JEE Advanced 2024 Question Paper

Joint Entrance Examination – Advanced

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Date: 26/05/2024

Time : 3 hrs. **Answers & Solutions** Max. Marks: 180

for

JEE (Advanced)-2024 (Paper-1)

PART-I : MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. Let $f(x)$ be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and

$$\lim_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$$

for each $x > 0$. Then, for all $x > 0$, $f(x)$ is equal to

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$

(B) $\frac{9}{11x}+\frac{13}{11}x^{10}$

(C) $\frac{-9}{11x}+\frac{31}{11}x^{10}$

(D) $\frac{13}{11x}+\frac{9}{11}x^{10}$

Answer (B)

Sol. $\lim\limits_{t\to x}\dfrac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

$\lim\limits_{t\to x}\dfrac{10t^9f(x)-f'(t)x^{10}}{9t^8}=1$

$\Rightarrow\ 10x^9f(x)-f'(x)x^{10}=9x^8$

$\Rightarrow\ f'(x)-\dfrac{10}{x}f(x)=-\dfrac{9}{x^2}$

IF = $e^{-\int\frac{10}{x}dx}=\dfrac{1}{x^{10}}$

$\therefore$ Sol^n

$\dfrac{y}{x^{10}}=\int-\dfrac{9}{x^{10}}\times\dfrac{1}{x^2}dx$

$=-9\int x^{-12}dx$

$\dfrac{y}{x^{10}}=\dfrac{9}{11}x^{-11}+C$

$\because\ y(1)=2\Rightarrow C=\dfrac{13}{11}$

$\Rightarrow\ y=\dfrac{9}{11x}+\dfrac{13}{11}x^{10}$

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\dfrac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\dfrac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is

(A) $\dfrac{1}{12}$ (B) $\dfrac{1}{7}$

(C) $\dfrac{5}{7}$ (D) $\dfrac{5}{12}$

Answer (C)

Sol. Let P(knows answer) = k

P(guesses) = 1 – k

$$P\left(\frac{\text{correct ans}}{\text{guessed}}\right)=\frac{1}{2}$$

$$P\left(\frac{\text{guessed}}{\text{correct answer}}\right)=\frac{P(\text{guessed})\,P\left(\frac{\text{correct ans}}{\text{guessed}}\right)}{P(\text{guessed})\,P\left(\frac{\text{correct ans}}{\text{guessed}}\right)+P(\text{knows})\,P\left(\frac{\text{correct ans}}{\text{knows}}\right)}$$

$$=\frac{(1-k)\left(\frac{1}{2}\right)}{(1-k)\left(\frac{1}{2}\right)+k(1)}=\frac{1}{6}$$

$$\Rightarrow\ (3-3k)=\frac{1}{2}+\frac{k}{2}$$

$$\Rightarrow\ \frac{5}{2}=\frac{7k}{2}\Rightarrow k=\frac{5}{7}$$

3. Let $\frac{\pi}{2}<x<\pi$ be such that $\cot x=\frac{-5}{\sqrt{11}}$. Then $\left(\sin\frac{11x}{2}\right)(\sin 6x-\cos 6x)+\left(\cos\frac{11x}{2}\right)(\sin 6x+\cos 6x)$ is equal to

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$

(B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$

(C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$

(D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

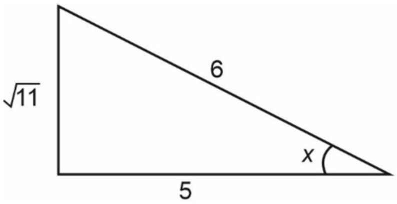
Answer (B)

Sol. Let $E=\sin 6x\cos\frac{11x}{2}-\cos 6x\sin\frac{11x}{2}+\cos 6x\cos\frac{11x}{2}+\sin 6x\sin\frac{11x}{2}$

$$E=\sin\frac{x}{2}+\cos\frac{x}{2}$$

Now, $E^2 = 1 + \sin x$ $\qquad\qquad \because \cot x=\frac{-5}{\sqrt{11}}$

$= 1 + \frac{\sqrt{11}}{6}$



$\therefore \quad E = \sqrt{\frac{6+\sqrt{11}}{6}}$

$= \sqrt{\frac{12+2\sqrt{11}}{12}}$

$= \frac{\sqrt{11}+1}{2\sqrt{3}}$

4. Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let $S(p, q)$ be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct?

(A) $q = 2, p = 3\sqrt{3}$

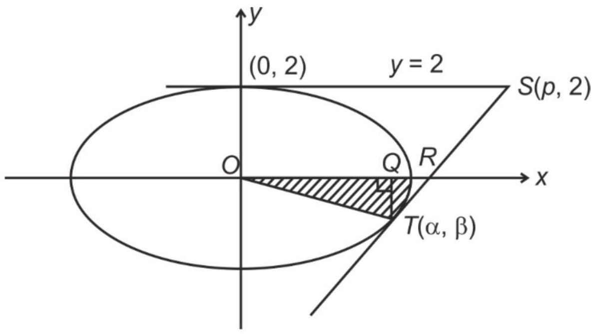
(B) $q = 2, p = 4\sqrt{3}$

(C) $q = 1, p = 5\sqrt{3}$

(D) $q = 1, p = 6\sqrt{3}$

Answer (A)

Sol.



$q = 2$

Area $(ORT) = \frac{3}{2}$

$\Rightarrow \left|\frac{1}{2} \times OR \times QT\right| = \frac{3}{2}$

$\Rightarrow \left|\frac{1}{2} \times 3 \times \beta\right| = \frac{3}{2}$

$\Rightarrow \beta = -1$

$\therefore \frac{\alpha^2}{9} + \frac{\beta^2}{4} = 1$

$\Rightarrow \frac{\alpha^2}{9} = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \alpha^2 = \frac{27}{4} \Rightarrow \alpha = \frac{3\sqrt{3}}{2}$

Tangent at T

$T = 0$

$$\left.\frac{x \cdot \frac{3\sqrt{3}}{2}}{9} + \frac{y(-1)}{4} = 1\right|_{(p,\,2)}$$

$\Rightarrow \frac{p\sqrt{3}}{6} - \frac{1}{2} = 1 \Rightarrow \frac{p\sqrt{3}}{6} = \frac{3}{2} \Rightarrow p = 3\sqrt{3}$

$\therefore p = 3\sqrt{3}, q = 2$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

5. Let $S=\{a+b\sqrt{2}: a,b\in\mathbb{Z}\}, T_1=\left\{\left(-1+\sqrt{2}\right)^n : n\in\mathbb{N}\right\}$ and $T_2=\left\{\left(1+\sqrt{2}\right)^n : n\in\mathbb{N}\right\}$. Then which of the following statements is (are) TRUE?

(A) $\mathbb{Z}\cup T_1\cup T_2\subset S$

(B) $T_1\cap\left(0,\dfrac{1}{2024}\right)=\phi$, where ϕ denotes the empty set

(C) $T_2\cap(2024,\infty)\neq\phi$

(D) For any given a, $b\in\mathbb{Z}$, $\cos\left(\pi\left(a+b\sqrt{2}\right)\right)+i\sin\left(\pi\left(a+b\sqrt{2}\right)\right)\in\mathbb{Z}$ if and only if $b = 0$, where $i=\sqrt{-1}$

Answer (A, C, D)

Sol. $S=\{a+b\sqrt{2}: a,b\in Z\}$

For $b = 0$; $Z\subset S$

$T_1=\left\{\left(-1+\sqrt{2}\right)^n : n\in N\right\}$ and $T_2=\left\{\left(1+\sqrt{2}\right)^n : n\in N\right\}$

For $n \in N$ elements of T_1 and T_2 are of the form $a+b\sqrt{2}$

Hence $Z \cup T_1 \cup T_2 \subset S$

- Now, $-1+\sqrt{2}<1$ and its higher powers decreases

$\Rightarrow (-1+\sqrt{2})^n < 1$ and can be made in $\left(0, \frac{1}{2024}\right)$ for some higher n.

- $1+\sqrt{2}>1$ and its higher power increases

$\Rightarrow \left(1+\sqrt{2}\right)^n$ can be made in (2024, ∞) for some higher n.

- $\cos\pi(a+b\sqrt{2})+i\sin\pi(a+b\sqrt{2}) \in Z$ if

$a+b\sqrt{2}$ is an integer $\Rightarrow b = 0$

6. Let $\mathbb{R}^2$ denote $\mathbb{R}\times\mathbb{R}$. Let

$S = \{(a, b, c) : a, b, c \in \mathbb{R}$ and $ax^2 + 2bxy + cy^2 > 0$ for all $(x, y) \in \mathbb{R}^2 - \{(0, 0)\}\}$.

Then which of the following statements is (are) TRUE?

(A) $\left(2, \frac{7}{2}, 6\right) \in S$

(B) If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$

(C) For any given $(a, b, c) \in S$, the system of linear equations $\begin{matrix} ax+by=1 \\ bx+cy=-1 \end{matrix}$ has a unique solution.

(D) For any given $(a, b, c) \in S$, the system of linear equations $\begin{matrix} (a+1)x+by=0 \\ bx+(c+1)y=0 \end{matrix}$ has a unique solution.

Answer (B, C, D)

Sol. $ax^2 + 2bxy + cy^2 > 0$

$y, x \in \mathbb{R} - \{(0, 0)\}$

$\Rightarrow c\left(\frac{y}{x}\right)^2 + 2b\left(\frac{y}{x}\right) + a > 0$

$\Rightarrow c > 0, D < 0$

$4b^2 - 4ac < 0$

$\Rightarrow b^2 < ac$

(A) $\left(2, \frac{7}{2}, 6\right)$

$\left(\frac{7}{2}\right)^2 > 2 \times 6$

$\therefore$ option A is incorrect

(B) If $\left(3, b, \frac{1}{12}\right) \in S$

$\Rightarrow b^2 < 3 \cdot \frac{1}{12}$

$\Rightarrow b^2 < \frac{1}{4}$

$\Rightarrow 4b^2 < 1$

$\Rightarrow |2b| < 1$ option B is correct

(C) $ax + by = 1$

$bx + cy = -1$

$D = \begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 \neq 0$

$\therefore$ unique solution option C is correct.

(D) $(a + 1)x + by = 0$

$bx + (c + 1)y = 0$

$D = \begin{vmatrix} (a+1) & b \\ b & (c+1) \end{vmatrix}$

$= (a + 1)(c + 1) - b^2$

$\Rightarrow ac - b^2 + a + c + 1$

$b^2 < ac \Rightarrow ac$ is +ve

$\Rightarrow a$ and c are positive then $(ac - b^2) + a + c + 1 > 0$ $\therefore$ unique solution

$\therefore$ option D is correct.

7. Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$. Let $dist(X, Y)$ denote the distance between two points X and Y in $\mathbb{R}^3$. Let

$S = \left\{X \in \mathbb{R}^3 : (dist(X, P))^2 - (dist(X, Q))^2 = 50\right\}$ and

$T = \left\{Y \in \mathbb{R}^3 : (dist(Y, Q))^2 - (dist(Y, P))^2 = 50\right\}$.

Then which of the following statements is (are) TRUE?

(A) There is a triangle whose area is 1 and all of whose vertices are from *S*.

(B) There are two distinct points *L* and *M* in *T* such that each point on the line segment *LM* is also in *T*.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from *S* and the other two vertices are from *T*.

(D) There is a square of perimeter 48, two of whose vertices are from *S* and the other two vertices are from *T*.

Answer (A, B, C)

Sol. $S : \{((x-1)^2 + (y-2)^2 + (z-3)^2) - ((x-4)^2 + (y-2)^2 + (z-7)^2) = 50\}$

$\Rightarrow \; S : \{6x + 8z - 105 = 0\}$

Similarly $T = \{6x + 8z - 5 = 0\}$

S represents a plane. So it will contain a triangle of area 1. So (A) is correct.

T represents a plane. So (B) is correct.

S & *T* are two parallel planes at a distance of 10 units from each other.

$\therefore$ (C) is correct and (D) is incorrect.

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

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8. Let $a = 3\sqrt{2}$ and $b = \dfrac{1}{5^{\frac{1}{6}}\sqrt{6}}$. If $x, y \in \mathbb{R}$ are such that

$3x + 2y = \log_a (18)^{\frac{5}{4}}$ and

$2x - y = \log_b \left(\sqrt{1080}\right)$,

then $4x + 5y$ is equal to ________.

Answer (8)

Sol. $a = 3\sqrt{2} \Rightarrow a^2 = 18$

Notice that $1080 = 5 \cdot 6^3 \Rightarrow$

$5^{\frac{1}{6}} \cdot 6^{\frac{1}{2}} = (1080)^{\frac{1}{6}} = \dfrac{1}{b} \Rightarrow 1080^{\frac{1}{2}} = \dfrac{1}{b^3}$

$\Rightarrow \; 3x + 2y = \log_a \left(a^2\right)^{\frac{5}{4}} = \dfrac{5}{2}$ ...(i)

$2x - y = \log_b \frac{1}{b^3} = \log_b b^{-3} = -3$ ...(ii)

$\Rightarrow$ Solving (i) & (ii)

$\Rightarrow$ $x = \frac{-1}{2}, y = 2 \Rightarrow 4x + 5y = 8$

9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If α_1, α_2, α_3, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ________.

Answer (20)

Sol. $\because$ $f(1) = -9 \Rightarrow 1 + a + b + c = -9$ ...(1)

$4x^3 + 3ax^2 + 2bx = 0$

$\Rightarrow$ $x = 0$, $4x^2 + 3ax + 2b = 0$ ...(2)

$\Rightarrow$ $\sqrt{3}i$ and $-\sqrt{3}i$ are roots of (2)

$\Rightarrow$ $\sqrt{3}i - \sqrt{3}i = \frac{-3a}{4}, \sqrt{3}i\left(-\sqrt{3}i\right) = \frac{2b}{4}$

$\Rightarrow$ $a = 0, b = 6, c = -16$

$\Rightarrow$ $f(x) = 0 \Rightarrow x^4 + 6x^2 - 16 = 0$

$\Rightarrow$ $x^2 = \frac{-6 \pm \sqrt{36 + 64}}{2} = -3 \pm 5 = 2, -8$

$x = -\sqrt{2}, +\sqrt{2}, -2\sqrt{2}i, 2\sqrt{2}i$

$\Rightarrow$ $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$

10. Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$, where $|A|$ denotes the determinant of A. Then the number of elements in S is _______.

Answer (16)

Sol. $|A| = -(e - d) + c(b - a) = \pm 1$

Case (i) : $c = 0 \Rightarrow (e, d) = (1, 0), (0, 1) \rightarrow 2$ ways

b and a can be each 2 ways

$\Rightarrow$ Total = 8 ways

Case (ii): $c \neq 0 \Rightarrow c = 1$

$\Rightarrow$ $d - e + b - a = \pm 1$

$$\left.\begin{matrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}\right\} \rightarrow 4 \times 2 = 8 \text{ ways}$$

Total = 16 ways

11. A group of 9 students, s_1, s_2,…,s_9, is to be divided to form three teams X, Y and, Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is______.

Answer (665)

Sol. Number of required ways

$$= \frac{9!}{2!3!4!} - (n(s_1 \in X) + n(s_2 \in Y) - n(s_1 \in X \text{ and } s_2 \in Y))$$

$$= \frac{9!}{2!3!4!} - \left(\frac{8!}{1!3!4!} + \frac{8!}{2!2!4!} - \frac{7!}{1!2!4!}\right)$$

= 665

12. Let $\overrightarrow{OP} = \frac{\alpha - 1}{\alpha}\hat{i} + \hat{j} + \hat{k}, \overrightarrow{OQ} = \hat{i} + \frac{\beta - 1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right) \cdot \overrightarrow{OR} = 0$ and the point (α, β, 2) lies on the plane $3x + 3y - z + l = 0$, then the value of l is _______.

Answer (5)

Sol. $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right) \cdot \overrightarrow{OR} = 0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0 \Rightarrow \frac{\alpha-1}{\alpha}\left(\frac{\beta-1}{2\beta} - 1\right) - \left(\frac{1}{2} - 1\right) + 1\left(1 - \frac{\beta-1}{\beta}\right) = 0$$

$$\frac{\alpha-1}{\alpha}\left(\frac{-\beta-1}{2\beta}\right) + \frac{1}{2} + \frac{1}{\beta} = 0$$

$$\Rightarrow \frac{\beta+2}{2\beta} = \frac{\alpha\beta + \alpha - \beta - 1}{2\alpha\beta}$$

$\Rightarrow \alpha\beta + 2\alpha = \alpha\beta + \alpha - \beta - 1 \Rightarrow \boxed{\alpha + \beta + 1 = 0}$ ... (1)

Now $(\alpha, \beta, 2)$ lies on $3x + 3y - z + l = 0$

$\Rightarrow 3(\alpha + \beta) - 2 + l = 0$ ... (2)

$\Rightarrow -3 - 2 + l = 0 \Rightarrow l = 5$

13. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x. Suppose that the points $(x, P(X = x))$, $x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy-plane, and $P(X = x) = 0$ for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is _______.

Answer (42)

Sol. $\sum_{x=0}^{4} xP(x) = \frac{5}{2}$

$\sum_{x=0}^{4} x^2P(x) = ?$

$(0, P(0)), (1, P(1)), (2, P(2)), (3, P(3)), (4, P(4))$

$K = P(1) - P(0) = P(2) - P(1) = P(3) - P(2) = P(4) - P(3)$

$P(1) = K + P(0)$

$P(2) = 2K + P(0)$

$P(3) = 3K + P(0)$

$P(4) = 4K + P(0)$

$P(0) + P(1) + P(2) + P(3) + P(4) = 1$

$\Rightarrow \quad 5P(0) + 10K = 1$

$K + P(0) + 4K + 2P(0) + 9K + 3P(0) + 16K + 4P(0) = \frac{5}{2}$

$30K + 10P(0) = \frac{5}{2}$

$\therefore \quad 10K = \frac{1}{2}$

$K = \frac{1}{20}, \ P(0) = \frac{1}{10}$

$P(1) = \frac{3}{20}, \ P(2) = \frac{4}{20}, \ P(3) = \frac{5}{20}, \ P(4) = \frac{6}{20}$

$\sum_{x=0}^{4} x^2P(x) = 8$

$\therefore$ Variance = $8-\frac{25}{4}=\frac{32-25}{4}=\frac{7}{4}$

$\therefore$ $24\alpha=\frac{24\times 7}{4}=42$

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

14. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times 3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$

Match each entry in **List-I** to the correct entry in **List-II**.

	List-I		**List-II**
(P)	The number of matrices $M = (a_{ij})_{3\times 3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times 3}$ with all entries in T such that $C_j = 0$ for all j is	(2)	12
(R)	Let $M = (a_{ij})_{3\times 3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in R, M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}\right\}$ is	(3)	Infinite
(S)	Let $M = (a_{ij})_{3\times 3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (5) (S) → (1)

(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (5)

(C) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

(D) (P) → (1) (Q) → (5) (R) → (3) (S) → (4)

Answer (C)

Sol. $x^2 + x - 1 = 0 \rightarrow$ roots are α and β

$\alpha + \beta = -1 \qquad \alpha\beta = -1$

Set $T = \{1, \alpha, \beta\}$ $M = (a_{ij})_{3 \times 3}$

$R_i = a_{i1} + a_{i2} + a_{i3} \quad C_j = a_{1j} + a_{2j} + a_{3j}$

(P) $R_i = C_j = 0$ for all i, j

$\alpha + \beta = -1 \qquad T = \{1, \alpha, \beta\}$

Number of matrices

$= \underline{|3} \times 2 \times 1 = 12$

Number of ways to arrange $1, \alpha, \beta$ in R_1 (for $\underline{|3}$)

Number of ways to arrange $1, \alpha, \beta$ in R_2 (for 2)

$$\begin{bmatrix} 1 & \alpha & \beta \\ & - & - \\ & & \end{bmatrix}$$

(Q) Number of symmetric matrices = ?

$C_j = 0 \ \forall \ j$

Number of symmetric matrices

$$= \underline{|3} \times 1 = 6 \quad \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

(R) $M \rightarrow$ skew symmetric of 3 × 3

$|M| = 0 \ \ a_{ij} \in T$ for $i > j$

$$M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}$$

$$\begin{bmatrix} 0 & -a_{21} & -a_{31} \\ a_{21} & 0 & -a_{32} \\ a_{31} & a_{32} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{12} \\ 0 \\ -a_{23} \end{bmatrix}$$

As $x, y, z \in R$ and a_{12} & $a_{23} \in R$

& $|M| = 0$

$\therefore$ System has infinite solutions

(S) $R_i = 0 \ \forall\ i$

$$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

$$C_1 \to C_1 + C_2 + C_3 \ |M| = \begin{vmatrix} 1+\alpha+\beta & \alpha & \beta \\ 1+\alpha+\beta & \beta & 1 \\ 1+\alpha+\beta & 1 & \alpha \end{vmatrix} = 0$$

(P) → (2) (Q) → (4) (R) → (3) (S) → (5)

15. Let the straight line $y = 2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

	List-I		**List-II**
(P)	α equals	(1)	(–2, 4)
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	(–2, 6)
(S)	B_1 equals	(4)	5
		(5)	(2, 4)

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

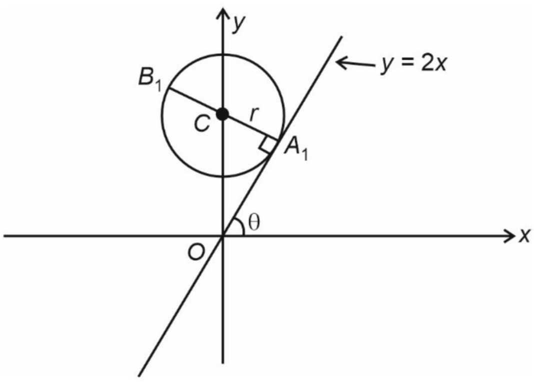
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (3)

(C) (P) → (4) (Q) → (2) (R) → (5) (S) → (3)

(D) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

Answer (C)

Sol.



