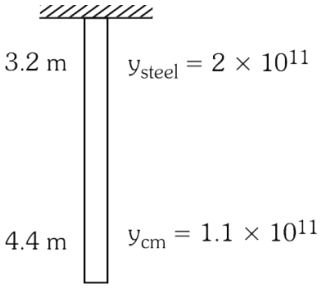
$$h = \frac{R}{1-\lambda^2}$$

7. A steel wire of length 3.2 m ($Y_S = 2.0 \times 10^{11}$ Nm^{-2}) and a copper wire of length 4.4 M ($Y_C = 1.1 \times 10^{11}$ Nm^{-2}), both of radius 1.4 mm are connected end to end. When stretched by a load, the net elongation is found to be 1.4 mm. The load applied, in Newton, will be: (Given $\pi = \frac{22}{7}$)

(A) 360 (B) 180 (C) 1080 (D) 154

Official Ans. by NTA (D)

Sol.

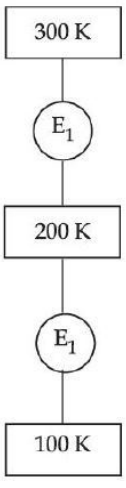


$\Delta\ell_1 + \Delta\ell_2 = \Delta\ell$

$$\frac{F\ell_1}{A_1y_1} + \frac{F\ell_2}{A_2y_2} = \Delta\ell$$

$$F = \frac{\Delta\ell}{\frac{\ell_1}{A_1y_1} + \frac{\ell_2}{A_2y_2}} = 1.54 \times 10^2 = 154$$

8. In 1st case, Carnot engine operates between temperatures 300 K and 100 K. In 2nd case, as shown in the figure, a combination of two engines is used. The efficiency of this combination (in 2nd case) will be :



2nd Case

(A) same as the 1st case

(B) always greater than the 1st case

(C) always less than the 1st case

(D) may increase or decrease with respect to the 1st case

Official Ans. by NTA (C)

Sol. First case : $\eta = 1 - \frac{100}{300} = \frac{2}{3}$

Second case : $\eta_{net} = \eta_1 + \eta_2 - \eta_1\eta_2$

$$\eta_1 = 1 - \frac{200}{300} = \frac{1}{3}$$

$$\eta_2 = 1 - \frac{100}{200} = \frac{1}{2}$$

$$\eta_{net} = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$$

η (first case) = η (second case)

9. Which statements are correct about degrees of freedom?

A. A molecule with n degrees of freedom has n^2 different ways of storing energy.

B. Each degree of freedom is associated with $\frac{1}{2}RT$ average energy per mole.

C. A monoatomic gas molecule has 1 rotational degree of freedom where as diatomic molecule has 2 rotational degrees of freedom

D. CH_4 has a total to 6 degrees of freedom

Choose the correct answer from the option given below:

(A) B and C only (B) B and D only

(C) A and B only (D) C and D only

Official Ans. by NTA (B)

Sol. Methane molecule is tetrahedron

Degree of freedom due to rotation = 3

Degree of freedom due to translation = 3

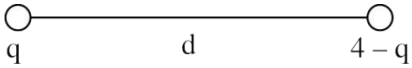
10. A charge of 4 μC is to be divided into two. The distance between the two divided charges is constant. The magnitude of the divided charges so that the force between them is maximum, will be:

(A) 1 μC and 3 μC (B) 2 μC and 2 μC

(C) 0 and 4 μC (D) 1.5 μC and 2.5 μC

Official Ans. by NTA (B)

Sol.



$$F = \frac{Kq(4-q)}{d^2}$$

$$\frac{dF}{dq} = \frac{K}{d^2}[4 - 2q] = 0$$

$q = 2$

11. A. The drift velocity of electrons decreases with the increase in the temperature of conductor.

B. The drift velocity is inversely proportional to the area of cross-section of given conductor.

C. The drift velocity does not depend on the applied potential difference to the conductor.

D. The drift velocity of electron is inversely proportional to the length of the conductor.

E. The drift velocity increases with the increase in the temperature of conductor.

Choose the correct answer from the options given below:

(A) A and B only (B) A and D only

(C) B and E only (D) B and C only

Official Ans. by NTA (B)

Sol. Drift velocity $= \left(\frac{e\tau}{m}\right)E$

$$v_d = \left(\frac{e\tau}{m}\right)\left(\frac{\Delta V}{\ell}\right)$$

ΔV = Potential difference applied across the wire

As temperature increases, relaxation time decreases, hence V_d decreases.

As per formula, $V_d \propto \frac{1}{\ell}$

$v_d = \frac{I}{neA}$, as it is not mentioned that current is at steady state neither it is mentioned that n is constant for given conductor. So it can't be said that v_d is inversely proportional to A.

$$I = neAv_d = \frac{V}{R} = \frac{V}{\rho\ell}A$$

$$v_d = \frac{V}{\rho\ell ne} \quad \left(E = \frac{V}{\ell}\right)$$

$$v_d = \frac{eE\tau}{m}$$

τ decrease with temperature increase.

First and fourth statements are correct.

12. A compass needle of oscillation magnetometer oscillates 20 times per minute at a place P of dip 30°. The number of oscillations per minute become 10 at another place Q of 60° dip. The ratio of the total magnetic field at the two places ($B_Q : B_P$) is:

(A) $\sqrt{3}:4$ (B) $4:\sqrt{3}$

(C) $\sqrt{3}:2$ (D) $2:\sqrt{3}$

Official Ans. by NTA (A)

Sol. $T = 2\pi\sqrt{\frac{I}{B_H M}}$

$$T_1 = 3\,\text{sec} = 2\pi\sqrt{\frac{I}{(B_P\cos 30°)M}}$$

$$T_2 = 6\,\text{sec} = 2\pi\sqrt{\frac{I}{(B_Q\cos 60°)M}}$$

$$\frac{3}{6} = \sqrt{\frac{1}{\left(B_P\frac{\sqrt{3}}{2}\right)}\times\left(B_Q/2\right)}$$

$$\frac{3}{6} = \sqrt{\left(\frac{B_Q}{\sqrt{3}B_P}\right)}$$

$$\frac{\sqrt{3}}{4} = \frac{B_Q}{B_P}$$

$$B_Q : B_P = \sqrt{3}:4$$

13. A cyclotron is used to accelerate protons. If the operating magnetic field is 1.0 T and the radius of the cyclotron 'dees' is 60 cm, the kinetic energy of the accelerated protons in MeV will be :

[use $m_p = 1.6 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C]

(A) 12 (B) 18

(C) 16 (D) 32

Official Ans. by NTA (B)

Sol. Kinetic energy of electron in cyclotron

$$= \left[\frac{q^2B^2r_0^2}{2m}\right]$$

= 18 MeV

14. A series LCR circuit has L = 0.01 H, R = 10 Ω and C = 1 μF and it is connected to ac voltage of amplitude (V_m) 50 V. At frequency 60% lower than resonant frequency, the amplitude of current will be approximately :

(A) 466 mA (B) 312 mA

(C) 238 mA (D) 196 mA

Official Ans. by NTA (C)

Sol. Resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$ rad/sec

$\omega' = .4 \times 10^4 = 4000$ rad/sec

$$i_0 = \frac{V_0}{\sqrt{R^2 + (X'_C - X'_L)^2}} = 238 \text{ mA}$$

15. Identify the correct statements from the following descriptions of various properties of electromagnetic waves.

A. In a plane electromagnetic wave electric field and magnetic field must be perpendicular to each other and direction of propagation of wave should be along electric field or magnetic field.

B. The energy in electromagnetic wave is divided equally between electric and magnetic fields.

C. Both electric field and magnetic field are parallel to each other and perpendicular to the direction of propagation of wave.

D. The electric field, magnetic field and direction of propagation of wave must be perpendicular to each other.

E. The ratio of amplitude of magnetic field to the amplitude of electric field is equal to speed of light.

Choose the most appropriate answer from the options given below:

(A) D only

(B) B and D only

(C) B, C and E only

(D) A, B and E only

Official Ans. by NTA (B)

Sol. Second and fourth statements are correct.

16. Two coherent sources of light interfere. The intensity ratio of two sources is 1 : 4. For this interference pattern if the value of $\frac{I_{max} + I_{min}}{I_{max} - I_{min}}$ is equal to $\frac{2\alpha + 1}{\beta + 3}$, then $\frac{\alpha}{\beta}$ will be :

(A) 1.5 (B) 2 (C) 0.5 (D) 1

Official Ans. by NTA (B)

Sol. $\frac{I_1}{I_2} = \frac{1}{4}$

$I_2 = 4I_1$

$I_{max} = I_1 + 4I_1 + 2\sqrt{I_1 4I_1} = 9I_1$

$I_{min} = I_1 + 4I_1 - 2\sqrt{I_1 4I_1} = I_1$

$\therefore \frac{9I_1 + I_1}{9I_1 - I_1} = \frac{10}{8} = \frac{5}{4} = \frac{2\alpha + 1}{\beta + 1}$

$\alpha = 2 \quad \beta = 1$

$\therefore \frac{\alpha}{\beta} = \frac{2}{1} = 2$

17. With reference to the observations in photo-electric effect, identify the correct statements from below:

A. The square of maximum velocity of photoelectrons varies linearly with frequency of incident light.

B. The value of saturation current increases on moving the source of light away from the metal surface.

C. The maximum kinetic energy of photo-electrons decreases on decreasing the power of LED (light emitting diode) source of light.

D. The immediate emission of photo-electrons out of metal surface can not be explained by particle nature of light/electromagnetic waves.

E. Existence of threshold wavelength can not be explained by wave nature of light/electromagnetic waves.

Choose the correct answer from the options given below:

(A) A and B only (B) A and E only

(C) C and E only (D) D and E only

Official Ans. by NTA (B)

Sol. $\frac{1}{2} m V_{max}^2 = hf - \phi$

Photoelectric effect can be explained by particle nature of light. Threshold λ is max wavelength at which emission takes place.

18. The activity of a radioactive material is 6.4×10^{-4} curie. Its half life is 5 days. The activity will become 5×10^{-6} curie after :

(A) 7 days (B) 15 days

(C) 25 days (D) 35 days

Official Ans. by NTA (D)

Sol. $A_0 = 6.4 \times 10^{-4}$ Curie

$T_{1/2} = 5 \text{ days} = \dfrac{\ln 2}{\lambda}$

$A = A_0 e^{-\lambda t}$

$5 \times 10^{-6} = 6.4 \times 10^{-4} e^{-\lambda t}$

$\dfrac{5}{6.4} \times 10^{-2} = e^{-\lambda t}$

$7.8 \times 10^{-3} = e^{-\lambda t}$

$\log(7.8 \times 10^{-3}) = -\lambda t \ln e$

$\ln(7.8 \times 10^{-3}) = -\dfrac{\lambda n 2}{5} \cdot t$

$\therefore \dfrac{5 \times 4.853}{0.693} = t = 35 \text{ days}$

19. For a constant collector-emitter voltage of 8V, the collector current of a transistor reached to the value of 6 mA from 4 mA, whereas base current changed from 20 μA to 25 μA value. If transistor is in active state, small signal current gain (current amplification factor) will be :

(A) 240 (B) 400

(C) 0.0025 (D) 200

Official Ans. by NTA (B)

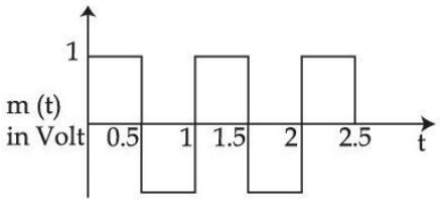
Sol. $V_{CE} = 8$ V = $I_C = 6$ mA from 4 mA,

$I_B = 20$ μA to 25 μA

Current gain $\beta_{av} = \dfrac{I_C' - I_C}{I_B' - I_B} = \dfrac{2\text{mA}}{5\mu\text{A}}$

$\beta_{av} = \dfrac{2}{5} \times 10^3 = \dfrac{2000}{5} = 400$

20. A square wave of the modulating signal is shown in the figure. The carrier wave is given by C(t) = 5 sin (8 πt) Volt. The modulation index is :



(A) 0.2 (B) 0.1

(C) 0.3 (D) 0.4

Official Ans. by NTA (A)

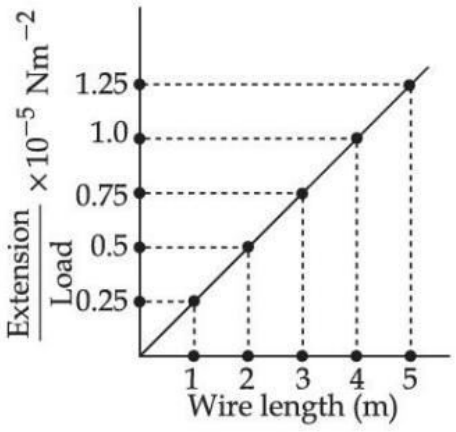
Sol. Modulation Index $\mu = \dfrac{A_m}{A_C} = \dfrac{1}{5} = 0.2$

A_m = amp. of modulating signal

A_C = amp. of carrier wave

SECTION-B

1. In an experiment to determine the Young's modulus, steel wires of five different lengths (1, 2, 3, 4 and 5 m) but of same cross section (2 mm^2) were taken and curves between extension and load were obtained. The slope (extension/load) of the curves were plotted with the wire length and the following graph is obtained. If the Young's modulus of given steel wires is x × 10^{11} Nm^{-2}, then the value of x is _____ .



Official Ans. by NTA (2)

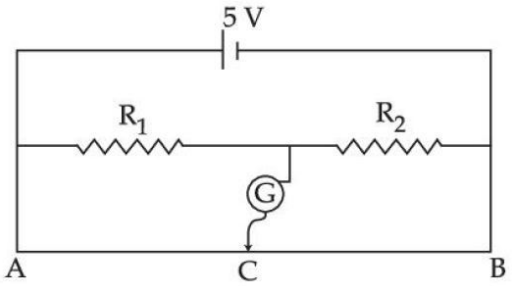
Sol. $\text{Slope} = \dfrac{\Delta l / w}{L} = \dfrac{\Delta l / L}{w} = \dfrac{1}{YA}$

$\Rightarrow Y = \dfrac{1}{(\text{slope})A}$

$Y = \dfrac{1}{2\times10^{-6}(0.25\times10^{-5})}$

$Y = 2\times10^{11}\,N/m^2$

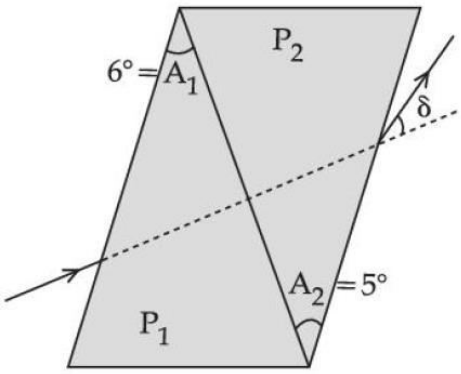
2. In the given figure of meter bridge experiment, the balancing length AC corresponding to null deflection of the galvanometer is 40 cm. The balancing length, if the radius of the wire AB is doubled, will be…….cm.



Official Ans. by NTA (40)

Sol. Independent of area in case of uniform wire.

3. A thin prism of angle 6º and refractive index for yellow light (n_Y)1.5 is combined with another prism of angle 5º and n_Y = 1.55. The combination produces no dispersion. The net average deviation (δ) produced by the combination is $\left(\dfrac{1}{x}\right)^\circ$. The value of x is…….



Official Ans. by NTA (4)

Sol. $\delta = A(\mu_y - 1) - A'(\mu_y' - 1)$

$= 6(1.5 - 1) - 5(1.55 - 1)$

$= \dfrac{1}{4}$

4. A conducting circular loop is placed in X – Y plane in presence of magnetic field $\vec{B} = \left(3t^3\hat{j} + 3t^2\hat{k}\right)$ in SI unit. If the radius of the loop is 1m, the induced emf in the loop, at time, t = 2s is nπV. The value of n is……..

Official Ans. by NTA (12)

Sol. $\phi = \vec{B}\cdot\vec{A}$

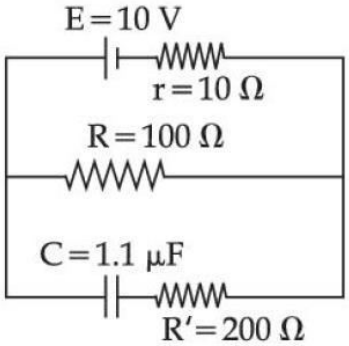
$= (3t^3\hat{j} + 3t^2\hat{k})\cdot(\pi(1)^2\hat{k})$

$\phi = 3t^2\pi$

$\varepsilon_{IND} = \left|\dfrac{d\phi}{dt}\right| = 6t\pi$

at t = 2, $\varepsilon_{IND} = 12$

5. As show in the figure, in steady state, the charge stored in the capacitor is……. × 10^{-6}C.



Official Ans. by NTA (10)

Sol. $q = C\,V_{100\Omega}$

$= (1.1\times10^{-6})\left(\dfrac{10}{R+r}R\right)$

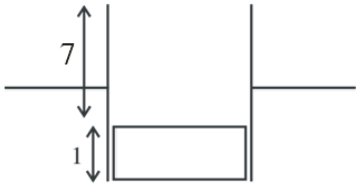
$= 1.1\times10^{-6}\left(\dfrac{10}{110}\times100\right)$

$= 10\ \mu C$

6. A parallel plate capacitor with width 4 cm, length 8 cm and separation between the plates of 4mm is connected to a battery of 20 V. A dielectric slab of dielectric constant 5 having length 1cm, width 4 cm and thickness 4 mm is inserted between the plates of parallel plate capacitor. The electrostatic energy of this system will be……$\in_0$ J. (Where $\in_0$ is the permittivity of free space)

Official Ans. by NTA (240)

Sol.



$$C_{eff} = \left[\frac{\varepsilon_0(7\times 4)}{4/10} + \frac{5\varepsilon_0(1\times 4)}{4/10}\right]\times 10^{-2}$$

$$C_{eff} = 1.2\varepsilon_0$$

$$\text{Energy} = \frac{1}{2}C_{eff}V^2$$

$$= \frac{1}{2}(1.2)\varepsilon_0(20)(20) = 240\varepsilon_0$$

7. A wire of length 30 cm, stretched between rigid supports, has it's n^{th} and $(n + 1)^{th}$ harmonics at 400 Hz and 450 Hz, respectively. If tension in the string is 2700 N, it's linear mass density is……..kg/m.

Official Ans. by NTA (3)

Sol. $\frac{nv}{0.6} = 400 \;\&\; \frac{(n+1)v}{0.6} = 450$

$$\Rightarrow \left[\frac{0.6\times 400}{v} + 1\right]\frac{v}{0.6} = 450$$

$$\Rightarrow = v = 30$$

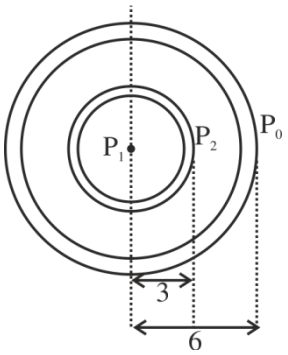
$$\Rightarrow \sqrt{\frac{T}{\mu}} = 30$$

$$\Rightarrow \frac{2700}{\mu} = 900 = \mu = 3$$

8. A spherical soap bubble of radius 3 cm is formed inside another spherical soap bubble of radius 6 cm. If the internal pressure of the smaller bubble of radius 3 cm in the above system is equal to the internal pressure of the another single soap bubble of radius r cm. The value of r is…….

Official Ans. by NTA (2)

Sol.

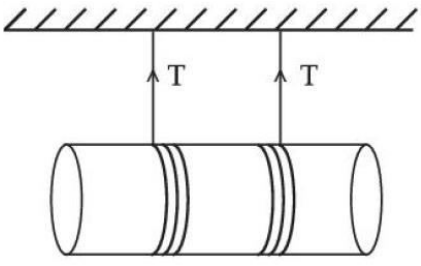


$$P_2 - P_0 = \frac{4T}{6} \;\&\; P_1 - P_2 = \frac{4T}{3}$$

$$\Rightarrow P_1 - P_0 = \frac{4T}{2} = 2$$

9. A solid cylinder length is suspended symmetrically through two massless strings, as shown in the figure. The distance from the initial rest position, the cylinder should by unbinding the strings to achieve a speed of 4 ms^{-1}, is…….cm.

(take g = 10 ms^{-2})



Official Ans. by NTA (120)

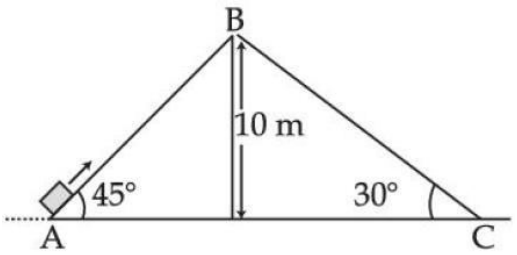
Sol. From energy conservation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

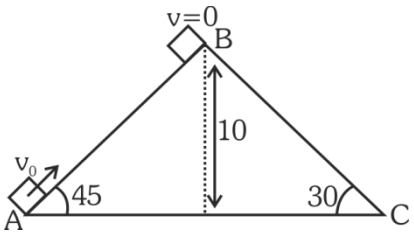
$$10h = \frac{16}{2} + \frac{16}{4} \Rightarrow h = 1.2m = 120cm$$

10. Two inclined planes are placed as shown in figure. A block is projected from the Point A of inclined plane AB along its surface with a velocity just sufficient to carry it to the top Point B at a height 10 m. After reaching the Point B the block slides down on inclined plane BC. Time it takes to reach to the point C from point A is $t\left(\sqrt{2}+1\right)s$. The value of t is……..(use g = 10 m/s^2)



Official Ans. by NTA (2)

Sol.



From E.C. $= \frac{1}{2}mv_0^2 = mgh$

$$v_0 = 10\sqrt{2}$$

For A $\rightarrow$ B

at B, $v = 0$

$$a = -g\sin 45° = \frac{-10}{\sqrt{2}}$$

$$v = u + at_1 \Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}}t_1 \Rightarrow t_1 = 2\sec$$

For B $\rightarrow$ C

$$s = ut_2 + \frac{1}{2}at_2^2$$

$$\frac{10}{\sin 30°} = \frac{1}{2}(10\sin 30°)t_2^2$$

$$t_2 = 2\sqrt{2}$$

So total time

$$T = t_1 + t_2$$

$$= 2\sqrt{2} + 2$$

$$= 2(\sqrt{2}+1)\sec$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. The correct decreasing order of energy, for the orbitals having, following set of quantum numbers:

(A) n = 3, l = 0, m = 0

(B) n = 4, l = 0, m = 0

(C) n = 3, l = 1, m = 0

(D) n = 3, l = 2, m = 1

(A) (D) > (B) > (C) > (A)

(B) (B) > (D) > (C) > (A)

(C) (C) > (B) > (D) > (A)

(D) (B) > (C) > (D) > (A)

Official Ans. by NTA (A)

Sol. (A) $n + \ell = 3 + 0 = 3$

(B) $n + \ell = 4 + 0 = 4$

(C) $n + \ell = 3 + 1 = 4$

(D) $n + \ell = 3 + 2 = 5$

Higher $n + \ell$ value, higher the energy & if same $n + \ell$ value, then higher n value, higher the energy.

Thus : D > B > C > A.

2. Match List-I with List-II

List-I	List-II
(A) $\Psi_{MO} = \Psi_A - \Psi_B$	(I) Dipole moment
(B) $\mu = Q \times r$	(II) Bonding molecular orbital
(C) $\frac{N_b - N_a}{2}$	(III) Anti-bonding molecualr orbital
(D) $\Psi_{MO} = \Psi_A + \Psi_B$	(IV) Bond order

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

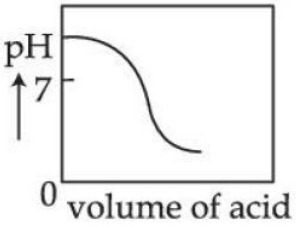
Official Ans. by NTA (C)

Sol.

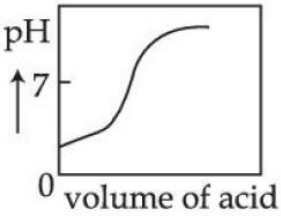
(A)	$\psi_{MO} = \psi_A - \psi_B$	(III) ABMO
(B)	$\mu = Q \times r$	(I) Dipole moment
(C)	$\frac{N_b - N_a}{2}$	(IV) Bond order
(D)	$\psi_{MO} = \psi_A + \psi_B$	(II) BMO

3. The Plot of pH-metric titration of weak base NH_4OH vs strong acid HCl looks like:

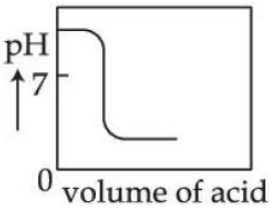
(A)



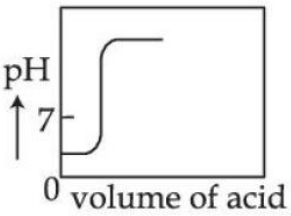
(B)



(C)

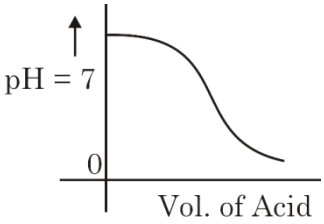


(D)



Official Ans. by NTA (A)

Sol. Titration curve of NH_4OH vs HCl (WB + SA).



4. Given below are two statements:

Statement I: For KI, molar conductivity increases steeply with dilution.

Statement II: For carbonic acid, molar conductivity increases slowly with dilution.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

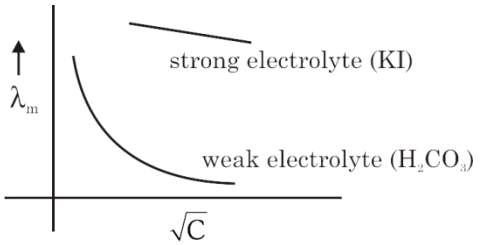
(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (B)

Sol. **Statement I:** KI is strong electrolyte thus almost constant on dilution.

Statement II: In weak electrolyte it increases, sharply.



5. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Dissolved substances can be removed from a colloidal solution by diffusion through a parchment paper.

Reason (R) : Particles in a true solution cannot pass through parchment paper but the collodial particles can pass through the parchment paper.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(B) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(C) **(A)** is correct but **(R)** is not correct

(D) **(A)** is not correct but **(R)** is correct

Official Ans. by NTA (C)

Sol. **Assertion (A):** Correct.

Reason(R): Incorrect.

Particles of true solution pass through parchment paper thus answer is (C).

6. Outermost electronic configurations of four elements A, B, C, D are given below:

(A) $3s^2$

(B) $3s^2 3p^1$

(C) $3s^2 3p^3$

(D) $3s^2 3p^4$

The **correct** order of first ionization enthalpy for them is:

(A) (A) < (B) < (C) < (D)

(B) (B) < (A) < (D) < (C)

(C) (B) < (D) < (A) < (C)

(D) (B) < (A) < (C) < (D)

Official Ans. by NTA (B)

Sol. (A) $3s^2 \rightarrow$ Mg

(B) $3s^2 3p^1 \rightarrow$ Al

(C) $3s^2 3p^3 \rightarrow$ P

(D) $3s^2 3p^4 \rightarrow$ S

$$\underbrace{P > S}_{\text{Half filled stability}} > \underbrace{Mg > Al}_{\text{Penetrating power of s>p.}}$$

C > D > A > B.

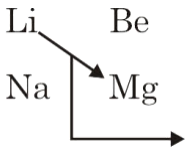
7. An element A of group 1 shows similarity to an element B belonging to group 2. If A has maximum hydration enthalpy in group 1 then B is:

(A) Mg (B) Be

(C) Ca (D) Sr

Official Ans. by NTA (A)

Sol.



Diagonal relationship

$Li^+ \rightarrow$ Maximum hydration enthalpy in group 1 due to small size.

So 'B' is Mg.

8. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Boron is unable to form BF_6^{3-}

Reason (R) : Size of B is very small.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(B) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(C) **(A)** is true but **(R)** is false

(D) **(A)** is false but **(R)** is true

Official Ans. by NTA (B)

Sol. **Assertion (A):** True

Reason (R): True but not correct explanation.

Correct explanation: Expansion of octet not possible for 'B'.

9. In neutral or alkaline solution, MnO_4^- oxidises thiosulphate to:

(A) $S_2O_7^{2-}$ (B) $S_2O_8^{2-}$

(C) SO_3^{2-} (D) SO_4^{2-}

Official Ans. by NTA (D)

Sol. $8MnO_4^- + 3S_2O_3^{2-} + H_2O \xrightarrow[\text{alk. solution}]{\text{neutral or}} 8MnO_2 + 6SO_4^{2-} + 2OH^-$

10. Low oxidation state of metals in their complexes are common when ligands:

(A) have good π -accepting character

(B) have good σ -donor character

(C) are havind good π -donating ability

(D) are havind poor σ -donating ability

Official Ans. by NTA (A)

Sol. When metal is in low oxidation state then it forms complexes when ligands have good π -accepting character.

11. Given below are two statements:

Statement I : The non bio-degradable fly ash and slag from steel industry can be used by cement industry.

Statement II : The fuel obtained from plastic waste is lead free.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(A) Both **Statement I** and **Statement II** are correct

(B) Both **Statement I** and **Statement II** are incorrect

(C) **Statement I** is correct but **Statement II** is incorrect

(D) **Statement I** is incorrect but **Statement II** is correct

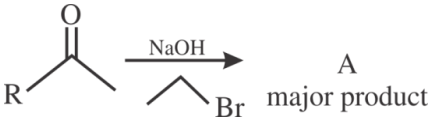
Official Ans. by NTA (A)

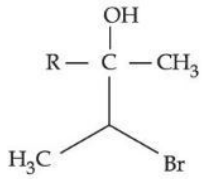
Sol. (I) Fly ash and slag from steel industry are utilised by cement industry.

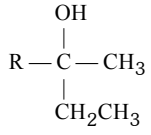
(II) Fuel obtained from plastic waste has high octane rating. It contains no lead and it is known as green fuel.

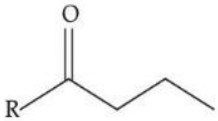
Both statement (I) & (II) are correct.

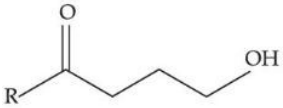
12. The structure of A in the given reaction is:



(A) 

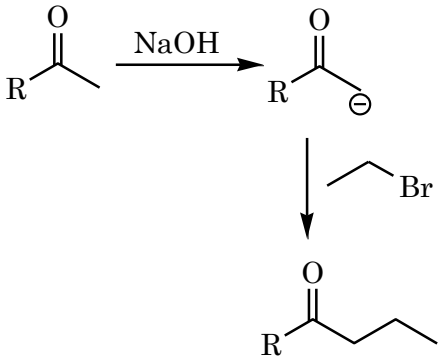
(B) 

(C) 

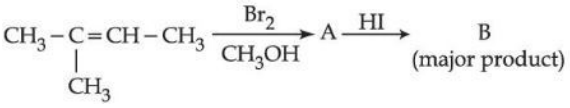
(D) 

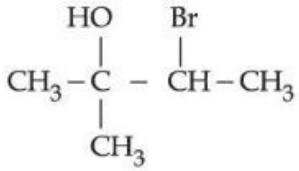
Official Ans. by NTA (C)

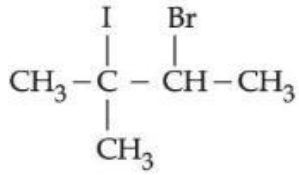
Sol.

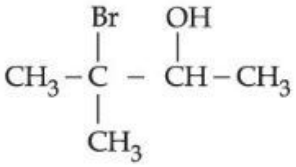


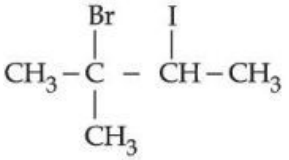
13. Major product 'B' of the following reaction sequence is:



(A) 

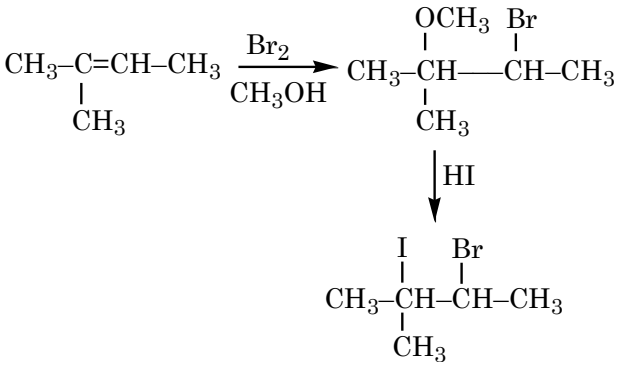
(B) 

(C) 

(D) 

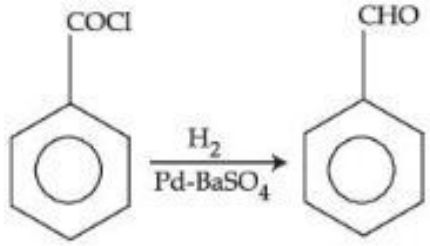
Official Ans. by NTA (B)

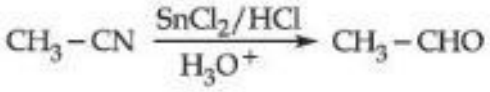
Sol

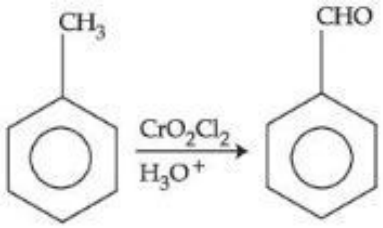


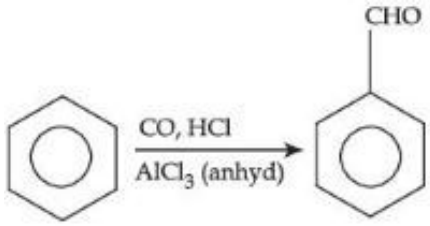
14. Match List-I with List-II.

List-I

(A) 

(B) 

(C) 

(D) 

Lits-II

(I) Gatterman Koch reaction

(II) Etard reaction

(III) Stephen reaction

(IV) Rosenmund reaction

Choose the **correct** answer from the options given below:

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

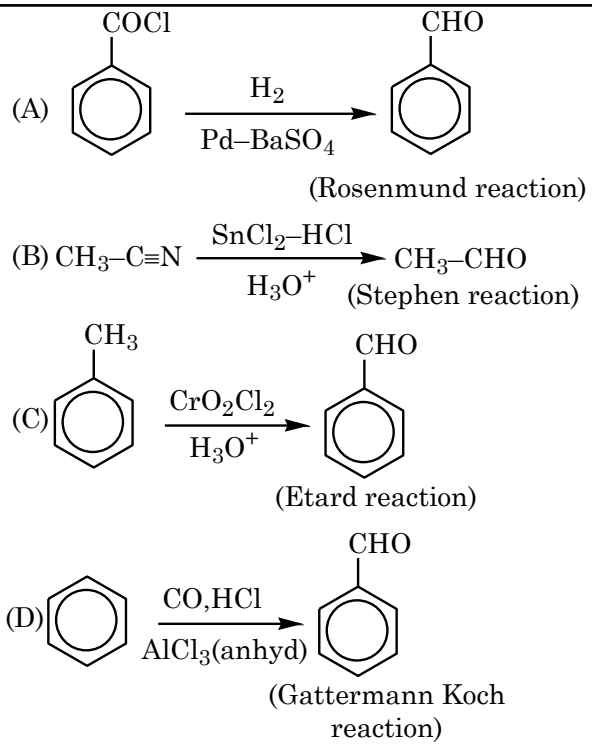
(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Official Ans. by NTA (A)

Sol.



15. Match List-I with List-II.

List-I **(Polymer)**	**List-II** **(Monomer)**
(A) Neoprene	(I) Acrylonitrile
(B) Teflon	(II) Chloroprene
(C) Acrilan	(III) Tetrafluoroethene
(D) Natural rubber	(IV) Isoprene

Choose the correct answer from the option given below:

(A) (A)-(II), (B)-(III), (C)-(I), (D-(IV)

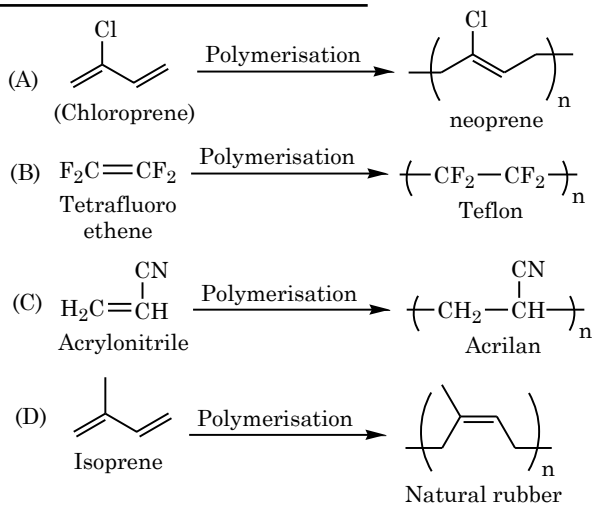
(B) (A)-(II), (B)-(I), (C)-(III), (D-(IV)

(C) (A)-(II), (B)-(I), (C)-(IV), (D-(III)

(D) (A)-(I), (B)-(II), (C)-(III), (D-(IV)

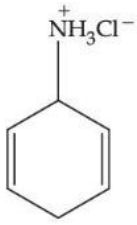
Official Ans. by NTA (A)

Sol.

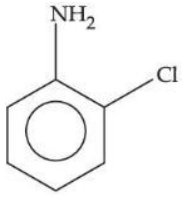


16. An organic compound 'A' contains nitrogen and chlorine. It dissolves readily in water to give a solution that turns litmus red. Titration of compound 'A' with standard base indicates that the molecular weight of 'A' is 131 ± 2. When a sample of 'A' is treated with aq. NaOH, a liquid separates which contains N but not Cl. Treatment of the obtained liquid with nitrous acid followed by phenol gives orange precipitate. The compound 'A' is :

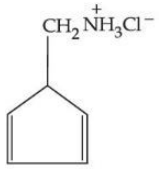
(A)



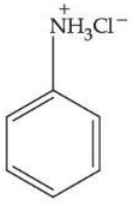
(B)



(C)

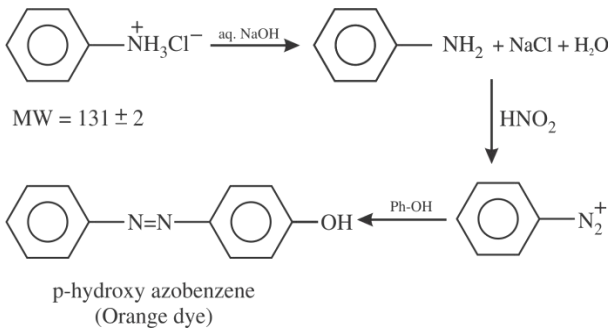


(D)



Official Ans. by NTA (D)

Sol.



p-hydroxy azobenzene
(Orange dye)

17. Match List-I with List-II

List-I

(A) Glucose + HI

(B) Glucose + Br_2 water

(C) Glucose + acetic anhydride

(D) Glucose + HNO_3

List-II

(I) Gluconic acid

(II) Glucose pentacetate

(III) Saccharic acid

(IV) Hexane

Choose the correct answer from the options given below:

(A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Official Ans. by NTA (A)

Sol.

(A) Glucose $\xrightarrow{HI}$ n-hexane

(B) Glucose $\xrightarrow[H_2O]{Br_2}$ $(CHOH)_4$ with $COOH$ and CH_2OH — Gluconic acid

(C) Glucose $\xrightarrow[\text{anhydride}]{5 \text{ acetic}}$ Glucose pentacetate

(D) Glucose $\xrightarrow{HNO_3}$ $(CHOH)_4$ with $COOH$ and $COOH$ — Saccharic acid

18. Which of the following enhances the lathering property of soap?

(A) Sodium stearate

(B) Sodium carbonate

(C) Sodium rosinate

(D) Trisodium phosphate

Official Ans. by NTA (C)

Sol. Rosin is added to soaps which forms sodium rosinate which lathers well.

19. Match List-I with List-II

List-I (Mixture)

(A) Chloroform & Aniline

(B) Benzoic acid & Napthalene

(C) Water & Aniline

(D) Napthalene & Sodium chloride

List-II (Purification Process)

(I) Steam distillation

(II) Sublimation

(III) Distillation

(IV) Crystallisation

(A) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Official Ans. by NTA (D)

Sol. (A) Chloroform + Aniline → (III) Distillation

(B) Benzoic acid + Napthalene → (IV) Crystallisation

(C) Water + Aniline → (I) Steam distillation

(D) Napthalene + Sodium chloride →(II) Sublimation

20. Fe^{3+} cation gives a prussian blue precipitate on addition of potassium ferrocyanide solution due to the formation of:

(A) $[Fe(H_2O)_6]_2\,[Fe(CN)_6]$

(B) $Fe_2[Fe(CN)_6]_2$

(C) $Fe_3[Fe(OH)_2(CN)_4]_2$

(D) $Fe_4[Fe(CN)_6]_3$

Official Ans. by NTA (D)

Sol. $4Fe^{3+} + 3[Fe(CN)_6]^{-4} \longrightarrow \underset{\text{Prussian Blue}}{Fe_4[Fe(CN)_6]_3}$

SECTION-B

1. The normality of H_2SO_4 in the solution obtained on mixing 100 mL of 0.1 M H_2SO_4 with 50 mL of 0.1 M NaOH is_____$\times 10^{-1}$ N. (Nearest Integer)

Official Ans. by NTA (1)

Sol. No. of equivalents of $H_2SO_4 = 100 \times 0.1 \times 2 = 20$

No. of equivalents of NaOH $= 50 \times 0.1 = 5$

No. of equivalents of H_2SO_4 left $= 20 - 5 = 15$

$\Rightarrow 150 \times x = 15$

$$x = \frac{1}{10} = 0.1\,N = 1 \times 10^{-1}\,N$$

2. for a real gas at 25°C temperature and high pressure (99 bar) the value of compressibility factor is 2, so the value of Vander Waal's constant 'b' should be_____$\times 10^{-2}$ L mol^{-1} (Nearest integer) (Given R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by N

Sol. For real gas under high pressure

$$Z = 1 + \frac{Pb}{RT} \qquad \Rightarrow b = \frac{RT}{P}$$

$$= \frac{0.083 \times 298}{99}$$

$$= 0.25 \times 10^{-2}\ L\ mol^{-1}$$

3. A gas (Molar mass = 280 g mol^{-1}) was burnt in excess O_2 in a constant volume calorimeter and during combustion the temperature of calorimeter increased from 298.0 K to 298.45 K. If the heat capacity of calorimeter is 2.5 kJ K^{-1} and enthalpy of combustion of gas is 9 kJ mol^{-1} then amount of gas burnt is_____g. (Nearest Integer)

Official Ans. by NTA (35)

Sol. Let x g is burnt

$$\text{moles} = \frac{x}{280}$$

heat released by $\frac{x}{280}$ mole $= 2.5 \times 0.45$ kJ

$$\text{heat released by 1 mole} = \frac{2.5 \times 0.45 \times 280}{x}\,kJ$$

$\Delta H = \Delta U + \Delta ngRT$

$\Delta H \simeq \Delta U$

$$9 = \frac{2.5 \times 280 \times 0.45}{x}$$

$x =$ **35 g**

4. When a certain amount of solid A is dissolved in 100 g of water at 25°C to make a dilute solution, the vapour pressure of the solution is reduced to one-half of that of pure water. The vapour pressure of pure water is 23.76 mmHg. The number of moles of solute A added is_____. (Nearest Integer)

Official Ans. by NTA (3)

Sol. $\because$ Diliute solution given:

$$\frac{P^0 - P_s}{P^0} \sim \frac{{}^n\text{solute}}{{}^n\text{solvent}}$$

$$\frac{P^0 - P^0/2}{P^0} = \frac{{}^n\text{solute}}{{}^n\text{solvent}}$$

$${}^n\text{solute} \sim \frac{{}^n\text{solvent}}{2} = \frac{100}{18 \times 2} = 2.78\,\text{mol}$$

More accurate approach:

$$\frac{P^0 - P_s}{P_s} = \frac{{}^n\text{solute}}{{}^n\text{solvent}}$$

$$\frac{P^0 - P^0/2}{P^0/2} = \frac{{}^n\text{solute}}{{}^n\text{solvent}}$$

$${}^n\text{solute} = {}^n\text{solvent} = \frac{100}{18} = 5.55\,\text{mol}$$

5. [A] → [B]

Reactant Product

If formation of compound [B] follows the first order of kinetics and after 70 minutes the concentration of [A] was found to be half of its initial concentration. Then the rate constant of the reaction is x × 10^{-6} s^{-1}. The value of x is_____. (Nearest Integer)

Official Ans. by NTA (165)

Sol. $K = \frac{0.693}{t_{1/2}} = \frac{0.693}{70 \times 60}$

$= \frac{6930}{7 \times 6} \times 10^{-6}$

$= 165 \times 10^{-6}\ s^{-1}$

6. Among the following ores Bauxite, Siderite, Cuprite, Calamine, Haematite, Kaolinite, Malachite, Magnetite, Sphalerite, Limonite, Cryolite, the number of principal ores if (of) iron is_____.

Official Ans. by NTA (4)

Sol. Bauxite — $AlO_X(OH)_{3-2x}$(where $0 < x < 1$)

✓Siderite — $FeCO_3$

Cuprite — Cu_2O

Calamine — $ZnCO_3$

✓Haematite — Fe_2O_3

Kaolinite — $Al_2(OH)_4Si_2O_5$

Malachite — $CuCO_3 . Cu(OH)_2$

✓Magnetite — Fe_3O_4

Sphalerite — ZnS

✓Limonite — $Fe_2O_3.3H_2O$

Cryolite — Na_3AlF_6

7. The oxidation state of manganese in the product obtained in a reaction of potassium permanganate and hydrogen peroxide in basic medium is_____.

Official Ans. by NTA (4)

Sol. $2KMnO_4 + 3H_2O_2 \xrightarrow{\text{basic medium}} 2\overset{+4}{MnO_2} + 3O_2 + 2H_2O + 2KOH$

8. The number of molecule(s) or ion(s) from the following having non-planar structure is_____.

$NO_3^-, H_2O_2, BF_3, PCl_3, XeF_4,$
$SF_4, XeO_3, PH_4^+, SO_3, [Al(OH)_4]^-$

Official Ans. by NTA (6)

Sol.

SO_3	—	sp^2	Planar
BF_3	—	sp^2	Planar
NO_3^-	—	sp^2	Planar
SF_4	—	sp^3d	Non-planar
H_2O_2	—	sp^3	Non-planar
PCl_3	—	sp^3	Non-planar
$[Al(OH)_4]^-$	—	sp^3	Non-planar
XeF_4	—	sp^3d^2	Planar
XeO_3	—	sp^3	Non-planar
PH_4^+	—	sp^3	Non-planar

9. The spin only magnetic moment of the complex present in Fehling's reagent is_____ B.M. (Nearest integer).

Official Ans. by NTA (2)

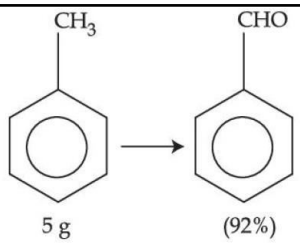
Sol. Fehling solution is a complex of Cu^{++}

$Cu^{++} = 3d^9$

No. of unpaired $e^- = 1$

$M.M = \sqrt{1(1+2)} = \sqrt{3} = 1.73$ BM

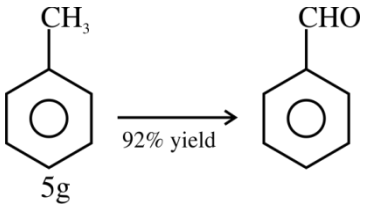
10.



In the above reaction, 5 g of toluene is converted into benzaldehyde with 92% yield. The amount of benzaldehyde produced is_____$\times 10^{-2}$ g. (Nearest integer)

Official Ans. by NTA (530)

Sol.



$$\text{moles} = \frac{5}{92}$$

$$\text{moles of } \text{C}_6\text{H}_5\text{CHO} = \frac{5}{92} \times \frac{92}{100} = 5 \times 10^{-2}$$

$$\text{mass of } \text{C}_6\text{H}_5\text{CHO} = 106 \times 5 \times 10^{-2} = 5.3 \text{ g}$$

$$= 530 \times 10^{-2} \text{ g}$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The domain of the function

$$f(x)=\sin^{-1}[2x^2-3]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right),$$

where [t] is the greatest integer function, is :

(A) $\left(-\sqrt{\frac{5}{2}},\frac{5-\sqrt{5}}{2}\right)$ (B) $\left(\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right)$

(C) $\left(1,\frac{5-\sqrt{5}}{2}\right)$ (D) $\left[1,\frac{5+\sqrt{5}}{2}\right)$

Official Ans. by NTA (C)

Sol. $f(x)=\sin^{-1}[2x^2-3]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$

$P_1: -1\le\left[2x^2-3\right]<1$

$\Rightarrow -1\le 2x^2-3<2$

$\Rightarrow 2<2x^2<5$

$\Rightarrow 1<x^2<\frac{5}{2}$

$\Rightarrow P_1: x\in\left(-\sqrt{\frac{5}{2}},-1\right)\cup\left(1,\sqrt{\frac{5}{2}}\right)$

$P_2: x^2-5x+5>0$

$\Rightarrow \left(x-\left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{5+\sqrt{5}}{2}\right)\right)>0$

$P_3: \log_{\frac{1}{2}}\left(x^2-5x+5\right)>0$

$\Rightarrow x^2-5x-5<1$

$\Rightarrow x^2-5x+4<0$

$\Rightarrow P_3: x\in(1,4)$

So, $P_1\cap P_2\cap P_3=\left(1,\frac{5-\sqrt{5}}{2}\right)$

2. Let S be the set of all (α,β), $\pi<\alpha,\beta<2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta}=\sin 2\alpha+i\cos 2\beta,(\alpha,\beta)\in S$.

Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta}+\frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to :

(A) 3 (B) 3 i

(C) 1 (D) 2 – i

Official Ans. by NTA (C)

Sol. $\pi<\alpha,\beta<2\pi$

$\frac{1-i\sin\alpha}{1+i(2\sin\alpha)}=$ Purely imaginary

$\Rightarrow \frac{(1-i\sin\alpha)(1-i(2\sin\alpha))}{1+4\sin^2\alpha}=$ Purely imaginary

$\Rightarrow \frac{1-2\sin^2\alpha}{1+4\sin^2\alpha}=0$

$\Rightarrow \sin^2\alpha=\frac{1}{2}$

$\Rightarrow \alpha=\left\{\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

& $\frac{1+i\cos\beta}{1+i(-2\cos\beta)}=$ Purely real

$\Rightarrow \frac{(1+i\cos\beta)(1+2i\cos\beta)}{1+4\cos^2\beta}=$ Purely real

$\Rightarrow 3\cos\beta=0$

$\Rightarrow \boxed{\beta=\frac{3\pi}{2}}$

$\Rightarrow Z_{\alpha\beta}=\sin\frac{5\pi}{2}+i\cos 3\pi=1-i$

or

$Z_{\alpha\beta}=\sin\frac{7\pi}{2}+i\cos 3\pi=-1-i$

Required value $=\left[i(1-i)+\frac{1}{i(1+i)}\right]+\left[i(-1-i)+\frac{1}{i(-1+i)}\right]$

$=i(-2i)+\frac{1}{i}\frac{2i}{(-2)}\Rightarrow 2-1=1$

3. If α, β are the roots of the equation

$$x^2-\left(5+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}}\right)+3\left(3^{(\log_3 5)^{\frac{1}{3}}}-5^{(\log_5 3)^{\frac{2}{3}}}-1\right)=0$$

then the equation, whose roots are

$\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$,

(A) $3x^2-20x-12=0$

(B) $3x^2-10x-4=0$

(C) $3x^2-10x+2=0$

(D) $3x^2-20x+16=0$

Official Ans. by NTA (B)

Sol. Bonus because 'x' is missing the correct will be,

$$x^2-\left(5+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}}\right)x+3\left(3^{(\log_3 5)^{\frac{1}{3}}}-5^{(\log_5 3)^{\frac{2}{3}}}-1\right)=0$$

$$3^{\sqrt{\log_3 5}}=3^{\sqrt{\log_3 5}.\sqrt{\log_3 5}.\sqrt{\log_5 3}}=3^{\log_3 5.\sqrt{\log_5 3}}$$

$$=\left(3^{\log_3 5}\right)^{\sqrt{\log_5 3}}=5^{\sqrt{\log_5 3}}$$

$$3^{\sqrt[3]{\log_3 5}}=3^{\log_3 5.\sqrt[3]{(\log_5 3)^2}}=\left(3^{\log_3 5}\right)^{(\log_5 3)^{2/3}}$$

$$=5^{(\log_5 3)^{2/3}}$$

So, equation is $x^2-5x-3=0$ and roots are α & β

$\{\alpha+\beta=5;\ \alpha\beta=-3\}$

New roots are $\alpha+\frac{1}{\beta}$ & $\beta+\frac{1}{\alpha}$

i.e., $\frac{\alpha\beta+1}{\beta}$ & $\frac{\alpha\beta+1}{\alpha}$ i.e., $\frac{-2}{\beta}$ & $\frac{-2}{\alpha}$

Let $\frac{-2}{\alpha}=t \Rightarrow \alpha=\frac{-2}{t}$

As $\alpha^2-5\alpha-3=0$

$\Rightarrow \left(\frac{-2}{t}\right)^2-5\left(\frac{-2}{t}\right)-3=0$

$\Rightarrow \frac{4}{t^2}+\frac{10}{t}-3=0$

$\Rightarrow 4+10t-3t^2=0$

$\Rightarrow 3t^2-10t-4=0$

i.e., $3x^2-10x-4=0$

4. Let $A=\begin{pmatrix}4 & -2\\ \alpha & \beta\end{pmatrix}$

If $A^2+\gamma A+18I=O$, then det (A) is equal to _____.

(A) –18 (B) 18 (C) –50 (D) 50

Official Ans. by NTA (B)

Sol. The characteristic equation for A is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}4-\lambda & -2\\ \alpha & \beta-\lambda\end{vmatrix}=0$$

$\Rightarrow (4-\lambda)(\beta-\lambda)+2\alpha=0$

$\Rightarrow \lambda^2-(\beta+4)\lambda+4\beta+2\alpha=0$

Put $\lambda = A$

$A^2-(\beta+4)A+(4\beta+2\alpha)I=0$

On comparison

$-9(\beta+4)=\gamma$ & $4\beta+2\alpha=18$

and $|A| = 4\beta + 2\alpha = 18$

5. If for $p\neq q\neq 0$, then function

$f(x)=\frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$ is continuous at x = 0, then:

(A) $7pq\ f(0)-1=0$ (B) $63q\ f(0)-p^2=0$

(C) $21q\ f(0)-p^2=0$ (D) $7pq\ f(0)-9=0$

Official Ans. by NTA (B)

Sol. $f(0)=\lim\limits_{x\to 0} f(x)$

Limit should be $\frac{0}{0}$ form

So, $\sqrt[7]{p.729}-3=0 \Rightarrow p.3^6=3^7 \Rightarrow p=3$

Now, $f(0)=\lim\limits_{x\to 0}\frac{\sqrt[7]{3(3^6+x)}-3}{\sqrt[3]{3^6+qx}-9}$

$$=\lim_{x\to 0}\frac{3\left[\left(1+\frac{x}{3^6}\right)^{1/7}-1\right]}{9\left[\left(1+\frac{qx}{3^6}\right)^{1/3}-1\right]}=\frac{3}{9}\times\frac{\frac{1}{7.3^6}}{\frac{q}{3.3^6}}$$

$\Rightarrow f(0)=\frac{1}{3}\times\frac{3}{7q}=\frac{1}{7q}$

$\Rightarrow 7qf(0)-1=0$

$\Rightarrow 7.p^2.qf(0)-p^2=0$ (for option)

$\Rightarrow 63qf(0)-p^2=0$

6. Let $f(x) = 2 + |x| - |x - 1| + |x + 1|$, $x \in \mathbf{R}$. Consider

$$(S1): f'\left(-\frac{3}{2}\right) + f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 2$$

$$(S2): \int_{-2}^{2} f(x)dx = 12$$

Then,

(A) both (S1) and (S2) are correct

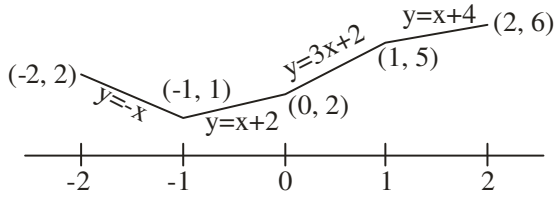
(B) both (S1) and (S2) are wrong

(C) only (S1) is correct

(D) only (S2) is correct

Official Ans. by NTA (D)

Sol.



$$(S1): f'\left(-\frac{3}{2}\right) + f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 4$$

$$(S2): \int_{-2}^{2} f(x)dx = 12$$

$\therefore$ (D)

7. Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to :

(A) $21\ a_{11}$ (B) $22\ a_{11}$

(C) $15\ a_{16}$ (D) $14\ a_{16}$

Official Ans. by NTA (A)

Sol. $S_{21} = \frac{21}{2}[20ar + 20.10ar^2]$

$= 21\ [10\ ar + 100\ ar^2]$

$= 21.\ a_{11}$

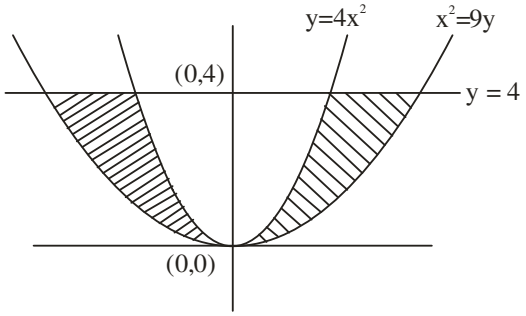
8. The area of the region enclosed by

$y \le 4x^2, x^2 \le 9y$ and $y \le 4$, is equal to :

(A) $\frac{40}{3}$ (B) $\frac{56}{3}$ (C) $\frac{112}{3}$ (D) $\frac{80}{3}$

Official Ans. by NTA (D)

Sol.



$$\Delta = 2 \cdot \int_{0}^{4} \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$$

$$= 2 \cdot \int_{0}^{4} \frac{5}{2}\sqrt{y}\, dy = \frac{80}{3}$$

9. $\int_{0}^{2} \left(\left|2x^2 - 3x\right| + \left[x - \frac{1}{2}\right]\right) dx$,

where [t] is the greatest integer function, is equal to:

(A) $\frac{7}{6}$ (B) $\frac{19}{12}$ (C) $\frac{31}{12}$ (D) $\frac{3}{2}$

Official Ans. by NTA (B)

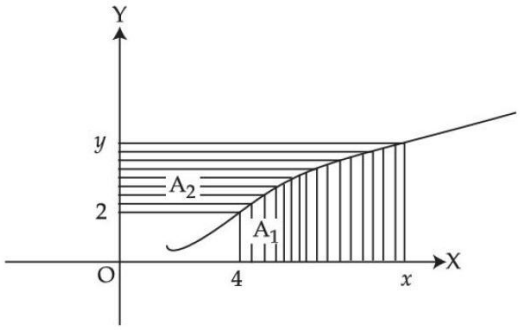
Sol. $\int_{0}^{2} \left|2x^2 - 3x\right| dx$

$$= \int_{0}^{\frac{3}{2}} (3x - 2x^2)dx + \int_{\frac{3}{2}}^{2} (2x^2 - 3x)dx = \frac{19}{12}.$$

$$\int_{0}^{2} \left[x - \frac{1}{2}\right] dx = \int_{\frac{-1}{2}}^{\frac{3}{2}} [t]dt$$

$$= \int_{\frac{-1}{2}}^{0} (-1)dt + \int_{0}^{1} 0 \cdot dt + \int_{1}^{\frac{3}{2}} 1 \cdot dt = 0.$$

10. Consider a curve y = y(x) in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line 2x – 12y = 15 does **NOT** pass through the point.



(1) (6, 21) (2) (8, 9)

(3) (10, –4) (4) (12, –15)

Official Ans. by NTA (C)

Sol. Given that $A_1 = 2A_2$

from the graph $A_1 + A_2 = xy - 8$

$$\Rightarrow \frac{3}{2}A_1 = xy - 8$$

$$\Rightarrow A_1 = \frac{2}{3}xy - \frac{16}{3}$$

$$\Rightarrow \int_4^x f(x)dx = \frac{2}{3}xy - \frac{16}{3}$$

$$\Rightarrow f(x) = \frac{2}{3}\left(x\frac{dy}{dx} + y\right)$$

$$\Rightarrow \frac{2}{3}x\frac{dy}{dx} = \frac{y}{3}$$

$$\Rightarrow 2\int\frac{dy}{y} = \int\frac{dx}{x}$$

$$\Rightarrow 2\ell ny = \ell nx + \ell nc$$

$$\Rightarrow y^2 = cx$$

As f(4) = 2 $\Rightarrow$ c = 1

so $y^2 = x$

slope of normal = –6

$$y = -6(x) - \frac{1}{2}(-6) - \frac{1}{4}(-6)^3$$

$\Rightarrow$ y = –6x + 3 + 54

$\Rightarrow$ y + 6x = 57

Now check options and (C) will not satisfy.

11. The equations of the sides AB, BC and CA of a triangle ABC are 2x + y = 0, x + py = 39 and x – y = 3 respectively and P(2, 3) is its circumcentre. Then which of the following is NOT true :

(A) $(AC)^2 = 9p$

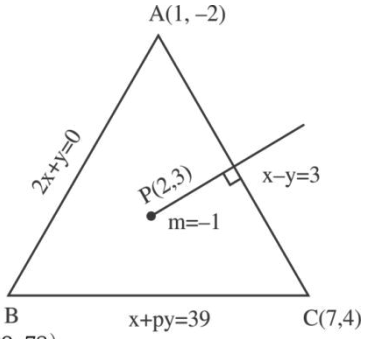
(B) $(AC)^2 + p^2 = 136$

(C) 32 < area (ΔABC) < 36

(D) 34 < area (ΔABC) < 38

Official Ans. by NTA (D)

Sol.



$\left(\frac{-39}{15}, \frac{78}{15}\right)$

Perpendicular bisector of AB

x + y = 5

Take image of A

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{-2(-6)}{2} = 6$$

(7, 4)

7 + 4p = 39

p = 8

solving x + 8y = 39 and y = –2x

$$x = \frac{-39}{15} \qquad y = \frac{78}{15}$$

$AC^2 = 72 = 9p$

$AC^2 + p^2 = 72 + 64 = 136$

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 7 & 4 & 1 \\ \frac{-39}{15} & \frac{78}{15} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[4 - \frac{78}{15} + 2\left(7 + \frac{39}{15}\right) + 7\left(\frac{78}{15}\right) + \frac{4\times 39}{15}\right]$$

$$= \frac{1}{2}\left[18 + 18\times\frac{13}{5}\right]$$

$$= 9\left[\frac{18}{5}\right] = \frac{162}{5} = 32.4$$

Ans. (D)

12. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y = 2x$ gives a chord OA of a circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then QA : AP is equal to :

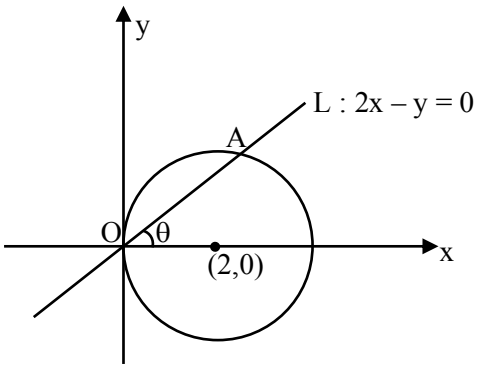
(A) 1 : 4 (B) 1 : 5

(C) 2 : 5 (D) 1 : 3

Official Ans. by NTA (A)

Sol. $C_1 : x^2 + y^2 - 4x = 0$

$\tan\theta = 2$



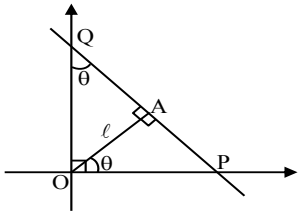
C_2 is a circle with OA as diameter.

So, tangent at A on C_2 is perpendicular to OR

Let $OA = \ell$

$$\therefore \frac{QA}{AP} = \frac{\ell\cot\theta}{\ell\tan\theta}$$

$$= \frac{1}{\tan^2\theta} = \frac{1}{4}$$



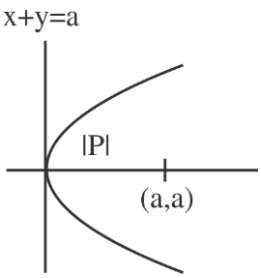
13. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to :

(A) $2\sqrt{2}$ (B) $2\sqrt{3}$

(C) $4\sqrt{2}$ (D) 4

Official Ans. by NTA (C)

Sol.



$$|P| = \left|\frac{a}{\sqrt{2}}\right| = \frac{16}{4} = 4$$

$$|a| = 4\sqrt{2}$$

Ans. (C)

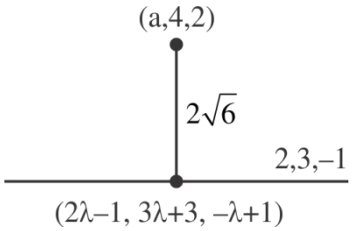
14. If the length of the perpendicular drawn from the point P(a, 4, 2), $a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $a + \sum_{i=1}^{3}\alpha_i$ is equal to :

(A) 7 (B) 8

(C) 12 (D) 14

Official Ans. by NTA (B)

Sol.



$(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$\Rightarrow 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$\Rightarrow 14\lambda - 4 - 2a = 0$

$\Rightarrow 7\lambda - 2 - a = 0$

and,

$(2\lambda - 1 - a)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow (5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow 35\lambda^2 - 14\lambda - 21 = 0$

$\Rightarrow (\lambda - 1)(35\lambda + 21) = 0$

For, $\lambda = 1 \quad \Rightarrow a = 5$

Let $(\alpha_1, \alpha_2, \alpha_3)$ be reflection of point P

$\alpha_1 + 5 = 2 \qquad \alpha_2 + 4 = 12 \qquad \alpha_3 + 2 = 0$

$\alpha_1 = -3 \qquad \alpha_2 = 8 \qquad \alpha_3 = -2$

$a + \alpha_1 + \alpha_2 + \alpha_3 = 8$

15. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$, $a > 0$ makes an angle $30°$ with the plane $y - z + 2 = 0$, then the direction cosines of the line are :

(A) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (B) $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

(C) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$ (D) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0$

Official Ans. by NTA (B)

Sol. $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a & b & 0\\ a & b & c\end{vmatrix}$

$= bc\hat{i}-ac\hat{j}$

Direction ratios of line are **b, –a, 0**

Direction ratios of normal of the plane are **0, 1, – 1**

$$\cos 60^\circ=\left|\frac{-a}{|\sqrt{2}|\sqrt{b^2+a^2}}\right|=\frac{1}{2}$$

$$\Rightarrow \left|\frac{a}{\sqrt{a^2+b^2}}\right|=\frac{1}{\sqrt{2}}$$

$\Rightarrow b=\pm a$

So, D.R.'s can be (±a, –a, 0)

$\therefore$ D.C.'s can be $\pm\left(\frac{\pm 1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\right)$

16. Let X have a binomial distribution B(n, p) such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X>n-3)=\frac{k}{2^n}$, then k is equal to

(A) 528 (B) 529 (C) 629 (D) 630

Official Ans. by NTA (B)

Sol. Let α = Mean & β = Variance $(\alpha>\beta)$

So, $\alpha+\beta=24$, $\alpha\beta=128$

$\Rightarrow \alpha=16$ & $\beta=8$

$\Rightarrow np=16$ $npq=8 \Rightarrow q=\frac{1}{2}$

$\therefore p=\frac{1}{2}, n=32$

$$p(x>n-3)=\frac{1}{2^n}({}^nC_{n-2}+{}^nC_{n-1}+{}^nC_n)$$

$$\therefore k={}^{32}C_{30}+{}^{32}C_{31}+{}^{32}C_{32}=\frac{32\times 31}{2}+32+1$$

$= 496 + 33 = 529$

17. A six faced die is biased such that 3 × P(a prime number) = 6 × P(a composite number) = 2 × P(1). Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :

(A) $\frac{3}{11}$ (B) $\frac{5}{11}$

(C) $\frac{7}{11}$ (D) $\frac{8}{11}$

Official Ans. by NTA (D)

Sol. Let $\frac{P(\text{a prime number})}{2}=\frac{P(\text{a composite})}{1}=\frac{P(1)}{3}=k$

So, P(a prime number) = 2k,

P(a composite number) = k,

& P(1) = 3k

& 3 × 2k + 2 × k + 3k = 1

$\Rightarrow k=\frac{1}{11}$

P(success) = P(1 or 4) = 3k + k = $\frac{4}{11}$

Number of trials, n = 2

$\therefore$ mean = np = $2\times\frac{4}{11}=\frac{8}{11}$

18. The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45º. Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60º. If ∠BAQ = 30º, AB = d and the area of the trapezium PQRB is α, then the ordered pair (d, α) is :

(A) $\left(10\left(\sqrt{3}-1\right),25\right)$ (B) $\left(10\left(\sqrt{3}-1\right),\frac{25}{2}\right)$

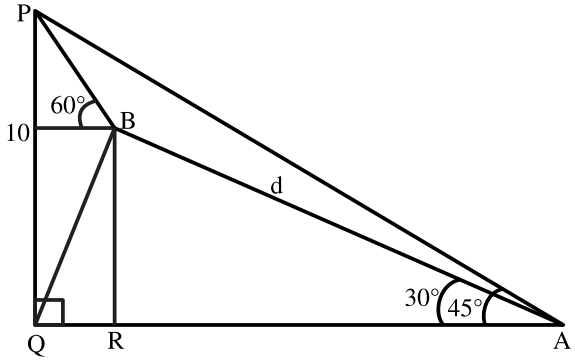
(C) $\left(10\left(\sqrt{3}+1\right),25\right)$ (D) $\left(10\left(\sqrt{3}+1\right),\frac{25}{2}\right)$

Official Ans. by NTA (A)

Sol. $QA = 10 \qquad RA = d\cos 30^\circ = \dfrac{\sqrt{3}d}{2}$

$$QR = 10 - \frac{\sqrt{3}d}{2}$$

$$BR = d\sin 30^\circ = \frac{d}{2}$$



$$\tan 60^\circ = \frac{PQ - BR}{QR} = \frac{10 - \dfrac{d}{2}}{10 - \dfrac{\sqrt{3}d}{2}}$$

$$\Rightarrow \sqrt{3} = \frac{20 - d}{20 - \sqrt{3}d}$$

$$\Rightarrow 20\sqrt{3} - 3d = 20 - d$$

$$\Rightarrow 2d = 20(\sqrt{3} - 1)$$

$$\Rightarrow d = 10(\sqrt{3} - 1)$$

$$\text{ar(PQRB)} = \alpha = \frac{1}{2}(PQ + BR)\cdot QR$$

$$= \frac{1}{2}\left(10 + \frac{d}{2}\right)\cdot\left(10 - \frac{\sqrt{3}d}{2}\right)$$

$$= \frac{1}{2}\left(10 + 5\sqrt{3} - 5\right)\left(10 - 15 + 5\sqrt{3}\right)$$

$$= \frac{1}{2}\left(5\sqrt{3} + 5\right)\left(5\sqrt{3} - 5\right) = \frac{1}{2}(75 - 25) = 25$$

19. Let $S = \left\{\theta \in \left(0, \dfrac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\dfrac{\pi}{6}\right)\sec\left(\theta + \dfrac{m\pi}{6}\right) = -\dfrac{8}{\sqrt{3}}\right\}$

Then

(A) $S = \left\{\dfrac{\pi}{12}\right\}$ (B) $S = \left\{\dfrac{2\pi}{3}\right\}$

(C) $\sum_{\theta \in S} \theta = \dfrac{\pi}{2}$ (D) $\sum_{\theta \in S} \theta = \dfrac{3\pi}{4}$

Official Ans. by NTA (C)

Sol. Let $\alpha = \theta + (m-1)\dfrac{\pi}{6}$

& $\beta = \theta + m\dfrac{\pi}{6}$

So, $\beta - \alpha = \dfrac{\pi}{6}$

Here, $\sum_{m=1}^{9} \sec\alpha \cdot \sec\beta = \sum_{m=1}^{9} \dfrac{1}{\cos\alpha \cdot \cos\beta}$

$$= 2\sum_{m=1}^{9} \frac{\sin(\beta - \alpha)}{\cos\alpha \cdot \cos\beta} = 2\sum_{m=1}^{9} (\tan\beta - \tan\alpha)$$

$$= 2\sum_{m=1}^{9} \left(\tan\left(\theta + m\frac{\pi}{6}\right) - \tan\left(\theta + (m-1)\frac{\pi}{6}\right)\right)$$

$$= 2\left(\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\theta\right) = 2(-\cot\theta - \tan\theta) = -\frac{8}{\sqrt{3}}$$

(Given)

$\therefore \quad \tan\theta + \cot\theta = \dfrac{4}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$ or $\sqrt{3}$

So, $S = \left\{\dfrac{\pi}{6}, \dfrac{\pi}{3}\right\}$

$$\sum_{\theta \in S} \theta = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$$

20. If the truth value of the statement $(P \wedge (\sim R)) \rightarrow ((\sim R) \wedge Q)$ is F, then the truth value of which of the following is F ?

(A) $P \vee Q \rightarrow \sim R$ (B) $R \vee Q \rightarrow \sim P$

(C) $\sim(P \vee Q) \rightarrow \sim R$ (D) $\sim(R \vee Q) \rightarrow \sim P$

Official Ans. by NTA (D)

Sol. $X \Rightarrow Y$ is a false

when X is true and Y is false

So, $P \rightarrow T$, $Q \rightarrow F$, $R \rightarrow F$

(A) $P \vee Q \rightarrow \sim R$ is T

(B) $R \vee Q \rightarrow \sim P$ is T

(C) $\sim(P \vee Q) \rightarrow \sim R$ is T

(D) $\sim(R \vee Q) \rightarrow \sim P$ is F

SECTION-B

1. Consider a matrix $A=\begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers.

If $\frac{\det\left(\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}A\right)\right)\right)\right)}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}\times 3^{16}$, then the number of such 3 – tuples (α, β, γ) is _____ .

Official Ans. by NTA (42)

Sol. $A=\begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$

$R_3 \to R_3 + R_1$

$\Rightarrow |A| = |\alpha+\beta+\gamma| \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$

$\Rightarrow |A| = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

$\because$ $|\text{adj } A| = |A|^{n-1}$

$|\text{adj (adj } A)| = |A|^{(n-1)^2}$

$\left|\text{adj}\left(\text{adj}\left(\text{adj}(\text{adj}A)\right)\right)\right| = |A|^{(n-1)^4} = |A|^{2^4} = |A|^{16}$

$\therefore (\alpha+\beta+\gamma)^{16} = 2^{32} \cdot 3^{16}$

$\Rightarrow (\alpha+\beta+\gamma)^{16} = (2^2 \cdot 3)^{16} = (12)^{16}$

$\Rightarrow \alpha+\beta+\gamma = 12$

$\because$ $\alpha, \beta, \gamma \in N$

$(\alpha-1)+(\beta-1)+(\gamma-1) = 9$

number all tuples $(\alpha, \beta, \gamma) = {}^{11}C_2 = 55$

1 case for $\alpha = \beta = \gamma$

& 12 case when any two of these are equal

So, No. of distinct tuples (α, β, γ)

$= 55 - 13 = 42$

2. The number of functions f, from the set $A=\{x\in N : x^2-10x+9\le 0\}$ to the set $B=\{n^2 : n\in N\}$ such that $f(x)\le (x-3)^2+1$, for every $x\in A$, is _____ .

Official Ans. by NTA (1440)

Sol. $(x^2 - 10x + 9) \le 0 \Rightarrow (x-1)(x-9) \le 0$

$\Rightarrow x \in [1, 9] \Rightarrow A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$f(x) \le (x-3)^2 + 1$

$x = 1 : f(1) \le 5 \Rightarrow 1^2, 2^2$

$x = 2 : f(2) \le 2 \Rightarrow 1^2$

$x = 3 : f(3) \le 1 \Rightarrow 1^2$

$x = 4 : f(4) \le 2 \Rightarrow 1^2$

$x = 5 : f(5) \le 5 \Rightarrow 1^2, 2^2$

$x = 6 : f(6) \le 10 \Rightarrow 1^2, 2^2, 3^2$

$x = 7 : f(7) \le 17 \Rightarrow 1^2, 2^2, 3^2, 4^2$

$x = 8 : f(8) \le 26 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2$

$x = 9 : f(9) \le 37 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

Total number of such function

$= 2(6!) = 2(720) = 1440$

3. Let for the 9^{th} term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of 6x, to be the greatest for $x=\frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to:

Official Ans. by NTA (24)

Sol. $(3+6x)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1}(6x)^1 + \ldots$

$T_{r+1}\ {}^nC_r 3^{n-r}.(6x)r = {}^nC_r 3^{n-r}.6^r.\ x^r$

$= {}^nC_r 3^{n-r}.3^r.2^r.\left(\frac{3}{2}\right)^r = {}^nC_r 3^n.3^r \qquad [\text{for } x = \frac{3}{2}]$

T_9 is greatest of $x = \frac{3}{2}$

So, $T_9 > T_{10}$ and $T_9 > T_8$

(concept of numerically greatest term)

Here, $\frac{T_9}{T_{10}} > 1$ and $\frac{T_9}{T_8} > 1$

$\Rightarrow \frac{{}^nC_8 3^n.3^8}{{}^nC_9 3^n.3^9} > 1$ and $\frac{{}^nC_8 3^n.3^8}{{}^nC_7 3^n.3^7} > 1$

and $\frac{{}^nC_8}{{}^nC_7} > \frac{1}{3}$

and $\frac{n-7}{8} > \frac{1}{3}$

$\Rightarrow \frac{29}{3} < n < 11 \Rightarrow n = 10 = n_0$

So, in $(3+6x)^n$ for $n = n_0 = 10$

i.e., in $(3+6x)^{10}$, here $T_{r+1} = {}^{10}C_r 3^{10-r}6^r x^r$

$T_7 = {}^{10}C_6 3^4.6^6.x^6 = 210.3^{10}.2^6x^6$

$T_4 = {}^{10}C_3 3^7 6^3 x^3 = 120.3^{10}.2^3x^3$

Ratio of coefficient of x^6 and coefficient of $x^3 = k$

$\therefore k = \frac{210.3.^{10}2^6}{120.3^{10}.2^3} = \frac{7}{4}\times 2^3 = 14$

So, $k + n_0 = 14 + 10 = 24$

4. $\frac{2^3-1^3}{1\times 7} + \frac{4^3-3^3+2^3-1^3}{2\times 11} + \frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times 15} + \ldots.. + \frac{30^3-29^3+28^3-27^3+\ldots+2^3-1^3}{15\times 63}$

is equal to _____.

Official Ans. by NTA (120)

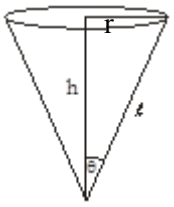
Sol. $T_n = \frac{2\sum_{r=1}^{n}(2r)^3 - (\sum_{r=1}^{2n} r^3)}{n(4n+3)}$

$\Rightarrow T_n = n$

So, $\sum_{n=1}^{15} T_n = 120$

5. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is _____.

Official Ans. by NTA (5)



$\tan\theta = \frac{3}{4} = \frac{r}{h}$

$\frac{dV}{dt} = 6$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3 \tan^2\theta = \frac{9\pi}{48}h^3 = \frac{3\pi}{16}h^3$

$\Rightarrow \frac{dV}{dt} = \frac{3\pi}{16}.3h^2.\frac{dh}{dt} = 6 \Rightarrow \left(\frac{dh}{dt}\right)_{h=4} = \frac{2}{3\pi}\text{m/hr}$

Now, $S = \pi r\ell = \frac{15}{16}\pi h^2$

$\Rightarrow \frac{dS}{dt} = \frac{15\pi}{16}.2h\frac{dh}{dt}$

$\Rightarrow \left(\frac{dS}{dt}\right)_{h=4} = 5\text{m}^2/\text{hr}$

6. For the curve C : $(x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α), $\alpha > 0$, on C, is equal to _____.

Official Ans. by NTA (16)

Sol. (α, α) lies on

$C: x^2 + y^2 - 3 + x^2 - y^2 - 1^{\ 5} = 0$

Put (α, α), $2\alpha^2 - 3 + -1^{\ 5} = 0$

$\Rightarrow \alpha = \sqrt{2}$

Now, differentiate C

$2x + 2y\cdot y' + 5\left(x^2 - y^2 - 1\right)^4\left(2x - 2yy'\right) = 0 \quad \ldots (1)$

At $\left(\sqrt{2}, \sqrt{2}\right)$

$$\sqrt{2}+\sqrt{2}y'+5(-1)^4\left(\sqrt{2}-\sqrt{2}y'\right)=0$$

$$\Rightarrow\ y'=\frac{3}{2} \qquad \ldots.\ (2)$$

Diff. (1) w.r.t. x

Again, Diff. (1) w.r.t. x

$$1+(y')^2+yy''+20\left(x^2-y^2-1\right)^3(x-yy')^2.2$$
$$+5\left(x^2-y^2-1\right)^4\left(1-(y')^2-yy''\right)=0$$

At $\left(\sqrt{2},\sqrt{2}\right)$ and $y'=\frac{3}{2}$

We have,

$$\left(1+\frac{9}{4}\right)+\sqrt{2}y''-40\left(\sqrt{2}-\sqrt{2}\cdot\frac{3}{2}\right)^2$$
$$+5(1)\left(1-\frac{9}{4}-\sqrt{2}y''\right)=0$$

$$\Rightarrow\ 4\sqrt{2}y''=-23$$

$$\therefore\ 3y'-y^3y''=\frac{9}{2}+\frac{23}{2}=16$$

7. Let $f(x)=\min\{[x-1],[x-2],\ldots,[x-10]\}$ where $[t]$ denotes the greatest integer $\le t$. Then $\int_0^{10} f(x)dx+\int_0^{10}(f(x))^2dx+\int_0^{10}|f(x)|dx$ is equal to ___.

Official Ans. by NTA (385)

Sol. $f(x)=[x]-10$

$$\int_0^{10} f(x)\cdot dx=-10-9-8-.....-1$$

$$=-\frac{10\cdot11}{2}=-55$$

$$\int_0^{10}\left(f(x)\right)^2dx=10^2+9^2+8^2+\ldots+1^2$$

$$=\frac{10\cdot11\cdot21}{6}=385$$

$$\int_0^{10}\left|f(x)\right|=10+9+8+....+1$$

$$=\frac{10\cdot11}{2}=55$$

$$=-55+385+55=385$$

8. Let f be a differentiable function satisfying $f(x)=\frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2x}{3}\right)d\lambda$, $x>0$ and $f(1)=\sqrt{3}$. If $y=f(x)$ passes through the point $(\alpha, 6)$, then α is equal to _____.

Official Ans. by NTA (12)

Sol. Let, $\frac{\lambda^2x}{3}=t$

$$\Rightarrow\ \frac{2\lambda x}{3}d\lambda=dt$$

$$\Rightarrow\ d\lambda=\frac{3}{2}\cdot\frac{1\sqrt{x}}{x\cdot\sqrt{3}\sqrt{t}}dt$$

$$\Rightarrow d\lambda=\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{x}}\cdot\frac{dt}{\sqrt{t}}$$

So, $f(x)=\frac{1}{\sqrt{x}}\int_0^x\frac{f(t)}{\sqrt{t}}dt$

$$\Rightarrow\ \sqrt{x}\cdot f'(x)+\frac{f(x)}{2\sqrt{x}}=\frac{f(x)}{\sqrt{x}}$$

$$\Rightarrow\ \sqrt{x}\cdot f'(x)=\frac{f(x)}{2\sqrt{x}}$$

$$\Rightarrow\ \frac{dy}{y}=\frac{dx}{2x}$$

$$\Rightarrow\ \ln y=\frac{1}{2}\ln x+c\Rightarrow\ f(x)=\sqrt{x}$$

$$\Rightarrow\ y=\sqrt{3x}\quad\{\text{as } f(1)=\sqrt{3}\}$$

So, $f(x)=\sqrt{3x}$

Now, $f(\alpha)=6\Rightarrow 36=3\alpha$

$$\Rightarrow\ \alpha=12$$

9. A common tangent T to the curves $C_1:\frac{x^2}{4}+\frac{y^2}{9}=1$ and $C_2:\frac{x^2}{42}-\frac{y^2}{143}=1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1+x_2|$ is equal to _____.

Official Ans. by NTA (20)

Sol. Let common tangents are

$T_1 : y = mx \pm \sqrt{4m^2 + 9}$

& $T_2 : y = mx \pm \sqrt{42m^2 - 13}$

So, $4m^2 + 9 = 42m^2 - 143$

$\Rightarrow 38m^2 = 152$

$\Rightarrow m = \pm 2$

& $c = \pm 5$

For given tangent not pass through 4th quadrant

$T : y = 2x + 5$

Now, comparing with $\frac{xx_1}{4} + \frac{yy_1}{9} = 1$

We get, $\frac{x_1}{8} = -\frac{1}{5} \Rightarrow x_1 = -\frac{8}{5}$

$\frac{xx_2}{42} - \frac{yy_2}{143} = 1$

$2x - y = -5$ we have

$x_2 = -\frac{84}{5}$

So, $|2x_1 + x_2| = \left|\frac{-100}{5}\right| = 20$

10. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that

$\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}, \alpha > 0$.

If $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \frac{1}{36}$, then α is equal to _____.

Official Ans. by NTA (36)

Sol. $\vec{a} \times \vec{b} = 4\vec{c} \Rightarrow \vec{a} \cdot \vec{c} = 0 = \vec{b} \cdot \vec{c}$

$\vec{b} \times \vec{c} = 9\vec{a} \Rightarrow \vec{a} \cdot \vec{b} = 0 = \vec{a} \cdot \vec{c}$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are mutually $\perp$ set of vectors.

$\Rightarrow |\vec{a}||\vec{b}| = 4|\vec{c}|$, $|\vec{b}||\vec{c}| = 9|\vec{a}|$ & $|\vec{c}||\vec{a}| = \alpha|\vec{b}|$

$\Rightarrow \frac{|\vec{a}|}{|\vec{c}|} = \frac{4}{9}\frac{|\vec{c}|}{|\vec{a}|}$

$\Rightarrow \frac{|\vec{c}|}{|\vec{a}|} = \frac{3}{2}$

$\therefore$ If $|a| = \lambda$, $|c| = \frac{3\lambda}{2}$ & $|b| = 6$

Now $|a| + |b| + |c| = \frac{1}{36}$

$\Rightarrow \frac{5}{2}\lambda + 6 = \frac{1}{36}$, $\lambda = \frac{-43}{18} = |a|$

which gives negative value of λ or $|a|$ which is NOT possible & hence data seems to be wrong.

But if $|\vec{a}| + |\vec{b}| + |\vec{c}| = 36$

$\frac{5}{2}\lambda + 6 = 36$

$\lambda = 12$

$\alpha = \frac{|\vec{c}||\vec{a}|}{|\vec{b}|} = \frac{3 \times 12}{2} \times \frac{12}{6}$

$\alpha = 36$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTON

SECTION-A

1. The dimensions of $\left(\frac{B^2}{\mu_0}\right)$ will be :

(if μ_0 : permeability of free space and

B : magnetic field)

(A) $[M L^2 T^{-2}]$ (B) $[M L T^{-2}]$

(C) $[M L^{-1} T^{-2}]$ (D) $[M L^2 T^{-2} A^{-1}]$

Official Ans. by NTA (C)

Sol. $u = \frac{B^2}{2\mu_0}$

$u \rightarrow$ Energy per unit volume

$$\left[\frac{B^2}{\mu_0}\right] = [u] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

2. A NCC parade is going at a uniform speed of 9 km/h under a mango tree on which a monkey is sitting at a height of 19.6 m. At any particular instant, the monkey drops a mango. A cadet will receive the mango whose distance from the tree at time of drop is :

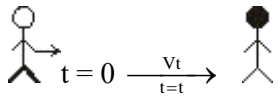
(Given g = 9.8 m/s^2)

(A) 5 m (B) 10 m

(C) 19.8 m (D) 24.5 m

Official Ans. by NTA (A)

Sol. Monkey



Time taken by mango $= \sqrt{\frac{2n}{g}}$

$$= \sqrt{\frac{2\times19.6}{9.8}} = 2 \text{ second}$$

Distance = vt

$$= 9\times\frac{5}{18}\times2 = 5m$$

3. In two different experiments, an object of mass 5 kg moving with a speed of 25 ms^{-1} hits two different walls and comes to rest within
(i) 3 second, (ii) 5 seconds, respectively.
Choose the correct option out of the following :

(A) Impulse and average force acting on the object will be same for both the cases.

(B) Impulse will be same for both the cases but the average force will be different.

(C) Average force will be same for both the cases but the impulse will be different.

(D) Average force and impulse will be different for both the cases.

Official Ans. by NTA (B)

Sol. Impulse = change in momentum

$$I = \Delta P$$

$$F_{aug} = \frac{\Delta P}{\Delta t}$$

$$\Delta t_1 = 3 \qquad \Delta t_2 = 5$$

$$\Delta P_1 = \Delta P_2$$

$$I_1 = I_2$$

F_{avg} in case (i) is more than (ii)

4. A balloon has mass of 10 g in air. The air escapes from the balloon at a uniform rate with velocity 4.5 cm/s. If the balloon shrinks in 5 s completely. Then, the average force acting on that balloon will be (in dyne).

(A) 3 (B) 9 (C) 12 (D) 18

Official Ans. by NTA (B)

Sol. $F = \frac{dm}{dt}v$

$$= \frac{10g}{5s}\left(4.5\frac{cm}{s}\right) = 9\frac{gcm}{s^2} = 9 \text{ dyne}$$

5. If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately :

(A) decrease by 2% (B) decrease by 4%

(C) increase by 2% (D) increase by 4%

Official Ans. by NTA (D)

Sol. $g = \frac{GM}{R^2}$

M = constant $g < \frac{1}{R^2}$

$100\ \frac{\Delta g}{g} = -2\frac{\Delta R}{R}100$

% change = – 2 (–2)

% change in g = 4%

increase by 4%

6. The force required to stretch a wire of cross-section 1 cm^2 to double its length will be :

(Given Yong's modulus of the wire = 2×10^{11} N/m^2)

(A) 1×10^7 N (B) 1.5×10^7 N

(C) 2×10^7 N (D) 2.5×10^7 N

Official Ans. by NTA (C)

Sol. $F = \gamma A\frac{\Delta \ell}{\ell}$

$= 2\times10^{11}\times10^{-4}\left(\frac{2\ell-\ell}{\ell}\right)$

$= 2\times10^7$ N

7. A Carnot engine has efficiency of 50%. If the temperature of sink is reduced by 40°C, its efficiency increases by 30%. The temperature of the source will be :

(A) 166.7 K (B) 255.1 K

(C) 266.7 K (D) 367.7 K

Official Ans. by NTA (C)

Sol. $\eta = 1-\frac{T_L}{T_H}$

$\frac{1}{2} = 1-\frac{T_L}{T_H}$

$\frac{1}{2}(1\cdot3) = 1-\left(\frac{T_L-40}{T_H}\right)$

$\frac{1}{2}(1\cdot3) = \frac{1}{2}+\frac{40}{T_H}$ $\qquad T_H = 266.7\,K$

8. Given below are two statements :

Statement I : The average momentum of a molecule in a sample of an ideal gas depends on temperature.

Statement II : The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become 2v.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (D)

Sol. [$P_{avg} = 0$] (due to random motion)

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$T_{new} = 2T$

$M_{new} = \frac{M}{2}$

$\frac{v_{new}}{v} = \frac{\sqrt{\frac{2T}{M/2}}}{\sqrt{\frac{T}{M}}}$

$v_{new} = 2v$

9. In the wave equation

$$y = 0.5\sin\frac{2\pi}{\lambda}(400t - x)m$$

the velocity of the wave will be :

(A) 200 m/s (B) $200\sqrt{2}$ m / s

(C) 400 m/s (D) $400\sqrt{2}$ m / s

Official Ans. by NTA (C)

Sol. $y = 0.5\sin\left(\frac{2\pi}{\lambda}400t - \frac{2\pi}{\lambda}x\right)$

$\omega = \frac{2\pi}{\lambda}400$

$K = \frac{2\pi}{\lambda}$

$v = \frac{\omega}{k}$ [v = 400 m/s]

10. Two capacitors, each having capacitance 40 μF are connected in series. The space between one of the capacitors is filled with dielectric material of dielectric constant K such that the equivalence capacitance of the system became 24 μF. The value of K will be :

(A) 1.5 (B) 2.5

(C) 1.2 (D) 3

Official Ans. by NTA (A)

Sol.

$C_{eq} = \frac{C(KC)}{C+KC} = \frac{KC}{K+1}$

$24 = \frac{K40}{K+1}$

$[K = 1{\cdot}5]$

11. A wire of resistance R_1 is drawn out so that its length is increased by twice of its original length. The ratio of new resistance to original resistance is:

(A) 9 : 1 (B) 1 : 9

(C) 4 : 1 (D) 3 : 1

Official Ans. by NTA (A)

Sol. $R_1 = \rho\frac{L_1}{A_1}$

$R_2 = \rho\left(\frac{3L_1}{A_1/3}\right) = 9\rho\frac{L_1}{A_1}$

$\therefore \frac{R_2}{R_1} = 9$

12. The current sensitivity of a galvanometer can be increased by :

(A) decreasing the number of turns

(B) increasing the magnetic field

(C) decreasing the area of the coil

(D) decreasing the torsional constant of the spring

Choose the most appropriate answer from the options given below :

(A) (B) and (C) only (B) (C) and (D) only

(C) (A) and (C) only (D) (B) and (D) only

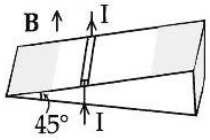
Official Ans. by NTA (D)

Sol. $i = \left(\frac{K}{NAB}\right)\theta$

$\therefore \frac{d\theta}{di} = \frac{NAB}{K}$

13. As shown in the figure, a metallic rod of linear density 0.45 kg m^{-1} is lying horizontally on a smooth incline plane which makes an angle of 45° with the horizontal. The minimum current flowing in the rod required to keep it stationary, when 0.15 T magnetic field is acting on it in the vertical upward direction, will be :

{Use g = 10 m/s^2}

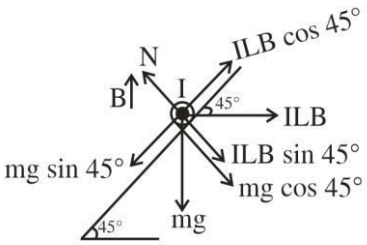


(A) 30 A (B) 15 A

(C) 10 A (D) 3 A

Official Ans. by NTA (A)

Sol.



$$mg\sin 45° = ILB\cos 45°$$

$$\therefore I = \left(\frac{m}{L}\right)\frac{g}{B}$$

$$= \frac{(0.45)(10)}{0.15} = 30\,A$$

14. The equation of current in a purely inductive circuit is $5\sin(49\pi t - 30°)$. If the inductance is 30 mH then the equation for the voltage across the inductor, will be :

$\left\{\text{Let } \pi = \frac{22}{7}\right\}$

(A) $1.47\sin(49\pi t - 30°)$ (B) $1.47\sin(49\pi t + 60°)$

(C) $23.1\sin(49\pi t - 30°)$ (D) $23.1\sin(49\pi t + 60°)$

Official Ans. by NTA (D)

Sol. $v_0 = i_0 x_L$

$= i_0(wL)$

$= (5)(49\pi)(30\times10^{-3})$

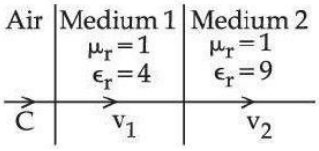
$= 23.1$

Voltage will lead current by 90°.

