# JEE Advanced 2023 Question Paper with Solution

Joint Entrance Examination – Advanced

# JEE(Advanced) EXAMINATION – 2023

**(Held On Sunday 04th June, 2023)**

**PAPER-1**

## PHYSICS

**SECTION-1 : (Maximum Marks : 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : −2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 marks;

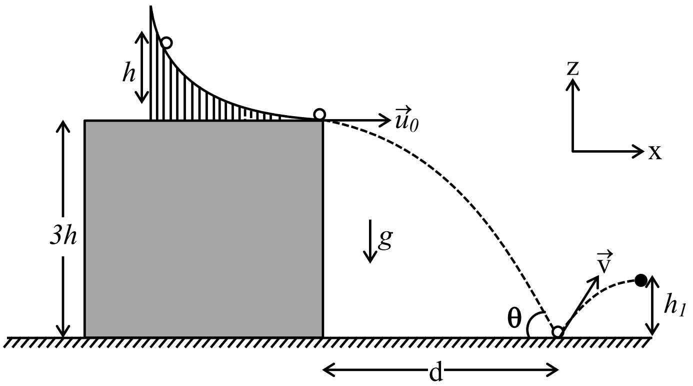
  choosing ONLY (B) will get +1 marks;

  choosing ONLY (D) will get +1 marks;

  choosing no option (i.e. the question is unanswered) will get 0 marks; and

  choosing any other combination of options will get –2 marks.

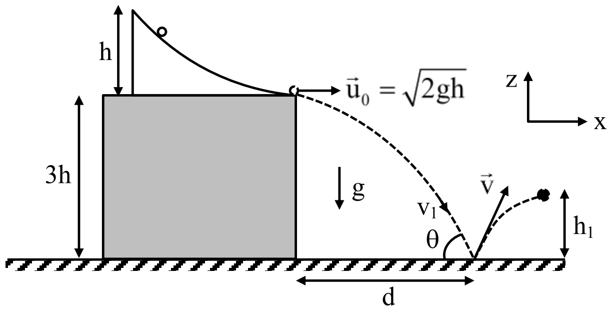
**1.** A slide with a frictionless curved surface, which becomes horizontal at its lower end,, is fixed on the terrace of a building of height $3h$ from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height $h$ from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance $d$ from the building making an angle $\theta$ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height $h_1$. The acceleration due to gravity is $g$ and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct?




(A) $\vec{u}_0 = \sqrt{2gh}\hat{x}$ (B) $\vec{v} = \sqrt{2gh}(\hat{x} - \hat{z})$ (C) $\theta = 60°$ (D) $d/h_1 = 2\sqrt{3}$

**Ans. (A,C,D)**

**Sol.**




$$\vec{v}_1 = \sqrt{2gh}\,\hat{i} - \sqrt{2g3h}\,\hat{k}$$

$$\vec{v} = \sqrt{2gh}\,\hat{i} + \sqrt{2g3h} \times \frac{1}{\sqrt{3}}\hat{k}$$

$$= \sqrt{2gh}\,\hat{i} + \sqrt{2gh}\,\hat{k}$$

$$\tan\theta = \frac{\sqrt{2g3h}}{\sqrt{2gh}} = \sqrt{3} \quad \theta = 60°$$

$$h_1 = \frac{v_{1y}^2}{2g} = \frac{2gh}{2g} = h$$

$$d = v_x t = \sqrt{2gh} \times \sqrt{\frac{2 \times 3h}{g}}$$

$$= \sqrt{2gh}\sqrt{\frac{6h}{g}} = 2\sqrt{3}h$$

$$= \frac{d}{h_1} = 2\sqrt{3}$$

**2.** A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60°$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{min} = 30°$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?

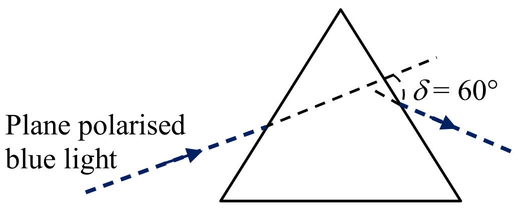



Figure-1

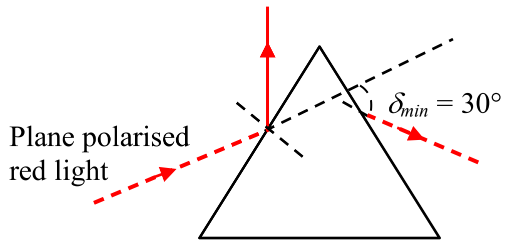



Figure-2

(A) The blue light is polarized in the plane of incidence.

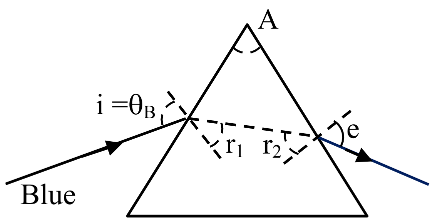
(B) The angle of the prism is 45°.

(C) The refractive index of the material of the prism for red light is $\sqrt{2}$.

(D) The angle of refraction for blue light in air at the exit plane of the prism is 60°.

**Ans. (A,C,D)**

**Sol.**




$\tan\theta_B = \mu_B = \sqrt{3}$

$i = \theta_B = 60°$

$1 \sin 60° = \sqrt{3}\sin r_1$

$r_1 = 30°$

$r_1 + r_2 = A$

$\delta = (i + e) - A$

$60° = 60° + e - A$

$e = A$

$\sqrt{3}\sin r_2 = 1 \sin e$

$\sqrt{3}\sin(A - 30) = \sin A$

Solving

$A = 60°$

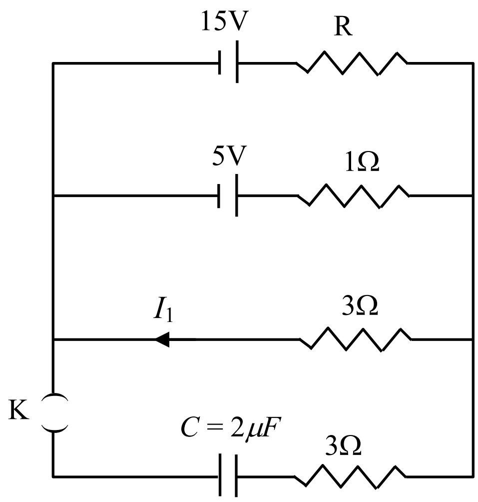
$\therefore\ e = 60°$

For red light

$$\mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}} = \sqrt{2}$$

**3.** In a circuit shown in the figure, the capacitor *C* is initially uncharged and the key *K* is open. In this condition, a current of 1 A flows through the 1 Ω resistor. The key is closed at time $t = t_0$. Which of the following statement(s) is(are) correct?

[Given: $e^{-1} = 0.36$]




(A) The value of the resistance *R* is 3Ω.

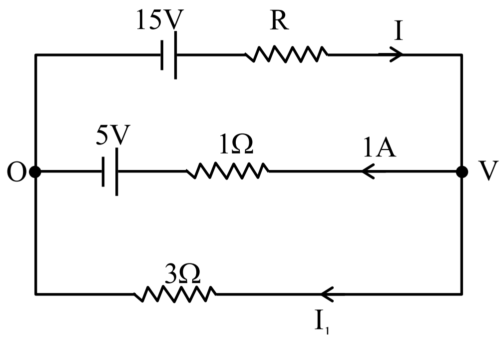
(B) For $t < t_0$, the value of current $I_1$ is 2A.

(C) At $t = t_0 + 7.2$ μs, the current in the capacitor is 0.6 A.

(D) For $t \to \infty$, the charge on the capacitor is 12 $\mu C$.

**Ans. (A,B,C,D)**

**Sol.**




By writing voltage drop across 1Ω

$\Rightarrow 0 + 5 + 1 \times 1 = V$

$V = 6$

$\Rightarrow$ Similarly across R

$0 + 15 - I \times R = 6$

$IR = 9$

$\Rightarrow$ across $3\Omega$

$6 - 3\,I_1 = 0$

$I_1 = 2A$

Hence option (B) is correct

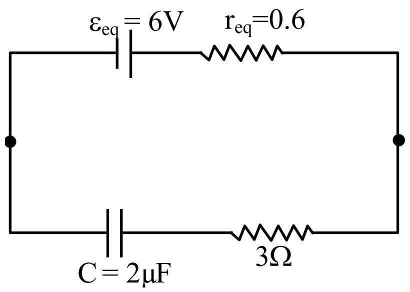
$\Rightarrow$ $I = 1 + 2$ (by KCL)

$I = 3$

$IR = 9$

$R = 3\Omega$

Option (A) is correct




$$\varepsilon = \frac{\frac{15}{3} + \frac{5}{1} + \frac{0}{3}}{\frac{1}{3} + \frac{1}{1} + \frac{1}{3}} = 10 \times \frac{3}{5} = 6V$$

$q_{max} = 2 \times 6 = 12\mu C$

$$i = \frac{6}{3.6} e^{-\frac{t}{\tau}}$$

$$= \frac{5}{3} e - \frac{7.2}{7.2} = \frac{5}{3} e^{-1} \approx 0.6A$$

## SECTION-2 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  *Full Marks* : +3 If **ONLY** the correct option is chosen;
  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
  *Negative Marks* : −1 In all other cases.

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**4.** A bar of mas $M = 1.00$ kg and length $L = 0.20$ m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass $m = 0.10$ kg is moving on the same horizontal surface with 5.00 m $s^{-1}$ speed on a path perpendicular to the bar. It hits the bar at a distance $L/2$ from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity $\omega$. Which of the following statement is correct?

(A) $\omega = 6.98$ rad $s^{-1}$ and v = 4.30 m $s^{-1}$

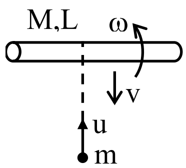
(B) $\omega = 3.75$ rad $s^{-1}$ and v = 4.30 m $s^{-1}$

(C) $\omega = 3.75$ rad $s^{-1}$ and v = 10.0 m $s^{-1}$

(D) $\omega = 6.80$ rad $s^{-1}$ and v = 4.10 m $s^{-1}$

**Ans. (A)**

**Sol.**




Applying angular momentum conservation about hinge

$$mv\frac{L}{2} + 0 = -mv\frac{L}{2} + \frac{ML^2}{3}\omega \quad \text{....(i)}$$

Also from eq. of restitution

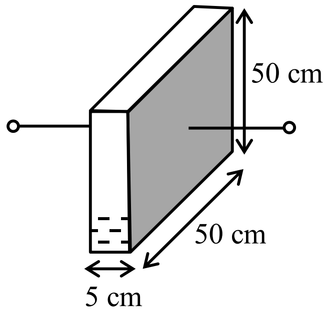
$$e = 1 = \frac{\omega\frac{L}{2} + V}{u} \Rightarrow u = \omega\frac{L}{2} + V \quad \text{....(ii)}$$

Solving (i) & (ii)

$\omega \approx 6.98$ rad/sec & v = 4.30 m/s

Hence option (A)

**5.** A container has a base of 50 cm × 5 cm and height 50 cm, as shown in the figure. It has two parallel electrically conducting walls each of area 50 cm × 50 cm. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of 250 $cm^3\ s^{-1}$. What is the value of the capacitance of the container after 10 seconds? [Given: Permittivity of free space $\epsilon_0 = 9 \times 10^{-12}\ C^2 N^{-1} m^{-2}$, the effects of the non-conducting walls on the capacitance are negligible]




(A) 27 pF (B) 63 pF (C) 81 pF (D) 135 pF

**Ans. (B)**

**Sol.** In t = 10 sec volume of liquid is

V = 2500 cc

$$h = \frac{2500}{50 \times 5} = 10\text{cm}$$

$$C_d = \frac{A_d \varepsilon_0 k}{d}$$

$$= \frac{50 \times 10^{-2} \times 10 \times 10^{-2} \varepsilon_0 \times 3}{5 \times 10^{-2}} = 3\varepsilon_0$$

$$C_a = \frac{A_a \varepsilon_0}{d} = \frac{50 \times 10^{-2} \times 40 \times 10^{-2} \varepsilon_0}{5 \times 10^{-2}} = 4\varepsilon_0$$

$C = C_a + C_d = 7\varepsilon_0$

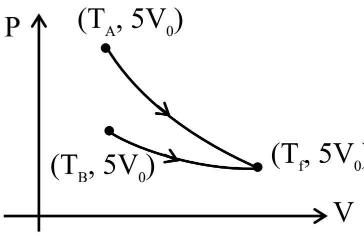
$= 7 \times 9 \times 10^{-12} = 63$ Pf

**6.** One mole of an ideal gas expands adiabatically from an initial state ($T_A$, $V_0$) to final state ($T_f$, $5V_0$). Another mole of the same gas expands isothermally from a different initial state ($T_B$, $V_0$) to the same final state ($T_f$, $5V_0$). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is $\gamma$. What is the ratio $T_A/T_B$?

(A) $5^{\gamma-1}$ (B) $5^{1-\gamma}$ (C) $5^{\gamma}$ (D) $5^{1+\gamma}$

**Ans. (A)**

**Sol.**




$$T_A V_0^{\gamma-1} = T_f (5V_0)^{\gamma-1}$$

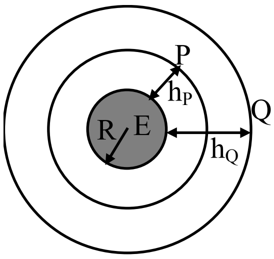
$$\frac{T_A}{T_f} = 5^{\gamma-1} = \frac{T_A}{T_B}$$

**7.** Two satellites P and Q are moving in different circular orbits around the Earth (radius *R*). The heights of P and Q from the Earth surface are $h_P$ and $h_Q$, respectively, where $h_p = R/3$. The accelerations of P and Q due to Earth's gravity are $g_P$ and $g_Q$, respectively. If $g_P/g_Q = 36/25$, what is the value of $h_Q$?

(A) $3R/5$ (B) $R/6$ (C) $6R/5$ (D) $5R/6$

**Ans. (A)**

**Sol.**




$$\frac{g_P}{g_Q} = \frac{\frac{GM}{r_P^2}}{\frac{GM}{r_Q^2}} = \left(\frac{r_Q}{r_P}\right)^2$$

$$\frac{36}{25} = \left(\frac{r_Q}{r_P}\right)^2$$

$$\frac{r_Q}{r_P} = \frac{6}{5}$$

$$r_Q = \frac{6}{5} r_P$$

$$R + h_Q = \frac{6}{5}\left(R + \frac{R}{3}\right)$$

$$h_Q = \frac{24}{15}R - R = \frac{9}{15}R = \frac{3}{5}R$$

## SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  *Full Marks* : +4 If **ONLY** the correct integer is entered;

  *Zero Marks* : 0 In all other cases.

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**8.** A Hydrogen-like atom has atomic number $Z$. Photons emitted in the electronic transitions from level $n = 4$ to level $n = 3$ in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1.95 eV. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of $Z$ is _______.

[Given: $hc$ = 1240 eV-nm and $Rhc$ = 13.6 eV, where $R$ is the Rydberg constant, $h$ is the Planck's constant and $c$ is the speed of light in vacuum]

**Ans. (3)**

**Sol.** n = 4 n = 3

$-1.51Z^2$eV $-0.85\ Z^2$ eV

$E = E_4 - E_3 = 0.66\ Z^2$ eV

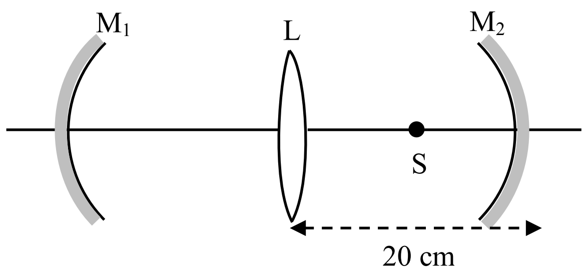
$K_{max} = E - W$

$0.66\ Z^2\ \ 1.95 + 4 = 5.95$

$$W = 0.66Z^2 - 1.95 = \frac{hc}{\lambda} = \frac{1240}{310}$$

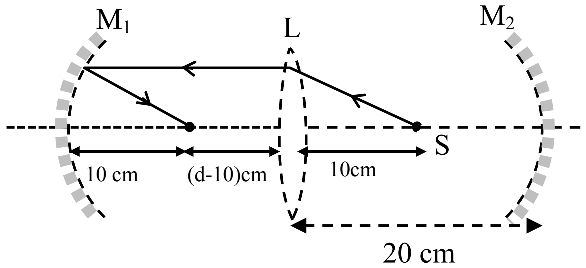
$\therefore$ Z = 3

**9.** An optical arrangement consists of two concave mirrors $M_1$ and $M_2$, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of $M_1$ and $M_2$ are 20 cm and 24 cm, respectively. The distance between L and $M_2$ is 20 cm. A point object S is placed at the mid-point between L and $M_2$ on the axis. When the distance between L and $M_1$ is $n/7$ cm, one of the images coincides with S. The value of $n$ is _______.




**Ans. (80 or 150 or 220)**

**Sol.**




**Two cases are possible if I[st] refraction on lens :**
Since object is at focus ⇒ light will become parallel.
I[st] reflection at $M_1$ :-
Light is parallel ⇒ Image will be at focus.
II[nd] refraction from L :-
$u = -(d - 10)$
$f = 10$ cm

$$\frac{1}{v} - \frac{1}{\mu} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{d-10} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{(d-10)} \quad \ldots \text{(i)}$$

This v will be object for $M_2$, and image should be at 10 cm

$$\frac{1}{\mu} + \frac{1}{v_1} = \frac{1}{f}$$

$$-\frac{1}{(20-v)} - \frac{1}{10} = -\frac{1}{12}$$

$$\frac{1}{12} - \frac{1}{10} = \frac{1}{20-v}$$

$$-\frac{2}{120} = \frac{1}{20-v}$$

$20 - v = -60$
$v = 80$ cm
From equation (i)

$$\frac{1}{80} = \frac{1}{10} - \frac{1}{d-10}$$

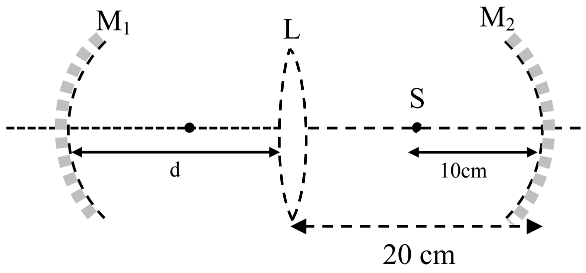
$$\frac{1}{d-10} = \frac{1}{10} - \frac{1}{80}$$

$$\frac{1}{d-10} = \frac{80-10}{800} = \frac{70}{800}$$

$$d - 10 = \frac{80}{7} \Rightarrow d = 10 + \frac{80}{7} = \frac{150}{7}$$

$\boxed{n = 150}$

**Case-2: If 1$^{st}$ reflection on mirror $m_2$**




For $m_2$

$$\frac{1}{V_1}+\frac{1}{-10}=\frac{1}{-12}$$

$V_1 = 60$ cm

Then refraction on lens L

$u_2 = -80$ cm

$$\frac{1}{V_2}-\frac{1}{-60}=\frac{1}{10}$$

$$V_2=\frac{80}{7}$$

Then reflection on $m_2$

Either $V_2$ is at centre (normal incidence)

$$d-\frac{80}{7}=20$$

$$d=\frac{220}{7}$$

$$\frac{n}{7}=\frac{220}{7},$$

$$\boxed{n=220}$$

$V_2$ is at pole of $m_2$

$$d-\frac{80}{7}=0$$

$$d=\frac{80}{7}$$

$$\frac{n}{7}=\frac{80}{7}$$

$$\boxed{n=80}$$

**10.** In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is $10 \pm 0.1$ cm and the distance of its real image from the lens is $20 \pm 0.2$ cm. The error in the determination of focal length of the lens is *n* %. The value of *n* is _______.

**Ans. (1)**

**Sol.** $u = 10 \pm 0.1$ cm, $\quad v = 20 \pm 0.2$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v^2}dv+\frac{1}{u^2}du=-\frac{1}{f^2}df$$

$$\frac{1}{20}+\frac{1}{10}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{3}{20} \Rightarrow f=\frac{20}{3}\text{cm}$$

$$\Rightarrow \frac{1}{(20)^2}(0.2)+\frac{1}{(10)^2}(0.1)=\frac{9}{400}df$$

$$df=\frac{1}{9}\left(\frac{400}{400}\times 0.2+\frac{400}{100}\times 0.1\right)$$

$$df=\frac{1}{9}(0.2+0.4) \Rightarrow df=\frac{0.6}{9}$$

$$\frac{df}{f}=\frac{0.6}{9}\times\frac{3}{20}=\frac{1}{100}$$

% error = 1 %

**Sol.** $\frac{1}{V}-\frac{1}{U}=\frac{1}{f}$; $\quad +\frac{1}{20}+\frac{1}{10}=\frac{1}{f}$

$-\frac{1}{V^2}dv+\frac{dU}{u^2}=-\frac{df}{f^2} \quad \frac{1+2}{20}=\frac{1}{f};\ f=\frac{20}{3}$

$$\frac{0.1}{100}+\frac{0.2}{400}=\frac{f\%}{f}$$

$$\frac{0.4+0.2}{400}=\frac{\Delta f}{f\left(\frac{20}{3}\right)}$$

$$\frac{0.6\times 20}{400\times 3}=\frac{\Delta f}{f}$$

$$\frac{1}{100}=\frac{\Delta f}{f}$$

% change in f is 1%

**11.** A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas ($\gamma = 5/3$) and one mole of an ideal diatomic gas ($\gamma = 7/5$). Here, $\gamma$ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is ________ Joule.