Slope of line = 2 ⇒ $\tan\theta = 2$

$C(0, \alpha)$ $\alpha > 0$

$\alpha + r = 5 + \sqrt{5}$ ...(1)

Line $y = 2x$ is tangent to the circle

$\therefore \quad \left|\frac{0-\alpha}{\sqrt{4+1}}\right| = r$

$\Rightarrow \quad |-\alpha| = r\sqrt{5}$

$\Rightarrow \quad \alpha = r\sqrt{5}$ as $\alpha > 0$

From equation (1) $r\sqrt{5} + r = 5 + \sqrt{5}$

$\Rightarrow \quad r\left(\sqrt{5}+1\right) = \sqrt{5}\left(\sqrt{5}+1\right)$

$\Rightarrow \quad r = \sqrt{5}$

And $\alpha = r\sqrt{5} = \sqrt{5} \times \sqrt{5} = 5$

Centre $C(0, 5)$

$OC = 5$ $\qquad A_1C = \sqrt{5}$

$\therefore \quad OA_1 = \sqrt{25-5} = \sqrt{20} = 2\sqrt{5}$

$\tan\theta = 2$ (from figure)

$\cos\theta = \frac{1}{\sqrt{5}} \qquad \sin\theta = \frac{2}{\sqrt{5}}$

$A_1(0 + OA_1 \cos\theta, 0 + OA_1 \sin\theta)$

$A_1\left(2\sqrt{5} \times \frac{1}{\sqrt{5}}, 2\sqrt{5} \times \frac{2}{\sqrt{5}}\right)$

$A_1(2, 4)$

Let $B_1(x_1, y_1)$

$\therefore \quad \frac{x_1+2}{2} = 0$ and $\frac{y_1+4}{2} = 5$

$x_1 = -2 \qquad y_1 = 6$

$B_1(-2, 6)$

$\alpha = 5 \qquad r = \sqrt{5} \qquad A_1(2, 4) \qquad B_1(-2, 6)$

16. Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let O = (0, 0, 0), and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.

Match each entry in **List-I** to the correct entry in **List-II**.

	List-I		**List-II**
(P)	γ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\sqrt{\frac{3}{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is

(A) (P) → (3) (Q) → (4) (R) → (1) (S) → (2)

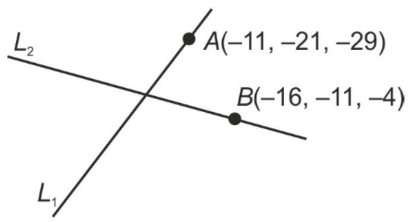
(B) (P) → (5) (Q) → (4) (R) → (1) (S) → (2)

(C) (P) → (3) (Q) → (4) (R) → (1) (S) → (5)

(D) (P) → (3) (Q) → (1) (R) → (4) (S) → (5)

Answer (C)

Sol. Vector parallel to the line L_1 (say $\vec{b}_1$) = $= \hat{i} + 2\hat{i} + 3\hat{k}$



Normal vector of plane $(\vec{n})$ containing L_1 and L_2 will be perpendicular to both $\vec{b}_1$ and $\overrightarrow{AB}$

$\Rightarrow \quad \vec{n} = p(\overrightarrow{AB} \times \vec{n}) = p(5\hat{i} - 10\hat{j} - 25\hat{k}) \times (i + 2\hat{j} + 3\hat{k})$

$= p(20\hat{i} - 40\hat{j} + 20\hat{k})$

$\Rightarrow \quad \hat{n} = \frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

Now, vector parallel to L_2 (say $\vec{b}_2$) is perpendicular to $\vec{n} \Rightarrow \vec{b}_2 \cdot \vec{n} = 0$

$(3\hat{i} + 2\hat{j} + \gamma\hat{k}) \cdot p(20\hat{i} - 40\hat{j} + 20\hat{k}) = 0$

$\Rightarrow \quad \gamma = 1$

Now, for point of intersection (*POI*)

$$L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = \lambda \text{ and } L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma} = u$$

Comparing x and y coordinates, $-11 + \lambda = -16 + 3u$ and $-21 + 2\lambda = -11 + 2u$

$\Rightarrow \quad \lambda = 10,\ u = 5$

$\Rightarrow \quad POI$ *i.e.*, $\overrightarrow{OR_1} : (-\hat{i} - \hat{j} + \hat{k})$ and $\overrightarrow{OR} \cdot \hat{n} = \sqrt{\frac{2}{3}}$

17. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x\,|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1-2x, & 0 \le x \le \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}$$

Let $a, b, c, d \in \mathbb{R}$. Define the function $h : \mathbb{R} \to \mathbb{R}$ by

$$h(x) = af(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c(x - g(x)) + d\,g(x),\ x \in \mathbb{R}$$

Match each entry in **List-I** to the correct entry in **List-II.**

List-I		**List-II**	
(P)	If $a = 0$, $b = 1$, $c = 0$ and $d = 0$, then	(1)	h is one-one
(Q)	If $a = 1$, $b = 0$, $c = 0$ and $d = 0$, then	(2)	h is onto.
(R)	If $a = 0$, $b = 0$, $c = 1$ and $d = 0$, then	(3)	h is differentiable on $\mathbb{R}$.
(S)	If $a = 0$, $b = 0$, $c = 0$ and $d = 1$, then	(4)	the range of h is [0, 1]
		(5)	the range of h is {0, 1}

The correct option is

(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)

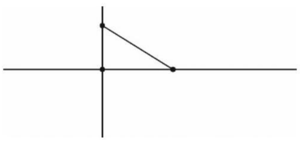
(B) (P) → (5) (Q) → (2) (R) → (4) (S) → (3)

(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(D) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

Answer (C)

Sol. $g(x) = \begin{cases} 1-2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$



$$g\left(\frac{1}{2}-x\right)=\begin{cases}1-2\left(\frac{1}{2}-x\right), & 0\le \frac{1}{2}-x\le \frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$$

$$=\begin{cases}2x, & 0\le x\le \frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$$

$$g(x)+g\left(\frac{1}{2}-x\right)=\begin{cases}1, & 0\le x\le \frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$$

Now, option (P), at $b = 1$

$h(x)=g(x)+g\left(\frac{1}{2}-x\right)$ has range {0, 1}

(P) $\rightarrow$ (5)

Option (Q) at $a = 1$, $h(x) = f(x)$

$$f'(0^+)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2\sin\frac{1}{h}}{h}$$

$$f'(0^-)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}h\sin h=0$$

$$\Rightarrow f'(0^+)=f'(0^-)$$

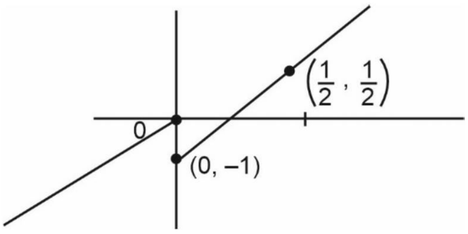
as $f'(0^+) = f'(0^-)$

$\Rightarrow h(x) = f(x)$ is differentiable at $x = 0$ and all other points $f(x) = h(x)$ is differentiable as product of two differentiable functions

(Q) $\rightarrow$ (3)

Option (R)

$$h(x) = x - g(x) = \begin{cases}3x-1, & 0\le x\le \frac{1}{2}\\ x, & \text{otherwise}\end{cases}$$



by graph $h(x)$ has range R and onto (R) $\rightarrow$ (2)

(S) at $d = 1$ $h(x) = g(x)$ has range [0, 1] (S) $\rightarrow$ (4)

PART-II : PHYSICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

1. A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$ is given by

(A) $(2n, -n, -n, -n)$ (B) $(n, -n, -2n, -n)$

(C) $(n, -n, -n, -2n)$ (D) $(2n, -n, -2n, -2n)$

Answer (A)

Sol. $[AT]^{\alpha}[M^{-1}L^{-3}T^{4}A^{2}]^{\beta}[ML^{2}T^{-1}]^{\gamma}[LT^{-1}]^{\delta} = 0$

$\Rightarrow \alpha + 2\beta = 0$

$-\beta + \gamma = 0$ $\Rightarrow \alpha = -2\beta$

$-3\beta + 2\gamma + \delta = 0$ $\gamma = \beta$

$\alpha + 4\beta - \gamma - \delta = 0$ $\delta = \beta$

$(-2\beta, \beta, \beta, \beta)$

2. An infinitely long wire, located on the z-axis, carries a current I along the $+z$-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B}\cdot\vec{dl}$ along a straight line from the point $\left(-\sqrt{3}a, a, 0\right)$ to $(a, a, 0)$ is given by

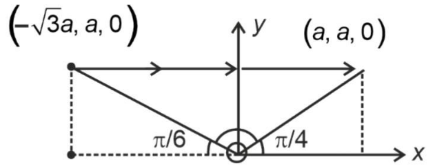
[μ_0 is the magnetic permeability of free space.]

(A) $7\mu_0 I / 24$ (B) $7\mu_0 I / 12$

(C) $\mu_0 I / 8$ (D) $\mu_0 I / 6$

Answer (A)

Sol.



$$\theta = \pi - \frac{\pi}{4} - \frac{\pi}{6}$$

$$\Rightarrow \quad \theta = \frac{12\pi - 3\pi - 2\pi}{12} = \frac{7\pi}{12}$$

So, $\int \vec{B} \cdot \vec{dl}$ along the line is

$$\int \vec{B} \cdot \vec{dl} = -\frac{\mu_0(I)}{2\pi} \cdot \theta = \frac{\mu_0 I}{2\pi} \cdot \frac{7\pi}{12}$$

$$\Rightarrow \quad \left|\int \vec{B} \cdot \vec{dl}\right| = \frac{7\mu_0 I}{24}$$

Option (A) is correct Answer.

3. Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by

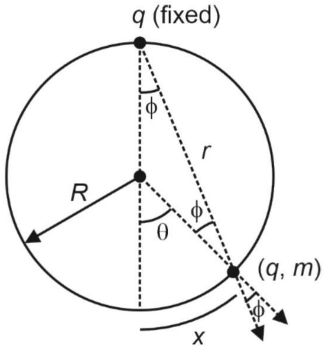
[ε_0 is the permittivity of free space.]

(A) $q^2 / (4\pi\varepsilon_0 R^3 m)$ (B) $q^2 / (32\pi\varepsilon_0 R^3 m)$

(C) $q^2 / (8\pi\varepsilon_0 R^3 m)$ (D) $q^2 / (16\pi\varepsilon_0 R^3 m)$

Answer (B)

Sol. As the hoop mass is not given so it must not move or else its inertia must have some effect.



Here $r = 2R\cos\phi$

Also $\theta = 2\phi$

$\Rightarrow \quad \theta = \dfrac{\phi}{2}$

And $\theta = \dfrac{x}{R}$

If θ is the small angular displacement of free charge, then $F(\phi) = \dfrac{Kq^2}{r^2}$

So, restoring force towards mean position is $F_{(R)} = \dfrac{Kq^2}{r^2}\sin\phi$

$$a_R = \frac{F_{(R)}}{m} = \frac{-Kq^2}{mr^2}\cdot\sin\phi = \frac{-Kq^2\cdot\sin\phi}{m\,4R^2\cos^2\phi}$$

$$\Rightarrow a_R = -\frac{Kq^2}{4mR^2\cos^2\left(\dfrac{\phi}{2}\right)}\cdot\sin\left(\frac{\theta}{2}\right) \approx \frac{-Kq^2}{4mR^2}\frac{1}{2}\cdot\frac{x}{R} = \omega^2\cdot x$$

So, $\boxed{\omega^2 = \dfrac{q^2}{32\pi\varepsilon_0 mR^3}}$

Option (B) is correct answer.

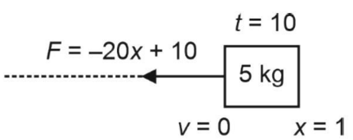
4. A block of mass 5 kg moves along the x-direction subject to the force $F = (-20x + 10)$ N, with the value of x in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = (\pi/4)$ s are

(A) –0.5 m, 5 kg m/s (B) 0.5 m, 0 kg m/s

(C) 0.5 m, –5 kg m/s (D) –1 m, 5 kg m/s

Answer (C)

Sol.



Now,

$$a = -\frac{20x + 10}{5} = -4x + 2$$

$a = \frac{vdv}{dx} = -4x + 2$

Hence $\int_0^v v\,dv = \int_1^x (-4x+2)dx \rightarrow \frac{v^2}{2} = \left[-2x^2 + 2x\right]_1^x$

$\boxed{v = -2\sqrt{x - x^2}}$ (as particle starts moving in –ve x-direction)

$\Rightarrow \frac{dx}{dt} = -2\sqrt{x - x^2} \rightarrow \int_{x=1}^{x=x} \frac{dx}{\sqrt{x - x^2}} = -2\int_0^{\pi/4} dt$

$\sin^{-1}[2x - 1]_1^x = -\frac{\pi}{2}$

$\boxed{x = \frac{1}{2} = 0.5 \text{ m}}$

Also, momentum $= mV = 5(V)_{x=\frac{1}{2}} = -5$ kg-m/s

Hence correct option is (C)

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

5. A particle of mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$, corresponding to the potential energy $V(r) = kr^2/2$, where k is a positive force constant and r is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, h is the Planck's constant, and n a positive integer. If v and E are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct?

(A) $r^2 = n\hbar\sqrt{\frac{1}{mk}}$

(B) $v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

(C) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$

(D) $E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}$

Answer (A, B, C)

Sol. $L = mvr = n\hbar$, also, $\frac{mv^2}{r} = kr$

$mv^2 = kr^2$

$m^2v^2 = mkr^2$

$mv = \sqrt{mkr^2}$

$mvr = r^2\sqrt{mk}$

$n\hbar = r^2\sqrt{mk}$

$$\boxed{\frac{n\hbar}{\sqrt{mk}} = r^2}$$

Option (A) is correct

Also, $v^2 = \frac{kr^2}{m}$

$= \frac{n\hbar}{\sqrt{mk}} \cdot \frac{k}{m}$

$$\boxed{v^2 = n\hbar\sqrt{\frac{k}{m^3}}}$$

Option (B) is correct

Now,

T.E = $E = \frac{1}{2}kr^2 + \frac{1}{2}kr^2$

$E = kr^2$

$= k\dfrac{nh}{\sqrt{mk}}$

$= nh\sqrt{\dfrac{k}{m}}$

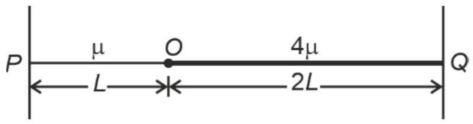
Option (D) is incorrect

$\Rightarrow \quad \dfrac{L}{mr^2} = \dfrac{h\sqrt{mk}}{mnh}$

$$\boxed{\dfrac{L}{mr^2} = \sqrt{\dfrac{k}{m}}}$$

Option (C) is correct

6. Two uniform strings of mass per unit length μ and 4μ, and length L and $2L$, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $v_0 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$, which of the following statement(s) is(are) correct?



(A) With a node at O, the minimum frequency of vibration of the composite string is v_0

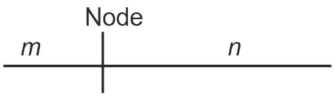
(B) With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$

(C) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes

(D) No vibrational mode with an antinode at O is possible for the composite string

Answer (A, C, D)

Sol. With node at O



$v = \sqrt{\dfrac{T}{\mu}} \qquad v' = \sqrt{\dfrac{T}{4\mu}} = \dfrac{1}{2}v$

$$\Rightarrow \quad m\frac{1}{2l}\sqrt{\frac{T}{\mu}} = n\frac{1}{2(2l)}\sqrt{\frac{T}{4\mu}}$$

$$\Rightarrow \quad m = \frac{n}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{1}{4}$$

$\therefore \quad m = 1, n = 4$

With antinode at O

$$m\frac{1}{4l}\sqrt{\frac{T}{\mu}} = n\frac{1}{4(2l)}\sqrt{\frac{T}{4\mu}}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{1}{4}$$

$\therefore \quad m = 1 \quad f_{\text{min}} = 1\frac{1}{4l}\sqrt{\frac{T}{\mu}} = \frac{v_0}{2}$

(B is wrong)

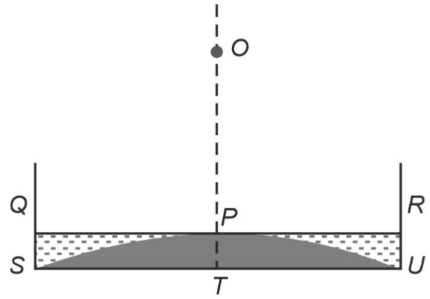
Also, when node at O.

Total nodes = 6

(C is correct)

A, C, D are correct

7. A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level *QPR*. If the image of a point object *O* at a height of h (*OT* in the figure) is formed onto itself, then, which of the following option(s) is (are) correct?



(A) For $n = 1.42$, $h = 50$ cm

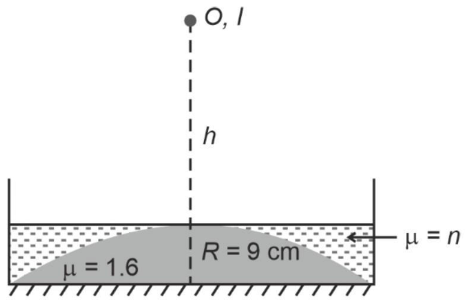
(B) For $n = 1.35$, $h = 36$ cm

(C) For $n = 1.45$, $h = 65$ cm

(D) For $n = 1.48$, $h = 85$ cm

Answer (A, B)

Sol.



For image to coincide with object

$-h = 2(f_{net})$

$$\Rightarrow \quad -\frac{1}{f_{net}} = 2\left(\frac{1}{f_{liq}}\right) + 2\left(\frac{1}{f_{lens}}\right) + \left(\frac{-1}{f_{mirror}}\right) \quad \text{...(i)}$$

$$-\frac{1}{f_{net}} = 2\left(\frac{n-1}{-9}\right) + 2\left(\frac{0.6}{9}\right) + \left(-\frac{1}{\infty}\right) \quad \text{...(ii)}$$

From (i) and (ii)

$$h = \frac{9}{(1.6 - n)}$$

For n = 1.42, h = 50 cm (A is correct)

For n = 1.35, h = 36 cm (B is correct)

For n = 1.45, h = 60 cm (C is incorrect)

For n = 1.48, h = 75 cm (D is incorrect)

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If **ONLY** the correct integer is entered;
Zero Marks	:	0	In all other cases.

---

8. The specific heat capacity of a substance is temperature dependent and is given by the formula $C = kT$, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from –73°C to 27 °C is nk, the value of n is ______. [Given: 0 K = –273° C.]

Answer (25000)

Sol. $C = \dfrac{dQ/m}{dT}$

$\Rightarrow dQ = m\cdot C\cdot dT$

$= 1\cdot kTdT$

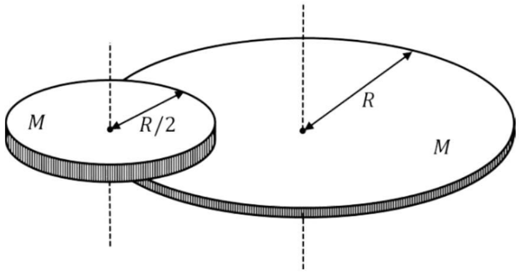
$$Q = \int_{200}^{300} kTdT = \frac{k}{2}\left[300^2 - 200^2\right]$$

$$= \frac{10^4}{2}\cdot k\cdot 5$$

$= 25000k$

$\Rightarrow n = 25000$

9. A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass M and radius $R/2$ is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω. If the angular speed at which the large disc rotates is ω/n, then the value of n is _____.



Answer (12)

Sol. Conserving angular momentum of the system (bigger disc + smaller disc) about the symmetric axis of bigger disc:

$$\frac{MR^2}{2}\omega' + M\cdot R\omega'\cdot R + \frac{M(R/2)^2}{2}\omega = 0$$

Where ω' : required angular speed.

$$\Rightarrow \frac{3}{2}MR^2\omega' = \frac{-MR^2\omega}{8}$$

$$\Rightarrow \omega' = -\frac{\omega}{12}$$

$\Rightarrow$ $n = 12$

10. A point source S emits unpolarized light uniformly in all directions. At two points A and B, the ratio $r = I_A/I_B$ of the intensities of light is 2. If a set of two polaroids having 45° angle between their pass-axes is placed just before point B, then the new value of r will be _____.

Answer (8)

Sol. $I \propto \dfrac{1}{l^2}$

Where l : distance from point source.

$$\Rightarrow \frac{I_A}{I_B} = \frac{l_B^2}{l_A^2} = 2$$

$$\Rightarrow l_B = \sqrt{2}\, l_A \qquad \ldots(1)$$

Also, due to polaroids:

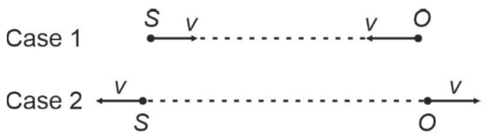
$$I_B' = \frac{I_B}{2}\cos^2 45^\circ = \frac{I_B}{4}$$

$$\Rightarrow \; I_B' = \frac{I_B}{4} \qquad \text{...(2)}$$

$\Rightarrow$ Ratio becomes 4 times.

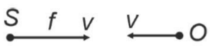
$\Rightarrow$ $r_{new} = 8$

11. A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is ________.



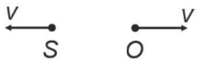
Answer (200)

Sol. For Case 1 :



$$f_{app} = f\left(\frac{c+v}{c-v}\right) \Rightarrow \; 288 = 240\left(\frac{c+v}{c-v}\right) \quad \text{...(i)}$$

For Case 2 :



$$f_{app} = f\left(\frac{c-v}{c+v}\right)$$

$$= 240\left(\frac{c-v}{c+v}\right) \qquad \text{...(ii)}$$

From (i) and (ii)

$$288 \times f_{app} = 240\left(\frac{c+v}{c-v}\right) \times 240\left(\frac{c-v}{c+v}\right)$$

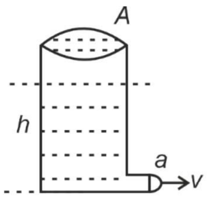
$288\ f_{app} = 240 \times 240$

$f_{app} = 200$ Hz

12. Two large, identical water tanks, 1 and 2, kept on the top of a building of height H, are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2, respectively. If $H = \left(\frac{16}{9}\right)h$, then the ratio t_1/t_2 is _______.

