JEE Main 2019 Question Paper with Answer

January & Other (All Shifts)

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

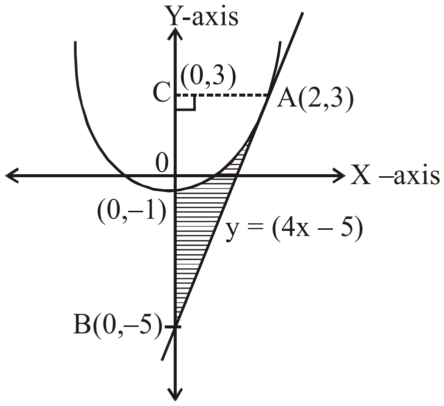
MATHEMATICS

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$

Ans. **(3)**

Sol.



Equation of tangent at (2,3) on $y = x^2 - 1$, is $y = (4x - 5)$ ....(i)

$\therefore$ Required shaded area

$$= \text{ar}(\Delta ABC) - \int_{-1}^{3} \sqrt{y+1}\, dy$$

$$= \frac{1}{2}.(8).(2) - \frac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$$

$$= 8 - \frac{16}{3} = \frac{8}{3} \text{ (square units)}$$

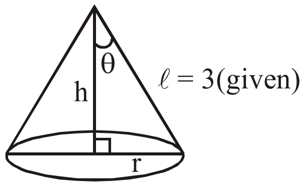
2. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

(1) $3\sqrt{3}\ \pi$ (2) $6\ \pi$

(3) $2\sqrt{3}\ \pi$ (4) $\frac{4}{3}\ \pi$

Ans. **(3)**

Sol.



$\therefore$ $h = 3\cos\theta$

$r = 3\sin\theta$

Now,

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}(9\sin^2\theta).(3\cos\theta)$$

$$\therefore \frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$$

Also, $\left.\frac{d^2V}{d\theta^2}\right]_{\sin\theta = \sqrt{\frac{2}{3}}}$ = negative

$\Rightarrow$ Volume is maximum,

when $\sin\theta = \sqrt{\frac{2}{3}}$

$$\therefore V_{max}\left(\sin\theta = \sqrt{\frac{2}{3}}\right) = 2\sqrt{3}\pi \text{ (in cu. m)}$$

3. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x \sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}\, dx$$

is equal to :

(where c is a constant of integration)

(1) $\log_e \left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

(2) $\log_e \left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

(3) $\frac{1}{2}\log_e \left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

(4) $\frac{1}{2}\log_e \left|\sec(x^2-1)\right| + c$

Ans. **(1)**

Sol. Put $(x^2 - 1) = 1$

$\Rightarrow 2x\,dx = dt$

$$\therefore I = \frac{1}{2}\int \sqrt{\frac{1-\cos t}{1+\cos t}}\, dt$$

$$= \frac{1}{2}\int \tan\left(\frac{t}{2}\right) dt$$

$$= \ln\left|\sec\frac{t}{2}\right| + c$$

$$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(1) 512 (2) –512 (3) –256 (4) 256

Ans. **(3)**

Sol. We have

$(x + 1)^2 + 1 = 0$

$\Rightarrow (x + 1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 + i)(x + 1 - i) = 0$

$\therefore \quad x = \underbrace{-(1+i)}_{\alpha(\text{let})}, \underbrace{-(1-i)}_{\beta(\text{let})}$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7 \alpha + (\beta^2)^7 \beta$

$= -128(-i + 1 + i + 1)$

$= -256$

5. If $y = y(x)$ is the solution of the differential equation,

$x\dfrac{dy}{dx} + 2y = x^2$ satisfying

$y(1) = 1$, then $y\left(\dfrac{1}{2}\right)$ is equal to :

(1) $\dfrac{7}{64}$ (2) $\dfrac{13}{16}$ (3) $\dfrac{49}{16}$ (4) $\dfrac{1}{4}$

Ans. **(3)**

Sol. $\dfrac{dy}{dx} + \left(\dfrac{2}{x}\right)y = x$

$\Rightarrow$ I.F. $= x^2$

$\therefore \quad yx^2 = \dfrac{x^4}{4} + \dfrac{3}{4}$ (As, $y(1) = 1$)

$\therefore \quad y\left(x = \dfrac{1}{2}\right) = \dfrac{49}{16}$

6. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(1) $2\sqrt{3}\,y = 12x + 1$

(2) $2\sqrt{3}\,y = -x - 12$

(3) $\sqrt{3}\,y = x + 3$

(4) $\sqrt{3}\,y = 3x + 1$

Ans. **(3)**

Sol. Let equation of tangent to the parabola $y^2 = 4x$ is

$y = mx + \dfrac{1}{m}$,

$\Rightarrow m^2x - ym + 1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

$\Rightarrow \dfrac{|3m^2 + 1|}{\sqrt{m^4 + m^2}} = 3$

$m = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow$ tangent are $x + \sqrt{3}y + 3 = 0$

and $x - \sqrt{3}y + 3 = 0$

7. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 200 (2) 300 (3) 500 (4) 350

Ans. **(2)**

Sol. Required number of ways

= Total number of ways – When A and B are always included.

$= {}^5C_2 \cdot {}^7C_3 - {}^5C_1{}^5C_2 = 300$

8. Three circles of radii a, b, c($a < b < c$) touch each other externally. If they have x-axis as a common tangent, then :

(1) $\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$

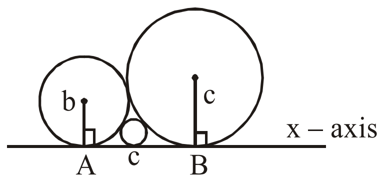
(2) a, b, c are in A. P.

(3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A. P.

(4) $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{c}}$

Ans. **(1)**

Sol.



AB = AC + CB

$\sqrt{(b+c)^2 - (b-c)^2}$

$= \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (a-c)^2}$

$\sqrt{bc} = \sqrt{ab} + \sqrt{ac}$

$\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{c}} + \dfrac{1}{\sqrt{b}}$

9. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(1) 14 (2) 6

(3) 4 (4) 8

Ans. (4)

Sol. $\frac{2^{403}}{15} = \frac{2^3.(2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$

$= \frac{8}{15}(15\lambda + 1) = 8\lambda + \frac{8}{15}$

$\because$ 8λ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

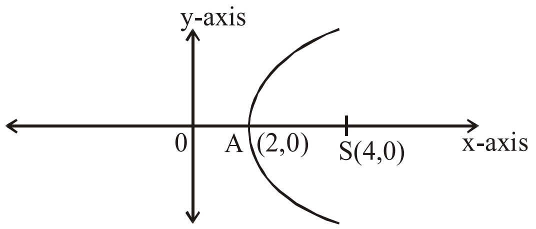
10. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(1) (4, –4) (2) (5, $2\sqrt{6}$)

(3) (8, 6) (4) $6, 4\sqrt{2}$

Ans. (3)

Sol.



equation of parabola is

$y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

11. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point :

(1) (–3, 0, –1) (2) (3, 3, –1)

(3) (3, 2, 1) (4) (–3, 1, 1)

Ans. (3)

Sol. Equation of plane

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$

dr's of normal of the plane are

$1 + 2\lambda,\ 1 + 3\lambda,\ 1 - \lambda$

Since plane is parallel to y - axis, $1 + 3\lambda = 0$

$\Rightarrow \quad \lambda = -1/3$

So the equation of plane is

$x + 4z - 7 = 0$

Point (3, 2, 1) satisfies this equation

Hence Answer is (3)

12. If a, b and c be three distinct real numbers in G. P. and a + b + c = xb, then x cannot be :

(1) 4 (2) –3 (3) –2 (4) 2

Ans. (4)

Sol. $\frac{b}{r}, b, br \rightarrow$ G.P. $\quad (|r| \neq 1)$

given a + b + c = xb

$\Rightarrow$ b/r + b + br = xb

$\Rightarrow$ b = 0 (not possible)

or $1 + r + \frac{1}{r} = x \quad \Rightarrow \quad x - 1 = r + \frac{1}{r}$

$\Rightarrow x - 1 > 2$ or $x - 1 < -2$

$\Rightarrow x > 3$ or $x < -1$

So x can't be '2'

13. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true ?

(1) The lines are all parallel.

(2) Each line passes through the origin.

(3) The lines are not concurrent

The lines are concurrent at the point

(4) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Ans. (4)

Sol. Given set of lines px + qy + r = 0

given condition 3p + 2q + 4r = 0

$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$

$\Rightarrow$ All lines pass through a fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

14. The system of linear equations.

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for a = 4

(2) is inconsistent when $|a| = \sqrt{3}$

(3) is inconsistent when a = 4

(4) has a unique solution for $|a| = \sqrt{3}$

Ans. (2)

Sol. $D=\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 2 & 3 & a^2-1\end{vmatrix}=a^2-3$

$D_1=\begin{vmatrix}2 & 1 & 1\\ 5 & 3 & 2\\ a+1 & 3 & a^2-1\end{vmatrix}=a^2-a+1$

$D_2=\begin{vmatrix}1 & 2 & 1\\ 2 & 5 & 2\\ 2 & a+1 & a^2-1\end{vmatrix}=a^2-3$

$D_3=\begin{vmatrix}1 & 1 & 2\\ 2 & 3 & 5\\ 2 & 3 & a+1\end{vmatrix}=a-4$

D = 0 at $|a| = \sqrt{3}$ but $D_3 = \pm\sqrt{3}-4\neq 0$

So the system is Inconsistant for $|a|=\sqrt{3}$

15. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to :-

(1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

Ans. (1)

Sol. $\vec{a}\times\vec{c}=-\vec{b}$

$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$\Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{a})\vec{c}-(\vec{c}.\vec{a})\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}$

$=3\hat{i}-5\hat{j}+2\hat{k}$

$\Rightarrow \vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$

$|\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$

$|\vec{c}|^2=\frac{19}{2}$

16. Let $a_1, a_2, \ldots\ldots\ldots, a_{30}$ be an A. P., $S=\sum_{i=1}^{30}a_i$ and $T=\sum_{i=1}^{15}a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :

(1) 57 (2) 47 (3) 42 (4) 52

Ans. (4)

Sol. $S = a_1 + a_2 + \ldots\ldots + a_{30}$

$S=\frac{30}{2}[a_1+a_{30}]$

$S = 15(a_1 + a_{30}) = 15(a_1 + a_1 + 29d)$

$T = a_1 + a_3 + \ldots\ldots + a_{29}$

$= (a_1) + (a_1 + 2d) \ldots\ldots + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + \ldots\ldots + 14)$

$T = 15a_1 + 210\ d$

Now use $S - 2T = 75$

$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210\ d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

17. 5 students of a class have an average height 150 cm and variance 18 cm². A new student, whose height is 156 cm, joined them. The variance (in cm²) of the height of these six students is:

(1) 22 (2) 20 (3) 16 (4) 18

Ans. (2)

Sol. Given $\bar{x}=\frac{\Sigma x_i}{5}=150$

$\Rightarrow \sum_{i=1}^{5}x_i=750$ .....(i)

$\frac{\Sigma x_i^2}{5}-(\bar{x})^2=18$

$\frac{\Sigma x_i^2}{5}-(150)^2=18$

$\Sigma x_i^2=112590$ ....(ii)

Given height of new student

$x_6 = 156$

Now, $\bar{x}_{new}=\frac{\sum_{i=1}^{6}x_i}{6}=\frac{750+156}{6}=151$

Also, New variance $=\frac{\sum_{i=1}^{6}x_i^2}{6}-(\bar{x}_{new})^2$

$=\frac{112590+(156)^2}{6}-(151)^2$

$=22821-22801 = 20$

18. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals :

(1) 52/169 (2) 25/169

(3) 49/169 (4) 24/169

Ans. (2)

Sol. Two cards are drawn successively with replacement

4 Aces 48 Non Aces

$$P(x=1)=\frac{{}^{4}C_1}{{}^{52}C_1}\times\frac{48C_1}{52C_1}+\frac{48C_1}{52C_1}\times\frac{4C_1}{52C_1}=\frac{24}{169}$$

$$P(x=2)=\frac{{}^{4}C_1}{{}^{52}C_1}\times\frac{{}^{4}C_1}{{}^{52}C_1}=\frac{1}{169}$$

$$P(x = 1) + P(x = 2) = \frac{25}{169}$$

19. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2{\circ}J{\circ}f_1)(x) = f_3(x)$ then J(x) is equal to :-

(1) $f_3(x)$ (2) $f_1(x)$

(3) $f_2(x)$ (4) $\frac{1}{x}f_3(x)$

Ans. (1)

Sol. Given $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x)=\frac{1}{1-x}$

$$(f_2 \circ J \circ f_1)(x) = f_3(x)$$

$$f_2 \circ (J(f_1(x))) = f_3(x)$$

$$f_2 \circ \left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$$

$$1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{-x}{1-x}=\frac{x}{x-1}$$

Now $x \to \frac{1}{x}$

$$J(x)=\frac{\frac{1}{x}}{\frac{1}{x}-1}=\frac{1}{1-x}=f_3(x)$$

20. Let

$$A=\left\{0\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$$

Then the sum of the elements in A is :

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$

(3) $\frac{3\pi}{4}$ (4) π

Ans. (2)

Sol. Given $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely img

so real part becomes zero.

$$z=\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)\times\left(\frac{1+2i\sin\theta}{1+2i\sin\theta}\right)$$

$$z=\frac{(3-4\sin^2\theta)+i(8\sin\theta)}{i+4\sin^2\theta}$$

Now Re(z) = 0

$$\frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = -\,-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$$

$$\because\ \theta \in \left(-\frac{\pi}{2},\pi\right)$$

then sum of the elements in A is

$$-\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3} = \frac{2\pi}{3}$$

21. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to :

(1) 4/9 (2) 7/17

(3) 8/17 (4) 8/15

Ans. (4)

Sol. Point of intersection is P(2,6).

Also, $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$

$$\therefore |\tan\theta| = \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \frac{8}{15}$$

22. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Ans. (1)

Sol. Here, $AA^T = I$

$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Also, $A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$

$\therefore A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

23. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(1) (2, 3] (2) (3, ∞)

(3) (3/2, 2] (4) (1, 3/2]

Ans. (2)

Sol. $e = \sqrt{1+\tan^2\theta} = \sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2}$

$\Rightarrow \theta \in (60°, 90°)$

Now, $\ell(L{\cdot}R) = \frac{2b^2}{a} = 2\frac{(1-\cos^2\theta)}{\cos\theta}$

$= 2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

ℓ (L.R) $\in (3, \infty)$.

24. The equation of the line passing through (–4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

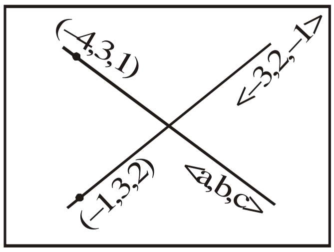
(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Ans. (2)

Sol.



Normal vector of plane containing two intersecting lines is parallel to vector.

$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$

$= -2\hat{i} + 6\hat{k}$

$\therefore$ Required line is parallel to vector

$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$

$\Rightarrow$ Required equation of line is

$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

25. For any $\theta \in \left(\frac{\pi}{4},\frac{\pi}{2}\right)$, the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(1) $13 - 4\cos^6\theta$

(2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

(3) $13 - 4\cos^2\theta + 6\cos^4\theta$

(4) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

Ans. (1)

Sol. We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3$

$= 13 - 4\cos^6\theta$

26. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ then x is equal to :

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

Ans. (1)

Sol. $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} \quad \left(x>\frac{3}{4}\right)$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}-\cos^{-1}\left(\frac{2}{3x}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \sin^{-1}\left(\frac{2}{3x}\right)$$

$$\cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\frac{2}{3x}\right)$$

$$\frac{3}{4x}=\frac{\sqrt{9x^2-4}}{3x}$$

$$\frac{81}{16}+4=9x^2$$

$$x^2 = \frac{145}{16\times 9} \Rightarrow x=\frac{\sqrt{145}}{12}$$

27. The value of $\int_0^{\pi}|\cos x|^3\,dx$

(1) 2/3 (2) 0

(3) –4/3 (4) 4/3

Ans. (4)

Sol. $\int_0^{\pi}|\cos x|^3\,dx = \int_0^{\pi/2}\cos^3 x\,dx - \int_{\pi/2}^{\pi}\cos^3 x\,dx$

$$= \int_0^{\pi/2}\left(\frac{\cos 3x+3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi}\left(\frac{\cos 3x+3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3}+3\sin x\right)_0^{\pi/2} - \left(\frac{\sin 3x}{3}+3\sin x\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3}+3\right)-(0+0)-\left\{(0+0)-\left(\frac{-1}{3}+3\right)\right\}\right]$$

$$= \frac{4}{3}$$

28. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge,\vee\}$, then the ordered pair $(\oplus, \odot)$ is:

(1) $(\wedge,\vee)$

(2) $(\vee,\vee)$

(3) $(\wedge,\wedge)$

(4) $(\vee,\wedge)$

Ans. (1)

Sol. $(p \oplus q) \wedge (\sim p \ \square \ q) \equiv p \wedge q$ (given)

p	q	$\sim p$	$p\wedge q$	$p\vee q$	$\sim p\vee q$	$\sim p\wedge q$	$(p\wedge q)\wedge(\sim p\vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

from truth table $(\oplus, \square) = (\wedge, \vee)$

29. $\lim\limits_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) exists and equals $\dfrac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\dfrac{1}{2\sqrt{2}}$

(4) exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

Ans. (1)

Sol. $\lim\limits_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$= \lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}=\frac{1}{4\sqrt{2}}$$

30. Let f : R → R be a function defined as :

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a+bx, & \text{if} \quad 1 < x < 3 \\ b+5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, f is :

(1) continuous if a = 5 and b = 5

(2) continuous if a = –5 and b = 10

(3) continuous if a = 0 and b = 5

(4) not continuous for any values of a and b

Ans. (4)

Sol. $f(x) = \begin{cases} 5 & \text{if} \quad x \le 1 \\ a+bx & \text{if} \quad 1 < x < 3 \\ b+5x & \text{if} \quad 3 \le x < 5 \\ 30 & \text{if} \quad x \ge 5 \end{cases}$

$f(1) = 5$, $f(1^-) = 5$, $f(1^+) = a + b$

$f(3^-) = a + 3b$, $f(3) = b + 15$, $f(3^+) = b + 15$

$f(5^-) = b + 25$; $f(5) = 30$ $f(5^+) = 30$

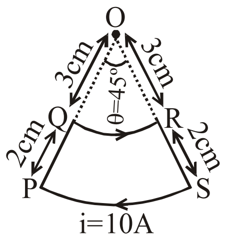
from above we concluded that f is not continuous for any values of a and b.

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS

1. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to :

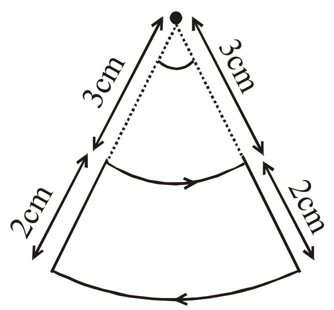


(1) 1.0×10^{-5} T (2) 1.5×10^{-5} T

(3) 1.0×10^{-7} T (4) 1.0×10^{-7} T

Ans. (1)

Sol.



$$\vec{B} = \frac{\mu_0 i}{4\pi}\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right]\hat{k}$$

$r_1 = 3$ cm $= 3\times10^{-2}$m

$r_2 = 5\text{cm} = 5\times10^{-2}$m

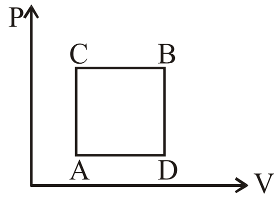
$\theta = \frac{\pi}{4}$, i = 10 A

$$\Rightarrow \vec{B} = \frac{4\pi\times10^{-7}}{16}\times10\left[\frac{1}{3\times10^{-2}} - \frac{1}{5\times10^{-2}}\right]\hat{k}$$

$$\Rightarrow |\vec{B}| = \frac{\pi}{3}\times10^{-5}\text{T}$$

$\approx 1\times10^{-5}$T

2. A gas can be taken from A to B via two different processes ACB and ADB.

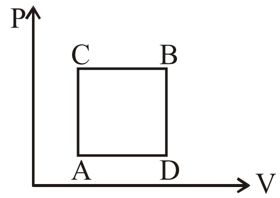


When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is :

(1) 80 J (2) 20 J (3) 100 J (4) 40 J

Ans. (4)

Sol.



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$\Rightarrow$ 60 J = 30 J + ΔU_{ACB}

$\Rightarrow \Delta U_{ACB} = 30$ J

$\Rightarrow \Delta U_{ADB} = \Delta U_{ACB} = 30$ J

$\Delta Q_{ACD} = \Delta U_{ACB} + \Delta W_{ADB}$

= 10 J + 30 J = 40 J

3. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}\,\text{V/m}$. The corresponding magnetic field $\vec{B}$, at that point will be:

(1) $18.9\times10^{-8}\hat{k}\text{T}$ (2) $6.3\times10^{-8}\hat{k}\text{T}$

(3) $2.1\times10^{-8}\hat{k}\text{T}$ (4) $18.9\times10^{8}\hat{k}\text{T}$

Ans. (3)

Sol. $|B| = \frac{|E|}{C} = \frac{6.3}{3\times10^8} = 2.1\times10^{-8}\text{T}$

and $\hat{E}\times\hat{B} = \hat{C}$

$\hat{j}\times\hat{B} = \hat{i}$

$\hat{B} = \hat{k}$

$\vec{B} = |B|\hat{B}$ = $2.1\times10^{8}\hat{k}\text{T}$

4. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 4 : 1 (2) 25 : 9

(3) 16 : 9 (4) 5 : 3

Ans. (2)

Sol. $\frac{I_{max}}{I_{min}} = 16$

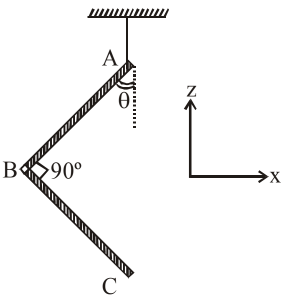
$$\Rightarrow \frac{A_{max}}{A_{min}} = 4$$

$$\Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$$

Using componendo & dividendo.

$$\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

5. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then :



(1) $\tan\theta = \frac{2}{\sqrt{3}}$

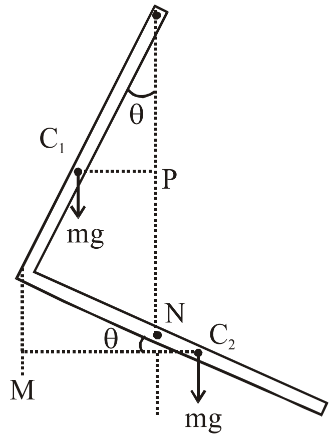
(2) $\tan\theta = \frac{1}{3}$

(3) $\tan\theta = \frac{1}{2}$

(4) $\tan\theta = \frac{1}{2\sqrt{3}}$

Ans. (2)

Sol.



Let mass of one rod is m.

Balancing torque about hinge point.

mg (C_1P) = mg (C_2N)

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$$

$$\Rightarrow \tan\theta = \frac{1}{3}$$

6. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}((\text{argon})}\right]$, is close to :

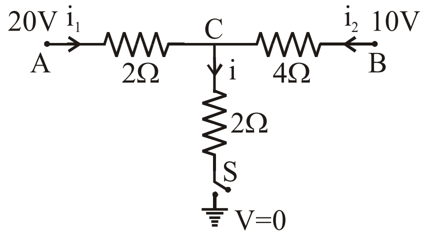
(1) 2.24 (2) 0.45

(3) 0.32 (4) 3.16

Ans. (4)

Sol. $\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}}$ = 3.16

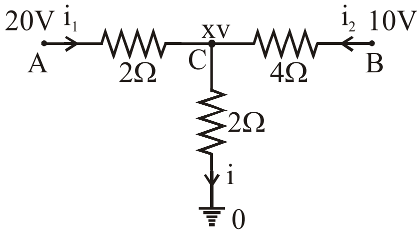
7. When the switch S, in the circuit shown, is closed, then the value of current *i* will be :



(1) 3 A (2) 5 A (3) 4 A (4) 2 A

Ans. (2)

Sol.



Let voltage at C = xv

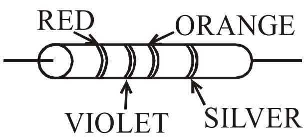
KCL : $i_1 + i_2 = i$

$$\frac{20-x}{2}+\frac{10-x}{4}=\frac{x-0}{2}$$

$\Rightarrow$ x = 10

and i = 5 amp.

8. A resistance is shown in the figure. Its value and tolerance are given respectively by:



(1) 27 KΩ, 20% (2) 270 KΩ, 5%

(3) 270 KΩ, 10% (4) 27 KΩ, 10%

Ans. (4)

Sol. Color code :

Red violet orange silver

R = 27 × 10^3 Ω ± 10%

= 27 KΩ ± 10%

9. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turas, and carrying a current of 5.2 A. The coercivitv of the bar magnet is :

(1) 1200 A/m (2) 2600 A/m

(3) 520 A/jm (4) 285 A/m

Ans. (2)

Sol. Coercivity = $H=\frac{B}{\mu_0}$

$$= ni = \frac{N}{\ell}i = \frac{100}{0.2}\times 5.2$$

= 2600 A/m

10. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on

each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is :

(1) $\frac{F}{2A\alpha\Delta T}$ (2) $\frac{F}{A\alpha(\Delta T-273)}$

(3) $\frac{F}{A\alpha\Delta T}$ (4) $\frac{2F}{A\alpha\Delta T}$

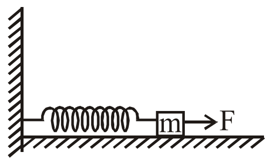
Ans. (3)

Sol. Young's modulus $y=\frac{\text{Stress}}{\text{Strain}}$

$$=\frac{F/A}{(\Delta\ell/\ell)}$$

$$=\frac{F}{A(\alpha\Delta T)}$$

11. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is :



(1) $\frac{\pi F}{\sqrt{mk}}$ (2) $\frac{2F}{\sqrt{mk}}$

(3) $\frac{F}{\sqrt{mk}}$ (4) $\frac{F}{\pi\sqrt{mk}}$

Ans. **(3)**

Sol. Maximum speed is at mean position (equilibrium). F = kx

$$x = \frac{F}{k}$$

$W_F + W_{sp} = \Delta KE$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

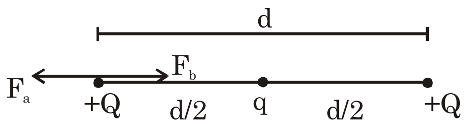
$$\Rightarrow v_{max} = \frac{F}{\sqrt{mk}}$$

12. Three charges +Q, q, + Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at x= 0, Ls zero, then value of q is :

(1) +Q/2 (2) –Q/2 (3) –Q/4 (4) +Q/4

Ans. **(3)**

Sol.



For equilibrium,

$$\vec{F}_a + \vec{F}_B = 0$$

$$\vec{F}_a = -\vec{F}_B$$

$$\frac{kQQ}{d^2} = -\frac{kQq}{(d/2)^2}$$

$$\Rightarrow q = -\frac{Q}{4}$$

13. A conducting circular loop made of a thill wire, has area 3.5×10^{-3} m² and resistance 10Ω . It is placed perpendicular to a time dependent magnetic field

B(t) = (0.4T)sin(50πt). The field is uniform in space. Then the net charge flowing through the loop during t = 0 s and t = 10 ms is close to:

(1) 14mC (2) 21 mC (3) 6 mC (4) 7 mC

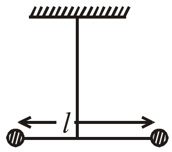
Ans. **(1)**

Sol. $Q = \frac{\Delta\phi}{R} = \frac{1}{10}A(B_f - B_i) = \frac{1}{10} \times 3.5 \times 10^{-3}\left(0.4\sin\frac{\pi}{2} - 0\right)$

$$= \frac{1}{10}(3.5 \times 10^{-3})(0.4 - 0)$$

$= 1.4 \times 10^{-4} = 0.14$ mC

14. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length *l*. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is τ=kθ for angular displacement 0. If the rod is rota ted by θ_0 and released, the tension in it when it passes through its mean position will be:



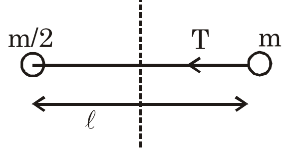
(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$

(3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

Ans. **(4)**

Sol. $\omega = \sqrt{\frac{k}{I}}$

$$\omega = \sqrt{\frac{3k}{m\ell^2}}$$



$\Omega = \omega\theta_0$ = average velocity

$T = m\Omega^2 r_1$

$$T = m\Omega^2 \frac{\ell}{3}$$

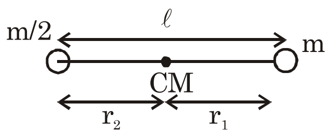
$$= m\omega^2\theta_0^2 \frac{\ell}{3}$$

$$= m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3}$$

$$= \frac{k\theta_0^2}{\ell}$$

$$I = \mu\ell^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}\ell^2$$

$$= \frac{m\ell^2}{3}$$



$$\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$$

15. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is:

(1) 2.5% (2) 0.5% (3) 1.0% (4) 2.0%

Ans. (3)

Sol. $R = \frac{\rho\ell}{A}$ and volume (V) = $A\ell$.

$$R = \frac{\rho\ell^2}{V}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 1\%$$

16. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure.

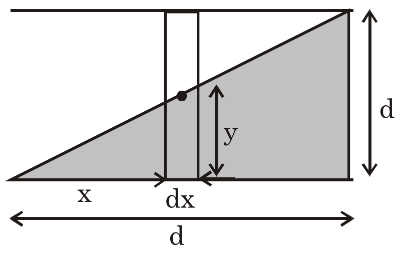
Capacitance of this capacitor is :

(1) $\frac{1}{2}\frac{k\in_0 a^2}{d}$ (2) $\frac{k\in_0 a^2}{d}\ln K$

(3) $\frac{k\in_0 a^2}{d(K-1)}\ln K$ (4) $\frac{k\in_0 a^2}{2d(K+1)}$

Ans. (3)

Sol.



$$\frac{y}{x} = \frac{d}{a}$$

$$y = \frac{d}{a}x$$

$$dy = \frac{d}{a}(dx)$$

$$\frac{1}{dc} = \frac{y}{KE \cdot adx} + \frac{(d-y)}{\in_0 adx}$$

$$\frac{1}{dc} = \frac{1}{\in_0 adx}\left(\frac{y}{k} + d - y\right)$$

$$\int dc = \int \frac{\in_0 adx}{\frac{y}{k} + d - y}$$

$$c = \in_0 a \cdot \frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)}$$

$$= \frac{\in_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$$

$$= \frac{k\in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$$

$$= \frac{k\in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k\in_0 a^2 \ell nk}{(k-1)d}$$

17. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19} m^{-3}$ and their mobility is $1.6\ m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor
contribution of holes is ignored) is close to:

(1) $2\Omega m$ (2) $0.4\Omega m$
(3) $4\Omega m$ (4) $0.2\Omega m$

Ans. **(2)**

Sol. $j = \sigma E = nev_d$

$$\sigma = ne\frac{v_d}{E}$$

$$= ne\mu$$

$$\frac{1}{\sigma} = \rho = \frac{1}{n_e e \mu_e}$$

$$= \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6}$$

$= 0.4\ \Omega m$

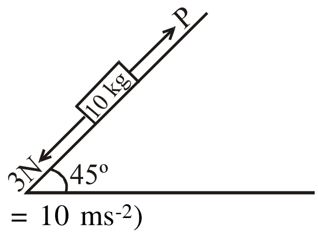
18. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is :

(1) $\frac{4L}{m}$ (2) $\frac{L}{m}$ (3) $\frac{L}{2m}$ (4) $\frac{2L}{m}$

Ans. **(3)**

Sol. $\frac{dA}{dt} = \frac{L}{2m}$

19. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward ?

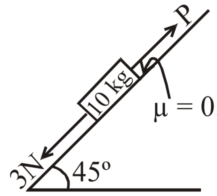


(take $g = 10\ ms^{-2}$)

(1) 32 N (2) 25 N (3) 23 N (4) 18 N

Ans. **(1)**

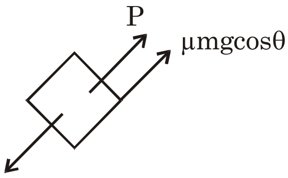
Sol.



$$mg\sin 45° = \frac{100}{\sqrt{2}} = 50\sqrt{2}$$

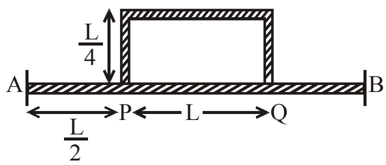
$$\mu mg\cos\theta = 0.6 \times mg \times \frac{1}{\sqrt{2}} = 0.6 \times 50\sqrt{2}$$

$$P = 31.28 \simeq 32N$$



73.7=3+mgsinθ

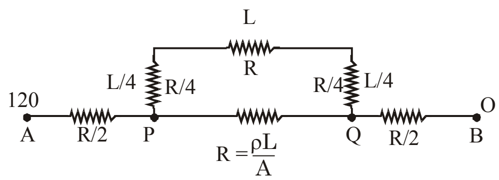
20. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to :



(1) 60ºC (2) 75ºC (3) 35ºC (4) 45ºC

Ans. **(4)**

Sol.



$$\frac{\Delta T}{R_{eq.}} = I = \frac{(120)5}{8R} = \frac{120 \times 5}{8R}$$

$$\Delta T_{PQ} = \frac{120 \times 5}{8R} \times \frac{3}{5} R = \frac{360}{8} = 45°C$$

21. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to :

(1) $\frac{g}{5}$ (2) $\frac{g}{20}$

(3) $\frac{g}{10}$ (4) $\frac{g}{30}$

Ans. **(1)**

Sol. $60=\sqrt{\frac{Mg}{\mu}}$

$$60.5=\sqrt{\frac{M\left(g^2+a^2\right)^{1/2}}{\mu}}\Rightarrow\frac{60.5}{60}=\sqrt{\sqrt{\frac{g^2+a^2}{g^2}}}$$

$$\left(1+\frac{0.5}{60}\right)^4=\frac{g^2+a^2}{g^2}=1+\frac{2}{60}$$

$$\Rightarrow g^2+a^2=g^2+g^2\times\frac{2}{60}$$

$$a=g\sqrt{\frac{2}{60}}=\frac{g}{\sqrt{30}}=\frac{g}{5.47}$$

$$\simeq\frac{g}{5}$$

22. A sample of radioactive material A, that has an activity of 10 mCi(l Ci = 3.7 × l0^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive maternal B which has an activity of 20 mCi. The correct choices for hall-lives of A and B would then be respectively :

(1) 20 days and 5 days (2) 20 days and 10 days (3) 5 days and 10 days (4) 10 days and 40 days

Ans. **(1)**

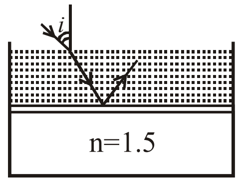
Sol. Activity A = λN

For A $10 = (2N_0)\lambda_A$

For B $20 = N_0\lambda_B$

$$\therefore \lambda_B=4\lambda_A\Rightarrow\left(T_{1/2}\right)_A=4\left(T_{1/2}\right)_B$$

23. Consider a tank made of glass(reiractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ,. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is :



(1) $\frac{3}{\sqrt{5}}$ (2) $\frac{5}{\sqrt{3}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $\frac{4}{3}$

Ans. **(1)**

Sol. $C < i_b$

here i_b is "brewester angle" and c is critical angle

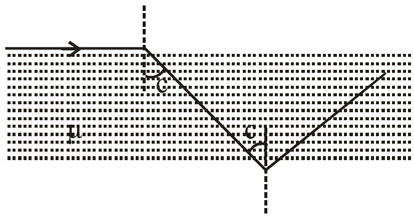
$\sin_c < \sin i_b$ since $\tan i_b=\mu_{0_{rel}}=\frac{1.5}{\mu}$

$\frac{1}{\mu}<\frac{1.5}{\sqrt{\mu^2+(1.5)^2}}$ $\therefore \sin i_b=\frac{1.5}{\sqrt{\mu^2+(1.5)^2}}$

$$\sqrt{\mu^2\times(1.5)^2}<1.5\times\mu$$

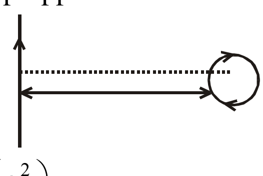
$$\mu^2 + (1.5)^2 < (\mu \times 1.5)^2$$

$$\mu<\frac{3}{\sqrt{5}}$$



slab μ = 1.5

24. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d»a). If the loop applies a force F on the wire then :

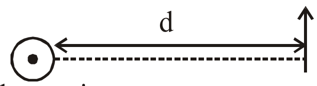


(1) $F\propto\left(\frac{a^2}{d^3}\right)$ (2) $F\propto\left(\frac{a}{d}\right)$

(3) $F\propto\left(\frac{a}{d}\right)^2$ (4) F = 0

Ans. (3)

Sol.



∞ long wire

Eqvilent dipole of given loop

$$F = m \cdot \frac{dB}{dr}$$

Now $\frac{dB}{dx} = \frac{d}{dx}\left(\frac{\mu_0 I}{2\pi x}\right)$

$$\propto \frac{1}{x^2}$$

$\Rightarrow$ So $F \propto \frac{M}{x^2}[\because M = NIA]$

$$\therefore F \propto \frac{a^2}{d^2}$$

25. Surface of certain metal is first illuminated with light of wavelength λ_1 =350 nm and then, by light of wavelength λ_2=54D nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to :

(Energjr of photon = $\frac{1240}{\lambda(\text{in nm})}$eV)

(1) 1.8 (2) 1.4 (3) 2.5 (4) 5.6

Ans. (1)

Sol. $\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v)^2$

$$\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\phi$$

$$\Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$

= 1.8 eV

26. A particle is moving with a velocity $\overline{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

(1) xy = constant (2) $y^2 = x^2$ + constant

(3) $y = x^2$ + constant (4) $y^2 = x$ + constant

Ans. (2)

Sol. $\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$

$\Rightarrow$ ydy = xdx

Integrating both side

$y^2 = x^2 + c$

27. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is :

(1) 0.55 cm away from the lens

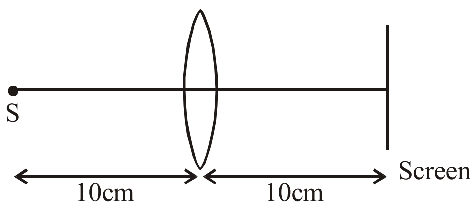
(2) 1.1 cm away from the lens

(3) 0.55 cm towards the lens

(4) 0

Ans. (1)

Sol.



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{10} - \frac{1}{-10} = \frac{1}{f} \Rightarrow f = 5cm$$

Shift due to slab = $t\left(1 - \frac{1}{\mu}\right)$ in the direction of incident ray

$$= 1.5\left(1 - \frac{2}{3}\right) = 0.5$$

again, $\frac{1}{v} - \frac{1}{-9.5} = \frac{1}{5}$

$$\Rightarrow \frac{1}{u} = \frac{1}{5} - \frac{2}{19} = \frac{9}{95}$$

$$\Rightarrow v = \frac{95}{9} = 10.55cm$$

28. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is :

(1) $\frac{R}{\sqrt{5}}$ (2) R (3) $\frac{R}{\sqrt{2}}$ (4) $R\sqrt{2}$

Ans. (3)

Sol. Electric field on axis of ring

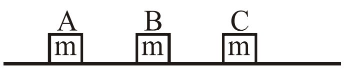
$$E = \frac{kQh}{\left(h^2 + R^2\right)^{3/2}}$$

for maximum electric field

$$\frac{dE}{dh} = 0$$

$$\Rightarrow h = \frac{R}{\sqrt{2}}$$

29. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an brutal speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m ?



(1) 4 (2) 5 (3) 3 (4) 2

Ans. (1)

Sol. $k_i = \frac{1}{2}mv_0^2$

From linear momentum conservation

$mv_0 = (2m + M)\ v_f$

$$\Rightarrow v_f = \frac{mv_0}{2m + M}$$

$$\frac{k_i}{k_f} = 6$$

$$\Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2} = 6$$

$$\Rightarrow \frac{2m + M}{m} = 6$$

$$\Rightarrow \frac{M}{m} = 4$$

30. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm2, is v. If the electron density in copper is 9×10^{28} /m^3 the value of v in mm/s is close to (Take charge of electron to be $=1.6 \times 10^{-19}$C)

(1) 0.2 (2) 3

(3) 2 (4) 0.02

Ans. (4)

Sol. $I = neAv_d$

$$\Rightarrow v_d = \frac{I}{neA} = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$$

= 0.02 m/s

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

CHEMISTRY

1. Which one of the following statements regarding Henry's law not correct ?

(1) The value of K_H increases with function of the nature of the gas

(2) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids.

(3) The partial of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

(4) Different gases have different K_H (Henry's law constant) values at the same temperature.

Ans. **(2)**

Sol. Liquid solution

$P_{gas} = K_H \times X_{gas}$

More is K_H less is solubility, lesser solubility is at higher temperature. So more is temperature more is K_H.

2. The correct decreasing order for acid strength is :-

(1) $NO_2CH_2COOH > NCCH_2COOH >$ $FCH_2COOH > ClCH_2COOH$

(2) $FCH_2COOH > NCCH_2COOH >$ $NO_2CHCOOH > ClCH_2COOH$

(3) $NO_2CH_2COOH > FCH_2COOH >$ $CNCH_2COOH > ClCH_2COOH$

(4) $CNCH_2COOH > O_2NCH_2COOH >$ $FCH_2COOH > ClCH_2COOH$

Ans. **(1)**

Sol. EWG increasea acidic strength

$NO_2CH_2COOH > NCCH_2COOH >$ $FCH_2COOH > ClCH_2COOH$

3. Two complexes $[Cr(H_2O_6)Cl_3]$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is :

(1) Δ_0 value of (A) is less than that of (B).

(2) Δ_0 value of (A) and (B) are calculated from the energies of violet and yellow light, respectively

(3) Bothe absorb energies corresponding to their complementary colors.

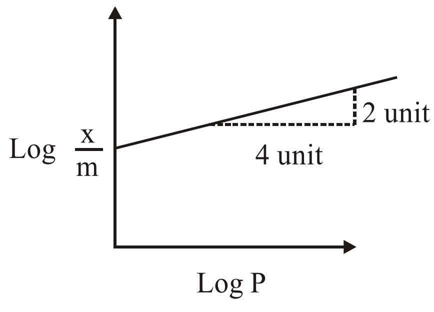
(4) Bothe are paramagnetic with three unpaired electrons.

Ans. **(2)**

Sol. Δ_0 order will be compared by spectro chemical series not by energies of violet & yellow light so Δ_0 order is

$[Cr(H_2O)_6]Cl_3 < [Cr(NH_3)_6]Cl_3$

4. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. $\frac{x}{m}$ is proportional to



(1) $P^{\frac{1}{4}}$ (2) P^2 (3) P (4) $P^{\frac{1}{2}}$

Ans. **(4)**

Sol. $\frac{x}{m} = K \times P^{1/n}$

$\log\frac{x}{m} = \log K + \frac{1}{n}\log P$

$m = \frac{1}{n} = \frac{2}{4} = \frac{1}{2} \Rightarrow n = 2$

So, $\frac{x}{m} = K \times P^{1/2}$

5. Correct statements among a to d regarding silicones are :

(a) They are polymers with hydrophobic character

(b) They are biocompatible.

(c) In general, they have high thermal stability and low dielectric strenth.

(d) Usually, they are resistant to oxidation and used as greases.

(1) (a), (b) and (c) only

(2) (a), and (b) only

(3) (a), (b), (c) and (d)

(4) (a), (b) and (d) only

Ans. **(3)**

Sol. These are properties and uses of silicones.

6. For emission line of atomic hydrogen from $n_i = 8$ to $n_f =$ the plot of wave number $(\bar{v})$ against $\left(\frac{1}{n^2}\right)$ will be (The Ry dberg constant, R_H is in wave number unit).

(1) Linear with slope - R_H

(2) Linear with intercept - R_H

(3) Non linear

(4) Linear with sslope R_H

Ans. **(4)**

Sol. $$\frac{1}{\lambda} = \bar{v} = R_H z^2\left(\frac{1}{\eta_1^2} - \frac{1}{\eta_2^2}\right)$$

$$\bar{v} = R_H \times \left(\frac{1}{\eta_1^2} - \frac{1}{8^2}\right)$$

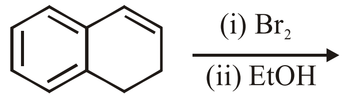
$$\bar{v} = R_H \times \frac{1}{\eta^2} - \frac{R_H}{8^2}$$

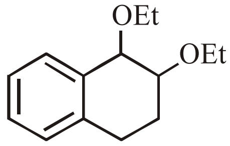
$$\bar{v} = R_H \times \frac{1}{\eta^2} - \frac{R_H}{64}$$

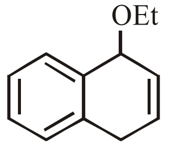
$m = R_H$

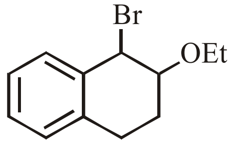
Linear with slope R_H

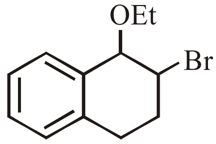
7. The major product the following reaction is :



(1) 

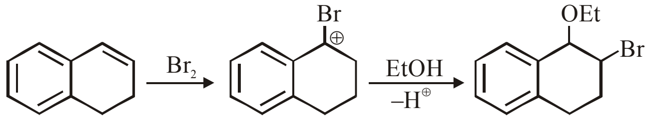
(2) 

(3) 

(4) 

Ans. **(4)**

Sol.



8. The alkaline earth metal nitrate that does not crystallise with water molecules, is :

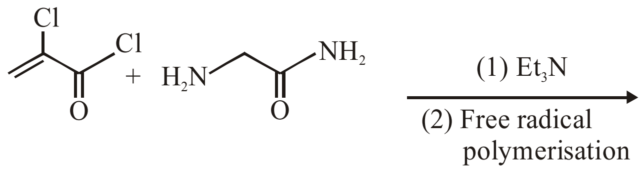
(1) $Sr(NO_3)_2$ (2) $Mg(NO_3)_2$

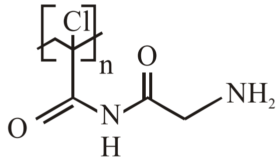
(3) $Ca(NO_3)_2$ (4) $Ba(NO_3)_2$

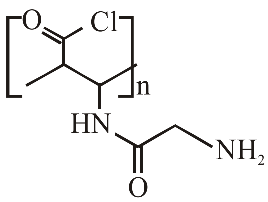
Ans. **(4)**

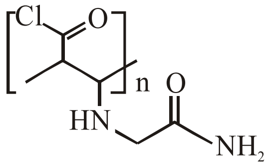
Sol. Smaller in size of center atoms more water molecules will crystallize hence $Ba(NO_3)_2$ is answer due to its largest size of '+ve' ion.

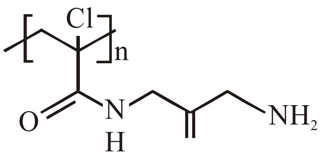
9. Major product of the following reaction is :



(1) 

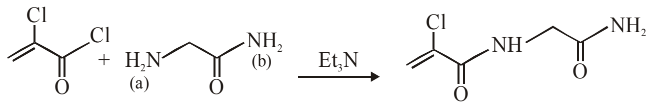
(2) 

(3) 

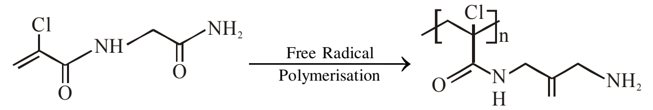
(4) 

Ans. **(4)**

Sol.



NH_2(a) will wact as nucleophile as (b) is having delocalised lonepair.



10. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexs is :

(1) 5.92 (2) 3.87 (3) 6.93 (4) 4.90

Ans. **(1)**

Sol. $\mu = \sqrt{n(n+2)}$ B.M.

n = Number of unpaired electrons

n = Maximum number of unpaired electron = 5

Ex : Mn^{2+} complex.

11. 20 mL of 0.1 MH_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resulatant mixture is : [pk_b of NH_4OH = 4.7].

(1) 9.4 (2) 5.0 (3) 9.0 (4) 5.2

Ans. **(3)**

Sol. 20 ml 0.1 M H_2SO_4 $\Rightarrow$ $\eta_{H^+} = 4$

30 ml 0.2 M NH_4OH $\Rightarrow$ $\eta_{NH_4OH} = 6$

NH_4OH	+ H^+ $\rightleftharpoons$	$NH_4^{\oplus}$	+ H_2O
$\Rightarrow$ 6	4	0	0
$\Rightarrow$ 2	0	4	4

Solution is basic buffer

$$pOH = pK_b + \log \frac{NH_4^+}{NH_4OH}$$

$$= 4.7 + \log 2$$

$$= 4.7 + 0.3 = 5$$

pH = 14 – 5 = 9

12. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a a container of volume 10 m^3 at 1000 K. given R is the gas constant in JK^{-1} mol^{-1}m, x is :

(1) $\frac{2R}{4+12}$ (2) $\frac{2R}{4-R}$ (3) $\frac{4-R}{2R}$ (4) $\frac{4+R}{2R}$

Ans. **(3)**

Sol. $n_T = (0.5 + x)$

PV = n × R × T

200 × 10 = (0.5 + x) × R × 1000

2 = (0.5 + x) R

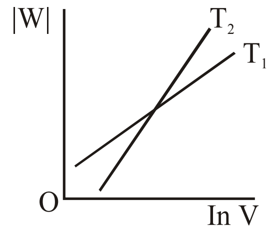
$$\frac{2}{R} = \frac{1}{2} + x$$

$$\frac{4}{R} - 1 = 2x$$

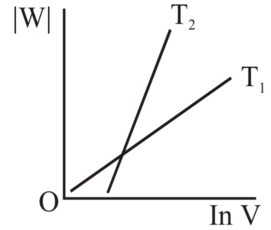
$$\boxed{\frac{4-R}{2R} = x}$$

13. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is:

(1)



(2)



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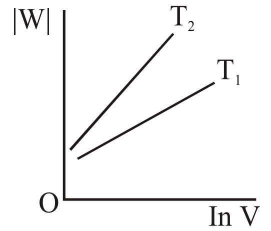

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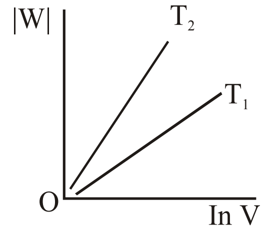


Ans. **(2)**

Sol. $w = -nRT\ln\frac{V_2}{V_1}$

$w = -nRT\ln\frac{V_b}{V_i}$

$|w| = nRT\ln\frac{V_b}{V_i}$

$|w| = nRT(\ln V_b - \ln V_i)$

$|w| = nRT\ln V_b - nRT\ln V_i$

$Y = mx - C$

So, slope of curve 2 is more than curve 1 and intercept of curve 2 is more negative then curve 1.

14. The major product of following reaction is :

$R-C\equiv N \xrightarrow[(2)H_2O]{(1)AlH(i-Bu_2)} ?$

(1) RCHO (2) RCOOH

(3) RCH_2NH_2 (4) $RCONH_2$

Ans. **(1)**

Sol. $R-C\equiv N \xrightarrow{AlH(i-Bu_2)} R-CH=N- \xrightarrow{H_2O} R-CH=O$

15. In general, the properties that decrease and increase down a group in the periodic table, respectively, are :

(1) electronegativity and electron gain enthalpy.

(2) electronegativity and atomic radius.

(3) atomic radius and electronegativity.

(4) electron gain enthalpy and electronegativity.

Ans. **(2)**

Sol. Electronegativity decreases as we go down the group and atomic radius increases as we go down the group.

16. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is:

(1) 16 (2) 8 (3) 4 (4) 12

Ans. **(4)**

Sol. $n_{Na^+} = \frac{92}{23} = 4$

So molality = 4

17. A water sample has ppm level concentration of the following metals: Fe= 0.2; Mn = 5.0; Cu = 3.0; Zn = 5.0. The metal that makes the water sample unsuitable drinking is :

(1) Zn (2) Fe (3) Mn (4) Cu

Ans. **(3)**

Sol. (i) Zn = 0.2 (ii) Fe = 0.2

(iii) Mn = 5.0 (iv) Cu = 3.0

18. The increasing order of pKa of the following amino acids in aqueous solution is :

Gly Asp Lys Arg

(1) Asp < Gly < Arg < Lys

(2) Arg < Lys < Gly < Asp

(3) Gly < Asp < Arg < Lys

(4) Asp < Gly < Lys < Arg

Ans. **(4)**

Sol. Order of acidic strength :

$HOOC-CH_2-CH(NH_2)-COOH$ > NH_2-CH_2-COOH >

Aspartic acid Glycine

$H_2N-C(=NH)-NH-CH_2CH_2CH_2-CH(NH_2)-C(=O)-OH$

Arginine

So, pK_a

Asp < Gly < Arg < Lys

19. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ?

(1) Both are unstable

(2) Li_2^+ is unstable and Li_2^- is stable

(3) Li_2^+ is stable and Li_2^- is unstable

(4) Both are stabel

Ans. **(4)**

Sol. Both Li_2^+ and Li_2^- has 0.5 bond order and hence both are stable.

20. The following results were obtained during kinetic studies of the reaction :
$2A + B \rightarrow$ Products

Experment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
(I)	0.10	0.20	6.93×10^{-3}
(II)	0.10	0.25	6.93×10^{-3}
(III)	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is :
(1) 10 (2) 5 (3) 100 (4) 1

Ans. **(2)**

Sol. $6.93 \times 10^{-3} = K \times (0.1)^x\ (0.2)^y$
$6.93 \times 10^{-3} = K \times (0.1)^x\ (0.25)^y$
So $y = 0$
and $1.386 \times 10^{-2} = K \times (0.2)^x\ (0.30)^y$

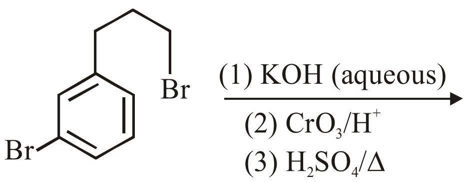
$$\frac{1}{2} = \left(\frac{1}{2}\right)^x \quad \boxed{x = 1}$$

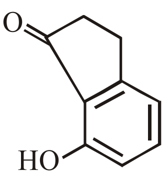
So $r = K \times (0.1) \times (0.2)^0$
$6.93 \times 10^{-3} = K \times 0.1 \times (0.2)^0$

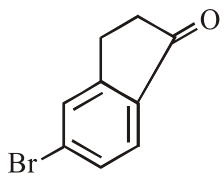
$$\boxed{K = 6.93 \times 10^{-2}}$$

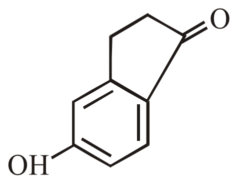
$$t_{1/2} = \frac{0.693}{2K} = \frac{0.693}{0.693 \times 10^{-1} \times 2} = \frac{10}{2} = 5$$

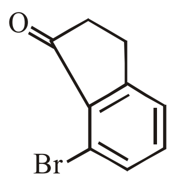
21. The major product of the following reaction is:



(1)


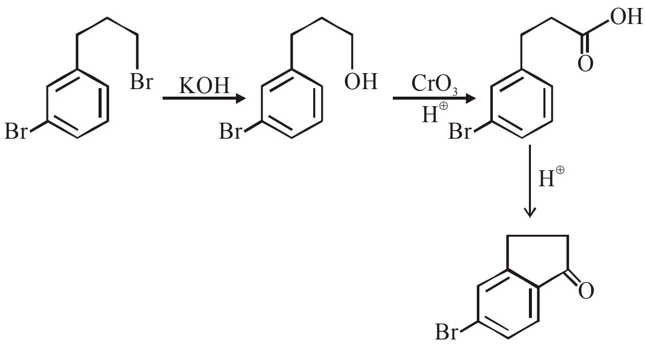
(2)


(3)


(4)


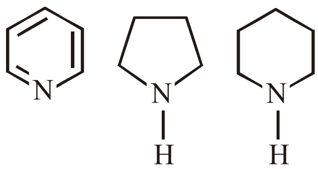
Ans. **(2)**

Sol.



During AES Br is o/p directing and major product will be formed on less hindrance p position :

22. Arrange the following amines in the decreasing order of basicity:

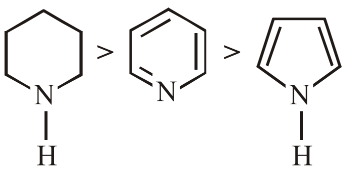


(I) (II) (III)

(1) I > II > III (2) III > II > I
(3) I > III > II (4) III > I > II

Ans. **(4)**

Sol. Order of basic strength :



sp^3N sp^2N delocalised ℓp of N

23. Which amongst the following is the strongest acid ?
(1) CHI_3 (2) $CHCI_3$
(3) $CHBr_3$ (4) $CH(CN)_3$

Ans. **(4)**

Sol. CN makes anino most stable so answer is $CH(CN)_3$

24. The anodic half-cell of lead-acid battery is recharged unsing electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process in : (Molar mass of $PbSO_4$ = 303 g mol^{-1})
(1) 22.8 (2) 15.2 (3) 7.6 (4) 11.4

Ans. **(2)**

Sol. (A) $PbSO_4(s) + 2OH^- \longrightarrow PbO_2 + H_2SO_4 + 2e^-$
0.05/2 mole 0.05F
(C) $PbSO_4 + 2e^- + 2H^+ \longrightarrow Pb(s) + H_2SO_4$
0.05/2 mole 0.05 F

$$n_T(PbSO_4) = 0.05 \text{ mole}$$

$$m_{PbSO_4} = 0.05 \times 303 = 15.2 \text{ gm}$$

25. The one that is extensively used as a piezoelectric material is :
(1) Quartz
(2) Amorphous silica
(3) Mica
(4) Tridymite

Ans. **(1)**

Sol. Quartz (Information)

26. Aluminium is usually found in +3 oxidation stagte. In contarast, thallium exists in +1 and +3 oxidation states. This is due to :
(1) lanthanoid contraction
(2) lattice effect
(3) diagonal relationship
(4) inert pair effect

Ans. **(4)**

Sol. Inert pair effect is promenent character of p-block element.

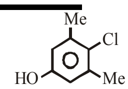
27. The correct match between Item -I and Item-II is :

Item – I (drug)		Item – II (test)	
(A)	Chloroxylenol	**(P)**	Carbylamine Test
(B)	Norethindrone	**(Q)**	Sodium Hydrogen carbonateTest
(C)	Sulphapyridine	**(R)**	Ferric chloride test
(D)	Penicillin	**(S)**	Bayer's test

(1) A→Q ; B→P ; C→S ; D→R
(2) A→R ; B→P ; C→S ; D→Q
(3) A→R ; B→S ; C→P ; D→Q
(4) A→Q ; B→S ; C→P ; D→R

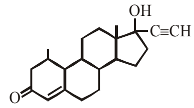
Ans. **(3)**

Sol. (A) Chloroxylenol



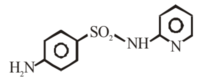
$FeCl_3$ test

(B) Norethindrone



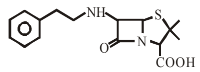
Bayer's test

(C) Sulphapyridine



Carbylamine test

(D) Penicllin



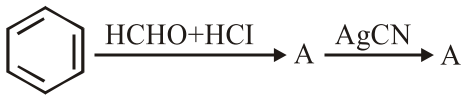
Sodium hydrogen carbonate test

28. The ore that contains both iron and copper is:
(1) malachite
(2) dolomite
(3) azurite
(4) copper pyrites

Ans. **(4)**

Sol. Copper pyrites : $CuFeS_2$
Malachite : $Cu(OH)_2 . CuCO_3$
Azurite $Cu(OH)_2 . 2CuCO_3$
Dolomite $CaCO_3 . MgCO_3$

29. The compounds A and B in the following reaction are, respectively:



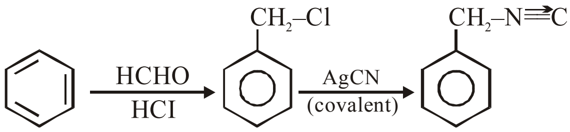
(1) A = Benzyl alcohol, B = Benzyl isocyanide

(2) A = Benzyl alcohol, B = Benzyl cyanide

(3) A = Benzyl chloride, B = Benzyl cyanide

(4) A = Benzyl chloride, B = Benzyl isocyanide

Ans. **(4)**

Sol. 

30. The isotopes of hydrogen are :

(1) Tritium and protium only

(2) Deuterium and tritium only

(3) Protium and deuterum only

(4) Protium, deuterium and tritium

Ans. **(4)**

Sol. Isotopes of hydrogen is :

Proteium Deuterium Tritium

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let f be a differentiable function from R to R such that $\left|f(x)-f(y)\right| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

Ans. **(4)**

Sol. $|f(x) - f(y)| \le 2|x - y|^{3/2}$

divide both sides by $|x - y|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2.|x-y|^{1/2}$$

apply limit $x \to y$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = c \Rightarrow f(x) = 1$

$$\int_0^1 1.dx = 1$$

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta = 1-\frac{1}{\sqrt{2}},(k>0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

Ans. **(1)**

Sol. $$\frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{\sec\theta}}d\theta = \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$$

$$= -\frac{1}{\sqrt{2k}}2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}}-1\right)$$

given it is $1-\frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(1) 12 (2) 15 (3) 10 (4) 14

Ans. **(2)**

Sol. $(1 - t^6)^3 (1 - t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

$\Rightarrow$ cofficient of t^4 in $(1 - t)^{-3}$ is

${}^{3+4-1}C_4 = {}^6C_2 = 15$

4. For each xεR, let [x] be the greatest integer less than or equal to x. Then

$\lim_{x\to0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0 (3) 1 (4) sin1

Ans. **(1)**

Sol. $$\lim_{x\to0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$$

$x \to 0^-$

$$[x]=-1 \Rightarrow \lim_{x\to0^-}\frac{x(-x-1)\sin(-1)}{-x} = -\sin 1$$

$|x| = -x$

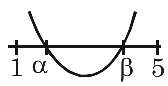
5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4) (3) (5,6) (4) (–5,–4)

Ans. **(Bonus/1)**

Sol. $x^2 - mx + 4 = 0$

$\alpha,\beta \in [1,5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$

$\Rightarrow m \in (-\infty,-4) \cup (4,\infty)$

(2) $f(1) \ge 0 \Rightarrow 5 - m \ge 0 \Rightarrow m \in (-\infty,5]$

(3) $f(5) \ge 0 \Rightarrow 29 - 5m \ge 0 \Rightarrow m \in \left(-\infty,\frac{29}{5}\right]$

(4) $1<\frac{-b}{2a}<5 \Rightarrow 1<\frac{m}{2}<5 \Rightarrow m \in (2,10)$

$\Rightarrow m \in (4,5)$

No option correct : Bonus

* If we consider $\alpha,\beta \in (1,5)$ then option (1) is correct.

6. If

$$A=\begin{bmatrix} e^{t} & e^{-t}\cos t & e^{-t}\sin t \\ e^{t} & -e^{-t}\cos t-e^{-t}\sin t & -e^{-t}\sin t+e^{-t}\cos t \\ e^{t} & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

Then A is-

(1) Invertible only if $t=\frac{\pi}{2}$

(2) not invertible for any tεR

(3) invertible for all tεR

(4) invertible only if t=π

Ans. (3)

Sol. $$|A|=e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t-\sin t & -\sin t+\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$

$= 5e^{-t} \neq 0\ \forall\ t \in R$

7. The area of the region

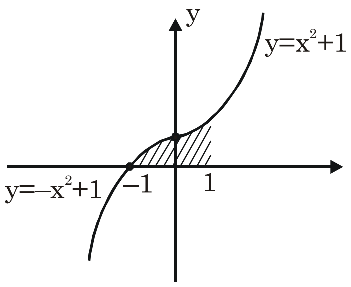
$A=\left[(x,y): 0 \le y \le x|x|+1 \text{ and } -1 \le x \le 1\right]$

in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Ans. (3)

Sol. The graph is a follows



$$\int_{-1}^{0}\left(-x^2+1\right)dx+\int_{0}^{1}\left(x^2+1\right)dx=2$$

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{6}$

Ans. (1)

Sol. $z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$\Rightarrow \arg z = \frac{\pi}{4}$

9. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

Ans. (4)

Sol. Projection of $\vec{b}$ on $\vec{a}=\frac{\vec{a}.\vec{b}}{|\vec{a}|}=|\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $(\vec{a}+\vec{b})\perp\vec{c}\Rightarrow(\vec{a}+\vec{b}).\vec{c}=0$

$\Rightarrow 5b_1 + b_2 = -10$ ...(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

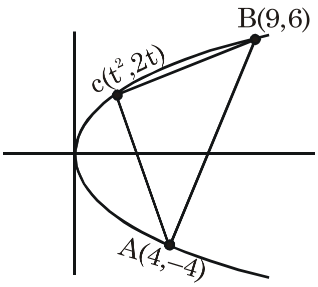
then $|\vec{b}|=\sqrt{b_1^2+b_2^2+2}=6$

10. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

Ans. (4)

Sol.



Area $= 5|t^2 - t - 6| = 5\left|\left(t-\frac{1}{2}\right)^2 - \frac{25}{4}\right|$

is maximum if $t = \frac{1}{2}$

11. The logical statement

$\left[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)\right]$ is equivalent to:

(1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$

(3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

Ans. (1)

Sol. $s\left[\sim(\sim p \vee q) \wedge (p \wedge r)\right] \cap (\sim q \wedge r)$

$\equiv \left[(p \wedge \sim q) \vee (p \wedge r)\right] \wedge (\sim q \wedge r)$

$\equiv \left[p \wedge (\sim q \vee r)\right] \wedge (\sim q \wedge r)$

$\equiv p \wedge (\sim q \wedge r)$

$\equiv (p \wedge r) \sim q$

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

Ans. (2)

Sol. E_1 : Event of drawing a Red ball and placing a green ball in the bag

E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$$

$$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$$

13. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which sin x-sin2x+sin3x = 0, is

(1) 2 (2) 1

(3) 3 (4) 4

Ans. (1)

Sol. sinx – sin2x + sin3x = 0

$\Rightarrow$ (sinx + sin3x) – sin2x = 0

$\Rightarrow$ 2sinx. cosx – sin2x = 0

$\Rightarrow$ sin2x(2 cosx – 1) = 0

$\Rightarrow$ sin2x = 0 or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{3}$

14. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:

(1) x + 2y – 2z = 0 (2) x – 2y + z = 0

(3) 5x + 2y – 4z = 0 (4) 3x + 2y – 3z = 0

Ans. (2)

Sol. Vector along the normal to the plane containing the lines

$$\frac{x}{3}=\frac{y}{4}=\frac{z}{2} \text{ and } \frac{x}{4}=\frac{y}{2}=\frac{z}{3}$$

is $\left(8\hat{i}-\hat{j}-10\hat{k}\right)$

vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is $26\hat{i}-52\hat{j}+26\hat{k}$

so, required plane is

$26x - 52y + 26z = 0$

$x - 2y + z = 0$

15. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

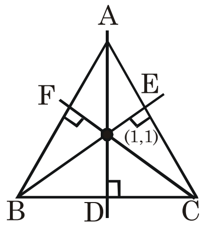
(1) $122y - 26x - 1675 = 0$

(2) $26x + 61y + 1675 = 0$

(3) $122y + 26x + 1675 = 0$

(4) $26x - 122y - 1675 = 0$

Ans. (4)



Sol. Equation of AB is

$3x - 2y + 6 = 0$

equation of AC is

$4x + 5y - 20 = 0$

Equation of BE is

$2x + 3y - 5 = 0$

Equation of CF is $5x - 4y - 1 = 0$

$\Rightarrow$ Equation of BC is $26x - 122y = 1675$

16. If $x = 3 \tan t$ and $y = 3 \sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is:

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$

Ans. (4)

Sol. $\frac{dx}{dt}=3\sec^2 t$

$\frac{dy}{dt}=3\sec t\tan t$

$\frac{dy}{dx}=\frac{\tan t}{\sec t}=\sin t$

$\frac{d^2y}{dx^2}=\cos t\frac{dt}{dx}$

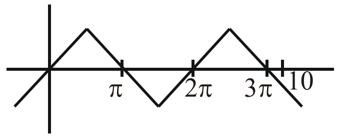
$=\frac{\cos t}{3\sec^2 t}=\frac{\cos^3 t}{3}=\frac{1}{3.2\sqrt{2}}=\frac{1}{6\sqrt{2}}$

17. If $x = \sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then $y–x$ is equal to:

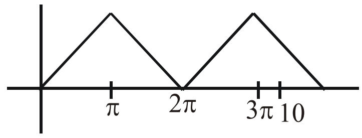
(1) π (2) 7π (3) 0 (4) 10

Ans. (1)

Sol.



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

18. If the lines $x = ay+b$, $z = cy + d$ and $x=a'z + b'$, $y = c'z + d'$ are perpendicular, then:

(1) $cc' + a + a' = 0$

(2) $aa' + c + c' = 0$

(3) $ab' + bc' + 1 = 0$

(4) $bb' + cc' + 1 = 0$

Ans. (2)

Sol. Line $x = ay + b$, $z = cy + d \Rightarrow \frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

Line $x = a'z + b'$, $y = c'z + d'$

$$\Rightarrow \frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2–11x+\alpha = 0$ are rational numbers is :

(1) 2 (2) 5 (3) 3 (4) 4

Ans. (3)

Sol. $6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow$ D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I$ $\Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

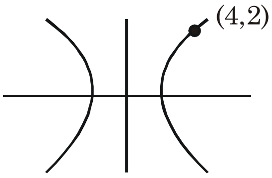
$\alpha = 5 \Rightarrow \lambda \in I$

20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) 2

Ans. (1)

Sol.



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$2a = 4 \qquad a = 2$

$$\frac{x^2}{4}-\frac{y^2}{b^2}=1$$

Passes through (4,2)

$$4-\frac{4}{b^2}=1 \Rightarrow b^2=\frac{4}{3} \Rightarrow e=\frac{2}{\sqrt{3}}$$

21. Let A={xεR:x is not a positive integer}

Define a function f :A→R as $f(x)=\frac{2x}{x-1}$ then f is

(1) injective but not surjective
(2) not injective
(3) surjective but not injective
(4) neither injective nor surjective

Ans. (1)

Sol. $f(x)=2\left(1+\frac{1}{x-1}\right)$

$$f'(x)=-\frac{2}{(x-1)^2}$$

$\Rightarrow f$ is one-one but not onto

22. If $f(x)=\int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx,(x\geq 0)$ and $f(0)=0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (4)

Sol. $\int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx$

$$=\int\frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx=\frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}}+C$$

As $f(0) = 0$, $f(x)=\frac{x^7}{2x^7+x^2+1}$

$$f(1)=\frac{1}{4}$$

23. If the circles $x^2 + y^2 - 16x-20y + 164 = r^2$ and $(x-4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then:

(1) $0 < r < 1$ (2) $1 < r < 11$
(3) $r > 11$ (4) $r = 11$

Ans. (2)

Sol. $x^2 + y^2 - 16x - 20y + 164 = r^2$
A(8,10), $R_1 = r$
$(x - 4)^2 + (y - 7)^2 = 36$
B(4,7), $R_2 = 6$
$|R_1 - R_2| < AB < R_1 + R_2$
$\Rightarrow 1 < r < 11$

24. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:

(1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

Sol. Let A(α,0) and B(0,β)
be the vectors of the given triangle AOB
$\Rightarrow |\alpha\beta| = 100$
$\Rightarrow$ Number of triangles
= 4 × (number of divisors of 100)
= 4 × 9 = 36

25. The sum of the follwing series

$$1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}+\frac{15\left(1^2+2^2+....+5^2\right)}{11}+.... \text{ up to 15 terms, is:}$$

(1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)

Sol. $T_n = \dfrac{\left(3+(n-1)\times 3\right)\left(1^2+2^2+....+n^2\right)}{(2n+1)}$

$$T_n = \dfrac{3.\dfrac{n(n+1)(2n+1)}{6}}{2n+1} = \dfrac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}\left(n^3+n^2\right) = \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times16\times31}{6}\right]$$

$= 7820$

26. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\dfrac{a}{c}$ is equal to:

(1) $\dfrac{1}{2}$ (2) 4

(3) 2 (4) $\dfrac{7}{13}$

Ans. (2)

Sol. a = A + 6d

b = A + 10d

c = A + 12d

a,b,c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)\ (a + 12d)$

$\Rightarrow \dfrac{A}{d} = -14$

$$\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{6+\frac{A}{d}}{12+\frac{A}{d}} = \frac{6-14}{12-14} = 4$$

27. If the system of linear equations

x–4y+7z = g

3y – 5z = h

–2x + 5y – 9z = k

is consistent, then :

(1) g + h + k = 0

(2) 2g + h + k = 0

(3) g + h + 2k = 0

(4) g + 2h + k = 0

Ans. (2)

Sol. $P_1 \equiv x - 4y + 7z - g = 0$

$P_2 \equiv 3x - 5y - h = 0$

$P_3 \equiv -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

28. Let f:[0,1]→R be such that $f(xy) = f(x).f(y)$ for all x,y,ε[0,1], and $f(0)\neq 0$. If y = y(x) satisfies the differential equation, $\dfrac{dy}{dx} = f(x)$ with y(0) = 1, then $y\left(\dfrac{1}{4}\right)+y\left(\dfrac{3}{4}\right)$ is equal to

(1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

Sol. $f(xy) = f(x).\ f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1$

$$\frac{dy}{dx} = f(x) = 1$$

$\Rightarrow y = x + c$

At, x = 0, y = 1 ⇒ c = 1

y = x + 1

$$\Rightarrow y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right) = \frac{1}{4}+1+\frac{3}{4}+1 = 3$$

29. A data consists of n observations:

x_1, x_2, ..., x_n. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Ans. (2)

Sol. $\sum(x_i+1)^2 = 9n$ ...(1)

$\sum(x_i-1)^2 = 5n$ ...(2)

(1) + (2) $\Rightarrow \sum(x_i^2+1) = 7n$

$\Rightarrow \frac{\sum x_i^2}{n} = 6$

(1) - (2) $\Rightarrow 4\Sigma x_i = 4n$

$\Rightarrow \Sigma x_i = n$

$\Rightarrow \frac{\Sigma x_i}{n} = 1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ Standard diviation $= \sqrt{5}$

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)

Sol.

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1$

= 374

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 2 : 30 PM To 05 : 30 PM

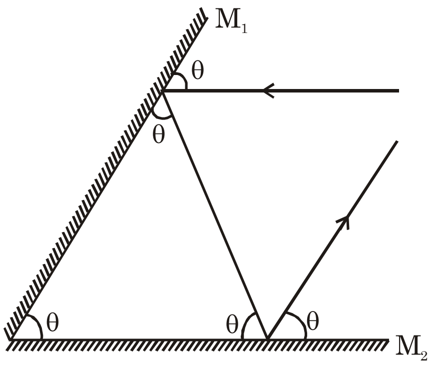
PHYSICS

1. Two plane mirrors arc inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be :

(1) 90° (2) 45° (3) 75° (4) 60°

Ans. **(4)**

Sol.



Assuming angles between two mirrors be θ as per geometry,

sum of anlges of Δ

$3\theta = 180°$

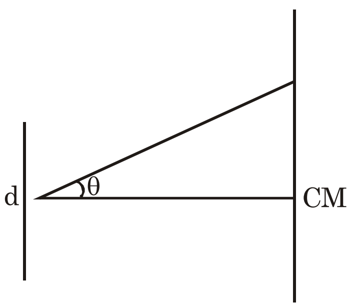
$\theta = 60°$

2. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is:

(1) 320 (2) 641 (3) 321 (4) 640

Ans. **(2)**

Sol.



Pam difference

$d\sin\theta = n\lambda$

where d = seperation of slits

λ = wave length

n = no. of maximas

$0.32 \times 10^{-3} \sin 30 = n \times 500 \times 10^{-9}$

n = 320

Hence total no. of maximas observed in angular range $-30° \le \theta \le 30°$ is

maximas = 320 + 1 + 320 = 641

3. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. [f the half-life of A is m_2, the half-life of B is :

(1) $\frac{ln2}{2}$ (2) $2ln2$ (3) $\frac{ln2}{4}$ (4) $4ln2$

Ans. **(3)**

Sol. Half life of A = $\ell n2$

$t_{1/2} = \frac{\ell n2}{\lambda}$

$\lambda_A = 1$

at t = 0 $R_A = R_B$

$N_A e^{-\lambda AT} = N_B e^{-\lambda BT}$

$N_A = N_B$ at t = 0

at t = t $\frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}}$

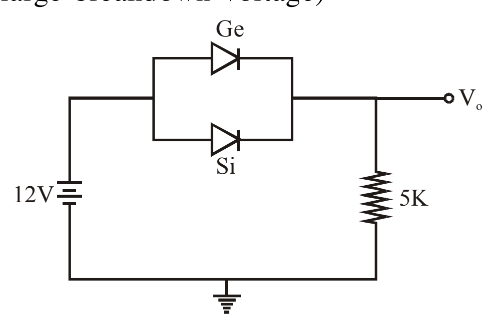
$e^{-(\lambda_B - \lambda_A)t} = e^{-t}$

$\lambda_B - \lambda_A = 3$

$\lambda_B = 3 + \lambda_A = 4$

$t_{1/2} = \frac{\ell n2}{\lambda_B} = \frac{\ell n2}{4}$

4. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by : (assume that the Ge diode has large breakdown voltage)



(1) 0.6 V (2) 0.8 V (3) 0.4 V (4) 0.2 V

Ans. **(3)**

Sol. Initially Ge & Si are both forward biased so current will effectivily pass through Ge diode with a drop of 0.3 V
if "Ge" is revesed then current will flow through "Si" diode hence an effective drop of (0.7 – 0.3) = 0.4 V is observed.

5. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :
(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

Ans. **(2)**

Sol. Frequency of torsonal oscillations is given by

$$f = \frac{k}{\sqrt{I}}$$

$$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$$

$$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$f_2 = 0.8\ f_1$

$$\frac{m}{M} = 0.375$$

6. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about :
[Take R = 8.3 J/ K mole]
(1) 10 kJ (2) 0.9 kJ (3) 6 kJ (4) 14 kJ

Ans. **(1)**

Sol. $Q = nC_v\Delta T$ as gas in closed vessel

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

Q = 10000 J = 10 kJ

7. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A

Ans. **(3)**

Sol. Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

x = $\pm\frac{A}{\sqrt{2}}$

∴ Correct answer is (3)

8. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to :
(1) 753 Hz (2) 500 Hz
(3) 333 Hz (4) 666 Hz

Ans. **(4)**

Sol. Frequency of the sound produced by flute,

$$f = 2\left(\frac{v}{2\ell}\right) = \frac{2 \times 330}{2 \times 0.5} = 660\text{Hz}$$

Velocity of observer, $v_0 = 10 \times \frac{5}{18} = \frac{25}{9}$ m/s

∴ frequency detected by observer, f' = $\left[\frac{v + v_0}{v}\right]f$

$$\therefore f' = \left[\frac{\frac{25}{9} + 330}{330}\right]660$$

= 335.56 × 2 = 671.12

∴ closest answer is (4)

9. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J-s)
(1) 3.75×10^6 (2) 4.87×10^5
(3) 3.86×10^6 (4) 6.25×10^5

Ans. **(4)**

Sol. $f = \dfrac{3\times10^{8}}{8\times10^{-7}} = \dfrac{30}{8}\times10^{14}\,\text{Hz}$

$= 3.75 \times 10^{14}$ Hz

1% of f = 0.0375×10^{14} Hz

$= 3.75 \times 10^{12}$ Hz $= 3.75 \times 10^{6}$ MHz

number of channels = $\dfrac{3.75\times10^{6}}{6}$ = 6.25×10^{5}

$\therefore$ correct answer is (4)

10. Two point charges $q_1(\sqrt{10}\,\mu C)$ and $q_2(-25\,\mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is,

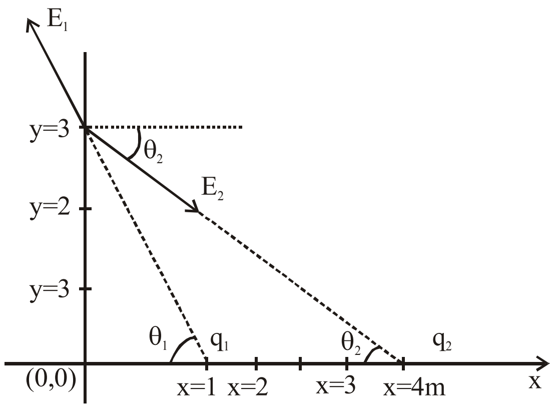
$\left[\text{take} \dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,\text{Nm}^2\text{C}^{-2}\right]$

(1) $(-63\hat{i}+27\hat{j})\times10^{2}$ (2) $(81\hat{i}-81\hat{j})\times10^{2}$

(3) $(63\hat{i}-27\hat{j})\times10^{2}$ (4) $(-81\hat{i}+81\hat{j})\times10^{2}$

Ans. **(3)**

Sol.



Let $\vec{E}_1$ & $\vec{E}_2$ are the vaues of electric field due to q_1 & q_2 respectively magnitude of

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}$$

$$E_2 = \frac{9\times10^{9}\times(25)\times10^{-6}}{(4^2+3^2)}\,\text{V/m}$$

$E_2 = 9 \times 10^{3}$ V/m

$$\therefore \vec{E}_2 = 9\times10^{3}\left(\cos\theta_2\hat{i}-\sin\theta_2\hat{j}\right)$$

$$\because \tan\theta_2 = \frac{3}{4}$$

$$\therefore \vec{E}_2 = 9\times10^{3}\left(\frac{4}{5}\hat{i}-\frac{3}{5}\hat{j}\right) = \left(72\hat{i}-54\hat{j}\right)\times10^{2}$$

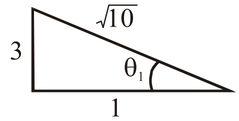
Magnitude of $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{\sqrt{10}\times10^{-6}}{(1^2+3^2)}$

$= (9\times10^{9})\times\sqrt{10}\times10^{-7}$

$= 9\sqrt{10}\times10^{2}$

$\therefore \vec{E}_1 = 9\sqrt{10}\times10^{2}\left[\cos\theta_1(-\hat{i})+\sin\theta_1\hat{j}\right]$

$\therefore \tan\theta_1 = 3$



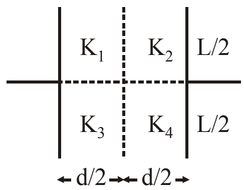
$$E_1 = 9\times\sqrt{10}\times10^{2}\left[\frac{1}{\sqrt{10}}(-\hat{i})+\frac{3}{\sqrt{10}}\hat{j}\right]$$

$$E_1 = 9\times10^{2}\left[-\hat{i}+3\hat{j}\right] = \left[-9\hat{i}+27\hat{j}\right]10^{2}$$

$$\therefore \vec{E} = \vec{E}_1+\vec{E}_2 = \left(63\hat{i}-27\hat{j}\right)\times10^{2}\,\text{V/m}$$

$\therefore$ correct answer is (3)

11. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :



(1) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

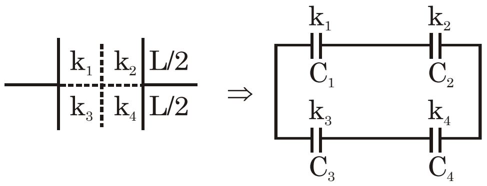
(2) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{(K_1+K_2+K_3+K_4)}$

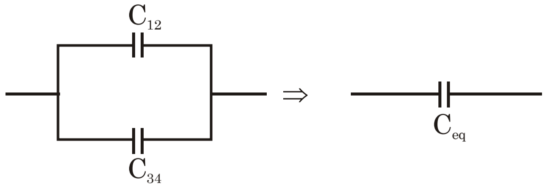
(3) $K = \dfrac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

(4) $K = \dfrac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

Ans. **(Bonus)**

Sol.





$$C_{12} = \frac{C_1C_2}{C_1+C_2} = \frac{\dfrac{k_1 \in_0 \frac{L}{2}\times L}{d/2}\cdot\dfrac{k_2\left[\in_0 \frac{L}{2}\times L\right]}{d/2}}{(k_1+k_2)\left[\dfrac{\in_0 \cdot \frac{L}{2}\times L}{d/2}\right]}$$

$$C_{12} = \frac{k_1k_2}{k_1+k_2}\frac{\in_0 L^2}{d}$$

in the same way we get, $C_{34} = \dfrac{k_3k_4}{k_3+k_4}\dfrac{\in_0 L^2}{d}$

$$\therefore\ C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2}+\frac{k_3k_4}{k_3+k_4}\right]\frac{\in_0 L^2}{d} \quad ..(i)$$

Now if $k_{eq} = k$, $C_{eq} = \dfrac{k\in_0 L^2}{d}$ .....(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so probably they have assumed the wrong combination

$$C_{13} = \frac{k_1 \in_0 L\frac{L}{2}}{d/2} + k_3 \in_0 \frac{L.\frac{L}{2}}{d/2}$$

$$= (k_1 + k_3)\frac{\in_0 L^2}{d}$$

$$C_{24} = (k_2 + k_4)\ \frac{\in_0 L^2}{d}$$

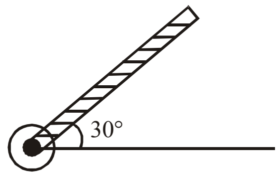
$$C_{eq} = \frac{C_{13}C_{24}}{C_{13}C_{24}} = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}\frac{\in_0 L^2}{d}$$

$$= \frac{k\in_0 L^2}{d}$$

$$k = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}$$

However this is one of the four options.
It must be a "Bonus" logically but of the given options probably they might go with (4)

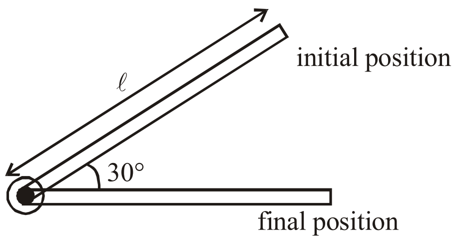
12. A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10ms^{-2}$)



(1) $\sqrt{30}$ (2) $\sqrt{\dfrac{30}{2}}$ (3) $\dfrac{\sqrt{30}}{2}$ (4) $\dfrac{\sqrt{20}}{3}$

Ans. **(2)**

Sol.



Work done by gravity from initial to final position is,

$$W = mg\frac{\ell}{2}\sin 30°$$

$$= \frac{mg\ell}{4}$$

According to work energy theorem

$$W = \frac{1}{2}I\omega^2$$

$$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{4}$$

$$\omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3\times 10}{2\times 0.5}}$$

$$\omega = \sqrt{30}\ \text{rad/sec}$$

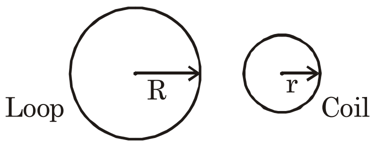
$\therefore$ correct answer is (1)

13. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. R $\frac{B_L}{B_C}$ will be :

(1) $\frac{1}{N}$ (2) N^2 (3) $\frac{1}{N^2}$ (4) N

Ans. **(3)**

Sol.



$L = 2\pi R$ $L = N \times 2\pi r$

$R = Nr$

$B_L = \frac{\mu_0 i}{2R}$ $B_C = \frac{\mu_0 Ni}{2r}$

$B_C = \frac{\mu_0 N^2 i}{2R}$

$$\frac{B_L}{B_C} = \frac{1}{N^2}$$

14. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is:

(1) 1.28×10^4 km (2) 6.4×10^3 km

(3) 3.2×10^3 km (4) 1.6×10^3 km

Ans. **(3)**

Sol. $U_{surface} + E_1 = U_h$

KE of satelite is zero at earth surface & at height h

$$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re + h)}$$

$$E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$$

$$E_1 = \frac{GM_e m}{(R_e + h)} \times \frac{h}{R_e}$$

Gravitational attraction $F_G = ma_C = \frac{mv^2}{(R_e + h)}$

$$E_2 \Rightarrow \frac{mv^2}{(R_e + h)} = \frac{GM_e m}{(R_e + h)^2}$$

$$mv^2 = \frac{GM_e m}{(R_e + h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e + h)}$$

$E_1 = E_2$

$$\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$$

15. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then :

(1) $U_E = \frac{U_B}{2}$ (2) $U_E < U_B$

(3) $U_E = U_B$ (4) $U_E > U_B$

Ans. **(3)**

Sol. Average energy density of magnetic field,

$u_B = \frac{B_0^2}{2\mu_0}$, B_0 is maximum value of magnetic field.

Average energy density of electric field,

$$u_E = \frac{\varepsilon_0 \in_0^2}{2}$$

now, $\in_0 = CB_0$, $C^2 = \frac{1}{\mu_0 \in_0}$

$$u_E = \frac{\in_0}{2} \times C^2 B_0^2$$

$= \frac{\epsilon_0}{2} \times \frac{1}{\mu_0 \epsilon_0} \times B_0^2 = \frac{B_0^2}{2\mu_0} = u_B$

$u_E = u_B$

since energy density of electric & magnetic field is same, energy associated with equal volume will be equal.

$u_E = u_B$

16. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is :

(1) 2.26×10^3 J (2) 3.39×10^3 J

(3) 5.65×10^2 J (4) 5.17×10^2 J

Ans. **(4)**

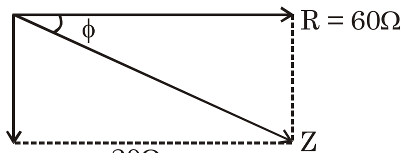
Sol. $R = 60\Omega$ $f = 50\text{Hz}, \omega = 2\pi f = 100\pi$

$x_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}}$

$x_C = 26.52\ \Omega$

$x_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$

$x_C - x_L = 20.24 \approx 20$



$x_C - x_L = 20\Omega$

$z = \sqrt{R^2 + (x_C - x_L)^2}$

$z = 20\sqrt{10}\Omega$

$\cos\phi = \frac{R}{z} = \frac{3}{\sqrt{10}}$

$P_{avg} = VI\cos\phi,\ I = \frac{v}{z}$

$= \frac{v^2}{z}\cos\phi$

$= 8.64$ watt

$Q = P.t = 8.64 \times 60 = 5.18 \times 10^2$

17. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :

(1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{hc^5}{G}}$

(3) $\sqrt{\frac{c^3}{Gh}}$ (4) $\sqrt{\frac{Gh}{c^5}}$

Ans. **(4)**

Sol. $F = \frac{GM^2}{R^2} \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$t \propto G^x h^y C^z$

$[T] = [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

on comparing the powers of M, L, T

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ....(i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ ...(ii)

on solving (i) & (ii) $x = y = \frac{1}{2}, z = -\frac{5}{2}$

$t \propto \sqrt{\frac{Gh}{C^5}}$

18. The magnetic field associated with a light wave is given, at the origin, by

$B = B_0[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons ?

($c = 3 \times 10^8 ms^{-1}$, $h = 6.6 \times 10^{-34}$ J-s)

(1) 7.72 eV (2) 8.52 eV

(3) 12.5 eV (4) 6.82 eV

Ans. **(1)**

Sol. $B = B_0\sin(\pi \times 10^7 C)t + B_0\sin(2\pi \times 10^7 C)t$

since there are two EM waves with different frequency, to get maximum kinetic energy we take the photon with higher frequency

$B_1 = B_0\sin(\pi \times 10^7 C)t$ $\nu_1 = \frac{10^7}{2} \times C$

$B_2 = B_0\sin(2\pi \times 10^7 C)t$ $\nu_2 = 10^7 C$

where C is speed of light $C = 3 \times 10^8$ m/s

$\nu_2 > \nu_1$

so KE of photoelectron will be maximum for photon of higher energy.

$\nu_2 = 10^7 C$ Hz

$h\nu = \phi + KE_{max}$

energy of photon

$E_{ph} = h\nu = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$

$E_{ph} = 6.6 \times 3 \times 10^{-19}$J

$= \frac{6.6 \times 3 \times 10^{-19}}{1.6 \times 10^{-19}} eV = 12.375 eV$

$KE_{max} = E_{ph} - \phi$

$= 12.375 - 4.7 = 7.675\ eV \approx 7.7\ eV$

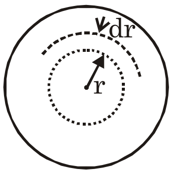
19. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

(1) $\frac{a}{2}\log\left(1 - \frac{Q}{2\pi aA}\right)$ (2) $a\log\left(1 - \frac{Q}{2\pi aA}\right)$

(3) $a\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$ (4) $\frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$

Ans. (4)

Sol.



$$Q = \int \rho dv$$

$$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= 4\pi A\int_0^R e^{-2r/a}dr$$

$$= 4\pi A\left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R$$

$$= 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a} - 1\right)$$

$$Q = 2\pi aA(1 - e^{-2R/a})$$

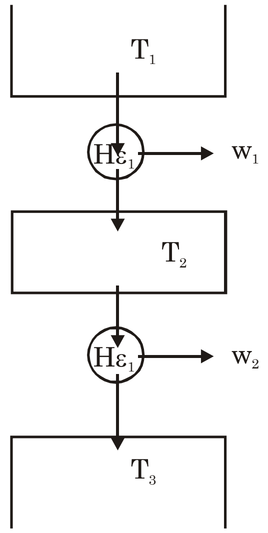
$$R = \frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$$

20. Two Carrnot engines A and B are operated in series. The first one, A, receives heat at T_1(= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3(= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal :

(1) 400 K (2) 600 K (3) 500 K (4) 300 K

Ans. (3)

Sol.



$w_1 = w_2$

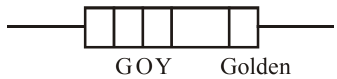
$\Delta u_1 = \Delta u_2$

$T_3 - T_2 = T_2 - T_1$

$2T_2 = T_1 + T_3$

$T_2 = 500$ K

21. A carbon resistance has a following colour code. What is the value of the resistance ?

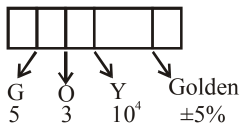


(1) 1.64 MΩ ± 5% (2) 530 kΩ ± 5%

(3) 64 kΩ ± 10% (4) 5.3 MΩ ± 5%

Ans. (2)

Sol.



$R = 53 \times 10^4 \pm 5\% = 530\ k\Omega \pm 5\%$

22. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

(1) 850 J (2) 900 J

(3) 950 J (4) 875 J

Ans. (2)

Sol. $x = 3t^2 + 5$

$$v = \frac{dx}{dt}$$

$v = 6t + 0$

at t = 0 v = 0

t = 5 sec v = 30 m/s

W.D. = ΔKE

$$W.D. = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900J$$

23. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is :

(1) $a\omega$ (2) $\sqrt{3}\,a\omega$

(3) $\sqrt{2}\,a\omega$ (4) $2a\omega$

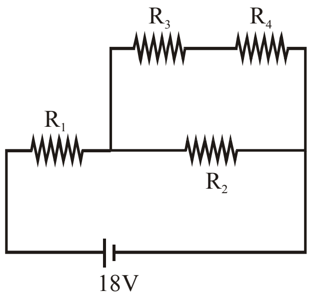
Ans. (3)

Sol. $v_x = -a\omega\sin\omega t \Rightarrow v_y = a\omega\cos\omega t$

$v_z = a\omega \quad \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

24. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5V, then the value R_2 will be :

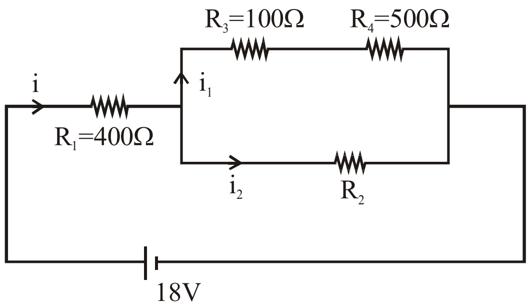


(1) 300 Ω (2) 230 Ω

(3) 450 Ω (4) 550 Ω

Ans. (1)

Sol.



$V_4 = 5V$

$i_1 = \frac{V_4}{R_4} = 0.01\ A$

$V_3 = i_1R_3 = 1V$

$V_3 + V_4 = 6V = V_2$

$V_1 + V_3 + V_4 = 18V$

$V_1 = 12\ V$

$i = \frac{V_1}{R_1} = 0.03 Amp.$

$i_2 = 0.02$ Amp $\qquad V_2 = 6V$

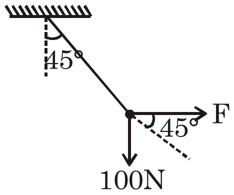
$R_2 = \frac{V_2}{i_2} = \frac{6}{0.02} = 300\Omega$

25. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$)

(1) 200 N (2) 100 N (3) 140 N (4) 70 N

Ans. (2)

Sol.



at equation

$\tan 45° = \frac{100}{F}$

F = 100 N

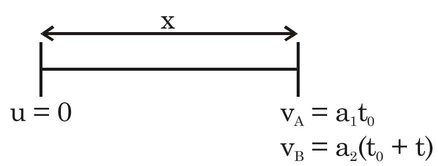
26. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to

(1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}\,t$

(3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}\,t$

Ans. (4)

Sol. For A & B let time taken by A is t_0



from ques.

$v_A - v_B = v = (a_1 - a_2)t_0 - a_2t \qquad ....(i)$

$x_B = x_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t)$

$\Rightarrow \left(\sqrt{a_2} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}t \qquad ....(ii)$

putting t_0 in equation

$$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1}-\sqrt{a_2}} - a_2 t$$

$$= \left(\sqrt{a_1}+\sqrt{a_2}\right)\sqrt{a_2}t - a_2 t \Rightarrow v = \sqrt{a_1 a_2}t$$

$$\Rightarrow \sqrt{a_1 a_2}t + a_2 t - a_2 t$$

27. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V bv the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

(1) 25 A (2) 50 A
(3) 35 A (4) 45 A

Ans. (4)

Sol. $\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P}$

$$\Rightarrow 0.9 = \frac{23 \times I_S}{230 \times 5}$$

$$\Rightarrow I_S = 45A$$

28. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :

(1) 9.6 m (2) 4.8 m (3) 2.9 m (4) 6.0 m

Ans. (2)

Sol. In flow volume = outflow volume

$$\Rightarrow \frac{0.74}{60} = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi}$$

$$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10}\left(\pi^2 = 10\right)$$

$$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24}$$

$$\Rightarrow h \approx 4.8m$$

29. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.
The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is :

(1) 5.755 m (2) 5.725 mm
(3) 5.740 m (4) 5.950 mm

Ans. (2)

Sol. $LC = \frac{\text{Pitch}}{\text{No. of division}}$

$LC = 0.5 \times 10^{-2}$ mm
+ve error = $3 \times 0.5 \times 10^{-2}$ mm
= 1.5×10^{-2} mm = 0.015 mm
Reading = MSR + CSR – (+ve error)
= 5.5 mm + $(48 \times 0.5 \times 10^{-2})$ – 0.015
= 5.5 + 0.24 – 0.015 = 5.725 mm

30. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron $=1.6 \times 10^{-19}$C)

(1) 2.0×10^{-24} kg
(2) 1.6×10^{-19} kg
(3) 1.6×10^{-27} kg
(4) 9.1×10^{-31} kg

Ans. (1)

Sol. $\frac{mv^2}{R} = qvB$

mv = qBR ....(i)
Path is straight line
it qE = qvB
E = vB ....(ii)
From equation (i) & (ii)

$$m = \frac{qB^2R}{E}$$

$m = 2.0 \times 10^{-24}$ kg

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 2 : 30 PM To 05 : 30 PM

CHEMISTRY

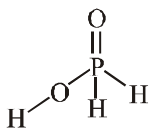
1. lood reducing nature of H_3PO_2 ttributed to the presence of:

(1) One P-OH bond (2) One P-H bond

(3) Two P-H bonds (4) Two P-OH bonds

Ans. (3)

Sol. H_3PO_2 is good reducing agent due to presence of two P–H bonds.



2. The complex thai has highest cry splitting energy (Δ), is :

(1) $K_3[Co(CN)_6]$

(2) $[Co(NH_3)_5(H_2O)]Cl_3$

(3) $K_2[CoCl_4]$

(4) $[Co(NH_3)_5Cl]Cl_2$

Ans. (1)

Sol. As complex $K_3[Co(CN)_6]$ have CN^- ligand which is strongfield ligand amongst the given ligands in other complexes.

3. The metal that forms nitride by reacting directly with N_2 of air, is :

(1) K (2) Cs (3) Li (4) Rb

Ans. (3)

Sol. Only Li react directly with N_2 out of alkali metals

$6Li + N_2 \rightarrow 2Li_3N$

4. In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic ?

(1) $N_2 \rightarrow N_2^+$ (2) $NO \rightarrow NO^+$

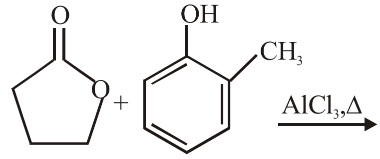
(3) $O_2 \rightarrow O_2^{2-}$ (4) $O_2 \rightarrow O_2^+$

Ans. (2)

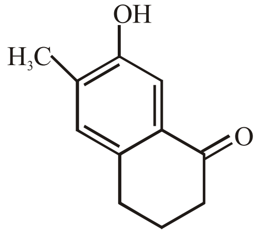
Sol.

Process	Change in magnetic nature	Bond Order Change
$N_2 \rightarrow N_2^+$	Dia $\rightarrow$ para	$3 \rightarrow 2.5$
$NO \rightarrow NO^+$	Para $\rightarrow$ Dia	$2.5 \rightarrow 3$
$O_2 \rightarrow O_2^{-2}$	Para $\rightarrow$ Dia	$2 \rightarrow 1$
$O_2 \rightarrow O_2^+$	Para $\rightarrow$ Para	$2 \rightarrow 2.5$

5. The major product of the following reaction is:



(1)



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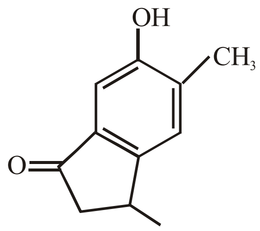

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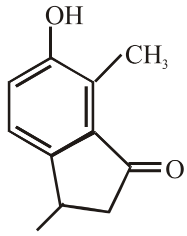

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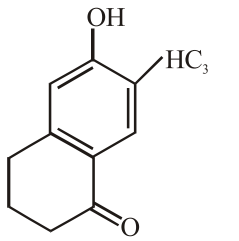
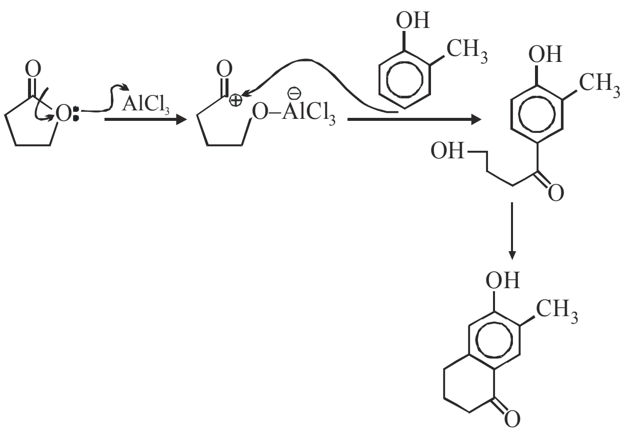


Ans. (4)

Sol.



6. The transition element that has lowest enthalpy of atomisation, is :

(1) Zn

(2) Cu

(3) V

(4) Fc

Ans. (2)

Sol. Since Zn is not a transition element so transition element having lowest atomisation energy out of Cu, V, Fe is Cu.

7. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals ?

(a) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.

(b) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number.

(c) According to wave mechanics, the ground state angular momentum is h equal to $\frac{h}{2\pi}$.

(d) The plot of ψ Vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value.

(1) (b), (c) (2) (a), (d) (3) (a), (b) (4) (a), (c)

Ans. (4)

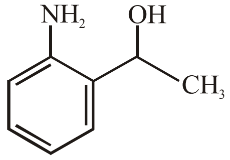
Sol. Refer Theory

8. The tests performed on compound X and their inferences are:

Test	Inference
(a) 2,4 - DNP test	Coloured precipitate
(b) Iodoform test	Yellow precipitate
(c) Azo-dye test	No dye formation

Compound 'X' is:

(1)



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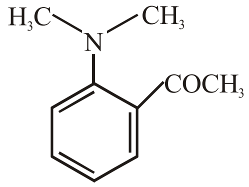

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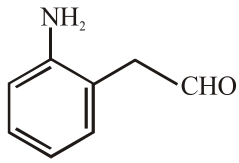
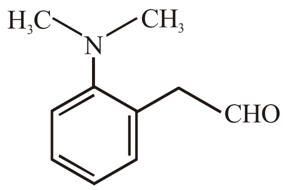


(4)

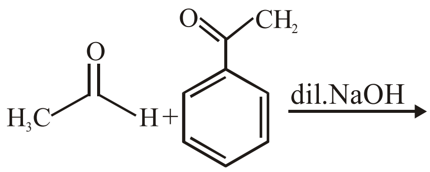


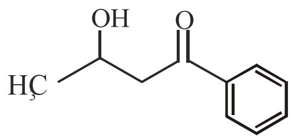
Ans. (2)

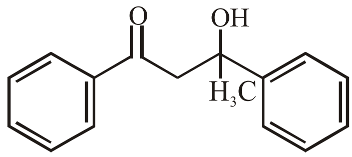
Sol. $\rightarrow$ 2,4 – DNP test is given by aldehyde on ketone

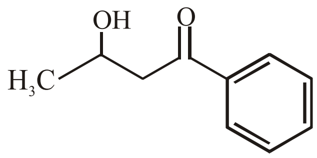
$\rightarrow$ Iodoform test is given by compound having $CH_3-\underset{\underset{O}{\|}}{C}-$ group.

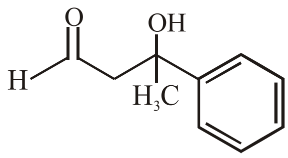
9. The major product formed in the following reaction is:



(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. Aldehyde reacts at a faster rate than keton during aldol and stericall less hindered anion will be a better nucleophile so sefl aldol at

$CH_3-\overset{\overset{\displaystyle O}{||}}{C}-H$ will be the major product.

10. For the reaction, 2A + B → products, when the concentrations of A and B both wrere doubled, the rate of the reaction increased from 0.3 mol $L^{-1}s^{-1}$ to 2.4 mol L^{-1} s^{-1}. When the concentration of A alone is doubled, the rate increased from 0.3 mol $L^{-1}s^{-1}$ to0.6 mol $L^{-1}s^{-1}$

Which one of the following statements is correct ?

(1) Order of the reaction with respect to Bis2

(2) Order of the reaction with respect to Ais2

(3) Total order of the reaction is 4

(4) Order of the reaction with respect to B is 1

Ans. **(1)**

Sol. $r = K[A]^x[B]^y$

$\Rightarrow 8 = 2^3 = 2^{x+y}$

$\Rightarrow x + y = 3$ ...(1)

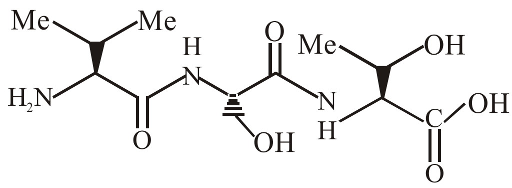
$\Rightarrow 2 = 2^x$

$\Rightarrow x = 1, y = 2$

Order w.r.t. A = 1

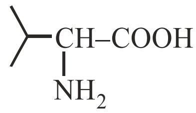
Order w.r.t. B = 2

11. The correct sequence of amino acids present in the tripeptide given below is :



(1) Leu - Ser - Thr (2) Thr - Ser- Leu
(3) Thr - Ser - Val (4) Val - Ser - Thr

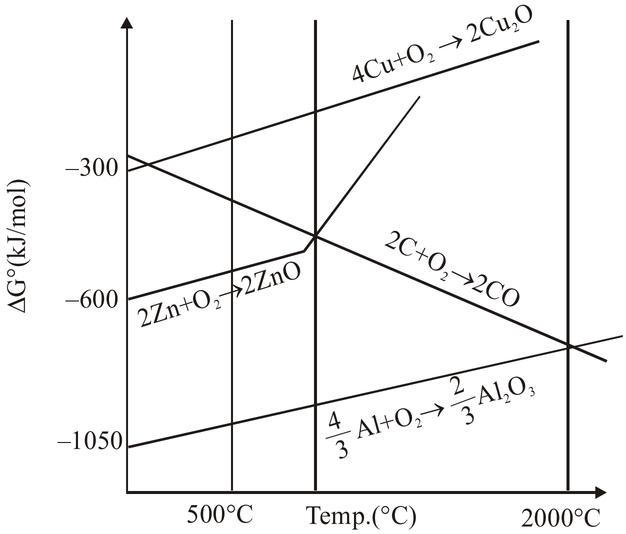
Ans. **(4)**

Sol. **Leusine** 

Serine $NO-CH_2-\underset{\underset{\displaystyle NH_2}{|}}{CH}-COOH$

Thrconine $H_3C-\underset{\underset{\displaystyle OH}{|}}{CH}-\underset{\underset{\displaystyle NH_2}{|}}{CH}-COOH$

12. The correct statement regarding the given Ellingham diagram is:



(1) At 800°C, Cu can be used for the extraction of Zn from ZnO

(2) At 500 C, coke can be used for the extraction of Zn from ZnO

(3) Coke cannot be used for the extraction of Cu from Ca_2O.

(4) At 1400°C, Al can be used for the extraction of Zn from ZnO

Ans. **(4)**

Sol. According to the given diagram Al can reduce ZnO.

$3ZnO + 2Al \rightarrow 3Zn + Al_2O_3$

13. For the following reaction, the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is :

$2C_{57}H_{110}O_6(s) + 163O_2(g) \rightarrow 114CO_2(g) + 110\ H_2OP(1)$

(1) 495 g (2) 490 g (3) 890 g (4) 445 g

Ans. **(1)**

Sol. moles of $C_{57}H_{110}O_6(s) = \frac{445}{890} = 0.5$ moles

$2C_{57}H_{110}O_6(s) + 163\ O_2(g) \rightarrow 114\ CO_2(g) + 110\ H_2O(l)$

$$n_{H_2O} = \frac{110}{4} = \frac{55}{2}$$

$$m_{H_2O} = \frac{55}{2} \times 18$$

$= 495$gm

14. The correct match between Item I and Item II is :

Item I	**Item II**
(A) Benzaldehyde	(P) Mobile phase
(B) Alumina	(Q) Adsorbent
(C) Acetonitrile	(R) Adsorbate

(1) (A) $\rightarrow$ (Q);(B) $\rightarrow$ (R);(C) $\rightarrow$ (P)

(2) (A) $\rightarrow$ (P); (B) $\rightarrow$ (R); (C) $\rightarrow$ (Q)

(3) (A) $\rightarrow$ (Q); (B) $\rightarrow$ (P); (C) $\rightarrow$ (R)

(4) (A) $\rightarrow$ (R); (B) $\rightarrow$ (Q); (C) $\rightarrow$ (P)

Ans. **(4)**

Sol.

15. The increasing basicity order of the following compounds is :

(A) $CH_3CH_2NH_2$

(B) $CH_3CH_2N(H)CH_2CH_3$

(C) $H_3C{-}N(CH_3){-}CH_3$

(D) $Ph{-}N(CH_3){-}H$

(1) (D)<(C)<(A)<(B) (2) (A)<(B)<(D)<(C)

(3) (A)<(B)<(C)< (D) (4) (D)<(C)<(B)<(A)

Ans. **(1)**

Sol.

$Ph-N(CH_3)-H < CH_3-N(CH_3)-CH_3 < CH_3-CH_2-NH(CH_2-CH_3) < CH_3-CH_2-NH_2$

lone pair delocalized

more steric hinderence less solutions energy

16. For coagulation of arscnious sulphide sol, which one of the following salt solution will be most effective ?

(1) $AlCl_3$ (2) NaCl

(3) $BaCl_2$ (4) Na_3PO_4

Ans. **(1)**

Sol. Sulphide is –ve charged colloid so cation with maximum charge will be most effective for coagulation.

$Al^{3+} > Ba^{2+} > Na^+$ coagulating power.

17. At 100°C, copper (Cu) has FCC unit cell structure with cell edge length of x Å. What is the approximate density of Cu (in g cm^{-3}) at this temperature ?

[Atomic Mass of Cu = 63.55u]

(1) $\frac{105}{x^3}$ (2) $\frac{211}{x^3}$ (3) $\frac{205}{x^3}$ (4) $\frac{422}{x^3}$

Ans. **(4)**

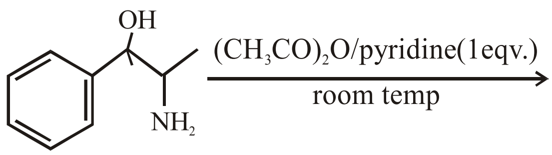
Sol. FCC unit cell Z = 4

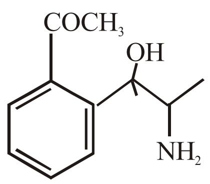
$$d = \frac{63.5\times4}{6\times10^{23}\times x^3\times10^{-24}}\ g/cm^3$$

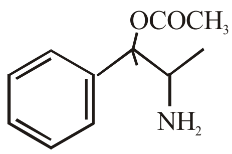
$$d = \frac{63.5\times4\times10}{6}\ g/cm^3$$

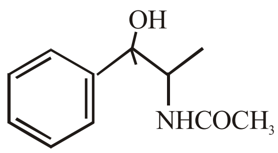
$$d = \frac{423.33}{x^3} \simeq \left(\frac{422}{x^3}\right)$$

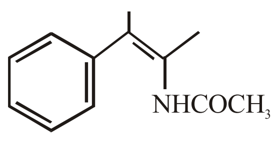
18. The major product obtained in the following reaction is :



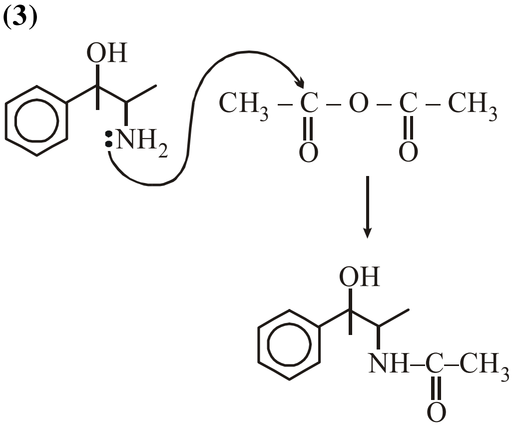
(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

Sol. 

19. Which of the following conditions in drinking water causes methemoglobinemia ?

(1) > 50ppm of load

(2) > 100 ppm of sulphate

(3) > 50 ppm of chloride

(4) > 50 ppm of nitrate

Ans. **(4)**

Sol. Concentration of nitrate >50 ppm in drinking water causes methemoglobinemia

20. Homoleptic octahedral complexes of a metal ion 'M^{3+}' with three monodentate ligands and L_1, L_2, L_3 absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is :

(1) $L_2 < L_1 < L_3$ (2) $L_3 < L_2 < L_1$

(3) $L_3 < L_1 < L_2$ (4) $L_1 < L_2 < L_3$

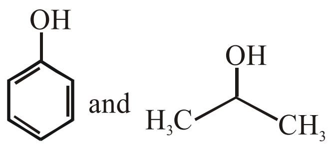
Ans. **(3)**

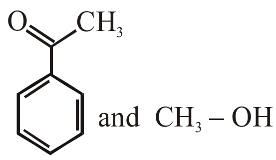
Sol. Order of λ_{abs} - $L_3 > L_1 > L_2$

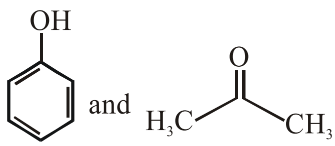
So Δ_O order will be $L_2 > L_1 > L_3$ (as $\Delta_O \propto \frac{1}{\lambda_{abs}}$)

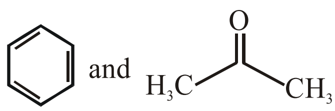
So order of ligand strength will be $L_2 > L_1 > L_3$

21. The product formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are :

(1) 

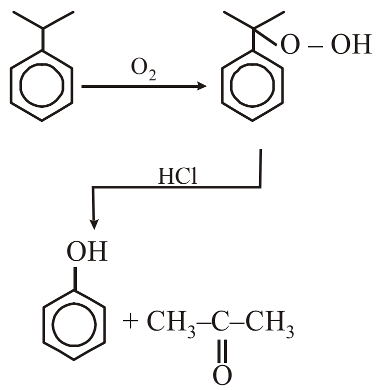
(2) 

(3) 

(4) 

Ans. **(3)**

Sol. Cummene hydroperoxide reaction



22. The temporary hardness of water is due to :-

(1) $Ca(HCO_3)_2$ (2) NaCl

(3) Na_2SO_4 (4) $CaCl_2$

Ans. (1)

Sol. $Ca(HCO_3)_2$ is reponsible for temporary hardness of water

23. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is :

(Specific heat of water liquid and water vapour are 4.2 kJ K^{-1} kg^{-1} and 2.0 kJ K^{-1} kg^{-1} ; heat of liquid fusion and vapourisation of water are 344 kJ kg^{-1} and 2491 kJ kg^{-1}, respectively). (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583)

(1) 7.90 kJ kg^{-1} K^{-1} (2) 2.64 kJ kg^{-1} K^{-1}

(3) 8.49 kJ kg^{-1} K^{-1} (4) 4.26 kJ kg^{-1} K^{-1}

Ans. (4)

Sol.

$$\underset{273K}{H_2O(s)} \xrightarrow[\Delta S_1]{} \underset{273K}{H_2O(\ell)} \xrightarrow[\Delta S_2]{} \underset{373K}{H_2O(\ell)} \xrightarrow[\Delta S_3]{} \underset{373K}{H_2O(g)} \xrightarrow[\Delta S_4]{} \underset{383K}{H_2O(g)}$$

$$\Delta S_1 = \frac{\Delta H_{fusion}}{273} = \frac{334}{273} = 1.22$$

$$\Delta S_2 = 4.2\ell N\left(\frac{363}{273}\right) = 1.31$$

$$\Delta S_3 = \frac{\Delta H_{vap}}{373} = \frac{2491}{373} = 6.67$$

$$\Delta S_4 = 2.0\ell n\left(\frac{383}{373}\right) = 0.05$$

$$\Delta S_{total} = 9.26 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

24. The pH of rain water, is approximately :

(1) 6.5 (2) 7.5 (3) 5.6 (4) 7.0

Ans. (3)

Sol. pH of rain water is approximate 5.6

25. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction

$$Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$$

at 300 K is approximately.

(R = 8 JK^{-1} mol^{-1} , F = 96000 C mol^{-1})

(1) e^{160} (2) e^{320}

(3) e^{-160} (4) e^{-80}

Ans. (1)

Sol. $\Delta G° = -RT\ \ln k = -nFE^{\circ}_{cell}$

$$\ln k = \frac{n \times F \times E°}{R \times T} = \frac{2 \times 96000 \times 2}{8 \times 300}$$

$\ln k = 160$

$k = e^{160}$

26. A solution containing 62 g ethylene glycol in 250 g water is cooled to –10°C. If K_f for water is 1.86 K kg mol^{-1}, the amount of water (in g) separated as ice is :

(1) 32 (2) 48 (3) 16 (4) 64

Ans. (4)

Sol. $\Delta T_f = K_f\ .\ m$

$$10 = 1.86 \times \frac{62/62}{W_{kg}}$$

W = 0.186 kg

$\Delta W = (250 - 186) = 64$ gm

27. When the first electron gain enthalpy ($\Delta_{eg}H$) of oxygen is –141 kJ/mol, its second electron gain enthalpy is :

(1) almost the same as that of the first

(2) negative , but less negative than the first

(3) a positive value

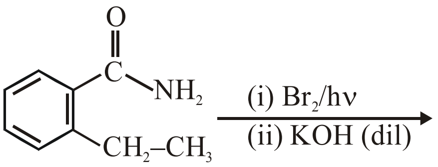
(4) a more negative value than the first

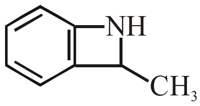
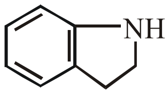
Ans. (3)

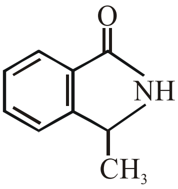
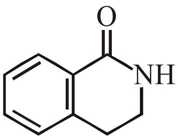
Sol. Second electron gain enthalpy is always positive for every element.

$O^{-}_{(g)} + e^{-} \rightarrow O^{-2}_{(g)}$; ΔH = positive

28. The major product of the following reaction is :

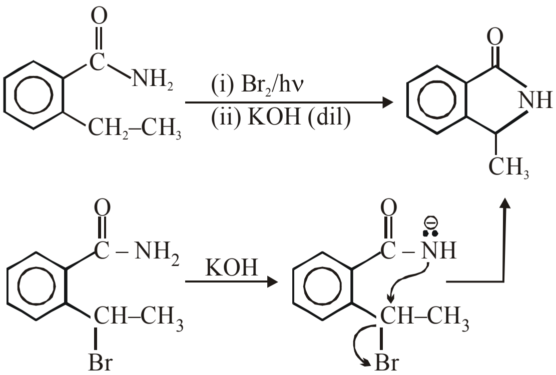


(1)  (2) 

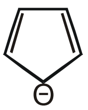
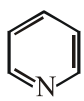
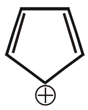
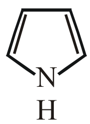
(3)  (4) 

Ans. (3)

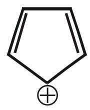
Sol.



29. Which of the following compounds is not aromatic ?

(1)  (2)  (3)  (4) 

Ans. (3)

Sol. 

Do not have (4n + 2) π electron It has 4n π electrons

So it is Anti aromatic.

