JEE Main 2021 Question Paper with Answer

February, March & August (Shift 1 & Shift 2)

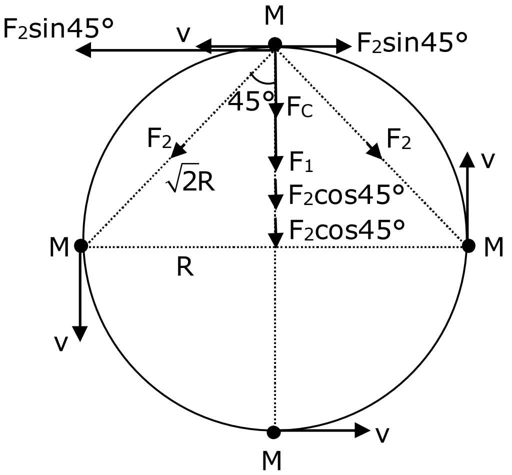
24^{th} Feb. 2021 | Shift - 1
PHYSICS

Section - A

1. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be -

(1) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$ (2) $\sqrt{G(1+2\sqrt{2})}$ (3) $\sqrt{\frac{G}{2}\left(2\sqrt{2}-1\right)}$ (4) $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$

Sol. **(1)**



$\Rightarrow$ By resolving force F_2, we get

$\Rightarrow F_1 + F_2\ \cos 45° + F_2 \cos 45°$

$\Rightarrow F_1 + 2F_2 \cos 45° = F_c$

F_c = centripital force = $\frac{MV^2}{R}$

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{\left(\sqrt{2}R\right)^2}\cos 45°\right] = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM^2}{4R^2} + \frac{2GM^2}{2\sqrt{2}R^2} = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM}{4R} + \frac{GM}{\sqrt{2}.R} = V^2$$

$$\Rightarrow V = \sqrt{\frac{GM}{4R}+\frac{GM}{\sqrt{2}.R}}$$

$$\Rightarrow V = \sqrt{\frac{GM}{R}\left[\frac{1+2\sqrt{2}}{4}\right]}$$

$$\Rightarrow V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

(given : mass = 1 kg, radius = 1 m)

$$\Rightarrow v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}$$

2. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits.The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is -

(1)8 : 1 (2)1 : 8 (3)2 : 1 (4) 1 : 4

Sol. **(1)**

We know that $\omega = \frac{2\pi}{T}$

given : Ratio of time period

$\frac{T_1}{T_2} = \frac{1}{8}$

$\Rightarrow \omega \propto \frac{1}{T}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$

$\Rightarrow \omega_1 : \omega_2 = 8 : 1$

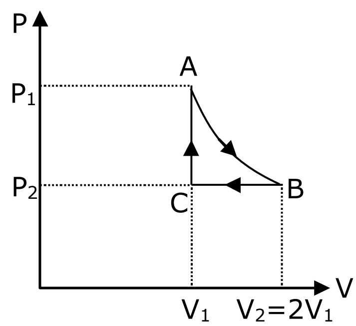
3. n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes –

$A \rightarrow B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

$B \rightarrow C$: Isobaric compression at pressure P_2 to initial volume V_1.

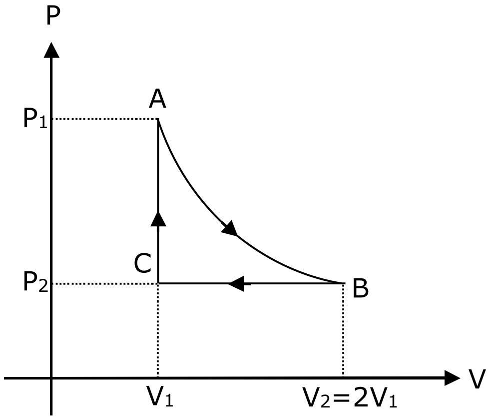
$C \rightarrow A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle ABCA is –



(1)0 (2)$nRT\left(\ln 2 + \frac{1}{2}\right)$ (3)$nRT\ln 2$ (4) $nRT\left(\ln 2 - \frac{1}{2}\right)$

Sol. **(4)**



$A \rightarrow B$ = isotheraml process

$B \rightarrow C$ = isobaric process

$C \rightarrow A$ = isochoric process

also, $V_2 = 2V_1$

work done by gas in the complete cycle ABCA is –

$\Rightarrow w = w_{AB} + w_{BC} + w_{CA}$ ....(1)

$\Rightarrow w_{CA} = 0$, as isochoric process

$\Rightarrow w_{AB} = 2P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 2\ nRT \ln(2)$

$\Rightarrow w_{BC} = P_2(V_1 - V_2) = P_2(V_1 - 2V_1) = -P_2V_1 = -nRT$

$\Rightarrow$Now put the value of w_{AB}, w_{BC} and w_{CA} in equation, we get

$\Rightarrow w = 2nRT \ln(2) - nRT + 0$

$\Rightarrow w = nRT\ [2\ln(2) - 1]$

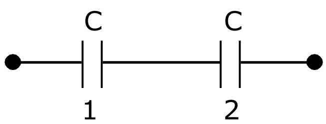
$\Rightarrow w = nRT\ [\ln(2) - \frac{1}{2}]$

4. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities capacities in the two cases will be -

(1)2 : 1 (2)1 : 4 (3)4 : 1 (4) 1 : 2

Sol. **(2)**

Given that first connection



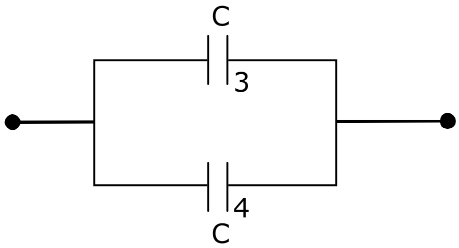
$\Rightarrow \frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_{12} = \frac{C}{2}$

Second connection

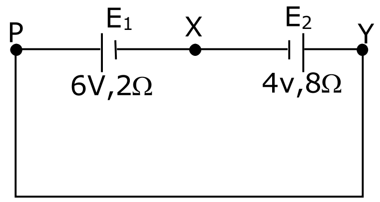
$C_{34} = C + C = 2\ C$

Now, the ratio of equivalent capacities in the two cases will be –

$$\Rightarrow \frac{C_{12}}{C_{34}} = \frac{C/2}{2C} \Rightarrow \frac{C_{12}}{C_{34}} = \frac{1}{4}$$

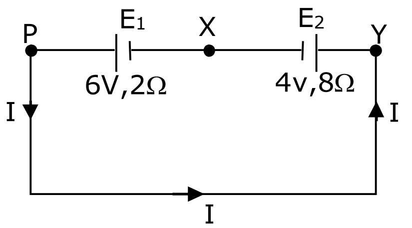


5. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is –



(1)3.6V (2)10.0V (3)5.6V (4) 2.0V

Sol. **(3)**



emf of E_1 = 6v

$r_1 = 2\ \Omega$

emf of E_2= 4 Ω

$r_2 = 8\Omega$

$|v_x - v_y|$ = potential difference across points x and y

$E_{eff} = 6 - 4 = 2$ V

$R_{eq} = 2 + 8 = 10\ \Omega$

So, current in the circuit will be

$$\Rightarrow I = \frac{E_{eff}}{R_{eq}} \Rightarrow I = \frac{2}{10} = 0.2 \text{ A}$$

Now, potential difference across points X and Y is

$|v_x - v_y| = E + iR$

$\Rightarrow |v_x - v_y| = 4 + 0.2 \times 8 = 5.6$ V

$\Rightarrow |v_x - v_y| = 5.6$ v

6. If Y,K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(1) $K = \dfrac{Y\eta}{9\eta - 3Y}$ N/m^2 (2) $\eta = \dfrac{3YK}{9K + Y}$ N/m^2

(3) $Y = \dfrac{9K\eta}{3K - \eta}$ N/m^2 (4) $Y = \dfrac{9K\eta}{2\eta + 3K}$ N/m^2

Sol. **(1)**

$\Rightarrow y = 3k\,(1 - 2\sigma)$

$\Rightarrow \sigma = \dfrac{1}{2}\left(1 - \dfrac{y}{3k}\right)$ ....(1)

$\Rightarrow y = 2\eta\,(1 + \sigma)$

$\Rightarrow \sigma = \dfrac{y}{2\eta} - 1$ ....(2)

by comparing equation (1) and (2), we get

$\Rightarrow \dfrac{y}{2\eta} - 1 = \dfrac{1}{2}\left(1 - \dfrac{y}{3k}\right)$

$\Rightarrow \dfrac{y}{\eta} - 2 = 1 - \dfrac{y}{3k}$

$\Rightarrow \dfrac{y}{\eta} = 1 + 2 - \dfrac{y}{3k} \Rightarrow \dfrac{y}{\eta} = 3 - \dfrac{y}{3k}$

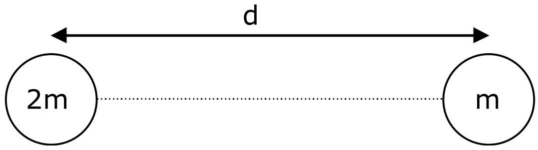
$\Rightarrow \dfrac{y}{3k} = 3 - \dfrac{y}{\eta} \Rightarrow \dfrac{y}{3k} = \dfrac{3\eta - y}{\eta}$

$\Rightarrow k = \dfrac{\eta y}{9\eta - 3y}$

7. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is -

(1) $2\pi\sqrt{\dfrac{d^3}{3Gm}}$ (2) $\dfrac{1}{2\pi}\sqrt{\dfrac{3Gm}{d^3}}$ (3) $\dfrac{1}{2\pi}\sqrt{\dfrac{d^3}{3Gm}}$ (4) $2\pi\sqrt{\dfrac{3Gm}{d^3}}$

Sol. **(1)**



$\Rightarrow \dfrac{G(m)(2m)}{d^2} = m\omega^2 \times \dfrac{2d}{3}$

$\Rightarrow \dfrac{2Gm}{d^2} = \omega^2 \times \dfrac{2d}{3}$

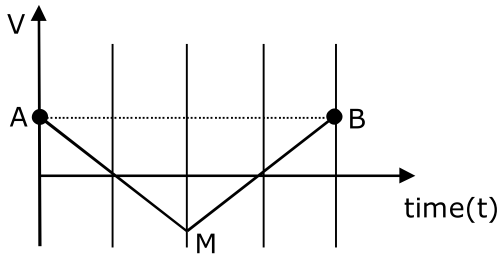
$\Rightarrow \omega^2 = \dfrac{3Gm}{d^3}$

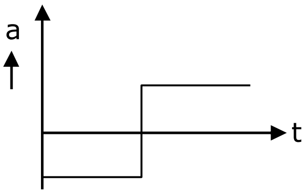
$\Rightarrow \omega = \sqrt{\dfrac{3Gm}{d^3}}$

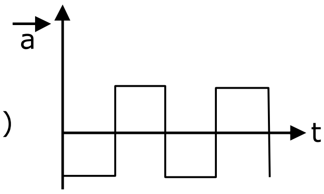
we know that, $\omega = \dfrac{2\pi}{T}$ so $T = \dfrac{2\pi}{\omega}$

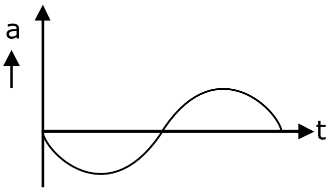
$\Rightarrow T = \dfrac{2\pi}{\sqrt{\dfrac{3Gm}{d^3}}} \Rightarrow T = 2\pi\sqrt{\dfrac{d^3}{3Gm}}$

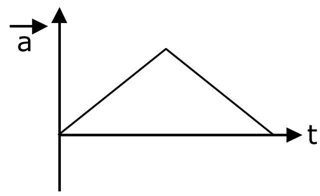
8. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



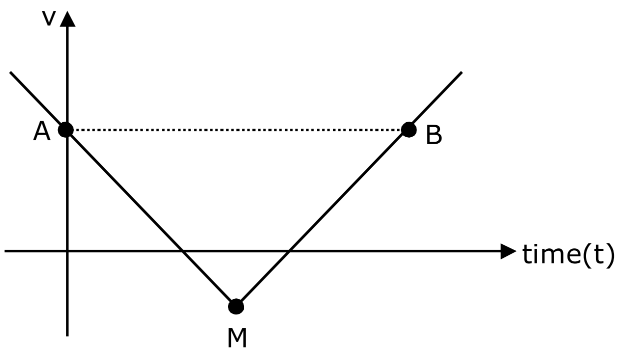
(1)


(2)


(3)


(4)


Sol. **(1)**



$a = \dfrac{dv}{dt}$ = slope of (v – t) curve

If m = +ve, then equation of straight line is

$y = mx + c \Rightarrow v = mt + c$ (for MB)

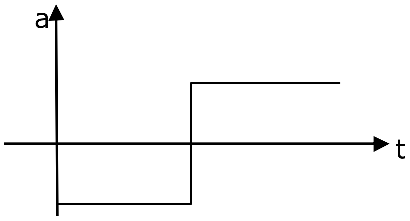
If m =-ve, then equation of straight line is

$y = -mx + c \Rightarrow v = -mt + c$ (for AM)

If we differentiate equation (1) and (2), we get

a_{MB} = +ve = m

a_{AM} =-ve = -m, so graph of (a-t) will be



9. Given below are two statements :

Statement – I: Two photons having equal linear momenta have equal wavelengths.

Statement-II: If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

(1)Statemnet-I is false but Statement-II is true

(2)Both Statement-I and Statement-II are true

(3)Both Statement-I and Statement-II are false

(4) Statement-I is true but Statement-II is false

Sol. **(4)**

By theory

10. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$

where $\alpha_0 = 20$ A/s and $\beta = 8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

(1)2100 C (2)260 C (3)2250 C (4) 11250 C

Sol. **(4)**

given :$i = \alpha_0 t + \beta t^2$

$\alpha = 20$ A/s and $\beta = 8$As^{-2}

t = 15 sec

we know that, $i = \dfrac{dq}{dt} \Rightarrow \int_0^t i dt = \int_0^Q dq$

$$\Rightarrow \int_0^{15} (\alpha_0 t + \beta t^2) dt = \int_0^Q dq$$

$$\Rightarrow Q = \left[\frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3}\right]_0^{15}$$

$\Rightarrow Q = \dfrac{20 \times 15 \times 15}{2} + \dfrac{8 \times 15 \times 15 \times 15}{3} - 0$

$\Rightarrow Q = 11250\ C$

11. match List I with List II

List-I	List-II
(a) Isothermal	(i) Pressure constant
(b) Isochoric	(ii) Temperature constant
(c) Adiabatic	(iii) Volume constant
(d) Isobaric	(iv) Heat content is constant

Choose the correct answer from the options given below –

(1)(a) – (ii), (b) – (iv), (c) – (iii), (d) – (i)

(2)(a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)

(3)(a) – (i), (b) – (iii), (c) – (ii), (d) – (iv)

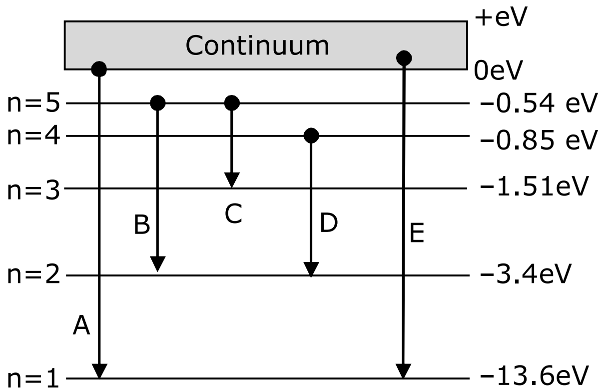
(4) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)

Sol. (**2**)

(a)$\rightarrow$(ii), (b)$\rightarrow$(iii), (c)$\rightarrow$(iv), (d)$\rightarrow$(i),
By theory
In isothermal process, temperature is constant.
In isochoric process, volume is constant.
In adiabatic process, heat content is constant.
In isobaric process, pressure is constant.

12. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A,B,C,D and E.

The transitions A,B and C respectively represents –



(1)The series limit of Lyman series, third member of balmer series and second member of paschen series

(2)The first member of the Lyman series, third member of Balmer series and second member of paschen series

(3)The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series

(4) The series limit of Lyman series, second memebr of Balmer series and second member of Paschen series.

Sol. **(1)**

A $\rightarrow$ series limit of lyman.

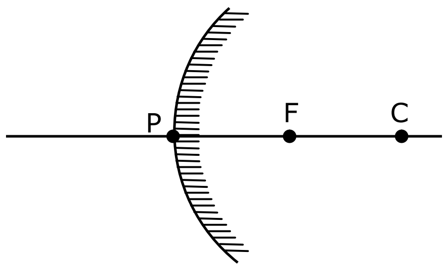
B $\rightarrow$ 3rd member of Balmer series.

C $\rightarrow$ 2nd member of Paschen series.

13. The focal length f is related to the radius of curvature r of the spherical convex mirror by -

(1) $f = r$ (2) $f = -\frac{1}{2}r$ (3) $f = +\frac{1}{2}r$ (4) $f = -r$

Sol. **(3)**



So, $\frac{R}{2} = f$

$F = +\frac{1}{2}R$

14. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as –

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then :-

(1) $I_1 = I_2 = I_3 < I_4$ (2) $I_1 + I_2 = I_3 + \frac{5}{2} I_4$

(3) $I_1 + I_3 < I_2 + I_4$ (4) $I_1 = I_2 = I_3 > I_4$

Sol. **(4)**

Given$\Rightarrow$ I_1 = M.I. of thin circular ring about its diameter

I_2 = M.I. circular disc about an axis perpendicular to disc and going through the centre.

I_3 = M.I. of solid cylinder about its axis

I_4 = M.I. of solid sphere about its diameter

we know that,

$I_1 = \frac{MR^2}{2}$, $I_2 = \frac{MR^2}{2}$, $I_3 = \frac{MR^2}{2}$

$I_4 = \frac{2}{5}\ MR^2$

So, $I_1 = I_2 = I_3 > I_4$

15. The workdone by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature.α and β are constants. Then the dimensions of β will be -

(1)$[M^0LT^0]$ (2)$[M^2LT^2]$ (3)$[MLT^{-2}]$ (4) $[ML^2T^{-2}]$

Sol. **(3)**

given : work = $\alpha.\beta^2.e^{-\frac{x^2}{\alpha.k.T}}$

k = boltzmann constant

T = temperature

x = displacement

we know that, $\frac{x^2}{\alpha.k.T}$ = dimensionless

$\left[\frac{x^2}{\alpha.k.T}\right] = [M^0L^0T^0]$

$[\alpha] = \left[\frac{L^2}{K.T}\right]$

$\Rightarrow [K] = [M^1L^2T^{-2}K^{-1}]$

$[T] = [K]$

$\Rightarrow [\alpha] = \left[\frac{L^2}{M^1L^2T^{-2}K^{-1} \times K}\right] \Rightarrow [\alpha] = [M^{-1}T^2]$

$\Rightarrow \omega = \alpha.\beta^2$

$\Rightarrow \frac{[M^1L^1T^{-2}][L^{-1}]}{[M^{-1}T^2]} = [\beta^2] = [M^2L^2T^{-4}]$

$[\beta] = [MLT^{-2}]$

16. If an emitter current is changed by 4mA, the collector current changes by 3.5 mA.The value of β will be -

(1)7 (2)0.875 (3)0.5 (4) 3.5

Sol. **(1)**

Given :

$\Delta I_E = 4$ mA

$\Delta I_C = 3.5$ mA

we know that, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

$\Rightarrow \alpha = \frac{3.5}{4} = \frac{7}{8}$

Also, $\beta = \frac{\alpha}{1-\alpha}$, so

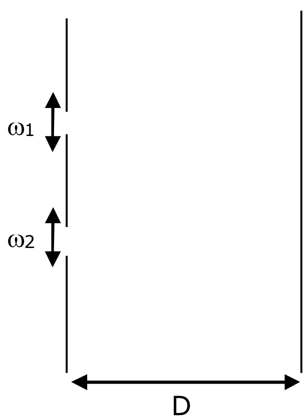
$$\beta = \frac{\frac{7}{8}}{1-\frac{7}{8}} = \frac{7}{1}$$

$\beta = 7$

17. In a Young's double slit experiment, the width ofthe one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

(1)4 : 1 (2)2 : 1 (3)3 : 1 (4) 1 : 4

Sol. **(1)**



given : $\omega_2 = 3\omega_1$

also, $A \propto \omega$

$\frac{\omega_1}{\omega_2} = \frac{1}{3}$ ...(1)

Assume $\omega_1 = x$, $\omega_2 = 3x$

we know that

$I_{max} = (A_1 + A_2)^2$, and

$I_{min} = (A_1 - A_2)^2$

$$\frac{A_1}{A_2} = \frac{\omega_1}{\omega_2} \qquad ....(2)$$

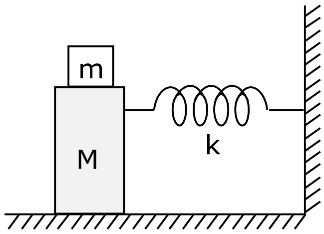
from equation (2) we can say that

$A_1 = A$ and $A_2 = 3A$

Now, $\frac{I_{max}}{I_{min}} = \frac{(A+3A)^2}{(A-3A)^2} = \frac{16A^2}{4A^2} = \frac{4}{1}$

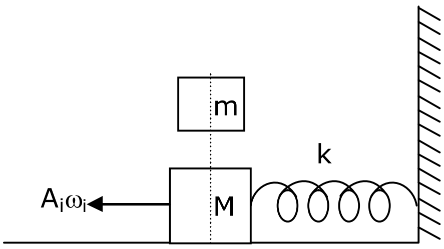
$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{4}{1}$

18. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be -

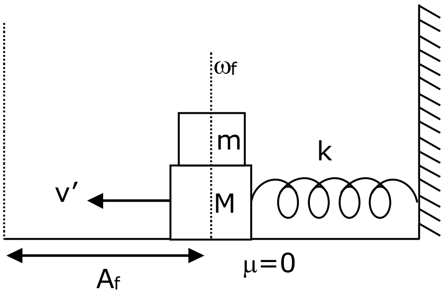


(1) $A\sqrt{\frac{M}{M+m}}$ (2) $A\sqrt{\frac{M}{M-m}}$ (3) $A\sqrt{\frac{M-m}{M}}$ (4) $A\sqrt{\frac{M+m}{M}}$

Sol. **(1)**



Before placing



After placing

We know that $\omega = \sqrt{\frac{k}{m}}$ and $\omega_i = \sqrt{\frac{k}{M}}$ $A_i = A$

Also, momentum is conserved just before and just after the block of mass (m) is placed because there is no implusive force. So -

$MA_i\omega_i = (M + m)\,v'$

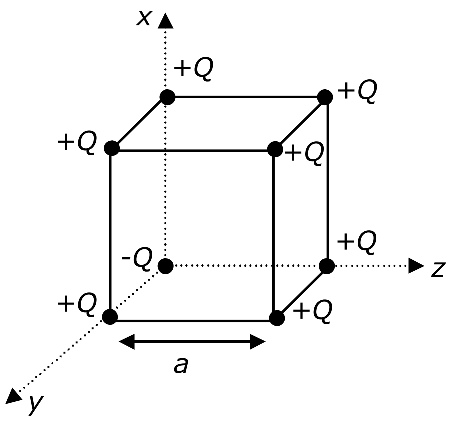
$v' = \frac{MA_i\omega_i}{(M+m)} \Rightarrow v' = A_f\omega_f$

$\frac{MA\omega_i}{(M+m)} = A_f\sqrt{\frac{K}{(M+m)}}$

$\Rightarrow \frac{MA\sqrt{\frac{K}{M}}}{M+m} \times \sqrt{\frac{M+m}{K}} = A_f$

$\Rightarrow A_f = A\sqrt{\frac{M}{(M+m)}}$

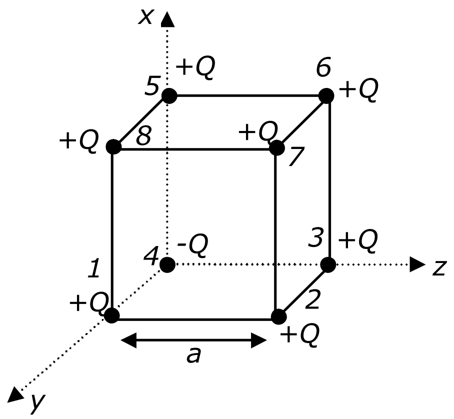
19. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is –Q. The electric field at the centre of cube is :



(1) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$ (2) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

(3) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$ (4) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

Sol. **(3)**



If only +Q charges are placedat the corners of cube of side a then electric field at the centre of the cube will be zero.

But in the given condition one (-Q) is placed at one corner of cube so here

$E_1 = E_6$, $E_2 = E_5$ and $E_3 = E_8$ (So it will cancel out each other so electric field at centre is due to Q_4 and Q_7.

Here electric field at centre = 2 $(E.f.)_4$

As, $|E_4| = |E_7|$

$(E.F)_C = \dfrac{2kQ}{\left(\dfrac{\sqrt{3}a}{2}\right)^2} = \dfrac{8KQ}{3a^2}$ $\left\{\because K = \dfrac{1}{4\pi\varepsilon_0}\right\}$

$(E.F)_C = \dfrac{2Q}{3a^2\pi\varepsilon_0}$

In vector form $\Rightarrow \vec{E} = \dfrac{-2Q}{3a^2\pi\varepsilon_0} \times \left(\dfrac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)$

20. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temeprature is $T+\Delta T$. Calculate theincrease in the volume of the metal box-

(1) $\dfrac{4}{3}\pi a^3\alpha\Delta T$ (2) $4\pi a^3\alpha\Delta T$ (3) $3a^3\alpha\Delta T$ (4) $4a^3\alpha\Delta T$

Sol. **(3)**

volume expansion $\gamma = 3\alpha$

$\dfrac{\Delta V}{V} = \gamma\Delta T$

$\Delta V = V.\gamma\Delta T$

$\Delta V = a^3.\ 3\alpha\Delta T$

SECTION-B

1. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is________.
[π = 3.14]

Sol. **2000**

Given : R = 6.28 Ω

f = 10 MHz

L = 2×10^{-4} Henry

we know that quality factor Q is given by

$\Rightarrow Q = \frac{X_L}{R} = \frac{\omega L}{R}$

also, $\omega = 2\pi f$, so

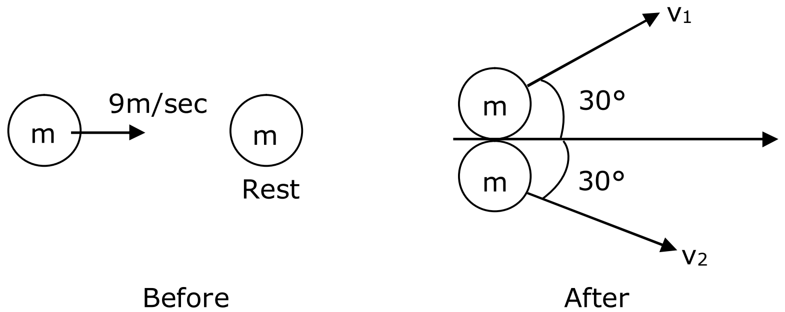
$\Rightarrow Q = \frac{2\pi f L}{R}$

$\Rightarrow Q = \frac{2\pi \times 10 \times 10^6 \times 2 \times 10^{-4}}{6.28} = 2000$

Q = 2000

2. A ball with a speed of 9 m/s collides with another identical ball at rest. After thecollision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is _________.

Sol. **1**



Momentum is conserved just before and just after the collision in both x-y direction.

In y-direction

$p_i = 0$

$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$

$p_i = p_f$, so

$= \frac{mv_1}{2} - \frac{mv_2}{2} = 0$

$\Rightarrow \frac{mv_1}{2} = \frac{mv_2}{2} \Rightarrow v_1 = v_2$

$\frac{v_1}{v_2} = 1$

3. An audio signal $\upsilon_m = 20\sin 2\pi(1500t)$ amplitude modulates a carrier $\upsilon_c = 80 \sin 2\pi (100{,}000t)$. The value of percent modulation is ________.

Sol. **25**

Given : $\upsilon_m = 20 \sin \left[100\pi t + \frac{\pi}{4}\right]$

$\upsilon_c = 80 \sin \left[10^4 \pi t + \frac{\pi}{6}\right]$

we know that, modulation index = $\frac{A_m}{A_c}$

from given equations, $A_m = 20$ and $A_c = 80$

percentage modulation index = $\frac{A_m}{A_c} \times 100$

$\Rightarrow \frac{20}{80} \times 100 = 25\%$

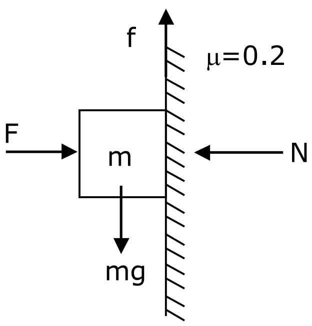
The value of percentage modulation index is

=25

4. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be ______ N.

[$g = 10\ ms^{-2}$]

Sol. **25**



Given : $\mu_s = 0.2$

$m = 0.5$ kg

$g = 10\ m/s^2$

we know that

$f_s = \mu N$ and ....(1)

To keep the block adhere to the wall

here $N = F$ ...(2)

$f_s = mg$ ....(3)

from equation (1), (2), and (3), we get

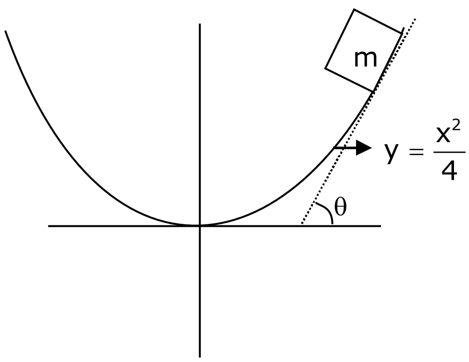
$\Rightarrow mg = \mu F$

$\Rightarrow F = \frac{mg}{\mu} \Rightarrow F = \frac{0.5 \times 10}{0.2}$

$F = 25$ N

5. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is ______ cm.

Sol. **25**



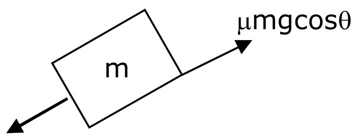
given

$y = \frac{x^2}{4}$

$\mu = 0.5$

condition for block will not slip downward

$mg \sin\theta = \mu mg \cos\theta$



$\Rightarrow \tan\theta = \mu$

and we know that

$\Rightarrow \tan\theta = \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \mu \Rightarrow \dfrac{x}{2} = 0.5$ $\left[\begin{array}{l} y = \dfrac{x^2}{4} \\ \dfrac{dy}{dx} = \dfrac{x}{2} \end{array}\right]$

$\Rightarrow x = 1,$

put $x = 1$ in equation $y = x^2/4$

$\Rightarrow y = \dfrac{(1)^2}{4} \Rightarrow y = \dfrac{1}{4} \Rightarrow y = 0.25$

y = 25 cm

6. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability bothare 2. Its velocity in this medium is _______ × 10^7 m/s.

Sol. 15

Given : f = 5 GHz

$\varepsilon_r = 2$

$\mu_r = 2$

velocity of wave $\Rightarrow v = \dfrac{c}{n}$ ....(1)

where, $n = \sqrt{\mu_r \varepsilon_r}$ and c = speed of light = 3×10^8 m/s

$n = \sqrt{2 \times 2} = 2$

put the value of n in we get

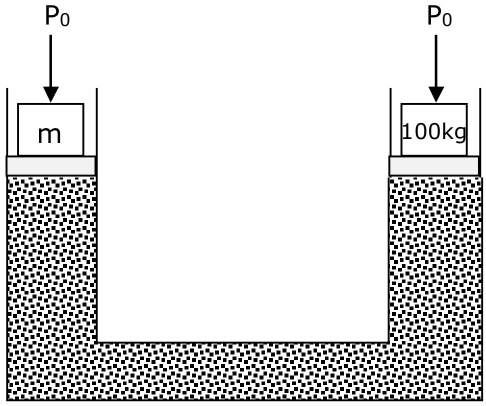
$\Rightarrow v = \dfrac{3 \times 10^8}{2} = 15 \times 10^7$ m/s

$\Rightarrow X \times 10^7 = 15 \times 10^7$

X = 15

7. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _______ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Sol. **25600**



Atmospheric pressure P_0 will be acting on both the limbs of hydraulic lift.

Applying pascal's law for same liquid level

$$\Rightarrow P_0 + \frac{mg}{A_1} = P_0 + \frac{(100)g}{A_2}$$

$$\Rightarrow \frac{Mg}{A_1} = \frac{(100)g}{A_2} \Rightarrow \frac{m}{100} = \frac{A_1}{A_2} \quad ...(1)$$

Diameter of piston on side of 100 kg is increased by 4 times so new area = $16A_2$

Diameter of piston on side of (m) kg is decreasing

$$A_1 = \frac{A_1}{16}$$

(In order to increasing weight lifting capacity, diameter of smaller piston must be reduced)

Again, $\dfrac{mg}{\left(\dfrac{A_1}{16}\right)} = \dfrac{M'g}{16A_2} \Rightarrow \dfrac{256m}{M'} = \dfrac{A_1}{A_2}$

From equation (1) = $\dfrac{256m}{M'} = \dfrac{m}{100} \Rightarrow \therefore M' = 25600$ kg

8. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are ______.

Sol. **440**

Given

Primary voltage, $V_p = 220$ V

Secondary voltage, $v_s = 12$ V

No. of turns in secondary coil is $N_s = 24$

no. of turns in primary coil, $N_p = ?$

We know that for a transformer

$\Rightarrow \frac{N_p}{N_s} = \frac{V_p}{V_s}$

$\Rightarrow N_p = \frac{V_p \times N_S}{V_s} = \frac{220 \times 24}{12}$

$\Rightarrow N_p = 440$

9. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging fromthe analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be________ Lumens.

Sol. 75

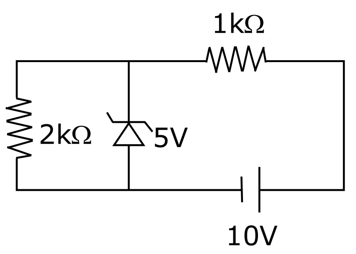
Given :I_0 = 100 lumens, $\theta = 30^{\circ}$

$I_{net} = I_0 \cos^2\theta$

$I_{net} = 100 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{100 \times 3}{4}$

I_{net} = 75 lumens

10. In connection with the circuit drawn below, the value of current flowing through 2kΩresistor is _______ × 10^{-4} A.



Sol. 25

In zener diode there will be o change in current after 5V zener diode breakdown

$\Rightarrow i = \frac{5}{2 \times 10^3}$

$\Rightarrow i = 2.5 \times 10^{-3}$ A

$\Rightarrow i = 25 \times 10^{-4}$ A

24^{th} Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. The gas released during anaerobic degradation of vegetation may lead to:
(1) Global warming and cancer (2) Acid rain
(3) Corrosion of metals (4) Ozone hole

Ans. **(1)**

Sol. Biogas is the mixtrue of gases produced by the breakdown of organic matter in the absence of oxygen (anaerobically), primary consisting of methane and carbondioxide. Biogas can be produced from raw material such as agricultural waste, manure, municiple waste, plant material, sewage, green waste or good waste. Due to release of CH_4 gas during anaerobic vegetative degradstion which caueses globle warming and cancer.

2. Out of the following, which type of interaction is responsible for the stabilisation α–helix structure of proteins ?
(1) Ionic bonding (2) Hydrogen bonding
(3) vander Waals forces (4) Covalent bonding

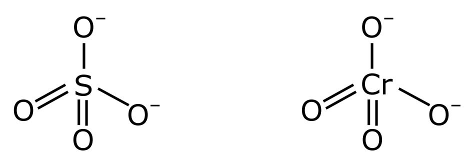
Ans. **(2)**

Sol. The α-helix is stabilized by hydrogen bond between the NH and CO group of the main chain.

3. Which of the following are isostructural pairs ?
(A) SO_4^{2-} and CrO_4^{2-}
(B) $SiCl_4$ and $TiCl_4$
(c) NH_3 and NO_3^-
(D) BCl_3 and $BrCl_3$
1. A and C only
2. A and B only
3. B and C only
4. C and D only

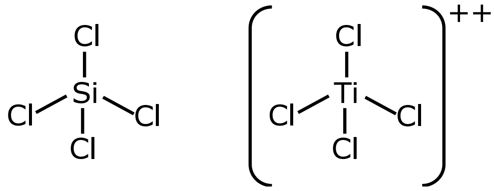
Ans. **(2)**

Sol. (1) SO_4^{-2} and CrO_4^{2-} both have tetrahedral structure.

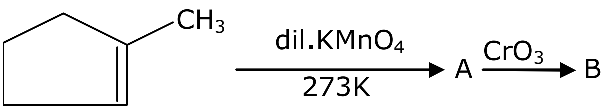


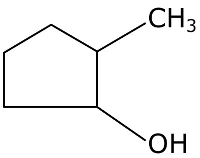
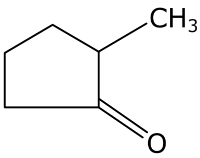
Tetrahedral Tetrahedral

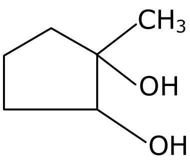
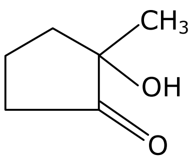
(2) $SiCl_4$ and $TiCl_4$ both have Tetrahedral structure also.



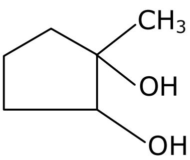
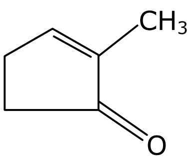
4. Identify products A and B.



(1) A :  B : 

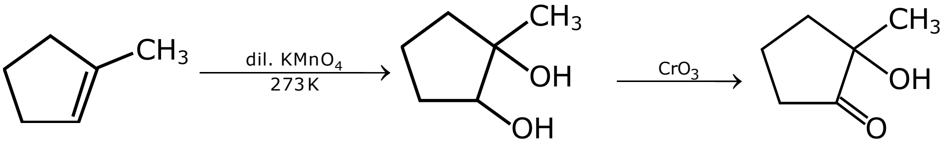
(2) A :  B : 

(3) A : $OHC-CH_2CH_2CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ B : $HOOC-CH_2CH_2CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$

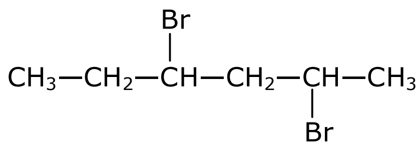
(4) A :  B : 

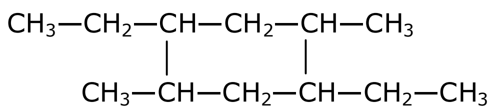
Ans. (2)

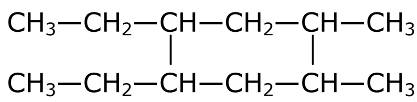
Sol.

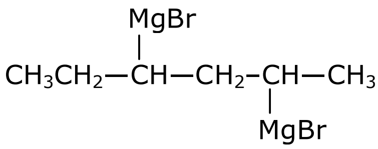


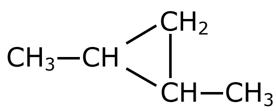
5. The product formed in the first step of the reaction of

 with excess $Mg/Et_2O(Et = C_2H_5)$ is :

(1) 

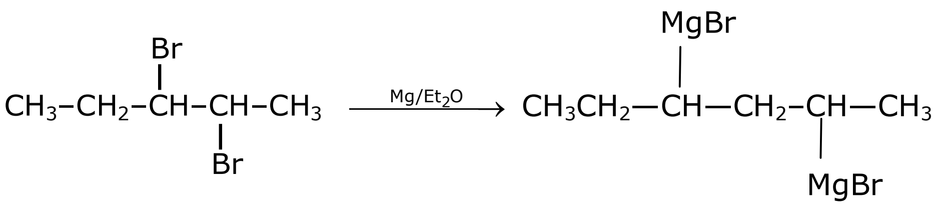
(2) 

(3) 

(4) 

Ans. (3)

Sol.



6. The electrode potential of M^{2+}/M of 3d– series elements shows positive value for:

(1) Zn (2) Co (3) Fe (4) Cu

Ans. (4)

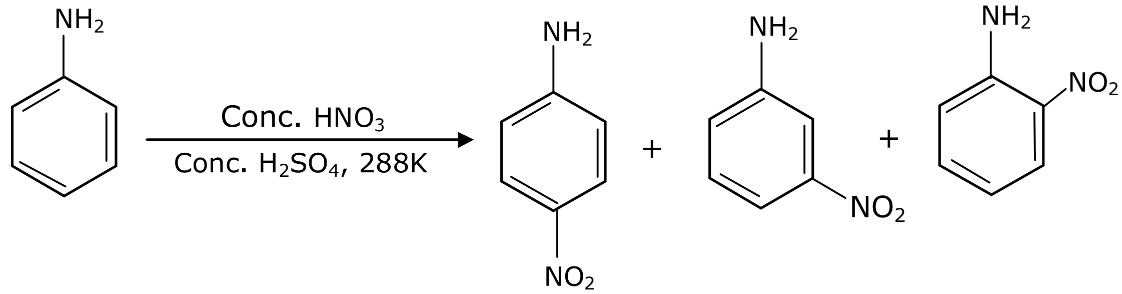
Sol. (A) Zn –0.76

(B) CO –0.28

(C) Fe –0.44

(D) Cu +0.34

7. In the following reaction the reason why meta–nitro product also formed is:



[A] 51%   [B] 47%   [C] 2%

(1) Formation of anilinium ion

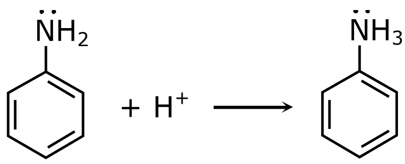
(2) $-NO_2$ substitution always takes place at meta–position

(3) low temperature

(4) $-NH_2$ group is highly meta–directive

Ans. **(1)**

Sol.



In acidic medium the $-NH_2$ group in aniline converts into anilinium ion which is meta directing.

8. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

(1) H_2O_2 act as oxidizing and reducing agent respectively in equations (A) and (B).

(2) H_2O_2 acts as oxidizing agent in equations (A) and (B).

(3) H_2O_2 acts as reducing agent in equations (A) and (B).

(4) H_2O_2 acts as reducing and oxidising agent respectively in equation (A) and (B).

Ans. **(3)**

Sol. When H_2O_2 acts a reducing agent it liberates the O_2.

$H_2O_2 \rightleftharpoons 2H^+ + O_2 + 2e^-$

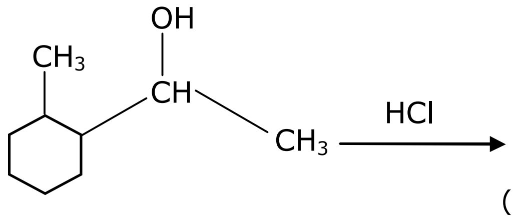
9. Which of the following ore is concentrated using group 1 cyanide salt ?
(1) Sphalerite (2) Siderite
(3) Malachite (4) Calamine

Ans. **(1)**

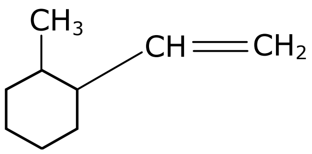
Sol. Conc. of sphalerite, first by cyanide salt as a depressant to remove the impurity of galena

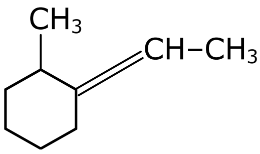
$$\text{Zns} + \text{Pbs} + \underset{\text{excess}}{\text{NaCN}} \longrightarrow \underset{\text{solution}}{Na_2[Zn(CN)_4]} + \text{PbS}\uparrow$$

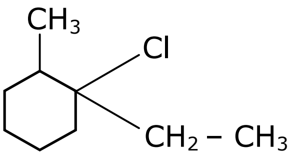
10. Which is the final product (major) 'A' in the given reaction?

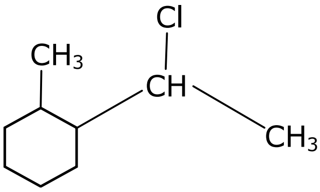


(major Product)

(1) 

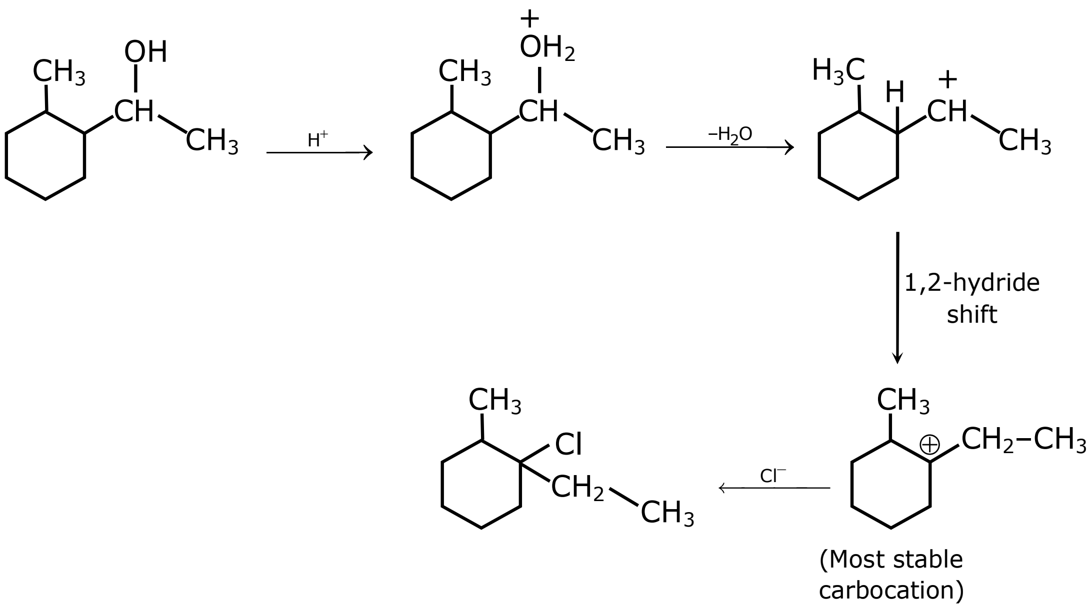
(2) 

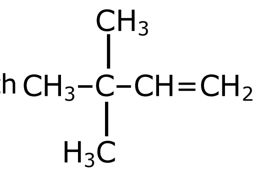
(3) 

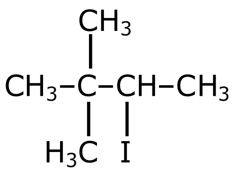
(4) 

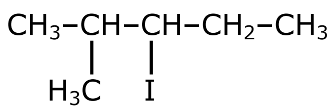
Ans. **(3)**

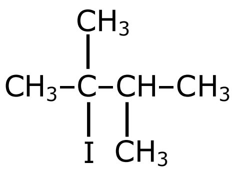
Sol.

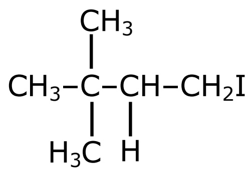


11. What is the major product formed by HI on reaction with  ?

(1) 

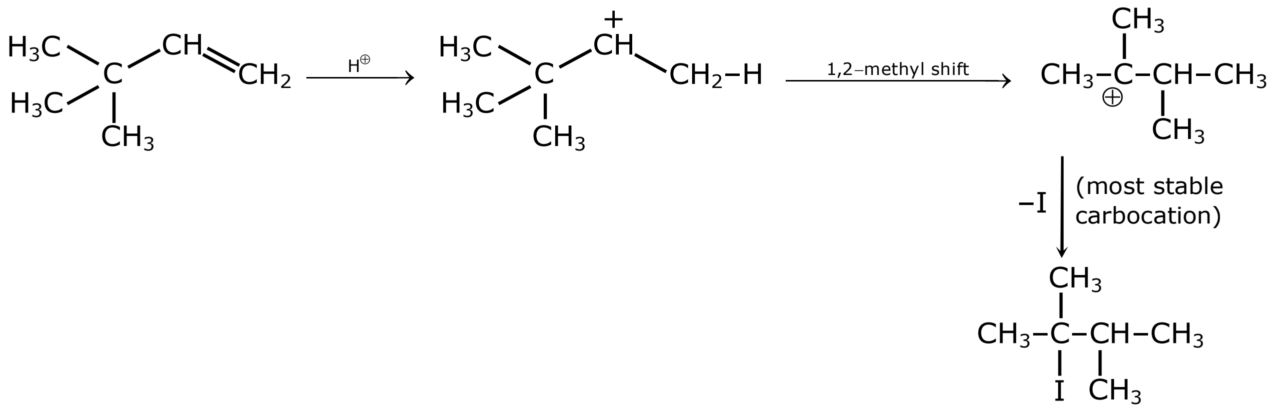
(2) 

(3) 

(4) 

Ans. (3)

Sol.



12. Which of the following reagent is used for the following reaction ?

$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$

(1) Potassium permanganate

(2) Molybdenum oxide

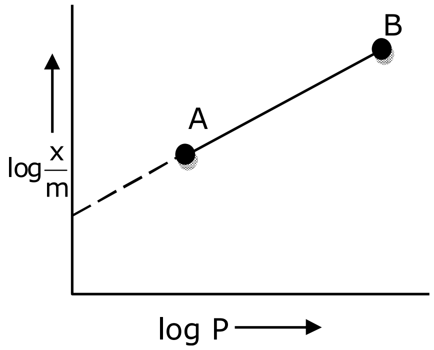
(3) Copper at high temperature and pressure

(4) Manganese acetate

Ans. (2)

Sol. $CH_3{-}CH_2{-}CH_3 \xrightarrow{MO_2O_3} CH_3{-}CH_2{-}CH{=}0$

13. In Freundlich adsorption isotherm, slope of AB line is :



(1) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$ (2) $\log \frac{1}{n}$ with (n<1)

(3) log n with (n>1) (4) n with (n, 0.1 to 0.5)

Ans. **(1)**

Sol. Freundlich adsorption isotherm is :

$\frac{x}{m} = kp^{1/n}$

x = mass of adsorbate

m = mass of adsorbent

P = eq. pressure

$k_1 n = \frac{1}{n} \log p + \log k$

y = mx + c

compairing

$m = \frac{1}{n}$ = slope $[\frac{1}{n} = 0 \text{ to } 1]$

n > 1

14. The major components in "Gun Metal" are:

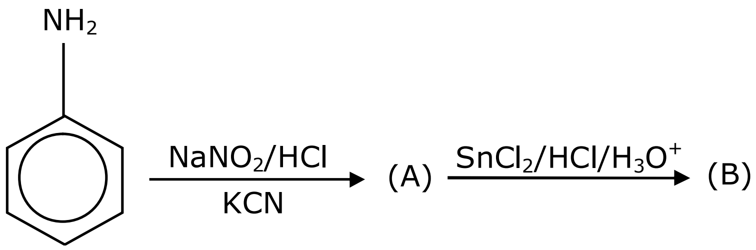
(1) Al, Cu, Mg and Mn (2) Cu, Sn and Zn

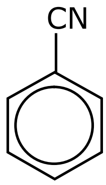
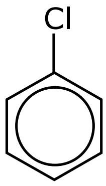
(3) Cu, Zn and Ni (4) Cu, Ni and Fe

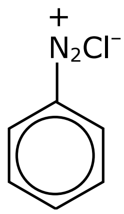
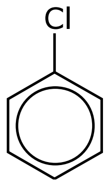
Ans. **(2)**

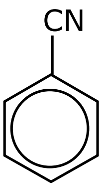
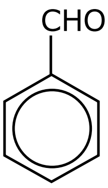
Sol. "Gun metal" is alloy of copper with tin and zinc.

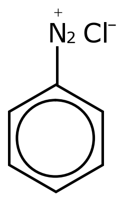
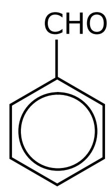
15. 'A' and 'B' in the following reactions are :



(1) (A) :  (B) : 

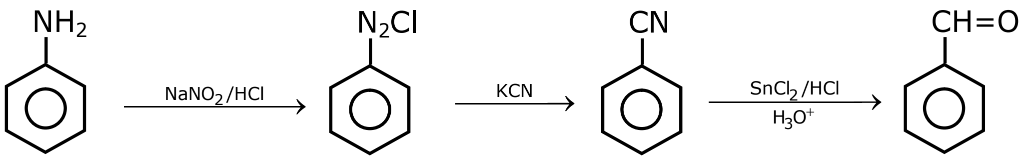
(2) (A) :  (B) : 

(3) (A) :  (B) : 

(4) (A) :  (B) : 

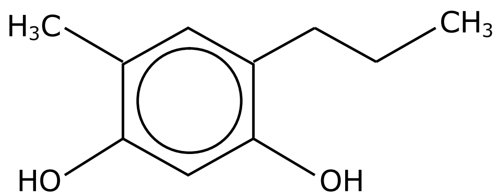
Ans. (3)

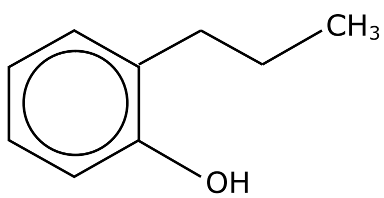
Sol.

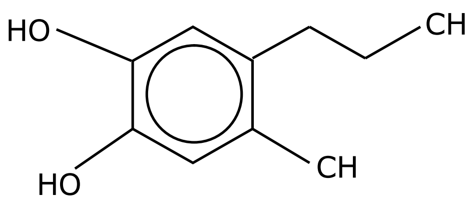


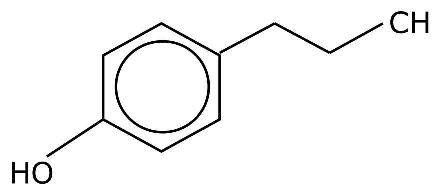
(A) (Stephen's rexn) (B)

16. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc.H_2SO_4 followed by treatment with NaOH ?

(1) 

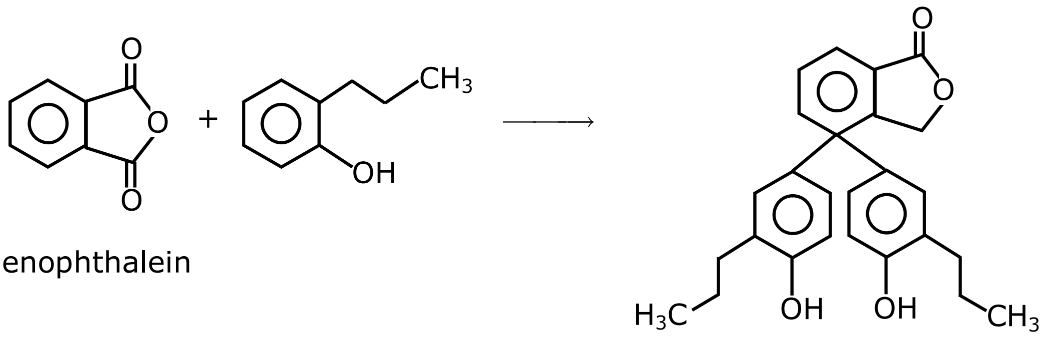
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



Phenophthalein

(Pink colour)

17. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is:

(1) Al < Mg < Si < S < P (2) Al < Mg < S < Si < P

(3) Mg < Al < Si < S < P (4) Mg < Al < Si < P < S

Ans. **(1)**

Sol. Order of IE, in 3[rd] period is

Na < Mg > Al < Si < P > S < Cl < Ar

Na < Al < Mg < Si < S < P < Cl < Ar

due to stable full filed 3s-orbital and more penetrating power

due to half filed $3p^3$-orbital of phosphorous

18. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is false but statement II is true.

(2) Statement I is true but Statement II is false.

(3) Both Statement I and Statement II are true.

(4) Both Statement I and Statement II are false.

Ans. **(4)**

Sol. Both are False

(1) Copper sulphate form copper meta boric with beric an hydride

$CuSO_4 \longrightarrow CuO + SO_3$

$CuO + B_2O_3 \longrightarrow Cu(BO_2)_2$

blue in cold oxidising flame (non luminous flame)

(2) Blue coloured metal borate is reduced to copper in a luminous flame.

19. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

(1) X = $Na[Al(OH)_4]$, Y=CO_2, Z = $Al_2O_3.xH_2O$

(2) X=$Na[Al(OH)_4]$, Y=SO_2, Z = Al_2O_3

(3) X=$Al(OH)_3$, Y=SO_2, Z = $Al_2O_3.xH_2O$

(4) X = $Al(OH)_3$, Y=CO_2, Z= Al_2O_3

Ans. **(1)**

Sol. (1) $Al_2O_3 + NaOH \longrightarrow \underset{\text{"X"}}{Na[Al(OH)_4]}$

(2) $Na[Al(OH)_4] \xrightarrow[\text{"Y"}]{CO_2} \underset{\text{"Z"}}{Al(OH)_3 \text{ or } Al_2O_3 \,.\, xH_2O}$

20. Match List I with List II.

List I	**List II**
(Monomer Unit)	(Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1,3-butadiene	(ii) Buna-N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below :

(1) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

(2) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

(3) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

(4) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

Ans. **(1)**

Sol. (1) Polymer of caprolactum is nylon-6

(2) Polymer of 2-chloro-1,3-butadiene is neoprene.

(3) Polymer of isoprene is natureal rubber

(4) Polymer of acrylonitrile and 1,3-butadiene is buna-N

SECTION – B

1. The stepwise formation of $[Cu(NH_3)]^{2+}$ is given below:

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} \left[Cu(NH_3)_4\right]^{2+}$$

$$\left[Cu(NH_3)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_2} \left[Cu(NH_3)_2\right]^{2+}$$

$$\left[Cu(NH_3)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_3} \left[Cu(NH_3)_3\right]^{2+}$$

$$\left[Cu(NH_3)_3\right]^{2+} + NH_3 \xrightleftharpoons{K_4} \left[Cu(NH_3)_4\right]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^2 , 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is ______. (Rounded off to the nearest integer)

Ans. **(1)**

Sol. $\left[Cu(NH_3)_4\right]^{+2} \xrightleftharpoons{k} cu^{+2} + 4NH_3$ .....(A)

For this :

$$Cu^{+2} + NH_3 \xrightleftharpoons{k1} \left[Cu(NH_3)\right]^{+2} \text{......(1)}$$

$$\left[Cu(NH_3)\right]^{+2} + NH_3 \xrightleftharpoons{k_2} \left[cu(NH_3)_2\right]^{+2} \text{.....(2)}$$

$$\left[Cu(NH_3)\right]^{+2} + NH_3 \xrightleftharpoons{k_3} \left[cu(NH_3)_3\right]^{+2} \text{.......(3)}$$

$$\left[Cu(NH_3)_3\right]^{+2} + NH_3 \xrightleftharpoons{k_4} \left[Cu(NH_3)_4\right]^{+2} \text{.......(4)}$$

(1) + (2) + (3) + (4)

$$Cu^{+2} + 4NH_3 \xrightleftharpoons{k_1.k_2.k_3.k_4} \left[C_4(NH_3)_4\right]^{+4} \text{.......(B)}$$

So for (A)

$$K = \frac{1}{k_1.K_2.K_3.K_4}$$

Putting the value of k_1,k_2,k_3 and k_4.

$$K = \frac{1}{(10)^4.(1.58 \times 10^3)(5 \times 10^2)(10)^2} = 1.26 \times 10^{-12}$$

x = 1.

2. At 1990 K and 1 atrm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is x × 10^{-1}. The value of x is _______. (Rounded off to the nearest integer)

Ans. **(5)**

Sol. $Cl_2 \rightleftharpoons 2Cl^-$

Lets mole of eq. x x

P.P. at eq. $\frac{x}{2x} \times 1$ $\frac{x}{2x} \times 1$

$\frac{1}{2}$ $\frac{1}{2}$

$$K_p = \frac{[P_{c1}]^2}{[P_{Cl_2}]} = \frac{\left[\frac{1}{2}\right]^2}{\frac{1}{2}} = \frac{1}{2} = 0.5 = 5 \times 10^{-1}$$

X = 5.

3. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is x × 10^{-1}. The value of x is _______. (Rounded off to the nearest integer)

Ans. **(2)**

Sol. Moles of A = $\frac{\text{Weight}}{\text{M.w}}$

$= \frac{4.5}{90} = \frac{1}{20} = 0.05$

Volume (Lit) = $= \frac{250}{1000} = 0.250 \text{ lit lit}$

Moles of A = $\frac{\text{Weight}}{\text{M.w}}$

$= \frac{4.5}{90} = \frac{1}{20} = 0.05$

Volume (Lit) = $= \frac{250}{1000} = 0.250 \text{ lit lit}$

Molarity (M) = $\frac{\text{Mole}}{\text{(Lit)volume}} = \frac{0.05}{0.250} = 0.2$

$= 2 \times 10^{-1} \frac{\text{mol}}{\text{Lit}}$

x = 2

Molarity (M) = $\frac{\text{Mole}}{\text{(Lit)volume}} = \frac{0.05}{0.250} = 0.2$

$= 2 \times 10^{-1} \frac{\text{mol}}{\text{Lit}}$ x = 2

4. The coordination number of an atom in a body-centered cubic structure is ________________.
[Assume that the lattice is made up of atoms]

Ans. (4)

Sol. Fact

5. Number of amphoteric compounds among the following is ________.
(A) BeO (B) BaO (C) $Be(OH)_2$ (D) $Sr(OH)_2$

Ans. (2)

Sol. BeO and $Be(OH)_2$ are amphoteric in nature

6. When 9.45g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is x × 10^{-3}. The value of x is _______. (Rounded off to the nearest integer)
$[K_{f(H_2O)} = 1.86\,K\,kg\,mol^{-1}]$

Ans. (35)

Sol.

	$ClCH_2COOH \longrightarrow$	$ClCH_2COO^{\ominus}$ +	$H^{\oplus}$
t = 0	C	O	O
t = t	$C - c\alpha$	$c\alpha$	$c\alpha$

Total no. of moles = c+ cα = c(1+ α)

$$i = \frac{\text{observed}}{\text{calculate}} = \frac{c(1+\alpha)}{c} = (1+\alpha)$$

M.W. = 94.5

$\Delta T_f = i \times k_f \times m$ $\qquad \Delta T_f = 0.5°C$

$i = 1+\alpha$

$$0.5 = (1+\alpha) \times 1.86 \times \frac{\frac{9.45}{94.5}}{\frac{500}{1000}}$$

$$m = \frac{\text{mole}}{\text{k.g(Solvent)}}$$

$k_t = 1.86k$ kg/mol

$$(1+\alpha) = \frac{2.5}{1.86}$$

$$\alpha = \frac{0.64}{1.86} = \frac{32}{93}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.2 \times 1024}{93 \times 93 \times \frac{61}{93}}$$

$K_a = 0.0351 = 35.1 \times 10^{-3}$

7. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is $x \times 10^{-1}$. The value of x is _______. [Rounded off to the nearest integer]
[Mass of Li^{3+} = 8.3 mass of proton]

Ans. (2)

Sol. De Brogir Davelength

$$\lambda = \frac{h}{\sqrt{2m\,k.E.}}$$

$$\frac{\lambda_{Li^{+3}}}{\lambda_p} = \sqrt{\frac{m_p \times \left(e^{-}v\right)_p}{m_{Li^{+3}} \times 3e_p v}}$$

$m_{Li^{+3}}$ = 8.3 mp

$$\frac{\lambda_{Li^{+3}}}{\lambda_p} = \sqrt{\frac{m_p}{3 \times 8.3 m_p}} = \sqrt{\frac{1}{25}}$$

$$= \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

x = 2.

8. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4}\ s^{-1}$ at 153°C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is ______. (Rounded off to the nearest integer)

Ans. (26)

Sol. For firdst order Rxn :-

$$t = \frac{2.303}{k} \log\left[\frac{100}{100 - x}\right]$$

X = 40, k = 3.3×10^{-4}

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log\left[\frac{100}{60}\right]$$

For firdst order Rxn :-

$$t = \frac{2.303}{k} \log\left[\frac{100}{100 - x}\right]$$

X = 40, k = 3.3×10^{-4}

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log\left[\frac{100}{60}\right]$$

$$t = \frac{2.303}{3.3 \times 10^{-4}} \times 0.22$$

t = $0.1535.3 \times 10^4$

t = 1535 sec.

t = $0.1535.3 \times 10^4$

t = 1535 sec = 25.6 Min.

9. For the reaction $A_{(g)} \rightarrow B_{(g),}$ the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is _____. (Rounded off to the nearest integer)
[R = 8.31 J $mol^{-1}K^{-1}$ and ln 10 = 2.3]

Ans. **(1380)**

Sol. $\Delta G° = -RT \ln Keq.$
$= -R \times 300 \times \ln(10^2)$
$= 300 \times 2\times 2.3 \times (-R)$
$= -1380R$
x = 1380 ans.

10. The reaction of sulphur in alkaline medium is given below:
$S_{8(s)} + a\ OH^-_{(aq)} \longrightarrow b\ S^{2-}_{(aq)} + c\ S_2O_3^{2-}{}_{(aq)} + d\ H_2O_{(\ell)}$
The values of 'a' is _________. (Integer answer)

Ans. **(12)**

Sol. $S_8 + aOH^- \longrightarrow bs^{-2} + CdS_2O_3^{-2} + dH_2O$
$S_8 + bOH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + dH_2O$
$S_8 + 12OH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + 6H_2O$
a = 12

24^{th} Feb. 2021 | Shift - 1
MATHEMATICS

1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:.

(1) $x = a$ (2) $x = 0$ (3) $x = -\frac{a}{2}$ (4) $x = \frac{a}{2}$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

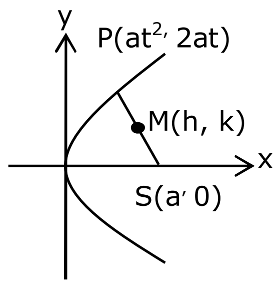
$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$

$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$



2. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

(1) 560 (2) 1050 (3) 1625 (4) 575

Ans. (3)

Sol. (2I, 4F)+ (3I, 6F) + (4I, 8F)

$= {}^6C_2{}^8C_4 + {}^6C_3{}^8C_6 + {}^6C_4{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

(1) $3x - 10y - 2z + 11 = 0$
(2) $6x - 5y - 2z - 2 = 0$
(3) $11x + y + 17z + 38 = 0$
(4) $6x - 5y + 2z + 10 = 0$

Ans. (3)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ $11(x - 1) + (y - 2) + 17(z + 3) = 0$

$11x + y + 17z + 38 = 0$

4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only
(2) A only
(3) All the three
(4) C only

Ans. (1)

Sol. $\frac{x}{a} + \frac{y}{b} = 1$

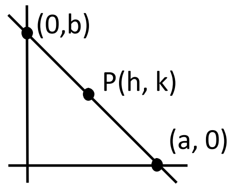
$\frac{h}{a} + \frac{k}{b} = 1$ ....(1)

$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ ....(ii)

$\therefore$ Line passes through fixed point B(2, 2)

(from (1) and (2))



5. The statement among the following that is a tautology is:

(1) $A \wedge (A \vee B)$ (2) $B \rightarrow [A \wedge (A \rightarrow B)]$ (3) $A \vee (A \wedge B)$ (4) $[A \wedge (A \rightarrow B)] \rightarrow B$

Ans. **(4)**

Sol. $A \wedge (\sim A \vee B) \rightarrow B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= \sim A \vee \sim B \vee B$

$= t$

6. Let f : R→ R be defined as $f(x) = 2x-1$ and g:R − {1} → R be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x-1}$.

Then the composition function $f(g(x))$ is :

(1) both one-one and onto (2) onto but not one-one

(3) neither one-one nor onto (4) one-one but not onto

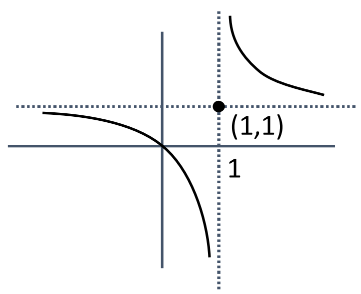
Ans. **(4)**

Sol. $f(g(x)) = 2g(x) - 1$

$= 2\dfrac{\left(x - \frac{1}{2}\right)}{x-1} = \dfrac{x}{x-1}$

$f(g(x)) = 1 + \dfrac{1}{x-1}$

one-one, into



7. If f:R $\rightarrow$ R is a function defined by f(x) = [x–1] $\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous only at x = 1

(2) discontinuous at all integral values of x except at x = 1

(3) continuous only at x = 1

(4) continuous for every real x

Ans. (4)

Sol. Doubtful points are x = n, n $\in$ I

$$\text{L.H.L} = \lim_{x\to n^-}\left[x-1\right]\cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

$$\text{R.H.L} = \lim_{x\to n^+}\left[x-1\right]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

f(n) = 0

Hence continuous.

8. The function $f(x) = \frac{4x^3-3x^2}{6} - 2\sin x + (2x-1)\cos x$:

(1) increases in $\left[\frac{1}{2},\infty\right)$ (2) decreases $\left(-\infty,\frac{1}{2}\right]$

(3) increases in $\left(-\infty,\frac{1}{2}\right]$ (4) decreases $\left[\frac{1}{2},\infty\right)$

Ans. (1)

Sol. $f'(x) = (2x-1)(x-\sin x)$

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2},\infty\right)$

and $f'(x) \leq 0$ in $x \in \left(-\infty,\frac{1}{2}\right]$

9. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is:

(1) $\sqrt{38}$ (2) $19\sqrt{2}$ (3) $2\sqrt{19}$ (4) 38

Ans. **(1)**

Sol. $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

$\Rightarrow$ x = λ+3, y = 2λ+4, z = 2λ+5

Which lines on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda=\frac{5}{5}=1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

10. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ (2) 0 (3) $\frac{1}{15}$ (4) $\frac{3}{2}$

Ans. **(1)**

Sol. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}dt}{x^3}=\lim\limits_{x\to 0}\frac{\left(\sin|x|\right)2x}{3x^2}=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$

11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

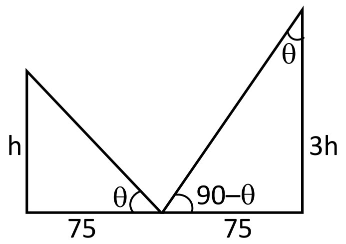
(1) 25 (2) $20\sqrt{3}$ (3) 30 (4) $25\sqrt{3}$

Ans. **(4)**

Sol. $\tan\theta = \frac{h}{75}=\frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$m



12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) $-t^3$ (3) 0 (4) $2t^3$

Ans. (1)

Sol. Equation of tangent at $P(t, t^3)$

$(y - t^3) = 3t^2(x - t)$ ……..(1)

Now solve the above equation with

$y = x^3$ …….(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point = $\dfrac{2t^3 + \left(-8t^3\right)}{3} = -2t^3$

13. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

(1) $24\pi + 3\sqrt{3}$

(2) $12\pi + 3\sqrt{3}$

(3) $12\pi - 3\sqrt{3}$

(4) $24\pi - 3\sqrt{3}$

Ans. (4)

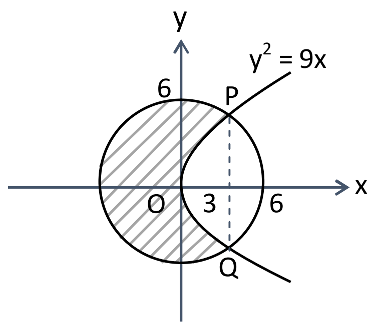
Sol. The curves intersect at point $(3, \pm 3\sqrt{3})$

Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9 - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$



14. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1, –3) (2) (1, 3) (3) (–1, 3) (4) (3, 1)

Ans. (2)

Sol. put $\sin x + \cos x = t \Rightarrow 1 + \sin 2x = t^2$

$\Rightarrow (\cos x - \sin x)\,dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C$$

$\Rightarrow a = 1$ and $b = 3$

15. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt}$ = 0.5P – 450. If P(0) = 850, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$ (2) $2\log_e 18$ (3) $\log_e 9$ (4) $\log_e 18$

Ans. **(2)**

Sol. $\frac{dp}{dt} = \frac{p-900}{2}$

$\int_{850}^{0} \frac{dp}{p-900} = \int_{0}^{t} \frac{dt}{2}$

$\ell n\left|P-900\right|\Big|_{850}^{0} = \frac{t}{2}$

$\ell n|900| - \ell n|50| = \frac{t}{2}$

$\frac{t}{2} = \ell n|18|$

$\Rightarrow t = 2\ell n18$

16. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is:

(1) 2^{14} (2) $2^{13} - 13$ (3) $2^{16} - 1$ (4) $2^{13} - 14$

Ans. **(4)**

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - .... - 15\ {}^{15}C_{15}$

$= \sum_{r=1}^{15} (-1)^r .r.{}^{15}C_r = 15\sum_{r=1}^{15} (-1)^r\ {}^{14}C_{r-1}$

$= 15\ (-{}^{14}C_0 + {}^{14}C_1 - ....- {}^{14}C_{14}) = 15\ (0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

$= S_1 + S_2 = 2^{13} - 14$

17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$ (3) $\frac{5}{16}$ (4) $\frac{1}{32}$

Ans. (2)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2 \left(\frac{1}{2}\right)^n = {}^nC_3 \left(\frac{1}{2}\right)^n \Rightarrow n = 5$

Success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

18. Let p and q be two positive number such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Ans. (4)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 + 16pq + 2p^2 q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

Now

$x^2 - (p + q)x + pq = 0$

$x^2 - 2x + 16 = 0$

19. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+...\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is :

(1) $\frac{3}{2}$ (2) $2\sqrt{3}$ (3) $\frac{1}{2}$ (4) $\sqrt{3}$

Ans. (3)

Sol. $e^{\left(\cos^2 x+\cos^4 x+......\infty\right)\ell n 2} = 2^{\cos^2 x+\cos^4 x+......\infty}$

$= 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1,8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

20. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3, m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3, m \neq \frac{4}{5}$ (4) $k = 3, m \neq \frac{4}{5}$

Ans. (4)

Sol. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 1 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$

$\Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$

$10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m) \neq 0$

$80 - 12 + 20m - 36 - 60m \neq 0$

$40m \neq 32 \Rightarrow m \neq \dfrac{4}{5}$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} \neq 0$$

$3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6) \neq 0$

$-18 + 30m - 30m + 18 \neq 0 \Rightarrow 0$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$$

$3(-20m - 12) + 2(10m - 6) + 10(4 + 4) - 40m + 32 \neq 0 \Rightarrow m \neq \dfrac{4}{5}$

Section – B

1. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\dfrac{k}{8}$ and $|Q| = \dfrac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ______

Ans. **17**

Sol. As $PQ = KI \Rightarrow Q = kP^{-1}I$

now $Q = \dfrac{k}{|P|}(adjP)\,I \Rightarrow Q = \dfrac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \dfrac{-k}{8} \Rightarrow \dfrac{k}{(20+12\alpha)}(-3\alpha - 4) = \dfrac{-k}{8} \Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \dfrac{k^3|I|}{|P|} \Rightarrow \dfrac{k^2}{2} = \dfrac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$

2. Let B_i(i=1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ____

Ans. **6**

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1 - y)(1 - z) = \alpha$

$\Rightarrow y(1 - x)(1 - z) = \beta$

$\Rightarrow z(1 - x)(1 - y) = \gamma$

$\Rightarrow (1 - x)(1 - y)(1 - z) = p$

$(\alpha - 2\beta)p = \alpha\beta$

$(x(1-y)(1-z) - 2y(1-x)(1-z))(1-x)(1-y)(1-z) = xy(1-x)(1-y)(1-z)$

$x - xy - 2y + 2xy = xy$

$x = 2y \qquad ...(1)$

Similarly $(\beta - 3r)\ p = 2\beta r$

$\Rightarrow y = 3z \qquad ...(2)$

From (1) & (2)

$x = 6z$

Now

$\frac{x}{z} = 6$

3. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____

Ans. **9**

Sol. $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

Let sinx = t $\quad \because\ x \in \left(0, \frac{\pi}{2}\right) \quad \Rightarrow 0 < t < 1$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$$

$$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$$

$$= \frac{(t-2(1-t))(t+2(1-t))}{t^2(1-t)^2}$$

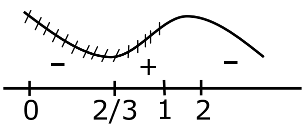
$$= \frac{(3t-2)(2-t)}{t^2(1-t)^2}$$

$$f_{min} \text{ at } t = \frac{2}{3}$$

$$\alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}}$$

= 6 + 3

= 9



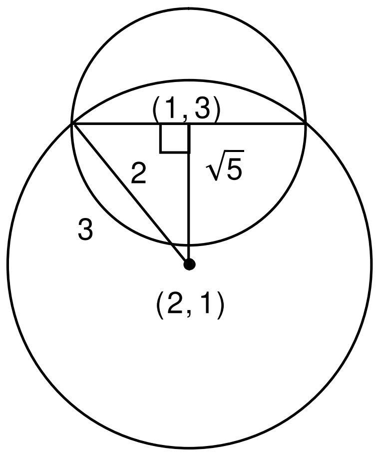
4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2,1), then its radius is _______

Ans. **3**

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$$

$$\Rightarrow r = 3$$



5. $\lim\limits_{x\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ______

Ans. **1**

Sol.
$$\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$$
$$=\tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$$
$$=\tan\left(\frac{\pi}{4}\right)=1$$

6. If $\int_{-a}^{a}(|x|+|x-2|)\,dx=22$,(a > 2) and [x] denotes the greatest integer $\le$ x, then $\int_{a}^{-a}(x+[x])\,dx$ is equal to _____

Ans. **3**

Sol.
$$\int_{-a}^{0}(-2x+2)\,dx+\int_{0}^{2}(x+2-x)\,dx+\int_{2}^{a}(2x-2)\,dx=22$$
$$x^2-2x|_0^{-a}+2x|_0^2+x^2-2x|_2^a=22$$
$$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$$
$$2a^2 = 18 \Rightarrow a = 3$$
$$\int_{3}^{-3}(x+[x])\,dx = -\left(\int_{-3}^{3}(x+[x])\,dx\right) = -\left(\int_{-3}^{3}[x]dx\right)$$
$$= -(-3 - 2 - 1 + 0 + 1 + 2) = 3$$

7. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}\,.\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is _______

Ans. **75**

Sol.
$$\vec{c}=\lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$$
$$=\lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$$
$$=\lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$$

$= \lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a}=7 \Rightarrow$ 3λ + 5λ + 6λ = 7

$\lambda=\frac{1}{2}$

$\therefore\ 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$= \ 2\left(\frac{1}{4}+\frac{49}{4}+25\right)=25+50=75$

8. Let A = {n∈N : n is a 3-digit number}

B = {9k + 2 : k∈N}

and C : {9k + ℓ : k∈N} for some ℓ (0 < ℓ < 9)

If the sum of all the elements of the set A ∩ (B ∪ C) is 274×400, then ℓ is equal to __

Ans. 5

Sol. 3 digit number of the form 9K + 2 are {101, 109, …………992}

⇒ Sum equal to $\frac{100}{2}$ (1093) = s_1 = 54650

274 × 400 = s_1 + s_2

274 × 400 = $\frac{100}{2}$ [101 + 992] + s_2

274 × 400 = 50 × 1093 + s_2

s_2 = 109600 – 54650

s_2 = 54950

$s_2 = 54950 = \frac{100}{2}\left[(99+\ell)+(990+\ell)\right]$

1099 = 2ℓ + 1089

ℓ = 5

9. If the least and the largest real values of α, for which the equation $z + \alpha |z - 1| + 2i = 0$ $\left(z \in \mathbb{C} \text{ and } i = \sqrt{-1}\right)$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ___

Ans. 10

Sol. $x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \dfrac{x^2}{x^2 - 2x + 5} \Rightarrow x^2(\alpha^2 - 1) - 2x\alpha^2 + 5\alpha^2 = 0$

$x \in R \Rightarrow D \geq 0$

$4\alpha^4 - 4(\alpha^2 - 1)5\alpha^2 \geq 0$

$\alpha^2\left[4\alpha^2 - 2\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[-16\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[\alpha^2 - \dfrac{5}{4}\right] \leq 0$

$0 \leq \alpha^2 \leq \dfrac{5}{4}$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

then $4[(q)^2 + (p)^2] = 4\left[\dfrac{5}{4} + \dfrac{5}{4}\right] = 10$

10. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is __

Ans. 540

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case II : One (2) and three (1's) and five (0's)

$$\frac{9!}{5!3!} = 504$$

$\therefore$ Total = 540

24th Feb. 2021 | Shift - 2
PHYSICS

1. Zener breakdown occurs in a p-n junction having p and n both :
(1) lightly doped and have wide depletion layer.
(2) heavily doped andhave narrow depletion layer.
(3) heavily doped and have wide depletion layer.
(4) lightly doped and have narrow depletion layer.

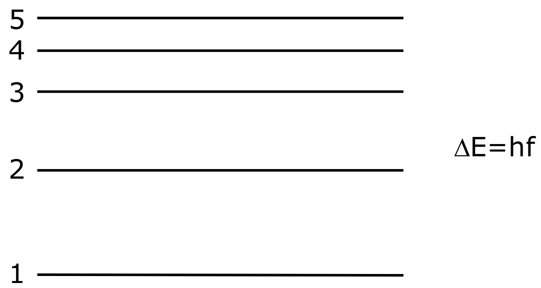
Ans. (2)

Sol. The zener breakdown occurs in the heavily doped p-n junction diode. Heavily doped p-n junction diodes have narrow depletion region.

2. According to Bohr atom model, in which of the following transitions will the frequency be maximum?
(1)n=2 to n=1 (2)n=4 to n =3 (3)n=5 to n=4 (4)n=3 to n=2

Ans. (1)

Sol.



f is more for transition from n = 2 to n = 1.

3. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be :
(1) 10^{-2} nm (2)10^{-3} nm (3)10^{-4} nm (4)10^{-1} nm

Ans. (2)

Sol. $\lambda_{min} = \dfrac{hc}{eV}$

$\lambda_{min} = \dfrac{1240nm - eV}{1.24 \times 10^6}$

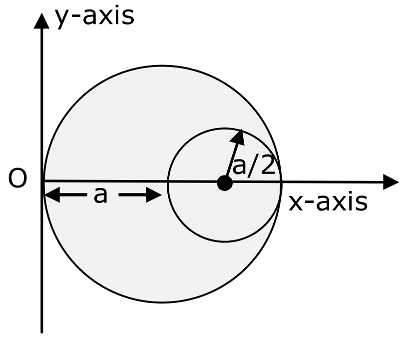
$\lambda_{min} = 10^{-3}nm$

4. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules:
(1) suffer change in momentum when impinge on the walls of container.
(2) continuously stick to the walls of container.
(3) continuously lose their energy till it reaches wall.
(4) are attracted by the walls of container.

Ans. (1)

Sol. On the basis of kinetic theory of gases, the gas pressure is due to the molecules suffering change in momentum when impinge on the walls of container.

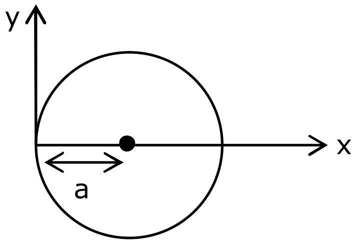
5. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be :

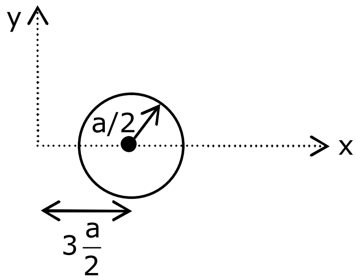


(1) $\frac{10}{11}a$ (2) $\frac{2}{3}a$ (3) $\frac{1}{6}a$ (4) $\frac{5}{6}a$

Ans. **(4)**

Sol. Let σ is the surface mass density of disc.





$$X_{com} = \frac{(\sigma \times \pi a^2 \times a) - (\sigma \frac{\pi a^2}{4} \times \frac{3a}{2})}{\sigma\pi a^2 - \frac{\sigma\pi a^2}{4}}$$

$$X_{com} = \frac{a - 3\frac{a}{8}}{1 - \frac{1}{4}}$$

$$X_{com} = \frac{\frac{5a}{8}}{\frac{3}{4}}$$

$$X_{com} = \frac{5a}{6}$$

6. Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are false

Ans. **(1)**

Sol. **S-1**

Statement 1 is false because in case of two discrete back to back connected diodes, there are four doped regions instead of three and there is nothing that resembles a thin base region between an emitter and a collector.

S-2

Statement-2 is true, as

$$\beta = \frac{I_C}{I_B}$$

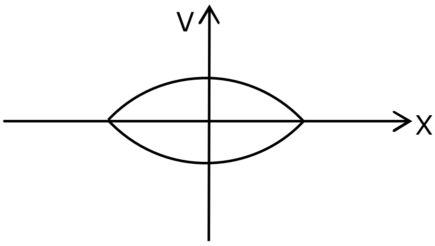
7. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

(1)elliptical (2)parabolic (3)straight line (4)circular

Ans. **(1)**

Sol. We know that is SHM;

$$V = \omega\sqrt{A^2 - x^2}$$



elliptical

8. Match List – I with List – II.

	List – I	List - II
(a)	Source of microwave frequency	(i) Radioactive decay of nucleus
(b)	Source of infrared frequency	(ii) Magnetron
(c)	Source of Gamma Rays	(iii) Inner shell electrons
(d)	Source of X-rays	(iv) Vibration of atoms and molecules
		(v) LASER
		(vi) RC circuit

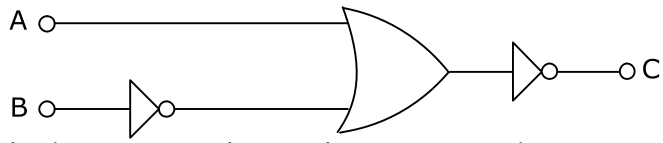
Choose the correct answer from the options given below :

(1) (a)-(ii),(b)-(iv),(c)-(i),(d)-(iii)
(2) (a)-(vi),(b)-(iv),(c)-(i),(d)-(v)
(3) (a)-(ii),(b)-(iv),(c)-(vi),(d)-(iii)
(4) (a)-(vi),(b)-(v),(c)-(i),(d)-(iv)

Ans. **(1)**

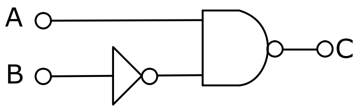
Sol. (a) Source of microwave frequency – (ii) Magnetron
(b) Source of infra red frequency – (iv) Vibration of atom and molecules
(c) Source of gamma ray – (i) Radio active decay of nucleus
(d) Source of X-ray – (iii) inner shell electron

9.

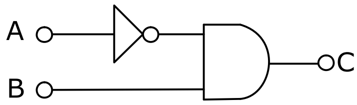


The logic circuit shown above is equivalent to :

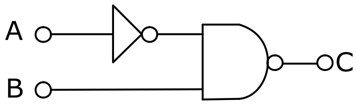
(1)



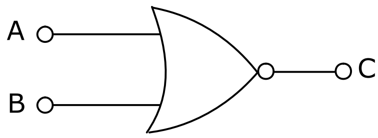
(2)



(3)

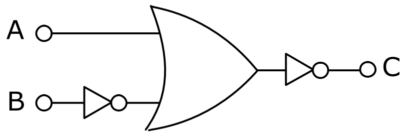


(4)



Ans. **(2)**

Sol.



$C = \overline{A + \overline{B}}$

$C = \overline{A}.B$

10. If the source of light used in a Young's double slit experiment is changed from red to violet:
(1)the fringes will become brighter.
(2)consecutive fringe lineswill come closer.
(3)the central bright fringe will become a dark fringe.
(4)the intensity of minima will increase.

Ans. **(2)**

Sol. $\beta = \frac{\lambda D}{d}$

As $\lambda_v < \lambda_R$

$\Rightarrow \beta_v < \beta_R$

$\therefore$ Consecutive fringe line will come closer.

$\therefore$ (2)

11. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

[Use $g = \frac{GM}{R^2}$ = 9.8 ms^{-2} and radius of earth, R = 6400 km.]

(1)49 N (2) 49.83 N (3) 49.17 N (4) 48.83 N

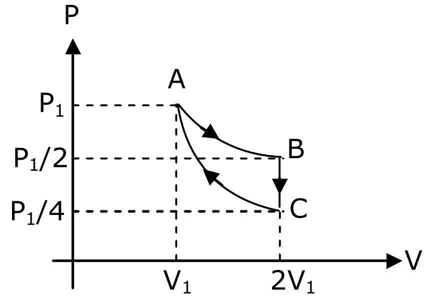
Ans. **(4)**

Sol. At north pole, weight
Mg = 49
Now, at equator
$g' = g - \underline{\omega}^2 R$

$\Rightarrow Mg' = M(g - \omega^2 R)$

$\Rightarrow$ weight will be less than Mg at equator.

12. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B→C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to :



(1)0 (2)-$\frac{RT}{2(\gamma-1)}$ (3)$RT\left[\ln 2-\frac{1}{2(\gamma-1)}\right]$ (4)RT ln2

Ans. **(3)**

Sol. AB → Isothermal process

$W_{AB}\rightarrow$ nRT ln2 = RT ln2

BC → Isochoric process

$W_{BC} = 0$

CA → Adiabatic process

$$W_{CA} = \frac{P_1V_1 - \frac{P_1}{4}X2V_1}{1-\gamma} = \frac{P_1V_1}{2(1-\gamma)} = \frac{RT}{2(1-\gamma)}$$

$$W_{ABCA} = RT\ell n2 + \frac{RT}{2(1-\gamma)}$$

$$= RT\left[\ell n2 - \frac{1}{2(\gamma-1)}\right]$$

13. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

(1)1.33 % (2)1.30 % (3)1.13 % (4)1.03 %

Ans. **(3)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T^2 = 4\pi^2\left[\frac{\ell}{g}\right]$$

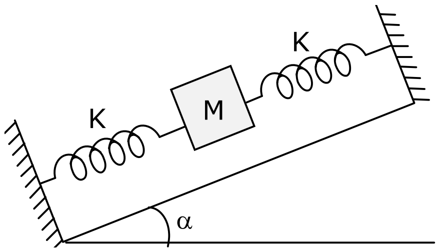
$$g = 4\pi^2\left[\frac{\ell}{T^2}\right]$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \left[\frac{1mm}{1m} + \frac{2(10\times10^{-3})}{1.95}\right]\times 100$$

= 1.13 %

14. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is :



(1) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

Ans. **(1)**

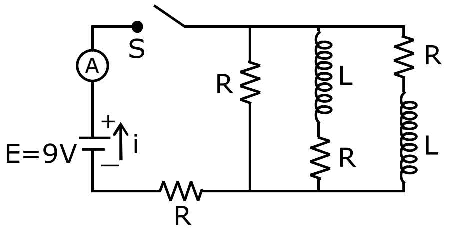
Sol. Equivalent K = K + K = 2K

Now, $T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{2k}}$

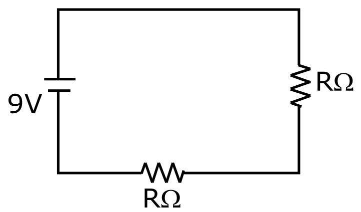
$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

15. Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω. Two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9.V. The current 'i' just after the switch 's' is closed will be :



(1) 9A (2) 3.0 A (3) 2.25 A (4) 3.37 A

Ans. **(3)**
Sol. When switch S is closed-



Given : v = 9v
From V = IR

$I = \frac{v}{R}$

$R_{eq.}$ = 2+2 = 4Ω

$I = \frac{9}{4}$ = 2.25 A

16. The de Broglie wavelength of a proton andα-particle are equal. The ratio of their velocities is :
(1) 4:2 (2) 4:1 (3) 1:4 (4) 4:3

Ans. **(2)**

Sol. From De-broglie's wavelength :-

$\lambda = \frac{h}{mv}$

Given $\lambda_P = \lambda_\alpha$

$v \alpha \frac{1}{m}$

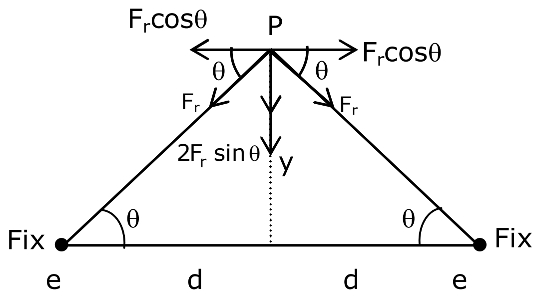
$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$

17. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x<<d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:
(m = mass of charged particle)

(1) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ (2) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$ (3) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$ (4) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

Ans. **(1)**

Sol.



Restoring force on proton :-

$$F_r = \frac{2Kq^2y}{\left[d^2 + y^2\right]^{\frac{3}{2}}}$$

Y <<< d

$$F_r = \frac{2kq^2y}{d^3} = \frac{q^2y}{2\pi\varepsilon_0 d^3} = ky$$

$$K = \frac{q^2}{2\pi\varepsilon_0 d^3}$$

Angular Frequency :-

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}$$

18. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

(1) decrease in size and changes orientation.
(2) may increase or decrease in size and change its orientation.
(3) increase in size but no change in orientation.
(4) have no relation with external magnetic field.

Ans. (2)

Sol. Atoms of ferromagnetic material in unmagnetised state form domains inside the ferromagnetic material. These domains have large magnetic moment of atoms. In the absence of magnetic field, these domains have magnetic moment in different directions. But when the magnetic field is applied, domains aligned in the direction of the field grow in size and those aligned in the direction opposite to the field reduce in size and also its orientation changes.

19. Which of the following equations represents a travelling wave?

(1) $y = Ae^{-x^2}(vt + \theta)$
(2) $y = A\sin(15x - 2t)$
(3) $y = Ae^{x}\cos(\omega t - \theta)$
(4) $y = A\sin x \cos \omega t$

Ans. (2)

Sol. Y = F(x,t)
For travelling wave y should be linear function of x and t and they must exist as (x±vt)
Y = A sin (15x-2t) $\rightarrow$ linear function in x and t.

20. A particle is projected with velocity v_0 along x-axis. A damping forceis acting on the particle which is proportional to the square of the distance from the origin i.e. ma = $-\alpha x^2$. The distance at which the particle stops :

(1) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$ (2) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$ (3) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$

Ans. Bonus

Sol. $a = \dfrac{vdv}{dx}$

$$\int_{v_i}^{v_f} Vdv = \int_{x_i}^{x_f} adx$$

Given :- $v_i = v_0$

$V_f = 0$

$X_i = 0$

$X_f = x$

From Damping Force : $a = -\dfrac{\alpha x^2}{m}$

$$\int_{V_0}^{O} VdV = -\int_{o}^{x} \frac{\alpha x^2}{m} dx$$

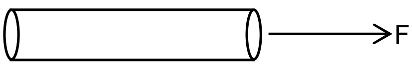
$$-\frac{v_0^2}{2} = \frac{-\alpha}{m}\left[\frac{x^3}{3}\right]$$

$$x = \left[\frac{3mv_0^2}{2\alpha}\right]^{\frac{1}{3}}$$

1. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled andsubjected to the same force will be ________ cm.

Ans. **2**

Sol.



$$y = \frac{F/A}{\Delta\ell/\ell}$$

$$\Rightarrow \frac{F}{A} = y\frac{\Delta\ell}{\ell}$$

$$\Rightarrow \frac{F}{A} = y \times \frac{0.04}{\ell} \qquad ...(1)$$

When length & diameter is doubled.

$$\Rightarrow \frac{F}{4A} = y \times \frac{\Delta\ell}{2\ell} \qquad ...(2)$$

$(1) \div (2)$

$$\frac{F/A}{F/4A} = \frac{y \times \frac{0.04}{\ell}}{y \times \frac{\Delta\ell}{2\ell}}$$

$$4 = \frac{0.04 \times 2}{\Delta\ell}$$

$\Delta\ell = 0.02$

$\Delta\ell = 2 \times 10^{-2}$

$\therefore x = 2$

2. A cylindrical wire of radius 0.5 mm and conductivity 5×10^{7} S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ____.

Ans. 5

Sol. We know that

$J = \sigma E$

$\Rightarrow J = 5\times10^{7}\times10\times10^{-3}$

$\Rightarrow J = 50\times10^{4}\ \text{A/m}^2$

Currentflowing ;

$I = J \times \pi R^2$

$I = 50\times10^{4}\times\pi(0.5\times10^{-3})^2$

$I = 5\times10^{4}\times\pi\times0.25\times10^{-6}$

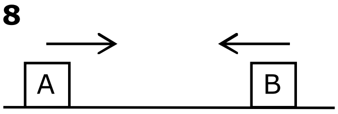
$I = 125\times10^{-3}\pi$

X = 5

3. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be ________ Hz. [Velocity of sound in air is 340 m/s.]

Ans. 8

Sol.



Speed = 7.2 km/h = 2 m/s

Frequency as heard by A

$$f'_A = f_B\left(\frac{v+v_0}{v-v_s}\right)$$

$$f'_A = 676\left(\frac{340+2}{340-2}\right)$$

$$f'_A = 684\text{Hz}$$

$$\therefore f_{Beat} = f'_A - f_B$$

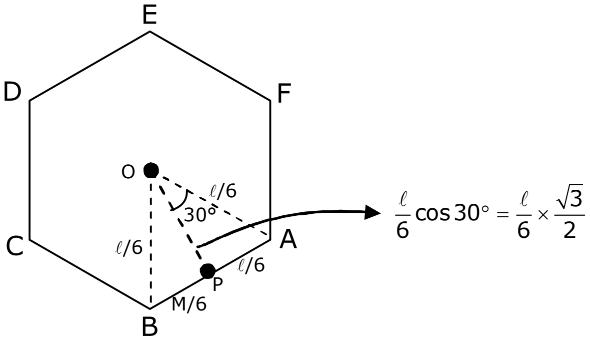
=684-676

= 8 Hz

4. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is ______$\times10^{-1}$ kg m^2.

Ans. 8

Sol.



MOI of AB about P : $I_{AB_P} = \dfrac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12}$

MOI of AB about O,

$$I_{AB_O} = \left[\frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12} + \frac{M}{6}\left(\frac{\ell}{6}\frac{\sqrt{3}}{2}\right)^2\right]$$

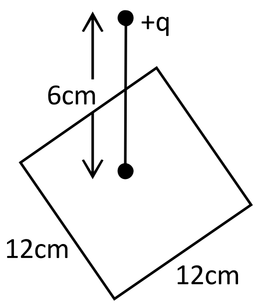
$$I_{Hexagon_0} = 6I_{AB_0} = M\left[\frac{\ell^2}{12\times 36} + \frac{\ell^2}{36}\times\frac{3}{4}\right]$$

$$= \frac{6}{100}\left[\frac{24\times 24}{12\times 36} + \frac{24\times 24}{36}\times\frac{3}{4}\right]$$

$= 0.8\ kgm^2$

$= 8\times 10^{-2}\ kg/m^2$

5. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be _____ $\times 10^3\ Nm^2/C$.



Ans. **226**

Sol. Using Gauss law, it is a part of cube of side 12 cm and charge at centre so;

$$\phi = \frac{Q}{6\varepsilon_0} = \frac{12\mu c}{6\varepsilon_0} = 2\times 4\pi\times 9\times 10^9\times 10^{-6}$$

$= 226\times 10^3\ Nm^2/C$

6. Two solids A and B of mass 1 kg and 2 kg respectively are moving withequal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$. So the value of A will be ________.

Ans. **2**

Sol. Given that, $\frac{M_1}{M_2} = \frac{1}{2}$

Also, $p_1 = p_2 = p$

$\Rightarrow M_1V_1 = M_2V_2 = p$

Also, we know that

$$K = \frac{p^2}{2M} \Rightarrow K_1 = \frac{p^2}{2M_1} \& \Rightarrow K_2 = \frac{p^2}{2M_2}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{p^2}{2M_1} \times \frac{2M_2}{p^2} \Rightarrow \frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$$

$$\Rightarrow \frac{A}{1} = \frac{2}{1} \Rightarrow \therefore A = 2$$

7. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be __________.

Ans. **400 m/s**

Sol. $V_{rms}\sqrt{\frac{3RT_1}{M_0}}$

$$200 = \sqrt{\frac{3R \times 300}{M_0}} \qquad ...(1)$$

Also, $\frac{x}{\sqrt{3}} = \sqrt{\frac{3R \times 400}{M_0}}$ ...(2)

$(1) \div (2)$

$$\frac{200}{x/\sqrt{3}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$\Rightarrow x = 400\,m/s$

8. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5 rad/s$. The circuit draws 16W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _______ Ω.

Ans. **900**

Sol. $P = \frac{V^2}{R}$

$$16 = \frac{120^2}{R} \Rightarrow R = \frac{14400}{16}$$

$\Rightarrow R = 900\Omega$

9. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium wil be _______ $\times 10^{-2}$ cm.

Ans. 667

Sol. f = 3GHz, ε_r = 2.25

$$v = \lambda f \Rightarrow \lambda = \frac{v}{f}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} \Rightarrow \lambda = \frac{1}{f.\sqrt{\mu_0 \varepsilon_0}.\sqrt{\mu_r \varepsilon_r}.f}$$

$$\Rightarrow \lambda = \frac{C}{f.\sqrt{\mu_r}.\sqrt{\varepsilon_r}} \Rightarrow \lambda = \frac{3\times 10^8}{3\times 10^9 \times \sqrt{1} \times \sqrt{2.25}}$$

$$\Rightarrow \lambda = 667 \times 10^{-2}\,\text{cm}$$

10. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. the power received at receiver is 10^{-x}W. The value of x is ______.

[Gain in dB = 10 $\log_{10}\left(\frac{P_0}{P_i}\right)$]

Ans. 8

Sol. Power of signal transmitted : P_i = 0.1 Kw = 100w

Rate of attenuation = -5 dB/Km

Total length of path = 20 km

Total loss suffered = -5×20=-100dB

Gain in dB = 10 $\log_{10}\frac{P_0}{P_i}$

$$-100=10\log_{10}\frac{P_0}{P_i}$$

$$\Rightarrow \log_{10}\frac{P_i}{P_0} = 10$$

$$\Rightarrow \log_{10}\frac{P_i}{P_0} = \log_{10} 10^{10}$$

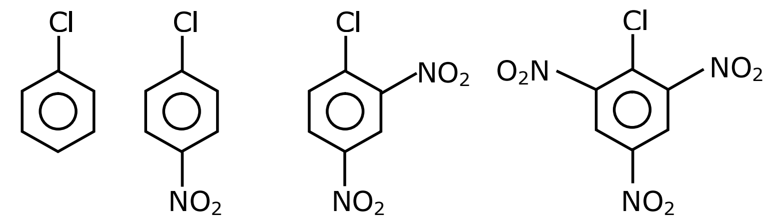
$$\Rightarrow \frac{100}{P_0} = 10^{10}$$

$$\Rightarrow P_0 = \frac{1}{10^8} = 10^{-8}$$

$\therefore$ x = 8

24th Feb. 2021 | Shift - 2
CHEMISTRY

1. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :

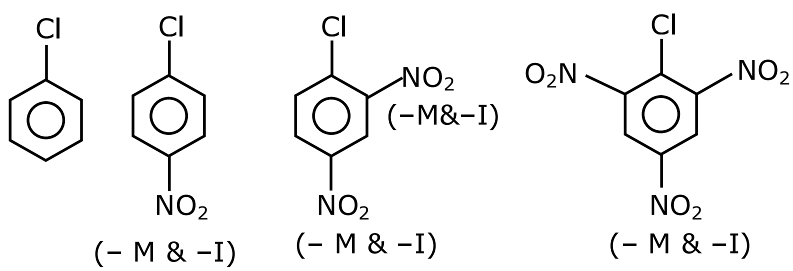


(i) (ii) (iii) (iv)

(1) (iv) < (i) < (iii) < (ii) (2) (iv) < (i) < (ii) < (iii)
(3) (i) < (ii) < (iii) < (iv) (4) (iv) < (iii) < (ii) < (i)

Ans. **(3)**

Sol.



(i) (ii) (iii) (iv)

Reactivity $\propto$ − m group present at O/P position.

2. Match List-I with List-II

List- I (Metal)	List-II (Ores)
(a) Aluminium	(i) Siderite
(b) Iron	(ii) Calamine
(c) Copper	(iii) Kaolinite
(d) Zinc	(iv) Malachite

Choose the correct answer from the options given below :

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i) (2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii) (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. **(3)**

Sol.

Siderite	$FeCO_3$
Calamine	$ZnCO_3$
Kaolinite	$Si_2Al_2O_5(OH)_4$ or $Al_2O_3.2SiO_2.2H_2O$
Malachite	$CuCO_3.Cu(OH)_2$

3. Match List-I with List-II

List- I	List-II
(Salt)	(Flame colour wavelength)
(a) LiCl	(i) 455.5 nm
(b) NaCl	(ii) 970.8 nm
(c) RbCl	(iii) 780.0 nm
(d) CsCl	(iv) 589.2 nm

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)   (2) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)   (4) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Ans. **(2)**

Sol. Range of visible region : - 390nm – 760nm
VIBGYOR
Violet   Red

LiCl	Crimson Red
NaCl	Golden yellow
RbCl	Violet
CsCl	Blue

So Licl Which is crimson have wave length closed to red in the spectrum of visible region which is as per given data is.

4. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the given below

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **(2)**

Sol. Hydrogen is most abundant element in universe because all luminous body of universe i.e. stars & nebulae are made up of hydrogen which acts as nuclear fuel & fusion reaction is responsible for their light.

5. Given below are two statements :

Statement I : The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

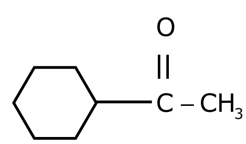
(4) Both Statement I and Statement II are true

Ans. (3)

Sol. For survival of aquatic life dissolved oxygen is responsible its optimum limit 6.5 ppm and optimum limit of BOD ranges from 10-20 ppm & BOD stands for biochemical oxygen demand.

6. Wich one of the following carbonyl compounds cannot be prepared by addition of wate on an alkyne in the presence of $HgSO_4$ and H_2SO_4 ?

(1) $CH_3-CH_2-\overset{\overset{\displaystyle O}{||}}{C}-H$

(2) 

(3) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-H$

(4) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_2CH_3$

Ans. (1)

Sol. Reaction of Alkyne with $HgSO_4$ & H_2SO_4 follow as

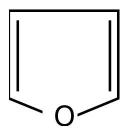
$$CH\equiv CH \xrightarrow[H_2O]{HgSO_4,H_2SO_4} CH_3CHO$$

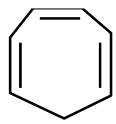
$$CH_3-C\equiv CH \xrightarrow[H_2O]{HgSO_4,H_2SO_4} CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_3$$

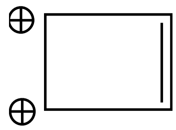
Hence, by this process preparation of CH_3CH_2CHO

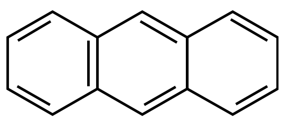
Cann't possible.

7. Which one of the following compounds is non-aromatic ?

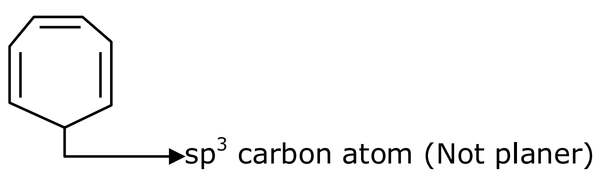
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

Hence It is non-aromatic.

8. The incorrect statement among the following is :

(1) $VOSO_4$ is a reducing agent

(2) Red colour of ruby is due to the presence of CO^{3+}

(3) Cr_2O_3 is an amphoteric oxide

(4) RuO_4 is an oxidizing agent

Ans. (2)

Sol. Red colour of ruby is due to presence of CrO_3 or Cr^{+6} not CO^{3+}

9. According to Bohr's atomic theory :

(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$

(B) The product of velocity (v) of electron and principal quantum number (n). 'vn' $\propto Z^2$.

(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.

(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.

Choose the most appropriate answer from the options given below:

(1) (C) only
(2) (A) and (D) only
(3) (A) only
(4) (A), (C) and (D) only

Ans. (2) Correction on NTA

Sol. (A) KE = –TE = 13.6 × $\frac{Z^2}{n^2}$eV

$KE \propto \frac{Z^2}{n^2}$

(B) $V = 2.188 \times 10^6 \times \frac{Z}{n}$ m/sec.

So, $Vn \propto Z$

(C) Frequency = $\frac{V}{2\pi r}$

So, $F \propto \frac{Z^2}{n^3}$ $\left[\because r \propto \frac{n^2}{z} \text{ and } v \propto \frac{Z}{n}\right]$

(D) Force $\propto \frac{z}{r^2}$

So, $F \propto \frac{Z^3}{n^4}$

So, only statement (A) is correct

10. Match List-I with List-II

List- I	List-II
(a) Valium	(i) Antifertility drug
(b) Morphine	(ii) Pernicious anaemia
(c) Norethindrone	(iii) Analgesic
(d) Vitamin B_{12}	(iv) Tranquilizer

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
(3) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
(4) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Ans. (4)

Sol.

(a) Valium	(iv) Tranquilizer
(b) Morphine	(iii) Analgesic
(c) Norethindrone	(i) Antifertility drug
(d) Vitamin B_{12}	(ii) Pernicious anaemia

11. The Correct set from the following in which both pairs are in correct order of melting point is :

(1) LiF > LiCl ; NaCl > MgO (2) LiF > LiCl ; MgO > NaCl

(3) LiCl > LiF ; NaCl > MgO (4) LiCl > LiF ; MgO > NaCl

Ans. **(2)**

Sol. Generally

M.P. $\propto$ Lattice energy = $\dfrac{KQ_1Q_2}{r^+ + r^-}$

$\propto$ (packing efficiency)

12. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are :

(1) 5.92, 4.90 and 0 BM (2) 5.82, O and 0 BM

(3) 4.90, 0 and 1.73 BM (4) 4.90, 0 and 2.83 BM

Ans. **(3)**

Sol. $[FeCl_4]^{2-}$ Fe^{2+} $3d^6 \rightarrow$ 4 unpaired electron. as Cl^- in a weak field liquid.

$\mu_{spin} = \sqrt{24}\,8M$

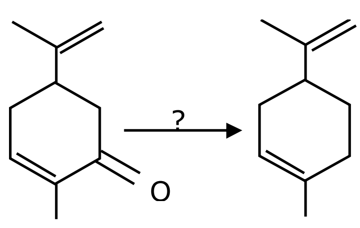
= 4.9 BM

$[Co(C_2O_4)_3]^{3-}$ Co^{3+} $3d^6 \rightarrow$ for Co^{3+} with coodination no. 6 $C_2O_4^{2-}$ is strong field ligend & causes pairing & hence no. unpaired electron

μ_{spin} = 0

$[MnO_4]^{2-}$ Mn^{+6} it has one unpaired electron.

μ_{spin} = $\sqrt{3}$ BM

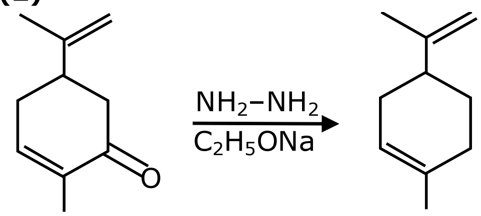
13. 

Which of the following reagent is suitable for the preparation of the product in the above reaction.

(1) Red P + Cl_2 (2) $NH_2{-}NH_2/C_2H_5\overset{\ominus\oplus}{ONa}$

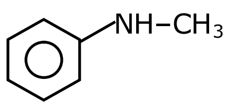
(3) Ni/H_2 (4) $NaBH_4$

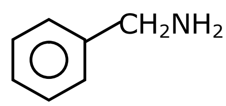
Ans. **(2)**

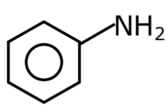
Sol. 

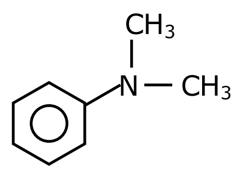
It is wolf-kishner reduction of carbonyl compounds.

14. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH ?

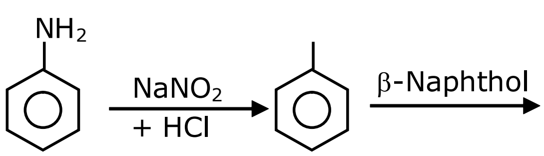
(1) 

(2) 

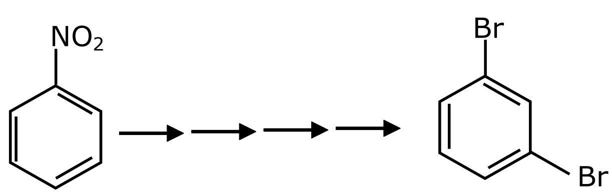
(2) 

(4) 

Ans. **(3)**

Sol.  Orange bright dye.

15. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene ?



(1) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

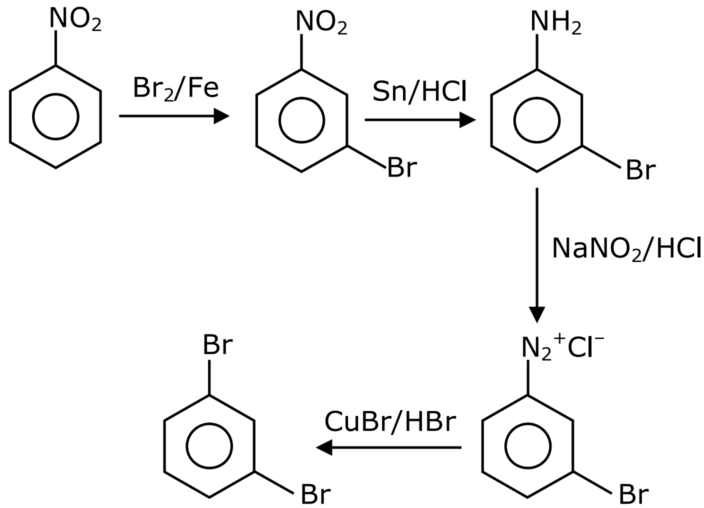
(2) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$

(3) $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{H^+}$

(4) $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

Ans. **(4)**

Sol.



16. The correct shape and I-I-I bond angles respectively in I_3^- ion are :

(1) Trigonal planar; 120º

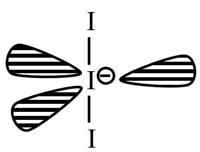
(2) Distorted trigonal planar; 135º and 90º

(3) Linear; 180º

(4) T-shaped; 180º and 90º

Ans. (3)

Sol. I_3^- sp^3d hybridisation (2BP + 3L.P.) Linear geometry



17. What is the correct order of the following elements with respect to their density ?

(1) Cr < Fe < Co < Cu < Zn

(2) Cr < Zn < Co < Cu < Fe

(3) Zn < Cu < Co < Fe < Cr

(4) Zn < Cr < Fe < Co < Cu

Ans. (4)

Sol. Fact Based

Density depend on many factor like atomic mass. atomic radius and packing efficiency.

18. Match List-I and List-II.

List – I	List-II
(a) $R-\overset{\overset{\displaystyle O}{\|}}{C}-Cl \rightarrow R-CHO$	(i) Br_2/NaOH
(b) $R-CH_2-COOH \rightarrow R-\underset{\underset{\displaystyle Cl}{\vert}}{CH}-COOH$	(ii) H_2/Pd-$BaSO_4$
(c) $R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 \rightarrow R-NH_2$	(iii) Zn(Hg)/Conc. HCl
(d) $R-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \rightarrow R-CH_2-CH_3$	(iv) Cl_2/Red P, H_2O

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
(4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. (4)

Sol. (a) $R-\overset{\overset{\displaystyle O}{\|}}{C}-Cl \xrightarrow{H_2/Pd-BaSO_4} R-CHO$ (Rosenmunt reaction)

(b) $R-CH_2-COOH \xrightarrow{Cl_2/Red\ P,\ H_2O} R-\underset{\underset{\displaystyle Cl}{\vert}}{CH}-COOH$ (HVZ reaction)

(c) $R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$ (Hoffmann Bromamide reaction)

(d) $R-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \xrightarrow{Zn(Hg)/conc.HCl} R-CH_2-CH_3$ (Clemmensen reaction)

19. In polymer Buna-S ; 'S' stands for :

(1) Styrene (2) Sulphur (3) Strength (4) Sulphonation

Ans. (1)

Sol. Buna-S is the co-polymer of buta- 1, 3 diene & styrene.

20. Most suitable salt which can be used for efficient clotting of blood will be :

(1) $Mg(HCO_3)_2$ (2) $FeSO_4$ (3) $NaHCO_3$ (4) $FeCl_3$

Ans. **(4)**

Sol. Blood is a negative sol. According to hardy-Schulz's rule, the cation with high charge has high coagulation power. Hence, $FeCl_3$ can be used for clotting blood.

Section -B

1. The magnitude of the change in oxidising power of the MnO_4^- / Mn^{2+} couple is x × 10^{-4} V, if the H^+ concentration is decreased from 1M to 10^{-4} M at 25ºC. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____.
(Rounded off to the nearest integer)

$$\left[\text{Given : } \frac{2303RT}{F} = 0.059\right]$$

Ans. **3776**

Sol. $5e^- + MnO_4^- + 8H^+ \longrightarrow Mn^{+2} + 4H_2O$

$$Q = \frac{\left[Mn^{+2}\right]}{\left[H^+\right]^8\left[MnO_4^-\right]} \quad \Rightarrow \quad E_1 = E^\circ - \frac{0.059}{5}\log(Q_1)$$

$$E_2 = E^\circ - \frac{0.059}{5}\log(Q_2) \quad \Rightarrow \quad E_2 - E_1 = \frac{0.059}{5}\log\left(\frac{Q_1}{Q_2}\right)$$

$$= \frac{0.059}{5}\log\left\{\frac{\left[H^+\right]_{II}}{\left[H^+\right]_{I}}\right\}^8 \quad \Rightarrow \quad = \frac{0.059}{5}\log\left(\frac{10^{-4}}{1}\right)^8$$

$$\left(E_2 - E_1\right) = \frac{0.059}{5}\times(-32) \quad \Rightarrow \quad \left|\left(E_2 - E_1\right)\right| = 32\times\frac{0.059}{5} = x\times10^{-4}$$

$$= \frac{32\times590}{5}\times10^{-4} = x\times10^{-4} \quad \Rightarrow \quad = 3776 \times 10^{-4} \qquad x = 3776$$

2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ______.
(1) α-sulphur (2) β-sulphur (3) S_2-form

Ans. **(1)**

Sol. S_2 is like O_2 i;e paramagnetic as per molecular orbital theory.

3. C_6H_6 freezes at 5.5ºC. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is ________ºC. (The molal freezing point depression constant of C_6H_6 is) 5.12ºC/m)

Ans. **1**

Sol. $\Delta T_f = i \times K_f \times m$

$= (1) \times 5.12 \times \dfrac{10/58}{200} \times 1000 \quad \Rightarrow \quad \Delta T_f = \dfrac{5.12 \times 50}{58} = 4.414$

$T_{f(\text{solution})} = T_{K(\text{ solvent})} - \Delta T_f$

$= 5.5 - 4.414$

$= 1.086°C$

$\approx 1.09°C = 1$ (nearest integer)

4. The volume occupied by 4.75 g of acetylene gas at 50ºC and 740 mmHg pressure is ________L. (Rounded off to the nearest integer)
(Given R = 0.0826 L atm K^{-1} mol^{-1})

Ans. **5**

Sol. T = 50ºC = 323.15 K

P = 740 mm of Hg = $\dfrac{740}{760}$ atm

V = ?

moles (n) = $\dfrac{4.75}{26}$

$V = \dfrac{4.75}{26} \times \dfrac{0.0821 \times 323.15}{740} \times 760$

V = 4.97 $\simeq$ 5 Lit

5. The solubility product of PbI_2 is 8.0×10^{-9} . The solubility of lead iodide in 0.1 molar solution of lead nitrate is x $\times 10^{-6}$ mol/L. The value of x is ________(Rounded off to the nearest integer)
[Given $\sqrt{2} = 1.41$]

Ans. **141**

Sol. $K_{SP}(PbI_2) = 8 \times 10^{-9}$

$PbI_2(s) \rightleftharpoons Pb^{+2}(aq) + 2I^-(aq)$

S + 0.1 2S

$K_{SP} = [Pb^{+2}][I^-]^2$

$8 \times 10^{-9} = (S + 0.1)(2S)^2 \Rightarrow 8 \times 10^{-9} \simeq 0.1 \times 4S^2$

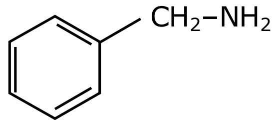
$\Rightarrow S^2 = 2 \times 10^{-8}$

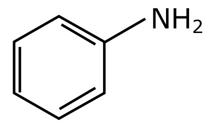
$S = 1.414 \times 10^{-4}$ mol/Lit

$= x \times 10^{-6}$ mol/Lit $\quad \therefore \; x = 141.4 \simeq 141$

6. The total number of amines among the following which can be synthesized by Gabriel synthesis is _______.

(1) $\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>CH-CH_2-NH_2$ (2) $CH_3CH_2NH_2$

(3) 

(4) 

Ans. (3)

Sol. Only aliphatic amines can be prepared by Gabriel synthesis.

7. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purificatiion. Amount of acetanilide obtained after purification (in g) is ____$\times 10^{-2}$.

Ans. 243

Sol.

$$\underset{\substack{(C_6H_7N)\\ \text{Molar mass} = 93}}{Ph-NH_2} \longrightarrow \underset{\substack{(\text{Acetanilide})(C_8H_9NO)\\ \text{Molar mass} = 135}}{Ph-NH-\overset{\overset{\Large O}{||}}{C}-CH_3}$$

93 g Aniline produce 135 g acetanilide

1.86 g produce $\frac{135 \times 1.86}{93} = 2.70\,g$

At 10% loss, 90% product will be formed after purification.

$\therefore$ Amount of product obtained = $\frac{2.70 \times 90}{100} = 2.43\,g$ = 243×10^{-2} g

8. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is …………

Ans. 8

Sol. $C_xH_y + 6O_2 \longrightarrow 4CO_2 + \frac{y}{2}\ H_2O$

Applying POAC on 'O' atoms

$6 \times 2 = 4 \times 2 + y/2 \times 1$

$y/2 = 4 \Rightarrow y = 8$

9. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25ºC. After 9h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _________ $\times 10^{-2}$ (Rounded off to the nearest integer)
[Assume: ln10 = 2.303, ln2 = 0.693]

Ans. **81**

Sol. Sucose $\xrightarrow{\text{Hydrolysis}}$ Glucose + Fructose

$t_{1/2} = 3.33\text{h} = \frac{10}{3}\text{h} \qquad \Rightarrow \qquad C_t = \frac{C_o}{2^{t/t_{1/2}}}$

Fraction of sucrose remaining = $f = \frac{C_t}{C_o} = \frac{1}{2^{t/t_{1/2}}}$

$\frac{1}{f} = 2^{t/t_{1/2}}$

$\log(1/f) = \log\left(2^{t/t_{1/2}}\right) = \frac{t}{t_{1/2}}\log(2)$

$= \frac{9}{10/3} \times 0.3 = \frac{8.1}{10} = 0.81 \quad = x \times 10^{-2} \qquad x = 81$

10. Assuming ideal behaviour, the magnitude of log K for the following reaction at 25ºC is $x \times 10^{-1}$. The value of x is __________.(Integer answer)

$$3HC \equiv CH_{(g)} \rightleftharpoons C_6H_{6(\ell)}$$

[Given : $\Delta_f G°(HC \equiv CH) = -2.04 \times 10^5$] mol^{-1}; $\Delta_f G°(C_6H_6) = -1.24 \times 10^5 \text{ J mol}^{-1}$; $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$]

Ans. **855**

Sol. $3HC \equiv CH(g) \rightleftharpoons C_6H_6(\ell)$

$\Delta G_r^\circ = \Delta G_f^\circ\left[C_6H_8(\ell)\right] - 3 \times \Delta G_f^\circ\left[HC \equiv CH\right]$

$= [-1.24 \times 10^5 - 3x(-2.04 \times 10^5)]$

$= 4.88 \times 10^5$ J/mol

$\Delta G_r^\circ = -RT\ln(K_{eq})$

$\log(K_{eq}) = \frac{-\Delta G^\circ}{2.303RT}$

$= \frac{-4.88 \times 10^5}{2.303 \times 8.314 \times 298}$

$= -8.55 \times 10^1 = 855 \times 10^{-1}$

24^{th} Feb. 2021 | Shift - 2
MATHEMATICS

1. Let $a,b \in R.$ If the mirror image of the point $P(a,6,9)$ with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is $(20,\text{b},-\text{a}-9)$, then $|\text{a}+\text{b}|$ is equal to :

(1) 86 (2) 88 (3)84 (4)90

Ans. (2)

Sol. P(a, 6, 9), Q (20, b, –a–9)

mid point of $\text{PQ}=\left(\frac{\text{a}+20}{2},\frac{\text{b}+6}{2},-\frac{\text{a}}{2}\right)$

lie on line

$$\frac{\frac{\text{a}+20}{2}-3}{7}=\frac{\frac{\text{b}+6}{2}-2}{5}=\frac{-\frac{\text{a}}{2}-1}{-9}$$

$$\frac{\text{a}+20-6}{14}=\frac{\text{b}+6-4}{10}=\frac{-\text{a}-2}{-18}$$

$$\frac{\text{a}+14}{14}=\frac{\text{a}+2}{18}$$

$$18\text{a}+252=14\text{a}+28$$

$$4\text{a}=-224$$

$$\boxed{\text{a}=-56}$$

$$\frac{\text{b}+2}{10}=\frac{\text{a}+2}{18}$$

$$\frac{\text{b}+2}{10}=\frac{-54}{18}$$

$$\frac{\text{b}+2}{10}=-3 \Rightarrow \text{b}=-32$$

$$|\text{a}+\text{b}|=|-56-32|=88$$

2. Let f be a twice differentiable function defined on R such that $f(0)=1, f'(0)=2$ and $f'(x)\neq 0$ for all x$\in$R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0,$ for all x$\in$R then the value of $f(1)$ lies in the interval:

(1) (9, 12) (2) (6, 9) (3) (3, 6) (4) (0, 3)

Ans. (2)

Sol. Given $f(x)\, f''(X) - (f'(x))^2 = 0$

Let $h(x) = \dfrac{f(x)}{f'(x)}$

$\Rightarrow h'(x) = 0 \qquad \Rightarrow h(x) = k$

$\Rightarrow \dfrac{f(x)}{f'(x)} = k \qquad \Rightarrow f(x) = k\, f'(x)$

$\Rightarrow f(0) = k\, f'(0) \qquad \Rightarrow 1 = k(2) \Rightarrow k = \dfrac{1}{2}$

Now $f(x) = \dfrac{1}{2} f'(x) \Rightarrow \int 2dx = \int \dfrac{f'(x)}{f(x)} dx$

$\Rightarrow 2x = \ln|f(x)| + C$

As $f(0) = 1 \Rightarrow C = 0$

$\Rightarrow 2x = \ln|f(X)| \Rightarrow f(x) = \pm e^{2x}$

As $f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$

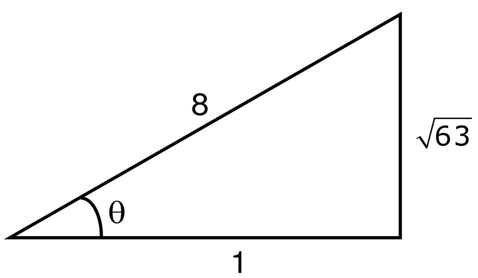
3. A possible value of $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$ is:

(1) $\dfrac{1}{2\sqrt{2}}$ (2) $\dfrac{1}{\sqrt{7}}$ (3) $\sqrt{7}-1$ (4) $2\sqrt{2}-1$

Ans. (2)

Sol. $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\dfrac{\sqrt{63}}{8}\right) = \theta \qquad \sin\theta = \dfrac{\sqrt{63}}{8}$



$\cos\theta = \dfrac{1}{8}$

$$2\cos^2\frac{\theta}{2}-1=\frac{1}{8}$$

$$\cos^2\frac{\theta}{2}=\frac{9}{16}$$

$$\cos\frac{\theta}{2}=\frac{3}{4}$$

$$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}}=\frac{3}{4}$$

$$\tan\frac{\theta}{4}=\frac{1}{\sqrt{7}}$$

4. The probability that two randomly selected subsets of the set {1,2,3,4,5} have exactly two elements in their intersection, is:

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$ (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

Ans. (2)

Sol. Required probability

$= \frac{{}^5C_2 \times 3^3}{4^5}$

$= \frac{10\times 27}{2^{10}}=\frac{135}{2^9}$

5. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j})=-2,$ and the point (1,0,2) is :

(1) $\vec{r}.\left(\hat{i}-7\hat{j}+3\hat{k}\right)=\frac{7}{3}$ (2) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

(3) $\vec{r}.\left(3\hat{i}+7\hat{j}+3\hat{k}\right)=7$ (4) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=\frac{7}{3}$

Ans. **(2)**

Sol. Plane passing through intersection of plane is

$$\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}+\lambda\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$$

Passes through $\hat{i}+2\hat{k}$, we get

$$(3-1)+\lambda(1+2)=0 \Rightarrow \lambda=-\frac{2}{3}$$

Hence, equation of plane is $3\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}-2\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$

$$\Rightarrow \quad \vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$$

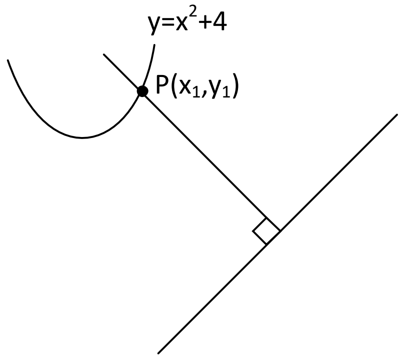
6. If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1,$ then the co-ordinates of P are :

(1) (–2, 8) (2) (1, 5) (3) (3, 13) (4) (2, 8)

Ans. **(4)**

Sol. $\frac{dy}{dx}\Big|_{p}=4$

$\therefore 2x_1 = 4$



$\Rightarrow x_1 = 2$

$\therefore$ Point will be (2, 8)

7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a,c),(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α,β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is:

(1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. (4)

Sol. 2b = a + c

$\frac{2a+2}{3}=\frac{10}{3}$ and $\frac{2b+c}{3}=\frac{7}{3}$

a = 4, $\left.\begin{matrix}2b+c=7\\2b-c=4\end{matrix}\right\}$, solving

$b=\frac{11}{4}$

$c=\frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2+\frac{11}{4}x+1=0$

$\therefore$ The value of $(\alpha+\beta)^2-3\alpha\beta=\frac{121}{256}-\frac{3}{4}=-\frac{71}{256}$

8. The value of the integral, $\int_1^3\left[x^2-2x-2\right]dx$, where [x] denotes the greatest integer less than or equal to x, is:

(1) –4 (2) –5 (3) $-\sqrt{2}-\sqrt{3}-1$ (4) $-\sqrt{2}-\sqrt{3}+1$

Ans. (3)

Sol. $I=\int_1^3 -3dx+\int_1^3\left[(x-1)^2\right]dx$

Put $x-1=t$; $dx=dt$

$I=(-6)+\int_0^2\left[t^2\right]dt$

$I=-6+\int_0^1 0\,dt+\int_1^{\sqrt{2}}1\,dt+\int_{\sqrt{2}}^{\sqrt{3}}2\,dt+\int_{\sqrt{3}}^{2}3\,dt$

$I=-6+\left(\sqrt{2}-1\right)+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$

$I=-1-\sqrt{2}-\sqrt{3}$

9. Let $f: \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\le x\le 4\\ 2x^3-3x^2-36x-336, & \text{if } x>4\end{cases}$$

Let $A=\{x\in R: f$ is increasing}. Then A is equal to :

(1) $(-5,-4)\cup(4,\infty)$ (2) $(-5,\infty)$

(3) $(-\infty,-5)\cup(4,\infty)$ (4) $(-\infty,-5)\cup(-4,\infty)$

Ans. (1)

Sol. $$f(x) = \begin{cases}-55 & ; \quad x<-5\\ 6\left(x^2-x-20\right) & ; \quad -5<x<4\\ 6\left(x^2-x-6\right) & ; \quad x>4\end{cases}$$

$$f(x) = \begin{cases}-55 & ; \quad x<-5\\ 6(x-5)(x+4) & ; \quad -5<x<4\\ 6(x-3)(x+2) & ; \quad x>4\end{cases}$$

Hence, f(x) is monotonically increasing in interval $(-5, -4)\cup(4, \infty)$

10. If the curve $y=ax^2+bx+c, x\in R,$ passes through the point (1,2) and the tangent line to this curve at origin is $y=x$, then the possible values of a, b, c are :

(1). a =1, b=1, c=0 (2) a = −1, b=1, c =1

(3) a =1, b=0, c =1 (4) $a=\frac{1}{2}, b=\frac{1}{2}, c=1$

Ans. (1)

Sol. 2 = a + b + c ......(i)

$\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)} = 1$

$\Rightarrow b = 1 \Rightarrow a + c = 1$

(0,0) lie on curve

$\therefore$ c=0, a=1

11. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim p \vee q$ (4) $p \vee \sim q$

Ans. (4)

Sol.

p	q	$\sim p$	$p \vee q$	$(\sim p) \wedge (p \vee q)$	$\sim q$	$p \vee \sim q$
T	T	F	T	F	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T

$\therefore \sim p \wedge (p \vee q) \equiv p \vee \sim q$

12. For the system of linear equations:

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbf{R}$

consider the following statements:

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

(1) (B) and (E) only (2)(C) and (D) only

(3) (A) and (D) only (4) (A) and (E) only

Ans. (3)

Sol. $x - 2y + 0.z = 1$

$x - y + kz = -2$

$0.x + ky + 4z = 6$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$$

For unique solution $4 - k^2 \neq 0$

$$\boxed{k \neq \pm 2}$$

For k=2

$x - 2y + 0.z = 1$

$x - y + 2z = -2$

$0.x + 2y + 4z = 6$

$$\Delta x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = (-8) + 2[-20]$$

$\Delta x = -48 \neq 0$

For k=2 $\Delta x \neq 0$

For K=2; The system has no solution

13. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$

(2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$

(4) $x^2 + y^2 = 7$

Ans. (1)

Sol. Tangent to $x^2 + 9y^2 = 9$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is $x\left|\frac{3\sqrt{3}}{2}\right| + 9y\left(\frac{1}{2}\right) = 9$

$3\sqrt{3}x + 9y = 18 \Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

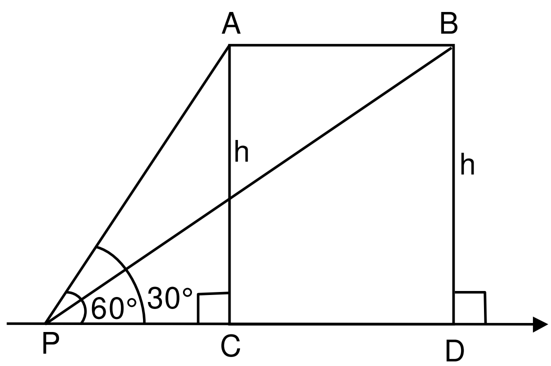
$\Rightarrow$ option (1) is true

14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/ hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:

(1) $1200\sqrt{3}\text{m}$ (2) $1800\sqrt{3}\text{m}$ (3) $3600\sqrt{3}\text{m}$ (4) $2400\sqrt{3}\text{m}$

Ans. (1)

Sol.



v = 432 × $\frac{1000}{60\times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In ΔPAC

$\tan 60° = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

$\tan 30° = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

h = $1200\sqrt{3}$ meter

15. For the statements p and q, consider the following compound statements:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q)) \wedge \sim P) \rightarrow P$

Then which of the following statements is correct?

(1) (a) is a tautology but not (b)
(2) (a) and (b) both are not tautologies.
(3) (a) and (b) both are tautologies.
(4) (b) is a tautology but not (a).

Ans. (3)

Sol. (a)

p	q	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim p$	$(\sim q) \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

(a) is tautologies

(b)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) is tautologies

$\therefore$ a & b are both tautologies.

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $\left(A^2 B^2 - B^2 A^2\right) X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

(1) a unique solution
(2) exactly two solutions
(3) infinitely many solutions
(4) no solution

Ans. (3)

Sol. $A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $ay + bz = 0$ ...(1)

$-ax + cz = 0$ ...(2)

$-bx - cy = 0$ ...(3)

From equation 1,2,3

$\Delta = 0$ & $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore$ equation have infinite number of solution

17. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$ is :

(1) $\frac{n(n+1)^2(n+2)}{12}$

(2) $\frac{n(n-1)(2n+1)}{6}$

(3) $\frac{n(n+1)(2n+1)}{6}$

(4) $\frac{n(2n+1)(3n+1)}{6}$

Ans. (3)

Sol. ${}^2C_2 = {}^3C_3$

$S = {}^3C_3 + {}^3C_2 + \ldots\ldots + {}^nC_2 = {}^{n+1}C_3$

$\because$ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$

$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)(2n+1)}{6}$

18. If a curve $y = f(x)$ passes through the point (1,2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of $b, \int_1^2 f(x)dx = \frac{62}{5}$?

(1)5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

Ans. (4)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$, I.F. $= e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$

Passes through (1,2), we get

$2 = \frac{b}{5} + C$ .....(i)

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x}\right)dx = \frac{62}{5}$

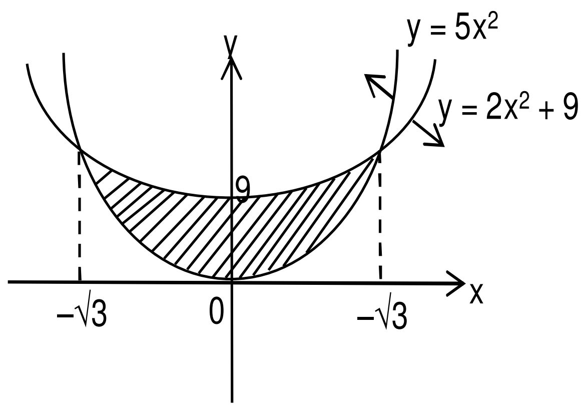
$\Rightarrow \frac{b}{25} \times 32 + C\ln 2 - \frac{b}{25} = \frac{62}{5} \Rightarrow C = 0 \ \& \ b = 10$

19. The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is:

(1) $9\sqrt{3}$ square units (2) $12\sqrt{3}$ square units (3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. (2)

Sol.



Required area

$$= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right) dx$$

$$= 2\int_0^{\sqrt{3}} \left(9 - 3x^2\right) dx$$

$$= 2\,|\,9x - x^3\,|_0^{\sqrt{3}} = 12\sqrt{3}$$

20. Let $f(x)$ be a differentiable function defined on [0,2] such that $f'(x) = f'(2-x)$ for all $x \in (0,2), f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)dx$ is:

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Ans. **(1)**

Sol. $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$f(0) + f(2) = c \quad \Rightarrow c = 1 + e^2$

$\Rightarrow f(x) + f(2-x) = 1 + e^2$ ......(i)

$$I = \int_0^2 f(x)\,dx = \int_0^1 \{f(x) + f(2-x)\}\,dx = \left(1 + e^2\right)$$

Section B

1. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is_______.

Ans. **2**

Sol. $x \geq 5$

$$\left(x+1\right)^2 + \left(x-5\right) = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$$

$$\Rightarrow 4x^2 + 12x - 43 = 0$$

$$x = \frac{-12 \pm \sqrt{144+688}}{8}$$

$$x = \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$$

$$= \frac{-3 \pm 7.2}{2}$$

$$= \frac{-3+7.2}{2}, \frac{-3-7.2}{2}$$ (Therefore no solution)

For $x \le 5$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$x^2 + x + 6 - \frac{27}{4} = 0$$

$$4x^2 + 4x - 3 = 0$$

$$x = \frac{-4 \pm \sqrt{16+48}}{8}$$

$$x = \frac{-4 \pm 8}{8} \Rightarrow x = -\frac{12}{8}, \frac{4}{8}$$

$\therefore$ 2 Real Root's

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is_______.

Ans. 31650

Sol.

$C \to 1 \qquad 9 \left[\begin{array}{l} A \\ B \end{array} \right.$

$C \to 2 \qquad 8 \left[\begin{array}{l} A \\ B \end{array} \right.$

$C \to 3 \qquad 7 \left[\begin{array}{l} A \\ B \end{array} \right.$

$= {}^{10}C_1\,[2^9 - 2] + {}^{10}C_2[2^8 - 2] + {}^{10}C_3\,[2^7 - 2]$

$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128\ [40 + 90 + 120\ ] - 350$

$= (128 \times 250) - 350$

$= 10[3165] = 31650$

3. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \neq 0,$ then the value of the expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is________.

Ans. 2

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ....(i)

$x \to \frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ....(ii)

(i) + (ii)

$(a + \alpha)\left[f(x) + f\left(\frac{1}{x}\right)\right] = \left(x + \frac{1}{x}\right)(b + \beta)$

$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$

4. If the variance of 10 natural numbers $1, 1, 1, \ldots, 1, k$ is less than 10, then the maximum possible value of k is**_________.**

Ans. 11

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\sigma^2 = \frac{\left(9+k^2\right)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$(90 + k^2)\ 10-(81 + k^2+8k) < 1000$

$90 + 10k^2-k^2 - 18k -81<1000$

$9k^2-18k + 9 < 1000$

$(k -1)^2 < \frac{1000}{9} \Rightarrow k-1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

5. Let λ be an integer. If the shortest distance between the lines $x-\lambda = 2y-1 = -2z$ and $x = y+2\lambda = z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

Ans. 1

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{2}{-1}$ .....(1) Point on line = $\left(\lambda, \frac{1}{2}, 0\right)$

$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$ .....(2) Point on line = $\left(0, -2\lambda, \lambda\right)$

Distance between skew lines = $\frac{\left[\vec{a}_2 - \vec{a}_1\ \vec{b}_1\ \vec{b}_2\right]}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}$$

$= \frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}}$ (given)

$= |10\lambda + 3| = 7 \Rightarrow \lambda = -1$

$\Rightarrow |\lambda| = 1$

6. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k,$ and $n = [|k|]$ be the greatest integral part of |k|. Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to______.

Ans. 310

Sol. $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}} + \frac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$

$\Rightarrow \frac{2^{21}.e^{i14\pi}}{2^{12}.e^{-i6\pi}} + \frac{2^{21}\left(e^{i7\pi}\right)}{2^{12}\left(e^{i6\pi}\right)}$

$\Rightarrow 2^9 e^{i(20\pi)} + 2^9 e^{i\pi}$

$\Rightarrow 2^9 + 2^9(-1) = 0$

n= 0

$\sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$

$\Rightarrow \left[5^2+6^2+7^2+8^2+9^2+10^2\right]-\left[5+6+7+8+9+10\right]$

$\Rightarrow \left[\left(1^2+2^2+....+10^2\right)-\left(1^2+2^2+3^2+4^2\right)\right]-\left[\left(1+2+3+.....+10\right)-\left(1+2+3+4\right)\right]$

$\Rightarrow (385-30)-[55-10]$

$\Rightarrow$ 355 – 45 $\Rightarrow$ 310 ans.

7. Let a point P be such that its distance from the point (5,0) is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

Ans. 56.25

Sol. Let P(h,k)

Given

PA = 3PB

$PA^2 = 9PB^2$

$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2+k^2]$

$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$

$\therefore$ Locus

$x^2 + y^2 + \left(\frac{25}{2}\right)x + 25 = 0$

$\therefore$ $c \equiv \left(\frac{-25}{4}, 0\right)$

$\therefore$ $r^2 = \left(\frac{-25}{4}\right)^2 - 25$

$= \frac{625}{16} - 25$

$= \frac{225}{16}$

$\therefore$ $4r^2 = 4\times\frac{225}{16} = \frac{225}{4} = 56.25$

8. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to________.

Ans. 12

Sol. $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \ldots\ldots + {}^{10}C_{10}x^{10}$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \ldots.{}^{15}C_{k-1}\, x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \ldots\ldots\ {}^{15}C_{15}x^{15}$

$\sum_{i=0}^{k} (10C_i)(15C_{k-i}) = {}^{10}C_0.\ {}^{15}C_k + {}^{10}C_1.{}^{15}C_{k-1} + \ldots..+{}^{10}C_k.{}^{15}C_0$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$\sum_{i=0}^{k+1} ({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0.{}^{13}C_{k+1} + {}^{12}C_1.\ {}^{13}C_k + \ldots\ldots + {}^{12}C_{k+1}.\ {}^{13}C_0$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

For maximum value

k+1= 13

K= 12

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is__________.

Ans. 3

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ …………(1)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$$\frac{1}{a}\left(\frac{r^3+r^2+r+1}{r^3}\right)=\frac{65}{18} \qquad \text{............(2)}$$

$$\frac{(i)}{(ii)}, a^2r^3=\frac{18}{12}=\frac{3}{2}$$

$$a^3r^3=1 \Rightarrow a\left(\frac{3}{2}\right)=1 \Rightarrow a=\frac{2}{3}$$

$$\frac{4}{9}r^3=\frac{3}{2} \Rightarrow r^3=\frac{3^3}{2^3} \Rightarrow r=\frac{3}{2}$$

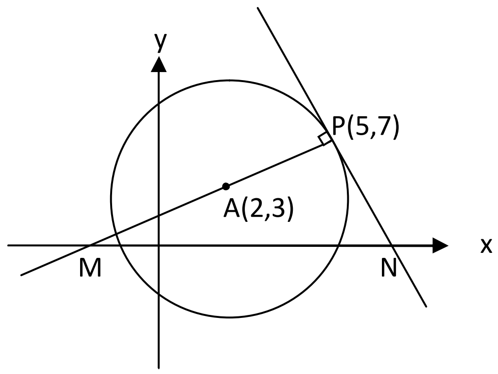
$$\alpha=ar^2=\frac{2}{3}.\left(\frac{3}{2}\right)^2=\frac{3}{2}$$

$2\alpha = 3$

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5,7) is A, then 24A is equal to______.

Ans. 1225

Sol.



Equation of normal at P

$$(y-7)=\left(\frac{7-3}{5-2}\right)(x-5)$$

3y −21 = 4x − 20

$\Rightarrow$ 4x − 3y + 1 = 0 ....(i)

$$\Rightarrow \quad M\left(-\frac{1}{4},0\right)$$

Equation of tangent at P

$$\left(y-7\right)=-\frac{3}{4}\left(x-5\right)$$

4y −28 = −3x + 15

$\Rightarrow$ 3x + 4y = 43 ………………(ii)

$$\Rightarrow \quad N\left(\frac{43}{3},0\right)$$

Hence ar (ΔPMN) = $\frac{1}{2}\times MN\times 7$

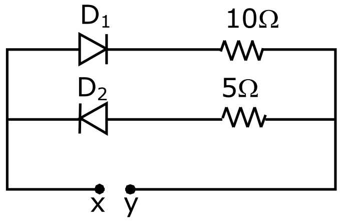
$$\lambda=\frac{1}{2}\times\frac{175}{12}\times 7$$

$\Rightarrow$ 24λ = 1225

25^{th} Feb. 2021 | Shift - 1
PHYSICS

Section - A

1. A 5V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



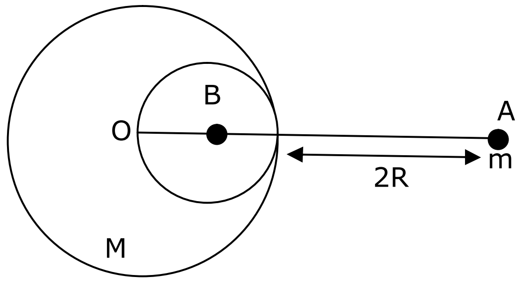
(1) ~0.86 A (2) ~0.5 A (3) ~ 0.43 A (4)~ 1.5 A

Sol. **3**

Since silicon diode is used so 0.7 Volt is drop across it, only D_1 will conduct so current through cell

$I = \frac{5-0.7}{10} = 0.43$ A

2. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :



(1)41 : 50 (2) 36 : 25 (3) 50 : 41 (4) 25 : 36

Sol. **1**

$$g_1 = \frac{GM}{(3R)^2} = \frac{GM}{9R^2}$$

$$g_2 = \frac{GM}{9R^2} - \frac{G\left(\frac{M}{8}\right)}{\left(3R - \frac{R}{2}\right)^2}$$

$$= \frac{GM}{9R^2} - \frac{GM}{R^2 50} = \frac{41}{9 \times 50}\frac{GM}{R^2}$$

$$\frac{g_1}{g_2} = \frac{41}{50}$$

Force $\Rightarrow \frac{F_1}{F_2} = \frac{mg_1}{mg_2} = \frac{41}{50}$

3. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

(1) 13 cm (2) 14.8 cm (3) 16.6 cm (4) 18.4 cm

Sol. **2**

$\lambda = \frac{v}{f} = \frac{336}{504} = 66.66\text{cm}$

$\frac{\lambda}{4} = l + e = l + 0.3d$

$= l + 1.8$

$16.66 = l + 1.8$ cm

$l = 14.86$ cm

4. A diatomic gas, having $C_p = \frac{7}{2}R$ and $C_v = \frac{5}{2}R,$ is heated at constant pressure.

The ratio dU : dQ : dW

(1) 3 : 7 : 2 (2) 5 : 7 : 2 (3) 5 : 7 : 3 (4) 3 : 5 : 2

Sol. **2**

$C_p = \frac{7}{2} R$

$C_v = \frac{5}{2} R$

$dU = nC_v dT$

$dQ = nC_p dT$

$dW = nRdT$

$dU : dQ : dW$

$C_v : C_p : R$

$\frac{5}{2}R : \frac{7}{2} R : R$

5 : 7 : 2

5. Given below are two statements :

Statement I : A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II : The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both statement I and statement II are false

(2) Statement I is false but statement II is true

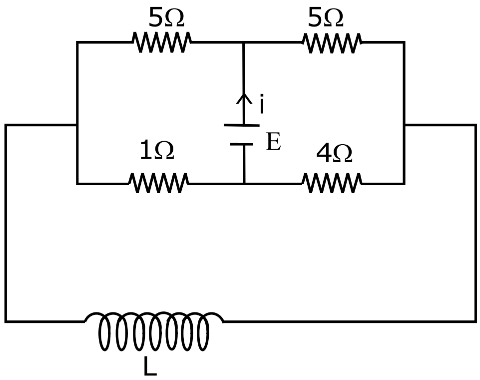
(3) Statement I is true but statement II is false

(4) Both statement I and statement II are true

Sol. **4**

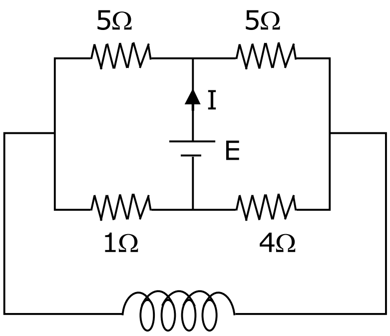
Side band = $(f_c - f_m)$ to $(f_c + f_m)$
= (1000 – 2) KHz to (1000 + 2) KHz
= 998 KHz to 1002 kHz
Band width = $2f_m$
= 2 ×2 KHz
= 4 KHz
Both statements are true

6. The current (i) at time t=0 and t = ∞ respectively for the given circuit is :

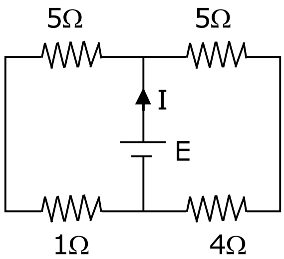


(1) $\frac{18E}{55}, \frac{5E}{18}$ (2) $\frac{5E}{18}, \frac{18E}{55}$ (3) $\frac{5E}{18}, \frac{10E}{33}$ (4) $\frac{10E}{33}, \frac{5E}{18}$

Sol. **3**



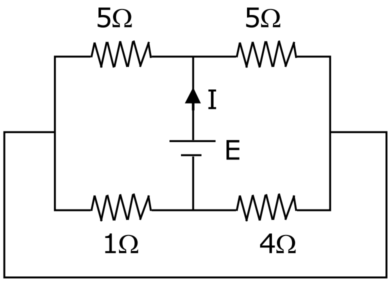
at t = 0, inductor is removed, so circuit will look like this
at t = 0



$R_{eq} = \frac{6 \times 9}{6 + 9} = \frac{54}{15}$

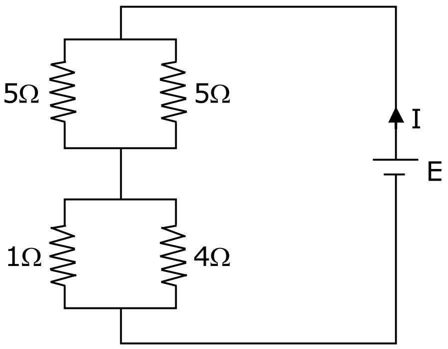
$I\ (t = 0) = \frac{E \times 15}{54} = \frac{5E}{18}$

at t = ∞, inductor is replaced by plane wire, so circuit will look like this
at t = ∞



$$I\ (t = \infty) = \frac{E}{\frac{5}{2}+\frac{4}{5}} = \frac{10E}{33}$$

Now,

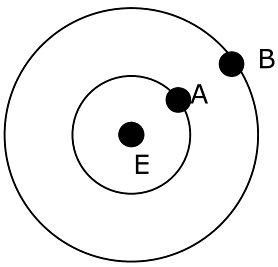


$$R_{eq} = \frac{1\times 4}{1+4} + \frac{5\times 5}{5+5}$$

$$= \frac{4}{5} + \frac{5}{2} = \frac{8+25}{10} = \frac{33}{10}$$

$$I = \frac{E}{R_{eq}} = \frac{10E}{33}$$

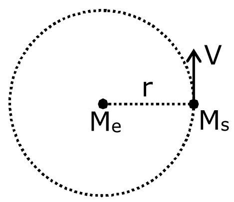
7. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.
If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:



[Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg]
(1) 4.24×10^2 s (2) 3.33×10^2 s (3) 1.33×10^3 s (4) 4.24×10^3 s

Sol. **3**

$V = \sqrt{\dfrac{GM_e}{r}}$



$T = \dfrac{2\pi r}{\sqrt{\dfrac{GM_e}{r}}} = 2\pi r\sqrt{\dfrac{r}{GM_e}}$

$T = \sqrt{\dfrac{4\pi^2 r^3}{GM_e}} = \sqrt{\dfrac{4\pi^2 r^3}{GM_e}}$

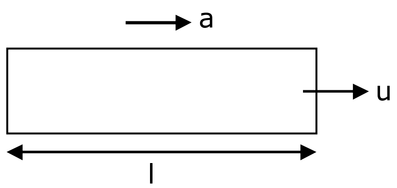
$T_2 - T_1 = \sqrt{\dfrac{4\pi^2(8000\times 10^3)^3}{G\times 6\times 10^{24}}} - \sqrt{\dfrac{4\pi^2(7000\times 10^3)^3}{G\times 6\times 10^{24}}}$

$\cong 1.33 \times 10^3 s$

8. An engine of a train, moving with uniform acceleration, passes the signal post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

(1) $\sqrt{\dfrac{v^2 - u^2}{2}}$ (2) $\dfrac{v-u}{2}$ (3) $\sqrt{\dfrac{v^2 + u^2}{2}}$ (4) $\dfrac{u+v}{2}$

Sol. **3**



a = uniform acceleration
u = velocity of first compartment
v = velocity of last compartment
l = length of train
$v^2 = u^2 + 2as$ (3rd equation of motion)
$v^2 = u^2 + 2al$ .....(1)

$v^2_{middle} = u^2 + 2a\ \dfrac{l}{2}$

$\therefore\ v^2_{middle} = u^2 + al$ ....(2)

from equation (1) and (2)

$v^2_{middle} = u^2 + \left(\dfrac{v^2 - u^2}{2}\right)$

$= \dfrac{v^2 + u^2}{2}$

$\therefore v_{middle} = \sqrt{\dfrac{v^2 + u^2}{2}}$

9. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces action on them is _______ and their speed is _____, in the ratio.

(1) 2 : 1 : 1 and 4 : 2 : 1 (2) 1 : 2 : 4 and 2 : 1 :1

(3) 1 : 2 : 4 and 1 : 1 : 2 (4) 4 : 2 : 1 and 2 : 1 : 1

Sol. **1**

As $v = \frac{p}{m}$ & $F = qvB$

$\therefore F = \frac{qp}{m}B$

$F_1 = \frac{qpB}{m}, v_1 = \frac{p}{m}$

$F_2 = \frac{qpB}{2m}, v_2 = \frac{p}{2m}$

$F_3 = \frac{2qpB}{4m}, v_3 = \frac{p}{4m}$

$F_1 : F_2 : F_3$ & $V_1 : V_2 : V_3$

$1 : \frac{1}{2} : \frac{1}{2}$ & $1 : \frac{1}{2} : \frac{1}{4}$

2 : 1 : 1 & 4 : 2 : 1

10. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.

Reason R : On heating, the length of the rod increases.

In the light of the above statements, choose the corect answer from the options given below :

(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is false but R is true

Sol. **3**

When a rod is free and it is heated then there is no thermal stress produced in it.

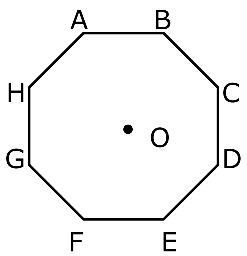
The rod will expand due to increase in temperature.

so both a & R are true.

11. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$

If, $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$



(1) $16\hat{i} + 24\hat{j} - 32\hat{k}$ (2) $-16\hat{i} - 24\hat{j} - 32\hat{k}$ (3) $-16\hat{i} - 24\hat{j} + 32\hat{k}$ (4) $-16\hat{i} + 24\hat{j} + 32\hat{k}$

Sol. **1**

$\overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{AB}$

$\overrightarrow{AO} + \overrightarrow{OC} = \overrightarrow{AC}$

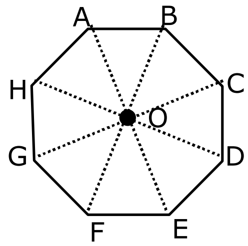
$\overrightarrow{AO} + \overrightarrow{OD} = \overrightarrow{AD}$

$\overrightarrow{AO} + \overrightarrow{OE} = \overrightarrow{AE}$

$\overrightarrow{AO} + \overrightarrow{OF} = \overrightarrow{AF}$

$\overrightarrow{AO} + \overrightarrow{OG} = \overrightarrow{AG}$

$\overrightarrow{AO} + \overrightarrow{OH} = \overrightarrow{AH}$



$8\ \overrightarrow{AO} = (\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH})$

$= 8(2\hat{i} + 3\hat{j} - 4\hat{k}).$

$= 16\hat{i} + 24\hat{j} - 32\hat{k}$

12. Two radioactive substances X and Y originally have N_1 and N_2nuclei respectively. Half life of X is half of the half life of Y. After there half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to :

(1) $\frac{8}{1}$ (2) $\frac{1}{8}$ (3) $\frac{3}{1}$ (4) $\frac{1}{3}$

Sol. **1**

After n half life no of nuclei undecayed $= \frac{N_o}{2^n}$

given $T_{\frac{1}{2}x} = \frac{T_{\frac{1}{2}y}}{2}$

So 3 half life of y = 6 half life of x

Given, $N_x = N_y\left(\text{after } 3T_{\frac{1}{2}y}\right)$

$\frac{N_1}{2^6} = \frac{N_2}{2^3}$

$\frac{N_1}{N_2} = \frac{2^6}{2^3} = 2^3 = \frac{8}{1}$

13. Match List –I with List- II :

List-I	List-II
(a)h (Planck's constant)	(i) $[M\,L\,T^{-1}]$
(b)E (kinetic energy)	(ii) $[M\,L^2\,T^{-1}]$
(c)V (electric potential)	(iii) $[M\,L^2\,T^{-2}]$
(d)P (linear momentum)	(iv) $[ML^2I^{-1}\,T^{-3}]$

Choose the correct answer from the options given below :

(1) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)

(2) (a) $\rightarrow$(i), (b) $\rightarrow$ (ii), (c) $\rightarrow$(iv), (d) $\rightarrow$ (iii)

(3) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$(iv), (d) $\rightarrow$(i)

(4) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

Sol. **1**

K.E. = $[ML^2T^{-2}]$

P (linear momentum) = $[MLT^{-1}]$

h (planck's constant) = $[ML^2T^{-1}]$

v (electric potential) = $[ML^2T^{-3}I^{-1}]$

14. The pitch of the screw guage is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lines 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line. The radius of the wire is :

(1) 1.64 mm (2) 1.80 mm (3) 0.82 mm (4) 0.90 mm

Sol. **3**

Least count. = $\frac{\text{pitch}}{\text{no. of div.}} = \frac{1\text{mm}}{100} = 0.01$ m

+ve error = 8 × L.C. = +0.08 mm

measured reading = 1mm + 72 × L.C.

= 1mm + 0.72 mm

= 1.72 mm

True reading = 1.72 – 0.08

= 1.64 mm

Radius = $\frac{1.64}{2}$ = 0.82 mm

15. If the time period of a two meter long simple pendulum is 2 s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :

(1) $2\pi^2 ms^{-2}$ (2) $16 m/s^2$ (3) $9.8 ms^{-2}$ (4) $\pi^2 ms^{-2}$

Sol. **1**

$T = 2\pi\sqrt{\frac{l}{g}}$

$T^2 = \frac{4\pi^2 l}{g}$

$g = \frac{4\pi^2 l}{T^2}$

$= \frac{4\pi^2 \times 2}{(2)^2} = 2\pi^2 ms^{-2}$

16. An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is :

(1) 8 (2) 2.8 (3) 3.8 (4) 7.8

Sol. **2**

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqv}}$

$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4 \times 2}{1 \times 1}}$

$= 2\sqrt{2} = 2.8$

17. Given below are two statements : one is labelled as Assertion A and the other is labelled as reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Sol. **2**

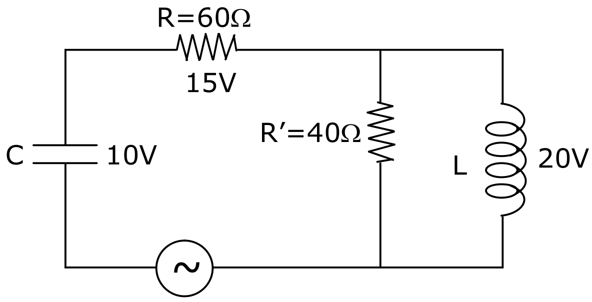
V_e = escape velocity

$v_e = \sqrt{\frac{2GM}{R}}$

so for same v_e, $\frac{M_1}{R_1} = \frac{M_2}{R_2}$

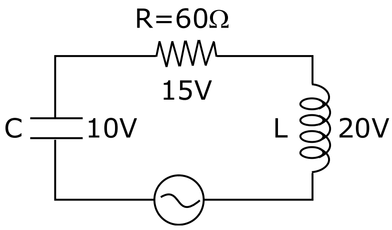
A is true but R is false

18. The angular frequency of alterlating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



(1) 0.8 H and 250 μF
(2) 0.8 H and 150 μF
(3) 1.33 H and 250 μF
(4) 1.33 H and 150 μF

Sol. **1**



Since key is open, circuit is series

$15 = i_{rms}\,(60)$

$\therefore i_{rms} = \frac{1}{4}$ A

Now, $20 = \frac{1}{4} X_L = \frac{1}{4}(\omega L)$

$\therefore L = \frac{4}{5} = 0.8$ H

& $10 = \frac{1}{4}\frac{1}{(100C)}$

$C = \frac{1}{4000}$ F = 250 μF

19. Two coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be :

(1) $\frac{2\sqrt{2x}}{x+1}$
(2) $\frac{\sqrt{2x}}{2x+1}$
(3) $\frac{2\sqrt{2x}}{2x+1}$
(4) $\frac{\sqrt{2x}}{x+1}$

Sol. **3**

Let $I_1 = 2x$

$I_2 = 1$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

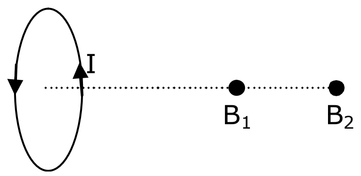
$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{2x}+1\right)^2 - \left(\sqrt{2x}-1\right)^2}{\left(\sqrt{2x}+1\right)^2 + \left(\sqrt{2x}-1\right)^2}$$

$$= \frac{4\sqrt{2x}}{2+4x} = \frac{2\sqrt{2x}}{1+2x}$$

20. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 02 m from the centre are in the rato 8 : 1. The radius of coil is _______

(1) 0.15 m (2) 0.2 m (3) 0.1 m (4) 1.0 m

Sol. **3**



$$B = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$$

at $x_1 = 0.05$m, $B_1 = \dfrac{\mu_0 NiR^2}{2(R^2 + (0.05)^2)^{3/2}}$

at $x_2 = 0.2$m, $B_2 = \dfrac{\mu_0 NiR^2}{2(R^2 + (0.2)^2)^{3/2}}$

$$\frac{B_1}{B_2} = \frac{(R^2 + 0.04)^{3/2}}{(R^2 + 0.0025)^{3/2}}$$

$$\left(\frac{8}{1}\right)^{2/3} = \frac{R^2 + 0.04}{R^2 + 0.0025}$$

$4\ (R^2 + 0.0025) = R^2 + 0.04$

$3R^2 = 0.04 - 0.0100$

$R^2 = \dfrac{0.03}{3} = 0.01$

$R = \sqrt{0.01} = 0.1$ m

Section - B

1. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is ______ cm.

Sol. **15**

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ...(1)

$m = \frac{v}{u}$ ....(2)

from (1) and (2) we get

$m = \frac{f}{f+u}$

given conditions

$m_1 = -m_2$

$\frac{f}{f-10} = \frac{-f}{f-20}$

f − 20 = -f + 10

2f = 30

f = 15 cm

2. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to y-z plane) to that of the surface of area 0.3 m^2(parallel to x-z plane) is a : b, where a = ________

[Here $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively]

Sol. **0.5**

$\phi = \vec{E}.\vec{A}$

$\vec{A}_a = 0.2\hat{i}$

$\vec{A}_b = 0.3\ \hat{j}$

$\phi_a = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).0.2\hat{i}$

$\phi_a = \frac{3}{5}\ E_0 \times 0.2$

$\phi_a = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).0.3\hat{j}$

$\phi_b = \frac{4}{5}\ E_0 \times 0.3$

$\frac{a}{b} = \frac{\phi_a}{\phi_b} = \frac{\frac{3}{5}E_0 \times 0.2}{\frac{4}{5}E_0 \times 0.3} = \frac{6}{12} = 0.5$

3. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is ____ V.

Sol. **128**

Let charge on each drop = q

radius = r

$$v = \frac{kq}{r}$$

$$2 = \frac{kq}{r}$$

radius of bigger

$$\frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3$$

R = 8r

$$v = \frac{k(512)q}{R} = \frac{512}{8}\frac{kq}{r} = \frac{512}{8} \times 2$$

= 128 V

4. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$ Where, a and b are positive constants. The equilibrium distance between two atoms will $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$. Where a =_______

Sol. **1**

$$F = -\frac{dU}{dr}$$

$$F = -\left[-\frac{10\alpha}{r^{11}} + \frac{5\beta}{r^6}\right]$$

for equilibrium, F = 0

$$\frac{10\alpha}{r^{11}} = \frac{5\beta}{r^6}$$

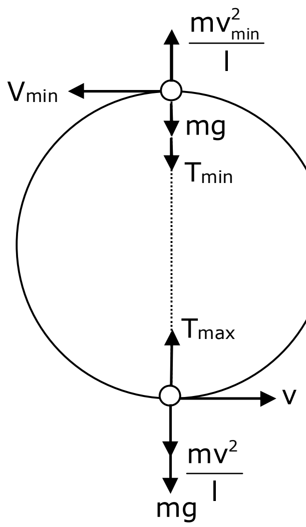
$$\frac{2\alpha}{\beta} = r^5$$

$$r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$$

a = 1

5. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the rato 5 : 1. The velocity of the bob at the highest position is _______ m/s. (take g=10 m/s^2)

Sol. **5**



by conservation of energy,

$v^2_{min} = V^2 - 4gl$ ....(1)

$T_{max} = mg + \frac{mv^2}{l}$ ....(2)

$T_{min} = \frac{mv^2_{min}}{l} - mg$ ....(3)

from equation (1) and (3)

$T_{min} = \frac{m}{l}(v^2 - 4gl) - mg$

$$\frac{T_{max}}{T_{min}} = \frac{\frac{v^2}{l} + g}{\frac{v^2}{l} - 5g}$$

$$\frac{5}{1} = \frac{\frac{v^2}{1} + 10}{\frac{v^2}{1} - 50}$$

$5v^2 - 250 = v^2 + 10$

$v^2 = 65$ ....(4)

from equation (4) and (1)

$v^2_{min} = 65 - 40 = 25$

$v_{min} = 5$

6. In a certain themodynamical process, the pressure ofa gas depends on its volume as kV^3 . The work done when the temperature changes from 100ºC to 300ºC will be ____ nR, where n denotes number of moles of a gas.

Sol. **50**

$P = kv^3$

$pv^{-3} = k$

$x = -3$

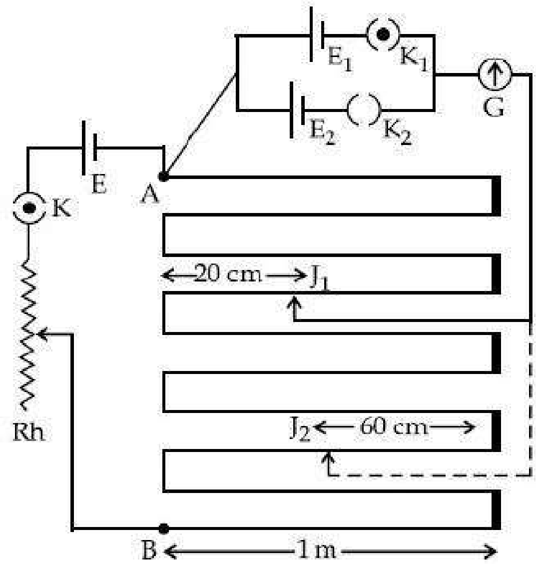
$$w = \frac{nR(T_1 - T_2)}{x - 1}$$

$$= \frac{nR(100 - 300)}{-3 - 1}$$

$$= \frac{nR(-200)}{-4}$$

$= 50nR$

7. In the given circuit of potentiometer, the potentital difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and E_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ____.



Sol. **1**

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$= \frac{3 \times 100cm + (100 - 20)cm}{7 \times 100cm + 60cm}$$

$$= \frac{380}{760} = \frac{1}{2} = \frac{a}{b}$$

$a = 1$

8. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R=gas constant) is $\frac{x}{3R}$. Value of x is _______ .

Sol. **3600**

$\Delta K_E = \Delta U$

$\Delta U = nC_V\Delta T$

$\frac{1}{2}mv^2 = \frac{3}{2}nR\Delta T$

$\frac{mv^2}{3nR} = \Delta T$

$\frac{4 \times (30)^2}{3 \times 1 \times R} = \Delta T$

$\Delta T = \frac{1200}{R}$

$\frac{x}{3R} = \frac{1200}{R}$

$x = 3600$

9. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by V =3t volt. (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4 s is ________ J.

Sol. **144**

$L\frac{di}{dt} = \varepsilon$

$= 3t$

$L\int di = 3\int tdt$

$Li = \frac{3t^2}{2}$

$i = \frac{3t^2}{2L}$

energy, $E = \frac{1}{2}Li^2$

$= \frac{1}{2}L\left(\frac{3t^2}{2L}\right)^2$

$= \frac{1}{2} \times \frac{9t^4}{4L}$

$= \frac{9}{8} \times \frac{(4)^4}{4 \times 2} = 144$ J

10. A transmitting station releases waves of wavelength 960 m. A capacitor of 256 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is _____ $\times 10^{-8}$H.

Sol. **10**

Since resonance

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore\ 2\pi f = \frac{1}{\sqrt{LC}}$$

$$\therefore\ 4\pi^2\frac{C^2}{\lambda^2} = \frac{1}{LC}$$

$$\therefore \frac{4\pi^2 \times 9 \times 10^8 \times 9 \times 10^8}{960 \times 960} = \frac{1}{L \times 2.56 \times 10^{-6}}$$

$$L = \frac{375 \times 960}{10^{-6} \times 4 \times \pi^2 \times 9 \times 10^{16}} = \frac{10^3}{10^{10}}$$

$= 10^{-7}$ H

$= 10 \times 10^{-8}$

25^{th} Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. Ellingham diagram is a graphical representation of:
(1) ΔG vs T (2) $(\Delta G - T\Delta S)$ vs T (3) ΔH vs T (4) ΔG vs P

Sol. (1)
Ellingham diagram tells us about the spontanity of a reaction with temperature.

2. Which of the following equation depicts the oxidizing nature of H_2O_2?
(1) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$ (2) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$
(3) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$ (4) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H2O + O_2$

Sol. (3)
$2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$
Oxygen reduces from -1 to -2
So, its reduction will takes place. Hence it will behave as oxidising agent or it shows oxidising nature.
While in other option it change from (-1) to 0.

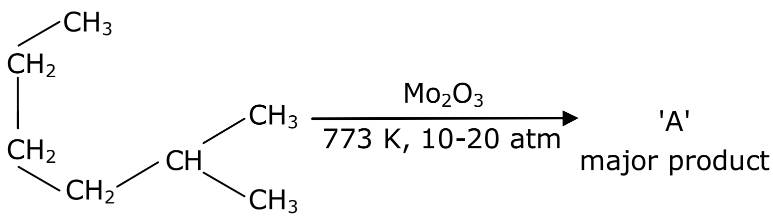
3. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is:
(1) ∞ (2) 1 (3) zero (4) $\frac{1}{n}$

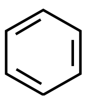
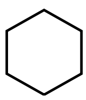
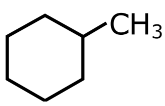
Sol. (4)
$\frac{x}{m} = p^x$
the formula is $\frac{x}{m} = p^{1/n}$
Hence $x = \frac{1}{n}$
The value of 'n' is any natural number.