$\therefore$ V = 23.1 sin (49 πt + 60°)

15. As shown in the figure, after passing through the medium 1. The speed of light v_2 in medium 2 will be :

(Given c = 3 × 10^8 ms^{-1})



(A) 1.0 × 10^8 ms^{-1} (B) 0.5 × 10^8 ms^{-1}

(C) 1.5 × 10^8 ms^{-1} (D) 3.0 × 10^8 ms^{-1}

Official Ans. by NTA (A)

Sol. $\frac{\mu_2}{\mu_{air}} = \frac{C}{v_2}$

$$\therefore \frac{\sqrt{\mu_{r_2}\varepsilon_{r_2}}}{(1)} = \frac{C}{v_2}$$

$$\therefore \sqrt{(1)(9)} = \frac{C}{v_2}$$

$$\therefore v_2 = \frac{C}{3}$$

16. In normal adjustment, for a refracting telescope, the distance between objective and eye piece is 30 cm. The focal length of the objective, when the angular magnification of the telescope is 2, will be:

(A) 20 cm (B) 30 cm

(C) 10 cm (D) 15 cm

Official Ans. by NTA (A)

Sol. $f_0 + f_e = 30$

$m = \frac{f_0}{f_e}$

$2 = \frac{f_0}{f_e} \Rightarrow f_0 = 2f_e$

So $f_0 + \frac{f_0}{2} = 30$

$f_0 = 20\,cm$

17. The equation $\lambda = \frac{1.227}{x}nm$ can be used to find the de-Brogli wavelength of an electron. In this equation x stands for :

Where,

m = mass of electron

P = momentum of electron

K = Kinetic energy of electron

V = Accelerating potential in volts for electron

(A) $\sqrt{mK}$ (B) $\sqrt{P}$

(C) $\sqrt{K}$ (D) $\sqrt{V}$

Official Ans. by NTA (D)

Sol. $\lambda = \frac{h}{m\nu}$ (de-Broglie's wavelength)

$\lambda \frac{h}{\sqrt{2m(K \cdot E)}}$

$h = \frac{h}{\sqrt{2mqV}}$

Putting the values of m ; q

We get $\lambda = \frac{1 \cdot 22}{\sqrt{V}}nm$

18. The half life period of a radioactive substance is 60 days. The time taken for $\frac{7}{8}$th of its original mass to disintegrate will be :

(A) 120 days (B) 130 days

(C) 180 days (D) 20 days

Official Ans. by NTA (C)

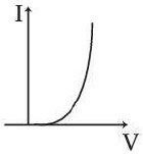
Sol. 7/8 disintegrates means 1/8 remains

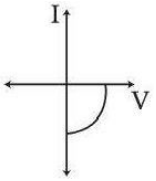
Or $\left(\frac{1}{2}\right)^3$

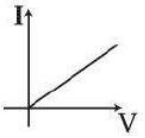
$\therefore$ 3 half lives

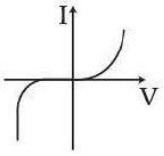
= 180 days

19. Identify the solar cell characteristics from the following options :

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. Conceptual / theory

20. In the case of amplitude modulation to avoid distortion the modulation index (μ) should be :

(A) $\mu \leq 1$ (B) $\mu \geq 1$

(C) $\mu = 2$ (D) $\mu = 0$

Official Ans. by NTA (A)

Sol. $\mu = \frac{A_m}{A_c}$

$\mu \leq 1$ to avoid distortion

because $\mu > 1$ will result in interference between career frequency & message frequency.

SECTION-B

1. If the projection of $2\hat{i}+4\hat{j}-2\hat{k}$ on $\hat{i}+2\hat{j}+\alpha\hat{k}$ is zero. Then, the value of α will be

Official Ans. by NTA (5)

Sol. $\vec{a}\cdot\vec{b}=0$

$\therefore\ \vec{a}\cdot\vec{b}=0$

$\therefore\ 2\times1+4\times2-2\times\alpha=0$

$\therefore\ \alpha=\boxed{5}$

2. A freshly prepared radioactive source of half life 2 hours 30 minutes emits radiation which is 64 times the permissible safe level. The minimum time, after which it would be possible to work safely with source, will be _____ hours.

Official Ans. by NTA (15)

Sol. $A=A_0\times2^{-t/T}$

$\frac{A_0}{64}=A_0\times2^{-t/T}$

$\therefore\ t=6T=6\times2\cdot5=\boxed{15}\text{ hours}$

3. In a Young's double slit experiment, a laser light of 560 nm produces an interference pattern with consecutive bright fringes' separation of 7.2 mm. Now another light is used to produce an interference pattern with consecutive bright fringes' separation of 8.1 mm. The wavelength of second light is _____ nm.

Official Ans. by NTA (630)

Sol. $\beta\propto\lambda$

$\lambda_2=\frac{9}{8}\lambda_1$

$\therefore\ \beta_2=\frac{9}{8}\beta_1=\frac{9}{8}\times560=\boxed{630}\text{ nm}.$

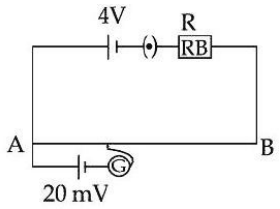
4. The frequencies at which the current amplitude in an LCR series circuit becomes $\frac{1}{\sqrt{2}}$ times its maximum value, are 212 rad s^{-1} and 232 rad s^{-1}. The value of resistance in the circuit is $R=5\Omega$. The self inductance in the circuit is _____ mH.

Official Ans. by NTA (250)

Sol. Band width $=232-212=\frac{R}{L}$

$\therefore\ L=\frac{5}{20}=\boxed{250}\text{ mH}$

5. As shown in the figure, a potentiometer wire of resistance 20Ω and length 300 cm is connected with resistance box (R.B.) and a standard cell of emf 4 V. For a resistance 'R' of resistance box introduced into the circuit, the null point for a cell of 20 mV is found to be 60 cm. The value of 'R' is _____ Ω.



Official Ans. by NTA (780)

Sol. $E=\frac{AC}{AB}(V_A-V_B)$

$\therefore\ 20\times10^{-3}=\frac{60}{300}\times\frac{4\times20}{R+20}$

$\therefore\ R=\boxed{780}\ \Omega$

6. Two electric dipoles of dipole moments 1.2×10^{-30} cm and 2.4×10^{-30} cm are placed in two difference uniform electric fields of strengths 5×10^4 NC^{-1} and 15×10^4 NC^{-1} respectively. The ratio of maximum torque experienced by the electric dipoles will be $\frac{1}{x}$. The value of x is _____.

Official Ans. by NTA (6)

Sol. $|\tau|_{max} = PE$

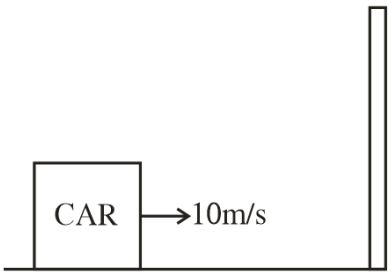
$$\frac{\tau_1}{\tau_2} = \frac{P_1E_1}{P_2E_2} = \frac{1\cdot2\times10^{-30}\times5\times10^4}{2\cdot4\times10^{-30}\times15\times10^4} = \frac{1}{6}$$

Hence x = 6

7. The frequency of echo will be _____ Hz if the train blowing a whistle of frequency 320 Hz is moving with a velocity of 36 km/h towards a hill from which an echo is heard by the train driver. Velocity of sound in air is 330 m/s.

Official Ans. by NTA (340)

Sol. The hill will be a secondary source.



f_1 = frequency of the car w.r.t. the hill

$$f_1 = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{320}\right) \times 320 = 330\,Hz$$

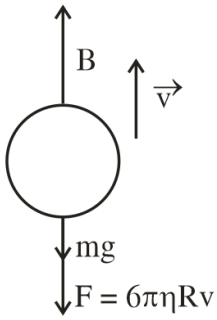
f_2 = Frequency of the sound reflected by hill w.r.t. the car (echo)

$$f_2 = \left(\frac{v + v_0}{v}\right) f_1 = \frac{(330+10)}{330} \times 330 = 340\,Hz$$

8. The diameter of an air bubble which was initially 2 mm, rises steadily through a solution of density 1750 kg m^{-3} at the rate of 0.35 cms^{-1}. The coefficient of viscosity of the solution is _____ poise (in nearest integer). (the density of air is negligible).

Official Ans. by NTA (11)

Sol. As the bubble is rising steadily the net force acting on it will be zero



(Because of density of air the value of mg can be neglected)

So $B = F \Rightarrow \frac{4\pi}{3} R^3 \rho g = 6\pi\eta Rv$

Putting $R = 1\,mm = 10^{-3}\,m$

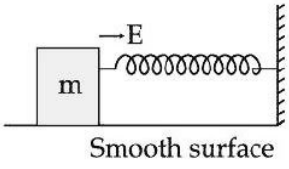
$\rho = 1\cdot75\times10^3\,kg/m^3$

$g = 10\,m/s^2$

$v = 0.35\times10^{-2}\,m/s$

$$\eta = \frac{10}{9} \simeq 1\cdot11\ \text{SI unit} = 11\,\text{poise}(CGS)$$

9. A block of mass 'm' (as shown in figure) moving with kinetic energy E compresses a spring through a distance 25 cm when, its speed is halved. The value of spring constant of used spring will be nE Nm^{-1} for n = _____.



Official Ans. by NTA (24)

Sol. Using work – energy theorem

$$W_{net} = (K_f - K_i)$$

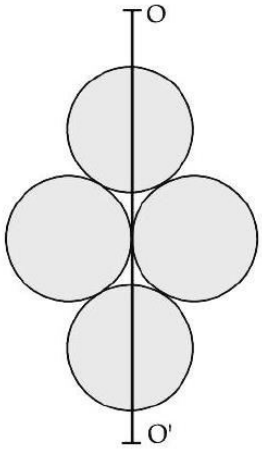
$$\Rightarrow -\frac{1}{2}Kx^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = \frac{E}{4} - E$$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{3E}{4} \Rightarrow K = \frac{3E}{2x^2}$$

$$\Rightarrow K = \frac{3E}{2\times\left(\frac{1}{4}\right)^2} = 24E$$

n = 24

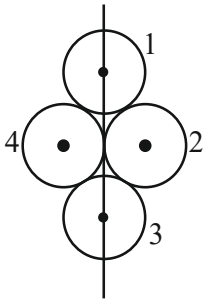
10. Four identical discs each of mass 'M' and diameter 'a' are arranged in a small plane as shown in figure. If the moment of inertia of the system about OO' is $\frac{x}{4}Ma^2$. Then, the value of x will be _____.



Official Ans. by NTA (3)

Sol. $I_1 = I_3 = \frac{MR^2}{4}$

$$I_2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2 = I_4$$



So $I = I_1 + I_2 + I_3 + I_4$

$$= \frac{MR^2}{2} + \frac{5}{2}MR^2$$

$= 3MR^2$, Putting $R = \frac{a}{2}$

$$I = \frac{3Ma^2}{4}, \text{So } x = 3$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Identify the incorrect statement from the following.

(A) A circular path around the nucleus in which an electron moves is proposed as Bohr's orbit.

(B) An orbital is the one electron wave function (Ψ) in an atom.

(C) The existence of Bohr's orbits is supported by hydrogen spectrum.

(D) Atomic orbital is characterised by the quantum numbers n and *l* only

Official Ans. by NTA (D)

Sol. Atomic orbital is characterised by n, *l*, m.

2. Which of the following relation is not correct ?

(A) $\Delta H = \Delta U - P\Delta V$ (B) $\Delta U = q + W$

(C) $\Delta S_{sys} + \Delta S_{surr} \geq 0$ (D) $\Delta G = \Delta H - T\Delta S$

Official Ans. by NTA (A)

Sol. If $U + Pv$ (By definition)

$\Delta 14 = \Delta U + \Delta(Pr)$ at constant pressure

$\Delta H = \Delta U + P\Delta V$

3. Match List-I with List-II.

	List-I		List-II
(A)	$Cd(s) + 2Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s) + H_2O(l)$	(I)	Primary battery
(B)	$Zn(Hg) + HgO(s) \rightarrow ZnO(s) + Hg(l)$	(II)	Discharging of secondary battery
(C)	$2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$	(III)	Fuel cell
(D)	$2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$	(IV)	Charging of secondary battery

Choose the correct answer from the options given below :

(A) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(B) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)

(C) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(D) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

Official Ans. by NTA (C)

Sol. (a) $Cd(s) + 2Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s) + H_2O(l)$

Discharge of secondary Battery

(b) $Zn(Hg) + HgO(s) \rightarrow ZnO(s) + Hg(l)$

(Primary Battery Mercury cell**)**

(c) $2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$

Charging of secondary Battery

(d) $2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$ – Fuel cell

4. Match List-I with List-II.

	List-I Reaction		List-II Catalyst
(A)	$4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(g)$	(I)	$NO(g)$
(B)	$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$	(II)	$H_2SO_4(l)$
(C)	$C_{12}H_{22}O_{11}(aq) + H_2O(l) \rightarrow C_6H_{12}O_6$ (Glucose) + $C_6H_{12}O_6$ (Fructose)	(III)	Pt(s)
(D)	$2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$	(IV)	Fe(s)

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(B) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

(C) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

(D) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)

Official Ans. by NTA (C)

Sol.

(a) $4NH_3(g) + 5O_2(g) \xrightarrow{Pt(s)} 4NO(g) + 6H_2O(g)$

Ostwald process 500 K

(b) $N_2 + 3H_2 \xrightarrow{Fe(s)} 2NH_3(g)$

Haber's process

(c) $C_{12}H_{22}O_{11}(aq.) + H_2O(\ell) \xrightarrow{H^+} \underset{(glucose)}{C_6H_{12}O_6} + \underset{(fructose)}{C_6H_{12}O_6}$

Inversion of sugar cane

(d) $2SO_2(g) + O_2(g) \xrightarrow{NO(g)} 2SO_3(g)$

5. In which of the following pairs, electron gain enthalpies of constituent elements are nearly the same or identical ?

(A) Rb and Cs (B) Na and K

(C) Ar and Kr (D) I and At

Choose the correct answer from the options given below :

(A) (A) and (B) only

(B) (B) and (C) only

(C) (A) and (C) only

(D) (C) and (D) only

Official Ans. by NTA (C)

Sol. Rb & Cs have nearly same electron gain enthalpy electron gain enthalpy = – 46 kj/ml

Ar & Kr have same ΔH_{eq}. Value is + 96 kj/ml

6. Which of the reaction is suitable for concentrating ore by leaching process ?

(A) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(B) $Fe_3O_4 + CO \rightarrow 3FeO + CO_2$

(C) $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na[Al(OH)_4]$

(D) $Al_2O_3 + 6Mg \rightarrow 6MgO + 4Al$

Official Ans. by NTA (C)

Sol. $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na,\left[Al(OH)_4\right]$

Leaching.

7. The metal salts formed during softening of hardwater using Clark's method are :

(A) $Ca(OH)_2$ and $Mg(OH)_2$

(B) $CaCO_3$ and $Mg(OH)_2$

(C) $Ca(OH)_2$ and $MgCO_3$

(D) $CaCO_3$ and $MgCO_3$

Official Ans. by NTA (B)

Sol. Clark's Method Reaction

$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$

$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + Mg(OH)_2 + 2H_2O$

8. Which of the following statement is incorrect ?

(A) Low solubility of LiF in water is due to its small hydration enthalpy.

(B) KO_2 is paramagnetic.

(C) Solution of sodium in liquid ammonia is conducting in nature.

(D) Sodium metal has higher density than potassium metal

Official Ans. by NTA (A)

Sol. Low solubility of LiF in water is due to high lattice enthalpy

9. Match List-I with List-II, match the gas evolved during each reaction.

	List-I		**List-II**
(A)	$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta}$	(I)	H_2
(B)	$KMnO_4 + HCl \rightarrow$	(II)	N_2
(C)	$Al + NaOH + H_2O \rightarrow$	(III)	O_2
(D)	$NaNO_3 \xrightarrow{\Delta}$	(IV)	Cl_2

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(B) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

(C) (A) – (II), (B) – (IV), (C) – (I), (D) – (III)

(D) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Official Ans. by NTA (C)

Sol. $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

$KMnO_4 + HCl \rightarrow MnCl_2 + KCl + Cl_2 + H_2O$

$Al + NaOH + H_2O \rightarrow H_2 + Na\left[Al(OH)_4\right]$

$NaNO_3 \longrightarrow NaNO_2 + O_2$

10. Which of the following has least tendency to liberate H_2 from mineral acids ?

(A) Cu (B) Mn

(C) Ni (D) Zn

Official Ans. by NTA (A)

Sol. Copper is least electropositive among the given metals and it lies below H in reactivity series

11. Given below are two statements :

Statement I : In polluted water values of both dissolved oxygen and BOD are very low.

Statement II : Eutrophication results in decrease in the amount of dissolved oxygen.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

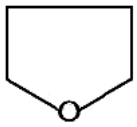
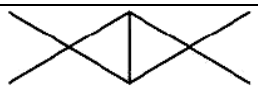
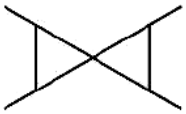
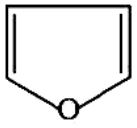
Official Ans. by NTA (D)

Sol. Since eutrophication is result of excessive growth of weed in water bodies, which consume dissolved oxygen of water bodies.

$\therefore$ Eutrophication decreases amount of dissolved oxygen in water bodies.

Polluted water has low value of dissolved oxygen, but high valueof BOD (Biological oxygen demand), since chemical and organic matter requires dissolved oxygen to get decompose.

12. Match List-I with List-II.

	List-I		List-II
(A)		(I)	Spiro compound
(B)		(II)	Aromatic compound
(C)		(III)	Non-planar Heterocyclic compound
(D)		(IV)	Bicyclo compound

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

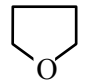
(B) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(C) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

(D) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Official Ans. by NTA (C)

Sol.

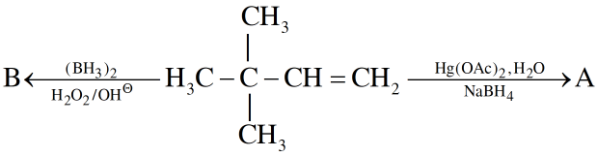
 : Non-planar heterocyclic Compound

 : Bicyclo Compound

 : Spiro Compound

 : Aromatic Compound

13. Choose the correct option for the following reactions.



(A) 'A' and 'B' are both Markovnikov addition products.

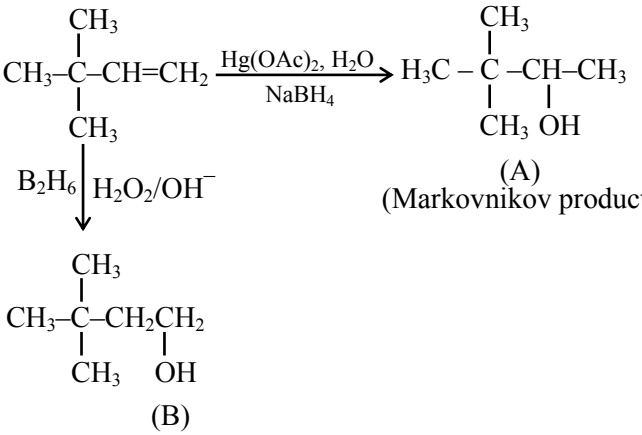
(B) 'A' is Markovnikov product and 'B' is anti-Markovnikov product.

(C) 'A' and 'B' are both anti-Markovnikov products.

(D) 'B' is Markovnikov and 'A' is anti-Markovnikov product.

Official Ans. by NTA (B)

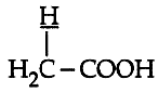
Sol.

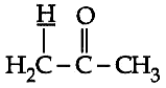


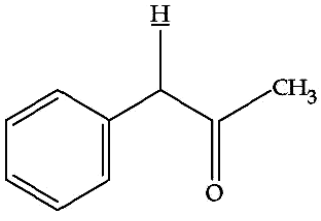
(A)
(Markovnikov product)

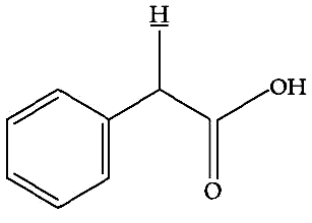
(B)
(Anti Markovnikov product)

14. Among the following marked proton of which compound shows lowest pK_a value ?

(A) 

(B) 

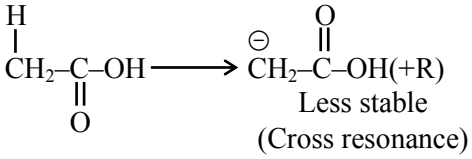
(C) 

(D) 

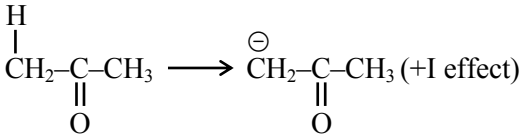
Official Ans. by NTA (C)

Sol.

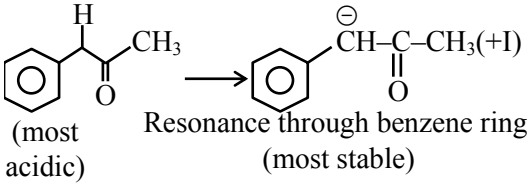
(A)



(B)

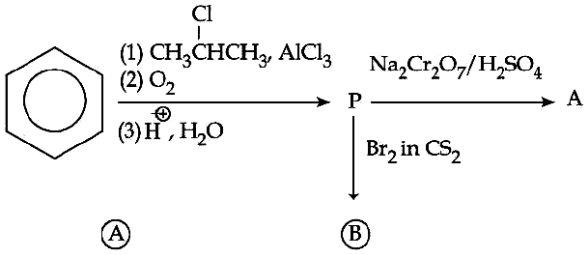


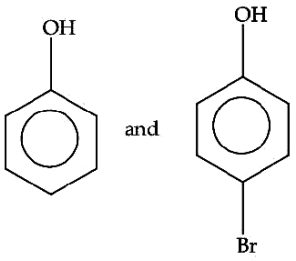
(C)

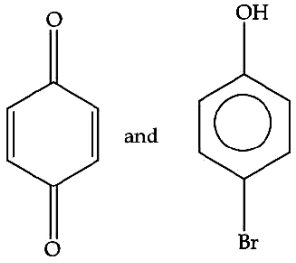


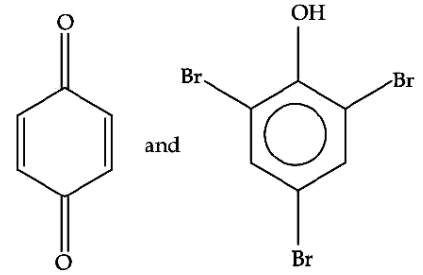
So it has least pK_a value.

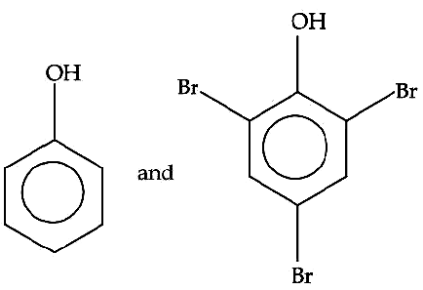
15. Identify the major product A and B for the below given reaction sequence.



(A) 

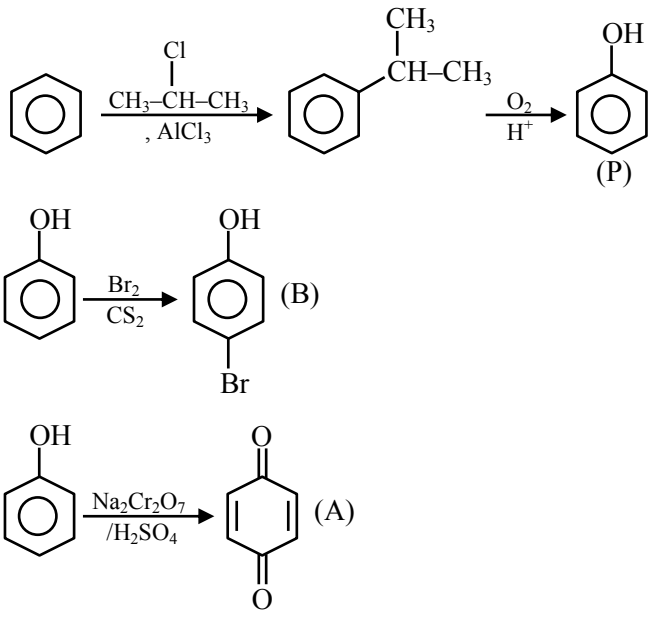
(B) 

(C) 

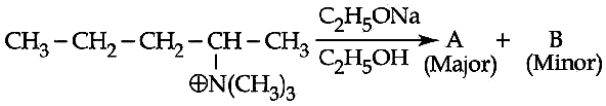
(D) 

Official Ans. by NTA (B)

Sol.



16. Identify the correct statement for the below given transformation.



(A) A - $CH_3CH_2CH = CH–CH_3$,
B – $CH_3CH_2CH_2CH = CH_2$,
Saytzeff products

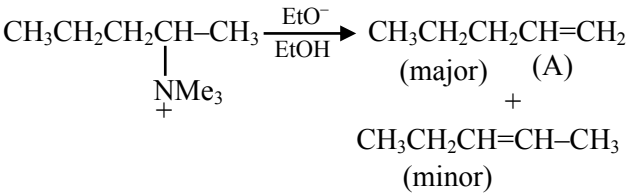
(B) A - $CH_3CH_2CH = CH–CH_3$,
B – $CH_3CH_2CH_2CH = CH_2$,
Hafmann products

(C) A - $CH_3CH_2CH_2CH = CH_2$,
B – $CH_3CH_2CH = CHCH_3$,
Hofmann products

(D) A - $CH_3CH_2CH_2CH = CH_2$,
B – $CH_3CH_2CH = CHCH_3$,
Saytzeff products

Official Ans. by NTA (C)

Sol.



17. Terylene polymer is obtained by condensation of :

(A) Ethane-1, 2-diol and Benzene-1, 3 dicarboxylic acid

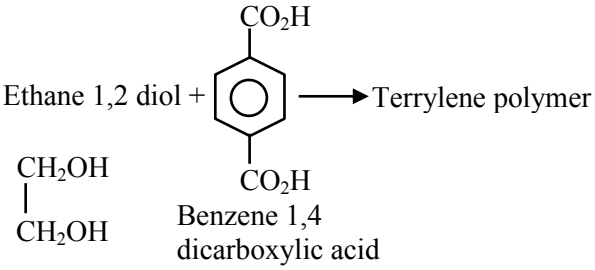
(B) Propane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

(C) Ethane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

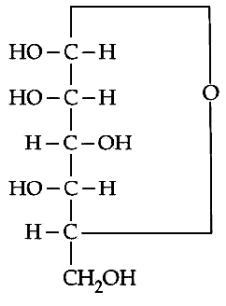
(D) Ethane-1, 2-diol and Benzene-1, 2 dicarboxylic acid

Official Ans. by NTA (C)

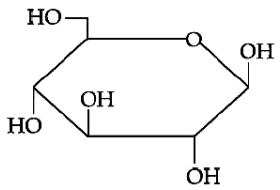
Sol.

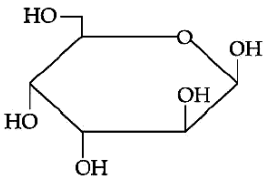


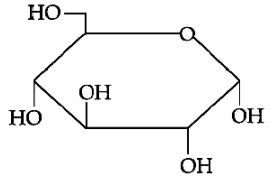
18. For the below given cyclic hemiacetal (X), the correct pyranose structure is :

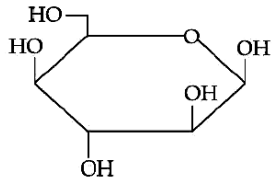


(X)

(A) 

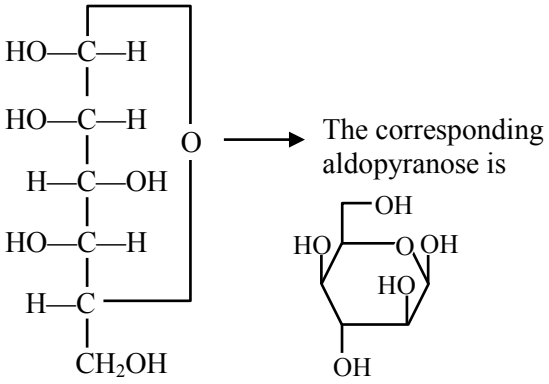
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

Sol. **Correct pyranose structure is**



X(Hemiacetal)

19. Statements about Enzyme Inhibitor Drugs are given below :

(A) There are Competitive and Non-competitive inhibitor drugs.

(B) These can bind at the active sites and allosteric sites.

(C) Competitive Drugs are allosteric site blocking drugs.

(D) Non-competitive Drugs are active site blocking drugs.

Choose the correct answer from the options given below :

(A) (A), (D) only (B) (A), (C) only

(C) (A), (B) only (D) (A), (B), (C) only

Official Ans. by NTA (C)

Sol. Enzyme inhibitors can be competitive inhibitors (inhibit the attachment of substrate on active site of enzyme) and non-competitive inhibitor (changes the active site of enzyme after binding at allosteric site.)

20. For kinetic study of the reaction of iodide ion with H_2O_2 at room temperature :

(A) Always use freshly prepared starch solution.

(B) Always keep the concentration of sodium thiosulphate solution less than that of KI solution.

(C) Record the time immediately after the appearance of blue colour.

(D) Record the time immediately before the appearance of blue colour.

(E) Always keep the concentration of sodium thiosulphate solution more than that of KI solution.

Choose the correct answer from the options given below :

(A) (A), (B), (C) only

(B) (A), (D), (E) only

(C) (D), (E) only

(D) (A), (B), (E) only

Official Ans. by NTA (A)

Sol. The is recorded immediately after the blue colour appears.

$Na_2S_2O_3$ is kept in limited amount.

SECTION-B

1. In the given reaction,

$$X + Y + 3Z \rightleftarrows XYZ_3$$

if one mole of each of X and Y with 0.05 mol of Z gives compound XYZ_3. (Given : Atomic masses of X, Y and Z are 10, 20 and 30 amu, respectively). The yield of XYZ_3 is _____ g.

(Nearest integer)

Official Ans. by NTA (2)

Sol. $\underset{1\text{mol}}{X} + \underset{1\text{mol}}{Y} + \underset{0.05\text{mol}}{3Z} \rightleftharpoons \mathbf{XYZ_3}$

Z is L.R.

$$\frac{0.05}{3} = 1 \text{ mole of } XYZ_3$$

$$\text{Mass of } XYZ_3 = \frac{0.05}{3} \times (10 + 20 + 30 \times 3)$$

$$= 2g$$

2. An element M crystallises in a body centred cubic unit cell with a cell edge of 300 pm. The density of the element is 6.0 g cm^{-3}. The number of atoms present in 180 g of the element is _____ $\times 10^{23}$.

(Nearest integer)

Official Ans. by NTA (22)

Sol. M is body certred cubic , $\therefore$ Z = 2

Let mass of 1 atom of M is A

Edge length = 300 pm

Density = $6g/cm^3$

$$\therefore 6g/cm^3 = \frac{Z \times A}{(300 \times 10^{-10})^3} = \frac{2 \times A}{27 \times 10^{-24}}$$

$$A = 81 \times 10^{-24} g$$

$\therefore$ Atomic mass = 48.6g

$$\therefore \text{Mole in 180g} = \frac{180}{48.6} = 3.7 \text{ moles}$$

Atoms of M = $3.7 \times 6 \times 10^{23}$

$= 22.22 \times 10^{23}$ atoms

3. The number of paramagnetic species among the following is _____.

B_2, Li_2, C_2, C_2^-, O_2^{2-}, O_2^+ and He_2^+

Official Ans. by NTA (4)

Sol. Paramagnetic $B_2, C_2^-, O_2^+, He_2^+$

4. 150 g of acetic acid was contaminated with 10.2 g ascorbic acid ($C_6H_8O_6$) to lower down its freezing point by ($x \times 10^{-1}$)°C. The value of x is _____. (Nearest integer) [Given $K_f = 3.9$ K kg mol^{-1}; Molar mass of ascorbic acid = 176 g mol^{-1}]

Official Ans. by NTA (15)

Sol. 150g CH_3COOH

10.2g ascorbic acid $\Rightarrow$ 0.058 moles

$$\Delta T_f = \left(x \times 10^{-1}\right)°C$$

$$\Delta T_f = k_f \cdot \text{molality}$$

$$= 3.9 \times \frac{0.058}{150} \times 1000$$

$$= 1.5°C$$

$$= 15 \times 10^{-1}°C$$

5. K_a for butyric acid (C_3H_7COOH) is 2×10^{-5}. The pH of 0.2 M solution of butyric acid is ___ $\times 10^{-1}$. (Nearest integer) [Given log 2 = 0.30]

Official Ans. by NTA (27)

Sol. K_a of Butyric acid $\Rightarrow 2 \times 10^{-5}$ PKa = 4.7

pH of 0.2 M solution

$$pH = \frac{1}{2} pK_a - \frac{1}{2} \log C$$

$$= \frac{1}{2}(4 \cdot 7) \frac{1}{2} \log(0.2)$$

$$= 2.35 + 0.35 = 2.7$$

$$pH = 27 \times 10^{-1}$$

6. For the given first order reaction

$A \rightarrow B$

the half life of the reaction is 0.3010 min. The ratio of the initial concentration of reactant to the concentration of reactant at time 2.0 min will be equal to _____. (Nearest integer)

Official Ans. by NTA (100)

Sol. $A \rightarrow B$ $\quad t_{1/2} = 0.3010$ min

A_0/A_t at time 2 min = ?

$$K = \frac{2.303}{t} \log\left[\frac{A_0}{A_t}\right]$$

$$\Rightarrow \frac{0 \cdot 693}{t_{\frac{1}{2}}} = \frac{2 \cdot 303}{2} \log\left(\frac{A_0}{A_t}\right)$$

$$\text{Or } \frac{2.303 \times 0.3010}{0.3010} = \frac{2.303}{2} \log \frac{A_0}{A_t}$$

$$\log \frac{A_0}{A_t} = 2$$

$$\therefore \frac{A_0}{A_t} = 10^2 = 100$$

7. The number of interhalogens from the following having square pyramidal structure is :

ClF_3, IF_7, BrF_5, BrF_3, I_2Cl_6, IF_5, ClF, ClF_5

Official Ans. by NTA (3)

Sol. Square pyramidal structures are

BrF_5, IF_5 and ClF_5.

8. The disproportionation of MnO_4^{2-} in acidic medium resulted in the formation of two manganese compounds A and B. If the oxidation state of Mn in B is smaller than that of A, then the spin-only magnetic moment (μ) value of B in BM is _____. (Nearest integer)

Official Ans. by NTA (4)

Sol. $MnO_4^{2-} \xrightarrow{H^+} MnO_4^- + MnO_2$

No. of unpaired $\bar{e} = 3$

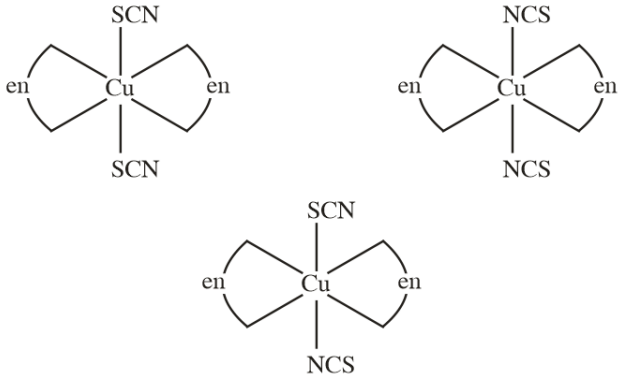
$\therefore \ \mu = \sqrt{15} = 3.877$

Nearest Integer = 4

9. Total number of relatively more stable isomer(s) possible for octahedral complex $[Cu(en)_2(SCN)_2]$ will be _____.

Official Ans. by NTA (3)

Sol. $[Cu(en)_2(SCN)_2]$



10. On complete combustion of 0.492 g of an organic compound containing C, H and O, 0.7938 g of CO_2 and 0.4428 g of H_2O was produced. The % composition of oxygen in the compound is _____.

Official Ans. by NTA (46)

Sol. 0.492g of $C_xH_yO_z$

Gives 0.7938 g CO_2 = 0.018 moles

0.4428g H_2O = 0.0246 moles

So moles of C = 0.018 $\Rightarrow$ 0.216 g

Moles of H = 0.049 $\Rightarrow$ 0.049g

$\therefore$ wt. of Oxygen = 0.492 – 0.216 – 0.049

= 0.227g

% of Oxygen = $\frac{0.227}{0.492} \times 100$ 46 (approx.)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let the solution curve of the differential equation $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$, $x>0$, intersect the line $x = 1$ at $y = 0$ and the line $x = 2$ at $y = \alpha$. Then the value of α is :

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) $\frac{5}{2}$

Official Ans. by NTA (B)

Sol. $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$

$$xdy - ydx = \sqrt{x^2+y^2}dx$$

$$\frac{xdy-ydx}{x^2} = \sqrt{1+\frac{y^2}{x^2}}\cdot\frac{dx}{x}$$

$$\frac{d(y/x)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$$

$$\ln\left(\frac{y}{x}+\sqrt{\left(\frac{y}{x}\right)^2+1}\right) = \ln x + R$$

$$\frac{y+\sqrt{y^2+x^2}}{x} = cx$$

$$y+\sqrt{y^2+x^2} = cx^2$$

$$x=1, y=0 \Rightarrow 0+1=C \Rightarrow C=1$$

Curve is $y+\sqrt{x^2+y^2} = x^2$

$$x=2, y=\alpha$$

$$2+\sqrt{4+\alpha^2} = 4$$

$$4+\alpha^2 = 16+\alpha^2 = 8\alpha$$

$$\alpha = \frac{3}{2}$$

2. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$ is :

(A) $\left(-\infty, \frac{1}{4}\right]$ (B) $\left[-\frac{1}{4}, \infty\right)$

(C) $\left(-\frac{1}{3}, \infty\right)$ (D) $\left(-\infty, \frac{1}{3}\right]$

Official Ans. by NTA (B)

Sol.

$$\left|\frac{x^2+4x+2}{x^2+3}\right| \le 1$$

$$\Leftrightarrow \left(x^2-4x+2\right)^2 \le \left(x^2+3\right)^2$$

$$\Leftrightarrow \left(x^2-4x+2\right)^2 - \left(x^2+3\right)^2 \le 0$$

$$\Leftrightarrow \left(2x^2-4x+5\right)\left(-4x-1\right) \le 0$$

$$\Leftrightarrow -4x-1 \le 0 \rightarrow x \ge -\frac{1}{4}$$

3. Let the vectors $\vec{a} = (1+t)\hat{i}+(1-t)\hat{j}+\hat{k}$, $\vec{b} = (1-t)\hat{i}+(1+t)\hat{j}+2\hat{k}$ and $\vec{c} = t\hat{i}-t\hat{j}+\hat{k}$, $t \in R$ be such that for $\alpha, \beta, \gamma \in R$, $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c} = \vec{0}$ $\Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is :

(A) a non-empty finite set

(B) equal to N

(C) equal to R – {0}

(D) equal to R

Official Ans. by NTA (C)

Sol. By its given condition

: $\vec{a},\vec{b},\vec{c}$ are linearly independent vectors

$\Rightarrow \left[\bar{a}\,\bar{b}\,\bar{c}\right] \neq 0$ ..(i)

Now, $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$$= \begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix}$$

$C_2 \to C_1 + C_2$

$$\begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1+t & 1 & 1 \\ 1-t & 1 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$= 2\left[(1+t)-(1-t)+t\right]$

$= 2[3t] = 6t$

$\left[\bar{a}\ \bar{b}\ \bar{c}\right] \neq 0 \Rightarrow t \neq 0$

4. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to :

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Official Ans. by NTA (A)

Sol. $\cos^{-1} x = 2\sin^{-1} x = \cos^{-1} 2x$

$\cos^{-1} x - 2\left(\frac{\pi}{2} - \cos^{-1} x\right) = \cos^{-1} 2x$

$\cos^{-1} x - \pi + 2\cos^{-1} x = \cos^{-1} 2x$

$3\cos^2 x = \pi + \cos^{-1} 2x$ **...(1)**

$\cos\left(3\cos^{-1} x\right) = \cos\left(\pi + \cos^{-1} 2x\right)$

$4x^3 - 3x = -2x$

$4x^3 = x \Rightarrow x = 0, \pm\frac{1}{2}$

All satisfy the original equation

$\text{sum} = -\frac{1}{2} \text{ to } +\frac{1}{2} = 0$

5. Let the operations $*, \odot \in \{\wedge, \vee\}$. If $(p * q) \odot (p \odot \sim q)$ is a tautology, then the ordered pair $(*, \odot)$ is :

(A) $(\vee, \wedge)$ (B) $(\vee, \vee)$ (C) $(\wedge, \wedge)$ (D) $(\wedge, \vee)$

Official Ans. by NTA (B)

Sol. Well check each option

For A $\pi = v$ of $0 = \Lambda$

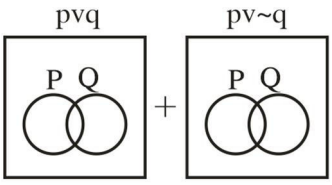
$(pvq) \wedge (pv \sim q)$

$\equiv pv(q \wedge \sim q)$

$\equiv pv(c) \equiv p$

For B : * = v, O = v

$(pvq) \vee (pv \sim q) \equiv t$ using Venn Diagrams



6. Let a vector $\vec{a}$ has a magnitude 9. Let a vector $\vec{b}$ be such that for every $(x,y) \in R \times R - \{(0,0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$. Then the value of $|\vec{a} \times \vec{b}|$ is equal to:

(A) $9\sqrt{3}$ (B) $27\sqrt{3}$ (C) 9 (D) 81

Official Ans. by NTA (B)

Sol. $|\vec{a}| = 9$ & $\left(x\vec{a} + y\vec{b}\right) \cdot \left(6y\vec{a} - 18x\vec{b}\right) = 0$

$\Rightarrow 6xy|\bar{a}|^2 - 18x^2(\bar{a}\cdot\bar{b}) + 6y^2(\bar{a}\cdot\bar{b}) - 18xy|\bar{b}|^2 = 0$

$\Rightarrow 6xy\left(|\vec{a}|^2 - 3|\bar{b}|^2\right) + (\bar{a}\cdot\bar{b})\left(y^2 - 3x^2\right) = 0$

This should hold $\forall\, x, y \in R \times R$

$\therefore\ |\bar{a}|^2 = 3|\bar{b}|^2$ & $(\bar{a}\cdot\bar{b}) = 0$

Now $|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2 - (\bar{a}\cdot\bar{b})^2$

$= |\bar{a}|^2 \cdot \frac{|\bar{a}|^2}{3}$

$\therefore\ |\bar{a}\times\bar{b}| = \frac{|\bar{a}|^2}{\sqrt{3}} = \frac{81}{\sqrt{3}} = 27\sqrt{3}$

7. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices A(sint, –cost), B(cost, sint) and C(a, b) such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to :

(A) $\frac{8}{3}$ (B) 8

(C) $\frac{77}{9}$ (D) $\frac{80}{9}$

Official Ans. by NTA (B)

Sol. $s \equiv \sin t, c \equiv \cos t$

Let orthocentre be (h,k)

Since it if an equilateral triangle hence orthocentre coincides with centroid.

$\therefore\ a + s + c = 3h, b + s - c = 3k$

$\therefore\ (3h - a)^2 + (3k - b)^2 = (s + c)^2 + (s - c)^2 = 2(s^2 + c^2) = 2$

$\therefore\ \left(h - \frac{a}{3}\right)^2 + \left(K - \frac{b}{3}\right)^2 = \frac{2}{9},$

circle centre at $\left(\frac{a}{3}, \frac{b}{3}\right)$

Gives, $\frac{a}{3} = 1, \frac{b}{3} = \frac{1}{3} \Rightarrow a = 3, b = 1$

$\therefore\ a^2 - b^2 = 8$

8. For $\alpha \in N$, consider a relation R on N given by $R = \{(x, y) : 3x + \alpha y$ is a multiple of $7\}$. The relation R is an equivalence relation if and only if :

(A) $\alpha = 14$

(B) α is a multiple of 4

(C) 4 is the remainder when α is divided by 10

(D) 4 is the remainder when α is divided by 7

Official Ans. by NTA (D)

Sol. For R to be reflexive $\Rightarrow$ x R x

$\Rightarrow\ 3x + \alpha x = 7x \Rightarrow (3 + \alpha)x = 7K$

$\Rightarrow\ 3 + \alpha = 7\lambda \Rightarrow \alpha = 7\lambda - 3 = 7N + 4, K, \lambda, N \in I$

$\therefore$ when α divided by 7, remainder is 4.

R to be symmetric $xRy \Rightarrow yRx$

$3x + \alpha y = 7N_1, 3y + \alpha x = 7N_2$

$\Rightarrow (3 + \alpha)(x + y) = 7(N_1 + N_2) = 7N_3$

Which holds when $3 + \alpha$ is multiple of 7

$\therefore\ \alpha = 7N + 4$ (as did earlier)

R to be transitive

$xRy\ \&\ yRz \Rightarrow xRz$.

$3x + \alpha y = 7N_1\ \&\ 3y + \alpha z = 7N_2$ and

$3x + \alpha z = 7N_3$

$\therefore\ 3x + 7N_2 - 3y = 7N_3$

$\therefore\ 7N_1 - \alpha y + 7N_2 - 3y = 7N_3$

$\therefore\ 7(N_1 + N_2) - (3 + \alpha)y = 7N_3$

$\therefore\ (3 + \alpha)y = 7N$

Which is true again when $3 + \alpha$ divisible by 7, i.e. when α divided by 7, remainder is 4.

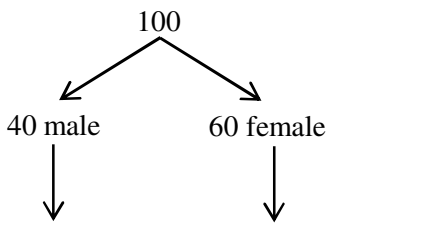
9. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is :

(A) $\frac{3}{4}$ (B) $\frac{11}{16}$

(C) $\frac{23}{32}$ (D) $\frac{13}{16}$

Official Ans. by NTA (A)

Sol.



Probability that chosen candidate is female = $\frac{40}{60} = \frac{2}{3}$

10. If $y = y(x), x \in \left(0, \frac{\pi}{2}\right)$ be the solution curve of the differential equation

$$\left(\sin^2 2x\right)\frac{dy}{dx} + \left(8\sin^2 2x + 2\sin 4x\right)y = 2e^{-4x}\left(2\sin 2x + \cos 2x\right), \quad \text{with} \quad y\left(\frac{\pi}{4}\right) = e^{-\pi},$$

then $y\left(\frac{\pi}{6}\right)$ is equal to :

(A) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$ (B) $\frac{2}{\sqrt{3}}e^{2\pi/3}$

(C) $\frac{1}{\sqrt{3}}e^{-2\pi/3}$ (D) $\frac{1}{\sqrt{3}}e^{2\pi/3}$

Official Ans. by NTA (A)

Sol. Given differential equation can be re-written as

$$\frac{dy}{dx} + \left(8 + 4\cot 2x\right)y = \frac{2e^{-4x}}{\sin^2 2x}\left(2\sin x + \cos 2x\right)$$

which is a linear diff. equation.