**Ans. (121)**

**Sol.** At constant pressure

$W = nR\Delta T = 66$

$\Delta U = n(C_V)_{mix}\Delta T$

$$(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$(C_V)_{mix} = \frac{2\times\frac{3}{2}R + 1\times\frac{5}{2}R}{3}$$

$$(C_V)_{mix} = \frac{11}{6}R$$

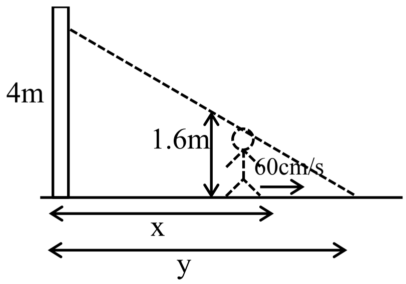
$$\Delta U = \frac{11}{6}(nR\Delta T)$$

$$\Delta U = \frac{11}{6}\times 66 = 121J$$

**12.** A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm $s^{-1}$, the speed of the tip of the person's shadow on the ground with respect to the person is _______ cm $s^{-1}$.

**Ans. (40)**

**Sol.**




$$\frac{4}{y} = \frac{1.6}{y-x}$$

$4y - 4x = 1.6y$

$2.4\ y = 4x$

$X = 0.6y$

$$\frac{dx}{dt} = 0.6\times\frac{dy}{dt}$$

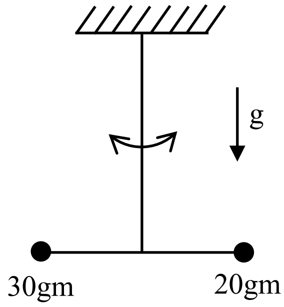
$$60 = 0.6\times\frac{dy}{dt}$$

$$\therefore \frac{dy}{dt} = 100cm/s$$

Speed of tip of person's

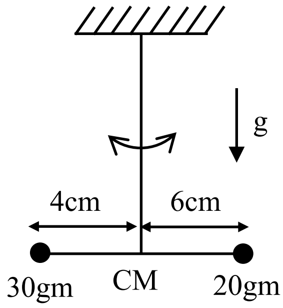
Shadow w.r.t person = 100 – 60 = 40 cm/s

**13.** Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is $1.2 \times 10^{-8}$ N m rad$^{-1}$. The angular frequency of the oscillations in $n \times 10^{-3}$ rad s$^{-1}$. The value of $n$ is _____.




**Ans. (10)**

**Sol.**




$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}$$

$$\Rightarrow \omega = \sqrt{\frac{C}{I}}$$

Where I = moment of inertia

$I = (30)(4)^2 + (20)(6)^2$

$= 1200$ gm-cm$^2$

$= 1.2 \times 10^{-4}$ kg-m$^2$

$$\Rightarrow \omega = \sqrt{\frac{1.2\times10^{-8}}{1.2\times10^{-4}}}$$

$$\Rightarrow \omega = \sqrt{10^{-4}}$$

$\omega = (10^{-2})$

$n \times 10^{-3} = 10^{-2} \Rightarrow n = 10$

## SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  *Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen;

  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

  *Negative Marks* : –1 In all other cases.

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**14.** List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

| **List-I** | | **List-II** | |
|---|---|---|---|
| (P) | ${}^{238}_{92}U \rightarrow {}^{234}_{91}Pa$ | (1) | one $\alpha$ particle and one $\beta^+$ particle |
| (Q) | ${}^{214}_{82}Pb \rightarrow {}^{210}_{82}Pb$ | (2) | three $\beta^-$ particles and one $\alpha$ particle |
| (R) | ${}^{210}_{81}Tl \rightarrow {}^{206}_{82}Pb$ | (3) | two $\beta^-$ particles and one $\alpha$ particle |
| (S) | ${}^{228}_{91}Pa \rightarrow {}^{224}_{88}Ra$ | (4) | one $\alpha$ particle and one $\beta^-$ particle |
| | | (5) | one $\alpha$ particle and two $\beta^+$ particles |

(A) $P \rightarrow 4,\ Q \rightarrow 3,\ R \rightarrow 2,\ S \rightarrow 1$
(B) $P \rightarrow 4,\ Q \rightarrow 1,\ R \rightarrow 2,\ S \rightarrow 5$
(C) $P \rightarrow 5,\ Q \rightarrow 3,\ R \rightarrow 1,\ S \rightarrow 4$
(D) $P \rightarrow 5,\ Q \rightarrow 1,\ R \rightarrow 3,\ S \rightarrow 2$

**Ans. (A)**

**Sol.** ${}_{Z_1}Z^{A_1} \rightarrow {}_{Z_2}Y^{A_2} + N_1\,{}_2He^4 + N_2\,{}_1e^0 + N_3\,{}_{-1}e^0$

Conservation of charge

$Z_1 = Z_2 + 2N_1 + N_2 - N_3$ … (i)

Conservation of nucleons.

$A_1 = A_2 + 4N_1$

$$N_1 = \frac{A_1 - A_2}{4} \quad \text{… (ii)}$$

From (i) and (ii)

$$N_2 - N_3 = Z_1 - Z_2 - \left(\frac{A_1 - A_2}{2}\right)$$

(P) ${}_{92}U^{238} \rightarrow {}_{91}Pa^{234}$

$$N_1 = \frac{238 - 234}{4} = 1 \rightarrow 1\alpha$$

$$N_2 - N_3 = (92 - 91) - \left(\frac{4}{2}\right) = -1 \rightarrow 1\beta^-$$

(Q) ${}_{82}Pb^{214} \rightarrow {}_{82}Pb^{210}$

$$N_1 = \frac{214-210}{4} = 1 \rightarrow 1\alpha$$

$$N_2 - N_3 = (82-82) - \left(\frac{4}{2}\right) = -2 \rightarrow 2\beta^-$$

(R) ${}_{81}T\ell^{210} \rightarrow {}_{82}Pb^{206}$

$$N_1 = \frac{210-206}{4} = 1 \rightarrow 1\alpha$$

$$N_2 - N_3 = (81-83) - \frac{4}{2} = -3 \rightarrow 3\beta^-$$

(S) ${}_{91}Pa^{228} \rightarrow {}_{88}Ra^{224}$

$$N_1 = \frac{228-224}{4} = 1\alpha$$

$$N_2 - N_3 = (91-88) - \frac{4}{2} = 1\beta^+$$

**15.** Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option.

[Given: Wien's constant as $2.9 \times 10^{-3}$ m-K and $\frac{hc}{e} = 1.24 \times 10^{-6}$ V-m]

| List-I | | List-II | |
|---|---|---|---|
| (P) | 2000 K | (1) | The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4 eV |
| (Q) | 3000 K | (2) | The radiation at peak wavelength is visible to human eye. |
| (R) | 5000 K | (3) | The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction. |
| (S) | 10000 K | (4) | The power emitted per unit area is 1/16 of that emitted by a blackbody at temperature 6000 K. |
| | | (5) | The radiation at peak emission wavelength can be used to image human bones. |

(A) $P \rightarrow 3,\ Q \rightarrow 5,\ R \rightarrow 2,\ S \rightarrow 3$

(B) $P \rightarrow 3,\ Q \rightarrow 2,\ R \rightarrow 4,\ S \rightarrow 1$

(C) $P \rightarrow 3,\ Q \rightarrow 4,\ R \rightarrow 2,\ S \rightarrow 1$

(D) $P \rightarrow 1,\ Q \rightarrow 2,\ R \rightarrow 5,\ S \rightarrow 3$

**Ans. (C)**

**Sol.** $\Rightarrow$ For option (P) temperature is minimum hence $\lambda m$ will be maximum $\beta = \frac{\lambda D}{d}$ $\Rightarrow$ $\beta$ will also be maximum

$\Rightarrow$ For option (Q) T = 3000

$$\lambda m = \frac{b}{T} = \frac{2.9\times10^{-3}}{30000}$$

$$\lambda m = \frac{2.9}{3}\times10^{-6}$$

$= 0.96 \times 10^{-6}$

= 966.6 nm

$P_{3000} = 6A\ (3000)^4$

$P_{6000} = 6A\ (6000)^4$

$$\frac{P_{3000}}{P_{6000}} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$P_{3000} = \frac{1}{16}P_{6000}$$

Q – 4

$\Rightarrow$ For (R) T = 5000 K

$$\lambda m = \frac{2.9\times10^{-3}}{5\times10^{3}} = 0.58\times10^{-6}$$

= 580 nm

Visible to human eyes

R – 2

$\Rightarrow$ For (S) T = 10,000 $\rightarrow$ maximum

Hence (3) is wrong as it has minimum ($\lambda m$)

**16.** A series LCR circuit is connected to a 45 sin ($\omega t$) Volt source. The resonant angular frequency of the circuit is $10^5$ rad $s^{-1}$ and current amplitude at resonance is $I_0$.. When the angular frequency of the source is $\omega = 8 \times 10^4$ rad $s^{-1}$, the current amplitude in the circuit is 0.05 $I_0$. If $L$ = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option.

| | **List-I** | | **List-II** |
|---|---|---|---|
| (P) | $I_0$ in mA | (1) | 44.4 |
| (Q) | The quality factor of the circuit | (2) | 18 |
| (R) | The bandwidth of the circuit in rad $s^{-1}$ | (3) | 400 |
| (S) | The peak power dissipated at resonance in Watt | (4) | 2250 |
| | | (5) | 500 |

(A) $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 5, S \rightarrow 1$

(B) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$

(C) $P \rightarrow 4, Q \rightarrow 5, R \rightarrow 3, S \rightarrow 1$

(D) $P \rightarrow 4, Q \rightarrow 2, R \rightarrow 1, S \rightarrow 5$

**Ans. (B)**

**Sol.** $V = 45 \sin \omega t$, $L = 50$ mH

$$\omega_0 = 10^5 \text{rad/s} = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega_0^2} = \frac{1}{5 \times 10^{-2} \times 10^{10}}$$

$= 2 \times 10^{-9}$ F

$$I_0 = \frac{45}{R}$$

$\omega = 8 \times 10^4$ rad/s $= 0.8\ \omega_0$

$$I = 0.05 I_0 = \frac{I_0}{20} \Rightarrow Z = 20R$$

$$X_L = 8 \times 10^4 \times 5 \times 10^{-2}\,\Omega = 4\text{k}\Omega$$

$$X_C = \frac{1}{8 \times 10^4 \times 2 \times 10^{-9}} = \frac{1}{16} \times 10^5\,\Omega = \frac{25}{4}\text{k}\Omega$$

$$Z^2 = R^2 + \left(X_C - X_L\right)^2$$

$$400R^2 = R^2 + \left(\frac{9}{4}\text{k}\Omega\right)^2$$

$$R = \frac{\frac{9}{4}\text{k}\Omega}{\sqrt{399}} \approx \frac{9}{80}\text{k}\Omega = \frac{900}{8}\Omega$$

$$I_0 = \frac{V_0}{R} = \frac{45 \times 8}{900} = \frac{8}{20}\text{A} \approx 0.4\text{A} = 400\text{mA}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{8}{900}\sqrt{\frac{5 \times 10^{-2}}{2 \times 10^{-9}}} = \frac{8}{900}\sqrt{25 \times 10^6}$$

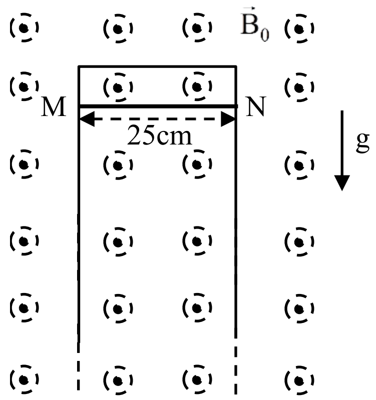
$$Q = \frac{8}{900} \times 5000 = 44.4$$

$$Q = \frac{\omega_0}{\Delta\omega} \Rightarrow \Delta\omega = \frac{\omega_0}{Q} = \frac{10^5}{44.4} = 2250.0$$

$$P_{max} = I_0^2 R = \frac{45^2}{R^2} \times R = \frac{45^2}{R} = \frac{45^2}{900} \times 8 = 18.4\text{W}$$

**17.** A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field $B_0$ = 4 T directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option.

[Given: The acceleration due to gravity $g = 10\ ms^{-2}$ and $e^{-1} = 0.4$]




| | List-I | | List-II |
|---|---|---|---|
| (P) | At t = 0.2 s, the magnitude of the induced emf in Volt | (1) | 0.07 |
| (Q) | At $t$ = 0.2 s, the magnitude of the magnetic force in Newton | (2) | 0.14 |
| (R) | At $t$ = 0.2 s, the power dissipated as heat in Watt | (3) | 1.20 |
| (S) | The magnitude of terminal velocity of the rod in m $s^{-1}$ | (4) | 0.12 |
| | | (5) | 2.00 |

(A) $P \to 5,\ Q \to 2,\ R \to 3,\ S \to 1$

(B) $P \to 3,\ Q \to 1,\ R \to 4,\ S \to 5$

(C) $P \to 4,\ Q \to 3,\ R \to 1,\ S \to 2$

(D) $P \to 3,\ Q \to 4,\ R \to 2,\ S \to 5$

**Ans. (D)**

**Sol.** From force equation

$$mg - Bi\ell = \frac{mdv}{dt}$$

$$mg - \frac{BBi\ell}{R} \times \ell = \frac{mdv}{dt}$$

$$\frac{mgR}{B^2\ell^2} - v = \frac{mR}{B^2\ell^2}\frac{dv}{dt}$$

$$\frac{B^2\ell^2}{mR}\int_{t=0}^{t} dt = \int_{0}^{v} \frac{dv}{\left(\frac{mgR}{B^2\ell^2} - v\right)}$$

Now $\frac{mgR}{B^2\ell^2} = \frac{20\times10^{-3}\times10\times10}{16\times\frac{1}{16}} = 2$

And $\frac{B^2\ell^2}{mR} = \frac{16\times\frac{1}{16}}{20\times10^{-3}\times10} = \frac{1}{0.2} = 5$

$\therefore 5t = \left[-\ell n(2-v)\right]_0^v$

$-5t = \ell n\left[\frac{2-v}{v}\right]$

$\therefore\ v = 2\ (1 - e^{-5t})$

At t = 0.2 sec $\quad v = 2\ (1 - e^{-5\times0.2})$

$v = 2\ (1 - 0.4)$

$v = 1.2$ m/s

(P) Now at t = 0.2 sec

The magnitude of the induced emf = E = $Bv\ell$

$= 4\times1.2\times\frac{1}{4} = 1.2\text{Volt}$

(Q) At t = 0.2 sec, the magnitude of magnetic force = $BI\ell\sin\theta$

$= B\times\frac{B\ell v}{R}\times\ell\times\sin 90°$

$= \frac{4\times4\times\frac{1}{4}\times1.3\times\frac{1}{4}}{10}$ = 0.12 Newton

(R) At t = 0.2 sec, the power dissipated as heat

$P = i^2R = \frac{v^2}{R} = \frac{1.2\times1.2}{10}$

P = 0.144 watt

(S) Magnitude of terminal velocity

At terminal velocity, the net force become zero

$\therefore\ mg = Bi\ell$

$mg = B\times\frac{B\ell v_t}{R}\times\ell$

$\therefore v_T = \frac{mgR}{B^2\ell^2} = \frac{20\times10^{-3}\times10\times10}{16\times\frac{1}{16}}$

$v_T = 2$ m/s

Hence, Answer is (D)

# CHEMISTRY

**SECTION-1 : (Maximum Marks : 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : −2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

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**1.** The correct statement(s) related to processes involved in the extraction of metals is(are)

(A) Roasting of Malachite produces Cuprite.

(B) Calcination of Calamine produces Zincite.

(C) Copper pyrites is heated with silica in a reverberatory furnace to remove iron.

(D) Impure silver is treated with aqueous KCN in the presence of oxygen followed by reduction with zinc metal.

**Ans. (B,C,D)**

**Sol.** $\Rightarrow$ Under roasting condition, the malachite will be converted into

$$CuCO_3.Cu(OH)_2 \rightarrow 2CuO + CO_2 + H_2O$$

$\Rightarrow$ $\underset{\text{(Calamine)}}{ZnCO_3} \rightarrow \underset{\text{(Zincite)}}{ZnO} + CO_2 \uparrow$

$\Rightarrow$ Copper pyrites is heated in a reverberatory furnace after mixing with silica. In the furnace, iron oxide 'slag of' as iron silicate and copper is produced in the form of copper matte.

$$FeO + SiO_2 \rightarrow \underset{\text{(Slag)}}{FeSiO_3}$$

$\Rightarrow$ $Ag + KCN + O_2 + H_2O \longrightarrow [Ag(CN)_2]^- + KOH$

$$[Ag(CN)_2]^- \xrightarrow{Zn} Ag\downarrow + [Zn(CN)_4]^{2-}$$

**2.** In the following reactions, **P**, **Q**, **R**, and **S** are the major products.

$$CH_3CH_2CH(CH_3)CH_2CN \xrightarrow[\text{(ii) PhMgBr, then } H_2O]{\text{(i) PhMgBr, then } H_3O^{\oplus}} \textbf{P}$$

$$Ph-H + CH_3\overset{\overset{\displaystyle O}{||}}{C}Cl \xrightarrow[\text{(ii) PhMgBr, then } H_2O]{\text{(i) anhyd. } AlCl_3} \textbf{Q}$$

$$CH_3CH_2\overset{\overset{\displaystyle O}{||}}{C}Cl \xrightarrow[\text{(ii) PhMgBr, then } H_2O]{\text{(i) } \frac{1}{2}(PhCH_2)_2Cd} \textbf{R}$$

$$PhCH_2CHO \xrightarrow[\substack{\text{(iii) HCN} \\ \text{(iv) } H_2SO_4, \Delta}]{\substack{\text{(i) PhMgBr, then } H_2O \\ \text{(ii) } CrO_3, \text{dil.} H_2SO_4}} \textbf{S}$$

The correct statement(s) about **P**, **Q**, **R**, and **S** is(are)

(A) Both **P** and **Q** have asymmetric carbon(s).

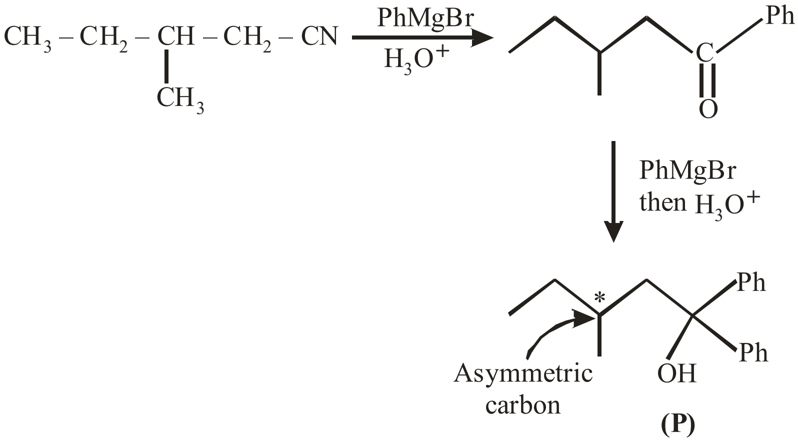
(B) Both **Q** and **R** have asymmetric carbon(s).

(C) Both **P** and **R** have asymmetric carbon(s).

(D) **P** has asymmetric carbon(s), **S** does **not** have any asymmetric carbon.