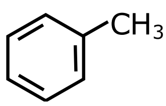
4. According to molecular orbital theory, the species among the following that does not exist is:
(1) He_2^- (2) He_2^+ (3) O_2^{2-} (4) Be_2

Sol. (4)
B.O. of Be_2 is zero, So it does not exist.

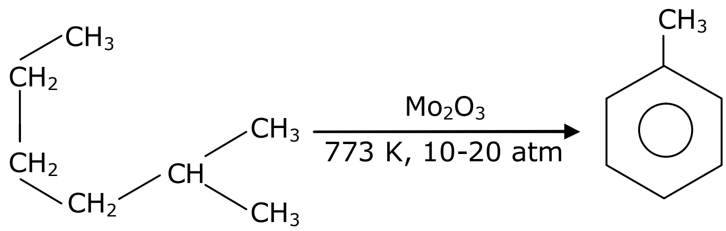
5. Identify A in the given chemical reaction.



(1)  (2)  (3) 

(4) 

Sol. **(4)**



Aromatization reaction or hydroforming reaction.

6. Given below are two statements:
Statement-I : CeO_2 can be used for oxidation of aldehydes and ketones.
Statement-II : Aqueous solution of $EuSO_4$ is a strong reducing agent.
(1) Statement I is true, statement II is false
(2) Statement I is false, statement II is true
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Sol. **(4)**
CeO_2 can be used as oxidising agent like seO_2.
Similarly $EuSO_4$ used as a reducing agent.

7. The major product of the following chemical reaction is:

$$CH_3CH_2CN \xrightarrow[\text{3) Pd/BaSO}_4\text{, H}_2]{\text{1) H}_3\text{O}^+\text{, }\Delta \quad \text{2) SOCl}_2} ?$$

(1) $(CH_3CH_2CO)_2O$ (2) CH_3CH_2CHO (3) $CH_3CH_2CH_3$ (4) $CH_3CH_2CH_2OH$

Sol. **(2)**

$$CH_3CH_2CN \xrightarrow{H_3O^+} CH_3-CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-OH \xrightarrow{SOCl_2} CH_3-CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-Cl$$

$$\xrightarrow{\text{Pd/Ba,SO}_4\text{,}} CH_3-CH_2-CH=O$$

8. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is:
(1) 63.53 (2) 53.33 (3) 51.63 (4) 50.33

Sol. **(2)**

$n_{CO_2} = \frac{2.64}{44} = 0.06$

$n_c = 0.06$

weight of carbon = 0.06 × 12 = 0.72 gm

$n_{H_2O} = \frac{1.08}{18} = 0.06$

$n_H = 0.06 \times 2 = 0.12$

weight of H = 0.12 gm

∴ Weight of oxygen in $C_xH_yO_z$

= 1.8 – (0.72 + 0.12)

= 0.96 gram

% weight of oxygen = $\frac{0.96}{1.8} \times 100$

= 53.3%

9. The correct statement about B_2H_6 is:

(1) All B–H–B angles are of 120°.

(2) Its fragment, BH_3, behaves as a Lewis base.

(3) Terminal B–H bonds have less p-character when compared to bridging bonds.

(4) The two B–H–B bonds are not of same length.

Sol. **(3)**

Terminal bond angle is greater than that of bridge bond angle

Bond angle $\propto$ S-character

$\propto \frac{1}{p - \text{character}}$

10. In which of the following pairs, the outer most electronic configuration will be the same?

(1) Fe^{2+} and Co^+

(2) Cr^+ and Mn^{2+}

(3) Ni^{2+} and Cu^+

(4) V^{2+} and Cr^+

Sol. **(2)**

$Cr^+ \rightarrow [Ar]3d^5$

$Mn^{2+} \Rightarrow [Ar]3d^5$

11. Which statement is correct?

(1) Buna-S is a synthetic and linear thermosetting polymer

(2) Neoprene is addition copolymer used in plastic bucket manufacturing

(3) Synthesis of Buna-S needs nascent oxygen

(4) Buna-N is a natural polymer

Sol. **(3)**

Synthesis of Buna-S needs nascent oxygen.

12. Given below are two statements:

Statement-I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement-II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.

In the light of the above statements, choose the correct answer from the options given below:

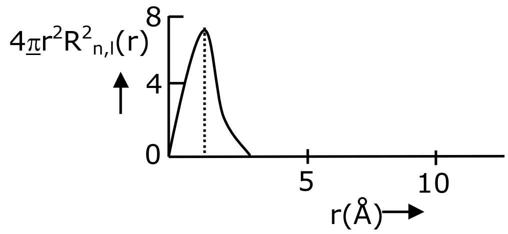
(1) Statement I and Statement II are true
(2) Statement I is true about Statement II is false
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Sol. **(3)**

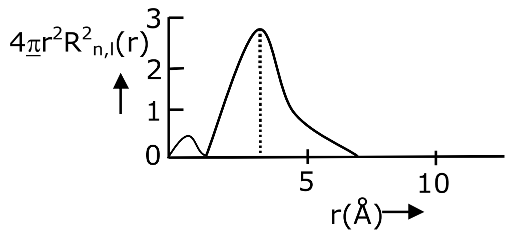
Reducing smog as is acts as reducing agent, the reducing character is due to presence of sulphur dioxide and carbon particles.

13. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below:

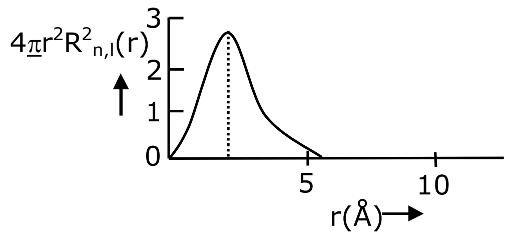
(A)



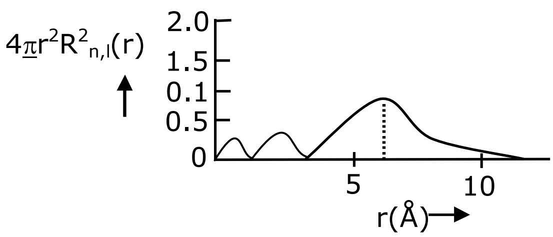
(B)



(C)



(D)



The correct plot for 3s orbital is:

(1) D (2) B (3) A (4) C

Sol. **(1)**

3s orbital

Number of radial nodes = $n - \ell - 1$

For 3s orbital $n = 3 \quad \ell = 0$

Number of radial nodes = $3 - 0 - 1 = 2$

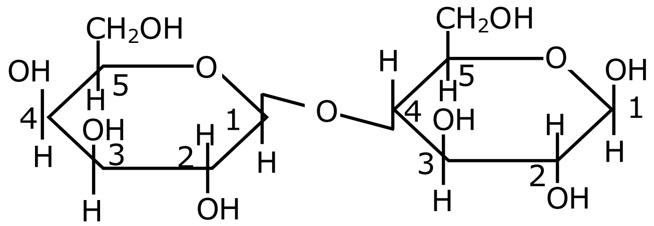
It is correctly represented in graph of option D

14. Which of the glycosidic linkage galactose and glucose is present in lactose?

(1) C-1 of glucose and C-6 of galactose (2) C-1 of galactose and C-4 of glucose

(3) C-1 of glucose and C-4 of galactose (4) C-1 of galactose and C-6 of glucose

Sol. **(2)**



β-D-Galactose β-D-Glucose

15. Which one of the following reactions will not form acetaldehyde?

(1) $CH_3CH_2OH \xrightarrow{CrO_3 - H_2SO_4}$

(2) $CH_3CN \xrightarrow[\text{ii) } H_2O]{\text{i) DIBAL-H}}$

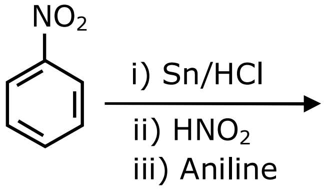
(3) $CH_2 = CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

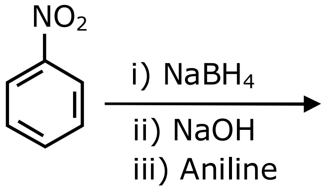
(4) $CH_3CH_2OH \xrightarrow[573\ K]{Cu}$

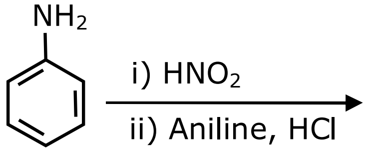
Sol. **(1)**

$$CH_3CH_2OH \xrightarrow{CrO_3 - H_2SO_4} CH_3 - COOH$$

16. Which of the following reaction/s will not give p-aminoazobenzene?

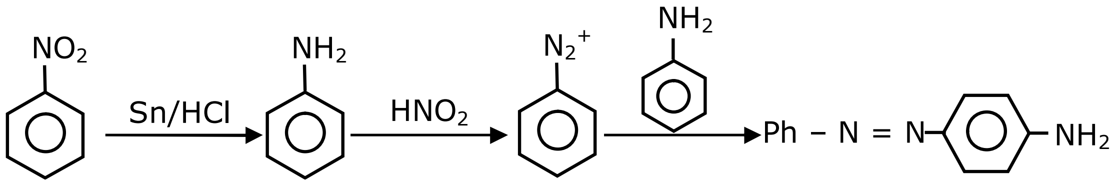
(A) 

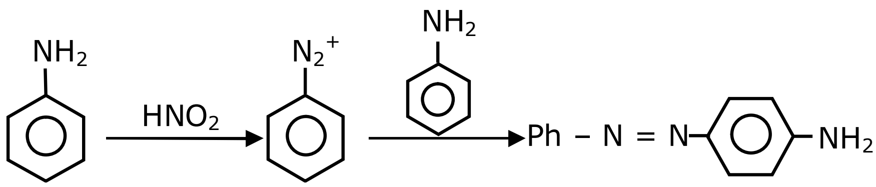
(B) 

(C) 

(1) B only (2) A and B (3) C only (4) A only

Sol. **(1)**

(A) 

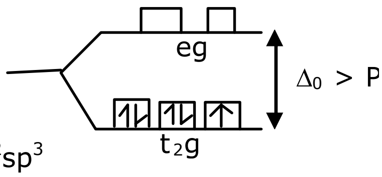
(C) 

17. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are:

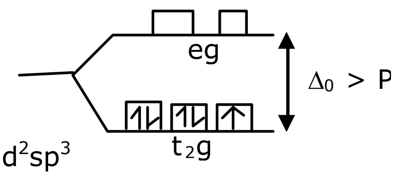
(1) d^2sp^3 and paramagnetic (2) sp^3d^2 and paramagnetic
(3) d^2sp^3 and diamagnetic (4) sp^3d^2 and diamagnetic

Sol. **(1)**

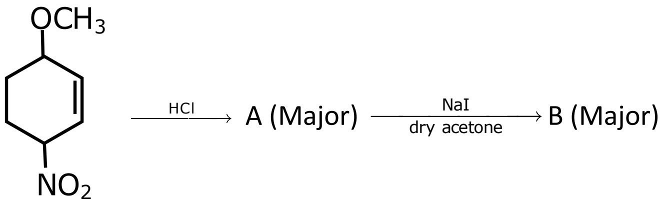
1. $(Mn(CN)_6)^{4-}$
$Mn^{++} = 3d^5$
$\mu = \sqrt{3}$
hybridization = d^2sp^3

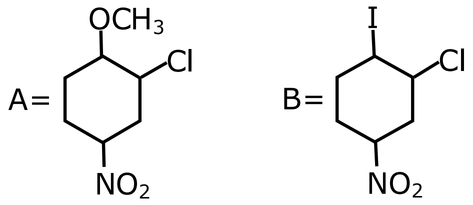


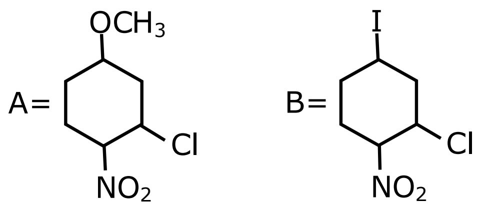
2. $[Fe(CN)_6]^{3-}$
$Fe^{3+} = 3d^5$
$\mu = \sqrt{3}$
Hybridization – d^2sp^3

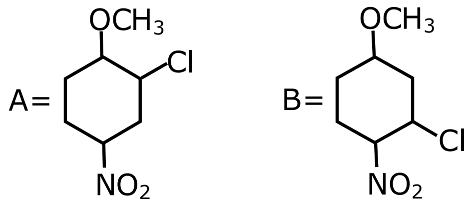


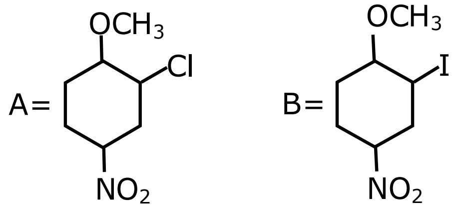
18. Identify A and B in the chemical reaction.



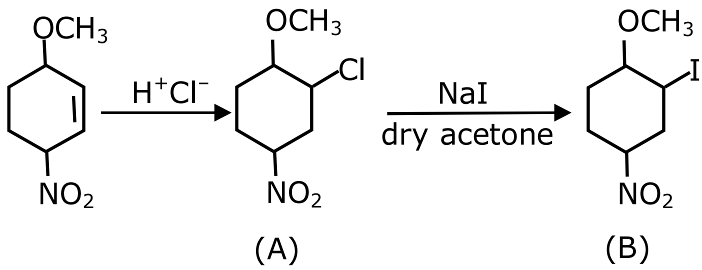
(1) 

(2) 

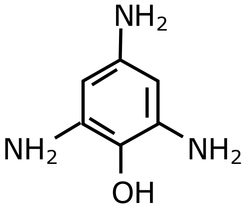
(3) 

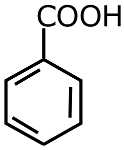
(4) 

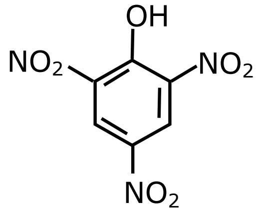
Sol. (4)



19. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are:

A = 

B = 

C = 

(1) B and C only (2) B only (3) A and B only (4) C only

Sol. (1)

Compounds which are more acidic then H_2CO_3, gives CO_2 gas on reaction with $NaHCO_3$. Compound B i.e. Benzoic acid and compound C i.e. picric acid both are more acidic than H_2CO_3.

20. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is:
[Assume: No cyano complex is formed; $K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and K_a (HCN) = 6.2×10^{-10}]
(1) 0.625×10^{-6} (2) 1.6×10^{-6} (3) 2.2×10^{-16} (4) 1.9×10^{-5}

Sol. (4)

Let solubility is x

$$\underset{}{AgCN} \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{CN^-} \qquad K_{sp} = 2.2 \times 10^{-16}$$

$$H^+ + CN^- \rightleftharpoons HCN \qquad K = \frac{1}{K_a} = \frac{1}{6.2\times10^{-10}}$$

$$K_{sp} \times \frac{1}{K_a} = [Ag^+]\,[CN^-] \times \frac{[HCN]}{[H^+][CN^-]}$$

$$2.2\times10^{-16} \times \frac{1}{6.2\times10^{-10}} = \frac{[S][S]}{10^{-3}}$$

$$S^2 = \frac{2.2}{6.2}\times10^{-9}$$

$$S^2 = 3.55 \times 10^{-10}$$

$$S = \sqrt{3.55\times10^{-10}}$$

$$S = 1.88 \times 10^{-5} \Rightarrow 1.9 \times 10^{-5}$$

SECTION - B

1. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$NH_2CN_{(s)} + \frac{3}{2}O_2(g) \rightarrow N_{2(g)} + O_{2(g)} + H_2O_{(l)}$ is ______ kJ. (Rounded off to the nearest integer)

[Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}]

Sol. 741 kJ/mol

$$NH_2CN(s) + \frac{3}{2}O_2(g) \longrightarrow N_2(g) + CO_2(g) + H_2O(\ell)$$

$$\Delta ng = (1 + 1) - \frac{3}{2} = \frac{1}{2}$$

$\Delta H = \Delta U + \Delta ng\ RT$

$= -742.24 + \frac{1}{2} \times \frac{8.314 \times 298}{1000}$

$= -742.24 + 1.24$

$= 741$ kJ/mol

2. In basic medium CrO_4^{2-} oxidizes $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is _______ mL. (Rounded-off to the nearest integer)

Sol. 173 mL

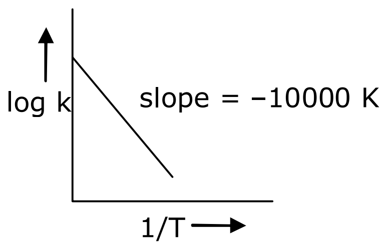
$17H_2O + 8CrO_4 + 3S_2O_3 \longrightarrow 6SO_4 + 8Cr(OH)_4^- + 2OH^-$

Applying mole-mole analysis

$$\frac{0.154 \times v}{8} = \frac{40 \times 0.25}{3}$$

V = 173 mL

3. For the reaction, $aA + bB \rightarrow cC + dD$, the plot of log k vs $\frac{1}{T}$ is given below:



The temperature at which the rate constant of the reaction is $10^{-4} s^{-1}$ is _________ K.
[Rounded off to the nearest integer)
[Given: The rate constant of the reaction is $10^{-5}\ s^{-1}$ at 500 K]

Sol. 526 K

$$\log_{10}K = \log_{10}A - \frac{E_a}{2.303RT}$$

$$\text{Slope} = \frac{E_a}{2.303R} = -10000$$

$$\log_{10}\frac{K_2}{K_1} = \frac{E_a}{2.303R} \times \left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log_{10}\frac{10^{-4}}{10^{-5}} = 10000 \times \left[\frac{1}{500} - \frac{1}{T}\right]$$

$$1 = 10000 \times \left[\frac{1}{500} - \frac{1}{T}\right]$$

$$\frac{1}{10000} = \frac{1}{500} - \frac{1}{T}$$

$$\frac{1}{T} = \frac{1}{500} - \frac{1}{10000}$$

$$\frac{1}{T} = \frac{20-1}{10000} = \frac{19}{10000}$$

$$T = \frac{10,000}{19} \Rightarrow 526 \text{ K}$$

4. 0.4g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is ______. (Rounded-off to the nearest integer)

Sol. **3%**

1st end point reaction

$NaOH + HCl \longrightarrow NaCl + H_2O$

nf = 1

$NaCO_3 + HCl \longrightarrow NaHCO_3$

nf = 1

Eq of HCl used $= n_{NaOH} \times 1 + n_{Na_2CO_3} \times 1$

$$17.5 \times \frac{1}{10} \times 10^{-3} = n_{NaOH} + n_{Na_2CO_3}$$

2nd end point

$NaHCO_3 + HCl \longrightarrow H_2CO_3$

$$1.5 \times \frac{1}{10} \times 10^{-3} = n_{NaHCO_3} \times 1 = n_{NaHCO_3}$$

0.15 mmol = $n_{Na_2CO_3}$

0.15 = $n_{Na_2CO_3}$

$$w_{Na_2CO_3} = \frac{0.15 \times 106 \times 10^{-3}}{0.5} \times 100 \times 10$$

$= 3 \times 106 \times 10^{-2}$

$= 3 \times 1.06 = 3.18\%$

5. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is −325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is −728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (−)_____ $\times 10^{-1}$ kJ mol^{-1}.

Sol. **5576 kJ**

$Na(s) \longrightarrow Na^+(g)$ $\quad \Delta H = 495.8$

$\frac{1}{2}Br_2(\ell) + e^- \longrightarrow Br^-(g)$ $\quad \Delta H = 325$

$Na^+(g) + Br^-(g) \longrightarrow NaBr(s)$ $\quad \Delta H = -728.4$

$Na(s) + \frac{1}{2}Br_2(\ell) \longrightarrow NaBr(s).$ $\quad \Delta H = ?$

$\Delta H = 495.8 - 325 - 728.4$

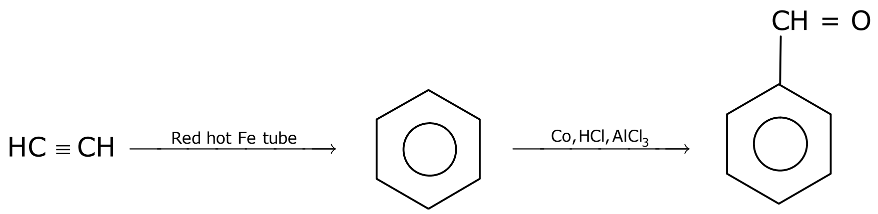
$-557.6 \text{ kJ} = -5576 \times 10^{-1} \text{ kJ}$

6. Consider the following chemical reaction.

$$HC \equiv CH \xrightarrow[(2)CO+HCl/AlCl_3]{(1)\text{Red hot Fe tube, }873K} \text{Product}$$

The number of sp^2 hybridized carbon atom(s) present in the product is _______.

Sol. **7**



benzaldehyde

All carbon atoms in benzaldehyde are sp^2 hybridised

7. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is ______. (Rounded-off to the nearest integer)

Sol. **69.85°C ≃ 70°C**

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{35}{300} = \frac{40}{T_2}$$

$$T_2 = \frac{40 \times 300}{35}$$

= 342.86 K

= 69.85°C ≃ 70°C

8. Among the following, the number of halide(s) which is/are inert to hydrolysis is ______.

(A) BF_3 (B) $SiCl_4$ (C) PCl_5 (D) SF_6

Sol. **1**

Due to crowding SF_6 is not hydrolysed.

9. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _____ K. (Rounded-off to the nearest integer)

[Given K_b for (H_2O) = 0.52 K kg mol^{-1}]

Sol. **375 K**

$A_2B_3 \longrightarrow 2A^{+3} + 3B^{-2}$

No. of ions = 2 + 3 = 5

$i = 1 + (n - 1) \propto$

$= 1 + (5 - 1) \times 0.6$

$= 1 + 4 \times 0.6 = 1 + 2.4 = 3.4$

$\Delta T_b = K_b \times m \times i$

$= 0.52 \times 1 \times 3.4 = 1.768°C$

$\Delta T_b = (T_b)_{solution} - [(T_b)_{H_2O}]_{Solution}$

$1.768 = (T_b)_{solution} - 100$

$(T_b)_{solution} = 101.768\ °C$

$= 375\ K$

10. Using the provided information in the following paper chromatogram:

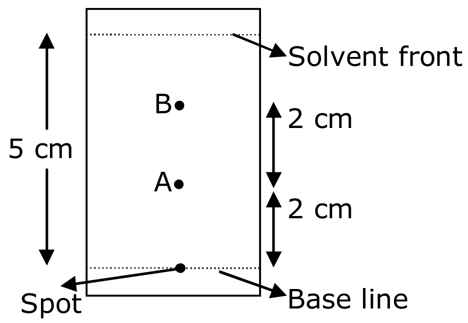


Fig: Paper chromatography for compounds A and B

the calculated R_f value of A ______ $\times 10^{-1}$.

Sol. **4**

$$R_f = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$$

On chromatogram distance travelled by compound is $\rightarrow$ 2 cm

Distance travelled by solvent = 5 cm

So $R_f = \frac{2}{5} = 4 \times 10^{-1} = 0.4$

25^{th} Feb. 2021 | Shift - 1
MATHEMATICS

Section : Mathematics Section A

1. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(1) $\frac{1}{54}$

(2) $\frac{1}{72}$

(3) $\frac{1}{36}$

(4) $\frac{5}{216}$

Ans. (4)

Sol. $ax^2 + bx + c = 0$

$a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2$, $ac = 1$ $\Rightarrow$ $a = 1, c = 1$

If $b = 4$, $ac = 4$ $\Rightarrow$ $a = 1, c = 4$

$a = 4, c = 1$

$a = 2, c = 2$

If $b = 6$, $ac = 9 \Rightarrow$ $a = 3, c = 3$

$\therefore$ probability $= \frac{5}{216}$

2. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

(3) $\frac{5}{8}$

(4) $\frac{3}{8}$

Ans. (3)

Sol. $l^2 + m^2 + n^2 = 1$

$\therefore 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 = \frac{1}{2}$ & $l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$

$\Rightarrow$l m = 0 or m = 0

$\therefore$ l = 0, m = $\frac{1}{\sqrt{2}}$ or l = $\frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}, \frac{3}{4} = \frac{5}{8}$

3. The value of the integral

$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$ is

(where c is a constant of integration)

(1) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

Ans. (3)

Sol. $\int \frac{2\sin^2\theta\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta} d\theta$

Let sinθ = t, cosθ dθ = dt

$= \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}\,dt = \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}\,dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)\,dt = dz$

$= \frac{1}{12}\int \sqrt{z}\,dz = \frac{1}{18}z^{3/2} + c$

$= \frac{1}{18}[(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta)^{3/2} + C$

$= \frac{1}{18}\left[\left(1 - \cos^2\theta\right)\left(2(1 - \cos^2\theta)^2 + 3 - 3\cos^2\theta + 6\right)\right]^{3/2} + C$

$= \frac{1}{18}[(1 - \cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + C$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

(1) $10(\sqrt{3}-1)$

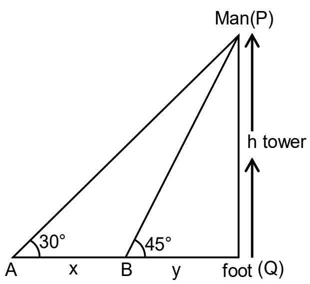
(2) $10\sqrt{3}$

(3) 10

(4) $10(\sqrt{3}+1)$

Ans. (4)

Sol.



$$\frac{h}{x+y}=\tan 30°$$

x + y = $\sqrt{3}h$ ....(1)

Also

$$\frac{h}{y}=\tan 45°$$

h = y ....(2)

put in (1)

x + y = $\sqrt{3}y$

x = $(\sqrt{3}-1)y$

$$\frac{x}{20}='v'\text{speed}$$

$\therefore$ time taken to reach

Foot from B

$$\Rightarrow \frac{y}{V}$$

$$\Rightarrow \frac{x}{(\sqrt{3}-1).x}\times 20$$

$\Rightarrow 10(\sqrt{3}+1)$

5. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta,\ y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta . \sin^{2n}\phi$ then :

(1) xyz = 4
(2) xy – z = (x + y)z
(3) xy + yz + zx = z
(4) xy + z = (x +y)z

Ans. (4)

Sol. $x = 1 + \cos^2\theta + \ldots\ldots\ldots\infty$

$x = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$ ....(1)

$y = 1 + \sin^2\phi + \ldots\ldots\ldots\infty$

$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$ ....(2)

$z = \frac{1}{1-\cos^2\theta.\sin^2\phi} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$

xz + yz – z = xy
xy + z = (x + y)z

6. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is :

(1) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

(2) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

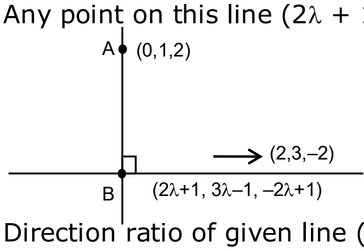
(3) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

(4) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

Ans. (1)

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$

Any point on this line (2λ + 1, 3λ – 1, –2λ+1)



Direction ratio of given line (2, 3, –2)

Direction ratio of line to be found $(2\lambda + 1, 3\lambda - 2, -2\lambda - 1)$

$\therefore\ \vec{d}_1.\vec{d}_2 = 0$

$\lambda = 2/17$

Direction ratio of line $(21, -28, -21) \equiv (3, -4, -3) \equiv (-3, 4, 3)$

7. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to:

(1) $A \rightarrow (A \wedge B)$

(2) $A \rightarrow (A \vee B)$

(3) $A \rightarrow (A \rightarrow B)$

(4) $A \rightarrow (A \leftrightarrow B)$

Ans. (2)

Sol. $A \rightarrow (B \rightarrow A)$

$\Rightarrow A \rightarrow (\sim B \vee A)$

$\Rightarrow \sim A \vee (\sim B \vee A)$

$\Rightarrow \sim B \vee (\sim A \vee A)$

$\Rightarrow \sim B \vee t$

$= t$ (tantology)

From options :

(2) $A \rightarrow (A \vee B)$

$\Rightarrow \sim A \vee (A \vee B)$

$\Rightarrow (\sim A \vee A) \vee B$

$\Rightarrow t \vee B$

$\Rightarrow t$

8. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R is :

(1) 3

(2) 2

(3) 4

(4) 0

Ans. (1)

Sol. $D < 0$

$(2(3k-1))^2 - 4(8k^2 - 7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k-4)(k-2) < 0$

$2 < k < 4$

then $k = 3$

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?
(1) (0, 3)
(2) (−6, 0)
(3) (4, 5)
(4) (5, 4)

Ans. (4)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2} + 3 \quad \Rightarrow x - 2y + 6 = 0$

10. Let f, g: $N \to N$ such that $f(n + 1) = f(n) + f(1)\ \forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?
(1) f is one-one
(2) If fog is one-one, then g is one-one
(3) If g is onto, then fog is one-one
(4) If f is onto, then $f(n) = n\ \forall n \in N$

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$
$f(2) = 2f(1)$
$f(3) = 3f(1)$
$f(4) = 4f(1)$
....
$f(n) = nf(1)$
f(x) is one-one

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is :

(1) $\frac{3}{\sqrt{2}}$

(2) $3\sqrt{2}$

(3) $\frac{3}{2\sqrt{2}}$

(4) $\frac{1}{2\sqrt{2}}$

Ans. (3)

Sol. $(2-i)z=(2+i)\bar{z}$

$\Rightarrow (2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow 2x - ix + 2iy + y = 2x + ix - 2\text{-}iy + y$

$\Rightarrow 2ix - 4iy = 0$

$L_1 : x - 2y = 0$

$\Rightarrow (2+i)z + (i-2)\bar{z} - 4i = 0.$

$\Rightarrow (2+i)(x+iy) + (i-2)(x-iy) - 4i = 0.$

$\Rightarrow 2x + ix + 2iy - y + ix - 2x + y + 2iy - 4i = 0$

$\Rightarrow 2ix + 4iy - 4i = 0$

$L_2 : x + 2y - 2 = 0$

Solve L_1 and L_2 $4y=2$, $y=\frac{1}{2}$

$\therefore x=1$

Centre $\left(1,\frac{1}{2}\right)$

$L_3 : iz + \bar{z} + 1 + i = 0$

$\Rightarrow i(x+iy) + x - iy + 1 + i = 0$

$\Rightarrow ix - y + x - iy + 1 + i = 0$

$\Rightarrow (x - y + 1) + i(x - y + 1) = 0$

Radius = distance from $\left(1,\frac{1}{2}\right)$ to $x - y + 1 = 0$

$r = \frac{1-\frac{1}{2}+1}{\sqrt{2}}$

$r = \frac{3}{2\sqrt{2}}$

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in:

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

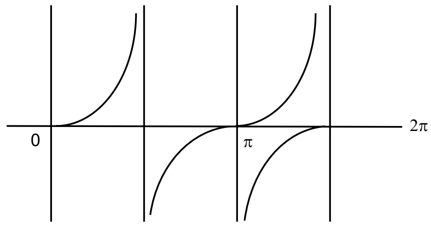
(3) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

1 y

Ans. (2)

Sol.



tan 2θ (1 + cos2θ) > 0

$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

13. The image of the point (3,5) in the line x – y + 1 = 0, lies on :

(1) $(x - 2)^2 + (y - 4)^2 = 4$

(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 2)^2 = 12$

Ans. (1)

Sol. Image of P(3, 5) on the line x – y + 1 = 0 is

$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$

x = 4, y = 4

∴ Image is (4, 4)

Which lies on

$(x - 2)^2 + (y - 4)^2 = 4$

14. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (–5, 8)

(2) (5, 8)

(3) (5, –8)

(4) (–5, –8)

Ans. (2)

Sol. f(1) = f(2)

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

3a – b = 7 ...(1)

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16$ ...(2)

a = 5, b = 8

15. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is true ?

(1) a + b = c + d
(2) a − b = c − d
(3) ab = $\frac{c+d}{a+b}$
(4) a − c = b + d

Ans. (2)

Sol. $\frac{x^2}{a}+\frac{y^2}{b}=1$ ....(1)

diff : $\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0 \Rightarrow \frac{y}{b}\frac{dy}{dx}=\frac{-x}{a}$

$\frac{dy}{dx}=\frac{-bx}{ay}$ ......(2)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ ....(3)

Diff : $\frac{dy}{dx}=\frac{-dx}{cy}$ ....(4)

$m_1m_2 = -1 \Rightarrow \frac{-bx}{ay}\times\frac{-dx}{cy} = -1$

$\Rightarrow bdx^2 = -acy^2$ ....(5)

(1)−(3) $\Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2 + \left(\frac{1}{b}-\frac{1}{d}\right)y^2 = 0$

$\Rightarrow \frac{c-a}{ac}x^2+\frac{d-b}{bd}\times\left(\frac{-bd}{ac}\right)x^2=0$ (using 5)

$\Rightarrow (c - a) - (d - b) = 0$

$\Rightarrow c - a = d - b$

$\Rightarrow c - d = a - b$

16. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$
(2) $\frac{1}{e}$
(3) 1
(4) 0

Ans. (3)

Sol. It is 1^{∞} form

$$L = e^{\lim_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)}$$

$$S = 1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\left(\frac{1}{8}+........+\frac{1}{15}\right)$$

$$S < 1+\left(\frac{1}{2}+\frac{1}{2}\right)\ +\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right).........+\underbrace{\left(\frac{1}{2^P}+........+\frac{1}{2^P}\right)}_{2^P \text{ times}}$$

$$S < 1+ 1 + 1 + 1 +......... +1$$

$$S < P + 1$$

$$\therefore \quad L = e^{\lim_{n\to\infty}\frac{(P+1)}{2^P}}$$

$$\Rightarrow L = e^{\circ} = 1$$

17. The total number of positive integral solutions (x, y, z) such that xyz = 24 is

(1) 36

(2) 45

(3) 24

(4) 30

Ans. (4)

Sol. x.y.z = 24

$x.y.z=2^3.3^1$

Now using beggars method.

3 things to be distributed among 3 persons

Each may receive none, one or more

$\therefore$ ${}^5C_2$ ways

Similarly for '1' $\therefore$ ${}^3C_2$ ways

Total ways = ${}^5C_2 \cdot {}^3C_2$ = 30 ways

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$,then this curve also passes through the point :

(1) (4, 5)

(2) (5, 4)

(3) (4, 4)

(4) (5, 5)

Ans. (4)

Sol. $\frac{dy}{dx}=\frac{(x-2)^2+y+4}{(x-2)}=(x-2)+\frac{y+4}{(x-2)}$

Let x – 2 = t $\Rightarrow$ dx = dt

and y + 4 = u $\Rightarrow$ dy = du

$\frac{dy}{dx} = \frac{du}{dt}$

$\frac{du}{dt}$ =t + $\frac{u}{t} \Rightarrow \frac{du}{dt} - \frac{u}{t} = t$

I.F = $e^{\int \frac{-1}{t} dt} = e^{-\ln t} = \frac{1}{t}$

u. $\frac{1}{t} = \int t.\frac{1}{t} dt \Rightarrow \frac{u}{t} = t + c$

$\frac{y+4}{x-2}$ = (x – 2) + c

Passing through (0, 0)

c = 0

$\Rightarrow$ (y + 4) = $(x - 2)^2$

19. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\leq$ t, is :

(1) $\frac{e+1}{3}$

(2) $\frac{e-1}{3e}$

(3) $\frac{e+1}{3e}$

(4) $\frac{1}{3e}$

Ans. (3)

Sol. I = $\int_{-1}^{0} x^2.e^{-1} dx + \int_{0}^{1} x^2 dx$

$\therefore$ I = $\left.\frac{x^3}{3e}\right|_{-1}^{0} + \left.\frac{x^3}{3}\right|_{0}^{1}$

$\Rightarrow$ I = $\frac{1}{3e} + \frac{1}{3}$

20. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:

(1) $\frac{1}{8}$

(2) $\frac{1}{27}$

(3) $\frac{3}{4}$

(4) $\frac{3}{8}$

Ans. (1)

Sol. Probability of not getting intercepted = $\frac{2}{3}$

Probability of missile hitting target = $\frac{3}{4}$

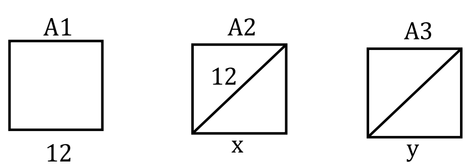
$\therefore$ Probability that all 3 hit the target = $\left(\frac{2}{3}\times\frac{3}{4}\right)^3=\frac{1}{8}$

Section : Mathematics Section B

1. Let A_1, A_2, A_3, …… be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ____________.

Ans. (9)

Sol.



$x = \frac{12}{\sqrt{2}}$ $y = \frac{12}{(\sqrt{2})^2}$

$\therefore$ Side lengths are in G.P.

$T_n = \frac{12}{(\sqrt{2})^{n-1}}$

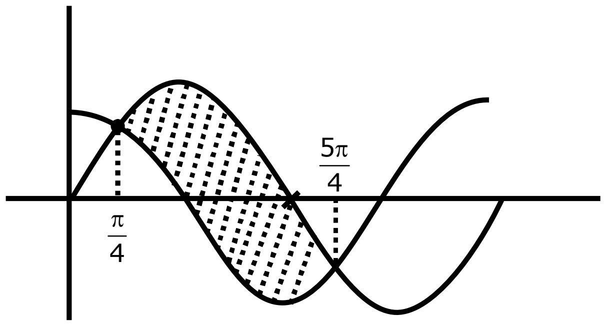
$\therefore$ Area = $\frac{144}{2^{n-1}}<1$ $\Rightarrow 2^{n-1} > 144$

Smallest n = 9

2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _________

Ans. (64)

Sol.



$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 64$$

3. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}\,x - y - 4(\sqrt{3})$ $k = 0$ is a conic, whose eccentricity is _________.

Ans. (2)

Sol. $\sqrt{3}kx + ky = 4\sqrt{3}$ .... (1)

$\sqrt{3}kx - ky = 4\sqrt{3}\,k^2$ ....(2)

Adding equation (1) & (2)

$2\sqrt{3}kx = 4\sqrt{3}(k^2 + 1)$

$x = 2\left(k + \frac{1}{k}\right)$ ....(3)

Substracting equation (1) & (2)

$y = 2\sqrt{3}\left(\frac{1}{k} - k\right)$ ....(4)

$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 4$

$$\frac{x^2}{16}-\frac{y^2}{48}=1 \quad \text{Hyperbola}$$

$\therefore e^2 = 1 + \frac{48}{16}$

e = 2

4. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ______.

Ans. (13)

Sol. $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$

$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ $\{\therefore |I - A| = \sec^2\theta/2\}$

$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow (I+A)(I-A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$

$$a = \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$b=\frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$\therefore$ $a^2 + b^2 = 1$

5. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1 and x = 1. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then 5.f(2) is equal to __________

Ans. (144)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore$ $f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots 1 & – 1

$\therefore$ 6 + 5a + 4b + 3 = 0 & – 6 + 5a – 4b + 3 = 0 solving

$a = -\frac{3}{5}$ $\quad$ $b = -\frac{3}{2}$

$\therefore$ $f(x) = x^6-\frac{3}{5}x^5-\frac{3}{2}x^4+x^3$

$\therefore$ $5.f(2) = 5\left[64-\frac{96}{5}-24+8\right]=144$

6. The number of points, at which the function $f(x) = |2x + 1| -3|x+2|+|x^2 + x-2|$, $x \in R$ is not differentiable, is _________.

Ans. (2)

Sol. $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$$f(x)=\begin{cases} x^2-7 \; ; & x>1 \\ -x^2-2x-3 \; ; & -\frac{1}{2}<x<1 \\ -x^2-6x-5 \; ; & -2<x<\frac{-1}{2} \\ x^2+2x+3; & x<-2 \end{cases}$$

$$\therefore f'(x)=\begin{cases} 2x \; ; & x>1 \\ -2x-3 \; ; & -\frac{1}{2}<x<1 \\ -2x-6; & -2<x<\frac{-1}{2} \\ 2x+2; & x<-2 \end{cases}$$

Check at 1, –2 and $\frac{-1}{2}$

Non. Differentiable at x = 1 and $\frac{-1}{2}$

7. If the system of equations

$$kx + y + 2z = 1$$
$$3x - y - 2z = 2$$
$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to __________.

Ans. (21)

Sol. D = 0

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k (4 – 4)– 1(–12 –4) +2(–6 –2)

$\Rightarrow$ 16 – 16 = 0

Also. $D_1 = D_2 = D_3 = 0$

$$\Rightarrow D_2 = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k(–8+6)–1(–12–4)+2(9+4)=0

$\Rightarrow$ –2k + 16 + 26 = 0

$\Rightarrow$ 2k = 42

$\Rightarrow$ k = 21

8. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to _______

Ans. (12)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\vec{r} \times \vec{a} - \vec{c} \times \vec{a} = 0$

$(\vec{r} - \vec{c}) \times \vec{a} = 0$

$\therefore \ \vec{r} - \vec{c} = \lambda \vec{a}$

$\vec{r} = \lambda \vec{a} + \vec{c}$

$\vec{r}.\vec{b} = \lambda \vec{a}.\vec{b} + \vec{c}.\vec{b} = 0$

$\Rightarrow \lambda$ (1–2) + 2 = 0

$\Rightarrow \lambda$ = 2

$\therefore \ \vec{r} = 2\vec{a} + \vec{c}$

$\vec{r}.\vec{a} = 2|\vec{a}|^2 + \vec{a}.\vec{c}$

= 2(1 + 4 + 1) + (1 – 2 + 1)

= 12

9. Let A = $\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that x + y + z > 0 and xyz = 2.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _______.

Ans. (7)

Sol. A = $\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ $\quad \therefore |A| = \left(x^3 + y^3 + z^3 - 3xyz\right)$

$A^2 = I_3$

$|A^2| = 1$

$\therefore (x^3 + y^3 + z^3 - 3xyz)^2 = 1$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$ only as (x+y+z>0)

$\Rightarrow x^3 + y^3 + z^3 = 6 + 1 = 7$

10. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4,5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 is ________.

Ans. (32)

Sol. $\boxed{}\boxed{}\boxed{}$ divisible by $\rightarrow$ 3

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

$\boxed{}\boxed{}\boxed{5} = 12$

4×3

Required No. = 24 + 12 − 4 = 32

25^{th} Feb. 2021 | Shift - 2
PHYSICS

SECTION - A

1. Match List I with List II.

	List I	**List II**
(a)	Rectifier	(i) Used either for stepping up or stepping down the a.c. Voltage
(b)	Stabilizer	(ii) Used to convert a.c. voltage into d.c. voltage
(c)	Transformer	(iii) Used to remove any ripple in the rectified output voltage
(d)	Filter	(iv) Used for constant output voltage even when the input voltage or load current change

Choose the correct answer form the options given below:

(1) (a)-(ii), (b)- (i), (c)-(iv), (d)-(iii)
(2) (a)-(ii), (b)- (iv), (c)-(i), (d)-(iii)
(3) (a)-(ii), (b)- (i), (c)-(iii), (d)-(iv)
(4) (a)-(iii), (b)- (iv), (c)-(i), (d)-(ii)

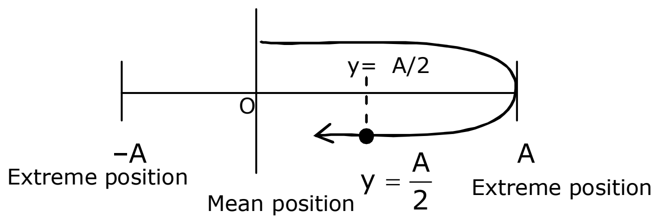
Sol. **2**

(a)Rectifier:- used to convert a.c voltage into d.c. Voltage.
(b) Stabilizer:- used for constant output voltage even when the input voltage or load current change
(c) Transformer:- used either for stepping up or stepping down the a.c. voltage.
(d) Filter:- used to remove any ripple in the rectified output voltage.

2. $Y = A \sin(\omega t + \phi_0)$ is the time – displacement equation of a SHM, At t = 0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be.

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

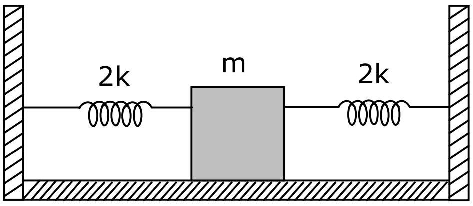
Sol. **4**



The initial phase angle $\phi_0 = \pi - \frac{\pi}{6}$

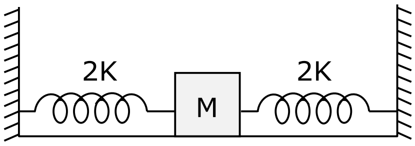
$= \frac{5\pi}{6}$

3. Two identical spring of spring constant '2K' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. Then time period of oscillations of this system is:



(1) $\pi\sqrt{\frac{m}{k}}$ (2) $\pi\sqrt{\frac{m}{2k}}$ (3) $2\pi\sqrt{\frac{m}{k}}$ (4) $2\pi\sqrt{\frac{m}{2k}}$

Sol. **1**



Dut to parallel combination $K_{eff} = 2k + 2k$

$= 4k$

$$\because T = 2\pi\sqrt{\frac{m}{k_{eff}}}$$

$$= 2\pi\sqrt{\frac{m}{4k}}$$

$$T = \pi\sqrt{\frac{m}{k}}$$

4. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2 to n = 1 state is:

(1) 194.8 nm (2) 490.7 nm (3) 913.3 nm (4) 121.8 nm

Sol. **4**

$\Delta E = 10.2$ eV

$\frac{hc}{\lambda} = 10.2$ ev

$$\lambda = \frac{hc}{(10.2)e}$$

$$= \frac{12400}{10.2} Å$$

= 121.56 nm

$\simeq$ 121.8 nm

5. In a ferromagnetic material, below the curie temperature, a domain is defined as:

(1) a macroscopic region with consecutive magnetic diploes oriented in opposite direction.

(2) a macroscopic region with zero magnetization.

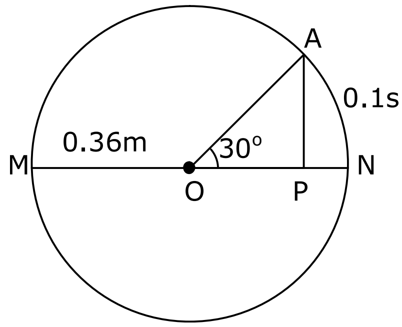
(3) a macroscopic region with saturation magnetization.

(4) a macroscopic region with randomly oriented magnetic dipoles.

Sol. **3**

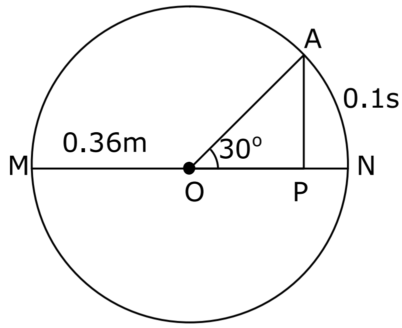
In a ferromagnetic material, below the curie temperature a domain is defined as a macroscopic region with saturation magnetization.

6. The point A moves with a uniform speed along the circumference of a circle of radius 0.36m and cover 30° in 0.1s. The perpendicular projection 'P' form 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be:



(1) 100 N (2) 50 N (3) 9.87 N (4) 0.49 N

Sol. **3**



The point a covers 30° in 0.1 sec.

Means $\frac{\pi}{6} \longrightarrow 0.1\,\text{sec}.$

$1 \longrightarrow \frac{0.1}{\frac{\pi}{6}}$

$2\pi = \longrightarrow \frac{0.1 \times 6}{\pi} \times 2\pi$

T = 1.2 sec.

We know that $\omega = \frac{2\pi}{T}$

$$\omega = \frac{2\pi}{1.2}$$

Restoration force (F) = $m\omega^2 A$

Then Restoration force per unit mass $\left(\frac{F}{m}\right) = \omega^2 A$

$$\left(\frac{F}{m}\right) = \left(\frac{2\pi}{1.2}\right)^2 \times 0.36$$

$$\cong 9.87 \text{ N}$$

7. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43V. The new wavelength is:

(1) 400 NM (2) 382 nm (3) 309 nm (4) 329 nm

Sol. 2

From the photoelectric effect equation

$$\frac{hc}{\lambda} = \phi + ev_s$$

so $ev_{s_1} = \frac{hc}{\lambda_1} - \phi$ ......(i)

$ev_{s_2} = \frac{hc}{\lambda_2} - \phi$ .....(ii)

Subtract equation (i) from equation (ii)

$$ev_{s_1} - ev_{s_2} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$v_{s_1} - v_{s_2} = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$(0.710 - 1.43) = 1240\left(\frac{1}{491} - \frac{1}{\lambda_2}\right)$$

$$\frac{-0.72}{1240} = \frac{1}{491} - \frac{1}{\lambda_2}$$

$$\frac{1}{\lambda_2} = \frac{1}{491} + \frac{0.72}{1240}$$

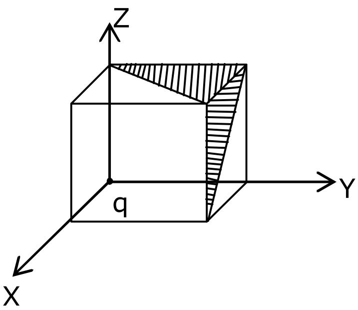
$$\frac{1}{\lambda_2} = 0.00203 + 0.00058$$

$$\frac{1}{\lambda_2} = 0.00261$$

$\lambda_2 = 383.14$

$\lambda_2 \simeq 382nm$

8. A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ though the shaded area is:



(1) $\frac{q}{48\varepsilon_0}$ (2) $\frac{q}{8\varepsilon_0}$ (3) $\frac{q}{24\varepsilon_0}$ (4) $\frac{q}{4\varepsilon_0}$

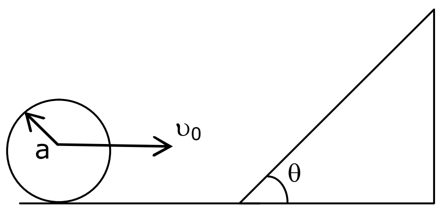
Sol. 3

$$\phi = \frac{q}{24\varepsilon_0}$$

$$\phi_T = \left(\frac{q}{24\varepsilon_0} + \frac{q}{24\varepsilon_0}\right) \times \frac{1}{2}$$

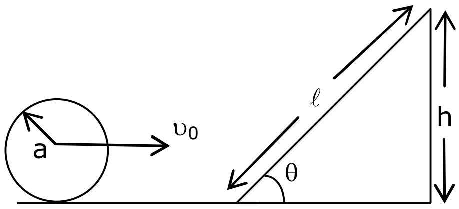
$$\phi_T = \frac{q}{24\varepsilon_0}$$

9. A sphere of radius 'a' and mass 'm' rolls along horizontal plane with constant speed υ_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping how far up the sphere will travel ?



(1) $\frac{2}{5}\frac{\upsilon_0^2}{g\sin\theta}$ (2) $\frac{10\upsilon_0^2}{7g\sin\theta}$ (3) $\frac{\upsilon_0^2}{5g\sin\theta}$ (4) $\frac{\upsilon_0^2}{2g\sin\theta}$

Sol. **Bonus, our answer $\left(\frac{7v_0^{\,2}}{10g\sin\theta}\right)$, NTA answer (2)**



From energy conservation

$$mgh = \frac{1}{2}m\upsilon_0^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}m\upsilon_0^2 + \frac{1}{2}\times\frac{2}{5}ma^2\times\frac{\upsilon_0^{\,2}}{a^2}$$

$$gh = \frac{1}{2}\upsilon_0^{\,2} + \frac{1}{5}\upsilon_0^{\,2}$$

$$gh = \frac{7}{10}\upsilon_0^{\,2}$$

$$h = \frac{7}{10}\frac{\upsilon_0^{\,2}}{g}$$

from triangle, $\sin\theta = \frac{h}{\ell}$

then $h = \ell\sin\theta$

$$\ell\sin\theta = \frac{7}{10}\frac{\upsilon_0^{\,2}}{g}$$

$$\ell = \frac{7}{10}\frac{\upsilon_0^{\,2}}{g\sin\theta}$$

10. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1 μm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that:

(1) its size decreases, but intensity increases

(2) its size increases, but intensity decreases

(3) its size increases, and intensity increases

(4) its size decreases, and intensity decreases

Sol. **1**

$$\sin\theta = \frac{1.22\lambda}{D}$$

If D is increased, then $\sin\theta$ will decreased

$\therefore$ size of circular fringe will decrease but intensity increases

11. An electron of mass m_e and a proton of mass m_p = 1836 m_e are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{Proton}}$ will be:

(1) 918 (2) 1836 (3) $\frac{1}{1836}$ (4) 1

Sol. **2**

Given mass of electron = m_e

Mass of proton = m_p

$\therefore$ given m_p = 1836 m_e

From de-Broglie wavelength

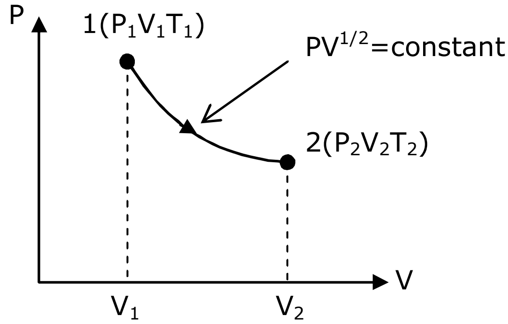
$\lambda = \frac{h}{p} = \frac{h}{mv}$

$\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e}$

$= \frac{1836m_e}{m_e}$

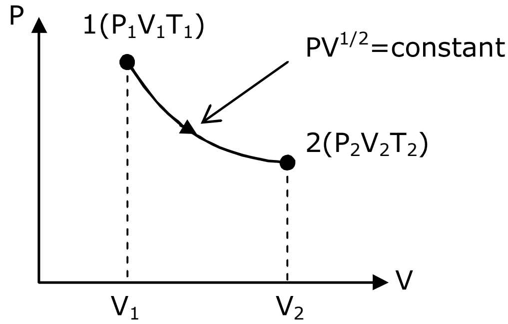
$\frac{\lambda_e}{\lambda_p} = 1836$

12. thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is:



(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) 2 (4) $\sqrt{2}$

Sol. **4**



From p-v diagram,

Given $Pv^{1/2}$ = constant .....(i)

We know that

Pv = nRT

$P \propto \left(\frac{T}{v}\right)$

Put in equation (i)

$\left(\frac{T}{v}\right)(v)^{1/2}$ = constant

$T \propto v^{1/2}$

$\frac{T_2}{T_1} = \sqrt{\frac{v_2}{v_1}}$

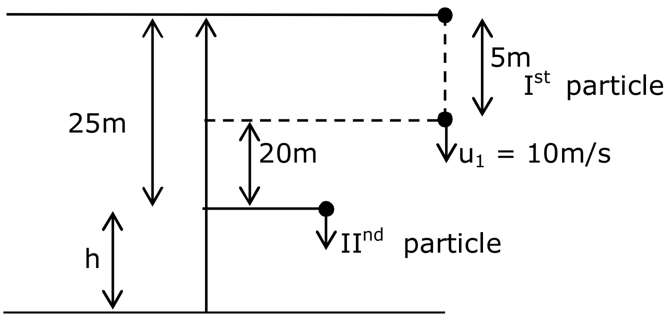
$\frac{T_2}{T_1} = \sqrt{\frac{2v_1}{v_1}}$

$\frac{T_2}{T_1} = \sqrt{2}$

13. A stone is dropped from the top of a building. When it crosses a point 5m below the top, another stone starts to fall from a point 25m below the top, Both stones reach the bottom of building simultaneously. The height of the building is:

(1) 45 m (2) 35 m (3) 25 m (4) 50 m

Sol. **1**



For particle (1)

$20+h = 10t + \frac{1}{2}\ gt^2$ .....(i)

For particle (2)

$h = \frac{1}{2}\ gt^2$ .....(ii)

put equation (ii) in equation (i)

$20 + \frac{1}{2}gt^2 = 10t + \frac{1}{2}gt^2$

t = 2sec.

Put in equation (ii)

$h = \frac{1}{2}\ gt^2$

$= \frac{1}{2} \times 10 \times 2^2$

h = 20m

the height of the building = 25 + 20 = 45m

14. if a message signal of frequency 'f_m' is amplitude modulated with a carrier signal of frequency 'f_c' and radiated through an antenna, the wavelength of the corresponding signal in air is:

(1) $\frac{c}{f_c + f_m}$ (2) $\frac{c}{f_c - f_m}$ (3) $\frac{c}{f_m}$ (4) $\frac{c}{f_c}$

Sol. **4**

Given frequency of massage signal = f_m

frequency of carrier signal = f_c

the wavelength of the corresponding signal in air is $\Rightarrow \lambda = \frac{c}{f_c}$

15. Given below are two statements:

Statement I: In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II: in a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and statement II are false.

(2) Both statement I and statement II are true.

(3) Statement I is false but statement II is true

(4) Statement I is true but statement II is false.

Sol. **4**

The translational kinetic energy & rotational kinetic energy both obey Maxwell's distribution independent of each other.

T.K.E of diatomic molecules = $\frac{3}{2}kT$

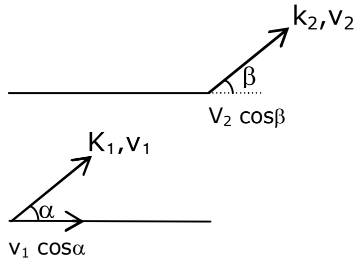
R.K.E. of diatomic molecules = $\frac{2}{2}kT$

So statement I is true but statement II is false.

16. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be:

(1) $\frac{\sin^2\beta}{\cos^2\alpha}$ (2) $\frac{\cos^2\beta}{\cos^2\alpha}$ (3) $\frac{\cos\beta}{\sin\alpha}$ (4) $\frac{\cos\beta}{\cos\alpha}$

Sol. **2**



$\because\ v_1 \cos\alpha = v_2\cos\beta$

$\frac{v_1}{v_2} = \frac{\cos\beta}{\cos\alpha}$

Then the ratio of kinetic energies

$\frac{k_1}{k_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\cos\beta}{\cos\alpha}\right)^2$

$\frac{k_1}{k_2} = \frac{\cos^2\beta}{\cos^2\alpha}$

17. An LCR circuit contains resistance of 110Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be:

(1) 2.5A (2) 2A (3) 1A (4) 1.5 A

Sol. **2**

Since ϕ remain same, circuit is in resonance

$\therefore I_{rms} = \frac{V_{rms}}{z}$

$= \frac{220}{110}$

$I_{rms} = 2A$

18. For extrinsic semiconductors: when doping level is increased;

(1) Fermi–level of p and n-type semiconductors will not be affected.

(2) Fermi–level of p-type semiconductors will go downward and Fermi–level of n-type semiconductor will go upward.

(3) Fermi–level of both p–type and n–type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F K$, where T_F is Fermi temperature.

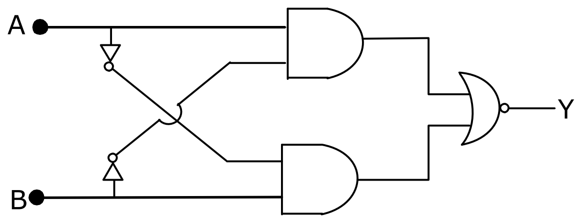
(4) Fermi–level of p-type semiconductor will go upward and Fermi–level of n–type semiconductors will go downward.

Sol. **2**

In n-type semiconductor pentavalent impurity is added. Each pentavalent impurity donates a free electron. So the Fermi-level of n-type semiconductor will go upward .

& In p-type semiconductor trivalent impurity is added. Each trivalent impurity creates a hole in the valence band. So the Fermi-level of p-type semiconductor will go downward.

19. The truth table for the following logic circuit is:



(1)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

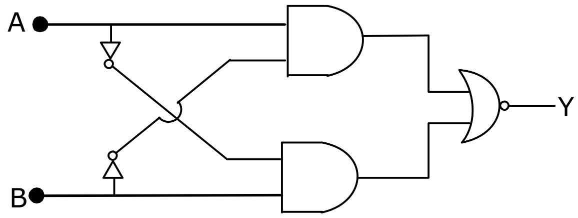
(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Sol. **4**



If A = B = 0 then output y = 1
If A = B = 1 then output y = 1

20. If e is the electronic charged, c is the speed of light in free space and h is planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of :

(1) [LC^{-1}] (2) [$M^0 L^0 T^0$] (3) [$M L T^0$] (4) [$M L T^{-1}$]

Sol. **2**

Given
e = electronic charge
c = speed of light in free space
h = planck's constant

$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{hc} = \frac{ke^2}{hc} \times \frac{\lambda^2}{\lambda^2}$

$= \frac{F \times \lambda}{E}$

$= \frac{E}{E}$

= dimensionless

= [$M^0 L^0 T^0$]

SECTION – B

1. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4% will be___________%.

Sol. **2**

Speed of transverse wave is

$V = \sqrt{\frac{T}{\mu}}$

$\ell n\ v = \frac{1}{2}\ell nT - \frac{1}{2}\ell n\mu$

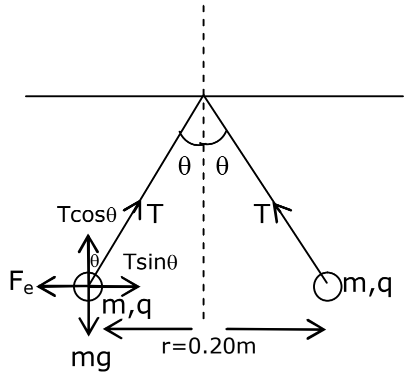
$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$

$= \frac{1}{2} \times 4$

$\frac{\Delta v}{v} = 2\%$

2. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m . Then charge on each of the sphere is $\frac{a}{21} \times 10^{-8} C$. The value of 'a' will be________.

Sol. **20**



$T \sin\theta = \frac{kq^2}{r^2}$

$T \cos\theta = mg$

$\tan\theta = \frac{kq^2}{mgr^2}$

$q^2 = \frac{\tan\theta \, mgr^2}{k}$

$\because \tan\theta = \frac{0.1}{0.5} = \frac{1}{5}$

$q^2 = \frac{1}{5} \times \frac{10 \times 10^{-6} \times 10 \times 0.2 \times 0.2}{9 \times 10^9}$

$q = \frac{2\sqrt{2}}{3} \times 10^{-8}$

after comparison from the given equation

a = 20

3. The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}} \frac{V}{m}$. The efficiency of the bulb is 10% and it is a point source. The value of x is ___.

Sol. **2**

$I = \frac{1}{2} c \in_0 E_0^2$

$\frac{8}{4\pi \times 10^2} = \frac{1}{2} \times c \times \frac{1}{\mu_0 c^2} \times E_0^2$

$E_0 = \frac{2}{10}\sqrt{\frac{\mu_0 c}{\pi}}$

$\Rightarrow x = 2$

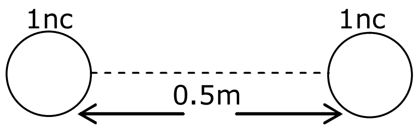
4. Two identical conducting spheres with negligible volume have 2.1nC and -0.1nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is _______$\times 10^{-9}$ N.

[Given : $4\pi\varepsilon_0 = \frac{1}{9\times10^9}$ SI unit]

Sol. **36**

When they are brought into contact & then separated by a distance = 0.5 m

Then charge distribution will be



The electrostatic force acting b/w the sphere is

$$F_e = \frac{kq_1q_2}{r^2}$$

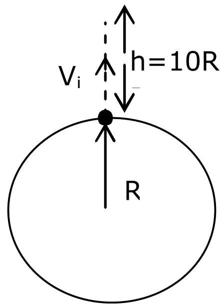
$$= \frac{9\times10^9\times1\times10^{-9}\times1\times10^{-9}}{(0.5)^2}$$

$$= \frac{900}{25}\times10^{-9}$$

$F_e = 36 \times 10^{-9}$ N

5. The initial velocity υ_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity υ_e such that $\upsilon_i = \sqrt{\frac{x}{y}}\times\upsilon_e$. The value of x will be_______.

Sol. **10**



Here R = radius of the earth

From energy conservation

$$\frac{-Gm_e m}{R}+\frac{1}{2}mv_i^2=\frac{-Gm_e m}{11R}+0$$

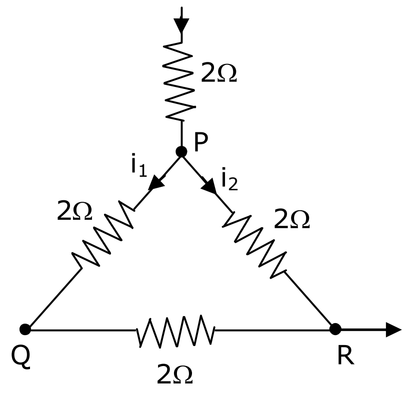
$$\frac{1}{2}mv_i^2=\frac{10}{11}\frac{Gm_e m}{R}$$

$$V_i = \sqrt{\frac{20}{11}\frac{Gm_e}{R}}$$

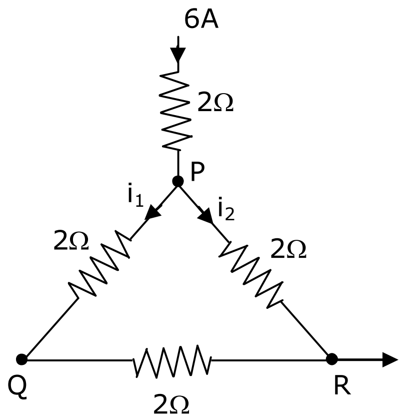
$$V_i = \sqrt{\frac{10}{11}}v_e \qquad \left\{\because \text{ escape velocity } v_e=\sqrt{\frac{2Gm_e}{R}}\right\}$$

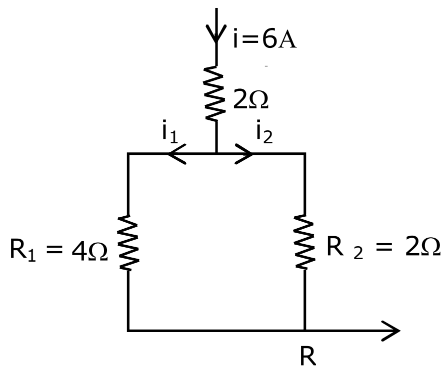
Then the value of x = 10

6. A current of 6A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The currents i_1 in ampere is______.



Sol. **2**





The current $i_1 = \left(\frac{R_2}{R_1 + R_2}\right)i$

$= \left(\frac{2}{4+2}\right) \times 6$

$i_1 = 2A$

7. The wavelength of an X-ray beam is 10 Å. The mass of a fictitious particle having the same energy as that of the X – ray photons is $\frac{x}{3}$h kg. The value of x is _______.

Sol. **10**

Given wavelength of an x-ray beam = 10Å

$\because \quad E = \frac{hc}{\lambda} = mc^2$

$m = \frac{h}{c\lambda}$

The mass of a fictitious particle having the same energy as that of the x-ray photons = $\frac{x}{3}$hkg

$\frac{x}{3}h = \frac{h}{c\lambda}$

$x = \frac{3}{c\lambda}$

$= \frac{3}{3 \times 10^8 \times 10 \times 10^{-10}}$

$x = 10$

8. A reversible heat engine converts one- fourth of the heat input into work. When the temperature of the sink is reduced by 52K, its efficiency is doubled. The temperature in Kelvin of the source will be_______.

Sol. **208**

$\because\ n = \frac{w}{Q_{in}} = \frac{1}{4}$

$\frac{1}{4} = 1 - \frac{T_1}{T_2}$

$\frac{T_1}{T_2} = \frac{3}{4}$

When the temperature of the sink is reduced by 52k then its efficiency is doubled.

$\frac{1}{2} = 1 - \frac{(T_1 - 52)}{T_2}$

$\frac{T_1 - 52}{T_2} = \frac{1}{2}$

$\frac{T_1}{T_2} \frac{-52}{T_2} = \frac{1}{2}$

$\frac{3}{4} - \frac{52}{T_2} = \frac{1}{2}$

$\frac{52}{T_2} = \frac{1}{4}$

T_2 = 208 k

9. Two particles having masses 4g and 16g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n:2. The value of n will be_____.

Sol. **1**

$\because$ relation b/w kinetic energy & momentum is

$P = \sqrt{2mKE}$ $\quad$ ($\because$ KE = same)

$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

$\frac{n}{2} = \sqrt{\frac{4}{16}}$

n = 1

10. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$. The value of '$\theta$' will be____.

Sol. **180**

If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

Only if $\vec{P} = 0$

Or $\vec{Q} = 0$

The angle b/w $\vec{P}$ & $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$

So $\theta = 180^\circ$

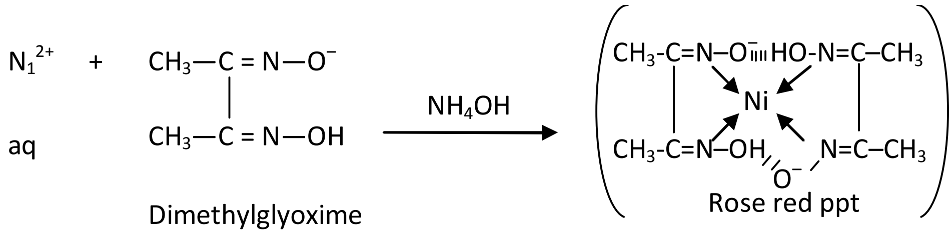
25th Feb. 2021 | Shift - 2
CHEMISTRY

Section -A

1. Given below are two statements :
Statement I :
The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH
Statement II :
The dimethyl glyoxime is a bidentate neutral ligand.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both statement I and statement II are true
(2) Both statement I and statement II are false
(3) Statement I is false but statement II is true
(4) Statement I is true but statement II is false

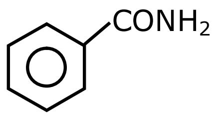
Ans. 4

Sol.

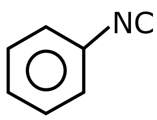


Dimethyl glyoxime is a negative bidentate legend.

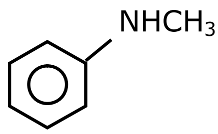
2. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline ?



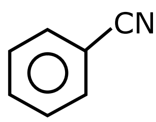
(1)



(2)



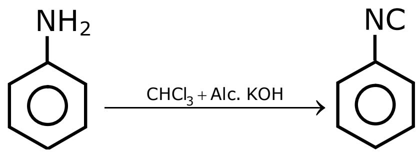
(3)



(4)

Ans. 2

Sol.



3. The correct order of bond dissociation enthalpy of halogen is :
(1) $F_2 > Cl_2 > Br_2 > I_2$
(2) $Cl_2 > F_2 > Br_2 > I_2$
(3) $Cl_2 > Br_2 > F_2 > I_2$
(4) $I_2 > Br_2 > Cl_2 > F_2$

Ans. 3

Sol. Fact based
F_2 has F — F, F_2 involves repulsion of non-bonding electrons & more over its size is small & hence due to high repulsion its bond dissociation energy in very low.

4. Which one of the following statements is FALSE for hydrophilic sols ?
(1) These sols are reversible in nature
(2) The sols cannot be easily coagulated
(3) They do not require electrolytes for stability.
(4) Their viscosity is of the order of that of H_2O

Ans. **4**

Sol. Fact base

5. Water does not produce CO on reacting with :
(1) C_3H_8
(2) C
(3) CH_4
(4) CO_2

Ans. **4**

Sol. $H_2O + CO_2 \rightarrow H_2CO_3$

6. What is 'X' in the given reaction ?

$$\begin{matrix} CH_2OH \\ | \\ CH_2OH \end{matrix} + \text{oxalic acid} \xrightarrow{210^0C} \underset{\text{(major product)}}{x}$$

(1) $\begin{matrix} CH_2 \\ \| \\ CH_2 \end{matrix}$ (2) $\begin{matrix} CH{-}OH \\ \| \\ CH_2 \end{matrix}$ (3) $\begin{matrix} CH_2OH \\ | \\ CHO \end{matrix}$ (4) $\begin{matrix} CHO \\ | \\ CHO \end{matrix}$

Ans. **1**

Sol.
$$\begin{matrix} CH_2-OH \\ | \\ CH_2-OH \end{matrix} + oxalic\ acid \xrightarrow{210^0C} CH_2 = CH_2$$

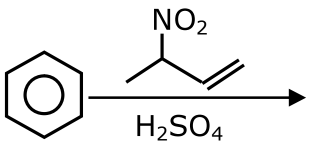
7. If which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment ?

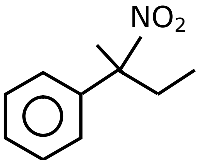
(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$
(iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$
(1) (ii)>(i)>(iii)>(iv) (2) (iii)>(iv)>(ii)>(i)
(3) (ii)>(iii)>(i)>(iv) (4) (i)>(iii)>(iv)>(ii)

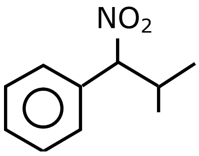
Ans. **4**

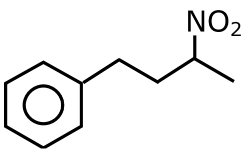
Sol. $[FeF_6]^{3-}$ Fe^{3+} $3d^5$ $\rightarrow$ 5-unpaired electrons as F^- is weal field legend
$[Co(NH_3)_6]^{3+}$ Co^{3+} $3d^6$ $\rightarrow$ No-unpaired electron as NH_3 is strong field light and causes pairing
$[NiCl4]^{2-}$ Ni^{2+} $3d^8$ $\rightarrow$ 2-unpaired electrons
$[Cu(NH_3)_4]^{2+}$ Cu^{2+} $3d^9$ $\rightarrow$ 1-unpaired electrons

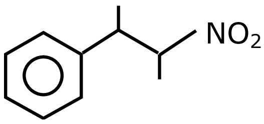
8. The major product of the following reaction is :



(1) 

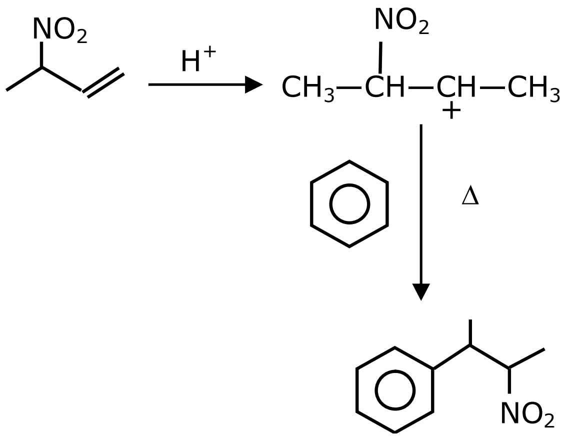
(2) 

(3) 

(4) 

Ans. **4**

Sol.



9. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benezene is :

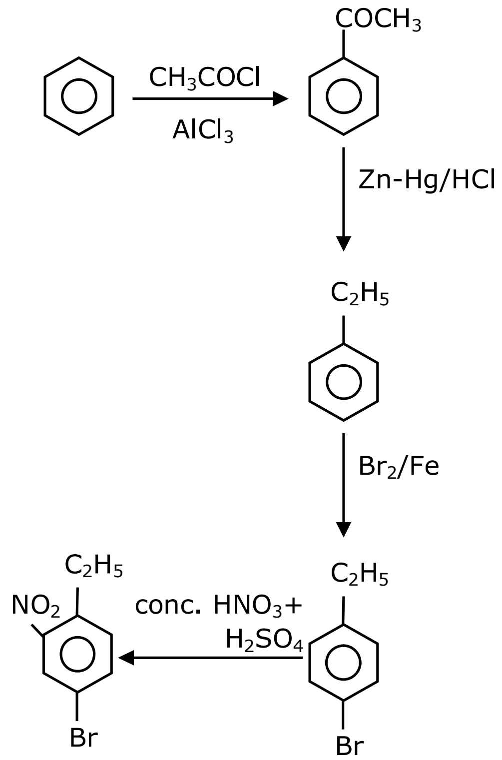
(1) $CH_3COCl/AlCl_3$, $Br_2/AlBr_3$, HNO_3/H_2SO_4, Zn/HCl

(2) $CH_3COCl/AlCl_3$, Zn-Hg/HCl, $Br_2/AlBr_3$, HNO_3/H_2SO_4

(3) $Br_2/AlBr_3$, $CH_3COCl/AlCl_3$, HNO_3/H_2SO_4, Zn/HCl

(4) HNO_3/H_2SO_4, $Br_2/AlCl_3$, $CH_3COCl/AlCl_3$, Zn-Hg/HCl

Ans. **2**



Sol.

10. The major components of German Silver are :

(1) Cu, Zn and Ag (2) Ge, Cu and Ag

(3) Zn, Ni and Ag (4) Cu, Zn and Ni

Ans. **4**

Sol. Fact

German silver is alloy which does not have silver.

Cu-50%; Ni-30%; Zn-20%

11. The method used for the purification of Indium is :

(1) van Arkel method (2) vapour phase refining

(3) zone refining (4) Liquation

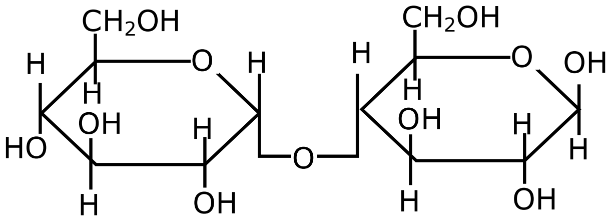
Ans. **3**

Sol. Fact

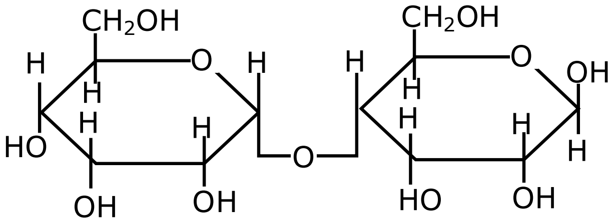
Ga, In, Si, Ge are refined by zone refining or vaccume refining.

12. Which of the following is correct structure of α-anomer of maltose :

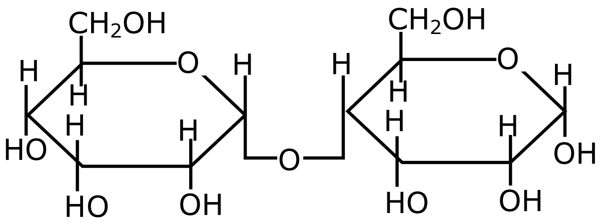
(1)



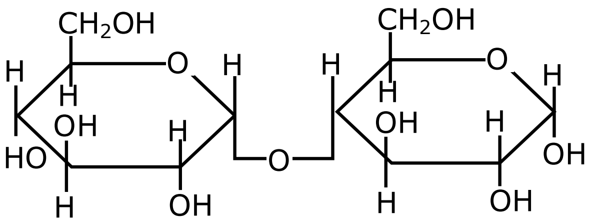
(2)



(3)

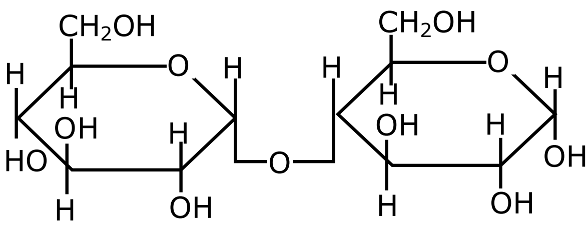


(4)



Ans. **4**

Sol.



[α-Anomer of maltose]

13. The major product of the following reaction is :

$$CH_3CH_2CH{=}CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

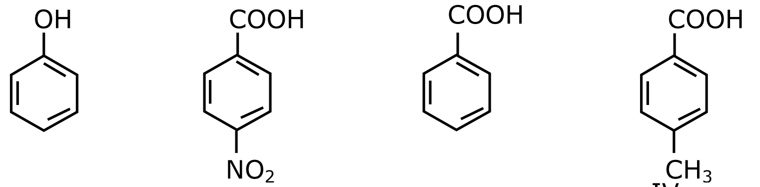
(1) $CH_3CH_2CH_2CHO$ (2) $CH_3CH_2CH{=}CH{-}CHO$

(3) $CH_3CH_2CH_2CH_2CHO$ (4) $CH_3CH_2\underset{\displaystyle CHO}{\underset{|}{C}}{=}CH_2$

Ans. **3**

Sol. $CH_3-CH_2-CH=CH_2 \xrightarrow[Rh\ catalyst]{H_2/CO} CH_3CH_2CH_2CHO$

14. The correct order of acid character of the following compounds is :



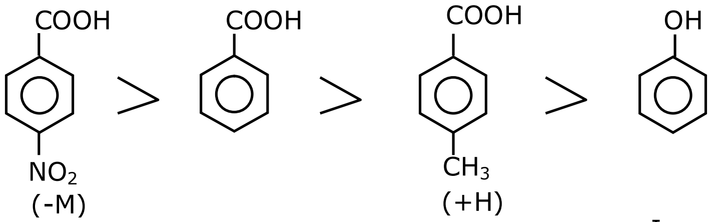
I II III IV

(1) II>III>IV>I (2) III>II>I>IV

(3) IV>III>II>I (4) I>II>III>IV

Ans. **1**

Sol. Acidity of carboxylic acid $\propto$ -R>-H>-I $\propto \dfrac{1}{+R>+H>+I}$

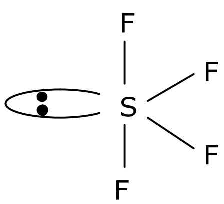


II III IV I

15. Which among the following species has unequal bond lengths ?

(1) XeF_4 (2) SiF_4 (3) BF_4^- (4) SF_4

Ans. **4**



Sp^{3}d Hybridisation Sea-saw shape & axial bond length is more than equitorial bond length

Sol.

16. Given below are two statements :

Statement I :

α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statement II :

At room temperature the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both statement I and statement II are false

(2) Statement I is true but statement II is false

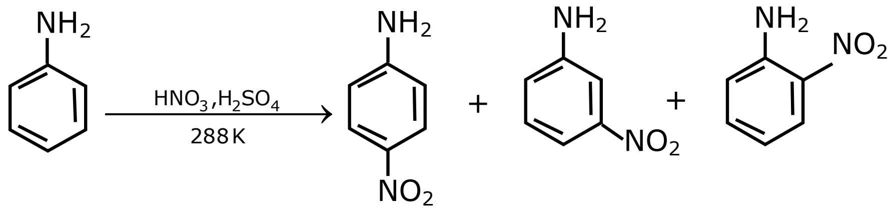
(3) Both statement I and statement II are true

(4) Statement I is false but statement II is true

Ans. **2**

Sol.
$$\underset{\alpha - sulphur}{S_{Rhambic}} \underset{95.6^0 C}{\rightleftharpoons} \underset{\beta - sulphur}{S_{Monoclinic}}$$

17.



(A) (B) (C)

Correct statement about the given chemical reaction is :

(1) Reaction is possible and compound (A) will be major product.

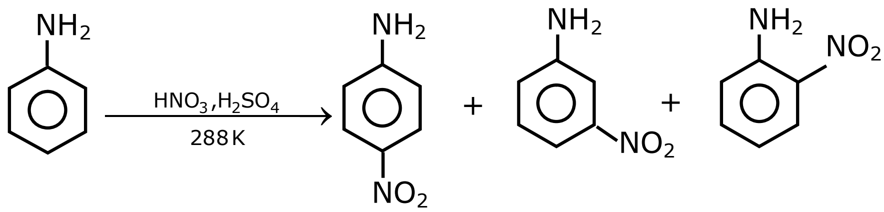
(2) The reaction will form sulphonated product instead of nitration.

(3) $-\ddot{N}H_2$ group is ortho and para directive, so product (B) is not possible.

(4) Reaction is possible and compound (B) will be the major product.

Ans. **1**

Sol.



(A) (B) (C)

51% 47% 2%

18. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens ?

(1) Nitric acid (2) Sodium hydroxide

(3) Hydrochloric acid (4) Ammonia

Ans. **1**

Sol. $NaCN + HNO_3 \rightarrow NaNO_3 + HCN \uparrow$

$Na_2S + HNO_3 \rightarrow NaNO_3 + H_2S \uparrow$

Nilnic acid decomposed NaCN & Na_2S, else they precipitate in test & misquite the resolve

19. Given below are two statements :

Statement I :

The pH of rain water is normally ~5.6.

Statement II :

If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, choose the correct answer from the option given below.

(1) Statement I is false but Statement II is true

(2) Both statement I and statement II are true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Ans. **2**

Sol. Both statements are correct

20. The solubility of $Ca(OH)_2$ in water is :

[Given : The solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}]

(1) 1.11×10^{-6} (2) 1.77×10^{-6}

(3) 1.77×10^{-2} (4) 1.11×10^{-2}

Ans. **4**

$$Ca(OH)_2 \rightleftharpoons \underset{s}{Ca^{+2}} + \underset{(2s+10^{-7})}{2OH^-}$$

$s(2s+10^{-7})^2 = 55\times10^{-7}$

$4s^3 = 55\times10^{-7}$

$s^3 = \frac{5500}{4}\times10^{-9}$

$s = \left(\frac{2250}{2}\right)^{1/3}\times10^{-3}$

$s = (1125)^{1/3}\times10^{-3}$

$s = 1.11\times10^{-2}$

Section -B

1. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is ______molal.
(Rounded-off to the nearest integer)
[K_b=0.52 K kg mol^{-1}]

Ans. **3**

$AB \rightarrow A^{+} + B^{-}$

$1-\alpha \quad \alpha \quad \alpha$

α=3/4

N = 2

i = [1+(2-1)α]

2.5 = [1+(2-1)3/4]×0.52 ×m

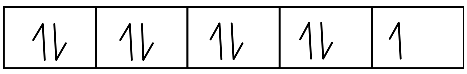
m = $\dfrac{2.5}{0.52\times 7/4} = \dfrac{10}{3.64} = 2.747$

m= 2.747 $\simeq 3$ mol/kg

2. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is ____BM.

Ans. **2**

Sol. ${}_{29}Cu^{+2} \rightarrow [Ar]^{18}\underline{3d^{9}}$

$d^9 \longrightarrow$ 

No. of unpaired e^- = 1

Magnetic moment = $\mu = \sqrt{n(n+2)}$

$\mu = \sqrt{(1)(1+2)} = \sqrt{3}B.M.$

= 1.73 Ans.

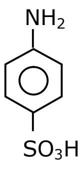
3. The number of compound/s given below which contain/s —COOH group is ______.

(1) Sulphanilic acid (2) Picric acid

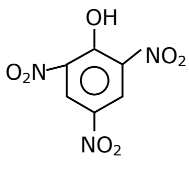
(3) Aspirin (4) Ascorbic acid

Ans. **1**

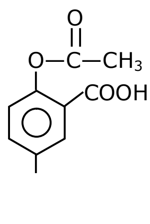
Sol.



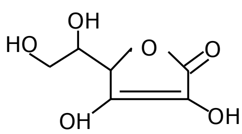
(A)



(B)



(C)



(D)

4. The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is ______.
[Molar mass of Cu : 63.54 g; Avogadro number = 6.022×10^{23}]

Ans. **9077**

Sol. a = 3.596 Å

d = $\frac{Z\times GMM}{N_A\times a^3}$

d = $\frac{4\times63.54\times10^{-3}}{6.022\times10^{23}\times(3.596\times10^{-10})^3}$

d = 0.9076×10^4 = 9076.2 kg/m^3

5. Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 ml. (ii) 4.5 ml. (iii) 4.4 ml. (iv) 4.4 ml (v) 4.4 ml

If the volume of oxalic acid taken was 10.0 ml. then the molarity of the NaOH solution is ____M. (Rounded-off to the nearest integer)

Ans. **6**

Eq. of NaOH = Eq. of oxalic acid

[NaOH]×1×4.4 = $\frac{5}{4}\times2\times10$

[NaOH] = $\frac{100}{4\times4.4}=\frac{25}{4.4}=5.68$

Nearest integer = 6M Ans.

6. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionize the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is____. (Rounded off to the nearest integer)
[$h=6.63\times10^{-34}$Js, c = $3.00\times10^8 ms^{-1}$, $N_A=6.02\times10^{23}$ mol^{-1}]

Ans. **180**

Sol. Energy req. to ionize an atom of metal 'A' = $\frac{hc}{\lambda}=\frac{hc}{663nm}$

for 1 mole atoms of 'A'

Total energy required = $N_A\times\frac{hc}{\lambda}$

= $\frac{6.023\times10^{23}\times6.63\times10^{-34}\times3\times10^8}{663\times10^{-9}}$

= $6.023\times3\times10^{23-34+8+7}$

= 18.04×10^4 J/mol

= 180.4 KJ/mol

Nearest Integer = 180 KJ/Mol.

7. The rate constant of a reaction increases by five times on increase in temperature from 27^0 C to 52^0C. The value of activation energy in kJ mol^{-1} is ______. (Rounded off to the nearest integer) [R=8.314 J K^{-1} mol^{-1}]

Ans. **52**

$$\frac{K_{52^0 C}}{K_{27^0 C}} = 5$$

$$\ln\left\{\frac{k_{T_2}}{k_{T_1}}\right\} = \frac{E_a}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\}$$

$$\ln(5) = \frac{E_a}{R}\left\{\frac{1}{300} - \frac{1}{325}\right\}$$

$$\frac{2.303 \times 0.7 \times 8.314 \times 300 \times 325}{25} = E_a$$

E_a = 51524.96 J/mol

E_a = 51.524 KJ/mol

52 Ans.

8. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed [Cu^{2+}] and $P_{NO}=P_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of 2x is ______. (Rounded-off to the nearest integer)

[Given : $E^0_{Cu^{2+}/Cu} = 0.34V, E^0_{NO_3^-/NO} = 0.96V, E^0_{NO_3^-/NO_2} = 0.79V$ and at 298 K, $\frac{RT}{F}(2.303) = 0.059$]

Ans. **1**

Sol. Anode

Cu(s) $\rightarrow$ Cu^{+2} + $2e^-$

Cathode (1)

$$\frac{3e^- + 4H^+ + NO_3^- \rightarrow NO + 2H_2O}{8H^- + 2N0_3^- + 3Cu(s) \rightarrow 3Cu^{+2} + 2NO + 4H_2O}$$

Q = $\dfrac{\left[Cu^{+2}\right]^3 \times (p_{NO})^2}{\left[NO_3^-\right]^2\left[H^+\right]^8}$

$$\in^0_{cell} = 1.3$$

$$\in_{cell} = 1.3 - \frac{0.059}{6}\log\frac{\left(Cu^{+2}\right)^3 (p_{NO})^2}{\left(NO_3^-\right)^2 \times \left(H^+\right)^8} \quad ......(1)$$

Anode Cu(s) $\rightarrow$ Cu^{+2} + $2e^-$

$$\text{Cathode}\ \frac{e^- + 2n^+ + NO_3^- \to NO_2 + H_2O}{Cu(s) + 4H^+ + 2NO_3^- \to 2NO_2 + 2H_2O + Cu^{+2}}$$

$$\in^0_{cell} = 1.13$$

$$Q = \frac{(Cu^{+2})(p_{NO_2})^2}{\left(NO_3^-\right)^2\left(H^+\right)^4}$$

$$\in_{cell} = 1.13 - \frac{0.059}{2}\log\frac{\left(Cu^{+2}\right)\left(p_{NO_2}\right)^2}{\left(NO_3^-\right)^2\left(H^+\right)^4}$$

$$\in_{cell_T} = \in_{cell_2}$$

$$1.3 - \frac{0.059}{6}\log\left(Q_1\right) = 1.13 - \frac{0.059}{2}\log\left(Q_2\right)$$

$$0.17 = \frac{0.059}{6}\left\{\log(Q_1) - 3\log(Q_2)\right\}$$

$$= \frac{0.059}{6}\left\{\log\frac{\left(Cu^{+2}\right)^3 \times \left(p_{NO}\right)^2 \times \left(NO_3^-\right)^6\left(H^+\right)^{12}}{\left(NO_3^-\right)^2\left(H^+\right)^8 \times \left(Cu^{+2}\right)^3 \times \left(p_{NO_2}\right)^6}\right\}$$

$$= \frac{0.059}{6}\left\{\log\frac{\left[NO_3^-\right]^4\left[H^+\right]^4}{\left(P_{NO_2}\right)^4}\right\}$$

$$0.17 = \frac{0.059}{6} \times 8\ \log\left(HNO_3\right)$$

log (HNO_3) = 2.16

[HNO_3] = $10^{2.16}$ = 10^x

x = 2.16 $\Rightarrow$ 2x = 4.32 ≈ 4

9. Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external 4.3 MPa. The heat transferred in this process is ____kJ mol^{-1}. (Rounded-off of the nearest integer)

[Use R = 8.314 J mol^{-1} K^{-1}]

Ans. **15**

Sol. Moles (n) = 5

T = 293k

Process = IsoT. $\to$ Irreversible

P_{ini} = 2.1 M Pa

P_t = 1.3 M Pa

P_{ext} = 4.3 mPa

Work = - P_{ext} Δv

$$= -4.3 \times \left(\frac{5 \times 293R}{1.3} - \frac{5 \times 293}{2.1} \right)$$

$$= -5 \times 293 \times 8.314 \times 43 \left(\frac{1}{13} - \frac{1}{21} \right)$$

$$= \frac{5 \times 293 \times 8.314 \times 43 \times 8}{21 \times 13}$$

= -15347.7049 J

= - 15.34 KJ

Isothermal process, so $\Delta U = 0$

w = - Q

Q = 15.34 KJ / mol

So answer is 15

10. Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _____(Integer answer).

(A) Li (B) Na (C) Rb (D) Cs

Ans. **1**

Sol. Cs is used in photoelectric cell due to its very low ionization potential.

25^{th} Feb. 2021 | Shift - 2
MATHEMATICS

SECTION-A

1. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overline{OP}$ on this plane is of length:

(1) $\sqrt{\frac{2}{5}}$

(2) $\sqrt{\frac{2}{3}}$

(3) $\sqrt{\frac{2}{11}}$

(4) $\sqrt{\frac{2}{7}}$

Ans. **(3)**

Sol. A(1, 2, 3), B(2, 3, 1), C(2, 4, 2), O(0, 0, 0)

Equation of plane passing through A, B, C will be

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 1-3 \\ 2-1 & 4-2 & 2-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

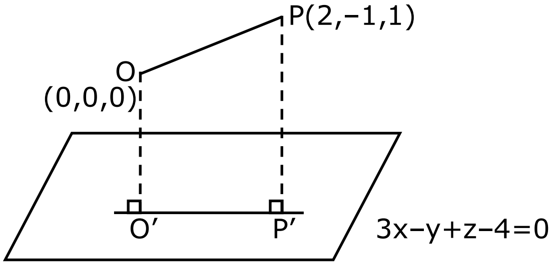
$\Rightarrow$ (x – 1)(–1 + 4) – (y – 2)(–1 + 2) + (z – 3)(2 – 1) = 0

$\Rightarrow$ (x – 1)(3) – (y – 2)(1) + (z – 3)(1) = 0

$\Rightarrow$ 3x – 3 – y + 2 + z – 3 = 0

$\Rightarrow$ 3x – y + z – 4 = 0, is the required plane.

Now, given O(0, 0, 0) & P(2, –1, 1)



Plane is 3x – y + z – 4 = 0

O′ & P′ are foot of perpendiculars.

for O'

$$\frac{x-0}{3}=\frac{y-0}{-1}=\frac{z-0}{1}=\frac{-(0-0+0-4)}{9+1+1}$$

$$\frac{x}{3}=\frac{y}{-1}=\frac{z}{1}=\frac{4}{11}$$

$$\Rightarrow \quad O'\left(\frac{12}{11},\frac{-4}{11},\frac{4}{11}\right)$$

for P'

$$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{1}=\frac{-(3(2)-(-1)+1-4)}{9+1+1}$$

$$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{1}=\left(\frac{-4}{11}\right)$$

$$P'\left(\frac{-12}{11}+2,\ \frac{4}{11}-1,\ \frac{-4}{11}+1\right)$$

$$\Rightarrow \quad P'\left(\frac{10}{11},\frac{-7}{11},\frac{7}{11}\right)$$

$$O'P'=\sqrt{\left(\frac{10}{11}-\frac{12}{11}\right)^2+\left(\frac{-7}{11}+\frac{4}{11}\right)^2+\left(\frac{7}{11}-\frac{4}{11}\right)^2}$$

$$\Rightarrow \quad O'P'=\frac{1}{11}\sqrt{4+9+9}$$

$$\Rightarrow \quad O'P'=\frac{\sqrt{22}}{11}$$

$$\Rightarrow \quad O'P'=\frac{\sqrt{2}\times\sqrt{11}}{11}$$

$$\Rightarrow \quad O'P'=\sqrt{\frac{2}{11}}$$

2. The contrapositive of the statement "If you will work, you will earn money" is:

(1) If you will not earn money, you will not work

(2) You will earn money, if you will not work

(3) If you will earn money, you will work

(4) To earn money, you need to work

Ans. (1)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$p \to$ you will work

$q \to$ you will earn money

$\sim q \to$ you will not earn money

$\sim p \to$ you will not work

$\sim q \to \sim p \Rightarrow$ if you will not earn money, you will not work.

3. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to:

(1)7

(2) −3

(3) 3

(4)−7

Ans. (4)

Sol. $(1-2i)^2 + \alpha(1-2i) + \beta = 0$

$1 - 4 - 4i + \alpha - 2i\alpha + \beta = 0$

$(\alpha + \beta - 3) - i(4 + 2\alpha) = 0$

$\alpha + \beta - 3 = 0 \ \ \& \ \ 4 + 2\alpha = 0$

$\alpha = -2 \qquad \beta = 5$

$\alpha - \beta = -7$

4. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$, then:

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(3) $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in A.P.

(4) $I_2 + I_4$, $(I_3 + I_5)^2$, $I_4 + I_6$ are in G.P.

Ans. (2)

Sol.

$$I_{n+2} + I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x.\operatorname{cosec}^2 x\, dx = \left[\frac{-(\cot x)^{n+1}}{n+1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$I_{n+2} + I_n = \frac{1}{n+1}$$

$$I_2 + I_4 = \frac{1}{3}, I_3 + I_5 = \frac{1}{4}, I_4 + I_6 = \frac{1}{5}$$

5. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is:

(1) 1

(2) 3

(3) 2

(4) 4

Ans. (1)

Sol. $\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$1 + \alpha^2 = 1$

$\alpha^2 = 0$

$\alpha^2 + \beta^2 = 1$

$\beta^2 = 1$

$\alpha^4 = 0$

$\beta^4 = 1$

$\alpha^4 + \beta^4 = 1$

6. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then:

(1)$y = 273x$

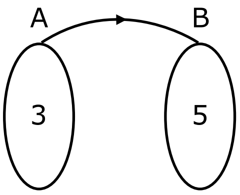
(2) $2y = 91x$

(3)$y = 91x$

(4)$2y = 273x$

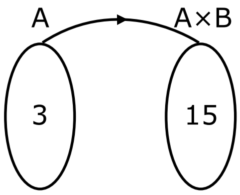
Ans. (2)

Sol. Number of elements in A = 3
Number of elements in B = 5
Number of elements in A × B = 15



Number of one-one function
$x = 5 \times 4 \times 3$

$x = 60$



Number of one-one function
$y = 15 \times 14 \times 13$

$y = 15 \times 4 \times \frac{14}{4} \times 13$

$y = 60 \times \frac{7}{2} \times 13$

$2y = (13)(7x)$

$2y = 91x$

7. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is:

(1)$\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

(2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

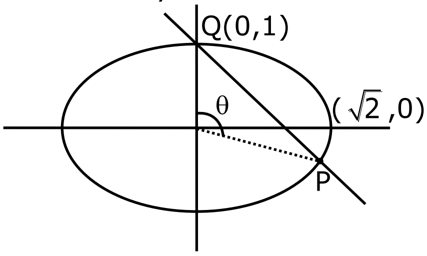
(3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$

(4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

Ans. (1)

Sol. Ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$

Line $x + y = 1$



Using homogenisation

$x^2 + 2y^2 = 2(1)^2$

$x^2 + 2y^2 = 2(x + y)^2$

$x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$

$x^2 + 4xy = 0$

for $ax^2 + 2hxy + by^2 = 0$

$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

$\tan\theta = \left|\frac{2\sqrt{(2)^2-0}}{1+0}\right|$

$\tan\theta = -4$

$\cot\theta = -\frac{1}{4}$

$\theta = \cot^{-1}\left(-\frac{1}{4}\right)$

$\theta = \pi - \cot^{-1}\left(\frac{1}{4}\right)$

$\theta = \pi - \left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)\right)$

$\theta = \frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

8. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, x > 0, is equal to:

(where c is a constant of integration)

(1) $\log_e|x^2 + 5x - 7| + c$

(2) $\frac{1}{4}\log_e|x^2 + 5x - 7| + c$

(3) $4\log_e|x^2 + 5x - 7| + c$

(4) $\log_e\sqrt{x^2 + 5x - 7} + c$

Ans. **(3)**

Sol. $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$

$= \int \frac{8x^3 + 5(4x^2)}{x^4 + 5x^3 - 7x^2}$

$= \int \frac{8x^3 + 20x^2}{x^4 + 5x^3 - 7x^2}$

$= \int \frac{8x + 20}{x^2 + 5x - 7}$

$= \int \frac{4(2x + 5)}{x^2 + 5x - 7}$ $\quad \left\{\begin{array}{l}\text{Let } x^2 + 5x - 7 = t \\ (2x + 5)dx = dt\end{array}\right\}$

$= \int \frac{4dt}{t}$

$= 4\ln|t| + C$

$= 4\ln|(x^2 + 5x - 7)| + c$

9. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:

(1) $\frac{x^2}{9}-\frac{y^2}{4}=1$

(2) $\frac{x^2}{9}-\frac{y^2}{16}=1$

(3) $x^2 - y^2 = 9$

(4) $\frac{x^2}{9}-\frac{y^2}{25}=1$

Ans. (2)

Sol.

$e_1=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$ foci $(\pm ae, 0)$

Foci $=(\pm 3, 0)$

Let equation of hyperbola be $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

Passes through $(\pm 3, 0)$

$A^2=9, A=3, e_2=\frac{5}{3}$

$e_2^2=1+\frac{B^2}{A^2}$

$\frac{25}{9}=1+\frac{B^2}{9}\Rightarrow B^2=16$

Ans $\frac{x^2}{9}-\frac{y^2}{16}=1$

10. $\lim\limits_{x\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+......+\frac{n}{(2n-1)^2}\right]$ is equal to:

(1) 1

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) $\frac{1}{4}$

Ans. (3)

Sol.

$$\operatorname*{Lim}_{x\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\operatorname*{Lim}_{x\to\infty}\sum_{r=0}^{n-1}\frac{n^2}{n^2\left(1+\frac{r}{n}\right)^2}=\int_0^1\frac{dx}{(1+x)^2}$$

$$=-\left[\frac{1}{1+x}\right]_0^1\Rightarrow-\left[\frac{1}{2}-1\right]=\frac{1}{2}$$

11. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(1) $\frac{7}{45}$

(2) $\frac{8}{45}$

(3) $\frac{14}{45}$

(4) $\frac{28}{45}$

Ans. **(4)**

Sol. Based on Baye's theorem

$$\text{Probability} = \frac{\left(160 \times \frac{35}{100}\right)}{\left(160 \times \frac{35}{100}\right) + \left(100 \times \frac{20}{100}\right) + \left(140 \times \frac{10}{100}\right)}$$

$$= \frac{5600}{9000}$$

$$= \frac{28}{45}$$

12. The following system of linear equations

$3x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) does not have any solution

(2) has a unique solution

(3) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(4) has infinitely many solutions

Ans. **(2)**

Sol. $\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = -20 \neq 0 \qquad \therefore$ unique solution

$$\Delta_x = \begin{vmatrix} 9 & 3 & 2 \\ 9 & 2 & 2 \\ 8 & -1 & 4 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 2 & 9 & 2 \\ 3 & 9 & 2 \\ 1 & 8 & 4 \end{vmatrix} = -20$$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 9 \\ 3 & 2 & 9 \\ 1 & -1 & 8 \end{vmatrix} = -40$$

$$\therefore \quad x = \frac{\Delta_x}{\Delta} = 0$$

$$y = \frac{\Delta_y}{\Delta} = 1$$

$$z = \frac{\Delta_z}{\Delta} = 2$$

Unique solution: (0, 1, 2)

13. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to:

(1) $a + \frac{1}{a}$

(2) a + 1

(3) 2a

(4) $2\sqrt{a}$

Ans. **(4)**

Sol. AM $\geq$ GM

$$\frac{a^{ax} + \frac{a}{a^{ax}}}{2} \geq \left(a^{ax} \cdot \frac{a}{a^{ax}}\right)^{1/2} \Rightarrow a^{ax} + a^{1-ax} \geq 2\sqrt{a}$$

14. A function f(x) is given by $f(x) = \dfrac{5^x}{5^x+5}$, then the sum of the series

$f\left(\dfrac{1}{20}\right) + f\left(\dfrac{2}{20}\right) + f\left(\dfrac{3}{20}\right) + ... + f\left(\dfrac{39}{20}\right)$

is equal to:

(1) $\dfrac{19}{2}$

(2) $\dfrac{49}{2}$

(3) $\dfrac{39}{2}$

(4) $\dfrac{29}{2}$

Ans. (3)

Sol.

$f(x) = \dfrac{5^x}{5^x+5}$ ....(i)

$f(2-x) = \dfrac{5^{2-x}}{5^{2-x}+5}$

$f(2-x) = \dfrac{5}{5^x+5}$......(ii)

Adding equation (i) and (ii)

$f(x) + f(2-x) = 1$

$f\left(\dfrac{1}{20}\right) + f\left(\dfrac{39}{20}\right) = 1$

$f\left(\dfrac{2}{20}\right) + f\left(\dfrac{38}{20}\right) = 1$

:

:

$f\left(\dfrac{19}{20}\right) + f\left(\dfrac{21}{20}\right) = 1$

and $f\left(\dfrac{20}{20}\right) = f(1) = \dfrac{1}{2}$

$\Rightarrow 19 + \dfrac{1}{2} \Rightarrow \dfrac{39}{2}$

15. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

(1) 4

(2) 1

(3) 2

(4) 3

Ans. (3)

Sol. $x^2 - 6x - 2 = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix} \quad \begin{matrix} \alpha + \beta = 6 \\ \alpha\beta = -2 \end{matrix}$

and $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$

$\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$

Now

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{\left(\alpha^{10} - \beta^{10}\right) - 2\left(\alpha^8 - \beta^8\right)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\left(\alpha^{10} - 2\alpha^8\right) - \left(\beta^{10} - 2\beta^8\right)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8\left(\alpha^2 - 2\right) - \beta^8\left(\beta^2 - 2\right)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} = \frac{6\left(\alpha^9 - \beta^9\right)}{3(\alpha^9 - \beta^9)} = \frac{6}{3} = 2$$

16. Let A be a 3 × 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to:

(1) 64

(2) 16

(3) 80

(4) 128

Ans. (1)

Sol.

$$A = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$2A = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 2R_{21} & 2R_{22} & 2R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$R_2 \to 2R_2 + 5R_3$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} + 10R_{31} & 4R_{22} + 10R_{32} & 4R_{23} + 10R_{33} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$R_2 \to R_2 - 5R_3$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{vmatrix}$$

$$|B| = 2 \times 2 \times 4 \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$= 16 \times 4$

$= 64$

17. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is:

(1) $\frac{1}{2}$

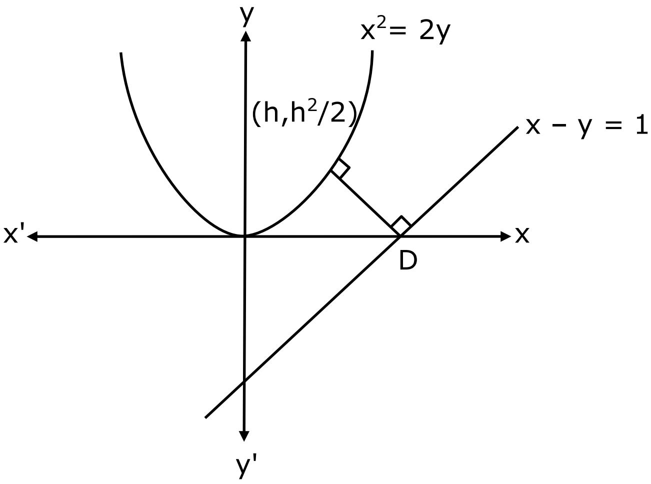
(2) 0

(3) $\frac{1}{2\sqrt{2}}$

(4) $\frac{1}{\sqrt{2}}$

Ans. **(3)**

Sol. Shortest distance must be along common normal



m_1 (slope of line x–y = 1) = 1 $\Rightarrow$ slope of perpendicular line =–1

$m_2 = \frac{2x}{2} = x \Rightarrow m_2 = h \Rightarrow$ slope of normal $-\frac{1}{h}$

$-\frac{1}{h} = -1 \Rightarrow h = 1$

so point is $\left(1, \frac{1}{2}\right)$

$D = \left| \frac{1 - \frac{1}{2} - 1}{\sqrt{1+1}} \right| = \frac{1}{2\sqrt{2}}$

18. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is:

(1) $\frac{1}{5}$

(2) $\frac{2}{9}$

(3) $\frac{97}{297}$

(4) $\frac{122}{297}$

Ans. **(3)**

Sol. Total cases

(4 × 9 × 9 × 9) – (3 × 9 × 9)

Probability = $\frac{(3 \times 9 \times 9) - (2 \times 9) + (8 \times 9 \times 9)}{(4 \times 9^3) - (3 \times 9^2)}$

$= \frac{97}{217}$

19. $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to:

(1) $\frac{75}{56}$

(2) $\frac{65}{56}$

(3) $\frac{56}{33}$

(4) $\frac{65}{33}$

Ans. **(2)**

Sol. $\cos ec\left(2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$\cos ec\left(2\tan^{-1}\left(\frac{1}{5}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$=\cos ec\left(\tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$=\cos ec\left(\tan^{-1}\left(\frac{5}{12}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

Let $\tan^{-1}(5/12)=\theta \Rightarrow \sin\theta=\frac{5}{13}, \cos\theta=\frac{12}{13}$

and $\cos^{-1}\left(\frac{4}{5}\right)=\phi \Rightarrow \cos\phi=\frac{4}{5}$ and $\sin\phi=\frac{3}{5}$

$=\cos ec(\theta+\phi)$

$=\frac{1}{\sin\theta\cos\phi+\cos\theta\sin\phi}$

$=\frac{1}{\frac{5}{13}.\frac{4}{5}+\frac{12}{13}.\frac{3}{5}}=\frac{65}{56}$

20. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to:

(1) $\frac{1+\sqrt{3}}{2}$

(2) $\frac{1-\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{2}$

(4) $\frac{1}{2}$

Ans. (1)

Sol.

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-\left[2\cos^2\left(\frac{x+y}{2}\right)-1\right]=\frac{3}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right)-\cos\left(\frac{x+y}{2}\right)\right]=\frac{1}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[2\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)\right]=\frac{1}{2}$$

$$\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)=\frac{1}{8}$$

Possible when $\frac{x}{2}=30^\circ$ & $\frac{y}{2}=30^\circ$

$x = y = 60^\circ$

$$\sin x+\cos y=\frac{\sqrt{3}}{2}+\frac{1}{2}=\frac{\sqrt{3}+1}{2}$$

SECTION-B

1. If $\lim\limits_{x\to 0}\dfrac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of a – 2b is ______.

Ans. **(5)**

$$\lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$$

Applying L' Hospital Rule

$$\lim_{x\to 0}\frac{a-4e^{4x}}{a\left(e^{4x}-1\right)+ax\left(4e^{4x}\right)}\quad \text{So } a=4$$

Sol. Applying L' Hospital Rule

$$\lim_{x\to 0}\frac{-16e^{4x}}{a\left(4e^{4x}\right)+a\left(4e^{4x}\right)+ax\left(16e^{4x}\right)}$$

$$\frac{-16}{4a+4a}=\frac{-16}{32}=-\frac{1}{2}=b$$

$$a-2b=4-2\left(\frac{-1}{2}\right)=4+1=5$$

2. A line is a common tangent to the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a+c) is equal to ______.

Ans. **(9)**

Sol. Circle: $(x-3)^2+y^2=9$

Parabola: $y^2=4x$

Let tangent $y=mx+\dfrac{a}{m}$

$$y=mx+\frac{1}{m}$$

$m^2x-my+1=0$

the above line is also tangent to circle

$(x-3)^2+y^2=9$

$\therefore \perp$ from (3, 0) = 3

$$\left|\frac{3m^2-0+1}{\sqrt{m^2+m^4}}\right|=3$$

$(3m^2+1)^2=9(m^2+m^4)$

$6m^2 + 1 + 9m^4 = 9m^2 + 9m^4$

$3m^2 = 1$

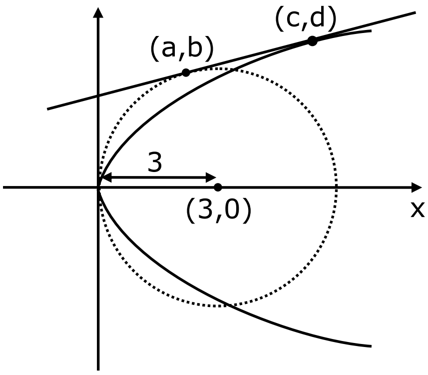
$m = \pm\frac{1}{\sqrt{3}}$

$\therefore$tangent is

$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ or $y = -\frac{1}{\sqrt{3}}x - \sqrt{3}$

(it will be used) (rejected)

$m = \frac{1}{\sqrt{3}}$



for Parabola $\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv (3, 2\sqrt{3})$

(c, d)

for Circle $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ & $(x - 3)^2 + y^2 = 9$

solving, $(x - 3)^2 + \left(\frac{1}{\sqrt{3}}x + \sqrt{3}\right)^2 = 9$

$x^2 + 9 - 6x + \frac{1}{3}x^2 + 3 + 2x = 9$

$\frac{4}{3}x^2 - 4x + 3 = 0$

$4x^2 - 12x + 9 = 0$

$4x^2 - 6x - 6x + 9 = 0$

$2x(2x - 3) - 3(2x - 3) = 0$

$(2x - 3)(2x - 3) = 0$

$x = \frac{3}{2}$

$\therefore \quad y = \frac{1}{\sqrt{3}}\left(\frac{3}{2}\right) + \sqrt{3}$

$y = \frac{\sqrt{3}}{2} + \sqrt{3}$

$y = \frac{3\sqrt{3}}{2}$

$(a, b) \equiv \left(\frac{3}{2}, \; \frac{3\sqrt{3}}{2}\right)$

$2(a + c) = 2\left(\frac{3}{2} + 3\right)$

$= 2\left(\frac{3}{2} + \frac{6}{2}\right)$

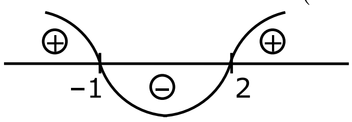
$= 9$

3. The value of $\int_{-2}^{2} \left|3x^2 - 3x - 6\right| \, dx$ is ______.

Ans. (19)

Sol. $3\int_{-2}^{2} \left|x^2 - x - 2\right| dx \qquad x^2 - x - 2$

$= (x-2)(x+1)$



$= 3\left\{\int_{-2}^{-1} \left(x^2 - x - 2\right) dx + \int_{-1}^{2} \left(-x^2 + x + 2\right) dx\right\}$

$= 3\left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-1}^{2}\right]$

$= 19$

4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is ______.

Ans. **(1)**

Sol. Let x = 4k + 3

$(2020 + x)^{2022}$

$= (2020 + 4k + 3)^{2022}$

$= (4(505) + 4k + 3)^{2022}$

$= (4P + 3)^{2022}$

$= (4P + 4 - 1)^{2022}$

$= (4A - 1)^{2022}$

${}^{2022}C_0(4A)^0(-1)^{2022} + {}^{2022}C_1(4A)^1(-1)^{2021} + \ldots\ldots$

$1 + 8\lambda$

Reminder is 1.

5. A line 'ℓ' passing through origin is perpendicular to the lines

$$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on 'ℓ_2' at the distance of $\sqrt{17}$ from the point of intersection of 'ℓ' and 'ℓ_1' are (a, b, c), then 18(a+b+c) is equal to ______.

Ans. (44)

Sol. $\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$\ell_1 : \dfrac{x-3}{1} = \dfrac{y+1}{2} = \dfrac{z-4}{2} \quad \Rightarrow \quad$ D.R. of ℓ_1 = 1, 2, 2

$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

$\ell_2 : \dfrac{x-3}{2} = \dfrac{y-3}{2} = \dfrac{z-2}{1} \quad \Rightarrow \quad$ D.R. of ℓ_2 = 2, 2, 1

D.R. of ℓ is $\perp$ to $\ell_1 \& \ell_2$

$\therefore$ D.R. of ℓ || $(\ell_1 \times \ell_2) \quad \Rightarrow \quad \langle -2, 3, -2 \rangle$

$\therefore$ Equation of $\ell : \dfrac{x}{2} = \dfrac{y}{-3} = \dfrac{z}{2}$

Solving ℓ & ℓ_1

$(2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + \mu)$

$\Rightarrow$ $2\lambda = \mu + 3$

$-3\lambda = 2\mu - 1$

$2\lambda = 2\mu + 4$

$\Rightarrow$ $\mu + 3 = 2\mu + 4$

$\mu = -1$

$\lambda = 1$

P(2, −3, 2) {intersection point}

Let, $Q(2\nu + 3, 2\nu + 3, \nu + 2)$ be point on ℓ_2

Now, PQ = $\sqrt{(2\nu+3-2)^2+(2\nu+3+3)^2+(\nu+2-2)^2} = \sqrt{17}$

$\Rightarrow$ $(2\nu + 1)^2 + (2\nu + 6)^2 + (\nu)^2 = 17$

$\Rightarrow$ $9\nu^2 + 28\nu + 36 + 1 - 17 = 0$

$\Rightarrow$ $9\nu^2 + 28\nu + 20 = 0$

$\Rightarrow$ $9\nu^2 + 18\nu + 10\nu + 20 = 0$

$\Rightarrow$ $(9\nu + 10)(\nu + 2) = 0$

$\Rightarrow$ $\nu = -2$ (rejected), $-\dfrac{10}{9}$ (accepted)

$Q\left(3-\dfrac{20}{9},\ 3-\dfrac{20}{9},\ 2-\dfrac{10}{9}\right)$

$\left(\dfrac{7}{9},\ \dfrac{7}{9},\ \dfrac{8}{9}\right)$

$\therefore$ $18(a + b + c)$

$= 18\left(\dfrac{7}{9}+\dfrac{7}{9}+\dfrac{8}{9}\right)$

$= 44$

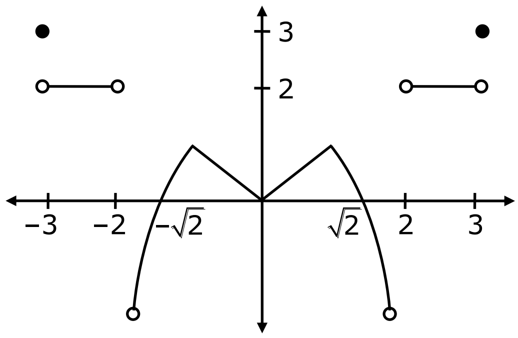
6. A function f is defined on [–3,3] as

$$f(x)=\begin{cases}\min\{|x|,2-x^2\}, & -2\le x\le 2\\ [|x|] & ,2<|x|\le 3\end{cases}$$

where [x] denotes the greatest integer $\le x$. The number of points, where f is not differentiable in (–3,3) is ______.

Ans. **(5)**

Sol.



Points of non-differentiability in (–3, 3) are at x = –2, –1, 0, 1, 2.

i.e. 5 points.

7. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to ______.

Ans. **4**

Sol. $4y^3\dfrac{dy}{dx}=1$ & $x\dfrac{dy}{dx}+y=0$

$m_1=\dfrac{1}{4y^3}$ $\quad\dfrac{dy}{dx}=\dfrac{-y}{x}=m_2$

$m_1m_2=-1$

$\dfrac{1}{4.y^3}\times\dfrac{-y}{x}=-1\quad\because x=y^4$

$\dfrac{1}{4.y^6}=1$ and $xy=k$

$y^6=\dfrac{1}{4}$ $\quad\Rightarrow k=y^5$

$\Rightarrow k^6=y^{30}$

$\Rightarrow k^6=\left(\dfrac{1}{4}\right)^5$

$\therefore (4k)^6=4^6\times k^6=4$

8. The total number of two digit numbers 'n', such that 3^n+7^n is a multiple of 10, is ______.

Ans. (45)

Sol. $\because 7^n = (10-3)^n = 10K + (-3)^n$

$\therefore 7^n + 3^n = 10K + (-3)^n + 3^n$ → 10K if n = odd

→ $10K + 2.3^n$ if n = even

Let n = 2t; $t \in N$

$\therefore 3^n = 3^{2t} = (10-1)^t$

$= 10p + (-1)^t$

$= 10p \pm 1$

$\therefore$ if n = even then $7^n + 3^n$ will not be multiply of 10

So if n is odd then only $7^n + 3^n$ will be multiply of 10

$\therefore$ n = 11, 13, 15, ..., 99

$\therefore$ Ans 45

9. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ______.

Ans. (2)

Sol. $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

Area of parallelogram = $|\vec{a} \times \vec{b}|$

$= \left|(\hat{i} + \alpha\hat{j} + 3\hat{k}) \times (3\hat{i} - \alpha\hat{j} + \hat{k})\right|$

$8\sqrt{3} = \left|(4\alpha)\hat{i} + 8\hat{j} - (4\alpha)\hat{k}\right|$

$(64)(3) = 16\alpha^2 + 64 + 16\alpha^2$

$(64)(3) = 32\alpha^2 + 64$

$6 = \alpha^2 + 2$

$\alpha^2 = 4$

$\therefore \quad \vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

$\vec{a}\cdot\vec{b} = 3 - \alpha^2 + 3$

$= 6 - \alpha^2$

$= 6 - 4$

$= 2$

10. If the curve $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdx = 0$, passes through the intersection of the lines, $2x - 3y=1$ and $3x+2y=8$, then $|y(1)|$ is equal to ______.

Ans. 1

Sol. Given,

$(2xy^2 - y)dx + xdx = 0$

$$\Rightarrow \frac{dy}{dx} + 2y^2 - \frac{y}{x} = 0$$

$$\Rightarrow -\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{1}{x}\right) = 2$$

$$\frac{1}{y} = z$$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{dx} + z\left(\frac{1}{x}\right) = 2$$

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = x$$

$$\therefore \ z(x) = \int 2(x)\,dx = x^2 + c$$

$$\Rightarrow \frac{x}{y} = x^2 + c$$

As it passes through P(2, 1)

[Point of intersection of $2x - 3y = 1$ and $3x + 2y = 8$]

$$\therefore \frac{2}{1} = 4 + c$$

$$\Rightarrow c = -2$$

$$\Rightarrow \frac{x}{y} = x^2 - 2$$

Put $x = 1$

$$\frac{1}{y} = 1 - 2 = -1$$

$$\Rightarrow y(1) = -1$$

$$\Rightarrow |y(1)| = 1$$

26^{th} Feb. 2021 | Shift - 1
PHYSICS

SECTION - A

1. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is :

(1) 1 : 3 (2) 1 : 9 (3) 7 : 135 (4) 7 : 108

Sol. **(3)**

For Lyman series

$n_1 = 1,\qquad n_2 = 4$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{1_1^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_1} = \frac{15Rz^2}{16}$$

$$\lambda_1 = \frac{16}{15Rz^2}$$

For paschen series

$n_1 = 3,\qquad n_2 = 4$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{16-9}{9\times 16}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{7}{9\times 16}\right)$$

$$\lambda_2 = \frac{9\times 16}{7Rz^2}$$

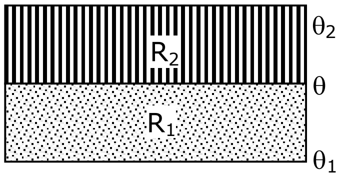
So,

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{16}{15Rz^2}}{\frac{9\times 16}{7Rz^2}}$$

$$= \frac{16\times 7}{15\times 9\times 16}$$

$$= \frac{7}{135}$$

2. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by :



(1) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$ (2) $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$ (3) $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$ (4) $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$

Sol. **(1)**

Temperature at the junction is θ.

so using the formula

$$\frac{T_2 - T}{R_1} = \frac{T - T_1}{R_2}$$

$$\frac{\theta_2 - \theta}{R_2} = \frac{\theta - \theta_1}{R_1}$$

$R_1(\theta_2 - \theta) = R_2(\theta - \theta_1)$

$R_1\theta_2 - R_1\theta = R_2\theta - R_2\theta_1$

$R_1\theta + R_2\theta = R_1\theta_2 + R_2\theta_1$

$$\theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$$

3. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be :

(1) 0.75 mm (2) 0.50 mm (3) 1 mm (4) 0.25 mm

Sol. **(4)**

Fringe width $(\beta) = \frac{\lambda D}{d}$

$d = 2\times10^{-3}$m

$\lambda = 500\times10^{-9}$m

$D = 1$m

Now

$$\beta = \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}}$$

$$\beta = \frac{5}{2} \times 10^{-4}$$

$\beta = 2.5 \times 10^{-4}$

$\beta = 0.25$ mm

4. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false.

(2) Both A and R are true but R is NOT the correct explanation of A.

(3) Both A and R are true and R is the correct explanation of A.

(4) A is false but R is true.

Sol. **(3)**

Resolution limit $(\Delta\theta) = \dfrac{1.22\lambda}{d}$

Resolution power $= \dfrac{1}{\text{Resolution limit}}$

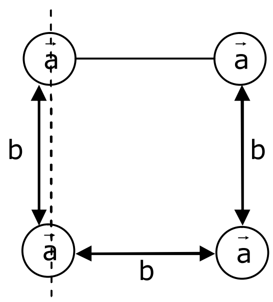
$\lambda \downarrow \Delta\theta \downarrow$

$\Delta\theta \downarrow$ Power $\uparrow$

5. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

(1) $\frac{4}{5}ma^2$ (2) $\frac{8}{5}ma^2 + mb^2$ (3) $\frac{4}{5}ma^2 + 2mb^2$ (4) $\frac{8}{5}ma^2 + 2mb^2$

Sol. **(4)**



$$I = \frac{2}{5}\ ma^2 + \frac{2}{5}\ ma^2 + \left[\frac{2}{5}ma^2 + mb^2\right] + \frac{2}{5}ma^2 + mb^2$$

$$I = 4 \times \frac{2}{5}\ ma^2 + 2mb^2$$

$$= \frac{8}{5}ma^2 + 2mb^2$$

6. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be :

(1) $\frac{\rho K}{P}$ (2) $\frac{K}{\rho P}$ (3) $\frac{PK}{\rho}$ (4) $\frac{\rho P}{K}$

Sol. **(4)**

Bulk modulus $K = \frac{-\Delta P}{\frac{\Delta v}{v}} = \frac{-\Delta p v}{\Delta v}$

We know, $\rho = \frac{M}{V}$

So, $\frac{-\Delta\rho}{\rho} = \frac{\Delta v}{v}$

$$K = \frac{-\Delta P}{\left(-\frac{\Delta\rho}{\rho}\right)} = \frac{\rho\Delta P}{\Delta\rho}$$

$$\Delta\rho = \frac{\rho\Delta P}{K}$$

$$\Delta\rho = \frac{\rho P}{K}$$

7. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.
[$h=6.63\times10^{-34}$ Js and $c=3\times10^{8}$ ms^{-1}]

(1) 654 nm and red colour (2) 1046 nm and blue colour
(3) 1046 nm and red colour (4) 654 nm and orange colour

Sol. **(1)**

We know that $E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} \Rightarrow \frac{1240(\text{in eV})}{E\ (\text{in eV})}$$

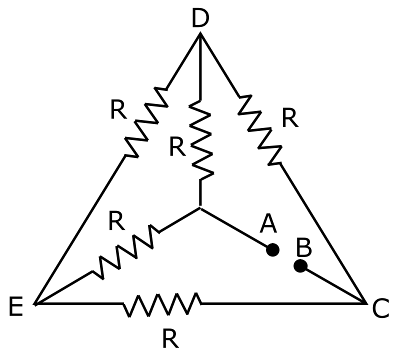
$$\lambda = \frac{1240}{1.9}$$

$= 652.63$ nm ≈ 654 nm

Wavelength of red light is 620 nm to 750 nm

So, answer is 1.

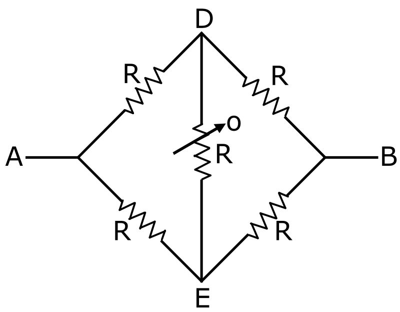
8. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :



(1) $\frac{3R}{2}$ (2) $\frac{R}{2}$ (3) R (4) 2R

Sol. **(3)**

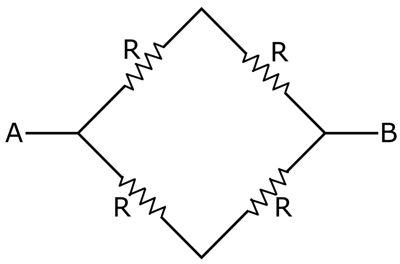
It is balanced wheat stone bridge

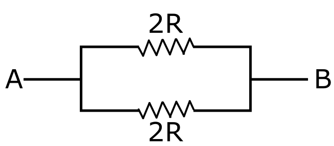


So, we know that

$$R_1R_4 = R_2R_3$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

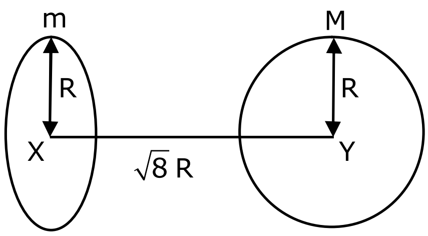




$$\text{Req} = \frac{2R \times 2R}{2R + 2R}$$

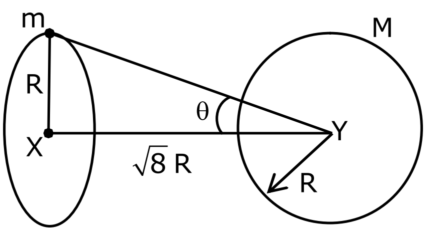
$$= \frac{4R^2}{4R} \Rightarrow R$$

9. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.



(1) $\frac{\sqrt{8}}{9}\cdot\frac{GmM}{R}$ (2) $\frac{\sqrt{8}}{27}\cdot\frac{GmM}{R^2}$ (3) $\frac{2\sqrt{2}}{3}\cdot\frac{GMm}{R^2}$ (4) $\frac{1}{3\sqrt{8}}\cdot\frac{GMm}{R^2}$

Sol. **(2)**



We know that

$$F = ME = M\left(\frac{GM\sqrt{8}R}{\left(R^2+(\sqrt{8}R)^2\right)^{3/2}}\right)$$

$$F = \frac{GMm\sqrt{8}R}{\left(9R^2\right)^{3/2}} \Rightarrow \frac{2\sqrt{2}GmM}{\left(9R^2\right)^{3/2}}$$

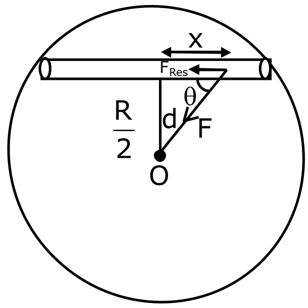
$$= \frac{2\sqrt{2}GmM}{27R^2}$$

$$F = \frac{\sqrt{8}GMm}{27R^2}$$

10. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

(1) $2\pi\sqrt{\frac{R}{g}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ (3) $\frac{2\pi R}{g}$ (4) $\frac{g}{2\pi R}$

Sol. **(1)**



$$\cos\theta = \frac{x}{d}$$

If displaced from equilibrium position,

$$F_{restoring} = \left(\frac{GMmd}{R^3}\right)\cos\theta$$

$$F_{Res.} = \frac{GMmd}{R^3}\cdot\frac{x}{d} = \frac{GMmx}{R^3}$$

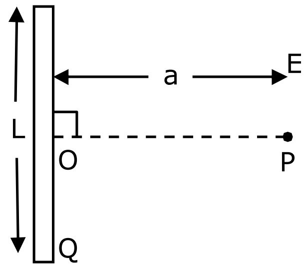
$$a_R = \frac{GMx}{R^3} \qquad GM_e = gR^2$$

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

$$T = 2\pi\sqrt{\frac{x}{\frac{GMx}{R^3}}}\sqrt{\frac{R^3}{gR^2}}$$

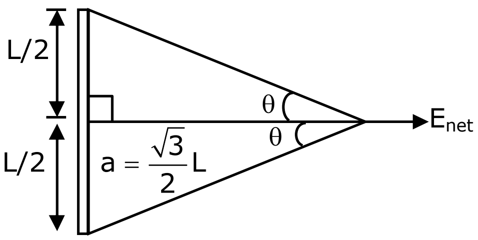
$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

11. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L$.



(1) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ (2) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$ (3) $\frac{Q}{3\pi\varepsilon_0 L^2}$ (4) $\frac{Q}{4\pi\varepsilon_0 L^2}$

Sol. **(1)**



$$\tan\theta = \frac{L/2}{\frac{\sqrt{3}}{2}L} \Rightarrow \frac{1}{\sqrt{3}}$$

$\theta = 30°$

$$E_{net} = \frac{K\lambda}{\frac{\sqrt{3}}{2}L}(\sin 30° + \sin 30°) \Rightarrow \frac{2KQ}{\sqrt{3}L^2}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\sqrt{3}L^2}$$

$$E_{net} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

12. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

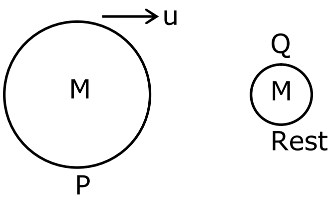
Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m<<M, body 'Q' will have a maximum speed equal to '2u' after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is correct but R is not correct.
(2) Both A and R are correct but R is NOT the correct explanation of A.
(3) A is not correct but R is correct.
(4) Both A and R are correct and R is the correct explanation of A.

Sol. **(4)**



$m << M$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

For elastic collision $\rightarrow e = 1$

$$1 = \frac{v_2 - u}{u - 0}$$

$u = v_2 - u$

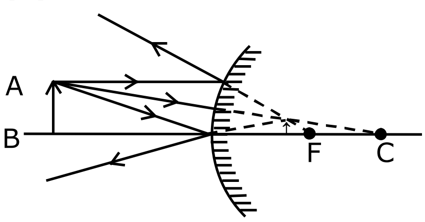
$\boxed{v_2 = 2u}$

In elastic collision kinetic energy & momentum are conserved.

13. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value $|f| = 40$ cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information :

(1) Image is real, same side of concave mirror.
(2) Image is virtual, opposite side of convex mirror.
(3) Image is virtual, opposite side of concave mirror.
(4) Image is real, same side of convex mirror.

Sol. **(2)**



Same orientation so image is virtual. It is combination of real object and virtual image using height it is possible only from convex mirror.

14. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by :

(1) $T \propto R^{\frac{5}{2}}$ (2) $T \propto R^2$ (3) $T \propto R^{\frac{4}{3}}$ (4) $T \propto R^{\frac{3}{2}}$

Sol. **(2)**

$$F \propto \frac{1}{R^3}$$

$$F = \frac{K}{R^3}$$

$$\frac{mv^2}{R} = \frac{K}{R^3}$$

$$m(\omega R)^2 = \frac{K}{R^2}$$

$$m\omega^2 R^2 = \frac{K}{R^2}$$

$$\omega^2 = \frac{K}{m}\left(\frac{1}{R^4}\right)$$

$$\left(\frac{2\pi}{T}\right)^2 \propto \frac{1}{R^4}$$

$$\frac{4\pi^2}{T^2} \propto \frac{1}{R^4}$$

$$\boxed{T \propto R^2}$$

15. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be :

(1) $\frac{2T}{rJ}$ (2) $\frac{3T}{rJ}$ (3) $\frac{2T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ (4) $\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

Sol. **(4)**

R is the radius of bigger drop.
r is the radius of n water drops.
Water drops are combined to make bigger drop.
So,

Volume of n drops = volume of bigger drop

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$R = rn^{1/3} \Rightarrow n = \left(\frac{R}{r}\right)^3$$

$\Delta U = T$(Change in surface area)
$\Delta U = T(n4\pi r^2 - 4\pi R^2)$

$$\Delta U = 4\pi T\left[\left(\frac{R}{r}\right)^3 r^2 - R^2\right] \Rightarrow \frac{4\pi T\left(\frac{R^3}{r} - R^2\right)}{J}$$

$$\frac{\Delta U}{V} = \frac{4\pi T\left(\frac{R^3}{r} - R^2\right)}{J \times \frac{4}{3}\pi R^3} = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

16. A planet revolving in elliptical orbit has :

A. a constant velocity of revolution.

B. has the least velocity when it is nearest to the sun.

C. its areal velocity is directly proportional to its velocity.

D. areal velocity is inversely proportional to its velocity.

E. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below :

(1) A only (2) E only (3) D only (4) C only

Sol. **(2)**

$$\boxed{\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}}$$

17. An alternating current is given by the equation $i=i_1\sin\omega t + i_2\cos\omega t$. The rms current will be :

(1) $\frac{1}{2}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$ (2) $\frac{1}{\sqrt{2}}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$ (3) $\frac{1}{\sqrt{2}}\left(i_1+i_2\right)^2$ (4) $\frac{1}{\sqrt{2}}\left(i_1+i_2\right)$

Sol. **(2)**

$$I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$$

$$I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos 90^\circ}$$

$$I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2(0)} \Rightarrow \sqrt{I_1^2 + I_2^2}$$

We , know that

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

So, $$I_{rms} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}}$$

18. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

(1) $\frac{15}{11}$ (2) $\frac{29}{15}$ (3) $\frac{15}{4}$ (4) $\frac{111}{80}$

Sol. **Bonus**

$$C_1 + C_2 = \frac{15}{4}\left(\frac{C_1C_2}{C_1+C_2}\right)$$

$$4(C_1 + C_2)^2 = 15C_1C_2$$

$$4C_1^2 + 4C_2^2 - 7C_1C_2 = 0$$

$$4 + 4\left(\frac{C_2}{C_1}\right)^2 - 7\frac{C_2}{C_1} = 0$$

$$4\left(\frac{C_2}{C_1}\right)^2 - 7\frac{C_2}{C_1} + 4 = 0$$

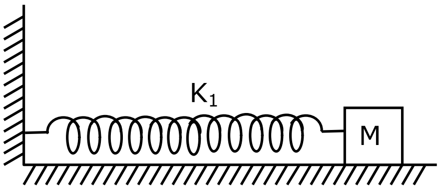
$\frac{C_2}{C_1}$ has not real value

$\frac{C_2}{C_1}$ = imaginary

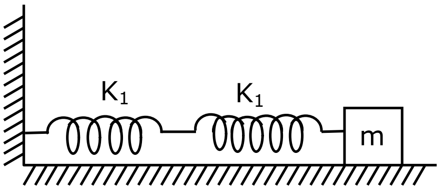
19. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor :

(1) $\frac{1}{2}, \sqrt{2}$ (2) $\frac{1}{4}, 2\sqrt{2}$ (3) $\frac{1}{2}, 2\sqrt{2}$ (4) $\frac{1}{4}, \sqrt{2}$

Sol. **(1)**



$T = 2\pi\sqrt{\frac{m}{K_1}}$



$$K_{eq} = \frac{K_1 \times K_1}{K_1 + K_1} = \frac{K_1^2}{2K_1}$$

$$K_{eq} = \frac{K_1}{2}$$

$$T' = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{\frac{k_1}{2}}}$$

$$= \sqrt{2}T$$

20. In a typical combustion engine the workdone by a gas molecule is given by $W = \alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be :

(1) $[M^0\ L\ T^0]$ (2) $[M^2\ L\ T^{-2}]$ (3) $[M\ L\ T^{-2}]$ (4) $[M\ L\ T^{-1}]$

Sol. **(1)**

$\frac{\beta x^2}{KT}$ is dimension less

so

$KT = \beta x^2 \quad \Rightarrow M^1L^2T^{-2}$

$\beta = \frac{M^1L^2T^{-2}}{L^2} \quad \Rightarrow M^1T^{-2}$

$M^1L^2T^{-2} = \alpha^2M^1T^{-2}$

$\alpha^2 = L^2$

$\alpha = L$

$\alpha = M^0L^1T^0$

SECTION - B

1. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y=-0.21\sin(x+30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is __________. (Round-off to the nearest integer)

Sol. **1215**

$y = -0.21\sin(x+30t)$

$$v = \frac{\omega}{K} = \frac{30}{1} = 30\text{m/s}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$T = v^2 \times \mu$

$T = (30)^2 \times 0.135 \times 10^{-1}$ $\qquad \mu = 0.135\text{gm/cm}$

$T = 900 \times 0.135 \times 10^{-1}$ $\qquad \mu = 0.135 \times \dfrac{10^{-3}}{10^{-2}}\dfrac{\text{kg}}{\text{m}}$

$T = 12.15\text{N}$

$T = 1215 \times 10^{-2}$ N

$\boxed{x = 1215}$

2. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is __________. (Rounded-off to the nearest integer)
[Take $\varepsilon_0 = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2} , $c=3\times10^8$ ms^{-1}]

Sol. **137**

Intensity of electro magnetic wave is,

$$I = \frac{1}{2}c\varepsilon_0E_0^2 = \frac{P}{4\pi r^2}$$

$$\frac{1}{2}4\pi\varepsilon_0 \times c \times E_0^2 = \frac{P}{r^2}$$

$$\frac{1}{2} \times \frac{3\times10^5 \times E_0^2}{9\times10^9} = \frac{1000\times1.25}{(2)^2} \times \frac{1}{100}$$

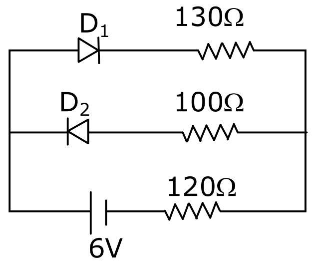
$$E_0^2 = \frac{60\times1000\times1.25}{4\times100} = \frac{125\times3}{2}$$

$$E_0^2 = \frac{375}{2} = 187.5$$

$\boxed{E_0 = 13.69}$

$E_0 \approx 137\times10^{-1}$ v/m

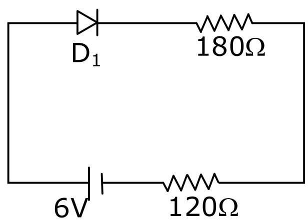
3. The circuit contains two diodes each with a forward resistance of 50Ω and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 Ω resistance is _________ mA.



Sol. **20**

D_2 is reverse bias so current does not flow through D_2.

D_1 is forward bias.

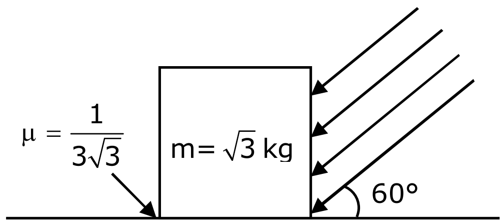


$I = \frac{6}{300}$ $\Rightarrow 0.02$ A

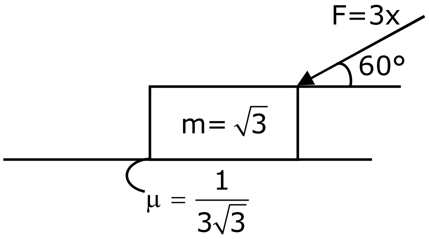
= 20 mA

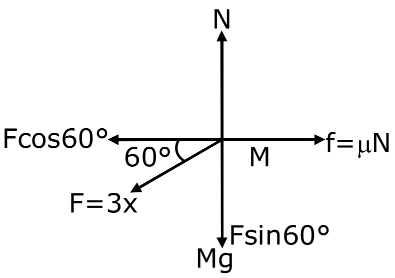
4. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be 3x. The value of x will be ________.

[g=10m/s^2; sin 60°=$\frac{\sqrt{3}}{2}$; cos 60°=$\frac{1}{2}$]



Sol. **3.33**





N = Mg + Fsin60°

$$N = \sqrt{3}g + \frac{F\sqrt{3}}{2}$$

For No slipping

Fcos60° = Friction

$$\frac{F}{2} = \mu N = \frac{1}{3\sqrt{3}}\left(\sqrt{3}g + \frac{F\sqrt{3}}{2}\right)$$

$$\frac{F}{2} = \frac{g}{3} + \frac{F}{6}$$

$$\frac{F}{2} - \frac{F}{6} = \frac{g}{3}$$

$$\frac{6F - 2F}{12} = \frac{g}{3}$$

4F = 4g

F = 10

F = 3x

$$x = \frac{F}{3} = \frac{10}{3} = 3.33$$

$$\boxed{x = 3.33}$$

5. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be __________.

Sol. **282.84**

Quality factor = $\frac{X_L}{R} = \frac{\omega L}{R}$

$$Q = \frac{1}{\sqrt{LC}}\frac{L}{R}$$

$$Q = \left(\frac{1}{\sqrt{C}}\right)\frac{\sqrt{L}}{R}$$

$$Q = \frac{XL}{R} = \frac{\omega L}{R} = \frac{1}{\sqrt{LC}}\frac{L}{R} = \frac{1}{R}\frac{\sqrt{L}}{\sqrt{C}}$$

$$Q' = \frac{\sqrt{2L}}{\left(\frac{R}{2}\right)\sqrt{C}} = 2\sqrt{2}Q$$

$$\boxed{Q' = 282.84}$$

6. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is __________ J.

Sol. **300**

Charge flown (Q) = 20 C

Potential difference (V) = 15V

Work done (w) = Q.V

= 20 × 15 = 300 J

$$\boxed{w = 300\ J}$$

7. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is x × 10^{-1} atm. Value of x is _______.

Sol. **25**

By energy Conservation

$$\frac{3}{2}n_1RT_1 + \frac{3}{2}n_2RT_2 = \frac{3}{2}(n_1 + n_2)RT$$

Using PV = nRT

$P_1V_1 + P_2V_2 = P(V_1 + V_2)$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = \frac{2\times 4.5 + 3\times 5.5}{4.5 + 5.5}$$

$$P = \frac{9 + 16.5}{10} = \frac{25.5}{10}$$

$\approx 25 \times 10^{-1}$ atm

8. A boy pushes a box of mass 2kg with a force $\vec{F} = (20\hat{i} + 10\hat{j})$ N on a frictionless surface. If the box was initially at rest, then __________m is displacement along the x-axis after 10 s.

Sol. **500**

$F = 20\hat{i} + 10\hat{j}$

$F_x = 20N$

$F_y = 10N$

$$a_x = \frac{F_x}{M} = \frac{20}{2} = 10 \text{ m/s}^2$$

$$a_y = \frac{F_y}{M} = \frac{10}{2} = 5 \text{ m/s}^2$$

displacement on x axis is

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$S = 0 \times 10 + \frac{1}{2} \times 10 \times (10)^2$$

$$\boxed{S = 500\,m}$$

9. The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V respectively. The modulation index for this amplitude modulated wave is x $\times 10^{-2}$. The value of x is ________.

Sol. **33**

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$A_C = \frac{A_{max} + A_{min}}{2} \qquad \begin{bmatrix} A_{max} = 16V \\ A_{min} = 8V \end{bmatrix}$$

$$\text{Modulation index (mi)} = \frac{A_m}{A_C} = \frac{\frac{A_{max} - A_{min}}{2}}{\frac{A_{max} + A_{min}}{2}} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

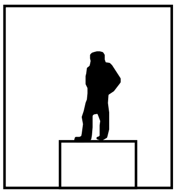
$$mi = \frac{16 - 8}{16 + 8} = \frac{8}{24} = \frac{1}{3} = 0.33$$

$$mi = 33 \times 10^{-2}$$

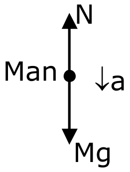
$$\boxed{x = 33}$$

10. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be ___________ N. [g =10m/s^2]

Sol. **492**



$\downarrow 1.8 m/s^2$



Mg − N = Ma

N = M(g−a)

N = 60(10−1.8)

N = 60 × 8.2 = 492N

$\boxed{N = 492}$

26th Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373\text{ K}]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B.

(1) 1, 1-Dichlorobutane and 2-Butanone

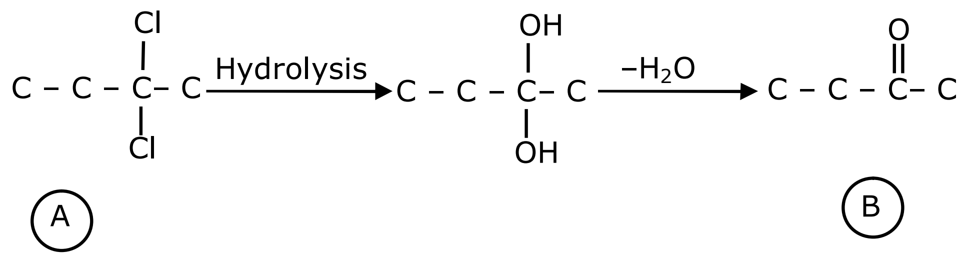
(2) 2, 2- Dichlorobutane and Butan-2-one

(3) 2, 2- Dichlorobutane and Butanal

(4) 1, 1- Dichlorobutane and Butanal

Ans. **(2)**

Sol.



Compound 'B' does not gives Tollen's test due to presence of kenotic group but react with hydroxyl amine

2. Match List-I with List-II.

List –I	**List-II**
(Ore)	**(Element Present)**
(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

Choose the most appropriate answer from the option given below :

(1) (a) – (ii), (b) – (iv), (c) – (i), (d) – (iii)

(2) (a) – (ii), (b) – (i), (c) – (iv), (d) – (iii)

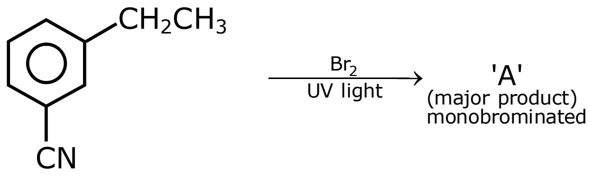
(3) (a) – (i), (b) – (iii), (c) – (iv), (d) – (ii)

(4) (a) – (iii), (b) – (i), (c) – (ii), (d) – (iv)

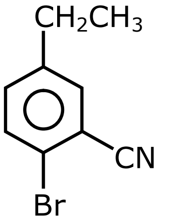
Ans. **(2)**

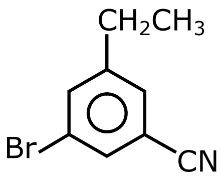
Sol. Fact

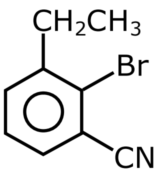
3. For the given reaction :

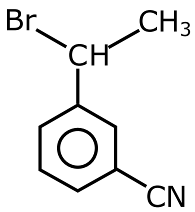


What is 'A' ?

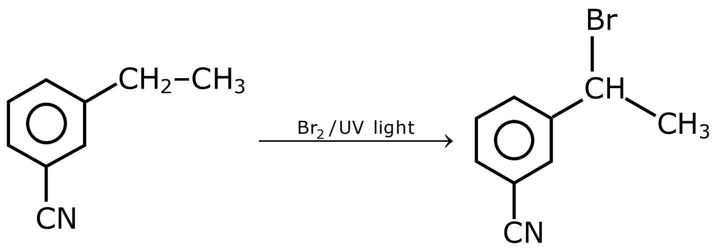
(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

Sol. 

It is bezylic substitution reaction

4. The orbital having two radial as well as two angular nodes is

(1) 5d (2) 4f (3) 3p (4) 4d

Ans. **(1)**

Sol. A.N. = ℓ

R.N = n – ℓ – 1

Orbital	Angular Node	Radial Node
5d	2	2
4f	3	0
3p	1	1
4d	0	1

5. Given below are two statement :

Statement I : o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding

Statement II : o-Nitrophenol has high melting point due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

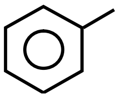
(4) Statement I is true but Statement II is false

Ans. (4)

Sol. o-Nitrophenol is steam volatile due to intramolecular hydrogen H-bonding. but m-Nitrophenol has more melting point due to its symmetry.

6. An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is :

(1) $CH_3CH_2CH_2\overset{H}{N}-CH_2CH_3$ (2) $CH_3CH_2CH_2NHCH_3$

(3)  $NH-CH_2CH_2CH_3$ (4) $CH_3CH_2NH_2$

Ans. (1)

Sol.

$$\underset{1^\circ\text{–amine}}{R-NH_2} \xrightarrow{C_6H_5SO_2Cl} \underset{\text{Soluble in alkalines}}{R-NH-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-C_6H_5}$$

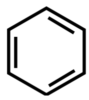
$$\underset{2^\circ\text{–amine}}{R-NH-R} \xrightarrow{C_6H_5SO_2Cl} \underset{\text{(In soluble in alkalines)}}{R-\overset{\overset{R}{|}}{N}-\underset{\underset{O}{\|}}{\overset{\overset{O}{\|}}{S}}-C_6H_5}$$

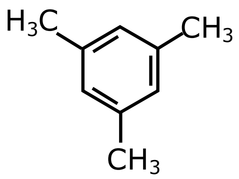
According to the question the amine should be 2°-amine, in which one of the alkyl group should be ethyl, because it can be formed by ammonolysis of ethyl chloride

7. For the given reaction :

$$\underset{\displaystyle CH_3}{\overset{\displaystyle CH{=}CHBr}{|}} \xrightarrow[\text{2. Red hot iron tube, 873 K}]{\text{1. } NaNH_2} \underset{\text{(major product)}}{(A)}$$

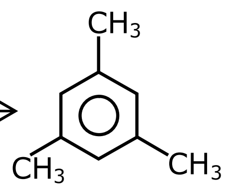
What is 'A'

(1) $\underset{\displaystyle CH_3}{\overset{\displaystyle CH = CH- NH_2}{|}}$ (2) 

(3)  (4) $CH_3CH_2CH_2NH_2$

Ans. (3)

Sol.

$$CH_3 - CH = \overset{\displaystyle Br}{\overset{|}{CH}} \xrightarrow{NaNH_2} CH_3 - CH \equiv CH \xrightarrow{\text{red hot Fe-tube}}$$ 

(mesitylene)

8. Statement about heavy water are given below

A. Heavy water is used in exchange reactions for the study of reaction mechanisms

B. Heavy water is prepared by exhaustive electrolysis of water

C. Heavy water has higher boiling point than ordinary water

D. Viscosity of H_2O is greater than D_2O

(1) A and B only (2) A and D only

(3) A, B and C only (4) A and C only

Ans. (3)

Sol. Fact

9. Which of the following is 'a' FALSE statement ?

(1) Carius tube used in the estimation of sulphur in an organic compound

(2) Kjedahl's method is used for the estimation of nitrogen in an organic compound

(3) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture

(4) Carius method is used for the estimation of nitrogen in an organic compound

Ans. (4)

Sol. Fact

10. Given below are two statements :

Statement I : A mixture of chloroform and aniline can be separated by simple distillation

Statement II : When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is true, statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false, Statement II is true

Ans. (2)

Sol. A suitable method for separating a mixture of aniline and chloro form would be steam distillation. Steam distillation is the process used to separate aromatic compound from a mixture because of their temperature sensitivity. Therefore, steam distillation is an ideal method for their separation

11. Which of the following vitamin is helpful in delaying the blood clotting ?

(1) Vitamin B (2) Vitamin C (3) Vitamin K (4) Vitamin E

Ans. (3)

Sol. Vitamin K is used by the body to help blood clot.

12. The presence of ozone in troposphere :

(1) generates photochemical smog (2) Protects us from the UV radiation

(3) Protects us from the X-ray radiation (4) Protects us from greenhouse effect

Ans. (2)

Sol. The presence of ozone in troposphere protect earth from ultra violet rays

13. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are :

(1) X = SO_2, Y = $Cr_2(SO_4)_3$ (2) X = SO_2, Y = Cr_2O_3

(3) X = SO_3, Y = Cr_2O_3 (4) X = SO_3, Y = $Cr_2(SO_4)_3$

Ans. (1)

Sol. $\underset{(X)}{SO_2} + K_2Cr_2O_7 + H_2SO_4 \longrightarrow \underset{(Y)}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$

14. Find A, B and C in the following reaction :

$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$

(1) A – H_2O ; B – CO_2 ; C – $NaHCO_3$
(2) A – H_2O ; B – O_2 ; C – Na_2CO_3
(3) A – O_2 ; B – CO_2 ; C – Na_2CO_3
(4) A – H_2O ; B – O_2 ; C – $NaHCO_3$

Ans. **(1)**

Sol. (1) $NH_3 + \underset{(A)}{H_2O} + CO_2 \rightarrow (NH_4)_2CO_3$

(2) $(NH_4)_2CO_3 + H_2O + \underset{(B)}{CO_2} \rightarrow NH_4HCO_3$

(3) $NH_4HCO_3 + NaCl \rightarrow NH_4Cl + \underset{(C)}{NaHCO_3}$

15. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation

Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true and R is not the correct explanation of A

Ans. **(3)**

Sol. Fact

16. Match List-I with List-II.

List –I Electronic configuration of elements	**List-II $\Delta_i H$ in kJ mol^{-1}**
(a) $1s^2 2s^2$	(i) 801
(b) $1s^2 2s^2 2p^4$	(ii) 899
(c) $1s^2 2s^2 2p^3$	(iii) 1314
(d) $1s^2 2s^2 2p^1$	(iv) 1402

(1) (a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)
(2) (a) – (iv), (b) – (i), (c) – (ii), (d) – (iii)
(3) (a) – (i), (b) – (iv), (c) – (iii), (d) – (ii)
(4) (a) – (i), (b) – (iii), (c) – (iv), (d) – (ii)

Ans. **(1)**

Sol. Order of I.E. in second period

$Li < \underset{2p^1}{B} < \underset{2s^2}{Be} < \underset{2p^2}{C} < \underset{2p^4}{O} < \underset{2p^3}{N} < F < Ne$

17. Which one of the following lanthanoids does not form MO_2 ?

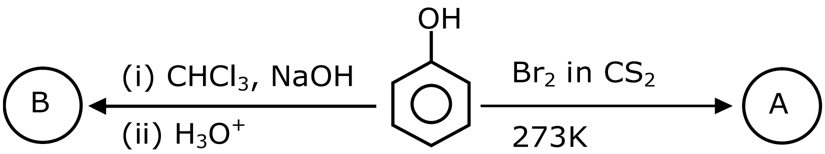
[M is lanthanoid metal]

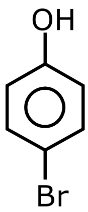
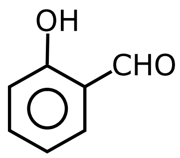
(1) Nd (2) Yb (3) Dy (4) Pr

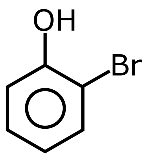
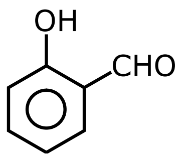
Ans. **(2)**

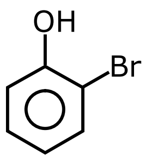
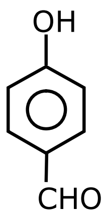
Sol. Fact

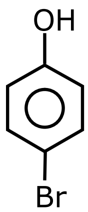
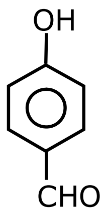
18. Identify the major products A and B respectively in the following reaction of phenol :



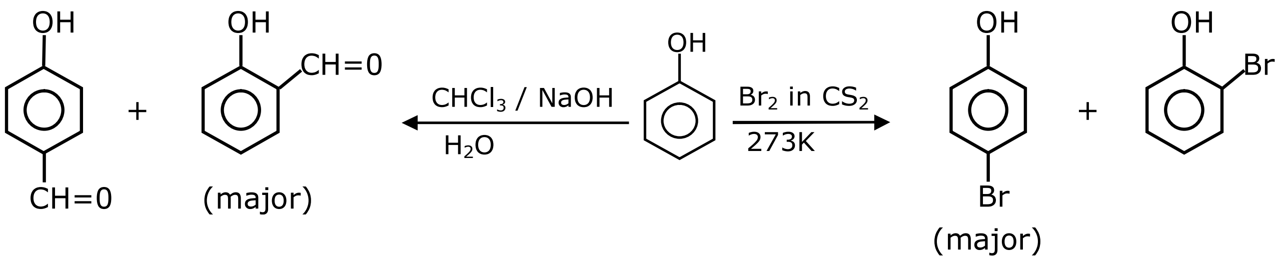
(1)  and 

(2)  and 

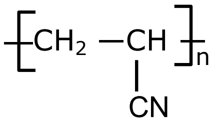
(3)  and 

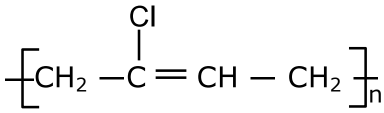
(4)  and 

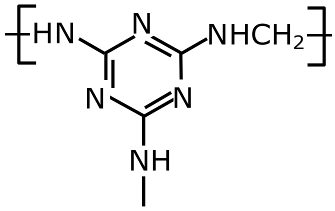
Ans. **(1)**

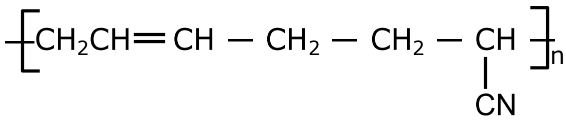
Sol. 

19. The structure of Neoprene is :

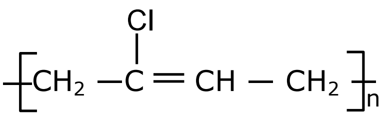
(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. 

neoprene

20. Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is :

(1) Pb_3O_4 (2) PbO_2 (3) $PbSO_4$ (4) PbO

Ans. **(2)**

Sol. lead storage batteries PbO_2 is used. In this O.S. of Pb is +4 so it is always reduced and behaves as oxidizing agent

SECTION - B

1. 224 mL of $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute) ($P^*_{(H_2O)}$ = 24 mm of Hg) is x × 10^{-2} mm of Hg, the value of x is _____ .

Ans. **(0.18)**

Sol. The balanced equation is

$SO_2 + 2NaOH \longrightarrow Na_2SO_3 + H_2O$

moles of NaOH = molarity × volume (in litre)

= 0.1 × 0.1

= 0.01 moles

Here NaOH is limiting Reagent

2 mole NaOH $\longrightarrow$ 1 mole Na_2SO_3

0.01 mole NaOH $\longrightarrow \frac{1}{2} \times 0.01$ mole Na_2SO_3

Moles of $Na_2SO_3 \longrightarrow 0.005$ mole

$Na_2SO_3 \longrightarrow 2Na^+ + SO_3^{2-}$

i = 3

Moles of $H_2O = \frac{36}{18}$ = 2 moles

Accoding to RLVP -

$$\frac{P_A^o - P_A}{P_A^o} = iX_B$$

$$\frac{P_A^o - P_A}{P_A^o} = \frac{in_A}{in_A + n_B} \quad (in_A \simeq 0)$$

$n_B << n_A$

$\{n_A + n_B \simeq n_A\}$

$$\frac{P_A^o - P_A}{P_A^o} = i \times \frac{n_B}{n_A}$$

$$\frac{2H - P_A}{2H} = 3 \times \frac{0.005}{2}$$

$\Rightarrow$ 2H − P_A = 0.18

Lowering in pressure = 0.18 mm of Hg

lowering in pressure = 18 × 10^{-12} mm of Hg

$\boxed{x = 18}$

2. Consider the following reaction

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$, E° = 1.51 V.

The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is____.

Ans. **(25)**

Sol. $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$

1 mole of MnO_4^- require 5 faraday charge

5 moles of MnO_4^- will require 25 faraday charge.

3. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The value of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is ________.

[R = 0.0821 L atm K^{-1} mol^{-1}]

Ans. **(2)**

Sol. Moles of $O_2 = \frac{3.12}{32}$ = 0.0975

volume of $O_2 = \frac{nRT}{p} = \frac{0.0975 \times 0.082 \times 300}{1}$

= 2.3985L $\simeq$ 2.4 L

volume of O_2 absorbed per gm of pt = $\frac{2.4}{1.2}$ = 2

4. The number of significant figures in 50000.020×10^{-3} is ________.

Ans. (7)

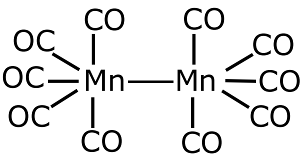
Sol. $\underline{50000.02}0 \times 10^{-3}$

Number of significant figure = 7

5. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is ______.

Ans. (0)

Sol. Fact



6. For a chemical reaction $A + B \rightleftharpoons C + D$

($\Delta_r H^\ominus$ = 80 kJ mol^{-1}) the entropy change $\Delta_r S^\ominus$ depends on the temperature T (in K) as

$\Delta_r S^\ominus = 2T$ (J K^{-1} mol^{-1}).

Minimum temperature at which it will become spontaneous is _________K.

Ans. (200)

Sol. $\Delta G° = \Delta H° - T\Delta S°$

To make the process spontaneous

$\Delta G° < 0$

$\Delta H° - T\Delta S° < 0$

$T > \frac{\Delta H°}{\Delta S°}$

$T > \frac{80000}{2T}$

$2T^2 > 80000$

$T^2 > 40000$

$T > 200$

The minimum temperature to make it spontaneous is 200 K.

7. An exothermic reaction $X \rightarrow Y$ has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ, then the activation energy for the reverse reaction in kJ is _______.

Ans. (50)

Sol. $\Delta H = E_{a,f} - E_{a,b}$

$-20 = 30 - E_{a,b}$

$E_{a,b} = 50$ kJ/mole

8. A certain gas obeys $P(V_m - b) = RT$. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is ______.

Ans. **(1)**

Sol. $P(v - b) = RT$

$PV - Pb = RT$

$\frac{PV}{RT} - \frac{Pb}{RT} = 1$

$Z = 1 + \frac{PV}{RT}$

$\frac{dz}{dp} = 0 + \frac{b}{RT}$

$\Rightarrow \frac{b}{RT} = \frac{xb}{RT}$

$x = 1$

9. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm.
The value of K_p is $x \times 10^{-2}$. The value of x is ______.
[$R = 0.08206\ dm^3\ atm\ K^{-1}\ mol^{-1}$]

Ans. **(74)**

Sol. $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$

initial $1-x$ $\quad x \quad 2x$

at eq. $\frac{1}{1+2x} \quad \frac{1}{1.9}$

By ratio of pressure & mole

$\frac{1}{1+2x} = \frac{0.985}{1.9}$

$1.9 = 0.985 + 1.9x$

$0.915 = 1.9x$

$\frac{0.915}{1.9} = x$; $K_p = \frac{4x^2.x}{(1-x)}\left[\frac{P_{total}}{n_{total}}\right]^2$

$\Rightarrow \frac{4x^3}{1-x}\left(\frac{RT}{V}\right)^2$

On substituting the values

$K_p = 74 \times 10^{-2}$

10. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is :

Ans. **(6)**

Sol. $Cr_2O_7^{2-} + 2OH^- \rightleftharpoons 2CrO_4^{2-} + H_2O$

$2CrO_7^{2-}$

$x + (-2 \times 4) = -2$

$x = 6$

26^{th} Feb. 2021 | Shift - 1
MATHEMATICS

1. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 77

(2) 42

(3) 35

(4) 82

Ans. (1)

Sol. CASE-I: 1, 1, 1, 1, 1, 2, 3

WAYS = $\frac{7!}{5!}$ = 42

CASE-II: 1, 1, 1, 1, 2, 2, 2

WAYS = $\frac{7!}{4!\cdot 3!} = 35$

TOTAL WAYS = 42 + 35 = 77

2. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is:

(1) $\frac{10!}{\sqrt{3}(5!)^2}$

(2) $\frac{2.10!}{3(5!)^2}$

(3) $\frac{10!}{3(5!)^2}$

(4) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

Ans. **(4)**

Sol. $T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r} \left[\frac{(1-x)^{1/10}}{t}\right]^r$

$= {}^{10}C_r t^{(10-2r)} \times x^{\frac{10-r}{5}} \times (1-x)^{\frac{r}{10}}$

$\Rightarrow$ 10 −2r = 0 $\Rightarrow$ r = 5

$T_6 = {}^{10}C_5 x\sqrt{1-x}$

$\frac{dT_6}{dx} = {}^{10}C_5 \left[\sqrt{1-x} - \frac{x}{2\sqrt{1-x}}\right] = 0$

= 1 − x = x/2 $\Rightarrow$ 3x = 2

$\Rightarrow$ x = 2/3

$T_6|_{max} = \frac{10!}{5!5!} \times \frac{2}{3\sqrt{3}}$

3. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le x$, is:

(1) 100 (e–1)
(2) 100e
(3) 100(1-e)
(4) 100 (1 + e)

Ans. **(1)**

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$

$= \int_0^1 e^{\{x\}}dx + \int_1^2 e^{\{x\}}dx + \int_2^3 e^{\{x\}}dx + ... \int_{99}^{100} e^{\{x\}}dx$ ($\because$ {x}= x − [x])

$= e^x |_0^1 + e^{(x-1)} |_1^2 + e^{(x-2)} |_2^3 + ... + e^{(x-99)} |_{99}^{100}$

$= (e-1) + (e-1) + (e-1) + ...(e-1)$

$= 100(e-1)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4
(2) 2
(3) 16
(4) 8

Ans. **(1)**

Sol. $\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = \lambda x$

$\int_{1000}^{x} \frac{dx}{x} = \int_{0}^{t} \lambda dt$

$\ln x - \ln 1000 = \lambda t$

$\ln\left(\frac{x}{1000}\right) = \lambda t$

Put t = 2, x = 1200

$\ln\left(\frac{12}{10}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2}\ln\frac{6}{5}$

Now $\ln\left(\frac{x}{1000}\right) = \frac{t}{2}\ln\left(\frac{6}{5}\right)$

$x = 1000e^{\frac{t}{2}\ln\left(\frac{6}{5}\right)}$

$x = 2000$ at $t = \frac{k}{\ln\left(\frac{6}{5}\right)}$

$\Rightarrow 2000 = 1000\ e^{\frac{k}{2\ln(6/5)} \times \ln(6/5)}$

$\Rightarrow 2 = e^{k/2}$

$\Rightarrow \ln 2 = \frac{k}{2}$

$\Rightarrow \frac{k}{\ln 2} = 2$

$\Rightarrow \left(\frac{k}{\ln 2}\right)^2 = 4$

5. If $\vec{a}$ & $\vec{b}$ are perpendicular vactors, then

$\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)$ is equal to

(1) $\frac{1}{2}|\vec{a}|^4\ \vec{b}$

(2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^4\ \vec{b}$

(4) $\vec{0}$

Ans. **(3)**

Sol. $\vec{a}\times\left(\vec{a}\times\left(\left(\vec{a}.\vec{b}\right)\vec{a}-|\vec{a}|^2\,\vec{b}\right)\right)$

$\vec{a}\times(-|\vec{a}|^2\,(\vec{a}\times\vec{b}))=-|\vec{a}|^2\,((\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\,\vec{b})$

$= -(\vec{a}.\vec{b})\vec{a}\,|\vec{a}|^2+|\vec{a}|^4\,\vec{b}$

$= |\vec{a}|^4\,\vec{b}\;\;\left(\because \vec{a}\cdot\vec{b}=0\right)$

6. In an increasing, geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(1) 35

(2)30

(3) 26

(4) 32

Ans **(1)**

Sol. $ar + ar^5 = \frac{25}{2}$

$ar^2 \times ar^4 = 25$

$a^2r^6 = 25$

$ar^3 = 5$

$\boxed{a=\frac{5}{r^3}}$ ....(1)

$\frac{5r}{r^3}+\frac{5r^5}{r^3}=\frac{25}{2}$

$\frac{1}{r^2}+r^2=\frac{5}{2}$

Put $r^2 = t$

$\frac{t^2+1}{t}=\frac{5}{2}$

$2t^2 - 5t + 2 = 0$

$2t^2 - 4t - t + 2 = 0$

$(2t - 1)(t - 2) = 0$

$t=\frac{1}{2},2 \quad \Rightarrow \boxed{r^2=\frac{1}{2},2}$

$\boxed{r = \sqrt{2}}$

$= ar^3 + ar^5 + ar^7$

$= ar^3 (1 + r^2 + r^4)$

$= 5[1 + 2 + 4] = 35$

7. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true?

(1) P_1 and P_3 are parallel.

(2) P_2 and P_3 are parallel.

(3) P_1 and P_2 are parallel.

(4) P_1, P_2 and P_3 all are parallel.

Ans. (1)

Sol. $P_1 = x + 5y + 7z = 3$

$P_2 = x - 3y - z = 5$

$P_3 = x + 5y + 7z = 5/2$

$\Rightarrow P_1 \| P_3$

8. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to

(1) $\frac{9}{4}$

(2) $\frac{15}{4}$

(3) $\frac{13}{4}$

(4) $\frac{11}{4}$

Ans. **(3)**

Sol. $s = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$

$\frac{s}{3} = \quad \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + ...\infty$

$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + ...\infty$

$\frac{2s}{3} = \frac{4}{3} + \frac{5}{3}\left\{\frac{1/3}{1-\frac{1}{3}}\right\} \quad = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$

$\boxed{s = \frac{13}{4}}$

9. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) −2

(2) (a+1) (a+2) (a+3)

(3) 0

(4) (a+2) (a+3) (a+4)

Ans. **(1)**

Sol. $c_1 \rightarrow c_1 - c_2$, $c_2 \rightarrow c_2 - c_3$

$= \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$

$R_2 \rightarrow R_2 - R_1$ & $R_3 \rightarrow R_3 - R_1$

$= \begin{vmatrix} a^2+2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$

$= 6 - 8 = -2$

10. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c};0<x<1,$ then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is:

(1) $\frac{1-y^2}{2y}$

(2) $\frac{1-y^2}{1+y^2}$

(3) $1-y^2$

(4) $\frac{1-y^2}{y\sqrt{y}}$

Ans. (2)

Sol. $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$

$\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\sin^{-1}x+\cos^{-1}x}{a+b}=\frac{\pi}{2(a+b)}$

Now, $\frac{\tan^{-1}y}{c}=\frac{\pi}{2(a+b)}$

$2\tan^{-1}y=\frac{\pi c}{a+b}$

$\Rightarrow\quad \cos\left(\frac{\pi c}{a+b}\right)=\cos\left(2\tan^{-1}y\right)=\frac{1-y^2}{1+y^2}$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

(1) 6

(2) 1

(3) 4

(4)12

Ans. (3)

Sol. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & c^2+b^2 \end{bmatrix}$$

$= a^2 + 2b^2 + c^2 = 1$

$a = 1, b = 0, c = 0$

$a = 0, b = 0, c = 1$

$a = -1, b = 0, c = 0$

$c = -1, b = 0, a = 0$

12. The intersection of three lines $x-y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a:

(1) Equilateral triangle

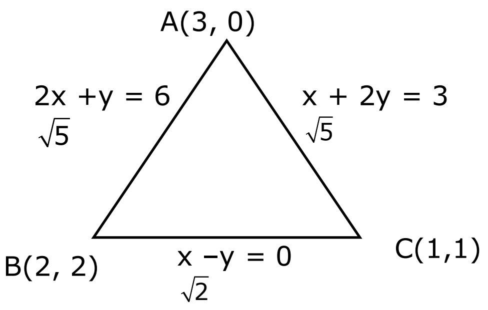
(2) None of the above

(3) Isosceles triangle

(4) Right angled triangle

Ans. (3)

Sol.



13. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point:

(1) (2, 9)

(2) (2,2)

(3) $\left(3, \frac{21}{2}\right)$

(4) (0, 0)

Ans. (2)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Let $f(x) = 2x^3 - 15x^2 + 36x - 19$

$f'(x) = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$\boxed{x = 2, 3}$

$f''(x) = 12x - 30$

$f''(x) < 0$ for $\boxed{x = 2}$

At $\boxed{x = 2}$

$y = 8 - 40 + 72 - 38$

$y = 72 - 70 = 2$

$\Rightarrow (2,2)$

14. Let *f* be any function defined on R and let it satisfy the condition :

$(|f(x)-f(y)| \le |(x-y)^2|, \forall (x,y) \in R$

If f(0) = 1, then:

(1) $f(x) < 0, \ \forall x \in R$

(2) f(x) can take any value in R

(3) $f(x) = 0, \ \forall \ x \in R$

(4) $f(x) > 0, \ \forall \ x \in R$

Ans. (4)

Sol. $|f(x)-f(y)| \le |(x-y)^2|, \forall (x,y) \in R$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$$

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le 0$$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$f(y) = C$

$\Rightarrow \boxed{c=1} \qquad \because \quad f(0) = 1$

$\Rightarrow f(x) = 1$

15. The value of $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is:

(1) 2π
(2) 4π
(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{4}$

Ans. **(4)**

Sol. Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$

$$I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$$

$$I = \int_{0}^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$$

16. The value of $\lim_{h\to 0} \left\{ \frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)} \right\}$ is:

(1) $\frac{3}{4}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{3}$

(4) $\frac{2}{3}$

Ans. (3)

Sol. $\lim_{h\to 0} 2 \times 2 \left\{ \frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{2\sqrt{3}h\left(\cos(h+\frac{\pi}{6}\right)} \right\}$

$$= \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{4}{3}$$

17. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{12}}$

(2) $\frac{15}{2^{13}}$

(3) $\frac{15}{2^{14}}$

(4) $\frac{15}{2^{8}}$

Ans. (2)

Sol. $p(x=9)=p(x=7)$

$${}^{n}C_9\left(\frac{1}{2}\right)^{n-9}\times\left(\frac{1}{2}\right)^{9}={}^{n}C_7\left(\frac{1}{2}\right)^{n-7}\times\left(\frac{1}{2}\right)^{7}$$

$${}^{n}C_9\times\left(\frac{1}{2}\right)^{2}=\left(\frac{1}{2}\right)^{2}\times{}^{n}C_7$$

$$x+y=n \quad \Rightarrow \quad n=16$$

$$p(x=2)={}^{16}C_2\times\left(\frac{1}{2}\right)^{14}\times\left(\frac{1}{2}\right)^{2}$$

$$={}^{16}C_2\times\left(\frac{1}{2}\right)^{16}=\frac{15}{2^{13}}$$

18. If (1,5,35), (7,5,5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is:

(1) $-\frac{44}{5}$

(2) $\frac{39}{5}$

(3) $-\frac{39}{5}$

(4) $\frac{44}{5}$

Ans. **(4)**

Sol. Let P(1,5,35), Q(7,5,5), R(1, λ,7), S(2λ,1,2)

$$\left[\vec{PQ}\ \ \vec{PR}\ \ \vec{PS}\right] = 0$$

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\{-33\lambda + 165 - 112\} + 5(\lambda - 5)(2\lambda - 1) = 0$$

$$53 - 33\lambda + 5\{2\lambda^2 - 11\lambda + 5\} = 0$$

$$16\lambda^2 - 88\lambda + 78 = 0$$

$$5\lambda^2 - 44\lambda + 39 = 0 <^{\lambda_1}_{\lambda_2}$$

$$\Rightarrow \lambda_1 + \lambda_2 = 44/5$$

19. Let R = {P,Q)|P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set:

(1) S = {(x,y)|x^2+y^2=1}

(2) S = {(x,y)|x^2+y^2=4}

(3) S = {(x,y)|x^2+y^2= $\sqrt{2}$ }

(4) S = {(x,y)|x^2+y^2= 2}

Ans. **(4)**

Sol. P(a, b) , Q(c, d), PO = QO

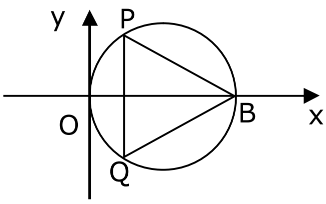
$\Rightarrow a^2 + b^2 = c^2 + d^2$

R(x,y)   s = (1, –1) $\Rightarrow$ RO = SO

($\because$ equivalence class)

$x^2 + y^2 = 2$

20. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is:



(1) $26\sqrt{3}$

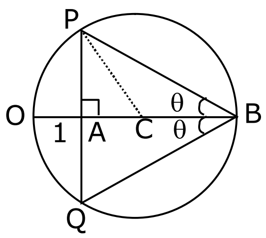
(2) $24\sqrt{2}$

(3) $24\sqrt{3}$

(4) $26\sqrt{2}$

Ans. **(3)**

Sol.



$OC = \frac{13}{2} = 6.5$

$AC = CO - AO$

$= 6.5 - 1$

$= 5.5$

In ΔPAC

$PA = \sqrt{6.5^2 - 5.5^2}$

$PA = \sqrt{12}$

$\Rightarrow PQ = 2PA = 2\sqrt{12}$

Now, area of $\Delta PQB = \frac{1}{2} \times PQ \times AB$

$= \frac{1}{2} \times 2\sqrt{12} \times 12$

$= 12\sqrt{12}$

$= 24\sqrt{3}$

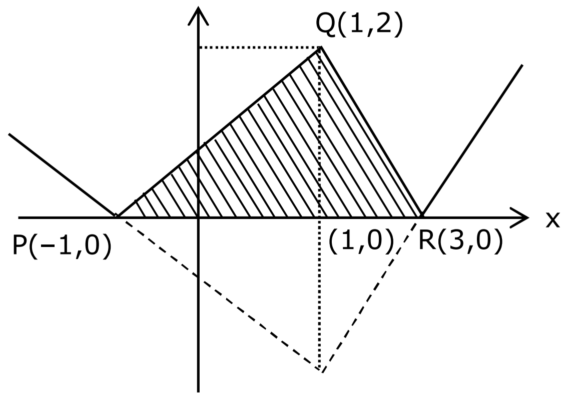
Section-B

1. The area bounded by the lines y = || x-1|-2| is…….

Ans. Bonus

NTA Ans. (8)

Sol.



2. The number of integral values of 'k' for which the equation 3sinx + 4cos x = k + 1 has a solution, $k \in R$ is ______.

Ans. (11)

Sol. 3 sinx + 4cosx = k+1

$-5 \le k+1 \le 5$

$-6 \le k \le 4$

$\boxed{-6,-5,-4,-3,-2,-1,0,1,2,3,4}$ $\Rightarrow$ 11 integral values

3. Let $m, n \in N$ and gcd (2, n) = 1. If $30\binom{30}{0} + 29\binom{30}{1} + ... + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m$,

then n + m is equal to_____.

Ans. (45)

Sol. Let $S = \sum_{r=0}^{30}(30-r)\ {}^{30}C_r$

$$= 30\sum_{r=0}^{30} {}^{30}C_r - \sum_{r=0}^{30} r\ {}^{30}C_r$$

$= 20 \times 2^{30} - \sum_{r=1}^{30} r.\frac{30}{4}.^{29}C_{r-1}$

$= 30 \times 2^{30} - 30.2^{29}$

$= (30\times 2 - 30).2^{29} = 30.2^{29} \Rightarrow 15.2^{30}$

$= n = 15, \; m = 30$

$n + m = 45$

4. If $y = y(x)$ is the solution of the equation $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x, y(0) = 0$; then

$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right)$ is equal to ______.

Ans. (1)

Sol. $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$

Put $e^{\sin y} = t$

$e^{\sin y} \times \cos y \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + t\cos x = \cos x$

I.F. $= e^{\int \cos x dx} = e^{\sin x}$

Solution of diff equation:

$t.e^{\sin x} = \int e^{\sin x}.\cos x dx$

$e^{\sin y}.e^{\sin x} = e^{\sin x} + c$

at $x = 0, y = 0$

$1 = 1 + c \qquad \Rightarrow c = 0$

$e^{\sin x + \sin y} = e^{\sin x}$

$\sin x + \sin y = \sin x$

$y = 0$

$\Rightarrow y\left(\frac{\pi}{6}\right) = 0, \quad y\left(\frac{\pi}{3}\right) = 0, \quad y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 1 + 0 + 0 + 0 = 1$

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is ______.

Ans. (1)

Sol. $\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$x-1 = (x-3)^2$

$x^2 - 6x + 9 = x - 1$

$x^2 - 7x + 10 = 0$

$x = 2, 5$

X = 2 Not possible as $\log_2(x-3)$ is not defined

$\Rightarrow$ No. of solution = 1

6. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1,$ the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is _______.

Ans. (1)

Sol. $\sqrt{3}t^2 - \left(\sqrt{3}-1\right)t - 1 = 0$ (where t = cosx)

Now, $t = \dfrac{\left(\sqrt{3}-1\right) \pm \sqrt{4+2\sqrt{3}}}{2\sqrt{3}}$

t = cos x = 1 or $-\frac{1}{\sqrt{3}} \rightarrow$ rejected as $x \in \left[0, \frac{\pi}{2}\right]$

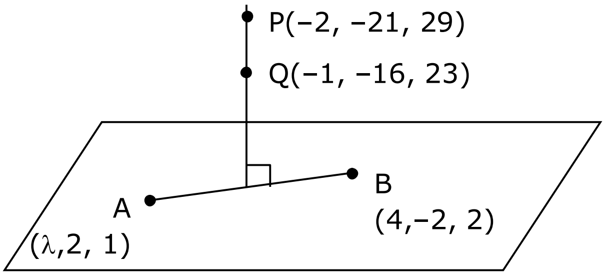
$\Rightarrow$ cos x = 1

$\Rightarrow$ No. of solution = 1

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the point $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _______.

Ans. **(8)**

Sol.



$\vec{AB} \perp \vec{PQ}$

$\left[(4-\lambda)\hat{i} - 4\hat{j} + \hat{k}\right].\left[+\hat{i} + 5\hat{j} - 6\hat{k}\right] = 0$

$4 - \lambda - 20 - 6 = 0$

$\boxed{\lambda = -22}$

Now , $\frac{\lambda}{11} = -2$

$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$

$\Rightarrow \quad 4 + 8 - 4 = 8$

8. The difference betweeen degree and order of differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _______.

Ans. **(2)**

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$

$2yy' = a$

$y^2 = 2yy'\left(x + \frac{\sqrt{2yy'}}{2}\right)$

$$y = 2y'\left(x + \frac{\sqrt{yy'}}{\sqrt{2}}\right)$$

$$y - 2xy' = \sqrt{2}y'\sqrt{yy'}$$

$$\left(y - 2x\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right)^3$$

D = 3 & O = 1

D– O = 3 – 1 = 2

9. The sum of 162[th] power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is______.

Ans. (3)

Sol. Let roots of $x^3 - 2x^2 + 2x - 1 = 0$ are α, β, γ

$(x - 1)(x^2 - x + 1) = 0$

$$x = \underset{\alpha}{\underset{\downarrow}{1}},\ \underset{\beta}{\underset{\downarrow}{-\omega}},\ \underset{\gamma}{\underset{\downarrow}{-\omega^2}}$$

Now $\alpha^{162} + \beta^{162} + \gamma^{162}$

$= 1 + \omega^{162} + (\omega^2)^{162}$

$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$

$= 3$

10. The value of the integeral $\int_0^{\pi} |\sin 2x|\,dx$ is ________

Ans. (2)

Sol. $$I = \int_0^{\pi} |\sin 2x|\,dx$$

$$I = 2\int_0^{\pi/2} |\sin 2x|\,dx = 2\int_0^{\pi/2} \sin 2x\,dx$$

$$I = 2\left[\frac{-\cos(2x)}{2}\right]_0^{\pi/2} = 2$$

26th Feb. 2021 | Shift - 2
PHYSICS

SECTION – A

1. A tuning fork A of unknown frequency produces 5beats/s with a fork of known frequency 340 HZ. When fork A filed, the beat frequency decreases to 2beats/s. What is the frequency of fork A?

(1) 342 Hz

(2) 335 Hz

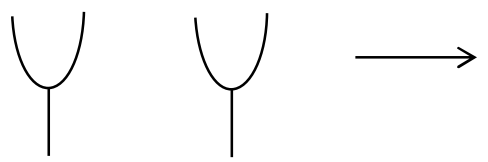
(3) 338 Hz

(4) 345 Hz

Sol. **(2)**

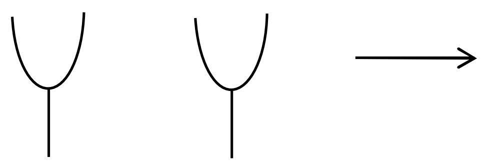
Given

BeforeFiled:



So answer should be 335 Hz or 345 Hz.

After Filed :



$$340 \nearrow 335 \xrightarrow{\text{Filed}} 336, 337, 338\ldots\ldots$$

$$340 \searrow 345 \xrightarrow{\text{Filed}} 346, 347, 348\ldots\ldots$$

After filed beat/sec decreases only in case of 335 Hz.

2. The trajectory a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distance of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by:

(1) $\tan^{-1}\alpha, \dfrac{\alpha^2}{4\beta}$

(2) $\tan^{-1}\beta, \dfrac{\alpha^2}{2\beta}$

(3) $\tan^{-1}\left(\dfrac{\beta}{\alpha}\right), \dfrac{\alpha^2}{\beta}$

(4) $\tan^{-1}\alpha, \dfrac{4\alpha^2}{\beta}$

Sol. **(1)**

Given :

$y = \alpha x - \beta x^2$ ....(1)

for maximum height, we should find out maximum value of y from equation (1)

so, for maximum value of y

$\dfrac{dy}{dx} = 0 \Rightarrow \alpha - 2\beta x = 0$

$x = \dfrac{\alpha}{2\beta}$ ....(2)

Now, put value of x from equation (2) in quation (1)

$y = \alpha\left(\dfrac{\alpha}{2\beta}\right) - \beta\left(\dfrac{\alpha^2}{4\beta^2}\right)$

$\Rightarrow \left(\dfrac{\alpha^2}{2\beta}\right) - \left(\dfrac{\alpha^2}{4\beta}\right) \Rightarrow \dfrac{\alpha^2}{4\beta}$

So, $H_{max} = \dfrac{\alpha^2}{4\beta}$ ....(3)

As we know maximum height $H_{max} = \dfrac{u^2 \sin^2\theta}{2g}$ ...(4)

from (3) and (4) $u^2 = \left(\dfrac{\alpha^2}{4\beta}\right)\left(\dfrac{2g}{\sin^2\theta}\right)$

and range (R) = 2x = $\dfrac{u^2 \times 2\sin\theta\cos\theta}{g}$

$$2\left(\dfrac{\alpha}{2\beta}\right) = \dfrac{\left(\dfrac{\alpha^2}{4\beta}\right)\left(\dfrac{2g}{\sin^2\theta}\right) \times 2\sin\theta\cos\theta}{g}$$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}(\alpha)$

3. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be:

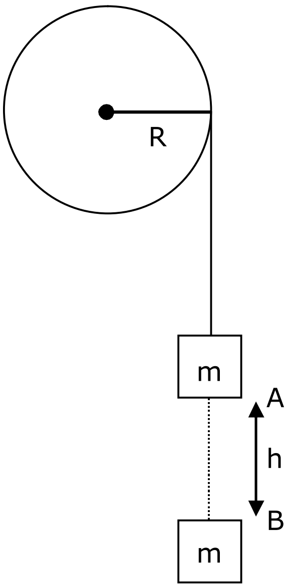
(1) $\frac{2gh}{I+mr^2}$

(2) 2gh

(3) $\frac{2mgh}{I+2mr^2}$

(4) $\frac{2mgh}{I+mr^2}$

Sol. **(4)**



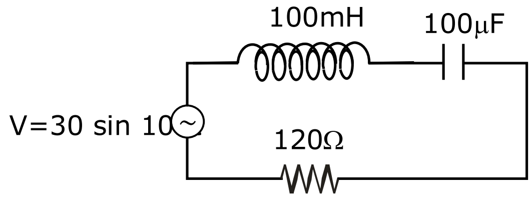
using energy conservation between A and B point

$$mgh = \frac{1}{2} m (wR)^2 + \frac{1}{2} I\omega^2$$

$$2mgh = (MR^2 + I)\,\omega^2$$

$$\omega^2 = \frac{2mgh}{I+MR^2}$$

4. Find the peak current and resonant frequency of the following circuit (as shown in figure)



(1) 0.2 A and 100 Hz
(2) 2 A and 50 Hz
(3) 2 A and 100 Hz
(4) 0.2 A and 50 Hz

Sol. **(4)**

Peak current in series LCR CKT

$$i = \frac{v_0}{z} \Rightarrow \frac{30}{\sqrt{(x_L - x_C)^2 + R^2}}$$

$$i = \frac{30}{\sqrt{(10-100)^2 + (120)^2}}$$

$$i \Rightarrow \frac{30}{150} \Rightarrow \frac{1}{5} \Rightarrow 0.2 Amp.$$

$$\therefore X_L = \omega \times L$$

$\Rightarrow$ (100) (100 × 10^{-3})$\Rightarrow$ 10

$$X_L = \frac{1}{\omega \times c} \Rightarrow \frac{1}{100 \times 100 \times 10^{-6}}$$

$$\Rightarrow \frac{10^6}{10^4} \Rightarrow 100$$

Resonance frequency $\omega = \dfrac{1}{\sqrt{LC}}$

$$\omega = \frac{1}{\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}} \Rightarrow \frac{1}{\sqrt{10^{-5}}}$$

$\therefore \omega = 2\pi F$

$$F = \frac{1}{2\pi} \times \frac{1}{\sqrt{10^{-5}}}$$

$$\Rightarrow \frac{1}{2\pi}\sqrt{10^5}$$

$$\Rightarrow \frac{100}{2\pi}\sqrt{10}$$

$\Rightarrow$ 50Hz

5. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

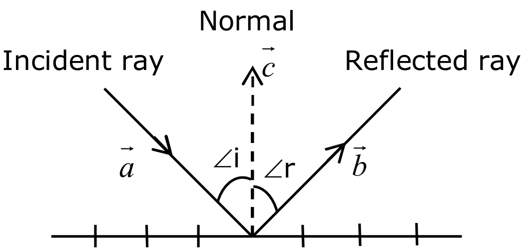
(1) $\vec{b} = 2\vec{a} + \vec{c}$

(2) $\vec{b} = \vec{a} - \vec{c}$

(3) $\vec{b} = \vec{a} + 2\vec{c}$

(4) $\vec{b} = \vec{a} - 2\left(\vec{a} \cdot \vec{c}\right)\vec{c}$

Sol. **(4)**



We see from the diagram that because of the law of reflection, the component of the unit vector $\vec{a}$ along $\vec{b}$ changes sign on reflection while the component parallel to the mirror remain unchanges.

$\vec{a} = \overrightarrow{a_{11}} + \overrightarrow{a_{\perp}}$

and $\overrightarrow{a_{\perp}} = \vec{c}\left(\vec{a}.\vec{c}\right)$

we see that the reflected unit vector is

$\vec{b} = \overrightarrow{a_{11}} - \overrightarrow{a_{\perp}} \Rightarrow \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}$

6. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample?

(1) $\frac{t_2 - t_1}{\ln 5}$

(2) $\frac{\ln(t_2 + t_1)}{2}$

(3) $\frac{t_1 - t_2}{\ln 5}$

(4) $\frac{\ln 5}{t_2 - t_1}$

Sol. **(1)**

For activity of radioactivesample

$A = A_0 e^{-\alpha t_1}$ ....(1)

$\frac{A}{5}$ $A_0 e^{-\alpha t_2}$ ....(2)

From (1)/(2)

$5 = e^{-\lambda(t_1 - t_2)}$

$\ln(5) = (t_2 - t_1)\lambda \Rightarrow \lambda = \frac{\ln(5)}{t_2 - t_1}$

avg. life = $\frac{1}{\lambda} \Rightarrow \frac{t_2 - t_1}{\ln(5)}$

7. A particle executes S.H.M., the graph of velocity as a function of displacement is:

(1) a circle

(2) a parabola

(3) an ellipse

(4) a helix

Sol. **(3)**

For a body performing SHM, relation between velocity and displacement

$v = \omega\sqrt{A^2 - x^2}$

now, square both side

$v^2 = w^2 (A^2 - x^2)$

$\Rightarrow v^2 = w^2A^2 - \omega^2x^2$

$v^2 + \omega^2x^2 = \omega^2A^2$

divide whole equation by ω^2A^2

$\frac{v^2}{\omega^2A^2} + \frac{\omega^2x^2}{\omega^2A^2} = \frac{\omega^2x^2}{\omega^2A^2}$

$\frac{v^2}{(\omega A)^2} + \frac{x^2}{(A)^2} = 1$

above equation is similar as standard equation of ellipes, so graph between velocity and displacement will be ellipes.

8. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be:

(1) $\frac{a_1 + a_2}{a_2}$

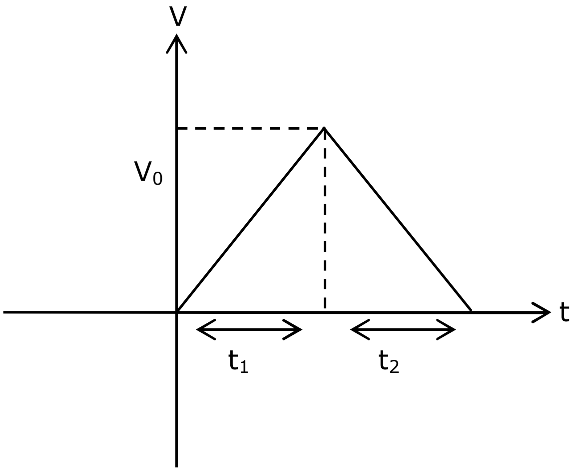
(2) $\frac{a_2}{a_1}$

(3) $\frac{a_1 + a_2}{a_1}$

(4) $\frac{a_1}{a_2}$

Sol. **(2)**

From given information:



For 1^{st} interval

$a_1 = \frac{v_0}{t_1}$

$v_0 = a_1 t_1$ .....(1)

For 2^{nd} interval

$a_2 = \frac{v_0}{t_2}$

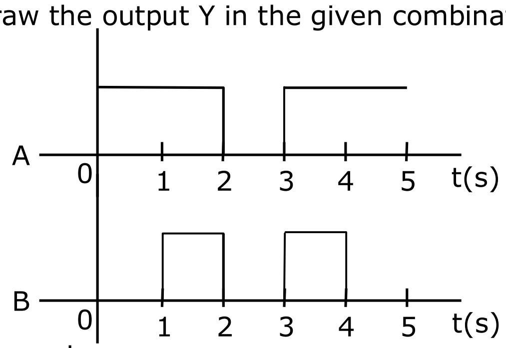
$v_0 = a_2 t_2$ .....(2)

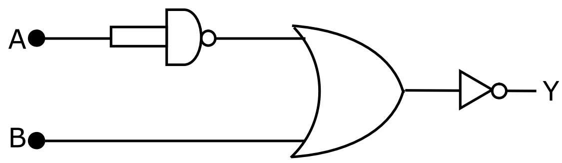
from (1) & (2)

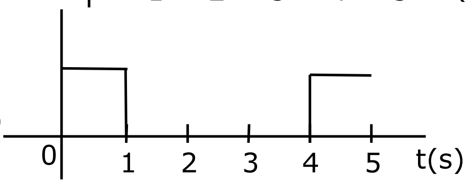
$a_1 t_1 = a_2 t2$

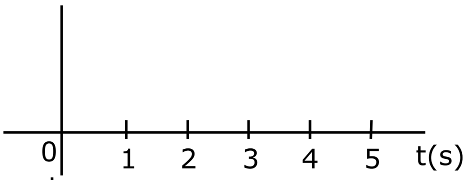
$\frac{t_1}{t_2} = \frac{a_2}{a_1}$

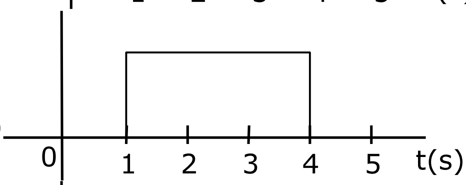
9. Draw the output Y in the given combination of gates.

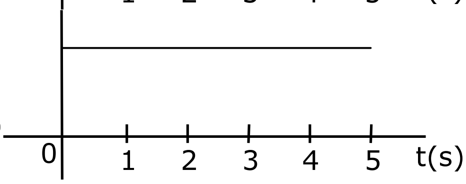




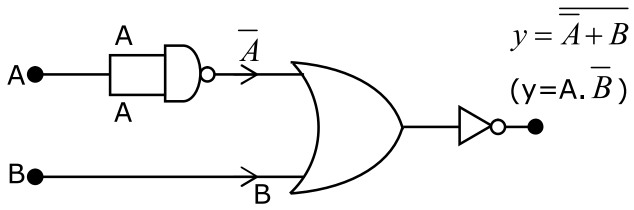
(1)


(2)


(3)


(4)


Sol. **(1)**



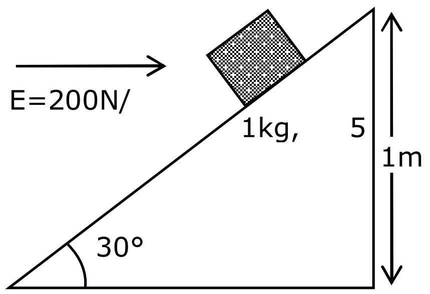
Find output expression $y = A \cdot \overline{B}$

Inputs

A	B	$y = A \cdot \overline{B}$
1	0	1
1	1	0
0	0	0
1	1	0
1	0	1

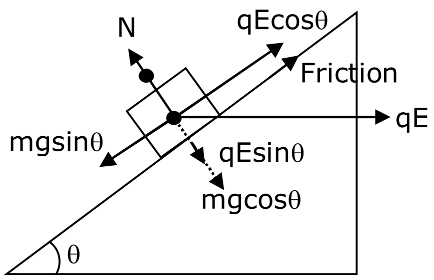
10. An inclined plane making an angle of 30º with horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1 kg and charge 5mC is allowed to slide down from rest at a height of 1m. If the coefficient of frication is 0.2, find the time taken by the body to reach the bottom.

$\left[g = 9.8 m/s^2, \sin 30° = \frac{1}{2}; \cos 30° = \frac{\sqrt{3}}{2}\right]$



(1) 2.3 s
(2) 0.46 s
(3) 1.3 s
(4) 0.92 s

Sol. **(3)**



F= mg sinθ −(μN + q E cosθ)

F= mg sinθ − μ(mg cosθ + qE sinθ) − qE cosθ

F = 1 × 10 × sin30 − 0.2 (1 × 10 × cos30° + 200 × 5 × 10^{-3} sin30°)

- 200 × 5 × 10^{-3} cos30°

F = 2.3 N

$a = \frac{F}{m} \Rightarrow \frac{2.3}{1} \Rightarrow 2.3 m/sec^2$

$t = \sqrt{\frac{25}{9}} \Rightarrow \sqrt{\frac{2 \times 2}{2.3}} \Rightarrow 1.3\,sec$

11. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where C/V = λ?

(1) $\left[M^{-2}L^{-4}I^{3}T^{7}\right]$

(2) $\left[M^{-2}L^{-3}I^{2}T^{6}\right]$

(3) $\left[M^{-1}L^{-3}I^{-2}T^{-7}\right]$

(4) $\left[M^{-3}L^{-4}I^{3}T^{7}\right]$

Sol. **(1)**

$\therefore$ v = $\frac{w}{q}$ and c= $\frac{q}{v}$

dimension of $\frac{c}{v}$

$\Rightarrow \frac{q}{v^2}$

$\Rightarrow \frac{q}{w^2} \times q^2 \Rightarrow \frac{q^3}{w^2}$

$\Rightarrow \frac{I^3T^3}{M^2L^4T^{-4}} \Rightarrow \left[M^{-2}L^{-4}T^{7}I^{3}\right]$

12. Given below are two statements: One is labeled as Assertion A and the other is labeled as Reason R.

Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below:

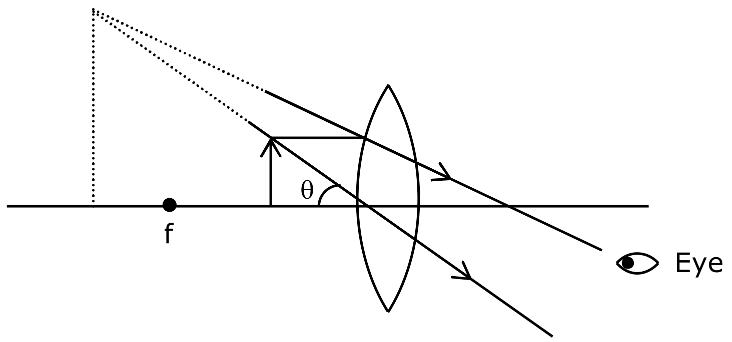
(1) Both A and R are true but R is NOT the correct explanationof A

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

Sol. **(2)**

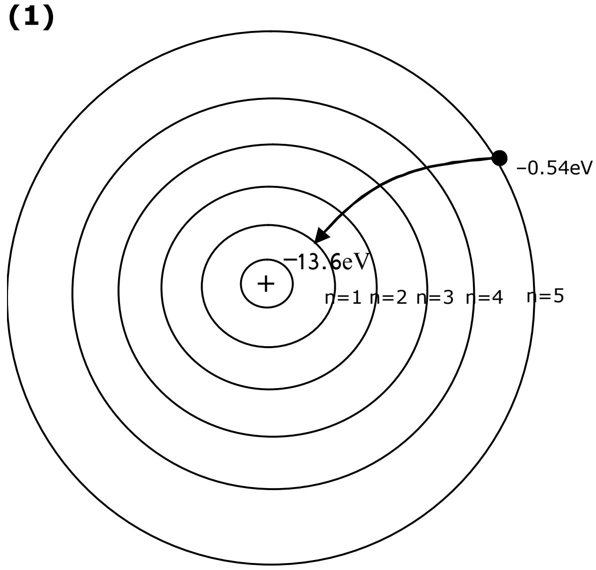


Both obtain same angle, since image can be at a distance greater than 25 cm, object can be moved closer to eye.

13. The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be:

(1) 4.17 m/s (2) 4.34 m/s (3) 219 m/s (4) 3.25 m/s

Sol. **(1)**

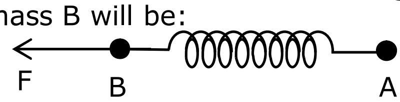


momentum (P) = $\frac{\Delta E}{C} \Rightarrow \frac{(13.6-0.54)eV}{3\times10^{8}}$

mv = $\frac{(13.06)\times1.6\times10^{-19}}{3\times10^{8}}$

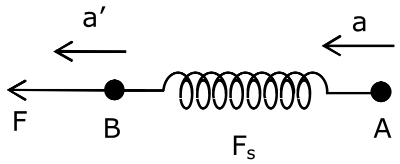
v = $\frac{(13.06)\times1.6\times10^{-19}}{3\times10^{8}\times1.67\times10^{-27}} \Rightarrow 4.17$ m/sec

14. Two masses A and B, each of mass M are fixed together by a massless springs. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', than the acceleration of mass B will be:



(1) $\frac{F+Ma}{M}$ (2) $\frac{F-Ma}{M}$ (3) $\frac{Ma-F}{M}$ (4) $\frac{MF}{F+Ma}$

Sol. (2)



$F - F_s = Ma'$

$a' = \frac{F}{M}$ –9

$\frac{F-ma}{M}$

15. A wire of 1Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in a resistance to the nearest integer is:

(1) 25%

(2) 12.5%

(3) 76%

(4) 56%

Sol. (4)

For stretched or compressed wire

$R \propto l^2$

$\frac{R_1}{R_2} = \frac{l_1^2}{l_2^2}$

$\Rightarrow \frac{R}{R_2} = \frac{l^2}{(1.25l)^2}$

$\Rightarrow R_2 = 1.5625$ R

% increase $\rightarrow$ 56.235%

16. Given below are two statements :

Statement (1) :- A second's pendulum has a time period of 1 second.

Statement (2) :- It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options give below.

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II is true

Sol. **(3)**

As we know time period of second's penduklum is 2 sec, so statement (1) is incorrect.

Time taken between two extreme points in second's pendulum is 1 sec.

Above statement is correct because time taken by particle performing SHM between two extreme position is T/2.

Here, T = 2 sec.

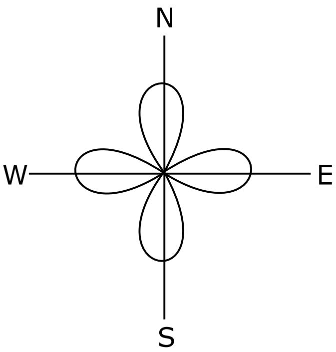
So, time = 2/2 = 1 sec

17. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be ________.

(1) 88.37 mV (2) 62.50 mV

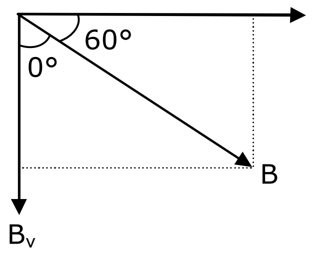
(3) 54.125 mV (4) 108.25 mV

Sol. **(4)**



$$\sum = B \perp v\ell$$

$$\sin 60^\circ = \frac{B_v}{B}$$



$$\frac{\sqrt{3}}{2} = \frac{B_v}{B}$$

$$Bv = \frac{\sqrt{3}}{2}B$$

$$E = \frac{\sqrt{3}}{2}B\ell v$$

$$= \frac{\sqrt{3}}{2} \times 2.5 \times 10^{-4} \times 10 \times 180 \times \frac{5}{18}$$

$$= \frac{\sqrt{3}}{2} \times 2.5 \times 5 \times 10^{-2} \quad = 10.825 \times 10^{-2} \qquad = 108.25 \text{ mV}$$

18. The length of metallic wire is l_1 when tension in it is T_1. It is l_2 when the tension is T_2. The original length of the wire will be :

(1) $\frac{l_1 + l_2}{2}$

(2) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$

(3) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$

(4) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Sol. **(4)**

From young's modulus relation $\left(y = \frac{\frac{F}{A}}{\left(\frac{\Delta l}{l}\right)} \right)$

we can write for 1st case

$$\frac{T_1}{A} = \frac{y(\ell_1 - \ell)}{\ell}$$

we can write for 2^{nd} case

$$\frac{T_2}{A}=\frac{y(\ell_2-\ell)}{\ell}$$

$$\frac{T_1}{T_2}=\frac{\ell_1-\ell}{\ell_2-\ell}$$

$$T_1\ell_2-T_1\ell=T_2\ell_1-T_2\ell$$

$$\frac{T_2 l_1-T_1 l_2}{T_2-T_1}=\ell$$

19. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U = 3PV + 4. The gas is :

(1) polyatomic only

(2) monoatomic only

(3) either monoatomic or diatomic

(4) diatomic only.

Sol. **(1)**

U = 3 PV + 4

$\frac{f}{2}$ PV = 3PV + 4 $\quad\quad \therefore u=\frac{f}{2}nRT$

$f = 6+\frac{8}{PV}$ $\quad\quad \therefore Pv = nRT$

f>6 $\therefore$ Polyatomic gas.

20. Given below are two statements :

Statement – I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement – II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but theelectric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements. Choose the correct answerfrom the option given below :

Option :

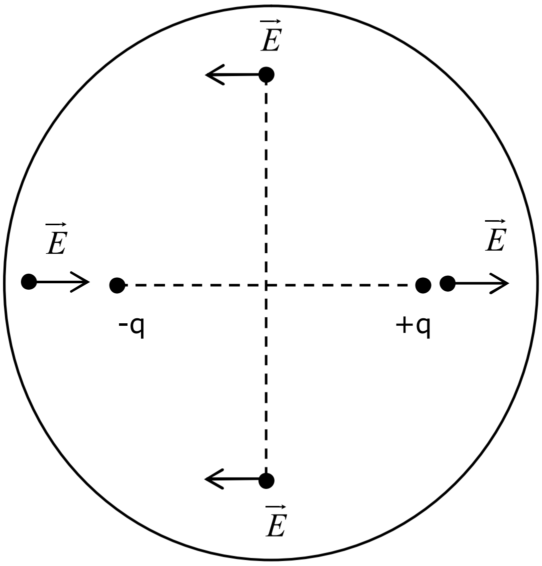
(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

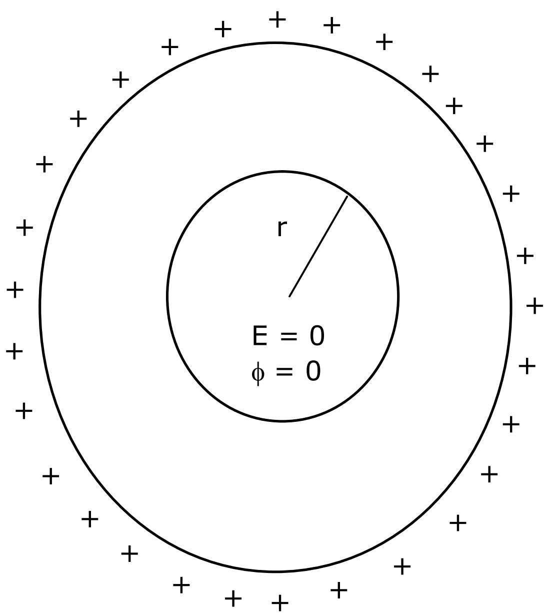
(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Sol. **(1)**



Statement – 1 $\rightarrow$ Correct



Statement – 2 $\rightarrow$ Incorrect

SECTION – B

1. If thehighest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are ________________________.

Sol. **(9)**

$$\text{No. of station} = \frac{\text{Band width}}{2 \times \text{Highest Band width}}$$

$$\Rightarrow \frac{90}{2 \times 5}$$

$\Rightarrow 9$

2. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of x is _________.

Sol. **(25)**

From thermodynamics law:

$\Delta Q = \Delta U + \Delta W$ .....(1)

$Q = nC_v\Delta T + \frac{Q}{5}$

$Q - \frac{Q}{5} = 1 \times \frac{5}{2}R \times \Delta T$

$Q = \frac{25}{8}R\Delta T$ ......(2) $\therefore Q = n\,c\,\Delta T$

$C = \frac{25}{8}R$ given $C = \frac{xR}{8}$

x = 25

3. A particle excutes S.H.M with amplitude 'a' and time period T. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}\,a}{2}$. The value of x is _______________

Sol. **(3)**

Fora particle excutes S.H.M

$V = \omega\sqrt{a^2 - x^2}$

Given $V = \frac{V_{max}}{2} \Rightarrow \frac{A\omega}{2}$

$\frac{A^2\omega^2}{4} = \omega^2 a^2 - \omega^2 x^2$

$x = \frac{\sqrt{3}}{2}a$

4. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is x : y. The value of x is ________.

Sol. **(1)**

Forphotoelectric effectk.E_{max} = E − ϕ

$E_1 = 2\phi$, $k_1 = \phi$

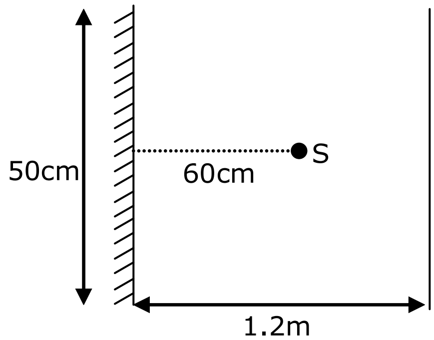
$E_2 = 10\phi$, $k_2 = 9\phi$

$\therefore V \propto \sqrt{k}$ $\left(k = \frac{1}{2}mv^2\right)$

$\frac{v_1}{v_2} = \sqrt{\frac{1}{9}} \Rightarrow \frac{1}{3} = \frac{x}{y}$

x = 1

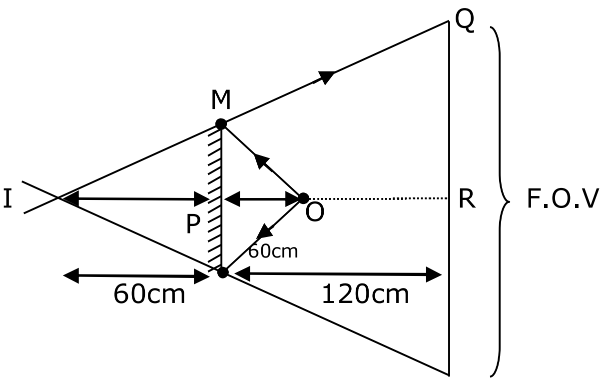
5. A point source of light S, placed at a distance 60 cm infront of the centre of plane mirror of width 50 cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is ______________cm



Sol. **(150)**

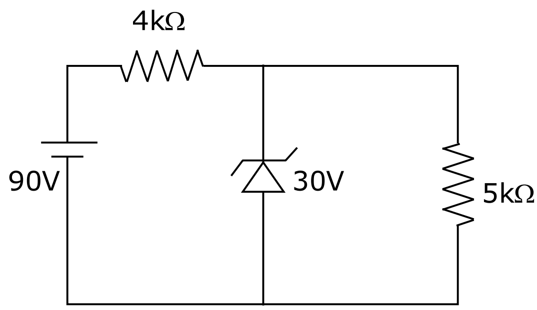
from similar triangle IMP and IQR

$\frac{QR}{25} = \frac{180}{60} \Rightarrow QR = 7$

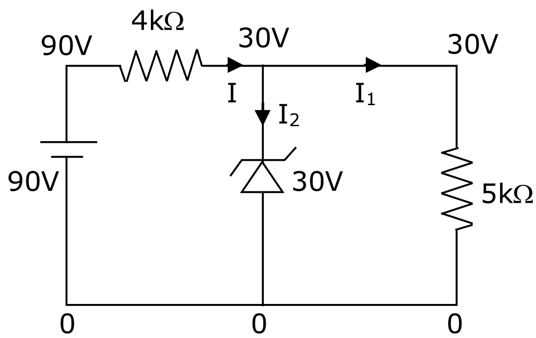


F.O.V. = 2 × 75 ⇒ 150 cm

6. The zener diode has a V_Z= 30 V. The current passing through the diode for the following ciruit is ________________mA.



Sol. **(9)**

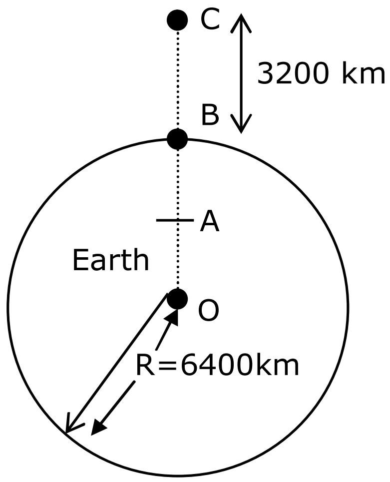


$$I = \frac{90-30}{4} = 15mA$$

$$I_I = \frac{30}{5K\Omega} = 6mA$$

$$I_2 = 15mA - 6mA = 9mA$$

7. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is ________________.



Sol. (4)

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GMr}{R^3}$$

$OA = \frac{4R}{9} = r$

$AB = R - \frac{4R}{9} = \frac{5R}{9}$

OA : AB

$\frac{4R}{9} : \frac{5R}{9} \Rightarrow 4:5 = x:y$

(x=4)

8. 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is ________________ times that of a smaller drop.

Sol. (243)

For self energy of sphere (conducting)

$$U = \frac{kq^2}{2r}$$

For small drop $\rightarrow U_i = \frac{kq^2}{2r}$ ....(1)

After combine small drops volume remains same as bigger drop

$$\therefore \ \frac{4}{3}\pi r^3 \times n = \frac{4}{3}\pi R^3$$

$R = (n)^{\frac{1}{3}} r$ ....(2)

For large drop $\rightarrow U_f = \frac{k(nq)^2}{2 \times 3R}$ ....(3)

From equation (1), (2), (3)

$$\frac{U_f}{U_i} = (n)^{5/3}$$

$\Rightarrow (27)^{5/3}$

$\Rightarrow 243$

9. The volume V of a given mass of monatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The work done when temperature changes by 90 K will be xR. The value of x is _________________.
[R =universal gas constant]

Sol. **(60)**
Given: $V = k\,T^{2/3}$
$V^{3/2} = (k)^{3/2}\,T$
$TV^{-3/2}$ = const. ......(1)
and $TV^{\gamma-1}$ = const. ......(2)
From (1) & (2)

$$-\frac{3}{2} = \gamma - 1$$

$$\gamma = -\frac{1}{2}$$

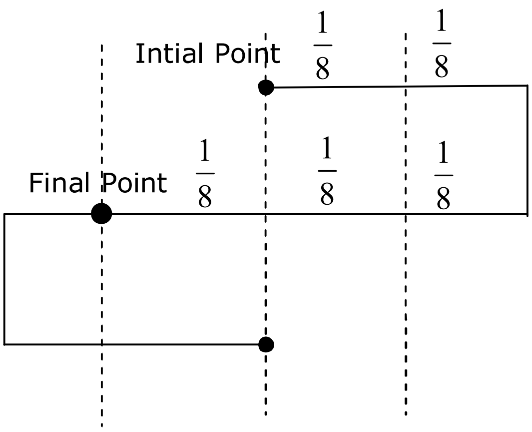
Work done (w) = $\dfrac{nR\Delta T}{\gamma - 1}$

$W = \dfrac{1 \times R \times 90}{-\frac{1}{2} - 1}$ $|W| = 60R$ x = 60

10. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}$T. The value of α is ___________.

Sol. **(7)**
For given $\left(\frac{5}{8}\right)$ oscillation, we can write it as $\rightarrow \left(\frac{1}{2} + \frac{1}{8}\right)$

And we know for half oscillations time $\rightarrow \frac{T}{2}$



For final point $\rightarrow \pi + \frac{\pi}{6}$ $\Rightarrow \frac{7\pi}{6}$

Time $\rightarrow \frac{7T}{12} \rightarrow$ given $\rightarrow \frac{\alpha}{\beta} T$ α = 7p

26^{th} Feb. 2021 | Shift - 2
CHEMISTRY

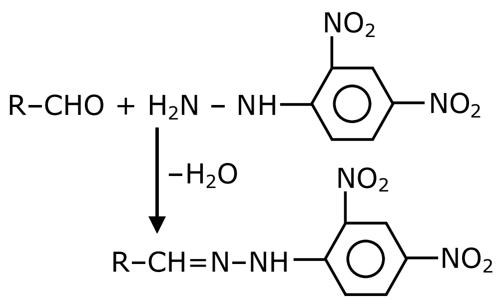
Section - A

1. 2,4-DNP test can be used to identify:

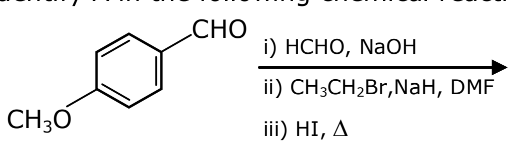
(1) aldehyde
(2) halogens
(3) ether
(4) amine

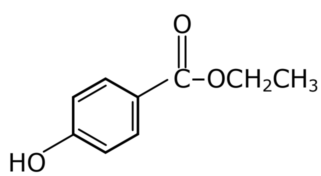
Ans. **(1)**

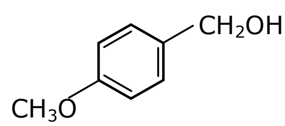
Sol.

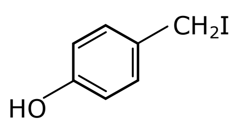


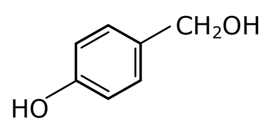
2. Identify A in the following chemical reaction.



(1) 

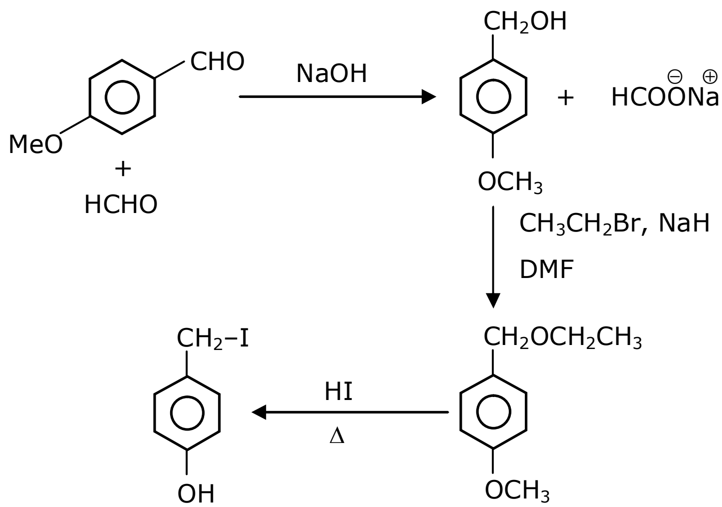
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



3. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is:

(1) positive

(2) neutral

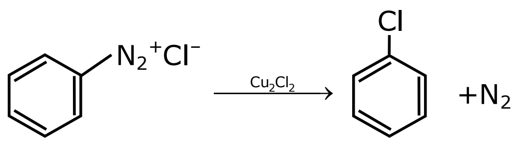
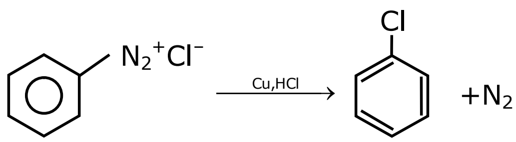
(3) sometimes positive and sometimes negative

(4) negative

Ans. **(1)**

Sol. If $FeCl_3$ is added to excess of hot water, a positively charged sol of hydrated ferric oxide is formed due to adsorption of Fe^{3+} ions.

4. Match **List-I** with **List-II**

List-I	List-II
(a) 	(i) Wurtz reaction
(b) 	(ii) Sandmeyer reaction
(c) $2CH_3CH_2Cl + 2Na \xrightarrow{\text{Ether}} C_2H_5-C_2H_5 + 2NaCl$	(iii) Fitting reaction
(d) $2C_2H_5Cl + 2Na \xrightarrow{\text{Ether}} C_6H_5-C_6H_5 + 2NaCl$	(iv) Gatterman reaction

Choose the correct answer from the option given below:

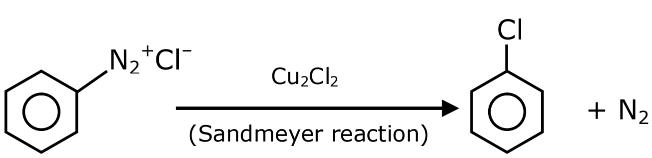
(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

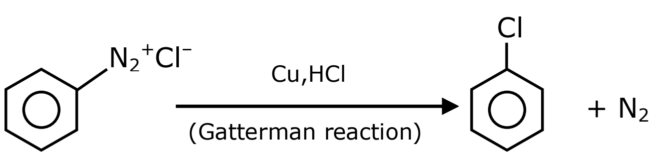
(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Ans. **(3)**

Sol. (a) 

(b) 

(c) $2CH_3-CH_2Cl + 2Na \xrightarrow[\text{(Wurtz reaction)}]{\text{Ether}} C_2H_5-C_2H_5+2NaCl$

(D) $2C_6H_5Cl + 2Na \xrightarrow[\text{(Fitting reaction)}]{\text{Ether}} C_6H_5-C_6H_5+2NaCl$

5. In $\overset{1}{C}H_2 = \overset{2}{C} = \overset{3}{C}H–\overset{4}{C}H_3$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively are:

(1) sp^2, sp, sp^2, sp^3

(2) sp^2, sp^2, sp^2, sp^3

(3) sp^2, sp^3, sp^2, sp^3

(4) sp^3, sp, sp^3, sp^3

Ans. **(1)**

Sol. $\underset{sp^2}{CH_2} = \underset{sp}{C} = \underset{sp^2}{CH} - \underset{sp^3}{CH_3}$

6. Match List-I with List-II.

List-I	**List-II**
(a) Sucrose	(i) β-D-Galactose and β-D-Glucose
(b) Lactose	(ii) α-D-Glucose and β-D-Fructose
(c) Maltose	(iii) α-D- Glucose and α-D-Glucose

Choose the correct answer from the options given below:

(1) (a)-(iii), (b)-(ii), (c)-(i)

(2) (a)-(iii), (b)-(i), (c)-(ii)

(3) (a)-(i), (b)-(iii), (c)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iii)

Ans. **(4)**

Sol. Sucrose $\rightarrow \alpha$-D- Glucose and β-D- Fructose

Lactose $\rightarrow \beta$-D- Galactose and β-D- Glucose

Maltose $\rightarrow \alpha$-D- Glucose and α-D- Glucose

7. Which pair of oxides is acidic in nature?

(1) N_2O, BaO

(2) CaO, SiO_2

(3) B_2O_3, CaO

(4) B_2O_3, SiO_2

Ans. **(4)**

Sol. B_2O_3 and SiO_2 both are oxides of non-metal and hence are acidic in nature.

8. Calgon is used for water treatment. Which of the following statement is NOT true about calgon?

(1) Calgon contains the 2^{nd} most abundant element by weight in the earth's crust.

(2) It is also known as Graham's salt.

(3) It is polymeric compound and is water soluble.

(4) It doesnot remove Ca^{2+} ion by precipitation.

Ans. **(1)**

Sol. $Na_6(PO_3)_6$ or $Na_6P_6O_{18}$

Order of abundance of element in earth crust is

O > Si > Al > Fe > Ca > Na > Mg > K

So second most abundant element in earth crust is Si not Ca.

9. Ceric ammonium nitrate and $CHCl_3$/alc. KOH are used for the identification of functional groups present in __________and_________respectively.

(1) alcohol, amine (2) amine, alcohol

(3) alcohol, phenol (4) amine, phenol

Ans. **(1)**

Sol. Alcohol give positive test with ceric ammonium nitrate and primary amines gives carbyl amine test with $CHCl_3$, KOH.

10. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: In TlI_3, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason R: Tl metals has fourteen *f* electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is not correct but R is correct

(3) Both A and R are correct R is NOT the correct explanation of A

(4) A is correct but R is not correct

Ans. **(3)**

Sol. $T\ell I_3$ is $T\ell^+ \ I_3^-$

CsI_3 is $Cs^+ \ I_3^-$

Thallium shows $T\ell^+$ state due to inert pair effect.

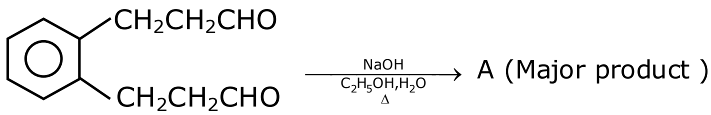
11. The correct order of electron gain enthalpy is:

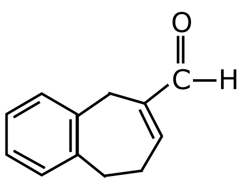
(1) S > Se > Te > O
(2) O > S > Se > Te
(3) S > O > Se > Te
(4) Te > Se > S > O

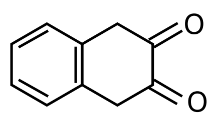
Ans. (1)

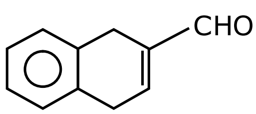
Sol. Electron gain enthalpy of O is very low due to small size.

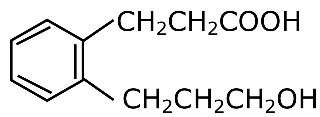
12. Identify A in the given chemical reaction.



(1) 

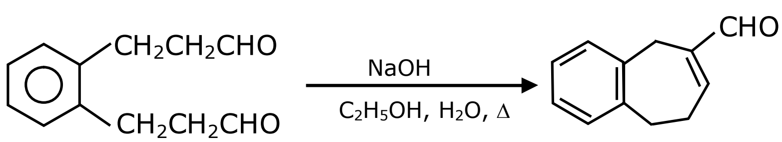
(2) 

(3) 

(4) 

Ans. (1)

Sol.



(Internal aldol condensation)

13. Match List-I with List-II

List-I	**List-II**
(a) Siderite	(i) Cu
(b) Calamine	(ii) Ca
(c) Malachite	(iii) Fe
(d) Cryolite	(iv) Al
	(v) Zn

Choose the correct answer from the options given below:

(1) (a)-(i), (b)-(ii), (c)-(v), (d)-(iii)
(2) (a)-(iii), (b)-(v), (c)-(i), (d)-(iv)
(3) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(4) (a)-(iii), (b)-(i), (c)-(v), (d)-(ii)

Ans. **(2)**

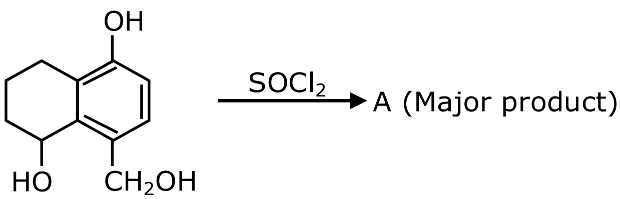
Sol. Siderite - $FeCO_3$

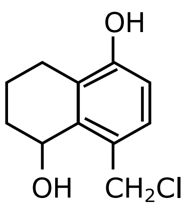
Calamine - $ZnCO_3$

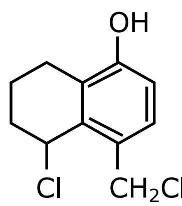
Malachite - $CuCO_3.Cu(OH)_2$

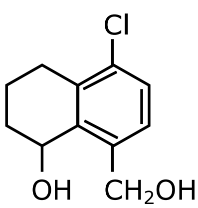
Cryolite - Na_3AlF_6

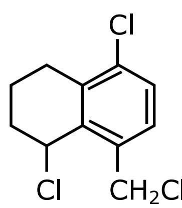
14. Identify A in the given reaction



(1) 

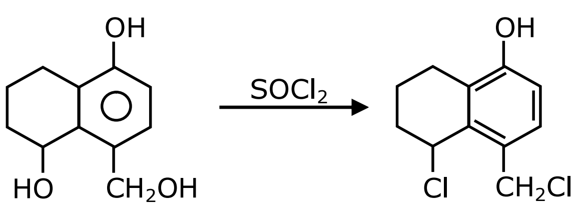
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



15. Match List-I with List-II.

List-I	**List-II**
(a) Sodium Carbonate	(i) Deacon
(b) Titanium	(ii) Caster-Kellner
(c) Chlorine	(iii) Van-Arkel
(d) Sodium hydroxide	(iv) Solvay

Choose the correct answer from the option given below:

(1) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(2) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(3) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(4) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

Ans. **(2)**

Sol. Sodium carbonate Na_2CO_3 & $NaHCO_3$

Titanium : Van arkel method

$$T_i + I_2 \xrightarrow{T1} T_i\underset{(g)}{I_4}$$

$$T_i\underset{(g)}{I_4} \xrightarrow[T_2 > T_1]{T_2} \underset{\text{Refined titanium}}{T_i} + 2\underset{(g)}{I_2}$$

Chlorine : Decon's process

$$HCl + O_2 \xrightarrow{CuCl_2} H_2O + Cl_2$$

Sodium hydroxide :- Caster-Kellner cell

16. Match List-I with List-II.

List-I (Molecule)	**List-II (Bond order)**
(a) Ne_2	(i) 1
(b) N_2	(ii) 2
(c) F_2	(iii) 0
(d) O_2	(iv) 3

Choose the correct answer from the options given below:

(1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii) (2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii) (4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Ans. **(1)**

Sol.

Ne_2O	BO = 0
N_2	BO = 3
F_2	BO = 1
O_2	BO = 2

As per molecular orbital theory

17. Which of the following forms of hydrogen emits low energy β^- particles?

(1) Proton H^+

(2) Deuterium ${}^2_1H$

(3) Protium ${}^1_1H$

(4) Tritium ${}^3_1H$

Ans. **(4)**

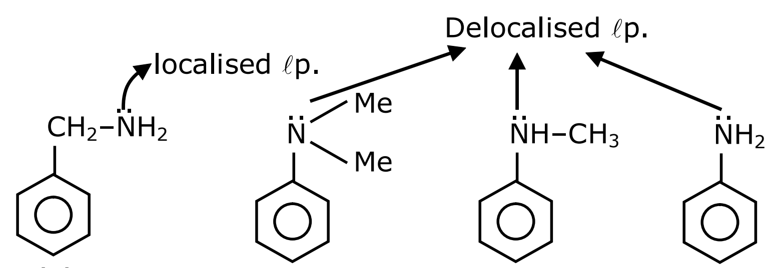
Sol. Tritium isotope of hydrogen is radioactive and emits low energy β^- particles. It is because of high n/p ratio of tritium which makes nucleus unstable.

18. A. Phenyl methanamine
B. N, N-Dimethylaniline
C. N-Methyl aniline
D. Benzenamine
Choose the correct order of basic nature of the above amines.

(1) D > C > B > A
(2) D > B > C > A
(3) A > C > B > D
(4) A > B > C > D

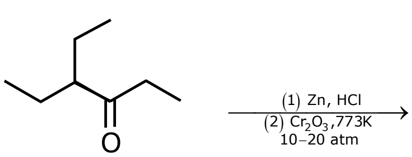
Ans. **(4)**

Sol.

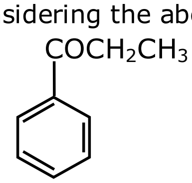


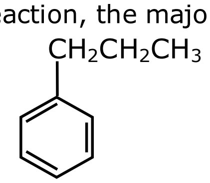
(a) (b) (c) (d)

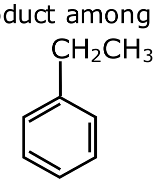
19.

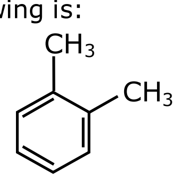


Considering the above reaction, the major product among the following is:

(1) 

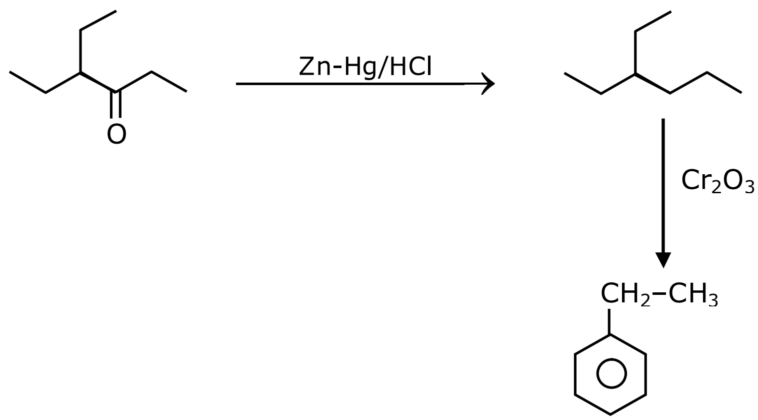
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



(Aromatisation)

20. Seliwanoff test and Xanthoproteic test are used for the identification of ___________ and ___________ respectively

(1) ketoses, proteins
(2) proteins, ketoses
(3) aldoses, ketoses
(4) ketoses, aldoses

Ans. **(1)**

Sol. Seliwanoff test and Xanthaproteic test are used for identification of 'Ketoses' and proteins respectively.

Section - B

1. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is_________g. (Rounded off to the nearest integer)
[Given: Atomic weight in g mol^{-1}. Na: 23; N: 14; O : 16]

Ans. **13**

Sol. Na^+ = 70 mg/mL

W_{Na^+} in 50mL solution $= 70 \times 50$mg

$= 3500$ mg

$= 3.5$ gm

Moles of Na^+ in 50 ml solution $= \frac{3.5}{23}$

Moles of $NaNO_3$ = moles of Na^+

$= \frac{3.5}{23}$ mol

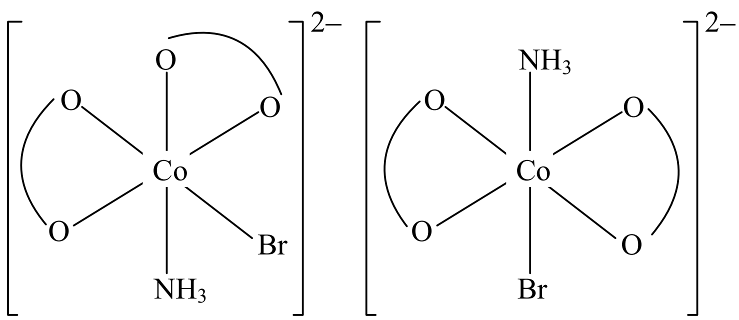
Mass of $NaNO_3 = \frac{3.5}{23} \times 85 = 12.934$

$\simeq$ 13gm Ans.

2. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is ____________[ox = oxalate]

Ans. 3

Sol. $\left[Co(ox)_2 Br(NH_3)\right]^{2-}$



Optically active   Optically inactive

Mirror image

Total stereoisomer = 2 (OI) + 1 POE (pair of enantiomers) = 3

3. The average S–F bond energy in kJ mol^{-1} of SF_6 is __________. (Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are - 1100, 275 and 80 kJ mol^{-1} respectively.]

Ans. **309**

Sol. $SF_6(g) \longrightarrow S(g) + 6F(g)$

$\Delta H^o_{reaction} = 6 \times E_{S-F} = \Delta H^o_f[S(g)] + 6 \times \Delta H^o_f[F(g)] - \Delta H^o_f[SF_6(g)]$

$6 \times E_{S-F} = 275 + 6 \times 80 - (-1100)$

$= 275 + 480 + 1100$

$6 \times E_{S-F} = 1855$

$E_{S-F} = \frac{1855}{6} = 309.1667$

$\simeq 309$ kJ/mol Ans.

4. Emf of the following cell at 298 K in V is x $\times 10^{-2}$.
$Zn|Zn^{2+}$ (0.1 M)||Ag^{+}(0.01 M)| Ag
The value of x is _________. (Rounded off to the nearest integer)
[Given: $E^0_{Zn^{2+}/Zn} = -0.76V; E^0_{Ag^+/Ag} = +0.80V; \dfrac{2.303RT}{F} = 0.059$]

Ans. **147**

Sol. $Zn(s)\left|Zn^{+2}(0.1M)\right|\left|Ag^{+}(0.01M)\right| Ag(s)$

$$Zn(s) + 2\,Ag^{+} \rightleftharpoons 2Ag(s) + Zn^{+2}$$

$$E^0 = 0.80 + 0.76 = 1.56 \;;\quad Q = \left\{\frac{Zn^{2+}}{(Ag^{+})^2}\right\}$$

$$E = E^0 - \frac{0.059}{n}\log(Q)$$

$$E = 1.56 - \frac{0.059}{2}\log\left[\frac{0.1}{(0.01)^2}\right]$$

$$E = 1.56 - \frac{0.059}{2}\log\left[(10)^3\right]$$

$$E = 1.4715 = 147.15 \times 10^{-2}\text{volt}$$

$= x \times 10^{-2}$

X = 147.15 $\simeq$ 147 Ans.

5. A ball weighing 10g is moving with a velocity of 90ms^{-1}. If the uncertainty in its velocity is 5%, then the uncertainty in its position is ________$\times 10^{-33}$m. (Rounded off to the nearest integer)
[Given : h = 6.63×10^{-34} Js]

Ans. **1**

Sol. m = 10 g = 10^{-2} Kg
v = 90 m/sec.

$$\Delta v = v \times 5\% \quad = 90 \times \frac{5}{100} = 4.5\,m/sec$$

$$m.\Delta v.\Delta x \geq \frac{h}{4\pi}$$

$$10^{-2} \times 4.5 \times \Delta x \geq \frac{6.63 \times 3 \times 10^{-34}}{4 \times \frac{22}{7}}$$

$$\Delta x \geq \frac{6.63 \times 7 \times 2 \times 10^{-34}}{9 \times 4 \times 22 \times 10^{-2}}$$

$$\Delta x \geq 1.17 \times 10^{-33} = x \times 10^{-33}$$

$$x = 1.17 \simeq 1$$

6. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is________.

Ans. **6**

Sol. $S_2O_3^{2-} + MnO_4^- \xrightarrow[\text{Medium}]{\text{Alkaline}} A$

$A \rightarrow SO_4^{-2}$

$\therefore$ Oxidation no. of 'S' = +6 Ans.

7. When 12.2 g of benzoic acid is dissolved in 100g of water, the freezing point of solution was found to be –0.93°C (K_f (H_2O) = 1.86 K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is_____________.

Ans. **2**

Sol. n PhCOOH $\rightarrow$ $(PhCOOH)_n$

$$N = \frac{1}{x} = i\{As \quad \alpha = 1\}$$

$$\Delta T_f = i \times k_f \times m$$

$$0.93 = \frac{1}{n} \times 1.86 \times \frac{12.2 \times 1000}{122 \times 100}$$

n = 2

8. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700K, having enough energy to react to form products is e^{-x}. The value of x is ______.

(Rounded off to the nearest integer)

[Use R = 8.31 JK^{-1} mol^{-1}]

Ans. **14**

Sol. $E_a = 80.9 kJ / mol$

Fraction of molecules able to cross energy barrier = $e^{-E_a/RT} = e^{-x}$

$$x = \frac{E_a}{RT} = \frac{80.9 \times 1000}{8.31 \times 700} = 13.91$$

$x \simeq 14$ Ans

9. The pH of ammonium phosphate solution, if pk_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is_____________.

Ans. **7**

Sol. $(NH_4)_3PO_4 \rightleftharpoons 3NH_4^+ + PO_4^{3-}$

$$\left[H^+\right] = K_a \times \sqrt{\frac{kw}{k_a \times k_b}}$$

$$pH = pk_a + \frac{1}{2}\left\{pk_w - pk_a - pk_b\right\}$$

$$pH = 5.23 + \frac{1}{2}\{14–5.23–4.75\}$$

$$pH = 5.23 + \frac{1}{2}\ (4.02) = 7.24 = 7 \text{(Nearest integer)}$$

10. The number of octahedral voids per lattice site in a lattice is __________.

(Rounded off to the nearest integer)

Ans. **1**

Sol. Assuming FCC

No of lactice sites = 6 face centre + 8 corner = 14

No. of octahedral voids = 13

Ratio $= \frac{13}{14} = 0.92857 = 1$ (Nearest integer)

26^{th} Feb. 2021 | Shift - 2
MATHEMATICS

1. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 142

(2) 68

(3) 136

(4) 102

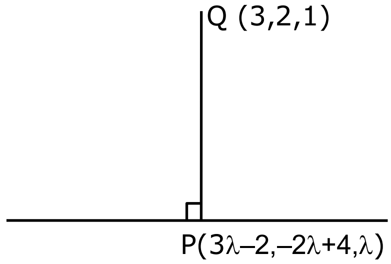
Ans. (4)

Sol. Dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$

Dr/s :- (3,–2, 1)

Points on the line (–2,4,0)

Equation of the line $\dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z}{1} = \lambda$



Dr's of PQ ; $3\lambda-5, -2\lambda+2. \lambda-1$

Dr's of y lines are (3, –2, 1)

Since PQ $\perp$ line

$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$

$\lambda = \dfrac{10}{7}$

$P\left(\dfrac{16}{7}, \dfrac{8}{7}, \dfrac{10}{7}\right)$

$21(\alpha + \beta + \gamma) = 21\left(\dfrac{34}{7}\right) = 102$

2. The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}$ e + $\frac{19}{8}$ e^{-1} – 10

(2) $-\frac{41}{8}$ e + $\frac{19}{8}$ e^{-1} – 10

(3) $\frac{41}{8}$ e – $\frac{19}{8}$ e^{-1} – 10

(4) $\frac{41}{8}$ e + $\frac{19}{8}$ e^{-1} + 10

Ans. (3)

Sol. $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$

Put 2n + 1 = r, where r = 3,5,7,...

$\Rightarrow n = \frac{r-1}{2}$

$$\frac{n^2-6n+10}{(2n+1)!}=\frac{\left(\frac{r-1}{2}\right)^2+3r-3+10}{r!}=\frac{r^2+10r+29}{4r!}$$

Now $\sum_{r=3,5,7}\frac{r(r-1)+11r+29}{4r!}=\frac{1}{4}\sum_{r=3,5,7,......}\left(\frac{1}{(r-2)!}+\frac{11}{(r-1)!}+\frac{29}{r!}\right)$

$= \frac{1}{4}\left\{\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+.....\right)+11\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+....\right)+29\left(\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+.....\right)\right\}$

$= \frac{1}{4}\left\{\frac{e-\frac{1}{e}}{2}+11\left(\frac{e+\frac{1}{e}-2}{2}\right)+29\left(\frac{e-\frac{1}{e}-2}{2}\right)\right\}$

$= \frac{1}{8}\left\{e-\frac{1}{e}+11e+\frac{11}{e}-22+29e-\frac{29}{e}-58\right\}$

$= \frac{1}{8}\left\{41e-\frac{19}{e}-80\right\}$

3. Let f(x) be a differentiable function at x = a with f′(a) = 2 and f(a) = 4. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals :

(1) 2a + 4

(2) 2a – 4

(3) $4 - 2a$
(4) $a + 4$

Ans. **(3)**

Sol. By L-H rule

$$L = \lim_{x \to a} \frac{f(a) - af'(x)}{1}$$

$\therefore L = 4 - 2a$

4. Let A (1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on :

(1) a parabola

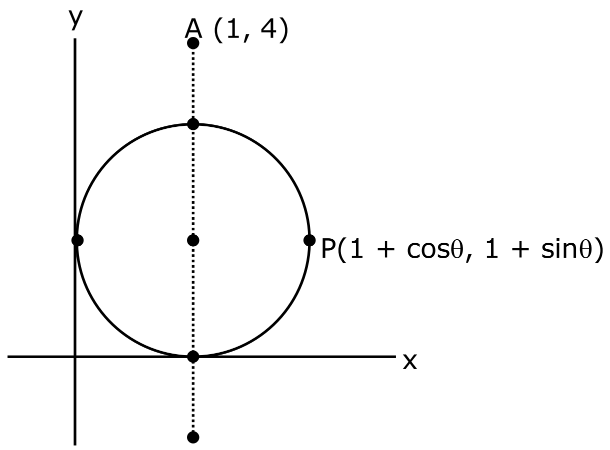
(2) a straight line

(3) a hyperbola

(4) an ellipse

Ans. **(2)**

Sol.



$\therefore PA^2 = \cos^2\theta + (\sin\theta - 3)^2 = 10 - 6\sin\theta$

$PB^2 = \cos^2\theta + (\sin\theta - 6)^2 = 37 - 12\sin\theta$

$PA^2 + PB^2 |_{max.} = 47 - 18\sin\theta|_{min.} \Rightarrow \theta = \frac{3\pi}{2}$

$\therefore$ P, A, B lie on a line x = 1

5. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of the radius r, then r is equal to :

(1) $\frac{1}{4}$

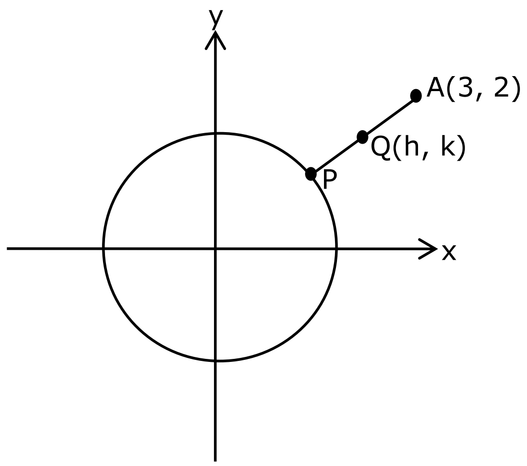
(2) $\frac{1}{2}$

(3)1

(4) $\frac{1}{3}$

Ans. (2)

Sol.



$\therefore$ P $\equiv$ (2h − 3, 2k − 2) $\rightarrow$on circle

$\therefore \left(h-\frac{3}{2}\right)^2+(k-1)^2=\frac{1}{4}$

$\Rightarrow$ radius = $\frac{1}{2}$

6. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = – 2, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $-\frac{18}{11}$

(2) $-\frac{18}{19}$

(3) $-\frac{4}{3}$

(4) $\frac{18}{35}$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{xy^2 + y}{x}$

$\Rightarrow \frac{xdy - ydx}{y^2} = xdx$

$\Rightarrow -d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right)$

$\Rightarrow \frac{-x}{y} = \frac{x^2}{2} + C$

Curve intersect the line x + 2y = 4 at x = – 2

So, –2 + 2y = 4 $\Rightarrow$ y = 3

So the curve passes through (–2, 3)

$\Rightarrow \frac{2}{3} = 2 + C$

$\Rightarrow C = \frac{-4}{3}$

$\therefore$ curve is $\frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$

It also passes through (3, y)

$\frac{-3}{y} = \frac{9}{2} - \frac{4}{3}$

$\Rightarrow \frac{-3}{y} = \frac{19}{6}$

$\Rightarrow y = -\frac{18}{19}$

7. Let A_1 be the area of the region bounded by the curves y = sinx, y = cos x and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sin x, y = cos x, x-axis and x= $\frac{\pi}{2}$ in the first quadrant. Then,

(1) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

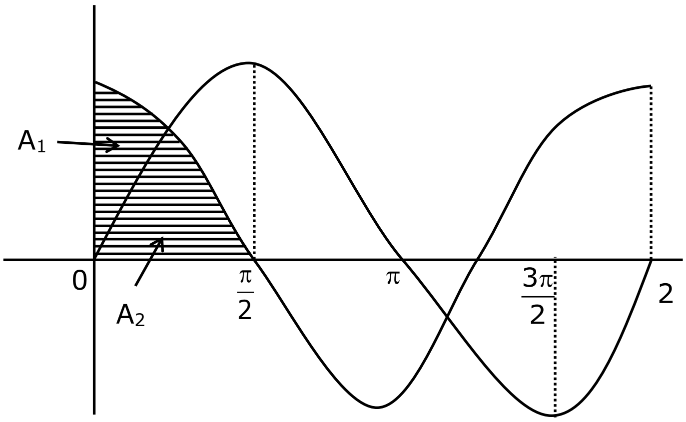
(2) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(4) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

Ans. **(4)**

Sol. $A_1 + A_2 = \int_0^{\pi/2} \cos x.dx = \sin x\Big|_0^{\pi/2} = 1$



$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\,dx = (\sin x + \cos x)\Big|_0^{\pi/4} = \sqrt{2} - 1$

$\therefore\ A_2 = 1 - \left(\sqrt{2} - 1\right) = 2 - \sqrt{2}$

$\therefore\ \frac{A_1}{A_2} = \frac{\sqrt{2} - 1}{\sqrt{2}\left(\sqrt{2} - 1\right)} = \frac{1}{\sqrt{2}}$

8. If 0 < a, b < 1, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + ..$ is :

(1) $\log_e 2$

(2) $\log_e \left(\frac{e}{2}\right)$

(3) e

(4) $e^2 - 1$

Ans. **(1)**

Sol. $\tan^{-1}\left(\frac{a + b}{1 - ab}\right) = \frac{\pi}{4} \Rightarrow a + b = 1 - ab \Rightarrow (1 + a)(1 + b) = 2$

Now, $(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) .... \infty$

$= \left(a - \frac{a^2}{2} + \frac{a^3}{3} ....\right) + \left(b - \frac{b^2}{2} + \frac{b^3}{3} ....\right)$

$\log_e (1 + a) + \log_e (1 + b) = \log_e (1 + a) (1 + b) = \log_e 2$

9. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then :

(1) F_1 is not a tautology but F_2 is a tautology

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 and F_2 both area tautologies

(4) Both F_1 and F_2 are not tautologies

Ans. (1)

Sol. Truth table for F_1

A	B	C	~A	~B	~C	$A \vee \sim B$	$A \vee B$	$\sim C \vee (A \vee B)$	$[\sim C \wedge (A \vee B)] \vee \sim A$	$(A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$
T	T	T	F	F	F	F	T	F	F	F
T	T	F	F	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	F	F	T
T	F	F	F	T	T	T	T	T	T	T
F	T	T	T	F	F	F	T	F	T	T
F	T	F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	F	T	T

Not a tautology

Truth table for F_2

A	B	$A \vee B$	$\sim A$	$B \rightarrow \sim A$	$(A \vee B) \vee (B \rightarrow \sim A)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

F_1 not shows tautology and F_2 shows tautology

10. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

Where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when $5a = 2b + c$

(2) has infinite number of solutions when $5a = 2b + c$

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

Ans. (2)

Sol. $D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$

$= 20 - 2(25) - 3(-10)$

$= 20 - 50 + 30 = 0$

$D_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$

$= 20a - 2(7b + 11c) - 3(-2b - 6c)$

$= 20a - 14b - 22c + 6b + 18c$

$= 20a - 8b - 4c$

$= 4(5a - 2b - c)$

$$D_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

$= 7b + 11c - a(25) - 3(2c - b)$

$= 7b + 11c - 25a - 6c + 3b$

$= -25a + 10b + 5c$

$= -5(5a - 2b - c)$

$$D_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

$= 6c + 2b - 2(2c - b) - 10a$

$= -10a + 4b + 2c$

$= -2(5a - 2b - c)$

for infinite solution

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow 5a = 2b + c$

11. A seven digit number is formed using digit 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$

(2) $\frac{4}{7}$

(3) $\frac{3}{7}$ (4) $\frac{1}{7}$

Ans. (3)

Sol. $n(s) = \frac{7!}{2!3!2!}$

$n(E) = \frac{6!}{2!2!2!}$

$P(E) = \frac{n(E)}{n(S)} = \frac{6!}{7!} \times \frac{2!3!2!}{2!2!2!}$

$\frac{1}{7} \times 3 = \frac{3}{7}$

12. If vectors $\vec{a_1} = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a_2} = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

(1) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

(2) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(4) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

Ans. (3)

Sol. $\frac{x}{1} = -\frac{1}{y} = \frac{1}{z} = \lambda(\text{let})$

Unit vector parallel to $x\hat{i} + y\hat{j} + z\hat{k} = \pm\frac{\left(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2 + \frac{2}{\lambda^2}}}$

For $\lambda = 1$, it is $\pm\frac{\left(\hat{i} - \hat{j} + \hat{k}\right)}{\sqrt{3}}$

13. For x>0,if $f(x) = \int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

(1) $\frac{1}{2}$

(2) −1

(3) 1

(4) 0

Ans. (1)

Sol. $f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\ell nt}{1+t}dt + \int_1^{1/e} \frac{\ell nt}{1+t}dt = I_1 + I_2$

$I_2 = \int_1^{1/e} \frac{\ell nt}{1+t}dt$ put $t = \frac{1}{z}, dt = -\frac{dz}{z^2}$

$= \int_1^e -\frac{\ell nz}{1+\frac{1}{z}} \times \left(-\frac{dz}{z^2}\right) = \int_1^e \frac{\ell nz}{z(z+1)}dz$

$$f(e)+f\left(\frac{1}{e}\right)=\int_1^e \frac{\ell nt}{1+t}dt+\int_1^e \frac{\ell nt}{t(t+1)}\ dt=\int_1^e \frac{\ell nt}{1+t}+\frac{\ell nt}{t(t+1)}dt$$

$$=\int_1^e \frac{\ell nt}{t}dt\ \{\ln t=u,\frac{1}{t}dt\}$$

$$=du=\int_0^1 u\ du=\left.\frac{u^2}{2}\right|_0^1=\frac{1}{2}$$

14. Let f : R $\rightarrow$ R be defined as $f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1 \\ |ax^2+x+b|, & \text{if } -1\le x\le 1 \\ \sin(\pi x) & \text{if } x>1 \end{cases}$

If f(x) is continuous on R, then a + b equals :

(1) 3

(2) −1

(3) −3

(4) 1

Ans. **(2)**

Sol. If f is continuous at x = −1, then

$f(-1^-)=f(-1)$

$\Rightarrow 2=|a-1+b|$

$\Rightarrow |a+b-1|=2$ ..... (i)

similarly

$f(1^-)=f(1)$

$\Rightarrow |a+b+1|=0$

$\Rightarrow a+b=-1$

15. Let A = {1,2,3……,10}and f: A$\rightarrow$ A be defined as $f(k)=\begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$ Then the number of possible functions g : A$\rightarrow$A such that gof = f is :

(1) 10^5

(2) ${}^{10}C_5$

(3) 5^5

(4) 5!

Ans. **(1)**

Sol. $g(f(x)) = f(x)$

$\Rightarrow g(x) = x$, when x is even.

5 elements in A can be mapped to any 10

So, $10^5 \times 1 = 10^5$

16. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :

(1) 11
(2) 6x
(3)12
(4) 6

Ans. **(3)**

Sol. $y + z = 5$ ...(1)

$$\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$$

$$\Rightarrow \frac{y+z}{yz} = \frac{5}{6}$$

$$\Rightarrow \frac{5}{yz} = \frac{5}{6}$$

$\Rightarrow yz = 6$

Also $(y - z)^2 = (y + z)^2 - 4yz$

$\Rightarrow (y - z)^2 = (y + z)^2 - 4yz$

$\Rightarrow (y - z)^2 = 25 - 4(6) = 1$

$\Rightarrow y - z = 1$ ...(2)

from (1) and (2), y = 3 and z = 2

for calculating odd divisor of $p = 2^x.3^y.5^z$

x must be zero

$P = 2^0.3^3.5^2$

$\therefore$ total odd divisors must be $(3 + 1)(2 + 1) = 12$

17. Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(2) $(-\infty, -1] \cup [2, \infty)$

(3) $(-\infty, -2] \cup [-1, \infty)$

(4) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

Ans. **(1)**

Sol. $g(2) = \lim_{x \to 2} \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \frac{3}{7}$

For domain of fog (x)

$\left|\frac{x^2 - x - 2}{2x^2 - x - 6}\right| \le 1 \Rightarrow (x + 1)^2 \le (2x + 3)^2 \Rightarrow 3x^2 + 10x + 8 \ge 0$

$\Rightarrow (3x + 4)(x + 2) \ge 0$

$x \in (-\infty, -2] \cup \left(-\frac{4}{3}, \infty\right]$

18. If the mirror image of the point (1,3,5) with respect to the plane $4x-5y+2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals:

(1) 47

(2) 39

(3) 43

(4) 41

Ans. **(1)**

Sol. Image of (1, 3, 5) in the plane $4x - 5y + 2z = 8$ is (α, β, γ)

$\Rightarrow \frac{\alpha - 1}{4} = \frac{\beta - 3}{-5} = \frac{\gamma - 5}{2} = -2 \frac{(4(1) - 5(3) + 2(5) - 8)}{4^2 + 5^2 + 2^2} = \frac{2}{5}$

$\therefore \alpha = 1 + 4\left(\frac{2}{5}\right) = \frac{13}{5}$

$\beta = 3 - 5\left(\frac{2}{5}\right) = 1 = \frac{5}{5}$

$\gamma = 5 + 2\left(\frac{2}{5}\right) = \frac{29}{5}$

Thus, $5(\alpha + \beta + \gamma) = 5\left(\frac{13}{5} + \frac{5}{5} + \frac{29}{5}\right) = 47$

19. Let $f(x)=\int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then $f(x)$equals.

(1) $2e^{(e^x-1)}-1$

(2) $e^{(e^x-1)}$

(3) $2e^{e^x}-1$

(4) $e^{e^x}-1$

Ans. (1)

Sol. Given, $f(x)=\int_0^x e^t f(t)dt + e^x$ ...(1)

Differentiating both sides w.r.t x

$f'(x) = e^x . f(x) + e^x$ (Using Newton Leibnitz Theorem)

$\Rightarrow \frac{f'(x)}{f(x)+1} = e^x$

Integrating w.r.t x

$\int \frac{f'(x)}{f(x)+1}dx = \int e^x dx$

$\Rightarrow \ell n (f(x) + 1) = e^x + c$

Put $x = 0$

$\ell n\ 2 = 1 + c$ ($\because f(0) = 1$, from equation (1))

$\therefore \ell n(f(x) + 1) = e^x + \ell n2 - 1$

$\Rightarrow f(x) + 1 = 2.\ e^{e^x-1}$

$\Rightarrow f(x) = 2e^{e^x-1} - 1$

20. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is:

(1) A right angle triangle having two of its sides of length 2r and r.

(2) An equilateral triangle of height $\frac{2r}{3}$.

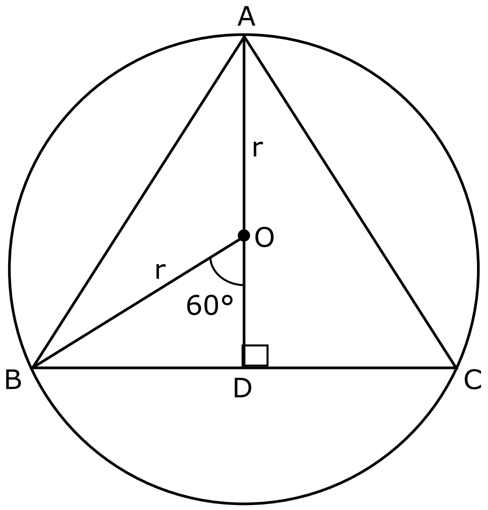
(3) An isosceles triangle with base equal to 2r.

(4) An equilateral triangle having each of its side of length $\sqrt{3}$ r.

Ans. (4)

Sol. Triangle of maximum area that can be inscribed in a circle is an equilateral triangle.

Let ΔABC be inscribed in circle,



Now, in ΔOBD

OD = r cos60° = $\frac{r}{2}$

Height = AD = $\frac{3r}{2}$

Again in ΔABD

Now sin 60° = $\frac{3\frac{r}{2}}{AB}$

$\Rightarrow$ AB = $\sqrt{3}$ r

Section - B

1. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is

Ans. 1000

Sol. Since, required number has G.C.D with 18 as 3. It must be odd multiple of '3' but not a multiple of '9'.

(i) Now, 4-digit number which are odd multiple of '3' are,
1005,1011,1017, ... 9999 $\rightarrow$ 1499

(ii) 4–digit number which are odd multiple of 9 are,
1017, 1035, ... 9999 $\rightarrow$ 499

$\because$ Required numbers = 1499–499 = 1000

2. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ and $P_{n+1} = 29$ for some integer n $\geq$ 1. Then, the value of P_n^2 is ______________.

Ans. 324

Sol. Given, $\alpha + \beta = 1$, $\alpha\beta = -1$

$\therefore$ Quadratic equation with roots α,β is $x^2 - x - 1 = 0$

$\Rightarrow \alpha^2 = \alpha + 1$

Multiplying both sides by α^{n-1}

$\alpha^{n+1} = \alpha^n + \alpha^{n-1}$ ________ (1)

Similarly,
$\beta^{n+1} = \beta^n + \beta^{n-1}$ ________ (2)
Adding (1) & (2)
$\alpha^{n+1} + \beta^{n+1} = (\alpha^n+\beta^n) + (\alpha^{n-1}+\beta^{n-1})$
$\Rightarrow P_{n+1} = P_n + P_{n-1}$
$\Rightarrow 29 = P_n + 11$ (Given, $P_{n+1} = 29$, $P_{n-1} = 11$)
$\Rightarrow P_n = 18$
$\therefore P_n^2 = 18^2 = 324$

3. Let X_1, X_2,.......... X_{18} be eighteen observation such that $\sum_{i=1}^{18}(X_i-\alpha)=36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ______________.

Ans. **4**

Sol. Given, $\sum_{i=1}^{18}(X_i-\alpha)=36$

$\Rightarrow \sum x_i - 18\alpha = 36$

$\Rightarrow \sum x_i - 18(\alpha+2)$ ...(1)

Also, $\sum_{i=1}^{18}(X_i-\beta)^2=90$

$\Rightarrow \sum x_i^2 + 18\beta^2 - 2\beta\sum x_i = 90$

$\Rightarrow \sum x_i^2 + 18\beta^2 + 2\beta\times 18(\alpha+2) = 90$ (using equation (1))

$\Rightarrow \sum x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$

$\sigma^2 = 1 \Rightarrow \frac{1}{18}\sum x_i^2 - \left(\frac{\sum x_i}{18}\right)^2 = 1$ ($\because \sigma = 1$, given)

$\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\alpha\beta + 72\beta) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\alpha\beta + 72\beta - 18(\alpha+2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (\alpha+2)^2 = 1$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4 - 4\alpha = 1$

$\Rightarrow \alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$

$\Rightarrow (\alpha-\beta)(\alpha-\beta+4) = 0$

$\Rightarrow \alpha - \beta = -4$

$\therefore |\alpha-\beta| = 4$ $(\alpha \neq \beta)$

4. In $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for m, n $\geq$ 1 and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n}$, $\alpha \in R$, then α equals______________ .

Ans. **1**

Sol. $I_{m,n}=\int_0^1 x^{m-1}.(1-x)^{n-1}\,dx$

Put $x=\frac{1}{y+1} \Rightarrow dx = \frac{-1}{(y+1)^2}dy$

$1-x=\frac{y}{y+1}$

$\therefore\ I_{m,n}=\int_{\infty}^{0}\frac{y^{n-1}}{(y+1)^{m+n}}(-1)\,dy=\int_0^{\infty}\frac{y^{n-1}}{(y+1)^{m+n}}\,dy$ ...(i)

Similarly $I_{m,n}=\int_0^1 x^{n-1}.(1-x)^{m-1}\,dx$

$\Rightarrow I_{m,n}=\int_0^{\infty}\frac{y^{m-1}}{(y+1)^{m+n}}dy$ ...(ii)

From (i) & (ii)

$2I_{m,n}=\int_0^{\infty}\frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy$

$\Rightarrow 2I_{m,n}=\underbrace{\int_0^1\frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy}_{I_1}+\underbrace{\int_1^{\infty}\frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy}_{I_2}$

Put $y=\frac{1}{z}$ in I_2

$dy=-\frac{1}{z^2}\,dz$

$\Rightarrow 2I_{m,n}=\int_0^1\frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy+\int_1^0\frac{z^{m-1}+z^{n-1}}{(z+1)^{m+n}}(-dz)$

$\Rightarrow I_{m,n}=\int_0^1\frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}dy \Rightarrow \alpha=1$

5. Let L be a common tangent line to the curves $4x^2+9y^2=36$ and $(2x)^2+(2y)^2=31$. Then the square of the slope of the line L is ______________.

Ans. **3**

Sol. $E:\frac{x^2}{9}+\frac{y^2}{4}=1$ $\quad C:x^2+y^2=\frac{31}{4}$

equation of tangent to ellipse is

$y=mx\pm\sqrt{9m^2+4}$ ...(i)

equation of tangent to circle is

$y = mx \pm \sqrt{\frac{31}{4}m^2 + \frac{31}{4}}$ ...(ii)

Comparing equation (i) & (ii)

$9m^2 + 4 = \frac{31}{4}m^2 + \frac{31}{4}$

$\Rightarrow 36m^2 + 16 = 31m^2 + 31$

$\Rightarrow 5m^2 = 15$

$\Rightarrow m^2 = 3$

6. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to______________.

Ans. **4**

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

.
.
.
.
.

$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}, A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{L.H.S} = A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$$

$$\text{R.H.S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \alpha+\beta = 0 \text{ and } 2^{20} + \alpha 2^{19} + 2\beta = 4$$

$\Rightarrow 2^{20} + \alpha\,(2^{19} - 2) = 4$

$\Rightarrow \alpha = \dfrac{4-2^{20}}{2^{19}-2} = -2$

$\Rightarrow \beta = 2$

$\therefore \beta - \alpha = 4$

7. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence −16, 8, −4, 2, ………… satisfy the equation $4x^2 - 9x+5 = 0$, then p+q is equal to ______________.

Ans. **10**

Sol. Given, $4x^2 - 9x + 5 = 0$

$\Rightarrow (x - 1)(4x - 5) = 0$

$\Rightarrow$ A.M = $\dfrac{5}{4}$, G.M = 1 (Q A.M > G.M)

Again, for the series

−16, 8, −4, 2 ......

p^{th} term $t_p = -16\left(\dfrac{-1}{2}\right)^{p-1}$

q^{th} term $t_p = -16\left(\dfrac{-1}{2}\right)^{q-1}$

Now, A.M = $\dfrac{t_p + t_q}{2} = \dfrac{5}{4}$ & $G.M = \sqrt{t_p t_q} = 1$

$\Rightarrow 16^2\left(-\dfrac{1}{2}\right)^{p+q-2} = 1$

$\Rightarrow (-2)^8 = (-2)^{(p+q-2)}$

$\Rightarrow p + q = 10$

8. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, -3) and $\left(4,-2\sqrt{2}\right)$, and given that $a-2\sqrt{2}$ b = 3, then (a^2+b^2+ab) is equal to______________.

Ans. **9**

Sol. Let the equation of normal is $Y - y = -\frac{1}{m}(X - x)$, where, $m = \frac{dy}{dx}$

As it passes through (a, b)

$b - y = -\frac{1}{m}(a - x) = -\frac{dx}{dy}(a - x)$

$\Rightarrow (b - y)dy = (x - a)dx$

by $-\frac{y^2}{2} = \frac{x^2}{2} - ax + c$ ...(i)

It passes through (3,-3) & $(4, -2\sqrt{2})$

$\therefore \quad -3b - \frac{9}{2} = \frac{9}{2} - 3a + c$

$\Rightarrow -6b - 9 = 9 - 6a + 2c$

$\Rightarrow 6a - 6b - 2c = 18$

$\Rightarrow 3a - 3b - c = 9$ ...(ii)

Also

$-2\sqrt{2}b - 4 = 8 - 4a + c$

$4a - 2\sqrt{2}\,b - c = 12$ ...(iii)

Also $a - 2\sqrt{2}\,b = 3$ ...(iv) (given)

(ii) − (iii) $\Rightarrow -a + \left(2\sqrt{2} - 3\right)b = -3$ ....(v)

(iv) + (v) $\Rightarrow b = 0, \quad a = 3$

$\therefore a^2 + b^2 + ab = 9$

9. Let z be those complex number which satisfy

$|z+5| \le 4$ and $z(1+i) + \overline{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ________________.

Ans. **48**

Sol. Given, $|z + 5| \le 4$

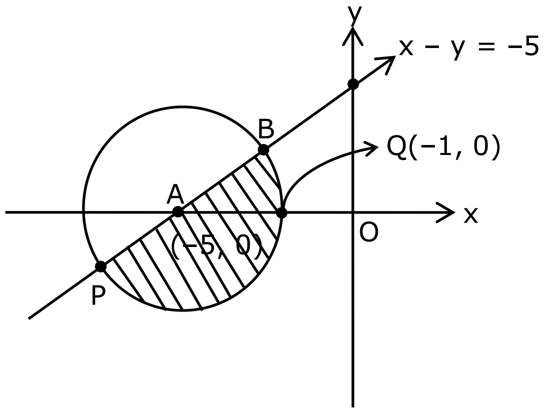
$\Rightarrow (x + 5)^2 + y^2 \le 16$ ...(1)

Also, $z(1+i) + \overline{z}(1-i) \ge -10$.

$\Rightarrow x - y \ge -5$ ...(2)

From (1) and (2)

Locus of z is the shaded region in the diagram.



$|z + 1|$ represents distance of 'z' from Q(−1, 0)

Clearly 'p' is the required position of 'z' when $|z + 1|$ is maximum.

$\therefore P \equiv \left(-5-2\sqrt{2}, -2\sqrt{2}\right)$

$\therefore (PQ)^2\Big|_{max} = 32 + 16\sqrt{2}$

$\Rightarrow \alpha = 32$

$\Rightarrow \beta = 16$

Thus, $\alpha + \beta = 48$

10. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4+10x^3+10x^2+10x+10$ lie in the interval $(a, a + 1)$.
Then, $|a|$ is equal to______________.

Ans. 2

Sol. Let, $f(x) = 2x^5+5x^4+10x^3+10x^2+10x+10$

$\Rightarrow f'(x) = 10\ (x^4 + 2x^3 + 3x^2 + 2x + 1)$

$$= 10\left(x^2 + \frac{1}{x^2} + 2\left(x + \frac{1}{x}\right) + 3\right)$$

$$= 10\left(\left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) + 1\right)$$

$$= 10\left(\left(x + \frac{1}{x}\right) + 1\right)^2 > 0; \forall x \in R$$

$\therefore$ f(x) is strictly increasing function. Since it is an odd degree polynomial it will have exactly one real root.

Now, by observation

$f(-1) = 3 > 0$

$f(-2) = -64 + 80 - 80 + 40 - 20 + 10$

$= -34 < 0$

$\Rightarrow$ f(x) has at least one root in $(-2,-1) \equiv (a, a + 1)$

$\Rightarrow a = -2$

$\Rightarrow |a| = 2$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS | TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n - 1)^{th}$ division of the main scale. The least count of the callipers in mm is :

(1) $\frac{10na}{(n-1)}$ (2) $\frac{10a}{(n-1)}$

(3) $\left(\frac{n-1}{10n}\right)a$ (4) $\frac{10a}{n}$

Official Ans. by NTA (4)

Sol. $(n - 1)a = n(a')$

$$a' = \frac{(n-1)a}{n}$$

$\therefore$ L.C. = 1 MSD – 1 VSD

$= (a - a')$cm

$= a - \frac{(n-1)a}{n}$

$= \frac{na - na + a}{n} = \frac{a}{n}$ cm

$= \left(\frac{10a}{n}\right)$ mm

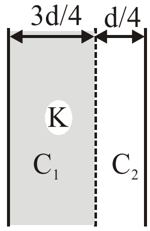
2. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}$d, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation :

(1) $C' = \frac{3+K}{4K}C_0$ (2) $C' = \frac{4+K}{3}C_0$

(3) $C' = \frac{4K}{K+3}C_0$ (4) $C' = \frac{4}{3+K}C_0$

Official Ans. by NTA (3)

Sol.



$$C_0 = \frac{\epsilon_0 A}{d}$$

C' = C_1 and C_2 in series.

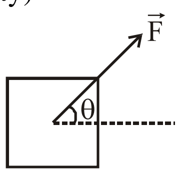
i.e. $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C'} = \frac{(3d/4)}{\epsilon_0 KA} + \frac{d/4}{\epsilon_0 A}$$

$$\frac{1}{C'} = \frac{d}{4\epsilon_0 A}\left(\frac{3+K}{K}\right)$$

$$C' = \frac{4KC_0}{(3+K)}$$

3. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_K. Then, the block's acceleration 'a' is given by : (g is acceleration due to gravity)



(1) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

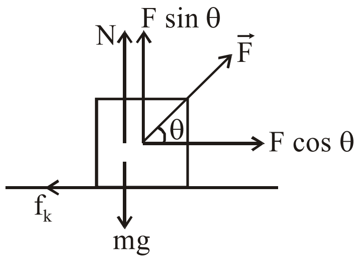
(2) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(3) $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$

(4) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

Official Ans. by NTA (2)

Sol.



$N = mg - f \sin \theta$

$F \cos \theta - \mu_k N = ma$

$F \cos \theta - \mu_k (mg - F \sin \theta) = ma$

$$a = \frac{F}{m}\cos\theta - \mu_K \left(g - \frac{F}{m}\sin\theta\right)$$

4. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be : (Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})

(1) $\frac{200}{3}\%$ (2) $\frac{200}{5}\%$

(3) $\frac{5}{200}\%$ (4) $\frac{3}{200}\%$

Official Ans. by NTA (1)

Sol. $P_1 = \rho g d + P_0 = 3 \times 10^5$ Pa

$\therefore \rho g d = 2 \times 10^5$ Pa

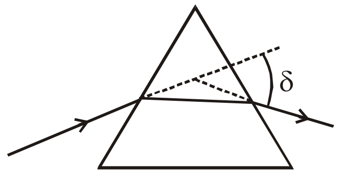
$P_2 = 2\rho g d + P_0$

$= 4 \times 10^5 + 10^5 = 5 \times 10^5$ Pa

$$\%\text{increase} = \frac{P_2 - P_1}{P_1} \times 100$$

$$= \frac{5\times10^5 - 3\times10^5}{3\times10^5}\times 100 = \frac{200}{3}\%$$

5. The angle of deviation through a prism is minimum when



(A) Incident ray and emergent ray are symmetric to the prism

(B) The refracted ray inside the prism becomes parallel to its base

(C) Angle of incidence is equal to that of the angle of emergence

(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below :

(1) Statements (A), (B) and (C) are true

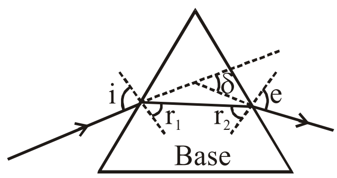
(2) Only statement (D) is true

(3) Only statements (A) and (B) are true

(4) Statements (B) and (C) are true

Official Ans. by NTA (1)

Sol. Deviation is minimum in a prism when :
$i = e$, $r_1 = r_2$ and ray (2) is parallel to base of prism.



6. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0 \times 10^{-8}\,\hat{z}T$. The value of electric field at this point is :
(speed of light = 3×10^8 ms^{-1})
$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z direction.

(1) $-24\hat{x}$ V/m (2) $2.6\hat{x}$ V/m

(3) $24\hat{x}$ V/m (4) $-2.6\hat{y}$ V/m

Official Ans. by NTA (1)

Sol. $f = 5 \times 10^8$ Hz

EM wave is travelling towards $+\hat{j}$

$\vec{B} = 8.0 \times 10^{-8}\hat{z}T$

$\vec{E} = \vec{B} \times \vec{C}$ = $(8 \times 10^{-8}\hat{z}) \times (3 \times 10^8\ \hat{y})$

$= -24\hat{x}$ V / m

7. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is :

(1) 1.5×10^3 m/s (2) 6.0×10^3 m/s

(3) 3.0×10^3 m/s (4) 4.5×10^3 m/s

Official Ans. by NTA (3)

Sol. By angular momentum conservation :

$mv_1r_1 = mv_2r_2$

$$v_1 = \frac{48 \times 10^{14}}{1.6 \times 10^{12}} = 3000 \text{ m/sec}$$

$= 3 \times 10^3$ m/sec.

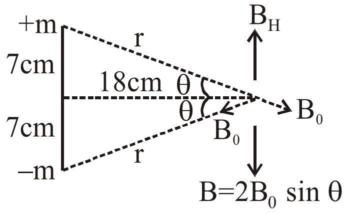
8. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is (1 G $= 10^{-4}$T)

(1) 2.880×10^3 J T^{-1} (2) 2.880×10^2 J T^{-1}

(3) 2.880 J T^{-1} (4) 28.80 J T^{-1}

Official Ans. by NTA (3)

Sol.



$$\text{i.e. } \frac{2\mu_0}{4\pi}\frac{m}{r^2} \times \frac{7}{r} = 0.4 \times 10^{-4}$$

$$\Rightarrow 2 \times 10^{-7} \times \frac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4$$

$= 0.4 \times 10^{-4}$

$$m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14}$$

$$M = m \times 14 \text{ cm} = m \times \frac{14}{100}$$

$$= \frac{0.04 \times (373)^{3/2}}{14} \times \frac{14}{100}$$

$= 4 \times 10^{-4} \times 7203.82 = 2.88$ J/T

9. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is :

(1) $\frac{88RT}{V}$ (2) $\frac{3RT}{V}$

(3) $\frac{5}{2}\frac{RT}{V}$ (4) $\frac{4RT}{V}$

Official Ans. by NTA (3)

Sol. $PV = (n_1 + n_2 + n_3)RT$

$$P \times V = \left[\frac{16}{32} + \frac{28}{28} + \frac{44}{44}\right]RT$$

$$PV = \left[\frac{1}{2} + 1 + 1\right]RT$$

$$P = \frac{5}{2}\frac{RT}{V}$$

10. In thermodynamics, heat and work are :

(1) Path functions

(2) Intensive thermodynamic state variables

(3) Extensive thermodynamic state variables

(4) Point functions

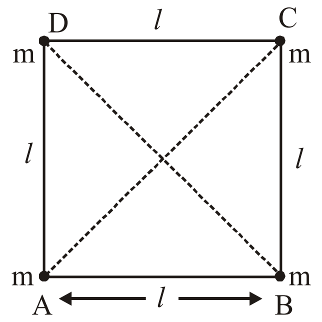
Official Ans. by NTA (1)

Sol. Heat and work are treated as path functions in thermodynamics.

$\Delta Q = \Delta U + \Delta W$

Since work done by gas depends on type of process i.e. path and ΔU depends just on initial and final states, so ΔQ i.e. heat, also has to depend on process is path.

11. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :



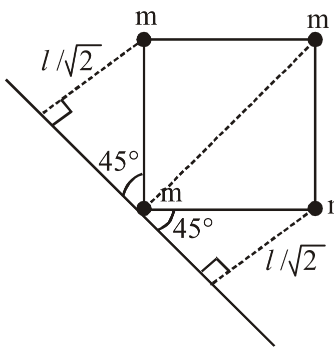
(1) ml^2 (2) $2\ ml^2$

(3) $3\ ml^2$ (4) $\sqrt{3}\,ml^2$

Official Ans. by NTA (3)

Sol. Moment of inertia of point mass

= mass × (Perpendicular distance from axis)^2



Moment of Inertia

$$= m(0)^2 + m(l\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$= 3\ ml^2$

12. A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

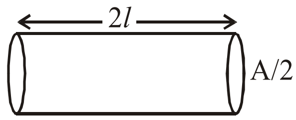
If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

(1) $\frac{1}{4}\frac{VA}{\rho l}$ (2) $\frac{3}{4}\frac{VA}{\rho l}$

(3) $\frac{1}{4}\frac{\rho l}{VA}$ (4) $4\frac{VA}{\rho l}$

Official Ans. by NTA (1)

Sol. As per the question



$$\text{Resistance} = \frac{\rho(2l)}{(A/2)} = \frac{4\rho l}{A}$$

$$\Rightarrow \text{Current} = \frac{V}{R} = \frac{VA}{4\rho l}$$

13. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be :

(1) $\sqrt{3}T$

(2) $\frac{T}{\sqrt{3}}$

(3) $\sqrt{\frac{3}{2}}T$

(4) $\sqrt{\frac{2}{3}}T$

Official Ans. by NTA (4)

Sol. When lift is stationary

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When lift is moving upwards

$\Rightarrow$ Pseudo force acts downwards

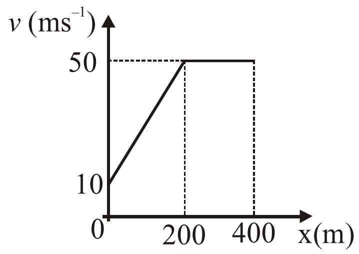
$$\Rightarrow g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$ New time period

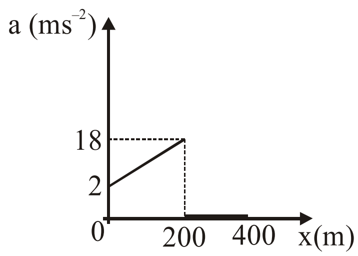
$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

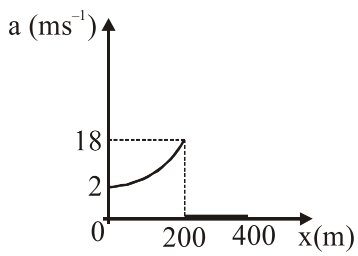
$$\boxed{T' = \sqrt{\frac{2}{3}}\,T}$$

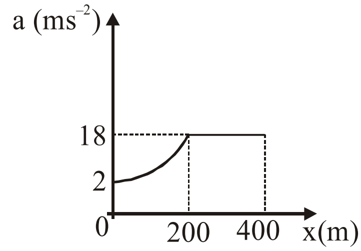
14. The velocity-displacement graph describing the motion of a bicycle is shown in the figure.

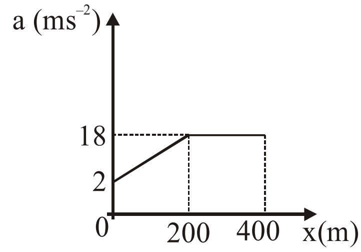


The acceleration-displacement graph of the bicycle's motion is best described by :

(1)


(2)


(3)


(4)


Official Ans. by NTA (1)

Official Ans. by ALLEN (1 or Bonus)

Sol. For $0 \le x \le 200$

$v = mx + C$

$$v = \frac{1}{5}x + 10$$

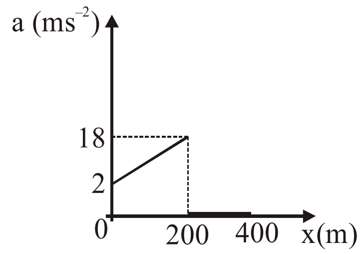
$$a = \frac{vdv}{dx} = \left(\frac{x}{5}+10\right)\left(\frac{1}{5}\right)$$

$$a = \frac{x}{25}+2 \Rightarrow \text{Straight line till } x = 200$$

for $x > 200$

v = constant

$\Rightarrow a = 0$

$\Rightarrow$


Hence most approriate option will be (1), otherwise it would be BONUS.

15. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :

(1) 300 (2) 400

(3) 200 (4) 100

Official Ans. by NTA (4)

Sol. Length of Antena = 25m = $\frac{\lambda}{4}$

$\Rightarrow \boxed{\lambda = 100\,m}$

16. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is :

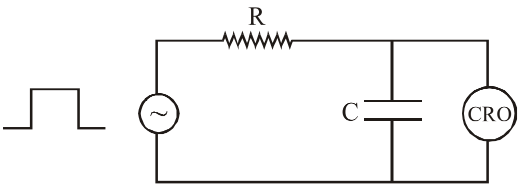
(1) $U_e = U_m$ (2) $U_e > U_m$

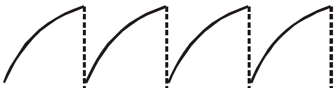
(3) $U_e < U_m$ (4) $U_e \neq U_m$

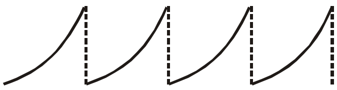
Official Ans. by NTA (1)

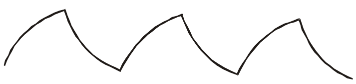
Sol. In EMW, Average energy density due to electric (U_e) and magnetic (U_m) fields is same.

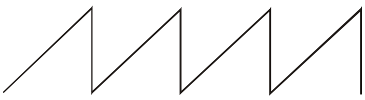
17. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :



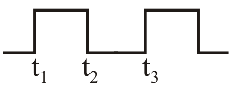
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

For $t_1 - t_2$ Charging graph

$t_2 - t_3$ Discharging graph

18. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation :

(1) Phase (2) Intensity

(3) Amplitude (4) Frequency

Official Ans. by NTA (4)

Sol. Stopping potential changes linearly with frequency of incident radiation.

19. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is :

(1) 0.0314 N

(2) 9.859×10^{-2} N

(3) 6.28×10^{-3} N

(4) 9.859×10^{-4} N

Official Ans. by NTA (4)

Sol. $N = m\omega^2 R$

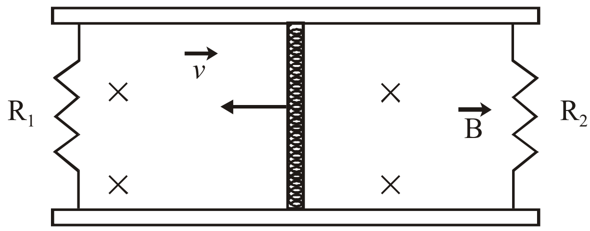
$$N = m\left[\frac{4\pi^2}{T^2}\right]R \qquad .....(1)$$

Given m = 0.2 kg, T = 40 S, R = 0.2 m

Put values in equation (1)

$N = 9.859 \times 10^{-4}$ N

20. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure



Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.

The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is :

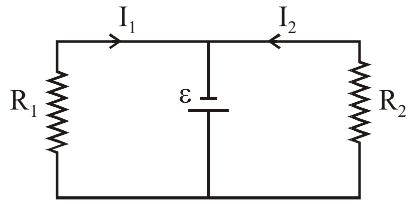
(1) Both I_1 and I_2 are in anticlockwise direction

(2) Both I_1 and I_2 are in clockwise direction

(3) I_1 is in clockwise direction and I_2 is in anticlockwise direction

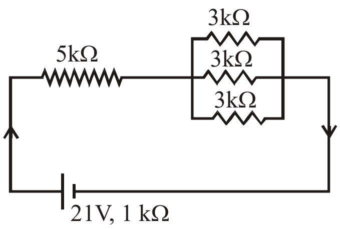
(4) I_1 is in anticlockwise direction and I_2 is in clockwise direction

Official Ans. by NTA (3)

Sol. 

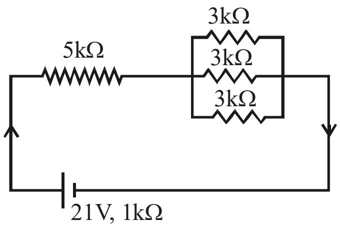
SECTION-B

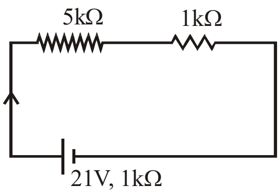
1. In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA.



The value of x to the nearest integer is _______.

Official Ans. by NTA (3)

Sol. 

= 

$$I = \frac{21}{5+1+1} = 3 \text{ mA}$$

2. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is ______.

Official Ans. by NTA (600)

Sol. $\beta = \dfrac{\lambda D}{d}$

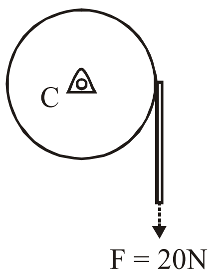
$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{6\times10^{-3}\times10^{-3}}{10}$$

$\lambda = 6 \times 10^{-7}$ m $= 600 \times 10^{-9}$ m

$\lambda = 600$ nm

3. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around its periphery as shown in the figure.



Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}. The value of n, to the nearest integer, is _______. [Given : In one complete revolution, the disk rotates by 6.28 rad]

Official Ans. by NTA (20)

Sol. $\alpha = \dfrac{\tau}{I} = \dfrac{F.R.}{mR^2/2} = \dfrac{2F}{mR}$

$$\alpha = \frac{2\times200}{20\times(0.2)} = 10 \text{ rad/s}^2$$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$(50)^2 = 0^2 + 2(10)\,\Delta\theta \Rightarrow \Delta\theta = \dfrac{2500}{20}$

$\Delta\theta = 125$ rad

No. of revolution $= \dfrac{125}{2\pi} \approx 20$ revolution

4. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' $\times 10^{-1}$. The value of x, to the nearest integer, is ________.

Official Ans. by NTA (15)

Sol. For 1st line

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = Rz^2\,\frac{5}{36} \quad .....(i)$$

For 3rd line

$$\frac{1}{\lambda_3} = Rz^2\left(\frac{1}{2^2}-\frac{1}{5^2}\right)$$

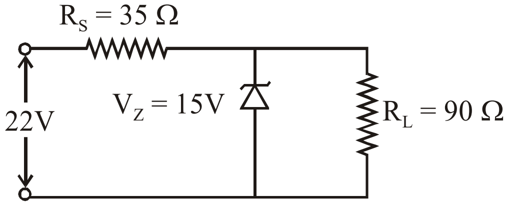
$$\frac{1}{\lambda_3} = Rz^2\,\frac{21}{100} \quad .....(ii)$$

(ii) + (i)

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100}\times\frac{36}{5} = 1.512 = 15.12\times10^{-1}$$

$x \approx 15$

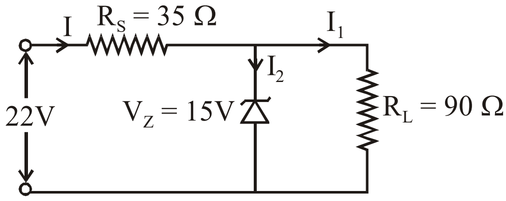
5. The value of power dissipated across the zener diode (V_z = 15 V) connected in the circuit as shown in the figure is x $\times 10^{-1}$ watt.



The value of x, to the nearest integer, is ______.

Official Ans. by NTA (5)

Sol.



Voltage across R_S = 22 – 15 = 7V

Current through $R_S = I = \frac{7}{35} = \frac{1}{5}$ A

Current through $90\Omega = I_2 = \frac{15}{90} = \frac{1}{6}$ A

Current through zener $= \frac{1}{5}-\frac{1}{6} = \frac{1}{30}$ A

Power through zener diode

P = VI

$P = 15\times\frac{1}{30} = 0.5$ watt

$P = 5\times10^{-1}$ watt

6. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which R = 8Ω, L = 24 mH and C = 60μF. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is ________.

Official Ans. by NTA (4)

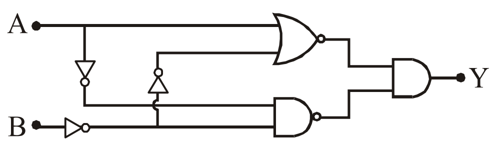
Sol. At resonance power (P)

$$P = \frac{(V_{rms})^2}{R}$$

$$P = \frac{(250/\sqrt{2})^2}{8} = 3906.25\ \text{W}$$

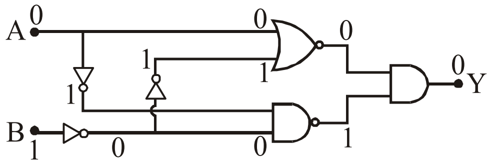
≈ 4 kW

7. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'. The value of x is _______.



Official Ans. by NTA (0)

Sol.



8. The resistance $R = \frac{V}{I}$, where V = (50 ± 2)V and I = (20 ± 0.2)A. The percentage error in R is 'x' %. The value of 'x' to the nearest integer is _________.

Official Ans. by NTA (5)

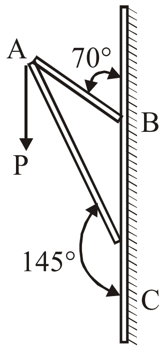
Sol. $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

% error in R = $\frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$

% error in R = 4 + 1

% error in R = 5%

9. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.

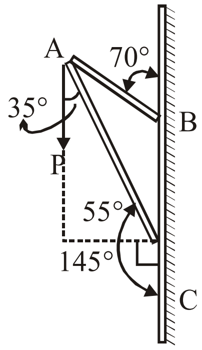


Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN. The value of x, to the nearest integer, is ________.

[Given : sin(35°) = 0.573, cos(35°) = 0.819
sin(110°) = 0.939, cos(110°) = –0.342]

Official Ans. by NTA (82)

Sol.



Component along AC

= 100 cos 35°N

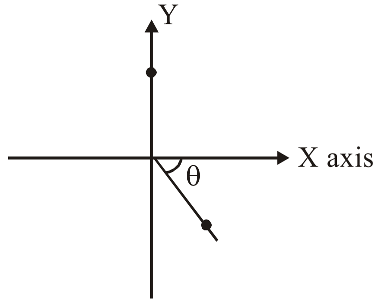
= 100 × 0.819 N

= 81.9 N

≈ 82 N

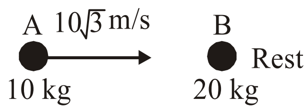
10. A ball of mass 10 kg moving with a velocity $10\sqrt{3}\,ms^{-1}$ along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is _______.

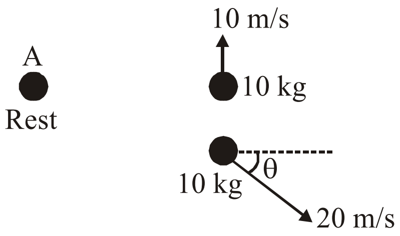


Official Ans. by NTA (30)

Sol. **Before Collision**



After Collision



From conservation of momentum along x axis;

$\vec{P}_i = \vec{P}_f$

$10 \times 10\sqrt{3} = 200 \cos\theta$

$\cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = 30^\circ$

SECTION-A

1. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason R : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true

(2) Both A and R are true but R is not the correct explanation of A

(3) Both A and R are true and R is the correct explanation of A

(4) A is true but R is false

Official Ans. by NTA (3)

Official Ans. by ALLEN (4)

Sol. Size of $_{97}Bk^{3+}$ ion is less than that of $_{93}Np^{3+}$ due to actinoid contraction.
As we know that in a period from left to right ionic radius decreases and in actinide series it is due to actinoid contraction.

2. Which among the following pairs of Vitamins is stored in our body relatively for longer duration ?

(1) Thiamine and Vitamin A

(2) Vitamin A and Vitamin D

(3) Thiamine and Ascorbic acid

(4) Ascorbic acid and Vitamin D

Official Ans. by NTA (2)

Sol. Vitamin-A & Vitamin-D

3. Given below are two statements :

Statement I : Both $CaCl_2.6H_2O$ and $MgCl_2.8H_2O$ undergo dehydration on heating.

Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but statement II is true

(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false

Official Ans. by NTA (2)

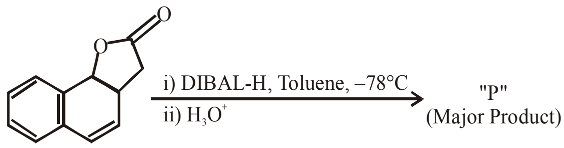
Sol. (a) $CaCl_2.6H_2O \xrightarrow{\Delta} \underset{\text{(Anhydrous)}}{CaCl_2} + 6H_2O$

(b) $MgCl_2.8H_2O \xrightarrow{\Delta} MgO + 2HCl + 6H_2O$

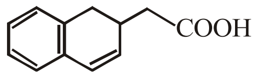
The dehydration of hydrated chloride of calcium can be achieved. The corresponding hydrated chloride of magnesium on heating suffer hydrolysis.

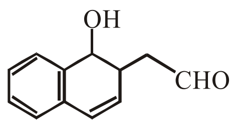
(c) BeO $\rightarrow$ Amphoteric

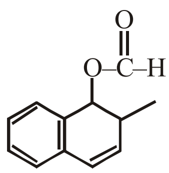
$$\left.\begin{array}{l} MgO \\ CaO \\ SrO \\ BaO \end{array}\right] \Rightarrow \text{All are basic oxide}$$

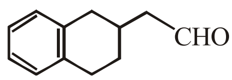
4. 

The product "P" in the above reaction is :

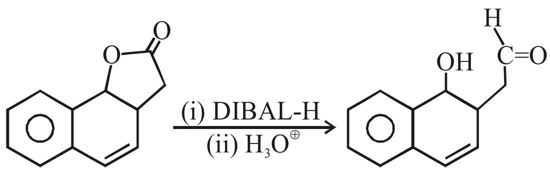
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

DIBAL can not reduce double bond
It can reduce cyclic ester.

5. Match List-I with List-II :

List-I	List-II
Industrial process	Application
(a) Haber's process	(i) HNO_3 synthesis
(b) Ostwald's process	(ii) Aluminium extraction
(c) Contact process	(iii) NH_3 synthesis
(d) Hall-Heroult process	(iv) H_2SO_4 synthesis

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

Official Ans. by NTA (3)

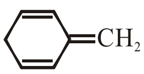
Sol. (a) Haber's process is used for NH_3 synthesis.

(b) Ostwald's process is used for HNO_3 synthesis.

(c) Contact process is used for H_2SO_4 synthesis.

(d) In Hall-Heroult process, electrolytic reduction of impure alumina can be done. (Aluminium extraction)

6. Among the following, the aromatic compounds are :

(A)  (B) 

(C)  (D) 

Choose the correct answer from the following options :

(1) (A) and (B) only

(2) (B) and (C) only

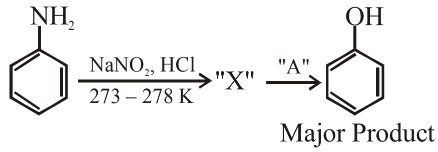
(3) (B), (C) and (D) only

(4) (A), (B) and (C) only

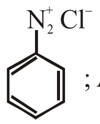
Official Ans. by NTA (2)

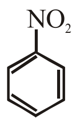
Sol. (A) Non-Aromatic (B) Aromatic

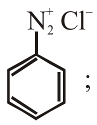
(C) Aromatic (D) Anti-Aromatic

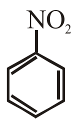
7. 

In the above chemical reaction, intermediate "X" and reagent/condition "A" are :

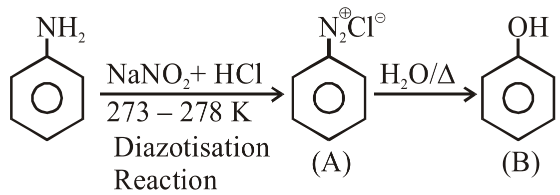
(1) X-  ; A- H_2O/NaOH

(2) X-  ; A- H_2O/Δ

(3) X-  ; A- H_2O/Δ

(4) X-  ; A- H_2O/NaOH

Official Ans. by NTA (3)

Sol. 

8. Given below are two statements :

Statement I : The E° value of Ce^{4+} / Ce^{3+} is + 1.74 V.

Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both statement I and statement II are correct

(2) Statement I is incorrect but statement II is correct

(3) Both statement I and statement II are incorrect

(4) Statement I is correct but statement II is incorrect

Official Ans. by NTA (4)

Sol. The E° value for Ce^{4+}/Ce^{3+} is +1.74 V because the most stable oxidation state of lanthanide series elements is +3.
It means Ce^{3+} is more stable than Ce^{4+}.

9. The functions of antihistamine are :

(1) Antiallergic and Analgesic

(2) Antacid and antiallergic

(3) Analgesic and antacid

(4) Antiallergic and antidepressant

Official Ans. by NTA (2)

10. Which of the following is Lindlar catalyst ?

(1) Zinc chloride and HCl

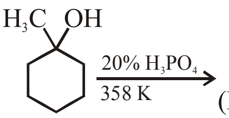
(2) Cold dilute solution of $KMnO_4$

(3) Sodium and Liquid NH_3

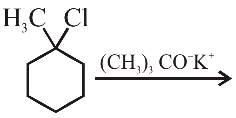
(4) Partially deactivated palladised charcoal

Official Ans. by NTA (4)

Sol. Partially deactivated palladised charcoal (H_2/pd/$CaCO_3$) is lindlar catalyst.

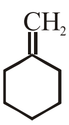
11. 

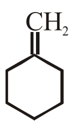
"A" (Major Product) ,

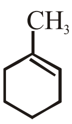


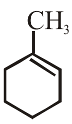
"B" (Major Product) ,

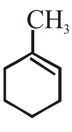
The product "A" and "B" formed in above reactions are :

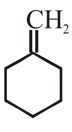
(1) A- 

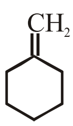
B- 

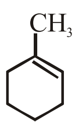
(2) A- 

B- 

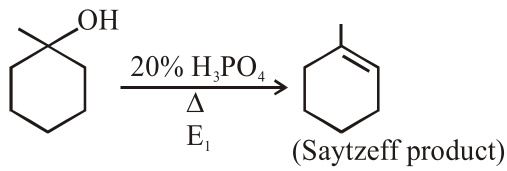
(3) A- 

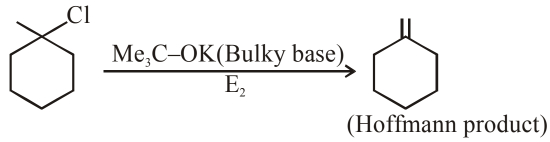
B- 

(4) A- 

B- 

Official Ans. by NTA (3)

Sol. 



12. Given below are two statements :
Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.
Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both statement I and statement II are false
(2) Both statement I and statement II are true
(3) Statement I is true but statement II is false
(4) Statement I is false but statement II is true
Official Ans. by NTA (2)

Sol. (a) H_2O_2 can acts as both oxidising and reducing agent in basic medium.
(i) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$
In this reaction, H_2O_2 acts as oxiding agent.
(ii) $2\overset{+7}{M}nO_4^- + 3H_2O_2 \rightarrow 2\overset{+4}{M}nO_2 + 3O_2 + 2H_2O + 2OH^-$
In this reaction, H_2O_2 acts as reducing agent.
(b) The basic principle of hydrogen economy is the transportation and storage of energy in the form of liquids or gaseous dihydrogen.
Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

13. The type of pollution that gets increased during the day time and in the presence of O_3 is :
(1) Reducing smog (2) Oxidising smog
(3) Global warming (4) Acid rain
Official Ans. by NTA (2)

Sol. In presence of ozone(O_3), oxidising smog gets increased during the day time because automobiles and factories produce main components of the photochemcial smog (oxidising smog) results from the action of sunlight on unsaturated hydrocarbon and nitrogen oxide.
Ozone is strong oxidising agent and can react with the unburnt hydrocarbons in the polluted air to produce chemicals.

14. Assertion A : Enol form of acetone $[CH_3COCH_3]$ exists in < 0.1% quantity. However, the enol form of acetyl acetone $[CH_3COCH_2OCCH_3]$ exists in approximately 15% quantity.

Reason R : enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement :

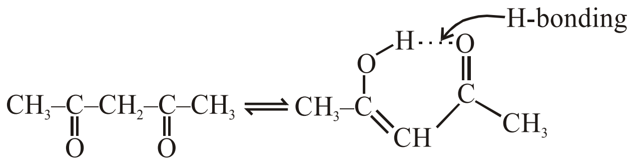
(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) Both A and R are true but R is not the correct explanation of A
(4) A is true but R is false

Official Ans. by NTA (2)

Sol.

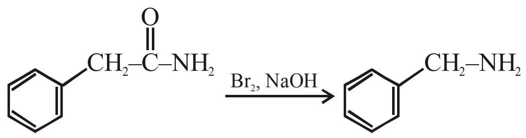
$$\underset{\text{(Keto form)}}{CH_3-\overset{\displaystyle O}{\overset{\|}{C}}-CH_3} \rightleftharpoons \underset{\text{(enol form)}}{CH_2=\overset{}{\underset{\displaystyle OH}{\underset{|}{C}}}-CH_3}$$

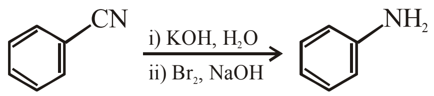
enol from of acetone is very less (< 0.1 %)

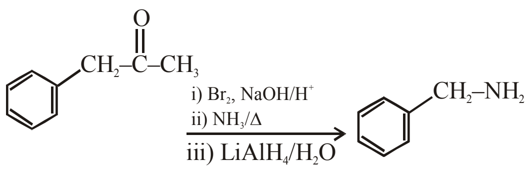


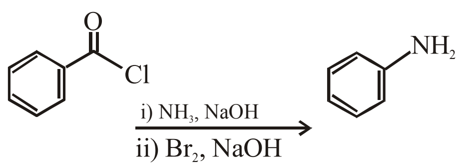
enol from (more than 50%)

15. Which of the following reaction DOES NOT involve Hoffmann Bromamide degradation ?

(1) 

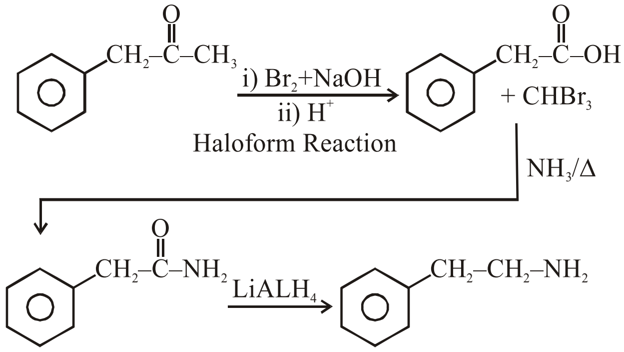
(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol.



⇒ This reaction does not involve haffmann bromanide degradation.

⇒ Rest all options involve haffmann bromamide degradation during the reaction of Br_2+NaOH with amide.

16. The process that involves the removal of sulphur from the ores is :

(1) Smelting
(2) Roasting
(3) Leaching
(4) Refining

Official Ans. by NTA (2)

Sol. In roasting process, metal sulphide (MS) ore are converted into metal oxide and sulphur is remove in the form of SO_2 gas.

$$2MS + 3O_2 \xrightarrow{\Delta} 2MO + 2SO_2\uparrow$$

17. Match List-I with List-II :

List-I		**List-II**
	Name of oxo acid	Oxidation state of 'P'
(a)	Hypophosphorous acid	(i) +5
(b)	Orthophosphoric acid	(ii) +4
(c)	Hypophosphoric acid	(iii) +3
(d)	Orthophosphorous acid	(iv) +2
		(v) +1

Choose the correct answer from the options given below :

(1) (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)
(2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
(3) (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)
(4) (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)

Official Ans. by NTA (1)

Sol. (a) Hypophosphorus acid : $H_3\underline{P}O_2$

$(+1)\ 3 + x + (-2)2 = 0$

$x = +1$

(b) Orthophosphoric acid : $H_3\underline{P}O_4$

$(+1)\ 3 + x + (-2)4 = 0$

$x = +5$

(c) Hypophosphoric acid : $H_4P_2O_6$

$(+1)\ 4 + 2x + (-2)6 = 0$

$x = +4$

(d) Orthophosphorous acid : $H_3\underline{P}O_3$

$(+1)3 + x + (-2)3 = 0$

$x = +3$

18. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The H–O–H bond angle in water molecule is 104.5°.

Reason R : The lone pair – lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

(1) A is false but R is true

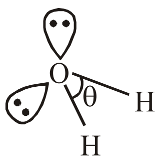
(2) Both A and R are true, but R is not the correct correct explanation of A

(3) A is true but R is false

(4) Both A and R are true, and R is the correct explanation of A

Official Ans. by NTA (4)

Sol. H_2O



$\theta = 104.5°$

the hybridisation of oxygen is water molecule is sp^3.

So electron geometry of water molecule is tetrahedral and the bond angle should be 109°28" but as we know that lone pair-lone pair repulsion of electrons is higher than the bond pair-bond pair repulsion because lone pair is occupied more space areound central atom than that of bond pair.

19. In chromotography technique, the purification of compound is independent of :

(1) Mobility or flow of solvent system

(2) Solubility of the compound

(3) Length of the column or TLC Plate

(4) Physical state of the pure compound

Official Ans. by NTA (4)

Sol. In chromotography technique, the purification of a compound is independent of the physical state of the pure compound.

20. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is :

(1) Sb (2) P (3) As (4) Bi

Official Ans. by NTA (4)

Sol. In group 15

$\left.\begin{matrix} N \\ P \end{matrix}\right] \rightarrow$ Non metal

$\left.\begin{matrix} As \\ Sb \end{matrix}\right] \rightarrow$ Metalloid

$\left.Bi\right] \rightarrow$ Metal

Hydrides of group 15 elements are

NH_3

PH_3

AsH_3

SbH_3

BiH_3

In NH_3, hydrogen atom gets partial positive charge due to less electronegativity.

But in BiH_3, hydrogen atom gets partial negative charge because hydrogen is more electronegative than bismuth.

i.e. BiH_3 is a strong reducing agent than others because we know that H^- is a strong reducing agent.

SECTION-B

1. For the reaction $A(g) \rightleftharpoons B(g)$ at 495 K, $\Delta_r G° = -9.478$ kJ mol$^{-1.}$

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B is the equilibrium mixture is _______ millimoles. (Round off to the Nearest Integer).

[$R = 8.314$ J mol^{-1} K^{-1}; $\ell n\ 10 = 2.303$]

Official Ans. by NTA (20)

Sol. $\Delta G° = -RT\ \ell n\ K_{eq}$

Given $\Delta G° = -9.478$ KJ/mole

$T = 495K \quad R = 8.314\ J\ mol^{-1}$

So $-9.478 \times 10^3 = -495 \times 8.314 \times \ell n\ K_{eq}$

$\ell n\ K_{eq} = 2.303$

$= \ell n\ 10$

So $K_{eq} = 10$

Now $A(g) \rightleftharpoons B(g)$

t = 0   22   0

t = t   22–x   x

$$K_{eq} = \frac{[B]}{[C]} = \frac{x}{22-x} = 10$$

or x = 20

So millmoles of B = 20

2. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and ________ respectively. (Round off to the Nearest Integer)

Official Ans. by NTA (3)

Sol. 44 gm CO_2 have 12 gm carbon

So, 420 gm $CO_2 \Rightarrow \frac{12}{44} \times 420$

$\Rightarrow \frac{1260}{11}$ gm carbon

$\Rightarrow$ 114.545 gram carbon

So, % of carbon $= \frac{114.545}{750} \times 100$

$\simeq 15.3\%$

18 gm $H_2O \Rightarrow$ 2 gm H_2

210 gm $\Rightarrow \frac{2}{18} \times 210$

= 23.33 gm H_2

So, % $H_2 \Rightarrow \frac{23.33}{750} \times 100 = 3.11\%$

$\approx 3\%$

3. $2\,MnO_4^- + b\ C_2O_4^{2-} + c\ H^+ \rightarrow x\ Mn^{2+} + y\ CO_2 + z\ H_2O$

If the above equation is balanced with integer coefficients, the value of c is _______. (Round off to the Nearest Integer).

Official Ans. by NTA (16)

Sol. Writting the half reaction

oxidation half reaction

$MnO_4^- \rightarrow Mn^{2+}$

balancing oxygen

$MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing Hydrogen

$8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing charge

$5e^- + 8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

Reduction half

$C_2O_4^{2-} \rightarrow CO_2$

Balancing carbon

$C_2O_4^{2-} \rightarrow 2CO_2$

Balancing charge

$C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-$

Net equation

$$16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$$

So c = 16

4. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _______°C. (Round off to the Nearest Integer).

[Given : Molal elevation constant of water $K_b = 0.5$ K kg mol^{-1} boiling point of pure water = 100°C]

Official Ans. by NTA (106)

Sol. $AB_2 \rightarrow A^{2+} + 2B^-$

t = 0   a   0   0

t = t   a –aα   aα   2aα

$n_T = a - a\alpha + a\alpha + 2a\alpha$

$= a(1 + 2\alpha)$

so $i = 1 + 2\alpha$

Now $\Delta T_b = i \times m \times K_b$

$\Delta T_b = (1 + 2\alpha) \times m \times K_b$

$\alpha = 0.1 \quad m = 10 \quad K_b = 0.5$

$\Delta T_b = 1.2 \times 10 \times 0.5$

$= 6$

So boiling point = 106

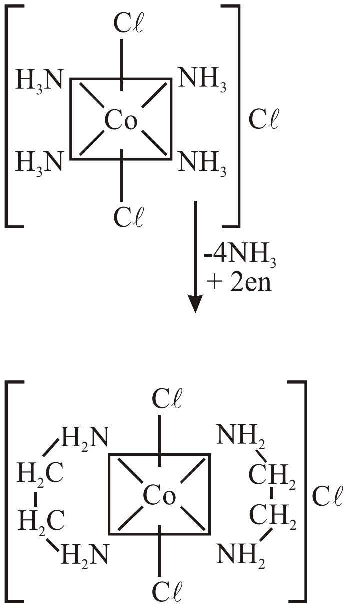
5. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3.4NH_3$ is _______. (Round off to the Nearest Integer).

Official Ans. by NTA (2)

Sol. trans - $CoCl_3.4NH_3$

or

trans-$[Co(NH_3)_4Cl_2]C\ell$



As we know that ethylene diamine is a bidentate ligand and ammonia is a mono dentate ligand.

It means overall two ethylene diamine is required to replace the all neutral ligands (four ammonia) from the coordination sphere of this complex.

6. A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm^{-3}. The molarity of the solution is ________ mol dm^{-3}. (Round off to the Nearest Integer).

[Atomic masses: K :39.0 u; O :16.0 u; H :1.0 u]

Official Ans. by NTA (9)

Sol. 6.5 molal KOH = 1000gm solvent has 6.5 moles KOH

so wt of solute = 6.5 × 56

= 364 gm

wt of solution = 1000 + 364 = 1364

Volume of solution = $\frac{1364}{1.89} m\ell$

$$\text{Molarity} = \frac{\text{mole of solute}}{V_{solution} \text{ in Litre}}$$

$$= \frac{6.5 \times 1.89 \times 1000}{1364}$$

= 9.00

7. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is _______ Å. (Round off to the Nearest Integer).

[Use : $\sqrt{3}$ = 1.73, h = 6.63 × 10^{-34} Js

m_e = 9.1 × 10^{-31} kg ; c = 3.0 × 10^8 ms^{-1} ; 1eV = 1.6 × 10^{-19}J]

Official Ans. by NTA (9)

Sol. Energy incident = $\frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{248 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{6.63 \times 3 \times 100}{248 \times 1.6}$$

= 0.05 eV × 100 = 5 eV

Now using

E = ϕ + K.E.

5 = 3 + K.E.

K.E. = 2eV = 3.2 × 10^{-19} J

for debroglie wavelength $\lambda = \frac{h}{mv}$

$$K.E = \frac{1}{2} mv^2$$

so $v = \sqrt{\frac{2KE}{m}}$

hence $\lambda = \frac{h}{\sqrt{2KE \times m}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$

$$= \frac{6.63}{7.6} \times \frac{10^{-34}}{10^{-25}} = \frac{66.3 \times 10^{-10} \text{ m}}{7.6}$$

= 8.72 × 10^{-10} m

$\approx 9 \times 10^{-10}$ m

$= 9$Å

8. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e. $\frac{S(A_2X)}{S(MX)}$ = _______.

(Round off to the Nearest Integer).

Official Ans. by NTA (50)

Sol. For A_2X

$A_2X \rightarrow 2A^+ + X^{2-}$

$\quad 2S_1 \quad S_1$

$K_{sp} = 4S_1^3 = 4 \times 10^{-12}$

$S_1 = 10^{-4}$

for MX

$MX \rightarrow M^+ + X^-$

$\quad S_2 \quad S_2$

$K_{sp} = S_2^2 = 4 \times 10^{-12}$

$S_2 = 2 \times 10^{-6}$

so $\frac{S_{A_2X}}{S_{MX}} = \frac{10^{-4}}{2 \times 10^{-6}} = 50$

9. A certain element crystallises in a bcc lattice of unit cell edge length 27 Å. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in Å will be ________. (Round off to the Nearest Integer).

[Assume each lattice point has a single atom]

[Assume $\sqrt{3}$ = 1.73, $\sqrt{2}$ = 1.41]

Official Ans. by NTA (33)

Sol. For BCC $\sqrt{3}\,a = 4r$

so $\quad r = \frac{\sqrt{3}}{4} \times 27$

for FCC $\quad a = 2\sqrt{2}\,r$

$$= 2 \times \sqrt{2} \times \frac{\sqrt{3}}{4} \times 27$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \times 27$$

$$= 33$$

10. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is $1.0 \times 10^{-3}\ s^{-1}$ and the activation energy $E_a = 11.488\ kJ\ mol^{-1}$, the rate constant at 200 K is _______ $\times 10^{-5}\ s^{-1}$. (Round of to the Nearest Integer).

(Given : $R = 8.314\ J\ mol^{-1}\ K^{-1}$)

Official Ans. by NTA (10)

Sol. $K_{300} = 10^{-4} \qquad K_{200} = ?$

$E_a = 11.488$ KJ/mole $\quad R = 8.314$ J/mole-K

so $\ell n\left(\frac{K_{300}}{K_{200}}\right) = \frac{E_a}{R}\left(\frac{1}{200} - \frac{1}{300}\right)$

$\ell n\left(\frac{K_{300}}{K_{200}}\right) = \frac{11.488 \times 1000 \times 100}{8.314 \times 200 \times 300}$

$= 2.303$

$= \ell n 10$

so $\frac{K_{300}}{K_{200}} = 10$

$K_{200} = \frac{1}{10} \times K_{300} = 10^{-4}$

$= 10 \times 10^{-5}\ sec^{-1}$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3)\,|x + 4| = 6\}$ is equal to

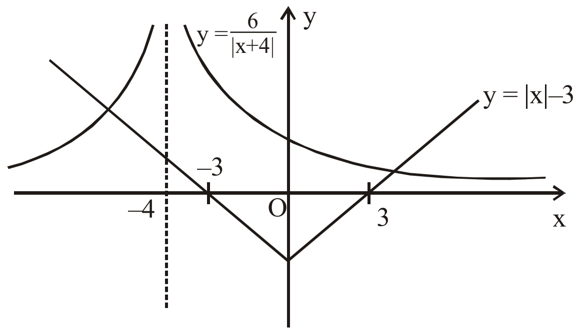
(1) 3 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (2)

Sol. $x \neq -4$

$(|x| - 3)(|x + 4|) = 6$

$\Rightarrow \quad |x| - 3 = \dfrac{6}{|x+4|}$



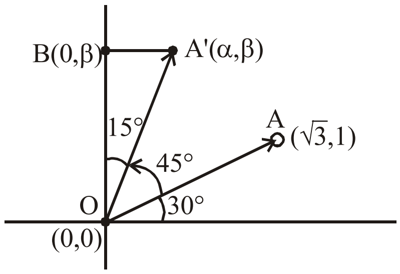
No. of solutions = 2

2. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to

(1) $\dfrac{1}{2}$ (2) 1 (3) $\dfrac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

Official Ans. by NTA (1)

Sol.



Area of $\Delta(OA'B) = \dfrac{1}{2}OA'\cos 15° \times OA'\sin 15°$

$= \dfrac{1}{2}(OA')^2 \dfrac{\sin 30°}{2}$

$= (3+1)\times\dfrac{1}{8} = \dfrac{1}{2}$

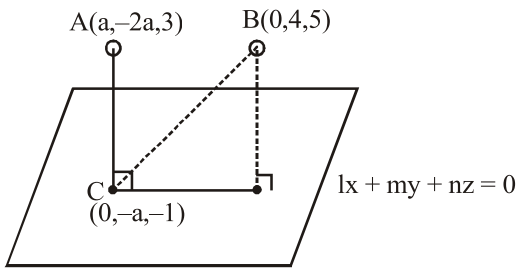
3. If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{31}$ (2) $\sqrt{41}$

(3) $\sqrt{55}$ (4) $\sqrt{66}$

Official Ans. by NTA (4)

Sol.



C lies on plane $\Rightarrow -ma - n = 0 \Rightarrow \dfrac{m}{n} = -\dfrac{1}{a}$ ....(1)

$\overrightarrow{CA} \parallel l\hat{i}+m\hat{j}+n\hat{k}$

$\dfrac{a-0}{l} = \dfrac{-a}{m} = \dfrac{4}{n} \Rightarrow \dfrac{m}{n} = -\dfrac{a}{4}$ ....(2)

From (1) & (2)

$-\dfrac{1}{a} = \dfrac{-a}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2$ (since $a > 0$)

From (2) $\dfrac{m}{n} = \dfrac{-1}{2}$

Let $m = -t \Rightarrow n = 2t$

$\dfrac{2}{l} = \dfrac{-2}{-t} \Rightarrow l = t$

So plane : $t(x - y + 2z) = 0$

$BD = \dfrac{6}{\sqrt{6}} = \sqrt{6}$ $\quad C \cong (0, -2, -1)$

$CD = \sqrt{BC^2 - BD^2}$

$= \sqrt{(0^2+6^2+6^2)-\left(\sqrt{6}\right)^2}$

$= \sqrt{66}$

4. Consider three observations a, b and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true ?

(1) $b^2 = 3(a^2 + c^2) + 9d^2$

(2) $b^2 = a^2 + c^2 + 3d^2$

(3) $b^2 = 3(a^2 + c^2 + d^2)$

(4) $b^2 = 3(a^2 + c^2) - 9d^2$

Official Ans. by NTA (4)

Sol. For a, b, c

mean $= \dfrac{a+b+c}{3} (= \bar{x})$

$b = a + c$

$\Rightarrow \quad \bar{x} = \dfrac{2b}{3}$ ......(1)

S.D. (a + 2, b + 2, c + 2) = S.D. (a, b, c) = d

$\Rightarrow \quad d^2 = \dfrac{a^2+b^2+c^2}{3} - (\bar{x})^2$

$\Rightarrow \quad d^2 = \dfrac{a^2+b^2+c^2}{3} - \dfrac{4b^2}{9}$

$\Rightarrow \quad 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$

$\Rightarrow \quad b^2 = 3(a^2 + c^2) - 9d^2$

5. If for $x \in \left(0, \dfrac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10}n - 1)$, $n > 0$, then the value of n is equal to :

(1) 20 (2) 12 (3) 9 (4) 16

Official Ans. by NTA (2)

Sol. $x \in \left(0, \dfrac{\pi}{2}\right)$

$\log_{10}\sin x + \log_{10}\cos x = -1$

$\Rightarrow \quad \log_{10}\sin x.\cos x = -1$

$\Rightarrow \quad \sin x.\cos x = \dfrac{1}{10}$ ......(1)

$\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \quad \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\dfrac{n}{10}}$

by squaring

$1 + 2\sin x.\cos x = \dfrac{n}{10}$

$\Rightarrow \quad 1 + \dfrac{1}{5} = \dfrac{n}{10} \quad \Rightarrow \quad n = 12$

6. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$ .Then, the system of linear equations $A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) A unique solution

(2) Infinitely many solutions

(3) No solution

(4) Exactly two solutions

Official Ans. by NTA (3)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad 128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad x - y = \dfrac{1}{16}$ .....(1)

& $\quad -x + y = \dfrac{1}{2}$ .....(2)

$\Rightarrow$ From (1) & (2) : No solution.

7. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than :

(1) $\dfrac{1}{2}$ (2) $-\dfrac{1}{2}$ (3) –1 (4) 1

Official Ans. by NTA (4)

Sol. For standard parabola

For more than 3 normals (on axis)

$x > \dfrac{L}{2}$ (where L is length of L.R.)

For $y^2 = 2x$
L.R. = 2
for (a, 0)

$$a > \frac{\text{L.R.}}{2} \Rightarrow a > 1$$

8. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{482}$ (2) $\sqrt{171}$

(3) $\sqrt{5}$ (4) $\sqrt{227}$

Official Ans. by NTA (2)

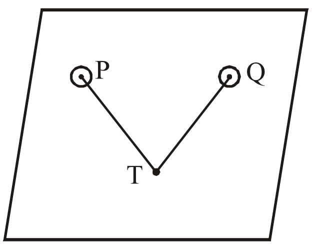
Sol. P(3, –1, 2)
Q(1, 2, –4)

$\overrightarrow{PR} \parallel 4\hat{i}-\hat{j}+2\hat{k}$

$\overrightarrow{QS} \parallel -2\hat{i}+\hat{j}-2\hat{k}$

dr's of normal to the plane containing P, T & Q will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$



$$\therefore \quad \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$$

For point, T : $\overrightarrow{PT} = \frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2} = \lambda$

$$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$$

T : (4λ + 3, –λ –1, 2λ + 2)

$$\cong (2\mu+1, \mu+2, -2\mu-4)$$

$4\lambda + 3 = -2\mu + 1 \quad \Rightarrow \quad 2\lambda + \mu = -1$
$\lambda + \mu = -3 \quad \Rightarrow \quad \lambda = 2$
& $\mu = -5 \qquad \lambda + \mu = -3 \quad \Rightarrow \quad \lambda = 2$
So point T : (11, –3, 6)

$$\overrightarrow{OA} = \left(11\hat{i}-3\hat{j}+6\hat{k}\right) \pm \left(\frac{2\hat{j}+\hat{k}}{\sqrt{5}}\right)\sqrt{5}$$

$$\overrightarrow{OA} = \left(11\hat{i}-3\hat{j}+6\hat{k}\right) \pm \left(2\hat{j}+\hat{k}\right)$$

$\overrightarrow{OA} = 11\hat{i}-\hat{j}+7\hat{k}$

or

$9\hat{i}-5\hat{j}+5\hat{k}$

$|\overrightarrow{OA}| = \sqrt{121+1+49} = \sqrt{171}$

or

$\sqrt{81+25+25} = \sqrt{131}$

9. Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} x+2, & x<0 \\ x^2, & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x<1 \\ 3x-2, & x \ge 1 \end{cases}$$

Then, the number of points in $\mathbb{R}$ where $(f\text{o}g)(x)$ is NOT differentiable is equal to :

(1) 3 (2) 1

(3) 0 (4) 2

Official Ans. by NTA (2)

Sol. $$f(g(x)) = \begin{cases} g(x)+2, & g(x)<0 \\ (g(x))^2, & g(x) \ge 0 \end{cases}$$

$$= \begin{cases} x^3+2, & x<0 \\ x^6, & x \in [0,1) \\ (3x-2)^2, & x \in [1,\infty) \end{cases}$$

$$(f\text{o}g(x))' = \begin{cases} 3x^2, & x<0 \\ 6x^5, & x \in (0,1) \\ 2(3x-2)\times 3, & x \in (1,\infty) \end{cases}$$

At 'O'
L.H.L. ≠ R.H.L. (Discontinuous)
At '1'
L.H.D. = 6 = R.H.D.
$\Rightarrow \quad f\text{o}g(x)$ is differentiable for $x \in \mathbb{R} - \{0\}$

10. Which of the following Boolean expression is a tautology ?

(1) $(p \wedge q) \vee (p \vee q)$

(2) $(p \wedge q) \vee (p \rightarrow q)$

(3) $(p \wedge q) \wedge (p \rightarrow q)$

(4) $(p \wedge q) \rightarrow (p \rightarrow q)$

Official Ans. by NTA (4)

Sol.

p	q	$p\wedge q$	$p\rightarrow q$	$(p\wedge q)\rightarrow(p\rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$ is tautology

11. Let a complex number z, $|z| \neq 1$,

satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____ .

(1) 8 (2) 7 (3) 6 (4) 5

Official Ans. by NTA (2)

Sol. $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$

$\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

$2|z| + 22 \geq (|z| - 1)^2$

$2|z| + 22 \geq |z|^2 + 1 - 2|z|$

$|z|^2 - 4|z| - 21 \leq 0$

$\Rightarrow$ $|z| \leq 7$

$\therefore$ Largest value of $|z|$ is 7

12. If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then$(n - 1)$ is divisible by :

(1) 26 (2) 30 (3) 8 (4) 7

Official Ans. by NTA (1)

Sol. $(3^{1/4} + 5^{1/8})^{60}$

${}^{60}C_r(3^{1/4})^{60-r}.(5^{1/8})^r$

${}^{60}C_r(3)^{\frac{60-r}{4}}.5^{\frac{r}{8}}$

For rational terms.

$\frac{r}{8} = k$; $0 \leq r \leq 60$

$0 \leq 8k \leq 60$

$0 \leq k \leq \frac{60}{8}$

$0 \leq k \leq 7.5$

$k = 0, 1, 2, 3, 4, 5, 6, 7$

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

Total terms = 61

irrational terms = 53

$n - 1 = 53 - 1 = 52$

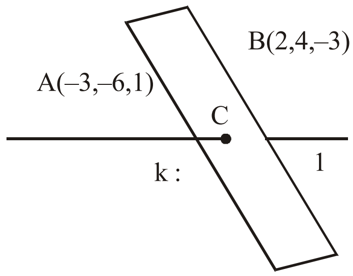
52 is divisible by 26.

13. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to :

(1) 1.5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (3)

Sol.



Point C is

$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$

Plane $lx + my + nz = 0$

$l(-1) + m(2) + n(3) = 0$

$-l + 2m + 3n = 0$ ....(1)

It also satisfy point (1, –4, –2)

$l - 4m - 2n = 0$ ....(2)

Solving (1) and (2)

$2m + 3n = 4m + 2n$

$n = 2m$

$l - 4m - 4m = 0$

$l = 8m$

$\frac{l}{8} = \frac{m}{1} = \frac{n}{2}$

$l : m : n = 8 : 1 : 2$

Plane is $8x + y + 2z = 0$

It will satisfy point C

$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28$ $\therefore$ $k = 2$

14. The range of $a \in \mathbb{R}$ for which the function

$$f(x) = (4a-3)(x+\log_e 5)+2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi, n \in \mathbb{N}$, has critical points, is :

(1) $(-3, 1)$ (2) $\left[-\frac{4}{3}, 2\right]$

(3) $[1, \infty)$ (4) $(-\infty, -1]$

Official Ans. by NTA (2)

Sol. $f(x) = (4a - 3)(x + \log_e 5) + (a - 7)\sin x$

$f(x) = (4a - 3)(1) + (a - 7)\cos x = 0$

$$\Rightarrow \quad \cos x = \frac{3-4a}{a-7}$$

$$-1 \leq \frac{3-4a}{a-7} < 1$$

$$\frac{3-4a}{a-7}+1 \geq 0 \qquad \frac{3-4a}{a-7} < 1$$

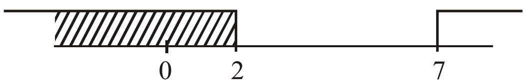
$$\frac{3-4a+a-7}{a-7} \geq 0 \qquad \frac{3-4a}{a-7}-1 < 0$$

$$\frac{-3a-4}{a-7} \geq 0 \qquad \frac{3-4a-a+7}{a-7} < 0$$

$$\frac{3a+4}{a-7} \leq 0 \qquad \frac{-5a+10}{a-7} < 0$$

$$\frac{5a-10}{a-7} > 0$$

$$\frac{5(a-2)}{a-7} > 0$$



$$\alpha \in \left[-\frac{4}{3}, 2\right)$$

Check end point $\left[-\frac{4}{3}, 2\right)$

15. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$ (2) $\frac{52}{867}$

(3) $\frac{39}{50}$ (4) $\frac{22}{425}$

Official Ans. by NTA (3)

Sol. E_1 : Event denotes spade is missing

$$P(E_1) = \frac{1}{4}; \; P(\bar{E}_1) = \frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A) = \frac{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right)+\frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)+\frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right)+\frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$= \frac{39}{50}$$

16. Let $[x]$ denote greatest integer less than or equal to x. If for $n \in \mathbb{N}$, $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$,

then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

(1) 2 (2) 2^{n-1}

(3) 1 (4) n

Official Ans. by NTA (3)

Sol. $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$(1 - x + x^3)^n = a_0 + a_1x + a_2x^2 \ldots\ldots + a_{3n}x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of $a_0 + a_2 + a_4 \ldots\ldots$

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ = Sum of $a_1 + a_3 + a_5 \ldots\ldots$

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 \ldots\ldots + a_{3n}$ ....(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 \ldots\ldots + (-1)^{3n}a_{3n}$ ....(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 \ldots = 1$

$a_1 + a_3 + a_5 \ldots\ldots = 0$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$$

17. If y = y(x) is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x,\ y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function y(x) over $\mathbb{R}$ is equal to :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

Official Ans. by NTA (4)

Sol. $\frac{dy}{dx} + 2y\tan x = \sin x$

I.F. = $e^{\int 2\tan x dx} = e^{2\ell n \sec x}$

I.F. = $\sec^2 x$

$y.(\sec^2 x) = \int \sin x.\sec^2 x dx$

$y.(\sec^2 x) = \int \sec x \tan x dx$

$y.(\sec^2 x) = \sec x + C$

$x = \frac{\pi}{3};\ y = 0$

$\Rightarrow$ C = –2

$\Rightarrow\quad y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2\cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt} = 1 - 4t = 0 \Rightarrow t = \frac{1}{4}$

$\therefore\quad \max = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$

18. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

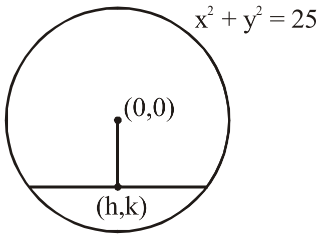
(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Official Ans. by NTA (4)

Sol.



Equation of chord

$y - k = -\frac{h}{k}(x - h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$

tangent to $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$c^2 = a^2m^2 - b^2$

$\left(\frac{h^2 + k^2}{k}\right)^2 = 9\left(-\frac{h}{k}\right)^2 - 16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

19. The number of roots of the equation,

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

in the interval [0, π] is equal to :

(1) 3 (2) 4 (3) 8 (4) 2

Official Ans. by NTA (2)

Sol. $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

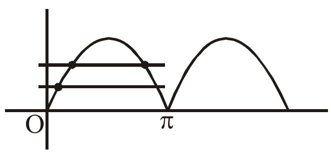
$t^2 - 30t + 81 = 0$

$(t - 3)(t - 27) = 0$

$(81)^{\sin^2 x} = 3^1$ or $(81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1$ or $3^{4\sin^2 x} = 3^3$

$\sin^2 x = \frac{1}{4}$ or $\sin^2 x = \frac{3}{4}$



Total sol. = 4

20. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\cot^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}(3)$

Official Ans. by NTA (3)

Sol. $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$$

Let $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\frac{2t}{3}\right)\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}\left(\frac{2}{3}\right)^r-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}(0)$$

$$\therefore \quad S_\infty = \tan^{-1}\left(\frac{2}{3}\right) = \cot^{-1}\left(\frac{3}{2}\right)$$

SECTION-B

1. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ______ .

Official Ans. by NTA (3)

Sol. **GP :** 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

AP : 11, 16, 21, 26, 31, 36

Common terms : 16, 256, 4096 only

2. Let $f : (0, 2) \to \mathbb{R}$ be defined as

$$f(x) = \log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$ is equal to _______ .

Official Ans. by NTA (1)

Sol. $E = 2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi x}{4}\right)dx \qquad .....(i)$$

replacing $x \to 1 - x$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\frac{1+\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1\left(\ell n2-\ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx \qquad .....(ii)$$

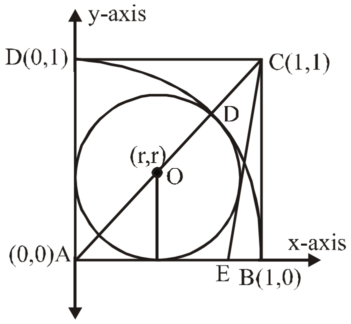
equation (i) + (ii)

E = 1

3. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to_______.

Official Ans. by NTA (1)

Sol.



Here AO + OD = 1 or $(\sqrt{2}+1)r = 1$

$\Rightarrow \quad r = \sqrt{2-1}$

equation of circle $(x - r)^2 + (y - r)^2 = r^2$

Equation of CE

$y - 1 = m(x - 1)$

$mx - y + 1 - M = 0$

It is tangent to circle

$$\therefore \quad \left|\frac{mr-r+1-m}{\sqrt{m^2+1}}\right| = r$$

$$\left|\frac{(m-1)r+1-m}{\sqrt{m^2+1}}\right| = r$$

$$\frac{(m-1)^2(r-1)^2}{m^2+1} = r^2$$

Put $r=\sqrt{2}-1$

On solving $m = 2 - \sqrt{3}, 2 + \sqrt{3}$

Taking greater slope of CE as $2 + \sqrt{3}$

$y - 1 = (2 + \sqrt{3})(x - 1)$

Put $y = 0$

$-1 = (2 + \sqrt{3})(x - 1)$

$$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x-1$$

$x - 1 = \sqrt{3} - 1$

$EB = 1 - x = 1 - (\sqrt{3} - 1)$

$EB = 2 - \sqrt{3}$

4. If $\lim\limits_{x\to0}\dfrac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then a + b + c is equal to _______.

Official Ans. by NTA (4)

Sol. $$\lim_{x\to0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$$

$$\Rightarrow \lim_{x\to0}\frac{a\left(1+x+\frac{x^2}{2!}...\right)-b\left(1-\frac{x^2}{2!}+...\right)+c\left(1-x+\frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x} = 2$$

$a - b + c = 0 \qquad .....(1)$

$a - c = 0 \qquad .....(2)$

$\& \quad \dfrac{a+b+c}{2} = 2$

$\Rightarrow \boxed{a+b+c=4}$

5. The total number of 3 × 3 matrices A having enteries from the set (0, 1, 2, 3) such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

Official Ans. by NTA (766)

Sol. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

diagonal elements of

AA^T, $a^2 + b^2 + c^2$, $d^2 + e^2 + f^2$, $g^2 + b^2 + c^2$

Sum $= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$

$a, b, c, d, e, f, g, h, i \in \{0, 1, 2, 3\}$

	Case	No. of Matrices
(1)	All – 1s	$\frac{9!}{9!}=1$
(2)	One → 3 remaining-0	$\frac{9!}{1!\times 8!}=9$
(3)	One-2 five-1s three-0s	$\frac{9!}{1!\times 5!\times 3!}=8\times 63$
(4)	two – 2's one-1 six-0's	$\frac{9!}{2!\times 6!}=63\times 4$

Total no. of ways = 1 + 9 + 8 × 63 + 63 × 4

= $\boxed{766}$

6. Let

$$P=\begin{bmatrix}-30 & 20 & 56\\ 90 & 140 & 112\\ 120 & 60 & 14\end{bmatrix} \text{ and } A=\begin{bmatrix}2 & 7 & \omega^2\\ -1 & -\omega & 1\\ 0 & -\omega & -\omega+1\end{bmatrix}$$

where $\omega=\frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ______ .

Official Ans. by NTA (36)

Sol. Let $M = (P^{-1}AP - I)^2$

$= (P^{-1}AP)^2 - 2P^{-1}AP + I$

$= P^{-1}A^2P - 2P^{-1}AP + I$

$PM = A^2P - 2AP + P$

$= (A^2 - 2A.I + I^2)P$

$\Rightarrow$ $Det(PM) = Det((A - I)^2 \times P)$

$\Rightarrow$ $DetP.DetM = Det(A - I)^2 \times Det(P)$

$\Rightarrow$ $Det\ M = (Det(A - I))^2$

$$\text{Now } A-I=\begin{bmatrix}1 & 7 & w^2\\ -1 & -w-1 & 1\\ 0 & -w & -w\end{bmatrix}$$

$Det(A - I) = (w^2 + w + w) + 7(-w) + w^3 = -6w$

$Det((A - I))^2 = 36w^2$

$\Rightarrow$ $\alpha = 36$

7. If the normal to the curve $y(x)=\int_0^x (2t^2-15t+10)\,dt$ at a point (a,b) is parallel to the line x + 3y = –5, a > 1, then the value of |a + 6b| is equal to ________ .

Official Ans. by NTA (406)

Sol. $y(x)=\int_0^x (2t^2-15t+10)\,dt$

$$y'(x)\big]_{x=a}=\left[2x^2-15x+10\right]_a=2a^2-15a+10$$

Slope of normal $=-\frac{1}{3}$

$\Rightarrow$ $2a^2-15a+10=3 \Rightarrow a=7$

& $a = \frac{1}{2}$ (rejected)

$b = y(7) = \int_0^7 (2t^2-15t+10)dt$

$$=\left[\frac{2t^3}{3}-\frac{15t^2}{2}+10t\right]_0^7$$

$\Rightarrow$ $6b=4\times 7^3-45\times 49+60\times 7$

|a + 6b| = 406

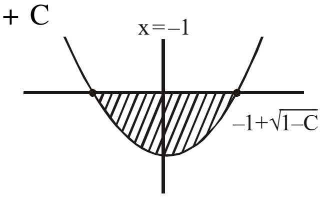
8. Let the curve y = y(x) be the solution of the differential equation, $\frac{dy}{dx}=2(x+1)$. If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to _______ .

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}=2(x+1)$

$\Rightarrow$ $\int dy=\int 2(x+1)dx$

$\Rightarrow$ $y(x) = x^2 + 2x + C$



Area $=\frac{4\sqrt{8}}{3}$

$-1+\sqrt{1-C}$

$$\Rightarrow\ 2\int_{-1}^{-1+\sqrt{1-C}} (-(x+1)^2-C+1)dx=\frac{4\sqrt{8}}{3}$$

$$\Rightarrow\ 2\left[-\frac{(x+1)^3}{3}-Cx+x\right]_{-1}^{-1+\sqrt{1-C}}=\frac{4\sqrt{8}}{3}$$

$\Rightarrow$ $-(\sqrt{1-C})^3+3c-3C\sqrt{1-C}$

$-3+3\sqrt{1-C}-3C+3=2\sqrt{8}$

$\Rightarrow$ C = –1

$\Rightarrow$ $f(x) = x^2 + 2x - 1$, $f(1) = 2$

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) + f(x+1) = 2$, for all $x \in \mathbb{R}$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^3 f(x)dx$, then the value of $I_1 + 2I_2$ is equal to ________ .

Official Ans. by NTA (16)

Sol. $f(x) + f(x+1) = 2$

$\Rightarrow$ $f(x)$ is periodic with period = 2

$I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$

$= 4\int_0^1 \left(f(x) + f(1+x)\right)dx = 8$

Similarly $I_2 = 2 \times 2 = 4$

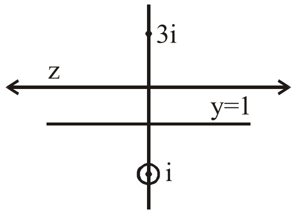
$I_1 + 2I_2 = 16$

10. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and $\text{Re}(w)$ has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which w^n is real, is equal to _____.

Official Ans. by NTA (4)

Sol. $\omega = z\bar{z} - 2z + 2$

$\left|\frac{z+i}{z-3i}\right| = 1$



$\Rightarrow$ $|z+i| = |z-3i|$

$\Rightarrow$ $z = x + i,\ x \in \mathbb{R}$

$\omega = (x+i)(x-i) - 2(x+i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min $(\text{Re}(\omega))$, $x = 1$

$\Rightarrow$ $\omega = 2 - 2i = 2(1-i) = 2\sqrt{2}\,e^{-i\frac{\pi}{4}}$

$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n,

n = 4

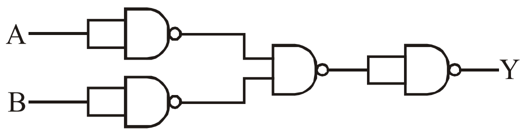
FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. The following logic gate is equivalent to :



(1) NOR Gate (2) OR Gate

(3) AND Gate (4) NAND Gate

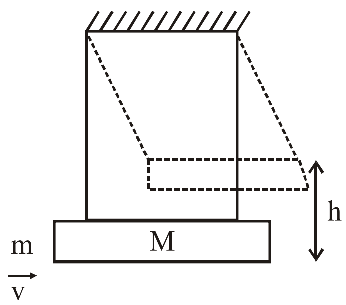
Official Ans. by NTA (1)

Sol. Truth table for the given logic gate :

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The truth table is similar to that of a NOR gate.