Answer (3)

Sol. In general case



$$a\sqrt{2gh} = -A\frac{dh}{dt}$$

$$dt = -\frac{A}{a}\frac{dh}{\sqrt{2gh}}$$

$$T = \int dt = \frac{-2A}{a\sqrt{2g}}\left(\sqrt{h_f} - \sqrt{h_i}\right)$$

$$T = \frac{2A}{a\sqrt{2g}}(\sqrt{h_i} - \sqrt{h_f})$$

For tank 1 :

$h_i = h$, $h_f = 0$

$$T_1 = \frac{2A}{a\sqrt{2g}}\left(\sqrt{h}\right)$$

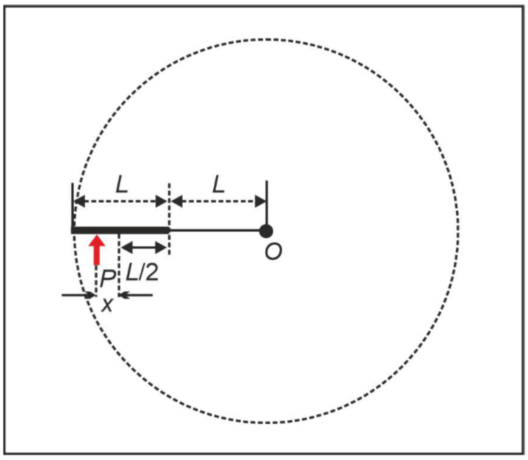
For tank 2 :

$$h_f = \frac{16h}{9},\ h_i = h + H = \frac{25h}{9}$$

$$T_2 = \frac{2A\sqrt{h}}{a\sqrt{2g}}\left(\frac{5}{3} - \frac{4}{3}\right) = \frac{2A\sqrt{h}}{a\sqrt{2g}} \times \frac{1}{3}$$

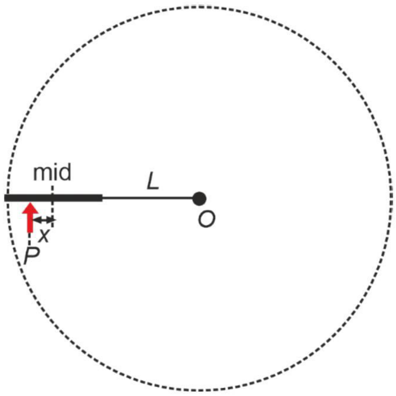
$$\frac{T_1}{T_2} = 3$$

13. A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance $x = L / n$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is ________.



Answer (18)

Sol.



M.I. of rod about centre (O) $= \dfrac{mL^2}{12} + m\left(L + \dfrac{L}{2}\right)^2$

$$I_0 = \frac{7mL^2}{3}$$

Since rod is in pure rotation about 'O'

So, angular impulse $= P\left(x + \frac{3L}{2}\right) = I_0\omega_0$ ...(i)

and linear impulse = mv_c ...(ii)

where $V_c = \frac{3L}{2}\omega_0$ ...(iii)

(20 m (i)) $m\left(\frac{3L}{2}\omega_0\right)\left(x + \frac{3L}{2}\right) = \frac{7}{3}mL^2 \cdot \omega_0$

$x + \frac{3L}{2} = \frac{14}{9}L$

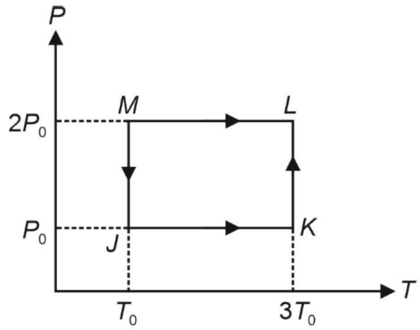
$\boxed{x = \frac{L}{18}}$

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

14. One mole of a monatomic ideal gas undergoes the cyclic process J $\rightarrow$ K $\rightarrow$ L $\rightarrow$ M $\rightarrow$ J, as shown in the P-T diagram.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

[R is the gas constant.]

	List-I		List-II
(P)	Work done in the complete cyclic process	(1)	$RT_0 - 4RT_0 \ln 2$
(Q)	Change in the internal energy of the gas in the process JK	(2)	0
(R)	Heat given to the gas in the process KL	(3)	$3RT_0$
(S)	Change in the internal energy of the gas in the process MJ	(4)	$-2RT_0 \ln 2$
		(5)	$-3RT_0 \ln 2$

(A) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 5; S $\rightarrow$ 4

(B) P $\rightarrow$ 4; Q $\rightarrow$ 3; R $\rightarrow$ 5; S $\rightarrow$ 2

(C) P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 2

(D) P $\rightarrow$ 2; Q $\rightarrow$ 5; R $\rightarrow$ 3; S $\rightarrow$ 4

Answer (B)

Sol. From $M \rightarrow J$, isothermal

$W = nRT \ln 2$

$= RT_0 \ln 2$

$\Delta U = 0$

From $J \rightarrow K$, isobaric

$W = nR(3T_0 - T_0)$

$W = 2RT_0$

$\Delta U = nC_v \Delta T$

$= \frac{3R}{2} \times 2T_0 = 3RT_0$

From $K \rightarrow L$, isothermal

$W = -nR(3T_0)\ln 2$

$W = -3RT_0 \ln 2$

$\Delta U = 0$

$Q = -3RT_0 \ln 2$

From $L \rightarrow M$, isobaric

$W = nR(T_0 - 3T_0)$

$W = -2RT_0$

$\Delta U = nC_v \Delta T$

$= n \times \frac{3R}{2} \times (-2T_0)$

$= -3RT_0$

For P :–

$W_{\text{net}} = RT_0 \ln 2 + 2RT_0 - 3RT_0 \ln 2 - 2RT_0$

$= -2RT_0 \ln 2$

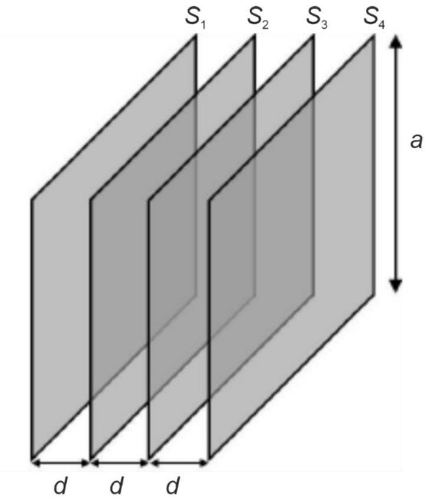
P $\rightarrow$ 4

for Q $\rightarrow$ 3

for R $\rightarrow$ 5

for S $\rightarrow$ 2

15. Four identical thin, square metal sheets, S_1, S_2, S_3, and S_4, each of side a are kept parallel to each other with equal distance d($<<a$) between them, as shown in the figure. Let $C_0 = \varepsilon_0 a^2/d$, where ε_0 is the permittivity of free space.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The capacitance between S_1 and S_4, with S_2 and S_3 not connected, is	(1)	$3C_0$
(Q)	The capacitance between S_1 and S_4, with S_2 shorted to S_3, is	(2)	$C_0/2$
(R)	The capacitance between S_1 and S_3, with S_2 shorted to S_4, is	(3)	$C_0/3$
(S)	The capacitance between S_1 and S_2, with S_3 shorted to S_1, and S_2 shorted to S_4, is	(4)	$2C_0/3$
		(5)	$2C_0$

(A) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 5

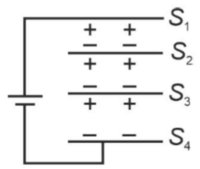
(B) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 1

(C) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1

(D) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 2; S $\rightarrow$ 5

Answer (C)

Sol. For *P*

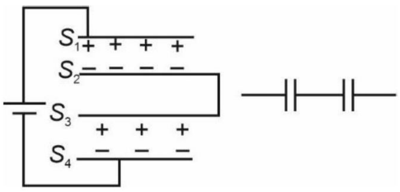


All are in series

$$C_{eq} = \frac{C_0}{3}$$

$P \rightarrow (3)$

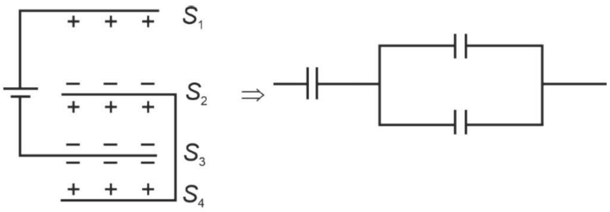
For Q



$$C_{eq} = \frac{C_0}{2}$$

$Q \rightarrow (2)$

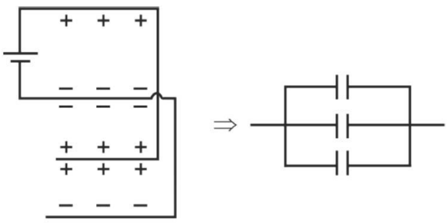
For R



$$C_{eq} = \frac{2C}{3}$$

$R \rightarrow (4)$

For S



$\Rightarrow \quad C_{eq} = 3C_0$

$S \rightarrow (1)$

16. A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0. The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	If n = 2 and α = 180°, then all the possible values of θ_0 will be	(1)	30° and 0°
(Q)	If $n = \sqrt{3}$ and α = 180°, then all the possible values of θ_0 will be	(2)	60° and 0°
(R)	If $n = \sqrt{3}$ and α = 180°, then all the possible values of ϕ_0 will be	(3)	45° and 0°
(S)	If $n = \sqrt{2}$ and θ_0 = 45°, then all the possible values of α will be	(4)	150°
		(5)	0°

(A) P → 5; Q → 2; R→ 1; S→ 4

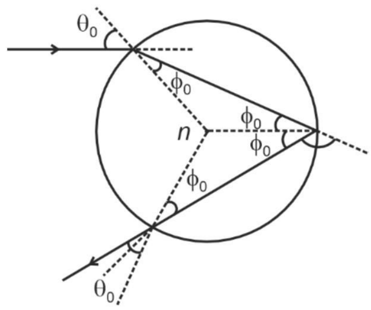
(B) P→5; Q→ 1; R→ 2; S→ 4

(C) P→ 3; Q→ 2; R→ 1; S→ 4

(D) P→ 3; Q→ 1; R→ 2; S→ 5

Answer (A)

Sol.



$\alpha = (\theta_0 - \phi_0) + (\pi - 2\phi_0) + (\theta_0 - \phi_0)$

$\alpha = \pi + 2\theta_0 - 4\phi_0$ ...(i)

$\sin\theta_0 = n\sin\phi_0$ ...(ii)

For (P)

$n = 2$, $\alpha = 180$

if $\alpha = \pi$, $2\theta_0 - 4\phi_0 = 0$

$\theta_0 = 2\phi_0$

$$\sin\theta_0 = 2\sin\left(\frac{\theta_0}{2}\right)$$

P → (5)

For Q, $n = \sqrt{3}$, α = 180

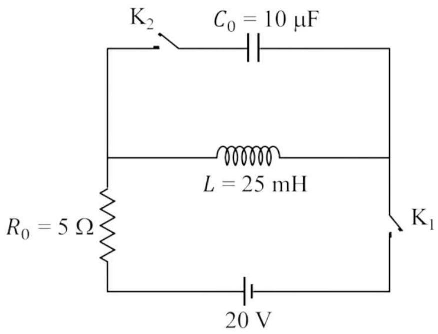
$\theta_0 = 2\phi_0$

Q → (2)

$$\sin\theta_0 = \sqrt{3}\sin\left(\frac{\theta_0}{2}\right)$$

$\theta_0 = 60, 0°$

17. The circuit shown in the figure contains an inductor L, a capacitor C_0, a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2. Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1. After a long time, the current attains a steady state value I_2. Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0.



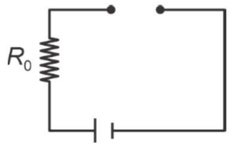
Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The value of I_1 in Ampere is	(1)	0
(Q)	The value of I_2 in Ampere is	(2)	2
(R)	The value of ω_0 in kilo-radians/s is	(3)	4
(S)	The value of V_0 in Volt is	(4)	20
		(5)	200

(A) P → 1; Q → 3; R → 2; S → 5
(B) P → 1; Q → 2; R → 3; S → 5
(C) P → 1; Q → 3; R → 2; S → 4
(D) P → 2; Q → 5; R → 3; S → 4

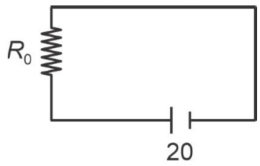
Answer (A)

Sol. Just after closing K_1



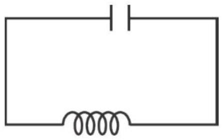
$i_1 = 0$, $P \to (1)$

After long time



$i_2 = \frac{20}{5} = 4A$, Q → (3)

After opening K_1 & closing K_2



$\omega = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-6}}}$

ω = 2000 rad/s

ω = 2 krad/s

$R \to (2)$

$i_0 = 4$

$Q_0 = \frac{i_0}{\omega} = \frac{4}{2 \times 10^3} = 2$ mC

$\therefore\ V_0 = \frac{Q_0}{C} = \frac{2 \times 10^3}{10} = 200$

S → (5)

PART-III : CHEMISTRY

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. A closed vessel contains 10 g of an ideal gas **X** at 300 K, which exerts 2 atm pressure. At the same temperature, 80 g of another ideal gas **Y** is added to it and the pressure becomes 6 atm. The ratio of root mean square velocities of **X** and **Y** at 300 K is

(A) $2\sqrt{2} : \sqrt{3}$ (B) $2\sqrt{2} : 1$

(C) 1 : 2 (D) 2 : 1

Answer (D)

Sol. Given,

$W_X = 10$ g

$P_X = 2$ atm

$W_Y = 80$ g

$P_Y = P_{total} - P_X$

$\Rightarrow$ 6 – 2 = 4 atm

As $V_{rms} = \sqrt{\dfrac{3RT}{M}}$,

$$\frac{(V_{rms})_X}{(V_{rms})_Y} = \sqrt{\frac{M_Y}{M_X}}$$

As we know,

PV = nRT

Volume and temperature remains same.

$$P_X V = \frac{W_X}{M_X} RT$$

$$P_Y V = \frac{W_Y}{M_Y} RT$$

$$M_X \propto \frac{W_X}{P_X}$$

$$M_Y \propto \frac{W_Y}{P_Y}$$

$$\frac{(V_{rms})_X}{(V_{rms})_Y} = \sqrt{\frac{W_Y}{P_Y} \cdot \frac{P_X}{W_X}} = \sqrt{\frac{80}{4} \times \frac{2}{10}} = \sqrt{4} = \frac{2}{1} = 2:1$$

2. At room temperature, disproportionation of an aqueous solution of *in situ* generated nitrous acid (HNO_2) gives the species

(A) H_3O^+, NO_3^- and NO

(B) H_3O^+, NO_3^- and NO_2

(C) H_3O^+, NO^- and NO_2

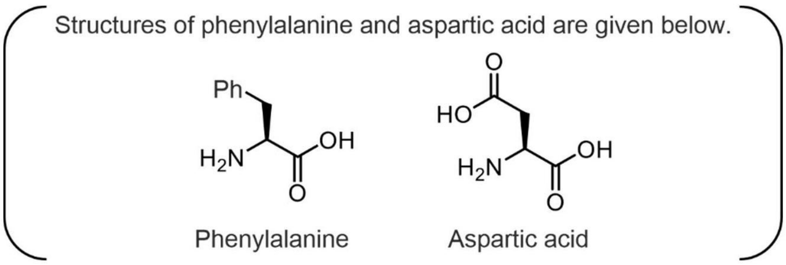
(D) H_3O^+, NO_3^- and N_2O

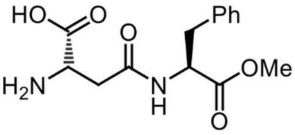
Answer (A)

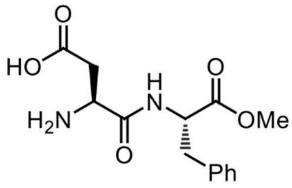
Sol. $\underset{+3}{3HNO_2} \rightleftharpoons \underset{+5}{HNO_3} + \underset{+1}{2NO} + H_2O$

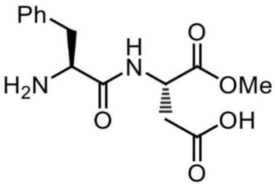
H_3O^+, NO_3^- and NO

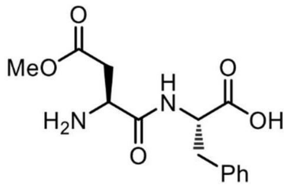
3. Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is



(A) 

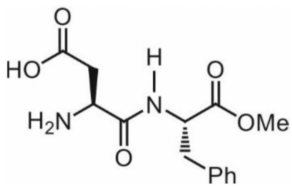
(B) 

(C) 

(D) 

Answer (B)

Sol. Aspartame is



4. Among the following options, select the option in which each complex in **Set-I** shows geometrical isomerism and the two complexes in **Set-II** are ionization isomers of each other.

[en = $H_2NCH_2CH_2NH_2$]

(A) **Set-I:** $[Ni(CO)_4]$ and $[PdCl_2(PPh_3)_2]$

Set-II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(B) **Set-I:** $[Co(en)(NH_3)_2Cl_2]$ and $[PdCl_2(PPh_3)_2]$

Set-II: $[Co(NH_3)_6][Cr(CN)_6]$ and $[Cr(NH_3)_6][Co(CN)_6]$

(C) **Set-I:** $[Co(NH_3)_3(NO_2)_3]$ and $[Co(en)_2Cl_2]$

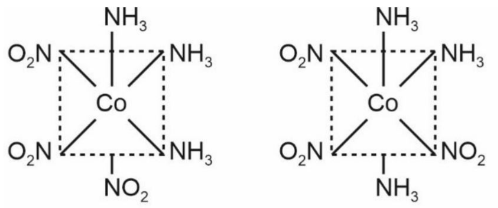
Set-II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(D) **Set-I:** $[Cr(NH_3)_5Cl]Cl_2$ and $[Co(en)(NH_3)_2Cl_2]$

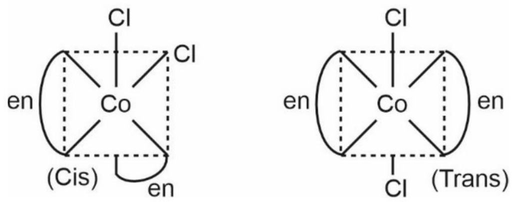
Set-II: $[Cr(H_2O)_6]Cl_3$ and $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$

Answer (C)

Sol. Set-I:



(Facial) (Meridional)



Set-II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5SO_4]Cl$ are ionisation isomers.

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

---

5. Among the following the correct statement(s) for electrons in an atom is(are)

(A) Uncertainty principle rules out the existence of definite paths for electrons.

(B) The energy of an electron in 2*s* orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.

(C) According to Bohr's model, the most negative energy value for an electron is given by n = 1, which corresponds to the most stable orbit.

(D) According to Bohr's model, the magnitude of velocity of electrons increases with increase in values of n.

Answer (A, B, C)

Sol. (A) Uncertainty principle rules out existence of definite paths or trajectories of electron and other similar particles.

So, option (A) is correct.

(B) Shell or orbit more near to nucleus has less energy than faraway.

So, option (B) is also correct.

(C) $E = -13.6 \frac{Z^2}{n^2}$ eV/atom

So, n = 1 has most negative energy.

So, option (C) is also correct.

(D) $V = V_0 \times \frac{Z}{n}$

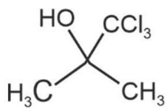
when n increases velocity decreases.

So, option (D) is incorrect.

6. Reaction of *iso*-propylbenzene with O_2 followed by the treatment with H_3O^+ forms phenol and a by-product **P**. Reaction of **P** with 3 equivalents of Cl_2 gives compound **Q**. Treatment of **Q** with $Ca(OH)_2$ produces compound **R** and calcium salt **S**.

The correct statement(s) regarding **P, Q, R** and **S** is(are)

(A) Reaction of **P** with **R** in the presence of KOH followed by acidification gives



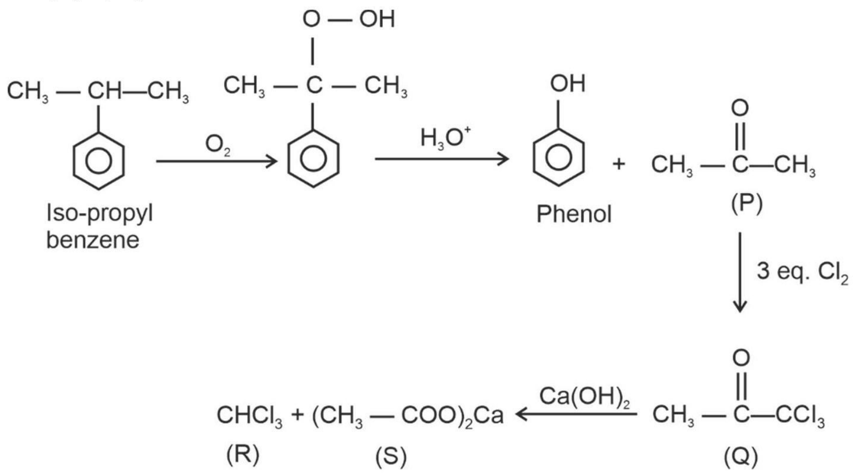
(B) Reaction of **R** with O_2 in the presence of light gives phosgene gas

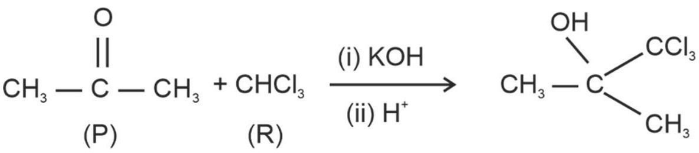
(C) **Q** reacts with aqueous NaOH to produce Cl_3CCH_2OH and $Cl_3CCOONa$

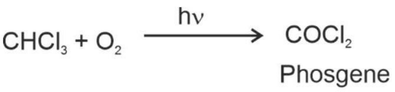
(D) **S** on heating gives **P**

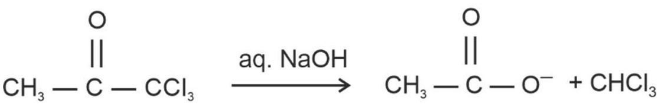
Answer (A, B, D)

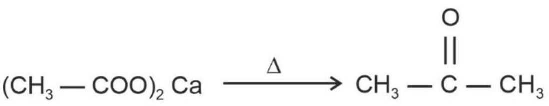
Sol.