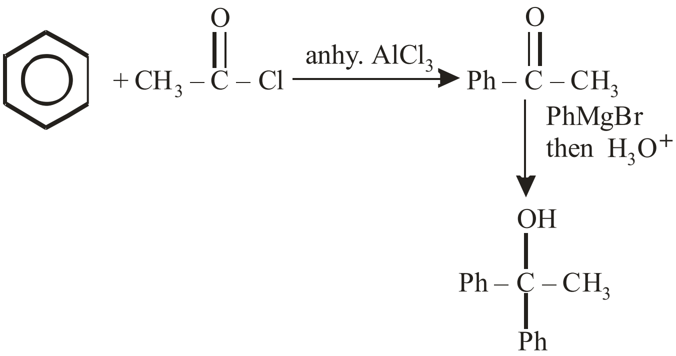
**Ans. (C,D)**

**Sol.** **Formation of P**




**(P)**

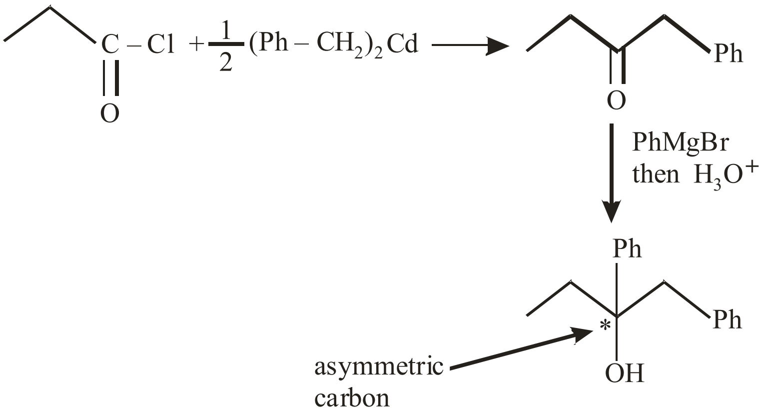
**Formation of Q**




**(Q)**

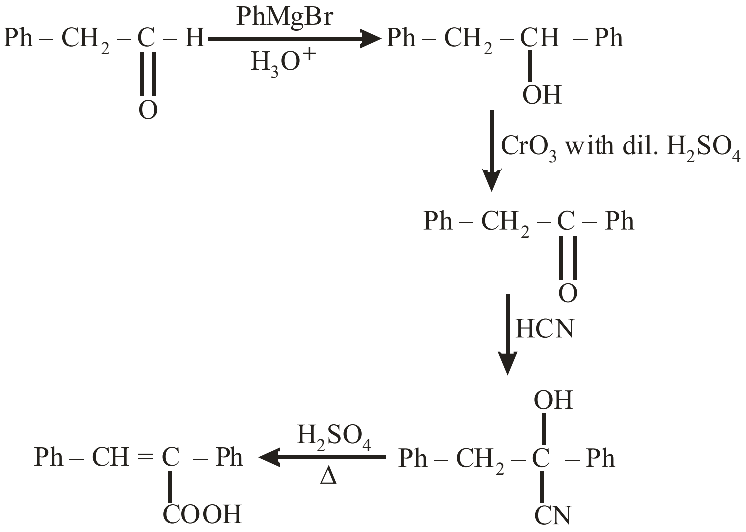
No asymmetric carbon

**Formation of R**




**(R)**

**Formation of S**

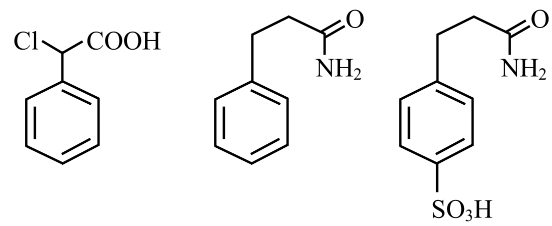


**(S)** No asymmetric carbon

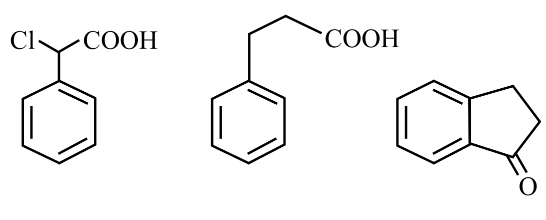
**3.** Consider the following reaction scheme and choose the correct option(s) for the major products **Q**, **R** and **S**.

$$\text{Styrene} \xrightarrow[\text{(ii) NaOH, } H_2O_2, H_2O]{\text{(i) } B_2H_6} \mathbf{P} \xrightarrow[\text{(ii) } Cl_2\text{, Red Phosphorus; (iii) } H_2O]{\text{(i) } CrO_3, H_2SO_4} \mathbf{Q}$$

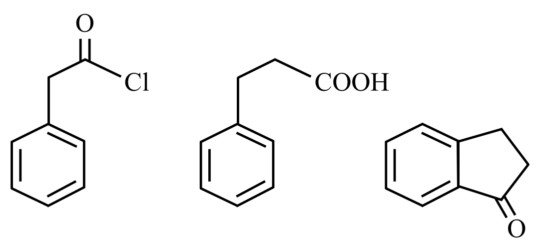
$$\mathbf{P} \xrightarrow[\text{(ii) NaCN; (iii) } H_3O^+, \Delta]{\text{(i) } SOCl_2} \mathbf{R} \xrightarrow{\text{conc. } H_2SO_4} \mathbf{S}$$

(A) 

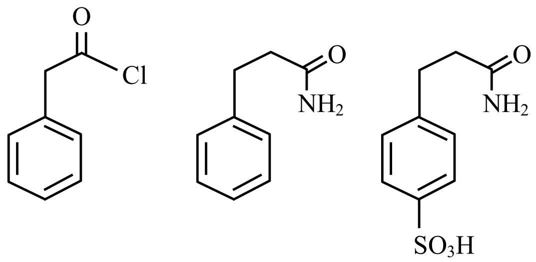
**Q** **R** **S**

(B) 

**Q** **R** **S**

(C) 

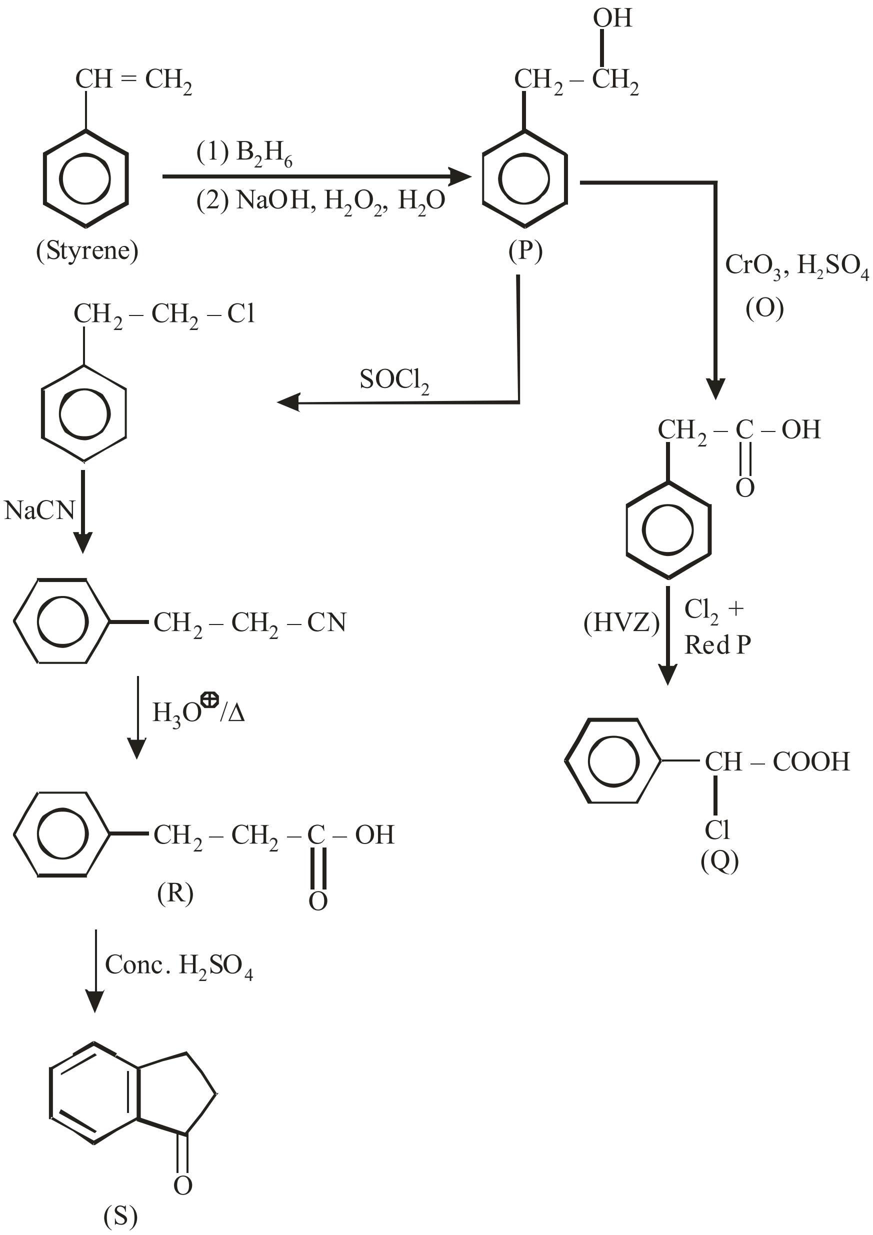
**Q** **R** **S**

(D) 

**Q** **R** **S**

**Ans. (B)**

**Sol.**



OH
CH = CH₂
CH₂ – CH₂
(1) B₂H₆
(2) NaOH, H₂O₂, H₂O
(Styrene)
(P)
CrO₃, H₂SO₄
(O)
CH₂ – CH₂ – Cl
SOCl₂
CH₂ – C – OH
O
NaCN
CH₂ – CH₂ – CN
(HVZ)
Cl₂ +
Red P
H₃O⊕/Δ
CH – COOH
Cl
(Q)
CH₂ – CH₂ – C – OH
(R)
O
Conc. H₂SO₄
O
(S)

## SECTION-2 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If **ONLY** the correct option is chosen;
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
*Negative Marks* : −1 In all other cases.

---

**4.** In the scheme given below, **X** and **Y**, respectively, are

$$\text{Metal halide} \xrightarrow{\text{aq. NaOH}} \text{White precipitate } \mathbf{(P)} + \text{Filtrate } \mathbf{(Q)}$$

$$\mathbf{P} \xrightarrow[\text{heat}]{\text{aq.}H_2SO_4,\ PbO_2(\text{excess})} \mathbf{X} \text{ (a coloured species in solution)}$$

$$\mathbf{Q} \xrightarrow[\text{warm}]{MnO(OH)_2,\ \text{Conc.}H_2SO_4} \mathbf{Y} \text{ (gives blue-coloration with KI-starch paper)}$$

(A) $CrO_4^{2-}$ and $Br_2$
(B) $MnO_4^{2-}$ and $Cl_2$
(C) $MnO_4^{-}$ and $Cl_2$
(D) $MnSO_4$ and HOCl

**Ans. (C)**

**Sol.** $$MnCl_2 + NaOH \rightarrow \underset{\text{(white ppt.)}}{\underset{\mathbf{(P)}}{Mn(OH)_2}}\downarrow + \underset{\text{(Filterate)}}{\underset{\mathbf{(Q)}}{NaCl}}$$

$$Mn(OH)_2 \xrightarrow[\text{heat}]{PbO_2+H^+(H_2SO_4)} \underset{\text{Purple}}{MnO_4^-} + Pb^{2+}$$

$$Cl^- \xrightarrow{MnO(OH)_2/\text{conc. } H_2SO_4/□} Cl_2 \xrightarrow{2I^-} \underbrace{(\text{Starch} + I_2)}_{\text{blue coloration}} + 2Cl^-$$

**5.** Plotting $1/\Lambda_m$ against $c\Lambda_m$ for aqueous solutions of a monobasic weak acid (HX) resulted in a straight line with y-axis intercept of P and slope of S. The ratio P/S is

[$\Lambda_m$ = molar conductivity

$\Lambda_m^\circ$ = limiting molar conductivity

c = molar concentration

$K_a$ = dissociation constant of HX]

(A) $K_a\ \Lambda_m^\circ$
(B) $K_a\ \Lambda_m^\circ / 2$
(C) $2\ K_a \Lambda_m^\circ$
(D) $1 / (K_a \Lambda_m^\circ)$

**Ans. (A)**

**Sol.** For weak acid, $\alpha = \frac{\Lambda_m}{\Lambda_0}$

$$K_a = \frac{C\alpha^2}{1-\alpha} \Rightarrow K_a\,(1-\alpha) = C\alpha^2$$

$$\Rightarrow K_a\left(1-\frac{\Lambda_m}{\Lambda_0}\right) = C\left(\frac{\Lambda_m}{\Lambda_0}\right)^2$$

$$\Rightarrow K_a - \frac{\Lambda_m K_a}{\Lambda_0} = \frac{C\Lambda_m^2}{(\Lambda_0)^2}$$

Divide by '$\Lambda_m$'

$$\Rightarrow \frac{K_a}{\Lambda_m} = \frac{C\Lambda_m}{(\Lambda_0)^2} + \frac{K_a}{\Lambda_0}$$

$$\Rightarrow \frac{1}{\Lambda_m} = \frac{C\Lambda_m}{K_a(\Lambda_0)^2} + \frac{1}{\Lambda_0}$$

Plot $\frac{1}{\Lambda_m}$ vs $C\,\Lambda_m$ has

Slope $= \frac{1}{K_a(\Lambda_0)^2} = S$

y-intercept $= \frac{1}{\Lambda_0} = P$

Then, $\frac{P}{S} = \frac{\frac{1}{\Lambda_0}}{\frac{1}{K_a(\Lambda_0)^2}} = K_a\Lambda_0$

**6.** On decreasing the $p$H from 7 to 2, the solubility of a sparingly soluble salt (MX) of a weak acid (HX) increased from $10^{-4}$ mol $L^{-1}$ to $10^{-3}$ mol $L^{-1}$. The $pK_a$ of HX is:

(A) 3 (B) 4 (C) 5 (D) 2

**Ans. (B)**

**Sol.** At pH = 7 $\Rightarrow$ pure water

solubility = $S_1 = \sqrt{K_{sp}}$

At pH = 2

$\Rightarrow$ $MX(s) + aq \xrightleftharpoons{K_{sp}} M^+(aq) + X^-(aq)$

$\qquad\qquad\qquad\qquad\qquad\qquad s \qquad\quad s-x$

$X^-(aq) + H^+(aq) \xrightleftharpoons{1/K_a} HX(aq)$

$s-x \qquad 10^{-2} \qquad\qquad\qquad x \simeq s$

Approximation : $s - x \simeq 0$ [$X^-$ is limiting reagent]

$\Rightarrow s \simeq x$

$\Rightarrow$ $s(s-x) = K_{sp}$ …… (1)

$$\frac{s}{(s-x)\,(10^{-2})} = \frac{1}{K_a} \qquad \text{…… (2)}$$

Multiply (1) × (2) $\Rightarrow \dfrac{s^2}{10^{-2}} = \dfrac{K_{sp}}{K_a}$

$$\Rightarrow s = \frac{\sqrt{K_{sp}}}{10\sqrt{K_a}}$$

Now given : $\dfrac{s}{s_1} = \dfrac{10^{-3}}{10^{-4}}$

$$\Rightarrow \frac{\dfrac{\sqrt{K_{sp}}}{10\sqrt{K_a}}}{\sqrt{K_{sp}}} = 10 \qquad \Rightarrow \frac{1}{10\sqrt{K_a}} = 10$$

$\Rightarrow \sqrt{K_a} = 10^{-2}$

$\Rightarrow K_a = 10^{-4}$

$\Rightarrow pK_a = 4$

**7.** In the given reaction scheme, **P** is a phenyl alkyl ether, **Q** is an aromatic compound; **R** and **S** are the major products.

$$\mathbf{P} \xrightarrow{HI} \mathbf{Q} \xrightarrow[\text{(iii)}\,H_3O^+]{\text{(i) NaOH, (ii)}\,CO_2} \mathbf{R} \xrightarrow[\text{(ii)}\,H_3O^+]{\text{(i)}\,(CH_3CO)_2O} \mathbf{S}$$

The correct statement about **S** is

(A) It primarily inhibits noradrenaline degrading enzymes.

(B) It inhibits the synthesis of prostaglandin.

(C) It is a narcotic drug.

(D) It is *ortho*-acetylbenzoic acid.

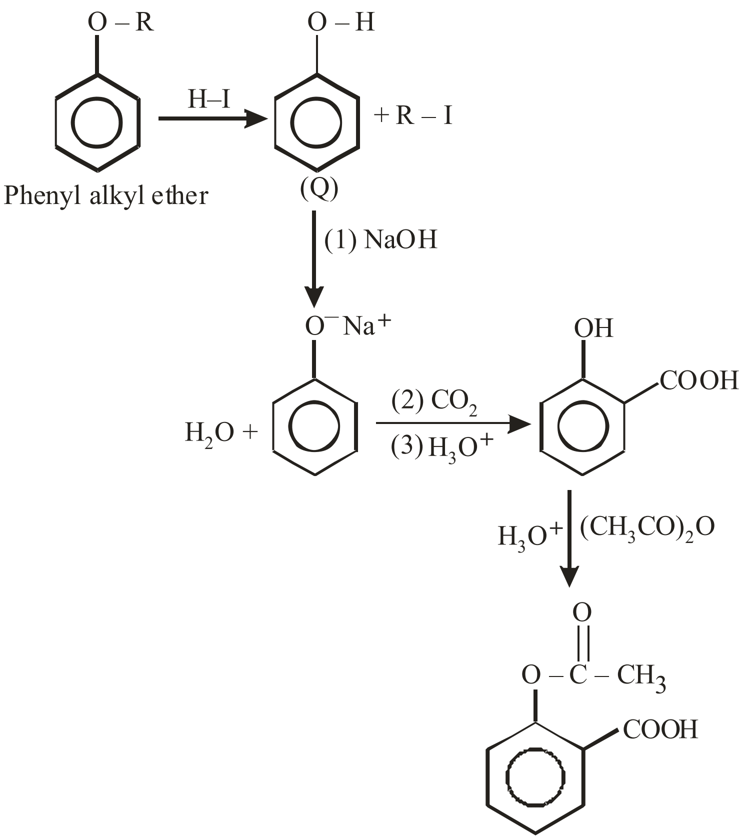
**Ans. (B)**

**Sol.** P is phenyl alkyl ether

Q is aromatic compound

R and S are the major product

i.e.




(Aspirin)
Pain killer
Acetyl salicylic acid
(Non-narcotic analgesic)

Correct ans is (B)

Aspirin inhibits the synthesis of chemicals known as prostaglandin's.

**SECTION-3 : (Maximum Marks : 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +4 **ONLY** If the correct integer is entered;
  *Zero Marks* : 0 In all other cases.

---

**8.** The stoichiometric reaction of 516 g of dimethyldichlorosilane with water results in a tetrameric cyclic product **X** in 75% yield. The weight (in g) of **X** obtained is___.

[Use, molar mass (g mol$^{-1}$): H = 1, C = 12, O = 16, Si = 28, Cl = 35.5]

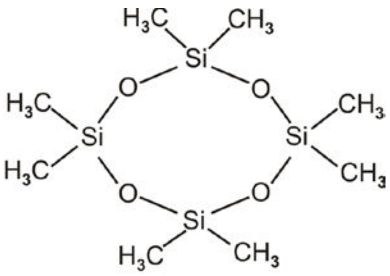
**Ans. (222)**

**Sol.** $4(CH_3)_2SiCl_2 + 4H_2O \xrightarrow{75\%} \underset{(X)}{(CH_3)_8Si_4O_4} + 8HCl$

$w = 516\,g$

$\underset{(moles)}{n} = \frac{516}{129}$

$= 4$



weight = 296 g

% yield = 75

The weight of $X\,(\text{in gram}) = 296 \times \frac{75}{100} = 222\,g$

**9.** A gas has a compressibility factor of 0.5 and a molar volume of 0.4 $dm^3\ mol^{-1}$ at a temperature of 800 K and pressure **x** atm. If it shows ideal gas behaviour at the same temperature and pressure, the molar volume will be **y** $dm^3\ mol^{-1}$. The value of **x**/**y** is ___.

[Use: Gas constant, $R = 8 \times 10^{-2}$ L atm $K^{-1}\ mol^{-1}$]

**Ans. (100)**

**Sol.** For gas : $Z = 0.5$, $V_m = 0.4$ L/mol

$T = 800$ K, $P = X$ atm.

$\Rightarrow\ Z = \frac{PV_m}{RT}$

$\Rightarrow\ \frac{X(0.4)}{0.08 \times 800} = 0.5$

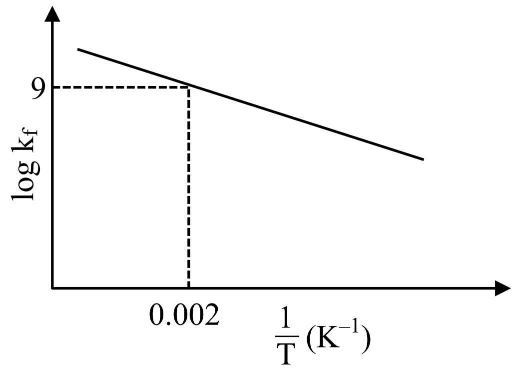
$\Rightarrow\ X = 80$

For ideal gas, $PV_m = RT$

$\Rightarrow\ V_m = \frac{RT}{P} = \frac{0.08 \times 800}{80} = 0.8\ L\ mol^{-1} = y$

Then, $\frac{x}{y} = \frac{80}{0.8} = 100.$

**10.** The plot of log $k_f$ versus $^1/_T$ for a reversible reaction A (g) $\rightleftharpoons$ P (g) is shown.




Pre-exponential factors for the forward and backward reactions are $10^{15}$ $s^{-1}$ and $10^{11}$ $s^{-1}$, respectively. If the value of log K for the reaction at 500 K is 6, the value of | log $k_b$ | at 250 K is ______.