2. A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is : (Take g = 9.8 ms^{-2})



(1) 841.4 m/s (2) 811.4 m/s

(3) 831.4 m/s (4) 821.4 m/s

Official Ans. by NTA (3)

Sol. From energy conservation,

$\left[\begin{array}{l}\text{after bullet gets embedded till the} \\ \text{system comes momentarily at rest}\end{array}\right]$

$$(M + m)g\,h = \frac{1}{2}(M+m)v_1^2$$

[v_1 is velocity after collision]

$\therefore\ v_1 = \sqrt{2gh}$

Applying momentum conservation, (just before and just after collision)

$mv = (M + m)v_1$

$$v = \left(\frac{M+m}{m}\right)v_1 = \frac{6}{10\times10^{-3}}\times\sqrt{2\times9.8\times9.8\times10^{-2}}$$

$\approx 831.55\ m/s$

3. A charge Q is moving $\overrightarrow{dl}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.

(1) 1 (2) Infinite

(3) Zero (4) –1

Official Ans. by NTA (3)

Sol. Since force on a point charge by magnetic field is always perpendicular to $\vec{v}\left[\vec{F} = q\vec{V}\times\vec{B}\right]$

$\therefore$ Work by magnetic force on the point charge is zero.

4. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min ? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.

(Density of water : 10^3 kg/m^3)

(1) Unsteady to steady flow

(2) Remains steady flow

(3) Remains turbulent flow

(4) Steady flow to unsteady flow

Official Ans. by NTA (4)

Sol. The nature of flow is determined by Reynolds Number.

$$R_e = \frac{\rho v D}{\eta}$$

$$\begin{bmatrix} \rho \rightarrow \text{density of fluid} \;\; ; & \eta \rightarrow \text{coefficient of} \\ v \rightarrow \text{velocity of flow} & \text{viscosity} \\ D \rightarrow \text{Diameter of pipe} & \end{bmatrix}$$

From NCERT

If $R_e < 1000$ $\rightarrow$ flow is steady

$1000 < R_e < 2000$ $\rightarrow$ flow becomes unsteady

$R_e > 2000$ $\rightarrow$ flow is turbulent

$$R_{e\,\text{initial}} = 10^3 \times \frac{0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 382.16$

$$R_{e\,\text{final}} = 10^3 \times \frac{0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 1019.09$

5. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s ?

(1) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

(2) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(4) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. Given :

$$\vec{v} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$$

$$\vec{v}_{\text{at } t=2} = 2\,\hat{i} + 6\,\hat{j} + 9\hat{k}$$

$\therefore$ Angle made by direction of motion of mosquito will be,

$\cos^{-1}\frac{2}{11}$ (from x-axis) $= \tan^{-1}\frac{\sqrt{117}}{2}$

$\cos^{-1}\frac{6}{11}$ (from y-axis) $= \tan^{-1}\frac{\sqrt{85}}{6}$

$\cos^{-1}\frac{9}{11}$ (from z-axis) $= \tan^{-1}\frac{\sqrt{40}}{9}$

None of the option is matching.

Hence this question should be bonus.

6. Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.

(1) 0.424 nC m^{-2} (2) 47.88 C/m

(3) 0.07 nC m^{-2} (4) 4.0 nC m^{-2}

Official Ans. by NTA (1)

Sol. $\frac{2K\lambda}{r} = \frac{\sigma}{\varepsilon_0}$ (x = 3m)

$$\sigma = 0.424 \times 10^{-9}\,\frac{C}{m^2}$$

7. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths ? (m_P = 1.00727 u, m_e = 0.00055u)

(1) 1860 : 1 (2) $(1860)^2$: 1

(3) 41.4 : 1 (4) 43 : 1

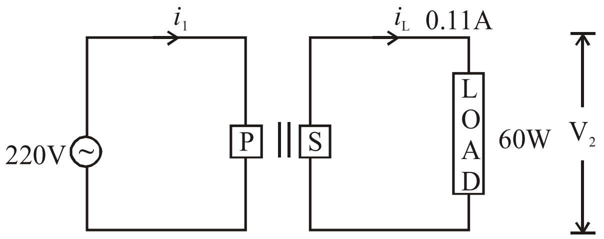
Official Ans. by NTA (4)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{\lambda_e}{\lambda_P} = \sqrt{\frac{m_P}{m_e}} = \sqrt{1831.4} = 42.79$$

8. For the given circuit, comment on the type of transformer used :



(1) Auxilliary transformer
(2) Auto transformer
(3) Step-up transformer
(4) Step down transformer

Official Ans. by NTA (3)

Sol. $V_S = \frac{P}{i} = \frac{60}{0.11} = 545.45$

$V_P = 220$

$V_S > V_P$

$\Rightarrow$ Step up transformer

9. The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, ($N_A = 6 \times 10^{23}$/mol)

(1) 240 Ci (2) 357 Ci
(3) 535 Ci (4) 252 Ci

Official Ans. by NTA (2)

Sol. $A = \lambda N$ $\left(t_{1/2} = \frac{\ln 2}{\lambda}\right)$

$N = nN_A$

$N = \left(\frac{1.5 \times 10^{-3}}{198}\right) N_A$

$A = \left(\frac{\ln 2}{t_{1/2}}\right) N$

1 Curie = 3.7×10^{10} Bq

A = 365 Bq

10. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}$ JK^{-1})

(1) 58 nm (2) 32 nm
(3) 86 nm (4) 102 nm

Official Ans. by NTA (4)

Sol. $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$

λ =102 nm

11. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? (Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively)

(1) 1 (2) Infinite

(3) $\frac{R_1R_2}{R_1 - R_2}$ (4) Zero

Official Ans. by NTA (2)

Sol. $\frac{1}{F} = \left[\frac{\mu_L}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

If $\mu_L = \mu_S \Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$

12. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment ?

(1) 0.14%
(2) 0.9%
(3) 9%
(4) 1.4%

Official Ans. by NTA (4)

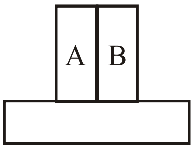
Sol. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{Al} = \frac{mg.L}{\pi R^2.\ell}$

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2.\frac{\Delta R}{R} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta Y}{Y} \times 100 = 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$$

$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$$

13. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold both, it will :



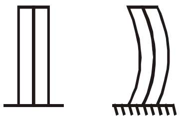
(1) Bend towards the right
(2) Not bend but shrink
(3) Neither bend nor shrink
(4) Bend towards the left

Official Ans. by NTA (4)

Sol. $\alpha_A > \alpha_B$

Length of both strips will decrease

$\Delta L_A > \Delta L_B$



14. A resistor develops 500 J of thermal energy in 20s when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3A, what will be the energy developed in 20 s.

(1) 1500 J (2) 1000 J
(3) 500 J (4) 2000 J

Official Ans. by NTA (4)

Sol. $500 = (1.5)^2 \times R \times 20$

$E = (3)^2 \times R \times 20$

$E = 2000$ J

15. **Statement I :** A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve ($g = 9.8$ m/s^2)

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is incorrect and statement II is correct
(2) Statement I is correct and statement II is incorrect
(3) Both statement I and statement II are false
(4) Both statement I and statement II are true

Official Ans. by NTA (4)

Sol. **Statement I :**

$$v_{max} = \sqrt{\mu Rg} = \sqrt{(0.2) \times 2 \times 9.8}$$

$v_{max} = 1.97$ m/s

7 km/h = 1.944 m/s

Speed is lower than v_{max}, hence it can take safe turn.

Statement II

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1 + 0.2}{1 - 0.2}\right]} = 5.42 \text{ m/s}$$

18.5 km/h = 5.14 m/s

Speed is lower than v_{max}, hence it can take safe turn.

16. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight ? (Assume radius of earth is 6400 km)

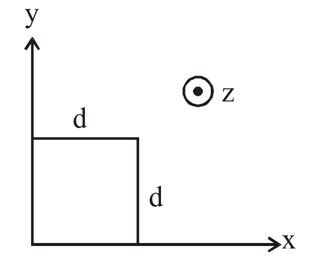
(1) 19.77 m (2) 39.55 m
(3) 79.1 m (4) 158.2 m

Official Ans. by NTA (2)

Sol. $D = 2\sqrt{2Rh}$

$$h = \frac{D^2}{8R} = \frac{45^2}{8 \times 6400} \text{km} \cong 39.55 \text{m}$$

17. The magnetic field in a region is given by $\vec{B} = B_0\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is :



(1) $\frac{B_0 v_0^2 d}{2a}$ (2) $\frac{B_0 v_0 d}{2a}$

(3) $\frac{B_0 v_0 d^2}{a}$ (4) $\frac{B_0 v_0 d^2}{2a}$

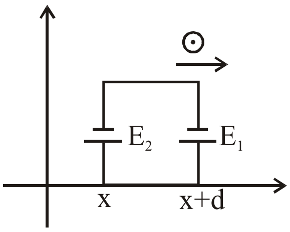
Official Ans. by NTA (3)

Sol. $E_1 = \frac{B_0(x+d)}{a}v_0 d$

$E_2 = \frac{B_0(x)}{a}v_0 d$

$E_{net} = E_1 - E_2$

$E_{net} = \frac{B_0 v_0 d^2}{a}$



18. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value ? (ln 2 = 0.693)

(1) 34.65 s (2) 17.32 s

(3) 0.034 s (4) 15.01 s

Official Ans. by NTA (1)

Sol. $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$

$\frac{A_0}{2} = A_0 e^{-\frac{bt}{2m}}$

$\frac{bt}{2m} = \ln 2$

$t = \frac{2m}{b}\ln 2 = \frac{2\times 500\times 0.693}{20}$

t = 34.65 second.

19. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

(1) 60 minutes (2) 20 minutes

(3) 40 minutes (4) 13 minutes

Official Ans. by NTA (2)

Sol. $N_1 = N_0 e^{-\lambda t_1}$

$\frac{N_1}{N_0} = e^{-\lambda t_1}$

$0.67 = e^{-\lambda t_1}$

$\ln(0.67) = -\lambda t_1$

$N_2 = N_0 e^{-\lambda t_2}$

$\frac{N_2}{N_0} = e^{-\lambda t_2}$

$0.33 = e^{-\lambda t_2}$

$\ln(0.33) = -\lambda t_2$

$\ln(0.67) - \ln(0.33) = \lambda t_1 - \lambda t_2$

$\lambda(t_1 - t_2) = \ln\left(\frac{0.67}{0.33}\right)$

$\lambda(t_1 - t_2) \cong \ln 2$

$t_1 - t_2 \simeq \frac{\ln 2}{\lambda} = t_{1/2}$

Half life = $t_{1/2}$ = 20 minutes.

20. Red light differs from blue light as they have :

(1) Different frequencies and different wavelengths

(2) Different frequencies and same wavelengths

(3) Same frequencies and same wavelengths

(4) Same frequencies and different wavelengths

Official Ans. by NTA (1)

Sol. Red light and blue light have different wavelength and different frequency.

SECTION-B

1. The energy dissipated by a resistor is 10 mJ in 1s when an electric current of 2 mA flows through it. The resistance is _______ Ω. (Round off to the Nearest Integer)

Official Ans. by NTA (2500)

Sol. **Ans. (2500)**

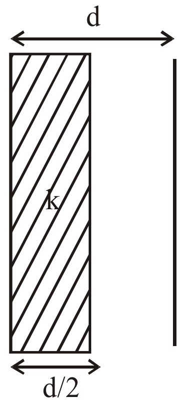
$Q = i^2 RT$

$R = \frac{Q}{i^2 t} = \frac{10\times 10^{-3}}{4\times 10^{-6}\times 1} = 2500\ \Omega$

2. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area $2m^2$ (see fig.) the capacitance of the set-up will be ______ ε_0.

(Dielectric constant of the material = 3.2)

(Round off to the Nearest Integer)



Official Ans. by NTA (3)

Sol. **Ans. (3)**

$$C = \frac{\varepsilon_0 A}{\frac{d}{2K}+\frac{d}{2}} = \frac{2\varepsilon_0 A}{\frac{d}{K}+d}$$

$$= \frac{2\times 2\varepsilon_0}{\frac{1}{3.2}+1} = \frac{4\times 3.2}{4.2}\varepsilon_0$$

$$= 3.04\ \varepsilon_0$$

3. A force $\vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$ is applied on an intersection point of x = 2 plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is ________. (Round off to the Nearest Integer)

Official Ans. by NTA (20)

Sol. **Ans. (20)**

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\vec{r} = (2\hat{i}) - (2\hat{i}+3\hat{j}+4\hat{k}) = -3\hat{j}-4\hat{k}$$

$$\& \ \vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$$

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12+12) - \hat{j}(0+16) + \hat{k}(0+12)$$

$$= -16\hat{i}+12\hat{k}$$

$$\therefore\ |\vec{\tau}| = \sqrt{16^2+12^2} = 20$$

4. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ____ (Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

Official Ans. by NTA (3)

Sol. **Ans. (3)**

Energy given = $U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

$$= \frac{3}{5}\frac{GM^2}{R}$$

x = 3

5. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be ________° (in degree)

(Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. Ans. (12)

$\omega_1 = 0.02\ ;\ \mu_1 = 1.5\ \ ;\ \ \omega_2 = 0.03\ ;\ \mu_2 = 1.6$

Achromatic combination

$\therefore\ \theta_{net} = 0$

$\theta_1 - \theta_2 = 0$

$\theta_1 = \theta_2$

$\omega_1\delta_1 = \omega_2\delta_2$

& $\delta_{net} = \delta_1 - \delta_2 = 2°$

$$\delta_1 - \frac{\omega_1\delta_1}{\omega_2} = 2°$$

$$\delta_1\left(1 - \frac{\omega_1}{\omega_2}\right) = 2°$$

$$\delta_1\left(1 - \frac{2}{3}\right) = 2°$$

$\delta_1 = 6°$

$\delta_1 = (\mu_1 - 1)\ A_1$

$6° = (1.5 - 1)\ A_1$

$A_1 = 12°$

6. A body of mass 2kg moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k})N$. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is __________. (Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. Ans. (12)

$$\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$$

$$= \hat{i} + 1.5\hat{j} + 2.5\hat{k}$$

$$\vec{\tau} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$= 0 + \frac{1}{2}(\hat{i} + 1.5\hat{j} + 2.5\hat{k})\ (16)$$

$$= 8\hat{i} + 12\hat{j} + 20\hat{k}$$

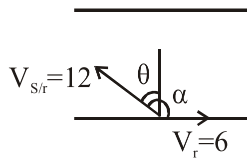
b = 12

7. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _________°. (Round off to the Nearest Integer) (find the angle in degree)

Official Ans. by NTA (120)

Sol. Ans. (12)

$12\sin\theta = v_r$



$$\sin\theta = \frac{1}{2}$$

$\theta = 30°$

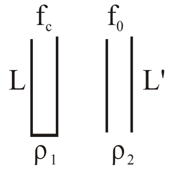
$\therefore\ \alpha = 120°$

8. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is ________. (Round off to the Nearest Integer)

Official Ans. by NTA (4)

Sol. Ans. (4)

$f_c = f_0$



$$\frac{3V_C}{4L} = \frac{2V_0}{2L'}$$

$$\frac{3V_C}{4L} = \frac{V_0}{L'}$$

$$L' = \frac{4L}{3}\frac{V_0}{V_C} = \frac{4L}{3}\sqrt{\frac{B.\rho_1}{\rho_2.B}}\ \text{(B is bulk modulus)}$$

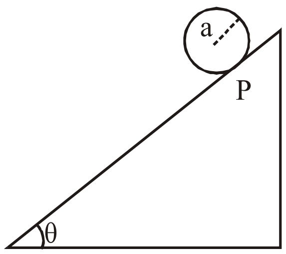
$$= \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

x = 4

9. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$ where b is ______. (Round off to the Nearest Integer)

(g = acceleration due to gravity)

(θ = angle as shown in figure)



Official Ans. by NTA (3)

Sol. Ans. (3)

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

b = 3

10. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _______°C. (Round off to the Nearest Integer)

Official Ans. by NTA (113)

Official Ans. by ALLEN (–113)

Sol. Ans. (–113)

$$n = 0.60 = 1 = \frac{T_L}{T_H}$$

$$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times 400$$

$$= 160 \text{ K}$$

$$= -113°C$$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

CHEMISTRY | **TEST PAPER WITH ANSWER & SOLUTION**

SECTION-A

1. The green house gas/es is (are) :

(A) Carbon dioxide
(B) Oxygen
(C) Water vapour
(D) Methane

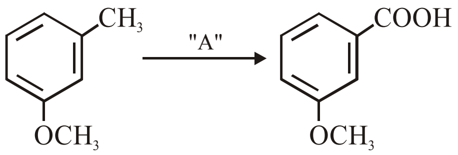
Choose the most appropriate answer from the options given below :

(1) (A) and (C) only
(2) (A) only
(3) (A), (C) and (D) only
(4) (A) and (B) only

Official Ans. by NTA (3)

Sol. The green house gases are CO_2, $H_2O_{(vapour)}$ & CH_4.

2.

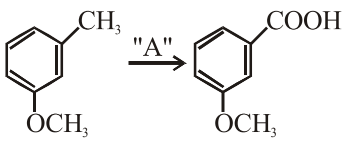


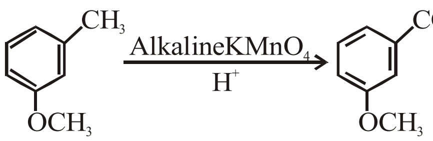
In the above reaction, the reagent "A" is :

(1) $NaBH_4$, H_3O^+
(2) $LiAlH_4$
(3) Alkaline $KMnO_4$, H^+
(4) HCl, Zn-Hg

Official Ans. by NTA (3)

Sol.





;

3. Which of the following reduction reaction CANNOT be carried out with coke ?

(1) $Al_2O_3 \rightarrow Al$
(2) $ZnO \rightarrow Zn$
(3) $Fe_2O_3 \rightarrow Fe$
(4) $Cu_2O \rightarrow Cu$

Official Ans. by NTA (1)

Sol. Reduction of $Al_2O_3 \rightarrow Al$ is carried out by electrolytic reduction of its fused salts.
ZnO, Fe_2O_3 & Cu_2O can be reduce by carbon.

4. Identify the elements X and Y using the ionisation energy values given below :

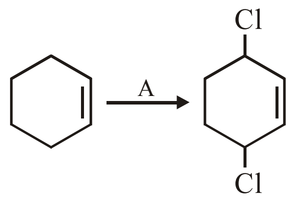
	Ionization energy	**(kJ/mol)**
	1^{st}	2^{nd}
X	495	4563
Y	731	1450

(1) X = Na ; Y = Mg
(2) X = Mg ; Y = F
(3) X = Mg ; Y = Na
(4) X = F ; Y = Mg

Official Ans. by NTA (1)

Sol. $Na \rightarrow [Ne]\ 3s^1$ IE_1 is very low but IE_2 is very high due to stable noble gas configuration of Na^+.
$Mg \rightarrow [Ne]\ 3s^2$ IE_1 & $IE_2 \rightarrow$ Low
IE_3 is very high.

5.

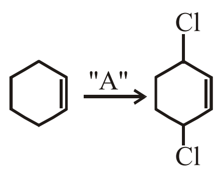


Identify the reagent(s) 'A' and condition(s) for the reaction :

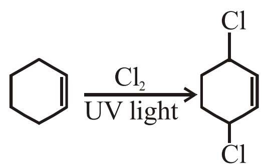
(1) A = HCl; Anhydrous $AlCl_3$
(2) A = HCl, $ZnCl_2$
(3) A = Cl_2; UV light
(4) A = Cl_2; dark, Anhydrous $AlCl_3$

Official Ans. by NTA (3)

Sol.



For substitution at allylic position in the given compound, the reagent used is Cl_2/uv light. The reaction is free radical halogenation.



6. The secondary structure of protein is stabilised by:
(1) Peptide bond
(2) glycosidic bond
(3) Hydrogen bonding
(4) van der Waals forces
Official Ans. by NTA (3)

Sol. The secondary structure of protein includes two type :
(a) α-Helix (b) β-pleated sheet
In α-Helix structure, the poly peptide chain is coil around due to presence of Intramolecular H-Bonding.

7. Fex_2 and Fey_3 are known when x and y are :
(1) x = F, Cl, Br, I and y = F, Cl, Br
(2) x = F, Cl, Br and y = F, Cl, Br, I
(3) x = Cl, Br, I and y = F, Cl, Br, I
(4) x = F, Cl, Br, I and y = F, Cl, Br, I
Official Ans. by NTA (1)

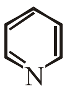
Sol. $\underset{\text{(Unstable)}}{2FeI_3} \longrightarrow \underset{\text{(Stable)}}{2FeI_2} + I_2$
Due to strong reducing nature of I^-
$2Fe^{3+} + 2I^- \longrightarrow 2Fe^{2+} + I_2$
remaining halides of Fe^{2+} & Fe^{3+} are stable.

8. Which of the following polymer is used in the manufacture of wood laminates ?
(1) *cis*-poly isoprene
(2) Melamine formaldehyde resin
(3) Urea formaldehyde resin
(4) Phenol and formaldehyde resin
Official Ans. by NTA (3)

Sol. Urea –HCHO resin is used in manufacture of wood laminates.

9. **Statement I :** Sodium hydride can be used as an oxidising agent.
Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.
Choose the CORRECT answer from the options given below :
(1) Both statement I and statement II are false
(2) Statement I is true but statement II is false
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are true
Official Ans. by NTA (3)

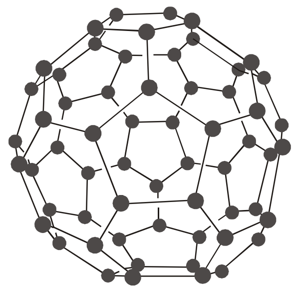
Sol. (1) NaH (sodium Hydride) is used as a reducing reagent.

(2) 

In pyridine, due to free electron on N atom, it is basic in nature.
Hence statement I is false & II is true.

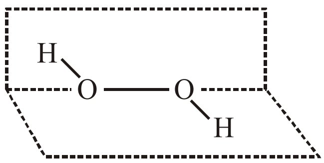
10. The INCORRECT statement regarding the structure of C_{60} is :
(1) The six-membered rings are fused to both six and five-membered rings.
(2) Each carbon atom forms three sigma bonds.
(3) The five-membered rings are fused only to six-membered rings.
(4) It contains 12 six-membered rings and 24 five-membered rings.
Official Ans. by NTA (4)

Sol. Structure of C_{60}



It contain 20 hexagons 20 and 12 pentagons 12 so option 4 is incorrect.

11. The correct statements about H_2O_2 are :
(A) used in the treatment of effluents.
(B) used as both oxidising and reducing agents.
(C) the two hydroxyl groups lie in the same plane.
(D) miscible with water.
Choose the correct answer from the options given below :
(1) (A), (B), (C) and (D)
(2) (A), (B) and (D) only
(3) (B), (C) and (D) only
(4) (A), (C) and (D) only
Official Ans. by NTA (2)

Sol. 

Structure of H_2O_2
(Open book type) → Non planar

H_2O_2 is used in the treatment of effluents.

$H_2O_2 \xrightarrow{R.A.} O_2$
$H_2O_2 \xrightarrow{O.A.} H_2O$
act as both O.A & R.A.

H_2O_2 is miscible in water due to hydrogen bonding.

12. Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is :

(1) to remove basic impurities

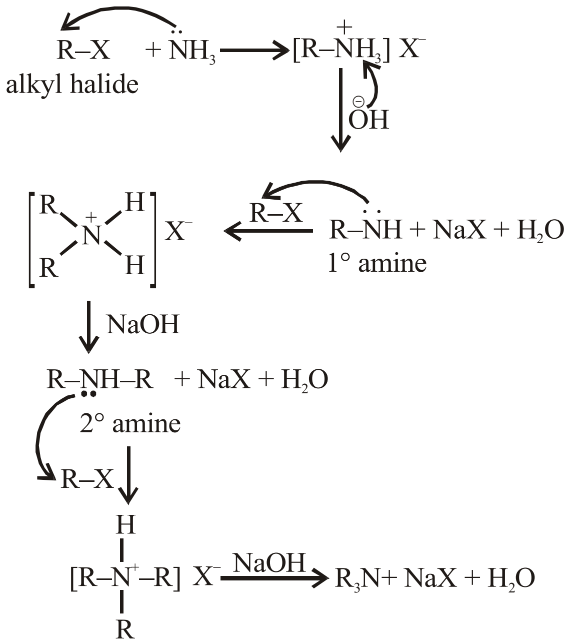
(2) to activate NH_3 used in the reaction

(3) to remove acidic impurities

(4) to increase the reactivity of alkyl halide

Official Ans. by NTA (3)

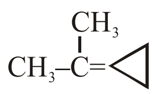
Sol.



So the purpose of NaOH in the above reactions in to remove acidic impurities.

13. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is :

(1) $CH_3–C(CH_3)=C(CH_3)–CH_3$

(2) 

(3) $HC{\equiv}C–CH_2–CH_3$

(4) $CH_3–C{\equiv}C–CH_3$

Official Ans. by NTA (3)

Sol. $\underset{\text{Unsaturated Hydrocarbon}}{(X)} \xrightarrow{\text{Ozonolysis}} (A) \xrightarrow[\text{(Tollen's regent)}]{\text{Ammonical } AgNO_3} \underset{\text{silver mirror}}{Ag\downarrow}$

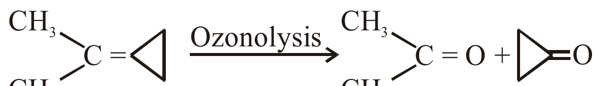
As (A) compound given positive tollen's test hence it may consist –CHO (aldehyde group). or it can be HCOOH

So for the given option :

(3) $CH_3–CH_2–C{\equiv}CH \xrightarrow{\text{Ozonolysis}} CH_3CH_2COOH + \underset{\text{+ve tollen's test}}{\underset{(A)}{HCOOH}}$

and for other compounds (options):

(1) $(CH_3)_2C=C(CH_3)_2 \xrightarrow{\text{Ozonolysis}} (CH_3)_2C=O$ (Does not show tollen's Test)

(2) 

(Both do not show tollen's test)

(3) $CH_3–C{\equiv}C–CH_3 \xrightarrow{\text{Ozonolysis}} 2CH_3–COOH$ (Does not show tollen's test)

14. Which of the following is least basic ?

(1) $(CH_3CO)\ddot{N}HC_2H_5$

(2) $(C_2H_5)_3\ddot{N}$

(3) $(CH_3CO)_2\ddot{N}H$

(4) $(C_2H_5)_2\ddot{N}H$

Official Ans. by NTA (3)

Sol. For the given compounds :

(1) $CH_3–\underset{}{\overset{O}{\overset{\|}{C}}}–\ddot{N}H–C_2H_5$; L.P. on Nitrogen is delocalised.

(2) $CH_3CH_2–\underset{CH_2CH_3}{\underset{|}{\ddot{N}}}–CH_2CH_3$; L.P. on Nitrogen is localised.

(3) $CH_3–\underset{O}{\underset{\|}{C}}–\ddot{N}H–\underset{O}{\underset{\|}{C}}–CH_3$; L.P. on Nitrogen is delocalised due to conjugation with both $–\underset{O}{\underset{\|}{C}}–$

(Hence least basic)

(4) $CH_3–CH_2–\ddot{N}H–CH_2–CH_3$; L.P. on Nitrogen is localised.

15. The characteristics of elements X, Y and Z with atomic numbers, respectively, 33, 53 and 83 are :

(1) X and Y are metalloids and Z is a metal.

(2) X is a metalloid, Y is a non-metal and Z is a metal.

(3) X, Y and Z are metals.

(4) X and Z are non-metals and Y is a metalloid

Official Ans. by NTA (2)

Sol. $X = {}_{33}As \rightarrow$ Metalloid

$Y = {}_{53}I \rightarrow$ Nonmetal

$Z = {}_{83}Bi \rightarrow$ Metal

16. Match List-I with List-II

List-I Test/Reagents/Observation(s)	List-II Species detected
(a) Lassaigne's Test	(i) Carbon
(b) Cu(II) oxide	(ii) Sulphur
(c) Silver nitrate	(iii) N, S, P, and halogen
(d) The sodium fusion extract gives black precipitate with acetic acid and lead acetate	(iv) Halogen Specifically

The correct match is :

(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(2) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Official Ans. by NTA (3)

Sol. Match list :-

(a) Lassaigne's Test	(iii) N, S, P and Halogen
(b) Cu(II) Oxide	(i) Carbon
(c) $AgNO_3$	(iv) Halogen specifically.
(d) Sodium fusion extract given black precipitate with acetic acid and lead acetate ($CH_3COOH/(CH_3COO)_2Pb$)	(ii) Sulphur

Option-(a)-(iii) ; (b)-(i) ; (c)-(iv) ; (d)-(ii)

17. The INCORRECT statements below regarding colloidal solutions is :

(1) A colloidal solution shows colligative properties.

(2) An ordinary filter paper can stop the flow of colloidal particles.

(3) The flocculating power of Al^{3+} is more than that of Na^+.

(4) A colloidal solution shows Brownian motion of colloidal particles.

Official Ans. by NTA (2)

Sol. * Colloidel solution exhibits colligative properties

* An ordinary filter can not stop the flow of colloidal particles.

* Flocculating power increases with increase the opposite charge of electrolyte.

* Colloidal particles show brownian motion.

18. Arrange the following metal complex/ compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.

(Atomic numbers Ce = 58, Gd = 64 and Eu = 63.)

(a) $(NH_4)_2[Ce(NO_3)_6]$ (b) $Gd(NO_3)_3$ and

(c) $Eu(NO_3)_3$

Answer is :

(1) (b) < (a) < (c)
(2) (c) < (a) < (b)
(3) (a) < (b) < (c)
(4) (a) < (c) < (b)

Official Ans. by NTA (4)

Sol. (a) ${}_{58}Ce \rightarrow [Xe]4f^2\ 5d^0\ 6s^2$

In complex $Ce^{4+} \rightarrow [Xe]\ 4f^0\ 5d^0\ 6s^0$

there is no unpaired electron so $\mu_m = 0$

(b) ${}_{64}Gd^{3+} \rightarrow [Xe]4f^7\ 5d^0\ 6s^0$

contain seven unpaired electrons so,

$\mu_m = \sqrt{7(7+2)} = \sqrt{63}$ B.M.

(c) ${}_{63}Eu^{3+} \rightarrow [{}_{54}Xe]4f^6\ 5d^0\ 6s^0$

contain six unpaired electron

so, $\mu_m = \sqrt{6(6+2)} = \sqrt{48}$ B.M.

Hence, order of spin only magnetic movement

$\boxed{b > c > a}$

19. The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M H_3PO_3 solution and 100 mL of 2 M H_3PO_2 solution, respectively, are :

(1) 100 mL and 100 mL
(2) 100 mL and 50 mL
(3) 100 mL and 200 mL
(4) 50 mL and 50 mL

Official Ans. by NTA (3)

Sol. $H_3PO_3 + 2NaOH \rightarrow Na_2HPO_3 + 2H_2O$

50 ml   1M
1M   V = ?

$$\Rightarrow \frac{n_{NaoH}}{n_{H_3PO_3}} = \frac{2}{1}$$

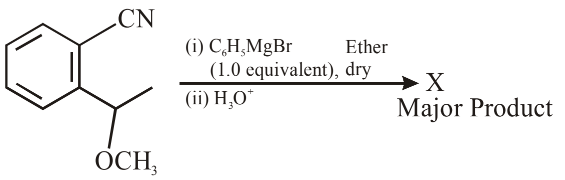
$$\Rightarrow \frac{1\times V}{50\times 1} = \frac{2}{1} \Rightarrow \boxed{V_{NaOH} = 100\,ml}$$

$H_3PO_2 + 2NaOH \rightarrow NaH_2PO_3 + H_2O$

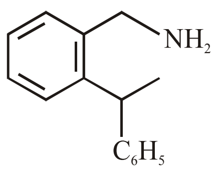
100 ml   1M
2M   V = ?

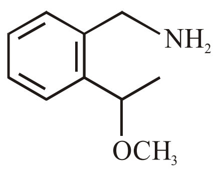
$$\Rightarrow \frac{n_{NaoH}}{n_{H_3PO_3}} = \frac{1}{1} \quad \Rightarrow \frac{1\times V}{2\times 100} = \frac{1}{1} \Rightarrow \boxed{V_{NaOH} = 200\,ml}$$

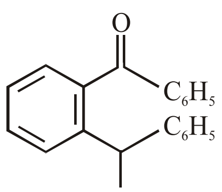
20.

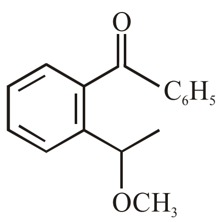


The structure of X is :

(1)


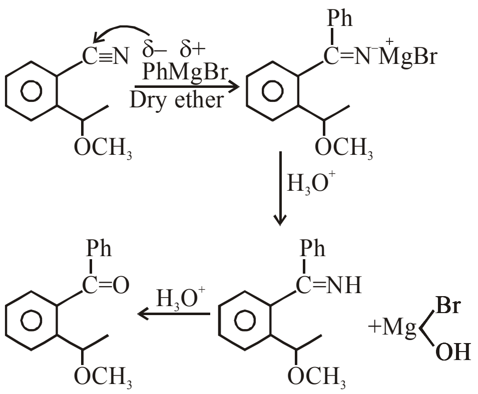
(2)


(3)


(4)


Official Ans. by NTA (4)

Sol.



SECTION-B

1. Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is _______ $\times 10^{21}$. (Round off to the Nearest Integer).

Official Ans. by NTA (15)

Sol. HCP structure : Per atom, there will be one octahedral void (OV) and two tetrahedral voids (TV).

Therefore total three voids per atom are present in HCP structure.

$\rightarrow$ therefore total no of atoms of Ga will be-

$$= \frac{\text{Mass}}{\text{Molar Mass}} \times N_A = \frac{0.581g}{70g/mol} \times 6.023\times 10^{23}$$

$\rightarrow$ Now, total Number of voids = 3 × total no. of atoms

$$= 3\times \frac{0.581}{70}\times 6.023\times 10^{23} = 14.99 \times 10^{21}$$

$$\simeq 15\times 10^{21}$$

2. A 5.0 m mol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell constant 1.3 cm^{-1}. The molar conductivity of this solution is _______ $mSm^2\ mol^{-1}$. (Round off to the Nearest Integer)

Official Ans. by NTA (143)
Official Ans. by ALLEN (14)

Sol. Given conc^n of KCl = $\frac{m.mol}{L}$

: Conductance (G) = 0.55 mS

: Cell constant $\left(\frac{\ell}{A}\right) = 1.3\ cm^{-1}$

To Calculate : Molar conductivity (λ_m) of sol.

$\rightarrow$ Since $\boxed{\lambda_m = \frac{1}{1000}\times\frac{k}{m}}$ ......(1)

$\rightarrow$ Molarity = $5\times 10^{-3}\ \frac{mol}{L}$

$$\rightarrow \text{Conductivity} = G\times\left(\frac{\ell}{A}\right) = 0.55\ mS\times\frac{1.3}{\frac{1}{100}}m^{-1}$$

$= 55 \times 1.3 \qquad mSm^{-1}$

$$\text{eq}^n\ (1) \quad \lambda_m = \frac{1}{1000}\times\frac{55\times 1.3}{\left(\frac{5}{1000}\right)}\ \frac{mSm^2}{mol}$$

$$\Rightarrow \lambda_m = 14.3\frac{mSm^2}{mol}$$

3. A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is ______ min. (Round off to the Nearest Integer).

Official Ans. by NTA (108)

Sol. Given $t_{\frac{1}{2}} = 54$ min $\quad T_{1/2} = 18$ min

A $\quad$ B

t = 0 'x' M $\quad$ t = 0 'x' M

$\Rightarrow$ To calculate : $[A_t] = 16 \times [B_t]$ ....(1) time = ?

$\Rightarrow$ For I order kinetic : $\boxed{[A_t] = \frac{A_0}{(2)^n}}$

n $\rightarrow$ no of Half lives

$\Rightarrow$ Now from the relation (1)

$[A_t] = 16 \times [B_t]$

$\Rightarrow \frac{x}{(2)^{n_1}} = \frac{x}{(2)^{n_2}} \times 16 \quad \Rightarrow \quad (2)^{n_2} = (2)^{n_1} \times (2)^4$

$\Rightarrow n_2 = n_1 + 4 \quad \Rightarrow \quad \frac{t}{(t_{1/2})_2} = \frac{t}{(t_{1/2})_1} + 4$

$\Rightarrow t\left(\frac{1}{18} - \frac{1}{54}\right) = 4 \Rightarrow t = \frac{4 \times 18 \times 54}{36}$

$\Rightarrow \boxed{t = 108\,\text{min}}$

4. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is ______. (Round off to the Nearest Integer).

[Given : Aqueous tension at 287 K = 14 mm of Hg]

Official Ans. by NTA (19)

Sol. In Duma's method of estimation of Nitrogen. 0.1840 gm of organic compound gave 30 mL of nitrogen which is collected at 287 K & 758 mm of Hg.

Given ;

Aqueous tension at 287 K = 14 mm of Hg.

Hence actual pressure = (758 – 14)

= 744 mm of Hg.

Volume of nitrogen at STP = $\frac{273 \times 744 \times 30}{287 \times 760}$

V = 27.935 mL

$\because$ 22400 mL of N_2 at STP weighs = 28 gm.

$\therefore$ 27.94 mL of N_2 at STP weighs =

$\left(\frac{28}{22400} \times 27.94\right)$gm

= 0.0349 gm

Hence % of Nitrogen = $\left(\frac{0.0349}{0.1840} \times 100\right)$

= 18.97 %

Rond off. Answer = 19 %

5. The number of orbitals with n = 5, $m_1 = +2$ is ______. (Round off to the Nearest Integer).

Official Ans. by NTA (3)

Sol. For, n = 5

ℓ = (0, 1, 2, 3, 4)

If ℓ = 0, m = 0

ℓ = 1, m = {–1, 0, +1}

ℓ = 2, m = {–2, –1, 0, +1, **+2**}

ℓ = 3, m = {–3, –2, –1, 0, +1, **+2**, +3}

ℓ = 4, m = {–4, –3, –2, –1, 0, +1, **+2**, +3, +4}

5d, 5f and 5g subshell contain one-one orbital having $m_\ell = +2$

6. At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is ________ kPa. (Round of to the Nearest Integer).

Official Ans. by NTA (19)

Sol. Given $P_A^0 = 21\text{kPa} \quad \Rightarrow \quad P_B^0 = 18\text{kPa}$

$\rightarrow$ An Ideal solution is prepared by mixing 1 mol A and 2 mol B.

$\rightarrow X_A = \frac{1}{3}$ and $X_B = \frac{2}{3}$

$\rightarrow$ Acc to Raoult's low

$P_T = X_A P_A^0 + X_B P_B^0$

$\Rightarrow P_T = \left(\frac{1}{3} \times 21\right) + \left(\frac{2}{3} \times 18\right)$

$\Rightarrow P_T$ = 7 +12 = 19 KPa

7. Sulphurous acid (H_2SO_3) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M H_2SO_3 is _________. (Round off to the Nearest Integer)

Official Ans. by NTA (1)

Sol. H_2SO_3 [Dibasic acid]

c = 0.588 M

$\Rightarrow$ pH of solution Þ due to First dissociation only since $K_{a_1} >> Ka_2$

$\Rightarrow$ First dissociation of H_2SO_3

$$H_2SO_3(aq) \rightleftharpoons H^{\oplus}(aq) + HSO_3^-(aq) : ka_1 = 1.7 \times 10^{-2}$$

t = 0 C

t C–x x x

$$\Rightarrow \quad Ka_1 = \frac{1.7}{100} = \frac{[H^{\oplus}][HSO_3^-]}{[H_2SO_3]}$$

$$\Rightarrow \frac{1.7}{100} = \frac{x^2}{(0.58 - x)}$$

$\Rightarrow$ $1.7 \times 0.588 - 1.7x = 100\,x^2$

$\Rightarrow$ $100x^2 + 1.7x - 1 = 0$

$$\Rightarrow \quad [H^{\oplus}] = x = \frac{-1.7 + \sqrt{(1.7)^2 + 4 \times 100 \times 1}}{2 \times 100} = 0.09186$$

Therefore pH of sol. is : pH = – log $[H^{\oplus}]$

$\Rightarrow$ pH = –log (0.09186) = 1.036 $\simeq$ 1

8. When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, ________ $\times 10^{-5}$ moles of lead sulphate precipitate out. (Round off to the Nearest Integer).

Official Ans. by NTA (525)

Sol. $3\,Pb(NO_3)_2 + Cr_2(SO_4)_3 \rightarrow 3PbSO_4 + 2Cr(NO_3)_3$

35 ml 20 ml

0.15 M 0.12 M

= 5.25 m.mol = 2.4 m.mol 5.25 m.mol

$= 5.25 \times 10^{-3}$ mol

therefore moles of $PbSO_4$ formed $= 5.25 \times 10^{-3}$

$= 525 \times 10^{-5}$

9. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is _______ J.

(Round off to the Nearest Integer)

[Given : R = 8.314 J mol^{-1} K^{-1}. Assume, hydrogen is an ideal gas]

[Atomic mass off Fe is 55.85 u]

Official Ans. by NTA (2218)

Sol. T = 298 K, R = 8.314 $\frac{J}{mol\,K}$

$\rightarrow$ Chemical reaction is

$Fe + 2HCl \rightarrow FeCl_2 + H_2(g)$

50g P = 1 bar

$$= \frac{50}{55.85} mol$$

$$\frac{50}{55.85} mol$$

$\rightarrow$ Work done for 1 mol gas

$= -P_{ext} \times \Delta V$

$= \Delta ng\ RT$

$= -1 \times 8.314 \times 298$ J

$\rightarrow$ Work done for $\frac{50}{55.85}$ mol of gas

$= -1.8314 \times 298 \times \frac{50}{55.85}$ J

= –2218.059 J

$\simeq$ –2218 J

10. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d – d transition. The octahedral splitting energy for the above complex is _____ $\times 10^{-19}$ J. (Round off to the Nearest Integer). h $= 6.626 \times 10^{-34}$ Js; c $= 3 \times 10^8$ ms^{-1}.

Official Ans. by NTA (4)

Sol. $\lambda_{absorbed}$ = 498 nm (given)

The octahedral spilitting energy

$$\Delta_0 \text{ or } E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{498 \times 10^{-9}}$$

$= 0.0399 \times 10^{-17}$ J

$= 3.99 \times 10^{-19}$ J

$= 4.00 \times 10^{-19}$ J (round off)

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The maximum value of

$$f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x\in R \text{ is:}$$

(1) $\sqrt{7}$ (2) $\frac{3}{4}$ (3) $\sqrt{5}$ (4) 5

Official Ans by NTA (3)

Sol. $C_1 + C_2 \to C_1$

$$\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 - R_2 \to R_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

Open w.r.t. R_1

$-(2 \sin 2x - \cos 2x)$

$\cos 2x - 2 \sin 2x = f(x)$

$$f(x)\big|_{max} = \sqrt{1+4} = \sqrt{5}$$

2. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1) $\frac{9}{56}$ (2) $\frac{4}{9}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

Official Ans by NTA (2)

Sol. Total cases :

$\underline{6} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2}$

$n(s) = 6 \cdot 6!$

Favourable cases :

Number divisible by 3 $\equiv$

Sum of digits must be divisible by 3

Case-I

1, 2, 3, 4, 5, 6

Number of ways = 6!

Case-II

0, 1, 2, 4, 5, 6

Number of ways = $5\cdot 5!$

Case-III

0, 1, 2, 3, 4, 5

Number of ways = $5\cdot 5!$

n(favourable) = $6! + 2\cdot 5\cdot 5!$

$$P = \frac{6!+2\cdot 5\cdot 5!}{6\cdot 6!} = \frac{4}{9}$$

3. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x\neq 0 \\ \alpha, & x=0 \end{cases}$$

is continuous at x = 0, where $\{x\} = x - [x]$, [x] is the greatest integer less than or equal to x. Then :

(1) $\alpha = \frac{\pi}{\sqrt{2}}$ (2) $\alpha = 0$

(3) no such α exists (4) $\alpha = \frac{\pi}{4}$

Official Ans by NTA (3)

Sol.

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} (x)$$

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\cdot \sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)}{x\cdot 1\cdot 1}\cdot\frac{\pi}{2}$$

Let $1 - x^2 = \cos\theta$

$$\frac{\pi}{2}\lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2}\lim_{\theta\to 0^+}\frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}}=\frac{\pi}{\sqrt{2}}$$

Now, $\lim\limits_{x\to 0^-}\dfrac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}\left(-\sin^{-1}x\right)}{(1+x)(2+x)(-x)}$$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}}{1\cdot 2}\cdot\frac{\sin^{-1}x}{x}=\frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

4. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}$$

$$+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

(1) 0 (2) 3 (3) 39 (4) –45

Official Ans by NTA (2)

Sol. Plane passing through (42, 0, 0), (0, 42, 0), (0, 0, 42)

From intercept from, equation of plane is

$x + y + z = 42$

$\Rightarrow (x - 11) + (y - 19) + (z - 12) = 0$

let $a = x - 11,\ b = y - 19,\ c = z - 12$

$a + b + c = 0$

Now, given expression is

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

$$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$$

If $a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow 3$

5. Consider the integral

$$I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(1) 9(e – 1) (2) 45(e + 1)
(3) 45(e – 1) (4) 9(e + 1)

Official Ans by NTA (3)

Sol. $I=\int_0^{10}[x]\cdot e^{[x]-x+1}$

$$I=\int_0^1 0dx+\int_1^2 1\cdot e^{2-x}+\int_2^3 2\cdot e^{3-x}+\ldots\ldots+\int_9^{10}9\cdot e^{10-x}dx$$

$$\Rightarrow\ I=\sum_{n=0}^{9}\int_n^{n+1}n\cdot e^{n+1-x}dx$$

$$=-\sum_{n=0}^{9}n\left(e^{n+1-x}\right)_n^{n+1}$$

$$=-\sum_{n=0}^{9}n\cdot(e^0-e^1)$$

$$=(e-1)\sum_{n=0}^{9}n$$

$$=(e-1)\cdot\frac{9\cdot 10}{2}$$

$= 45(e - 1)$

6. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2,1) is :

(1) $x - y = 1$ (2) $2x + y = 5$

(3) $x + 3y = 5$ (4) $x + 2y = 4$

Official Ans by NTA (1)

Sol. Given $y^2 = 4x$

Mirror image on $y = x \Rightarrow$ C : $x^2 = 4y$

$$2x = 4\cdot\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2, 1)

$\Rightarrow$ $y - 1 = 1(x - 2)$

$\Rightarrow$ $x - y = 1$

7. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)\, y = \sin x,\ 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\frac{1}{4}\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

Official Ans by NTA (2)

Sol. $\frac{dy}{dx} + (\tan x)y = \sin x;\ 0 \le x \le \frac{\pi}{3}$

$$\text{I.F.} = e^{\int \tan x\,dx} = e^{\ell n \sec x} = \sec x$$

$$y \sec x = \int \tan x\, dx$$

$$y \sec x = \int \tan x dx$$

$$y \sec x = \ell n |\sec x| + C$$

$x = 0,\ y = 0 \Rightarrow \therefore c = 0$

$$y \sec x = \ell n |\sec x|$$

$$y = \cos x \cdot \ell n |\sec x|$$

$$y|_{x=\frac{\pi}{4}} = \left(\frac{1}{\sqrt{2}}\right)\cdot \ell n\sqrt{2}$$

$$y|_{x=\frac{\pi}{4}} = \frac{1}{2\sqrt{2}}\log_e 2$$

8. Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 6 (3) 8 (4) 7

Official Ans by NTA (4)

Sol. A = {2, 3, 4, 5, ... , 30}

(a, b) $\simeq$ (c, d) $\Rightarrow$ ad = bc

(4, 3) $\simeq$ (c, d) $\Rightarrow$ 4d = 3c

$$\Rightarrow \frac{4}{3} = \frac{c}{d}$$

$\frac{c}{d} = \frac{4}{3}$ & c, d $\in$ {2, 3, ..., 30}

(c, d) = {(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)}

No. of ordered pair = 7

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is euqal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$ (3) $\sqrt{6}$ (4) $\sqrt{10}$

Official Ans by NTA (3)

Sol. $x^2 + y^2 + ax + 2ay + c = 0$

$$2\sqrt{g^2 - c} = 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$$

$$\Rightarrow \frac{a^2}{4} - c = 2 \quad ...(1)$$

$2\sqrt{f^2-c}=2\sqrt{a^2-c}=2\sqrt{5}$

$\Rightarrow\ a^2 - c = 5 \quad ...(2)$

(1) & (2)

$\frac{3a^2}{4}=3 \Rightarrow a = -2 \quad (a < 0)$

$\therefore\ c = -1$

Circle $\Rightarrow x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\frac{1}{2}$

$m_{tangent} = 2$

Equation of tangent

$\Rightarrow (y-2) = 2(x-1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from (0, 0) = $\left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right|=\sqrt{6}$

10. The least value of |z| where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{\left||z|+1\right|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|,$$

$i=\sqrt{-1}$, is equal to :

(1) 3 (2) $\sqrt{5}$ (3) 2 (4) 8

Official Ans by NTA (1)

Sol. $\exp\left(\frac{(|z|+3)(|z|-1)}{\left||z|+1\right|}\ell n\,2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}}(16)$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$

$\Rightarrow \frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$

$\Rightarrow (|z| + 3)(|z| - 1) \geq 3(|z| + 1)$

$|z|^2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow |z|^2 + |z| - 6 \geq 0$

$\Rightarrow (|z| - 3)(|z| + 2) \geq 0 \Rightarrow |z| - 3 \geq 0$

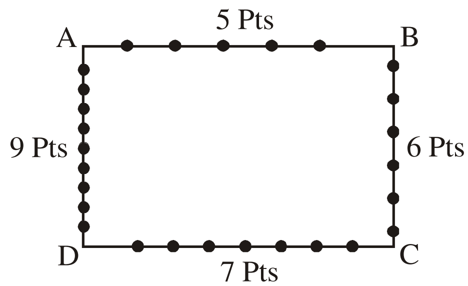
$\Rightarrow |z| \geq 3 \quad \Rightarrow |z|_{min} = 3$

11. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

(1) 795 (2) 1173 (3) 1890 (4) 717

Official Ans by NTA (4)

Sol.



α = Number of triangles

$\alpha = 5{\cdot}6{\cdot}7 + 5{\cdot}7{\cdot}9 + 5{\cdot}6{\cdot}9 + 6{\cdot}7{\cdot}9$

$= 210 + 315 + 270 + 378$

$= 1173$

β = Number of Quadrilateral

$\beta = 5{\cdot}6{\cdot}7{\cdot}9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

12. If the point of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(1) 12 (2) 5 (3) 6 (4) 10

Official Ans by NTA (1)

Sol. $y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so $x^2 = b$

$\frac{x^2}{16}+\frac{3x^2}{b^2}=1$

$$\frac{b}{16}+\frac{3}{b}=1$$

$b^2 - 16b + 48 = 0$

$(b - 12)(b - 4) = 0$

$b = 12,\ b > 4$

13. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 0

Official Ans by NTA (3)

Sol. $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$

$$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$$

$$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$$

$x = 0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get

$16(25-9x^2) = 625+9(25-16x^2)-150\sqrt{25-16x^2}$

$400 = 625 + 225 - 150\sqrt{25-16x^2}$

$\sqrt{25-16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$

$\Rightarrow x^2 = 1$

Put x = 0, 1, –1 in the original equation

We see that all values satisfy the original equation.

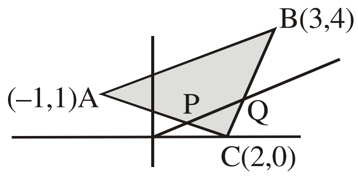
Number of solution = 3

14. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

Official Ans by NTA (2)

Sol.



$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$$A_1=\frac{1}{2}\begin{vmatrix}3 & 4 & 1\\ 2 & 0 & 1\\ -1 & 1 & 1\end{vmatrix}=\frac{13}{2}$$

$$A_2=\frac{1}{2}\begin{vmatrix}x_1 & mx_1 & 1\\ x_2 & mx_2 & 1\\ 2 & 0 & 1\end{vmatrix}$$

$$A_2=\frac{1}{2}|2(mx_1-mx_2)|=m|x_1-x_2|$$

$A_1 = 3A_2 \Rightarrow \frac{13}{2} = 3m|x_1-x_2|$

$\Rightarrow |x_1-x_2|=\frac{16}{6m}$

AC : x + 3y = 2

BC : y = 4x – 8

P : x + 3y = 2 & y = mx $\Rightarrow x_1 = \frac{2}{1+3m}$

Q : y = 4x – 8 & y = mx $\Rightarrow x_2 = \frac{8}{4-m}$

$$|x_1-x_2|=\left|\frac{2}{1+3m}-\frac{8}{4-m}\right|$$

$$=\left|\frac{-26m}{(1+3m)(4-m)}\right| = \frac{26m}{(3m+1)|m-4|}$$

$= \dfrac{26m}{(3m+1)(4-m)}$

$|x_1 - x_2| = \dfrac{13}{6m}$

$\dfrac{26m}{(3m+1)(4-m)} = \dfrac{13}{6m}$

$\Rightarrow\ 12m^2 = -(3m + 1)(m - 4)$

$\Rightarrow\ 12m^2 = -(3m^2 - 11m - 4)$

$\Rightarrow\ 15m^2 - 11m - 4 = 0$

$\Rightarrow\ 15m^2 - 15m + 4m - 4 = 0$

$\Rightarrow\ (15m + 4)(m - 1) = 0$

$\Rightarrow\ m = 1$

15. Let f be a real valued function, defined on $R - \{-1, 1\}$ and given by

$f(x) = 3\log_e\left|\dfrac{x-1}{x+1}\right| - \dfrac{2}{x-1}$.

Then in which of the following intervals, function f(x) is increasing?

(1) $(-\infty,-1)\cup\left(\left[\frac{1}{2},\infty\right)-\{1\}\right)$

(2) $(-\infty, \infty) - \{-1, 1\}$

(3) $\left(-1,\frac{1}{2}\right]$

(4) $\left(-\infty,\frac{1}{2}\right]-\{-1\}$

Official Ans by NTA (1)

Sol. $f(x) = 3\ell n(x-1) - 3\ell n(x+1) - \dfrac{2}{x-1}$

$f'(x) = \dfrac{3}{x-1} - \dfrac{3}{x+1} + \dfrac{2}{(x-1)^2}$

$f'(x) = \dfrac{4(2x-1)}{(x-1)^2(x+1)}$

$f'(x) \geq 0$

$\Rightarrow\ x \in (-\infty,-1)\cup\left[\frac{1}{2},1\right)\cup(1,\infty)$

16. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x + 1) = xf(x)$. If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

(1) $\dfrac{205}{144}$ (2) $\dfrac{197}{144}$ (3) $\dfrac{187}{144}$ (4) 1

Official Ans by NTA (1)

Sol. $\ell nf(x + 1) = \ell n(xf(x))$

$\ell nf(x + 1) = \ell nx + \ell nf(x)$

$\Rightarrow\ g(x + 1) = \ell nx + g(x)$

$\Rightarrow\ g(x + 1) - g(x) = \ell nx$

$\Rightarrow\ g''(x + 1) - g''(x) = -\dfrac{1}{x^2}$

Put x = 1, 2, 3, 4

$g''(2) - g''(1) = -\dfrac{1}{1^2}$ ...(1)

$g''(3) - g''(2) = -\dfrac{1}{2^2}$ ...(2)

$g''(4) - g''(3) = -\dfrac{1}{3^2}$ ...(3)

$g''(5) - g''(4) = -\dfrac{1}{4^2}$ ...(4)

Add all the equation we get

$g''(5) - g''(1) = -\dfrac{1}{1^2} - \dfrac{1}{2^2} - \dfrac{1}{3^2} - \dfrac{1}{4^2}$

$|g''(5) - g''(1)| = \dfrac{205}{144}$

17. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by $(x - 2)$. Then the value of $9(b + c)$ is equal to:

(1) 9 (2) 15 (3) 7 (4) 11

Official Ans by NTA (3)

Sol. $\int_0^1 (x^2+bx+c)dx = 1$

$\frac{1}{3}+\frac{b}{2}+c=1 \quad \Rightarrow \quad \frac{b}{2}+c=\frac{2}{3}$

$3b+6c=4$ ...(1)

P(2) = 5

4 + 2b + c = 5

2b + c = 1 ...(2)

From (1) & (2)

$b = \frac{2}{9}$ & $c = \frac{5}{9}$

9(b + c) = 7

18. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l}=\frac{y-2}{3}=\frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans by NTA (2)

Sol. (3,5,7) satisfy the line L_1

$\frac{3-a}{\ell}=\frac{5-2}{3}=\frac{7-b}{4}$

$\frac{3-a}{\ell}=1$ & $\frac{7-b}{4}=1$

$a + \ell = 3$ ...(1) & b = 3 ...(2)

$\vec{v}_1 = <4,3,8> - <3,5,7>$

$\vec{v}_1 = <1,-2,1>$

$\vec{v}_2 = <\ell,3,4>$

$\vec{v}_1.\vec{v}_2 = 0 \quad \Rightarrow \quad \ell-6+4=0 \quad \Rightarrow \quad \ell = 2$

$a+\ell=3 \quad \Rightarrow \quad a = 1$

$L_1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

$L_2 : \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

A = <1, 2, 3>

B = <2, 4, 5>

$\overrightarrow{AB} = <1,2,2>$

$\vec{p} = 2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{q} = 3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{p}\times\vec{q} = -\hat{i}+2\hat{j}-\hat{k}$

Shortest distance = $\left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \frac{1}{\sqrt{6}}$

19. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2$, x > 0. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1,1), then the area enclosed by the curves C_1 and C_2 is equal to :

(1) $\pi - 1$ (2) $\frac{\pi}{2}-1$ (3) $\pi + 1$ (4) $\frac{\pi}{4}+1$

Official Ans by NTA (2)

Sol. $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$, $x \in (0, \infty)$

put y = vx

$x\frac{dv}{dx}+v=\frac{v^2-1}{2v}$

$\frac{2v}{v^2+1}dv=-\frac{dx}{x}$

Integrate,

$ln(v^2 + 1) = - ln x + C$

$ln\left(\frac{y^2}{x^2}+1\right)=-ln x+C$

put x = 1, y = 1, C = $ln2$

$$ln\left(\frac{y^2}{x^2}+1\right)=-lnx+ln2$$

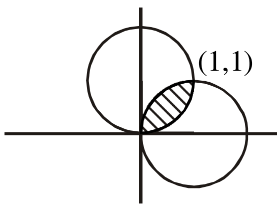
$\Rightarrow$ $x^2 + y^2 - 2x = 0$ (Curve C_1)

Similarly,

$$\frac{dy}{dx}=\frac{2xy}{x^2-y^2}$$

Put y = vx

$x^2 + y^2 - 2y = 0$



required area = $2\int_0^1\left(\sqrt{2x-x^2}-x\right)dx$ = $\frac{\pi}{2}$ – 1

20. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1$, $\alpha \in$ R, then the value of $\alpha+\left|\vec{r}\right|^2$ is equal to :

(1) 9 (2) 15 (3) 13 (4) 11

Official Ans by NTA (2)

Sol. $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$ $\Rightarrow$ $\vec{r}\times(\vec{a}+\vec{b})=0$

$\vec{r}=\vec{\lambda}(\vec{a}+\vec{b})$ $\Rightarrow$ $\vec{r}=\vec{\lambda}(\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-3\hat{j}+5\hat{k})$

$\vec{r}=\vec{\lambda}(3\hat{i}-\hat{j}+2\hat{k})$ ...(1)

$\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k})=3$

Put $\vec{r}$ from (1) $\alpha\lambda=1$ ...(2)

$\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\hat{k})=-1$

Put $\vec{r}$ from (1) $2\lambda\alpha-\lambda=1$ ...(3)

Solve (2) & (3)

α = 1, $\lambda=1$

$\Rightarrow$ $\vec{r}=3\hat{i}-\hat{j}+2\hat{k}$

$\left|\vec{r}\right|^2=14$ & α = 1

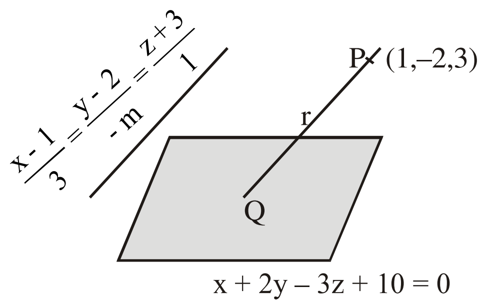
$\alpha+\left|\vec{r}\right|^2$ = 15

SECTION-B

1. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3}=\frac{2-y}{m}=\frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to ______.

Official Ans by NTA (2)

Sol.



$$\text{DC of line} \equiv \left(\frac{3}{\sqrt{m^2+10}},\frac{-m}{\sqrt{m^2+10}},\frac{1}{\sqrt{m^2+10}}\right)$$

$$Q\equiv\left(1+\frac{3r}{\sqrt{m^2+10}},-2+\frac{-mr}{\sqrt{m^2+10}},3+\frac{r}{\sqrt{m^2+10}}\right)$$

Q lies on x + 2y – 3z + 10 = 0

$$1+\frac{3r}{\sqrt{m^2+10}}-4-\frac{2mr}{\sqrt{m^2+10}}-9-\frac{3r}{\sqrt{m^2+10}}+10=0$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(3-2m-3)=2$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(-2m)=2$$

$r^2m^2 = m^2 + 10$

$\frac{7}{2}m^2 = m^2 + 10 \Rightarrow \frac{5}{2}m^2 = 10 \Rightarrow m^2 = 4$

$|m| = 2$

2. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to __________.

Official Ans by NTA (5)

Sol. $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1n_2}{(n_1+n_2)}(\bar{x}_1 - \bar{x}_2)^2$

$n_1 = 10$, $n_2 = n$, $\sigma_1^2 = 2$, $\sigma_2^2 = 1$

$\bar{x}_1 = 2$, $\bar{x}_2 = 3$, $\sigma^2 = \frac{17}{9}$

$\frac{17}{9} = \frac{10\times 2 + n}{n+10} + \frac{10n}{(n+10)^2}(3-2)^2$

$\Rightarrow \frac{17}{9} = \frac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340\,n = 90n + 9(n^2+30n+200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow (2n + 5)(n - 5) = 0 \Rightarrow n = \underset{\substack{\downarrow \\ \text{(Rejected)}}}{\frac{-5}{2}}, 5$

Hence n = 5

3. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $(k^2+1)b_2^2 \neq -2b_1b_2$, then the value of k is _______.

Official Ans by NTA (1)

Sol. A = XB

$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}a_1 \\ \sqrt{3}a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$b_1 - b_2 = \sqrt{3}a_1$ ...(1)

$b_1 + kb_2 = \sqrt{3}a_2$ ...(2)

Given, $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$(1)^2 + (2)^2$

$(b_1 + b_2)^2 + (b_1 + kb_2)^2 = 3(a_1^2 + a_2^2)$

$a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{(1+k^2)}{3}b_2^2 + \frac{2}{3}b_1b_2(k-1)$

Given, $a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2$

On comparing we get

$\frac{k^2+1}{3} = \frac{2}{3} \Rightarrow k^2 + 1 = 2$

$\Rightarrow k = \pm 1$ ....(3)

& $\frac{2}{3}(k-1) = 0 \Rightarrow k = 1$ ...(4)

From both we get k = 1

4. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$= \alpha \log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)$

$+\beta \tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right) + \delta \tan^{-1}\left(\frac{x^2+1}{x}\right) + C$

where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to __________.

Official Ans by NTA (6)

Sol. $\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)} + \int \frac{dx}{x^4+3x^2+1}$

$$\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)} + \frac{1}{2}\int \frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right) = t$

$$\int \frac{dt}{t} + \frac{1}{2}\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5} - \frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x-\frac{1}{x} = y$, $x+\frac{1}{x} = z$

$$\log_e t + \frac{1}{2}\int \frac{dy}{y^2+5} - \frac{1}{2}\int \frac{dz}{z^2+1}$$

$$= \log_e \tan^{-1}\left(x+\frac{1}{x}\right) + \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

$\alpha = 1$, $\beta = \frac{1}{2\sqrt{5}}$, $\gamma = \frac{1}{\sqrt{5}}$, $\delta = \frac{-1}{2}$

or

$\alpha = 1$, $\beta = \frac{-1}{2\sqrt{5}}$, $\gamma = \frac{-1}{\sqrt{5}}$, $\delta = \frac{-1}{2}$

$10(\alpha + \beta\gamma + \delta) = 10\left(1+\frac{1}{10}-\frac{1}{2}\right) = 6$

5. Let $f : R \to R$ and $g : R \to R$ be defined as

$f(x) = \begin{cases} x+a, & x<0 \\ |x-1|, & x \ge 0 \end{cases}$ and

$g(x) = \begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x \ge 0 \end{cases}$

where a, b are non-negative real numbers. If (gof)(x) is continuous for all $x \in R$, then a + b is equal to ___________.

Official Ans by NTA (1)

Sol. $g[f(x)] = \left[\begin{array}{ll} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x) \ge 0 \end{array}\right.$

$$g[f(x)] = \left[\begin{array}{ll} x+a+1 & x+a<0 \ \&\ x<0 \\ |x-1|+1 & |x-1|<0 \ \&\ x \ge 0 \\ (x+a-1)^2+b & x+a \ge 0 \ \&\ x<0 \\ (|x-1|-1)^2+b & |x-1| \ge 0 \ \&\ x \ge 0 \end{array}\right.$$

$$g[f(x)] = \left[\begin{array}{ll} x+a+1 & x \in (-\infty,-a) \ \&\ x \in (-\infty,0) \\ |x-1|+1 & x \in \phi \\ (x+a-1)^2+b & x \in [-a,\infty) \ \&\ x \in (-\infty,0) \\ (|x-1|-1)^2+b & x \in R \ \&\ x \in [0,\infty) \end{array}\right.$$

$$g[f(x)] = \left[\begin{array}{ll} x+a+1 & x \in (-\infty,-a) \\ (x+a-1)^2+b & x \in [-a,0) \\ (|x-1|-1)^2+b & x \in [0,\infty) \end{array}\right.$$

g(f(x)) is continuous

at $x = -a$ & at $x = 0$

$1 = b + 1$ & $(a-1)^2 + b = b$

$b = 0$ & $a = 1$

$\Rightarrow a + b = 1$

6. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a},\frac{1}{b},6$ be in A.P., where a, b > 0. Then 72(a + b) is equal to ________.

Official Ans by NTA (14)

Sol. $a^2 = \frac{b}{16} \Rightarrow \frac{1}{b} = \frac{1}{16a^2}$

$$\frac{2}{b} = \frac{1}{a} + 6$$

$$\frac{1}{8a^2} = \frac{1}{a} + 6$$

$$\frac{1}{a^2} - \frac{8}{a} - 48 = 0$$

$$\frac{1}{a} = 12, -4 \quad \Rightarrow \quad a = \frac{1}{12}, -\frac{1}{4}$$

$$a = \frac{1}{12},\ a > 0$$

$$b = 16a^2 = \frac{1}{9}$$

$\Rightarrow 72(a + b) = 6 + 8 = 14$

7. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _______.

Official Ans by NTA (15)

Sol. $\Delta = \frac{1}{2}.5.12.\sin A = 30$

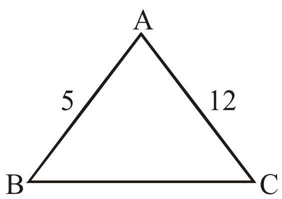
$\sin A = 1$

$A = 90° \Rightarrow BC = 13$

$BC = 2R = 13$

$r = \frac{\Delta}{S} = \frac{30}{15} = 2$

$2R + r = 15$



8. Let n be a positive integer. Let

$$A = \sum_{k=0}^{n} (-1)^k\, {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to _______.

Official Ans by NTA (6)

Sol. $A = \sum_{k=0}^{n} {}^nC_k \left[\left(-\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k + \left(\frac{-7}{8}\right)^k + \left(\frac{-15}{16}\right)^k + \left(\frac{-37}{32}\right)^k\right]$

$$A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$$

$$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$$

$$A = \frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2^n}\right)^5}{1-\frac{1}{2^n}}\right) \Rightarrow A = \frac{\left(1-\frac{1}{2^{5n}}\right)}{(2^n-1)}$$

$(2^n - 1)A = 1 - \frac{1}{2^{5n}}$, Given $63A = 1 - \frac{1}{2^{30}}$

Clearly $5n = 30$

$n = 6$

9. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

If $\vec{c}.\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$ then the value of $\vec{c}.\left(\vec{a} \times \vec{b}\right)$ is equal to _________.

Official Ans by NTA (28)

Sol. $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$(\vec{a} \times \vec{b}) = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{c}\cdot(\hat{i} + \hat{j} + 3\hat{k}) = \lambda(3\hat{i} - 2\hat{j} + \hat{k}) . (\hat{i} + \hat{j} + 3\hat{k})$$

$\Rightarrow \lambda(4)=8 \Rightarrow \lambda=2$

$\vec{c}=2(\vec{a}\times\vec{b})$

$\vec{c}.(\vec{a}\times\vec{b})=2|\vec{a}\times\vec{b}|^2=28$

10. Let

$S_n(x)=\log_{a^{1/2}}x+\log_{a^{1/3}}x+\log_{a^{1/6}}x$

$+\log_{a^{1/11}}x+\log_{a^{1/18}}x+\log_{a^{1/27}}x+.....$

up to n-terms, where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _______.

Official Ans by NTA (16)

Sol. $S_n(x) = (2+3+6+11+18+27+.......+\text{n-terms})\log_a x$

Let $S_1 = 2 + 3 + 6 + 11 + 18 + 27 + ... + T_n$

$S_1 = 2 + 3 + 6 + ... + T_n$

$T_n = 2 + 1 + 3 + 5 + ... + \text{n terms}$

$T_n = 2 + (n-1)^2$

$S_1 = \Sigma T_n = 2n + \dfrac{(n-1)n(2n-1)}{6}$

$\Rightarrow S_n(x)=\left(2n+\dfrac{n(n-1)(2n-1)}{6}\right)\log_a x$

$S_{24}(x) = 1093$ (Given)

$\log_a x\left(48+\dfrac{23.24.47}{6}\right)=1093$

$\log_a x = \dfrac{1}{4}$ .... (1)

$S_{12}(2x) = 265$

$S_{12}(2x) = 265$

$\log_a(2x)\left(24+\dfrac{11.12.23}{6}\right)=265$

$\log_a 2x = \dfrac{1}{2}$ .... (2)

(2) – (1)

$\log_a 2x - \log_a x = \dfrac{1}{4}$

$\log_a 2 = \dfrac{1}{4} \Rightarrow a = 16$

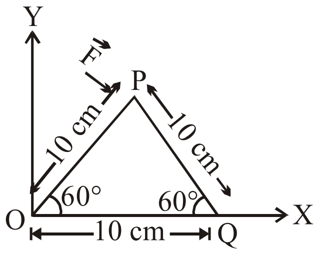
FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. A triangular plate is shown. A force $\vec{F}=4\hat{i}-3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are :



(1) $-15-20\sqrt{3}$, $15-20\sqrt{3}$

(2) $15+20\sqrt{3}$, $15-20\sqrt{3}$

(3) $15-20\sqrt{3}$, $15+20\sqrt{3}$

(4) $-15+20\sqrt{3}$, $15+20\sqrt{3}$

Official Ans. by NTA (1)

Sol. $\vec{F}=4\hat{i}-3\hat{j}$

$\vec{r}_1=5\hat{i}+5\sqrt{3}\hat{j}$ & $\vec{r}_2=-5\hat{i}+5\sqrt{3}\hat{j}$

Torque about 'O'

$$\vec{\tau}_O=\vec{r}_1\times\vec{F}=\left(-15-20\sqrt{3}\right)\hat{k}=\left(15+20\sqrt{3}\right)\left(-\hat{k}\right)$$

Torque about 'Q'

$$\vec{\tau}_Q=\vec{r}_2\times\vec{F}=\left(-15+20\sqrt{3}\right)\hat{k}=\left(15-20\sqrt{3}\right)\left(-\hat{k}\right)$$

2. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is :

(1) $\frac{ab}{b-a}$ (2) $\frac{a+b}{ab}$

(3) $\frac{b-a}{ab}$ (4) $\frac{ab}{a+b}$

Official Ans. by NTA (1)

Sol. Excess pressure at common surface is given by

$$P_{ex}=4T\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{4T}{r}$$

$$\therefore \frac{1}{r}=\frac{1}{a}-\frac{1}{b}$$

$$r=\frac{ab}{b-a}$$

3. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ?

(1) 1.03 (2) 1.30

(3) 1.37 (4) 10.3

Official Ans. by NTA (1)

Sol. Since each vibrational mode has 2 degrees of freedom hence total vibrational degrees of freedom = 48

f = 3 + 3 + 48 = 54

$$\gamma=1+\frac{2}{f}=\frac{28}{27}=1.03$$

4. If an electron is moving in the n^{th} orbit of the hydrogen atom, then its velocity (v_n) for the n^{th} orbit is given as :

(1) $v_n\propto n$ (2) $v_n\propto\frac{1}{n}$

(3) $v_n\propto n^2$ (4) $v_n\propto\frac{1}{n^2}$

Official Ans. by NTA (2)

Sol. We know velocity of electron in n^{th} shell of hydrogen atom is given by

$$v=\frac{2\pi kZe^2}{nh}$$

$$\therefore v\propto\frac{1}{n}$$

5. An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light)

(1) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

(3) $\left(\frac{E}{2m}\right)^{1/2}$ (4) $c\,(2mE)^{1/2}$

Official Ans. by NTA (2)

Sol. $\lambda_1 = \frac{h}{\sqrt{2mE}}$

$\lambda_2 = \frac{hc}{E}$

$\frac{\lambda_1}{\lambda_2} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

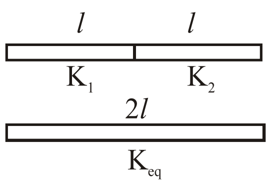
6. Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is :

(1) $\frac{2K_1K_2}{K_1+K_2}$ (2) $\frac{K_1+K_2}{2K_1K_2}$

(3) $\frac{K_1+K_2}{K_1K_2}$ (4) $\frac{K_1K_2}{K_1+K_2}$

Official Ans. by NTA (1)

Sol.



$R_{eff} = \frac{l}{K_1A} + \frac{l}{K_2A} = \frac{2l}{K_{eq}A}$

$K_{eq} = \frac{2K_1K_2}{K_1+K_2}$

7. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is_____ cm.
(least count = 0.01 cm)

(1) 8.36 cm (2) 8.54 cm
(3) 8.58 cm (4) 8.56 cm

Official Ans. by NTA (2)

Sol. Positive zero error = 0.2 mm
Main scale reading = 8.5 cm
Vernier scale reading = 6 × 0.01 = 0.06 cm
Final reading = 8.5 + 0.06 – 0.02 = 8.54 cm

8. An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading :

(1) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$ (2) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$

(3) $\frac{I_1+I_2}{\sqrt{2}}$ (4) $\frac{I_1+I_2}{2\sqrt{2}}$

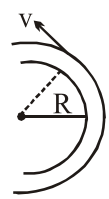
Official Ans. by NTA (2)

Sol. $I = I_1 \sin \omega t + I_2 \cos \omega t$

$\therefore I_0 = \sqrt{I_1^2 + I_2^2}$

$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2+I_2^2}{2}}$

9. A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is :
(Assume forces on the four tyres are identical and g = acceleration due to gravity)



(1) $m\left(\frac{v^2}{\mu_s R} + g\right)$ (2) $m\left(\frac{v^2}{\mu_s R} - g\right)$

(3) $m\left(g - \frac{v^2}{\mu_s R}\right)$ (4) $-m\left(g + \frac{v^2}{\mu_s R}\right)$

Official Ans. by NTA (2)

Sol. $\mu_s N = \dfrac{mv^2}{R}$

$N = \dfrac{mv^2}{\mu_s R} = mg + F_L$

$F_L = \dfrac{mv^2}{\mu_s R} - mg$

10. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is :

(1) $\dfrac{4\alpha\beta}{(\alpha+\beta)}t^2$ (2) $\dfrac{2\alpha\beta}{(\alpha+\beta)}t^2$

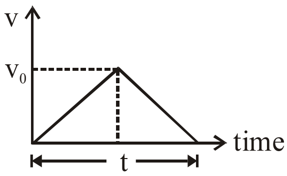
(3) $\dfrac{\alpha\beta}{2(\alpha+\beta)}t^2$ (4) $\dfrac{\alpha\beta}{4(\alpha+\beta)}t^2$

Official Ans. by NTA (3)

Sol. $v_0 = \alpha t_1$ and $0 = v_0 - \beta t_2 \Rightarrow v_0 = \beta t_2$

$t_1 + t_2 = t$

$v_0\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right) = t$



$\Rightarrow v_0 = \dfrac{\alpha\beta t}{\alpha+\beta}$

Distance = area of v-t graph

$= \dfrac{1}{2} \times t \times v_0 = \dfrac{1}{2} \times t \times \dfrac{\alpha\beta t}{\alpha+\beta} = \dfrac{\alpha\beta t^2}{2(\alpha+\beta)}$

11. A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5A. The magnetic flux density produced by the solenoid is :

(permeability of free space = $4\pi \times 10^{-7}$ H/m)

(1) πT (2) $2 \times 10^{-3}\ \pi T$

(3) $\dfrac{\pi}{5} T$ (4) $10^{-4}\pi T$

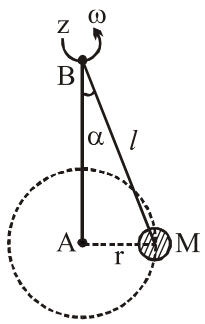
Official Ans. by NTA (1)

Sol. $B = \mu n I = \mu_0 \mu_r n I$

$B = 4\pi \times 10^{-7} \times 500 \times 1000 \times 5$

$B = \pi$ Tesla

12. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about B is L_B. The correct statement for this system is :



(1) L_A and L_B are both constant in magnitude and direction

(2) L_B is constant in direction with varying magnitude

(3) L_B is constant, both in magnitude and direction

(4) L_A is constant, both in magnitude and direction

Official Ans. by NTA (4)

Sol. We know, $\vec{L} = m(\vec{r}\times\vec{v})$

Now with respect to A, we always get direction of $\vec{L}$ along +ve z-axis and also constant magnitude as mvr. But with respect to B, we get constant magnitude but continuously changing direction.

13. For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal ?

(1) x = 0 (2) x = ± A

(3) $x = \pm \dfrac{A}{\sqrt{2}}$ (4) $x = \dfrac{A}{2}$

Official Ans. by NTA (3)

Sol. KE = PE

$$\frac{1}{2}m\omega^2 (A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$A^2 - x^2 = x^2$

$2x^2 = A^2$

$$x = \pm\frac{A}{\sqrt{2}}$$

14. A Carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is :

(1) 3200 J (2) 1800 J

(3) 1600 J (4) 2400 J

Official Ans. by NTA (4)

Sol. $\eta = \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{Q_1 - W}{Q_1}$ $(\because W = Q_1 - Q_2)$

$$\frac{400}{800} = 1 - \frac{W}{Q_1}$$

$$\frac{W}{Q_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

$Q_1 = 2W = 2400$ J

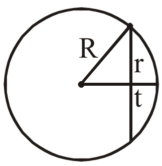
15. The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}. The focal length of the lens is _____.

(1) 0.30 cm (2) 15 cm

(3) 1.5 cm (4) 30 cm

Official Ans. by NTA (4)

Sol. $R^2 = r^2 + (R - t)^2$



$R^2 = r^2 + R^2 + t^2 - 2Rt$

Neglecting t^2, we get

$$R = \frac{r^2}{2t}$$

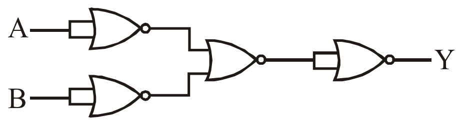
$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

$$f = \frac{R}{\mu - 1} = \frac{r^2}{2t(\mu - 1)} = \frac{(3\times10^{-2})^2}{2\times3\times10^{-3}\times\left(\frac{3}{2}-1\right)}$$

$$= \frac{9\times10^{-4}}{6\times10^{-3}\times1} \times 2$$

$f = 0.3$ m $= 30$ cm

16. The output of the given combination gates represents :

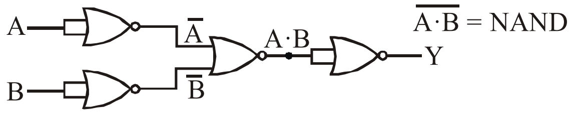


(1) XOR Gate (2) NAND Gate

(3) AND Gate (4) NOR Gate

Official Ans. by NTA (2)

Sol. By De Morgan's theorem, we have



17. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now ?

(1) 19.0 ms^{-1} (2) 4.47 ms^{-1}

(3) 14.41 ms^{-1} (4) 1.00 ms^{-1}

Official Ans. by NTA (2)

Sol. Given, m = 0.5 kg and u = 20 m/s

Initial kinetic energy $(k_i) = \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.5 \times 20 \times 20 = 100 \text{ J}$$

After deflection it moves with 5% of k_i

$$\therefore k_f = \frac{5}{100} \times k_i \Rightarrow \frac{5}{100} \times 100$$

$\Rightarrow k_f = 5$ J

Now, let the final speed be 'v' m/s, then :

$$k_f = 5 = \frac{1}{2}mv^2$$

$\Rightarrow v^2 = 20$

$\Rightarrow v = \sqrt{20} = 4.47$ m/s

18. Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom?

(1) 1 (2) 6

(3) 4 (4) 8

Official Ans. by NTA (2)

Sol. Energy of H-atom is $E = -13.6\ Z^2/n^2$

for H-atom Z = 1 & for ground state, n = 1

$$\Rightarrow E = -13.6 \times \frac{1^2}{1^2} = -13.6\ eV$$

Now for carbon atom (single ionised), Z = 6

$$E = -13.6\ \frac{Z^2}{n^2} = -13.6 \qquad \text{(given)}$$

$\Rightarrow n^2 = 6^2 \Rightarrow n = 6$

19. Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2, m_1 and m_2, n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is :

(1) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ (2) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$

(3) $\frac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$ (4) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$

Official Ans. by NTA (2)

Sol. Let the final temperature of the mixture be T. Since, there is no loss in energy.

$\Delta U = 0$

$$\Rightarrow \frac{F_1}{2} n_1R\ \Delta T + \frac{F_2}{2}\ n_2R\Delta T = 0$$

$$\Rightarrow \frac{F_1}{2} n_1R\ (T_1 - T) + \frac{F_2}{2} n_2R\ (T_2 - T) = 0$$

$$\Rightarrow T = \frac{F_1n_1RT_1 + F_2n_2RT_2}{F_1n_1R + F_2n_2R} \Rightarrow \frac{F_1n_1T_1 + F_2n_2T_2}{F_1n_1 + F_2n_2}$$

20. A current of 10A exists in a wire of crosssectional area of 5 mm^2 with a drift velocity of $2 \times 10^{-3}\ ms^{-1}$. The number of free electrons in each cubic meter of the wire is ___.

(1) 2×10^6 (2) 625×10^{25}

(3) 2×10^{25} (4) 1×10^{23}

Official Ans. by NTA (2)

Sol. $i = 10A$, $A = 5\ mm^2 = 5 \times 10^{-6}\ m^2$

and $v_d = 2 \times 10^{-3}$ m/s

We know, $i = neAvd$

$\therefore\ 10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}$

$\Rightarrow n = 0.625 \times 10^{28} = 625 \times 10^{25}$

SECTION-B

1. For VHF signal broadcasting, ____ km^2 of maximum service area will be covered by an antenna tower of height 30m, if the receiving antenna is placed at ground. Let radius of the earth be 6400 km. (Round off to the Nearest Integer) (Take π as 3.14)

Official Ans. by NTA (1206)

Sol. $d = \sqrt{2Rh}$

$A = \pi d^2$

$A = \pi 2Rh$

$$= 3.14 \times 2 \times 6400 \times \frac{30}{1000}$$

$A = 1205.76\ km^2$

$A \simeq 1206\ km^2$

2. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is _______.

(Assuming the acceleration to be uniform).

Official Ans. by NTA (728)

Sol. We know, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N

$$\therefore\ 2\pi N = 2\pi\ \left(\frac{900 + 2460}{60 \times 2}\right) \times 26$$

N = 728

3. The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If s = np, then the minimum value for n is ___. (Round off to the Nearest Integer)

Official Ans. by NTA (4)

Sol. $R_1 + R_2 = s$ ... (1)

$$\frac{R_1R_2}{R_1+R_2} = p \quad ... (2)$$

$R_1R_2 = sp$

$R_1R_2 = np^2$

$$R_1 + R_2 = \frac{nR_1R_2}{(R_1+R_2)}$$

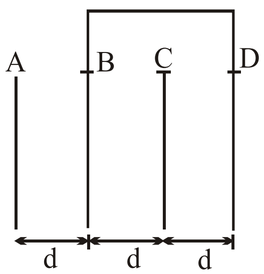
$$\frac{(R_1+R_2)^2}{R_1R_2} = n$$

for minimum value of n

$R_1 = R_2 = R$

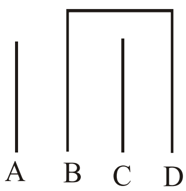
$$\therefore \ n = \frac{(2R)^2}{R^2} = 4$$

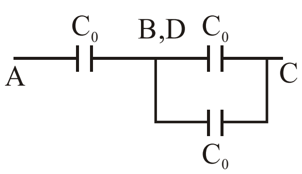
4. Four identical rectangular plates with length, l = 2 cm and breadth, b = $\frac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\varepsilon_0}{d}$. The value of x is ___.
(Round off to the Nearest Integer)



Official Ans. by NTA (2)

Sol.





$$C_{eq} = \frac{2C_0}{3} = \frac{2}{3}\frac{\epsilon_0 A}{d}$$

$$C_{eq} = \frac{2\epsilon_0}{3d}\times\left(2\times\frac{3}{2}\right) = 2 \quad (\because \ A = lb = 2 \times \frac{3}{2})$$

5. The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 time is _______.

Official Ans. by NTA (64)

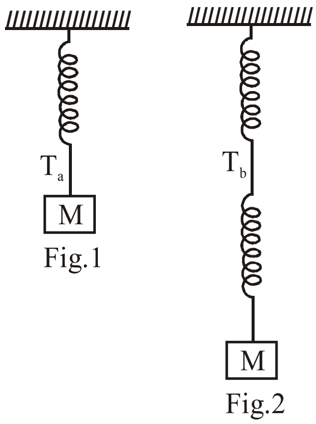
Sol. $V_e = \sqrt{\frac{2Gm}{R}}$ .... (1)

$$10V_e = \sqrt{\frac{2Gm}{R'}} \quad ... (2)$$

$$\therefore \ 10 = \sqrt{\frac{R}{R'}}$$

$$\Rightarrow \ R' = \frac{R}{100} = \frac{6400}{100} = 64 \text{ km}$$

6. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig.1 shows one of them and Fig.2 shows their series combination. The ratios of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is ______.
(Round off to the Nearest Integer)



Official Ans. by NTA (2)

Sol. $T_a = 2\pi\sqrt{\frac{M}{K}}$

$T_b = 2\pi\sqrt{\frac{M}{K/2}}$

$\frac{T_b}{T_a} = \sqrt{2} = \sqrt{x}$

$\Rightarrow x = 2$

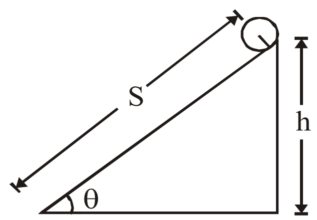
7. The following bodies,

(1) a ring (2) a disc

(3) a solid cylinder (4) a solid sphere,

of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ______.

[Mark the body as per their respective numbering given in the question]



Official Ans. by NTA (4)

Sol. Mg sinθ R = $(mk^2 + mR^2)\,\alpha$

$\alpha = \frac{Rg\sin\theta}{k^2+R^2} \Rightarrow a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}\left(1+\frac{k^2}{R^2}\right)}$

for least time, k should be least & we know k is least for solid sphere.

8. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12V between its plates. The charging battery is now disconnected and a porcelin plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of ______ pJ.

(Assume no friction)

Official Ans. by NTA (864)

Sol. $U_i = \frac{1}{2} \times 14 \times 12 \times 12$ pJ $\left(\because U = \frac{1}{2}CV^2\right)$

= 1008 pJ

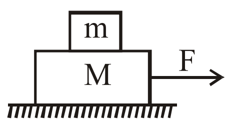
$U_f = \frac{1008}{7}$ pJ = 144 pJ $(\because C_m = kC_0)$

Mechanical energy = ΔU

= 1008 – 144

= 864 pJ

9. Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is ______ N. (Round off to the Nearest Integer) [Take g as 9.8 ms^{-2}]



Official Ans. by NTA (21)

Sol. $a_{max} = \mu g = \frac{3}{7} \times 9.8$

$F = (M + m)\, a_{max} = 5\, a_{max}$

= 21 Newton

10. If 2.5×10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is ____ W/cm^2.

(Round off to the Nearest Integer)

(Assume complete absorption and normal incidence conditions are there)

Official Ans. by NTA (25)

Sol. $F = \frac{IA}{C}$

$$I = \frac{FC}{A} = \frac{2.5\times10^{-6}\times3\times10^{8}}{30} = 25\ \text{W/cm}^2$$

CHEMISTRY | **TEST PAPER WITH ANSWER & SOLUTION**

SECTION-A

1. With respect to drug-enzyme interaction, identify the wrong statement:

(1) Non-Competitive inhibitor binds to the allosteric site

(2) Allosteric inhibitor changes the enzyme's active site

(3) Allosteric inhibitor competes with the enzyme's active site

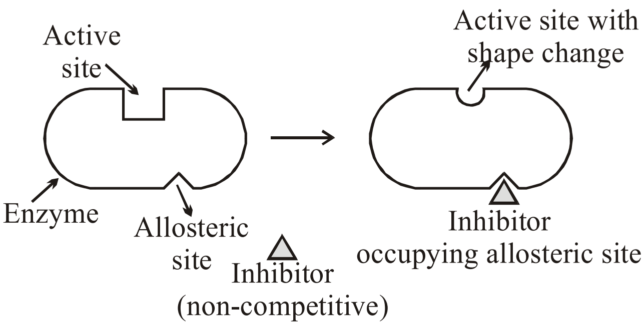
(4) Competitive inhibitor binds to the enzyme's active site

Official Ans. by NTA (3)

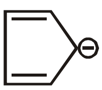
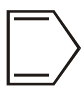
Sol. Some durg do not bind to the Enzyme's active site. These bind to a different site of enzyme which called **allosteric site**.

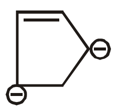
This binding of inhibitor at allosteric site changes the shape of the active site in such a way that substrate can not recognise it.

Such inhibitor is known as **Non-competitive inhibitor**.

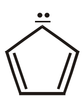


2. Which of the following is an aromatic compound?

(1) 
(2) 

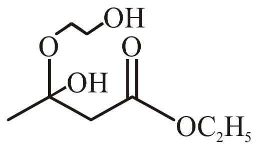
(3) 
(4) 

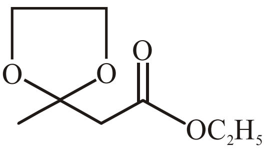
Official Ans. by NTA (1)

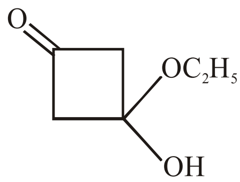
Sol.  $\rightarrow$ Aromatic compound

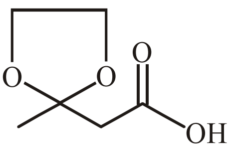
3. 

The product "A" in the above reaction is:

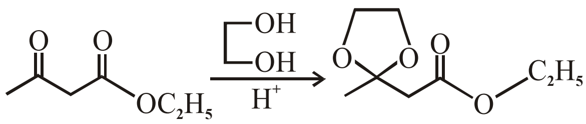
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

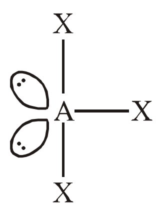
Sol. 

4. A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is:

(1) see-saw (2) planar triangular

(3) T-shaped (4) trigonal pyramidal

Official Ans. by NTA (3)

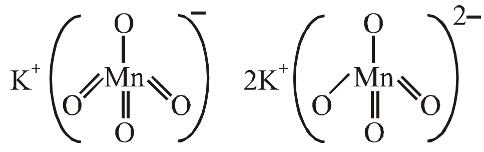
Sol. 

sp^3d hybridised

T-shaped

5. Given below are two statements:
Statement I : Potassium permanganate on heating at 573 K forms potassium manganate.
Statement II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Statement I is true but statement II is false
(2) Both statement I and statement II are true
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are false
Official Ans. by NTA (1)

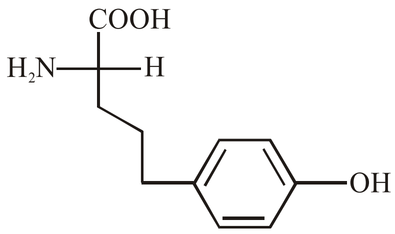
Sol. $2KMnO_4 \xrightarrow{573K} K_2MnO_4 + MnO_2 + O_2$
Potassium permanganate — Potassium manganate

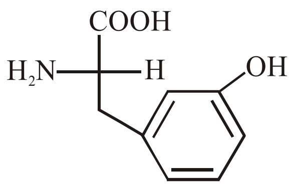


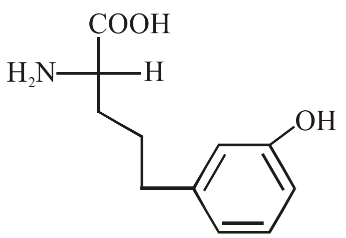
Tetrahedral unit diamagnetic — Tetrahedral unit paramagnetic

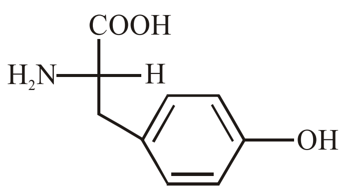
Statement-I is correct.
Statement-II is incorrect.

6. Which of the following is correct structure of tyrosine?

(1)


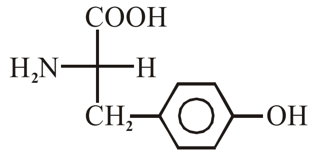
(2)


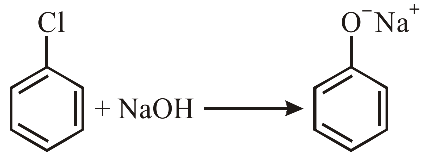
(3)


(4)


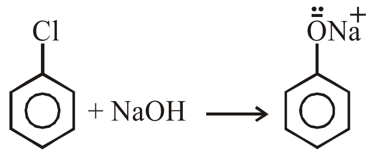
Official Ans. by NTA (4)

Sol. The structure of Tyrosine amino acid is



7.


The above reaction requires which of the following reaction conditions?
(1) 573 K, Cu, 300 atm
(2) 623 K, Cu, 300 atm
(3) 573 K, 300 atm
(4) 623 K, 300 atm
Official Ans. by NTA (4)

Sol.

Dow process

Temperature = 623 K
Pressure = 300 atm

8. The absolute value of the electron gain enthalpy of halogens satisfies:
(1) I > Br > Cl > F (2) Cl > Br > F > I
(3) Cl > F > Br > I (4) F > Cl > Br > I
Official Ans. by NTA (3)

Sol. Order of electron gain enthalpy (Absolute value)
Cl > F > Br > I

9. Which of the following compound CANNOT act as a Lewis base?
(1) NF_3 (2) PCl_5 (3) SF_4 (4) ClF_3
Official Ans. by NTA (2)

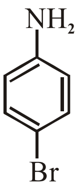
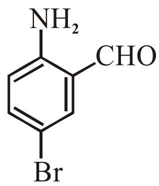
Sol. Lewis base : Chemical species which has capability to donate electron pair.
In NF_3, SF_4, ClF_3 central atom (i.e. N, S, Cl) having lone pair therefore act as lewis base.

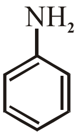
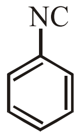
In PCl_5 central atom (P) does not have lone pair therefore does not act as lewis base.

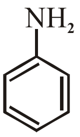
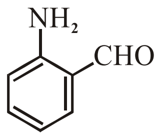
10. Reducing smog is a mixture of:
(1) Smoke, fog and O_3
(2) Smoke, fog and SO_2
(3) Smoke, fog and $CH_2{=}CH{-}CHO$
(4) Smoke, fog and N_2O_3
Official Ans. by NTA (2)

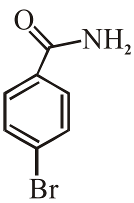
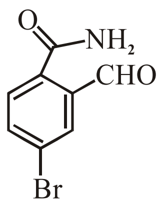
Sol. Reducing or classical smog is the combination of smoke, fog and SO_2.

11. Hoffmann bromomide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B. The structures of A and B are :

(1) A -  B - 

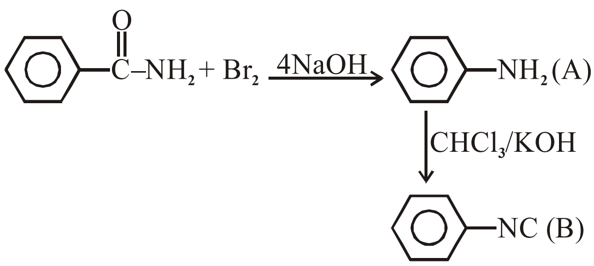
(2) A -  B - 

(3) A -  B - 

(4) A -  B - 

Official Ans. by NTA (2)

Sol. Hoffmann bromamide degradation reaction :



Carbylamine reaction :

12. Mesityl oxide is a common name of :

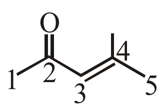
(1) 2,4-Dimethyl pentan-3-one

(2) 3-Methyl cyclohexane carbaldehyde

(3) 2-Methyl cyclohexanone

(4) 4-Methyl pent-3-en-2-one

Official Ans. by NTA (4)

Sol. 

Mesityloxide

IUPAC [4-Methylpent-3-en-2-one]

13. Which of the following reaction is an example of ammonolysis?

(1) $C_6H_5COCl + C_6H_5NH_2 \longrightarrow C_6H_5CONHC_6H_5$

(2) $C_6H_5CH_2CN \xrightarrow{[H]} C_6H_5CH_2CH_2NH_2$

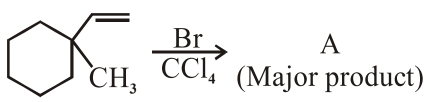
(3) $C_6H_5NH_2 \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3Cl^-$

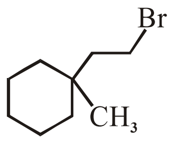
(4) $C_6H_5CH_2Cl + NH_3 \longrightarrow C_6H_5CH_2NH_2$

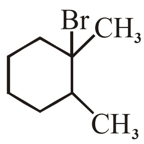
Official Ans. by NTA (4)

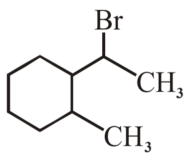
Sol. The process of cleavage of the C–X bond by Ammonia molecule is known as ammonolysis.

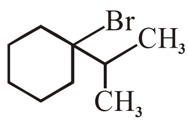
Ex : $R–CH_2–Cl + \ddot{N}H_3 \longrightarrow R–CH_2–NH_2$

14. 

(1) 

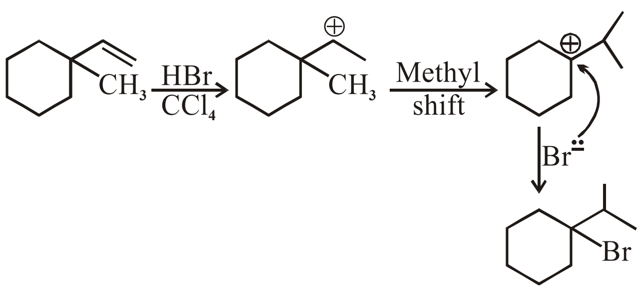
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



15. A colloidal system consisting of a gas dispersed in a solid is called a/an :

(1) solid sol (2) gel

(3) aerosol (4) foam

Official Ans. by NTA (1)

Sol. Colloid of gas dispersed in solid is called solid sol.

16. The INCORRECT statement(s) about heavy water is (are)

(A) used as a moderator in nuclear reactor

(B) obtained as a by-product in fertilizer industry.

(C) used for the study of reaction mechanism

(D) has a higher dielectric constant than water

Choose the correct answer from the options given below :

(1) (B) only (2) (C) only

(3) (D) only (4) (B) and (D) only

Official Ans. by NTA (3)

Sol. The dielectric constant of H_2O is greater than heavy water.

17. The correct order of conductivity of ions in water is :

(1) $Na^+ > K^+ > Rb^+ > Cs^+$

(2) $Cs^+ > Rb^+ > K^+ > Na^+$

(3) $K^+ > Na^+ > Cs^+ > Rb^+$

(4) $Rb^+ > Na^+ > K^+ > Li^+$

Official Ans. by NTA (2)

Sol. $Li^+ \; Na^+ \; K^+ \; Rb^+ \; Cs^+$

$\xrightarrow{\hspace{3cm}}$ Hydration energy ↑

$\xrightarrow{\hspace{3cm}}$ Ionic mobility ↓

$\xrightarrow{\hspace{3cm}}$ Conductivity ↓

$\therefore$ Correct option is $Na^+ > K^+ > Rb^+ > Cs^+$.

OR

Sol. As the size of gaseous ion decreases, it get more hydrated in water and hence, the size of aqueous ion increases. When this bulky ion move in solution, it experience greater resistance and hence lower conductivity.

Size of gasesous ion : $Cs^+ > Rb^+ > K^+ > Na^+$

Size of aqueous ion : $Cs^+ < Rb^+ < K^+ < Na^+$

Conductivity : $Cs^+ > Rb^+ > K^+ > Na^+$

18. What is the spin-only magnetic moment value (BM) of a divalent metal ion with atomic number 25, in it's aqueous solution?

(1) 5.92

(2) 5.0

(3) zero

(4) 5.26

Official Ans. by NTA (1)

Sol. Electronic configuration of divalent metal ion having atomic number 25 is

$$Mn^{+2}_{(aq)} \Rightarrow \underset{1s^2}{\boxed{\uparrow\downarrow}}\ \underset{2s^2}{\boxed{\uparrow\downarrow}}\ \underset{2p^6}{\boxed{\uparrow\downarrow|\uparrow\downarrow|\uparrow\downarrow}}\ \underset{3s^2}{\boxed{\uparrow\downarrow}}\ \underset{3p^6}{\boxed{\uparrow\downarrow|\uparrow\downarrow|\uparrow\downarrow}}\ \underset{3d^5}{\boxed{\uparrow|\uparrow|\uparrow|\uparrow|\uparrow}}$$

Total number of unpaired electrons = 5

μ (Magnetic moment) = $\sqrt{n(n+2)}$ BM

where n = number of unpaired e^-

$\therefore \; \mu = \sqrt{5(5+2)} = \sqrt{35}$ BM $= 5.92$ BM

19. Given below are two statements :

Statement-I : Retardation factor (R_f) can be measured in meter/centimeter.

Statement-II : R_f value of a compound remains constant in all solvents.

Choose the most appropriate answer from the options given below:

(1) Statement-I is true but statement-II is false

(2) Both statement-I and statement-II are true

(3) Both statement-I and statement-II are false

(4) Statement-I is false but statement-II is true

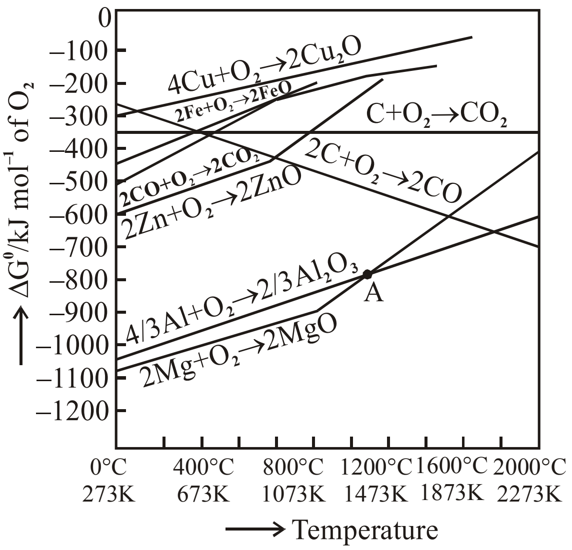
Official Ans. by NTA (3)

Sol. R_f = retardation factor

$$R_f = \frac{\text{Distance travelled by the substance from reference line (c.m)}}{\text{Distance travelled by the solvent from reference line (c.m)}}$$

Note : R_f value of different compounds are different.

20. The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates :



(1) $\Delta G = 0$ and melting or boiling point of the metal oxide

(2) $\Delta G > 0$ and decomposition of the metal oxide

(3) $\Delta G < 0$ and decomposition of the metal oxide

(4) $\Delta G = 0$ and reduction of the metal oxide

Official Ans. by NTA (1)

Official Ans. by **ALLEN (Bonus)**

Sol. At intersection point $\Delta G = 0$ and sudden increase in slope is due to melting or boiling point of the metal.

SECTION-B

1. The reaction of white phosphorus on boiling with alkali in inert atmosphere resulted in the formation of product 'A'. The reaction 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives _______ mol(s) of Ag. (Round off to the Nearest Integer).

Official Ans. by NTA (4)

Sol. $P_4 + 3OH^- + 3H_2O \rightarrow PH_3 + \underset{(A)}{3H_2PO_2^-}$

$$\underset{1\text{ mole}}{H_2PO_2^-} + \underset{\text{excess}}{4Ag^+} + 2H_2O \rightarrow \underset{4\text{ mole}}{4Ag} + H_3PO_4 + 3H^+$$

2. 0.01 moles of a weak acid HA($K_a = 2.0 \times 10^{-6}$) is dissolved in 1.0 L of 0.1 M HCl solution. The degree of dissociation of HA is _______ $\times 10^{-5}$ (Round off to the Nearest Integer).
[Neglect volume change on adding HA. Assume degree of dissociation <<1]

Official Ans. by NTA (2)

Sol.

	HA	$\rightleftharpoons$ H^+	+ A^-
Initial conc.	0.01M	0.1M	0
Equ. conc.	(0.01 – x)	(0.1 + x)	xM
	≈ 0.01M	≈ 0.1M	

Now, $K_a = \dfrac{[x^+][A^-]}{[HA]} \Rightarrow 2\times10^{-6} = \dfrac{0.1\times x}{0.01}$

$\therefore$ $x = 2 \times 10^{-7}$

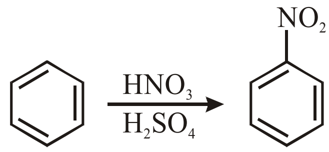
Now, $\alpha = \dfrac{x}{0.01} = \dfrac{2\times10^{-7}}{0.01} = 2\times10^{-5}$

3. A certain orbital has n = 4 and $m_L = -3$. The number of radial nodes in this orbital is _______. (Round off to the Nearest Integer).

Official Ans. by NTA (0)

Sol. n = 4 and $m_\ell = -3$
Hence, ℓ value must be 3.
Now, number of radial nodes $= n - \ell - 1$
$= 4 - 3 - 1 = 0$

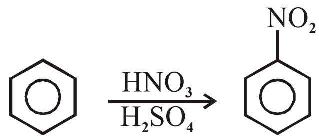
4.



In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is _______ %. (Round off to the Nearest Integer).
(Given atomic mass : C : 12.0 u, H : 1.0u, O : 16.0 u, N : 14.0 u)

Official Ans. by NTA (80)

Sol.



1 mole   1 mole
78gm   123gm

3.9gm $\dfrac{123}{78} \times 3.9 = 6.15$gm

But actual amount of nitrobenzene formed is 4.92 gm and hence.

Percentage yield $= \dfrac{4.92}{6.15}\times100 = 80\%$

5. The mole fraction of a solute in a 100 molal aqueous solution _______ $\times 10^{-2}$. (Round off to the Nearest Integer).
[Given : Atomic masses : H : 1.0 u, O : 16.0 u]
Official Ans. by NTA (64)

Sol. 100 molal aqueous solution means there is 100 mole solute in 1 kg = 1000 gm water.
Now,

$$\text{mole-fraction of solute} = \frac{n_{solute}}{n_{solute} + n_{solvent}}$$

$$= \frac{100}{100 + \frac{1000}{18}} = \frac{1800}{2800} = 0.6428$$

$= 64.28 \times 10^{-2}$

6. For a certain first order reaction 32% of the reactant is left after 570 s. The rate constant of this reaction is ______ $\times 10^{-3}$ s^{-1}. (Round off to the Nearest Integer).
[Given : $\log_{10}2 = 0.301$, $\ln 10 = 2.303$]
Official Ans. by NTA (2)

Sol. For 1^{st} order reaction,

$$K = \frac{2.303}{t}.\log\frac{[A_0]}{[A_t]} = \frac{2.303}{570\,\text{sec}}.\log\left(\frac{100}{32}\right)$$

$= 1.999 \times 10^{-3}\ \text{sec}^{-1} \approx 2 \times 10^{-3}\ \text{sec}^{-1}$

7. The standard enthalpies of formation of Al_2O_3 and CaO are –1675 kJ mol^{-1} and –635 kJ mol^{-1} respectively.
For the reaction
$3CaO + 2Al \rightarrow 3Ca + Al_2O_3$ the standard reaction enthalpy Δ_rH^0= _______ kJ.
(Round off to the Nearest Integer).
Official Ans. by NTA (230)

Sol. Given reaction:
$3CaO + Al \rightarrow Al_2O_3 + 3Ca$
Now, $\Delta_rH° = \Sigma\, \Delta_fH°_{Products} - \Sigma\Delta_fH°_{Reactants}$
$= [1 \times (-1675) + 3 \times 0] - [3 \times (-635) + 2 \times 0]$
$= + 230$ kJ mol^{-1}

8. 15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20 mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is _______ $\times 10^{-2}$ M (Round off to the Nearest Integer).
Official Ans. by NTA (24)

Sol. $n_{eq}\ Fe^{2+} = n_{eq}\ Cr_2O_7^{2-}$

$$\text{or, } \left(\frac{15 \times M_{Fe^{2+}}}{1000}\right) \times 1 = \left(\frac{20 \times 0.03}{1000}\right) \times 6$$

$\therefore\ M_{Fe^{2+}} = 0.24\ M = 24 \times 10^{-2}$ M

9. The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is _______ $\times 10^{-5}$ mol dm^{-3}.
(Round off to the Nearest Integer).
[Given : Henry's law constant
$= K_H = 8.0 \times 10^4$ kPa for O_2.
Density of water with dissolved oxygen = 1.0 kg dm^{-3}]
Official Ans. by NTA (25)
Official Ans. by ALLEN (1389)

Sol. $P = K_H \cdot x$

$$\text{or, } 20 \times 10^3 = (8 \times 10^4 \times 10^3) \times \frac{n_{O_2}}{n_{O_2} + n_{water}}$$

$$\text{or, } \frac{1}{4000} = \frac{n_{O_2}}{n_{O_2} + n_{water}} = \frac{n_{O_2}}{n_{water}}$$

Means 1 mole water (= 18 gm = 18 ml) dissolves $\frac{1}{4000}$ moles O_2. Hence, molar solubility

$$= \frac{\left(\frac{1}{4000}\right)}{18} \times 1000 = \frac{1}{72}\,\text{mol dm}^{-3}$$

$= 1388.89 \times 10^{-5}$ mol $dm^{-3} \approx 1389$ mol dm^{-3}

10. The pressure exerted by a non-reactive gaseous mixture of 6.4 g of methane and 8.8 g of carbon dioxide in a 10 L vessel at 27°C is ______ kPa. (Round off to the Nearest Integer).
[Assume gases are ideal, R = 8.314 J mol^{-1} K^{-1}
Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]
Official Ans. by NTA (150)

Sol. Total moles of gases, $n = n_{CH_4} + n_{CO_2}$

$$= \frac{6.4}{16} + \frac{8.8}{44} = 0.6$$

$$\text{Now, } P = \frac{nRT}{V} = \frac{0.6 \times 8.314 \times 300}{10 \times 10^{-3}}$$

$= 1.49652 \times 10^5$ Pa $= 149.652$ kPa
≈ 150 kPa

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The inverse of $y = 5^{\log x}$ is :

(1) $x = 5^{\log y}$ (2) $x = y^{\log 5}$

(3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

Official Ans. by NTA (3)

Official Ans. by ALLEN (1 or 2 or 3)

Sol. $y = 5^{\log x}$

$y = x^{\log 5}$

$y^{\frac{1}{\log x}} = x$

Replying $x \to y$ and $y \to x$

2. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}, \vec{r} \cdot \left(\hat{i} + 2\hat{j} + \hat{k}\right) = -3$, then $\vec{r} \cdot \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

is equal to :

(1) 12 (2) 8

(3) 13 (4) 10

Official Ans. by NTA (1)

Sol. $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r}.(2\hat{i} - 3\hat{j} + \hat{k})$

$= -10 + 12 + 10 = 12$

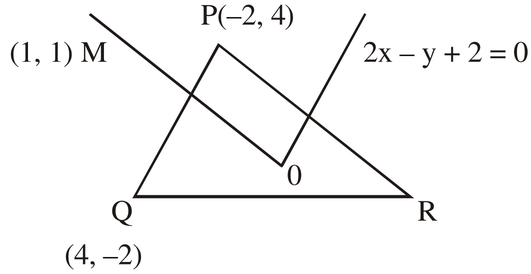
3. In a triangle PQR, the co-ordinates of the points P and Q are $(-2, 4)$ and $(4, -2)$ respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is :

(1) $(-1, 0)$ (2) $(-2, -2)$

(3) $(0, 2)$ (4) $(1, 4$

Official Ans. by NTA (2)

Sol.



Equation of perpendicular bisector of PR is

$y = x$

Solving with $2x - y + 2 = 0$ will give

$(-2, 2)$

4. The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution if k is equal to :

(1) 0 (2) 1 (3) –1 (4) –2

Official Ans. by NTA (4)

Sol. $kx + y + z = 1$

$x + ky + z = k$

$x + y + zk = k^2$

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K(K^2 - 1) - 1(K - 1) + 1(1 - K)$$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K - 1)^2 (K + 2)$

For $K = 1$

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for $K = -2$, at least one out of $\Delta_1, \Delta_2, \Delta_3$ are not zero

Hence for no sol^n, $K = -2$

5. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots$ upto 100 terms, then α is :

(1) 1.01 (2) 1.00

(3) 1.02 (4) 1.03

Official Ans. by NTA (1)

Sol. $\cot^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + \ldots$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

$$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$= \tan^{-1} 201 - \tan^{-1} 1$

$$= \tan^{-1}\left(\frac{200}{202}\right)$$

$$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$$

$\alpha = 1.01$

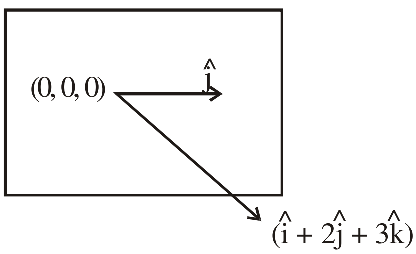
6. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :

(1) $x + 3z = 10$ (2) $x + 3z = 0$

(3) $3x + z = 6$ (4) $3x - z = 0$

Official Ans. by NTA (4)

Sol.



$$\vec{n} = \hat{j} \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -3\hat{i} + 0\hat{j} + \hat{k}$$

So, $(-3)(x - 1) + 0(y - 2) + (1)(z - 3) = 0$

$\Rightarrow -3x + z = 0$

Option 4

Alternate :

Required plane is

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 3x - z = 0$

7. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Official Ans. by NTA (3)

Sol. $A^2 = \sin^2\alpha \, I$

So, $\left|A^2 - \frac{I}{2}\right| = \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{2}}$

8. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $\sim q \Rightarrow p$

(3) $p \Rightarrow \sim q$ (4) $p \Rightarrow q$

Official Ans. by NTA (1)

Sol. $\because p \to q \equiv \sim p \vee q$

So, $* \equiv \vee$

Thus, $p * (\sim q) \equiv p \vee (\sim q)$

$\equiv q \to p$

9. Two dices are rolled. If both dices have six faces numbered 1,2,3,5,7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(1) $\frac{4}{9}$ (2) $\frac{17}{36}$ (3) $\frac{5}{12}$ (4) $\frac{1}{2}$

Official Ans. by NTA (2)

Sol. n(E) = 5 + 4 + 4 + 3 + 1 = 17

So, P (E) = $\frac{17}{36}$

10. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is equal to :

(1) 2

(2) 4

(3) 3

(4) 1

Official Ans. by NTA (1)

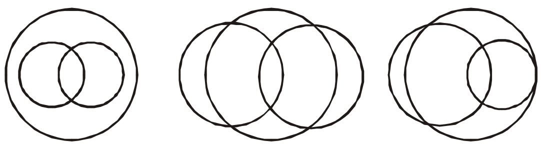
Sol. ${}^7C_3x^4\ x^{(3\log_2^x)} = 4480$

$\Rightarrow\ x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow\ (4+3t)t = 7; t = \log_2^x$

$\Rightarrow\ t = 1, \frac{-7}{3} \Rightarrow x = 2$

11. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



P Q R

(1) P and Q

(2) P and R

(3) None of these

(4) Q and R

Official Ans. by NTA (3)

Sol. $A \cap B \cap C$ is visible in all three venn diagram

Hence, Option (3)

12. The sum of possible values of x for $\tan^{-1}(x + 1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is :

(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$

(3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

Official Ans. by NTA (1)

Sol. $\tan^{-1}(x + 1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$

Taking tangent both sides :-

$$\frac{(x+1)+(x-1)}{1-(x^2-1)} = \frac{8}{31}$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$$

$\Rightarrow\ 4x^2 + 31x - 8 = 0$

$\Rightarrow\ x = -8, \frac{1}{4}$

But, if x = $\frac{1}{4}$

$$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$$

$$\& \ \cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$$

$\Rightarrow$ LHS > $\frac{\pi}{2}$ & RHS < $\frac{\pi}{2}$

(Not possible)

Hence, x = – 8

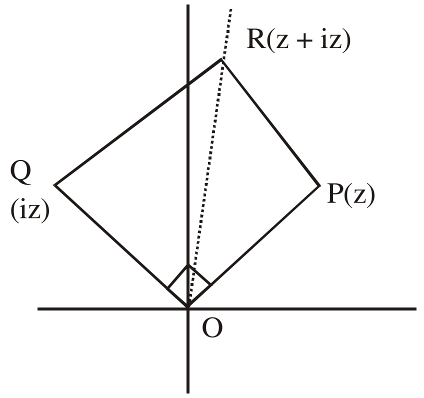
13. The area of the triangle with vertices A(z), B(iz) and C (z + iz) is :

(1) 1 (2) $\frac{1}{2}|z|^2$

(3) $\frac{1}{2}$ (4) $\frac{1}{2}|z+iz|^2$

Official Ans. by NTA (2)

Sol.



$A=\frac{1}{2}|z|\,|iz|$

$=\frac{|z|^2}{2}$

14. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is:

(1) $3\sqrt{5}$

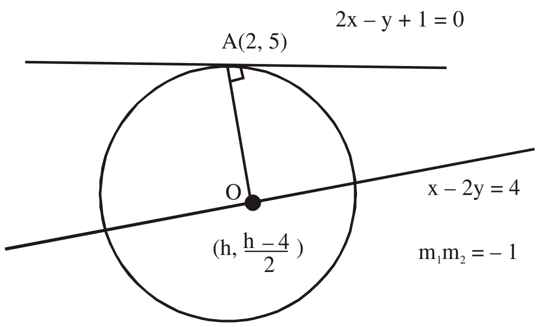
(2) $5\sqrt{3}$

(3) $5\sqrt{4}$

(4) $4\sqrt{5}$

Official Ans. by NTA (1)

Sol.



$$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2)=-1$$

h = 8

center (8, 2)

$$\text{Radius}=\sqrt{(8-2)^2+(2-5)^2}=3\sqrt{5}\ )$$

15. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(1) 5 (2) 2 (3) 4 (4) 6

Official Ans. by NTA (3)

Sol. Total matches between boys of both team

$= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both team $= {}^nC_1\,{}^6C_1 = 6n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

16. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\ldots\infty}}}}$ is :

(1) $2+\frac{2}{5}\sqrt{30}$ (2) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(3) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (4) $5+\frac{2}{5}\sqrt{30}$

Official Ans. by NTA (1)

Sol. $y=4+\frac{1}{\left(5+\frac{1}{y}\right)}$

$y-4=\frac{y}{(5y+1)}$

$5y^2 - 20y - 4 = 0$

$y=\frac{20+\sqrt{480}}{10}$

$y=\frac{20-\sqrt{480}}{10}\rightarrow$ rejected

$y=2+\sqrt{\frac{480}{100}}$

Correct with Option (A)

17. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Both circles' centres lie inside region of one another.

(3) Both circles pass through the centre of each other.

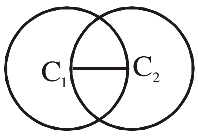
(4) Circles have two intersection points.

Official Ans. by NTA (2)

Sol. $r_1 = 3$, c_1 (5, 5)

$r_2 = 3$, c_2 (8, 5)

$C_1C_2 = 3$, $r_1 = 3$, $r_2 = 3$



18. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^{\alpha} x}{\cos^{\alpha} x+\sin^{\alpha} x}dx$$

(1) $g(\alpha)$ is a strictly increasing function

(2) $g(\alpha)$ has an inflection point at $\alpha = -\frac{1}{2}$

(3) $g(\alpha)$ is a strictly decreasing function

(4) $g(\alpha)$ is an even function

Official Ans. by NTA (4)

Allen Answer (1 or 2 or 3/Bonus)

Sol. $g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3}\frac{\sin^{\alpha} x}{(\sin^{\alpha} x+\cos^{\alpha} x)}$ .....(i)

$g(\alpha) = \int_{\frac{\pi}{6}}^{\pi/3}\frac{\cos^{\alpha} x}{(\sin^{\alpha} x+\cos^{\alpha} x)}$ .....(ii)

(1) + (2)

$2g(\alpha) = \frac{\pi}{6}$

$g(\alpha)=\frac{\pi}{12}$

Constant and even function

Due to typing mistake it must be bonus.

19. Which of the following is true for y(x) that satisfies the differential equation

$$\frac{dy}{dx}=xy-1+x-y\ ;\ y(0)=0:$$

(1) $y(1)=e^{-\frac{1}{2}}-1$

(2) $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$

(3) $y(1) = 1$

(4) $y(1)=e^{\frac{1}{2}}-1$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}=(1+y)(x-1)$

$$\frac{dy}{(y+1)}=(x-1)dx$$

Integrate $\ln(y + 1) = \frac{x^2}{2}-x+c$

$(0,0) \Rightarrow c = 0 \Rightarrow y=e^{\left(\frac{x^2}{2}-x\right)}-1$

20. The value of

$$\lim_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)}{x-x^3},$$ where

$[x]$ denotes the greatest integer $\leq x$ is :

(1) π

(2) 0

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{2}$

Official Ans. by NTA (4)

Sol. $\lim_{x\to 0^+}\frac{\cos^{-1}x}{(1-x^2)}\times\frac{\sin^{-1}x}{x}=\frac{\pi}{2}$

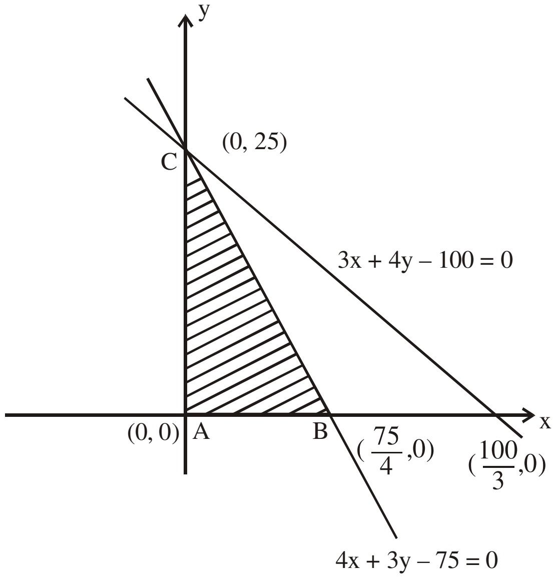
SECTION-B

1. The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \le 100$ and $4x + 3y \le 75$ for $x \ge 0$ and $y \ge 0$ is _______ .

Official Ans. by NTA (904)

Allen Answer (904 or 904.01 or 904.02)

Sol.



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \le 100$ .....(i)

$4x + 3y \le 75$ ......(ii)

$x \ge 0$

$y \ge 0$

$x \le \dfrac{75-3y}{4}$

$Z = y(6x + y)$

$Z \le y\left(6.\left(\dfrac{75-3y}{4}\right)+y\right)$

$Z \le \dfrac{1}{2}(225y-7y^2) \le \dfrac{(225)^2}{2\times4\times7}$

$= \dfrac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attained at $y = \dfrac{225}{14}$

2. If the function $f(x)=\dfrac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0)=\dfrac{1}{k}$, then k is _________ .

Official Ans. by NTA (6)

Sol. $\lim\limits_{x\to0}\dfrac{\cos(\sin x)-\cos x}{x^4}=f(0)$

$\Rightarrow \lim\limits_{x\to0}\dfrac{2\sin\left(\dfrac{\sin x+x}{2}\right)\sin\left(\dfrac{x-\sin x}{2}\right)}{x^4}=\dfrac{1}{K}$

$\Rightarrow \lim\limits_{x\to0}2\left(\dfrac{\sin x+x}{2x}\right)\left(\dfrac{x-\sin x}{2x^3}\right)=\dfrac{1}{K}$

$\Rightarrow 2\times\dfrac{(1+1)}{2}\times\dfrac{1}{2}\times\dfrac{1}{6}=\dfrac{1}{K}$

$\Rightarrow K = 6$

3. If $f(x)=\sin\left(\cos^{-1}\left(\dfrac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\dfrac{b}{a}\log_e 2$ when $x = 1$, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is _______ .

Official Ans. by NTA (481)

Sol. $f(x)=\sin\left(\cos^{-1}\left(\dfrac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at x= 1 ; $2^{2x} = 4$

for $\sin\left(\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)\right)$;

Let $\tan^{-1} x = \theta$; $\theta\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

$\therefore \sin\left(\cos^{-1}\cos2\theta\right)=\sin2\theta$

$$\left\{\begin{array}{l}\text{If} \quad x>1 \Rightarrow \frac{\pi}{2}>\theta>\frac{\pi}{4} \\ \therefore \quad \pi>2\theta>\frac{\pi}{2}\end{array}\right\}$$

$$=2\sin\theta\cos\theta=\frac{2\tan\theta}{1+\tan^2\theta}$$

$$=\frac{2x}{1+x^2}$$

Hence, $f(x) = \frac{2\cdot 2^x}{1+2^{2x}}$

$$\therefore \ f'(x)=\frac{(1+2^{2x})(2.2^x\ln 2)-2^{2x}\cdot 2\cdot\ln 2\cdot 2\cdot 2^x}{(1+2^{2x})}$$

$$\therefore \quad f^1(1)=\frac{20\ln 2-32\ln 2}{25}=-\frac{12}{25}\ln 2$$

So, $a = 25$, $b = 12 \Rightarrow |a^2 - b^2| = 25^2 - 12^2$

$= 625 - 144$

$= 481$

4. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)\ p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to _____.

Official Ans. by NTA (6)

Sol. Let $P(E_1) = P_1$; $P(E_2) = P_2$; $P(E_3) = P_3$

$$P(E_1\cap\bar{E}_2\cap\bar{E}_3)=\alpha=P_1(1-P_2)(1-P_3)......(1)$$

$$P(\bar{E}_1\cap E_2\cap\bar{E}_3)=\beta=(1-P_1)P_2(1-P_3)......(2)$$

$$P(\bar{E}_1\cap\bar{E}_2\cap E_3)=\gamma=(1-P_1)(1-P_2)P_3......(3)$$

$$P(\bar{E}_1\cap\bar{E}_2\cap\bar{E}_3)=P=(1-P_1)(1-P_2)(1-P_3)......(4)$$

Given that, $(\alpha - 2\beta)\ P = \alpha\beta$

$\Rightarrow (P_1(1-P_2)(1-P_3)-2(1-P_1)P_2(1-P_3))P=P_1P_2$
$(1-P_1)\ (1 - P_2)\ (1 - P_3)^2$

$\Rightarrow (P_1(1-P_2) - 2(1-P_1)\ P_2) = P_1P_2$

$\Rightarrow (P_1 - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ......(1)

and similarly, $(\beta - 3\gamma)P = 2B\gamma$

$P_2 = 3P_3$ ......(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\frac{P_1}{P_3}=6}$

5. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then

$\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to _______.

Official Ans. by NTA (2)

Sol. $\vec{a}.\vec{b}=1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \boxed{\alpha\beta=-2}$ ....(1)

$\vec{b}.\vec{c}=-3 \Rightarrow -\beta+2\alpha+1=-3$

$\boxed{\beta-2\alpha=4}$ ....(2)

Solving (1) & (2), $(\alpha,\beta) = (-1, 2)$

$$\frac{1}{3}[\vec{a}\ \vec{b}\ \vec{c}]=\frac{1}{3}\begin{vmatrix}\alpha & \beta & 3\\ -\beta & -\alpha & -1\\ 1 & -2 & -1\end{vmatrix}$$

$$=\frac{1}{3}\begin{vmatrix}-1 & 2 & 3\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}$$

$$=\frac{1}{3}\begin{vmatrix}0 & 0 & 2\\ -2 & 1 & -1\\ 1 & -2 & -1\end{vmatrix}=\frac{1}{3}[2(4-1)]=2$$

6. If $A=\begin{bmatrix}2 & 3\\ 0 & -1\end{bmatrix}$, then the value of

$\det(A^4)$ + det $\left(A^{10}-(\text{Adj}(2A))^{10}\right)$ is equal to

_________ .

Official Ans. by NTA (16)

Sol. 2A adj (2A) = |2A|I

$\Rightarrow$ A adj (2A) = –4I ....(i)

Now, E = $|A^4| + |A^{10} - (\text{adj}(2A))^{10}|$

$= (-2)^4 + \dfrac{|A^{20} - A^{10}(\text{adj } 2A)^{10}|}{|A|^{10}}$

$= 16 + \dfrac{|A^{20} - (A\,\text{adj}(2A))^{10}|}{|A|^{10}}$

$= 16 + \dfrac{|A^{20} - 2^{10}I|}{2^{10}}$ (from (1))

Now, characteristic roots of A are 2 and –1.

So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} - 2^{10}\ I)\ (A^{20} - I) = 0$

$\Rightarrow |A^{20} - 2^{10}I| = 0$ (as $A^{20} \neq I$)

$\Rightarrow$ E = 16 Ans.

7. If $[\cdot]$ represents the greatest integer function, then the value of

$\left|\displaystyle\int_0^{\sqrt{\frac{\pi}{2}}}\left[\left[x^2\right]-\cos x\right]dx\right|$ is ________ .

Official Ans. by NTA (1)

Sol. $I = \displaystyle\int_0^{\sqrt{\pi/2}}\left([x^2]+[-\cos x]\right)dx$

$= \displaystyle\int_0^1 0\ dx + \int_1^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}}(-1)\,dx$

$= \sqrt{\dfrac{\pi}{2}} - 1 - \sqrt{\dfrac{\pi}{2}} = -1$

$\Rightarrow |I| = 1$

8. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

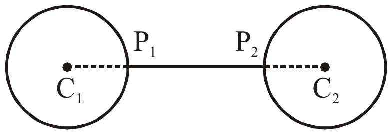
$x^2 + y^2 - 24x - 10y + 160 = 0$ is ______ .

Official Ans. by NTA (1)

Sol. Given $C_1(5, 5)$, $r_1 = 3$ and C_2 (12, 5), $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$



9. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point (–2, 1, 3) is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is _____ .

Official Ans. by NTA (4)

Sol. Required plane is

$p_1 + \lambda p_2 = (2 + 3\lambda)\ x - (7 + 5\lambda)\ y$

$+ (4 + 4\lambda)z - 3 + 11\lambda = 0$;

which is satisfied by (–2, 1, 3).

Hence, $\lambda = \dfrac{1}{6}$

Thus, plane is $15x - 47y + 28z - 7 = 0$

So, $2a + b + c - 7 = 4$

10. If $(2021)^{3762}$ is divided by 17, then the remainder is _______ .

Official Ans. by NTA (4)

Sol. $(2023 - 2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4\ (17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder = 4

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS **TEST PAPER WITH ANSWER & SOLUTION**

SECTION-A

1. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take g = 10 ms^{-2})

(1) 3.0 ms^{-1} (2) 3.50 ms^{-1}

(3) 2.0 ms^{-1} (4) 2.50 ms^{-1}

Official Ans. by NTA (4)

Sol. (4) $v_0 = \sqrt{2gh}$

$v = e\sqrt{2gh} = \sqrt{2gh}$

$\Rightarrow e = 0.9$

$S = h + 2e^2h + 2e^4 h + ...$

$$t = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + ...$$

$$v_{av} = \frac{s}{t} = 2.5 \text{ m/s}$$

2. If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is :

(1) 1.02 (2) 1.2

(3) 1.25 (4) 1.35

Official Ans. by NTA (2)

Sol. (2) f = 4 + 3 + 3 = 10

assuming non linear

$$\beta = \frac{C_p}{C_v} = 1 + \frac{2}{f} = \frac{12}{10} = 1.2$$

3. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take In 2 = 0.693)

(1) 0.69 × 10^2 kg s^{-1} (2) 3.3 × 10^2 kg s^{-1}

(3) 1.16 × 10^2 kg s^{-1} (4) 5.7 × 10^{-3} kg s^{-1}

Official Ans. by NTA (NA)

Official Ans. by ALLEN (Bonus)

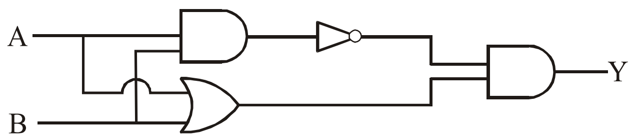
Sol. $A = A_0e^{-\gamma t}$

$$\ln 2 = \frac{b}{2m} \times 120$$

$$\frac{0.693 \times 2 \times 1}{120} = b$$

1.16 × 10^{-2} kg/sec.

4. Which one of the following will be the output of the given circuit ?



(1) NOR Gate (2) NAND Gate

(3) AND Gate (4) XOR Gate

Official Ans. by NTA (4)

Sol. (4) Conceptual

5. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36% , the ratio of hydraulic stress to the corresponding hydraulic strain will be ____________.

[Given : density of water is 1000 kg m^{-3} and g = 9.8 ms^{-2}.]

(1) 1.96 × 10^7 Nm^{-2} (2) 1.44 × 10^7 Nm^{-2}

(3) 2.26 × 10^9 Nm^{-2} (4) 1.44 × 10^9 Nm^{-2}

Official Ans. by NTA (4)

Sol. (4) $P = h\rho g$

$$\beta = \frac{p}{\frac{\Delta V}{V}} = \frac{2\times10^3\times10^3\times9.8}{1.36\times10^{-2}}$$

$= 1.44 \times 10^9 \ \ N/m^2$

6. A geostationary satellite is orbiting around an arbitary planet 'P' at a height of 11R above the surface of 'P' , R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is__________.'P' has the time period of 24 hours.

(1) $6\sqrt{2}$ (2) $\frac{6}{\sqrt{2}}$ (3) 3 (4) 5

Official Ans. by NTA (3)

Sol. (3) $T \propto R^{3/2}$

$$\frac{24}{T} = \left(\frac{12R}{3R}\right)^{3/2} \Rightarrow T = 3hr$$

7. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

(1) $Y(x,t) = 0.03\ [\sin 5.1\ x - (0.2 \times 10^3)t]$

(2) $Y(x,t) = 0.06\ [\sin 5.1\ x - (1.5 \times 10^3)t]$

(3) $Y(x,t) = 0.06\ [\sin 0.8\ x - (0.5 \times 10^3)t]$

(4) $Y(x,t) = 0.03\ [\sin 5.1\ x - (1.5 \times 10^3)t]$

Official Ans. by NTA (4)

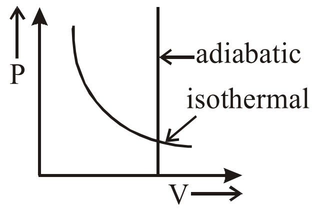
Sol. (4) $\omega = 2\pi\ f$

$= 1.5 \times 10^3$

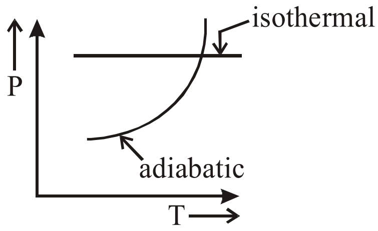
$A = \frac{6}{2} = 3\ cm = 0.03\ m$

8. Which one is the correct option for the two different thermodynamic processes ?

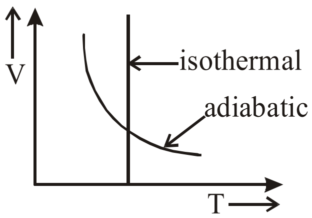
(a)



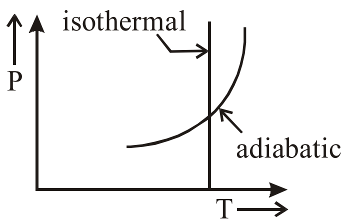
(b)



(c)



(d)



(1) (c) and (a) (2) (c) and (d)

(3) (a) only (4) (b) and (c)

Official Ans. by NTA (2)

Sol. (2) Option (a) is wrong ; since in adiabatic process V $\neq$ constant.

Option (b) is wrong, since in isothermal process T = constant

Option (c) & (d) matches isothermes & adiabatic formula :

$TV^{\gamma-1}$= constant & $\frac{T^\gamma}{p^{\gamma-1}}$ = constant

9. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time $(t = 1)$ is :

(1) $v_0 + g + F$ (2) $v_0 + \frac{g}{2} + \frac{F}{3}$

(3) $v_0 + \frac{g}{2} + F$ (4) $v_0 + 2g + 3F$

Official Ans. by NTA (2)

Sol. (2) $v = v_0 + gt + Ft^2$

$$\frac{ds}{dt} = v_0 + gt + Ft^2$$

$$\int ds = \int_0^1 (v_0 + gt + Ft^2)dt$$

$$s = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_0^1$$

$$s = v_0 + \frac{g}{2} + \frac{F}{3}$$

10. A carrier signal $C(t) = 25 \sin (2.512 \times 10^{10} t)$ is amplitude modulated by a message signal $m(t) = 5 \sin (1.57 \times 10^8 t)$ and transmitted through an antenna. What will be the bandwidth of the modulated signal ?

(1) 8 GHz

(2) 2.01 GHz

(3) 1987.5 MHz

(4) 50 MHz

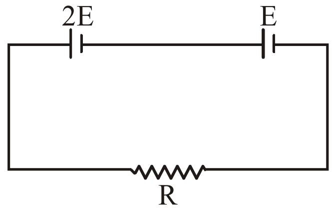
Official Ans. by NTA (4)

Sol. (4) Band width = $2 f_m$

$\omega_m = 1.57 \times 10^8 = 2\pi f_m$

$$BW = 2f_m = \frac{10^8}{2}\text{Hz} = 50 \text{ MHz}$$

11. Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is

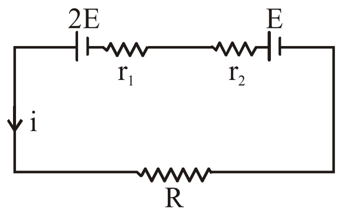


(1) $r_1 + r_2$ (2) $\frac{r_1}{2} - r_2$

(3) $\frac{r_1}{2} + r_2$ (4) $r_1 - r_2$

Official Ans. by NTA (2)

Sol. (2)



$$i = \frac{3E}{R + r_1 + r_2}$$

$TPD = 2E - ir_1 = 0$

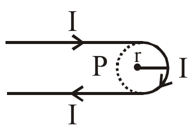
$2E = ir_1$

$$2E = \frac{3E \times r_1}{R + r_1 + r_2}$$

$2R + 2r_1 + 2r_2 = 3r_1$

$$R = \frac{r_1}{2} - r_2$$

12. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle ?



(1) $\frac{\mu_0 I}{4\pi r}(2 - \pi)$ (2) $\frac{\mu_0 I}{4\pi r}(2 + \pi)$

(3) $\frac{\mu_0 I}{2\pi r}(2 + \pi)$ (4) $\frac{\mu_0 I}{2\pi r}(2 - \pi)$

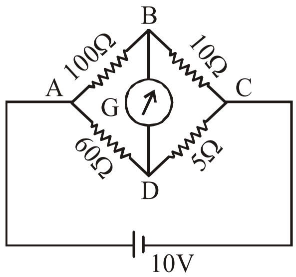
Official Ans. by NTA (2)

Sol. (2) $B = 2 \times B_{st.wire} + B_{loop}$

$$B = 2 \times \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{2r}\left(\frac{\pi}{2\pi}\right)$$

$$B = \frac{\mu_0 i}{4\pi r}(2+\pi)$$

13. The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10V is maintained across AC.

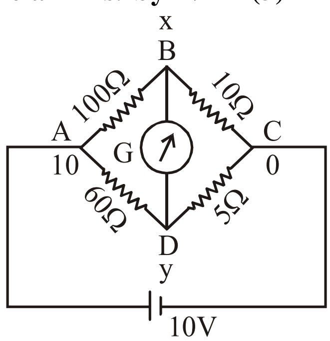


(1) 2.44 μA (2) 2.44 mA

(3) 4.87 mA (4) 4.87 μA

Official Ans. by NTA (3)

Sol. (3)



$$\frac{x-10}{100} + \frac{x-y}{15} + \frac{x-0}{10} = 0$$

$53x - 20y = 30$ .....(1)

$$\frac{y-10}{60} + \frac{y-x}{15} + \frac{y-0}{5} = 0$$

$17y - 4x = 10$ ......(2)

on solving (1) & (2)

$x = 0.865$

$y = 0.792$

$\Delta V = 0.073$ $R = 15\Omega$

$i = 4.87$ mA

14. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is

(1) $\frac{K_2}{K_1}$ (2) $\frac{K_1}{K_2}$ (3) $\sqrt{\frac{K_1}{K_2}}$ (4) $\sqrt{\frac{K_2}{K_1}}$

Official Ans. by NTA (4)

Sol. (4) $A_1\omega_1 = A_2\omega_2$

$$A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

15. Match List-I with List-II

List-I	**List-II**
(a) Phase difference between current and voltage in a purely resistive AC circuit	(i) $\frac{\pi}{2}$; current leads voltage
(b) Phase difference between current and voltage in a pure inductive AC circuit	(ii) zero
(c) Phase difference between current and voltage in a pure capacitive AC circuit	(iii) $\frac{\pi}{2}$; current lags voltage
(d) Phase difference between current and voltage in an LCR series circuit	(iv) $\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below :

(1) (a)–(i),(b)–(iii),(c)–(iv),(d)–(ii)

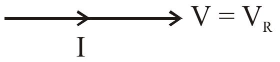
(2) (a)–(ii),(b)–(iv),(c)–(iii),(d)–(i)

(3) (a)–(ii),(b)–(iii),(c)–(iv),(d)–(i)

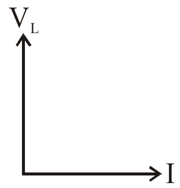
(4) (a)–(ii),(b)–(iii),(c)–(i),(d)–(iv)

Official Ans. by NTA (4)

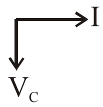
Sol. (4) (a)



(b)

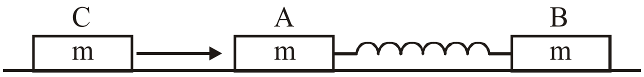


(c)



(d) $\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$

16. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A.The maximum compression in the spring is



(1) $v\sqrt{\frac{M}{2K}}$ (2) $\sqrt{\frac{mv}{2K}}$

(3) $\sqrt{\frac{mv}{K}}$ (4) $\sqrt{\frac{m}{2K}}$

Official Ans. by NTA (1)

Sol. (1) C comes to rest

V_{cm} of A & B = $\frac{v}{2}$

$\Rightarrow \frac{1}{2}\text{is } v_{ret}^2 = \frac{1}{2}kx^2$

$x = \sqrt{\frac{\mu \times v^2}{k}} = \sqrt{\frac{m}{2k}}v$

17. The atomic hydrogen emits a line spectrum consisting of various series.Which series of hydrogen atomic spectra is lying in the visible region ?

(1) Brackett series (2) Paschen series

(3) Lyman series (4) Balmer series

Official Ans. by NTA (4)

Sol. (4) Conceptual

18. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then

(1) $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$

(2) $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$

(3) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(4) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

Official Ans. by NTA (1)

Sol. (1) $\frac{1}{2}mv_1^2 = hf_1 - \phi$

$\frac{1}{2}mv_2^2 = hf_2 - \phi$

$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

19. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved ?

(1) Both, inductive reactance and current will be halved.

(2) Inductive reactance will be halved and current will be doubled.

(3) Inductive reactance will be doubled and current will be halved.

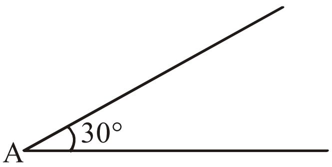
(4) Both, inducting reactance and current will be doubled.

Official Ans. by NTA (2)

Sol. (2) $X_L = \omega L$

$i = \frac{v_0}{\omega L}$

20. A sphere of mass 2kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How low will the sphere take to return to the starting point A ?



(1) 0.60 s (2) 0.52 s
(3) 0.57 s (4) 0.80 s

Official Ans. by NTA (3)

Sol. (3) $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}} = \dfrac{5}{7}\times\dfrac{10}{2} = \dfrac{25}{7}$

$t = \dfrac{2v_0}{a} = \dfrac{2\times1\times7}{25}$

$= 0.56$

SECTION-B

1. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is $\sqrt{\dfrac{x}{5}}E$. Where the value of x =_________.

Official Ans. by NTA (3)

Sol. $c\in_0 E^2 = \dfrac{100}{4\pi\times3^2}$

$c\in_0\left(\sqrt{\dfrac{x}{5}}E\right)^2 = \dfrac{60}{4\pi\times3^2}$

$\Rightarrow \dfrac{x}{5} = \dfrac{3}{5}$

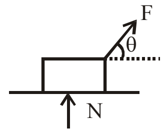
$\Rightarrow x = 3$

2. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\dfrac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be __________. (Round off to the Nearest Integer)

[Take g = 10 ms^{-2}]

Official Ans. by NTA (5)

Sol.



$F\cos\theta = \mu N$

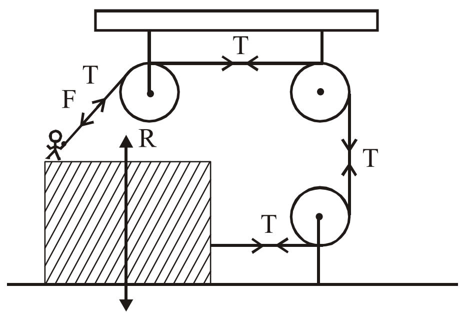
$F\sin\theta + N = mg$

$\Rightarrow F = \dfrac{\mu mg}{\cos\theta+\mu\sin\theta}$

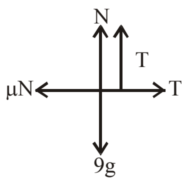
$F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}} = \dfrac{\dfrac{1}{\sqrt{3}}\times10}{\dfrac{2}{\sqrt{3}}} = 5$

3. A boy of mass 4 kg is standing on a piece of wood having mass 5kg . If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is ________N.(Round off to the Nearest Integer)

[Take g = 10 ms^{-2}]



Official Ans. by NTA (30)

Sol. 

N + T = 90

T = μ N = 0.5 (90–T)

1.5 T = 45

T = 30

4. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic acid layer will be x × 10^{-14} m. Where x is__________.

Official Ans. by NTA (25)

Sol. $4t_T = 100 \times \frac{4}{3}\pi r^3$

$= 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9} = 10^{-8}\ cm^3$

$t_T = 25 \times 10^{-10}$ cm

$= 25 \times 10^{-12}$ m

$t_0 = 0.01\ t_T = 25 \times 10^{-14}$ m

= 25

5. A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{1/\alpha}$, where α is___________.

Official Ans. by NTA (3)

Sol. $F = \frac{-dU}{dr} = -4U_0 r^3 = \frac{mv^2}{r}$

$mv^2 = 4U_0 r^4$

$v \propto r^2$

$mvr = \frac{nh}{2\pi}$

$r^3 \propto n$

$r \propto n1/3$

= 3

6. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3\ \frac{N}{C}$ .The flux of this field through a rectangular surface area 0.4 m^2 parallel to the Y – Z plane is _________Nm^2C^{-1}.

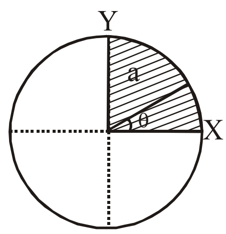
Official Ans. by NTA (640)

Sol. $\phi = E_x A \Rightarrow \frac{2}{5} \times 4 \times 10^3 \times 0.4 = 640$

7. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is ________.

(Round off to the Nearest Integer)

[a is an area as shown in the figure]



Official Ans. by NTA (4)

Sol. C.O.M of quarter disc is at $\frac{4a}{3\pi}, \frac{4a}{3\pi}$

= 4

8. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2^{rd}}{3}$ of the distance of the object from the surface .The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. the value of 'x' is__________.

Official Ans. by NTA (30)

Sol. $\lambda_m = \frac{\lambda_a}{\mu} \Rightarrow \mu = \frac{3}{2}$

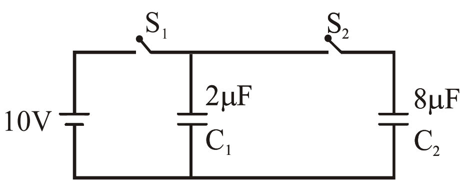
$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

$$\frac{3}{2\times 10} + \frac{1}{15} = \frac{\frac{3}{2} - 1}{R}$$

$$R = \frac{30}{13}$$

$= 30$

9. A 2 μF capacitor C_1 is first charged to a potential difference of 10 V using a battery.Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8μF. The charge in C_2 on equilibrium condition is_______μC. (Round off to the Nearest Integer)



Official Ans. by NTA (16)

Sol. $20 = (C_1 + C_2)\ V \Rightarrow V = 2$ volt.

$Q_2 = C_2V = 16\mu C$

$= 16$

10. Seawater at a frequency $f = 9 \times 10^2$ Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\ \Omega$m. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t)=V_0 \sin(2\pi ft)$. Then the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is ________

$$\left(\text{Given}: \frac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \text{Nm}^2\text{C}^{-2}\right)$$

Official Ans. by NTA (6)

Sol. $J_c = \frac{E}{\rho} = \frac{V}{\rho d}$

$$J_d = \frac{1}{A}\frac{dq}{dt}$$

$$= \frac{C}{A}\frac{dV_c}{dt}$$

$$= \frac{\in}{d}\frac{dV_c}{dt}$$

$$\Rightarrow \frac{V_0 \sin 2\pi ft}{\rho d} = 10^x \times \frac{80\varepsilon_0}{d} V_0 (2\pi f) \cos 2\pi ft$$

$$\tan\left(2\pi \times \frac{900}{800}\right) = 10^x \times \frac{40}{9\times 10^9} \times 900$$

$= x = 6$

SECTION-A

1. Fructose is an example of :-

(1) Pyranose

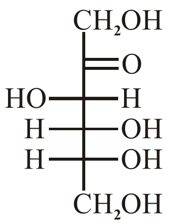
(2) Ketohexose

(3) Aldohexose

(4) Heptose

Official Ans. by NTA (2)

Sol. Fructose is a ketohexose.



2. The set of elements that differ in mutual relationship from those of the other sets is :

(1) Li – Mg (2) B – Si

(3) Be – Al (4) Li – Na

Official Ans. by NTA (4)

Sol. Li–Mg, B–Si, Be–Al show diagonal relationship but Li and Na do not show diagonal relationship as both belongs to same group and not placed diagonally.

3. The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are :

(1) $-SO_3H$ and $-NH_2$

(2) $-SO_3H$ and $-COOH$

(3) $-NH_2$ and $-COOH$

(4) $-NH_2$ and $-SO_3H$

Official Ans. by NTA (1)

Sol. Cation exchanger contains $-SO_3H$ or $-COOH$ groups while anion exchanger contains basic groups like $-NH_2$.

4. Match List-I and List-II :

	List-I		List-II
(a)	Haematite	(i)	$Al_2O_3.xH_2O$
(b)	Bauxite	(ii)	Fe_2O_3
(c)	Magnetite	(iii)	$CuCO_3.Cu(OH)_2$
(d)	Malachite	(iv)	Fe_3O_4

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(3) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Official Ans. by NTA (4)

Sol.

	Ore	Formula
(a)	Haematite	Fe_2O_3
(b)	Bauxite	$Al_2O_3.xH_2O$
(c)	Magnetite	Fe_3O_4
(d)	Malachite	$CuCO_3.Cu(OH)_2$

5. The correct pair(s) of the ambident nucleophiles is (are) :

(A) AgCN/KCN

(B) RCOOAg/RCOOK

(C) $AgNO_2/KNO_2$

(D) AgI/KI

(1) (B) and (C) only

(2) (A) only

(3) (A) and (C) only

(4) (B) only

Official Ans. by NTA (3)

Sol. Ambident nucleophile

(A) KCN & AgCN

(C) $AgNO_2$ & KNO_2

6. The set that represents the pair of neutral oxides of nitrogen is :

(1) NO and N_2O (2) N_2O and N_2O_3

(3) N_2O and NO_2 (4) NO and NO_2

Official Ans. by NTA (1)

Sol. N_2O and NO are neutral oxides of nitrogen NO_2 and N_2O_3 are acidic oxides.

7. Match List-I with List-II :

	List-I		List-II
(a)	$[Co(NH_3)_6]\ [Cr(CN)_6]$	(i)	Linkage isomerism
(b)	$[Co(NH_3)_3\ (NO_2)_3]$	(ii)	Solvate isomerism
(c)	$[Cr(H_2O)_6]Cl_3$	(iii)	Co-ordination isomerism
(d)	*cis*-$[CrCl_2(ox)_2]^{3-}$	(iv)	Optical isomerism

Choose the correct answer from the options given below :

(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(2) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(3) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Official Ans. by NTA (1)

Sol.	Complex	Type of Isomerism
(a)	$[Co(NH_3)_6]\ [Cr(CN)_6]$	Co-ordination isomerism
(b)	$[Co(NH_3)_3\ (NO_2)_3]$	Linkage isomerism
(c)	$[Cr(H_2O)_6]Cl_3$	Solvate isomerism
(d)	*cis*-$[CrCl_2(ox)_2]^{3-}$	Optical isomerism

8. Primary, secondary and tertiary amines can be separated using :-
(1) Para-Toluene sulphonyl chloride
(2) Chloroform and KOH
(3) Benzene sulphonic acid
(4) Acetyl amide

Official Ans. by NTA (1)

Sol. Primary amines react with Para Toluene sulfonyl chloride to form a precipitate that is soluble in NaOH.

Secondary amines reacts with para toluene sulfonyl chloride to give a precipitate that is insoluble in NaOH.

Tertiary amines do not react with para toluen.

9. The common positive oxidation states for an element with atomic number 24, are :
(1) +2 to +6 (2) +1 and +3 to +6
(3) +1 and +3 (4) +1 to +6

Official Ans. by NTA (1)

Sol. Cr(Z=24)
[Ar] $4s^1 3d^5$ Cr shows common oxidation states starting from +2 to +6.

10. Match List-I with List-II :

	List-I Chemical Compound		List-II Used as
(a)	Sucralose	(i)	Synthetic detergent
(b)	Glyceryl ester of stearic acid	(ii)	Artificial sweetener
(c)	Sodium benzoate	(iii)	Antiseptic
(d)	Bithionol	(iv)	Food preservative

Choose the correct match :
(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Official Ans. by NTA (2)

Sol. Artificial sweetner : Sucralose

Antiseptic : Bithional

Preservative : Sodium Benzoate

Glyceryl ester of stearic acid : Sodium steasate

11. Given below are two statements :

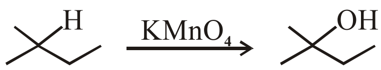
Statement-I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.

Statement-II : n-alkanes can be easily oxidised to corresponding alcohol with $KMnO_4$.

Choose the correct option :
(1) Both statement I and statement II are correct
(2) Both statement I and statement II are incorrect
(3) Statement I is correct but Statement II is incorrect
(4) Statement I is incorrect but Statement II is correct

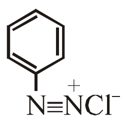
Official Ans. by NTA (3)

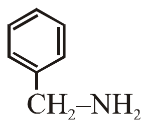
Sol. Alkane are very less reactive, tertiary hydrogen can oxidise to alcohal with $KMnO_4$.

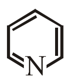


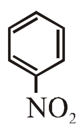
2-methyl-butane

12. Nitrogen can be estimated by Kjeldahl's method for which of the following compound ?

(1) 

(2) 

(3) 

(4) 

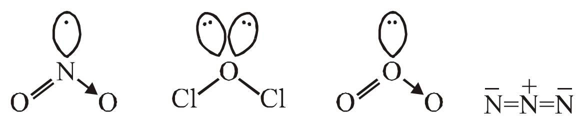
Official Ans. by NTA (2)

Sol. Kjeldahl method is not applicable to compounds containing nitrogen in nitrogroup, Azo groups and nitrogen present in the ring (e.g Pyridine) as nitrogen of these compounds does not change to Ammonium sulphate under these conditions.

13. Amongst the following, the linear species is :
(1) NO_2 (2) Cl_2O
(3) O_3 (4) N_3^-

Official Ans. by NTA (4)

Sol.



Bent shape Bent shape Bent shape Linear

14. $C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Enzyme A}} C_6H_{12}O_6 + C_6H_{12}O_6$

Sucrose Glucose Fructose

$C_6H_{12}O_6 \xrightarrow{\text{Enzyme B}} 2C_2H_5OH + 2CO_2$

Glucose

In the above reactions, the enzyme A and enzyme B respectively are :-

(1) Amylase and Invertase
(2) Invertase and Amylase
(3) Invertase and Zymase
(4) Zymase and Invertase

Official Ans. by NTA (3)

Sol. Informative

OR

$C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} C_6H_{12}O_6 + C_6H_{12}O_6$

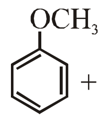
Glucose Fructose

$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$

15. One of the by-products formed during the recovery of NH_3 from Solvay process is :

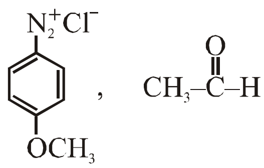
(1) $Ca(OH)_2$ (2) $NaHCO_3$
(3) $CaCl_2$ (4) NH_4Cl

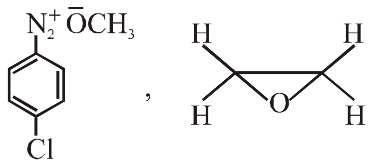
Official Ans. by NTA (3)

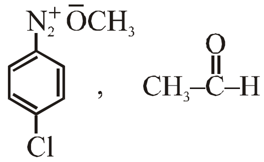
16. $C_7H_7N_2OCl + C_2H_5OH \rightarrow$  $+ N_2 + "X" + "Y"$

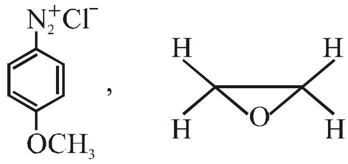
(A)

In the above reaction, the structural formula of (A), "X" and "Y" respectively are :

(1)  , HCl

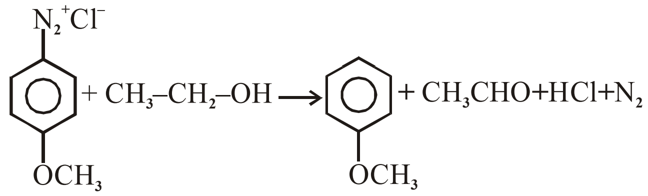
(2)  , HCl

(3)  , H_2O

(4)  , H_2O

Official Ans. by NTA (1)

Sol.



Anisole

17. For the coagulation of a negative sol, the species below, that has the highest flocculating power is :

(1) SO_4^{2-} (2) Ba^{2+} (3) Na^+ (4) PO_4^{3-}

Official Ans. by NTA (2)

Sol. To coagulate negative sol, cation with higher charge has higher coagulation value.

18. Which of the following statement(s) is (are) incorrect reason for eutrophication ?

(A) excess usage of fertilisers
(B) excess usage of detergents
(C) dense plant population in water bodies
(D) lack of nutrients in water bodies that prevent plant growth

Choose the most appropriate answer from the options given below :

(1) (A) only (2) (C) only
(3) (B) and (D) only (4) (D) only

Official Ans. by NTA (4)

Sol. The process in which nutrient enriched water bodies support a dense plant population which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as eutrophication.

19. Choose the correct statement regarding the formation of carbocations A and B given :-

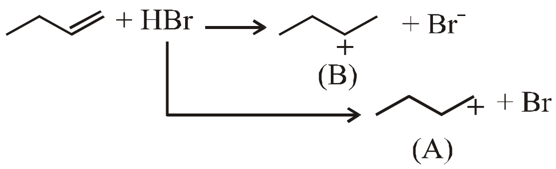
$CH_3-CH_2-CH=CH_2 + HBr \rightarrow CH_3-CH_2-CH_2-\overset{+}{C}H_2 + Br^-$ "A"

$CH_3-CH_2-CH=CH_2 + HBr \rightarrow CH_3-CH_2-\overset{+}{C}H-CH_3 + Br^-$ "B"

(1) Carbocation B is more stable and formed relatively at faster rate
(2) Carbocation A is more stable and formed relatively at slow rate
(3) Carbocation B is more stable and formed relatively at slow rate
(4) Carbocation A is more stable and formed relatively at faster rate

Official Ans. by NTA (1)

Sol.



This is more stable due to secondary cation formation and formed with faster rate due to low activation energy.

20. During which of the following processes, does entropy decrease ?

(A) Freezing of water to ice at 0°C
(B) Freezing of water to ice at –10°C
(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
(D) Adsorption of CO(g) and lead surface
(E) Dissolution of NaCl in water

Official Ans. by NTA (1)

(1) (A), (B), (C) and (D) only
(2) (B) and (C) only
(3) (A) and (E) only
(4) (A), (C) and (E) only

Sol. (A) Water $\xrightarrow{0°C}$ ice; $\Delta S = -ve$

(B) Water $\xrightarrow{-10°C}$ ice; $\Delta S = -ve$

(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$; $\Delta S = -ve$

(D) Adsorption; $\Delta S = -ve$

(E) $NaCl(s) \rightarrow Na^+(aq) + Cl^-(aq)$; $\Delta S = +ve$

SECTION-B

1. A KCl solution of conductivity 0.14 S m^{-1} shows a resistance of 4.19 Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03 Ω. The conductivity of the HCl solution is ____ $\times 10^{-2}$ S m^{-1}. (Round off to the Nearest Integer).

Official Ans. by NTA (57)

Sol. $\kappa = \frac{1}{R} \cdot G^*$

For same conductivity cell, G* is constant and hence κ.R. = constant.

$\therefore\ 0.14 \times 4.19 = \kappa \times 1.03$

or, κ of HCl solution $= \frac{0.14 \times 4.19}{1.03}$

$= 0.5695\ Sm^{-1}$

$= 56.95 \times 10^{-2}\ Sm^{-1} \approx 57 \times 10^{-2}\ Sm^{-1}$

2. On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is ____.

(Round off to the Nearest Integer).

Official Ans. by NTA (6)

Sol. $Fe^{3+} + 3K^+ + 3C_2O_4^{2-} \rightarrow \underset{(A)}{K_3[Fe(C_2O_4)_3]}$

Secondary valency of Fe in 'A' is 6.

3. The reaction $2A + B_2 \rightarrow 2AB$ is an elementary reaction.

For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of _____. (Round off to the Nearest Integer).

Official Ans. by NTA (27)

Sol. Reaction : $2A + B_2 \longrightarrow 2AB$

As the reaction is elementary, the rate of reaction is

$r = K \cdot [A]^2 [B_2]$

on reducing the volume by a factor of 3, the concentrations of A and B_2 will become 3 times and hence, the rate becomes $3^2 \times 3 = 27$ times of initial rate.

4. The total number of C–C sigma bond/s in mesityl oxide ($C_6H_{10}O$) is_____. (Round off to the Nearest Integer).

Official Ans. by NTA (5)

Sol. Mesityle oxide

$$H_3C \overset{\sigma}{-} \underset{\underset{CH_3}{|\sigma}}{C} \overset{\sigma}{=} CH \overset{\sigma}{-} \underset{\underset{O}{\|}}{C} \overset{\sigma}{-} CH_3$$

$\therefore\quad C \underset{\sigma}{-} C = 5$

5. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is_____ u. (Round off to the Nearest Integer).

[Density of water = 1.0 g cm^{-3}]

Official Ans. by NTA (85)

Sol. $K_4Fe(CN)_6 \rightleftharpoons 4K^+ + Fe(CN)_6^{4-}$

Initial conc. 1 m   0   0

Final conc. $(1 - 0.4)$m 4×0.4 0.4m

= 0.6 m   = 1.6 m

Effective molality = 0.6 + 1.6 + 0.4 = 2.6m

For same boiling point, the molality of another solution should also be 2.6 m.

Now, 18.1 weight percent solution means 18.1 gm solute is present in 100 gm solution and hence, (100 – 18.1 =) 81.9 gm water.

Now, $2.6 = \dfrac{18.1/M}{81.9/1000}$

$\therefore$ Molar mass of solute, M = 85

6. In the ground state of atomic Fe(Z = 26), the spin-only magnetic moment is ______ $\times 10^{-1}$ BM. (Round off to the Nearest Integer).

[Given : $\sqrt{3}$ = 1.73, $\sqrt{2}$ = 1.41]

Official Ans. by NTA (49)

Sol. $Fe \rightarrow [Ar]\ 4s^2 3d^6$ $\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

Number of unpaired e^- = 4

$\mu = \sqrt{4(4+2)}$ B.M.

$\mu = \sqrt{24}$ B.M.

$\mu = 4.89$ B.M.

$\mu = 48.9 \times 10^{-1}$ B.M.

Nearest integer value will be 49.

7. The number of chlorine atoms in 20 mL of chlorine gas at STP is_____10^{21}. (Round off to the Nearest Integer).

[Assume chlorine is an ideal gas at STP

R = 0.083 L bar mol^{-1} K^{-1}, $N_A = 6.023 \times 10^{23}$]

Official Ans. by NTA (1)

Sol. PV = nRT

$$1.0 \times \frac{20}{1000} = \frac{N}{6.023\times10^{23}} \times 0.083 \times 273$$

$\therefore$ Number of Cl_2 molecules, $N = 5.3 \times 10^{20}$

Hence, Number of Cl-atoms = 1.06×10^{21}

$\approx 1 \times 10^{21}$

8. KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1 g of KBr crystal is _____10^{14}. (Round off to the Nearest Integer).

[Atomic Mass : K : 39.1 u, Br : 79.9 u,

$N_A = 6.023 \times 10^{23}$]

Official Ans. by NTA (5)

Sol. 1 mole KBr (= 119 gm) have $\dfrac{10^{-5}}{100}$ moles $SrBr_2$

and hence, 10^{-7} moles cation vacancy (as 1 Sr^{2+} will result 1 cation vacancy)

$\therefore$ Required number of cation vacancies

$$= \frac{10^{-7}\times6.023\times10^{23}}{119} = 5.06\times10^{14} \simeq 5\times10^{14}$$

9. Consider the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$. The temperature at which K_C = 20.4 and K_P = 600.1, is_____K. (Round off to the Nearest Integer).

[Assume all gases are ideal and R = 0.0831 L bar K^{-1} mol^{-1}]

Official Ans. by NTA (354)

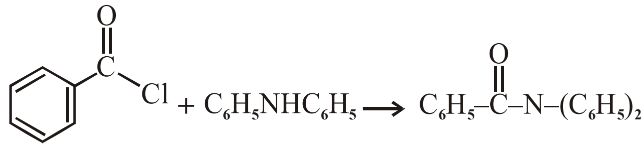
Sol. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$; $\Delta n_g = 2 - 1 = 1$

Now, $K_p = K_c\ .\ (RT)^{\Delta ng}$

or, $600.1 = 20.4 \times (0.0831 \times T)^1$

$\therefore$ T = 353.99 K = 354K

10.



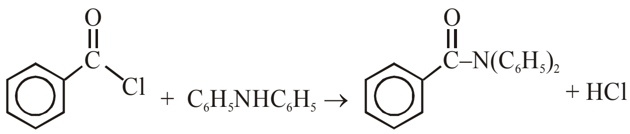
0.140g   0.388g   0.210g

Consider the above reaction. The percentage yield of amide product is _______. (Round off to the Nearest Integer).

(Given : Atomic mass : C : 12.0 u, H : 1.0u, N : 14.0 u, O : 16.0 u, Cl : 35.5 u)

Official Ans. by NTA (77)

Sol.



1 mole   1 mole   1 mole

= 140.5 gm   = 169 gm   = 273 gm

$\therefore$ 0.140 gm $\dfrac{169}{140.5}\times0.140$

L.R.   = 0.168 gm < 0.388 gm

excess

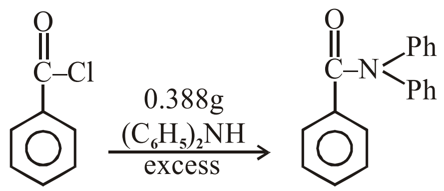
$\therefore$ Theoretical amount of given product formed

$$= \frac{273}{140.5}\times0.140 = 0.272\text{gm}$$

But its actual amount formed is 0.210 gm. Hence, the percentage yield of product.

$$= \frac{0.210}{0.272}\times100 = 77.20 \approx 77$$

OR



0.140g 0.210g

Mole of Ph – CoCl = $\frac{0.140}{140} = 10^{-3}$mol

Mole of $Ph-\overset{O}{\overset{\|}{C}}-N(Ph)_2$, that should be obtained by mol-mol analysis = 10^{-3} mol.