I.f. $= e^{\int (8+4\cot 2x)dx} = e^{8x+2Cu(\sin 2x)}$

$= e^{8x} \cdot \sin^2 2x$

$\therefore$ solution is

$$y\left(e^{8x} \cdot \sin^2 2x\right) = \int 2e^{4x}\left(2\sin 2x + \cos 2x\right)dx + C$$

$$= e^{4x} \cdot \sin 2x + C$$

Given $y\left(\frac{\pi}{4}\right) = e^{-\pi} \Rightarrow C = 0$

$$\therefore \; y = \frac{e^{-4x}}{\sin 2x}$$

$$\therefore \; y\left(\frac{\pi}{6}\right) = \frac{e^{-4\cdot\frac{\pi}{6}}}{\sin\left(2\cdot\frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}}e^{-\frac{2\pi}{3}}$$

11. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point R(0, 1), then the orthocentre of the triangle PQR is :

(A) (0, 1) (B) (2, –1)

(C) (6, 3) (D) (2, 1)

Official Ans. by NTA (B)

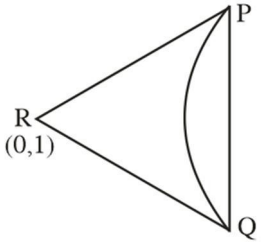
Sol. $y^2 = 2x - 3$ **…(i)**

Equation of chord of contact

PQ : r = 0

yx1 = (x + 0) – 3

$y = x - 3$ …(2)



from (1) and (2)

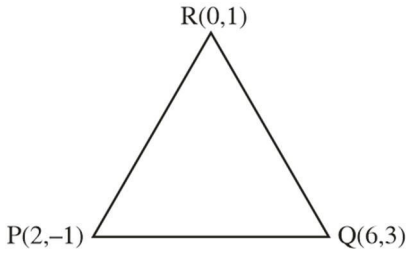
$$(x \cdot 3)^2 = 2x - 3$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$x = 2 \text{ or } 6$$

$$y = -1 \text{ or } 3$$



$$MPQ = \frac{1}{4} = 1$$

$$MQR = \frac{2}{6} = \frac{1}{3}$$

$$MPR = \frac{2}{6} = \frac{1}{3}$$

$$MPR = \frac{2}{-2} = -1$$

$MPQ \times MPR = - \Rightarrow PQ \perp PR$

Orthocentre = P (2,–1)

12. Let C be the centre of the circle $x^2 + y^2 - x + 2y = \frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R. Then the area of the triangle PQR (in unit^2) is :

(A) 2 (B) $2\sqrt{2}$

(C) $8\sin\left(\frac{\pi}{8}\right)$ (D) $8\cos\left(\frac{\pi}{8}\right)$

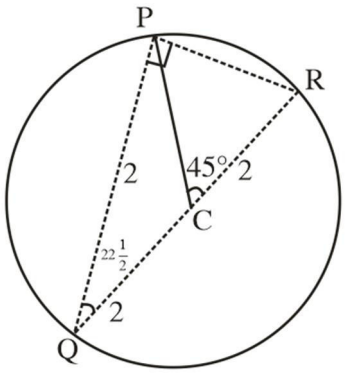
Official Ans. by NTA (B)

Sol. $x^2 + y^2 - x + 2y = \frac{11}{4}$

$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = (2)^2$

Or Δ PQR

$PR = QK \sin 2 \geq \frac{1}{3}$



$= 4 \cdot 6 \sin\frac{\pi}{8}$

$PQ = QR \cos 22\frac{1}{2}$

$= 4\cos\frac{\pi}{8}$

As $\Delta PQR = \frac{1}{2} PR \times PQ$

$= \frac{1}{2}\left(4^2 \sin\frac{\pi}{6}\right)\left(4\cos\frac{\pi}{8}\right)$

$= 4\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

13. The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is:

(A) 0 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (C)

Sol. $7^{2022} + 3^{2022}$

$= (49)^{1011} + (9)^{1011}$

$= (50-1)^{1011} + (10-1)^{1011}$

$= 5\lambda - 1 + 5K - 1$

$= 5m - 2$

Remainder $= 5 - 2 = 3$

14. Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If $B_n = \text{Adj}(B_{n-1})$ for all $n \geq 1$, then $\det(B_4)$ is equal to :

(A) 3^{28} (B) 3^{30} (C) 3^{32} (D) 3^{36}

Official Ans. by NTA (C)

Sol. $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$a \leftrightarrow R_2$

$-\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$B_0 = A^{49} + 2A^{98}$

$= A + 2I$

$B_n = \text{Adj}(B_n - 1)$

$$B_4 = \text{Adj}(\text{Adj}(\text{Adj}(\text{Adj}\,B_0))$$

$$= |B_0|^{(n-1)^4}$$

$$= |B_0|^{16}$$

$$B_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= 2(4-0) - 1(0-1)$$

$$= 9$$

$$B_4 (9)^{16} = (3)^{32}$$

15. Let $S_1 = \left\{ z_1 \in C : |z_1 - 3| = \frac{1}{2} \right\}$ and $S_2 = \left\{ z_2 \in C : \left| z_2 - |z_2 + 1| \right| = \left| z_2 + |z_2 - 1| \right| \right\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is :

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

Official Ans. by NTA (C)

Sol. $\left| z_2 + |z_2 - 1| \right|^2 = \left| z_2 - |z_2 + 1| \right|^2$

$$\Rightarrow \left| z_2 + |z_2 - 1| \right| \left(\bar{z}_2 + |z_2 - 1| \right) = \left(z_2 - |z_2 + 1| \right) \left(\bar{z}_2 - (z_2 + 1) \right)$$

$$\Rightarrow z_2 |\bar{z}_2 + 12_2 - 1| - \left(\bar{z}_2 - |z_2 + 1| \right) + \bar{z}_2 \left(|z_2 - 1| + |z_2 + 1| \right)$$

$$= |z_2 + 1|^2 = |z_2 - 1|^2$$

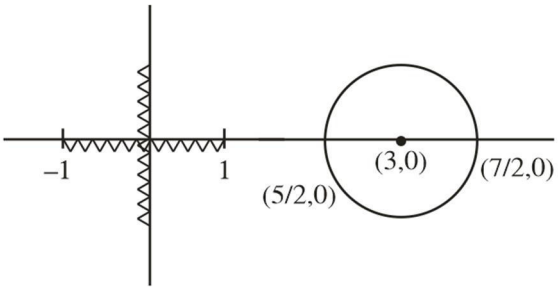
$$\Rightarrow \left[z_2 + \bar{z}_2 \right) \left(|z_2 - 1| \right) + \left(z_2 + 1 \right) = 2 \left(z_2 + \bar{z}_2 \right)$$

$$\Rightarrow \left(z_2 + \bar{z}_2 \right) \left(|z_2 - 1| + |z_2 + 1| - 2 \right) = 0$$

$\therefore\ z_2 + \bar{z}_2 = 0$ or $|z_2 - 1| + |z_2 + 1| - 2 = 0$

$\therefore\ z_2$ lie on imaginary axis. Or on real axis with in $[-1,1]$

Also $|z_1 - 3| = \frac{1}{2}$ lie on circle having centre 3 and radius $\frac{1}{2}$.



Clearly $|z_1 - z_2|\min = \frac{5}{2} - 1 = \frac{3}{2}$

16. The foot of the perpendicular from a point on the circle $x^2 + y^2 = 1$, $z = 0$ to the plane $2x + 3y + z = 6$ lies on which one of the following curves ?

(A) $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1$, $z = 6 - 2x - 3y$

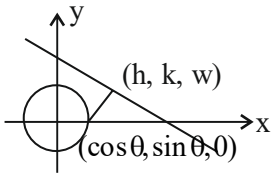
(B) $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1$, $z = 6 - 2x - 3y$

(C) $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1$, $z = 6 - 2x - 3y$

(D) $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1$, $z = 6 - 2x - 3y$

Official Ans. by NTA (B)

Sol.



$$\frac{h - \cos\theta}{2} = \frac{k - \sin\theta}{3} = \frac{w - 0}{1}$$

$$= \frac{-1(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$h = \cos\frac{-2(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$= \frac{10\cos\theta - 6\sin\theta + 12}{14}$$

$$k = \sin\theta - \frac{3}{14}(2\cos\theta + 3\sin\theta - 6)$$

$$k = \frac{5\sin\theta - 6\cos\theta + 18}{14}$$

Elementary $\sin\theta$ and $\cos\theta$

$$(5h + 6k - 12)^2 + 4(3h + 5k - 9)^2 = 1$$

17. If the minimum value of $f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5}$, $x>0$, is 14, then the value of α is equal to :

(A) 32 (B) 64

(C) 128 (D) 256

Official Ans. by NTA (C)

Sol. $\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{\alpha}{2x^5}+\frac{\alpha}{2x^5}$

$\geq 7\left(\frac{\alpha^2}{2^7}\right)^{\frac{1}{7}}$

$\frac{7\cdot(\alpha)^{2/7}}{2}=14$

$(\alpha^2)^{1/7}=2^2$

$\alpha=\left(2^2\right)^{7/2}=2^7$

$\alpha=128$

18. Let α, β and γ be three positive real numbers. Let $f(x)=\alpha x^5+\beta x^3+\gamma x$, $x\in R$ and $g:R\to R$ be such that g(f(x)) = x for all $x\in R$. If $a_1, a_2, a_3,..., a_n$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n}f(a_i)\right)\right)$ is equal to :

(A) 0 (B) 3

(C) 9 (D) 27

Official Ans. by NTA (A)

Sol. Consider a case when $\alpha=\beta=0$ then

f(x) = yx

$g(x)=\frac{x}{y}$

$\frac{1}{n}\sum_{i=1}^{n}f(a_i)\Rightarrow\frac{y}{n}(a_1+a_2+.....+a_n)$

$=0$

$\Rightarrow f(g(0))\Rightarrow f(0)$

$\Rightarrow 0$

19. Consider the sequence $a_1, a_2, a_3, \ldots\ldots$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2}=\frac{2}{a_{n+1}}+a_n$ for n = 1, 2, 3, … .

If $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdot\left(\frac{a_3+\frac{1}{a_4}}{a_5}\right).....\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)=2^{\alpha}\left({}^{61}C_{31}\right)$,

then α is equal to :

(A) –30 (B) –31

(C) –60 (D) –61

Official Ans. by NTA (C)

Sol. $a_{n+2}\,a_{n+1}-a_{n+1}.a_n=2$

Series will satisfy

$a_1a_2, a_2a_3, a_3a_4, a_4a_5,$

1.2 2.2 2.3 2.4

$\frac{a_n+\frac{1}{a_{n+1}}}{a_{n+2}}=\frac{a_{n+2}-\frac{1}{a_{n+1}}}{a_{n+2}}$

$=1-\frac{1}{a_{n+1}a_{n+2}}$

$=1-\frac{1}{2(r+1)}$

$=\frac{2r+1}{2(r+1)}$

Now proof is given by

$=\prod_{r=1}^{30}\frac{(2r+1)}{2(r+1)}$

$=\frac{(1\cdot3\cdot5\cdot.....\cdot61)}{2^{30}\cdot(2\cdot3\cdot.....\cdot31)}$

$\Rightarrow\frac{(1\cdot3\cdot5\cdot.....\cdot61)}{\lfloor31\cdot2^{30}}\times\frac{2^{30}\times\lfloor30}{2^{30}\times\lfloor30}$

$=\frac{\lfloor61}{2^{60}\lfloor31\cdot\lfloor30}$

$\alpha=-60$

20. The minimum value of the twice differentiable function $f(x)=\int_0^x e^{x-t}f'(t)\,dt-(x^2-x+1)e^x,\ x\in R,$ is :

(A) $-\frac{2}{\sqrt{e}}$ (B) $-2\sqrt{e}$

(C) $-\sqrt{e}$ (D) $\frac{2}{\sqrt{e}}$

Official Ans. by NTA (A)

Sol. $f(x)=e^x\cdot\int_0^x\frac{f'(t)}{e^t}dt$

$f'(x)=e^x\cdot\int_0^x\frac{f'(t)}{e^t}dt+e^x\cdot\frac{f'(x)}{e^x}$

$-\left[(2x-1)\cdot e^x+(x^2-x+1)\cdot e^x\right]$

$\int_0^x\frac{f'(t)}{e^t}dt=x^2+x$

$\frac{f'(x)}{e^x}=2x+1$

$f'(x)=(2x+1)\cdot e^x$

$f'(x)=0\Rightarrow x=-\frac{1}{2}$

$f(x)=(2x+1)\cdot e^x-2e^x+C$

$\Big|\ f(0)=-1$

$-1=1-2+C$

$C=0$

$f(x)=e^x(2x-1)$

$f\left(-\frac{1}{2}\right)=\frac{-2}{\sqrt{e}}$

SECTION-B

1. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from {A, B, C, D, E} or a number from {1, 2, 3, 4, 5} with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from {1, 2, 3, 4, 5} is $\alpha\times5^6$, then α is equal to _____.

Official Ans. by NTA (7073)

Sol. Required no. = Total – no character from {1, 2, 3, 4, 5}

$=(10^6-5^6)+(10^7-5^7)+(10^8-5^8)$

$=10^6(1+10+100)-5^6(1+5+25)$

$=10^6\times111-5^6\times31$

$=2^6\times5^6\times111-5^6\times31$

$=5^6(2^6\times111-31)$

$=5^6\times\underbrace{7073}_{\alpha}$

$\therefore\alpha=7073$

2. Let P(–2, –1, 1) and $Q\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ be the vertices of the rhombus PRQS. If the direction ratios of the diagonal RS are $\alpha,-1,\beta$, where both α and β are integers of minimum absolute values, then $\alpha^2+\beta^2$ is equal to _____.

Official Ans. by NTA (450)

Sol. $RS\equiv(\alpha,-1,\beta)$

DR of PQ $\equiv\left(\frac{56}{17}+2,\frac{43}{17}+1,\frac{111}{17}-1\right)$

$\equiv\left(\frac{90}{17},\frac{60}{17},\frac{94}{17}\right)$

$\frac{90}{17}\alpha+\frac{60}{17}(-1)+\frac{94}{17}\beta=0$

$90\alpha+94\beta=60$

$\beta=\frac{60-90\alpha}{94}$

$\beta=\frac{30(2-3\alpha)}{94}$

$\beta=-30\frac{(3\alpha-2)}{94}$

$\beta=\frac{-15}{47}(3\alpha-2)$

$\Rightarrow\frac{\beta}{-15}=\frac{3\alpha-2}{47}$

$\Rightarrow\beta=-15,\alpha=-15$

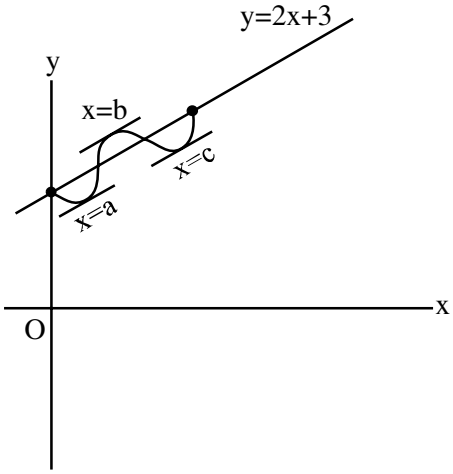
$\alpha^2+\beta^2=225+225$

$=450$

3. Let $f:[0,1]\to R$ be a twice differentiable function in (0, 1) such that f(0) = 3 and f(1) = 5. If the line y = 2x + 3 intersects the graph of f at only two distinct points in (0, 1), then the least number of points $x\in(0,1)$, at which $f''(x)=0$, is _____.

Official Ans. by NTA (2)

Sol.



$f'(a) = f'(b) = f'(c) = 2$

$\Rightarrow f''(x)$ is zero

for atleast $x_1 \in (a, b)$ & $x_2 \in (b, c)$

4. If $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}dx = \alpha\sqrt{2}+\beta\sqrt{3}$, where α, β are integers, then $\alpha+\beta$ is equal to

Official Ans. by NTA (10)

Sol. Put $1 + x^2 = t^2$

$2x\,dx = 2t\,dt$

$X\,dx = t\,dt$

$$\therefore \int_1^2 \frac{15(t^2-1)t\,dt}{\sqrt{t^2+t^3}}$$

$$15\int_1^2 \frac{t(t^2-1)}{t\sqrt{1+t}}dt$$

Put $1 + t = u^2$

$dt = 2u\,du$

$$15\int_{\sqrt{2}}^{\sqrt{3}} \frac{(u^2-1)^2-1}{u}\times 2u\,du$$

$$30\int_{\sqrt{2}}^{\sqrt{3}} (u^4-2u^2)du$$

$$30\left(\frac{u^5}{5}-\frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}}$$

$$30\left[\frac{1}{5}\left(\sqrt{3}^5-\sqrt{2}^5\right)-\frac{2}{3}\left(\sqrt{3}^3-\sqrt{2}^3\right)\right]$$

$$30\left[\frac{1}{5}\left(9\sqrt{3}-4\sqrt{2}\right)-\frac{2}{3}\left(3\sqrt{3}-2\sqrt{2}\right)\right]$$

$$30\left[-\frac{1}{5}\times\sqrt{3}+\frac{8}{15}\sqrt{2}\right]$$

$$-6\sqrt{3}+16\sqrt{2}=\alpha\sqrt{2}+\beta\sqrt{3}$$

$\alpha = 16, \beta = -6$

$\therefore \alpha+\beta = 10$

5. Let $A=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$ and $B=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}$, $\alpha,\beta\in R$. Let α_1 be the value of α which satisfies $(A+B)^2 = A^2+\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ and α_2 be the value of α which satisfies $(A+B)^2 = B^2$. Then $|\alpha_1-\alpha_2|$ is equal to _____.

Official Ans. by NTA (2)

Sol. $A+B=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$

$$(A+B)^2=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$$

$$=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1-\alpha\\ 2+2\alpha & \alpha^2-2\end{bmatrix}$$

$$\therefore\begin{bmatrix}1 & -\alpha+1\\ 2\alpha+4 & \alpha^2\end{bmatrix}=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\alpha+\beta+1) & \alpha^2\end{bmatrix}$$

$\boxed{\alpha=1}=\alpha_1$

$$B^2=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\beta^2+1 & \beta\\ \beta & 1\end{bmatrix}=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$$

$\therefore \beta = 0, \alpha = -1 = \alpha_2$

$|\alpha_1-\alpha_2| = |1-(-1)| = 2$

6. For $p,q \in R$, consider the real valued function $f(x) = (x-p)^2 - q$, $x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all i = 1, 2, 3, 4, then the absolute difference between the roots of f(x) = 0 is

Official Ans. by NTA (50)

Sol. $f(x) = 0 \Rightarrow (x - p)^2 - q = 0.$

Roots are $p + \sqrt{q}$, $p - \sqrt{q}$ absolute difference between roots $2\sqrt{q}$.

Now, $|f(a_i)| = 500$

Let a_1, a_2, a_3, a_4 are a_1 a + d, a + 2d, a + 3d

$|f(a_4)| = 500$

$|(a_1 - p)^2 - q| = 500$

$\Rightarrow (a_1 - p)^2 - q = 500$

$\Rightarrow \frac{9}{4}d^2 - q = 500$ _____(1)

and $|f(a_1)|^2 = |f(a_2)|^2$

$((a_1 - p)^2 - q)^2 = ((a_2 - p)^2 - q)^2$

$\Rightarrow ((a_1 - p)^2 - (a_2 - p)^2)\,((a_1 - p)^2 - q + (a_2 - p)^2 - q) = 0$

$\Rightarrow \frac{9}{4}d^2 - q + \frac{d^2}{4} - q = 0$

$2q = \frac{10d^2}{4} \Rightarrow q = \frac{5d^2}{4}$

$\Rightarrow d^2 = \frac{4q}{5}$

From equation (1) $\frac{9}{4}.\frac{4.q}{5} - q = 500$

$\frac{4q}{5} = 500$

$\frac{4q}{5} = 500$

and $2\sqrt{q} = 2 \times \frac{50}{2} = 50$

7. For the hyperbola H : $x^2 - y^2 = 1$ and the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0,$ let the

(1) eccentricity of E be reciprocal of the eccentricity of H, and

(2) the line $y = \sqrt{\frac{5}{2}}x + K$ be a common tangent of E and H.

Then $4(a^2 + b^2)$ is equal to _____.

Official Ans. by NTA (3)

Sol. $e_E = \sqrt{1 - \frac{b^2}{a^2}}$, $e_H = \sqrt{2}$

If $\Rightarrow e_E = \frac{1}{e_H}$

$\Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{2}$

$2a^{2-2b}2 = a^2$

$a^2 = 2b^2$

and $y = \sqrt{\frac{5}{2}}x + k$ is tangent to ellipse then

$K^2 = a^2 \times \frac{5}{2} + b^2 = \frac{3}{2}$

$6b^2 = \frac{3}{2} \Rightarrow b^2 = \frac{1}{4}$ and $a^2 = \frac{1}{2}$

$\therefore 4.(a^2 + b^2) = 3$

8. Let $x_1, x_2, x_3, \ldots\ldots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ration $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data, then the greatest integer less than or equal to $\bar{x}$ is _____.

Official Ans. by NTA (142)

Sol. $\sum x_0^1 = \dfrac{3\left(1-\left(\frac{1}{2}\right)\right)^{20}}{1\frac{-1}{2}} = 6\left(1-\frac{1}{2^{20}}\right)$

$= \sum_{i=1}^{20}\left(x_{i-i}\right)^2$

$= \sum_{i=1}^{20}\left(x_i\right)^2 + (i)^2 - 2x_i i$

Now $= \sum_{i=1}^{20}\left(x_i\right)^2 = \dfrac{9\left(1-\left(\frac{1}{4}\right)\right)^{20}}{1-\frac{1}{4}} = 12\left(1-\frac{1}{2^{40}}\right)$

$\sum_{i=1}^{20} i^2 = \frac{1}{6}\times 20\times 21\times 41 = 2870$

$\sum_{i=1}^{20} x_i\ i = s = 3 + 2.3\frac{1}{2} + 3.3\frac{1}{2^2} + 4.3\frac{1}{2^3} + \ldots\ldots\text{AGP}$

$= 6\left(2-\frac{22}{2^{20}}\right)$

$\bar{x} = \dfrac{12-\frac{12}{2^{40}}+2870-12\left(2-\frac{22}{2^{20}}\right)}{20}$

$\bar{x} = \frac{2858}{20} + \left(\frac{-12}{2^{40}} + \frac{22}{2^{20}}\right)\times\frac{1}{20}$

$\left[\bar{x}\right] = 142$

9. $\lim\limits_{x\to 0}\left(\dfrac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{\frac{100}{x}}$

is equal to _____.

Official Ans. by NTA (1)

Sol.

$\lim\limits_{x\to 10}\left(\dfrac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{x}$

Form 1^{∞}

$= e^{\lim\limits_{x\to 0}\left[\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)-1\right]\times\frac{100}{x}}$

$= e^{\lim\limits_{x\to 0}\left[\frac{100}{x}\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)-\left((x+2)^3+2(x+2)^2+3\sin(x+2)\right)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)\right]}$

$= e^{\lim\limits_{x\to 0}\frac{100}{x}\left[\left(\frac{(x+2\cos x)^3+(x+2)^3+2(x+2\cos x)^2-2(x+2)^2+3\sin(x+2\cos x)-3\sin(x+2)}{8+8+3\sin^2}\right)\right]}$

$= e^{\frac{100}{16+3\sin^2}\ \lim\limits_{x\to 0}\frac{3(x+2\cos x)^2\times(1+2\sin x)-3(x+2)^2-4(x+2\cos x)}{x(1-2\sin x)-4(x+2)+3\cos(x+2\cos x)\times(1-2\sin x)-3\cos(x+2)}}$

$= e^{\frac{100}{16+3\sin 2}\left(\frac{12-3(4)+8\times 1-8+3\cos 2-3\cos 2}{1}\right)}$

Using L'H rule.

$= e^{o} = 1$

10. The sum of all real values of x for which $\dfrac{3x^2-9x+17}{x^2+3x+10} = \dfrac{5x^2-7x+19}{3x^2+5x+12}$ is equal to ___.

Official Ans. by NTA (6)

Sol. $\dfrac{3x^2-9x+17}{x^2+3x+10} = \dfrac{5x^2-7x+19}{3x^2+5x+12}$

$\dfrac{x^2+3x+10+2x^2-12x+7}{x^2+3x+10} = \dfrac{3x^2+5x+12+2x^2-12x+7}{3x^2+5x+12}$

$1+\dfrac{2x^2-12x+7}{x^2+3x+10} = 1+\dfrac{2x^2-12x+7}{3x^2+5x+12}$

$\left(2x^2-12x+7\right)\left(\dfrac{1}{x^2+3x+10}-\dfrac{1}{3x^2+5x+12}\right) = 0$

$2x^2-12x+7=0$ OR $3x^2+5x+12 = x^2+3x+10$

$x = \dfrac{12\pm\sqrt{D}}{4}$ $\qquad$ $2x^2+2x+2=0$

$x^2+x+1=0$

Sum of Roots = 6 $\qquad$ No solution.

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the efficiency of Carnot's engine is given by $\eta = \frac{\alpha\beta}{\sin\theta}\log_e\frac{\beta x}{kT}$, where α and β are constants. If T is temperature, k is Boltzman constant, θ is angular displacement and x has the dimensions of length. Then, choose the **incorrect** option.

(A) Dimensions of β is same as that of force.

(B) Dimensions of α^{-1} x is same as that of energy.

(C) Dimensions of $\eta^{-1}\sin\theta$ is same as that of $\alpha\beta$

(D) Dimensions of α is same as that of β

Official Ans. by NTA (D)

Sol. $[\alpha\beta] = [\eta] = [\sin\theta] = \text{Dimensionless}$

$\left[\eta^{-1}\sin\theta\right] = [\alpha\beta] = \text{D.L.}$

2. At time t = 0 a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the x and y directions are defined as 3t and $5t^3$ respectively. At t = 1s acceleration of the particle will be

(A) $-30y$ (B) $30y$

(C) $3x + 15y$ (D) $3x + 15y + 7\hat{z}$

Official Ans. by NTA (B)

Sol. $\vec{r} = 3t\hat{i} + 5t^3\hat{j} + 7k$

$\frac{d^2\vec{r}}{dt^2} = 30t\,\hat{j}$

At $t = 1 \Rightarrow \boxed{\frac{d^2\vec{r}}{dt^2} = 30\hat{j}}$

3. A pressure-pump has a horizontal tube of cross-sectional area 10 cm^2 for the outflow of water at a speed of 20 m/s. The force exerted on the vertical wall just in front of the tube which stops water horizontally flowing out of the tube, is: [given : density of water = 1000 kg/m^3]

(A) 300 N (B) 500 N

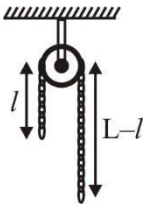
(C) 250 N (D) 400 N

Official Ans. by NTA (D)

Sol. $F = \rho a \upsilon^2 = 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$F = 400$

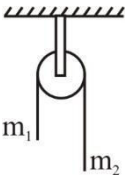
4. A uniform metal chain of mass m and length 'L' passes over a massless and frictionless pulley. It is released from rest with a part of its length 'l' is hanging on one side and rest of its length 'L – l' is hanging on the other side of the pulley. At a certain point of time, when $l = \frac{L}{x}$, the acceleration of the chain is $\frac{g}{2}$. The value of x is .....



(A) 6 (B) 2 (C) 1.5 (D) 4

Official Ans. by NTA (D)

Sol.

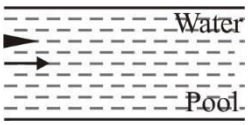


$a = \frac{(m_2 - m_1)}{(m_2 + m_1)}g$

$\frac{g}{2} = \frac{(\lambda(L-\ell) - \lambda\ell)g}{\lambda L} \Rightarrow L = \frac{L}{4} = \frac{L}{x}$

$x = 4$

5. A bullet of mass 200 g having initial kinetic energy 90 J is shot inside a long swimming pool as shown in the figure. If it's kinetic energy reduces to 40 J within 1s, the minimum length of the pool, the bullet has a to travel so that it completely comes to rest is



(A) 45 m (B) 90 m

(C) 125 m (D) 25 m

Official Ans. by NTA (A)

Sol. Using $mv = \sqrt{2mk}$

$$u = \frac{1}{0.2}\sqrt{2\times0.2\times90} = 30 \text{ m/s}$$

$$v = \frac{1}{0.2}\sqrt{2\times0.2\times40} = 20 \text{ m/s}$$

$$a = \frac{20-30}{1} = -10 \text{ m/s}^2$$

$$s = \frac{-u^2}{2a} = 45 \text{ m}$$

6. Assume there are two identical simple pendulum Clocks-1 is placed on the earth and Clock-2 is placed on a space station located at a height h above the earth surface. Clock-1 and Clock-2 operate at time periods 4s and 6s respectively. Then the value of h is –

(consider radius of earth R_E = 6400 km and g on earth 10 m/s^2)

(A) 1200 km

(B) 1600 km

(C) 3200 km

(D) 4800 km

Official Ans. by NTA (C)

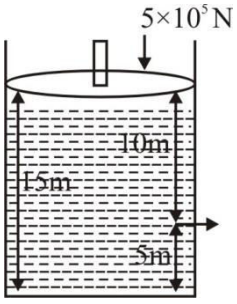
Sol. $t \propto \frac{1}{\sqrt{g}}$ and $g \propto \frac{1}{(R+h)^2}$

$$\frac{t_1}{t_2} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{R^2}{(R+h)^2}}$$

$$\frac{t_1}{t_2} = \frac{4}{6} = \frac{R}{(R+h)} \Rightarrow h = 3200\,\text{km}$$

7. Consider a cylindrical tank of radius 1m is filled with water. The top surface of water is at 15 m from the bottom of the cylinder. There is a hole on the wall of cylinder at a height of 5m from the bottom. A force of 5×10^5 N is applied an the top surface of water using a piston. The speed of efflux from the hole will be :

(given atmospheric pressure $P_A = 1.01 \times 10^5$ Pa, density of water $\rho_w = 1000$ kg/m^3 and gravitational acceleration g = 10 m/s^2)



(A) 11.6 m/s (B) 10.8 m/s

(C) 17.8 m/s (D) 14.4 m/s

Official Ans. by NTA (C)

Sol. Apply Bernoulli's theorem between Piston and hole $P_A + \rho gh = P_0 + \frac{1}{2}\rho v_e^2$

Assuming there is no atmospheric pressure on piston

$$\frac{5\times10^5}{\pi} + 10^3\times10\times10 = 1.01\times10^5 + \frac{1}{2}\times10^3\times v_e^2$$

v_e = 17.8 m/s

8. A vessel contains 14 g of nitrogen gas at a temperature of 27°C. The amount of heat to be transferred to the gap to double the r.m.s. speed of its molecules will be : (Take R = 8.32 J mol^{-1}k^{-1})

(A) 2229 J (B) 5616 J

(C) 9360 J (D) 13,104 J

Official Ans. by NTA (C)

Sol. $v_{rms} \propto \sqrt{T}$

$v_{rms} \propto \sqrt{300K}$, $v_{rms_f} = 2v_{rms_i}$

$v_{rms_f} \propto \sqrt{1200\,K}$

$T_f = 1200K,\ T_i = 300\,K,\ n = \frac{14}{28} = \frac{1}{2}$

$Q = nC_v\Delta T = \frac{1}{2} \times \frac{5R}{2} \times 900$

Q = 9360 J

9. A slab of dielectric constant K has the same cross-sectional area as the plates of a parallel plate capacitor and thickness $\frac{3}{4}d$, where d is the separation of the plates. The capacitance of the capacitor when the slab is inserted between the plates will be :

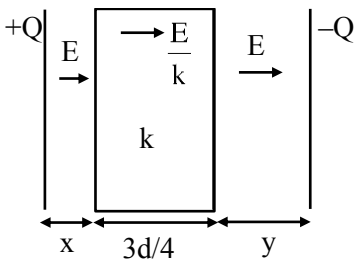
(Given C_o = capacitance of capacitor with air as medium between plates.)

(A) $\frac{4KC_0}{3+K}$ (B) $\frac{3KC_0}{3+K}$

(C) $\frac{3+K}{4KC_0}$ (D) $\frac{K}{4+K}$

Official Ans. by NTA (A)

Sol.



$x + y + \frac{3d}{4} = d$

$x + y = \frac{d}{4}$

$\frac{A\epsilon_0}{d} = C_0$

$\Delta V = Ex + \frac{E}{k} \times \frac{3d}{4} + Ey$

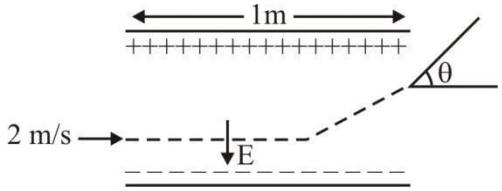
$= \frac{3Ed}{4k} + E(x+y)$

$\Delta V = E\left[\frac{3d}{4k} + \frac{d}{4}\right]$

$\Delta V = \frac{\sigma}{\epsilon_0}\left[\frac{3d+dk}{4k}\right] = \frac{Qd}{A\epsilon_0}\left[\frac{3+k}{4k}\right]$

$\frac{Q}{\Delta V} = C = \frac{A\epsilon_0}{d}\left[\frac{4k}{3+k}\right] = \frac{4kC_0}{k+3}$

10. A uniform electric field E = (8m/e) V/m is created between two parallel plates of length 1m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be .....

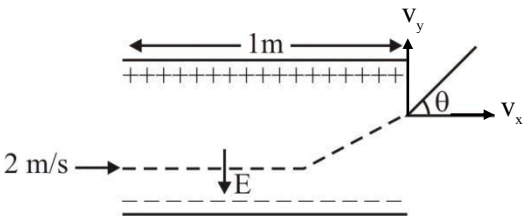


(A) $\tan^{-1}(4)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$ (D) $\tan^{-1}(3)$

Official Ans. by NTA (B)

Sol.



$a_y = \frac{F_y}{m} = \frac{e(E)}{m} = \frac{e\left(\frac{8m}{e}\right)}{m} = 8m/s^2$

$s_x = u_x t$

$1 = 2 \times t$

$t = \frac{1}{2}\,sec$

$v_y = u_y + a_y t$

$v_y = 0 + 8 \times \frac{1}{2}$

$v_y = 4$ m/s

$\tan\theta = \frac{v_y}{v_x} = \frac{4}{2} = 2 \Rightarrow \theta = \tan^{-1}(2)$

11. Given below are two statements :

Statement I : A uniform wire of resistance 80Ω is cut into four equal parts. These parts are now connected in parallel. The equivalent resistance of the combination will be 5 Ω.

Statement II : Two resistance 2R and 3R are connected in parallel in a electric circuit. The value of thermal energy developed in 3R and 2R will be in the ratio 3 : 2.

In the light of the above statements, choose the most appropriate answer from the options given below

(A) Both statement I and statement II are correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

(D) Statement I is incorrect but statement II is correct.

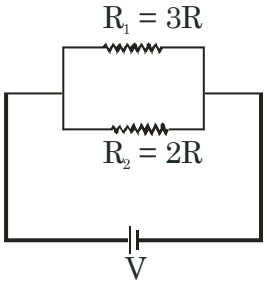
Official Ans. by NTA (C)

Sol. **Statement 1 -** $R = 80\Omega$

$R_1 = R_2 = R_3 = R_4 = 20\ \Omega$

In parallel $R_{eq} = \frac{20}{4} = 5\Omega$

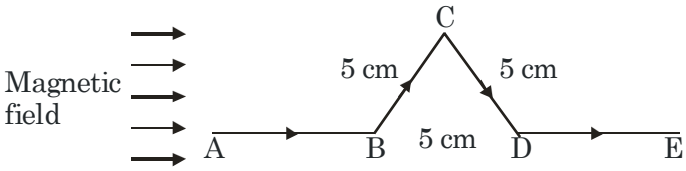
Statement 2 –



$P_{th} = \frac{v^2}{R}$

$\frac{P_1}{P_2} = \left(\frac{R_2}{R_1}\right) = \frac{2}{3}$ (where P is power)

12. A triangular shaped wire carrying 10A current is placed in a uniform magnetic field of 0.5T, as shown in figure. The magnetic force on segment CD is (Given BC = CD = BD = 5 cm).



(A) 0.126 N (B) 0.312 N

(C) 0.216 N (D) 0.245 N

Official Ans. by NTA (C)

Sol. $F_M(CD) = BI\ell_{eff}$

$= 0.5\times(10) \times (5\sin 60 \times 10^{-2})$

$= 0.216$ N

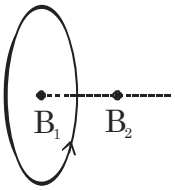
13. The magnetic field at the center of current carrying circular loop is B_1. The magnetic field at a distance of $\sqrt{3}$ times radius of the given circular loop from the center on its axis is B_2. The value of B_1/B_2 will be

(A) 9 : 4 (B) $12:\sqrt{5}$

(C) 8 : 1 (D) $5:\sqrt{3}$

Official Ans. by NTA (C)

Sol.



$B_1 = \frac{\mu_0 I}{2R}$

$B_2 = \frac{\mu_0 I R^2}{2\left(R^2+3R^2\right)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right) = \frac{B_1}{8}$

$\frac{B_1}{B_2} = \frac{8}{1}$

14. A transformer operating at primary voltage 8 kV and secondary voltage 160 V serves a load of 80 kW. Assuming the transformer to be ideal with purely resistive load and working on unity power factor, the loads in the primary and secondary circuit would be

(A) 800 Ω and 1.06 Ω (B) 10 Ω and 500 Ω

(C) 800 Ω and 0.32 Ω (D) 1.06 Ω and 500 Ω

Official Ans. by NTA (C)

Sol. $\frac{\left(8\times10^{3}\right)^{2}}{R_P}=80\times10^{3}$

$R_P=800\Omega$

$\frac{(160)^{2}}{R_S}=80\times10^{3}$

$R_S=0.32\Omega$

15. Sun light falls normally on a surface of area 36 cm^2 and exerts an average force of 7.2×10^{-9} N within a time period of 20 minutes. Considering a case of complete absorption, the energy flux of incident light is

(A) 25.92×10^{2} W/cm^2 (B) 8.64×10^{-6} W/cm^2

(C) 6.0 W/cm^2 (D) 0.06 W/cm^2

Official Ans. by NTA (D)

Sol. $\frac{I}{C}\times\text{area}=\text{force}$

$\frac{I}{C}\times36\times10^{-4}=7.2\times10^{-9}$

$I=\frac{7.2\times10^{-9}\times3\times10^{8}}{36\times10^{-9}\times10}$

$=\frac{6\times10^{-1}}{10^{-3}}$

$I=6\times10^{2}\frac{w}{m^2}$

$=0.06\frac{w}{cm^2}$

16. The power of a lens (biconvex) is 1.25 m^{-1} in particular medium. Refractive index of the lens is 1.5 and radii of curvature are 20 cm and 40 cm respectively. The refractive index of surrounding medium :

(A) 1.0 (B) $\frac{9}{7}$

(C) $\frac{3}{2}$ (D) $\frac{4}{3}$

Official Ans. by NTA (B)

Sol. $P=\frac{\mu_2}{f}=(\mu_1-\mu_2)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (For this formula refer to NCERT Part-2, Chapter-9, Page no. 328, solved example 8)

(μ_1 is refractive index of lens and μ_2 is of surrounding medium)

$1.25=(1.5-\mu_2)\left(\frac{1}{0.2}+\frac{1}{0.4}\right)$

$\frac{1.25\times0.08}{0.6}=(1.5-\mu_2)$

$\Rightarrow\mu_2=\frac{4}{3}$

17. Two streams of photons, possessing energies to five and ten times the work function of metal are incident on the metal surface successively. The ratio of the maximum velocities of the photoelectron emitted, in the two cases respectively, will be

(A) 1 : 2 (B) 1 : 3

(C) 2 : 3 (D) 3 : 2

Official Ans. by NTA (C)

Sol. $\frac{1}{2}mv_1^2 = 4\phi$

$\frac{1}{2}mv_2^2 = 9\phi$

$\frac{v_1}{v_2} = \frac{2}{3}$

18. A radioactive sample decays $\frac{7}{4}$ times its original quantity in 15 minutes. The half-life of the sample is

(A) 5 min (B) 7.5 min

(C) 15 min (D) 30 min

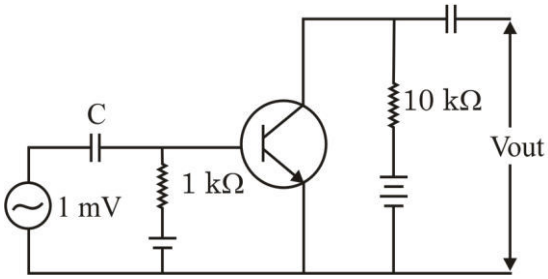
Official Ans. by NTA (A)

Sol. Remaining $= \frac{1}{8}$

$3t_{1/2} = 15\,min$

$t_{1/2} = 5\,min$

19. An n.p.n transistor with current gain β = 100 in common emitter configuration is shown in figure. The output voltage of the amplifier will be



(A) 0.1 V (B) 1.0 V

(C) 10 V (D) 100 V

Official Ans. by NTA (B)

Sol. $\frac{v_{out}}{v_{in}} = \beta\frac{R_{out}}{R_{in}}$

$v_{out} = \frac{100\times10\times10^3}{10^3}\times10^{-3}$

= 1 V

20. A FM Broad cast transmitter, using modulating signal of frequency 20 kHz has a deviation ratio of 10. The Bandwidth required for transmission is :

(A) 220 kHz

(B) 180 kHz

(C) 360 kHz

(D) 440 kHz

Official Ans. by NTA (D)

Sol. Given

FM broadcast

Modulating frequency = 20 k Hz = f

$\text{Deviation ratio} = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta f}{f}$

$\Rightarrow$ frequency deviation – $\Delta f = f \times 10$

= 20 kHz × 10 = 200 kHz

$\Rightarrow$ Bandwidth = 2(f + Δf)

= 2 (20 + 200) kHz

= 440 kHz

SECTION-B

1. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)$m. The value of k is ….. (use g = 9.8 m/s^2)

Official Ans. by NTA (392)

Sol. $t_a = \frac{u}{g} = \frac{19.6}{9.8} = 2s$

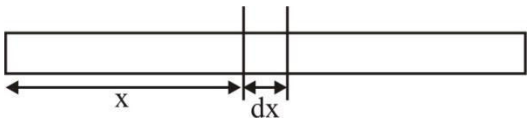
$t_d = 6 - 2s = \sqrt{\frac{2h_{max}}{g}}$

$\Rightarrow h_{max} = \frac{16 \times 9.8}{2} = \frac{392}{5}$

2. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1 - \frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$m. The value of α is ……. (where x is the distance form end A)

Official Ans. by NTA (8)

Sol.



$dm = \lambda \cdot dx = \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)$

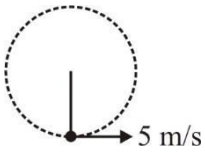
$X_{cm} = \frac{\int x dm}{\int dm_\ell}$

$= \frac{\lambda_0\int_0^\ell\left(x - \frac{x^3}{\ell^2}\right)dx}{\int_0^\ell \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)dx} = \frac{\frac{\ell^2}{2} - \frac{\ell^4}{4\ell^2}}{\ell - \frac{\ell^3}{3\ell^2}} = \frac{3\ell}{8}$

3. A string of area of cross-section 4 mm^2 and length 0.5 is connected with a rigid body of mass 2 kg. The body is rotated in a vertical circular path of radius 0.5 m. The body acquires a speed of 5 m/s at the bottom of the circular path. Strain produced in the string when the body is at the bottom of the circle is …… × 10^{-5}. (Use Young's modulus 10^{11} N/m^2 and g = 10 m/s^2)

Official Ans. by NTA (30)

Sol.



Strain = F/AY

$= \frac{mg + \frac{mv^2}{R}}{AY}$

$= \frac{20 + \frac{2(5)^2}{0.5}}{3 \times 10^{-6} \times 10^{11}} = 30 \times 10^{-5}$

4. At a certain temperature, the degrees of freedom per molecule for gas is 8. The gas performs 150 J of work when it expands under constant pressure. The amount of heat absorbed by the gas will be …….. J.

Official Ans. by NTA (750)

Sol. $W = nR\Delta T = 150$ J

$Q = \left(\frac{f}{2} + 1\right)nR\Delta T = \left(\frac{8}{2} + 1\right)150 = 750$ J

5. The potential energy of a particle of mass 4 kg in motion along the x-axis is given by U = 4(1 – cos 4x) J. The time period of the particle for small oscillation ($\sin\theta \simeq \theta$) is $\left(\frac{\pi}{K}\right)$s. The value of K is ……..

Official Ans. by NTA (2)

Sol. $U = 4(1 - \cos 4x)$

$F = -\frac{dU}{dx} = -4(+\sin 4x)4 = -16\sin(4x)$

For small θ

$\sin\theta \approx \theta$

$F = -64x$

$a = -64x/m = -16x$

$\omega^2 = 16$

$T = \frac{2\pi}{\omega} = \frac{\pi}{2}$

6. An electrical bulb rated 220 V, 100 W, is connected in series with another bulb rated 220 V, 60 W. If the voltage across combination is 220 V, the power consumed by the 100 W bulb will be about ……. W.

Official Ans. by NTA (14)

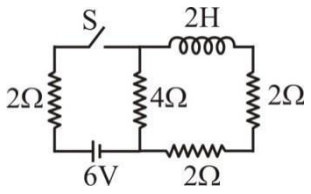
Sol. $R_1 = \frac{V^2}{P} = \frac{220^2}{100} = 484$

$R_2 = \frac{V^2}{P} = \frac{220^2}{60} = 484\left(\frac{10}{6}\right)$

$I = \frac{220}{484 + 484 \times \frac{10}{6}}$

$P_1 = I^2R_1 = 14.06\,W$

7. For the given circuit the current through battery of 6 V just after closing the switch 'S' will be …..…… A.

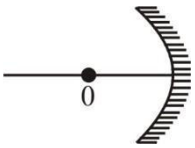


Official Ans. by NTA (1)

Sol. Just after closing the switch S, inductor behaves like an open circuit.

$I = \frac{6}{2+4} = 1A$

8. An object 'o' is placed at a distance of 100 cm in front of a concave mirror of radius of curvature 200 cm as shown in the figure. The object starts moving towards the mirror at a speed 2 cm/s. The position of the image from the mirror after 10s will be at ……. cm.



Official Ans. by NTA (400)

Sol. After 10 sec.

u = –80 cm

f = –100 cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

v = 400 cm

9. In an experiment with a convex lens. The plot of the image distance (v') against the object distance (μ') measured from the focus gives a curve v' μ' = 225. If all the distances are measured in cm. The magnitude of the focal length of the lens is …….. cm.

Official Ans. by NTA (15)

Sol. $vu = f^2$ (by Newton's formula)

$f^2 = 225$

$f = 15$ cm

10. In an experiment to find acceleration due to gravity (g) using simple pendulum, time period of 0.5 s is measured from time of 100 oscillation with a watch of 1s resolution. If measured value of length is 10 cm known to 1mm accuracy. The accuracy in the determination of g is found to be x %. The value of x is

Official Ans. by NTA (5)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$g = \frac{1}{4\pi^2}\frac{T^2}{\ell}$$

$$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta g}{g} = 2\cdot\frac{1}{100\times 0.5} + \frac{1\text{mm}}{10\text{cm}}$$

$$\frac{\Delta g}{g} = \frac{5}{100}$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Zero orbital overlap is an out of phase overlap.

Reason : It results due to different orientation/ direction of approach of orbitals.

In the light of the above statements. Choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

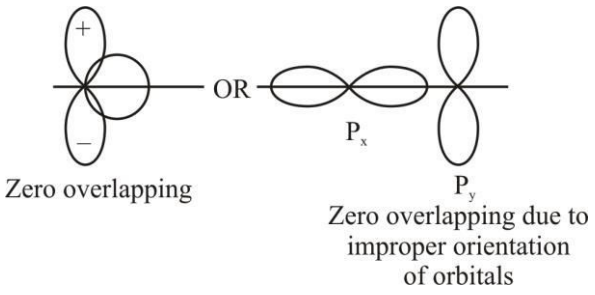
(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol.



2. The correct decreasing order for metallic character is

(A) Na > Mg > Be > Si > P

(B) P > Si > Be > Mg > Na

(C) Si > P > Be > Na > Mg

(D) Be > Na > Mg > Si > P

Official Ans. by NTA (A)

Sol. Across a period metallic character decreases

3. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The reduction of a metal oxide is easier if the metal formed is in liquid state than solid state.

Reason R : The value of $\Delta G^{\ominus}$ becomes more on negative side as entropy is higher in liquid state than solid state.

In the light of the above statements. Choose the most appropriate answer from the options given below

(A) Both A and R are correct and R is the correct explanation of A

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol. $\Delta G = \Delta H - T\Delta S$

$\because$ Entropy of liquid is more than solid

$\therefore$ on melting the entropy increases and ΔG becomes more negative and hence it becomes easier to reduce metal

4. The products obtained during treatment of hard water using Clark's method are:

(A) $CaCO_3$ and $MgCO_3$

(B) $Ca(OH)_2$ and $Mg(OH)_2$

(C) $CaCO_3$ and $Mg(OH)_2$

(D) $Ca(OH)_2$ and $MgCO_3$

Official Ans. by NTA (C)

Sol. In Clark's method lime water is used

$$Ca(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$$

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + Mg(OH)_2 + 2H_2O$$

5. **Statement I:** An alloy of lithium and magnesium is used to make aircraft plates.

Statement II : The magnesium ions are important for cell-membrane integrity.

In the light the above statements, choose the *correct* answer from the options given below

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (B)

Sol. Alloy of Li and Mg is used to make armour plates and not aircraft plates.

Calcium plays important roles in neuromuscular function, interneuronal transmission and cell membrane integrity

6. White phosphorus reacts with thionyl chloride to give

(A) PCl_5, SO_2 and S_2Cl_2 (B) PCl_3. SO_2 and S_2Cl_2

(C) PCl_3, SO_2 and Cl_2 (D) PCl_5, SO_2 and Cl_2

Official Ans. by NTA (B)

Sol. $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

7. Concentrated HNO_3 reacts with Iodine to give

(A) HI, NO_2 and H_2O (B) HIO_2, N_2O and H_2O

(C) HIO_3, NO_2 and H_2O (D) HIO_4, N_2O and H_2O

Official Ans. by NTA (C)

Sol. $I_2 + 10HNO_{3(conc)} \Rightarrow 2HIO_3 + 10NO_2 + 4H_2O$

8. Which of the following pair is not isoelectronic species?

(At. no. Sm, 62; Er, 68: Yb, 70: Lu, 71; Eu, 63: Tb, 65; Tm, 69)

(A) Sm^{2+} and Er^{3+} (B) Yb^{2+} and Lu^{3+}

(C) Eu^{2+} and Tb^{4+} (D) Tb^{2+} and Tm^{4+}

Official Ans. by NTA (D)

Sol. $Sm^{2+} \rightarrow$ electron = 60

$Er^{3+} \rightarrow$ electron = 65

$Tb^{2+} \rightarrow$ electron = 63

$Tm^{4+} \rightarrow$ electron = 65

(not isoelectronic)

9. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Permanganate titrations are not performed in presence of hydrochloric acid.

Reason R : Chlorine is formed as a consequence of oxidation of hydrochloric acid.

In the light of the above statements, choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol. $2KMnO_4 + 16HCl \rightarrow 2MnCl_2 + 2KCl + 8H_2O + Cl_2$

HCl gets oxidised by $KMnO_4$ into Cl_2

10. Match **List I** with **List II**

	List I (Complex)		**List II (Hybridization)**
A	$Ni(CO)_4$	I	sp^3
B	$[Ni(CN)_4]^{2-}$	II	sp^3d^2
C	$[Co(CN)_6]^{3-}$	III	d^2sp^3
D	$[CoF_6]^{3-}$	IV	dsp^2

Choose the correct answer from the options given below:

(A) A-IV, B-I, C-III, D-II

(B) A-I. B-IV, C-III, D-II

(C) A-I. B-IV, C-II, D-III

(D) A-IV, B-I, C-II. D-III

Official Ans. by NTA (B)

Sol. $Ni(CO)_4$ Hybridisation sp^3

$[Ni(CN)_4]^{2-}$ Hybridisation dsp^2

$[Co(CN)_6]^{3-}$ Hybridisation d^2sp^3

$[Co(F)_6]^{3-}$ Hybridisation sp^3d^2

11. Dinitrogen and dioxygen. the main constituents of air do not react with each other in atmosphere to form oxides of nitrogen because

(A) N_2 is unreactive in the condition of atmosphere.