[K = equilibrium constant of the reaction
$k_f$ = rate constant of forward reaction
$k_b$ = rate constant of backward reaction]

**Ans. (5)**

**Sol.** For reaction A(g) $\rightleftharpoons$ P(g)

$$\log k_f = \frac{-E_f}{2.303RT} + \log A_f$$ [Arrhenius equation for forward reaction]

From plot when, $\frac{1}{T} = 0.002$, log $k_f = 9$

$$\Rightarrow 9 = \frac{-E_f}{2.303R}(0.002) + \log(A_f)$$

Given : $A_f = 10^{15}$ $s^{-1}$

$$\Rightarrow 9 = \frac{-E_f}{2.303R}(0.002) + 15$$

$$\Rightarrow \frac{E_f}{2.303R} = \frac{6}{0.002} = 3000$$

Now, $K = \frac{k_f}{k_b} = \frac{A_f}{A_b}e^{-(E_f - E_b)/RT}$

$$\log K = -\frac{1}{2.303}\frac{(E_f - E_b)}{RT} + \log\left(\frac{10^{15}}{10^{11}}\right)$$

At 500 K

$$\Rightarrow 6 = \frac{-(E_f - E_b)}{500R\ (2.303)} + 4$$

$\Rightarrow$ (1000 R) (2.303) = $E_b - E_f$

$\Rightarrow$ (1000 R) (2.303) = $E_b$ – 3000 (2.303 R)

$\Rightarrow$ $E_b$ = 4000 R (2.303) ……… (1)

Now $k_b = A_b e^{-E_b/RT}$

$$\Rightarrow \log k_b = \frac{-E_b}{2.303\,RT} + \log A_b$$

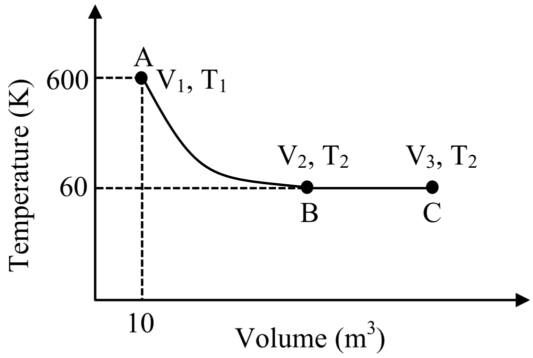
At 250 K

$$\Rightarrow \log k_b = -\frac{4000}{250} + \log(10^{11}) \qquad \text{[From equation (1)]}$$

$$= -16 + 11 = -5$$

$|\log k_b| = 5$

**11.** One mole of an ideal monoatomic gas undergoes two reversible processes ($A \to B$ and $B \to C$) as shown in the given figure :




$A \to B$ is an adiabatic process. If the total heat absorbed in the entire process ($A \to B$ and $B \to C$) is $RT_2$ ln 10, the value of 2 log $V_3$ is _________.

[Use, molar heat capacity of the gas at constant pressure, $C_{p,m} = \frac{5}{2}R$]

**Ans. (7)**

**Sol.** For $A \to B$ $\qquad 600\,V_1^{\gamma-1} = 60V_2^{\gamma-1}$ $(\gamma = 5/3)$

(Reversible adiabatic)

$$\Rightarrow 600\,(V_1)^{2/3} = 60\,(V_2)^{2/3}$$

$$\Rightarrow 10 = \left(\frac{V_2}{V_1}\right)^{2/3}$$

$$\Rightarrow 10 = \left(\frac{V_2}{10}\right)^{2/3}$$

$$\Rightarrow V_2 = 10(10)^{3/2} = 10^{5/2}$$

Now, $q_{net} = RT_2\,ln\,10 = 60\,R\,ln\,10 = q_{AB} + q_{BC}$

$\because \; q_{AB} = 0$

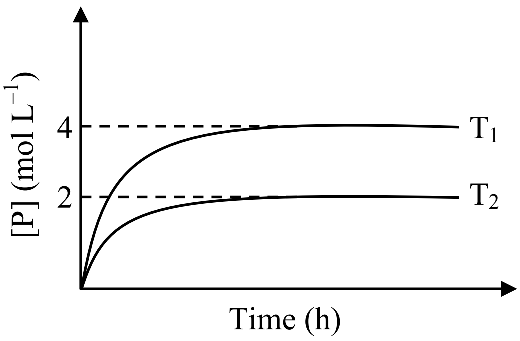
$$\Rightarrow q_{BC} = 60\,R\,ln\,10 = 60\,R\,ln\,\frac{V_3}{V_2} \qquad [\because B \to C \text{ is reversible isothermal}]$$

$$\Rightarrow 60\,R\,ln\,10 = 60\,R\,ln\left(\frac{V_3}{10^{5/2}}\right)$$

$$\Rightarrow \log 10 = \log V_3 - \frac{5}{2}$$

$$\Rightarrow \log V_3 = \frac{7}{2} \Rightarrow 2\log V_3 = 7$$

**12.** In a one-litre flask, 6 moles of A undergoes the reaction A (g) $\rightleftharpoons$ P (g). The progress of product formation at two temperatures (in Kelvin), $T_1$ and $T_2$, is shown in the figure:




If $T_1 = 2T_2$ and $(\Delta G_2^{\ominus} - \Delta G_1^{\ominus}) = RT_2 \ln x$, then the value of x is ______.

[$\Delta G_1^{\ominus}$ and $\Delta G_2^{\ominus}$ are standard Gibb's free energy change for the reaction at temperatures $T_1$ and $T_2$, respectively.]

**Ans. (8)**

**Sol.** At $T_1$ K : $\quad A(g) \rightleftharpoons P(g)$

t = 0 $\quad$ 6

t = ∞ $\quad$ 6 – x $\quad$ x = 4 (from plot)

$\Rightarrow$ At $T_1$ K : $K_{P_1} = \frac{4}{2} = 2$

At $T_2$ K : $\quad A(g) \rightleftharpoons P(g)$

t = 0 $\quad$ 6

t = ∞ $\quad$ 6 – y $\quad$ y = 2 (from plot)

$\Rightarrow$ At $T_2$ K : $K_{P_2} = \frac{2}{4} = \frac{1}{2}$

Now, $\Delta G_2^{o} = -RT_2\, ln\, K_{P_2} = -RT_2\, ln\, \frac{1}{2}$

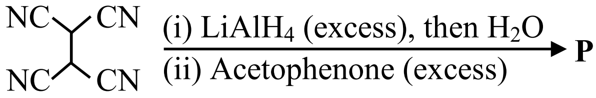
$\Rightarrow \Delta G_2^{o} = RT_2\, ln\, 2$

$\Delta G_1^{o} = -RT_1 \ln K_{P_1} = -RT_1\, ln\, 2 = -2RT_2\, ln\, 2$

Given : $\Delta G_2^{o} - \Delta G_1^{o} = RT_2\, ln\, 2 + 2RT_2\, ln\, 2 = 3RT_2\, ln\, 2 = RT_2\, ln\, x$

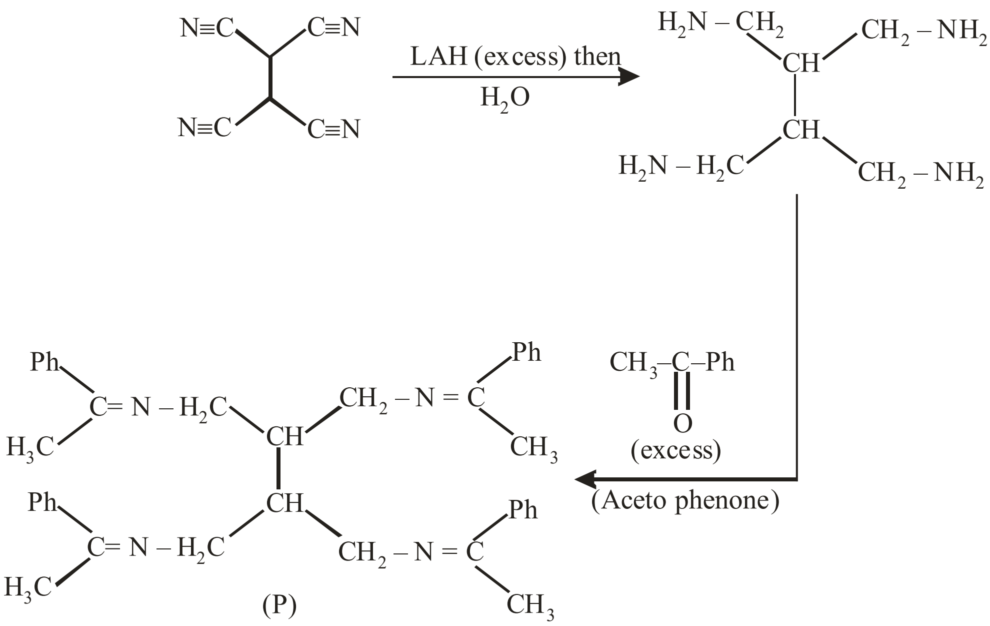
$\Rightarrow x = 2^3 = 8$

**13.** The total number of $sp^2$ hybridised carbon atoms in the major product **P** (a non-heterocyclic compound) of the following reaction is _______.




**Ans. (28)**

**Sol.**




Total number of $sp^2$ hybridised C-atom in P = 28

## SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
  *Negative Marks* : –1 In all other cases.

---

**14.** Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in List-II and choose the correct option.

| List-I | | List-II | |
|---|---|---|---|
| (P) | $P_2O_3 + 3H_2O \rightarrow$ | (1) | $P(O)(OCH_3)Cl_2$ |
| (Q) | $P_4 + 3NaOH + 3H_2O \rightarrow$ | (2) | $H_3PO_3$ |
| (R) | $PCl_5 + CH_3COOH \rightarrow$ | (3) | $PH_3$ |
| (S) | $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow$ | (4) | $POCl_3$ |
| | | (5) | $H_3PO_4$ |

(A) P → 2; Q → 3; R → 1; S → 5 (B) P → 3; Q → 5; R → 4; S → 2

(C) P → 5; Q → 2; R → 1; S → 3 (D) P → 2; Q → 3; R → 4; S → 5

**Ans. (D)**

**Sol.** (P) $P_2O_3 + 3H_2O \rightarrow 2\underline{H_3PO_3}$

(Q) $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + \underline{PH_3}$

(R) $PCl_5 + CH_3COOH \rightarrow CH_3COCl + \underline{POCl_3} + HCl$

(S) $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow 4Ag + 4HNO_3 + \underline{H_3PO_4}$

**15.** Match the electronic configurations in List-I with appropriate metal complex ions in List-II and choose the correct option.

[Atomic Number: Fe = 26, Mn = 25, Co = 27]

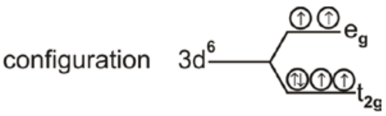
| List-I | | List-II | |
|---|---|---|---|
| (P) | $t_{2g}^6e_g^0$ | (1) | $[Fe(H_2O)_6]^{2+}$ |
| (Q) | $t_{2g}^3e_g^2$ | (2) | $[Mn(H_2O)_6]^{2+}$ |
| (R) | $e^2t_2^3$ | (3) | $[Co(NH_3)_6]^{3+}$ |
| (S) | $t_{2g}^4e_g^2$ | (4) | $[FeCl_4]^-$ |
| | | (5) | $[CoCl_4]^{2-}$ |

(A) P → 1; Q → 4; R → 2; S → 3 (B) P → 1; Q → 2; R → 4; S → 5

(C) P → 3; Q → 2; R → 5; S → 1 (D) P → 3; Q → 2; R → 4; S → 1

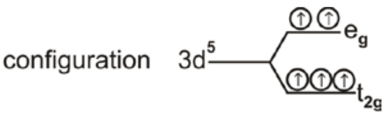
**Ans. (D)**

**Sol.** 1. $\underset{\text{WFL}}{[\overset{\text{II}}{\text{Fe}}(\text{H}_2\text{O})_6]^{+2}}$



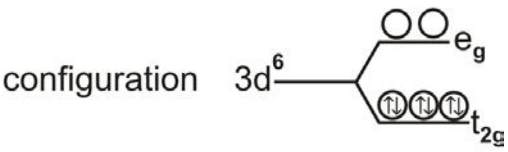

$t_{2g}^4\, e_g^2$ (S)

2. $\underset{\text{WFL}}{[\overset{\text{II}}{\text{Mn}}(\text{H}_2\text{O})_6]^{+2}}$



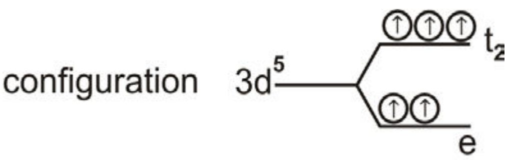

$t_{2g}^3\, e_g^2$ (Q)

3. $\underset{\text{SFL}}{[\overset{\text{III}}{\text{Co}}(\text{NH}_3)_6]^{+3}}$



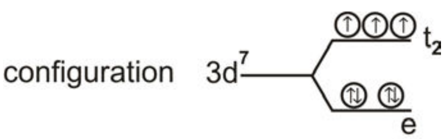

$t_{2g}^6\, e_g^0$ (P)

4. $\underset{\text{WFL}}{[\overset{\text{III}}{\text{Fe}}\text{Cl}_4]^{\ominus}}$



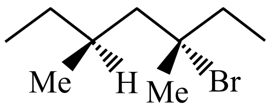

$e^2\, t_2^3$ (R)

5. $\underset{\text{WFL}}{[\overset{\text{II}}{\text{Co}}\text{Cl}_4]^{-2}}$




$e^4\, t_2^3$ (None)

**16.** Match the reactions in List-I with the features of their products in List-II and choose the correct option.

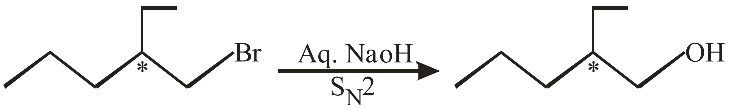
| List-I | List-II |
|---|---|
| (P) (-)-1-Bromo-2-ethylpentane (single enantiomer) $\xrightarrow[\text{S}_N2 \text{ reaction}]{\text{aq. NaOH}}$ | (1) Inversion of configuration |
| (Q) (-)-2-Bromopentane (single enantiomer) $\xrightarrow[\text{S}_N2 \text{ reaction}]{\text{aq. NaOH}}$ | (2) Retention of configuration |
| (R) (-)-3-Bromo-3-methylhexane (single enantiomer) $\xrightarrow[\text{S}_N1 \text{ reaction}]{\text{aq. NaOH}}$ | (3) Mixture of enantiomers |
| (S)  (Single enantiomer) $\xrightarrow[\text{S}_N1 \text{ reaction}]{\text{aq. NaOH}}$ | (4) Mixture of structural isomers |
| | (5) Mixture of diastereomers |

(A) P → 1; Q → 2; R → 5; S → 3 (B) P → 2; Q → 1; R → 3; S → 5

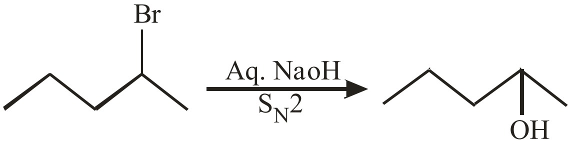
(C) P → 1; Q → 2; R → 5; S → 4 (D) P → 2; Q → 4; R → 3; S → 5

**Ans. (B)**

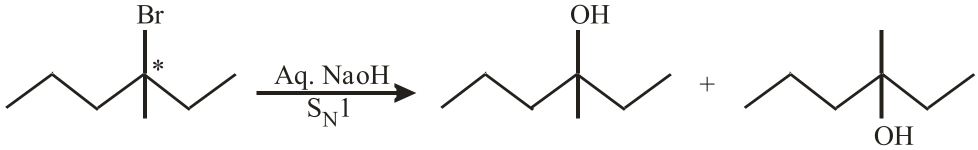
**Sol.** **P → 2, Q → 1, R → 3, S → 5**

(P) 

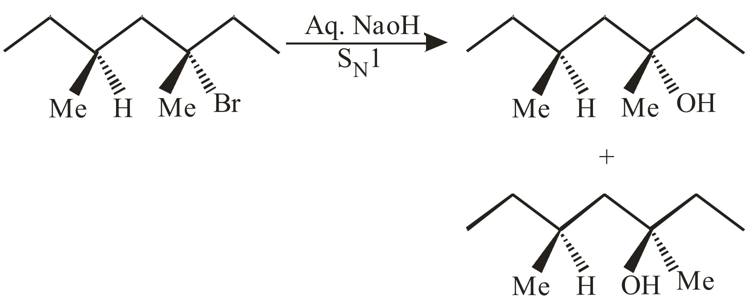

Retention of configuration

(Q) 


Inversion of configuration

(R) 


Mixture of enantiomers




(S) Diastereomeric mixture

**17.** The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option.

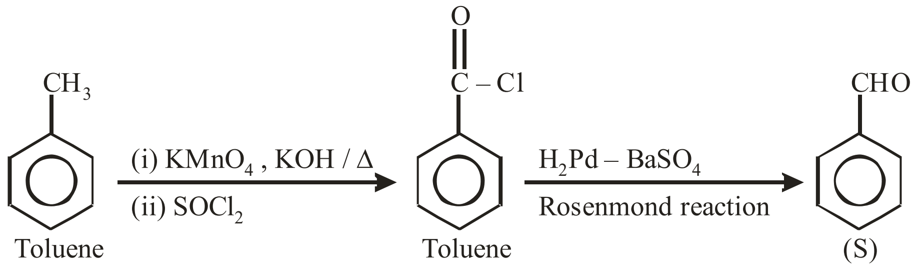
| | **List-I** | | **List-II** |
|---|---|---|---|
| (P) | Etard reaction | (1) | $\text{Acetophenone} \xrightarrow{\text{Zn–Hg, HCl}}$ |
| (Q) | Gattermann reaction | (2) | $\text{Toluene} \xrightarrow[\text{(ii) SOCl}_2]{\text{(i) KMnO}_4\text{,KOH, }\Delta}$ |
| (R) | Gattermann-Koch reaction | (3) | $\text{Benzene} \xrightarrow[\text{anhyd. AlCl}_3]{\text{CH}_3\text{Cl}}$ |
| (S) | Rosenmund reduction | (4) | $\text{Aniline} \xrightarrow[\text{273–278 K}]{\text{NaNO}_2\text{/HCl}}$ |
| | | (5) | $\text{Phenol} \xrightarrow{\text{Zn, }\Delta}$ |

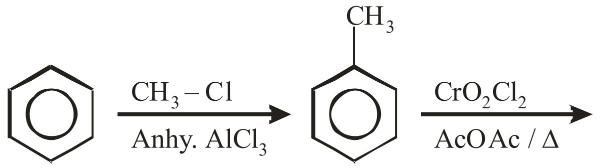
(A) P → 2; Q → 4; R → 1; S → 3
(B) P → 1; Q → 3; R → 5; S → 2
(C) P → 3; Q → 2; R → 1; S → 4
(D) P → 3; Q → 4; R → 5; S → 2

**Ans. (D)**

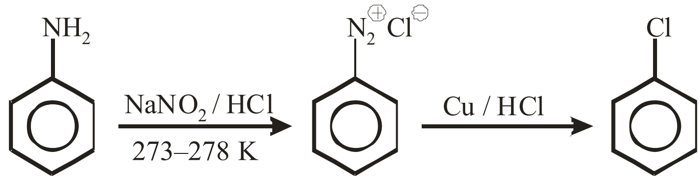
**Sol. P → 3, Q → 4, R → 5, S → 2**

(i) $\text{Ph}-\overset{\overset{\displaystyle O}{\|}}{C}-\text{CH}_3 \xrightarrow{\text{Zn–Hg/HCl}} \text{Ph}-\text{CH}_2-\text{CH}_3$
Acetophenone

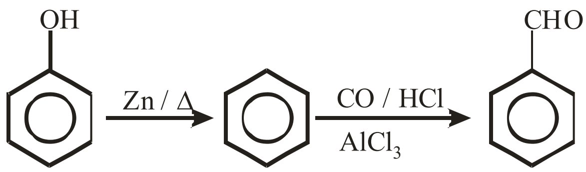
(ii)



(iii)


Etard reaction
(P)

(iv)


Gattermann reaction
(Q)

(v)


Gattermann Koch reaction
(R)

# MATHEMATICS

## SECTION-1 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : −2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

**1.** Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is(are) true ?

(A) There are infinitely many functions from S to T

(B) There are infinitely many strictly increasing functions from S to T

(C) The number of continuous functions from S to T is at most 120

(D) Every continuous function from S to T is differentiable

**Ans. (ACD)**

**Sol.** $S = (0, 1) \cup (1, 2) \cup (3, 4)$

$T = \{0, 1, 2, 3\}$

Number of functions :

Each element of S have 4 choice

Let n be the number of element in set S.