(A) 

(B) 

(C) 

(D) 

7. The option(s) in which at least three molecules follow Octet Rule is(are)

(A) CO_2, C_2H_4, NO and HCl

(B) NO_2, O_3, HCl and H_2SO_4

(C) BCl_3, NO, NO_2 and H_2SO_4

(D) CO_2, BCl_3, O_3 and C_2H_4

Answer (A, D)

Sol. (A) CO_2, C_2H_4 and HCl follow octet rule.

(B) O_3 and HCl and follow octet rule.

(C) None of them follow octet rule.

(D) CO_2, O_3 and C_2H_4 follow octet rule.

Correct answer is (A) and (D)

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

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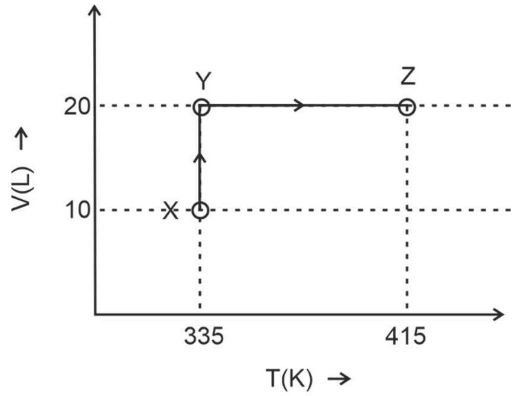
8. Consider the following volume−temperature (V−T) diagram for the expansion of 5 moles of an ideal monoatomic gas.



Considering only P-V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence X → Y→ Z is ______.

[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m}$ = 12 J K^{-1} mol^{-1} and gas constant, R = 8.3 J K^{-1} mol^{-1}]

Answer (8120)

Sol. X → Y is an isothermal process an ideal gas:

$\Delta H = 0$

Y → Z is an isochoric process

$\therefore$ w = 0

$\Delta U = nC_{V,m}(T_2 - T_1)$

$= 5 \times 12 (415 - 335)$

$= 4800$ J

$\Delta H = \Delta U + \Delta(PV)$

$= \Delta U + nR\Delta T$

$= 4800 + 5 \times 8.3 \times (415 - 335)$

$= 8120$ J

9. Consider the following reaction,

$$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$$

which follows the mechanism given below:

$$2NO(g) \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} N_2O_2(g) \quad \text{(fast equilibrium)}$$

$$N_2O_2(g) + H_2(g) \xrightarrow{k_2} N_2O(g) + H_2O(g) \quad \text{(slow reaction)}$$

$$N_2O(g) + H_2(g) \xrightarrow{k_3} N_2(g) + H_2O(g) \quad \text{(fast reaction)}$$

The order of the reaction is_____?

Answer (3)

Sol. Rate of reaction (according to slowest step)

$$\Rightarrow \quad r = k_2[N_2O_2]\,[H_2] \quad ...(1)$$

Now for intermediate $[N_2O_2]$,

$$\frac{k_1}{k_{-1}} = \frac{[N_2O_2]}{[NO]^2}$$

$$\Rightarrow \quad [N_2O_2] = \frac{k_1}{k_{-1}}[NO]^2 \quad ...(2)$$

from equation (1) and (2)

$$r = \frac{k_2k_1}{k_{-1}}[NO]^2[H_2]$$

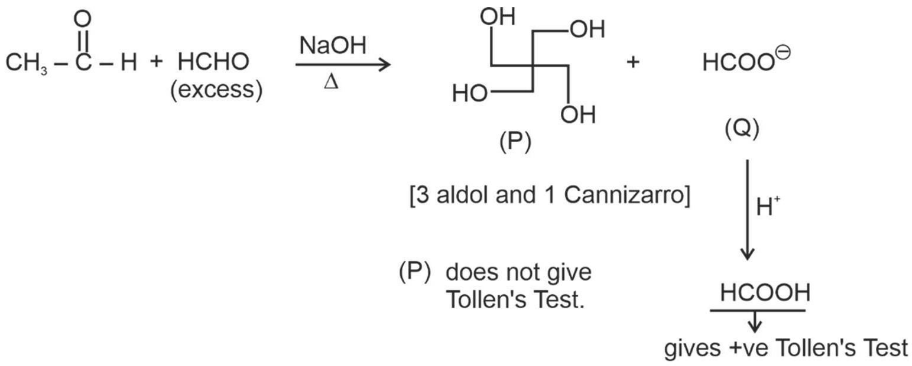
Overall order of reaction = 2 + 1 = 3

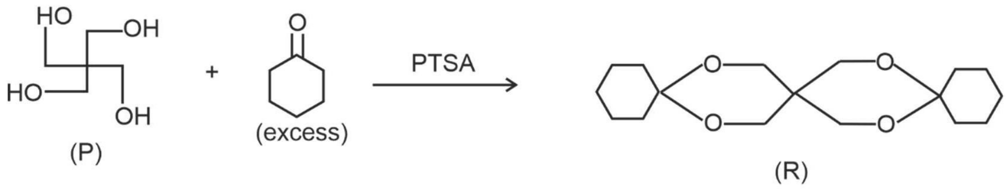
10. Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives **P** and **Q**. Compound **P** does not give Tollens' test, whereas **Q** on acidification gives positive Tollens' test. Treatment of P with excess cyclohexanone in the presence of catalytic amount of p-toluenesulfonic acid (PTSA) gives product **R**.

Sum of the number of methylene groups ($-CH_2-$) and oxygen atoms in **R** is____.

Answer (18)

Sol.





Number of CH_2 groups in R = 14

Number of O-atoms = 4

Required Answer = 14 + 4 = 18

11. Among $V(CO)_6$, $Cr(CO)_5$, $Cu(CO)_3$, $Mn(CO)_5$, $Fe(CO)_5$, $[Co(CO)_3]^{3-}$, $[Cr(CO)_4]^{4-}$, and $Ir(CO)_3$, the total number of species isoelectronic with $Ni(CO)_4$ is _______.

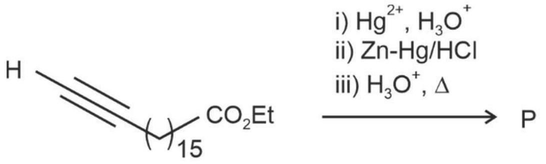
[Given atomic number : V = 23, Cr, = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

Answer (1)

Sol. Total number of electron in $Ni(CO)_4$ = 84

Species		**Total electron**
$V(CO)_6$	–	107
$Cr(CO)_5$	–	94
$Cu(CO)_3$	–	71
$Mn(CO)_5$	–	95
$Fe(CO)_5$	–	96
$[Co(CO)_3]^{3-}$	–	72
$[Cr(CO)_4]^{4-}$	–	84
$Ir(CO)_3$	–	119

12. In the following reaction sequence, the major product **P** is formed.

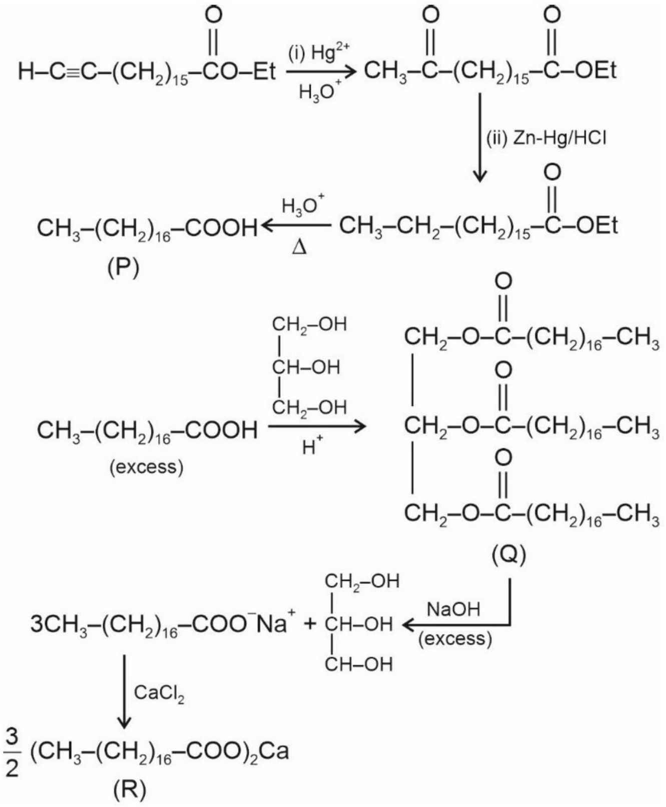


Glycerol reacts completely with excess **P** in the presence of an acid catalyst to form **Q**. Reaction of **Q** with excess NaOH followed by the treatment with $CaCl_2$ yields Ca-soap **R**, quantitatively. Starting with one mole of **Q**, the amount of **R** produced in gram is ______.

[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

Answer (909)

Sol.



1 mole of Q will give 1.5 mole of R.

So, mass of R produced = 606 g × 1.5

= 909 g

13. Among the following complexes, the total number of diamagnetic species is ______.

$[Mn(NH_3)_6]^{3+}$, $[MnCl_6]^{3-}$, $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Fe(NH_3)_6]^{3+}$ and $[Co(en)_3]^{3+}$

[Given, atomic number: Mn = 25, Fe = 26, Co = 27; en = $H_2NCH_2CH_2NH_2$]

Answer (1)

Sol. $[Mn(NH_3)_6]^{3+}$: Paramagnetic

$[MnCl_6]^{3-}$: Paramagnetic

$[FeF_6]^{3-}$: Paramagnetic

$[CoF_6]^{3-}$: Paramagnetic

$[Fe(NH_3)_6]^{3+}$: Paramagnetic

$[Co(en)_3]^{3+}$: Diamagnetic

Only 1 complex is diamagnetic.

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

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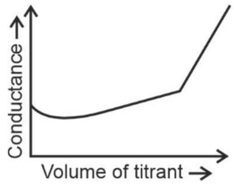
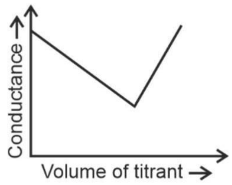
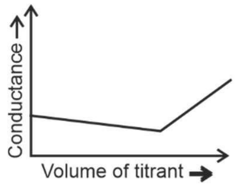
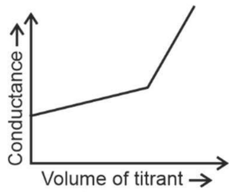
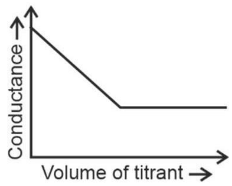
14. In a conductometric titration, small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.

The limiting ionic conductivity (Λ_0) values (in mS m^2 mol^{-1}) for different ions in aqueous solutions are given below:

Ions	Ag^+	K^+	Na^+	H^+	NO_3^-	Cl^-	SO_4^{2-}	OH^-	CH_3COO^-
Λ_0	6.2	7.4	5.0	35.0	7.2	7.6	16.0	19.9	4.1

For different combinations of titrates and titrants given in **List-I**, the graphs of 'conductance' versus 'volume of titrant' are given in **List-II**.

Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		List-II
(P)	Titrate: KCl Titrant: $AgNO_3$	(1)	
(Q)	Titrate: $AgNO_3$ Titrant: KCl	(2)	
(R)	Titrate: NaOH Titrant: HCl	(3)	
(S)	Titrate: NaOH Titrant: CH_3COOH	(4)	
		(5)	

(A) P-4, Q-3, R-2, S-5

(B) P-2, Q-4, R-3, S-1

(C) P-3, Q-4, R-2, S-5

(D) P-4, Q-3, R-2, S-1

Answer (C)

Sol. (P) $KCl + AgNO_3 \longrightarrow AgCl\downarrow + KNO_3$

Cl^- is replaced by NO_3^-

Conductance will first decrease and then after equivalence point, it will increase

$P \longrightarrow 3$

(Q) $AgNO_3 + KCl \longrightarrow AgCl + KNO_3$

Ag^+ is replaced by K^+

Conductance will first increase slightly and then will increase further

(R) $NaOH + HCl \longrightarrow NaCl + H_2O$

OH^- is replaced by Cl^-

(S) $NaOH + CH_3COOH \longrightarrow CH_3CCONa + H_2O$ OH^- is replaced by CH_3COO^- conductance will first decrease and them become almost constant due to buffer formation.

15. Based on VSEPR model, match the xenon compounds given in **List-I** with the corresponding. geometries and the number of lone pairs on xenon given in **List-II** and choose the correct option.

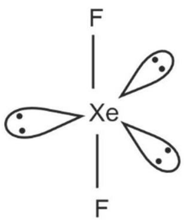
	List-I		**List-II**
(P)	XeF_2	(1)	Trigonal bipyramidal and two lone pair of electrons
(Q)	XeF_4	(2)	Tetrahedral and one lone pair of electrons
(R)	XeO_3	(3)	Octahedral and two lone pair of electrons
(S)	XeO_3F_2	(4)	Trigonal bipyramidal and no lone pair of electrons
		(5)	Trigonal bipyramidal and three lone pair of electrons

(A) P-5, Q-2, R-3, S-1 (B) P-5, Q-3, R-2, S-4

(C) P-4, Q-3, R-2, S-1 (D) P-4, Q-2, R-5, S-3

Answer (B)

Sol. XeF_2 :

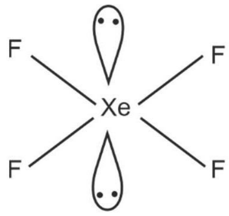


Hybridisation – sp^3d

Geometry:-

Trigonal bipyramidal and three lone pair of electrons

XeF$_4$:

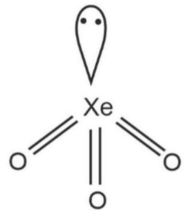


Hybridisation – sp^3d^2

Geometry:-

Octahedral and two lone pair of electrons.

XeO$_3$:

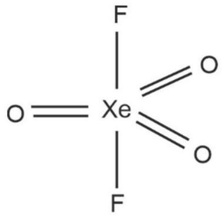


Hybridisation - sp^3

Geometry :-

tatrahedral & one lone pair of electrons.

XeO$_3$F$_2$:



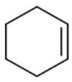
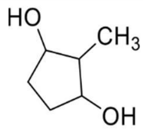
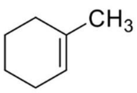
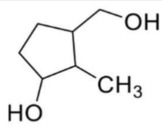
Hybridisation - sp^3d

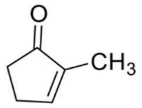
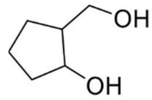
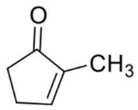
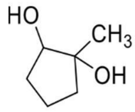
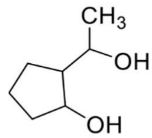
Geometry :-

Trigonal bipyramidal and no lone pair of electrons.

Correct match : P $\rightarrow$ 5; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4

16. **List-I** contains various reaction sequences and **List-II** contains the possible products. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		**List-II**
(P)	 i) O_3, Zn; ii) aq. NaOH, Δ; iii) ethylene glycol, PTSA $\longrightarrow$ iv) a) BH_3, b) H_2O_2, NaOH; v) H_3O^+; vi) $NaBH_4$	(1)	
(Q)	 i) O_3, Zn; ii) aq. NaOH, Δ $\longrightarrow$ iii) ethylene glycol, PTSA; iv) a) BH_3, b) H_2O_2, NaOH; v) H_3O^+; vi) $NaBH_4$	(2)	

(R)	 i) ethylene glycol, PTSA; ii) a) $Hg(OAc)_2$, H_2O, b) $NaBH_4$; iii) H_3O^+; iv) $NaBH_4$	(3)	
(S)	 i) ethylene glycol, PTSA; ii) a) BH_3, b) H_2O_2, NaOH; iii) H_3O^+; iv) $NaBH_4$	(4)	
		(5)	

(A) P-3, Q-5, R-4, S-1

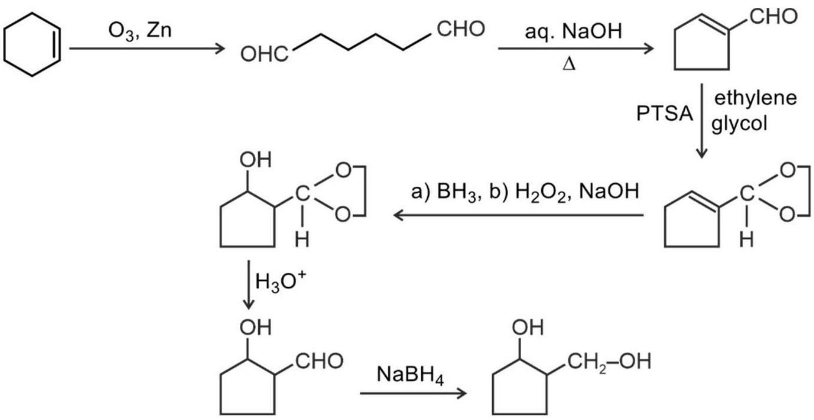
(B) P-3, Q-2, R-4, S-1

(C) P-3, Q-5, R-1, S-4

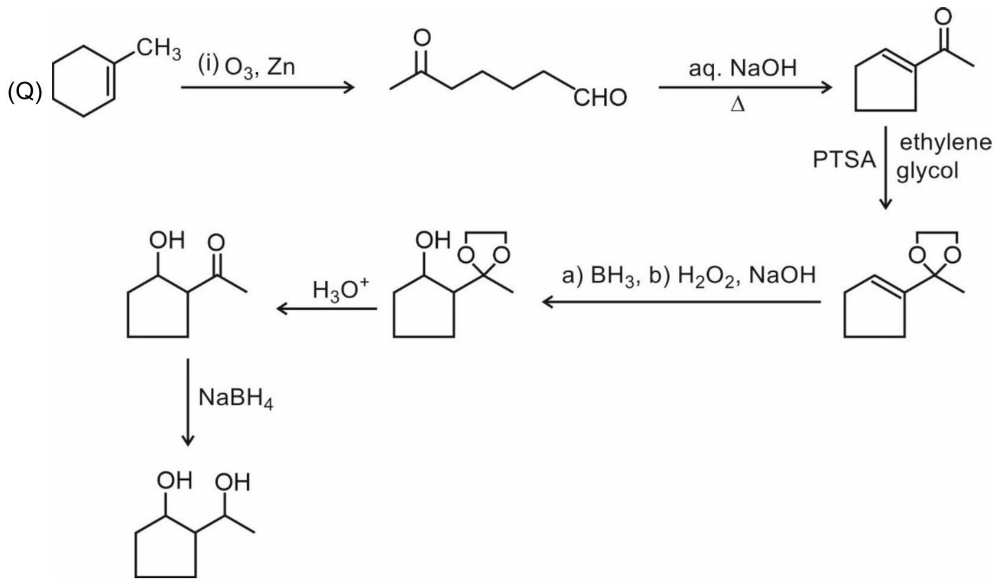
(D) P-5, Q-2, R-4, S-1

Answer (A)

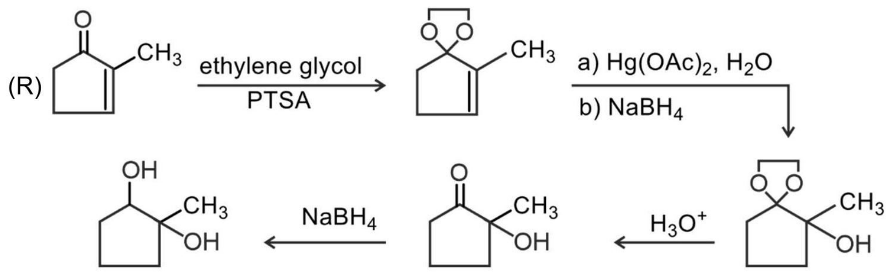
Sol. (P)



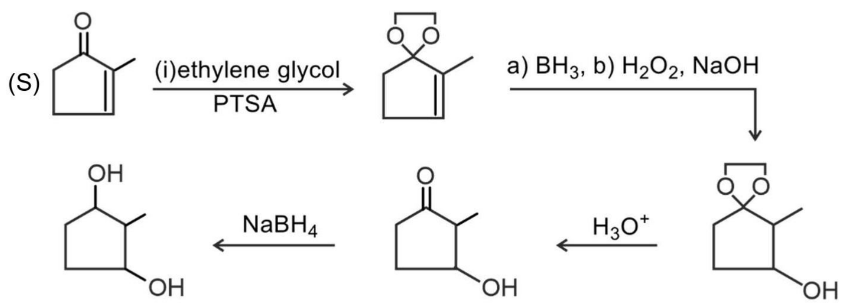
P-3


(Q)
CH_3
(i) O_3, Zn
O
CHO
aq. NaOH
Δ
O
PTSA
ethylene
glycol
OH
O
H_3O^+
OH
O O
a) BH_3, b) H_2O_2, NaOH
O O
$NaBH_4$
OH OH