30. Consider the following reversible chemical reactions :

$A_2(g) + Br_2(g) \xrightleftharpoons{K_1} 2AB(g)$ .....(1)

$6AB(g) \xrightleftharpoons{K_2} 3A_2(g) + 3B_2(g)$ .....(2)

The relation between K_1 and K_2 is :

(1) $K_2 = K_1^3$ (2) $K_2 = K_1^{-3}$

(3) $K_1K_2 = 3$ (4) $K_1K_2 = \frac{1}{3}$

Ans. (2)

Sol. $A_2(g) + B_2(g) \xrightleftharpoons{k_1} 2AB$ ...(1)

$\Rightarrow$ eq. (1) $\times$ 3

$6\ AB(g) \rightleftharpoons 3A_2(g) + 3B_2(g)$

$\Rightarrow \left(\frac{1}{k_1}\right)^3 = k_2 \Rightarrow k_2 = (k_1)^{-3}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

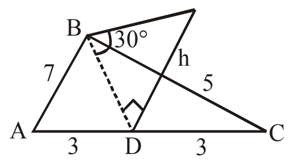
MATHEMATICS

1. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

(1) $7\sqrt{3}$ (2) $\frac{2}{3}\sqrt{21}$ (3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

Ans. **(2)**

Sol.



BD = hcot30° = $h\sqrt{3}$

So, $7^2 + 5^2 = 2((h\sqrt{3})^2 + 3^2)$

$\Rightarrow$ 37 = 3h² + 9.

$\Rightarrow$ $3h^2 = 28$

$\Rightarrow$ $h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$

2. Let f : R→R be a function such that $f(x) = x^3+x^2f'(1) + xf''(2)+f'''(3)$, $x\in R$. Then f(2) equal :

(1) 8 (2) –2 (3) –4 (4) 30

Ans. **(2)**

Sol. $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$\Rightarrow$ $f'(x) = 3x^2 + 2xf'(1) + f''(x)$ .....(1)

$\Rightarrow$ $f''(x) = 6x + 2f'(1)$ .....(2)

$\Rightarrow$ $f'''(x) = 6$ .....(3)

put x = 1 in equation (1) :

$f'(1) = 3 + 2f'(1) + f''(2)$ .....(4)

put x = 2 in equation (2) :

$f''(2) = 12 + 2f'(1)$ .....(5)

from equation (4) & (5) :

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow$ $3f'(1) = -15$

$\Rightarrow$ $f'(1) = -5$ $\Rightarrow$ $f''(2) = 2$ ....(2)

put x = 3 in equation (3) :

$f'''(3) = 6$

$\therefore$ $f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

3. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is :

(1) $\sqrt{57}$ (2) 4 (3) $2\sqrt{5}$ (4) 5

Ans. **(4)**

Sol. $x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through (4, 0) :

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow$ $20 + \lambda(5) = 0$

$\Rightarrow$ $\lambda = -4$

$\therefore$ $(x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$

or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius = $\sqrt{16+25-16} = 5$

4. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and theose whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

(1) 102 (2) 42 (3) 1 (4) 38

Ans. **(4)**

Sol. Let n(A) = number of students opted Mathematics = 70,

n(B) = number of students opted Physics = 46,

n(C) = number of students opted Chemistry = 28,

$n(A \cap B) = 23$,

$n(B \cap C) = 9$,
$n(A \cap C) = 14$,
$n(A \cap B \cap C) = 4$,
Now $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$
So number of students not opted for any course
$= \text{Total} - n(A \cup B \cup C)$
$= 140 - 102 = 38$

5. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :

(1) 1365 (2) 1256 (3) 1465 (4) 1356

Ans. (4)

Sol. $\sum_{r=2}^{13}(7r+2) = 7.\frac{2+13}{2}\times 6 + 2\times 12$

$= 7 \times 90 + 24 = 654$

$\sum_{r=1}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right)\times 13 + 5\times 13 = 702$

Total = 654 + 702 = 1356

6. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is :-

(1) $\left(\frac{1}{2}, 4, -2\right)$ (2) $\left(-\frac{1}{2}, 4, 0\right)$

(3) (1,3,1) (4) (1,5,1)

Ans. (2)

Sol. $4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$

$\Rightarrow 3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3$ ....(1)

Given $\vec{a}.\vec{c} = 0$

$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow 2\lambda_1 + \lambda_3 = -1$ .....(2)

Now $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$

Now check the options, option (2) is correct

7. The equation of a tangent to the hyperbola $4x^2-5y^2 = 20$ parallel to the line $x-y = 2$ is :

(1) $x-y+9 = 0$
(2) $x-y+7 = 0$
(3) $x-y+1 = 0$
(4) $x-y-3 = 0$

Ans. (3)

Sol. Hyperbola $\frac{x^2}{5} - \frac{y^2}{4} = 1$

slope of tangent = 1

equation of tangent $y = x \pm \sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$ or $y = x - 1$

8. If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, $(k>0)$, is 1 square unit. Then k is:

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

Ans. (1)

Sol. Area bounded by $y^2 = 4ax$ & $x^2 = 4by$, $a, b \neq 0$

is $\left|\frac{16ab}{3}\right|$

by using formula : $4a = \frac{1}{k} = 4b, k > 0$

Area $= \left|\frac{16.\frac{1}{4k}.\frac{1}{4k}}{3}\right| = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\Rightarrow k = \frac{1}{\sqrt{3}}$

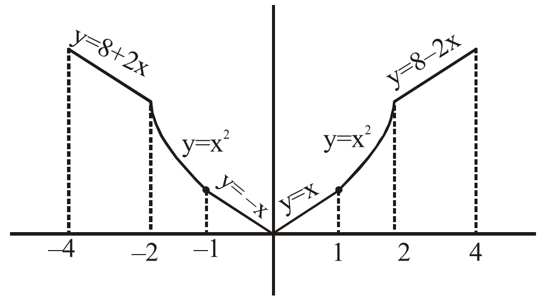
9. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (–4,4) at which f is not differentiable. Then S:

(1) is an empty set
(2) equals {–2, –1, 1, 2}
(3) equals {–2, –1, 0, 1, 2}
(4) equals {–2, 2}

Ans. (3)

Sol. $f(x) = \begin{cases} 8+2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8-2x, & 2 < x \le 4 \end{cases}$



$f(x)$ is not differentiable at x = {–2,–1,0,1,2}

$\Rightarrow$ S = {–2, –1, 0, 1, 2}

10. If the parabolas $y^2=4b(x-c)$ and $y^2=8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

(1) (1, 1, 0) (2) $\left(\frac{1}{2},2,3\right)$

(3) $\left(\frac{1}{2},2,0\right)$ (4) (1, 1, 3)

Ans. (1,2,3,4)

Sol. Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b)\, m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b)\, m = (b - 2a)m^3$

We get all options are correct for $m = 0$

(common normal x-axis)

Ans. (1), (2), (3), (4)

Remark :

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$$m^2 = \frac{c}{2a-b} - 2 > 0 \Rightarrow \frac{c}{2a-b} > 2$$

Now according to options, option 4 is correct

11. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

Ans. (1)

Sol. $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$, $\theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} = \cos^2 \frac{\pi}{4}$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}$, $n \in I$

$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{\pi}{2} - \frac{\pi}{8}$

Sum of solutions $\frac{\pi}{2}$

12. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4\,|z_2|$.

If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then :

(1) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (2) $\mathrm{Re}(z) = 0$

(3) $|z| = \sqrt{\frac{5}{2}}$ (4) $\mathrm{Im}(z) = 0$

Ans. (Bonus)

Sol. $3|z_1| = 4|z_2|$

$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$

$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$

Let $\frac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a}$

$= \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$

Now all options are incorrect

Remark :

There is a misprint in the problem actual problem should be :

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| = 2|z_2|$.

If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\frac{3z_1}{2z_2}\right| = 1$

Let $\frac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$

$= a + \frac{1}{a} = 2\cos\theta$

$\therefore$ Im(z) = 0

Now option (4) is correct.

13. If the system of equations

x+y+z = 5

x+2y+3z = 9

x+3y+αz = β

has infinitely many solutions, then $\beta-\alpha$ equals:

(1) 5 (2) 18 (3) 21 (4) 8

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = (\alpha-1)-4 = (\alpha-5)$

for infinite solutions D = 0 $\Rightarrow$ α = 5

$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta-15 & -2 & 5 \end{vmatrix} = 0$

$\Rightarrow 2+\beta-15 = 0 \Rightarrow \beta-13 = 0$

on β = 13 we get $D_y = D_z = 0$

α = 5, β = 13

14. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$ is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{5}{4}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{3}}{2}$

Ans. (1)

Sol. Let points $\left(\frac{3}{2}, 0\right)$, (t^2, t), t > 0

Distance $= \sqrt{t^2 + \left(t^2 - \frac{3}{2}\right)^2}$

$= \sqrt{t^4 - 2t^2 + \frac{9}{4}} = \sqrt{(t^2-1)^2 + \frac{5}{4}}$

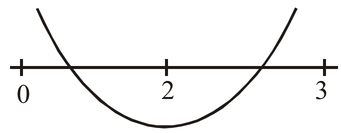
So minimum distance is $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

15. Consider the quadratic equation $(c-5)x^2-2cx + (c-4) = 0$, $c\neq5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of elements in S is :

(1) 11 (2) 18 (3) 10 (4) 12

Ans. (1)

Sol.



Let $f(x) = (c-5)x^2 - 2cx + c - 4$

$\therefore$ $f(0)f(2) < 0$ .....(1)

& $f(2)f(3) < 0$ .....(2)

from (1) & (2)

$(c-4)(c-24) < 0$

& $(c-24)(4c-49) < 0$

$\Rightarrow \frac{49}{4} < c < 24$

$\therefore$ s = {13, 14, 15, ... 23}

Number of elements in set S = 11

16. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right) = \frac{k}{21}$, then k equals :

(1) 200 (2) 50 (3) 100 (4) 400

Ans. (3)

Sol. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$

$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21}$

$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$

$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$

$\Rightarrow$ 100 = k

17. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(1) –7 (2) $2\left(\sqrt{2}+2\right)$

(3) –5 (4) $2\left(\sqrt{2}+1\right)$

Ans. **(3)**

Sol. $$\det A = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$$

$(R_1 \to R_1 + R_3 - 2R_2)$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & 2+2d-\sin\theta \end{vmatrix}$$

$= (2+\sin\theta)(2+2d-\sin\theta) - d(2\sin\theta - d)$

$= 4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$= d^2 + 4d + 4 - \sin^2\theta$

$= (d+2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d+2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5

18. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is :

(1) $2\sqrt{2}$ (2) $\frac{1}{8}$ (3) $4\sqrt{2}$ (4) $\frac{1}{4}$

Ans. **(4)**

Sol. $(1+x^{\log_2 x})^5$

$T_3 = {}^5C_2 . (x^{\log_2 x})^2 = 2560$

$\Rightarrow 10.x^{2\log_2 x} = 2560$

$\Rightarrow x^{2\log_2 x} = 256$

$\Rightarrow 2(\log_2 x)^2 = \log_2 256$

$\Rightarrow 2(\log_2 x)^2 = 8$

$\Rightarrow (\log_2 x)^2 = 4 \quad \Rightarrow \log_2 x = 2$ or -2

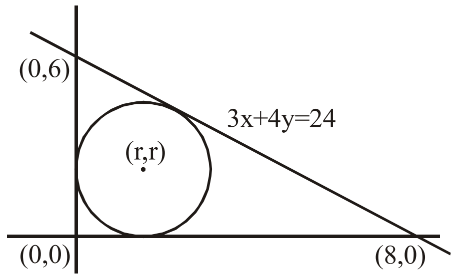
$$x = 4 \text{ or } \frac{1}{4}$$

19. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

Ans. **(2)**

Sol.



$$\left|\frac{3r+4r-24}{5}\right| = r$$

$7r - 24 = \pm 5r$

$2r = 24$ or $12r + 24$

$r = 14, \quad r = 2$

then incentre is (2, 2)

20. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :

(1) 4 : 9 (2) 6 : 7

(3) 5 : 8 (4) 10 : 3

Ans. **(1)**

Sol. Let two observations are x_1 & x_2

mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ....(1)

variance $(\sigma^2) = \frac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ .....(2)

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

21. A point P moves on the line 2x – 3y + 4 = 0. IfQ(1,4) and R(3,–2) are fixed points, then the locus of the centroid of ΔPQR is a line :

(1) parallel to x-axis (2) with slope $\frac{2}{3}$

(3) with slope $\frac{3}{2}$ (4) parallel to y-axis

Ans. **(2)**

Sol. Let the centroid of ΔPQR is (h, k) & P is (α, β), then

$\frac{\alpha+1+3}{3}=h$ and $\frac{\beta+4-2}{3}=k$

α = (3h – 4) β = (3k – 4)

Point P(α, β) lies on line 2x – 3y + 4 = 0

$\therefore$ 2(3h – 4) – 3(3k – 2) + 4 = 0

$\Rightarrow$ locus is 6x – 9y + 2 = 0

22. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and

$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals :

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}$

(3) $-\frac{4}{3}$ (4) $\frac{1}{3}+e^3$

Ans. **(1)**

Sol. $\frac{dy}{dx}+3\sec^2 x.y=\sec^2 x$

I.F. = $e^{3\int \sec^2 x dx}=e^{3\tan x}$

or $y.e^{3\tan x}=\int \sec^2 x.e^{3\tan x}dx$

or $y.e^{3\tan x}=\frac{1}{3}e^{3\tan x}+C$ ....(1)

Given

$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$

$\therefore \frac{4}{3}.e^3=\frac{1}{3}e^3+C$

$\therefore$ C = e^3

Now put x = $-\frac{\pi}{4}$ in equation (1)

$\therefore y.e^{-3}=\frac{1}{3}e^{-3}+e^3$

$\therefore y=\frac{1}{3}+e^6$

$\therefore y\left(-\frac{\pi}{4}\right)=\frac{1}{3}+e^6$

23. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point :

(1) (–1, –1, –1) (2) (–1, –1, 1)

(3) (1, 1, –1) (4) (1, 1, 1)

Ans. **(4)**

Sol. Let $\vec{n}$ be the normal vector to the plane passing through (4, –1, 2) and parallel to the lines L_1 & L_2

then $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 1 & 2 & 3\end{vmatrix}$

$\therefore \vec{n}=-7\hat{i}-7\hat{j}+7\hat{k}$

$\therefore$ Equation of plane is

–1(x – 4) – 1(y + 1) + 1(z – 2) = 0

$\therefore$ x + y – z – 1 = 0

Now check options

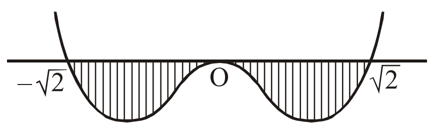
24. Let $I=\int_a^b (x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is :

(1) $\left(-\sqrt{2},0\right)$ (2) $\left(-\sqrt{2},\sqrt{2}\right)$

(3) $\left(0,\sqrt{2}\right)$ (4) $\left(\sqrt{2},-\sqrt{2}\right)$

Ans. **(2)**

Sol. Let f(x) = $x^2(x^2 - 2)$



As long as f(x) lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2},\sqrt{2})$ for minimum of I

25. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) $\frac{7}{4}$

Ans. **(4)**

Sol. r = 1 is obviously true.

Let 0 < r < 1

$\Rightarrow$ $r + r^2 > 1$

$\Rightarrow$ $r^2 + r - 1 > 0$

$$\left(r-\frac{-1-\sqrt{5}}{2}\right)\left(r-\left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow\ r-\frac{-1-\sqrt{5}}{2} \text{ or } r>\frac{-1+\sqrt{5}}{2}$$

$$r\in\left(\frac{\sqrt{5}-1}{2},1\right)$$

$$\frac{\sqrt{5}-1}{2}<r<1$$

When r > 1

$$\Rightarrow\ \frac{\sqrt{5}+1}{2}>\frac{1}{r}>1$$

$$\Rightarrow\ r\in\left(\frac{\sqrt{5}-1}{2},\frac{\sqrt{5}+1}{2}\right)$$

Now check options

26. Consider the statement : "P(n): $n^2 - n + 41$ is prime." Then which one of the following is true?

(1) P(5) is false but P(3) is true

(2) Both P(3) and P(5) are false

(3) P(3) is false but P(5) is true

(4) Both P(3) and P(5) are true

Ans. **(4)**

Sol. P(n) : $n^2 - n + 41$ is prime

P(5) = 61 which is prime

P(3) = 47 which is also prime

27. Let A be a point on the line $\vec{r}=(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x-4y+3z=1$ is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

Ans. **(3)**

Sol. Let point A is

$(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$

and point B is (3, 2, 6)

then $\overrightarrow{AB}=(2+3\mu)\hat{i}+(3-\mu)\hat{j}+(4-5\mu)\hat{k}$

which is parallel to the plane x – 4y + 3z = 1

$\therefore$ 2 + 3μ – 12 + 4μ + 12 – 15μ = 0

8μ = 2

$$\mu=\frac{1}{4}$$

28. For each t∈R, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) equals –1 (2) equals 1

(3) does not exist (4) equals 0

Ans. **(4)**

Sol. $$\lim_{x\to1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$=\lim_{x\to1^+}\frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$$

$$=\lim_{x\to1^+}\left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1)=(1-1)(-1)=0$$

29. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

Ans. (3)

Sol. $\boxed{\text{Start}}$

$\xrightarrow{1/2}$ $H \rightarrow \text{Sum } 7 \text{ or } 8 \Rightarrow \frac{11}{36}$

$\xrightarrow{1/2}$ $T \rightarrow \text{Number is } 7 \text{ or } 8 = \frac{2}{9}$

$$P(A) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$$

30. Let $n \geq 2$ be a natural number and $0<\theta<\pi/2$.

Then $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos\theta}{\sin^{n+1}\theta} d\theta$ is equal to :

(Where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

Ans. (3)

Sol. $\int \frac{(\sin^n \theta - \sin \theta)^{1/n} \cos\theta}{\sin^{n+1}\theta} d\theta$

$$= \int \frac{\sin\theta\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{1/n}}{\sin^{n+1}\theta} d\theta$$

Put $1 - \frac{1}{\sin^{n-1}\theta} = t$

So $\frac{(n-1)}{\sin^n\theta}\cos\theta d\theta = dt$

Now $\frac{1}{n-1}\int (t)^{1/n} dt$

$$= \frac{1}{(n-1)} \frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{(n-1)}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{1}{n}+1} + C$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 09 : 30 AM To 12 : 30 PM

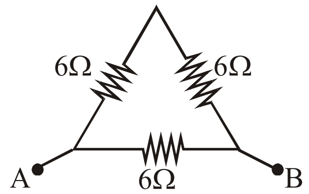
PHYSICS

1. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is :

(1) 8 Ω (2) 12 Ω (3) 4 Ω (4) 2Ω

Ans. **(3)**

Sol.



R_{eq} between any two vertex will be

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq.} = 4\Omega$$

2. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is :

(1) $\frac{3}{2}mv^2$ (2) mv^2

(3) $2mv^2$ (4) $\frac{1}{2}mv^2$

Ans. **(2)**

Sol. At height r from center of earth. orbital velocity

$$= \sqrt{\frac{GM}{r}}$$

$\therefore$ By energy conservation

$$\text{KE of 'm'} + \left(-\frac{GMm}{r}\right) = 0 + 0$$

(At infinity, PE = KE = 0)

$$\Rightarrow \text{KE of 'm'} = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$$

3. A solid metal cube of edge length 2 cm is moving in a positive y direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is :

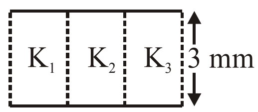
(1) 6 mV (2) 1 mV (3) 12 mV (4) 2 mV

Ans. **(3)**

Sol. Potential difference between two faces perpendicular to x-axis will be

$$\ell.(\vec{V} \times \vec{B}) = 12\text{mV}$$

4. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1, = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :



(1) 12 (2) 4 (3) 36 (4) 14

Ans. **(1)**

Sol. Let dielectric constant of material used be K.

$$\therefore \frac{10 \in_0 A/3}{d} + \frac{12 \in_0 A/3}{d} + \frac{14 \in_0 A/3}{d} = \frac{K \in_0 A}{d}$$

$\Rightarrow$ K = 12

5. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is :

(1) 63 mA (2) 0.4 mA

(3) 100 mA (4) 20 mA

Ans. **(4)**

Sol. $P = i^2R$.

$\therefore$ for i_{max}, R must be minimum

from color coding R = $50 \times 10^2 \Omega$

$\therefore$ i_{max} = 20mA

6. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are :

(1) 380 nm, 500 nm (2) 625 nm, 500 nm

(3) 380 nm, 525 nm (4) 400 nm, 500 nm

Ans. **(2)**

Sol. Path difference = d $\sin\theta \approx d\theta$

$= 0.1 \times \frac{1}{40}$ mm = 2500nm

or bright fringe, path difference must be integral multiple of λ.

$\therefore$ $2500 = n\lambda_1 = m\lambda_2$

$\therefore$ $\lambda_1 = 625$, $\lambda_2 = 500$ (from m=5)

(for n = 4)

7. A magnet of total magnetic moment $10^{-2}\,\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\,\hat{i}\,(\cos\omega t)$ where B = 1 Tesla and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is :

(1) 0.007 J (2) 0.014 J (3) 0.01 J (4) 0.028 J

Ans. **(2)**

Sol. Work done, $W = (\Delta\vec{\mu}) \cdot \vec{B}$

$= 2 \times 10^{-2} \times 1 \cos(0.125)$

$= 0.02$ J

$\therefore$ correct answer is (2)

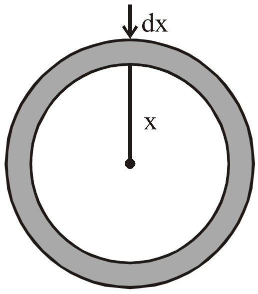
8. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is :

(1) $\frac{2}{3}\mu FR$ (2) $\mu FR/3$

(3) $\mu FR/2$ (4) $\mu FR/6$

Ans. **(1)**

Sol.



Consider a strip of radius x & thickness dx,

Torque due to friction on this strip.

$$\int d\tau = \int_0^R \frac{x\mu F \cdot 2\pi x dx}{\pi R^2}$$

$$\tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

$\therefore$ correct answer is (1)

9. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to :

(1) 150 (2) 360

(3) 200 (4) 400

Ans. **(3)**

Sol. at t = 0, $A_0 = \frac{dN}{dt} = 1600 C/s$

at t = 8s, A = 100 C/s

$\frac{A}{A_0} = \frac{1}{16}$ in 8 sec

Therefor half life is $t_{1/2} = 2$ sec

$\therefore$ Activity at t = 6 will be $1600\left(\frac{1}{2}\right)^3$

= 200 C/s

$\therefore$ correct answer is (3)

10. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8/m/s)

$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is :

(1) 4×10^4 N/C (2) 4.5×10^4 N/C

(3) 6×10^4 N/C (4) 3×10^4 N/C

Ans. (4)

Sol. $E_0 = B_0 \times C$

$= 100 \times 10^{-6} \times 3 \times 10^8$

$= 3 \times 10^4$ N/C

$\therefore$ correct answer is 3×10^4 N/C

11. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be :

(1) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$

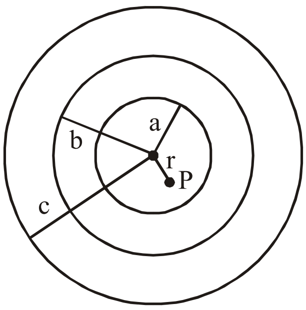
(2) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

(3) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$

(4) $\frac{Q}{4\pi\varepsilon_0}\frac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$

Ans. (2)

Sol.



Potential at point P, $V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$

$\because Q_a : Q_b : Q_c : : a^2 : b^2 : c^2$

{since $\sigma_a = \sigma_b = \sigma_c$}

$\therefore Q_a = \left[\frac{a^2}{a^2+b^2+c^2}\right]Q$

$Q_b = \left[\frac{b^2}{a^2+b^2+c^2}\right]Q$

$Q_c = \left[\frac{c^2}{a^2+b^2+c^2}\right]Q$

$V = \frac{Q}{4\pi\in_0}\left[\frac{(a+b+c)}{a^2+b^2+c^2}\right]$

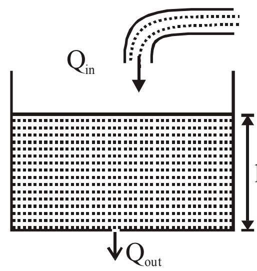
$\therefore$ correct answer is (2)

12. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is:

(1) 4 cm (2) 2.9 cm (3) 1.7 cm (4) 5.1 cm

Ans. (4)

Sol.



Since height of water column is constant therefore, water inflow rate (Q_{in})

= water outflow rate

$Q_{in} = 10^{-4}$ m^3s^{-1}

$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4}\sqrt{20 \times h}$

$h = \frac{1}{20}$ m

h = 5cm

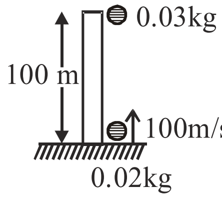
$\therefore$ correct answer is (4)

13. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : (g =10ms^{-2})

(1) 30 m (2) 10 m (3) 40 m (4) 20 m

Ans. **(3)**

Sol.



Time taken for the particles to collide,

$$t=\frac{d}{V_{rel}}=\frac{100}{100}=1\sec$$

Speed of wood just before collision = gt = 10 m/s

& speed of bullet just before collision v-gt

= 100 – 10 = 90 m/s

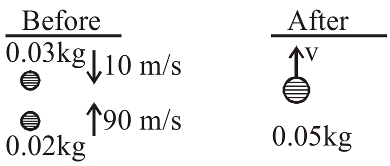
Now, conservation of linear momentum just before and after the collision -

–(0.02) (1v) + (0.02) (9v) = (0.05)v

⇒ 150 = 5v

⇒ v = 30 m/s

Max. height reached by body h = $\frac{v^2}{2g}$



$$h=\frac{30\times 30}{2\times 10}=45m$$

∴ Height above tower = 40 m

14. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is :

(1) 410 (2) 640 (3) 16 (4) 40

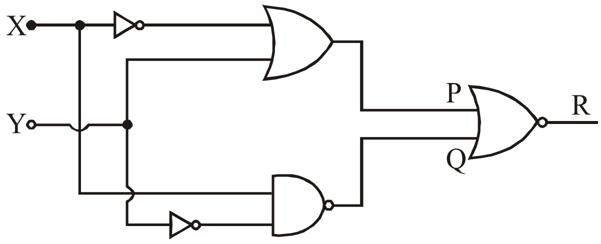
Ans. **(4)**

Sol. $$\frac{128kg}{m^3}=\frac{125(50g)(20)}{(25cm)^3(4)^3}$$

$$=\frac{128}{64}(20)\text{units}$$

= 40 units

15. To get output '1' at R, for the given logic gate circuit the input values must be :



(1) X = 0, Y = 1 (2) X =1 , Y = 1

(3) X = 0, Y = 0 (4) X = 1, Y = 0

Ans. **(4)**

Sol. $p=\overline{x}+y$

$Q=\overline{y.x}=y+\overline{x}$

$O/P=\overline{P+Q}$

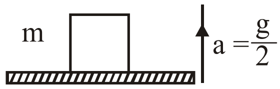
To make O/P

P + Q must be 'O'

SO, y = 0

x = 1

16. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. Work done bv normal reaction on block in time t is :



(1) 0 (2) $\frac{3mg^2t^2}{8}$

(3) $-\frac{mg^2t^2}{8}$ (4) $\frac{mg^2t^2}{8}$

Ans. **(2)**

Sol. $$N-mg=\frac{mg}{2}\Rightarrow N=\frac{3mg}{2}$$

Now, work done $W=\vec{N}\vec{S}=\left(\frac{3mg}{2}\right)\left(\frac{1}{2}gt^2\right)$

$$\Rightarrow W=\frac{3mg^2t^2}{4}$$

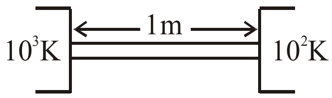
17. A heat source at T= 10^3 K is connected to another heat reservoir at T=10^2 K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is :

(1) 90 Wm^{-2} (2) 200 Wm^{-2}

(3) 65 Wm^{-2} (4) 120 Wm^{-2}

Ans. **(1)**

Sol.



$$\left(\frac{dQ}{dt}\right)=\frac{kA\Delta T}{\ell}$$

$$\Rightarrow \frac{1}{A}\left(\frac{dQ}{dt}\right)=\frac{(0.1)(900)}{1}=90W/m^2$$

18. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS(Line of Sight) mode ? (Given : radius of earth = 6.4×10^6m).

(1) 80 km (2) 48 km

(3) 40 km (4) 65 km

Ans. **(4)**

Sol. Maximum distance upto which signal can be broadcasted is

$$d_{max}=\sqrt{2Rh_T}+\sqrt{2Rh_R}$$

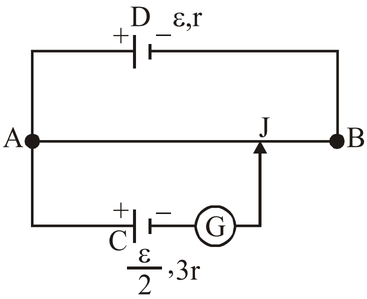
where h_T and h_R are heights of transmiter tower and height of receiver respectively.

Putting all values -

$$d_{max}=\sqrt{2\times 6.4\times 106}\left[\sqrt{104}+\sqrt{40}\right]$$

on solving, d_{max} = 65 km

19. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is :



(1) $\frac{5}{12}L$ (2) $\frac{11}{24}L$ (3) $\frac{11}{12}L$ (4) $\frac{13}{24}L$

Ans. **(4)**

Sol. $i=\frac{\varepsilon}{13r}$

$$i\left(\frac{x}{L}12r\right)=\frac{\varepsilon}{2}$$

$$\frac{\varepsilon}{13r}\left[\frac{x}{L}.12r\right]=\frac{\varepsilon}{2}\Rightarrow \boxed{x=\frac{13L}{24}}$$

20. An insulating thin rod of length ℓ has a x linear charge density $p(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is :

(1) $\frac{\pi}{4}n\rho\ell^3$ (2) $n\rho\ell^3$ (3) $\pi n\rho\ell^3$ (4) $\frac{\pi}{3}n\rho\ell^3$

Ans. **(1)**

Sol. $\because$ M = NIA

dq = λdx & A = πx^2

$$\int dm=\int (x)\frac{\rho_0 x}{\ell}dx.\pi x^2$$

$$M=\frac{n\rho_0\pi}{\ell}.\int_0^{\ell}x^3.dx=\frac{n\rho_0\pi}{\ell}.\left[\frac{L^4}{4}\right]$$

$$M=\frac{n\rho_0\pi\ell^3}{4} \text{ or } \frac{\pi}{4}n\rho\ell^3$$

21. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is :

(1) 1 : 2 (2) 1 : 4 (3) 1 : 8 (4) 1 : 16

Ans. **(4)**

Sol. $R = \frac{u^2 \sin 2\theta}{g}$

$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$

22. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The siring is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to :

(1) 16.6 cm (2) 20.0 cm

(3) 10.0 cm (4) 33.3 cm

Ans. **(2)**

Sol. Velocity of wave on string

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{5} \times 1000} = 40 \text{m/s}$

Now, wavelength of wave $\lambda = \frac{v}{n} = \frac{40}{100} \text{m}$

Separation b/w successive nodes, $\frac{\lambda}{2} = \frac{20}{100}$ m

= 20 cm

23. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is :

(1) 18/17 (2) 19/18 (3) 20/19 (4) 21/20

Ans. **(2)**

Sol. $f_{app} = f_0 \left[\frac{v_2 \pm v_0}{v_2 \mp v_s}\right]$

$f_1 = f_0 \left[\frac{340}{340 - 34}\right]$

$f_2 = f_0 \left[\frac{340}{340 - 17}\right]$

$\frac{f_1}{f_2} = \frac{340 - 17}{340 - 34} = \frac{323}{306} \Rightarrow \frac{f_1}{f_2} = \frac{19}{18}$

24. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12}m, the minimum electron energy required is close to :

(1) 100 keV (2) 500 keV

(3) 25 keV (4) 1 keV

Ans. **(3)**

Sol. $\lambda = \frac{h}{p}$ $\{\lambda = 7.5 \times 10^{-12}\}$

$P = \frac{h}{\lambda}$

$KE = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left\{\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right\}}{2 \times 9.1 \times 10^{-31}} J$

KE = 25 Kev

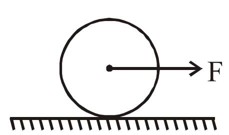
25. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is:

(1) $\frac{3F}{2mR}$ (2) $\frac{F}{3mR}$

(3) $\frac{2F}{3mR}$ (4) $\frac{F}{2mR}$

Ans. **(3)**

Sol.



$FR = \frac{3}{2} MR^2 \alpha$

$\alpha = \frac{2F}{3MR}$

26. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as :

(1) $\mu_1 + \mu_2 = 3$

(2) $2\mu_1 - \mu_2 = 1$

(3) $2\mu_2 - \mu_1 = 1$

(4) $3\mu_2 - 2\mu_1 = 1$

Ans. (2)

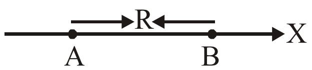
Sol. $\frac{1}{2f_2} = \frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$

$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$\frac{(\mu_1 - 1)}{R} = \frac{(\mu_2 - 1)}{2R}$

$2\mu_2 - \mu_2 = 1$

27. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ placed on the x-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is :

(1) $\frac{\sqrt{2}R}{\sqrt{2}+1}$

(2) $\frac{R}{\sqrt{2}+1}$

(3) $\frac{\sqrt{2}\,R}{\sqrt{2}-1}$

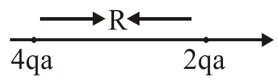
(4) $\frac{R}{\sqrt{2}-1}$

Ans. (3)

Sol. $V = \frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$

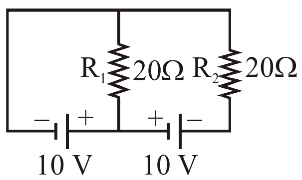
$\sqrt{2}x = R + x$

$x = \frac{R}{\sqrt{2}-1}$



dist $= \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$

28. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1, and R_2 respectively, are:



(1) 2, 2 (2) 0,1

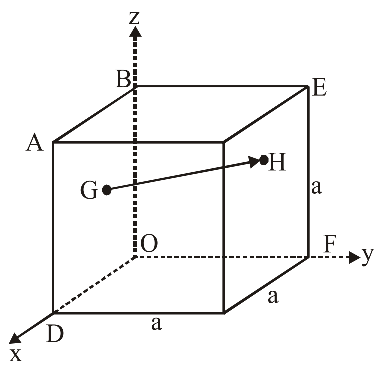
(3) 1,2 (4) 0.5,0

Ans. (4)

Sol. $i_1 = \frac{10}{20} = 0.5A$

$i_2 = 0$

29. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be:



(1) $\frac{1}{2}a(\hat{i} - \hat{k})$ (2) $\frac{1}{2}a(\hat{j} - \hat{i})$

(3) $\frac{1}{2}a(\hat{k} - \hat{i})$ (4) $\frac{1}{2}a(\hat{j} - \hat{k})$

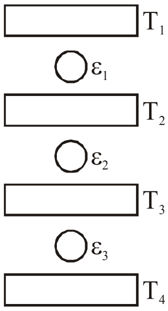
Ans. (2)

Sol. $\vec{r}_g = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}$

$\vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$\vec{r}_H - \vec{r}_g = \frac{a}{2}\left(\hat{j} - \hat{i}\right)$

30. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2, and T_3, as shown, with $T_2 > T_2 > T_3 > T_4$. The three engines are equally efficient if:



(1) $T_2 = \left(T_1^2 T_4\right)^{1/3}; T_3 = \left(T_1 T_4^2\right)^{1/3}$

(2) $T_2 = \left(T_1 T_4^2\right)^{1/3}; T_3 = \left(T_1^2 T_4\right)^{1/3}$

(3) $T_2 = \left(T_1^3 T_4\right)^{1/4}; T_3 = \left(T_1 T_4^3\right)^{1/4}$

(4) $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$

Ans. (1)

Sol. $t_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{T_2} = 1 - \frac{T_4}{T_3}$

$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_4}{T_3}$

$\Rightarrow T_2 = \sqrt{T_1 T_3} = \sqrt{T_1\sqrt{T_2 T_4}}$

$T_3 = \sqrt{T_2 T_4}$

$T_2^{3/4} = \sqrt{T_1^{1/2}} T_4^{1/4}$

$T_2 = T_1^{2/3} T_4^{1/3}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

CHEMISTRY

1. Two pi and half sigma bonds are present in:

(1) N_2^+ (2) N_2 (3) O_2^+ (4) O_2

Ans. (1)

Sol.

$N_2^{\oplus} \Rightarrow BO = 2.5 \Rightarrow \left[\pi - \text{Bond} = 2\ \&\ \sigma - \text{Bond} = \frac{1}{2}\right]$

$N_2 \Rightarrow$ B.O. $= 3.0 \Rightarrow$ [π-Bond = 2 & σ-Bond = 1]

$O_2^{\oplus}$ = B.O. $\Rightarrow 2.5 \Rightarrow$ [π-Bond = 1.5 & σ-Bond = 1]

$O_2 \Rightarrow$ B.O. $\Rightarrow 2 \Rightarrow$ [π-Bond $\Rightarrow$ 1 & σ-Bond = 1]

2. The chemical nature of hydrogen preoxide is :-

(1) Oxidising and reducing agent in acidic medium, but not in basic medium.

(2) Oxidising and reducing agent in both acidic and basic medium

(3) Reducing agent in basic medium, but not in acidic medium

(4) Oxidising agent in acidic medium, but not in basic medium.

Ans. (2)

Sol. H_2O_2 act as oxidising agent and reducing agent in acidic medium as well as basic medium.

H_2O_2 Act as oxidant :-

$H_2O_2 + 2H^{\oplus} + 2e^{\ominus} \rightarrow 2H_2O$ (In acidic medium)

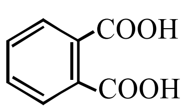
$H_2O + 2e^{\ominus} \rightarrow 2OH^{\ominus}$ (In basic medium)

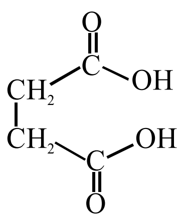
H_2O_2 Act as reductant :-

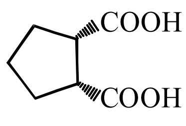
$H_2O_2 \rightarrow 2H^+ + O_2 + 2e^{\ominus}$ (In acidic medium)

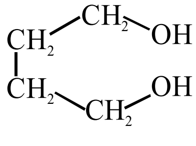
$H_2O_2 + 2OH^{\ominus} \rightarrow 2H_2O + O_2 + 2e^{\ominus}$ (In basic medium)

3. Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride :

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. Adipic acid $CO_2H–(CH_2)_4–CO_2H \xrightarrow[\text{agent}]{\text{dehydrating}}$ 7 membered cyclic anhydride (Very unstable)

4. Which premitive unit cell has unequal edge lenghs (a ≠ b ≠ c) and all axial angles differetn from 90° ?

(1) Tetragonal (2) Hexagonal

(3) Monoclinic (4) Triclinic

Ans. (4)

Sol. In Triclinic unit cell

$a \neq b \neq c\ \&\ \alpha \neq \beta \neq \gamma \neq 90°$

5. Wilkinson catalyst is :

(1) $[(Ph_3P)_3RhCl]$ (Et = C_2H_5)

(2) $[Et_3P)_3IrCl]$

(3) $[Et_3P)_3RhCl]$

(4) $[Ph_3P)_3IrCl]$

Ans. (1)

Sol. Wilkinsion catalyst is $[(ph_3P)_3RhCl]$

6. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are :

(1) 2 and 0 (2) 3 and 2

(3) 3 and 1 (4) 2 and 1

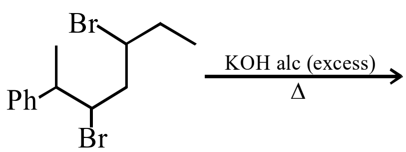
Ans. (3)

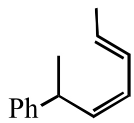
Sol. Total number of isotopes of hydrogen is 3

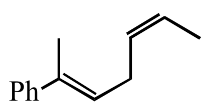
$\Rightarrow\ {}_1^1H,\ {}_1^2H$ or ${}_1^2D,\ {}_1^3H$ or ${}_1^3T$

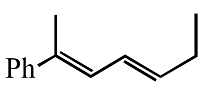
and only ${}_1^3H$ or ${}_1^3T$ is an Radioactive element.

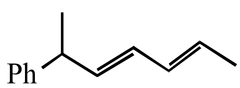
7. The major product of the following reaction is



(1) 

(2) 

(3) 

(4) 

Ans. (3)

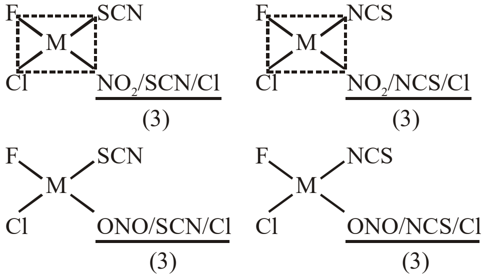
Sol. Example of E_2 elimination and conjugated diene is formed with phenyl ring in conjugation which makes it very stable.

8. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is :

(1) 12 (2) 8 (3) 16 (4) 4

Ans. **(1)**

Sol. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is 12.



9. Hall-Heroult's process is given by "

(1) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(2) $Cu^{2+}(aq.) + H_2(g) \rightarrow Cu(s) + 2H^+(aq)$

(3) $ZnO + C \xrightarrow{\text{Coke,1673K}} Zn + CO$

(4) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Ans. **(4)**

Sol. In Hall-Heroult's process is given by

$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$

$2Al_2O_3(\ell) \rightleftharpoons 4Al^{3+}(\ell) + 6O^{2\ominus}(\ell)$

At cathode :- $4Al^{3+}_{(\ell)} + 12e^{\ominus} \rightarrow 4Al(\ell)$

At Anode : $6O^{2\ominus}_{(\ell)} \rightarrow 3O_2(g) + 12e^{\ominus}$

$3C + 3O_2 \rightarrow 3CO_2\ (\uparrow)$

10. The value of K_p/K_C for the following reactions at 300K are, respectively :

(At 300K, RT = 24.62 $dm^3atm\ mol^{-1}$)

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

(1) 1, 24.62 $dm^3atm\ mol^{-1}$, 606.0 $dm^6atm^2mol^{-2}$

(2) 1, 4.1×10^{-2} $dm^{-3}atm^{-1}\ mol^{-1}$, 606.0 $dm^6\ atm^2\ mol^{-2}$

(3) 606.0 $dm^6atm^2mol^{-2}$, 1.65×10^{-3} $dm^3atm^{-2}\ mol^{-1}$

(4) 1, 24.62 $dm^3atm\ mol^{-1}$, 1.65×10^{-3} $dm^{-6}atm^{-2}\ mol^2$

Ans. **(4)**

Sol. $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$$\frac{k_p}{k_c} = (RT)^{\Delta n_g} = (RT)^0 = 1$$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$$\frac{k_p}{k_c} = (RT)^1 = 24.62$$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$$\frac{k_p}{k_c} = (RT)^{-2} = \frac{1}{(RT)^2} = 1.65 \times 10^{-3}$$

11. If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct ?

(1) DCM and H_2O would stay as lower and upper layer respectively in the S.F.

(2) DCM and H_2O will be miscible clearly

(3) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)

(4) DCM and H_2O will make trubid/colloidal mixture

Ans. **(1)**

12. The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$, respectively, are :

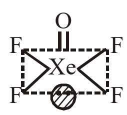
(1) sp^3d and 1

(2) sp^3d and 2

(3) sp^3d^2 and 1

(4) sp^3d^2 and 2

Ans. **(3)**

Sol.  $sp^3d^2 \Rightarrow$ [5σ-bond +1 l.p.]

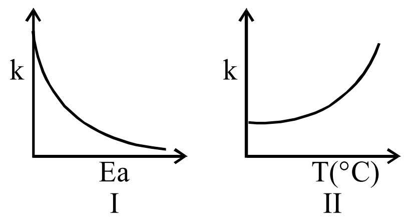
13. The metal used for making X-ray tube window is :

(1) Mg (2) Na (3) Ca (4) Be

Ans. **(4)**

Sol. "Be" Metal is used in x-ray window is due to transparent to x-rays.

14. Consider the given plots for a reaction obeying Arrhenius equation (0°C < T < 300°C) : (k and E_a are rate constant and activation energy, respectively)



Choose the correct option :

(1) Both I and II are wrong

(2) I is wrong but II is right

(3) Both I and II are correct

(4) I is right but II is wrong

Ans. **(4)**

Sol. On increasing E_a, K decreases

15. Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is :

(1) A is more polluted than B

(2) A is suitable for drinking, whereas B is not

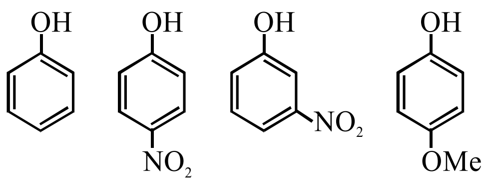
(3) B is more polluted than A

(4) Both A and B are suitable for drinking

Ans. **(3)**

Sol. Two glasses "A" and "B" have BOD values 10 and "20", respectively.
Hence glasses "B" is more polluted than glasses "A".

16. The increasing order of the pKa values of the following compounds is :



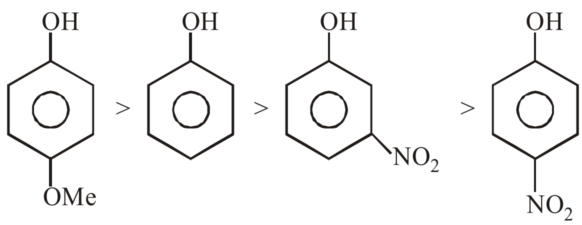
A B C D

(1) D < A < C < B (2) B < C < D < A

(3) C < B < A < D (4) B < C < A < D

Ans. **(4)**

Sol. Acidic strength is inversely proportional to pka.



17. Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vapor pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapor in equilibrium with a solution containing 40 mole percent of A at this temperature is :

(1) $x_A = 0.37$; $x_B = 0.63$

(2) $x_A = 0.28$; $x_B = 0.72$

(3) $x_A = 0.76$; $x_B = 0.24$

(4) $x_A = 0.4$; $x_B = 0.6$

Ans. **(2)**

Sol. $y_A = \dfrac{P_A}{P_{Total}} = \dfrac{P_A^o x_A}{P_A^o x_A \times p_B^o x_B}$

$$= \frac{7 \times 10^3 \times 0.4}{7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6}$$

$$= \frac{2.8}{10} = 0.28$$

$y_B = 0.72$

18. Consider the following reduction processes :

$Zn^{2+} + 2e^- \rightarrow Zn(s)$; E° = – 0.76 V

$Ca^{2+} + 2e^- \rightarrow Ca(s)$; E° = – 2.87 V

$Mg^{2+} + 2e^- \rightarrow Mg(s)$; E° = – 2.36 V

$Ni^{2+} + 2e^- \rightarrow Ni(s)$; E° = – 0.25 V

The reducing power of the metals increases in the order :

(1) Ca < Zn < Mg < Ni

(2) Ni < Zn < Mg < Ca

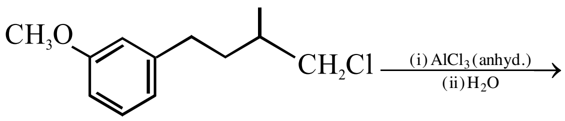
(3) Zn < Mg < Ni < Ca

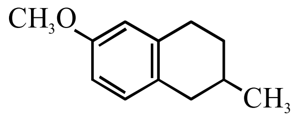
(4) Ca < Mg < Zn < Ni

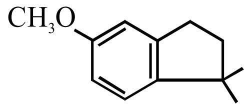
Ans. **(2)**

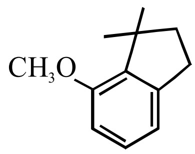
Sol. Higher the oxidation potential better will be reducing power.

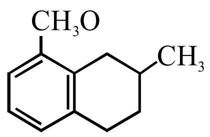
19. The major product of the following reaction is:



(1) 

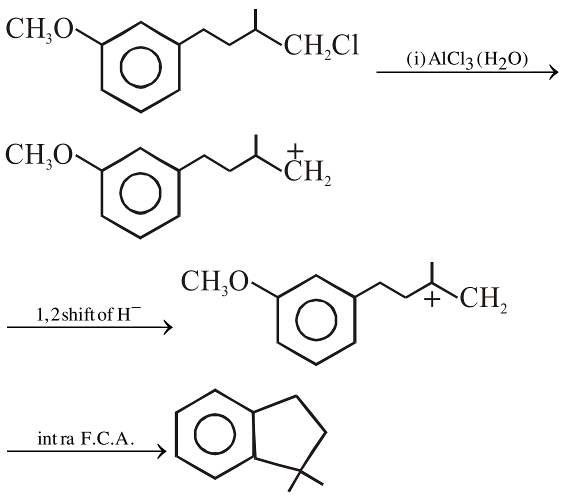
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



20. The electronegativity of aluminium is similar to :

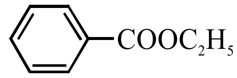
(1) Boron (2) Carbon

(3) Lithium (4) Beryllium

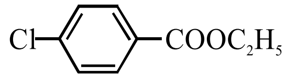
Ans. **(4)**

Sol. E.N. of Al = (1.5) $\approxeq$ Be (1.5)

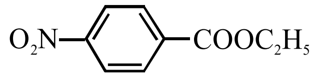
21. The decreasing order of ease of alkaline hydrolysis for the following esters is :



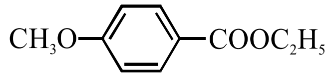
I



II



III



IV

(1) IV > II > III > I

(2) III > II > I > IV

(3) III > II > IV > I

(4) II > III > I > IV

Ans. **(2)**

Sol. More is the electrophilic character of carbonyl group of ester faster is the alkaline hydrolysis.

22. A process has $\Delta H = 200\ \text{Jmol}^{-1}$ and $\Delta S = 40\ \text{JK}^{-1}\text{mol}^{-1}$. Out of the values given below, choose the minimum temperature above which the process will be spontaneous :

(1) 5 K

(2) 4 K

(3) 20 K

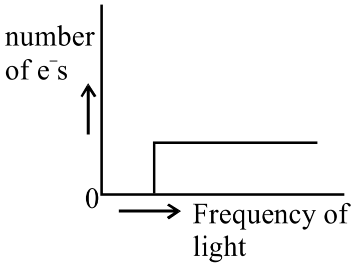
(4) 12 K

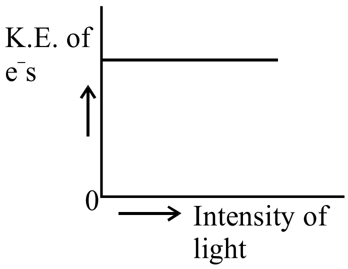
Ans. **(1)**

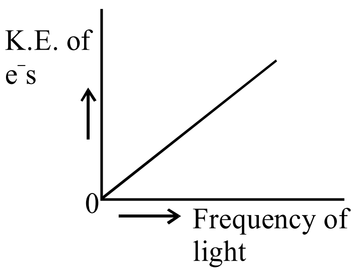
Sol. $\Delta G = \Delta H - T\Delta S$

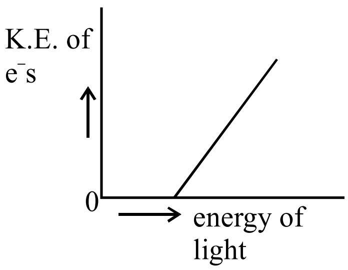
$$T = \frac{\Delta H}{\Delta S} = \frac{200}{40} = 5\text{K}$$

23. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected form metal surface ?

(1)


(2)


(3)


(4)


Ans. **(3)**

Sol. $E = W + \frac{1}{2}mv^2$

K.E. = $h\nu - 4\nu_0$

K.E. = $h\nu + (-h\nu_0)$

$y = mx + \underline{C}$

24. Which of the following is not and example of heterogeneous catalytic reaction ?

(1) Ostwald's process

(2) Haber's process

(3) Combustion of coal

(4) Hydrogenation of vegetable oils

Ans. **(3)**

Sol. Then is no catalyst is required for combustion of coal.

25. The effect of lanthanoid contraction in the lanthanoid series of elements by and large means :

(1) decrease in both atomic and ionic radii

(2) increase in atomic radii and decrease in ionic radii

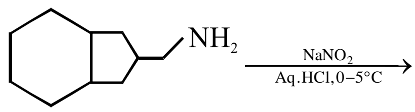
(3) increase in both atomic and ionic radii

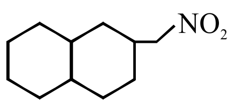
(4) decrease in atomic radii and increase in ionic radii

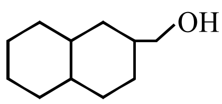
Ans. **(1)**

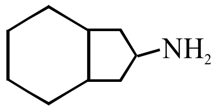
Sol. Due to Lanthanoid contraction both atomic radii and ionic radii decreases gradually in the lanthanoid series.

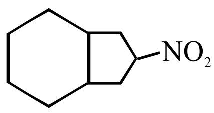
26. The major product formed in the reaction given below will be :



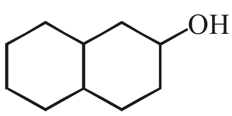
(1)


(2)


(3)


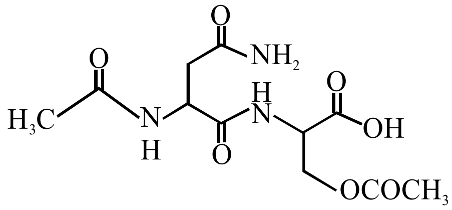
(4)


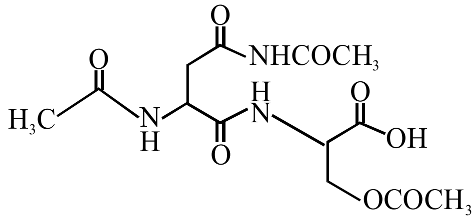
Ans. **(Bonus)**

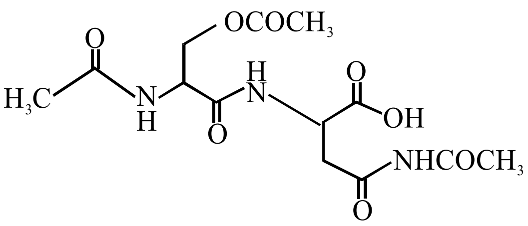
Sol. Answer should be


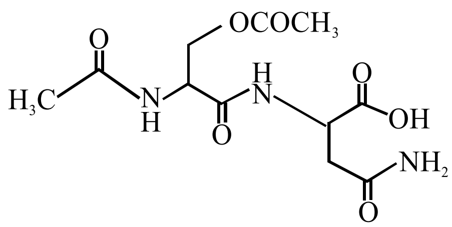
27. The correct structure of product 'P' in the following reaction is :

Asn-Ser + $\underset{\text{(excess)}}{(CH_3CO)_2O} \xrightarrow{NEt_3} P$

(1) 

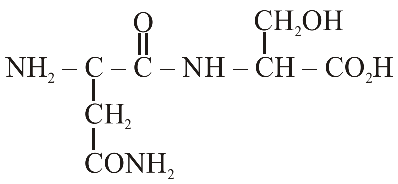
(2) 

(3) 

(4) 

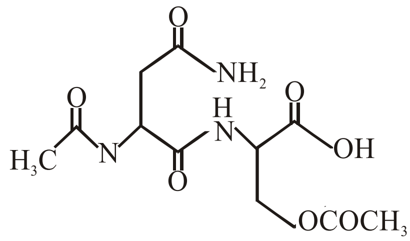
Ans. **(1)**

Sol. Asn–Ser is dipeptide having following structure



$\text{Asn} - \text{Ser} + \underset{\text{excess}}{(CH_3CO)_2\,O} \xrightarrow{NEt_3} P$

P is



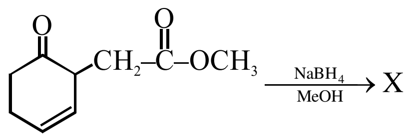
28. Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light :

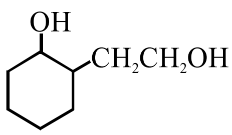
$\underset{\delta}{CH_3} - \underset{\gamma}{CH_2} - \underset{\beta}{CH} = \underset{\alpha}{CH_2}$

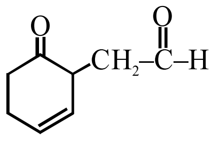
(1) β – hydrogen
(2) γ – hydrogen
(3) δ – hydrogen
(4) α – hydrogen

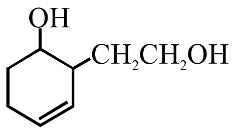
Ans. **(2)**

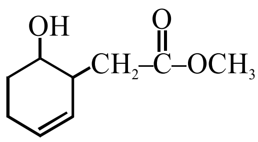
29. The major product 'X' formed in the following reaction is :



(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

30. A mixture of 100 m mol of $Ca(OH)_2$ and 2g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are : (Molar mass of $Ca(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1}, respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

(1) 1.9 g, 0.14 mol L^{-1}

(2) 13.6 g, 0.14 mol L^{-1}

(3) 1.9 g, 0.28 mol L^{-1}

(4) 13.6 g, 0.28 mol L^{-1}

Ans. **(3)**

Sol.

$Ca(OH)_2$	+ Na_2SO_4	$\longrightarrow$ $CaSO_4$	+ 2NaOH
100 m mol	14 m mol	—	—
—	—	14 m mol	28 m mol

$$w_{CaSO_4} = 14 \times 10^{-3} \times 136 = 1.9 gm$$

$$[OH^-] = \frac{28}{100} = 0.28M$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

1. Let $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I[z] respectively denote the real and imaginary parts of z, then :

(1) R(z) > 0 and I(z) > 0

(2) R(z) < 0 and I(z) > 0

(3) R(z) = –3

(4) I(z) = 0

Ans. **(4)**

Sol. $z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$

$z = \left(e^{i\pi/6}\right)^5 + \left(e^{-i\pi/6}\right)^5$

$= e^{i5\pi/6} + e^{-i5\pi/6}$

$= \cos\frac{5\pi}{6} + i\frac{\sin 5\pi}{6} + \cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)$

$= 2\cos\frac{5\pi}{6} < 0$

I(z) = 0 and Re(z) < 0

Option (4)

2. Let $a_1, a_2, a_3, ..., a_{10}$ be in G.P. with $a_i > 0$ for i = 1,2,...., 10 and S be the set of pairs (r,k), r k∈N (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :

(1) Infinitely many (2) 4

(3) 10 (4) 2

Ans. **(1)**

Sol. Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get D = 0

Option (1)

3. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :

(1) $\sqrt{5}$ (2) 4

(3) $2\sqrt{2}$ (4) 3

Ans. **(2)**

Sol. $x^2\left({}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$

$x^2\left[{}^{10}C_r (x)^{\frac{10-r}{2}} (\lambda)^r (x)^{-2r}\right]$

$x^2\left[{}^{10}C_r\ \lambda^r\ x^{\frac{10-5r}{2}}\right]$

∴ r = 2

Hence, ${}^{10}C_2\ \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

Option (2)

4. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot .....\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

Ans. **(3)**

Sol. $2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}......\cos\frac{\pi}{2^2}$

$\frac{1}{2^9}\sin\frac{\pi}{2} = \frac{1}{512}$

Option (3)

5. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$ (4) $\frac{3}{20}(4\pi-3)$

Ans. (4)

Sol. $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$

$$= \int_{\frac{-\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^{0} \frac{dx}{-1-1+4}$$

$$+\int_{0}^{1} \frac{dx}{0+0+4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{1+0+4}$$

$$\int_{\frac{-\pi}{2}}^{-1} \frac{dx}{1} + \int_{-1}^{0} \frac{dx}{2} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{5}$$

$$\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20}+\frac{6\pi}{10}$$

$$\frac{-9}{20}+\frac{3\pi}{5}$$

Option (4)

6. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5

(3) 4 (4) 3

Ans. (2)

Sol. $1 - {}^nC_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n > \frac{5}{6}$

$$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$$

$n_{min} = 5 \Rightarrow$ Option (2)

7. If mean and standard deviation of 5 observations x_1, x_2, x_3,x_4,x_5 are 10 and 3, respectively, then the variance of 6 observations x_1,x_2, ...,x_5 and –50 is equal to :

(1) 582.5 (2) 507.5

(3) 586.5 (4) 509.5

Ans. (2)

Sol. $\bar{x} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 8$$

$$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 109$$

$$\text{variance} = \frac{\sum_{i=1}^{5}(x_i)^2 + (-50)^2}{6} - \left(\sum_{i=1}^{5} \frac{x_i - 50}{6}\right)$$

$= 507.5$

Option (2)

8. The length of the chord of the parabola $x^2 = 4y$ having equation $x-\sqrt{2}y+4\sqrt{2}=0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

Ans. (3)

Sol. $x^2 = 4y$

$x-\sqrt{2}y+4\sqrt{2}=0$

Solving together we get

$$x^2=4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$$

$\sqrt{2}x^2+4x+16\sqrt{2}$

$\sqrt{2}x^2-4x-16\sqrt{2}=0$

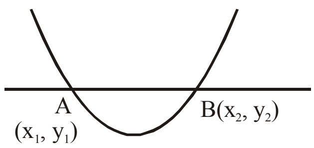
$x_1 + x_2 = 2\sqrt{2}$; $\quad x_1x_2 = \dfrac{-16\sqrt{2}}{\sqrt{2}} = -16$

Similarly,

$\left(\sqrt{2}y-4\sqrt{2}\right)^2=4y$

$2y^2 + 32 - 16y = 4y$

$2y^2 - 20y + 32 = 0 \quad \begin{cases} y_1+y_2=10 \\ y_1\,y_2=16 \end{cases}$



$\ell_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{\left(2\sqrt{2}\right)^2+64+(10)^2-4(16)}$

$=\sqrt{8+64+100-64}$

$=\sqrt{108}=6\sqrt{3}$

Option (3)

9. Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ where $b > 0$. Then the minimum value of $\dfrac{\det(A)}{b}$ is :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$

(3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Ans. **(4)**

Sol. $A = \begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ $(b > 0)$

$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1\ (b^2 - b^2 - 1)$

$|A| = 2(b^2 + 2) - b^2 - 1$

$|A| = b^2 + 3$

$\dfrac{|A|}{b}=b+\dfrac{3}{b} \Rightarrow \dfrac{b+\frac{3}{b}}{2}\ge\sqrt{3}$

$b+\dfrac{3}{b}\ge2\sqrt{3}$

Option (4)

10. The tangent to the curve, $y=xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\dfrac{4}{3},2e\right)$ (2) (2,3e)

(3) $\left(\dfrac{5}{3},2e\right)$ (4) (3,6e)

Ans. **(1)**

Sol. $y=xe^{x^2}$

$\left.\dfrac{dy}{dx}\right|_{(1,\,e)}=\left.\left(e\cdot e^{x^2}\cdot2x+e^{x^2}\right)\right|_{(1,\,e)} = 2 \cdot e + e = 3e$

T : $y - e = 3e\ (x - 1)$

$y = 3ex - 3e + e$

$y = (3e)x - 2e$

$\left(\dfrac{4}{3},\ 2e\right)$ lies on it

Option (1)

11. The number of values of $\theta\in(0,\pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)\ y + 2z = 0$

has a non-trivial solution, is :

(1) One (2) Three

(3) Four (4) Two

Ans. **(4)**

Sol. $\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta)$
$+ 7(-\cos 2\theta - 4\sin 3\theta) = 0$
$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta$
$- 28\sin 3\theta = 0$
$14 - 7\sin 3\theta - 14\cos 2\theta = 0$
$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$
$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$
$7\sin\theta[-3 + 4\sin^2\theta + 4\sin\theta] = 0$
$\sin\theta,$
$4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$
$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = \frac{-3}{2}$; $\sin\theta = \frac{1}{2}$

Hence, 2 solutions in $(0, \pi)$
Option (4)

12. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then $f'(1/2)$ is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{18}{25}$ (4) $\frac{4}{5}$

Ans. **(2)**

Sol. $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$ $f'\left(\frac{1}{2}\right) = ?$

Differentiate w.r.t. 'x'
$f(x) = 2x + 0 - x^2 f(x)$

$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$

$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

13. Let $f : (-1,1) \to R$ be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

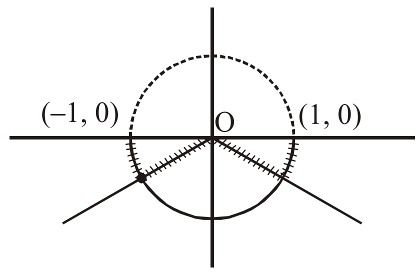
(1) Three elements (2) One element

(3) Five elements (4) Two elements

Ans. **(1)**

Sol. $f : (-1, 1) \to R$

$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$



Non-derivable at 3 points in (–1, 1)
Option (1)

14. Let $S = \left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

Ans. **(4)**

Sol. $\frac{y^2}{1+r}-\frac{x^2}{1-r}=1$

for r > 1, $\frac{y^2}{1+r}+\frac{x^2}{r-1}=1$

$e=\sqrt{1-\left(\frac{r-1}{r+1}\right)}$

$=\sqrt{\frac{(r+1)-(r-1)}{(r+1)}}$

$=\sqrt{\frac{2}{r+1}}=\sqrt{\frac{2}{r+1}}$

Option (4)

15. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to :

(1) $2^{25}-1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

Ans. **(3)**

Sol. $\sum_{r=0}^{25}{}^{50}C_r\cdot{}^{50-r}C_{25-r}$

$=\sum_{r=0}^{25}\frac{50!}{r!\,(50-r)!}\times\frac{(50-r)!}{(25)!\,(25-r)!}$

$=\sum_{r=0}^{25}\frac{50!}{25!\,25!}\times\frac{25!}{(25-r)!\,(r!)}$

$={}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r=\left(2^{25}\right){}^{50}C_{25}$

$\therefore$ $K=2^{25}$

Option (3)

16. Let N be the set of natural numbers and two functions f and g be defined as f,g : N→N such that : $f(n)=\begin{cases}\frac{n+1}{2} & \text{if } n \text{ is odd}\\ \frac{n}{2} & \text{if } n \text{ is even}\end{cases}$

and $g(n) = n-(-1)^n$. The fog is :

(1) Both one-one and onto

(2) One-one but not onto

(3) Neither one-one nor onto

(4) onto but not one-one

Ans. **(4)**

Sol. $f(x)=\begin{cases}\frac{n+1}{2} & n \text{ is odd}\\ n/2 & n \text{ is even}\end{cases}$

$g(x) = n-(-1)^n \begin{cases}n+1\ ; & n \text{ is odd}\\ n-1\ ; & n \text{ is even}\end{cases}$

$f(g(n))=\begin{cases}\frac{n}{2}; & n \text{ is even}\\ \frac{n+1}{2}; & n \text{ is odd}\end{cases}$

$\therefore$ many one but onto

Option (4)

17. The values of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3-\lambda)x + 2 = \lambda$ has the least value is :

(1) 2 (2) $\frac{4}{9}$

(3) $\frac{15}{8}$ (4) 1

Ans. **(1)**

Sol. $\alpha+\beta=\lambda-3$

$\alpha\beta=2-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2+9-6\lambda-4+2\lambda$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

$\therefore$ $\lambda=2$

Option (1)

18. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

(1) Fourth

(2) Second

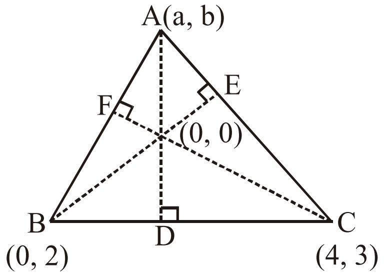
(3) Third

(4) First

Ans. **(2)**

Sol. $m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right)\times\left(\frac{b-0}{a-0}\right)=-1$

$\Rightarrow$ b + 4a = 0 ……(i)



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{(b-2)}{a-0}\right)\times\left(\frac{3}{4}\right)=-1$

$\Rightarrow$ 3b – 6 = –4a $\Rightarrow$ 4a + 3b = 6 ……(ii)

From (i) and (ii)

$a = \frac{-3}{4}$, b = 3

$\therefore$ IInd quadrant.

Option (2)

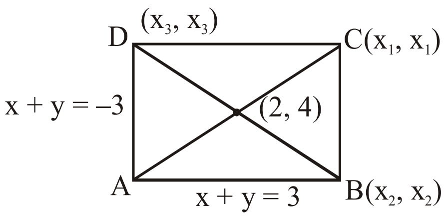
19. Two sides of a parallelogram are along the lines, x + y = 3 and x – y + 3 = 0. If its diagonals intersect at (2,4), then one of its vertex is :

(1) (2,6) (2) (2,1)

(3) (3,5) (4) (3,6)

Ans. **(4)**

Sol.



Solving x + y = 3 and x – y = –3 $\Rightarrow$ A(0, 3)

$\frac{x_1+0}{2} = 2$; $x_i = 4$ similarly $y_1 = 5$

C $\Rightarrow$ (4, 5)

Now equation of BC is x – y = –1

and equation of CD is x + y = 9

Solving x + y = 9 and x – y = –3

Point D is (3, 6)

Option (4)

20. Let $\vec{\alpha} = (\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(1) –3 (2) 4

(3) 3 (4) –4

Ans. **(4)**

Sol. $\vec{\alpha} = (\lambda-2)\vec{\alpha}+\vec{b}$

$\vec{\beta} = (4\lambda-2)\vec{\alpha}+3\vec{b}$

$\frac{\lambda-2}{4\lambda-2}=\frac{1}{3}$

3λ – 6 = 4λ – 2

$\boxed{\lambda=-4}$

$\therefore$ Option (4)

21. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Ans. **(3)**

Sol. $\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))\right)$

$\cot\left(\sum_{n=1}^{19}\cot^{-1}(n^2+n+1)\right) = \cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$

$\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}n)$

$\cot(\tan^{-1}20 - \tan^{-1}1) = \frac{\cot A\cot\beta+1}{\cot\beta-\cot A}$

(Where tanA=20 , tanB=1) $\frac{1\left(\frac{1}{20}\right)+1}{1-\frac{1}{20}}=\frac{21}{19}$

$\therefore$ Option (3)

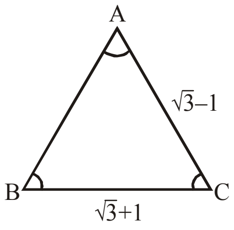
22. With the usual notation, in ΔABC, if $\angle A+\angle B=120°$, $a=\sqrt{3}+1$ and $b=\sqrt{3}-1$, then the ratio $\angle A:\angle B$, is :

(1) 7 : 1 (2) 5 : 3

(3) 9 : 7 (4) 3 : 1

Ans. **(1)**

Sol. A + B = 120°



$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$=\frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30°) = \frac{1}{\sqrt{3}}.\sqrt{3}=1$$

$$\frac{A-B}{2}=45° \quad \Rightarrow \quad A-B=90°$$

$$A+B=120°$$

$$2A = 210°$$

$$A = 105°$$

$$B = 15°$$

$\therefore$ Option (1)

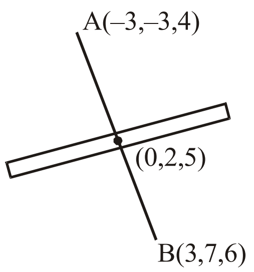
23. The plane which bisects the line segment joining the points (–3,–3,4) and (3,7,6) at right angles, passes through which one of the following points ?

(1) (4, –1,7) (2) (4,1,–2)

(3) (–2,3,5) (4) (2,1,3)

Ans. **(2)**

Sol.



$\vec{n}=3\hat{i}+5\hat{j}+\hat{k}$

p : 3(x – 0) + 5 (y – 2) + 1 (z – 5) = 0

3x + 5y + z = 15

$\therefore$ Option (2)

24. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

(1) $(P \wedge Q) \vee (\sim R)$

(2) $(\sim P) \wedge (\sim Q \wedge R)$

(3) $(\sim P) \vee (Q \wedge R)$

(4) $P \vee (\sim Q \wedge R)$

Ans. **(4)**

Sol. It is obvious

$\therefore$ Option (4)

25. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, x + y + z = 2 ?

(1) $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

(2) $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

(3) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

(4) $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$

Ans. **(3)**

Sol. General point on the given line is

x = 2λ + 4

y = 2λ + 5

z = λ + 3

Solving with plane,

2λ + 4 + 2λ + 5 + λ + 3 = 2

5λ + 12 = 2

5λ = –10

$\boxed{\lambda=-2}$

$\therefore$ Option (3)

26. Let f be a differentiable function such that

$f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}, (x > 0)$ and $f(1) \neq 4$.

Then $\lim\limits_{x \to 0^+} x f\left(\frac{1}{x}\right)$:

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

Ans. (1)

Sol. $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x} \quad (x > 0)$

Given $f(1) \neq 4 \qquad \lim\limits_{x \to 0^+} xf\left(\frac{1}{x}\right) = ?$

$\frac{dy}{dx} + \frac{3}{4}\frac{y}{x} = 7$ (This is LDE)

IF = $e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln|x|} = x^{\frac{3}{4}}$

$y.x^{\frac{3}{4}} = \int 7.x^{\frac{3}{4}}dx$

$y.x^{\frac{3}{4}} = 7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$

f(x) = $4x + C.x^{-\frac{3}{4}}$

$f\left(\frac{1}{x}\right) = \frac{4}{x} + C.x^{\frac{3}{4}}$

$\lim\limits_{x \to 0^+} xf\left(\frac{1}{x}\right) = \lim\limits_{x \to 0^+}\left(4 + C.x^{\frac{7}{4}}\right) = 4$

$\therefore$ Option (1)

27. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$

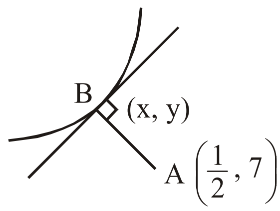
(3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

Ans. (3)

Sol. $y - x^{3/2} = 7 \ (x \geq 0)$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$



$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$

$\frac{3}{2} \cdot x^2 = \frac{1}{2} - x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x + 1)(3x - 1) = 0$

$\therefore$ x = –1 (rejected)

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{3+4}{9 \times 12}}$

$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

Option (3)

28. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

(1) $-4x^3 - 1$ (2) $4x^3 + 1$

(3) $-2x^3 - 1$ (4) $-2x^3 + 1$

Ans. (1)

Sol. $\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$

Put $x^3 = t$

$3x^2\,dx = dt$

$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$

$\frac{1}{3}\int t \cdot e^{-4t}\,dt$

$\frac{1}{3}\left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt\right]$

$-\frac{e^{-4t}}{48}[4t+1]+c$

$\frac{-e^{-4x^3}}{48}[4x^3+1]+c$

$\therefore$ $f(x) = -1 - 4x^3$

Option (1)

(From the given options (1) is most suitable)

29. The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy\,dy = 0$ which passes through (1,1) is :

(1) A circle with centre on the y-axis

(2) A circle with centre on the x-axis

(3) An ellipse with major axis along the y-axis

(4) A hyperbola with transverse axis along the x-axis

Ans. (2)

Sol. $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Solving we get,

$\int \frac{2v}{v^2+1} dv = \int -\frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$\boxed{y^2 + x^2 = 2x}$

$\therefore$ Option (2)

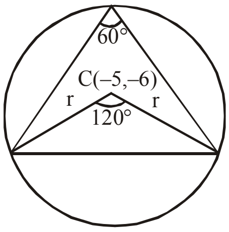
30. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20 (2) 25

(3) 13 (4) –25

Ans. (2)

Sol. $3\left(\frac{1}{2}r^2.\sin 120°\right) = 27\sqrt{3}$



$$\frac{r^2}{2}\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$$

$$r^2 = \frac{108}{3} = 36$$

Radius = $\sqrt{25+36-C} = \sqrt{36}$

$\boxed{C = 25}$

$\therefore$ Option (2)

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

PHYSICS

1. Two forces P and Q of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle is :

(1) 30º (2) 60º (3) 90º (4) 120º

Ans. **(4)**

Sol. $4F^2 + 9F^2 + 12F^2 \cos\theta = R^2$

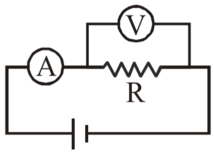
$4F^2 + 36F^2 + 24F^2 \cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2 \cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$$\cos\theta = -\frac{12F^2}{24F^2} = -\frac{1}{2}$$

2. The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :



(1) 350Ω (2) 570Ω (3) 35 Ω (4) 600 Ω

Ans. **(2)**

Sol. $0.95\,R = \frac{R R_\upsilon}{R + R_\upsilon}$

$0.95 \times 30 = 0.05\,R_\upsilon$

$R_\upsilon = 19 \times 30 = 570\ \Omega$

3. An unknown metal of mass 192 g heated to a temperature of 100ºC was immersed into a brass calorimeter of mass 128 g containing 240 g of water a temperature of 8.4ºC Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5ºC (Specific heat of brass is 394 J kg^{-1} K^{-1})

(1) 1232 J kg^{-1} K^{-1} (2) 458 J kg^{-1} K^{-1}

(3) 654 J kg^{-1} K^{-1} (4) 916 J kg^{-1} K^{-1}

Ans. **(4)**

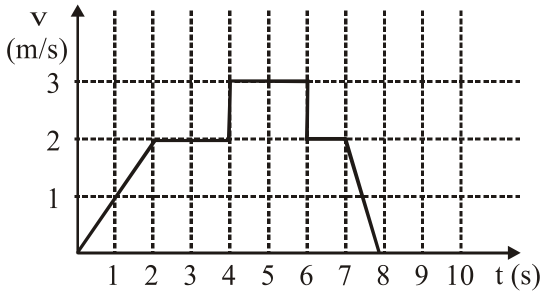
Sol. $192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4)$

$+ 240 \times 4200 \times (21.5 - 8.4)$

$\Rightarrow S = 916$

4. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?



(1) 6 m (2) 9 m (3) 3 m (4) 10 m

Ans. **(2)**

S = Area under graph

$$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9\text{ m}$$

5. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is :

(1) 437.5 J (2) 637.5 J

(3) 740 J (4) 540 J

Ans. **(1)**

$$L\frac{di}{dt} = 25$$

$$L \times \frac{15}{1} = 25$$

$$L = \frac{5}{3}H$$

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5\text{ J}$$

6. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11V is connected across it is :

(1) 11×10^{-5} W (2) 11×10^{-4} W

(3) 11×10^{5} W (4) 11×10^{-3} W

Ans. (1)

$P = I^2R$

$4.4 = 4 \times 10^{-6}$ R

$R = 1.1 \times 10^6\ \Omega$

$$P' = \frac{11^2}{R} = \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5}\,W$$

7. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

(1) 4260 ± 80 cm^3 (2) 4300 ± 80 cm^3

(3) 4264.4 ± 81.0 cm^3 (4) 4264 ± 81 cm^3

Ans. (1)

$$\frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta h}{h} = 2\left(\frac{0.1}{12.6}\right) + \frac{0.1}{34.2}$$

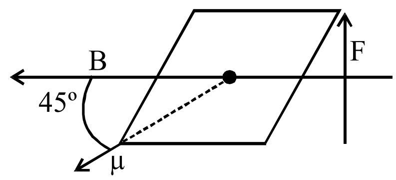
$$V = 12.6 \times \frac{\pi}{4} \times 314.2$$

8. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic neeedle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

(1) 3.6×10^{-5} N (2) 6.5×10^{-5} N

(3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

Ans. (2)



$$\mu B \sin 45° = F\frac{\ell}{2}\sin 45°$$

$F = 2\mu B$

9. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ?

(1) 2750 kHz (2) 2000 kHz

(3) 2250 kHz (4) 2900 kHz

Ans. (2)

$$f_{carrier} = \frac{250}{0.1} = 2500\,KHZ$$

∴ Range of signal = 2250 Hz to 2750 Hz

Now check all options : for 2000 KHZ

f_{mod} = 200 Hz

∴ Range = 1800 KHZ to 2200 KHZ

10. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then :

(1) $T_h = 0.5\ T_c$ (2) $T_h = 2\ T_c$

(3) $T_h = 1.5\ T_c$ (4) $T_h = T_c$

Ans. (4)

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

$$T_h = 2\pi\sqrt{\frac{mR^2}{(2\mu)B}}$$

$$T_C = 2\pi\sqrt{\frac{1/2mR^2}{\mu B}}$$

11. The electric field of a plane polarized electromagnetic wave in free space at time t= 0 is given by an expression

$$\vec{E}(x,y) = 10\hat{j}\ \cos\ [(6x + 8z)]$$

The magnetic field $\vec{B}$ (x, z, t) is given by : (c is the velocity of light)

(1) $\frac{1}{c}\left(6\hat{k} + 8\hat{i}\right)\cos\left[\left(6x - 8z + 10ct\right)\right]$

(2) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos\left[\left(6x + 8z - 10ct\right)\right]$

(3) $\frac{1}{c}\left(6\hat{k} + 8\hat{i}\right)\cos\left[\left(6x + 8z - 10ct\right)\right]$

(4) $\frac{1}{c}\left(6\hat{k} - 8\hat{i}\right)\cos\left[\left(6x + 8z + 10ct\right)\right]$

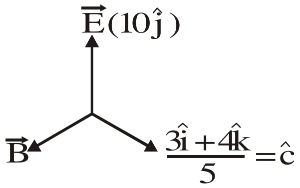
Ans. (2)

$$\vec{E} = 10\hat{j}\cos\left[\left(6\hat{i}+8\hat{k}\right)\cdot\left(x\hat{i}+z\hat{k}\right)\right]$$

$$= 10\hat{j}\cos[\vec{K}\cdot\vec{r}]$$

$\therefore\ \vec{K} = 6\hat{i}+8\hat{k}$; direction of waves travel.

i.e. direction of 'c'.



$\therefore$ Direction of $\hat{B}$ will be along

$$\hat{C}\times\hat{E} = \frac{-4\hat{i}+3\hat{k}}{5}$$

Mag. of $\vec{B}$ will be along $\hat{C}\times\hat{E} = \frac{-4\hat{i}+3\hat{k}}{5}$

Mag. of $\vec{B} = \frac{E}{C} = \frac{10}{C}$

$$\therefore\ \vec{B} = \frac{10}{C}\left(\frac{-4\hat{i}+3\hat{k}}{5}\right) = \frac{\left(-8\hat{i}+6\hat{k}\right)}{C}$$

12. Condiser the nuclear fission

$Ne^{20} \rightarrow 2He^{4} + C^{12}$

Given that the binding energy/nucleon of Ne^{20}, He^{4} and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement :

(1) 8.3 MeV energy will be released

(2) energy of 12.4 MeV will be supplied

(3) energy of 11.9 MeV has to be supplied

(4) energy of 3.6 MeV will be released

Ans. (3)

$Ne^{20} \rightarrow 2He^{4} + C^{12}$

$8.03 \times 20 \qquad 2 \times 7.07 \times 4 + 7.86 \times 12$

$\therefore\ E_B = (BE)_{react} - (BE)_{product} = 9.72$ MeV

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $\left(\vec{A}+\vec{B}\right)$ is 'n' times the magnitude of $\left(\vec{A}-\vec{B}\right)$. The angle between $\vec{A}$ and $\vec{B}$ is :

(1) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (2) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$

(3) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (4) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

Ans. (3)

$$|\vec{A}+\vec{B}| = 2a\cos\theta/2 \qquad \text{\_\_\_(1)}$$

$$|\vec{A}-\vec{B}| = 2a\cos\frac{(\pi-\theta)}{2} = 2a\sin\theta/2 \qquad \text{\_\_\_(2)}$$

$$\Rightarrow\ n\left(2a\cos\frac{\theta}{2}\right) = 2a\frac{\sin\theta}{2}$$

$$\Rightarrow\ \tan\frac{\theta}{2} = n$$

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is :

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$

(3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

Ans. (4)

$$v = \omega\sqrt{A^2-x^2} \qquad \text{\_\_\_(1)}$$

$$a = -\omega^2 x \qquad \text{\_\_\_(2)}$$

$$|v| = |a| \qquad \text{\_\_\_(3)}$$

$$\omega\sqrt{A^2-x^2} = \omega^2 x$$

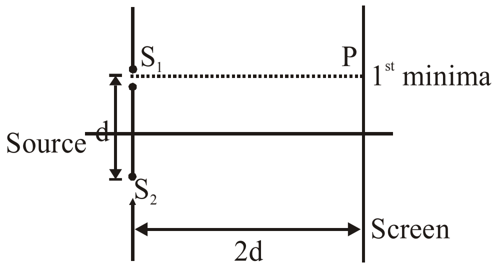
$$A^2 - x^2 = \omega^2 x^2$$

$$5^2 - 4^2 = \omega^2(4^2)$$

$$\Rightarrow\ 3 = \omega \times 4$$

$$T = 2\pi/\omega$$

15. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?



(1) $\frac{\lambda}{2\left(5-\sqrt{2}\right)}$ (2) $\frac{\lambda}{\left(5-\sqrt{2}\right)}$

(3) $\frac{\lambda}{\left(\sqrt{5}-2\right)}$ (4) $\frac{\lambda}{2\left(\sqrt{5}-2\right)}$

Ans. (4)

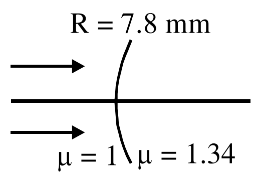
$$\sqrt{5}d - 2d = \frac{\lambda}{2}$$

16. The eye can be regarded as a single refracting surface . The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(1) 2 cm (2) 1 cm

(3) 3.1 cm (4) 4.0 cm

Ans. (3)



$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34-1}{7.8}$$

$\therefore$ V = 30.7 mm

17. Half mole of an ideal monoatomic gas is heated at constant pressure of 1atm from 20 ºC to 90ºC. Work done by gas is close to : (Gas constant R = 8.31 J /mol.K)

(1) 73 J (2) 291 J (3) 581 J (4) 146 J

Ans. (2)

$$WD = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70$$

18. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2.The work function of the metal is 5eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be : [1 eV = 1.6×10^{-19}J]

(1) 10^{10} and 5 eV (2) 10^{14} and 10 eV

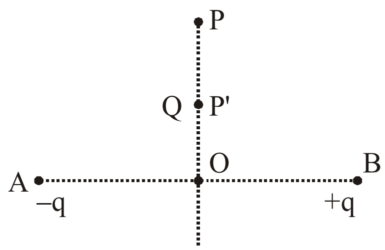
(3) 10^{12} and 5 eV (4) 10^{11} and 5 eV

Ans. (4)

$$I = \frac{nE}{At}$$

$$16\times10^{-3} = \left(\frac{n}{t}\right)_{Photon}\frac{10\times1.6\times10^{-19}}{10^{-4}} = 10^{12}$$

19. Charges –q and +q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences and electrostatic force F. If Q is now moved along the equatorial line to P' such that OP'=$\left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3} >> 2a\right)$



(1) $\frac{F}{3}$ (2) 3F (3) 9F (4) 27F

Ans. **(4)**

Sol. Electric field of equitorial plane of dipole

$$= -\frac{K\vec{P}}{r^3}$$

$$\therefore \text{ At P, F} = -\frac{K\vec{P}}{r^3}Q.$$

$$\text{At P}^1\text{ , F}^1 = -\frac{K\vec{P}Q}{(r/3)^3} = 27F.$$

20. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11}m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (Take Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 1.4×10^5 m/s (2) 24×10^4 m/s

(3) 3.8×10^4 m/s (4) 2.8×10^5 m/s

Ans. **(4)**

By energy convervation between 0 & ∞.

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV^2 = 0 + 0$$

[M is mass of star m is mass of meteroite)

$$\Rightarrow \; v = \sqrt{\frac{4GM}{r}} = 2.8 \times 10^5 \, m/s$$

21. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

(1) 7 (2) 5 (3) 6 (4) 4

Ans. **(1)**

Sol. For closed organ pipe, resonate frequency is odd multiple of fundamental frequency.

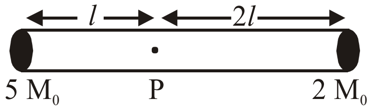
$\therefore$ $(2n + 1) f_0 \le 20{,}000$

(f_o is fundamental frequency = 1.5 KHz)

$\therefore$ n = 6

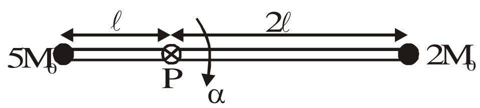
$\therefore$ Total number of overtone that can be heared is 7. (0 to 6).

22. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(1) $\frac{g}{2l}$ (2) $\frac{7g}{3l}$ (3) $\frac{g}{13l}$ (4) $\frac{g}{3l}$

Ans. **(3)**



Applying torque equation about point P.

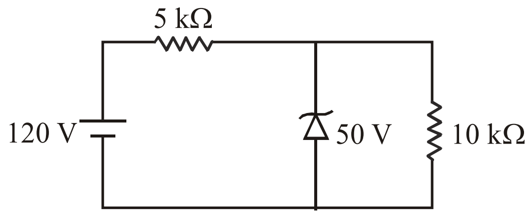
$2M_0\,(2l) - 5\,M_0\,gl = I\alpha$

$I = 2M_0\,(2l)^2 + 5M_0\,l^2 = 13\,M_0 l^2 d$

$$\therefore \quad \alpha = -\frac{M_0 g\ell}{13M_0\ell^2} \;\Rightarrow\; \alpha = -\frac{g}{13\ell}$$

$$\therefore \quad \alpha = \frac{g}{13\ell} \text{ anticlockwise}$$

23. For the circuit shown below, the current through the Zener diode is :



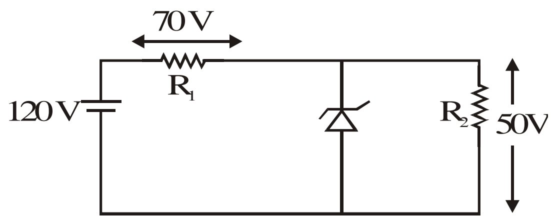
(1) 5 mA (2) Zero (3) 14 mA (4) 9 mA

Ans. **(4)**

Assuming zener diode doesnot undergo breakdown, current in circuit = $\frac{120}{15000} = 8\text{mA}$

$\therefore$ Voltage drop across diode = 80 V > 50 V.

The diode undergo breakdown.



$$\text{Current is } R_1 = \frac{70}{5000} = 14\text{mA}$$

$$\text{Current is } R_2 = \frac{50}{10000} = 5\text{mA}$$

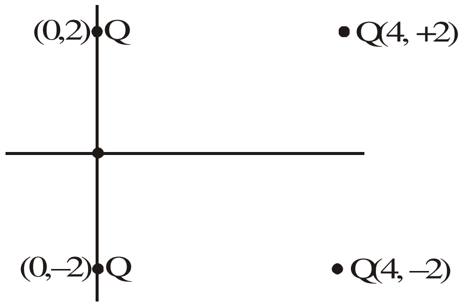
$\therefore$ Current through diode = 9mA

24. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be :

(1) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (2) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

(3) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{Q^2}{4\pi\varepsilon_0}$

Ans. (2)



Potential at origin = $\frac{KQ}{2}+\frac{KQ}{2}+\frac{KQ}{\sqrt{20}}+\frac{KQ}{\sqrt{20}}$

(Potential at ∞ = 0)

$= KQ\left(1+\frac{1}{\sqrt{5}}\right)$

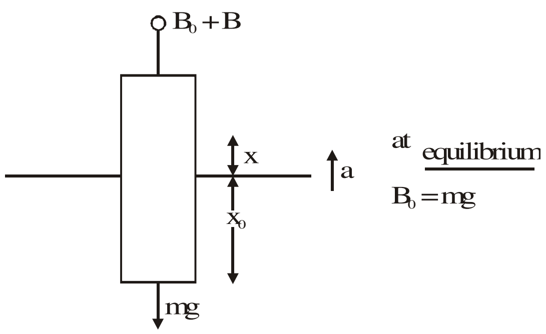
$\therefore$ Work required to put a fifth charge Q at origin is equal to $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

25. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω close to : (density of water = 10^3 kg / m^3)

(1) 5.00 rad s^{-1} (2) 1.25 rad s^{-1}

(3) 3.75 rad s^{-1} (4) 2.50 rad s^{-1}

Ans. (Bonus)



Extra Boyant force = δAxg

$B_0 + B \times mg = ma$

B = ma

$a=\left(\frac{\delta Ag}{m}\right)^x$

$w^2=\frac{\delta Ag}{m}$

$w=\sqrt{\frac{10^3\times\pi(2.5)^2\times10^{-4}\times10}{310\times10^{-6}\times10^3}}$

$=\sqrt{63.30}$ = 7.95

26. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

(1) 692 pJ (2) 60 pJ

(3) 508 pJ (4) 560 pJ

Ans. (3)

Intial energy of capacitor

$U_i=\frac{1}{2}\frac{v^2}{c}$

$=\frac{1}{2}\times\frac{120\times120}{12}=600\text{ J}$

Since battery is disconnected so charge remain same.

Final energy of capacitor

$U_f=\frac{1}{2}\frac{v^2}{c}$

$=\frac{1}{2}\times\frac{120\times120}{12\times6.5}=92$

$W + U_f = U_i$

W = 508 J

27. Two kg of a monoatomic gas is at a pressure of 4 × 10^4 N/m^2 . The density of the gas is 8 kg /m^3. What is the order of energy of the gas due to its thermal motion ?

(1) 10^3 J (2) 10^5 J

(3) 10^6 J (4) 10^4 J

Ans. (4)

Thermal energy of N molecule

$= N\left(\frac{3}{2}kT\right)$

$$=\frac{N}{N_A}\frac{3}{2}RT$$

$$=\frac{3}{2}(nRT)$$

$$=\frac{3}{2}PV$$

$$=\frac{3}{2}P\left(\frac{m}{8}\right)$$

$$=\frac{3}{2}\times 4\times 10^4\times\frac{2}{8}$$

$= 1.5 \times 10^4$

order will 10^4

28. A particle which is experiencing a force, given by $\vec{F}=3\vec{i}-12\vec{j}$, undergoes a displacement of $\vec{d}=4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(1) 15 J (2) 10 J (3) 12 J (4) 9 J

Ans. (1)

Work done = $\vec{F}\cdot\vec{d}$

= 12J

work energy theorem

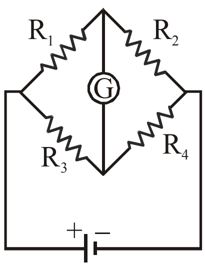
$w_{net} = \Delta K.E.$

$12 = K_f - 3$

$K_f = 15J$

29. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω, respectively.

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be :



(1) Red, Green, Brown
(2) Brown, Blue, Brown
(3) Grey, Black, Brown
(4) Brown, Blue, Black

Ans. (2)

$R_1 = 32 \times 10 = 320$

for wheat stone bridge

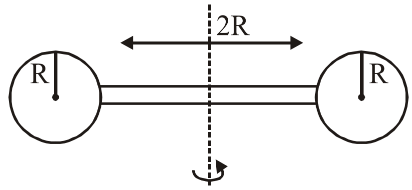
$$\Rightarrow \frac{R_1}{R_3}=\frac{R_2}{R_4}$$

$$\frac{320}{R_3}=\frac{80}{40}$$

$R_3 = 160$

Brown Blue Brown

30. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$

(3) $\frac{137}{15}MR^2$ (4) $\frac{209}{15}MR^2$

Ans. (3)

For Ball

using parallel axis theorem.

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2$$

$$= \frac{22}{5}MR^2$$

2 Balls so $\frac{44}{5}MR^2$

Irod = for rod $\frac{M(2R)^2}{R}=\frac{MR^2}{3}$

$I_{system} = I_{Ball} + I_{rod}$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3}$$

$$= \frac{137}{15}MR^2$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thrusday 10th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

CHEMISTRY

1. An ideal gas undergoes isothermal compression from 5 m^3 against a constant external pressure of 4 Nm^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J mol^{-1} K^{-1}, the temperature of Al increases by :

(1) $\frac{3}{2}$ K (2) $\frac{2}{3}$ K (3) 1 K (4) 2 K

Ans. **(2)**

Sol. Work done on isothermal irreversible for ideal gas

$= -P_{ext}(V_2 - V_1)$

$= -4\ N/m^2\ (1m^3 - 5m^3)$

$= 16$ Nm

Isothermal process for ideal gas

$\Delta U = 0$

$q = -w$

$= -16$ Nm

$= -16$ J

Heat used to increase temperature of $A\ell$

$q = n\ C_m\ \Delta T$

$$16\ J = 1 \times 24\ \frac{J}{mol.K} \times \Delta T$$

$$\Delta T = \frac{2}{3} K$$

2. The 71st electron of an element X with an atomic number of 71 enters into the orbital :

(1) 4f (2) 6p (3) 6s (4) 5d

Ans. **(1)**

3. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B_2H_6, respectively, are :

(1) 2 and 4 (2) 2 and 1

(3) 2 and 2 (4) 4 and 2

Ans. **(4)**

4. The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2 L of its 0.1 M aqueous solution is :

(1) 68.4 g (2) 17.1 g (3) 34.2 g (4) 136.8 g

Ans. **(1)**

Sol. $$\text{Molarity} = \frac{(n)_{solute}}{V_{solution}\ (\text{in lit})}$$

$$0.1 = \frac{wt./342}{2}$$

wt ($C_{12}H_{22}O_{11}$) = 68.4 gram

5. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is :

(1) $H_2 + I_2 \rightarrow 2HI$ (2) $H_2 + F_2 \rightarrow 2HF$

(3) $H_2 + Cl_2 \rightarrow 2HCl$ (4) $H_2 + Br_2 \rightarrow 2HBr$

Ans. **(1)**

6. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of:

(1) sodium ion-ammonia complex

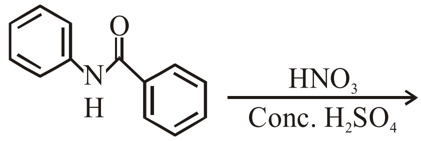
(2) sodamide

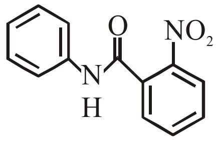
(3) sodium-ammonia complex

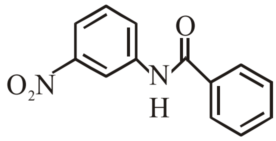
(4) ammoniated electrons

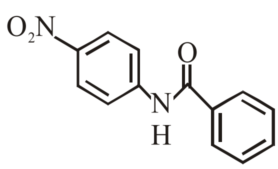
Ans. **(4)**

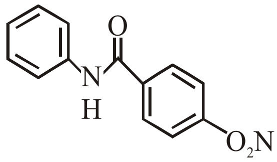
7. What will be the major product in the following mononitation reaction ?



(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

8. In the cell Pt(s)|H_2(g, 1bar|HCl(aq)|Ag(s)|Pt(s) the cell potential is 0.92 when a 10^{-6} molal HCl solution is used. THe standard electrode potential of ($AgCl/Ag,Cl^-$) electrode is :

$$\left\{\text{given}, \frac{2.303RT}{F} = 0.06\text{V at } 298\text{K}\right\}$$

(1) 0.20 V (2) 0.76 V (3) 0.40 V (4) 0.94 V

Ans. (1)

Sol. $Pt(s)|H_2(g, 1bar)|HCl(aq)|AgCl(s)|Ag(s)|Pt(s)$
10^{-6} m

Anode: $H_2 \longrightarrow 2H^+ + 2e \times 1$

Cathode : $e^- + AgCl(s) \longrightarrow Ag(s) + Cl^-(aq) \times 2$

$H_2(g)l + AgCl(s) \longrightarrow 2H^+ + 2Ag(s) + 2Cl^-(aq)$

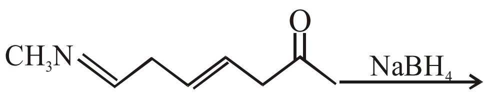
$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log_{10}\left((H^+)^2 \cdot (Cl^-)^2\right)$$

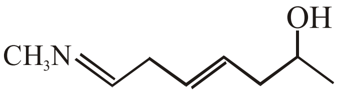
$$.925 = \left(E^0_{H_2/H^+} + E^0_{AgCl/Ag,Cl^-}\right) - \frac{0.06}{2}\log_{10}\left((10^{-6})^2(10^{-6})^2\right)$$

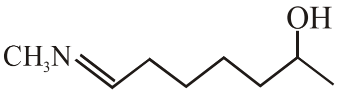
$$.92 = 0 + E^0_{AgCl/Ag,Cl^-} - 0.03\log_{10}(10^{-6})^4$$

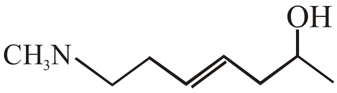
$$E^0_{AgCl}/Ag, Cl^- = .92 + .03 \times -24 = 0.2 \text{ V}$$

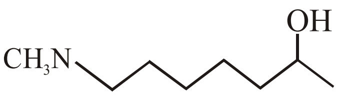
9. The major product of the following recation is:



(1) 

(2) 

(3) 

(4) 

Ans. (3)

10. The pair that contains two P–H bonds in each of the oxoacids is :

(1) H_3PO_2 nad $H_4P_2O_5$

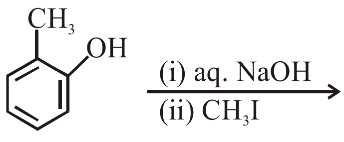
(2) $H_4P_2O_5$ and $H_4P_2O_6$

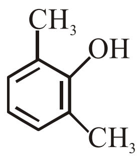
(3) H_3PO_3 and H_3PO_2

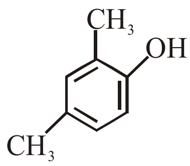
(4) $H_4P_2O_5$ nad H_3PO_3

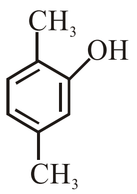
Ans. (1)

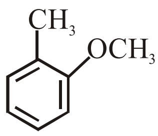
11. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

Ans. (4)

12. The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is :

(1) Fe^{2+} (2) Co^{2+} (3) Mn^{2+} (4) Ni^{2+}

Ans. (2)

13. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms :

(1) hcp lattice-A, $\frac{2}{3}$ Tetrachedral voids-B

(2) hcp lattice-B, $\frac{1}{3}$ Tetrachedral voids-A

(3) hcp lattice-B, $\frac{2}{3}$ Tetrachedral voids-A

(4) hcp lattice-A $\frac{1}{3}$ Tetrachedral voids-B

Ans. (2)

Sol. A_2B_3 has HCP lattice

If A form HCP, then $\frac{3}{4}^{th}$ of THV must occupied by B to form A_2B_3

If B form HCP, then $\frac{1}{3}^{th}$ of THV must occupied by A to form A_2B_3

14. The reaction that is NOT involved in the ozone layer depletion mechanism is the stratosphere is:

(1) $HOCl(g) \xrightarrow{h\upsilon} \dot{O}H(g) + \dot{C}l(g)$

(2) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

(3) $CH_4 + 2O_3 \rightarrow 3CH_2{=}O + 3H_2OP$

(4) $Cl\dot{O}(g) + O(g) \rightarrow \dot{C}l(g) + O_2(g)$

Ans. **(3)**

15. The process with negative entropy change is :

(1) Dissolution of iodine in water

(2) Synthesis of ammonia from N_2 and H_2

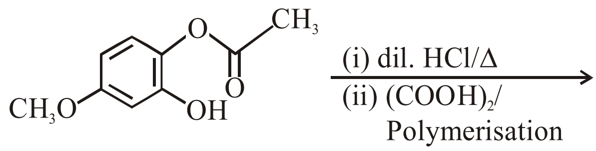
(3) Dissolution of $CaSO_4(s)$ to $CaO(s)$ and $SO_3(g)$

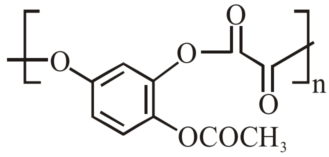
(4) Subimation of dry ice

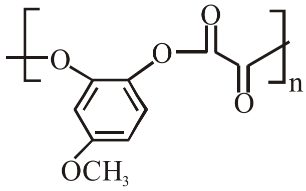
Ans. **(2)**

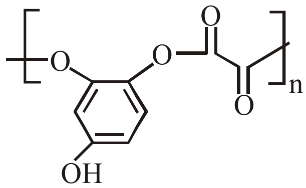
Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$; $\Delta n_g < 0$

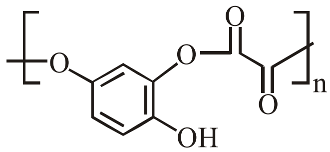
16. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

17. A reaction of cobalt(III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) B (green coloured). A can show optial actively, B is optically inactive. What type of isomers does A and B represent ?

(1) Geometrical isomers

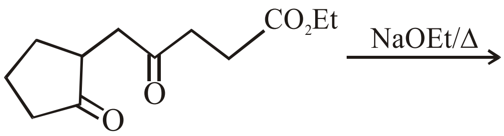
(2) Ionisation isomers]

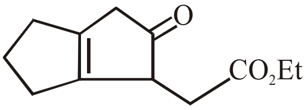
(3) Coordination isomers

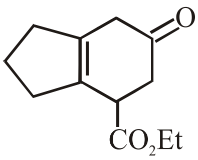
(4) Linkage isomers

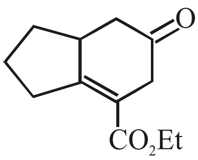
Ans. **(1)**

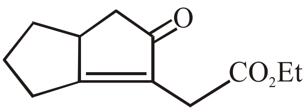
18. The major product obtained in the following reaction is :



(1) 

(2) 

(3) 

(4) 

Ans. (4)

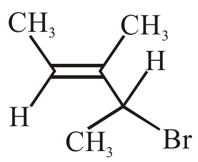
19. Which of the following tests cannot be used for identifying amino acids ?

(1) Biuret test (2) Xanthoproteic test

(3) Barfoed test (4) Ninhydrin test

Ans. (3)

20. What is the IUPAC name of the following compound ?



(1) 3-Bromo-1, 2-dimethylbut-1-ene]

(2) 4-Bromo-3-methylpent-2-ene

(3) 2-Bromo-3-methylpent-3-ene

(4) 3-Bromo-3-methyl-1, 2-dimethylprop-1-ene

Ans. (2)

21. Which is the most suitable reagent for the following transformation ?

$$CH_3–CH=CH–CH_2–\overset{\displaystyle OH}{\overset{|}{C}}H–CH_3 \longrightarrow$$

$$CH_3–CH=CH–CH_2CO_2H$$

(1) alkaline $KMnO_4$ (2) I_2/NaOH

(3) Tollen's reagent (4) CrO_2/CS_2

Ans. (2)

22. The correct match between item 'I' and item 'II' is :

Item 'I' (compound)	Item 'II' (reagent)
(A) Lysine	(P) 1-naphthol
(B) Furfural	(Q) ninhydrin
(C) Benzyl alcohol	(R) $KMnO_4$
(D) Styrene	(S) Ceric ammonium nitrate

(1) (A)→(Q), (B)→(P), (C)→(S), (D)→(R)

(2) (A)→(Q), (B)→(R), (C)→(S), (D)→(P)

(3) (A)→(Q), (B)→(P), (C)→(R), (D)→(S)

(4) (A)→(R), (B)→(P), (C)→(Q), (D)→(S)

Ans. (1)

23. In the reaction of oxalate with permaganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is :

(1) 10 (2) 2 (3) 1 (4) 5

Ans. (3)

Sol. $$2\overset{+7}{M}nO_4 + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2\overset{+2}{M}n^{2+} + 10CO_2 + 8H_2O$$

10 e^- trans for 10 molecules of CO_2 so per molecule of CO_2 transfer of e^- is '1'

24. 5.1g NH_4SH is introduced in 3.0 L evacuated flask at 327°C. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at 327°C is (R = 0.082 L atm $mol^{-1}K^{-1}$, Molar mass of S = 32 g $mol^{/01}$, molar mass of N = 14g mol^{-1})

(1) 1×10^{-4} atm^2 (2) 4.9×10^{-3} atm^2

(3) 0.242 atm^2 (4) 0.242×10^{-4} atm^2

Ans. (3)

$$NH_4SH(s) \rightleftharpoons NH_3(g) + H_2S(g)$$

Sol. $n = \frac{5.1}{51} = .1\text{mole} \quad 0 \quad 0$

$.1(-1-\alpha) \quad .1\alpha \quad .1\alpha$

α = 30% = .3

so number of moles at equilibrium

$.1\ (1 - .3) \quad .1 \times .3 \quad .1 \times .3$

$= \quad .07 \quad =.03 \quad =.03$

Now use PV = nRT at equilibrium

$P_{total} \times 3$ lit = (.03 + .03) × .082 × 600

P_{total} = .984 atm

At equilibrium

$$P_{NH_3} = P_{H_2S} = \frac{P_{total}}{2} = .492$$

So $k_p = P_{NH_3} \cdot P_{H_2S} = (.492)\ (.492)$

k_p = .242 atm^2

25. The electrolytes usually used in the electroplating of gold and silver, respectively, are :

(1) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$

(2) $[Au(CN)_2]^-$ and $[Ag\ CI_2]^-$

(3) $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$

(4) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$

Ans. **(4)**

26. Elevation in the boiling point for 1 molal solution of glucose is 2 K. The depression in the freezing point of 2 molal solutions of glucose in the same solvent is 2 K. The relation between K_b and K_f is:

(1) $K_b = 0.5\ K_f$ (2) $K_b = 2\ K_f$

(3) $K_b = 1.5\ K_f$ (4) $K_b = K_f$

Ans. **(2)**

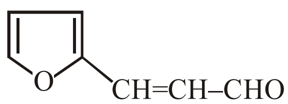
Sol. **Ans.(2)**

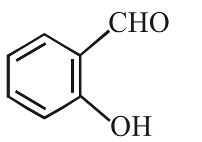
$$\frac{\Delta T_b}{\Delta T_f} = \frac{i.m \times k_b}{i \times m \times k_f}$$

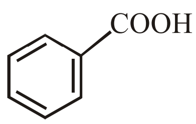
$$\frac{2}{2} = \frac{1 \times 1 \times k_b}{1 \times 2 \times k_f}$$

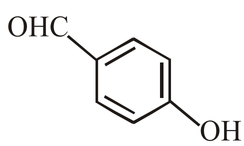
$k_b = 2k_f$

27. An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N. The structure of 'A' is :

(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

28. The ground state energy of hydrogen atom is –13.6 eV. The energy of second excited state He^+ ion in eV is :

(1) –6.04 (2) –27.2 (3) –54.4 (4) –3.4

Ans. **(1)**

Sol. $(E)_{n^{th}} = (E_{GND})_H \cdot \frac{Z^2}{n^2}$

$$E_{3^{rd}}(He^+) = (-13.6eV) \cdot \frac{2^2}{3^2} = -6.04\ eV$$

29. For an elementary chemical reaction,

$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is :

(1) $2k_1[A_2]-k_{-1}[A]^2$ (2) $k_1[A_2]-k_{-1}[A]^2$

(3) $2k_1[A_2]-2k_{-1}[A]^2$ (4) $k_1[A_2]+k_{-1}[A]^2$

Ans. **(3)**

Sol. **Ans.(3)**

$$A_2 \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} 2A$$

$$\frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$$

30. Haemoglobin and gold sol are examples of :

(1) negatively charged sols

(2) positively charged sols]

(3) negatively and positively charged sols, respectively

(4) positively and negatively charged sols, respectively

Ans. **(4)**

Sol. **Ans.(4)**

Haemoglobin $\longrightarrow$ positive sol

Ag – sol $\longrightarrow$ negative sol

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

1. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. It $AA^T = I_3$, then $|p|$ is :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{\sqrt{5}}$

(3) $\frac{1}{\sqrt{6}}$

(4) $\frac{1}{\sqrt{3}}$

Ans. **(1)**

Sol. A is orthogonal matrix

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$

2. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

(1) $\frac{5}{4}$

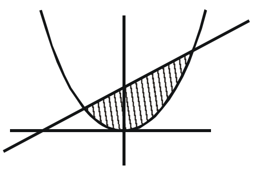
(2) $\frac{9}{8}$

(3) $\frac{3}{4}$

(4) $\frac{7}{8}$

Ans. **(2)**

Sol.



$x = 4y - 2$ & $x^2 = 4y$

$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$x = 2, -1$

So, $\int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx = \frac{9}{8}$

3. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :-

(1) 2 (2) $\frac{\sqrt{5}}{2}$ (3) $\frac{2}{3}$ (4) $\sqrt{2}$

Ans. **(4)**

Sol. Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

4. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :

(1) $\frac{4}{9}$ (2) $\frac{2}{9}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans. **(3)**

Sol. $\frac{a}{1-r} = 3$ ...(1)

$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow r = \frac{2}{3}$ as $|r| < 1$

5. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is :

(1) $-14\hat{i} - 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$

(3) $-10\hat{i} + 5\hat{j}$ (4) $-14\hat{i} + 5\hat{j}$

Ans. **(3)**

Sol. $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$

$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, 3, -3$

So, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

Hence $\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix}$

$= -10\hat{i} + 5\hat{j}$

6. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27} (i = \sqrt{-1})$, where x and y are real numbers, then y – x equals :

(1) –85 (2) 85

(3) –91 (4) 91

Ans. **(4)**

Sol. $\left(-2 - \frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27}$

$= \frac{-198 - 107i}{27} = \frac{x + iy}{27}$

Hence, y – x = 198 – 107 = 91

7. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval (–2, 2), g is :-

(1) differentiable at all points

(2) not differentiable at two points

(3) not continuous

(4) not differentiable at one point

Ans. **(4)**

Sol. $|f(x)| = \begin{cases} 1 & , & -2 \le x < 0 \\ 1 - x^2 & , & 0 \le x < 1 \\ x^2 - 1 & , & 1 \le x \le 2 \end{cases}$

and $f(|x|) = x^2 - 1$, $x \in [-2, 2]$

Hence $g(x) = \begin{cases} x^2 & , & x \in [-2, 0) \\ 0 & , & x \in [0, 1) \\ 2(x^2 - 1) & , & x \in [1, 2] \end{cases}$

It is not differentiable at x = 1

8. Let f : R → R be defined by $f(x) = \frac{x}{1 + x^2}$, $x \in R$. Then the range of f is :

(1) (–1, 1) – {0} (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) R – [–1, 1]

Ans. **(2)**

Sol. $f(0) = 0$ & $f(x)$ is odd.

Further, if x > 0 then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$$

Hence, $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

9. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :

(1) 6 (2) 8 (3) 0 (4) 4

Ans. **(3)**

Sol. $T_5 = {}^8C_4 \frac{x^{12}}{81}\times\frac{81}{x^4} = 5670$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x = \pm\sqrt{3}$

10. The value of r for which

${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + ... {}^{20}C_0\,{}^{20}C_r$ is maximum, is

(1) 20 (2) 15

(3) 11 (4) 10

Ans. **(1)**

Sol. Given sum = coefficient of x^r in the expansion of $(1 + x)^{20}(1 + x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when r = 20

11. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals :

(1) $2(5^2)$ (2) $4(5^2)$

(3) 5^4 (4) 5^3

Ans. **(3)**

Sol. $a_1, a_2, ..., a_{10}$ are in G.P.,

Let the common ratio be r

$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1r^2}{a_1} = 25 \Rightarrow r^2 = 25$

$\frac{a_9}{a_5} = \frac{a_1r^8}{a_1r^4} = r^4 = 5^4$

12. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$

(3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

Ans. **(2)**

Sol. $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$

$\int \frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx$,

Put $\frac{1}{x^2}-1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

Case-1 $x \geq 0$

$-\frac{1}{2}\int\sqrt{t}\ dt \Rightarrow -\frac{t^{3/2}}{3}+C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2}$

$\Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2}+C$

$A(x) = -\frac{1}{3x^3}$ and m = 3

$(A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$

Case-II $x \leq 0$

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x) = \frac{1}{-3x^3}$, m = 3

$(A(x))^m = \frac{-1}{27x^9}$

13. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$ (3) $\frac{c}{3}$ (4) $\frac{3}{2}y$

Ans. **(2)**

Sol. Given $a + b = x$ and $ab = y$

If $x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

14. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$

(where [x] denotes the greatest integer less than ^{20}Cr or equal to x) is :

(1) 4 (2) 4 – sin4

(3) sin 4 (4) 0

Ans. **(4)**

Sol. $$I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$$

$$I = \int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right] + \left[-\frac{x}{\pi}\right] = -1 \text{ as } \quad x \neq n\pi\right)$$

$$I = \int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx = 0$$

15. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more then one solution, then :

(1) $b - c - a = 0$ (2) $a + b + c = 0$

(3) $b + c - a = 0$ (4) $b - c + a = 0$

Ans. **(1)**

Sol. $P_1 : 2x + 2y + 3z = a$

$P_2 : 3x - y + 5z = b$

$P_3 : x - 3y + 2z = c$

We find

$P_1 + P_3 = P_2 \Rightarrow a + c = b$

16. A square is inscribed inthe circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the corrdinate axes. Then the distance of the vertex of this square which is nearest to the origin is :-

(1) 13 (2) $\sqrt{137}$

(3) 6 (4) $\sqrt{41}$

Ans. **(4)**

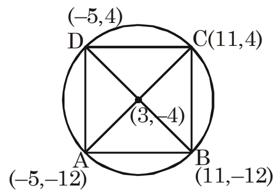
Sol. $R = \sqrt{9+16+103} = 8\sqrt{2}$

OA = 13

$OB = \sqrt{265}$

$OC = \sqrt{137}$

$OD = \sqrt{41}$



17. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to :-

(1) $\frac{5}{12}$ (2) $\frac{-1}{12}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

Ans. **(4)**

Sol. $f_4(x) - f_6(x)$

$$= \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$$

18. Let [x] denote the greatest integer less than or equal to x. Then :-

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$$

(1) equals π

(2) equals 0

(3) equals $\pi + 1$

(4) does not exist

Ans. **(4)**

Sol. R.H.L. $= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$

(as $x \to 0^+ \Rightarrow [x] = 0$)

$$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+x^2}{x^2}$$

$$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)}{\left(\pi\sin^2 x\right)}+1=\pi+1$$

L.H.L. $= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

(as $x \to 0^- \Rightarrow [x] = -1$)

$$\lim_{x\to 0+}\frac{\tan\left(\pi\sin^2 x\right)}{\pi\sin^2 x}.\frac{\pi\sin^2 x}{x^2}+\left(-1+\frac{\sin x}{x}\right)^2 \Rightarrow \pi$$

R.H.L. $\neq$ L.H.L.

19. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an anlge $\frac{\pi}{4}$ with the plane y–z+5=0 are:

(1) $2\sqrt{3}$, 1, –1

(2) 2, $\sqrt{2}$, $-\sqrt{2}$

(3) 2, –1, 1

(4) $\sqrt{2}$, 1, –1

Ans. **(2, 4)**

Sol. Let the equation of plane be

$a(x - 0) + b(y + 1) + c(z - 0) = 0$

It passes through (0,0,1) then

$b + c = 0$ ...(1)

Now $\cos\frac{\pi}{4}=\frac{a(0)+b(1)+c(-1)}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$

we get $a^2 = 2c^2$

$\Rightarrow a=\pm\sqrt{2}\,c$

$\Rightarrow$ direction ratio (a, b, c) = $\left(\sqrt{2}, -1, 1\right)$ or $\left(\sqrt{2}, 1, -1\right)$

20. If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then dy/dx at x = e is equal to :

(1) $\frac{e}{\sqrt{4+e^2}}$

(2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(4) $\frac{(1+2e)}{\sqrt{4+e^2}}$

Ans. **(3)**

Sol. Differentiating with respect to x,

$$x.\frac{1}{\ell nx}.\frac{1}{x}+\ell n(\ell nx)-2x+2y.\frac{dy}{dx}=0$$

at x = e we get

$$1-2e+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2e-1}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2e-1}{2\sqrt{4+e^2}} \quad \text{as } y(e)=\sqrt{4+e^2}$$

21. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

(1) $\frac{\sqrt{5}}{4}$

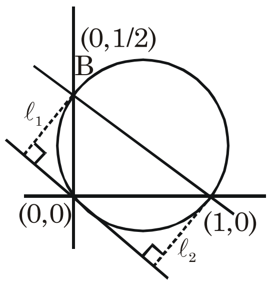
(2) $\frac{\sqrt{5}}{2}$

(3) $2\sqrt{5}$

(4) $4\sqrt{5}$

Ans. **(2)**

Sol.