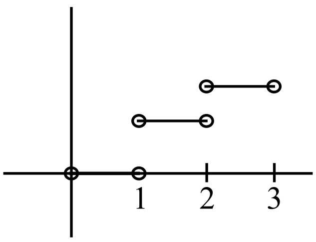
Number of function = $4^n$

Here $n \to \infty$

$\Rightarrow$ Option (A) is correct.

Option (B) is incorrect (obvious)

(C)




For continuous function

Each interval will have 4 choices.

$\Rightarrow$ Number of continuous functions

$= 4\times4\times4 = 64$

$\Rightarrow$ Option (C) is correct.

(D) Every continuous function is piecewise constant functions

$\Rightarrow$ Differentiable.

Option (D) is correct.

**2.** Let $T_1$ and $T_2$ be two distinct common tangents to the ellipse E : $\frac{x^2}{6}+\frac{y^2}{3}=1$ and the parabola P : $y^2 = 12x$. Suppose that the tangent $T_1$ touches P and E at the point $A_1$ and $A_2$, respectively and the tangent $T_2$ touches P and E at the points $A_4$ and $A_3$, respectively. Then which of the following statements is(are) true?

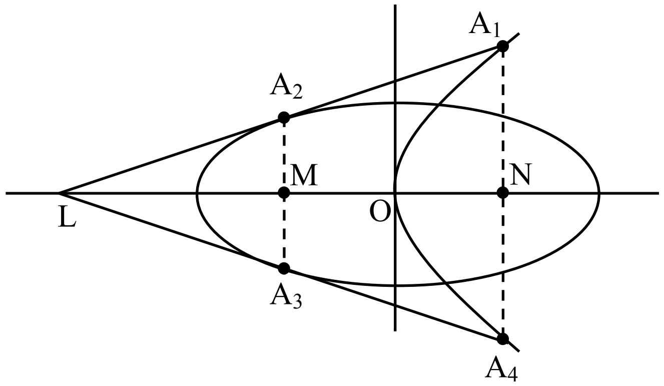
(A) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units

(B) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units

(C) The tangents $T_1$ and $T_2$ meet the x-axis at the point (–3, 0)

(D) The tangents $T_1$ and $T_2$ meet the x-axis at the point (–6, 0)

**Ans. (AC)**

**Sol.** 


$y = mx + \dfrac{3}{m}$

$C^2 = a^2m^2 + b^2$

$\dfrac{9}{m^2} = 6m^2 + 3 \qquad \Rightarrow m^2 = 1$

$T_1$ & $T_2$

$y = x + 3$, $y = -x - 3$

Cuts x-axis at $(-3, 0)$

$A_1(3, 6) \qquad A_4(3, -6)$

$A_2(-2, 1) \qquad A_3(-2, -1)$

$A_1A_4 = 12$, $A_2A_3 = 2$, $MN = 5$

Area = $\dfrac{1}{2}(12+2)\times 5 = 35$ sq.unit

Ans. (A, C)

**3.** Let $f : [0, 1] \to [0, 1]$ be the function defined by $f(x) = \dfrac{x^3}{3} - x^2 + \dfrac{5}{9}x + \dfrac{17}{36}$. Consider the square region $S = [0, 1]\times[0,1]$. Let $G = \{(x, y)\in S : y > f(x)\}$ be called the green region and $R = \{(x, y)\in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h)\in S : x\in[0, 1]\}$ be the horizontal line drawn at a height $h \in[0, 1]$. Then which of the following statements is(are) ture?

(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line $L_h$ equals the area of the green region below the line $L_h$

(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line $L_h$ equals the area of the red region below the line $L_h$

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line $L_h$ equals the area of the red region below the line $L_h$

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line $L_h$ equals the area of the green region below the line $L_h$

**Ans. (BCD)**

**Sol.** $f(x)=\frac{x^3}{3}-x^2+\frac{5x}{9}+\frac{17}{36}$

$f'(x)=x^2-2x+\frac{5}{9}$

$f'(x)=0$ at $x=\frac{1}{3}$ in $[0, 1]$

$A_R$ = Area of Red region

$A_G$ = Area of Green region

$A_R=\int_0^1 f(x)dx=\frac{1}{2}$

Total area = 1

$\Rightarrow A_G=\frac{1}{2}$

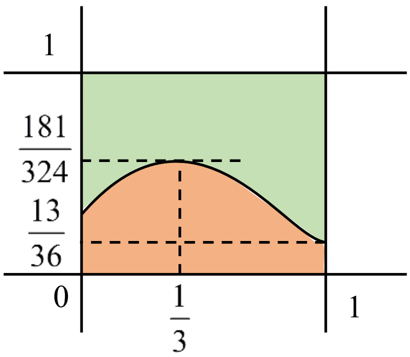
$\int_0^1 f(x)dx=\frac{1}{2}$

$A_G = A_R$

$f(0)=\frac{17}{36}$

$f(1)=\frac{13}{36}\approx 0.36$

$f\left(\frac{1}{3}\right)=\frac{181}{324}\approx 0.558$




(A) Correct when $h=\frac{3}{4}$ but $h\in\left[\frac{1}{4},\frac{2}{3}\right]$

$\Rightarrow$ (A) is incorrect

(B) Correct when $h=\frac{1}{4}$

$\Rightarrow$ (B) is correct

(C) When $h=\frac{181}{324}, A_R=\frac{1}{2}, A_G<\frac{1}{2}$

$h=\frac{13}{36}, A_R<\frac{1}{2}, A_G=\frac{1}{2}$

$\Rightarrow A_R = A_G$ for some $h\in\left(\frac{13}{36},\frac{181}{324}\right)$

$\Rightarrow$ (C) is correct

(D) Option (D) is remaining coloured part of option (C), hence option (D) is also correct.

## SECTION-2 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  *Full Marks* : +3 If **ONLY** the correct option is chosen;

  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

  *Negative Marks* : −1 In all other cases.

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**4.** Let f : (0, 1) $\to \mathbb{R}$ be the functions defined as f(x) = $\sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where n∈N. Let g : (0, 1) $\to \mathbb{R}$ be a function such that $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x}$ for all x∈(0, 1). Then $\lim_{x\to 0} f(x)g(x)$

(A) does **NOT** exist (B) is equal to 1

(C) is equal to 2 (D) is equal to 3

**Ans. (C)**

**Sol.** $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt.\sqrt{n} \le f(x)g(x) \le 2\sqrt{x}\sqrt{n}$

$\because \int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt = \sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x} - \sin^{-1}x - x\sqrt{1-x^2}$

$\Rightarrow \lim_{x\to 0}\left(\frac{\sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x} - \sin^{-1}x - x\sqrt{1-x^2}}{\sqrt{x}} \le f(x)g(x) \le \frac{2\sqrt{x}}{\sqrt{x}}\right)$

$\Rightarrow 2 \le \lim_{x\to 0} f(x)g(x) \le 2$

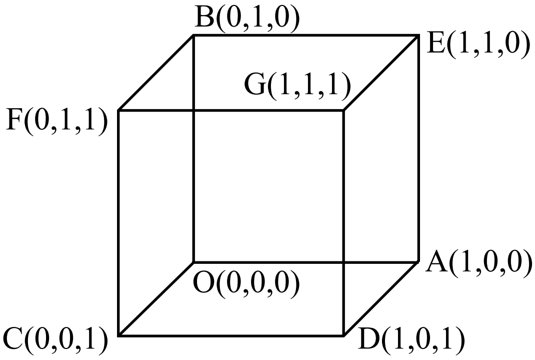
$\Rightarrow \lim_{x\to 0} f(x)g(x) = 2$

**5.** Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3\{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines $\ell_1$ and $\ell_2$, let d($\ell_1$, $\ell_2$) denote the shortest distance between them. Then the maximum value of d($\ell_1$, $\ell_2$), as $\ell_1$ varies over F and $\ell_2$ varies over S, is

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

**Ans. (A)**

**Sol.**




DR'S of OG = 1, 1, 1

DR'S of AF = –1, 1, 1

DR'S of CE = 1, 1, –1

DR'S of BD = 1, –1, 1

Equation of OG $\Rightarrow \frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

Equation of AB $\Rightarrow \frac{x-1}{1}=\frac{y}{-1}=\frac{z}{0}$

Normal to both the line's

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=\hat{i}+\hat{j}-2\hat{k}$$

$\overrightarrow{OA}=\hat{i}$

$$\text{S.D.} = \frac{\left|\hat{i}.(\hat{i}+\hat{j}-2\hat{k})\right|}{\left|\hat{i}+\hat{j}-2\hat{k}\right|}=\frac{1}{\sqrt{6}}$$

Ans. (A)

**6.** Let $X=\left\{(x,y)\in\mathbb{Z}\times\mathbb{Z}:\frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

**Ans. (B)**

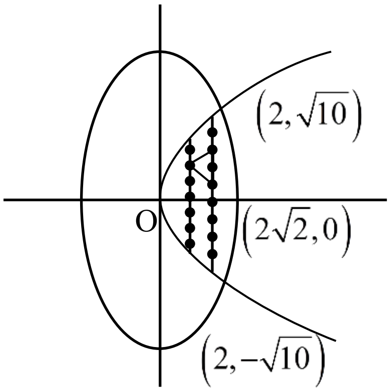
**Sol.** $\frac{x^2}{8}+\frac{y^2}{20}<1$ & $y^2 < 5x$

Solving corresponding equations

$\frac{x^2}{8}+\frac{y^2}{20}=1$ & $y^2 = 5x$

$\Rightarrow \begin{Bmatrix} x=2 \\ y=\pm\sqrt{10} \end{Bmatrix}$

X = {(1,1), (1,0), (1,–1), (1,2), (1,–2), (2, 3), (2,2), (2,1), (2,0), (2,–1), (2,–2), (2,–3)}




Let S be the sample space & E be the event n(S) = ${}^{12}C_3$

For E

Selecting 3 points in which 2 points are either or x = 1 & x = 2 but distance b/w then is even

Triangles with base 2 :

= 3×7 + 5×5 = 46

Triangles with base 4 :

= 1×7 + 3×5 = 22

Triangles with base 6 :

= 1×5 = 5

$P(E)=\frac{46+22+5}{{}^{12}C_3}=\frac{73}{220}$

Ans. (B)

**7.** Let P be a point on the parabola $y^2 = 4ax$, where a > 0. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a,m) is

(A) (2, 3) (B) (1, 3) (C) (2, 4) (D) (3, 4)

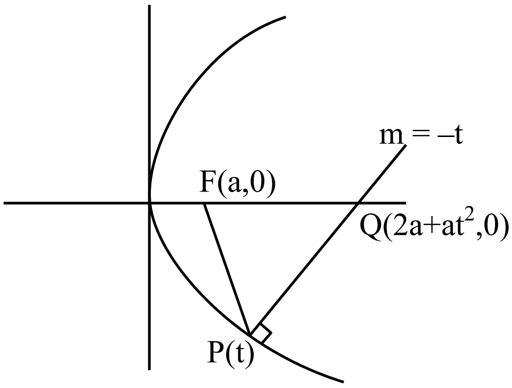
**Ans. (A)**

**Sol.** Let point P $(at^2, 2at)$

normal at P is $y = -tx + 2at + at^3$

$y = 0, x = 2a + at^2$

$Q(2a + at^2, 0)$




Area of $\Delta PFQ = \left|\frac{1}{2}(a + at^2)(2at)\right| = 120$

$\because m = -t$

$\because a^2[1 + m^2]m = 120$

$(a, m) = (2, 3)$ will satisfy

## SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 **ONLY** If the correct integer is entered;
*Zero Marks* : 0 In all other cases.

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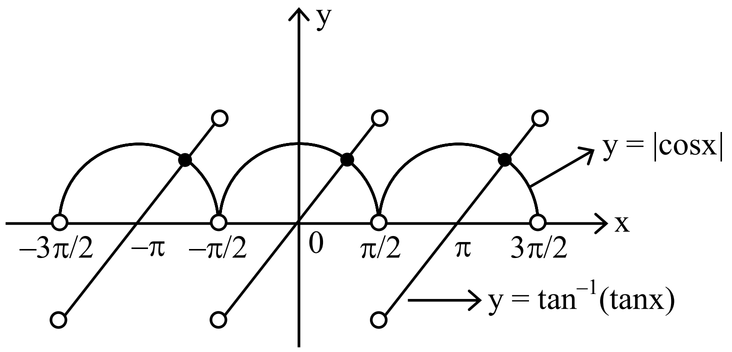
**8.** Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in \mathbb{R}$. Then the number of real solutions of the equation

$\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

**Ans. (3)**

**Sol.** $\sqrt{2}|\cos x| = \sqrt{2}.\tan^{-1}(\tan x)$

$|\cos x| = \tan^{-1}\tan x$




No. of solutions = 3

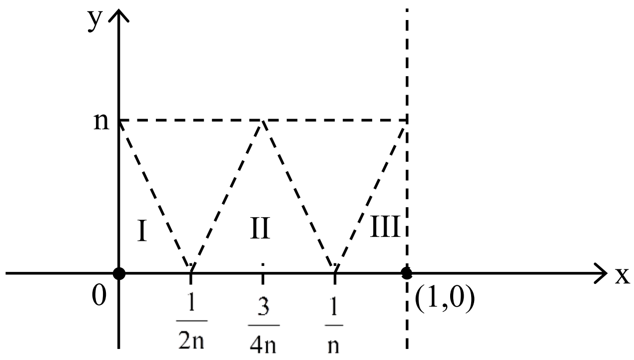
**9.** Let $n \geq 2$ be a natural number and $f : [0,1] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves x = 0 , x = 1 , y = 0 and y = f (x) is 4 , then the maximum value of the function f is

**Ans. (8)**

**Sol.**




Area = Area of (I + II + III) = 4

$$=\frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\left(1-\frac{1}{n}\right)\times n$$

$$=\frac{1}{4}+\frac{1}{4}+\frac{n-1}{2}=4$$

$$\boxed{n=8}$$

$\therefore$ maximum value of f(x) = 8

**10.** Let $7\overset{r}{5...5}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + …+ $7\overset{98}{5...5}7$. If $S=\frac{7\overset{99}{5...5}7+m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

**Ans. (1219)**

**Sol.** S = 77 + 757 + 7557 + … + $7\underset{98}{5.....5}7$

10S = 770 + 7570 + … + 75 … 570 + 755 ….. 570

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$9S = -77 + \underbrace{13+13+.....+13}_{98\text{ times}} + 7\underset{98}{5.....5}70$

$= -77 + 13 \times 98 + 7\underset{99}{5.....5}7 + 13$

$$S=\frac{7\underset{99}{5.....5}7+1210}{9}$$

m = 1210

n = 9

m + n = 1219

**11.** Let $A=\left\{\dfrac{1967+1686i\sin\theta}{7-3i\cos\theta}:\theta\in\mathbb{R}\right\}$. If A contains exactly one positive integer n, then the value of n is

**Ans. (281)**

**Sol.** $A=\dfrac{1967+1686i\sin\theta}{7-3i\cos\theta}$

$=\dfrac{281(7+6i\sin\theta)}{7-3i\cos\theta}\times\dfrac{7+3i\cos\theta}{7+3i\cos\theta}$

$=\dfrac{281(49-18\sin\theta\cos\theta+i(21\cos\theta+42\sin\theta))}{49+9\cos^2\theta}$

for positive integer

Im(A) = 0

$21\cos\theta + 42\sin\theta = 0$

$\tan\theta=\dfrac{-1}{2}$ ; $\sin 2\theta=\dfrac{-4}{5}$ , $\cos^2\theta=\dfrac{4}{5}$

$\text{Re}(A)=\dfrac{281(49-9\sin 2\theta)}{49+9\cos^2\theta}$

$=\dfrac{281\left(49-9\times\dfrac{-4}{5}\right)}{49+9\times\dfrac{4}{5}}$ = 281 (+ve integer)

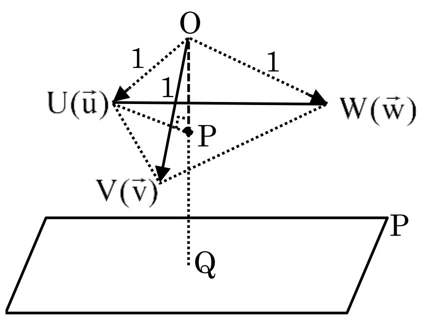
**12.** Let P be the plane $\sqrt{3}x+2y+3z=16$ and let

$S=\left\{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}:\alpha^2+\beta^2+\gamma^2=1 \text{ and the distance of } (\alpha,\beta,\gamma) \text{ from the plane P is } \dfrac{7}{2}\right\}$.

Let $\vec{u},\vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u},\vec{v}$ and $\vec{w}$. Then the value of $\dfrac{80}{\sqrt{3}}V$ is

**Ans. (45)**

**Sol.**




Given $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$

$\Rightarrow \Delta UVW$ is an equilateral $\Delta$

Now distances of U, V, W from $P = \frac{7}{2}$

$\Rightarrow\ PQ = \frac{7}{2}$

Also, Distance of plane P from origin

$\Rightarrow OQ = 4$

$\therefore\ OP = OQ - PQ \Rightarrow\ OP = \frac{1}{2}$

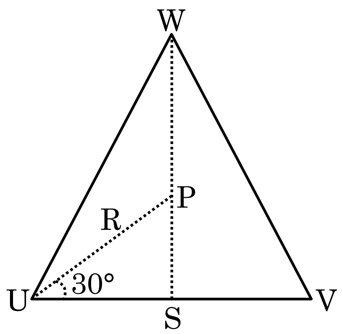
Hence, $PU = \sqrt{OU^2 - OP^2}\ \Rightarrow\ PU = \frac{\sqrt{3}}{2} = R$

Also, for $\Delta UVW$, P is circumcenter

$\therefore$ for $\Delta UVW$ : $US = R\cos 30°$

$\Rightarrow UV = 2R\cos 30°$

$\Rightarrow\ UV = \frac{3}{2}$




$\therefore\ Ar(\Delta UVW) = \frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2 = \frac{9\sqrt{3}}{16}$

$\therefore$ Volume of tetrahedron with coterminous edges $\vec{u}, \vec{v}, \vec{w}$

$= \frac{1}{3}(Ar.\Delta UVW) \times OP = \frac{1}{3} \times \frac{9\sqrt{3}}{16} \times \frac{1}{2} = \frac{3\sqrt{3}}{32}$

$\therefore$ parallelopiped with coterminous edges

$\vec{u}, \vec{v}, \vec{w} = 6 \times \frac{3\sqrt{3}}{32} = \frac{9\sqrt{3}}{16} = V$

$\therefore\ \frac{80}{\sqrt{3}} V = 45$

**13.** Let a and b be two nonzero real numbers. If the coefficient of $x^5$ in the expansion of $\left(ax^2 + \frac{70}{27bx}\right)^4$ is equal to the coefficient of $x^{-5}$ is equal to the coefficient of $\left(ax - \frac{1}{bx^2}\right)^7$, then the value of 2b is

**Ans. (3)**

**Sol.** $T_{r+1} = {}^4C_r \left(a.x^2\right)^{4-r} . \left(\frac{70}{27bx}\right)^r$

$= {}^4C_r.a^{4-r}.\frac{70^r}{(27b)^r}.x^{8-3r}$

here $8 - 3r = 5$

$8 - 5 = 3r \Rightarrow r = 1$

$\therefore$ coeff. $= 4.a^3.\frac{70}{27b}$

$T_{r+1} = {}^7C_r(ax)^{7-r}\left(\frac{-1}{bx^2}\right)^r$

$= {}^7C_r.a^{7-r}\left(\frac{-1}{b}\right)^r.x^{7-3r}$

$7 - 3r = -5 \Rightarrow 12 = 3r \Rightarrow r = 4$

coeff. : ${}^7C_4.a^3.\left(\frac{-1}{b}\right)^4 = \frac{35a^3}{b^4}$

now $\frac{35a^3}{b^4} = \frac{280a^3}{27b}$

$b^3 = \frac{35 \times 27}{280} = b = \frac{3}{2} \Rightarrow 2b = 3$

## SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  *Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen;

  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

  *Negative Marks* : –1 In all other cases.