(B) Oxides of nitrogen are unstable.

(C) Reaction between them can occur in the presence of a catalyst.

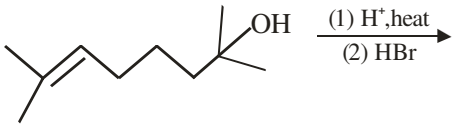
(D) The reaction is endothermic and require very high temperature.

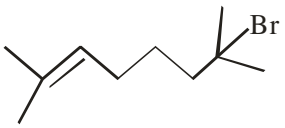
Official Ans. by NTA (D)

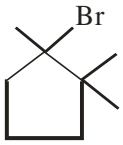
Sol. $N_2 + O_2 \underset{}{\overset{(1483-2000\ K)}{\rightleftharpoons}} 2NO$

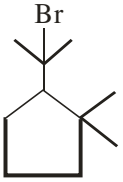
(Endothermic and feasible at high temperature)

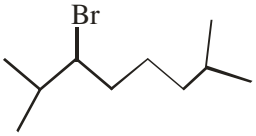
12. The major product in the given reaction is



(A) 

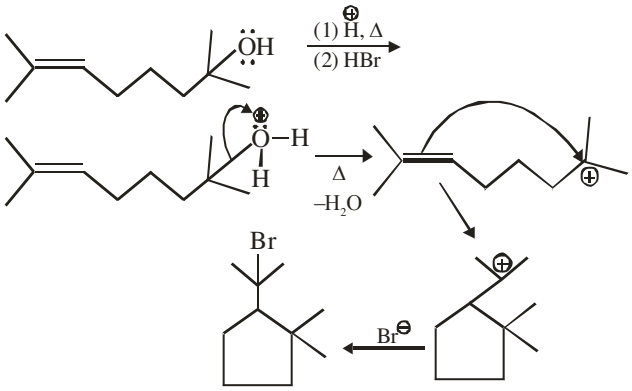
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol.



13. Arrange the following in increasing order of reactivity towards nitration

A. p-xylene

B. bromobenzene

C. mesitylene

D, nitrobenzene

E. benzene

Choose the correct answer from the options given below

(A) C < D < E < A < B

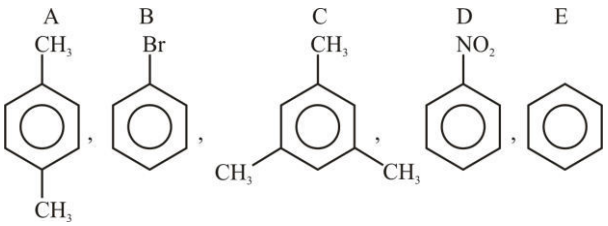
(B) D < B < E < A < C

(C) D < C < E < A < B

(D) C < D < E < B < A

Official Ans. by NTA (B)

Sol.



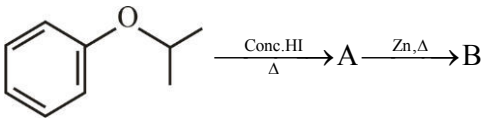
$-NO_2$ is strongly deactivating

–Br – deactivating

$-CH_3$–activating group

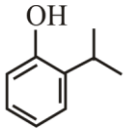
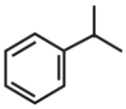
D < B < E < A < C

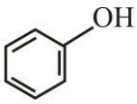
14. Compound I is heated with Conc. HI to give a hydroxy compound A which is further heated with Zn dust to give compound B. Identify A and B.



(A) A =  , B = 

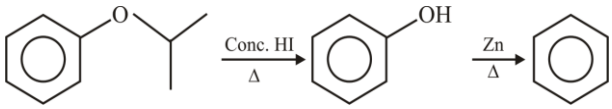
(B) A =  , B = 

(C) A =  , B = 

(D) A =  , B = 

Official Ans. by NTA (D)

Sol.



15. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Aniline on nitration yields ortho, meta & para nitro derivatives of aniline.

Reason R: Nitrating mixture is a strong acidic mixture.

In the light of the above statements, choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

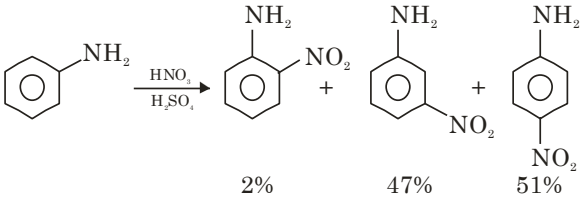
(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

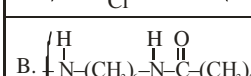
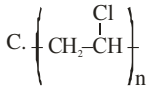
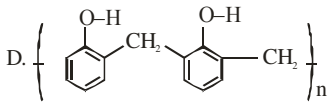
Official Ans. by NTA (A)

Sol.



Due to formation of anilinium ion in acidic medium meta product is also obtained in significant amount

16. Match **List I** with **List II**

List (Polymer)	List II(Nature)
A. 	I. Thermosetting polymer
B.	II. Fibers
C. 	III. Elastomer
D. 	IV. Thermoplastic polymer

Choose the correct answer from the options given below:

(A) A-II, B-III, C-IV, D-I

(B) A-III, B-II, C-IV, D-I

(C) A-III, B-I, C-IV, D-II

(D) A-I. B-III, C-IV, D-II

Official Ans. by NTA (B)

Sol. Neoprene is elastomer

Nylon-6, 6 is fiber

PVC is thermoplastic

Novolac is thermosetting

17. Two statements in respect of drug-enzyme interaction are given below

Statement I : Action of an enzyme can be blocked only when an inhibitor blocks the active site of the enzyme.

Statement II : An inhibitor can form a strong covalent bond with the enzyme.

In the light of the above statements. Choose the ***correct*** answer from the options given below

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (D)

Sol. Some drugs do not bind to active sites. These bind to different site of enzyme called allosteric sites.

18. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Thin layer chromatography is an adsorption chromatography.

Reason : A thin layer of silica gel is spread over a glass plate of suitable size in thin layer chromatography which acts as an adsorbent.

In the light of the above statements, choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

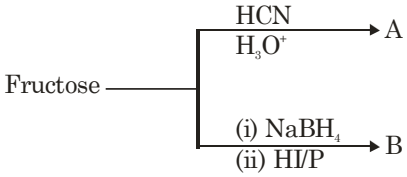
Official Ans. by NTA (A)

Sol. Theory based

Thin layer chromoatography (TLC) is another type of adsorption chromatography, which involve sepration of substance of a mixture ovel a thin layer of an adsorbent coated on glass plate.

A thin layer (about 0.2 mm thick) of an adsorbent (silica gel) or (Alumina) in spread overa glass plate of suitable size. Hence Assertion (A) is correct and Reason (R) is correct explanation of (A)

19. The formulas of A and B for the following reaction sequence are



(A) A = $C_7H_{14}O_8$, B = C_6H_{14}

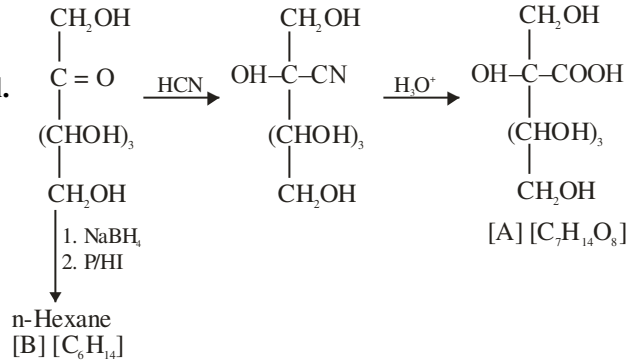
(B) A = $C_7H_{13}O_7$, B = $C_7H_{14}O$

(C) A = $C_7H_{12}O_8$, B = C_6H_{14}

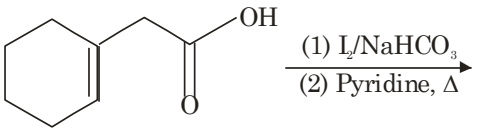
(D) A = $C_7H_{14}O_8$, B = $C_6H_{14}O_6$

Official Ans. by NTA (A)

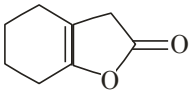
Sol.

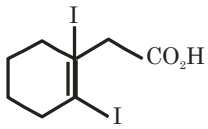


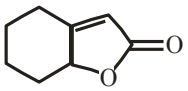
20.

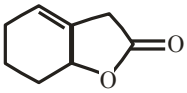


Find out the major product for the above reaction.

(A) 

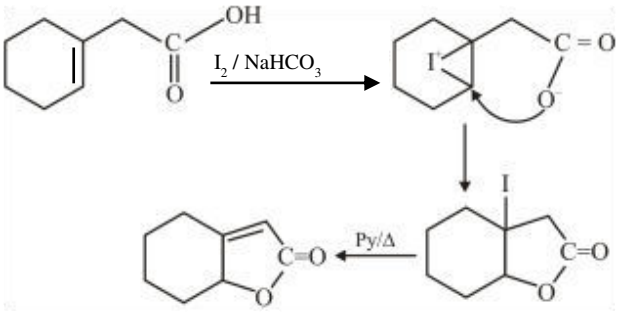
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol.



SECTION-B

1. 2L of 0.2 M H_2SO_4 is reacted with 2L of 0.1 M NaOH solution, the molarity of the resulting product Na_2SO_4 in the solution is ____ millimolar. (Nearest integer).

Official Ans. by NTA (25)

Sol. $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$

0.4 mol	0.2 mol	-
0.3 mol	-	0.1 mol

Molarity of Na_2SO_4 is $\frac{0.1}{4} = 0.025M$

= 25 mM.

2. Metal M crystallizes into a FCC lattice with the edge length of 4.0×10^{-8} cm. The atomic mass of the metal is _____ g/mol. (Nearest integer).

(Use : $N_A = 6.02\times10^{23}$ mol^{-1}, density of metal, M = 9.03 g cm^{-3})

Official Ans. by NTA (87)

Sol. $a = 4 \times 10^{-8}$ cm

d = 9.03g/ml

$$d = \frac{ZM}{N_A a^3}$$

$$M = \frac{9.03\times6.02\times10^{23}\times64\times10^{-24}}{4} \quad 86.97$$

3. If the wavelength for an electron emitted from H-atom is 3.3×10^{-10}m, then energy absorbed by the electron in its ground state compared to minimum energy required for its escape from the atom, is _____ times. (Nearest integer).

[Given : h = 6.626×10^{-34} Js,

Mass of electron = 9.1×10^{-31}**]**

Official Ans. by NTA (2)

Sol. $$\lambda = \frac{h}{\sqrt{2mK}}$$

$$K = \frac{h^2}{2m\lambda^2}$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{43.9\times10^{-68}}{2\times9.1\times10^{-31}\times10.89\times10^{-20}}$$

$K = 2.215 \times 10^{-18}$

$E_{abs} = E_{req} + K$

$$\frac{E_{abs}}{E_{req}} = 1 + \frac{K}{E_{req}} = 1 + \frac{2.215\times10^{-18}}{13.6\times1.602\times10^{-19}} = 2.0166$$

4. A gaseous mixture of two substances A and B, under a total pressure of 0.8 atm is in equilibrium with an ideal liquid solution. The mole fraction of substance A is 0.5 in the vapour phase and 0.2 in the liquid phase. The vapour pressure of pure liquid A is _____ atm. (Nearest integer)

Official Ans. by NTA (2)

Sol. **$Y_A = 0.5 \Rightarrow Y_B = 0.5$**

$P_A = P_B = 0.4$ atm

$$P_A = P_A^0 X_A$$

$$P_A^0 = 2$$

5. At 600K, 2 mol of NO are mixed with 1 mol of O_2.

$$2NO_{(g)} + O_2(g) \rightleftharpoons 2NO_2(g)$$

The reaction occurring as above comes to equilibrium under a total pressure of 1 atom. Analysis of the system shows that 0.6 mol of oxygen are present at equilibrium. The equilibrium constant for the reaction is _____. (Nearest integer).

Official Ans. by NTA (2)

Sol.

$2NO +$	$O_2 \rightarrow$	$2NO$
2	1	-
2-2x	1-x	2x
1.2	0.6	0.8

$$K_p = \frac{\left(\frac{0.8}{2.6}\right)^2}{\left(\frac{1.2}{2.6}\right)^2\left(\frac{0.6}{2.6}\right)} = 1.925$$

6. A sample of 0.125 g of an organic compound when analysed by Duma's method yields 22.78 mL of nitrogen gas collected over KOH solution at 280K and 759 mm Hg. The percentage of nitrogen in the given organic compound is ____. (Nearest integer).

(a) The vapour pressure of water at 280 K is 14.2 mm Hg

(b) R = 0.082 L atm K^{-1} mol^{-1}

Official Ans. by NTA (22)

Sol. V = 22.78 ml, T = 280 K

P_{total} = 759 mmHg

$$P_{N_2} = 759 - 14.2 = 744.8 mmHg$$

$$n_{N_2} = \frac{744.8 \times 22.78}{760 \times 1000 \times 0.082 \times 280} = 0.00097$$

$$W_{Nitrogen} = 0.02716$$

$$\%N = \frac{0.02716}{0.125} \times 1000 = 21.728$$

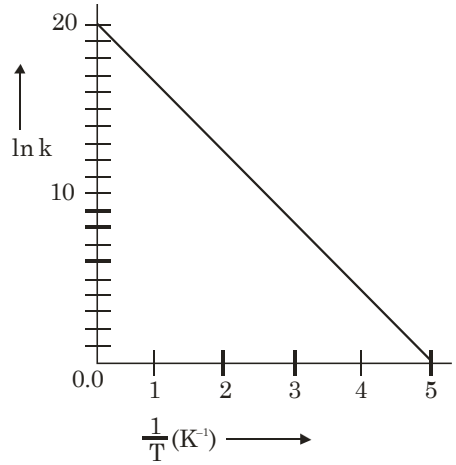
7. On reaction with stronger oxidizing agent like KIO_4, hydrogen peroxide oxidizes with the evolution of O_2. The oxidation number of I in KIO_4 changes to _____.

Official Ans. by NTA (5)

Sol. $IO_4^- + H_2O_2 \rightarrow IO_3^- + O_2$

8. For a reaction, given below is the graph of ln k vs $\frac{1}{T}$. The activation energy for the reaction is equal to _____ cal mol^{-1}. (Nearest integer).

(Given : R = 2 cal K^{-1} mol^{-1}**)**



Official Ans. by NTA (8)

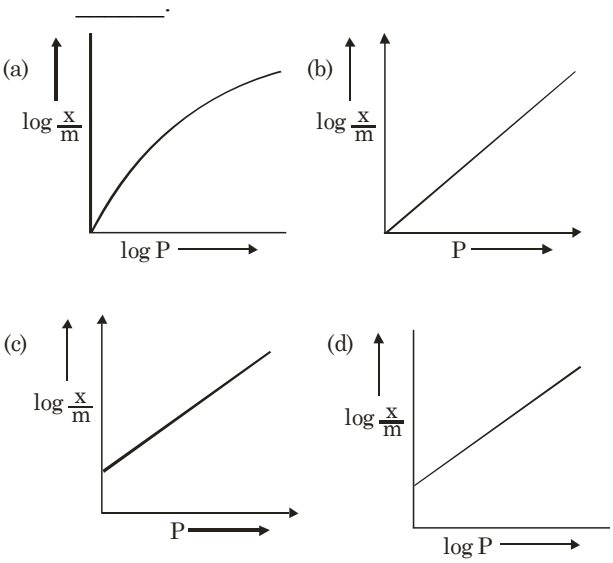
Sol. $K = Ae^{-Ea/RT}$

$$\ln k = \frac{-Ea}{RT} + \ln A$$

$$Slope = \frac{Ea}{R} = \frac{20}{5}$$

E_a = 4R = 8 Cal/mol

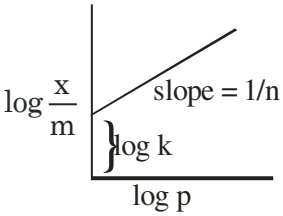
9. Among the following the number of curves not in accordance with Freundlich adsorption isotherm is _____.



Official Ans. by NTA (3)

Sol. $\frac{X}{m} = KP^{\frac{1}{n}}$

$$\log \frac{x}{m} = \frac{1}{n} \log p + \log k$$



10. Among the following the number of state variable is _____.

Internal energy (U)

Volume (V)

Heat (q)

Enthalpy (H)

Official Ans. by NTA (3)

Sol. Internal energy, volume enthalpy are state variable

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $S=\left\{x\in[-6,3]-\{-2,2\}:\frac{|x+3|-1}{|x|-2}\geq 0\right\}$ and $T=\{x\in Z:x^2-7|x|+9\leq 0\}$. Then the number of elements in $S\cap T$ is

(A) 7 (B) 5

(C) 4 (D) 3

Official Ans. by NTA (D)

Sol. $S\cap T=\{-5,-4,3\}$

2. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1, \frac{1}{\beta^2}+1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

(A) non-real complex numbers

(B) real and both negative

(C) real and both positive

(D) real and exactly one of them is positive

Official Ans. by NTA (B)

Sol. $a=\frac{-1}{\alpha^2}-\frac{1}{\beta^2}-2$

$b=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+1+\frac{1}{\alpha^2\beta^2}$

$a+b=\frac{1}{(\alpha\beta)^2}-1=\frac{1}{6}-1=-\frac{5}{6}$

$x^2-\left(-\frac{5}{6}-2\right)x+\left(2-\frac{5}{6}\right)=0$

$6x^2 + 17x + 7 = 0$

$x=-\frac{7}{3}$, $x=-\frac{1}{2}$ are the roots

Both roots are real and negative.

3. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is **NOT** true?

(A) $A^4 - B^4$ is a symmetric matrix

(B) AB – BA is a symmetric matrix

(C) $B^5 - A^5$ is a skew-symmetric matrix

(D) AB + BA is a skew-symmetric matrix

Official Ans. by NTA (C)

Sol. Given that $A^T = A$, $B^T = -B$

(A) $C = A^4 - B^4$

$C^T = (A^4 - B^4) = (A^4)^T - (B^4)^T = A^4 - B^4 = C$

(B) $C = AB - BA$

$C^T = (AB - BA)^T = (AB)^T - (BA)^T$

$= B^TA^T - A^TB^T = -BA + AB = C$

(C) $C = B^5 - A^5$

$C^T = (B^5 - A^5)^T = (B^5)^T - (A^5)^T = -B^5 - A^5$

(D) $C = AB + BA$

$C^T = (AB + BA)^T = (AB)^T + (BA)^T$

$= -BA - AB = -C$

$\therefore$ Option C is not true.

4. Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$, then λ is equal to

(A) – 4 (B) $\frac{13}{2}$

(C) $\frac{23}{2}$ (D) 4

Official Ans. by NTA (D)

Sol. $f(0) + 3 + \lambda + 4 = 14$

$\therefore$ $f(0) = 7 - \lambda = c$

$f(1) = a + b + c = 3$ …(i)

$f(3) = 9a + 3b + c = 4$ …(ii)

$f(-2) = 4a - 2b + c = \lambda$ …(iii)

(ii) – (iii)

$a + b = \frac{4-\lambda}{5}$ put in equation (i)

$\frac{4-\lambda}{5}+7-\lambda=3$

$6\lambda = 24$; $\lambda = 4$

5. The function $f : R \to R$ defined by

$$f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$$ is

continuous for all x in

(A) $R - \{-1\}$ (B) $R - \{-1, 1\}$

(C) $R - \{1\}$ (D) $R - \{0\}$

Official Ans. by NTA (B)

Note : n should be given as a natural number.

Sol. $$f(x) = \begin{cases} \frac{-\sin(x-1)}{x-1} & x < -1 \\ -(\sin 2+1) & x = -1 \\ \cos 2\pi x & -1 < x < 1 \\ 1 & x = 1 \\ \frac{-\sin(x-1)}{x-1} & x > 1 \end{cases}$$

f(x) is discontinuous at x = –1 and x = 1

6. The function $f(x) = xe^{x(1-x)}, x \in R,$ is

(A) increasing in $\left(-\frac{1}{2}, 1\right)$

(B) decreasing in $\left(\frac{1}{2}, 2\right)$

(C) increasing in $\left(-1, -\frac{1}{2}\right)$

(D) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Official Ans. by NTA (A)

Sol. $f(x) = x\ e^{x(1-x)}$

$f'(x) = -e^{x(1-x)}\ (2x + 1)\ (x - 1)$

f(x) is increasing in $\left(-\frac{1}{2}, 1\right)$

7. The sum of the absolute maximum and absolute minimum values of the function

$f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is

(A) 0 (B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$ (D) $\frac{-\pi}{12}$

Official Ans. by NTA (C)

Sol. $f(x) = \tan^{-1}(\sin x - \cos x)$

$$f'(x) = \frac{\cos x + \sin x}{(\sin x - \cos x)^2 + 1} = 0$$

$\therefore\ x = \frac{3\pi}{4}$

x	0	$\frac{3\pi}{4}$	π
f(x)	$-\frac{\pi}{4}$	$\tan^{-1}\sqrt{2}$	$\frac{\pi}{4}$

$$\therefore \begin{matrix} (f(x))_{max} = \tan^{-1}\sqrt{2} \\ (f(x))_{min} = -\frac{\pi}{4} \end{matrix}$$

$$\text{sum} = \tan^{-1}\sqrt{2} - \frac{\pi}{4}$$

$$= \cos^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4}$$

8. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then

$\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to

(A) $\frac{-2\sqrt{2}}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{-2}{3}$

Official Ans. by NTA (D)

Sol. $x = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$$

$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$

$$\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\frac{dy}{dx} = \tan 3t$$

$$\frac{dy}{dx} = -1 \text{ at } t = \frac{\pi}{4}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{2}}\sec^3 3t \cdot \sqrt{\sin 2t} = -3 \text{ at } t = \frac{\pi}{4}$$

$$\therefore \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{1+1}{-3} = -\frac{2}{3}$$

9. Let $I_n(x)=\int_0^x \frac{1}{(t^2+5)^n}dt$, n = 1, 2, 3, …. Then

(A) $50I_6-9I_5=xI_5'$ (B) $50I_6-11I_5=xI_5'$

(C) $50I_6-9I_5=I_5'$ (D) $50I_6-11I_5=I_5'$

Official Ans. by NTA (A)

Sol. $I_n(x)=\int_0^x \frac{dt}{(t^2+5)^n}$

Applying integral by parts

$$I_n(x)=\left[\frac{t}{(t^2+5)^n}\right]_0^x-\int_0^x n(t^2+5)^{-n-1}\cdot 2t^2$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int_0^x\frac{t^2}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int_0^x\frac{(t^2+5)-5}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\,I_n(x)-10n\,I_{n+1}(x)$$

$$10n\,I_{n+1}(x)+(1-2n)I_n(x)=\frac{x}{(x^2+5)^n}$$

Put n = 5

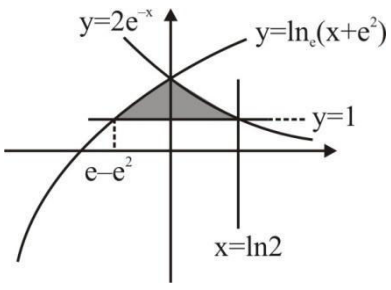
10. The area enclosed by the curves $y=\log_e(x+e^2)$, $x=\log_e\left(\frac{2}{y}\right)$ and $x=\log_e 2$, above the line y = 1 is

(A) $2+e-\log_e 2$ (B) $1+e-\log_e 2$

(C) $e-\log_e 2$ (D) $1+\log_e 2$

Official Ans. by NTA (B)

Sol.



Required area is

$$=\int_{e-e^2}^{0}\ell n\left(x+e^2\right)-1dx+\int_0^{\ell n2}2e^{-x}-1dx=1+e-\ell n2$$

11. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, x > 1 passing through the point $\left(2,\sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to

(A) $11+6\log_e 3$ (B) 19

(C) $12-2\log_e 3$ (D) $19-6\log_e 3$

Official Ans. by NTA (D)

Sol. $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$,

$\frac{dy}{dx}+Py=Q$

$$\text{I.F.}=e^{\int Pdx}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}=\int\left(\frac{x-1}{x+1}\right)^{1}dx$$

$$=x-2\log_e|x+1|+C$$

Curve passes through $\left(2,\frac{1}{\sqrt{3}}\right)$

$$\Rightarrow C=2\log_e 3-\frac{5}{3}$$

at x = 8,

$$\sqrt{7}y(8)=19-6\log_e 3$$

12. The differential equation of the family of circles passing through the points (0, 2) and (0, – 2) is

(A) $2xy\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$

(B) $2xy\frac{dy}{dx}+\left(x^2+y^2-4\right)=0$

(C) $2xy\frac{dy}{dx}+\left(y^2-x^2+4\right)=0$

(D) $2xy\frac{dy}{dx}-\left(x^2-y^2+4\right)=0$

Official Ans. by NTA (A)

Sol. Equation of circle passing through (0, –2) and (0, 2) is

$$x^2+\left(y^2-4\right)+\lambda x=0,\ (\lambda\in R)$$

Divided by x we get

$$\frac{x^2+\left(y^2-4\right)}{x}+\lambda=0$$

Differentiating with respect to x

$$\frac{x\left[2x+2y\cdot\frac{dy}{dx}\right]-\left[x^2+y^2-4\right]\cdot1}{x^2}=0$$

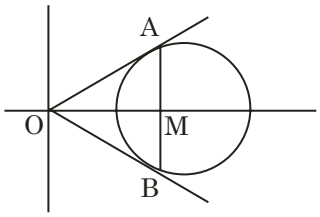
$$\Rightarrow 2xy\cdot\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$$

13. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin O (0, 0). Then the area of the triangle of OAB is

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$

(C) $\frac{3}{2\sqrt{3}}$ (D) $\frac{3}{4\sqrt{3}}$

Official Ans. by NTA (B)



Sol. C : $(x-2)^2 + y^2 = 1$

Equation of chord AB : 2x = 3

OA = OB = $\sqrt{3}$

$$AM=\frac{\sqrt{3}}{2}$$

$$\text{Area of triangle OAB}=\frac{1}{2}(2AM)(OM)$$

$$=\frac{3\sqrt{3}}{4}\text{ sq. units}$$

14. Let the hyperbola H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ pass through the point $\left(2\sqrt{2},-2\sqrt{2}\right)$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

(A) $\left(2\sqrt{3},3\sqrt{2}\right)$ (B) $\left(3\sqrt{3},-6\sqrt{2}\right)$

(C) $\left(\sqrt{3},-\sqrt{6}\right)$ (D) $\left(3\sqrt{6},6\sqrt{2}\right)$

Official Ans. by NTA (B)

Sol. H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Foci : S (ae, 0), S' (–ae, 0)

Foot of directrix of parabola is (–ae, 0)

Focus of parabola is (ae, 0)

Now, semi latus rectum of parabola $=|SS'|=2ae$

Given, $4ae=e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2=2a^2$ ..... (1)

Given, $\left(2\sqrt{2},-2\sqrt{2}\right)$ lies on H

$\Rightarrow \frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{8}$ ..... (2)

From (1) and (2)

$a^2 = 4, b^2 = 8$

$\because b^2=a^2\left(e^2-1\right)$

$\therefore e=\sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2=8\sqrt{3}x$

15. Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does **NOT** lies on P?

(A) (0, – 2, –2) (B) (– 5, 0, –1)

(C) (3, –1, 0) (D) (0, 4, 5)

Official Ans. by NTA (D)

Sol. Given, $L_1:\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$

and $L_2:\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$

are coplanar

$$\Rightarrow\begin{vmatrix}27 & 20 & 31\\ \lambda & 1 & 2\\ -2 & 3 & \lambda\end{vmatrix}=0$$

$\Rightarrow\lambda=3$

Now, normal of plane P, which contains L_1 and L_2

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ -2 & 3 & 3\end{vmatrix}$$

$=-3\hat{i}-13\hat{j}+11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3x + 13y – 11z + 4 = 0$

(0, 4, 5) does not lie on plane P.

16. A plane P is parallel to two lines whose direction ratios are –2, 1, –3, and –1, 2, –2 and it contains the point (2, 2, –2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin and $p = \alpha+\beta+\gamma$, then the ordered pair (V, p) is equal to

(A) (48, –13) (B) (24, –13)

(C) (48, 11) (D) (24, –5)

Official Ans. by NTA (B)

Sol. Normal of plane P :

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & -3\\ -1 & 2 & -2\end{vmatrix}=4\hat{i}-\hat{j}-3\hat{k}$$

Equation of plane P which passes through (2, 2,–2) is $4x – y – 3z – 12 = 0$

Now, A (3, 0, 0), B (0, – 12, 0), C (0, 0, –4)

$\Rightarrow\alpha=3,\beta=-12,\gamma=-4$

$\Rightarrow p=\alpha+\beta+\gamma=-13$

Now, volume of tetrahedron OABC

$$V=\left|\frac{1}{6}\overrightarrow{OA}\cdot\left(\overrightarrow{OB}\times\overrightarrow{OC}\right)\right|=24$$

(V, p) = (24, –13)

17. Let S be the set of all $a \in R$ for which the angle between the vectors $\vec{u}=a\left(\log_e b\right)\hat{i}-6\hat{j}+3\hat{k}$ and $\vec{v}=\left(\log_e b\right)\hat{i}+2\hat{j}+2a\left(\log_e b\right)\hat{k},(b>1)$ is acute. Then S is equal to

(A) $\left(-\infty,-\frac{4}{3}\right)$ (B) Φ

(C) $\left(-\frac{4}{3},0\right)$ (D) $\left(\frac{12}{7},\infty\right)$

Official Ans. by NTA (C)

Sol. For angle to be acute

$\vec{u}\cdot\vec{v}>0$

$\Rightarrow a\left(\log_e b\right)^2-12+6a\left(\log_e b\right)>0$

$\forall\ b>1$

let $\log_e b=t\Rightarrow t>0$ as $b>1$

$y=at^2+6at-12\ \&\ y>0,\ \forall\ t>0$

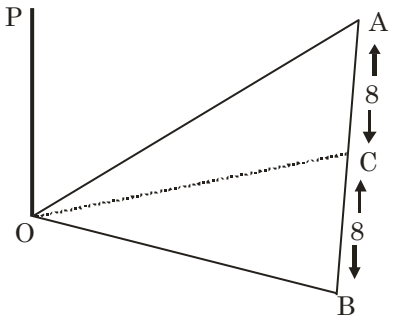
$\Rightarrow a\in\phi$

18. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(A) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (B) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$

(C) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (D) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

Official Ans. by NTA (B)



Sol.

$$\frac{OP}{OA}=\tan 15^\circ$$

$$\Rightarrow OA = OP\cot 15^\circ$$

$$\frac{OP}{OC}=\tan 45^\circ \Rightarrow OP = OC$$

Now, $OP=\sqrt{OA^2-8^2}$

$$\Rightarrow OP^2=(OP)^2\cot^2 15^\circ-64$$

$$\Rightarrow OP^2=\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$$

19. Let A and B be two events such that $P(B|A)=\frac{2}{5}$, $P(A|B)=\frac{1}{7}$ and $P(A\cap B)=\frac{1}{9}$. Consider

$(S1)\ P(A'\cup B)=\frac{5}{6}$,

$(S2)\ P(A'\cap B')=\frac{1}{18}$. Then

(A) Both (S1) and (S2) are true

(B) Both (S1) and (S2) are false

(C) Only (S1) is true

(D) Only (S2) is true

Official Ans. by NTA (A)

Sol. $P(A|B)=\frac{1}{7}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{1}{7}$

$$\Rightarrow P(B)=\frac{7}{9}$$

$$P(B|A)=\frac{2}{5}\Rightarrow\frac{P(A\cap B)}{P(A)}=\frac{2}{5}$$

$$\Rightarrow P(A)=\frac{5}{18}$$

Now, $P(A'\cup B)=1-P(A\cup B)+P(B)$

$$=1-P(A)+P(A\cap B)\ =\frac{5}{6}$$

$$P(A'\cap B')=1-P(A\cup B)$$

$$=1-P(A)-P(B)+P(A\cap B)=\frac{1}{18}$$

$\Rightarrow$ Both (S1) and (S2) are true.

20. Let

p : Ramesh listens to music.

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

(A) $((\sim q)\wedge(r\vee s))\Rightarrow p$

(B) $(q\wedge(r\vee s))\Rightarrow p$

(C) $p\Rightarrow(q\wedge(r\vee s))$

(D) $p\Rightarrow((\sim q)\wedge(r\vee s))$

Official Ans. by NTA (D)

Sol. $p\equiv$ Ramesh listens to music

$\sim q\equiv$ He is in village.

$r\vee s\equiv$ Saturday or sunday

$p\Rightarrow((\sim q)\wedge(r\vee s))$

SECTION-B

1. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta>0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to _____

Official Ans. by NTA (57)

Sol. ${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$$

$$\beta = \frac{12}{5} \text{ or } \beta = -2 \ \therefore \beta = \frac{12}{5}$$

$$d = -\frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$$

$$\therefore 50 - \frac{2d}{\beta^2} = 57$$

2. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3 g is equal to

Official Ans. by NTA (17)

Sol. ${}^bC_3 \times {}^gC_2 = 168$

b(b – 1)(b – 2) (g)(g – 1) = 8 × 7 × 6 × 3 × 2

b + 3 g = 17

3. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ meet at the point $R(\sqrt{2}, 2\sqrt{2} - 2)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to

Official Ans. by NTA (13)

Sol. Ellipse is

$$\frac{x^2}{2} + \frac{y^2}{4} = 1; \ e = \frac{1}{\sqrt{2}}; \ S \equiv (0, -\sqrt{2})$$

Chord of contact is

$$\frac{x}{\sqrt{2}} + \frac{(2\sqrt{2} - 2)y}{4} = 1$$

$$\Rightarrow \frac{x}{\sqrt{2}} = 1 - \frac{(\sqrt{2} - 1)y}{2} \text{ solving with ellipse}$$

$$\Rightarrow y = 0, \sqrt{2} \ \therefore x = \sqrt{2}, 1$$

$$P \equiv (1, \sqrt{2}) \ Q \equiv (\sqrt{2}, 0)$$

$$\therefore (SP)^2 + (SQ)^2 = 13$$

4. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \ldots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50})$ is equal to $2^n.m$, where m is odd, then n + m is equal to _____

Official Ans. by NTA (99)

Sol. $1 + (1 + 2^{49})(2^{49} - 1) = 2^{98}$

$m = 1, \ n = 98$

m + n = 99

5. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x - 5)^2 + y^2 = r$, then 17r is equal to

Official Ans. by NTA (9)

Sol. $y^2 = -\frac{x}{2}$

$$y = mx - \frac{1}{8m}$$

this tangent pass through (2, 0)

$m = \pm\frac{1}{4}$ i.e., one tangent is x – 4y – 2 = 0

17r = 9

6. If $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then m.n is equal to _____

Official Ans. by NTA (12)

Sol. $\frac{6}{3^{12}} + 10\left(\frac{1}{3^{11}} + \frac{2}{3^{10}} + \frac{2^2}{3^9} + \frac{2^3}{3^8} + \ldots + \frac{2^{10}}{3}\right)$

$$\frac{6}{3^{12}} + \frac{10}{3^{11}}\left(\frac{6^{11} - 1}{6 - 1}\right)$$

$= 2^{12} \cdot 1; \ m.n = 12$

7. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$. Then the number of elements in the set

$A=\left\{\theta\in S:\tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$

is _____

Official Ans. by NTA (5)

Sol. $\tan\theta+\sqrt{5}\tan2\theta\tan\theta=\sqrt{5}-\tan2\theta$

$\tan3\theta=\sqrt{5}$

$\theta=\frac{n\pi}{3}+\frac{\alpha}{3};\quad\tan\alpha=\sqrt{5}$

Five solution

8. Let z = a + ib, b ≠ 0 be complex numbers satisfying $z^2=\overline{z}\cdot2^{1-|z|}$. Then the least value of n ∈ N, such that $z^n=(z+1)^n$, is equal to _____

Official Ans. by NTA (6)

Sol. $|z^2|=|\overline{z}|\cdot2^{1-|z|}\Rightarrow|z|=1$

$z^2=\overline{z}\Rightarrow z^3=1\therefore z=\omega\text{ or }\omega^2$

$\omega^n=(1+\omega)^n=(-\omega^2)^n$

Least natural value of n is 6.

9. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then 100 σ^2 is equal to

Official Ans. by NTA (56)

Sol.

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$\sigma^2=\sum X^2P(X)-\left(\sum XP(X)\right)^2=\frac{56}{100}$

$100\ \sigma^2=56$

10. The value of the integral $\int_0^{\frac{\pi}{2}}60\frac{\sin(6x)}{\sin x}dx$ is equal to

Official Ans. by NTA (104)

Sol.

$I=60\int_0^{\pi/2}\left(\frac{\sin6x-\sin4x}{\sin x}+\frac{\sin4x-\sin2x}{\sin x}+\frac{\sin2x}{\sin x}\right)dx$

$I=60\int_0^{\pi/2}(2\cos5x+2\cos3x+2\cos x)dx$

$I=60\left(\frac{2}{5}\sin5x+\frac{2}{3}\sin3x+2\sin x\right)\Big|_0^{\pi/2}=104$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements : One is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Time period of oscillation of a liquid drop depends on surface tension (S), if density of the liquid is p and radius of the drop is r, then $T = k\sqrt{pr^3/s^{3/2}}$ is dimensionally correct, where K is dimensionless.

Reason (R) : Using dimensional analysis we get R.H.S. having different dimension than that of time period.

In the light of above statements, choose the correct answer from the options given below.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Official Ans. by NTA (D)

Sol. $T = k\sqrt{\dfrac{\rho r^3}{s^{3/2}}}$

Dimensions of RHS $= \dfrac{\left[M^{1/2}L^{-3/2}\right]\left[L^{3/2}\right]}{\left[MT^{-2}\right]^{3/4}} = M^{1/8}\,L^{0}\,T^{3/2}$

Dimensions of L.H.S $\neq$ Dimensions of R.H.S

$\therefore$ option (D)

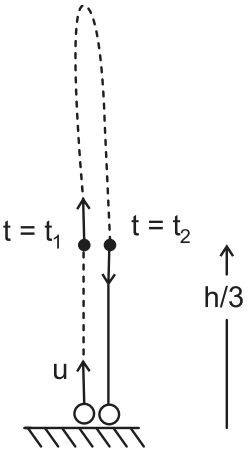
2. A ball is thrown up vertically with a certain velocity so that, it reaches a maximum height h. Find the ratio of the times in which it is at height $\dfrac{h}{3}$ while going up and coming down respectively.

(A) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (B) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(C) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ (D) $\dfrac{1}{3}$

Official Ans. by NTA (B)

Sol.



Max. Height $= h = \dfrac{u^2}{2g}$

$\Rightarrow u = \sqrt{2gh}$

$S = ut + \dfrac{1}{2}at^2$

$\dfrac{h}{3} = \sqrt{2gh}\,t + \dfrac{1}{2}(-g)t^2$

$\dfrac{gt^2}{2} - \sqrt{2gh}\,t + \dfrac{h}{3} = 0$ (Roots are t_1 & t_2)

$\dfrac{t_2}{t_1} = \dfrac{\sqrt{2gh} + \sqrt{2gh - 4\times\frac{g}{2}\times\frac{h}{3}}}{\sqrt{2gh} - \sqrt{2gh - 4\times\frac{g}{2}\times\frac{h}{3}}} = \dfrac{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}} = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

3. If $t=\sqrt{x}+4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is:

(A) 4 (B) Zero

(C) 8 (D) 16

Official Ans. by NTA (B)

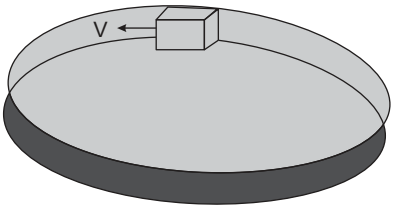
Sol. $t=\sqrt{x}+4$

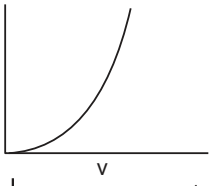
$\Rightarrow x=(t-4)^2=t^2-8t+16$

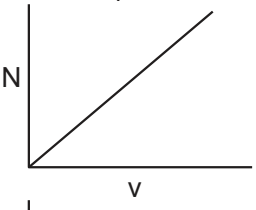
$\Rightarrow \frac{dx}{dt}=2t-8$

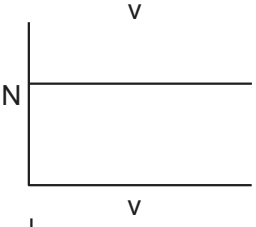
$\Rightarrow \left.\frac{dx}{dt}\right|_{t=4}=2\times4-8=0$

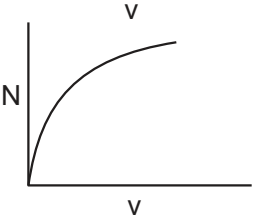
4. A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed v. Which of the following curve represents the correct relation between the normal reaction on the block by the wall (N) and speed of the block (v) ?



(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. $N=\frac{mv^2}{r}$

Curve is parabola

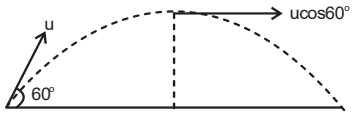
$Y=kx^2$

5. A ball is projected with kinetic energy E, at an angle of 60^0 to the horizontal. The kinetic energy of this ball at the highest point of its flight will become :

(A) Zero (B) $\frac{E}{2}$

(C) $\frac{E}{4}$ (D) E

Official Ans. by NTA (C)

Sol. 

$E=\frac{1}{2}mu^2$

At Highest point, Velocity $V = u\cos60^\circ = \frac{u}{2}$

$\therefore$ K.E at topmost point $= \frac{1}{2}m\left(\frac{u}{2}\right)^2=\frac{E}{4}$

6. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector :

(A) $\hat{i}-2\hat{j}+\hat{k}$ (B) $-3\hat{i}-2\hat{j}+\hat{k}$

(C) $-2\hat{i}+2\hat{k}$ (D) $-2\hat{i}-\hat{j}+2\hat{k}$

Official Ans. by NTA (A)

Sol. $\overrightarrow{r_{com}}=\frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}=\frac{1(\hat{i}+2\hat{j}+\hat{k})+3(-3\hat{i}-2\hat{j}+\hat{k})}{1+3}$

$=-2\hat{i}-\hat{j}+\hat{k}$

$|2\hat{i}-\hat{j}+\hat{k}|=\sqrt{(2)^2+(1)^2+(1)^2}=\sqrt{6}$

7. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Clothes containing oil or grease stains cannot be cleaned by water wash.

Reason (R) : Because the angle of contact between the oil/ grease and water is obtuse. In the light of the above statements, choose the correct answer from the option given below.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

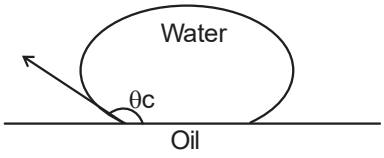
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is true but (R) is true

Official Ans. by NTA (A)

Sol.



$\theta c > 90^{o}$

For water oil interface

8. If the length of a wire is made double and radius is halved of its respective values. Then, the Young's modules of the material of the wire will :

(A) Remains same

(B) Become 8 times its initial value

(C) Become $\frac{1^{th}}{4}$ of its initial value

(D) Become 4 times its initial value

Official Ans. by NTA (A)

Sol. Y depends on material of wire

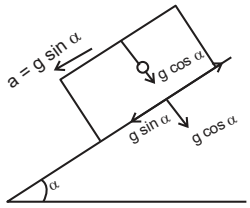
9. The time period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α, is given by :

(A) $2\pi\sqrt{L/(g\cos\alpha)}$ (B) $2\pi\sqrt{L/(g\sin\alpha)}$

(C) $2\pi\sqrt{L/g}$ (D) $2\pi\sqrt{L/(g\tan\alpha)}$

Official Ans. by NTA (A)

Sol. $g_{eff} = g\cos\alpha$

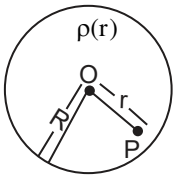


$$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

10. A spherically symmetric charge distribution is considered with charge density varying as

$$\rho(r) = \begin{cases} \rho_0\left(\frac{3}{4} - \frac{r}{R}\right) & \text{for } r \le R \\ \text{Zero} & \text{for } r > R \end{cases}$$

Where, r(r < R) is the distance from the centre O (as shown in figure). The electric field at point P will be :

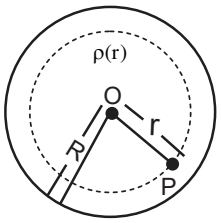


(A) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$ (B) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$

(C) $\frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$ (D) $\frac{\rho_0 r}{5\varepsilon_0}\left(1 - \frac{r}{R}\right)$

Official Ans. by NTA (C)

Sol. By Gauss law



$$\oint \vec{E}.d\vec{s} = \frac{Q_{in}}{\varepsilon_o}$$

$$E.4\pi r^2 = \frac{\int_0^r \rho_o\left(\frac{3}{4} - \frac{r}{R}\right)4\pi r^2 dr}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{\rho_o 4\pi}{\varepsilon_o}\left(\frac{3}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$Er^2 = \frac{\rho_o r^3}{4\varepsilon_o}\left\{1 - \frac{r}{R}\right\}$$

$$E = \frac{\rho_o r}{4\varepsilon_o}\left\{1 - \frac{r}{R}\right\}$$

11. Given below are two statements.

Statement I : Electric potential is constant within and at the surface of each conductor.

Statement II : Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

In the light of the above statements, choose the most appropriate answer from the options give below.

(A) Both statement I and statement II are correct
(B) Both statement I and statement II are incorrect
(C) Statement I is correct but statement II is incorrect
(D) Statement I is incorrect but and statement II is correct

Official Ans. by NTA (A)

Sol. (Properties of conductor)

Statement – I, true as body of conductor acts as equipotential surface.

Statement – 2 True, as conductor is equipotential. Tangential component of electric field should be zero. Therefore electric field should be perpendicular to surface.

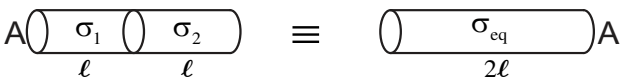
12. Two metallic wires of identical dimensions are connected is series. If σ_1 and σ_2 are the conductivities of the these wires respectively, the effective conductivity of the combination is :

(A) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ (B) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

(C) $\frac{\sigma_1+\sigma_2}{2\sigma_1\sigma_2}$ (D) $\frac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$

Official Ans. by NTA (B)

Sol.



Let length of wire be ' ℓ '

Area of wire as 'A'

For equivalent wire length = 2ℓ & area will be A

Thermal resistance

$R_{eq} = R_1 + R_2$

$$\frac{2\ell}{\sigma_{eq}A} = \frac{\ell}{\sigma_1 A} + \frac{\ell}{\sigma_1 A}$$

$$\frac{2\ell}{\sigma_{eq}} = \frac{\ell}{\sigma_1} + \frac{\ell}{\sigma_2} \Rightarrow \sigma_{eq} = \frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$$

13. An alternating emf E = 440 sin 100πt is applited to a circuit containing an inductance of $\frac{\sqrt{2}}{\pi}$ H. If an a.c. ammeter is connected in the circuit, its reading will be :

(A) 4.4 A (B) 1.55 A
(C) 2.2 A (D) 3.11 A

Official Ans. by NTA (C)

Sol. $E = 440\ \text{Sin}100\pi t$, $L = \frac{\sqrt{2}}{\pi}H$

$$X_L = \omega L = 100\pi\frac{\sqrt{2}}{\pi} = 100\sqrt{2}\ \Omega$$

$$\text{Peak current } I_0 = \frac{E_0}{X_L} = \frac{440}{100\sqrt{2}} = 2.2\sqrt{2}\ \text{A}$$

AC ammeter reads RMS value therefore reading will be I_{rms}

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 2.2\ \text{A}$$

14. A coil of inductance 1 H and resistance 100 Ω is connected to a battery of 6 V. Determine approximately :

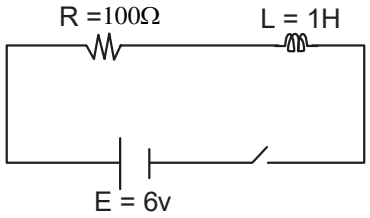
(a) The time elapsed before the current acquires half of its steady – state value

(b) The energy stored in the magnetic field associated with the coil at an instant 15 ms after the circuit is switched on. (Given In2 = 0.693, $e^{-3/2} = 0.25$)

(A) t = 10 ms; U = 2 mJ
(B) t = 10 ms; U = 1 mJ
(C) t = 7 ms; U = 1 mJ
(D) t = 7 ms; U = 2 mJ

Official Ans. by NTA (C)

Sol. Given circuit is R – L growth circuit



$i = \frac{E}{R}(1-e^{-t/\tau})$

$i=\frac{E}{2R}=\frac{E}{R}(1-e^{-t/\tau})$

Solving $t = \tau \ln 2$

$t=\frac{1}{R}\ln 2 = \frac{1}{100}0.693 = 0.00693$

$= 7$ ms

$i(15ms)=\frac{E}{R}(1-e^{-\frac{15}{10}})$

$i=\frac{6}{100}(1-1/4) = \frac{3}{4}\times\frac{6}{100}$

$U = \frac{1}{2}LI^2,$

by solving we get U = 1 mJ.