Equation of circle

$$(x-1)(x-0)+(y-0)\left(y-\frac{1}{2}\right)=0$$

$$\Rightarrow x^2+y^2-x-\frac{y}{2}=0$$

Equation of tangent of origin is 2x + y = 0

$$\ell_1+\ell_2=\frac{2}{\sqrt{5}}+\frac{1}{2\sqrt{5}}$$

$$=\frac{4+1}{2\sqrt{5}}=\frac{\sqrt{5}}{2}$$

22. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(1) $(p \vee r) \rightarrow (p \wedge r)$

(2) $p \vee r$

(3) $p \wedge r$

(4) $(p \wedge r) \rightarrow (p \vee r)$

Ans. **(4)**

Sol. Given q is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow$ q is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

23. If y(x) is the solution of the differential equation

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x},\ x>0,$$

where $y(1)=\frac{1}{2}e^{-2}$, then :

(1) y(x) is decreasing in (0, 1)

(2) y(x) is decreasing in $\left(\frac{1}{2},1\right)$

(3) $y(\log_e 2)=\frac{\log_e 2}{4}$

(4) $y(\log_e 2)=\log_e 4$

Ans. **(2)**

Sol. $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$

$$\text{I.F.}=e^{\int\left(\frac{2x+1}{x}\right)dx}=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\ell nx}=e^{2x}.x$$

So, $y(xe^{2x})=\int e^{-2x}.xe^{2x}+C$

$\Rightarrow xye^{2x}=\int x\,dx+C$

$\Rightarrow 2xye^{2x}=x^2+2C$

It passess through $\left(1,\frac{1}{2}e^{-2}\right)$ we get C = 0

$$y=\frac{xe^{-2x}}{2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}e^{-2x}(-2x+1)$$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2},1\right)$

$$y(\log_e 2)=\frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$$

$$=\frac{1}{8}\log_e 2$$

24. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is :

(1) 122 (2) –222
(3) –122 (4) 222

Ans. **(1)**

Sol. $S = \{x \in R, x^2 + 30 - 11x \le 0\}$
$= \{x \in R, 5 \le x \le 6\}$
Now $f(x) = 3x^3 - 18x^2 + 27x - 40$
$\Rightarrow f'(x) = 9(x-1)(x-3)$,
which is positive in [5, 6]
$\Rightarrow f(x)$ increasing in [5, 6]
Hence maximum value $= f(6) = 122$

25. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(1) –81 (2) 100 (3) –300 (4) 144

Ans. **(3)**

Sol. $81x^2 + kx + 256 = 0$; $x = \alpha, \alpha^3$

$$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$$

Now $-\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27}$

$\Rightarrow k = \pm 300$

26. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) 1 (2) $\sqrt{2}$

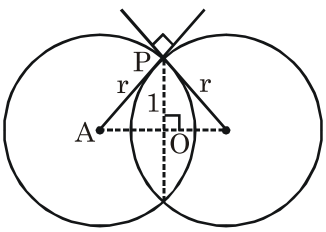
(3) $2\sqrt{2}$ (4) 2

Ans. **(4)**

Sol. In ΔAPO

$$\left(\frac{\sqrt{2}\,r}{2}\right)^2 + 1^2 = r^2$$

$$\Rightarrow \boxed{r = \sqrt{2}}$$



So distance between centres $= \sqrt{2}\,r = 2$

27. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbole $xy = 2$ is :

(1) $x + 2y + 4 = 0$

(2) $x - 2y + 4 = 0$

(3) $x + y + 1 = 0$

(4) $4x + 2y + 1 = 0$

Ans. **(1)**

Sol. Let the equation of tangent to parabola

$y^2 = 4x$ be $y = mx + \frac{1}{m}$

It is also a tangent to hyperbola $xy = 2$

$$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$$

$$\Rightarrow x^2m + \frac{x}{m} - 2 = 0$$

$D = 0 \Rightarrow m = -\frac{1}{2}$

So tangent is $2y + x + 4 = 0$

28. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

(1) (2, 0, –2) (2) (–2, 2, 2)
(3) (0, –2, 2) (4) (2, 2, 0)

Ans. **(1)**

Sol. The normal vector of required plane

$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is (–1, 1, 1)
So required plane is
$-(x-3) + (y+2) + (z-1) = 0$
$\Rightarrow -x + y + z + 4 = 0$
Which is satisfied by (2, 0, –2)

29. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted betwen the coordinate axes lie on the curve :

(1) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (2) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

(3) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (4) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

Ans. **(3)**

Sol. Equation of general tangent on ellipse

$$\frac{x}{a\sec\theta} + \frac{y}{b\operatorname{cosec}\theta} = 1$$

$a = \sqrt{2}$, b = 1

$$\Rightarrow \frac{x}{\sqrt{2}\sec\theta} + \frac{y}{\operatorname{cosec}\theta} = 1$$

Let the midpoint be (h, k)

$$h = \frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$$

and $k = \frac{\operatorname{cosec}\theta}{2} \Rightarrow \sin\theta = \frac{1}{2k}$

$\because \sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

30. Two integers are selected at random from the set {1, 2,...., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(1) $\frac{2}{5}$

(2) $\frac{1}{2}$

(3) $\frac{3}{5}$

(4) $\frac{7}{10}$

Ans. **(1)**

Sol. Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {}^5C_2 + {}^6C_2$

so required probability $= \frac{{}^5C_2}{{}^5C_2 + {}^6C_2}$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS

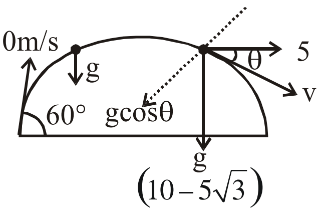
1. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is :

(1) 2.5 m (2) 10.3 m

(3) 2.8 m (4) 5.1 m

Ans. **(3)**

Sol.



v_x = 10cos60° = 5 m/s

v_y = 10cos30° = $5\sqrt{3}$ m/s

velocity after t = 1 sec.

v_x = 5 m/s

$v_y = \left|\left(5\sqrt{3}-10\right)\right| m/s = 10 - 5\sqrt{3}$

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v_x^2+v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10\cos\theta}$$

$$\tan\theta = \frac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100\left(2-\sqrt{3}\right)}{10\cos 15} = 2.8m$$

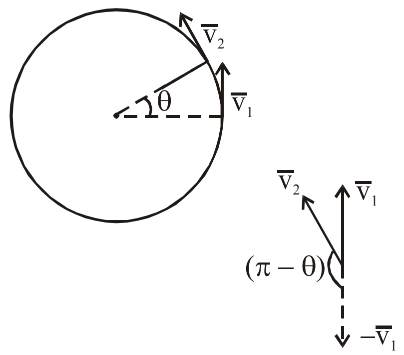
2. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(1) zero (2) 10 m/s

(3) $10\sqrt{3}$ m/s (4) $10\sqrt{2}$ m/s

Ans. **(2)**

Sol.



$$|\Delta\overline{v}| = \sqrt{v_1^2+v_2^2+2v_1v_2\cos(\pi-\theta)}$$

$$= 2v\sin\frac{\theta}{2} \quad \text{since}\left[|\overline{v}_1| = |\overline{v}_2|\right]$$

= (2 × 10) × sin(30°)

= 10 m/s

3. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980Å. The radius of the atom in the excited state, it terms of Bohr radius a_0, will be :

(h_c = 12500 eV – Å)

(1) $9a_0$ (2) $25a_0$ (3) $4a_0$ (4) $16a_0$

Ans. **(4)**

Sol. Energy of photon = $\frac{12500}{980} = 12.75\text{eV}$

∴ Electron will excite to n= 4

Since 'R' ∝ n^2

∴ Radius of atom will be $16a_0$

4. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be :

(1) pv^2 (2) $\frac{3}{4}pv^2$

(3) $\frac{1}{2}pv^2$ (4) $\frac{1}{4}pv^2$

Ans. **(2)**

Sol. Momentum per second carried by liquid per second is $\rho a v^2$

net force due to reflected liquid = $2\times\left[\frac{1}{4}\rho a v^2\right]$

net force due to stopped liquid = $\frac{1}{4}\rho a v^2$

Total force = $\frac{3}{4}\rho a v^2$

net pressure = $\frac{3}{4}\rho v^2$

5. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electrric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by :

(1) $\left(\frac{1}{\sqrt{n}},\frac{1}{\sqrt{n}}\right)$

(2) $\left(\sqrt{n},\frac{1}{\sqrt{n}}\right)$

(3) $\left(\sqrt{n},\sqrt{n}\right)$

(4) $\left(\frac{1}{\sqrt{n}},\sqrt{n}\right)$

Ans. (2)

Sol. $C=\frac{1}{\sqrt{\mu_0 \in_0}}$

$V = \frac{1}{\sqrt{k \in_0 \mu_0}}$ [For transparent medium $\mu_r \approx \mu_0$]

$\therefore\ \frac{C}{V}=\sqrt{k}=n$

$\frac{1}{2}\in_0 E_0^2 C$ = intensity = $\frac{1}{2}\in_0 kE^2 v$

$\therefore\ E_0^2 C = kE^2 v$

$\Rightarrow \frac{E_0^2}{E^2}=\frac{kV}{C}=\frac{n^2}{n} \Rightarrow \frac{E_0}{E}=\sqrt{n}$

similarly

$\frac{B_0^2 C}{2\mu_0}=\frac{B^2 v}{2\mu_0} \Rightarrow \frac{B_0}{B}=\frac{1}{\sqrt{n}}$

6. An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband frequencies (in kHz) are, [Given π = 22/7]

(1) 1785 and 1715

(2) 892.5 and 857.5

(3) 89.25 and 85.75

(4) 178.5 smf 171.5

Ans. (3)

Sol. $V(t)=10+\frac{3}{2}[2\cos A \sin B]$

$=10+\frac{3}{2}[\sin(A+B)-\sin(A-B)]$

$=10+\frac{3}{2}[\sin(57.2\times10^4 t)-\sin(52.8\times10^4 t)]$

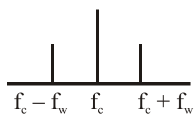
$\omega_1 = 57.2 \times 10^4 = 2\pi f_1$

$f_1=\frac{57.2\times10^4}{2\times\left(\frac{22}{7}\right)} = 9.1\times10^4$

$\simeq 91\text{KHz}$

$f_2=\frac{52.8\times10^4}{2\times\left(\frac{22}{7}\right)}$

$\simeq 84\text{KHz}$



Side band frequency are

$f_1 = f_c - f_w = \frac{52.8\times10^4}{2\pi} \approx 85.00$ kHz

$f_2 = f_c + f_w = \frac{57.2\times10^4}{2\pi} \approx 90.00$ kHz

7. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is :

(1) $M^2L^2T^{-2}$

(2) M^2LT^{-4}

(3) $M^0L^2T^{-4}$

(4) MLT^{-2}

Ans. (2)

Sol. $F=\alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$

$$\left[\frac{x^2}{\alpha KT}\right]=M^\circ L^\circ T^\circ$$

$$\frac{L^2}{[\alpha]ML^2T^{-2}}=M^\circ L^\circ T^\circ$$

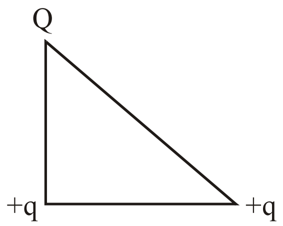
$\Rightarrow$ $[\alpha] = M^{-1}T^2$

$[F] = [\alpha]\ [\beta]$

$MLT^{-2} = M^{-1}T^2[\beta]$

$\Rightarrow [\beta] = M^2LT^{-4}$

8. The charges Q + q and +q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, it the value of Q is:



(1) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (2) $-2q$

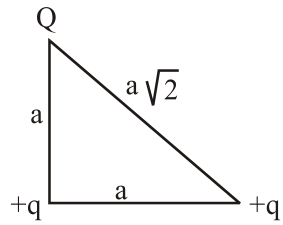
(3) $\frac{-q}{1+\sqrt{2}}$ (4) $+q$

Ans. (1)

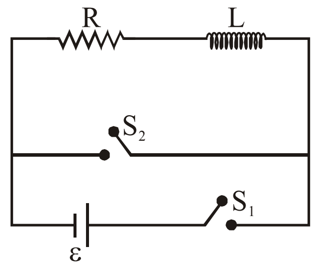
Sol. $U = K\left[\frac{q^2}{a}+\frac{Qq}{a}+\frac{Qq}{a\sqrt{2}}\right]=0$

$\Rightarrow$ $q=-Q\left[1+\frac{1}{\sqrt{2}}\right]$

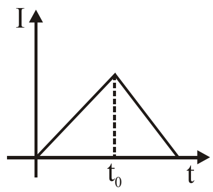
$\Rightarrow$ $Q=\frac{-q\sqrt{2}}{\sqrt{2}+1}$

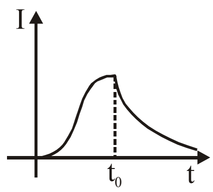


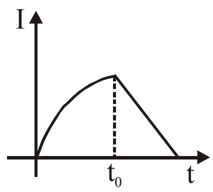
9. In the circuit shown,

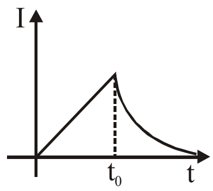


the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behavious of the current I as a function of time 't' is given by :

(1)


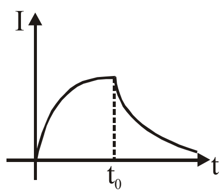
(2)


(3)


(4)


Ans. (2)

Sol. From time t = 0 to t = t_0, growth of current takes place and after that decay of current takes place.



most appropriate is (2)

10. Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin(450 t – 9x) where distance and time are measured is SI units. The tension in the string is :
(1) 10 N (2) 12.5 N (3) 7.5 N (4) 5 N

Ans. **(2)**

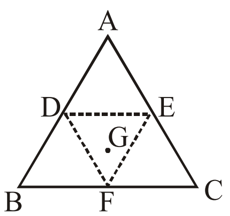
Sol. y = 0.03 sin(450 t – 9x)

$$v=\frac{\omega}{k}=\frac{450}{9}=50\,m/s$$

$$v=\sqrt{\frac{T}{\mu}}\Rightarrow\frac{T}{\mu}=2500$$

$\Rightarrow$ T = 2500 × 5 × 10^{-3}
= 12.5 N

11. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. It the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



(1) $I=\frac{9}{16}I_0$ (2) $I=\frac{3}{4}I_0$

(3) $I=\frac{I_0}{4}$ (4) $I=\frac{15}{16}I_0$

Ans. **(4)**

Sol. Suppose M is mass and a is side of larger triangle, then $\frac{M}{4}$ and $\frac{a}{2}$ will be mass and side length of smaller triangle.

$$\frac{I_{removed}}{I_{original}}=\frac{\frac{M}{4}\cdot\left(\frac{a}{2}\right)^2}{M\cdot(a)^2}$$

$$I_{removed}=\frac{I_0}{16}$$

$$\text{So, } I=I_0-\frac{I_0}{16}=\frac{15I_0}{16}$$

12. There are two long co-axial solenoids of same length l. the inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The rate of mutual inductance to the self-inductance of the inner-coil is :

(1) $\frac{n_2}{n_1}\cdot\frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1}\cdot\frac{r_1}{r_2}$ (3) $\frac{n_1}{n_2}$ (4) $\frac{n_2}{n_1}$

Ans. **(4)**

Sol. $M=\mu_0 n_1 n_2 \pi r_1^2$

$L=\mu_0 n_1^2 \pi r_1^2$

$$\Rightarrow\ \frac{M}{L}=\frac{n_2}{n_1}$$

13. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature,. The relation between temperature and volume of this process is TV^x = constant, then x is :

(1) $\frac{5}{3}$ (2) $\frac{2}{5}$ (3) $\frac{2}{3}$ (4) $\frac{3}{5}$

Ans. **(2)**

Sol. For adiabatic process : $TV^{\gamma-1}$ = constant

For diatomic process : $\gamma-1=\frac{7}{5}-1$

$$\therefore x=\frac{2}{5}$$

14. The gas mixture constists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total inernal energy of the system is:
(1) 12 RT (2) 20 RT (3) 15 RT (4) 4 RT

Ans. **(3)**

Sol. $U=\frac{f_1}{2}n_1RT+\frac{f_2}{2}n_2RT$

$$=\frac{5}{2}(3RT)+\frac{3}{2}\times5RT$$

U = 15RT

15. In a Young's double slit experiment, the path different, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a brigth fringe is close to :

(1) 0.94 (2) 0.74 (3) 0.85 (4) 0.80

Ans. (3)

Sol. $\Delta x = \frac{\lambda}{8}$

$$\Delta\phi = \frac{(2\pi)}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

$$I = I_0 \cos^2\left(\frac{\pi}{8}\right)$$

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

16. If the deBronglie wavelenght of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to :

(Speed of light = 3×10^8 m/s

Planck's constant = 6.63×10^{-34} J.s

Mass of electron = 9.1×10^{-31} kg)

(1) 1.45×10^6 m/s (2) 1.7×10^6 m/s

(3) 1.8×10^6 m/s (4) 1.1×10^6 m/s

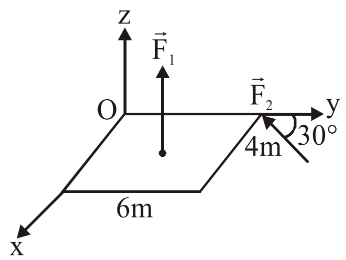
Ans. (1)

Sol. $\frac{h}{mv} = 10^{-3}\left(\frac{3\times10^8}{6\times10^{14}}\right)$

$$v = \frac{6.63\times10^{-34}\times6\times10^{14}}{9.1\times10^{-31}\times3\times10^5}$$

$v = 1.45 \times 10^6$ m/s

17. A slob is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z-axis at the point $\left(2\vec{i} + 3\vec{j}\right)$. The moment of these forces about point O will be :



(1) $\left(3\hat{i} - 2\hat{j} - 3\hat{k}\right)F$ (2) $\left(3\hat{i} + 2\hat{j} + 3\hat{k}\right)F$

(3) $\left(3\hat{i} + 2\hat{j} - 3\hat{k}\right)F$ (4) $\left(3\hat{i} - 2\hat{j} + 3\hat{k}\right)F$

Ans. (4)

Sol. Torque for F_1 force

$$\vec{F}_1 = \frac{F}{2}\left(-\hat{i}\right) + \frac{F\sqrt{3}}{2}\left(-\hat{j}\right)$$

$$\vec{r}_1 = 0\hat{i} + 6\hat{j}$$

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 3F\hat{k}$$

Torque for F_2 force

$$\vec{F}_2 = F\hat{k}$$

$$\vec{r}_2 = 2\hat{i} + 3\hat{j}$$

$$\vec{\tau}_{F_2} = \vec{r}_2 \times \vec{F}_2 = 3F\hat{i} + 2F\left(-\hat{j}\right)$$

$$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2}$$

$$= 3F\hat{i} + 2F\left(-\hat{j}\right) + 3F\left(\hat{k}\right)$$

18. A satellite is revolving in a circular orbit at a height h from the earth surface, such that h << R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed requried so that the satellite could escapte from the gravitational field of earth is :

(1) $\sqrt{gR}\left(\sqrt{2}-1\right)$

(2) $\sqrt{2gR}$

(3) $\sqrt{gR}$

(4) $\sqrt{\frac{gR}{2}}$

Ans. **(1)**

Sol. $v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$

$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR}$

$\Delta v = v_e - v_0 = \left(\sqrt{2}-1\right)\sqrt{gR}$

19. In an experiment electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied.
[Charge of the electron = 1.6×10^{-19} C
Mass of the electron = 9.1×10^{-31} kg]

(1) 7.5×10^{-4} m (2) 7.5×10^{-3} m
(3) 7.5 m (4) 7.5×10^{-2} m

Ans. **(1)**

Sol. $r = \dfrac{\sqrt{2mk}}{eB} = \dfrac{\sqrt{2me\Delta v}}{eB}$

$$r = \frac{\sqrt{\frac{2m}{e}.\Delta v}}{B} = \frac{\sqrt{\frac{2\times9.1\times10^{-31}}{1.6\times10^{-19}}(500)}}{100\times10^{-3}}$$

$$r = \frac{\sqrt{\frac{9.1}{0.16}\times10^{-10}}}{10^{-1}} = \frac{3}{.4}\times10^{-4} = 7.5\times10^{-4}$$

20. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be :

(1) 2 (2) $\frac{1}{9}$ (3) 3 (4) 1

Ans. **(3)**

Sol. $k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

$U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$

Hence ratio is 3 (most appropriate)

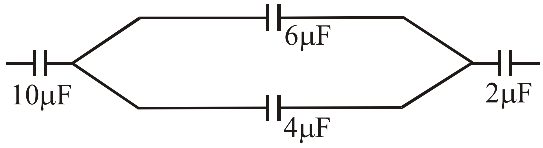
21. Ice at –20° C os added tp 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to
(Specific heat of water = 4.2 J/g/°C)
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334 J/g)

(1) 50 g (2) 40 g
(3) 60 g (4) 100 g

Ans. **(2)**

Sol. Let amount of ice is m gm.
According to principal of calorimeter
heat taken by ice = heat given by water
$\therefore$ 20 × 2.1 × m + (m – 20) × 334
= 50 × 4.2 × 40
376 m = 8400 + 6680
m = 40.1
$\therefore$ correct answer is (2)

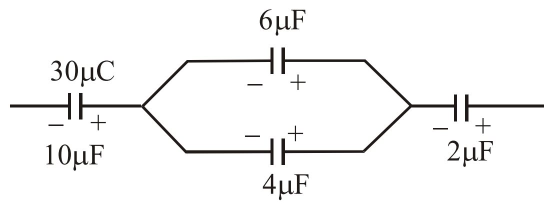
22. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30 μC. ?The charge on the right plate of the 6 μF capacitor is :



(1) –18 μC (2) –12 μC
(3) +12 μC (4) +18 μC

Ans. **(4)**

Sol.



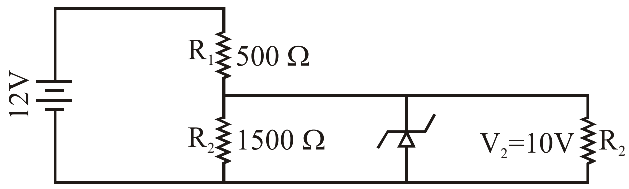
6μF & 4μF are in parallel & total charge on this combination is 30 μC

$\therefore$ Charge on 6μF capacitor = $\frac{6}{6+4}\times30$

= 18 μC

Since charge is asked on right plate therefore is +18μC
Correct answer is (4)

23. In the given circuit the current through Zener Diode is close to :



(1) 6.0 mA (2) 4.0 mA

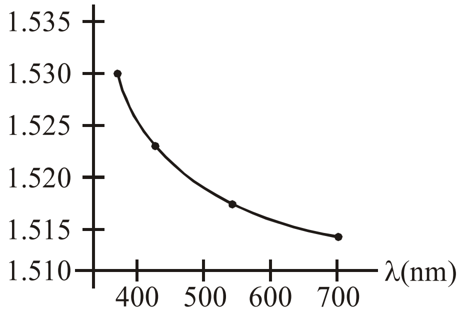
(3) 6.7 mA (4) 0.0 mA

Ans. **(4)**

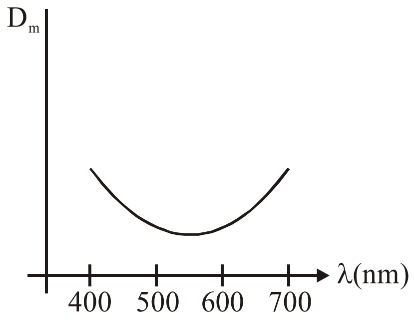
Sol. Since voltage across zener diode must be less than 10V therefore it will not work in breakdown region, & its resistance will be infinite & current through it = 0

$\therefore$ correct answer is (4)

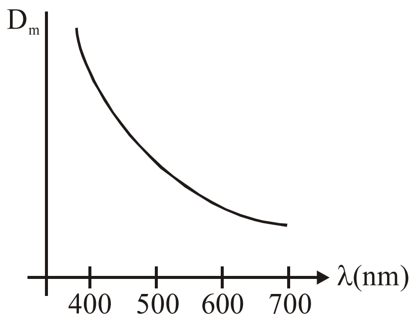
24. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



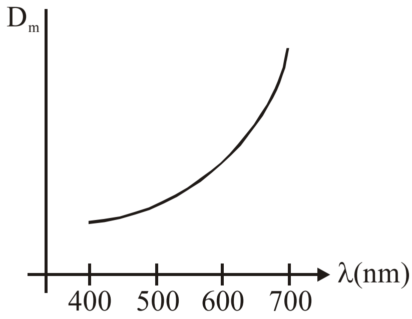
(1)



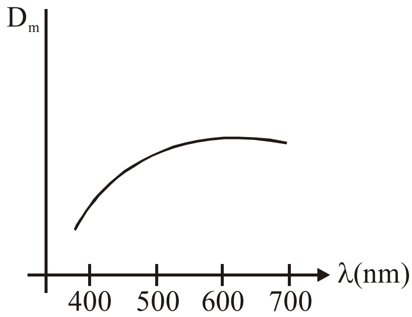
(2)



(3)



(4)

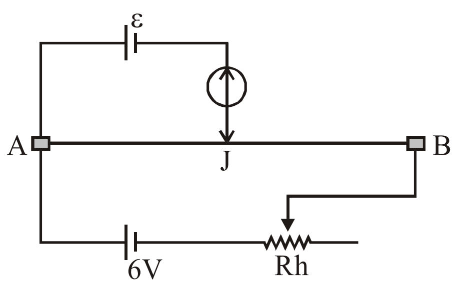


Ans. **(2)**

Sol. Since $D_m = (\mu - 1)A$

& on increasing the wavelength, μ decreases & hence D_m decreases. Therefore correct answer is (2)

25. The resistance of the meter bridge AB is given figure is 4Ω. With a cell of emf $\varepsilon = 0.5$ V and rheostat resistance $R_h = 2\ \Omega$ the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for $R_h = 6\ \Omega$. The emf ε_2 is;



(1) 0.6 V (2) 0.5 V

(3) 0.3 V (4) 0.4 V

Ans. **(3)**

Sol. Potential gradient with $R_h = 2\Omega$

is $\left(\frac{6}{2+4}\right)\times\frac{4}{L}=\frac{dV}{dL}$; $L = 100\,\text{cm}$

Let null point be at ℓ cm

thus $\varepsilon_1 = 0.5V = \left(\frac{6}{2+4}\right)\times\frac{4}{L}\times\ell$ ...(1)

Now with $R_h = 6\Omega$ new potential gradient is

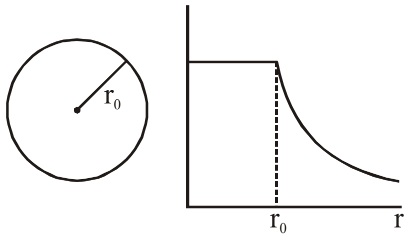
$\left(\frac{6}{4+6}\right)\times\frac{4}{L}$ and at null point

$$\left(\frac{6}{4+6}\right)\left(\frac{4}{L}\right)\times\ell=\varepsilon_2 \quad ...(2)$$

dividing equation (1) by (2) we get

$\frac{0.5}{\varepsilon_2}=\frac{10}{6}$ thus $\varepsilon_2 = 0.3$

26. The given graph shows variation (with distance r from centre) of :



(1) Potential of a uniformly charged sphere
(2) Potential of a uniformly charged spherical shell
(3) Electric field of uniformly charged spherical shell
(4) Electric field of uniformly charged sphere

Ans. (2)

27. Two equal resistance when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be :

(1) 60 W (2) 240 W
(3) 30 W (4) 120 W

Ans. (2)

Sol. In series condition, equivalent resistance is 2R

thus power consumed is $60W = \frac{\varepsilon^2}{2R}$

In parallel condition, equivalent resistance is R/2 thus new power is

$P'=\frac{\varepsilon^2}{(R/2)}$

or $P'=4P=240W$

28. An object is at a distacen of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be :

(1) 0.92×10^{-3} m/s away from the lens
(2) 2.26×10^{-3} m/s away from the lens
(3) 1.16×10^{-3} m/s towards the lens
(4) 3.22×10^{-3} m/s towards the lens

Ans. (3)

Sol. From lens equation

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{(-20)}=\frac{1}{(.3)}=\frac{10}{3}$$

$$\frac{1}{v}=\frac{10}{3}-\frac{1}{20}$$

$$\frac{1}{v}=\frac{197}{60};v=\frac{60}{197}$$

$$m=\left(\frac{v}{u}\right)=\frac{\left(\frac{60}{197}\right)}{20}$$

velocity of image wrt. to lens is given by

$v_{I/L} = m^2 v_{O/L}$

direction of velocity of image is same as that of object

$v_{O/L} = 5$ m/s

$$v_{I/L} = \left(\frac{60\times 1}{197\times 20}\right)^2 (5)$$

$= 1.16 \times 10^{-3}$ m/s towards the lens

29. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\ ms^{-2}$, the value of x will be close to :

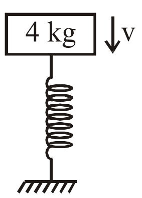
(1) 4 cm
(2) 8 cm
(3) 80 cm
(4) 40 cm

Ans. **(1)**

Sol. Velocity of 1 kg block just before it collides with 3kg block = $\sqrt{2gh} = \sqrt{2000}\ m/s$

Applying momentum conversation just before and just after collision.

$$1\times\sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4} m/s$$



initial compression of spring

$1.25 \times 10^6\ x_0 = 30 \Rightarrow x_0 \approx 0$

applying work energy theorem,

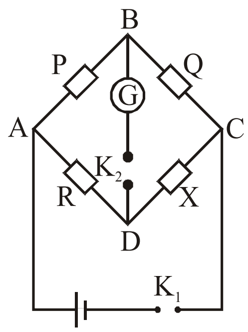
$W_g + W_{sp} = \Delta KE$

$$\Rightarrow 40 \times x + \frac{1}{2}\times 1.25\times 10^6 (0^2 - x^2)$$

$$= 0 - \frac{1}{2}\times 4\times v^2$$

solving $x \approx 4$ cm

30. In a Wheatstone bridge (see fig.), Resistances P and Q are approximately equal. When R = 400 Ω, the bridge is equal. When R = 400 Ω, the bridge is balanced. On inter-changing P and Q, the value of R, for balance, is 405 Ω. The value of X is close to :



(1) 403.5 ohm (2) 404.5 ohm
(3) 401.5 ohm (4) 402.5 ohm

Ans. **(4)**

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

CHEMISTRY

1. For the cell Zn(s) | Zn^{2+}(aq) || M^{x+} (aq) | M(s), different half cells and their standard electrode potentials are given below :

$M^{x+}(aq/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^{o}_{M^{x+}/M^{(v)}}$	1.40	0.80	0.77	–0.44

If $E^{o}_{Zn^{2+}/Zn} = -0.76V$, which cathode will give a mximum value of E^{o}_{cell} per electron transferred ?

(1) Fe^{3+} / Fe^{2+} (2) Ag^{+} / Ag

(3) Au^{3+} / Au (4) Fe^{2+} / Fe

Ans. (2)

2. The correct match between items-I and II is :

Item-I	***Item-II***
(Mixture)	***(Separation method)***
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

(1) A-Q, B-R, C-S (2) A-R, B-P, C-S

(3) A-S, B-R, C-P (4) A-Q, B-R, C-P

Ans. (1)

Sol. (Mixture) (Seperation method)

H_2O : Sugar $\Rightarrow$ Recrystallization

H_20 : Aniline $\Rightarrow$ Steam distillation

H_2O : Toluene $\Rightarrow$ Differential extraction

3. If a reaction follows the Arrhenius equation, the plot lnk vs $\frac{1}{(RT)}$ gives straight line with a gradient (–y) unit. The energy required to activate the reactant is :

(1) y unit (2) –y unit

(3) yR unit (4) y/R unit

Ans. (1)

4. The concentration of dissolved oxygen (DO) in cold water can go upto :

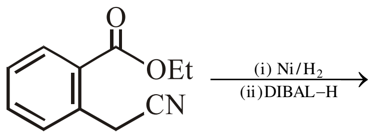
(1) 10 ppm (2) 14 ppm

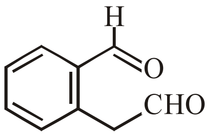
(3) 16 ppm (4) 8 ppm

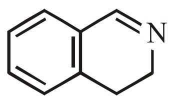
Ans. (1)

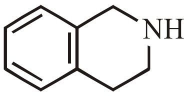
Sol. In cold water, dissolved oxygen (DO) can reach a concentration upto 10 ppm

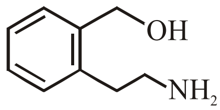
5. The major product of the following reaction is:



(1) 

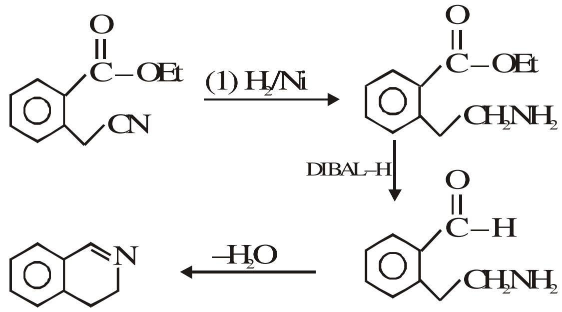
(2) 

(3) 

(4) 

Ans. (2)

Sol.



6. Th correct statements among (a) to (d) regarding H_2 as a fuel are :

(a) It produces less pollutant than petrol

(b) A cylinder of compressed dihydrogen weighs ~ 30 times more than a petrol tank producing the same amount of energy

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively

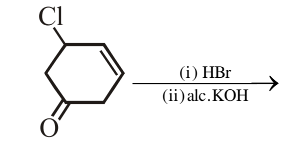
(1) b and d only (2) a, b and c only

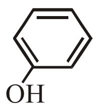
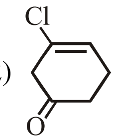
(3) b, c and d only (4) a and c only

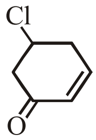
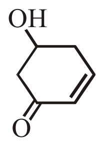
Ans. (2)

Sol. Option (a), (b) & (c) are correct answer (NCERT THEORY BASED)

7. The major poduct of the following reaction is:

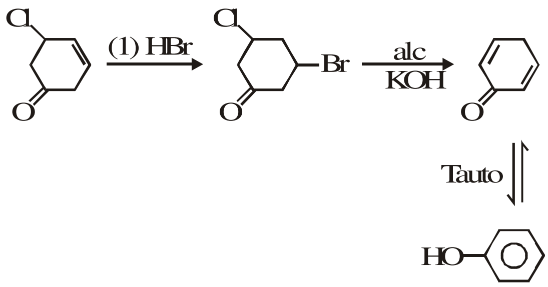


(1)  (2) 

(3)  (4) 

Ans. **(1)**

Sol.



8. The element that usually does not show variable oxidation states is :

(1) V (2) Ti (3) Sc (4) Cu

Ans. **(3)**

Sol. Usally Sc(Scandium) does not show variable oxidation states.

Most common oxidation states of :

(i) Sc : +3

(ii) V : +2, +3, +4, +5

(iii) Ti : +2, +3, +4

(iv) Cu : +1, +2

9. An organic compound is estimated through Dumus method and was found to evolve 6 moles of CO_2. 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is :

(1) $C_{12}H_8N$ (2) $C_{12}H_8N_2$

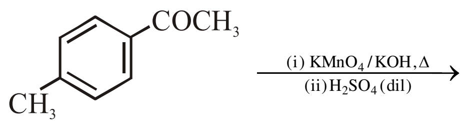
(3) C_6H_8N (4) $C_6H_8N_2$

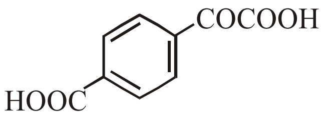
Ans. **(4)**

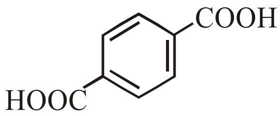
Sol. $[C_xH_yN_z] \xrightarrow[\text{Method}]{\text{Duma}} 6CO_2 + 4H_2O + N_2$

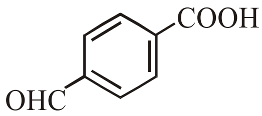
Hence, $C_6H_8N_2$

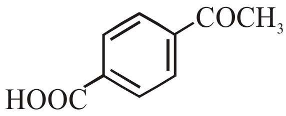
10. The major product of the following reaction is :



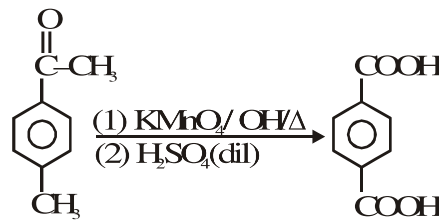
(1) 

(2) 

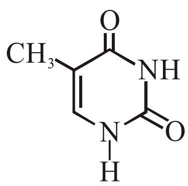
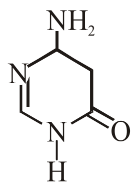
(3) 

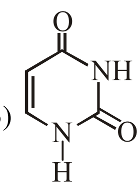
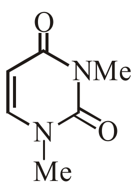
(4) 

Ans. **(2)**

Sol. 

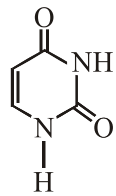
11. Among the following compound which one is found in RNA ?

(1)  (2) 

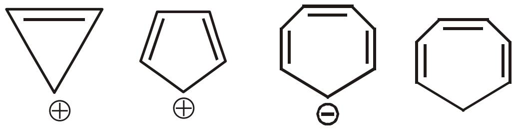
(3)  (4) 

Ans. **(3)**

Sol. For the given structure 'uracil' is found in RNA



12. Which compound(s) out of the following is/are not aromatic ?

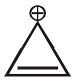


(A) (B) (C) (D)

(1) C and D (2) B, C and D

(3) A and C (4) B

Ans. (2)

Sol. out of the given options only  is aromatic.

Hence (B), (C) and (D) are not aromatic

13. The correct match between Item(I) and Item(II) is :

Item-I	**Item-II**
(A) Nortehindrone	(P) Anti-biotic
(B) Ofloxacin	(Q) Anti-fertility
(C) Equanil	(R) Hypertension
	(S) Analgesics

(1) A-R, B-P, C-S (2) A-Q, B-P, C-R

(3) A-R, B-P, C-R (4) A-Q, B-R, C-S

Ans. (2)

Sol. (A) Norethindrone – Antifertility

(B) Ofloaxacin – Anti-Biotic

(C) Equanil – Hypertension (traiquilizer)

14. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose ?

[$R_H = 1 \times 10^5\ cm^{-1}$, $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8\ ms^{-1}$]

(1) Paschen, $5 \rightarrow 3$ (2) Paschen, $\infty \rightarrow 3$

(3) Lyman, $\infty \rightarrow 1$ (4) Balmer, $\infty \rightarrow 2$

Ans. (2)

15. Consider the reaction,

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The equilibrium constant of the above reaction is K_P. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{NH_3} << P_{total}$ at equilibrium)

(1) $\frac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{4}$ (2) $\frac{3^{\frac{3}{2}} K_P^{\frac{1}{2}} P^2}{16}$

(3) $\frac{K_P^{\frac{1}{2}} P^2}{16}$ (4) $\frac{K_P^{\frac{1}{2}} P^2}{4}$

Ans. (2)

16. Match the ores(Column A) with the metals (column B) :

Column-A Ores	***Column-B Metals***
(I) Siderite	(a) Zinc
(II) Kaolinite	(b) Copper
(III) Malachite	(c) Iron
(IV) Calamine	(d) Aluminium

(1) I-b ; II-c ; III-d ; IV-a
(2) I-c ; II-d ; III-a ; IV-b
(3) I-c ; II-d ; III-b ; IV-a
(4) I-a ; II-b ; III-c ; IV-d

Ans. (3)

Sol. Siderite : $FeCO_3$

Kaolinite : $Al_2(OH)_4Si_2O_5$

Malachite : $Cu(OH)_2.CuCO_3$

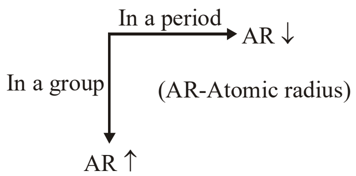
Calamine : $ZnCO_3$

17. The correct order of the atomic radii of C, Cs, Al and S is :

(1) S < C < Al < Cs (2) S < C < Cs < Al
(3) C < S < Cs < Al (4) C < S < Al < Cs

Ans. (4)

Sol.



Atomic radii order : C < S < Al < Cs

Atomic radius of C : 170 pm

Atomic radius of S : 180 pm

Atomic radius of Al : 184 pm

Atomic radius of Cs : 300 pm

18. Match the metals (Column I) with the coordination compound(s) / enzyme(s) (Column II)

Column-I ***Metals***	***Column-II*** ***Coordination compound(s) / Enzyme(s)***
(A) Co	(i) Wilkinson catalyst
(B) Zn	(ii) Chlorophyll
(C) Rh	(iii) Vitamin B_{12}
(D) Mg	(iv) Carbonic anhydrase

(1) A-ii ; B-i ; C-iv ; D-iii

(2) A-iii ; B-iv ; C-i ; D-ii

(3) A-iv ; B-iii ; C-i ; D-ii

(4) A-i ; B-ii ; C-iii ; D-iv

Ans. (2)

Sol. (i) Wilkinson catalyst : $RhCl(PPh_3)_3$

(ii) Chlorophyll : $C_{55}H_{72}O_5N_4Mg$

(iii) Vitamin B_{12}(also known as cyanocobalamin) contain cobalt.

(iv) Carbonic anhydrase contains a zinc ion.

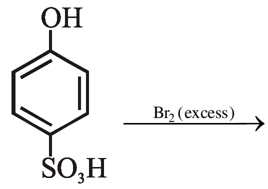
19. A 10 mg effervescent tablet contianing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet ? [Molar mass of $NaHCO_3$ = 84 g mol^{-1}]

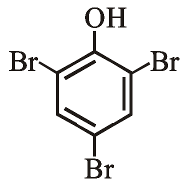
(1) 16.8 (2) 8.4

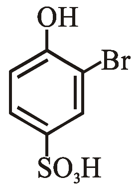
(3) 0.84 (4) 33.6

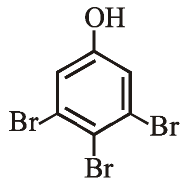
Ans. (1)

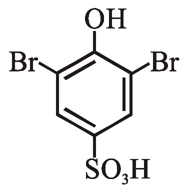
20. The major product of the following reaction is :



(1)


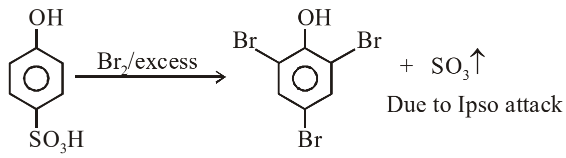
(2)


(3)


(4)


Ans. (1)

Sol.



21. Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively. are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is :

(1) $2C_P \ln\left(\dfrac{T_1+T_2}{4T_1T_2}\right)$ (2) $2C_P \ln\left[\dfrac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$

(3) $C_P \ln\left[\dfrac{(T_1+T_2)^2}{4T_1T_2}\right]$ (4) $2C_P \ln\left[\dfrac{T_1+T_2}{2T_1T_2}\right]$

Ans. (3)

22. The chloride that CANNOT get hydrolysed is :

(1) $SiCl_4$ (2) $SnCl_4$

(3) $PbCl_4$ (4) CCl_4

Ans. (4)

Sol. CCl_4 cannot get hydrolyzed due to the absence of vacant orbital at carbon atom.

23. For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as :

$$\Delta_r G^o \text{ (in kJ mol}^{-1}) = 120 - \frac{3}{8}T$$

The major component of the reaction mixture at T is :

(1) X if T = 315 K

(2) X if T = 350 K

(3) Y if T = 300 K

(4) Y if T = 280 K

Ans. (1)

24. The freezing point of a diluted milk sample is found to be –0.2°C, while it should have been –0.5°C for pure milk. How much water has been added to pure milk to make the diluted sample ?

(1) 2 cups of water to 3 cups of pure milk

(2) 1 cup of water to 3 cups of pure milk

(3) 3 cups of water to 2 cups of pure milk

(4) 1 cup of water to 2 cups of pure milk

Ans. (3)

25. A solid having density of 9×10^3 kg m^{-3} forms face centred cubic crystals of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid ?

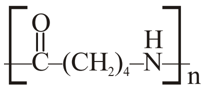
(Avogadro constant $\cong 6 \times 10^{23}$ mol^{-1}, $\pi \cong 3$)

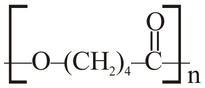
(1) 0.0216 kg mol^{-1} (2) 0.0305 kg mol^{-1}

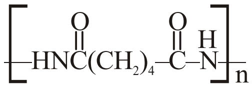
(3) 0.4320 kg mol^{-1} (4) 0.0432 kg mol^{-1}

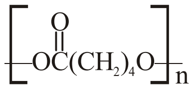
Ans. (2)

26. The polymer obtained from the following reactions is :

(1) 

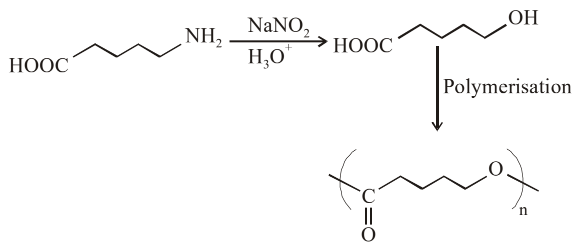
(2) 

(3) 

(4) 

Ans. (2)

Sol.



27. An example of solid sol is :

(1) Butter (2) Gem stones

(3) Paint (4) Hair cream

Ans. (2)

28. Peoxyacetyl nitrate (PAN), an eye irritant is produced by :

(1) Acid rain (2) Photochemical smog

(3) Classical smog (4) Organic waste

Ans. (2)

Sol. Photochemical smog produce chemicals such as formaldehyde, acrolein and peroxyacetyl nitrate (PAN).

29. NaH is an example of :

(1) Electron-rich hydride (2) Molecular hydride

(3) Saline hydride (4) Metallic hydride

Ans. (3)

Sol. NaH is an example of ionic hydride which is also known as saline hydride.

30. The amphoteric hydroxide is :

(1) $Ca(OH)_2$ (2) $Be(OH)_2$

(3) $Sr(OH)_2$ (4) $Mg(OH)_2$

Ans. (2)

Sol. $Be(OH)_2$ is amphoteric in nature while rest all alkaline earth metal hydroxide are basic in nature.

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

1. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

(1) 5 (2) 17 (3) 12 (4) 7

Ans. (4)

Sol. Normal vector of plane

$$=\begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix} = -4\left(5\hat{i}+2\hat{j}-3\hat{k}\right)$$

equation of plane is $5(x-7)+2y-3(z-6)=0$

$5x + 2y - 3z = 17$

2. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :-

(1) $\frac{1}{1-\cos\theta}+\frac{1}{1+\sin\theta}$

(2) $\frac{1}{1+\cos\theta}+\frac{1}{1-\sin\theta}$

(3) $\frac{1}{1-\cos\theta}-\frac{1}{1+\sin\theta}$

(4) $\frac{1}{1+\cos\theta}-\frac{1}{1-\sin\theta}$

Ans. (1)

Sol. $D = (1+\sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta = (1-\sin\theta\cos\theta)^2$

$\Rightarrow$ roots are $\beta = \operatorname{cosec}\theta$ and $\alpha = \cos\theta$

$$\Rightarrow \sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{n}(-\sin\theta)^n$$

$$= \frac{1}{1-\cos\theta}+\frac{1}{1+\sin\theta}$$

3. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x-\pi)\cos|x|$ is not differentiable. Then the set K is equal to :-

(1) $\{\pi\}$ (2) $\{0\}$

(3) ϕ (an empty set) (4) $\{0, \pi\}$

Ans. (3)

Sol. $f(x) = \sin|x| - |x| + 2(x-\pi)\cos x$

$\because$ $\sin|x| - |x|$ is differentiable function at $x=0$

$\therefore$ $k = \phi$

4. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$ (4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

Ans. (2)

Sol. $\frac{2b^2}{a} = 8$ and $2ae = 2b$

$\Rightarrow \frac{b}{a} = e$ and $1 - e^2 = e^2 \Rightarrow e = \frac{1}{\sqrt{2}}$

$\Rightarrow b = 4\sqrt{2}$ and $a = 8$

so equation of ellipse is $\frac{x^2}{64}+\frac{y^2}{32}=1$

5. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is :-

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$ (3) $5(2^{1/3})$ (4) 5

Ans. (4)

Sol. Vertex is $(a^2, 0)$

$y^2 = -(x - a^2)$ and $x = 0 \Rightarrow (0, \pm 2a)$

Area of triangle is $= \frac{1}{2}.4a.\left(a^2\right) = 250$

$\Rightarrow a^3 = 125$ or $a = 5$

6. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals :-

(1) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(3) $\frac{\pi}{10}$

(4) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

Ans. (1)

Sol. $I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$

$I = \frac{1}{2}\int_{\pi/6}^{\pi/4} \frac{\tan^4 x \sec^2 x\, dx}{\left(1 + \tan^{10} x\right)}$ Put $\tan^5 x = t$

$I = \frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1} \frac{dt}{1+t^2} = \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\frac{1}{9\sqrt{3}}\right)$

7. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{50} x^{50}$, for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to:-

(1) 12.50 (2) 12.00 (3) 12.75 (4) 12.25

Ans. (4)

Sol. $(10 + x)^{50} + (10 - x)^{50}$

$\Rightarrow a_2 = 2.{}^{50}C_2 10^{48}$, $a_0 = 2.10^{50}$

$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$

8. Let a function $f : (0, \infty) \rightarrow (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is :-

(1) Injective only

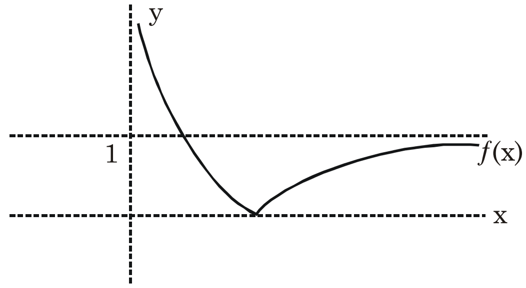
(2) Not injective but it is surjective

(3) Both injective as well as surjective

(4) Neither injective nor surjective

Ans. (Bonus)

Sol. $f(x) = \left|1 - \frac{1}{x}\right| = \frac{|x-1|}{x} = \begin{cases} \frac{1-x}{x} & 0 < x \le 1 \\ \frac{x-1}{x} & x \ge 1 \end{cases}$



$\Rightarrow f(x)$ is not injective

but range of function is $[0,\infty)$

Remark : If co-domain is $[0,\infty)$, then $f(x)$ will be surjective

9. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :-

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

Ans. (2)

Sol. 7,

1,6

2,5

3,4

1,2,4

$\boxed{P = \frac{5}{2^{20}}}$

10. Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

(1) (2, 4, 7) (2) (–2, 4, 7)

(3) (2, –4, –7) (4) (2, –4, 7)

Ans. (3)

Sol. Point on L_1 $(\lambda + 3, 3\lambda - 1, -\lambda + 6)$

Point on L_2 $(7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\Rightarrow \lambda + 3 = 7\mu - 5$ ...(i)

$3\lambda - 1 = -6\mu + 2$ ...(ii) $\Rightarrow \lambda = -1, \mu = 1$

point R(2,–4,7)

Reflection is (2,–4,–7)

11. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :-

(1) (15)! × 6! (2) $5^6 \times 15$

(3) 5! × 6! (4) $6^5 \times (15)!$

Ans. (1)

Sol. $f(k) = 3m\ (3,6,9,12,15,18)$

for $k = 4,8,12,16,20$ 6.5.4.3.2 ways

For rest numbers 15! ways

Total ways = 6!(15!)

12. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is :-

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

Ans. (1)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

13. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when y(1) = 1, is :-

(1) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(2) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$

(3) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(4) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

Ans. (4)

Sol. $x - y = t \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$$\Rightarrow 1 - \frac{dt}{dx} = t^2 \Rightarrow \int \frac{dt}{1-t^2} = \int 1dx$$

$$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+t}{1-t}\right) = x + \lambda$$

$$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+x-y}{1-x+y}\right) = x + \lambda \quad \text{given } y(1) = 1$$

$$\Rightarrow \frac{1}{2}\ell n(1) = 1 + \lambda \Rightarrow \lambda = -1$$

$$\Rightarrow \ell n\left(\frac{1+x-y}{1-x+y}\right) = 2(x-1)$$

$$\Rightarrow -\ell n\left(\frac{1-x+y}{1+x-y}\right) = 2(x-1)$$

14. Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :-

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

Ans. (2)

Sol. $|A|^2.|B| = 8$ and $\frac{|A|}{|B|} = 8 \Rightarrow |A| = 4$ and $|B| = \frac{1}{2}$

$\therefore \det(BA^{-1}.B^T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

15. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to :-

(1) $\frac{1}{3}(x+4)$ (2) $\frac{1}{3}(x+1)$

(3) $\frac{2}{3}(x+2)$ (4) $\frac{2}{3}(x-4)$

Ans. (1)

Sol. $\sqrt{2x-1} = t \Rightarrow 2x - 1 = t^2 \Rightarrow 2dx = 2t.dt$

$$\int \frac{x+1}{\sqrt{2x-1}}dx = \int \frac{\frac{t^2+1}{2}+1}{t}t\,dt = \int \frac{t^2+3}{2}dt$$

$$= \frac{1}{2}\left(\frac{t^3}{3} + 3t\right) = \frac{t}{6}\left(t^2 + 9\right) + c$$

$$= \sqrt{2x-1}\left(\frac{2x-1+9}{6}\right) + c = \sqrt{2x-1}\left(\frac{x+4}{3}\right) + c$$

$$\Rightarrow f(x) = \frac{x+4}{3}$$

16. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, the $\left(\dfrac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to :-

(1) 4 (2) $\dfrac{4\sqrt{3}}{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{2}$

Ans. **(3)**

Sol. p (probability of getting white ball) = $\dfrac{30}{40}$

$q=\dfrac{1}{4}$ and n = 16

mean =np = $16.\dfrac{3}{4}=12$

and standard diviation

$=\sqrt{npq}=\sqrt{16.\dfrac{3}{4}.\dfrac{1}{4}}=\sqrt{3}$

17. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:-

(1) $5x + 3y - 11 = 0$ (2) $3x - 5y + 7 = 0$
(3) $3x + 5y - 13 = 0$ (4) $5x - 3y + 1 = 0$

Ans. **(4)**

Sol. co-ordinates of point D are (4,7)
$\Rightarrow$ line AD is $5x - 3y + 1 = 0$

18. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :-

(1) 2 (2) $\dfrac{13}{6}$ (3) $\dfrac{13}{8}$ (4) $\dfrac{13}{12}$

Ans. **(4)**

Sol. 2b = 5 and 2ae = 13

$b^2 = a^2(e^2 - 1) \Rightarrow \dfrac{25}{4}=\dfrac{169}{4}-a^2$

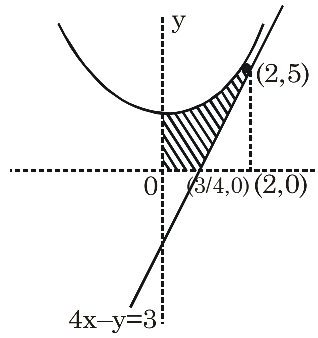
$\Rightarrow a=6 \quad \Rightarrow e=\dfrac{13}{12}$

19. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :-

(1) $\dfrac{14}{3}$ (2) $\dfrac{187}{24}$ (3) $\dfrac{37}{24}$ (4) $\dfrac{8}{3}$

Ans. **(3)**

Sol.



Area $=\int_0^2\left(x^2+1\right)dx-\dfrac{1}{2}\left(\dfrac{5}{4}\right)(5)=\dfrac{37}{24}$

20. Let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\dfrac{3}{\sqrt{2}}$, then the sum of all possible values of β is :-

(1) 2 (2) 1 (3) 3 (4) 4

Ans. **(2)**

Sol. Angle bisector is $x - y = 0$

$\Rightarrow \dfrac{|\beta-(1-\beta)|}{\sqrt{2}}=\dfrac{3}{\sqrt{2}}$

$\Rightarrow |2\beta-1|=3$

$\Rightarrow \beta=2 \text{ or } -1$

21. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $= (a + b + c)(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to :-

(1) $-(a + b + c)$ (2) $2(a + b + c)$
(3) abc (4) $-2(a + b + c)$

Ans. **(4)**

Sol. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$=\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$$=(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 2c & c-a-b \end{vmatrix}$$

$= (a + b + c)(a + b + c)^2$
$\Rightarrow x = -2(a + b + c)$

22. Let $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1+\left(\frac{q+1}{2}\right)+\left(\frac{q+1}{2}\right)^2+........+\left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + ... + {}^{101}C_{101}.S_{100} = \alpha T_{100}$, then α is equal to :-

(1) 2^{100} (2) 200 (3) 2^{99} (4) 202

Ans. (1)

Sol. ${}^{101}C_1 + {}^{101}C_2S_1 + ... + {}^{101}C_{101}S_{100}$
$= \alpha T_{100}$
${}^{101}C_1 + {}^{101}C_2(1 + q) + {}^{101}C_3(1 + q + q^2) + ... + {}^{101}C_{101}(1 + q + ... + q^{100})$

$$=2\alpha\frac{\left(1-\left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$$

$\Rightarrow {}^{101}C_1(1 - q) + {}^{101}C_2(1 - q^2) + ... + {}^{101}C_{101}(1 - q^{101})$

$$=2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow (2^{101} - 1) - ((1 + q)^{101} - 1)$

$$=2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow 2^{101}\left(1-\left(\frac{1+q}{2}\right)^{101}\right)=2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow \alpha = 2^{100}$

23. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :-

(1) A hyperbola (2) A parabola
(3) A straight line (4) An ellipse

Ans. (2)

Sol. Let equation of circle is

$x^2 + y^2 + 2fx + 2fy + e = 0$, it passes through (0, 2b)

$\Rightarrow 0 + 4b^2 + 2g \times 0 + 4f + c = 0$

$\Rightarrow 4b^2 + 4f + c = 0$ ...(i)

$2\sqrt{g^2-c}=4a$ ...(ii)

$g^2 - c = 4a^2 \Rightarrow c=\left(g^2-4a^2\right)$

Putting in equation (1)

$\Rightarrow 4b^2 + 4f + g^2 - 4a^2 = 0$

$\Rightarrow x^2 + 4y + 4(b^2 - a^2) = 0$, it represent a parabola.

24. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :-

(1) 3 : 1 (2) 4 : 1 (3) 2 : 1 (4) 1 : 3

Ans. (1)

Sol. $a + 18d = 0$ ...(1)

$$\frac{a+48d}{a+28d}=\frac{-18d+48d}{-18d+28d}=\frac{3}{1}$$

25. Let $f(x)=\frac{x}{\sqrt{a^2+x^2}}-\frac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x\in R$, where a, b and d are non-zero real constants. Then :-

(1) f is a decreasing function of x
(2) f is neither increasing nor decreasing function of x
(3) f' is not a continuous function of x
(4) f is an increasing function of x

Ans. (4)

Sol. $$f(x)=\frac{x}{\sqrt{a^2+x^2}}-\frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$f'(x)=\frac{a^2}{\left(a^2+x^2\right)^{3/2}}+\frac{b^2}{\left(b^2+(d-x)^2\right)^{3/2}}>0\ \forall x\in R$$

$f(x)$ is an increasing function.

26. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to :-

(1) $\frac{5}{4}$ (2) $\frac{\sqrt{41}}{4}$ (3) $\frac{\sqrt{34}}{3}$ (4) $\frac{5}{3}$

Ans. (4)

Sol. $|z| + z = 3 + i$

$z = 3 - |z| + i$

Let $3 - |z| = a \Rightarrow |z| = (3 - a)$

$\Rightarrow z=a+i\Rightarrow|z|=\sqrt{a^2+1}$

$\Rightarrow 9 + a^2 - 6a = a^2 + 1 \Rightarrow a=\frac{8}{6}=\frac{4}{3}$

$\Rightarrow |z|=3-\frac{4}{3}=\frac{5}{3}$

27. All x satisfying the inequality $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$, lie in the interval:-

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(2) $(\cot 5, \cot 4)$

(3) $(\cot 2, \infty)$

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

Ans. **(4)**

Sol. $\cot^{-1}x > 5, \quad \cot^{-1}x < 2$

$\Rightarrow x < \cot 5,\ x > \cot 2$

28. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value :-

(1) (3, 4, 5) (2) (19, 7, 25)

(3) (7, 19, 25) (4) (5, 12, 13)

28. **Ans. (3)**

Sol. $b + c = 11\lambda,\ c + a = 12\lambda,\ a + b = 13\lambda$

$\Rightarrow a = 7\lambda,\ b = 6\lambda,\ c = 5\lambda$

(using cosine formula)

$$\cos A = \frac{1}{5}, \cos B = \frac{19}{35}, \cos C = \frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

29. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :-

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1

Ans. **(2)**

Sol. $$\frac{x^m y^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)} = \frac{1}{\left(x^m + \frac{1}{x^m}\right)\left(y^n + \frac{1}{y^n}\right)} \le \frac{1}{4}$$

using AM $\ge$ GM

30. $\lim_{x\to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :-

(1) 2 (2) 0 (3) 4 (4) 1

Sol. $$\lim_{x\to 0} \frac{x \tan^2 2x}{\tan 4x \sin^2 x} = \lim_{x\to 0} \frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

PHYSICS

1. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is :-

(1) 2.3×10^{-2} (2) 3.3×10^{-2}

(3) 3.3×10^{-4} (4) 4.3×10^{-2}

Ans. (3)

Sol. $\chi = \frac{I}{H}$

$$I = \frac{\text{Magnetic moment}}{\text{Volume}}$$

$$I = \frac{20 \times 10^{-6}}{10^{-6}} = 20\ \text{N/m}^2$$

$$\chi = \frac{20}{60 \times 10^{+3}} = \frac{1}{3} \times 10^{-3}$$

$= 0.33 \times 10^{-3} = 3.3 \times 10^{-4}$

2. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is :-

(1) $2\sqrt{\frac{p}{k}}$ (2) $\sqrt{\frac{2p}{k}}$ (3) $\sqrt{\frac{2k}{p}}$ (4) $2\sqrt{\frac{k}{p}}$

Ans. (1)

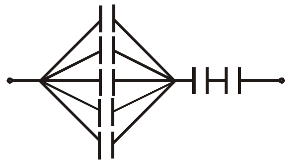
Sol. $\frac{dp}{dt} = F = kt$

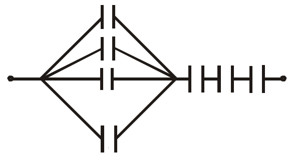
$$\int_{P}^{3P} dP = \int_{O}^{T} kt\ dt$$

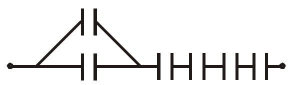
$$2p = \frac{KT^2}{2}$$

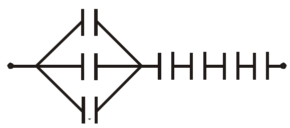
$$T = 2\sqrt{\frac{P}{K}}$$

3. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value ?

(1) 

(2) 

(3) 

(4) 

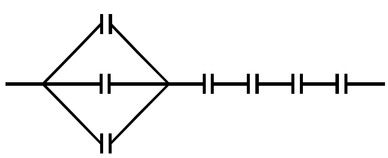
Ans. (4)

Sol. $C_{eq} = \frac{6}{13}\mu F$

Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$$



4. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole ?

(1) -9×10^{-20} J
(2) -7×10^{-27} J
(3) -10×10^{-29} J
(4) -20×10^{-18} J

Ans. **(2)**

Sol. $U = -\vec{P}.\vec{E}$

$= -PE \cos\theta$

$= -(10^{-29})(10^{3}) \cos 45°$

$= -0.707 \times 10^{-26}$ J

$= -7 \times 10^{-27}$ J.

5. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :-

(1) 10^{-3} rad/s
(2) 10^{-1} rad/s
(3) 1 rad/s
(4) 10^{-5} rad/s

Ans. **(1)**

Sol. Angular frequency of pendulum

$$\omega = \sqrt{\frac{g_{eff}}{\ell}}$$

$$\therefore \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

[ω_s = angular frequency of support]

$$\Delta\omega = \frac{1}{2} \times \frac{2A\omega_s^2}{100} \times 100$$

$\Delta\omega = 10^{-3}$ rad/sec.

6. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is :-

(1) 270°C (2) 230°C
(3) 250°C (4) 200°C

Ans. **(2)**

Sol. $\Delta\ell_1 = \Delta\ell_2$

$\ell\alpha_1\Delta T_1 = \ell\alpha_2\Delta T_2$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2}$$

$$\frac{4}{3} = \frac{T-30}{180-30}$$

$$\boxed{T = 230°C}$$

7. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is :-

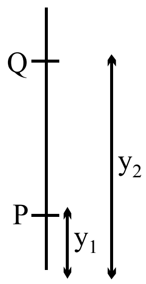
(1) 09 (2) 10
(3) 04 (4) 05

Ans. **(4)**

Sol. For diffraction

location of 1st minime

$$y_1 = \frac{D\lambda}{a} = 0.2469 D\lambda$$



location of 2nd minima

$$y_2 = \frac{2D\lambda}{a} = 0.4938 D\lambda$$

Now for interference

Path difference at P.

$$\frac{dy}{D} = 4.8\lambda$$

path difference at Q

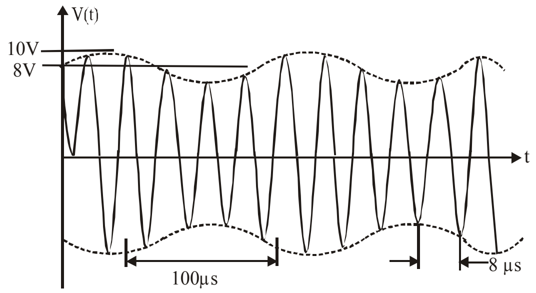
$$\frac{dy}{D} = 9.6\lambda$$

So orders of maxima in between P & Q is

5, 6, 7, 8, 9

So 5 bright fringes all present between P & Q.

8. An amplitude modulated signal is plotted below :-



Which one of the following best describes the above signal ?

(1) $(9 + \sin(2.5\pi \times 10^5\ t))\ \sin(2\pi \times 10^4 t)$V

(2) $(9 + \sin(4\pi \times 10^4\ t))\ \sin(5\pi \times 10^5 t)$V

(3) $(1 + 9\sin(2\pi \times 10^4\ t))\ \sin(2.5\pi \times 10^5 t)$V

(4) $(9 + \sin(2\pi \times 10^4\ t))\ \sin(2.5\pi \times 10^5 t)$V

Ans. (4)

Sol. Analysis of graph says

(1) Amplitude varies as 8 – 10 V or 9 ± 1

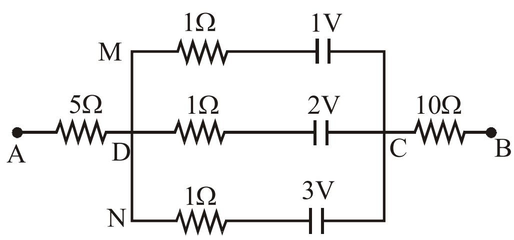
(2) Two time period as

100 μs (signal wave) & 8 μs (carrier wave)

Hence signal is $\left[9 \pm 1\sin\left(\frac{2\pi t}{T_1}\right)\right]\sin\left(\frac{2\pi t}{T_2}\right)$

$= 9 \pm 1\sin(2\pi \times 10^4 t)\ \sin 2.5\pi \times 10^5\ t$

9. In the circuit , the potential difference between A and B is :-



(1) 6 V (2) 1 V (3) 3 V (4) 2 V

Ans. (4)

Sol. Potential difference across AB will be equal to battery equivalent across CD

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{\frac{1}{1}+\frac{2}{1}+\frac{3}{1}}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}}$$

$$= \frac{6}{3} = 2V$$

10. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Given permittivity of space $\in_0 = 9 \times 10^{-12}$ SI units, Speed of light $c = 3 \times 10^8$ m/s]:-

(1) 1 kV/m (2) 2 kV/m

(3) 1.4 kV/m (4) 0.7 kV/m

Ans. (3)

Sol. Intensity of EM wave is given by

$$I = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$= \frac{27 \times 10^{-3}}{10 \times 10^{-6}} = \frac{1}{2} \times 9 \times 10^{-12} \times E^2 \times 3 \times 10^8$$

$$E = \sqrt{2} \times 10^3 \text{ kv/m}$$

$$= 1.4 \text{ kv/m}$$

11. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :-

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$

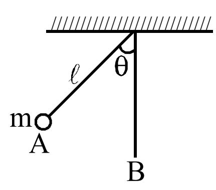
(3) $K_2 = 2K_1$ (4) $K_2 = K_1$

Ans. (3)

Sol. Maximum kinetic energy at lowest point B is given by

$K = mgl(1 - \cos\theta)$

where θ = angular amp.



$K_1 = mg\ell(1 - \cos\theta)$

$K_2 = mg(2\ell)(1 - \cos\theta)$

$K_2 = 2K_1$.

12. In a hydrogen like atom, when an electron jumps from the M - shell to the L - shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be :-

(1) $\frac{27}{20}\lambda$ (2) $\frac{16}{25}\lambda$ (3) $\frac{20}{27}\lambda$ (4) $\frac{25}{16}\lambda$

Ans. **(3)**

Sol. For M $\rightarrow$ L steel

$$\frac{1}{\lambda} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{K \times 5}{36}$$

for N $\rightarrow$ L

$$\frac{1}{\lambda'} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{K \times 3}{16}$$

$$\lambda' = \frac{20}{27}\lambda$$

13. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be :-

(1) $V^{-2} A^2 F^2$ (2) $V^{-4} A^2 F$

(3) $V^{-4} A^{-2} F$ (4) $V^{-2} A^2 F^{-2}$

Ans. **(2)**

Sol. $\frac{F}{A} = y.\frac{\Delta \ell}{\ell}$

$$[Y] = \frac{F}{A}$$

Now from dimension

$$F = \frac{ML}{T^2}$$

$$L = \frac{F}{M}.T^2$$

$$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because T = \frac{V}{A}$$

$$L^2 = \frac{F^2}{M^2A^2}\frac{v^4}{A^2} \qquad F = MA$$

$$L^2 = \frac{V^4}{A^2}$$

$$[Y] = \frac{[F]}{[A]} = F^1 V^{-4} A^2$$

14. A particle moves from the point $\left(2.0\hat{i} + 4.0\hat{j}\right)$ m, at t = 0, with an initial velocity $\left(5.0\hat{i} + 4.0\hat{j}\right)$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i} + 4.0\hat{j}\right)$ ms^{-2}. What is the distance of the particle from the origin at time 2 s ?

(1) $20\sqrt{2}$ m (2) $10\sqrt{2}$ m

(3) 5 m (4) 15 m

Ans. **(1)**

Sol. $\vec{S} = \left(5\hat{i} + 4\hat{j}\right)2 + \frac{1}{2}\left(4\hat{i} + 4\hat{j}\right)4$

$$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$$

$$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$$

$$\vec{r}_f = 20\hat{i} + 20\hat{j}$$

$$|\vec{r}_f| = 20\sqrt{2}$$

15. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is :-

(1) 30° (2) 45°

(3) 90° (4) 60°

Ans. **(4)**

Sol. i = e

$$r_1 = r_2 = \frac{A}{2} = 30^o$$

by Snell's law

$$1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

i = 60

16. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is :-

(1) 80 Ω (2) 120 Ω

(3) 125 Ω (4) 100 Ω

Ans. (1)

Sol. $R_g = 20\Omega$

$N_L = N_R = N = 30$

$\text{FOM} = \frac{I}{\phi} = 0.005$ A/Div.

Current sentivity $= CS = \left(\frac{1}{0.005}\right) = \frac{\phi}{I}$

$Ig_{max} = 0.005 \times 30$

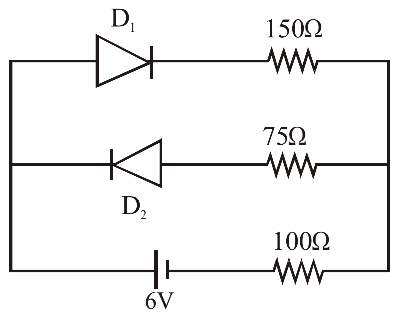
$= 15 \times 10^{-2} = 0.15$

$15 = 0.15\ [20 + R]$

$100 = 20 + R$

$R = 80$

17. The circuit shown below contains two ideal diodes, each with a forward resistance of 50Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in Amperes) is :-



(1) 0.027 (2) 0.020

(3) 0.030 (4) 0.036

Ans. (2)

Sol. $I = \frac{6}{300} = 0.002$ (D_2 is in reverse bias)

18. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be :-

(1) 80°C (2) 60°C

(3) 70°C (4) 85°C

Ans. (1)

Sol. $100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5\ S_B$

$S_A = \frac{3}{4} S_B$

Now, $100 \times S_A \times [100 - T] = 50 \times S_B\ (T - 50)$

$2 \times \left(\frac{3}{4}\right)(100 - T) = (T - 50)$

$300 - 3T = 2T - 100$

$400 = 5T$

$T = 80$

19. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be :-

(1) $\frac{2}{\sqrt{3}}$s (2) $2\sqrt{3}$ s

(3) $\frac{\sqrt{3}}{2}$s (4) $\frac{3}{2}$s

Ans. (2)

Sol. $\because g = \frac{GM}{R^2}$

$\frac{g_p}{g_e} = \frac{M_e}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$

Also $T \propto \frac{1}{\sqrt{g}}$

$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$

$\Rightarrow T_p = 2\sqrt{3}$s

20. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is :-

(1) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(2) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{\sqrt{2}}\hat{j}\right)$

(3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$

(4) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$

Ans. (BONUS)

21. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice.

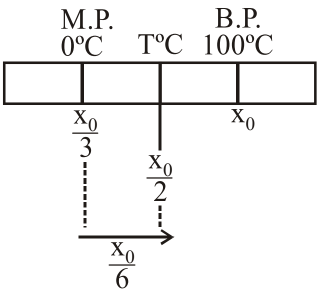
What is the temperature of an object in 0 ºC, if this thermometer in the contact with the object reads $x_0/2$?

(1) 35 (2) 25

(3) 60 (4) 40

Ans. (2)

Sol.

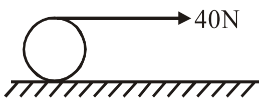


$$\Rightarrow T^{o}C=\frac{x_0}{6} \quad \& \quad \left(x_0-\frac{x_0}{3}\right)=(100-0^{o}C)$$

$$x_0=\frac{300}{2}$$

$$\Rightarrow T^{o}C=\frac{150}{6}=25^{o}C$$

22. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :-

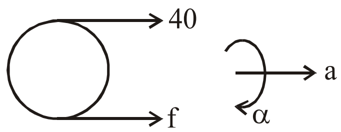


(1) 12 rad/s^2 (2) 16 rad/s^2

(3) 10 rad/s^2 (4) 20 rad/s^2

Ans. (2)

Sol.



$40 + f = m(R\alpha)$ .....(i)

$40 \times R - f \times R = mR^2\alpha$

$40 - f = mR\alpha$ ...... (ii)

From (i) and (ii)

$$\alpha=\frac{40}{mR}=16$$

23. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature of the gas is incresed by ΔT. The amount of heat absorbed by gas is (R is gas constant) :

(1) $\frac{1}{2}R\Delta T$ (2) $\frac{3}{2}R\Delta T$

(3) $\frac{1}{2}KR\Delta T$ (4) $\frac{2K}{3}\Delta T$

Ans. (1)

Sol. VT = K

$$\Rightarrow V\left(\frac{PV}{nR}\right)=k \Rightarrow PV^2 = K$$

$$\because \quad C=\frac{R}{1-x}+C_V \text{ (For polytropic process)}$$

$$C=\frac{R}{1-2}+\frac{3R}{2}=\frac{R}{2}$$

$\therefore \Delta Q = nC\,\Delta T$

$= \frac{R}{2} \times \Delta T$

24. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to : $\left(\frac{hc}{e} = 1240\,nm - V\right)$

(1) 0.5 V (2) 1.0 V
(3) 2.0 V (4) 1.5 V

Ans. (2)

Sol. $\frac{hc}{\lambda_1} = \phi + eV_1$ .... (i)

$\frac{hc}{\lambda_2} = \phi + eV_2$ .... (ii)

(i) – (ii)

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e(V_1 - V_2)$$

$$\Rightarrow V_1 - V_2 = \frac{hc}{e}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 - \lambda_2}\right)$$

$$= (1240nm - V)\frac{100nm}{300nm \times 400nm}$$

= 1V

25. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water ? [Specific Heat Capacities of water and metal are, respectively, 4200 $Jkg^{-1}K^{-1}$ and 400 $JKg^{-1}K^{-1}$]

(1) 30% (2) 20%
(3) 25% (4) 15%

Ans. (2)

Sol. $0.1 \times 400 \times (500 - T) = 0.5 \times 4200 \times (T - 30) + 800\,(T - 30)$

$\Rightarrow 40(500 - T) = (T - 30)\,(2100 + 800)$

$\Rightarrow 20000 - 40T = 2900\,T - 30 \times 2900$

$\Rightarrow 20000 + 30 \times 2900 = T(2940)$

$T = 30.4°C$

$$\frac{\Delta T}{T} \times 100 = \frac{6.4}{30} \times 100$$

$\simeq 20\%$

26. The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) :-

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

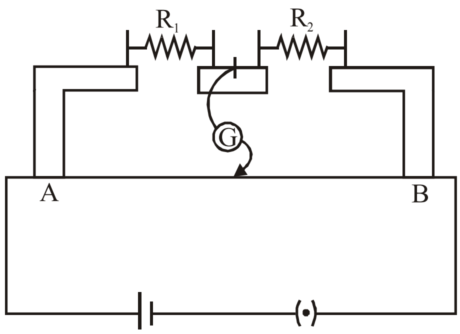
Ans. (2)

Sol. $2.5 = 1 \times 5 \sin\theta$

$\sin\theta = 0.5 = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

27. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is



(1) 40 Ω (2) 60 Ω (3) 20 Ω (4) 30Ω

Ans. **(2)**

Sol. $\frac{R_1}{R_2} = \frac{2}{3}$ ......(i)

$\frac{R_1+10}{R_2} = 1 \Rightarrow R_1 + 10 = R_2$ ......(ii)

$\frac{2R_2}{3} + 10 = R_2$

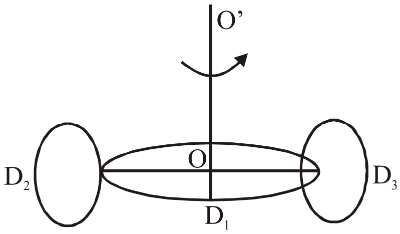
$10 = \frac{R_2}{3} \Rightarrow R_2 = 30\Omega$

& $R_1 = 20\Omega$

$\frac{\frac{30\times R}{30+R}}{30} = \frac{2}{3}$

$R = 60\ \Omega$

28. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be:-



(1) $3MR^2$ (2) $\frac{2}{3}MR^2$

(3) MR^2 (4) $\frac{4}{5}MR^2$

Ans. **(1)**

Sol. $I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2$

$= 3\ MR^2$

29. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil :

(1) Decreases by a factor of $9\sqrt{3}$

(2) Increases by a factor of 3

(3) Decreases by a factor of 9

(4) Increases by a factor of 27

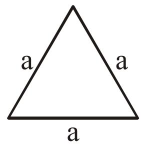
Ans. **(2)**

Sol. Total length L will remain constant

L = (3a) N (N = total turns)

and length of winding = (d) N

(d = diameter of wire)



self inductance $= \mu_0 n^2 A\ell$

$= \mu_0 n^2 \left(\frac{\sqrt{3}a^2}{4}\right) dN$

$\propto a^2 N \propto a$

So self inductance will become 3 times

30. A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i} + 3\hat{j}$; $\vec{B} = 4\hat{j} + 6\hat{k}$.

The charged particle is shifted from the origin to the point P(x = 1 ; y = 1) along a straight path. The magnitude of the total work done is :-

(1) (0.35)q (2) (0.15)q

(3) (2.5)q (4) 5q

Ans. **(4)**

Sol. $\vec{F}_{net} = q\vec{E} + q(\vec{v}\times\vec{B})$

$= (2q\hat{i} + 3q\hat{j}) + q(\vec{v}\times\vec{B})$

$W = \vec{F}_{net}.\vec{S}$

$= 2q + 3q$

$= 5q$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

CHEMISTRY

1. The correct option with respect to the Pauling electronegativity values of the elements is :-

(1) Ga < Ge (2) Si < Al

(3) P > S (4) Te > Se

Ans. (1)

Sol.

B C

Al Si

$\boxed{Ga < Ge}$

Along the period electronegativity increases

2. The homopolymer formed from 4-hydroxy-butanoic acid is :-

(1) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_3-O-\right]_n$

(2) $\left[-O\overset{O}{\overset{\|}{C}}(CH_2)_3-O-\right]_n$

(3) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}-O-\right]_n$

(4) $\left[-\overset{O}{\overset{\|}{C}}(CH_2)_2\overset{O}{\overset{\|}{C}}-\right]_n$

Ans. (1)

Sol.

3. The correct match between Item I and Item II is :-

Item I		Item II	
(A)	Ester test	(P)	Tyr
(B)	Carbylamine test	(Q)	Asp
(C)	Phthalein dye test	(R)	Ser
		(S)	Lys

(1) (A)→(Q); (B)→(S); (C)→(P)

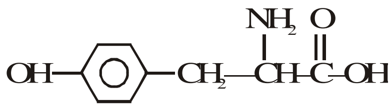
(2) (A)→(R); (B)→(Q); (C)→(P)

(3) (A)→(Q); (B)→(S); (C)→(R)

(4) (A)→(R); (B)→(S); (C)→(Q)

Ans. (1)

Sol.

(P) Tyrosine Tyr 

(Q) Aspartic Acid ASP $H-O-\overset{O}{\overset{\|}{C}}-CH_2-\overset{NH_2}{\overset{|}{CH}}-\overset{O}{\overset{\|}{C}}-OH$

(R) Serine Ser $HO-CH_2-CH\begin{matrix} NH_2 \\ COOH \end{matrix}$

(S) Lysine $NH_2-CH_2-CH_2-CH_2-CH_2-CH\begin{matrix} NH_2 \\ COOH \end{matrix}$

(A) Ester test (Q) Aspartic acid (Acidic amino acid)

(B) Carbylamine (S) Lysine [NH_2 group present]

(C) Phthalein dye (P) Tyrosine {Phenolic group present)

4. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to :-

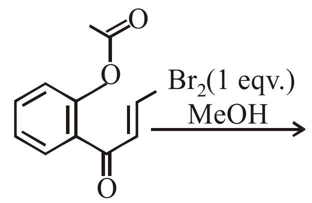
(1) Water pollution (2) Global warming

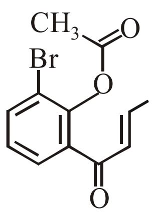
(3) Soil pollution (4) Acid rain

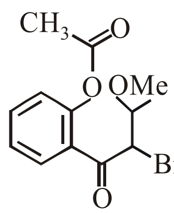
Ans. (4)

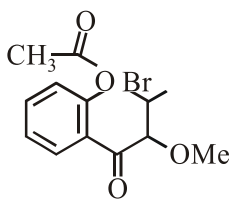
Sol. Taj mahal is slowely disfigured and discoloured due to acid rain.

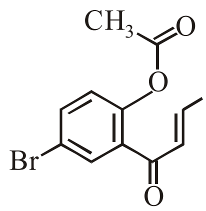
5. The major product obtained in the following conversion is :-



(1) 

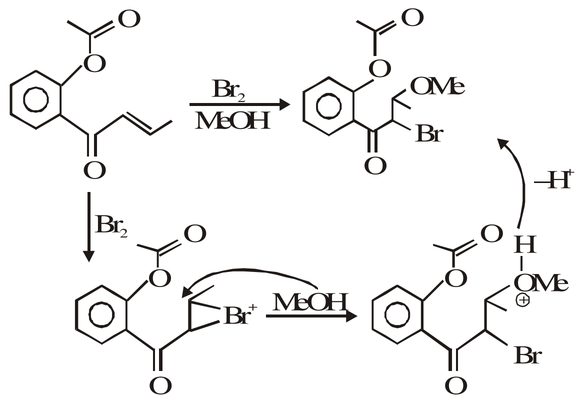
(2) 

(3) 

(4) 

Ans. (2)

Sol.



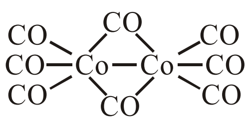
6. The number of bridging CO ligand (s) and Co-Co bond (s) in $CO_2(CO)g$, respectively are :-

(1) 0 and 2 (2) 2 and 0

(3) 4 and 0 (4) 2 and 1

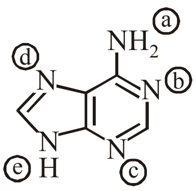
Ans. **(4)**

Sol.



Bridging CO are 2 and Co – Co bond is 1.

7. In the following compound,



the favourable site/s for protonation is/are :-

(1) (b), (c) and (d) (2) (a)

(3) (a) and (e) (4) (a) and (d)

Ans. **(1)**

Sol. Localised lone pair e^-.

8. The higher concentration of which gas in air can cause stiffness of flower buds ?

(1) SO_2 (2) NO_2

(3) CO_2 (4) CO

Ans. **(1)**

Sol. Due to acid rain in plants high concentration of SO_2 makes the flower buds stiff and makes them fall.

9. The correct match between item I and item II is :-

Item I		Item II	
(A)	Allosteric effect	(P)	Molecule binding to the active site of enzyme
(B)	Competitive inhibitor	(Q)	Molecule crucial for communication in the body
(C)	Receptor	(R)	Molecule binding to a site other than the active site of enzyme
(D)	Poison	(S)	Molecule binding to the enzyme covalently

(1) (A)→(P); (B)→(R); (C)→(S); (D)→(Q)

(2) (A)→(R); (B)→(P); (C)→(S); (D)→(Q)

(3) (A)→(P); (B)→(R); (C)→(Q); (D)→(S)

(4) (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

Ans. **(4)**

10. The radius of the largest sphere which fits properly at the centre of the edge of body centred cubic unit cell is : (Edge length is represented by 'a') :-

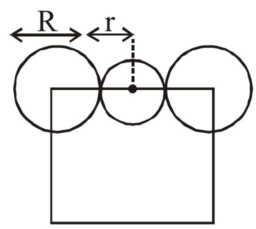
(1) 0.134 a (2) 0.027 a

(3) 0.067 a (4) 0.047 a

Ans. **(3)**

Sol.

$a = 2(R + r)$



$\frac{a}{2} = (R + r)$ ...(1)

$a\sqrt{3} = 4R$ ...(2)

Using (1) & (2)

$\frac{a}{2} = \frac{a\sqrt{3}}{4} = r$

$a\left(\frac{2-\sqrt{3}}{4}\right) = r$

$r = 0.067$ a

11. Among the colloids cheese (C), milk (M) and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is :-

(1) C : solid in liquid; M : solid in liquid; S : solid in gas

(2) C : solid in liquid; M : liquid in liquid; S : gas in solid

(3) C : liquid in solid; M : liquid in solid; S : solid in gas

(4) C : liquid in solid; M : liquid in liquid; S : solid in gas

Ans. (4)

Sol.

	Dispersed Phase	**Dispersion Medium**
Cheese	Liquid	Solid
Milk	Liquid	Liquid
Smoke	Solid	Gas

12. The reaction that does NOT define calcination is:-

(1) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$

(2) $Fe_2O_3 \cdot XH_2O \xrightarrow{\Delta} Fe_2O_3 + XH_2O$

(3) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2\ CO_2$

(4) $2\ Cu_2S + 3\ O_2 \xrightarrow{\Delta} 2\ Cu_2O + 2\ SO_2$

Ans. (4)

Sol. Calcination in carried out for carbonates and oxide ores in absence of oxygen. Roasting is carried out mainly for sulphide ores in presence of excess of oxygen.

13. The reaction,

$MgO(s) + C(s) \rightarrow Mg(S) + CO(g)$, for which $\Delta_r H^o = +\ 491.1$ kJ mol^{-1} and $\Delta_r S^o = 198.0$ JK^{-1} mol^{-1}, is not feasible at 298 K. Temperature above which reaction will be feasible is :-

(1) 1890.0 K (2) 2480.3 K

(3) 2040.5 K (4) 2380.5 K

Ans. (2)

Sol. $T_{eq} = \dfrac{\Delta H}{\Delta S}$

$= \dfrac{491.1 \times 1000}{198}$

= 2480.3 K

14. Given the equilibrium constant :

KC of the reaction :

$Cu(s) + 2Ag^+(aq) \rightarrow Cu^{2+}(aq) + 2Ag(s)$ is 10×10^{15}, calculate the E^0_{cell} of this reaction at 298 K

$$\left[2.303\frac{RT}{F} \text{at } 298\text{K} = 0.059\text{V}\right]$$

(1) 0.04736 V (2) 0.4736 V

(3) 0.4736 mV (4) 0.04736 mV

Ans. (2)

Sol. $E_{cell} = E^o_{cell} - \dfrac{0.059}{n}\log Q$

At equilibrium

$E^\circ_{Cell} = \dfrac{0.059}{2}\log 10^{16}$

$= 0.059 \times 8$

= 0.472 V

15. The hydride that is NOT electron deficient is:-

(1) B_2H_6 (2) AlH_3

(3) SiH_4 (4) GaH_3

Ans. (3)

Sol. (1) B_2H_6 : Electron deficient

(2) AlH_3 : Electron deficient

(3) SiH_4 : Electron precise

(4) GaH_3 : Electron deficient

16. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by

$$\Delta_r G^o = A - Bt$$

Where A and B are non-zero constants. Which of the following is TRUE about this reaction ?

(1) Exothermic if $B < 0$

(2) Exothermic if $A > 0$ and $B < 0$

(3) Endothermic if $A < 0$ and $B > 0$

(4) Endothermic if $A > 0$

Ans. (4)

Sol. Theory

17. K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is :-

(1) 1.8 (2) 2.2 (3) 2.0 (4) 1.6

Ans. (1)

Sol. For $K_2[HgI_4]$

$i = 1 + 0.4\ (3-1)$

$= 1.8$

18. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (v) of the incident radiation as, [v_0 is thrshold frequency] :

(1) $\lambda \propto \dfrac{1}{(v - v_0)^{\frac{3}{2}}}$ (2) $\lambda \propto \dfrac{1}{(v - v_0)^{\frac{1}{2}}}$

(3) $\lambda \propto \dfrac{1}{(v - v_0)^{\frac{1}{4}}}$ (4) $\lambda \propto \dfrac{1}{(v - v_0)}$

Ans. (2)

Sol. For electron

$\lambda_{DB} = \dfrac{\lambda}{\sqrt{2mK.E.}}$ (de broglie wavelength)

By photoelectric effect

$h\nu = h\nu_0 + KE$

$KE = h\nu - h\nu_0$

$$\lambda_{DB} = \frac{h}{\sqrt{2m \times (h\nu - h\nu_0)}}$$

$$\lambda_{DB} \propto \frac{1}{(\nu - \nu_0)^{1/2}}$$

19. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2 M, the half-life is 6 h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be :-

(1) 18.0 h (2) 7.2 h (3) 9.0 h (4) 12.0 h

Ans. (1)

Sol. For zero order

$[A_0]-[A_t] = kt$

$0.2 - 0.1 = k \times 6$

$$k = \frac{1}{60}\ \text{M/hr}$$

and $0.5 - 0.2 = \dfrac{1}{60} \times t$

t = 18 hrs.

20. A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is :-

(1) $CH_3COCH_2NHCH_3$

(2) $CH_3CH_2COCH_2NH_2$

(3) $CH_3CH_2CH_2CONH_2$

(4) $CH_3CON(CH_3)_2$

Ans. (3)

Sol.

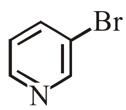
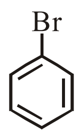
$$[X] \xrightarrow[\text{NaOH}]{Br_2} C_3H_9N \xrightarrow[\text{KOH}]{CHCl_3} CH_3CH_2CH_2\text{–}NC$$

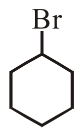
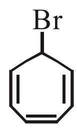
Hoff mann's Bromaide degradation   Carbylamine Reaction

Thus [X] must be aride with oen carbon more than is amine.

Thus [X] is $CH_3CH_2CH_2CONH_2$

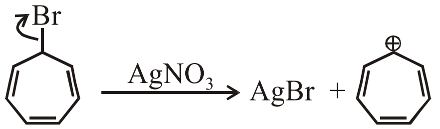
21. Which of the following compounds will produce a precipitate with $AgNO_3$?

(1) 
(2) 

(3) 
(4) 

Ans. (4)

Sol.



aromatic cation

as it can produce aromatic cation so will produce precipitate with $AgNO_3$.

22. The relative stability of +1 oxidation state of group 13 elements follows the order :-

(1) Al < Ga < Tl < In (2) Tl < In < Ga < Al

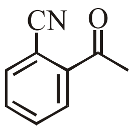
(3) Al < Ga < In < Tl (4) Ga < Al < In < Tl

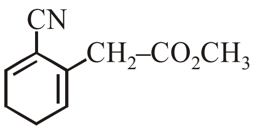
Ans. (3)

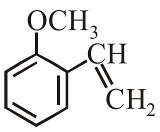
Sol. Due to inert pair effect as we move down the group in 13th group lower oxidation state becomes more stable.

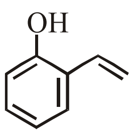
Al < Ga < In < Tℓ

23. Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution :-

(1) 

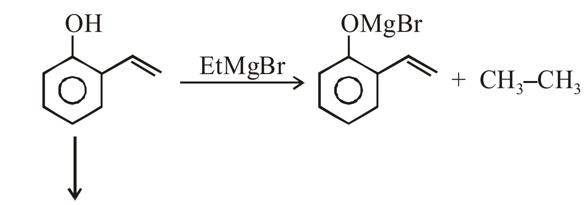
(2) 

(3) 

(4) 

Ans. (4)

Sol.



declolourizes Bromin water

24. Match the following items in column I with the corresponding items in column II.

Column I		Column II	
(i)	$Na_2CO_3 \cdot 10\ H_2O$	(P)	Portland cement ingredient
(ii)	$Mg(HCO_3)_2$	(Q)	Castner-Keller process
(iii)	NaOH	(R)	Solvay process
(iv)	$Ca_3Al_2O_6$	(S)	Temporary hardness

(1) (i)→(C); (ii)→(B); (iii)→(D); (iv)→(A)

(2) (i)→(C); (ii)→(D); (iii)→(B); (iv)→(A)

(3) (i)→(D); (ii)→(A); (iii)→(B); (iv)→(C)

(4) (i)→(B); (ii)→(C); (iii)→(A); (iv)→(D)

Ans. (2)

Sol. $Na_2CO_3.10H_2O \rightarrow$ Solvay process

$Mg(HCO_3)_2 \rightarrow$ Temporary hardness

NaOH → Castner-kellner cell

$Ca_3Al_2O_6 \rightarrow$ Portland cement

25. 25 ml of the given HCl solution requires 30 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.2 M aqueous NaOH solution?

(1) 25 mL (2) 50 mL (3) 12.5 mL (4) 75 mL

Ans. (1)

Sol. HCl with Na_2CO_3

Eq. of HCl = Eq. of Na_2CO_3

$$\frac{25}{1000}\times M\times 1 = \frac{30}{1000}\times 0.1\times 2$$

$$M = \frac{6}{25}M$$

Eq of HCl = Eq. of NaOH

$$\frac{6}{25}\times 1\times\frac{V}{1000} = \frac{30}{1000}\times 0.2\times 1$$

V = 25 ml

26. $\underline{A} \xrightarrow{4\ KOH,\ O_2} 2\underline{B} + 2\ H_2O$
(Green)

$3\ \underline{B} \xrightarrow{4\ HCl} 2\ \underline{C} + MnO_2 + 2\ H_2O$
(Purple)

$2\ \underline{B} \xrightarrow{H_2O,\ KI} 2\ \underline{A} + 2KOH + \underline{D}$

In the above sequence of reactions,

$\underline{A}$ and $\underline{D}$ respectively, are :-

(1) KIO_3 and MnO_2 (2) KI and K_2MnO_4

(3) MnO_2 and KIO_3 (4) KI and $KMnO_4$

Ans. **(3)**

Sol. $MnO_2(A) \xrightarrow{4KOH, O_2} 2K_2MnO_4(B) + 2H_2O$
(Green)

$3K_2MnO_4(B) \xrightarrow{4HCl} 2KMnO_4(C) + 2H_2O$
(Purple)

$2KMnO_4(C) \xrightarrow{H_2O, KI} 2MnO_2(A) + 2KOH + KIO_3(D)$

$A \rightarrow MnO_2$

$D \rightarrow KIO_3$

27. The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is :-

$\left(C_2O_4^{2-} = \text{Oxalato}\right)$

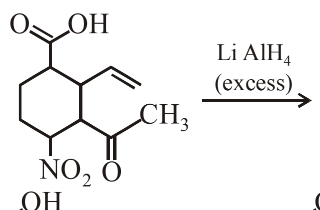
(1) 6 (2) 10 (3) 14 (4) 8

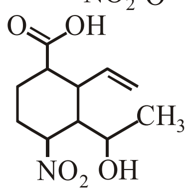
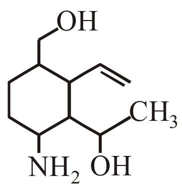
Ans. **(2)**

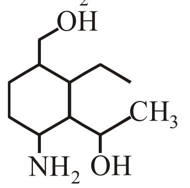
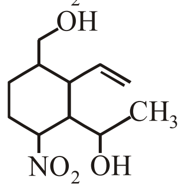
Sol. $C_2O_4^{2-}$ (oxalato) : bidentate

H_2O (aqua) : Monodentate

28. The major product obtained in the following reaction is :-

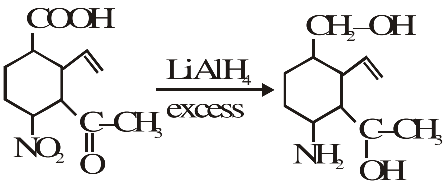


(1)  (2) 

(3)  (4) 

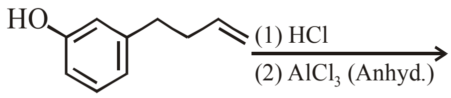
Ans. **(2)**

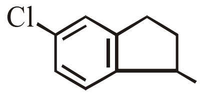
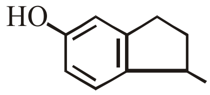
Sol.

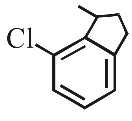
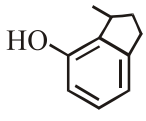


$LiAlH_4$ will not affect C=C in this compound.

29. The major product of the following reaction is :-

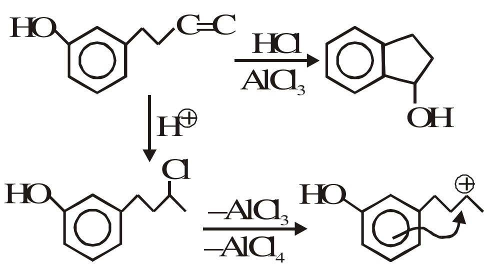


(1)  (2) 

(3)  (4) 

Ans. **(2)**

Sol.



30. For the equilibrium,

$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of ΔG° at 298 K is approximately :-

(1) $-80\ kJ\ mol^{-1}$ (2) $-100\ kJ\ mol^{-1}$

(3) $100\ kJ\ mol^{-1}$ (4) $80\ kJ\ mol^{-1}$

Ans. **(4)**

Sol.

$2H_2O = H_3O^+ + OH^-$ $K = 10^{-14}$

$\Delta G^\circ = -RT\ \ell n\ K$

$= \frac{-8.314}{1000} \times 298 \times \ell n 10^{-14}$

$= 80$ KJ/Mole

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On SATURDAY 12th JANUARY., 2019) TIME : 09 : 30 AM To 12 : 30 PM

MATHEMATICS

1. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $\left(1+\log_e 2x\right)^2 \frac{dy}{dx}$ is equal to :

(1) $\log_e 2x$

(2) $\frac{x\log_e 2x + \log_e 2}{x}$

(3) $x\log_e 2x$

(4) $\frac{x\log_e 2x - \log_e 2}{x}$

Ans. **(4)**

Sol. $(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y = \frac{x+\ell n2}{1+\ell n2x}$$

$$y' = \frac{(1+\ell n2x)-(x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$

$$y'(1+\ell n2x)^2 = \left[\frac{x\ell n2x - \ell n2}{x}\right]$$

2. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are co-planer, is :

(1) 2 (2) 0 (3) –1 (4) 1

Ans. **(3)**

Sol. $$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$\mu(\mu^2 - 1)-1(\mu-1) + 1(1-\mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1\ \mu = 0$

$\mu^3 - 3\mu + 2 = 0$

$\mu^3 - 1 - 3(\mu-1) = 0$

$\mu = 1,\ \mu^2 + \mu - 2 = 0$

$\mu = 1,\ \mu = -2$

sum of distinct solutions = –1

3. Let S be the set of all points in $(-\pi,\pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following?

(1) $\left\{-\frac{3\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{4}\right\}$

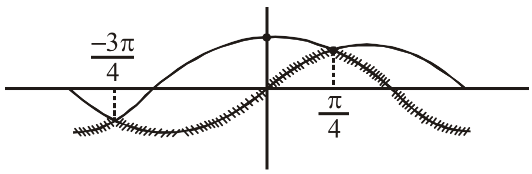
(2) $\left\{-\frac{3\pi}{4},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2},-\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4},0,\frac{\pi}{4}\right\}$

Ans. **(1)**

Sol.



4. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now from an A.P. Then the sum of the original three terms of the given G.P. is

(1) 36 (2) 24

(3) 32 (4) 28

Ans. **(4)**

Sol. Let terms are $\frac{a}{r}, a, ar \rightarrow$ G.P

$\therefore\ a^3 = 512 \Rightarrow a = 8$

$\frac{8}{r}+4, 12, 8r \rightarrow$ A.P.

$24 = \frac{8}{r}+4+8r$

$r = 2,\ r = \frac{1}{2}$

$r = 2\ (4,\ 8,\ 16)$

$r = \frac{1}{2}\ (16, 8, 4)$

Sum = 28

5. The integral $\int \cos(\log_e x)dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x}{2}[\sin(\log_e x)-\cos(\log_e x)]+C$

(2) $\frac{x}{2}[\cos(\log_e x)+\sin(\log_e x)]+C$

(3) $x[\cos(\log_e x)+\sin(\log_e x)]+C$

(4) $x[\cos(\log_e x)-\sin(\log_e x)]+C$

Ans. (2)

Sol. $I=\int \cos(\ell n\, x)dx$

$I=\cos(\ln x).x+\int \sin(\ell n\, x)\, dx$

$\cos(\ell n\, x)x+[\sin(\ell n\, x).x-\int \cos(\ell n\, x)dx]$

$I=\frac{x}{2}[\sin(\ell n\, x)+\cos(\ell n\, x)]+C$

6. Let $S_k = \frac{1+2+3+....+k}{k}$. If

$S_1^2+S_2^2+....+S_{10}^2=\frac{5}{12}A$, then A is equal to :

(1) 303 (2) 283 (3) 156 (4) 301

Ans. (1)

Sol. $S_K=\frac{K+1}{2}$

$\Sigma S_k^2=\frac{5}{12}A$

$\sum_{K=1}^{10}\left(\frac{K+1}{2}\right)^2=\frac{2^2+3^2+--+11^2}{4}=\frac{5}{12}A$

$\frac{11\times 12\times 23}{6}-1=\frac{5}{3}A$

$505=\frac{5}{3}A$, A = 303

7. Let S = {1,2,3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :-

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$

(3) $2^{50}-1$ (4) $2^{50}+1$

Ans. (1)

Sol. S = {1,2,3------100}

= Total non empty subsets-subsets with product of element is odd

$= 2^{100}-1-1[(2^{50}-1)]$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50}-1)$

8. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observation is :

(1) 50 (2) 51 (3) 30 (4) 31

Ans. (4)

Sol. $\sum_{i=1}^{50}(x_i-30)=50$

$\Sigma x_i=50\times 30=50$

$\Sigma x_i=50+50+30$

Mean = $\bar{x}=\frac{\Sigma x_i}{n}=\frac{50\times 30+50}{50}=30+1=31$

9. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ are on its opposite sides, then the set of all values of λ is the interval :-

(1) [12, 21] (2) (2, 17)

(3) (23, 31) (4) [13, 23]

Ans. (1)

Sol. Centre of circles are opposite side of line

$(3+4-\lambda)(27+4-\lambda)<0$

$(\lambda-7)(\lambda-31)<0$

$\lambda \in (7, 31)$

distance from S_1

$\left|\frac{3+4-\lambda}{5}\right|\ge 1 \Rightarrow \lambda\in(-\infty, 2] \cup[(12,\infty]$

distance from S_2

$$\left|\frac{27+4-\lambda}{5}\right| \geq 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$$

so $\lambda \in [12, 21]$

10. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is :

(1) $1 : 4(16)^{\frac{1}{3}}$ (2) $1 : 2(6)^{\frac{1}{3}}$

(3) $2(36)^{\frac{1}{3}} : 1$ (4) $4(36)^{\frac{1}{3}} : 1$

Ans. (4)

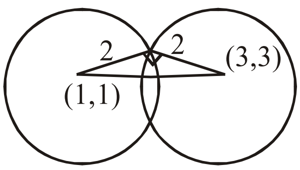
Sol. $$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4 (2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4} = 4.(36)^{1/3}$$

11. let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2 = 0$ and $x^2+y^2-6x-6y+14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

(1) 8 (2) 6 (3) 9 (4) 4

Ans. (4)

Sol.



Area = 2 × $\frac{1}{2}.4 = 2$

12. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

Ans. (2)

Sol. $\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{4}\ \underline{4}$

$$\frac{1}{6^2}\left(\frac{5^3}{6^3} + \frac{2C_1.5^2}{6^3}\right) = \frac{175}{6^5}$$

13. If the straight line, $2x-3y+17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals :-

(1) –5 (2) $-\frac{35}{3}$

(3) $\frac{35}{3}$ (4) 5

Ans. (4)

Sol. $\frac{17-\beta}{-8} \times \frac{2}{3} = -1$

$\beta = 5$

14. Let f and g be continuous functions on [0, a] such that $f(x) = f(a-x)$ and $g(x)+g(a-x)=4$, then $\int_0^a f(x)g(x)dx$ is equal to :-

(1) $4\int_0^a f(x)dx$ (2) $2\int_0^a f(x)dx$

(3) $-3\int_0^a f(x)dx$ (4) $\int_0^a f(x)dx$

Ans. (2)

Sol. $I = \int_0^a f(x)g(x)dx$

$I = \int_0^a f(a-x)g(a-x)dx$

$I = \int_0^a f(x)(4-g(x)dx$

$I = 4\int_0^a f(x)dx - I$

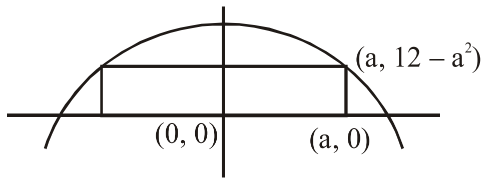
$\Rightarrow\ I = 2\int_0^a f(x)dx$

15. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12-x^2$ such that the rectangle lies inside the parabola, is :-

(1) $20\sqrt{2}$ (2) $18\sqrt{3}$ (3) 32 (4) 36

Ans. (3)

Sol. $f(a) = 2a(12 - a)^2$



$f'(a) = 2(12 - 3a^2)$

maximum at a = 2

maximum area = f(2) = 32

16. The Boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to:

(1) $p \wedge (\sim q)$ (2) $p \vee (\sim q)$

(3) $(\sim p) \wedge (\sim q)$ (4) $p \wedge q$

Ans. (3)

17. $\lim_{x \to \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \pi/4\right)}$ is :

(1) 4 (2) $8\sqrt{2}$ (3) 8 (4) $4\sqrt{2}$

Ans. (3)

Sol. $\lim_{x \to \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$\lim_{x \to \pi/4} \frac{(1 - \tan^4 x)}{\cos(x + \pi/4)}$

$2 \lim_{x \to \pi/4} \frac{(1 - \tan^2 x)}{\cos(x + \pi/4)}$

$R \lim_{x \to \pi/4} \frac{\cos^2 x - \sin^2 x}{\frac{\cos x - \sin x}{\sqrt{2}}} \frac{1}{\cos^2 x}$

$4\sqrt{2} \lim_{x \to \pi/4} (\cos x + \sin x) = 8$

18. Considering only the principal values of inverse functions, the set

$A = \left\{ x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4} \right\}$

(1) is an empty set

(2) Contains more than two elements

(3) Contains two elements

(4) is a singleton

Ans. (4)

Sol. $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\Rightarrow \frac{5x}{1 - 6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$x = -1$ or $x = \frac{1}{6}$

$x = \frac{1}{6} \quad \because x > 0$

19. An ordered pair(α,β) for which the system of linear equations

$(1+\alpha)x + \beta y + z = 2$

$\alpha x + (1+\beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$ has a unique solution is

(1) (1,–3) (2) (–3,1)

(3) (2, 4) (4) (–4, 2)

Ans. (3)

Sol. For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

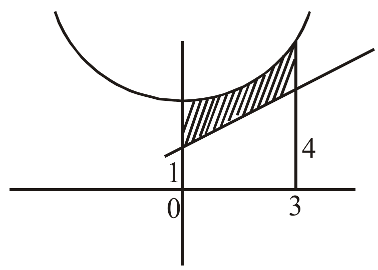
$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

20. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

(1) $\frac{15}{4}$ (2) $\frac{15}{2}$ (3) $\frac{21}{2}$ (4) $\frac{17}{4}$

Ans. (2)

Sol.



Req. area $= \int_0^3 (x^2 + 2)dx - \frac{1}{2}.5.3 = 9 + 6 - \frac{15}{2} = \frac{15}{2}$

21. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2 = 0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is :

(1) $2-\sqrt{3}$ (2) $4-3\sqrt{2}$

(3) $-2+\sqrt{2}$ (4) $4-2\sqrt{3}$

Ans. (2)

Sol. $3m^2x^2 + m(m - 4)x + 2 = 0$

$\lambda+\frac{1}{\lambda}=1$, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=1$, $\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3m^2}\right)^2=\frac{3(2)}{3m^2}$, $\frac{(m-4)^2}{9m^2}=\frac{6}{3m}$

$(m - 4)^2 = 18$, $m = 4\pm\sqrt{18},\ 4\pm3\sqrt{2}$

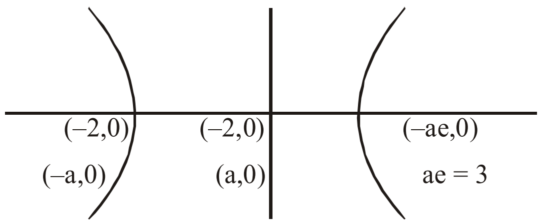
22. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola?

(1) $\left(4,\sqrt{15}\right)$ (2) $\left(-6,2\sqrt{10}\right)$

(3) $\left(6,5\sqrt{2}\right)$ (4) $\left(2\sqrt{6},5\right)$

Ans. (3)

Sol.



$ae = 3$, $e=\frac{3}{2}$, $b^2=4\left(\frac{9}{4}-1\right)$, $b^2 = 5$

$\frac{x^2}{4}-\frac{y^2}{5}=1$

23. If $\frac{z-\alpha}{z+\alpha}(\alpha\in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is :

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $\frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$

$z\bar{z}+z\alpha-\alpha\bar{z}-\alpha^2+z\bar{z}-z\alpha+\bar{z}\alpha-\alpha^2=0$

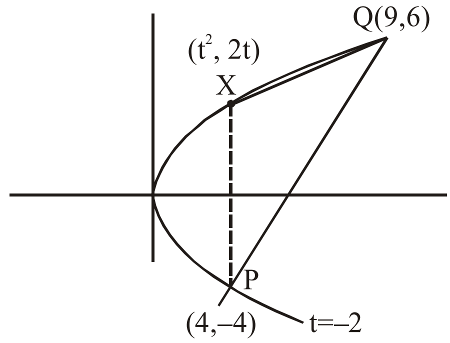
$|z|^2 = \alpha^2$, a = ±2

24. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2= 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is :

(1) $\frac{125}{4}$ (2) $\frac{125}{2}$ (3) $\frac{625}{4}$ (4) $\frac{75}{2}$

Ans. (1)

Sol.



$y^2 = 4x$

$2yy' = 4$

$y'=\frac{1}{t}=2$, $t=\frac{1}{2}$

Area $=\frac{1}{2}\begin{vmatrix}\frac{1}{4} & 1 & 1\\ 9 & 6 & 1\\ 4 & -4 & 1\end{vmatrix}=\frac{125}{4}$

25. the perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

(1) $11/\sqrt{6}$ (2) $6\sqrt{11}$ (3) 11 (4) $11\sqrt{6}$

Ans. (1)

Sol. $\begin{vmatrix} i & j & k\\ 3 & 5 & 7\\ 1 & 4 & 7\end{vmatrix}$

$\hat{i}(35-28)-\hat{j}(21.7)+\hat{k}(12-5)$

$7\hat{i}-14\hat{j}+7\hat{k}$

$\hat{i}-2\hat{j}+\hat{k}$

$1(x + 2) - 2(y - 2) + 1(z+15) = 0$

$x - 2y + z + 11 = 0$

$\frac{11}{\sqrt{4+1+1}}=\frac{11}{\sqrt{6}}$

26. The maximum value of $3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$ for any real value of θ is :

(1) $\sqrt{19}$ (2) $\frac{\sqrt{79}}{2}$ (3) $\sqrt{31}$ (4) $\sqrt{34}$

Ans. (1)

Sol. $y=3\cos\theta+5\left(\sin\theta\frac{\sqrt{3}}{2}-\cos\theta\frac{1}{2}\right)$

$\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$

$y_{max}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$

27. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is :

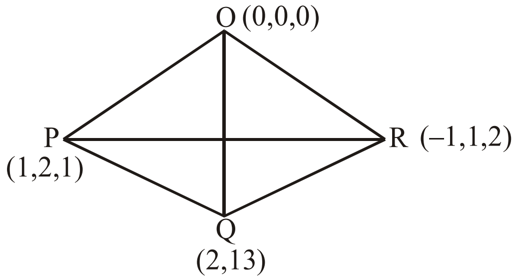
(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$

(3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

Ans. (1)

Sol. $\overline{OP}\times\overline{OQ}=(\hat{i}+2\hat{j}+\hat{k})\times(2\hat{i}+\hat{j}+3\hat{k})$

$5\hat{i}-\hat{j}-3\hat{k}$



$\overline{PQ}\times\overline{PR}=(\hat{i}-\hat{j}+2\hat{k})\times(-2\hat{i}-\hat{j}+\hat{k})$

$\hat{i}-5\hat{j}-3\hat{k}$

$\cos\theta=\frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2}=\frac{19}{35}$

28. Lety = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x,(x>1)$. If $2y(2)=\log_e 4-1$, then y(e) is equal to :-

(1) $\frac{e^2}{4}$ (2) $\frac{e}{4}$ (3) $-\frac{e}{2}$ (4) $-\frac{e^2}{2}$

Ans. (2)

Sol. $\frac{dy}{dx}=\frac{y}{x}=\ell n\,x$

$e^{\int\frac{1}{x}dx}=x$

$xy=\int x\ell nx+C$

$\ell n\,x\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}$

$xy=\frac{x}{2}\ell n\,x-\frac{x^2}{4}$ +C, for 2y(2)= 2ℓn2 – 1

$\Rightarrow$ C = 0

$y=\frac{x}{2}\ell n\,x-\frac{x}{4}$

$y(e)=\frac{e}{4}$

29. Let P = $\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$ and Q = $[q_{ij}]$ be two 3×3 matrices such that $Q-P^5=I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to:

(1) 15 (2) 9 (3) 135 (4) 10

Ans. (4)

Sol. $P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$

$P^2=\begin{bmatrix}1&0&0\\3+3&1&0\\9+9+9&3+3&1\end{bmatrix}$

$P^3=\begin{bmatrix}1&0&0\\3+3+3&1&0\\6.9&3+3+3&1\end{bmatrix}$

$P^n=\begin{bmatrix}1&0&0\\3n&1&0\\\frac{n(n+1)}{2}3^2&3n&1\end{bmatrix}$

$P^5=\begin{bmatrix}1&0&0\\5.3&1&0\\15.9&5.3&1\end{bmatrix}$

$Q = P^5 + I_3$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$P = I + X$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

30. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

(1) 82 (2) 240 (3) 164 (4) 120

Ans. **(4)**

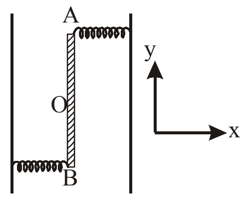
Sol. No. of ways = $^{10}C_3$ = 120

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 09 : 30 AM To 12 : 30 PM

PHYSICS

1. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

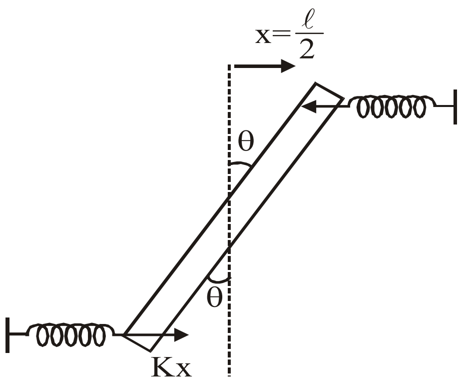
(2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

Ans. **(1)**

Sol.



$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$

$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

2. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:

(1) $K_1 + K_2$

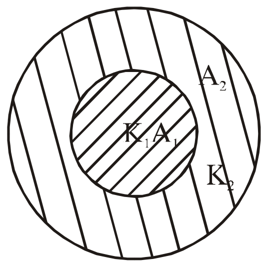
(2) $\frac{K_1+K_2}{2}$

(3) $\frac{2K_1+3K_2}{5}$

(4) $\frac{K_1+3K_2}{4}$

Ans. **(4)**

Sol.



$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

$$= \frac{K_1\left(\pi R^2\right) + K_2\left(3\pi R^2\right)}{4\pi R^2}$$

$$= \frac{K_1 + 3K_2}{4}$$

3. A travelling harmonic wave is represented by the equation y (x, t) = 10^{-3} sin (50 t + 2x), where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave?

The wave is propagating along the

(1) negative x-axis with speed $25ms^{-1}$

(2) The wave is propagating along the positive x-axis with speed 25 ms^{-1}

(3) The wave is propagating along the positive x-axis with speed 100 ms^{-1}

(4) The wave is propagating along the negative x-axis with speed 100 ms^{-1}

Ans. **(1)**

Sol. y= a sin(ωt + kx)

⇒ wave is moving along –ve x-axis with speed

$$v = \frac{\omega}{K} \Rightarrow v = \frac{50}{2} = 25\text{m/sec.}$$

4. A straight rod of length L extends from x = a to x=L + a. The gravitational force is exerts on a point mass 'm' at x = 0, if the mass per unit length of the rod is A + Bx², is given by:

(1) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

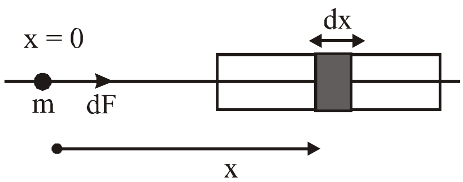
(2) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(3) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(4) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

Ans. **(2)**

Sol.



dm= (A + Bx²)dx

$$dF = \frac{GM\,dm}{x^2}$$

$$= F = \int_a^{a+L} \frac{GM}{x^2}\left(A + Bx^2\right)dx$$

$$= GM\left[-\frac{A}{x} + Bx\right]_a^{a+L}$$

$$= GM\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$$

5. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be:

(1) 10 V/m (2) 24 V/m

(3) 30 V/m (4) 6 V/m

Ans. **(2)**

Sol. $P_{refracted} = \frac{96}{100}P_I$

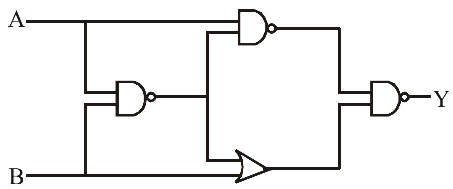
$$\Rightarrow K_2A_t^2 = \frac{96}{100}K_1A_i^2$$

$$\Rightarrow r_2A_t^2 = \frac{96}{100}r_1A_i^2$$

$$\Rightarrow A_t^2 = \frac{96}{100} \times \frac{1}{\frac{3}{2}} \times (30)^2$$

$$A_t\sqrt{\frac{64}{100}\times(30)^2} = 24$$

6. The output of the given logic circuit is :



(1) $\overline{A}B$

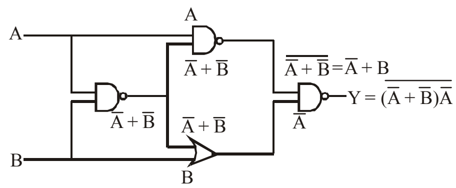
(2) $A\overline{B}$

(3) $AB + \overline{AB}$

(4) $A\overline{B} + \overline{A}B$

Ans. **(2)**

Sol.



$$Y = \overline{\left(\overline{A}+\overline{B}\right)A}$$

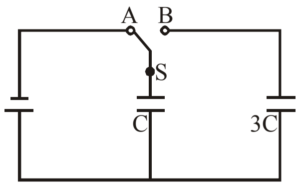
$$= \overline{\overline{A} + \overline{A}B}$$

$$= A\left(\overline{\overline{A}B}\right)$$

$$= A\left(A + \overline{B}\right)$$

$$= A + A\overline{B} = A\overline{B}$$

7. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is:



(1) $\frac{3}{8}\frac{Q^2}{C}$ (2) $\frac{3}{4}\frac{Q^2}{C}$ (3) $\frac{1}{8}\frac{Q^2}{C}$ (4) $\frac{5}{8}\frac{Q^2}{C}$

Ans. **(1)**

Sol. $V_i = \frac{1}{2}CE^2$

$$V_f = \frac{(CE)^2}{2\times 4c} = \frac{1}{2}\frac{CE^2}{4}$$

$$\Delta E = \frac{1}{2}CE^2 \times \frac{3}{4} = \frac{3}{8}CE^2$$

8. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as:

(1) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$ (2) $r_n \propto \sqrt{n}, E_n \propto \frac{1}{n}$

(3) $r_n \propto n$, $E_n \propto n$ (4) $r_n \propto \sqrt{n}$, $E_n \propto n$

Ans. **(4)**

Sol. $F = \frac{dV}{dr} = kr = \frac{mv^2}{r}$

$$mvr = \frac{nh}{2\pi}$$

$$r^2 \propto n$$

$$r^2 \propto \sqrt{n}$$

$$E = \frac{1}{2}kr^2 + \frac{1}{2}mv^2 \propto r^2$$

$$\propto n$$

9. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then:

(1) P1 = 9 W, P_2 = 16 W

(2) P_1 = 4 W, P_2 = 16W

(3) P_1 = 16 W, P_2 = 4W

(4) P_1 16 W, P_2 = 9W

Ans. **(3)**

Sol. $R_1 = \frac{220^2}{25}$

$$R_2 = \frac{220^2}{100}$$

$$L = \frac{220}{R_1 + R_2}$$

$P_1 = i^2 R_1$

$P_2 = i^2$ (R_2 = 4W)

$$= \frac{220^2}{\left(\frac{220^2}{25} + \frac{220^2}{100}\right)} \times \frac{220^2}{25}$$

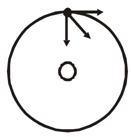
$$= \frac{400}{25} = 16W$$

10. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be :

(1) in a circular orbit of a different radius
(2) in the same circular orbit of radius R
(3) in an elliptical orbit
(4) such that it escapes to infinity

Ans. (3)

Sol. $m v\hat{i} + m v\hat{j}$



$= 2m\vec{v}^1$

$$\vec{v} = \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$$

11. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is:
(1) 12 cm (2) 18 cm (3) 16 cm (4) 14 cm

Ans. (3)

12. A passenger train of length 60m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction, and (ii) in the opposite directions is :

(1) $\frac{5}{2}$ (2) $\frac{25}{11}$ (3) $\frac{3}{2}$ (4) $\frac{11}{5}$

Ans. (4)

13. An ideal gas occupies a volume of $2m^3$ at a pressure of 3×10^6 Pa. The energy of the gas is:
(1) 3×10^2 (2) 10^8 J
(3) 6×10^4 J (4) 9×10^6 J

Ans. (4)

Sol. Energy $= \frac{1}{2}nRT = \frac{f}{2}PV$

$= \frac{f}{2}(3\times10^6)(2)$

$= f \times 3 \times 10^6$

Considering gas is monoatomic i.e. f = 3

E. $= 9 \times 10^6$ J

Option-(4)

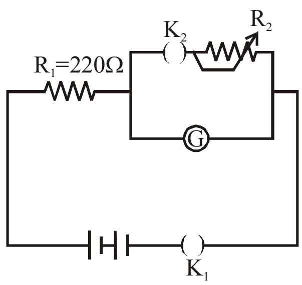
14. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?
(1) 0.6 (2) 0.5 (3) 0.3 (4) 0.4

Ans. (1)

Sol. $E_m + E_c = 160$
$E_m + 100 = 160$
$E_m = 60$

$\mu = \frac{E_m}{E_C} = \frac{60}{100}$
$\mu = 0.6$

15. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]:



(1) 12Ω (2) 25Ω (3) 5Ω (4) 22Ω

Ans. (4)

Sol. case I $\quad i_g = \dfrac{E}{220 + R_g} = C\theta_0 \quad$ ..(i)

Case II

$$i_g = \left(\dfrac{E}{220 + \dfrac{5R_g}{5 + R_g}}\right) \times \dfrac{5}{(R_g + 5)} = \dfrac{C\theta_0}{5} \quad ..(ii)$$

$$\Rightarrow \dfrac{5E}{225R_g + 1100} = \dfrac{C\theta_0}{5} \quad ..(ii)$$

$$\dfrac{E}{220 + R_g} = C\theta \quad ...(i)$$

$$\Rightarrow \dfrac{225R_g + 1100}{1100 + 5R_g} = 5$$

$\Rightarrow$ 5500 + 25R_g = 225R_g + 1100

200R_g = 4400

R_g = 22Ω

Ans. – 4

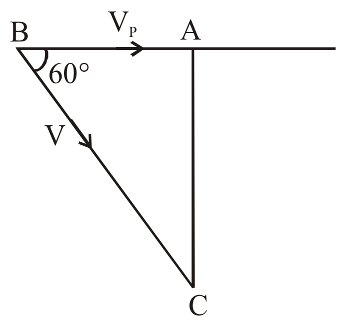
16. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If υ is the speed of sound, speed of the plane is :

(1) $\dfrac{2\upsilon}{\sqrt{3}}$ (2) υ

(3) $\dfrac{\upsilon}{2}$ (4) $\dfrac{\sqrt{3}}{2}\upsilon$

Ans. **(3)**

Sol.