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**14.** Let $\alpha$, $\beta$ and $\gamma$ be real numbers. consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List - I** to the correct entries in **List-II**

| **List-I** | **List-II** |
|---|---|
| (P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has | (1) a unique solution |
| (Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has | (2) no solution |
| (R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has | (3) infinitely many solutions |
| (S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has | (4) $x = 11$, $y = -2$ and $z = 0$ as a solution |
| | (5) $x = -15$, $y = 4$ and $z = 0$ as a solution |

The correct option is :

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (4)

(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (5) (S) $\rightarrow$ (4)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (1) (S) $\rightarrow$ (3)

**Ans. (A)**

**Sol.** Given x + 2y + z = 7 .... (1)

x + $\alpha$z = 11 .... (2)

2x – 3y + $\beta$z = $\gamma$ .... (3)

Now, $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 7\alpha - 2\beta - 3$

$\therefore$ if $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \boxed{\Delta = 0}$

Now, $\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}$

$= 21\alpha - 22\beta + 2\alpha\gamma - 33$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_x = 0$

$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}$

$\Delta_y = 4\beta + 14\alpha - \alpha\gamma + \gamma - 22$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_y = 0$

Now, $\Delta_z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 56 - 2\gamma$

If $\gamma = 28$

$\Rightarrow \boxed{\Delta_z = 0}$

$\therefore$ if $\gamma = 28$ and $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow$ system has infinite solution

and if $\gamma \neq 28$

$\Rightarrow$ system has no solution

$\Rightarrow$ P $\rightarrow$ (3) ; Q $\rightarrow$ (2)

Now if $\beta \neq \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \Delta \neq 0$

and for $\alpha = 1$ clearly

y = –2 is always be the solution

$\therefore$ if $\gamma \neq 28$

System has a unique solution

if $\gamma = 28$

$\Rightarrow$ x = 11, y = –2 and z = 0 will be one of the solution

$\therefore$ R $\rightarrow$ 1 ; S $\rightarrow$ 4

$\therefore$ option 'A' is correct

**15.** Consider the given data with frequency distribution

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_i$ | 3 | 8 | 11 | 10 | 5 | 4 |
| $f_i$ | 5 | 2 | 3 | 2 | 4 | 4 |

Match each entry in **List-I** to the correct entries in **List-II.**

| **List-I** | **List-II** |
|---|---|
| (P) The mean of the above data is | (1) 2.5 |
| (Q) The median of the above data is | (2) 5 |
| (R) The mean deviation about the mean of the above data is | (3) 6 |
| (S) The mean deviation about the median of the above data is | (4) 2.7 |
| | (5) 2.4 |

The correct option is :

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

(B) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (4) (S) $\rightarrow$ (1)

(D) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (5) (S) $\rightarrow$ (5)

**Ans. (A)**

**Sol.**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_i$ | 3 | 4 | 5 | 8 | 10 | 11 |
| $f_i$ | 5 | 4 | 4 | 2 | 2 | 3 |

(P) Mean

(Q) Median

(R) Mean deviation about mean

(S) Mean deviation about median

| $x_i$ | $f_i$ | $x_if_i$ | C.F. | $\lvert x_i - \text{Mean}\rvert$ | $f_i\lvert x_i - \text{Mean}\rvert$ | $\lvert x_i - \text{Median}\rvert$ | $f_i\lvert x_i - \text{Median}\rvert$ |
|---|---|---|---|---|---|---|---|
| 3 | 5 | 15 | 5 | 3 | 15 | 2 | 10 |
| 4 | 4 | 16 | 9 | 2 | 8 | 1 | 4 |
| 5 | 4 | 20 | 13 | 1 | 4 | 0 | 0 |
| 8 | 2 | 16 | 15 | 2 | 4 | 3 | 6 |
| 10 | 2 | 20 | 17 | 4 | 8 | 5 | 10 |
| 11 | 3 | 33 | 20 | 5 | 15 | 6 | 18 |
| | $\Sigma f_i = 20$ | $\Sigma x_if_i = 120$ | | | $\Sigma f_i\lvert x_i - \text{Mean}\rvert = 54$ | | $\Sigma f_i\lvert x_i - \text{Median}\rvert = 48$ |

(P) Mean $= \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{120}{20} = 6$

(Q) Median $= \left(\dfrac{20}{2}\right)^{th}$ observation $= 10^{th}$ observation $= 5$

(R) Mean deviation about mean $= \dfrac{\Sigma f_i \left|x_i - \text{Mean}\right|}{\Sigma f_i} = \dfrac{54}{20} = 2.70$

(S) mean deviation about median $= \dfrac{\Sigma f_i \left|x_i - \text{Median}\right|}{\Sigma f_i} = \dfrac{48}{20} = 2.40$

**16.** Let $\ell_1$ and $\ell_2$ be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = (\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line $\ell_1$. For a plane H, let d(H) denote the smallest possible distance between the points of $\ell_2$ and H. Let $H_0$ be plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II.**

| List-I | List-II |
|---|---|
| (P) The value of $d(H_0)$ is | (1) $\sqrt{3}$ |
| (Q) The distance of the point (0,1,2) from $H_0$ is | (2) $\frac{1}{\sqrt{3}}$ |
| (R) The distance of origin from $H_0$ is | (3) 0 |
| (S) The distance of origin from the point of intersection of planes y = z, x = 1 and $H_0$ is | (4) $\sqrt{2}$ |
| | (5) $\frac{1}{\sqrt{2}}$ |

The correct option is :

(A) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)

(B) (P) → (5) (Q) → (4) (R) → (3) (S) → (1)

(C) (P) → (2) (Q) → (1) (R) → (3) (S) → (2)

(D) (P) → (5) (Q) → (1) (R) → (4) (S) → (2)

**Ans. (B)**

**Ans. ( )**

**Sol.** $L_1 : \vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$

$L_2 : \vec{r}_2 = \hat{j}-\hat{k}+\mu(\hat{i}+\hat{k})$

Let system of planes are

$ax + by + cz = 0$ …. (1)

$\because$ It contain $L_1$

$\therefore\ a + b + c = 0$ …. (2)

For largest possible distance between plane (1) and $L_2$ the line $L_2$ must be parallel to plane (1)

$\therefore\ a + c = 0$ …. (3)

$\Rightarrow \boxed{b = 0}$

$\therefore$ Plane $H_0$ : $\boxed{x - z = 0}$

Now $d(H_0) = \perp$ distance from point (0, 1, –1) on $L_2$ to plane.

$$\Rightarrow d\left(H_0\right)=\left|\frac{0+1}{\sqrt{2}}\right|=\frac{1}{\sqrt{2}}$$

$\therefore$ P $\rightarrow$ 5

for 'Q' distance = $\left|\frac{2}{\sqrt{2}}\right| = \sqrt{2}$

$\therefore$ Q $\rightarrow$ 4

$\therefore$ (0, 0, 0) lies on plane

$\therefore$ R $\rightarrow$ 3

for 'S' x = z ; y = z ; x = 1

$\therefore$ point of intersection p(1, 1, 1).

$\therefore$ $OP = \sqrt{1+1+1} = \sqrt{3}$

$\therefore$ S $\rightarrow$ 2

$\therefore$ option [B] is correct

**17.** Let z be complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be nonzero.

Match each entry in **List-I** to the correct entries in **List-II.**

| **List-I** | **List-II** |
|---|---|
| (P) $|z|^2$ is equal to | (1) 12 |
| (Q) $|z-\bar{z}|^2$ is equal to | (2) 4 |
| (R) $|z|^2 + |z+\bar{z}|^2$ is equal to | (3) 8 |
| (S) $|z+1|^2$ is equal to | (4) 10 |
| | (5) 7 |

The correct option is :

(A) (P) $\rightarrow$ (1) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (5) (S) $\rightarrow$ (4)

(B) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (3) (S) $\rightarrow$ (5)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (5) (S) $\rightarrow$ (1)

(D) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (5) (S) $\rightarrow$ (4)

**Ans. (B)**

**Sol.** $\because |z|^3 + 2z^2 + 4\bar{z} - 8 = 0$ …. (1)

Take conjugate both sides

$\Rightarrow |z|^3 + 2\bar{z}^2 + 4z - 8 = 0$ …. (2)

By (1) – (2)

$\Rightarrow 2(z^2 - \bar{z}^2) + 4(\bar{z} - z) = 0$

$\Rightarrow \boxed{z + \bar{z} = 2}$ …. (3)

$\Rightarrow |z + \bar{z}| = 2$ …. (4)

Let z = x + iy

$\therefore \boxed{x = 1}$ $\qquad \therefore z = 1 + iy$

Put in (1)

$\Rightarrow (1 + y^2)^{3/2} + 2(1 - y^2 + 2iy) + 4(1 - iy) - 8 = 0$

$\Rightarrow (1 + y^2)^{3/2} = 2(1 + y^2)$

$\Rightarrow \sqrt{1 + y^2} = 2 = |z|$

Also $\boxed{y = \pm\sqrt{3}}$

$\therefore z = 1 \pm i\sqrt{3}$

$\Rightarrow z - \bar{z} = \pm 2i\sqrt{3}$

$\Rightarrow |z - \bar{z}| = 2\sqrt{3}$

$\Rightarrow |z - \bar{z}|^2 = 12$

Now $z + 1 = 2 + i\sqrt{3}$

$|z + 1|^2 = 4 + 3 = 7$

$\therefore P \to 2$ ; $Q \to 1$ ; $R \to 3$ ; $S \to 5$

$\therefore$ Option [B] is correct.

# JEE(Advanced) EXAMINATION – 2023

**(Held On Sunday 04th June, 2023)**

**PAPER-2**

## PHYSICS

**SECTION-1 : (Maximum Marks : 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If **ONLY** the correct option is chosen;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : –1 In all other cases.

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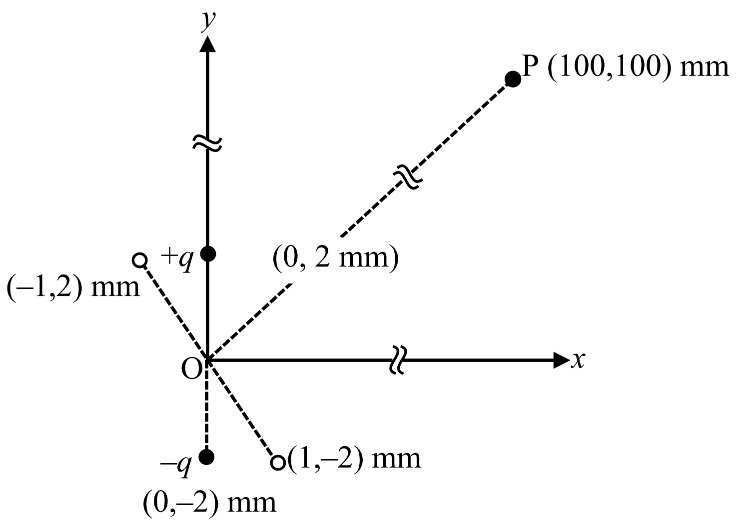
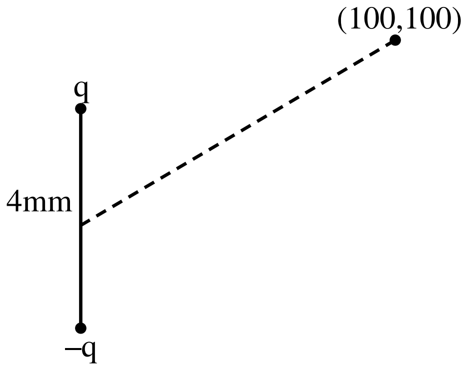
**1.** An electric dipole is formed by two charges $+q$ and $-q$ located in xy-plane at (0, 2) mm and (0, –2) mm, respectively, as shown in the figure. The electric potential at point P(100, 100) mm due to the dipole is $V_0$. The charges $+q$ and $-q$ are then moved to the points (–1, 2) mm and (1, –2) mm, respectively. What is the value of electric potential at P due to the new dipole ?




(A) $V_0/4$ (B) $V_0/2$ (C) $V_0/\sqrt{2}$ (D) $3V_0/4$

**Ans. (B)**

**Sol.**

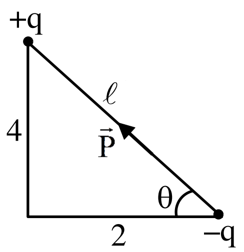



$P_1 = q(4)$

$\vec{P}_1 = P_1\hat{j}$

$\vec{r} = 100(\hat{i}+\hat{j})\text{mm}$

$$v_0 = \frac{KP_1.\vec{r}}{r^3} = \frac{K(100P_1)}{(100\sqrt{2})^3}$$




$\tan\theta = 2$

$\vec{P}_2 = P_2\left[-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$

$\vec{r} = 100(\hat{i}+\hat{j})\text{mm}$

$P_2 = q\ell$

$$v = \frac{K\vec{P}_2.\vec{r}}{r^3}$$

$$v = \frac{K(100P_2) - (-\cos\theta + \sin\theta)}{(100\sqrt{2})^3}$$

$$\frac{v_0 P_2}{P_1} = (-\cos\theta + \sin\theta)$$

$$v = v_0\,\frac{q\ell}{q(4)}[-\cos\theta + \sin\theta]$$

$$= \frac{v_0}{4}[-2+4] = \frac{v_0}{2}$$

**2.** Young's modulus of elasticity $Y$ is expressed in terms of three derived quantities, namely, the gravitational constant $G$, Planck's constant $h$ and the speed of light c, as $Y = c^{\alpha} h^{\beta} G^{\gamma}$. Which of the following is the correct option ?

(A) $\alpha = 7, \beta = -1, \gamma = -2$
(B) $\alpha = -7, \beta = -1, \gamma = -2$
(C) $\alpha = 7, \beta = -1, \gamma = 2$
(D) $\alpha = -7, \beta = 1, \gamma = -2$

**Ans. (A)**

**Sol.** $Y = c^{\alpha}h^{\beta}G^{\gamma}$

$$ML^{-1}T^{-2} = (LT^{-1})^{\alpha}(ML^{2}T^{-1})^{\beta}(M^{-1}L^{3}T^{-2})^{\gamma}$$

$1 = \beta - \gamma$ ...(1)

$-1 = \alpha + 2\beta + 3\gamma$ ...(2)

$-2 = -\alpha - \beta - 2\gamma$ ...(3)

$-3 = \beta + \gamma$

$1 = \beta - \gamma$

$-2 = 2\beta \quad \Rightarrow \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6 \quad \therefore \ \alpha = 7$

**3.** A particle of mass m is moving in the $xy$-plane such that its velocity at a point $(x, y)$ is given as $\vec{v} = \alpha\left(y\hat{x} + 2x\hat{y}\right)$, where $\alpha$ is a non-zero constant. What is the force $\vec{F}$ acting on the particle ?

(A) $\vec{F} = 2m\alpha^{2}\left(x\hat{x} + y\hat{y}\right)$
(B) $\vec{F} = m\alpha^{2}\left(y\hat{x} + 2x\hat{y}\right)$
(C) $\vec{F} = 2m\alpha^{2}\left(y\hat{x} + x\hat{y}\right)$
(D) $\vec{F} = m\alpha^{2}\left(x\hat{x} + 2y\hat{y}\right)$

**Ans. (A)**

**Sol.** $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$

$v_x = \alpha y \qquad v_y = 2\alpha x$

$$\frac{dv_x}{dt} = \alpha\frac{dy}{dt} = 2\alpha^{2}x \qquad \frac{dv_y}{dt} = 2\alpha v_x = 2\alpha^{2}y$$

$\therefore \ \vec{F} = m\vec{a} = 2m\alpha^{2}(x\hat{x} + y\hat{y})$

**4.** An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas in $n$. The internal energy of one mole of the gas is $U_n$ and the speed of sound in the gas is $v_n$. At a fixed temperature and pressure, which of the following is the correct option ?

(A) $v_3 < v_6$ and $U_3 > U_6$
(B) $v_5 > v_3$ and $U_3 > U_5$
(C) $v_5 > v_7$ and $U_5 < U_7$
(D) $v_6 < v_7$ and $U_6 < U_7$

**Ans. (C)**

**Sol.** $U = \frac{1}{2}fnrT = \frac{frT}{2}$

$\therefore$ A and B are wrong.

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\left(\frac{2}{f}+1\right)\frac{RT}{M}}$$

$\Rightarrow$ more 'f', less 'v'

$\therefore \ v_5 > v_7$

**SECTION-2 : (Maximum Marks : 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If unanswered; |
| *Negative Marks* | : −2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get –2 marks.

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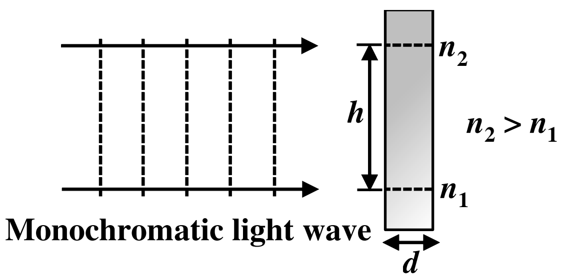
**5.** A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from $n_1$ to $n_2$ over the height h. Which of the following statement(s) is (are) true about the light wave emerging out of the slab ?




(A) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2^2 - n_1^2)d}{2h}\right]$

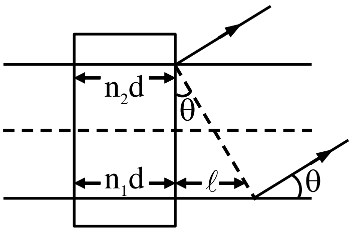
(B) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2 - n_1)d}{h}\right]$

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of $n_1$ and $n_2$.

**Ans. (B, D)**

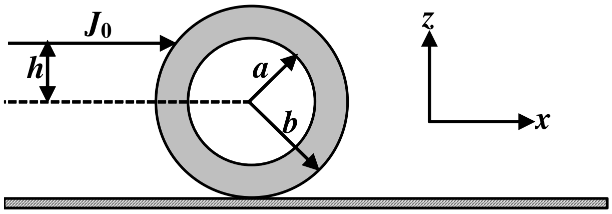
**Sol.**




$n_1d + \ell = n_2 d$

$$\tan\theta = \frac{\ell}{h} = \frac{(n_2 - n_1)d}{h}$$

**6.** An annular disk of mass $M$, inner radius $a$ and outer radius $b$ is placed on a horizontal surface with coefficient of friction $\mu$, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the $x$-axis. Which of the following statement(s) is(are) correct ?




(A) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$

(B) For $\mu \neq 0$ and $a \to b$, $h_m = b$

(C) For $h = h_m$, the initial angular velocity does **not** depend on the inner radius $a$.

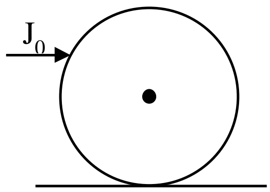
(D) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

**Ans. (A,B,C,D)**

**Sol.** $J_0 = mv$ .....(1)

$J_0h_m = I_c\omega$ ......(2)

$v = \omega R$ .......(3)




$$\Rightarrow h_m = \frac{I_C}{mR}$$

(A) If a = 0 $I_c = \frac{1}{2}mb^2$ & R = b $\therefore h_m = \frac{b}{2}$

(B) If a = b $I_C = mb^2$ & R = b $\therefore h_m = b$

(C) $v = \frac{J_0}{m} \Rightarrow 100 = \frac{V}{R} = \frac{J_0}{mR}$

(D) Force is acting on COM $\therefore$ No rotation.

**7.** The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ V m$^{-1}$. Which of the following option(s) is(are) correct?

[Given: The speed of light in vacuum, $c = 3 \times 10^8\ ms^{-1}$]

(A) $B_x = -2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm$^{-2}$.

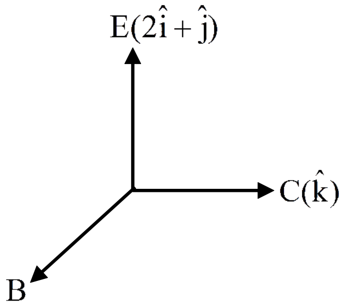
(B) $B_y = 2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm$^{-2}$

(C) The wave is polarized in the *xy*-plane with polarization angle 30° with respect to the *x*-axis.

(D) The refractive index of the medium is 2.