Theoritical mass of product = $10^{-3} \times 273 = 273 \times 10^{-3}$g

Observed mass of product = 210×10^{-3}g

%yield of product = $\frac{210\times10^{-3}}{273\times10^{-3}}\times100 = 76.9\% = 77$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let f : R → R be defined as $f(x) = e^{-x} \sin x$. If F : [0, 1] → R is a differentiable function such that $F(x)=\int_0^x f(t)\,dt$, then the value of $\int_0^1 (F'(x)+f(x))e^x dx$ lies in the interval

(1) $\left[\frac{327}{360},\frac{329}{360}\right]$ (2) $\left[\frac{330}{360},\frac{331}{360}\right]$

(3) $\left[\frac{331}{360},\frac{334}{360}\right]$ (4) $\left[\frac{335}{360},\frac{336}{360}\right]$

Official Ans. by NTA (2)

Sol. $f(x)=e^{-x}\sin x$

Now, $F(x)=\int_0^x f(t)dt \quad \Rightarrow F'(x) = f(x)$

$$I=\int_0^1 (F'(x)+f(x))e^x\,dx = \int_0^1 (f(x)+f(x))\cdot e^x\,dx$$

$$= 2\int_0^1 f(x)\cdot e^x\,dx = 2\int_0^1 e^{-x}\sin x\cdot e^x\,dx$$

$$= 2\int_0^1 \sin x\,dx$$

$$= 2(1 - \cos 1)$$

$$I=2\left\{1-\left(1-\frac{1}{\lfloor 2}+\frac{1}{\lfloor 4}+\frac{1}{\lfloor 6}+\frac{1}{\lfloor 8}.........\right)\right\}$$

$$I=1-\frac{2}{\lfloor 4}+\frac{2}{\lfloor 6}-\frac{2}{\lfloor 9}+......$$

$$1-\frac{2}{\lfloor 4}<I<1-\frac{2}{\lfloor 4}+\frac{2}{\lfloor 6}$$

$$\frac{11}{12}<I<\frac{331}{360}$$

$$\Rightarrow I\in\left[\frac{11}{12},\frac{331}{360}\right]$$

$$\Rightarrow I\in\left[\frac{330}{360},\frac{331}{360}\right]$$ **Ans. (2)**

2. If the integral $\int_0^{10}\frac{[\sin 2\pi x]}{e^{x-[x]}}dx=\alpha e^{-1}+\beta e^{-\frac{1}{2}}+\gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of α + β + γ is equal to :

(1) 0 (2) 20 (3) 25 (4) 10

Official Ans. by NTA (1)

Sol. Let $I=\int_0^{10}\frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \int_0^{10}\frac{[\sin 2\pi x]}{e^{\{x\}}}dx$

Function $f(x)=\frac{[\sin 2\pi x]}{e^{\{x\}}}$ is periodic with period '1'

Therefore

$$I=10\int_0^1\frac{[\sin 2\pi x]}{e^{\{x\}}}dx$$

$$=10\int_0^1\frac{[\sin 2\pi x]}{e^{x}}dx$$

$$=10\left(\int_0^{1/2}\frac{[\sin 2\pi x]}{e^{x}}dx+\int_{1/2}^1\frac{[\sin 2\pi x]}{e^{x}}dx\right)$$

$$=10\left(0+\int_{1/2}^1\frac{(-1)}{e^{x}}dx\right)$$

$$=-10\int_{1/2}^1 e^{-x}\,dx$$

$$=10\left(e^{-1}-e^{-1/2}\right)$$

Now,

$$10\cdot e^{-1}-10\cdot e^{-1/2}= \alpha e^{-1}+\beta e^{-1/2}+\gamma \text{ (given)}$$

⇒ α = 10, β = –10, γ = 0

⇒ α + β + γ = 0 **Ans. (1)**

3. Let y = y(x) be the solution of the differential equation

cosx (3sinx + cosx + 3)dy = (1 + y sinx (3sinx + cosx + 3))dx,

$0 \le x \le \frac{\pi}{2}$, y(0) = 0. Then , $y\left(\frac{\pi}{3}\right)$ is equal to:

(1) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

(3) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

Official Ans. by NTA (2)

Sol. $\cos x(3\sin x+\cos x+3)dy$

$$= (1+y\sin x(3\sin x+\cos x+3))dx$$

$$\frac{dy}{dx}-(\tan x)y = \frac{1}{(3\sin x+\cos x+3)\cos x}$$

$$\text{I.F.} = e^{\int -\tan x\,dx} = e^{\ell n|\cos x|} = |\cos x|$$

$$= \cos x \;\forall\; x \in \left[0,\frac{\pi}{2}\right)$$

Solution of D.E.

$$y(\cos x) = \int(\cos x)\cdot\frac{1}{\cos x(3\sin x+\cos x+3)}dx + C$$

$$y(\cos x) = \int\frac{dx}{3\sin x+\cos x+3}dx + C$$

$$y(\cos x) = \int\frac{\left(\sec^2\frac{x}{2}\right)}{2\tan^2\frac{x}{2}+6\tan\frac{x}{2}+4}dx + C$$

Now

$$\text{Let } I_1 = \int\frac{\left(\sec^2\frac{x}{2}\right)}{2\left(\tan^2\frac{x}{2}+3\tan\frac{x}{2}+2\right)}dx + C$$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$I_1 = \int\frac{dt}{t^3+3t+2} = \int\frac{dt}{(t+2)(t+1)}$$

$$= \int\left(\frac{1}{t+1}-\frac{1}{t+2}\right)dt$$

$$= \ell n\left|\left(\frac{t+1}{t+2}\right)\right| = \ell n\left|\left(\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right)\right|$$

So solution of D.E.

$$y(\cos x) = \ell n\left|\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right| + C$$

$$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + C \quad \text{for } 0 \le x < \frac{\pi}{2}$$

Now, it is given y(0) = 0

$$\Rightarrow 0 = \ell n\left(\frac{1}{2}\right) + C \qquad \Rightarrow \boxed{C = \ell n 2}$$

$$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + \ell n 2$$

For $x = \frac{\pi}{3}$

$$y\left(\frac{1}{2}\right) = \ell n\left(\frac{1+\frac{1}{\sqrt{3}}}{2+\frac{1}{\sqrt{3}}}\right) + \ell n 2$$

$$y = 2\ell n\left(\frac{2\sqrt{3}+10}{11}\right)$$

Ans.(2)

4. The value of $\sum_{r=0}^{6}\left({}^6C_r\cdot{}^6C_{6-r}\right)$ is equal to :

(1) 1124 (2) 1324 (3) 1024 (4) 924

Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{6}{}^6C_r\cdot{}^6C_{6-r}$

$= {}^6C_0\cdot{}^6C_6 + {}^6C_1\cdot{}^6C_5 + \ldots\ldots + {}^6C_6\cdot{}^6C_0$

Now,

$(1+x)^6(1+x)^6$

$= \left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \ldots\ldots + {}^6C_6x^6\right)$

$\left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + \ldots\ldots + {}^6C_6x^6\right)$

Comparing coefficeint of x^6 both sides

${}^6C_0\cdot{}^6C_6 + {}^6C_1 + {}^6C_5 + \ldots\ldots. + {}^6C_6\cdot{}^6C_0 = {}^{12}C_6$

= 924

Ans.(4)

5. The value of $\lim\limits_{n\to\infty}\dfrac{[r]+[2r]+.....+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

(1) $\dfrac{r}{2}$ (2) r (3) 2r (4) 0

Official Ans. by NTA (1)

Sol. We know that

$$r \le [r] < r+1$$

and

$$2r \le [2r] < 2r+1$$
$$3r \le [3r] < 3r+1$$
$$\vdots \quad \vdots \quad \vdots$$
$$nr \le [nr] < nr+1$$

r + 2r + ...+ nr

$\le$ [r] + [2r] +....+ [nr] < (r + 2r +....+ nr) + n

$$\frac{\frac{n(n+1)}{2}\cdot r}{n^2} \le \frac{[r]+[2r]+......+[nr]}{n^2} < \frac{\frac{n(n+1)}{2}r+n}{n^2}$$

Now,

$$\lim_{n\to\infty}\frac{n(n+1)\cdot r}{2\cdot n^2}=\frac{r}{2}$$

and $$\lim_{n\to\infty}\frac{\frac{n(n+1)r}{2}+n}{n^2}=\frac{r}{2}$$

So, by Sandwich Theorem, we can conclude that

$$\lim_{n\to\infty}\frac{[r]+[2r]+......+[nr]}{n^2}=\frac{r}{2}$$

Ans. (1)

6. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for x ∈ [–1, 1], and [x] denotes the greatest integer less than or equal to x, is :

(1) 2 (2) 0

(3) 4 (4) Infinite

Official Ans. by NTA (2)

Sol. Given equation

$$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if

$$-1\le x^2+\frac{1}{3}<2 \Rightarrow \frac{-4}{3}\le x^2<\frac{5}{3}$$

$$\Rightarrow \boxed{0\le x^2<\frac{5}{3}} \qquad ....(1)$$

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$$-1\le x^2-\frac{2}{3}<2 \Rightarrow \frac{-1}{3}\le x^2<\frac{8}{3}$$

$$\Rightarrow \boxed{0\le x^2<\frac{8}{3}} \qquad ....(2)$$

So, form (1) and (2) we can conclude

$$\boxed{0\le x^2<\frac{5}{3}}$$

Case - I if $0\le x^2<\frac{2}{3}$

$\sin^{-1}(0)+\cos^{-1}(-1)=x^2$

$\Rightarrow x + \pi = x^2$

$\Rightarrow x^2 = \pi$

but $\pi\notin\left[0,\frac{2}{3}\right)$

$\Rightarrow$ No value of 'x'

Case - II if $\frac{2}{3}\le x^2<\frac{5}{3}$

$\sin^{-1}(1)+\cos^{-1}(0)=x^2$

$\Rightarrow \frac{\pi}{2}+\frac{\pi}{2}=x^2$

$\Rightarrow x^2 = \pi$

but $\pi\notin\left[\frac{2}{3},\frac{5}{3}\right)$

$\Rightarrow$ No value of 'x'

So, number of solutions of the equation is zero.

Ans.(2)

7. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Official Ans. by NTA (4)

Sol. $\underset{\text{odd place}}{1}\ \underset{\text{even place}}{0}\ \underset{\text{odd place}}{0}\ \underset{\text{even place}}{1}$

or $\underset{\text{even place}}{1}\ \underset{\text{odd place}}{0}\ \underset{\text{even place}}{0}\ \underset{\text{odd place}}{1}$

$$\Rightarrow \left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right)+\left(\frac{2}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right)$$

$$\Rightarrow \frac{1}{9}$$

8. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

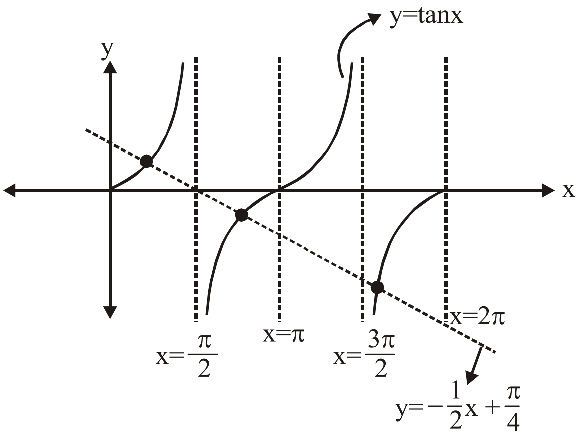
(1) 3 (2) 4 (3) 2 (4) 5

Official Ans. by NTA (1)

Sol. $x+2\tan x=\frac{\pi}{2}$

$$\Rightarrow 2\tan x=\frac{\pi}{2}-x$$

$$\Rightarrow \tan x=-\frac{1}{2}x+\frac{\pi}{4}$$



Number of soluitons of the given eauation is '3'.

Ans. (1)

9. Let S_1, S_2 and S_3 be three sets defined as

$S_1=\{z\in\mathbb{C}:|z-1|\le\sqrt{2}\}$

$S_2=\{z\in\mathbb{C}:\text{Re}((1-i)z)\ge 1\}$

$S_3=\{z\in\mathbb{C}:\text{Im}(z)\le 1\}$

Then the set $S_1\cap S_2\cap S_3$

(1) is a singleton

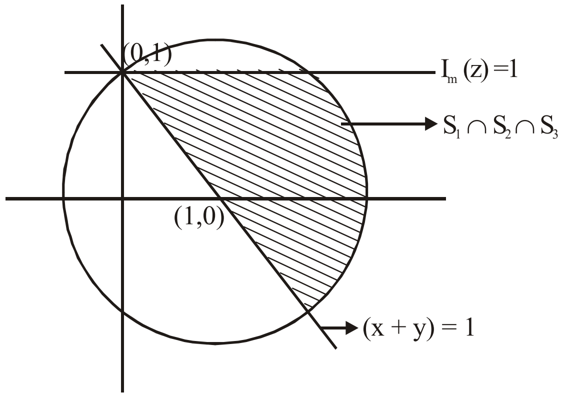
(2) has exactly two elements

(3) has infinitely many elements

(4) has exactly three elements

Official Ans. by NTA (3)

Sol. For $|z-1|\le\sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre (1, 0).



For S_2

Let z = x + iy

Now, (1 – i) (z) = (1 – i) (x + iy)

Re((1 – i)z) = x + y

$\Rightarrow x + y \ge 1$

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinity many elements

Ans. (3)

10. If the curve y = y(x) is the solution of the differential equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}\,dx$, $x > 0$

which passes through the point $\left(1, 1-\frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to :

(1) $4\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$ (4) $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx}-\frac{y}{2x}=\frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$

$$\text{IF}=e^{-\int\frac{dx}{2d}}=e^{-\frac{1}{2}\ln x}=\frac{1}{x^{1/2}}$$

$$y.x^{-1/2}=\int\frac{x^{9/4}\cdot x^{-1/2}}{x^{5/4}\left(x^{3/4}+1\right)}dx$$

$$\int\frac{x^{1/2}}{(x^{3/4}+1)}dx$$

$x = t^4 \Rightarrow dx = 4t^3\,dt$

$$\int \frac{t^2 \cdot 4t^3\,dt}{(t^3+1)}$$

$$4\int \frac{t^2\left(t^3+1-1\right)}{(t^3+1)}dt$$

$$4\int t^2dt - 4\int \frac{t^2}{t^3+1}dt$$

$$\frac{4t^3}{3} - \frac{4}{3}\ln\left(t^3+1\right) + C$$

$$yx^{-1/2} = \frac{4x^{3/4}}{3} - \frac{4}{3}\ln\left(x^{3/4}+1\right) + C$$

$$1 - \frac{4}{3}\log_e 2 = \frac{4}{3} - \frac{4}{3}\log_e 2 + C$$

$$\Rightarrow C = -\frac{1}{3}$$

$$y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\ln\left(x^{3/4}+1\right) - \frac{\sqrt{x}}{3}$$

$$y(16) = \frac{4}{3}\times 32 - \frac{4}{3}\times 4\ln 9 - \frac{4}{3}$$

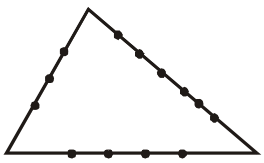
$$= \frac{124}{3} - \frac{32}{3}\ln 3 = 4\left(\frac{31}{3} - \frac{8}{3}\ln 3\right)$$

11. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

(1) 364 (2) 240 (3) 333 (4) 360

Official Ans. by NTA (3)

Sol.



Total Number of triangles formed

$= {}^{14}C_3 - {}^{3}C_3 - {}^{5}C_3 - {}^{6}C_3$

$= 333$ **Option (3)**

12. If x, y, z are in arithmetic progression with common difference d, $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is

(1) 72 (2) 12 (3) 36 (4) 6

Official Ans. by NTA (1)

Sol. $\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$R_2 \to R_1 + R_3 - 2R_2$

$$\Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k-6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$$

$\Rightarrow \left(k-6\sqrt{2}\right)(3z-5x) = 0$

if $3z - 5x = 0 \Rightarrow 3(x + 2d) - 5x = 0$

$\Rightarrow x = 3d$ (Not possible)

$\Rightarrow k = 6\sqrt{2} \qquad \Rightarrow k^2 = 72$ **Option (1)**

13. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, x, y $\in$ R, x > 0, be such that $\left|\overrightarrow{PQ}\right| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, z $\in$ R, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to

(1) 7 (2) 9 (3) 2 (4) 1

Official Ans. by NTA (2)

Sol. $\overrightarrow{OP} \perp \overrightarrow{OQ}$

$\Rightarrow -x + 2y - 3x = 0$

$\Rightarrow y = 2x$ .....(i)

$\left|\overrightarrow{PQ}\right|^2 = 20$

$\Rightarrow (x + 1)^2 + (y - 2)^2 + (1 + 3x)^2 = 20$

$\Rightarrow x = 1$

$\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}$ are coplanar.

$$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0$$

$\Rightarrow 1(-14 - 3z) - 2(7 - 9) - 1(-z - 6) = 0$

$\Rightarrow z = -2$

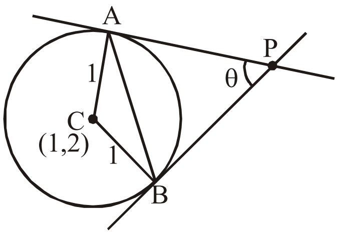
$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$ **Option (2)**

14. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(1) 11 : 4 (2) 9 : 4 (3) 3 :1 (4) 2 : 1

Official Ans. by NTA (2)

Sol.



$$\tan\theta = \frac{12}{5}$$

$$PA = \cot\frac{\theta}{2}$$

$$\therefore \text{ area of } \Delta PAB = \frac{1}{2}(PA)^2 \sin\theta = \frac{1}{2}\cot^2\frac{\theta}{2}\sin\theta$$

$$= \frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta$$

$$= \frac{1}{2}\left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)\left(\frac{12}{13}\right) = \frac{1}{2}\frac{18}{18}\times\frac{2}{13} = \frac{27}{26}$$

$$\text{area of } \Delta CAB = \frac{1}{2}\sin\theta = \frac{1}{2}\left(\frac{12}{13}\right) = \frac{6}{13}$$

$$\therefore \frac{\text{area of } \Delta PAB}{\text{area of } \Delta CAB} = \frac{9}{4}$$ **Option (2)**

15. Consider the function $f : R \to R$ defined by $f(x) = \begin{cases} \left(2-\sin\left(\frac{1}{x}\right)\right)|x|, & x \neq 0 \\ 0 & , x = 0 \end{cases}$. Then f is :

(1) monotonic on $(-\infty, 0) \cup (0, \infty)$

(2) not monotonic on $(-\infty, 0)$ and $(0, \infty)$

(3) monotonic on $(0, \infty)$ only

(4) monotonic on $(-\infty, 0)$ only

Official Ans. by NTA (2)

Sol. $f(x) = \begin{cases} -x\left(2-\sin\left(\frac{1}{x}\right)\right) & x<0 \\ 0 & x=0 \\ x\left(2-\sin\left(\frac{1}{x}\right)\right) & \end{cases}$

$$f'(x) = \begin{cases} -\left(2-\sin\frac{1}{x}\right) - x\left(-\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)\right) & x<0 \\ \left(2-\sin\frac{1}{x}\right) + x\left(-\cos\frac{1}{x}\left(-\frac{1}{x^2}\right)\right) & x>0 \end{cases}$$

$$f'(x) = \begin{cases} -2+\sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x} & x<0 \\ 2-\sin\frac{1}{x} + \frac{1}{x}\cos\frac{1}{x} & x>0 \end{cases}$$

$f'(x)$ is an oscillating function which is non-monotonic in $(-\infty, 0) \cup (0, \infty)$.

Option (2)

16. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to :

(1) 11 (2) 14 (3) 16 (4) 20

Official Ans. by NTA (2)

Sol. Tangent to parabola

$2y = 2(x + 6) - 20$

$\Rightarrow y = x - 4$

Condition of tangency for ellipse.

$16 = 2(1)^2 + b$

$\Rightarrow b = 14$ **Option (2)**

17. The value of the limit $\lim_{\theta\to 0}\frac{\tan\left(\pi\cos^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$ is equal to :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) 0 (4) $\frac{1}{4}$

Official Ans. by NTA (1)

Sol. $\lim_{\theta\to 0}\frac{\tan\left(\pi\left(1-\sin^2\theta\right)\right)}{\sin\left(2\pi\sin^2\theta\right)}$

$$= \lim_{\theta\to 0}\frac{-\tan\left(\pi\sin^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$= \lim_{\theta\to 0} -\left(\frac{\tan\left(\pi\sin^2\theta\right)}{\pi\sin^2\theta}\right)\left(\frac{2\pi\sin^2\theta}{\sin\left(2\pi\sin^2\theta\right)}\right)\times\frac{1}{2}$$

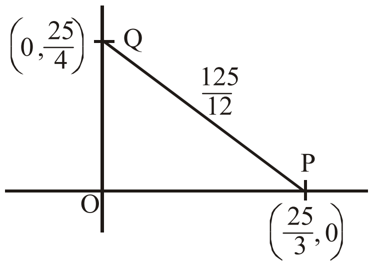
$$= \frac{-1}{2}$$ **Option (1)**

18. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(1) $\frac{529}{64}$ (2) $\frac{125}{72}$ (3) $\frac{625}{72}$ (4) $\frac{585}{66}$

Official Ans. by NTA (3)

Sol. Tangent to circle $3x + 4y = 25$



OP + OQ + OR = 25

$$\text{Incentre} = \left(\frac{\frac{25}{4}\times\frac{25}{3}}{25}, \frac{\frac{25}{4}\times\frac{25}{3}}{25}\right)$$

$$= \left(\frac{25}{12}, \frac{25}{12}\right)$$

$$\therefore\ r^2 = 2\left(\frac{25}{12}\right)^2 = 2\times\frac{625}{144} = \frac{625}{72}$$

Option (3)

19. If the Boolean expression $(p\wedge q)\circledast(p\otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

Official Ans. by NTA (1)

Sol. **Option (1)**

$(p\wedge q)\longrightarrow(p\rightarrow q)$

$= \sim(p\wedge q)\vee(\sim p\vee q)$

$= (\sim p\vee \sim q)\vee(\sim p\vee q)$

$= \sim p\vee(\sim q\vee q)$

$= \sim p\vee t$

$= t$

Option (2)

$(p\wedge q)\wedge(p\vee q) = (p\wedge q)$ (Not a tautology)

Option (3)

$(p\wedge q)\vee(p\rightarrow q)$

$= (p\wedge q)\vee(\sim p\vee q)$

$= \sim p\vee q$ (Not a tautology)

Option (4)

$(p\wedge q)\wedge(p\rightarrow q)$

$= (p\wedge q)\wedge(\sim p\vee q)$

$= p\wedge q$ (Not a tautology)

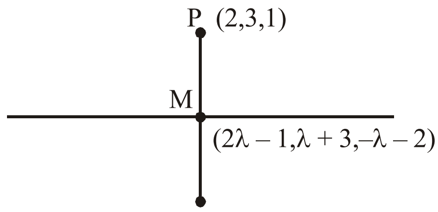
Option (1)

20. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ and containing the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

(1) 20 (2) 19 (3) 18 (4) 21

Official Ans. by NTA (2)

Sol. Line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$



$\overrightarrow{PM} = (2\lambda-3, \lambda, -\lambda-3)$

$\overrightarrow{PM} \perp (2\hat{i}+\hat{j}-\hat{k})$

$4\lambda - 6 + \lambda + \lambda + 3 = 0 \Rightarrow \lambda = \frac{1}{2}$

$\therefore\ M \equiv \left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore$ Reflection (–2, 4, –6)

$$\text{Plane : } \begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(-10+3)-(y-1)(15-4)+(z+1)(-1)=0$

$\Rightarrow -7x + 14 - 11y + 11 - z - 1 = 0$

$\Rightarrow 7x + 11y + z = 24$

$\therefore\ \alpha = 7, \beta = 11, \gamma = 1$

$\alpha + \beta + \gamma = 19$ **Option (2)**

SECTION-B

1. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to :

Official Ans. by NTA (2)

Sol. $2\log_{10}(4^x-2)=1+\log_{10}\left(4^x+\frac{18}{5}\right)$

$(4^x-2)^2=10\left(4^x+\frac{18}{5}\right)$

$(4^x)^2 + 4 - 4(4^x) - 32 = 0$

$(4^x - 16)(4^x + 2) = 0$

$4^x = 16$

$x = 2$

$\begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 3(-2) - 1(0-4) + 4(1)$

$= -6 + 4 + 4 = 2$

2. Let $f : [-1, 1] \to R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where $a, b, c \in R$ such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to ______.

Official Ans. by NTA (5)

Sol. $f : [-1, 1] \to R$

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c = 2 \quad ...(1)$

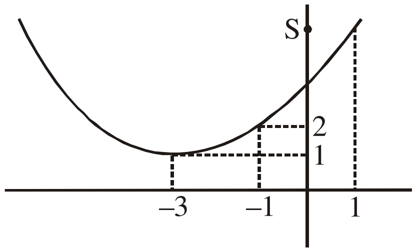
$f'(-1) = -2a + b = 1 \quad ...(2)$

$f''(x) = 2a$

$\Rightarrow$ Max. value of $f''(x) = 2a = \frac{1}{2}$

$\Rightarrow a = \frac{1}{4};\quad b = \frac{3}{2};\quad c = \frac{13}{4}$

$\therefore\ f(x) = \frac{x^2}{4}+\frac{3}{2}x+\frac{13}{4}$



For, $x \in [-1, 1] \Rightarrow 2 \le f(x) \le 5$

$\therefore$ Least value of α is 5

3. Let $f : [-3, 1] \to R$ be given as

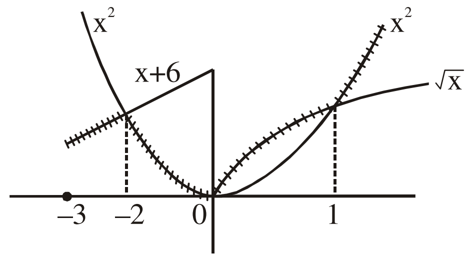
$$f(x)=\begin{cases}\min\{(x+6),x^2\}, & -3\le x\le 0\\ \max\{\sqrt{x},x^2\}, & 0\le x\le 1.\end{cases}$$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6A is equal to ______.

Official Ans. by NTA (41)

Sol. $f : [-3, 1] \to R$

$$f(x)=\begin{cases}\min\{(x+6),x^2\}, & -3\le x\le 0\\ \max\{\sqrt{x},x^2\}, & 0\le x\le 1\end{cases}$$



area bounded by $y = f(x)$ and x-axis

$= \int_{-3}^{-2}(x+6)dx+\int_{-2}^{0}x^2dx+\int_{0}^{1}\sqrt{x}\,dx$

$A = \frac{41}{6}$

$6A = 41$

4. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \ne \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin.If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos3\alpha+\cos3\beta+\cos3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to :

Official Ans. by NTA (144)

Sol. Since orthocentre and circumcentre both lies on y-axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow \Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$$

$= 12$

5. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

Official Ans. by NTA (68)

Sol. Let number be $a_1, a_2, a_3, \ldots\ldots a_{2n}, b_1, b_2, b_3 \ldots b_n$

$$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2$$

$$\Rightarrow \sum a^2 + \sum b^2 = 87n$$

Now, distribution becomes

$a_1 + 1, a_2 + 1, a_3 + 1, \ldots\ldots a_{2n} + 1, b_1 - 1, b_2 - 1 \ldots\ldots b_n - 1$

Variance

$$= \frac{\sum(a+1)^2 + \sum(b-1)^2}{3n} - \left(\frac{12n + 2n + 3n - n}{3n}\right)^2$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n}$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{87n + 3n + 2(12n) - 2(3n)}{3n} - \left(\frac{16}{3}\right)^2$$

$$\Rightarrow k = \frac{108}{3} - \left(\frac{16}{5}\right)^2$$

$\Rightarrow 9k = 3(108) - (16)^2 = 324 - 256 = 68$

Ans. 68.00

6. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _______.

Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^nC_r (x)^{n-r}\left(\frac{a}{x^2}\right)^r$

$= {}^nC_r\, a^r x^{n-3r}$

${}^nC_2\, a^2 : {}^nC_3\, a^3 : {}^nC_4\, a^4 = 12 : 8 : 3$

After solving

$n = 6,\ a = \frac{1}{2}$

For term independent of 'x' $\Rightarrow n = 3r$

$r = 2$

$\therefore$ Coefficient is ${}^6C_2\left(\frac{1}{2}\right)^2 = \frac{15}{4}$

Nearest integer is 4.

Ans. 4

7. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ________.

Official Ans. by NTA (2020)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

AB = B

$\Rightarrow (A - I)B = O$

$\Rightarrow |A - I| = O$, since $B \neq O$

$$\begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix} = 0$$

ad – bc = 2020

8. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ________.

Sol. Let $\vec{x} = \lambda\vec{a} + \mu\vec{b}$ (λ and μ are scalars)

$\vec{x} = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$

Since $\vec{x}\cdot(3\hat{i} + 2\hat{j} - \hat{k}) = 0$

$3\lambda + 8\mu = 0$ .....(1)

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2}$

$6\lambda - \mu = 51$ .....(2)

From (1) and (2)

$\lambda = 8,\ \mu = -3$

$\vec{x} = 13\hat{i} - 14\hat{j} + 11\hat{k}$

$|\vec{x}|^2 = 486$ **Ans.**

Official Ans. by NTA (486)

9. Let $I_n = \int_1^e x^{19}(\log|x|)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equal to ________.

Official Ans. by NTA (1)

Sol. Let $\vec{x} = \lambda\vec{a} + \mu\vec{b}$ (λ and μ are scalars)

$\vec{x} = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$

Since $\vec{x}\cdot(3\hat{i} + 2\hat{j} - \hat{k}) = 0$

$3\lambda + 8\mu = 0$ .....(1)

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2}$

$6\lambda - \mu = 51$ .....(2)

From (1) and (2)

$\lambda = 8,\ \mu = -3$

$\vec{x} = 13\hat{i} - 14\hat{j} + 11\hat{k}$

$|\vec{x}|^2 = 486$ **Ans.**

10. Let P be an arbitrary point having sum of the squares of the distance from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to _______.

Official Ans. by NTA (0)

Sol. Let point P is (α, β, γ)

$$\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2 + \left(\frac{\ell\alpha - n\gamma}{\sqrt{\ell^2+n^2}}\right)^2 + \left(\frac{\alpha - 2\beta + \gamma}{\sqrt{6}}\right)^2 = 9$$

Locus is

$$\frac{(x+y+z)^2}{3} + \frac{(\ell x - nz)^2}{\ell^2+n^2} + \frac{(x-2y+z)^2}{6} = 9$$

$$x^2\left(\frac{1}{2} + \frac{\ell^2}{\ell^2+n^2}\right) + y^2 + z^2\left(\frac{1}{2} + \frac{n^2}{\ell^2+n^2}\right) + 2zx\left(\frac{1}{2} - \frac{\ell n}{\ell^2+n^2}\right) - 9 = 0$$

Since its given that $x^2 + y^2 + z^2 = 9$

After solving $\ell = n$

SECTION-A

1. An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field 3.55×10^5 V m^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess ? (consider g = 9.81 m/s^2)

(1) 48.8×10^{11}

(2) 1.73×10^{10}

(3) 17.3×10^{10}

(4) 1.73×10^{12}

Official Ans. by NTA (2)

Sol. qE = Mg

$$neE = \rho\left(\frac{4}{3}\pi r^3\right) \times g$$

$$n \times 1.6 \times 10^{-19} \times 3.55 \times 10^5$$

$$= 3 \times 10^3 \times \frac{4}{3} \times \pi \times (2 \times 10^{-3})^3 \times 9.81$$

$$n = 173 \times 10^{(3 - 9 - 5 + 19)}$$

$$n = 1.73 \times 10^{10}$$

2. Match List–I with List–II.

List–I

(a) 10 km height over earth's surface

(b) 70 km height over earth's surface

(c) 180 km height over earth's surface

(d) 270 km height over earth's surface

List–II

(i) Thermosphere

(ii) Mesosphere

(iii) Stratosphere

(iv) Troposphere

(1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)

(2) (a)–(i), (b)–(iv), (c)–(iii), (d)–(ii)

(3) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)

(4) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

Official Ans. by NTA (1)

Sol. Order of atmosphere stratification from bottom Troposphere, stralospherre, Mesosphre, Thermosphere

(a) $\rightarrow$ (iv)

(b) $\rightarrow$ (iii)

(c) $\rightarrow$ (ii)

(d) $\rightarrow$ (i)

3. Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be :-

(1) 13.6 eV (2) 2815.2 eV

(3) 331.2 eV (4) 27.2 eV

Official Ans. by NTA (2)

Sol. $E \propto \frac{1}{r}$ $\qquad r \propto \frac{1}{m}$

$E \propto m$

$$\text{Ionization potential} = 13.6 \times \frac{(Mass_\mu)eV}{(Mass_e)}$$

$= 13.6 \times 207$ eV $= 2815.2$ eV

4. A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B} = 2.0 \times 10^{-8}\hat{k}T$. (where, $\hat{k}$ is unit vector along z-direction) What is $\vec{E}$ at this point ?

(1) $0.6\hat{j}$ V/m (2) $6.0\hat{k}$ V/m

(3) $6.0\hat{j}$ V/m (4) $0.6\hat{k}$ V/m

Official Ans. by NTA (3)

Sol. $E = BC = 6$

(Dir. of wave) $\parallel \left(\vec{E}\times\vec{B}\right)$

$\hat{i} = \hat{j}\times\hat{k}$

$\vec{E} = 6\hat{j}\ \text{V/m}$

5. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become :

(1) $\omega\dfrac{M}{M+m}$ (2) $\omega\dfrac{M+2m}{M}$

(3) $\omega\dfrac{M}{M+2m}$ (4) $\omega\dfrac{M-2m}{M+2m}$

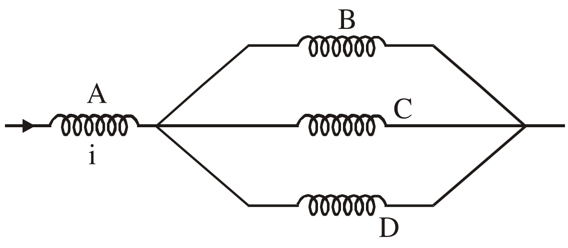
Official Ans. by NTA (3)

Sol. Using conservation of angular momentum

$(Mr^2)\omega = (Mr^2 + 2mr^2)\omega'$

$$\omega' = \frac{M\omega}{M+2m}$$

6. Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3T, the field at the center of C would be : (Assume that the magnetic field is confined with in the volume of respective solenoid).

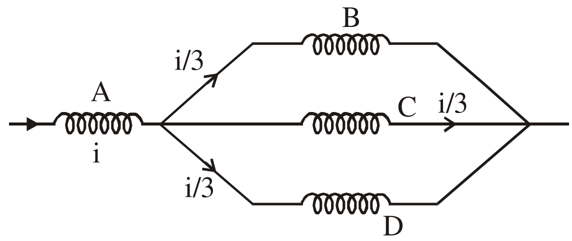


(1) 12T (2) 6T

(3) 9T (4) 1T

Official Ans. by NTA (4)

Sol.



$\phi \propto i$

$\Rightarrow B \propto i$

so, field at centre of C = $\dfrac{3}{3} = 1\text{T}$

7. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\dfrac{\ell}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to the nearest integer is:-

(1) 2% (2) 3%

(3) 5% (4) 4%

Official Ans. by NTA (2)

Sol. $g = \dfrac{4\pi^2\ell}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + 2\frac{\Delta T}{T} = \frac{0.1}{10} + 2\left(\frac{\frac{1}{200}}{0.5}\right)$$

$$\frac{\Delta g}{g} = \frac{1}{100} + \frac{1}{50}$$

$$\frac{\Delta g}{g}\times 100 = 3\%$$

8. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to :-

(1) $t^{2/3}$ (2) $t^{3/2}$

(3) t (4) $t^{1/2}$

Official Ans. by NTA (2)

Sol. P = C

FV = C

$$M\frac{dV}{dt}V = C$$

$$\frac{V^2}{2} \propto t$$

$V \propto t^{1/2}$

$$\frac{dx}{dt} \propto t^{1/2}$$

$x \propto t^{3/2}$

9. What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T ? (k_B is Boltzmann constant)

(1) $\frac{1}{2}k_BT$ (2) $\frac{2}{3}k_BT$

(3) $\frac{3}{2}k_BT$ (4) k_BT

Official Ans. by NTA (1)

Sol. Energy associated with each degree of freedom per molecule = $\frac{1}{2}k_BT$.

10. A radioactive sample disintegrates via two independent decay processes having half lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$ respectively. The effective half-life $T_{1/2}$ of the nuclei is :

(1) None of the above (2) $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$

(3) $T_{1/2} = \frac{T_{1/2}^{(1)}T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$ (4) $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$

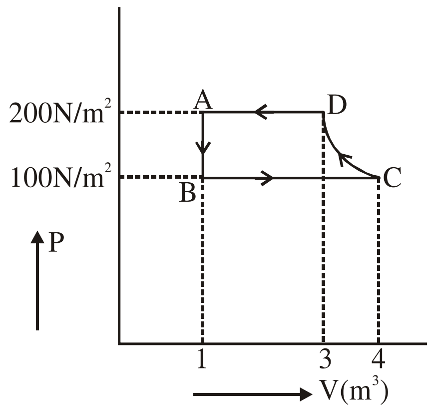
Official Ans. by NTA (3)

Sol. $\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{1}{T_{1/2}} = \frac{1}{T_{1/2}^{(1)}} + \frac{1}{T_{1/2}^{(2)}}$$

$$T_{1/2} = \frac{T_{1/2}^{(1)}T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$$

11. The P-V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use $\gamma = 1.4$) :



(1) –500 J (2) –400 J

(3) 400 J (4) 200 J

Official Ans. by NTA (1)

Sol. Adiabatic process is from C to D

$$WD = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$= \frac{P_DV_D - P_CV_C}{1-\gamma}$$

$$= \frac{200(3) - (100)(4)}{1-1.4}$$

= –500 J Ans. (1)

12. In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is :-

(1) 1178×10^{-9} m (2) 1178×10^{-6} m

(3) 1178×10^{-12} m (4) 5890×10^{-7} m

Official Ans. by NTA (2)

Sol. $\beta = \frac{\lambda D}{d} = \frac{5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$

= 589×10^{-6} m

Distance between first and third bright fringe is $2\beta = 2 \times 589 \times 10^{-6}$ m

= 1178×10^{-6} m Ans. (2)

13. A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of the particle is :-

(1) $\frac{1}{16}$ times the mass of e^-

(2) 8 times the mass of e^-

(3) 16 times the mass of e^-

(4) $\frac{1}{8}$ times the mass of e^-

Official Ans. by NTA (4)

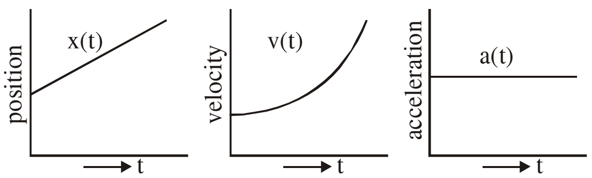
Sol. $\lambda = \frac{h}{p}$

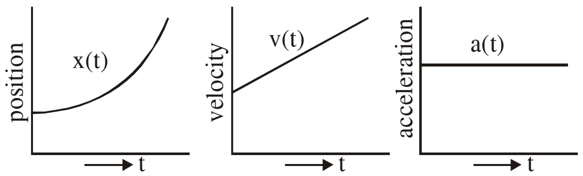
$$\frac{\lambda_p}{\lambda_e} = \frac{p_e}{p_p} = \frac{m_e v_e}{m_p v_p}$$

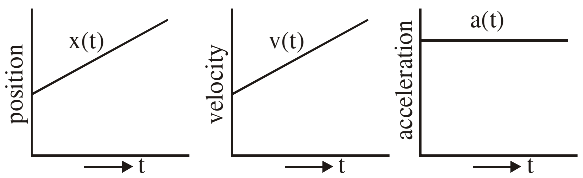
$$2 = \frac{m_e}{m_p}\left(\frac{v_e}{4v_e}\right)$$

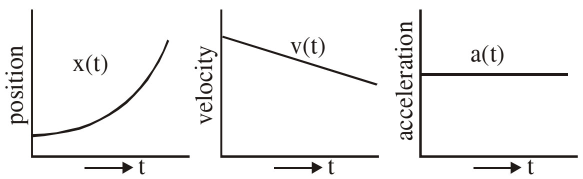
$\therefore\ m_p = \frac{m_e}{8}$ Ans. (4)

14. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. Option (2) represent correct graph for particle moving with constant acceleration, as for constant acceleration velocity time graph is straight line with positive slope and x-t graph should be an opening upward parabola.

15. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is :-

(1) 3.9 (2) 8.4

(3) 7.5 (4) 3.0

Official Ans. by NTA (1)

Sol. $R = \frac{\rho\ell}{A} = \frac{V}{I}$

$$\rho = \frac{AV}{I\ell} = \frac{\pi d^2 V}{4I\ell} \qquad \left(A = \frac{\pi d^2}{4}\right)$$

$$\therefore\ \frac{\Delta\rho}{\rho} = \frac{2\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta\ell}{\ell}$$

$$\frac{\Delta\rho}{\rho} = 2\left(\frac{0.01}{5.00}\right) + \frac{0.1}{5.0} + \frac{0.01}{2.00} + \frac{0.1}{10.0}$$

$$\frac{\Delta\rho}{\rho} = 0.004 + 0.02 + 0.005 + 0.01$$

$$\frac{\Delta\rho}{\rho} = 0.039$$

$$\%\ \text{error} = \frac{\Delta\rho}{\rho} \times 100 = 0.039 \times 100 = 3.90\%$$

Ans. (1)

16. In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value :

(1) The bandwidth of resonance circuit will increase.

(2) The resonance frequency will increase.

(3) The quality factor will increase.

(4) The quality factor and the resonance frequency will remain constant.

Official Ans. by NTA (1)

Sol. Bandwidth = R/L

Bandwidth $\propto$ R

So bandwidth will increase

17. An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is :

(1) 2.5 ms (2) 25 ms

(3) 2.5 s (4) 0.25 ms

Official Ans. by NTA (1)

Sol. $i = i_0 \cos(\omega t)$

$i = i_0$ at $t = 0$

$$i = \frac{i_0}{\sqrt{2}} \text{ at } \omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4(2\pi f)} = \frac{1}{8f}$$

$$t = \frac{1}{400} = 2.5\,\text{ms}$$

18. Your friend is having eye sight problem. She is not able lo see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as :

(1) Astigmatism

(2) Myopia with Astigmatism

(3) Presbyopia with Astigmatism

(4) Myopia and hypermetropia

Official Ans. by NTA (2)

Sol. If distant objects are blurry then problem is Myopia.

If objects are distorted then problem is Astigmatism

19. A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.

(1) Loop assumes circular shape with its plane normal to the field.

(2) Loop assumes circular shape with its plane parallel to the field.

(3) Wire gets stretched to become straight.

(4) Shape of the loop remains unchanged.

Official Ans. by NTA (1)

Sol. Every part ($d\ell$) of the wire is pulled by force $i(d\ell)B$ acting perpendicular to current & magnetic field giving it a shape of circle.

20. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is :

(1) 9 T (2) 27 T

(3) 12 T (4) 3 T

Official Ans. by NTA (2)

Sol. $T^2 \propto R^3$

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$$

$T'^2 = T^2 \times 9^3$

$T' = T \times 3^3$

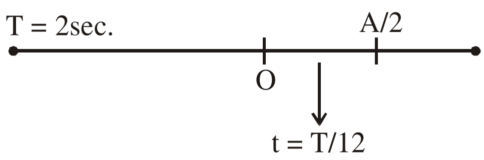
$T' = 27\ T$

SECTION-B

1. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$s. The value of 'a' to the nearest integer is _______ .

Official Ans. by NTA (6)

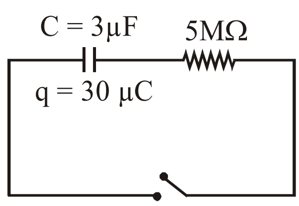
Sol.



$$t = \frac{2}{12} = \frac{1}{6}$$

$\therefore$ Correct answer = 6.00

2. The circuit shown in the figure consists of a charged capacitor of capacity 3 μF and a charge of 30 μC. At time t = 0, when the key is closed, the value of current flowing through the 5 MΩ resistor is 'x' μ-A. The value of 'x to the nearest integer is _______.

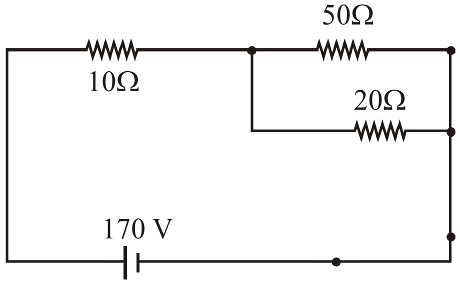


Official Ans. by NTA (2)

Sol. $i_0 = \frac{V}{R} = \frac{30/3}{5\times10^6} = 2\times10^{-6}$

$\therefore$ Ans. = 2.00

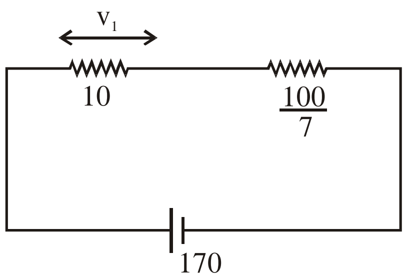
3. The voltage across the 10Ω resistor in the given circuit is x volt.



The value of 'x' to the nearest integer is _____.

Official Ans. by NTA (70)

Sol. $R_{eq_1} = \dfrac{50\times 20}{70} = \dfrac{100}{7}$



$R_{eq} = \dfrac{170}{7}$

$v_1 = \left[\dfrac{170}{\frac{170}{7}}\right]\times 10 = 70v$

Ans. = 70.00

4. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a : b. Then a/b can be expressed as 1/x where x is _______ .

Official Ans. by NTA (32)

Sol. For A $\dfrac{E}{\pi r^2} = y\dfrac{2mm}{a}$ ....(1)

For B $\dfrac{E}{\pi .16r^2} = y\dfrac{4mm}{b}$ ....(2)

$\therefore$ (1)/(2)

$16 = \dfrac{2b}{4a}$

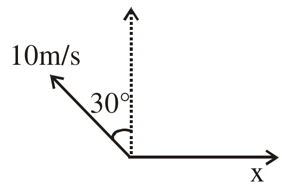
$\dfrac{a}{b} = \dfrac{1}{32}$

$\therefore$ Answer = 32

5. A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is ______.

Official Ans. by NTA (5)

Sol.



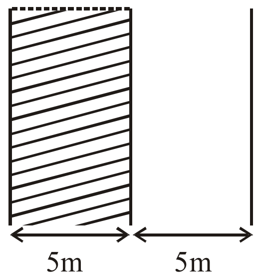
$10 \sin 30° = x$

x = 5 m/s

6. A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m. The space between the plates is filled up to a thickness 5 m with a material of dielectric constant of 10. The resultant capacitance of the system is 'x' pF. The value of $\varepsilon_0 = 8.85 \times 10^{-12}$ $F.m^{-1}$.
The value of 'x' to the nearest integer is_____.

Official Ans. by NTA (161)

Sol.



A = 100 m^2

Using $C = \dfrac{k \in_0 A}{d}$

$C_1 = \dfrac{10 \in_0 (100)}{5}$

$= 200 \in_0$

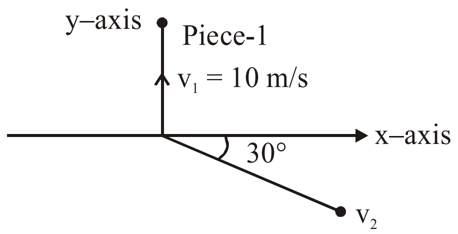
$C_2 = \dfrac{\in_0 (100)}{5} = 20 \in_0$

C_1 & C_2 are in series so $C_{eqv.} = \dfrac{C_1C_2}{C_1 + C_2}$

$= \dfrac{4000 \in_0}{220}$

$= 160.9 \times 10^{-12} \simeq 161$ pF

7. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m/s along the x-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of 10 m/s. The second piece starts moving at an angle of 30° with respect to the x-axis. The velocity of the ball moving at 30° with x-axis is x m/s. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is ______ .



Official Ans. by NTA (20)

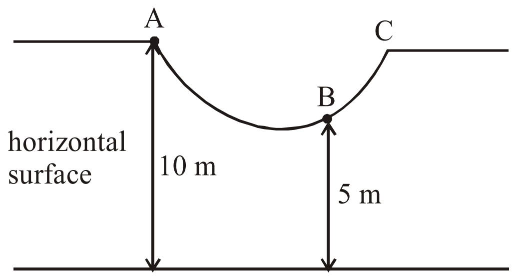
Sol. Let velocity of 2nd fragment is $\vec{v}$ then by conservation of linear momentum

$$10\left(10\sqrt{3}\right)\hat{i} = (10)\left(10\hat{j}\right) + 10\vec{v}$$

$$\Rightarrow \vec{v} = 10\sqrt{3}\hat{i} - 10\hat{j}$$

$$|\vec{v}| = \sqrt{300+100} = \sqrt{400} = 20\text{m/s}$$

8. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take g = 10 m/s^2)
The value of 'x' to the nearest integer is_____.



Official Ans. by NTA (10)

Sol. Using work energy theorem,
$W_g = \Delta K.E.$

$$(10)\,(g)\,(5) = \frac{1}{2}(10)v^2 - 0$$

v = 10 m/s

9. An npn transistor operates as a common emitter amplifier with a power gain of 10^6. The input circuit resistance is 100Ω and the output load resistance is 10 KΩ. The common emitter current gain 'β' will be ______. (Round off to the Nearest Integer)

Official Ans. by NTA (100)

Sol. $$10^6 = \beta^2 \times \frac{R_0}{R_i}$$

$$10^6 = \beta^2 \times \frac{10^4}{10^2}$$

$$\beta^2 = 10^4 \Rightarrow \beta = 100$$

10. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N. The value of 'x' to the nearest integer is ______ .

Official Ans. by NTA (10)

Sol. $v^2 = u^2 + 2as$

$$0 = (10)^2 + 2\,(-a)\left(\frac{1}{2}\right)$$

a = 100 m/s^2

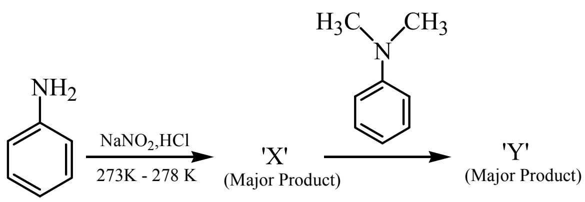
F = ma = (0.1) (100) = 10 N

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

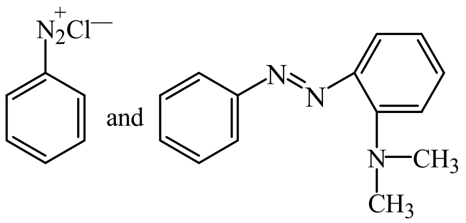
(Held On Thursday 18th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

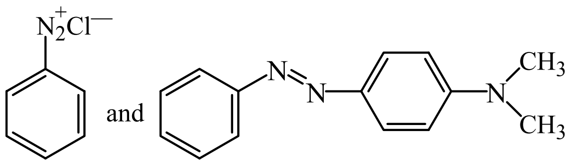
CHEMISTRY | **TEST PAPER WITH ANSWER & SOLUTION**

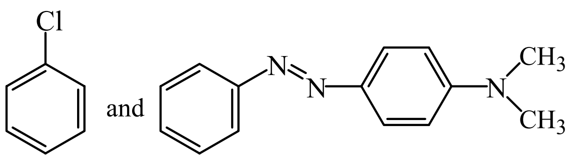
SECTION-A

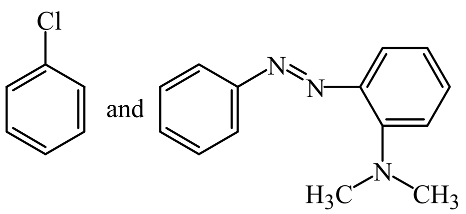
1. 

Considering the above reaction, X and Y respectively are :

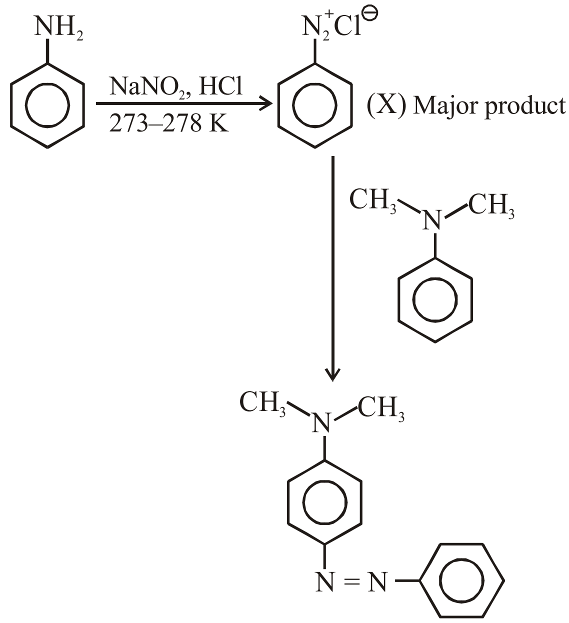
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

(Y)
Major product

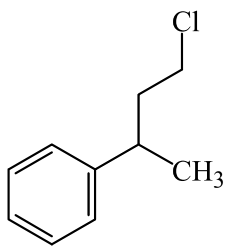
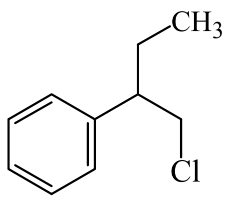
2. The ionic radius of Na^+ ions is 1.02 Å. The ionic radii (in Å) of Mg^{2+} and Al^{3+}, respectively, are-

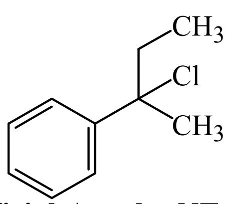
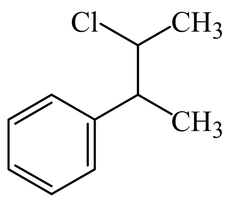
(1) 1.05 and 0.99 (2) 0.72 and 0.54
(3) 0.85 and 0.99 (4) 0.68 and 0.72

Official Ans. by NTA (2)

Sol. The ionic radii order is
$Na^+ > Mg^{2+} > Al^{3+}$

3. Reaction of Grignard reagent, C_2H_5MgBr with C_8H_8O followed by hydrolysis gives compound "A" which reacts instantly with Lucas reagent to give compound B, $C_{10}H_{13}Cl$.
The Compound B is :

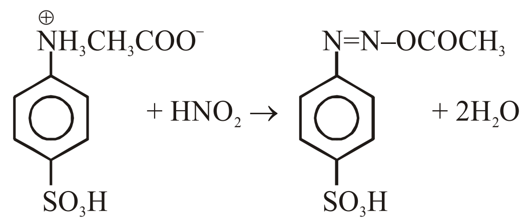
(1)  (2) 

(3)  (4) 

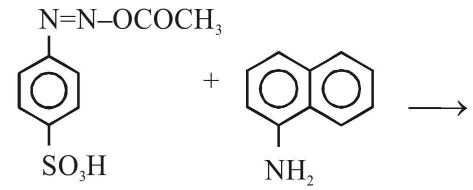
Official Ans. by NTA (3)

4. Reagent, 1-naphthylamine and sulphanilic acid in acetic acid is used for the detection of
(1) N_2O (2) NO_3^- (3) NO (4) NO_2^-
Official Ans. by NTA (4)

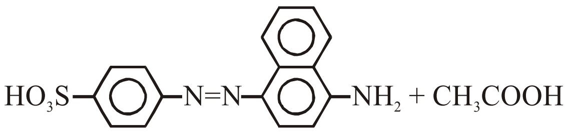
Sol. For detection of NO_2^-, the following test is used.
$NO_2^- + CH_3COOH \rightarrow HNO_2 + CH_3COO^-$



(Sulphanilic acid solution)



Diazotized acid   1-napthyl amine



(Red azo dye)

5. A non-reducing sugar "A" hydrolyses to give two reducing mono saccharides. Sugar A is-
(1) Fructose (2) Galactose
(3) Glucose (4) Sucrose
Official Ans. by NTA (4)

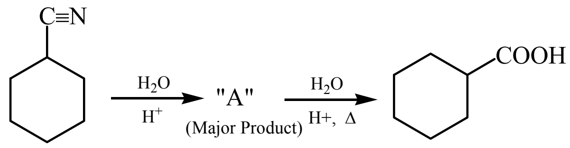
Sol. Sucrose $\xrightarrow{H_2O}$ glucose + Fructose
(Non reducing sugar)   (Reducing sugar)   (Reducing sugar)

6. Match the list -I with list - II

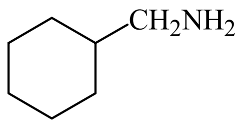
List-I (Class of Drug)	**List-II (Example)**
(a) Antacid	(i) Novestrol
(b) Artificial sweetener	(ii) Cimetidine
(c) Antifertility	(iii) Valium
(d) Tranquilizers	(iv) Alitame

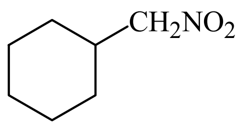
(1) (a) – (ii), (b) – (iv),(c) – (i), (d) – (iii)
(2) (a) – (iv), (b) – (i),(c) – (ii), (d) – (iii)
(3) (a) – (iv), (b) – (iii),(c) – (i), (d) – (ii)
(4) (a) – (ii), (b) – (iv),(c) – (iii), (d) – (i)
Official Ans. by NTA (1)

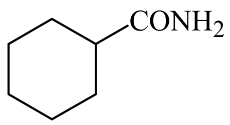
Sol. (a) Antacid : Cimetidine
(b) Artifical Sweetener : Alitame
(c) Antifertility : Novestrol
(d) Tranquilizers : Valium

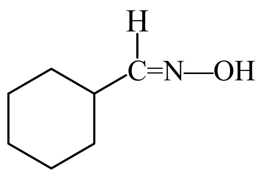
7. 

Consider the above chemical reaction and identify product "A"

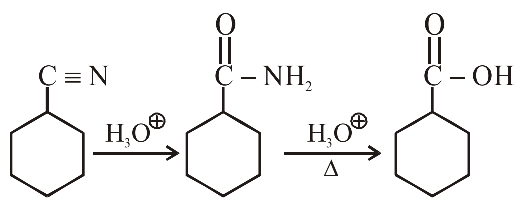
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

'A' Major product (partial hydrolysis)   Complete hydrolysis

8. Match List-I with List-II

List-I	List-II
(a) Chlorophyll	(i) Ruthenium
(b) Vitamin-B_{12}	(ii) Platinum
(c) Anticancer drug	(iii) Cobalt
(d) Grubbs catalyst	(iv) Magnesium

Choose the most appropriate answer from the options given below :

(a) a-iii, b-ii, c-iv, d-i

(b) a-iv, b-iii), c-ii, d-i

(c) a-iv, b-iii, c-i, d-ii

(d) a-iv, b-ii, c-iii, d-i

Official Ans. by NTA (2)

Sol. Chlorophyll is a coordination compound of magnesium.

Vitamin B-12, cyanocobalamine is a coordination compound of cobalt.

Cisplatin is used as an anti-cancer drug and is a coordination compound of platinum.

Grubbs catalyst is a compound of Ruthenium.

9. Match List-I with List-II :

List-I

(Chemicals)

(a) Alcoholic potassium hydroxide

(b) Pd/ $BaSO_4$

(c) BHC (Benzene hexachloride)

(d) Polyacetylene

List-II

(Use / Preparation / Constituent)

(i) Electrodes in batteries

(ii) Obtained by addition reaction

(iii) Used for β - elimination reaction

(iv) Lindlar's catalyst

Choose the most appropriate match :

(1) a-ii, b-i, c-iv, d-iii

(2) a-iii, b-iv, c-ii, d-i

(3) a-iii, b-i, c-iv, d-ii

(4) a-ii, b-iv, c-i, d-iii

Official Ans. by NTA (2)

Sol. (a) Alcoholic potassium hydroxide $\rightarrow$ used for β-elimination

(b) Pd/ $BaSO_4$ $\rightarrow$ Lindlar's catalyst

(c) BHC (Benzene hexachloride) $\rightarrow$ Obtained by addition reactions

(d) Polyacetylene $\rightarrow$ Electrodes in batteries

10. The satements that are TRUE :

(A) Methane leads to both global warming and photochemical smog

(B) Methane is generated from paddy fields

(C) Methane is a stronger global warming gas than CO_2

(D) Methane is a part of reducing smog

Choose the most appropriate answer from the options given below :

(1) (A), (B), (C) only

(2) (A) and (B) only

(3) (B), (C), (D) only

(4) (A), (B), (D) only

Official Ans. by NTA (1)

Sol. Methane leads to both global warming & photochemical smog.

Methane is generated in large amounts from paddy fields.

CO_2 can be absorbed by photosynthesis, or by formation of acid rain etc., while no such activities are there for methane.

Hence methane is stronger global warming gas than CH_4.

Methane is not a part of reducing smog.

11. Match List-I with List-II

List-I	**List-II**
(a) $Ca(OCI)_2$	(i) Antacid
(b) $CaSO_4 \cdot \frac{1}{2}H_2O$	(ii) Cement
(c) CaO	(iii) Bleach
(d) $CaCO_3$	(iv) Plaster of paris

Choose the most appropriate answer from the options given below :

(1) a-i, b-iv, c-iii, d-ii

(2) a-iii, b-ii, c-iv, d-i

(3) a-iii, b-iv, c-ii, d-i

(4) a-iii, b-ii, c-i, d-iv

Official Ans. by NTA (3)

Sol. $Ca(OCl)_2$ is Bleach.

$CaSO_4 \cdot \frac{1}{2}H_2O$ is plaster of paris.

$CaCO_3$ is used as an antacid.

CaO is major component of cement.

12. Compound with molecular formula C_3H_6O can show :

(1) Positional isomerism
(2) Both positional isomerism and metamerism
(3) Metamerism
(4) Functional group isomerism

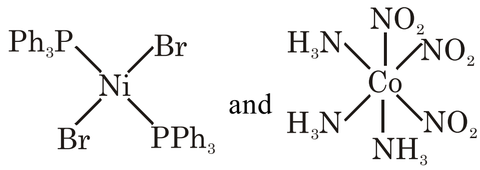
Official Ans. by NTA (4)

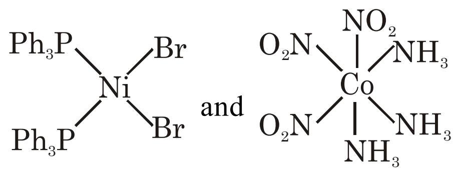
Sol. $C_3H_6O \Rightarrow CH_3–CH_2–CH=O$

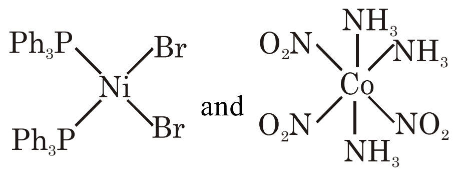
& $CH_3-\underset{\underset{O}{||}}{C}-CH_3$

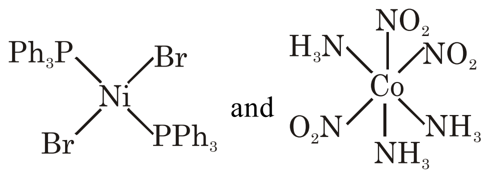
They are functional group isomerism.

13. The correct structures of trans-$[NiBr_2(PPh_3)_2]$ and meridonial-$[Co(NH_3)_3(NO_2)_3]$, respectively, are

(1) 

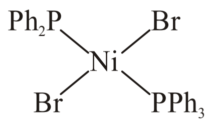
(2) 

(3) 

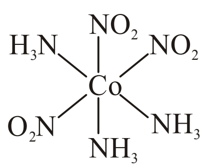
(4) 

Official Ans. by NTA (4)

Sol. trans-$[Ni\ Br_2(PPh_3)_2]$ is



meridional - $[Co(NH_3)_3(NO_2)_3]$ is



14. A certain orbital has no angular nodes and two radial nodes. The orbital is :

(1) 2s (2) 3s (3) 3p (4) 2p

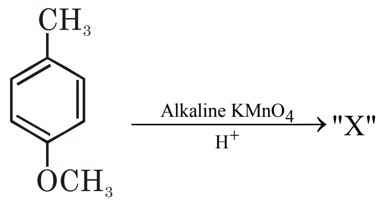
Official Ans. by NTA (2)

Sol. $l = 0 \Rightarrow$'s' orbital

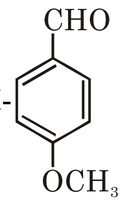
$n - l - 1 = 2$

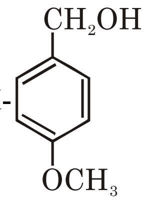
$n - 1 = 2$

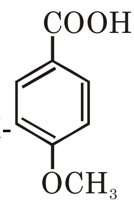
$n = 3$

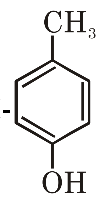
15. 

Considering the above chemical reaction, identify the product "X" :

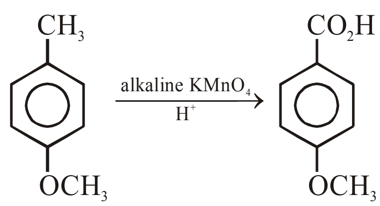
(1) X- 

(2) X- 

(3) X- 

(4) X- 

Official Ans. by NTA (3)

Sol. 

(X)

16. Match **List-I** with **List-II**

List-I (process)	**List-II (catalyst)**
(a) Deacron's process	(i) ZSM-5
(b) Contact process	(ii) $CuCl_2$
(c) Cracking of hydrocarbons	(iii) Particles 'Ni'
(d) Hydrogenation of vegetable oils	(iv) V_2O_5

Choose the most appropriate answer from the options given below -

(1) a-ii, b-iv, c-i, d-iii (2) a-i, b-iii, c-ii, d-iv
(3) a-iii, b-i, c-iv, d-ii (4) a-iv, b-ii, c-i, d-iii

Official Ans. by NTA (1)

Sol. In manufacture of H_2SO_4 (contact process), V_2O_5 is used as a catalyst.
Ni catalysts enables the hydrogenation of fats.
$CuCl_2$ is used as catalyst in Deacon's process.
ZSM-5 used as catalyst in cracking of hydrocarbons.

17. Given below are two statements : One is labelled as Assertion A and the other labelled as reason R

Assertion A : During the boiling of water having temporary hardness, $Mg(HCO_3)_2$ is converted to $MgCO_3$.

Reason R : The solubility product of $Mg(OH)_2$ is greater than that of $MgCO_3$.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are true but R is not the correct explanation of A
(2) A is true but R is false
(3) Both A and R are true and R is the correct explanation of A
(4) A is false but R is true

Official Ans. by NTA (4)

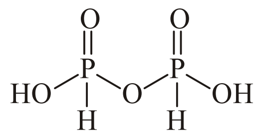
Sol. For temporary hardness,
$Mg(HCO_3)_2 \xrightarrow{\text{heating}} Mg(OH)_2\downarrow + 2CO_2\uparrow$
Assertion is false.
$MgCO_3$ has high solubility product than $Mg(OH)_2$.
According to data of NCERT table 7.9 (Equilibrium chapter), the solubility product of magnesium carbonate is 3.5×10^{-8} and solubility product of $Mg(OH)_2$ is 1.8×10^{-11}.
Hence Reason is incorrect.
The question should be Bonus.

18. The number of ionisable hydrogens present in the product obtained from a reaction of phosphorus trichloride and phosphonic acid is:

(1) 3 (2) 0 (3) 2 (4) 1

Official Ans. by NTA (3)

Sol. $PCl_3 + H_3PO_3 \rightarrow H_4P_2O_5$



(Two ionisable H)

19. In a binary compound, atoms of element A form a hcp structure and those of element M occupy 2/3 of the tetrahedral voids of the hcp structure. The formula of the binary compound is :

(1) M_2A_3 (2) M_4A_3 (3) M_4A (4) MA_3

Official Ans. by NTA (2)

Sol. $M_{12\times\frac{2}{3}}A_6$

M_8A_6

M_4A_3

20. The chemical that is added to reduce the melting point of the reaction mixture during the extraction of aluminium is :

(1) Cryolite (2) Bauxite
(3) Calamine (4) Kaolite

Official Ans. by NTA (1)

Sol. To reduce the melting point of reaction mixture, cryolite is added.

SECTION-B

1. AX is a covalent diatomic molecule where A and X are second row elements of periodic table. Based on Molecular orbital theory, the bond order of AX is 25. The total number of electrons in AX is ______. (Round off to the Nearest Integer).

Official Ans. by NTA (15)

Sol. AX is a covalent diatomic molecule.
The molecule is NO.
Total no. of electrons is 15.

2. In order to prepare a buffer solution of pH 5.74, sodium acetate is added to acetic acid. If the concentration of acetic acid in the buffer is 1.0 M, the concentration of sodium acetate in the buffer is ________ M. (Round off to the Nearest Integer).

[Given : pKa (acetic acid) = 4.74]

Official Ans. by NTA (10)

Sol. $pH = pKa + \log \frac{[CB]}{[WA]}$

$5.74 = 4.74 + \log \frac{[CB]}{1}$

$\Rightarrow [CB] = 10\ M$

3. $2\ NO(g) + Cl_2(g) \rightleftharpoons 2\ NOCl(s)$

This reaction was studied at –10°C and the following data was obtained

run	$[NO]_0$	$[Cl_2]_0$	r_0
1	0.10	0.10	0.18
2	0.10	0.20	0.35
3	0.20	0.20	1.40

$[NO]_0$ and $[Cl_2]_0$ are the initial concentrations and r_0 is the initial reaction rate.

The overall order of the reaction is ______. (Round off to the Nearest Integer).

Official Ans. by NTA (3)

Sol. $r = k[NO]^m\ [Cl_2]^n$

$= k(0.1)^m\ (0.1)^n$ .....(1)

$= k(0.1)^m\ (0.2)^n$ .....(2)

$= k(0.2)^m\ (0.2)^n$ .....(3)

$n = 1$

$m = 2$

$m + n = 3$

4. For the reaction

$C_2H_6 \rightarrow C_2H_4 + H_2$

the reaction enthalpy Δ_rH = ______ kJ mol^{-1}. (Round off to the Nearest Integer).

[Given : Bond enthalpies in kJ mol^{-1} : C–C : 347, C=C : 611; C–H : 414, H–H : 436]

Official Ans. by NTA (128)

Sol. $\Delta_rH = [\epsilon_{C-C} + 2\epsilon_{C-H}] - [\epsilon_{C=C} + \epsilon_{H-H}]$

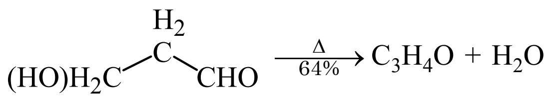
$= [347 + 2 \times 414] - [611 + 436]$

$= 128$

5. _____ grams of 3-Hydroxy propanal (MW=74) must be dehydrated to produce 7.8 g of acrolein (MW = 56) (C_3H_4O) if the percentage yield is 64. (Round off to the Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

Official Ans. by NTA (16)

Sol. 

$\frac{x}{74}$ mol $\qquad \frac{x}{74} \times 0.64 = \frac{7.8}{56}$

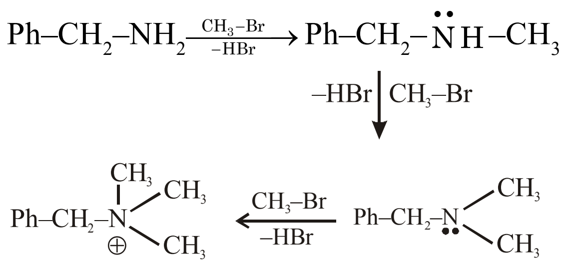
$x = 16.10$

$\simeq 16.00$

6. A reaction of 0.1 mole of Benzylamine with bromomethane gave 23 g of Benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are n × 10^{-1}, when n = _______. (Round off to the Nearest Integer).

(Given : Atomic masses : C : 12.0 u, H : 1.0 u, N : 14.0 u, Br : 80.0 u]

Official Ans. by NTA (3)

Sol. 

no of moles = 3

7. The total number of unpaired electrons present in the complex $K_3[Cr(oxalate)_3]$ is ______.

Official Ans. by NTA (3)

Sol. $K_3[Cr(oxalate)_3]$

Chromium is in +3 oxidation state.

Number of unpaired electrons in Cr^{+3} will be 3.

8. 2 molal solution of a weak acid HA has a freezing point of 3.885°C. The degree of dissociation of this acid is _____ × 10^{-3}. (Round off to the Nearest Integer).

[Given : Molal depression constant of water = 1.85 K kg mol^{-1} Freezing point of pure water = 0°C]

Official Ans. by NTA (50)

Sol. $\Delta T_f = (1 + \alpha)\, K_f.m$

$\alpha = 0.05 = 50 \times 10^{-3}$

9. For the reaction

$2Fe^{3+}(aq) + 2I^{-}(aq) \rightarrow 2Fe^{2+}(aq) + I_2(s)$

the magnitude of the standard molar free energy change, $\Delta_r G^{\circ}_m$ = – ______ kJ (Round off to the Nearest Integer).

$$\begin{bmatrix} E^{o}_{Fe^{2+}/Fe(s)} = -0.440\ V;\ E^{o}_{Fe^{3+}/Fe(s)} = -0.036\ V \\ E^{o}_{I_2/2I^-} = 0.539\ V; \qquad F = 96500\ C \end{bmatrix}$$

Official Ans. by NTA (46)
Official Ans. by ALLEN (45)

Sol. $Fe^{3+} \xrightarrow{E_1^0} Fe^{2+} \xrightarrow{E_2^0} Fe$; overall $Fe^{3+} \xrightarrow{E_3^0} Fe$

$$E_1^0 + 2E_2^0 = 3E_3^0$$

$$E_1^0 = 3E_3^0 - 2E_2^0$$

= 3 (–0.036) – 2(–0.44)

= + 0.772 V

$$E^0_{cell} = E^0_{Fe^{3+}/Fe^{2+}} + E^0_{I^-/I_2} = 0.233$$

$\Delta_r G^0$ = – 2 × 96.5 × 0.233 = –45 kJ

10. Complete combustion of 3 g of ethane gives x × 10^{22} molecules of water. The value of x is ______. (Round off to the Nearest Integer).

[Use : N_A = 6.023 × 10^{23}; Atomic masses in u : C : 12.0 ; O : 16.0 ; H : 1.0]

Official Ans. by NTA (18)

Sol. $C_2H_6 \rightarrow 3H_2O$

0.1 mol → 0.3 mol; $0.3 = 0.3 \times 6 \times 10^{23} = 18 \times 10^{22}$

No. of molecules = 0.3 × 6.023 × 10^{23}

= 18.069 × 10^{22}

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

Official Ans. by NTA (3)

Sol. $y^2 = 4ax + 4a^2$

differentiate with respect to x

$$\Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow a = \left(\frac{y}{2}\frac{dy}{dx}\right)$$

so, required differential equation is

$$y^2 = \left(4\times\frac{y}{2}\frac{dy}{dx}\right)x + 4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$$

$$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

2. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1 (2) 2 (3) 3 (4) 0

Official Ans. by NTA (2)

Sol. $3x + 4y = 9$

$y = mx + 1$

$\Rightarrow 3x + 4mx + 4 = 9$

$\Rightarrow (3 + 4m)x = 5$

$\Rightarrow$ x will be an integer when

$3 + 4m = 5, -5, 1, -1$

$$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$$

so, number of integral values of m is 2

3. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to

(1) $2^{20}(2^{20} - 21)$

(2) $2^{19}(2^{20} - 21)$

(3) $2^{19}(2^{20} + 21)$

(4) $2^{20}(2^{20} + 21)$

Official Ans. by NTA (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + ...+ a_{40}x^{40}$ put $x = 1, -1$

$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$

$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20}$

$$\Rightarrow a_1 + a_3 + ... + a_{39} = \frac{4^{20} - 2^{20}}{2}$$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$

$$\text{here } a_{39} = \frac{20!(2)^{19}\times 1}{19!} = 20\times 2^{19}$$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{19}(2^{20} - 1 - 20)$

$= 2^{19}(2^{20} - 21)$

4. The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi), \text{ are}$$

(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$

(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Official Ans. by NTA (4)

Sol. $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

use $R_1 \to R_1 + R_2 + R_3$

$\Rightarrow (2+4\sin 2x)\begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

$\Rightarrow \sin 2x = -\frac{1}{2}$

$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{2} + \frac{\pi}{12}, \pi - \frac{\pi}{12}$

5. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have same centre

(2) circles have no meeting point

(3) circles have only one meeting point

(4) circles have two meeting points

Official Ans. by NTA (3)

Sol. $x^2 + y^2 - 10x - 10y + 41 = 0$

$A(5,5)$, $R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

$B(11,5)$, $R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point.

6. Let α,β,γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, ($a,b,c \in R$ and $a,b \neq 0$). If the system of equations (in, u,v,w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is

(1) 5 (2) 3 (3) 1 (4) 0

Official Ans. by NTA (2)

Sol. $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \Sigma\alpha\beta) = 0$

$\Rightarrow -(-a)(a^2 - 2b - b) = 0$

$\Rightarrow a(a^2 - 3b) = 0$

$\Rightarrow a^2 = 3b \Rightarrow \frac{a^2}{b} = 3$

7. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(2) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

(4) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

Official Ans. by NTA (1)

Sol. $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$

$(2x - 1)^2 + 5 = t^2$

$2(2x - 1)\, 2dx = 2t\, dt$

$2\sqrt{t^2-5}dx = t\,dt$

So $\int \frac{\sqrt{t^2-5}\cos t}{2\sqrt{t^2-5}}dt = \frac{1}{2}\sin t + c$

$= \frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

8. The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y+1=3\sqrt{2}\,x$ is

(1) $4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$

(2) $5\sqrt{2}x+4y-\left(15+4\sqrt{2}\right)=0$

(3) $4\sqrt{2}x+5y-4\sqrt{2}=0$

(4) $4\sqrt{2}x-5y-\left(5+4\sqrt{2}\right)=0$

Official Ans. by NTA (1)

Sol. $y=mx+c$

$3 = m + c$

$$\sqrt{2}=\left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|$$

$$=6m+\sqrt{2}=m-3\sqrt{2}$$

$$=\sin=-4\sqrt{2} \rightarrow m=\frac{-4\sqrt{2}}{5}$$

$$= =6m-\sqrt{2}=m-3\sqrt{2}$$

$$=7m-2\sqrt{2} \rightarrow m=\frac{2\sqrt{2}}{7}$$

According to options take $m=\dfrac{-4\sqrt{2}}{5}$

So $y=\dfrac{-4\sqrt{2}x}{5}+\dfrac{3+4\sqrt{2}}{5}$

$$4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$$

9. If $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{2}$

(3) 6 (4) 2

Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3!}\ldots\right)-\left(x-\frac{x^3}{3}\ldots\right)}{3x^3}=\frac{1}{6}$$

So 6L + 1 = 2

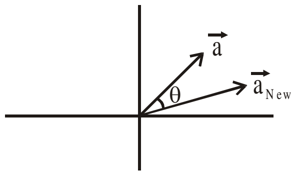
10. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1 (2) $-\dfrac{5}{4}$ (3) $\dfrac{4}{5}$ (4) –1

Official Ans. by NTA (4)

Sol. $\vec{a}_{Old}=3p\hat{i}+\hat{j}$

$\vec{a}_{New}=\left(p+1\right)\hat{i}+\sqrt{10}\hat{j}$



$\Rightarrow\left|\vec{a}_{Old}\right|=\left|\vec{a}_{New}\right|$

$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0 \rightarrow p = \dfrac{5}{4}, -1$

11. If the equation $a\left|z\right|^2+\overline{\alpha z+\alpha\bar{z}}+d=0$ represents a circle where a,d are real constants then which of the following condition is correct ?

(1) $\left|\alpha\right|^2-ad\neq 0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $|\alpha|^2 - ad \geq 0$ and $a \in R$

(4) $\alpha = 0$, $a,d \in R^+$

Official Ans. by NTA (2)

Sol. $az\bar{z}+\alpha\bar{z}+\bar{\alpha}z+d=0 \rightarrow$ Circle

centre $=\dfrac{-\alpha}{a}$ $\qquad 2=\sqrt{\dfrac{\alpha\bar{\alpha}}{a^2}-\dfrac{d}{a}}=\sqrt{\dfrac{\alpha\bar{\alpha}-ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in R - \{0\}$

12. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

(1) Rhombus

(2) Square

(3) Rectangle

(4) Parallelogram

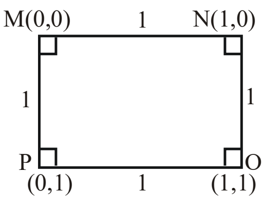
Official Ans. by NTA (2)

Sol. M : $x^2 + y^2 = 1$ (0,0)

N : $x^2 + y^2 - 2x = 0$ (1,0)

O : $x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

P : $x^2 + y^2 - 2y = 0$ (0,1)



13. If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through (α,β) and origin is :

(1) 540

(2) 550

(3) 530

(4) 510

Official Ans. by NTA (2)

Sol. $S = (100)(100) + (99)(101) + (98)(102) \ldots\ldots$

$$\ldots(2)(198) + (1)(199)$$

$$S = \sum_{x=0}^{99}(100-x)(100+x) = \sum 100^2 - x^2$$

$$= 100^3 - \frac{99 \times 100 \times 199}{6}$$

$\alpha = 3$ $\beta = 1650$

$$\text{slope} = \frac{1650}{3} = 550$$

14. The real valued function $f(x) = \dfrac{\operatorname{cosec}^{-1}x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) all reals except integers

(2) all non-integers except the interval [–1,1]

(3) all integers except 0,–1,1

(4) all reals except the Interval [–1,1]

Official Ans. by NTA (2)

Sol. $f(x) = \dfrac{\operatorname{cosec}^{-1}x}{\sqrt{\{x\}}}$

Domain $\in (-\infty, -1] \cup [1, \infty)$

$\{x\} \neq 0$ so $x \neq$ integers

15. $\dfrac{1}{3^2-1} + \dfrac{1}{5^2-1} + \dfrac{1}{7^2-1} + \ldots + \dfrac{1}{(201)^2-1}$ is equal to

(1) $\dfrac{101}{404}$ (2) $\dfrac{25}{101}$ (3) $\dfrac{101}{408}$ (4) $\dfrac{99}{400}$

Official Ans. by NTA (2)

Sol. $T_n = \dfrac{1}{(2n+1)^2 - 1} \dfrac{1}{(2n+2)2n} = \dfrac{1}{4(n)(n+1)}$

$$= \frac{(n+1)-n}{4n(n+1)} = \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S = \frac{1}{4}\left(1 - \frac{1}{101}\right) = \frac{1}{4}\left(\frac{100}{101}\right) = \frac{25}{101}$$

16. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

$f+g$, $f-g$, f/g, g/f, $g-f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \dfrac{f(x)}{g(x)}$

(1) $0 \le x \le 1$
(2) $0 \le x < 1$
(3) $0 < x < 1$
(4) $0 < x \le 1$

Official Ans. by NTA (3)

Sol. $f(x) + g(x) = \sqrt{x}+\sqrt{1-x}$, domain [0, 1]

$f(x) - g(x) = \sqrt{x}-\sqrt{1-x}$, domain [0, 1]

$g(x) - f(x) = \sqrt{1-x}-\sqrt{x}$, domain [0, 1]

$\dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{\sqrt{1-x}}$, domain [0, 1)

$\dfrac{g(x)}{f(x)} = \dfrac{\sqrt{1-x}}{\sqrt{x}}$, domain (0, 1]

So, common domain is (0, 1)

17. If $f(x)=\begin{cases} \dfrac{1}{|x|} & ; \ |x| \ge 1 \\ ax^2+b & ; \ |x|<1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\dfrac{1}{2}, \dfrac{1}{2}$

(2) $\dfrac{1}{2}, -\dfrac{3}{2}$

(3) $\dfrac{5}{2}, -\dfrac{3}{2}$

(4) $-\dfrac{1}{2}, \dfrac{3}{2}$

Official Ans. by NTA (4)

Sol. $f(x)=\begin{cases} \dfrac{1}{|x|}, & |x| \ge 1 \\ ax^2+b, & |x|<1 \end{cases}$

at x = 1 function must be continuous

So, $1 = a + b$ ...(1)

differentiability at x = 1

$$\left(-\frac{1}{x^2}\right)_{x=1} = (2ax)_{x=1}$$

$\Rightarrow -1 = 2a \Rightarrow a = -\dfrac{1}{2}$

(1) $\Rightarrow b = 1 + \dfrac{1}{2} = \dfrac{3}{2}$

18. Let $A+2B=\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A-B=\begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to

(1) 1 (2) 2 (3) 0 (4) 3

Official Ans. by NTA (2)

Sol. $A+2B=\begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}$ ...(1)

$2A-B=\begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$\Rightarrow 4A - 2B = \begin{pmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{pmatrix}$ ...(2)

$(1) + (2) \Rightarrow 5A = \begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$ and $2A = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$

$\therefore\ B = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$B = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$

$tr(A) = 1 - 1 + 1 = 1$
$tr(B) = -1$

tr(A) = 1 and tr(B) = –1

$\therefore$ tr(A) – tr(B) = 2

19. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(1) 26664 (2) 122664

(3) 122234 (4) 22264

Official Ans. by NTA (1)

Sol. Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when 1 at unit place is 3.
2 at unit place is 6
3 at unit place is 3.

So, sum $= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$
$= (1111) \times 24$
$= 26664$

20. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$ is equal to

(1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

Official Ans. by NTA (1)

Sol. Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$

So, $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}} = 3 + \cfrac{1}{\cfrac{4x+1}{x}}$

$\Rightarrow (x - 3) = \frac{x}{(4x+1)}$

$\Rightarrow (4x + 1)(x - 3) = x$

$\Rightarrow 4x^2 - 12x + x - 3 = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8}$

$= \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$

$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$.

But only positive value is accepted

So, $x = 1.5 + \sqrt{3}$

SECTION-B

1. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

Official Ans. by NTA (300)

Sol. 3_ _ = 10 × 10 = 100

_3 _ = 10 × 10 = 100

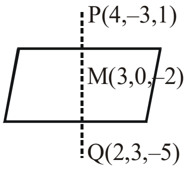
_ _ 3 = 10 × 10 = $\underline{100}$

300

2. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points $(4,-3,1)$ and $(2, 3, -5)$ at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

Official Ans. by NTA (28)

Sol.



Plane is $1(x - 3) - 3(y - 0) + 3(z + 2) = 0$

$x - 3y + 3z + 3 = 0$

$(a^2 + b^2 + c^2 + d^2)_{min} = 28$

3. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x)^2 \, dx$ is

Official Ans. by NTA (512)

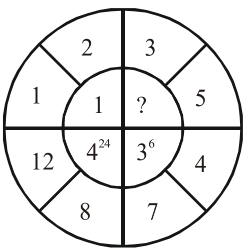
Sol. $I = 2\int_0^4 f(x^2)dx$ {Even funtion}

$$= 2\int_0^4 \left(4x^3 - g(4-x)\right)dx$$

$$= 2\left(\left.\frac{4x^4}{4}\right|_0^4 - \int_0^4 g(4-x)dx\right)$$

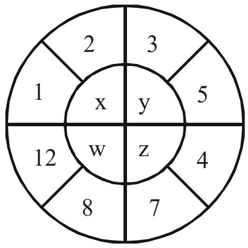
$= 2(256 - 0) = 512$

4. The missing value in the following figure is



Official Ans. by NTA (4)

Sol. $x = (2 - 1)^{1!} = 1$



$w = (12 - 8)^{4!} = 4^{24}$

$z = (7 - 4)^{3!} = 3^6$

hence $y = (5 - 3)^{2!} = 2^2$

5. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is

Official Ans. by NTA (6)

Sol. If $0, z, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0\ (z_1 + z_2) + z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow a^2 = 3 \times 12$

$\Rightarrow |a| = 6$

6. The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point (1, 2, 3) is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

Official Ans. by NTA (4)

Sol. Let plane is $x - 2y + 2z + \lambda = 0$

distance from (1,2,3) = 1

$$\Rightarrow \frac{|\lambda + 3|}{5} = 1 \Rightarrow \lambda = 0, -6$$

$\Rightarrow a = 1, b = -2, c = 2, d = -6$ or 0

$b - d = 4$ or -2, $c - a = 1$

$\Rightarrow k = 4$ or -2

7. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is_.

Official Ans. by NTA (35)

Sol. $\dfrac{\sum x_i}{25} = 40$ & $\dfrac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow$ 1000 – 60 + N = 975

$\Rightarrow$ N = 35 years

8. If $f(x) = \int \dfrac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx, (x \geq 0), f(0) = 0$

and $f(1) = \dfrac{1}{K}$, then the value of K is

Official Ans. by NTA (4)

Sol. $f(x) = \int \dfrac{\left(5x^8 + 7x^6\right)dx}{x^{14}\left(x^{-5} + x^{-7} + 2\right)^2}$

Let $x^{-5} + x^{-7} + 2 = t$

$(-5x^{-6} - 7x^{-8})dx = dt$

$\Rightarrow f(x) = \int -\dfrac{dt}{t^2} = \dfrac{1}{t} + c$

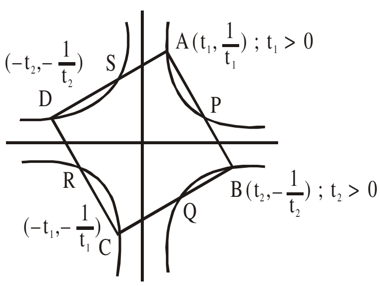
$f(x) = \dfrac{x^7}{x^2 + 1 + 2x^7}$

$f(1) = \dfrac{1}{4}$

9. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

Official Ans. by NTA (80)

Sol. xy = 1, –1



$$\frac{t_1 + t_2}{2} \cdot \frac{\frac{1}{t_1} - \frac{1}{t_2}}{2} = 1$$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$\dfrac{1}{t_1^2} \times \left(-\dfrac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_2^2 = 4$

$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2 + 4} = 2\sqrt{5}$

$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \dfrac{1}{t_1^2} = \sqrt{5} - 2$

$AB^2 = (t_1 - t_2)^2 + \left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)^2$

$= 2\left(t_1^2 + \dfrac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80$

10. The number of solutions of the equation $|\cot x| = \cot x + \dfrac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

Official Ans. by NTA (1)

Sol. If cotx > 0 $\Rightarrow \dfrac{1}{\sin x} = 0$ (Not possible)

If cotx < 0 $\Rightarrow 2\cot x + \dfrac{1}{\sin x} = 0$

$\Rightarrow$ 2cosx = –1

$\Rightarrow x = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$ (reject)

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18th March, 2021)   TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS | **TEST PAPER WITH ANSWER & SOLUTION**

SECTION-A

1. Which of the following statements are correct?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

Choose the correct answer from the options given below :

(1) (C) and (E) only

(2) (B) and (D) only

(3) (A) and (B) only

(4) (B) and (C) only

Official Ans. by NTA (1)

Sol. Statement (C) is correct because, the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement (E) is correct because we know that super conductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

$\mu_r = 1 + \chi$

$\chi = -1$

$\mu_r = 0$

For superconductors.

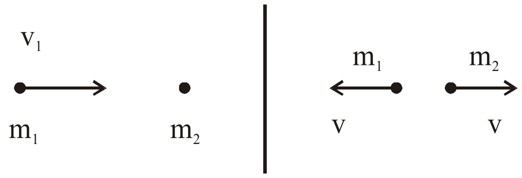
2. An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

(1) 3 : 1 (2) 2 : 1

(3) 1 : 2 (4) 1 : 1

Official Ans. by NTA (1)

Sol.



$$m_1v_1 = -m_1v + m_2v$$

$$v_1 = -v + \frac{m_2}{m_1}v$$

$$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$$

$$e = \frac{2v}{v_1} = 1$$

$$v = \frac{v_1}{2}$$

$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$$

$$3 = \frac{m_2}{m_1}$$

3. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats):

(1) $-\gamma\frac{dV}{V}$ (2) $-\gamma\frac{V}{dV}$

(3) $-\frac{1}{\gamma}\frac{dV}{V}$ (4) $\frac{dV}{V}$

Official Ans. by NTA (1)

Sol. PV^γ = constant

Differentiating

$$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$\frac{dP}{P} = -\frac{\gamma dV}{V}$$

4. A proton and an α-particle, having kinetic energies K_p and K_α, respectively, enter into a magnetic field at right angles.
The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of $K_p : K_\alpha$ is :

(1) 1 : 8 (2) 8 : 1
(3) 1 : 4 (4) 4 : 1

Official Ans. by NTA (4)

Sol. $r = \frac{mv}{qB} = \frac{p}{qB}$ $\quad \frac{m_\alpha}{m_p} = 4$

$$\frac{r_p}{r_\alpha} = \frac{p_p}{q_p}\frac{q_\alpha}{p_\alpha} = \frac{2}{1}$$

$$\frac{p_p}{p_\alpha} = \frac{2q_p}{q_\alpha} = 2\left(\frac{1}{2}\right)$$

$$\frac{p_p}{p_\alpha} = 1$$

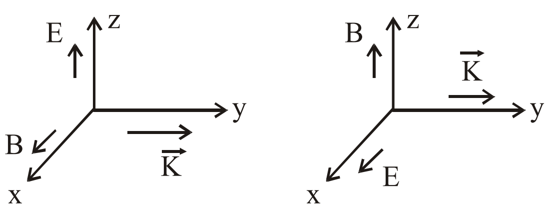
$$\frac{K_p}{K_\alpha} = \frac{p_p^2}{p_\alpha^p}\frac{m_\alpha}{m_p} = (1)\ (4)$$

5. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field $(\vec{E})$ and magnetic field $(\vec{B})$ components.

(1) E_y, B_y or E_z, B_z
(2) E_y, B_x or E_x, B_y
(3) E_x, B_z or E_z, B_x
(4) E_x, B_y or E_y, B_x

Official Ans. by NTA (3)

Sol.



6. Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is :

(1) $\frac{1}{4}\frac{ML^2}{\pi^2}$ (2) $\frac{2}{5}\frac{ML^2}{\pi^2}$

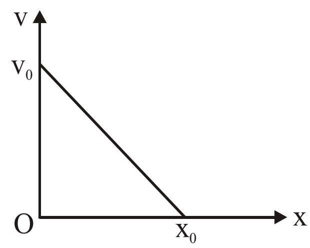
(3) $\frac{ML^2}{\pi^2}$ (4) $\frac{1}{2}\frac{ML^2}{\pi^2}$

Official Ans. by NTA (3)

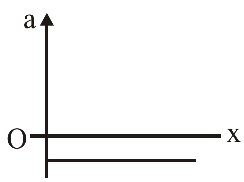
Sol. $\pi r = L \Rightarrow r = \frac{L}{\pi}$

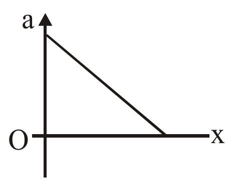
$$I = Mr^2 = \frac{ML^2}{\pi^2}$$

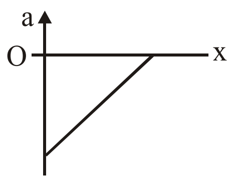
7. The velocity-displacement graph of a particle is shown in the figure.

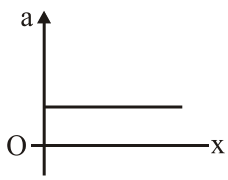


The acceleration-displacement graph of the same particle is represented by :

(1)


(2)


(3)


(4)


Official Ans. by NTA (3)

Sol. $v = -\left(\frac{v_0}{x_0}\right)x + v_0$

$$a = \frac{vdv}{dx}$$

$$a = \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$$

$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

8. The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is :

(1) $\beta = \frac{\alpha}{1+\alpha}$

(2) $\alpha = \frac{\beta}{1-\alpha}$

(3) $\beta = \frac{1}{1-\alpha}$

(4) $\alpha = \frac{\beta}{1+\beta}$

Official Ans. by NTA (4)

Sol. $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B}$

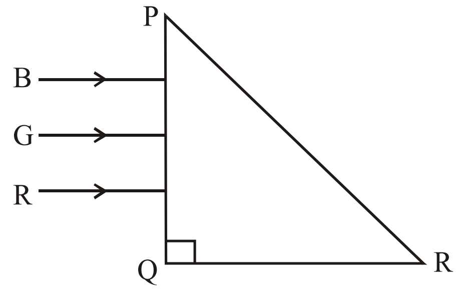
$I_E = I_B + I_C$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C}+1}$$

$$\alpha = \frac{1}{\frac{1}{\beta}+1}$$

$$\alpha = \frac{\beta}{1+\beta}$$

9. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in figure.



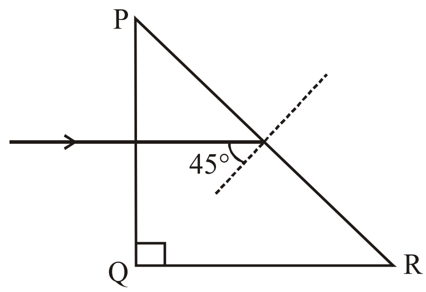
The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

(1) green (2) red

(3) blue and green (4) blue

Official Ans. by NTA (2)

Sol.



Assuming that the right angled prism is an isoceles prism, so the other angles will be 45° each.

$\Rightarrow$ Each incident ray will make an angle of 45° with the normal at face PR.

$\Rightarrow$ The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

$\Rightarrow$ θ_C = critical angle

$$\Rightarrow \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$\Rightarrow$ If $\theta_C \geq 45°$

the light ray will pass

$$\Rightarrow (\theta_C)_{Red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94°$$

Red will pass.

$$\Rightarrow (\theta_C)_{Green} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76°$$

Green will not pass

$$\Rightarrow (\theta_C)_{Blue} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15°$$

Blue will not pass

$\Rightarrow$ So only red will pass through PR.

10. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

(Take : g = 10 ms^{-2}, the radius of earth, R = 6400 × 10^3 m, Take π = 3.14)

(1) 60 minutes

(2) does not change

(3) 1200 minutes

(4) 84 minutes

Official Ans. by NTA (4)

Sol. For objects to float

$mg = m\omega^2 R$

ω = angular velocity of earth.

R = Radius of earth

$$\omega = \sqrt{\frac{g}{R}} \quad \text{... (1)}$$

Duration of day = T

$$T = \frac{2\pi}{\omega} \quad \text{... (2)}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$$

$$\Rightarrow \frac{T}{60} = 83.775 \text{ minutes}$$

$\simeq$ 84 minutes

11. The decay of a proton to neutron is :

(1) not possible as proton mass is less than the neutron mass

(2) possible only inside the nucleus

(3) not possible but neutron to proton conversion is possible

(4) always possible as it is associated only with β^+ decay

Official Ans. by NTA (2)

Sol. It is possible only inside the nucleus and not otherwise.

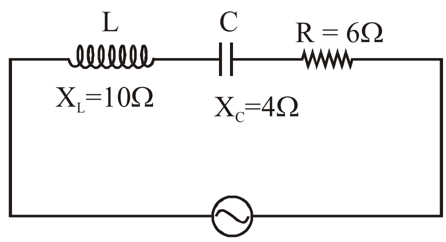
12. In a series LCR circuit, the inductive reactance (X_L) is 10 Ω and the capacitive reactance (X_C) is 4 Ω. The resistance (R) in the circuit is 6 Ω. The power factor of the circuit is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Official Ans. by NTA (3)

Sol.

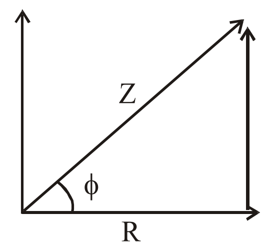


We know that power factor is cosφ,

$$\cos\phi = \frac{R}{Z} \quad \text{... (1)}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{... (2)}$$

(ωL–1/ωC)



$$\Rightarrow Z = \sqrt{6^2 + (10-4)^2}$$

$$\Rightarrow Z = 6\sqrt{2} \quad | \quad \cos\phi = \frac{6}{6\sqrt{2}}$$

$$\cos\phi = \frac{1}{\sqrt{2}}$$

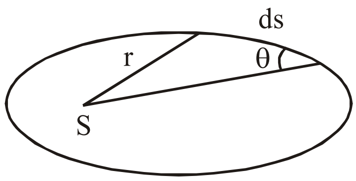
13. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

(1) $\frac{4L}{M}$ (2) $\frac{L}{M}$

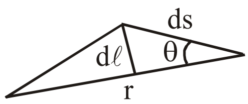
(3) $\frac{2L}{M}$ (4) $\frac{L}{2M}$

Official Ans. by NTA (4)

Sol.



For small displacement ds of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell$$

$$= \frac{1}{2} r ds \sin\theta$$

$$\text{A.vel} = \frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr\sin\theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2m}$$

14. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is :

(1) $\sin(\omega t) + \cos(\omega t)$
(2) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$
(3) $\sin^2(\omega t)$
(4) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Official Ans. by NTA (4)

Sol. Time period $T = \frac{2\pi}{\omega'}$

$$\frac{\pi}{\omega} = \frac{2\pi}{\omega'}$$

$\omega' = 2\omega \rightarrow$ Angular frequency of SHM
Option (3)

$$\sin^2\omega t = \frac{1}{2}\left(2\sin^2\omega t\right) = \frac{1}{2}\left(1 - \cos 2\omega t\right)$$

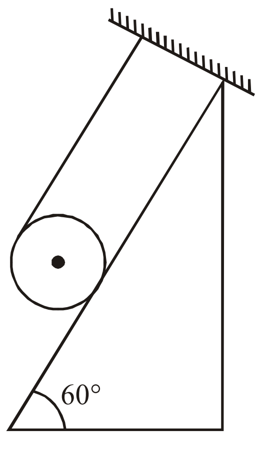
Angular frequency of $\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$ is 2ω

Option (4)
Angular frequency of SHM

$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ is 2ω.

So option (3) & (4) both have angular frequency 2ω but option (4) is direct answer.

15. A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :



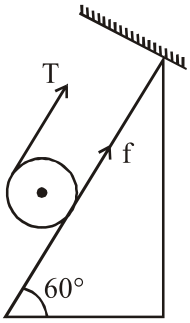
[The coefficient of static friction, μ_s, is 0.4]

(1) $\frac{7}{2}mg$ (2) 5 mg

(3) $\frac{mg}{5}$ (4) 0

Official Ans. by NTA (3)

Sol.



Let's take solid cylinder is in equilibrium

T + f = mg sin60 ....(i)

TR – fR = 0 ....(ii)

Solving we get

$$T = f_{req} = \frac{mg\sin\theta}{2}$$

But limiting friction < required friction

$$\mu mg\cos 60° < \frac{mg\sin 60°}{2}$$

$\therefore$ Hence cylinder will not remain in equilibrium

Hence f = kinetic

$= \mu_k N$

$= \mu_k mg\cos 60°$

$= \dfrac{mg}{5}$

16. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

(1) $\dfrac{L}{R}\ell n5$ (2) infinite

(3) $\dfrac{L}{R}\ell n2$ (4) $\dfrac{L}{R}\ell n10$

Official Ans. by NTA (3)

Sol. Magnetic energy = $\dfrac{1}{2}Li^2 = 25\%$

$$ME \Rightarrow 25\% \Rightarrow i = \frac{i_0}{2}$$

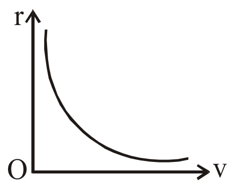
$i = i_0(1 - R^{-Rt/L})$ for charging

$$t = \frac{L}{R}\ell n2$$

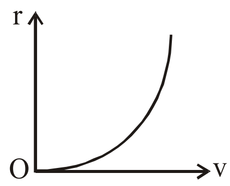
17. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \dfrac{-C}{r}$, where C is a positive constant.

The correct radius – velocity graph of the particle's motion is :

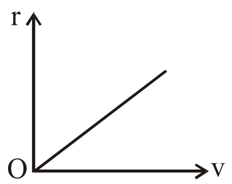
(1)



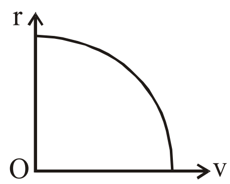
(2)



(3)



(4)



Official Ans. by NTA (1)

Sol. $U = -\dfrac{C}{r}$

$$F = -\frac{dU}{dr} = -\frac{C}{r^2}$$

$$|F| = \frac{mv^2}{r}$$

$$\frac{C}{r^2} = \frac{mv^2}{r}$$

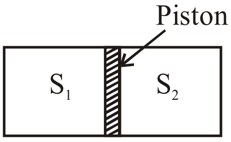
$$v^2 \propto \frac{1}{r}$$

18. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

(1) $S_1 \times S_2$ (2) $S_1 - S_2$

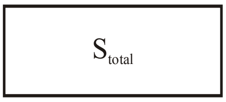
(3) $\dfrac{S_1}{S_2}$ (4) $S_1 + S_2$

Official Ans. by NTA (4)

Sol. 

; $S_1 > S_2$

After piston is removed



; $S_{total} = S_1 + S_2$

19. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32 g/mol; R = 8.3 J K^{-1} mol^{-1})

(1) $\sqrt{\frac{3}{3}}$ (2) $\sqrt{\frac{8}{3}}$

(3) $\sqrt{\frac{3\pi}{8}}$ (4) $\sqrt{\frac{8\pi}{3}}$

Official Ans. by NTA (3)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$

$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}}$

20. The speed of electrons in a scanning electron microscope is 1×10^7 ms^{-1}. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of:

(1) 1837 (2) $\frac{1}{1837}$

(3) $\sqrt{1837}$ (4) $\frac{1}{\sqrt{1837}}$

Official Ans. by NTA (1)

Sol. Resolving power (RP) $\propto \frac{1}{\lambda}$

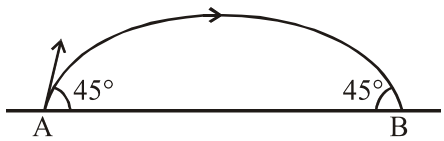
$\lambda = \frac{h}{P} = \frac{h}{mv}$

So (RP) $\propto \frac{mv}{h}$

RP $\propto$ P
RP $\propto$ mv
RP $\propto$ m

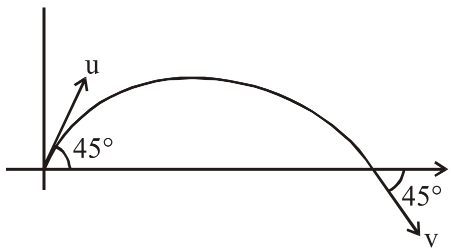
SECTION-B

1. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is x × 10^{-2} $kgms^{-1}$. The value of x, to the nearest integer, is _______.

Official Ans. by NTA (5)

Sol. 

$|\vec{u}| = |\vec{v}|$ ... (1)

$\vec{u} = u\cos 45\hat{i} + u\sin 45\hat{j}$ ... (2)

$\vec{v} = v\cos 45\hat{i} - v\sin 45\hat{j}$ ... (3)

$|\overrightarrow{\Delta P}| = |m(\vec{v} - \vec{u})|$ ... (4)

$\Delta P = 2mu \sin 45°$

$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50 \times 10^{-3}$
$= 5 \times 10^{-2}$

2. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is_____.

Official Ans. by NTA (6)

Sol. Let's say the compression in the spring by : y. So, by work energy theorem we have

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$$

$$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$$

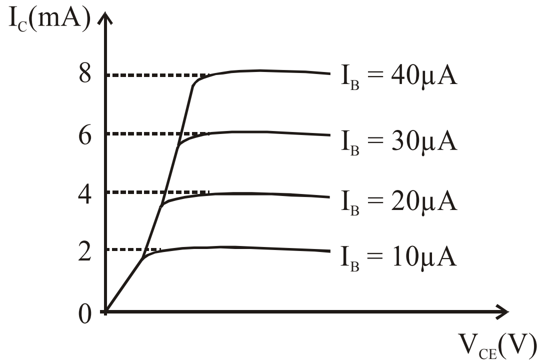
$$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$$

$\Rightarrow$ y = 2m

$\Rightarrow$ final length of spring

= 8 – 2 = 6m

3. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is

Official Ans. by NTA (200)

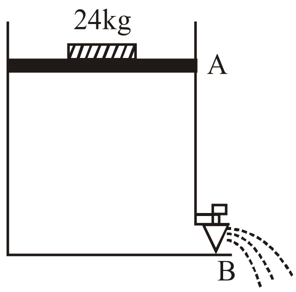
Sol. $$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2\times10^{-3}}{10\times10^{-6}}$$

$$\beta = \frac{1}{5}\times10^3$$

β = 2 × 10^2

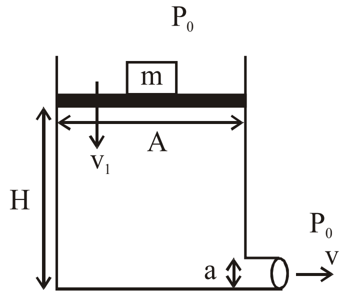
β = 200

4. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}. The value of v, to the nearest integer, is___. [Take value of g to be 10 ms^{-2}]



Official Ans. by NTA (3)

Sol.



m = 24 kg

A = 0.4 m^2

a = 1 cm^2

H = 40cm

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho g H + \cancelto{0}{\frac{1}{2}\rho v_1^2}$$

$$= P_0 + 0 + \frac{1}{2}\rho v^2 \qquad \text{... (1)}$$

$\Rightarrow$ Neglecting v_1

$\Rightarrow \quad v = \sqrt{2gH + \frac{2mg}{A\rho}}$

$\Rightarrow \quad v = \sqrt{8 + 1.2}$

$\Rightarrow$ v = 3.033 m/s

$\Rightarrow \quad v \simeq 3 \text{m/s}$

5. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

(i) ground level

(ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is .

Official Ans. by NTA (50)

Sol. Range = $\sqrt{2Rh}$

Range (i) = $\sqrt{2Rh}$

Range (ii) = $\sqrt{2Rh} + \sqrt{2Rh'}$

where h = 20 m & h' = 5m

Ans = $\frac{\sqrt{2Rh'}}{\sqrt{2Rh}} \times 100\% = \frac{\sqrt{5}}{\sqrt{20}} \times 100\%$ = 50%

6. The radius of a sphere is measured to be (7.50 ± 0.85) cm. Suppose the percentage error in its volume is x. The value of x, to the nearest x, is_______.

Official Ans. by NTA (34)

Sol. $\because$ v = $\frac{4}{3}\pi r^3$

taking log & then differentiate

$\frac{dV}{V} = 3\frac{dr}{r}$

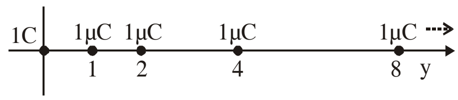
= $\frac{3 \times 0.85}{7.5} \times 100\%$ = 34 %

7. An infinite number of point charges, each carrying 1 μC charge, are placed along the y-axis at y = 1 m, 2 m, 4 m, 8 m................. The total force on a 1 C point charge, placed at the origin, is x × 10^3 N. The value of x, to the nearest integer, is________.

[Take $\frac{1}{4\pi \in_0}$ = 9 × 10^9 Nm^2/C^2]

Official Ans. by NTA (12)

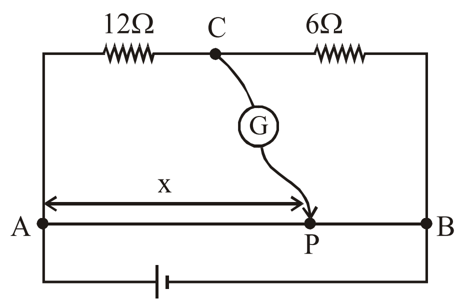
Sol.



F = $k(1C)(1\mu C)\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + ...\right]$

= 9 × 10^3 $\left[\frac{1}{1 - \frac{1}{4}}\right]$ = 12 × 10^3N

8. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is

Official Ans. by NTA (48)

Sol. In Balanced conditions

$\frac{12}{6} = \frac{x}{72 - x}$

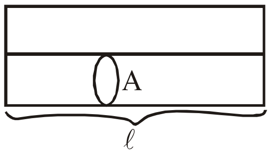
x = 48 cm

9. Two wires of same length and thickness having specific resistances 6Ω cm and 3Ω cm respectively are connected in parallel. The effective resistivity is ρ Ω cm. The value of ρ, to the nearest integer, is_____.

Official Ans. by NTA (4)

Sol. ∵ in parallel

$R_{net} = \dfrac{R_1R_2}{R_1 + R_2}$



$$\frac{\rho\ell}{2A} = \frac{\rho_1\frac{\ell}{A} \times \rho_2\frac{\ell}{A}}{\rho_1\frac{\ell}{A} + \rho_2\frac{\ell}{A}}$$

$$\frac{\rho}{2} = \frac{6 \times 3}{6+3} = 2$$

$\rho = 4$

10. A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the wavelength of a red line at 630 nm is $x \times 10^{-10}$ m. The value of x, to the nearest integer, is_____.

[Take the value of speed of light c, as 3×10^8 ms^{-1}]

Official Ans. by NTA (6)

Sol. $\dfrac{\Delta\lambda}{\lambda}c = v$

$$\Delta\lambda = \frac{v}{c} \times \lambda = \frac{286}{3 \times 10^5} \times 630 \times 10^{-9} = 6 \times 10^{-10}$$

SECTION-A

1. The oxidation states of nitrogen in NO, NO_2, N_2O and NO_3^- are in the order of :

(1) $NO_3^- > NO_2 > NO > N_2O$

(2) $NO_2 > NO_3^- > NO > N_2O$

(3) $N_2O > NO_2 > NO > NO_3^-$

(4) $NO > NO_2 > N_2O > NO_3^-$

Official Ans. by NTA (1)

Sol. The oxidation states of Nitrogen in following molecules are as follows

$NO_3^- \rightarrow +5$

$NO_2 \rightarrow +4$

$NO \rightarrow +2$

$N_2O \rightarrow +1$

2. In basic medium, H_2O_2 exhibits which of the following reactions ?

(A) $Mn^{2+} \rightarrow Mn^{4+}$

(B) $I_2 \rightarrow I^-$

(C) $PbS \rightarrow PbSO_4$

Choose the most appropriate answer from the options given below :

(1) (A), (C) only

(2) (A) only

(3) (B) only

(4) (A), (B) only

Official Ans. by NTA (4)

Sol. In basic medium, oxidising action of H_2O_2.

$Mn^{2+} + H_2O_2 \rightarrow Mn^{+4} + 2OH^-$

In basic medium, reducing action of H_2O_2

$I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

In acidic medium, oxidising action of H_2O_2.

$PbS(s) + 4H_2O_2(aq) \rightarrow PbSO_4(s) + 4H_2O(\ell)$

Hence correct option (4)

3. In the reaction of hypobromite with amide, the carbonyl carbon is lost as :

(1) CO_3^{2-}

(2) HCO_3^-

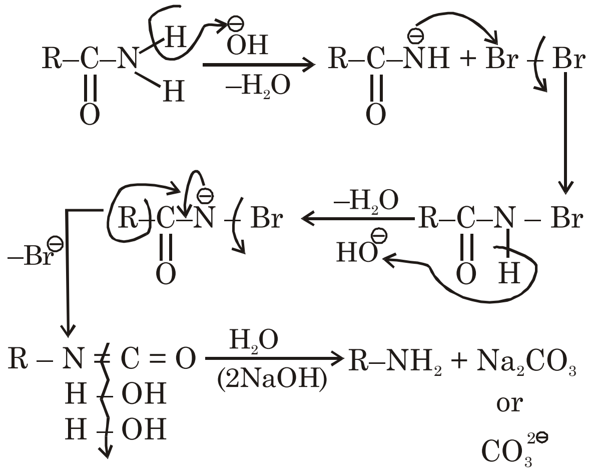
(3) CO_2

(4) CO

Official Ans. by NTA (1)

Sol. $R{-}\underset{\underset{O}{\|}}{C}{-}NH_2 + Br_2 + 4NaOH \longrightarrow R{-}NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$

Mechanism



4. The oxide that shows magnetic property is :

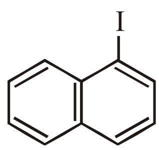
(1) SiO_2 (2) Mn_3O_4

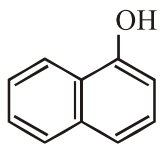
(3) Na_2O (4) MgO

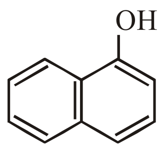
Official Ans. by NTA (2)

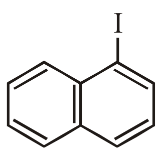
Sol. Mn_3O_4 shows magnetic properties.

5. Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are:

(1)  and CH_3OH

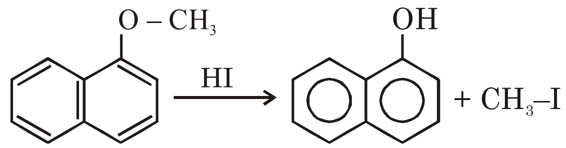
(2)  and CH_3I

(3)  and CH_3OH

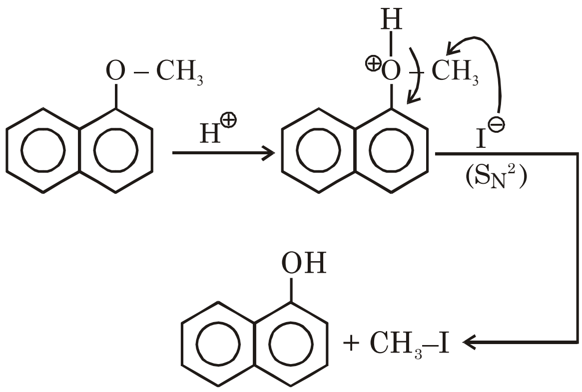
(4)  and CH_3I

Official Ans. by NTA (2)

Sol.



Mechanism

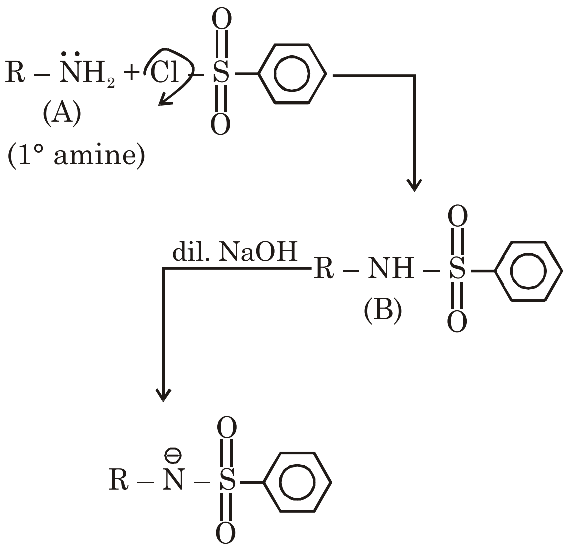


6. Deficiency of vitamin K causes :
(1) Increase in blood clotting time
(2) Increase in fragility of RBC's
(3) Cheilosis
(4) Decrease in blood clotting time
Official Ans. by NTA (1)

Sol. Due to deficiency of Vitmain K causes increases in blood clotting time.
Note : Vitamin K related to blood factor.

7. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is :
(1) $C_6H_5–N–(CH_3)_2$ (2) $C_6H_5–NHCH_2CH_3$
(3) $C_6H_5–CH_2\ NHCH_3$ (4) $C_6H_5–\underset{\substack{|\\ CH_3}}{CH}–NH_2$
Official Ans. by NTA (4)

Sol. Hinsberg reagent (Benzene sulphonyl chloride) gives reaction product with 1° amine and it is soluble in dil. NaOH.

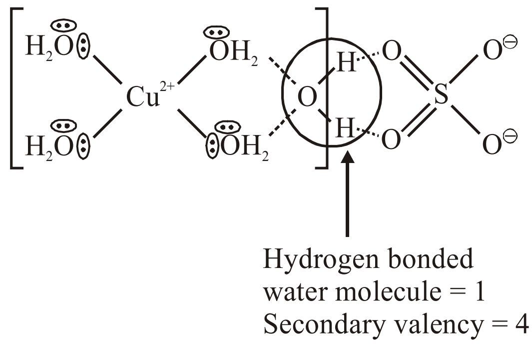


8. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. the elements X, Y and Z, respectively, are :
(1) chlorine, lithium and sodium
(2) argon, lithium and sodium
(3) argon, chlorine and sodium
(4) neon, sodium and chlorine
Official Ans. by NTA (3)

Sol. The 1st IE order of 3rd period is
Na < Al < Mg < Si < S < P < Cl < Ar
X & Y are Ar & Cl
Z is sodium (Na).

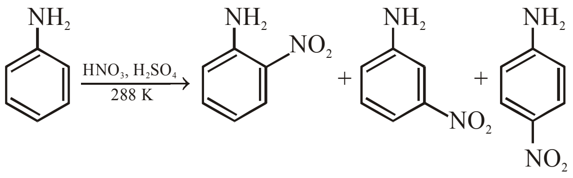
9. The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4·5H_2O$, respectively, are :
(1) 6 and 4 (2) 4 and 1
(3) 6 and 5 (4) 5 and 1
Official Ans. by NTA (2)

Sol.



10. Given below are two statements :
Statement I : Bohr's theory accounts for the stability and line spectrum of Li^+ ion.
Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Both statement I and statement II are true.
(2) Statement I is false but statement II is true.
(3) Both statement I and statement II are false.
(4) Statement I is true but statement II is false.
Official Ans. by NTA (2)

Sol. Statement-I is false since Bohr's theory accounts for the stability and spectrum of single electronic species (eg : He^+, Li^{2+} etc)
Statement II is true.

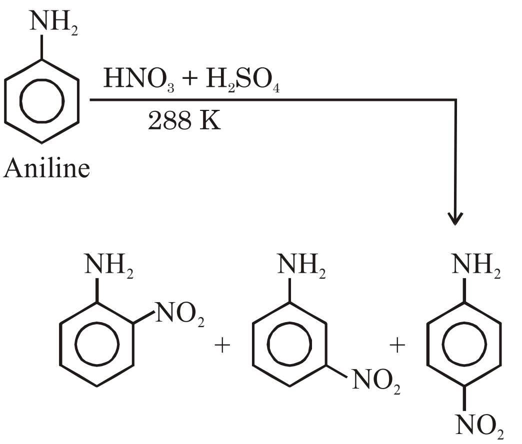
11. 

A B C

Consider the given reaction, percentage yield of :

(1) C > A > B (2) B > C > A
(3) A > C > B (4) C > B > A

Official Ans. by NTA (4)

Sol. 

Aniline

(A) (B) (C)
(2%) (47%) (51%)

% yield order ⇒ C > B > A

12. The charges on the colloidal CdS sol and TiO_2 sol are, respectively :

(1) positive and positive
(2) positive and negative
(3) negative and negative
(4) negative and positive

Official Ans. by NTA (4)

Sol. CdS sol → –ve sol
TiO_2 sol → +ve sol

13. Match List - I with List - II :

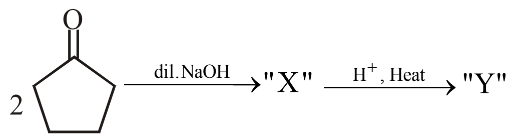
List - I (Class of Chemicals)	List - II (Example)
(a) Antifertility drug	(i) Meprobamate
(b) Antibiotic	(ii) Alitame
(c) Tranquilizer	(iii) Norethindrone
(d) Artificial Sweetener	(iv) Salvarsan

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
(2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

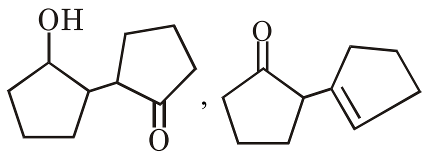
Official Ans. by NTA (3)

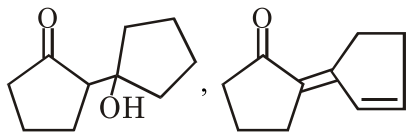
Sol. (A) Antifertility drug → (iii) Nor ethindrone
(B) Antibiotic → (iv) Salvarsan
(C) Tranquilizer → (i) Meprobamate
(D) Artificial sweetener → (ii) Alitame

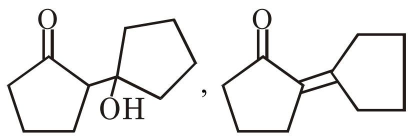
Ans. A–iii, B-iv, C-i, D-ii

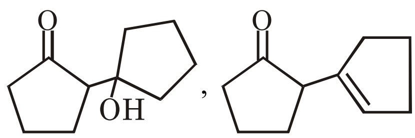
14. 

Consider the above reaction, the product 'X' and 'Y' respectively are :

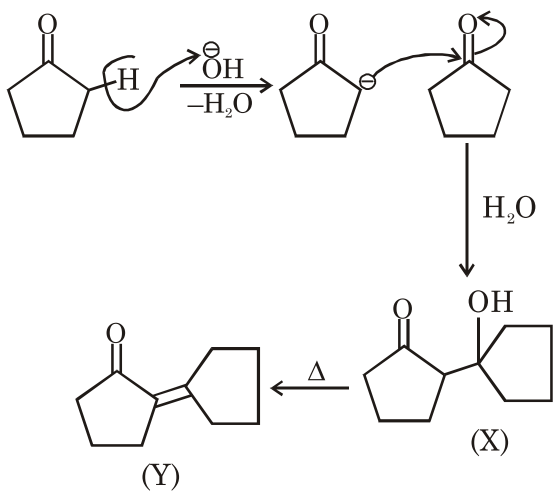
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

15. Match list-I with list-II :

List-I	List-II
(a) Be	(i) Treatment of cancer
(b) Mg	(ii) Extraction of metals
(c) Ca	(iii) Incendiary bombs and signals
(d) Ra	(iv) Windows of X-ray tubes
	(v) Bearings for motor engines.

Choose the most appropriate answer the option given below :

(1) a-iv, b-iii, c-i, d-ii

(2) a-iv, b-iii, c-ii, d-i

(3) a-iii, b-iv, c-v, d-ii

(4) a-iii, b-iv, c-ii, d-v

Official Ans. by NTA (2)

Sol. (a) Be $\rightarrow$ it is used in the Windows of X-ray tubes

(b) Mg $\rightarrow$ it is used in the Incendiary bombs and signals

(c) Ca $\rightarrow$ it is used in the Extraction of metals

(d) Ra $\rightarrow$ it is used in the Treatment of cancer

16. Given below are two statements :

Statement I : C_2H_5OH and AgCN both can generate nucleophile.

Statement II : KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

Choose the most appropriate option :

(1) Statement I is true but statement II is false

(2) Both statement I and statement II are true

(3) Statement I is false but statement II is true

(4) Both statement I and statement II are false

Official Ans. by NTA (1)

17. Given below are two statements :

Statement I : Non-biodegradable wastes are generated by the thermal power plants.

Statement II : Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the option given below :

(1) Both statement I and statement II are false

(2) Statement I is true but statement II is false

(3) Statement I is false but statement II is true

(4) Both statement I and statement II are true.

Official Ans. by NTA (4)

Sol. Non-biodegradable wastes are generated by the thermal power plants which produces fly ash. Detergents which are biodegradable causes problem called eutrophication which kills animal life by deprieving it of oxygen.

18. Match list-I with list-II :

List-I	List-II
(a) Mercury	(i) Vapour phase refining
(b) Copper	(ii) Distillation refining
(c) Silicon	(iii) Electrolytic refining
(d) Nickel	(iv) Zone refining

Choose the most appropriate answer from the option given below :

(1) a-i, b-iv, c-ii, d-iii (2) a-ii, b-iii, c-i, d-iv

(3) a-ii, b-iii, c-iv, d-i (4) a-ii, b-iv, c-iii, d-i

Official Ans. by NTA (3)

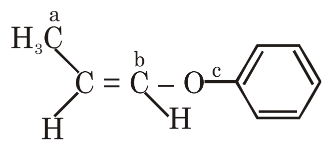
Sol. (a) Mercury $\rightarrow$ Distillation refining

(b) Copper $\rightarrow$ Electrolytic refining

(c) Silicon $\rightarrow$ Zone refining

(d) Nickel $\rightarrow$ Vapour phase refining

19. In the following molecules,



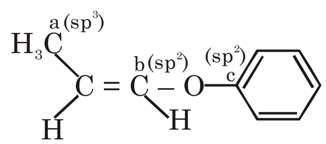
Hybridisation of carbon a, b and c respectively are :

(1) sp^3, sp, sp (2) sp^3, sp^2, sp

(3) sp^3, sp^2, sp^2 (4) sp^3, sp, sp^2

Official Ans. by NTA (3)

Sol.



20. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a / an :

(1) Ionic solid (2) Molecular solid

(3) Metallic solid (4) Covalent solid

Official Ans. by NTA (4)

Sol. Covalent or network solid have very high melting point and they are insulators in their solid and molten form.

SECTION-B

1. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is ______ min. (Round off to the Nearest integer)

[Use : ln 2= 0.69, ln 10 = 2.3]

Official Ans. by NTA (10)

Sol. $$\frac{t_{99.9\%}}{t_{50\%}} = \frac{\frac{1}{K}\ln\frac{100}{0.1}}{\frac{1}{K}\ln 2}$$

$$= \frac{\ln 1000}{\ln 2} \times t_{50\%}$$

$$= \frac{3\ln 10}{\ln 2} \times 1$$

$$= \frac{3 \times 2.3}{0.69} = 10$$

2. The molar conductivities at infinite dilution of barium chloride, sulphuric arid and hydrochloric acid are 280, 860 and 426 Scm^2 mol^{-1} respectively. The molar conductivity at infinite dilution of barium sulphate is ________S cm^2 mol^{-1}(Round off to the Nearest Integer).

Official Ans. by NTA (288)

Sol. From Kohlrausch's law

$$\Lambda_m^\infty(BaSO_4) = \lambda_m^\infty(Ba^{2+}) + \lambda_m^\infty(SO_4^{2-})$$

$$\Lambda_m^\infty(BaSO_4) = \Lambda_m^\infty(BaCl_2) + \Lambda_m^\infty(H_2SO_4) - 2\,\Lambda_m^\infty(HCl)$$

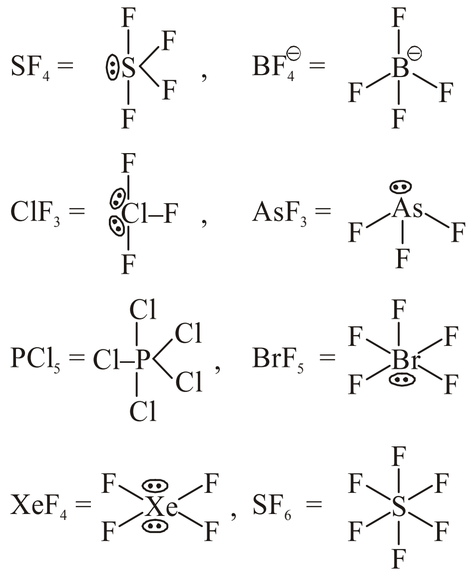
$= 280 + 860 - 2\ (426)$
$= 288\ Scm^2mol^{-1}$

3. The number of species below that have two lone pairs of electrons in their central atom is ____(Round off to the Nearest integer)

SF_4, BF_4^- , ClF_3, AsF_3, PCl_5, BrF_5, XeF_4, SF_6

Official Ans. by NTA (2)

Sol.

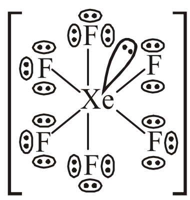


Two l.p. on central atom is = ClF_3, XeF_4

4. A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2. The number of lone pair of electrons present in compound A is _______(Round off to the Nearest integer)

Official Ans. by NTA (19)

Sol. $XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$
(A) (Limited water)
Structure of 'A'



Total l.p. on (A) = 19

5. The gas phase reaction

$$2A(g) \rightleftharpoons A_2(g)$$

at 400 K has $\Delta G^o = +\ 25.2$ kJ mol^{-1}.
The equilibrium constant K_C for this reaction is ______$\times\ 10^{-2}$. (Round off to the Nearest integer)
[Use : R = 8.3 J mol^{-1}K^{-1} , ln 10 = 2.3
$\log_{10} 2 = 0.30$, 1 atm = 1 bar]
[antilog (–0.3) = 0.501]

Official Ans. by NTA (166)
Official Ans. by ALLEN (2)

Sol. Using formula

$\Delta_r G^0 = -RT\ln K_p$

$25200 = -2.3 \times 8.3 \times 400 \log(K_p)$

$K_p = 10^{-3.3} = 10^{-3} \times 0.501$

$= 5.01 \times 10^{-4}\ \text{Bar}^{-1}$

$= 5.01 \times 10^{-9}\ \text{Pa}^{-1}$

$= \dfrac{K_C}{8.3 \times 400}$

$K_C = 1.66 \times 10^{-5}\ \text{m}^3/\text{mole}$

$= 1.66 \times 10^{-2}$ L/mol

Ans = 2

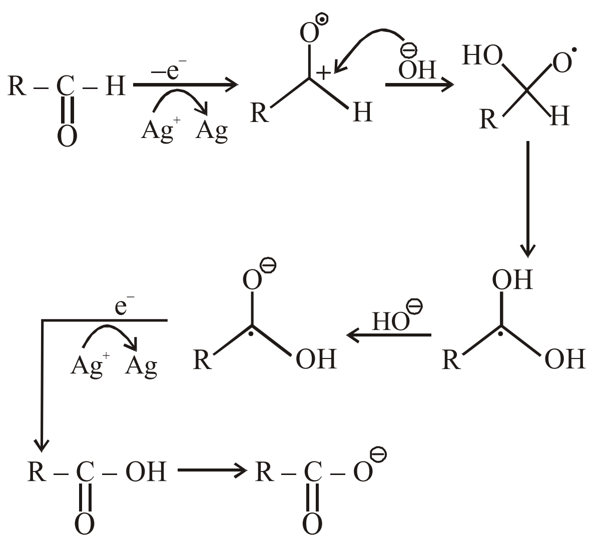
6. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is_______.(Round off to the Nearest integer)

Official Ans. by NTA (2)

Sol. $AgNO_3 + NaOH \rightarrow AgOH + NaNO_3$

$2AgOH \rightarrow Ag_2O + H_2O$

$Ag_2O + 4NH_3 + H_2O \rightarrow 2Ag(NH_3)_2^+ + 2\overset{\ominus}{O}H$



Total $2e^-$ transfer to Tollen's reagent

7. The solubility of $CdSO_4$ in water is 8.0×10^{-4} mol L^{-1}. Its solubility in 0.01 M H_2SO_4 solution is ______$\times 10^{-6}$ mol L^{-1}. (Round off to the Nearest integer) (Assume that solubility is much less than 0.01 M)

Official Ans. by NTA (64)

Sol. In pure water,

$K_{sp} = S^2 = (8 \times 10^{-4})^2$

$= 64 \times 10^{-8}$

In 0.01 M H_2SO_4

$H_2SO_{4(aq)} \rightarrow 2H^+_{(aq)} + SO_4^{2-}(aq.)$

$\qquad\qquad\quad 0.02 \qquad 0.01$

$BaSO_{4(s)} \rightleftharpoons Ba^{2+}_{(aq.)} + SO_{4\ (aq)}^{2-}$

$x \qquad\qquad x \qquad\qquad (x + 0.01)$

$K_{sp} = x\,(x + 0.01)$

$= 64 \times 10^{-8}$

$x + 0.01 \cong 0.01$ M

So, $x\,(0.01) = 64 \times 10^{-8}$

$x = 64 \times 10^{-6}$ M

8. A solute a dimerizes in water. The boiling point of a 2 molar solution of A is 100.52ºC. The percentage association of A is.____.

(Round off to the Nearest integer)

[Use : K_b for water = 0.52 K kg mol^{-1}

Boiling point of water = 100°C]

Official Ans. by NTA (50)
Official Ans. by ALLEN (100)

Sol. $\Delta T_b = T_b - T_b^0$

$100.52 - 100$

$= 0.52ºC$

$i = \left(1 - \dfrac{\alpha}{2}\right)$

$\because\ \Delta T_b = i\,K_b \times m$

$0.52 = \left(1 - \dfrac{\alpha}{2}\right) \times 0.52 \times 2$

$\alpha = 1$

So, percentage association = 100%.

9. 10.0 ml of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings.

4.8 ml, 4.9 ml, 5.0 ml, 5.0 ml and 5.0 ml

Based on these readings, and convention of titrimetric estimation of concentration of Na_2CO_3 solution is ________mM.

(Round off to the Nearest integer)

Official Ans. by NTA (50)

Sol. Most precise volume of HCl = 5 ml at equivalence point

Meq. of Na_2CO_3 = meq. of HCl.

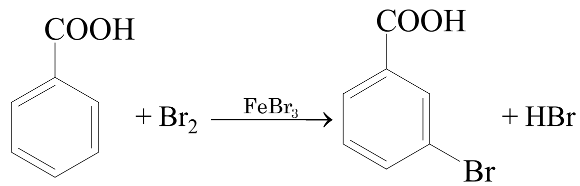
Let molarity of $Na2CO_3$ solution = M, then

$M \times 10 \times 2 = 0.2 \times 5 \times 1$

$M = 0.05$ mol / L

$= 0.05 \times 1000$

$= 50$ mM

10. 

Consider the above reaction where 6.1 g of benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is_____.

(Round off to the Nearest integer)

[Given : Atomic masses : C = 12.0u, H : 1.0u, O : 16.0u, Br = 80.0 u]

Official Ans. by NTA (78)

Sol. Moles of Benzoic acid $= \frac{6.1}{122}$

= moles of m-bromobenzoic acid

So, weight of m-bromobenzoic acid

$= \frac{6.1}{122} \times 201 \text{gm}$

$= 10.05$ gm

$\%\ \text{yield} = \frac{\text{Actual weight}}{\text{Theoretical weight}} \times 100$

$= \frac{7.8}{10.05} \times 100$

$= 77.61\%$

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right), 0<x<2.1,$ with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$

(3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

Official Ans. by NTA (1)

Sol. Let y + 1 = Y

$\therefore \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\frac{1}{Y}=k$

$\Rightarrow \frac{dk}{dx}+k(-x)=e^{\frac{x^2}{2}}$

I.F. = $e^{-\frac{x^2}{2}}$

$\therefore k=(x+c)e^{x^2/2}$

Put $k=-\frac{1}{y+1}$

$\therefore y+1=-\frac{1}{(x+c)e^{x^2/2}}$ ...(i)

when x = 2, y = 0, then c = $-2-\frac{1}{e^2}$

diffentiate equation (i) & put x = 1

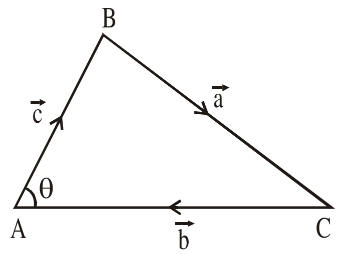
we get $\left(\frac{dy}{dx}\right)_{x=1}=-\frac{e^{3/2}}{(1+e^2)^2}$

2. In a triangle ABC, if $|\overrightarrow{BC}|=8$, $|\overrightarrow{CA}|=7$, $|\overrightarrow{AB}|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Official Ans. by NTA (2)

Sol.



$|\vec{a}|=8, |\vec{b}|=7, |\vec{c}|=10$

$\cos\theta=\frac{|\vec{b}|^2+|\vec{c}|^2-|\vec{a}|^2}{2|\vec{b}||\vec{c}|}=\frac{17}{28}$

Projection of $\vec{c}$ on $\vec{b}$

$=|\vec{c}|\cos\theta$

$=10\times\frac{17}{28}$

$=\frac{85}{14}$

3. Let the system of linear equations

4x + λy + 2z = 0

2x – y + z = 0

μx + 2y + 3z = 0, λ, μ ∈ R.

has a non-trivial solution. Then which of the following is true ?

(1) μ = 6, λ ∈ R (2) λ = 2, μ ∈ R

(3) λ = 3, μ ∈ R (4) μ = –6, λ ∈ R

Official Ans. by NTA (1)

Sol. For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$

when $\mu = 6$, $\quad 12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ

4. Let $f : R - \{3\} \to R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

Official Ans. by NTA (3)

Sol. $f(x) = y = \frac{x-2}{x-3}$

$\therefore\ x = \frac{3y-2}{y-1}$

$\therefore\ f^{-1}(x) = \frac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore\ x = \frac{y+3}{2}$

$\therefore\ g^{-1}(x) = \frac{x+3}{2}$

$\because\ f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\therefore\ x^2 - 5x + 6 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$\therefore$ sum of roots

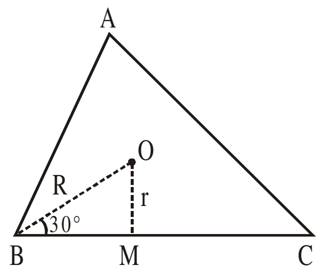
$x_1 + x_2 = 5$

5. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Official Ans. by NTA (1)

Sol.



$r = OM = \frac{3}{\sqrt{2}}$

& $\sin 30° = \frac{1}{2} = \frac{r}{R} \Rightarrow R = \frac{6}{\sqrt{2}}$

$\therefore r + R = \frac{9}{\sqrt{2}}$

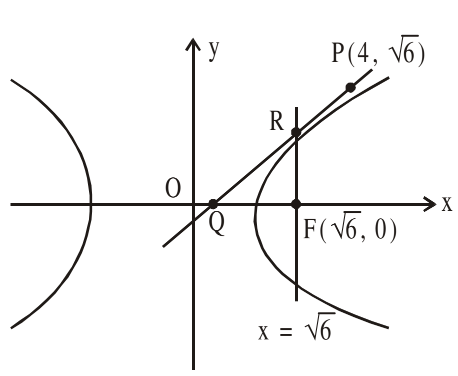
6. Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6} - 1$

(3) $\frac{7}{\sqrt{6}} - 2$ (4) $4\sqrt{6} - 1$

Official Ans. by NTA (3)

Sol.



$$\frac{x^2}{4}-\frac{y^2}{2}=1$$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$$

$\therefore$ Focus F(ae, 0) $\Rightarrow$ $F\left(\sqrt{6},0\right)$

equation of tangent at P to the hyperbola is

$2x - y\sqrt{6} = 2$

tangent meet x-axis at Q(1, 0)

& latus rectum x = $\sqrt{6}$ at $R\left(\sqrt{6},\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)\right)$

$\therefore$ Area of Δ_{QFR} = $\frac{1}{2}\left(\sqrt{6}-1\right).\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)$

$$=\frac{7}{\sqrt{6}}-2$$

7. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$
(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$
(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$
(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

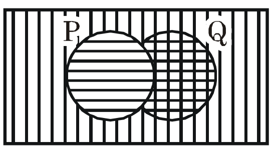
Official Ans. by NTA (2)

Sol. LHS of all the options are some i.e.

$((P \to Q) \wedge \sim Q)$
$\equiv (\sim P \vee Q) \wedge \sim Q$
$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$
$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \to Q$
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$
$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \to \sim P$
$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$
$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \to P$
$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \to (P \wedge Q)$
$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	$P\vee Q$	$P\vee Q$	$\sim P$	$(P\vee Q)\vee\sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

8. Let $g(x)=\int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3}\le f(t)\le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which g(3) lies is :

(1) $\left[-1,-\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2},-1\right]$

(3) $\left[\frac{1}{3},2\right]$ (4) [1, 3]

Official Ans. by NTA (3)

Sol. $\frac{1}{3}\le f(t)\le 1\ \forall\ t\in[0,1]$

$0\le f(t)\le\frac{1}{2}\ \forall\ t\in(1,3]$

Now, $g(3)=\int_0^3 f(t)\,dt=\int_0^1 f(t)\,dt+\int_1^3 f(t)\,dt$

$\because \int_0^1 \frac{1}{3}dt \le \int_0^1 f(t)\,dt \le \int_0^1 1.\,dt$ ....(1)

and $\int_1^3 0\,dt \le \int_1^3 f(1)\,dt \le \int_1^3 \frac{1}{2}dt$ ....(2)

Adding, we get

$$\frac{1}{3}+0\le g(3)\le 1+\frac{1}{2}(3-1)$$

$$\frac{1}{3}\le g(3)\le 2$$

9. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:
(1) 1000 (2) 7000 (3) 5000 (4) 3000
Official Ans. by NTA (4)

Sol. $S_{2n} = \frac{2n}{2}[2a + (2n - 1)d]$, $S_{4n} = \frac{4n}{2}[2a + (4n - 1)d]$

$\Rightarrow S_2 - S_1 = \frac{4n}{2}[2a + (4n - 1)d] - \frac{2n}{2}[2a + (2n - 1)d]$

$= 4an + (4n - 1)2nd - 2na - (2n - 1)dn$

$= 2na + nd[8n - 2 - 2n + 1]$

$\Rightarrow 2na + 2n[6n - 1] = 1000$

$2a + (6n - 1)d = \frac{1000}{n}$

Now, $S_{6n}=\frac{6n}{2}\left[2a+(6n-1)d\right]$

$=3n.\frac{1000}{n}=3000$

10. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

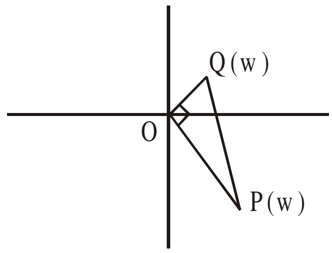
(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Official Ans. by NTA (2)

Sol. $w = 1 - \sqrt{3}.i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and $\text{amp}(z) = \frac{\pi}{2} + \text{amp}(w)$



$\Rightarrow$ Area of triangle $= \frac{1}{2}.OP.OQ$

$=\frac{1}{2}.2.\frac{1}{2}=\frac{1}{2}$

11. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :
(1) 425 (2) 650 (3) 250 (4) 925
Official Ans. by NTA (1)

Sol. Let observations are denoted by x_i for $1 \le i < 2n$

$$\bar{x}=\frac{\sum x_i}{2n}=\frac{(a+a+...+a)-(a+a+...+a)}{2n}$$

$\Rightarrow \bar{x}=0$

and $\sigma_x^2=\frac{\sum x_i^2}{2n}-(\bar{x})^2=\frac{a^2+a^2+...+a^2}{2n}-0=a^2$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y}=\bar{x}+b=5$

$\Rightarrow b = 5$

and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$

12. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(1) $\left(0, \pm\sqrt{3}\right)$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

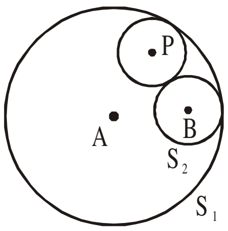
(3) $\left(2, \pm\frac{3}{2}\right)$ (4) $\left(1, \pm 2\right)$

Official Ans. by NTA (3)

Sol. $S_1 : x^2 + y^2 = 9 \begin{cases} r_1 = 3 \\ A(0, 0) \end{cases}$

$S_2 : (x - 2)^2 + y^2 = 1 \begin{cases} r_2 = 1 \\ B(2, 0) \end{cases}$

$\because\ c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally

Let a veriable circle with centre P and radius r

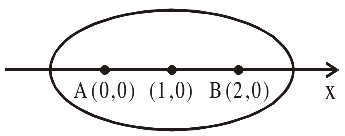
$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2$

$\Rightarrow PA + PB = 4 \ \ (> AB)$

$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is 2a = 4, $e = \frac{1}{2}$

$\Rightarrow$ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$ if x-ellipse



$$\Rightarrow E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

13. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Official Ans. by NTA (2)

Sol. $|\vec{a}| = |\vec{b}|$, $|\vec{a} \times \vec{b}| = |\vec{a}|$, $\vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90° = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{i}$, $\vec{b} = \hat{j} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$$\cos\theta = \frac{\left(\hat{i} + \hat{j} + \hat{k}\right).\hat{i}}{\sqrt{3}\ \sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$ (3) $\frac{40}{243}$ (4) $\frac{128}{625}$

Official Ans. by NTA (1)

Sol. $P(X = 1) = {}^5C_1.p.q^4 = 0.4096$

$P(X = 2) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$$P(X = 3) = {}^5C_3.\left(\frac{1}{5}\right)^3.\left(\frac{4}{5}\right)^2 = \frac{10 \times 16}{125 \times 25} = \frac{32}{625}$$

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta\in\left(0,\frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Official Ans. by NTA (3)

Sol. Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}}+\frac{y.\sin\theta}{1}=1, \qquad \theta\in\left(0,\frac{\pi}{2}\right)$$

intercept on x-axis

OA = $3\sqrt{3}\sec\theta$

intercept on y-axis

OB = $\text{cosec}\theta$

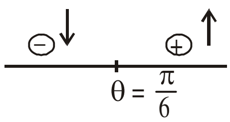
Now, sum of intercept

= $3\sqrt{3}\sec\theta + \text{cosec}\theta = f(\theta)$ let

$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\theta\cot\theta$

$$=3\sqrt{3}\frac{\sin\theta}{\cos^2\theta}-\frac{\cos\theta}{\sin^2\theta}$$

$$=\underbrace{\frac{\cos\theta}{\sin^2\theta}.3\sqrt{3}}_{\oplus}\left[\tan^2\theta-\frac{1}{3\sqrt{3}}\right]=0\Rightarrow\theta=\frac{\pi}{6}$$



$\Rightarrow$ at $\theta = \frac{\pi}{6}$, $f(\theta)$ is minimum

16. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,
(2) R is reflexive, symmetric but not transitive
(3) R is an equivalence relation
(4) R is reflexive, transitive but not symmetric

Official Ans. by NTA (3)

Sol. A and B are matrices of n × n order & ARB iff there exists a non singular matrix $P(\det(P) \neq 0)$ such that $PAP^{-1} = B$

For reflexive

ARA $\Rightarrow PAP^{-1} = A$ ...(1) must be true
for P = I, Eq.(1) is true so 'R' is reflexive

For symmetric

ARB $\Leftrightarrow PAP^{-1} = B$ ...(1) is true
for BRA iff $PBP^{-1} = A$ ...(2) must be true
$\because PAP^{-1} = B$
$P^{-1}PAP^{-1} = P^{-1}B$
$IAP^{-1}P = P^{-1}BP$
$A = P^{-1}BP$ ...(3)
from (2) & (3) $PBP^{-1} = P^{-1}BP$
can be true some $P = P^{-1} \Rightarrow P^2 = I$ $(\det(P) \neq 0)$
So 'R' is symmetric

For trnasitive

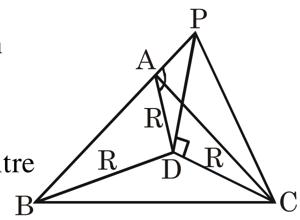
ARB $\Leftrightarrow PAP^{-1} = B$... is true
BRC $\Leftrightarrow PBP^{-1} = C$... is true
now $PPAP^{-1}P^{-1} = C$
$P^2A(P^2)^{-1} = C \Rightarrow$ ARC
So 'R' is transitive relation
$\Rightarrow$ Hence R is equivalence

17. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle ot ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans. by NTA (2)

Sol. Let PD = h, R = 2
As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC



$$\tan\left(\frac{\pi}{3}\right)=\frac{PD}{R}=\frac{h}{R}$$

$$h=R\tan\left(\frac{\pi}{3}\right)=2\sqrt{3}$$

18. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\text{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

Official Ans. by NTA (4)

Sol. $15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$$\tan^2\alpha = \frac{2}{3},\ \cot^2\alpha = \frac{3}{2}$$

$\Rightarrow 27\sec^6\alpha + 8\text{cosec}^6\alpha$

$= 27(\sec^6\alpha)^3 + 8(\text{cosec}^6\alpha)^3$

$= 27(1 + \tan^2\alpha)3 + 8(1 + \cot^2\alpha)^3$

$= 250$

19. The area bounded by the curve $4y^2 = x^2(4 - x)(x - 2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

Official Ans. by NTA (3)

Sol. $4y^2 = x^2(4 - x)(x - 2)$

$$|y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$$

$$\Rightarrow\ y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)}$$

$$\text{and}\ \ y_2 = \frac{-x}{2}\sqrt{(4-x)(x-2)}$$

D : $x \in [2, 4]$

Required Area

$$= \int_2^4 (y_1 - y_2)\,dx = \int_2^4 x\sqrt{(4-x)(x-2)}\,dx \quad ...(1)$$

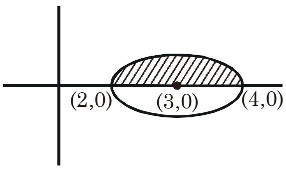
$$\text{Applying } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\text{Area} = \int_2^4 (6-x)\sqrt{(4-x)(x-2)}\,dx \quad ...(2)$$

(1) + (2)

$$2A = 6\int_2^4 \sqrt{(4-x)(x-2)}\,dx$$

$$A = 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$$



$$A = 3.\frac{\pi}{2}.1^2 = \frac{3\pi}{2}$$

20. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x} & , \text{if } x < 0 \\ b & , \text{if } x = 0 \\ \dfrac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{if } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal to :

(1) $-\frac{5}{2}$ (2) -2 (3) -3 (4) $-\frac{3}{2}$

Official Ans. by NTA (4)

Sol. $f(x)$ is continuous at $x = 0$

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x) \quad ...(1)$$

$$f(0) = b \quad ...(2)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$$

$$= \frac{a+1}{2} + 1 \quad ...(3)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to 0^+} \frac{(x+bx^3-x)}{bx^{5/2}\left(\sqrt{x+bx^3}+\sqrt{x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+bx^2}+1\right)} = \frac{1}{2} \quad ...(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow\ b = \frac{1}{2},\ a = -2$$

$$a + b = \frac{-3}{2}$$

SECTION-B

1. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to______.

Official Ans. by NTA (0)

Sol. $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now $P(x)$ is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x^2 + x + 1)$

$P(w) = 0 = P(w^2)$ where w, w^2 are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w - w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$

2. Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to _______.

Official Ans. by NTA (6)

Sol. $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$$

$\Rightarrow n = 6$

3. If $\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right) = \alpha(11!)$, then the value of α is equal to _______.

Official Ans. by NTA (160)

Sol. $\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$

$$= \sum_{r=1}^{10}\left[\{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\}\right]$$

$= (13! + 12! - 2! - 3!) - 8(11! - 1)$

$= (12.13 + 12 - 8).11! - 8 + 8$

$= (160)(11)!$

Hence $\alpha = 160$

4. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to _______.

Official Ans. by NTA (210)

Sol. $\left((x^{1/3} + 1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow r = 4$

$$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

5. Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x = -3$. Let $P(x)$ have local minima at $x = 1$, local maxima at $x = -1$ and $\int_{-1}^{1} P(x)dx = 18$, then the sum of all the coefficients of the polynomial $P(x)$ is equal to _______.

Official Ans. by NTA (8)

Sol. Let p'(x) = a(x – 1) (x + 1) = a(x² – 1)

$$p(x) = a\int\left(x^2-1\right)dx + c$$

$$= a\left(\frac{x^3}{3}-x\right)+c$$

Now p(–3) = 0

$$\Rightarrow a\left(-\frac{27}{3}+3\right)+c=0$$

⇒ –6a + c = 0 ...(1)

$$\text{Now } \int_{-1}^{1}\left(a\left(\frac{x^3}{3}-x\right)+c\right)dx = 18$$

= 2c = 18 ⇒ c = 9 ...(2)

⇒ from (1) & (2) ⇒ –6a + 9 = 0 ⇒ a = $\frac{3}{2}$

$$\Rightarrow p(x) = \frac{3}{2}\left(\frac{x^3}{3}-x\right)+9$$

sum of coefficient

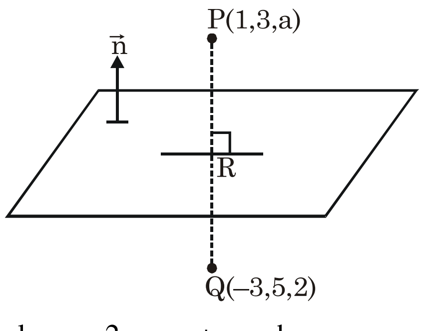
$$=\frac{1}{2}-\frac{3}{2}+9$$

= 8

6. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)-b=0$ be (–3, 5, 2). Then the value of |a + b| is equal to _______.

Official Ans. by NTA (1)

Sol.



plane = 2x – y + z = b

$$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow \text{on plane}$$

$$\therefore\ -2-4+\frac{a+2}{2}=b$$

⇒ a + 2 = 2b + 12 ⇒ a = 2b + 10 ...(i)

<PQ> = <4, –2, a – 2>

$$\therefore\ \frac{2}{4}=\frac{-1}{-2}=\frac{1}{a-2}$$

⇒ a – 2 = 2 ⇒ a = 4, b = –3

∴ |a + b| = 1

7. Let f : R → R satisfy the equation f(x + y) = f(x).f(y) for all x, y ∈ R and f(x) ≠ 0 for any x ∈ R. If Ihe function f is differentiable at x = 0 and $f'(0)$ = 3, then

$$\lim_{h\to 0}\frac{1}{h}\left(f(h)-1\right) \text{ is equal to \_\_\_\_\_\_.}$$

Official Ans. by NTA (3)

Sol. If f(x + y) = f(x).f(y) & $f'(0)$ = 3 then

$f(x) = a^x \Rightarrow f'(x) = a^x.\ell na$

$\Rightarrow f'(0) = \ell na = 3 \Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)^x = e^{3x}$

$$\lim_{x\to 0}\frac{f(x)-1}{x}=\lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times 3\right)=1\times 3=3$$

8. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10}\left(2^2+3k\right){}^nC_k=\alpha.3^{10}+\beta.2^{10}$, α, β ∈ R, then α + β is equal to ______.

Official Ans. by NTA (19)

Allen Answer (Bonus)

Sol. **BONUS**

Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$$\sum_{k=0}^{10}\left(2^2+3k\right){}^{10}C_k=\alpha.3^{10}+\beta.2^{10}$$

$$\text{LHS} = 4\sum_{k=0}^{10}{}^{10}C_k+3\sum_{k=0}^{10}k.\frac{10}{k}.{}^9C_{k-1}$$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

⇒ α = 0, β = 19 ⇒ α + β = 19

9. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point (1, –1, α) lies on the plane P, then the value of |5α| is equal to ______.

Official Ans. by NTA (38)

Sol. Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$

Now (1, –1, α) lies on it so

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5\alpha + 38 = 0 \Rightarrow |5\alpha| = 38$

10. Let y = y(x) be the solution of the differential equation $xdy-ydx=\sqrt{(x^2-y^2)}\,dx$, x ≥ 1, with y(1) = 0. If the area bounded by the line x = 1, x = e^{π}, y = 0 and y = y(x) is $\alpha e^{2\pi}+\beta$, then the value of 10(α + β) is equal to ______.

Official Ans. by NTA (4)

Sol. $xdy-ydx=\sqrt{x^2-y^2}\,dx$

$\Rightarrow \frac{xdy-ydx}{x^2}=\frac{1}{x}\sqrt{1-\frac{y^2}{x^2}}\,dx$

$\Rightarrow \int\frac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}}=\int\frac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right)=\ell n|x|+c$

at x = 1, y = 0 ⇒ c = 0

y = xsin(ℓnx)

$A=\int_1^{e^{\pi}} x\sin(\ell n x)dx$

$x = e^t$, $dx = e^t dt \Rightarrow \int_0^{\pi} e^{2t}\sin(t)\,dt = A$

$\alpha e^{2\pi}+\beta=\left(\frac{e^{2t}}{5}(2\sin t-\cos t)\right)_0^{\pi}=\frac{1+e^{2\pi}}{5}$

$\alpha=\frac{1}{5}, \beta=\frac{1}{5}$ so 10(α + β) = 4

PHYSICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is : (Take r < a)

(1) $\frac{3}{2}\frac{a^2}{r^2}$ (2) $\frac{2}{3}\frac{a^2}{r^2}$

(3) $\frac{2}{3}\frac{r^2}{a^2}$ (4) $\frac{3}{2}\frac{r^2}{a^2}$

Official Ans. by NTA (4)

Sol. $$B_{axis} = \frac{\mu_0 i R^2}{2\left(R^2 + x^2\right)^{3/2}}$$

$$B_{centre} = \frac{\mu_0 i}{2R}$$

$$\therefore B_{centre} = \frac{\mu_0 i}{2a}$$

$$\therefore B_{axis} = \frac{\mu_0 i a^2}{2\left(a^2 + r^2\right)^{3/2}}$$

$\therefore$ fractional change in magnetic field =

$$\frac{\frac{\mu_0 i}{2a} - \frac{\mu_0 i a^2}{2\left(a^2+r^2\right)^{3/2}}}{\frac{\mu_0 i}{2a}} = 1 - \frac{1}{\left[1+\left(\frac{r^2}{a^2}\right)\right]^{3/2}}$$

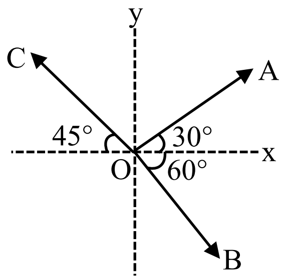
$$\approx 1 - \left[1 - \frac{3}{2}\frac{r^2}{a^2}\right] = \frac{3}{2}\frac{r^2}{a^2}$$

Note : $\left(1+\frac{r^2}{a^2}\right)^{-3/2} \approx \left(1 - \frac{3}{2}\frac{r^2}{a^2}\right)$

[True only if r << a]

Hence option (4) is the most suitable option

2. The magnitude of vectors OA , OB and OC in the given figure are equal. The direction of $OA + OB - OC$ with x-axis will be :-



(1) $\tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

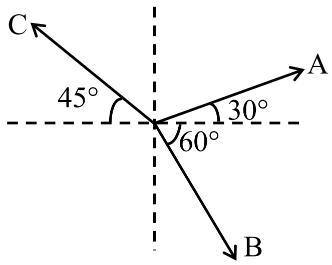
(2) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1+\sqrt{3}-\sqrt{2}\right)}$

(3) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1-\sqrt{3}+\sqrt{2}\right)}$

(4) $\tan^{-1}\frac{\left(1+\sqrt{3}-\sqrt{2}\right)}{\left(1-\sqrt{3}-\sqrt{2}\right)}$

Official Ans. by NTA (1)

Sol.



Let magnitude be equal to λ.

$$OA = \lambda\left[\cos 30°\hat{i} + \sin 30\hat{j}\right] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$$OB = \lambda\left[\cos 60°\hat{i} - \sin 60\hat{j}\right] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$OC = \lambda\left[\cos 45°\left(-\hat{i}\right) + \sin 45\hat{j}\right] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$\therefore OA + OB - OC$

$$= \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}\right]=\tan^{-1}\left[\frac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$$

$$=\tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$$

Hence option (1)

3. Car B overtakes another car A at a relative speed of 40 ms^{-1}. How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?

(1) 4 ms^{-1} (2) 0.2 ms^{-1}

(3) 40 ms^{-1} (4) 0.1 ms^{-1}

Official Ans. by NTA (4)

Sol.

Mirror used is convex mirror (rear-view mirror)

$\therefore V_{I/m}=-m^2V_{O/m}$

Given,

$V_{O/m}=40m/s$

$$m=\frac{f}{f-u}=\frac{10}{10+190}=\frac{10}{200}$$

$$\therefore V_{I/m}=-\frac{1}{400}\times40=-0.1m/s$$

$\therefore$ Car will appear to move with speed 0.1 m/s.

Hence option (4)

4. Inside a uniform spherical shell :

(a) the gravitational field is zero

(b) the gravitational potential is zero

(c) the gravitational field is same everywhere

(d) the gravitation potential is same everywhere

(e) all of the above

Choose the most appropriate answer from the options given below :

(1) (a), (c) and (d) only

(2) (e) only

(3) (a), (b) and (c) only

(4) (b), (c) and (d) only

Official Ans. by NTA (1)

Sol. Inside a spherical shell, gravitational field is zero and hence potential remains same everywhere

Hence option (1)

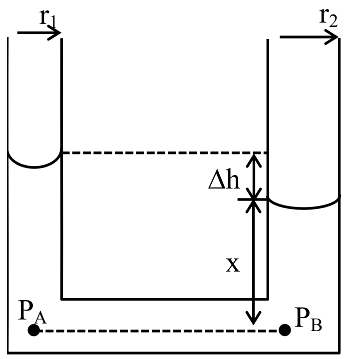
5. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water T = 7.3 × 10^{-2} Nm^{-1}, angle of contact = 0, g = 10 ms^{-2} and density of water = 1.0 × 10^3 kg m^{-3}]

(1) 3.62 mm (2) 2.19 mm

(3) 5.34 mm (4) 4.97 mm

Official Ans. by NTA (2)

Sol.



We have $P_A = P_B$. [Points A & B at same horizontal level]

$$\therefore P_{atm}-\frac{2T}{r_1}+\rho g(x+\Delta h)=P_{atm}-\frac{2T}{r_2}+\rho gx$$

$$\therefore \rho g\Delta h=2T\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

$$=2\times7.3\times10^{-2}\left[\frac{1}{2.5\times10^{-3}}-\frac{1}{4\times10^{-3}}\right]$$

$$\therefore \Delta h=\frac{2\times7.3\times10^{-2}\times10^3}{10^3\times10}\left[\frac{1}{2.5}-\frac{1}{4}\right]$$

= 2.19 × 10^{-3}m = 2.19 mm

Hence option (2)

6. An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90W. How long it would take to increase the internal energy by 2.5 × 10^3 J ?

(1) 2.5 × 10^2 s (2) 4.1 × 10^1s

(3) 2.4 × 10^3 s (4) 2.5 × 10^1s

Official Ans. by NTA (1)

Sol. $\Delta Q = \Delta U + \Delta W$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

$$\frac{6000}{60}\frac{J}{sec} = \frac{2.5\times10^{3}}{\Delta t} + 90$$

$\Delta t = 250$ sec

Option (1)

7. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds :

(1) 0.4 (2) 0.8 (3) 0.125 (4) 0.2

Official Ans. by NTA (4)

Sol. $U = \frac{1}{2}Li^2 = 64 \Rightarrow L = 2$

$i^2R = 640$

$$R = \frac{640}{(8)^2} = 10$$

$$\tau = \frac{L}{R} = \frac{1}{5} = 0.2$$

Option (4)

8. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance X_L = 250 πΩ and an unknown capacitor. The value of capacitance to maximize the average power should be : (Take $\pi^2 = 10$)

(1) 4 μF (2) 25 μF (3) 400 μF (4) 40 μF

Official Ans. by NTA (1)

Sol. For maximum average power

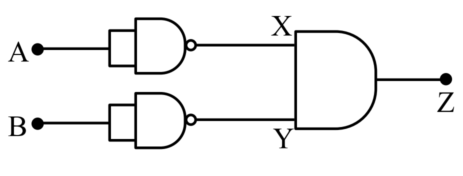
$$X_L = X_C$$

$$250\pi = \frac{1}{2\pi(50)C}$$

$$C = 4\times10^{-6}$$

Option (1)

9. Identify the logic operation carried out by the given circuit :-



(1) OR (2) AND (3) NOR (4) NAND

Official Ans. by NTA (3)

Sol.

A	B	X	Y	Z
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

Option (3)

10. A particular hydrogen like ion emits radiation of frequency 2.92×10^{15} Hz when it makes transition from n = 3 to n = 1. The frequency in Hz of radiation emitted in transition from n = 2 to n = 1 will be :

(1) 0.44×10^{15} (2) 6.57×10^{15}
(3) 4.38×10^{15} (4) 2.46×10^{15}

Official Ans. by NTA (4)

Sol. $nf_1 = k\left(\frac{1}{1} - \frac{1}{3^2}\right)$

$$nf_2 = k\left(1 - \frac{1}{2^2}\right)$$

$$\frac{f_1}{f_2} = \frac{8/9}{3/4} \Rightarrow f_2 = 2.46\times10^{15}$$

Option (4)

11. In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function ϕ = 2.5 eV. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. (h = 6.63×10^{-34} Js, c = 3×10^{8} ms^{-1})

(1) 1.3 V (2) 1.1 V (3) 1.9 V (4) 0.6 V

Official Ans. by NTA (1)

Sol. $KE_{max} = eV_S = \frac{hc}{\lambda} - \phi$

$$\Rightarrow eV_S = \frac{1240}{280} - 2.5 = 1.93eV$$

$$\to V_{S_1} = 1.93V \ \dots (i)$$

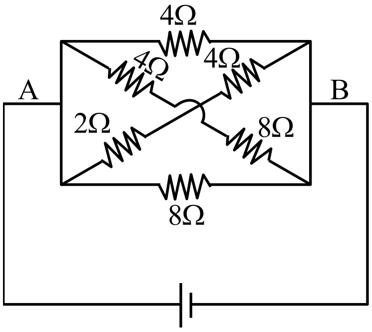
$$\to eV_{S_2} = \frac{1240}{400} - 2.5 = 0.6eV$$

$$\Rightarrow V_{S_2} = 0.6V \ \dots (ii)$$

$$\Delta V = V_{S_1} - V_{S_2} = 1.93 - 0.6 = 1.33V$$

Option (1)

12. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6Ω. Calculate the power dissipated in the whole circuit :



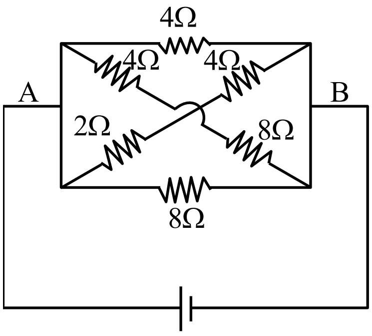
2.2 V, r=0.6Ω

(1) 1.32 W (2) 0.65 W

(3) 2.2 W (4) 4.4 W

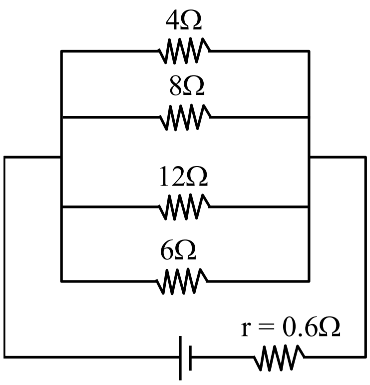
Official Ans. by NTA (3)

Sol.



2.2 V, r=0.6Ω

⇒



2.2 V

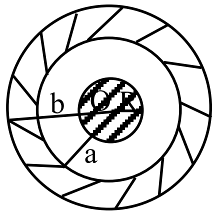
$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6} = \frac{6+3+2+4}{24} = \frac{15}{24}$$

$$R_{eq} = \frac{24}{15} = 1.6 \Rightarrow R_T = 1.6 + 0.6 = 2.2\Omega$$

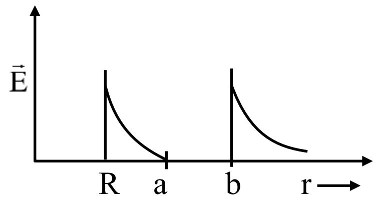
$$P = \frac{V^2}{R_T} = \frac{(2.2)^2}{2.2} = 2.2W$$

Option (3)

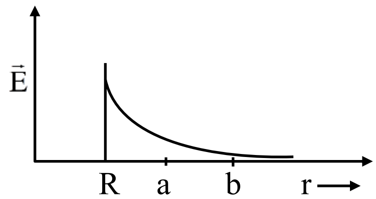
13. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation electric field E as a function of distance r from centre O is given by



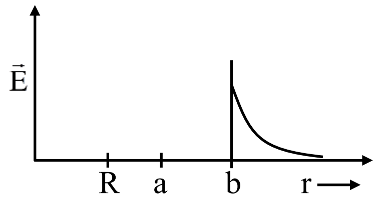
(1)



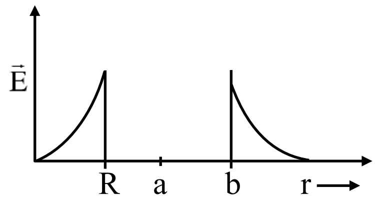
(2)



(3)



(4)



Official Ans. by NTA (1)

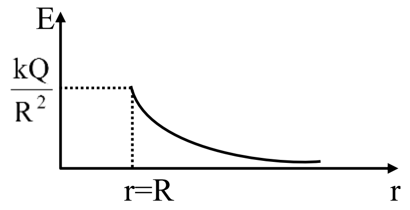
Official Ans. by ALLEN (1 or 2)

Sol. Considering outer spherical shell is non-conducting

Electric field inside a metal sphere is zero.

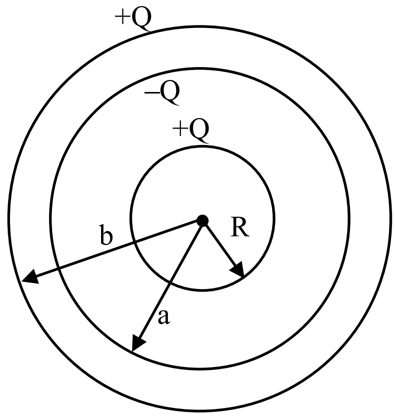
$r < R \Rightarrow E = 0$

$r > R \Rightarrow E = \frac{kQ}{r^2}$



Option (2)

Considering outer spherical shell is conducting

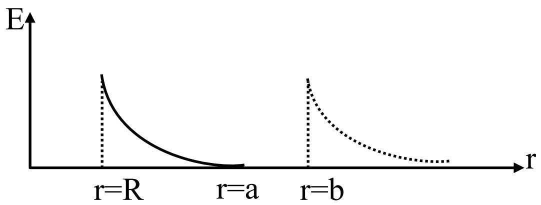


$r < R$, $E = 0$

$R \le r < a$ $\qquad E = \frac{kQ}{r^2}$

$a \le r < b$, $\qquad E = 0$

$r \ge b$ $\qquad E = \frac{kQ}{r^2}$



Option (1)

14. The rms speeds of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are V_H, V_O and V_C respectively then :

(1) $V_H > V_O > V_C$ (2) $V_C > V_O > V_H$

(3) $V_H = V_O > V_C$ (4) $V_H = V_O = V_C$

Official Ans. by NTA (1)

Sol. $V_{RMS} = \sqrt{\frac{3RT}{M_W}}$

At the same temperature $V_{RMS} \propto \frac{1}{\sqrt{M_w}}$

$\Rightarrow V_H > V_O > V_C$

Option (1)

15. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20^{th} division of the circular scale coincides with reference line. Calculate the true reading.

(1) 5.00 mm (2) 5.25 mm

(3) 5.15 mm (4) 5.20 mm

Official Ans. by NTA (3)

Sol. Least count (L.C) = $\frac{0.5}{50}$

True reading $= 5 + \frac{0.5}{50}\times 20 - \frac{0.5}{50}\times 5$

$= 5 + \frac{0.5}{50}(15) = 5.15\text{mm}$

Option (3)

16. What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω?

(Given resistivities of iron and copper-nickel alloy wire are 12 μΩ cm and 51 μΩ cm respectively)

(1) 82 m (2) 97 m

(3) 110 m (4) 90 m

Official Ans. by NTA (2)

Sol. $\frac{R_1R_2}{R_1+R_2} = 3$

$$\frac{\frac{(12\times10^{-6}\times10^{-2})\ell\times4}{\pi(2)^2\times10^{-6}}\times\frac{(51\times10^{-6}\times10^{-2})\ell\times4}{\pi(2)^2\times10^{-6}}}{\frac{63\times10^{-6}\times10^{-2}\times\ \times4}{\pi(2)^2\times10^{-6}}}$$

$\Rightarrow\ = 97\text{m}$

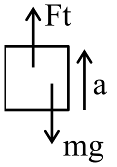
Option (2)

17. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket :[Use g = 10 m/s^2]

(1) 6.0×10^2 kg s^{-1} (2) 500 kg s^{-1}

(3) 10 kg s^{-1} (4) 60 kg s^{-1}

Official Ans. by NTA (4)

Sol. 

$$F_{thrust} = \left(\frac{dm}{dt} \cdot V_{rel}\right)$$

$$\left(\frac{dm}{dt} V_{rel} - mg\right) = ma$$

$$\Rightarrow \left(\frac{dm}{dt}\right) \times 500 - 10^3 \times 10 = 10^3 \times 20$$

$$\frac{dm}{dt} = (60 kg/s)$$

Option (4)

18. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :-

(1) $[M^0 L^1 T^0]$ (2) $[M^{-1} L^{-1} T^2]$

(3) $[M^1 L^1 T^{-2}]$ (4) $[M^0 L^0 T^0]$

Official Ans. by NTA (4)

Sol. $E = ML^2T^{-2}$

$L = ML^2T^{-1}$

m = M

$G = M^{-1}L^{+3}T^{-2}$

$$P = \frac{EL^2}{M^5G^2}$$

$$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$$

Option (4)

19. The material filled between the plates of a parallel plate capacitor has resistivity 200 Ωm. The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is : (given the value of relative permitivity of material is 50)

(1) 9.0 μA (2) 9.0 mA

(3) 0.9 mA (4) 0.9 μA

Official Ans. by NTA (3)

Sol. ρ = 200 Ωm

$C = 2 \times 10^{-12}$ F

V = 40 V

K = 56

$$i = \frac{q}{\rho k\varepsilon_0} = \frac{q_0}{\rho k\varepsilon_0} e^{-\frac{t}{\rho k\varepsilon_0}}$$

$$i_{max} = \frac{2 \times 10^{-12} \times 40}{200 \times 50 \times 8.85 \times 10^{-12}}$$

$$= \frac{80}{10^4 \times 8.85} = 903\mu A = 0.9mA$$

Option (3)

20. **Statement-I :** By doping silicon semiconductor with pentavalent material, the electrons density increases.