AB = V_p × t

BC = Vt

$$\cos 60° = \dfrac{AB}{BC}$$

$$\dfrac{1}{2} = \dfrac{V_p \times t}{Vt}$$

$$V_P = \dfrac{V}{2}$$

17. A proton and an α-particle (with their masses in the ratio of 1:4 and charges in the ratio of 1:2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be :

(1) $1:\sqrt{2}$ (2) 1 : 2 (3) 1 : 3 (4) $1:\sqrt{3}$

Ans. **(1)**

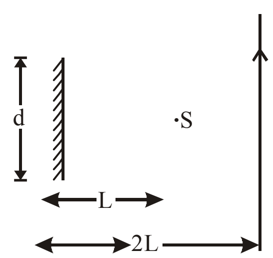
Sol. KE = qΔV

$$r = \dfrac{\sqrt{2mq\Delta V}}{qB}$$

$$r \propto \sqrt{\dfrac{m}{q}}$$

$$\dfrac{r_p}{r_\propto} = \dfrac{1}{\sqrt{2}}$$

18. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance 2L as shown below. The distance over which the man can see the image of the light source in the mirror is :

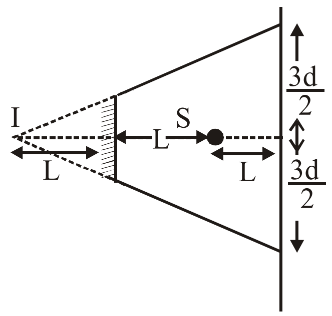


(1) 3d (2) $\frac{d}{2}$

(3) d (4) 2d

Ans. **(1)**

Sol.



3d

19. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5μm diameter of wire is :

(1) 50 (2) 100

(3) 200 (4) 500

Ans. **(3)**

Sol. Least count = $\frac{\text{Pitch}}{\text{Number of division on circular scale}}$

$$5\times10^{-6} = \frac{10^{-3}}{N}$$

N = 200

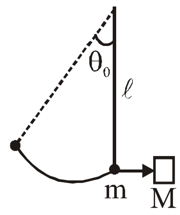
20. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by :

(1) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$ (2) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

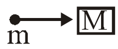
(3) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (4) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

Ans. **(3)**

Sol.



Before colision After collision



$v = \sqrt{2g\ell(1-\cos\theta_0)}$ $v_1 = \sqrt{2g\ell(1-\cos\theta_1)}$

By momentum conservation

$$m\sqrt{2g\ell(1-\cos\theta_0)} = MV_m - m\sqrt{2gl(1-\cos\theta}$$

$$\Rightarrow m\sqrt{2g\ell}\left\{\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right\} = MV_m$$

and $e = 1 = \frac{V_m + \sqrt{2g\ell(1-\cos\theta_1)}}{\sqrt{2g\ell(1-\cos\theta_0)}}$

$$\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right) = V_m \quad ..(I)$$

$$m\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right) = MV_M \quad ..(II)$$

Dividing

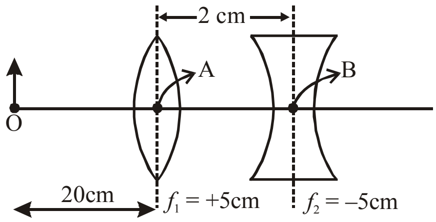
$$\frac{\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)} = \frac{M}{m}$$

By componendo divided

$$\frac{m-M}{m+M} = \frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{M}{m} = \frac{\theta_0-\theta_1}{\theta_0+\theta_1} \Rightarrow M = \frac{\theta_0-\theta_1}{\theta_0+\theta_1}$$

21. What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths)



(1) 70 cm from point B at left; virtual

(2) 40 cm from point B at right; real

(3) $\frac{20}{3}$ cm from point B at right , real

(4) 70 cm from point B at right, real

Ans. **(4)**

Sol. For first lens

$$\frac{1}{V} - \frac{1}{-20} = \frac{1}{5}$$

$$V = \frac{20}{3}$$

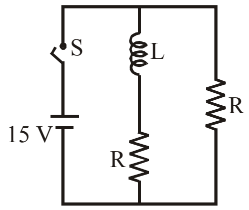
For second lens

$$V = \frac{20}{3} - 2 = \frac{14}{3}$$

$$\frac{1}{V} - \frac{1}{\frac{14}{3}} = \frac{1}{-5}$$

V = 70cm

22. In the figure shown, a circuit contains two identical resistors with resistance R = 5Ω and an inductance with L = 2mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed?

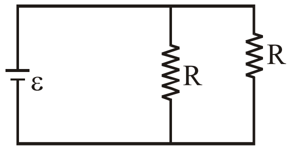


(1) 6A (2) 7.5A

(3) 5.5A (4) 3A

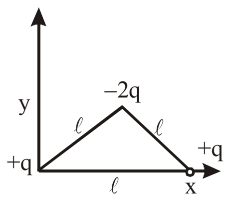
Ans. **(1)**

Sol. Ideal inductor will behave like zero resistance long time after switch is closed



$$I = \frac{2\varepsilon}{R} = \frac{2 \times 15}{5} = 6A$$

23. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure:

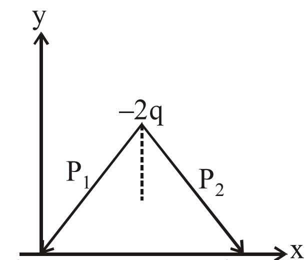


(1) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (2) $\sqrt{3}q\ell\frac{\hat{j}-\hat{i}}{\sqrt{2}}$

(3) $-\sqrt{3}q\ell\hat{j}$ (4) $2q\ell\hat{j}$

Ans. **(3)**

Sol.



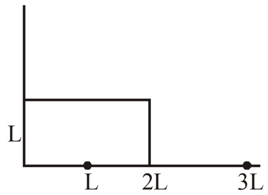
$|P_1| = q(d)$

$|P_2| = qd$

$|\text{Resultant}| = 2\,P\cos 30^\circ$

$$2\,qd\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,qd$$

24. The position vector of the centre of mass $\vec{r}\,cm$ of an symmetric uniform bar of negligible area of cross-section as shown in figure is :



(1) $\vec{r}\,cm = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

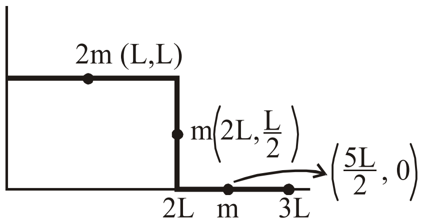
(2) $\vec{r}\,cm = \frac{11}{8}L\,\hat{x} + \frac{3}{8}L\hat{y}$

(3) $\vec{r}\,cm = \frac{3}{8}L\,\hat{x} + \frac{11}{8}L\hat{y}$

(4) $\vec{r}\,cm = \frac{5}{8}L\,\hat{x} + \frac{13}{8}L\hat{y}$

Ans. (1)

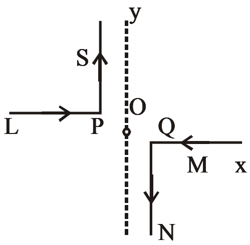
Sol.



$$X_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

$$Y_{cm} = \frac{2m \times L + m \times \left(\frac{L}{2}\right) + m \times 0}{4m} = \frac{5L}{8}$$

25. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) :



(1) 40 A, perpendicular into the page

(2) 40 A, perpendicular out of the page

(3) 20 A, perpendicular out of the page

(4) 20 A, perpendicular into the page

Ans. (4)

Sol. Magnetic field at 'O' will be done to 'PS' and 'QN' only

i.e. $B_0 = B_{PS} + B_{QN} \rightarrow$ Both inwards

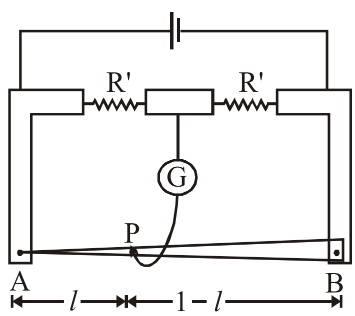
Let current in each wire = i

$$\therefore \quad B_0 = \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d}$$

$$\text{or} \quad 10^{-4} = \frac{\mu_0 i}{2\pi d} = \frac{2\times10^{-7}\times i}{4\times10^{-2}}$$

$$\therefore \quad i = 20\text{ A}$$

26. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?



(1) 0.25 m (2) 0.3m

(3) 0.35 m (4) 0.2 m

Ans. (1)

Sol. For the given wire : $dR = C\frac{d\ell}{\sqrt{\ell}}$, where C = constant.

Let resistance of part AP is R_1 and PB is R_2

$\therefore \quad \frac{R'}{R'} = \frac{R_1}{R_2}$ or $R_1 = R_2$ By balanced WSB concept.

Now $\int dR = c\int \frac{d\ell}{\sqrt{\ell}}$

$$\therefore R_1 = C\int_0^{\ell} \ell^{-1/2} d\ell = C.2.\sqrt{\ell}$$

$$R_2 = C\int_{\ell}^{1} \ell^{-1/2} d\ell = C.(2 - 2\sqrt{\ell})$$

Putting $R_1 = R_2$

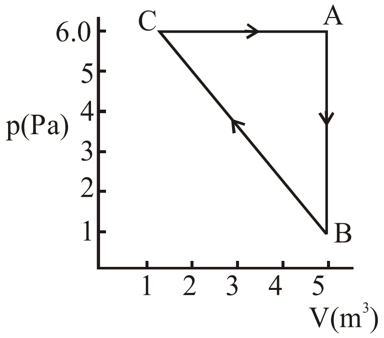
$C2\sqrt{\ell} = C\,(2 - 2\sqrt{\ell})$

$\therefore 2\sqrt{\ell} = 1$

$\sqrt{\ell} = \frac{1}{2}$

i.e. $\ell = \frac{1}{4}$ m $\Rightarrow$ 0.25 m

27. For the given cyclic process CAB as shown for a gas, the work done is :



(1) 1 J (2) 5 J

(3) 10 J (4) 30 J

Ans. (3)

Sol. Since P–V indicator diagram is given, so work done by gas is area under the cyclic diagram.

$\therefore \quad \Delta W$ = Work done by gas = $\frac{1}{2} \times 4 \times 5$ J

= 10 J

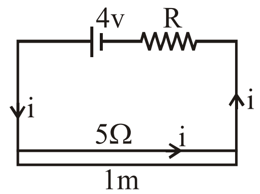
28. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire, is :

(1) 490 Ω (2) 480 Ω

(3) 395 Ω (4) 495 Ω

Ans. (3)

Sol.



Let current flowing in the wire is i.

$\therefore \quad i = \left(\frac{4}{R+5}\right)$A

If resistance of 10 m length of wire is x

then $x = 0.5\ \Omega = 5 \times \frac{0.1}{1}\Omega$

$\therefore \quad \Delta V$ = P. d. on wire = i. x

$5 \times 10^{-3} = \left(\frac{4}{R+5}\right)\cdot(0.5)$

$\therefore \quad \frac{4}{R+5} = 10^{-2}$ or $R + 5 = 400\ \Omega$

$\therefore$ R = 395 Ω

29. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass '4 m' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to :

(1) 10.00 (2) 14.14 (3) 4.47 (4) 0.07

Ans. (2)

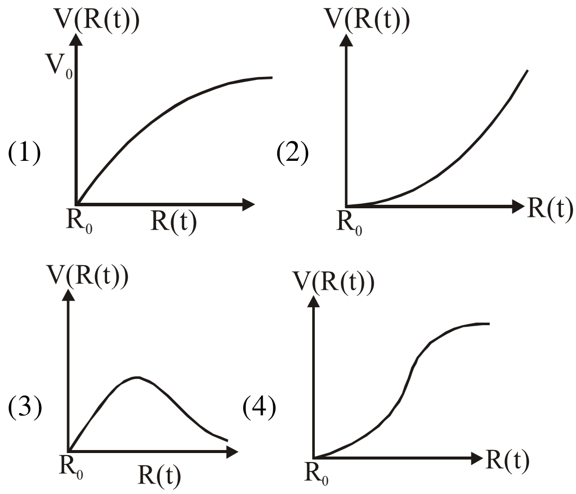
Sol. K.E. acquired by charge = K = qV

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \frac{\sqrt{2m_Bq_BV_B}}{\sqrt{2m_Aq_AV_A}} = \sqrt{\frac{4m.q.2500}{m.q.50}} = 2\sqrt{50}$$

= 2 × 7.07 = 14.14

30. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V(R(t)) of the distribution as a function of its instantaneous radius R (t) is :



Ans. (1)

Sol. At any instant 't'

Total energy of charge distribution is constant

i.e. $\frac{1}{2}mV^2 + \frac{KQ^2}{2R} = 0 + \frac{KQ^2}{2R_0}$

$\therefore \ \frac{1}{2}mV^2 = \frac{KQ^2}{2R_0} - \frac{KQ^2}{2R}$

$\therefore \ V = \sqrt{\frac{2}{m}\frac{KQ^2}{2}.\left(\frac{1}{R_0} - \frac{1}{R}\right)}$

$\therefore \ V = \sqrt{\frac{KQ^2}{m}\left(\frac{1}{R_0} - \frac{1}{R}\right)} = C\sqrt{\frac{1}{R_0} - \frac{1}{R}}$

Also the slope of v-s curve will go on decreasing

$\therefore$ Graph is correctly shown by option(1)

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 09 : 30 AM To 12 : 30 PM

CHEMISTRY

1. Iodine reacts with concentrated HNO_3 to yield Y along with other products. The oxidation state of iodine in Y, is :-

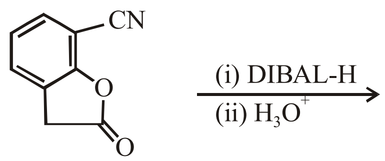
(1) 5 (2) 3 (3) 1 (4) 7

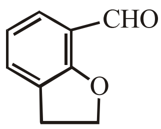
Ans. (1)

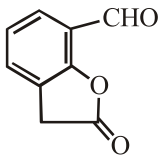
Sol. $I_2 + 10HNO_3 \longrightarrow 2HIO_3 + 10NO_2 + 4H_2O$

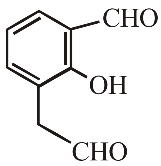
In HIO_3 oxidation state of iodine is +5.

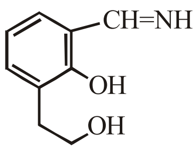
2. The major product of the following reaction is:



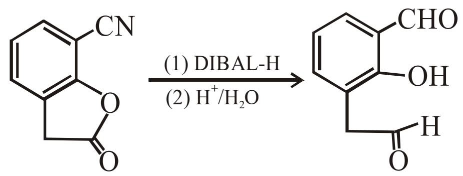
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

DIBAL-H will reduce cyanides & esters to aldehydes.

3. In a chemical reaction, $A + 2B \overset{K}{\rightleftharpoons} 2C + D$, the initial concentration of B was 1.5 times of the concentration of A, but the equilibrium concentrations of A and B were found to be equal. The equilibrium constant(K) for the aforesaid chemical reaction is :

(1) 16 (2) 4 (3) 1 (4) $\frac{1}{4}$

Ans.(2)

Sol.

	A	+	$2B$	$\rightleftharpoons$	$2C$	+	D
$t=0$	a_0		$1.5a_0$		0		0
$t = t_{eq}$	$a_0 - x$		$1.5a_0 - 2x$		$2x$		x

At equilibrium [A] = [B]

$a_0 - x = 1.5a_0 - 2x \Rightarrow x = 0.5a_0$

$t = t_{eq}$ $0.5a_0$ $0.5a_0$ a_0 $0.5a_0$

$$K_C = \frac{[C]^2[D]}{[A][B]^2} = \frac{(a_0)^2\,(0.5a_0)}{(0.5a_0)\,(0.5a_0)^2} = 4$$

4. Two solids dissociate as follows

$A(s) \rightleftharpoons B(g) + C(g)$; $K_{P_1} = x\ atm^2$

$D(s) \rightleftharpoons C(g) + E(g)$; $K_{P_2} = y\ atm^2$

The total pressure when both the solids dissociate simultaneously is :-

(1) $x^2 + y^2$ atm (2) $x^2 + y^2$ atm

(3) $2\left(\sqrt{x+y}\right)$atm (4) $\sqrt{x+y}$ atm

Ans. (3)

Sol. $A(s) \rightleftharpoons B(g) + C(g)$ $K_{P_1} = x = P_B \cdot P_C$

P_1 P_1 $x = P_1(P_1 + P_2)$ ...(1)

$D(s) \rightleftharpoons C(g) + E(g)$ $K_{P_2} = y = P_C \cdot P_E$

P_2 P_2 $y = (P_1+P_2)(P_2)$ ...(2)

Adding (1) and (2)

$x + y = (P_1 + P_2)^2$

Now total pressure

$P_T = P_C + P_B + P_E$

$= (P_1 + P_2) + P_1 + P_2 = 2(P_1 + P_2)$

$P_T = 2\left(\sqrt{x+y}\right)$

5. Freezing point of a 4% aqueous solution of X is equal to freezing point of 12% aqueous solution of Y. If molecular weight of X is A, then molecular weight of Y is :-

(1) A

(2) 3A

(3) 4A

(4) 2A

Ans. (2)

Sol. For same freezing point, molality of both solution should be same.

$m_x = m_y$

$$\frac{4\times1000}{96\times M_x}=\frac{12\times1000}{88\times M_y}$$

or, $M_y = \frac{96\times12}{4\times88}M_x = 3.27$ A

Closest option is 3A.

6. Poly-β-hydroxybutyrate-co-β-hydroxyvalerate(PHBV) is a copolymer of___.

(1) 3-hydroxybutanoic acid and 4-hydroxypentanoic acid

(2) 2-hydroxybutanoic acid and 3-hydroxypentanoic acid

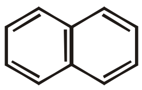
(3) 3-hydroxybutanoic acid and 2-hydroxypentanoic acid

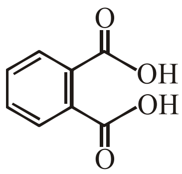
(4) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid

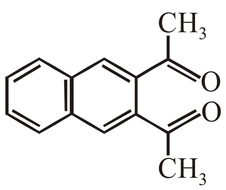
Ans. **(4)**

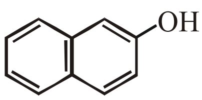
Sol. PHBV is a polymer of 3-hydroxybutanoic acid and 3-Hydroxy pentanoic acid.

7. Among the following four aromatic compounds, which one will have the lowest melting point ?

(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. M.P. of Napthalene $\simeq$ 80°C

8. $CH_3CH_2-\underset{\substack{|\\ Ph}}{\overset{\substack{OH\\ |}}{C}}-CH_3$ cannot be prepared by :

(1) $HCHO + PhCH(CH_3)CH_2MgX$

(2) $PhCOCH_2CH_3 + CH_3MgX$

(3) $PhCOCH_3 + CH_3CH_2MgX$

(4) $CH_3CH_2COCH_3 + PhMgX$

Ans. **(1)**

Sol. (1) $H-\overset{\substack{O\\ \|}}{C}-H + Ph-\underset{\substack{|\\ CH_3}}{CH}-CH_2MgX$

$$\downarrow$$

$Ph-\underset{\substack{|\\ CH_3}}{CH}-CH_2-CH_2-OH$

9. The volume of gas A is twice than that of gas B. The compressibility factor of gas A is thrice than that of gas B at same temperature. The pressures of the gases for equal number of moles are :

(1) $2P_A = 3P_B$

(2) $P_A = 3P_B$

(3) $P_A = 2P_B$

(4) $3P_A = 2P_B$

Ans. **(1)**

Sol. $V_A = 2V_B$

$Z_A = 3Z_B$

$$\frac{P_AV_A}{n_ART_A}=\frac{3\cdot P_B\cdot V_B}{n_B.RT_B}$$

$2P_A = 3P_B$

10. The element with Z = 120 (not yet discovered) will be an/a :

(1) transition metal

(2) inner-transition metal

(3) alkaline earth metal

(4) alkali metal

Ans. **(3)**

Sol. Z = 120

Its general electronic configuration may be represented as [Nobal gas] ns^2, like other alkaline earth metals.

11. Decomposition of X exhibits a rate constant of 0.05 μg/year. How many years are required for the decomposition of 5 μg of X into 2.5 μg ?

(1) 50 (2) 25 (3) 20 (4) 40

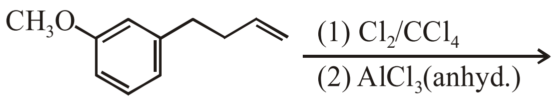
Ans.(1)

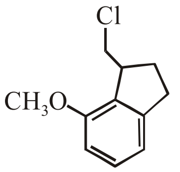
Sol. Rate constant (K) = 0.05 μg/year

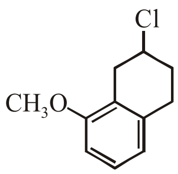
means zero order reaction

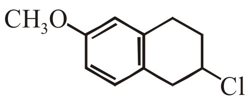
$$t_{1/2} = \frac{a_0}{2K} = \frac{5\mu g}{2\times 0.05\mu g/\text{year}} = 50 \text{ year}$$

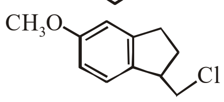
12. The major product of the following reaction is :



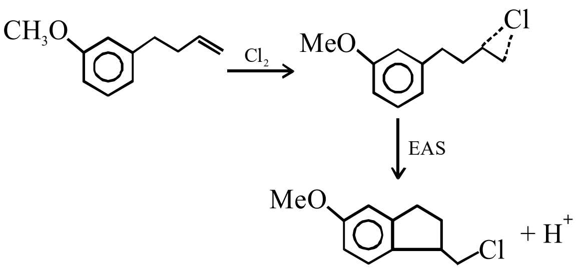
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

13. Given

Gas	H_2	CH_3	CO_2	SO_2
Critical Temperature/K	33	190	304	630

On the basis of data given above, predict which of the following gases shows least adsorption on a definite amount of charcoal ?

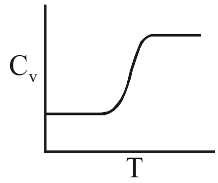
(1) H_2 (2) CH_4 (3) SO_2 (4) CO_2

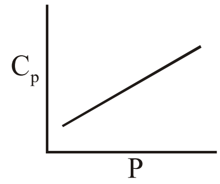
Ans. (1)

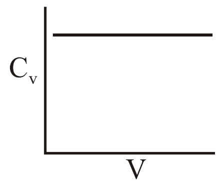
Sol. Smaller the value of critical temperature of gas, lesser is the extent of adsorption.

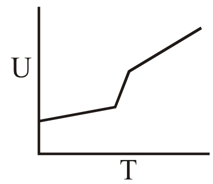
so least adsorbed gas is H_2

14. For diatomic ideal gas in a closed system, which of the following plots does not correctly describe the relation between various thermodynamic quantities ?

(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. At higher temperature, rotational degree of freedom becomes active.

$$C_P = \frac{7}{2}R \quad \text{(Independent of P)}$$

$$C_V = \frac{5}{2}R \quad \text{(Independent of V)}$$

Variation of U vs T is similar as C_V vs T.

15. The standard electrode potential $E^{\ominus}$ and its temeprature coefficient $\left(\frac{dE^{\ominus}}{dT}\right)$ for a cell are 2V and -5×10^{-4} VK^{-1} at 300 K respectively. The cell reaction is

$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$

The standard reaction enthalpy $(\Delta_r H^{\ominus})$ at 300 K in kJ mol^{-1} is,

[Use R = 8jK^{-1} mol^{-1} and F = 96,000 Cmol^{-1}]

(1) –412.8 (2) –384.0

(3) 206.4 (4) 192.0

Ans. (1)

Sol. Chiefly NO_2, O_3 and hydrocarbon are responsible for build up smog.

16. The molecule that has minimum/no role in the formation of photochemical smog, is :

(1) $CH_2 = O$

(2) N_2

(3) O_3

(4) NO

Ans. (2)

Sol. Chiefly NO_2, O_3 and hydrocarbon are responsible for build up smog.

17. In the Hall-Heroult process, aluminium is formed at the cathode. The cathode is made out of :

(1) Platinum

(2) Carbon

(3) Pure aluminium

(4) Copper

Ans. (2)

17. Ans.(2) Carbon

Sol. In the Hall-Heroult process the cathode is made of carbon.

18. Water samples with BOD values of 4 ppm and 18 ppm, respectively, are :

(1) Highly polluted and Clean

(2) Highly polluted and Highly polluted

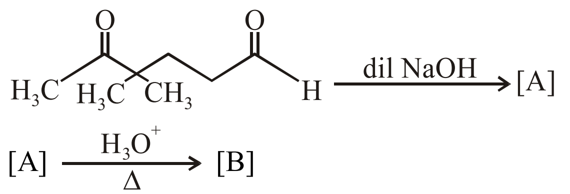
(3) Clean and Highly polluted

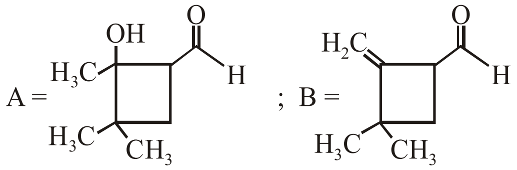
(4) Clean and Clean

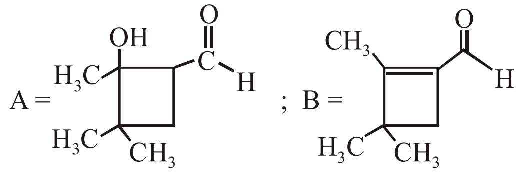
Ans. (3)

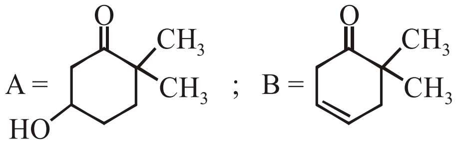
Sol. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

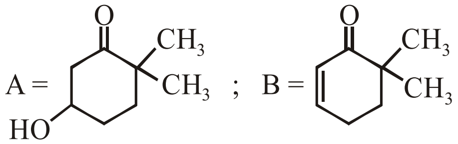
19. In the following reactions, products A and B are :



(1) 

(2) 

(3) 

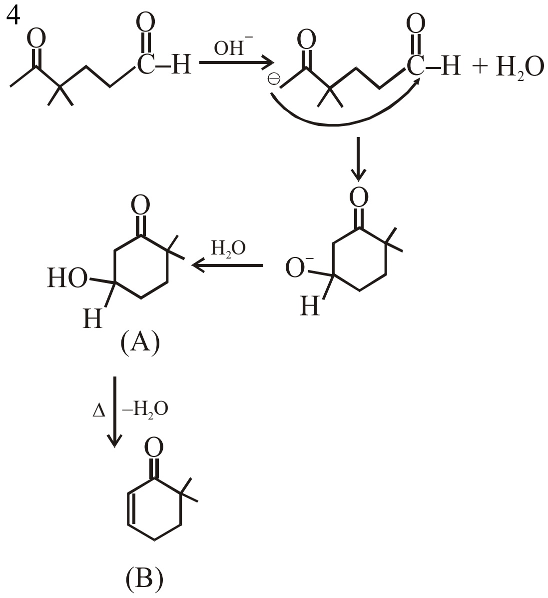
(4) 

Ans. (4)

19.

Ans. 4

Sol.



20. What is the work function of the metal if the light of wavelength 4000 Å generates photoelectrons of velocity 6×10^5 ms^{-1} form it ?

(Mass of electron = 9×10^{-31} kg

Velocity of light = 3×10^8 ms^{-1}

Planck's constant = 6.626×10^{-34} Js

Charge of electron = 1.6×10^{-19} JeV^{-1})

(1) 0.9 eV

(2) 4.0 eV

(3) 2.1 eV

(4) 3.1 eV

Ans. (3)

Sol. $h\nu = \phi + h\nu^\circ$

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)$$

$$h\nu=\phi+\frac{1}{2}mv^2$$

$$\phi=\frac{6.626\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}-\frac{1}{2}\times9\times10^{-31}\times(6\times10^{5})^2$$

$\phi = 3.35 \times 10^{-19}$ J $\Rightarrow$ $\phi \simeq 2.1$ eV

21. Among the following compounds most basic amino acid is :
(1) Lysine
(2) Asparagine
(3) Serine
(4) Histidine

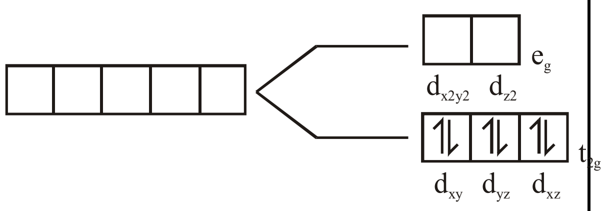
Ans. (4)

Sol. Histidine

22. The metal d-orbitals that are directly facing the ligands in $K_3[Co(CN)_6]$ are :
(1) d_{xz}, d_{yz} and d_{z^2}
(2) d_{xy}, d_{xz} and d_{yz}
(3) d_{xy} and $d_{x^2-y^2}$
(4) $d_{x^2-y^2}$ and d_{z^2}

Ans. (4)

Sol. $K_3[Co(CN)_6]$
$Co^{+3} \rightarrow [Ar]_{18}\ 3d^6$



d^2sp^3

23. The hardness of a water sample (in terms of equivalents of $CaCO_3$) containing 10^{-3} M $CaSO_4$ is :
(molar mass of $CaSO_4$ = 136 g mol^{-1})
(1) 100 ppm
(2) 50 ppm
(3) 10 ppm
(4) 90 ppm

Ans. (1)

Sol. ppm of $CaCO_3$
$(10^{-3} \times 10^3) \times 100 = 100$ ppm

24. The correct order for acid strength of compounds $CH\equiv CH$, $CH_3-C\equiv CH$ and $CH_2=CH_2$ is as follows :
(1) $CH \equiv CH > CH_2 = CH_2 > CH_3-C \equiv CH$
(2) $HC \equiv CH > CH_3-C \equiv CH > CH_2 = CH_2$
(3) $CH_3-C \equiv CH > CH_2 = CH_2 > HC \equiv CH$
(4) $CH_3-C \equiv CH > CH \equiv CH > CH_2 = CH_2$

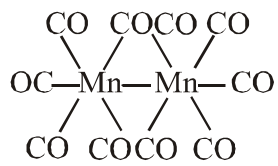
Ans. (2)

Sol. $CH\equiv CH > CH_3-C\equiv CH > CH_2=CH_2$
(Acidic strength order)

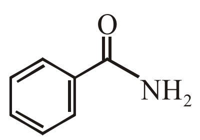
25. $Mn_2(CO)_{10}$ is an organometallic compound due to the presence of :
(1) Mn – Mn bond (2) Mn – C bond
(3) Mn – O bond (4) C – O bond

Ans. (2)

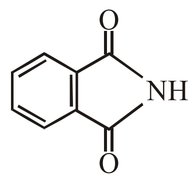
Sol. Compounds having at least one bond between carbon and metal are known as organometallic compounds.



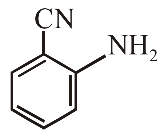
26. The increasing order of reactivity of the following compounds towards reaction with alkyl halides directly is :



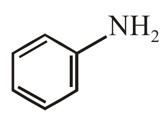
(A)



(B)



(C)

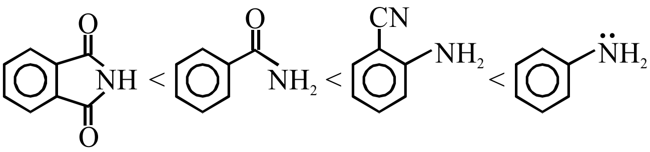


(D)

(1) (B) < (A) < (D) < (C)
(2) (B) < (A) < (C) < (D)
(3) (A) < (C) < (D) < (B)
(4) (A) < (B) < (C) < (D)

Ans. (2)

Sol. Nucleophilicity order



B A C D

27. The pair of metal ions that can give a spin only magnetic moment of 3.9 BM for the complex $[M(H_2O)_6]Cl_2$, is :

(1) Cr^{2+} and Mn^{2+} (2) V^{2+} and Co^{2+}

(3) V^{2+} and Fe^{2+} (4) Co^{2+} and Fe^{2+}

Ans. **(2)**

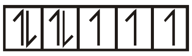
27. **Ans.(2) V^{2+} and Co^{2+}**

Sol. $V^{2+} \rightarrow [V(H_2O)_6]Cl_2$; $[Ar]_{18}$ 

$3d^3$

3 unpaired e^-, spin only magnetic moment

= 3.89 B.M.

$Co^{2+} \rightarrow [Co(H_2O)_6]Cl_2$; $[Ar]_{18}$ 

$3d^7$

3 unpaired e^-, spin only magnetic moment

= 3.89 B.M.

28. In the following reaction

Aldehyde + Alcohol $\xrightarrow{HCl}$ Acetal

Aldehyde	Alcohol
HCHO	^tBuOH
CH_3CHO	MeOH

The best combinations is :

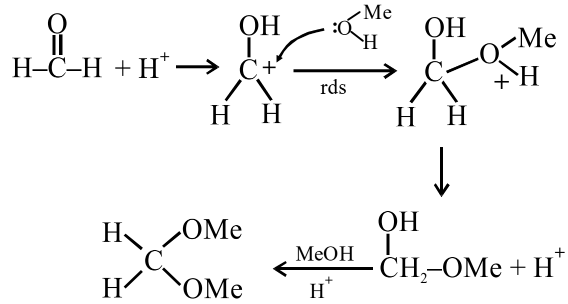
(1) HCHO and MeOH

(2) HCHO and ^tBuOH

(3) CH_3CHO and MeOH

(4) CH_3CHO and ^tBuOH

Ans. **(1)**

Sol. 

acetal

$$\text{rate} \propto \frac{1}{\text{steric crowding of aldehyde}}$$

t-butanol can show formation of carbocation in acidic medium.

29. 50 mL of 0.5 M oxalic acid is needed to neutralize 25 mL of sodium hydroxide solution. The amount of NaOH in 50 mL of the given sodium hydroxide solution is :

(1) 40 g (2) 20 g (3) 80 g (4) 10 g

BONUS

$H_2C_2O_4 + 2NaOH \longrightarrow Na_2C_2O_4 + 2H_2O$

m_{eq} of $H_2C_2O_4$ = m_{eq} NaOH

$50 \times 0.5 \times 2 = 25 \times M_{NaOH} \times 1$

$\therefore$ $M_{NaOH} = 2$ M

Now 1000 ml solution = 2×40 gram NaOH

$\therefore$ 50 ml solution = 4 gram NaOH

30. A metal on combustion in excess air forms X, X upon hydrolysis with water yields H_2O_2 and O_2 along with another product. The metal is :

(1) Rb (2) Na (3) Mg (4) Li

Ans. **(1)**

Sol. $Rb + O_{2(excess)} \longrightarrow RbO_2$

$2RbO_2 + 2H_2O \longrightarrow 2RbOH + H_2O_2 + O_2$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On SATURDAY 12th JANUARY., 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A × B, is :

(1) 2^{18} (2) 2^{10} (3) 2^{15} (4) 2^{12}

Ans (3)

Sol. $A = \{x \in z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = \{x \in Z : -3 < 2x - 1 < 9\}$

$B = \{0, 1, 2, 3, 4\}$

A × B has is 15 elements so number of subsets of A × B is 2^{15}.

2. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to :

(1) 0 (2) $-\sqrt{2}$ (3) -1 (4) $\sqrt{2}$

Ans (2)

Sol. A.M. ≥ G.M.

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha . 4\cos^4\beta . 1 . 1)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^2\beta + 2 \geq 4\sqrt{2}\sin\alpha\cos\beta$

given that $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$

$\Rightarrow$ A.M. = G.M. $\Rightarrow \sin^4\alpha = 1 = 4\cos^4\beta$

$\sin\alpha = 1, \cos\beta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\beta = \frac{1}{\sqrt{2}}$ as $\beta \in [0, \pi]$

$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2\sin\alpha\sin\beta$

$= -\sqrt{2}$

3. If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is:

(1) $-\frac{5}{3}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $-\frac{3}{5}$

Ans (3)

Sol. DR's of line are 2, 1, –2

normal vector of plane is $\hat{i} - 2\hat{j} - k\hat{k}$

$$\sin\alpha = \frac{(2\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} - 2\hat{j} - k\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin\alpha = \frac{2k}{3\sqrt{k^2+5}} \quad ....(1)$$

$$\cos\alpha = \frac{2\sqrt{2}}{3} \quad ....(2)$$

$(1)^2 + (2)^2 = 1 \Rightarrow k^2 = \frac{5}{3}$

4. If a straight line passing thourgh the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :

(1) $x - y + 7 = 0$

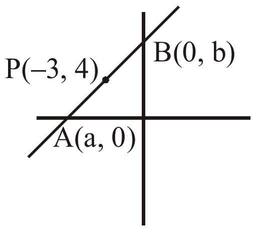
(2) $3x - 4y + 25 = 0$

(3) $4x + 3y = 0$

(4) $4x - 3y + 24 = 0$

Ans (4)

Sol.



Let the line be $\frac{x}{a}+\frac{y}{b} = 1$

$(-3, 4) = \left(\frac{a}{2},\frac{b}{2}\right)$

$a = -6,\ b = 8$

equation of line is $4x - 3y + 24 = 0$

5. The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

(2) $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3}+C$

(3) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

(4) $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

Ans (2)

Sol. $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$

$$\int \frac{\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}$$

Let $\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right) = t$

$$-\frac{1}{2}\int\frac{dt}{t^4}=\frac{1}{6t^3}+C \Rightarrow \frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$$

6. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

(1) 9 (2) 11 (3) 12 (4) 7

Ans (3)

Sol. Let m-men, 2-women

${}^mC_2 \times 2 = {}^mC_1\ {}^2C_1 \cdot 2 + 84$

$m^2 - 5m - 84 = 0 \Rightarrow (m - 12)(m + 7) = 0$

$m = 12$

7. If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is :

(1) 6 (2) 5 (3) 7 (4) –7

Ans (3)

Sol. $f'(x) = 3x^2 - 6(a - 2)x + 3a$

$f'(x) \geq 0\ \forall\ x \in (0, 1]$

$f'(x) \leq 0\ \forall\ x \in [1, 5)$

$\Rightarrow f'(x) = 0$ at $x = 1 \Rightarrow a = 5$

$f(x) - 14 = (x - 1)^2 (x - 7)$

$\frac{f(x)-14}{(x-1)^2} = x - 7$

8. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to :

(1) 4e (2) $4e^2$ (3) 2e (4) $2e^2$

Ans (1)

Sol. $\frac{f'(x)}{f(x)} = 1\ \forall\ x \in R$

Intergrate & use $f(1) = 2$

$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$

$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))\ f'(x)$

$h'(1) = f'(f(1))\ f'(1)$

$= f'(2)\ f'(1)$

$= 2e \cdot 2 = 4e$

9. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point.

(1) $\left(\frac{1}{4},\frac{7}{2}\right)$ (2) $\left(\frac{7}{2},\frac{1}{4}\right)$

(3) $\left(-\frac{1}{8},7\right)$ (4) $\left(\frac{1}{8},-7\right)$

Ans (4)

Sol. $y = x^2 - 5x + 5$

$$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$$

at $x = \frac{7}{2}$, $y = \frac{-1}{4}$

Equation of tangent at $\left(\frac{7}{2},\frac{-1}{4}\right)$ is $2x - y - \frac{29}{4} = 0$

Now check options

$x = \frac{1}{8}$, $y = -7$

10. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to :

(1) $\{\sqrt{3}\}$ (2) $\{\sqrt{3}-\sqrt{3}\}$

(3) $\{1 , -1\}$ (4) $\{3, -3\}$

Ans (2)

Sol. All four points are coplaner so

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & -\lambda^2+1 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$(\lambda^2 + 1)^2 (3 - \lambda^2) = 0$

$\lambda = \pm\sqrt{3}$

11. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$

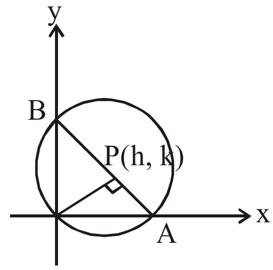
(2) $(x^2 + y^2)(x + y) = R^2xy$

(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

Ans (3)

Sol.



Slope of AB $= \frac{-h}{k}$

Equation of AB is $hx + ky = h^2 + k^2$

$A\left(\frac{h^2+k^2}{h},0\right)$, $B\left(0,\frac{h^2+k^2}{k}\right)$

AB = 2R

$\Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$

$\Rightarrow (x^2 + y^2)^3 = 4R^2x^2y^2$

12. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is :

(1) $x = y\cot\theta + 2\tan\theta$ (2) $x = y\cot\theta - 2\tan\theta$

(3) $y = x\tan\theta - 2\cot\theta$ (4) $y = x\tan\theta + 2\cot\theta$

Ans (1)

Sol. $x^2 = 8y$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan\theta$$

$\therefore$ $x_1 = 4\tan\theta$

$y_1 = 2\tan^2\theta$

Equation of tangent :-

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

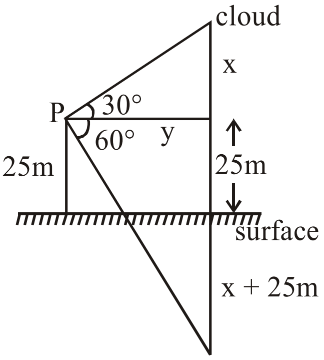
$\Rightarrow x = y\cot\theta + 2\tan\theta$

13. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is :

(1) 42 (2) 50 (3) 45 (4) 60

Ans (2)

Sol.



$\tan 30° = \frac{x}{y} \Rightarrow y = \sqrt{3}\,x$ .....(i)

$\tan 60° = \frac{25+x+25}{y}$

$\Rightarrow \sqrt{3}\,y = 50 + x$

$\Rightarrow 3x = 50 + x$

$\Rightarrow x = 25$ m

$\therefore$ Height of cloud from surface = 25 + 25 = 50m

14. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\}\log_e x\,dx$ is equal to :

(1) $\frac{1}{2} - e - \frac{1}{e^2}$ (2) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

(3) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$ (4) $\frac{3}{2} - e - \frac{1}{2e^2}$

Ans. (4)

$$\int_1^e \left(\frac{x}{e}\right)^{2x}\log_e x.dx - \int_1^e \left(\frac{e}{x}\right)\log_e x.dx$$

Let $\left(\frac{x}{e}\right)^{2x} = t$, $\left(\frac{e}{x}\right)^{x} = v$

$$= \frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1} dt + \int_e^1 dv$$

$$= \frac{1}{2}\left(1 - \frac{1}{e^2}\right) + (1-e) = \frac{3}{2} - \frac{1}{2e^2} - e$$

15. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + ... + \frac{1}{5n}\right)$

is equal to :

(1) $\frac{\pi}{4}$ (2) $\tan^{-1}(2)$

(3) $\tan^{-1}(3)$ (4) $\frac{\pi}{2}$

Ans. (2)

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)} = \int_0^2 \frac{dx}{1+x^2} = \tan^{-1}2$$

16. The set of all values of λ for which the system of linear equations.

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution.

(1) contains more than two elements

(2) is a singleton

(3) is an empty set

(4) contains exactly two elements

Ans. (2)

$$\begin{vmatrix} \lambda-1 & 2 & 2 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow (\lambda-1)^3 = 0 \Rightarrow \lambda = 1$$

17. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be:

(1) 14 (2) 11 (3) 9 (4) 12

Ans. (1)

$2.{}^nC_5 = {}^nC_4 + {}^nC_6$

$$2.\frac{\underline{|n}}{\underline{|5}\,\underline{|n-5}} = \frac{\underline{|n}}{\underline{|4}\,\underline{|n-4}} + \frac{\underline{|n}}{\underline{|6}\,\underline{|n-6}}$$

$$\frac{2}{5}.\frac{1}{n-5} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

n = 14 satisfying equation.

18. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to :

(1) 60º (2) 30º (3) 90º (4) 45º

Ans. (2)

$$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b}).\vec{c} = \frac{1}{2}\vec{b}$$

$\because$ $\vec{b}$ & $\vec{c}$ are linearly independent

$\therefore$ $\vec{a}.\vec{c} = \frac{1}{2}$ & $\vec{a}.\vec{b} = 0$

(All given vectors are unit vectors)

$\therefore$ $\vec{a} \wedge \vec{c} = 60°$ & $\vec{a} \wedge \vec{b} = 90°$

$\therefore$ $|\alpha - \beta| = 30°$

19. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det(A) lies in the interval :

(1) $\left[\frac{5}{2}, 4\right)$ (2) $\left(\frac{3}{2}, 3\right]$

(3) $\left(0, \frac{3}{2}\right]$ (4) $\left(1, \frac{5}{2}\right]$

Ans (2)

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 2(1 + \sin^2\theta)$

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \le \sin^2\theta < \frac{1}{2}$$

$\therefore$ $|A| \in [2, 3)$

20. $\lim\limits_{x\to1-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ ie equal to :

(1) $\frac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\frac{\pi}{2}}$ (3) $\sqrt{\frac{2}{\pi}}$ (4) $\sqrt{\pi}$

Ans. (3)

$$\lim_{x\to1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi} + \sqrt{2\sin^{-1}x}}{\sqrt{\pi} + \sqrt{2\sin^{-1}x}}$$

$$\lim_{x\to1^-} \frac{2\left(\frac{\pi}{2} - \sin^{-1}x\right)}{\sqrt{1-x}\cdot\left(\sqrt{\pi} + \sqrt{2\sin^{-1}x}\right)}$$

$$\lim_{x\to1^-} \frac{2\cos^{-1}x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

Put x = cosθ

$$\lim_{\theta\to0^+} \frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \cdot \frac{1}{2\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

21. The expression $\sim(\sim p \to q)$ is logically equvalent to :

(1) $\sim p \wedge \sim q$ (2) $p \wedge q$

(3) $\sim p \wedge q$ (4) $p \wedge \sim q$

Ans. (1)

p	q	~p	~p→q	~(~p→q)	(~p ^ ~q)
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

22. The total number of irrational terms in the binomial expansion of $\left(7^{1/5} - 3^{1/10}\right)^{60}$ is :

(1) 55 (2) 49 (3) 48 (4) 54

Ans. (4)

General term $T_{r+1} = {}^{60}C_r \; 7^{\frac{60-r}{5}} \; 3^{\frac{r}{10}}$

$\therefore$ for rational term, r = 0, 10, 20, 30, 40, 50, 60

$\Rightarrow$ no of rational terms = 7

$\therefore$ number of irrational terms = 54

23. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is :

(1) 1 (2) 3 (3) 7 (4) 5

Ans. (3)

mean $\overline{x} = 4, \sigma^2 = 5.2, n = 5,. x_1 = 3\ x_2 = 4 = x_3$

$\sum x_i = 20$

$x_4 + x_5 = 9$ ....(i)

$\frac{\sum x_i^2}{x} - (\overline{x})^2 = \sigma^2 \Rightarrow \sum x_i^2 = 106$

$x_4^2 + x_5^2 = 65$ ....(ii)

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

24. If the sum of the first 15 tems of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225 k, then k is equal to :

(1) 9 (2) 27 (3) 108 (4) 54

Ans. (2)

$$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + ...15 \text{ term}$$

$$= \frac{27}{64} \sum_{r=1}^{15} r^3$$

$$= \frac{27}{64} \cdot \left[\frac{15(15+1)}{2}\right]^2$$

= 225 K (Given in question)

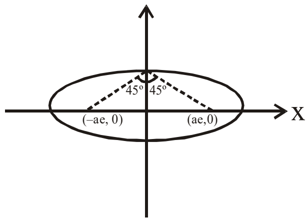
K = 27

25. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If ΔS'BS is a right angled triangle with right angle at B and area (ΔS'BS) = 8 sq. units, then the length of a latus rectum of the ellipse is :

(1) $2\sqrt{2}$ (2) 2 (3) 4 (4) $4\sqrt{2}$

Ans. (3)

$m_{SB} \cdot m_{S'B} = -1$



$b^2 = a^2e^2$ .... (i)

$\frac{1}{2} S'B \cdot SB = 8$

S'B. SB = 16

$a^2e^2 + b^2 = 16$ ..... (ii)

$b^2 = a^2\ (1 - e^2)$ ..... (iii)

using (i),(ii), (iii) a = 4

$b = 2\sqrt{2}$

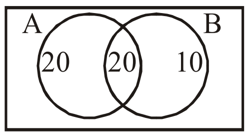
$e = \frac{1}{\sqrt{2}}$

$\therefore \ell$ (L.R) $= \frac{2b^2}{a} = 4$ $\boxed{\text{Ans.3}}$

26. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

(1) $\frac{2}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{3}$ (4) $\frac{5}{6}$

Ans. (2)



A $\rightarrow$ opted NCC

B $\rightarrow$ opted NSS

$\therefore$ P (nither A nor B) $= \frac{10}{60} = z\frac{1}{6}$

27. The number of integral values of m for which the quadratic expression.
$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive, is :
(1) 8 (2) 7 (3) 6 (4) 3

Ans. (2)

Exprsssion is always positve it

$$2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$$

&

$D < 0 \Rightarrow m^2 - 6m - 3 < 0$

$$3 - \sqrt{12} < m < 3 + \sqrt{12} \quad .... \text{(iii)}$$

$\therefore$ Common interval is

$$3 - \sqrt{12} < m < 3 + \sqrt{12}$$

$\therefore$ Intgral value of m $\{0,1,2,3,4,5,6\}$

28. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/ loss (in rupees) is :

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss

(3) 0 (4) $\frac{400}{9}$ loss

Ans. (3)

Expected Gain/ Loss =
$= w \times 100 + Lw(-50 + 100) + L^2w(-50 - 50 + 100) + L^3(-150)$

$$= \frac{1}{3}\times 100 + \frac{2}{3}.\frac{1}{3}(50) + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0) + \left(\frac{2}{3}\right)^3(-150) = 0$$

here w denotes probability that outcome 5 or 6 ($w = \frac{2}{6} = \frac{1}{3}$)

here L denotes probability that outcome 1,2,3,4 ($L = \frac{4}{6} = \frac{2}{3}$)

29. If a cuver passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point :

(1) $\left(-\sqrt{2}, 1\right)$ (2) $\left(\sqrt{3}, 0\right)$

(3) (–1, 2) (4) (3, 0)

Ans. (2)

$$\frac{dy}{dx} = \frac{x^2 - 2y}{x} \qquad \text{(Given)}$$

$$\frac{dy}{dx} + 2\frac{y}{x} = x$$

$$\text{I.F} = e^{\int \frac{2}{x}dx} = x^2$$

$$\therefore \ y.x^2 = \int x.x^2 dx + C$$

$$= \frac{x^4}{y} + C$$

hence bpasses through (1, –2) $\Rightarrow C = -\frac{9}{4}$

$$\therefore \ yx^2 = \frac{x^4}{4} - \frac{9}{4}$$

Now check option(s) , Which is satisly by option (ii)

30. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2 - 3 - 4i| = 4$. Then the minimum value of $|Z_1 - Z_2|$ is :
(1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2

Ans. (1)

$$|z_1| = 9 \ , \ |z_2 - (3 + 4i)| = 4$$

C_1 (0, 0) radius $r_1 = 9$
C_2(3, 4), radius $r_2 = 4$
$C_1C_2 = |r_1 - r_2| = 5$
$\therefore$ Circle touches internally

$$\therefore \ |z_1 - z_2|_{min} = 0$$

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

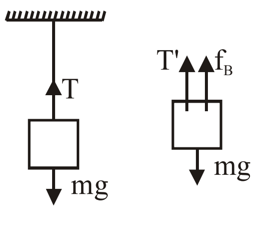
PHYSICS

1. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is :

(1) 4.0mm (2) 3.0mm (3) 5.0mm (4) zero

Ans (2)

Sol.



$$\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$$

$\Delta\ell \propto F$ .....(i)

$T = mg$

$$T = mg - f_B = mg - \frac{m}{\rho_b}.\rho_\ell.g$$

$$= \left(1 - \frac{\rho_\ell}{\rho_b}\right)mg$$

$$= \left(1 - \frac{2}{8}\right) mg$$

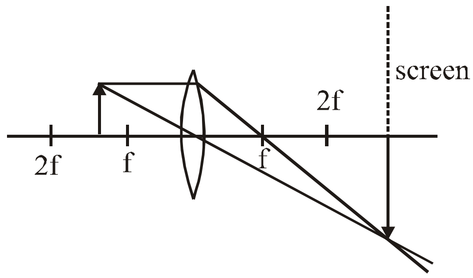
$$T' = \frac{3}{4}mg$$

From (i)

$$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4}$$

$$\Delta\ell' = \frac{3}{4} \cdot \Delta\ell = 3 \text{ mm}$$

2. Formation of real image using a biconvex lens is shown below :



If the whole set up is immersed in water without disturbing the object and the screen position, what will one observe on the screen ?

(1) Image disappears
(2) No change
(3) Erect real image
(4) Magnified image

Ans (1)

Sol. From $\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Focal length of lens will change hence image disappears from the screen.

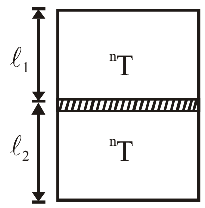
3. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is ℓ_1, and that below the piston is ℓ_2, such that $\ell_1 > \ell_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by :
(R is universal gas constant and g is the acceleration due to gravity)

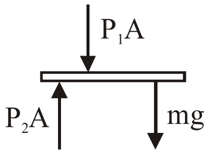
(1) $\frac{nRT}{g}\left[\frac{1}{\ell_2} + \frac{1}{\ell_1}\right]$ (2) $\frac{nRT}{g}\left[\frac{\ell_1 - \ell_2}{\ell_1\ell_2}\right]$

(3) $\frac{RT}{g}\left[\frac{2\ell_1 + \ell_2}{\ell_1\ell_2}\right]$ (4) $\frac{RT}{ng}\left[\frac{\ell_1 - 3\ell_2}{\ell_1\ell_2}\right]$

Ans (2)

Sol.





$P_2A = P_1A + mg$

$$\frac{nRT.A}{A\ell_2} = \frac{nRT.A}{A\ell_1} + mg$$

$$nRT\left(\frac{1}{\ell_2} - \frac{1}{\ell_1}\right) = mg$$

$$m = \frac{nRT}{g}\left(\frac{\ell_1 - \ell_2}{\ell_1.\ell_2}\right)$$

4. A simple harmonic motion is represented by:

$y = 5(\sin 3\pi t + \sqrt{3}\ \cos 3\pi t)$ cm

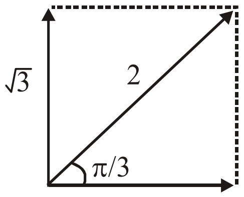
The amplitude and time period of the motion are:

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s

(3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

Ans. (4)

Sol.



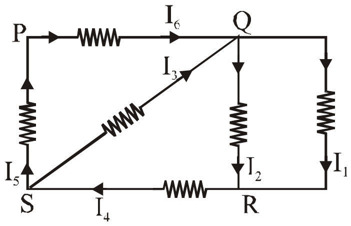
$$y = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$$

$$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Amplitude = 10 cm

$$T = \frac{2\pi}{w} = \frac{2\pi}{3\pi} = \frac{2}{3}\text{sec}$$

5. In the given circuit diagram, the currents, $I_1 = -0.3A$, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6, respectively, are :



(1) 1.1 A, 0.4 A, 0.4 A

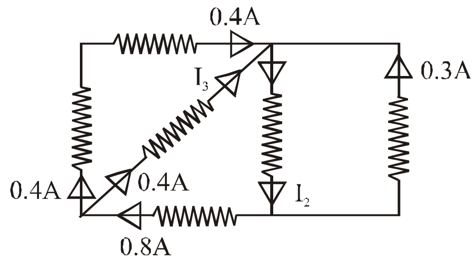
(2) –0.4 A, 0.4 A, 1.1 A

(3) 0.4 A, 1.1 A, 0.4 A

(4) 1.1 A, –0.4 A, 0.4 A

Ans. (1)

Sol.



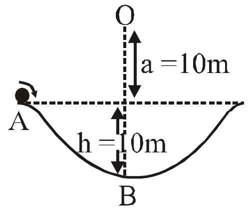
From KCL, $I_3 = 0.8 - 0.4 = 0.4A$

$I_2 = 0.4 + 0.4 + 0.3$

$= 1.1$ A

$I_6 = 0.4A$

6. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g= 10 m/s^2)



(1) 8kg-m^2/s (2) 6kg-m^2/s

(3) 3kg-m^2/s (4) 2kg-m^2/s

Ans. (2)

Sol. Work Energy Theorem from A to B

$$mgh = \frac{1}{g}mv_B^2 - \frac{1}{g}mv_A^2$$

$$2gh = v_B^2 - v_A^2$$

$$2\times10\times10 = v_B^2 - 5^2$$

$$v_B = 15\,m/s$$

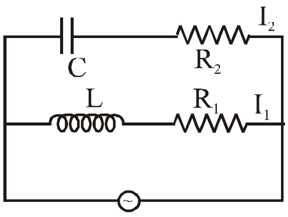
Angular momentum about 0

$L_0 = mvr$

$= 20\times10^{-3}\times20$

$L_0 = 6$ kg.m^2/s

7.

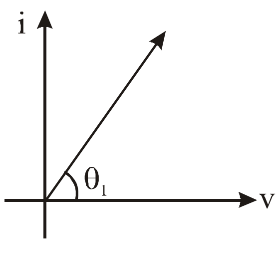


In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is :

(1) 30° (2) 0° (3) 90° (4) 60°

Ans. (Bonus)

Sol.



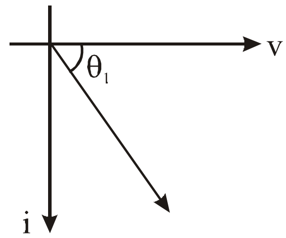
$$x_e = \frac{1}{\omega_c} = \frac{4}{10^{-6}\times\sqrt{3}\times100} = \frac{2\times10^4}{\sqrt{3}}$$

$$\tan\theta/2\ \frac{x_e}{R_e} = \frac{10^3}{\sqrt{3}}$$

θ_1 is close to 90

For L–R circuit

$$x_L = w_L = 100\times\frac{\sqrt{3}}{10} = \sqrt{3}$$



$R_1 = 10$

$$\tan\theta_2 = \frac{x_e}{R}$$

$\tan\theta_2 = \sqrt{3}$

$\theta_2 = 60$

So phase difference comes out 90 + 60 = 150.

Therefore Ans. is Bonus

If R_2 is 20 KΩ

then phase difference comes out to be 60+30 = 90°

8. A paramagnetic material has 10^{28}atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8 $\times10^{-4}$. Its susceptibility at 300 K is :

(1) 3.672×10^{-4} (2) 3.726×10^{-4}

(3) 3.267×10^{-4} (4) 2.672×10^{-4}

Ans (3)

Sol. $x\ \alpha\ \frac{1}{T_C}$

curie law for paramagnetic substane

$$\frac{x_1}{x_2} = \frac{T_{C_2}}{T_{C_1}}$$

$$\frac{2.8\times10^{-4}}{x_2} = \frac{300}{350}$$

$$x_2 = \frac{2.8\times350\times10^{-4}}{300}$$

$= 3.266\times10^{-4}$

9. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of $5.0 ms^{-1}$, at right angles to the horizontal component of the earth's magnetic field, of $0.3 \times 10^{-4} Wb/m^2$. The value of the induced emf in wire is :

(1) 2.5×10^{-3}V (2) 1.1×10^{-3}V

(3) 0.3×10^{-3}V (4) 1.5×10^{-3}V

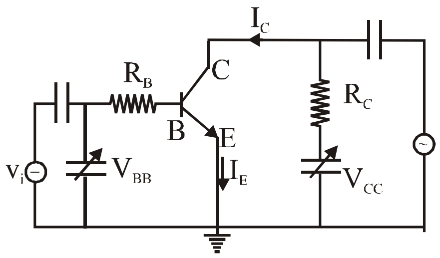
Ans (4)

Sol. Induied emf = $Bv\ell$

$= 0.3 \times 10^{-4} \times 5 \times 10$

$= 1.5 \times 10^{-3}$ V

10.



In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5V$, $\beta_{dc} = 200$, $R_B = 100 k\Omega$, $R_C = 1 k\Omega$ and $V_{BE} = 1.0$ V, The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively :

(1) 20μA and 3.5V (2) 25μA and 3.5V

(3) 25μA and 2.5V (4) 20μA and 2.8V

Ans (2)

Sol. At saturation, $V_{CE} = 0$

$V_{CE} = V_{CC} - I_C R_C$

$\Rightarrow I_C = \dfrac{V_{CC}}{R_C} = 5 \times 10^{-3}$ A

Given

$\beta_{dc} = \dfrac{I_C}{I_B}$

$I_B = \dfrac{5 \times 10^{-3}}{200}$

$I_B = 25\ \mu A$

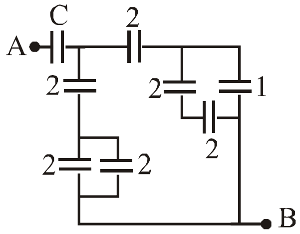
At input side

$V_{BB} = I_B R_B + V_{BE}$

$= (25mA)(100k\Omega) + 1V$

$V_{BB} = 3.5$ V

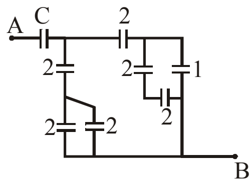
11. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF.



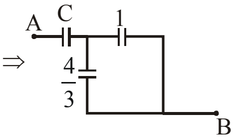
(1) $\dfrac{7}{10}$μF (2) $\dfrac{7}{11}$μF (3) $\dfrac{6}{5}$μF (4) 4μF

Ans (2)

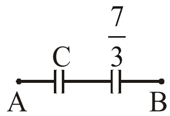
Sol.



$\Rightarrow$



$\Rightarrow$



From equs.

$$\frac{\frac{7C}{3}}{\frac{7}{3}+C} = \frac{1}{2}$$

$\Rightarrow$ 14 C = 7 + 3 C

$\Rightarrow$ C = $\dfrac{7}{11}$

12. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is:

(1) 2 (2) $\sqrt{\dfrac{1}{2}}$ (3) 1 (4) $\dfrac{1}{2}$

Ans (3)

Sol. Orbital velocity $V = \sqrt{\dfrac{GMe}{r}}$

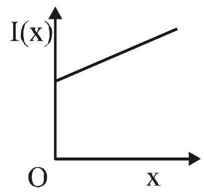
$T_A = \dfrac{1}{2} m_A V_A^2$

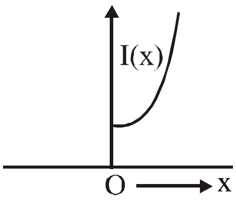
$T_B = \dfrac{1}{2} m_B V_B^2$

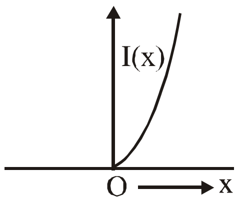
$\Rightarrow \dfrac{T_A}{T_B} = \dfrac{m \times \frac{Gm}{R}}{2m \times \frac{Gm}{2R}}$

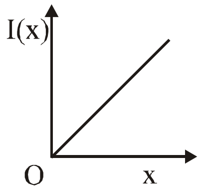
$\Rightarrow \dfrac{T_A}{T_B} = 1$

13. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is I(x)'. Which one of the graphs represents the variation of I(x) with x correctlv?

(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. $I = \frac{2}{5}mR^2 + mx^2$

14. When a certain photosensistive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photo current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is:

(1) $\frac{3\nu}{2}$ (2) 2ν (3) $\frac{4}{3}\nu$ (4) $\frac{5\nu}{3}$

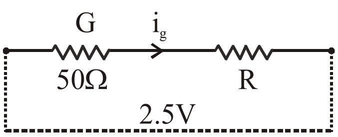
Ans. (BONUS)

15. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of:

(1) 6250 ohm (2) 250 ohm
(3) 200 ohm (4) 6200 ohm

Ans. (3)

Sol. $I_g = 4 \times 10^{-4} \times 25 = 10^{-2}$ A



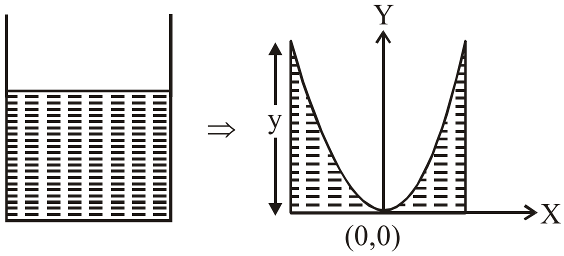
$2.5 = (50 + R)\ 10^{-2} \quad \therefore \quad R = 200\ \Omega$

16. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be:

(1) 1.2 (2) 0.1 (3) 2.0 (4) 0.4

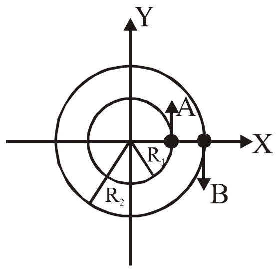
Ans. (3)

Sol.



$$y = \frac{\omega^2 x^2}{2g} = \frac{(2\times 2\pi)^2 \times (0.05)^2}{20} \simeq 2 \text{ cm}$$

17. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure :



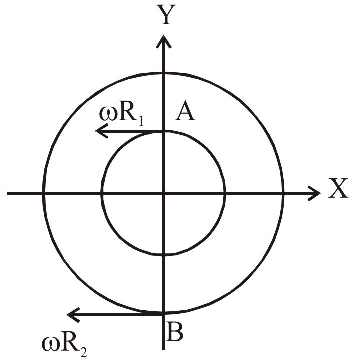
The relative velocity $\vec{\upsilon}_A - \vec{\upsilon}_B$ at $t = \frac{\pi}{2\omega}$ is given by :

(1) $-\omega(R_1 + R_2)\hat{i}$ (2) $\omega(R_1 + R_2)\hat{i}$

(3) $\omega(R_1 - R_2)\hat{i}$ (4) $\omega(R_2 - R_1)\hat{i}$

Ans. (4)

Sol. $\theta = \omega t = \omega\dfrac{\pi}{2\omega} = \dfrac{\pi}{2}$



$\vec{V}_A - \vec{V}_S = \omega R_1\left(-\hat{i}\right) - \omega R_2\left(-i\right)$

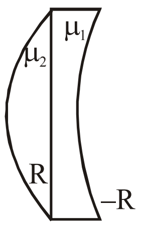
18. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be :

(1) $f_1 - f_2$ (2) $f_1 + f_2$

(3) $\dfrac{R}{\mu_2 - \mu_1}$ (4) $\dfrac{2f_1f_2}{f_1 + f_2}$

Ans. **(3)**

Sol.



$\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{1-\mu_1}{R} + \dfrac{\mu_2 - 1}{R}$

19. Let ℓ, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\dfrac{\ell}{rcv}$ in SI units will be:

(1) [LTA] (2) $[LA^{-2}]$ (3) $[A^{-1}]$ (4) $[LT^2]$

Ans. **(3)**

Sol. $\left[\dfrac{\ell}{r}\right] = T$

$[CV] = AT$

So, $\left[\dfrac{\ell}{rCV}\right] = \dfrac{T}{AT} = A^{-1}$

20. In a radioactive decay chain, the initial nucleus is $^{232}_{90}Th$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus, If $^{A}_{Z}X$, A and Z are given by :

(1) A = 208; Z = 80

(2) A = 202; Z = 80

(3) A = 200; Z = 81

(4) A = 208; Z = 82

Ans. **(4)**

Sol. $^{232}_{90}Th \longrightarrow {}^{208}_{78}Y + {}^{4}_{2}He$

$^{208}_{78}Y \longrightarrow {}^{208}_{82}X + 4\beta$ praticle

21. The mean intensity of radiation on the surface of the Sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closest to :

(1) 10^2T (2) 10^{-4}T

(3) 1T (4) 10^{-2}T

Ans. **(2)**

Sol. $I = \varepsilon_0 C E_{rms}^2$

$\& E_{rms} = cB_{rms}$

$I = \varepsilon_0 C^3 B_{rms}^2$

$B_{rms} = \sqrt{\dfrac{I}{\epsilon_0 C^3}}$

$B_{rms} \approx 10^{-4}$

22. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to:

(1) 328ms^{-1} (2) 322ms^{-1}

(3) 341ms^{-1} (4) 335ms^{-1}

Ans. **(1)**

23. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to:

(1) 4×10^{-8}s (2) 3×10^{-6}s

(3) 2×10^{-7}s (4) 0.5×10^{-8}s

Ans. **(1)**

Sol. $t \propto \dfrac{\text{Volume}}{\text{velocity}}$

$\text{volume} \propto \dfrac{T}{P}$

$\therefore\ t \propto \dfrac{\sqrt{T}}{P}$

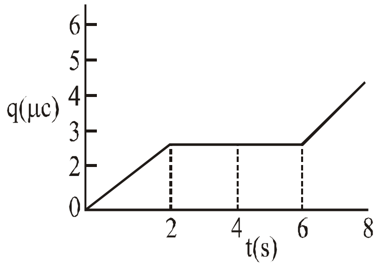
$\dfrac{t_1}{6\times10^{-8}} = \dfrac{\sqrt{500}}{2P} \times \dfrac{P}{\sqrt{300}}$

$t_1 = 3.8 \times 10^{-8}$

$\approx 4 \times 10^{-8}$

24. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure:

What is the value of current at t = 4 s ?

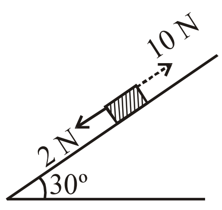


(1) 3μA (2) 2μA (3) zero (4) 1.5μA

Ans. **(3)**

25. A block kept on a rough inclined plane, as shown in the figure, remaias at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction betwreen the block and the plane is :

[Take g = 10 m/s²]



(1) $\dfrac{2}{3}$ (2) $\dfrac{\sqrt{3}}{2}$ (3) $\dfrac{\sqrt{3}}{4}$ (4) $\dfrac{1}{2}$

Ans. **(2)**

Sol. $2 + mg\sin30 = \mu mg\cos30°$

$10 = mg\sin 30 + \mu\, mg\cos30°$

$= 2\mu mg\cos30 - 2$

$6 = \mu mg\cos 30$

$4 = mg\sin 30$

$\dfrac{3}{2} = \mu \times \sqrt{3}$

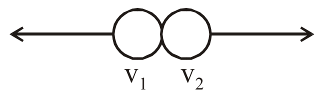
$\mu = \dfrac{\sqrt{3}}{2}$

26. An alpha-particle of mass m suffers 1-dimensional elastic coolision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is :-

(1) 4 m (2) 3.5 m (3) 2 m (4) 1.5 m

Ans. **(1)**

Sol.



$mv_0 = mv_2 - mv_1$

$\dfrac{1}{2}mV_1^2 = 0.36 \times \dfrac{1}{2}mV_0^2$

$v_1 = 0.6v_0$

$\dfrac{1}{2}MV_2^2 = 0.64 \times \dfrac{1}{2}mV_0^2$

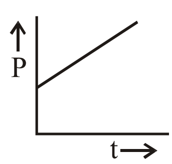
$V_2 = \sqrt{\dfrac{m}{M}} \times 0.8V_0$

$mV_0 = \sqrt{mM} \times 0.8V_0 - m \times 0.6V_0$

$\Rightarrow 1.6m = 0.8\sqrt{mM}$

$4m^2 = mM$

27. A soap bubble, blown by a mechanical pump at the mough of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by :-

(1)


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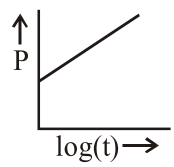

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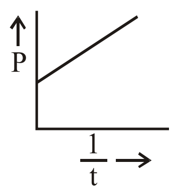
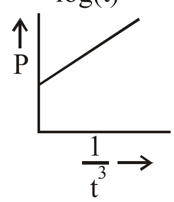


(4)


Ans. **(4)**

28. To double the coverging range of a TV transmittion tower, its height should be multiplied by :-

(1) $\frac{1}{\sqrt{2}}$ (2) 4 (3) $\sqrt{2}$ (4) 2

Ans. (2)

29. A parallel plate capacitor with plates of area $1m^2$ each, area t a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge each plate is :-

(Take $\varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N-m^2}$)

(1) 7.85×10^{-10} C (2) 6.85×10^{-10} C
(3) 9.85×10^{-10} C (4) 8.85×10^{-10} C

Ans. (4)

Sol. $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$Q = AE\epsilon_0$
$Q = (1)(100)(8.85\times10^{-12})$
$Q = 8.85 \times 10^{-10}C$

30. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to :-

(1) 2020 nm Y:\node05\JEE-Main 2019-(On line)\12-01-2019\Evening\PDF (2) 220 nm
(3) 250 nm (4) 1700 nm

Ans. (3)

Sol. $\lambda = \frac{1240}{5.6-0.7}$ nm

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Saturday 12th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

CHEMISTRY

1. 8g of NaOH is dissolved in 18g of H_2O. Mole fraction of NaOH in solution and molality (in mol kg^{-1}) of the solutions respectively are :

(1) 0.167, 11.11 (2) 0.2, 22.20

(3) 0.2, 11.11 (4) 0.167,22.20

Ans. (1)

Sol. 8g NaOH, mol of NaOH = $\frac{8}{40} = 0.2\text{mol}$

18g H_2O, mol of H_2O = $\frac{18}{18} = 1\text{mol}$

$\therefore$ $X_{NaOH} = \frac{0.2}{1.2} = 0.167$

Molality = $\frac{0.2 \times 1000}{18} = 11.11$ m

2. The correct statement(s) among I to III with respect to potassium ions that are abundant within the cell fluids is/are :

I. They activate many enzymes

II. They participate in the oxidation of glucose to produce ATP

III. Along with sodium ions, they are responsible for the transmission of nerve signals

(1) I, II and III (2) I and III only

(3) III only (4) I and II only

Ans. (1)

Sol. All the three statements are correct a/c to NCERT (s-block)

3. The magnetic moment of an octahedral homoleptic Mn(II) complex is 5.9 BM. The suitable ligand for this complex is :

(1) CN^- (2) NCS^-

(3) CO (4) ethylenediamine

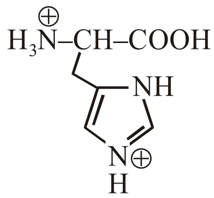
Ans. (2)

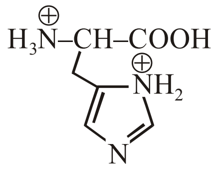
Sol. μ = 5.9 BM $\therefore$ n (no of unpaired.e^-) = 5

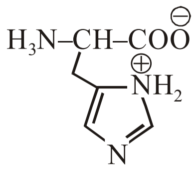
Cation Mn^{II} – $3d^5$ confn only possible for relatively weak ligand.

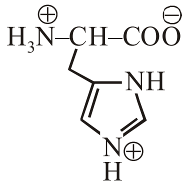
$\therefore$ NCS^-

4. The correct structure of histidine in a strongly acidic solution (pH=2) is

(1) 

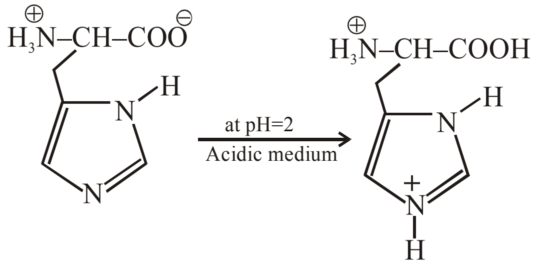
(2) 

(3) 

(4) 

Ans. (1)

Sol. Histidine is



Zwitter ionic form

pIn = 7.59

5. The compound that is NOT a common component of photochemical smog is :

(1) O_3 (2) CH_2=CHCHO

(3) CF_2Cl_2 (4) $H_3C-\underset{\underset{O}{\|}}{C}-OONO_2$

Ans. (3)

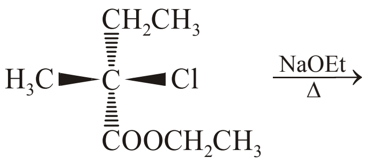
Sol. Freons (CFC's) are not common components of photo chemical smog.

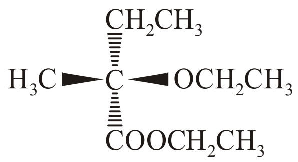
6. The upper stratosphere consisting of the ozone layer protects us from the sun's radiation that falls in the wavelength region of :
(1) 600-750 nm
(2) 0.8-1.5 nm
(3) 400-550 nm
(4) 200-315 nm

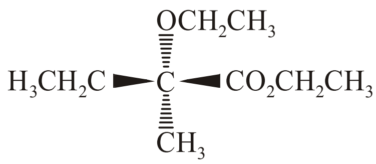
Ans. (4)

Sol. Ozone protects most of the medium freequnecies ultravoilet light from 200 - 315 nm wave length.

7. The major product of the following reaction is:



(1) 

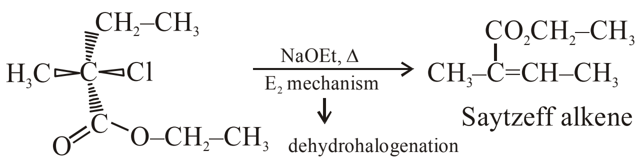
(2) 

(3) $CH_3\underset{}{\overset{CO_2CH_2CH_3}{\overset{|}{C}}}{=}CHCH_3$

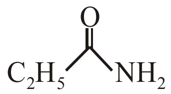
(4) $CH_3CH_2\underset{\underset{CO_2CH_2CH_3}{|}}{C}{=}CH_2$

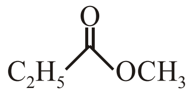
Ans. (3)

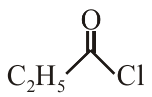
Sol.

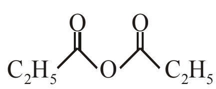


8. The increasing order of the reactivity of the following with $LiAlH_4$ is :

(A) 

(B) 

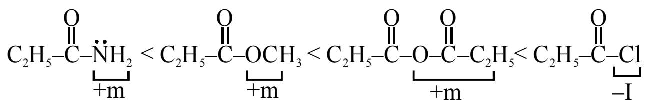
(C) 

(D) 

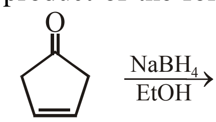
(1) (A) < (B) < (D) < (C)
(2) (A) < (B) < (C) < (D)
(3) (B) < (A) < (D) < (C)
(4) (B) < (A) < (C) < (D)

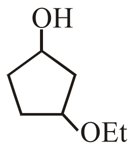
Ans. (1)

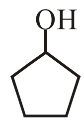
Sol. Rate of nucleophilic attack on carbonyl $\propto$ Electrophilicity of carbonyl group

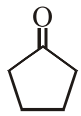


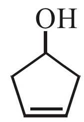
9. The major product of the following reaction is:



(1) 

(2) 

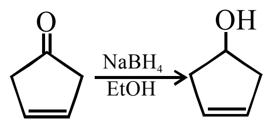
(3) 

(4) 

Ans. (4)

Sol. $NaBH_4$ can not reduce C=C

but can reduce $-\underset{\underset{O}{\|}}{C}-$ into OH.



10. Molecules of benzoic acid (C_6H_5COOH) dimerise in benzene. 'w' g of the acid dissolved in 30 g of benzene shows a depression in freezing point equal to 2K. If the percentage association of the acid to form dimer in the solution is 80, then w is :
(Given that K_f = 5 K kg mol^{-1}, Molar mass of benzoic acid = 122 g mol^{-1})
(1) 1.8 g (2) 2.4 g (3) 1.0 g (4) 1.5 g

Ans. (2)

Sol. $$\underset{Wg}{2(C_6H_5COOH)} \xrightarrow[(30g)]{C_6H_6} \underset{diner}{(C_6H_5COOH)_2}$$

$\Delta_f T = i\, k_f\, m$

$2 = 0.6 \times 5 \times \dfrac{w \times 1000}{122 \times 30}$

(i = 1 – 0.8 +0.4 = 0.6)

w = 2.44 g

11. Given :
(i) C(graphite) + O_2(g) $\rightarrow$ CO_2(g);
$\Delta rH°$ = x kJ mol^{-1}

(ii) C(graphite)+ $\frac{1}{2}O_2$(g) $\rightarrow$ CO_2(g);

$\Delta rH° = y$ kJ mol^{-1}

(iii) $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$;

$\Delta rH° = z$ kJ mol^{-1}

Based on the above thermochemical equations, find out which one of the following algebraic relationships is correct ?

(1) $z = x + y$ (2) $x = y - z$

(3) $x = y + z$ (4) $y = 2z - x$

Ans. **(3)**

Sol.

$$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)\Delta_r H° = x kJ/mol \ ...(1)$$

$$C_{(graphite)} + \frac{1}{2}O_2(g) \rightarrow CO(g)\Delta_r H° = y kJ/mol \ ...(2)$$

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)\Delta_r H° = z kJ/mol \ .....(3)$$

(1) = (2) + (3)

$x = y + z$

12. An open vessel at 27°C is heated until two fifth of the air (assumed as an ideal gas) in it has escaped from the vessel. Assuming that the volume of the vessel remains constant, the temperature at which the vessel has been heated is :

(1) 750°C (2) 500°C

(3) 750 K (4) 500 K

Ans. **(4)**

Sol. $\frac{2}{5}$ air escaped from vessel, $\therefore \frac{3}{5}$ air remain is vessel. P, V constant

$n_1T_1 = n_2T_2$

$$n_1(300) = \left(\frac{3}{5}n_1\right)T_2 \Rightarrow T_2 = 500 \text{ K}$$

13. $\wedge_m^\circ$ for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S cm^{2}mol^{-1}, respectively. If the conductivity of 0.001 M HA is 5×10^{-5} S cm^{-1}, degree of dissociation of HA is :

(1) 0.75 (2) 0.125 (3) 0.25 (4) 0.50

Ans. **(2)**

Sol.

$$\Lambda_m^0(HA) = \Lambda_m^0(HCl) + \Lambda_m^0(NaA) - \Lambda_m^0(NaCl)$$

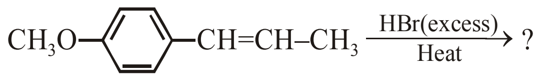
= 425.9 + 100.5 - 126.4

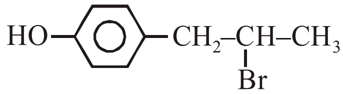
= 400 S cm^2 mol^{-1}

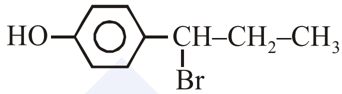
$$\Lambda_m = \frac{1000K}{M} = \frac{1000 \times 5 \times 10^{-5}}{10^{-3}} = 50 \text{ S cm}^2 \text{ mol}^{-1}$$

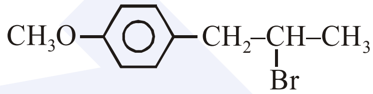
$$\alpha = \frac{\Lambda_m}{\Lambda_m^0} = \frac{50}{400} = 0.125$$

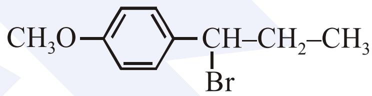
14. The major product in the following conversion is :



(1) 

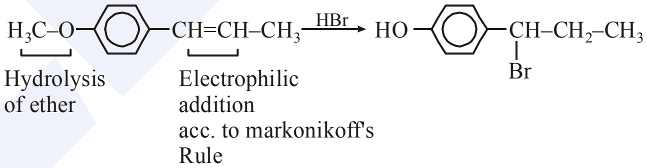
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



15. If K_{sp} of Ag_2CO_3 is 8×10^{-12}, the molar solubility of Ag_2CO_3 in 0.1M $AgNO_3$ is :

(1) 8×10^{-12} M (2) 8×10^{-10} M

(3) 8×10^{-11} M (4) 8×10^{-13} M

Ans. **(2)**

Sol.

$$Ag_2CO_3\ (s) \rightleftharpoons \underset{(0.1+2S)\ M}{2Ag^+(aq.)} + \underset{S\ M}{CO_3^{-2}(aq)}$$

$Ksp = [Ag^+]^2[CO_3^{-2}]$

$8 \times 10^{-12} = (0.1 + 2S)^2\ (S)$

$S = 8 \times 10^{-10}$ M

16. Among the following, the false statement is :

(1) Latex is a colloidal solution of rubber particles which are positively charged

(2) Tyndall effect can be used to distingush between a colloidal solution and a true solution.

(3) It is possible to cause artificial rain by throwing electrified sand carrying charge opposite to the one on clouds from an aeroplane.

(4) Lyophilic sol can be coagulated by adding an electrolyte.

Ans. **(1)**

Sol. Colloidal solution fo rubber are negatively charged.

17. The pair that does NOT require calcination is:

(1) ZnO and MgO

(2) Fe_2O_3 and $CaCO_3.MgCO_3$

(3) ZnO and $Fe_2O_3.xH_2O$

(4) $ZnCO_3$ and CaO

Ans. **(1)**

Sol. ZnO & MgO both are in oxide form therefore no change on calcination.

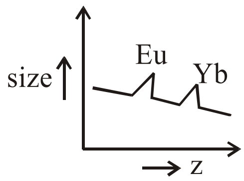
18. The correct order of atomic radii is :

(1) Ce > Eu > Ho > N (2) N > Ce > Eu > Ho

(3) Eu > Ce > Ho > N (4) Ho > N > Eu > Ce

Ans. **(3)**

Sol.



Eu > Ce > Ho > N.

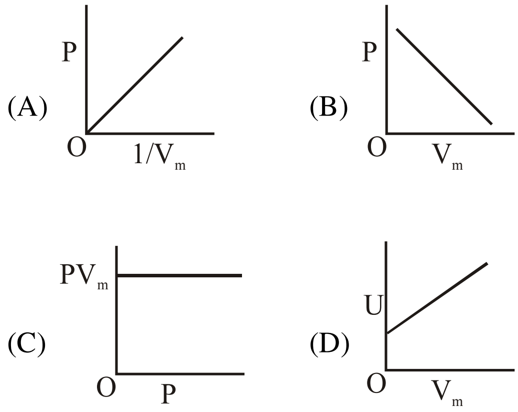
19. The element that does NOT show catenation is:

(1) Sn (2) Ge (3) Si (4) Pb

Ans. **(4)**

Sol. Catenation is not shown by lead.

20. The combination of plots which does not represent isothermal expansion of an ideal gas is:



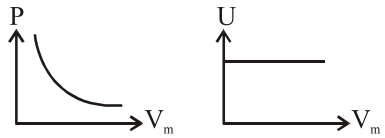
(1) (A) and (C) (2) (A) and (D)

(3) (B) and (D) (4) (B) and (C)

Ans. **(3)**

Sol. Isothermal expansion $PV_m = K$(Graph-C)

$$P = \frac{K}{V_m} \text{ (Graph-A)}$$



21. The volume strength of 1M H_2O_2 is:

(Molar mass of H_2O_2 = 34 g mol^{-1})

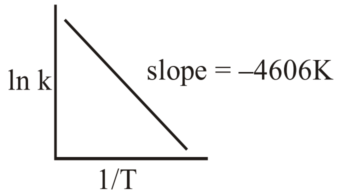
(1) 16.8 (2) 11.35

(3) 22.4 (4) 5.6

Ans. **(2)**

Sol. 1L – 1M H_2O_2 solution will produce 11.35 L O_2 gas at STP.

22. For a reaction consider the plot of ln k versus 1/T given in the figure. If the rate constant of this reaction at 400 K is 10^{-5} s^{-1}, then the rate constant at 500 K is :



(1) 2×10^{-4} s^{-1} (2) 10^{-4} s^{-1}
(3) 10^{-6} s^{-1} (4) 4×10^{-4} s^{-1}

Ans. **(2)**

Sol. $$\ell n\frac{K_2}{K_1}=\frac{E_a}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$2.303\log\frac{K_2}{10^{-5}}=4606\left[\frac{1}{400}-\frac{1}{500}\right]$$

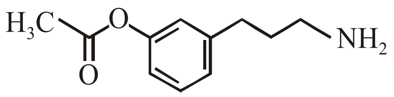
$\Rightarrow K_2 = 10^{-4}$ s^{-1}

23. The element that shows greater ability to form pπ-pπ multiple bonds, is :
(1) Si (2) Ge (3) Sn (4) C

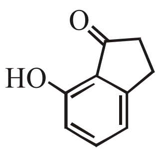
Ans. **(4)**

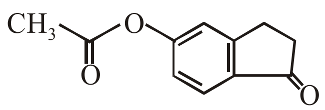
Sol. carbon atom have 2p orbitals able to form strongest pπ - pπ bonds

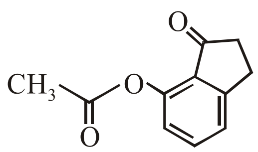
24. The major product of the following reaction is:

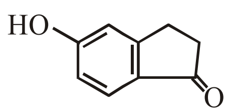


(i) $NaNO_2/H^+$
(ii) $CrCO_3/H^+$
(iii) H_2SO_4 (conc.),Δ

(1) 

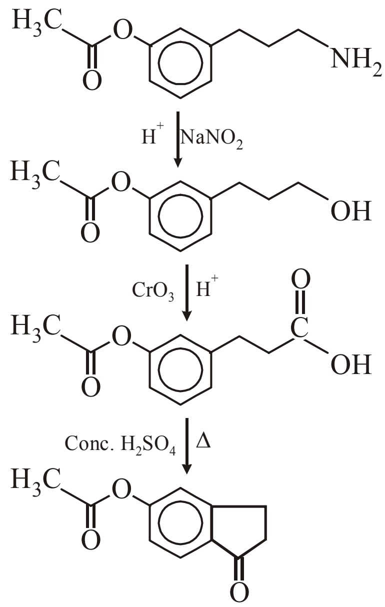
(2) 

(3) 

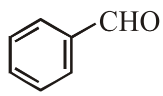
(4) 

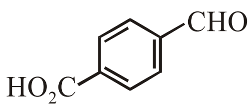
Ans. **(2)**

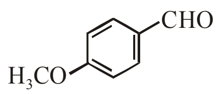
Sol.

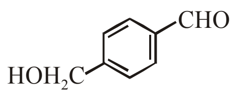


25. The aldehydes which will not form Grignard product with one equivalent Grignard reagents are :

(A) 

(B) 

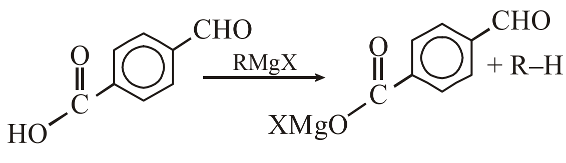
(C) 

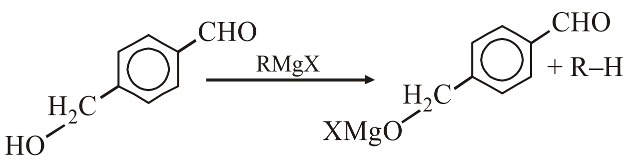
(D) 

(1) (B), (C), (D) (2) (B), (D)
(3) (B), (C) (4) (C), (D)

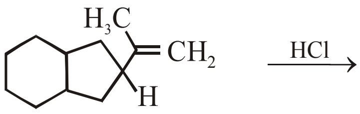
Ans. **(2)**

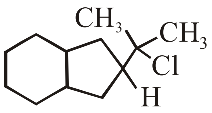
Sol. Acid-base reaction of G.R are fast.

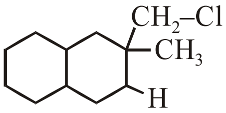


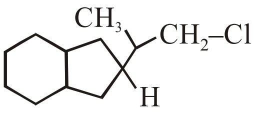


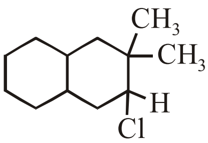
26. The major product of the following reaction is:



(1) 

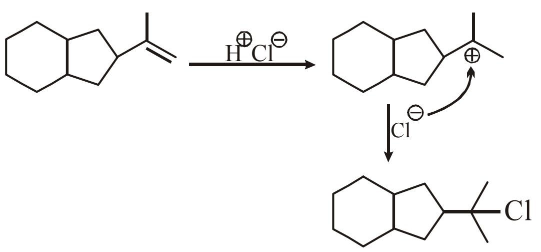
(2) 

(3) 

(4) 

Ans. **(1)**

Sol.



27. Chlorine on reaction with hot and concentrated sodium hydroxide gives :

(1) Cl^- and ClO_2^- (2) Cl^- and ClO_3^-

(3) Cl^- and ClO^- (4) ClO_3^- and ClO_2^-

Ans. **(2)**

Sol. $3Cl_2 + 6\,OH^- \rightarrow 5Cl^- + ClO_3^- + 3H_2O$

28. The major product of the following reaction is:

$$CH_3CH_2\underset{\large Br}{\underset{|}{C}H}-\underset{\large Br}{\underset{|}{C}H_2} \xrightarrow[\text{(ii) NaNH}_2 \text{ in liq NH}_3]{\text{(i) KOH alc.}}$$

(1) $CH_3CH_2C\equiv CH$

(2) $CH_3CH_2\underset{NH_2}{\underset{|}{C}H}-\underset{NH_2}{\underset{|}{C}H_2}$

(3) $CH_3CH=C=CH_2$

(4) $CH_3CH=CHCH_2NH_2$

Ans. **(1)**

Sol.

$$CH_3-CH_2-\underset{Br}{\underset{|}{C}H}-\underset{Br}{\underset{|}{C}H_2}$$

$$\Big\downarrow \text{Alc. KOH}$$

$$CH_3-CH_2-\underset{Br}{\underset{|}{C}}=CH_2$$

$$\text{NaNH}_2\ \Big\downarrow \text{ in liq. NH}_3$$

$$CH_3-CH_2-C\equiv CH$$

29. If the de Broglie wavelength of the electron in n^{th} Bohr orbit in a hydrogenic atom is equal to $1.5\ \pi a_0$(a_0 is Bohr radius), then the value of n/z is :

(1) 1.0 (2) 0.75 (3) 0.40 (4) 1.50

Ans. **(2)**

Sol. According to de-broglie's hypothesis

$$2\pi r_n = n\lambda \Rightarrow 2\pi\ .\ a_0 = \frac{n^2}{z} = n\times 1.5\pi a_0$$

$$\frac{n}{z} = 0.75$$

30. The two monomers for the synthesis of Nylone 6, 6 are :

(1) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_6NH_2$

(2) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_4NH_2$

(3) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_4NH_2$

(4) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_6NH_2$

Ans. **(4)**

Sol. Nylon-6,6 is polymer of

Hexamethylene diamine & Adipic acid

$\downarrow$ $\downarrow$

$H_2N-(CH_2)_6-NH_2$ $HOOC-(CH_2)_4-COOH$

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

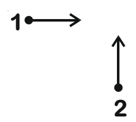
1. Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ, of the final particle, is given by:

(1) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (2) $\lambda = \dfrac{\lambda_1 + \lambda_2}{2}$

(3) $\dfrac{2}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$ (4) $\dfrac{1}{\lambda^2} = \dfrac{1}{\lambda_1^2} + \dfrac{1}{\lambda_2^2}$

Answer (4)

Sol.



$$p_1 = \frac{h}{\lambda_1}$$

$$p_2 = \frac{h}{\lambda_2}$$

$$\therefore \quad p_f = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \quad \frac{h}{\lambda} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$$

$$\Rightarrow \quad \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

2. In SI units, the dimensions of $\sqrt{\dfrac{\epsilon_0}{\mu_0}}$ is:

(1) $AT^2M^{-1}L^{-1}$ (2) $AT^{-3}ML^{3/2}$

(3) $A^{-1}TML^3$ (4) $A^2T^3M^{-1}L^{-2}$

Answer (4)

Sol. $\left[\sqrt{\dfrac{\epsilon_0}{\mu_0}}\right] = \left[\dfrac{\epsilon_0}{\sqrt{\mu_0 \epsilon_0}}\right] = [LT^{-1}] \times [\epsilon_0]$

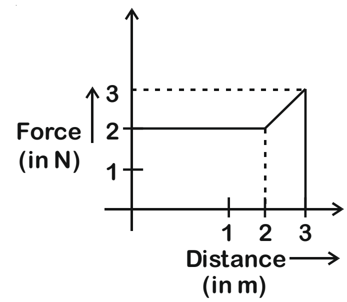
$$\because \quad F = \frac{q^2}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow \quad [\epsilon_0] = \frac{[AT]^2}{[MLT^{-2}] \times [L^2]}$$

$$\therefore \quad \left[\sqrt{\frac{\epsilon_0}{\mu_0}}\right] = [LT^{-1}] \times [A^2M^{-1}L^{-3}T^4]$$

$$= [M^{-1}L^{-2}T^3A^2]$$

3. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is



(1) 4 J (2) 2.5 J

(3) 5 J (4) 6.5 J

Answer (4)

Sol. = Area under F–x graph

$$\Delta K.E = W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2$$

= 2.5 + 4

= 6.5 J

4. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is

(1) n = 2 $\rightarrow$ n = 4

(2) n = 2 $\rightarrow$ n = 5

(3) n = 2 $\rightarrow$ n = 3

(4) n = 1 $\rightarrow$ n = 4

Answer (1)

Sol. Energy released by hydrogen atom

$$\Delta E_1 = 13.6 \times \left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4} \times 13.6 \text{ eV}$$

= 10.2 eV

Also, energy absorbed by H_e^+ ion in transition n = 2 $\rightarrow$ n = 4

$$\Delta E_2 = 13.6 \times 4 \times \left(\frac{1}{4} - \frac{1}{16}\right) = 10.2 \text{ eV}$$

So, possible transition is n = 2 $\rightarrow$ n = 4 is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1 mPa s)

(1) 10^2 (2) 10^4

(3) 10^3 (4) 10^6

Answer (2)

Sol. Flow rate of water (Q) = 100 lit/min

$$= \frac{100\times10^{-3}}{60} = \frac{5}{3}\times10^{-3}\,m^3$$

$\therefore$ Velocity of flow (v) = $\frac{Q}{A} = \frac{5\times10^{-3}}{3\times\pi\times(5\times10^{-2})^2}$

$$= \frac{10}{15\pi} = \frac{2}{3\pi}\,m/s$$

= 0.2 m/s

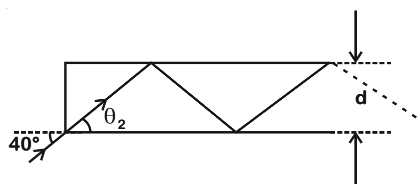
$\therefore$ Reynold number (R_e) $= \frac{Dv\rho}{\eta}$

$$= \frac{(10\times10^{-2})\times\frac{2}{3\pi}\times1000}{1} \simeq 2\times10^4$$

Order of $R_e = 10^4$

6. In figure, the optical fiber is l = 2 m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle θ_1 = 40°, the number of reflections it makes before emerging from the other end is close to:

(refractive index of fiber is 1.31 and sin 40° = 0.64)



(1) 66000 (2) 55000

(3) 45000 (4) 57000

Answer (4)

Sol. 1 × sin 40° = 1.31 sinθ

$$\Rightarrow \sin\theta = \frac{0.64}{1.31} \Rightarrow \theta \simeq 30°$$

l = 20 μm × cotθ

$$\therefore N = \frac{2}{20\times10^{-6}\times\cot\theta}$$

$$= \frac{2\times10^6}{20\times\sqrt{3}} = 57735$$

$N \approx 57000$

code, the new resistance will be:

(1) 400 Ω

(2) 500 Ω

(3) 300 Ω

(4) 100 Ω

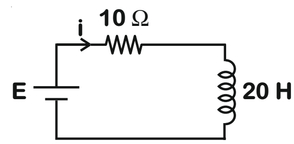
Answer (2)

Sol. 200 Ω = Red + Black + Brown

Green $\equiv$ 5

So, Green + Black + Brown $\equiv$ 500 Ω

8. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is:



(1) $\frac{2}{\ln 2}$

(2) 2 ln2

(3) ln2

(4) $\frac{1}{2}\ln 2$

Answer (2)

Sol. $P_R = i^2 \times R$

$P_B = V \times i$

$\therefore P_L = Vi - i^2R$

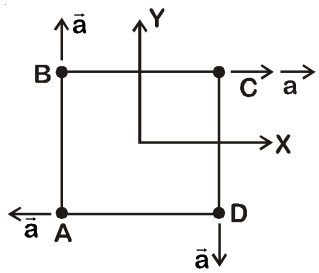
$\Rightarrow Vi - i^2R = i^2R$

$$\Rightarrow i = \frac{V}{2R} \text{ and } i = \frac{V}{R}\cdot\left(1-e^{-t/\tau}\right)$$

$$\therefore \frac{V}{2R} = \frac{V}{R}\left(1-e^{-t/\tau}\right)$$

$$\Rightarrow t = \tau\ln(2) = \frac{20}{10}\ln(2) = 2\ln(2)$$

the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



(1) Zero (2) $a(\hat{i}+\hat{j})$

(3) $\frac{a}{5}(\hat{i}+\hat{j})$ (4) $\frac{a}{5}(\hat{i}-\hat{j})$

Answer (4)

Sol. $a_{CM} = \dfrac{(2m)a\hat{j}+3m\times a\hat{i}+ma(-\hat{i})+4m\times a(-\hat{j})}{2m+3m+4m+m}$

$$= \frac{2a\hat{i}-2a\hat{j}}{10} = \frac{a}{5}(\hat{i}-\hat{j})$$

10. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is E = 6 Vm^{-1} along y-direction. Its corresponding magnetic field component, B would be :

(1) 2×10^{-8} T along y-direction

(2) 6×10^{-8} T along z-direction

(3) 2×10^{-8} T along z-direction

(4) 6×10^{-8} T along x-direction

Answer (3)

Sol. $B_0 = \dfrac{E_0}{c} = \dfrac{6}{3\times10^{8}} = 2\times10^{-8}\,T$

Propagation direction = $\hat{E}\times\hat{B}$

$\hat{i} = \hat{j}\times\hat{B}$

$\Rightarrow \quad \hat{B} = \hat{k}$

11. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant if the capacitance is 15 pF?

(given $\varepsilon_0 = 8.86 \times 10^{-12}$ C^2/Nm^2)

(1) 3.8 (2) 4.5

(3) 8.5 (4) 6.2

Sol. $C = \dfrac{k\epsilon_0 A}{d}$

$$E = \frac{V}{d}$$

$$15\times10^{-12} = \frac{k\times8.86\times10^{-12}\times10^{-4}\times10^{6}}{500}$$

k = 8.5

12. The wavelength of the carrier waves in a modern optical fiber communication network is close to :

(1) 600 nm (2) 900 nm

(3) 1500 nm (4) 2400 nm

Answer (3)

Sol. Fact Based

Wavelength of carrier waves in modern optical fiber communication is most widely used near about 1500 nm.

13. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1 π ms^{-2}, what will be the tensile stress that would be developed in the wire?

(1) 4.8×10^6 Nm^{-2}

(2) 3.1×10^6 Nm^{-2}

(3) 5.2×10^6 Nm^{-2}

(4) 6.2×10^6 Nm^{-2}

Answer (2)

Sol. Stress $= \dfrac{F}{A} = \dfrac{4\times3.1\pi}{\pi\times(2\times10^{-3})^2} = 3.1\times10^{6}\ N/m^2$

14. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is I = a MR^2. The value of the coefficient a is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{8}{5}$

Answer (4)

$$I_C = \int_0^R \rho_0 r \times 2\pi r dr \times r^2 = \frac{2\pi\rho_0 R^5}{5}$$

$$\therefore\ I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3}+\frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$$

$$= \frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right]R^2 = \frac{8}{5}MR^2$$

15. An alternating voltage v(t) = 220 sin 100πt volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is

(1) 2.2 ms (2) 7.2 ms

(3) 5 ms (4) 3.3 ms

Answer (4)

Sol. $I = I_m \sin(100\pi t)$

$$\Rightarrow \frac{I_m}{2} = I_m \sin(100\pi t_1)$$

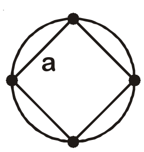
$$\Rightarrow \frac{\pi}{6} = 100\pi t_1$$

$$\Rightarrow t_1 = \frac{1}{600}\text{ s}$$

$$T = \frac{2\pi}{100\pi} = \frac{1}{50}\text{ s}$$

$$\therefore\ t_{req} = \frac{T}{4} - t_1 = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300}\text{ s} = 3.3\text{ ms}$$

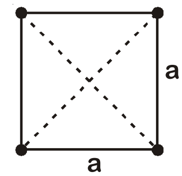
16. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square?



(1) $1.41\sqrt{\frac{GM}{a}}$ (2) $1.16\sqrt{\frac{GM}{a}}$

(3) $1.21\sqrt{\frac{GM}{a}}$ (4) $1.35\sqrt{\frac{GM}{a}}$

Answer (2)



$$\therefore\ \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)} = \frac{GM^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v = \sqrt{\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\frac{GM}{a}}$$

17. The bob of a simple pendulum has mass 2 g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is

(take g = 10 m/s^2)

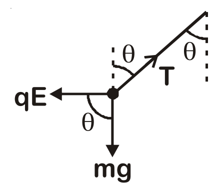
(1) $\tan^{-1}(0.2)$ (2) $\tan^{-1}(5.0)$

(3) $\tan^{-1}(2.0)$ (4) $\tan^{-1}(0.5)$

Answer (4)

Sol. $T\cos\theta = mg$

$T\sin\theta = qE$

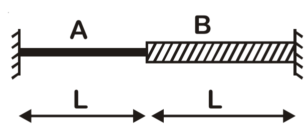


$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

18.



A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is

(1) 4 : 9 (2) 1 : 2

(3) 3 : 5 (4) 1 : 4

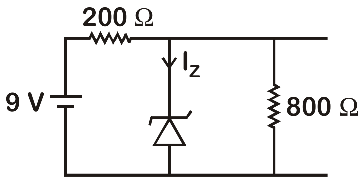
Answer (2)

Sol. V_B $\sqrt{u_A}$ r_A λ_B

$\Rightarrow \lambda_A = 2\lambda_B$

$\Rightarrow \frac{p}{q} = \frac{1}{2}$

19. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



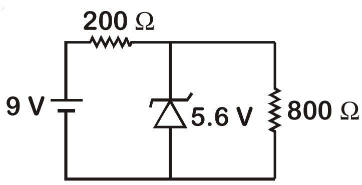
The current I_Z through the Zener is

(1) 15 mA (2) 7 mA

(3) 10 mA (4) 17 mA

Answer (3)

Sol.

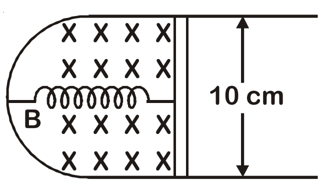


$$I_{800\,\Omega} = \frac{5.6}{800}\,A = 7\,mA$$

$$I_{200\,\Omega} = \frac{9-5.6}{200} = 17\,mA$$

$\therefore$ $I_Z = 17 - 7 = 10$ mA

20. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1}(see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10 Ω and air drag negligible, N will be close to



(1) 1000 (2) 5000

(3) 50000 (4) 10000

Answer (2)

$$F = -kx - \frac{B^2l^2}{R} \times v$$

So, it is case of damped oscillation

$$\Rightarrow A = A_0 e^{-\frac{bt}{2m}}$$

$$\Rightarrow \frac{A_0}{e} = A_0 e^{-\frac{bt}{2m}}$$

$$\Rightarrow t = \frac{2m}{\left(\frac{B^2l^2}{R}\right)} = \frac{2 \times 50 \times 10^{-3} \times 10}{0.01 \times 0.01} = 10000\,s$$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} \simeq 2\,s$

$\therefore$ Number, $N = \frac{10000}{2} = 5000$

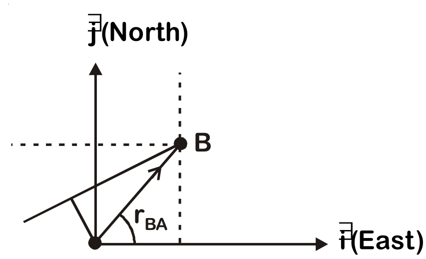
21. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in:

(1) 2.2 hrs. (2) 4.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.

Answer (4)

Sol.



$\vec{v}_A = 30i + 50j$ km/hr

$\vec{v}_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_B = (-10i)$ km/hr

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = -40\hat{i} - 50\hat{j}$

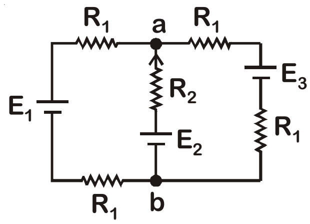
Projection of

$$(\vec{r}_{BA}) \text{ on } \vec{v}_{BA} = \frac{(\vec{r}_{BA})\cdot(\vec{v}_{BA})}{(\vec{v}_{BA})}$$

$$= \frac{(80\hat{i} + 150\hat{j})(-40\hat{i} - 50\hat{j})}{10\sqrt{41}} = \frac{\sqrt{mk}}{qB}$$

$$\therefore\ t = \frac{10 \times 107}{\sqrt{41} \times 10\sqrt{41}} = \frac{107}{41} = 2.6\,\text{Hrs.}$$

potential difference between the points ‘a’ and ‘b’ is approximately (in V)



(1) 2.7 (2) 3.7

(3) 2.3 (4) 3.3

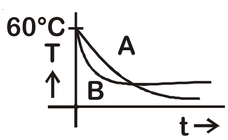
Answer (4)

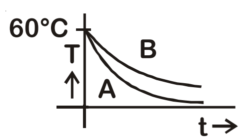
Sol. $v = \dfrac{E_1r_2r_3 + E_2r_3r_1 + E_3r_1r_2}{r_1r_2 + r_2r_3 + r_3r_1}$

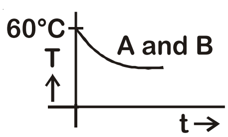
$= \dfrac{2(2\times2)+4(2\times2)+4(2\times2)}{4+4+4}$

$= \dfrac{40}{12} = \dfrac{10}{3} = 3.3\text{ Volt}$

23. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 Jkg^{-1}K^{-1} while liquid in B has density of 10^3 kgm^{-3} and specific heat of 4000 Jkg^{-1}K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

(1)


(2)


(3)


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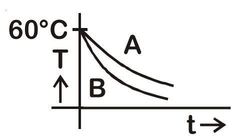


Answer (2)

Sol. $mS\left(-\dfrac{dT}{dt}\right) = e\sigma AT^4$

$-\dfrac{dT}{dt} = \dfrac{e\sigma \times A \times T^4}{\rho \times Vol. \times S}$

$\dfrac{\left(-\dfrac{dT}{dt}\right)_A}{\left(-\dfrac{dT}{dt}\right)_B} = \dfrac{\rho_B}{\rho_A} \times (2) > 1$

So, A cools down at faster rate.

hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4 Q, the new potential difference between the same two surfaces is:

(1) –2 V (2) V

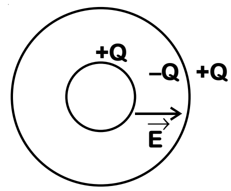
(3) 2 V (4) 4 V

Answer (2)

Sol. In case-1

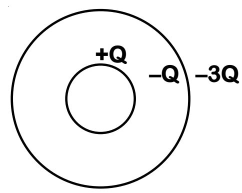
Electric field between spherical surface

$\vec{E} = \dfrac{KQ}{r^2}$



In case-2

Electric field between surfaces remain unchanged.



$\therefore$ Potential difference between them remain unchanged too.

25. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is:

(1) $\dfrac{B\pi r^2 I}{N}$ (2) $\dfrac{Br^2 I}{\pi N}$

(3) $B\pi r^2 I N$ (4) Zero

Answer (3)

Sol. $|\vec{\tau}| = |\vec{\mu} \times \vec{B}|$

$= NIA \times B$ $[A = \pi r^2]$

$\Rightarrow \tau = NI\pi r^2 B$

length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be:

(1) 20 cm from the convergent mirror, twice the size of the object

(2) 20 cm from the convergent mirror, same size as the object

(3) 40 cm from the convergent lens, twice the size of the object

(4) 40 cm from the convergent mirror, same size as the object

Answer (Bonus)

Sol. $v_1 = \dfrac{40 \times 20}{(40-20)} = 40 \text{ cm}$

$u_2 = 60 - 40 = 20$ cm

$\therefore \quad v_2 = \dfrac{20 \times 10}{(20-10)} = 20 \text{ cm}$

$\therefore$ Image traces back to object itself as image formed by lens is a centre of curvature of mirror.

27. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of:

(1) 10^8 N/m^2 (2) 10^3 N/m^2

(3) 10^{16} N/m^2 (4) 10^4 N/m^2

Answer (Bonus)

Sol. $P = \dfrac{N \times (2mv)}{\Delta t \times A}$

$= \dfrac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1 \times 1} = 2 \text{ N/m}^2$

28. A thermally insulated vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to:

(Latent heat of vaporization of water = 2.10 × 10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36 × 10^5 J kg^{-1})

(3) 20 g (4) 30 g

Answer (3)

Sol. Let amount of water evaporated be m gram.

$\therefore \quad m \times L_v = (150 - m) \times L_s$

$m \times 540 = (150 - m) \times 80$

$\Rightarrow \quad m \simeq 20$ g

29. In an interference experiment the ratio of amplitudes of coherent waves is $\dfrac{a_1}{a_2} = \dfrac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be:

(1) 4 (2) 18

(3) 9 (4) 2

Answer (1)

Sol. $\dfrac{a_1}{a_2} = \dfrac{3}{1}$

$\therefore \quad \dfrac{I_{max}}{I_{min}} = \left(\dfrac{a_2 + a_1}{a_2 - a_1}\right)^2 = \left(\dfrac{3+1}{3-1}\right)^2 = 4$

30. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to:

(1) 10^4 Nm^{-2} (2) 10^3 Nm^{-2}

(3) 10^8 Nm^{-2} (4) 10^6 Nm^{-2}

Answer (4)

Sol. $\dfrac{1}{2} \cdot \left(\dfrac{YA}{L}\right)(\Delta l)^2 = \dfrac{1}{2} mv^2$

$\Rightarrow \quad Y = \dfrac{mv^2 L}{A(\Delta l)^2}$

$= \dfrac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04}$

$= 2.3 \times 10^6$ N/m^2

So, order is 10^6.

1. With respect to an ore, Ellingham diagram helps to predict the feasibility of its

   (1) Zone refining

   (2) Vapour phase refining

   (3) Thermal reduction

   (4) Electrolysis

Answer (3)

Sol. Ellingham diagram is used to select reducing agent so it help to predict feasibility of its thermal reduction.

2. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas? (Assume non-expansion work is zero)

   (1) Isothermal process : $q = -w$

   (2) Cyclic process : $q = -w$

   (3) Isochoric process : $\Delta U = q$

   (4) Adiabatic process : $\Delta U = -w$

Answer (4)

Sol. $\Delta U = q + W$

Adiabatic process $q = 0$

$\Delta U = W$

For isothermal, $\Delta U = 0$

For cyclic, $\Delta U = 0$

For isochoric, $W = 0$

3. The quantum number of four electrons are given below:

   (I) $n = 4, l = 2, m_l = -2, m_s = -\frac{1}{2}$

   (II) $n = 3, l = 2, m_l = 1, m_s = +\frac{1}{2}$

   (III) $n = 4, l = 1, m_l = 0, m_s = +\frac{1}{2}$

   (IV) $n = 3, l = 1, m_l = 1, m_s = -\frac{1}{2}$

   The correct order of their increasing energies will be:

   (1) IV < II < III < I

   (2) I < III < II < IV

   (3) IV < III < II < I

   (4) I < II < III < IV

Answer (1)

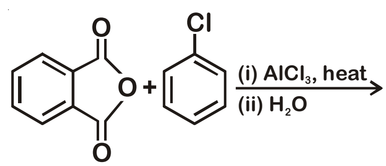
Sol.

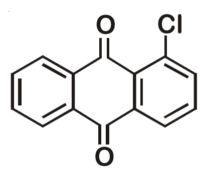
				n + l
(I)	n = 4	l = 2	4d	6
(II)	n = 3	l = 2	3d	5
(III)	n = 4	l = 1	4p	5
(IV)	n = 3	l = 1	3p	4

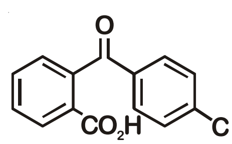
more is n + l value, more is energy

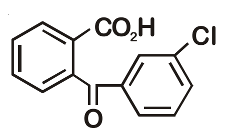
3p < 3d < 4p < 4d

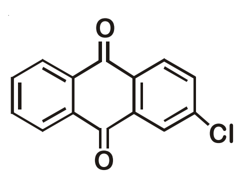
4. The major product of the following reaction is :

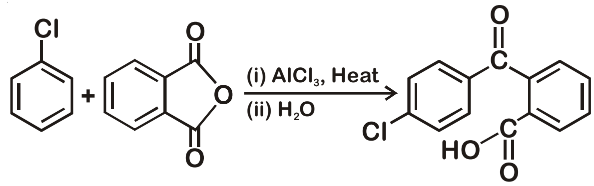


(1) 

(2) 

(3) 

(4) 

Sol. 

5. The correct order of the spin-only magnetic moment of metal ions in the following low-spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$ and $[Cr(NH_3)_6]^{2+}$, is:

(1) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

(2) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

(3) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$

(4) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$

Answer (1)

Sol.

		No. of unpaired electrons
$[V(CN)_6]^{4-}$	V^{+2}	3
$[Ru(NH_3)_6]^{3+}$	Ru^{+3}	1
$[Fe(CN)_6]^{4-}$	Fe^{+2}	0
$[Cr(NH_3)_6]^{2+}$	Cr^{+2}	2

$\therefore$ Order of spin magnetic moment

$V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

6. Which is wrong with respect to our responsibility as a human being to protect our environment?

(1) Using plastic bags

(2) Restricting the use of vehicles

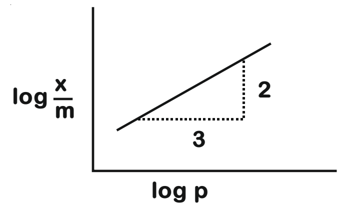
(3) Avoiding the use of floodlighted facilities

(4) Setting up compost tin in gardens

Answer (1)

Sol. Use of plastic bags is hazardous to our environment

adsorbed on mass m of the adsorbent. The plot of log $\frac{x}{m}$ versus log p is shown in the given graph. $\frac{x}{m}$ is proportional to



(1) $p^{3/2}$ (2) p^3

(3) $p^{2/3}$ (4) p^2

Answer (3)

Sol. $\frac{x}{m} \propto p^{\frac{1}{n}}$ $\quad \frac{x}{m} = kp^{\frac{1}{n}}$

Slope = $\frac{2}{3}$

$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

Slope = $\frac{1}{n} = \frac{2}{3}$

$\frac{x}{m} \propto p^{\frac{2}{3}}$

8. Given that $E^{\ominus}_{O_2/H_2O} = +1.23\ V$;

$E^{\ominus}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05\ V$

$E^{\ominus}_{Br_2/Br^-} = +1.09\ V$;

$E^{\ominus}_{Au^{3+}/Au} = +1.4\ V$

The strongest oxidizing agent is

(1) Br_2

(2) Au^{3+}

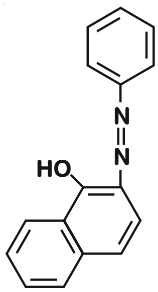
(3) $S_2O_8^{2-}$

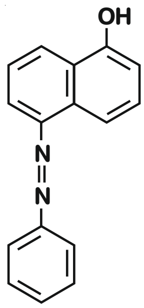
(4) O_2

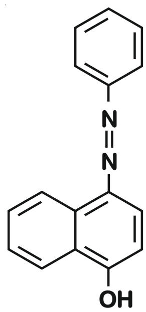
Answer (3)

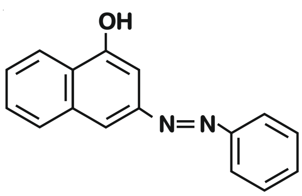
Sol. More positive is the reduction potential stronger is the oxidising agent.