15. Match List – I with List – II

List – I	List - II
(a) UV rays	(i) Diagnostic tool in medicine
(b) X-rays	(ii) Water purification
(c) Microwave	(iii) Communication, Radar
(d) Infrared wave	(iv) Improving visibility in foggy days

Choose the correct answer from the options given below :

(A) (a)–(iii), (b)-(ii), (c)-(i), (d)-(iv)

(B) (a)–(ii), (b)-(i), (c)-(iii), (d)-(iv)

(C) (a)–(ii), (b)-(iv), (c)-(iii), (d)-(i)

(D) (a)–(iii), (b)-(i), (c)-(ii), (d)-(iv)

Official Ans. by NTA (B)

Sol. (a) uv rays – used for water purification

(b) x-rays used for diagnosing fracture

(c) Microwaves are used for mobile and radar communication

(d) Infrared waves show less scattering therefore used in foggy days

(a – ii), (b – i), (c – iii), (d – iv)

16. The kinetic energy of emitted electron is E when the light incident on the metal has wavelength λ. To double the kinetic energy, the incident light must have wavelength :

(A) $\frac{hc}{E\lambda - hc}$ (B) $\frac{hc\lambda}{E\lambda + hc}$

(C) $\frac{h\lambda}{E\lambda + hc}$ (D) $\frac{hc\lambda}{E\lambda - hc}$

Official Ans. by NTA (B)

Sol. $E = \frac{hc}{\lambda} - \phi \quad - (i)$

$2E = \frac{hc}{\lambda'} - \phi \quad - (ii)$

(ii) – (i)

$E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$

$\Rightarrow \lambda' = \frac{hc\lambda}{E\lambda + hc}$

17. Find the ratio of energies of photons produced due to transition of an election of hydrogen atom from its(i) second permitted energy level to the first level, and (ii) the highest permitted energy level to the first permitted level.

(A) 3 : 4 (B) 4 : 3

(C) 1 : 4 (D) 4 : 1

Official Ans. by NTA (A)

Sol. $E_n = \frac{-13.6}{n^2}ev$

$\Rightarrow \frac{E_2 - E_1}{E_\infty - E_1} = \frac{13.6(1-\frac{1}{4})}{13.6} = \frac{3}{4}$

18. Find the modulation index of an AM wave having 8 V variation where maximum amplitude of the AM wave is 9 V.

(A) 0.8 (B) 0.5

(C) 0.2 (D) 0.1

Official Ans. by NTA (A)

Sol. Modulation index: $m = \frac{A_m}{A_c}$

Given $2A_m = 8$

$A_m + A_c = 9 \Rightarrow A_c = 5$

$\therefore m = \frac{4}{5} = 0.8$

19. A travelling microscope has 20 divisions per cm on the main scale while its Vernier scale has total 50 divisions and 25 Vernier scale divisions are equal to 24 main scale divisions, what is the least count of the travelling microscope ?

(A) 0.001 cm (B) 0.002 mm

(C) 0.002 cm (D) 0.005 cm

Official Ans. by NTA (C)

Sol. $1\ \text{MSD} = \frac{1}{20}\text{cm}$

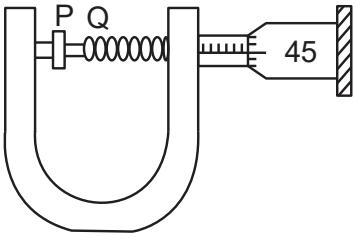
$1\ \text{VSD} = \frac{24}{25}\ \text{MSD} = \frac{24}{25} \times \frac{1}{20}\text{cm}$

$\therefore \text{Least count} = \frac{1}{20}\left(1 - \frac{24}{25}\right)\text{cm}$

$= \frac{1}{20} \times \frac{1}{25} = \frac{1}{500}\text{cm}$

$= 0.002$ cm

20. In an experiment to find out the diameter of wire using screw gauge, the following observation were noted :



(a) Screw moves 0.5 mm on main scale in one complete rotation

(b) Total divisions on circular scale = 50

(c) Main scale reading is 2.5 mm

(d) 45^{th} division of circular scale is in the pitch line

(e) Instrument has 0.03 mm negative error

Then the diameter of wire is :

(A) 2.92 mm (B) 2.54 mm

(C) 2.98 mm (D) 3.45 mm

Official Ans. by NTA (C)

Sol. MSR = 2.5 mm

$\text{CSR} = 45 \times \frac{0.5}{50}\text{mm}$

$= 0.45$ mm

Diameter reading = MSR + CSR – zero error

$= 2.5 + 0.45 - (-0.03)$

$= 2.98$ mm

SECTION-B

1. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be _____degree.

Official Ans. by NTA (15)

Sol. $R_{max} = \frac{u^2 \sin 2(45^\circ)}{g} = \frac{u^2}{g}$

$\frac{R}{2} = \frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \frac{1}{2}$

$2\theta = 30^\circ, 150^\circ$

$\theta = 15^\circ, 75^\circ$

Ans. 15, 75

2. If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth αh ($h << R_e$) from the earth surface. The value of α will be _____.

(use $R_e = 6400$ km)

Official Ans. by NTA (2)

Sol. $g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

$\frac{2h}{R} = \frac{d}{R}$

$\alpha h = d$

$\alpha = 2$

3. The pressure P_1 and density d_1 of diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes suddenly to $P_2(>P_1)$ and d_2 respectively during an adiabatic process. The temperature of the gas increases and becomes _____ times of its initial temperature.

(given $\frac{d_2}{d_1} = 32$)

Official Ans. by NTA (4)

Sol. $PV^{\gamma} = \text{const}$ $\qquad d = \frac{m}{v}$

$p\left(\frac{m}{d}\right)^{\gamma} = \text{const}$

$\frac{p}{d^{\gamma}} = \text{const}$ $\qquad \frac{d_2}{d_1} = 32$

$\frac{p_1}{p_2} = \left(\frac{d_1}{d_2}\right)^{\gamma} = \left(\frac{1}{32}\right)^{7/5} = \frac{1}{128}$

$\frac{T_1}{T_2} = \frac{P_1V_1}{P_2V_2} = \frac{1}{128}32 = \frac{1}{4}$

4. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. The molecular specific heat of mixture at constant volume is $\frac{\alpha^2}{4}R$ J/mol K; then the value of α will be _____. (Assume that the given diatomic gas has no vibrational mode.)

Official Ans. by NTA (3)

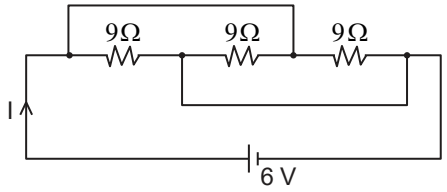
Sol. $C_V / \text{mix} = \frac{n_1Cv_1 + n_2Cv_2}{n_1 + n_2}$

$= \frac{1.\frac{3R}{2} + 3.\frac{5R}{2}}{1+3}$

$= \frac{9R}{4} = \frac{\alpha^2}{4}R$

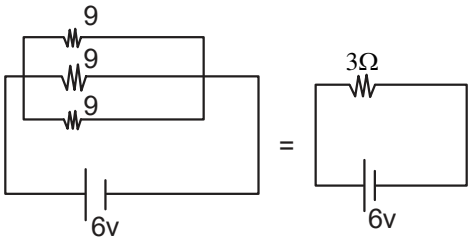
$\alpha = 3$

5. The current I flowing through the given circuit will be _____ A.



Official Ans. by NTA (2)

Sol. Equivalent circuit



$I = \frac{6}{3} = 2$ A

6. A closely wounded circular coil of radius 5 cm produces a magnetic field of 37.68 x 10^{-4} T at its center. The current through the coil is _____ A. [Given, number of turns in the coil is 100 and $\pi = 3.14$]

Official Ans. by NTA (3)

Sol. $B_{centre} = \frac{N\mu_0 I}{2R}$

$37.68 \times 10^{-4} = \frac{100 \times 4\pi \times 10^{-7} \times I}{2 \times 5 \times 10^{-2}}$

$I = 3A$

7. Two light beams of intensities 4I and 9I interfere on a screen. The phase difference between these beams on the screen at point A is zero and at point B is π. The difference of resultant intensities, at the point A and B, will be _____ I.

Official Ans. by NTA (24)

Sol. $I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

I_{max} for $\phi = 0$ & I_{min} for $\phi = \pi$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{9I} + \sqrt{4I}\right)^2 = 25\,I$

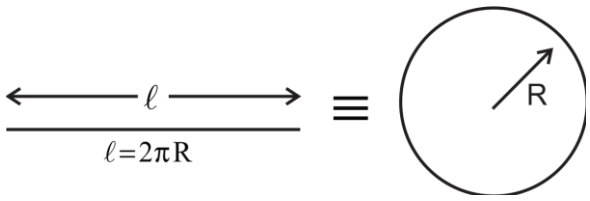
$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{9I} - \sqrt{4I}\right)^2 = I$

$I_{max} - I_{min} = 25\,I - I = 24\,I$

8. A wire of length 314 cm carrying current of 14 A is bent to form a circle. The magnetic moment of the coil is _____ A-m^2. [Given $\pi = 3.14$]

Official Ans. by NTA (11)

Sol.



$\frac{314}{100} = 2\pi R$ $\qquad$ R = 0.5 m

Magnetic Moment = IA

$= 14 \times \pi R^2$

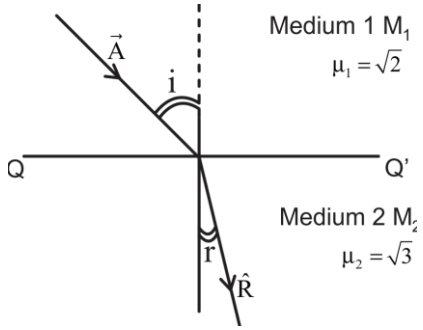
$= 14 \times (3.14) \times \frac{1}{4}$

$= 10.99 \approx 11.00$

9. The X-Y plane be taken as the boundary between two transparent media M_1 and M_2. M_1 in $Z \geq 0$ has a refractive index of $\sqrt{2}$ and M_2 with $Z < 0$ has a refractive index of $\sqrt{3}$. A ray of light travelling in M_1 along the direction given by the vector $\vec{A} = 4\sqrt{3}\hat{i} - 3\sqrt{3}\hat{j} - 5\hat{k}$, is incident on the plane of separation. The value of difference between the angle of incident in M_1 and the angle of refraction in M_2 will be _____ degree.

Official Ans. by NTA (15)

Sol. $\vec{A} = 4\sqrt{3}\hat{i} - 3\sqrt{3}\hat{j} - 5\hat{k}$



$\mu_1 \sin i = \mu_2 \sin r$

As incident vector A makes i angle with normal z-axis & refracted vector R makes r angle with normal z – axis with help of direction cosine

$$i = \cos^{-1}\left(\frac{A_z}{A}\right) = \cos^{-1}\left(\frac{5}{\sqrt{(4\sqrt{3})^2 + (3\sqrt{3})^2 + 5^2}}\right)$$

$$= \cos^{-1}\left(\frac{5}{10}\right) \Rightarrow i = 60^\circ$$

$$\sqrt{2}\sin 60 = \sqrt{3} \times \sin r$$

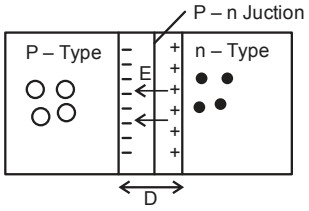
$$r = 45^\circ$$

Difference between i & r = 60 – 45 = 15

10. If the potential barrier across a p-n junction is 0.6 V. Then the electric field intensity, in the depletion region having the width of 6×10^{-6}m, will be _____ $\times 10^5$ N/C.

Official Ans. by NTA (1)

Sol.



$$E = \frac{V}{d} = \frac{\text{Potential barrier Across Junction}}{\text{width of Depletion layer}}$$

$$= \frac{0.6V}{6 \times 10^{-6} m} = 1 \times 10^5 V/m$$

$= 1 \times 10^5$ N/C

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Which of the following pair of molecules contain odd electron molecule and an expanded octet molecule?

(A) BCl_3 and SF_6 (B) NO and H_2SO_4

(C) SF_6 and H_2SO_4 (D) BCl_3 and NO

Official Ans. by NTA (B)

Sol. **(A)** $BCl_3 \rightarrow$ Even Electron molecule

$SF_6 \rightarrow$ Expanded octet molecule

(B) NO $\rightarrow$ Odd Electron molecule

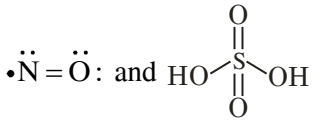
$H_2SO_4 \rightarrow$ Expanded octet.

(C) $SF_6 \rightarrow$ Even Electron molecule

$H_2SO_4 \rightarrow$ Expanded octet.

(D) $BCl_3 \rightarrow$ Even Electron molecule

NO $\rightarrow$ Odd Electron molecule



$S \rightarrow 12e^-$ in outer orbit.

2. $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$

20 g 5 g

Consider the above reaction, the limiting reagent of the reaction and number of moles of NH_3 formed respectively are:

(A) H_2, 1.42 moles (B) H_2, 0.71 moles

(C) N_2, 1.42 moles (D) N_2, 0.71 moles

Official Ans. by NTA (C)

Sol.

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

$W_2 =$ 20g 5g.

$$n = \frac{20}{28} \quad \frac{5}{2}$$

Stoichiometric Amount:

$$N_2 \rightarrow \frac{20/28}{1} = \frac{20}{28} \qquad H_2 \rightarrow \frac{5/2}{3} = \frac{5}{6}$$

$\therefore$ N_2 is the Limiting Reagent.

$$\therefore \quad n(NH_3) = 2 \times n(N_2) = 2 \times \frac{20}{28}$$

$$= 1.42$$

3. 100 mL of 5% (w/v) solution of NaCl in water was prepared in 250 mL beaker. Albumin from the egg was poured into NaCl solution and stirred well. This resulted in a/ an :

(A) Lyophilic sol (B) Lyophobic sol

(C) Emulsion (D) Precipitate

Official Ans. by NTA (A)

Sol. Standard method for the preparation of lyophilic sol. (Discussed in lab Manual)

4. The first ionization enthalpy of Na, Mg and Si, respectively, are: 496, 737 and 786 kJ mol^{-1}. The first ionization enthalpy (kJ mol^{-1}) of Al is:

(A) 487 (B) 768

(C) 577 (D) 856

Official Ans. by NTA (C)

Sol. I. E : Na < Al < Mg < Si

$\therefore$ 496 < IE (Al) < 737

Option (C), matches the condition.

i.e $IE(Al) = 577\ kJmol^{-1}$

5. In metallurgy the term "gangue" is used for:

(A) Contamination of undesired earthy materials.

(B) Contamination of metals, other than desired metal

(C) Minerals which are naturally occuring in pure form

(D) Magnetic impurities in an ore.

Official Ans. by NTA (A)

Sol. Earthy and undesired materials present in the ore, other then the desired metal, is known as gangue.

6. The reaction of zinc with excess of aqueous alkali, evolves hydrogen gas and gives :

(A) $Zn(OH)_2$ (B) ZnO

(C) $[Zn(OH)_4]^{2-}$ (D) $[ZnO_2]^{2-}$

Official Ans. by NTA (D)

Sol. Zinc dissolves in excess of aqueous alkali

$$Zn + 2OH^- + 2H_2O \rightarrow \underset{\text{Tetrahydroxozincate(II) ion}}{[Zn(OH)_4]^{2-}} + H_2\uparrow$$

However, this reaction in NCERT is given as

$$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2\uparrow$$

ZnO_2^{2-} is anhydrous form of $[Zn(OH)_4]^{2-}$

So in aqueous medium best answer of this question is $[Zn(OH)_4]^{2-}$

7. Lithium nitrate and sodium nitrate, when heated separately, respectively, give :

(A) $LiNO_2$ and $NaNO_2$

(B) Li_2O and Na_2O

(C) Li_2O and $NaNO_2$

(D) $LiNO_2$ and Na_2O

Official Ans. by NTA (C)

Sol. Li_2O, $NaNO_2$

As per NCERT Lithium nitrate when heated gives lithium oxide, Li_2O, whereas other alkali metal nitrates decompose to give the corresponding nitrite.

$$4LiNO_3 \longrightarrow 2Li_2O + 4NO_2 + O_2$$

$$2NaNO_3 \longrightarrow 2NaNO_2 + O_2$$

However, the decomposition product of $NaNO_3$ are temperature dependent process as shown in the below reaction.

$$NaNO_3 \xrightarrow[500^\circ C]{\Delta} NaNO_2(s) + \frac{1}{2}O_2(g)$$

$$\downarrow \Delta\ 800^\circ C$$

$$Na_2O(s) + N_2(g) + O_2(g)$$

As temperature is not mentioned, we can go by

Ans. (C)

8. Number of lone pairs of electrons in the central atom of SCl_2, O_3, ClF_3 and SF_6, respectively, are :

(A) 0, 1, 2 and 2

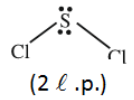
(B) 2, 1, 2 and 0

(C) 1, 2, 2 and 0

(D) 2, 1, 2 and 0

Official Ans. by NTA (B)

Sol.



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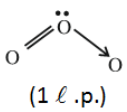

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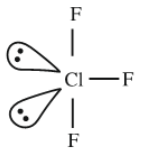

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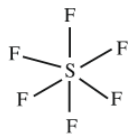


(0 ℓ .p.)

9. In following pairs, the one in which both transition metal ions are colourless is :

(A) Sc^{3+}, Zn^{2+}

(B) Ti^{4+}, Cu^{2+}

(C) V^{2+}, Ti^{3+}

(D) Zn^{2+}, Mn^{2+}

Official Ans. by NTA (A)

Sol. (A) Sc^{3+}, Zn^{2+} (B) Ti^{4+}, Cu^{2+}

$3d^0$ $3d^{10}$ $3d^0$ $3d^9$

(C) V^{2+}, Ti^{3+} (D) Zn^{2+}, Mn^{2+}

$3d^3$ $3d^1$ $3d^{10}$ $3d^5$

No d-d transitions in ions with d^o & d^{10} configuration. Therefore they are colourless.

10. In neutral or faintly alkaline medium, $KMnO_4$ being a powerful oxidant can oxidize, thiosulphate almost quantitatively, to sulphate. In this reaction overall change in oxidation state of manganese will be :

(A) 5 (B) 1 (C) 0 (D) 3

Official Ans. by NTA (D)

Sol. $$8\overset{+7}{Mn}O_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8\overset{+4}{Mn}O_2 + 6SO_4^{2-} + 2OH^-$$

Change in oxidation state of Mn is from +7 to +4 which is 3.

11. Which among the following pairs has only herbicides ?

(A) Aldrin and Dieldrin

(B) Sodium chlorate and Aldrin

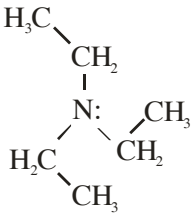
(C) Sodium arsinate and Dieldrin

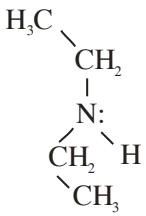
(D) Sodium chlorate and sodium arsinite.

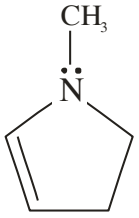
Official Ans. by NTA (D)

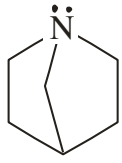
Sol. Both sodium chlorate and sodium arsenite behave as herbicide.

12. Which among the following is the strongest Bronsted base ?

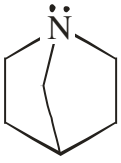
(A) 

(B) 

(C) 

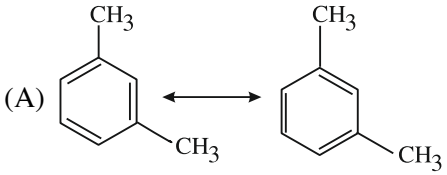
(D) 

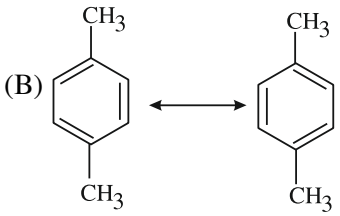
Official Ans. by NTA (D)

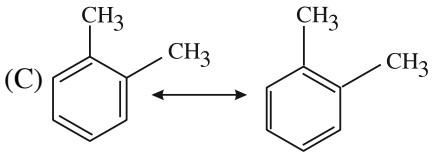
Sol. 

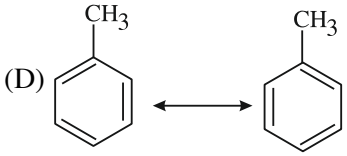
It is most basic because there is no amine inversion.

13. Which among the following pairs of the structures will give different products on ozonolysis? (Consider the double bonds in the structures are rigid and not delocalized.)

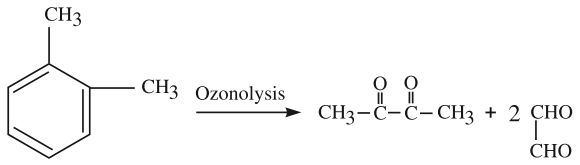
(A) 

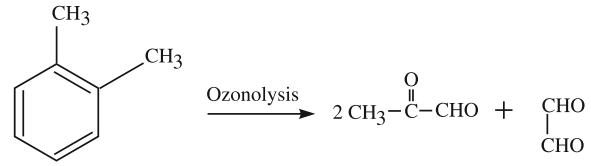
(B) 

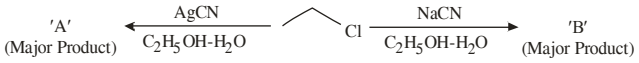
(C) 

(D) 

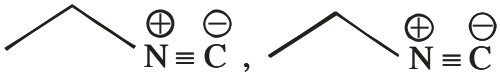
Official Ans. by NTA (C)

Sol. 





14. Considering the above reactions, the compound 'A' and compound 'B' respectively are :

(A) 

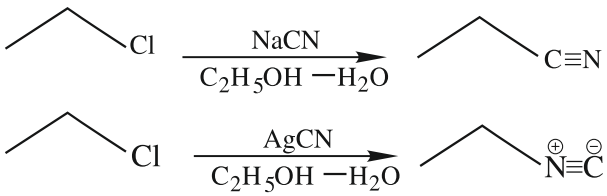
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

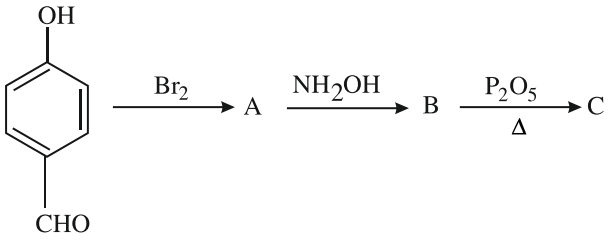
Sol.



In NaCN; carbon is more nucleophilic atom.

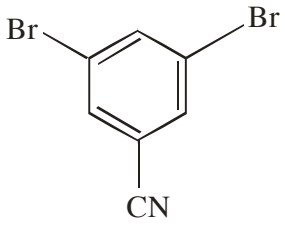
Whereas in AgCN; Ag – C has covalent bond.

15.

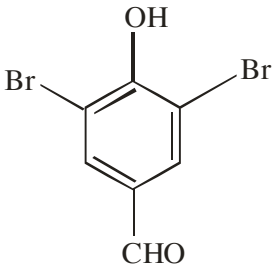


Consider the above reaction sequence, the Product 'C' is :

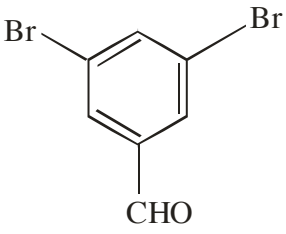
(A)



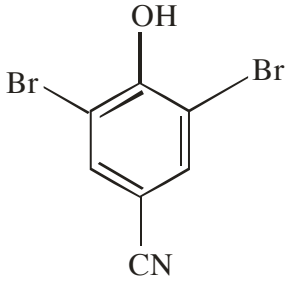
(B)



(C)

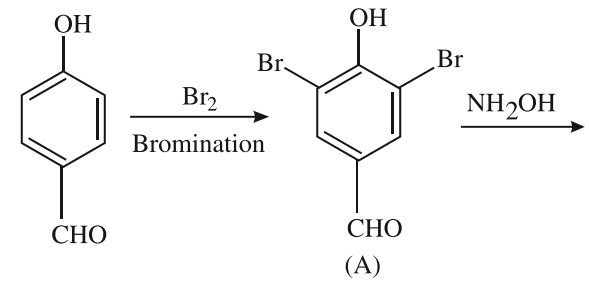


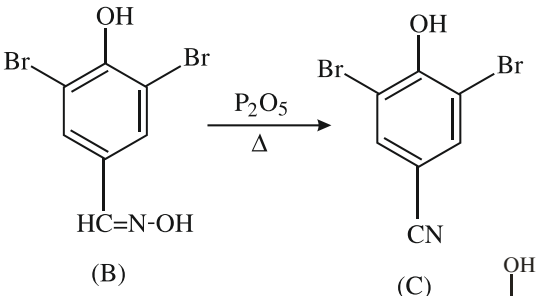
(D)



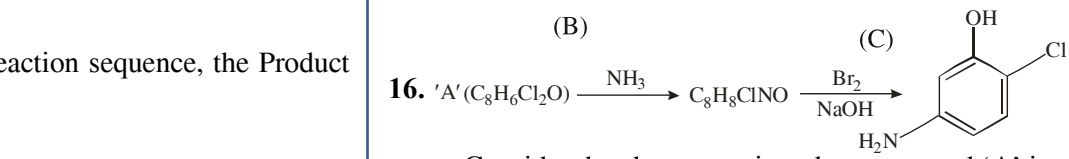
Official Ans. by NTA (D)

Sol.



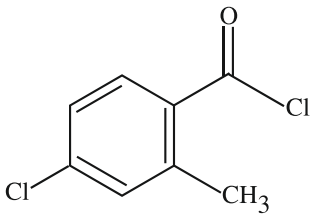


16.

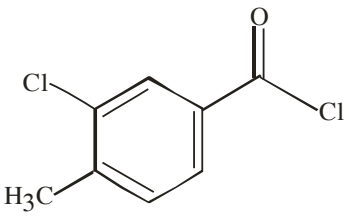


Consider the above reaction, the compound 'A' is :

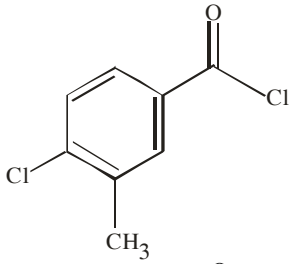
(A)



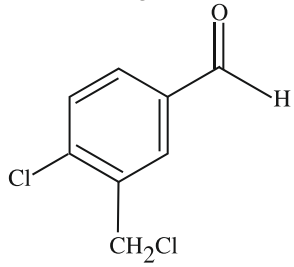
(B)



(C)

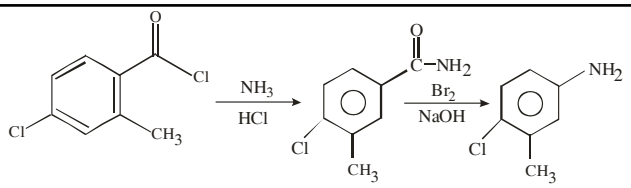


(D)

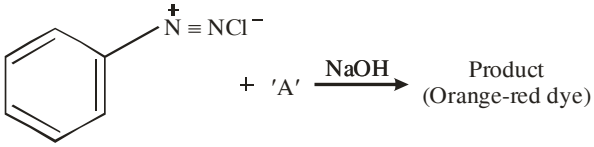


Official Ans. by NTA (C)

Sol.

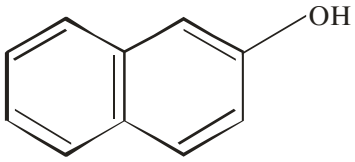


17.

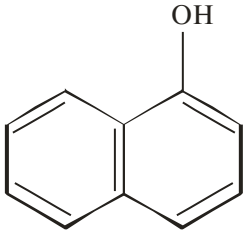


Which among the following represent reagent 'A'?

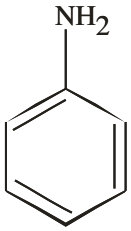
(A)



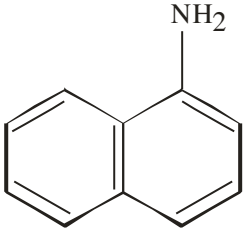
(B)



(C)

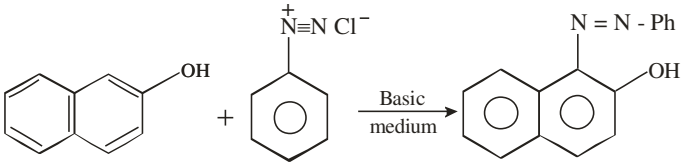


(D)

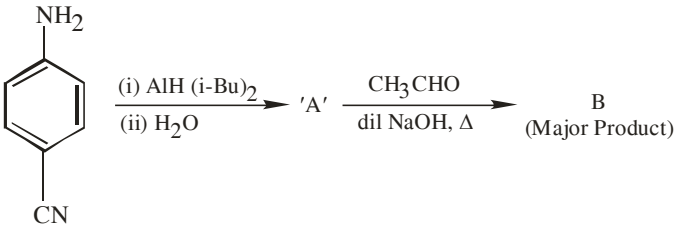


Official Ans. by NTA (A)

Sol.

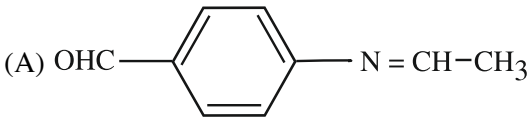


18. Consider the following reaction sequence :

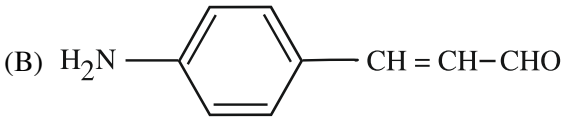


The product 'B' is :

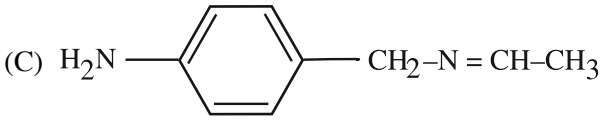
(A)



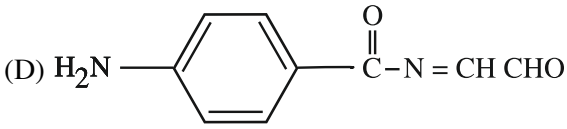
(B)



(C)

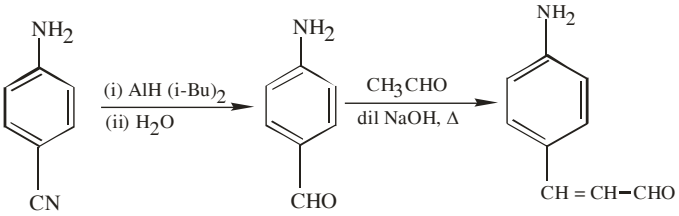


(D)



Official Ans. by NTA (B)

Sol.



Cross aldol condensation

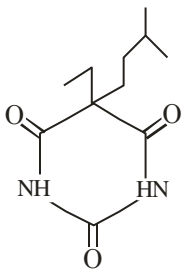
19. Which of the following compounds is an example of hypnotic drug ?

(A) Seldane (B) Amytal

(C) Aspartame (D) Prontosil

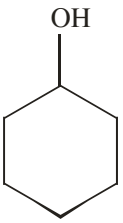
Official Ans. by NTA (B)

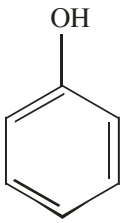
Sol. Amytal is hypnotic drug used to treat sleeping disorder.

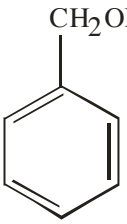


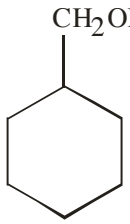
Amytal

20. A compound 'X' is acidic and it is soluble in NaOH solution, but insoluble in $NaHCO_3$ solution. Compound 'X' also gives violet colour with neutral $FeCl_3$ solution. The compound 'X' is :

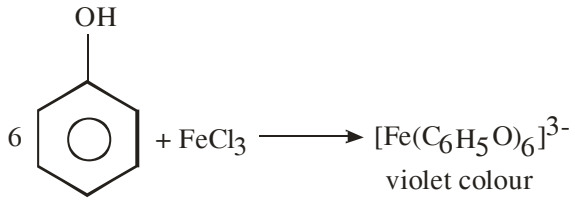
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. 

SECTION-B

1. Resistance of a conductivity cell (cell constant 129 m^{-1}) filled with 74.5 ppm solution of KCl is 100 Ω (labelled as solution 1). When the same cell is filled with KCl solution of 149 ppm, the resistance is 50 Ω (labelled as solution 2). The ratio of molar conductivity of solution 1 and solution 2 is i.e. $\frac{\wedge_1}{\wedge_2} = x \times 10^{-3}$. The value of x is _____.

(Nearest integer)

Given, molar mass of KCl is 74.5 g mol^{-1}

Official Ans. by NTA (1000)

Sol. $\frac{\ell}{A} = 129 m^{-1}$

KCl solution 1 :

74.5 ppm, $R_1 = 100\,\Omega$

KCl solution 2 :

149 ppm, $R_2 = 50\,\Omega$

149 ppm, $R_2 = 50\,\Omega$

Here, $\frac{ppm_1}{ppm_2} = \frac{M_1}{M_2}\left(= \frac{W_1/M_0}{V} \times \frac{V}{W_2/M_0}\right)$

$$\frac{\wedge_1}{\wedge_2} = \frac{\kappa_1 \times \frac{1000}{M_1}}{\kappa_2 \times \frac{1000}{M_2}}$$

$$= \frac{\kappa_1}{\kappa_2} \times \frac{M_2}{M_1}$$

$$= \frac{50}{100} \times 2$$

$$= \frac{\wedge_1}{\wedge_2} = 1{,}000 \times 10^{-3}$$

Ans. 1,000

2. Ionic radii of cation A^+ and anion B^- are 102 and 181 pm respectively. These ions are allowed to crystallize into an ionic solid. This crystal has cubic close packing for B^-. A^+ is present in all octahedral voids. The edge length of the unit cell of the crystal AB is _____ pm. (Nearest Integer)

Official Ans. by NTA (512)

Sol. $a = 2(r_+ + r_-)$

$a = 2\,(102 + 181)$

$a = 2(283)$

$a = 566$ pm

3. The minimum uncertainty in the speed of an electron in an one dimensional region of length $2a_O$ (Where a_0 = Bohr radius 52.9 pm) is _____km s^{-1}.

(Given : Mass of electron = 9.1×10^{-31} kg, Planck's constant h = 6.63×10^{-34} Js)

Official Ans. by NTA (548)

Sol. **Heisenberg's uncertainty principle**

$$\Delta x \times \Delta p_x \geq \frac{h}{4\pi}$$

$$\Rightarrow 2a_0 \times m\Delta v_x = \frac{h}{4\pi} (\text{minimum})$$

$$\Rightarrow \Delta v_x = \frac{h}{4\pi} \times \frac{1}{2a_0} \times \frac{1}{m}$$

$$= \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 2 \times 52.9 \times 10^{-12} \times 9.1 \times 10^{-31}}$$

$= 548273\ ms^{-1}$

$= 548.273\ km\ s^{-1}$

$= \boxed{548}\ km\ s^{-1}$

4. When 600 mL of 0.2 M HNO_3 is mixed with 400 mL of 0.1M NaOH solution in a flask, the rise in temperature of the flask is _____ $\times 10^{-2}\ ^0C$.

(Enthalpy of neutralisation = 57 kJ mol^{-1} and Specific heat of water = 4.2 $JK^{-1}\ g^{-1}$)

(Neglect heat capacity of flask)

Official Ans. by NTA (54)

Sol.

	HNO_3	NaOH	
	600 mL × 0.2 M	400 mL × 0.1 M	
	= 120 m mol	= 40 m mol	
	$HNO_3 + NaOH \rightarrow NaNO_3 + H_2O$		
Bef.	120	40	
Aft.	80	0	40 m mol

$$\Delta_r H = 40\,\text{m mol} \times (57 \times 10^3) \frac{J}{mol}$$

$$= 40 \times 10^{-3}\ mol \times 57 \times 10^3 \frac{J}{mol}$$

$= 2280\ J$

$$m\ S\Delta T = 2280$$

$$\Rightarrow 1000\ mL \times \frac{1gm}{mL} \times 4,2 \times \Delta T = 2280$$

$$\Delta T = \frac{2280}{4.2} \times 10^{-3}$$

$$= \frac{22800}{42} \times 10^{-3}$$

$= 542.86 \times 10^{-3}$

$$\Delta T = 54.286 \times 10^{-2} K$$

$$\Delta T = 54.286 \times 10^{-2\,0} C$$

Ans. $\boxed{54.286}$

Answer mentioned as 54 (Closest integer)

5. If O_2 gas is bubbled through water at 303 K, the number of millimoles of O_2 gas that dissolve in 1 litre of water is_____. (Nearest Integer)

(Given : Henry's Law constant for O_2 at 303 K is 46.82 k bar and partial pressure of O_2 = 0.920 bar)

(Assume solubility of O_2 in water is too small, nearly negligible)

Official Ans. by NTA (1)

Sol. $p = K_H \times x$

$$0.920_{bar} = 46.82 \times 10^3\ bar \times \frac{\text{mol of } O_2}{\text{mol of } H_2O}$$

$$0.920 = 46.82 \times \cancel{10^3} \times \frac{\text{mol of } O_2}{\cancel{1000}/18}$$

$$0.920 = 46.82 \times n_{O_2}$$

$$p = \frac{0.920}{46.82 \times 18} = n_{O_2}$$

$$\Rightarrow 1.09 \times 10^{-3} = n_{O_2}$$

$$\Rightarrow m\,\text{mol of } O_2 = 1$$

6. If the solubility product of PbS is 8×10^{-28}, then the solubility of PbS in pure water at 298 K is $x \times 10^{-16}$ mol L^{-1}. The value of x is _____. (Nearest Integer)

[Given $\sqrt{2} = 1.41$]

Official Ans. by NTA (282)

Sol. $K_{sp} = S^2$

$$S = \sqrt{K_{sp}} = \sqrt{8\times10^{-28}} = 2\sqrt{2}\times10^{-14}$$

$$= 2.82 \times 10^{-14}$$

$$= 282 \times 10^{-16}$$

Ans. = 282

7. The reaction between X and Y is first order with respect to X and zero order with respect to Y.

Experiment	[X] / mol L^{-1}	[Y] / mol L^{-1}	Initial rate / mol L^{-1} min^{-1}
I.	0.1	0.1	2×10^{-3}
II.	L	0.2	4×10^{-3}
III.	0.4	0.4	$M\times10^{-3}$
IV.	0.1	0.2	2×10^{-3}

Examine the data of table and calculate ratio of numerical values of M and L. (Nearest Inetger)

Official Ans. by NTA (40)

Sol. $r = k\,[x]\,[y]^0 = k\,[x]$

Using I & II

$$\frac{4\times10^{-3}}{2\times10^{-3}} = \left(\frac{L}{0.1}\right) \Rightarrow L = 0.2$$

Using I & III

$$\frac{M\times10^{-3}}{2\times10^{-3}} = \frac{0.4}{0.1} \Rightarrow M = 8$$

$$\frac{M}{L} = \frac{8}{0.2} = 40$$

Ans. 40

8. In a linear tetrapeptide (Constituted with different amino acids), (number of amino acids) - (number of peptide bonds) is_____.

Official Ans. by NTA (1)

Sol. In Tetrapeptide,

No. of Amino Acids = 4

No. of Peptide bonds = 3

Hence

Ans. = 1

9. In bromination of Propyne, with Bromine 1, 1, 2, 2-tetrabromopropane is obtained in 27% yield. The amount of 1, 1, 2, 2 tetrabromopropane obtained from 1 g of Bromine in this reaction is _____ $\times 10^{-1}$ g. (Nearest integer)

(Molar Mass : Bromine = 80 g/mol)

Official Ans. by NTA (3)

Sol. $CH_3 - C \equiv CH + 2Br_2 \rightarrow CH_3 - \underset{\underset{Br}{|}}{\overset{\overset{Br}{|}}{C}} - \underset{\underset{Br}{|}}{\overset{\overset{Br}{|}}{CH}}$

$$= \frac{1}{160}\times\frac{1}{2}\times360\times0.27$$

$$= 0.30375$$

$$= 3.0375 \times 10^{-1}$$

Ans. = 3

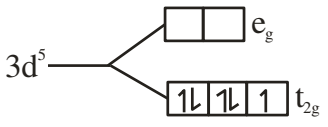
10. $[Fe(CN)_6]^{3-}$ should be an inner orbital complex. Ignoring the pairing energy, the value of crystal field stabilization energy for this complex is (–) _____ Δ_o. (Nearest integer)

Official Ans. by NTA (2)

Sol. $[Fe(CN)_6]^{3-}$

CN^- is strong field ligand

Fe^{+3} $3d^5$ (t_{2g}^5 e_g^0)



CFSE = 5 (–0.4 Δ_0) = –2.0 Δ_0

Ans. (2)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let R be a relation from the set {1, 2, 3……,60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers}. Then, the number of elements in R is :

(A) 600 (B) 660

(C) 540 (D) 720

Official Ans. by NTA (B)

Sol. Number of possible values of a = 60, for b = pq,

If p = 3, q = 3, 5, 7, 11, 13, 17, 19

If p = 5 q = 5, 7, 11

If p = 7 q = 7

Total cases = 60 × 11 = 660

2. If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to :

(A) 244 (B) 224

(C) 245 (D) 265

Official Ans. by NTA (A)

Sol. $z^5 + (\bar{z})^5 = (2+3i)^5 + (2-3i)^5$

$= 2({}^5C_0 2^5 + {}^5C_2 2^3 (3i)^2 + {}^5C_4 2^1 (3i)^4)$

$= 2\,(32 + 10 \times 8\,(-9) + 5 \times 2 \times 81) = 244$

3. Let A and B be two 3 × 3 non-zero real matrices such that AB is a zero matrix. Then

(A) The system of linear equations AX = 0 has a unique solution

(B) The system of linear equations AX = 0 has infinitely many solutions

(C) B is an invertible matrix

(D) adj (A) is an invertible matrix

Official Ans. by NTA (B)

Sol. $AB = 0 \Rightarrow |AB| = 0$

$|A|\ |B| = 0$

$|A| = 0 \qquad |B| = 0$

If $|A| \neq 0$, B = 0 (not possible)

If $|B| \neq 0$, A = 0 (not possible)

Hence $|A| = |B| = 0$

$\Rightarrow$ AX = 0 has infinitely many solutions

4. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \ldots\ldots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is :

(A) 198 (B) 202

(C) 212 (D) 218

Official Ans. by NTA (C)

Sol. By splitting

$$\frac{1}{20}\left[\left(\frac{1}{20-a} - \frac{1}{40-a}\right) + \left(\frac{1}{40-a} - \frac{1}{60-a}\right) + \ldots + \left(\frac{1}{180-a} - \frac{1}{200-a}\right)\right]$$

$$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a} - \frac{1}{200-a}\right) = \frac{1}{256}$$

$(20 - a)(200 - a) = 256 \times 9$

$a^2 - 220a + 1696 = 0$

a = 8, 212

Hence maximum value of a is 212.

5. If $\lim\limits_{x\to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$,

where $\alpha, \beta, \gamma \in R$, then which of the following is NOT correct ?

(A) $\alpha^2 + \beta^2 + \gamma^2 = 6$

(B) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

(C) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

(D) $\alpha^2 - \beta^2 + \gamma^2 = 4$

Official Ans. by NTA (C)

Sol.

$$\lim_{x\to 0}\frac{\alpha\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\right)+\beta\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\right)+\gamma\left(x-\frac{x^3}{3!}+\ldots\right)}{x^3}$$

constant terms should be zero

$\Rightarrow a+\beta=0$

coeff of x should be zero

$\Rightarrow \alpha-\beta+\gamma=0$

coeff of x^2 should be zero

$$\lim_{x\to 0}\frac{x^3\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)+x^4\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)}{x^3}=-\frac{}{3}$$

$$\Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}=0$$

$$\frac{\alpha}{6}-\frac{\beta}{6}-\frac{\gamma}{6}=2/3$$

$\Rightarrow \alpha=1, \beta=-1, \gamma=-2$

6. The integral $\int_0^{\frac{\pi}{2}}\frac{1}{3+2\sin x+\cos x}dx$ is equal to:

(A) $\tan^{-1}(2)$ (B) $\tan^{-1}(2)-\frac{\pi}{4}$

(C) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$ (D) $\frac{1}{2}$

Official Ans. by NTA (B)

Sol.

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}.dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Put $\tan\frac{x}{2}=t$, so

$$I=\int_0^1\frac{dt}{(t+1)^2+1}=\tan^{-1}(x+1)\Big|_0^1=\tan^{-1}2-\frac{\pi}{4}$$

7. Let the solution curve y = y (x) of the differential equation $(1+e^{2x})\left(\frac{dy}{dx}+y\right)=1$ pass through the point $\left(0,\frac{\pi}{2}\right)$. Then, $\lim_{x\to\infty}e^x y(x)$ is equal to :

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$

So integrating factor is $e^{\int 1.dx}=e^x$

So solution is $y\cdot e^x=\tan^{-1}\left(e^x\right)+c$

Now as curve is passing through $\left(0,\frac{\pi}{2}\right)$ so

$\Rightarrow c=\frac{\pi}{4}$

$$\Rightarrow \lim_{x\to\infty}\left(y\cdot e^x\right)=\lim_{x\to\infty}\left(\tan^{-1}\left(e^x\right)+\frac{\pi}{4}\right)=\frac{3\pi}{4}$$

8. Let a line L pass through the point of intersection of the lines bx + 10y – 8 = 0 and 2x – 3y = 0, $b\in R-\left\{\frac{4}{3}\right\}$. If the line L also passes through the point (1, 1) and touches the circle $17(x^2+y^2)=16$, then the eccentricity of the ellipse $\frac{x^2}{5}+\frac{y^2}{b^2}=1$ is :

(A) $\frac{2}{\sqrt{5}}$ (B) $\sqrt{\frac{3}{5}}$

(C) $\frac{1}{\sqrt{5}}$ (D) $\sqrt{\frac{2}{5}}$

Official Ans. by NTA (B)

Sol. Line is passing through intersection of $bx+10y-8=0$ and $2x-3y=0$ is $(bx+10y-8)+\lambda(2x-3y)=0$. As line is passing through $(1,1)$ so $\lambda=b+2$

Now line $(3b+4)x-(3b-4)y-8=0$ is tangent to circle $17(x^2+y^2)=16$

So $\frac{8}{\sqrt{(3b+4)^2+(3b-4)^2}}=\frac{4}{\sqrt{17}}$

$\Rightarrow b^2=2 \Rightarrow e=\sqrt{\frac{3}{5}}$

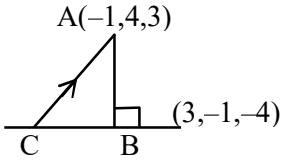
9. If the foot of the perpendicular from the point A(– 1, 4, 3) on the plane P : 2x + my + nz = 4, is $\left(-2,\frac{7}{2},\frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios 3, – 1, – 4, is equal to :

(A) 1 (B) $\sqrt{26}$

(C) $2\sqrt{2}$ (D) $\sqrt{14}$

Official Ans. by NTA (B)

Sol.