Q-5

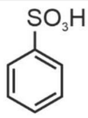
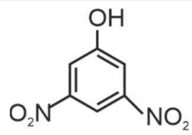
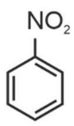
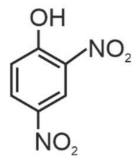
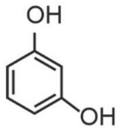
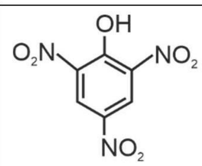
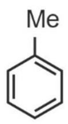
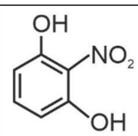
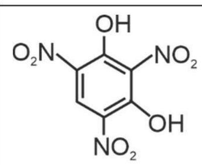

(R)
O
CH_3
ethylene glycol
PTSA
O O
CH_3
a) $Hg(OAc)_2$, H_2O
b) $NaBH_4$
OH
CH_3
OH
$NaBH_4$
O
CH_3
OH
H_3O^+
O O
CH_3
OH

R-4


(S)
O
(i)ethylene glycol
PTSA
O O
a) BH_3, b) H_2O_2, NaOH
OH
OH
$NaBH_4$
O
OH
H_3O^+
O O
OH

S-1

17. **List-I** contains various reaction sequences and **List-II** contains different phenolic compounds. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

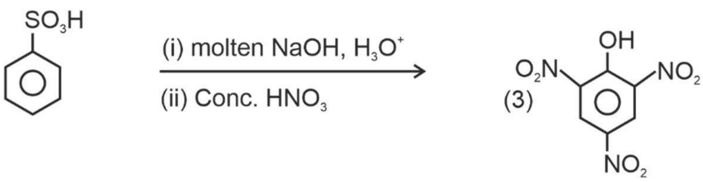
	List-I		List-II
(P)	 (i) molten NaOH, H_3O^+ → (ii) Conc. HNO_3	(1)	
(Q)	 (i) Conc. HNO_3/ Conc. H_2SO_4 (ii) Sn/HCl (iii) $NaNO_2$/HCl, 0-5°C, → (iv) H_2O (v) Conc. HNO_3/ Conc. H_2SO_4	(2)	
(R)	 (i) Conc. H_2SO_4 (ii) Conc. HNO_3 → (iii) H_3O^+, Δ	(3)	
(S)	 (i) (a) $KMnO_4$/KOH, Δ; (b) H_3O^+ (ii) Conc. HNO_3/ Conc. H_2SO_4, Δ (iii) (a) $SOCl_2$, (b) NH_3 → (iv) Br_2, NaOH (v) $NaNO_2$/HCl, 0-5°C (vi) H_2O	(4)	
		(5)	

(A) P-2, Q-3, R-4, S-5 (B) P-2, Q-3, R-5, S-1

(C) P-3, Q-5, R-4, S-1 (D) P-3, Q-2, R-5, S-4

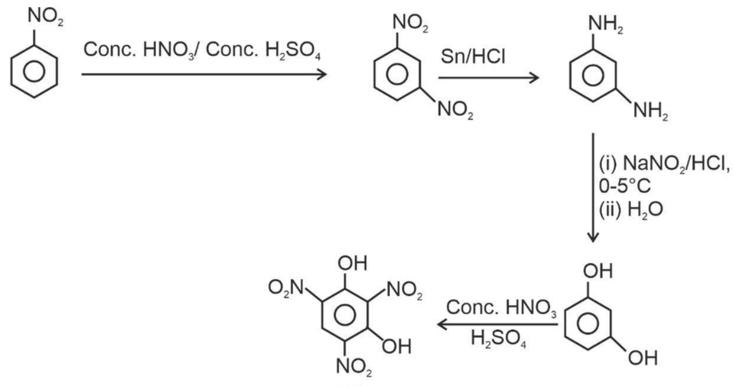
Answer (C)

Sol.

(P) 

(3)

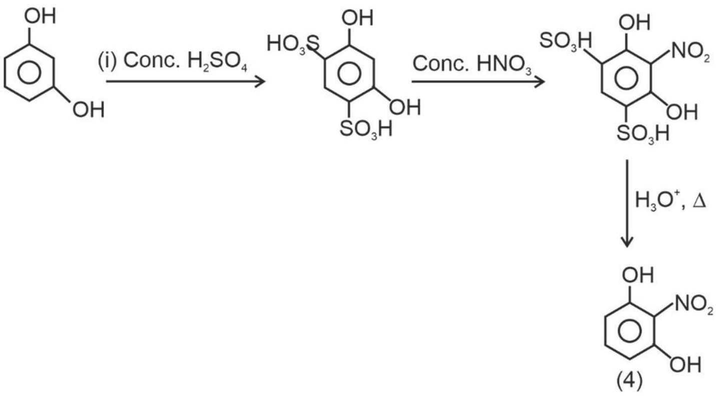
(Q)



NO2
Conc. HNO3/ Conc. H2SO4
NO2
NO2
Sn/HCl
NH2
NH2
(i) NaNO2/HCl, 0-5°C
(ii) H2O
OH
OH
Conc. HNO3
H2SO4
OH
O2N
NO2
OH
NO2

(5)

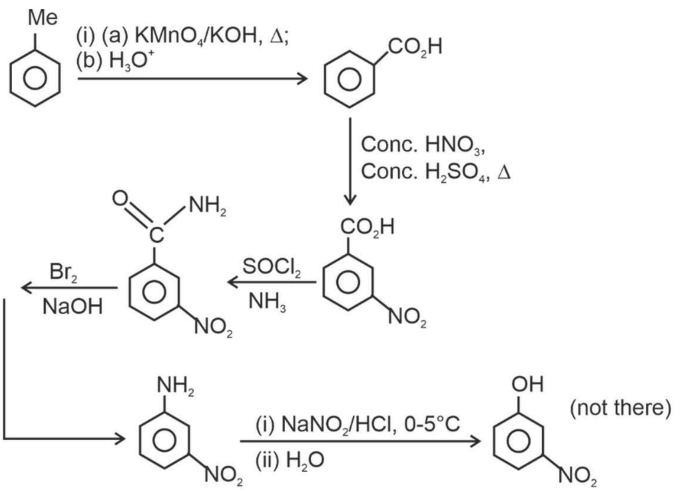
(R)



OH
OH
(i) Conc. H2SO4
HO3S
OH
OH
SO3H
Conc. HNO3
SO3H
OH
NO2
OH
SO3H
H3O+, Δ
OH
NO2
OH

(4)

(S)



Me
(i) (a) KMnO4/KOH, Δ;
(b) H3O+
CO2H
Conc. HNO3,
Conc. H2SO4, Δ
CO2H
NO2
SOCl2
NH3
O
NH2
C
NO2
Br2
NaOH
NH2
NO2
(i) NaNO2/HCl, 0-5°C
(ii) H2O
OH
NO2
(not there)

Date: 26/05/2024

Time : 3 hrs.

Answers & Solutions

for

JEE (Advanced)-2024 (Paper-2)

Max. Marks: 180

PART-I : MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of

$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is

(A) $\frac{7}{24}$

(B) $\frac{-7}{24}$

(C) $\frac{-5}{24}$

(D) $\frac{5}{24}$

Answer (B)

Sol. $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$

Let $\sin^{-1}\frac{3}{5}=\alpha$, $\quad 2\cos^{-1}\frac{2}{\sqrt{5}}=\beta \quad \Rightarrow \cos\frac{\beta}{2}=\frac{2}{\sqrt{5}}$

$\because \sin\alpha=\frac{3}{5} \quad \Rightarrow \tan\alpha=\frac{3}{4} \quad \tan\beta=\frac{2\tan\frac{\beta}{2}}{1-\tan^2\frac{\beta}{2}}=\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$

$\Rightarrow \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{\frac{3}{4}-\frac{4}{3}}{1+1}=-\frac{7}{24}$

2. Let $S=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x\ge 0, y\ge 0, y^2\le 4x, y^2\le 12-2x \text{ and } 3y+\sqrt{8}x\le 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

(A) $\frac{17}{2}$

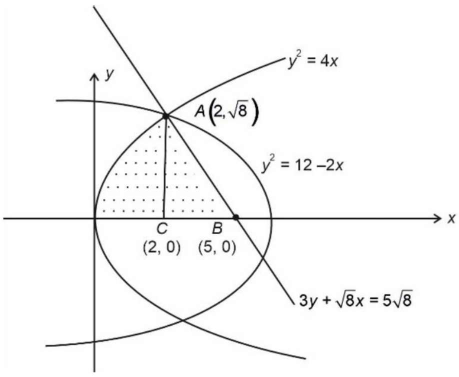
(B) $\frac{17}{3}$

(C) $\frac{17}{4}$

(D) $\frac{17}{5}$

Answer (B)

Sol. $y^2=4x,\ y^2=12-2x \Rightarrow x=2,\ y=\sqrt{8}$



$A=\int_0^2 2\sqrt{x}dx+\frac{1}{2}\times 3\times\sqrt{8}$

$=\left[2\times\frac{2}{3}x^{\frac{3}{2}}\right]_0^2+3\sqrt{2}=\frac{4}{3}\times 2\sqrt{2}+3\sqrt{2}=\frac{17}{3}\sqrt{2}$

$\because A=\alpha\sqrt{2}\Rightarrow \alpha=\frac{17}{3}$

Option (B) is correct.

3. Let $k\in\mathbb{R}$. If $\lim_{x\to 0+}\left(\sin(\sin kx)+\cos x+x\right)^{\frac{2}{x}}=e^6$, then the value of k is

(A) 1

(B) 2

(C) 3

(D) 4

Answer (B)

Sol. $I = \lim_{x\to 0^+} \left(\sin(\sin kx) + \cos x + x\right)^{\frac{2}{x}} = e^6$

$$\Rightarrow \quad \ln I = \lim_{x\to 0^+} \frac{2}{x}\left(\sin(\sin kx) + \cos x + x - 1\right)$$

$$\Rightarrow \quad \ln I = \lim_{x\to 0^+} 2\left(\frac{\sin(\sin kx)}{\sin kx}\cdot\frac{\sin kx}{kx}\cdot\frac{kx}{x} + 1 - \frac{(1-\cos x)}{x^2}\cdot x\right)$$

$\Rightarrow \quad \ln I = 2(k + 1) \qquad \Rightarrow \quad I = e^{2(k+1)} = e^6$

$k + 1 = 3 \qquad \Rightarrow \quad k = 2$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

(A) $f(x) = 0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.

(B) $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$.

(C) The set of solutions of $f(x) = 0$ in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite.

(D) $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$.

Answer (D)

Sol. $f(x) = \begin{cases} x^2 \sin\left(\dfrac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$

$f(x) = 0 \Rightarrow \sin\left(\dfrac{\pi}{x^2}\right) = 0$

$\Rightarrow \dfrac{\pi}{x^2} = n\pi$

$\Rightarrow x^2 = \dfrac{1}{n}$

$\Rightarrow \boxed{x = \dfrac{1}{\sqrt{n}}}$

If $x \in \left[\frac{1}{10^{10}}, \infty\right)$	If $x \in \left[\frac{1}{\pi}, \infty\right)$	If $x \in \left(0, \frac{1}{10^{10}}\right)$
$\frac{1}{\sqrt{n}} \in \left[\frac{1}{10^{10}}, \infty\right)$	$\frac{1}{\sqrt{n}} \in \left[\frac{1}{\pi}, \infty\right)$	$\sqrt{n} \in (10^{10}, \infty)$
$\sqrt{n} \in \left(0, 10^{10}\right]$	$\sqrt{n} \in (0, \pi]$	n infinite
$n \in \left(0, (10^{10})^2\right]$	$n \in \left(0, \pi^2\right]$	If $x \in \left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$
Finite values of n	$n = 1, 2, 3 \ldots 9$	$\sqrt{n} \in (\pi, \pi^2)$
		$n \in (\pi^2, \pi^4)$
		$n \in (9.8,\ 97.2 \ldots)$
		More than 25 solutions

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

5. Let S be the set of all $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$ such that

$$\lim_{x\to\infty} \frac{\sin(x^2)(\log_e x)^{\alpha} \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^{\beta}} = 0$$

Then which of the following is (are) correct?

(A) $(-1, 3) \in S$

(B) $(-1, 1) \in S$

(C) $(1, -1) \in S$

(D) $(1, -2) \in S$

Answer (B, C)

Sol. $\lim_{x\to\infty} \dfrac{\sin(x^2)\sin\left(\dfrac{1}{x^2}\right)(\ln x)^{\alpha}}{x^{\alpha\beta}(\ln(1+x))^{\beta}} = 0$

$= \lim_{x\to\infty} \dfrac{(\sin x^2)\sin\left(\dfrac{1}{x^2}\right)\dfrac{1}{x^2}}{\left(\dfrac{1}{x^2}\right)x^{\alpha\beta}(\ln(1+x))^{\beta}} = 0$

It is possible if $\alpha\beta + 2 > 0$

$\alpha\beta > -2$

(A) $\alpha\beta = -3$

(B) $\alpha\beta = -1$

(C) $\alpha\beta = -1$

(D) $\alpha\beta = -2$

6. A straight line drawn from the point $P(1,3,2)$, parallel to the line $\dfrac{x-2}{1} = \dfrac{y-4}{2} = \dfrac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R. Then which of the following statements is(are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are (1,6,3)

(C) The centroid of the triangle PQR is $\left(\dfrac{4}{3},\dfrac{14}{3},\dfrac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

Answer (A, C)

Sol. Equation of line parallel to $\dfrac{x-2}{1} = \dfrac{y-4}{2} = \dfrac{z-6}{1}$ through $P(1,3,2)$ is $\dfrac{x-1}{1} = \dfrac{y-3}{2} = \dfrac{z-2}{1} = \lambda$ (let)

Now, putting any point $(\lambda + 1, 2\lambda + 3, \lambda + 2)$ in L_1

$\boxed{\lambda = 1}$

$\Rightarrow$ Point $Q(2,5,3)$

Equation of line through $Q(2,5,3)$ perpendicular to L_1 is

$\dfrac{x-2}{1} = \dfrac{y-5}{-1} = \dfrac{z-3}{3} = \mu$ (Let)

Putting any point $(\mu + 2, -\mu + 5, 3\mu + 3)$ in L_2

$\boxed{\mu = -1}$

$\Rightarrow$ Point R (1, 6, 0)

(A) $PQ = \sqrt{1+4+1} = \sqrt{6}$

(B) $R(1, 6, 0)$

(C) Centroid $\left(\dfrac{4}{3},\dfrac{14}{3},\dfrac{5}{3}\right)$

(D) $PQ + QR + PR = \sqrt{6}+\sqrt{11}+\sqrt{13}$

7. Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If $O = (0,0)$ and $C_1 = (-4,0)$, then which of the following statements is (are) TRUE?

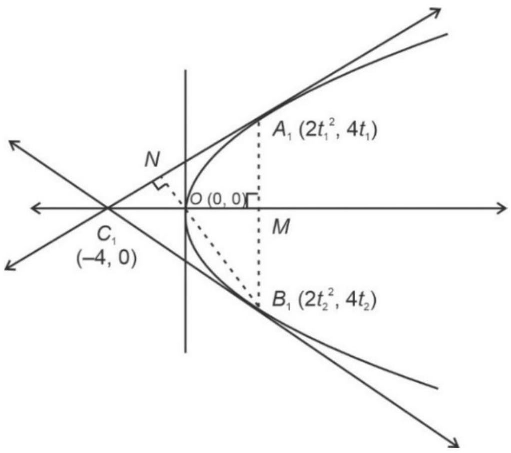
(A) The length of the line segment OA_1 is $4\sqrt{3}$

(B) The length of the line segment A_1B_1 is 16

(C) The orthocentre of the triangle $A_1B_1C_1$ is (0, 0)

(D) The orthocentre of the triangle $A_1B_1C_1$ is (1, 0)

Answer (A, C)



Let $A_1 = (2t_1^2, 4t_1)$ and $B_1 = (2t_2^2, 4t_2)$

$C \equiv (-4, 0) \equiv (2t_1t_2, 2(t_1 + t_2))$

$\Rightarrow$ $t_2 = -t_1$ and $t_1(-t_1) = -2$

$t_1 = \sqrt{2},\ t_2 = -\sqrt{2}$

$A_1 \equiv (4, 4\sqrt{2}),\ B_1 \equiv (4, -4\sqrt{2})$

$\therefore$ $OA_1 = \sqrt{4^2 + (4\sqrt{2})^2} = 4\sqrt{3}$

$A_1B_1 = 8\sqrt{2}$

Altitude $C_1M : y = 0$ ...(i)

Altitude $B_1N : \sqrt{2}x + y = 0$ ...(ii)

$\therefore$ Orthocentre $\equiv (0, 0)$

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, and $g : \mathbb{R} \to (0,\infty)$ be a function such that $g(x+y) = g(x)\,g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is ______.

Answer (51)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(x) = kx$

$$f\left(\frac{-3}{5}\right) = 12 \Rightarrow k = -20$$

$\therefore \quad f(x) = -20x$

$g(x + y) = g(x)\, g(y) \Rightarrow g(x) = a^x$

$$g\left(\frac{-1}{3}\right) = 2 \Rightarrow a = \frac{1}{8}$$

$$\therefore \quad g(x) = \left(\frac{1}{8}\right)^x$$

$$\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0) = (-5 + 64 - 8) \times 1 = 51$$

9. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For i = 1, 2, 3, let W_i, G_i, and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively, If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 \mid W_1 \cap G_2) = \frac{2}{9}$, then N equals ________.

Answer (11)

Sol. N Balls = $3W + 6G + (N - 9)B$

$$P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$$

$$\Rightarrow \quad \frac{3}{N} \times \frac{6}{N-1} \times \frac{N-9}{N-2} = \frac{2}{5N}$$

$\Rightarrow \quad N^2 - 48N + 407 = 0$

$\Rightarrow \quad N = 11$ or 37

$$P(B_3 \mid W_1 \cap G_2) = \frac{2}{9}$$

$$\Rightarrow \quad \frac{P(W_1 \cap G_2 \cap B_3)}{P(W_1 \cap G_2)} = \frac{2}{9}$$

$$\Rightarrow \quad \frac{\frac{2}{5N}}{\frac{3}{N} \times \frac{6}{N-1}} = \frac{2}{9}$$

$$\Rightarrow \quad \frac{N-1}{45} = \frac{2}{9}$$

$\Rightarrow \quad N = 11$

10. Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}.$$

Then the number of solutions of $f(x) = 0$ in $\mathbb{R}$ is ______.

Answer (01)

Sol. $f(x) = 0$

$$\Rightarrow \frac{x^{2023} + 2024x + 2025}{(x^2 - x + 3)} \left[\frac{\sin x + 2}{e^{\pi x}} \right] = 0$$

$\Rightarrow$ $x^{2023} + 2024x + 2025 = 0$

Let $g(x) = x^{2023} + 2024x + 2025$

$g'(x) = 2023x^{2022} + 2024 > 0 \ \forall x \in \mathbb{R}$

$\therefore$ $f(x) = 0$ has only one solution

11. Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β and γ, we have $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$, then the value of γ is _____.

Answer (2)

Sol. $2\vec{p} + \vec{q} = 5\hat{i} + \hat{j} + 7\hat{k}$

$\vec{p} - 2\vec{q} = 0\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(4) - \hat{j}(-1) + \hat{k}(-3)$$

$= 4\hat{i} + \hat{j} - 3\hat{k}$

$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(5\hat{i} + \hat{j} + 7\hat{k}) + \beta(3\hat{j} + \hat{k}) + \gamma(4\hat{i} + \hat{j} - 3\hat{k})$

$\therefore$ $15 = 5\alpha + 4\gamma$

$10 = \alpha + 3\beta + \gamma$

$6 = 7\alpha + \beta - 3\gamma$

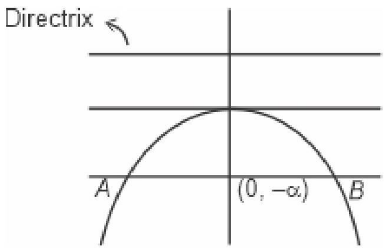
$\therefore$ $\alpha = \frac{7}{5}, \beta = \frac{11}{5}, \gamma = 2$

$\therefore$ $\gamma = 2$

12. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r : s = 1 : 16$, then the value of $24a$ is __________.

Answer (12)

Sol. $x^2 = -4ay$



Equation of normal

$$y = mx - 2a - \frac{a}{m^2}$$

$$-\alpha = -2a - \frac{a}{\frac{1}{6}} = -8a$$

$\Rightarrow \quad \alpha = 8a$

Equation of required line

$y = -\alpha$

$\Rightarrow \quad y = -8a$, solving with $x^2 = -4ay$

$\Rightarrow \quad x^2 = 32a^2$

$\Rightarrow \quad x = \pm 4\sqrt{2}a$

$$= \pm \frac{\alpha}{\sqrt{2}}$$

$$A\left(\frac{\alpha}{\sqrt{2}}, -\alpha\right), B\left(\frac{-\alpha}{\sqrt{2}}, -\alpha\right) \Rightarrow AB = \sqrt{2}\alpha$$

$$\Rightarrow \frac{r}{s} = \frac{4a}{2\alpha^2} = \frac{1}{16} \Rightarrow \frac{4a}{2 \times 64a^2} = \frac{1}{16}$$

$$\Rightarrow a = \frac{1}{2}$$

$$\Rightarrow \boxed{24a = 12}$$

13. Let the function $f:[1,\infty)\to\mathbb{R}$ be defined by

$$f(t)=\begin{cases}(-1)^{n+1}2, & \text{if } t=2n-1,\ n\in\mathbb{N},\\ \dfrac{(2n+1-t)}{2}f(2n-1)+\dfrac{(t-(2n-1))}{2}f(2n+1), & \text{if } 2n-1<t<2n+1,\ n\in\mathbb{N}.\end{cases}$$

Define $g(x)=\int_1^x f(t)dt, x\in(1,\infty)$. Let α denote the number of solutions of the equation $g(x)=0$ in the interval $(1, 8]$ and $\beta=\lim_{x\to 1+}\frac{g(x)}{x-1}$. Then the value of $\alpha+\beta$ is equal to _______.