Reduction potential is maximum for $S_2O_8^{2-}$

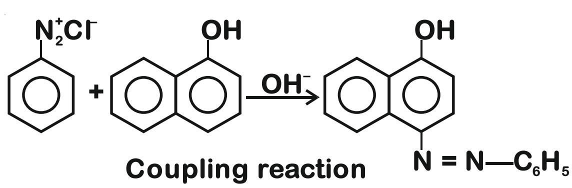
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

10. Maltose on treatment with dilute HCl gives

(1) D-Galactose

(2) D-Glucose and D-Fructose

(3) D-Glucose

(4) D-Fructose

Sol. Hydrolysis of maltose give glucose as maltose is composed of two α-D glucose units.

11. The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298 K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are :

(1) 500 mmHg, 0.4, 0.6

(2) 500 mmHg, 0.5, 0.5

(3) 450 mmHg, 0.4, 0.6

(4) 450 mmHg, 0.5, 0.5

Answer (1)

Sol. $P = x_B p_B^\circ + x_A p_A^\circ$

$= 0.5 \times 600 + 0.5 \times 400 = 300 + 200 = 500$

$p_B = y_B P_{Total}$

$$y_B = \frac{p_B}{P_{Total}} = \frac{300}{500} = \frac{3}{5} = 0.6$$

$$y_A = \frac{p_A}{P_{Total}} = \frac{200}{500} = \frac{2}{5} = 0.4$$

12. For the reaction $2A + B \rightarrow C$, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is

$[A](mol\ L^{-1})$	$[B](mol\ L^{-1})$	Initial Rate $(mol\ L^{-1}s^{-1})$
0.05	0.05	0.045
0.10	0.05	0.090
0.20	0.10	0.72

(1) Rate = $k[A]^2[B]^2$ (2) Rate = $k[A][B]$

(3) Rate = $k[A]^2[B]$ (4) Rate = $k[A][B]^2$

Answer (4)

Sol. $2A + B \longrightarrow P$

Rate = $k[A]^x [B]^y$

Exp-1, $0.045 = k[0.05]^x [0.05]^y$ ...(i)

Exp-3, $0.72 = k[0.2]^x [0.1]^y$ ...(iii)

Divide equation (i) by equation (ii)

$$\frac{0.045}{0.090} = \left(\frac{1}{2}\right)^x \Rightarrow x = 1$$

Divide equation (i) by equation (iii)

$$\frac{0.045}{0.72} = \left(\frac{0.05}{0.1}\right)^y \left(\frac{0.05}{0.2}\right)^1$$

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^y \Rightarrow y = 2$$

Rate law = $k[A]^1 [B]^2$.

13. For silver, $C_p(JK^{-1} mol^{-1}) = 23 + 0.01T$. If the temperature (T) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of ΔH will be close to

(1) 21 kJ (2) 13 kJ

(3) 62 kJ (4) 16 kJ

Answer (3)

Sol. n = 3

$T_1 = 300$

$T_2 = 1000$

$C_p = 23 + 0.01T$

$$\Delta H = \int_{T_1}^{T_2} nC_p dT$$

$$= n \int_{300}^{1000} (23 + 0.01T)dT$$

$$= 3\left[23T + \frac{0.01T^2}{2}\right]_{300}^{1000}$$

$= 3[16100 + 4550]$

$= 3 \times 20650 = 61950$ J

$= 61.95$ kJ

≈ 62

(1) Gd^{3+} (2) Lu^{3+}

(3) La^{3+} (4) Sm^{3+}

Answer (4)

Sol. Sm^{+3} = Partially filled f orbital = $4f^5$

$Sm = 4f^6 6s^2$

Sm^{+3} = Yellow.

$Lu^{+3} = 4f^{14}$ colourless.

15. The correct order of hydration enthalpies of alkali metal ions is

(1) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$

(2) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$

(3) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

(4) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

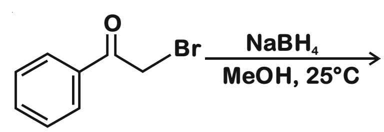
Answer (4)

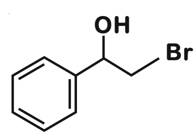
Sol. Smaller is size more is hydration energy.

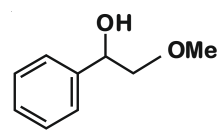
$Li^+ < Na^+ < K^+ < Rb^+ < Cs^+$ Size

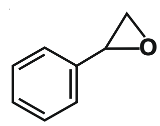
$Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$ Hydration energy

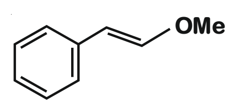
16. The major product of the following reaction is



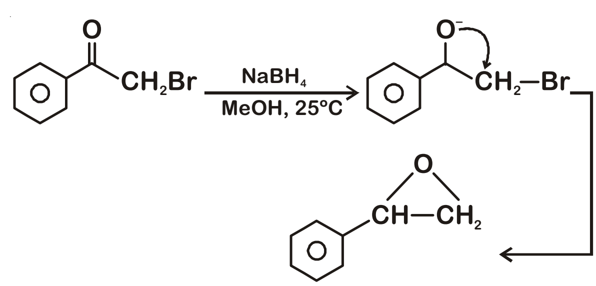
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is

(1) A_2B_2O

(2) AB_2O_4

(3) A_4B_2O

(4) A_2BO_4

Answer (2)

Sol. Lattice formed by B(ccp) = 4

A = 50% of octahedral voids = 2

O = tetrahedral voids = 8

Formula = AB_2O_4

18. If solubility product of $Zr_3(PO_4)_4$ is denoted by K_{sp} and its molar solubility is denoted by S, then which of the following relation between S and K_{sp} is correct?

(1) $S=\left(\frac{K_{sp}}{929}\right)^{\frac{1}{9}}$ (2) $S=\left(\frac{K_{sp}}{216}\right)^{\frac{1}{7}}$

(3) $S=\left(\frac{K_{sp}}{144}\right)^{\frac{1}{6}}$ (4) $S=\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$

Answer (4)

Sol. $Zr_3(PO_4)_4 \rightleftharpoons \underset{3S}{3Zr^{+4}} + \underset{4S}{4PO_4^{3-}}$

$$K_{sp}=\left[Zr^{+4}\right]^3\left[PO_4^{3-}\right]^4=(3S)^3(4S)^4$$

K_{sp} = 6912 S^7

$$S=\left(\frac{K_{sp}}{6912}\right)^{1/7}$$

19. In order to oxidise a mixture of one mole of each of FeC_2O_4, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is

(1) 1.5

(2) 2

(3) 3

(4) 1

Sol. $5e + MnO_4^- \longrightarrow Mn^{+2}$

(i) $FeC_2O_4 \longrightarrow Fe^{3+} + 2CO_2 + 3e$

1 mole of FeC_2O_4 react with $\frac{3}{5}$ moles of acidified $KMnO_4$

(ii) $Fe_2(C_2O_4)_3 \longrightarrow Fe^{3+} + CO_2 + 6e$

1 mole of $Fe_2(C_2O_4)_3$ react with $\frac{6}{5}$ moles of $KMnO_4$

(iii) $FeSO_4 \longrightarrow Fe^{3+} + e$

1 mole of $FeSO_4$ react with $\frac{1}{5}$ moles of $KMnO_4$

$\therefore$ Total moles required = $\frac{3}{5}+\frac{6}{5}+\frac{1}{5}$ = 2

20. 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is

(molar mass of calcium bicarbonate is 162 g mol^{-1} and magnesium bicarbonate is 146 g mol^{-1})

(1) 5,000 ppm

(2) 100 ppm

(3) 10,000 ppm

(4) 1,000 ppm

Answer (3)

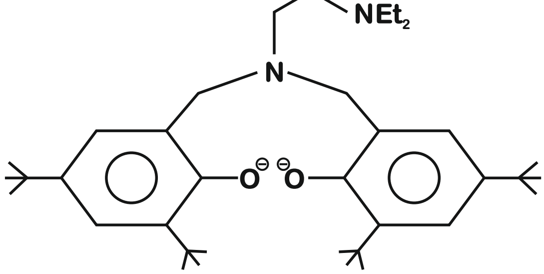
Sol. Moles of $Ca(HCO_3)_2$ = 0.005

Moles of $Mg(HCO_3)_2$ = 0.005

Hardness in terms of $CaCO_3$ pm

$$= \frac{(0.005+0.005)\times 100}{100}\times 10^6$$

= 10^4 ppm



(1) Tetradentate (2) Tridentate

(3) Bidentate (4) Hexadentate

Answer (1)

Sol. It has four lone pairs but maximum it will be able to donate three lone pairs.

Maximum denticity is 3.

22. Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively:

(1) H_3BO_3 and B_2O_3 (2) HBO_2 and H_3BO_3

(3) B_2O_3 and H_3BO_3 (4) B_2O_3 and $[BH_4]^-$

Answer (3)

Sol. $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

$B_2H_6 + 6H_2O \longrightarrow 2H_3BO_3 + 6H_2$

23. The size of the iso-electronic species Cl^-, Ar and Ca^{2+} is affected by

(1) Nuclear charge

(2) Principal quantum number of valence shell

(3) Azimuthal quantum number of valence shell

(4) Electron-electron interaction in the outer orbitals

Answer (1)

Sol. Iso-electronic species differ in size due to different effective nuclear charge.

24. The IUPAC name of the following compound is:

$$H_3C-\underset{|}{\overset{CH_3}{CH}}-\overset{OH}{\underset{|}{CH}}-CH_2-COOH$$

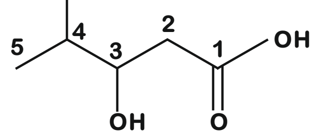
(1) 3-Hydroxy-4-methylpentanoic acid

(2) 4-Methyl-3-hydroxypentanoic acid

(3) 2-Methyl-3-hydroxypentan-5-oic acid

(4) 4,4-Dimethyl-3-hydroxybutanoic acid

Answer (1)



3-Hydroxy-4-methyl pentanoic acid

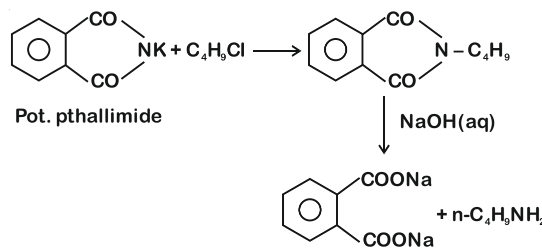
25. Which of the following amines can be prepared by Gabriel phthalimide reaction?

(1) Neo-pentylamine (2) n-butylamine

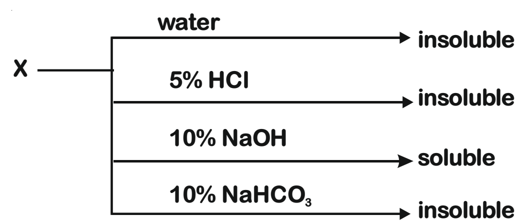
(3) t-butylamine (4) Triethylamine

Answer (2)

Sol. Primary amines are prepared by Gabriel pthallimide synthesis



26. An organic compound 'X' showing the following solubility profile is

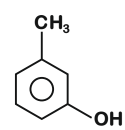


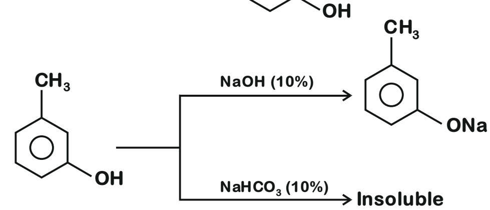
(1) Benzamide (2) Oleic acid

(3) o-Toluidine (4) m-Cresol

Answer (4)

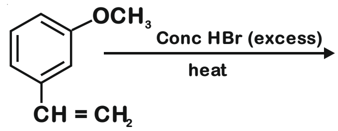
Sol. X is Meta cresol

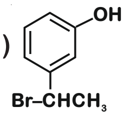


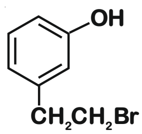
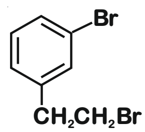


Benzamide is amphoteric

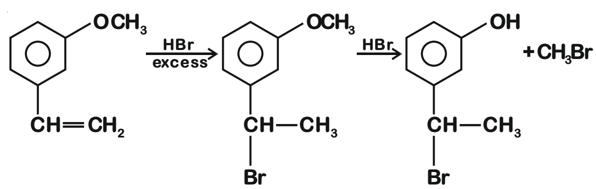
Oleic acid will dissolve in NaOH as well as $NaHCO_3$ due to acidic nature.



(1)  (2) 

(3)  (4) 

Answer (1)

Sol. 

28. Assertion : Ozone is destroyed by CFCs in the upper stratosphere.

    Reason : Ozone holes increase the amount of UV radiation reaching the earth.

    (1) Assertion and reason are both correct, and the reason is the correct explanation for the assertion.

    (2) Assertion is false, but the reason is correct.

    (3) Assertion and reason are correct, but the reason is not the explanation for the assertion.

    (4) Assertion and reason are incorrect.

Answer (3)

Sol. CFC's are responsible for depletion of ozone layer

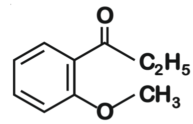
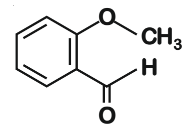
$$CF_2Cl_2 \xrightarrow{uv} \dot{C}l_{(g)} + F_2\dot{C}Cl$$

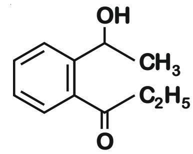
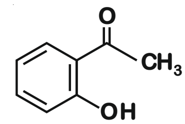
$$\dot{C}l + O_3 \longrightarrow ClO\!\!\!\dot{}\;\; + O_2$$

$$Cl\dot{O} + O \longrightarrow \dot{C}l + O_2$$

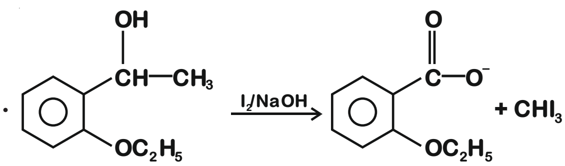
Both statements are correct.

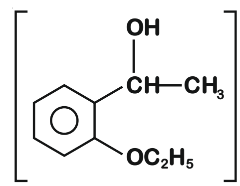
Fehling solution. It however, reacts with Grignard reagent and gives positive iodoform test. The compound is

(1)  (2) 

(3)  (4) 

Answer (3)

Sol. 



→ Reacts with Grignard's reagent due to acidic hydrogen.

→ Fehling solution test is negative as there is no – CHO group.

→ Neutral $FeCl_3$ test is negative as there is no phenolic group

30. In the following compounds, the decreasing order of basic strength will be

    (1) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

    (2) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$

    (3) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$

    (4) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$

Answer (4)

Sol. Correct order of K_b value

$$(C_2H_5)_2\ddot{N} > (C_2H_5)_3N > NH_3$$

In aqueous medium sec. amines are most basic.

3°amines are more basic than NH_3 as +I factor dominate over steric factor.

1. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is :

(1) $\frac{11}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) 2 (4) $\frac{7}{4\sqrt{2}}$

Answer (4)

Sol. The shortest distance between line y = x and parabola = the distance between line y = x and tangent of parabola having slope 1.

Let equation of tangent of parabola having slope 1 is,

$y = m(x-2) + \frac{a}{m}$

where m = 1 and $a = \frac{1}{4}$

Equation of tangent $y = x - \frac{7}{4}$

Distance between the line y = x and the tangent

$$= \left| \frac{\frac{7}{4} - 0}{\sqrt{1^2 + 1^2}} \right| = \frac{7}{4\sqrt{2}}$$

2. The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6, (x > 1)$ is equal to :

(1) 24 (2) 32

(3) 26 (4) 29

Answer (1)

Sol. $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$$= 2\left[{}^6C_0 x^6 + {}^6C_2 x^4 (x^3 - 1) + {}^6C_4 x^2 (x^3 - 1)^2 + {}^6C_6 (x^3 - 1)^3\right]$$

$= 2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1]$

Sum of coefficients of even powers of x = 2[1 – 15 + 15 + 15 – 3 – 1] = 24

3. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25, x \in R$, then :

(1) $S_1 = \{-2\}; S_2 = \{0, 1\}$

(2) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(3) $S_1 = \{-1\}; S_2 = \{0, 2\}$

(4) $S_1 = \{-2, 0\}; S_2 = \{1\}$

Answer (2)

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 72$

$f'(x) = 36[x^3 + x^2 - 2x] = 36x(x - 1)(x + 2)$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -2 & & 0 & & 1 & \end{array}$$

Whenever derivative changes sign from negative to positive, we get local minima, and whenever derivative changes sign from positive to negative, we get local maxima (while moving left to right on x-axis)

$S_1 = \{-2, 1\}$

$S_2 = \{0\}$

4. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2, x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to :

(1) $2x - \frac{\pi}{3}$ (2) $x - \frac{\pi}{6}$

(3) $\frac{\pi}{3} - x$ (4) $\frac{\pi}{6} - x$

Answer (2)

Sol. $$2y = \left[\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x}\right)\right]^2$$

$$\Rightarrow 2y = \left[\cot^{-1}\left(\frac{\cos\left(\frac{\pi}{6} - x\right)}{\sin\left(\frac{\pi}{6} - x\right)}\right)\right]^2$$

$$\Rightarrow 2y=\begin{cases}\left(\frac{7\pi}{6}-x\right)^2 & \text{if } \frac{\pi}{6}-x\in\left(-\frac{\pi}{3},0\right)\\ \left(\frac{\pi}{6}-x\right)^2 & \text{if } \frac{\pi}{6}-x\in\left(0,\frac{\pi}{6}\right)\end{cases}$$

$$\Rightarrow \frac{dy}{dx}=\begin{cases}x-\frac{7\pi}{6} & \text{if } x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)\\ x-\frac{\pi}{6} & \text{if } x\in\left(0,\frac{\pi}{6}\right)\end{cases}$$

Note: Only one given option is correct.

5. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :

(1) 4^{th} quadrant

(2) 1^{st} quadrant

(3) 1^{st}, 2^{nd} and 4^{th} quadrants

(4) 1^{st} and 2^{nd} quadrants

Answer (4)

Sol. A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$.

As it is given that the point lies on the line $3x + 5y = 15$

So the required point is :

$$\begin{array}{l} 3x+5y=15\\ \underline{x+y=0}\\ x=-\frac{15}{2},\quad y=\frac{15}{2}\Rightarrow\left(\frac{15}{2},\frac{15}{2}\right)\left\{2^{nd}\text{ quadrant}\right\}\end{array}$$

$$\begin{array}{l}\text{or } 3x+5y=15\\ \underline{x=y}\\ x=\frac{15}{8},\quad y=\frac{15}{8}\Rightarrow\left(\frac{15}{8},\frac{15}{8}\right)\left\{1^{st}\text{ quadrant}\right\}\end{array}$$

6. $\displaystyle\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration)

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

Answer (3)

Sol. $\displaystyle\int\frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}dx$

$$=\int\frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}}dx$$

$$=\int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

Now use $\sin 2x = 2\sin x\cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$

$$=\int(1+2\cos x+2\cos 2x)dx$$

$= x + 2\sin x + \sin 2x + c$

7. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{4}{17}$ (4) $\frac{128}{17}$

Answer (2)

Sol. Equation of tangent at $A(1, 2)$;

$4x + 2y = 8 \Rightarrow 2x + y = 4$

So tangent at $B(a, b)$ can be assumed as

$x - 2y = c \quad\Rightarrow\quad y=\frac{1}{2}x-\frac{c}{2}$

Condition for tangency;

$$-\frac{c}{2}=\pm\sqrt{2\left(\frac{1}{2}\right)^2+8}=\pm\sqrt{\frac{17}{2}}$$

$$\Rightarrow c=\pm\sqrt{34}$$

Equation of tangent; $x - 2y = \pm\sqrt{34}$ ...(i)

Equation of tangent at $P(a, b)$; $4ax + by = 8$...(ii)

Comparing both the equations ;

$$\frac{4a}{1}=\frac{b}{-2}=\frac{8}{\pm\sqrt{34}}$$

$$\Rightarrow a=\pm\frac{2}{\sqrt{34}}\quad\Rightarrow\quad a^2=\frac{2}{17}$$

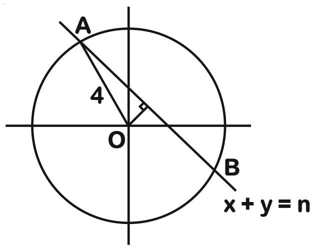
by the lines, x + y = n, $n \in N$, where N is the set of all natural numbers, is

(1) 105 (2) 160

(3) 320 (4) 210

Answer (4)

Sol.



Let the chord x + y = n cuts the circle $x^2 + y^2 = 16$ at A and B length of perpendicular from

$$O \text{ on } AB = \left|\frac{0+0-n}{\sqrt{1^2+1^2}}\right| = \frac{n}{\sqrt{2}}$$

$$\text{Length of chord } AB = 2\sqrt{4^2 - \left(\frac{n}{\sqrt{2}}\right)^2}$$

$$= 2\sqrt{16 - \frac{n^2}{2}}$$

Here possible values of n are 1, 2, 3, 4, 5.

Sum of square of length of chords

$$= \sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right)$$

$$= 64 \times 5 - 2.\frac{5 \times 6 \times 11}{6} = 210$$

9. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

(1) 2f(x) (2) $2f(x^2)$

(3) –2f(x) (4) $(f(x))^2$

Answer (1)

Sol. $\because \quad f(x) = \ln\left(\frac{1-x}{1+x}\right)$

$$f\left(\frac{2x}{1+x^2}\right) = \ln\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+2x^2}}\right)$$

$$\left(1+x^2+2x\right)$$

$$= \ln\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\ln\left(\frac{1-x}{1+x}\right)$$

$$= 2f(x)$$

10. Let y = y(x) be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx} + 2x(x^2+1)y = 1$ such that y(0) = 0. If $\sqrt{a}\, y(1) = \frac{\pi}{32}$, then the value of ‘a’ is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 1 (4) $\frac{1}{16}$

Answer (4)

Sol. $(1+x^2)\frac{2dy}{dx} + 2x(1+x^2)y = 1$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{1}{(1+x^2)^2}$$

It is a linear differential equation

$$I{\cdot}F{\cdot} = e^{\int\frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1+x^2$$

$$\Rightarrow y{\cdot}(1+x^2) = \int\frac{dx}{1+x^2} + c$$

$\Rightarrow$ y $(1 + x^2) = \tan^{-1} x + c$

If x = 0 then y = 0

So, 0 = 0 + c

$\Rightarrow$ c = 0

$\Rightarrow$ y $(1 + x^2) = \tan^{-1} x$

put x = 1

$$2y = \frac{\pi}{4}$$

$$\Rightarrow 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \sqrt{a} = \frac{1}{4}$$

$$\Rightarrow a = \frac{1}{16}$$

$\beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to

(1) $\tan^{-1}\left(\frac{9}{14}\right)$ (2) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(3) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Answer (3)

Sol. $\because \cos\alpha = \frac{3}{5}$

$\Rightarrow \tan\alpha = \frac{4}{3}$

$\Rightarrow$ and $\tan\beta = \frac{1}{3}$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\cdot\tan\beta}$$

$$= \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{9}} = \frac{1}{\frac{13}{9}}$$

$$= \frac{9}{13}$$

$$\alpha-\beta = \tan^{-1}\left(\frac{9}{13}\right) = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right) = \cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$$

12. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is

(1) Decreasing on (0, 2)

(2) Increasing on (0, 2)

(3) Decreasing on (0, 1) and increasing on (1, 2)

(4) Increasing on (0, 1) and decreasing on (1, 2)

Answer (3)

Sol. $\phi(x) = f(x) + f(2 - x)$

differentiating w.r.t. x

$\phi'(x) = f'(x) - f'(2 - x)$

For $\phi(x)$ to be increasing $\phi'(x) > 0$

$\Rightarrow f'(x) > f'(2 - x)$

($\because f''(x) > 0$ then $f'(x)$ is an increasing function)

$\Rightarrow x > 2 - x$

$\Rightarrow x > 1$

So $\phi(x)$ is increasing in (1, 2) and decreasing in (0, 1)

(1) 2^{23} (2) 2^{25}

(3) 2^{24} (4) 2^{26}

Answer (2)

Sol. $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + \ldots\ldots 62.{}^{20}C_{20}$

$$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r = 3\sum_{r=0}^{20} r.\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$$

$$= 60\sum_{r=1}^{20}{}^{19}C_{r-1} + 2\sum_{r=0}^{20}{}^{20}C_r$$

$= 60 \times 2^{19} + 2 \times 2^{20}$

$= 2^{21}[15 + 1] = 2^{25}$

14. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

(1) 180 (2) 175

(3) 162 (4) 160

Answer (1)

Sol. There are total 9 digits; out of which only 3 digits are odd.

$$\_\ \underset{\uparrow}{\_}\ \_\ \underset{\uparrow}{\_}\ \_\ \underset{\uparrow}{\_}\ \_\ \underset{\uparrow}{\_}\ \_$$

Number of ways to arrange odd digits first

$$= {}^4C_3\cdot\frac{\lfloor 3}{\lfloor 2}$$

Total number of 9 digit numbers $= \left({}^4C_3\cdot\frac{\lfloor 3}{\lfloor 2}\right)\cdot\frac{\lfloor 6}{\lfloor 2\,\lfloor 4}$

$= 180$

15. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is :

(1) –1 (2) 0

(3) 2 (4) $\frac{1}{2}$

Answer (4)

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow \quad (1-c^2)+c(-c-c^2)-c(c^2+c)=0$

$\Rightarrow \quad (1+c)(1-c)-2c^2(1+c)=0$

$\Rightarrow \quad (1+c)(1-c-2c^2)=0$

$\Rightarrow \quad (1+c)^2(1-2c)=0$

$\Rightarrow \quad c=-1 \text{ or } \dfrac{1}{2}$

16. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $8x^2-9y^2+9y=18$ (2) $9x^2+8y^2-8y=16$

(3) $9x^2-8y^2+8y=16$ (4) $8x^2+9y^2-9y=18$

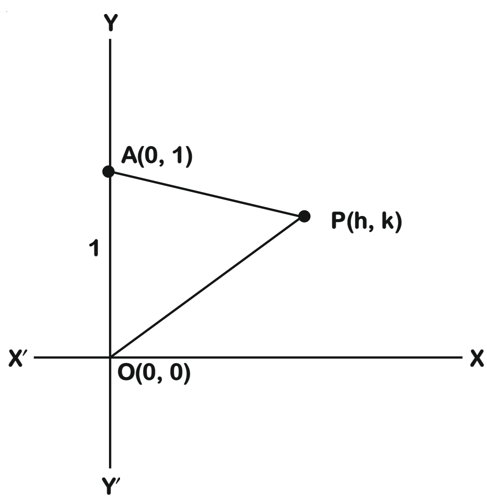
Answer (2)

Sol. Let point P(h, k)

$\because$ OA = 1

So, OP + AP = 3

$\sqrt{h^2+k^2}+\sqrt{h^2+(k-1)^2}=3$



$\Rightarrow \quad h^2+(k-1)^2=9+h^2+k^2-6\sqrt{h^2+k^2}$

$\Rightarrow \quad 6\sqrt{h^2+k^2}=2k+8$

$\Rightarrow \quad 9(h^2+k^2)=k^2+16+8k$

$\Rightarrow \quad 9h^2+8k^2-8k-16=0$

Locus of point P will be,

$9x^2+8y^2-8y-16=0$

is always correct?

(1) $P(A|B)=P(B)-P(A)$

(2) $P(A|B)\le P(A)$

(3) $P(A|B)\ge P(A)$

(4) $P(A|B)=1$

Answer (3)

Sol. $\because$ $A\subset B$; so $A\cap B=A$

Now, $P\left(\dfrac{A}{B}\right)=\dfrac{P(A\cap B)}{P(B)}$

$\Rightarrow \quad P\left(\dfrac{A}{B}\right)=\dfrac{P(A)}{P(B)}$

$\because \quad P(B)\le 1$

So, $P\left(\dfrac{A}{B}\right)\ge P(A)$

18. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\dfrac{\alpha}{\beta}\right)^n=1$ is :

(1) 4 (2) 5

(3) 3 (4) 2

Answer (1)

Sol. $x^2-2x+2=0$

roots of this equation are $\dfrac{2\pm\sqrt{-4}}{2}=1\pm i$

Then $\dfrac{\alpha}{\beta}=\dfrac{1+i}{1-i}=\dfrac{(1+i)^2}{1-i^2}=i$

or $\dfrac{\alpha}{\beta}=\dfrac{1-i}{1+i}=\dfrac{(1-i)^2}{1-i^2}=-i$

So, $\dfrac{\alpha}{\beta}=\pm i$

Now, $\left(\dfrac{\alpha}{\beta}\right)^n=1 \Rightarrow (\pm i)^n=1$

$\Rightarrow$ n must be a multiple of 4.

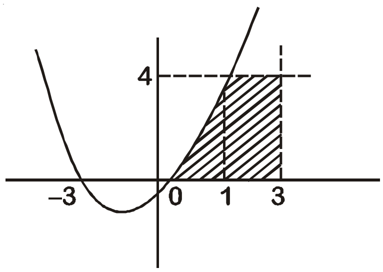
minimum value of n = 4

(1) $\dfrac{26}{3}$ (2) $\dfrac{59}{6}$

(3) 8 (4) $\dfrac{53}{6}$

Answer (2)

Sol. $y \le x^2 + 3x$ represents region below the parabola.



Area of the required region

$$= \int_0^1 (x^2 + 3x)dx + \int_1^3 4 \cdot dx$$

$$= \frac{1}{3} + \frac{3}{2} + 8$$

$$= \frac{59}{6}$$

20. If $f(x) = \dfrac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\displaystyle\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is :

(1) $\log_e 1$ (2) $\log_e 3$

(3) $\log_e 2$ (4) $\log_e e$

Answer (1)

Sol. $g(f(x)) = \ln\left(\dfrac{2 - x\cos x}{2 + x\cos x}\right)$

Let $I = \displaystyle\int_{-\pi/4}^{\pi/4} \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx$ ...(i)

$\left(\text{Using property } \displaystyle\int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right)$

$I = \displaystyle\int_{-\pi/4}^{\pi/4} \ln\left(\frac{2 + x\cos x}{2 - x\cos x}\right)dx$ ...(ii)

Adding (i) and (ii)

$$2I = \int_{-\pi/2}^{\pi/2} \ln(1)\,dx = 0$$

$\Rightarrow$ $I = 0 = \ln 1$

$y + 2z - 4 = 0$ and passing through the point $(1, 1, 0)$ is :

(1) $2x - z = 2$ (2) $x - 3y - 2z = -2$

(3) $x - y - z = 0$ (4) $x + 3y + z = 4$

Answer (3)

Sol. Let the equation of required plane be;

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

$\because$ This plane passes through $(1, 1, 0)$ then

$(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0 \Rightarrow \lambda = -1$

Equation of required plane will be

$(2x - y - 4) - (y + 2z - 4) = 0$

$\Rightarrow$ $2x - 2y - 2z = 0$

$\Rightarrow$ $x - y - z = 0$

22. If $\cos(\alpha + \beta) = \dfrac{3}{5}$, $\sin(\alpha - \beta) = \dfrac{5}{13}$ and $0 < \alpha, \beta < \dfrac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

(1) $\dfrac{21}{16}$ (2) $\dfrac{63}{52}$

(3) $\dfrac{33}{52}$ (4) $\dfrac{63}{16}$

Answer (4)

Sol. $\because$ $\alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$$\cos(\alpha + \beta) = \frac{3}{5}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{4}{3}$$

And $\sin(\alpha - \beta) = \dfrac{5}{13}$

$$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

Now, $\tan 2\alpha = \tan\left((\alpha + \beta) + (\alpha - \beta)\right)$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$$

$$= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$$

23. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, $(\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is :

(1) $\dfrac{\pi}{32}$ (2) $\dfrac{\pi}{64}$

(3) 0 (4) $\dfrac{\pi}{16}$

Answer (2)

$\sin\alpha \qquad \cos\alpha$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

Then $A^4 = A^2.A^2 = \begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix}$

similarly $A^8 = A^4.A^4 \begin{bmatrix} \cos 8\alpha & -\sin 8\alpha \\ \sin 8\alpha & \cos 8\alpha \end{bmatrix}$

and so on $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow \quad 32\alpha = 2n\pi + \frac{\pi}{2} \Rightarrow \alpha = \frac{n\pi}{16} + \frac{\pi}{64}$ where $n \in Z$

put n = 0, $\alpha = \frac{\pi}{64}$

24. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is:

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not born in India, then you are not a citizen of India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not a citizen of India, then you are not born in India.

Answer (4)

Sol. S: "If you are born in India, then you are a citizen of India."

Contrapositive of $p \to q$ is $\sim q \to \sim p$

So contrapositive of statement S will be :

"If you are not a citizen of India, then you are not born in India."

25. The sum of the solutions of the equation $\left|\sqrt{x} - 2\right| + \sqrt{x}\left(\sqrt{x} - 4\right) + 2 = 0, (x > 0)$ is equal to:

(1) 4	(2) 10
(3) 9	(4) 12

Answer (2)

$|t - 2| + t(t - 4) + 2 = 0$

$\Rightarrow |t - 2| + t^2 - 4t + 4 - 2 = 0$

$\Rightarrow |t - 2| + (t - 2)^2 - 2 = 0$

Let $|t - 2| = z$ (Clearly $z \geq 0$)

$\Rightarrow z + z^2 - 2 = 0$

$\Rightarrow z = 1$ or -2 (rejected)

$\Rightarrow |t - 2| = 1 \Rightarrow t = 1, 3$

If $\sqrt{x} = 1 \Rightarrow x = 1$

If $\sqrt{x} = 3 \Rightarrow x = 9$

Sum of solutions = 10

26. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is:

(1) $\sqrt{\frac{3}{2}}$	(2) $3\sqrt{6}$
(3) $\frac{\sqrt{3}}{2}$	(4) $\sqrt{6}$

Answer (1)

Sol. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ vector perpendicular to $\bar{a}$ and $\bar{b}$ is $\bar{a} \times \bar{b}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

Projection of vector $\bar{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ on $\bar{a} \times \bar{b}$ is

$$= \left|\frac{\bar{c}.(\bar{a} \times \bar{b})}{|\bar{a} \times \bar{b}|}\right| = \left|\frac{2 - 6 + 1}{\sqrt{6}}\right|$$

$$= \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

27. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is:

(1) 45	(2) 40
(3) 48	(4) 49

Answer (3)

Mean $(\bar{x}) = \frac{\sum x_i}{N} = \frac{2+4+10+12+14+x+y}{7} = 8$

$\Rightarrow x + y = 14$ ....(i)

Variance $(\sigma^2) = \frac{\sum x_i^2}{N} - (\bar{x})^2 = 16$

$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+x^2+y^2}{7} - (8)^2 = 16$

$\Rightarrow x^2 + y^2 = 100$ ....(ii)

From (i) and (ii) (x, y) = (6, 8) or (8, 6)

xy = 48

28. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals:

(1) $\sqrt{2}$ (2) $2\sqrt{2}$

(3) 4 (4) $4\sqrt{2}$

Answer (4)

Sol. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}} = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{2\cos^2 \frac{x}{2}}}$ $\left[\frac{0}{0}\right]$

$= \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}\left[1-\cos\frac{x}{2}\right]}$

$= \lim_{x\to 0} \frac{\sin^2 x}{2\sqrt{2}\sin^2\frac{x}{4}}$

$= \lim_{x\to 0} \frac{\left(\frac{\sin x}{x}\right)^2 .16}{2\sqrt{2}\left(\frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2}$

$= \frac{16}{2\sqrt{2}} = 4\sqrt{2}$

29. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :

(1) 3303 (2) 3121

(3) 3203 (4) 3221

Answer (2)

So the required numbers are either divisible by 7 or 13

Sum of such numbers = Sum of no. divisible by 7 + sum of the no. divisible by 13 – Sum of the numbers divisible by 91

= (105 + 112 + ... + 196) + (104 + 117 + ... + 195) – 182

= 2107 + 1196 – 182

= 3121

30. The length of the perpendicular from the point (2, –1, 4) on the straight line,

$\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :

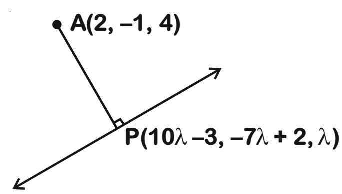
(1) Greater than 3 but less than 4

(2) Greater than 2 but less than 3

(3) Greater than 4

(4) Less than 2

Answer (1)

Sol. 

Let P be the foot of perpendicular from point A(2, –1, 4) on the given line. So P can be assumed as P(10λ – 3, –7λ + 2, λ)

DR's of AP∝ to 10λ – 5, –7λ + 3, λ – 4

∵ AP and given line are perpendicular, so

10(10λ – 5) – 7(–7λ + 3) + 1(λ – 4) = 0

$\Rightarrow \lambda = \frac{1}{2}$

$AP = \sqrt{(10\lambda-5)^2+(-7\lambda+3)^2+(\lambda-4)^2}$

$= \sqrt{0+\frac{1}{4}+\frac{49}{4}}$

$= \sqrt{12.5}\,;\ \sqrt{12.5} \in (3, 4)$

❑ ❑ ❑

Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :

(1) $R = 1000\ r$ (2) $R = r$

(3) $R = 2r$ (4) $R = 0.001\ r$

Answer (2)

Sol. For maximum power in external resistance, Internal resistance = External resistance

$\Rightarrow R = r$

2. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.

If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - v_2$

(3) $v_4 + v_2$ (4) $v_4 - \frac{v_2}{2}$

Answer (2)

Sol. $m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$

$m_1v_1 + 0.5m_1v_2 = 0.5m_1v_1 + 0.5m_1v_4$

$v_1 = v_4 - v_2$

3. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be :

(Radius of the Earth = 6.4×10^6 m).

(1) 20 m (2) 51 m

(3) 32 m (4) 40 m

Answer (3)

Sol. $\sqrt{2\times 70\times R_E} + \sqrt{2\times h_R\times R_E} = 50\times 10^3$

Putting $R_E = 6.4 \times 10^6$ m and solving we get

$h_R = 32$ m

4. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = – 5 is V_2, then $V_1 - V_2$ is :

(1) 180 V (2) –520 V

(3) 320 V (4) –48 V

Answer (1)

Sol. $dV = -\vec{E}\cdot\vec{dr} = -(Ax + B)dx$

$$\int_{V_2}^{V_1} dV = \int_{-5}^{1} -(Ax + B)dx$$

$$V_1 - V_2 = \left(-A\frac{x^2}{2} - Bx\right)_{-5}^{1}$$

$$= \left(-\frac{A}{2} - B\right) + \left(\frac{A}{2}25 + B(-5)\right)$$

$= 12A - 6B = 240 - 60 = 180$ V

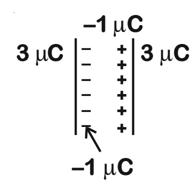
5. A parallel plate capacitor has 1 μF capacitance. One of its two plates is given +2 μC charge and the other plate, +4 μC charge. The potential difference developed across the capacitor is :

(1) 5 V (2) 1 V

(3) 3 V (4) 2 V

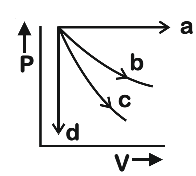
Answer (2)

Sol. $V = \frac{q}{C} = \frac{1\mu C}{1\mu F}$



= 1 V

6. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :

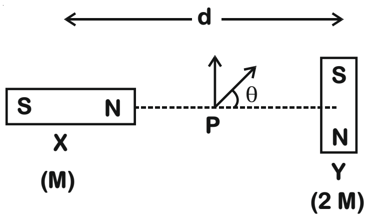


(1) a d c b (2) a d b c

(3) d a b c (4) d a c b

Answer (3)

7. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle θ = 45° with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)



(1) 0

(2) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3}\times qv$

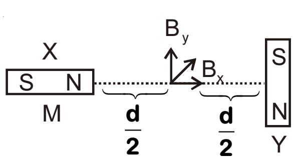
(3) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3}\times qv$

(4) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3}\times qv$

Answer (1)

Sol. $\vec{F}_m = q(\vec{V}\times\vec{B})$

$\vec{B} = \vec{B}_x + \vec{B}_y$



Since $M_y = 2M_x$

$\Rightarrow |\vec{B}_x| = |\vec{B}_y|$

$\vec{B}_{net}$ is parallel to $\vec{V}$

$\Rightarrow \vec{F} = 0$

drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to:

(1) 100 s

(2) 10 s

(3) 50 s

(4) 20 s

Answer (4)

Sol. Time for 10 oscillations = $\frac{10}{5} = 2$ s

$A = A_0 e^{-kt}$

$\frac{1}{2} = e^{-2k} \quad \Rightarrow \quad \ln 2 = 2k$

$10^{-3} = e^{-kt} \quad \Rightarrow \quad 3\ln 10 = kt$

$t = \frac{3\ln 10}{k} = \frac{3\ln 10}{\ln 2}\times 2$

$= 6\times\frac{2.3}{0.69} \approx 20$ s

9. If Surface tension (S), Moment of Inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be:

(1) $S^{1/2}I^{3/2}h^{-1}$

(2) $S^{3/2}I^{1/2}h^{0}$

(3) $S^{1/2}I^{1/2}h^{-1}$

(4) $S^{1/2}I^{1/2}h^{0}$

Answer (4)

Sol. $[p] = MLT^{-1} = [I^x h^y S^z]$

$= M^xL^{2x}(ML^2T^{-1})^y(MT^{-2})^z$

$= M^{x+y+z}L^{2x+2y}T^{-y-2z}$

$x + y + z = 1$

$2(x + y) = 1 \quad \Rightarrow \quad x + y = \frac{1}{2} \quad \Rightarrow z = \frac{1}{2}$

$y + 2z = 1 \quad \Rightarrow \quad y = 0 \quad \Rightarrow x = \frac{1}{2}$

$[p] = \sqrt{IS}$

difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this?

(1) RL circuit with R = 1 kΩ and L = 10 mH

(2) RL circuit with R = 1 kΩ and L = 1 mH

(3) RC circuit with R = 1 kΩ and C = 10 μF

(4) RC circuit with R = 1 kΩ and C = 1 μF

Answer (3)

Sol. As $\phi = \frac{\pi}{4}$, $x_c = R$

11. The magnetic field of an electromagnetic wave is given by:

$$\vec{B} = 1.6\times10^{-6}\cos(2\times10^{7}z+6\times10^{15}t)(2\hat{i}+\hat{j})\frac{\text{Wb}}{\text{m}^2}$$

The associated electric field will be:

(1) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z+6\times10^{15}t\right)(-\hat{i}+2\hat{j})\frac{V}{m}$

(2) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z-6\times10^{15}t\right)(2\hat{i}+\hat{j})\frac{V}{m}$

(3) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z-6\times10^{15}t\right)(-2\hat{j}+\hat{i})\frac{V}{m}$

(4) $\vec{E} = 4.8\times10^{2}\cos\left(2\times10^{7}z+6\times10^{15}t\right)(\hat{i}-2\hat{j})\frac{V}{m}$

Answer (1)

Sol. Amplitude of electric field, $E = B_0C$

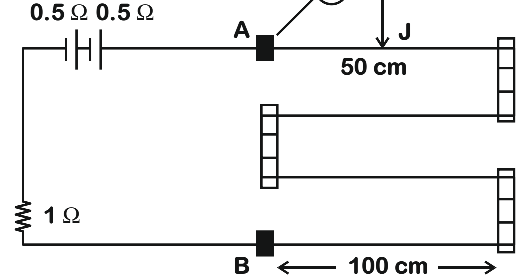
$$= 1.6\times10^{-6}\times\sqrt{5}\times3\times10^{8}$$

$$= 4.8\times10^{2}\sqrt{5}\ \text{V/m}$$

Also $\vec{E}\times\vec{B}$ is along $-\hat{k}$ (the direction of propagation)

$$\Rightarrow\ \vec{E} = 4.8\times10^{2}\cos(2\times10^{7}z+6\times10^{15}t)(-\hat{i}+2\hat{j})\frac{V}{m}$$

12. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be:



(1) 0.75 V (2) 0.50 V

(3) 0.20 V (4) 0.25 V

Answer (4)

Sol. Resistance of potentiometer wire, $R_p = 400 \times 0.01$

$= 4\ \Omega$

$\Rightarrow\ I = \frac{3}{6} = 0.5\ \text{A}$

$\Rightarrow$ Reading of voltmeter = $I R_{AJ}$

$= 0.5 \times 50 \times 0.01$

$= 0.25$ V

13. A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelength λ_B and λ_C of B and C are respectively:

(1) $2\lambda_A, \lambda_A$ (2) $\lambda_A, \frac{\lambda_A}{2}$

(3) $\frac{\lambda_A}{2}, \lambda_A$ (4) $\lambda_A, 2\lambda_A$

Answer (3)

Sol. $\lambda_A = \frac{h}{mV_A}$

Conservation of linear momentum

$$\Rightarrow\ mV_A = \frac{m}{2}V - \frac{m}{2}\cdot\frac{V}{2} = \frac{mV}{4}$$

$$\Rightarrow\ \lambda_A = \frac{4h}{mV}$$

$$\therefore\ V_A = \frac{V}{4}$$

$$\lambda_B = \frac{h}{\frac{m}{2}V} = \frac{2h}{mV} = \frac{\lambda_A}{2}$$

$$\lambda_c = \frac{h}{\frac{m}{2}\cdot\frac{V}{2}} = \frac{4h}{mV} = \lambda_A$$

energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

(1) $\frac{E}{64}$ (2) $\frac{E}{4}$

(3) $\frac{E}{16}$ (4) $\frac{E}{32}$

Answer (3)

Sol. $E = \frac{GM_E m}{R_E}$

$$E' = \frac{GM_m m}{R_M}$$

$$\rho R_E^3 = 64\rho R_M^3$$

$\Rightarrow$ $R_E = 4\ R_M$

$$\frac{E'}{E} = \frac{M_M}{M_E} \cdot \frac{R_E}{R_M} = \frac{1}{64} . 4 = \frac{1}{16}$$

$$\Rightarrow \quad E' = \frac{E}{16}$$

15. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to:

(1) 6.8%

(2) 0.2%

(3) 3.5%

(4) 0.7%

Sol. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = 4\pi^2 \frac{l}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} = \left(\frac{0.1}{55} + \frac{2\times 1}{30}\right)\times 100$$

$\approx 6.8\%$

16. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is:

(1) $\sqrt{\frac{2qE}{md}}$

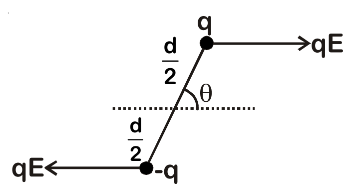
(2) $2\sqrt{\frac{qE}{md}}$

(3) $\sqrt{\frac{qE}{2md}}$

(4) $\sqrt{\frac{qE}{md}}$

Answer (1)

Sol.

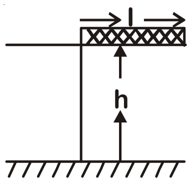


$$-Eqd\theta = I\frac{d^2\theta}{dt^2} = 2\frac{md^2}{4} \cdot \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} - \frac{2Eq\theta}{md}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2Eq}{md}}$$

of a platform of height 5 m. When released, it slips off the table in a very short time τ = 0.01 s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to:



(1) 0.3

(2) 0.02

(3) 0.28

(4) 0.5

Answer (4)

Sol. Initial angular acceleration before it slips off

$$mg\frac{l}{2} = I\alpha$$

$$\alpha = \frac{3g}{2l}$$

$\therefore$ Angular speed acquire by the box in time τ = 0.01 s

$$\omega = \alpha t = \frac{3g}{2l} \times 0.01 = \frac{3 \times 10 \times 0.01}{2 \times 0.3} = \frac{1}{2} \text{ rad/sec}$$

$\therefore$ The angle by which it would rotate when hits the ground

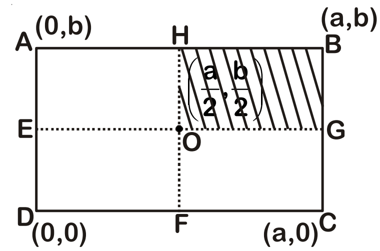
$\theta = \omega t'$

[Assuming ω = constant and t' = time of fall =

$$\sqrt{\frac{2H}{g}} = 1 \text{ sec}$$

$$\therefore \quad \theta = \frac{1}{2} \text{ radians}$$

in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be:



(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (2) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (4) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

Answer (2)

Sol. X- coordinate of CM of remaining sheet

$$X_{cm} = \frac{MX - mx}{M - m}$$

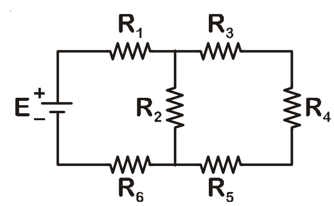
$$= \frac{(4m) \times \left(\frac{a}{2}\right) - M\left(\frac{3a}{4}\right)}{4m - m} = \frac{5a}{12}$$

Similarly $y_{cm} = \dfrac{5b}{12}$

$$\therefore \quad CM\left(\frac{5a}{12}, \frac{5b}{12}\right)$$

19. In the figure shown, what is the current (in Ampere) drawn from the battery? You are given :

R_1 = 15 Ω, R_2 = 10 Ω, R_3 = 20 Ω, R_4 = 5 Ω, R_5 = 25 Ω, R_6 = 30 Ω, E = 15 V



(1) $\frac{13}{24}$ (2) $\frac{9}{32}$

(3) $\frac{20}{3}$ (4) $\frac{7}{18}$

Sol. Equivalent resistance of the given circuit

$$R_{eq} = 45 + \frac{10 \times 50}{10+50} = \left(45 + \frac{50}{6}\right)\Omega = \frac{160}{3}\Omega$$

$$\therefore \quad I = \frac{15}{\frac{160}{3}} = \frac{9}{32}\text{A}$$

20. Let $|\overrightarrow{A_1}| = 3, |\overrightarrow{A_2}| = 5$ and $|\overrightarrow{A_1} + \overrightarrow{A_2}| = 5.$ The value of $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \cdot (3\overrightarrow{A_1} - 2\overrightarrow{A_2})$ is :

(1) – 106.5 (2) – 118.5

(3) – 99.5 (4) – 112.5

Answer (2)

Sol. $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \cdot (3\overrightarrow{A_1} - 2\overrightarrow{A_2}) = 6|\overrightarrow{A_1}|^2 + 5\overrightarrow{A_1} \cdot \overrightarrow{A_2} - 6|\overrightarrow{A_2}|^2$

$$= (6 \times 9) + 5\overrightarrow{A_1} \cdot \overrightarrow{A_2} - (6 \times 25) \quad \text{...(i)}$$

As $25 = 9 + 25 + 2\overrightarrow{A_1} \cdot \overrightarrow{A_2}$

$\Rightarrow \quad 2\overrightarrow{A_1} \cdot \overrightarrow{A_2} = -9$ ...(ii)

From (i) and (ii),

$$\Rightarrow \quad (2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \cdot (3\overrightarrow{A_1} - 2\overrightarrow{A_2}) = 54 - 22.5 - 150$$

$$= -118.5$$

21. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.

(1) 457.5×10^{-9} radian

(2) 305×10^{-9} radian

(3) 152.5×10^{-9} radian

(4) 610×10^{-9} radian

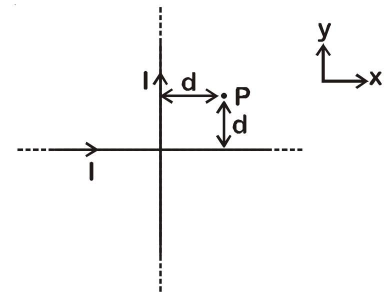
Answer (2)

Sol. $\theta = \dfrac{1.22\lambda}{D}$

$$\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{2}$$

$\theta = 305 \times 10^{-9}$ radian

xy-plane as shown in the figure.



These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be

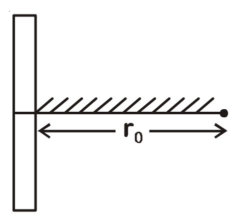
(1) Zero (2) $\dfrac{+\mu_0 I}{\pi d}(\hat{z})$

(3) $-\dfrac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$ (4) $\dfrac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$

Answer (1)

Sol. $\vec{B} = \overrightarrow{B_1} + \overrightarrow{B_2} = \dfrac{\mu_0 1}{2\pi d}\left[\hat{k} - \hat{k}\right] = 0$

23. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (ν) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :



(1) $\nu \propto \sqrt{\ln\left(\dfrac{r}{r_0}\right)}$

(2) $\nu \propto e^{+r/r_0}$

(3) $\nu \propto \ln\left(\dfrac{r}{r_0}\right)$

(4) $\nu \propto \left(\dfrac{r}{r_0}\right)$

Answer (1)

$2\pi\varepsilon_0 r$

$$\int_{V_G}^{V_P} dV = \int -\frac{\lambda}{2\pi\varepsilon_0 r} dr$$

$$\Rightarrow \quad V_P - V_G = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r}{r_0}$$

$$\frac{1}{2}mv^2 = q(V_P - V_G)$$

$$\Rightarrow \quad v \propto \left[\ln\left(\frac{r}{r_0}\right)\right]^{\frac{1}{2}}$$

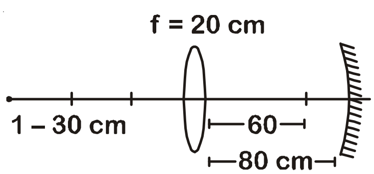
24. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :

(1) 30 cm (2) 25 cm

(3) 20 cm (4) 10 cm

Answer (4)

Sol.



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v} + \frac{1}{30} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{30-20}{20 \times 30} = \frac{10}{20 \times 30}$$

v = 60 cm

So clearly radius of curvature of mirror is 20 cm. Now if the object is placed within focal plane i.e. 10 cm then image formed by mirror is virtual.

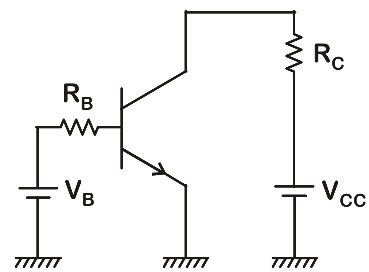
(1) 0.1 (2) 1

(3) 2 (4) 5

Answer (2)

Sol. Densities of nucleus happens to be constant, irrespective of mass number.

26. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, R_C = 1 kΩ and V_{CC} = 10 V. What is the minimum base current for V_{CE} to reach saturation?



(1) 10 μA (2) 100 μA

(3) 7 μA (4) 40 μA

Answer (4)

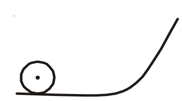
Sol. For saturation, $V_{CC} - i_c \times R_c = 0$

$$\Rightarrow \quad i_c = \frac{V_{CC}}{R_C} = \frac{10}{10^3} = 10^{-2} \text{ A}$$

$$\because \quad \beta = \frac{I_C}{I_B} = 250$$

$$\therefore \quad I_B = \frac{I_C}{250} = \frac{10^{-2}}{250} = 40\ \mu\text{A}$$

27. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by



(1) $\frac{4}{5}$ (2) $\frac{2}{\sqrt{5}}$

(3) 1 (4) $\frac{14}{15}$

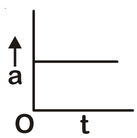
Sol. $mgh_{sph} = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\left(\frac{v}{R}\right)^2$

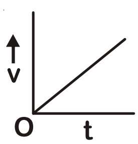
$$= \frac{7}{10}mv^2 \qquad ...(i)$$

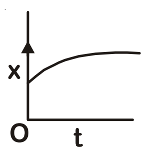
$$mgh_{cylinder} = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2$$

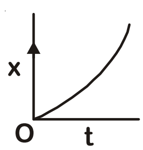
$$\Rightarrow \frac{h_{sph}}{h_{cylinder}} = \frac{7\times4}{10\times3} = \frac{14}{15}$$

28. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

(A) 

(B) 

(C) 

(D) 

(1) (A) (2) (A), (B), (C)

(3) (A), (B), (D) (4) (B), (C)

Answer (3)

Sol. a = Constant

v = at

$x = \frac{1}{2}at^2$ [Particle starts from the origin]

A, B and D are correct graphs.

Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to

(1) 1.3 mm (2) 1.9 mm

(3) 1.5 mm (4) 1.7 mm

Answer (4)

Sol. $\Delta L = \frac{FL}{YA}$

$$\Rightarrow \frac{L_A}{Y_A r_A^2} = \frac{L_B}{Y_B r_B^2}$$

$$\Rightarrow r_A^2 = \sqrt{\frac{L_A}{L_B}\cdot\frac{Y_B}{Y_A}}\cdot r_B$$

$$= \sqrt{\frac{2\times2\times4}{3\times7}}\times 2\text{ mm}$$

$$= \frac{4}{4.58}\times 2 = 1.7\text{ mm}$$

30. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to

[Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K

Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg

Radius of Earth : 6.4×10^6 m

Gravitational acceleration on Earth = 10 ms^{-2}]

(1) 10^4 K

(2) 650 K

(3) 800 K

(4) 3×10^5 K

Answer (1)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}} = 11.2\times10^3$ m/s

$$\Rightarrow T = \frac{M}{3R}\times(11.2\times10^3)^2$$

$$= \frac{2\times10^{-3}}{3\times8.3}\times125.44\times10^6 \simeq 10^4\text{ K}$$

1. 0.27 g of a long chain fatty acid was dissolved in 100 cm^3 of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?

[Density of fatty acid = 0.9 g cm^{-3}; π = 3]

(1) 10^{-8} m (2) 10^{-4} m

(3) 10^{-2} m (4) 10^{-6} m

Answer (4)

Sol. 0.27 gm in 100 ml of hexane

$\therefore$ in 10 ml of aqueous solution only 0.027 gm acid is present

volume of 0.027 g acid $= \frac{0.027}{0.9}$ ml

$\therefore \quad \pi r^2 h = \frac{0.027}{0.9}$ (given r = 10 cm, π = 3)

$\therefore$ h = 10^{-4} cm

= 10^{-6} m

2. The Mond process is used for the :

(1) purification of Zr and Ti

(2) extraction of Mo

(3) purification of Ni

(4) extraction of Zn

Answer (3)

Sol. Nickel is purified by Mond's process

$$\underset{\text{(impure)}}{Ni} + 4CO \xrightarrow{330-350\text{ k}} Ni(CO)_4$$

$$Ni(CO)_4 \xrightarrow{450-470\text{ k}} \underset{\text{(pure)}}{Ni} + 4CO$$

3. The correct statement about ICl_5 and ICl_4^- is :

(1) ICl_5 is square pyramidal and ICl_4^- is tetrahedral.

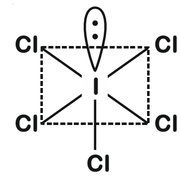
(2) both are isostructural.

(3) ICl_5 is square pyramidal and ICl_4^- is square planar.

(4) ICl_5 is trigonal bipyramidal and ICl_4^- is tetrahedral.

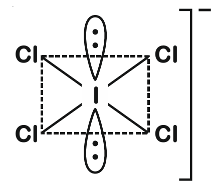
Answer (3)

Sol. ICl_5 is sp^3d^2 hybridised (5 bond pairs, 1 lone pair)



square pyramidal

ICl_4^- is sp^3d^2 hybridised (4 bond pairs, 2 lone pairs)



square planar

4. For the following reactions, equilibrium constants are given :

$$S(s) + O_2(g) \rightleftharpoons SO_2(g); K_1 = 10^{52}$$

$$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g); K_2 = 10^{129}$$

The equilibrium constant for the reaction,

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) \text{ is :}$$

(1) 10^{154}

(2) 10^{25}

(3) 10^{77}

(4) 10^{181}

Answer (2)

Sol. eq^n 1 $\quad S + O_2 \rightleftharpoons SO_2 \quad K_1 = 10^{52}$

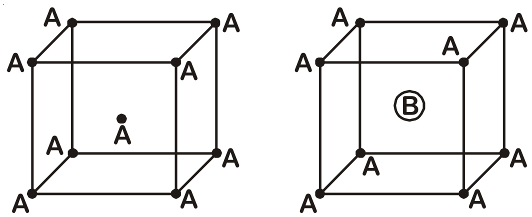
eq^n 2 $\quad 2S + 3O_2 \rightleftharpoons 2SO_3 \quad K_2 = 10^{129}$

eq^n 3 $\quad 2SO_2 + O_2 \rightleftharpoons 2SO_3$

eq^n 3 = eq^n 2 – 2 (eq^n 1)

$$= \frac{10^{129}}{(10^{52})^2} = 10^{25}$$

The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2?



Solid 1 Solid 2

(1) 45% (2) 65%

(3) 75% (4) 90%

Answer (4)

Sol. Volume occupied by atoms in solid 2

$$= \frac{4}{3}\pi r^3 + \frac{4}{3}\pi(2r)^3 = 12\ \pi r^3$$

relationship between edge length (a) and radius of atom (r)

$$= 6r = \sqrt{3}a \Rightarrow a = \frac{6r}{\sqrt{3}}$$

packing efficiency $= \dfrac{12\pi r^3}{\left(\dfrac{6r}{\sqrt{3}}\right)^3} \times 100 = 90\%$

6. The compound that inhibits the growth of tumors is

(1) cis-$[Pt(Cl)_2(NH_3)_2]$

(2) trans-$[Pt(Cl)_2(NH_3)_2]$

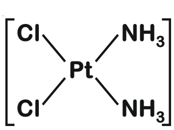
(3) cis-$[Pd(Cl)_2(NH_3)_2]$

(4) trans-$[Pd(Cl)_2(NH_3)_2]$

Answer (1)

Sol. Cis-platin is used as an anti-cancer drug.

Cis $[PtCl_2(NH_3)_2]$



(1) $\left[(CH_2)_4 - \overset{O}{\overset{\|}{C}} - \overset{H}{\overset{|}{N}} \right]_n$

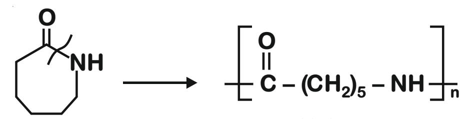
(2) $\left[\overset{O}{\overset{\|}{C}} - (CH_2)_6 - \overset{H}{\overset{|}{N}} \right]_n$

(3) $\left[(CH_2)_6 - \overset{O}{\overset{\|}{C}} - \overset{H}{\overset{|}{N}} \right]_n$

(4) $\left[\overset{O}{\overset{\|}{C}} - (CH_2)_5 - \overset{H}{\overset{|}{N}} \right]_n$

Answer (4)

Sol. Nylon-6 is produced from caprolactam



Nylon-6

8. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product ?

(1) $F_3C–CH=CH_2$

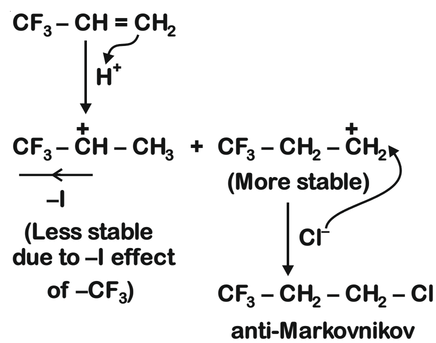
(2) $CH_3O–CH=CH_2$

(3) $H_2N–CH=CH_2$

(4) $Cl–CH=CH_2$

Answer (1)

Sol.



9. The strength of 11.2 volume solution of H_2O_2 is [Given that molar mass of H = 1 g mol^{-1} and O = 16 g mol^{-1}]

(1) 13.6%

(2) 1.7%

(3) 3.4%

(4) 34%

Sol. 11.2 V of H_2O_2

$$H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$$

11.2 L of O_2 at STP = 0.5 mol

It means 1 L of given H_2O_2 solution consist 1 mole of H_2O_2 (i.e., 34 g)

strength = $\frac{34}{1000} \times 100 = 3.4\%$

10. The covalent alkaline earth metal halide (X = Cl, Br, I) is

(1) BeX_2

(2) SrX_2

(3) CaX_2

(4) MgX_2

Answer (1)

Sol. According to Fajan's rule, graeter the polarising power of cation greater would be the covalent character.

Since Be^{2+} has maximum polarising power among given cation, therefore, BeX_2 would be most covalent among given alkaline with metal halides.

11. Polysubstitution is a major drawback in :

(1) Reimer Tiemann reaction

(2) Acetylation of aniline

(3) Friedel Craft's acylation

(4) Friedel Craft's alkylation

Answer (4)

Sol. Polysubstitution is a major drawback in Friedel Craft's alkylation

12. The IUPAC symbol for the element with atomic number 119 would be :

(1) une (2) uun

(3) uue (4) unh

Answer (3)

Sol. Symbol for 1 is u

and for 9 is e

$\therefore$ IUPAC symbol for 119 is uue

temperature becomes 200 K. If C_V = 28 J K^{-1} mol^{-1}, calculate ΔU and ΔpV for this process. (R = 8.0 J K^{-1} mol^{-1})

(1) ΔU = 14 kJ; $\Delta(pV)$ = 18 kJ

(2) ΔU = 2.8 kJ; $\Delta(pV)$ = 0.8 kJ

(3) ΔU = 14 J; $\Delta(pV)$ = 0.8 J

(4) ΔU = 14 kJ; $\Delta(pV)$ = 4 kJ

Answer (4)

Sol. $\Delta U = n\ c_{vm}\ \Delta T = 5 \times 28 \times 100$

= 14 kJ

$\Delta(PV) = nR(T_2 - T_1)$

= 5 × 8 × 100

= 4 kJ

14. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are:

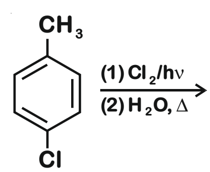
(1) 2.84 and 5.92 (2) 4.9 and 0

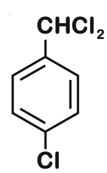
(3) 0 and 5.92 (4) 0 and 4.9

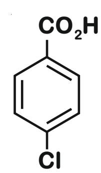
Answer (4)

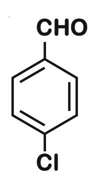
Sol. $[Fe(H_2O)_6]_2$	$[Fe(CN)_6]$
$[Fe(H_2O)_6]^{2+}$	$[Fe(CN)_6]^{4-}$
4 unpaired electrons	no unpaired electron
μ = 4.9	μ = 0

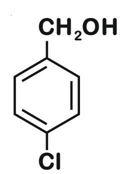
15. The major product of the following reaction is:



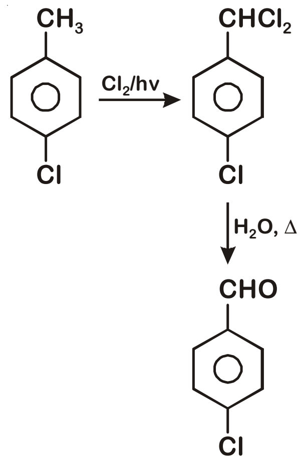
(1) 

(2) 

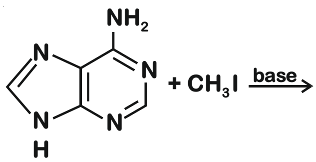
(3) 

(4) 

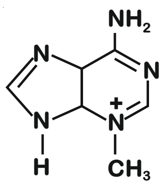
Sol.



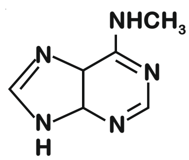
16. The major product in the following reaction is:



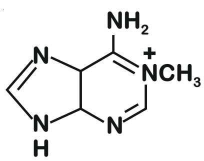
(1)



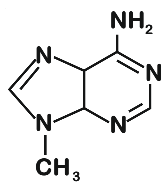
(2)



(3)

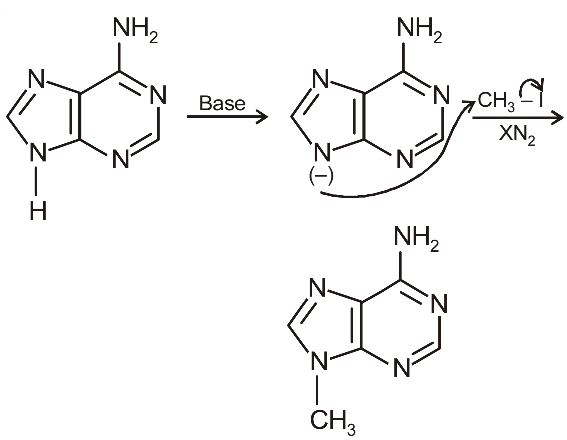


(4)



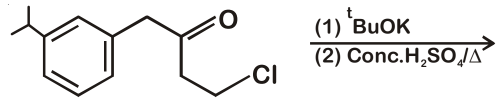
Answer (4)

Sol.

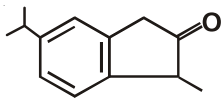


removes the more acidic H-atom and the conjugate base of the given compound attacks at CH_3 group to give the final product shown above.

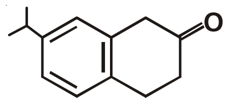
17. The major product of the following reaction is :



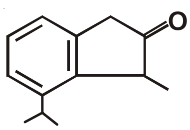
(1)



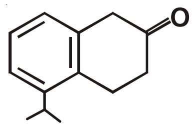
(2)



(3)

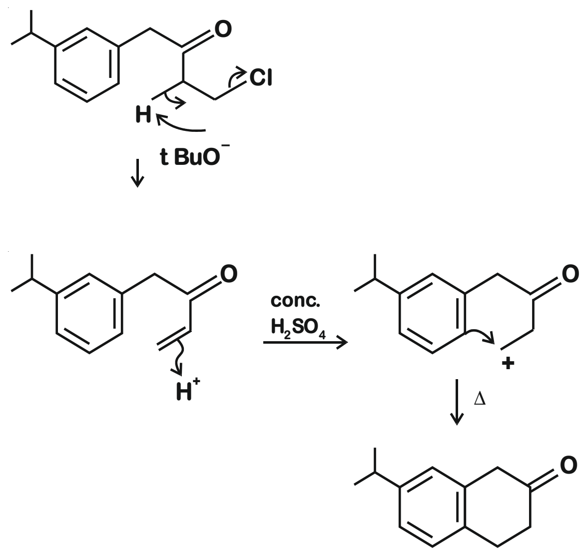


(4)



Answer (2)

Sol.



18. The ion that has sp^3d^2 hybridization for the central atom, is :

(1) $[ICl_2]^-$ (2) $[IF_6]^-$

(3) $[BrF_2]^-$ (4) $[ICl_4]^-$

Answer (4)

Sol. Species	**Hybridisation**
ICl_2^-	sp^3d
ICl_4^-	sp^3d^2
BrF_2^-	sp^3d
IF_6^-	sp^3d^3

place :

$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

Given that

$E^{o}_{Ag^{+}/Ag} = xV$

$E^{o}_{Fe^{2+}/Fe} = yV$

$E^{o}_{Fe^{3+}/Fe} = zV$

(1) $x - y$ (2) $x + y - z$

(3) $x + 2y - 3z$ (4) $x - z$

Answer (3)

Sol.

$$\underset{(aq)}{Ag^{+}} + \underset{(aq)}{Fe^{+2}} \longrightarrow \underset{(aq)}{Fe^{3+}} + \underset{(s)}{Ag}$$

$$E^{o}_{cell} = E^{o}_{Ag^{+}/Ag} - E^{o}_{Fe^{3+}/Fe^{2+}}$$

To calculate

$E^{o}_{Fe^{3+}/Fe^{2+}}$

$$Fe^{3+} \rightarrow Fe^{2+} \xrightarrow{y} Fe, \quad Fe^{3+} \xrightarrow{E^{o} = z} Fe$$

$$E^{o}_{Fe^{3+}/Fe^{2+}} = 3z - 2y$$

$$E^{o}_{Ag^{+}/Ag} = x$$

$$E^{o}_{cell} = x - 3z + 2y$$

20. Which of the following compounds will show the maximum 'enol' content?

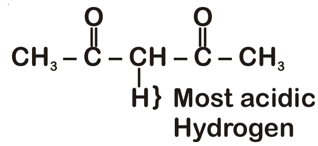
(1) $CH_3COCH_2COOC_2H_5$

(2) CH_3COCH_3

(3) $CH_3COCH_2COCH_3$

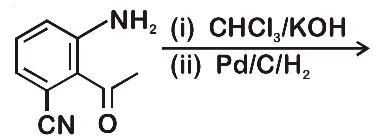
(4) $CH_3COCH_2CONH_2$

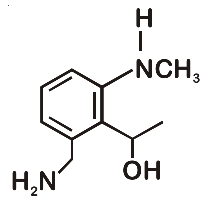
Answer (3)

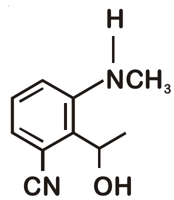


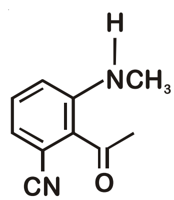
Maximum enol content

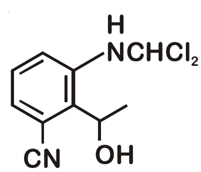
21. The major product obtained in the following reaction is:



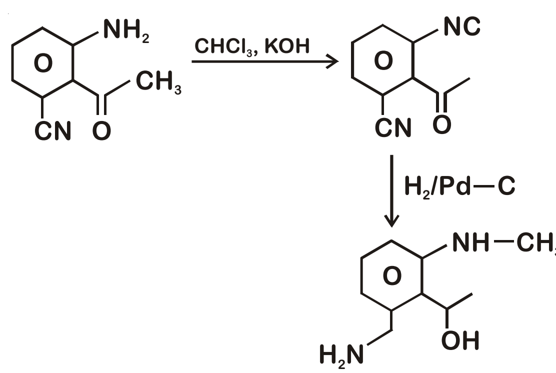
(1) 

(2) 

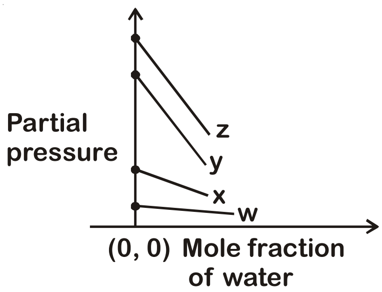
(3) 

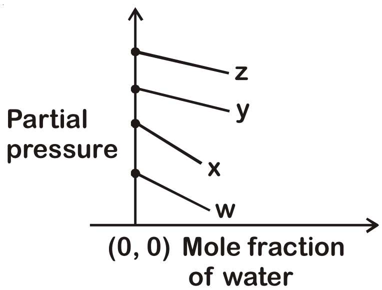
(4) 

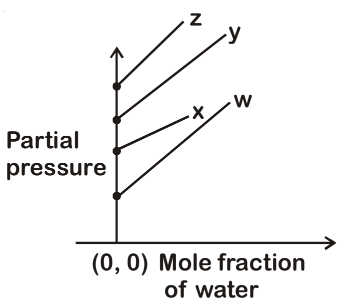
Answer (1)

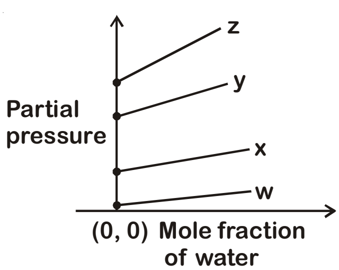
Sol. 

are 0.5, 2, 35 and 40 kbar, respectively. The correct plot for the given data is :

(1) 

(2) 

(3) 

(4) 

Answer (1)

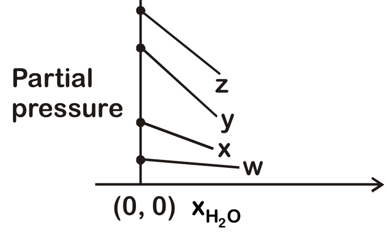
Sol. According to Henry's law

$P = K_H \cdot X_{gas}$

$\because \quad x_{gas} = 1 - x_{H_2O}$

$\therefore \quad P = K_H - K_H \cdot X_{H_2O}$

$y = C + mx$



gas	K_H
w	0.5
x	2
y	35
z	50

23. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ, then for 1.5 p momentum of the photoelectron, the wavelength of the light should be :

    (Assume kinetic energy of ejected photoelectron to be very high in comparison to work function):

    (1) $\frac{3}{4}\lambda$ (2) $\frac{4}{9}\lambda$

    (3) $\frac{2}{3}\lambda$ (4) $\frac{1}{2}\lambda$

Answer (2)

Sol. In photoelectric effect,

$$\frac{hc}{\lambda} = w + \text{KE of electron}$$

It is given that KE of ejected electron is very high in comparison to w.

$$\frac{hc}{\lambda} = KE \Rightarrow \frac{hc}{\lambda} = \frac{P^2}{2m}$$

New wavelength

$$\frac{hc}{\lambda_1} = \frac{(1.5P)^2}{2m} \Rightarrow \lambda' = \frac{4}{9}\lambda$$

24. Fructose and glucose can be distinguished by:

    (1) Fehling's test (2) Seliwanoff's test

    (3) Barfoed's test (4) Benedict's test

Sol. Seliwanoff's test is used to distinguish aldose and ketose

25. Among the following molecules/ions,

C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2

Which one is diamagnetic and has the shortest bond length?

(1) O_2 (2) O_2^{2-}

(3) N_2^{2-} (4) C_2^{2-}

Answer (4)

Sol. Bond length $\propto \dfrac{1}{\text{bond order}}$

and diamagnetic species has no unpaired electron in their molecular orbitals.

	Bond order	Magnetic character
C_2^{2-}	3	diamagnetic
N_2^{2-}	2	paramagnetic
O_2^{2-}	1	diamagnetic
O_2	2	paramagnetic

$\therefore$ C_2^{2-} has least bond length and is diamagnetic

26. For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by :

(1) $k_1k_2[A]$ (2) $(k_1 - k_2)[A]$

(3) $\left(\dfrac{k_1}{k_2}\right)[A]$ (4) $(k_1 + k_2)[A]$

Answer (3)

Sol. $A \xrightarrow{k_1} B \xrightarrow{k_2} C$

$$\frac{d[B]}{dt} = k_1[A] - k_2[B] = 0$$

$$[B] = \frac{k_1[A]}{k_2}$$

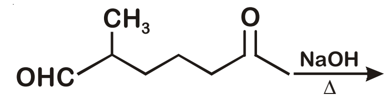
27. The maximum prescribed concentration of copper in drinking water is :

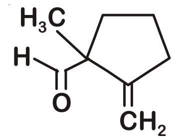
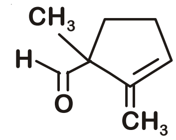
(1) 3 ppm (2) 0.05 ppm

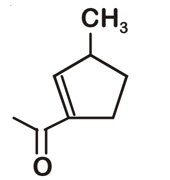
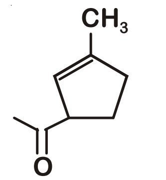
(3) 0.5 ppm (4) 5 ppm

Answer (1)

Sol. Maximum prescribed concentration of Cu in drinking water is 3 ppm.

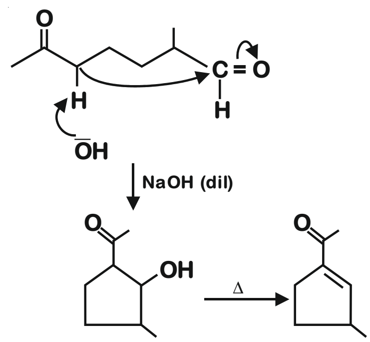


(1)  (2) 

(3)  (4) 

Answer (3)

Sol.



29. The percentage composition of carbon by mole in methane is

(1) 80% (2) 75%

(3) 20% (4) 25%

Answer (3)

Sol. In CH_4

one atom of carbon among 5 atoms (1C + 4H atoms)

$$\therefore \quad \text{Mole \% of C} = \frac{1}{5} \times 100$$

$$= 20\%$$

30. The statement that is INCORRECT about the interstitial compounds is

(1) They are chemically reactive.

(2) They are very hard.

(3) They have high melting points.

(4) They have metallic conductivity.

Answer (1)

Sol. Interstitial compounds are inert.

1. The vector equation of plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0 is :

(1) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$ (2) $\vec{r}\cdot(\hat{i}-\hat{k})-2=0$

(3) $\vec{r}\times(\hat{i}-\hat{k})+2=0$ (4) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

Answer (1)

Sol. Equation of the plane passing through the line of intersection of x + y + z = 1 and 2x + 3y + 4z = 5 is

$(2x + 3y + 4z - 5) + \lambda\,(x + y + z - 1) = 0$

$(2 + \lambda)\,x + (3 + \lambda)\,y + (4 + \lambda)\,z + (-5 - \lambda) = 0$ ...(i)

(i) is perpendicular to x – y + z = 0

$\Rightarrow (2 + \lambda)(1) + (3 + \lambda)(-1) + (4 + \lambda)(1) = 0$

$2 + \lambda - 3 - \lambda + 4 + \lambda = 0$

$\lambda = -3$

$\Rightarrow$ Equation of required plane is

$-x + z - 2 = 0$

$\Rightarrow x - z + 2 = 0$

$\Rightarrow \vec{r}\cdot(\hat{i}-\hat{k})+2=0$

2. Which one of the following statements is not a tautology?

(1) $(p \wedge q) \to (\sim p) \vee q$

(2) $(p \wedge q) \to p$

(3) $(p \vee q) \to (p \vee (\sim q))$

(4) $p \to (p \vee q)$

Answer (3)

Sol. By help of truth table :

p	q	~q	pv~q	~p	p∧~q	pvq	p→pvq	p∧q	(p∧q)→p	~pvq
T	T	F	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	F	T	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	T	T

$(p\wedge q)\to(\sim p)vq$	$(pvq)\to(pv(\sim q))$
T	T
T	T
T	F
T	T

3. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is :

(1) 360 (2) 306

(3) 288 (4) 310

Answer (4)

Sol. 0, 1, 2, 3, 4, 5

| 5 | _ | _ | _ | = 6 × 6 × 6 = 216

6 6 6

| 4 | 5 | _ | _ | = 6 × 6 = 36

6 6

| 4 | 4 | _ | _ | = 6 × 6 = 36

6 6

| 4 | 3 | _ | _ | = 3 × 6 = 18

(5/4/3) 3 6

| 4 | 3 | 2 | _ | = 4

4

$\Rightarrow$ Required numbers = 216 + 36 + 36 + 18 + 4 = 310

4. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0, 5\sqrt{3}\right)$, then the length of its latus rectum is :

(1) 5 (2) 6

(3) 8 (4) 10

Answer (1)

Sol. $\because$ Focus is $\left(0, 5\sqrt{3}\right) \Rightarrow |b| > |a|$

Let b > a > 0

foci (0, ± be)

$a^2 = b^2 - b^2e^2 \Rightarrow b^2e^2 = b^2 - a^2$

$be = \sqrt{b^2 - a^2}$

$2b - 2a = 10$

$\Rightarrow$ b – a = 5 ...(ii)

From (i) and (ii)

b + a = 15 ...(iii)

$\Rightarrow$ b = 10, a = 5

Length of L·R· = $\frac{2a^2}{b} = \frac{50}{10} = 5$

5. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is :

(1) $\frac{100}{\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$

(3) $\frac{100}{3}$ (4) $\frac{10}{3}$

Answer (2)

Sol. $\bar{x} = \frac{41+45+54+57+43+x}{6} = 48$

x + 240 = 288

x = 48

$$\sigma^2 = \frac{1}{6}\begin{bmatrix}(48-41)^2 + (48-45)^2 + (48-54)^2 \\ +(48-57)^2 + (48-43)^2 + (48-48)^2\end{bmatrix}$$

$$= \frac{1}{6}(49+9+36+81+25)$$

$$= \frac{200}{6} = \frac{100}{3}$$

$$\sigma = \frac{10}{\sqrt{3}}$$

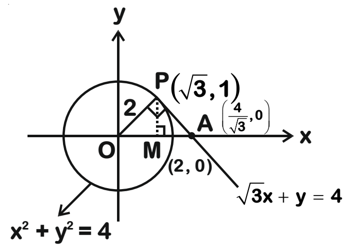
6. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{4}{\sqrt{3}}$

(3) $\frac{1}{3}$ (4) $\frac{1}{\sqrt{3}}$

Answer (1)

$\sqrt{3}x + y = 4$



$\therefore$ Coordinate of $A = \left(\frac{4}{\sqrt{3}}, 0\right)$

Area = $\frac{1}{2} \times OA \times PM$

$$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}} \text{ Square units}$$

7. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is :

(1) 10 (2) 10^3

(3) 100 (4) 10^4

Answer (1)

Sol. $T_4 = {}^6C_3 \left(\sqrt{x^{\left(\frac{1}{1+\log_{10} x}\right)}}\right)^3 \left(x^{\frac{1}{12}}\right)^3 = 200$

$\Rightarrow$ $20x^{\frac{3}{2(1+\log_{10} x)}} \cdot x^{\frac{1}{4}} = 200$

$x^{\frac{1}{4} + \frac{3}{2(1+\log_{10} x)}} = 10$

Taking $\log_{10}$ on both sides and put $\log_{10} x = t$

$$\left(\frac{1}{4} + \frac{3}{2(1+t)}\right)t = 1$$

$\left(\frac{(1+t)+6}{4(1+t)}\right) \times t = 1$ $\Rightarrow$ $t^2 + 7t = 4 + 4t$

$t^2 + 3t - 4 = 0$ $\Rightarrow$ $t^2 + 4t - t - 4 = 0$

$\Rightarrow$ t(t + 4) –1 (t + 4) = 0

$\Rightarrow$ t = 1 or t = – 4

$\log_{10} x = 1$

$\Rightarrow$ x = 10 or if $\log_{10} x = -4$

$\Rightarrow$ $x = 10^{-4}$

Note: There seems a printing error in this question in the original question paper.

first quadrant, passes through the point :

(1) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (2) $\left(-\frac{1}{3}, \frac{4}{3}\right)$

(3) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

Answer (1)

Sol. Intersection point of

$x^2 + y^2 = 5, \quad y^2 = 4x$

$\Rightarrow x^2 + 4x - 5 = 0$

$\Rightarrow x^2 + 5x - x - 5 = 0$

$\Rightarrow x(x + 5) -1(x + 5) = 0$

$\therefore x = 1, -5$

Intersection point in 1st quadrant be (1, 2)

equation of tangent to $y^2 = 4x$ at (1, 2) is

$y \times 2 = 2(x + 1)$

$\Rightarrow y = x + 1$

$\Rightarrow x - y + 1 = 0 \quad \ldots(i)$

$\left(\frac{3}{4}, \frac{7}{4}\right)$ lies on (i)

9. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d, e, f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d, e, f are in G.P.

Answer (3)

Sol. Since a, b, c are in G.P.

$\Rightarrow b^2 = ac$

Given, $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}\,x + c = 0$

$\Rightarrow \left(\sqrt{a}\,x + \sqrt{c}\right)^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}$

$\because$ $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have common root

$\Rightarrow d \cdot \frac{c}{a} + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$

$\frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0$

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

10. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\ (i = \sqrt{-1})$, then

$\left(1 + iz + z^5 + iz^8\right)^9$ is equal to :

(1) 0 (2) $(-1 + 2i)^9$

(3) -1 (4) 1

Answer (3)

Sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega$

where ω is not real cube root of unity

$\Rightarrow (1 + iz + z^5 + iz^8)^9 = (1 + \omega - i\omega^2 + i\omega^2)^9$

$= (1 + \omega)^9$

$= (-\omega^2)^9$

$= -\omega^{18}$

$= -1$

11. The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is :

(1) Infinitely many (2) 3

(3) 2 (4) 1

Answer (1)

Sol. $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

equation has no real solution

$\Rightarrow D < 0$

$4(1 + 3m)^2 < 4(1 + m^2)(1 + 8m)$

$1 + 9m^2 + 6m < 1 + 8m + m^2 + 8m^3$

$2m(4m^2 - 4m + 1) > 0$

$2m(2m - 1)^2 > 0$

$m > 0,\ m \neq \frac{1}{2}$

$\Rightarrow$ number of integral values of m are infinitely many.

12. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A) $\in$ [2, 16], then c lies in the interval :

(1) [2, 3] (2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$ (4) [4, 6]

Answer (4)

Sol. $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$c_2 \rightarrow c_2 - c_1,\ c_3 \rightarrow c_3 - c_1$

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$

$= (b-2)(c-2)(c-b)$

2, b, c are in A.P. $\Rightarrow (b - 2) = (c - b) = d,\ c - 2 = 2d$

$\Rightarrow |A| = d.2d.d = 2d^3$

$\because |A| \in [2, 16] \Rightarrow 1 \leq d^3 \leq 8 \Rightarrow 1 \leq d \leq 2$

$4 \leq 2d + 2 \leq 6 \ \Rightarrow 4 \leq c \leq 6$

13. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

(1) 5 (2) 2

(3) 4 (4) 3

Answer (3)

$P(x \geq 1) \geq \frac{9}{10}$

$1 - P(x = 0) \geq \frac{9}{10}$

$1 - {}^nC_0 \left(\frac{1}{2}\right)^n \geq \frac{9}{10}$

$\frac{1}{2^n} \leq 1 - \frac{9}{10} \Rightarrow \frac{1}{2^n} \leq \frac{1}{10}$

$2^n \geq 10 \quad \Rightarrow n \geq 4$

$\Rightarrow n_{min} = 4$

14. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if :

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

(2) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$

(3) $0 < r \leq \sqrt{\frac{3}{2}}$

(4) $r \geq 5\sqrt{\frac{3}{2}}$

Answer (4)

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2 + x)\hat{i} + (x - 3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = r = \sqrt{(2+x)^2 + (x-3)^2 + (-5)^2}$

$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$

$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}}$

$\Rightarrow r \geq \sqrt{\frac{75}{2}} \Rightarrow r \geq 5\sqrt{\frac{3}{2}}$

(1) $2-\frac{3}{2^{17}}$

(2) $1-\frac{11}{2^{20}}$

(3) $2-\frac{21}{2^{20}}$

(4) $2-\frac{11}{2^{19}}$

Answer (4)

Sol. $S=\sum_{k=1}^{20} k.\frac{1}{2^k}$

$$S=\frac{1}{2}+2.\frac{1}{2^2}+3.\frac{1}{2^3}+....+20.\frac{1}{2^{20}}$$

$$\frac{1}{2}S=\frac{1}{2^2}+2.\frac{1}{2^3}+....+19\frac{1}{2^{20}}+20\frac{1}{2^{21}}$$

On subtracting

$$\frac{S}{2}=\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^{20}}\right)-20\frac{1}{2^{21}}$$

$$=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-20.\frac{1}{2^{21}}=1-\frac{1}{2^{20}}-10.\frac{1}{2^{20}}$$

$$\frac{S}{2}=1-11.\frac{1}{2^{20}}\Rightarrow S=2-11.\frac{1}{2^{19}}=2-\frac{11}{2^{19}}$$

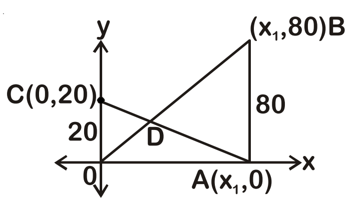
16. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is :

(1) 16 (2) 18

(3) 15 (4) 12

Answer (1)

Sol.



$$y=\frac{80}{x_1}x \quad ...(i)$$

$$\frac{x}{x_1}+\frac{y}{20}=1 \quad ...(ii)$$

For intersection point, from equations (i) and (ii)

$$\frac{y}{80}+\frac{y}{20}=1$$

$\Rightarrow$ y + 4y = 80

$\Rightarrow$ y = 16 m

$\Rightarrow$ Height of intersection point is 16 m

17. Let f : [–1, 3] $\rightarrow$ R be defined as

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3,\end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at :

(1) Only one point

(2) Only two points

(3) Only three points

(4) Four or more points

Answer (3)

Sol. $f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$

$$=\begin{cases}-x-1, & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ 6, & x=3\end{cases}$$

$\Rightarrow$ $f(-1) = 0$, $f(-1^+) = 0$

$f(0^-) = -1$, $f(0) = 0$, $f(0^+) = 0$

$f(1^-) = 1$, $f(1) = 2$, $f(1^+) = 2$

$f(2^-) = 4$, $f(2) = 4$, $f(2^+) = 4$

$f(3^-) = 5$, $f(3) = 6$

f(x) is discontinuous at x = {0, 1, 3}

$y = y(x)$ at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is

(1) $x \log_e |y| = -2(x - 1)$

(2) $x \log_e |y| = x - 1$

(3) $x \log_e |y| = 2(x - 1)$

(4) $x^2 \log_e |y| = -2(x - 1)$

Answer (3)

Sol. $\frac{dy}{dx} = \frac{2y}{x^2} \Rightarrow \int \frac{dy}{y} = 2\int \frac{dx}{x^2}$

$\Rightarrow \ln|y| = -\frac{2}{x} + C$ ...(i)

(i) passes through (1, 1)

$\Rightarrow C = 2$

$\Rightarrow \ln|y| = -\frac{2}{x} + 2$

$x\ln|y| = -2 + 2x$

$x\ln|y| = -2(1 - x) = 2(x - 1)$

19. Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals

(1) 3 (2) $-\frac{1}{7}$

(3) 0 (4) $\frac{1}{3}$

Answer (4)

Sol. (h, k), (1, 2) and (–3, 4) and collinear

$$\begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \Rightarrow -2h - 4k + 10 = 0$$

$\Rightarrow h + 2k = 5$ ...(i)

$m_{L_1} = \frac{4-2}{-3-1} = \frac{2}{-4} = -\frac{1}{2} \Rightarrow m_{L_2} = 2$

$\Rightarrow m_{L_2} = \frac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$

$2h - k = 5$ ...(ii)

from (i) and (ii)

$h = 3, k = 1 \Rightarrow \frac{k}{h} = \frac{1}{3}$

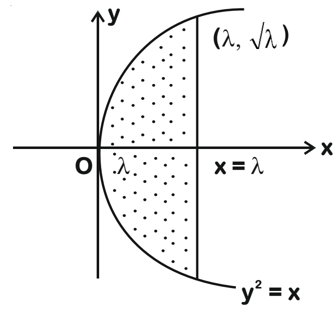
$A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (2) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

Answer (4)

Sol.



$$A(\lambda) = 2 \times \frac{2}{3}(\lambda \times \sqrt{\lambda}) = \frac{4}{3}\lambda^{3/2}$$

$$\Rightarrow \frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{8} = \frac{2}{5}$$

$$\lambda = \left(\frac{16}{5}\right)^{2/3} = 4.\left(\frac{4}{25}\right)^{1/3}$$

21. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then the distance of R from the origin is

(1) $\sqrt{53}$ (2) $2\sqrt{21}$

(3) 6 (4) $2\sqrt{14}$

Answer (4)

Sol. P, Q, R are collinear

$\Rightarrow \overrightarrow{PR} = \lambda\overrightarrow{PQ}$

$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$

$\Rightarrow 6\lambda = 2, y + 3 = 3\lambda, z - 4 = 6\lambda$

$\Rightarrow \lambda = \frac{1}{3}, y = -2, z = 6$

$\Rightarrow$ point R (4, –2, 6)

$\Rightarrow OR = \sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{16 + 4 + 36}$

$= \sqrt{56} = 2\sqrt{14}$

smallest, then a ratio of lengths of the sides of this triangle is

(1) 4 : 5 : 6

(2) 3 : 4 : 5

(3) 5 : 9 : 13

(4) 5 : 6 : 7

Answer (1)

Sol. Let a > b > c

$\because$ A = 2C

$\Rightarrow$ A + B + C = π

$\Rightarrow$ B = π – 3C ...(i)

$\because$ a + c = 2b

$\Rightarrow$ sinA + sinC = 2sinB ...(ii)

$\Rightarrow$ sinA = sin(2C), sinB = sin3C

$\Rightarrow$ From (ii), sin2C + sinC = 2sin3C

(2cosC + 1)sinC = 2sinC (3 – 4 $\sin^2$C)

$\Rightarrow$ 2cosC + 1 = 6 – 8(1 – $\cos^2$C)

$\Rightarrow$ $8\cos^2C - 2\cos C - 3 = 0$

$\Rightarrow$ $\cos C = \frac{3}{4}$ or $\cos C = -\frac{1}{2}$

$\because$ C is acute angle

$\Rightarrow$ $\cos C = \frac{3}{4}, \sin A = 2\sin C \cos C = 2 \times \frac{\sqrt{7}}{4} \times \frac{3}{4}$

$\Rightarrow$ $\sin C = \frac{\sqrt{7}}{4}, \sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4} \times \frac{7}{16} = \frac{5\sqrt{7}}{16}$

$\Rightarrow$ sinA : sinB : sinC :: a : b : c is 6 : 5 : 4

23. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $\frac{2}{3}\sqrt{3}$

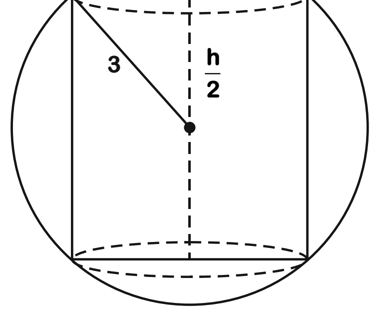
(2) $2\sqrt{3}$

(3) $\sqrt{3}$

(4) $\sqrt{6}$

Answer (2)

Sol. Let radius of base and height of cylinder be r and h respectively.



$\therefore$ $r^2 + \frac{h^2}{4} = 9$ ...(i)

$\because$ Volume of cylinder

V = $\pi r^2 h$

$V = \pi h\left(9 - \frac{h^2}{4}\right)$

$V = 9\pi h - \frac{\pi}{4}h^3$

$\therefore$ $\frac{dV}{dh} = 9\pi - \frac{3}{4}\pi h^2$

For maxima/minima

$\frac{dV}{dh} = 0$

$\Rightarrow$ $h = \sqrt{12}$

and $\frac{d^2V}{dh^2} = -\frac{3}{2}\pi h$

$\therefore$ $\left(\frac{d^2V}{dh^2}\right)_{h=\sqrt{12}} < 0$

$\Rightarrow$ Volume is maximum when $h = 2\sqrt{3}$

24. If the system of linear equations

x – 2y + kz = 1

2x + y + z = 2

3x – y – kz = 3

has a solution (x, y, z), z $\neq$ 0, then (x, y) lies on the straight line whose equation is

(1) 3x – 4y – 4 = 0 (2) 3x – 4y – 1 = 0

(3) 4x – 3y – 1 = 0 (4) 4x – 3y – 4 = 0

Answer (4)

$$\Delta = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = 1(-k+1)+2(-2k-3)+k(-2-3)$$

$= -k + 1 - 4k - 6 - 5k = -10k - 5 = -5(2k + 1)$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = -5(2k+1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 1 \\ 3 & 3 & -k \end{vmatrix} = 0, \Delta_3 = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$\because$ $z \neq 0$

$\Rightarrow$ $\Delta = 0$

$\Rightarrow$ $k = -\dfrac{1}{2}$

$\therefore$ System of equation has infinite many solutions.

Let $z = \lambda \neq 0$ then $x = \dfrac{10-3\lambda}{10}$ and $y = -\dfrac{2\lambda}{5}$

$\therefore$ (x, y) must lie on line $4x - 3y - 4 = 0$

25. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is :

(1) 33 (2) 12

(3) 9 (4) 15

Answer (1)

Sol. Let $g(x) = f(f(f(x))) + (f(x))^2$

On differentiating both sides w.r.t. x we get

$g'(x) = f'(f(f(x)))\, f'(f(x))\, f'(x) + 2f(x)\, f'(x)$

$g'(1) = f'(f(f(1)))\, f'(f(1))\, f'(1) + 2f(1)\, f'(1)$

$= f'(f(1))\, f'(1)\, f'(1) + 2f(1)\, f'(1)$

$= 3 \times 3 \times 3 + 2 \times 1 \times 3 = 27 + 6 = 33$

26. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x + 5) = g(x)$, then $\int_0^x f(t)dt$, equals:

(1) $\int_{x+5}^{5} g(t)dt$ (2) $2\int_{5}^{x+5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $5\int_{x+5}^{5} g(t)dt$

Answer (1)

$g(-x) = g(x)$, ...(ii)

$f(x + 5) = g(x)$ ...(iii)

From (i)

$f'(x) = g(x)$

Let $I = \int_0^x f(t)dt$,

Put $t = \lambda - 5$

$\Rightarrow$ $I = \int_5^{x+5} f(\lambda - 5)d\lambda$

$\because$ $f(x + 5) = g(x)$

$\Rightarrow$ $f(-x + 5) = g(-x) = g(x)$ ...(iv)

$I = \int_5^{x+5} f(\lambda - 5)d\lambda$,

$I = \int_5^{x+5} -f(5-\lambda)d\lambda$

($\because$ $f(0) = 0$, g(x) is even $\Rightarrow$ f(x) is odd)

$I = -\int_5^{x+5} g(\lambda)d\lambda = \int_{x+5}^{5} g(t)dt$ (from (iv))

27. Let $f(x) = a^x$ ($a > 0$) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals:

(1) $2f_1(x)f_1(y)$ (2) $2f_1(x + y)f_1(x - y)$

(3) $2f_1(x + y)f_2(x - y)$ (4) $2f_1(x)f_2(y)$

Answer (1)

Sol. $f(x) = a^x = \left(\dfrac{a^x + a^{-x}}{2}\right) + \left(\dfrac{a^x - a^{-x}}{2}\right)$

where $f_1(x) = \dfrac{a^x + a^{-x}}{2}$ is even function

$f_2(x) = \dfrac{a^x - a^{-x}}{2}$ is odd function

$\Rightarrow$ $f_1(x + y) + f_1(x - y)$

$= \left(\dfrac{a^{x+y} + a^{-x-y}}{2}\right) + \left(\dfrac{a^{x-y} + a^{-x+y}}{2}\right)$

$= \dfrac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})\right]$

$= \dfrac{(a^x + a^{-x})(a^y + a^{-y})}{2}$

$= 2f_1(x).f_1(y)$

$\int \overline{x^3(1+x^6)^{2/3}}$

constant of integration, then the function f(x) is equal to:

(1) $-\frac{1}{2x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{6x^3}$ (4) $-\frac{1}{2x^2}$

Answer (1)

Sol. $I = \int \frac{dx}{x^3(1+x^6)^{2/3}} = \int \frac{dx}{x^7(1+x^{-6})^{2/3}}$

Put $1 + x^{-6} = t^3$

$\Rightarrow -6x^{-7}dx = 3t^2\,dt$

$\Rightarrow \frac{dx}{x^7} = -\frac{1}{2}t^2dt$

$\Rightarrow I = \int -\frac{1}{2}\frac{t^2dt}{t^2}$

$= -\frac{1}{2}t + C$

$= -\frac{1}{2}(1+x^{-6})^{\frac{1}{3}} + C$

$= -\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2} + C$

$= -\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}} + C$

$\Rightarrow f(x) = -\frac{1}{2x^3}$

29. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is:

(1) $2x - 3y + 10 = 0$ (2) $x - 2y + 8 = 0$

(3) $3x - 2y = 0$ (4) $2x - y - 2 = 0$

Answer (4)

Sol. Let equation of hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(i)

$\Rightarrow b^2 = a^2(e^2 - 1)$

$\because$ e = 2 $\Rightarrow$ $\boxed{b^2 = 3a^2}$ ...(ii)

$\Rightarrow \frac{16}{a^2} - \frac{36}{b^2} = 1$ ...(iii)

From (ii) and (iii)

$a^2 = 4, b^2 = 12$

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Equation of tangent to the hyperbola at (4, 6) is

$\frac{4x}{4} - \frac{6y}{12} = 1$

$\Rightarrow x - \frac{y}{2} = 1$

$\Rightarrow 2x - y = 2$

30. Let f : R $\rightarrow$ R be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim_{x\to0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to:

(1) e (2) 1

(3) e^2 (4) e^{-1}

Answer (2)

Sol. $I = \lim_{x\to0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$

form : 1^∞

$\Rightarrow I = e^{I_1}$, where

$I_1 = \lim_{x\to0}\left(\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)} - 1\right)\frac{1}{x}\right)$

$= \lim_{x\to0}\left(\left(\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)\frac{1}{x}\right)$

form : $\frac{0}{0}$

Using L.H. Rule

$I_1 = \lim_{x\to0}\left(\frac{f'(3+x)+f'(2-x)}{1}\right).\lim_{x\to0}\left(\frac{1}{1+f(2-x)-f(2)}\right)$

$= f'(3) + f'(2) = 0$

$\Rightarrow I = e^{I_1} = 1$

Morning

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

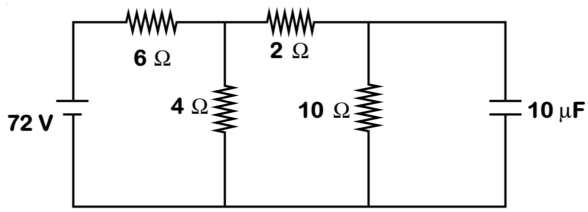
(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. Determine the charge on the capacitor in the following circuit :



(1) 200 μC (2) 60 μC

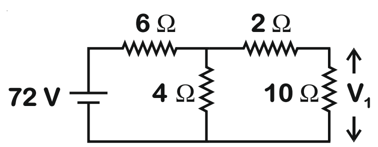
(3) 10 μC (4) 2 μC

Answer (1)

Sol. At steady state current through capacitor is zero.

$V_C = V_1$

$$V_1 = \frac{5}{6} \times V_0 \times \frac{3}{9}$$



$$V_1 = \frac{5 \times 72 \times 3}{6 \times 9} = 20 \text{ V}$$

$Q_1 = CV_1$

$= 200 \ \mu C$

2. The magnetic field of a plane electromagnetic wave is given by :

$$\vec{B} = \vec{B}_0 \hat{i} \, [\cos(kz - \omega t) + B_1 \hat{j} \cos(kz + \omega t)$$

where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T.

The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z =0 is closest to:

(1) 0.6 N (2) 0.9 N

(3) 3×10^{-2} N (4) 0.1 N

Answer (1)

Sol. $|\vec{E}_1| = CB_1$

$|\vec{E}_2| = CB_2$

Also $\vec{E}_1 \perp E_2$

$$F_{net} = \frac{\theta}{\sqrt{2}} \sqrt{E_1^2 + E_2^2}$$

$$= \frac{10^{-4}}{\sqrt{2}} \times 3 \times 10^8 \times 30 \times 10^{-6}$$

$$= \frac{90 \times 10^8 \times 10^{-10}}{\sqrt{2}}$$

$\simeq 0.6$ N

3. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is:

(1) $\frac{2k}{I}\theta$ (2) $\frac{k}{I}\theta$

(3) $\frac{k}{2I}\theta$ (4) $\frac{k}{4I}\theta$

Answer (1)

Sol. $\tau = \frac{dE}{d\theta}$

$2k\theta = I\alpha$

$$\alpha = \frac{2k\theta}{I}$$

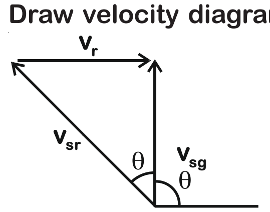
4. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

(1) 60° (2) 90°

(3) 150° (4) 120°

Answer (4)

Sol. Draw velocity diagram



$$\sin\theta = \frac{V_r}{V_{sr}} = \frac{1}{2}$$

$\theta = 30°$

$\phi = 90 + \theta = 120°$

$\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

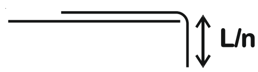
(1) $\frac{MgL}{n^2}$ (2) nMgL

(3) $\frac{MgL}{2n^2}$ (4) $\frac{2MgL}{n^2}$

Answer (3)

Sol. $m_1 = \frac{M}{n}$

$$U_i = \frac{-M}{n} \times g \frac{L}{2n}$$



$$W = \frac{MgL}{2n^2}$$

6. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is:

(1) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

(2) $(v_0 + A) \cos\omega t \sin\omega_0 t$

(3) $v_0 \sin\omega_0 t + A\cos\omega t$

(4) $v_0 \sin[\omega_0(1 + 0.01 A\sin\omega t)t]$

Answer (1)

Sol. $A = (v_o + A\cos\omega t)\sin\omega_o t$

$$= v_0 \sin(\omega_0 t) + \frac{A}{2}[\sin(\omega_0 - \omega)t + \sin(\omega_0 + \omega)t]$$

7. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(1) 14 : 15 : 20 (2) 10 : 15 : 7

(3) 4 : 3 : 2 (4) 2 : 3 : 4

Answer (Bonus)

For ring $\rightarrow h_1 = \frac{1}{2}\frac{(2mr'^2)}{mg}\frac{v^2}{(r')^2} = \frac{v^2}{g}$

For cylinder $\rightarrow h_2 = \frac{1}{2}\left(\frac{3}{2}mr'^2\right)\frac{v^2}{(r')^2} = \frac{3}{4}\frac{v^2}{g}$

For sphere $\rightarrow h_3 = \frac{1}{2} \times \frac{\frac{7}{5}m(r')^2}{g}\frac{v^2}{(r')^2} = \frac{7}{10}\frac{v^2}{g}$

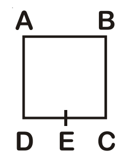
$h_1 : h_2 : h_3$

$2 : \frac{3}{2} : \frac{14}{10}$

20 : 15 : 14

8. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is

(E is mid-point of arm CD)



(1) $\frac{3}{4}R$ (2) R

(3) $\frac{1}{16}R$ (4) $\frac{7}{64}R$

Answer (4)

Sol. $R_1 = \frac{R}{8}$, $R_2 = \frac{7R}{8}$

$$\frac{1}{R_{eq}} = \frac{8}{R} + \frac{8}{7R}$$

$$R_{eq} = \frac{7R}{64}$$

9. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is

(1) 0.31 kg/m^3 (2) 0.01 kg/m^3

(3) 0.10 kg/m^3 (4) 0.07 kg/m^3

Answer (Bonus)

$(0.1)^3$

$$\frac{d\rho}{\rho} = \left[\frac{dM}{M} + \frac{dV}{V}\right]$$

$$\frac{d\rho}{10{,}000} = \frac{0.1}{10} + \frac{0.03}{0.1}$$

$d\rho = 3100$ kg/m^3

10. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is

(1) 0.55 Nm (2) 0.27 Nm

(3) 0.42 Nm (4) 0.38 Nm

Answer (2)

Sol. $\tau = \vec{M} \times \vec{B}$

$$\tau = 100 \times 3 \times 5 \times 2.5 \times 10^{-4} \times 1 \times \frac{1}{\sqrt{2}}$$

$$= 0.27 \text{ Nm}$$

11. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $2T\sqrt{\frac{1}{14}}$ (2) $4T\sqrt{\frac{1}{15}}$

(3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{10}}$

Answer (2)

Sol. $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

$$\frac{T'}{T} = \sqrt{\frac{g_{eff}}{g'_{eff}}}$$

$$g'_{eff} = g - \frac{g}{16} = \frac{15}{16}$$

$$\frac{T'}{T} = \sqrt{\frac{16}{15}}$$

L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to

(1) 1/L (2) L

(3) $1/L^2$ (4) L^2

Answer (1)

Sol. $B = \mu_0 ni$

$\phi = \mu_0 ni(nL)\ A$

(Inductance) $L' = \mu_0 n^2 L\ A$ $\left[\eta = \frac{N}{L}\right]$

$$L' = \mu_0 NA\left(\frac{N}{L}\right)$$

$$L' \propto \frac{1}{L}$$

13. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(1) $100\sqrt{5}$ m/s (2) 100 m/s

(3) $80\sqrt{5}$ m/s (4) 80 m/s

Answer (1)

Sol. $V \propto \sqrt{T}$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{200}{V_2} = \sqrt{\frac{400}{500}}$$

$$V_2 = 200\sqrt{\frac{5}{4}} = 100\sqrt{5} \text{ m/s}$$

14. A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?

(1) 2.55×10^{-6} J (2) 6.25×10^{-6} J

(3) 3.75×10^{-6} J (4) 2.16×10^{-6} J

Answer (3)

Sol. $U_i = \frac{Q^2}{2C_1}$

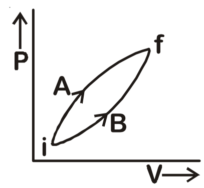
$$U_f = \frac{Q^2}{2C_2}$$

$$W = \frac{Q^2}{2}\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$$

$$= \frac{(5)^2}{2}\left[\frac{1}{2} - \frac{1}{5}\right]$$

W = 3.75 μJ

15. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then:



(1) $\Delta Q_A > \Delta Q_B$, $\Delta U_A = \Delta U_B$

(2) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$

(3) $\Delta Q_A > \Delta Q_B$, $\Delta U_A > \Delta U_B$

(4) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$

Answer (1)

Sol. $\Delta W_A > \Delta W_B$

$\Delta U_A > \Delta U_B$

From first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\Delta Q_A > \Delta Q_B$

16. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:

(1) 0.32 m (2) 0.24 m

(3) 1.60 m (4) 0.16 m

Answer (1)

Sol. $$\frac{1}{V} + \frac{1}{U} = -\frac{1}{40}$$

$$\frac{V}{U} = -5$$

$$\frac{1}{5U} - \frac{1}{U} = -\frac{1}{40}$$

U = – 32 cm

17. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be:

(1) $\frac{GM}{9a^2}$ (2) $\frac{2GM}{9a^2}$

(3) $\frac{GM}{3a^2}$ (4) $\frac{2GM}{3a^2}$

Answer (3)

Sol. $$E = \frac{GM}{(3a)^2} + \frac{2GM}{(3a)^2}$$

$$E = \frac{GM}{3a^2}$$

18. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

(1) 10 V (2) 20 V

(3) 15 V (4) 40 V

Answer (2)

Sol. $V = I_g (R_s + R_g)$

$= 4 \times 10^{-3}$ [5050]

≈ 20 V

19. The pressure wave, $P = 0.01\sin[1000\,t - 3x]\,Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is :

(1) 4°C (2) 12°C

(3) 15°C (4) 11°C

Answer (1)

Sol. $V_1 = \frac{1000}{3}$ m/s

$$\frac{dV}{V}=\frac{1}{2}\frac{dT}{T}$$

$$\frac{8\times 3}{3\times 1000}=\frac{1}{2}\times\frac{dT}{273}$$

$$dT=\frac{273\times 2\times 8}{1000}=4.36°C$$

20. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 889.2 nm (2) 488.9 nm

(3) 388.9 nm (4) 642.7 nm

Answer (2)

Sol. $\frac{1}{\lambda_1}=R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$

$$\frac{1}{\lambda_2}=R\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

$$\frac{5\lambda_1}{36}=\frac{12\lambda_2}{4\times 16}$$

$$\lambda_2=\frac{5\times 660\times 64}{36\times 12}$$

= 489 nm

21. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.5 kg (2) 1.8 kg

(3) 1.0 kg (4) 1.2 kg

Answer (4)

Sol. $2V=\frac{2V}{4}+m_2V_2$

$$V=V_2-\frac{V}{4}$$

$$\frac{3V}{2}=m_2\left(V+\frac{V}{4}\right)$$

$$m_2=\frac{6}{5}\text{ kg}$$

HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be :

(1) $\frac{m\bar{v}^2}{5k_B}$

(2) $\frac{m\bar{v}^2}{6k_B}$

(3) $\frac{m\bar{v}^2}{7k_B}$

(4) $\frac{m\bar{v}^2}{3k_B}$

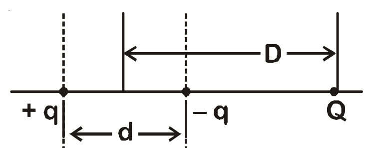
Answer (4)

Sol. $\bar{v}=\sqrt{\frac{3RT}{M}}$

$$\bar{v}=\sqrt{\frac{3RT}{mN_A}}$$

$$\bar{v}=\sqrt{\frac{3k_BT}{m}}$$

23. A system of three charges are placed as shown in the figure :



If D >> d, the potential energy of the system is best given by :

(1) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

(2) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}+\frac{2qQd}{D^2}\right]$

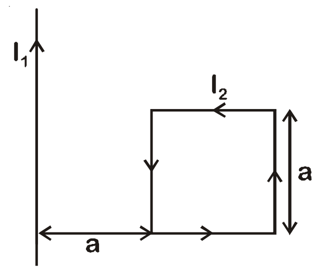
(4) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{2D^2}\right]$

Sol. $U = -\dfrac{Kq^2}{d} + QV$

$$V = -\frac{KP}{D^2} = -\frac{Kqd}{D^2}$$

$$U = -\frac{Kq^2}{d} - \frac{KQqd}{D^2}$$

24. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be :



(1) Repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{4\pi}$

(2) Repulsive and equal to $\dfrac{\mu_0 I_1 I_2}{2\pi}$

(3) Zero

(4) Attractive and equal to $\dfrac{\mu_0 I_1 I_2}{3\pi}$

Answer (1)

Sol. $F = I_2 a\,(B_1 - B_2)$

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

$$B_2 = \frac{\mu_0 I_1}{4\pi a}$$

$$F = \frac{\mu_0 I_1 I_2}{4\pi}$$

the stationary wave is Y = 0.3 sin (0.157x) cos (200πt). The length of the string is : (All quantities are in SI units)

(1) 60 m

(2) 20 m

(3) 40 m

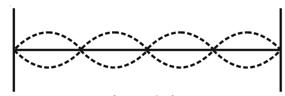
(4) 80 m

Answer (4)

Sol. λ = 0.157

$$= \frac{3.14}{20} = \frac{\pi}{20}$$

In 4th harmonic



$$\therefore\ \frac{2\pi}{\lambda} = \pi/20$$

λ = 40 m

$$\therefore\ L = \frac{4\lambda}{2} = 80\text{ m}$$

26. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are :

(1) 0.33 kΩ, 1.5

(2) 0.33 kΩ, 300

(3) 0.67 kΩ, 200

(4) 0.67 kΩ, 300

Answer (4)

I_B

$$R_i = \frac{10\times10^{-3}}{15\times10^{-6}} = 0.67\,K\Omega$$

$$\text{Voltage Gain } = \beta\left(\frac{R_o}{R_i}\right) = 300$$

27. The electric field of light wave is given as $\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\,\frac{N}{C}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is :

Given, E (in eV) = $\frac{12375}{\lambda\,(\text{in Å})}$

(1) 0.48 V

(2) 2.48 V

(3) 0.72 V

(4) 2.0 V

Answer (1)

Sol. $\lambda = 5 \times 10^{-7}$ m

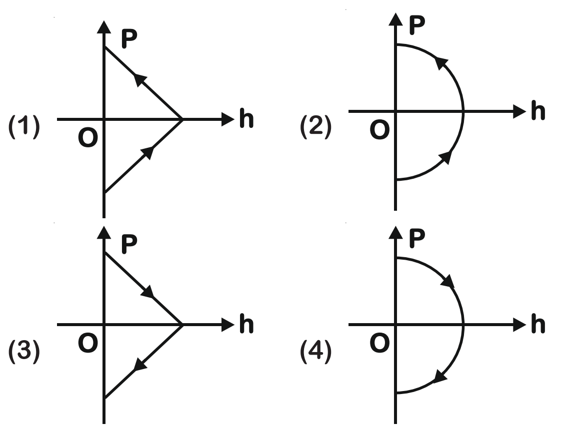
$$\nu = \frac{3\times10^{8}}{5\times10^{-7}} = 6\times10^{14}\,Hz$$

$$E = \frac{12375}{5000} = 2.475\,eV$$

$K_{max} = E - \phi$

$= 0.48$ eV

28. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is:



Sol. $V = \sqrt{V_0^2 - 2gh}$

Direction of velocity changes at top most point

29. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is

(1) 2 M

(2) M

(3) 4 M

(4) $\frac{M}{2}$

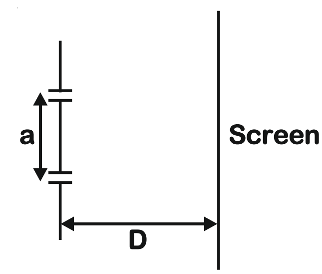
Answer (1)

Sol. $h \propto 1/r$

$M \propto \pi r^2 h$

$M \propto r$

30. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be



(1) $\frac{2nD\lambda}{a(\mu-1)}$

(2) $\frac{2D\lambda}{a(\mu-1)}$

(3) $\frac{nD\lambda}{a(\mu-1)}$

(4) $\frac{D\lambda}{a(\mu-1)}$

Answer (Bonus)

Sol. $(\mu - 1)\,t = n\,\lambda$

$$t = \frac{n\lambda}{\mu-1}$$

1. C_{60}, an allotrope of carbon contains

(1) 16 hexagons and 16 pentagons

(2) 18 hexagons and 14 pentagons

(3) 20 hexagons and 12 pentagons

(4) 12 hexagons and 20 pentagons

Answer (3)

Sol. Fullerene (C_{60}) contains 20 six membered rings and 12 five membered rings.

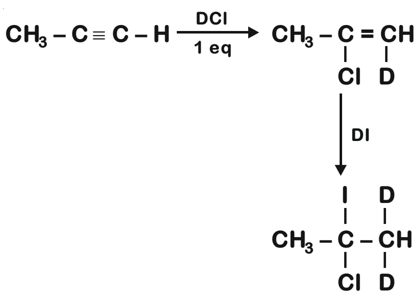
2. The major product of the following reaction is

$$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1 equiv.)}}$$

(1) $CH_3C(I)(Cl)CHD_2$ (2) $CH_3CD(I)CHD(Cl)$

(3) $CH_3CD(Cl)CHD(I)$ (4) $CH_3CD_2CH(Cl)(I)$

Answer (1)

Sol. 