Let B be foot of $\perp$ coordinates of $B=\left(-2,\frac{7}{2},\frac{3}{2}\right)$

Direction ratio of line AB is $<2,1,3>$ so $m=1,\ n=3$

So equation of AC is $\frac{x+1}{3}=\frac{y-4}{-1}=\frac{z-3}{-4}=\lambda$

So point C is $(3\lambda-1,-\lambda+4,-4\lambda+3)$. But C lies on the plane, so

$6\lambda-2-\lambda+4-12\lambda+9=4$

$\Rightarrow \lambda=1 \Rightarrow C(2,3,-1)$

$\Rightarrow AC=\sqrt{26}$

10. Let $\vec{a}=3\hat{i}+\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is :

(A) – 5 (B) 5

(C) 1 (D) – 1

Official Ans. by NTA (A)

Sol. $a=3\hat{i}+\hat{j},\ \vec{b}=\hat{i}+2\hat{j}+\hat{k}$

As $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$

$\Rightarrow \vec{a}.\vec{c}(\vec{b})-(\vec{a}.\vec{b})\vec{c}=\vec{b}+\lambda\vec{c}$

$\Rightarrow \vec{a}\cdot\vec{c}=1, \vec{a}\cdot\vec{b}=-\lambda$

$\Rightarrow (3\hat{i}+\hat{j}).(\hat{i}+2\hat{j}+\hat{k})=-\lambda$

$\Rightarrow \lambda=-5$

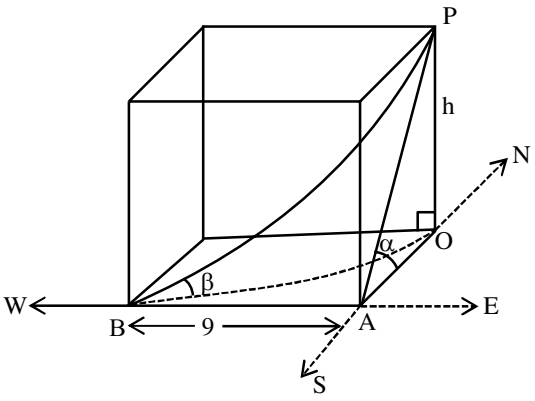
11. The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then cot α is equal to :

(A) $\frac{6}{5}$ (B) $\frac{9}{5}$

(C) $\frac{4}{3}$ (D) $\frac{7}{3}$

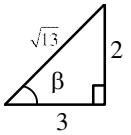
Official Ans. by NTA (A)

Sol.

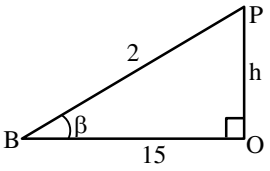


given OB = 15

$\cos\beta=\frac{3}{\sqrt{13}}$



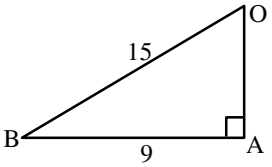
$\tan\beta=\frac{2}{3}$



$$\tan\beta = \frac{h}{15}$$

$$\frac{2}{3} = \frac{h}{15}$$

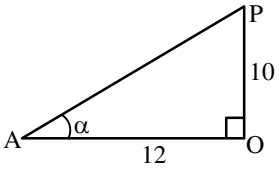
$$\boxed{10 = h}$$



$$OA^2 + AB^2 = 225$$

$$OA^2 + 81 = 225$$

$$\boxed{OA = 12}$$



$$\tan\alpha = \frac{10}{12}$$

$$\cot\alpha = \frac{12}{10} = \frac{6}{5}$$

12. The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to :

(A) $q \Rightarrow (p \wedge r)$ (B) $p \Rightarrow (p \wedge r)$

(C) $(p \wedge r) \Rightarrow (p \wedge q)$ (D) $(p \wedge q) \Rightarrow r$

Official Ans. by NTA (D)

Sol. $(p \wedge q) \Rightarrow (p \wedge r)$

$\sim(p \wedge q) \vee (p \wedge r)$

$(\sim p \vee \sim q) \vee (p \wedge r)$

$(\sim p \vee (p \wedge r)) \vee \sim q$

$(\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$

$(\sim p \vee r) \vee \sim q$

$(\sim p \vee \sim q) \vee r$

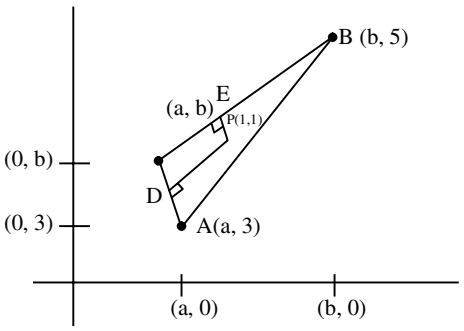
$\sim(p \wedge q) \vee r$

$(p \wedge q) \Rightarrow r$

13. Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), ab > 0 be P(1, 1). If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to :

(A) 2 (B) $\frac{4}{7}$ (C) $\frac{2}{7}$ (D) 4

Official Ans. by NTA (B)



$m_{AC} \longrightarrow \infty$

$m_{PD} = 0$

$$D\left(\frac{a+a}{2}, \frac{b+3}{2}\right)$$

$$D\left(a, \frac{b+3}{2}\right)$$

$m_{PD} = 0$

$$\frac{b+3}{2} - 1 = 0$$

$b + 3 - 2 = 0$

$$\boxed{b = -1}$$

$$E\left(\frac{b+a}{2}, \frac{5+b}{2}\right) = \left(\frac{af}{2}, 2\right)$$

$m_{CB} \cdot m_{EP} = -1$

$$\left(\frac{5-b}{b-a}\right) = \left(\frac{2-1}{\frac{a-1}{2} - 1}\right) = -1$$

$$\left(\frac{6}{-1-a}\right) = \left(\frac{2}{a-3}\right) = -1$$

$12 = (1 + a)(a - 3)$

$12 = a^2 - 3a + a - 3$

$\Rightarrow$ $a^2 - 2a - 15 = 0$

$(a - 5)(a + 3) = 0$

a = 5 or a = –3

Given ab > 0

a(–1) > 0

–a > 0

a < 0

$\boxed{a=-3}$ Accept

AP line A (–3, 3) P(1, 1)

$y-1=\left(\frac{3-1}{-3-1}\right)(x-1)$

–2y + 2 = x – 1

$\Rightarrow \boxed{x+2y=3}$ Appling .....(1)

Line BC B(–1, 5)

C(–3, –1)

$(y-5)=\frac{6}{2}(x+1)$

y –5 = 3x + 3

$\boxed{y=3x+8}$ .....(2)

Solving (1) & (2)

x + 2 (3x + 8) = 3

$\Rightarrow$ 7x + 16 = 3

7x = –13

$x=-\frac{13}{7}$

$y=3\left(-\frac{13}{7}\right)+8$

$=\frac{-39+56}{7}$

$y=\frac{17}{7}$

$x+y=\frac{-13+17}{7}=\frac{4}{7}$

14. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $\left(\hat{a}+\hat{b}\right)$ and $\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)$, then the value of $164\cos^2\theta$ is equal to :

(A) $90+27\sqrt{2}$ (B) $45+18\sqrt{2}$

(C) $90+3\sqrt{2}$ (D) $54+90\sqrt{2}$

Official Ans. by NTA (A)

Sol. $\hat{a}\wedge\hat{b}=\frac{\pi}{4}=\phi$

$\hat{a}.\hat{b}=|\hat{a}||\hat{b}|\cos\phi$

$\hat{a}.\hat{b}=\cos\phi=\frac{1}{\sqrt{2}}$

$$\cos\theta=\frac{\left(\hat{a}+\hat{b}\right).\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)}{\left|\hat{a}+\hat{b}\right|\left|\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right|}$$

$\left|\hat{a}+\hat{b}\right|^2=\left(\hat{a}+\hat{b}\right).\left(\hat{a}+\hat{b}\right)$

$\left|\hat{a}+\hat{b}\right|^2=2+2\hat{a}.\hat{b}$

$=2+\sqrt{2}$

$\hat{a}\times\hat{b}=|\hat{a}||\hat{b}|\sin\phi\,\hat{n}$

$\hat{a}\times\hat{b}=\frac{\hat{n}}{\sqrt{2}}$ when $\hat{n}$ is vector $\perp$ $\hat{a}$ and $\hat{b}$

let $\vec{c}=\hat{a}\times\hat{b}$

We know.

$\vec{c}.\vec{a}=0$

$\vec{c}.\vec{b}=0$

$\left|\hat{a}+2\hat{b}+2\vec{c}\right|^2$

$=1+4+\frac{(4)}{2}+4\,\hat{a}.\hat{b}+8\hat{b}.\vec{c}+4\vec{c}.\hat{a}$

$=7+\frac{4}{\sqrt{2}}=7+2\sqrt{2}$

Now

$\left(\hat{a}+\hat{b}\right).\left(\hat{a}+2\hat{b}+2\vec{c}\right)$

$=|\hat{a}|^2+2\hat{a}.\hat{b}+0+\hat{b}.\hat{a}+2|\hat{b}|^2+0$

$=1+\frac{2}{\sqrt{2}}+\frac{1}{\sqrt{2}}+2$

$=3+\frac{3}{\sqrt{2}}$

$$\cos\theta=\frac{3+\frac{3}{\sqrt{2}}}{\sqrt{2+\sqrt{2}}\sqrt{7+2\sqrt{2}}}$$

$$\cos^2\theta=\frac{9\left(\sqrt{2}+1\right)^2}{2\left(2+\sqrt{2}\right)\left(7+2\sqrt{2}\right)}$$

$$\cos^2\theta = \left(\frac{9}{2\sqrt{2}}\right)\frac{\left(\sqrt{2}+1\right)}{\left(7+2\sqrt{2}\right)}$$

$$164\cos^2\theta = \frac{(82)(9)}{\sqrt{2}}\frac{\left(\sqrt{2}+1\right)}{\left(7+2\sqrt{2}\right)}\frac{\left(7-2\sqrt{2}\right)}{\left(7-2\sqrt{2}\right)}$$

$$= \frac{(82)}{\sqrt{2}}\ \frac{(9)\left[7\sqrt{2}-4+7-2\sqrt{2}\right]}{(41)}$$

$$= \left(9\sqrt{2}\right)\left[5\sqrt{2}+3\right]$$

$$= 90 + 27\sqrt{2}$$

15. If $f(\alpha) = \int_1^{\alpha}\frac{\log_{10} t}{1+t}dt, \alpha > 0$, then $f(e^3) + f(e^{-3})$ is equal to :

(A) 9 (B) $\frac{9}{2}$

(C) $\frac{9}{\log_e(10)}$ (D) $\frac{9}{2\log_e(10)}$

Official Ans. by NTA (D)

Sol. $f(e^3) = \int_1^{e^3}\frac{\ell n\, t}{\ell n 10(1+t)}dt.....(1)$

$f(\alpha) = \int_1^{\alpha}\frac{\ell n\, t}{(\ell n 10)(1+t)}dt$

$t = \frac{1}{x} \Rightarrow x = \frac{1}{t}$

$dt = \frac{-1}{x^2}dx$

$$= \int_1^{\frac{1}{\alpha}}\frac{-\ell n\, x}{(\ell n 10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$$

$$f(\alpha) = \frac{1}{\ell n 10}\int_1^{\frac{1}{\alpha}}\frac{\ell n x}{x(x+1)}dx$$

$$f(e^{-3}) = \frac{1}{\ell n 10}\int_1^{e^3}\frac{\ell n t}{t(t+1)}dt.....(2)$$

Add (1) & (2)

$f(e^3) + f(e^{-3})$

$$= \left(\frac{1}{\ell n 10}\right)\int_1^{e^3}\frac{\ell n t}{(1+t)}\left[1+\frac{1}{t}\right]dt$$

$$= \left(\frac{1}{\ell n 10}\right)\int_1^{3}\frac{\ell n t}{t}dt$$

$\ell n t = r$

$\frac{dt}{t} = dr$

$$= \frac{1}{\ell n 10}\int_0^3 r\,dr$$

$$= \left(\frac{1}{\ell n 10}\right)\left(\frac{r^2}{2}\right)\Bigg|_0^3$$

$$= \left(\frac{1}{\log 10}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{2\log_e 10}$$

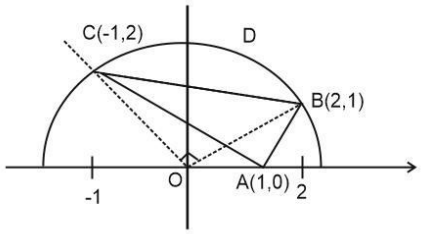
16. The area of the region $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$ is equal to :

(A) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$ (B) $\frac{5\pi}{4}-\frac{3}{2}$

(C) $\frac{3\pi}{4}+\frac{3}{2}$ (D) $\frac{5\pi}{4}-\frac{1}{2}$

Official Ans. by NTA (D)

Sol.



$|x-1| < y < \sqrt{5-x^2}$

When $|x-1| = \sqrt{5-x^2}$

$\Rightarrow (x-1)^2 = 5-x^2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x = 2, -1$

Required Area = Area of ΔABC + Area of region BCD

$$= \frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ 2 & 1 & 1\\ -1 & 2 & 1\end{vmatrix} + \frac{\pi}{4}\left(\sqrt{5}\right)^2 - \frac{1}{2}\left(\sqrt{5}\right)^2$$

$$= \frac{5\pi}{4} - \frac{1}{2}$$

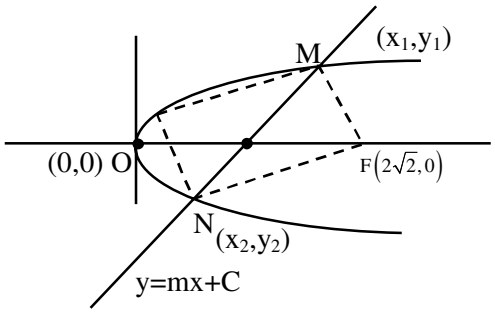
17. Let the focal chord of the parabola P : $y^2 = 4x$ along the line L : $y = mx + c$, $m > 0$ meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola H : $x^2 - y^2 = 4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is :

(A) $2\sqrt{6}$ (B) $2\sqrt{14}$

(C) $4\sqrt{6}$ (D) $4\sqrt{14}$

Official Ans. by NTA (B)

Sol.



$H : \frac{x^2}{4} - \frac{y^2}{4} = 1$

Focus (ae, 0)

$F(2\sqrt{2}, 0)$

Line L : y = mx + c pass (1,0)

$\boxed{o = m + C}$ .....(1)

Line L is tangent to Hyperbola. $\frac{x^2}{4} - \frac{y^2}{4} = 1$

$C = \pm\sqrt{a^2m^2 - \ell^2}$

$C = \pm\sqrt{4m^2 - 4}$

From (1)

$-m = \pm\sqrt{4m^2 - 4}$

Squaring

$m^2 = 4m^2 - 4$

$4 = 3m^2$

$\boxed{\frac{2}{\sqrt{3}} = m}$ (as $m > 0$)

$C = -m$

$C = \frac{-2}{\sqrt{3}}$

$y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$

$y^2 = 4x$

$\Rightarrow \left(\frac{2x-2}{\sqrt{3}}\right)^2 = 4x$

$\Rightarrow x^2 + 1 - 2x = 3x$

$\Rightarrow \boxed{x^2 - 5x + 1 = 0}$

$y^2 = 4\left(\frac{\sqrt{3}y + 2}{2}\right)$

$y^2 = 2\sqrt{3}y + 4$

$\Rightarrow \boxed{y^2 - 2\sqrt{3}y - 4 = 0}$

Area

$\left|\frac{1}{2}\begin{vmatrix} 0 & x_1 & 2\sqrt{2} & x_2 & 0 \\ 0 & y_1 & 0 & y_2 & 0 \end{vmatrix}\right|$

$= \left|\frac{1}{2}\left[-2\sqrt{2}y_1 + 2\sqrt{2}y_2\right]\right|$

$= \sqrt{2}|y_2 - y_1| = \frac{(\sqrt{2})\sqrt{12+16}}{111}$

$= \sqrt{56}$

$= 2\sqrt{14}$

18. The number of points, where the function $f : \mathbf{R} \to \mathbf{R}$, $f(x) = |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3)|x^2 - 5x + 4|$, is **NOT** differentiable, is :

(A) 1 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (B)

Sol. $f(x) = |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3)|x^2 - 5x + 4|$

$= |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3)|x - 1||x - 4|$

$= |x - 1| [\cos |x - 2| \sin |x - 1| + (x - 3)|x - 4|]$

Non differentiable at x = 1 and x = 4.

19. Let S = {1, 2, 3, …, 2022}. Then the probability, that a randomly chosen number n from the set S such that HCF (n, 2022) = 1, is :

(A) $\frac{128}{1011}$ (B) $\frac{166}{1011}$

(C) $\frac{127}{337}$ (D) $\frac{112}{337}$

Official Ans. by NTA (D)

Sol. Total number of elements = 2022

$2022 = 2\times3\times337$

HCF (n, 2022) = 1

is feasible when the value of 'n' and 2022 has no common factor.

A = Number which are divisible by 2 from {1,2,3.....2022}

n(A) = 1011

B = Number which are divisible by 3 by 3 from {1,2,3.....2022}

n(B) = 674

$A\cap B$ = Number which are divisible by 6

from {1,2,3.....2022}

6,12,18....., 2022

$\boxed{337 = n(A\cap B)}$

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

= 1011+ 674 –337

= 1348

C= Number which divisible by 337 from {1,.....1022}

C= {337,674,1011,1348,1685,20222}

Already counted in Set (A∪B) | Already counted in Set (A∪B) | Already counted in Set (A∪B)

Total elements which are divisible by 2 or 3 or 337

= 1348 +2 = 1350

Favourable cases = Element which are neither divisible by 2, 3 or 337

= 2022 – 1350

= 672

Required probability = $\frac{672}{2022} = \frac{112}{337}$

20. Let $f(x) = 3^{(x^2-2)^3+4}$, $x \in \mathbf{R}$. Then which of the following statements are true ?

P : x = 0 is a point of local minima of f

Q : $x = \sqrt{2}$ is a point of inflection of f

R : f' is increasing for $x > \sqrt{2}$

(A) Only P and Q (B) Only P and R

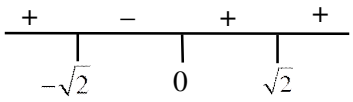
(C) Only Q and R (D) All, P, Q and R

Official Ans. by NTA (D)

Sol. $f(x) = 81.3^{(x^2-2)^3}$

$f'(x) = 81.3^{(x^2-2)^3}.\ell n3.3(x^2-2)^2.2x$

$= (81\times6)3^{(x^2-2)^3}\, x(x^2-2)^2\,\ell n3$



x = 6 is point of local min

$f'(x) = \underbrace{(486.\ell n3)}_{k}\underbrace{3^{(x^2-2)^3}\, x(x^2-2)^2}_{g(x)}$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)^2 + x.3^{(x^2-2)^3}.4x.(x^2-2)$

$+x.(x^2-2)^2.3^{(x^2-2)^3}\ell n3.3(x^2-2)^2.2x$

$= 3^{(x^2-2)^3}(x^2-2)\left[x^2-2+4x^2+6x^2\ell n3(x^2-2)^3\right]$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)\left[5x^2-2+6x^2\ell n3(x^2-2)^3\right]$

$f''(x) = k.g'(x)$

$f''(\sqrt{2}) = 0, f''(\sqrt{2}^+) > 0, f''(\sqrt{2}^-) < 0$

$x = \sqrt{2}$ is point of inflection

f''(x) > 0 for $x > \sqrt{2}$ so f'(x) is increasing

SECTION-B

1. Let S = {θ ∈ (0, 2π) : $7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$}. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$ θ ∈ S, is_____.

Official Ans. by NTA (16)

Sol. $7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2(1 + \cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos 2\theta = 0$

$\cos 2\theta(2\cos 2\theta - 5) = 0$

$\cos 2\theta = 0$

$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

$S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

For all four values of θ

$x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

Sum of roots of all four equations = 4 × 4 = 16.

2. Let the mean and the variance of 20 observations $x_1, x_2, \ldots x_{20}$ be 15 and 9, respectively. For $\alpha \in R$, if the mean of $(x_1+\alpha)^2, (x_2+\alpha)^2, \ldots, (x_{20}+\alpha)^2$ is 178, then the square of the maximum value of α is equal to _____.

Official Ans. by NTA (4)

Sol. $\sum x_1 = 15 \times 20 = 300 \quad ...( )$

$\frac{\sum x_1^2}{20} - (15)^2 = 9 \quad ...(ii)$

$\sum x_1^2 = 234 \times 20 = 4680$

$\frac{\sum (x_1+\alpha)^2}{20} = 178 \Rightarrow \sum (x_1+\alpha)^2 = 3560$

$\Rightarrow \sum x_1^2 + 2\alpha\sum x_1 + \sum \alpha^2 = 3560$

$4680 + 600\alpha + 20\alpha^2 = 3560$

$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$

$\Rightarrow (\alpha+28)(\alpha+2) = 0$

$\alpha = -2, -28$

Square of maximum value of α is 4

3. Let a line with direction ratios a, –4a, –7 be perpendicular to the lines with direction ratios 3, –1, 2b and b, a, –2. If the point of intersection of the line $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ and the plane x – y + z = 0 is (α, β, γ), then α + β + γ is equal to _____.

Official Ans. by NTA (10)

Sol. (a,–4a,–7) ⊥ to (3,–1,2b)

a = 2b …(i)

(a,–4a,–7) ⊥ to (b,a,–2)

3a + 4a – 14b = 0

$ab - 4a^2 + 14 = 0$ ….(ii)

From Equations (i) and (ii)

$2b^2 - 16b^2 + 14 = 0$

$b^2 = 1$

$a^2 = 4b^2 = 4$

$\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = k$

α = 5k – 1, β = 3k + 2, γ = k

As (α, β, γ) satisfies x – y + z = 0

5k – 1 – (3k + 2) + k = 0

k = 1

$\therefore\ \alpha + \beta + \gamma = 9k + 1 = 10$

4. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty}\frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _____.

Official Ans. by NTA (16)

Sol. $S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots.$

$\frac{S}{2} = \quad \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots$

$\frac{S}{2} = \frac{a_1}{2} + d\left(\frac{1}{2^2} + \frac{1}{2^3} + \ldots\right)$

$\frac{S}{2} = \frac{a_1}{2} + d\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)$

$\therefore S = a_1 + d = a_2 = 4$

Or $4a_2 = 16$

5. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}$: 1. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to _____.

Official Ans. by NTA (84)

Sol. $\frac{T_5}{T_{n-3}} = \frac{{}^nC_4(2^{1/4})^{n-4}(3^{-1/4})^4}{{}^nC_{n-4}(2^{1/4})^4(3^{-1/4})^{n-4}} = \frac{\sqrt[4]{6}}{1}$

$\Rightarrow 2^{\frac{n-8}{4}}3^{\frac{n-8}{4}} = 6^{1/4}$

$\Rightarrow 6^{n-8} = 6$

$\Rightarrow n-8=1 \Rightarrow n=9$

$T_6 = {}^9C_5(2^{1/4})^4(3^{-1/4})^5 = \frac{84}{\sqrt[4]{3}}$

$\therefore \alpha = 84$

6. The number of matrices of order 3 × 3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is_____.

Official Ans. by NTA (282)

Sol. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} a_{ij} \in \{0,1\}$

$\sum a_{ij} = 2,3,5,7$

Total matrix = ${}^9C_2 + {}^9C_3 + {}^9C_5 + {}^9C_7$

= 282

7. Let p and p + 2 be prime numbers and let

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

Then the sum of the maximum values of α and β, such that p^α and $(p+2)^\beta$ divide Δ, is _____.

Official Ans. by NTA (4)

Sol. $\Delta = \begin{vmatrix} P! & (P+1)! & (P+2)! \\ (P+1)! & (P+2)! & (P+3)! \\ (P+2)! & (P+3)! & (P+4)! \end{vmatrix}$

$\Delta = P!(P+1)!(P+2)!\begin{vmatrix} 1 & 1 & 1 \\ P+1 & P+2 & P+3 \\ (P+2)(P+1) & (P+3)(P+2) & (P+4)(P+3) \end{vmatrix}$

$\Delta = 2P!(P+1)!(P+2)!$

Which is divisible by P^α & $(P+2)^\beta$

$\therefore \alpha = 3, \beta = 1$

Ans. 4

8. If $\frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + \ldots + \frac{1}{100\times101\times102} = \frac{k}{101}$, then 34 k is equal to _____.

Official Ans. by NTA (286)

Sol. $\frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{100.101.102} = \frac{k}{101}$

$\frac{4-2}{2.3.4} + \frac{5-3}{3.4.5} + \ldots + \frac{102-100}{100.101.102} = \frac{2k}{101}$

$\frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \ldots + \frac{1}{100.101} - \frac{1}{101.102} = \frac{2k}{101}$

$\frac{1}{2.3} - \frac{1}{101.102} = \frac{2k}{101}$

$\therefore 2k = \frac{101}{6} - \frac{1}{102}$

$\therefore 34k = 286$

9. Let S = {4, 6, 9} and T = {9, 10, 11, …, 1000}. If $A = \{a_1 + a_2 + \ldots + a_k : k \in N, a_1, a_2, a_3, \ldots, a_k \in S\}$, then the sum of all the elements in the set T – A is equal to _____.

Official Ans. by NTA (11)

Sol. S={4,6,9} T = {9,10,11….1000}

$A\{a_1+a_2+.....+a_k : K\in N\}$ & $a_i \in S$

Here by the definition of set 'A'

$A=\{a : a=4x+6y+9z\}$

Except the element 11, every element of set T is of of the form 4x + 6y + 9z for some x, y, z ∈W

∴ T – A = {11}

Ans. 11

10. Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line y = x + 1 be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If r is the radius of circle c_2, then $\alpha + 6r^2$ is equal to _____

Official Ans. by NTA (12)

Sol. Image of centre $c_1 \equiv (1,3)$ in x – y + 1 = 0 is given by

$$\frac{x_1-1}{1}=\frac{y_1-3}{-1}=\frac{-2(1-3+1)}{1^2+1^2}$$

$\Rightarrow x_1=2, y_1=2$

∴ Centre of circle $c_2 \equiv (2,2)$

∴ Equation of c_2 be $x^2+y^2-4x-4y+\frac{38}{5}=0$

Now radius of c_2 is $\sqrt{4+4-\frac{38}{5}}=\sqrt{\frac{2}{5}}=r$

(radius of c_1)^2 = (radius of c_2)^2

$\Rightarrow 10-\alpha=\frac{2}{5} \Rightarrow \alpha=\frac{48}{5}$

$\therefore \alpha+6r^2=\frac{48}{5}+\frac{12}{5}=12$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Two identical metallic spheres A and B when placed at certain distance in air repel each other with a force of F. Another identical uncharged sphere C is first placed in contact with A and then in contact with B and finally placed at midpoint between spheres A and B. The force experienced by sphere C will be :

(A) 3F/2 (B) 3F/4

(C) F (D) 2F

Official Ans. by NTA (B)

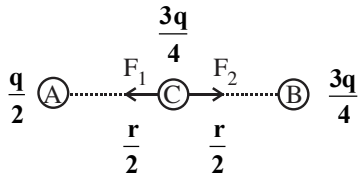
Sol. Let $q_A = q_B = q$

Ⓐ ..... r ..... Ⓑ $F = \frac{Kq^2}{r^2}$

When C is placed in contact with A, charge on A & C will be $= \frac{q}{2}$

Now C is placed in contact with B, charge on B & C will be $= \frac{q+\frac{q}{2}}{2} = \frac{3q}{4}$

Now,



$$F' = F_2 - F_1 = \frac{\left(K\frac{3q}{4} - K\frac{q}{2}\right)}{\frac{r^2}{4}} \cdot \frac{3q}{4}$$

$$= \frac{3Kq^2}{4r^2} = \frac{3F}{4} \quad \text{(B)}$$

2. Match List I with List II.

List I		List II	
A.	Torque	I.	Nms^{-1}
B.	Stress	II.	$J\,kg^{-1}$
C.	Latent Heat	III.	Nm
D.	Power	IV.	Nm^{-2}

Choose the correct answer from the options given below:

(A) A-III, B-II, C-I, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-I, C-III, D-II

(D) A-II, B-III, C-I, D-IV

Official Ans. by NTA (B)

Sol. Torque $= F \times r_{\perp}$ Nm

Stress $= \frac{\text{Force}}{\text{Area}}$ N/m^2

Latent heat $= \frac{\text{Energy}}{\text{Mass}}$ $J\,Kg^{-1}$

Power $= \frac{\text{Work}}{\text{Time}}$ $N\,ms^{-1}$

A-III, B-IV, C-II, D-I

3. Two identical thin metal plates has charge q_1 and q_2 respectively such that $q_1 > q_2$. The plates were brought close to each other to form a parallel plate capacitor of capacitance C. The potential difference between them is :

(A) $\frac{(q_1+q_2)}{C}$ (B) $\frac{(q_1-q_2)}{C}$

(C) $\frac{(q_1-q_2)}{2C}$ (D) $\frac{2(q_1-q_2)}{C}$

Official Ans. by NTA (C)

Sol. Electric field between plates $E = \frac{q_1-q_2}{2A\epsilon_0}$

$$V = Ed = \frac{q_1-q_2}{2A\epsilon_0}d$$

$$V = \frac{q_1-q_2}{2C}$$

4. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Alloys such as constantan and manganin are used in making standard resistance coils.

Reason R: Constantan and manganin have very small value of temperature coefficient of resistance.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (A)

Sol. Theory based

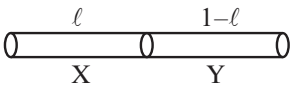
5. A 1 m long wire is broken into two unequal parts X and Y The X part of the wire is streched into another wire W. Length of W is twice the length of X and the resistance of W is twice that of Y. Find the ratio of length of X and Y.

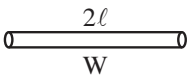
(A) 1 : 4 (B) 1 : 2

(C) 4 :1 (D) 2 : 1

Official Ans. by NTA (B)

Sol.





$$\frac{R_X}{R_Y}=\frac{\ell_X}{\ell_Y}$$

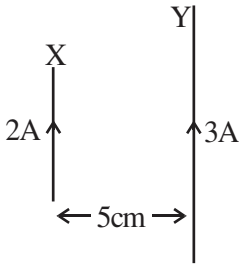
When wire is stretched to double of its length, then resistance becomes 4 times

$R_W = 4R_X = 2R_Y$

$$\frac{R_X}{R_Y}=\frac{1}{2}$$

So. $\frac{\ell_X}{\ell_y}=\frac{1}{2}$

6. A wire X of length 50 cm carrying a current of 2 A is placed parallel to a long wire Y of length 5 m. The wire Y carries a current of 3 A. The distance between two wires is 5 cm and currents flow in the same direction. The force acting on the wire Y is :



(A) 1.2×10^{-5} N directed towards wire X.

(B) 1.2×10^{-4} N directed away from wire X.

(C) 1.2×10^{-4} N directed towards wire X.

(D) 2.4×10^{-5} N directed towards wire X.

Official Ans. by NTA (A)

Sol. Force of interaction = $I_1\ \ell_1\ B_{12}$

$$=\frac{\mu_0 I_1 I_2}{2\pi r}\ell_1$$

$$=\frac{4\pi\times10^{-7}\times6\times0.5}{2\pi\times5\times10^{-2}}$$

$= 1.2 \times 10^{-5}$ towards X

7. A juggler throws balls vertically upwards with same initial velocity in air. When the first ball reaches its highest position, he throws the next ball. Assuming the juggler throws n balls per second, the maximum height the balls can reach is

(A) g/2n (B) g/n

(C) 2gn (D) $g/2n^2$

Official Ans. by NTA (D)

Sol. Time taken by ball to reach highest point = $\frac{u}{g}$

Frequency of throw = $\frac{g}{u}=n$

$$\Rightarrow u=\frac{g}{n}$$

$$H_{max}=\frac{u^2}{2g}=\frac{\left(\frac{g}{n}\right)^2}{2g}$$

$$\frac{g}{2n^2}$$

8. A circuit element X when connected to an a.c. supply of peak voltage 100 V gives a peak current of 5 A which is in phase with the voltage. A second element Y when connected to the same a.c. supply also gives the same value of peak current which lags behind the voltage by $\frac{\pi}{2}$. If X and Y are connected in series to the same supply, what will be the rms value of the current in ampere ?

(A) $\frac{10}{\sqrt{2}}$ (B) $\frac{5}{\sqrt{2}}$ (C) $5\sqrt{2}$ (D) $\frac{5}{2}$

Official Ans. by NTA (D)

Sol. Element X should be resistive with R = 20Ω

Element Y should be inductive with $X_L = 20\ \Omega$

When X and Y are connector in series

$$Z = \sqrt{X_L^2 + R^2} = 20\sqrt{2}$$

$$I_0 = \frac{E_0}{Z} = \frac{100}{20\sqrt{2}} = \frac{5}{\sqrt{2}}A$$

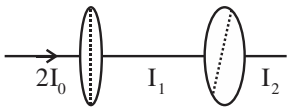
$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{2}A$$

9. An unpolarised light beam of intensity $2I_0$ is passed through a polaroid P and then through another polaroid Q which is oriented in such a way that its passing axis makes an angle of 30° relative to that of P. The intensity of the emergent light is

(A) $\frac{I_0}{4}$ (B) $\frac{I_0}{2}$ (C) $\frac{3I_0}{4}$ (D) $\frac{3I_0}{2}$

Official Ans. by NTA (C)

Sol.



$$I_1 = \frac{1}{2}(2I_0) = I_0$$

$$I_2 = I_1 \cos^2 30^\circ$$

$$= I_0 \cdot \frac{3}{4} = \frac{3I_0}{4}$$

10. An α particle and a proton are accelerated from rest through the same potential difference. The ratio of linear momenta acquired by above two particals will be :

(A) $\sqrt{2}:1$ (B) $2\sqrt{2}:1$

(C) $4\sqrt{2}:1$ (D) 8 : 1

Official Ans. by NTA (B)

Sol. $p = \sqrt{2mE} = \sqrt{2mqV}$

$$\frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4}{1} \times \frac{2}{1}}$$

$$= \frac{2\sqrt{2}}{1}$$

11. Read the following statements:

(A) Volume of the nucleus is directly proportional to the mass number.

(B) Volume of the nucleus is independent of mass number.

(C) Density of the nucleus is directly proportional to the mass number.

(D) Density of the nucleus is directly proportional to the cube root of the mass number.

(E) Density of the nucleus is independent of the mass number.

Choose the correct option from the following options.

(A) (A) and (D) only. (B) (A) and (E) only.

(C) (B) and (E) only. (D) (A) and (C) only

Official Ans. by NTA (B)

Sol. $R \propto A^{1/3}$

$$V = \frac{4}{3}\pi R^3 \propto A$$

Mass ∝ A

So density is independent of A.

12. An object of mass 1 kg is taken to a height from the surface of earth which is equal to three times the radius of earth. The gain in potential energy of the object will be

[If, $g=10ms^{-2}$ and radius of earth = 6400 km]

(A) 48 MJ (B) 24 MJ

(C) 36 MJ (D) 12 MJ

Official Ans. by NTA (A)

Sol. $U_i = \frac{-GMm}{R}$

$U_f = -\frac{GMm}{4R}$

$\Delta U = U_f - U_i = \frac{3GMm}{4R}$

$= \frac{3}{4}mgR$

$= \frac{3}{4} \times 1 \times 10 \times 64 \times 10^5$

$= 48$ MJ

13. A ball is released from a height h. If t_1 and t_2 be the time required to complete first half and second half of the distance respectively. Then, choose the correct relation between t_1 and t_2.

(A) $t_1 = (\sqrt{2})t_2$ (B) $t_1 = (\sqrt{2}-1)t_2$

(C) $t_2 = (\sqrt{2}+1)t_1$ (D) $t_2 = (\sqrt{2}-1)t_1$

Official Ans. by NTA (D)

Sol. For first $\frac{h}{2}$

$\frac{h}{2} = \frac{1}{2}gt_1^2$

For total height h

$h = \frac{1}{2}g(t_1+t_2)^2$

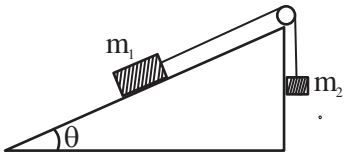
$\frac{1}{\sqrt{2}} = \frac{t_1}{t_1+t_2}$

$1 + \frac{t_2}{t_1} = \sqrt{2}$

$\frac{t_1}{t_2} = \frac{1}{\sqrt{2}-1}$

$t_2 = (\sqrt{2}-1)t_1$

14. Two bodies of masses $m_1 = 5$ kg and $m_2 = 3$ kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be :[Take $g = 10\ ms^{-2}$]



(A) 30 N (B) 40 N

(C) 50 N (D) 60 N

Official Ans. by NTA (B)

Sol. For equilibrium $m_2g = m_1g\sin\theta$

$\sin\theta = \frac{m_2}{m_1} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Normal force on $m_1 = 5g\cos\theta$

$= 5 \times 10 \times \frac{4}{5} = 40$ N

15. If momentum of a body is increased by 20%, then its kinetic energy increases by :

(A) 36% (B) 40%

(C) 44% (D) 48%

Official Ans. by NTA (C)

Sol. $P' = P + \frac{20}{100}P = 1.2\ P$

% change in KE = $\frac{K'-K}{K} \times 100$

$= \left(\frac{\frac{P'^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}}\right) \times 100$

$= [(1.2)^2 - 1] \times 100$

$= 44\ \%$

16. The torque of a force $5\hat{i}+3\hat{j}-7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2\hat{i}+2\hat{j}+\hat{k}$, then the value of τ will be :

(A) $11\hat{i}+19\hat{j}-4\hat{k}$ (B) $-11\hat{i}+9\hat{j}-16\hat{k}$

(C) $-17\hat{i}+19\hat{j}-4\hat{k}$ (D) $17\hat{i}+9\hat{j}+16\hat{k}$

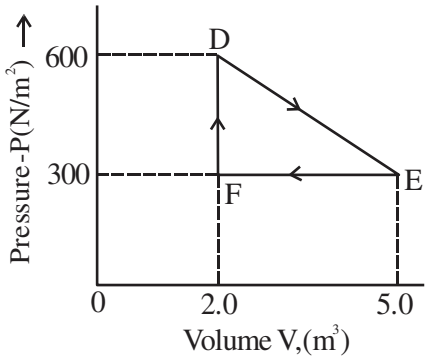
Official Ans. by NTA (C)

Sol. $\vec{\tau}=\vec{r}\times\vec{F}=\begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$

$=i(-14-3)-j(-14-5)+\hat{k}(6-10)$

$=-17\hat{i}+19\hat{j}-4\hat{k}$

17. A thermodynamic system is taken from an original state D to an intermediate state E by the linear process shown in the figure. Its volume is then reduced to the original volume from E to F by an isobaric process. The total work done by the gas from D to E to F will be



(A) –450 J (B) 450 J

(C) 900 J (D) 1350 J

Official Ans. by NTA (B)

Sol. $W_{DE}=\frac{1}{2}(600+300)3\ J$

$=1350\ J$

$W_{EF}=-300\times3=-900\,J$

$W_{DEF}=450\,J$

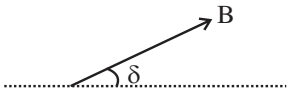
18. The vertical component of the earth's magnetic field is 6×10^{-5} T at any place where the angle of dip is 37°. The earth's resultant magnetic field at that place will be (Given $\tan37^\circ=\frac{3}{4}$)

(A) 8×10^{-5} T (B) 6×10^{-5} T

(C) 5×10^{-4} T (D) 1×10^{-4} T

Official Ans. by NTA (D)

Sol.



$(\delta=37^\circ)$

$B_V=B\sin\delta$

$6\times10^{-5}=B\frac{3}{5}$

$B=10\times10^{-5}$ T

$=10^{-4}$ T

19. The root mean square speed of smoke particles of mass 5×10^{-17} kg in their Brownian motion in air at NTP is approximately. [Given $k=1.38\times10^{-23}\ JK^{-1}$]

(A) 60 mm s^{-1} (B) 12 mm s^{-1}

(C) 15 mm s^{-1} (D) 36 mm s^{-1}

Official Ans. by NTA (C)

Sol. $V_{rms}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3\times1.38\times10^{-23}\times293}{5\times10^{-17}}}$

$\approx15\,mm/s$

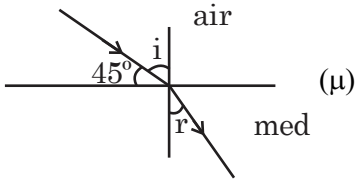
20. Light enters from air into a given medium at an angle of 45° with interface of the air-medium surface. After refraction, the light ray is deviated through an angle of 15° from its original direction. The refractive index of the medium is :

(A) 1.732 (B) 1.333

(C) 1.414 (D) 2.732

Official Ans. by NTA (C)

Sol.



$i = 45^\circ$

$D = i - r$

$15^\circ = 45 - r \Rightarrow r = 30^\circ$

$n_1 \sin i = n_2 \sin r$

$1 \sin 45^\circ = \mu \sin 30^\circ$

$$\frac{1}{\sqrt{2}} = \mu \frac{1}{2}$$

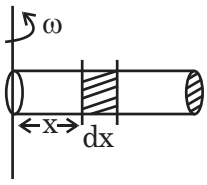
$$\mu = \sqrt{2} = 1.414$$

SECTION-B

1. A tube of length 50 cm is filled completely with an incompressible liquid of mass 250 g and closed at both ends. The tube is then rotated in horizontal plane about one of its ends with a uniform angular velocity $x\sqrt{F}$ rad s^{-1}. If F be the force exerted by the liquid at the other end then the value of x will be _____.

Official Ans. by NTA (4)

Sol.



$$F = \int (dm)\omega^2 x$$

$$= \int_0^L \left(\frac{m}{L}dx\right)\omega^2 x$$

$$= \frac{m}{L}\omega^2 \frac{L^2}{2}$$

$$= \frac{m\omega^2 L}{2}$$

$$\omega = \sqrt{\frac{2}{mL}}\sqrt{F}$$

$$= \sqrt{\frac{2}{0.25 \times 0.5}}\sqrt{F}$$

$$= \sqrt{16}\sqrt{F}$$

$$= 4\sqrt{F}$$

2. Nearly 10% of the power of a 110 W light bulb is converted to visible radiation. The change in average intensities of visible radiation, at a distance of 1 m from the bulb to a distance of 5 m is $a \times 10^{-2}$ W/m^2. The value of 'a' will be

Official Ans. by NTA (84)

Sol. **P' = 10% of 110 W**

$$= \frac{10}{100} \times 110\,W$$

= 11 W

$$I_1 - I_2 = \frac{P'}{4\pi r_1^2} - \frac{P'}{4\pi r_2^2}$$

$$= \frac{11}{4\pi}\left[\frac{1}{1} - \frac{1}{25}\right]$$

$$= \frac{11}{4\pi} \times \frac{24}{25}$$

$$= \frac{264}{\pi} \times 10^{-2} = 84 \times 10^{-2}\,W/m^2$$

3. A metal wire of length 0.5 m and cross-sectional area 10^{-4} m^2 has breaking stress 5×10^8 Nm^{-2}. A block of 10 kg is attached at one end of the string and is rotating in a horizontal circle. The maximum linear velocity of block will be____ ms^{-1}.

Official Ans. by NTA (50)

Sol. $$T = \frac{mv^2}{\ell} = \frac{10 \times v^2}{0.5} = 20v^2$$

T_{max} = Breaking stress × Area

$= 5 \times 10^8 \times 10^{-4} = 5 \times 10^4$

$20V^2 = 5 \times 10^4$

$$V = \sqrt{\frac{1}{4}10^4} = 50\,m/s$$

4. The velocity of a small ball of mass 0.3g and density 8g/cc when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is 1.3 g/cc, then the value of viscous force acting on the ball will be $x \times 10^{-4}$ N, the value of x is____. [use $g = 10 m/s^2$]

Official Ans. by NTA (25)

Sol. $F_V + F_B = mg$ (v = constant)

$F_V = mg - F_B$

$= \rho_B Vg - \rho_L Vg$

$= (\rho_B - \rho_L) Vg$

$= (8 - 1.3) \times 10^{+3} \times \frac{0.3 \times 10^{-3}}{8 \times 10^3} \times 10$

$= \frac{6.7 \times 0.3}{8} \times 10^{-2}$ $(g = 10)$

$= \frac{67 \times 3}{8} \times 10^{-4} = 25.125 \times 10^{-4}$

Ans. 25.125

5. A modulating signal $2\sin(6.28 \times 10^6)t$ is added to the carrier signal $4\sin(12.56 \times 10^9)t$ for amplitude modulation. The combined signal is passed through a non-linear square law device. The output is then passed through a band pass filter. The bandwidth of the output signal of band pass filter will be___MHz.

Official Ans. by NTA (2)

Sol. Frequencies present in output of square law device

$2f_c$, $f_c + f_m$, f_c, $f_c - f_m$, $2f_m$, f_m

After passing through band bass filte.

$f_c + f_m$, f_c, $f_c - f_m$

Band width $= 2f_m$

$= \frac{2\omega_m}{2\pi} = \frac{6.28 \times 10^6}{3.14}$

$= 2$ MHz

6. The speed of a transverse wave passing through a string of length 50 cm and mass 10 g is 60 ms^{-1}. The area of cross-section of the wire is 2.0 mm^2 and its Young's modulus is 1.2×10^{11} Nm^{-2}. The extension of the wire over its natural length due to its tension will be $x \times 10^{-5}$ m. The value of x is_____.

Official Ans. by NTA (15)

Sol. $V_w = \sqrt{\frac{T}{\mu}}$

$60 = \sqrt{\frac{T}{10 \times 10^{-3}} \times 0.5}$

$T = \frac{(60)^2 \times 10^{-2}}{0.5} = 72$ N

$\Delta\ell = \frac{F\ell}{AY} = \frac{72 \times 0.5}{2 \times 10^{-6} \times 1.2 \times 10^{11}}$

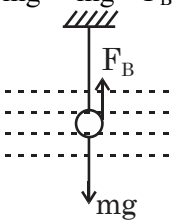
$= \frac{72 \times 5}{24} \times 10^{-5} = 15 \times 10^{-5}$

Ans. 15

7. The metallic bob of simple pendulum has the relative density 5. The time period of this pendulum is 10 s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}$ s. The value of x will be _____.

Official Ans. by NTA (5)

Sol. $mg' = mg - F_B$



$g' = \frac{mg - F_B}{m}$

$= \frac{\rho_B Vg - \rho_w Vg}{\rho_B V}$

$= \left(\frac{\rho_B - \rho_w}{\rho_B}\right) g$ $\quad T = 2\pi\sqrt{\frac{\ell}{g}}$

$= \frac{5-1}{5} \times g$

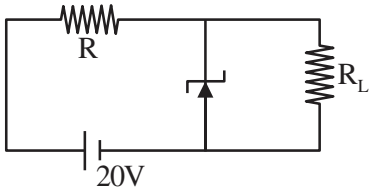
$= \frac{4}{5} g$

$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{\frac{4}{5}g}} = \sqrt{\frac{5}{4}}$

$T' = T\sqrt{\frac{5}{4}} = \frac{10}{2}\sqrt{5}$

$T' = 5\sqrt{5}$

8. A 8 V Zener diode along with a series resistance R is connected across a 20 V supply (as shown in the figure). If the maximum Zener current is 25 mA, then the minimum value of R will be _____Ω.



Official Ans. by NTA (480)

Sol. $\varepsilon - IR - V_z = 0$

$20 - IR - 6 = 0$

$IR = 12$

$25 \times 10^{-3} R = 12$

$$R = \frac{12}{25\times10^{-3}} = 480\Omega$$

9. Two radioactive materials A and B have decay constants 25λ and 16λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of B to that of A will be "e" after a time $\frac{1}{a\lambda}$. The value of a is_____.

Official Ans. by NTA (9)

Sol. $N = N_0 e^{-\lambda t}$

$$\frac{N_B}{N_A} = \frac{e^{-\lambda_2 t}}{e^{-\lambda_1 t}} = e^{-\lambda_2 t}.e^{\lambda_1 t}$$

$$e^1 = e^{(\lambda_1-\lambda_2)t}$$

$$(\lambda_1 - \lambda_2)t = 1$$

$$t = \frac{1}{\lambda_1 - \lambda_2} = \frac{1}{25\lambda - 16\lambda} = \frac{1}{9\lambda}$$

10. A capacitor of capacitance 500 μF is charged completely using a dc supply of 100 V. It is now connected to an inductor of inductance 50 mH to form an LC circuit. The maximum current in LC circuit will be _____ A.

Official Ans. by NTA (10)

Sol. Energy stored in capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}500\times10^{-6}\times10^4$$

$$= \frac{5}{2}J$$

Current will be maximum when whole energy of capacitor becomes energy of inductor.

$$\frac{1}{2}LI^2 = \frac{5}{2}$$

$$I = \sqrt{\frac{5}{L}} = \sqrt{\frac{5}{50\times10^{-3}}} = 10\,A.$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the reaction

$4HNO_3(l) + 3KCl(s) \rightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(s)$

The amount of HNO_3 required to produce 110.0 g of KNO_3 is :

(Given : Atomic masses of H, O, N and K are 1, 16, 14 and 39, respectively.)

(A) 32.2 g (B) 69.4 g

(C) 91.5 g (D) 162.5 g

Official Ans. by NTA (C)

Sol. $4HNO_3(\ell) + 3KCl(s) \rightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(g)$

x gm **110 gm**

$\frac{x}{63}$ Mole $= \frac{110}{101}$

$4 \rightarrow 3$

$1 \rightarrow \frac{3}{4}$

$\frac{x}{63} \rightarrow \frac{3}{4} \times \frac{x}{63} = \frac{110}{101}$

$x = \frac{110 \times 63 \times 4}{101 \times 3} = 91.5\,gm$

2. Given below are the quantum numbers for 4 electrons.

A. $n = 3, l = 2, m_l = 1, m_s = +1/2$

B. $n = 4, l = 1, m_l = 0, m_s = +1/2$

C. $n = 4, l = 2, m_l = -2, m_s = -1/2$

D. $n = 3, l = 1, m_l = -1, m_s = +1/2$

The correct order of increasing energy is :

(A) D < B < A < C (B) D < A < B < C

(C) B < D < A < C (D) B < D < C < A

Official Ans. by NTA (B)

Sol. Energy order of subshell decided by (n+λ) rule.