Answer (5)

Sol. $f(t)=\left(\frac{(2n+1)-t}{2}\right)(-1)^{n+1}2+\left(\frac{t-(2n-1)}{2}\right)(-1)^{n+2}2, t\in(2n-1, 2n+1)$

$\Rightarrow f(t)=2(-1)^{n+1}(2n-t),\ t\in(2n-1, 2n+1)$

$\Rightarrow g(x)=\int_1^x f(t)dt,\ x\in(1, 8]$

$$=\begin{cases}\int_1^x 2(2-t)dt,\ 1<x\le 3,\ n=1\\ \int_1^3 2(2-t)dt+\int_3^x(2t-8)dt,\ 3<x\le 5,\ n=2\\ \int_1^3 2(2-t)dt+\int_3^5(2t-8)dt+\int_5^x 2(6-t)dt,\ 5<x\le 7,\ n=3\\ \int_1^3 2(2-t)dt+\int_3^5(2t-8)dt+\int_5^7 2(6-t)dt+\int_7^x(2t-16)dt,\ x\in(7, 8],\ n=4\end{cases}$$

$$=\begin{cases}-x^2+4x-3,\ 1<x\le 3,\\ x^2-8x+15,\ 3<x\le 5\\ -x^2+12x-35,\ 5<x\le 7\\ x^2-16x+63,\ 7<x\le 8\end{cases}=\begin{cases}-(x-1)(x-3),\ 1<x\le 3\\ (x-3)(x-5),\ 3<x\le 5\\ -(x-5)(x-7),\ 5<x\le 7\\ (x-7)(x-9),\ 7<x\le 8\end{cases}$$

$\Rightarrow g(x)=0\Rightarrow x=3, 5, 7\Rightarrow \alpha=3$

$\beta=\lim_{x\to 1^+}\left(\frac{g(x)}{x-1}\right)=\lim_{x\to 1^+}-\frac{(x-1)(x-3)}{x-1}=2$

$\Rightarrow \alpha+\beta=5$

SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

PARAGRAPH I

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X :$ The range of R has exactly one element$\}$ and

$Z = \{R \in X : R$ is a function from S to $S\}$.

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

14. If $n(X) = {}^{m}C_6$, then the value of m is ________.

Answer (20)

Sol. $S = \{1, 2, 3, 4, 5, 6\}$ $R : S \to S$

Number of elements in $R = 6$

and for each $(a, b) \in R$; $|a - b| \geq 2$

$X \to$ set of all relation $R : S \to S$

If $a = 1, b = 3, 4, 5, 6 \to$ ④

$a = 2, b = 4, 5, 6 \to$ ③

$a = 3, b = 1, 5, 6 \to$ ③

$a = 4, b = 1, 2, 6 \to$ ③

$a = 5, b = 1, 2, 3 \to$ ③

$a = 6, b = 1, 2, 3, 4 \to$ ④

Total number of ordered pairs (a, b) s.t. $|a - b| \geq 2$

$= 20$

$\therefore$ $n(X)$ = number of elements in X

$= {}^{20}C_6$ $\therefore$ $m = 20$

PARAGRAPH I

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X$: The range of R has exactly one element$\}$ and

$Z = \{R \in X : R$ is a function from S to $S\}$.

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

15. If the value of $n(Y) + n(Z)$ is k^2, then $|k|$ is ____________.

Answer (36)

Sol. $S = \{1, 2, 3, 4, 5, 6\}$ $\quad\quad R : S \to S$

Number of elements in $R = 6$

and for each $(a, b) \in R$; $|a - b| \geq 2$

$X \to$ set of all relation $R : S \to S$

If

$a = 1$	$b = 3, 4, 5, 6$	$\to$	4
$a = 2$	$b = 4, 5, 6$	$\to$	3
$a = 3$	$b = 1, 5, 6$	$\to$	3
$a = 4$	$b = 1, 2, 6$	$\to$	3
$a = 5$	$b = 1, 2, 3$	$\to$	3
$a = 6$	$b = 1, 2, 3, 4$	$\to$	4

Total number of ordered pairs (a, b) s. t. $|a - b| \geq 2 = 20$

$\therefore$ $n(X)$ = number of elements in X

$= {}^{20}C_6 \quad\quad \therefore m = 20$

$Y = \{R \in X$: The range of R has exactly one element$\}$

From above, if range of R has exactly one element, then maximum number of elements in R will be 4.

$\therefore$ $n(Y) = 0$

$Z = \{R \in X : R$ is a function from S to $S\}$

$n(Z) = {}^4C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^4C_1$

$= (36)^2$

$n(y) + n(z) = 0 + (36)^2 = k^2$

$\Rightarrow$ $|k| = 36$

PARAGRAPH II

Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

16. The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx-\int_0^{\frac{\pi}{2}} g(x)dx$ is ________.

Answer (0)

Sol. $f(x)=\sin^2 x,\ g(x)=\sqrt{\frac{\pi}{2}x-x^2}$

Here $f\left(\frac{\pi}{2}-x\right)=\cos^2 x,\ g\left(\frac{\pi}{2}-x\right)=g(x)$

Let $I_1=2\int_0^{\frac{\pi}{2}} f(x)g(x)=2\int_0^{\frac{\pi}{2}} \sin^2 x\cdot g(x)dx$ ...(1)

as $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$\Rightarrow\ I_1=2\int_0^{\frac{\pi}{2}} \cos^2 x\, g(x)dx$ ...(2)

(1) + (2)

$\Rightarrow\ 2I_1=2\int_0^{\frac{\pi}{2}} g(x)dx$

$\Rightarrow\ I_1=\int_0^{\frac{\pi}{2}} g(x)dx$

$\Rightarrow\ 2\int_0^{\frac{\pi}{2}} f(x)g(x)-\int_0^{\frac{\pi}{2}} g(x)dx=0$

PARAGRAPH II

Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

17. The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is __________.

Answer (0.25)

Sol. According to Q.16

$$2\int_0^{\frac{\pi}{2}} f(x)g(x)dx = \int_0^{\frac{\pi}{2}} g(x)dx = I_1 \text{ (let)}$$

Now, $I_1 = \int_0^{\frac{\pi}{2}} g(x)dx = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\pi}{2}x - x^2}\,dx$

$$I_1 = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{\pi}{4}\right)^2 - \left(\frac{\pi}{4} - x\right)^2}\,dx$$

Put $\frac{\pi}{4} - x = t$

$\Rightarrow$ $dx = -dt$

$$I_1 = -\int_{\frac{\pi}{4}}^{\frac{-\pi}{4}} \sqrt{\left(\frac{\pi}{4}\right)^2 - t^2}\,dt$$

$$I_1 = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sqrt{\left(\frac{\pi}{4}\right)^2 - t^2}\,dt$$

$$I_1 = 2\int_0^{\frac{\pi}{4}} \sqrt{\left(\frac{\pi}{4}\right)^2 - t^2}\,dt = 2\left[\frac{t}{2}\sqrt{\left(\frac{\pi}{4}\right)^2 - t^2} + \frac{\pi^2}{32}\sin^{-1}\left(\frac{4t}{\pi}\right)\right]_0^{\frac{\pi}{4}}$$

$$I_1 = \frac{\pi^3}{32}$$

Now, $I = \frac{8}{\pi^3}I_1$

$$I = \frac{1}{4} = 0.25$$

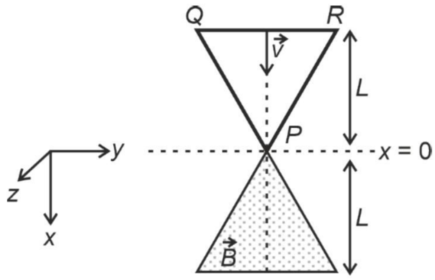
PART-II : PHYSICS

SECTION 1 (Maximum Marks : 12)

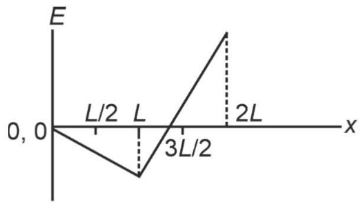
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

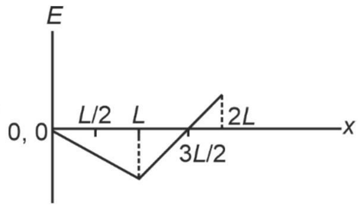
Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

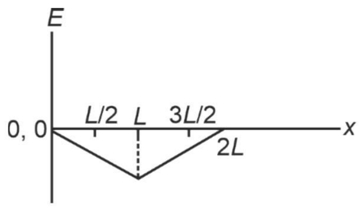
1. A region in the form of an equilateral triangle (in $x - y$ plane) of height L has a uniform magnetic field $\vec{B}$ pointing in the $+z$-direction. A conducting loop PQR, in the form of an equilateral triangle of the same height L, is placed in the $x - y$ plane with its vertex P at $x = 0$ in the orientation shown in the figure. At $t = 0$, the loop starts entering the region of the magnetic field with a uniform velocity $\vec{v}$ along the $+x$-direction. The plane of the loop and its orientation remain unchanged throughout its motion.

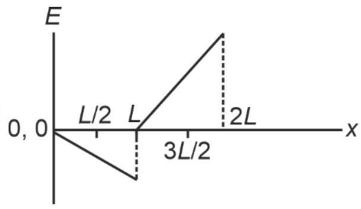


Which of the following graph best depicts the variation of the induced emf (E) in the loop as a function of the distance (x) starting from $x = 0$?

(A)


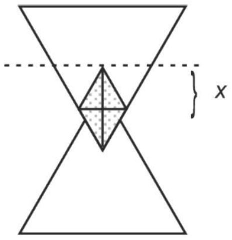
(B)


(C)


(D)


Answer (A)

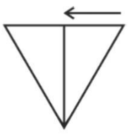
Sol. For $x < L$



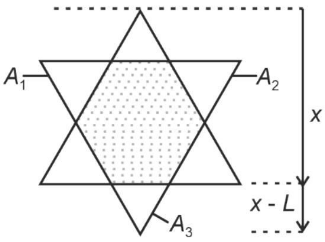
$$\text{Area} = \frac{x}{2}\ \frac{x}{2}\tan 30 \times 4 \times \frac{1}{2} = \frac{1}{2}x^2 \tan 30$$

$\phi' = B_0 x \tan 30 V$ $\boxed{\varepsilon \propto x}$

$L \tan 30°$



$x \geq L$



$$\begin{aligned}\text{Area} &= A_0 - A_1 - A_2 - A_3 \\ &= A_0 - 2A_1 - (x - L)(x - L)\tan 30 \\ &= A_0 - (x - L)^2 \tan 30 - \{L \tan 30 - (x - L)\tan 30°\}^2 \frac{1}{2} \times \frac{1}{2} \tan 60° \times 2 \\ &= A_0 - (x - L)^2 \tan 30° - \tan 30° \{2L - x\}^2 \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\varepsilon' &= -2(x - L)\tan 30 V - \tan 30\ 2(2L - x)(-)V \\ &= (4L - x - 2x + 2L)\tan 30° V \\ &= (4L - 3x)V\end{aligned}$$

$$\boxed{= 0 \text{ at } x = \frac{4L}{3}}$$

From 1 & 2

1.33 < 1.5

2. A particle of mass m is under the influence of the gravitational field of a body of mass M (>> m). The particle is moving in a circular orbit of radius r_0 with time period T_0 around the mass M. Then, the particle is subjected to an additional central force, corresponding to the potential energy $V_c(r) = m\alpha/r^3$, where α is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius r_0 in the combined gravitational potential due to M and $V_c(r)$, but with a new time period T_1, then $(T_1^2 - T_0^2)/T_1^2$ is given by

[G is the gravitational constant.]

(A) $\dfrac{3\alpha}{GMr_0^2}$

(B) $\dfrac{\alpha}{2GMr_0^2}$

(C) $\dfrac{\alpha}{GMr_0^2}$

(D) $\dfrac{2\alpha}{GMr_0^2}$

Answer (A)

Sol. $\dfrac{Gmm}{r_0^2} - \dfrac{3\alpha m}{r_0^4} = \dfrac{mv^2}{r_0}$

$$T = \frac{2\pi r_0}{\sqrt{\dfrac{Gmr_0^2 - 3\alpha}{r_0^3}}}$$

$$T_0^2 = \frac{4\pi^2}{Gm} r_0^3$$

$$\frac{T^3 - T_0^2}{T_1^2} = 1 - \frac{T_0^2}{T_1^2}$$

$$= 1 - \frac{4\pi^2}{Gm}\frac{r_0^3}{4\pi^2 r_0^2}\frac{Gmr_0^2 - 3\alpha}{r_0^3}$$

$$= 1 - 1 + \frac{3\alpha}{Gmr_0^2}$$

$$= \frac{3\alpha}{GMr_0^2}$$

3. A metal target with atomic number $Z = 46$ is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the K_α-line and the cut-off is found to be $r = 2$. If the same electron beam bombards another metal target with $Z = 41$, the value of r will be

(A) 2.53 (B) 1.27

(C) 2.24 (D) 1.58

Answer (A)

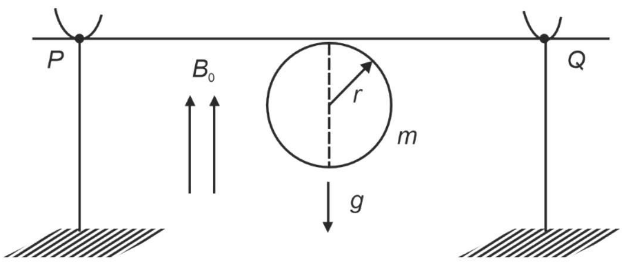
Sol. $\frac{1}{\lambda_\alpha} = \frac{3}{4}R(Z-1)^2 p$

$\lambda_{cut} = \frac{hc}{eV}$

$\Rightarrow$ Ratio $\propto \frac{1}{(Z-1)^2}$ for same beam

$\frac{Z}{x} = \frac{40^2}{45^2} \Rightarrow x = \frac{45^2}{40^2} \cdot 2 \approx 2.53$

4. A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and it is in a uniform vertical magnetic field B_0, as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity g, on two conducting supports at P and Q. When a current I is passed through the loop, the loop turns about the line PQ by an angle θ given by



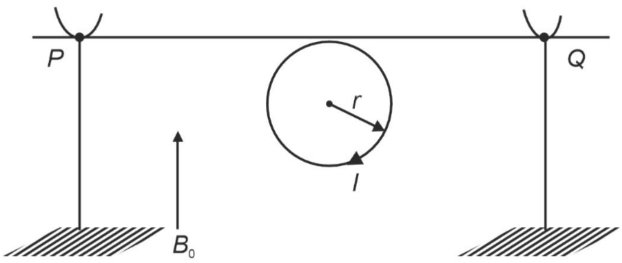
(A) $\tan\theta = \pi r I B_0/(mg)$

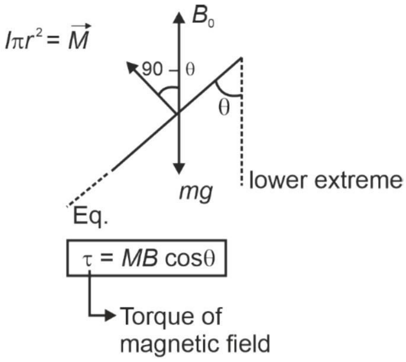
(B) $\tan\theta = 2\pi r I B_0/(mg)$

(C) $\tan\theta = \pi r I B_0/(2mg)$

(D) $\tan\theta = mg/(\pi r I B_0)$

Answer (A)

Sol. 



Now $\frac{\text{For equilibrium}}{\tau = mg\ r\sin\theta}$

$I\pi r^2 B_0 \cos\theta = mgr\sin\theta$

$$\boxed{\tan\theta = \frac{I\pi r B_0}{mg}}$$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

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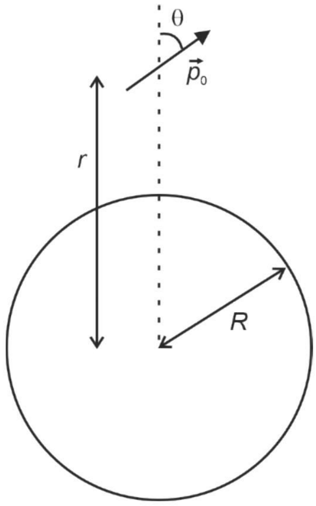
5. A small electric dipole $\vec{p}_0$, having a moment of inertia *I* about its center, is kept at a distance *r* from the center of a spherical shell of radius *R*. The surface charge density σ is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle θ as shown in the figure. While staying at a distance *r*, the dipole is free to rotate about its center.



If released from rest, then which of the following statement(s) is (are) correct?

[ε_0 is the permittivity of free space.]

(A) The dipole will undergo small oscillations at any finite value of r.

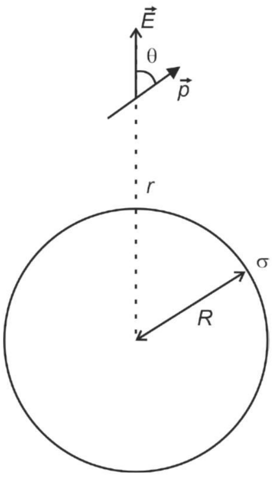
(B) The dipole will undergo small oscillations at any finite value of $r > R$.

(C) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{2\sigma p_0}{\varepsilon_0 I}}$ at $r = 2R$

(D) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{\sigma p_0}{100\ \varepsilon_0 I}}$ at $r = 10R$

Answer (B, D)

Sol.



$\tau = |\vec{p} \times \vec{E}|$

$I\alpha = p_0 E \sin\theta$

$\alpha = \frac{p.\theta}{I}\left(\frac{1}{4\pi\,\varepsilon_0}\frac{\sigma 4\pi R^2}{r^2}\right)$

$\alpha = \left(\frac{p_0 \sigma R^2}{I\,\varepsilon_0 r^2}\right)\cdot\theta$

$$\boxed{\therefore\ \omega = \sqrt{\frac{p_0 \sigma R^2}{I\,\varepsilon_0 r^2}}}$$

For $r = 2R$

$\omega = \sqrt{\frac{p_0\sigma}{4I\,\varepsilon_0}}$ (C is incorrect)

Also, for $r = 10R$

$\omega = \sqrt{\frac{p_0\sigma}{4I(100)}}$ (D is correct)

It will oscillate for any finite value of $r > R$. (B is correct)

6. A table tennis ball has radius (3/2) × 10^{-2} m and mass (22/7) × 10^{-3} kg. It is slowly pushed down into a swimming pool to a depth of d = 0.7 m below the water surface and then released from rest. It emerges from the water surface at speed v, without getting wet, and rises up to a height H. Which of the following option(s) is (are) correct?

[Given: π = 22/7, g = 10 ms^{-2}, density of water = 1 × 10^3 kg m^{-3}, viscosity of water = 1 × 10^{-3} Pa-s.]

(A) The work done in pushing the ball to the depth d is 0.077 J.

(B) If we neglect the viscous force in water, then the speed v = 7 m/s.

(C) If we neglect the viscous force in water, then the height H = 1.4 m.

(D) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is 500/9.

Answer (A, B, D)

Sol. Work done in pushing the ball

$W = (v\rho g)d - (v\sigma g)d$

Where $\rho \rightarrow$ Density of water

$\sigma \rightarrow$ Density of ball

$\Rightarrow \quad W = \frac{4}{3}\pi R^3 \times 10 \times 0.7\left[1000 - \frac{3}{4} \times \frac{10^{-3}}{R^3}\right]$

$\boxed{W = 0.077\text{ J}}$ [A is correct]

$\Rightarrow$ When ball is released at bottom same work (*i.e.* 0.077 J) is done on ball.

$\therefore \quad \frac{1}{2}mv^2 = 0.077$

$v = \sqrt{\frac{0.077 \times 2}{\frac{22}{7} \times 10^{-3}}}$

= 7 m/s [B is correct]

$\Rightarrow$ also, $H = \frac{v^2}{2g} = \frac{7 \times 7}{2 \times 10} = 2.45$ m [C is incorrect]

$\Rightarrow$ Net force $F_{net} = v\sigma g - v\sigma g$ = 0.11 N

Also, viscous force is maximum when v = 7 m/s.

$\therefore \quad (F_v)_{\max} = 6\pi\eta rv$

$= 6 \times \frac{22}{7} \times 10^{-3}\left(\frac{3}{2} \times 10^{-2}\right) \times 7$

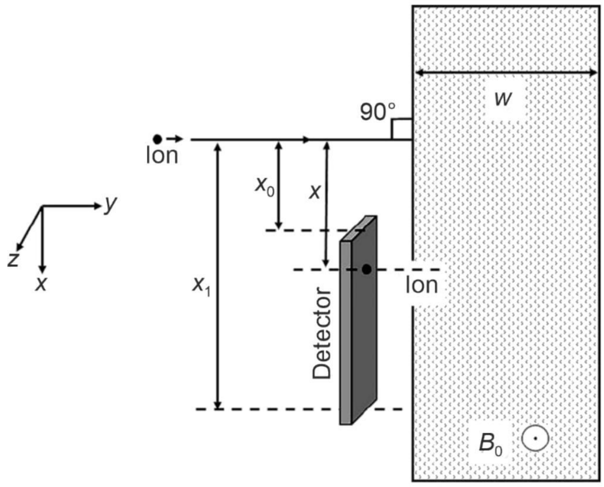
= 18 × 11 × 10^{-5} N

Now,

$\frac{F_{\text{net}}}{(F_v)_{\max}} = \frac{500}{9}$ [D is correct]

7. A positive, singly ionized atom of mass number A_M is accelerated from rest by the voltage 192 V. Thereafter, it enters a rectangular region of width w with magnetic field $\vec{B}_0 = 0.1\hat{k}$ Tesla, as shown in the figure. The ion finally hits a detector at the distance x below its starting trajectory.