Both addition follow Markownikov's rule.

3. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is

$Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$,

$E° = 2$ V at 298 K

(Faraday's constant, F = 96000 C mol^{-1})

(1) 192

(2) 384

(3) –384

(4) –192

Answer (3)

Sol. $\Delta G° = -nFE°_{cell}$

$= -2 \times (96000) \times 2$ V

$= -384$ kJ/mole

4. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is

(1) 4 (2) 1

(3) 2 (4) 3

Answer (2)

Sol. In $CuSO_4 \cdot 5H_2O$, four H_2O molecules are directly coordinated to the central metal ion while one H_2O molecule is hydrogen bonded.

5. Magnesium powder burns in air to give

(1) $Mg(NO_3)_2$ and Mg_3N_2

(2) MgO and $Mg(NO_3)_2$

(3) MgO and Mg_3N_2

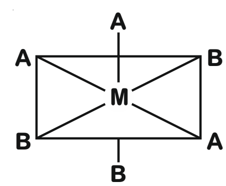
(4) MgO only

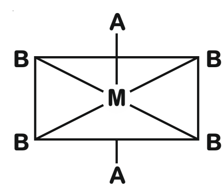
Answer (3)

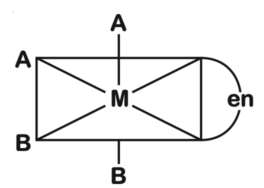
Sol. Mg burn in air and produces a mixture of nitride and oxide.

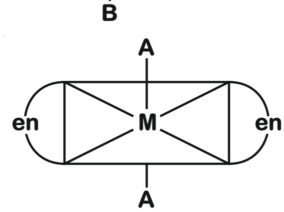
So, Mg_3N_2 and MgO are formed.

6. The one that will show optical activity is (en = ethane-1,2-diamine)

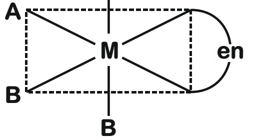
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

has no plane of symmetry or centre of symmetry, hence it is optically active.

7. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is

(x_M = Mole fraction of 'M' in solution;

x_N = Mole fraction of 'N' in solution;

y_M = Mole fraction of 'M' in vapour phase;

y_N = Mole fraction of 'N' in vapour phase)

(1) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ (2) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

(3) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$ (4) $(x_M - y_M) < (x_N - y_N)$

Answer (2)

Sol. $P_M^o = 450$ mmHg, $P_N^o = 700$ mmHg

$P_M = P_M^o X_M = Y_M P_T$

$\Rightarrow P_M^o = \frac{Y_M}{X_M}(P_T)$

Similarly, $P_N^o = \frac{Y_N}{X_N}(P_T)$

Given, $P_M^o < P_N^o$

$\Rightarrow \frac{Y_M}{X_M} < \frac{Y_N}{X_N}$

$\Rightarrow \frac{Y_M}{Y_N} < \frac{X_M}{X_N}$

8. The major product of the following reaction is

$$CH_3CH = CHCO_2CH_3 \xrightarrow{LiAlH_4}$$

(1) $CH_3CH_2CH_2CH_2OH$

(2) $CH_3CH_2CH_2CHO$

(3) $CH_3CH_2CH_2CO_2CH_3$

(4) $CH_3CH = CHCH_2OH$

Answer (4)

$$CH_3 - CH = CH - CO_2Me \xrightarrow{LiAlH_4} CH_3 - CH = CH - CH_2 - OH$$

9. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman} / \Delta\bar{v}_{Balmer}$ is

(1) 9 : 4 (2) 27 : 5

(3) 4 : 1 (4) 5 : 4

Answer (1)

Sol. $\bar{v} \propto \Delta E$

For H-atom

$$\bar{v} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series,

$$\bar{v}(\text{max}) \propto 13.6\left(1 - \frac{1}{\infty}\right)$$

$$\bar{v}(\text{min}) \propto 13.6\left(1 - \frac{1}{4}\right)$$

$$\therefore \quad \bar{v}_{max} - \bar{v}_{min} \propto 13.6\left(\frac{1}{4}\right)$$

For Balmer series,

$$\bar{v}(\text{max}) \propto 13.6\left(\frac{1}{4} - \frac{1}{\infty}\right)$$

$$\bar{v}(\text{min}) \propto 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\therefore \quad \bar{v}_{min} - \bar{v}_{min} \propto 13.6\left(\frac{1}{9}\right)$$

$$\frac{v_{Lyman}}{v_{Balmer}} = \frac{9}{4}$$

10. Among the following, the set of parameters that represents path functions, is

(A) q + w (B) q

(C) w (D) H – TS

(1) (A), (B) and (C) (2) (B) and (C)

(3) (B), (C) and (D) (4) (A) and (D)

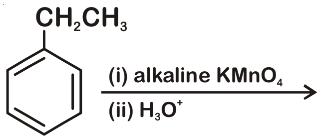
Answer (2)

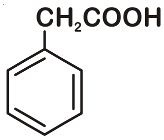
(B) q, path function

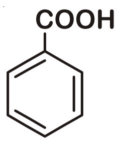
(C) w, path function

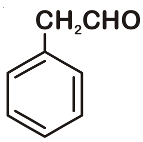
(D) H – TS = G, state function

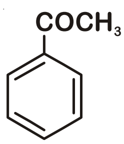
11. The major product of the following reaction is



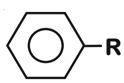
(1) 

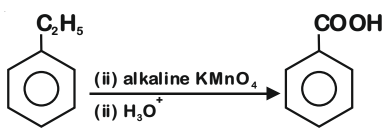
(2) 

(3) 

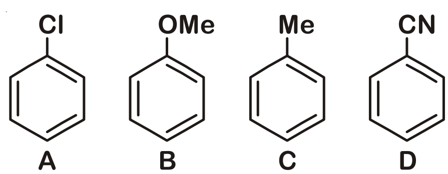
(4) 

Answer (2)

Sol. Alkaline $KMnO_4$ converts  with a benzylic hydrogen into benzoic acid.



12. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is



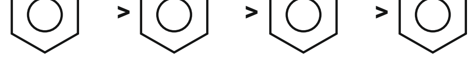
(1) D < B < A < C

(2) D < A < C < B

(3) B < C < A < D

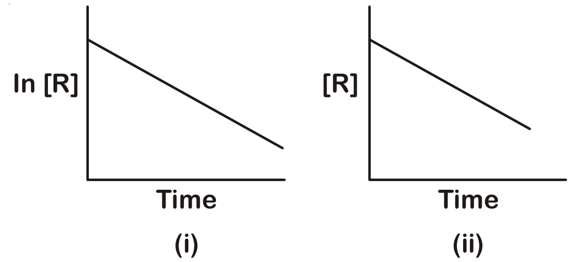
(4) A < B < C < D

Answer (2)

Sol. 

(+M) (+H) (–I > +M) (–M, –I)

13. The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are



(1) 0, 1 (2) 1, 1

(3) 1, 0 (4) 0, 2

Answer (3)

Sol. Graph-(i) : In[Reactant] vs time is linear

Hence, 1st order

Graph-(ii) : [Reactant] vs time is linear

Hence, zero order

14. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is

(1) 16×10^{-4} (2) 4×10^{-4}

(3) 6×10^{-2} (4) 4×10^{-2}

Answer (3)

Sol. $\pi_{XY} = 4\pi_{BaCl_2}$

$\therefore$ 2[XY] = $4 \times (0.01) \times 3$

[XY] = 0.06

$= 6\times10^{-2}\ \dfrac{mol}{L}$

15. Consider the van der Waals constants, a and b, for the following gases.

Gas	Ar	Ne	Kr	Xe
a/(atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/(10^{-2} dm^3 mol^{-1})	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?

(1) Ne (2) Kr

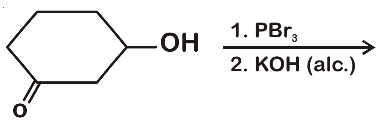
(3) Xe (4) Ar

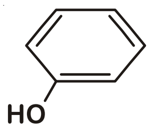
Answer (2)

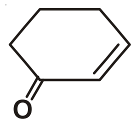
27.Rb

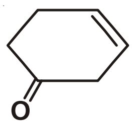
So, species with greatest value of $\frac{a}{b}$ has greatest value of critical temperature i.e. Kr.

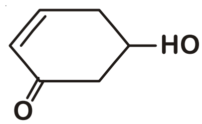
16. The major product of the following reaction is



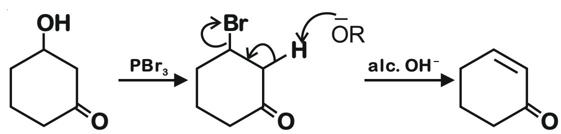
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

More stable product due to conjugation.

17. Among the following, the molecule expected to be stabilized by anion formation is

C_2, O_2, NO, F_2

(1) F_2 (2) NO

(3) C_2 (4) O_2

Answer (3)

Sol. C_2 has s-p mixing and the HOMO is $\pi 2p_x = \pi 2p_y$ and LUMO is $\sigma 2p_z$. So, the extra electron will occupy bonding molecular orbital and this will lead to an increase in bond order.

C_2^- has more bond order than C_2.

18. Excessive release of CO_2 into the atmosphere results in

(1) Depletion of ozone

(2) Polar vortex

(3) Formation of smog

(4) Global warming

Answer (4)

Sol. CO_2 causes global warming.

$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$,

Identify dihydrogen (H_2) as a limiting reagent in the following reaction mixtures.

(1) 35 g of N_2 + 8 g of H_2

(2) 28 g of N_2 + 6 g of H_2

(3) 56 g of N_2 + 10 g of H_2

(4) 14 g of N_2 + 4 g of H_2

Answer (3)

Sol. 28 g N_2 react with 6 g H_2

$$\underset{\substack{\text{1 mole} \\ \text{28 g}}}{N_2} + \underset{\substack{\text{3 moles} \\ \text{6 g}}}{3H_2} \longrightarrow 2NH_3$$

For 56 g of N_2, 12 g of H_2 is required.

20. Match the catalysts (Column I) with products (Column II).

Column I	Column II
Catalyst	Product
(A) V_2O_5	(i) Polyethylene
(B) $TiCl_4/Al(Me)_3$	(ii) Ethanal
(C) $PdCl_2$	(iii) H_2SO_4
(D) Iron Oxide	(iv) NH_3

(1) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

(2) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(3) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

(4) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)

Answer (3)

Sol. (A) V_2O_5 → Preparation of H_2SO_4 in contacts process

(B) $TiCl_4 + Al(Me)_3$ → Polyethylene (Ziegler-Natta catalyst)

(C) $PdCl_2$ → Ethanal (Wacker's process)

(D) Iron oxide → NH_3 in Haber's process

21. The element having greatest difference between its first and second ionization energies, is

(1) K (2) Sc

(3) Ca (4) Ba

Answer (1)

Sol. Alkali metals have high difference in the first ionisation and the second ionisation energy as they achieve stable noble gas configuration after first ionisation.

(1) $NO_2 < NO < N_2O_3 < N_2O$

(2) $N_2O < NO < N_2O_3 < NO_2$

(3) $NO_2 < N_2O_3 < NO < N_2O$

(4) $N_2O < N_2O_3 < NO < NO_2$

Answer (2)

Sol.

(oxide)	(oxidation state)
N_2O	+ 1
NO	+ 2
N_2O_3	+ 3
NO_2	+ 4

So, $N_2O < NO < N_2O_3 < NO_2$

23. Which of the following statements is not true about sucrose?

(1) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose

(2) On hydrolysis, it produces glucose and fructose

(3) It is a non-reducing sugar

(4) It is also named as invert sugar

Answer (1)

Sol. Sucrose contains glycosidic link between C_1 of α-D glucose and C_2 of β-D-Fructose.

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow \text{Glucose} + \text{Fructose}$$

24. The ore that contains the metal in the form of fluoride is

(1) malachite (2) sphalerite

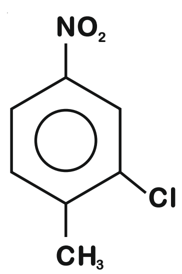
(3) magnetite (4) cryolite

Answer (4)

Sol.

Magnetite	Fe_3O_4
Sphalerite	ZnS
Cryolite	Na_3AlF_6
Malachite	$CuCO_3 \cdot Cu(OH)_2$

25. The correct IUPAC name of the following compound is



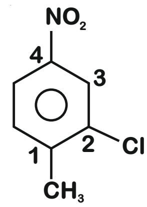
(2) 3-chloro-4-methyl-1-nitrobenzene

(3) 2-methyl-5-nitro-1-chlorobenzene

(4) 2-chloro-1-methyl-4-nitrobenzene

Answer (4)

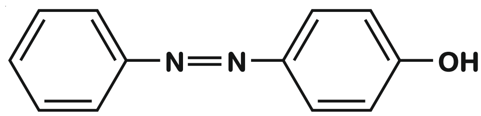
Sol.

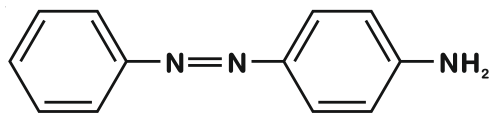


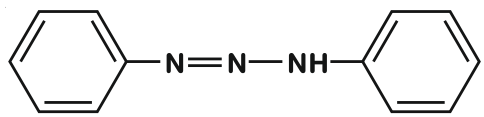
All Groups attached are to be treated as substituents and lowest set of locant rule is followed.

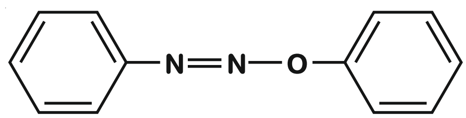
2-Chloro-1-methyl-4-nitrobenzene

26. Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is

(1) 

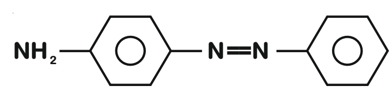
(2) 

(3) 

(4) 

Answer (2)

Sol. In acidic medium aniline is more reactive than phenol that's why electrophilic aromatic substitution of $Ph—\overset{+}{N}_2$ takes place with aniline



27. The aerosol is a kind of colloid in which

(1) solid is dispersed in gas

(2) gas is dispersed in solid

(3) liquid is dispersed in water

(4) gas is dispersed in liquid

Answer (1)

28. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are

(1) d_{xz} and d_{yz}

(2) $d_{x^2-y^2}$ and d_{xy}

(3) d_{z^2} and d_{xz}

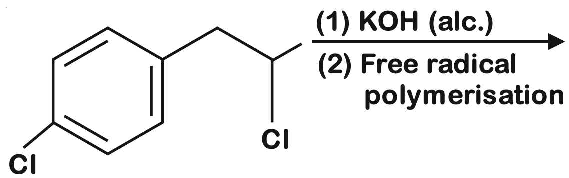
(4) d_{yz} and d_{z^2}

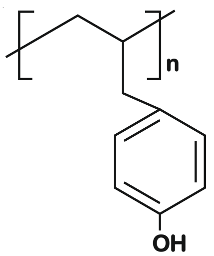
Answer (1)

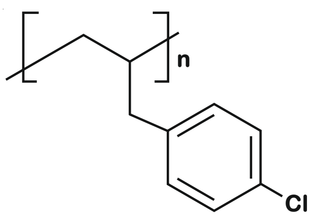
Sol. Cr^{3+} has d^3 configuration and forms an octahedral inner orbitals complex.

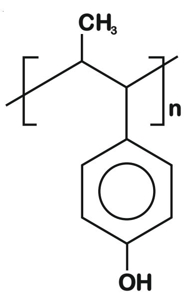
The set of degenerate orintals are (d_{xy}, d_{yz} and d_{xz}) and ($d_{x^2-y^2}$ and d_{z^2}).

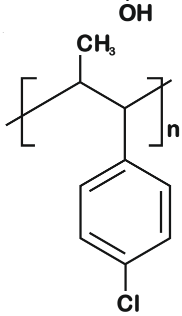
29. The major product of the following reaction is

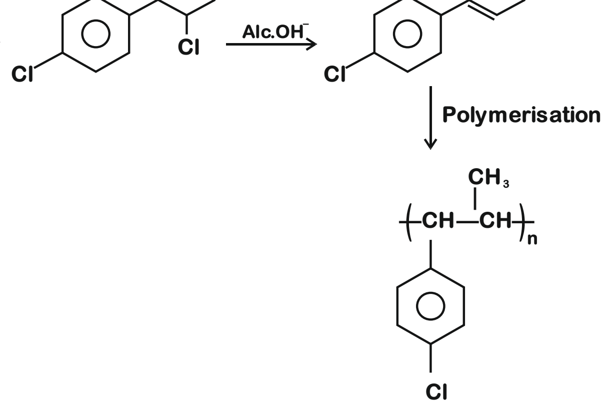


(1) 

(2) 

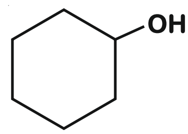
(3) 

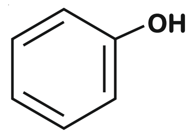
(4) 

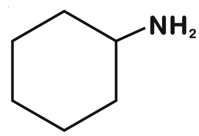
Sol. 

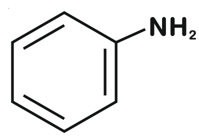
30. The organic compound that gives following qualitative analysis is

	Test	Inference
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	Soluble
(c)	Br_2/water	Decolourization

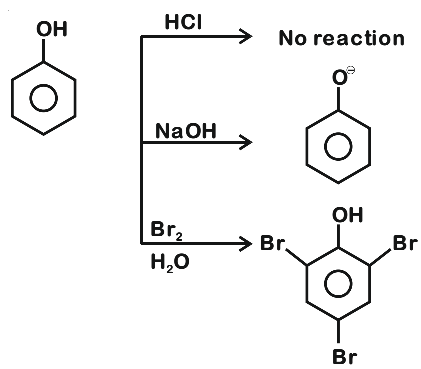
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

1. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $q^2 - 4p - 16 = 0$ (2) $p^2 - 4q + 12 = 0$

(3) $p^2 - 4q - 12 = 0$ (4) $q^2 + 4p + 14 = 0$

Answer (3)

Sol. p, q are rational numbers

$\therefore$ $2+\sqrt{3}$ in the other root

Now, p = –4, q = 1

$\Rightarrow$ $p^2 - 4q - 12 = 16 - 4 - 12$

$= 0$

Note:- (Erratum) p, q, should be given as rational numbers instead of real numbers

2. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is :

(1) $2\sqrt{5}$ (2) $\frac{9}{2}$

(3) $\frac{7}{2}$ (4) $\frac{\sqrt{5}}{2}$

Answer (2)

Sol. Let point on line is p(2r + 1, 3r – 1, 4r + 2)

It lies on the plane x + 2y + 3z = 15

$\therefore$ $2r + 1 + 6r - 2 + 12r + 6 = 15$

$\Rightarrow$ $r=\frac{1}{2}$

$\therefore$ $P=\left(2,\frac{1}{2},4\right)$

$\therefore$ $OP=\sqrt{4+\frac{1}{4}+16}=\sqrt{\frac{81}{4}}=\frac{9}{2}$

3. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :

(1) $x^2 + y^2 - 16x^2y^2 = 0$

(2) $x^2 + y^2 - 2x^2y^2 = 0$

(3) $x^2 + y^2 - 4x^2y^2 = 0$

(4) $x^2 + y^2 - 2xy = 0$

Answer (3)

Sol. Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

$\therefore$ $P\left(\frac{1}{\cos\theta},0\right)$; $Q\left(0,\frac{1}{\sin\theta}\right)$

$\therefore$ Mid-point of PQ let $M\left(\frac{1}{2\cos\theta},\frac{1}{2\sin\theta}\right)=(h,k)$

$\Rightarrow$ $\cos\theta=\frac{1}{2h}$; $\sin\theta=\frac{1}{2k}$

$\therefore$ On squaring and adding

$\frac{1}{h^2}+\frac{1}{k^2}=4$ $\Rightarrow x^2 + y^2 = 4x^2y^2$

4. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set

$S = \{x \in R : f(x) = f(0)\}$

contains exactly :

(1) Four irrational numbers

(2) Four rational numbers

(3) Two irrational and one rational number

(4) Two irrational and two rational numbers

Answer (3)

Sol. $f'(x) = A(x + 1)\, x\, (x - 1)$

$= A(x^3 - x)$

$\Rightarrow$ $f(x)=A\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$

Now f(0) = C

$\therefore$ f(x) = f(0) $\Rightarrow$ $A\left(\frac{x^4}{4}-\frac{x^2}{2}\right)=0$

$\Rightarrow$ $\frac{x^2}{2}\left(\frac{x^2}{2}-1\right)=0$

$\Rightarrow$ $x=0, 0,-\sqrt{2},\sqrt{2}$

$\therefore$ $S=\{0,-\sqrt{2},\sqrt{2}\}$

1, k is $\sqrt{5}$ where k > 0, then k is equal to :

(1) $\sqrt{6}$ (2) $2\sqrt{6}$

(3) $2\sqrt{\frac{10}{3}}$ (4) $4\sqrt{\frac{5}{3}}$

Answer (2)

Sol. Mean of given observation = $\frac{k}{4}$

$\therefore$ $\sigma^2 = 5$(given) ...(i)

Also $\sigma^2 = \frac{\left(\frac{k}{4}+1\right)^2+\left(\frac{k}{4}\right)^2+\left(\frac{k}{4}-1\right)^2+\left(\frac{3k}{4}\right)^2}{4}$ ...(ii)

$\therefore$ from (i) and (ii)

$$\frac{\frac{12k^2}{16}+2}{4}=5 \Rightarrow \frac{12k^2}{16}=18$$

$\Rightarrow$ $k^2 = 24$ $\Rightarrow k = 2\sqrt{6}$

6. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is :

(1) 8^3 (2) 8

(3) 8^{-2} (4) 8^2

Answer (4)

Sol. $T_4 = 20\times 8^7 = {}^6C_3\left(\frac{2}{x}\right)^3\times\left(x^{\log_8 x}\right)^3$

$$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7$$

$$\Rightarrow \left(\frac{x^{\log_8 x}}{x}\right)^3 = \left(8^2\right)^3$$

$\Rightarrow \frac{x^{\log_8 x}}{x} = 64$ Take $\log_8$ both side

$\Rightarrow$ $(\log_8 x)^2 - (\log_8 x) = 2$

$\Rightarrow$ $\log_8 x = -1$ or $\log_8 x = 2$

$\Rightarrow$ $x = \frac{1}{8}$ or $x = 8^2$

(Here C is a constant of integration)

(1) $-3\cot^{-1/3}x + C$

(2) $-3\tan^{-1/3}x + C$

(3) $-\frac{3}{4}\tan^{-4/3}x + C$

(4) $3\tan^{-1/3}x + C$

Answer (2)

Sol. $I = \int \sec^{\frac{2}{3}} x \cdot \operatorname{cosec}^{\frac{4}{3}} dx$

$I = \int \frac{\sec^2 x\,dx}{\tan^{\frac{4}{3}} x}$ Put tan x = t

$\Rightarrow \sec^2 x\, dx = dt$

$\Rightarrow$ $I = \frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+C$ $\Rightarrow I = -3(\tan x)^{\frac{-1}{3}}+C$

8. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :

(1) 24

(2) 20

(3) 22

(4) 25

Answer (4)

Sol. $\because$ $y^2 = 16x$ $\Rightarrow a = 4$

One end of focal chord (1, 4) $\therefore$ 2 at = 4

$\Rightarrow t = \frac{1}{2}$

Length of focal chord $= a\left(t+\frac{1}{t}\right)^2$

$= 4\times\left(2+\frac{1}{2}\right)^2$

= 25

9. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x)=\begin{cases}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\dfrac{\pi}{4}\\ k, & x=\dfrac{\pi}{4}\end{cases}$$

is continuous, then k is equal to :

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

Answer (2)

Sol. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}=k$ $\quad\therefore$ By L hospital rule.

$\Rightarrow \lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x}=k$ $\quad\Rightarrow k=\dfrac{1}{2}$

10. All the points in the set

$$S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in R\right\}(i=\sqrt{-1})$$

lie on a :

(1) Straight line whose slope is 1

(2) Circle whose radius is $\sqrt{2}$

(3) Circle whose radius is 1

(4) Straight line whose slope is –1

Answer (3)

Sol. $\because$ $S=\dfrac{\alpha+i}{\alpha-i}$ $\quad$ Let S = x + iy

$\Rightarrow x+iy=\dfrac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$\Rightarrow x+iy=\dfrac{(\alpha^2-1)}{\alpha^2+1}+\dfrac{i(2\alpha)}{\alpha^2+1}$ (On comparing both sides)

$\Rightarrow x=\dfrac{\alpha^2-1}{\alpha^2+1}$ ... (i) $\quad y=\dfrac{2\alpha}{\alpha^2+1}$ ... (ii)

By squaring and adding

$\Rightarrow x^2+y^2=1$

where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to :

(1) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$ (2) $\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$

(3) $-3\hat{i}+9\hat{j}+5\hat{k}$ (4) $3\hat{i}-9\hat{j}-5\hat{k}$

Answer (2)

Sol. $\vec{\beta}=\vec{\beta_1}-\vec{\beta_2}$ $\quad$ ...(i)

$\therefore\ \vec{\beta_2}\cdot\vec{\alpha}=0$

and Let $\vec{\beta_1}=\lambda\vec{\alpha}$

$\vec{\alpha}\cdot\vec{\beta}=\vec{\alpha}\cdot\vec{\beta_1}-\vec{\alpha}\cdot\vec{\beta_2}$

$\Rightarrow 5=\lambda\alpha^2$

$\Rightarrow 5=\lambda\times 10$

$\Rightarrow \lambda=\dfrac{1}{2}$

$\therefore\ \boxed{\vec{\beta_1}=\dfrac{\vec{\alpha}}{2}}$

Cross product with $\vec{\beta_1}$ in equation (i)

$\Rightarrow \vec{\beta}\times\vec{\beta_1}=-\vec{\beta_2}\times\vec{\beta_1}$

$\Rightarrow \boxed{\vec{\beta}\times\vec{\beta_1}=\vec{\beta_1}\times\vec{\beta_2}}=\dfrac{(\vec{\beta}\times\vec{\alpha})}{2}$

$\Rightarrow \vec{\beta_1}\times\vec{\beta_2}=\dfrac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 3\\ 3 & 1 & 0\end{vmatrix}$

$=\dfrac{1}{2}\left[-3\hat{i}-\hat{j}(-9)+\hat{k}(5)\right]$

$=\dfrac{1}{2}\left[-3\hat{i}+9\hat{j}+5\hat{k}\right]$

12. Let $\sum\limits_{k=1}^{10}f(a+k)=16(2^{10}-1)$, where the function f satisfies f(x + y) = f(x) f(y) for all natural numbers x, y and f(1) = 2. Then the natural number 'a' is :

(1) 2

(2) 3

(3) 16

(4) 4

Answer (2)

$\therefore$ Let $\boxed{f(x) = b^x}$

$\because$ $f(1) = 2$

$\therefore$ $b' = 2$

$\Rightarrow$ $\boxed{f(x) = 2^x}$

Now, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10} - 1)$

$\Rightarrow$ $2^a \sum_{k=1}^{10} 2^k = 16(2^{10} - 1)$

$\Rightarrow$ $2^a \times \frac{((2^{10}) - 1) \times 2}{(2 - 1)} = 16 \times (2^{10} - 1)$

$\Rightarrow$ $2^a = 8$

$\Rightarrow$ $\boxed{a = 3}$

13. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots\cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$,

then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is :

(1) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

Answer (2)

Sol. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 1 & 1+2+3+\ldots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\frac{(n-1)n}{2} = 78$

$\Rightarrow$ $n = 13$

Now, inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

14. Let $S = \{\theta \in [-2\pi, 2\pi] ; 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

(1) π (2) 2π

(3) $\frac{13\pi}{6}$ (4) $\frac{5\pi}{3}$

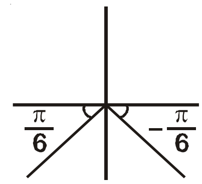
Answer (2)

$\Rightarrow$ $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow$ $(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow$ $\sin\theta = -\frac{1}{2}$; $\boxed{\sin\theta = 2} \rightarrow$ Not Possible

$\Rightarrow$



$\therefore$ Sum of all solutions in $[-2\pi, 2\pi]$ is

$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$

15. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) (10, 15)

(2) {5, 10, 15, 20}

(3) {10}

(4) {5, 10, 15}

Answer (4)

Sol. Given $f(x) = 15 - |(10 - x)|$

$\Rightarrow$ $f(f(x)) = 15 - ||10 - x| - 5|$

$\therefore$ Non-differentiable at points where

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow$ $x = 10$ and $x - 10 = \pm 5$

$\Rightarrow$ $x = 10$ and $x = 15, 5$

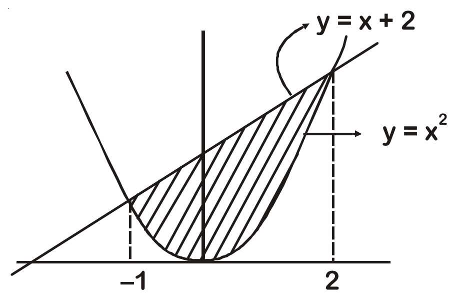
16. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is :

(1) $\frac{31}{6}$ (2) $\frac{10}{3}$

(3) $\frac{9}{2}$ (4) $\frac{13}{6}$

Answer (3)

Sol.



$$=\left(\frac{x^2}{2}+2x-\frac{x^3}{3}\right)_{-1}^{2}$$

$$=\left(2+4-\frac{8}{3}\right)-\left(+\frac{1}{2}-2+\frac{1}{3}\right)$$

$$=8-3-\frac{1}{2}$$

$$=5-\frac{1}{2}=\frac{9}{2}$$

17. If the line $y=mx+7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is :

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{\sqrt{15}}{2}$ (4) $\frac{\sqrt{5}}{2}$

Answer (1)

Sol. $mx-y+7\sqrt{3}$ is normal to hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$

then $\frac{24}{m^2}-\frac{18}{(-1)^2}=\frac{(24+18)^2}{(7\sqrt{3})^2}$

$$\Rightarrow \frac{24}{m^2}-18=\frac{42\times 42}{7\times 7\times 3}$$

$$\Rightarrow m=\frac{2}{\sqrt{5}}$$

If $lx + my + n = 0$ is a normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

then $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$

18. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$ is equal to :

(1) $y(y^2 - 1)$ (2) $y^3 - 1$

(3) $y(y^2 - 3)$ (4) y^3

Sol. Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

$$\therefore \begin{vmatrix} y+\omega & 1 & \omega^2 \\ 1 & y+\omega^2 & \omega \\ \omega^2 & \omega & 1+y \end{vmatrix} \text{ operate } c_1 \to c_1 + c_2 + c_3$$

$$=y\begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & y+\omega^2 & \omega \\ 1 & \omega & 1+y \end{vmatrix} \left(\text{By } \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}\right)$$

$$=y\begin{vmatrix} 1 & 1 & \omega^2 \\ 0 & y+\omega^2-1 & \omega-\omega^2 \\ 0 & \omega-1 & 1+y-\omega^2 \end{vmatrix}$$

$= y\{(y + \omega^2 - 1)(1 + y - \omega^2) - \omega(\omega - 1)(1 - \omega)\}$

$= y(y^2 - (\omega^2 - 1)^2) + y\omega(\omega - 1)^2$

$= y^3 + y(\omega - 1)^2 (\omega - (\omega + 1)^2) = y^3$

19. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{2}(1+\cos 20°)$

(3) $\frac{3}{2}$ (4) $\frac{3}{4}+\cos 20°$

Answer (1)

Sol. $\left(\frac{1+\cos 20°}{2}\right)+\left(\frac{1+\cos 100°}{2}\right)-\frac{1}{2}(2\cos 10° \cos 50°)$

$$=1+\frac{1}{2}(\cos 20°+\cos 100°)-\frac{1}{2}[\cos 60°+\cos 40°]$$

$$=\left(1-\frac{1}{4}\right)+\frac{1}{2}[\cos 20°+\cos 100°-\cos 40°]$$

$$=\frac{3}{4}+\frac{1}{2}[2\cos 60°\times\cos 40°-\cos 40°]$$

$$=\frac{3}{4}$$

20. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m = n = 68 (2) m + n = 68

(3) m = n = 78 (4) n = m – 8

Answer (3)

$n = C_3 \; C_8 + C_4 \; C_7 + C_5 \; C_6 = 78$

So m = n = 78

21. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is :

(1) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (2) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(3) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (4) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

Answer (2)

Sol. Point at 4 units from P(2, 3) will be

A(4cosθ + 2, 4sinθ + 3) will satisfy x + y = 7

$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$ on squaring

$\Rightarrow \boxed{\sin 2\theta = \frac{-3}{4}} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6}$ (Ignoring –ve sign)

$\Rightarrow \tan\theta = \frac{-8+2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$

22. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is :

(1) $\frac{\pi-2}{4}$ (2) $\frac{\pi-2}{8}$

(3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-1}{2}$

Answer (3)

Sol. $I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x \, dx}{\sin x + \cos x}$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^3 x \, dx}{\sin x + \cos x}$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2}\sin(2x)\right) dx$

$\Rightarrow I = \frac{1}{2}\left[x + \frac{1}{4}\cos 2x\right]_0$

$I = \frac{1}{2}\left(\frac{\pi-1}{2}\right) = \frac{\pi-1}{4}$

23. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f(–1)), then S is equal to :

(1) $\left\{\frac{1}{3}, -1\right\}$ (2) $\left\{-\frac{1}{3}, 1\right\}$

(3) $\left\{-\frac{1}{3}, -1\right\}$ (4) $\left\{\frac{1}{3}, 1\right\}$

Answer (2)

Sol. $y = f(x) = x^3 - x^2 - 2x$

$\frac{dy}{dx} = 3x^2 - 2x - 2$

f(1) = 1 – 1 – 2 = –2, f(–1) = –1 – 1 + 2 = 0

According to question, $3x^2 - 2x - 2$

$= \frac{f(1) - f(-1)}{1-(-1)}$

$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2}$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow x = \frac{2 \pm 4}{6} = 1, \frac{-1}{3}$

So, $S = \left\{\frac{-1}{3}, 1\right\}$

24. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point :

(1) $\left(\sqrt{2}, 1, 4\right)$ (2) $\left(\sqrt{2}, -1, 4\right)$

(3) $\left(-\sqrt{2}, -1, -4\right)$ (4) $\left(-\sqrt{2}, 1, -4\right)$

Answer (1)

plane is y – z + 5 = 0

$$\therefore \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{\frac{1}{a^2}+1+1}\ \sqrt{2}}$$

$$\Rightarrow a^2 = \frac{1}{2}$$

$$\Rightarrow \boxed{\frac{1}{a} = \pm\sqrt{2}}$$

$$\Rightarrow \boxed{\pm\sqrt{2}x - y + z = 1}$$

$\therefore$ $(\sqrt{2}, 1, 4)$ satisfies $-\sqrt{2}\,x - y + z = 1$

25. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$ with y(1) = 1, is :

(1) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (2) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(3) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (4) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

Answer (3)

Sol. $\frac{dy}{dx} + \frac{2}{x}y = x$ y(1) = 1 (given)

$$I \cdot F = e^{\int \frac{2}{x}dx} = x^2$$

$$y \times x^2 = \int x^3\, dx$$

$$\Rightarrow yx^2 = \frac{x^4}{4} + c$$

$\because$ at x = 1 ; y = 1

$$\Rightarrow c = \frac{3}{4}$$

$$\therefore y = \frac{x^2}{4} + \frac{3}{4x^2}$$

constant A.P., a_1, a_2, a_3, ... be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to :

(1) (50, 50 + 46A) (2) (A, 50 + 45A)

(3) (A, 50 + 46A) (4) (50, 50 + 45A)

Answer (3)

Sol. $\because$ $S_n = \left(50 - \frac{7A}{2}\right)n + n^2 \times \frac{A}{2}$

$\therefore$ Common difference = $\frac{A}{2} \times 2 = \boxed{A}$

$a_{50} = a_1 + 49 \times d$

$= (50 - 3A) + 49\,A$

$= 50 + 46\,A$

So, $(d, a_{50}) = (A, 50 + 46\,A)$

27. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is :

(1) $\sim p \wedge \sim q$ (2) $\sim p \vee \sim q$

(3) $p \wedge q$ (4) $p \leftrightarrow q$

Answer (1)

Sol. $\sim(p \vee (\sim p \wedge q)) = \sim(\sim p \wedge q) \wedge \sim p$

$= (\sim q \vee p) \wedge \sim p$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim q \wedge \sim p) \vee (p \wedge \sim p)$

$= (\sim p \wedge \sim q)$

28. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, –x + y + 4 = 0, then which one of the following points lies on curve?

(1) (–2, 1) (2) (2, –2)

(3) (2, –1) (4) (–2, 2)

Answer (2)

Sol. $f(x) = x^3 + ax - b \Rightarrow f'(x) = 3x^2 + a$

f(1) = –5 and f′(1) = 3 + a

$\Rightarrow$ 1 + a – b = –5

$\Rightarrow$ a – b = –6 ...(1)

Also slope of tangent P(1, –5) = –1 = f′(1)

$\therefore$ 3 + a = –1

$\Rightarrow$ a = –4

∴ Equation of the curve is $f(x) = x^2 - 4x + 2$

∴ (2, –2) lies on the curve

29. If the function $f : R - \{1, -1\} \rightarrow A$ defined by $f(x) = \dfrac{x^2}{1-x^2}$, is surjective, then A is equal to :

(1) $[0, \infty)$ (2) $R - \{-1\}$

(3) $R - (-1, 0)$ (4) $R - [-1, 0)$

Answer (4)

Sol. $f(x) = \dfrac{x^2}{1-x^2}$

$$\Rightarrow \quad f(-x) = \frac{x^2}{1-x^2} = f(x)$$

$$\Rightarrow \quad f'(x) = \frac{2x}{(1-x^2)^2}$$

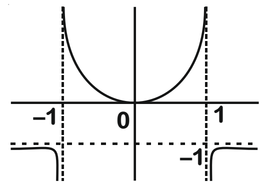
∴ f(x) increases in $x \in (0, \infty)$

Also $f(0) = 0$

$\lim\limits_{x \to \pm\infty} f(x) = -1$

and F(x) is even function

∴ Set $A \rightarrow R - [-1, 0)$



30. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

(1) $\dfrac{7}{32}$ (2) $\dfrac{25}{192}$

(3) $\dfrac{1}{192}$ (4) $\dfrac{25}{32}$

Answer (4)

Sol. P(at least one) = 1 – P(none)

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8}$$

$$= 1 - \frac{7}{32} = \boxed{\frac{25}{32}}$$

❑ ❑ ❑

Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T :

(1) T/R is a constant (2) TR is a constant

(3) T/R^2 is a constant (4) T^2/R^3 is a constant

Answer (1)

Sol. $M_{(in)} = \int_0^R 4\pi r^2 dr \cdot \frac{k}{r^2}$

$$M_{(in)} = 4\pi KR$$

$$\therefore \quad G \cdot \frac{4\pi KR}{R^2} = \frac{V^2}{R} \Rightarrow v = \sqrt{4\pi GK}$$

$$\therefore \quad T = \frac{2\pi R}{v} = \frac{2\pi \cdot R}{\sqrt{4\pi GK}}$$

$$\therefore \quad \frac{T}{R} = \text{constant}$$

2. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then :

(1) A is rigid but B has a vibrational mode.

(2) A has a vibrational mode but B has none.

(3) Both A and B have a vibrational mode each.

(4) A has one vibrational mode and B has two.

Answer (2)

Sol. For (A) $C_P = 29$, $C_V = 22$

For (B) $C_P = 30$ $C_V = 21$

$$\therefore \quad \gamma_A = \frac{C_{P_A}}{C_{V_A}} = \frac{29}{22} = 1.31$$

When A has vibrational degree of freedom, then $\gamma_A = 9/7 \simeq 1.29$.

$$\gamma_B = \frac{C_{P_B}}{C_{V_B}} = \frac{30}{21} = 1.42$$

$\Rightarrow$ B has no vibrational degree of freedom

3. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at t = 1?

(1) 50 (2) 100

(3) 40 (4) 25

Answer (1)

Sol. $\vec{r} = 15t^2\hat{i} + 4\hat{j} - 20t^2\hat{j}$

$$\frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$$

$$\frac{d^2\vec{r}}{dt^2} = 30\hat{i} - 40\hat{j}$$

$$\therefore \quad \left|\frac{d^2\vec{r}}{dt^2}\right| = 50\text{ m/s}^2$$

4. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3 \times 10^8$ m/s) :

(1) 20×10^{-8} N

(2) 35×10^{-8} N

(3) 15×10^{-8} N

(4) 10×10^{-8} N

Answer (1)

Sol. $\because \quad P = \frac{W}{c}$ and pressure = I/c

$$\therefore \quad \frac{h}{\lambda} + \frac{h}{4\lambda} \Rightarrow \Delta P = \frac{5h}{4\lambda} \text{ for one photon}$$

$$\therefore \quad \frac{5}{4} \cdot \frac{Nh}{\lambda \Delta t A} = \text{pressure}$$

$$\text{But } \left(\frac{Nhc}{\lambda \cdot \Delta t A}\right) = 50\text{ W/m}^2$$

$$\therefore \quad \frac{5}{4} \times \frac{50}{c} = \text{pressure}$$

$$\therefore \quad F_n = \frac{5 \times 50 \times 1\text{m}^2}{4 \times c} = 20 \times 10^{-8}\text{ N}$$

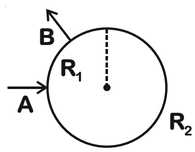
length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be :

(1) $\frac{5}{3}\ \Omega$ (2) $\frac{5}{2}\ \Omega$

(3) $\frac{7}{2}\ \Omega$ (4) $\frac{12}{5}\ \Omega$

Answer (1)

Sol.



$$R_0 = \rho\frac{l}{A} = 3\ \Omega$$

Now if l = 2l

Then A = $\frac{A}{2}$

$$\therefore \quad R = \frac{\rho 2l \times 2}{A} = 12\ \Omega$$

$$R_1 = \frac{12}{6} = 2\ \Omega$$

$R_2 = 10\ \Omega$

$$\therefore \quad \frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{10} = \frac{6}{10}$$

$$\therefore \quad R_{eq} = \frac{5}{3}\ \Omega$$

6. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be :

(1) $\frac{M\omega_0}{M+3m}$

(2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+6m}$

(4) $\frac{M\omega_0}{M+2m}$

Sol. Initial angular momentum = Final Angular Momentum

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + 2\frac{mL^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{M\omega_0}{M+6m}$$

7. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of

(1) 2.5 s (2) 2 s

(3) 5 s (4) 3 s

Answer (2)

Sol. I = 150; α = 20 rad/s^2

$\therefore \quad \omega = \alpha t$

$$E = \frac{1}{2}I\omega^2 = 1200$$

$$\frac{1}{2} \times 1.5 \times (20t)^2 = 1200\ \text{J}$$

$\Rightarrow$ t = 2 s

8. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A?

(1) 0.2 ohm

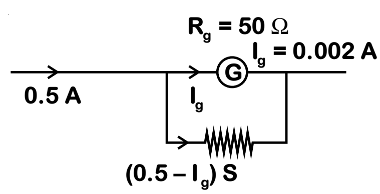
(2) 0.002 ohm

(3) 0.5 ohm

(4) 0.02 ohm

Answer (1)

Sol.



We have

$I_g R_g = (0.5 - I_g)$ S

$\Rightarrow$ (0.002) (50) = $(0.5 - I_g)$ S

$$\Rightarrow \quad S \simeq \frac{0.002 \times 50}{0.5} = 0.2\ \Omega$$

(1) 13.60 eV (2) 6.04 eV

(3) 48.36 eV (4) 54.40 eV

Answer (1)

Sol. $E_n: \dfrac{-E_0 z^2}{n^2} = \dfrac{-E_0 \times 4}{4} = -E_0$

To ionise it E_0 energy must be supplied.

$\therefore \quad E_0 = 13.6$ eV.

10. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B?

(speed of sound in air = 340 ms^{-1})

(1) 2150 Hz (2) 2300 Hz

(3) 2060 Hz (4) 2250 Hz

Answer (4)

Sol.

$$f = \frac{(v \pm u_0)}{(v \pm u_s)} \cdot f_0 = \frac{(v-20)}{(v+20)} \cdot f_0$$

$$\Rightarrow \quad 2000 = \frac{320}{360} \cdot f_0$$

$$\Rightarrow \quad \frac{2000 \times 9}{8} = f_0 = 2250\,\text{Hz}.$$

11. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :

(1) 3 : 1 (2) 2 : 1

(3) 4 : 3 (4) 5 : 3

Answer (1)

Sol. $\therefore \quad \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\therefore \quad v = (\pm 2u)$

$\therefore \quad \dfrac{1}{2u} + \dfrac{1}{u} = \dfrac{1}{20} \quad \Rightarrow \dfrac{3}{2u} = \dfrac{1}{20}$

$\therefore \quad u_1 = 30$ cm

And $\quad \dfrac{1}{u} - \dfrac{1}{2u} = \dfrac{1}{20}$

$\therefore \quad \dfrac{30}{10} = 3.$

12. The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be :

(1) $a + \dfrac{b^2}{4c}$ (2) $a + \dfrac{b^2}{3c}$

(3) $a + \dfrac{b^2}{2c}$ (4) $a + \dfrac{b^2}{c}$

Answer (2)

Sol. $x = at + bt^2 - ct^3$

$\dot{x} = a + 2bt - 3ct^2$

$\ddot{x} = 2b - 6ct$

For $\ddot{x} = 0 \quad t = +\dfrac{b}{3c}$

$$\therefore \quad v = \dot{x} = a + 2b\left(\frac{+b}{3c}\right) - 3c\left(\frac{b^2}{3c \times 3c}\right)$$

$$\Rightarrow \quad v = -\frac{b^2}{3c} + \frac{2b^2}{3c} + a = a + \frac{b^2}{3c}$$

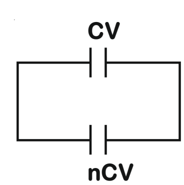
13. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :

(1) $\dfrac{(n+1)V}{(K+n)}$ (2) $\dfrac{V}{K+n}$

(3) V (4) $\dfrac{nV}{K+n}$

Answer (1)

Sol. Initially



$Q = CV(1 + n)$

$\therefore \quad C_{eq} = (K + n)C$

$$\therefore \quad V = \frac{CV(1+n)}{(K+n)C} = \frac{V(1+n)}{(K+n)}$$

are:

(1) Inversely proportional to modulation frequency

(2) Inversely proportional to carrier frequency

(3) Proportional to carrier frequency

(4) Independent of both carrier and modulation frequency

Answer (2)

Sol. Size of antenna depends on wavelength of carrier wave.

15. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :

(1) $\frac{v^2}{g}$ (2) $\frac{2v^2}{5g}$

(3) $\frac{2v^2}{7g}$ (4) $\frac{v^2}{2g}$

Answer (2)

Sol. mv = (4m + m)v′

$\therefore$ Common speed $v' = \frac{v}{5}$

$$mgh + \frac{1}{2}5m\cdot\frac{v^2}{25} = \frac{1}{2}mv^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2\left(1-\frac{1}{5}\right) = \frac{1}{2}mv^2\cdot\frac{4}{5}$$

$$\therefore \quad h = \frac{2mv^2}{5\times mg} = \frac{2v^2}{5g}$$

16. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is

(1) 0.5 (2) 0.8

(3) 0.7 (4) 0.6

Answer (4)

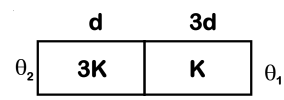
$$V\sigma g = \frac{v}{2}\rho_\omega g + \frac{v}{2}\rho_0 g$$

$$\Rightarrow \left(\frac{\rho_\omega}{2} + \frac{\rho_{oil}}{2}\right) = \frac{4}{5}\rho_\omega$$

$$\Rightarrow \frac{\rho_{oil}}{2} = \rho_\omega\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{10}\rho_\omega$$

$$\Rightarrow \rho_{oil} = \frac{3}{5}\rho_\omega = 0.6\rho_\omega$$

17. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is:



(1) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ (2) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$

(3) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (4) $\frac{\theta_2 + \theta_1}{2}$

Answer (2)

Sol. $H = \frac{3KA}{d}(\theta_2 - \theta) = \frac{KA}{3d}(\theta - \theta_1)$

$$\Rightarrow \theta = \frac{9\theta_2}{10} + \frac{\theta_1}{10}$$

18. The area of a square is 5.29 cm^2. The area of 7 such squares taking into account the significant figures is :

(1) 37.03 cm^2 (2) 37.0 cm^2

(3) 37.030 cm^2 (4) 37 cm^2

Answer (2)

Sol. 5.29 × 7 = 37.0 cm^2

Answer should be in 3 significant digits.

19. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm. coming from a distant object, the limit of resolution of the telescope is close to :

(1) 1.5 × 10^{-7} rad

(2) 3.0 × 10^{-7} rad

(3) 2.0 × 10^{-7} rad

(4) 4.5 × 10^{-7} rad

Sol. $\theta = \frac{1.22\lambda}{D}$

$\Rightarrow \quad \theta = \frac{1.22 \times 600 \times 10^{-9}}{250} \times 100 = 2.92 \times 10^{-7}$

$\Rightarrow \quad \theta = 3 \times 10^{-7}$ rad

20. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is

(1) $\frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$ (2) $\lambda_x - \lambda_y$

(3) $\lambda_x + \lambda_y$ (4) $\frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y}$

Answer (1)

Sol. $P_1 = \frac{h}{\lambda_x}$ $\qquad P_2 = \frac{h}{\lambda_y}$

$P = P_1 - P_2 = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right)$

$\therefore \quad P = \frac{h}{\lambda}$

$\frac{h}{\lambda} = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right)$

$\frac{1}{\lambda} = \frac{|\lambda_y - \lambda_x|}{\lambda_x \lambda_y}$

$\lambda = \frac{\lambda_x . \lambda_y}{|\lambda_x - \lambda_y|}$

21. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately

(1) 10^{-4} (2) 10^{-2}

(3) 10^{-1} (4) 10^{-3}

Answer (4)

Sol. NIAB = KQ

$175 \times 1 \times 10^{-3} \times 1 \times 10^{-4} \times B = \frac{10^{-6} \times \pi}{180}$

$\Rightarrow \quad B = \frac{\pi}{180} \times \frac{10}{175} \approx 9.97 \times 10^{-4}$ T.

$\Rightarrow \quad B = 10^{-3}$ T.

mean free time is 25 fs (femto second), it's approximate resistivity is

($m_e = 9.1 \times 10^{-31}$ kg)

(1) 10^{-5} Ωm (2) 10^{-6} Ωm

(3) 10^{-7} Ωm (4) 10^{-8} Ωm

Answer (4)

Sol. $J = \eta e v_d$ $\qquad \because \quad v_d = \frac{eE\tau}{m}$

$\therefore \quad J = \frac{\eta e \cdot eE \cdot \tau}{m} = \sigma \vec{E}$

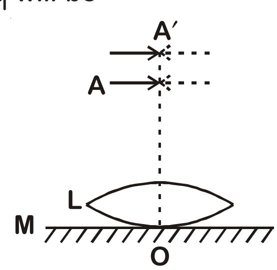
$J = \frac{\eta e^2 \tau}{m} \cdot E = \sigma \vec{E}$

$\therefore \quad \rho = \frac{m}{\eta e^2 \tau}$

$= \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}} \, \Omega \, m$

$= \frac{9.1}{8.5 \times 2.56 \times 25} \times 10^{(-59+53)} = 1.67 \times 10^{-8} \, \Omega \, m$

23. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A′, such that OA′ = 27 cm, to get its inverted real image at A′ itself. The value of μ_l will be



(1) $\sqrt{2}$ (2) $\frac{4}{3}$

(3) $\sqrt{3}$ (4) $\frac{3}{2}$

Answer (2)

Sol. f_L = 18 cm

$\frac{1}{18} = 0.5 \times \frac{2}{R} \quad \Rightarrow \quad R = 18$ cm

$\frac{1}{f_2} = (\mu_l - 1)\left(-\frac{1}{18}\right)$

$\Rightarrow$ $2 = 3(2 - \mu_l) = 6 - 3\mu_l$

$\Rightarrow$ $\mu_l = \dfrac{4}{3}$

24. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is

(1) 6.67×10^{-3} Wb (2) 6.67×10^{-4} Wb

(3) 3.67×10^{-3} Wb (4) 3.67×10^{-4} Wb

Answer (2)

Sol. $\phi_Q = Mi$

$\Rightarrow$ $10^{-3} = M(3)$

$\phi_P = M(2)$

$\therefore$ $\dfrac{10^{-3}}{3} = \dfrac{\phi_P}{2}$

$\phi_P = \dfrac{20}{3} \times 10^{-4} = 6.67 \times 10^{-4}$ Wb.

25. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)

(1) 10^{-5} K (2) 10^{-1} K

(3) 10^{-3} K (4) 10^{-4} K

Answer (1)

Sol. $\dfrac{1}{2}k\Delta x^2 = E(\text{dissipated})$

$\therefore$ $\dfrac{1}{2} \times 800 \times \left(\dfrac{2 \times 2}{100 \times 100}\right) = \dfrac{16}{100}$ J

$\dfrac{16}{100} = \dfrac{1}{2} \times 400 \times \Delta T + 1 \times 4184 \times \Delta T$

$\Rightarrow$ $\dfrac{16}{100} = (200 + 4184)\Delta T = 4384\ \Delta T$

$\therefore$ $\Delta T = \dfrac{16}{4384 \times 100} = 3.6 \times 10^{-5}$ K

speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be

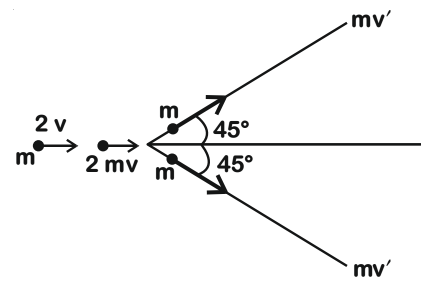
(1) $v/(2\sqrt{2})$ (2) $v/\sqrt{2}$

(3) $\sqrt{2}v$ (4) $2\sqrt{2}v$

Answer (4)

Sol. Initial momentum · $P_i = 2mv + 2mv = 4\ mv$

Let v' be the speed of I particle



$\therefore$ $2\dfrac{mv'}{\sqrt{2}} = 4mv$

$\Rightarrow$ $v' = 2\sqrt{2}\,v$

27. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is

(1) 320 m/s, 120 Hz

(2) 320 m/s, 80 Hz

(3) 180 m/s, 80 Hz

(4) 180 m/s, 120 Hz

Answer (2)

Sol. $\dfrac{\lambda}{2} = \dfrac{2}{3}$

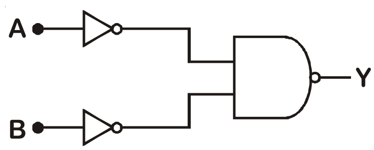
$\Rightarrow$ $\lambda = \dfrac{4}{3}$m

$\therefore$ $v = \dfrac{4}{3} \times 240 = 320$ m/s

3rd harmonic

$f_n = nf_0$

$f_0 = \dfrac{240}{3} = 80$ Hz



(1) OR

(2) NAND

(3) AND

(4) NOR

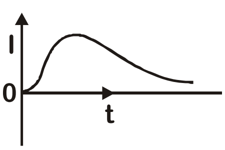
Answer (1)

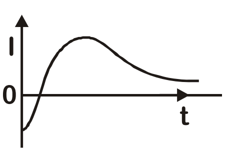
Sol. $y = \overline{\overline{A} \cdot \overline{B}} = A + B$

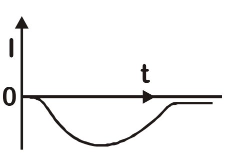
$y = A + B$

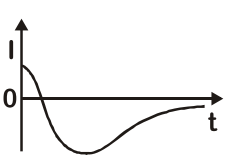
$\Rightarrow$ OR gate

29. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ $(k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by:

(1) t = 0 

(2) t = 0 

(3) t = 0 

(4) t = 0 

$\therefore \quad \phi = k_1 i = k_1 te^{-\alpha t} \qquad [k_1 = k\pi(2R)^2]$

$$\therefore \quad E = -\left(\frac{d\phi}{dt}\right) = -k_1 e^{-\alpha t} + k_1 \alpha t e^{-\alpha t}$$

$$= -k_1 e^{-\alpha t}(1 - \alpha t)$$

$$\therefore \quad \text{Induced current } I = \frac{E}{r} = -k_1 e^{-\alpha t}(1 - \alpha t)$$

30. For point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y =+2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as:

(1) $E \propto \frac{1}{D^3}$

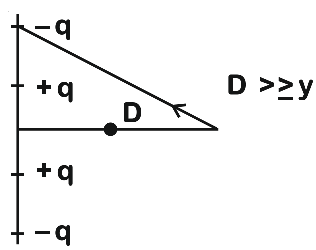
(2) $E \propto \frac{1}{D}$

(3) $E \propto \frac{1}{D^4}$

(4) $E \propto \frac{1}{D^2}$

Answer (3)

Sol.

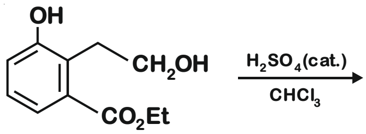


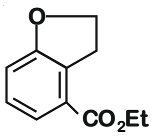
$$\vec{E} = \frac{2qD}{4\pi\varepsilon_0 \left(D^2 + 4y^2\right)^{\frac{3}{2}}} - \frac{2qD}{4\pi\varepsilon_0 (D^2 + y^2)^{\frac{3}{2}}} \quad (-\text{ve } \hat{x})$$

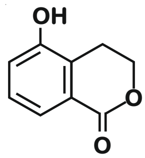
$$|\vec{E}| = \frac{2qD}{4\pi\varepsilon_0 D^3}\left[\frac{1}{\left(1 + \left(\frac{2y}{D}\right)^2\right)^{\frac{3}{2}}} - \frac{1}{\left(1 + \left(\frac{y}{D}\right)^2\right)^{\frac{3}{2}}}\right]$$

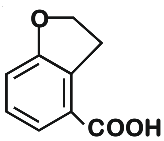
$$\Rightarrow \quad \vec{E} = \frac{2q}{4\pi\varepsilon_0 D^2}\left(1 - \frac{3}{2} \cdot \frac{4y^2}{D^2} - 1 + \frac{3}{2}\frac{y^2}{D^2}\right) = \frac{9qy^2}{4\pi\varepsilon_0 D^4}$$

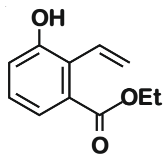
1. The major product of the following reaction is :



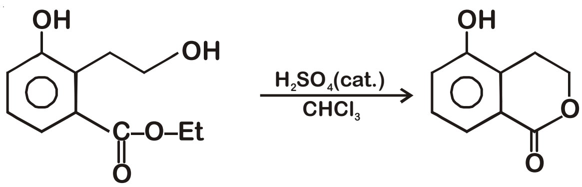
(1) 

(2) 

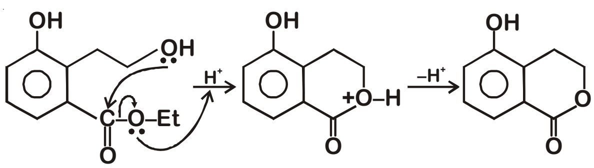
(3) 

(4) 

Answer (2)

Sol. 

Acid catalysed intramolecular esterification



2. The peptide that gives positive ceric ammonium nitrate and carbylamine tests is

(1) Ser - Lys (2) Lys - Asp

(3) Gln - Asp (4) Asp - Gln

Answer (1)

Sol. Ceric ammonium nitrate test is given by alcohol. Only serine(ser) contain –OH group.

3. Which of the following compounds is a constituent of the polymer $\left[HN - \overset{O}{\overset{||}{C}} - NH - CH_2 \right]_n$?

(1) Formaldehyde (2) Ammonia

(3) Methylamine (4) N-Methyl urea

Answer (1)

Sol. $\left[NH - \overset{O}{\overset{||}{C}} - NH - CH_2 \right]$ -urea-formaldehyde resin

$\therefore$ Monomers : Urea and formaldehyde

4. During compression of a spring the work done is 10 kJ and 2 kJ escaped to the surroundings as heat. The change in internal energy, ΔU (in kJ) is

(1) 12 (2) –12

(3) 8 (4) –8

Answer (3)

Sol. w = 10 kJ

q = – 2 kJ

$\Delta U = q + w = 10 - 2 = 8$ kJ

5. What would be the molality of 20% (mass/mass) aqueous solution of KI? (molar mass of KI = 166 g mol^{-1})

(1) 1.48 (2) 1.51

(3) 1.08 (4) 1.35

Answer (2)

Sol. 20% W/W KI solution

i.e. 100 g solution contains 20 g KI

$\therefore$ Mass of solvent = 100 – 20 = 80 g

$\therefore$ Molality = $\dfrac{20 \times 1000}{166 \times 80}$

$\simeq$ 1.51 molar

6. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?

(1) 0.20

(2) 0.15

(3) 0.10

(4) 0.05

Answer (4)

Sol. 0.1 F of electricity is passed through $Ni(NO_3)_2$ solution

$\therefore$ Amount of Ni deposited = 0.1 eq

$\therefore$ Moles = $\dfrac{0.1}{2} = 0.05$

(1) Strongest hydrogen bonding

(2) Lowest dissociation enthalpy

(3) Strongest van der Waals' interactions

(4) Lowest ionic character

Answer (1)

Sol. HF has highest boiling point among the hydrogen halides due to strong H-bonding between HF molecules.

8. The one that is not a carbonate ore is

(1) Bauxite (2) Calamine

(3) Siderite (4) Malachite

Answer (1)

Sol. Bauxite $\rightarrow$ $AlO_x(OH)_{3-2x}$

Calamine $\rightarrow$ $ZnCO_3$

Siderite $\rightarrow$ $FeCO_3$

Siderite $\rightarrow$ $FeCO_3$

Malachite $\rightarrow$ $CuCO_3 \cdot Cu(OH)_2$

9. Noradrenaline is a / an

(1) Neurotransmitter (2) Antihistamine

(3) Antacid (4) Antidepressant

Answer (1)

Sol. Noradrenaline is neurotransmitter.

10. Among the following species, the diamagnetic molecule is

(1) CO (2) NO

(3) O_2 (4) B_2

Answer (1)

Sol.

Molecule	No. of unpaired electrons
NO	1
CO	Zero
O_2	2
B_2	2

$\therefore$ Diamagnetic species is CO

11. The correct statements among I to III regarding group 13 element oxides are,

(I) Boron trioxide is acidic.

(II) Oxides of aluminium and gallium are amphoteric.

(III) Oxides of indium and thallium are basic.

(1) (II) and (III) only (2) (I) and (II) only

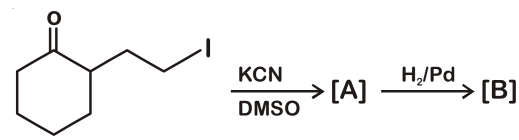
(3) (I), (II) and (III) (4) (I) and (III) only

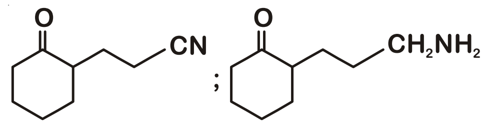
Sol. B_2O_3 is an acidic oxide

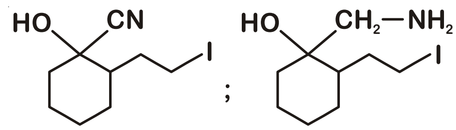
Al_2O_3 and Ga_2O_3 are amphoteric oxide

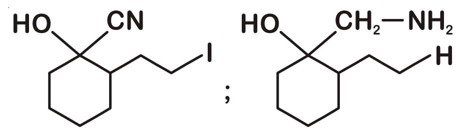
In_2O_3 and Tl_2O are basic oxide

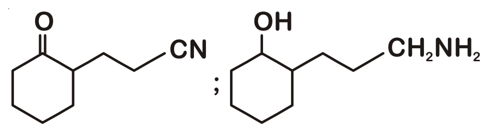
12. The major products A and B for the following reactions are, respectively



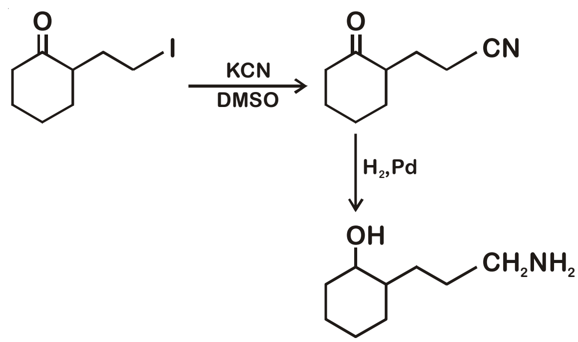
(1) 

(2) 

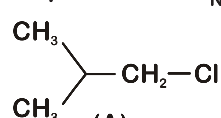
(3) 

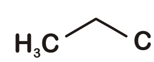
(4) 

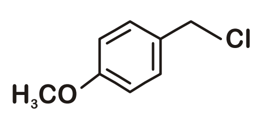
Answer (4)

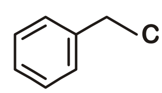
Sol. 

13. Increasing order of reactivity of the following compounds for S_N1 substitution is


(A)


(B)


(C)

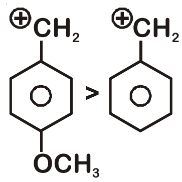

(D)

(3) (B) < (A) < (D) < (C)

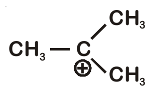
(4) (A) < (B) < (D) < (C)

Answer (3)

Sol. S_N1 reaction proceeds via formation of carbocation.

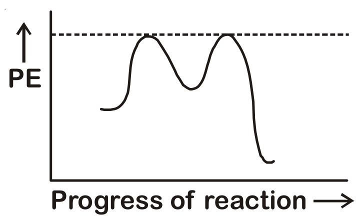


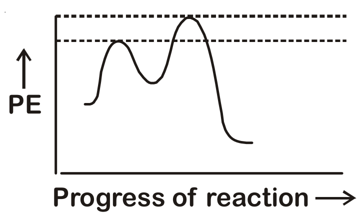
(C) (D)

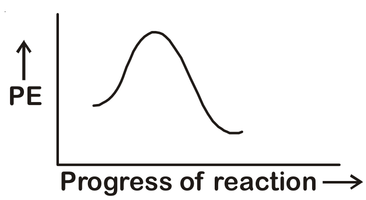
On comparing (A) and (B), in (A) there is formation of tertiary carbocation  after rearrangement while (B) is primary.

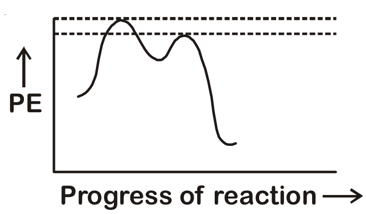
So, (C) > (D) > (A) > (B).

14. Which of the following potential energy (PE) diagrams represents the S_N1 reaction?

(1) 

(2) 

(3) 

(4) 

Sol. In S_N1 reaction, formation of carbocation (1^{st} step) is rate determining step (RDS)

$\therefore$ Correct graph is given in option-4.

15. Molal depression constant for a solvent is 4.0 K kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is

(Assume complete dissociation of the electrolyte)

(1) 0.36 K

(2) 0.18 K

(3) 0.12 K

(4) 0.24 K

Answer (1)

Sol. $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

i (Van't Hoff Factor) = 3

$\therefore\ \Delta T_f = iK_fm$

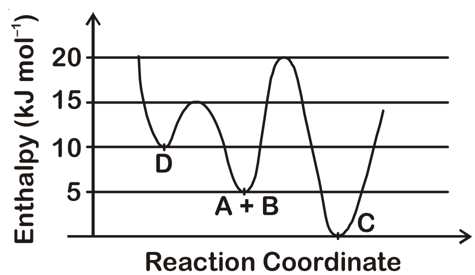
$= 3 \times 4 \times 0.03$

$= 0.36$ K

16. Consider the given plot of enthalpy of the following reaction between A and B.

$A + B \rightarrow C + D$

Identify the incorrect statement.



(1) Activation enthalpy to form C is 5 kJ mol^{-1} less than that to form D

(2) D is kinetically stable product

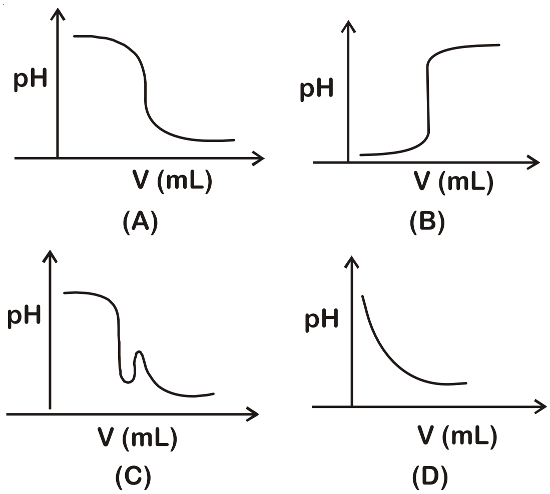
(3) Formation of A and B from C has highest enthalpy of activation

(4) C is the thermodynamically stable product

Answer (1)

Sol. Activation enthalpy to form C is 5 kJ more than that to form D.

strength. Which of the following correctly shows the change of pH of the titration mixture in this experiment?



(1) (A) (2) (C)

(3) (B) (4) (D)

Answer (1)

Sol. The pH of NaOH is more than 7 and during the titration it decreases so graph (1) is correct

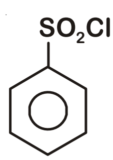
18. Hinsberg's reagent is

(1) $C_6H_5SO_2Cl$ (2) $(COCl)_2$

(3) C_6H_5COCl (4) $SOCl_2$

Answer (1)

Sol. Hinsberg's reagent is benzenesulphonyl chloride



19. The layer of atmosphere between 10 km to 50 km above the sea level is called as

(1) Stratosphere (2) Mesosphere

(3) Thermosphere (4) Troposphere

Answer (1)

Sol. Between 10-50 km above sea level lies stratosphere.

20. Assertion : For the extraction of iron, haematite ore is used.

Reason : Haematite is a carbonate ore of iron.

(1) Only the reason is correct

(2) Only the assertion is correct

(3) Both the assertion and reason are correct and the reason is the correct explanation for the assertion

(4) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion

Answer (2)

Sol. For the extraction of iron, haematite ore in used.

Haematite = Fe_2O_3

21. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as

$P = \frac{RT}{V - b}$ at T.

Here, b is the van der Waal's constant. Which gas will exhibit steepest increase in the plot of Z (compression factor) vs P?

(1) Kr

(2) Ar

(3) Xe

(4) Ne

Answer (3)

Sol. $P = \frac{RT}{V_m - b}$

$\Rightarrow PV_m - Pb = RT$

$\Rightarrow \frac{PV_m}{RT} = 1 + \frac{Pb}{RT}$

$\Rightarrow Z = 1 + \frac{Pb}{RT}$

Slope of Z vs P curve (straight line) = $\frac{b}{RT}$

$\therefore$ Higher the value of b, more steep will be the curve and b $\propto$ size of gas molecules

22. The amorphous form of silica is

(1) Quartz

(2) Tridymite

(3) Kieselguhr

(4) Cristobalite

Answer (3)

Sol. Quartz, tridymite and cristobalite are crystalline forms of silica.

Kieselguhr is an amorphous form of silica.

(I) Valence bond theory cannot explain the color exhibited by transition metal complexes.

(II) Valence bond theory can predict quantitatively the magnetic properties of transition metal complexes.

(III) Valence bond theory cannot distinguish ligands as weak and strong field ones.

(1) (II) and (III) only (2) (I), (II) and (III)

(3) (I) and (II) only (4) (I) and (III) only

Answer (4)

Sol. Valence bond theory cannot predict quantitatively the magnetic properties of transition metal complex.

24. In the following reaction

$$\text{carbonyl compound} + \text{MeOH} \xrightleftharpoons{\text{HCl}} \text{acetal}$$

Rate of the reaction is the highest for

(1) Acetone as substrate and methanol in excess

(2) Propanal as substrate and methanol in stoichiometric amount

(3) Propanal as substrate and methanol in excess

(4) Acetone as substrate and methanol in stoichiometric amount

Answer (3)

Sol. $CH_3-\overset{O}{\overset{\|}{C}}-CH_3$ $\quad$ CH_3-CH_2-CHO

Generally, aldehydes are more reactive than ketones in nucleophilic addition reactions.

∴ Rate of reaction with alcohol to form acetal and ketal is

$$CH_3-CH_2-CHO > CH_3-\overset{O}{\overset{\|}{C}}-CH_3$$

25. The structures of beryllium chloride in the solid state and vapour phase, respectively, are

(1) Chain and dimeric (2) Dimeric and dimeric

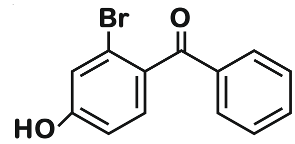
(3) Dimeric and chain (4) Chain and chain

Answer (1)

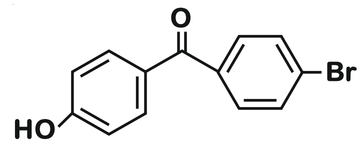
Sol. $BeCl_2$ in vapour phase exist as dimer (below 1200 K temperature)

$BeCl_2$ in solid state has chain structure.

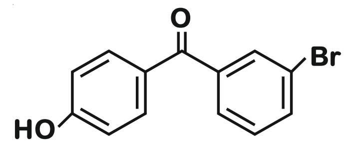
(1)



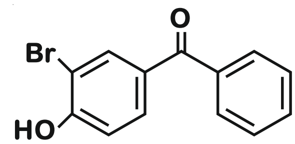
(2)



(3)

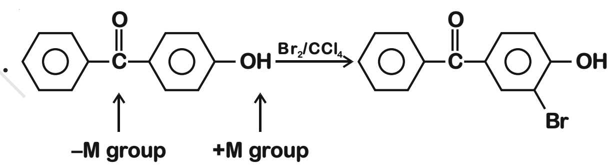


(4)



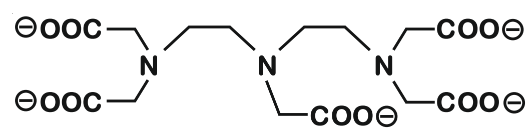
Answer (4)

Sol.



Product will formed as per –OH group (+M group)

27. The maximum possible denticities of a ligand given below towards a common transition and inner-transition metal ion, respectively, are



(1) 6 and 8 (2) 8 and 6

(3) 8 and 8 (4) 6 and 6

Answer (1)

Sol. The maximum possible denticities of the given ligand towards transition metal ion is 6 and towards inner transition metal ion is 8.

28. 10 mL of 1 mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as a cube, what is its edge length?

(1) 2.0 pm (2) 2.0 nm

(3) 0.1 nm (4) 1.0 pm

Sol. No. of surfactant molecule $6\times10^{23}\times\frac{10}{1000}\times10^{-3}$

$\Rightarrow$ 6 × 10^{18} molecule

Let edge length = a cm

Total surface area of surfactant = 6 × 10^{18} a^2

$\Rightarrow$ 0.24

a = 2 × 10^{-10} cm = 2 pm

29. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect? (The Bohr radius is represented by a_0)

(1) The probability density of finding the electron is maximum at the nucleus

(2) The electron can be found at a distance $2a_0$ from the nucleus

(3) The magnitude of the potential energy is double that of its kinetic energy on an average

(4) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

Answer (4)

1 s orbital.

30. The maximum number of possible oxidation states of actinoides are shown by

(1) Berkelium (Bk) and californium (Cf)

(2) Neptunium (Np) and plutonium (Pu)

(3) Actinium (Ac) and thorium (Th)

(4) Nobelium (No) and lawrencium (Lr)

Answer (2)

Sol.

Actinoids	Oxidation state shown
Th	+4
Ac	+3
Pu	+3, +4, +5, +6, +7
Np	+3, +4, +5, +6, +7
Bk	+3, +4
Cm	+3, +4
Lr	+3

$\therefore$ Maximum oxidation state is shown by (Np and Pu)

PART–C : MATHEMATICS

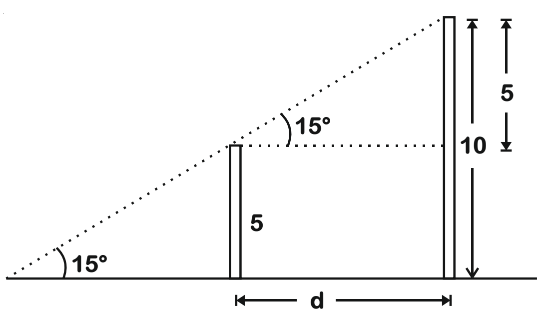
1. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is:

(1) $5\left(2+\sqrt{3}\right)$ (2) $10\left(\sqrt{3}-1\right)$

(3) $5\left(\sqrt{3}+1\right)$ (4) $\frac{5}{2}\left(2+\sqrt{3}\right)$

Answer (1)

Sol.



$$\tan15^\circ=\frac{5}{d}\Rightarrow d=\frac{5}{\tan15^\circ}=\frac{5\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$$

$$=\frac{5\left(4+2\sqrt{3}\right)}{2}$$

$$=5\left(2+\sqrt{3}\right)$$

2. If the system of equations 2x + 3y – z = 0, x + ky – 2z = 0 and 2x – y + z = 0 has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:

(1) $\frac{1}{2}$ (2) –4

(3) $\frac{3}{4}$ (4) $-\frac{1}{4}$

Answer (1)

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & k & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \boxed{k = \frac{[illegible]}{2}}$

$\therefore$ Equations are $2x + 3y - z = 0$ ...(i)

$2x - y + z = 0$ ...(ii)

$2x + 9y - 4z = 0$ ...(iii)

By (i) – (ii) $\boxed{2y = z}$ $\therefore$ $\boxed{z = -4x}$ and $\boxed{2x + y = 0}$

$$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2}$$

$$= \frac{1}{2}$$

3. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

(1) $8\sqrt{3}$ (2) $10\sqrt{5}$

(3) $4\sqrt{3}$ (4) $8\sqrt{5}$

Answer (4)

Sol. Sum of roots = $\frac{3}{m^2 + 1}$

For maximum m = 0

Hence equation becomes $x^2 - 3x + 1 = 0$

$\alpha + \beta = 3, \quad \alpha\beta = 1, \quad |\alpha - \beta| = \sqrt{5}$

$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)| = \sqrt{5}(9 - 1) = 8\sqrt{5}$

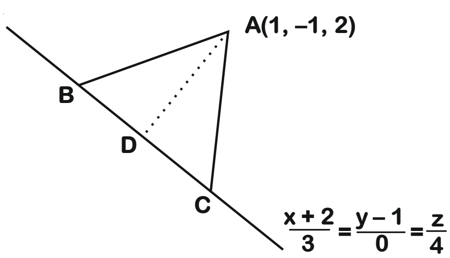
4. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is:

(1) $5\sqrt{17}$ (2) $\sqrt{34}$

(3) 6 (4) $2\sqrt{34}$

Answer (2)

Sol.



Given BC = 5 so we need perpendicular distance of A from line BC.

Let a point D on BC = $(3\lambda - 2, 1, 4\lambda)$

$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$

Also $\overrightarrow{AD}$ & $\overrightarrow{BC}$ should be perpendicular

$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$

$(3\lambda - 3)3 + 2(0) + (4\lambda - 2)4 = 0$

$9\lambda - 9 + 16\lambda - 8 = 0 \Rightarrow \lambda = \frac{17}{25}$

Hence, D = $\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$$|\overrightarrow{AD}| = \sqrt{\left(\frac{1}{25} - 1\right)^2 + (2)^2 + \left(\frac{68}{25} - 2\right)^2}$$

$$= \sqrt{\left(\frac{-24}{25}\right)^2 + 4 + \left(\frac{18}{25}\right)^2}$$

$$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25^2}}$$

$$= \sqrt{\frac{576 + 2500 + 324}{25^2}}$$

$$= \sqrt{\frac{3400}{25^2}}$$

$$= \frac{\sqrt{34} \cdot 10}{25} = \frac{2\sqrt{34}}{5}$$

Area of triangle = $\frac{1}{2} \times |\overrightarrow{BC}| \times |\overrightarrow{AD}|$

$$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$$

5. If $\cos x \frac{dy}{dx} - y \sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{\pi^2}{2}$ (2) $-\frac{\pi^2}{4\sqrt{3}}$

(3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

Answer (4)

$$\Rightarrow \int d(y\cos x) = \int 6x\,dx$$

$$\Rightarrow y\cos x = 3x^2 + C$$

As $y\left(\frac{\pi}{3}\right) = 0 \;\Rightarrow\; (0)\times\left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C = \frac{-\pi^2}{3}$

$$\Rightarrow y\cos x = 3x^2 - \frac{\pi^2}{3}$$

For $y\left(\frac{\pi}{6}\right)$

$$y\frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$$

$$\frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$$

6. If $\int e^{\sec x}\left(\sec x \tan x f(x) + \sec x \tan x + \sec^2 x\right)dx$

$= e^{\sec x} f(x) + C$,then a possible choice of f(x) is:

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $\sec x + \tan x + \frac{1}{2}$

(3) $\sec x + x\tan x - \frac{1}{2}$ (4) $x\sec x + \tan x + \frac{1}{2}$

Answer (2)

Sol. $\int e^{\sec x}\left(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)\right)dx$

$$= e^{\sec x} f(x) + C$$

$\because$ We know that

$$\int e^{g(x)}\left((g'(x)f(x)) + f'(x)\right)dx = e^{g(x)} \times f(x) + C$$

$$\therefore\; f(x) = \int\left((\sec x \tan x) + \sec^2 x\right)dx$$

$$\therefore\; \boxed{f(x) = \sec x + \tan x + C}$$

7. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2:15:70, then the average of these three coefficients is:

(1) 625 (2) 964

(3) 232 (4) 227

Answer (3)

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15} \;\&\; \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{2}{15} \;\&\; \frac{r+1}{n-r} = \frac{3}{14}$$

$\Rightarrow$ 15r = 2n – 2r + 2 & 14r + 14 = 3n – 3r

$\Rightarrow$ 17r = 2n + 2 & 17r = 3n – 14

i.e., 2n + 2 = 3n – 14 $\Rightarrow$ n = 16 & r = 2

Mean $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16 + 120 + 560}{3}$

$$= \frac{696}{3} = 232$$

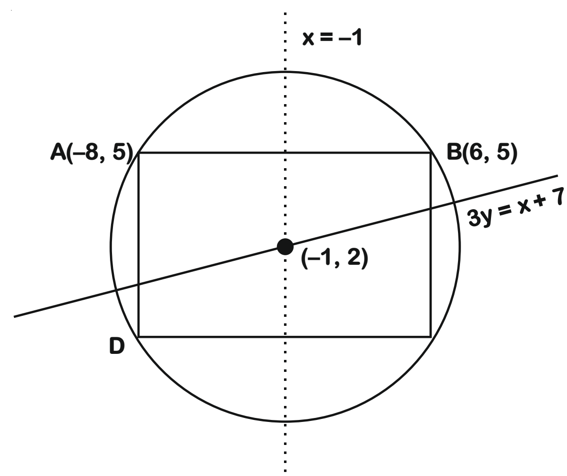
8. A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :

(1) 56 (2) 84

(3) 72 (4) 98

Answer (2)

Sol. Given situation



Perpendicular bisector of AB will pass from centre.

$\therefore$ Equation of perpendicular bisector x = –1

Hence centre (–1, 2)

Let

$D = (\alpha, \beta) \Rightarrow \frac{\alpha+6}{2} = -1 \;\&\; \frac{\beta+5}{2} = 2$

$\alpha = -8$ & $\beta = -1$ $D \equiv (-8, -1)$

|AD| = 6 & |AB| = 14

Area = 6 × 14 = 84

$\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is :

(1) $\frac{5\pi}{12}$ (2) $\frac{2\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{5\pi}{6}$

Answer (2)

Sol. Let $\cos\alpha, \cos\beta, \cos\gamma$ be direction cosines of $\vec{a}$

Hence, by given data

$$\cos\alpha = \cos\frac{\pi}{3}, \ \cos\beta = \cos\frac{\pi}{4} \ \& \ \cos\gamma = \cos\theta$$

$$\therefore \ \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}, \quad \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

10. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$ is continuous at x = 5, then the value of a – b is:

(1) $\frac{2}{\pi - 5}$ (2) $\frac{-2}{\pi + 5}$

(3) $\frac{2}{\pi + 5}$ (4) $\frac{2}{5 - \pi}$

Answer (4)

Sol. L.H.L $\lim\limits_{x \to 5} b|\pi - 5| + 3 = (5 - \pi)b + 3$

f(5) = R.H.L. $\lim\limits_{x \to 5} a|5 - \pi| + 1 = a(5 - \pi) + 1$

For continuity LHL = RHL

$(5 - \pi)b + 3 = (5 - \pi)a + 1$

$\Rightarrow 2 = (a - b)(5 - \pi)$

$\Rightarrow a - b = \frac{2}{5 - \pi}$

11. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5 + 3z}{5(1 - z)}$, then

(1) 5 Re(ω) > 4 (2) 5 Re(ω) > 1

(3) 4 Im(ω) > 5 (4) 5 Im(ω) < 1

Answer (2)

$\Rightarrow 5\omega - 5 = z(3 + 5\omega)$

$\Rightarrow z = \frac{5(\omega - 1)}{3 + 5\omega}$

Given $|z| < 1$

$\Rightarrow 5|\omega - 1| < |3 + 5\omega|$

$\Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$

(using $|z|^2 = z\bar{z}$)

$\Rightarrow 16 < 40\omega + 40\bar{\omega}$

$\Rightarrow \omega + \bar{\omega} > \frac{2}{5}$

$\Rightarrow 2\text{Re}(\omega) > \frac{2}{5} \quad \Rightarrow \quad \text{Re}(\omega) > \frac{1}{5}$

12. Let P be the plane, which contains the line of intersection of the planes, x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :

(1) $63\sqrt{5}$ (2) $\frac{17}{\sqrt{5}}$

(3) $205\sqrt{5}$ (4) $\frac{11}{\sqrt{5}}$

Answer (4)

Sol. Let the plane be

P ≡ (2x + 3y + z + 5) + λ(x + y + z – 6) = 0

As the above plane is perpendicular to xy plane

$\Rightarrow ((2 + \lambda)\hat{i} + (3 + \lambda)\hat{j} + (1 + \lambda)\hat{k}).\hat{k} = 0$

$\Rightarrow \lambda = -1$

P ≡ x + 2y + 11 = 0

Distance from (0, 0, 256)

$$\left|\frac{0 + 0 + 11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$$

13. If f(x) = [x] – $\left[\frac{x}{4}\right]$, $x \in R$, where [x] denotes the greatest integer function, then :

(1) $\lim\limits_{x \to 4+} f(x)$ exists but $\lim\limits_{x \to 4-} f(x)$ does not exist

(2) f is continuous at x = 4

(3) $\lim\limits_{x \to 4-} f(x)$ exists but $\lim\limits_{x \to 4+} f(x)$ does not exist

(4) Both $\lim\limits_{x \to 4-} f(x)$ and $\lim\limits_{x \to 4+} f(x)$ exist but are not equal

Sol. L.H.L $\lim\limits_{x\to4^-}[x]-\left[\dfrac{x}{4}\right]=3-0=3$

$$\left(x<4\Rightarrow[x]=3\ \&\ \frac{x}{4}<1\Rightarrow\left[\frac{x}{4}\right]=0\right)$$

R.H.L $\lim\limits_{x\to4^+}[x]-\left[\dfrac{x}{4}\right]=4-1=3$

$$\left(x>4\Rightarrow[x]=4\ \&\ \frac{x}{4}>1\Rightarrow\left[\frac{x}{4}\right]=1\right)$$

$f(4)=[4]-\left[\dfrac{4}{4}\right]=4-1=3$

LHL = f(4)= RHL

Hence f(x) is continuous at x = 4

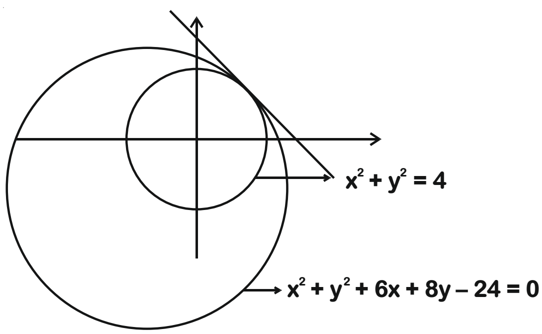
14. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :

(1) (–6, 4) (2) (–4, 6)

(3) (4, –2) (4) (6, –2)

Answer (4)

Sol. In given situation $d_{c_1c_2}=|r_1-r_2|$



Common tangent

$S_1 - S_2 = 0$

$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$

Hence (6, –2) lies on it

15. The domain of the definition of the function

$f(x)=\dfrac{1}{4-x^2}+\log_{10}(x^3-x)$ is :

(1) $(-1, 0)\cup(1, 2)\cup(3, \infty)$

(2) $(-1, 0)\cup(1, 2)\cup(2, \infty)$

(3) $(1, 2)\cup(2, \infty)$

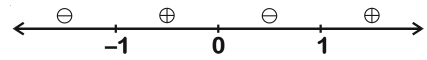
(4) $(-2, -1)\cup(-1, 0)\cup(2, \infty)$

Answer (2)

$4 - x^2 \neq 0 \Rightarrow x \neq \pm 2$ ...(1)

and $x^3 - x > 0$

$\Rightarrow x(x - 1)(x + 1) > 0$



$x\in(-1, 0) \cup (1, \infty)$ ...(2)

Hence domain is intersection of (1) & (2) i.e.

$x\in(-1, 0) \cup (1, 2) \cup (2, \infty)$

16. The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 + ... upto 11th term is :

(1) 916 (2) 946

(3) 945 (4) 915

Answer (2)

Sol. 1 + 2.3 + 3.5 + 4.7 + ...

Lets break the sequence as shown

$$1+\underbrace{(2.3+3.5+4.7+......)}_{S}$$

We find $s_{10}=\sum\limits_{n=1}^{10}(n+1)(2n+1)$

$$=\sum_{n=1}^{10}(2n^2+3n+1)$$

$$=\frac{2n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}+n(n=10)$$

$$=\frac{2.10.11.21}{6}+\frac{3.10.11}{2}+10$$

= 770 + 165 + 10 = 945

Hence required sum = 1 + 945 = 946

17. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :

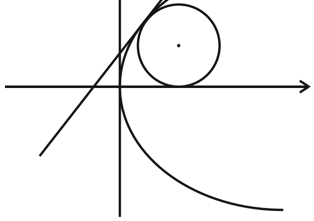
(1) $4\pi\left(3+\sqrt{2}\right)$

(2) $8\pi\left(2-\sqrt{2}\right)$

(3) $8\pi\left(3-2\sqrt{2}\right)$

(4) $4\pi\left(2-\sqrt{2}\right)$

Answer (3)



The circle and parabola will have common tangent at P(1, 2)

$\therefore$ Equation of tangent to parabola

$$\equiv y\times(2) = 4\frac{(x+1)}{2} \Rightarrow 2y = 2x+2$$

$$y = x + 1$$

Let equation of circle be (using family of circles)

$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$

$\Rightarrow$ $c \equiv (x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

Also circle touches x-axis $\Rightarrow$ y-coordinate of centre = radius

$\Rightarrow$ $c \equiv x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$$\frac{\lambda+4}{2} = \sqrt{\left(\frac{\lambda-2}{2}\right)^2 + \left(\frac{-\lambda-4}{2}\right)^2 - (\lambda+5)}$$

$$\Rightarrow \quad \frac{\lambda^2-4\lambda+4}{4} = \lambda+5 \Rightarrow \lambda^2-4\lambda+4 = 4\lambda+20$$

$\Rightarrow$ $\lambda^2 - 8\lambda - 16 = 0$

$$\Rightarrow \quad \lambda = \frac{8\pm\sqrt{64+64}}{2}$$

$$= 4\pm4\sqrt{2}$$

$\lambda = 4-4\sqrt{2}$ (Other value forms bigger circle)

Hence centre of circle $(2\sqrt{2}-2, 4-2\sqrt{2})$

Radius $= 4-2\sqrt{2}$

Area $= \pi(4-2\sqrt{2})^2 = 8\pi(3-2\sqrt{2})$

18. The total number of matrices

$$A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}, \ (x, y \in R, x \neq y) \text{ for}$$

Which $A^TA = 3I_3$ is :

(1) 6 (2) 3

(3) 4 (4) 2

Answer (3)

Sol. $A^TA = \begin{bmatrix} 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = 3I$

$$= \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow$ $8x^2 = 3, 6y^2 = 3$

$$x = \pm\sqrt{\frac{3}{8}}, y = \pm\sqrt{\frac{1}{2}}$$

Total combinations of (x, y) = 2 × 2 = 4

19. If $p \Rightarrow (q \vee r)$ is false, then truth values of p, q, r are respectively :

(1) T, F, F (2) F, F, F

(3) T, T, F (4) F, T, T

Answer (1)

Sol. For $p \to q \vee r$ to be F

r must be F & $p \to q$ must be F

for $p \to q$ to be F $\quad p \to T$ & $q \to F$

p, q, r $\equiv$ T, F, F

20. The mean and the median of the following ten numbers in increasing order

10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :

(1) $\frac{7}{3}$

(2) $\frac{8}{3}$

(3) $\frac{7}{2}$

(4) $\frac{9}{4}$

Answer (1)

Sol. Mean $= \frac{\sum x_i}{n} = \frac{x+y+300}{10} = 42 \Rightarrow x+y = 120$

Median $= \frac{T_5+T_6}{2} = 35 = \frac{34+x}{2} \Rightarrow x = 36$ & $y = 84$

Hence $\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :

(1) 157 (2) 225

(3) 262 (4) 190

Answer (4)

Sol. Balls used in equilateral triangle $=\frac{n(n+1)}{2}$

Here side of equilateral triangle has n-balls

No. of balls in each side of square is = (n – 2)

Given $\frac{n(n+1)}{2}+99=(n-2)^2$

$\Rightarrow$ $n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow$ $n^2 - 9n - 190 = 0$

$\Rightarrow$ $n^2 - 19n + 10n - 190 = 0$

$\Rightarrow$ $(n - 19)(n + 10) = 0$

$\Rightarrow$ $n = 19$

Balls used to form triangle

$=\frac{n(n+1)}{2}=\frac{19\times 20}{2}=190$

22. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is :

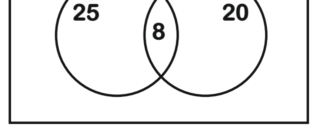
(1) 13.9

(2) 13

(3) 12.8

(4) 13.5

Answer (1)



n(A only) = 25 – 8 = 17%

n(B only) = 20 – 8 = 12%

% of people from A only who read advertisement = 17 × 0.3 = 5.1%

% of people from B only who read advertisement = 12 × 0.4 = 4.8%

% of people from A&B both who read advertisement = 8 × 0.5 = 4%

Total % of people who read advertisement

= 5.1 + 4.8 + 4 = 13.9%

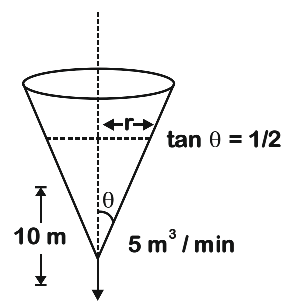
23. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

(1) $\frac{2}{\pi}$ (2) $\frac{1}{15\pi}$

(3) $\frac{1}{10\pi}$ (4) $\frac{1}{5\pi}$

Answer (4)

Sol.



Given $\frac{dv}{dt}=$ 5m³/min

$V=\frac{1}{3}\pi r^2 h$ ...(i) (where r is radius and h is height at any time)

Differentiate eq^n. (i), we get

$$\frac{dV}{dt}=\frac{1}{3}\left(\pi 2r\frac{dr}{dt}h+\pi r^2\frac{dh}{dt}\right)=\frac{1}{3}\left(100\pi\frac{1}{2}+25\pi\right)\frac{dh}{dt}$$

at h = 10, r = 5

$$5=\frac{75\pi}{3}\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt}=\frac{1}{5\pi}\text{m/mim}$$

24. The value of the integral

$$\int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx \text{ is}$$

(1) $\frac{\pi}{4}-\log_e 2$ (2) $\frac{\pi}{2}-\log_e 2$

(3) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$

Answer (4)

Sol. $\int_0^1 x\cot^{-1}\left(1-x^2+x^4\right)dx=\int_0^1 x\tan^{-1}\left(\frac{1}{1+x^4-x^2}\right)$

$$\Rightarrow \int_0^1 x\tan^{-1}\left(\frac{x^2-\left(x^2-1\right)}{1+x^2\left(x^2-1\right)}\right)dx$$

$$\Rightarrow \int_0^1 x\tan^{-1}x^2dx-\int_0^1 x\tan^{-1}\left(x^2-1\right)dx$$

Put $x^2 = t \Rightarrow 2xdx = dt$, (For 1st integration)

put $x^2 - 1 = k \Rightarrow 2xdx = dk$ (For 2nd integration)

$$\Rightarrow \frac{1}{2}\int_0^1 1\tan^{-1}t\,dt-\frac{1}{2}\int_{-1}^0 1\tan^{-1}k\,dk$$

$$\Rightarrow \frac{1}{2}\left(\int_0^1 t\tan^{-1}t\Big|_0^1-\int_0^1\frac{t}{1+t^2}dt\right)-\frac{1}{2}$$

$$\left(k\tan^{-1}k\Big|_{-1}^0-\int_{-1}^0\frac{k}{1+k^2}dk\right)$$

$$\left(0-\frac{\pi}{4}-\left(\frac{1}{2}\ln\left(1+k^2\right)\right)\Big|_{-1}^0\right)$$

$$\Rightarrow \left(\frac{\pi}{8}-\frac{1}{4}\ln 2\right)-\left(\frac{-\pi}{8}-\frac{1}{4}10-\ln 2\right)$$

$$\Rightarrow \frac{\pi}{4}-\frac{1}{2}\ln 2$$

25. The area (in sq. units) of the region

$$A=\left\{(x,y):\frac{y^2}{2}\le x\le y+4\right\} \text{ is}$$

(1) 18

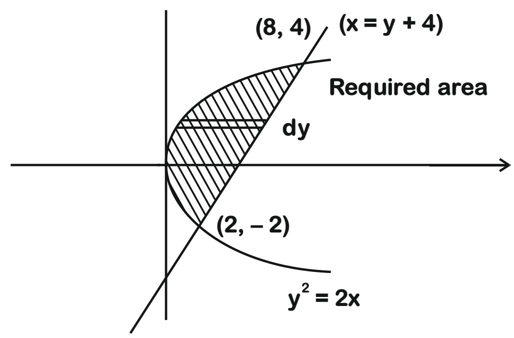
(2) 16

(3) $\frac{53}{3}$

(4) 30

Answer (1)

Sol.



Hence area $=\int_{-2}^{4} x\,dy$

$$=\int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right)dy$$

$$=\frac{y^2}{2}+4y-\frac{y^3}{6}\Big|_{-2}^{4}=\left(8+16-\frac{64}{6}\right)-\left(2-8+\frac{8}{6}\right)$$

$$=\left(24-\frac{32}{3}\right)-\left(-6+\frac{4}{3}\right)=\frac{40}{3}+\frac{14}{3}=\frac{54}{3}=18$$

of their point of intersection from the origin is

(1) $\sqrt{\frac{2}{5}}$ (2) $\frac{\sqrt{2}}{5}$

(3) $\frac{2}{5}$ (4) $\frac{2}{\sqrt{5}}$

Answer (1)

Sol. For perpendicular $m_1m_2 = -1$

$$\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$$

$\Rightarrow$ $2 = a^2(1-a)$

$\Rightarrow$ $a^3 - a^2 + 2 = 0$

$\Rightarrow$ $(a + 1)(a^2 + 2a + 2) = 0$

$\boxed{a = -1}$

Hence lines are $x - 2y = 1$ and $2x + y = 1$

$\therefore$ Intersection point $\left(\frac{3}{5}, \frac{-1}{5}\right)$

Distance from origin $= \sqrt{\frac{9}{25} + \frac{1}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

27. The value of sin10° sin30° sin50° sin70° is

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$

(3) $\frac{1}{16}$ (4) $\frac{1}{36}$

Answer (3)

Sol. $\sin(60° + A).\sin(60° - A)\sin A = \frac{1}{4}\sin 3A$

Hence, sin10° sin50° sin70°

$= \sin 10° \sin(60° - 10°)\sin(60° + 10°) = \frac{1}{4}\sin 30°$

Hence, $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4}\sin^2 30°$

$= \frac{1}{16}$

$x^2 + 2y^2 = 1$, then α is equal to

(1) $\sqrt{2} - 1$

(2) $\sqrt{2} + 1$

(3) $2\sqrt{2} + 1$

(4) $2\sqrt{2} - 1$

Answer (2)

Sol. Let tangent in terms of m to parabola and ellipse

i.e $y = mx + \frac{1}{4m}$ for parabola at point $\left(\frac{1}{4m^2}, \frac{-1}{2m}\right)$

and $y = mx \pm \sqrt{m^2 + \frac{1}{2}}$ for ellipse on comparing

$$\Rightarrow \frac{1}{4m} = \pm\sqrt{m^2 + \frac{1}{2}} \Rightarrow \frac{1}{16m^2} = m^2 + \frac{1}{2}$$

$\Rightarrow$ $16m^4 + 8m^2 - 1 = 0$

$$m^2 = \frac{-8 \pm \sqrt{64 + 64}}{2(16)} = \frac{-8 \pm 8\sqrt{2}}{2(16)} = \frac{\sqrt{2} - 1}{4}$$

$$\alpha = \frac{1}{4m^2} = \frac{1}{4\frac{\sqrt{2}-1}{4}} = \sqrt{2} + 1$$

29. If $f : R \to R$ is a differentiable function and $f(2) = 6$, then $\lim\limits_{x\to 2} \int_6^{f(x)} \frac{2t\,dt}{(x-2)}$ is

(1) 0 (2) $2f'(2)$

(3) $12f'(2)$ (4) $24f'(2)$

Answer (3)

Sol. Using L' Hospital rule and Leibnitz theorem

$$\lim_{x\to 2} \frac{\int_6^{f(x)} 2t\,dt}{(x-2)}$$

$$\lim_{x\to 2} \frac{2f(x)f'(x) - 0}{1}$$

$2f(2)f'(2) = 12f'(2)$

value of its 11th term is

(1) –36 (2) 25

(3) –25 (4) –35

Answer (3)

Sol. Let terms be a – d, a, a + d

$\Rightarrow$ 3a = 33 $\Rightarrow$ 11

Product of terms

(a – d) a (a + d) = 11 (121 – d^2) = 1155

$\Rightarrow$ 121 – d^2 = 105 $\Rightarrow$ d = $\pm$ 4

$$\left.\begin{matrix} T_1 = 7 \\ T_2 = 11 \\ T_3 = 15 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 7 + 10(4) = 47$$

if d = – 4

$$\left.\begin{matrix} T_1 = 15 \\ T_2 = 11 \\ T_3 = 7 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 15 + 10(-4) = -25$$

❑ ❑ ❑

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question.

1. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$ about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

(1) $-\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $-\frac{I_1\omega_1^2}{24}$

Answer (4)

Sol. By applying conservation of angular momentum

$(I_1 + I_2)\,\omega_{common} = I_1\omega_1 + I_2\omega_2$

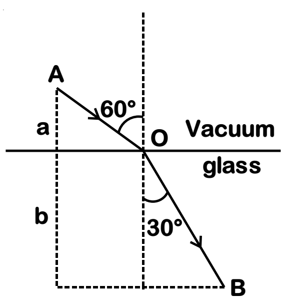
$$\omega_{common} = \frac{I_1\omega_1 + \frac{I_1\omega_1}{4}}{I_1 + \frac{I_1}{2}} = \left(\frac{5}{4}\times\frac{2}{3}\right)\omega_1$$

$$\omega_c = \frac{5\omega_1}{6}$$

$\therefore$ Loss in KE = $\left(\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2\right) - \frac{1}{2}(I_1 + I_2)\,\omega_c^2$

$\therefore$ $\Delta KE = -\frac{I_1\omega_1^2}{24}$

2. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is :

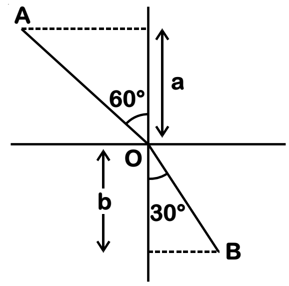


(1) $\frac{2\sqrt{3}}{a} + 2b$ (2) $2a + \frac{2b}{\sqrt{3}}$

(3) $2a + \frac{2b}{3}$ (4) 2a + 2b

Answer (4)

Sol. From the given figure



$$\frac{a}{AO} = \cos 60°$$

AO = 2a

$$\frac{b}{BO} = \cos 30°$$

$$BO = \frac{2b}{\sqrt{3}}$$

$\therefore$ Length of optical path = AO + BO × $\sqrt{3}$

= 2a + 2b

3. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is :

(1) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(2) $\frac{1}{\sqrt{\gamma g}}\ln\left(1 + \sqrt{\frac{\gamma}{g}}V_0\right)$

(3) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(4) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

Answer (3)

Sol. Retardation of the particle

$a = -(g + \gamma v^2)$

$$\int_{v_0}^{0}\frac{-dv}{g + \gamma v^2} = \int_0^t dt \quad \text{[for } H_{max}\ v = 0]$$

$$\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\frac{\sqrt{\gamma}\,v_0}{\sqrt{g}}\right) = t$$

$x = x_0 + a \cos\omega_1 t$

$y = y_0 + b \sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is :

(1) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

(2) $m(-x_0 b + y_0 a)\omega_1^2 \hat{k}$

(3) $+my_0 a\omega_1^2 \hat{k}$

(4) Zero

Answer (3)

Sol. $x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

$\Rightarrow$ $v_x = -a\omega_1 \sin(\omega_1 t)$, $v_y = b\omega_2 \cos(\omega_2 t)$

$a_x = -a\omega_1^2 \cos(\omega_1 t)$, $a_y = -b\omega_2^2 \sin(\omega_2 t)$

At t = 0, $x = x_0 + a$, $y = y_0$

$a_x = -a\omega_1^2$, $a_y = 0$

$\therefore$ $\vec{\tau} = m(-a\omega_1^2) \times y_0(-\hat{k})$

$= +my_0 a\omega_1^2 \hat{k}$.

5. In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be :

Given E (in eV) = $\dfrac{1237}{\lambda(\text{in nm})}$

(1) 4.5 eV (2) 15.1 eV

(3) 3.0 eV (4) 1.5 eV

Answer (4)

Sol. Wavelength of incident wave (λ) = 260 nm

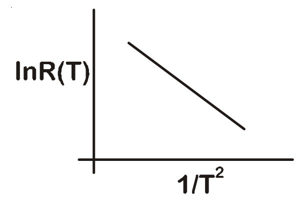
Cut off (threshold) wavelength (λ_0) = 380 nm

Then $KE_{max} = \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0}$

$= 1237\left[\dfrac{1}{260} - \dfrac{1}{380}\right]$

$= \dfrac{1237 \times 120}{380 \times 260} = 1.5 \text{ eV}$

range). As shown in the figure, it is a straight line.



One may conclude that :

(1) $R(T) = \dfrac{R_0}{T^2}$

(2) $R(T) = R_0 e^{-T_0^2/T^2}$

(3) $R(T) = R_0 e^{-T^2/T_0^2}$

(4) $R(T) = R_0 e^{T^2/T_0^2}$

Answer (2)

Sol. $\ln R(T) = a - \dfrac{a}{b} - \dfrac{1}{T^2}$

a, b are constant

$R(T) = R_0 e^{\frac{-T_0^2}{T^2}}$

7. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is :

(1) 10^4

(2) 6×10^2

(3) 10^2

(4) 60

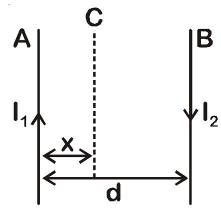
Answer (3)

Sol. $P_{gain} = \beta^2\left(\dfrac{R_{out}}{R_{in}}\right)$ & $I_{gain} = \beta$

$\therefore$ $10^6 = \beta^2\left(\dfrac{10000}{100}\right)$

$\beta = 100$

between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are :



(1) $x = \pm\dfrac{I_1 d}{(I_1 - I_2)}$

(2) $x = \left(\dfrac{I_1}{I_1 + I_2}\right)d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right)d$

(3) $x = \left(\dfrac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\dfrac{I_2}{I_1 - I_2}\right)d$

(4) $x = \left(\dfrac{I_1}{I_1 - I_2}\right)d$ and $x = \left(\dfrac{I_2}{I_1 + I_2}\right)d$

Answer (1)

Sol. $\Sigma\vec{F} = 0$

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(x-d)} \quad \text{since } (x > d)$$

$I_1 x - I_1 d = I_2 x$

$$x = \frac{I_1 d}{I_1 - I_2}$$

9. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1},

(Given speed of sound = 300 m/s)

(1) 12, 16 (2) 16, 14

(3) 8, 18 (4) 12, 18

Answer (4)

Sol. Frequency of sound source (f_0) = 500 Hz

When observer is moving away from the source

Apparent frequency $f_1 = 480 = f_0\left(\dfrac{v - v_0'}{v}\right)$ ...(i)

And when observer is moving towards the source $f_2 = 530 = f_0\left(\dfrac{v + v_0''}{v}\right)$ ...(ii)

$$480 = 500\left(\frac{300 - v_0'}{300}\right)$$

v_0' = 12 m/s

From equation (ii)

$$530 = 500\left(1 + \frac{v_0''}{v}\right)$$

$\therefore$ v_0'' = 18 m/s

10. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are :

(1) 440 V and 5 A (2) 220 V and 20 A

(3) 220 V and 10 A (4) 440 V and 20 A

Answer (1)

Sol. Power output ($V_2 I_2$) = 2.2 kW

$$\therefore \quad V_2 = \frac{2.2\,\text{kW}}{(10\text{A})} = 220 \text{ volts}$$

$\therefore$ Input voltage for step-down transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 2$$

$V_{input} = 2 \times V_{output} = 2 \times 220$

$= 440$ V

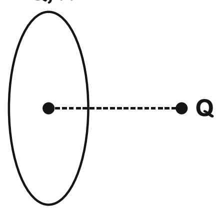
Also $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$

$$\therefore \quad I_1 = \frac{1}{2} \times 10 = 5\text{ A}$$

11. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at z = 4a. The minimum value of v such that it crosses the origin is :

(1) $\sqrt{\dfrac{2}{m}}\left(\dfrac{1}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (2) $\sqrt{\dfrac{2}{m}}\left(\dfrac{1}{5}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(3) $\sqrt{\dfrac{2}{m}}\left(\dfrac{4}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (4) $\sqrt{\dfrac{2}{m}}\left(\dfrac{2}{15}\dfrac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$



Potential at any point of the charged ring

$$V_P = \frac{KQ}{\sqrt{R^2 + x^2}}$$

The minimum velocity (v_0) should just sufficient to reach the point charge at the center, therefore

$$\frac{1}{2}mv_0^2 = Q[V_C - V_P]$$

$$= Q\left[\frac{KQ}{3a} - \frac{KQ}{5a}\right]$$

$$v_0^2 = \frac{4KQ^2}{15\,ma} = \frac{4}{15}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{ma}$$

$$\therefore \quad v_0 = \sqrt{\frac{2}{m}}\left(\frac{2q^2}{15 \times 4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$$

12. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

(1) $\frac{4nR}{C_V + nR}$ (2) $\frac{nR}{C_V + nR}$

(3) $\frac{4nR}{C_V - nR}$ (4) $\frac{nR}{C_V - nR}$

Answer (Bonus)

Sol. For Isobaric process

Work done (W) = $nR\Delta T$

and Heat given (Q) = $nC_P\Delta T$

$$\therefore \quad \frac{W}{Q} = \frac{R}{C_P} = \frac{R}{C_V + R}$$

13. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule ?

(1) ~10^{12} (2) ~10^{10}

(3) ~10^{11} (4) ~10^{13}

Sol. $V = 25 \times 10^{-3}$ m^3, N = 1 mole of O_2

T = 300 K

V_{rms} = 200 m/s

$$\because \quad \lambda = \frac{1}{\sqrt{2}N\pi r^2}$$

Average time $\frac{1}{\tau} = \frac{<V>}{\lambda} = 200 \cdot N\pi r^2 \cdot \sqrt{2}$

$$= \frac{\sqrt{2} \times 200 \times 6.023 \times 10^{23}}{25 \times 10^{-3}} \cdot \pi \times 10^{-18} \times 0.09$$

Average no. of collision $\approx 10^{10}$

14. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time :

(1) $\frac{1}{10\lambda}$ (2) $\frac{11}{10\lambda}$

(3) $\frac{1}{9\lambda}$ (4) $\frac{1}{11\lambda}$

Answer (3)

Sol. Number of nuclei present at any time t

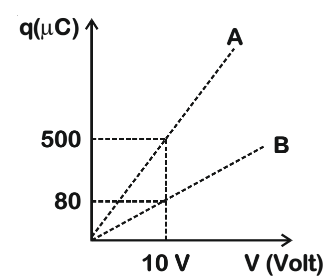
$N = N_0e^{-\lambda t}$

$$\therefore \quad \frac{N_A}{N_B} = e^{(\lambda_B - \lambda_A)t} = \frac{1}{e}$$

$(\lambda_A - \lambda_B) \cdot t = 1$

$$\therefore \quad t = \frac{1}{-\lambda + 10\lambda} = \frac{1}{9\lambda}$$

15. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :



(1) 40 μF and 10 μF (2) 20 μF and 30 μF

(3) 60 μF and 40 μF (4) 50 μF and 30 μF

Sol. Equivalent capacitance for series combination

$$C' = \frac{C_1C_2}{C_1+C_2}$$

For parallel combination $C'' = C_1 + C_2$

Also $C'' > C'$

$$C_1 + C_2 = \frac{500}{10} = 50\ \mu F$$

$$\text{and } \frac{C_1C_2}{C_1+C_2} = \frac{80}{10} = 8\ \mu F$$

$\therefore\quad C_1C_2 = 400\ \mu F$

Solving $C_1 = 40\ \mu F$ $C_2 = 10\ \mu F$

16. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :

(1) 4 ; 2×10^8 Hz (2) 4 ; 1×10^8 Hz

(3) 0.25 ; 1×10^8 Hz (4) 0.25 ; 2×10^8 Hz

Answer (4)

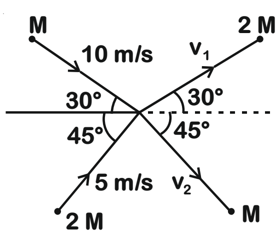
Sol. Range of frequency = $(f_c - f_m)$ to $(f_c + f_m)$

$\therefore$ Band width = $2f_m = 2 \times 100 \times 10^6$ Hz

$= 2 \times 10^8$ Hz

and Modulation index = $\frac{A_m}{A_C} = \frac{100}{400} = 0.25$

17. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly :



(1) 3.2 m/s and 12.6 m/s

(2) 3.2 m/s and 6.3 m/s

(3) 6.5 m/s and 3.2 m/s

(4) 6.5 m/s and 6.3 m/s

Sol. Apply conservation of linear momentum in X and Y direction for the system then

$M(10\cos 30°) + 2M(5\cos 45°) = 2M(v_1\cos 30°) + M(v_2\cos 45°)$

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}\,v_1 + \frac{v_2}{\sqrt{2}} \qquad ...(1)$$

Also

$2M(5\sin 45°) - M(10\sin 30°) = 2Mv_1\sin 30° - Mv_2\sin 45°$

$$5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \qquad ...(2)$$

Solving equation (1 and 2)

$(\sqrt{3}+1)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5 \quad \Rightarrow \quad v_1 = 6.5$ m/s

$v_2 = 6.3$ m/s

18. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0 – 5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0–10 mA is :

(1) 200 Ω

(2) 500 Ω

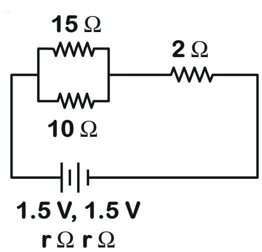
(3) 100 Ω

(4) 10 Ω

Answer (Bonus)

Sol. Data contradictory

19. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2 V. The internal resistance r, of each cell is :



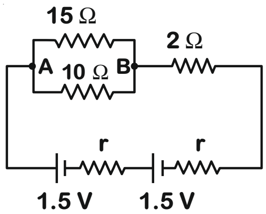
(1) 0.5 Ω

(2) 0 Ω

(3) 1.5 Ω

(4) 1 Ω

Sol. For the given circuit



Given that $V_{AB} = 2$ V

$$\therefore \quad I = \frac{2}{15} + \frac{2}{10} = \frac{1}{3}\text{ A}$$

Also $\quad I(2r + 2) = 1.5 + 1.5 - V_{AB}$

$\Rightarrow \quad 2r + 2 = (3-2)3$

$$\Rightarrow \quad r = \frac{1}{2}\,\Omega$$

20. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m)

(1) 9.0×10^6 m (2) 6.4×10^6 m

(3) 1.6×10^6 m (4) 2.6×10^6 m

Answer (4)

Sol. Given

$$g_{height} = \frac{g_{surface}}{2} = 4.9\text{ m/s}^2$$

$$\text{As} \quad g_h = g\left(1 + \frac{h}{R_e}\right)^{-2}$$

$$h = R_e\left(\sqrt{2} - 1\right)$$

$h = 6400 \times 0.414$

$h = 2649.6$ km

$h = 2.6 \times 10^6$ m

21. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :

(1) 4/5 (2) 2/3

(3) 3/5 (4) 2/5

Sol. Ratio of surface tension

$$\frac{S_{Hg}}{S_{Water}} = 7.5$$

$$\frac{\rho_{Hg}}{\rho_w} = 13.6 \;\&\; \frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135^\circ}{\cos 0^\circ} = \frac{1}{\sqrt{2}}$$

$$\frac{R_{Hg}}{R_{Water}} = \left(\frac{S_{Hg}}{S_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$= 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

22. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(1) $\frac{MR^2}{3}$ (2) $\frac{MR^2}{6}$

(3) $\frac{MR^2}{2}$ (4) $\frac{2MR^2}{3}$

Answer (4)

Sol. Surface mass density $(\sigma) = kr^2$

$$\text{Mass of disc } M = \int_0^R (kr^2)2\pi r dr$$

$$= 2\pi k\frac{R^4}{4} = \frac{\pi k R^4}{2}$$

$\therefore$ Moment of inertia about the axis of the disc.

$$I = \int dI = \int (dm)r^2 = \int \sigma dA r^2$$

$$= \int (Kr^2)(2\pi r dr)r^2$$

$$= \int_0^R 2\pi k\, r^5 dr = \frac{\pi k R^6}{3} = \frac{2}{3}MR^2$$

23. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J $mol^{-1}K^{-1}$]

(1) 700 J

(2) 350 J

(3) 374 J

(4) 748 J

Sol. No. of moles of He at STP $= \dfrac{67.2}{22.4} = 3$

As the volume is constant $\rightarrow$ Isochoric proces

$$Q = nC_V\Delta T = 3 \times \frac{3R}{2} \times 20 = 90R = 90 \times 8.31 \simeq 748\text{ J.}$$

24. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(1) $r_e < r_p < r_{He}$ (2) $r_e > r_p = r_{He}$

(3) $r_e < r_p = r_{He}$ (4) $r_e > r_p > r_{He}$

Answer (3)

Sol. Radius of circular path (r) in a perpendicular uniform magnetic field $= \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB}$

For proton, electron and α-particle,

$m_\alpha = 4m_p$ and $m_p >> m_e$

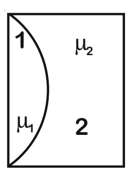
Also $q_\alpha = 2q_p$ and $q_p = q_e$

$\therefore$ As KE of all the particles is same then,

$$r \propto \frac{\sqrt{m}}{q}$$

$\therefore \quad r_\alpha = r_p > r_e$

25. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(1) $\dfrac{R}{2(\mu_1 - \mu_2)}$

(2) $\dfrac{R}{2 - (\mu_1 - \mu_2)}$

(3) $\dfrac{R}{\mu_1 - \mu_2}$

(4) $\dfrac{2R}{\mu_1 - \mu_2}$

Sol. Focal length of plano convex lens

$$f_1 = \frac{R}{(\mu_1 - 1)}$$

Focal length of plano concave lens-

$$f_2 = \frac{-R}{(\mu_2 - 1)}$$

For the combination of two lens-

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R}$$

$$= \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \quad f_{eq} = \frac{R}{\mu_1 - \mu_2}$$

26. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0 \hat{i} \cos(kz) \cos(\omega t)$

The corresponding magnetic field $\vec{B}$ is then given by :

(1) $\vec{B} = \dfrac{E_0}{C} \hat{j} \sin(kz) \cos(\omega t)$

(2) $\vec{B} = \dfrac{E_0}{C} \hat{j} \cos(kz) \sin(\omega t)$

(3) $\vec{B} = \dfrac{E_0}{C} \hat{j} \sin(kz) \sin(\omega t)$

(4) $\vec{B} = \dfrac{E_0}{C} \hat{k} \sin(kz) \cos(\omega t)$

Answer (3)

Sol. $\dfrac{E_0}{B_0} = C$

$\therefore \quad B_0 = \dfrac{E_0}{C}$

Given that $\vec{E} = E_0 \cos(kz)\cos(\omega t)\hat{i}$

$$\vec{E} = \frac{E_0}{2}\left[\cos(kz - \omega t)\hat{i} - \cos(kz + \omega t)\hat{i}\right]$$

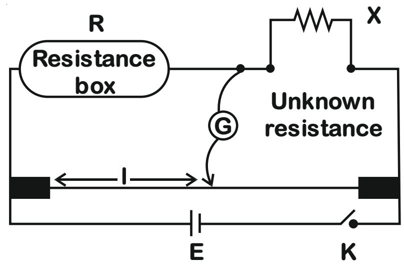
Correspondingly

$$\vec{B} = \frac{B_0}{2}\left[\cos(kz - \omega t)\hat{j} - \cos(kz + \omega t)\hat{j}\right]$$

$$\vec{B} = \frac{B_0}{2} \times 2\sin kz \sin \omega t$$

$$\vec{B} = \left(\frac{E_0}{C} \sin kz \sin \omega t\right)\hat{j}$$

table are shown in figure.