**Ans. (A, D)**

**Sol.**

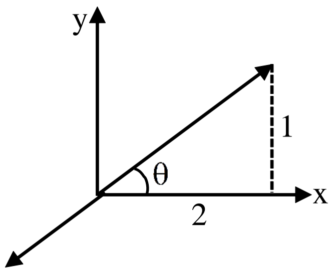



$$C_{\text{medium}} = \frac{5\times10^{14}}{10^7/3} = 1.5 \times 10^8 \text{ m/s} \quad \therefore \mu = 2$$

$$C_{\text{medium}} = \frac{E}{B} \Rightarrow B = \frac{E}{C_m} = \frac{30\sqrt{5}}{1.5\times10^8} = 2\sqrt{5}\times10^{-7}$$

$$\vec{B}_{\text{direction}} \equiv \hat{k}\times(2\hat{i}+\hat{j}) \equiv \frac{2\hat{j}-\hat{i}}{\sqrt{5}}$$

$$\therefore\ \vec{B} = 2\times10^{-7}(-\hat{i}+2\hat{j})\sin\left[27\left(5\times10^{17}t - \frac{10^7}{3}z\right)\right]$$




$$\tan\theta = \frac{1}{2}$$

**SECTION-3 : (Maximum Marks : 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +4 If **ONLY** the correct integer is entered;
  *Zero Marks* : 0 In all other cases

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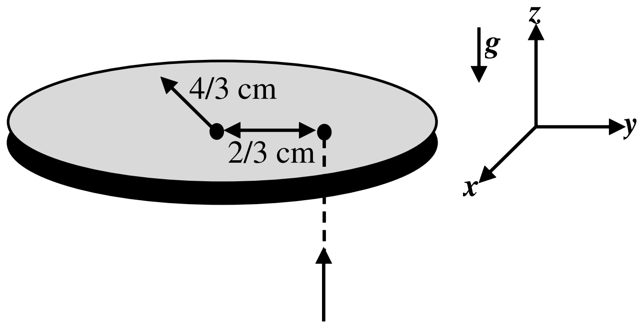
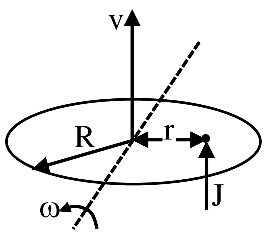
**8.** A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal *xy*-plane. The coin is tossed vertically up (+*z* direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N-s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the +*z* direction. By the time the coin reaches back to its initial position, it completes *n* rotations. The value of *n* is ____.
[Given: The acceleration due to gravity g = 10 ms$^{-2}$]




**Ans. (30)**
**Sol.**




$J = mv$ ......(1)

$Jr = I_c\omega$ ........(2)

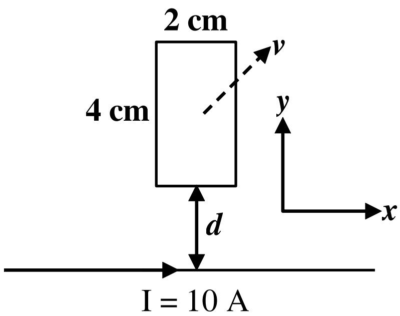
$J_C = \frac{1}{4}mR^2$ ........(3)

$t = \frac{2v}{g}$ ........(4)

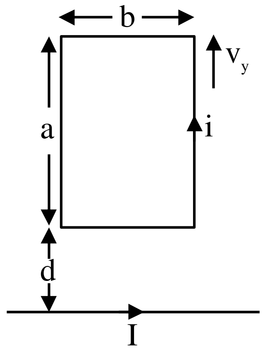
$\theta = 2\pi N = \omega t$ .......(5) $\therefore$ N = 30

**9.** A rectangular conducting loop of length 4 cm and width 2 cm is in the *xy*-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{\sqrt{3}}{2}\hat{x}+\frac{1}{2}\hat{y}$ with a constant speed $v$. The wire is carrying a steady current I = 10 A in the positive $x$-direction. A current of 10 $\mu$A flows through the loop when it is at a distance $d$ = 4 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of $v$ is __________ $ms^{-1}$.

[Given: The permeability of free space $\mu_0 = 4\pi \times 10^{-7}\, NA^{-2}$]




**Ans. (4)**




**Sol.** R = 0.1Ω

$\varepsilon = (B_1 - B_2)bv_y$

$$i = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi R}\left(\frac{1}{d} - \frac{1}{d+a}\right)bv_y$$

$$\Rightarrow 10^{-5} = \frac{2\times10^{-7}\times10}{0.1}\left[\frac{1}{4}-\frac{1}{8}\right]\times 2.v_y$$

$\therefore\ v_y = 2$

$$\tan\theta = \frac{v_y}{v_x} = \frac{1}{\sqrt{3}}$$

θ

$\therefore\ v_x = 2\sqrt{3}$

$\therefore\ v = \sqrt{v_x^2 + v_y^2} = 4$

**10.** A string of length 1 m and mass 2 × $10^{-5}$ kg is under tension T. when the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is _______ Newton.

**Ans. (5)**

**Sol.** $f = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

$750 = \frac{P}{2}\sqrt{\frac{T}{\mu}}$ ......(1)

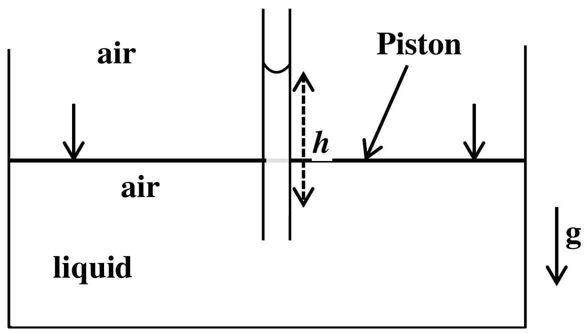
$1000 = \frac{P+1}{2}\sqrt{\frac{T}{\mu}}$ .....(2)

$\frac{4}{3} = \frac{P+1}{P} \quad \therefore P = 3$

$\Rightarrow 1000 = \frac{4}{2}\sqrt{\frac{T}{2\times10^{-5}}} \quad \therefore T = 5N$

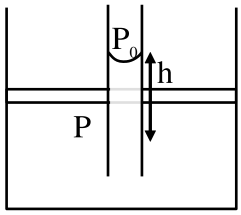
**11.** An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume $V_0$ to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height ($h$) of the liquid column in the capillary above the liquid level in cm is_______.

[Given: Surface tension of the liquid is 0.075 $Nm^{-1}$, atmospheric pressure is $10^5$ N $m^{-2}$, acceleration due to gravity ($g$) is 10 m $s^{-2}$, density of the liquid is $10^3$ kg $m^{-3}$ and contact angle of capillary surface with the liquid is zero]




**Ans. (25)**

**Sol.**




$$h_0 = \frac{2T\cos\theta}{\rho g r} = \frac{2\times 0.075\times 1}{10^3\times 10\times 10^{-4}} = 15\text{cm}$$

$$P_0V_0 = P\frac{100V_0}{101} \Rightarrow P = \frac{101}{100}P_0$$

$$P_0 - \frac{2T\cos\theta}{r} + \rho g h = P = \frac{101}{100}P_0$$

$$\Rightarrow -\rho g h_0 + \rho g h = \frac{P_0}{100}$$

$$\Rightarrow h = h_0 + \frac{P_0}{100\rho g}$$

$$= 15 \text{ cm} + \frac{10^5}{100\times 10^3\times 10} = 25\text{cm}$$

**12.** In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where $N(t)$ is the number of radioactive nuclei at time $t$. Two radioactive sources, $S_1$ and $S_2$ have same activity at time $t = 0$. At a later time, the activities of $S_1$ and $S_2$ are $A_1$ and $A_2$, respectively. When $S_1$ and $S_2$ have just completed their 3rd and 7th half-lives, respectively, the ratio $A_1/A_2$ is __________.

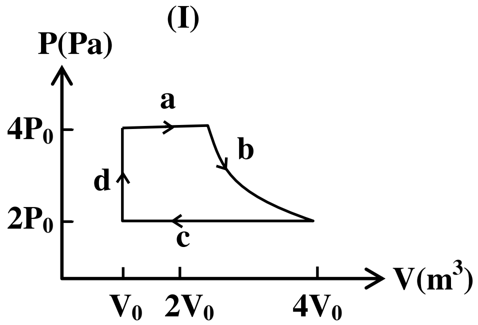
**Ans. (16)**

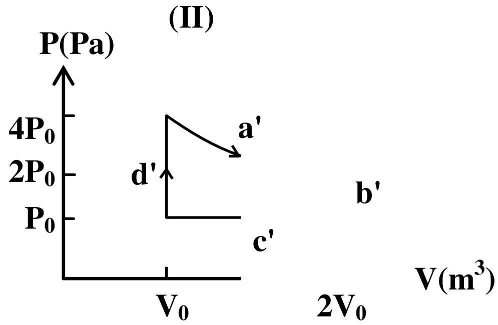
**Sol.**

| | $S_1$ | $S_2$ |
|---|---|---|
| $t=0$ | $A_0$ | $A_0$ |
| $t=\tau$ | $A_1$ | $A_2$ |

$$\frac{A_1}{A_2} = \frac{A_0(0.5)^{t/(t_{1/2})_1}}{A_0(0.5)^{t/(t_{1/2})_2}} = \frac{(0.5)^3}{(0.5)^7} = 2^4 = 16$$

**13.** One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the *P-V* diagrams below. In cycle I, processes *a, b, c* and *d* are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes *a′*, *b′*, *c′* and *d′* are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is $W_I$ and that during cycle II is $W_{II}$. The ratio $W_I / W_{II}$ is _______.




**Ans.** (2)

**Sol.** $\dfrac{W_I}{W_{II}} = \dfrac{4P_0V_0 + 8P_0V_0\ell n2 - 6P_0V_0 - 0}{4P_0V_0\ell n2 - 0 - P_0V_0 + 0}$

$= \dfrac{8\ell n2 - 2}{4\ell n2 - 1} = 2$

**SECTION-4 : (Maximum Marks : 12)**

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

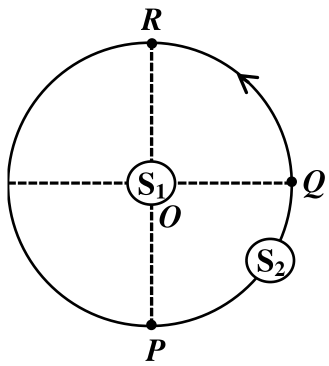
  *Full Marks* : +3 If ONLY the correct numerical value is entered in the designated place;

  *Zero Marks* : 0 In all other cases.

## PARAGRAPH-I

$S_1$ and $S_1$ are two identical sound sources of frequency 656 Hz. The source $S_1$ is located at $O$ and $S_2$ moves anti-clockwise with a uniform speed $4\sqrt{2}\ \text{ms}^{-1}$ on a circular path around $O$, as shown in the figure. There are three points $P$, $Q$ and $R$ on this path such that $P$ and $R$ are diametrically opposite while $Q$ is equidistant from them. A sound detector is placed at point $P$. The source $S_1$ can move along direction $OP$.

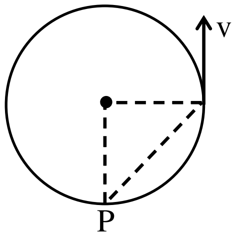
[Given: The speed of sound in air is 324 $\text{ms}^{-1}$]




**14.** When only $S_2$ is emitting sound and it is $Q$, the frequency of sound measured by the detector in Hz is __________.

**Ans. (648)**

**Sol.**




$$f' = \frac{C}{C + v\cos 45^\circ} f$$

$$= \frac{324}{324 + 4\sqrt{2} \times \frac{1}{\sqrt{2}}} \times 656 = 648 \text{ Hz}$$

## PARAGRAPH-I

$S_1$ and $S_1$ are two identical sound sources of frequency 656 Hz. The source $S_1$ is located at $O$ and $S_2$ moves anti-clockwise with a uniform speed $4\sqrt{2}\ \text{ms}^{-1}$ on a circular path around $O$, as shown in the figure. There are three points $P$, $Q$ and $R$ on this path such that $P$ and $R$ are diametrically opposite while $Q$ is equidistant from them. A sound detector is placed at point $P$. The source $S_1$ can move along direction $OP$.

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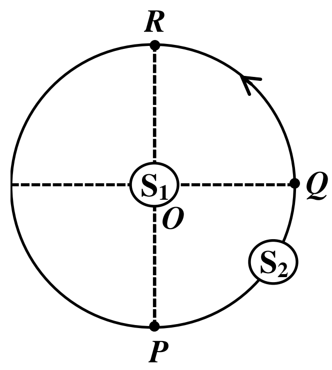



**15.** Consider both sources emitting sound. When $S_2$ is at $R$ and $S_1$ approaches the detector with a speed 4 $\text{ms}^{-1}$, the beat frequency measured by the detector is _______Hz.

**Ans. (8.2)**

**Sol.** $f_{\text{P from } S_2} = 656\text{Hz}$

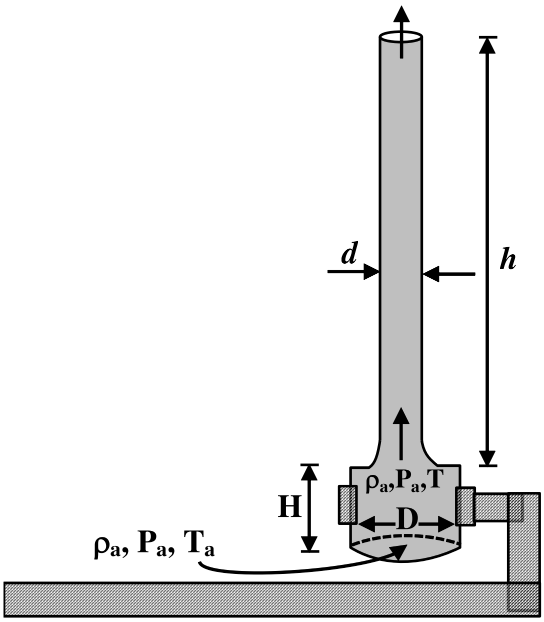
$$f_{\text{P from } S_1} = \frac{C}{C-V}f = \frac{656\times 324}{324-4} = 664.2$$

$\Delta f = 664.2 – 656 = 8.2\text{Hz}$

## PARAGRAPH-II

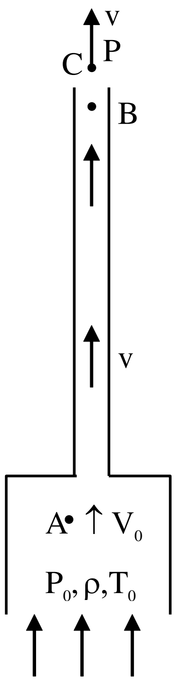
A cylindrical furnace has height ($H$) and diameter ($D$) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure $P_a$ and its temperature becomes $T = 360$ K. The hot air with density $\rho$ rises up a vertical chimney of diameter $d = 0.1$ m and height $h = 9$ m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density $\rho_a = 1.2$ kg m$^{-3}$, pressure $P_a$ and temperature $T_a = 300$ K enters the furnace. Assume air as an ideal gas, neglect the variations in $\rho$ and $T$ inside the chimney and the furnace. Also ignore the viscous effects.

[Given: The acceleration due to gravity $g = 10$ ms$^{-2}$ and $\pi = 3.14$]




**16.** Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is _________ gm s$^{-1}$.

**Ans. (60.80, 60.81)**




**Sol.**

$\rho_0 T_0 = \rho T$

$\Rightarrow 1.2 \times 300 = \rho(360) \therefore \rho = 1$

Between A & B

$$P_0 + \frac{1}{2}\rho V_0^2 = P + \frac{1}{2}\rho V^2 + \rho gh \text{ .......(1)}$$

$$\frac{\pi D^2}{4} V_0 = \frac{\pi d^2}{4} V \text{ ........(2)}$$

Between B & C

$$P + \frac{1}{2}\rho V^2 = P_0 - \rho_0 g(H + h) + \frac{1}{2}\rho V^2 \text{ .....(3)}$$

from (1) & (2) :

$$\Rightarrow P_0 + \frac{1}{2}\rho\left(V\frac{d^2}{D^2}\right)^2 = P + \frac{1}{2}\rho V^2 + \rho gh$$

$$\Rightarrow \rho_0 g(H + h) = \frac{1}{2}\rho V^2\left[1 - \frac{d^4}{D^4}\right] + \rho gh$$

$$\Rightarrow V^2 \simeq \frac{2\rho_0}{\rho} g(H + h) - 2gh$$

$= 2 \times 1.2 \times 10 \times 10 - 2 \times 10 \times 9$

$= 240 - 180 = 60 \quad \therefore V = \sqrt{60}\,\text{m/s}$

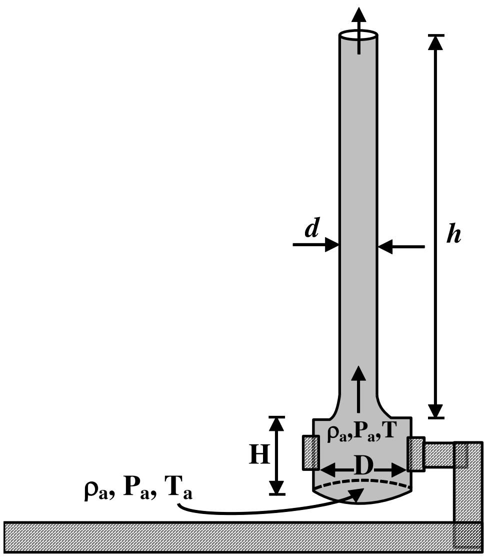
$$Q_m = \rho\frac{\pi d^2}{4}V = 1 \times \frac{\pi}{4} \times 10^{-2} \times \sqrt{60} \simeq 60.80$$

**Ans. 60.80 to 60.81**

**PARAGRAPH-II**

A cylindrical furnace has height ($H$) and diameter ($D$) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure $P_a$ and its temperature becomes $T = 360$ K. The hot air with density $\rho$ rises up a vertical chimney of diameter $d = 0.1$ m and height $h = 9$ m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density $\rho_a = 1.2$ kg m$^{-3}$, pressure $P_a$ and temperature $T_a = 300$ K enters the furnace. Assume air as an ideal gas, neglect the variations in $\rho$ and $T$ inside the chimney and the furnace. Also ignore the viscous effects.

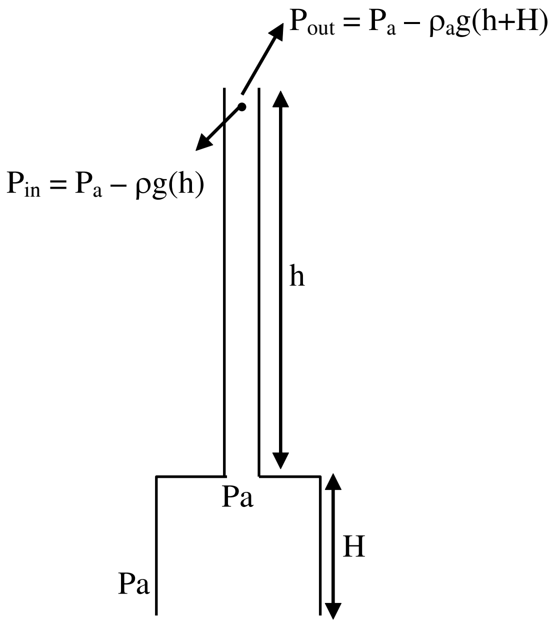
[Given: The acceleration due to gravity $g = 10$ ms$^{-2}$ and $\pi = 3.14$]



**17.** When the chimney is closed using a cap at the top, a pressure difference $\Delta P$ develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of $\Delta P$ is ______ $Nm^{-2}$.

**Ans. (30)**

**Sol.**




P = constant

$\Rightarrow \rho_a T_a = \rho T$

$1.2 \times 300 = \rho \times 360$

$\rho = 1 \text{ kg/m}^3$

$\Delta P = \rho_a g\,(h + H) - \rho g h$

$= 1.2 \times 10 \times 10 - 1 \times 10 \times 9$

$= 120 - 90 = 30 \text{ N/m}^2$

# CHEMISTRY

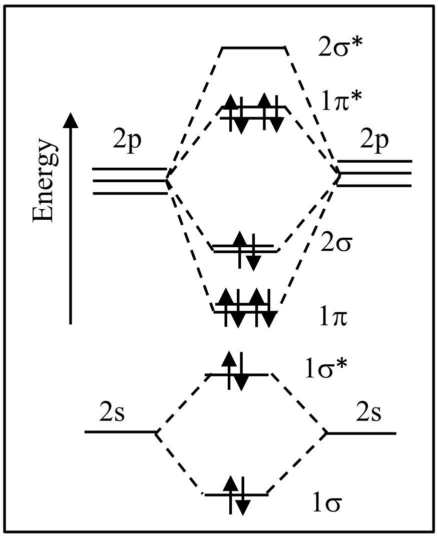
**SECTION-1 : (Maximum Marks : 12)**

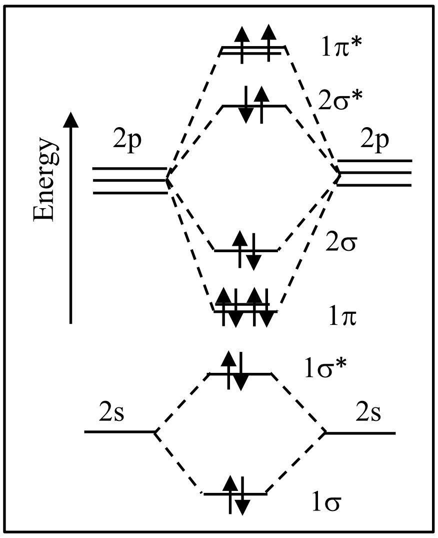
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

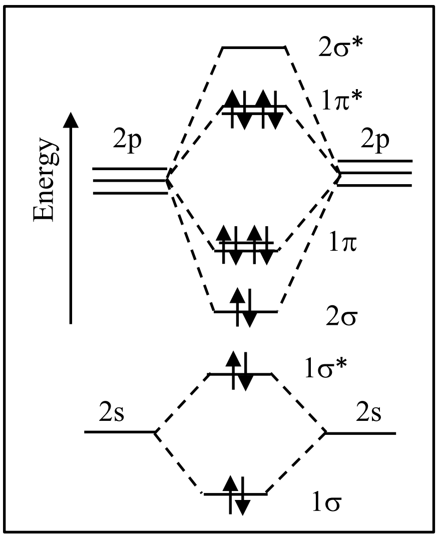
| | | |
|---|---|---|
| *Full Marks* | : +3 | If **ONLY** the correct option is chosen; |
| *Zero Marks* | : 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : −1 | In all other cases. |

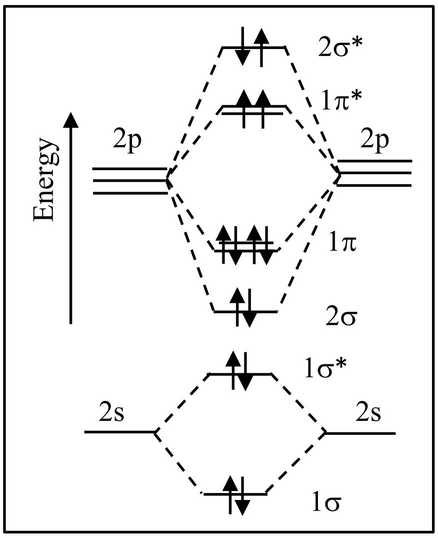
---

**1.** The correct molecular orbital diagram for $F_2$ molecule in the ground state is

(A)


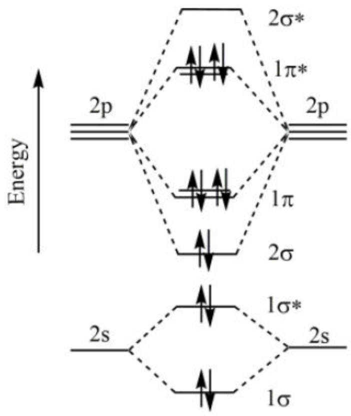

(B)



(C)



(D)



**Ans. (C)**

**Sol.** $F_2$ (18 $e^-$)




Naming of molecular orbitals are as per preference of formation of σ & π bonds respectively.