[Given: Mass of neutron/proton = (5/3) × 10^{-27} kg, charge of the electron = 1.6 × 10^{-19} C.]



Which of the following option(s) is(are) correct?

(A) The value of x for H^+ ion is 4 cm.

(B) The value of x for an ion with A_M = 144 is 48 cm.

(C) For detecting ions with $1 \le A_M \le 196$, the minimum height $(x_1 - x_0)$ of the detector is 55 cm.

(D) The minimum width w of the region of the magnetic field for detecting ions with A_M = 196 is 56 cm.

Answer (A, B)

Sol. $x = 2R$

$$= 2\frac{mv}{qB}$$

$$2\frac{\sqrt{2m(e\Delta V)}}{qB}$$

For H$^+$ ion

$x = 3.91$ cm

$\simeq$ 4 cm (A is correct)

For $m = 144\ (m_P)$

$$= 12\left(x_{H^+}\right)$$

= 48 cm (B is correct)

For $1 \le A_M \le 196$

$\Rightarrow$ $(x_1 - x_0)_{min} = 2R_{196} - 2R_1$

$= (14 \times 4) - 4$

$= 52$ cm (C is incorrect)

For $A_M = 196$

$w_{min} = R_{196} = 28$ cm (D is incorrect)

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

---

8. The dimensions of a cone are measured using a scale with a least count of 2 mm. The diameter of the base and the height are both measured to be 20.0 cm. The maximum percentage error in the determination of the volume is ______.

Answer (3)

Sol. $V = \frac{1}{3}\pi R^2 H$

$\Rightarrow \frac{dV}{V} = 2 \cdot \frac{dR}{R} + \frac{dH}{H}$

$\Rightarrow$ % error in measuring volume

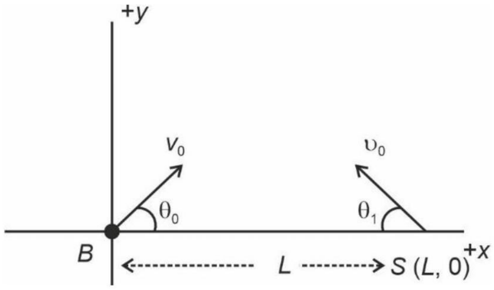
$= \left[2 \times \frac{0.2}{20} + \frac{0.2}{20}\right] \times 100$

$= 3$

9. A ball is thrown from the location $(x_0, y_0) = (0,0)$ of a horizontal playground with an initial speed υ_0 at an angle θ_0 from the $+x$-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the +x-direction with a suitable initial speed. For a fixed υ_0, when $(\theta_0, \theta_1) = (45°, 45°)$, the stone hits the ball after time T_1, and when $(\theta_0, \theta_1) = (60°, 30°)$, it hits the ball after time T_2. In such a case, $(T_1/T_2)^2$ is ________.

Answer (2)

Sol.



Let B : Ball

S : Stone

υ_0 : Initial speed of stone.

Since relative acceleration = zero

$\Rightarrow$ Path seen would be straight line

$\Rightarrow$ To meet, $v_0 \sin\theta_0 = v_0 \sin\theta_1$

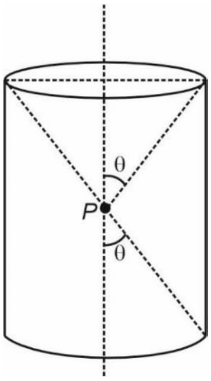
And $\Delta t = \dfrac{L}{\upsilon_0 \cos\theta_1 + v_0 \cos\theta_0}$

Case I : $\upsilon_0 = v_0 \Rightarrow \Delta t_1 = T_1 = \dfrac{L}{v_0\left[\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\right]} = \dfrac{L}{\sqrt{2}v_0}$

Case II : $\sqrt{3}v_0 = \upsilon_0 \Rightarrow \Delta t_2 = T_2 = \dfrac{L}{\sqrt{3}v_0 \cdot \dfrac{\sqrt{3}}{2} + \dfrac{v_0}{2}} = \dfrac{L}{2v_0}$

$\Rightarrow \left(\dfrac{T_1}{T_2}\right)^2 = \left(\sqrt{2}\right)^2 = 2$

10. A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle θ at P, as shown in the figure. When θ = 30°, then the electric flux through the curved surface of the cylinder is Φ If θ = 60°, then the electric flux through the curved surface becomes $\Phi / \sqrt{n}$, where the value of n is ______.



Answer (3)

Sol. For any θ, let us first find the flux inside a cone of half angle θ. we know that for such a cone, solid angle subtended at centre is

$\Omega = 2\pi\,[1 - \cos\theta]$

$\Rightarrow$ Flux through 1 cone = $\phi_0 = \dfrac{\Omega}{4\pi} \cdot \dfrac{Q}{\varepsilon_0} = \dfrac{Q}{2\varepsilon_0}[1 - \cos\theta]$

$\Rightarrow$ Flux through curved surface

$= \dfrac{Q}{\varepsilon_0} - 2\phi_0$

$= \dfrac{Q}{\varepsilon_0} - \dfrac{Q}{\varepsilon_0}[1 - \cos\theta] = \dfrac{Q}{\varepsilon_0}\cos\theta$

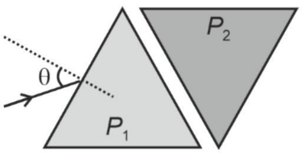
$\Rightarrow \phi = \dfrac{Q}{\varepsilon_0} \cdot \dfrac{\sqrt{3}}{2}$

And $\dfrac{\phi}{\sqrt{n}} = \dfrac{Q}{\varepsilon_0} \cdot \dfrac{1}{2}$

$\Rightarrow \sqrt{n} = \sqrt{3}$

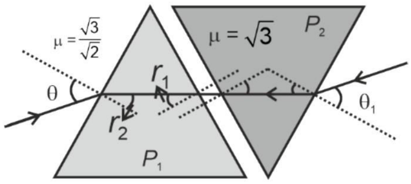
$\Rightarrow n = 3$

11. Two equilateral-triangular prisms P_1 and P_2 are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism P_1 at an angle of incidence θ such that the outgoing ray undergoes minimum deviation in prism P_2. If the respective refractive indices of P_1 and P_2 are $\sqrt{\dfrac{3}{2}}$ and $\sqrt{3}$, $\theta = \sin^{-1}\left[\sqrt{\dfrac{3}{2}}\sin\left(\dfrac{\pi}{\beta}\right)\right]$, where the value of β is ______.



Answer (12)

Sol. By using optical reversibility principle



For prism P_2

→ Minimum deviation

$1 \times \sin\theta_1 = \sqrt{3}\ \sin r$ $\qquad r_1 = r_2 = \frac{A}{2}$

$\sin\theta_1 = \sqrt{3} \times \frac{1}{2}$ $\qquad r_1 = r_2 = 30°$

$\Rightarrow \quad i = e = 60°$

For prism P_1

Incident angle will be 60°

$$1 \times \sin 60° = \frac{\sqrt{3}}{\sqrt{2}} \sin r_1$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}} \sin r_1$$

$r_1 + r_2 = 60°$

$$\sin r_1 = \frac{1}{\sqrt{2}}$$

$\boxed{r_1 = 45°}$

$r_2 = 15°$

$$\frac{\sqrt{3}}{\sqrt{2}} \sin(45°) = 1 \times \sin\theta$$

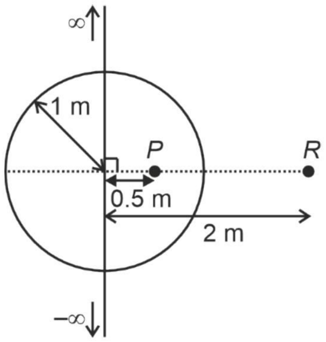
$$15° = \frac{\pi \times 15}{180} \text{ rad} = \frac{\pi}{12} \text{ rad}$$

$$\theta = \sin^{-1}\left[\frac{\sqrt{3}}{\sqrt{2}} \sin\left(\frac{\pi}{12}\right)\right]$$

$\boxed{\beta = 12}$

12. An infinitely long thin wire, having a uniform charge density per unit length of 5 nC/m, is passing through a spherical shell of radius 1 m, as shown in the figure. A 10 nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points *P* and *R*, in Volt, is ______.

[Given: In SI units $\frac{1}{4\pi \in_0} = 9 \times 10^9$, ln 2 = 0.7. Ignore the area pierced by the wire.]



Answer (171)

Sol. $E_{\text{Line charge}} = \frac{\lambda}{2\pi \in_0 r}$

$$\Rightarrow \quad \Delta V_{\text{Line charge}} = \int_{0.5}^{2} \frac{\lambda}{2\pi \in_0 r} dr = \frac{\lambda}{2\pi \in_0} \ln 4 \quad \text{...(i)}$$

$$\Delta V_{\text{Sphere}} = \frac{1}{4\pi \in_0} \frac{Q}{R} - \frac{1}{4\pi \in_0} \frac{Q}{2R}$$

$$= \frac{1}{4\pi \in_0} \frac{Q}{2} \quad \text{...(ii)}$$

$$\Rightarrow \quad \Delta V_{\text{Net}} = \frac{\lambda}{2\pi \in_0} \ln 4 + \frac{1}{4\pi \in_0} \frac{Q}{2}$$

= 171 Volts

13. A spherical soap bubble inside an air chamber at pressure $P_0 = 10^5$ Pa has a certain radius so that the excess pressure inside the bubble is ΔP = 144 Pa. Now, the chamber pressure is reduced to $8P_0/27$ so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure ΔP in both the cases to be much smaller than the chamber pressure. The new excess pressure ΔP in Pa is

Answer (96)

Sol. Since the situation follow isothermal condition.

$P_1V_1 = P_2V_2$

$$V_1 = \frac{4}{3}\pi R_1^3, V_2 = \frac{4}{3}\pi R_2^3$$

$$P_1 = P_0 + \Delta P_1, \Delta P_1 = \frac{4T}{R_1}$$

and $P_2 = \frac{8P_0}{27} + \Delta P_2, \Delta P_2 = \frac{4T}{R_2}$

So for isothermal condition

$$(P_0 + \Delta P_1) \times \frac{4}{3}\pi R_1^3 = \left(\frac{8P_0}{27} + \Delta P_2\right) \times \frac{4}{3}\pi R_2^3$$

here $P_0 = 10^5$ Pa

$\Delta P_1 = 144$ Pa

and $\Delta P_1 << P_0$

So $(P_0 + \Delta P_1)\left(\frac{4T}{\Delta P_1}\right)^3 = \left(\frac{8P_0}{27} + \Delta P_2\right)\left(\frac{4T}{\Delta P_2}\right)^3$

$$\frac{P_0}{(\Delta P_1)^3} \approx \frac{8P_0}{27} \times \frac{1}{(\Delta P_2)^3}$$

$$\Delta P_2 = \frac{2}{3}\ \Delta P_1 = \frac{2}{3} \times (144\ \text{Pa})$$

$\Delta P_2 = 96$ Pa

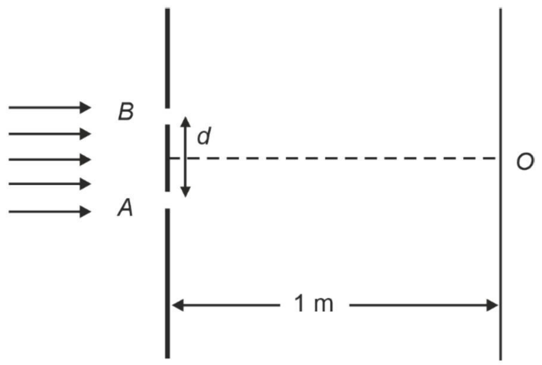
SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

PARAGRAPH I

In a Young's double slit experiment, each of the two slits *A* and *B*, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time *t* is given by $d = (0.8 + 0.04 \sin \omega t)$ mm, where $\omega = 0.08$ rad s^{-1}. The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is 6000 Å. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point *O*.



14. The 8^{th} bright fringe above the point *O* oscillates with time between two extreme positions. The separation between these two extreme positions, in micrometer (μm), is ______.

Answer (601.50)

Sol. As central bright fringe position is not changing, the two slits are oscillating with a phase diff of π.

For 8^{th} bright fringe

$$y = \frac{8\lambda D}{(0.8 + 0.04\sin\omega t)} \times 10^3$$

$$= \frac{8 \times 6000 \times 10^{-10} \times 10^3}{(0.8 + 0.04\sin\omega t)}$$

$$y = \frac{48 \times 10^{-4}}{(0.8 + 0.04\sin\omega t)}$$

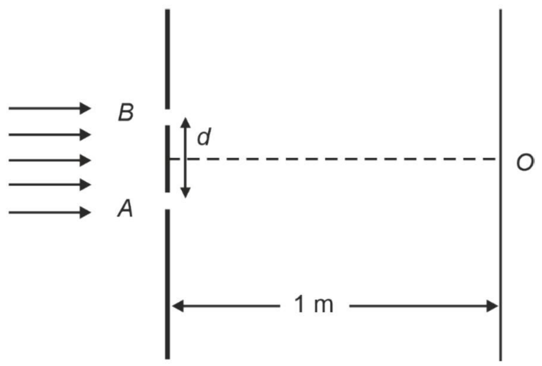
d varies from 0.84 mm to 0.76 mm

$\Delta y = 6.015 \times 10^{-4}$

$= 601.50\ \mu m$

PARAGRAPH I

In a Young's double slit experiment, each of the two slits *A* and *B*, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time *t* is given by $d = (0.8 + 0.04 \sin \omega t)$ mm, where $\omega = 0.08$ rad s^{-1}. The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is 6000 Å. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point *O*.



15. The maximum speed in μ m/s at which the 8th bright fringe will move is __________.

Answer (24.00)

Sol. Finding speed

$$\frac{\delta y}{\delta t}=\frac{\delta}{\delta t}\left(\frac{8\lambda D}{d}\right)$$

$$=-\frac{8\lambda D}{d^2}\frac{\delta d}{(\delta t)}$$

$$v=-\frac{8\lambda D}{d^2}(0.04\omega\cos\omega t)\times 10^{-3}$$

$$v_{max}=\frac{8\lambda D}{d^2}\times 4\omega\times 10^{-5}$$

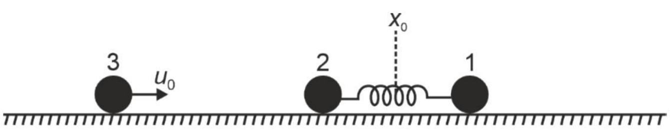
$$=\frac{8\times 6\times 10^{-7}\times 1\times 4\times 8\times 10^{-7}}{64\times 10^{-8}}$$

$$= 24\times 10^{-6}$$

$$= 24\ \mu\text{m/s}$$

PARAGRAPH II

Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a\sin\omega t$ and $x_2(t) = (x_0 - d) - a\sin\omega t$, respectively, where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega/2$, and undergoes instantaneous elastic collision with particle 2, at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω.



16. If the collision occurs at time $t_0 = 0$, the value of $v_{cm}/(a\omega)$ will be _________.

Answer (00.75)

Sol. At $t = 0$, 2 is at mean position

$\therefore \quad u_2 = a\omega$ towards left after collision, velocity will exchange

$\therefore \quad v_2 = \frac{a\omega}{2}$ towards right

$u_1 = a\omega$ towards right

$\therefore \quad v_{cm} = \frac{3a\omega}{4}$

$\frac{v_{cm}}{a\omega} = \frac{3}{4} = 0.75$

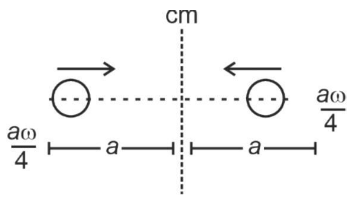
At $t = \frac{\pi}{2\omega}$, $u_2 = 0$

After collision, $v_2 = \frac{a\omega}{2}$ towards right

$u_1 = 0$

$\therefore \quad v_{cm} = \frac{a\omega}{4}$ towards right

w.r.t. centre of mass



$v = \omega\sqrt{A^2 - x^2}$

$\frac{a\omega}{4} = \omega\sqrt{A^2 - a^2}$

$\frac{a^2}{16} + a^2 = A^2$

$\frac{17}{16}a^2 = A^2 = b^2$

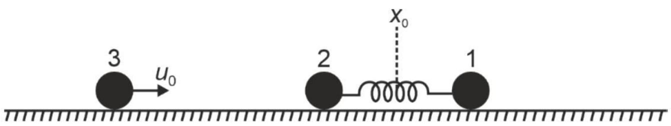
$\therefore \quad b^2 = \frac{17}{16}a^2$

$\frac{4b^2}{a^2} = \frac{17}{4}$

$= 4.25$

PARAGRAPH II

Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a \sin \omega t$ and $x_2(t) = (x_0 - d) - a \sin \omega t$, respectively, where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega/2$, and undergoes instantaneous elastic collision with particle 2, at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω.



17. If the collision occurs at time $t_0 = \pi/(2\omega)$, then the value of $4b^2/a^2$ will be __________.

Answer (04.25)

Sol. At $t = 0$, 2 is at mean position

$\therefore \quad u_2 = a\omega$ towards left after collision, velocity will exchange

$\therefore \quad v_2 = \frac{a\omega}{2}$ towards right

$u_1 = a\omega$ towards right

$\therefore \quad v_{cm} = \frac{3a\omega}{4}$

$\frac{v_{cm}}{a\omega} = \frac{3}{4} = 0.75$

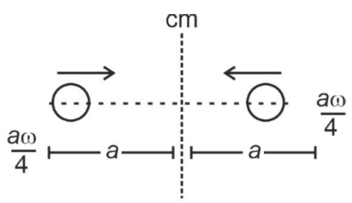
At $t = \frac{\pi}{2\omega}$, $u_2 = 0$

After collision, $v_2 = \frac{a\omega}{2}$ towards right

$u_1 = 0$

$\therefore \quad v_{cm} = \frac{a\omega}{4}$ towards right

w.r.t.



$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{a\omega}{4} = \omega\sqrt{A^2 - a^2}$$

$$\frac{a^2}{16} + a^2 = A^2$$

$$\frac{17}{16}a^2 = A^2 = b^2$$

$$\therefore \quad b^2 = \frac{17}{16}a^2$$

$$\frac{4b^2}{a^2} = \frac{17}{4}$$

$$= 4.25$$

PART-III : CHEMISTRY

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. According to Bohr's model, the highest kinetic energy is associated with the electron in the

(A) First orbit of H atom

(B) First orbit of He^{+}

(C) Second orbit of He^{+}

(D) Second orbit of Li^{2+}

Answer (B)

Sol. K.E. of electron in n^{th} Bohr's orbit,

$$K.E. = 13.6\ \frac{Z^2}{n^2}\ \text{eV/atom}$$

$$n = 1\ (\text{H-atom}) \rightarrow K.E. \propto \frac{1^2}{1^2} = 1$$

$$n = 1\ (He^{+}\ \text{ion}) \rightarrow K.E. \propto \frac{2^2}{1^2} = 4$$

$$n = 2\ (He^{+}\ \text{ion}) \rightarrow K.E. \propto \frac{2^2}{2^2} = 1$$

$$n = 2\ (Li^{2+}\ \text{ion}) \rightarrow K.E. \propto \frac{3^2}{2^2} = \frac{9}{4}$$

Highest for $\rightarrow$ n = 1 of He^{+} ion.

2. In a metal deficient oxide sample, $\mathbf{M_XY_2O_4}$ (**M** and **Y** are metals), **M** is present in both +2 and +3 oxidation states and **Y** is in +3 oxidation state. If the fraction of $\mathbf{M^{2+}}$ ions present in **M** is $\frac{1}{3}$, the value of **X** is _____.

(A) 0.25
(B) 0.33
(C) 0.67
(D) 0.75

Answer (D)

Sol. $M_XY_2O_4$

$M^{+2} = \frac{X}{3}, \; M^{+3} = \frac{2X}{3}$

So, total of O.N. of all atoms

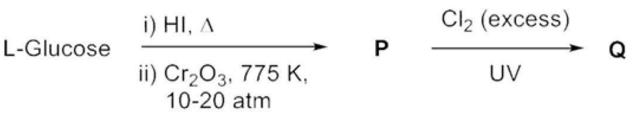
$$\frac{2X}{3} + 3\left(\frac{2X}{3}\right) + 2(+3) + 4(-2) = 0$$

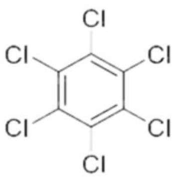
$$\frac{2X}{3} + 2X + 6 - 8 = 0$$

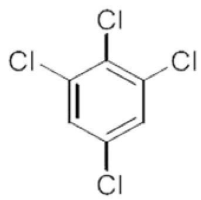
$$\frac{8X}{3} = 2$$

$$X = \frac{6}{8} = \frac{3}{4} = 0.75$$

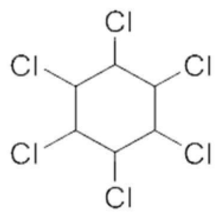
3. In the following reaction sequence, the major product **Q** is



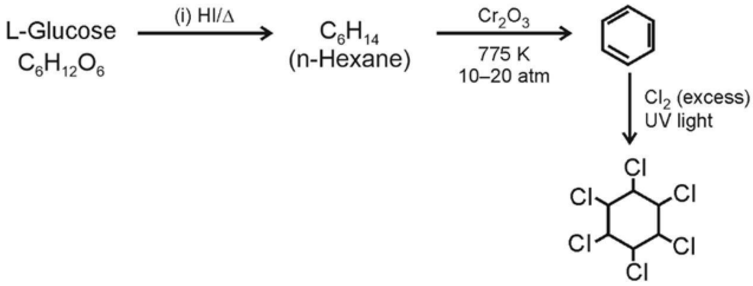
(A)


(B)


(C)


(D)


Answer (D)

Sol. 