Sl. No	R (Ω)	l (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent ?

(1) 3 (2) 2

(3) 1 (4) 4

Answer (4)

Sol. $\frac{R}{X} = \frac{l}{100-l}$

Using the above expression

$$X = \frac{R(100-l)}{l}$$

for case (a) $x = \frac{100\times40}{60} = \frac{2000}{3}\ \Omega$

for case (b) $x = \frac{100\times87}{13} = \frac{8700}{13}\ \Omega$

for case (c) $x = \frac{10\times98.5}{1.5} = \frac{1970}{3}\ \Omega$

for case (d) $x = \frac{1\times99}{1} = 99\ \Omega$

Clearly we can see that the value of x calculate in case (d) is inconsistent than other cases.

28. A current of 5 A passes through a copper conductor (resistivity = $1.7 \times 10^{-8}\ \Omega$ m) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1×10^{-3} m/s.

(1) 1.3 m^2/Vs

(2) 1.8 m^2/Vs

(3) 1.5 m^2/Vs

(4) 1.0 m^2/Vs

Sol. Mobility $(\mu) = \frac{v_d}{E}$

and resistivity $(\rho) = \frac{E}{j} = \frac{EA}{I}$

$$\therefore \quad \mu = \frac{v_d A}{i\rho}$$

$$= \frac{1.1\times10^{-3}\times\pi\times(5\times10^{-3})^2}{5\times1.7\times10^{-8}}$$

$$\mu = 1.0\frac{m^2}{Vs}$$

29. Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A.	Optical Fibre Communication	P.	Ultrasound
B.	Radar	Q.	Infrared Light
C.	Sonar	R.	Microwaves
D.	Mobile Phones	S.	Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(1) A-Q, B-S, C-P, D-R

(2) A-Q, B-S, C-R, D-P

(3) A-S, B-Q, C-R, D-P

(4) A-R, B-P, C-S, D-Q

Answer (1)

Sol. Optical Fibre Communication – Infrared Light

Radar – Radio Waves

Sonar – Ultrasound

Mobile Phones – Microwaves

30. The displacement of a damped harmonic oscillator is given by

$x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to :

(1) 7 s

(2) 27 s

(3) 13 s

(4) 4 s

Answer (1)

Sol. Amplitude at (t = 0) $A_0 = e^{-0.1\times0} = 1$

$\therefore$ at t = t if $A = \frac{A_0}{2}$

$$\Rightarrow \frac{1}{2} = e^{-0.1t}$$

$t = 10\ \ln 2 \simeq 7$ s

1. Consider the following statements

(a) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.

(b) Ionic product of water is temperature dependent.

(c) A monobasic acid with $K_a = 10^{-5}$ has a pH = 5. The degree of dissociation of this acid is 50%.

(d) The Le Chatelier's principle is not applicable to common-ion effect.

The correct statements are :

(1) (a), (b) and (d)

(2) (b) and (c)

(3) (a) and (b)

(4) (a), (b) and (c)

Answer (4)

Sol. (a) $H_2SO_4 + NaOH \rightarrow NaHSO_4 + H_2O$

	H_2SO_4	$NaOH$	$NaHSO_4$	H_2O
Initial moles	0.04	0.04		
	0	0	0.04	0.04

$NaHSO_4 \rightarrow Na^+ + H^+ + SO_4^{2-}$

$$[H^+] = \frac{0.04}{0.80} = 0.05\text{ M; pH} = 1.3$$

(b) Ionic product of water increases with increase of temperature because ionisation of water is endothermic.

(c) $HA \rightleftharpoons H^+ + A^-$

$C(1-\alpha) \quad C\alpha \quad C\alpha \quad pH = 5 \;\&\; K_a = 10^{-5}$

$$10^{-5} = \frac{C\alpha^2}{1-\alpha}; C = 2\times10^{-5} \text{ and } \alpha = 0.5$$

2. The oxoacid of sulphur that does not contain bond between sulphur atoms is :

(1) $H_2S_4O_6$

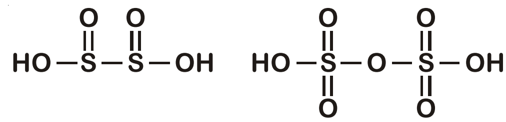
(2) $H_2S_2O_4$

(3) $H_2S_2O_7$

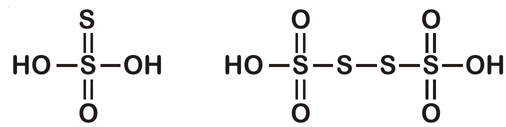
(4) $H_2S_2O_3$

Answer (3)

Sol.



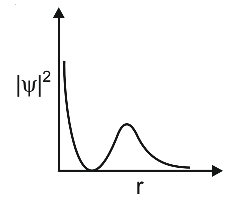
($H_2S_2O_4$) ($H_2S_2O_7$)



($H_2S_2O_3$) ($H_2S_4O_6$)

$H_2S_2O_7$ does not have S – S linkage

3. The graph between $|\psi|^2$ and r(radial distance) is shown below. This represents :

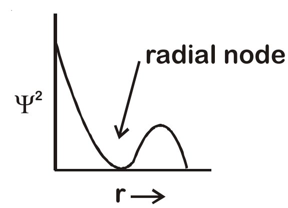


(1) 1s orbital (2) 2s orbital

(3) 2p orbital (4) 3s orbital

Answer (2)

Sol. The given probability density curve is for 2s orbital because it has only one radial node. Among other given orbitals, 1s and 2p do not have any radial node and 3s has two radial nodes.



4. Which of the following is a condensation polymer ?

(1) Nylon 6, 6

(2) Teflon

(3) Buna - S

(4) Neoprene

Answer (1)

Sol. Nylon 6, 6 is obtained by condensation polymerisation of hexamethylenediamine and adipic acid

$$\longrightarrow \left[NH-(CH_2)_6-NH-CO-(CH_2)_4-CO \right]_n$$

Nylon 6, 6

So, Nylon 6, 6 is a condensation polymer. Other polymers given, i.e., Buna-S, Teflon and Neoprene are addition polymers.

5. The principle of column chromatography is :

(1) Differential adsorption of the substances on the solid phase.

(2) Gravitational force.

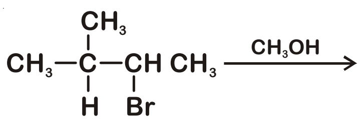
(3) Differential absorption of the substances on the solid phase.

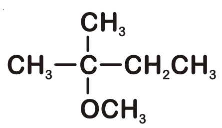
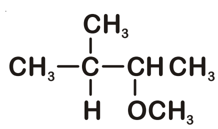
(4) Capillary action.

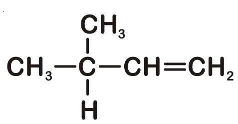
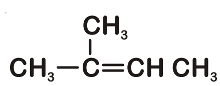
Answer (1)

Sol. In column chromatograph a solid adsorbent is packed in a column and a solution containing number of solute particles is allowed to flow down the column. The solute molecules get adsorbed on the surface of adsorbent. So it is differential adsorption of the substances on the solid phase.

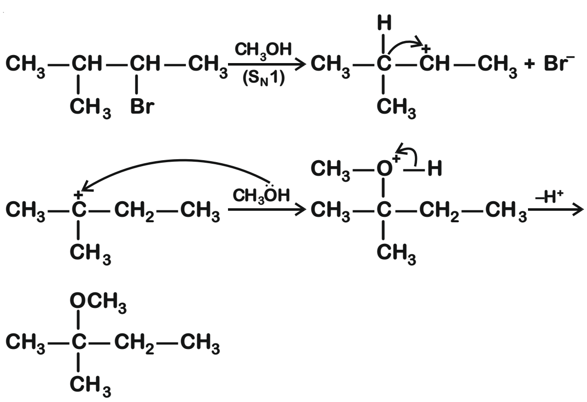
6. The major product of the following reaction is :



(1)  (2) 

(3)  (4) 

Answer (1)

Sol. 

S1 : Conductivity always increases with decrease in the concentration of electrolyte.

S2 : Molar conductivity always increases with decrease in the concentration of electrolyte.

The correct option among the following is :

(1) S1 is wrong and S2 is correct

(2) S1 is correct and S2 is wrong

(3) Both S1 and S2 are wrong

(4) Both S1 and S2 are correct

Answer (1)

Sol. Conductivity of an electrolyte is the conductance of 1 cm^3 of the given electrolyte. So, it increases with the increase of concentration of electrolyte. Molar conductivity (λ_m) is the conductance of a solution containing 1 mole of the electrolyte. It increases with the decrease of concentration (i) due to increase in interionic attraction for strong electrolytes and (ii) due to decrease in degree of ionisation for weak electrolytes. Therefore, (S_1) is wrong and (S_2) is correct.

8. Consider the following table :

Gas	a/(k Pa dm^6 mol^{-1})	b/(dm^3 mol^{-1})
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are van der Waals constants. The correct statement about the gases is :

(1) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

(2) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

(3) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

(4) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

Answer (2)

Sol. If two gases have same value of 'b' but different values of 'a', then the gas having a larger value of 'a' will occupy lesser volume. This is because the gas having larger value of

If two gases have same value of 'a' but different values of 'b', then the smaller value of 'b' will occupy lesser volume and hence will be more compressible.

9. Amylopectin is composed of

(1) β-D-glucose, $C_1 — C_4$ and $C_2 — C_6$ linkages

(2) α-D-glucose, $C_1 — C_4$ and $C_2 — C_6$ linkages

(3) β-D-glucose, $C_1 — C_4$ and $C_1 — C_6$ linkages

(4) α-D-glucose, $C_1 — C_4$ and $C_1 — C_6$ linkages

Answer (4)

Sol. Starch is a polymer of α-D-glucose. It has two components, namely

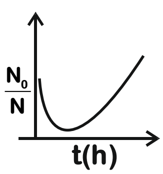
(i) Amylose and

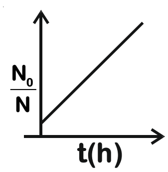
(ii) Amylopectin

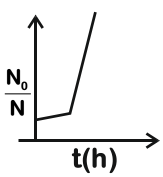
Amylose has only α-1,4-glycosidic linkage and is a linear polymer

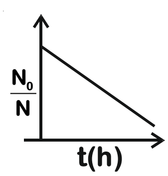
Amylopectin has α-1, 6-glycosidic linkage in addition to α-1,4-glycosidic linkage and is a cross-linked polymer.

10. A bacterial infection in an internal wound grows as $N'(t) = N_0\exp(t)$, where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be the plot of $\frac{N_0}{N}$ vs. t after 1 hour ?

(1) 

(2) 

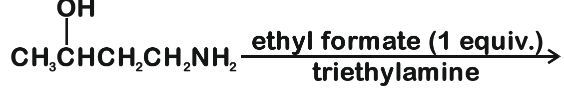
(3) 

(4) 

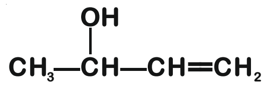
Answer (2)

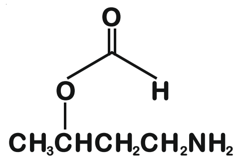
Sol. When drug is administered bacterial growth is given by $\frac{dN}{dt} = -5N^2$

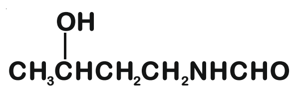
$\Rightarrow \frac{N_0}{N_t} = 1 + 5t\ N_0$. Thus $\frac{N_0}{N_t}$ increases linearly with t.



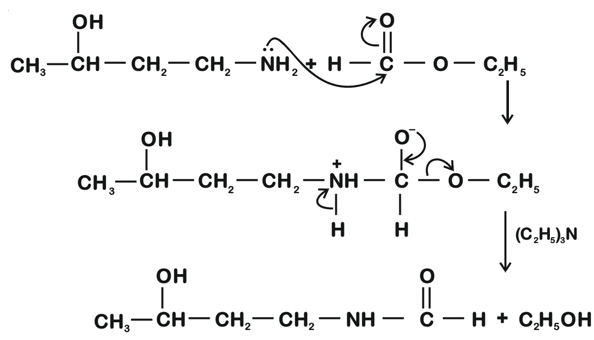
(1) $CH_3CH{=}CH—CH_2NH_2$

(2) 

(3) 

(4) 

Answer (4)

Sol. 

The acylation of NH_2 group takes place and not of OH group due to lower electronegativity of N-atom.

12. Consider the hydrated ions of Ti^{2+}, V^{2+}, Ti^{3+}, and Sc^{3+}. The correct order of their spin-only magnetic moments is :

(1) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

(2) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+}$

(3) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$

(4) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

Answer (1)

Sol. Electronic configuration of the given transition metal ions are

Sc^{3+} (Z = 21) $1s^2 2s^2 2p^6 3s^2 3p^6$

Ti^{2+} (Z = 22) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^2$

Ti^{3+} (Z = 22) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$

V^{2+} (Z = 23) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^3$

increasing order of magnetic moment is

$Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

0 1 2 3 unpaired electrons

13. The isoelectronic set of ions is :

(1) N^{3-}, Li^{+}, Mg^{2+} and O^{2-}

(2) Li^{+}, Na^{+}, O^{2-} and F^{-}

(3) N^{3-}, O^{2-}, F^{-} and Na^{+}

(4) F^{-}, Li^{+}, Na^{+} and Mg^{2+}

Answer (3)

Sol. Atomic numbers of N, O, F and Na are 7, 8, 9 and 11 respectively. Therefore, total number of electrons in each of N^{3-}, O^{2-}, F^{-} and Na^{+} is 10 and hence they are isoelectronic.

14. The alloy used in the construction of aircrafts is :

(1) Mg - Zn (2) Mg - Sn

(3) Mg - Mn (4) Mg - Al

Answer (4)

Sol. An alloy of Mg and Al called magnalium is used in manufacturing of aircraft due to its light weight and high strength.

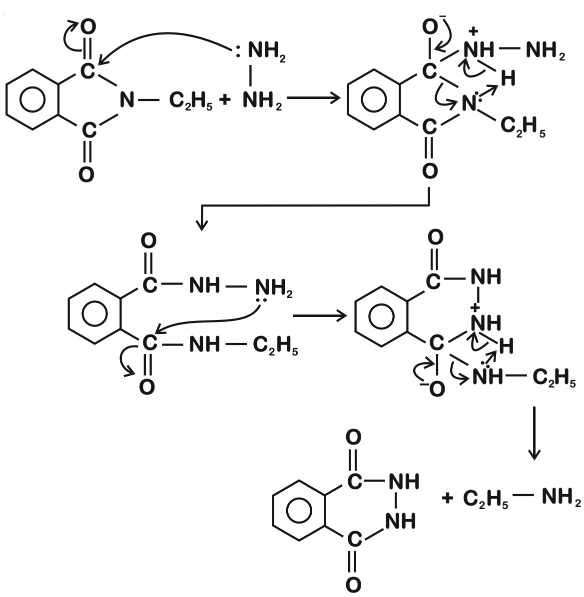
15. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphthalimide on treatment with :

(1) NH_2NH_2 (2) $NaBH_4$

(3) H_2O (4) CaH_2

Answer (1)

Sol. N-ethyl phthalimide on treatment with $NH_2—NH_2$ gives ethylamine.



16. A process will be spontaneous at all temperatures if :

(1) $\Delta H < 0$ and $\Delta S > 0$

(2) $\Delta H > 0$ and $\Delta S < 0$

(3) $\Delta H > 0$ and $\Delta S > 0$

(4) $\Delta H < 0$ and $\Delta S < 0$

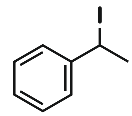
Answer (1)

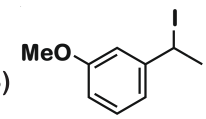
Sol. A reaction is spontaneous if ΔG_{sys} is negative.

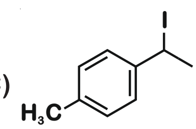
$\Delta G_{sys} = \Delta H_{sys} - T\Delta S_{sys}$

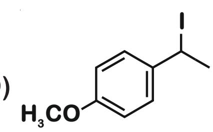
A reaction will be spontaneous at all temperatures if ΔH_{sys} is negative and ΔS_{sys} = +ve

17. Increasing rate of S_N1 reaction in the following compounds is :

(A) 

(B) 

(C) 

(D) 

(1) (B) < (A) < (C) < (D)

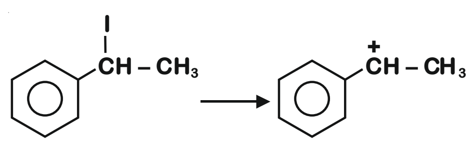
(2) (A) < (B) < (D) < (C)

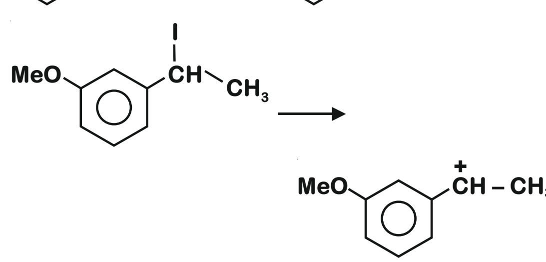
(3) (B) < (A) < (D) < (C)

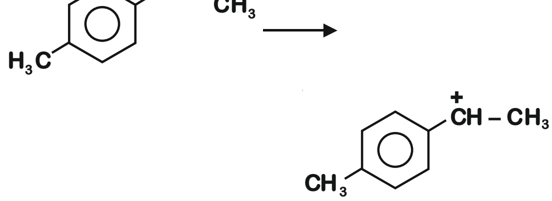
(4) (A) < (B) < (C) < (D)

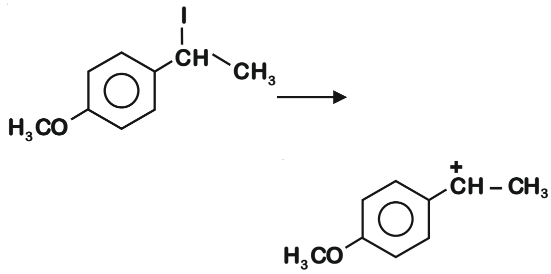
Answer (1)

Sol. The rate of S_N1 is decided by the stability of carbocation formed in the rate determining step.

(A) 

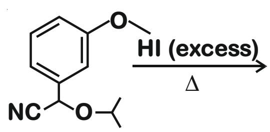
(B) 

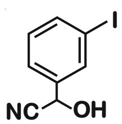
(C) 

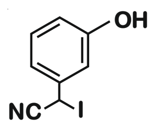
(D) 

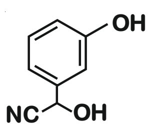
Carbocation (D) is most stable due to +R effect of CH_3O group, (C) is stabilised by +I and +H effect of CH_3 group; (B) is least stable due to –I effect of MeO group. So increasing order of rate of S_N1 is (B) < (A) < (C) < (D)

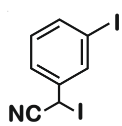
18. The major product of the following reaction is :



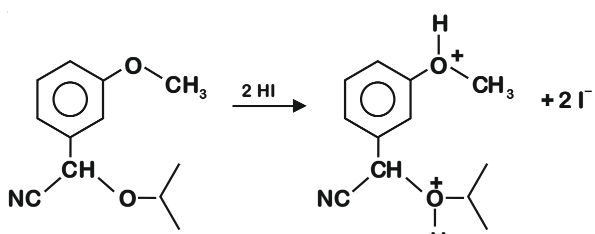
(1) 

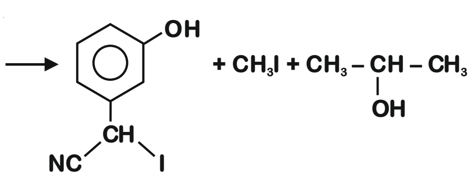
(2) 

(3) 

(4) 

Answer (2)

Sol. 



(en = ethane-1, 2-diamine, ox = oxalate)

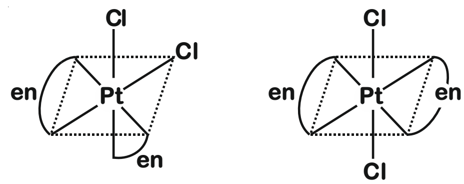
(1) $[Zn(en)Cl_2]$

(2) $[Pt(en)Cl_2]$

(3) $[Cr(en)_2(ox)]^+$

(4) $[Pt(en)_2Cl_2]^{2+}$

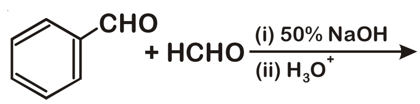
Answer (4)

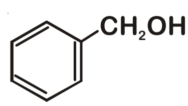
Sol. 

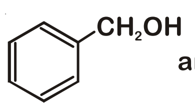
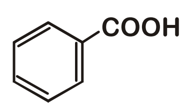
Cis Trans

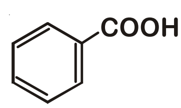
Cis-trans isomerism is possible with $[Pt(en)_2Cl_2]^{2+}$. $[Cr(en)_2Ox]^+$ shows optical isomerism but not geometrical isomerism. The other two complexes, i.e. $[Pt(en)Cl_2]$ and $[Zn(en)Cl_2]$ do not show stereoisomerism.

20. Major products of the following reaction are :



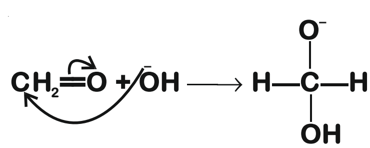
(1) HCOOH and 

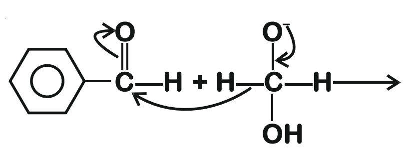
(2)  and 

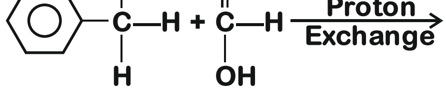
(3) CH_3OH and 

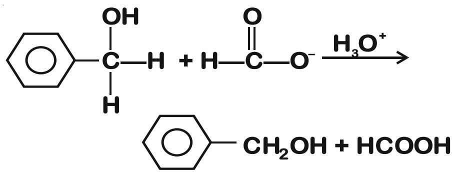
(4) CH_3OH and HCO_2H

Answer (1)

Sol. 







21. The correct order of catenation is :

(1) C > Si > Ge ≈ Sn

(2) C > Sn > Si ≈ Ge

(3) Si > Sn > C > Ge

(4) Ge > Sn > Si > C

Answer (1)

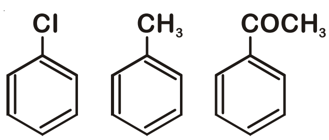
Sol. The order of catenation property amongst 14th group elements is based on bond enthalpy values of identical atoms of the same element. The decreasing order of bond enthalpy values is

$$\text{Bond enthalpy}\quad \underset{348\ \text{kJ/mol}}{\text{C–C}} > \underset{297}{\text{Si–Si}} > \underset{260}{\text{Ge–Ge}} \approx \underset{240\ \text{kJ/mol}}{\text{Sn–Sn}}$$

∴ Decreasing order of catenation is

C > Si > Ge ≈ Sn

22. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :



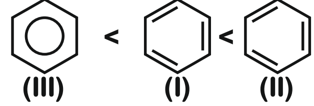
(I) (II) (III)

(1) I < III < II (2) III < II < I

(3) II < I < III (4) III < I < II

Answer (4)

Sol. CH_3 group when bonded to benzene increases the electron density of benzene by +I and hyper conjugation effects and hence makes the compound more reactive towards EAS. Cl group decreases the electron density of benzene by –I effect, and CH_3CO group strongly decreases the electron density of benzene by –I and –R effects. Therefore, correct increasing order the given compounds towards EAS is



23. During the change of O_2 to O_2^-, the incoming electron goes to the orbital :

(1) $\pi 2p_x$ (2) $\pi^* 2p_x$

(3) $\pi 2p_y$ (4) $\sigma^* 2p_z$

Answer (2)

Sol. Electronic configuration of O_2 is

$$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2p_z}^2\pi_{2p_x}^2=\pi_{2p_y}^2\pi_{2p_x}^{*1}=\pi_{2p_y}^{*1}$$

When O_2 gains an electron to form O_2^-, the incoming electron goes to $\pi_{2p_x}^*$ or $\pi_{2p_y}^*$

24. The regions of the atmosphere, where clouds form and where we live, respectively, are :

(1) Troposphere and Troposphere

(2) Stratosphere and Troposphere

(3) Troposphere and Stratosphere

(4) Stratosphere and Stratosphere

Answer (1)

Sol. The lowest region of atmosphere in which human beings live is troposphere. It extends up to a height of 10 km from sea level. Clouds are also formed in this layer.

25. At room temperature, a dilute solution of urea is prepared by dissolving 0.60 g of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mmHg, lowering of vapour pressure will be : (molar mass of urea = 60 g mol^{-1})

(1) 0.031 mmHg (2) 0.017 mmHg

(3) 0.028 mmHg (4) 0.027 mmHg

Answer (2)

Sol. Relative lowering of VP is given by

$$\frac{P_B^\circ - P_B}{P_B^\circ} = x_A = \frac{n_A}{n_A + n_B} \simeq \frac{n_A}{n_B}$$

$$\frac{P_B^\circ - P_B}{35} = \frac{0.6 \times 18}{60 \times 360} = \frac{1}{2000}$$

On solving, $\Delta P_B = P_B^\circ - P_B = 0.017$

adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increases with :

(1) Increase in p and decrease in T

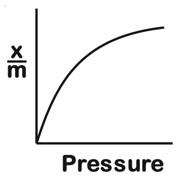
(2) Decrease in p and decrease in T

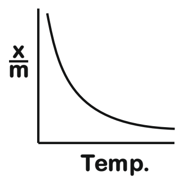
(3) Increase in p and increase in T

(4) Decrease in p and increase in T

Answer (1)

Sol. Freundlich adsorption is applicable for physical adsorption. The variation of extent of adsorption with (i) Pressure and (ii) Temp is given by the following curves.





So, extent of adsorption increases with increase of pressure and decrease of temperature.

27. The synonym for water gas when used in the production of methanol is :

(1) fuel gas

(2) syn gas

(3) laughing gas

(4) natural gas

Answer (2)

Sol. When steam is passed over red hot coke, an equimolar mixture of CO and H_2 is obtained

$H_2O(g) + C \longrightarrow CO + H_2$

Steam Red hot

The gaseous mixture thus obtained is called water gas or syn. gas.

28. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion, and 40 mL of CO_2 is formed. The formula of the hydrocarbon is :

(1) C_4H_{10} (2) C_4H_8

(3) C_4H_6 (4) C_4H_7Cl

Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

10 ml 55 ml

0 $55 - 10\left(x+\frac{y}{4}\right)$ 10 x

Vol. of CO_2, 10x = 40 ; x = 4

$55 - 10\left(x+\frac{y}{4}\right) = 0$; y = 6

$\therefore$ Hydrocarbon is C_4H_6

29. Three complexes, $[CoCl(NH_3)_5]^{2+}$(I), $[Co(NH_3)_5H_2O]^{3+}$(II) and $[Co(NH_3)_6]^{3+}$(III) absorb light in the visible region. The correct order of the wavelength of light absorbed by them is :

(1) (I) > (II) > (III) (2) (II) > (I) > (III)

(3) (III) > (I) > (II) (4) (III) > (II) > (I)

Answer (1)

Sol. In a co-ordination compound, the strong field ligand causes higher splitting of the d-orbitals. Wavelength of the energy absorbed by the co-ordination compound is inversely proportional to ligand field strength of the given co-ordination compound. The decreasing order of ligand field strength is $NH_3 > H_2O > Cl$. Therefore decreasing order of wavelength absorbed is (I) > (II) > (III).

30. Match the refining methods (Column I) with metals (Column II).

Column I (Refining methods)	Column II (Metals)
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

(1) (I)-(c); (II)-(a); (III)-(b); (IV)-(d)

(2) (I)-(b); (II)-(d); (III)-(a); (IV)-(c)

(3) (I)-(c); (II)-(d); (III)-(b); (IV)-(a)

(4) (I)-(b); (II)-(c); (III)-(d); (IV)-(a)

Answer (3)

Sol. Mond's process is used for refining of Ni, Van Arkel method is used for Zr, Liquation is used for Sn and zone refining is used for Ga.

So, correct match is

(I)-(c); (II)-(d); (III)-(b); (IV)-(a)

1. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x-2\sin x+5}}\cdot\frac{1}{4^{\sin^2 y}}\le 1$ also satisfy the equation :

(1) sin x = |sin y|

(2) sin x = 2 sin y

(3) 2 sin x = sin y

(4) 2|sin x| = 3 sin y

Answer (1)

Sol. $2^{\sqrt{\sin^2 x-2\sin x+5}}\le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x-2\sin x+5}\le 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x-1)^2+4}\le 2\sin^2 y$

it is true when sinx = 1

|siny| = 1

so sinx = |siny|

2. If a > 0 and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{1}{5}+\frac{3}{5}i$ (2) $-\frac{3}{5}-\frac{1}{5}i$

(3) $\frac{1}{5}-\frac{3}{5}i$ (4) $-\frac{1}{5}-\frac{3}{5}i$

Answer (4)

Sol. $z=\frac{(1+i)^2}{a-i}\times\frac{a+i}{a+i}$

$z=\frac{(1-1+2i)(a+i)}{a^2+1}=\frac{2ai-2}{a^2+1}$ ...(i)

$|z|=\sqrt{\left(\frac{-2}{a^2+1}\right)^2+\left(\frac{2a}{a^2+1}\right)^2}=\sqrt{\frac{4+4a^2}{(a^2+1)^2}}$

$=\sqrt{\frac{4(1+a^2)}{(1+a^2)^2}}=\frac{2}{\sqrt{1+a^2}}$

given $|z|=\sqrt{\frac{2}{5}}$

so $\sqrt{\frac{2}{5}}=\frac{2}{\sqrt{1+a^2}}$ from equation (i)

(square both side)

$\Rightarrow \frac{2}{5}=\frac{4}{1+a^2}$

$\Rightarrow 1+a^2=10$

$a^2=9$

$\Rightarrow$ a ± 3 $\because$ (a > 0) $\therefore$ a = 3

Hence $z=\frac{1+i^2+2i}{3-i}=\frac{2i}{3-i}=\frac{2i(3+i)}{10}=\frac{-1+3i}{5}$

$\bar{z}=\frac{-1}{5}-\frac{3}{5}i$

3. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

(1) 72 (2) 48

(3) 60 (4) 36

Answer (3)

Sol.

a_1	a_2	a_3	a_4	a_5	a_6

digit 0, 1, 2, 5, 7, 9

$(a_1+a_3+a_5)-(a_2+a_4+a_6)=11K$

so (1, 2, 9) (0, 5, 7)

Now number of ways to arranging them

= 3! × 3! + 3! × 2 × 2

= 6 × 6 + 6 × 4

= 6 × 10

= 60

4. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx,$ where [t] denotes the greatest integer function, is :

(1) π (2) $-\pi$

(3) -2π (4) 2π

Sol. $I=\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$ ...(i)

$\because \int_0^a f(x)=\int_0^a f(a-x)dx$

$\therefore \quad I=\int_0^{2\pi}\left[-\sin 2x(1+\cos 3x)\right]dx$ ...(ii)

By (i) + (ii)

$2I=\int_0^{2\pi}(-1)dx$

$2I=-(x)_0^{2\pi}$

$\Rightarrow$ I = $-\pi$

5. If $\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2=\begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x\neq 0;$ then

for all $\theta\in\left(0,\frac{\pi}{2}\right)$:

(1) $\Delta_1+\Delta_2=-2x^3$

(2) $\Delta_1-\Delta_2=-2x^3$

(3) $\Delta_1+\Delta_2=-2(x^3+x-1)$

(4) $\Delta_1-\Delta_2=x(\cos 2\theta-\cos 4\theta)$

Answer (1)

Sol. $\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x(-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)$

$+\cos\theta(-\sin\theta+x\cos\theta)$

$=-x^3-x+x\sin^2\theta+\sin\theta\cos\theta-\cos\theta\sin\theta$

$+x\cos^2\theta$

$=-x^3-x+x=-x^3$

Similarly $\Delta_1=-x^3$

$\Delta_1+\Delta_2=-2x^3$

6. If $f(x)=\begin{cases} \dfrac{\cdots}{x} & , x<0 \\ q & , x=0 \\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , x>0 \end{cases}$

is continuous at x = 0, then the ordered pair (p, q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},\frac{1}{2}\right)$

(3) $\left(-\frac{3}{2},-\frac{1}{2}\right)$ (4) $\left(-\frac{1}{2},\frac{3}{2}\right)$

Answer (2)

Sol. $f(x)=\begin{cases} \dfrac{\sin(p+1)x+\sin x}{x} & x<0 \\ q & x=0 \\ \dfrac{\sqrt{x^2+x}-\sqrt{x}}{x^{\frac{3}{2}}} & x>0 \end{cases}$

is continuous at x = 0

So $f(0^-)=f(0)=f(0^+)$ ...(1)

$f(0^-)=\lim\limits_{h\to 0} f(0-h)$

$=\lim\limits_{h\to 0}\dfrac{\sin(p+1)(-h)+\sin(-h)}{-h}$

$=\lim\limits_{h\to 0}\left[\dfrac{-\sin(p+1)h}{-h}+\dfrac{\sinh}{h}\right]$

$=\lim\limits_{h\to 0}\dfrac{\sin(p+1)h}{h(p+1)}\times(p+1)+\lim\limits_{h\to 0}\dfrac{\sinh}{h}$

$=(p+1)+1=p+2$ ...(2)

Now $f(0^+)=\lim\limits_{h\to 0}f(0+h)=\lim\limits_{h\to 0}\dfrac{\sqrt{h^2+h}-\sqrt{h}}{h^{3/2}}$

$=\lim\limits_{h\to 0}\dfrac{(h)^{\frac{1}{2}}\left[\sqrt{h+1}-1\right]}{h\left(h^{\frac{1}{2}}\right)}$

$=\lim\limits_{h\to 0}\dfrac{\sqrt{h+1}-1}{h}\times\dfrac{\sqrt{h+1}+1}{\sqrt{h+1}+1}$

$=\lim\limits_{h\to 0}\dfrac{h+1-1}{h\left(\sqrt{h+1}+1\right)}$

$=\lim\limits_{h\to 0}\dfrac{1}{\sqrt{h+1}+1}=\dfrac{1}{1+1}=\dfrac{1}{2}$ ...(3)

$p + 2 = q = \frac{1}{2}$

So, $q = \frac{1}{2}$ and $p = \frac{1}{2} - 2 = \frac{-3}{2}$

$(p, q) \equiv \left(-\frac{3}{2}, \frac{1}{2}\right)$

7. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) – 1 (2) – 2

(3) 1 (4) 2

Answer (1)

Sol. $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = p$ $\quad P(\beta, 0, \beta)$

any point on line A = (p, 1, – p – 1)

Now, DR of AP $\equiv$ < p – β, 1 – 0, – p – 1 – β >

Which is perpendicular to line so

$(p - \beta).\ 1 + 0.1 - 1(-p - 1 - \beta) = 0$

$\Rightarrow p - \beta + p + 1 + \beta = 0$

$p = \frac{-1}{2}$

Point $A\left(\frac{-1}{2}, 1 - \frac{1}{2}\right)$

Now, distance AP = $\sqrt{\frac{3}{2}}$

$\Rightarrow AP^2 = \frac{3}{2}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 + 1 + \left(\beta + \frac{1}{2}\right)^2 = \frac{3}{2}$

$2\left(\beta + \frac{1}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow \left(\beta + \frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow \beta = 0, -1, (\beta \neq 0)$

$\therefore \boxed{\beta = -1}$

to :

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$

(3) $\frac{4}{3}(2)^{3/4}$ (4) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

Answer (2)

Sol. $\lim_{n \to \infty} \frac{(n+1)^{\frac{1}{3}} + (n+2)^{\frac{1}{3}} + \ldots + (n+n)^{\frac{1}{3}}}{n(n)^{\frac{1}{3}}}$

$= \lim_{n \to \infty} \sum_{r=1}^{n} \frac{(n+r)^{\frac{1}{3}}}{n.n^{\frac{1}{3}}}$ $\qquad \frac{r}{n} \to x$ and $\frac{1}{n} \to dx$

$= \int_0^1 (1+x)^{\frac{1}{3}} dx$

$= \left[\frac{3}{4}(1+x)^{\frac{4}{3}}\right]_0^1 = \frac{3}{4}(2)^{\frac{4}{3}} - \frac{3}{4}$

9. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

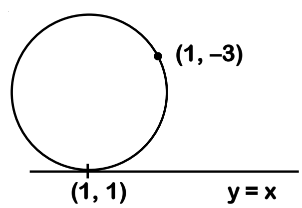
(1) $3\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 3

Answer (3)

Sol. Equation of circle = $(x - 1)^2 + (y - 1)^2 + \lambda(y - x) = 0$

Which passes through (1, –3)



So, $0 + 16 + \lambda(-3 - 1) = 0$

$16 + \lambda(-4) = 0$

$\boxed{\lambda = 4}$

Now equation of circle

$(x - 1)^2 + (y - 1)^2 + 4y - 4x = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

radius = $\sqrt{9 + 1 - 2} = 2\sqrt{2}$

(1) $(p \vee q) \vee (p \vee \sim q)$

(2) $(p \wedge q) \vee (p \wedge \sim q)$

(3) $(p \vee q) \wedge (p \vee \sim q)$

(4) $(p \vee q) \wedge (\sim p \vee \sim q)$

Answer (1)

Sol. $(p \vee q) \vee (p \vee \sim q)$

$= p \vee (q \vee p) \vee \sim q$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$ so first statement is tautology

11. If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1} = \lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$, then k is :

(1) $\dfrac{4}{3}$

(2) $\dfrac{3}{2}$

(3) $\dfrac{8}{3}$

(4) $\dfrac{3}{8}$

Answer (3)

Sol. If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1} = \lim\limits_{x\to K}\left(\dfrac{x^3-k^3}{x^2-k^2}\right)$

L.H.S.

$\mathop{\mathrm{Lt}}\limits_{x\to 1}\dfrac{x^4-1}{x-1} = \left(\dfrac{0}{0}\text{ form}\right)$

$\mathop{\mathrm{Lt}}\limits_{x\to 1}\dfrac{4x^3}{1} = 4$

Now, $\lim\limits_{x\to K}\dfrac{x^3-k^3}{x^2-k^2} = 4$

$\Rightarrow \lim\limits_{x\to K}\dfrac{3x^2}{2x} = 4$

$\Rightarrow \dfrac{3}{2}k = 4$

$k = \dfrac{8}{3}$

origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

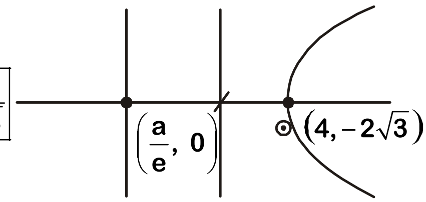
(1) $4e^4 + 8e^2 - 35 = 0$ (2) $4e^4 - 24e^2 + 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$ (4) $4e^4 - 24e^2 + 27 = 0$

Answer (2)

Sol. $x = \dfrac{4}{\sqrt{5}}$

$\because \boxed{\dfrac{a}{e} = \dfrac{4}{\sqrt{5}}}$



Equation of hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ it passes through $(4, -2\sqrt{3})$

$\because \quad e^2 = 1 + \dfrac{b^2}{a^2}$

$\Rightarrow \boxed{a^2e^2 - a^2 = b^2}$

$\Rightarrow \dfrac{16}{a^2} - \dfrac{12}{a^2e^2 - a^2} = 1$

$\Rightarrow \dfrac{4}{a^2}\left[\dfrac{4}{1} - \dfrac{3}{e^2-1}\right] = 1$

$\Rightarrow 4e^2 - 4 - 3 = (e^2 - 1)\left(\dfrac{a^2}{4}\right)$

$\Rightarrow 4(4e^3 - 7) = (e^2 - 1)\left(\dfrac{4e}{\sqrt{5}}\right)^2$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

13. If the line $x - 2y = 12$ is tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the point $\left(3, \dfrac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

(1) 5 (2) $8\sqrt{3}$

(3) $12\sqrt{2}$ (4) 9

Sol. Equation of tangent at $\left(3, -\frac{9}{2}\right)$ to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{3x}{a^2} - \frac{y9}{2b^2} = 1 \text{ which is equivalent to } x - 2y = 12$$

$$\frac{3}{a^2} = \frac{-9}{2b^2\cdot(-2)} = \frac{1}{12} \text{ (On comparing)}$$

$$a^2 = 3 \times 12 \text{ and } b^2 = \frac{9\times 12}{4}$$

$$\boxed{a=6} \Rightarrow b = 3\sqrt{3}$$

So latus rectum = $\frac{2b^2}{a} = \frac{2\times 27}{6} = 9$

14. Let A(3, 0 –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the mid point of AC. If G divides BM in the ratio, 2 : 1 then cos(∠ GOA) (O being the origin) is equal to :

(1) $\frac{1}{6\sqrt{10}}$

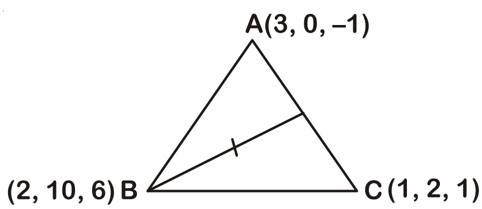
(2) $\frac{1}{\sqrt{30}}$

(3) $\frac{1}{2\sqrt{15}}$

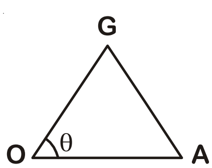
(4) $\frac{1}{\sqrt{15}}$

Answer (4)

Sol. G is the centroid of ΔABC



G (2, 4, 2)



$$OG = \sqrt{4+16+4}, \quad OA = \sqrt{9+1}$$

$$AG = \sqrt{1+16+9}$$

$2\sqrt{24}\sqrt{10}$

$$= \frac{8}{2\sqrt{8\times 3\times 2\times 5}}$$

$$= \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$$

15. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 10 (2) 12

(3) 7 (4) 9

Answer (1)

Sol. $x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$ have infinite solution

$\Delta = 0, \quad \Delta x = \Delta y = \Delta z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 2) + 1(3 - 2) = 0$

$\Rightarrow 2\lambda - 6 - \lambda + 2 + 1 = 0$

$$\boxed{\lambda = 3}$$

Now, $\Delta x = \begin{vmatrix} 5 & 1 & 1 \\ 6 & 2 & 2 \\ \mu & 3 & 3 \end{vmatrix} = 0$, $\Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu-5 & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(2 - \mu + 5) = 0$

$$\boxed{\mu = 7}$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix} = \begin{vmatrix} 1 & 1 & 5 \\ 0 & -1 & -1 \\ 0 & 2 & \mu-5 \end{vmatrix}$$

$\Rightarrow 1(5 - \mu + 2) = 0$

$\Rightarrow \mu = 7$

So, $\lambda + \mu = 10$

children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{12}$

(3) $\frac{1}{10}$ (4) $\frac{1}{17}$

Answer (1)

Sol. A = At least two girls

B = All girls

$$P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$$

$$=\frac{P(B)}{P(A)}=\frac{\left(\frac{1}{4}\right)^4}{1-{}^4C_0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^4}$$

$$=\frac{1}{16-1-4}=\frac{1}{11}$$

17. If for some $x\in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

Then the mean of the marks is :

(1) 3.2 (2) 3.0

(3) 2.5 (4) 2.8

Answer (4)

Sol. Number of students

$\Rightarrow$ $(x+1)^2+(2x-5)+(x^2-3x)+x=20$

$\Rightarrow$ $2x^2+2x-4=20$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=3$

So,

Marks	2	3	5	7
No. of students	16	1	0	3

Average marks = $\frac{32+3+21}{20}=\frac{56}{20}=2.8$

18. If the circles $x^2+y^2+5Kx+2y+K=0$ and $2(x^2+y^2)+2Kx+3y-1=0$, $(K\in R)$, intersect at the points P and Q, then the line $4x+5y-K=0$ passes through P and Q, for :

(1) Infinitely many values of K

(2) Exactly one value of K

(3) Exactly two values of K

(4) No value of K

Answer (4)

Sol. $S_1\equiv x^2+y^2+5Kx+2y+K=0$

$S_2\equiv x^2+y^2+Kx+\frac{3}{2}y-\frac{1}{2}=0$

Equation of common chord is

$S_1-S_2=0$

$\Rightarrow$ $4Kx+\frac{y}{2}+K+\frac{1}{2}=0$ ...(1)

$4x+5y-K=0$ ...(2) (given)

On comparing (1) and (2)

$$\frac{4K}{4}=\frac{1}{10}=\frac{2K+1}{-2K}$$

$\Rightarrow$ $\boxed{K=\frac{1}{10}}$ and $-2K=20K+10$

$\Rightarrow$ $22K=-10$

$\boxed{K=\frac{-5}{11}}$

$\therefore$ No value of K exists

19. The region represented by $|x-y|\le 2$ and $|x+y|\le 2$ is bounded by a :

(1) Square of side length $2\sqrt{2}$ units

(2) Square of area 16 sq. units

(3) Rhombus of side length 2 units

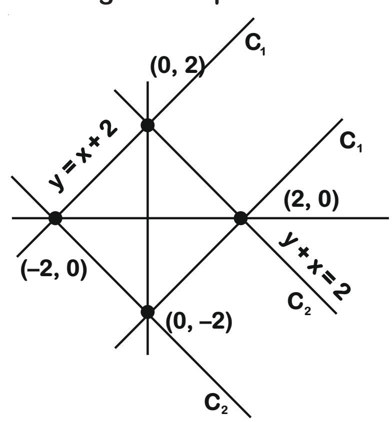
(4) Rhombus of area $8\sqrt{2}$ sq. units

Answer (1)

Sol. $C_1 : |y-x|\le 2$

$C_2 : |y+x|\le 2$

Now region is square



Length of side = $\sqrt{2^2+2^2}=2\sqrt{2}$

equal to :

(1) 98 (2) 38

(3) 64 (4) 76

Answer (4)

Sol. $3(a_1 + a_{16}) = 114$

$\boxed{a_1 + a_{16} = 38}$

Now $a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16})$

$= 2 \times 38 = 76$

21. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$

(3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

Answer (4)

Sol. Image of Q in plane

$$\frac{(x-0)}{3} = \frac{(y+1)}{-1} = \frac{z+3}{+4} = \frac{-2(1-12-2)}{9+1+16} = 1$$

$x = 3, y = -2, z = 1$

P(3, –2, 1), Q(0, –1, –3), R(3, –1, –2)

Now area of ΔPQR is

$$\frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{QR}| = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}\left|\left\{\hat{i}(-1) - \hat{j}(3-12) + \hat{k}(3)\right\}\right|$$

$$= \frac{1}{2}\sqrt{(1+81+9)}$$

$$= \frac{\sqrt{91}}{2}$$

22. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If S = [0, 4], then which one of the following statements is not true ?

(1) $f(g(S)) = S$ (2) $g(f(S)) = g(S)$

(3) $g(f(S)) \neq S$ (4) $f(g(S)) \neq f(S)$

Sol. $f(x) = x^2$ $x \in R$

$g(A) = \{x \in R : f(x) \in A\}$ $S \equiv [0, 4]$

$g(S) = \{x \in R : f(x) \in S\}$

$= \{x \in R : 0 \le x^2 \le 4\}$

$= \{x \in R : -2 \le x \le 2\}$

$\therefore$ $g(S) \neq S$

$\therefore$ $f(g(S)) \neq f(S)$

$g(f(S)) = \{x \in R : f(x) \in f(S)\}$

$= \{x \in R : x^2 \in S^2\}$

$= \{x \in R : 0 \le x^2 \le 16\}$

$= \{x \in R : -4 \le x \le 4\}$

$\therefore$ $g(f(S)) \neq g(S)$

$\therefore$ $g(f(S)) = g(S)$ is incorrect

23. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (– 54, 315)

(2) (28, 861)

(3) (–21, 714)

(4) 28, 315

Answer (4)

Sol. $(1 + ax + bx^2)(1 - 3x)^{15}$

Co-eff. of $x^2 = 1.{}^{15}C_2(-3)^2 + a.{}^{15}C_1(-3) + b.{}^{15}C_0$

$$= \frac{15 \times 14}{2} \times 9 - 15 \times 3a + b = 0 \text{ (Given)}$$

$\Rightarrow$ $945 - 45a + b = 0$ ...(i)

Now co-eff. of $x^3 = 0$

$\Rightarrow$ ${}^{15}C_3(-3)^3 + a.{}^{15}C_2(-3)^2 + b.{}^{15}C_1(-3) = 0$

$$\Rightarrow \frac{15 \times 14 \times 13}{3 \times 2} \times (-3 \times 3 \times 3) + a \times \frac{15 \times 14 \times 9}{2} - b \times 3 \times 15 = 0$$

$\Rightarrow$ $15 \times 3[-3 \times 7 \times 13 + a \times 7 \times 3 - b] = 0$

$\Rightarrow$ $21a - b = 273$ ...(ii)

From (i) and (ii)

$a = +28, b = 315 \equiv (a, b) \equiv (28, 315)$

$$\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+....$$

upto 10^{th} term, is :

(1) 620 (2) 600

(3) 680 (4) 660

Answer (4)

Sol. $T_r=\frac{(2r+1)(1^3+2^3+3^3+...+r^3)}{1^2+2^2+3^2+...+r^2}$

$$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2\times\frac{6}{r(r+1)(2r+1)}$$

$$T_r=\frac{3r(r+1)}{2}$$

Now,

$$S=\sum_{r=1}^{10}T_r=\frac{3}{2}\sum_{r=1}^{10}(r^2+r)$$

$$=\frac{3}{2}\left\{\frac{10\times(10+1)(2\times10+1)}{6}+\frac{10\times11}{2}\right\}$$

$$=\frac{3}{2}\left\{\frac{10\times11\times21}{6}+5\times11\right\}$$

$$=\frac{3}{2}\times5\times11\times8=660$$

25. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\ x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is :

(1) $\left[-1,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ (2) $[0,\infty)$

(3) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$ (4) $\left[\frac{-1}{2},0\right]\cup[1,\infty)$

Answer (3)

Sol. $f(x) = e^x - x$, $g(x) = x^2 - x$

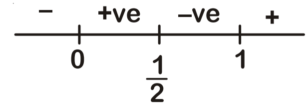
$f(g(x)) = e^{(x^2-x)}-(x^2-x)$

If $f(g(x))$ is increasing function

$(f(g(x)))' = e^{(x^2-x)}\times(2x-1)-2x+1$

$= (2x-1)e^{(x^2-x)}+1-2x$

$= \underbrace{(2x-1)}_{A}\underbrace{[e^{(x^2-x)}-1]}_{B}$



for $(f(g(x)))' \geq 0$,

$$x\in\left[0,\frac{1}{2}\right]\cup[1,\infty)$$

26. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta\in\left(0,\frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to :

(1) $\frac{2^{12}}{(\sin\theta-8)^6}$ (2) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

(3) $\frac{2^6}{(\sin\theta+8)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

Answer (4)

Sol. Given $\alpha + \beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$

$$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$$

Hence required quantity

$$\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$$

27. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx}=(\tan x-y)\sec^2 x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{1}{e}-2$

(2) $\frac{1}{2}-e$

(3) $e - 2$

(4) $2+\frac{1}{e}$

Sol. $\frac{dy}{dx} + y\sec^2 x = \sec^2 x \tan x$ $\rightarrow$ This is linear differential equation

$IF = e^{\int \sec^2 x dx} = e^{\tan x}$

Now solution is

$$y \cdot e^{\tan x} = \int e^{\tan x} \sec^2 x \tan x dx$$

$\because$ Let tanx = t

$\sec^2 x dx = dt$

$$ye^{\tan x} = \int \underset{\text{II}}{e^t}\, \underset{\text{I}}{t}\, dt$$

$ye^{\tan x} = te^t - e^t + c$

$ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$

$y = (\tan x - 1) + c \cdot e^{-\tan x}$

Given y(0) = 0

$\Rightarrow$ $0 = -1 + c$

$\Rightarrow$ $c = 1$

$$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$$

28. Let $f : R \rightarrow R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is :

(1) not differentiable if $f'(c) = 0$

(2) differentiable if $f'(c) = 0$

(3) not differentiable

(4) differentiable if $f'(c) \neq 0$

Answer (2)

Sol. $\because$ $g'(c) = \lim\limits_{x \to c} \frac{g(x) - g(c)}{x - c}$

$\Rightarrow$ $g'(c) = \lim\limits_{x \to c} \frac{|f(x)| - |f(c)|}{x - c}$

$\because$ $f(c) = 0$

$\Rightarrow$ $g'(c) = \lim\limits_{x \to c} \frac{|f(x)|}{x - c}$

$\Rightarrow$ $g'(c) = \lim\limits_{x \to c} \frac{f(x)}{x - c}$ if $f(x) > 0$

$x \to c \quad x - c$

$\Rightarrow$ $g'(c) = f'(c) = -f'(c)$

$\Rightarrow$ $2f'(c) = 0$

$\Rightarrow$ $f'(c) = 0$

29. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

(1) 20

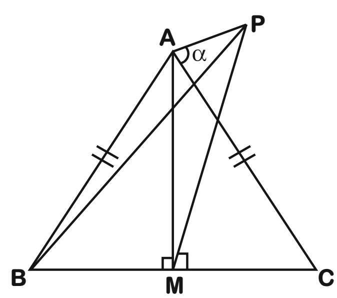
(2) $10\sqrt{5}$

(3) 25

(4) $\frac{100}{3\sqrt{3}}$

Answer (1)

Sol.



ΔAPM

$$\frac{h}{AM} = \frac{1}{3\sqrt{2}}$$

ΔBPM

$$\frac{h}{BM} = \frac{1}{\sqrt{7}}$$

ΔABM

$\therefore$ $AM^2 + MB^2 = (100)^2$

$\Rightarrow$ $18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow$ $h^2 = 4 \times 100$

$\Rightarrow$ $h = 20$

$(x^2-2x+10)$

$$=A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$$

where C is a constant of integration, then :

(1) $A=\frac{1}{81}$ and $f(x)=3(x-1)$

(2) $A=\frac{1}{54}$ and $f(x)=3(x-1)$

(3) $A=\frac{1}{27}$ and $f(x)=9(x-1)$

(4) $A=\frac{1}{54}$ and $f(x)=9(x-1)^2$

Answer (2)

Sol. $\displaystyle\int\frac{dx}{(x^2-2x+10)^2}=\int\frac{dx}{((x-1)^2+9)^2}$

Let $(x-1)^2 = 9\tan^2\theta$ ...(i)

$$\Rightarrow \quad \tan\theta=\frac{x-1}{3}$$

$$2(x-1)dx = 18\tan\theta\sec^2\theta\, d\theta$$

$$\therefore\ I=\int\frac{18\tan\theta\sec^2\theta\, d\theta}{2\times 3\tan\theta\times 81\sec^4\theta}$$

$$I=\frac{1}{27}\int\cos^2\theta\, d\theta=\frac{1}{27}\times\frac{1}{2}\int(1+\cos 2\theta)d\theta$$

$$I=\frac{1}{54}\left\{\theta+\frac{\sin 2\theta}{2}\right\}+c$$

$$I=\frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right]+c$$

$$I=\frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3(x-1)}{x^2-2x+10}\right]+c$$

So $A=\frac{1}{54}$

$f(x) = 3(x-1)$

❑ ❑ ❑

Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes from V_0 to $2V_0$.

(1) $\frac{1}{4}\frac{P_0V_0}{R}$ (2) $\frac{5}{4}\frac{P_0V_0}{R}$

(3) $\frac{1}{2}\frac{P_0V_0}{R}$ (4) $\frac{3}{4}\frac{P_0V_0}{R}$

Answer (2)

Sol. If $V_1 = V_0 \Rightarrow P_1 = P_0\left[1-\frac{1}{2}\right] = \frac{P_0}{2}$

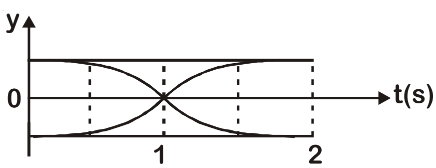
If $V_2 = 2V_0 \Rightarrow P_2 = P_0\left[1-\frac{1}{2}\left(\frac{1}{4}\right)\right] = \left(\frac{7P_0}{8}\right)$

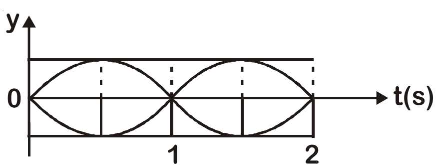
$\left(T = \frac{PV}{nR}\right) \Rightarrow \Delta T = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right|$

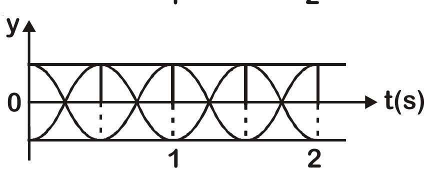
$\Delta T = \left|\left(\frac{1}{nR}\right)(P_1V_1 - P_2V_2)\right| = \left(\frac{1}{nR}\right)\left|\left(\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right)\right|$

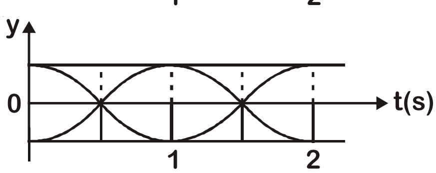
$= \frac{5P_0V_0}{4nR}$

2. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is :

(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. Beat frequency = $|f_1 - f_2| = 11 - 9 = 2$ Hz

3. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Given: Mass of planet = 8×10^{22} kg,

Radius of planet = 2×10^6 m,

Gravitational constant G = 6.67×10^{-11} Nm^2/kg^2]

(1) 17 (2) 13

(3) 11 (4) 9

Answer (3)

Sol. $T = \frac{2\pi r}{v}, v = \sqrt{\frac{GM}{r}}$

$T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$

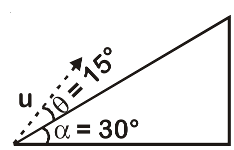
$T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{6.67 \times 10^{-11} \times 8 \times 10^{22}}}$ sec

T = 7812.2 s

$T \simeq 2.17\text{hr} \Rightarrow 11\,\text{revolutions}$

4. A plane is inclined at an angle α = 30° with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1}, from the base of the plane, making an angle θ = 15° with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :

(Take g = 10 ms^{-2})



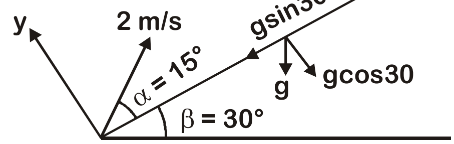
(1) 18 cm (2) 20 cm

(3) 14 cm (4) 26 cm

Answer (2)

Sol. Time of flight (T) $= \frac{2u\sin\alpha}{g\cos\beta}$

$T = \frac{(2)(2\sin15)}{g\cos30} = \frac{4}{10}\frac{\sin15}{\cos30}$



$$\text{Range (R)} = (2\cos 15)T - \frac{1}{2}g\sin 30(T)^2$$

$$= (2\cos 15)\frac{4}{10}\frac{\sin 15}{\cos 30} - \left(\frac{1}{3}\times 10\sin 30\right)\frac{16}{100}\frac{\sin^2 15}{\cos^2 30}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\text{ m} \simeq 20\text{ cm}$$

5. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to :

(1) 0.4 ms^{-1} (2) 0.7 ms^{-1}

(3) 0.3 ms^{-1} (4) 0.1 ms^{-1}

Answer (2)

Sol. $v^2 = u^2 - 2aS$

$$v^2 = (1)^2 - (2)\left[\frac{2.5\times 10^{-2}}{20\times 10^{-3}}\right]\frac{20}{100}$$

$$v^2 = 1 - \frac{1}{2}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{2}}\text{ m/s} = 0.7\text{ m/s}$$

6. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :

(1) $-34\left(\hat{k} - \hat{i}\right)$ (2) $48\left(\hat{i} + \hat{j}\right)$

(3) $36\hat{k}$ (4) $-48\hat{k}$

Answer (4)

Sol. $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$

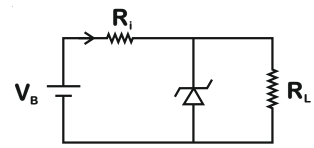
$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$$

$$\vec{a} = \frac{\overline{dv}}{dt} = -6\hat{j}$$

$$\vec{F} = m\vec{a} = -12\hat{j}$$

$$\vec{L} = m(\vec{r}\times\vec{v}) = 2(4\hat{i} - 12\hat{j})\times(2\hat{i} - 12\hat{j}) = -48\hat{k}$$

7. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, R_L = 4 kΩ. The series resistance of the circuit is R_i = 1 kΩ. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?

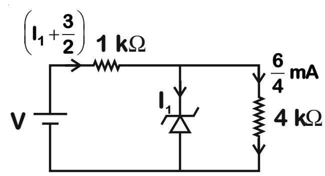


(1) 0.5 mA; 6 mA (2) 0.5 mA; 8.5 mA

(3) 1.5 mA; 8.5 mA (4) 1 mA; 8.5 mA

Answer (2)

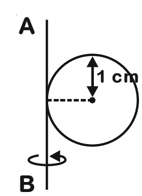
Sol.



$$I_1 = \left(8 - 6 - \frac{3}{2}\right) = \frac{1}{2} = 0.5\text{ mA}$$

$$I_2 = \left(16 - 6 - \frac{3}{2}\right) = 8.5\text{ mA}$$

8. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is closed to :



(1) 4.0×10^{-6} Nm (2) 7.9×10^{-6} Nm

(3) 2.0×10^{-5} Nm (4) 1.6×10^{-5} Nm

Answer (3)

$\omega = \omega_0 + \alpha t$

$\Rightarrow$ $25 \times 2\pi = (\alpha)5$

$\alpha = 10\pi$

$\Rightarrow$ $\tau = \left(\dfrac{5}{4}mR^2\right)\alpha$

$\approx \left(\dfrac{5}{4}\right)(5\times10^{-3})(10^{-4})10\pi$

$= 2.0 \times 10^{-5}$ Nm

9. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be :

(1) $(\sqrt{3}+1)^4 : 16$ (2) 4 : 1

(3) 25 : 9 (4) 9 : 1

Answer (4)

Sol. $I_1 = 4\,I_0$

$I_2 = I_0$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{9}{1}\right)$$

10. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\dfrac{1}{e}\right)^2$ will be :

(1) $\dfrac{1}{2\lambda}$ (2) $\dfrac{1}{4\lambda}$

(3) $\dfrac{1}{\lambda}$ (4) $\dfrac{2}{\lambda}$

Answer (1)

Sol. $N_x(\text{at } t) = N_0 e^{-5\lambda t}$

$N_y(\text{at } t) = N_0 e^{-\lambda t}$

$$\frac{N_x}{N_y} = \frac{1}{e^2} = e^{-4\lambda t}$$

$\Rightarrow$ $4\lambda t = 2$

$\Rightarrow$ $t = \dfrac{2}{4\lambda} = \left(\dfrac{1}{2\lambda}\right)$

maximum weight that can be put on the block without fully submerging it under water?

[Take, density of water = 10^3 kg/m^3]

(1) 30.1 kg (2) 46.3 kg

(3) 87.5 kg (4) 65.4 kg

Answer (3)

Sol. Given $(50)^3 \times \dfrac{30}{100} \times (1) \times g = M_{cube}\,g$ ...(i)

Let m mass should be placed

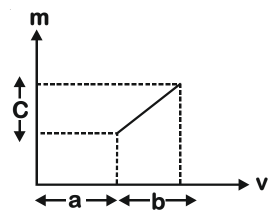
Hence $(50)^3 \times (1) \times g = (M_{cube} + m)g$ ...(ii)

equation (ii) – equation (i)

$\Rightarrow$ $mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3\,g$

$\Rightarrow$ m = 87.5 kg

12. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used?



(1) $\dfrac{b^2}{ac}$ (2) $\dfrac{b^2c}{a}$

(3) $\dfrac{a}{c}$ (4) $\dfrac{b}{c}$

Answer (4)

Sol. As the graph between magnification (m) and image distance (v) varies linearly, then

$m = k_1 v + k_2$

$\Rightarrow$ $\dfrac{v}{u} = k_1 v + k_2$

$\Rightarrow$ $\dfrac{1}{u} = k_1 + \dfrac{k_2}{v}$

$\Rightarrow$ $\dfrac{k_2}{v} - \dfrac{1}{u} = k_1$

Clearly, $k_1 = \dfrac{1}{f}$ and $k_2 = 1$ here

$\therefore$ $f = \dfrac{1}{\text{slope of } m - v \text{ graph}} = \dfrac{b}{c}$

ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ?

(Given: h = 6.63 × 10^{-34} Js; c = 3 × 10^{8} ms^{-1})

(1) 11.4 nm

(2) 12.3 nm

(3) 9.4 nm

(4) 10.8 nm

Answer (4)

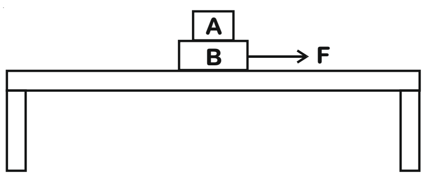
Sol. $\Delta E = \dfrac{hc}{\lambda}$

$$= (13.4)(3)^2\left[1-\frac{1}{16}\right]\text{eV}$$

$$\Rightarrow \quad \lambda = \frac{1242\times16}{(13.4)\times(9)(15)}\text{nm} \simeq 10.8 \text{ nm}$$

14. Two blocks A and B of masses m_A = 1 kg and m_B = 3 kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is:

[Take g = 10 m/s^2]



(1) 40 N

(2) 12 N

(3) 16 N

(4) 8 N

Answer (3)

Sol. $a_c = \left(\dfrac{F-f}{M+m}\right)$

$$a = \frac{F-(0.2)4\times10}{4} = \left(\frac{F-8}{4}\right)$$

We have $\dfrac{F-8}{4} \le (0.2)10$

$\Rightarrow \quad F - 8 \le 8$

$\Rightarrow \quad F \le 16$

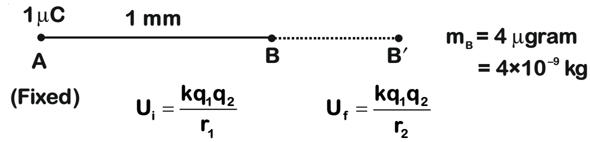
same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is :

$$\left[\text{Take } \frac{1}{4\pi\epsilon_0} = 9\times10^9 \text{ Nm}^2\text{C}^{-2}\right]$$

(1) 2.0 × 10^3 m/s (2) 3.0 × 10^4 m/s

(3) 1.5 × 10^2 m/s (4) 1.0 m/s

Answer (Bonus)



Conservation of energy

$$\frac{kq_1q_2}{r_1} = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv^2$$

$$v^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

$$= \frac{2\times9\times10^9\times10^{-12}}{4\times10^{-9}\times10^{-3}}\left[1-\frac{1}{9}\right] = 4 \times 10^9$$

$$v = \sqrt{40}\times10^4 \text{ m/s} = 6.32\times10^4 \text{ m/s}$$

16. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of stream, 0.15 m below the tap would be:

(Take g = 10 ms^{-2})

(1) 1 × 10^{-5} m^2 (2) 5 × 10^{-5} m^2

(3) 5 × 10^{-4} m^2 (4) 2 × 10^{-5} m^2

Answer (2)

Sol. Using Bernoullie's equation $v_2 = \sqrt{v_1^2 + 2gh}$

Equation of continuity

$A_1V_1 = A_2V_2$

$$(1\text{ cm}^2)(1\text{ m/s}) = (A_2)\left(\sqrt{(1)^2 + 2\times10\times\frac{15}{100}}\right)$$

$$\Rightarrow \quad A_2\left(\text{In cm}^2\right) = \frac{1}{2}$$

$\Rightarrow$ A_2 = 5 × 10^{-5} m^2

temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is:

(1) $\frac{3}{2}Q$ (2) $\frac{2}{3}Q$

(3) $\frac{7}{5}Q$ (4) $\frac{5}{3}Q$

Answer (3)

Sol. Heat supplied at constant volume

$Q = nC_V\Delta T$

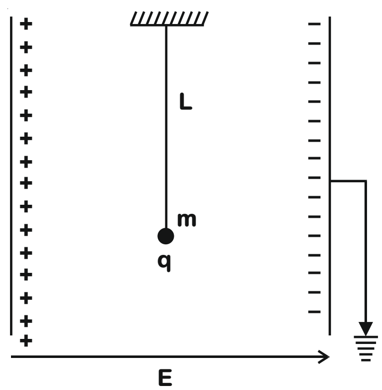
and heat supplied at constant pressure

$Q_1 = nC_p\Delta T$

$$\therefore \frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$\Rightarrow Q_1 = (Q)\left(\frac{7}{5}\right)$$

18. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by:



(1) $2\pi\sqrt{\frac{L}{\left(g-\frac{qE}{m}\right)}}$ (2) $2\pi\sqrt{\frac{L}{\left(g+\frac{qE}{m}\right)}}$

(3) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$ (4) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$

Answer (3)

$\sqrt{g_{eff}}$

$$\Rightarrow g_{eff} = \sqrt{g^2+\left(\frac{gE}{m}\right)^2}$$

$$\Rightarrow t = 2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$$

19. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crosssing him? (Take velocity of sound in air is 350 m/s)

(1) 857 Hz (2) 1143 Hz

(3) 807 Hz (4) 750 Hz

Answer (4)

Sol. $f_{app} = f_{act}\left(\frac{V \pm V_0}{V \mp V_s}\right)$

$$1000 = f_{act}\left(\frac{350-0}{350+(-50)}\right) \text{ and } f' = f_{act}\left(\frac{350}{350+50}\right)$$

$$\Rightarrow f_{act} = \frac{1000\times300}{400}$$

$f_{act} \approx 750\text{Hz}$

20. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is:

[Given Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^{8}$ m/s]

(1) 2×10^{16}

(2) 1.5×10^{16}

(3) 1×10^{16}

(4) 5×10^{15}

Answer (4)

Sol. $E = \frac{hc}{\lambda}$

Let no. of photons per sec. is N

$$\Rightarrow N\frac{hc}{\lambda} = 2\text{ mW}$$

$$\Rightarrow N = \frac{2\times\lambda}{hC} = \frac{2\times5000\times10^{-3}}{12420\times1.6\times10^{-19}}$$

$N = 5 \times 10^{15}$

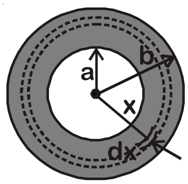
medium of resistivity ρ. The resistance between the two spheres will be:

(1) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (2) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

(3) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (4) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

Answer (1)

Sol.



$$dR=\frac{(\rho)(dx)}{4\pi x^2}$$

$$R=\int dR$$

$$=\left(\frac{\rho}{4\pi}\right)=\int_a^b \frac{dx}{x^2}$$

$$=\left(\frac{\rho}{4\pi}\right)\cdot\left(\frac{1}{a}-\frac{1}{b}\right)$$

22. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is,

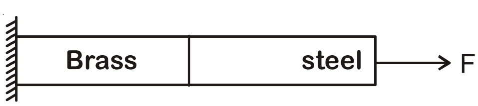
[Given, the Young's Modulus for steel and brass are, respectively, 120 × 10^9 N/m^2 and 60 × 10^9 N/m^2]

(1) 1.8 × 10^6 N/m^2 (2) 1.2 × 10^6 N/m^2

(3) 4.0 × 10^6 N/m^2 (4) 0.2 × 10^6 N/m^2

Answer (Bonus)

Sol.



Corresponding to the stress (σ)

Total elongation $\Delta l_{net}=\frac{\sigma L_1}{Y_1}+\frac{\sigma L_2}{Y_2}$

$$\sigma=\Delta l\left(\frac{Y_1 Y_2}{Y_1+Y_2}\right)$$

$$=8\times10^6\ \frac{N}{m^2}$$ (Answer is not matching)

23. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is: [take ln5 = 1.6]

(1) 0.002 s (2) 0.324 s

(3) 0.103 s (4) 0.016 s

Answer (4)

Sol. $I=I_{sat}\left(1-e^{-\frac{Rt}{L}}\right)$ Here $R=R_L+r=1\Omega$

$$0.8I_{sat}=I_{sat}\left(1-e^{-\frac{t}{.01}}\right)$$

$$\Rightarrow \frac{4}{5}=1-e^{-100t}$$

$$\Rightarrow e^{-100t}=\left(\frac{1}{5}\right)$$

$\Rightarrow$ 100t = ln5

$$\Rightarrow t=\frac{1}{100}\ln 5$$

= 0.016 sec

24. The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is:

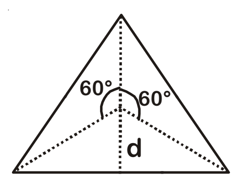
[Take μ_0 = 4π × 10^{-7} NA^{-2}]

(1) 18 μT (2) 1 μT

(3) 3 μT (4) 9 μT

Answer (1)

Sol.



$$d=\left(\frac{1}{3}\right)(a\sin 60)$$

$$d=\frac{a}{3}\times\frac{\sqrt{3}}{2}=\left(\frac{a}{2\sqrt{3}}\right)$$

$$= \frac{3\mu_0 I}{4\pi\left(\frac{a}{2\sqrt{3}}\right)} = (2)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)$$

$$= \frac{9\times 2\times 10^{-7}\times 10}{1}$$

$= 18\ \mu T$

$$d = \left(\frac{1}{3}\right)(a\sin 60)$$

$$d = \frac{a}{3}\times\frac{\sqrt{3}}{2}$$

$$= \left(\frac{a}{2\sqrt{3}}\right)$$

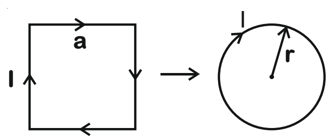
25. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be :

(1) $\frac{2m}{\pi}$ (2) $\frac{4m}{\pi}$

(3) $\frac{m}{\pi}$ (4) $\frac{3m}{\pi}$

Answer (2)

Sol.



$2\pi r = 4a \quad \Rightarrow \quad r = \left(\frac{2a}{\pi}\right)$

$m = (I)\ a^2$

$m_1 = (I)\ \pi r^2$

$$m_1 = (I)(\pi)\left(\frac{4a^2}{\pi^2}\right)$$

$$m_1 = \frac{4Ia^2}{\pi}$$

$$m_1 = \frac{4m}{\pi}$$

26. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2}) :

(1) 600 m (2) 500 m

(3) 300 m (4) 400 m

Answer (3)

$\Rightarrow \Delta H \simeq 300$ m

27. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(1) 0.90 mm

(2) 1.16 mm

(3) 1.00 mm

(4) 1.36 mm

Answer (2)

Sol. Stress = $\frac{400}{\pi r^2} \leq 379\times 10^6\ N/m^2$

$$\Rightarrow r^2 \geq \frac{400}{379\times 10^6\pi}$$

$2r \geq 1.15$ mm

28. Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be :

(1) 3.5×10^{-6} Ns

(2) 6.3×10^{-4} Ns

(3) 5.0×10^{-3} Ns

(4) 1.4×10^{-6} Ns

Answer (3)

Sol. $P = \frac{\Delta E}{C}$

$$= \frac{(25\times 25)\times 40\times 60}{3\times 10^8}\ \text{N-s}$$

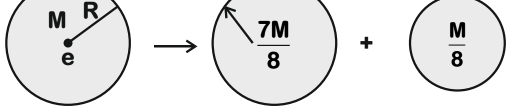
$= 5 \times 10^{-3}$ N-s

29. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by :

(1) 140 (2) 185

(3) 285 (4) 65

Answer (1)



$V=\frac{4}{3}\pi R^3 \qquad \left(\frac{7V}{8}\right) \qquad \frac{V}{8}$

$$I_1=\left(\frac{7M}{8}\right)(2R)^2\frac{1}{2}=\frac{14}{8}MR^2$$

$$I_2=\frac{2}{5}\left(\frac{M}{8}\right)r^2$$

$$\left[\begin{array}{l}\text{We have}\\ \frac{4}{3}\pi r^3\rho=\frac{M}{8}\\ \Rightarrow\ r=\left(\frac{R}{2}\right)\end{array}\right]$$

$$\Rightarrow\ I_2=\frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right)=\frac{MR^2}{80}$$

$$\frac{I_1}{I_2}=\frac{14\times 80}{(8)}=140$$

30. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ?

Answer (4)

Sol. $X = 5YZ^2$

$$Y\propto\frac{X}{Z^2}$$

$$X=C=\frac{Q^2}{E}=\frac{[A^2T^2]}{[ML^2T^{-2}]}$$

$$X=[M^{-1}L^{-2}T^4A^2]$$

$$Z=B=\frac{F}{IL}$$

$$Z=[MT^{-2}A^{-1}]$$

$$Y=\frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$$

$$Y=[M^{-3}L^{-2}T^8A^4]$$

PART–B : CHEMISTRY

1. The correct statement is :
   (1) Zincite is a carbonate ore.
   (2) Zone refining process is used for the refining of titanium.
   (3) Aniline is a froth stabilizer.
   (4) Sodium cyanide cannot be used in the metallurgy of silver.

Answer (3)

Sol. Ti is refined by Van Arkel method. Ag is leached by dilute solution of NaCN. Zincite is ZnO. Aniline is a froth stabilizer.

2. The pH of a 0.02 M NH_4Cl solution will be [given $K_b(NH_4OH) = 10^{-5}$ and log2 = 0.301]
   (1) 2.65
   (2) 5.35
   (3) 4.35
   (4) 4.65

Answer (2)

Sol. $NH_4^+ + H_2O \rightleftharpoons NH_4OH + H^+$

$0.02 - x \qquad x \qquad x \qquad K_h=\frac{10^{-14}}{10^{-5}}=10^{-9}$

≈ 0.02

$$K_h=\frac{x^2}{0.02}$$

$10^{-9} \times 2 \times 10^{-2} = x^2$

$$x=\sqrt{20}\times 10^{-6}$$

$$pH=-\log\left(\sqrt{20}\times 10^{-6}\right)$$

pH = 5.35

3. The INCORRECT statement is :
   (1) The spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.
   (2) The gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.
   (3) The spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 BM.
   (4) The color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

Answer (2)

Sol. Ruby is aluminium oxide (Al_2O_3) containing about 0.5 – 1% Cr^{3+} ions which are randomly distributed in the position normally occupied by Al^{3+} ions.

(1) 5 & 5

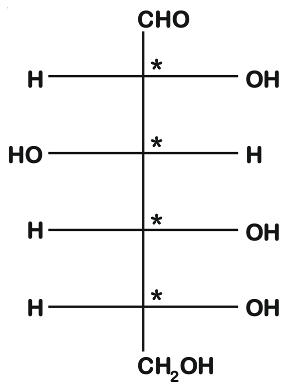
(2) 4 & 4

(3) 5 & 4

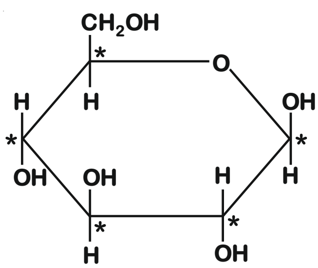
(4) 4 & 5

Answer (4)

Sol.



4 stereogenic centres



5 stereogenic centres

5. The difference between ΔH and ΔU ($\Delta H - \Delta U$), when the combustion of one mole of heptane(l) is carried out at a temperature T, is equal to :

(1) –3RT

(2) 4RT

(3) 3RT

(4) –4RT

Answer (4)

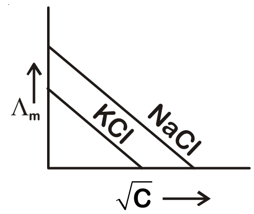
Sol. $\underset{(l)}{C_7H_{16}} + \underset{(g)}{11O_2} \xrightarrow{\Delta} \underset{(g)}{7CO_2} + \underset{(l)}{8H_2O}$

$\Delta H - \Delta U = \Delta n_g RT$

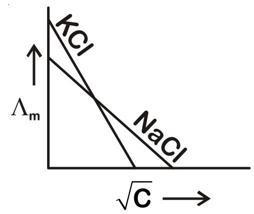
$\because \quad \Delta n_g = -4$

$\therefore \quad \Delta H - \Delta U = -4RT$

(1)



(2)



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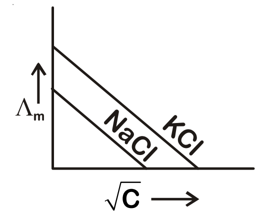

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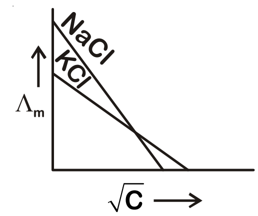
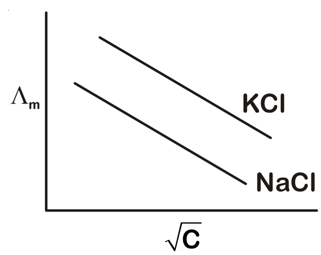


Answer (3)

Sol. KCl is more conducting than NaCl



7. The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$, respectively, are :

(1) $-2.4\Delta_o$ and $-1.2\Delta_t$

(2) $-0.6\Delta_o$ and $-0.8\Delta_t$

(3) $-0.4\Delta_o$ and $-0.8\Delta_t$

(4) $-0.4\Delta_o$ and $-1.2\Delta_t$

Answer (3)

Sol. $[Fe(H_2O)_6]^{2+}$ $\quad t_{2g}^4 e_g^2 \quad$ CFSE = $-0.4\Delta_0$

$[NiCl_4]^{2-}$ $\quad e^4 t_2^4 \quad$ CFSE = $-0.8\Delta_t$

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$,

$\Delta H = -57.2$ kJ mol^{-1} and $K_c = 1.7 \times 10^{16}$.

Which of the following statement is INCORRECT?

(1) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

(2) The addition of inert gas at constant volume will not affect the equilibrium constant.

(3) The equilibrium will shift in forward direction as the pressure increases.

(4) The equilibrium constant decreases as the temperature increases.

Answer (1)

Sol. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$K_c = 1.7 \times 10^{16}$ i.e. reaction goes to completion. Equilibrium constant has no relation with catalyst. Catalyst only affects the rate with which a reaction proceeds.

For the given reaction, catalyst V_2O_5 is used to speed up the reaction (Contact process).

9. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373 K leads to an anhydrous white powder Z. X and Z, respectively are :

(1) Baking soda and dead burnt plaster.

(2) Baking soda and soda ash.

(3) Washing soda and soda ash.

(4) Washing soda and dead burnt plaster.

Answer (3)

Sol. $\underset{(X)}{Na_2CO_3 \cdot 10H_2O} \longrightarrow \underset{(Y)}{Na_2CO_3 \cdot H_2O} + 9H_2O$

$\underset{(Y)}{Na_2CO_3 \cdot H_2O} \longrightarrow \underset{(Z)}{Na_2CO_3} + H_2O$

X = Washing soda

Z = Soda ash

10. Which of these factors does not govern the stability of a conformation in acyclic compounds?

(1) Angle strain

(2) Steric interactions

(3) Electrostatic forces of interaction

(4) Torsional strain

Answer (1)

11. The highest possible oxidation states of uranium and plutonium, respectively, are :

(1) 6 and 7 (2) 7 and 6

(3) 6 and 4 (4) 4 and 6

Answer (1)

Sol. Maximum oxidation state shown by

Uranium = + 6

Plutonium = + 7

12. The correct option among the following is :

(1) Colloidal medicines are more effective because they have small surface area.

(2) Colloidal particles in lyophobic sols can be precipitated by electrophoresis.

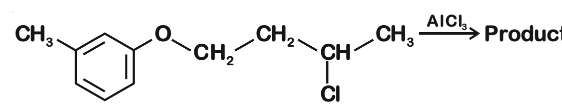
(3) Brownian motion in colloidal solution is faster if the viscosity of the solution is very high.

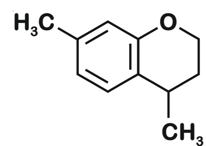
(4) Addition of alum to water makes it unfit for drinking.

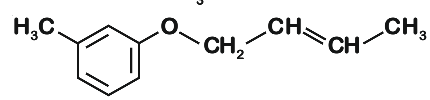
Answer (2)

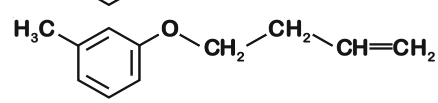
Sol. Electrophoresis is used to coagulate lyophobic colloids.

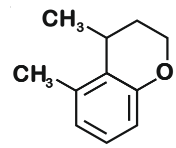
13. The major product obtained in the given reaction is :



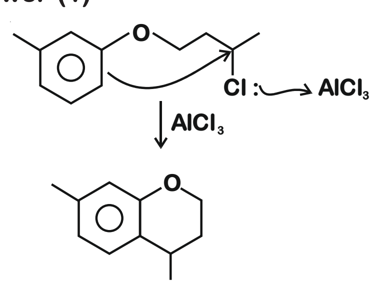
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

are :

(1) 20 and 3

(2) 12 and 3

(3) 12 and 4

(4) 20 and 4

Answer (3)

Sol. Pentagons in C_{60} = 12

Triangles in P_4 = 4

15. For the reaction of H_2 with I_2, the rate constant is 2.5×10^{-4} dm^3 mol^{-1} s^{-1} at 327°C and 1.0 dm^3 mol^{-1} s^{-1} at 527°C. The activation energy for the reaction, in kJ mol^{-1} is :

($R = 8.314$ J K^{-1} mol^{-1})

(1) 150 (2) 59

(3) 72 (4) 166

Answer (4)

Sol. $\log \dfrac{K_2}{K_1} = \dfrac{E_a}{2.303R}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)$

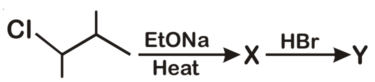
$$\log \frac{1}{2.5\times10^{-4}} = \frac{E_a}{8.314\times2.303}\left(\frac{1}{600} - \frac{1}{800}\right)$$

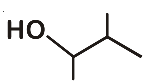
$$3.6 = \frac{E_a}{8.314\times2.303} \times \frac{200}{600\times800}$$

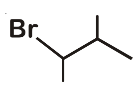
E_a = 165.4 kJ/mol

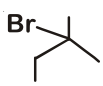
$\approx$ 166 kJ/mol

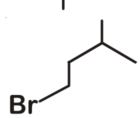
16. The major product 'Y' in the following reaction is :



(1) 

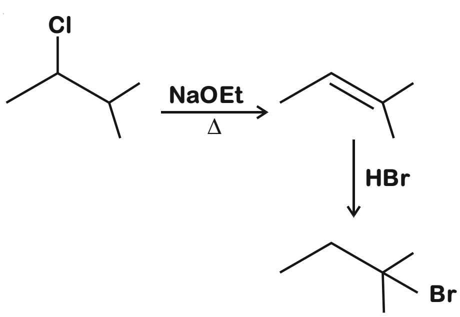
(2) 

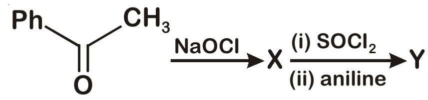
(3) 

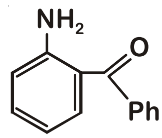
(4) 

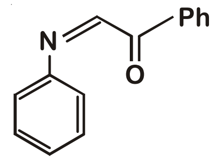
Answer (3)

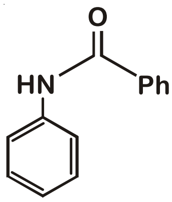
Sol.

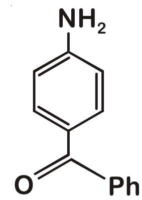




(1) 

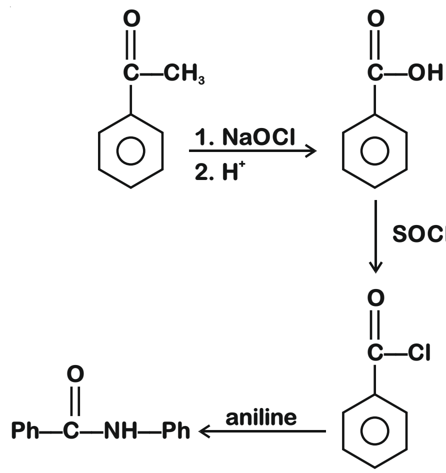
(2) 

(3) 

(4) 

Answer (3)

Sol.



18. 1 g of a non-volatile non-electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\dfrac{\Delta T_b(A)}{\Delta T_b(B)}$, is :

(1) 1 : 5 (2) 10 : 1

(3) 5 : 1 (4) 1 : 0.2

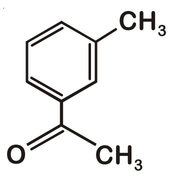
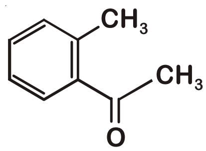
Answer (1)

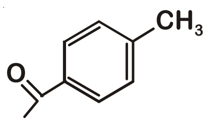
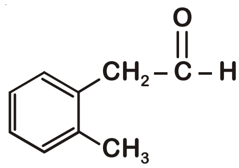
Sol. $\Delta T_b = k_b \times m$

$$\frac{(k_b)_A}{(k_b)_B} = \frac{1}{5}$$

$$\therefore \quad \frac{(\Delta T_b)_A}{(\Delta T_b)_B} = \frac{(k_b)_A}{(k_b)_B} = \frac{1}{5}$$

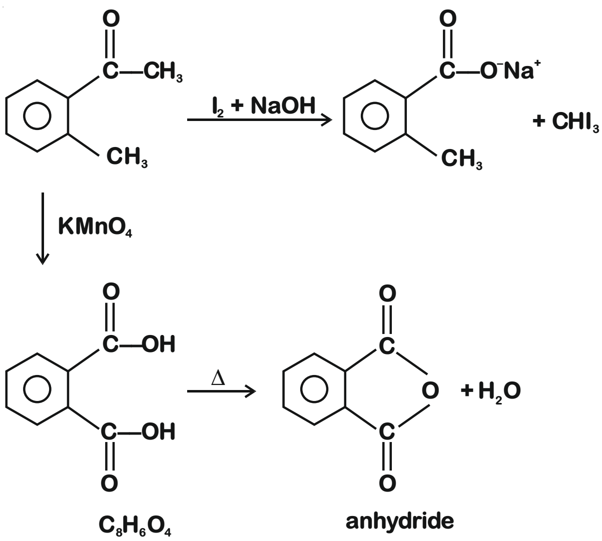
$B(C_8H_6O_4)$. Anhydride of B is used for the preparation of phenolphthalein. Compound A is:

(1)  (2) 

(3)  (4) 

Answer (2)

Sol.



20. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are :

(1) Paschen and Pfund

(2) Brackett and Pfund

(3) Lyman and Paschen

(4) Balmer and Brackett

Answer (3)

Sol. Shortest wavelength means $n_2 = \infty$

Lyman series $\bar{\nu}_L = \frac{1}{\lambda_L} = -1312\times\frac{1}{1^2}\times 1^2$

Paschen series $\bar{\nu}_P = \frac{1}{\lambda_P} = -1312\times\frac{1}{3^2}\times 1^2$

$$\frac{\bar{\nu}_L}{\bar{\nu}_P} = \frac{\lambda_P}{\lambda_L} = 9$$

	Item – I		Item – II
(a)	High density polythene	(I)	Peroxide catalyst
(b)	Polyacrylonitrile	(II)	Condensation at high temperature and pressure
(c)	Novolac	(III)	Ziegler – Natta Catalyst
(d)	Nylon 6	(IV)	Acid or base catalyst

(1) (a) → (III), (b) → (I), (c) → (IV), (d) → (II)

(2) (a) → (III), (b) → (I), (c) → (II), (d) → (IV)

(3) (a) → (IV), (b) → (II), (c) → (I), (d) → (III)

(4) (a) → (II), (b) → (IV), (c) → (I), (d) → (III)

Answer (1)

Sol. a. HDPE – Ziegler-Natta Catalyst

b. Polyacrylonitrile Peroxide Catalyst

c. Novolac – Catalysed by acid or base

d. Nylon-6 – Condensation at High T and P

22. Air pollution that occurs in sunlight is :

(1) Fog (2) Oxidising smog

(3) Acid rain (4) Reducing smog

Answer (2)

Sol. Air pollution caused by sunlight is photochemical smog and it is oxidising.

23. The increasing order of nucleophilicity of the following nucleophiles is :

(a) $CH_3CO_2^{\ominus}$ (b) H_2O

(c) $CH_3SO_3^{\ominus}$ (d) $\overset{\ominus}{O}H$

(1) (d) < (a) < (c) < (b)

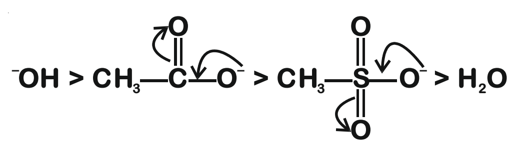
(2) (b) < (c) < (d) < (a)

(3) (a) < (d) < (c) < (b)

(4) (b) < (c) < (a) < (d)

Answer (4)

Sol. Greater the negative charge Present on a nucleophilic centre greater would be its nucleophilicity.



(1) Mn < Ti < Zn < Ni (2) Ti < Mn < Zn < Ni

(3) Ti < Mn < Ni < Zn (4) Zn < Ni < Mn < Ti

Answer (3)

Sol. Order for I.E. is

Ti < Mn < Ni < Zn

25. The noble gas that does NOT occur in the atmosphere is :

(1) Ne (2) Kr

(3) He (4) Ra

Answer (4)

Sol. Radon is not present in atmosphere.

26. Which of the following is NOT a correct method of the preparation of benzylamine from cyanobenzene?

(1) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$

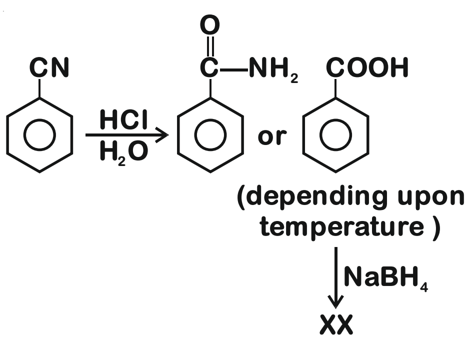
(2) H_2/Ni

(3) (i) $LiAlH_4$ (ii) H_3O^+

(4) (i) HCl/H_2O (ii) $NaBH_4$

Answer (4)

Sol.



27. The minimum amount of O_2(g) consumed per gram of reactant is for the reaction:

(Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)

(1) $2Mg(s) + O_2(g) \rightarrow 2MgO(s)$

(2) $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$

(3) $C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$

(4) $P_4(s) + 5O_2(g) \rightarrow P_4O_{10}(s)$

Answer (2)

Sol. (1) $2\,Mg + O_2 \longrightarrow 2\,MgO$

1 g requires $\frac{32}{48}$ g = 0.66 g of O_2

1 g Fe requires = 0.43 g of oxygen

(3) $C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$

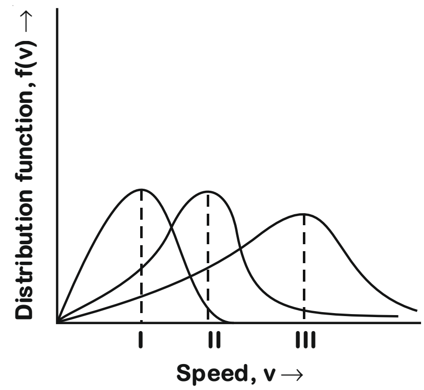
1 g of C_3H_8 requires = 3.6 g of O_2

(4) $P_4 + 5O_2 \longrightarrow P_4O_{10}$

1 g of P requires = 1.3 g of oxygen

28. Points I, II and III in the following plot respectively correspond to

(V_{mp} : most probable velocity)



(1) V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K); V_{mp} of H_2 (300 K)

(2) V_{mp} of H_2 (300 K); V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K)

(3) V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K); V_{mp} of O_2 (400 K)

(4) V_{mp} of O_2 (400 K); V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K)

Answer (1)

Sol. $V_{mp} = \sqrt{\frac{2RT}{M}}$

$\therefore$ as $\frac{T}{M}$ increases, V_{mp} increases

From curve

$(V_{mp})_I < (V_{mp})_{II} < (V_{mp})_{III}$

$(V_{mp})_{N_2} \propto \sqrt{\frac{300}{28}}, (V_{mp})_{O_2} \propto \sqrt{\frac{400}{32}}, (V_{mp})_{H_2} \propto \sqrt{\frac{300}{2}}$

$\therefore \quad (V_{mp})_{N_2} < (V_{mp})_{O_2} < (V_{mp})_{H_2}$

(Under given Condition)

(a) Saline hydrides produce H_2 gas when reacted with H_2O.

(b) Reaction of $LiAlH_4$ with BF_3 leads to B_2H_6.

(c) PH_3 and CH_4 are electron - rich and electron - precise hydrides, respectively.

(d) HF and CH_4 are called as molecular hydrides.

(1) (a), (c) and (d) only.

(2) (c) and (d) only.

(3) (a), (b) and (c) only.

(4) (a), (b), (c) and (d).

Answer (4)

Sol. – With water saline hydrides produce H_2 gas

– $3LiAlH_4 + 4BF_3 \longrightarrow 2B_2H_6 + 3LiF + 3AlF_3$

– HF and CH_4 are molecular hydrides

30. In chromatography, which of the following statements is INCORRECT for R_f ?

(1) The value of R_f cannot be more than one.

(2) Higher R_f value means higher adsorption.

(3) R_f value is dependent on the mobile phase.

(4) R_f value depends on the type of chromatography.

Answer (2)

Sol. R_f represents retardation factor in chromatography.

$$R_f = \frac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from baseline}}$$

– Higher R_f value means lower adsorpation

PART–C : MATHEMATICS

1. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

(1) $a\alpha^2 - b\alpha - a = 1$

(2) $a\alpha^2 + b\alpha + a = 0$

(3) $a\alpha^2 - b\alpha - a = 0$

(4) $a\alpha^2 + b\alpha - a = -2\alpha^2$

Answer (1)

Sol. $f(x) = \ln(\sin x)$, $g(x) = \sin^{-1}(e^{-x})$

$f(g(x)) = \ln(\sin(\sin^{-1} e^{-x}))$

$= -x$

$\Rightarrow -\alpha = b$

$f'(g(\alpha)) = a$

i.e., $a = -1$

$\therefore a\alpha^2 - b\alpha + 1 = -\alpha^2 + \alpha^2 + 1 = -a$

2. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is :

(1) $\dfrac{2}{\sqrt{3}}$

(2) $4\sqrt{3}$

(3) $2\sqrt{3}$

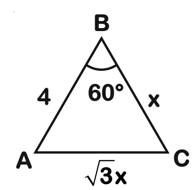
(4) $\dfrac{4}{\sqrt{3}}$

Answer (3)

Sol. $\because$ A, B, C, are in A.P

$\Rightarrow 2B = A + C$

$\Rightarrow B = \dfrac{\pi}{3}$



$\text{Area} = \dfrac{1}{2}(4x)\sin 60°$

$= \sqrt{3}x$

Now $\cos 60° = \dfrac{16 + x^2 - 3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2$

$x = 2$ (as -4 is rejected)

Hence, area = $2\sqrt{3}$ sq. cm

3. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to :}$$

(1) 6

(2) 0

(3) -4

(4) 1

Answer (2)

Sol. $\begin{vmatrix} 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

$\Rightarrow$ x($-3x^2$ – 6x – $2x^2$ + 6x) –6(–3x + 9 – 2x – 4)

–(4x – 9xA) = 0

$\Rightarrow$ x($-5x^2$) –6(–5x + 5) – 4x + 9x = 0

$\Rightarrow$ $x^3 - 7x + 6 = 0$

All the roots are real

$\therefore$ Sum of real roots = $\frac{0}{1}$ = 0

4. If z and w are two complex numbers such that $|zw| = 1$ and arg(z) – arg(w) $= \frac{\pi}{2}$, then :

(1) $z\bar{w} = \frac{1-i}{\sqrt{2}}$ (2) $\bar{z}w = i$

(3) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$ (4) $\bar{z}w = -i$

Answer (4)

Sol. |zw| = 1 ...(i)

$\arg\left(\frac{z}{w}\right) = \frac{\pi}{2}$ ...(ii)

$\therefore$ $\frac{z}{w} + \frac{\bar{z}}{\bar{w}} = 0$ $\Rightarrow z\bar{w} = -\bar{z}w$

from (i) $z\bar{z}w\bar{w} = 1$

$(\bar{z}w)^2 = -1$ $\Rightarrow \bar{z}w = \pm i$

from (ii) $-\arg(\bar{z}) - \arg w = \frac{\pi}{2}$

$\Rightarrow$ $\arg(\bar{z}w) = \frac{-\pi}{2}$

Hence, $\bar{z}w = -i$

5. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

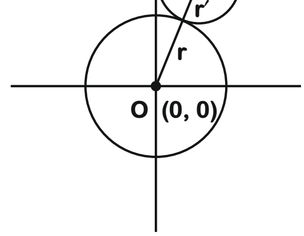
(1) $y = \sqrt{1+2x}, x \ge 0$ (2) $x = \sqrt{1+4y}, y \ge 0$

(3) $x = \sqrt{1+2y}, y \ge 0$ (4) $y = \sqrt{1+4x}, x \ge 0$

Answer (1)

Sol. Let centre of required circle is (h, k).

$\therefore$ OO′ = r + r′



$\Rightarrow$ $\sqrt{h^2+k^2} = 1+h$

$h^2 + k^2 = 1 + h^2 + 2h$

$k^2 = 1 + 2h$

Locus is $y = \sqrt{1+2x}$

6. The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots$

$+ \frac{1^3+2^3+3^3+\ldots+15^3}{1+2+3+\ldots+15} - \frac{1}{2}(1+2+3+\ldots+15)$

is equal to :

(1) 1860 (2) 620

(3) 660 (4) 1240

Answer (2)

Sol. $S = 1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots 15 \text{ terms}$

$$T_n = \frac{1^3+2^3+\ldots n^3}{1+2+\ldots n} = \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2}$$

$$S = \frac{1}{2}\left(\sum_{n=1}^{15} n^2 + \sum_{n=1}^{15} n\right) = \frac{1}{2}\left(\frac{15(16)(31)}{6} + \frac{15(16)}{2}\right)$$

= 680

$\Rightarrow$ $680 - \frac{1}{2}\frac{15(16)}{2} = 680 - 60 = 620$

7. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

(1) 58 (2) 35

(3) 38 (4) 23

Answer (3)

General term $T_{r+1} = {}^nC_r\left(x^2\right)^{n-r}\left(\frac{1}{x^3}\right)^r$

${}^nC_r \cdot x^{2n-5r}$

for coefficiant of x, $2n - 5r = 1$

Given ${}^nC_r = {}^nC_{23}$

$r = 23$ or $n - r = 23$

$\Rightarrow n = 58$ or $n = 38$

Minimum value is n = 38

8. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \ldots (x_{50} - 4)^2$ is :

(1) 380 (2) 480

(3) 400 (4) 525

Answer (3)

Sol. $\frac{x_1 + x_2 + \ldots x_{50}}{50} = 16$

$$16^2 = \frac{x_1^2 + x_2^2 \ldots x_{50}^2}{50} - 16^2$$

$$2(16)^2\,50 = x_1^2 + x_2^2 + \ldots x_{50}^2$$

Required mean = $\frac{(x_1 - 4)^2 + (x_2 - 4)^2 + \ldots (x_{50} - 4)^2}{50}$

$$= \frac{16^2(100) + 4^2(50) - 8(16 \times 50)}{50}$$

$= 16^2(2) + 16 - 8(16) = 400$

9. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then g(–1) is equal to :

(1) –1 (2) $-\frac{1}{2}$

(3) 1 (4) $-\frac{5}{2}$

Answer (4)

Sol. $I = \int x^5 \cdot e^{-x^2} dx$

Put $-x^2 = t$ $\Rightarrow -2x dx = dt$

$$I = \int \frac{t^2 \cdot e^t dt}{(-2)} = \frac{-1}{2} e^t\left(t^2 - 2t + 2\right) + c$$

$$\therefore \quad g(x) = \frac{-1}{2}\left(x^4 + 2x^2 + 2\right)$$

$$g(-1) = \frac{-5}{2}$$

where $a \neq 0$ and $0 < r \le \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4th term of this A.P. is :

(1) $\frac{2}{3}a$ (2) a

(3) $\frac{7}{3}a$ (4) 5a

Answer (2)

Sol. Let $b = ar$, $c = ar^2$

AP : $3a, 7ar, 15ar^2$

$14ar = 3a + 15ar^2$

$\Rightarrow 15r^2 - 14r + 3 = 0$

$\Rightarrow r = \frac{1}{3}$ or $\frac{3}{5}$ (rejected)

Fourth term = $15ar^2 + 7ar - 3a$

$= a(15r^2 + 7r - 3)$

$$= a\left(\frac{15}{9} + \frac{7}{3} - 3\right)$$

= a

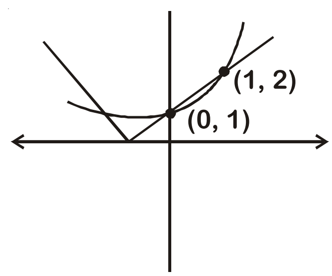
11. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

Answer (1)

Sol.



$$\text{Area} = \int_0^1 \left((x+1) - 2^x\right) dx$$

$$= \left[\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right]_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(\frac{-1}{\ln 2}\right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

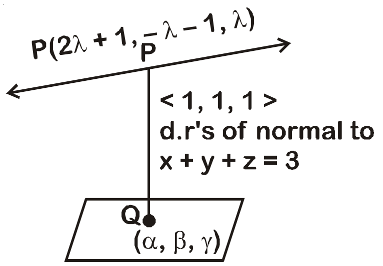
line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are :

(1) (1, 0, 2) (2) (2, 0, 1)

(3) (4, 0, –1) (4) (–1, 0, 4)

Answer (2)

Sol.



Let Q be (α, β, γ)

$\alpha + \beta + \gamma = 3$ ...(i)

$\alpha - \beta + \gamma = 3$ ...(ii)

$\therefore$ $\alpha + \gamma = 3$ and $\beta = 0$

Equating DR's of PQ :

$$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{\gamma-\lambda}{1}$$

$\Rightarrow$ $\alpha = 3\lambda + 2$, $\gamma = 2\lambda + 1$

Substituting in equation (i) we get

$\Rightarrow$ $5\lambda + 3 = 3$

$\lambda = 0$

Point is Q(2, 0, 1)

13. If the tangent to the curve $y=\frac{x}{x^2-3}$, $x \in R$, $(x \neq \pm\sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line 2x + 6y – 11 = 0, then :

(1) $|6\alpha + 2\beta| = 19$ (2) $|2\alpha + 6\beta| = 19$

(3) $|6\alpha + 2\beta| = 9$ (4) $|2\alpha + 6\beta| = 11$

Answer (1)

Sol. $y=\frac{x}{x^2-3}$

$$\frac{dy}{dx}=\frac{(x^2-3)-x(2x)}{(x^2-3)^2}=\frac{-x^2-3}{(x^2-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}=\frac{-\alpha^2-3}{(\alpha^2-3)^2}=-\frac{1}{3}$$

$3(\alpha^2 + 3) = (\alpha^2 - 3)^2$ ...(i)

Also, $\beta=\frac{\alpha}{\alpha^2-3}\Rightarrow \alpha^2-3=\frac{\alpha}{\beta}\Rightarrow\frac{\alpha}{\beta}=6$

$\Rightarrow$ $\alpha=\pm 3,\ \beta=\pm\frac{1}{2}$

Which satisfies $|6\alpha + 2\beta| = 19$

14. Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx}+y\tan x=2x+x^2\tan x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y(0) = 1.Then :

(1) $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

(2) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

(3) $y'\left(\frac{\pi}{4}\right)+y'\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

(4) $y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

Answer (4)

Sol. $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$

P = tanx, Q = 2x + x^2tan x

I.F. = $e^{\int \tan x\,dx}=e^{\ln|\sec x|}=|\sec x|$

$$y(\sec x)=\int(2x+x^2\tan x)\sec x\,dx$$

$$=\int x^2\tan x\sec x\,dx+\int 2x\sec x\,dx$$

$$=x^2\sec x-\int 2x\sec x\,dx+\int 2x\sec x\,dx$$

$= x^2\sec x + c$

As y(0) = 1, c = 1

$\therefore$ $y = x^2 + \cos x$

At $x=\frac{\pi}{4}$, $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$

$$y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$$

$$y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=0$$

$$\frac{dy}{dx}=2x-\sin x$$

$$y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}},\ y'\left(-\frac{\pi}{4}\right)=-\frac{\pi}{2}+\frac{1}{\sqrt{2}}$$

$$y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$$

maximises the product $a_1 a_4 a_5$, is :

(1) $\frac{2}{3}$ (2) $\frac{8}{5}$

(3) $\frac{3}{2}$ (4) $\frac{6}{5}$

Answer (1)

Sol. $a + 5d = 2$

Let $A = a_1 a_4 a_5 = a(a + 3d)(a + 4d)$

$= a(2 - 2d)(2 - d)$

$A = (2 - 5d)(4 - 6d + 2d^2)$

$\frac{dA}{dd} = 0$

$(2 - 5d)(-6 + 4d) + (4 - 6d + 2d^2)(-5) = 0$

$\Rightarrow 15d^2 - 34d + 16 = 0$

$d = \frac{8}{5}, \frac{2}{3}$

For $d = \frac{2}{3}, \frac{d^2A}{dd^2} < 0$

Hence $d = \frac{2}{3}$

16. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(1) 13 (2) 15

(3) 5 (4) 9

Answer (1)

Sol. $P_1 : 2x - y + 2z + 3 = 0$

$P_2 : 2x - y + 2z + \frac{\lambda}{2} = 0$

$P_3 : 2x - y + 2z + \mu = 0$

Given $\frac{1}{3} = \frac{\left|3 - \frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3 - \frac{\lambda}{2}\right| = 1$

$\lambda_{max} = 8$

Also, $\frac{2}{3} = \frac{|\mu - 3|}{\sqrt{9}} \Rightarrow \mu_{max} = 5$

$(\lambda + \mu)_{max} = 13$

(1) 5 (2) –4

(3) 1 (4) –7

Answer (4)

Sol. $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$

As limit is finite, $1 - a + b = 0$

$\Rightarrow \lim_{x \to 1} \frac{2x - a}{1} = 5 \quad \left(\frac{0}{0} \text{ form}\right)$

i.e., $2 - a = 5$

or $a = -3$

$\therefore b = -4$

$a + b = -3 - 4 = -7$

18. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is :

(1) 4 (2) 2

(3) 1 (4) 3

Answer (3)

Sol. Let $2^x - 1 = t$

$5 + |t| = (t + 1)(t - 1)$

$\Rightarrow |t| = t^2 - 6$

For $t > 0$, $t^2 - t - 6 = 0$

i.e., $t = 3$ or -2 (rejected)

For $t < 0$, $t^2 + t - 6 = 0$

i.e., $t = -3$ or 2 (both rejected)

$\therefore 2^x - 1 = 3$

$\Rightarrow x = 2$

19. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?

(1) $\left(\frac{1}{4}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$

(3) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (4) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

Answer (3)

Given $\frac{3}{5} = \left|\frac{\alpha}{5}\right|$

$\Rightarrow \alpha = \pm 3$

Line is $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$

Clearly $\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfies $4x - 3y + 3 = 0$

20. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is :

(1) 210 (2) 180

(3) 170 (4) 190

Answer (3)

Sol. Required number of beams = $^{20}C_2 - 20$

$= 190 - 20 = 170$

21. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{5}{6\pi}$ (2) $\frac{1}{36\pi}$

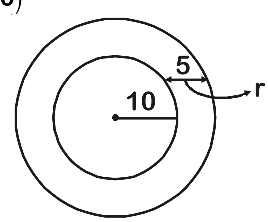
(3) $\frac{1}{9\pi}$ (4) $\frac{1}{18\pi}$

Answer (4)

Sol. $\frac{dV_{ice}}{dt} = 50$

$V_{ice} = \frac{4}{3}\pi(10+r)^3 - \frac{4}{3}\pi(10)^3$

$\frac{dV}{dt} = \frac{4}{3}\pi\, 3(10+r)^2 \frac{dr}{dt}$



$= 4\pi(10+r)^2 \frac{dr}{dt}$

At $r = 5, 50 = 4\pi(225)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{50}{4\pi(225)}$

$= \frac{1}{18\pi}$ cm/min

one head is more than 99% is :

(1) 8 (2) 6

(3) 5 (4) 7

Answer (4)

Sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\therefore \quad n \geq 7$

Minimum value is 7.

23. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(\frac{5}{3}, 0\right)$ (2) $\left(-\frac{5}{3}, 0\right)$

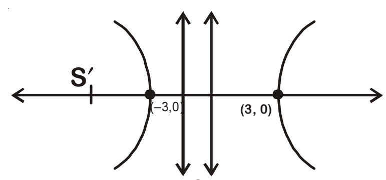
(3) (–5, 0) (4) (5, 0)

Answer (3)

Sol. $16x^2 - 9y^2 = 144$

i. e. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Focus $S'(-ae, 0)$



$a = 3, b = 4$

$e^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$S' \equiv \left(-3 \times \frac{5}{3}, 0\right) \equiv (-5, 0)$

24. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation :

(1) $\lambda^2 + 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + \lambda - 6 = 0$ (4) $\lambda^2 - 3\lambda - 4 = 0$

Answer (2)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4-\lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \qquad \Rightarrow \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda-2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda-2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda-2 \\ 3 & 2 & -5 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$\therefore$ For $\lambda = 3$, infinitely many solutions is obtained.

25. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to :

(1) $3^{7/6} - 3^{5/6}$ (2) $3^{5/3} - 3^{1/3}$

(3) $3^{5/6} - 3^{2/3}$ (4) $3^{4/3} - 3^{1/3}$

Answer (1)

Sol. $I = \int_{\pi/6}^{\pi/3} \sec^{2/3} x . \operatorname{cosec}^{4/3} x \, dx$

$$= \int_{\pi/6}^{\pi/3} \frac{1.dx}{\cos^{2/3} x . \sin^{4/3} x}$$

$$= \int_{\pi/6}^{\pi/3} \frac{1 \quad dx}{\cos^2 x . \tan^{4/3} x} = \int_{\pi/6}^{\pi/3} \frac{\sec^2 x dx}{\tan^{4/3} x}$$

Let tan x = t

$$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} t^{-4/3} dt = \frac{3\left[t^{-1/3}\right]_{1/\sqrt{3}}^{\sqrt{3}}}{-1}$$

$$= -3\left[3^{-1/6} - \frac{1}{3^{-1/6}}\right]$$

$$= -3(3^{-1/6} - 3^{1/6})$$

$$= 3(3^{1/6} - 3^{-1/6})$$

$$= 3^{7/6} - 3^{5/6}$$

$x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{2}$

(3) 2

(4) $\sqrt{2}$

Answer (4)

Sol. Tangent on $y^2 = 4\sqrt{2}x$ is $yt = x + \sqrt{2}t^2$

As it is tangent on circle also,

$$\left|\frac{\sqrt{2}t^2}{\sqrt{1+t^2}}\right| = 1$$

$2t^4 = 1 + t^2$ i.e. $t^2 = 1$

Equation is $\pm y = x + \sqrt{2}$

Hence $|c| = \sqrt{2}$

27. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\frac{16}{3}$

(2) $\frac{14}{3}$

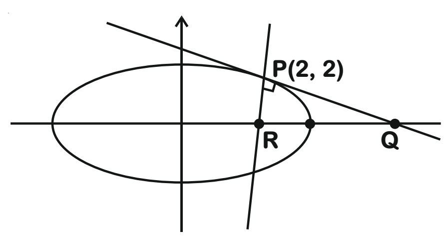
(3) $\frac{34}{15}$

(4) $\frac{68}{15}$

Answer (4)

Sol. For $\frac{3x^2}{32} + \frac{5y^2}{32} = 1$

Tangent at P is



$$\frac{3(2)x}{32} + \frac{5(2)y}{32} = 1$$

16 16

$Q \equiv \left(\frac{16}{3}, 0\right)$

Normal at P is $\frac{32x}{3(2)} - \frac{32y}{5(2)} = \frac{32}{3} - \frac{32}{5}$

$R \equiv \left(\frac{4}{5}, 0\right)$

area of $\Delta PQR = \frac{1}{2}$ (PQ) (PR) $= \frac{1}{2}\sqrt{\frac{136}{3}} \cdot \sqrt{\frac{136}{5}}$

$= \frac{68}{15}$

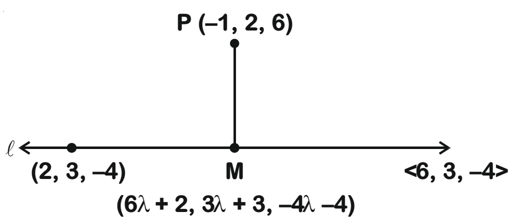
28. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

(1) 7 (2) $4\sqrt{3}$

(3) 6 (4) $2\sqrt{13}$

Answer (1)

Sol. Equation of l is $\frac{x-2}{6} = \frac{y-3}{3} = \frac{z+4}{-4}$



Let M $(6\lambda + 2, 3\lambda + 3, -4\lambda - 4)$

DR's of PM is $< 6\lambda + 3, 3\lambda + 1, -4\lambda - 10 >$

$\Rightarrow (6\lambda + 3)(6) + (3\lambda + 1)(3) + (-4\lambda - 10)(-4) = 0$

$\Rightarrow \lambda = -1$

i.e. $M \equiv (-4, 0, 0)$

$\therefore$ PM $= \sqrt{9+4+36} = 7$

$x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to :

(1) $2\sin^2\alpha$

(2) $4\sin^2\alpha - 2x^2y^2$

(3) $4\cos^2\alpha + 2x^2y^2$

(4) $4\sin^2\alpha$

Answer (4)

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{1-x^2}\cdot\sqrt{1-\frac{y^2}{4}}\right) = \alpha$

$\Rightarrow \frac{xy}{2} + \frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2} = \cos\alpha$

$\Rightarrow xy + \sqrt{1-x^2}\sqrt{4-y^2} = 2\cos\alpha$

$(xy - 2\cos\alpha)^2 = (1 - x^2)(4 - y^2)$

$x^2y^2 + 4\cos^2\alpha - 4xy\cos\alpha = 4 - y^2 - 4x^2 + x^2y^2$

$4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

30. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge r$

(2) r

(3) $\sim s \wedge \sim r$

(4) $s \vee r$

Answer (1)

Sol. $\sim s \vee (\sim r \wedge s)$

$\equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \sim r)$ ($\because$ $(\sim s \vee s)$ is tautology)

$\equiv \sim(s \wedge r)$

Hence its negation is $s \wedge r$

❑ ❑ ❑

Morning

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

(1) 0.24 μm (2) 0.38 μm

(3) 0.48 μm (4) 0.12 μm

Answer (1)

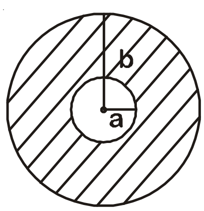
Sol. $\theta_{min} = \dfrac{1.22\lambda}{D}$

$\dfrac{D}{2f} = 1.25$

$d_{min} = \dfrac{1.22\lambda f}{D} = \dfrac{1.22 \times 5000 \times 10^{-10}}{2.50}$

$= 0.24\ \mu m$

2. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\dfrac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :



(1) $\dfrac{a+b}{2}$ (2) $\sqrt{\dfrac{a^2+b^2+ab}{2}}$

(3) $\dfrac{a+b}{3}$ (4) $\sqrt{\dfrac{a^2+b^2+ab}{3}}$

Answer (4)

Sol. $\Rightarrow \quad \sigma = \dfrac{\sigma_0}{r} \qquad \therefore \dfrac{\sigma_0}{r} 2\pi r dr = dm$

$\Rightarrow$ m = $\sigma_0 2\pi$ (b – a)

$I = \sigma_0 2\pi \int_a^b r^2 dr = \dfrac{2\pi\sigma_0}{3}(b^3 - a^3)$

$\therefore \quad mk^2 = I \Rightarrow 2\pi\sigma_0(b-a)k^2 = \dfrac{2\pi\sigma_0}{3}(b^3 - a^3)$

$\Rightarrow \quad k^2 = \dfrac{1}{3}(b^2 + ab + a^2)$

$\therefore \quad k = \sqrt{\dfrac{1}{3}\dfrac{(b^3 - a^3)}{b-a}}$

3. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is :

(1) $\dfrac{R}{2g}$ (2) $\dfrac{2R}{g}$

(3) $\dfrac{R}{g}$ (4) $\dfrac{R}{4g}$

Answer (2)

Sol. For same horizontal range.