Statement-II : The n-type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement-I is true but Statement-II is false.

(2) Statement-I is false but Statement-II is true.

(3) Both Statement-I and Statement-II are true.

(4) Both Statement-I and Statement-II are false.

Official Ans. by NTA (1)

Sol. Pentavalent activities have excess free e^-

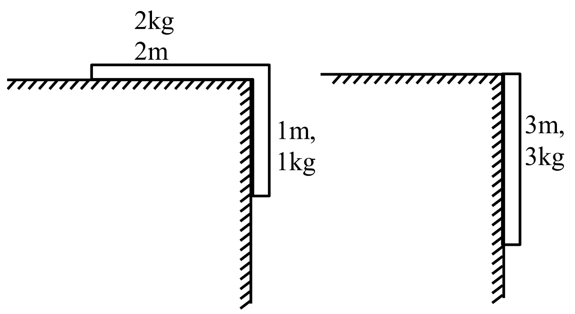
So e^- density increases but overall semiconductor is neutral.

Option (1)

SECTION-B

1. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is .... (Take g = 10 m/s^2)

Official Ans. by NTA (40)

Sol. 

From energy conservation

$K_i + U_i = k_f + U_f$

$$0+\left(-1\times10\times\frac{1}{2}\right)=k_f+\left(-3\times10\times\frac{3}{2}\right)$$

$-5 = k_f - 45$

$k_f = 40$ J

Ans. 40.00

2. The electric field in a plane electromagnetic wave is given by

$$E=200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x-\left(1.5\times10^{11}\frac{rad}{s}\times t\right)\right]\frac{V}{m}\hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2. If the radiation pressure exerted by the E.M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\frac{N}{m^2}$. Find the value of x.

Official Ans. by NTA (354)

Sol. $E_0 = 200$

$$I=\frac{1}{2}\varepsilon_0E_0^2\cdot C$$

Radiation pressure

$$P=\frac{2I}{C}$$

$$=\left(\frac{2}{C}\right)\left(\frac{1}{2}\varepsilon_0E_0^2C\right)$$

$$=\varepsilon_0E_0^2$$

$$=8.85\times10^{-12}\times200^2$$

$$=8.85\times10^{-8}\times4$$

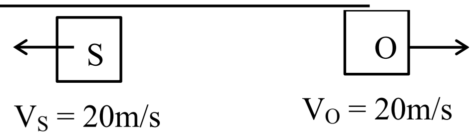
$$=\frac{354}{10^9}$$

Ans. 354.0

3. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be .... Hz.

Official Ans. by NTA (2025)

Sol.



$$f'=f\left(\frac{C-V_0}{C+V_s}\right)$$

$$1800=f\left(\frac{340-20}{340+20}\right)$$

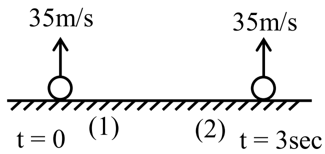
f = 2025 Hz

Ans. 2025

4. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of .... m. (Take g = 10 m/s^2)

Official Ans. by NTA (50)

Sol.



When both balls will collied

$y_1 = y_2$

$$35t-\frac{1}{2}\times10\times t^2=35(t-3)-\frac{1}{2}\times10\times(t-3)^2$$

$$35t-\frac{1}{2}\times10\times t^2=35t-105-\frac{1}{2}\times10\times t^2$$

$$-\frac{1}{2}\times10\times3^2+\frac{1}{2}\times10\times6t$$

0 = 150 – 30 t

t = 5 sec

$\therefore$ Height at which both balls will collied

$$h=35t-\frac{1}{2}\times10\times t^2$$

$$=35\times5-\frac{1}{2}\times10\times5^2$$

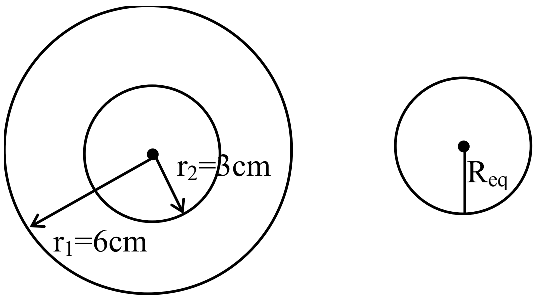
h = 50 m

Ans. 50.00

5. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is .... cm.

Official Ans. by NTA (2)

Sol.



Excess pressure inside the smaller soap bubble

$$\Delta P = \frac{4S}{r_1} + \frac{4S}{r_2} \quad \text{.... (i)}$$

The excess pressure inside the equivalent soap bubble

$$\Delta P = \frac{4S}{R_{eq}} \quad \text{... (ii)}$$

From (i) & (ii)

$$\frac{4S}{R_{eq}} = \frac{4S}{r_1} + \frac{4S}{r_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$= \frac{1}{6} + \frac{1}{3}$$

$R_{eq} = 2$ cm

Ans. 2.00

6. An amplitude modulated wave is represented by

$C_m(t) = 10(1 + 0.2\cos 12560t)\sin(111 \times 10^4 t)$ volts.

The modulating frequency in kHz will be ....

Official Ans. by NTA (2)

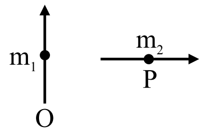
Sol. $W_m = 12560 = 2\pi f_m$

$$f_m = \frac{12560}{2\pi}$$

= 2000 Hz

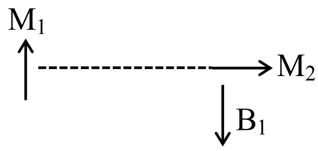
Ans. 2.00

7. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is ...... $\times 10^{-7}$ Nm.



Official Ans. by NTA (1)

Sol.



$$\tau = M_2 \times B_1$$

$$\tau = M_2 B_1 \sin 90°$$

$$= 1 \times \frac{\mu_0}{4\pi}\frac{M_1}{(1)^3}1$$

$= 10^{-7}$ N.m

Ans. 1.00

8. Two travelling waves produces a standing wave represented by equation,

$y = 1.0$ mm $\cos(1.57\text{ cm}^{-1})$ x $\sin(78.5\text{ s}^{-1})$t.

The node closest to the origin in the region x > 0 will be at x = ... cm.

Official Ans. by NTA (1)

Sol. For node

$$\cos\left(1.57\text{cm}^{-1}\right)x = 0$$

$$\left(1.57\text{cm}^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(1.57)}\text{cm} = 1\text{cm}$$

Ans. 1.00

9. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and voilet light is .... nm.

Official Ans. by NTA (300)

Sol. Position of bright fringe $y = n\frac{D\lambda}{d}$

y_1 of red $= \frac{D\lambda_r}{d} = 3.5\text{mm}$

$\lambda_r = 3.5\times10^{-3}\frac{d}{D}$

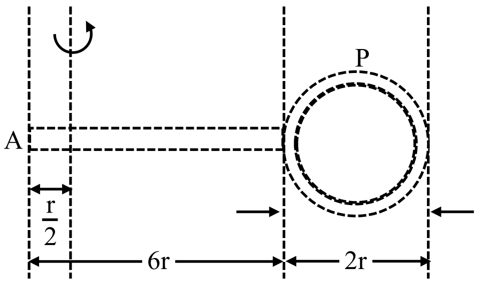
Similarly $\lambda_v = 2\times10^{-3}\frac{d}{D}$

$\lambda_r - \lambda_v = \left(1.5\times10^{-3}\right)\left(\frac{0.3\times10^{-3}}{1.5}\right)$

$= 3 \times 10^{-7} = 300$ nm

Ans. 300.0

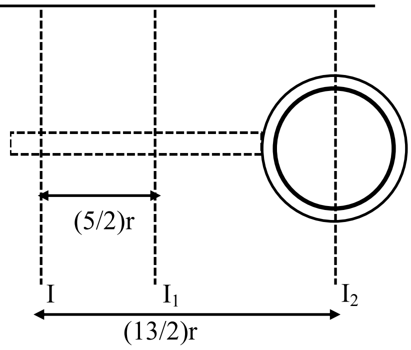
10. Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be .... Mr^2.

Official Ans. by NTA (52)

Sol.



$$I = \left[I_1 + M\left(\frac{5}{2}r\right)^2\right] + \left[I_2 + M\left(\frac{13r}{2}\right)^2\right]$$

$$= \left[\frac{M\left(36r^2\right)}{12} + \frac{M\left(25r^2\right)}{4}\right] + \left[\frac{Mr^2}{2} + \frac{169Mr^2}{4}\right]$$

$= 52\ Mr^2$

Ans. 52.00

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. Which one of the following complexes is violet in colour?

(1) $[Fe(CN)_6]^{4-}$ (2) $[Fe(SCN)_6]^{4-}$

(3) $Fe_4[Fe(CN_6)]_3 \cdot H_2O$ (4) $[Fe(CN)_5NOS]^{4-}$

Official Ans. by NTA (4)

Sol. (1) $[Fe(CN)_6]^{4-} \rightarrow$ Pale yellow solution

(2) $[Fe(SCN)_6]^{4-} \rightarrow$ Blood red colour

(3) $Fe_4[Fe(CN_6)]_3 \cdot H_2O \rightarrow$ Prussian blue

(4) $[Fe(CN)_5NOS]^{4-} \rightarrow$ Violet colour

2. Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface?

(1) $\Delta H > 0, \Delta S > 0$ (2) $\Delta H > 0, \Delta S < 0$

(3) $\Delta H < 0, \Delta S < 0$ (4) $\Delta H < 0, \Delta S > 0$

Official Ans. by NTA (3)

Sol. (i) Adsorption of gas at metal surface is an exothermic process so $\Delta H < 0$

(ii) As the adsorption of gas on metal surface reduces the free movement of gas molecules thus restricting its randomness hences $\Delta S < 0$

3. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere?

(1) $CuCl_2$ (2) AgCl

(3) $ZnCl_2$ (4) Cu_2Cl_2

Official Ans. by NTA (1)

Sol. (1) $CuCl_2 + nH_2O \rightarrow Cu^{+2}_{(aq.)}$

blue colour

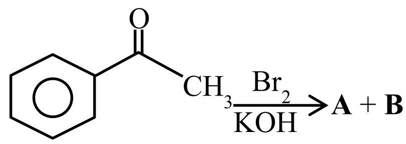
(2) $AgCl + nH_2O \rightarrow$ Insoluble

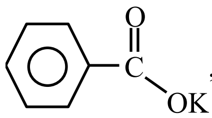
(3) $ZnCl_2 + nH_2O \rightarrow Zn^{+2}_{(aq.)}$

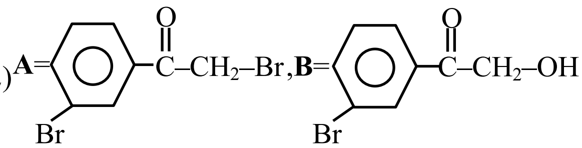
Colourless

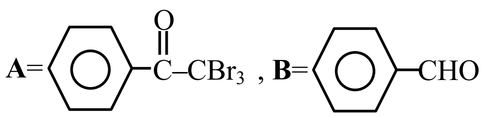
(4) $Cu_2Cl_2 + nH_2O \rightarrow$ Insoluble

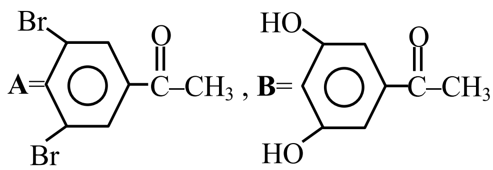
4. The major products formed in the following reaction sequence **A** and **B** are :



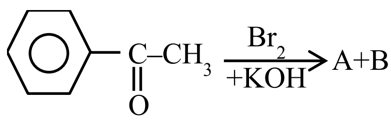
(1) **A** =  , **B**=$CHBr_3$

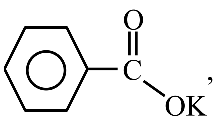
(2) 

(3) 

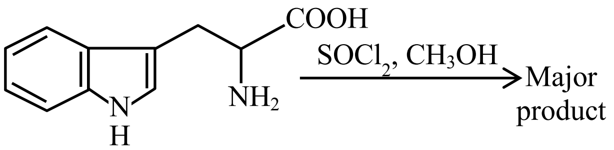
(4) 

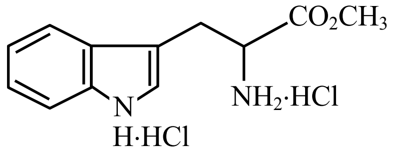
Official Ans. by NTA (1)

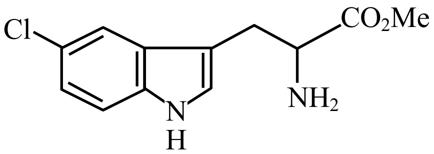
Sol. 

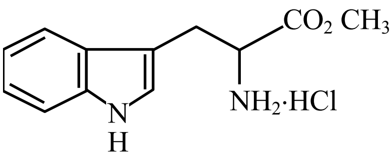
(1)  , B=$CHBr_3$

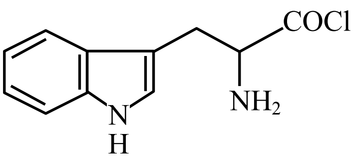
5. The major product formed in the following reaction is :



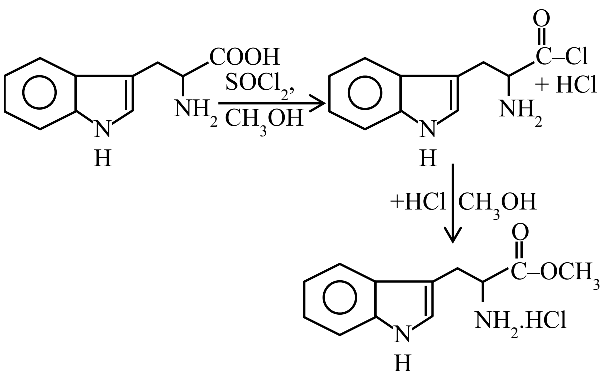
(1) 

(2) 

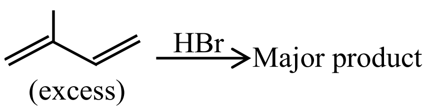
(3) 

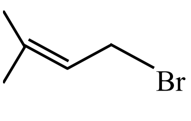
(4) 

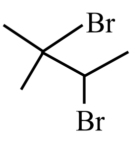
Official Ans. by NTA (3)

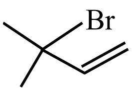
Sol. 

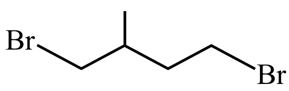
6. The major product formed in the following reaction is :



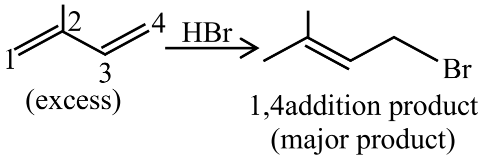
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

7. The polymer formed on heating Novolac with formaldehyde is :

(1) Bakelite (2) Polyester

(3) Melamine (4) Nylon 6,6

Official Ans. by NTA (1)

Sol. Novolac + formaldehyde $\rightarrow$ Bakelite

8. Given below are two statements :

Statement I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte).

Statement II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

(2) **Statement I** is false but **Statement II** is true.

(3) Both **Statement I** and **Statement II** are true.

(4) Both **Statement I** and **Statement II** are false.

Official Ans. by NTA (4)

Sol.

Ion	H^+	K^+	Cl^-	CH_3COO^-
$\Lambda^{\infty}_{m\ Scm^2/mole}$	349.8	73.5	76.3	40.9

So $\Lambda^{\infty}_{m\ CH_3COOH} = \Lambda^{\infty}_{m\ (H^+)} + \Lambda^{\infty}_{m\ CH_3COO^-}$

$= 349.8 + 40.9$

$= 390.7$ Scm^2/mole

$\Lambda^{\infty}_{m\ KCl} = \Lambda^{\infty}_{m\ (K^+)} + \Lambda^{\infty}_{m\ (Cl^-)}$

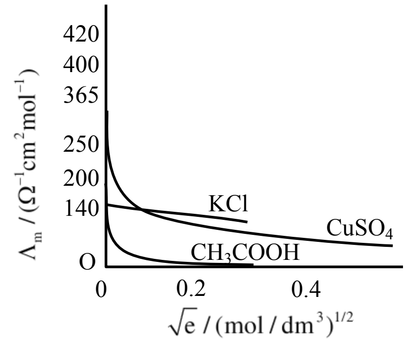
$= 73.5 + 76.3$

$= 149.3$ Scm^2/mole

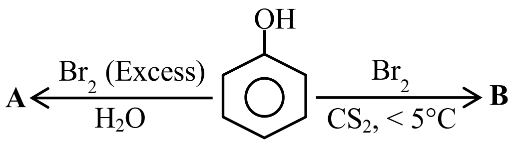
So statement-I is wrong or False.

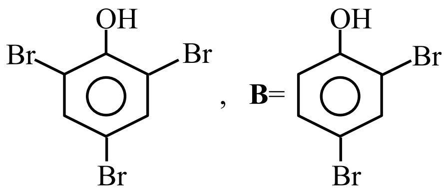
As the concentration decreases, the dilution increases which increases the degree of dissociation, thus increasing the no. of ions, which increases the molar conductance.

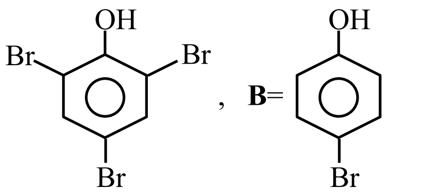
So statement-II is false.

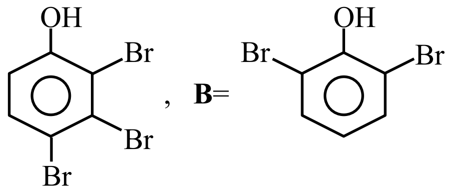


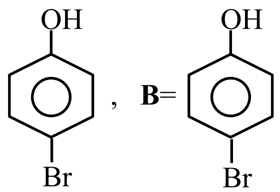
9. The correct options for the products **A** and **B** of the following reactions are :



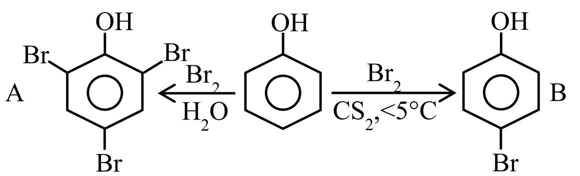
(1) A= , B=


(2) A= , B=


(3) A= , B=


(4) A= , B=


Official Ans. by NTA (2)

Sol.


10. The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is:

(1) $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$

(2) $[3Ca(OH)_2 \cdot CaF_2]$

(3) $[Ca_3(PO_4)_2 \cdot CaF_2]$

(4) $[3Ca_3(PO_4)_2 \cdot CaF_2]$

Official Ans. by NTA (1)

Sol. The $F^\ominus$ ions make the enamel on teeth much harder by converting hydroxyapatite,$[3(Ca_3(PO_4)_2].Ca(OH)_2]$, the enamel on the surface of the teeth into much harder fluroappatite. $[3Ca_3(PO_4)_2.CaF_2]$

11. Given below are two statements.

Statement I: In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II: For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is false but **Statement II** is true

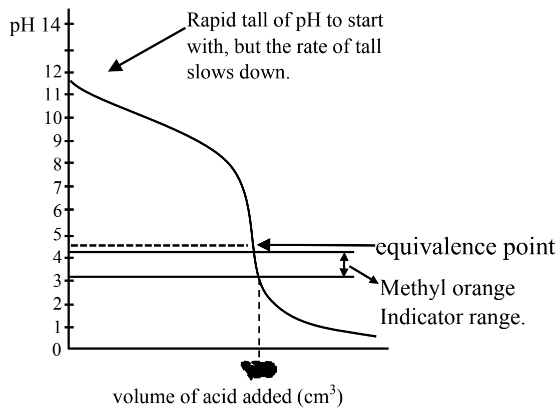
(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are true

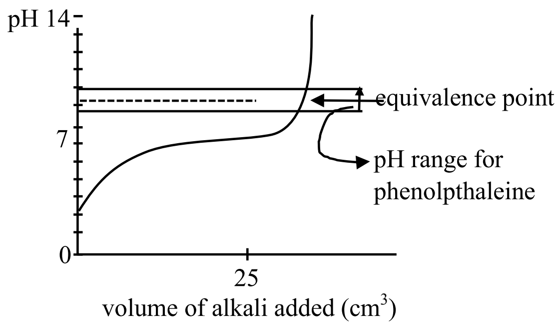
(4) Both **Statement I** and **Statement II** are false

Official Ans. by NTA (2)

Sol. Titration curve for strong acid and weak base initially a buffer of weak base and conjugate acid is :



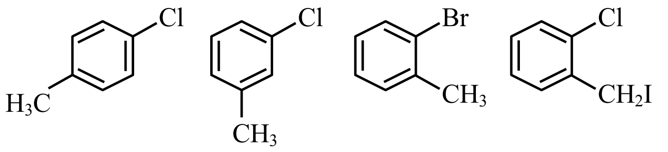
Formed, thus pH falls slowly and after equivalence point, so the pH falls sharply so methyl arrange, having pH range of 3.2 to 4.4 will weak as indicator. So statement-I is correct.



Titration curve for weak acid and strong base (NaOH)

Initially weak acid will form a buffer so pH increases slowly but after equivalence point. it rises sharply covering range of phenolphthalein so it will be suitable indicator so statement-II is false.

12. Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO_3 (iii) $AgNO_3$?



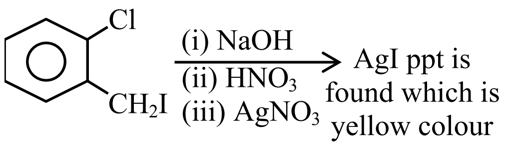
I II III IV

(1) II (2) IV

(3) I (4) III

Official Ans. by NTA (2)

Sol.



Other compounds halide can't be removed because corresponding C^+ is highly unstable.

13. Which one of the following methods is most suitable for preparing deionized water?

(1) Synthetic resin method

(2) Clark's method

(3) Calgon's method

(4) Permutit method

Official Ans. by NTA (1)

Sol. Pure demineralised (de-ionized) water free from all soluble mineral salts is obtained by passing water successively through a cation exchange (in the H^+ form) and an anion exchange (in the OH^- form) resins.

14. Given below are two statements.

Statement I: The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement II: The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true but **Statement II** is false

Official Ans. by NTA (4)

Sol. Given statement-I is true as in a number of processes, one element is used to reduce the oxide of another metal. Any element will reduce the oxide of other metal which lie above it in the Ellingham diagram because the free energy change will become more negative.

Given statement-II is false as the value of ΔS is decreases from left to right in Ellingham diagram.

15. What are the products formed in sequence when excess of CO_2 is passed in slaked lime?

(1) $Ca(HCO_3)_2$, $CaCO_3$

(2) $CaCO_3$, $Ca(HCO_3)_2$

(3) CaO, $Ca(HCO_3)_2$

(4) CaO, $CaCO_3$

Official Ans. by NTA (2)

Sol. $Ca(OH)_2 + CO_2 \longrightarrow CaCO_3\downarrow + H_2O$

$CaCO_3\downarrow + CO_2 + H_2O \rightarrow Ca(HCO_3)_2$

16. Given below are two statements.

Statement I: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is false but **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

Official Ans. by NTA (3)

Sol. Velocity of electron in Bohr's atom is given by

$$V \propto \frac{Z}{n}$$

Z = atomic number of atom, corresponds to +ve charge so as Z increase velocity increases so statement-I is wrong.

and as 'n' decreases velocity increases so statement-II is correct.

17. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is:

(1) $Br_2/AlBr_3$, HNO_3/H_2SO_4, Mg/ether, CO_2, H_3O^+

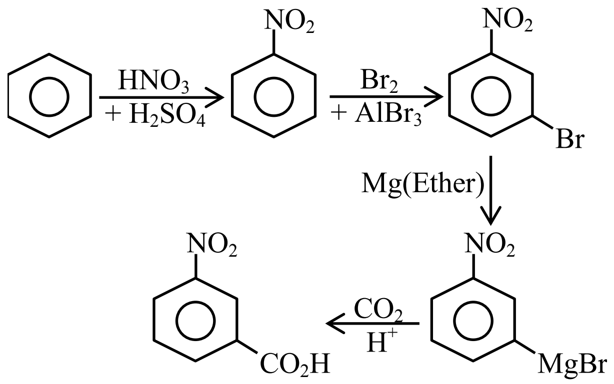
(2) $Br_2/AlBr_3$, NaCN, H_3O^+, HNO_3/H_2SO_4

(3) $Br_2/AlBr_3$, HNO_3/H_2SO_4, NaCN, H_3O^+

(4) HNO_3/H_2SO_4, $Br_2/AlBr_3$, Mg/ether, CO_2, H_3O^+

Official Ans. by NTA (4)

Sol.



3–nitrobenzoic acid

18. Given below are two statements.

Statement I: Frenkel defects are vacancy as well as interstitial defects.

Statement II: Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the **most appropriate** answer for the statements from the options given below:

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are false

Official Ans. by NTA (3)

Sol. Theory based.

19. The **incorrect** statement is:

(1) Cl_2 is more reactive than ClF.

(2) F_2 is more reactive than ClF.

(3) On hydrolysis ClF froms HOCl and HF.

(4) F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution

Official Ans. by NTA (1)

Sol. (i) Reactivity order :

$F_2 >$ ClF (inter halogen) $> Cl_2$

(ii) $ClF + H_2O \rightarrow HOCl + HF$

(iii) Oxidizing power in aqueous solution

$F_2 > Cl_2 > Br_2 > I_2$

20. Excess of isobutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product?

(1) $CH_3-\underset{CH_3}{\overset{Br}{C}}-CH_2-Br$

(2) $CH_3-\underset{CH_2Br}{CH}-CH_2Br$

(3) $CH_3-\underset{CH_3}{CH}-CH_2Br$

(4) $CH_3-\underset{CH_3}{\overset{CH_3}{C}}-Br$

Official Ans. by NTA (4)

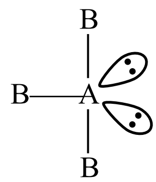
Sol. $\underset{\text{(excess isobutane)}}{CH_3-\overset{CH_3}{CH}-CH_3} \xrightarrow[h\nu/\Delta]{Br_2} CH_3-\underset{Br}{\overset{CH_3}{C}}-CH_3$

SECTION-B

1. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _______. (Integer answer)

Official Ans. by NTA (2)

Sol. T-shaped molecule means 3 sigma bond and 2 lone pairs of electron on central atom.



2. These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is _________ (Integer answer)

Official Ans. by NTA (3)

Sol. Sublimation enthalpy, Ionisation enthalpy and hydration enthalpy affect the reduction potential.

3. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of 0.10 M C_2H_5OH is ________ (Integer answer)

(i) 0.10 M $Ba_3(PO_4)_2$

(ii) 0.10 M Na_2SO_4

(iii) 0.10 M KCl

(iv) 0.10 M Li_3PO_4

Official Ans. by NTA (4)

Sol. As 0.1 M C_2H_5OH is non-dissociative and rest all salt given are electrolyte so in each case effective molarity > 0.1 so each will have lower freezing point.

4. The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is x × 10^{-6} M. The value of x is _______. (Nearest integer)

[Given $K_w = 1 \times 10^{-14}$ and $K_b = 1.8 \times 10^{-5}$]

Official Ans. by NTA (3)

Sol. $\left[NH_4^+\right] = 0.0504 \;\&\; \left[NH_3\right] = 0.0210$

So $K_b = \dfrac{[NH_4^+][HO^-]}{[NH_3]}$

$[HO^-] = \dfrac{K_b \times [NH_3]}{[NH_4^+]} = 1.8 \times 10^{-5} \times \dfrac{2}{5} \times \dfrac{210}{504}$

$= 3 \times 10^{-6}$

5. The number of 4*f* electrons in the ground state electronic configuration of Gd^{2+} is ________.

[Atomic number of Gd = 64]

Official Ans. by NTA (7)

Sol. The electronic configuration of

$_{64}Gd$: [Xe] $4f^7\ 5d^1\ 6s^2$

So the electronic configuration of

$_{64}Gd^{2+}$: [Xe] $4f^7\ 5d^1\ 6s^0$

i.e. the number of 4f electrons in the ground state electronic configuration of Gd^{2+} is 7.

6. The ratio of number of water molecules in Mohr's salt and potash alum is ________ × 10^{-1}. (Integer answer)

Official Ans. by NTA (5)

Sol. **(5)**

Mohr's salt : $(NH_4)_2\ Fe(SO_4)_2.6H_2O$

The number of water molecules in Mohr's salt = 6

Potash alum : $KAl(SO_4)_2.12H_2O$

The number of water molecules in potash alum = 12

So ratio of number of water molecules in Mohr's salt and potash alum = $\dfrac{6}{12}$

$= \dfrac{1}{2}$

$= 0.5$

$= 5 \times 10^{-1}$

7. The following data was obtained for chemical reaction given below at 975 K.

$$2NO_{(g)} + 2H_{2(g)} \rightarrow N_{2(g)} + 2H_2O_{(g)}$$

	[NO] mol L^{-1}	[H_2] mol L^{-1}	Rate mol $L^{-1}s^{-1}$
(A)	8×10^{-5}	8×10^{-5}	7×10^{-9}
(B)	24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C)	24×10^{-5}	32×10^{-5}	8.4×10^{-8}

The order of the reaction with respect to NO is _______. [Integer answer]

Official Ans. by NTA (1)

Sol. $7 \times 10^{-9} = K \times (8 \times 10^{-5})^x\ (8 \times 10^{-5})^y$ …….(1)

$2.1 \times 10^{-8} = K \times (24 \times 10^{-5})^x\ (8 \times 10^{-5})^y$ …….(2)

$\dfrac{1}{3} = \left(\dfrac{1}{3}\right)^x \Rightarrow x = 1$

8. The Born-Haber cycle for KCl is evaluated with the following data :

$\Delta_f H$ for $KCl = -436.7\,kJ\,mol^{-1}$;

$\Delta_{sub}H$ for K = 89.2 kJ mol^{-1};

$\Delta_{ionization}\ H^\ominus$ for K = 419.0 kJ mol^{-1}; $\Delta_{electron\ gain}\ H^\ominus$ for $Cl_{(g)}$ = –348.6 kJ mol^{-1}; $\Delta_{bond}\ H^\ominus$ for Cl_2 = 243.0 kJ mol^{-1}

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is _______ (Nearest integer)

Official Ans. by NTA (718)

Sol. $\Delta_f H_{KCl} = \Delta_{sub}H_{(K)} + \Delta_{ionization}H_{(K)} + \dfrac{1}{2}\Delta_{bond}H_{(Cl_2)}$

$+ \Delta_{\text{electron gain}}H_{(Cl)} + \Delta_{\text{lattice}}H_{(KCl)}$

$$\Rightarrow -436.7 = 89.2 + 419.0 + \frac{1}{2}(243.0) + \{-348.6\}$$

$$+ \Delta_{\text{lattice}}H_{(KCl)}$$

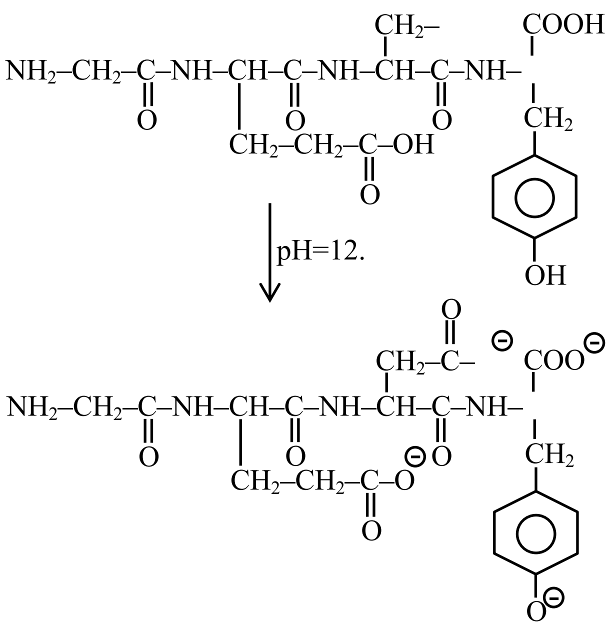
$$\Rightarrow \Delta_{\text{lattice}}H_{(KCl)} = -717.8 \text{ kJ mol}^{-1}$$

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is 718 (Nearest integer).

9. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be ________. (Integer answer)

Official Ans. by NTA (4)

Sol.



Total negative charge produced = 4.

10. An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1}. The molarity of the solution in mol L^{-1} is ________ (Nearest integer)

[Molar mass of KCl = 74.5]

Official Ans. by NTA (3)

Sol. 1000 kg solvent has 3.3 moles of KCl

1000 kg solvent $\longrightarrow$ 3.3 × 74.5 gm KCl

$\longrightarrow$ 245.85

Weight of solution = 1245.85 gm

Volume of solution = $\frac{1245.85}{1.2}$ ml

So molarity = $\frac{3.3 \times 1.2}{1245.85} \times 1000 = 3.17$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$

(3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

Official Ans. by NTA (1)

Sol. $\frac{\cos x}{1+\sin x} = |\tan 2x|$

$$\Rightarrow \frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$$

$$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$$

or sum $= \frac{-11\pi}{6}$.

2. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

(1) (11, 26) (2) (10.5, 25)

(3) (11, 25) (4) (10.5, 26)

Official Ans. by NTA (4)

Sol. Given :

Mean $(\bar{x}) = \frac{\Sigma x_i}{20} = 10$

or $\Sigma x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \Sigma x_i$ (Correct)

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given S.D = 2.5 (σ)

$$\sigma^2 = \frac{\Sigma x_i^2}{20} - (10)^2 = (2.5)^2$$

or $\Sigma x_i^2 = 2125$ (incorrect)

or $\Sigma x_i^2 = 2125 - 25^2 + 35^2$

$= 2725$ (Correct)

$$\therefore \text{ correct } \sigma^2 = \frac{2725}{20} - (10.5)^2$$

$\underline{\sigma}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

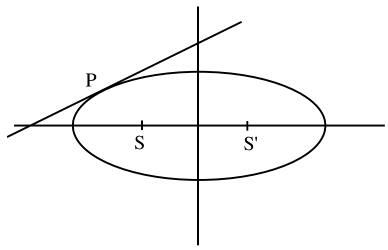
3. On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2)$. A is :

(1) 6 (2) 12

(3) 14 (4) 24

Official Ans. by NTA (1)

Sol.



Equation of tangent : $y = 2x + 6$

at P

$\therefore$ P(–8/3, 2/3)

$e = \frac{1}{\sqrt{2}}$

S & S' = (–2, 0) & (2, 0)

Area of Δ SPS' = $\frac{1}{2}\times 4\times\frac{2}{3}$

A = $\frac{4}{3}$

$\therefore$ $(5 - e^2)A = (5 - \frac{1}{2})\frac{4}{3} = 6$

4. Let y = y(x) be a solution curve of the differential equation $(y + 1)\tan^2x\, dx + \tan x\, dy + y\, dx = 0$, $x\in\left(0,\frac{\pi}{2}\right)$. If $\lim\limits_{x\to 0+} xy(x)=1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{4}-1$

(3) $\frac{\pi}{4}+1$ (4) $\frac{\pi}{4}$

Official Ans. by NTA (4)

Sol. $(y + 1)\tan^2x\, dx + \tan x\, dy + y\, dx = 0$

or $\frac{dy}{dx}+\frac{\sec^2 x}{\tan x}.y=-\tan x$

IF = $e^{\int\frac{\sec^2 x}{\tan x}dx}=e^{\ln\tan x}=\tan x$

$\therefore$ $y\tan x = -\int \tan^2 x dx$

or $y \tan x = -\tan x + x + C$

or $y = -1 + \frac{x}{\tan x}+\frac{C}{\tan x}$

or $\lim\limits_{x\to 0} xy=-x+\frac{x^2}{\tan x}+\frac{Cx}{\tan x}=1$

or C = 1

$y(x) = \cot x + x\cot x - 1$

$y\left(\frac{\pi}{4}\right)=\frac{\pi}{4}$

5. Let A and B be independent events such that P(A) = p, P(B) = 2p. The largest value of p, for which P (exactly one of A, B occurs) = $\frac{5}{9}$, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{5}{12}$

Official Ans. by NTA (4)

Sol. P(Exactly one of A or B)

$= P\left(A\cap\bar{B}\right)+P\left(\bar{A}\cap B\right)=\frac{5}{9}$

$= P(A)P(\bar{B})+P(\bar{A})P(B)=\frac{5}{9}$

$\Rightarrow P(A)(1-P(B)) + (1-P(A))P(B) = \frac{5}{9}$

$\Rightarrow p(1 - 2p) + (1 - p)\, 2p = \frac{5}{9}$

$\Rightarrow 36p^2 - 27p + 5 = 0$

$\Rightarrow p = \frac{1}{3}$ or $\frac{5}{12}$

$p_{max} = \frac{5}{12}$

6. Let $\theta\in\left(0,\frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$

$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4\sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4\sin 3\theta)\, z = 0$

has a non–trivial solution, then the value of θ is :

(1) $\frac{4\pi}{9}$ (2) $\frac{7\pi}{18}$ (3) $\frac{\pi}{18}$ (4) $\frac{5\pi}{18}$

Official Ans. by NTA (2)

Sol. Case–I

$$\begin{vmatrix}1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta\\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta\\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta\end{vmatrix}=0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix}2 & \sin^2\theta & 4\sin 3\theta\\ 2 & 1+\sin^2\theta & 4\sin 3\theta\\ 1 & \sin^2\theta & 1+4\sin 3\theta\end{vmatrix}=0$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix}0 & -1 & 0\\ 1 & 1 & -1\\ 1 & \sin^2\theta & 1+4\sin^3\theta\end{vmatrix}=0$$

or $4\sin 3\theta = -2$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

7. Let $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$,

$0 < x < 1$. Then :

(1) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$

(2) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$

(3) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

(4) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

Official Ans. by NTA (3)

Sol. $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$

$\cot^{-1}\sqrt{\frac{1-x}{x}} = \sin^{-1}\sqrt{x}$

or $f(x) = \cos(2\tan^{-1}\sqrt{x})$

$= \cos\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$

$f(x) = \frac{1-x}{1+x}$

Now $f'(x) = \frac{-2}{(1+x)^2}$

or $f'(x)(1-x)^2 = -2\left(\frac{1-x}{1+x}\right)^2$

or $(1-x)^2 f'(x) + 2(f(x))^2 = 0.$

8. The sum of the series

$\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+.....+\frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is :

(1) $1+\frac{2^{101}}{4^{101}-1}$

(2) $1+\frac{2^{100}}{4^{101}-1}$

(3) $1-\frac{2^{100}}{4^{100}-1}$

(4) $1-\frac{2^{101}}{4^{101}-1}$

Official Ans. by NTA (4)

Allen Ans. (BONUS)

Sol. $S = \frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+....\frac{2^{100}}{x^{2^{100}}+1}$

$S+\frac{1}{1-x} = \frac{1}{1-x}+\frac{1}{x+1}+.... = \frac{2}{1-x^2}+\frac{2}{1+x^2}+...$

$S+\frac{1}{1-x} = \frac{2^{101}}{1-x^{2^{101}}}$

Put $x = 2$

$S = 1-\frac{2^{101}}{2^{2^{101}}-1}$

Not in option (BONUS)

9. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\,{}^{20}C_r$ is equal to :

(1) 420×2^{19}

(2) 380×2^{19}

(3) 380×2^{18}

(4) 420×2^{18}

Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{20} r^2.{}^{20}C_r$

$\sum(4(r-1)+r).{}^{20}C_r$

$\sum r(r-1).\frac{20\times19}{r(r-1)}.{}^{18}C_r + r.\frac{20}{r}.\sum{}^{19}C_{r-1}$

$\Rightarrow 20\times 19.2^{18} + 20.2^{19}$

$\Rightarrow 420\times 2^{18}$

10. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

(1) {80, 83, 86, 89}

(2) {84, 86, 88, 90}

(3) {79, 81, 83, 85}

(4) {84, 87, 90, 93}

Official Ans. by NTA (3)

Sol. $n(A\cup B) \ge n(A) + n(B) - n(A\cap B)$

$100 \ge 89 + 98 - n(A\cup B)$

$n(A\cup B) \ge 87$

$87 \le n(A\cup B) \le 89$

Option (3)

11. The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with:

(1) centre at (0, –1) and radius $\sqrt{2}$

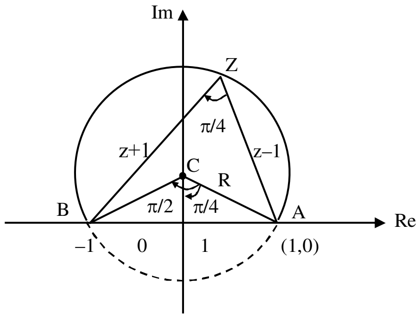
(2) centre at (0, 1) and radius $\sqrt{2}$

(3) centre at (0,0) and radius $\sqrt{2}$

(4) centre at (0,1) and radius 2

Official Ans. by NTA (2)

Sol.



In ΔOAC

$$\sin\left(\frac{\pi}{4}\right)=\frac{1}{AC}$$

$$\Rightarrow AC=\sqrt{2}$$

Also, $\tan\frac{\pi}{4}=\frac{OA}{OC}=\frac{1}{OC}$

$\Rightarrow OC = 1$

$\therefore$ centre (0, 1) ; Radius = $\sqrt{2}$

12. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$, then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to :

(1) –2 (2) –6 (3) 6 (4) 2

Official Ans. by NTA (1)

Sol. $|\vec{a}|=\sqrt{3};\ \vec{a}.\vec{c}=3;\ \vec{a}\times\vec{b}=-2\hat{i}+\hat{j}+\hat{k}$, $\vec{a}\times\vec{c}=\vec{b}$

Cross with $\vec{a}$.

$$\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$$

$$\Rightarrow (\vec{a}.\vec{c})\vec{a}-a^2\vec{c}=\vec{a}\times\vec{b}$$

$$\Rightarrow 3\vec{a}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$$

$$\Rightarrow 3\hat{i}+3\hat{j}+3\hat{k}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$$

$$\Rightarrow \vec{c}=\frac{5\hat{i}}{3}+\frac{2\hat{j}}{3}+\frac{2\hat{k}}{3}$$

$$\therefore \vec{a}.(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b}).\vec{c}=\frac{-10}{3}+\frac{2}{3}+\frac{2}{3}=-2$$

13. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point (–30, 0) and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :

(1) 5 (2) 7 (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

Official Ans. by NTA (4)

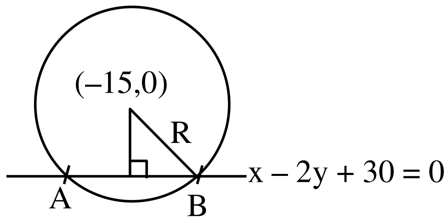
Sol. Equation of tangent to $y^2 = 30\,x$

$$y=mx+\frac{30}{4m}$$

Pass thru (–30, 0) : $a=-30m+\frac{30}{4m}\ \Rightarrow m^2=1/4$

$$\Rightarrow m=\frac{1}{2}\ \text{or}\ m=-\frac{1}{2}$$

At $m=\frac{1}{2}$: $y=\frac{x}{2}+15\ \Rightarrow x-2y+30=0$



$P=\frac{15}{\sqrt{5}}$

$$\ell_{AB}=2\sqrt{R^2-P^2}=2\sqrt{\frac{225}{4}-\frac{225}{5}}$$

$$\Rightarrow \ell_{AB}=30.\sqrt{\frac{1}{20}}=\frac{15}{\sqrt{5}}=3\sqrt{5}$$

14. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{1/2}dx$ is:

(1) $\log_e 4$ (2) $\log_e 16$

(3) $2\log_e 16$ (4) $4\log_e(3+2\sqrt{2})$

Official Ans. by NTA (2)

Sol. $$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}-\frac{x-1}{x+1}\right)^2\right)^{1/2}dx$$

$$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left|\frac{4x}{x^2-1}\right|dx\ \Rightarrow I=2.4\int_0^{1/\sqrt{2}}\left|\frac{x}{x^2-1}\right|dx$$

$$\Rightarrow I=-4\int_0^{1/\sqrt{2}}\frac{2x}{x^2-1}dx\ \Rightarrow I=-4\ln\left|x^2-1\right|_0^{1/\sqrt{2}}$$

$\Rightarrow I = 4 \ln 2 \Rightarrow I = \ln 16$

15. A plane P contains the line

$x + 2y + 3z + 1 = 0 = x - y - z - 6$,

and is perpendicular to the plane $-2x + y + z + 8 = 0$.

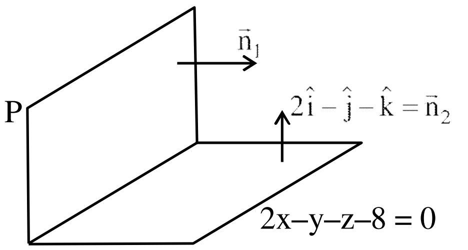
Then which of the following points lies on P ?

(1) (–1, 1, 2) (2) (0, 1, 1)

(3) (1, 0, 1) (4) (2, –1, 1)

Official Ans. by NTA (2)

Sol. Equation of plane P can be assumed as



$P : x + 2y + 3z + 1 + \lambda\,(x - y - z - 6) = 0$

$\Rightarrow P : (1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + 1 - 6\lambda = 0$

$\Rightarrow \vec{n}_1 = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (3-\lambda)\hat{k}$

$\therefore \vec{n}_1.\vec{n}_2 = 0$

$\Rightarrow 2(1 + \lambda) - (2 - \lambda) - (3 - \lambda) = 0$

$\Rightarrow 2 + 2\lambda - 2 + \lambda - 3 + \lambda = 0 \Rightarrow \lambda = \frac{3}{4}$

$\Rightarrow P : \frac{7x}{4} + \frac{5}{4}y + \frac{9z}{4} - \frac{14}{4} = 0$

$\Rightarrow 7x + 5y + 9z = 14$

(0, 1, 1) lies on P

16. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and

$Q = A^TBA$, then the inverse of the matrix $A\,Q^{2021}\,A^T$ is equal to :

(1) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Official Ans. by NTA (2)

Sol. $AA^T = \begin{pmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$Q^2 = A^TBA\,A^TBA = A^TBIBA$

$\Rightarrow Q^2 = A^T\,B^2A$

$Q^3 = A^TB^2AA^TBA \Rightarrow Q^3 = A^TB^3A$

Similarly : $Q^{2021} = A^T B^{2021}\;A$ ...... (1)

Now $B^2 = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}$

$B^3 = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} \Rightarrow B^3 = \begin{pmatrix} 1 & 0 \\ 3i & 1 \end{pmatrix}$

Similarly $B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore AQ^{2021}\,A^T = AA^T\,B^{2021}\,AA^T = IB^{2021}I$

$\Rightarrow AQ^{2021}\,A^T = B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore (AQ^{2021}\,A^T)^{-1} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

17. If the sum of an infinite GP a, ar, ar^2, ar^3,... is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, ... is :

(1) $\frac{5}{2}$ (2) $\frac{1}{2}$

(3) $\frac{25}{2}$ (4) $\frac{9}{2}$

Official Ans. by NTA (2)

Sol. Sum of infinite terms :

$\frac{a}{1-r} = 15$ ....(i)

Series formed by square of terms:

a^2, a^2r^2, a^2r^4, a^2r^6 .....

Sum $= \frac{a^2}{1-r^2} = 150$

$\Rightarrow \frac{a}{1-r}.\frac{a}{1+r} = 150 \Rightarrow 15.\frac{a}{1+r} = 150$

$\Rightarrow \frac{a}{1+r} = 10$ .... (ii) by (i) and (ii) a = 12 ; $r=\frac{1}{5}$

Now series : ar^2, ar^4, ar^6

$$\text{Sum}=\frac{ar^2}{1-r^2}=\frac{12.\left(\frac{1}{25}\right)}{1-\frac{1}{25}}=\frac{1}{2}$$

18. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is:

(1) $\frac{1}{2}\tan^{-1}(2)$ (2) $\frac{1}{2}\tan^{-1}(4)$

(3) $\tan^{-1}(4)$ (4) $\frac{1}{4}\tan^{-1}(4)$

Official Ans. by NTA (2)

Sol. $L=\lim\limits_{n\to\infty}\frac{1}{n}.\sum\limits_{r=0}^{2n-1}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$$\Rightarrow L=\int_0^2\frac{1}{1+4x^2}dx$$

$$\Rightarrow L=\frac{1}{2}\tan^{-1}(2x)\Big|_0^2 \Rightarrow L=\frac{1}{2}\tan^{-1}4$$

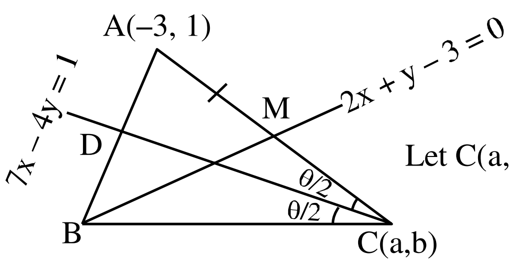
19. Let ABC be a triangle with A(–3, 1) and $\angle ACB=\theta,\ 0<\theta<\frac{\pi}{2}$. If the equation of the median through B is 2x + y – 3 = 0 and the equation of angle bisector of C is 7x – 4y – 1 = 0, then tanθ is equal to:

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) 2

Official Ans. by NTA (3)

Sol.



Let C(a, b)

$\therefore$ $M\left(\frac{a-3}{2},\frac{b+1}{2}\right)$ lies on 2x + y – 3 = 0

$\Rightarrow$ 2a + b = 11 .... (i)

$\because$ C lies on 7x – 4y = 1

$\Rightarrow$ 7a – 4b = 1 ......(ii)

$\therefore$ by (i) and (ii) : a = 3, b = 5

$\Rightarrow$ C(3, 5)

$\therefore$ $m_{AC}=2/3$

Also, $m_{CD}=7/4$

$$\Rightarrow \tan\frac{\theta}{2}=\left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right| \Rightarrow \tan\frac{\theta}{2}=\frac{1}{2}$$

$$\Rightarrow \tan\theta=\frac{2.\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$$

20. If the truth value of the Boolean expression $((p\vee q)\wedge(q\to r)\wedge(\sim r))\to(p\wedge q)$ is false, then the truth values of the statements p, q, r respectively can be:

(1) T F T (2) F F T

(3) T F F (4) F T F

Official Ans. by NTA (3)

Sol.

p	q	r	$\underbrace{p\vee q}_{a}$	$\underbrace{q\to r}_{b}$	$a\wedge b$	$\sim r$	$\underbrace{a\wedge b\wedge(\sim r)}_{c}$	$\underbrace{p\wedge q}_{d}$	$c\to d$
T	F	T	T	T	T	F	F	F	T
F	F	T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F	F	T

SECTION-B

1. Let $z=\frac{1-i\sqrt{3}}{2}$, $i=\sqrt{-1}$. Then the value of

$$21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+...+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$

is ________.

Official Ans. by NTA (13)

Sol. $Z = \dfrac{1-\sqrt{3}i}{2} = e^{-i\frac{\pi}{3}}$

$$z^r + \frac{1}{z^r} = 2\cos\left(-\frac{\pi}{3}\right)r = 2\cos\frac{r\pi}{3}$$

$$\Rightarrow\ 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3 = 8\left(\cos^3\frac{r\pi}{3}\right) = 2\left(\cos r\pi + 3\cos\frac{r\pi}{3}\right)$$

$$\Rightarrow\ 21 + \left(z + \frac{1}{2}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \ldots\ldots\left(z^{21} + \frac{1}{z^{21}}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(2\cos r\pi + 6\cos\frac{r\pi}{3}\right)$$

$= 21 - 2 - 6$

$= 13$

2. The sum of all integral values of k ($k \neq 0$) for which the equation $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$ in x has no real roots, is ________.

Official Ans. by NTA (66)

Sol. $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k}$

$x \in R - \{1, 2\}$

$\Rightarrow\ k(2x - 4 - x + 1) = 2\,(x^2 - 3x + 2)$

$\Rightarrow\ k(x - 3) = 2(x^2 - 3x + 2)$

for $x \neq 3$, $\quad k = 2\left(x - 3 + \dfrac{2}{x-3} + 3\right)$

$$x - 3 + \frac{2}{x-3} \geq 2\sqrt{2},\ \forall x > 3$$

$$\&\ x - 3 + \frac{2}{x-3} \leq -2\sqrt{2},\ \forall x < -3$$

$$\Rightarrow 2\left(x - 3 + \frac{2}{x-3} + 3\right) \in \left(-\infty, 6 - 4\sqrt{2}\right] \cup \left[6 + 4\sqrt{2}, \infty\right)$$

for no real roots

$k \in (6 - 4\sqrt{2}, 6 + 4\sqrt{2}) - \{0\}$

Integral $\ k \in \{1, 2\ldots..\ 11\}$

Sum of k = 66

3. Let the line L be the projection of the line

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$$

in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to ________.

Official Ans. by NTA (26)

Sol. $L_1 : \dfrac{x-1}{2} = \dfrac{y-3}{1} = \dfrac{z-4}{2}$

for foot of $\perp r$ of (1, 3, 4) on $x - 2y - z - 3 = 0$

$(1 + t) - 2(3 - 2t) - (4 - t) - 3 = 0$

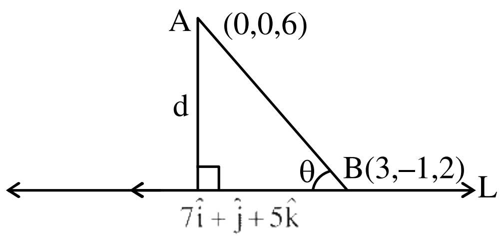
$\Rightarrow\ t = 2$

So foot of $\perp r \triangleq (3, -1, 2)$

& point of intersection of L_1 with plane is $(-11, -3, -8)$

dr's of L is <14, 2, 10>

$\cong$ <7, 1, 5>



$$d = AB\sin\theta = \left|\frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ 7 & 1 & 5 \end{vmatrix}}{\sqrt{7^2 + 1^2 + 5^2}}\right|$$

$$\Rightarrow\ d^2 = \frac{1^2 + (43)^2 + (10)^2}{49 + 1 + 25} = 26$$

4. If ${}^1P_1 + 2\cdot{}^2P_2 + 3\cdot{}^3P_3 + \ldots + 15\cdot{}^{15}P_{15} = {}^qP_r - s,\ 0 \leq s \leq 1$, then ${}^{q+s}C_{r-s}$ is equal to ________.

Official Ans. by NTA (136)

Sol. $^{1}P_1 + 2\cdot{}^{2}P_2 + 3\cdot{}^{3}P_3 + \ldots + 15\cdot{}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + \ldots.15 \times 15!$

$= \sum_{r=1}^{15}(r+1-1)r!$

$= \sum_{r=1}^{15}(r+1)!-(r)!$

$= 16! - 1$

$= {}^{16}P_{16} - 1$

$\Rightarrow q = r = 16, s = 1$

${}^{q+s}C_{r-s} = {}^{17}C_{15} = 136$

5. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to _________.

Official Ans. by NTA (36)

Sol. Let x + y = 36

x is perimeter of square and y is perimeter of circle

side of square = x/4

radius of circle = $\frac{y}{2\pi}$

Sum Areas = $\left(\frac{x}{4}\right)^2 + \pi\left(\frac{y}{2\pi}\right)^2$

$= \frac{x^2}{16} + \frac{(36-x)^2}{4\pi}$

For min Area :

$x = \frac{144}{\pi+4}$

$\Rightarrow$ Radius = $y = 36 - \frac{144}{\pi+4}$

$\Rightarrow k = \frac{36\pi}{\pi+4}$

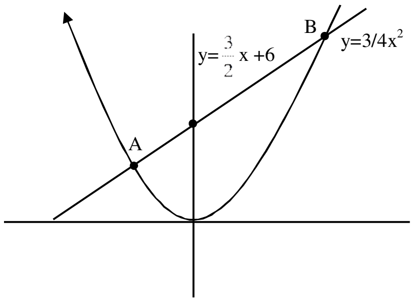
$\left(\frac{4}{\pi}+1\right)k = 36$

6. The area of the region

$S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is __________.

Official Ans. by NTA (27)

Sol.



For A & B

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

Area = $\int_{-2}^{4}\left(\frac{3}{2}x + 6 - \frac{3}{4}x^2\right)dx$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = 27$

7. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1) (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to ________.

Official Ans. by NTA (16)

Sol. Let P(x, y)

$x^2 + y^2 + x^2 + (y-1)^2 + (x-1)^2 + y^2 + (x-1)^2 + (y-1)^2;$

$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$

$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$

$d = 2\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{7}{2}}$

$\Rightarrow d^2 = 16$

8. If y = y(x) is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to _______.

Official Ans. by NTA (40)

Sol. $\ln(x + y) = 4xy$ (At $x = 0, y = 1$)

$x + y = e^{4xy}$

$$\Rightarrow 1+\frac{dy}{dx}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)$$

At $x = 0$ $\boxed{\dfrac{dy}{dx}=3}$

$$\frac{d^2y}{dx^2}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)^2+e^{4xy}\left(4x\frac{d^2y}{dx^2}+4y\right)$$

At $x = 0$, $\dfrac{d^2y}{dx^2} = e^0(4)^2 + e^0(24)$

$$\Rightarrow \frac{d^2y}{dx^2}=40$$

9. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

Official Ans. by NTA (52)

Sol. (i) When '0' is at unit place

$\boxed{\ \ }\boxed{\ \ }\boxed{0}$

5 4

Number of numbers = 20

(ii) When 4 or 6 are at unit place

$\boxed{\text{OX}}\boxed{\ \ }\boxed{4,6}$

4 × 4 2

Number of numbers = 32

So number of numbers = 52

10. Let $a, b \in \mathbf{R}$, $b \neq 0$, Define a function

$$f(x)=\begin{cases} a\sin\dfrac{\pi}{2}(x-1), & \text{for } x\le 0 \\ \dfrac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0. \end{cases}$$

If f is continuous at $x = 0$, then $10 - ab$ is equal to ________.

Official Ans. by NTA (14)

Sol. $$f(x)=\begin{cases} a\sin\dfrac{\pi}{2}(x-1), & x\le 0 \\ \dfrac{\tan 2x-\sin 2x}{bx^3}, & x>0 \end{cases}$$

For continuity at '0'

$$\lim_{x\to 0^+} f(x)=f(0)$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\tan 2x-\sin 2x}{bx^3}=-a$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\frac{8x^3}{3}+\frac{8x^3}{3!}}{bx^3}=-a$$

$$\Rightarrow 8\left(\frac{1}{3}+\frac{1}{3!}\right)=-ab$$

$\Rightarrow 4 = -ab$

$\Rightarrow 10 - ab = 14$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

1. The temperature of equal masses of three different liquids x,y and z are 10ºC, 20ºC and 30ºC respectively. The temperature of mixture when x is mixed with y is 16ºC and that when y is mixed with z is 26ºC. The temperature of mixture when x and z are mixed will be :

(1) 28.32º C (2) 25.62º C

(3) 23.84ºC (4) 20.28ºC

Official Ans. by NTA (3)

Sol.

X	Y	Z
$m_1 = m$	$m_2 = m$	$m_3 = m$
$T_1 = 10°C$	$T_2 = 20°C$	$T_3 = 30°C$
s_1	s_2	s_3

when x & y are mixed, $T_{f_1} = 16°C$

$m_1s_1T + m_2s_2T_2 = (m_1s_1 + m_2s_2)Tf_1$

$s_1 \times 10 + s_2 \times 20 = (s_1 + s_2) \times 16$

$s_1 = \frac{2}{3}s_2$ ....(i)

when y & z are mixex, $T_{f_2} = 26°C$

$m_2s_2T + m_3s_3T_3 = (m_3s_3 + m_3s_3)Tf_2$

$s_2 \times 20 + s_3 \times 30 = (s_2 + s_3) \times 26$

$s_3 = \frac{3}{2}s_2$ .....(ii)

when x & z are mixex

$m_1s_1T_1 + m_3s_3T_3 = (m_1s_1 + m_3s_3)Tf$

$$\frac{2}{3}s_2 \times 10 + \frac{2}{3}s_2 \times 20 = \left(\frac{2}{3}s_2 + \frac{3}{2}s_2\right)T_f$$

$T_f = 23.84°C$

Ans (3)

2. The de-Broglie wavelength of a particle having kinetic energy E is λ. How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value ?

(1) $\frac{1}{9}E$ (2) $\frac{7}{9}E$

(3) E (4) $\frac{16}{9}E$

Official Ans. by NTA (2)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$, $mv = \sqrt{2mE}$

$\lambda \propto \frac{1}{\sqrt{E}}$

$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \frac{3}{4}$, $\lambda_2 = 0.75\,\lambda_1$

$\frac{E_1}{E_2} = \left(\frac{3}{4}\right)^2$

$E_2 = \frac{16}{9}E_1 = \frac{16}{9}E$ $(E_1 = E)$

Extra energy given $= \frac{16}{9}E - E = \frac{7}{9}E$

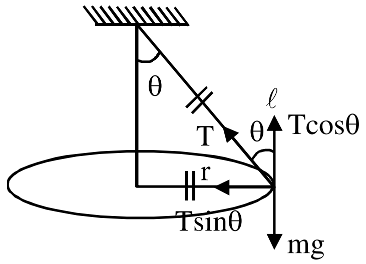
Ans. 2

3. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be :

(1) $\sqrt{rg}$ (2) $\sqrt{2rg}$ (3) $2\sqrt{rg}$ (4) $\sqrt{\frac{rg}{2}}$

Official Ans. by NTA (1)

Sol. Conical pendulum



$r = \frac{\ell}{\sqrt{2}}$

$\sin\theta = \frac{r}{\ell} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

Ans. 1

4. A cylindrical container of volume 4.0×10^{-3} m^3 contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is :

[Take gas constant as 8.3 J mol^{-1} K^{-1}]

(1) 249×10^1 Pa (2) 24.9×10^3 Pa

(3) 24.9×10^5 Pa (4) 24.9 Pa

Official Ans. by NTA (3)

Sol. $V = 4 \times 10^{-3}$ m^3

n = 3 moles

T = 400K

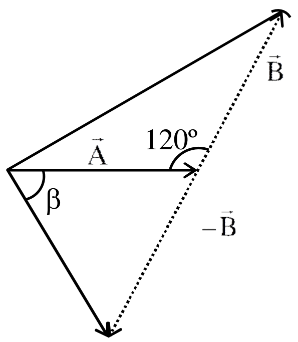
$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$P = \frac{3 \times 8.3 \times 400}{4 \times 10^{-3}}$$

$= 24.9 \times 10^5$ Pa

Ans 3

5. The angle between vector $(\vec{A})$ and $(\vec{A} - \vec{B})$ is :



(1) $\tan^{-1}\left(\dfrac{-\dfrac{B}{2}}{A - B\dfrac{\sqrt{3}}{2}}\right)$

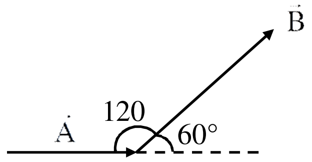
(2) $\tan^{-1}\left(\dfrac{A}{0.7B}\right)$

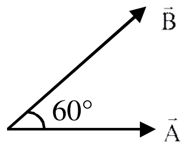
(3) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A - B}\right)$

(4) $\tan^{-1}\left(\dfrac{B\cos\theta}{A - B\sin\theta}\right)$

Official Ans. by NTA (3)

Sol.





Angle between $\vec{A}$ and $\vec{B}$, $\theta = 60°$

Angle betwenn $\vec{A}$ and $\vec{A} - \vec{B}$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$= \frac{B\sqrt{\frac{3}{2}}}{A - B \times \frac{1}{2}2}$$

$$\tan\alpha = \frac{\sqrt{3}B}{2A - B}$$

Ans 3

6. A light beam is described by $E = 800 \sin\omega\left(t - \frac{x}{c}\right)$ .An electron is allowed to move normal to the propagation of light beam with a speed of 3×10^7 ms^{-1}. What is the maximum magnetic force exerted on the electron ?

(1) 1.28×10^{-18} N

(2) 1.28×10^{-21} N

(3) 12.8×10^{-17} N

(4) 12.8×10^{-18} N

Official Ans. by NTA (4)

Sol. $\frac{E_0}{C} = B_0$

$F_{max} = eB_0V$

$$= 1.6 \times 10^{-19} \times \frac{800}{3 \times 10^8} \times 3 \times 10^7$$

$= 12.8 \times 10^{-18}$N

Ans. 4

7. The two thin coaxial rings, each of radius 'a' and having charges +Q and –Q respectively are separated by a distance of 's'. The potential difference between the centres of the two rings is :

(1) $\dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{a}+\dfrac{1}{\sqrt{s^2+a^2}}\right]$

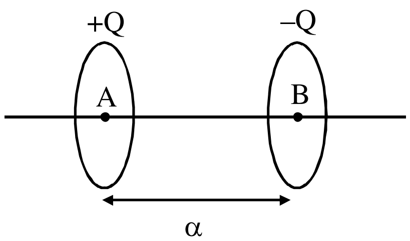
(2) $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{a}+\dfrac{1}{\sqrt{s^2+a^2}}\right]$

(3) $\dfrac{Q}{4\pi\varepsilon_0}\left[\dfrac{1}{a}-\dfrac{1}{\sqrt{s^2+a^2}}\right]$

(4) $\dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{a}-\dfrac{1}{\sqrt{s^2+a^2}}\right]$

Official Ans. by NTA (4)

Sol.



$$V_A = \frac{KQ}{a} - \frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_B = \frac{-KQ}{a} + \frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_A - V_B = \frac{2KQ}{a} - \frac{2KQ}{\sqrt{a^2+s^2}}$$

$$= \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{s^2+a^2}\right)$$

Ans 4

8. If you are provided a set of resistances 2Ω, 4Ω, 6Ω and 8Ω. Connect these resistances so as to obtain an equivalent resistance of $\dfrac{46}{3}\Omega$.

(1) 4Ω and 6Ω are in parallel with 2Ω and 8 Ω in series

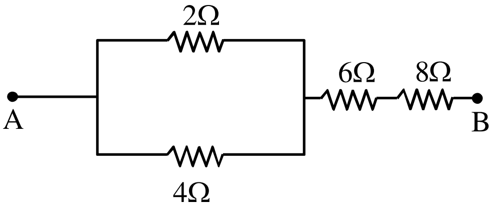
(2) 6Ω and 8Ω are in parallel with 2Ω and 4Ω in series

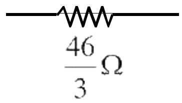
(3) 2Ω and 6Ω are in parallel with 4Ω and 8 Ω in series

(4) 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series

Official Ans. by NTA (4)

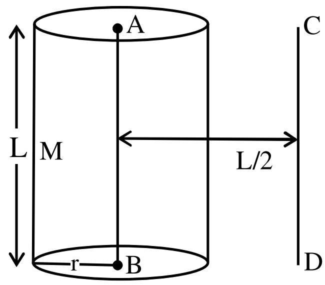
Sol.





Ans 4

9. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m^2.



(1) 14.9 kg / m^3 (2) 7.5×10^1 kg / m^3

(3) 7.5×10^2 kg/m^3 (4) 1.49×10^2 kg / m^3

Official Ans. by NTA (4)

Sol. Parallel axis theorem

$I = I_{CM} + Md^2$

$$I = \frac{Mr^2}{2} + M\left(\frac{L}{2}\right)^2$$

$$2.7 = M\frac{(0.2)^2}{2} + M\left(\frac{0.8}{2}\right)^2$$

$$2.7 = M\left[\frac{2}{100} + \frac{16}{100}\right]$$

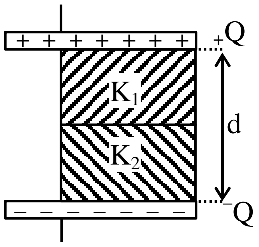
M = 15kg

$$\Rightarrow \rho = \frac{M}{\pi r^2 L} = \frac{15}{\pi(0.2)^2 \times 0.8}$$

$= 0.1492 \times 10^3$

Ans. 4

10. A parallel - plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by :



(1) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$

(2) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{2(K_1+K_2)}\right)$

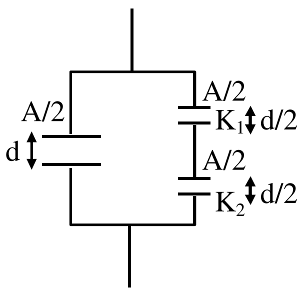
(3) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1+K_2}{K_1K_2}\right)$

(4) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{2(K_1+K_2)}{K_1K_2}\right)$

Official Ans. by NTA (1)

Sol. $C_{eq}=\frac{\frac{A}{2}\varepsilon_0}{d}+\frac{A\varepsilon_0}{d}\frac{K_1K_2}{K_1+K_2}$

$=\frac{A\varepsilon_0}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$



Ans.1

11. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a :

(1) hyperbola

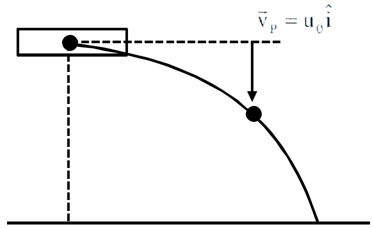
(2) parabola in the direction of motion of plane

(3) straight line vertically down the plane

(4) parabola in a direction opposite to the motion of plane

Official Ans. by NTA (3)

Sol.



$v_B = u_0\hat{i} - gt\hat{j}$

$\vec{v}_{B/P} = \vec{v}_B - \vec{v}_P$

$\vec{v}_{B/P} = -8t\hat{j}$

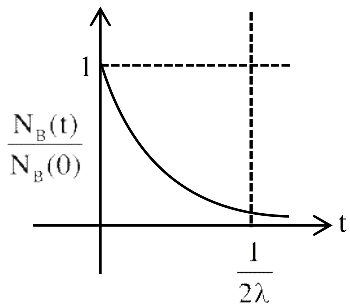
straight line vertically down

Ans.3

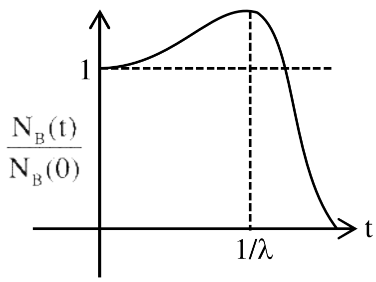
12. At time t = 0, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ. However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)$ / $N_B(0)$ with respect to time t ?

$$\begin{bmatrix} N_A(0) = \text{No. of A atoms at } t=0 \\ N_B(0) = \text{No. of B atoms at } t=0 \end{bmatrix}$$

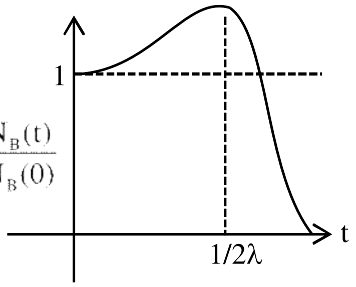
(1)



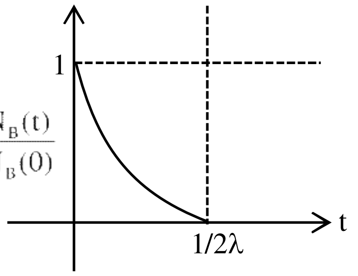
(2)



(3) $\frac{N_B(t)}{N_B(0)}$



(4) $\frac{N_B(t)}{N_B(0)}$



Official Ans. by NTA (3)

Sol.

$A \to B,\ B \to C$

$$\frac{dN_B}{dt} = \lambda N_A - \lambda N_B$$

$$\frac{dN_B}{dt} = 2\lambda N_{B_0} e^{-\lambda t} - \lambda N_B$$

$$e^{-\lambda t}\left(\frac{dN_B}{dt} + \lambda N_B\right) = 2\lambda N_{B_0} e^{-\lambda t} \times e^{\lambda t}$$

$$\frac{d}{dt}\left(N_B e^{\lambda t}\right) = 2\lambda N_{B_0}\text{, on integrating}$$

$$N_B e^{\lambda t} = 2\lambda t N_{B_0} + N_{B_0}$$

$$N_B = N_{B_0}[1 + 2\lambda t]e^{-\lambda t}$$

$$\frac{dN_B}{dt} = 0 \text{ at } -\lambda[1 + 2\lambda t)e^{-\lambda t} + 2\lambda e^{-\lambda t} = 0$$

$$N_{B_{max}} \text{ at } t = \frac{1}{2\lambda}$$

13. A transmitting antenna at top of a tower has a height of 50 m and the height of receiving antenna is 80 m. What is range of communication for Line of Sight (LoS) mode ?

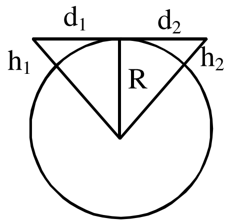
[use radius of earth = 6400 km]

(1) 45.5 km (2) 80.2 km

(3) 144.1 km (4) 57.28 km

Official Ans. by NTA (4)

Sol.



$$d_t = \sqrt{2Rh_1} + \sqrt{2Rh_2}$$

$$= \sqrt{2R}\left(\sqrt{h_1} + \sqrt{h_2}\right)$$

$$= (2 \times 6400 \times 10^3)^{1/2}\left(\sqrt{50} + \sqrt{80}\right)$$

= 3578 (7.07 + 8.94)

= 57.28 Km

14. A refrigerator consumes an average 35 W power to operate between temperature –10°C to 25°C. If there is no loss of energy then how much average heat per second does it transfer ?

(1) 263 J/s (2) 298 J/s (3) 350 J/s (4) 35 J/s

Official Ans. by NTA (1)

Sol. $$\frac{T_L}{T_H - T_L} = \text{C.O.P.} = \frac{\frac{dH}{dt}}{\frac{dW}{dt}}$$

$$\frac{263}{35} \times 35 = \frac{dH}{dt}$$

$$\frac{dH}{dt} = 263 \text{ watts}$$

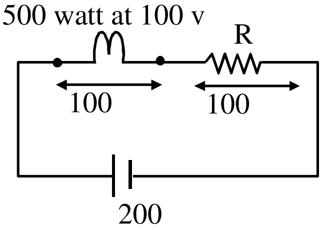
Ans.1

15. An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.

(1) 20 Ω (2) 30 Ω (3) 5 Ω (4) 10 Ω

Official Ans. by NTA (1)

500 watt at 100 v



Sol.

P = Vi

500 = Vi

i = 5 Amp

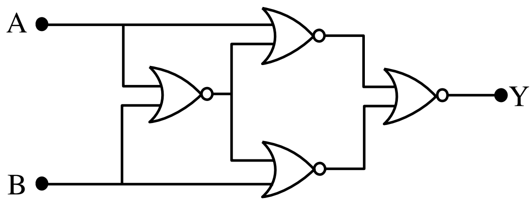
V = i × R

R = 20

Ans. 1

16. Four NOR gates are connected as shown in figure.

The truth table for the given figure is :



(1)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

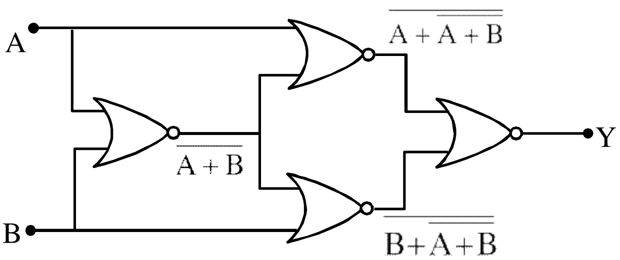
A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Official Ans. by NTA (4)

Sol.



$$y=\overline{\overline{(A+\overline{A+B})}+\overline{(B+\overline{A+B})}}$$

$$y=(A+\overline{A+B}).(B+\overline{A+B})$$

A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

Ans.4

17. Match List–I with List–II.

List-I		**List-II**	
(a)	Magnetic Induction	(i)	$ML^2T^{-2}A^{-1}$
(b)	Magnetic Flux	(ii)	$M^0L^{-1}A$
(c)	Magnetic Permeability	(iii)	$MT^{-2}A^{-1}$
(d)	Magnetization	(iv)	$MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below :

(1) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Official Ans. by NTA (4)

Sol. (a) Magnetic Induction = $MT^{-2}A^{-1}$

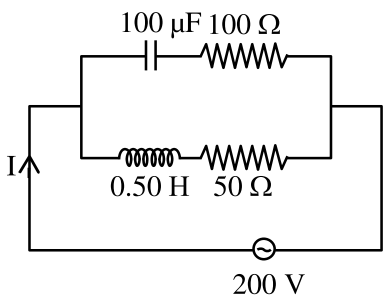
(b) Magnetic Flux = $ML^2T^{-2}A^{-1}$

(c) Magnetic Permeability = $MLT^{-2}A^{-2}$

(d) Magnetization = $M^0L^{-1}A$

Ans. 4

18. In the given circuit the AC source has ω = 100 rad s^{-1}. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?



(1) 5.9 A

(2) 4.24 A

(3) 0.94 A

(4) 6 A

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. $Z_C = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

$= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2}$

$Z_C = \sqrt{(100)^2 + (100)^2}$

$= 100\sqrt{2}$

$Z_L = \sqrt{(\omega L)^2 + R^2}$

$\sqrt{(100 \times 0.5)^2 + 50^2}$

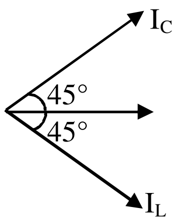
$= 50\sqrt{2}$

$i_C = \frac{200}{z_C} = \frac{200}{100\sqrt{2}} = \sqrt{2}$

$i_L = \frac{200}{z_L} = \frac{200}{50\sqrt{2}} = 2\sqrt{2}$

$\cos\phi_1 = \frac{100}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_1 = 45°$

$\cos\phi_2 = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45°$



$I = \sqrt{I_C^2 + I_L^2}$

$= \sqrt{2+8}$

$= \sqrt{10}$

I = 3.16 A

Ans. 3.16

19. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is:

(1) 86.4 s

(2) 4.32 s

(3) 43.2 s

(4) 8.64 s

Official Ans. by NTA (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

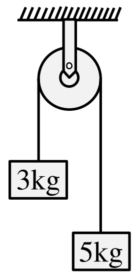
$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$

$\Delta T = \frac{1}{2} \times \frac{0.1}{100} \times 24 \times 3600$

$\Delta T = 43.2$

Ans. 3

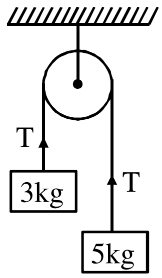
20. Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2$ Nm^{-2}. What is the minimum radius of the wire? (Take $g = 10$ ms^{-2})



(1) 125 cm (2) 1250 cm

(3) 12.5 cm (4) 1.25 cm

Official Ans. by NTA (3)



Sol.

$T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2 \times 3 \times 5 \times 10}{8}$

$= \frac{75}{2}$

$\text{Stress} = \frac{T}{A}$

$\frac{24}{\pi} \times 10^2 = \frac{75}{2 \times \pi R^2}$

$R^2 = \frac{75}{2 \times 24 \times 100} = \frac{3}{8 \times 24}$

$\Rightarrow R = 0.125$ m

R = 12.5 cm

SECTION-B

1. Two waves are simultaneously passing through a string and their equations are :

$y_1 = A_1 \sin k(x-vt)$, $y_2 = A_2 \sin k(x-vt + x_0)$. Given amplitudes $A_1 = 12$ mm and $A_2 = 5$ mm, $x_0 = 3.5$ cm and wave number $k = 6.28\ cm^{-1}$. The amplitude of resulting wave will be .... mm.

Official Ans. by NTA (7)

Sol. $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28\ (x - vt)$

$y_2 = 5 \sin 6.28\ (x - vt + 3.5)$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$= K(\Delta x)$

$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$A_{net} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$$

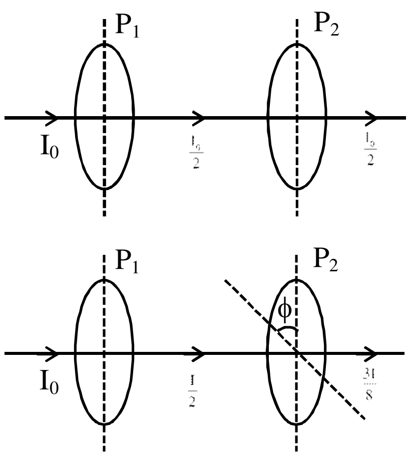
$$= \sqrt{144 + 25 - 120}$$

Ans. 7

2. A source of light is placed in front of a screen. Intensity of light on the screen is I. Two Polaroids P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is I/2. P_2 should be rotated by an angle of .... (degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$.

Official Ans. by NTA (30)

Sol. $I = \frac{I_0}{2}\cos^2\phi$



$$\frac{I}{2}\cos^2\phi = \frac{3I}{8}$$

$$\cos^2\phi = \frac{3}{4}$$

$$\cos^2\phi = \frac{\sqrt{3}}{2}$$

$\Rightarrow \phi = 30$

Ans. 30

3. If the maximum value of accelerating potential provided by a ratio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is ....

[$m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C,

Speed of light $= 3 \times 10^8$ m/s]

Official Ans. by NTA (543)

Sol. V = 12 kV

Number of revolution = n

$$n[2 \times q_P \times V] = \frac{1}{2}m_P \times v_P^2$$

$n[2\times 1.6 \times 10^{-19} \times 12 \times 10^3$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times \left[\frac{3\times10^8}{6}\right]^2$$

$n(38.4\times10^{-16}) = 0.2087 \times 10^{-11}$

n = 543.4

Ans. 543

4. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as ....%

Official Ans. by NTA (14)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \ell = \frac{T^2 g}{4\pi^2}$

$$E = mg\ell\frac{\theta^2}{2} = mg^2\frac{T^2\theta^2}{8\pi^2}$$

$$\frac{dE}{E} = 2\left(\frac{dg}{g} + \frac{dT}{T}\right)$$

$= (4 + 3) = 14\%$

5. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of 3.0×10^{-2} T. The maximum emf induced the coil will be .... $\times 10^{-2}$ volt (rounded off to the nearest integer)

Official Ans. by NTA (60)

Sol. Maximum emf $\varepsilon = N\,\omega AB$

$N = 20,\ \omega = 50,\ B = 3 \times 10^{-2}T$

$\varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 60.28 \times 10^{-2}$

Rounded off to nearest integer = 60

Ans.60

6. Two simple harmonic motions are represented by the equations

$x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}\,(\sin 2\pi t + \cos 2\pi t)$.

The amplitude of second motion is .... times the amplitude in first motion.

Official Ans. by NTA (2)

Sol. $x_2 = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 2\pi t + \frac{1}{\sqrt{2}}\cos 2\pi t\right)\sqrt{2}$

$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$

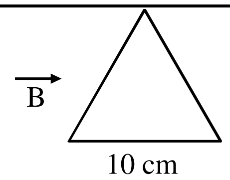
$\therefore \frac{A_2}{A_1} = \frac{10}{5} = 2$

Ans. 2

7. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5}$ Nm. The value of x is...........

Official Ans. by NTA (3)

Sol.



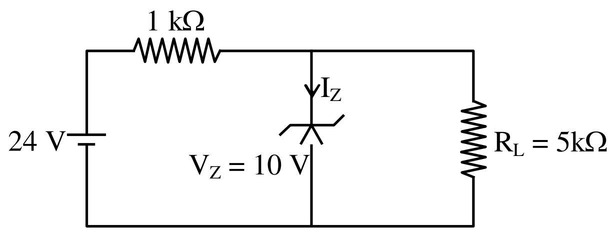
$\vec{\tau} = \vec{M} \times \vec{B} = MB\sin 90°$

$= MB = \frac{i\sqrt{3}\ell^2}{4}B$

$= \sqrt{3} \times 10^{-5}\,N-m$

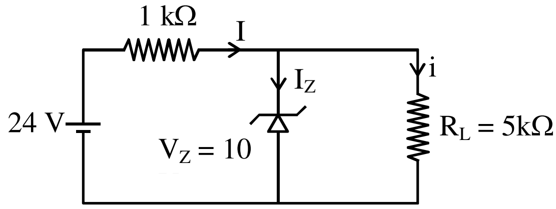
Ans. 3

8. For the given circuit, the power across zener diode is .... mW.



Official Ans. by NTA (120)

Sol.



$i = \frac{10V}{5k\Omega} = 2mA$

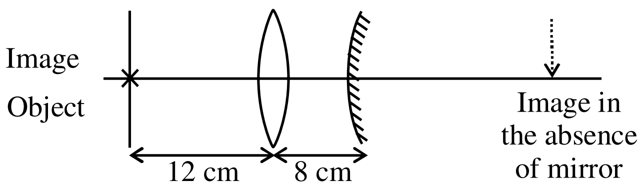
$I = \frac{14V}{1k\Omega} = 14mA$

$\therefore\ I_z = 12mA$

$\therefore\ P = I_z V_z = 120\ \ mW$

Ans. 120

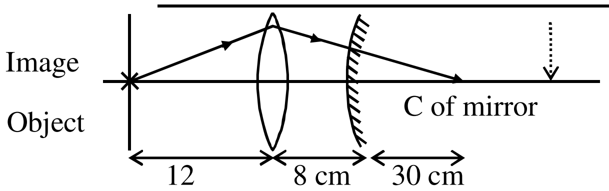
9. An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.



When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be ....(cm)

Official Ans. by NTA (50)

Sol.



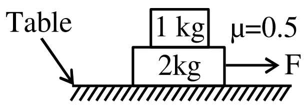
For the object to coincide with image, the light must fall perpendicularly to mirror. Which means that the light will have to converge at C of mirror.

Without the mirror also, the light would coverage at C.

So the distance is : 12 + 8 + 30 = 50 cm

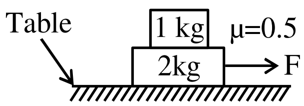
10. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is ....N.

(take g = 10 ms^{-2})

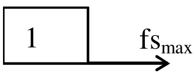


Official Ans. by NTA (15)

Sol.



F = 3a (For system) ….(i)



$fs_{max} = 1a$ (for 1kg block) ….(ii)

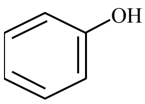
$\mu \times 1 \times g = a$

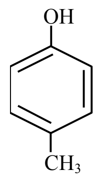
$\Rightarrow 5 = a$

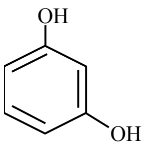
F = 15N

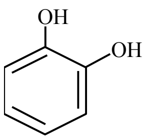
SECTION-A

1. Which one of the following phenols does not give colour when condensed with phthalic anhydride in presence of conc. H_2SO_4 ?

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. Only p-methyl, phenol does not give any colour with phthalic anhydroxide with cons. H_2SO_4.

2. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Photochemical smog causes cracking of rubber.

Reason (R) : Presence of ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate in photochemical smog makes it oxidizing.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Official Ans. by NTA (4)

Sol. Photochemical smog causes cracking of rubber, the common component of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyle nitrate (PAN).

3. The interaction energy of London forces between two particles is proportional to r^x, where r is the distance between the particles. The value of x is :

(1) 3 (2) –3

(3) –6 (4) 6

Official Ans. by NTA (3)

Sol. For london dispersion forces.

$E \propto \frac{1}{r^6}$

Hence x = –6

4. The number of non-ionisable hydrogen atoms present in the final product obtained from the hydrolysis of PCl_5 is :

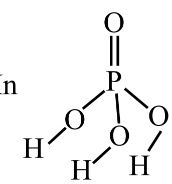
(1) 0 (2) 2 (3) 1 (4) 3

Official Ans. by NTA (1)

Sol. $PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$

$$\downarrow + 3H_2O$$

$H_3PO_4 + 3HCl$

In  + HCl

all hydrogens are ionisable

∴ Ans is zero.

5. The bond order and magnetic behaviour of O_2^- ion are, respectively :

(1) 1.5 and paramagnetic

(2) 1.5 and diamagnetic

(3) 2 and diamagnetic

(4) 1 and paramagnetic

Official Ans. by NTA (1)

Sol. $O_2^- = (\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2$

$\left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right)\left(\pi_{2p_x}^{*2} = \pi_{2p_y}^{*1}\right)$

Bond order = $\dfrac{10-7}{2} = 1.5$

and paramagnetic

6. Given below are two statements : one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (A) : Sucrose is a disaccharide and a non-reducing sugar.

Reason (R) : Sucrose involves glycosidic linkage between C_1 of β-glucose and C_2 of α-fructose.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Official Ans. by NTA (3)

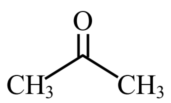
Sol. Surcrose is example of disaccharide & non reducing sugar

Assertion : correct

Sucrose involves glycosidic linkage between C_1 of α-D-glucose C_2 of β-D-fructose

Reason : Incorrect

7. Match **List-I** with **List-II** :

List-I (Chemical Reaction)	**List-II (Reagent used)**
(a) $CH_3COOCH_2CH_3 \rightarrow CH_3CH_2OH$	(i) CH_3MgBr / H_3O^+ (1.equivalent)
(b) $CH_3COOCH_3 \rightarrow CH_3CHO$	(ii) H_2SO_4 / H_2O
(c) $CH_3C \equiv N \rightarrow CH_3CHO$	(iii) DIBAL-H/H_2O
(d) $CH_3C \equiv N \rightarrow$ 	(iv) $SnCl_2$, HCl/H_2O

Choose the most appropriate match :

(1) a-ii, b-iv, c-iii, d-i

(2) a-iv, b-ii, c-iii, d-i

(3) a-ii, b-iii, c-iv, d-i

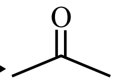
(4) a-iii, b-ii, c-i, d-iv

Official Ans. by NTA (3)

Sol. $CH_3-\underset{\underset{O}{\|}}{C}-O-CH_2CH_3 \xrightarrow{H_3O^+} CH_3CO_2H + CH_3CH_2OH$

$CH_3-\underset{\underset{O}{\|}}{C}-O-CH_3 \xrightarrow[-78^\circ C]{DIBALH/H_2O} CH_3CHO$

$CH_3-CN \xrightarrow[/H_2O]{SnCl_2+HCl} CH_3CH=O$

$CH_3-C\equiv N \xrightarrow[/H_3O^+]{CH_3MgBr\ (1eq)}$ 

8. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Barium carbonate is insoluble in water and is highly stable.

Reason (R) : The thermal stability of the carbonates increases with increasing cationic size.

(1) Both **(A)** and **(R)** are true but **(R)** is the true explanation of **(A)**

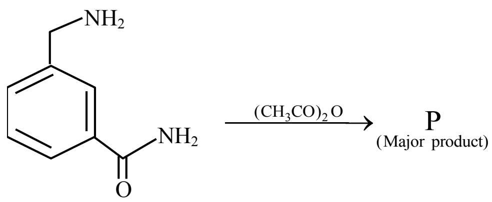
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is not the true explanation of **(A)**

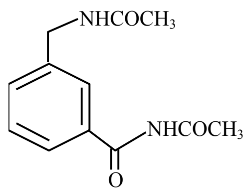
(4) **(A)** is false but **(R)** is true.

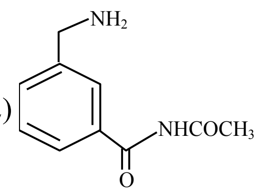
Official Ans. by NTA (1)

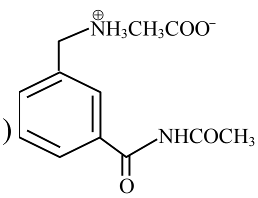
Sol. In IIA group on moving down the group size of cation increases and show thermal stability of carbonate increases.

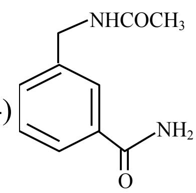
9. 

The major product in the above reaction is :

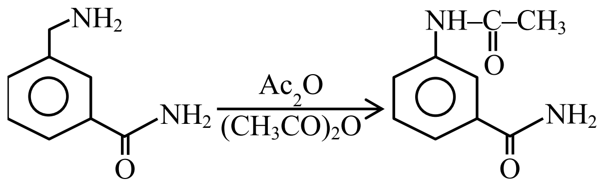
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. 

10. Indicate the complex/complex ion which did not show any geometrical isomerism :

(1) $[CoCl_2(en)_2]$ (2) $[Co(CN)_5(NC)]^{3-}$

(3) $[Co(NH_3)_3(NO_2)_3]$ (4) $[Co(NH_3)_4Cl_2]^+$

Official Ans. by NTA (2)

Sol. (1) $[CoCl_2(en)_2]$ show
Cis-trans isomerism

(2) $[Co(CN)_5(NC)]^{-3}$ can't
Show G.I.

(3) $[Co(NH_3)_3(NO_2)_3]$
Show fac & mer isomerism

(4) $[Co(NH_3)_4Cl_2]^{\oplus}$ show cis & trans isomerism

11. The sol given below with negatively charged colloidal particles is :

(1) $FeCl_3$ added to hot water

(2) KI added to $AgNO_3$ solution

(3) $AgNO_3$ added to KI solution

(4) $Al_2O_3.xH_2O$ in water

Official Ans. by NTA (3)

Sol.

12. Given below are two statements :

Statement I : Sphalerite is a sulphide ore of zinc and copper glance is a sulphide ore of copper.

Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a froth flotation method.

Choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

(2) Both **Statement I** and **Statement II** are true.

(3) **Statement I** is false but **Statement II** is true.

(4) Both **Statement I** and **Statement II** are false.

Official Ans. by NTA (2)

Sol. Sphalerite-ZnS, copper glance - Cu_2S two sulphide ores can be separated by adjusting proportions of oil to water or by using ' Depressants '

13. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Heavy water is used for the study of reaction mechanism.

Reason (R) : The rate of reaction for the cleavage of O - H bond is slower than that of O-D bond.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**.

(2) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**.

(3) **(A)** is false but **(R)** is true.

(4) **(A)** is true but **(R)** is false.

Official Ans. by NTA (4)

Sol. D_2O in used for the study of reaction mechanism.
Rate of reaction for the cleavage of O–H bond > O-D bond.

14. Arrange the following Cobalt complexes in the order of increasing Crystal Field Stabilization Energy (CFSE) value.

Complexes : $\underset{A}{[CoF_6]^{3-}}, \underset{B}{[Co(H_2O)_6]^{2+}}, \underset{C}{[Co(NH_3)_6]^{3+}}$

and $[Co(en)_3]^{3+}$

Choose the **correct** option :

(1) A < B < C < D (2) B < A < C < D

(3) B < C < D < A (4) C < D < B < A

Official Ans. by NTA (2)

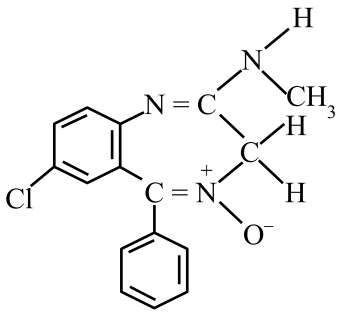
Sol. (i) CFSE $\propto$ charge or oxidation no. of central metal ion.

(ii) CFSE $\propto$ strength of ligand
en > NH_3 > H_2O > F^-

$\therefore$ order of CFSE

$$\overset{III}{[Co(en)_3]^{+3}} > \overset{III}{Co(NH_3)_6]^{+3}} > \overset{III}{[CoF_6]^{-3}} > \overset{II}{[Co(H_2O)_6]^{+2}}$$

15.



Chlordiazepoxide

The class of drug to which chlordiazepoxide with above structure belongs is :

(1) Antacid (2) Analgesic

(3) Tranquilizer (4) Antibiotic

Official Ans. by NTA (3)

Sol. The drug named chlordiate poxide is example of tranquilizer.

16. Chalcogen group elements are :

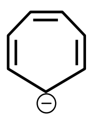
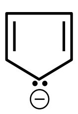
(1) Se, Tb and Pu. (2) Se, Te and Po.

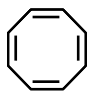
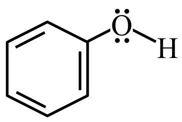
(3) S, Te and Pm. (4) O, Ti and Po.

Official Ans. by NTA (2)

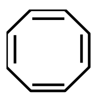
Sol. Group 16/oxygen family is known as Chalcogens the members are O, S, Se, Te, Po

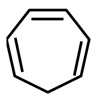
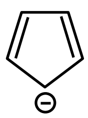
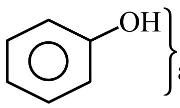
17. Which one of the following compounds is not aromatic ?

(1)  (2) 

(3)  (4) 

Official Ans. by NTA (3)

Sol.  : Non aromatic


(+)
;  ;  All are aromatic

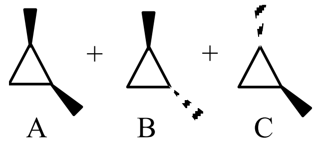
18. The number of stereoisomers possible for 1,2-dimethyl cyclopropane is :

(1) One (2) Four

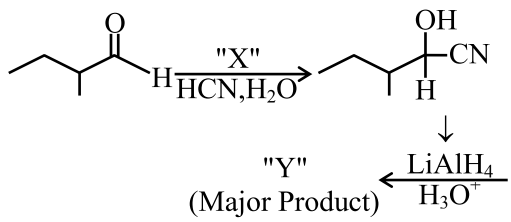
(3) Two (4) Three

Official Ans. by NTA (4)

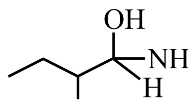
Sol.

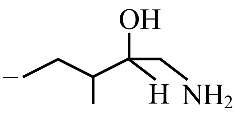


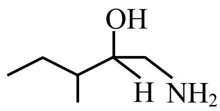
19.

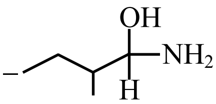


Consider the given reaction, Identify **'X'** and **'Y'** :

(1) X – NaOH Y – 

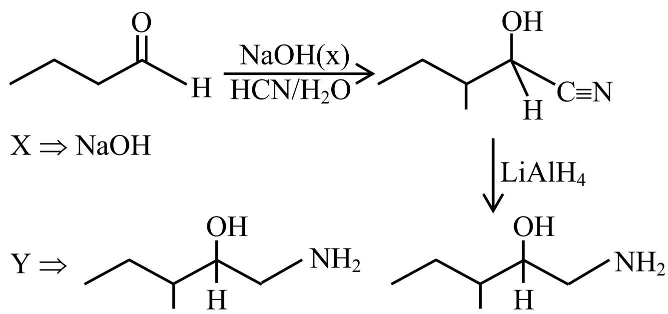
(2) X – HNO_3 Y – 

(3) X – NaOH Y – 

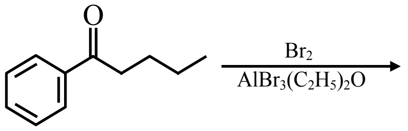
(4) X – HNO_3 Y – 

Official Ans. by NTA (3)

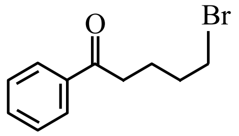
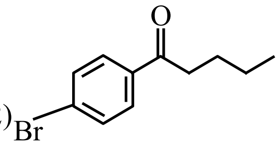
Sol.

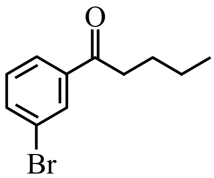
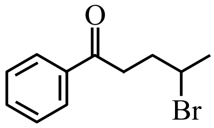


γ(major product)

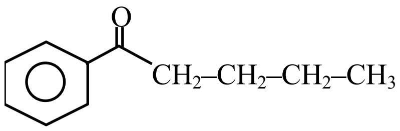
20.  'A' (Major product)

Consider the given reaction, the product **A** is :

(1)  (2) 

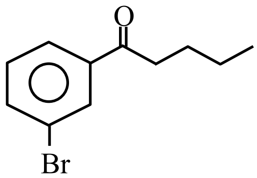
(3)  (4) 

Official Ans. by NTA (3)

Sol. 

(m–directing)

$\downarrow Br_2 + AlBr_3(El_2O)$



SECTION-B

1. In the sulphur estimation, 0.471 g of an organic compound gave 1.44 g of barium sulphate. The percentage of sulphur in the compound is _____%. (Nearest integer)

(Atomic Mass of Ba = 137 u)

Official Ans. by NTA (42)

Sol. Molecular mass of $BaSO_4$ = 233 g

$\because$ 233 $BaSO_4$ contain $\rightarrow$ 32 g sulphur

$\therefore$ 1.44 g $BaSO_4$ contain $\rightarrow \dfrac{32}{233} \times 1.44$ g sulphur

given : 0.471 g of organic compound

$$\% \text{ of S} = \frac{32\times1.44}{233\times0.471} \times 100 = 41.98\% \approx 42\%$$

OR

$$\underset{W_{OC}=0.471\,g}{\boxed{O.C.}} \longrightarrow \underset{1.44\,g}{BaSO_4}$$

$$\Rightarrow n_s = n_{BaSO_4} = \frac{1.44}{233}$$

$$\Rightarrow w_s = \frac{1.44}{233}\times 32g$$

therefore $\%S = \dfrac{W_s}{W_{O.C.}}\times100 = \dfrac{1.44\times32}{233\times0.471}\times100$

$$= \frac{46.08}{109.743}\times100 = 41.98 \quad 42$$

2. The equilibrium constant K_c at 298 K for the reaction $A + B \rightleftharpoons C + D$ is 100. Starting with an equimolar solution with concentrations of A, B, C and D all equal to 1M, the equilibrium concentration of D is_____ $\times 10^{-2}$ M. (Nearest integer)

Official Ans. by NTA (182)

Sol. $A + B \rightleftharpoons C + D : K_{eq} = 100$

1M 1M 1M 1M

First check direction of reversible reaction.

Since $Q_C = \dfrac{[C][D]}{[A][B]} = 1 < K_{eq.} \Rightarrow$ reaction will move in forward direction to attain equilibrium state.

$\Rightarrow A + B \rightleftharpoons C + D : K_{eq} = 100$

	A	B	C	D
to	1	1	1	1
$t_{eq.}$	1–x	1–x	1+x	1+x

Now : $K_{eq} = 100 = \dfrac{(1+x)(1+x)}{(1-x)(1-x)}$

$$\Rightarrow \boxed{100 = \left(\frac{1+x}{1-x}\right)^2}$$

(i) $10 = \left(\dfrac{1+x}{1-x}\right)$

$\Rightarrow 10 - 10x = 1+x$

$\Rightarrow 11x = 9$

$\Rightarrow \boxed{x = \dfrac{9}{11}}$

(ii) $-10 = \dfrac{1+x}{1-x}$

$\Rightarrow -10 + 10x = 1 + x$

$\Rightarrow -9x = -11$

$\Rightarrow \boxed{x = \frac{11}{9}}$

$\rightarrow$ 'x' cannot be more than one, therefore not valid.

therefore equation concretion of (D) = 1 + x

$$= 1 + \frac{9}{11} = \frac{20}{11}$$

$$= 1.8181 = 181.81 \times 10^{-2}$$

$$182 \times 10^{-2}$$

3. For water $\Delta_{vap} H = 41$ kJ mol^{-1} at 373 K and 1 bar pressure. Assuming that water vapour is an ideal gas that occupies a much larger volume than liquid water, the internal energy change during evaporation of water is_____kJ mol^{-1}

[Use : R = 8.3 J mol^{-1} K^{-1}]

Official Ans. by NTA (38)

Sol. Given equation is

$$H_2O(\ell) \longrightarrow H_2O(g) : \Delta H = 41 \ \frac{kJ}{mol}$$

$\Rightarrow$ From the relation : $\Delta H = \Delta U + \Delta n_g RT$

$$\Rightarrow 41\frac{kJ}{mol} = \Delta U + (1) \times \frac{8.3}{1000} \times 373$$

$\Rightarrow$ DU = 41 – 3.0959

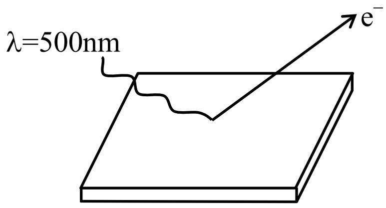
= 38 kJ/mol

4. A metal surface is exposed to 500 nm radiation. The threshold frequency of the metal for photoelectric current is 4.3×10^{14} Hz. The velocity of ejected electron is ________$\times 10^5$ ms^{-1} (Nearest integer)

[Use : h = 6.63×10^{-34} Js, $m_e = 9.0 \times 10^{-31}$ kg]

Official Ans. by NTA (5)

Sol.



υ : speed of electron having max. K.E.

$\Rightarrow$ from Einstein equation : E = ϕ + K.E.$_{max}$

$$\Rightarrow \frac{hc}{\lambda} = h\upsilon_0 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}} = 6.63 \times 10^{-34} \times 4.3 \times 10^{14} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{6.63 \times 30 \times 10^{-20}}{5} = 6.63 \times 4.3 \times 10^{-20} + \frac{1}{2}mv^2$$

$$\Rightarrow 11.271 \times 10^{-20} \text{ J} = \frac{1}{2} \times 9 \times 10^{-31} \times \upsilon^2$$

$$\Rightarrow \boxed{\upsilon = 5 \times 10^5 \text{ m/sec.}}$$

5. For the galvanic cell,

$Zn(s) + Cu^{2+}$ (0.02 M) $\rightarrow Zn^{2+}$ (0.04 M) + Cu(s),

E_{cell} =_______ $\times 10^{-2}$ V. (Nearest integer)

[Use : $E^0_{Cu/Cu^{2+}}$ = –0.34 V, $E^0_{Zn/Zn^{2+}}$ =+0.76 V,

$\frac{2.303 \text{ RT}}{F}$ = 0.059 V]

Official Ans. by NTA (109)

Sol. Galvanic cell:

$$Zn_{(s)} + \underset{0.02M}{Cu^{+2}_{(aq.)}} \rightarrow \underset{0.04M}{Zn^{+2}} + Cu(s)$$

Nernst equation = F_{cell} = $E^o_{cell} - \frac{0.059}{2}\log\frac{[2n^{+2}]}{[Cu^{+2}]}$

$$\Rightarrow E_{cell}\left[E^o_{cell} - E^o_{Zn^{+2}/Zn}\right] - \frac{0.059}{2}\log\frac{0.04}{0.02}$$

$$\Rightarrow E_{cell}[0.34 - (-0.76)] - \frac{0.059}{2}\log^2$$

$$\Rightarrow E_{cell} 1 - 1 - \frac{0.059}{2} \times 0.3010$$

$$= 1.0911 = 109.11 \times 10^{-2}$$

$$= 109$$

6. 100 mL of Na_3PO_4 solution contains 3.45 g of sodium. The molarity of the solution is _____$\times 10^{-2}$ mol L^{-1} . (Nearest integer)

[Atomic Masses - Na : 23.0 u, O : 16.0 u, P : 31.0 u]

Official Ans. by NTA (50)

Sol. $\boxed{Na_3PO_4}$ 100ml Sol.

$$\underset{\frac{1}{3} \times \frac{3.45}{23} \text{mol}}{Na_3PO_4} \longrightarrow \underset{\substack{3.45g \\ \frac{3.45}{23} \text{mol}}}{3Na}$$

therefore molarity of Na_3PO_4 Solution =

$$\frac{n_{Na_3PO_4}}{\text{volume of solution in L}}$$

$$=\frac{\frac{1}{3}\times\frac{3.45}{23}\text{mol}}{0.1\,\text{L}}$$

$= 0.5 = 50 \times 10^{-2}$

7. The overall stability constant of the complex ion $[Cu(NH_3)_4]^{2+}$ is 2.1×10^{13}. The overall dissociations constant is $y \times 10^{-14}$. Then y is ____.(Nearest integer)

Official Ans. by NTA (5)

Sol. Given $k_f = 2.1 \times 10^{13}$

$K_d = \frac{1}{k_f} = 4.7\times10^{-14}$

$\therefore$ y = 4.7 $\approx$ 5

8. 83 g of ethylene glycol dissolved in 625 g of water. The freezing point of the solution is _______K. (Nearest integer)

[Use : Molal Freezing point depression constant of water = 1.86 K kg mol^{-1}]

Freezing Point of water = 273 K

Atomic masses : C : 12.0 u, O : 16.0 u, H : 1.0 u]

Official Ans. by NTA (269)

Sol. k_f = 1.86 k. kg/mol

T_f^o = 273 k

solvent : H_2O(625 g)

Solute : 83 g $\begin{pmatrix} CH_2 - CH_2 \\ | \qquad | \\ OH \quad OH \end{pmatrix}$ $\Rightarrow$ Non dissociative solute

$\Rightarrow \Delta T_f = k_f \times m$

$\Rightarrow\left(T_f^o - T_f^1\right)=1.86\times\frac{83/62}{624/1000}$

$\Rightarrow 273 - T_f^1 = \frac{1.86\times83\times1000}{62\times625}=\frac{154380}{38750}$

$\Rightarrow 273 - T_f^1 = 4$

$\Rightarrow \boxed{T_f^1 = 259\text{ K}}$

9. The reaction rate for the reaction

$[PtCl_4]^{2-} + H_2O \qquad [Pt(H_2O)Cl_3]^- + Cl^-$

was measured as a function of concentrations of different species. It was observed that

$$\frac{-d\left[\left[PtCl_4\right]^{2-}\right]}{dt} = 4.8 \times 10^{-5}\left[\left[PtCl_4\right]^{2-}\right] -2.4 \times 10^{-3}\left[\left[Pt(H_2O)Cl_3\right]^-\right]\left[Cl^-\right].$$

where square brackets are used to denote molar concentrations. The equilibrium constant K_c=_____. (Nearest integer)

Official Ans. by NTA (50)

Sol. $[PtCl_4]^{-2} + H_2O \qquad [Pt(H_2O)Cl_3]^- + Cl^-$

$$\frac{-d[Pt\ Cl_4]^{-2}}{dt} = 4.8 \times 10^{-5}\ [PtCl_4^{-2}] - 2.4 \times 10^{3}$$

$[Pt(H_2O)Cl_3][\overset{o}{u}]$

$\Rightarrow K_{eq} = \frac{k_f}{k_b} = \frac{4.8\times10^{-5}}{2.4\times10^{-3}} = 0.02$

10. A chloro compound "A".

(i) forms aldehydes on ozonolysis followed by the hydrolysis.

(ii) when vaporized completely 1.53 g of A, gives 448 mL of vapour at STP.

The number of carbon atoms in a molecule of compound **A** is _______.

Official Ans. by NTA (3)

Sol. 448 ml of A $\Rightarrow$ 1.53 gm A

22400 ml of A $\Rightarrow \frac{1.53}{445} \times 22400$ gm A = 7650

$H_3CHC\text{–}CH\text{–}Cl \xrightarrow[Zn/H_2O]{O_3} CH_3\text{–}CH{=}O$
Aldehyde

It has 3 carbon atoms

& mm is 36 + 5 + 35.5 = 76.5

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x – [x], g(x) = 1 – x + [x], and h(x) = min{f(x), g(x)}, $x \in [-2, 2]$. Then h is :

(1) continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

(2) not continuous at exactly three points in [–2, 2]

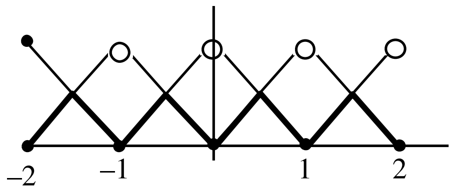
(3) continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)

(4) not continuous at exactly four points in [–2, 2]

Official Ans. by NTA (1)

Sol. min{x – [x], 1 – x + [x]}

h(x) = min{x – [x], 1 – [x – [x])}



$\Rightarrow$ always continuous in [–2, 2]

but non differentiable at 7 Points

2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :

(1) $A^6 - A$ (2) A^5

(3) $A^5 - A$ (4) A^6

Official Ans. by NTA (1)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^6 - A = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. The local maximum value of the function $f(x) = \left(\frac{2}{x}\right)^{x^2}$, $x > 0$, is

(1) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$ (2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) $(e)^{\frac{2}{e}}$ (4) 1

Official Ans. by NTA (3)

Sol. $f(x) = \left(\frac{2}{x}\right)^{x^2}$; $x > 0$

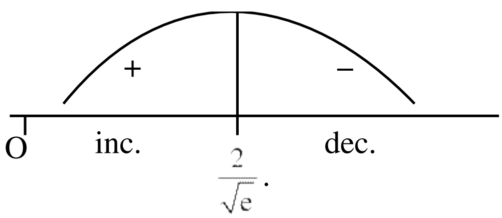
$\ell n\, f(x) = x^2 (\ell n 2 - \ell n x)$

$f'(x) = f(x)\, \{-x + (\ell n\, 2 - \ell n\, x) 2x\}$

$f'(x) = \underbrace{f(x)}_{+}.\underbrace{x}_{+}\underbrace{(2\ell n 2 - 2\ell n x - 1)}_{g(x)}$

$g(x) = 2\ell\, n^2 - 2\ell\, n\, x - 1$

$= \ell n \frac{4}{x^2} - 1 = 0 \Rightarrow x = \frac{2}{\sqrt{e}}$



$LM = \frac{2}{\sqrt{e}}$

Local maximum value $= \left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$

4. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to :

(1) 100 (2) 25

(3) 16 (4) 36

Official Ans. by NTA (2)

Sol. $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}}dx$

$$\int_0^1 \frac{x}{e^x}dx + \int_1^2 \frac{x+1}{e^{x-1}}dx + \int_2^3 \frac{x+2}{e^{x-2}}dx + ...\int_4^5 \frac{x+4}{e^{x-4}}dx$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$x = t + 1 \qquad x = z + 2 \qquad x = y + 4$$

$$\int_0^1 \frac{t+2}{e^t}dt + \int_0^1 \frac{z+4}{e^z}dz + ... + \int_0^1 \frac{y+8}{e^y}dx$$

$$\Rightarrow \int_0^5 \frac{5x+20}{e^x}dt = 5\int_0^1 \frac{x+4}{e^x}dx$$

$$\Rightarrow 5\int_0^1 (x+4)e^{-x}dx$$

$$\Rightarrow 5e^{-x}(-x-5)|_0^1 \Rightarrow -\frac{30}{e} + 25$$

$\alpha = -30$

$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

5. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :

(1) $4\sqrt{3}$ (2) 6

(3) $6\sqrt{3}$ (4) $3\sqrt{6}$

Official Ans. by NTA (3)

Sol. $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

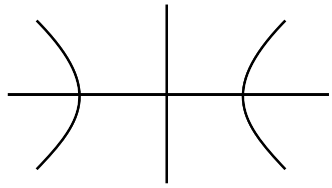
$$\Rightarrow \frac{24}{a^2} - \frac{3}{b^2} = 1 \quad ....(i)$$

$$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4} - 1\right) \Rightarrow 4b^2 = a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2} - \frac{3}{b^2} = 1 \Rightarrow b = \sqrt{3}$

$$\Rightarrow a = \sqrt{12}$$

$$\frac{x^2}{12} - \frac{y^2}{3} = 1$$



Tangent at P :

$$\frac{-x}{\sqrt{6}} - \frac{y}{\sqrt{3}} = 1 \Rightarrow Q\ (0, \sqrt{3})$$

Slope of $T = -\frac{1}{\sqrt{2}}$

Normal at P :

$$y - \sqrt{3} = \sqrt{2}(x + 2\sqrt{6})$$

$$\Rightarrow R = (0, 5\sqrt{3})$$

$QR = 6\sqrt{3}$

6. Let $y(x)$ be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to :

(1) 0 (2) 2

(3) $\log_e 2$ (4) $\log_e(2e)$

Official Ans. by NTA (3)

Sol. $2x^2dy + (e^y - 2x)dx = 0$

$$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$$

$$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow \text{Put } e^{-y} = z$$

$$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$$

$$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through $(e,1)$

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

7. Consider the two statements :

$(S1) : (p \to q) \vee (\sim q \to p)$ is a tautology.

$(S2) : (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

(1) only (S1) is true.

(2) both (S1) and (S2) are false.

(3) both (S1) and (S2) are true.

(4) only (S2) is true.

Official Ans. by NTA (3)

Sol. $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t =$ tautology

$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

$=$ fallacy

8. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (2) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(3) $\left(-\frac{1}{2}, \infty\right) - \{0\}$ (4) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

Official Ans. by NTA (4)

Sol. $\frac{1+x}{x} \in (-\infty, -1] \cup [1, \infty)$

$\frac{1}{x} \in (-\infty, -2] \cup [0, \infty)$

$x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$

$x \in \left[-\frac{1}{2}, \infty\right) - \{0\}$

9. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is :

(1) $\frac{125}{216}$ (2) $\frac{11}{36}$ (3) $\frac{5}{6}$ (4) $\frac{25}{36}$

Official Ans. by NTA (4)

Sol. $P(x \geq 5 | x > 2) = \frac{P(x \geq 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots + \infty}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \ldots\ldots + \infty}$$

$$\frac{\dfrac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1-\frac{5}{6}}}{\dfrac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1-\frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

10. If $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is :

(1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50} \tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{50} \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

$\tan^{-1}(101) - \tan^{-1} 1 \Rightarrow \tan^{-1}\frac{50}{51}$

11. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$ (2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{5}{6}$ and $q = \frac{1}{36}$ (4) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

Official Ans. by NTA (2)

Sol. $D \neq 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \neq 0 \Rightarrow \mu \neq 3$

$p = \frac{5}{6}$

$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

Option (2)

12. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :

(1) $y^3(x-2) = x^2$ (2) $x^3(x-2) = y^2$

(3) $y^2(x-2) = x^3$ (4) $x^2(x-2) = y^3$

Official Ans. by NTA (3)

Sol. $T = S_1$

$xh - yk = h^2 - k^2$

$y = \frac{xh}{k} - \frac{(h^2-k^2)}{k}$

this touches $y^2 = 8x$ then $c = \frac{a}{m}$

$\left(\frac{k^2-h^2}{k}\right) = \frac{2k}{h}$

$2y^2 = x(y^2 - x^2)$

$y^2(x-2) = x^3$

13. The value of

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

is :

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}$

(3) $\frac{1}{8}$ (4) $\frac{1}{8\sqrt{2}}$

Official Ans. by NTA (3)

Sol. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$

14. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation :

(1) $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

(2) $x^2 + \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

(3) $x^2 + \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

(4) $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

Official Ans. by NTA (1)

Sol. $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p+iq)$

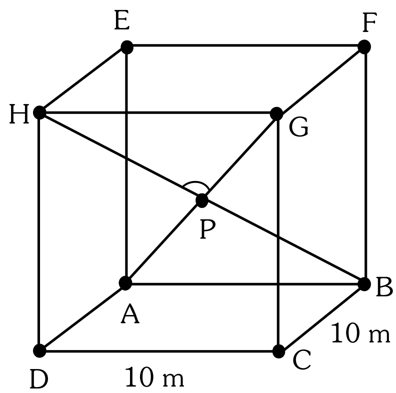
$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p+iq)$

$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$p = -1, q = \sqrt{3}$

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0.$

15. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



(1) 5 (2) $2\sqrt{10}$

(3) $5\sqrt{3}$ (4) $5\sqrt{2}$

Official Ans. by NTA (4)

Sol. $A(\hat{j})$. $B(10\hat{i})$

H $(h\hat{j} + 10\hat{k})$

G $(10\hat{i} + h\hat{j} + 10\hat{k})$

$\overrightarrow{AG} = 10\hat{i} + h\hat{j} + 10\hat{k}$

$\overrightarrow{BH} = -10\hat{i} + h\hat{j} + 10\hat{k}$

$\cos\theta = \dfrac{\overrightarrow{AG}\,\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$

$\dfrac{1}{5} = \dfrac{h^2}{h^2 + 200}$

$4h^2 = 200 \Rightarrow h = 5\sqrt{2}$

16. Let P be the plane passing through the point (1,2,3) and the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+4\hat{k}\right)=16$ and $\vec{r}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)=6$. Then which of the following points does **NOT** lie on P ?

(1) (3, 3, 2) (2) (6, –6, 2)

(3) (4, 2, 2) (4) (–8, 8, 6)

Official Ans. by NTA (3)

Sol. $(x+y+4z-16)+\lambda(-x+y+z-6)=0$

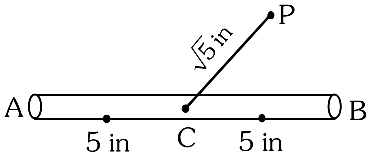
Passes through (1,2,3)

$-1+\lambda(-2) \Rightarrow \lambda = -\dfrac{1}{2}$

$2(x+y+4z-16)-(-x+y+z-6)=0$

$3x + y + 7z - 26 = 0$

17. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

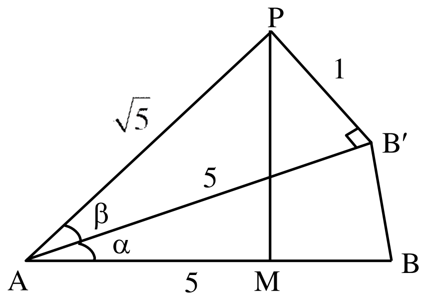


(1) $\tan^{-1}\left(\dfrac{3}{4}\right)$ (2) $\tan^{-1}(1)$

(3) $\tan^{-1}\left(\dfrac{4}{3}\right)$ (4) $\tan^{-1}\left(\dfrac{1}{2}\right)$

Official Ans. by NTA (1)

Sol.



From figure.

$\sin\beta = \dfrac{1}{\sqrt{5}}$

$\therefore \tan\beta = \dfrac{1}{2}$

$\tan(\alpha+\beta) = 2$

$\dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta} = 2$

$\dfrac{\tan\alpha + \dfrac{1}{2}}{1-\tan\alpha\left(\dfrac{1}{2}\right)} = 2$

$\tan\alpha = \dfrac{3}{4}$

$\boxed{\alpha = \tan^{1}\left(\dfrac{3}{4}\right)}$

18. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is

(1) $\dfrac{\pi}{2}$ (2) $\dfrac{5\pi}{4}$

(3) $\dfrac{3\pi}{4}$ (4) $\dfrac{3\pi}{2}$

Official Ans. by NTA (3)

Sol. $I = \displaystyle\int_0^{\pi/2}\frac{(1+\sin^2 x)}{(1+\pi^{\sin x})} + \frac{\pi^{\sin x}(1+\sin^2 x)}{(1+\pi^{\sin x})}dx$

$I = \int_0^{\pi/2}(1+\sin^2 x)dx$

$I = \dfrac{\pi}{2} + \dfrac{\pi}{2}\cdot\dfrac{1}{2} = \dfrac{3\pi}{4}$

19. A circle C touches the line x = 2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $7\sqrt{5}$ (2) 15

(3) $\sqrt{285}$ (4) $4\sqrt{15}$

Official Ans. by NTA (1)

Sol. $(x-2)^2+(y-1)^2+\lambda(x-2y)=0$

$C : x^2+y^2+x(\lambda-4)+y(-2-2\lambda)+5=0$

$C_1 : x^2+y^2+2y-5=0$

$S_1-S_2=0$ (Equation of PQ)

$(\lambda-4)x-(2\lambda+4)y+10=0$ Passes through $(0,-1)$

$\Rightarrow \lambda=-7$

$C : x^2+y^2-11x+12y+5=0$

$=\dfrac{\sqrt{245}}{4}$

Diometer $=7\sqrt{5}$

20. $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to :

(1) $\dfrac{9}{44}$ (2) $\dfrac{5}{24}$

(3) $\dfrac{1}{5}$ (4) $\dfrac{7}{36}$

Official Ans. by NTA (1)

Sol. $S=\lim\limits_{x\to 2}\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}$

$$S=\sum_{n=1}^{9}\frac{2}{4(n^2+3n+2)}=\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$S=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{9}{44}$$

SECTION-B

1. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is ________.

Official Ans. by NTA (7744)

Sol. 209, 220, 231, ..., 495

Sum $=\dfrac{27}{2}(209+495)=9504$

Number containing 1 at unit place
$\underline{2}\ \underline{3}\ \underline{1}$
$\underline{3}\ \underline{4}\ \underline{1}$
$\underline{4}\ \underline{5}\ \underline{1}$

Number containing 1 at 10^{th} place
$\underline{3}\ \underline{1}\ \underline{9}$
$\underline{4}\ \underline{1}\ \underline{8}$

Required $=9501-(231+341+451+319+418)$

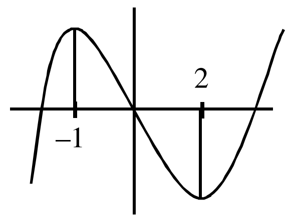
7744

2. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$. If A is the total area of the region bounded by $y=f(x)$, the x-axis and the lines $x=a$ and $x=b$, then 4A is equal to ________.

Official Ans. by NTA (114)

Sol. $f'(x)=6x^2-6x-12=6(x-2)(x+1)$

Point $=(2,-20)$ & $(-1,7)$



$$A=\int_{-1}^{0}(2x^3-3x^2-12x)\,dx+\int_{0}^{2}(12x+3x^2-2x^3)\,dx$$

$$A=\left(\frac{x^4}{2}-x^3-6x^2\right)_{-1}^{0}+\left(6x^2+x^3-\frac{x^4}{2}\right)_{0}^{2}$$

$4A=114$

3. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ______.

Official Ans. by NTA (5)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1,\ \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100\Rightarrow\lambda=5.$

4. Let $a_1, a_2,\ldots,a_{10}$ be an AP with common difference -3 and $b_1, b_2,\ldots, b_{10}$ be a GP with common ratio 2. Let $c_k=a_k+b_k$, $k=1,2,\ldots,10$. If $c_2=12$ and $c_3=13$, then $\sum\limits_{k=1}^{10}c_k$ is equal to _____.

Official Ans. by NTA (2021)

Sol. $c_2=a_2+b_2=a_1-3+2b_1=12$

$a_1 + 2b_1 = 15$ ____(1)

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$a_1 + 4b_1 = 19$ ____(2)

from (1) & (2) $b_1 = 2$, $a_1 = 11$

$$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$$= \frac{10}{2}(2\times 11 + 9\times(-3)) + \frac{2(2^{10}-1)}{2-1}$$

$= 5(22 - 27) + 2(1023)$
$= 2046 - 25 = 2021$

5. Let Q be the foot of the perpendicular from the point P(7,–2,13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$. Then $(PQ)^2$, is equal to ________.

Official Ans. by NTA (96)

Sol. Containing the line $\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$

$9(x + 1) - 18(y - 1) + 9(z - 3) = 0$

$x - 2y + z = 0$

$PQ = \left|\frac{7+4+13}{\sqrt{6}}\right| = 4\sqrt{6}$

$PQ^2 = 96$

6. Let $\begin{pmatrix} n \\ k \end{pmatrix}$ denotes ${}^nC_k$ and $\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \begin{pmatrix} n \\ k \end{pmatrix}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$

If $A_k = \sum_{i=0}^{9} \begin{pmatrix} 9 \\ i \end{pmatrix}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8} \begin{pmatrix} 8 \\ i \end{pmatrix}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$

and $A_4 - A_3 = 190$ p, then p is equal to :

Official Ans. by NTA (49)

Sol. $A_k = \sum_{i=0}^{9} {}^9C_i \, {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^8C_i \, {}^{13}C_{k-i}$

$A_k = {}^{21}C_k + {}^{21}C_k = 2\cdot{}^{21}C_k$

$A_4 - A_3 = 2\left({}^{21}C_4 - {}^{21}C_3\right) = 2(5985 - 1330)$

$190\text{ p} = 2(5985 - 1330) \Rightarrow p = 49.$

7. Let $\lambda \ne 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _________.

Official Ans. by NTA (18)

Sol. $3\alpha^2 - 10\alpha + 27\lambda = 0$ _____(1)

$\alpha^2 - \alpha + 2\lambda = 0$ _____(2)

(1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda - 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \ne 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\times 3}{\frac{1}{9}} = 18$$

8. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is ________.

Official Ans. by NTA (12)

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x + y = 10$

$\text{Var}(x) = 10 = \frac{3^2+7^2+x^2+y^2}{4} - 25$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

$x + y = 10$

$\Rightarrow (x,y) = (9,1)$

Four numbers are 21,9,10,8

Mean $= \frac{48}{4} = 12$

9. Let A be a 3 × 3 real matrix.

If $\det(2\text{Adj}(2\text{ Adj}(\text{Adj}(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equal _____.

Official Ans. by NTA (4)

Sol. adj (2A) = 2^2 adjA

$\Rightarrow$ adj(adj (2A)) = adj(4 adjA) = 16 adj (adj A)

= 16 |A| A

$\Rightarrow$ adj (32 |A| A) = $(32\ |A|)^2$ adj A

$12(32|A|)^2$ |adj A| = 2^3 $(32|A|)^6$ |adj A|

$2^3.2^{30}$ $|A|^6$. $|A|^2$ = 2^{41}

$|A|^8 = 2^8$ $\Rightarrow$ |A| = ±2

$|A|^2 = |A|^2 = 4$

10. The least positive integer n such that

$\frac{(2i)^n}{(1-i)^{n-2}}, i=\sqrt{-1}$ is a positive integer, is ______.

Official Ans. by NTA (6)

Sol. $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}} \cdot i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$

This is positive integer for n = 6

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

1. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin :-

(1) $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{(Z^2 + R^2)^{1/2}}\right)$

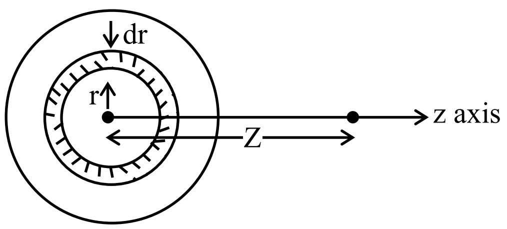
(2) $E = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{Z}{(Z^2 + R^2)^{1/2}}\right)$

(3) $E = \frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2 + R^2)^{1/2}} + Z\right)$

(4) $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{(Z^2 + R^2)} + \frac{1}{Z^2}\right)$

Official Ans. by NTA (1)

Sol. Consider a small ring of radius r and thickness dr on disc.



area of elemental ring on disc

$dA = 2\pi r dr$

charge on this ring $dq = \sigma dA$

$$dE_z = \frac{kdqz}{\left(z^2 + r^2\right)^{3/2}}$$

$$E = \int_0^R dE_z = \frac{\sigma}{2\in_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

2. There are 10^{10} radioactive nuclei in a given radioactive element, Its half-life time is 1 minute. How many nuclei will remain after 30 seconds ?

$\left(\sqrt{2} = 1.414\right)$

(1) 2×10^{10}

(2) 7×10^9

(3) 10^5

(4) 4×10^{10}

Official Ans. by NTA (2)

Sol. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$

$$\frac{N}{10^{10}} = \left(\frac{1}{2}\right)^{\frac{30}{60}}$$

$$\Rightarrow \quad N = 10^{10} \times \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{10^{10}}{\sqrt{2}} \approx 7 \times 10^9$$

3. Which of the following is not a dimensionless quantity ?

(1) Relative magnetic permeability (μ_r)

(2) Power factor

(3) Permeability of free space (μ_0)

(4) Quality factor

Official Ans. by NTA (3)

Sol. $[\mu_r] = 1$ as $\mu_r = \frac{\mu}{\mu_m}$

[power factor ($\cos\phi$)] = 1

$\mu_0 = \frac{B_0}{H}$ (unit = NA^{-2}) : Not dimensionless

$[\mu_0] = [MLT^{-2}A^{-2}]$

quality factor (Q) = $\frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$

So Q is unitless & dimensionless.

4. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be :

(1) ohm (2) mho

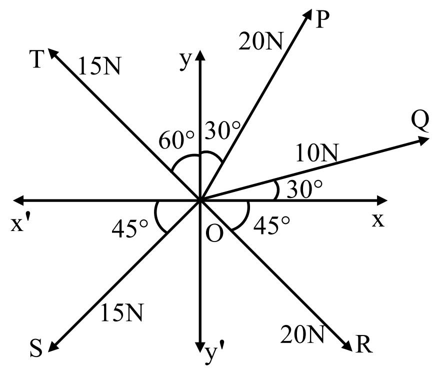
(3) joule (4) newton

Official Ans. by NTA (1)

Sol. Unit of $\frac{E}{H}$ is $\frac{\text{volt / metre}}{\text{Ampere / metre}} = \frac{\text{volt}}{\text{Ampere}}$ = ohm

5. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ...... N.

[Take $\sqrt{3} = 1.7, \sqrt{2} = 1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis]

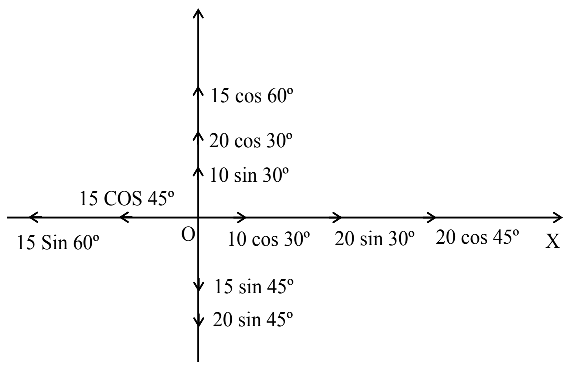


(1) $9.25\hat{i} + 5\hat{j}$ (2) $3\hat{i} + 15\hat{j}$

(3) $2.5\hat{i} - 14.5\hat{j}$ (4) $-1.5\hat{i} - 15.5\hat{j}$

Official Ans. by NTA (1)

Sol.



$$F_x = \left(10 \times \frac{\sqrt{3}}{2} + 20\left(\frac{1}{2}\right) + 20\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{\sqrt{3}}{2}\right)\right)i$$

= 9.25 i

$$F_y = \left(15\left(\frac{1}{2}\right) + 20\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 20\left(\frac{1}{\sqrt{2}}\right)\right)j$$

= 5 j

6. A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is –7°C. Assuming the volume constant ?

(1) 181.46 kg (2) 214.15 kg.

(3) 219.07 kg (4) 123.54 kg

Official Ans. by NTA (4)

Sol. $P_m = \rho RT$

$$\therefore \frac{P_1}{P_2} = \frac{\rho_1 T_1}{\rho_2 T_2}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{P_1 T_2}{P_2 T_1} = \left(\frac{76}{45}\right) \times \frac{266}{300}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{M_1}{M_2} = \frac{76 \times 266}{45 \times 300}$$

$$\therefore M_2 \Rightarrow \frac{45 \times 300 \times 185}{76 \times 266} = 123.54 \text{ kg}$$

7. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is :

(1) $\frac{2d_1d_2}{d_1 - d_2}$

(2) $\frac{2d_1d_2}{d_1 + d_2}$

(3) $\frac{d_1d_2}{d_1 + d_2}$

(4) $\frac{d_1d_2}{d_1 - d_2}$

Official Ans. by NTA (1)

Sol. Using Newton"s formula

$$(f + d_1)(f - d_2) = f^2$$

$$f^2 + fd_1 - fd_2 - d_1d_2 = f^2$$

$$f = \frac{d_1d_2}{d_1 - d_2}$$

$$\therefore R = \frac{2d_1d_2}{d_1 - d_2}$$

8. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second ?
Given : 1 ly = 9.46×10^{15} m
1 AU = 1.5×10^{11} m

(1) 4.1×10^{8} s (2) 4.5×10^{10} s
(3) 3.5×10^{6} s (4) 7.2×10^{8} s

Official Ans. by NTA (2)

Sol. $R = \frac{}{\theta}$

$\text{Time} = \frac{4\times 2\pi R}{v} = \frac{4\times 2\pi}{v}\left(\frac{}{\theta}\right)$

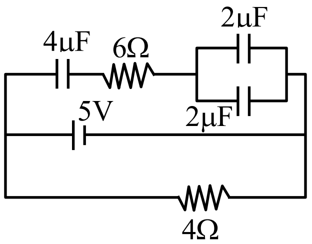
put $\ell = 4.4 \times 9.46 \times 10^{15}$

$v = 8 \times 1.5 \times 10^{11}$

$\theta = \frac{4}{3600} \times \frac{\pi}{180}$ rad.

we get time = 4.5×10^{10} sec

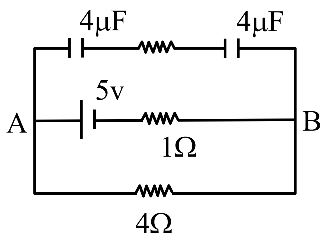
9. Calculate the amount of charge on capacitor of 4 μF. The internal resistance of battery is 1Ω :



(1) 8 μC (2) zero
(3) 16 μC (4) 4 μC

Official Ans. by NTA (1)

Sol. On simplifying circuit we get



No current in upper wire.

$\therefore\ V_{AB} = \frac{5}{4+1} \times 4 = 4$ v.

$\therefore\ \theta = (C_{eq})v$

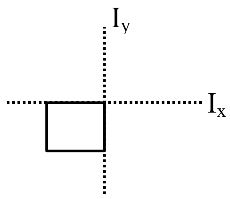
$\Rightarrow 2 \times 4 = 8\mu C$

10. Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by :

(1) $\frac{Ml^2}{6}$ (2) Ml^2 (3) $\frac{Ml^2}{12}$ (4) $\frac{2}{3}Ml^2$

Official Ans. by NTA (4)

Sol. According to perpendicular Axis theorem.



$I_x + I_y = I_z$

$I_z \Rightarrow \frac{m\ {}^2}{3} + \frac{m\ {}^2}{3}$

$= \frac{2m\ {}^2}{3}$

11. For a transistor in CE mode to be used as an amplifier, it must be operated in :
(1) Both cut-off and Saturation
(2) Saturation region only
(3) Cut-off region only
(4) The active region only

Official Ans. by NTA (4)

Sol. Active region of the CE transistor is linear region and is best suited for its use as an amplifier

12. An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is :

(1) $\frac{1}{T}$ (2) $\frac{2}{T}$ (3) $\frac{4}{T}$ (4) $\frac{3}{T}$

Official Ans. by NTA (3)

Sol. PT^3 = constant

$\left(\frac{nRT}{v}\right)T^3$ = constant

$T^4 V^{-1}$ = constant

$T^4 = kV$

$\Rightarrow 4\frac{\Delta T}{T} = \frac{\Delta V}{V}$ ....(1)

$\Delta V = V\gamma\Delta T$.............(2)

comparing (1) and (2)
we get

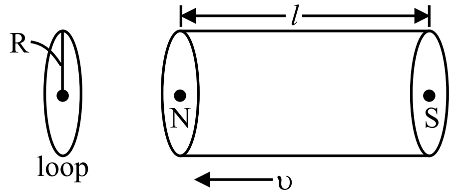
$\gamma = \frac{4}{T}$

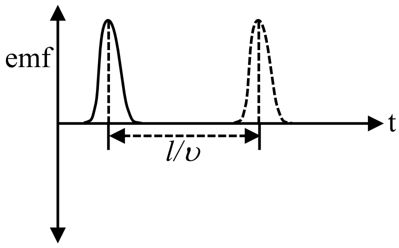
13. In a photoelectric experiment, increasing the intensity of incident light :
(1) increases the number of photons incident and also increases the K.E. of the ejected electrons
(2) increases the frequency of photons incident and increases the K.E. of the ejected electrons.
(3) increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged
(4) increases the number of photons incident and the K.E. of the ejected electrons remains unchanged

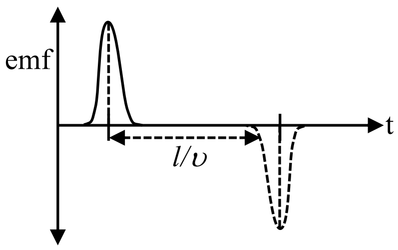
Official Ans. by NTA (4)

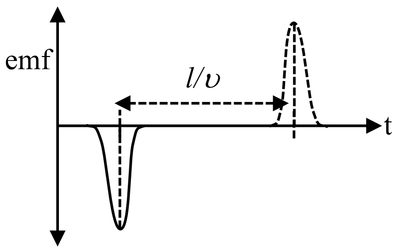
Sol. → Increasing intensity means number of incident photons are increased.
→Kinetic energy of ejected electrons depend on the frequency of incident photons, not the intensity.

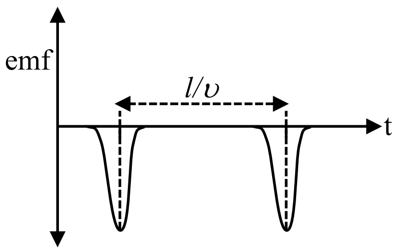
14. A bar magnet is passing through a conducting loop of radius R with velocity υ. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve :



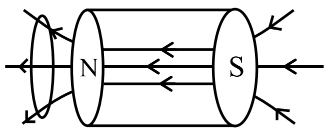
(1)


(2)


(3)


(4)


Official Ans. by NTA (3)

Sol.


$\rightarrow$ When magnet passes through centre region of solenoid , no current / Emf is induced in loop.

$\rightarrow$ While entering flux increases so negative induced emf

$\rightarrow$ While leaving flux decreases so positive induced emf.

15. Two ions of masses 4 amu and 16 amu have charges +2e and +3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then :

(1) lighter ion will be deflected less than heavier ion

(2) lighter ion will be deflected more than heavier ion

(3) both ions will be deflected equally

(4) no ion will be deflected.

Official Ans. by NTA (2)

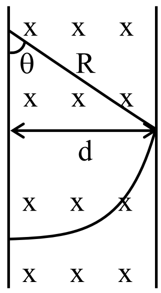
Sol. $r = \frac{P}{qB} = \frac{\sqrt{2mk}}{qB}$

Given they have same kinetic energy

$r \propto \frac{\sqrt{m}}{q}$

$\frac{r_1}{r_2} = \frac{\sqrt{4}}{2} \times \frac{3}{\sqrt{16}} = \frac{3}{4}$

$\boxed{r_2 = \frac{4r_1}{3}}$ (r_2 is for hearier ion and r_1 is for lighter ion)



$\sin\theta = \frac{d}{R}$

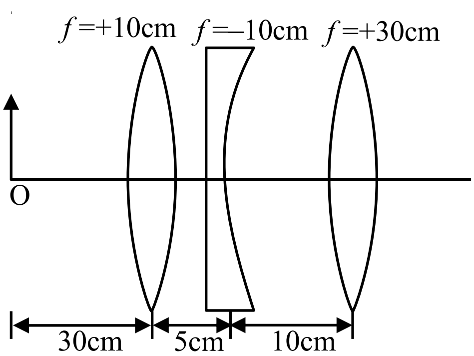
$\theta \rightarrow$ Deflection

$\theta \propto \frac{1}{R}$

($R \rightarrow$ Radius of path)

$\because R_2 > R_1 \Rightarrow \theta_2 < \theta_1$

16. Find the distance of the image from object O, formed by the combination of lenses in the figure :



(1) 75 cm (2) 10 cm

(3) 20 cm (4) infinity

Official Ans. by NTA (1)

Sol. $\frac{1}{V_1} + \frac{1}{30} = \frac{1}{10}$

$\frac{1}{V_1} = \frac{2}{30} \Rightarrow V_1 = 15$ cm

$\frac{1}{V_2} - \frac{1}{10} = -\frac{1}{10}$

$\frac{1}{V_2} = 0$ $\qquad$ $V_2 = \infty$

$V_3 = 30$ cm

$OV_3 = 75$ cm

17. In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^{3} kgm^{-3} ? Take viscosity of liquid = 1.8×10^{-5} Nsm^{-2}. (Neglect buoyancy due to air).

(1) 3.8×10^{-11} N $\qquad$ (2) 3.9×10^{-10} N

(3) 1.8×10^{-10} N $\qquad$ (4) 5.8×10^{-10} N

Official Ans. by NTA (2)

Sol. Viscous force = Weight

$= \rho \times \left(\frac{4}{3}\pi r^3\right)g$

$= 3.9 \times 10^{-10}$

18. Electric field in a plane electromagnetic wave is given by $E = 50 \sin(500x - 10 \times 10^{10}t)$ V/m

The velocity of electromagnetic wave in this medium is :

(Given C = speed of light in vacuum)

(1) $\frac{3}{2}C$ $\qquad$ (2) C $\qquad$ (3) $\frac{2}{3}C$ $\qquad$ (4) $\frac{C}{2}$

Official Ans. by NTA (3)

Sol. $V = \frac{\omega}{K} = \frac{10 \times 10^{10}}{500} = 2 \times 10^{8}$

$\mathbf{V} = \frac{2C}{3}$.

19. Five identical cells each of internal resistance 1Ω and emf 5V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same ?

(1) 1 Ω $\qquad$ (2) 25 Ω

(3) 5 Ω $\qquad$ (4) 10 Ω

Official Ans. by NTA (1)

Sol. $i_1 = \frac{25}{5+R}$

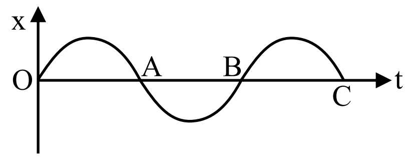
$i_2 = \frac{5}{R+\frac{1}{5}}$

$i_1 = i_2 \Rightarrow 5\left(R+\frac{1}{5}\right) = 5 + R$

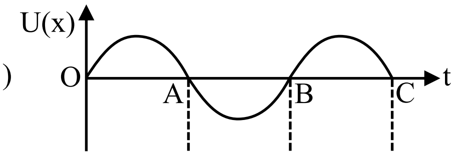
$4R = 4$

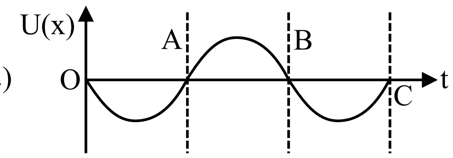
$R = 1\Omega$

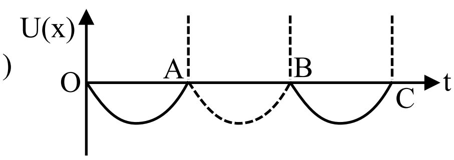
20. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

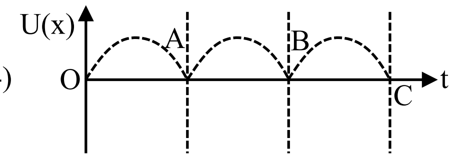


The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :

(1)


(2)


(3)


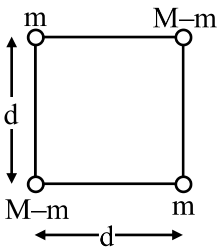
(4)


Official Ans. by NTA (4)

Sol. Potential energy is maximum at maximum distance from mean.

SECTION-B

1. A body of mass (2M) splits into four masses {m, M – m, m, M – m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ....



Official Ans. by NTA (2)

Sol. Energy is maximum when mass is split equally so

$$\frac{M}{m} = 2$$

2. The alternating current is given by

$$i = \left\{\sqrt{42}\sin\left(\frac{2\pi}{T}t\right) + 10\right\}A$$

The r.m.s. value of this current is .... A.

Official Ans. by NTA (11)

Sol. $f_{rms}^2 = f_{1rms}^2 + f_{2rms}^2$

$$= \left(\frac{\sqrt{42}}{\sqrt{2}}\right)^2 + 10^2$$

$= 121 \Rightarrow f_{rms} = 11$ A

3. A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1 : \sqrt{y}$ where y is ....

Official Ans. by NTA (3)

Sol. In triangle shape $N_t = \frac{24a}{3a} = 8$

In square $N_s = \frac{24a}{4a} = 6$

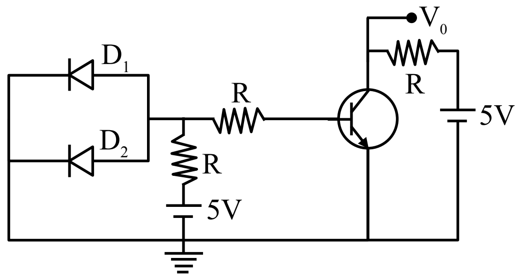
$\frac{M_t}{M_3} = \frac{N_t I A_t}{N_s I A_s}$ [I will be same in both]

$$= \frac{8 \times \frac{\sqrt{3}}{4} \times a^2}{6 \times a^2}$$

$$\frac{M_t}{M_s} = \frac{1}{\sqrt{3}}$$

$\boxed{y = 3}$

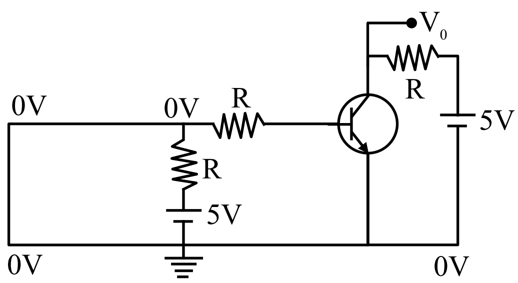
4. A circuit is arranged as shown in figure. The output voltage V_0 is equal to .... V.



Official Ans. by NTA (5)

Sol. As diodes D_1 and D_2 are in forward bias, so they acted as neligible resistances

$\Rightarrow$ Input voltage become zero



$\Rightarrow$ Input current is zero

$\Rightarrow$ Output current is zero

$\Rightarrow V_0 = 5$ volt

5. First, a set of n equal resistors of 10 Ω each are connected in series to a battery of emf 20V and internal resistance 10 Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is ....

Official Ans. by NTA (20)

Sol. In series

$R_{eq} = nR = 10\,n$

$$i_s = \frac{20}{10+10n} = \frac{2}{1+n}$$

in parallel

$$R_{eq} = \frac{10}{n}$$

$$i_p = \frac{20}{\frac{10}{n}+10} = \frac{2n}{1+n}$$

$$\frac{i_p}{i_s} = 20$$

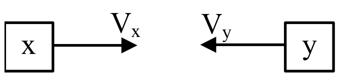
$$\frac{\left(\frac{2n}{1+n}\right)}{\left(\frac{2}{1+n}\right)} = 20$$

$n = 20$

6. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is .... Hz.

Official Ans. by NTA (1210)

Sol.



V_x = 36 km/hr = 10 m/s

V_y = 72 km/hr = 20 m/s

by doppler's effect

$$F' = F_0\left(\frac{V \pm V_0}{V \pm V_s}\right)$$

$$1320 = F_0\left(\frac{340+20}{340-10}\right) \Rightarrow F_0 = 1210\ \text{Hz}$$

7. If the velocity of a body related to displacement x is given by $\upsilon = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is ...... m/s^2.

Official Ans. by NTA (12)

Sol. $V = \sqrt{5000+24x}$

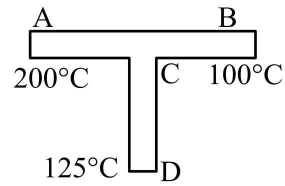
$$\frac{dV}{dx} = \frac{1}{2\sqrt{5000+24x}} \times 24 = \frac{12}{\sqrt{5000+24x}}$$

now $a = V\dfrac{dV}{dx}$

$$= \sqrt{5000+24x} \times \frac{12}{\sqrt{5000+24x}}$$

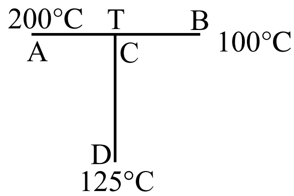
$$\boxed{a = 12\text{m/s}^2}$$

8. A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure, The end A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is ....



Official Ans. by NTA (2)

Sol.



Rods are identical so

$R_{AB} = R_{CD} = 10$ Kw^{-1}

C is mid-point of AB, so

$R_{AC} = R_{CB} = 5$ Kw^{-1}

at point C

$$\frac{200-T}{5} = \frac{T-125}{10} + \frac{T-100}{5}$$

2(200 – T) = T – 125 + 2(T – 100)

400 – 2 T = T – 125 + 2T – 200

$$T = \frac{725}{5} = 145°\text{C}$$

$$I_h = \frac{145-125}{10}\text{w} = \frac{20}{10}\text{w}$$

$$\boxed{I_h = 2\text{w}}$$

9. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ....

Official Ans. by NTA (2)

Sol. Given $W_A = W_B$

$F_A d\cos 45° = F_B d\cos 60°$

$$F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2}$$

$$\frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\boxed{x = 2}$$

10. A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is .... km.

Official Ans. by NTA (224)

Sol. $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

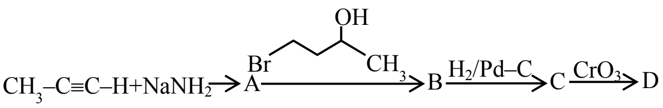
$$d_m = \left(\sqrt{2 \times 6400 \times 10^3 \times 320} + \sqrt{2 \times 6400 \times 10^3 \times 2000}\right)m$$

$$\boxed{d_m = 224km}$$

CHEMISTRY | **TEST PAPER WITH SOLUTION**

SECTION-A

1. In the following sequence of reactions, the final product D is :



(1) $H_3C-CH_2-CH_2-CH_2-CH_2-\overset{\overset{O}{\|}}{C}-H$

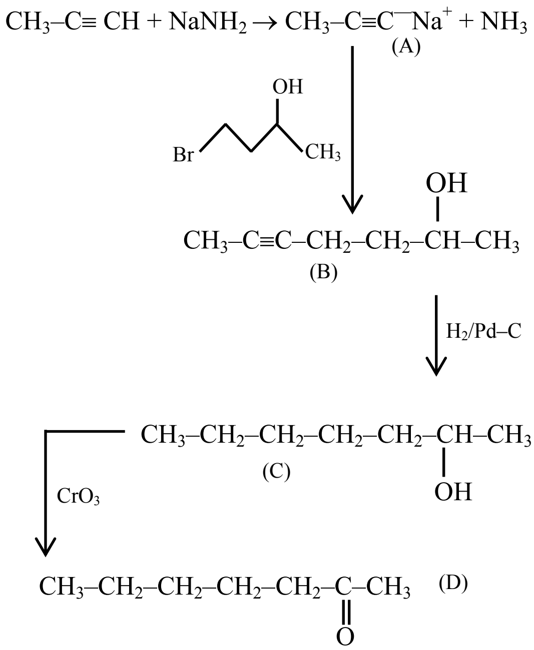
(2) $CH_3-CH=CH-CH_2-CH_2-CH_2-COOH$

(3) $H_3C-CH=CH-CH(OH)-CH_2-CH_2-CH_3$

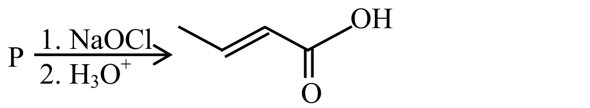
(4) $CH_3-CH_2-CH_2-CH_2-CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

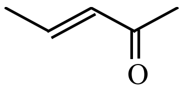
Official Ans. by NTA (4)

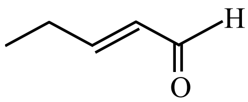
Sol.

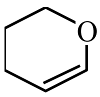


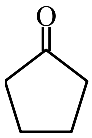
2. The structure of the starting compound **P** used in the reaction given below is :



(1) 

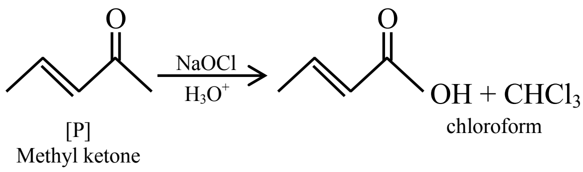
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



NaOCl is used in haloform reaction as reagent.

3. Match **List–I** with **List–II** :

List–I **(Species)**	**List–II** **(Number of lone pairs of electrons on the central atom)**
(a) XeF_2	(i) 0
(b) XeO_2F_2	(ii) 1
(c) XeO_3F_2	(iii) 2
(d) XeF_4	(iv) 3

Choose the **most appropriate** answer from the options given below :

(1) (a)–(iv), (b)–(i), (c)–(ii), (d)–(iii)

(2) (a)–(iii), (b)–(iv), (c)–(ii), (d)–(i)

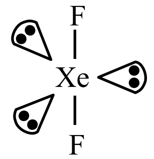
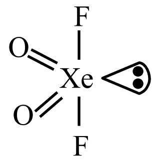
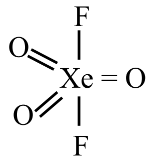
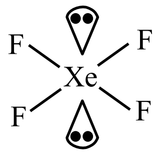
(3) (a)–(iii), (b)–(ii), (c)–(iv), (d)–(i)

(4) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)

Official Ans. by NTA (4)

Sol. **Species** **(Number of lone pairs of electrons**

on the central atom)

XeF_2	3	
XeO_2F_2	1	
XeO_3F_2	0	
XeF_4	2	

4. In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected?

(1) BCl_3 (2) BF_3

(3) BBr_3 (4) BI_3

Official Ans. by NTA (2)

Sol. Type of back bonding

BF_3	BCl	BBr_3	BI_3
(2pπ–2pπ)	(2pπ–3pπ)	(2pπ–4pπ)	(2pπ–5pπ)

Therefore back bonding strength is as follows

BF_3 > BCl > BBr_3 > BI_3

5. Deuterium resembles hydrogen in properties but :

(1) reacts slower than hydrogen

(2) reacts vigorously than hydrogen

(3) reacts just as hydrogen

(4) emits β^+ particles

Official Ans. by NTA (1)

Sol. The bond dissociation energy of D_2 is greater than H_2 and therefore D_2 reacts slower than H_2.

6. Which refining process is generally used in the purification of low melting metals ?

(1) Chromatographic method

(2) Liquation

(3) Electrolysis

(4) Zone refining

Official Ans. by NTA (2)

Sol. Liquation method is used to purify those impure metals which has lower melting point than the melting point of impurities associated.

∴ This method is used for metal having low melting point.

7. Match items of **List–I** with those of **List–II** :

List–I (Property)	**List–II (Example)**
(a) Diamagnetism	(i) MnO
(b) Ferrimagnetism	(ii) O_2
(c) Paramagnetism	(iii) NaCl
(d) Antiferromagnetism	(iv) Fe_3O_4

Choose the **most appropriate** answer from the options given below :

(1) (a)–(ii), (b)–(i), (c)–(iii), (d)–(iv)

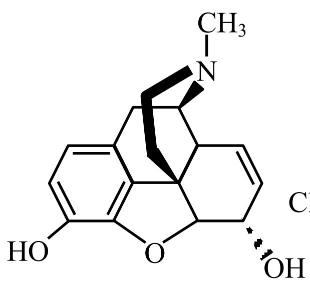
(2) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

(3) (a)–(iii), (b)–(iv), (c)–(ii), (d)–(i)

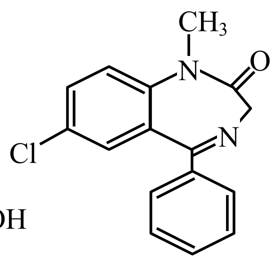
(4) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)

Official Ans. by NTA (3)

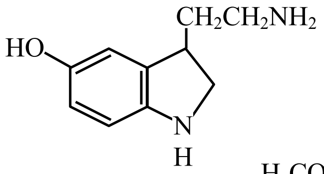
8.



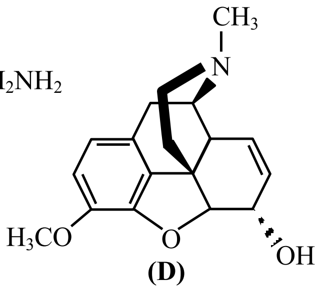
(A)



(B)



(C)



(D)

The correct statement about (A), (B), (C) and (D) is :

(1) (A), (B) and (C) are narcotic analgesics

(2) (B), (C) and (D) are tranquillizers

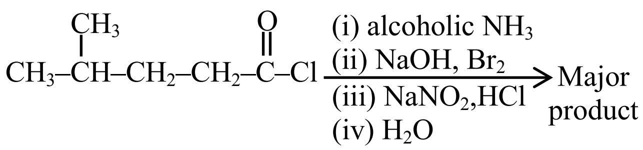
(3) (A) and (D) are tranquillizers

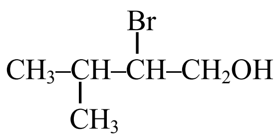
(4) (B) and (C) are tranquillizers

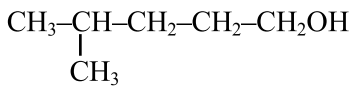
Official Ans. by NTA (4)

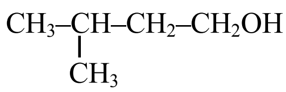
Sol. B and C are tranquilizers

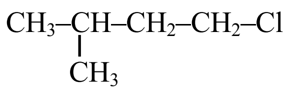
9. The major product of the following reaction is :



(1) 

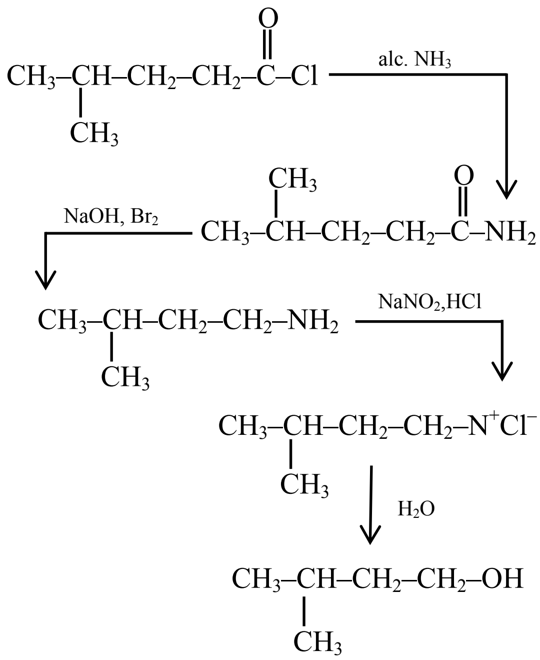
(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol.



10. Which of the following is **not** a correct statement for primary aliphatic amines?

(1) The intermolecular association in primary amines is less than the intermolecular association in secondary amines.

(2) Primary amines on treating with nitrous acid solution form corresponding alcohols except methyl amine.

(3) Primary amines are less basic than the secondary amines.

(4) Primary amines can be prepared by the Gabriel phthalimide synthesis.

Official Ans. by NTA (1)

Sol. The intermolecular association is more prominent in case of primary amines as compared to secondary, due to the availability of two hydrogen atom.

11. Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is :

(1) $Fe_4[Fe(CN)_6]_3$ (2) $K_5Fe[Fe(CN)_6]_2$

(3) $HFe[Fe(CN)_6]$ (4) $KFe[Fe(CN)_6]$

Official Ans. by NTA (4)

Sol. $FeCl_3 + K_4[Fe(CN)_6]$ (excess)

$\downarrow$

$K\,Fe[Fe(CN)_6]$

Colloidal species

12. The gas 'A' is having very low reactivity reaches to stratosphere. It is non–toxic and non–flammable but dissociated by UV—radiations in stratosphere. The intermediates formed initially from the gas 'A' are :

(1) $Cl\dot{O} + \dot{C}F_2Cl$ (2) $Cl\dot{O} + \dot{C}H_3$

(3) $\dot{C}H_3 + \dot{C}F_2Cl$ (4) $\dot{C}l + \dot{C}F_2Cl$

Official Ans. by NTA (4)

Sol. In stratosphere CFCs get broken down by powerful UV radiations releasing $Cl^{\bullet}$

$$CF_2Cl_2(g) \xrightarrow{U.V.} Cl^{\bullet}(g) + {}^{\bullet}CF_2Cl(g)$$

13. The number of water molecules in gypsum, dead burnt plaster and plaster of paris, respectively are:

(1) 2, 0 and 1 (2) 0.5, 0 and 2

(3) 5, 0 and 0.5 (4) 2, 0 and 0.5

Official Ans. by NTA (4)

Sol. Gypsum $CaSO_4.2H_2O$

Plaster of Paris $CaSO_4.\frac{1}{2}H_2O$

Dead burnt plaster $CaSO_4$

14. The nature of oxides V_2O_3 and CrO is indexed as 'X' and 'Y' type respectively. The correct set of X and Y is:

(1) X = basic Y = amphoteric

(2) X = amphoteric Y = basic

(3) X = acidic Y = acidic

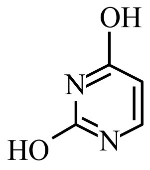
(4) X = basic Y = basic

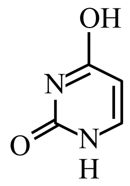
Official Ans. by NTA (4)

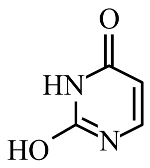
Sol. V_2O_3 basic

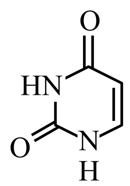
CrO basic

15. Out of following isomeric forms of uracil, which one is present in RNA ?

(1) 

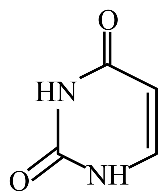
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. Isomeric form of uracil present in RNA



16. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A): Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

Reason (R): Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

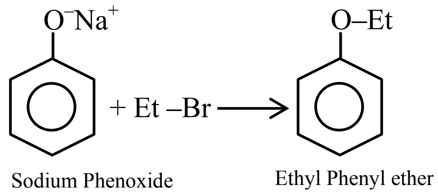
(2) **(A)** is correct but **(R)** is not correct

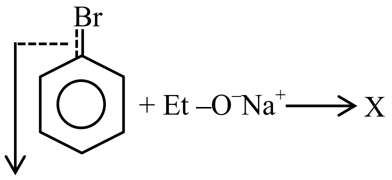
(3) **(A)** is not correct but **(R)** is correct

(4) Both **(A)** and **(R)** are correct but **(R)** is NOT the correct explanation of **(A)**

Official Ans. by NTA (2)

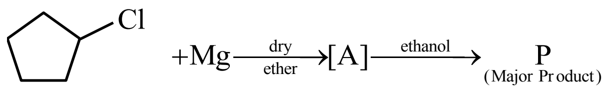
Sol.

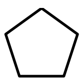


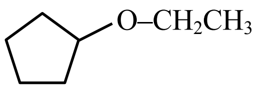


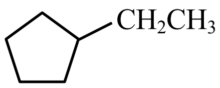
Partial double bond character

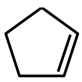
17. In the following sequence of reactions the P is :



(1) 

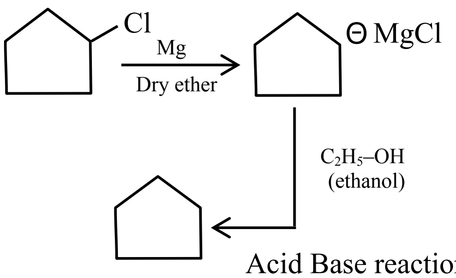
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



Acid Base reaction

18. The unit of the van der Waals gas equation parameter 'a' in $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$ is :

(1) kg m s^{-2} (2) dm^3 mol^{-1}

(3) kg m s^{-1} (4) atm dm^6 mol^{-2}

Official Ans. by NTA (4)

Sol. $\frac{an^2}{V^2}$ = atm $\Rightarrow$ a = atm $\times$ $\frac{dm^6}{mol^2}$

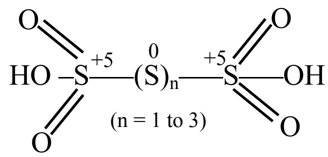
19. In polythionic acid, $H_2S_xO_6$(x = 3 to 5) the oxidation state(s) of sulphur is/are :

(1) + 5 only (2) + 6 only

(3) + 3 and + 5 only (4) 0 and + 5 only

Official Ans. by NTA (4)

Sol.



20. Tyndall effect is more effectively shown by :

(1) true solution (2) lyophilic colloid

(3) lyophobic colloid (4) suspension

Official Ans. by NTA (3)

Sol. Tyndall effect is observed in lyophobic colloids

SECTION-B

1. In Carius method for estimation of halogens, 0.2 g of an organic compound gave 0.188 g of AgBr. The percentage of bromine in the compound is _______ . (Nearest integer)

[Atomic mass : Ag = 108, Br = 80]

Official Ans. by NTA (40)

Sol. $n_{AgBr}=\frac{0.188g}{188g/mol}=10^{-3}mol$

$\Rightarrow n_{Br} = n_{AgBr}$ = 0.001 mol

$\Rightarrow mass_{Br}$ = (0.001 × 80) gm = 0.08 gm

$\Rightarrow$ mass % = $\frac{0.08\times100}{0.2}=40\%$

2. The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person's blood stream is

$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \rightarrow 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$

If the rate of appearance of $Cr_2(SO_4)_3$ is 2.67 mol min^{-1} at a particular time, the rate of disappearance of C_2H_6O at the same time is _______ mol min^{-1}. (Nearest integer)

Official Ans. by NTA (4)

Sol. $\left(\dfrac{\text{Rate of disappearance of } C_2H_6O}{3}\right)$

$= \left(\dfrac{\text{Rate of appearance of } Cr_2(SO_4)_3}{2}\right)$

$\Rightarrow \left(\dfrac{2.67 \text{mol}/\text{min} \times 3}{2}\right)$ = rate of disappearance of C_2H_6O.

$\Rightarrow$ Rate of disappearance of C_2H_6O = 4.005 mol/min

3. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is equal to $\dfrac{h^2}{xma_0^2}$. The value of 10x is _______ . (a_0 is radius of Bohr's orbit) (Nearest integer) [Given : π = 3.14]

Official Ans. by NTA (3155)

Sol. $mvr = \dfrac{nh}{2\pi}$

$$K.E. = \frac{n^2h^2}{8\pi^2mr^2} = \frac{4h^2}{8\pi^2m(4a_0)^2}$$

$$= \left(\frac{4}{8\pi^2 \times 16}\right)\frac{h^2}{ma_0^2}$$

$\Rightarrow x = 315.507$

$\Rightarrow 10x = 3155$ (nearest integer)

4. 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to –4°C before freezing. The amount of ice (in g) that will be separated out is _______ . (Nearest integer)

[Given : $K_f(H_2O) = 1.86$ K kg mol^{-1}]

Official Ans. by NTA (518)

Sol. Let mass of water initially present = x gm

$\Rightarrow$ Mass of sucrose = (1000 - x) gm

$\Rightarrow$ moles of sucrose = $\left(\dfrac{1000-x}{342}\right)$

$$\Rightarrow 0.75 = \frac{\left(\frac{1000-x}{342}\right)}{\left(\frac{x}{1000}\right)} \Rightarrow \frac{x}{1000} = \frac{1000-x}{342 \times 0.75}$$

$\Rightarrow 256.5\,x = 10^6 - 1000x$

$\Rightarrow x = 795.86$ gm

$\Rightarrow$ moles of sucrose = 0.5969

New mass of H_2O = a kg

$\Rightarrow 4 = \dfrac{0.5969}{a} \times 1.86 \Rightarrow a = 0.2775$ kg

$\Rightarrow$ ice separated = (795.86 – 277.5) = 518.3 gm

5. 1 mol of an octahedral metal complex with formula $MCl_3 \cdot 2L$ on reaction with excess of $AgNO_3$ gives 1 mol of AgCl. The denticity of Ligand L is _______ . (Integer answer)

Official Ans. by NTA (2)

Sol. $MCl_3.2L$ octahedral

$$\underset{1\text{mole}}{MCl_3.2L} \xrightarrow{\text{Ex.}AgNO_3} 1 \text{ mole of } AgCl$$

Its means that one Cl^- ion present in ionization sphere.