$A \Rightarrow 3d \Rightarrow n + l = 5$

$B \Rightarrow 4p \Rightarrow n + \lambda = 5$

$C \Rightarrow 4d \Rightarrow n + \ell \Rightarrow 6$

$D \Rightarrow 3s \Rightarrow (n + \ell) = 4$

D < A < B < C

3. $C(s) + O_2(g) \rightarrow CO_2(g) + 400\ kJ$

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g) + 100\ kJ$

When coal of purity 60% is allowed to burn in presence of insufficient oxygen, 60% of carbon is converted into 'CO' and the remaining is converted into 'CO_2'.

The heat generated when 0.6 kg of coal is burnt is _____.

(A) 1600 kJ (B) 3200 kJ

(C) 4400 kJ (D) 6600 kJ

Official Ans. by NTA (D)

Sol. $C(S) + O_2(g) \rightarrow CO_2(g) + 400\ kJ$

1 g mole

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g) + 100kJ$ .... (II)

0.6×1000

$= 600\ gm$

$600 \times \frac{60}{100}$ (Pure Carbon)

$= 360gm = \frac{360}{12} = 30mole$ (Pure Carbon)

Carbon converted into $CO_2 = \left(30 - 30 \times \frac{60}{100}\right)$

$= 12$ mole

and carbon converted in CO $= 30 \times \frac{60}{100} = 18mole$

Energy generated during II equation

$= 18 \times 100$

$= 1800$ kJ

Energy generated during I[st] reaction.

$= 12 \times 400$

$= 4800$

Total $= 1800 + 4800 = 6600$ kJ

4. 200 mL of 0.01 M HCl is mixed with 400 mL of 0.01M H_2SO_4. The pH of the mixture is ____.

(A) 1.14 (B) 1.78

(C) 2.34 (D) 3.02

Official Ans. by NTA (B)

Sol. $HCl + H_2SO_4$

$$[H^+] = \frac{(0.01\times 200)+(0.01\times 2\times 400)}{600}$$

$$= \frac{2+8}{600} = \frac{10}{600} = \frac{1}{60}$$

$$pH = -\log\left[\frac{1}{60}\right]$$

= 1.78

5. Given below are the critical temperatures of some of the gases :

Gas	Critical temperature (K)
He	5.2
CH_4	190
CO_2	304.2
NH_3	405.5

The gas showing least adsorption on a definite amount of charcoal is :

(A) He (B) CH_4

(C) CO_2 (D) NH_3

Official Ans. by NTA (A)

Sol. More the critical temp. of gas greater is the ease of liquefaction hence greater is the adsorption.

6. In liquation process used for tin (Sn), the metal :

(A) is reacted with acid

(B) is dissolved in water

(C) is brought to molten form which is made to flow on a slope

(D) is fused with NaOH.

Official Ans. by NTA (C)

Sol. Liquation process is used for metal having low melting point such as tin in which they are heated and brought to molten state and made to flow down the slope while impurities with higher melting point left on the top.

7. Given below are two statements.

Statement I : Stannane is an example of a molecular hydride.

Statement II : Stannane is a planar molecule.

In the light of the above statement, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are true.

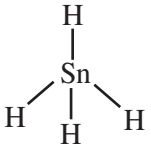
(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (C)

Sol. SnH_4 is non planar molecular hydride



Tetrahedral shape, sp^3 hybridisation

8. Portland cement contains 'X' to enhance the setting time. What is 'X'?

(A) $CaSO_4.\frac{1}{2}H_2O$ (B) $CaSO_4.2H_2O$

(C) $CaSO_4$ (D) $CaCO_3$

Official Ans. by NTA (B)

Sol. Gypsum ($CaSO_4.2H_2O$) is used to enhance setting time in portland cement.

9. When borax is heated with CoO on a platinum loop, blue coloured bead formed is largely due to :

(A) B_2O_3 (B) $Co(BO_2)_2$

(C) CoB_4O_7 (D) $Co[B_4O_5(OH)_4]$

Official Ans. by NTA (B)

Sol. $Na_2B_4O_7{\cdot}10H_2O \xrightarrow{\Delta} Na_2B_4O_7 + 10H_2O$

$Na_2B_4O_7 \xrightarrow{\Delta} 2NaBO_2$(sodium meta borate) $+ B_2O_3$

$B_2O_3 + CoO \rightarrow Co(BO_2)_2$(cobalt (II) meta borate)

Blue Bead

10. Which of the following 3d-metal ion will give the lowest enthalpy of hydration ($\Delta_{hyd}H$) when dissolved in water ?

(A) Cr^{2+} (B) Mn^{2+}

(C) Fe^{2+} (D) Co^{2+}

Official Ans. by NTA (B)

Sol.

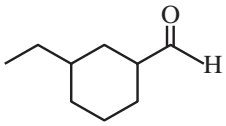
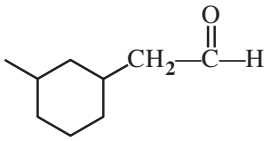
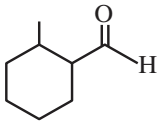
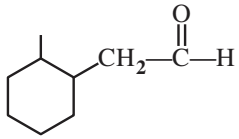
Ion	$\Delta H^{o}_{Hyd.}$ (kJ/mole)
Cr^{2+}	–1925
Mn^{2+}	–1862
Fe^{2+}	–1998
Co^{2+}	–2079

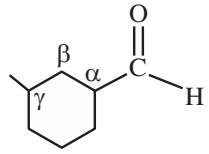
11. Octahedral complexes of copper (II) undergo structural distortion (Jahn-Teller). Which one of the given copper (II) complexes will show the maximum structural distortion ?
(en–ethylenediamine; $H_2N\text{-}CH_2\text{-}CH_2\text{-}NH_2$)
(A) $[Cu(H_2O)_6]SO_4$ (B) $[Cu(en)(H_2O)_4]SO_4$
(C) cis-$[Cu(en)_2Cl_2]$ (D) trans-$[Cu(en)_2Cl_2]$
Official Ans. by NTA (A)

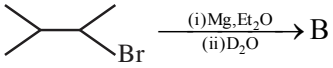
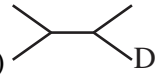
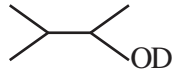
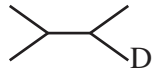
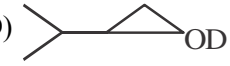
Sol. There is unsymmetric filling of e_g subset of Cu^{+2} ion, while there is symmetrical distribution in t_{2g} set, if the complex has same ligand there will be equal repulsion which leads to symmetrical bond length along t_{2g} , but due to uneven filling of electron in e_g subset, either octahedral will be elongated or compressed.

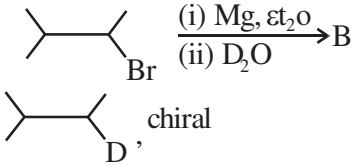
12. Dinitrogen is a robust compound, but reacts at high altitude to form oxides. The oxide of nitrogen that can damage plant leaves and retard photosynthesis is :
(A) NO (B) NO_3^-
(C) NO_2 (D) NO_2^-
Official Ans. by NTA (C)

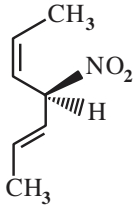
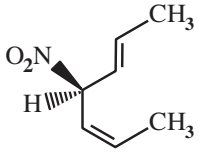
Sol. $N_2(g) + O_2(g) \rightarrow 2NO(g)$
$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$
NO_2 damage plant leaves

13. Correct structure of γ-methylcyclohexane carbaldehyde is :
(A) 
(B) 
(C) 
(D) 
Official Ans. by NTA (A)

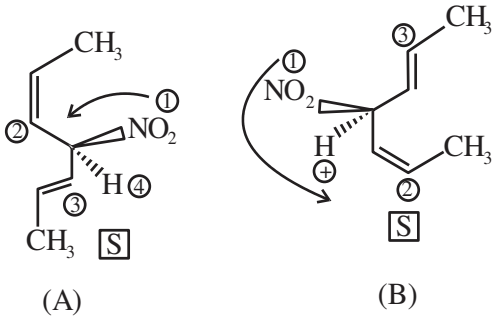
Sol. 
γ-methyl cyclohexane carbaldehyde

14. Compound 'A' undergoes following sequence of reactions to give compound 'B'. The correct structure and chirality of compound 'B' is:
[where Et is $-C_2H_5$]

Compound 'A'
(A)  , Achiral
(B)  , Chiral
(C)  , Chiral
(D)  , Achiral
Official Ans. by NTA (C)

Sol. 
chiral

15. Given below are two statements.
Statement I : The compound  is optically active.
Statement II :  is mirror image of above compound A.
In the light of the above statement, choose the **most appropriate** answer from the options given below.
(A) Both Statement I and Statement II are correct
(B) Both Statement I and Statement II are incorrect.
(C) Statement I is correct but Statement II is incorrect.
(D) Statement I is incorrect but Statement II is correct.
Official Ans. by NTA (C)

Sol.



(A) (B)

Having same configuration.

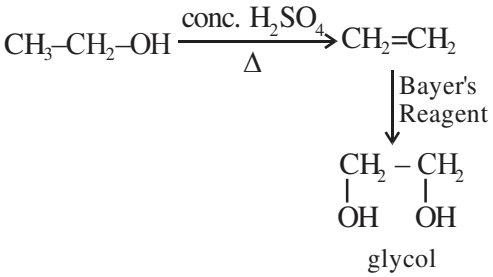
16. When enthanol is heated with conc. H_2SO_4, a gas is produced. The compound formed, when this gas is treated with cold dilute aqueous solution of Baeyer's reagent, is :

(A) Formaldehyde (B) Formic acid

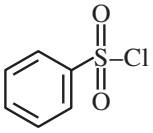
(C) Glycol (D) Ethanoic acid

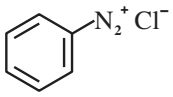
Official Ans. by NTA (C)

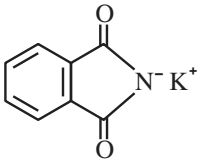
Sol.

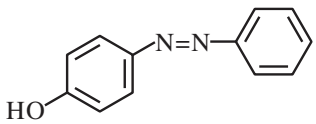


17. The Hinsberg reagent is :

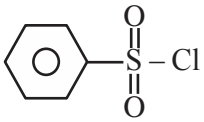
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. 

B.S.C (Benzene sulphonyl chloride) is known's Hinsberg Reagent

18. Which of the following is **NOT** a natural polymer?

(A) Protein

(B) Starch

(C) Rubber

(D) Rayon

Official Ans. by NTA (D)

Sol. Rayon is semisynthetic polymer.

19. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Amylose is insoluble in water.

Reason R : Amylose is a long linear molecule with more than 200 glucose units.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of A.

(B) Both **A** and **R** are correct and **R** is NOT the correct explanation of A.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (D)

Sol. Amylose is water soluble.

20. A compound 'X' is a weak acid and it exhibits colour change at pH close to the equivalence point during neutralization of NaOH with CH_3COOH. Compound 'X' exists in ionized form in basic medium. The compound 'X' is :

(A) methyl orange (B) methyl red

(C) phenolphthalein (D) erichrome Black T

Official Ans. by NTA (C)

Sol. Phenolphthalein is weak acid give colour in basic medium.

SECTION-B

1. 'x' g of molecular oxygen (O_2) is mixed with 200 g of neon (Ne). The total pressure of the non-reactive mixture of O_2 and Ne in the cylinder is 25 bar. The partial pressure of Ne is 20 bar at the same temperature and volume. The value of 'x' is_____.

[Given: Molar mass of O_2 = 32 g mol^{-1}.

Molar mass of Ne = 20 g mol^{-1}]

Official Ans. by NTA (80)

Sol. O_2 + Ne

Xgm 200gm

P_{total} = 25 bar ; P_{Ne} = 20

$$P_{O_2} + P_{Ne} = 25$$

$$P_{O_2} = 25 - 20 = 5\,\text{bar}$$

$$5 = \frac{\frac{x}{32}}{\frac{x}{32}+\frac{200}{20}} \times 25$$

$$\frac{1}{5} = \frac{\frac{x}{32}}{\frac{x}{32}+10}$$

$$\frac{1}{5} = \frac{x \times 32}{32(x+320)}$$

$$5x = x + 320$$

$$4x = 320$$

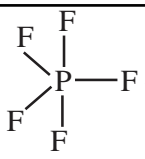
$$x = \frac{320}{4} = 80\,\text{gm}$$

2. Consider, PF_5, BrF_5, PCl_3, SF_6, $[ICl_4]^-$, ClF_3 and IF_5.

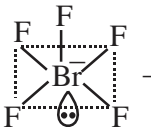
Amongst the above molecule(s)/ion(s), the number of molecule(s)/ion(s) having sp^3d^2 hybridisation is____.

Official Ans. by NTA (4)

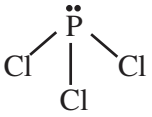
Sol.



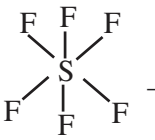
$\rightarrow sp^3d$ Hybridisation



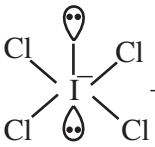
$\rightarrow sp^3d^2$ Hybridisation



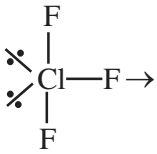
$\rightarrow sp^3$ Hybridisation



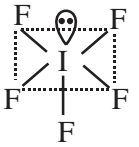
$\rightarrow sp^3d^2$ Hybridisation



$\rightarrow sp^3d^2$ Hybridisation



$\rightarrow sp^3d$ Hybridisation



$\rightarrow sp^3d^2$ Hybridisation

3. 1.80 g of solute A was dissolved in 62.5 cm^3 of ethanol and freezing point of the solution was found to be 155.1 K. The molar mass of solute A is __g mol^{-1}.

[Given: Freezing point of ethanol is 156.0 K.

Density of ethanol is 0.80 g cm^{-3}.

Freezing point depression constant of ethanol is 2.00 K kg mol^{-1}]

Official Ans. by NTA (80)

Sol. Mass of C_2H_5OH = 62.5 × 0.8 = 50 g

$$\Delta T_f = K_f \times m$$

$$0.9 = 2 \times \frac{1.8 \times 1000}{M_w \times 50}$$

$$M_w = \frac{2 \times 1.8 \times 1000}{0.9 \times 50} = 80$$

4. For a cell, Cu(s) $|Cu^{2+}(0.001M)|$ $|Ag^{+}(0.01M)|$ Ag(s) the cell potential is found to be 0.43 V at 298 K. The magnitude of standard electrode potential for Cu^{2+}/Cu is ___ $\times 10^{-2}$ V.

$$\left[\text{Given : } E^{\Theta}_{Ag^{+}/Ag} = 0.80V \text{ and } \frac{2.303RT}{F} = 0.06V\right]$$

Official Ans. by NTA (34)

Sol. At anode

$Cu \rightarrow Cu^{2+} + 2e^{-}$

At cathode

$2Ag^{+} + 2e^{-} \rightarrow 2Ag$

Cell reaction $\rightarrow Cu + 2Ag^{+} \rightarrow Cu^{2+} + 2Ag$

$$E_{cell} = E^{0}_{cell} - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^{+}]^2}$$

$$0.43 = E^{0}_{cell} - \frac{0.06}{2}\log\frac{(0.001)}{(0.01)^2}$$

$$E^{0}_{cell} = 0.46$$

$$E^{0}_{cell} = E^{0}_{Ag^{+}/Ag} - E^{0}_{Cu^{2+}/Cu}$$

$$0.46 = 0.80 - E^{0}_{Cu^{2+}/Cu}$$

$$E^{0}_{Cu^{2+}/Cu} = 0.34 \text{ volt}$$

$$E^{0}_{Cu^{2+}/Cu} = 34 \times 10^{-2}$$

5. Assuming 1μg of trace radioactive element X with a half life of 30 years is absorbed by a growing tree. The amount of X remaining in the tree after 100 years is___$\times 10^{-1}$μg.

[Given : ln 10 = 2.303; log2 = 0.30]

Official Ans. by NTA (1)

Sol. $$t = \frac{1}{\lambda}\ell n\left(\frac{a}{a-x}\right)$$

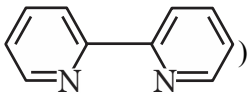
$$100 = \frac{30}{\ell n 2}\ell n\left(\frac{1}{w}\right)$$

$$\frac{1}{w} = 10$$

$W = 0.1 \times \mu g$

Ans. 1×10^{-1} μg

6. Sum of oxidation state (magnitude) and coordination number of cobalt in $Na[Co(bpy)Cl_4]$ is__.

(Given bpy = )

Official Ans. by NTA (9)

Sol. Coordination no. = 6

Oxidation state = 3

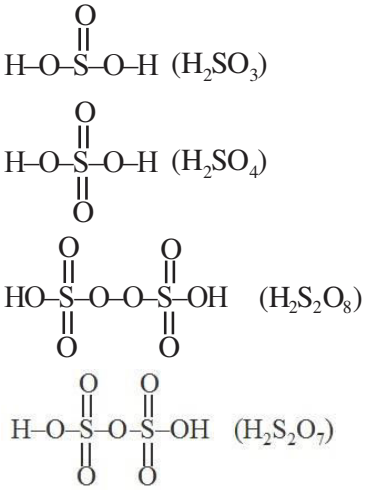
6 + 3 = 9

7. Consider the following sulphure based oxoacids.

H_2SO_3, H_2SO_4, $H_2S_2O_8$ and $H_2S_2O_7$.

Amongst these oxoacids, the number of those with peroxo(O-O) bond is_____.

Official Ans. by NTA (1)

Sol. 

8. A 1.84 mg sample of polyhydric alcoholic compound 'X' of molar mass 92.0 g/mol gave 1.344 mL of H_2 gas at STP. The number of alcoholic hydrogens present in compound 'X' is____.

Official Ans. by NTA (3)

Sol. $R(OH)_x \rightarrow H_2$

PoAC on H –

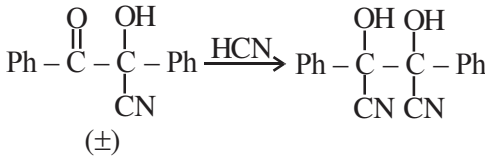
$$x\left(\frac{1.84\times10^{-3}}{92}\right) = \frac{1.344}{22.4}\times 2$$

$$x = \frac{1.344\times2\times92\times1000}{1.84\times22400} = 6$$

$x = 6$

9. The number of stereoisomers formed in a reaction of (±) Ph(C=O) C(OH)(CN)Ph with HCN is_____.

Official Ans. by NTA (3)

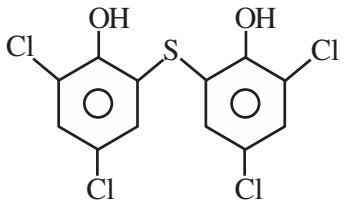
Sol. 

3 stereoisomers

10. The number of chlorine atoms in bithionol is____.

Official Ans. by NTA (4)

Sol. Bithinol



Chlorine atoms = 4

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If $z \neq 0$ be a complex number such that $|z - \frac{1}{z}| = 2$, then the maximum value of $|z|$ is:

(A) $\sqrt{2}$ (B) 1

(C) $\sqrt{2}-1$ (D) $\sqrt{2}+1$

Official Ans. by NTA (D)

Sol. $|z - 1/z| = 2$

$$\left||z| - \frac{1}{|z|}\right| \le \left|z - \frac{1}{z}\right| \le |z| + \frac{1}{|z|} \qquad \text{Let } |z| = r$$

$$\left|r - \frac{1}{r}\right| \le 2 \le r + \frac{1}{r}$$

$$\left|r - \frac{1}{r}\right| \le 2 \;\&\; r + \frac{1}{r} \ge 2 \text{ always true}$$

$$r - \frac{1}{r} \ge -2 \;\&\; r - \frac{1}{r} \le 2$$

$r^2 - 1 \le 2r$

$r^2 - 2r \le 1$

$(r-1)^2 \le 2$

$r - 1 \le \sqrt{2}$

$\therefore |z|_{max} = 1 + \sqrt{2}$

2. Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

(A) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Official Ans. by NTA (C)

Sol. $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

(1) $R_1 \to R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \to R_2 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

3. If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to :

(A) 8 (B) 36

(C) 44 (D) 48

Official Ans. by NTA (C)

Sol. $x + y + z = 6$ ___(1)

$2x + 5y + \alpha z = \beta$ ___(2)

$x + 2y + 3z = 14$ ____(3)

$x + y = 6 - z$

$x + 2y = 14 - 3z$

On solving

$x = z - 2 \Rightarrow y = 8 - 2z$ in (2)

$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$

$(\alpha - 8) z = \beta - 36$ For having infinite solutions

$\alpha - 8 = 0$ & $\beta - 36 = 0$

$\alpha = 8, \beta = 36$ $(\alpha + \beta = 44)$

4. Let the function

$$f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\text{if } x\neq 0\\ 10 & ;\text{if } x=0\end{cases}$$

be continuous at x = 0.

The α is equal to :

(A) 10 (B) –10

(C) 5 (D) –5

Official Ans. by NTA (D)

Sol. $$f(x)=\begin{cases}\dfrac{\ln(1+5x)-\ln(1+\alpha x)}{x} & ;x\neq 0\\ 10 & ;x=0\end{cases}$$

$$\lim_{x\to 0}\frac{\ln(1+5x)-\ln(1+\alpha x)}{x}=10$$

Using expension

$$\lim_{x\to 0}\frac{(5x+.....)-(\alpha x+.....)}{x}=10$$

$5-\alpha=10\Rightarrow\alpha=-5$

5. If [t] denotes the greatest integer ≤ t, then the value of $\int_0^1\left[2x-|3x^2-5x+2|+1\right]dx$ is:

(A) $\dfrac{\sqrt{37}+\sqrt{13}-4}{6}$ (B) $\dfrac{\sqrt{37}-\sqrt{13}-4}{6}$

(C) $\dfrac{-\sqrt{37}-\sqrt{13}+4}{6}$ (D) $\dfrac{-\sqrt{37}+\sqrt{13}+4}{6}$

Official Ans. by NTA (A)

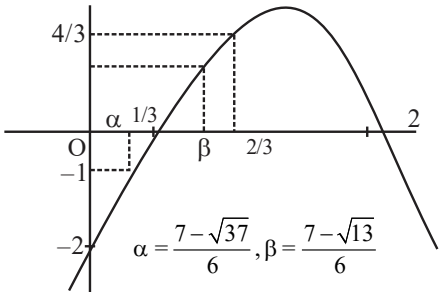
Sol. $$I=\int_0^1\left[2x-|3x^2-3x-2x+2|+1\right]dx$$

$$I=\int_0^1\left[2x-|(3x-2)(x-1)|\right]dx+\int_0^1 1dx$$

$$I=\int_0^{2/3}\left[\left(2x-\left(3x^2-5x+2\right)\right)\right]dx+\int_{2/3}^1\left(2x+\left(3x^2-5x+2\right)\right)dx+1$$

$$I=\int_0^{2/3}\left[-3x^2+7x-2\right]dx+\int_{2/3}^1\left(3x^2-3x+2\right)dx+1$$

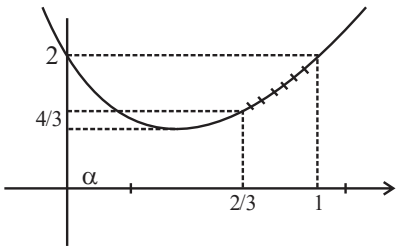
$y=-3x^2+7x-2$



$$\int_0^{\alpha}(-2)dx+\int_{\alpha}^{1/3}(-1)dx+\int_{1/3}^{\beta}0dx+\int_{\beta}^{2/3}1.dx$$

$$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$$

$y=3x^2-3x+2$



When $x\in\left(\frac{2}{3},1\right)$

$3x^2-3x+2\in\left(\frac{4}{3},2\right)$

$[3x^2-3x+2]=1$

$$\therefore\int_{2/3}^1\left[3x^2-3x+2\right]dx=1\left(1-\frac{2}{3}\right)=\frac{1}{3}$$

Hence $I=\left(\frac{1}{3}-(\alpha+\beta)\right)+\left(\frac{1}{3}\right)+1$

$$=\frac{5}{3}-\left(\frac{7-\sqrt{37}}{6}+\frac{7-\sqrt{13}}{6}\right)$$

$$=\frac{-2}{3}+\frac{\sqrt{37}+\sqrt{13}}{6}$$

$$=\frac{\sqrt{37}+\sqrt{13}-4}{6}$$

6. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall\ n \geq 0$.

Then $a_{25}\,a_{23} - 2\,a_{25}\,a_{22} - 2\,a_{23}\,a_{24} + 4\,a_{22}\,a_{24}$ is equal to:

(A) 483 (B) 528 (C) 575 (D) 624

Official Ans. by NTA (B)

Sol. $a_0 = 0,\ a_1 = 0$

$a_{n+2} = 3\,a_{n+1} - 2\,a_{n+1} : n \geq 0$

$a_{n+2} - a_{n+1} = 2\,(a_{n+1} - a_n) + 1$

$n = 0 \qquad a_2 - a_1 = 2\,(a_1 - a_0) + 1$

$n = 1 \qquad a_3 - a_2 = 2\,(a_2 - a_1) + 1$

$n = 2 \qquad a_4 - a_3 = 2\,(a_3 - a_2) + 1$

$n = n \qquad a_{n+2} - a_{n+1} = 2\,(a_{n+1} - a_n) + 1$

$(a_{n+2} - a_1) - 2\,(a_{n+1} - a_0) - (n+1) = 0$

$a_{n+2} = 2a_{n+1} + (n+1)$

$n \to n - 2$

$a_n - 2a_{n-1} = n - 1$

Now $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$= (a_{25} - 2a_{24})\,(a_{23} - 2a_{22}) = (24)\,(22) = 528$

7. $\sum_{r=1}^{20}\left(r^2+1\right)(r!)$ is equal to:

(A) $22! - 21!$ (B) $22! - 2\,(21!)$

(C) $21! - 2\,(20!)$ (D) $21! - 20!$

Official Ans. by NTA (B)

Sol. $\sum_{x=1}^{20}\left(r^2+1\right)r!$

$\sum_{x=1}^{20}\left((r+1)^2 - 2r\right)r!$

$\sum_{x=1}^{20}\left((r+1)(r+1)! - r.r!\right) - \sum_{r=1}^{20} r.r!$

$\sum_{x=1}^{20}\left((r+1)(r+1)! - r.r!\right) - \sum_{r=1}^{20}\left((r+1)! - r!\right)$

$= (21.\underline{|21} - 1) - (\underline{|21} - 1)$

$= 20.21! = 22! - 2.21!$

8. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then

(A) $3^{1010}I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(B) $3^{1010}I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(D) $3^{1011}I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

Official Ans. by NTA (A)

Sol. $I(x) = \int \sec^2 x.\sin^{-2022} x\,dx - 2022\int \sin^{-2022} x\,dx$

$\qquad\qquad\quad II \qquad\qquad I$

$= \tan x.(\sin x)^{-2022} + \int (2022)\tan x.(\sin x)^{-2023}\cos x dx$

$-2022\int (\sin x)^{-2022}dx$

$I(x) = (\tan x)\,(\sin x)^{-2022} + C$

At $X = \pi/4,\ 2^{1011} = \left(\frac{1}{\sqrt{2}}\right)^{-2022} + C \therefore C = 0$

Hence $I(x) = \frac{\tan x}{(\sin x)^{2022}}$

$I(\pi/6) = \frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}} = \frac{2^{2022}}{\sqrt{3}}$

$I(\pi/3) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}} = \frac{2^{2022}}{\left(\sqrt{3}\right)^{2021}} = \frac{1}{3^{1010}}I\left(\frac{\pi}{6}\right)$

$3^{1010}I(\pi/3) = I(\pi/6)$

9. If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passes through the point (2,1) and (k + 1,2), k > 0, then

(A) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

(B) $\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(k^2+1\right)$

(C) $2\tan^{-1}\left(\frac{1}{k+1}\right) = \log_e\left(k^2+2k+2\right)$

(D) $2\tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(\frac{k^2+1}{k^2}\right)$

Official Ans. by NTA (A)

Sol. $\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

$x-1=X,\ y-1=Y$

$\frac{dY}{dX}=\frac{X+Y}{X-Y}$

$Y=VX \qquad \frac{dY}{dX}=V+X\frac{dV}{dX}$

$V+X\frac{dV}{dX}=\frac{1+V}{1-V} \qquad X\frac{dV}{dX}=\frac{V^2+1}{1-V}$

$\int\frac{1-V}{1+V^2}dV=\int\frac{dX}{X}$

$\int\frac{dV}{1+V^2}-\frac{1}{2}\int\frac{2VdV}{1+V^2}=\int\frac{dX}{X}$

$\tan^{-1}V-\frac{1}{2}\ln\left(1+V^2\right)=\ln X+c$

$\tan^{-1}\left(\frac{Y}{X}\right)-\frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right)=\ln(X)+c$

$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{2}\ln\left(1+\frac{(y-1)^2}{(x-1)^2}\right)=\ln(x-1)+c$

Passes through (2,1)

$0-\frac{1}{2}\ln1=\ln1+c\ \therefore\ c=0$

Passes through (k + 1, 2)

$\therefore\ \tan^{-1}\left(\frac{1}{k}\right)-\frac{1}{2}\ln\left(1+\frac{1}{k^2}\right)=\ln k$

$2\tan^{-1}\left(\frac{1}{k}\right)=\ln\left(\frac{1+k^2}{k^2}\right)+2\ln k$

$2\tan^{-1}\left(\frac{1}{k}\right)=\ln(1+k^2)$

10. Let y = y (x) be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1}$, $x>-1$, which passes through the point (0,1). Then y (1) is equal to:

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{x+3}{x+1}$

$\int p(x)dx \qquad \text{I.F.}=e^{\int p(x)dx}$

$\int p(x)dx=\int\frac{(2x^2+11x+13)dx}{(x+1)(x+2)(x+3)}$

Using partial fraction

$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$

$A=\frac{4}{2}=2$

$B=1$

$C=-1$

$\because\int p(x)dx=A\ln(x+1)+B\ln(x+2)+c\ln(x+3)$

$=\ln\left(\frac{(x+1)^2(x+2)}{x+3}\right)$

$\text{I.F.}=e^{\int p(x)dx}=\frac{(x+1)^2(x+2)}{(x+3)}$

Solution $y(\text{IF})=\int Q.(\text{IF})dx$

$y\left(\frac{(x+1)^2(x+2)}{x+3}\right)=\int\left(\frac{\cancel{x+3}}{\cancel{x+1}}\right)\frac{(x+1)\cancel{^1}(x+2)}{\cancel{(x+3)}}dx$

$y\left(\frac{(x+1)^2(x+2)}{x+3}\right)=\frac{x^3}{3}+\frac{3x^2}{2}+2x+c$

Passes through (0, 1) $C=\frac{2}{3}$

Now put x = 1

$\Rightarrow y(1)=\frac{3}{2}$

11. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3\left(m_2^2+m_2^2\right)=220$. If one vertex of the square is (10(cosα − sinα), 10 (sinα + cosα)), where $\alpha\in\left(0,\frac{\pi}{2}\right)$ and the equation of one diagonal is (cosα − sinα) x + (sinα + cosα) y = 10, then 72 $(\sin^4\alpha+\cos^4\alpha)+a^2-3a+13$ is equal to:

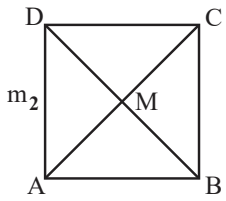
(A) 119 (B) 128

(C) 145 (D) 155

Official Ans. by NTA (B)

Sol. $m_1m_2 = -1$

$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$



Eq. of AC

AC = $(\cos\alpha - \sin\alpha) + (\sin\alpha + \cos\alpha)\, y = 10$

BD = $(\sin\alpha - \cos\alpha)\, x + (\sin\alpha - \cos\alpha)\, y = 0$

$(10(\cos\alpha - \sin\alpha), 10(\sin\alpha - \cos\alpha))$

Slope of AC = $\left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right) = \tan\theta = M$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m \pm \tan\frac{\pi}{4}}{1 \pm m\tan\frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} + 1}{1 - \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)}, \frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} - 1}{1 + \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}}$$

$m_1, m_2 = \tan\alpha, \cot\alpha$

mid point of AC & BD

= M $(5(\cos\alpha - \sin\alpha), 5(\cos\alpha + \sin\alpha))$

B $(10(\cos\alpha - \sin\alpha), 10(\cos\alpha + \sin\alpha))$

$a = AB = \sqrt{2}\,BM = \sqrt{2}\left(5\sqrt{2}\right) = 10$

a = 10

$\because a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1 2}\right) = 220$

$100 + 110 + 3(\tan^2\alpha + \cot^2\alpha) = 220$

Hence $\tan^2\alpha = 3$, $\tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3} \text{ or } \frac{\pi}{6}$

Now $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$

$$= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13$$

$$= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128$$

12. The number of elements in the set $S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\}$ is:

(A) 1 (B) 3

(C) 0 (D) infinite

Official Ans. by NTA (A)

Sol. $2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}$

L.H.S ≤ 2. & R.H.S. ≥ 2

Hence L.H.S = 2 & R.H.S = 2

$2\cos\left(\frac{x^2 + x}{6}\right) = 2 \quad 4^x + 4^{-x} = 2$

Check x = 0 Possible hence only one solution.

13. Let A $(\alpha, -2)$, B $(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is NOT correct about ΔABC:

(A) ares is 24 (B) perimeter is 25

(C) circumradius is 5 (D) inradius is 2

Official Ans. by NTA (B)

Sol. A $(\alpha, -2)$: B $(\alpha, 6)$: $C\left(\frac{\alpha}{4}, -2\right)$

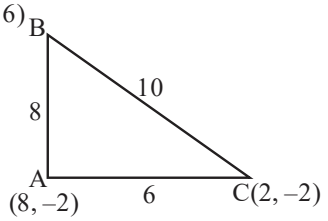
since AC is perpendicular to AB.

So, ΔABC is right angled at A.

Circumcentre = mid point of BC. = $\left(\frac{5\alpha}{8}, 2\right)$

$\therefore \frac{5\alpha}{8} = 5 \;\&\; \frac{\alpha}{4} = 2$

$\alpha = 8$



Area = $\frac{1}{2}(6)(8) = 24$

Perimeter = 24

Circumradius = 5

Inradius = $\frac{\Delta}{s} = \frac{24}{12} = 2$

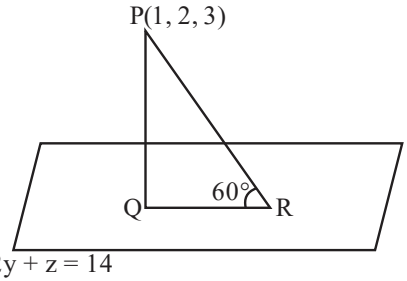
14. Let Q be the foot of perpendicular drawn from the point P (1, 2, 3) to the plane x + 2y + z = 14. If R is a point on the plane such that $\angle PRQ = 60°$, then the area of ΔPQR is equal to:

(A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{3}$

(C) $2\sqrt{3}$ (D) 3

Official Ans. by NTA (B)

Sol.



Length of perpendicular

$$PQ = \left|\frac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$$

$QR = (PQ)\cot 60° = \sqrt{2}$

$\therefore$ Area of $\Delta PQR = \frac{1}{2}(PQ)(QR) = \sqrt{3}$

15. If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is:

(A) $\frac{21}{2}$ (B) $\frac{59}{8}$

(C) $\frac{57}{8}$ (D) $\frac{95}{8}$

Official Ans. by NTA (D)

Sol. A(2, 3, 9) ; B(5, 2, 1) ; C(1, λ, 8) ; D(λ, 2, 3)

$$\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix} = 0$$

$\Rightarrow [-6(\lambda - 3) - 1] - 8(1 - (\lambda - 3)(\lambda - 2)) + (6 + (\lambda - 2)) = 0$

$3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$

$8\lambda^2 - 57\lambda + 95 = 0$

$\lambda_1 \lambda_2 = \frac{95}{8}$

16. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is draw from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:

(A) $\frac{4}{9}$ (B) $\frac{5}{18}$ (C) $\frac{1}{6}$ (D) $\frac{3}{10}$

Official Ans. by NTA (B)

Sol.

3R	2R
4B	5B
3W	2W

A : Drown ball from boy II is black

B : Red ball transferred

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{9}\times\frac{5}{10}}{\frac{3}{9}\times\frac{5}{10} + \frac{4}{9}\times\frac{6}{10} + \frac{3}{9}\times\frac{5}{10}}$$

$$= \frac{15}{15+24+15} = \frac{15}{54} = \frac{5}{18}$$

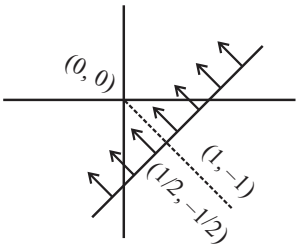
17. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

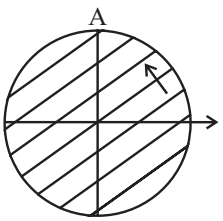
(A) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

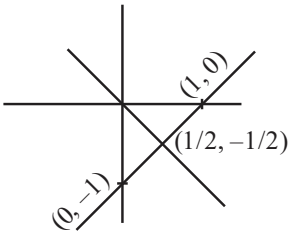
(C) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

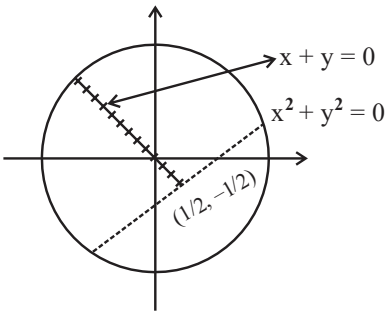
Official Ans. by NTA (B)

Sol. $|z - 1 + i| \geq |z|$; $|z| < 2$; $|z + i| = |z - 1|$









Hence

$w = 2x + iy \in S$

$2x \leq \frac{1}{2} \ \therefore x \leq \frac{1}{4}$

Now

$(2x)^2 + (2x)^2 < 4$

$x^2 < \frac{1}{2} \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore \ x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{4}\right]$

18. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a} \times \vec{b}).(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}).(\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}).(\vec{a} \times \vec{b}) = 168$ then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to:

(A) 10 (B) 14

(C) 16 (D) 18

Official Ans. by NTA (C)

Sol. $|\vec{a}||\vec{b}||\vec{c}| = 14$

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \frac{2\pi}{3}$

So, $\vec{a}.\vec{b} = -\frac{1}{2}ab$, $\vec{b}.\vec{c} = -\frac{1}{2}bc$, $\vec{a}.\vec{c}. = -\frac{1}{2}ac$

(let)

$(\vec{a} \times \vec{b}).(\vec{b} \times \vec{c}) = (\vec{a}.\vec{b})(\vec{b}.\vec{c}) - (\vec{a}.\vec{c})(\vec{b}.\vec{b})$

$= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c$

Similarly

$(\vec{b} \times \vec{c}).(\vec{c} \times \vec{a}) = \frac{3}{4}abc^2$

$(\vec{c} \times \vec{a}).(\vec{a} \times \vec{b}) = \frac{3}{4}a^2bc$

$168 = \frac{3}{4}abc(a + b + c)$

So, $(a + b + c) = 16$

19. The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is :

(A) $[1, \infty)$ (B) $(-1, 2]$

(C) $[-1, \infty)$ (D) $(-\infty, 2]$

Official Ans. by NTA (C)

Sol. $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ Domain

$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \geq -1$ and $\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$

$2x^2 - x + 9 \geq 0$ and $5x \geq -5 \Rightarrow x \geq -1$

$x \in R$

Hence Domain $x \in [-1, \infty)$

20. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to:

(A) $(p \wedge (\sim r)) \Rightarrow q$ (B) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(C) $p \Rightarrow (q \vee r)$ (D) $(p \wedge (\sim q)) \Rightarrow r$

Official Ans. by NTA (B)

Sol. $(p \to q) \vee (p \to r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$= \sim p \vee (q \vee r)$

$= p \to (q \vee r) \equiv (3)$ is true.

Now (1) $(p \wedge \sim r) \to q$

$\sim(p \wedge \sim r) \vee q = (\sim p \vee r) \vee q$

$= \sim p \vee (r \vee q) = p \to (q \vee r)$

(4) $(p \wedge \sim q) \to r = p \to (q \vee r)$

SECTION-B

1. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. They the number of trials in the binomial distribution is:

Official Ans. by NTA (96)

Sol. Let, mean = m = np

& variance = v = npq, p + q = 1

Sum = $m + v = \frac{165}{2}$

Product = mv = 1350

On solving,

$m = np = 60$ & $v = npq = \frac{45}{2}$ $\therefore q = \frac{3}{8}$ $\therefore P = \frac{5}{8}$

Hence n = 96

2. Let α, β $(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _____.

Official Ans. by NTA (16)

Sol. $Pn = \alpha^n - \beta^n$ $\quad x^2 - x - 4 = 0$

$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ __(1)

As $P_n - P_{n-1} = (\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})$

$= \alpha^{n-2}(\alpha^2 - \alpha) - \beta^{n-2}(\beta^2 - \beta)$

$= 4(\alpha^{n-2} - \beta^{n-2})$

$P_n - P_{n-1} = 4 P_{n-2}$

Hence Expression (1)

$\frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$

$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}} = 16$

3. Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if $X' A^k X = 33$, then k is equal to:

Official Ans. by NTA (10)

Sol. $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$; $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$X^T A^K X = 33$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

As $A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{10}=\begin{bmatrix}1&0&6\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&24\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&30\\0&1&0\\0&0&1\end{bmatrix}$$

for $K \to$ Even $A^K=\begin{bmatrix}1&0&3K\\0&1&0\\0&0&1\end{bmatrix}$

$X^TA^KX = 33$ (This is not correct)

$$[1\ 1\ 1]\begin{bmatrix}1&0&3K\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$=[1\ 1\ 3K+1]\begin{bmatrix}1\\1\\1\end{bmatrix}=[3K+3]$$

$\therefore$ 3K + 3 = 33 $\therefore$ K = 10

But it should be dropped as 33 is not matrix

If K is odd

$X^TA^KX = 33$

$X^TAA^{K-1}X = 33$

$$\begin{bmatrix}1&1&1\end{bmatrix}\begin{bmatrix}-1&2&3\\0&1&6\\0&0&-1\end{bmatrix}\begin{bmatrix}1&0&3k-3\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=33$$

$$\begin{bmatrix}-1&3&8\end{bmatrix}\begin{bmatrix}3k-2\\1\\1\end{bmatrix}=[33]$$

[–3k + 13] = [33]

k = 20/3 (not possible)

4. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is____,

Official Ans. by NTA (6)

Sol. 4 digit numbers

For divisibility by 55, no. should be div. by 5 and 11 both

Also, for divisibility by 11

			5
a	b	c	d

a + c = b + 5

for b = 1 a = 2, c = 4

a = 4, c = 2

for b = 2 a = 3, c = 4

a = 4, c = 3

for b = 3 a = 6, c = 2

a = 2, c = 6

$\therefore$ 6 possible four digit no.s are div. by 55

(II) 5 digit number is not possible

(Not possible)

5. If $\sum_{k=1}^{10} K^2\left(10_{C_K}\right)^2 = 22000L$, then L is equal to ___.

Official Ans. by NTA (221)

Sol. $\sum_{K=1}^{10} K^2\left({}^{10}C_K\right)^2$

$$\sum_{K=1}^{10}\left(K.{}^{10}C_K\right)^2=\sum_{K=1}^{10}\left(10.{}^{9}C_{K-1}\right)^2$$

$$=100\sum_{K=1}^{10}{}^{9}C_{K-1}.{}^{9}C_{10-K}$$

$$=100\left({}^{18}C_9\right)=100\left(\frac{18!}{9!9!}\right)$$

$\Rightarrow$ 4862000 = 22000L

Hence L = 221

6. If [t] denotes the greatest integer $\le$ t, then number of points, at which the function $f(x) = 4|2x + 3| + 9\left[x+\frac{1}{2}\right] - 12[x + 20]$ is not differentiable in the open interval (–20, 20), is___.

Official Ans. by NTA (79)

Sol. $f(x) = 4|2x + 3| + 9\left[x+\frac{1}{2}\right] - 12[x + 20]$

$x \in (-20, 20)$

f(x) is not Diff. at $x = I \in \{-19, -18, ...0,...19\} = 39$

at $x = I + \frac{1}{2}$, f(x) Non diff. at 39 points

Check at $x = \frac{-3}{2}$ Discount at $x = \frac{-3}{2}$ $\therefore$ N. R(1)

No. of point of non-differentiabilty

= 39 + 39 + 1 = 79

7. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a , b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, –1), then $|2a + 9b|$ is equal to _____.

Official Ans. by NTA (195)

Sol. $y = 5x^2 + 2x - 25$ $\quad P(2, -1)$

$y' = 10x + 2$

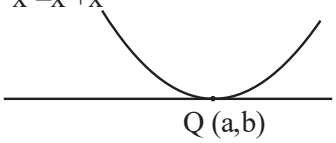
$y'_P = 22$

$\therefore$ tangent to curve at P

$y + 1 = 22\,(x - 2)$

$y = 22x - 45$

$y = x^3 - x^2 + x$



$\left.\frac{dy}{dx}\right|_{C_2} = 3x^2 - 2x + 1$

$\left.\frac{dy}{dx}\right|_{Q} = 3a^2 - 2a + 1$

Hence $3a^2 - 2a + 1 = 22$

$\therefore\ 3a^2 - 2a - 21 = 0$

$3a^2 - 9a + 7a - 21 = 0$

$(3a + 7)\,(a - 3) = 0 \Rightarrow a = 3,\ a = -7/3$

from curve $b = a^3 - a^2 + a$

$a = 3$

$b = 21$ $\quad |2a + 9b| = 195$

at $a = -7/3$ tangent will be parallel

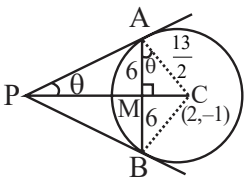
Hence it is rejected

8. Let AB be a chord of length 12 of the circle $(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$.

If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ___.

Official Ans. by NTA (72)

Sol.



$\cos\theta = \frac{6}{\frac{13}{2}} = \frac{12}{13}$

$\sin\theta = \frac{5}{13}$

$PM = AM \cot\theta$

$PM = 6\left(\frac{12}{5}\right)\ \therefore\ 5(PM) = 72$

9. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2, \vec{a}.\vec{b} = 3$ and $|\vec{a} \times \vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to____.

Official Ans. by NTA (14)

Sol. $\left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2\ ;\ \vec{a}.\vec{b} = 3$

As $\cancel{|\vec{a}|^2} + |\vec{b}|^2 + 2\vec{a}.\vec{b} = \cancel{|\vec{a}|^2} + 2|\vec{b}|^2$

$|\vec{b}|^2 = 2\vec{a}.\vec{b} = 6$

$|\vec{a} \times \vec{b}|^2 = 75$

$|\vec{a}|^2|\vec{b}|^2 - \left(\vec{a}.\vec{b}\right)^2 = 75$

$6|\vec{a}|^2 - 9 = 75 \Rightarrow |\vec{a}|^2 = 14$

10. Let

$S=\left\{(x,y)\in\mathbb{N}\times\mathbb{N}:9(x-3)^2+16(y-4)^2\le 144\right\}$

and $T=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}:(x-7)^2+(y-4)^2\le 36\right\}$.

The $n(S\cap T)$ is equal to_____.

Official Ans. by NTA (27)

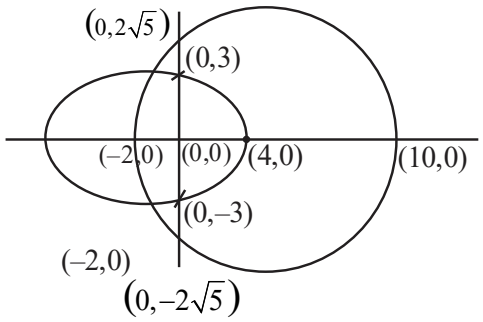
Sol. $S:\frac{(x-3)^2}{16}+\frac{(y-4)^2}{9}\le 1$; x, y ∈ {1, 2, 3, ...}

$T:(x-7)^2+(y-4)^2\le 36$ x, y ∈ R

Let x – 3 = x : y – 4 = y

$S:\frac{x^2}{16}+\frac{y^2}{9}\le 1$; x ∈ {–2, –1, 0, 1, ...}

$T:(x-4)^2+y^2\le 36$; y ∈ {–3, –2, –1, 0, ...}



S ∩ T = (–2, 0), (–1, 0), ...(4, 0) → (7)

(–1, 1), (0, 1), ...(3, 1) → (5)

(–1, –1), (0, –1), ...(3, –1) → (5)

(–1, 2), (0, 2), (1, 2), (2, 2) → (4)

(–1, –2), (0, –2), (1, –2), (2, –2) → (4)

(0, 3) (0, –3) → (2)