$\theta_1 = \theta \qquad R = \dfrac{u^2 \sin 2\theta}{g}$

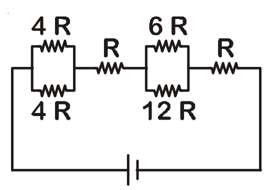
θ_2 = (90 – θ)

so $t_1 = \dfrac{2u\sin\theta}{g}$ and $t_2 = \dfrac{2u\cos\theta}{g}$

$\therefore \quad t_1t_2 = \dfrac{u^2 4\sin\theta\cos\theta}{g^2}$

$\Rightarrow \quad t_1t_2 = \dfrac{2u^2 \sin 2\theta}{g^2} = \dfrac{2R}{g}$

4. The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is :



(1) 8 Ω (2) 1 Ω

(3) 16 Ω (4) 6 Ω

Sol. $R_{eq} = 2R + R + 4R + R = 8R$

$$P = \frac{v^2}{R_{eq}} \Rightarrow \frac{16\times16}{8R} = 4 \text{ watt}$$

$$\frac{16\times16}{4\times8} = R \Rightarrow R = 8\,\Omega$$

5. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(1) $\frac{2\lambda}{(\mu-1)}$

(2) $\frac{\lambda}{2(\mu-1)}$

(3) $\frac{\lambda}{(2\mu-1)}$

(4) $\frac{\lambda}{(\mu-1)}$

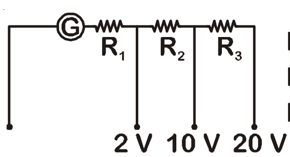
Answer (4)

Sol. $\mu t - t = \lambda$

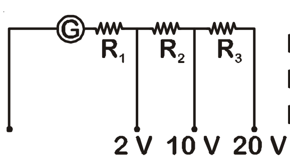
$\Rightarrow t(\mu - 1) = \lambda$

$$\Rightarrow t = \frac{\lambda}{(\mu-1)}$$

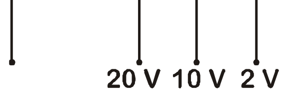
6. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is :

(1) 

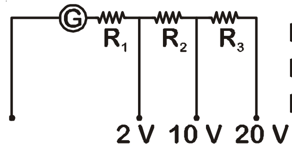
$R_1 = 1900\,\Omega$
$R_2 = 8000\,\Omega$
$R_3 = 10000\,\Omega$

(2) 

$R_1 = 1900\,\Omega$
$R_2 = 9900\,\Omega$
$R_3 = 19900\,\Omega$

(3) 

$R_2 = 9900\,\Omega$
$R_3 = 1900\,\Omega$

(4) 

$R_1 = 2000\,\Omega$
$R_2 = 8000\,\Omega$
$R_3 = 10000\,\Omega$

Answer (1)

Sol. For R_1

$\because \; I_g = 10^{-3}$ A

$\therefore \; 10^{-3}(R_1 + 100) = 2 \text{ V} \Rightarrow R_1 = 1900\,\Omega$

For R_2

$10^{-3}(R_1 + R_2 + 100) = 10$ V

$\Rightarrow R_1 + R_2 + 100 = 10000$

$\Rightarrow R_2 = 8000\,\Omega$

For R_3

$10^{-3}(R_1 + R_2 + R_3 + 100) = 20$ V

$\Rightarrow R_1 + R_2 + R_3 + 100 = 20 \times 1000$

$\Rightarrow R_3 = 10000\,\Omega$

7. A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively : (Take V = 0 at infinity)

(1) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

(2) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(3) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(4) $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

Answer (3)

Sol. $\vec{E} = K\frac{\vec{p}}{r^3}\sqrt{3\cos^2\theta+1}$

$\Rightarrow \theta = \pi/2 \; (0, d, 0)$

$$\therefore \; \vec{E} = \frac{-k\vec{p}}{d^3}$$

is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by :

(1) 2.2 (2) 0.7

(3) 3.6 (4) 1.8

Answer (2)

Sol. T_1 = 2 sec., T_2 = 3/2

For place (1), $B_{H_1} = B_1 \cos 45° - \frac{B_1}{\sqrt{2}}$

For place (2), $B_{H_2} = B_2 \cos 30° = \frac{B_2\sqrt{3}}{2}$

$\therefore \quad T = 2\pi\sqrt{\frac{I}{MB_H}} \qquad \therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}}$

$\frac{4\times4}{9} = \frac{B_2\sqrt{3}\times\sqrt{2}}{2\,B_1} \quad \Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{6}\times9}{2\times16}$

$\Rightarrow \quad \frac{B_1}{B_2} = 0.68 \approx 0.7.$

9. An electromagnetic wave is represented by the electric field

$\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propogation $\hat{s}$, is :

(1) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$

(2) $\hat{s} = \left(\frac{3\hat{i}-4\hat{j}}{5}\right)$

(3) $\hat{s} = \left(\frac{-4\hat{k}+3\hat{j}}{5}\right)$

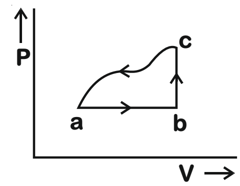
(4) $\hat{s} = \left(\frac{4\hat{j}-3\hat{k}}{5}\right)$

Answer (1)

Sol. $E = E_0 \sin(\omega t + 6y - 8z)$

$\hat{s} = \frac{8\hat{k}-6\hat{j}}{10} = \left(\frac{4\hat{k}-3\hat{j}}{5}\right)$

change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is :



(1) 130 J (2) 100 J

(3) 140 J (4) 120 J

Answer (1)

Sol. For the process (c – a), ΔU_{ca} = – 180 J

For process (b – c) $\rightarrow$ Isochoric (W_{bc} = 0)

$\therefore$ ΔU = 60 J

Heat absorbs along (a – b), Q_{ab} = 250 J

Also $\therefore$ ΔU_{cycle} = 0

$\therefore$ ΔU_{ab} = 120 J

So $W_{a \rightarrow b}$ = 130 J

Total work done from (a $\rightarrow$ b $\rightarrow$ c)

= W_{ab} + W_{bc} = 130 J

11. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to :

(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively ; g = 10 ms^{-2})

(1) 0.5 kg (2) 0.9 kg

(3) 1.5 kg (4) 9 kg

Answer (Bonus)

Sol. $\frac{Mgl}{A\,\Delta l} = Y$

$\therefore \quad \Delta l_{Mechanical} = \frac{Mgl}{AY}$

$\Delta l_{Thermal} = l\alpha\Delta T = l\alpha\times20$

$\frac{Mgl}{AY} = 20\,\alpha\, l$

$M = \frac{20\times10^{-5}\times\pi\times1\times10^{-6}\times10^{11}}{10} = 6.28\text{ kg}$

constant angular speed of 40π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2).

(1) 4×10^{-5} C (2) 3×10^{-5} C

(3) 7×10^{-6} C (4) 2×10^{-6} C

Answer (2)

Sol. $B = \frac{\mu_0 i}{2a}$ and $\frac{\omega q}{2\pi} = i$

$\therefore \quad B = \frac{\mu_0}{2a} \cdot \frac{\omega q}{2\pi}$

$B = \frac{10^{-7} \times 40}{0.1} \times q \times \pi \Rightarrow q = 3 \times 10^{-5}$ C

13. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \frac{1240 \text{ eV}}{\lambda(\text{in nm})}$):

(1) n = 5 (2) n = 7

(3) n = 4 (4) n = 6

Answer (1)

Sol. $\Rightarrow \quad \Delta E_n = -\frac{E_0 Z^2}{n^2}$

Let it start from n to m and from m to ground.

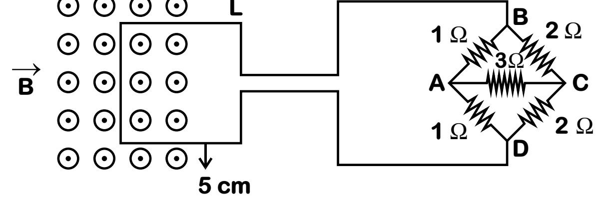
Then $13.6 \times 4 \left| 1 - \frac{1}{m^2} \right| = \frac{hc}{30.4 \text{ nm}}$

$\Rightarrow \quad 1 - \frac{1}{m^2} = 0.7498 \quad \Rightarrow 0.25 = \frac{1}{m^2}$

$\therefore \quad m = 2$, and now $13.6 \times 4 \left(\frac{1}{4} - \frac{1}{n^2} \right) = \frac{hc}{108.5 \times 10^{-9}}$

$n \approx 5$.

14. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to:



(1) 170 μA (2) 60 μA

(3) 150 μA (4) 115 μA

Answer (1)

Sol. $VBl = iR_{eq}$

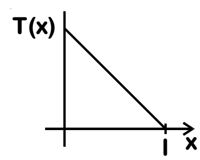
$\because \quad R_{eq} = \frac{4}{3}\Omega + 1.7 = 3\,\Omega$

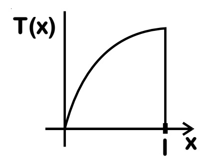
$i = \frac{(BLV)}{R_{eq}} = \frac{(1)(5 \times 10^{-2}) \times 10^{-2}}{3}$

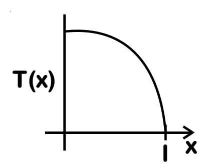
$= \frac{5}{3} \times 10^{-4}$ A $\simeq 1.7 \times 10^{-4}$ A

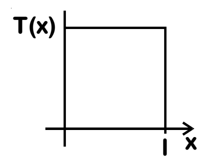
$= 170\ \mu$A

15. A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

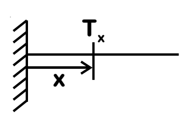
(1)


(2)


(3)


(4)


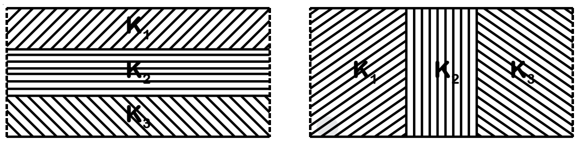
Answer (3)

Sol.


$T_x = \frac{M}{L}(L - x)\left\{x + \frac{L - x}{2}\right\}\omega^2 = \frac{M\omega^2}{2L}(L^2 - x^2)$

separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)):



(I) (II)

(1) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(2) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

(3) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(4) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

Answer (3)

Sol. $\frac{1}{C_1} = \frac{d}{3A\varepsilon_0}\left(\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}\right)$

$$C_1 = \frac{3A\varepsilon_0(K_1K_2K_3)}{d(K_1K_2+K_2K_3+K_3K_1)}$$

$$C_2 = \frac{A\varepsilon_0}{3d}(K_1+K_2+K_3)$$

$$\frac{E_1}{E_2} = \frac{C_1}{C_2} = \frac{3K_1K_2K_3}{(K_1K_2+K_2K_3+K_3K_1)} \times \frac{3}{(K_1+K_2+K_3)}$$

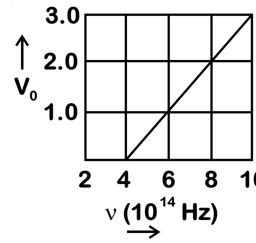
$$\Rightarrow \frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}$$

in the figure. The work function of sodium, from the data plotted in the figure, will be:

(Given: Planck's constant)

(h) = 6.63×10^{-34} Js, electron

charge e = 1.6×10^{-19} C)



(1) 1.95 eV (2) 2.12 eV

(3) 1.82 eV (4) 1.66 eV

Answer (4)

Sol. $\phi = \frac{hc}{\lambda} = h\nu$

$\therefore$ $\phi = h \times 4 \times 10^{14}$ Hz = 1.654 eV

$\Rightarrow$ $\phi \approx 1.66$ eV

18. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$):

(1) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(2) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

(3) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

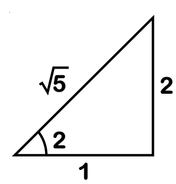
(4) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

Answer (3)

Sol. $y = 2x - 9x^2$

Comparing it with equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{24^2\cos^2\theta}$$



$\therefore$ tan θ = 2

And $9 = \frac{10 \times 5}{2v_0^2}$

$\Rightarrow$ $v_0 = \frac{5}{3}$ m/s

submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to : (Speed of sound in water = 1500 ms^{-1})

(1) 499 Hz (2) 504 Hz

(3) 507 Hz (4) 502 Hz

Answer (4)

Sol. f_1 (frequency received by A)

$$= \nu_0 \left[\frac{1500-5}{1500-7.5}\right]$$

f_2 [frequency received by B]

$$= \nu_0 \times \frac{1495}{1492.5} \times \frac{1507.5}{1505}$$

= 502 Hz.

20. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to :

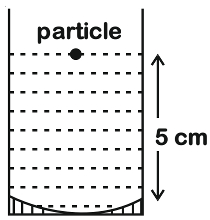
(Refractive index of water = 1.33)

(1) 11.7 cm

(2) 6.7 cm

(3) 13.4 cm

(4) 8.8 cm



Answer (4)

Sol. $\frac{1}{V} + \frac{1}{U} = \frac{-1}{20}$

$$\frac{1}{V} - \frac{1}{5} = \frac{-1}{20}$$

$$\frac{1}{V} = \frac{-1}{20} + \frac{1}{5}$$

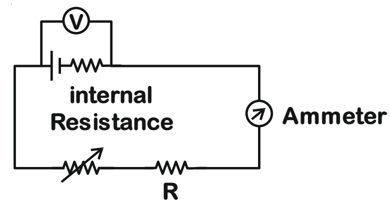
$$\frac{1}{V} = \frac{3}{20}$$

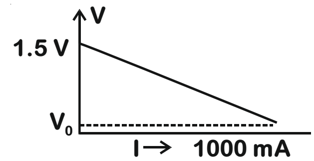
$$d = \left(\frac{20}{3} + 5\right) \times 3/4$$

$$= \frac{35}{4}$$

d = 8.8 cm

figure. The measured voltage is plotted as a function of the current, and the following graph is obtained :





If V_0 is almost zero, identify the correct statement :

(1) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(2) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

(3) The value of the resitance R is 1.5 Ω

(4) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

Answer (1)

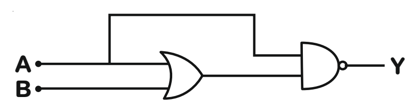
Sol. $V = \frac{ER}{R+r}$

for R = ∞, V = E = 1.5 V

for R = 0, I = E/r = 1

r = 1.5 Ω

22. The truth table for the circuit given in the fig. is:



(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(3)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(4)

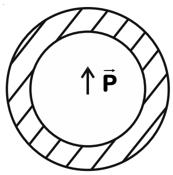
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

Sol. $\Rightarrow y = \overline{A\cdot(A+B)} = \overline{A} + \overline{(A+B)}$

$\Rightarrow y = \overline{A} + \overline{A}\cdot\overline{B} = \overline{A}(1+\overline{B})$

$\Rightarrow y = \overline{A}$

23. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case:



(1) Surface charge density on the outer surface depends on $|\vec{P}|$

(2) Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

(3) Electric field outside the shell is the same as that of point charge at the centre of the shell

(4) Surface charge density on the inner surface of the shell is zero everywhere

Answer (3)

Sol. Since dipole is having zero net charge. So inside surface shall have non-zero non-uniform charge distribution. And net field outside the region would be same as that would have been for point charge at surface.

24. Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

(1) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (2) $\frac{\varepsilon_0}{\mu_0}$

(3) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (4) $\frac{\mu_0}{\varepsilon_0}$

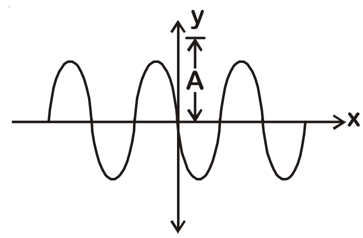
Sol. As we know t = RC

$t = \frac{L}{R}$

$R^2\frac{C}{L} = 1$

$R = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

25. A progressive wave travelling along the positive x-direction is represented by $y(x,t) = A\sin(kx-\omega t+\phi)$. Its snapshot at t = 0 is given in the figure.



For this wave, the phase ϕ is :

(1) π (2) $-\frac{\pi}{2}$

(3) $\frac{\pi}{2}$ (4) 0

Answer (1)

Sol. $y = A\sin(\omega t - kx + \phi)$

At t = 0 and x = 0 particle is at mean position and will proceed in positive y direction

26. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(1) 0.20 ms^{-1} (2) 0.14 ms^{-1}

(3) 0.47 ms^{-1} (4) 0.28 ms^{-1}

Answer (1)

Sol. $50\ V_1 = 20\ V_2$

$V_1 + V_2 = 0.70$

$V_1 = 0.20$

water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is :

(1) $\frac{50M_2}{M_1}-5$

(2) $\frac{5M_1}{M_2}-50$

(3) $\frac{50M_2}{M_1}$

(4) $\frac{5M_2}{M_1}-5$

Answer (1)

Sol. $M_1 \times 5 + M_1L = M_2\ 50$

$$L=\frac{50M_2}{M_1}-5$$

28. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume?

(R = 8.3 J/mol K)

(1) 19.7 J/mol K

(2) 21.6 J/mol K

(3) 15.7 J/mol K

(4) 17.4 J/mol K

Answer (4)

Sol. $5C_V = 2\times\frac{3R}{2}+3\times\frac{5R}{2}$

$$C_V=\frac{21R}{10}$$

29. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l (l<<L), is close to :

(1) Mgl

(2) $Mgl\left(1+\theta_0^2\right)$

(3) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$

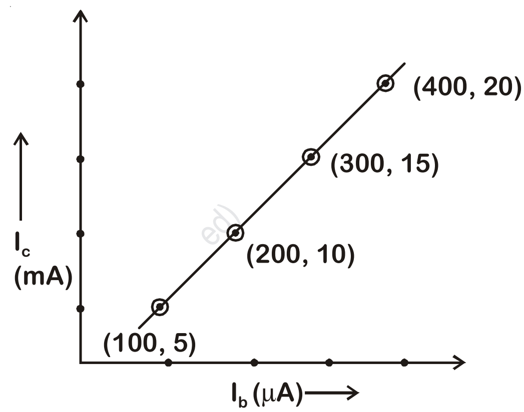
(4) $Mgl\left(1-\theta_0^2\right)$

Sol. $W_{man} = Mg_{eff}\ l$

$$g_{eff} = g(1+\theta_0^2)$$

$$W_{man} = Mgl(1+\theta_0^2)$$

30. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The Voltage and Power gain, are respectively :



(1) 5×10^4, 2.5×10^6

(2) 2.5×10^4, 2.5×10^6

(3) 5×10^4, 5×10^6

(4) 5×10^4, 5×10^5

Answer (1)

Sol. $\beta=\frac{I_c}{I_b}$

$$=\frac{5\times10^{-3}}{100\times10^{-6}}$$

= 50

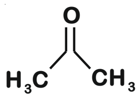
Voltage gain $=\beta\frac{R_O}{R_i}=5\times10^4$

Power gain = β (voltage gain)

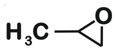
$= 250 \times 10^4 = 2.5 \times 10^6$

1. The major product of the following addition reaction is

$$H_3C-CH=CH_2 \xrightarrow{Cl_2/H_2O}$$

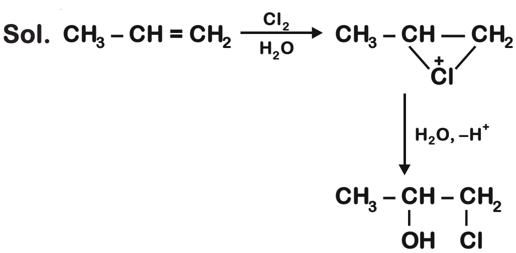
(1) 

(2) $\underset{Cl}{\underset{|}{CH_3-CH}}-\underset{OH}{\underset{|}{CH_2}}$

(3) 

(4) $\underset{OH}{\underset{|}{H_3C-CH}}-\underset{Cl}{\underset{|}{CH_2}}$

Answer (4)

Sol. 

2. But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give :

(1) 2 molecules of CH_3CHO

(2) 2 molecules of CH_3COOH

(3) $\underset{OH}{\underset{|}{CH_3-CH}}-\underset{OH}{\underset{|}{CH}}-CH_3$

(4) One molecule of CH_3CHO and one molecule of CH_3COOH

Answer (2)

Sol. $CH_3-CH=CH-CH_3 \xrightarrow[OH^-,\Delta]{KMnO_4} 2CH_3COOH$

3. The correct sequence of thermal stability of the following carbonates is :

(1) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

(2) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$

(3) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

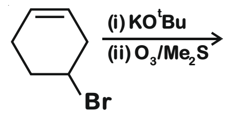
(4) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$

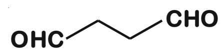
Answer (1)

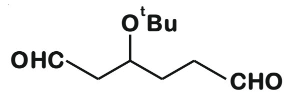
Sol. Stability of alkaline earth metal carbonates increases down the group :

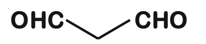
$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

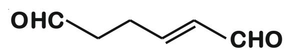
4. The major product(s) obtained in the following reaction is/are



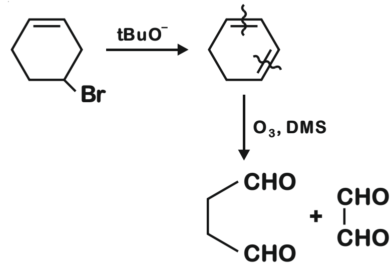
(1)  and OHC–CHO

(2) 

(3) 

(4) 

Answer (1)

Sol. 

5. Which of the following statements is not true about RNA?

(1) It usually does not replicate

(2) It is present in the nucleus of the cell

(3) It controls the synthesis of protein

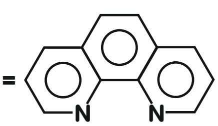
(4) It has always double stranded α-helix structure

Answer (4)

Sol. RNA has a single helix structure.

DNA has a double helix structure.

6. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is :

(Phen =  and ignore pairing energy)

(1) $[Ni(phen)_3]^{2+}$ (2) $[Co(phen)_3]^{2+}$

(3) $[Zn(phen)_3]^{2+}$ (4) $[Fe(phen)_3]^{2+}$

Answer (4)

$$Co^{2+}(d^7) \longrightarrow Co^{3+}(d^6)$$
$$t_{2g}^6 e_g^1 \qquad t_{2g} e_g^0$$

$$Zn^{2+}(d^{10}) \longrightarrow Zn^{2+}(d^9)$$
$$t_{2g}^6 e_g^4 \qquad t_{2g}^6 e_g^3$$

$$Fe^{2+}(d^6) \longrightarrow Fe^{3+}(d^5)$$
$$t_{2g}^6 e_g^0 \qquad t_{2g}^5 e_g^0$$

So, only Fe^{2+} will lose crystal field stabilisation upon oxidation to +3, others will gain crystal field stabilisation

7. An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is :

(1) $\frac{3}{2}a$ (2) $\frac{a}{2}$

(3) a (4) $\sqrt{2}a$

Answer (2)

Sol. In FCC, tetrahedral voids are located on the body diagonal at a distance of $\frac{\sqrt{3}a}{4}$ from the corner. Together they form a smaller cube of edge length $\frac{a}{2}$.

8. Glucose and Galactose are having identical configuration in all the positions except position.

(1) C – 2 (2) C – 5

(3) C – 3 (4) C – 4

Answer (4)

Sol. Galactose and Glucose are C_4 epimers.

9. The metal that gives hydrogen gas upon treatment with both acid as well as base is :

(1) Zinc (2) Magnesium

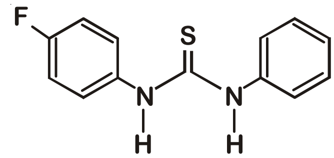
(3) Iron (4) Mercury

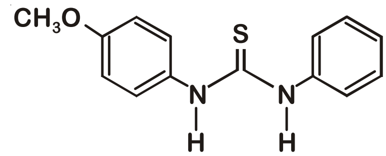
Answer (1)

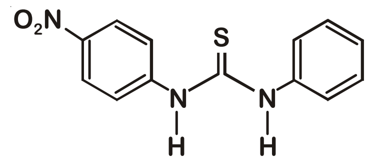
Sol. $Zn + NaOH \longrightarrow Na_2ZnO_2 + H_2$

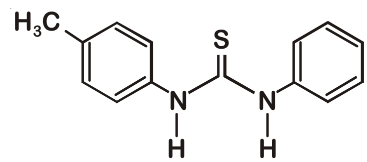
$Zn + H_2SO_4 \longrightarrow ZnSO_4 + H_2$

Zn is amphoteric.

(A) 

(B) 

(C) 

(D) 

(1) (B) < (D) < (A) < (C)

(2) (A) < (C) < (D) < (B)

(3) (B) < (D) < (C) < (A)

(4) (C) < (A) < (D) < (B)

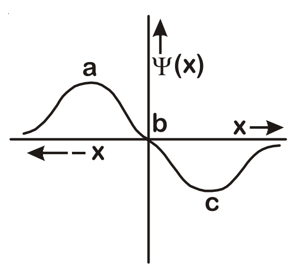
Answer (1)

Sol. EWG attached to benzene ring will reduce the basic strength and increase pK_b while EDG decreases pK_b.

Correct order of pK_b

(C) > (A) > (D) > (B)

11. The electrons are more likely to be found :



(1) In the region a and c

(2) Only in the region c

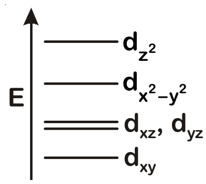
(3) Only in the region a

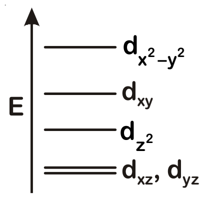
(4) In the region a and b

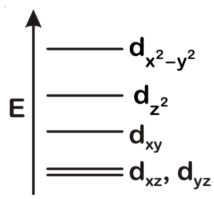
Answer (1)

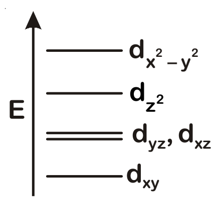
both 'a' and 'c'.

12. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. The field becomes square planar and the order of energy is $d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{zx} = d_{yz}$.

13. An example of a disproportionation reaction is:

(1) $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

(2) $2CuBr \rightarrow CuBr_2 + Cu$

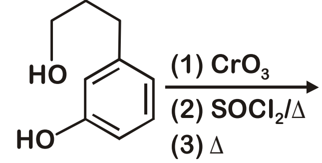
(3) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

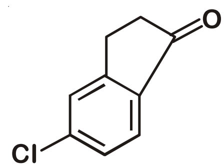
(4) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$

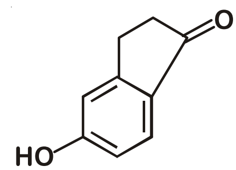
Answer (2)

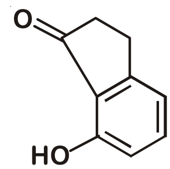
Sol. $\underset{Cu^{+}}{CuBr} \longrightarrow \underset{Cu^{0}}{Cu} + \underset{Cu^{2+}}{CuBr_2}$

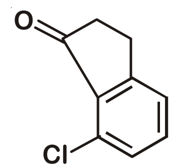
It is an example of disproportionation reaction.



(1) 

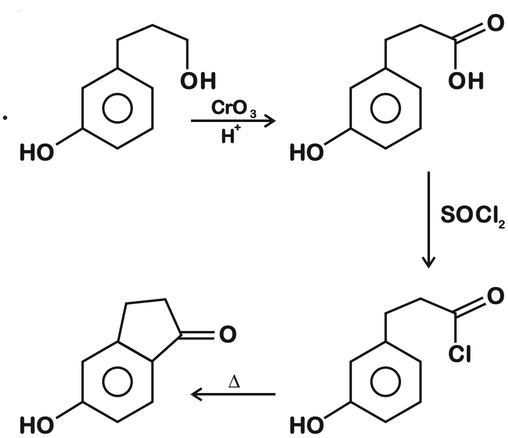
(2) 

(3) 

(4) 

Answer (2)

Sol.



15. An organic compound 'A' is oxidized with Na_2O_2 followed by boiling with HNO_3. The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate

Based on above observation, the element present in the given compound is :

(1) Fluorine

(2) Nitrogen

(3) Phosphorus

(4) Sulphur

Answer (3)

Sol. Phosphorus is detected in the form of canary yellow ppt on reaction with ammonium molybdate.

(1) $(SiO_4)^{4-}$

(2) $\left(\begin{array}{c} R \\ | \\ -Si-O- \\ | \\ R \end{array}\right)_n$ (R = Me)

(3) SiO_2

(4) $(SiO_3)^{2-}$

Answer (1)

Sol. These are examples of silicates, the basic unit being SiO_4^{4-} in each of them.

17. The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol kg^{-1}) of the aqueous solution is :

(1) 13.88×10^{-2} (2) 13.88×10^{-3}

(3) 13.88 (4) 13.88×10^{-1}

Answer (3)

Sol. Let, total 1 moles be present

$n_{solute} = 0.2$

$n_{solvent} = 0.8 \Rightarrow g_{solvent} = 0.8 \times 18$

$\therefore \quad m = \dfrac{0.2 \times 1000}{0.8 \times 18}$

$= \dfrac{1000}{4 \times 18} \approx 13.88$

18. The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are :

(1) 15, 5 and 3 (2) 15, 6 and 2

(3) 16, 5 and 2 (4) 16, 6 and 3

Answer (1)

Sol. Phosphorus has atomic number equal to 15. Its group number is 15, it has 5 valence electrons and valency equal to 3.

$Al(OH)_3 = 2.4 \times 10^{-24}$:

(1) 3×10^{-19}

(2) 12×10^{-21}

(3) 12×10^{-23}

(4) 3×10^{-22}

Answer (4)

Sol. $Al(OH)_3 \rightleftharpoons \underset{s}{Al^{3+}} + \underset{\substack{0.2+3s \\ \approx 0.2}}{3OH^-}$, $K_{sp} = 2.4 \times 10^{-24}$

$s(0.2)^3 = 2.4 \times 10^{-24}$

$s = \dfrac{24 \times 10^{-25}}{8 \times 10^{-3}} = 3 \times 10^{-22} \dfrac{mol}{L}$

20. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1 bar. The work done in kJ is :

(1) –9.0 (2) –0.9

(3) –2.0 (4) +10.0

Answer (2)

Sol. $w = -P\Delta V$

$= -(1 \text{ bar}) \times (9 \text{ L})$

$= -(10^5 \text{ Pa}) \times (9 \times 10^{-3})\ m^3$

$= -9 \times 10^2$ N-m

= –900 J

= –0.9 kJ

21. The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are :

(1) Fisher woman and concentration

(2) Washer woman and concentration

(3) Washer man and reduction

(4) Fisher man and reduction

Answer (2)

Sol. Froth floatation is a method of concentration and it was discovered by a washer women.

(1) PVC (2) Buna-N

(3) Bakelite (4) Nylon 6

Answer (3)

Sol. Bakelite is an example of thermosetting polymer.

23. Peptization is a :

(1) Process of converting a colloidal solution into precipitate

(2) Process of converting precipitate into colloidal solution

(3) Process of converting soluble particles to form colloidal solution

(4) Process of bringing colloidal molecule into solution

Answer (2)

Sol. Peptisation is the process of converting a precipitate into a colloidal sol by shaking it with dispersion medium in the presence of small amount of electrolyte.

24. The correct set of species responsible for the photochemical smog is :

(1) CO_2, NO_2, SO_2 and hydrocarbons

(2) N_2, O_2, O_3 and hydrocarbons

(3) NO, NO_2, O_3 and hydrocarbons

(4) N_2, NO_2 and hydrocarbons

Answer (3)

Sol. Photochemical smog contains oxides of nitrogen, ozone and hydrocarbons.

25. Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200°C. If specific heat of $I_2(s)$ and $I_2(vap)$ are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250°C in cal g^{-1} is :

(1) 11.4 (2) 2.85

(3) 5.7 (4) 22.8

Answer (4)

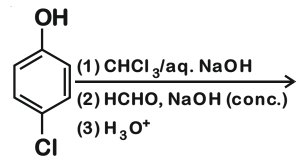
$(\Delta H)_{T_2} - (\Delta H)_{T_1} = (\Delta C_P)(T_2 - T_1)$

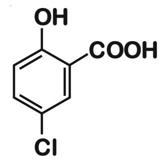
$\therefore \quad (\Delta H)_{250} = (\Delta H)_{200} + (0.031 - 0.055)\ 50$

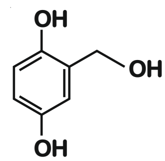
$= 24 - 50 \times 0.024$

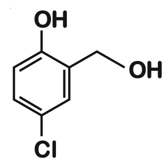
$= 22.8$

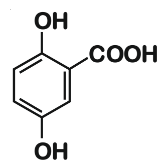
26. The major products of the following reaction are :



(1)  and Methanol

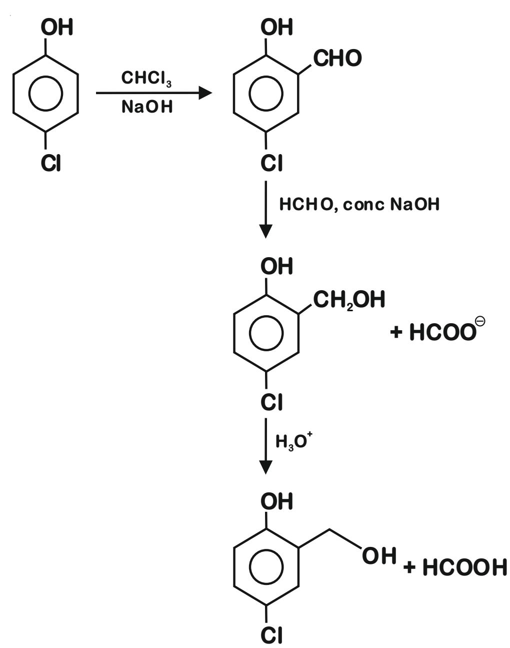
(2)  and Formic acid

(3)  and Formic acid

(4)  and Methanol

Answer (3)

Sol.



$Co^{3+} + e^- \rightarrow Co^{2+}$; E° = +1.81 V

$Pb^{4+} + 2e^- \rightarrow Pb^{2+}$; E° = +1.67 V

$Ce^{4+} + e^- \rightarrow Ce^{3+}$; E° = +1.61 V

$Bi^{3+} + 3e^- \rightarrow Bi$; E° = +0.20 V

Oxidizing power of the species will increase in the order :

(1) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$

(2) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$

(3) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$

(4) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

Answer (4)

Sol. Greater the reduction potential, greater is the oxidising power.

So, $Co^{3+} > Pb^{4+} > Ce^{4+} > Bi^{3+}$

28. The correct statement among the following is :

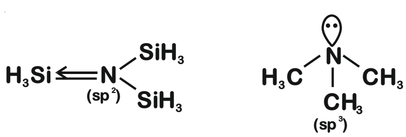
(1) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

(2) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$

(3) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$

(4) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$

Answer (1)

Sol. 

Trisilylamine is planar, due to backbonding of lone pairs of nitrogen into vacant d-orbitals of Si. In trimethylamine, there is no such delocalisation and hence it is more basic.

29. In the following reaction; $xA \rightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be :

(1) N_2O_4 and NO_2

(2) NO_2 and N_2O_4

(3) n-Butane and Iso-butane

(4) C_2H_2 and C_6H_6

Answer (1)

Sol. $xA \rightarrow yB$

$$\therefore \quad \frac{-dA}{xdt} = \frac{1}{y}\frac{dB}{dt}$$

$$\frac{-dA}{dt} = \frac{dB}{dt} \times \frac{x}{y}$$

$$\log\left[\frac{-dA}{dt}\right] = \log\left[\frac{dB}{dt}\right] + \log\left(\frac{x}{y}\right)$$

$$\frac{x}{y} = 2$$

The reaction is of type $2A \rightarrow B$.

30. 5 moles of AB_2 weigh 125 × 10^{-3} kg and 10 moles of A_2B_2 weigh 300 × 10^{-3} kg. The molar mass of $A(M_A)$ and molar mass of $B(M_B)$ in kg mol^{-1} are :

(1) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$

(2) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$

(3) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$

(4) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$

Answer (3)

Sol. 5 mol AB_2 weighs 125 g

$\therefore$ AB_2 = 25 g/mol

10 mol A_2B_2 weighs 300 g

$\therefore$ A_2B_2 = 30 g/mol

$\therefore$ Molar mass of A = 5

Molar mass of B = 10

1. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix}4 & -2\\ 1 & -4\end{bmatrix}$ (2) $\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$

(3) $\begin{bmatrix}-4 & -2\\ -1 & 4\end{bmatrix}$ (4) $\begin{bmatrix}-4 & 2\\ 1 & 4\end{bmatrix}$

Answer (2)

Sol. Let $A=\begin{bmatrix}a & c\\ c & b\end{bmatrix}$ and $B=\begin{bmatrix}0 & d\\ -d & 0\end{bmatrix}$

$\Rightarrow \ A+B=\begin{bmatrix}a & c+d\\ c-d & b\end{bmatrix}=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$

$\Rightarrow$ a = 2, b = –1, c – d = 5, c + d = 3

$\Rightarrow$ a = 2, b = –1, c = 4, d = –1

$\Rightarrow \ AB=\begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$=\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$

2. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r + \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to :

(1) $\frac{21}{346}$

(2) $\frac{7}{116}$

(3) $\frac{29}{358}$

(4) $\frac{1}{12}$

Answer (4)

Sol. $375x^2 - 25x - 2 = 0$

$\alpha+\beta=\frac{25}{375},\ \alpha\beta=\frac{-2}{375}$

$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left(\alpha^r+\beta^r\right)=\left(\alpha+\alpha^2+\alpha^3+\ldots\infty\right)+\left(\beta+\beta^2+\beta^3+\ldots\infty\right)$

$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}$

$=\frac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$

$=\dfrac{\frac{25}{375}+\frac{4}{375}}{1-\frac{25}{375}-\frac{2}{375}}=\frac{29}{375-25-2}$

$=\frac{29}{348}=\frac{1}{12}$

3. If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all value of α for which det (A) + 1 = 0, is :

(1) –1 (2) 2

(3) 0 (4) 1

Answer (4)

Sol. As $B = A^{-1}$

$|B|=\frac{1}{|A|}$

Now $|B|=\begin{vmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{vmatrix}=2\alpha^2-2\alpha-25$

Given |A| + 1 = 0

$\frac{1}{2\alpha^2-2\alpha-25}+1=0$

$\Rightarrow \ \frac{2\alpha^2-2\alpha-24}{2\alpha^2-2\alpha-25}=0$

α = 4, –3

Sum of values = 1

4. For $x \in R$, let [x] denote the greatest integer $\le x$, then the sum of the series

$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots.+\left[-\frac{1}{3}-\frac{99}{100}\right]$

is:

(1) – 135 (2) –153

(3) –133 (4) –131

Answer (3)

As $[x] + [-x] = -1$ $(x \notin z)$

Required value

$$= -100 - \left\{\left[\frac{1}{3}\right] + \left[\frac{1}{3} + \frac{1}{100}\right] + \ldots \left[\frac{1}{3} + \frac{99}{100}\right]\right\}$$

$$= -100 - \left[\frac{100}{3}\right]$$

$= -133$

5. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(4, 3\sqrt{2})$ (2) $(3, 3\sqrt{3})$

(3) $(5, 3\sqrt{6})$ (4) $(4, 3\sqrt{3})$

Answer (4)

Sol. $f(x) = x\sqrt{kx - x^2} = \sqrt{kx^3 - x^4}$

$$f'(x) = \frac{(3kx^2 - 4x^3)}{2\sqrt{kx^3 - x^4}} \geq 0 \text{ for } x \in [0, 3]$$

$\Rightarrow$ $3k - 4x \geq 0$

$3k \geq 4x$

$3k \geq 4x$ for $x \in [0, 3]$

Hence $k \geq 4$

i.e., $m = 4$

For $k = 4$,

$\Rightarrow$ $f(x) = x\sqrt{4x - x^2}$

For max. value, $f'(x) = 0$

$\Rightarrow$ $x = 3$

i.e., $y = 3\sqrt{3}$

Hence $M = 3\sqrt{3}$

6. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to:

(1) $2\sqrt{14}$ (2) 14

(3) $2\sqrt{7}$ (4) $\sqrt{14}$

Answer (1)

$Q(3\mu + 2, 2\mu - 1, -\mu + 1)$

As P lies on $2x + 3y - z + 13 = 0$

$6\lambda + 4 + 6\lambda - 3 + \lambda - 1 + 13 = 0$

$\Rightarrow$ $13\lambda = -13$

$\Rightarrow$ $\lambda = -1$

$\therefore$ $P(-1, -3, 2)$

Q lies on $3x + y + 4z = 16$

$9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$

$\Rightarrow$ $7\mu = 7$

$\Rightarrow$ $\mu = 1$

Q is (5, 1, 0)

$PQ = \sqrt{36 + 16 + 4} = \sqrt{56} = 2\sqrt{14}$

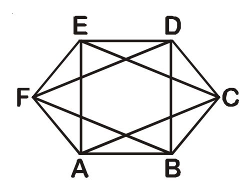
7. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is:

(1) $\frac{3}{20}$ (2) $\frac{1}{5}$

(3) $\frac{3}{10}$ (4) $\frac{1}{10}$

Answer (4)

Sol. Only two equilateral triangles are possible i.e. ΔAEC and ΔBDF.



Hence, required probability

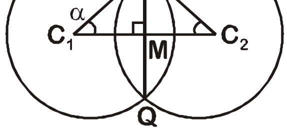
$$= \frac{2}{{}^6C_3} = \frac{1}{10}$$

8. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is:

(1) $\frac{120}{13}$ (2) $\frac{13}{2}$

(3) $\frac{13}{5}$ (4) $\frac{60}{13}$

Answer (1)



In ΔPC_1C_2,

$$\tan\alpha = \frac{5}{12}$$

$$\Rightarrow \quad \sin\alpha = \frac{5}{13}$$

In ΔPC_1M, $\sin\alpha = \frac{PM}{12}$

$$\Rightarrow \quad \frac{5}{13} = \frac{PM}{12}$$

$$\Rightarrow \quad PM = \frac{60}{13}$$

Length of common chord $(PQ) = \frac{120}{13}$

9. If the volume of parallelopiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}, \hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to :

(1) $\frac{1}{\sqrt{3}}$ (2) $-\sqrt{3}$

(3) $\sqrt{3}$ (4) $-\frac{1}{\sqrt{3}}$

Answer (1*) Vector are coplanar for $\lambda = \lambda_1$ where $\lambda_1^3 - \lambda_1 + 1 = 0 \Rightarrow$ volume is minimum when $\lambda = \lambda_1$.

Sol. $V = \left\| \begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix} \right\|$

$= |1(1) + \lambda(\lambda^2) + 1(-\lambda)|$

$= |\lambda^3 - \lambda + 1|$

Let $f(x) = x^3 - x + 1$

$f'(x) = 3x^2 - 1$

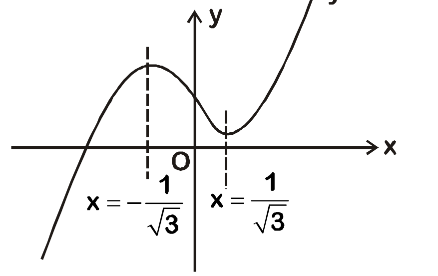
For maxima/minima, $f'(x) = 0$

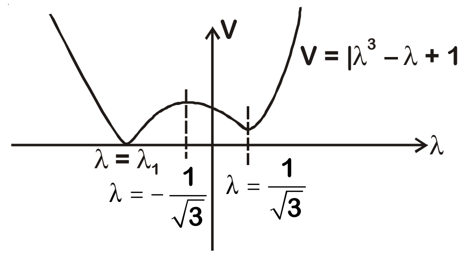
$$x = \pm\frac{1}{\sqrt{3}}$$

$f''(x) = 6x$

$$\because \quad f''\left(\frac{1}{\sqrt{3}}\right) > 0$$

$x = \frac{1}{\sqrt{3}}$ is point of local minima





When $\lambda = \lambda_1$, volume of parallelopiped is zero (vectors are coplanar)

10. The integral $\int \frac{2x^3 - 1}{x^4 + x}dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e\left|\frac{x^3+1}{x}\right| + C$ (2) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|} + C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2} + C$ (4) $\log_e\frac{|x^3+1|}{x^2} + C$

Answer (1)

Sol. $I = \int\frac{(2x^3-1)dx}{x^4+x} = \int\frac{(2x-x^{-2})dx}{x^2+x^{-1}}$

Put $x^2 + x^{-1} = t$

$(2x - x^{-2})dx = dt$

$$I = \int\frac{dt}{t} = \ln|t| + c$$

$$= \ln|x^2 + x^{-1}| + c$$

$$= \ln\left|\frac{x^3+1}{x}\right| + c$$

11. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{5\sqrt{5}}{2}$

(3) $\frac{\sqrt{157}}{2}$ (4) $\frac{\sqrt{221}}{2}$

Answer (2)

$3x^2 + 4y^2 = 12 \quad \Rightarrow \quad \frac{x^2}{2^2} + \frac{y^2}{(\sqrt{3})^2} = 1$

Let point $P(2\cos\theta, \sqrt{3}\sin\theta)$

$\Rightarrow$ Equation of tangent at P is

$$\frac{x}{2}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$$

$$\Rightarrow \quad m_T = -\frac{\sqrt{3}}{2}\cot\theta = \frac{1}{2}$$

$$\tan\theta = -\sqrt{3} \quad \Rightarrow \theta = \pi - \frac{\pi}{3} \quad \text{or } \theta = 2\pi - \frac{\pi}{3}$$

If $\theta = \frac{2\pi}{3}$, then $P\left(-1, \frac{3}{2}\right)$ and $PQ = \frac{5\sqrt{5}}{2}$

If $\theta = \frac{5\pi}{3}$, then tangent does not pass through Q(4, 4)

12. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) –260 (2) –380

(3) –320 (4) –410

Answer (3)

Sol. $S_4 = 16, \quad S_6 = -48$

$2(2a + 3d) = 16$

$\Rightarrow 2a + 3d = 8$

Also $3[2a + 5d] = -48$

$\Rightarrow 2a + 5d = -16$

$2d = -24$

$d = -12 \quad \Rightarrow a = 22$

$S_{10} = 5(44 + 9(-12))$

$= -320$

13. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

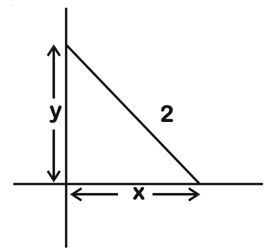
(3) $\frac{25}{\sqrt{3}}$ (4) 25

Answer (3)

Sol. Given $\frac{dy}{dt} = -25$ at y = 1

$x^2 + y^2 = 4$

When y = 1, $\quad x = \sqrt{3}$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$\Rightarrow \quad \sqrt{3}\frac{dx}{dt} + (-25) = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{25}{\sqrt{3}} \text{ cm/s}$$

14. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(1) $2^{20} + 1$ (2) 2^{21}

(3) $2^{20} - 1$ (4) 2^{20}

Answer (4)

Sol. Number of ways of selecting 10 objects

= (10I, 0D) or (9I, 1D) or (8I, 1D) or ... (0I, 10D)

where D signifies distinct object and I indicates identical object

$= 1 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}$

$= \frac{2^{21}}{2} = 2^{20}$

15. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S′ denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS′ in a ratio :

(1) 13 : 11 (2) 14 : 13

(3) 5 : 4 (4) 2 : 1

Answer (3)

m

Equation of tangent $\frac{x^2}{1}-\frac{y^2}{8}=1$ is $y=mx\pm\sqrt{m^2-8}$

for common tangent,

$\frac{3}{m}=\pm\sqrt{m^2-8} \qquad \Rightarrow \frac{9}{m^2}=m^2-8$

Put $m^2 = t$

$t^2 - 8t - 9 = 0 \qquad \Rightarrow t^2 - 9t + t - 9 = 0$

$\qquad \Rightarrow (t + 1)(t - 9) = 0$

$\because \quad t = m^2 \geq 0 \qquad \Rightarrow t = m^2 = 9$

$\qquad \Rightarrow m = \pm 3$

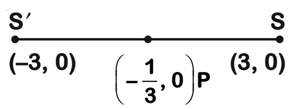
$\Rightarrow$ Equation of tangent is $y = 3x + 1$

or $y = -3x - 1$

Intersection point $P\left(-\frac{1}{3}, 0\right)$

$8 = 1(e^2 - 1) \qquad \Rightarrow e = 3$

foci $(\pm 3, 0) \qquad \Rightarrow$



$$\frac{S'P}{SP}=\frac{3-\frac{1}{3}}{3+\frac{1}{3}}=\frac{8}{10}=\frac{4}{5}$$

16. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to :

(1) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (2) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

(3) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

Answer (3)

Sol. $e^y + xy = e$ ...(i)

Put $x = 0$ in (i)

$\Rightarrow \quad e^y = e \Rightarrow y = 1$

Differentiate (i) w.r. to x

$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$ ...(ii)

Put y = 1 in (ii)

$e\frac{dy}{dx}+0+1=0 \Rightarrow \frac{dy}{dx}=-\frac{1}{e}$

$e^y\frac{d^2y}{dx^2}+\frac{dy}{dx}.e^y\frac{dy}{dx}+x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$ ...(iii)

Put $y = 1$, $x = 0$, $\frac{dy}{dx}=-\frac{1}{e}$

$e\frac{d^2y}{dx^2}+\frac{1}{e}-\frac{2}{e}=0 \Rightarrow \frac{d^2y}{dx^2}=\frac{1}{e^2}$

$\Rightarrow \left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right) \equiv \left(-\frac{1}{e}, \frac{1}{e^2}\right)$

17. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) $2\sqrt{2}$ (2) 4

(3) 2 (4) $\sqrt{2}$

Answer (3)

Sol. $\frac{x_1+x_2+x_3+x_4}{4}=11$ and $x_1+x_2+x_3+x_4=44$

$\frac{x_5+x_6+...+x_{10}}{6}=16 \Rightarrow x_5+x_6+...+x_{10}=96$

$x_1^2+x_2^2+...+x_{10}^2=2000$

$\sigma^2 = \frac{\Sigma x_i^2}{N}-(\bar{x})^2$

$= \frac{2000}{10}-\left(\frac{140}{10}\right)^2=4$

$\Rightarrow \sigma = 2$

18. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is :

(1) 84 (2) –126

(3) –84 (4) 126

Answer (1)

Sol. $(1-x)^{10}(1+x+x^2)^9(1+x)$

$= (1-x^3)^9(1-x^2)$

$= (1-x^3)^9 - x^2(1-x^3)^9$

$\Rightarrow$ Coefficient of x^{18} in $(1-x^3)^9$ – coeff. of x^{16} in $(1-x^3)^9$.

$= {}^9C_6 = \frac{9!}{6!3!}=\frac{7\times8\times9}{6}=84$

statements p, q, r are respectively :

(1) F, T, T (2) T, T, F

(3) T, F, F (4) T, F, T

Answer (2)

Sol. $P \rightarrow (\sim q \vee r)$ is a fallacy

$\Rightarrow$ P is True and $\sim q \vee r$ is False

$\Rightarrow$ P is True and $\sim q$ is False and r is False

$\Rightarrow$ Truth values of p, q, r are

T, T, F respectively.

20. Let a random variable X have a binomial distribution with mean 8 and variance 4.

If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to :

(1) 121 (2) 1

(3) 17 (4) 137

Answer (4)

Sol. $\mu = 8,\ \sigma^2 = 4$

$\Rightarrow$ $\mu = np = 8$, $\sigma^2 = npq = 4$, $p + q = 1$

$\Rightarrow\ q = \frac{1}{2}, p = \frac{1}{2}, n = 16$

$P(X \le 2) = \frac{k}{2^{16}}$

$^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{k}{2^{16}}$

$\Rightarrow$ k = (1 + 16 + 120) = 137

21. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is :

(1) $4(-2\hat{i} - 2\hat{j} + \hat{k})$ (2) $4(2\hat{i} + 2\hat{j} - \hat{k})$

(3) $4(2\hat{i} + 2\hat{j} + \hat{k})$ (4) $4(2\hat{i} - 2\hat{j} - \hat{k})$

Answer (4)

Sol. Let vector be $\lambda\left[(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right]$

$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$

$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$

$= \lambda\left[16\hat{i} - 16\hat{j} - 8\hat{k}\right]$

$= 8\lambda\left[2\hat{i} - 2\hat{j} - \hat{k}\right]$

$\Rightarrow\ 12 = 8|\lambda|\sqrt{4+4+1}$

$|\lambda| = \frac{1}{2}$

Hence required vector is $\pm 4(2\hat{i} - 2\hat{j} - \hat{k})$

22. Consider the differential equation, $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when x = 1, then the value of x for which y = 2, is :

(1) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (2) $\frac{3}{2} - \sqrt{e}$

(3) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (4) $\frac{1}{2} + \frac{1}{\sqrt{e}}$

Answer (3)

Sol. $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$

$\frac{dx}{dy} + \left(\frac{1}{y^2}\right)x = \frac{1}{y^3}$

I.F. $= e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

its solution is

$x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3}dy + c$

put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}dy = dt$

$\Rightarrow\ x.e^{-\frac{1}{y}} = -\int te^t dt + c = -te^t + e^t + c$

$x.e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y} + 1\right) + c$ passes through (1, 1)

$\Rightarrow$ 1 = 2 + ce $\Rightarrow c = -\frac{1}{e}$

$\Rightarrow\ x = \left(1 + \frac{1}{y}\right) - \frac{1}{e}e^{\frac{1}{y}}$ passes through (k, 2)

$\Rightarrow\ k = \frac{3}{2} - \frac{1}{\sqrt{e}}$

function such that f(2) = 6 and $f'(2)=\frac{1}{48}$. If

$\int_{6}^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to :

(1) 18 (2) 36

(3) 24 (4) 12

Answer (1)

Sol. $\int_{6}^{f(x)} 4t^3 dt = (x-2)g(x)$

$4(f(x))^3.f'(x) = g'(x)(x-2) + g(x)$

put x = 2,

$$\frac{4(6)^3.1}{48} = g(2)$$

$$\lim_{x\to 2} g(x) = 18$$

24. The equation $y = \sin x \sin(x+2) - \sin^2(x+1)$ represents a straight line lying in :

(1) Third and fourth quadrants only

(2) First, third and fourth quadrants

(3) First, second and fourth quadrants

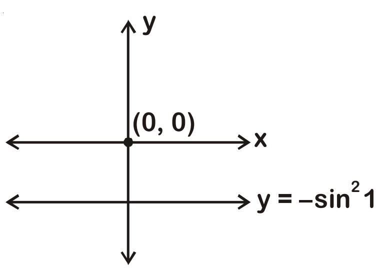
(4) Second and third quadrants only

Answer (1)

Sol. $y = \sin x \cdot \sin(x+2) - \sin^2(x+1)$

$$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$$

$$= \frac{(\cos 2) - 1}{2} = -\sin^2 1$$



Graph of y lies in

III and IV Quadrant

$\le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$ is $a\sqrt{2}+b$, then a – b is equal to :

(1) $-\frac{2}{3}$ (2) 6

(3) $\frac{10}{3}$ (4) $\frac{8}{3}$

Answer (2)

Sol. $y^2 = 4x$

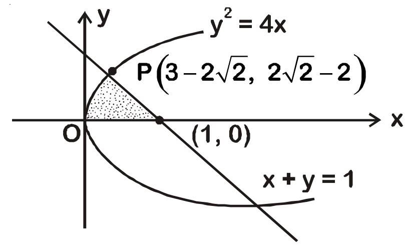
$x + y = 1$

$y^2 = 4(1 - y)$

$y^2 + 4y - 4 = 0$

$(y + 2)^2 = 8$

$y + 2 = \pm 2\sqrt{2}$



required area

$$= \int_{0}^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2}\times\left(2\sqrt{2}-2\right)\times\left(2\sqrt{2}-2\right)$$

$$= \left[2\times\frac{2}{3}x^{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}\left(8+4-8\sqrt{2}\right)$$

$$= \frac{4}{3}\times\left(3-2\sqrt{2}\right)\sqrt{3-2\sqrt{2}} + 6 - 4\sqrt{2}$$

$$= \frac{4}{3}\left(3-2\sqrt{2}\right)\left(\sqrt{2}-1\right) + 6 - 4\sqrt{2}$$

$$= \frac{4}{3}\left(3\sqrt{2}-3-4+2\sqrt{2}\right) + 6 - 4\sqrt{2}$$

$$= \left(6-\frac{28}{3}\right) + \left(\frac{20}{3}-4\right)\sqrt{2}$$

$$= -\frac{10}{3} + \frac{8}{3}\sqrt{2}$$

$$\Rightarrow\ a - b = \frac{10}{3} + \frac{8}{3} = 6$$

26. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to :

(1) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$ (2) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(3) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$ (4) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

Answer (1)

Sol. $-\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{12}{13}\right) = -\sin^{-1}\left(\frac{3}{5}\times\frac{5}{13} - \frac{12}{13}\times\frac{4}{5}\right)$

($\because$ $xy \ge 0$ and $x^2 + y^2 \le 1$)

$$= \sin^{-1}\left(\frac{33}{65}\right)$$

$$= \cos^{-1}\left(\frac{56}{65}\right)$$

$$= \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$$

27. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to :

(1) $\frac{1}{2}$ (2) 1

(3) –1 (4) $-\frac{1}{2}$

Answer (3)

Sol. $\int_0^{\pi/2} \frac{\cot x \, dx}{\cot x + \operatorname{cosec} x}$

$$= \int_0^{\pi/2} \frac{\cos x \, dx}{1 + \cos x} = \int_0^{\pi/2} \left(1 - \frac{1}{1+\cos x}\right) dx$$

$$= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx$$

$$= \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$$

$$= \frac{\pi}{2} - \left[\tan\frac{x}{2}\right]_0^{\pi/2}$$

$$= \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right)$$

$m = \frac{1}{2}, n = -2$

$\Rightarrow$ mn = –1

28. For $x \in \left(0, 3/2\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

(1) $\tan\frac{5\pi}{12}$ (2) $\tan\frac{\pi}{12}$

(3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{7\pi}{12}$

Sol. $\phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right)$

$$= h\left(f\left(\sqrt{3}\right)\right) = h(3^{\frac{1}{4}})$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}} = -\frac{1}{2}(1+3-2\sqrt{3}) = \sqrt{3} - 2 = -(-\sqrt{3}+2)$$

$$= -\tan 15^\circ = \tan(180^\circ - 15^\circ) = \tan\left(\pi - \frac{\pi}{12}\right)$$

$$= \tan\frac{11\pi}{12}$$

29. The equation $|z - i| = |z - 1|, i = \sqrt{-1}$, represents :

(1) The line through the origin with slope –1

(2) A circle of radius $\frac{1}{2}$

(3) A circle of radius 1

(4) The line through the origin with slope 1

Answer (4)

Sol. $|z - 1| = |z - i|$

Let $z = x + iy$

$(x - 1)^2 + y^2 = x^2 + (y - 1)^2$

$1 - 2x = 1 - 2y$

$\Rightarrow$ $x - y = 0$

Locus is straight line with slope 1

30. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(1) 3 (2) 5

(3) 4 (4) 7

Answer (2)

Sol. $1 + \sin^4 x = \cos^2 3x$

L.H.S = $1 + \sin^4 x$, R.H.S = $\cos^2 3x$

L.H.S ≥ 1 R.H.S ≤ 1

$\Rightarrow$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$ and $\cos^2 3x = 1$

$\sin x = 0$ and $(4\cos^2 x - 3)^2 \cos^2 x = 1$

$\Rightarrow$ $\sin x = 0$ and $\cos^2 x = 1$

$\Rightarrow$ $x = 0, \pm\pi, \pm 2\pi$

$\Rightarrow$ Total number of solutions is 5

Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to:

(1) $9F/(\pi r^2 YT)$

(2) $3F/(\pi r^2 YT)$

(3) $F/(3\pi r^2 YT)$

(4) $6F/(\pi r^2 YT)$

Answer (2)

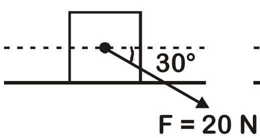
Sol. $Y = \frac{FL}{A|\Delta L|} \Rightarrow \Delta L = \frac{FL}{AY} = L\alpha T$

$\alpha = \frac{F}{AYT}$

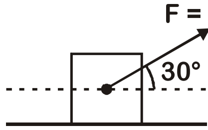
$\gamma = 3\alpha = \frac{3F}{\pi r^2 YT}$

2. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is μ = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be :

(g = 10 ms^{-2})



(A)



(B)

(1) 0.4 ms^{-2}

(2) 3.2 ms^{-2}

(3) 0 ms^{-2}

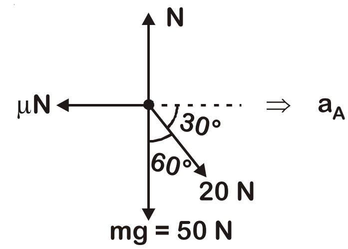
(4) 0.8 ms^{-2}

Answer (4)

Sol. N = 60 N

F = 0.2 × 60 = 12 N

$a_A = \frac{\left(\frac{20\sqrt{3}}{2} - 12\right)}{5} = \frac{5.3}{5} = 1.06 \text{ m/s}^2$



For B N = 40 N

$F = 8\text{ N} \Rightarrow \frac{20\sqrt{3}}{2} - 8 = 5a_B$

$a_B = \frac{17.3 - 8}{5} = \frac{9.3}{5} = 1.86 \text{ m/s}^2$

$a_B - a_A = 0.8 \text{ m/s}^2$

3. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is

(1) 7/5

(2) 27/5

(3) 9/7

(4) 20/7

Answer (4)

Sol. $\frac{1}{\lambda_1} = R\left[\frac{1}{9} - \frac{1}{16}\right] = R\frac{7}{144}$ ...(i)

$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{9}\right] = R\frac{5}{36}$ ...(ii)

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{5 \times 144}{36 \times 7} = \frac{20}{7}$

4. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V(V = GI_0) by connecting a series resistance R_V to it. Then,

(2) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(3) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

(4) $R_A R_V = G^2\left(\frac{(I_0 - I_g)}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

Answer (1)

Sol. $V = I_g(R_V + G) = GI_0$ ...(1)

$(I_0 - I_g)R_A = I_g G$ ...(2)

From (1), $R_V = \frac{G(I_0 - I_g)}{I_g}$

From (2), $R_A = \frac{I_g G}{I_0 - I_g}$

$\Rightarrow R_A R_V = G^2$

$$\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$$

5. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be

(1) n^2 (2) $\frac{1}{n^2}$

(3) n (4) $\frac{1}{n}$

Answer (4)

Sol. $l_1 = nl_2$

$\because k \propto \frac{1}{l}$

$\therefore \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{1}{n}$

6. One kg of water, at 20°C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to

[Specific heat of water = 4200 J/(kg °C), Latent heat of water = 2260 kJ/kg]

Answer (3)

Sol. $\Delta Q = 1 \times 4200 \times 80 + 2260 \times 10^3$ J

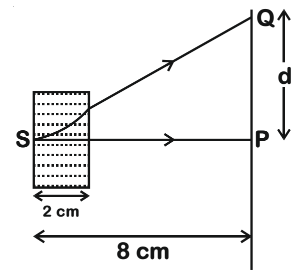
$= (336 + 2260) \times 10^3 \text{ J} = 2596 \times 10^3$ J

$\Delta Q = I_{rms} V_{rms} t = 200 \times \frac{200}{20} t = 2000t$

$\Rightarrow t = \frac{2596}{2} \text{ s} \approx 21.6 \text{ minutes} \approx 22 \text{ minutes}$

7. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3} \text{ T})\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

(electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg)



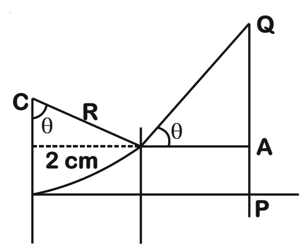
(1) 11.65 cm (2) 12.87 cm

(3) 2.25 cm (4) 1.22 cm

Answer (2)

Sol. Radius of path, $R = \frac{mv}{eB} = \frac{\sqrt{2m\,KE}}{eB}$

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 100e}}{e \times 1.5 \times 10^{-3}} = \frac{\sqrt{2 \times 9.1 \times 10^{-29}}}{\sqrt{1.6 \times 10^{-19}} \times 1.5 \times 10^{-3}} \text{ m}$$



$$= \frac{3.37 \times 10^{-5}}{1.5 \times 10^{-3}} \times 100 \text{ cm} = 2.25 \text{ cm}$$

$$\sin\theta = \frac{2}{2.25} = \frac{8}{9}$$

= 1.22 + 11.64 = 12.86 cm

8. A Carnot engine has an efficiency of $\frac{1}{6}$. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively,

(1) 99°C, 37°C (2) 37°C, 99°C

(3) 124°C, 62°C (4) 62°C, 124°C

Answer (1)

Sol. $\frac{1}{6}=1-\frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H}=\frac{5}{6}$ ...(i)

$\frac{1}{3}=1-\frac{(T_C-62)}{T_H} \Rightarrow \frac{T_C-62}{T_H}=\frac{2}{3}$ ...(ii)

$\frac{T_C-62}{T_C}=\frac{2\times 6}{3\times 5}=\frac{4}{5}$

$\Rightarrow T_C = 310\text{ K} = 37°\text{C}$

$T_H = 372\text{ K} = 99°\text{C}$

9. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound?

[Given reference intensity of sound as 10^{-12} W/m^2]

(1) 30 cm (2) 40 cm

(3) 10 cm (4) 20 cm

Answer (2)

Sol. $120=10\log_{10}\frac{I}{10^{-12}}$

$\Rightarrow \frac{I}{10^{-12}}=10^{12} \Rightarrow I = 1\text{ W/m}^2$

$\frac{2}{4\pi r^2}=1 \Rightarrow r=\sqrt{\frac{2}{4\pi}}\text{ m}=0.399\text{ m}$

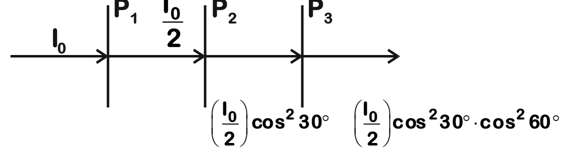
$= 40$ cm

10. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60°C to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) :

(1) 1.80 (2) 5.33

(3) 10.67 (4) 16.00

Answer (3)



$I=\frac{I_0}{2}\cdot\cos^2 30°\cdot\cos^2 60°=\frac{I_0}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3I_0}{32}$

$\Rightarrow \frac{I_0}{I}=\frac{32}{3}=10.67$

11. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio $\left(\frac{v_1}{v_2}\right)$ equals :

(1) $\frac{1}{27}$ (2) 9

(3) $\frac{1}{9}$ (4) 27

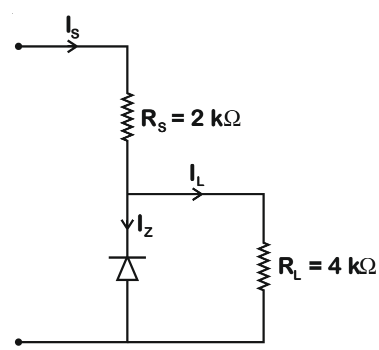
Answer (2)

Sol. $R^3 = 27r^3$

$\Rightarrow R = 3r$

$\frac{V_1}{V_2}=\left(\frac{R}{r}\right)^2=9$

12. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(1) 7.5 mA (2) 1.5 mA

(3) 2.5 mA (4) 3.5 mA

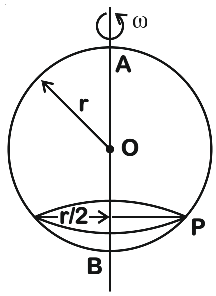
Answer (4)

$I_S = \dfrac{10}{2\times10^3} = 5\ \text{mA}$

$I_L = \dfrac{6}{4\times10^3} = 1.5\ \text{mA}$

$I_{Z(max)} = I_S - I_L = 3.5$ mA

13. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to:

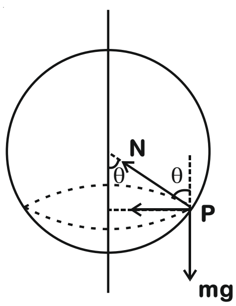


(1) $\dfrac{(g\sqrt{3})}{r}$ (2) $\dfrac{2g}{r}$

(3) $\dfrac{2g}{(r\sqrt{3})}$ (4) $\dfrac{\sqrt{3}g}{2r}$

Answer (3)

Sol.



$\theta = \sin^{-1}\dfrac{1}{2} = 30^\circ$

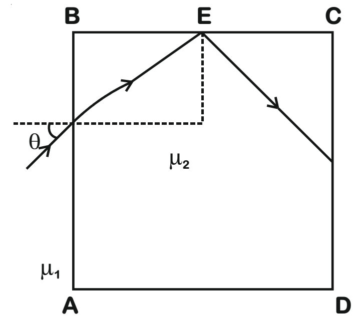
$N\cos\theta = mg$ ...(i)

$N\sin\theta = \dfrac{m\omega^2 r}{2}$ ...(ii)

$\tan\theta = \dfrac{\omega^2 r}{2g}$

$\Rightarrow \omega^2 = \dfrac{2g\tan\theta}{r} = \dfrac{2g}{r\sqrt{3}}$

a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.



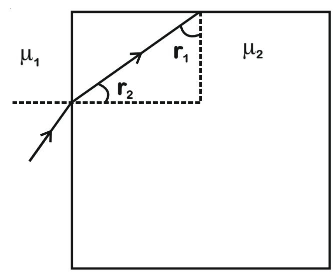
Then θ must satisfy:

(1) $\theta > \sin^{-1}\dfrac{\mu_1}{\mu_2}$ (2) $\theta < \sin^{-1}\dfrac{\mu_1}{\mu_2}$

(3) $\theta > \sin^{-1}\sqrt{\dfrac{\mu_2^2}{\mu_1^2}-1}$ (4) $\theta < \sin^{-1}\sqrt{\dfrac{\mu_2^2}{\mu_1^2}-1}$

Answer (4)

Sol. $\mu_2 \sin r_1 > \mu_1$



$\Rightarrow \sin r_1 > \dfrac{\mu_1}{\mu_2}$

$r_1 > \sin^{-1}\left[\dfrac{\mu_1}{\mu_2}\right]$

$\mu_1 \sin\theta = \mu_2 \sin r_2 = \mu_2 \sin(90^\circ - r_1)$

$\sin\theta = \dfrac{\mu_2}{\mu_1}\sin(90^\circ - r_1) = \dfrac{\mu_2}{\mu_1}\cos r_1$

$\Rightarrow \theta < \sin^{-1}\left[\dfrac{\mu_2}{\mu_1}\dfrac{\sqrt{\mu_2^2-\mu_1^2}}{\mu_2}\right]$

$\Rightarrow \theta < \sin^{-1}\left[\sqrt{\dfrac{\mu_2^2}{\mu_1^2}-1}\right]$

15. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = τ (assume that the particle is at origin at t = 0).

(1) $b^2\tau$ (2) $\dfrac{b^2\tau}{4}$

(3) $\dfrac{b^2\tau}{2}$ (4) $\dfrac{b^2\tau}{\sqrt{2}}$

Answer (3)

$$\Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^\tau b\,dt$$

$$\Rightarrow 2\sqrt{x} = b\tau \qquad \ldots\text{(ii)}$$

$$\Rightarrow v = b\cdot\frac{b\tau}{2} = \frac{b^2\tau}{2}$$

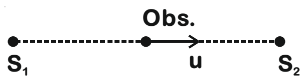
16. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals:

(1) 10.0 m/s (2) 5.5 m/s

(3) 2.5 m/s (4) 15.0 m/s

Answer (3)

Sol.



$$n_1 = \frac{v-u}{v}\times n_0$$

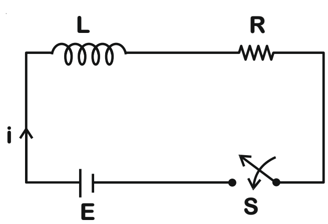
$$n_2 = \frac{v+v}{v}\times n_0$$

$$\Rightarrow n_2 - n_1 = \frac{2u\times n_0}{v}$$

$$\Rightarrow 10 = \frac{2\times u\times 660}{330}$$

$\Rightarrow$ u = 2.5 m/s

17. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is:



(1) $\frac{2.7EL}{R^2}$ (2) $\frac{EL}{2.7R^2}$

(3) $\frac{7.3\,EL}{R^2}$ (4) $\frac{EL}{7.3\,R^2}$

Answer (2)

$$\Rightarrow \int dq = \frac{E}{R}\int\left(1-e^{-\frac{t}{\tau}}\right)dt$$

$$\Rightarrow Q = \frac{E}{R}\times\left[t+e^{-\frac{t}{\tau}}\times\tau\right]_0^t$$

$$= \frac{E}{R}\times\left[\frac{L}{R}+\frac{L}{R}\cdot e^{-1}-\frac{L}{R}\right]$$

$$\Rightarrow Q = \frac{EL}{eR^2} = \frac{EL}{2.7R^2}$$

18. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

(1) $\frac{R}{4}$ (2) $\frac{R}{3}$

(3) $\frac{R}{9}$ (4) $\frac{R}{2}$

Answer (4)

Sol. $\frac{g_e}{g_p} = \frac{9}{4}$

$$\Rightarrow \frac{M_e\times R_P^2}{R_e^2\times M_p} = \frac{9}{4}$$

$$\Rightarrow 9\times\left(\frac{R_p}{R_e}\right)^2 = \frac{9}{4}$$

$$\Rightarrow R_p = \frac{R_e}{2}$$

$$\therefore R_p = \frac{R}{2}$$

19. In an amplitude modulator circuit, the carrier wave is given by,

C(t) = 4 sin (20000 πt) while modulating signal is given by, m(t) = 2 sin (2000 πt). The values of modulation index and lower side band frequency are :

(1) 0.5 and 10 kHz

(2) 0.3 and 9 kHz

(3) 0.4 and 10 kHz

(4) 0.5 and 9 kHz

Answer (4)

$$f_c = \frac{\omega_c}{2\pi} = \frac{20000\pi}{2\pi} = 10000 \text{ Hz}$$

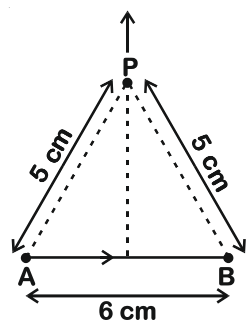
$$f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}$$

$\therefore$ Lower band side = (10000 – 1000) Hz

= 9 kHz

20. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure)

($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2})

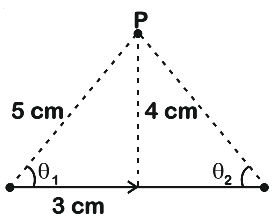


(1) 2.5×10^{-5} T (2) 1.5×10^{-5} T

(3) 3.0×10^{-5} T (4) 2.0×10^{-5} T

Answer (2)

Sol.



$$B_P = \frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$$

$$= \frac{10^{-7} \times 5}{0.04} \times \left(2 \times \frac{3}{5}\right) = 1.5 \times 10^{-5} \text{ T}$$

21. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is :

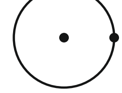
(1) 3.5 Å

(2) 12.9 Å

(3) 9.7 Å

(4) 6.6 Å

Answer (3)



$2\pi r = 3 \times \lambda$

$$\Rightarrow \lambda = \frac{2\pi \times 4.65}{3} \text{ Å} = 9.7 \text{ Å}$$

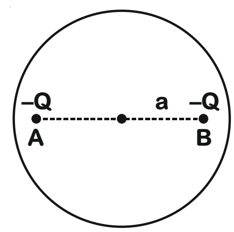
22. Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then :

(1) $a = \frac{3R}{2^{1/4}}$ (2) $a = R/\sqrt{3}$

(3) $a = 2^{-1/4}R$ (4) $a = 8^{-1/4}R$

Answer (4)

Sol. $E \times 4\pi a^2 = \frac{q_{in}}{\varepsilon_0}$



$$q_{in} = \int_0^a (kr) \times 4\pi r^2 dr$$

$$= 4\pi k \cdot \left(\frac{a^4}{4}\right)$$

$$\therefore \quad E \times 4\pi a^2 = \frac{4\pi k a^4}{4\varepsilon_0} \Rightarrow E = \frac{ka^2}{4\varepsilon_0}$$

$$\therefore \quad \frac{ka^2}{4\varepsilon_0} \times Q = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2}$$

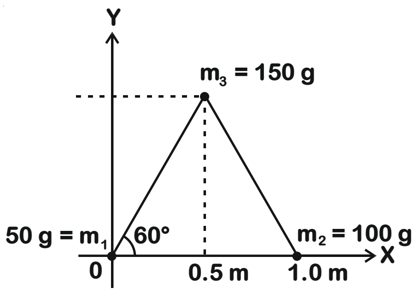
$$\frac{2Q}{\pi R^4} \cdot \frac{a^2 Q}{4\varepsilon_0} = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2} \Rightarrow 8a^4 = R^4$$

$$\Rightarrow \quad 8^{1/4} \cdot a = R$$

$$\Rightarrow \quad a = \frac{R}{8^{1/4}}$$

As, $2Q = \frac{4\pi k R^2}{4} \Rightarrow k = \frac{2Q}{\pi R^4}$

triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :

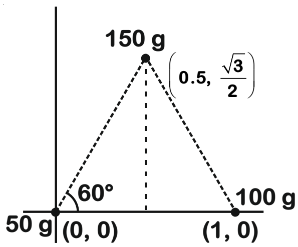


(1) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$ (2) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$

(3) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$ (4) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$

Answer (4)

Sol.



$$X_{CM} = \frac{m\times 0 + 2m\times 1 + 3m\times\left(\frac{1}{2}\right)}{6\text{ m}} = \frac{7}{12}\text{ m}$$

$$Y_{CM} = \frac{m\times 0 + 2m\times 0 + 3m\times\left(\frac{\sqrt{3}}{2}\right)}{6\text{ m}} = \frac{3\sqrt{3}}{12}\text{ m}$$

$$= \frac{\sqrt{3}}{4}\text{ m}$$

24. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1) 9 : 8 (2) 3 : 8

(3) 8 : 1 (4) 1 : 8

Answer (1)

Sol. $N_A = \dfrac{N_0}{2^{\left(\frac{60}{20}\right)}} = \dfrac{N_0}{64}$

$$N_B = \frac{N_0}{2^{\left(\frac{60}{20}\right)}} = \frac{N_0}{8}$$

$\therefore$ Required ratio = $\dfrac{\quad}{N_0 - \dfrac{N_0}{8}}$

$$= \frac{63\times 8}{64\times 7} = \frac{9}{8}$$

25. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, l_1 = 30 cm and l_2 = 70 cm. Then, v is equal to

(1) 332 ms^{-1} (2) 384 ms^{-1}

(3) 379 ms^{-1} (4) 338 ms^{-1}

Answer (2)

Sol. l_1 = 30 cm, l_2 = 70 cm

$$\therefore \frac{\lambda}{2} = (l_2 - l_1) = 40\text{ cm}$$

$$\Rightarrow \lambda = 80\text{ cm}$$

$$\therefore U = \nu\lambda = 480 \times (0.8)\text{ m/s}$$

$$= 384\text{ m/s}$$

26. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to

(1) $n_0\alpha^{-3/4}$ (2) $\sqrt{n_0}\,\alpha^{1/2}$

(3) $n_0\alpha^{1/4}$ (4) $n_0\alpha^{-3}$

Answer (1)

Sol. $n = n_0 e^{-\alpha r^4}$

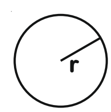
$$\Rightarrow \int dN = \int n_0 e^{-\alpha r^4} \times 4\pi r^2 dr$$

$$\Rightarrow N = 4\pi n_0 \cdot \int_0^\infty r^2 e^{-\alpha r^4} dr$$

Put $\sqrt{\alpha}\, r^2 = t$

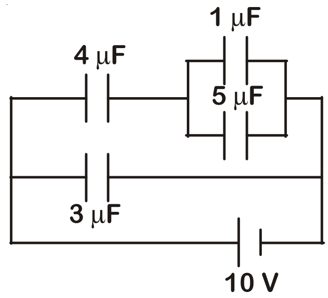
$2\sqrt{\alpha}\, r\, dr = dt$

$$N = \frac{4\pi n_0}{2\sqrt{\alpha}} \int_0^\infty \frac{t^{\frac{1}{2}} e^{-t^2}}{\alpha^{\frac{1}{4}}} dt$$



$$= \frac{4\pi n_0}{2\alpha^{\frac{3}{4}}} \int t^{\frac{1}{2}} e^{-t^2} dt$$

$$N \propto n_0 \alpha^{-\frac{3}{4}}$$

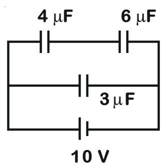


(1) 5.4 μC (2) 9.6 μC

(3) 13.4 μC (4) 24 μC

Answer (4)

Sol.



$$Q_{(4\,\mu F)} = \left(\frac{4\times6}{4+6}\right)\times 10\,\mu C$$

$= 24\ \mu C$

28. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?

(1) $R^2 = 4\,h_1h_2$ (2) $R^2 = 16\,h_1h_2$

(3) $R^2 = 2\,h_1h_2$ (4) $R^2 = h_1h_2$

Answer (2)

Sol. At complementary angles, ranges are equal.

$$\therefore\quad h_1 = \frac{u^2\sin^2\theta}{2g}, h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$\therefore\quad h_1\times h_2 = \left(\frac{2u^2\sin\theta\cos\theta}{g}\right)^2\times\left(\frac{1}{16}\right)$$

$\Rightarrow\ 16h_1h_2 = R^2$

What would be the heat energy absorbed by the gas, in this process?

(1) 30 J (2) 35 J

(3) 25 J (4) 40 J

Answer (2)

Sol. W = $nR\Delta T$ = 10 J

$\Delta Q = (\Delta Q)_P = nC_P\Delta T$

$$\therefore\quad \Delta Q = n\times\frac{7}{2}R\times\Delta T$$

$$= \frac{7}{2}\times(10)$$

$= 35$ J

30. A plane electromagnetic wave having a frequency ν = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

(1) $\vec{\ } = 0\sin(0.5\times10^3x+1.5\times10^{11}t)\hat{k}$

(2) $\vec{B} = 2\times10^7\sin(0.5\times10^3z+1.5\times10^{11}t)\hat{i}$

(3) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^3z-1.5\times10^{11}t)\hat{i}$

(4) $\vec{B} = 2\times10^{-7}\sin(1.5\times10^2x+0.5\times10^{11}t)\hat{j}$

Answer (3)

Sol. $B_0 = \frac{E_0}{c} = \frac{60}{3\times10^8} = 2\times10^{-7}\,T$

$\nu = 23.9\times10^9$ Hz

$\therefore\ \omega = 2\pi\nu = 2\times3.142\times23.9\times10^9$

$= 1.5\times10^{11}\,S^{-1}$

$$\because\quad c = \frac{\omega}{k}\quad\Rightarrow\quad k = \frac{\omega}{c} = \frac{1.5\times10^{11}}{3\times10^8}$$

$= 0.5\times10^3$

$$\therefore\quad \vec{B} = 2\times10^{-7}\sin\left(0.5\times10^3z-1.5\times10^{11}t\right)\hat{i}$$

PART–B : CHEMISTRY

1. The INCORRECT statement is :

(1) Lithium is least reactive with water among the alkali metals.

(2) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2.

(3) Lithium is the strongest reducing agent among the alkali metals.

(4) LiCl crystallises from aqueous solution as $LiCl.2H_2O$.

Answer (2)

Sol. $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

2. The pair that has similar atomic radii is :

(1) Mo and W (2) Ti and Hf

(3) Sc and Ni (4) Mn and Re

Answer (1)

Due to lanthanoid contraction, radius of Mo and W are almost same.

3. The C–C bond length is maximum in :

(1) graphite (2) C_{60}

(3) diamond (4) C_{70}

Answer (3)

Sol. Carbon-carbon bond length is maximum in diamond

Species	C – C bond length
Diamond	154 pm
Graphite	141.5 pm
C_{60}	138.3 pm and 143.5 pm
	(double bond) (single bond)

4. The decreasing order of electrical conductivity of the following aqueous solutions is:

0.1 M Formic acid (A),

0.1 M Acetic acid (B),

0.1 M Benzoic acid (C),

(1) A > B > C (2) A > C > B

(3) C > B > A (4) C > A > B

Answer (2)

Sol. HCOOH CH_3COOH C_6H_5COOH
(A) (B) (C)

Order of acidic strength

$$\begin{array}{lccccc} & HCOOH & > & C_6H_5COOH & > & CH_3COOH \\ pK_a & 3.8 & & 4.2 & & 4.8 \end{array}$$

More the acidic strength more will be the dissociation of acid into ions and more will be conductivity.

∴ Order of conductivity:

$$HCOOH > C_6H_5COOH > CH_3COOH$$
$$\Rightarrow \quad A > C > B$$

5. The compound used in the treatment of lead poisoning is :

(1) EDTA (2) desferrioxime B

(3) Cis-platin (4) D-penicillamine

Answer (1)

Sol. EDTA is used in the treatment of lead poisoning.

options :

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held π-electrons, the cleavage is difficult because of strong bonding.

(1) Both (A) and (R) are correct statements and (R) is the correct explanation of (A).

(2) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A).

(3) Both (A) and (R) are wrong statements.

(4) (A) is a correct statement but (R) is a wrong statement.

Answer (4)

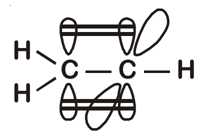
Sol. $CH_2 = CH - Cl$ $\quad CH_2 = \overset{\oplus}{C} - H$ or 

Fig. (1) Fig. (2) Fig. (3)

Also,

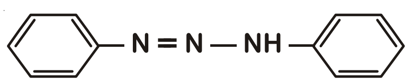
$$CH_2 = CH - \ddot{Cl} \longleftrightarrow \overset{\ominus}{C}H_2 - CH = \overset{+}{Cl}$$

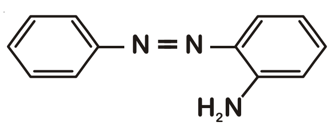
Due to partial double bond character of C – Halogen bond, Halogen leaves with great difficulty, if at all it does. Hence, vinyl halides do not undergo nucleophilic substitution easily.

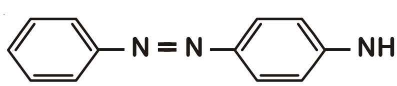
Assertion is correct.

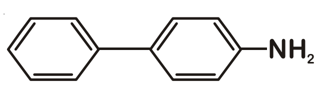
Intermediate carbocation is not stabilised by loosely held π-electrons because empty orbital (see Fig. (3)), being at 90°, cannot overlap with p-orbitals of π bond. Reason is wrong.

7. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

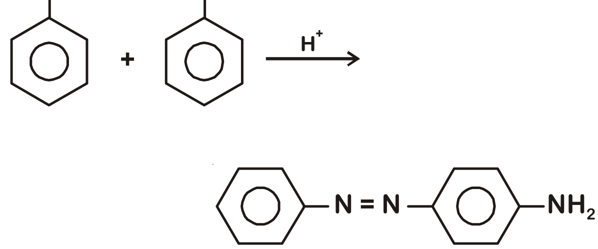
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

p-aminoazobenzene

8. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains :

(1) 22 g of carbon and 3 g of hydrogen

(2) 24 g of carbon and 1 g of hydrogen

(3) 20 g of carbon and 5 g of hydrogen

(4) 18 g of carbon and 7 g of hydrogen

Answer (2)

Sol. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 \quad + \quad \frac{y}{2}H_2O$

(excess)

25 g   88 gm   9 g

2 moles $\frac{1}{2}$ moles

$\Rightarrow x = 2$ $\Rightarrow y = 1$

$\therefore \quad x = 2 \quad$ and $\quad y = 1$

Hydrocarbon : $(C_2H)_n$

2 mol carbon contains 24 g

1 mol hydrogen contains 1 g

9. In which one of the following equilibria, $K_P \neq K_C$?

(1) $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$

(2) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$

(3) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$

(4) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

Answer (4)

Sol. $\because \quad K_P = K_C \cdot (RT)^{\Delta n_g}$

$\therefore \quad$ If $\Delta n_g \neq 0$ then $K_P \neq K_C$

$2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

$\Delta n_g = +1$

$\Rightarrow \quad K_P = K_C \cdot (RT)^1$

(1) They can scatter light.

(2) The range of diameters of colloidal particles is between 1 and 1000 nm.

(3) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

(4) They are larger than small molecules and have high molar mass.

Answer (3)

Sol. The osmotic pressure of a colloidal solution is of lower order than that of true solution at the same concentration due to association of solute molecule till they acquire colloidal dimensions.

$\pi = iCRT$

i is less in colloidal solution than true solution

11. The primary pollutant that leads to photochemical smog is

(1) Nitrogen oxides   (2) Sulphur dioxide

(3) Ozone   (4) Acrolein

Answer (1)

Sol. In photochemical smog :

Primary pollutants $\Rightarrow$ NO_2, hydrocarbons

Secondary pollutants $\Rightarrow$ Ozone, acrolein

12. The INCORRECT match in the following is

(1) $\Delta G° = 0, K = 1$ (2) $\Delta G° < 0, K < 1$

(3) $\Delta G° > 0, K < 1$ (4) $\Delta G° < 0, K > 1$

Answer (2)

Sol. $\Delta G° = -RT \ln K$

$\therefore \quad$ If K > 1 then $\Delta G° < 0$

If K < 1 then $\Delta G° > 0$

If K = 1 then $\Delta G° = 0$

13. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation,

$2N_2O_5(g) \rightarrow 4\,NO_2(g) + O_2(g)$.

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is

(1) 1.667×10^{-2} mol L^{-1} min^{-1}

(2) 4.167×10^{-3} mol L^{-1} min^{-1}

(3) 8.333×10^{-3} mol L^{-1} min^{-1}

(4) 2.083×10^{-3} mol L^{-1} min^{-1}

Answer (1)

$$\text{rate} = \frac{-1}{2}\frac{d[N_2O_5]}{dt} = \frac{1}{4}\frac{d[NO_2]}{dt} = \frac{d[O_2]}{dt}$$

Since, instant of finding rate of formation of NO_2 is not mentioned, hence

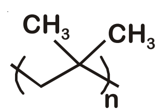
$$\because \quad \frac{-\Delta[N_2O_5]}{\Delta t} = -\frac{(2.75-3)}{30} = \frac{0.25}{30}\ \text{M min}^{-1}$$

$$\therefore \quad \frac{\Delta[NO_2]}{\Delta t} = 2\times\frac{-\Delta[N_2O_5]}{\Delta t}$$

$$= 2\times\frac{0.25}{30}$$

$$= 1.67\times10^{-2}\ \text{M min}^{-1}$$

14. The correct name of the following polymer is

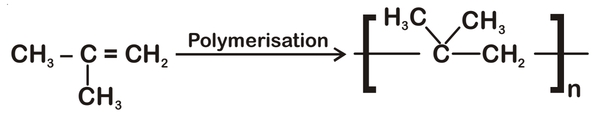


(1) Polyisobutane (2) Polyisoprene

(3) Polyisobutylene (4) Polytert-butylene

Answer (3)

Sol.



iso-butylene Poly isobutylene

15. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27°C. The osmotic pressure of the solution is

($R = 0.08206$ L atm K^{-1} mol^{-1})

(1) 1.64 atm (2) 2.46 atm

(3) 8.2 atm (4) 4.92 atm

Answer (4)

Sol. Osmotic pressure (π) = CRT

Solute : urea and glucose

$$\therefore \quad \pi = (C_1 + C_2)RT$$

$$= \left(\frac{0.6}{60\times0.1}+\frac{1.8}{180\times0.1}\right)\times 0.0821\times 300$$

$$= 0.2\times0.0821\times300$$

$$= 4.926\ \text{atm}$$

centered cubic structure are, respectively

(1) 4 : 2 : 3 (2) 4 : 2 : 1

(3) 8 : 1 : 6 (4) 1 : 2 : 4

Answer (4)

Sol. No. of atoms in

Simple cubic unit cell $= \frac{1}{8}\times 8 = 1$

B.C.C. unit cell $= \frac{1}{8}\times 8 + 1\times 1 = 2$

FCC unit cell $= \frac{1}{8}\times 8 + \frac{1}{2}\times 6 = 4$

17. In comparison to boron, berylium has

(1) Greater nuclear charge and lesser first ionisation enthalpy.

(2) Greater nuclear charge and greater first ionisation enthalpy.

(3) Lesser nuclear charge and greater first ionisation enthalpy.

(4) Lesser nuclear charge and lesser first ionisation enthalpy.

Answer (3)

Sol. Nuclear charge B > Be

Ionisation energy Be > B

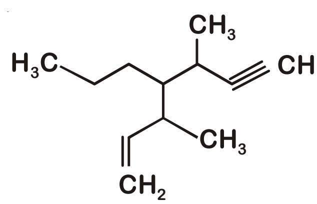
↓

(due to ns^2 outer electronic configuration)

Be = $1s^2\ 2s^2$ (more stable)

B = $1s^2\ 2s^2\ 2p^1$

18. The IUPAC name for the following compound is



(1) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene

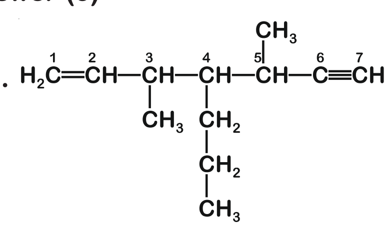
(2) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

(3) 3,5-dimethyl-4-propylhept-1-en-6-yne

(4) 3,5-dimethyl-4-propylhept-6-en-1-yne

Answer (3)

Sol.



3, 5-dimethyl-4-propylhept-1-en-6-yne

a buffer solution of pH = 12 is:

(1) 1.84×10^{-9} M (2) 6.23×10^{-11} M

(3) $\frac{2.49}{1.84} \times 10^{-9}$ M (4) 2.49×10^{-10} M

Answer (4)

Sol. $Cd(OH)_2 \rightleftharpoons \underset{S}{Cd^{2+}} + \underset{2S}{2OH^-}$

At equilibrium, $K_{sp} = S(2S)^2$

$= 4S^3$

$\Rightarrow K_{sp} = 4 \times (1.84 \times 10^{-5})^3$

Solubility in buffer solution having pH = 12

$[OH^-] = 10^{-2}$

$Cd(OH)_2 \rightleftharpoons \underset{S'}{Cd^{2+}} + \underset{2S'+10^{-2} \approx 10^{-2}}{2OH^-}$

$\therefore K_{sp} = 4 \times (1.84 \times 10^{-5})^3 = S'(10^{-2})^2$

$$\Rightarrow S' = \frac{2.49 \times 10^{-15}}{10^{-4}} = 2.49 \times 10^{-10} \text{ M}$$

20. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively are:

(1) $Ca(HCO_3)_2$ and $Ca(OH)_2$

(2) $Mg(HCO_3)_2$ and $Mg(OH)_2$

(3) $Mg(HCO_3)_2$ and $MgCO_3$

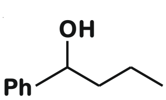
(4) $Ca(HCO_3)_2$ and CaO

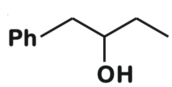
Answer (2)

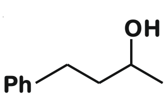
Sol. Temporary hardness is caused by bicarbonates of calcium and magnesium

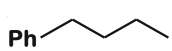
$$\underset{(X)}{Mg(HCO_3)_2(aq)} \xrightarrow{\text{Boiling}} \underset{(Y)}{Mg(OH)_2} \downarrow + 2CO_2 \uparrow$$

21. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is:

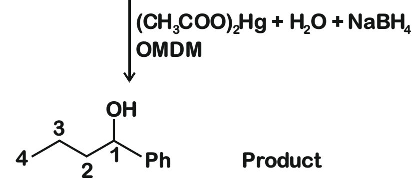
(1) 

(2) 

(3) 

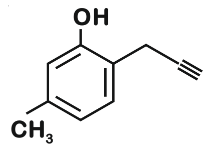
(4) 

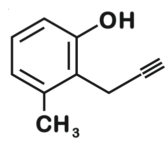
Answer (1)

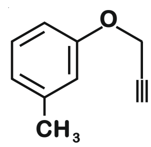


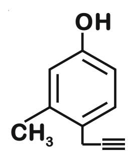
1-phenylbutanol

22. What will be the major product when m-cresol is reacted with propargyl bromide ($HC \equiv C{-}CH_2Br$) in presence of K_2CO_3 in acetone ?

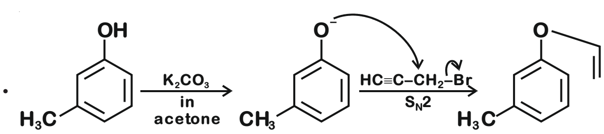
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

Meta cresol is neutralised by K_2CO_3. The phenoxide ion attacks at the C-atom carrying Br-atom to give ether following S_N2 mechanism.

23. Among the following, the energy of 2s orbital is lowest in:

(1) Li (2) K

(3) H (4) Na

Answer (2)

Sol. As the value of Z (atomic number) increases, energy of orbitals decreases (becomes more –ve value)

$\therefore$ order of energy of 2s orbital is

H > Li > Na > K

gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z respectively are:

(1) K_2MnO_4, $KMnO_4$ and Cl_2

(2) K_3MnO_4, K_2MnO_4 and Cl_2

(3) K_2MnO_4, $KMnO_4$ and SO_2

(4) $KMnO_4$, K_2MnO_4 and Cl_2

Answer (4)

Sol. $\underset{(X)}{2KMnO_4} \xrightarrow{513K} \underset{(Y)}{K_2MnO_4} + MnO_2 + \underset{gas}{O_2}$

$$MnO_2 + 4NaCl + \underset{Conc.}{4H_2SO_4} \rightarrow MnCl_2 + 4NaHSO_4 + \underset{\substack{(Z)\\ \text{pungent gas}}}{Cl_2} + 2H_2O$$

25. The correct statement is

(1) Pig iron is obtained from cast iron.

(2) Leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate.

(3) The blistered appearance of copper during the metallurgical process is due to the evolution of CO_2.

(4) The Hall-Heroult process is used for the production of aluminium and iron.

Answer (2)

Sol. During metallurgy of aluminium, when bauxite (powdered ore) is treated with NaOH (conc.), sodium aluminate ($Na[Al(OH)_4]$) is formed and impurity SiO_2 dissolves by forming sodium silicate (Na_2SiO_3)

$$Al_2O_3(s) + 2NaOH(aq) + 3H_2O(l) \rightarrow 2Na[Al(OH)_4](aq)$$

26. Which of the given statements is INCORRECT about glycogen?

(1) It is present in some yeast and fungi.

(2) It is a straight chain polymer similar to amylose.

(3) It is present in animal cells.

(4) Only α-linkages are present in the molecule.

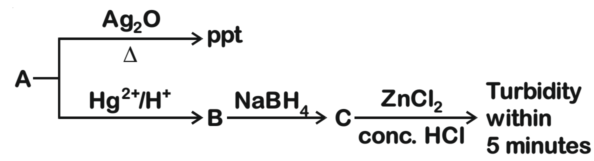
Answer (2)

Sol. Structure of glycogen is similar to amylopectin glycogen

– is stored in animal body

– is found in yeast and fungi

27. Consider the following reactions :



'A' is

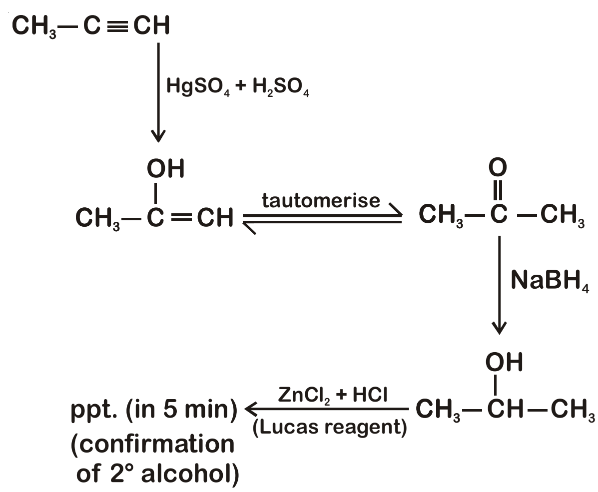
(1) $CH_3 - C \equiv CH$

(2) $CH_3 - C \equiv C - CH_3$

(3) $CH_2 = CH_2$

(4) $CH \equiv CH$

Answer (1)

Sol. 

28. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are (en = ethane-1, 2-diamine)

(1) 3 and 3

(2) 5 and 3

(3) 5 and 6

(4) 6 and 6

Answer (3)

Sol. $[Co(en)_2Cl]Cl$

Cl^- - monodentate ligand

en - bidentate ligand

$\therefore$ Co-ordination No. of Co = (2 × 2) + 1 = 5

$K_3[Al(C_2O_4)_3]$

$C_2O_4^{2-}$- bidentate ligand

$\therefore$ Co-ordination No. of Al = 2 × 3 = 6

(1) $CH_2 = CH - Cl$ (2) $CHCl_3$

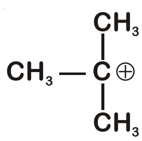
(3) CCl_4 (4) $(CH_3)_3CCl$

Answer (4)

Sol. Carbocation is formed on reaction with Ag^+

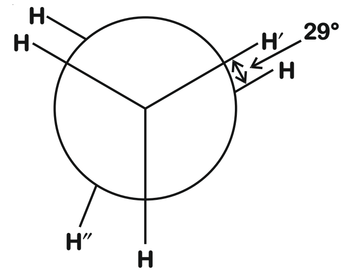
$$R—Cl + Ag^+ \longrightarrow R^+ + \underset{\text{ppt.}}{AgCl\downarrow}$$

The more the stability of R^+, the more R — Cl is likely to give precipitate



is most stable carbocation compared to other options:

$CH_2 = \overset{\oplus}{C}H$, $\overset{\oplus}{C}HCl_2$, $\overset{\oplus}{C}Cl_3$

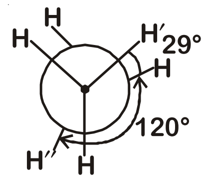


(1) 120° (2) 58°

(3) 151° (4) 149°

Answer (4)

Sol.



$\therefore$ Angle between H′ and H″ = 120° + 29°

= 149°

PART–C : MATHEMATICS

1. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?

(1) $\left(\sqrt{2}, 2\right)$

(2) $\left(2, 2\sqrt{2}\right)$

(3) $\left(1, 2\sqrt{2}\right)$

(4) $\left(2, \sqrt{2}\right)$

Answer (1)

Sol. Let the equation of ellipse : $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\because$ be = 2 and a = 2 (Given)

$\because$ $a^2 = b^2(1 - e^2)$

$\Rightarrow$ $4 = b^2 - 4$

$\Rightarrow$ $b = 2\sqrt{2}$

Equation of ellipse will be $\dfrac{x^2}{4} + \dfrac{y^2}{8} = 1$

Only $\left(\sqrt{2}, 2\right)$ satisfies this equation.

2. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\dfrac{4}{5}$, then the probability that he is unable to solve less than two problems is:

(1) $\dfrac{316}{25}\left(\dfrac{4}{5}\right)^{48}$

(2) $\dfrac{54}{5}\left(\dfrac{4}{5}\right)^{49}$

(3) $\dfrac{201}{5}\left(\dfrac{1}{5}\right)^{49}$

(4) $\dfrac{164}{25}\left(\dfrac{1}{5}\right)^{48}$

Answer (2)

Sol. Let p is the probability that candidate can solve a problem.

$p = \dfrac{4}{5}$; $q = \dfrac{1}{5}$ $(\because\ p + q = 1)$

$$= p^{49}\,[p + 50q]$$

$$= \left(\frac{4}{5}\right)^{49} \cdot \left(\frac{4}{5} + \frac{50}{5}\right)$$

$$= \frac{54}{5} \cdot \left(\frac{4}{5}\right)^{49}$$

3. The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$

(2) $\frac{2}{3}$

(3) 2

(4) 1

Answer (3)

Sol. $f(x) = \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right) = -\tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$

$\because \ \frac{\pi}{4} - x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

So, $f(x) = -\left(\frac{\pi}{4} - x\right) = x - \frac{\pi}{4}$

Let $y = \frac{x}{2}$

$$\frac{d}{dy} f(x) = \frac{\frac{d}{dx} f(x)}{\frac{d}{dx} y} = \frac{1}{1/2} = 2$$

4. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(\frac{5}{2}, 1\right)$ (2) $\left(-\frac{5}{2}, 1\right)$

(3) $\left(\frac{5}{2}, -1\right)$ (4) $\left(-\frac{5}{2}, -1\right)$

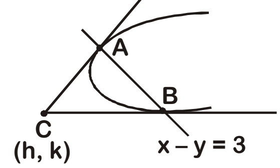
Answer (3)

Sol. Equation of chord is T = 0

$\Rightarrow \frac{1}{2}(y + k) = (x - 2)(h - 2) - 1$

$\Rightarrow \frac{y + k}{2} = xh - 2x - 2h + 3$

Given $x - y - 3 = 0$



$\Rightarrow \frac{2h - 4}{1} = \frac{4h - 6 + k}{3} = 1$

$h = \frac{5}{2}, \ k = -1$

5. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 48 (2) 24

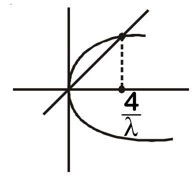
(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

Answer (2)

Sol. $y^2 = 4\lambda x$ and $y = \lambda x$

On solving ; $(\lambda x)^2 = 4\lambda x$

$x = 0, \ \frac{4}{\lambda}$



Required area $= \int_0^{\frac{4}{\lambda}} \left(2\sqrt{\lambda x} - \lambda x\right) dx$

$$= \left.\frac{2\sqrt{\lambda} \cdot x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\right|_0^{4/\lambda}$$

$$= \frac{32}{3\lambda} - \frac{8}{\lambda} = \frac{8}{3\lambda} = \frac{1}{9}$$

$\lambda = 24$

6. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 24 (2) 27

(3) 25 (4) 28

Answer (3)

Sol. Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$= {}^{n+5}C_3 - {}^{n}C_3 - {}^{5}C_3 = 1750$

$\Rightarrow \frac{(n+5)(n+4)(n+3)}{6} - \frac{n(n-1)(n-2)}{6} = 1760$

$\Rightarrow n^2 + 3n - 700 = 0$

$\Rightarrow n = -28$ (rejected) or $n = 25$

this A.P. is :

(1) 150

(2) 280

(3) 200

(4) 120

Answer (3)

Sol. Let the common difference is 'd'.

$a_1 + a_7 + a_{16} = 40$

$\Rightarrow a_1 + a_1 + 6d + a_1 + 15d = 40$

$\Rightarrow 3a_1 + 21d = 40$

$\Rightarrow a_1 + 7d = \dfrac{40}{3}$

$S_{15} = \dfrac{15}{2}[2a_1 + 14d]$

$= 15(a_1 + 7d)$

$= 15\left(\dfrac{40}{3}\right)$

$= 200$

8. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is :

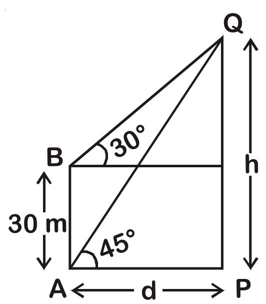
(1) $15(3+\sqrt{3})$ (2) $15(1+\sqrt{3})$

(3) $15(3-\sqrt{3})$ (4) $15(5-\sqrt{3})$

Answer (1)

Sol. Let the height of the tower be h.

Refer to diagram ;



$\tan 45° = \dfrac{h}{d} = 1$

$h = d$ ...(i)

$\tan 30° = \dfrac{h-30}{d}$

$\sqrt{3}(h-30) = d$ ...(ii)

from (i) and (ii)

$\sqrt{3}d = d + 30\sqrt{3}$

$d = \dfrac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$

$[\cot\theta]x + y = 0$

(1) Has a unique solution if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$.

(2) Has a unique solution if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right) \cup \left(\pi, \dfrac{7\pi}{6}\right)$.

(3) Have infinitely many solutions if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$.

(4) Have infinitely many solutions if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right) \cup \left(\pi, \dfrac{7\pi}{6}\right)$.

Answer (3)

Sol. There are two cases.

Case 1 : $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$

So ; $[\sin\theta] = 0$, $[-\cos\theta] = 0$, $[\cot\theta] = -1$

The system of equations will be ;

$0\cdot x + 0\cdot y = 0$ and $-x + y = 0$

(Infinitely many solutions)

Case 2 : $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

So ; $[\sin\theta] = -1$, $[-\cos\theta] = 0$,

The system of equations will be ;

$-x + 0\cdot y = 0$ and $[\cot\theta]x + y = 0$

Clearly $x = 0$ and $y = 0$ (unique solution)

10. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is:}$$

(1) $\dfrac{\pi}{18}$

(2) $\dfrac{7\pi}{36}$

(3) $\dfrac{7\pi}{24}$

(4) $\dfrac{\pi}{9}$

Answer (4)

$$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & (1+4\cos 6\theta) \end{vmatrix} = 0$$

$\Rightarrow$ $2 + 4\cos 6\theta = 0$

$$\cos 6\theta = \frac{-1}{2}$$

$\because$ $6\theta \in (0, 2\pi)$

So, $6\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$\Rightarrow$ $\theta = \frac{\pi}{9}$ or $\frac{2\pi}{9}$

11. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $p \wedge q$ (4) $q \Rightarrow \sim p$

Answer (3)

Sol. $\sim(p \Rightarrow (\sim q))$ $\{\because p \Rightarrow q$ is same as $\sim p \vee q\}$

$\equiv$ $\sim((\sim p) \vee (\sim q))$

$\equiv$ $p \wedge q$

12. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is :

(1) $x + y + 4 = 0$ (2) $2x - y + 2 = 0$

(3) $x - 2y + 16 = 0$ (4) $x - y + 4 = 0$

Answer (4)

Sol. $y^2 = 16x$ and $xy = -4$

Equation of tangent to the given parabola;

$$y = mx + \frac{4}{m}$$

If this is common tangent, then

$$x\left(mx + \frac{4}{m}\right) + 4 = 0$$

$\Rightarrow$ $mx^2 + \frac{4}{m}x + 4 = 0$

$D = 0$

$$\frac{16}{m^2} = 16m$$

$\Rightarrow$ $m^3 = 1$ $\Rightarrow m = 1$

Equation of common tangent is $y = x + 4$

(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 - 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 + 2x^2 + cx^3 = 0$

Answer (1)

Sol. $y^2dx - xydy = x^3dx$

$\Rightarrow$ $\frac{(ydx - xdy)y}{x^2} = xdx$

$\Rightarrow$ $-yd\left(\frac{y}{x}\right) = xdx$

$\Rightarrow$ $-\frac{y}{x}\cdot d\left(\frac{y}{x}\right) = dx$

$\Rightarrow$ $-\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow$ $2x^3 + cx^2 + y^2 = 0$

14. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :

(1) (1, –4, 1) (2) (2, –4, 1)

(3) (1, 4, –1) (4) (2, 4, 1)

Answer (2)

Sol. Equation of angle bisectors;

$$\frac{x+2y+2z-2}{3} = \pm\frac{2x-y+2z-4}{3}$$

$\Rightarrow$ $x - 3y - 2 = 0$ or $3x + y + 4z - 6 = 0$

Only (2, –4, 1) lies on the second plane

15. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha\sin x = 2\alpha - 7$ has a solution. Then S is equal to :

(1) [1, 4] (2) R

(3) [2, 6] (4) [3, 7]

Answer (3)

Sol. $\cos 2x + \alpha\sin x = 2\alpha - 7$

$1 - 2\sin^2 x + \alpha\sin x = 2\alpha - 7$

$\Rightarrow$ $2\sin^2 x - \alpha\sin x + (2\alpha - 8) = 0$

$\Rightarrow$ $\sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$

$\Rightarrow$ $\sin x = \frac{\alpha \pm (\alpha - 8)}{4}$

$\Rightarrow$ $\sin x = 2$ (rejected) or $\sin x = \frac{\alpha - 4}{2}$

$\because$ Equation has solution, then $\frac{\alpha - 4}{2} \in [1, 1]$

$\Rightarrow \alpha \in [2, 6]$

$\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and

$\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set

$S = \{\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$

(1) contains exactly two numbers only one of which is positive

(2) is singleton

(3) contains exactly two positive numbers

(4) is empty

Answer (4)

Sol. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha^2 + 6 = 0$

No value of 'α' exist

Set S is an empty set.

17. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is :

(1) $\frac{1}{3}$

(2) 3

(3) $\frac{1}{\sqrt{3}}$

(4) $\sqrt{3}$

Answer (4)

Sol. Equation of plane containing two given lines;

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$\Rightarrow x - y - z = 0$

The length of perpendicular from (2, 1, 4) to this plane $= \left|\frac{2-1-4}{\sqrt{1^2+1^2+1^2}}\right| = \sqrt{3}$

18. If $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$

A(x) $\cos 2\alpha$ + B(x) $\sin 2\alpha$ + C, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) $x - \alpha$ and $\log_e|\cos(x - \alpha)|$

(2) $x + \alpha$ and $\log_e|\sin(x - \alpha)|$

(3) $x - \alpha$ and $\log_e|\sin(x - \alpha)|$

(4) $x + \alpha$ and $\log_e|\sin(x + \alpha)|$

Answer (3)

Sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx$

$= \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$ let $x - \alpha = t$

$\Rightarrow dx = dt$

$= \int \frac{\sin(t+2\alpha)}{\sin t} dt$

$= [\cos 2\alpha + \sin 2\alpha \cdot \cot t]dt$

$= \cos 2\alpha \cdot t + \sin 2\alpha \cdot \ln|\sin t| + c$

$= (x - \alpha) \cdot \cos 2\alpha + \sin 2\alpha \cdot \ln|\sin(x - \alpha)| + c$

19. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + \dots\dots + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to :

(1) (420, 19)

(2) (380, 19)

(3) (420, 18)

(4) (380, 18)

Answer (3)

Sol. ${}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \dots + 20^2 \cdot {}^{20}C_{20}$

$$= \sum_{r=1}^{20} r^2 \cdot {}^{20}C_r$$

$$= 20\sum_{r=1}^{20} r \cdot {}^{19}C_{r-1}$$

$$= 20\left[\sum_{r=1}^{20}(r-1)\,{}^{19}C_{r-1} + \sum_{r=1}^{20}{}^{19}C_{r-1}\right]$$

$$= 20\left[19\sum_{r=2}^{20}{}^{18}C_{r-2} + 2^{19}\right]$$

$= 20[19 \cdot 2^{18} + 2^{19}]$

$= 20 \times 21 \times 2^{18}$

$= 420 \times 2^{18}$

So; A = 420 and $\beta = 18$

and (2, 3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3}, 2\right)$ (2) $\left(\frac{1}{3}, \frac{5}{3}\right)$

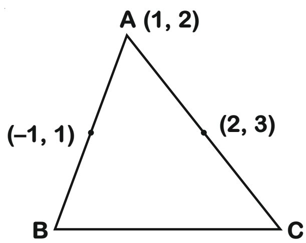
(3) $\left(\frac{1}{3}, 1\right)$ (4) $\left(1, \frac{7}{3}\right)$

Answer (1)

Sol. Co-ordinates of vertex B and C are B(–3, 0) and C(3, 4)

Centroid $G\left(\frac{3-3+1}{3}, \frac{0+4+2}{3}\right)$

$G\left(\frac{1}{3}, 2\right)$



21. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) 0 (2) $\alpha\gamma$

(3) $\beta\gamma$ (4) $\alpha\beta$

Answer (3)

Sol. $\beta^2 = \alpha\gamma$ so roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$

are $\frac{-2\beta \pm 2\sqrt{\beta^2 - \alpha\gamma}}{2\alpha} = -\frac{\beta}{\alpha}$

This root satisfy the equation $x^2 + x - 1 = 0$

$\beta^2 - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \alpha\gamma - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \alpha + \beta = \gamma$

Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma = \alpha\beta + \beta^2$

$= (\alpha + \beta)\beta = \beta\gamma$

22. $\lim\limits_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is :

(1) 3

(2) 6

(3) 1

(4) 2

Answer (4)

$\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}$

$$= \lim_{x\to 0} \frac{(x + 2\sin x)\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{(x^2 - \sin^2 x) + (x + 2\sin x)}$$

$$= \lim_{x\to 0} \frac{\left[1 + 2\left(\frac{\sin x}{x}\right)\right]\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{\left(x - \frac{\sin^2 x}{x}\right) + \left(1 + 2\left(\frac{\sin x}{x}\right)\right)}$$

$$= \frac{3 \times 2}{3} = 2$$

23. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is :

(1) $\left(\sqrt{3} + 1\right)x + \left(\sqrt{3} - 1\right)y = 8\sqrt{2}$

(2) $\left(\sqrt{3} - 1\right)x + \left(\sqrt{3} + 1\right)y = 8\sqrt{2}$

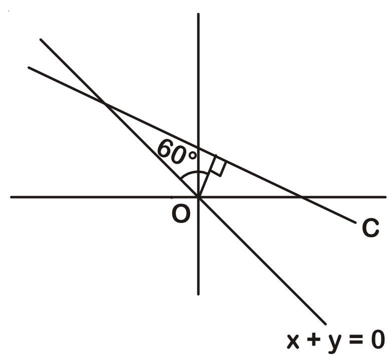
(3) $\sqrt{3}x + y = 8$

(4) $x + \sqrt{3}y = 8$

Answer (1) or (2)

Sol. If perpendicular makes an angle of 60° with the line x + y = 0.

Then the perpendicular makes an angle of 15° or 75° with x-axis. So the equation of line will be



$x\cos 75° + y\sin 75° = 4$ or $x\cos 15° + y\sin 15° = 4$

$\left(\sqrt{3} - 1\right)x + \left(\sqrt{3} + 1\right)y = 8\sqrt{2}$ or

$\left(\sqrt{3} + 1\right)x + \left(\sqrt{3} - 1\right)y = 8\sqrt{2}$

By rotating the normal towards the line x + y = 0 in anticlockwise sense we get the answer (2).

$\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) –108 (2) –36

(3) –72 (4) 36

Answer (2)

Sol. $\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6-\frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$

Coefficient of x° in $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6-\frac{1}{81}$ coefficient of x^{-8} in $\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{-1}{60}\,{}^6C_3\,(2)^3(3)^3+\frac{1}{81}\,{}^6C_5\,(2)(3)^5$

= –72 + 36

= –36

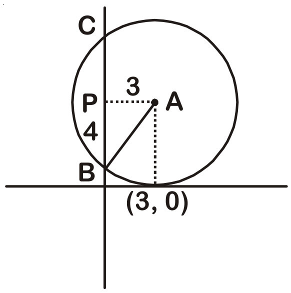
25. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (2, 3) (2) (1, 5)

(3) (3, 5) (4) (3, 10)

Answer (4)

Sol. Let centre of circle is A and circle cuts the y axis at B and C. Let mid point of chord BC is P.



$AB=\sqrt{PA^2+PB^2}$

$= 5 =$ radius of circle

Equation of circle is : $(x – 3)^2 + (y – 5)^2 = 5^2$

Only (3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation $(x – 3)^2 + (y – 5)^2 = 5^2$

two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/ loss (in Rs.) of the person is :

(1) $\frac{1}{2}$ loss (2) 2 gain

(3) $\frac{1}{2}$ gain (4) $\frac{1}{4}$ loss

Answer (1)

Sol. Let X be the random variable which denotes the Rs gained by the person.

$P(X=15)=\frac{6}{36}=\frac{1}{6}$ $\left\{\begin{matrix}\text{Total cases} = 36 \\ \text{favourable cases are} \\ (1,1),(2,2),(3,3),(4,4), \\ (5,5),(6,6)\end{matrix}\right\}$

$P(X=12)=\frac{4}{36}=\frac{1}{9}$ $\left\{\begin{matrix}\text{Favourable cases are} \\ (6,3),(5,4),(4,5),(3,6)\end{matrix}\right\}$

$P(X=-6)=\frac{26}{36}=\frac{13}{18}$

X	15	12	–6
$P(X)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
$X\cdot P(X)$	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{-13}{3}$

$E(X)=\sum X\cdot P(X)=\frac{5}{2}+\frac{4}{3}-\frac{13}{3}=-\frac{1}{2}$

27. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

(1) $B \cap C \neq \phi$

(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A – C) \subseteq B$, then $A \subseteq B$

(4) If $(A – B) \subseteq C$, then $A \subseteq C$

Answer (3)

Sol. $\because$ $A \cap B \subseteq C$ and $A \cap B \neq \phi$

(1) $B \cap C \neq \phi$ is correct

(2) $(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C$ (correct) (becasue $A \cap B \subseteq C$)

(3) If A = C then $A – C = \phi$

Clearly $\phi \subseteq B$ but $A \subseteq B$ is not always true.

(4) $\because$ $A – B \subseteq C$ and $A \cap B \subseteq C$ so $A \subseteq C$ (correct)

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right) \text{ is :}$$

(1) $\frac{1}{2}$ (2) -2

(3) $-\frac{1}{2}$ (4) 2

Answer (2)

Sol. $$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \int_{\alpha}^{\alpha+1}\left[\frac{1}{x+\alpha} - \frac{1}{x+\alpha+1}\right]dx$$

$$= \ln\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg|_{\alpha}^{\alpha+1}$$

$$= \ln\left(\frac{2\alpha+1}{2\alpha+2}\cdot\frac{2\alpha+1}{2\alpha}\right) = \ln\frac{9}{8}$$

So, $\frac{(2\alpha+1)^2}{\alpha(\alpha+1)} = \frac{9}{2}$

$\Rightarrow$ $8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$

$\Rightarrow$ $\alpha^2 + \alpha - 2 = 0$

$\Rightarrow$ $\alpha = 1, -2$

29. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then :

(1) n = 20 and Re(z) = 10

(2) n = 20 and Re(z) = –10

(3) n = 40 and Re(z) = –10

(4) n = 40 and Re(z) = 10

Answer (3)

Sol. Let $z = x + 10i$

$2z - n = (2i - 1)(2z + n)$

$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$

Comparing real and imaginary part

$-(2x + n) - 40 = 2x - n$ and $20 = 4x + 2n - 20$

$\Rightarrow$ $4x = -40$ $\quad$ $40 = -40 + 2n$

$\Rightarrow$ $x = -10$ $\quad$ $n = 40$

$\Rightarrow$ $\text{Re}(z) = -10$

minimum value at β, then

$$\lim_{x\to-\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8} \text{ is equal to :}$$

(1) 1/2

(2) –1/2

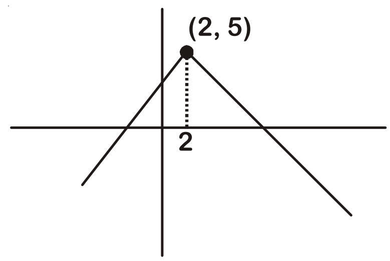
(3) –3/2

(4) 3/2

Answer (1)

Sol. $f(x) = 5 - |x - 2|$

Graph of $y = f(x)$

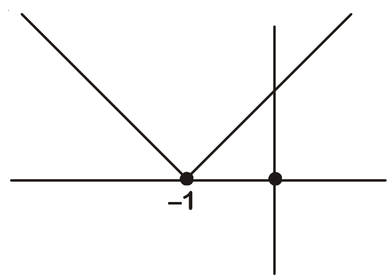


f(x) is maximum at x = 2

$\alpha = 2$

$g(x) = |x + 1|$

Graph of $y = g(x)$



g(x) is minimum at x = –1

$\beta = -1$

$$\lim_{x\to2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \lim_{x\to2}\frac{(x-1)(x-3)}{x-4}$$

$$= \frac{1}{2}$$