$\therefore$ formula = $[MCl_2L_2]Cl$

For octahedral complex coordination no. is 6

$\therefore$ L act as bidentate ligand

6. The number of moles of CuO, that will be utilized in Dumas method for estimation nitrogen in a sample of 57.5g of N, N-dimethylaminopentane is _______ $\times 10^{-2}$. (Nearest integer)

Official Ans. by NTA (1125)

Sol. Moles of N in N,N - dimethylaminopentane

$= \left(\dfrac{57.5}{115}\right) = 0.5 \text{mol}$

$$\Rightarrow C_7H_{17}N + \frac{45}{2}CuO \rightarrow 7CO_2 + \frac{17}{2}H_2O + \frac{1}{2}N_2 + \frac{45}{2}Cu$$

$$\frac{n_{CuO}\text{ reacted}}{\left(\frac{45}{2}\right)} = \frac{n_{C_7H_{17}N}\text{ reacted}}{1}$$

$$\Rightarrow n_{CuO}\text{ reacted} = \left(\frac{45}{2}\right)\times 0.5 = 11.25$$

7. The number of *f* electrons in the ground state electronic configuration of Np (Z = 93) is _______. (Nearest integer)

Official Ans. by NTA (4)

Allen Ans. (18)

Sol. Np = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ \boxed{4f^{14}}\ 5d^{10}\ 6p^6\ 7s^2\ \boxed{5f^4}\ 6d^1$

Total no. of 'f' electron = $14\ e^- + 4e^- = 18$

8. 200 mL of 0.2 M HCl is mixed with 300 mL of 0.1 M NaOH. The molar heat of neutralization of this reaction is –57.1 kJ. The increase in temperature in °C of the system on mixing is $x \times 10^{-2}$. The value of x is _______. (Nearest integer)

[Given : Specific heat of water = 4.18 J g^{-1} K^{-1}

Density of water = 1.00 g cm^{-3}]

(Assume no volume change on mixing)

Official Ans. by NTA (82)

Sol. ⇒ Millimoles of HCl = 200 × 0.2 = 40

⇒ Millimoles of NaOH = 300 × 0.1 = 30

$$\Rightarrow \text{ Heat released} = \left(\frac{30}{1000}\times 57.1\times 1000\right) = 1713\text{ J}$$

⇒ Mass of solution = 500 ml × 1 gm/ml = 500 gm

$$\Rightarrow \Delta T = \frac{q}{m\times C} = \frac{1713\text{J}}{500\text{g}\times 4.18\frac{\text{J}}{\text{g}-\text{K}}} = 0.8196\text{K}$$

$= 81.96 \times 10^{-2}$ K

9. The number of moles of NH_3, that must be added to 2 L of 0.80 M $AgNO_3$ in order to reduce the concentration of Ag^+ ions to 5.0×10^{-8} M ($K_{formation}$ for $[Ag(NH_3)_2]^+ = 1.0 \times 10^8$) is _______ . (Nearest integer)

[Assume no volume change on adding NH_3]

Official Ans. by NTA (4)

Sol. Let moles added = a

$$Ag^+_{(aq.)} + 2NH_{3(aq.)} \qquad Ag(NH_3)^+_{2(aq.)}$$

t = 0	0.8	$\left(\frac{a}{2}\right)$	
t = ∞	5×10^{-8}	$\left(\frac{a}{2}-1.6\right)$	0.8

$$\frac{0.8}{(5\times 10^{-8})\left(\frac{a}{2}-1.6\right)^2} = 10^8$$

$$\Rightarrow \quad \frac{a}{2}-1.6 = 0.4 \Rightarrow a = 4$$

10. When 10 mL of an aqueous solution of $KMnO_4$ was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of $KMnO_4$ in grams per litre is _______ $\times 10^{-2}$. (Nearest integer)

[Atomic mass of K = 39, Mn = 55, O = 16]

Official Ans. by NTA (316)

Sol. Let molarity of $KMnO_4$ = x

$KMnO_4 + FeSO_4 \rightarrow Fe_2(SO_4)_3 + Mn^{2+}$

n = 5 n = 1

(Equivalents of $KMnO_4$ reacted) = (Equivalents of $FeSO_4$ reacted)

⇒ (5 × x × 10 ml) = 1 × 0.1 × 10 ml

⇒ x = 0.02 M

Molar mass of $KMnO_4$ = 158 gm/mol

⇒ Strength = (x × 158) = 3.16 g/ℓ

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. If $0 < x < 1$, then $\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+.....$, is equal to :

(1) $x\left(\frac{1+x}{1-x}\right)+\log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right)+\log_e(1-x)$

(3) $\frac{1-x}{1+x}+\log_e(1-x)$

(4) $\frac{1+x}{1-x}+\log_e(1-x)$

Official Ans. by NTA (1)

Sol. Let $t=\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+....\infty$

$$=\left(2-\frac{1}{2}\right)x^2+\left(2-\frac{1}{3}\right)x^3+\left(2-\frac{1}{4}\right)x^4 +....\infty$$

$$=2\left(x^2+x^3+x^4+...\infty\right)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+...\infty\right)$$

$$=\frac{2x^2}{1-x}-\left(\ell n(1-x)-x\right)$$

$$\Rightarrow t=\frac{2x^2}{1-x}+x-\ell n(1-x)$$

$$\Rightarrow t=\frac{x(1+x)}{1-x}-\ell n(1-x)$$

2. If for $x, y \in \mathbf{R}$, $x > 0$,

$y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + ...$ upto ∞ terms

and $\frac{2+4+6+....+2y}{3+6+9+....+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to :

(1) $(10^6, 6)$ (2) $(10^4, 6)$

(3) $(10^2, 3)$ (4) $(10^6, 9)$

Official Ans. by NTA (4)

Sol. $\frac{2(1+2+3+...+y)}{3(1+2+3+...+y)}=\frac{4}{\log_{10}x}$

$\Rightarrow \log_{10}x = 6 \Rightarrow x = 10^6$

Now,

$$y=\left(\log_{10}x\right)+\left(\log_{10}x^{\frac{1}{3}}\right)+\left(\log_{10}x^{\frac{1}{9}}\right)+..\infty$$

$$=\left(1+\frac{1}{3}+\frac{1}{9}+...\infty\right)\log_{10}x$$

$$=\left(\frac{1}{1-\frac{1}{3}}\right)\log_{10}x=9$$

So, $(x,y) = (10^6,9)$

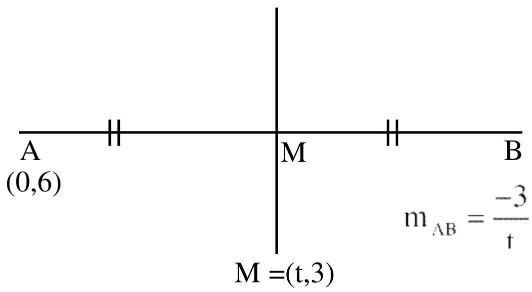
3. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

(1) $3x^2 - 2y - 6 = 0$ (2) $3x^2 + 2y - 6 = 0$

(3) $2x^2 + 3y - 9 = 0$ (4) $2x^2 - 3y + 9 = 0$

Official Ans. by NTA (3)

Sol. A(0,6) and B(2t,0)



Perpendicular bisector of AB is

$$(y-3)=\frac{t}{3}(x-t)$$

So, $C=\left(0,3-\frac{t^2}{3}\right)$

Let P be (h,k)

$$h=\frac{t}{2};k=\left(3-\frac{t^2}{6}\right)$$

$$\Rightarrow k=3-\frac{4h^2}{6}\Rightarrow 2x^2+3y-9=0 \text{ option (3)}$$

4. If $(\sin^{-1}x)^2-(\cos^{-1}x)^2=a;\ 0<x<1,\ a\neq 0$, then the value of $2x^2-1$ is :

(1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$

(3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

Official Ans. by NTA (2)

Sol. Given $a=\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2$

$=(\sin^{-1}x+\cos^{-1}x)(\sin^{-1}x-\cos^{-1}x)$

$=\frac{\pi}{2}\left(\frac{\pi}{2}-2\cos^{-1}x\right)$

$\Rightarrow 2\cos^{-1}x=\frac{\pi}{2}-\frac{2a}{\pi}$

$\Rightarrow \cos^{-1}\left(2x^2-1\right)=\frac{\pi}{2}-\frac{2a}{\pi}$

$\Rightarrow 2x^2-1=\cos\left(\frac{\pi}{2}-\frac{2a}{\pi}\right)$ option (2)

5. If the matrix $A=\begin{pmatrix}0 & 2\\ K & -1\end{pmatrix}$ satisfies $A(A^3+3I)=2I$, then the value of K is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

Official Ans. by NTA (1)

Sol. Given matrix $A=\begin{bmatrix}0 & 2\\ k & -1\end{bmatrix}$

$A^4+3\,IA=2I$

$\Rightarrow A^4=2I-3A$

Also characteristic equation of A is

$|A-\lambda I|=0$

$\Rightarrow \begin{vmatrix}0-\lambda & 2\\ k & -1-\lambda\end{vmatrix}=0$

$\Rightarrow \lambda+\lambda^2-2k=0$

$\Rightarrow A+A^2=2K.I$

$\Rightarrow A^2=2KI-A$

$\Rightarrow A^4=4K^2I+A^2-4AK$

Put $A^2=2KI-A$

and $A^4=2I-3A$

$2I-3A=4K^2I+2KI-A-4AK$

$\Rightarrow I(2-2K-4K^2)=A(2-4K)$

$\Rightarrow -2I\left(2K^2+K-1\right)=2A(1-2K)$

$\Rightarrow -2I(2K-1)(K+1)=2A(1-2K)$

$\Rightarrow (2K-1)(2A)-2I(2K-1)(K+1)=0$

$\Rightarrow (2K-1)[2A-2I(K+1)]=0$

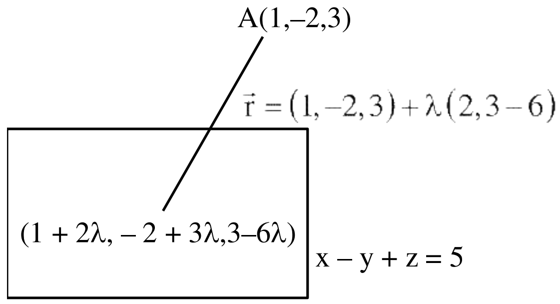
$\Rightarrow K=\frac{1}{2}$

6. The distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to a line, whose direction ratios are $2, 3, -6$ is :

(1) 3 (2) 5 (3) 2 (4) 1

Official Ans. by NTA (4)

Sol.



$(1+2\lambda)+2-3\lambda+3-6\lambda=5$

$\Rightarrow 6-7\lambda=5\Rightarrow\lambda=\frac{1}{7}$

so, $P=\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

$AP=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$

$AP=\sqrt{\left(\frac{4}{49}\right)+\frac{9}{49}+\frac{36}{49}}=1$

7. If $S=\left\{z\in\mathbb{C}:\frac{z-i}{z+2i}\in\mathbb{R}\right\}$, then :

(1) S contains exactly two elements

(2) S contains only one element

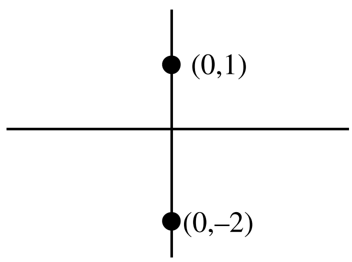
(3) S is a circle in the complex plane

(4) S is a straight line in the complex plane

Official Ans. by NTA (4)

Sol. Given $\frac{z-i}{z+2i}\in R$

Then $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or Π



$\Rightarrow$ S is straight line in complex

8. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)\,x - 2\cos x$ such that y(0) = 7. Then y(π) is equal to :

(1) $2e^{\pi^2}+5$ (2) $e^{\pi^2}+5$

(3) $3e^{\pi^2}+5$ (4) $7e^{\pi^2}+5$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}-2xy=2(2\sin x-5)x-2\cos x$

$IF=e^{-x^2}$

so, $y.e^{-x^2}=\int e^{-x^2}\left(2x(2\sin x-5)-2\cos x\right)dx$

$\Rightarrow y.e^{-x^2}=e^{-x^2}(5-2\sin x)+c$

$\Rightarrow y=5-2\sin x+c.e^{x^2}$

Given at x = 0, y = 7

$\Rightarrow 7=5+c\Rightarrow c=2$

So, $y=5-2\sin x+2e^{x^2}$

Now at x = π,

$y=5+2e^{\pi^2}$

9. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes x – y – z – 1 = 0 and 2x + y – 3z + 4 = 0, is :

(1) 3x – y – 5z + 2 = 0 (2) 3x – 4z + 3 = 0

(3) –x + 2y + 2z – 3 = 0 (4) 4x – y – 5z + 2 = 0

Official Ans. by NTA (4)

Sol. Required equation of plane

$P_1+\lambda P_2=0$

(x – y – z – 1) + λ(2x + y – 3z + 4) = 0

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$

Thus $\frac{|4\lambda-1|}{\sqrt{(2\lambda+1)^2+(\lambda-1)^2+(-3\lambda-1)^2}}=\frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda-1)^2=2(14\lambda^2+8\lambda+3)$

$\Rightarrow 336\lambda^2-168\lambda+21=28\lambda^2+16\lambda+6$

$\Rightarrow 308\lambda^2-184\lambda+15=0$

$\Rightarrow 308\lambda^2-154\lambda-30\lambda+15=0$

$\Rightarrow (2\lambda-1)(154\lambda-15)=0$

$\Rightarrow \lambda=\frac{1}{2}$ or $\frac{15}{154}$

for $\lambda=\frac{1}{2}$ reqd. plane is

4x – y – 5z + 2 = 0

10. If $U_n=\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2\ldots\left(1+\frac{n^2}{n^2}\right)^n$, then $\lim\limits_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to :

(1) $\frac{e^2}{16}$ (2) $\frac{4}{e}$ (3) $\frac{16}{e^2}$ (4) $\frac{4}{e^2}$

Official Ans. by NTA (1)

Sol. $U_n=\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$

$L=\lim\limits_{n\to\infty}(U_n)^{-4/n^2}$

$\log L=\lim\limits_{n\to\infty}\frac{-4}{n^2}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)^r$

$\Rightarrow \log L=\lim\limits_{n\to\infty}\sum_{r=1}^{n}-\frac{4r}{n}.\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)$

$\Rightarrow \log L\Rightarrow -4\int_0^1 x\log(1+x^2)dx$

put $1+x^2=t$

Now, 2xdx = dt

$=-2\int_1^2\log(t)dt=-2[t\log t-t]_1^2$

$\Rightarrow \log L=-2(2\log 2-1)$

$\therefore L=e^{-2(2\log 2-1)}$

$=e^{-2\left(\log\left(\frac{4}{e}\right)\right)}$

$=e^{\log\left(\frac{4}{e}\right)^{-2}}$

$=\left(\frac{e}{4}\right)^2=\frac{e^2}{16}$

11. The statement $(p\wedge(p\to q)\wedge(q\to r))\to r$ is :

(1) a tautology

(2) equivalent to $p\to\sim r$

(3) a fallacy

(4) equivalent to $q\to\sim r$

Official Ans. by NTA (1)

Sol. $(p\wedge(p\to q)\wedge(q\to r))\to r$

$\equiv(p\wedge(\sim p\vee q)\vee(\sim q\vee r))\to r$

$\equiv((p\wedge q)\wedge(\sim p\vee r))\to r$

$\equiv(p\wedge q\wedge r)\to r$

$\equiv\sim(p\wedge q\wedge r)\vee r$

$\equiv(\sim p)\vee(\sim q)\vee(\sim r)\vee r$

$\Rightarrow$ tautology

12. Let us consider a curve, y = f(x) passing through the point (–2, 2) and the slope of the tangent to the curve at any point (x, f(x)) is given by $f(x) + xf'(x) = x^2$. Then :

(1) $x^2 + 2xf(x) - 12 = 0$

(2) $x^3 + xf(x) + 12 = 0$

(3) $x^3 - 3xf(x) - 4 = 0$

(4) $x^2 + 2xf(x) + 4 = 0$

Official Ans. by NTA (3)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$

If $= e^{\int \frac{1}{x}dx} = x$

Solution of DE

$\Rightarrow y.x = \int x.x\ dx$

$\Rightarrow xy = \frac{x^3}{3} + \frac{c}{3}$

Passes through (–2,2), so

$-12 = -8 + c \Rightarrow c = -4$

$\therefore\ 3xy = x^3 - 4$

ie. $3x.f(x) = x^3 - 4$

13. $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ is equal to :

(1) ${}^{40}C_{21}$ (2) ${}^{40}C_{19}$ (3) ${}^{40}C_{20}$ (4) ${}^{41}C_{20}$

Official Ans. by NTA (3)

Sol. $\sum_{k=0}^{20} {}^{20}C_k . {}^{20}C_{20-k}$

sum of suffix is const. so summation will be

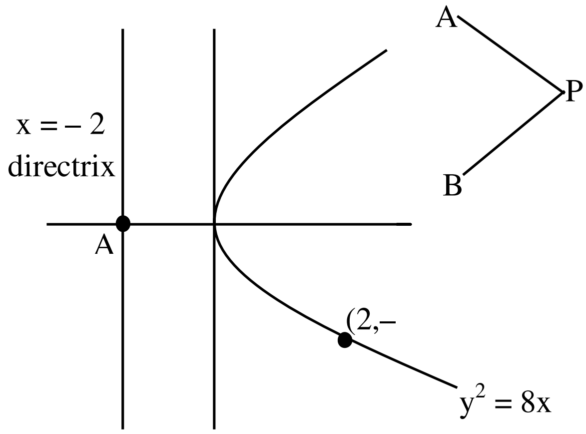
${}^{40}C_{20}$

14. A tangent and a normal are drawn at the point P(2, – 4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q(a, b) is a point such that AQBP is a square, then 2a + b is equal to :

(1) –16 (2) –18 (3) –12 (4) –20

Official Ans. by NTA (1)

Sol.



Equation of tangent at (2,–4) (T = 0)

$-4y = 4(x + 2)$

$x + y + 2 = 0$ …(1)

equation of normal

$x - y + \lambda = 0$

$\downarrow$(2,–4)

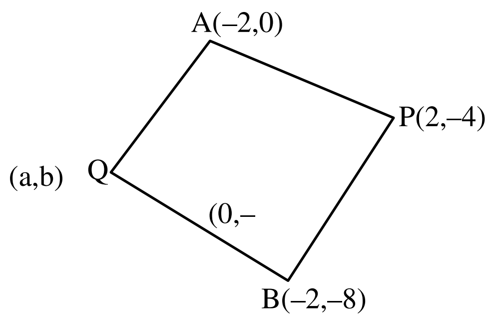
$\lambda = -6$

thus x – y = 6 …(2) equation of normal

POI of (1) & x = – 2 is A(–2,0)

POI of (2) & x = – 2 is A(–2,8)

Given AQBP is a sq.



$\Rightarrow m_{AQ}.m_{AP} = -1$

$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a + 2 = b$ …(1)

Also PQ must be parallel to x-axis thus

$\Rightarrow b = -4$

$\therefore\ a = -6$

Thus 2a + b = – 16

15. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

(1) $b^2 - a^2 = a^2 + c^2$
(2) b^2, c^2, a^2 are in A.P.
(3) c^2, a^2, b^2 are in A.P.
(4) a^2, b^2, c^2 are in A.P.

Official Ans. by NTA (2)

Sol. $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$

As A,B,C are angles of triangle

$A + B + C = \pi$

$A = \pi - (B + C)$

So, $\sin A = \sin(B + C)$ ...(1)

Similarly $\sin B = \sin(A + C)$ ...(2)

From (1) and (2)

$\frac{\sin(B+C)}{\sin(A+C)}=\frac{\sin(A-C)}{\sin(C-B)}$

$\sin(C + B).\sin(C - B) = \sin(A - C)\sin(A + C)$

$\sin^2 C-\sin^2 B=\sin^2 A-\sin^2 C$

$\{\because \sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y\}$

$2\sin^2 C=\sin^2 A+\sin^2 B$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

16. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim\limits_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to:

(1) $b^2 + 4c$ (2) $2(b^2 + 4c)$
(3) $2(b^2 - 4c)$ (4) $b^2 - 4c$

Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim\limits_{x\to\beta}\frac{1\left(1+\frac{2(x^2+bx+c)}{1!}+\frac{2^2(x^2+bx+c)^2}{2!}+...\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim\limits_{x\to\beta}\frac{2(x^2+bx+1)^2}{(x-\beta)^2}$

$\Rightarrow \lim\limits_{x\to\beta}\frac{2(x-\alpha)^2(x-\beta)^2}{(x-\beta)^2}$

$\Rightarrow 2(\beta-\alpha)^2=2(b^2-4c)$

17. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is:

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{9}$ (4) $\frac{1}{12}$

Official Ans. by NTA (2)

Sol. Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$=2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]$

$\Rightarrow 2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]=\frac{13}{96}$

(given)

$x=\frac{1}{8}$

18. If $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbb{R}$, then x and y respectively lie in the intervals:

(1) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

(2) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and [1, 3]

(3) [1, 3] and [1, 3]

(4) [1, 3] and $\left[-\frac{1}{3},\frac{1}{3}\right]$

Official Ans. by NTA (4)

Sol. $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x - 2)^2 + (3y)^2 = 1$

$\frac{(x-2)^2}{(1)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x-2\in[-1,1]$ and $y\in\left[-\frac{1}{3},\frac{1}{3}\right]$

$x\in[1,3]$ $y\in\left[-\frac{1}{3},\frac{1}{3}\right]$

19. $\int_{6}^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e(x^2 - 44x + 484)} dx$ is equal to:

(1) 6 (2) 8

(3) 5 (4) 10

Official Ans. by NTA (3)

Sol. Let $I = \int_{6}^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e(x^2 - 44x + 484)} dx$

$$I = \int_{6}^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x-22)^2} dx \quad \ldots(1)$$

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \text{ (king)}$$

So $I = \int_{6}^{16} \frac{\log_e (22-x)^2}{\log_e (22-x)^2 + \log_e (22-(22-x))^2}$

$$I = \int_{0}^{16} \frac{\log_e (22-x)^2}{\log_e x^2 + \log_e (22-x)^2} dx \quad \ldots(2)$$

(1) + (2)

$$2I = \int_{6}^{16} 1.dx = 10$$

$I = 5$

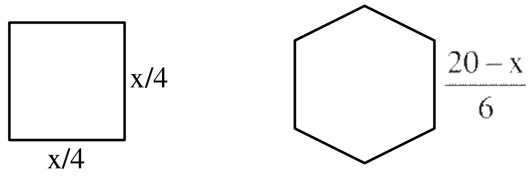
20. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is:

(1) $\frac{5}{2+\sqrt{3}}$ (2) $\frac{10}{2+3\sqrt{3}}$

(3) $\frac{5}{3+\sqrt{3}}$ (4) $\frac{10}{3+2\sqrt{3}}$

Official Ans. by NTA (4)

Sol. Let the wire is cut into two pieces of length x and 20 – x.



Area of square $= \left(\frac{x}{4}\right)^2$ Area of regular hexagon

$$= 6 \times \frac{\sqrt{3}}{4}\left(\frac{20-x}{6}\right)^2$$

Total area $= A(x) = \frac{x^2}{16} + \frac{3\sqrt{3}}{2}\frac{(20-x)^2}{36}$

$$A'(x) = \frac{2x}{16} + \frac{3\sqrt{3}\times 2}{2\times 36}(20-x)(-1)$$

$A'(x) = 0$ at $x = \frac{40\sqrt{3}}{3+2\sqrt{3}}$

Length of side of regular Hexagon $= \frac{1}{6}(20-x)$

$$= \frac{1}{6}\left(20 - \frac{4.\sqrt{3}}{3+2\sqrt{3}}\right)$$

$$= \frac{10}{2+2\sqrt{3}}$$

SECTION-B

1. Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _______.

Official Ans. by NTA (90)

Sol. since, $\vec{a}.\vec{b} = 0$

$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16 \quad \ldots(1)$

Also,

$$|\vec{b}\times\vec{c}|^2 = 75 \Rightarrow (10+\beta^2)14 - (5-3\beta)^2 = 75$$

$$\Rightarrow 5\beta^2 + 30\beta + 40 = 0$$

$$\Rightarrow \beta = -4, -2$$

$$\Rightarrow \alpha = 4, 8$$

$$\Rightarrow |\vec{a}|^2_{max} = (26+\alpha^2)_{max} = 90$$

2. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _________.

Official Ans. by NTA (4)

Sol. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

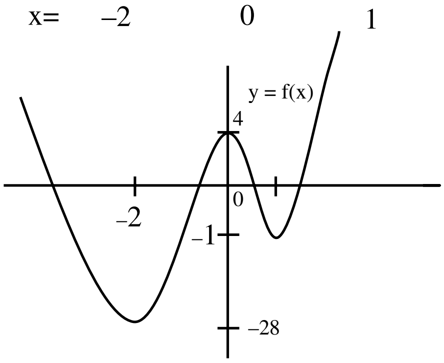
So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\therefore\ f'(x) = 12x(x^2 + x - 2)$

$= 12x\ (x + 2)\ (x - 1)$

$f'(x) = -\quad +\quad -\quad +$

$x = \quad -2 \quad 0 \quad 1$



3. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is _________.

Official Ans. by NTA (61)

Sol. $r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4} - 5} = \frac{\sqrt{2p^2 - 2p - 19}}{2}$

Since, $r \in (0, 5]$

So, $0 < 2p^2 - 2p - 19 \le 100$

$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$ so, number of integral values of p^2 is 61

4. If $A = \{x \in \mathbf{R} : |x - 2| > 1\}$, $B = \left\{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\right\}$, $C = \left\{x \in \mathbf{R} : |x - 4| \ge 2\right\}$ and $\mathbf{Z}$ is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap \mathbf{Z}$ is ___________.

Official Ans. by NTA (256)

Sol. $A = (-\infty, 1) \cup (3, \infty)$

$B = (-\infty, -2) \cup (2, \infty)$

$C = (-\infty, 2] \cup [6, \infty)$

So, $A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$

$z \cap (A \cap B \cap C)' = \{-2, -1, 0, -1, 2, 3, 4, 5\}$

Hence no. of its subsets $= 2^8 = 256$.

5. If $\int \frac{dx}{(x^2 + x + 1)^2} = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, $x > 0$ where C is the constant of integration, then the value of $9\left(\sqrt{3}a + b\right)$ is equal to ________.

Official Ans. by NTA (15)

Sol. $I = \int \frac{dx}{\left[\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right]^2}$

$\int \frac{dt}{\left(t^2 + \frac{3}{4}\right)^2} \quad \left(\text{Put } x + \frac{1}{2} = t\right)$

$= \frac{\sqrt{3}}{2} \int \frac{\sec^2\theta\, d\theta}{\frac{9}{16}\sec^4\theta} \quad \left(\text{Put } t = \frac{\sqrt{3}}{2}\tan\theta\right)$

$= \frac{4\sqrt{3}}{9} \int (1 + \cos 2\theta)\, d\theta$

$= \frac{4\sqrt{3}}{9}\left[\theta + \frac{\sin 2\theta}{2}\right] + c$

$= \frac{4\sqrt{3}}{9}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{\sqrt{3}(2x+1)}{3 + (2x+1)^2}\right] + c$

$= \frac{4\sqrt{3}}{9}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right) + c$

Hence, $9\left(\sqrt{3}a + b\right) = 15$

6. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _________.

Official Ans. by NTA (5)

Sol. $2 \times (i) - (ii) - (iii)$ gives :

$-(1 + \beta)z = 3 - \alpha$

For infinitely many solution

$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha, \beta) = (3, -1)$

Hence, $\alpha + \beta - \alpha\beta = 5$

7. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to ________.

Official Ans. by NTA (13)

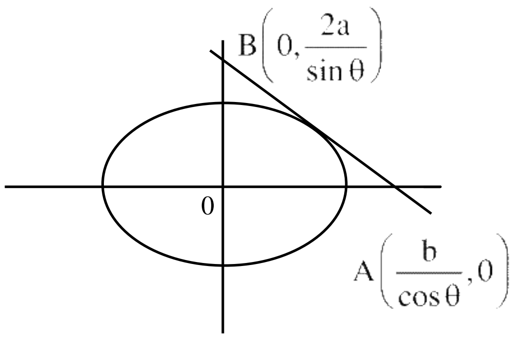
Sol. $\frac{n^2-1}{12}=14 \Rightarrow n=13$

8. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2}+\frac{y^2}{4a^2}=1$ and the co-ordinate axis is kab, then k is equal to _______.

Official Ans. by NTA (2)

Sol. Tangent

$$\frac{x\cos\theta}{b}+\frac{y\sin\theta}{2a}=1$$



So, $\text{area}(\Delta OAB)=\frac{1}{2}\times\frac{b}{\cos\theta}\times\frac{2a}{\sin\theta}$

$$=\frac{2ab}{\sin 2\theta}\geq 2ab$$

$$\Rightarrow k=2$$

9. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _________.

Official Ans. by NTA (100)

Sol.

5	a	b	b	a	5

It is always divisible by 5 and 11.

So, required number = 10 × 10 = 100

10. If $y^{1/4}+y^{-1/4}=2x$, and $(x^2-1)\frac{d^2y}{dx^2}+\alpha x\frac{dy}{dx}+\beta y=0$, then $|\alpha-\beta|$ is equal to __________.

Official Ans. by NTA (17)

Sol. $y^{\frac{1}{4}}+\frac{1}{y^{\frac{1}{4}}}=2x$

$$\Rightarrow\left(y^{\frac{1}{4}}\right)^2-2xy^{\left(\frac{1}{4}\right)}+1=0$$

$$\Rightarrow y^{\frac{1}{4}}=x+\sqrt{x^2-1}\text{ or }x-\sqrt{x^2-1}$$

So, $\frac{1}{4}\frac{1}{y^{\frac{3}{4}}}\frac{dy}{dx}=1+\frac{x}{\sqrt{x^2-1}}$

$$\Rightarrow\frac{1}{4}\frac{1}{y^{3/4}}\frac{dy}{dx}=\frac{y^{\frac{1}{4}}}{\sqrt{x^2-1}}$$

$$\Rightarrow\frac{dy}{dx}=\frac{4y}{\sqrt{x^2-1}}\quad\ldots(1)$$

Hence, $\frac{d^2y}{dx^2}=4\frac{\left(\sqrt{x^2-1}\right)y'-\frac{yx}{\sqrt{x^2-1}}}{x^2-1}$

$$\Rightarrow(x^2-1)y''=4\frac{(x^2-1)y'-xy}{\sqrt{x^2-1}}$$

$$\Rightarrow(x^2-1)y''=4\left(\sqrt{x^2-1}y'-\frac{xy}{\sqrt{x^2-1}}\right)$$

$$\Rightarrow(x^2-1)y''=4\left(4y-\frac{xy'}{4}\right)\text{ (from I)}$$

$$\Rightarrow(x^2-1)y''+xy'-16y=0$$

So, $|\alpha-\beta|=17$

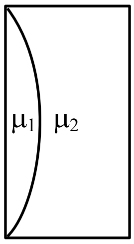
FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses.

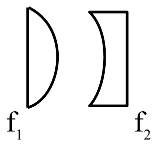


(1) $\dfrac{1}{\mu_2-\mu_1}$ (2) $\mu_1-\mu_2$

(3) $\dfrac{1}{\mu_1-\mu_2}$ (4) $\mu_2-\mu_1$

Official Ans. by NTA (2)

Sol.



$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R}\right)$$

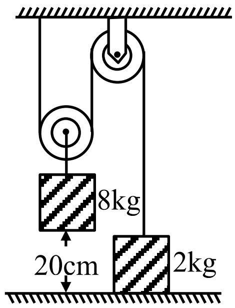
$$\frac{1}{f_2}=(\mu_2-1)\left(-\frac{1}{R}\right)$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f_{eq}}=\frac{(\mu_1-1)-(\mu_2-1)}{R}$$

$$\frac{1}{f_{eq}}=\frac{(\mu_1-\mu_2)}{R}$$

$$\frac{R}{f_{eq}}=(\mu_1-\mu_2)$$

2. The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use g = 10 m/s^2)

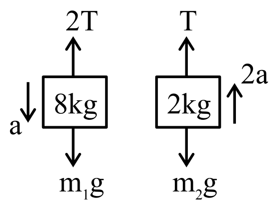


(1) 0.34 s (2) 0.2 s

(3) 0.25 s (4) 0.4 s

Official Ans. by NTA (4)

Sol.



$(m_1g-2T)=m_1a-(1)$

$T-m_2g=m_2(2a)$

$2T-2m_2g=4m_2\,a-(2)$

$m_1g-2m_2g=(m_1+4m_2)\,a$

$$a=\frac{(8-4)g}{(8+8)}=\frac{4}{16}g=\frac{g}{4}$$

$$a=\frac{10}{4}\ \text{m/s}^2$$

$$S=\frac{1}{2}at^2$$

$$\frac{0.2\times2\times4}{10}=t^2$$

$t=0.4$ sec

3. For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be :

(1) $\alpha = \frac{1-\beta}{\beta}$

(2) $\beta = \frac{\alpha}{1-\alpha}$

(3) $\alpha\beta = 1$

(4) $\alpha = \frac{\beta}{1-\beta}$

Official Ans. by NTA (2)

Sol. $\alpha = \frac{I_C}{I_E}$, $\beta = \frac{I_C}{I_B}$; $I_E = I_C + I_B$

$$\alpha = \frac{I_C}{I_C + I_B} = \frac{I_C / I_B}{\frac{I_C}{I_B} + 1} = \frac{\beta}{\beta + 1} +$$

$$1 + \frac{1}{\beta} = \frac{1}{\alpha}$$

$$\frac{1}{\beta} = \frac{1-\alpha}{\alpha}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

4. Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

(1) 4.18 m

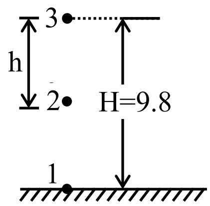
(2) 2.94 m

(3) 2.45 m

(4) 7.35 m

Official Ans. by NTA (4)

Sol.



$$H = \frac{1}{2}gt^2$$

$$\frac{9.8 \times 2}{9.8} = t^2$$

$t = \sqrt{2}$ sec

Δt: time interval between drops

$$h = \frac{1}{2}g(\sqrt{2} - \Delta t)^2$$

$$0 = \frac{1}{2}g(\sqrt{2} - 2\Delta t)^2$$

$$\Delta t = \frac{1}{\sqrt{2}}$$

$$h = \frac{1}{2}g\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \times 9.8 \times \frac{1}{2} = \frac{9.8}{4} = 2.45\text{m}$$

$H - h = 9.8 - 2.45$

$= 7.35$ m

5. Two discs have moments of intertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by :

(1) $\frac{I_1 I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$

(2) $\frac{(I_1 - I_2)^2 \omega_1 \omega_2}{2(I_1 + I_2)}$

(3) $\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$

(4) $\frac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

Official Ans. by NTA (3)

Sol. From conservation of angular momentum we get

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$k_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

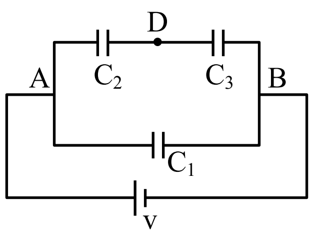
$$k_f = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$k_i - k_f = \frac{1}{2}\left[I_1\omega_1^2 + I_2\omega_2^2 - \frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}\right]$$

Solving above we get

$$k_i - k_f = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

6. Three capacitors $C_1 = 2\mu F$, $C_2 = 6\ \mu F$ and $C_3 = 12\ \mu F$ are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively :

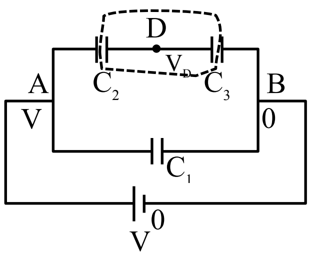


(1) 2 : 1 : 1 (2) 2 : 3 : 3

(3) 1 : 2 : 2 (4) 3 : 4 : 4

Official Ans. by NTA (3)

Sol.



$(V_D - V)\ C_2 + (V_D - 0)\ C_3 = 0$

$(V_D - V)\ 6 + (V_D - 0)\ 12 = 0$

$V_D - V + 2V_D = 0$

$$V_D = \frac{V}{3}$$

$$q_2 = (V - V_D)\ C_2 = \left(V - \frac{V}{3}\right)(6\ \mu F)$$

$q_2 = (4V)\ \mu F$

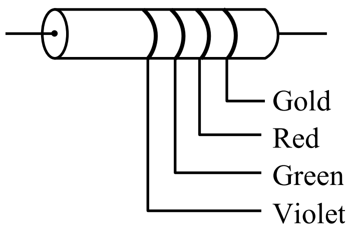
$$q_3 = (V_D - 0)\ C_3 = \frac{V}{3} \times 12\mu F = 4V\mu F$$

$q_1 = (V - 0)\ C_1 = V(2\mu F)$

$q_1 : q_2 : q_3 = 2 : 4 : 4$

$q_1 : q_2 : q_3 = 1 : 2 : 2$

7. The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is :



(1) $(5700 \pm 285)\ \Omega$

(2) $(7500 \pm 750)\ \Omega$

(3) $(5700 \pm 375)\ \Omega$

(4) $(7500 \pm 375)\ \Omega$

Official Ans. by NTA (4)

Sol. $R = 75 \times 10^2 \pm 5\%$ of 7500

$R = (7500 \pm 375)\Omega$

8. An antenna is mounted on a 400 m tall building. What will be the wavelength of signal of signal that can be radiated effectively by the transmission tower upto a range of 44 km?

(1) 37.8 m

(2) 605 m

(3) 75.6 m

(4) 302 m

Official Ans. by NTA (2)

Sol. h : height of antenna

λ : wavelength of signal

$h < \lambda$

$\lambda > h$

$\lambda > 400$ m

9. If the rms speed of oxygen molecules at 0°C is 160 m/s, find the rms speed of hydrogen molecules at 0°C.

(1) 640 m/s (2) 40 m/s

(3) 80 m/s (4) 332 m/s

Official Ans. by NTA (1)

Sol. $V_{rms} = \sqrt{\frac{3KT}{M}}$

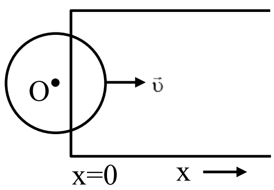
$$\frac{(V_{rms})_{O_2}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$$

$$(V_{rms})_{H_2} = 4 \times (V_{rms})_{O_2}$$

$= 4 \times 160$

$= 640$ m/s

10. A constant magnetic field of 1 T is applied in the x > 0 region. A metallic circular ring of radius 1m is moving with a constant velocity of 1 m/s along the x-axis. At t = 0s, the centre of O of the ring is at x = – 1m. What will be the value of the induced emf in the ring at t = 1s? (Assume the velocity of the ring does not change.)



(1) 1 V (2) 2π V

(3) 2 V (4) 0 V

Official Ans. by NTA (3)

Sol. emf = BLV

= 1.(2R) .1

= 2 V

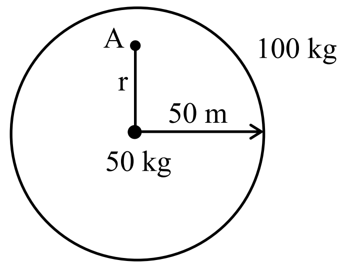
11. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is :

(1) – 60 G (2) + 2 G

(3) – 20 G (4) – 4 G

Official Ans. by NTA (4)

Sol.



$$V_A = \left[-\frac{GM_1}{r} - \frac{GM_2}{R}\right]$$

$$= \left[-\frac{50}{25}G - \frac{100}{50}G\right]$$

$= -4G$

12. For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be :

(1) 1 Ω (2) 5 Ω (3) 4 Ω (4) 2 Ω

Official Ans. by NTA (4)

Sol. $I_{max} = \frac{50}{2} = 25mA$

$$\mathbf{R} = \frac{V}{I} = \frac{50mV}{25mA} = 2\Omega$$

13. A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm?

(1) 0.96 V (2) 1.25 V (3) 0.24 V (4) 1.5 V

Official Ans. by NTA (2)

Sol. $kE_{max} = \frac{hc}{\lambda_i} + \phi$

or $eV_o = \frac{hc}{\lambda_i} + \phi$

when $\lambda_i = 670.5$ nm ; $V_o = 0.48$

when $\lambda_i = 474.6$ nm ; $V_o = ?$

So, $e(0.48) = \frac{1240}{670.5} + \phi$ ...(1)

$e(V_o) = \frac{1240}{474.6} + \phi$ ...(2)

(2) – (1)

$$e(V_o - 0.48) = 1240\left(\frac{1}{474.6} - \frac{1}{670.5}\right)eV$$

$$V_o = 0.48 + 1240\left(\frac{670.5 - 474.6}{474.6 \times 670.5}\right) \text{ Volts}$$

$V_o = 0.48 + 0.76$

$V_o = 1.24$ V 1.25 V

14. Match List-I with List-II.

List-I	List-II
(a) R_H (Rydberg constant)	(i) $kg\ m^{-1}\ s^{-1}$
(b) h(Planck's constant)	(ii) $kg\ m^2\ s^{-1}$
(c) μ_B (Magnetic field energy density)	(iii) m^{-1}
(d) η(coefficient of viscocity)	(iv) $kg\ m^{-1}\ s^{-2}$

Choose the most appropriate answer from the options given below :

(1) (a)–(ii), (b)–(iii), (c)–(iv), (d)–(i)

(2) (a)–(iii), (b)–(ii), (c)–(iv), (d)–(i)

(3) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)

(4) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)

Official Ans. by NTA (2)

Sol. SI unit of Rydberg const. = m^{-1}

SI unit of Plank's const. = $kg\ m^2 s^{-1}$

SI unit of Magnetic field energy density= $kg\ m^{-1}s^{-2}$

SI unit of coeff. of viscosity = $kg\ m^{-1}s^{-1}$

15. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density :

(1)$[FL^{-4} T^2]$

(2) $[FL^{-3} T^2]$

(3) $[FL^{-5} T^2]$

(4) $[FL^{-3} T^3]$

Official Ans. by NTA (1)

Sol. Density = $[F^aL^bT^c]$

$[ML^{-3}] = [M^aL^aT^{-2a}L^bT^c]$

$[M^1L^{-3}] = [M^aL^{a+b}\,T^{-2a+c}]$

$a = 1$; $a + b = -3$; $-2a + c = 0$

$1 + b = -3$ $\quad$ $c = 2a$

$b = -4$ $\quad$ $c = 2$

So, density = $[F^1L^{-4}T^2]$

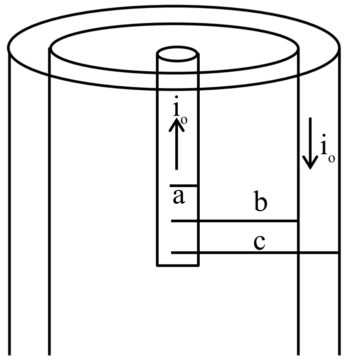
16. A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) x < a and (ii) a < x < b ?

(1) $\frac{x^2}{a^2}$ $\qquad$ (2) $\frac{a^2}{x^2}$

(3) $\frac{x^2}{b^2-a^2}$ $\qquad$ (4) $\frac{b^2-a^2}{x^2}$

Official Ans. by NTA (1)

Sol.



when x < a

$$B_1\,(2\pi x) = \mu_o\left(\frac{i_o}{\pi a^2}\right)\pi x^2$$

$$B(2\pi x) = \frac{\mu_o i_o x^2}{a^2}$$

$$B_1 = \frac{\mu_o i_o x}{2\pi a^2} \qquad ...(1)$$

when a < x < b

$$B_2(2\pi x) = \mu_o i_o$$

$$B_2 = \frac{\mu_o i_o}{2\pi x} \qquad ...(2)$$

$$\frac{B_1}{B_2} = \frac{\mu_o i_o \frac{x}{2\pi a^2}}{\frac{\mu_o i_o}{2\pi x}} = \frac{x^2}{a^2}$$

17. The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall ?

[Given 1 cal = 4.2 J and specific heat of water = 1 cal $g^{-1}\,{}^{o}C^{-1}$]

(1) 0.147º C

(2) 14.76º C

(3) 1.476º

(4) 0.014º C

Official Ans. by NTA (1)

Sol. Change in P.E. = Heat energy

mgh = mSΔT

$$\Delta T = \frac{gh}{S}$$

$$= \frac{10 \times 63}{4200 J / kgC}$$

= 0.147°C

18. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45º from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion ? (Take g = 10 ms^{-2})

(1) h_{max} = 10 m   T = 2.5 s

(2) h_{max} = 15.625 m   T = 3.54 s

(3) h_{max} = 15.625 m   T = 1.77 s

(4) h_{max} = 3.54 m   T = 0.125 s

Official Ans. by NTA (3)

Sol. $$H = \frac{U^2 \sin^2\theta}{2g}$$

$$= \frac{(25)^2.(\sin 45)^2}{2 \times 10}$$

= 15.625 m

$$T = \frac{U \sin\theta}{g}$$

$$= \frac{25 \times \sin 45°}{10}$$

= 2.5 × 0.7

= 1.77 s

19. The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero.What will be the intensity of light at maxima ?

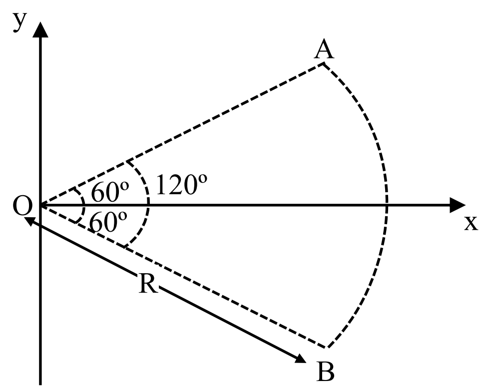
(1) I_0 (2) 2 I_0 (3) 5 I_0 (4) 4 I_0

Official Ans. by NTA (4)

Sol. $$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$= 4I_o$

20. Figure shows a rod AB, which is bent in a 120º circular arc of radius R. A charge (–Q) is uniformly distributed over rod AB. What is the electric field E at the centre of curvature O ?



(1) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}\left(i\right)$ (2) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}\left(i\right)$

(3) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}\left(i\right)$ (4) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}\left(-i\right)$

Official Ans. by NTA (2)

Sol. $$\varepsilon = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)(-\hat{i})$$

$$\lambda = \left(\frac{-Q}{R\theta}\right) = \left(\frac{-Q}{R.\frac{2\pi}{3}}\right)$$

$$\lambda = \frac{-3Q}{2\pi R}$$

$$\varepsilon = \frac{2k}{R}.\frac{-3Q}{2\pi R}.\sin(60°)(-\hat{i})$$

$$\varepsilon = \frac{3\sqrt{3}Q}{8\pi^2 \epsilon_o R^2}(+\hat{i})$$

SECTION-B

1. A heat engine operates between a cold reservoir at temperature T_2 = 400 K and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _________K.

Official Ans. by NTA (500)

Sol. Q_{in} = 300 J ; Q_{out} = 240 J

Work done = $Q_{in} - Q_{out}$ = 300 – 240 = 60 J

Efficiency = $\frac{W}{Q_{in}} = \frac{60}{300} = \frac{1}{5}$

efficiency = $1 - \frac{T_2}{T_1}$

$\frac{1}{5} = 1 - \frac{400}{T_1} \Rightarrow \frac{400}{T_1} = \frac{4}{5}$

T_1 = 500 k

2. Two simple harmonic motion, are represented by the equations $y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

Ratio of amplitude of y_1 to y_2 = x : 1. The value of x is ________.

Official Ans. by NTA (1)

Sol. $y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow$ Amplitude= 10

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$

$y_2 = 10\left(\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right)$

$y_2 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow$ Amplitude = 10

So ratio of amplitudes = $\frac{10}{10}$ = 1

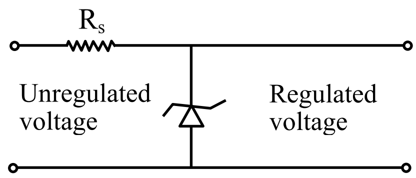
3. X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are exited to states with principal quantum number n = 6 ? The value of X is _______.

Official Ans. by NTA (15)

Sol. No. of different wavelengths = $\frac{n(n-1)}{2}$

$= \frac{6 \times (6-1)}{2} = \frac{6 \times 5}{2} = 15$

4. A zener diode of power rating 2W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_s for safe operation should be _______ Ω.



Official Ans. by NTA (20)

Sol. When unregulated voltage is 14 V voltage across zener diode must be 10 V So potential difference across resistor ΔV_{Rs} = 4V

and P_{zener} = 2W

VI = 2

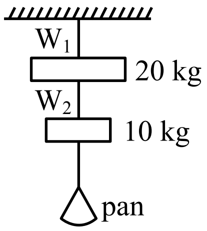
I = $\frac{2}{10}$ = 0.2 A

$\Delta V_{Rs} = I R_s$

$4 \times 0.2 R_s \Rightarrow R_s = \frac{40}{2} = 20\Omega$

5. Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7} m^2 and 4×10^{-7} m^2, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is ____ kg.

(Use g = 10 m/s^2)



Official Ans. by NTA (40)

Sol. $B.S_1 = \dfrac{T_{1max}}{8\times10^{-7}} \Rightarrow T_{1\,max} = 8 \times 1.25 \times 100$

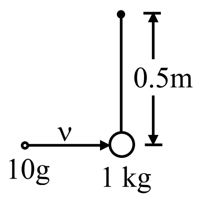
$= 1000$ N

$B.S_2 = \dfrac{T_{2max}}{4\times10^{-7}} \Rightarrow T_{2max} = 4\times1.25\times100$

$= 500$ N

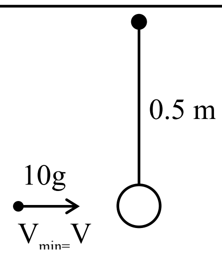
$m = \dfrac{500-100}{10} = 40$ kg

6. A bullet of 10 g, moving with velocity ν, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of ν = _____ m/s so that the pendulum describes a circle. (Assume the string to be inextensible and g = 10 m/s^2)



Official Ans. by NTA (400)

Sol.



$V' = \sqrt{5gR} = \sqrt{5\times10\times0.5}$

$V' = 5$ m/s

$m_1V = m_2 \times 5 - m_1 \times 100$

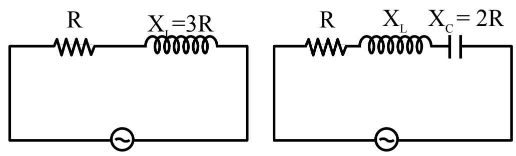
$\dfrac{10}{1000}\times V = 5 - \dfrac{10}{1000}\times100$

$V = 400$ m/s

7. An ac circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$. The value of x is _____.

Official Ans. by NTA (1)

Sol.

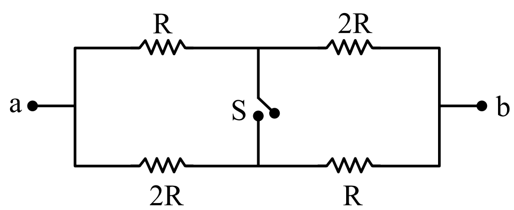


$\cos\phi = \dfrac{R}{\sqrt{R^2+3R^2}}$ $\qquad$ $\cos\phi' = \dfrac{R}{\sqrt{R^2+R^2}}$

$= \dfrac{1}{\sqrt{10}}$ $\qquad$ $= \dfrac{1}{\sqrt{2}}$

$\dfrac{\cos\phi'}{\cos\phi} = \dfrac{\sqrt{10}}{\sqrt{2}} = \dfrac{\sqrt{5}}{1}$ $\qquad$ $\therefore x = 1$

8. The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is _______.



Official Ans. by NTA (9)

Sol. $R_{eq\ open} = \frac{3R}{2}$

$R_{eq\ closed} = 2 \times \frac{R \times 2R}{3R} = \frac{4R}{3}$

$\frac{R_{eq\ open}}{R_{eq\ closed}} = \frac{3R}{2} \times \frac{3}{4R} = \frac{9}{8}$

$\therefore x = 9$

9. A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}$ T. The value of x is _____.

Official Ans. by NTA (2)

Sol. $|B| = \frac{|E|}{C} = \frac{6}{3 \times 10^{8}}$

$= 2 \times 10^{-8}$ T

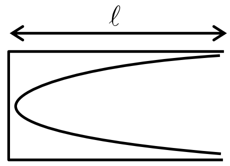
$\therefore x = 2$

10. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _____ cm.

(Take speed of sound in air as 340 ms^{-1})

Official Ans. by NTA (34)

Sol.



$\frac{\lambda}{4} = \quad \Rightarrow \lambda = 4$

$f = \frac{V}{\lambda} = \frac{V}{4}$

$\Rightarrow 250 = \frac{340}{4}$

$\Rightarrow \quad = \frac{34}{4 \times 25} = 0.34m$

$= 34cm$

SECTION-A

1. Choose the **correct** statement from the following :

(1) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.

(2) The low solubility of CsI in water is due to its high lattice enthalpy.

(3) Among the alkali metal halides, LiF is least soluble in water.

(4) LiF has least negative standard enthalpy of formation among alkali metal fluorides.

Official Ans. by NTA (3)

Sol. 1. Standard enthalpy of formation for alkali metal bromides becomes more negative on desending down the group.

2. In case of CsI, lattice energy is less, but Cs^+ is having less hydration enthalpy due to which it is less soluble in water.

3. For alkali metal fluorides, the solubility in water increases from lithium to caesium. LiF is least soluble in water.

4. Standard enthalpy of formation for LiF is most negative among alkali metal fluorides.

2. The addition of dilute NaOH to Cr^{3+} salt solution will give :

(1) a solution of $[Cr(OH)_4]^-$

(2) precipitate of $Cr_2O_3(H_2O)_n$

(3) precipitate of $[Cr(OH)_6]^{3-}$

(4) precipitate of $Cr(OH)_3$

Official Ans. by NTA (2)

Sol. $Cr^{3} \quad \underset{\text{dil.}}{NaOH} \quad \underset{\text{precipitate}}{Cr_2O_3.(H_2O)_n}$

3. Given below are two statements :

Statement I : Ethyl pent–4–yn–oate on reaction with CH_3MgBr gives a 3°–alcohol.

Statement II : In this reaction one mole of ethyl pent–4–yn–oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are false.

(2) **Statement I** is false but **Statement II** is true.

(3) **Statement I** is true but **Statement II** is false.

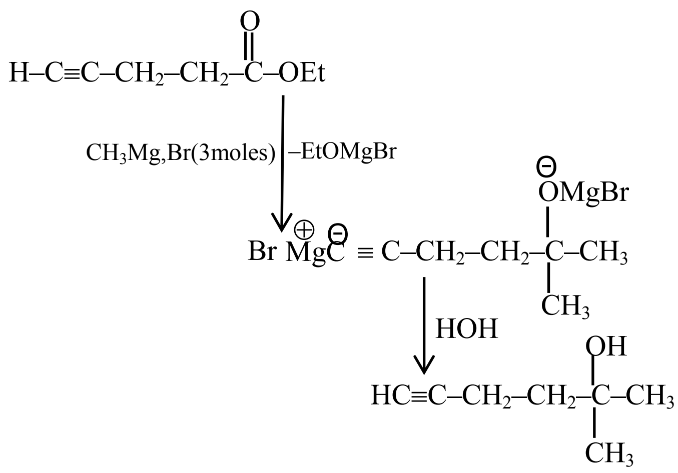
(4) Both **Statement I** and **Statement II** are true.

Official Ans. by NTA (3)

Sol. Statement 1 is true

But it consume 3 moles of G R

So statement 2 is false.



4. In stratosphere most of the ozone formation is assisted by :

(1) cosmic rays. (2) γ–rays.

(3) ultraviolet radiation. (4) visible radiations.

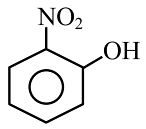
Official Ans. by NTA (3)

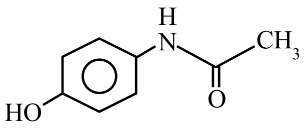
Sol. Ozone in the stratosphere is a product of UV radiations acting on dioxygen (O_2) molecules.

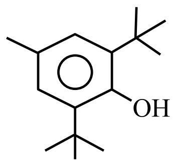
$$O_2(g) \xrightarrow{UV} O(g) \quad O(g)$$

$$O(g) \quad O_2(g) \xrightarrow{UV} O_3(g)$$

5. The compound/s which will show significant intermolecular H–bonding is/are :


(a)


(b)


(c)

(1) (b) only (2) (c) only
(3) (a) and (b) only (4) (a), (b) and (c)

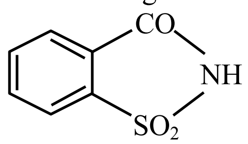
Official Ans. by NTA (1)

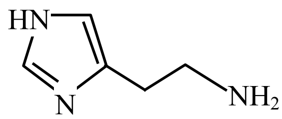
Sol. (a) Shows intra molecular H-bonding

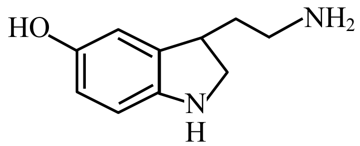
(b) Shows significant intermolecular H-bonding

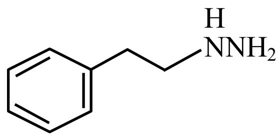
(c) It do not show intermolecular H-bonding due to steric hindrance.

6. Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain?

(1) 

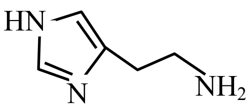
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. Histamine stimulate the secretion of HCl



Histamine structure

7. The oxide that gives H_2O_2 most readily on treatment with H_2O is :

(1) PbO_2 (2) Na_2O_2
(3) SnO_2 (4) $BaO_2{\cdot}8H_2O$

Official Ans. by NTA (2)

Sol. 1. $PbO_2 + 2H_2O \rightarrow Pb(OH)_4$

2. $Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$

this reaction is possible at room temperature

3. $SnO_2 + 2H_2O \rightarrow Sn(OH)_4$

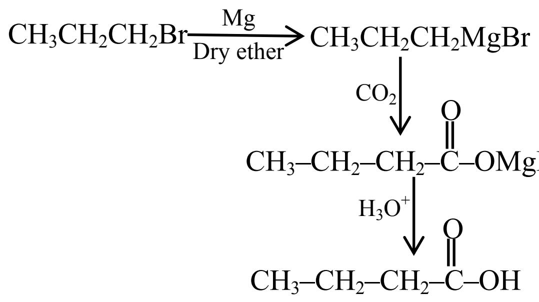
4. Acidified $BaO_2.8H_2O$ gives H_2O_2 after evaporation.

8. Which one of the following reactions will **not** yield propionic acid?

(1) $CH_3CH_2COCH_3 + OI^-/H_3O^+$
(2) $CH_3CH_2CH_3 + KMnO_4$ (Heat),OH^-/H_3O^+
(3) $CH_3CH_2CCl_3 + OH^-/H_3O^+$
(4) $CH_3CH_2CH_2Br$ + Mg, CO_2 dry ether/H_3O^+

Official Ans. by NTA (4)

Sol. All gives propanoic acid as product but option 4 gives butanoic as product



Butanoic acid

9. The correct order of ionic radii for the ions, P^{3-}, S^{2-}, Ca^{2+}, K^+, Cl^- is :

(1) $P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$
(2) $Cl^- > S^{2-} > P^{3-} > Ca^{2+} > K^+$
(3) $P^{3-} > S^{2-} > Cl^- > Ca^{2+} > K^+$
(4) $K^+ > Ca^{2+} > P^{3-} > S^{2-} > Cl^-$

Official Ans. by NTA (1)

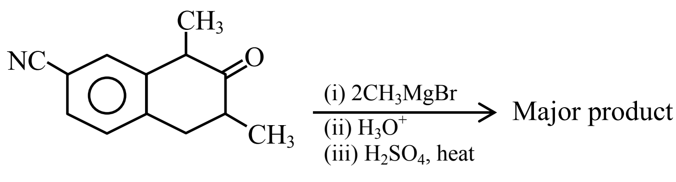
Sol. $P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$

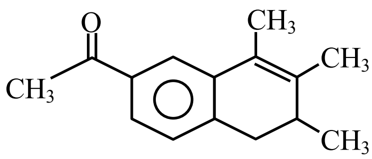
(Correct order of ionic radii)

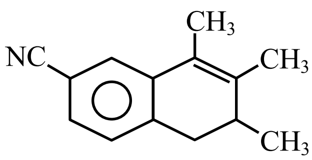
all the given species are isoelectronic species.

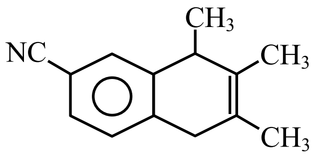
In isoelectronic species size increases with increase of negative charge and size decreases with increase in positive charge.

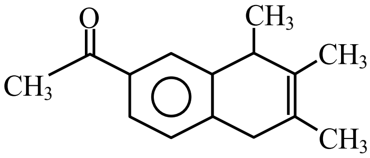
10. Which one of the following is the major product of the given reaction?



(1) 

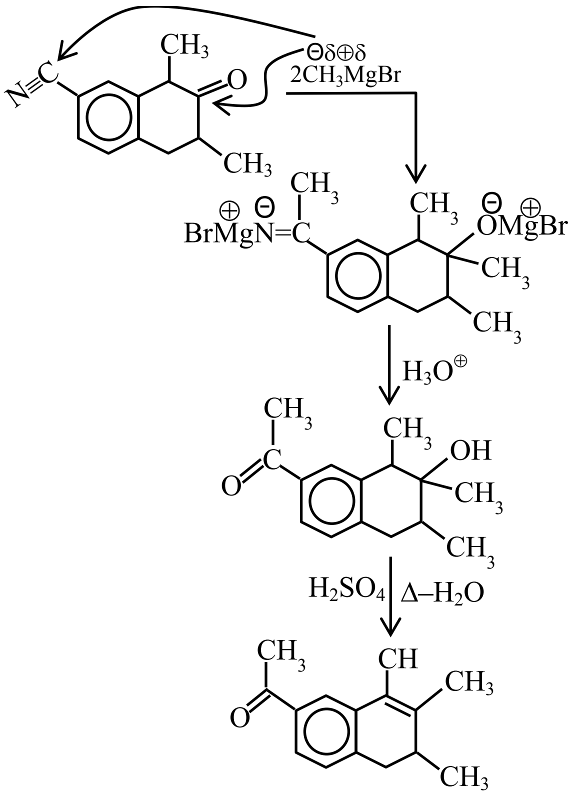
(2) 

(3) 

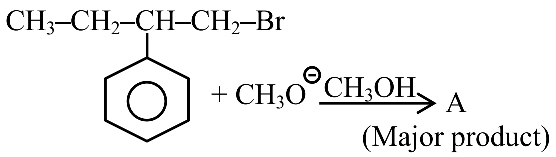
(4) 

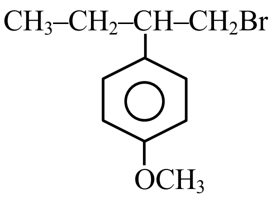
Official Ans. by NTA (1)

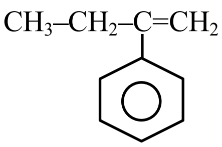
Sol.

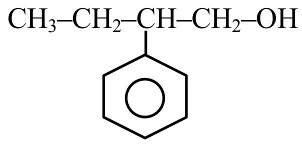


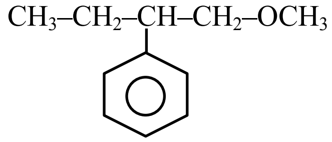
11. The major product (A) formed in the reaction given below is :



(1) 

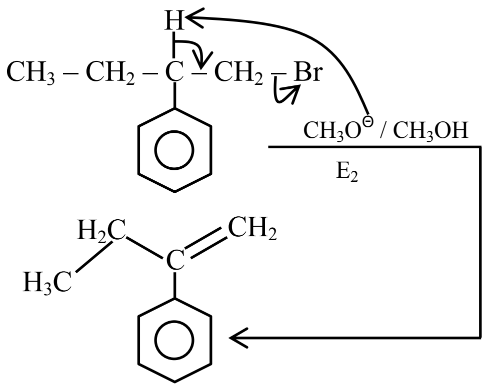
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



Major product (A)

12. Which one of the following is used to remove most of plutonium from spent nuclear fuel?

(1) ClF_3 (2) O_2F_2 (3) I_2O_5 (4) BrO_3

Official Ans. by NTA (2)

Sol. O_2F_2 oxidises plutonium to PuF_6 and the reaction is used in removing plutonium as PuF_6 from spent nuclear fuel.

13. Lyophilic sols are more stable than lyophobic sols because :

(1) there is a strong electrostatic repulsion between the negatively charged colloidal particles.

(2) the colloidal particles have positive charge.

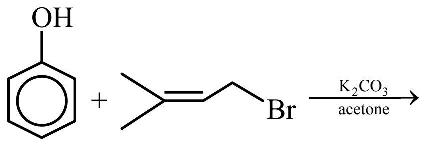
(3) the colloidal particles have no charge.

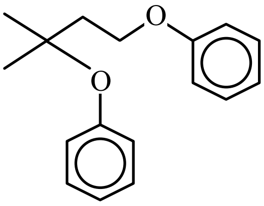
(4) the colloidal particles are solvated.

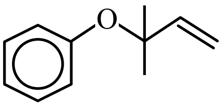
Official Ans. by NTA (4)

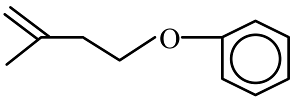
Sol. In the lyophilic colloids, the colloidal particles are extensively solvated.

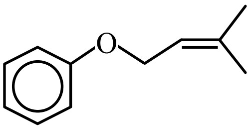
14. The major product of the following reaction, if it occurs by S_N2 mechanism is :



(1) 

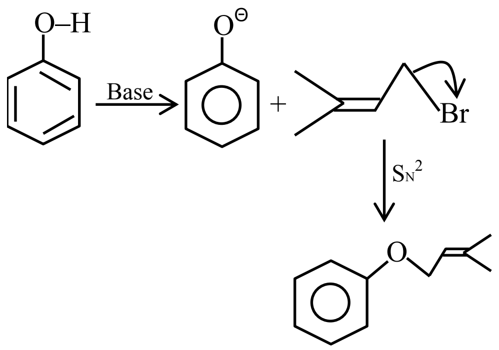
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



15. Potassium permanganate on heating at 513 K gives a product which is :

(1) paramagnetic and colourless

(2) diamagnetic and green

(3) diamagnetic and colourless

(4) paramagnetic and green

Official Ans. by NTA (4)

Sol. $2KMnO_4 \xrightarrow{200\ C} \underset{\text{Green}}{K_2MnO_4} + \underset{\text{Black}}{MnO_2} + O_2$

In K_2MnO_4, manganese oxidation state is +6 and hence it has one unpaired e^-.

16. Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent ?

(1) Barfoed's test

(2) Seliwanoff's test

(3) Benedict's test

(4) Biuret test for peptide bond

Official Ans. by NTA (2)

Sol. In Seliwanoff's reagent, Cu is not present.

In Barfoed, Biuret and in Benediet reagent Cu is present.

17. Hydrolysis of sucrose gives :

(1) α-D-(–)-Glucose and β-D-(–)-Fructose

(2) α-D-(+)-Glucose and α-D-(–)-Fructose

(3) α-D-(–)-Glucose and α-D-(+)-Fructose

(4) α-D-(+)-Glucose and β-D-(–)-Fructose

Official Ans. by NTA (4)

Sol. Sucrose is formed by α–D(+) . Glucose + β–D (–) Fructose.

we obtain these monomers on hydrolysis.

18. Match **List-I** with **List – II** :

List-I (Name of ore/mineral)	**List-II** (Chemical formula)
(a) Calamine	(i) Zns
(b) Malachite	(ii) $FeCO_3$
(c) Siderite	(iii) $ZnCO_3$
(d) Sphalerite	(iv) $CuCO_3 \cdot Cu(OH)_2$

Choose the **most appropriate** answer from the options given below :

(1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(4) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Official Ans. by NTA (1)

Sol. (Name of ore/mineral)

(a) Calamine $ZnCO_3$

(b) Malachite $CuCO_3.Cu(OH)_2$

(c) Siderite $FeCO_3$

(d) Sphalerite ZnS

19. Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K ?

(1) White phosphorus

(2) Yellow phosphorus

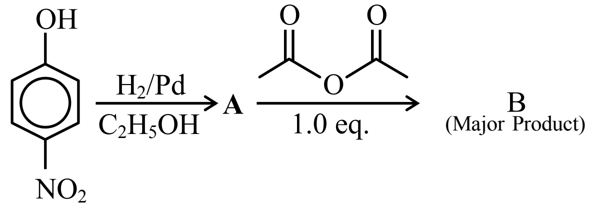
(3) β-Black phosphorus

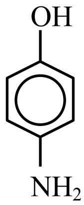
(4) α-Black phosphorus

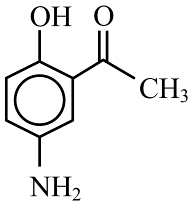
Official Ans. by NTA (4)

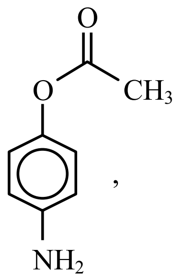
Sol. When red phosphorus is heated in a sealed tube at 803 K, α-black phosphorus is formed.

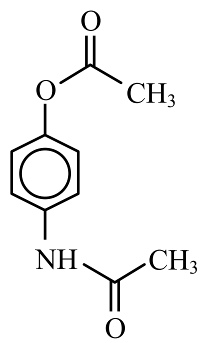
20. The correct structures of **A** and **B** formed in the following reactions are :

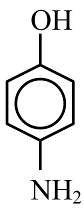


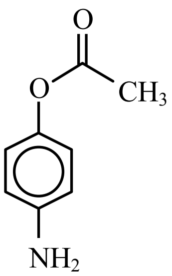
(1) **A** : 

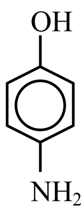
, **B** : 

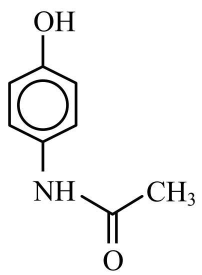
(2) **A** : 

, **B** : 

(3) **A** : 

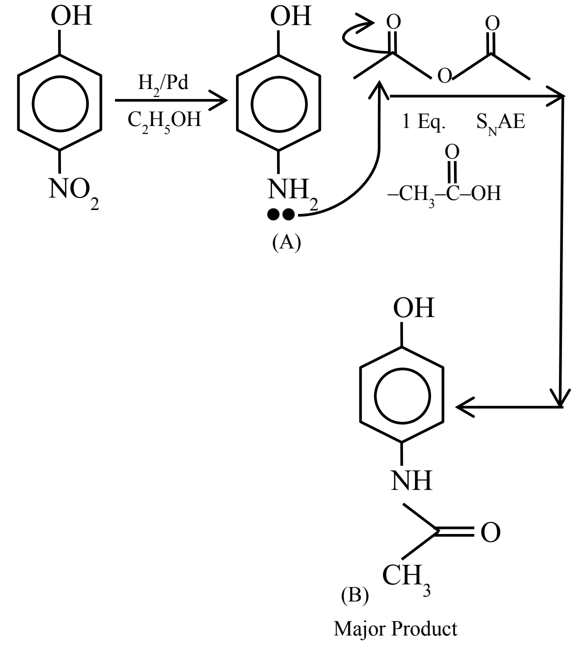
, **B** : 

(4) **A** : 

, **B** : 

Official Ans. by NTA (4)

Sol.



SECTION-B

1. The first order rate constant for the decomposition of $CaCO_3$ at 700 K is $6.36 \times 10^{-3} s^{-1}$ and activation energy is 209 kJ mol^{-1}. Its rate constant (in s^{-1}) at 600 K is $x \times 10^{-6}$. The value of x is _______. (Nearest integer)

[Given R = 8.31 J K^{-1} mol^{-1}; log $6.36 \times 10^{-3} = -2.19$, $10^{-4.79} = 1.62 \times 10^{-5}$]

Official Ans. by NTA (16)

Sol. $K_{700}\quad 6.36\quad 10^{-3}s^{-1}$;

$K_{600}\quad x\quad 10^{-6}s^{-1}$

$E_a = 209$ kJ/mol

Applying ;

$$\log \frac{K_{T_2}}{K_{T_1}} \quad \frac{-E_a}{2.303R} \quad \frac{1}{T_2} \quad \frac{1}{T_1}$$

$$\log \frac{K_{700}}{K_{600}} \quad \frac{-E_a}{2.303R} \quad \frac{1}{700} \quad \frac{1}{600}$$

$$\log \frac{6.36 \quad 10^{-3}}{K_{600}} \quad \frac{209 \quad 1000}{2.303 \quad 8.31} \quad \frac{100}{700 \quad 600}$$

$\log(6.36 \times 10^{-3}) - \log K_{600} = 2.6$

$\Rightarrow \log K_{600} = -2.19 - 2.6 = -4.79$

$\Rightarrow K_{600} = 10^{-4.79} = 1.62 \times 10^{-5}$

$= 16.2 \times 10^{-6}$

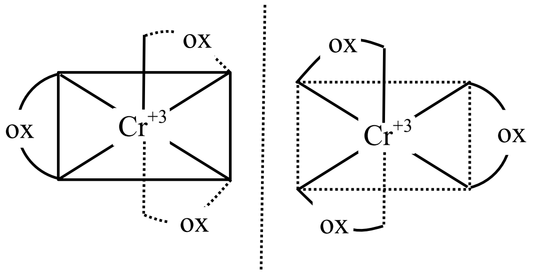
$= x \times 10^{-6}$

$\Rightarrow x = 16$

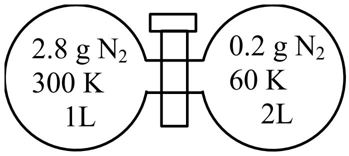
2. The number of optical isomers possible for $[Cr(C_2O_4)_3]^{3-}$ is ______.

Official Ans. by NTA (2)

Sol. The number of optical isomers for $[Cr(C_2O_4)_3]^{3-}$ is two.



3. Two flasks I and II shown below are connected by a valve of negligible volume.



I II

When the valve is opened, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is ______. (Integer answer)

[Assume–Ideal gas; 1 bar = 10^5 Pa; Molar mass of N_2 = 28.0 g mol^{-1} ; R = 8.31 J $mol^{-1}K^{-1}$]

Official Ans. by NTA (84)

Sol. Applying ; $(n_I + n_{II})_{initial} = (n_I + n_{II})_{final}$

$\Rightarrow$ Assuming the system attains a final temperature of T (such that 300 < T < 60)

$$\Rightarrow \begin{pmatrix} \text{Heat lost by} \\ N_2 \text{ of container} \\ \text{I} \end{pmatrix} \begin{pmatrix} \text{Heat gained by} \\ N_2 \text{ of container} \\ \text{II} \end{pmatrix}$$

$\Rightarrow n_I C_m(300-T) = n_{II} C_m(T-60)$

$$\Rightarrow \frac{2.8}{28}(300 \quad T) \quad \frac{0.2}{28}(T-60)$$

$\Rightarrow 14(300-T) = T-60$

$$\Rightarrow \frac{(14 \quad 300 \quad 60)}{15} \quad T$$

$\Rightarrow$ T = 284 K (final temperature)

$\Rightarrow$ If the final pressure = P

$$\Rightarrow (n_I \quad n_{II})_{final} \quad \frac{3.0}{28}$$

$$\Rightarrow \frac{P}{RT}(V_I \quad V_{II}) \quad \frac{3.0gm}{28gm/mol}$$

$$P = \frac{3}{28}mol \times 8.31\frac{J}{mol \quad K} \times \frac{284K}{3 \quad 10^{-3}m^3} \times 10^{-5}\frac{bar}{Pa}$$

$\Rightarrow$ 0.84287 bar

$\Rightarrow 84.28 \times 10^{-2}$ bar

$\Rightarrow$ 84

4. 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is $x \times 10^{-2}$. The value of x is _______. (Nearest integer)

[Atomic weight : H = 1.008; C = 12.00; O = 16.00]

Official Ans. by NTA (19)

Sol. $C_3H_{8(g)} + 5O_{2(g)} \longrightarrow 3CO_{2(g)} + 4H_2O_{(\ell)}$

t = 0 2.27 mole 31.25 mol

t = ∞ 0 19.9 mol 6.81 mol 9.08 mol

mole fraction of CO_2 in the final reaction mixture (heterogenous)

$$X_{CO_2} \quad \frac{6.81}{19.9 \quad 6.81 \quad 9.08}$$

$= 0.1902 = 19.02 \times 10^{-2}$

$\Rightarrow$ 19

5. 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is _____ K. (Nearest integer)

[Given : K_f = 1.86 K kg mol^{-1} ; Density of water = 1.00 g cm^{-3}; Freezing point of water = 273.15 K]

Official Ans. by NTA (271)

Sol. $$\text{molality} \quad \frac{\frac{40}{180}mol}{0.2Kg} \quad \frac{10}{9} \text{ molal}$$

$$T_f \quad T_f \quad T_f' \quad 1.86 \quad \frac{10}{9}$$

$$T_f' \quad 273.15 - 1.86 \quad \frac{10}{9}$$

= 271.08 K

≃ 271 K (nearest-integer)

6. The resistance of a conductivity cell with cell constant 1.14 cm^{-1}, containing 0.001 M KCl at 298 K is 1500 Ω. The molar conductivity of 0.001 M KCl solution at 298 K in S cm^2 mol^{-1} is_______. (Integer answer)

Official Ans. by NTA (760)

Sol. $K \quad \frac{1}{R} \quad \ell/A \quad \frac{1}{1500} \quad 1.14 \ \text{S cm}^{-1}$

$$_m \quad 1000 \quad \frac{\frac{1.14}{1500}}{0.001} \ \text{S cm}^2\text{mol}^{-1}$$

= 760 S cm^2 mol^{-1}

⇒ 760

7. The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is x × 10^{13}. The value of x is _____. (Nearest integer)

($h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 10^8$ ms^{-1})

Official Ans. by NTA (50)

Sol. Energy emitted in 0.1 sec.

$$0.1 \text{ sec}. \quad 10^{-3} \frac{J}{s}$$

= 10^{-4} J

If 'n' photons of λ = 1000 nm are emitted,

then ; $10^{-4} \quad n \quad \frac{hc}{}$

$$10^{-4} \quad \frac{n \quad 6.63 \quad 10^{-34} \quad 3 \quad 10^{8}}{1000 \quad 10^{-9}}$$

⇒ $n = 5.02 \times 10^{14} = 50.2 \times 10^{13}$

⇒ 50 (nearest integer)

8. When 5.1 g of solid NH_4HS is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27°C is x × 10^{-2}. The value of x is _______. (Integer answer)

[Given R = 0.082 L atm K^{-1} mol^{-1}]

Official Ans. by NTA (6)

Sol. moles of NH_4HS initially taken = $\frac{5.1\text{g}}{51\text{g/mol}}$

= 0.1 mol

volume of vessel = 2ℓ

$$NH_4HS_{(s)} \qquad NH_{3(g)} \quad H_2S_{(g)}$$

t = 0	0.1 mol		
t = ∞	0.1(1–0.2)	0.1×0.2	0.1×0.2

⇒ partial pressure of each component

$$P \quad \frac{nRT}{V} \quad \frac{0.1 \quad 0.2 \quad 0.082 \quad 300}{2}$$

= 0.246 atm

$$\Rightarrow k_P = P_{NH_3} \times P_{H_2S} = (0.246)^2 = 0.060516$$

= 6.05 × 10^{-2}

⇒ 6

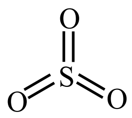
9. The number of species having non–pyramidal shape among the following is_______.

(A) SO_3 (B) NO_3^-

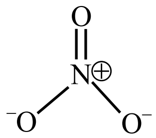
(C) PCl_3 (D) CO_3^{2-}

Official Ans. by NTA (3)

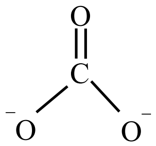
Sol.



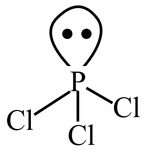
Trigonal planar



Trigonal planar



Trigonal planar



Pyramidal

Hence non-pyramidal species are SO_3, NO_3 and CO_3^2 .

10. Data given for the following reaction is as follows:

$FeO_{(s)} + C_{(graphite)} \longrightarrow Fe_{(s)} + CO_{(g)}$

Substance	$\Delta H°$ (kJ mol^{-1})	$\Delta S°$ (J mol^{-1}K^{-1})
$FeO_{(s)}$	–266.3	57.49
$C_{(graphite)}$	0	5.74
$Fe_{(s)}$	0	27.28
$CO_{(g)}$	–110.5	197.6

The minimum temperature in K at which the reaction becomes spontaneous is _______ .

(Integer answer)

Official Ans. by NTA (964)

Sol. $T_{min} \quad \frac{{}^0H}{{}^0S}$

$$\Delta^0 H_{rxn} = \left[\Delta_f^0 H(Fe) + \Delta_f^0 H(CO)\right] -$$

$$\left[\quad {}_f^0H(FeO) \qquad {}_f^0H(C_{(graphite)})\right]$$

= [0 – 110.5] – [–266.3+0]

= 155.8 kJ/mol

$$\Delta^0 S_{rxn} = \left[\Delta^0 S(Fe) + \Delta^0 S(CO)\right] -$$

$$\left[\quad {}^0S(FeO) \qquad {}^0S(C_{(graphite)})\right]$$

= [27.28 + 197.6] – [57.49 + 5.74]

= 161.65 J/mol-K

$$T_{min} = \frac{155.8 \quad 10^3 J/mol}{161.65 J/mol \quad K} = 963.8K$$

≃ 964 k (nearest integer)

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is :

(1) $\frac{\pi}{2}$ (2) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

(3) $\cos^{-1}\left(\frac{8}{9}\right)$ (4) $\frac{\pi}{3}$

Official Ans. by NTA (1)

Sol. $n = 2(\ell + m)$

$\ell m + n(\ell + m) = 0$

$\ell m + 2(\ell + m)^2 = 0$

$2\ell^2 + 2m^2 + 5m\ell = 0$

$2\left(\frac{\ell}{m}\right)^2 + 2 + 5\left(\frac{\ell}{m}\right) = 0.$

$2t^2 + 5t + 2 = 0$

$(t + 2)(2t + 1) = 0$

$\Rightarrow t = -2; -\frac{1}{2}$

(i) $\frac{\ell}{m} = -2$

$\frac{n}{m} = -2$

$(-2m, m, -2m)$

$(-2, 1, -2)$

(ii) $\frac{\ell}{m} = -\frac{1}{2}$

$n = -2\ell$

$(\ell, -2\ell, -2\ell)$

$(1, -2, -2)$

$\cos\theta = \frac{-2-2+4}{\sqrt{9}\sqrt{9}} = 0 \Rightarrow 0 = \frac{\pi}{2}$

2. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where $[t]$ denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval:

(1) [68, 69] (2) [62, 63)

(3) [65, 66) (4) [60, 61)

Official Ans. by NTA (2)

Sol. $\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$

$R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

3. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$, Then the value of $\tan(M - m)$ is equal to:

(1) $2+\sqrt{3}$ (2) $2-\sqrt{3}$

(3) $3+2\sqrt{2}$ (4) $3-2\sqrt{2}$

Official Ans. by NTA (4)

Sol. Let $g(x) = \sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

$g(x) \in [1, \sqrt{2}]$ for $x \in [0, \pi/2]$

$f(x) = \tan^{-1}(\sin x + \cos x) \in \left[\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$

$\tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right) = \frac{\sqrt{2}-1}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 3 - 2\sqrt{2}$

4. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{5}{16}$ (4) 1

Official Ans. by NTA (3)

Sol. C – I '0' Head

T T T $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3=\frac{1}{64}$

C – II '1' head

H T T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right)=\frac{9}{64}$

C – III '2' Head

H H T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right)=\frac{9}{64}$

C–IV '3' Heads

H H H $\left(\frac{1}{8}\right)\left(\frac{1}{8}\right)=\frac{1}{64}$

Total probability = $\frac{5}{16}$.

5. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) form the line 3x + 4y = 5, is given by :

(1) $10\frac{d^2y}{dx^2}=11$ (2) $11\frac{d^2x}{dy^2}=10$

(3) $10\frac{d^2x}{dy^2}=11$ (4) $11\frac{d^2y}{dx^2}=10$

Official Ans. by NTA (4)

Sol. α. R = $\frac{|3(2)+4(-3)-5|}{5}=\frac{11}{5}$

$$(x-h)^2=\frac{11}{5}(y-k)$$

differentiate w.r.t 'x' : –

$$2(x-h)=\frac{11}{5}\frac{dy}{dx}$$

again differentiate

$$2=\frac{11}{5}\frac{d^2y}{dx^2}$$

$$\frac{11d^2y}{dx^2}=10.$$

6. If two tangents drawn from a point P to the parabola $y^2 = 16(x – 3)$ are at right angles, then the locus of point P is :

(1) x + 3 = 0 (2) x + 1 = 0

(3) x + 2 = 0 (4) x + 4 = 0

Official Ans. by NTA (2)

Sol. Locus is directrix of parabola

$x - 3 + 4 = 0 \Rightarrow x + 1 = 0.$

7. The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is:

(1) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$ (2) $\vec{r}.\left(\hat{i}+3\hat{k}\right)+6=0$

(3) $\vec{r}.\left(\hat{i}-3\hat{k}\right)+6=0$ (4) $\vec{r}.\left(\hat{j}-3\hat{k}\right)-6=0$

Official Ans. by NTA (1)

Sol. Equation of planes are

$\vec{r}\,.\left(\hat{i}+\hat{j}+\hat{k}\right)-1=0 \Rightarrow x + y + z – 1 = 0$

and $\vec{r}\,.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0 \Rightarrow 2x + 3y – z + 4 = 0$

equation of planes through line of intersection of these planes is :–

$(x + y + z – 1) + \lambda (2x + 3y – z + 4) = 0$

$\Rightarrow (1 + 2\lambda) x + (1 + 3\lambda) y + (1–\lambda) z – 1 + 4\lambda = 0$

But this plane is parallel to x–axis whose direction are (1, 0, 0)

$\therefore$ $(1 + 2\lambda)1 + (1 + 3\lambda) 0 + (1 – \lambda) 0 = 0$

$\lambda=-\frac{1}{2}$

$\therefore$ Required plane is

$$0\,x+\left(1-\frac{3}{2}\right)y+\left(1+\frac{1}{2}\right)z-1+4\left(\frac{-1}{2}\right)=0$$

$$\Rightarrow \frac{-y}{2}+\frac{3}{2}z-3=0$$

$\Rightarrow y – 3z + 6 = 0$

$\Rightarrow \boxed{\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0}$ Ans.

8. If the solution curve of the differential equation $(2x - 10y^3)\,dy + y\,dx = 0$, passes through the points $(0, 1)$ and $(2, \beta)$, then β is a root of the equation:

(1) $y^5 - 2y - 2 = 0$ (2) $2y^5 - 2y - 1 = 0$

(3) $2y^5 - y^2 - 2 = 0$ (4) $y^5 - y^2 - 1 = 0$

Official Ans. by NTA (4)

Sol. $(2x - 10y^3)\,dy + y\,dx = 0$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{2}{y}\right)x = 10y^2$$

$$\text{I. F.} = e^{\int \frac{2}{y}dy} = e^{2\ell n(y)} = y^2$$

Solution of D.E. is

$$\therefore \quad x.\,y = \int (10y^2)\,y^2.dy$$

$$xy^2 = \frac{10y^5}{5} + C \Rightarrow xy^2 = 2y^5 + C$$

It passes through $(0, 1) \rightarrow 0 = 2 + C \Rightarrow C = -2$

$\therefore$ Curve is $\boxed{xy^2 = 2y^5 - 2}$

Now, it passes through $(2,\beta)$

$2\beta^2 = 2\beta^5 - 2 \Rightarrow \beta^5 - \beta^2 - 1 = 0$

$\therefore$ β is root of an equation $\boxed{y^5 - y^2 - 1 = 0}$ Ans.

9. Let $A(a, 0)$, $B(b, 2b + 1)$ and $C(0, b)$, $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

(1) $\dfrac{-2b}{b+1}$ (2) $\dfrac{2b}{b+1}$

(3) $\dfrac{2b^2}{b+1}$ (4) $\dfrac{-2b^2}{b+1}$

Official Ans. by NTA (4)

Sol.
$$\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$$

$$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 2$$

$$\Rightarrow a\,(2b + 1 - b) - 0 + 1\,(b^2 - 0) = \pm 2$$

$$\Rightarrow a = \frac{\pm 2 - b^2}{b+1}$$

$$\therefore \quad a = \frac{2 - b^2}{b+1} \text{ and } a = \frac{-2 - b^2}{b+1}$$

sum of possible values of 'a' is

$$= \frac{-2b^2}{a+1} \text{ Ans.}$$

10. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is:

(1) **R**

(2) $(-\infty, -9) \cup (-9, \infty)$

(3) $[-9, -8)$

(4) $(-\infty, -9) \cup [-8, \infty)$

Official Ans. by NTA (1)

Sol.
$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9$$

if $[\lambda] + 9 \neq 0$ then unique solution

if $[\lambda] + 9 = 0$ then $D_1 = D_2 = D_3 = 0$

so infinite solutions

Hence λ can be any red number.

11. The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is :

(1) $\left(1, \dfrac{5}{2}\right]$ (2) $[2, 3)$

(3) $\left[-\dfrac{1}{2}, 1\right)$ (4) $\left(\dfrac{1}{2}, \dfrac{3}{2}\right] - \{1\}$

Official Ans. by NTA (1)

Sol. $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2)$
$+ k\,(3x^2 + 4x + 2)^2 = 0$

Let $3x^2 + 4x + 3 = a$

and $3x^2 + 4x + 2 = b \Rightarrow b = a - 1$

Given equation becomes

$\Rightarrow a^2 - (k + 1)\,ab + k\,b^2 = 0$

$\Rightarrow a\,(a - kb) - b\,(a - kb) = 0$

$\Rightarrow (a - kb)(a - b) = 0 \Rightarrow a = kb$ or $a = b$ (reject)

$\because a = kb$

$\Rightarrow 3x^2 + 4x + 3 = k\,(3x^2 + 4x + 2)$

$\Rightarrow 3\,(k - 1)\,x^2 + 4\,(k - 1)\,x + (2k - 3) = 0$

for real roots

$D \geq 0$

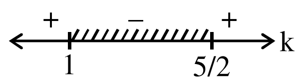
$\Rightarrow 16\,(k - 1)^2 - 4\,(3(k - 1))\,(2k - 3) \geq 0$

$\Rightarrow 4\,(k - 1)\,\{4\,(k - 1) - 3\,(2k - 3)\} \geq 0$

$\Rightarrow 4\,(k - 1)\,\{-2k + 5\} \geq 0$

$\Rightarrow -4\,(k - 1)\,\{2k - 5\} \geq 0$

$\Rightarrow (k - 1)\,(2k - 5) \leq 0$



$\therefore \quad k \in \left[1, \dfrac{5}{2}\right]$

$\because \quad k \neq 1$

$\therefore \quad k \in \left(1, \dfrac{5}{2}\right]$ Ans.

12. A box open from top is made from a rectangular sheet of dimension a × b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to :

(1) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

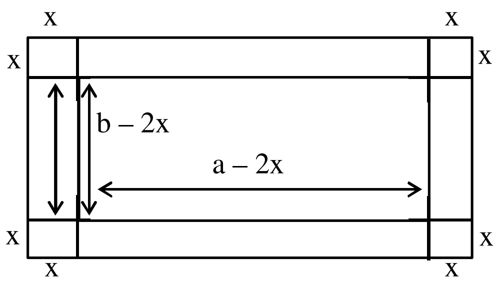
(2) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

(3) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(4) $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$

Official Ans. by NTA (3)

Sol.



$V = \ell.\,b.\,h \quad = (a-2x)(b-2x)\,x$

$\Rightarrow V(x) = (2x-a)(2x-b)\,x$

$\Rightarrow V(x) = 4x^3 - 2(a+b)x^2 + abx$

$\Rightarrow \frac{d}{dx}v(x) = 12x^2 - 4(a+b)x + ab$

$\frac{d}{dx}(v(x)) = 0 \Rightarrow 12x^2 - 4(a+b)x + ab = 0 <^{\alpha}_{\beta}$

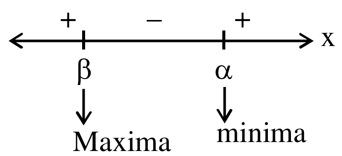
$\Rightarrow x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{2(12)}$

$= \frac{(a+b) \pm \sqrt{a^2+b^2-ab}}{6}$

Let $x = \alpha = \frac{(a+b)+\sqrt{a^2+b^2-ab}}{6}$

$\beta = \frac{(a+b)-\sqrt{a^2+b^2-ab}}{6}$

Now, $12(x-\alpha)(x-\beta) = 0$



$\therefore\ x = \beta$

$= \frac{a+b-\sqrt{a^2+b^2-ab}}{b}$

13. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to :

(1) $(p \wedge q) \Rightarrow (r \wedge q)$ (2) $(q \wedge r) \Rightarrow (p \wedge q)$

(3) $(p \wedge q) \Rightarrow (r \vee q)$ (4) $(p \wedge r) \Rightarrow (p \wedge q)$

Official Ans. by NTA (1)

Sol. $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim(p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim(p \wedge q) \vee ((r \wedge p) \wedge (p \wedge q))$

$\Rightarrow [\sim(p \wedge q) \vee (p \wedge q)] \wedge (\sim(p \wedge q) \vee (r \wedge p))$

$\Rightarrow t \wedge [\sim(p \wedge q) \vee (r \wedge p)]$

$\Rightarrow \sim(p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

Aliter :

given statement says

" if p and q both happen then

p and q and r will happen"

it Simply implies

" If p and q both happen then

'r' too will happen "

i.e.

" if p and q both happen then r and p too will happen

i.e.

$(p \wedge q) \Rightarrow (r \wedge p)$

14. Let $\mathbb{Z}$ be the set of all integers,

$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \le 4\}$,

$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\}$ and

$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y-2)^2 \le 4\}$

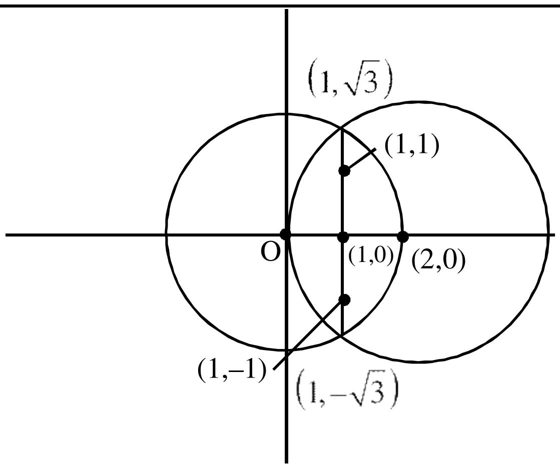
If the total number of relation from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is :

(1) 16 (2) 25

(3) 49 (4) 9

Official Ans. by NTA (2)

Sol.



$(x-2)^2 + y^2 \le 4$

$x^2 + y^2 \le 4$

No. of points common in C_1 & C_2 is 5.

(0, 0) , (1, 0) , (2, 0) , (1, 1), (1, –1)

Similarly in C_2 & C_3 is 5.

No. of relations = $2^{5\times5} = 2^{25}$.

15. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

(1) 9 (2) 10 (3) 4 (4) 6

Official Ans. by NTA (1)

Sol. $y = 3 \Rightarrow x = 2$

Point is (2,3)

Diff. w.r.t x

$2(y-2)y' = 1$

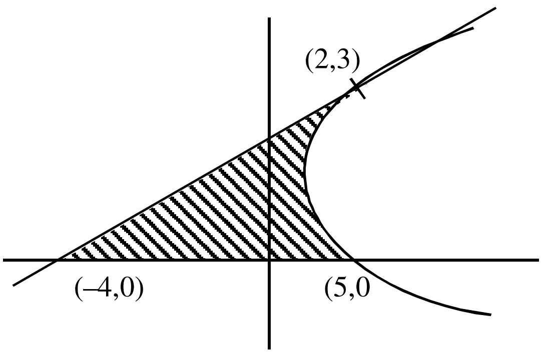
$$\Rightarrow y' = \frac{1}{2(y-2)}$$

$$\Rightarrow y'_{(2,3)} = \frac{1}{2}$$

$$\Rightarrow \frac{y-3}{x-2} = \frac{1}{2} \Rightarrow x - 2y + 4 = 0$$

$$\text{Area } = \int_0^3 \left((y-2)^2 + 1 - (2y-4)\right) dy$$

= 9 sq. units



16. If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, $x \in \left(\frac{\pi}{2}, \pi\right)$,

then $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$ is:

(1) $-\frac{1}{2}$ (2) -1 (3) $\frac{1}{2}$ (4) 0

Official Ans. by NTA (1)

Sol. $$y(x) = \cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right]$$

$$y(x) = \cot^{-1}\left(\tan\frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$$

$$y'(x) = \frac{-1}{2}$$

17. Two poles, AB of length a metres and CD of length a + b ($b \ne a$) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\angle ACB = \frac{1}{2}$, then:

(1) $x^2 + 2(a + 2b)x - b(a + b) = 0$

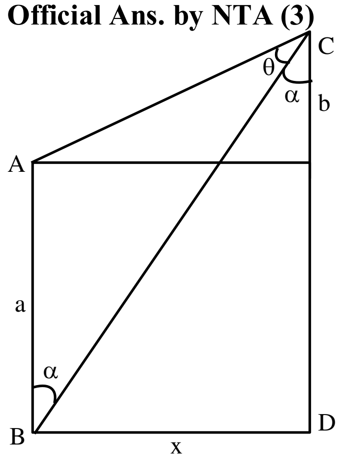
(2) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(3) $x^2 - 2ax + b(a + b) = 0$

(4) $x^2 - 2ax + a(a + b) = 0$

Official Ans. by NTA (3)

Sol.



$$\tan\theta = \frac{1}{2}$$

$$\tan(\theta+\alpha) = \frac{x}{b}, \ \tan\alpha = \frac{x}{a+b}$$

$$\Rightarrow \frac{1}{2} + \frac{x}{a+b}$$

$$\Rightarrow \frac{\frac{1}{2}+\frac{x}{a+b}}{1-\frac{1}{2}\times\frac{x}{a+b}} = \frac{x}{b}.$$

$\Rightarrow x^2 - 2ax + ab + b^2 = 0$

18. If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots$, then the value of e^{1+y} at $x = \frac{1}{2}$ is:

(1) $\frac{1}{2}e^2$ (2) $2e$

(3) $\frac{1}{2}\sqrt{e}$ (4) $2e^2$

Official Ans. by NTA (1)

Sol. $y = \left(1-\frac{1}{2}\right)x^2 + \left(1-\frac{1}{3}\right)x^3 + \ldots..$

$= (x^2 + x^3 + x^4 + \ldots\ldots.) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots.\right)$

$= \frac{x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^2}{3} + \ldots\right)$

$= \frac{x}{1-x} + \ell n(1-x)$

$x = \frac{1}{2} \Rightarrow y = 1 - \ell n2$

$e^{1+y} = e^{1+1-\ell n2}$

$= e^{2-\ell n2} = \frac{e^2}{2}$

19. The value of the integral $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is:

(1) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$ (2) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

(3) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$ (4) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

Official Ans. by NTA (1)

Sol. $I = \int_0^1 \frac{\sqrt{x}}{(1+x)(1+3x)(3+x)}dx$

Let $x = t^2 \Rightarrow dx = 2t.dt$

$I = \int_0^1 \frac{t(2t)}{(t^2+1)(1+3t^2)(3+t^2)}dt$

$I = \int_0^1 \frac{(3t^2+1)-(t^2+1)}{(3t^2+1)(t^2+1)(3+t^2)}dt$

$I = \int_0^1 \frac{dt}{(t^2+1)(3+t^2)} - \int_0^1 \frac{dt}{(1+3t^2)(3+t^2)}$

$= \frac{1}{2}\int_0^1 \frac{(3+t^2)-(t^2+1)}{(t^2+1)(3+t^2)}dt + \frac{1}{8}\int_0^1 \frac{(1+3t^2)-3(3+t^2)}{(1+3t^2)(3+t^2)}dt$

$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} - \frac{1}{2}\int_0^1 \frac{dt}{t^2+3} + \frac{1}{8}\int_0^1 \frac{dt}{t^2+3} - \frac{3}{8}\int_0^1 \frac{dt}{(1+3t^2)}$

$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+1} - \frac{3}{8}\int_0^1 \frac{dt}{t^2+3} - \frac{3}{8}\int_0^1 \frac{dt}{1+3t^2}$

$= \frac{1}{2}\left(\tan^{-1}(t)\right)_0^1 - \frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)_0^1$

$- \frac{3}{8\sqrt{3}}\left(\tan^{-1}(\sqrt{3}t)\right)_0^1$

$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{\sqrt{3}}{8}\left(\frac{\pi}{6}\right) - \frac{\sqrt{3}}{8}\left(\frac{\pi}{3}\right)$

$= \frac{\pi}{8} - \frac{\sqrt{3}}{16}\pi$

$= \frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

20. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right) = b$, then the ordered pair (a, b) is:

(1) $\left(1,\frac{1}{2}\right)$ (2) $\left(1,-\frac{1}{2}\right)$

(3) $\left(-1,\frac{1}{2}\right)$ (4) $\left(-1,-\frac{1}{2}\right)$

Official Ans. by NTA (2)]

Sol. (2)

$$\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \qquad (\infty-\infty)$$

$\Rightarrow a > 0$

Now, $\lim_{x\to\infty}\dfrac{(x^2-x+1-a^2x^2)}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\dfrac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\dfrac{(1-a^2)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow 1-a^2=0 \Rightarrow a=1$

Now, $\lim_{x\to\infty}\dfrac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow \dfrac{-1}{1+a}=b \Rightarrow b=-\dfrac{1}{2}$

$(a,b)=\left(1,-\dfrac{1}{2}\right)$

SECTION-B

1. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$ in $[0, 4\pi]$. Then $\dfrac{8S}{\pi}$ is equal to ________.

Official Ans. by NTA (56)

Sol. Given equation

$\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$

$\Rightarrow 1-\sin^2\theta\cos^2\theta-\sin\theta\cos\theta=0$

$\Rightarrow 2-(\sin 2\theta)^2-\sin 2\theta=0$

$\Rightarrow (\sin 2\theta)^2+(\sin 2\theta)-2=0$

$\Rightarrow (\sin 2\theta+2)(\sin 2\theta-1)=0$

$\Rightarrow \sin 2\theta=1$ or $\underset{\text{(not possible)}}{\boxed{\sin 2\theta=-2}}$

$\Rightarrow 2\theta=\dfrac{\pi}{2},\dfrac{5\pi}{2},\dfrac{9\pi}{2},\dfrac{13\pi}{2}$

$\Rightarrow \theta=\dfrac{\pi}{4},\dfrac{5\pi}{4},\dfrac{9\pi}{4},\dfrac{13\pi}{4}$

$\Rightarrow S=\dfrac{\pi}{4}+\dfrac{5\pi}{4}+\dfrac{9\pi}{4}+\dfrac{13\pi}{4}=7\pi$

$\Rightarrow \dfrac{8S}{\pi}=\dfrac{8\times 7\pi}{\pi}=56.00$

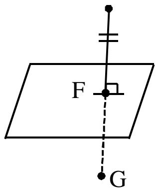
2. Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane $2x-y+z+3=0$ and let R $(3, 5, \gamma)$ be a point of this plane. Then the square of the length of the line segment SR is ________.

Official Ans. by NTA (72)

Sol. Since R $(3,5,\gamma)$ lies on the plane $2x-y+z+3=0$.

Therefore, $6-5+\gamma+3=0$

$\Rightarrow \gamma=-4$



Now,

dr's of line QS are $2, -1, 1$

equation of line QS is

$\dfrac{x-1}{2}=\dfrac{y-3}{-1}=\dfrac{z-4}{1}=\lambda$ (say)

$\Rightarrow F(2\lambda+1, -\lambda+3, \lambda+4)$

F lies in the plane

$\Rightarrow 2(2\lambda+1)-(-\lambda+3)+(\lambda+4)+3=0$

$\Rightarrow 4\lambda+2+\lambda-3+\lambda+7=0$

$\Rightarrow 6\lambda+6=0 \Rightarrow \lambda=-1.$

$\Rightarrow F(-1,4,3)$

Since, F is mid-point of QS.

Therefore, co-ordinated of S are $(-3,5,2)$.

So, $SR=\sqrt{36+0+36}=\sqrt{72}$

$SR^2=72.$

3. The probability distribution of random variable X is given by:

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p=P(1<X<4 \mid X<3)$. If $5p=\lambda K$, then λ equal to ________.

Official Ans. by NTA (30)

Sol. $\sum P(X) = 1 \Rightarrow k + 2k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{kX<4}{X<3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k}+\frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Now, $5p = \lambda k$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

4. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to ________.

Official Ans. by NTA (6)

Sol. $|z - 3| = \text{Re}(z)$

let $Z = x = iy$

$\Rightarrow (x-3)^2 + y^2 = x^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2$

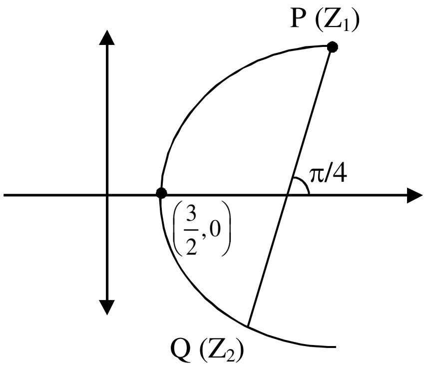
$\Rightarrow y^2 = 6x - 9$

$\Rightarrow y^2 = 6\left(x - \frac{3}{2}\right)$

$\Rightarrow z_1$ and z_2 lie on the parabola mentioned in eq.(1)

$\arg(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow$ Slope of PQ = 1.



Let $P\left(\frac{3}{2}+\frac{3}{2}t_1^2, 3t_1\right)$ and $Q\left(\frac{3}{2}+\frac{3}{2}t_2^2, 3t_2\right)$

Slope of PQ = $\frac{3(t_2 - t_1)}{\frac{3}{2}(t_2^2 - t_1^2)} = 1$

$\Rightarrow \frac{2}{t_2 + t_2} = 1$

$\Rightarrow t_2 + t_1 = 2$

$\text{Im}(z_1 + z_2) = 3t_1 + 3t_2 = 3(t_1 + t_2) = 3\ (2)$

Ans. 6.00

Aliter :

Let $z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\therefore \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ ______(1)

$|z_1 - 3| = \text{Re}(z_1) \Rightarrow (x_1 - 3)^2 + y_1^2 = x_1^2$ __ (2)

$|z_2 - 3| = \text{Re}(z_2) \Rightarrow (x_2 - 3)^2 + y_2^2 = x_2^2$ __(2)

sub (2) & (3)

$(x_1 - 3)^2 - (x_2 - 3)^2 + y_1^2 - y_2^2 = x_1^2 - x_2^2$

$(x_1 - x_2)(x_1 + x_2 - 6) + (y_1 - y_2)(y_1 + y_2)$

$= (x_1 - x_2)(x_1 + x_2)$

$x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2 \Rightarrow y_1 + y_2 = 6.$

5. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3} is ________.

Official Ans. by NTA (80)

Sol. 3n type $\rightarrow 3, 6, 9 = P$

$3n - 1$ type $\rightarrow 2, 5 = Q$

$3n - 2$ type $\rightarrow 1, 4 = R$

number of subset of S containing one element which are not divisible by 3 = ${}^2C_1 + {}^2C_1 = 4$

number of subset of S containing two numbers whose some is not divisible by 3

$= {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$

number of subsets containing 3 elements whose sum is not divisible by 3

$= {}^3C_2 \times {}^4C_1 + \left({}^2C_2 \times {}^2C_1\right)2 + {}^3C_1\left({}^2C_2 + {}^2C_2\right) = 22$

number of subsets containing 4 elements whose sum is not divisible by 3

$= {}^3C_3 \times {}^4C_1 + {}^3C_2\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_1{}^2C_1 \times {}^2C_2\right)2$

$= 4 + 6 + 12 = 22.$

number of subsets of S containing 5 elements whose sum is not divisible by 3.

$= {}^3C_3\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_2{}^2C_1 \times {}^2C_2\right) \times 2 = 2 + 12 = 14$

number of subsets of S containing 6 elements whose sum is not divisible by 3 = 4

$\Rightarrow$ Total subsets of Set A whose sum of digits is not divisible by 3 = 4 + 14 + 22 + 22 + 14 + 4 = 80.

6. Let A (secθ, 2tanθ) and B (secϕ, 2tanϕ), where $\theta + \phi = \pi/2$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ___________.

Official Ans. by NTA (36)

ALLEN Ans. (Bonus)

Sol. Since, point A (sec θ, 2 tan θ)

lies on the hyperbola

$2x^2 - y^2 = 2$

Therefore, $2\sec^2\theta - 4\tan^2\theta = 2$

$\Rightarrow 2 + 2\tan^2\theta - 4\tan^2\theta = 2$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$

Similarly, for point B, we will get $\phi = 0$.

but according to question $\theta + \phi = \dfrac{\pi}{2}$

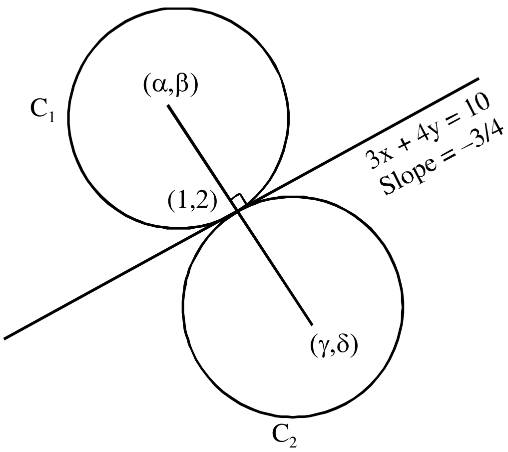
which is not possible.

Hence it must be a 'BONUS'.

7. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to ___________.

Official Ans. by NTA (40)

Sol. Slope of line joining centres of circles $= \dfrac{4}{3} = \tan\theta$



$\Rightarrow \cos\theta = \dfrac{3}{5}, \sin\theta = \dfrac{4}{5}$

Now using parametric form

$\dfrac{x-1}{\cos\theta} = \dfrac{y-2}{\sin\theta} = \pm 5$

$\oplus \quad (x, y) = (1 + 5\cos\theta, 2 + 5\sin\theta)$

$(\alpha, \beta) = (4, 6)$

$\ominus \quad (x, y) = (\gamma, \delta) = (1 - 5\cos\theta, 2 - 5\sin\theta)$

$(\gamma, s) = (-2, -2)$

$\Rightarrow |(\alpha + \beta)(\gamma + \delta)| = |10x - 4| = 40$

8. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder __________.

Official Ans. by NTA (15)

Sol. $3(1 + 6)^{22} + 2\cdot(1 + 9)^{22} - 44 = (3 + 2 - 44) = 18\,.I$

$= -39 + 18.I$

$= (54 - 39) + 18(I - 3)$

$= 15 + 18\,I_1$

$\Rightarrow$ Remainder = 15.

9. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _________.

Official Ans. by NTA (25)

Sol. $\sigma_b^2 = 2$ (variance of boys) $\quad n_1$ = no. of boys

$\bar{x}_b = 12 \quad n_2$ = no. of girls

$\sigma_g^2 = 2$

$\bar{x}_g = \dfrac{50 \times 15 - 12 \times \sigma_b}{30} = \dfrac{750 - 12 \times 20}{30} = 17 = \mu$

variance of combined series

$$\sigma^2 = \frac{n_1\sigma_b^2 + n_2\sigma_g^2}{n_1 + n_2} + \frac{n_1 \cdot n_2}{(n_1 + n_2)^2}(\bar{x}_b - \bar{x}_g)^2$$

$$\sigma^2 = \frac{20 \times 2 + 30 \times 2}{20 + 30} + \frac{20 \times 30}{(20 + 30)^2}(12 - 17)^2$$

$\sigma^2 = 8.$

$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$

10. If $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}(ux + v \log_e(4e^x + 7e^{-x})) + C$, where C is a constant of integration, then u + v is equal to __________.

Official Ans. by NTA (7)

Sol. $\int \frac{2e^x}{4e^x + 7e^{-x}} dx + 3\int \frac{e^{-x}}{4e^x + 7e^{-x}} dx$

$= \int \frac{2e^{2x}}{4e^{2x} + 7} dx + 3\int \frac{e^{-2x}}{4 + 7e^{-2x}} dx$

Let $4e^{2x} + 7 = T$ Let $4 + 7e^{-2x} = t$

$8\, e^{2x}\, dx = dT$ $-14\, e^{-2x}\, dx = dt$

$2e^{2x}dx = \frac{dT}{4}$ $e^{-2x}dx = -\frac{dt}{14}$

$\int \frac{dT}{4T} - \frac{3}{14}\int \frac{dt}{t}$

$= \frac{1}{4}\log T - \frac{3}{14}\log t + C$

$= \frac{1}{4}\log(4e^{2x} + 7) - \frac{3}{14}\log\left(4 + 7e^{-2x}\right) + C$

$= \frac{1}{14}\left[\frac{1}{2}\log\left(4e^x + 7e^{-x}\right) + \frac{13}{2}x\right] + C$

$u = \frac{13}{2}, v = \frac{1}{2} \Rightarrow u + v = 7$

Aliter :

$2e^x + 3e^{-x} = A\,(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x}) + \lambda$

$2 = 4A + 4B$; $3 = 7A - 7B$; $\lambda = 0$

$A + B = \frac{1}{2}$

$A - B = \frac{3}{7}$

$A = \frac{1}{2}\left(\frac{1}{2} + \frac{3}{7}\right) = \frac{7+6}{28} = \frac{13}{28}$

$B = A - \frac{3}{7} = \frac{13}{28} - \frac{3}{7} = \frac{13-12}{28} = \frac{1}{28}$

$\int \frac{13}{28}dx + \frac{1}{28}\int \frac{4e^x - 7e^{-x}}{4e^x + 7e^{-x}}dx$

$\frac{13}{28}x + \frac{1}{28}\ell n\,|\,4e^x + 7e^{-x}\,| + C$

$u = \frac{13}{2}; v = \frac{1}{2}$

$\Rightarrow u + v = 7$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Tuesday 31st August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

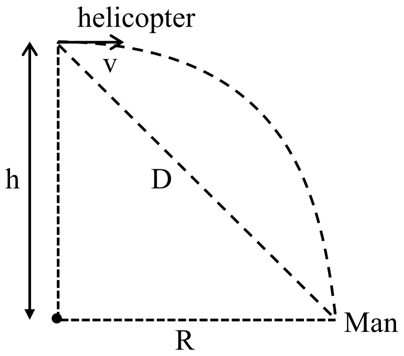
1. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

(1) $\sqrt{\dfrac{2ghv^2+1}{h^2}}$ (2) $\sqrt{2ghv^2+h^2}$

(3) $\sqrt{\dfrac{2v^2h}{g}+h^2}$ (4) $\sqrt{\dfrac{2gh}{v^2}+h^2}$

Official Ans. by NTA (3)

Sol.



$$R=\sqrt{\frac{2h}{g}}\cdot v$$

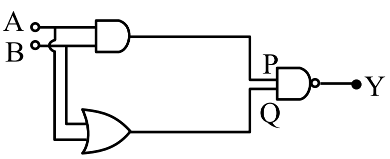
$$D=\sqrt{R^2+h^2}$$

$$=\sqrt{\left(\sqrt{\frac{2h}{g}}\cdot v\right)^2+h^2}$$

$$D=\sqrt{\frac{2hv^2}{g}+h^2}$$

Option (3) is correct

2. In the following logic circuit the sequence of the inputs A, B are (0, 0), (0,1), (1, 0) and (1, 1). The output Y for this sequence will be :

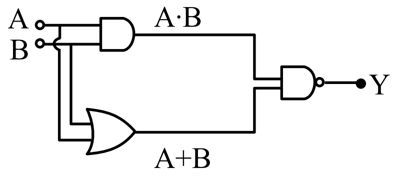


(1) 1, 0, 1, 0 (2) 0, 1, 0, 1

(3) 1, 1, 1, 0 (4) 0, 0, 1, 1

Official Ans. by NTA (3)

Sol.



$$Y=\overline{(A\cdot B)\cdot(A+B)}$$

$$Y)_{(0,0)}=1$$

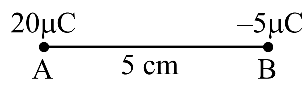
$$Y)_{(0,1)}=1$$

$$Y)_{(1,0)}=1$$

$$Y)_{(1,1)}=0$$

Option (3) is correct

3. Two particles A and B having charges 20 μC and –5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force?



(1) At 5 cm from 20 μC on the left side of system

(2) At 5 cm from – 5 μC on the right side

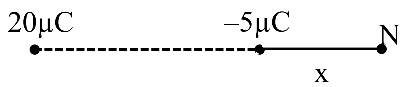
(3) At 1.25 cm from – 5 μC between two charges

(4) At midpoint between two charges

Official Ans. by NTA (2)

Sol.

Null point is possible only right side of –5μC



$$E_N=+\frac{k(-5\mu C)}{x^2}+\frac{k(20\mu C)}{(5+x)^2}=0$$

x = 5 cm

∴ option (2) is correct

4. A reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink :

(1) 174°C (2) 280°C

(3) 180.4°C (4) 382°C

Official Ans. by NTA (1)

Official Ans. by ALLEN (Bonus)

Sol. T_2 = sink temperature

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{4} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = \frac{3}{4} \quad \dots \text{(i)}$$

$$\frac{1}{2} = 1 - \frac{T_2 - 58}{T_1}$$

$$\frac{T_2}{T_1} - \frac{58}{T_1} = \frac{1}{2}$$

$$\frac{3}{4} = \frac{58}{T_1} + \frac{1}{2}$$

$$\frac{1}{4} = \frac{58}{T_1} \Rightarrow T_1 = 232$$

$$T_2 = \frac{3}{4} \times 232$$

T_2 = 174 K

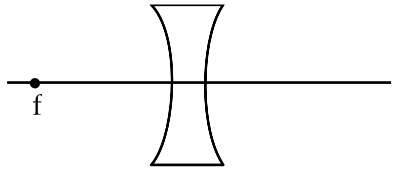
5. An object is placed at the focus of concave lens having focal length *f*. What is the magnification and distance of the image from the optical centre of the lens?

(1) 1, ∞ (2) Very high, ∞

(3) $\frac{1}{2}, \frac{f}{2}$ (4) $\frac{1}{4}, \frac{f}{4}$

Official Ans. by NTA (3)

Sol.



U = – f

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{-f} \Rightarrow \frac{1}{V} = -\frac{2}{f}$$

$$V = \frac{-f}{2}$$

$$m = \frac{V}{U} = \frac{1}{2}$$

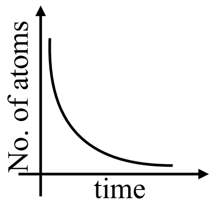
distance = $\frac{f}{2}$

Option (3)

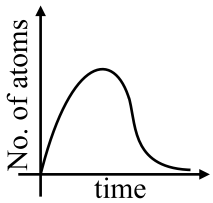
6. A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B vesus time is :

(Assume that at t = 0, there are no B atoms in the sample)

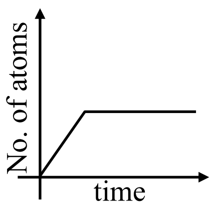
(1)



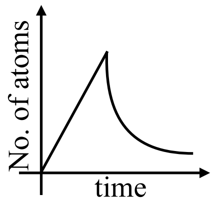
(2)



(3)



(4)



Official Ans. by NTA (2)

Sol. A $\longrightarrow$ B $\longrightarrow$ C (stable)

Initially no. of atoms of B = 0 after t = 0, no. of atoms of B will starts increasing & reaches maximum value when rate of decay of B = rate of formation of B.

After that maximum value, no. of atoms will starts decreasing as growth & decay both are exponential functions, so best possible graph is (2)

Option (2)

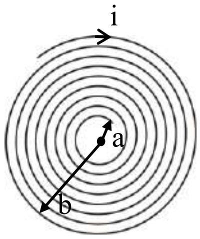
7. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil :

(1) $\frac{\mu_0 IN}{2(b-a)} \log_e\left(\frac{b}{a}\right)$ (2) $\frac{\mu_0 I}{8}\left[\frac{a+b}{a-b}\right]$

(3) $\frac{\mu_0 I}{4(a-b)}\left[\frac{1}{a}-\frac{1}{b}\right]$ (4) $\frac{\mu_0 I}{8}\left(\frac{a-b}{a+b}\right)$

Official Ans. by NTA (1)

Sol.



No. of turns in dx width = $\frac{N}{b-a}dx$

$$\int dB = \int_a^b \left(\frac{N}{b-a}\right) dx \frac{\mu_0 i}{2x}$$

$$B = \frac{N\mu_0 i}{2(b-a)} \ n\left(\frac{b}{a}\right)$$

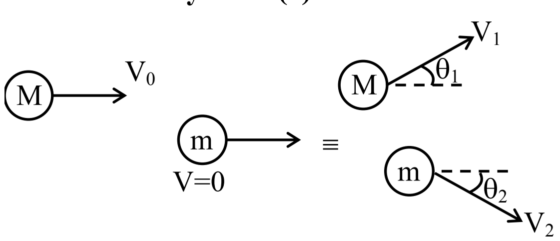
Option (1)

8. A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

(1) 4 (2) 1 (3) 3 (4) 2

Official Ans. by NTA (3)

Sol.



given $\theta_1 = \theta_2 = \theta$

from momentum conservation

in x-direction $MV_0 = MV_1\cos\theta + mV_2\cos\theta$

in y-direction $0 = MV_1\sin\theta - mV_2\sin\theta$

Solving above equations

$$V_2 = \frac{MV_1}{m}, V_0 = 2V_1\cos\theta$$

From energy conservation

$$\frac{1}{2}MV_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2$$

Substituting value of V_2 & V_0, we will get

$$\frac{M}{m} + 1 = 4\cos^2\theta \le 4$$

$$\frac{M}{m} \le 3$$

Option (3)

9. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses:

(1) $V = \frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

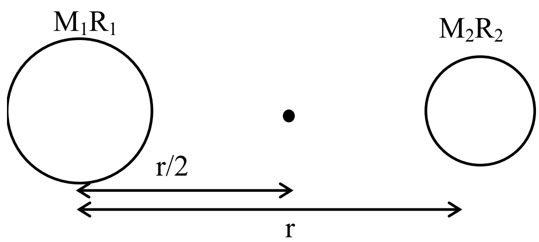
(2) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

(3) $V = \frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

(4) $V = \frac{\sqrt{2G}(M_1+M_2)}{r}$

Official Ans. by NTA (2)

Sol.



$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$\frac{1}{2}mV^2 = \frac{2Gm}{r}(M_1 + M_2)$$

$$V = \sqrt{\frac{4G(M_1+M_2)}{r}}$$

Option (2)

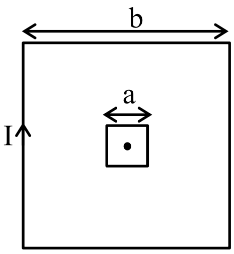
10. A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b >> a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is :

(1) $\frac{\mu_0}{4\pi}8\sqrt{2}\frac{a^2}{b}$ (2) $\frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{a}$

(3) $\frac{\mu_0}{4\pi}8\sqrt{2}\frac{b^2}{a}$ (4) $\frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{b}$

Official Ans. by NTA (1)

Sol.



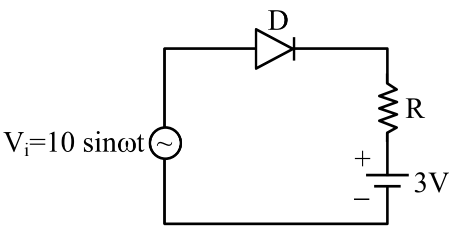
$$B=\left[\frac{\mu_0}{4\pi}\frac{I}{b/2}\times 2\sin 45\right]\times 4$$

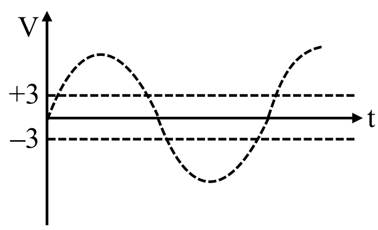
$$\phi = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{I}{b}\times a^2$$

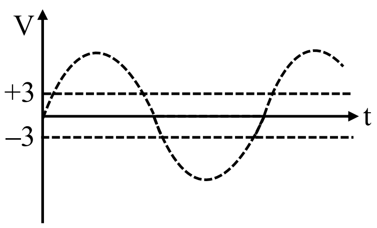
$$\therefore M=\frac{\phi}{I}=\frac{2\sqrt{2}\mu_0 a^2}{\pi b}=\frac{\mu_0}{4\pi}8\sqrt{2}\frac{a^2}{b}$$

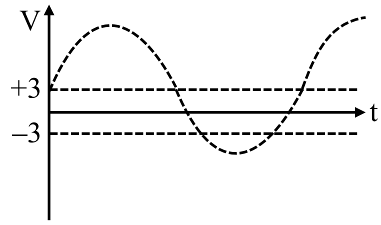
Option (1)

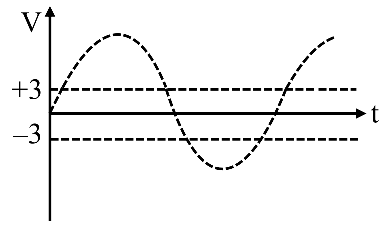
11. Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one:



(1)


(2)


(3)


(4)


Official Ans. by NTA (3)

Official Ans. by ALLEN (1)

Sol. When $V_i > 3$ volt, $V_R > 0$

Because diode will be in forward biased state

When $V_i \le 3$volt ; $V_R = 0$

Because diode will be in reverse biased state.

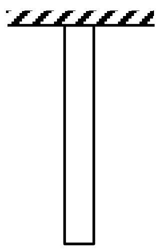
12. A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight.

(1) 2×10^{-9} m (2) 5×10^{-8} m

(3) 4×10^{-8} m (4) 5×10^{-10} m

Official Ans. by NTA (4)

Sol.



We know,

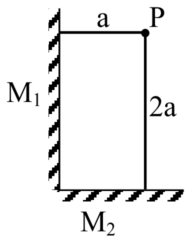
$$\Delta = \frac{WL}{2AY}$$

$$\Delta = \frac{10\times 1}{2\times 5}\times 100\times 10^{-4}\times 2\times 10^{11}$$

$$\Delta = \frac{1}{2}\times 10^{-9} = 5\times 10^{-10}\,\text{m}$$

Option (4)

13. Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : (Take $\sqrt{5}$ = 2.3)

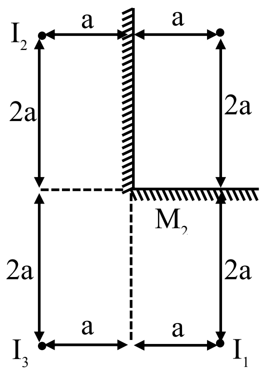


(1) 3a (2) 4.6 a

(3) 2.3 a (4) $2\sqrt{10}$ a

Official Ans. by NTA (2)

Sol.



Shortest distance is 2a between I_1 & I_3

But answer given is for I_1 & I_2

$$\sqrt{(4a)^2+(2a)^2}$$

$$a\sqrt{20}$$

4.47 a

Option (2)

14. Match **List-I** with **List-II**.

List-I	**List-II**
(a) Torque	(i) MLT^{-1}
(b) Impulse	(ii) MT^{-2}
(c) Tension	(iii) ML^2T^{-2}
(d) Surface Tension	(iv) MLT^{-2}

Choose the **most appropriate** answer from the option given below :

(1) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

(2) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

(3) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

(4) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

Official Ans. by NTA (1)

Sol. torque $\tau \rightarrow ML^2T^{-2}$ (III)

Impulse $I \Rightarrow MLT^{-1}$ (I)

Tension force $\Rightarrow MLT^{-2}$ (IV)

Surface tension $\Rightarrow MT^{-2}$ (II)

Option (1)

15. For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If $p = p_0$ at v = 0 is the given boundary condition, then the maximum temperature one mole of gas can attain is :

(Here R is the gas constant)

(1) $\frac{p_0}{aeR}$ (2) $\frac{ap_0}{eR}$

(3) infinity (4) 0ºC

Official Ans. by NTA (1)

Sol. $$\int_{p_0}^{p}\frac{dp}{P} = -a\int_0^v dv$$

$$\ell n\left(\frac{p}{p_0}\right) = -av$$

$$p = p_0e^{-av}$$

For temperature maximum p-v product should be maximum

$$T = \frac{pv}{nR} = \frac{p_0ve^{-av}}{R}$$

$$\frac{dT}{dv} = 0 \Rightarrow \frac{p_0}{R}\left\{e^{-av} + ve^{-av}(-a)\right\}$$

$$\frac{p_0e^{-av}}{R}\{1-av\} = 0$$

$$v = \frac{1}{a}, \infty$$

$$T = \frac{p_0 1}{Rae} = \frac{p_0}{Rae}$$

at $v = \infty$

$T = 0$

Option (1)

16. Which of the following equations is dimensionally incorrect ?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume , p = pressure, W = work done, Γ = torque, $\in$= permittivity, E = electric field, J = current density, L = length.

(1) $v = \frac{\pi p a^4}{8\eta L}$ (2) $h = \frac{2s\cos\theta}{\rho r g}$

(3) $J = \in \frac{\partial E}{\partial t}$ (4) $W = \Gamma\theta$

Official Ans. by NTA (1)

Sol. (i) $\frac{\pi p a^4}{8\eta L} = \frac{dv}{dt}$ = Volumetric flow rate (poiseuille's law)

(ii) $h\rho g = \frac{2s}{r}\cos\theta$

(iii) RHS $\Rightarrow \varepsilon \times \frac{1}{4\pi\varepsilon_0}\frac{a}{r^2} \times \frac{1}{\varepsilon} = \frac{q}{t} \times \frac{1}{r^2}$

$= \frac{I}{L^2} = IL^{-2}$

LHS

$T = \frac{I}{A} = IL^{-2}$

(iv) $W = \tau\theta$

Option (1)

17. Angular momentum of a single particle moving with constant speed along circular path :

(1) changes in magnitude but remains same in the direction

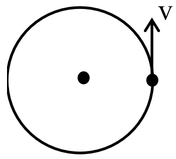
(2) remains same in magnitude and direction

(3) remains same in magnitude but changes in the direction

(4) is zero

Official Ans. by NTA (2)

Sol.



$|L| = mvr$

And direction will be upward & remain constant

Option (2)

18. In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

(1) $2R^2$ (2) Zero

(3) R (4) $R\sqrt{2}$

Official Ans. by NTA (3)

Sol. $Z = \sqrt{(X_L - X_C)^2 + R^2} = R \quad X_L = X_C$

Option (3)

19. A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

(1) $K_p < K_e$ and $P_p = P_e$

(2) $K_p = K_e$ and $P_p = P_e$

(3) $K_p < K_e$ and $P_p < P_e$

(4) $K_p > K_e$ and $P_p = P_e$

Official Ans. by NTA (1)

Sol. $\lambda_P = \frac{h}{P_P} \qquad \lambda_e = \frac{h}{P_e}$

$\lambda_P = \lambda_e$

$\Rightarrow P_P = P_e$

$(K)_P = \frac{P_P^2}{2m_P}$

$(K)_e = \frac{P_e^2}{2m_e}$

$K_P < K_e$ as $m_P > m_e$

Option (1)

20. Consider a galvanometer shunted with 5Ω resistance and 2% of current passes through it.

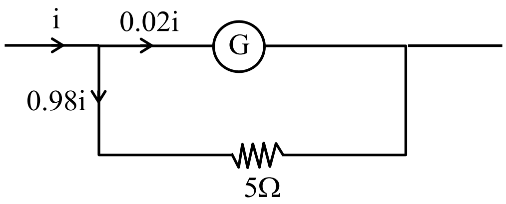
What is the resistance of the given galvanometer ?

(1) 300 Ω (2) 344 Ω

(3) 245 Ω (4) 226 Ω

Official Ans. by NTA (3)

Sol.



$0.02i\ Rg = 0.98i \times 5$

$Rg = 245\ \Omega$

Option (3)

SECTION-B

1. When a rubber ball is taken to a depth of __________m in deep sea, its volume decreases by 0.5%.

(The bulk modulus of rubber = 9.8×10^{8} Nm^{-2}

Density of sea water = 10^{3} kgm^{-3}

$g = 9.8\ m/s^{2}$)

Official Ans. by NTA (500)

Sol. $$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = -\frac{\rho g h}{\left(\frac{\Delta V}{V}\right)}$$

$$-\frac{B\frac{\Delta V}{V}}{\rho g} = h$$

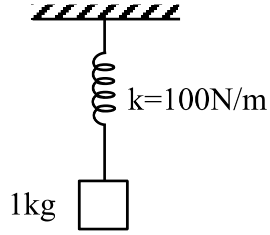
$$\frac{9.8\times10^{8}\times0.5}{100\times10^{3}\times9.8} = h$$

$h = 500$

2. A particle of mass 1 kg is hanging from a spring of force constant 100 $Nm^{-1.}$ The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$ .The value of x is ________.

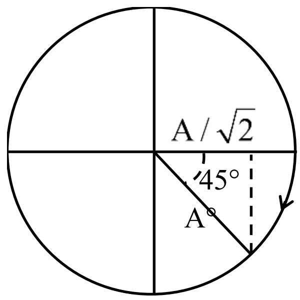
Official Ans. by NTA (8)

Sol.



KE = PE

$$y = \frac{A}{\sqrt{2}} = A\sin\omega t$$



$$t = \frac{T}{8} = \frac{T}{x}$$

$x = 8$

3. If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is _____km.

(Take radius of Earth = 6400 km)

Official Ans. by NTA (64)

Sol. $h_T = h_R = 160$ … (i)

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

$$d = \sqrt{2R}\left[\sqrt{h_T} + \sqrt{h_R}\right]$$

$$d = \sqrt{2R}\left[\sqrt{x} + \sqrt{160 - x}\right]$$

$$\frac{d(d)}{dx} = 0$$

$$\frac{1}{2\sqrt{x}} + \frac{1(-1)}{2\sqrt{160-x}} = 0$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{160-x}}$$

x = 80 m

$$d_{max} = \sqrt{2\times 6400}\left[\sqrt{\frac{80}{1000}} + \sqrt{\frac{20}{1000}}\right]$$

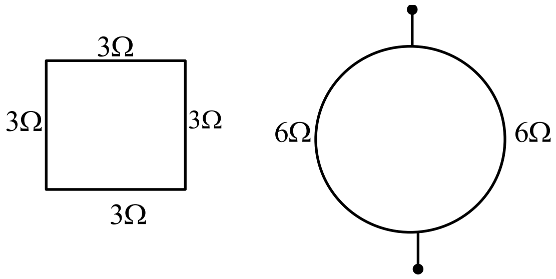
$$= \frac{80\sqrt{2}\times 2\sqrt{80}}{10\sqrt{10}}$$

$$= 8\times 2\times \sqrt{2}\times 2\sqrt{2}$$ = 64 km

4. A square shaped wire with resistance of each side 3Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be ______.

Official Ans. by NTA (3)

Sol.



$R_{eq} = 3\Omega$

5. A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is ______m.

Official Ans. by NTA (10)

Sol. $\mu = 9.0\times 10^{-4}\,\frac{kg}{m}$

T = 900 N

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9\times 10^{-4}}} = 1000\,m/s$$

f_1 = 500 Hz

f = 550

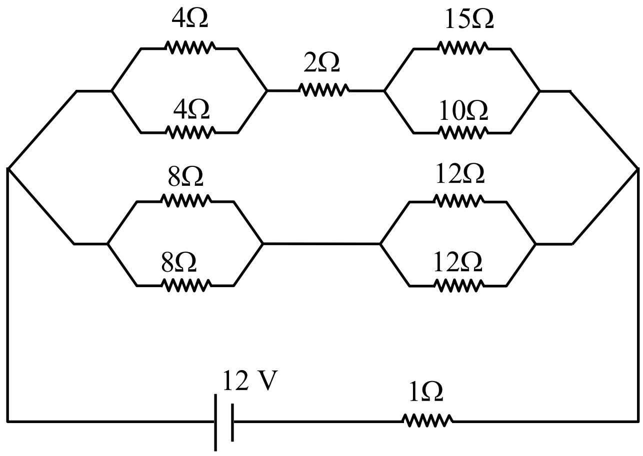
$$\frac{nV}{2} = 500 \;\dots.\,(i)$$

$$\frac{(n+1)V}{2} = 500 \;\dots\,(ii)$$

(ii) (i) $\frac{V}{2} = 50$

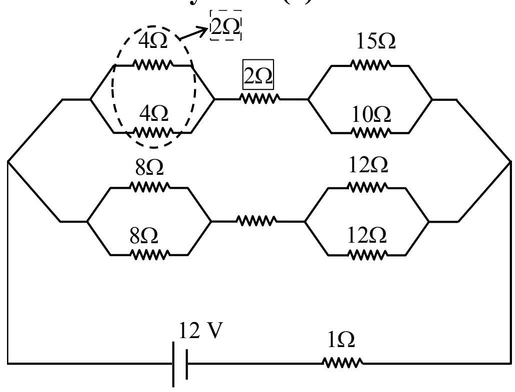
$$= \frac{1000}{2\times 50} = 10$$

6. The voltage drop across 15Ω resistance in the given figure will be______V.

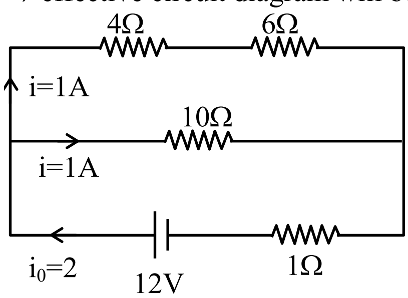


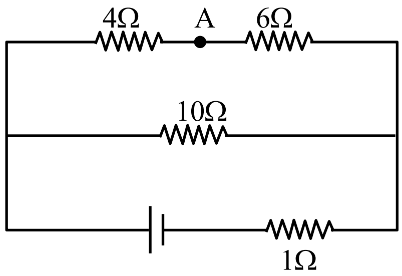
Official Ans. by NTA (6)

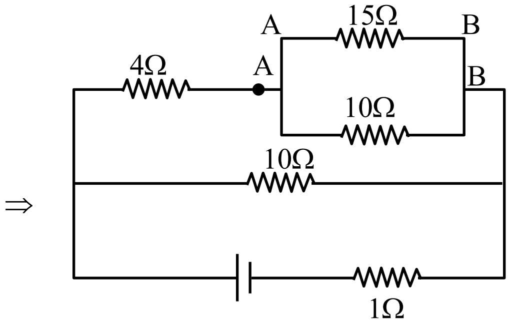
Sol.



⇒ effective circuit diagram will be







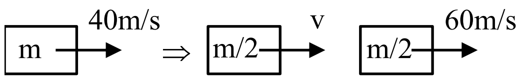
Point drop across $6\Omega = 1 \times 6 = 6 = V_{AB}$

$\Rightarrow$ Hence point drop across 15Ω = 6 volt = V_{AB}

7. A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x : 4 where x =__________.

Official Ans. by NTA (1)

Sol.



$P_i = P_f$

$$m \times 40 = \frac{m}{2} \times v + \frac{m}{2} \times 60$$

$$40 = \frac{v}{2} + 30$$

$\Rightarrow v = 20$

$$(K.E.)_I = \frac{1}{2} m \times (40)^2 = 800m$$

$$(K.E.)_f = \frac{1}{2}\frac{m}{2}.(20)^2 + \frac{1}{2}.\frac{m}{2}(60)^2 = 1000\ m$$

$$|\Delta K.E.| = |1000m - 800m| = 200m$$

$$\frac{\Delta K.E}{(K.E.)_i} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4}$$

x = 1

8. The electric field in an electromagnetic wave is given by $E = (50\ NC^{-1}) \sin\omega\ (t-x/c)$

The energy contained in a cylinder of volume V is 5.5×10^{-12} J. The value of V is __________cm^3.

$\left(\text{given} \in_0 = 8.8 \times 10^{-12} C^2N^{-1}m^{-2}\right)$

Official Ans. by NTA (500)

Sol. $E = 50 \sin\left(\omega t - \frac{\omega}{c}.x\right)$

Energy density = $\frac{1}{2} \in_0 E_0^2$

Energy for volume V = $\frac{1}{2} \in_0 E_0^2.V = 5.5 \times 10^{-12}$

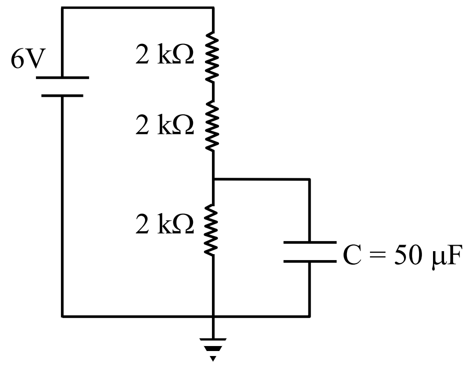
$$\frac{1}{2} 8.8 \times 10^{-12} \times 2500V = 5.5 \times 10^{-12}$$

$$V = \frac{5.5 \times 2}{2500 \times 8.8} = .0005m^3$$

$= .0005 \times 10^6\ (c.m)^3$

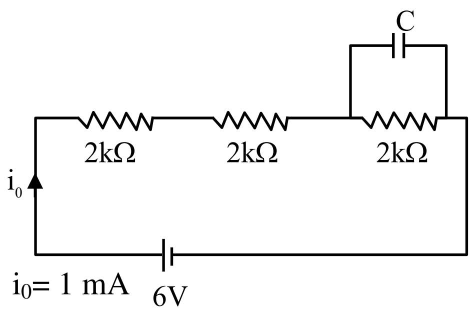
$= 500\ (c.m)^3$

9. A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is__________μC.



Official Ans. by NTA (100)

Sol.



Pot. Diff. across each resistor = 2V

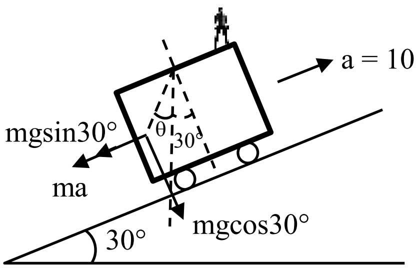
q = CV

$= 50 \times 10^{-6} \times 2 = 100 \times 10^{-6} = 100\ \mu C$

10. A car is moving on a plane inclined at 30º to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is______. (Take g = 10 ms^{-2})

Official Ans. by NTA (30)

Sol.



$$\tan(30+\theta) = \frac{mg\sin 30° + ma}{mg\cos 30°}$$

$$\tan(30+\theta) = \frac{5+10}{5\sqrt{3}} = \frac{1+2}{\sqrt{3}}$$

$$\frac{\tan\theta + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}\tan\theta} = \sqrt{3}$$

$$\sqrt{3}\tan\theta + 1 = 3 - \sqrt{3}\tan\theta$$

$$2\sqrt{3}\tan\theta = 2$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$\theta = 30°$

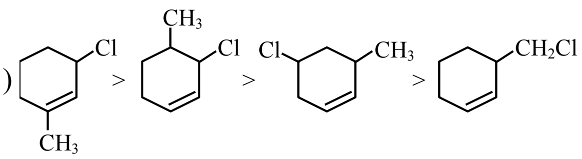
FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

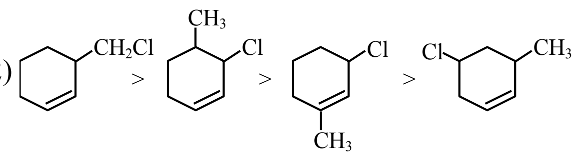
(Held On Tuesday 31st August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

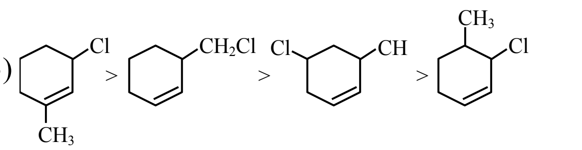
CHEMISTRY | **TEST PAPER WITH SOLUTION**

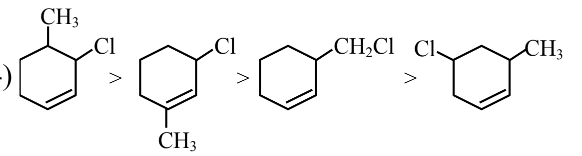
SECTION-A

1. The **correct** order of reactivity of the given chlorides with acetate in acetic acid is :

(1) 

(2) 

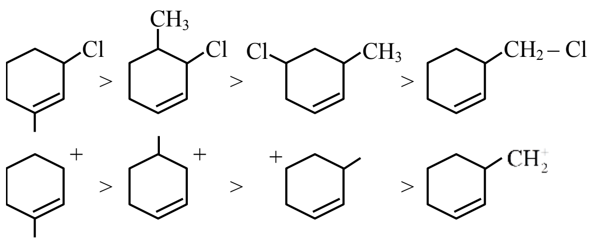
(3) 

(4) 

Official Ans. by NTA (1)

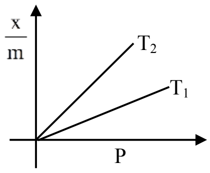
Sol. As it is example of SN^1.

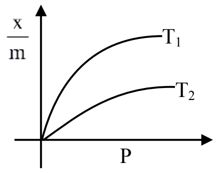
so carbocation stability ↑, reaction rate ↑

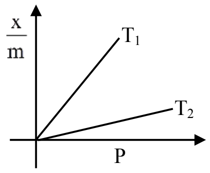


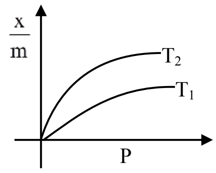
2. Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and T_2 $(T_1 > T_2)$ for a gas :

(x – mass of the gas adsorbed ; m – mass of adsorbent ; P – pressure)

(1) 

(2) 

(3) 

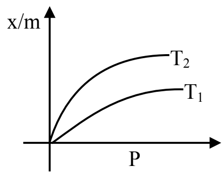
(4) 

Official Ans. by NTA (4)

Sol. $\frac{x}{m} \alpha P^{1/n} \left(0 < \frac{1}{n} < 1\right)$

On Increasing temperature $\frac{x}{m}$ decreases.

∵ adsorption is generally exothermic



$(T_1 > T_2)$

3. The major component/ingredient of Portland Cement is :

(1) tricalcium aluminate

(2) tricalcium silicate

(3) dicalcium aluminate

(4) dicalcium silicate

Official Ans. by NTA (2)

Sol. Major component of portland cement is "Tricalcium silicate (51% , $3CaO.SiO_2$)

4. In the structure of the dichromate ion, there is a :

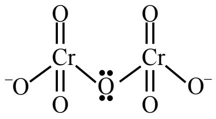
(1) linear symmetrical Cr–O–Cr bond.

(2) non-linear symmetrical Cr–O–Cr bond.

(3) linear unsymmetrical Cr–O–Cr bond.

(4) non-linear unsymmetrical Cr–O–Cr bond.

Official Ans. by NTA (2)

Sol. 

dichromate ion contain non-linear symmetrical Cr–O–Cr Bond

5. Which one of the following compounds contains β-C_1-C_4 glycosidic linkage ?

(1) Lactose (2) Sucrose

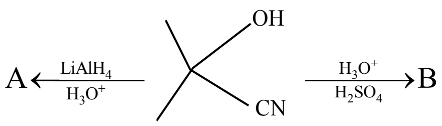
(3) Maltose (4) Amylose

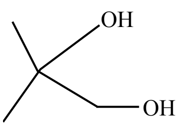
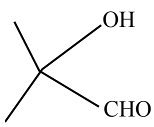
Official Ans. by NTA (1)

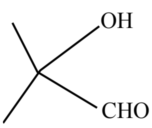
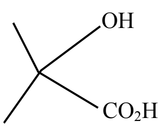
Sol. In Lactose it is β $C_1 - C_4$ glycosidic linkage.

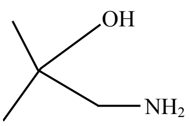
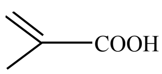
In Maltose, Amylose α $C_1 - C_4$ glycosidic linkage is present

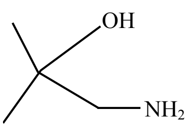
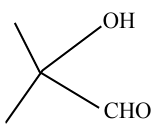
6. The major products A and B in the following set of reactions are :



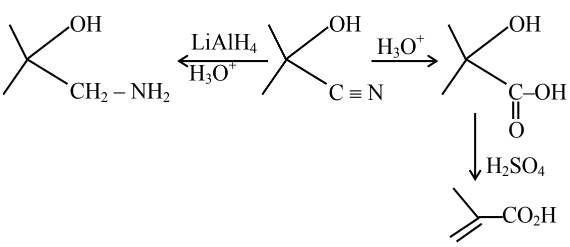
(1) A =  , B = 

(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Official Ans. by NTA (3)

Sol. 

7. Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature ? (Given Z for Nd = 60, Yb = 70, La = 57, Ce =58)

(1) Nd (2) Yb (3) La (4) Ce

Official Ans. by NTA (2)

Sol. Ytterbium shows +2 oxidation state with diamagnetic nature

So ans is 2

8. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **(A)** is true but **(R)** is false

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

Official Ans. by NTA (4)

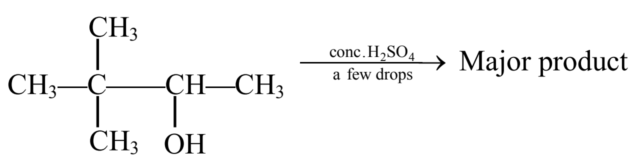
Sol. (A) Aluminium is reactive metal so Aluminium is extracted by electrolysis of Alumina with molten mixture of Cryolite

(B) Cryolite, Na_3AlF_6

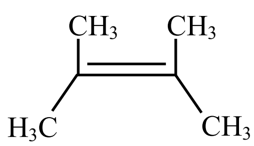
Here Al is in +3 O.S.

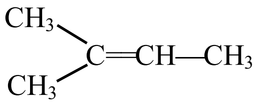
So Answer is 4

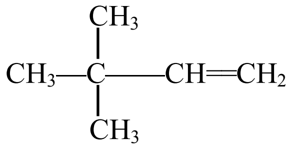
9. The major product formed in the following reaction is :



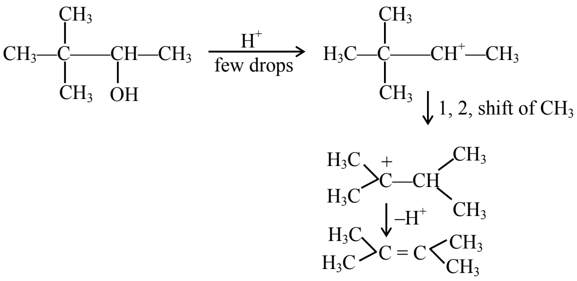
(1) $CH_3-\underset{\underset{CH_3}{|}}{C}=CH-CH_2CH_3$

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

10. Monomer of Novolac is :

(1) 3-Hydroxybutanoic acid

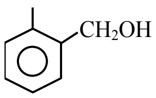
(2) phenol and melamine

(3) o-Hydroxymethylphenol

(4) 1,3-Butadiene and styrene

Official Ans. by NTA (3)

Sol. Monomer of Novolac is

 O-hydroxy methyl phenol

11. Given below are two statements :

Statement-I : The process of producing syn-gas is called gasification of coal.

Statement-II : The composition of syn-gas is $CO + CO_2 + H_2$ (1 : 1 : 1)

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement-I** is false but **Statement-II** is true

(2) **Statement-I** is true but **Statement-II** is false

(3) Both **Statement-I** and **Statement-II** are false

(4) Both **Statement-I** and **Statement-II** are true

Official Ans. by NTA (2)

Sol. The process of producing syn-gas from coal is called gasification of coal.

Syn-gas having composition of CO & H_2 in 1 : 1

12. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

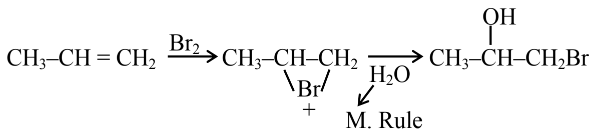
(1) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(4) **(A)** is true but **(R)** is false

Official Ans. by NTA (3)

Sol. 

Its IUPAC name 1-bromopropan–2-ol

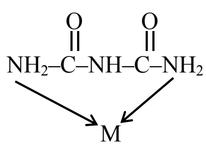
A and R are true and (R) is the correct explanation of (A)

13. The denticity of an organic ligand, biuret is :

(1) 2 (2) 4

(3) 3 (4) 6

Official Ans. by NTA (1)

Sol. 

Biuret :- Bidentate ligand

The denticity of organic ligand is 2.

14. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **(A)** is false but **(R)** is true.

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

Official Ans. by NTA (2)

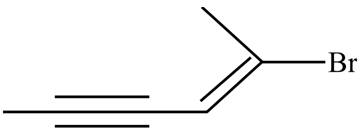
Sol. From left to right in periodic table :-

Metallic character decreases

Non-metallic character increases

$\Rightarrow$ It is due to increase in ionization enthalpy and increase in electron gain enthalpy.

15. Choose the **correct** name for compound given below :



(1) (4E)-5-Bromo-hex-4-en-2-yne

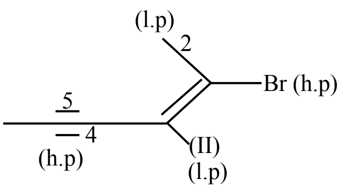
(2) (2E)-2-Bromo-hex-4-yn-2-ene

(3) (2E)-2-Bromo-hex-2-en-4-yne

(4) (4E)-5-Bromo-hex-2-en-4-yne

Official Ans. by NTA (3)

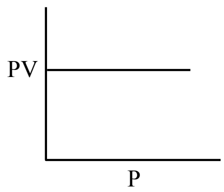
Sol.

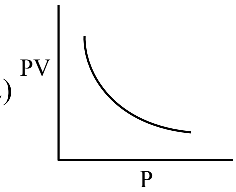


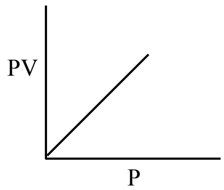
h.p. $\Rightarrow$ higher priority
l.p. $\Rightarrow$ lower priority

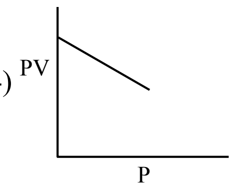
2E –2- bromo hex –2- en–4-yne

16. Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas ? (P and V stand for pressure and volume of the gas respectively)

(1)


(2)


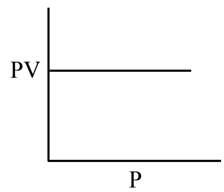
(3)


(4)


Official Ans. by NTA (1)

Sol. PV = nRT (n,T constant)

PV = constant



17. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)** :

Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reason (R) : Two liquids with a difference of more than 20°C in their boiling points can be separated by simple distillations.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **(A)** is false but **(R)** is true.

(2) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

Official Ans. by NTA (4)

Sol. Both assertion & reason are correct & (R) is the correct explanation of (A)

18. Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression ?

(1) hydrazine (2) glucose

(3) glycine (4) $KHSO_4$

Official Ans. by NTA (4)

18. $\because$ Van't Hoff factor is highest for $KHSO_4$

$\therefore$ colligative property (ΔT_f) will be highest for $KHSO_4$

19. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively :

(1) A > 50, B < 27 (2) A > 25, B < 17

(3) A < 5, B > 17 (4) A > 15, B > 47

Official Ans. by NTA (3)

Sol. BOD values of clean water (A) is less than 5 ppm

So A < 5

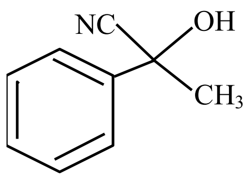
BOD values of polluted water (B is greater than 17 ppm

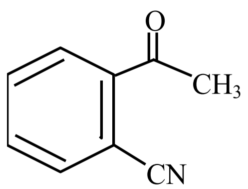
So B > 17

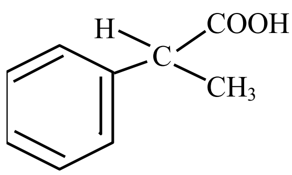
So Ans. is 3

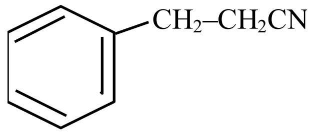
20. The structure of product C, formed by the following sequence of reactions is :

$$CH_3COOH + SOCl_2 \longrightarrow A \xrightarrow[AlCl_3]{Benzene} B \xrightarrow[-OH]{KCN} C$$

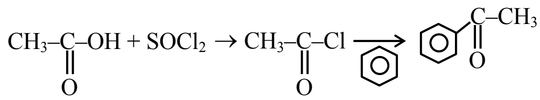
(1) 

(2) 

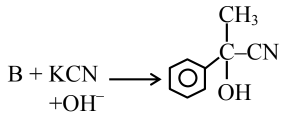
(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

(A) (B)



SECTION-B

1. Consider the following cell reaction :

$$Cd_{(s)} + Hg_2SO_{4(s)} + \frac{9}{5}H_2O_{(l)} \quad CdSO_4.\frac{9}{5}H_2O_{(s)} + 2Hg_{(l)}$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H° = -825.2$ kJ mol^{-1}, the standard entropy change $\Delta S°$ in J K^{-1} is ______. (Nearest integer)

[Given : Faraday constant = 96487 C mol^{-1}]

Official Ans. by NTA (25)

Sol. $\Delta G^o = -nFE^o = \Delta H^o - T\Delta S^o$

$$= \frac{\Delta H^o + nFE^o}{T}$$

$$= \frac{(-825.2\times10^3) + (2\times96487\times4.315)}{298}$$

$$= \frac{-825.2\times10^3 + 832.682\times10^3}{298}$$

$$= \frac{7.483\times10^3}{298} = 25.11\ JK^{-1}mol^{-1}$$

$\therefore$ Nearest integer answer is 25

2. The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4.2H_2O$) in 250 mL of water in mol L^{-1} is x × 10^{-2}. The value of x is ________. (Nearest integer)

[Atomic mass : H : 1.0, C : 12.0, O : 16.0]

Official Ans. by NTA (20)

Sol. $[H_2C_2O_4.2H_2O] = \frac{weight/M_W}{V(L)}$

$$\Rightarrow x \times 10^{-2} = \frac{6.3/126}{250/1000}$$

x = 20

3. Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is ________.

Official Ans. by NTA (4)

Sol. Pbs, CuS, As_2S_3, CdS are soluble in 50% HNO_3

HgS, Sb_2S_3 are insoluble in 50% HNO_3

So Answer is 4.

4. The total number of reagents from those given below, that can convert nitrobenzene into aniline is ______. (Integer answer)

I. Sn – HCl

II. Sn – NH_4OH

III. Fe – HCl

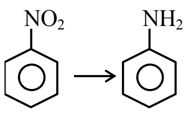
IV. Zn – HCl

V. H_2 – Pd

VI. H_2 – Raney Nickel

Official Ans. by NTA (5)

Sol.



Reagents used can be

(i) Sn + HCl

(ii) Fe + HCl

(iii) Zn + HCl

(iv) H_2 – Pd

(v) H_2 (Raney Ni)

5. The number of halogen/(s) forming halic (V) acid is __________.

Official Ans. by NTA (3)

Sol. The number of halogen forming halic (V) acid

$HClO_3$

$HBrO_3$

HIO_3

So Answer is 3

6. For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is _______. (Integer answer)

Official Ans. by NTA (2)

Sol. $k = \frac{2.303}{t}\log\frac{a}{a-x}$

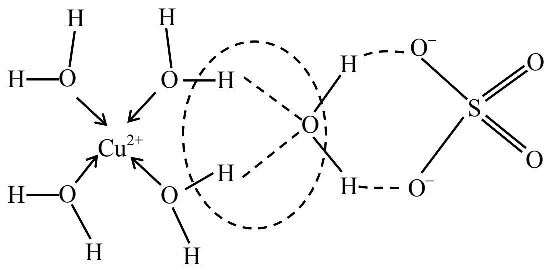
$$\frac{2.303}{t_{50\%}}\log\frac{100}{100-50} = \frac{2.303}{t_{75\%}}\log\frac{100}{100-75}$$

$t_{75\%} = 2\ t_{50\%}$

7. The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4.5H_2O$ is _______.

Official Ans. by NTA (1)

Sol.

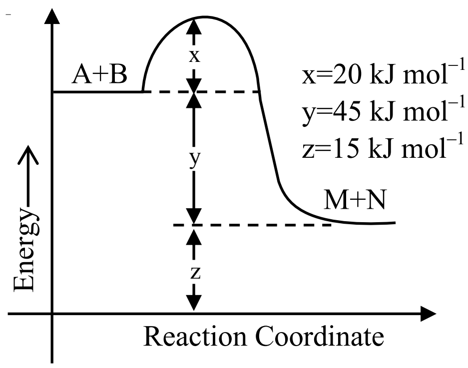


One hydrogen bonded H_2O molecule

8. According to the following figure, the magnitude of the enthalpy change of the reaction

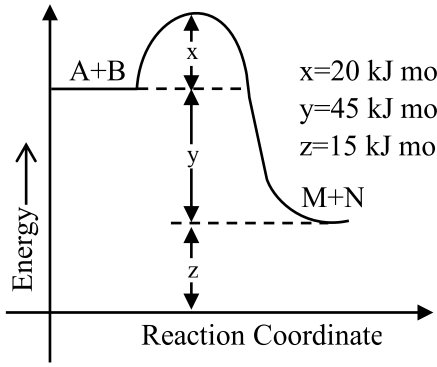
$A + B \rightarrow M + N$ in kJ mol^{-1}

is equal to _________. (Integer answer)



Official Ans. by NTA (45)

Sol.



$\Delta H = E_{a_f} - E_{a_b}$

$= 20 - 65$

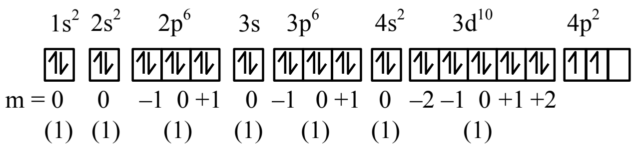
$= -45$ KJ/ mol

$|\Delta H| = 45$ KJ/mol

9. Ge(Z = 32) in its ground state electronic configuration has x completely filled orbitals with $m_l = 0$. The value of x is _______.

Official Ans. by NTA (7)

Sol.



Completely filled orbital with $m_\ell = 0$ are

= 1+1+1+1+1+1+1

= 7

So Answer is 7

10. A_3B_2 is a sparingly soluble salt of molar mass M (g mol^{-1}) and solubility x g L^{-1} . The solubility product satisfies $K_{sp} = a\left(\frac{x}{M}\right)^5$. The value of a is ________. (Integer answer)

Official Ans. by NTA (108)

Sol. $A_3B_2(s) \rightleftharpoons 3A^{+2}_{(aq)} + 2B^{-3}_{(aq)}$

3s 2s

$K_{SP} = (3s)^3 (2s)^2$

$K_{SP} = 108\ S^5$ & s = (X/M)

$$K_{SP} = 108\left(\frac{x}{m}\right)^5$$

given $K_{SP} = a\left(\frac{x}{m}\right)^5$

comparing a = 108

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Tuesday 31st August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \,\square\, q)$ is a tautology. Then :

(1) $* = \vee,\ \square = \vee$ (2) $* = \wedge,\ \square = \wedge$

(3) $* = \wedge,\ \square = \vee$ (4) $* = \vee,\ \square = \wedge$

Official Ans. by NTA (3)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

p	q	$\sim q$	$p \wedge \sim q$	$p \vee q$	$(p \wedge \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

2. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^{x} - 6 = 0$ is :

(1) 2 (2) 4

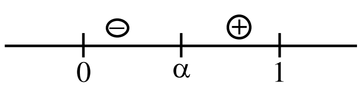
(3) 1 (4) 0

Official Ans. by NTA (3)

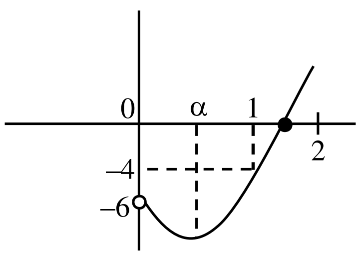
Sol. Let $e^{x} = t > 0$

$f(t) = t^4 + 2t^3 - t - 6 = 0$

$f'(t) = 4t^3 + 6t^2 - 1$



$f''(t) = 12t^2 + 12t > 0$



$f(0) = -6,\ f(1) = -4,\ f(2) = 24$

$\Rightarrow$ Number of real roots = 1

3. The sum of 10 terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots \text{ is :}$$

(1) 1 (2) $\frac{120}{121}$

(3) $\frac{99}{100}$ (4) $\frac{143}{144}$

Official Ans. by NTA (2)

Sol. $S = \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \ldots$

$= \left[\frac{1}{1^2} - \frac{1}{2^2}\right] + \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + \left[\frac{1}{3^2} - \frac{1}{4^2}\right] + \ldots + \left[\frac{1}{10^2} - \frac{1}{11^2}\right]$

$= 1 - \frac{1}{121}$

$= \frac{120}{121}$

4. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) –23 (2) – 15

(3) 23 (4) 15

Official Ans. by NTA (1)

Sol. Equation of plane is

$3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$

$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$

passing through (1, 4, –3)

$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$

$\Rightarrow \lambda = \frac{2}{3}$

$\Rightarrow$ equation of plane is

$-11x - 4y - 8z + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = -23$

5. Let f be a non–negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)dt$, $0 \le x \le 1$ and f(0) = 0, then $\lim\limits_{x\to 0}\frac{1}{x^2}\int_0^x f(t)dt$:

(1) equals 0 (2) equals 1

(3) does not exist (4) equals $\frac{1}{2}$

Official Ans. by NTA (4)

Sol. $\int\limits_0^x \sqrt{1-(f'(t))^2}\,dt = \int\limits_0^x f(t)dt \quad 0 \le x \le 1$

differentiating both the sides

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1 - (f'(x))^2 = f^2(x)$

$\frac{f'(x)}{\sqrt{1-f^2(x)}} = 1$

$\sin^{-1} f(x) = x + C$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

Now $\lim\limits_{x\to 0}\frac{\int\limits_0^x \sin t\,dt}{x^2}\left(\frac{0}{0}\right) = \frac{1}{2}$

6. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}| = |3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 4 (2) 6

(3) 5 (4) 8

Official Ans. by NTA (3)

Sol. $|3\vec{a}+\vec{b}|^2 = |2\vec{a}+3\vec{b}|^2$

$(3\vec{a}+\vec{b}).(3\vec{a}+\vec{b}) = (2\vec{a}+3\vec{b}).(2\vec{a}+3\vec{b})$

$9\vec{a}.\vec{a}+6\vec{a}.\vec{b}+\vec{b}.\vec{b} = 4\vec{a}.\vec{a}+12\vec{a}.\vec{b}+9.\vec{b}.\vec{b}$

$5|\vec{a}|^2 - 6\vec{a}.\vec{b} = 8|\vec{b}|^2$

$5(8)^2 - 6.8.|\vec{b}|\cos 60° = 8|\vec{b}|^2 \quad \left(\begin{array}{l}\because \frac{1}{8}|\vec{a}| = 1\\ \Rightarrow |\vec{a}| = 8\end{array}\right)$

$40 - 3|\vec{b}| = |\vec{b}|^2$

$\Rightarrow |\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$|\vec{b}| = -8, \quad |\vec{b}| = 5$

(rejected)

7. The function $f(x) = |x^2-2x-3|\cdot e^{|9x^2-12x+4|}$ is not differentiable at exactly :

(1) four points (2) three points

(3) two points (4) one point

Official Ans. by NTA (3)

Sol. $f(x) = |(x-3)(x+1)|.\ e^{(3x-2)^2}$

$$f(x) = \begin{cases} (x-3)(x+1).e^{(3x-2)^2} & ;\ x \in (3,\infty) \\ -(x-3)(x+1).e^{(3x-2)^2} & ;\ x \in [-1,3] \\ (x-3).(x+1).e^{(3x-2)^2} & ;\ x \in (-\infty,-1) \end{cases}$$

Clearly, non-differentiable at x = –1 & x = 3.

8. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is 3 r^2, then r^2 – d is equal to :

(1) $7-7\sqrt{3}$ (2) $7+\sqrt{3}$

(3) $7-\sqrt{3}$ (4) $7+3\sqrt{3}$

Official Ans. by NTA (2)

Sol. Let numbers be $\frac{a}{r}$, a, ar $\rightarrow$ G.P

$\frac{a}{r}$, 2a, ar $\rightarrow$ A.P $\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$r = 2 \pm \sqrt{3}$

4^{th} form of G.P = $3r^2 \Rightarrow ar^2 = 3r^2 \Rightarrow a = 3$

$r = 2+\sqrt{3}$, a = 3, d = $2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = (2+\sqrt{3})^2 - 3\sqrt{3}$

$= 7+4\sqrt{3}-3\sqrt{3}$

$= 7+\sqrt{3}$

9. Which of the following is **not** correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x-y| \le 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow |x-y| \le 1$ is reflexive and symmetric.

Official Ans. by NTA (2)

Sol. Note that (1,2) and (2,3) satisfy $0 < |x-y| \le 1$ but (1,3) does not satisfy it so

$0 \le |x-y| \le 1$ is symmetric but not transitive

So, (2) is correct.

10. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$ (2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

(3) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

Official Ans. by NTA (3)

Sol. $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$= \int \frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4}.(x-1)^2}$

put $\frac{x+2}{x-1} = t$

$= -\frac{1}{3}\int \frac{dt}{t^{5/4}}$

$= \frac{4}{3}.\frac{1}{t^{1/4}}+C$

$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$

11. If p and q are the lengths of the perpendiculars from the origin on the lines,

$x \operatorname{cosec}\alpha - y \sec\alpha = k\cot 2\alpha$ and

$x \sin\alpha + y\cos\alpha = k\sin 2\alpha$

respectively, then k^2 is equal to :

(1) $4p^2 + q^2$ (2) $2p^2 + q^2$

(3) $p^2 + 2q^2$ (4) $p^2 + 4q^2$

Official Ans. by NTA (1)

Sol. First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ ...(i)

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ ...(ii)

Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

12. $\operatorname{cosec}18°$ is a root of the equation :

(1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$

(3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$

Official Ans. by NTA (4)

Sol. $\operatorname{cosec}18° = \frac{1}{\sin 18°} = \frac{4}{\sqrt{5}-1} = \sqrt{5}+1$

Let $\operatorname{cosec}18° = x = \sqrt{5}+1$

$\Rightarrow x - 1 = \sqrt{5}$

Squaring both sides, we get

$x^2 - 2x + 1 = 5$

$\Rightarrow x^2 - 2x - 4 = 0$

13. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

(1) $a = -\frac{1}{3}, b \ne \frac{7}{3}$ (2) $a \ne \frac{1}{3}, b = \frac{7}{3}$

(3) $a \ne -\frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \ne \frac{7}{3}$

Official Ans. by NTA (4)

Sol. Here $D=\begin{vmatrix}2 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & a\end{vmatrix} \begin{aligned}&=2(-a-1)-1(a-1)+1+1\\ &=1-3a\end{aligned}$

$D_3=\begin{vmatrix}2 & 1 & 5\\ 1 & -1 & 3\\ 1 & 1 & b\end{vmatrix} \begin{aligned}&=2(-b-3)-1(b-3)+5(1+1)\\ &=7-3b\end{aligned}$

for $a=\frac{1}{3},\ b\neq\frac{7}{3}$, system has no solutions

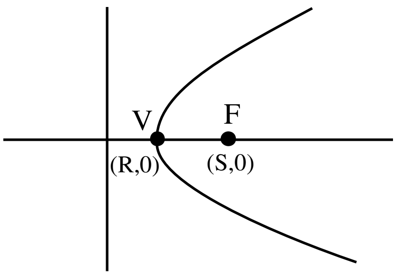
14. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x–axis at a distance R and S (>R) respectively from the origin, is :

(1) 4(S + R) (2) 2(S – R)

(3) 4(S – R) (4) 2(S + R)

Official Ans. by NTA (3)

Sol.



V $\rightarrow$ Vertex

F $\rightarrow$ focus

VF = S – R

So latus rectum = 4(S – R)

15. If the function $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , \quad x<0\\ k & , \quad x=0\\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , \quad x>0\end{cases}$

is continuous at x = 0, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(1) –5 (2) 5

(3) – 4 (4) 4

Official Ans. by NTA (1)

Sol. If $f(x)$ is continuous at x = 0, RHL = LHL = $f(0)$

$\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^+}\frac{\cos^2x-\sin^2x-1}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$ (Rationalisation)

$\lim\limits_{x\to0^+}-\frac{2\sin^2x}{x^2}\cdot\left(\sqrt{x^2+1}+1\right)=-4$

$\lim\limits_{x\to0^-} f(x)=\lim\limits_{x\to0^-}\frac{1}{x}\ell n\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$

$\lim\limits_{x\to0^-}\frac{\ell n\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right).a}+\frac{\ell n\left(1-\frac{x}{b}\right)}{\left(-\frac{x}{b}\right).b}$

$=\frac{1}{a}+\frac{1}{b}$

So $\frac{1}{a}+\frac{1}{b}=-4=k$

$\Rightarrow\frac{1}{a}+\frac{1}{b}+\frac{4}{k}=-4-1=-5$

16. If $\frac{dy}{dx}=\frac{2^{x+y}-2^x}{2^y}$, y(0) = 1, then y(1) is equal to :

(1) $\log_2(2 + e)$ (2) $\log_2(1 + e)$

(3) $\log_2(2e)$ (4) $\log_2(1 + e^2)$

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}=\frac{2^x2^y-2^x}{2^y}$

$2^y\frac{dy}{dx}=2^x\left(2^y-1\right)$

$\int\frac{2^y}{2^y-1}dy=\int 2^x\,dx$

$\frac{\ell n\left(2^y-1\right)}{\ell n\,2}=\frac{2^x}{\ell n\,2}+C$

$\Rightarrow \log_2(2^y - 1) = 2^x\log_2e + C$

$\because$ y(0) = 1 $\Rightarrow$ 0 = $\log_2e$ + C

C = $-\log_2e$

$\Rightarrow \log_2(2^y - 1) = (2^x - 1)\log_2e$

put x = 1, $\log_2(2^y - 1) = \log_2e$

$2^y = e + 1$

$y = \log_2(e + 1)$ Ans.

17. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $4\pi^2$ (4) 4π

Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 0}\dfrac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$

$\lim\limits_{x\to 0}\dfrac{1-\cos\left(2\pi\cos^4 x\right)}{2x^4}$

$\lim\limits_{x\to 0}\dfrac{1-\cos\left(2\pi-2\pi\cos^4 x\right)}{\left[2\pi\left(1-\cos^4 x\right)\right]^2}4\pi^2.\dfrac{\sin^4 x}{2x^4}\left(1+\cos^2 x\right)^2$

$=\dfrac{1}{2}.4\pi^2.\dfrac{1}{2}(2)^2=4\pi^2$

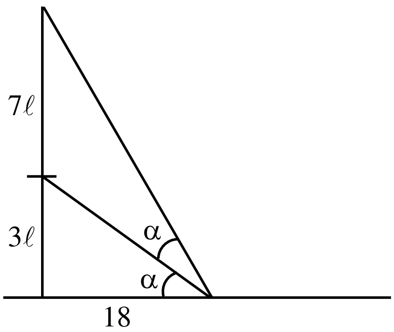
18. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

(1) $12\sqrt{15}$ (2) $12\sqrt{10}$

(3) $8\sqrt{10}$ (4) $6\sqrt{10}$

Official Ans. by NTA (2)

Sol.



Let height of pole = 10ℓ

$\tan\alpha = \dfrac{3\ell}{18}=\dfrac{\ell}{6}$

$\tan 2\alpha = \dfrac{10\ell}{18}$

$\dfrac{2\tan\alpha}{1-\tan^2\alpha}=\dfrac{10\ell}{18}$

use $\tan\alpha = \dfrac{\ell}{6} \Rightarrow \ell = \sqrt{\dfrac{72}{5}}$

height of pole = 10ℓ = $12\sqrt{10}$

19. If $a_r=\cos\dfrac{2r\pi}{9}+i\sin\dfrac{2r\pi}{9}$, r = 1, 2, 3,…, $i=\sqrt{-1}$,

then the determinant $\begin{vmatrix} a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}$ is equal to :

(1) $a_2a_6-a_4a_8$ (2) a_9

(3) $a_1a_9-a_3a_7$ (4) a_5

Official Ans. by NTA (3)

Sol. $a_r = e^{\frac{i2\pi r}{9}}$, r = 1, 2, 3, … a_1, a_2, a_3, … are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3\\ a_n & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}=\begin{vmatrix} a_1 & a_2^2 & a_1^3\\ a_1^4 & a_1^5 & a_1^6\\ a_1^7 & a_1^8 & a_1^9\end{vmatrix}=a_1.a_1^4.a_1^7\begin{vmatrix} 1 & a_1 & a_1^2\\ 1 & a_1 & a_1^2\\ 1 & a_1 & a_1^2\end{vmatrix}=0$$

Now $a_1a_9-a_3a_7=a_1^{10}-a_1^{10}=0$

20. The line $12x\cos\theta+5y\sin\theta=60$ is tangent to which of the following curves?

(1) $x^2+y^2=169$

(2) $144x^2+25y^2=3600$

(3) $25x^2+12y^2=3600$

(4) $x^2+y^2=60$

Official Ans. by NTA (2)

Sol. $12x\cos\theta+5y\sin\theta=60$

$\dfrac{x\cos\theta}{5}+\dfrac{y\sin\theta}{12}=1$

is tangent to $\dfrac{x^2}{25}+\dfrac{y^2}{144}=1$

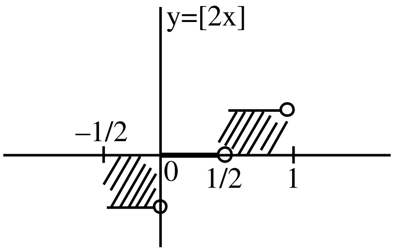
$144x^2+25y^2=3600$

SECTION-B

1. Let [t] denote the greatest integer $\le$ t. Then the value of $8\cdot\displaystyle\int_{-\frac{1}{2}}^{1}\left([2x]+|x|\right)dx$ is _________.