(BHC)

4. The species formed on fluorination of phosphorus pentachloride in a polar organic solvent are

(A) $[PF_4]^+[PF_6]^-$ and $[PCl_4]^+[PF_6]^-$

(B) $[PCl_4]^+[PCl_4F_2]^-$ and $[PCl_4]^+[PF_6]^-$

(C) PF_3 and PCl_3

(D) PF_5 and PCl_3

Answer (B)

Sol. If PCl_5 is fluorinated in a polar solvent, ionic isomers are formed. e.g.:-

$[PCl_4]^+[PCl_4F_2]^-$ (colourless crystals)

and $[PCl_4]^+[PF_6]^-$ (white crystals)

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

5. An aqueous solution of hydrazine (N_2H_4) is electrochemically oxidized by O_2, thereby releasing chemical energy in the form of electrical energy. One of the products generated from the electrochemical reaction is $N_2(g)$.

Choose the correct statement(s) about the above process

(A) OH^- ions react with N_2H_4 at the anode to form $N_2(g)$ and water, releasing 4 electrons to the anode.

(B) At the cathode, N_2H_4 breaks to $N_2(g)$ and nascent hydrogen released at the electrode reacts with oxygen to form water.

(C) At the cathode, molecular oxygen gets converted to OH^-.

(D) Oxides of nitrogen are major by-products of the electrochemical process.

Answer (A, C)

Sol. $\underset{\substack{\text{Oxidation}\\(\text{Anode})}}{\overset{-2}{N_2}H_4} + \underset{\substack{\text{Reduction}\\(\text{Cathode})}}{\overset{0}{O_2}} \longrightarrow \overset{0}{N_2} + H_2O^{-2}$

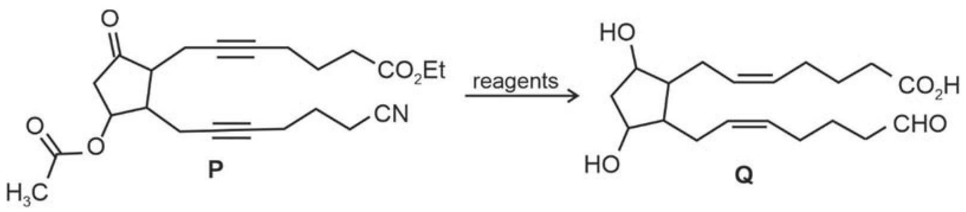
At anode: $N_2H_4 + 4OH^- \longrightarrow N_2 + 4H_2O + 4e^-$

At cathode: $O_2 + 2H_2O + 4e^- \longrightarrow 4OH^-$

Complete reaction: $N_2H_4 + O_2 \longrightarrow N_2 + 2H_2O$

Statements (A) and (C) are correct.

6. The option(s) with correct sequence of reagents for the conversion of **P** to **Q** is(are)



(A) i) Lindlar's catalyst, H_2; ii) $SnCl_2/HCl$; iii) $NaBH_4$; iv) H_3O^+

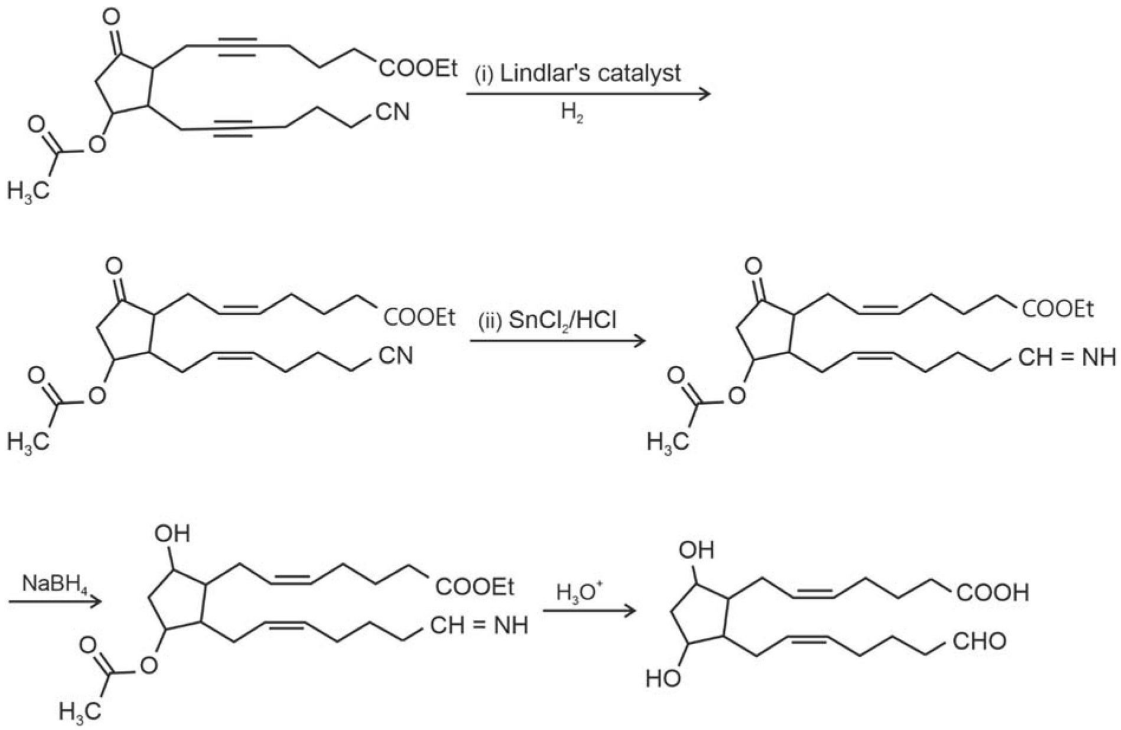
(B) i) Lindlar's catalyst, H_2; ii) H_3O^+; iii) $SnCl_2/HCl$; iv) $NaBH_4$

(C) i) $NaBH_4$; ii) $SnCl_2/HCl$; iii) H_3O^+; iv) Lindlar's catalyst, H_2

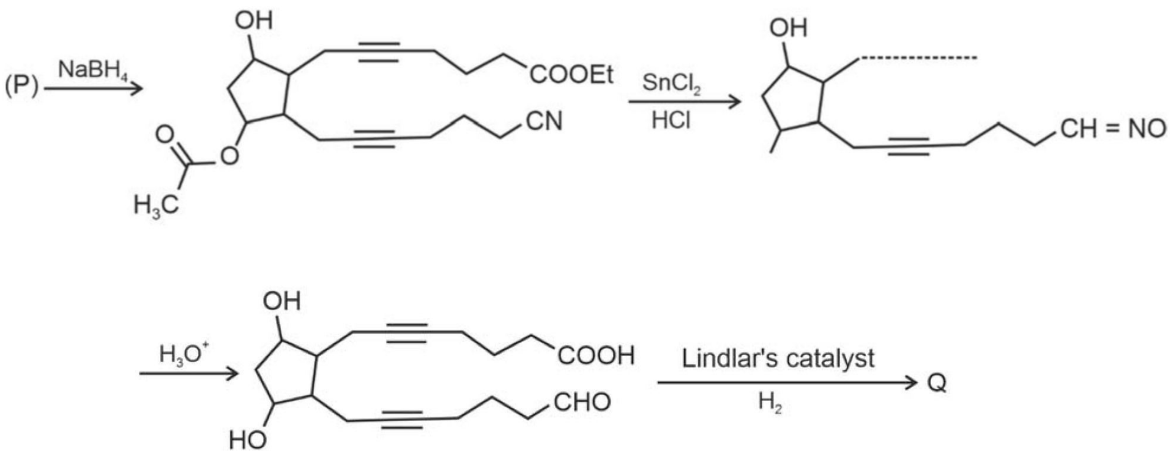
(D) i) Lindlar's catalyst, H_2; ii) $NaBH_4$; iii) $SnCl_2/HCl$; iv) H_3O^+

Answer (A, C, D)

Sol. (A)



(C)



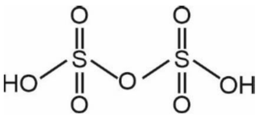
(D) $(P) \xrightarrow{\text{(i) Lindlar's catalyst, } H_2;\ \text{(ii) } NaBH_4;\ \text{(iii) } SnCl_2/HCl;\ \text{(iv) } H_3O^+} (Q)$

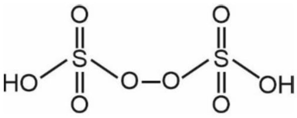
7. The compound(s) having peroxide linkage is(are)

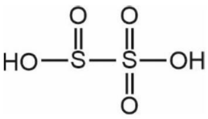
(A) $H_2S_2O_7$ (B) $H_2S_2O_8$

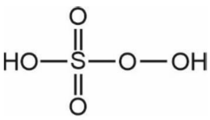
(C) $H_2S_2O_5$ (D) H_2SO_5

Answer (B, D)

Sol. $H_2S_2O_7$ 

$H_2S_2O_8$ 

$H_2S_2O_5$ 

H_2SO_5 

SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

8. To form a complete monolayer of acetic acid on 1 g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies $P \times 10^{-23}$ m^2 surface area on charcoal, the value of P is________.

[Use given data : Surface area of charcoal = 1.5×10^2 m^2g^{-1}; Avogadro's number (N_A) = 6.0×10^{23} mol^{-1}]

Answer (2500)

Sol. Number of moles of unadsorbed CH_3COOH = $\frac{40\times1}{1000}=4\times10^{-2}$ mol

Number of moles of adsorbed CH_3COOH = $\frac{100\times0.5}{1000}-4\times10^{-2}$

= 10^{-2} mol

Surface area occupied by one molecule of

$CH_3COOH=\frac{1.5\times10^{2}}{10^{-2}\times6\times10^{23}}=\frac{150\times10^{2}\times10^{-23}}{6}$

= 2500×10^{-23} m^2

$\therefore$ As per question P = 2500

9. Vessel-1 contains **w_2** g of a non-volatile solute **X** dissolved in **w_1** g of water. Vessel-2 contains **w_2** g of another non-volatile solute **Y** dissolved in **w_1** g of water. Both the vessels are at the same temperature and pressure. The molar mass of **X** is 80% of that of **Y**. The van't Hoff factor for **X** is 1.2 times of that of **Y** for their respective concentrations.

The elevation of boiling point for solution in Vessel-1 is _____ % of the solution in Vessel-2.

Answer (150)

Sol. Vessel-I

$$\left(\Delta T_B\right)_I=i_X\frac{w_2}{M_X}.\frac{1}{w_1}\times1000\times K_b$$

M_X = Molar mass of 'X'

Vessel-II

$$\left(\Delta T_B\right)_{II}=i_Y\frac{w_2}{M_Y}.\frac{1}{w_1}\times1000\times K_B$$

M_Y = Molar mass of 'Y'

$$\frac{\left(\Delta T_b\right)_I}{\left(\Delta T_b\right)_{II}}\times100=\frac{i_X}{i_Y}.\frac{M_Y}{M_X}\times100$$

$$=\ 1.2\times\frac{100}{80}\times100$$

= 150%

10. For a double strand DNA, one strand is given below:

5′ A G T C A C G T A A G T C 3′

The amount of energy required to split the double strand DNA into two single strands is _____ kcal mol^{-1}.

[Given: Average energy per H-bond for A-T base pair = 1.0 kcal mol^{-1}, G-C base pair = 1.5 kcal mol^{-1}, and A-U base pair = 1.25 kcal mol^{-1}. Ignore electrostatic repulsion between the phosphate groups.]

Answer (41)

Sol.

A	G	T	C	A	C	G	T	A	A	G	T	C
||	|||	||	|||	||	|||	|||	||	||	||	|||	||	|||
T	C	A	G	T	G	C	A	T	T	C	A	G
(1)	(i)	(2)	(ii)	(3)	(iii)	(iv)	(4)	(5)	(6)	(v)	(7)	(vi)

Total energy = [BE H-bond A – T × No. of A = T pair × 2] + [BE H-bond G – C × No. of G ≡ C pair × 3]

= [1 × 7 × 2] + [1.5 × 6 × 3]

= 14 + 27

= 41 kcal

11. A sample initially contains only U-238 isotope of uranium. With time, some of the U-238 radioactively decays into Pb-206 while the rest of it remains undisintegrated.

When the age of the sample is **P** × 10^8 years, the ratio of mass of Pb-206 to that of U-238 in the sample is found to be 7. The value of **P** is______.

[Given : Half-life of U-238 is 4.5 × 10^9 years; $\log_e 2$ = 0.693]

Answer (143)

Sol. Life of sample $\rightarrow$ t years

$[A]_0 \propto$ Initial mole of U-238

$[A]_t \propto$ Final mole of U-238

$$\frac{[A]_0}{[A]_t} = \frac{\frac{1}{238}+\frac{7}{206}}{\frac{1}{238}}$$

$$= \frac{0.0042+0.0340}{0.0042}$$

$= 9.1$

$$= \frac{2.303\log 2\times t}{4.5\times 10^9} = 2.303\log 9.1$$

$t = 14.27 \times 10^9$ years

$= 142.7 \times 10^9$ years

P = 142.7

P $\simeq$ 143

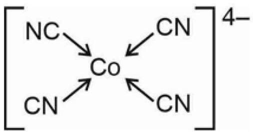
12. Among $[Co(CN)_4]^{4-}$, $[Co(CO)_3(NO)]$, XeF_4, $[PCl_4]^+$, $[PdCl_4]^{2-}$, $[ICl_4]^-$, $[Cu(CN)_4]^{3-}$ and P_4 the total number of species with tetrahedral geometry is _____.

Answer (3)

Sol. $[Co(CN)_4]^{4-} \Rightarrow Co^0 \Rightarrow 3d^74s^2$

Due to SFL, CN^- pairing and transference of electron takes place and hybridisation is dsp^2

Geometry $\Rightarrow$ Square planer

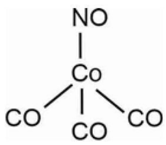


$[Co(CO)_3NO]$

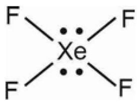
$Co^{-1} \Rightarrow 3d^{10}$ due to SFL CO and NO

sp^3 hybridisation

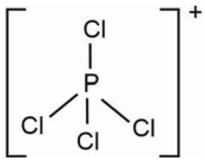
Geometry = Tetrahedral



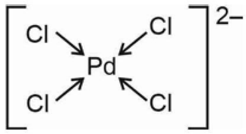
$XeF_4 \Rightarrow$ 4bp + 2lp $\Rightarrow sp^3d^2$

Square planer 

$PCl_4^+ \Rightarrow$ 4pb + 0lp

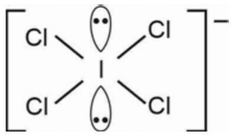
$sp^3 \Rightarrow$ tetrahedral 

$[PdCl_4]^{2-} \Rightarrow Pd^{2+}$, Cl^- behaves as SFL

$Pd^{2+} \Rightarrow 4d^8 \Rightarrow dsp^2 \Rightarrow$ square planer 

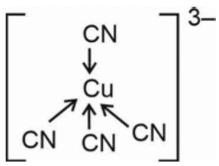
$ICl_4^- \Rightarrow$ 4bp + 2lp

sp^3d^2

square planer 

$[Cu(CN)_4]^{3-} \Rightarrow Cu^{+1} \Rightarrow 3d^{10}$

$\Rightarrow sp^3$

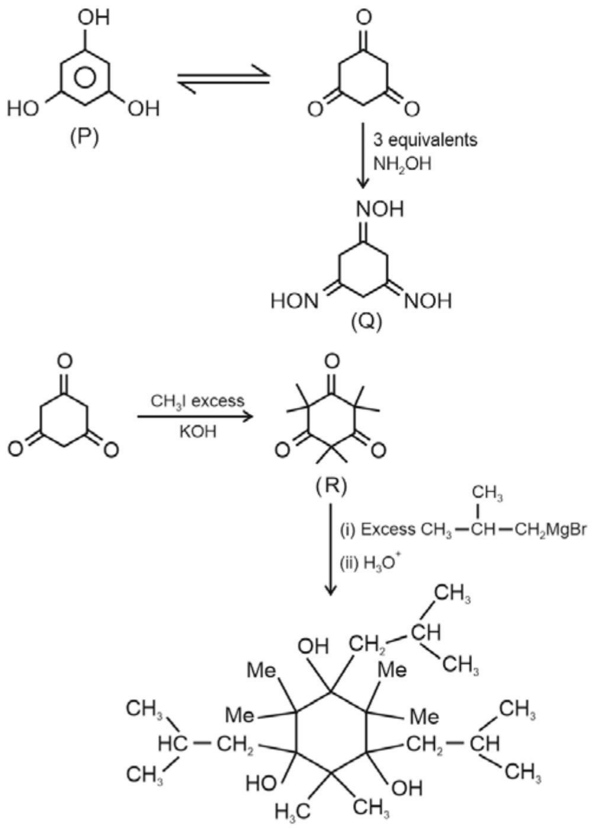
Tetrahedral 

13. An organic compound **P** having molecular formula $C_6H_6O_3$ gives ferric chloride test and does not have intramolecular hydrogen bond. The compound **P** reacts with 3 equivalents of NH_2OH to produce oxime **Q**. Treatment of **P** with excess methyl iodide in the presence of KOH produces compound **R** as the major product. Reaction of **R** with excess *iso*-butylmagnesium bromide followed by treatment with H_3O^+ gives compound **S** as the major product.

The total number of methyl ($-CH_3$) group(s) in compound **S** is _____.

Answer (12)

Sol.



Number of CH_3 groups = 12

SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

PARAGRAPH I

An organic compound **P** with molecular formula $C_9H_{18}O_2$ decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with H_2O_2 gives **Q** and **R**. While compound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

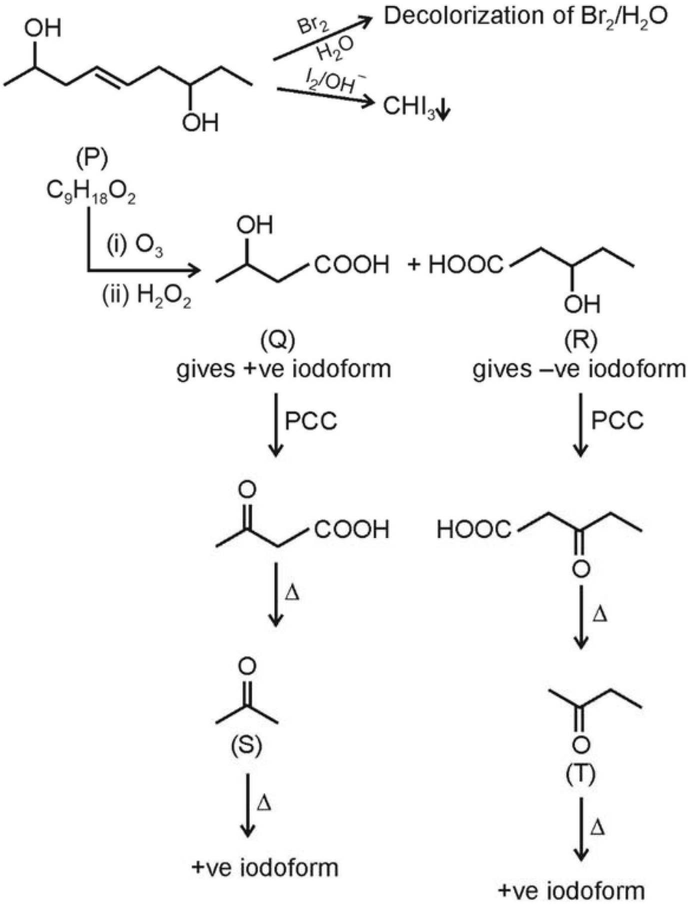
Complete copolymerization of 500 moles of **Q** and 500 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass: H =1, C = 12, O =16]

14. Sum of number of oxygen atoms in **S** and **T** is _____.

Answer (2)

Sol.



Sum of number of O-atoms in **S** and **T** = 1 + 1 = 2

PARAGRAPH I

An organic compound **P** with molecular formula $C_9H_{18}O_2$ decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with H_2O_2 gives **Q** and **R**. While compound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

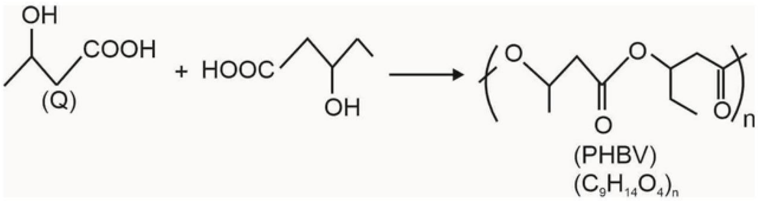
Complete copolymerization of 500 moles of **Q** and 500 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass: H = 1, C = 12, O = 16]

15. The molecular weight of **U** is _____.

Answer (102018)

Sol.



Mol. wt. of polymer = (104 × 500) + (118 × 500) – 18 × 499

= 52000 + 59000 – 8982

= 102018 g

PARAGRAPH II

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of zinc chloride to **P** in a slightly acidic medium results in a sparingly soluble complex **Q**.

16. The number of moles of potassium iodide required to produce two moles of **P** is ______.

Answer (2)

Sol. From this equation we need 2 mol of KI

$2KI + 2K_3[Fe(CN)_6] \rightarrow I_2 + 2K_4[Fe(CN)_6]$

$2K_4[Fe(CN)_6] + 3ZnCl_2 \rightarrow K_2Zn_3[Fe(CN)_6]_2 + 6KCl$

PARAGRAPH II

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of zinc chloride to **P** in a slightly acidic medium results in a sparingly soluble complex **Q**.

17. The number of zinc ions present in the molecular formula of **Q** is _____.

Answer (3)

Sol. From this equation we need 2 mol of KI

$$2KI + 2K_3[Fe(CN)_6] \rightarrow I_2 + 2K_4[Fe(CN)_6]$$

$$2K_4[Fe(CN)_6] + 3ZnCl_2 \rightarrow K_2Zn_3[Fe(CN)_6]_2 + 6KCl$$