Official Ans. by NTA (5)

Sol. $I = \int_{-1/2}^{1} \left([2x] + |x|\right) dx$



$= \int_{-1/2}^{1} [2x]\, dx + \int_{-1/2}^{1} |x|\, dx$

$= 0 + \int_{-1/2}^{0} (-x)\, dx + \int_{0}^{1} x\, dx$

$= \left(-\frac{x^2}{2}\right)_{-1/2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{1}$

$= \left(0 + \frac{1}{8}\right) + \frac{1}{2}$

$= \frac{5}{8}$

$8I = 5$

2. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $|z - 9\sqrt{2} - 2i|^2$ is equal to _________.

Official Ans. by NTA (98)

Sol. Let z = x + iy

$\arg\left(\frac{x-2+iy}{x+2+iy}\right) = \frac{\pi}{4}$

$\arg(x - 2 + iy) - \arg(x + 2 + iy) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

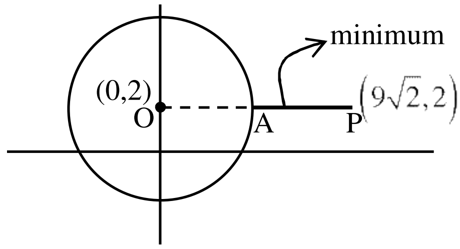
$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \left(\frac{y}{x-2}\right)\cdot\left(\frac{y}{x+2}\right)} = \tan\frac{\pi}{4} = 1$

$\frac{xy + 2y - xy + 2y}{x^2 - 4 + y^2} = 1$

$4y = x^2 - 4 + y^2$

$x^2 + y^2 - 4y - 4 = 0$

locus is a circle with center (0, 2) & radius = $2\sqrt{2}$



min. value = $(AP)^2 = (OP - OA)^2$

$= \left(9\sqrt{2} - 2\sqrt{2}\right)^2$

$= \left(7\sqrt{2}\right)^2 = 98$

3. The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point (–1, –1, 2) is _________.

Official Ans. by NTA (61)

Sol. $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$

$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 6\lambda - 1$

for point of intersection of line & plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

point : (3, 5, 5)

$(\text{distance})^2 = (3 + 1)^2 + (5 + 1)^2 + (5 - 2)^2$

$= 16 + 36 + 9 = 61$

4. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is _________.

Official Ans. by NTA (2)

Sol. $f(x) = x^2 + ax + 1$

$f'(x) = 2x + a$

when $f(x)$ is increasing on [1, 2]

$2x + a \geq 0 \quad \forall\ x \in [1, 2]$

$a \geq -2x\ \forall\ x \in [1, 2]$

$R = -4$

when $f(x)$ is decreasing on [1, 2]

$2x + a \leq 0\ \ \forall\ x \in [1, 2]$

$a \leq -2\ \ \forall\ x \in [1, 2]$

$S = -2$

$|R - S| = |-4 + 2| = 2$

5. The mean of 10 numbers

$7 \times 8,\ 10 \times 10,\ 13 \times 12,\ 16 \times 14, \ldots$ is __________.

Official Ans. by NTA (398)

Sol. $7 \times 8,\ 10 \times 10,\ 13 \times 12,\ 16 \times 14 \ldots\ldots$

$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$

$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$

$$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$$

$$= \frac{6(10\times11\times21)}{6} + 26\times\frac{10\times11}{2} + 24\times10$$

$= 10 \times 11\ (21 + 13) + 240$

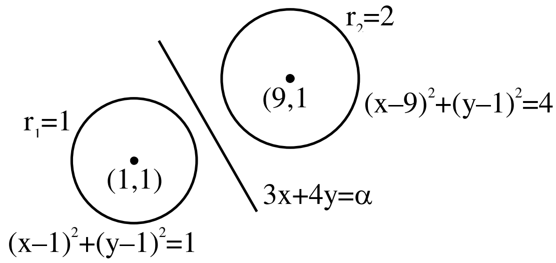
$= 3980$

$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$

6. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is __________.

Official Ans. by NTA (165)

Sol.



Both centres should lie on either side of the line as well as line can be tangent to circle.

$(3 + 4 - \alpha) \cdot (27 + 4 - \alpha) < 0$

$(7 - \alpha) \cdot (31 - \alpha) < 0 \Rightarrow \alpha \in (7, 31) \quad \ldots(1)$

d_1 = distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$$d_1 \geq r_1 \Rightarrow \frac{|7-\alpha|}{5}\ 1 \Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty) \quad \ldots(2)$$

$$d_2 \geq r_2 \Rightarrow \frac{|31-\alpha|}{5} \geq 2 \Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty)$$

$\ldots(3)$

$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$

Sum of integers = 165

7. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is __________.

Official Ans. by NTA (576)

Sol. VOWELS → 2 Vowels

VOWELS → 4 Consonants

All Consonants should not be together

= Total – All consonants together,

$= 6! - 3!\,4! = 576$

8. If $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right) dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is __________.

Official Ans. by NTA (4)

Sol. $x\phi(x) = \int_5^x 3t^2 - 2\phi'(t)\,dt$

$x\phi(x) = x^3 - 125 - 2[\phi(x) - \phi(5)]$

$x\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

$\phi(0) = 4 \Rightarrow \phi(5) = -\frac{133}{2}$

$\phi(x) = \frac{x^3+8}{x+2}$

$\phi(2) = 4$

9. If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to __________.

Official Ans. by NTA (55)

Sol. $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\frac{x}{4}\right)^{12-r}\left(\frac{12}{x^2}\right)^r$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\frac{1}{4}\right)^{12-r}(12)^r.(x)^{12-3r}$

Term independent of x $\Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$T_5 = (-1)^4 \cdot {}^{12}C_4 \left(\frac{1}{4}\right)^8 (12)^4 = \frac{3^6}{4^4}.k$

$\Rightarrow k = 55$

10. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to __________.

Official Ans. by NTA (28)

Sol. I_1 = first unit is functioning

I_2 = second unit is functioning

$P(I_1) = 0.9$, $P(I_2) = 0.8$

$P(\bar{I}_1) = 0.1$, $P(\bar{I}_2) = 0.2$

$P = \frac{0.8\times 0.1}{0.1\times 0.2 + 0.9\times 0.2 + 0.1\times 0.8} = \frac{8}{28}$

$98P = \frac{8}{28}\times 98 = 28$

(Held On Tuesday 31st August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50×10^3 kg, The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column. [Use $Y = 2.0 \times 10^{11}$ Pa, $g = 9.8\ m/s^2$]

(1) 3.60×10^{-8} (2) 2.60×10^{-7}

(3) 1.87×10^{-3} (4) 7.07×10^{-4}

Official Ans. by NTA (2)

Sol. Force on each column = $\frac{mg}{4}$

Strain = $\frac{mg}{4AY}$

$$= \frac{50 \times 10^3 \times 9.8}{4 \times \pi(1-0.25) \times 2 \times 10^{11}}$$

$= 2.6 \times 10^{-7}$

2. A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is :

(Assume that the current is flowing in the clockwise direction.)

(1) 3×10^{-7} T, outside the plane of triangle

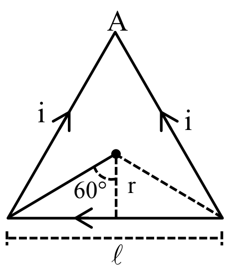
(2) $2\sqrt{3} \times 10^{-7}$ T, outside the plane of triangle

(3) $2\sqrt{3} \times 10^{-5}$ T, inside the plane of triangle

(4) 3×10^{-5} T, inside the plane of triangle

Official Ans. by NTA (4)

Sol.



$$B = 3\left[\frac{\mu_0 i}{4\pi r}(\sin 60° + \sin 60°)\right]$$

$$\tan 60° = \frac{/2}{r}$$

Where $r = \frac{9 \times 10^{-2}}{2\sqrt{3}}$ M

$\therefore B = 3 \times 10^{-5}$ T

Current is flowing in clockwise direction so, B is inside plane of triangle by right hand rule.

3. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the end of light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its midpoint ?

(1) 18.75 kgm^2

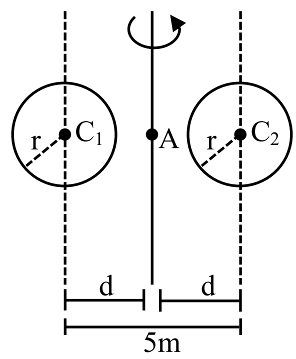
(2) 1.905×10^5 kgm^2

(3) 19.05 kgm^2

(4) 1.875×10^5 kgm^2

Official Ans. by NTA (3)

Sol.



$M = 1.5$ kg, $r = 0.5$ m, $d = \frac{5}{2}$ m

$$I = 2\left(\frac{2}{5}Mr^2 + Md^2\right)$$

$= 19.05\ kgm^2$

4. **Statement I :**

Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P}\perp\vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II :

In the situation given above.

$\theta_1 = 60°$ and $\theta_2 = 90°$

In the light of the above statements, choose the most appropriate answer from the options given below :-

(1) Statement-I is false but Statement-II is true

(2) Both Statement-I and Statement-II are true

(3) Statement-I is true but Statement-II is false

(4) Both Statement-I and Statement-II are false.

Official Ans. by NTA (2)

Sol. $\vec{A}=\vec{P}+\vec{Q}$

$\vec{B}=\vec{P}-\vec{Q}$ $\qquad\qquad \vec{P}\perp\vec{Q}$

$|\vec{A}|=|\vec{B}|=\sqrt{P^2+Q^2}$

$|\vec{A}+\vec{B}|=\sqrt{2(P^2+Q^2)(1+\cos\theta)}$

For $|\vec{A}+\vec{B}|=\sqrt{3(P^2+Q^2)}$

$\theta_1 = 60°$

For $|\vec{A}+\vec{B}|=\sqrt{2(P^2+Q^2)}$

$\theta_2 = 90°$

5. A free electron of 2.6 eV energy collides with a H^+ ion. This results in the formation of a hydrogen atom in the first excited state and a photon is released. Find the frequency of the emitted photon. ($h = 6.6 \times 10^{-34}$ Js)

(1) 1.45×10^{16} MHz

(2) 0.19×10^{15} MHz

(3) 1.45×10^{9} MHz

(4) 9.0×10^{27} MHz

Official Ans. by NTA (3)

Sol. For every large distance P.E. = 0

& total energy = 2.6 + 0 = 2.6 eV

Finally in first excited state of H atom total energy = –3.4 eV

Loss in total energy = 2.6 – (–3.4)

= 6eV

It is emitted as photon

$$\lambda = \frac{1240}{6} = 206 \text{ nm}$$

$$f = \frac{3\times10^8}{206\times10^{-9}} = 1.45 \times 10^{15} \text{ Hz}$$

$$= 1.45 \times 10^{9} \text{ MHz}$$

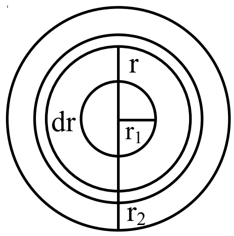
6. Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature $\theta_2(\theta_1<\theta_2)$. The rate at which heat flows radially through the material is :-

(1) $\dfrac{4\pi K r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$ (2) $\dfrac{\pi r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$

(3) $\dfrac{K(\theta_2-\theta_1)}{r_2-r_1}$ (4) $\dfrac{K(\theta_2-\theta_1)(r_2-r_1)}{4\pi r_1 r_2}$

Official Ans. by NTA (1)

Sol.



Thermal resistance of spherical sheet of thickness dr and radius r is

$$dR = \frac{dr}{K(4\pi r^2)}$$

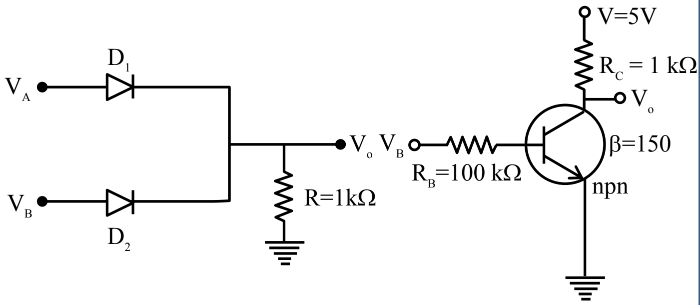
$$R = \int_{r_1}^{r_2} \frac{dr}{K(4\pi r^2)}$$

$$R = \frac{1}{4\pi K}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{1}{4\pi K}\left(\frac{r_2-r_1}{r_1 r_2}\right)$$

$$\text{Thermal current (i)} = \frac{\theta_2-\theta_1}{R}$$

$$i = \frac{4\pi K r_1 r_2}{r_2-r_1}(\theta_2-\theta_1)$$

7. If V_A and V_B are the input voltages (either 5V or 0V) and V_o is the output voltage then the two gates represented in the following circuit (A) and (B) are:-



(A) (B)

(1) AND and OR Gate

(2) OR and NOT Gate

(3) NAND and NOR Gate

(4) AND and NOT Gate

Official Ans. by NTA (2)

Sol. $V_A = 5V \Rightarrow A = 1$

$V_A = 0\ V \Rightarrow A = 0$

$V_B = 5\ V \Rightarrow B = 1$

$V_B = 0\ V \Rightarrow B = 0$

If A = B = 0, there is no potential anywhere here $V_0 = 0$

If A = 1, B = 0, Diode D_1 is forward biased, here $V_0 = 5V$

If A = 0, B = 1, Diode D_2 is forward biased hence $V_0 = 5V$

If A = 1, B = 1, Both diodes are forward biased hence $V_0 = 5V$

Truth table for I^{st}

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

$\therefore$ Given circuit is OR gate

For II^{nd} circuit

$V_B = 5V,\quad A = 1$

$V_B = 0V,\quad A = 0$

When A = 0, E–B junction is unbiased there is no current through it

$\therefore V_0 = 1$

When A = 1, E–B junction is forward biased

$V_0 = 0$

$\therefore$ Hence this circuit is not gate.

8. Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to :-

(1) $\left(\frac{m_p}{m_e}\right)^{3/2}$ (2) $\sqrt{\frac{m_e}{m_p}}$ (3) $\sqrt{\frac{m_p}{m_e}}$ (4) $\frac{m_p}{m_e}$

Official Ans. by NTA (3)

Sol. $\Delta x.\Delta p \geq \frac{h}{4\pi}$

$\Delta x = \frac{h}{4\pi m \Delta v}$ $\qquad v = \sqrt{\frac{3KT}{m}}$

$\frac{\Delta x_e}{\Delta x_p} = \sqrt{\frac{m_p}{m_e}}$

9. A bob of mass 'm' suspended by a thread of length *l* undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by $1/3^{rd}$ of the original length, then the time period of the simple harmonic oscillations will be :-

(1) T (2) $\frac{3}{2}$T (3) $\frac{3}{4}$T (4) $\frac{4}{3}$T

Official Ans. by NTA (4)

Sol. $T = 2\pi\sqrt{\ /g}$

When bob is immersed in liquid

$mg_{eff} = mg$ – Buoyant force

$mg_{eff} = mg - v\sigma g$ $\qquad$ (σ = density of liquid)

$= mg - v\frac{\rho}{4}g$

$= mg - \frac{mg}{4} = \frac{3mg}{4}$

$\therefore g_{eff} = \frac{3g}{4}$

$T_1 = 2\pi\sqrt{\frac{\ _1}{g_{eff}}}$ $\qquad {}_1 = \ + \frac{}{3} = \frac{4}{3}$, $\qquad {}_{eff} = \frac{3g}{4}$

By solving

$T_1 = \frac{4}{3}2\pi\sqrt{\ /g}$

$T_1 = \frac{4T}{3}$

10. **Statement :1**

If three forces F_1, F_2 and F_3 are represented by three sides of a triangle and $F_1 + F_2 = -F_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement : II

A triangle made up of three forces F_1, F_2 and F_3 as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement-I is false but Statement-II is true

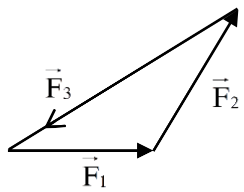
(2) Statement-I is true but Statement-II is false

(3) Both Statement-I and Statement-II are false

(4) Both Statement-I and Statement-II are true.

Official Ans. by NTA (4)

Sol.



Here $F_1 + F_2 + F_3 = 0$

$F_1 + F_2 = -F_3$

Since $F_{net} = 0$ (equilibrium)

Both statements correct

11. If velocity [V], time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be :

(1) $[FT^{-1} V^{-1}]$

(2) $[FTV^{-1}]$

(3) $[FT^2 V]$

(4) $[FVT^{-1}]$

Official Ans. by NTA (2)

Sol. $[M] = K[F]^a [T]^b [V]^c$

$[M^1] = [M^1L^1T^{-2}]^a [T^1]^b [L^1T^{-1}]^c$

$a = 1, b = 1, c = -1$

$\therefore [M] = [FTV^{-1}]$

12. The magnetic field vector of an electromagnetic wave is given by $B = B_o \dfrac{\hat{i}+\hat{j}}{\sqrt{2}} \cos(kz - \omega t)$; where $\hat{i}, \hat{j}$ represents unit vector along x and y-axis respectively. At t = 0 s, two electric charges q_1 of 4π coulomb and q_2 of 2π coulomb located at $\left(0,0,\dfrac{\pi}{k}\right)$ and $\left(0,0,\dfrac{3\pi}{k}\right)$, respectively, have the same velocity of $0.5\,c\,\hat{i}$, (where c is the velocity of light). The ratio of the force acting on charge q_1 to q_2 is :-

(1) $2\sqrt{2} : 1$

(2) $1 : \sqrt{2}$

(3) $2 : 1$

(4) $\sqrt{2} : 1$

Official Ans. by NTA (3)

Sol. $F = q\left(V \times B\right)$

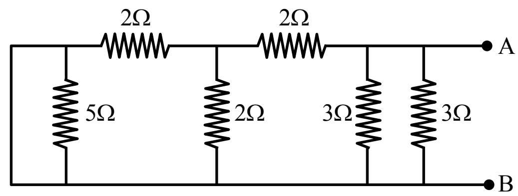
$$F_1 = 4\pi\left[0.5c\hat{i} \times B_0\left(\frac{\hat{i}+\hat{j}}{2}\right)\cos\left(K.\frac{\pi}{K} - 0\right)\right]$$

$$F_2 = 2\pi\left[0.5c\hat{i} \times B_0\left(\frac{\hat{i}+\hat{j}}{2}\right)\cos\left(K.\frac{3\pi}{K} - 0\right)\right]$$

$\cos\pi = -1, \quad \cos 3\pi = -1$

$\therefore \dfrac{F_1}{F_2} = 2$

13. The equivalent resistance of the given circuit between the terminals A and B is :

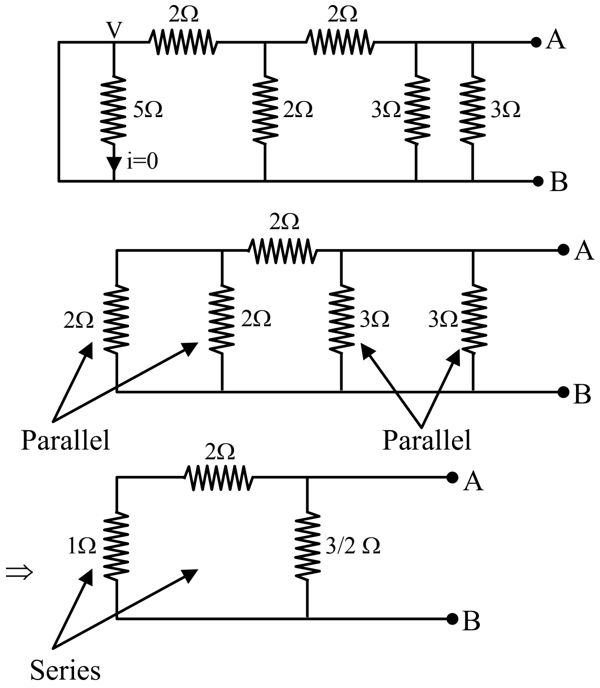


(1) 0Ω (2) 3Ω

(3) $\dfrac{9}{2}\Omega$ (4) 1Ω

Official Ans. by NTA (4)

Sol.



$$R_{eq} = \frac{3 \times 3/2}{3 + 3/2} = \frac{9/2}{9/2} = 1\Omega$$

14. Choose the **incorrect** statement :

(a) The electric lines of force entering into a Gaussian surface provide negative flux.

(b) A charge 'q' is placed at the centre of a cube. The flux through all the faces will be the same.

(c) In a uniform electric field net flux through a closed Gaussian surface containing no net charge, is zero.

(d) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.

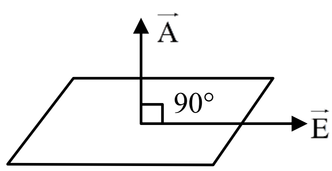
Choose the most appropriate answer from the options given below

(1) (c) and (d) only　　(2) (b) and (d) only

(3) (d) only　　(4) (a) and (c) only

Official Ans. by NTA (3)

Sol. Since $\phi = E.A = EA\cos\theta$



$\theta = 90°$

$\therefore \phi = 0$

15. A mixture of hydrogen and oxygen has volume 500 cm^3, temperature 300 K, pressure 400 kPa and mass 0.76 g. The ratio of masses of oxygen to hydrogen will be :-

(1) 3 : 8　　(2) 3 : 16

(3) 16 : 3　　(4) 8 : 3

Official Ans. by NTA (3)

Sol. PV = nRT

$$400 \times 10^3 \times 500 \times 10^{-6} = n\left(\frac{25}{3}\right)(300)$$

$$n = \frac{2}{25}$$

$n = n_1 + n_2$

$$\frac{2}{25} = \frac{M_1}{2} + \frac{M_2}{32}$$

Also $M_1 + M_2 = 0.76$ gm

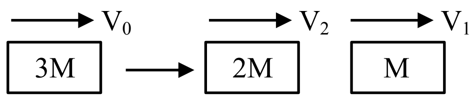
$$\frac{M_2}{M_1} = \frac{16}{3}$$

16. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is :-

(1) $\frac{1}{3}$　(2) $\frac{2}{3}$　(3) $\frac{1}{8}$　(4) $\frac{1}{4}$

Official Ans. by NTA (3)

Sol.



$3MV_0 = 2MV_2 + MV_1$

$3V_0 = 2V_2 + V_1$

$120 = 2V_2 + 60 \Rightarrow V_2 = 30$ m/s

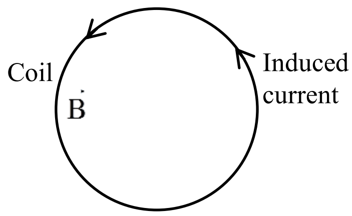
$$\frac{\Delta K.E.}{K.E.} = \frac{\frac{1}{2}MV_1^2 + \frac{1}{2}2MV_2^2 - \frac{1}{2}3MV_0^2}{\frac{1}{2}3MV_0^2}$$

$$= \frac{V_1^2 + 2V_2^2 - 3V_0^2}{3V_0^2}$$

$$= \frac{3600 + 1800 - 4800}{4800}$$

$$= \frac{1}{8}$$

17. A coil is placed in a magnetic field $\vec{B}$ as shown below :



A current is induced in the coil because $\vec{B}$ is :

(1) Outward and decreasing with time

(2) Parallel to the plane of coil and decreasing with time

(3) Outward and increasing with time

(4) Parallel to the plane of coil and increasing with time

Official Ans. by NTA (1)

Sol. $\vec{B}$ must not be parallel to the plane of coil for non zero flux and according to lenz law if B is outward it should be decreasing for anticlockwise induced current.

18. For a body executing S.H.M. :

(a) Potential energy is always equal to its K.E.

(b) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below :

(1) (c) and (d)

(2) only (c)

(3) (b) and (c)

(4) only (b)

Official Ans. by NTA (1)

Sol. In S.H.M. total mechanical energy remains constant and also $\langle K.E.\rangle = \langle P.E\rangle = \frac{1}{4}KA^2$

(for 1 time period)

19. **Statement–I :**

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load R_L.

Statement–II :

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with R_L.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) Both **Statement I** and **Statement II** are true

Official Ans. by NTA (4)

Sol. To convert pulsating dc into steady dc both of mentioned method are correct.

20. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is :

(Given : $r < R_E$)

(1) $1-\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$

(2) $1+\frac{r}{R_E}+\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

(3) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

(4) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$

Official Ans. by NTA (4)

Sol. $g_{up} = \frac{g}{\left(1+\frac{r}{R}\right)^2}$

$$g_{down} = g\left(1-\frac{r}{R}\right)$$

$$\frac{g_{down}}{g_{up}} = \left(1-\frac{r}{R}\right)\left(1+\frac{r}{R}\right)^2$$

$$= \left(1-\frac{r}{R}\right)\left(1+\frac{2r}{R}+\frac{r^2}{R^2}\right)$$

$$= 1+\frac{r}{R}-\frac{r^2}{R^2}-\frac{r^3}{R^3}$$

SECTION-B

1. A bandwidth of 6 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this band simultaneously without interfering with each other will be______.

Official Ans. by NTA (500)

Sol. Signal bandwidth = 2 fm

= 12 kHz

$$\therefore\ N = \frac{6MHZ}{12kHZ} = \frac{6\times10^6}{12\times10^3} = 500$$

2. A parallel plate capacitor of capacitance 200 μF is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be_____J.

Official Ans. by NTA (4)

Sol.
$$\Delta U = \frac{1}{2}(\Delta C)V^2$$

$$\Delta U = \frac{1}{2}(KC - C)V^2$$

$$\Delta U = \frac{1}{2}(2-1)CV^2$$

$$\Delta U = \frac{1}{2}\times 200\times10^{-6}\times200\times200$$

$\Delta U = 4$ J

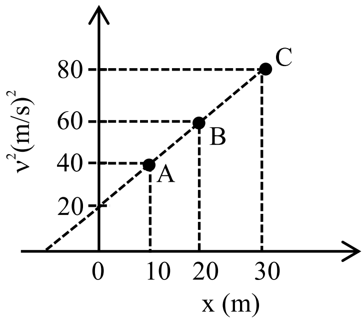
3. A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume 10^3 cm^3. If the core material is replaced by another material having relative permeability of 750 with same volume maintaining same current of 0.75 A in the solenoid, the fractional change in the magnetic moment of the core would be approximately $\left(\frac{x}{499}\right)$. Find the value of x.

Official Ans. by NTA (250)

Sol.
$$\frac{\Delta M}{M} = \frac{\Delta\mu}{\mu} = \frac{250}{500} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{x}{499} \Rightarrow x \quad 250$$

4. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x(displacement) plot. The acceleration of the particle is___m/s^2.



Official Ans. by NTA (1)

Sol. $y = mx + C$

$$v^2 = \frac{20}{10}x + 20$$

$$v^2 = 2x + 20$$

$$2v\frac{dv}{dx} = 2$$

$$\therefore\ a = v\frac{dv}{dx} = 1$$

5. In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright is 2.4 cm. The frequency of light used is_____$\times 10^{14}$ Hz.

Official Ans. by NTA (5)

Sol. $8\beta = 2.4$ cm

$$\frac{8\lambda\Delta}{d} = 2.4 \text{ cm}$$

$$\frac{8\times1.5\times c}{0.3\times10^{-3}\times f} = 2.4\times10^{-2}$$

$f = 5\times10^{14}$ Hz

6. The diameter of a spherical bob is measured using a vernier callipers. 9 divisions of the main scale, in the vernier callipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8^{th} division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is_____ $\times 10^{-2}$ cm.

Official Ans. by NTA (52)

Sol. 9 MSD = 10 VSD

9×1 mm = 10 VSD

$\therefore$ 1 VSD = 0.9 mm

LC = 1 MSD – 1 VSD = 0.1 mm

Reading = MSR + VSR × LC

$10 + 8 \times 0.1 = 10.8$ mm

Actual reading = 10.8 – 0.4 = 10.4 mm

$$\text{radius} = \frac{d}{2} = \frac{10.4}{2} = 5.2 \text{ mm}$$

$= 52 \times 10^{-2}$ cm

7. A sample of gas with $\gamma = 1.5$ is taken through an adiabatic process in which the volume is compressed from 1200 cm^3 to 300 cm^3. If the initial pressure is 200 kPa. The absolute value of the workdone by the gas in the process =_____J.

Official Ans. by NTA (480)

Sol. $\nu = 1.5$

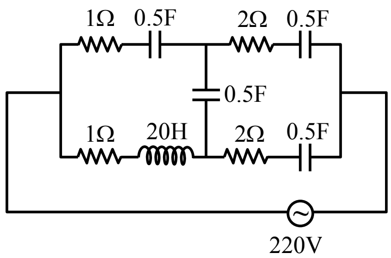
$p_1 v_1^{\nu} = p_2 v_2^{\nu}$

$(200)\,(1200)^{1.5} = P^2\,(300)^{1.5}$

$P_2 = 200\,[4]^{3/2} = 1600$ kPa

$$|\text{W.D.}| = \frac{p_2 v_2 - p_1 v_1}{\nu - 1} = \left(\frac{480 - 240}{0.5}\right) = 480 \text{ J}$$

8. At very high frequencies, the effective impendance of the given circuit will be______Ω.



Official Ans. by NTA (2)

Sol. $X_L = 2\pi f L$

f is very large

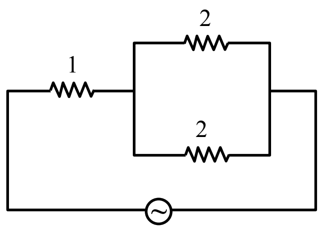
$\therefore X_L$ is very large hence open circuit.

$$X_C = \frac{1}{2\pi f C}$$

f is very large.

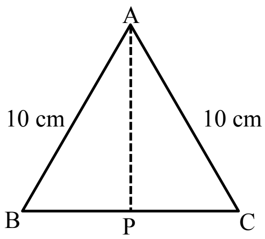
$\therefore X_C$ is very small, hence short circuit.

Final circuit



$$Z_{eq} = 1 + \frac{2 \times 2}{2 + 2} = 2$$

9. Cross–section view of a prism is the equilateral triangle ABC in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (midpoint of BC) to A is______ $\times 10^{-10}$ s. (Given, speed of light in vacuum = 3×10^8 m/s and $\cos 30° = \frac{\sqrt{3}}{2}$)



Official Ans. by NTA (5)

Sol. $i = A = 60°$

$\delta_{min} = 2i - A$

$= 2 \times 60° - 60° = 60°$

$$\mu = \frac{\sin^{-1}\left(\frac{\delta_{min} + A}{2}\right)}{\sin^{-1}\left(\frac{A}{2}\right)}$$

$= \sqrt{3}$

$$V_{prism} = \frac{3 \times 10^8}{\sqrt{3}}$$

$$AP = 10 \times 10^{-2} \times \frac{\sqrt{3}}{2}$$

$$\text{time} = \frac{5 \times 10^{-2}}{3 \times 10^8} \times \sqrt{3} \times \sqrt{3}$$

$= 5 \times 10^{-10}$ sec

Ans = 5

10. A resistor dissipates 192 J of energy in 1 s when a current of 4A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5 s in______J.

Official Ans. by NTA (3840)

Sol. $E = i^2Rt$

$192 = 16 (R) (1)$

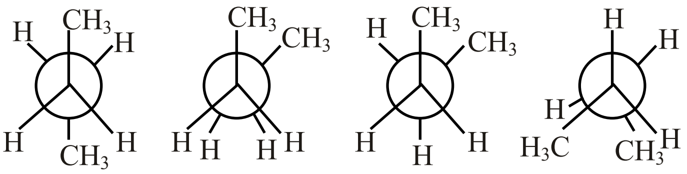
$R = 12\ \Omega$

$E^1 = (8)^2 (12) (5)$

$= 3840$ J

SECTION-A

1. Arrange the following conformational isomers of n-butane in order of their increasing potential energy :



I II III IV

(1) II < III < IV < I (2) I < IV < III < II

(3) II < IV < III < I (4) I < III < IV < II

Official Ans. by NTA (4)

Sol. More stable less potential energy.

Stability order : I > III > IV > II

So

Potential energy : II > IV > III > I

2. The Eu^{2+} ion is a strong reducing agent in spite of its ground state electronic configuration (outermost) : [Atomic number of Eu = 63]

(1) $4f^7 6s^2$ (2) $4f^6$

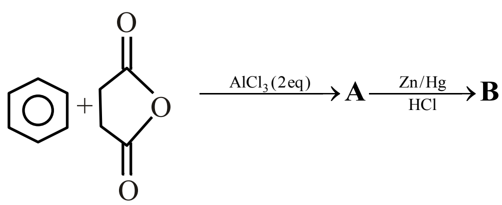
(3) $4f^7$ (4) $4f^6 6s^2$

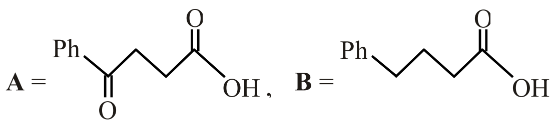
Official Ans. by NTA (3)

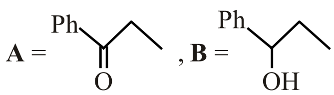
Sol. $Eu \rightarrow [Xe]4f^7 6s^2$

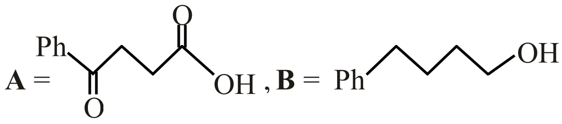
$Eu^{2+} \rightarrow [Xe]4f^7$

3. The structures of **A** and **B** formed in the following reaction are : [Ph = – C_6H_5]



(1) 

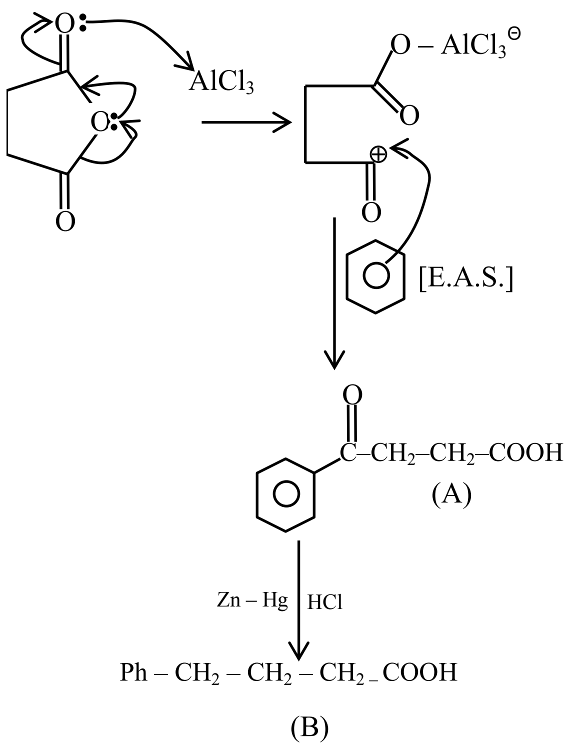
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



4. In which one of the following sets all species show disproportionation reaction ?

(1) ClO_2^-, F_2, MnO_4^- and $Cr_2O_7^{2-}$

(2) $Cr_2O_7^{2-}, MnO_4^-, ClO_2^-$ and Cl_2

(3) MnO_4^-, ClO_2^-, Cl_2 and Mn^{3+}

(4) $ClO_4^-, MnO_4^-, ClO_2^-$ and F_2

Official Ans. by NTA (3)

Allen Ans. (Bonus)

Sol. No option contains all species that show disproportionation reaction.

MnO_4^-

Mn is in +7 oxidation state (highest) hence cannot be simultaneously oxidized or reduced.

5. Match **List-I** with **List-II**

	List-I (Parameter)	**List-II (Unit)**
(a)	Cell constant	(i) $S\ cm^2\ mol^{-1}$
(b)	Molar conductivity	(ii) Dimensionless
(c)	Conductivity	(iii) m^{-1}
(d)	Degree of dissociation of electrolyte	(iv) $\Omega^{-1}\ m^{-1}$

Choose the **most appropriate** answer from the options given below :

(1) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(2) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(3) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Official Ans. by NTA (1)

Sol. Cell constant = $\left(\frac{\ }{A}\right) \Rightarrow$ Units = m^{-1}

Molar conductivity $(\Lambda_m) \Rightarrow$ Units = $Sm^2\ mole^{-1}$

Conductivity (K) $\Rightarrow$ Units = $S\ m^{-1}$

Degree of dissociation $(\alpha) \rightarrow$ Dimensionless

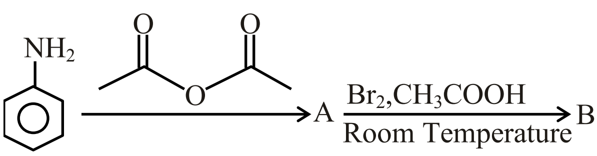
$\therefore$ (a) – (iii)

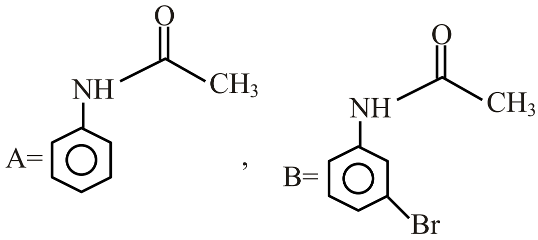
(b) – (i)

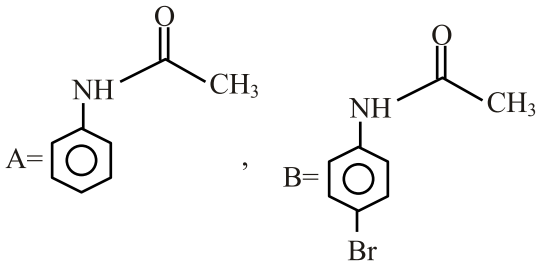
(c) – (iv)

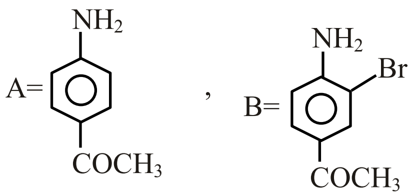
(d) – (ii)

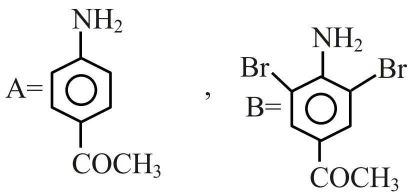
6. The major products A and B formed in the following reaction sequence are :



(1)


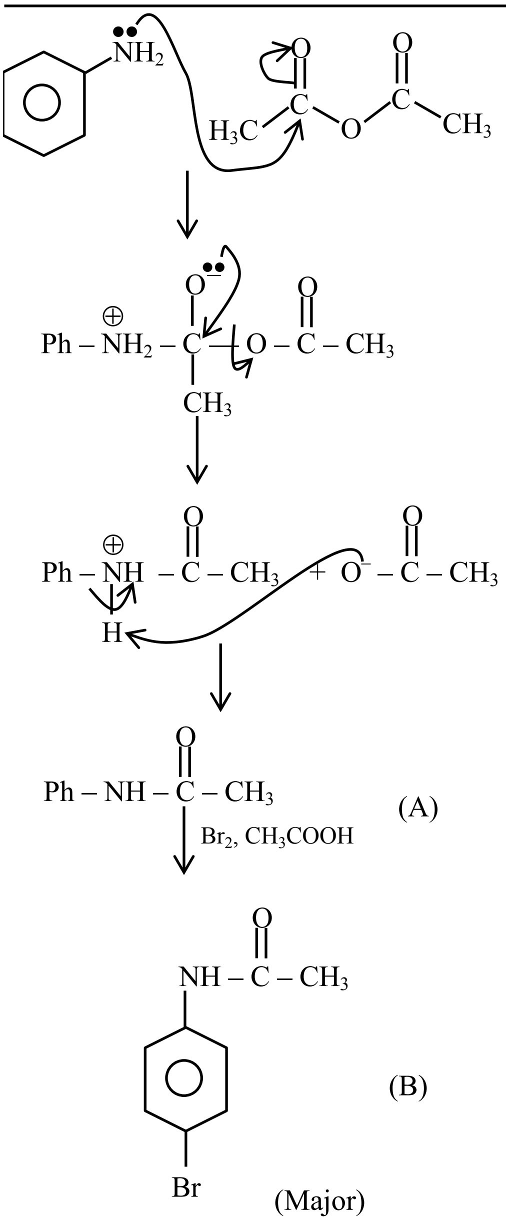
(2)


(3)


(4)


Official Ans. by NTA (2)

Sol.



7. Which of the following is NOT an example of fibrous protein ?

(1) Keratin (2) Albumin
(3) Collagen (4) Myosin

Official Ans. by NTA (2)

Sol. Keratin, collagen and myosin are example of fibrous protein.

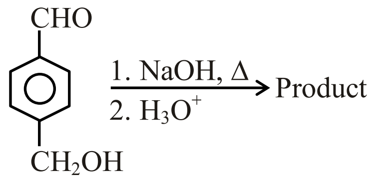
8. The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively. X and Y are :

(1) X = Ammonium salts, Y = CO_2
(2) X = SO_2, Y = Ammonium salts
(3) X = Ammonium salts, Y = SO_2
(4) X = CO_2, Y = SO_2

Official Ans. by NTA (3)

Sol. Oxides of nitrogen and sulphur are acidic and settle down on ground as dry deposition.
Ammonium salts in rain drops result in wet deposition

9. For the reaction given below :

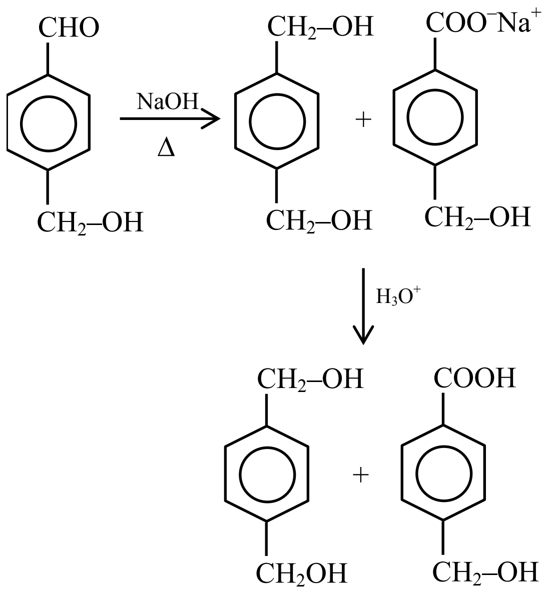


The compound which is **not** formed as a product in the reaction is a :

(1) compound with both alcohol and acid functional groups
(2) monocarboxylic acid
(3) dicarboxylic acid
(4) diol

Official Ans. by NTA (3)

Sol.

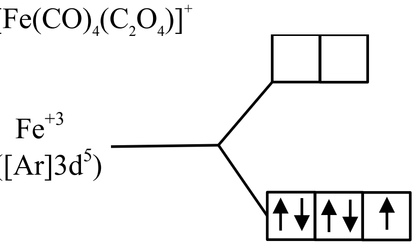


10. Spin only magnetic moment in BM of $[Fe(CO)_4(C_2O_4)]^+$ is :

(1) 5.92 (2) 0 (3) 1 (4) 1.73

Official Ans. by NTA (4)

Sol. $[Fe(CO)_4(C_2O_4)]^+$



One unpaired electron
Spin only magnetic moment
$= \sqrt{3}\,\text{B.M.} = 1.73\,\text{BM}$

11. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Lithium salts are hydrated.

Reason (R) : Lithium has higher polarising power than other alkali metal group members.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are correct but **(R)** is NOT the correct explanation of **(A).**

(2) **(A)** is correct but **(R)** is not correct.

(3) **(A)** is not correct but **(R)** is correct.

(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.

Official Ans. by NTA (1)

Sol. Lithium salts are hydrated due to high hydration energy of Li^+
Li^+ due to smallest size in IA group has highest polarizing power.

12. The **incorrect** expression among the following is:

(1) $\dfrac{\Delta G_{\text{System}}}{\Delta S_{\text{Total}}} = -T(\text{at constant P})$

(2) $\ln K = \dfrac{\Delta H^\circ - T\Delta S^\circ}{RT}$

(3) $K = e^{-\Delta G^\circ/RT}$

(4) For isothermal process $w_{\text{reversible}} = -\ nRT\ln\dfrac{V_f}{V_i}$

Official Ans. by NTA (2)

Sol. Option (2) is incorrect
$\Delta G^\circ = -RT\ \ell n\ K$
$\Delta H^\circ - T\Delta S^\circ = -RT\ \ell n\ K$
$nK = -\left[\dfrac{\Delta H^\circ - \Delta S^\circ}{RT}\right]$

13. Which one of the following statements is **incorrect ?**

(1) Atomic hydrogen is produced when H_2 molecules at a high temperature are irradiated with UV radiation.

(2) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%.

(3) Bond dissociation enthalpy of H_2 is highest among diatomic gaseous molecules which contain a single bond .

(4) Dihydrogen is produced on reacting zinc with HCl as well as $NaOH_{(aq)}$.

Official Ans. by NTA (2)

Sol. Atomic hydrogen is produced at high temperature in an electric are or under ultraviolet radiations
The dissociation of dihydrogen at 2000 K is only 0.081%
H–H bond dissociation enthalpy is highest for a single bond for any diatomic molecule.
Dihydrogen can be produced on reacting Zn with dil. HCl as well as NaOH (aq.)

14. Which among the following is not a polyester ?

(1) Novolac (2) PHBV

(3) Dacron (4) Glyptal

Official Ans. by NTA (1)

Sol. Novalac is a linear polymer of [Ph–OH + HCHO].
So ester linkage not present.
So novalac is not a polyester.

15. Which one of the following correctly represents the order of stability of oxides, X_2O; (X = halogen) ?

(1) Br > Cl > I (2) Br > I > Cl

(3) Cl > I > Br (4) I > Cl > Br

Official Ans. by NTA (4)

Sol. Stability of oxides of Halogens is

I > Cl > Br

16. Match **List-I** with **List-II** :

List-I (Metal Ion)			**List-II** (Group in Qualitative analysis)
(a)	Mn^{2+}	(i)	Group - III
(b)	As^{3+}	(ii)	Group - IIA
(c)	Cu^{2+}	(iii)	Group - IV
(d)	Al^{3+}	(iv)	Group - IIB

Choose the **most appropriate** answer from the options given below :

(1) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(2) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(3) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

(4) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

Official Ans. by NTA (2)

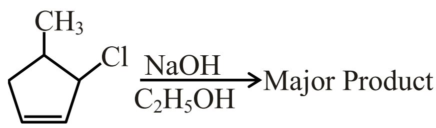
Sol. $Mn^{2+} \rightarrow$ III group

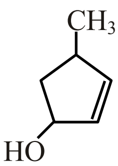
$As^{3+} \rightarrow$ II B group

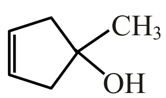
$Cu^{2+} \rightarrow$ II A group

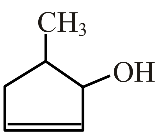
$Al^{3+} \rightarrow$ IV group

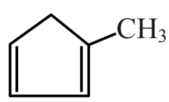
17. The major product of the following reaction is :



(1) 

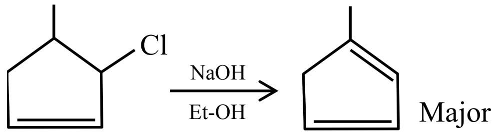
(2) 

(3) 

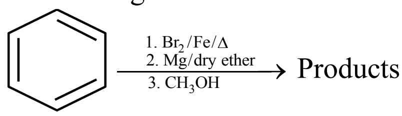
(4) 

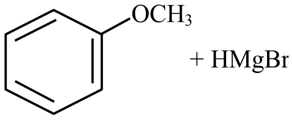
Official Ans. by NTA (3)
Allen Ans. (4)

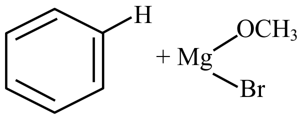
Sol. NaOH + EtOH is known as alcoholic NaOH, so it give E^2 reaction with given alkyl halide.

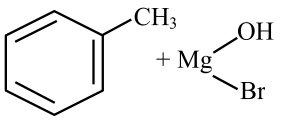


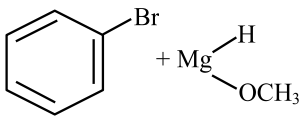
18. For the following :



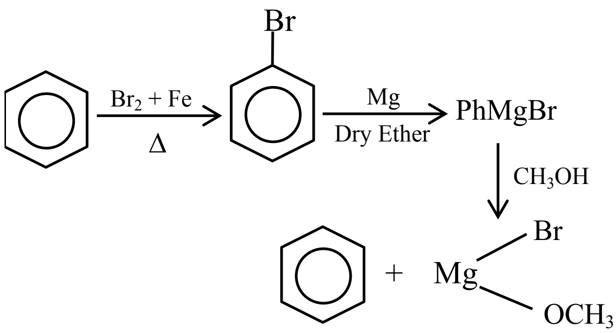
(1) 

(2) 

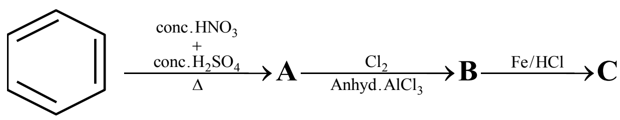
(3) 

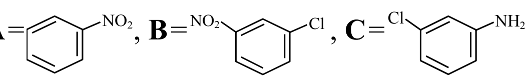
(4) 

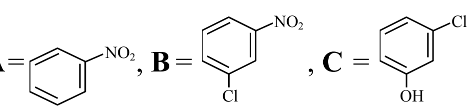
Official Ans. by NTA (2)

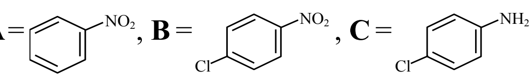


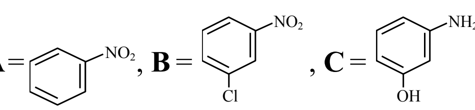
19. Identify correct **A, B** and **C** in the reaction sequence given below :



(1) 

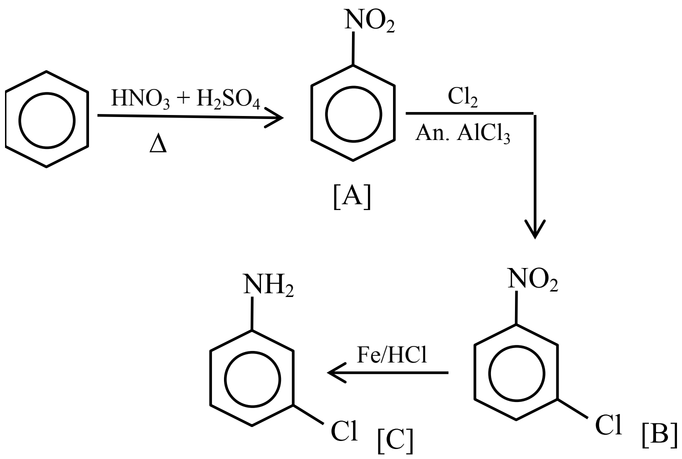
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



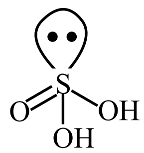
20. The number of S=O bonds present in sulphurous acid, peroxodisulphuric acid and pyrosulphuric acid, respectively are :

(1) 2, 3 and 4 (2) 1, 4 and 3

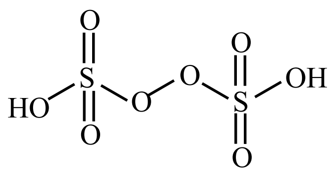
(3) 2, 4 and 3 (4) 1, 4 and 4

Official Ans. by NTA (4)

Sol.

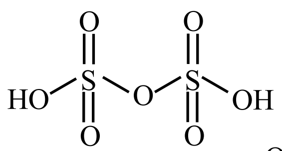


Sulphurous acid



Peroxodisulphuric acid

1 $S=O$ Bond. 4 $S=O$ Bond.



Pyrosulphuric acid

4 $S=O$ Bond.

SECTION-B

1. CH_4 is adsorbed on 1 g charcoal at 0°C following the Freundlich adsorption isotherm. 10.0 mL of CH_4 is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of CH_4 adsorbed at 300 mm of Hg is 10^x mL. The value of x is _______ $\times 10^{-2}$.

(Nearest integer)

[Use $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

Official Ans. by NTA (128)

Sol. We know

$$\frac{x}{m} = KP^{1/n} \text{ ; using } (x \propto V)$$

$$\Rightarrow \quad \frac{10}{1} = K \times (100)^{1/n} \qquad \ldots(1)$$

$$\frac{15}{1} = K \times (200)^{1/n} \qquad \ldots(2)$$

$$\frac{V}{1} = K \times (300)^{1/n} \qquad \ldots(3)$$

Divide

(2) / (1)

$$\frac{15}{10} = 2^{1/n}$$

$$\log\left(\frac{3}{2}\right) = \frac{1}{n}\log 2$$

$$\frac{1}{n} = \frac{\log 3 - \log 2}{\log 2} = \frac{0.4771 - 0.3010}{0.3010}$$

$$\frac{1}{n} = 0.585$$

Divide

(3) / (1)

$$\frac{V}{10} = 3^{1/n}$$

$$\log\left(\frac{V}{10}\right) = \frac{1}{n}\log 3$$

$$\log\left(\frac{V}{10}\right) = 0.585 \times 0.4771 = 0.2791$$

$$\frac{V}{10} = 10^{0.279} \quad \Rightarrow V = 10 \times 10^{0.279}$$

$$\Rightarrow V = 10^{1.279} = 10^x$$

$$\Rightarrow x = 1.279$$

$$\Rightarrow x = 128 \times 10^{-2} \text{ (Nearest integer)}$$

2. 1.22 g of an organic acid is separately dissolved in 100 g of benzene ($K_b = 2.6$ K kg mol^{-1}) and 100 g of acetone ($K_b = 1.7$ K kg mol^{-1}). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C.

The increase in boiling point of solution in benzene in °C is x × 10^{-2}. The value of x is _______.(Nearest integer)

[Atomic mass : C = 12.0, H = 1.0, O= 16.0]

Official Ans. by NTA (13)

Sol. With benzene as solvent

$\Delta T_b = i\ K_b\ m$

$$\Delta T_b = \frac{1}{2} \times 2.6 \times \frac{1.22/M_w}{100/1000} \quad \ldots(1)$$

With Acetone as solvent

$\Delta T_b = i\ K_b\ m$

$$0.17 = 1 \times 1.7 \times \frac{1.22/M_w}{100/1000} \quad \ldots(2)$$

(1) / (2)

$$\frac{\Delta T_b}{0.17} = \frac{\frac{1}{2} \times 2.6 + \frac{1.22/M_w}{100/1000}}{1 \times 1.7 \times \frac{1.22/M_w}{100/1000}}$$

$$\Delta T_b = \frac{0.26}{2}$$

$\Delta T_b = 13 \times 10^{-2}$

$\Rightarrow x = 13$

3. The value of magnetic quantum number of the outermost electron of Zn^+ ion is _________.

Official Ans. by NTA (0)

Sol. $Zn^+ \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1$

Outermost electron is in 4s subshell

m = 0

4. The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites in A_xB. The value of x is _________

(Integer answer)

Official Ans. by NTA (1)

Sol. Anions froms CCP or FCC (A^-) = 4 A^- per unit cell

Cations occupy all octahedral voids (B^+) = 4 B^+ per unit cell

cell formula $\rightarrow A_4B_4$

Empirical formula $\rightarrow$ AB

$\rightarrow$ (x = 1)

5. In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is ________

Pb, Sb, Se, Te, Ru, Ag, Au and Pt

Official Ans. by NTA (6)

Sol. Anode mud contains Sb, Se, Te, Ag, Au and Pt

6. The pH of a solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is x × 10^{-4}. The value of x is __________. (Nearest integer)

[log 2.5 = 0.3979]

Official Ans. by NTA (6021)

Sol. HCl (aq.) + NaOH (aq.) $\rightarrow$ NaCl(aq.) + $H_2O(\ell)$

	HCl	NaOH	NaCl	H_2O
	50 ml,1M	30ml, 1M	–	–
t = 0	50 mm	30 mm		
t = ∞	20 mm	–		

$$[HCl] = \frac{20}{80} = \frac{1}{4} M = 2.5 \times 10^{-1} M$$

pH = –log 2.15 × 10^{-1} = 1 – 0.3979 = 0.6021

pH = 6021 × 10^{-4}

7. For the reaction A $\rightarrow$ B, the rate constant k(in s^{-1}) is given by

$$\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}$$

The energy of activation in kJ mol^{-1} is _______. (Nearest integer)

[Given : R = 8.314 J K^{-1} mol^{-1}]

Official Ans. by NTA (47)

Sol. Given $\quad \log K = 20.35 - \frac{2.47 \times 10^3}{T}$

We know $\quad \log K = \log A - \frac{E_a}{2.303 RT}$

$$\Rightarrow \quad \frac{E_a}{2.303 RT} = 2.47 \times 10^3$$

$$E_a = 2.47 \times 10^3 \times 2.303 \times \frac{8.314}{1000} \text{KJ / mole}$$

= 47.29 = 47 (Nearest integer)

8. Sodium oxide reacts with water to produce sodium hydroxide. 20.0 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is ____________ × 10^{-1} M. (Nearest integer)

[Atomic mass : Na = 23.0, O = 16.0, H = 1.0]

Official Ans. by NTA (13)

Sol. $Na_2O + H_2O \rightarrow 2NaOH$

$\frac{20}{62}$ moles

Moles of NaOH formed = $\frac{20}{62} \times 2$

$$[NaOH] = \frac{\frac{40}{62}}{\frac{500}{1000}} = 1.29\,M = 13 \times 10^{-1}\ M$$

(Nearest integer)

9. According to molecular orbital theory, the number of unpaired electron(s) in O_2^{2-} is :

Official Ans. by NTA (0)

Sol. Molecular orbital configuration of O_2^{2-} is

$$\sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} \left(\pi 2p_x^2 = \pi 2p_y^2\right)\left(\pi_{2px}^{*2} = \pi_{2py}^{*2}\right)$$

Zero unpaired electron

10. The transformation occurring in Duma's method is given below :

$$C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$$

The value of *y* is ________. (Integer answer)

Official Ans. by NTA (7)

Sol.

$$C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$$

On balancing

$$C_2H_7N + \frac{15}{2}CuO \rightarrow 2CO_2 + \frac{7}{2}H_2O + \frac{1}{2}N_2 + \frac{15}{2}Cu$$

On comparing

y = 7

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Tuesday 31st August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

(1) no solution

(2) infinitely many solution

(3) exactly two solutions

(4) a unique solution

Official Ans. by NTA (2)

Sol. $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= 1 + 2\cos\alpha.\cos\beta.\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + (\cos(\alpha + \beta) + \cos(\alpha - \beta))\cos\gamma$

$= \sin^2\gamma - \cos^2\alpha - \cos^2\beta + \cos^2\gamma + \cos(\alpha - \beta)\cos\gamma$

$= \sin^2\alpha - \cos^2\beta + \cos(\alpha - \beta)\cos(\alpha + \beta)$

$= \sin^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta = 0$

2. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies.

$$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0},$$

then $\vec{r}$ is equal to :

(1) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$ (2) $\frac{1}{3}(2\vec{a}+\vec{b}-\vec{c})$

(3) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$ (4) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$

Official Ans. by NTA (3)

Sol. Suppose $\vec{r} = x\vec{a} + y\vec{b} + 2\vec{c}$

and $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$

$$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0}$$

$$\Rightarrow k^2(\vec{r}-\vec{b}) - k^2x\vec{a} + k^2(\vec{r}-\vec{c}) - k^2y\vec{b} + k^2(\vec{r}-\vec{a}) - k^2z\vec{c} = \vec{0}$$

$$\Rightarrow 3\vec{r} - (\vec{a}+\vec{b}+\vec{c}) - \vec{r} = \vec{0}$$

$$\Rightarrow \vec{r} = \frac{\vec{a}+\vec{b}+\vec{c}}{2}$$

3. The domain of the function

$$f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right) \text{ is :}$$

(1) $\left[0, \frac{1}{4}\right]$ (2) $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

(3) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$ (4) $\left[0, \frac{1}{2}\right]$

Official Ans. by NTA (3)

Sol. $$f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$$

$$-1 \le \frac{x-1}{x+1} \le 1 \Rightarrow 0 \le x < \infty \quad \ldots(1)$$

$$-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1 \Rightarrow x \in \left[\frac{-1}{4}, \frac{1}{2}\right] \cup \{0\} \quad \ldots(2)$$

(1) & (2)

$$\Rightarrow \text{Domain} = \left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$$

4. Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is :

(1) $\frac{1}{10}$ (2) $\frac{1}{15}$

(3) $\frac{1}{5}$ (4) $\frac{1}{30}$

Official Ans. by NTA (1)

Sol. $g(3) = 2g(1)$ can be defined in 3 ways

number of onto functions in this condition $= 3 \times 4!$

Total number of onto functions $= 6!$

Required probability $= \dfrac{3 \times 4!}{6!} = \dfrac{1}{10}$

5. Let $f : \mathbf{N} \to \mathbf{N}$ be a function such that

$f(m + n) = f(m) + f(n)$ for every $m, n \in \mathbf{N}$. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to :

(1) 6 (2) 54

(3) 18 (4) 36

Official Ans. by NTA (2)

Sol. $f(m + n) = f(m) + f(n)$

Put $m = 1, n = 1$

$f(2) = 2f(1)$

Put $m = 2, n = 1$

$f(3) = f(2) + f(1) = 3f(1)$

Put $m = 3, n = 3$

$f(6) = 2f(3) \Rightarrow f(3) = 9$

$\Rightarrow f(1) = 3, f(2) = 6$

$f(2).f(3) = 6 \times 9 = 54$

6. The distance of the point $(-1, 2, -2)$ from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is :

(1) $\dfrac{1}{\sqrt{2}}$ (2) $\dfrac{5}{2}$

(3) $\dfrac{\sqrt{42}}{2}$ (4) $\dfrac{\sqrt{34}}{2}$

Official Ans. by NTA (4)

Sol. $P_1 : 2x + 3y + 2z = 0$

$\Rightarrow \vec{n}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$P_2 : x - 2y + z = 0$

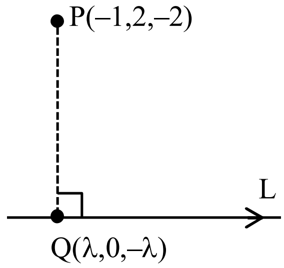
$\Rightarrow \vec{n}_2 = \hat{i} - 2\hat{j} + k$

Direction vector of line L which is line of intersection of P_1 & P_2

$\vec{r} = \vec{n}_1 \times \vec{n}_2 = 7\hat{i} - 7\hat{k}$

DR's of L are $(1, 0, -1)$

$\Rightarrow$ Equation of L : $\dfrac{x}{1} = \dfrac{y}{0} = \dfrac{z}{-1} = \lambda$



DR's of $\overrightarrow{PQ} = (\lambda + 1, -2, 2 - \lambda)$

$\because \overrightarrow{PQ} \perp \vec{r}$

$\Rightarrow (\lambda + 1)(1) + (-2)(0) + (2 - \lambda)(-1) = 0$

$\Rightarrow \lambda = \dfrac{1}{2} \Rightarrow Q\left(\dfrac{1}{2}, 0, \dfrac{-1}{2}\right)$

$\Rightarrow PQ = \dfrac{\sqrt{34}}{2}$

7. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is :

(1) $p \wedge \sim q \wedge \sim r$ (2) $\sim p \wedge q \wedge \sim r$

(3) $\sim p \wedge q \wedge r$ (4) $p \wedge q \wedge r$

Official Ans. by NTA (1)

Sol. $\because \sim(A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim((p \vee r) \Rightarrow (q \vee r))$

$= (p \vee r) \wedge (\sim q \wedge \sim r)$

$= ((p \vee r) \wedge (\sim r)) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

8. If $\alpha = \lim\limits_{x \to \pi/4} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x \to 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

(1) $(1, -3)$ (2) $(-1, 3)$

(3) $(-1, -3)$ (4) $(1, 3)$

Official Ans. by NTA (4)

Sol. $\alpha = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$; $\dfrac{0}{0}$ form

Using L Hopital rule

$\alpha = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{3\tan^2 x \sec^2 x - \sec^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$

$\Rightarrow \alpha = -4$

$\beta = \lim\limits_{x\to 0}(\cos x)^{\cot x} = e^{\lim\limits_{x\to 0}\frac{(\cos x - 1)}{\tan x}}$

$\beta = e^{\lim\limits_{x\to 0}\frac{-(1-\cos x)}{x^2}\cdot\frac{x^2}{\left(\frac{\tan x}{x}\right)^x}}$

$\beta = e^{\lim\limits_{x\to 0}\left(\frac{-1}{2}\right)\cdot\frac{x}{1}} = e^0 \Rightarrow \beta = 1$

$\alpha = -4$; $\beta = 1$

If $ax^2 + bx - 4 = 0$ are the roots then

$16a - 4b - 4 = 0$ & $a + b - 4 = 0$

$\Rightarrow a = 1$ & $b = 3$

9. The locus of mid–points of the line segments joining $(-3, -5)$ and the points on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :

(1) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(2) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(3) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

(4) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

Official Ans. by NTA (3)

Sol. General point on $\frac{x^2}{4}+\frac{y^2}{9}=1$ is $A(2\cos\theta, 3\sin\theta)$

given $B(-3, -5)$

midpoint $C\left(\frac{2\cos\theta-3}{2}, \frac{3\sin\theta-5}{2}\right)$

$h = \frac{2\cos\theta-3}{2}$; $k = \frac{3\sin\theta-5}{2}$

$\Rightarrow \left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$

$\Rightarrow 36x^2 + 16y^2 + 108x + 80y + 145 = 0$

10. If $\frac{dy}{dx} = \frac{2^x y + 2^y\cdot 2^x}{2^x + 2^{x+y}\log_e 2}$, $y(0) = 0$, then for $y = 1$, the value of x lies in the interval:

(1) $(1, 2)$ (2) $\left(\frac{1}{2}, 1\right]$

(3) $(2, 3)$ (4) $\left(0, \frac{1}{2}\right]$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = \frac{2^x(y+2^y)}{2^x(1+2^y \ell n2)}$

$\Rightarrow \int\frac{(1+2^y)\ell n2}{(y+2^y)}dy = \int dx$

$\Rightarrow \ell n|y + 2^y| = x + c$

$x = 0$; $y = 0 \Rightarrow c = 0$

$\Rightarrow x = \ell n|y + 2^y|$

$\Rightarrow$ at $y = 1$, $x = \ell n3$

$\because 3 \in (e, e^2) \Rightarrow x \in (1, 2)$

11. An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2 + y^2 = ab$, $a > b$, is :

(1) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

(3) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(2\sqrt{ab}\right)$

Official Ans. by NTA (3)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $x^2 + y^2 = ab$

$\frac{2x_1}{a^2}+\frac{2y_1y'}{b^2}=0$

$\Rightarrow y_1' = \frac{-x_1}{a^2}\frac{b^2}{y_1}$ ...(1)

$\therefore 2x_1 + 2y_1y' = 0$

$\Rightarrow y_2' = \frac{-x_1}{y_1}$ ...(2)

Here (x_1y_1) is point of intersection of both curves

$\therefore x_1^2 = \frac{a^2b}{a+b}$, $y_1^2 = \frac{ab^2}{a+b}$

$\therefore \tan\theta = \left|\frac{y_1' - y_2'}{1 + y_1'y_2'}\right| = \left|\frac{\frac{-x_1b^2}{a^2y_1}+\frac{x_1}{y_1}}{1+\frac{x_1^2b^2}{a^2y_1^2}}\right|$

$$\tan\theta = \left|\frac{-b^2x_1y_1 + a^2x_1y_1}{a^2y_1^2 + b^2x_1^2}\right|$$

$$\tan\theta = \left|\frac{a-b}{\sqrt{ab}}\right|$$

12. If $y\dfrac{dy}{dx} = x\left[\dfrac{y^2}{x^2} + \dfrac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x > 0, \phi > 0$, and $y(1) = -1$,

then $\phi\left(\dfrac{y^2}{4}\right)$ is equal to :

(1) $4\phi(2)$ (2) $4\phi(1)$

(3) $2\phi(1)$ (4) $\phi(1)$

Official Ans. by NTA (2)

Sol. Let, $y = tx$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$\therefore\ tx\left(t + x\frac{dt}{dx}\right) = x\left(t^2 + \frac{\varphi(t^2)}{\varphi'(t^2)}\right)$$

$$t^2 + xt\frac{dt}{dx} = t^2 + \frac{\varphi(t^2)}{\varphi'(t^2)}$$

$$\int\frac{t\varphi'(t^2)}{\varphi(t^2)}dt = \int\frac{dx}{x}$$

Let $\varphi(t^2) = p$

$\therefore \varphi'(t^2)2tdt = dp$

$$\Rightarrow \int\frac{dy}{2p} = \int\frac{dx}{x}$$

$\frac{1}{2}\ell n\varphi(t^2) = \ell nx + \ell nc$

$\varphi(t^2) = x^2k$

$\varphi\left(\dfrac{y^2}{x^2}\right) = kx^2$, $\varphi(1) = k$

$\varphi\left(\dfrac{y^2}{4}\right) = 4\varphi(1)$

13. The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is :

(1) $\log_2 14$ (2) $\log_2 11$

(3) $\log_2 12$ (4) $\log_2 13$

Official Ans. by NTA (2)

Sol. $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$\log_2(2^{x+1}) - \log_2(3 + 2^x)^2 + \log_2(10 - 2^{-x}) = 0$

$$\log_2\left(\frac{2^{x+1}.(10 - 2^{-x})}{(3 + 2^x)^2}\right) = 0$$

$$\frac{2(10.2^x - 1)}{(3 + 2^x)^2} = 1$$

$\Rightarrow 20.2^x - 2 = 9 + 2^{2x} + 6.2^x$

$\therefore (2^x)^2 - 14(2^x) + 11 = 0$

Roots are 2^{x_1} & 2^{x_2}

$\therefore\ 2^{x_1}.2^{x_2} = 11$

$x_1 + x_2 = \log_2(11)$

14. If z is a complex number such that $\dfrac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is :

(1) $2\sqrt{2} - 1$ (2) $3\sqrt{2}$

(3) $6\sqrt{2}$ (4) $2\sqrt{2}$

Official Ans. by NTA (4)

Sol. $\dfrac{z-i}{z-1}$ is purely Imaginary number

Let $z = x + iy$

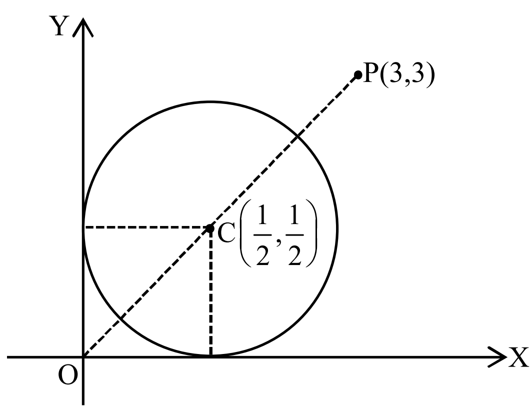
$$\therefore \frac{x + i(y-1)}{(x-1) + i(y)} \times \frac{(x-1) - iy}{(x-1) - iy}$$

$$\Rightarrow \frac{x(x-1) + y(y-1) + i(-y - x + 1)}{(x-1)^2 + y^2}$$ is purely

Imaginary number

$\Rightarrow x(x - 1) + y(y - 1) = 0$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$$



$\therefore |z-(3+3i)|_{min} = |PC| - \frac{1}{\sqrt{2}}$

$= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}$

15. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots+a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

(1) $\frac{19}{21}$ (2) $\frac{100}{121}$

(3) $\frac{21}{19}$ (4) $\frac{121}{100}$

Official Ans. by NTA (3)

Sol. $\frac{\frac{10}{2}(2a_1+9d)}{\frac{p}{2}(2a_1+(p-1)d)} = \frac{100}{p^2}$

$(2a_1 + 9d)p = 10(2a_1 + (p-1)d)$

$9dp = 20a_1 - 2pa_1 + 10d(p-1)$

$9p = (20-2p)\frac{a_1}{d} + 10(p-1)$

$\frac{a_1}{d} = \frac{(10-p)}{2(10-p)} = \frac{1}{2}$

$\therefore \frac{a_{11}}{a_{10}} = \frac{a_1+10d}{a_1+9d} = \frac{\frac{1}{2}+10}{\frac{1}{2}+9} = \frac{21}{19}$

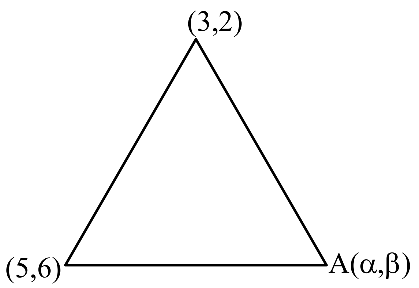
16. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

(1) $\frac{4}{\sqrt{5}}$ (2) $\frac{16}{\sqrt{5}}$

(3) $\frac{8}{\sqrt{5}}$ (4) $\frac{12}{\sqrt{5}}$

Official Ans. by NTA (3)

Sol.



$$\left|\frac{1}{2}\begin{vmatrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{vmatrix}\right| = 12$$

$4\alpha - 2\beta = \pm 24 + 8$

$\Rightarrow 4\alpha - 2\beta = +24 + 8 \Rightarrow 2\alpha - \beta = 16$

$2x - y - 16 = 0$ ...(1)

$\Rightarrow 4\alpha - 2\beta = -24 + 8 \Rightarrow 2\alpha - \beta = -8$

$2x - y + 8 = 0$ ...(2)

perpendicular distance of (1) from (0, 0)

$\left|\frac{0-0-16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$

perpendicular distance of (2) from (0, 0) is

$\left|\frac{0-0+8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$

17. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ is :

(1) 3 (2) 1

(3) 0 (4) 2

Official Ans. by NTA (2)

Sol. $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

In interval $\left[0, \frac{\pi}{4}\right]$ only one solution

18. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) = 0, f(1) = 1 and f(2) = 2, then

(1) $f''(x) = 0$ for all $x \in (0, 2)$

(2) $f''(x) = 0$ for some $x \in (0, 2)$

(3) $f'(x) = 0$ for some $x \in [0, 2]$

(4) $f''(x) > 0$ for all $x \in (0, 2)$

Official Ans. by NTA (2)

Sol. $f(0) = 0$ $f(1) = 1$ and $f(2) = 2$

Let $h(x) = f(x) - x$ has three roots

By Rolle's theorem $h'(x) = f'(x) - 1$ has at least two roots

$h''(x) = f''(x) = 0$ has at least one roots

19. If [x] is the greatest integer $\leq x$, then

$$\pi^2 \int_0^2 \left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}\,dx \text{ is equal to :}$$

(1) $2(\pi - 1)$ (2) $4(\pi - 1)$

(3) $4(\pi + 1)$ (4) $2(\pi + 1)$

Official Ans. by NTA (2)

Sol. $$\pi^2\left[\int_0^1 \sin\frac{\pi x}{2}dx + \int_1^2 \sin\frac{\pi x}{2}(x-1)\,dx\right]$$

$$= \pi^2\left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2}\right) + \left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)\right)_1^2 - \int_1^2 -\frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$$

$$= \pi^2\left[0 + \frac{2}{\pi} + \frac{2}{\pi} + \frac{2}{\pi}\cdot\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_1^2\right]$$

$= 4\pi - 4 = 4(\pi - 1)$

20. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

(1) $\frac{92}{5}$ (2) $\frac{134}{5}$

(3) $\frac{536}{25}$ (4) $\frac{112}{5}$

Official Ans. by NTA (3)

Sol. Let 8, 16, x_1, x_2, x_3, x_4, x_5 be the observations.

Now $\dfrac{x_1 + x_2 + ... + x_5 + 14}{7} = 8$

$\Rightarrow \sum_{i=1}^{5} x_i = 42$ ...(1)

Also $\dfrac{x_1^2 + x_2^2 + ... x_5^2 + 8^2 + 6^2}{7} - 64 = 16$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 560 - 100 = 460$ ...(2)

So variance of $x_1, x_2, \ldots, x_5$

$$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{2300-1764}{25} = \frac{536}{25}$$

SECTION-B

1. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K.2^{16}$, then K is equal to ______.

Official Ans. by NTA (315)

Sol. $$\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha}(2b)^{\beta}.(4ab)^{\gamma}$$

$$\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha+\gamma}.b^{\beta+\gamma}.2^{\beta}.4^{\gamma}$$

$\alpha + \beta + \gamma = 10$ ...(1)

$\alpha + \gamma = 7$ ...(2)

$\beta + \gamma = 8$ ...(3)

$(2) + (3) - (1) \Rightarrow \gamma = 5$

$\alpha = 2$

$\beta = 3$

so coefficients $= \dfrac{10!}{2!3!5!}2^3.2^{10}$

$= \dfrac{10\times9\times8\times7\times6\times5}{2\times3\times2\times5!}\times 2^{13}$

$= 315 \times 2^{16} \Rightarrow k = 315$

2. Suppose the line $\dfrac{x-2}{\alpha} = \dfrac{y-2}{-5} = \dfrac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to ____.

Official Ans. by NTA (7)

Sol. Point (2, 2, –2) also lies on given plane

So $2 + 3 \times 2 - 2(-2) + \beta = 0$

$\Rightarrow 2 + 6 + 4 + \beta = 0 \Rightarrow \beta = -12$

Also $\alpha \times 1 - 5 \times 3 + 2 \times -2 = 0$

$\Rightarrow \alpha - 15 - 4 = 0 \Rightarrow \alpha = 19$

$\therefore\ \alpha + \beta = 19 - 12 = 7$

3. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is ____.

Official Ans. by NTA (5143)

Sol. A = 4 – digit numbers divisible by 3

A = 1002, 1005, …, 9999.

$9999 = 1002 + (n - 1)3$

$\Rightarrow (n-1)3 = 8997 \Rightarrow n = 3000$

B = 4 – digit numbers divisible by 7

B = 1001, 1008, …, 9996

$\Rightarrow 9996 = 1001 + (n-1)7$

$\Rightarrow n = 1286$

$A \cap B = 1008, 1029, \ldots, 9996$

$9996 = 1008 + (n-1)21$

$\Rightarrow n = 429$

So, no divisible by either 3 or 7

$= 3000 + 1286 - 429 = 3857$

total 4-digits numbers = 9000

required numbers = 9000 – 3857 = 5143

4. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx =$

$\alpha \log_e |1+\tan x| + \beta \log_e |1-\tan x+\tan^2 x| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C,$

when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is ________.

Official Ans. by NTA (3)

Sol. $= \int \frac{\frac{\sin x}{\cos^3 x}}{1+\tan^3 x} dx = \int \frac{\tan x . \sec^2 x}{(\tan x + 1)(1+\tan^2 x - \tan x)} dx$

Let $\tan x = t \Rightarrow \sec^2 x . dx = dt$

$= \int \frac{t}{(t+1)(t^2-t+1)} dt$

$= \int \left(\frac{A}{t+1} + \frac{B(2t-1)}{t^2-t+1} + \frac{C}{t^2-t+1}\right) dx$

$\Rightarrow A(t^2 - t + 1) + B(2t - 1)(t^2 - t + 1) + C(t + 1) = t$

$\Rightarrow t^2(A + 2B) + t(-A + B + C) + A - B + C = 1$

$\therefore A + 2B = 0 \quad \ldots(1)$

$-A + B + C = 1 \quad \ldots(2)$

$A - B + C = 0 \quad \ldots(3)$

$\Rightarrow C = \frac{1}{2} \Rightarrow A - B = -\frac{1}{2} \quad \ldots(4)$

$A + 2B = 0$

$A - B = -\frac{1}{2}$

$\Rightarrow 3B = \frac{1}{2} \Rightarrow B = \frac{1}{6}$

$A = -\frac{1}{3}$

$I = -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{6}\int \frac{2t-1}{t^2-t+1} dt + \frac{1}{2}\int \frac{dt}{t^2-t+1}$

$= -\frac{1}{3} \ell n|(1+\tan x)| + \frac{1}{6} \ell n|\tan^2 x - \tan x + 1|$

$+ \frac{1}{2} . \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\left(\tan x - \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right)$

$= -\frac{1}{3} \ell n|(1+\tan x)| + \frac{1}{6} \ell n|\tan^2 x - \tan x + 1|$

$+ \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$

$\alpha = -\frac{1}{3}, \ \beta = \frac{1}{6}, \ \gamma = \frac{1}{\sqrt{3}}$

$18(\alpha + \beta + \gamma^2) = 18\left(-\frac{1}{3} + \frac{1}{6} + \frac{1}{3}\right) = 3$

5. A tangent line L is drawn at the point (2, –4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to________.

Official Ans. by NTA (2)

Sol. tangent of $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$P(2, -4) \Rightarrow -4 = 2m + \frac{2}{m}$

$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$

$\therefore$ tangent is $y = -x - 2$

$\Rightarrow x + y + 2 = 0 \quad \ldots(1)$

(1) is also tangent to $x^2 + y^2 = a$

So $\frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$

$\Rightarrow a = 2$

6. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$, then 160 S is equal to _______.

Official Ans. by NTA (305)

Sol.

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$$

$$\frac{1}{5}S = \quad \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + ...$$

On subtracting

$$\frac{4}{5}S = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + ...$$

$$S = \frac{7}{4} + \frac{1}{10}\left(1 + \frac{2}{5} + \frac{3}{5^2} + ...\right)$$

$$S = \frac{7}{4} + \frac{1}{10}\left(1 - \frac{1}{5}\right)^{-2}$$

$$= \frac{7}{4} + \frac{1}{10} \times \frac{25}{16} = \frac{61}{32}$$

$\Rightarrow 160S = 5 \times 61 = 305$

7. The number of elements in the set

$$\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1\} \text{ and } (I - A)^3 = I - A^3\right\},$$

where I is 2×2 identity matrix, is :

Official Ans. by NTA (8)

Sol. $(I - A)^3 = I^3 - A^3 - 3A(I - A) = I - A^3$

$\Rightarrow 3A(I - A) = 0$ or $A^2 = A$

$$\Rightarrow \begin{bmatrix} a^2 & ab + bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$\Rightarrow a^2 = a,\ b(a + d - 1) = 0,\ d^2 = d$

If $b \neq 0$, $a + d = 1 \Rightarrow 4$ ways

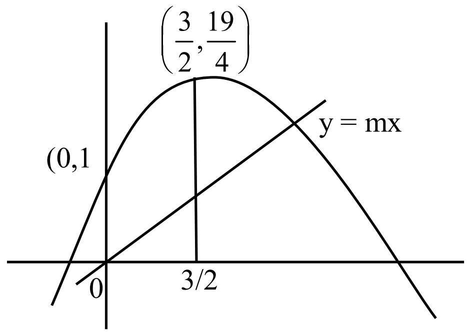
If $b = 0$, $a = 0, 1$ & $d = 0, 1 \Rightarrow 4$ ways

$\Rightarrow$ Total 8 matrices

8. If the line $y = mx$ bisects the area enclosed by the lines $x = 0$, $y = 0$, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12 m is equal to _______.

Official Ans. by NTA (26)

Sol.



Total area $= \int_0^{3/2} (1 + 4x - x^2)\,dx$

$$= x + 2x^2 - \frac{x^3}{3}\Big|_0^{3/2} = \frac{39}{8}$$

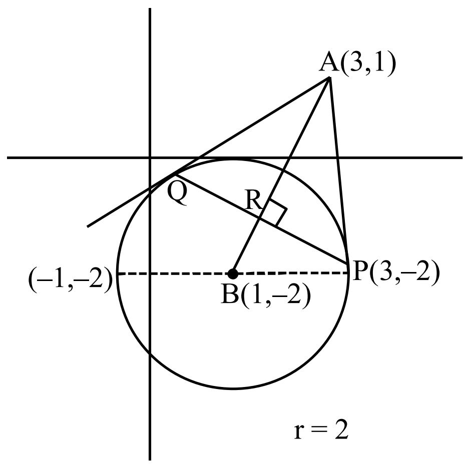
$$\& \ \frac{39}{16} = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}m$$

$$\Rightarrow 3m = \frac{13}{2} \Rightarrow 12m = 26$$

9. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8.\left(\frac{\text{area } \Delta APQ}{\text{area } \Delta BPQ}\right)$ is equal to _____.

Official Ans. by NTA (18)

Sol.



$\tan\theta = \frac{3}{2}$

$$\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$$

$$8\left(\frac{\text{Area } \Delta APQ}{\text{Area } \Delta BPQ}\right) = 18$$

10. Let f(x) be a cubic polynomial with f(1) = –10, f(–1) = 6, and has a local minima at x = 1, and f′(x) has a local minima at x = –1. Then f(3) is equal to ______.

Official Ans. by NTA (22)

Sol. $F'(x) = a(x-1)(x+3)$

$F''(x) = 6a(x+1)$

$F'(x) = 3a(x+1)^2 + b$

$F'(1) = 0 \Rightarrow b = -12a$

$F(x) = a(x+1)^3 - 12ax + c$

$= (x+1)^3 - 12x - 6$

$F(3) = 64 - 36 - 6 = 22$

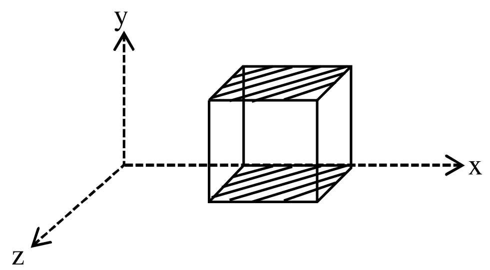
FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Wednesday 01st September, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is :



(1) 3.8×10^{-11} C (2) 8.3×10^{-11} C

(3) 3.8×10^{-12} C (4) 8.3×10^{-12} C

Official Ans. by NTA (2)

Sol. As electric field is in y-direction so electric flux is only due to top and bottom surface

Bottom surface y = 0

$\Rightarrow E = 0 \Rightarrow \phi = 0$

Top surface y = 0.5 m

$\Rightarrow E = 150\,(.5)^2 = \dfrac{150}{4}$

Now flux $\phi = EA = \dfrac{150}{4}(.5)^2 = \dfrac{150}{16}$

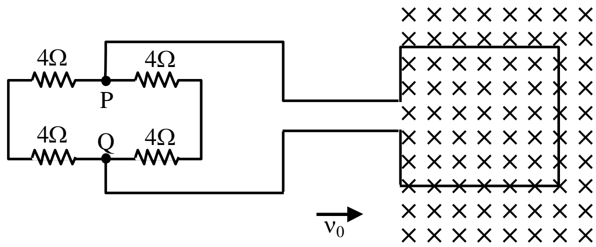
By Gauss's law $\phi = \dfrac{Q_{in}}{\epsilon_0}$

$\dfrac{150}{16} = \dfrac{Q_{in}}{\epsilon_0}$

$Q_{in} = \dfrac{150}{16} \times 8.85 \times 10^{-12} = 8.3 \times 10^{-11}$ C

Option (2)

2. A square loop of side 20 cm and resistance 1Ω is moved towards right with a constant speed v_0. The right arm of the loop is in a uniform magnetic field of 5T. The field is perpendicular to the plane of the loop and is going into it. The loop is connected to a network of resistors each of value 4Ω. What should be the value of v_0 so that a steady current of 2 mA flows in the loop ?

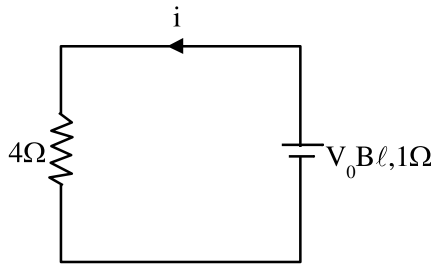


(1) 1 m/s (2) 1 cm/s

(3) 10^2 m/s (4) 10^{-2} cm/s

Official Ans. by NTA (2)

Sol. Equivalent circuit



$i = \dfrac{V_0 B\ell}{4+1} \Rightarrow V_0 = \dfrac{5(2mA)}{5 \times .2} = 10^{-2}$ m/s = 1 cm/s

Option (2)

3. The temperature of an ideal gas in 3-dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is :

[m_e = mass of electron = 9×10^{-31} kg

h = Planck constant = 6.6×10^{-34} Js

k_B = Boltzmann constant = 1.38×10^{-23} JK^{-1}]

(1) 6.26 nm (2) 8.46 nm

(3) 2.26 nm (4) 3.25 nm

Official Ans. by NTA (1)

Sol. De-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Where E is kinetic energy

$$E = \frac{3kT}{2} \text{ for gas}$$

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}}$$

$\lambda = 6.26 \times 10^{-9}$ m = 6.26 nm

Option (1)

4. A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is :

(1) –0.68 mgh (2) mgh

(3) 1.64 mgh (4) 0.64 mgh

Official Ans. by NTA (1)

Sol. Work done = Change in kinetic energy

$$W_{mg} + W_{air\text{-}friction} = \frac{1}{2}m\left(.8\sqrt{gh}\right)^2 - \frac{1}{2}m(0)^2$$

$$W_{air-friction} = \frac{.64}{2}mgh - mgh = -0.68mgh$$

Option (1)

5. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option :

(1) $R_1 > R_2$ and $H_1 = H_2$ (2) $R_1 = R_2$ and $H_1 < H_2$

(3) $R_1 < R_2$ and $H_1 < H_2$ (4) $R_1 = R_2$ and $H_1 = H_2$

Official Ans. by NTA (2)

Sol. Range $R = \frac{u^2 \sin 2\theta}{g}$ and same for θ and $90 - \theta$

So same for 42° and 48°

Maximum height $H = \frac{u^2 \sin^2 \theta}{2g}$

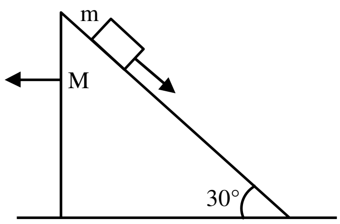
H is high for higher θ

So H for 48° is higher than H for 42°

Option (2)

6. A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is : Given m = 8 kg, M = 16 kg

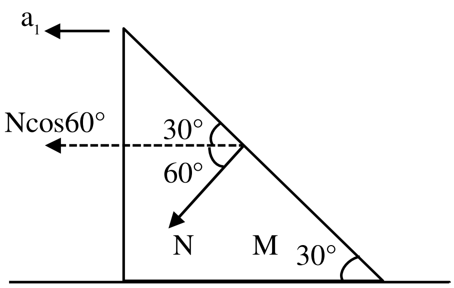
Assume all the surfaces shown in the figure to be frictionless.



(1) $\frac{4}{3}g$ (2) $\frac{6}{5}g$ (3) $\frac{3}{5}g$ (4) $\frac{2}{3}g$

Official Ans. by NTA (4)

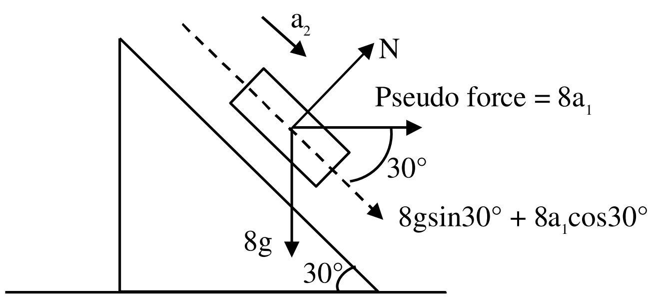
Sol. Let acceleration of wedge is a_1 and acceleration of block w.r.t. wedge is a_2



$$N\cos 60° = Ma_1 = 16a_1$$

$\Rightarrow N = 32a_1$

F.B.D. of block w.r.t wedge



$\perp$ to incline

$N = 8g \cos 30° - 8a_1 \sin 30° \Rightarrow 32a_1 = 4\sqrt{3}g - 4a_1$

$$\Rightarrow a_1 = \frac{\sqrt{3}}{9}g$$

Along incline

$8g\sin30° + 8a_1\cos30° = ma_2 = 8a_2$

$$a_2 = g \times \frac{1}{2} + \frac{\sqrt{3}}{9} g . \frac{\sqrt{3}}{2} = \frac{2g}{3}$$

Option (4)

7. Due to cold weather a 1 m water pipe of cross–sectional area 1 cm^2 is filled with ice at –10°C. Resistive heating is used to melt the ice. Current of 0.5 A is passed through 4 kΩ resistance. Assuming that all the heat produced is used for melting, what is the minimum time required ?
(Given latent heat of fusion for water/ice $= 3.33 \times 10^5$ J kg^{-1}, specific heat of ice $= 2 \times 10^3$ J kg^{-1} and density of ice $= 10^3$ kg / m^3

(1) 0.353 s (2) 35.3 s
(3) 3.53 s (4) 70.6 s

Official Ans. by NTA (2)

Sol. mass of ice $m = \rho A\ell = 10^3 \times 10^{-4} \times 1 = 10^{-1}$ kg

Energy required to melt the ice

$Q = ms\Delta T + mL$

$= 10^{-1} (2 \times 10^3 \times 10 + 3.33 \times 10^5) = 3.53 \times 10^4$ J

$$Q = i^2RT \Rightarrow 3.53 \times 10^4 = \left(\frac{1}{2}\right)^2 \left(4\times10^3\right)(t)$$

Time = 35.3 sec

Option (2)

8. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2, without any significant error, his observation are as following.

Physical Quantity	Least count of the Equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is :

(1) 0.0083 (2) 0.0155
(3) 0.155 (4) 0.083

Official Ans. by NTA (2)

Sol. $$y = \frac{MgL^3}{4bd^3\delta}$$

$$\frac{\Delta y}{y} = \frac{\Delta M}{M} + \frac{3\Delta L}{L} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta}$$

$$\frac{\Delta y}{y} = \frac{10^{-3}}{2} + \frac{3\times10^{-3}}{1} + \frac{10^{-2}}{4} + \frac{3\times10^{-2}}{4} + \frac{10^{-2}}{5}$$

$$= 10^{-3}\left[0.5+3+2.5+7.5+2\right] = 0.0155$$

Option (2)

9. Two resistors $R_1 = (4 \pm 0.8)\ \Omega$ and $R_2 = (4 \pm 0.4)\ \Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be :

(1) (4 ± 0.4) Ω
(2) (2 ± 0.4) Ω
(3) (2 ± 0.3) Ω
(4) (4 ± 0.3) Ω

Official Ans. by NTA (3)

Sol. $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} \Rightarrow R_{eq} = 2\Omega$$

Also $$\frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\frac{\Delta R_{eq}}{4} = \frac{.8}{16} + \frac{.4}{16} = \frac{1.2}{16}$$

$\Delta R_{eq} = 0.3\Omega$

$R_{eq} = (2 \pm 0.3)\Omega$

Option (3)

10. The half life period of radioactive element x is same as the mean life time of another radioactive element y. Initially they have the same number of atoms. Then :

(1) x–will decay faster than y.
(2) y– will decay faster than x.
(3) x and y have same decay rate initially and later on different decay rate.
(4) x and y decay at the same rate always.

Official Ans. by NTA (2)

Sol. $(t_{1/2})_x = (\tau)_y$

$\Rightarrow \dfrac{\ell n2}{\lambda_x} = \dfrac{1}{\lambda_y} \Rightarrow \lambda_x = 0.693\ \lambda_y$

Also initially $N_x = N_y = N_0$

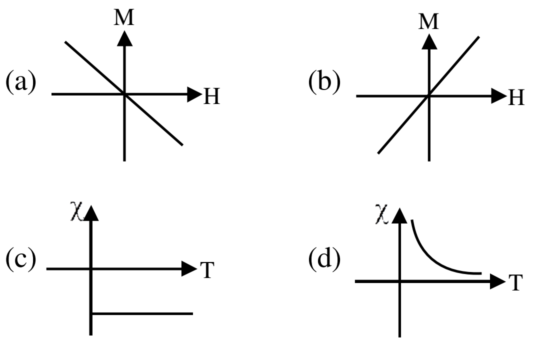
Activity $A = \lambda N$

As $\lambda_x < \lambda_y \Rightarrow A_x < A_y$

$\Rightarrow$ y will decay faster than x

Option (2)

11. Following plots show Magnetization (M) vs Magnetising field (H) and Magnetic susceptibility (χ) vs temperature (T) graph :



Which of the following combination will be represented by a diamagnetic material?

(1) (a), (c) (2) (a), (d)

(3) (b), (d) (4) (b), (c)

Official Ans. by NTA (1)

Sol. Conceptual question

Option (1)

12. A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water (μ = 4/3) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled ?

(1) 11.7 cm

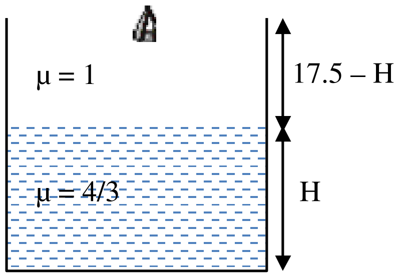
(2) 10 cm

(3) 7.5 cm

(4) 8.75 cm

Official Ans. by NTA (2)

Sol.



Height of water observed by observer

$= \dfrac{H}{\mu_w} = \dfrac{H}{(4/3)} = \dfrac{3H}{4}$

Height of air observed by observer = 17.5 – H

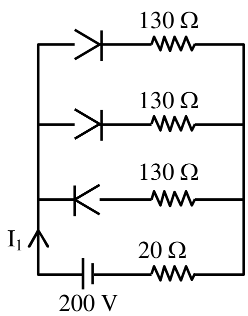
According to question, both height observed by observer is same.

$\dfrac{3H}{4} = 17.5 - H$

$\Rightarrow$ H = 10 cm

Option (2)

13. In the given figure, each diode has a forward bias resistance of 30Ω and infinite resistance in reverse bias. The current I_1 will be :

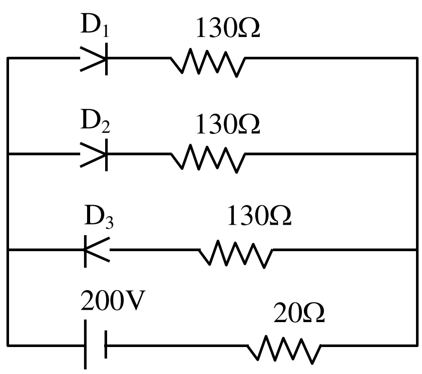


(1) 3.75 A (2) 2.35 A

(3) 2 A (4) 2.73 A

Official Ans. by NTA (3)

Sol.



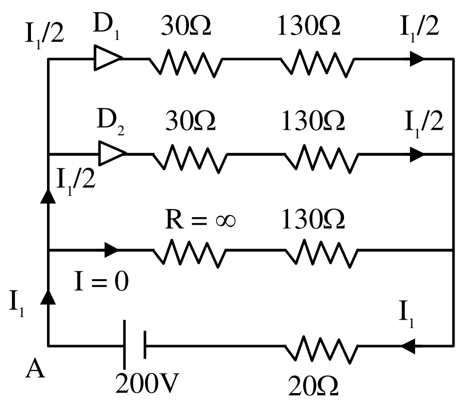
As per diagram,

Diode D_1 & D_2 are in forward bias i.e. R = 30Ω

whereas diode D_3 is in reverse bias i.e. R = infinite

$\Rightarrow$ Equivalent circuit will be

Applying KVL starting from point A



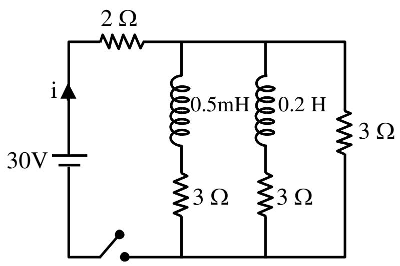
$$-\left(\frac{I_1}{2}\right)\times 30-\left(\frac{I_1}{2}\right)\times 130-I_1\times 20+200=0$$

$\Rightarrow -\ 100\ I_1 + 200 = 0$

$I_1 = 2$

Option (3)

14. For the given circuit the current i through the battery when the key in closed and the steady state has been reached is__________.

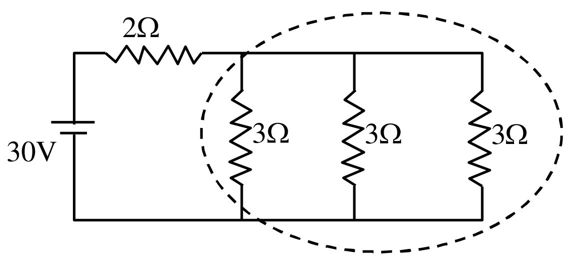


(1) 6 A (2) 25 A (3) 10 A (4) 0 A

Official Ans. by NTA (3)

Sol. In steady state, inductor behaves as a conducting wire.

So, equivalent circuit becomes

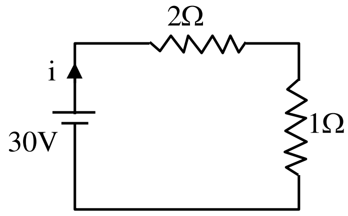


Parallel (R_{eq})

$$\frac{1}{R_{eq}}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

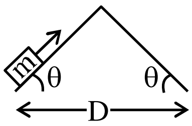
$\Rightarrow R_{eq} = 1\Omega$

$\Rightarrow$ Circuit becomes



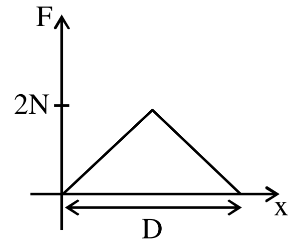
$$\Rightarrow\ i=\frac{30}{3}=10A$$

15. An object of mass 'm' is being moved with a constant velocity under the action of an applied force of 2N along a frictionless surface with following surface profile.

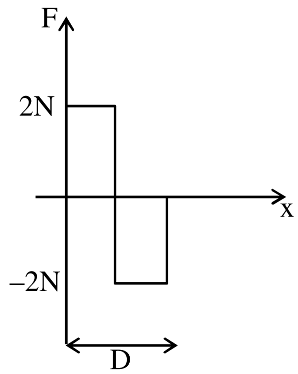


The correct applied force vs distance graph will be:

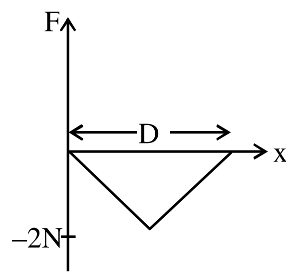
(1)

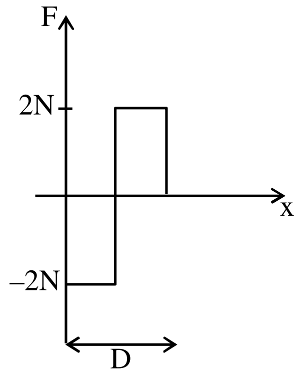


(2)



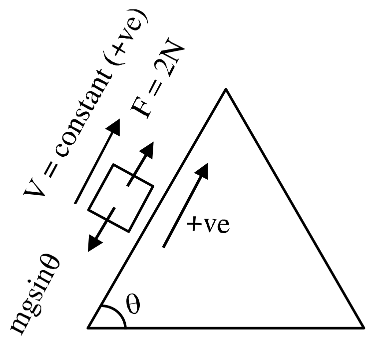
(3)



(4) 

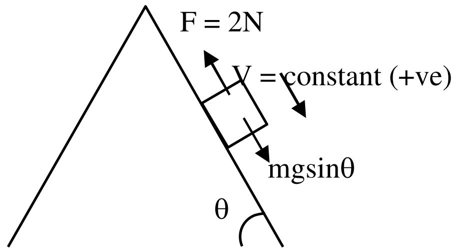
Official Ans. by NTA (2)

Sol. During upward motion



F = 2N = (+ve) constant

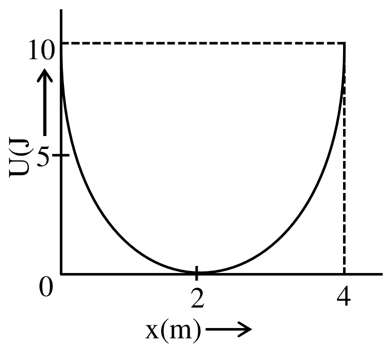
During downward motion



$\Rightarrow$ F = 2N = (–ve) constant

$\Rightarrow$ Best possible answer is option (2)

16. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?



(1) 10 m/s^2 (2) 5 m/s^2

(3) 4 m/s^2 (4) 9.8 m/s^2

Official Ans. by NTA (3)

Sol. From potential energy curve

$$U_{max} = \frac{1}{2}kA^2 \Rightarrow 10 = \frac{1}{2}k(2)^2$$

$$\Rightarrow k = 5$$

Now $T_{spring} = T_{pendulum}$

$$2\pi\sqrt{\frac{5}{5}} = 2\pi\sqrt{\frac{4}{g}}$$

$$\Rightarrow 1 = \sqrt{\frac{4}{g}} \Rightarrow g = 4 \text{ on planet}$$

Option (3)

17. A capacitor is connected to a 20 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 2 V in 1 μs. The capacitance of the capacitor is ....μF.

Given : $\ln\left(\frac{10}{9}\right) = 0.105$

(1) 9.52 (2) 0.95

(3) 0.105 (4) 1.85

Official Ans. by NTA (2)

Sol. $V = V_0\left(1 - e^{-t/RC}\right)$

$$2 = 20\left(1 - e^{-t/RC}\right)$$

$$\frac{1}{10} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = \frac{9}{10}$$

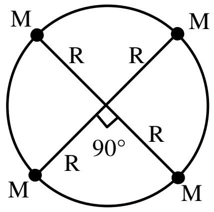
$$e^{t/RC} = \frac{10}{9}$$

$$\frac{t}{RC} = \ell n\left(\frac{10}{9}\right) \Rightarrow C = \frac{t}{R\ell n\left(\frac{10}{9}\right)}$$

$$C = \frac{10^{-6}}{10 \times .105} = .95\mu F$$

Option (2)

18. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is :

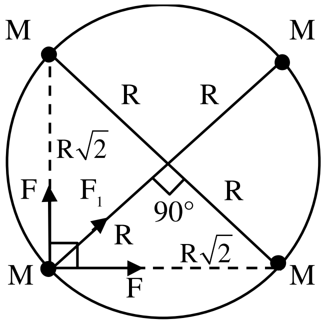


(1) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$ (2) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

(3) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{GM}{R}}$

Official Ans. by NTA (2)

Sol.



$$F_{net} = \frac{MV^2}{R}$$

$$\sqrt{2}F + F_1 = \frac{MV^2}{R}$$

$$\sqrt{2}\frac{GMM}{(\sqrt{2}R)^2} + \frac{GMM}{(2R)^2} = \frac{MV^2}{R}$$

$$\frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = V^2$$

$$\frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = V^2$$

$$V = \sqrt{\frac{GM(4+\sqrt{2})}{R4\sqrt{2}}}$$

$$V = \frac{1}{2}\sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$$

Option (2)

19. Electric field of plane electromagnetic wave propagating through a non–magnetic medium is given by E = 20cos(2 × 10^{10} t–200x) V/m. The dielectric constant of the medium is equal to : (Take $\mu_r = 1$)

(1) 9 (2) 2 (3) $\frac{1}{3}$ (4) 3

Official Ans. by NTA (1)

Sol. Speed of wave = $\frac{2\times10^{10}}{200} = 10^8 \text{ m/s}$

Refractive index = $\frac{3\times10^8}{10^8} = 3$

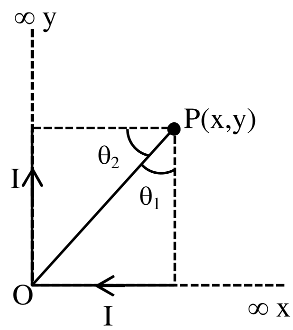
Now refractive index = $\sqrt{\varepsilon_r\mu_r}$

$3 = \sqrt{\varepsilon_r(1)}$

$\Rightarrow \varepsilon_r = 9$

Option (1)

20. There are two infinitely long straight current carrying conductors and they are held at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ratio of current in both conductor is 1 : 1. The magnetic field at point P is _____.



(1) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

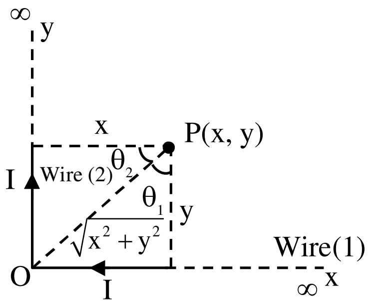
(2) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(3) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(4) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}+(x+y)\right]$

Official Ans. by NTA (1)

Sol.



$$B_{\text{due to wire (1)}} = \frac{\mu_0 I}{4\pi y}[\sin 90 + \sin\theta_1]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{y}\left(1+\frac{x}{\sqrt{x^2+y^2}}\right) \quad ......(1)$$

$$B_{\text{due to wire (2)}} = \frac{\mu_0}{4\pi}\frac{I}{x}(\sin 90^\circ + \sin\theta_2)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{x}\left(1+\frac{y}{\sqrt{x^2+y^2}}\right) \quad .....(2)$$

Total magnetic field

$B = B_1 + B_2$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{y}+\frac{x}{y\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{y}{x\sqrt{x^2+y^2}}\right]$$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{x+y}{xy}+\frac{x^2+y^2}{xy\sqrt{x^2+y^2}}\right]$$

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{x+y}{xy}+\frac{\sqrt{x^2+y^2}}{xy}\right]$$

$$B = \frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$$

Option (1)

SECTION-B

1. The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0 °C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of workdone by the gas is $\frac{x}{10}$. Then the value of x (round off to the nearest integer) is _______.

(Given R = 8.31 J mol^{-1} K^{-1})

Official Ans. by NTA (25)

Sol. Pressure is not changing ⇒ isobaric process

$$\Rightarrow \Delta U = nC_v\Delta T = \frac{5nR\Delta T}{2}$$

and W = nRΔT

$$\frac{\Delta U}{W} = \frac{5}{2} = \frac{x}{10} \Rightarrow x = 25.00$$

2. The width of one of the two slits in a Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the slit-width, the ratio of minimum to maximum intensity in the interference pattern is x : 4 where x is _______.

Official Ans. by NTA (1)

Sol. Given amplitude ∝ slit width

Also intensity ∝ $(\text{Amplitude})^2$ ∝ $(\text{Slit width})^2$

$$\frac{I_1}{I_2} = \left(\frac{3}{1}\right)^2 = 9 \Rightarrow I_1 = 9I_2$$

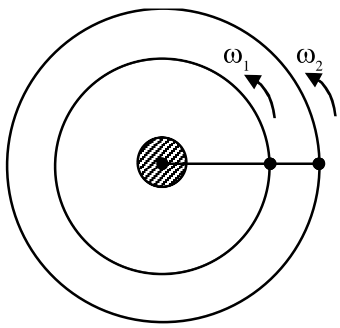
$$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1}-\sqrt{I_2}}{\sqrt{I_1}+\sqrt{I_2}}\right)^2 = \left(\frac{3-1}{3+1}\right)^2 = \frac{1}{4} = \frac{x}{4}$$

⇒ x = 1.00

3. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2 × 10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1} where x is ....

Official Ans. by NTA (3)

Sol.



T_1 = 1 hour

⇒ ω_1 = 2π rad/hour

T_2 = 8 hours

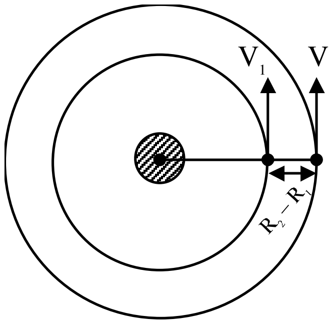
$\Rightarrow \omega_2 = \frac{\pi}{4}$ rad / hour

$R_1 = 2 \times 10^3$ km

As $T^2 \propto R^3$

$$\Rightarrow \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2$$

$$\Rightarrow \frac{R_2}{R_1} = \left(\frac{8}{1}\right)^{2/3} = 4 \Rightarrow R_2 = 8\times10^3\,\text{km}$$



$$V_1 = \omega_1 R_1 = 4\pi\times10^3\,\text{km / h}$$

$$V_2 = \omega_2 R_2 = 2\pi\times10^3\,\text{km / h}$$

$$\text{Relative } \omega = \frac{V_1 - V_2}{R_2 - R_1} = \frac{2\pi\times10^3}{6\times10^3}$$

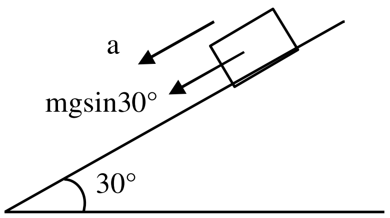
$= \frac{\pi}{3}$ rad / hour

x = 3

4. When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2-1}{\alpha^2}\right)$ where x = ...

Official Ans. by NTA (3)

Sol.

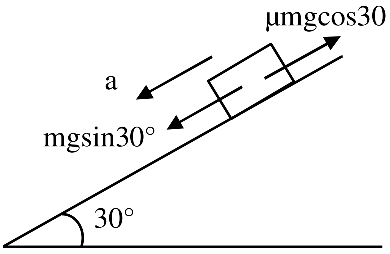


On smooth incline

a = g sin30°

by $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{2}\frac{g}{2}T^2 = \frac{g}{4}T^2 \ ....(i)$$



On rough incline

a = g sin30° – μg cos 30°

by $S = ut + \frac{1}{2}at^2$

$$S = \frac{1}{4}g\left(1-\sqrt{3}\mu\right)\left(\alpha T\right)^2 \ ...(ii)$$

By (i) and (ii)

$$\frac{1}{4}gT^2 = \frac{1}{4}g\left(1-\sqrt{3}\mu\right)\alpha^2T^2$$

$$\Rightarrow 1-\sqrt{3}g = \frac{1}{\alpha^2} \Rightarrow g = \left(\frac{\alpha^2-1}{\alpha^2}\right).\frac{1}{\sqrt{3}}$$

$\Rightarrow$ x = 3.00

5. The average translational kinetic energy of N_2 gas molecules at ....°C becomes equal to the K.E. of an electron accelerated from rest through a potential difference of 0.1 volt.

(Given $k_B = 1.38 \times 10^{-23}$ J/K)

(Fill the nearest integer).

Official Ans. by NTA (500)

Sol. Given

Translation K.E. of N_2 = K.E. of electron

$$\frac{3}{2}kT = eV$$

$$\frac{3}{2}\times1.38\times10^{-23}T = 1.6\times10^{-19}\times0.1$$

$$\Rightarrow T = 773k$$

T = 773 – 273 = 500°C

6. A uniform heating wire of resistance 36 Ω is connected across a potential difference of 240 V. The wire is then cut into half and potential difference of 240 V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be 1 : x, where x is..........

Official Ans. by NTA (4)

Sol. First case $P_1 = \frac{V^2}{R} = \frac{(240)^2}{36}$

Second case Resistance of each half = 18 Ω

$$P_2 = \frac{(240)^2}{18} + \frac{(240)^2}{18} = \frac{(240)^2}{9}$$

$$\frac{P_1}{P_2} = \frac{1}{4}$$

x = 4.00

7. A steel rod with y = 2.0×10^{11} Nm^{-2} and $\alpha = 10^{-5}$ $°C^{-1}$ of length 4 m and area of cross-section 10 cm^2 is heated from 0° C to 400°C without being allowed to extend. The tension produced in the rod is $x \times 10^5$ N where the value of x is ....

Official Ans. by NTA (8)

Sol. Thermal force $F = Ay\alpha\Delta T$

$F = (10 \times 10^{-4})(2 \times 10^{11})(10^{-5})(400)$

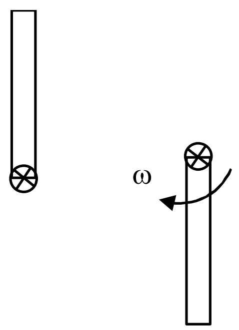
$F = 8 \times 10^5$ N

$\Rightarrow x = 8$

8. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity, Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is ....ms^{-1}. (Take g = 10 ms^{-2})

Official Ans. by NTA (6)

Sol.



by energy conservation $mg\ell = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2\omega^2}{3}$

$$\Rightarrow \omega = \sqrt{\frac{6g}{\ell}}$$

Speed $v = \omega r = \omega\ell = \sqrt{6g\ell}$

$$v = \sqrt{6 \times 10 \times .6} = 6 m/s$$

9. A carrier wave with amplitude of 250 V is amplitude modulated by a sinusoidal base band signal of amplitude 150 V. The ratio of minimum amplitude to maximum amplitude for the amplitude modulated wave is 50 : x, then value of x is ....

Official Ans. by NTA (200)

Sol. $A_{max} = A_C + A_m = 250 + 150 = 400$

$A_{min} = A_C - A_m = 250 - 150 = 100$

$$\frac{A_{min}}{A_{max}} = \frac{100}{400} = \frac{1}{4} = \frac{50}{200}$$

x = 200

10. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is .... $\times 10^5$ N/m.

Official Ans. by NTA (16)

Sol. Work = ΔK.E.

$$W_{friction} + W_{spring} = 0 - \frac{1}{2}mv^2$$

$$-\frac{90}{100}\left(\frac{1}{2}mv^2\right) + W_{Spring} = -\frac{1}{2}mv^2$$

$$W_{spring} = -\frac{10}{100} \times \frac{1}{2}mv^2$$

$$-\frac{1}{2}kx^2 = -\frac{1}{20}mv^2$$

$$\Rightarrow k = \frac{40000 \times (20)^2}{10 \times (1)^2} = 16 \times 10^5$$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Wednesday 01st September, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Water sample is called cleanest on the basis of which one of the BOD values given below

(1) 11 ppm (2) 15 ppm

(3) 3 ppm (4) 21 ppm

Official Ans. by NTA (3)

Sol. Clean water could have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

2. Calamine and Malachite, respectively, are the ores of :

(1) Nickel and Aluminium

(2) Zinc and Copper

(3) Copper and Iron

(4) Aluminium and Zinc

Official Ans. by NTA (2)

Sol. Calamine $\Rightarrow ZnCO_3$

Malachite $\Rightarrow Cu(OH)_2 \cdot CuCO_3$

3. Experimentally reducing a functional group **cannot** be done by which one of the following reagents ?

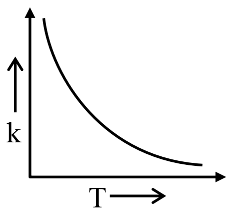
(1) Pt-C/H_2 (2) Na/H_2

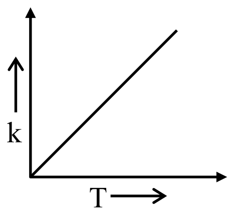
(3) Pd-C/H_2 (4) Zn/H_2O

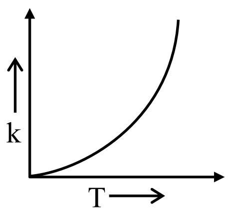
Official Ans. by NTA (2)

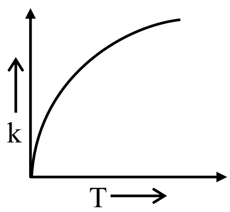
Sol. Solution NaH_2 is not reducing agent

4. Which one of the following given graphs represents the variation of rate constant (k) with temperature (T) for an endothermic reaction ?

(1) 

(2) 

(3) 

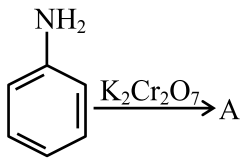
(4) 

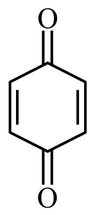
Official Ans. by NTA (3)

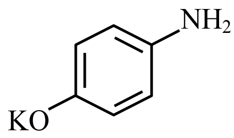
Sol. By observation we get this plot during measurable temperatures

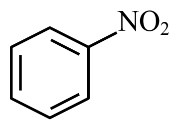
Ans. 3rd Option.

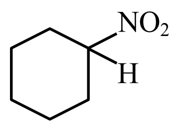
5. Identify A in the following reaction.



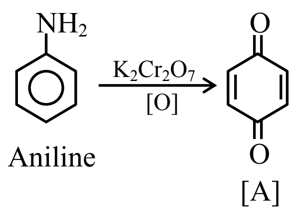
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

6. In the following sequence of reactions a compound **A**, (molecular formula $C_6H_{12}O_2$) with a straight chain structure gives a C_4 carboxylic acid. **A** is :

$$\mathbf{A} \xrightarrow[H_3O^+]{LiAlH_4} \mathbf{B} \xrightarrow{\text{Oxidation}} C_4-\text{carboxylic acid}$$

(1) $CH_3-CH_2-COO-CH_2-CH_2-CH_3$

(2) $CH_3-CH_2-\underset{}{\overset{OH}{\overset{|}{C}H}}-CH_2-O-CH=CH_2$

(3) $CH_3-CH_2-CH_2-COO-CH_2-CH_3$

(4) $CH_3-CH_2-CH_2-O-CH=CH-CH_2-OH$

Official Ans. by NTA (3)

$CH_3–CH_2–CH_2–\underset{\underset{O}{\|}}{C}–O–CH_2–CH_3$ (A) $[C_6H_{12}O_2]$

$\downarrow$ (1) $LiAlH_4$ (2) H_3O^+

$CH_3–CH_2–CH_2–CH_2–OH + CH_3–CH_2–$

(B)

$\downarrow$ [O]

$CH_3–CH_2–CH_2–\underset{\underset{O}{\|}}{C}–OH$ [C_4 carboxylic acid]

7. Match **List – I** with **List - II**.

List -I (Colloid Preparation Method)		**List -II (Chemical Reaction)**	
(a)	Hydrolysis	(i)	$2AuCl_3 + 3HCHO + 3H_2O \rightarrow 2Au(sol) + 3HCOOH + 6HCl$
(b)	Reduction	(ii)	$As_2O_3 + 3H_2S \rightarrow As_2S_3(sol) + 3H_2O$
(c)	Oxidation	(iii)	$SO_2 + 2H_2S \rightarrow 3S(sol) + 2H_2O$
(d)	Double Decomposition	(iv)	$FeCl_3 + 3H_2O \rightarrow Fe(OH)_3(sol) + 3HCl$

Choose the most appropriate answer from the options given below.

(1) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(2) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)

(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Official Ans. by NTA (2)

Sol. According to type of reactions for preparation, colloids have been classified

8. The Crystal Field Stabilization Energy (CFSE) and magnetic moment (spin-only) of an octahedral aqua complex of a metal ion (M^{z+}) are $–0.8\ \Delta_0$ and 3.87 BM, respectively. Identify (M^{Z+}) :

(1) V^{3+} (2) Cr^{3+}

(3) Mn^{4+} (4) Co^{2+}

Official Ans. by NTA (4)

Sol. $V^{3+} \Rightarrow \boxed{\ \ }\boxed{\ \ }_{e_g}$ $= 2 \times 0.4\ \Delta_0$

$\boxed{1}\boxed{1}\boxed{\ \ }_{t_{2g}}$ $= –0.8\ \Delta_0$

$= 2$ unpaired e^-

$\mu = 2.89$ Bm

$Co^{2+} \Rightarrow \boxed{1}\boxed{1}_{e_g}$ $[2 \times 0.6\ \Delta_0 – 5 \times 0.4\ \Delta_0]$

$= –0.8\ \Delta_0$

$\boxed{1\!\downarrow}\boxed{1\!\downarrow}\boxed{1}_{t_{2g}}$ 3 unpaired $e^- \Rightarrow \mu = 3.87$ BM

hence d^7 configuration is of Co^{2+} Ans.

9. Monomer units of Dacron polymer are :

(1) ethylene glycol and phthalic acid

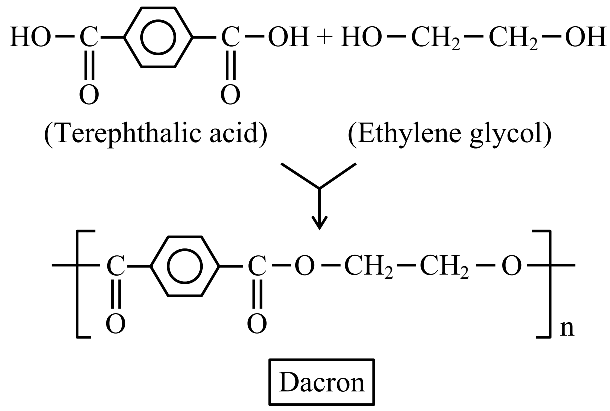
(2) ethylene glycol and terephthalic acid

(3) glycerol and terephthalic acid

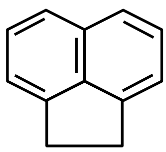
(4) glycerol and phthalic acid

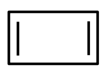
Official Ans. by NTA (2)

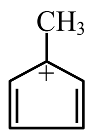
Sol.

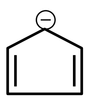


10. Which one of the following compounds is aromatic in nature ?

(1) 

(2) 

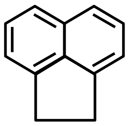
(3) 

(4) 

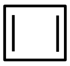
Official Ans. by NTA (4)

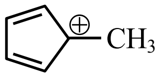
Allen Ans. (1,4)

Sol. (1) (Acenaphthene)

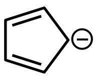
 $\rightarrow$ 10 πe^- in cyclic conjugation

$\rightarrow$ $\boxed{\text{Aromatic}}$

(2)  $\rightarrow$ $4\pi e^-$ in ring conjugation $\Rightarrow$ Anti Aromatic

(3)  $\Rightarrow$ $4\pi e^-$ in ring conjugation $\Rightarrow$ Antiaromatic

(4)

 $\Rightarrow$ $6\pi e^-$ in ring conjugation $\Rightarrow$ $\boxed{\text{Aromatic}}$

Cyclopentadienyl anion

11. In the given chemical reaction, colors of the Fe^{2+} and Fe^{3+} ions, are respectively :

$5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$

(1) Yellow, Orange (2) Yellow, Green

(3) Green, Orange (4) Green, Yellow

Official Ans. by NTA (4)

Sol. Colour of Fe^{2+} is observed green and Fe^{3+} is yellow

12. The stereoisomers that are formed by electrophilic addition of bromine to trans-but-2-ene is/are :

(1) 2 enantiomers and 2 mesomers

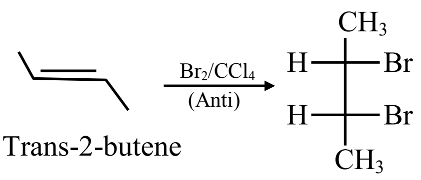
(2) 2 identical mesomers

(3) 2 enantiomers

(4) 1 racemic and 2 enantiomers

Official Ans. by NTA (2)

Sol.



Trans-2-butene

[I]

meso product

13. Hydrogen peroxide reacts with iodine in basic medium to give :

(1) IO_4^- (2) IO^- (3) I^- (4) IO_3^-

Official Ans. by NTA (3)

Sol. $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$

14. In the following sequence of reactions,

$C_3H_6 \xrightarrow{H^+/H_2O} \mathbf{A} \xrightarrow[\text{dil KOH}]{\text{KIO}} \mathbf{B} + \mathbf{C}$

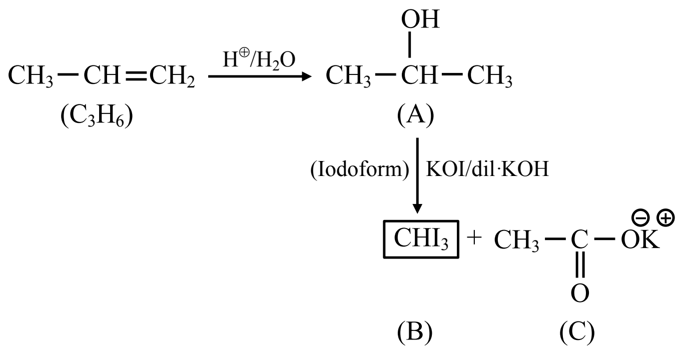
The compounds **B** and **C** respectively are :

(1) CI_3COOK, HCOOH (2) CI_3COOK, CH_3I

(3) CH_3I, HCOOK (4) CHI_3, CH_3COOK

Official Ans. by NTA (4)

Sol.



15. Given below are **two** statements :

Statement I : The nucleophilic addition of sodium hydrogen sulphite to an aldehyde or a ketone involves proton transfer to form a stable ion.

Statement II : The nucleophilic addition of hydrogen cyanide to an aldehyde or a ketone yields amine as final product.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true.

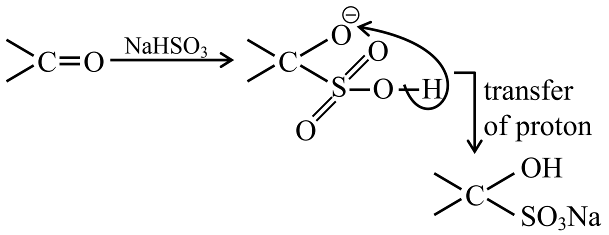
(2) **Statement I** is true but **Statement II** is false.

(3) **Statement I** is false but **Statement II** is true.

(4) Both **Statement I** and **Statement II** are false.

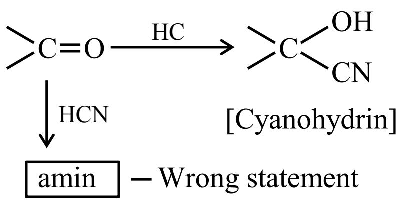
Official Ans. by NTA (2)

Sol. **Statement I :** Correct



(White crystalline soluble ppt)

Statement II :

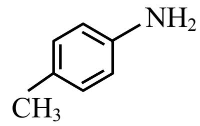


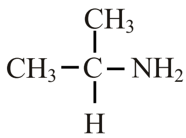
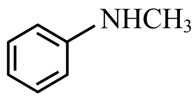
[Cyanohydrin]

— Wrong statement

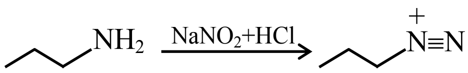
(Amine not formed)

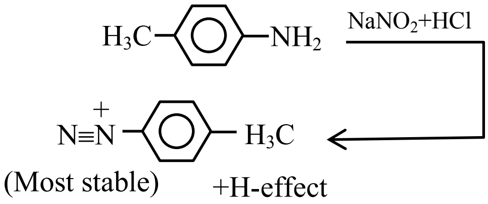
16. Which one of the following gives the most stable Diazonium salt ?

(1) $CH_3–CH_2–CH_2–NH_2$ (2) 

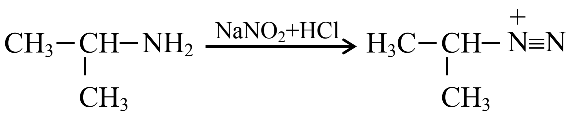
(3)  (4) 

Official Ans. by NTA (2)

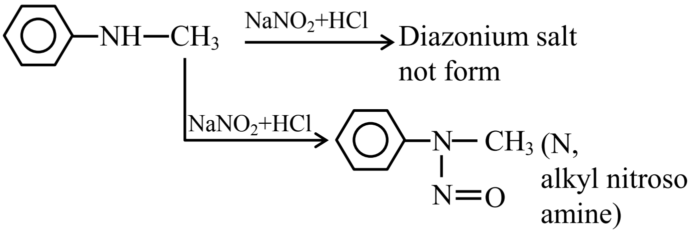
Sol. (1) 

(2) 

(3)



(4)



17. The potassium ferrocyanide solution gives a Prussian blue colour, when added to :

(1) $CoCl_3$ (2) $FeCl_2$

(3) $CoCl_2$ (4) $FeCl_3$

Official Ans. by NTA (4)

Sol. $FeCl_3 + K_4[Fe(CN)_6] \rightarrow \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3}$

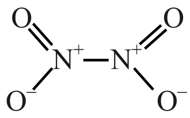
18. The oxide **without** nitrogen-nitrogen bond is :

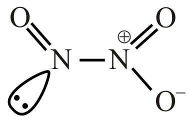
(1) N_2O (2) N_2O_4

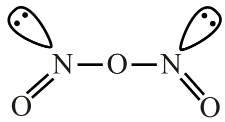
(3) N_2O_3 (4) N_2O_5

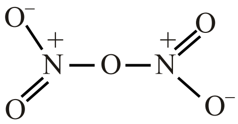
Official Ans. by NTA (4)

Sol. (1) $N{\equiv}N^+{-}O^-$

(2) 

(3)  unsymmetrical

 symmetrical

(4) 

19. Number of paramagnetic oxides among the following given oxides is __________.

Li_2O, CaO, Na_2O_2, KO_2, MgO and K_2O

(1) 1 (2) 2

(3) 3 (4) 0

Official Ans. by NTA (1)

Sol. $Li_2O \Rightarrow 2Li^+ \quad O^{2-}$ $\qquad MgO \Rightarrow Mg^{2+} \quad O^{2-}$

$CaO \Rightarrow Ca^{2+} \quad O^{2-}$ $\qquad K_2O \Rightarrow 2K^+ \quad O^{2-}$

$Na_2O_2 \Rightarrow 2Na^+ \quad O_2^{2-}$

$KO_2 \Rightarrow K^+ \quad O_2^-$

$O_2^- \Rightarrow$ Complete octet, diamagnetic

$O^{2-} \Rightarrow \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2px}^2\ \pi_{2py}^2\ \ \pi_{2pz}^2\ \pi_{2py}^{*2}\ \ \pi_{2pz}^{*2}$ (dia)

$O_2^- \Rightarrow \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \sigma_{2px}^2\ \pi_{2py}^2\ \ \pi_{2pz}^2\ \pi_{2py}^{*2}\ \ \pi_{2pz}^{*1}$ (para)

20. Identify the element for which electronic configuration in +3 oxidation state is $[Ar]3d^5$:

(1) Ru (2) Mn

(3) Co (4) Fe

Official Ans. by NTA (4)

Sol. Fe^{3+} $[Ar]\ 3d^5$

SECTION-B

1. An empty LPG cylinder weighs 14.8 kg. When full, it weighs 29.0 kg and shows a pressure of 3.47 atm. In the course of use at ambient temperature, the mass of the cylinder is reduced to 23.0 kg. The final pressure inside of the cylinder is ______atm. (Nearest integer)

(Assume LPG of be an ideal gas)

Official Ans. by NTA (2)

Sol. Initial mass of gas = 29 –14.8 = 14.2 Kg
mass of gas used = 29 –23 = 6 Kg
gas left = 14.2 – 6 = 8.2 Kg

$$(1)\ 3.47 \times V = \left(\frac{14.2\times10^3}{M}\right) \times R \times T$$

$$(2)\ p \times V = \left(\frac{8.2\times10^3}{M}\right) \times R \times T$$

Divide :

$$\frac{(1)}{(2)} \Rightarrow \frac{3.47}{P} = \frac{14.2}{8.2}$$

P = 2.003

2. The molar solubility of $Zn(OH)_2$ in 0.1 M NaOH solution is $x \times 10^{-18}$ M. The value of x is _____(Nearest integer)

(Given : The solubility product of $Zn(OH)_2$ is 2×10^{-20})

Official Ans. by NTA (2)

Sol. $Zn(OH)_2\ (s) \quad\quad Zn^{+2}\ (aq) + 2OH^-\ (aq)$

$\quad\quad\quad\quad\quad\quad\quad\quad S \quad\quad (0.1 + 2s) \quad 0.1$

$K_{sp} = S(0.1)^2$

$2 \times 10^{-20} = s \times 10^{-2} \Rightarrow s = 2 \times 10^{-18}$

$= x \times 10^{-18}$

x = 2

3. For the reaction $2NO_2(g) \quad\quad N_2O_4(g)$, when $\Delta S = -176.0\ JK^{-1}$ and $\Delta H = -57.8\ kJ\ mol^{-1}$, the magnitude of ΔG at 298 K for the reaction is_____ $kJ\ mol^{-1}$. (Nearest integer)

Official Ans. by NTA (5)

Sol. $\Delta G = \Delta H - T\Delta S$

$$\Delta G = 57.8 - \frac{298(-176)}{1000}$$

$\Delta G = -5.352$ kJ/mole

|Nearest integer value| = 5

4. The sum of oxidation states of two silver ions in $[Ag(NH_3)_2]\ [Ag(CN)_2]$ complex is _______.

Official Ans. by NTA (2)

Sol. $[Ag(NH_3)_2]^+\ [Ag(CN)_2]^-$

+1 +1

5. The number of atoms in 8 g of sodium is $x \times 10^{23}$. The value of x is________.(Nearest integer)

[Given : $N_A = 6.02 \times 10^{23}\ mol^{-1}$

Atomic mass of Na = 23.0 u]

Official Ans. by NTA (2)

Sol. No. of atoms $= \frac{8}{23} \times 6.02 \times 10^{23} = 2.09 \times 10^{23}$

2×10^{23}

$= x \times 10^{23}$

x = 2

6. If 80 g of copper sulphate $CuSO_4{\cdot}5H_2O$ is dissolved in deionised water to make 5 L of solution. The concentration of the copper sulphate solution is $x \times 10^{-3}\ mol\ L^{-1}$. The value of x is _________.

[Atomic masses Cu : 63.54 u, S : 32 u, O : 16 u, H : 1 u]

Official Ans. by NTA (64)

Sol. Moles of $CuSO_4{\cdot}5H_2O = \frac{80}{249.54}$

$$\text{Molarity} = \frac{\frac{80}{249.54}}{5} = 64.117 \times 10^{-3}$$

Nearest integer, x = 64

7. A 50 watt bulb emits monochromatic red light of wavelength of 795 nm. The number of photons emitted per second by the bulb is $x \times 10^{20}$. The value of x is _________.

[Given : $h = 6.63 \times 10^{-34}$ Js and $c = 3.0 \times 10^8\ ms^{-1}$]

Official Ans. by NTA (2)

Sol. Total energy per sec. = 50 J

$$50 = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{795 \times 10^{-9}}$$

$n = 1998.49 \times 10^{17}$ [n = no. of photons per second]

$= 1.998 \times 10^{20}$

2×10^{20}

$= x \times 10^{20}$

$x = 2$

8. The spin-only magnetic moment value of B_2^+ species is ________$\times 10^{-2}$ BM. (Nearest integer)

[Given : $\sqrt{3} = 1.73$]

Official Ans. by NTA (173)

Sol. $B_2^+ \Rightarrow \sigma_{1s}^2\ \sigma_{1s}^{*2}\ \sigma_{2s}^2\ \sigma_{2s}^{*2}\ \pi_{2py}^1\ \ \ \pi_{2pz}^0$

$\Rightarrow 9e^-$

$\mu = \sqrt{1(1+2)} = \sqrt{3}$ BM

$= 1.73$ BM

$= 1.73 \times 10^{-2}$ BM

9. If the conductivity of mercury at 0°C is 1.07×10^6 S m^{-1} and the resistance of a cell containing mercury is 0.243 Ω, then the cell constant of the cell is $x \times 10^4$ m^{-1}. The value of x is _______.(Nearest integer)

Official Ans. by NTA (26)

Sol. $k = 1.07 \times 10^6\ Sm^{-1}$, $R = 0.243\ \Omega$

$$G = \frac{1}{R} = \frac{1}{0.243}\Omega^{-1}$$

$k = G \times G^*$

$$G^* = \frac{k}{G} = \frac{1.07 \times 10^6}{\frac{1}{0.243}} \quad 26 \times 10^4\ m^{-1}$$

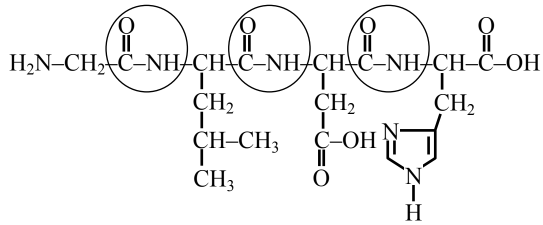
10. A peptide synthesized by the reactions of one molecule each of Glycine, Leucine, Aspartic acid and Histidine will have ________ peptide linkages.

Official Ans. by NTA (3)

Sol. Glycine —leucine—Aspartic acid—Histidine

peptide link   peptide link   peptide link

Total (3) peptide linkages are present



3 peptide linkage

Ans. (3)

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Wednesday 01st September, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let f : **R** → **R** be a continuous function. Then $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)dx}{x^2-\frac{\pi^2}{16}}$ is equal to :

(1) f (2) (2) 2f (2)

(3) $2f\left(\sqrt{2}\right)$ (4) 4 f (2)

Official Ans. by NTA (2)

Sol. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)dx}{x^2-\frac{\pi^2}{16}}$

$$\lim_{x\to\frac{\pi}{4}} \frac{\pi}{4}.\frac{\left[f\left(\sec^2 x\right).2\sec x.\sec x\tan x\right]}{2x}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\pi}{4}f\left(\sec^2 x\right).\sec^3 x.\frac{\sin x}{x}$$

$$\frac{\pi}{4}f(2).\left(\sqrt{2}\right)^3.\frac{1}{\sqrt{2}}\times\frac{4}{\pi}$$

⇒ 2f(2)

2. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to :

(The inverse trigonometric functions take the principal values)

(1) 3π – 11 (2) 4 π – 9

(3) 4 π – 11 (4) 3π + 1

Official Ans. by NTA (3)

Sol. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$

⇒ (2π – 5) + (6 – 2 π) – (12 – 4 π)

⇒ 4π – 11.

3. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all a ∈ **R** for which the system is inconsistent and S_2 be the set of all a ∈ **R** for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

(1) $n(S_1) = 2$, $n(S_2) = 2$ (2) $n(S_1) = 1$, $n(S_2) = 0$

(3) $n(S_1) = 2$, $n(S_2) = 0$ (4) $n(S_1) = 0$, $n(S_2) = 2$

Official Ans. by NTA (3)

Sol. $\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$

$= -1(a^2 + 10) -1(-3a-10) + 2(-6 + 2a)$

$= -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Delta = -a^2 + 7a - 12$

$\Delta = -[a^2 - 7a + 12]$

$\Delta = -[(a-3)(a-4)]$

$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$

$= 0 - 1(-a - 35) + 2(-2 + 7a)$

$\Rightarrow a + 35 - 4 + 14a$

$15a + 31$

Now $\Delta_1 = 15a + 31$

For inconsistent $\Delta = 0$ ∴ a = 3, a = 4

and for a = 3 and 4 $\Delta_1 \neq 0$

$n(S_1) = 2$

For infinite solution : $\Delta = 0$

and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

∴ $n(S_2) = 0$

4. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P?

(1) $\left(3,1,-\frac{1}{2}\right)$ (2) $\left(-2,0,-\frac{1}{2}\right)$

(3) (0, 2, –4) (4) (4, 0, – 2)

Official Ans. by NTA (2)

Sol. $P_1 : x - 2y - 2z + 1 = 0$

$P_2 : 2x - 3y - 6z + 1 = 0$

$$\left|\frac{x-2y-2z+1}{\sqrt{1+4+4}}\right| = \left|\frac{2x-3y-6z+1}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\frac{x-2y-2z+1}{3} = \pm\frac{2x-3y-6z+1}{7}$$

Since $a_1a_2 + b_1b_2 + c_1c_2 = 20 > 0$

$\therefore$ Negative sign will give

acute bisector

$7x - 14y - 14z + 7 = -[6x - 9y - 18z + 3]$

$\Rightarrow 13x - 23y - 32z + 10 = 0$

$\left(-2, 0, -\frac{1}{2}\right)$ satisfy it $\therefore$ Ans (2)

5. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

(1) $\sim (q \to p)$ (2) $\sim p \to \sim q$

(3) $\sim (p \to \sim q)$ (4) $\sim (p \to q)$

Official Ans. by NTA (4)

Sol.

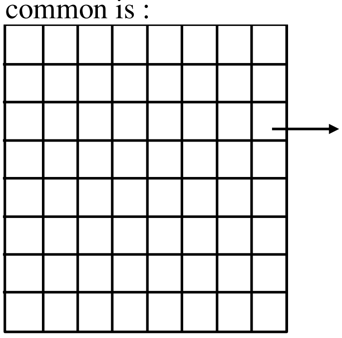
p	q	$\sim p$	$\sim q$	$p - q$	$\sim(p \to q)$	$q \to p$	$\sim(q \to p)$
T	T	F	F	T	F	T	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	F	T	F

$p \wedge \sim q$	$\sim p \to \sim q$	$p \to \sim q$	$\sim(p \to \sim q)$
F	T	F	T
T	T	T	F
F	F	T	F
F	T	T	F

$p \wedge \sim q \equiv \sim (p \to q)$

Option (4)

6. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :



64 square

(1) $\frac{2}{7}$ (2) $\frac{1}{18}$

(3) $\frac{1}{7}$ (4) $\frac{1}{9}$

Official Ans. by NTA (2)

Sol. Total ways of choosing square = ${}^{64}C_2$

$$= \frac{64 \times 63}{2 \times 1} = 32 \times 63$$

ways of choosing two squares having common side $= 2\,(7 \times 8) = 112$

Required probability $= \frac{112}{32 \times 63} = \frac{16}{32 \times 9} = \frac{1}{18}$.

Ans. (2)

7. If $y = y\,(x)$ is the solution curve of the differential equation $x^2dy + \left(y - \frac{1}{x}\right)dx = 0$; $x > 0$ and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (2) $3 + \frac{1}{\sqrt{e}}$

(3) $3 + e$ (4) $3 - e$

Official Ans. by NTA (4)

Sol. $x^2dy + \left(y - \frac{1}{x}\right)dx = 0 : x > 0, y(1) = 1$

$$x^2dy + \frac{(xy-1)}{x}dx = 0$$

$$x^2dy = \frac{(xy-1)}{x}dx$$

$$\frac{dy}{dx} = \frac{1-xy}{x^3}$$

$$\frac{dy}{dx} = \frac{1}{x^3} - \frac{y}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2}.y = \frac{1}{x^3}$$

$$\text{If } e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}} = \int \frac{1}{x^3}.e^{-\frac{1}{x}}$$

$$ye^{-\frac{1}{x}} = e^{-x}\left(1 + \frac{1}{x}\right) + C$$

$$1.e^{-1} = e^{-1}(2) + C$$

$$C = -e^{-1} = -\frac{1}{e}$$

$$ye^{-\frac{1}{x}} = e^{-\frac{1}{x}}\left(1 + \frac{1}{x}\right) - \frac{1}{e}$$

$$y\left(\frac{1}{2}\right) = 3 - \frac{1}{e} \times e^2$$

$$y\left(\frac{1}{2}\right) = 3 - e$$

8. If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0, \pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is :

(1) $(3, 13\pi / 9)$ (2) $(2, 2\pi / 3)$

(3) $(2, 8\pi / 9)$ (4) $(3, 5\pi / 3)$

Official Ans. by NTA (1)

Sol. $$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$$

$$2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(2-4\sin^2 x-1\right)=1$$

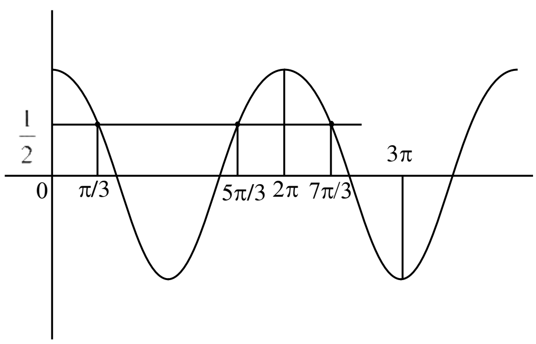
$$2\cos x\left(1-4\sin^2 x\right)=1$$

$$2\cos x\left(4\cos^2 x-3\right)=1$$

$$4\cos^3 x-3\cos x=\frac{1}{2}$$

$$\cos 3x=\frac{1}{2}$$

$x \in [0, \pi]$ $\therefore$ $3x \in [0, 3\pi]$



9. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

Then

(1) both (S1) and (S2) are true

(2) (S1) is false and (S2) is true

(3) both (S1) and (S2) are false

(4) (S1) is true and (S2) is false

Official Ans. by NTA (1)

Sol. $f(x) = x^3 - 6x^2 + ax + b$

$f(2) = 8 - 24 + 2a + b = 0$

$2a + b = 16 \ldots(1)$

$f(4) = 64 - 96 + 4a + b = 0$

$4a + b = 32 \ \ldots(2)$

Solving (1) and (2)

$a = 8, b = 0$

$$\boxed{f(x) = x^3 - 6x^2 + 8x}$$

$f(x) = x^3 - 6x^2 + 8x$

$f'(x) = 3x^2 - 12x + 8$

$f''(x) = 6x - 12$

$\Rightarrow f'(x)$ is $\uparrow$ for $x > 2$, and $f'(x)$ is $\downarrow$ for $x < 2$

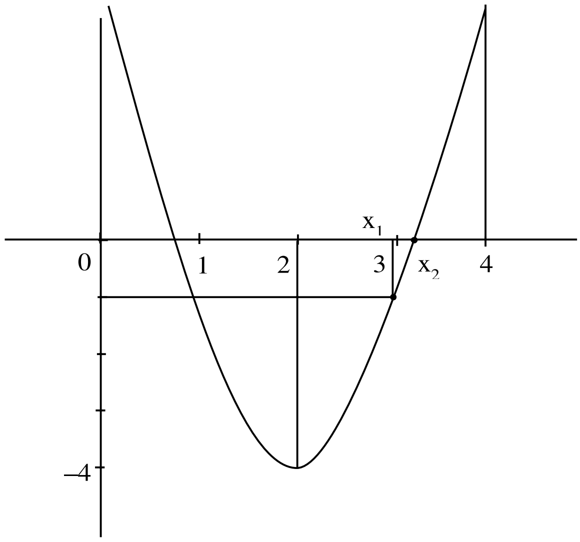
$f'(2) = 12 - 24 + 8 = -4$

$f'(4) = 48 - 48 + 8 = 8$

$f'(x) = 3x^2 - 12x + 8$

vertex $(2, -4)$

$f'(2) = -4$, $f'(4) = 8$, $f'(3) = 27 - 36 + 8$



$f'(x_1) = -1$, then $x_1 = 3$

$f'(x_2) = 0$

Again $\quad f'(x) < 0$ for $x \in (2, x_4)$

$\quad f'(x) > 0$ for $x \in (x_4, 4)$

$x_4 \in (3, 4)$

$f(x) = x^3 - 6x^2 + 8x$

$f(3) = 27 - 54 + 24 = -3$

$f(4) = 64 - 96 + 32 = 0$

For $x_4(3, 4)$

$f(x_4) < -3\sqrt{3}$

and $f'(x_3) > -4$

$2f'(x_3) > -8$

So, $2f'(x_3) = \sqrt{3}\ f(x_4)$

Correct Ans. (1)

10. Let $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx, \quad \forall\ n > m$ and $n, m \in N$.

Consider a matrix $A = [a_{ij}]_{3\times 3}$ where

$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0 \quad , & i > j \end{cases}$. Then $\left|\text{adj}A^{-1}\right|$ is :

(1) $(15)^2 \times 2^{42}$ (2) $(15)^2 \times 2^{34}$

(3) $(105)^2 \times 2^{38}$ (4) $(105)^2 \times 2^{36}$

Official Ans. by NTA (3)

Sol. $\begin{bmatrix} \overset{\surd}{a_{11}} & \overset{\surd}{a_{12}} & \overset{\surd}{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$J_{6+i,3} - J_{i+3,3}$; $i \le j$

$\Rightarrow \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3-1} - \int_0^{\frac{1}{2}} \frac{x^{i+3}}{x^3-1}$

$\Rightarrow \int_0^{1/2} \frac{x^{i+3}\left(x^3-1\right)}{x^3-1}$

$\Rightarrow \frac{x^{3+i+1}}{3+i+1} = \left(\frac{x^{4+i}}{4+i}\right)_0^{1/2}$

$a_{ij} = j_{6+i,3} - j_{i+3,3} = \frac{\left(\frac{1}{2}\right)^{4+i}}{4+i}$

$a_{11} = \frac{\left(\frac{1}{2}\right)^5}{5} = \frac{1}{5.2^5}$

$a_{12} = \frac{1}{5.2^5}$

$a_{13} = \frac{1}{5.2^5}$

$a_{22} = \frac{1}{6.2^6}$

$a_{23} = \frac{1}{6.2^6}$

$a_{33} = \frac{1}{7.2^7}$

$A = \begin{bmatrix} \frac{1}{5.2^5} & \frac{1}{5.2^5} & \frac{1}{5.2^5} \\ 0 & \frac{1}{6.2^6} & \frac{1}{6.2^6} \\ 0 & 0 & \frac{1}{7.2^7} \end{bmatrix}$

$|A| = \frac{1}{5.2^5}\left[\frac{1}{6.2^6} \times \frac{1}{7.2^7}\right]$

$|A| = \frac{1}{210.2^{18}}$

$\left|\text{adj}A^{-1}\right| = \left|A^{-1}\right|^{n-1} = \left|A^{-1}\right|^2 = \frac{1}{\left(|A|\right)^2}$

$\Rightarrow (210.2^{18})^2$

$(105)^2 \times 2^{38}$

11. The area, enclosed by the curves y = sin x + cos x and $y = |\cos x - \sin x|$ and the lines $x = 0,\ x = \frac{\pi}{2}$, is :

(1) $2\sqrt{2}\left(\sqrt{2}-1\right)$ (2) $2\left(\sqrt{2}+1\right)$

(3) $4\left(\sqrt{2}-1\right)$ (4) $2\sqrt{2}\left(\sqrt{2}+1\right)$

Official Ans. by NTA (1)

Sol. $A = \int_0^{\pi/2}\left((\sin x + \cos x) - |\cos x - \sin x|\right)dx$

$A = \int_0^{\pi/4}\left((\sin x + \cos x) - (\cos x - \sin x)\right)dx$

$+\int_{\pi/4}^{\pi/2}\left((\sin x + \cos x) - (\sin x - \cos x)\right)dx$

$A = 2\int_0^{\pi/2}\sin x\,dx + 2\int_{\pi/4}^{\pi/2}\cos x\,dx$

$A = -2\left(\frac{1}{\sqrt{2}}-1\right) + 2\left(1-\frac{1}{\sqrt{2}}\right)$

$A = 4 - 2\sqrt{2} = 2\sqrt{2}\,(\sqrt{2} - 1)$

Option (1)

12. The distance of line 3y – 2z – 1 = 0 = 3x – z + 4 from the point (2, – 1, 6) is :

(1) $\sqrt{26}$ (2) $2\sqrt{5}$

(3) $2\sqrt{6}$ (4) $4\sqrt{2}$

Official Ans. by NTA (3)

Sol. $3y - 2z - 1 = 0 = 3x - z + 4$

$3y - 2z - 1 = 0$ D.R′s ⇒ (0, 3, –2)

$3x - z + 4 = 0$ D.R′s ⇒ (3, –1, 0)

Let DR′s of given line are a, b, c

Now 3b – 2c = 0 & 3a – c = 0

$\therefore$ 6a = 3b = 2c

a : b : c = 3 : 6 : 9

Any pt on line

3K – 1, 6K + 1, 9K + 1

Now 3(3K – 1) + 6(6K + 1)1 + 9(9K + 1) =0

$\Rightarrow K = \frac{1}{3}$

Point on line ⇒ (0, 3, 4)

Given point (2, –1, 6)

⇒ Distance = $\sqrt{4+16+4} = 2\sqrt{6}$

Option (3)

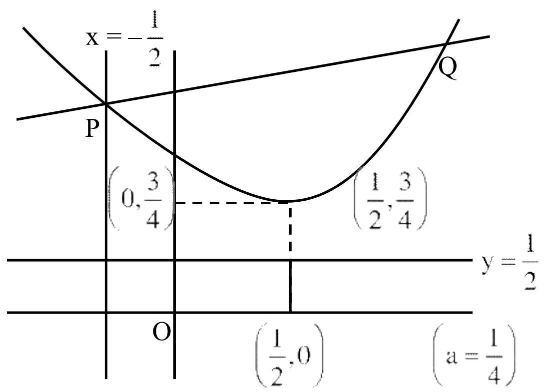
13. Consider the parabola with vertex $\left(\frac{1}{2},\frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to :

(1) $\frac{75}{8}$ (2) $\frac{125}{16}$

(3) $\frac{25}{2}$ (4) $\frac{15}{2}$

Official Ans. by NTA (2)

Sol.



$\left(y-\frac{3}{4}\right) = \left(x-\frac{1}{2}\right)^2$ ... (1)

For $x = -\frac{1}{2}$

$y - \frac{3}{4} = 1 \Rightarrow y = \frac{7}{4} \Rightarrow P\left(-\frac{1}{2},\frac{7}{4}\right)$

Now $y' = 2\left(x-\frac{1}{2}\right)$ At $x = -\frac{1}{2}$

$\Rightarrow m_T = -2, m_N = \frac{1}{2}$

Equation of Normal is

$$y - \frac{7}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$$

$$y = \frac{x}{2} + 2$$

Now put y in equation (1)

$$\frac{x}{2} + 2 - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$\Rightarrow x = 2 \ \& -\frac{1}{2}$$

$\Rightarrow Q(2, 3)$

Now $(PQ)^2 = \frac{125}{16}$

Option (2)

14. The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is :

(1) 6 (2) 2

(3) 4 (4) 8

Official Ans. by NTA (1)

Sol. Consider the equation $x^2 + ax + b = 0$

If has two roots (not necessarily real α & β)

Either $\alpha = \beta$ or $\alpha \neq \beta$

Case (1) If $\alpha = \beta$, then it is repeated root. Given that $\alpha^2 - 2$ is also a root

So, $\alpha = \alpha^2 - 2 \Rightarrow (\alpha + 1)(\alpha - 2) = 0$

$\Rightarrow \alpha = -1$ or $\alpha = 2$

When $\alpha = -1$ then (a, b) = (2, 1)

$\alpha = 2$ then (a, b) = (–4, 4)

Case (2) If $\alpha \neq \beta$ Then

(I) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$

Here $(\alpha, \beta) = (2, -1)$ or $(-1, 2)$

Hence $(a, b) = (-(\alpha + \beta), \alpha\beta)$

= (–1, –2)

(II) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$

Since $\alpha \neq \beta$ we get $\alpha + \beta = \beta^2 + \alpha^2 - 4$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$

Thus $-1 = 1 - 2\alpha\beta - 4$ which implies

$\alpha\beta = -1$ Therefore $(a, b) = (-(\alpha + \beta), \alpha\beta)$

= (1, –1)

(III) $\alpha = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$

$\Rightarrow \alpha = -\beta$

Thus $\alpha = 2, \beta = -2$

$\alpha = -1, \beta = 1$

Therefore (a, b) = (0, –4) & (0, –1)

(IV) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$ is same as (III)

Therefore we get 6 pairs of (a, b)

Which are (2, 1), (–4, 4), (–1, –2), (1, –1) (0, –4)

Option (1)

15. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \ldots + (n-1) \cdot 1$, $n \geq 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to :

(1) $\frac{e-1}{3}$ (2) $\frac{e-2}{6}$

(3) $\frac{e}{3}$ (4) $\frac{e}{6}$

Official Ans. by NTA (1)

Sol. Let $T_r = r(n-r)$

$T_r = nr - r^2$

$$\Rightarrow S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n}\left(nr - r^2\right)$$

$$S_n = \frac{n.(n)(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n(n-1)(n+1)}{6}$$

Now $\displaystyle\sum_{r=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$

$$= \sum_{r=4}^{\infty}\left(2.\frac{n(n-1)(n+1)}{6.n(n-1)(n-2)!} - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty}\left(\frac{1}{3}\left(\frac{n-2+3}{(n-2)!}\right) - \frac{1}{(n-2)!}\right)$$

$$= \sum_{r=4}^{\infty}\frac{1}{3}\cdot\frac{1}{(n-3)!} = \frac{1}{3}(e-1)$$

Option (1)

16. Let $P_1, P_2, \ldots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is :

(1) 12 (2) 419 (3) 443 (4) 455

Official Ans. by NTA (3)

Sol. Total Number of Triangles = ${}^{15}C_3$

$i + j + k = 15$ (Given)

5 Cases

i	j	k
1	2	12
1	3	11
1	4	10
1	5	9
1	6	8

4 Cases

i	j	k
2	3	10
2	4	9
2	5	8
2	6	7

3 Cases

i	j	k
3	4	8
3	5	7

1 Cases

i	j	k
4	5	6

Number of Possible triangles using the vertices P_i, P_j, P_k such that $i + j + k \neq 15$ is equal to ${}^{15}C_3 - 12 = 443$

Option (3)

17. The range of the function,

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

is :

(1) $\left(0, \sqrt{5}\right)$ (2) $[-2, 2]$

(3) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$ (4) $[0, 2]$

Official Ans. by NTA (4)

Sol. $f(x) = \log_{\sqrt{5}}$

$$\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

$$f(x) = \log_{\sqrt{5}}\left[3 + 2\cos\left(\frac{\pi}{4}\right)\cos(x) - 2\sin\left(\frac{3\pi}{4}\right)\sin(x)\right]$$

$$f(x) = \log_{\sqrt{5}}\left[3 + \sqrt{2}(\cos x - \sin x)\right]$$

Since $-\sqrt{2} \leq \cos x - \sin x \leq \sqrt{2}$

$$\Rightarrow \log_{\sqrt{5}}\left[3 + \sqrt{2}\left(-\sqrt{2}\right) \leq f(x) \leq \log_{\sqrt{5}}\left[3 + \sqrt{2}\left(\sqrt{2}\right)\right]\right]$$

$$\Rightarrow \log_{\sqrt{5}}(1) \leq f(x) \leq \log_{\sqrt{5}}(5)$$

So Range of f(x) is [0, 2]

Option (4)

18. Let $a_1, a_2, \ldots\ldots, a_{21}$ be an AP such that $\displaystyle\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this AP is 189, then $a_6 a_{16}$ is equal to :

(1) 57 (2) 72

(3) 48 (4) 36

Official Ans. by NTA (2)

Sol. $$\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20}\frac{1}{a_n(a_n + d)}$$

$$= \frac{1}{d}\sum_{n=1}^{20}\left(\frac{1}{a_n} - \frac{1}{a_n + d}\right)$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{21}}\right) = \frac{4}{9} \text{ (Given)}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_{21} - a_1}{a_1 a_{21}}\right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_1 + 20d - a_1}{a_1 a_2}\right) = \frac{4}{9} \Rightarrow a_1 a_2 = 45 \text{ .... (1)}$$

Now sum of first 21 terms = $\frac{21}{2}$ $(2a_1 + 20d) = 189$

$\Rightarrow a_1 + 10d = 9$ ... (2)

For equation (1) & (2) we get

$a_1 = 3$ & $d = \frac{3}{5}$

OR

$a_1 = 15$ & $d = -\frac{3}{5}$

So, $a_6.a_{16} = (a_1 + 5d)(a_1 + 15d)$

$\Rightarrow a_6 a_{16} = 72$

Option (2)

19. The function f(x), that satisfies the condition $f(x)=x+\int_{0}^{\pi/2}\sin x\cdot\cos y f(y)dy$, is :

(1) $x+\frac{2}{3}(\pi-2)\sin x$ (2) $x + (\pi + 2) \sin x$

(3) $x+\frac{\pi}{2}\sin x$ (4) $x + (\pi - 2) \sin x$

Official Ans. by NTA (4)

Sol. $f(x) = x+\int_0^{\pi/2}\sin x\cos y f(y)dy$

$f(x)=x+\sin x\underbrace{\int_0^{\pi/2}\cos y f(y)dy}_{K}$

$\Rightarrow f(x) = x + K \sin x$

$\Rightarrow f(y) = y + K \sin y$

Now $K = \int_0^{\pi/2}\cos y(y+K\sin y)dy$

$K = \underbrace{\int_0^{\pi/2} y\cos dy}_{\text{Apply IBP}} + \underbrace{\int_0^{\pi/2}\cos y\sin ydy}_{\text{Put } \sin y = t}$

$K=(y\sin y)_0^{\pi/2}-\int_0^{\pi/2}\sin dy+K\int_0^1 t\,dt$

$\Rightarrow K = \frac{\pi}{2}-1+K\left(\frac{1}{2}\right)$

$\Rightarrow K = \pi - 2$

So $f(x) = x + (\pi - 2) \sin x$

Option (4)

20. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to :

(1) $\frac{5}{2\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{\sqrt{3}}$ (4) 2

Official Ans. by NTA (2)

Sol. The point of intersection of the curves $\frac{x^2}{9}+\frac{y^2}{1}=1$ and $x^2 + y^2 = 3$ in the first quadrant is

$\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$

Now slope of tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$ is

$m_1 = -\frac{1}{3\sqrt{3}}$

And slope of tangent to the circle at $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right)$ is m_2

$= -\sqrt{3}$

So, if angle between both curves is θ then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-\frac{1}{3\sqrt{3}}+\sqrt{3}}{1+\left(-\frac{1}{3\sqrt{3}}\right)(-\sqrt{3})}\right|$$

$=\frac{2}{\sqrt{3}}$

Option (2)

SECTION-B

1. Let X be a random variable with distribution.

x	–2	–1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\ \sigma^2$ is equal to :

Official Ans. by NTA (781)

Sol.

x	–2	–1	3	4	6
P(X = x)	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

$\overline{X} = 2.3$

$-a + 6b = \frac{9}{10}$ .... (1)

$\sum P_i = \frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$

$a + b = \frac{4}{15}$ .... (2)

From equation (1) and (2)

$a = \frac{1}{10}, \quad b = \frac{1}{6}$

$\sigma^2 = \Sigma p_i x_i^2 - (\overline{X})^2$

$\frac{1}{5}(4) + a(1) + \frac{1}{3}(9) + \frac{1}{5}(16) + b(36) - (2.3)^2$

$= \frac{4}{5} + a + 3 + \frac{16}{5} + 36b - (2.3)^2$

$= 4 + a + 3 + 36b - (2.3)^2$

$= 7 + a + 36b - (2.3)^2$

$= 7 + \frac{1}{10} + 6 - (2.3)^2$

$= 13 + \frac{1}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{131}{10} - \left(\frac{23}{10}\right)^2$

$= \frac{1310 - (23)^2}{100}$

$= \frac{1310 - 529}{100}$

$\sigma^2 = \frac{781}{100}$

$100\sigma^2 = 781$

2. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$, $x \in \mathbf{R}$. Then the natural number n for which $\lim_{x \to 1} \frac{x^n f(1) - f(x)}{x - 1} = 44$ is________.

Official Ans. by NTA (7)

Sol. $f(n) = x^6 + 2x^4 + x^3 + 2x + 3$

$\lim_{x \to 1} \frac{x^n f(1) - f(x)}{x - 1} = 44$

$\lim_{x \to 1} \frac{9x^n - (x^6 + 2x^4 + x^3 + 2x + 3)}{x - 1} = 44$

$\lim_{x \to 1} \frac{9nx^{n-1} - (6x^5 + 8x^3 + 3x^2 + 2)}{1} = 44$

$\Rightarrow 9n - (19) = 44$

$\Rightarrow 9n = 63$

$\Rightarrow n = 7$

3. If for the complex numbers z satisfying $|z - 2 - 2i| \le 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to ________.

Official Ans. by NTA (5)

Sol. $|z - 2 - 2i| \le 1$

$|x + iy - 2 - 2i| \le 1$

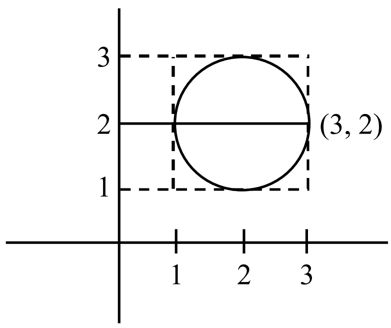
$|(x - 2) + i(y - 2)| \le 1$

$(x - 2)^2 + (y - 2)^2 \le 1$

$|3iz + 6|_{max}$ at $a + ib$

$|3i| \left|z + \frac{6}{3i}\right|$

$3|z - 2i|_{max}$



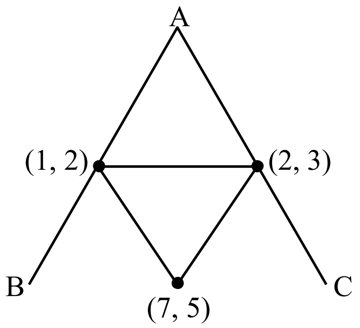
From Figure maximum distance at 3 + 2i

a + ib = 3 + 2i = a + b = 3 + 2 = 5 **Ans.**

4. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is _______.

Official Ans. by NTA (6)

Sol. intersection point of give lines are (1, 2), (7, 5), (2,3)



$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(5-3) - 2(7-2) + 1(21-10)]$$

$$= \frac{1}{2}[2 - 10 + 11]$$

$$\Delta DEF = \frac{1}{2}(3) = \frac{3}{2}$$

$$\Delta ABC = 4\ \Delta DEF = 4\left(\frac{3}{2}\right) = 6$$

5. Let f(x) be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of $52 - 10\,f(10)$ is equal to :

Official Ans. by NTA (26)

Sol. $k\,f(k) + 2 = \lambda\,(x-2)(x-3)(x-4)(x-5)$ ...(1)

put x = 0

we get $\lambda = \frac{1}{60}$

Now put λ in equation (1)

$\Rightarrow kf(k) + 2 = \frac{1}{60}\,(x-2)(x-3)(x-4)(x-5)$

Put x = 10

$\Rightarrow 10f(10) + 2 = \frac{1}{60}(8)(7)(6)(5)$

$\Rightarrow 52 - 10f(10) = 52 - 26 = 26$

6. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ______.

Official Ans. by NTA (77)

Sol. FARMER (6)

A, E, F, M, R, R

A					
E					
F	A	E			
F	A	M			
F	A	R	E		
F	A	R	M	E	R

$\frac{\underline{|5}}{\underline{|2}} - \underline{|4} = 60 - 24 = 36$

$\frac{\underline{|3}}{\underline{|2}} - \underline{|2} = 3 - 2 = 1$

= 1

= 2

= 1

77

7. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is _______.

Official Ans. by NTA (924)

Sol. $(x + y)^n \Rightarrow 2^n = 4096$ $\quad 2^{10} = 1024 \times 2$

$\Rightarrow 2^n = 2^{12}$ $\quad 2^{11} = 2048$

$n = 12$ $\quad 2^{12} = \underline{4096}$

$$^{12}C_6 = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$= 11 \times 3 \times 4 \times 7$

$= 924$

8. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____.

Official Ans. by NTA (1494)

Sol. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\vec{v} = x\vec{a} + y\vec{b}$ $\quad\quad$ $\vec{v}\left(3\hat{i} + 2\hat{j} - k\right) = 0$

$\vec{v}\cdot\hat{a} = 19$

$\vec{v} = \lambda\vec{c} \times \left(\vec{a} \times \vec{b}\right)$

$\vec{v} = \lambda\left[\left(\vec{c}.\vec{b}\right)\vec{a} - \left(\vec{c}\cdot\vec{a}\right)\vec{b}\right]$

$= \lambda[(3+4+1)\left(2\hat{i} - \hat{j} + 2\hat{k}\right) - \left(\frac{6-2-2}{2}\right)\left(\hat{i} + 2\hat{j} + \hat{k}\right)$

$= \lambda[16\hat{i} - 8\hat{j} + 16\hat{k} - 2\hat{i} - 4\hat{j} + 2\hat{k}]$

$\vec{v} = \lambda\left[14\hat{i} - 12\hat{j} + 18\hat{k}\right]$

$\lambda[14\hat{i} - 12\hat{j} + 18\hat{k}] \cdot \frac{\left(2\hat{i} - \hat{j} + 2\hat{k}\right)}{\sqrt{4+1+4}} = 19$

$\lambda\frac{[28+12+36]}{3} = 19$

$\lambda\left(\frac{76}{3}\right) = 19$

$4\lambda = 3 \Rightarrow \lambda = \frac{3}{4}$

$|2v^2| = \left|2 \times \frac{3}{4}\left(14\hat{i} - 12\hat{j} + 18\hat{k}\right)\right|^2$

$\frac{9}{4} \times 4\left(7\hat{i} - 6\hat{j} + 9\hat{k}\right)^2$

$= 9\ (49 + 36 + 81)$

$= 9\ (166)$

$= 1494$

9. Let [t] denote the greatest integer $\le$ t. The number of points where the function

$$f(x) = [x]\left|x^2 - 1\right| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1], x \in (-2, 2)$$

is not continuous is ______.

Official Ans. by NTA (2)

Sol. $f(x) = [x]\ |x^2 - 1| + \sin\frac{\pi}{[x+3]} - [x+1]$

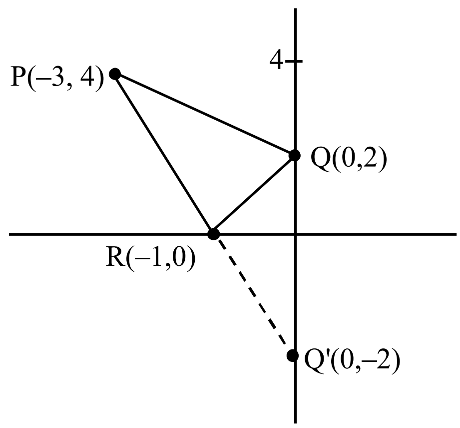
$$f(x) = \begin{cases} 3 - 2x^2, & -2 < x < -1 \\ x^2, & -1 \le x < 0 \\ \frac{\sqrt{3}}{2} + 1 & 0 \le x < 1 \\ x^2 + 1 + \frac{1}{\sqrt{2}}, & 1 \le x < 2 \end{cases}$$

discontinuous at x=0 , 1

10. A man starts walking from the point P(–3,4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2 + (RQ)^2\right)$ is equal to _____.

Official Ans. by NTA (1250)

Sol.



$50(PR^2 + RQ^2)$

$50\ (20 + 5)$

$50(25)$

$= 1250$