**2.** Consider the following statements related to colloids.

(I) Lyophobic colloids are **not** formed by simple mixing of dispersed phase and dispersion medium.

(II) For emulsions, both the dispersed phase and the dispersion medium are liquid.

(III) Micelles are produced by dissolving a surfactant in any solvent at any temperature.

(IV) Tyndall effect can be observed from a colloidal solution with dispersed phase having the same refractive index as that of the dispersion medium.

The option with the correct set of statements is

(A) (I) and (II) (B) (II) and (III) (C) (III) and (IV) (D) (II) and (IV)

**Ans.** **(A)**

**Sol.** (I) As in Lyophobic colloids there is no interaction between dispersed phase and dispersion medium, special methods are used for preparation, simple mixing will not form colloid.

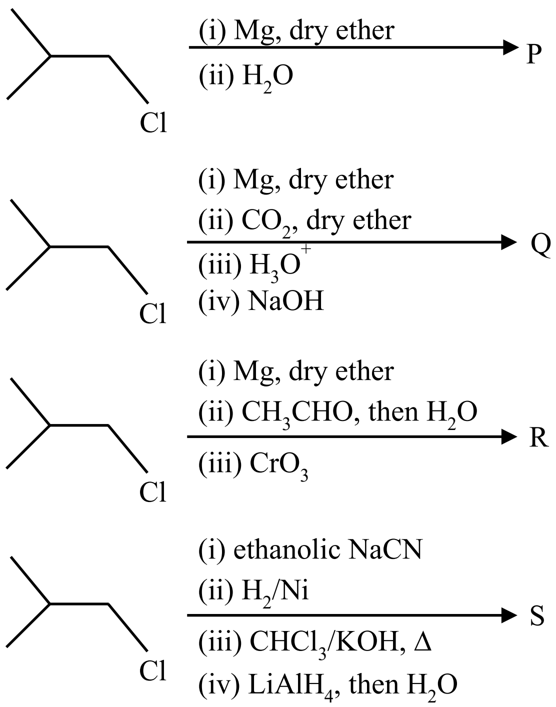
(II) Emulsions are liquid in liquid type colloids.

(III) Dissolving surfactant in a proper solvent will only form micelles at temperature above Kraft's temperature.

(IV) For Tyndall effect there must be a large difference in refractive index between dispersed phase and dispersion medium in order to have diffraction of light.

Hence ans (I) & (II) are correct.

**3.** In the following reactions, **P**, **Q**, **R**, and **S** are the major products.




The correct statement about **P**, **Q**, **R**, and **S** is

(A) **P** is a primary alcohol with four carbons.

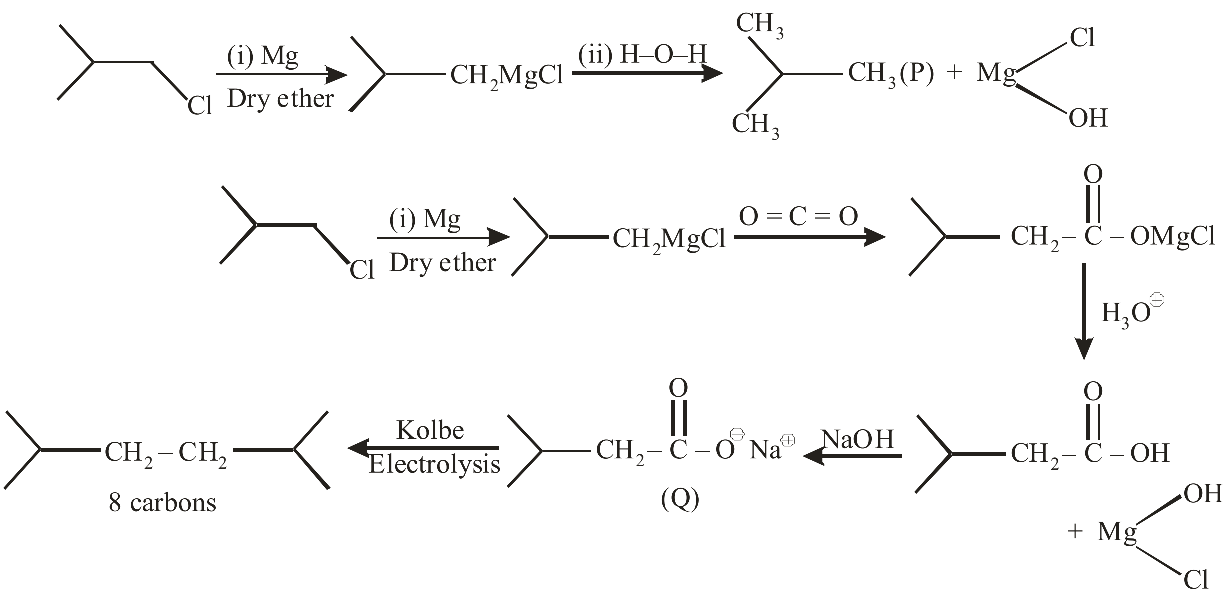
(B) **Q** undergoes Kolbe's electrolysis to give an eight-carbon product.

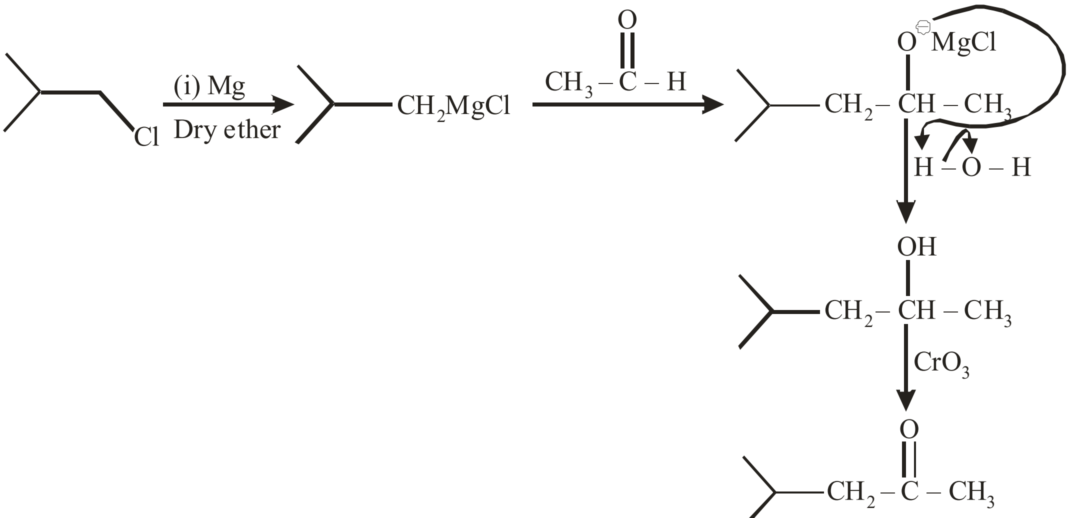
(C) **R** has six carbons and it undergoes Cannizzaro reaction.

(D) **S** is a primary amine with six carbons.

**Ans.** **(B)**

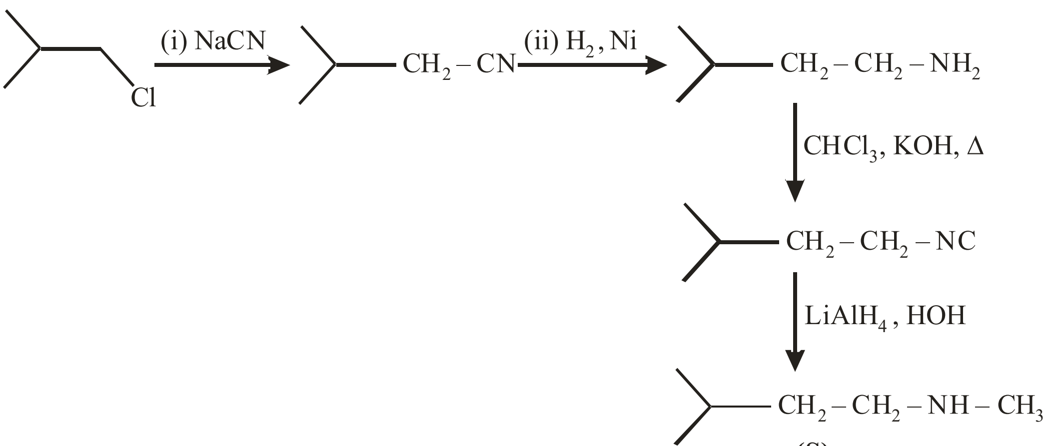
**Sol.**





**(R)**

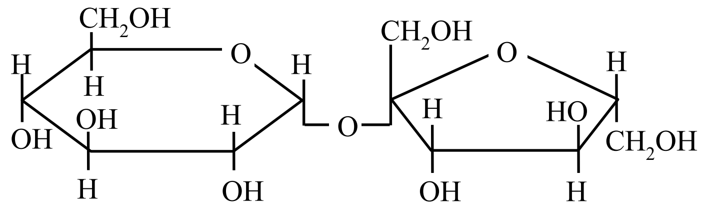
It does not give Cannizaro reaction

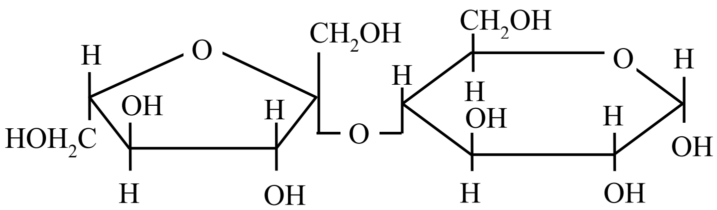


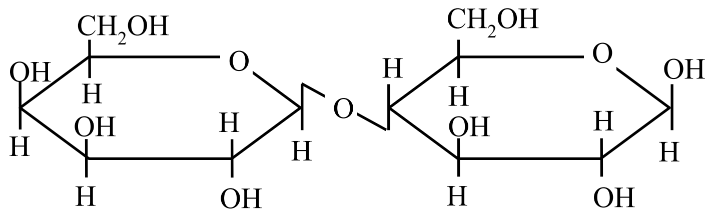
**(S)**

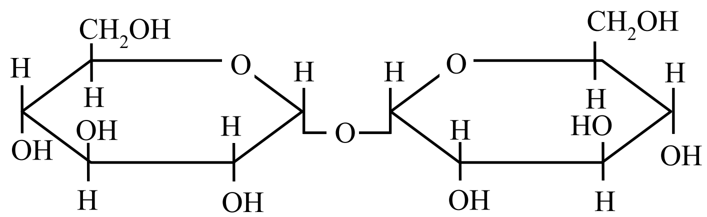
It's secondary amine

**4.** A disaccharide **X** cannot be oxidised by bromine water. The acid hydrolysis of **X** leads to a laevorotatory solution. The disaccharide **X** is

(A) 

(B) 

(C) 

(D) 

**Ans. (A)**

**Sol.** Sucrose $\xrightarrow{H_3O^+}$ Glucose + Fructose

Specific rotation + 52.5° –92° (mixture of products is laevorotatory)

Sucrose $\xrightarrow{Br_2+H_2O}$ No reaction

BCD ⇒ reducing sugars, will get oxidized by $Br_2 + H_2O$

## SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;

*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

*Zero Marks* : 0 If unanswered;

*Negative Marks* : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get –2 marks.

---

**5.** The complex(es), which can exhibit the type of isomerism shown by $[Pt(NH_3)_2Br_2]$, is(are) $[en = H_2NCH_2CH_2NH_2]$

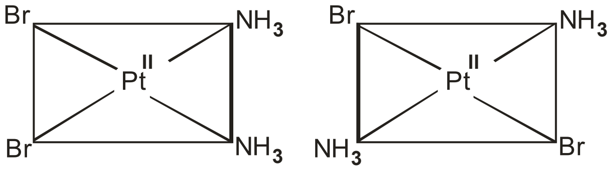
(A) $[Pt(en)(SCN)_2]$ (B) $[Zn(NH_3)_2Cl_2]$

(B) (C) $[Pt(NH_3)_2Cl_4]$ (D) $[Cr(en)_2(H_2O)(SO_4)]^+$

**Ans. (C,D)**

**Sol.** $\overset{+2}{[Pt(NH_3)_2Br_2]}$

Hybridisation : $dsp^2$, geometry : square planar

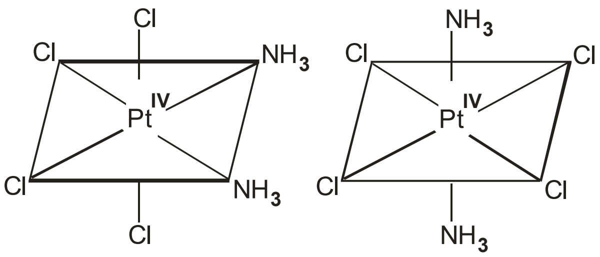



cis trans

(A) $[Pt(en)(SCN)_2]$ : square planar, cis–trans not possible

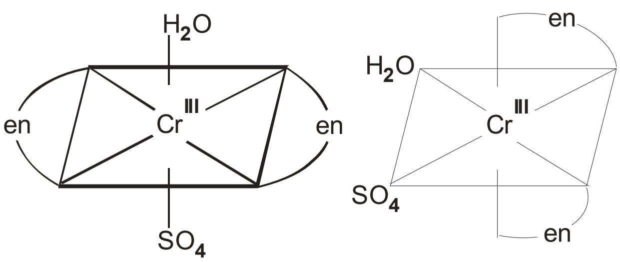
(B) $[Zn(NH_3)_2Cl_2]$ : tetrahedral, cis–trans not possible

(C) $[Pt(NH_3)_2Cl_4]$ : octahedral, cis–trans possible




cis trans

(D) $[Cr(en)_2(H_2O)SO_4]^+$ : Octahedral




trans cis

**6.** Atoms of metals x, y, and z form face-centred cubic (fcc) unit cell of edge length $L_x$, body-centred cubic (bcc) unit cell of edge length $L_y$, and simple cubic unit cell of edge length $L_z$, respectively.

If $r_z = \frac{\sqrt{3}}{2} r_y; r_y = \frac{8}{\sqrt{3}} r_x; M_z = \frac{3}{2} M_y$ and $M_z = 3M_x$, then the correct statement (s) is (are)

[Given : $M_x$, $M_y$, and $M_z$ are molar masses of metals x, y, and z, respectively.

$r_x$, $r_y$, and $r_z$ are atomic radii of metals x, y, and z, respectively.]

(A) Packing efficiency of unit cell of x > Packing efficiency of unit cell of y > Packing efficiency of unit cell of z

(B) $L_y > L_z$

(C) $L_x > L_y$

(D) Density of x > Density of y

**Ans.** **(A,B,D)**

**Sol.**

| Element | X | Y | Z |
|---|---|---|---|
| Packing | FCC | BCC | Primitive |
| Edge | $L_x$ | $L_y$ | $L_z$ |
| Relation between edge length and radius | $L_x = 2\sqrt{2}r_x$ | $L_y = \frac{4}{\sqrt{3}}r_y$ | $L_z = 2r_z$ |
| Packing fraction | $\frac{\pi}{3\sqrt{2}}$ | $\frac{\sqrt{3}\pi}{8}$ | $\frac{\pi}{6}$ |

Now, $r_y = \frac{8}{\sqrt{3}} r_x$ & $r_z = \frac{\sqrt{3}}{2} r_y = \frac{\sqrt{3}}{2} \times \frac{8}{\sqrt{3}} r_x \Rightarrow r_z = 4r_x$

So, $L_x = 2\sqrt{2}\, r_x$, $L_y = \frac{4}{\sqrt{3}} \times \frac{8}{\sqrt{3}} r_x$, $L_z = 8r_x$

$L_x = 2\sqrt{2}\, r_x$, $L_y = \frac{32}{3} r_x$, $L_z = 8r_x$

So $L_y > L_z > L_x$

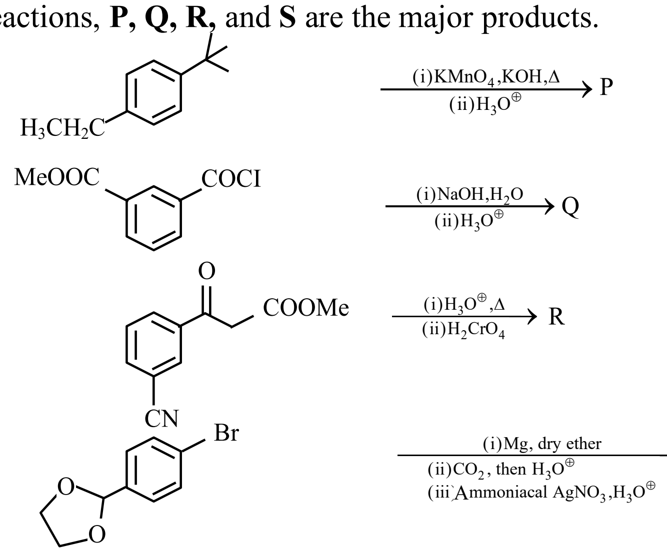
Density $\frac{4M_x}{L_x^3}, \frac{2 \times M_y}{L_y^3}$

Now, $3M_x = \frac{3M_y}{2}$ or $M_x \times 2 = M_y$

$$\frac{\text{density}(x)}{\text{density}(y)} = \frac{4M_x}{2M_y} \times \frac{L_y^3}{L_x^3} = \frac{4M_x}{4M_x} \times \frac{\left(\frac{32}{3}\right)^3}{\left(2\sqrt{2}\right)^3}$$

Hence d(x) > d(y)

**7.** In the following reactions, **P, Q, R,** and **S** are the major products.




The correct statement (s) about **P, Q, R,** and **S** is (are)

(A) **P** and **Q** are monomers of polymers dacron and glyptal, respectively.

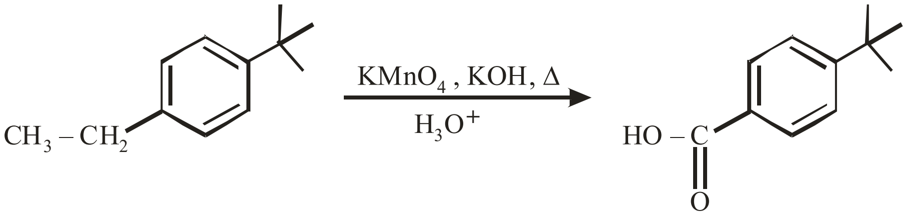
(B) **P, Q,** and **R** are dicarboxylic acids.

(C) Compounds **Q** and **R** are the same.

(D) **R** does **not** undergo aldol condensation and **S** does **not** undergo Cannizzaro reaction.

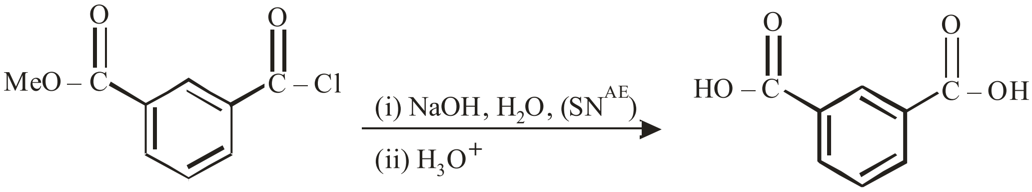
**Ans. (C,D)**

**Sol.**



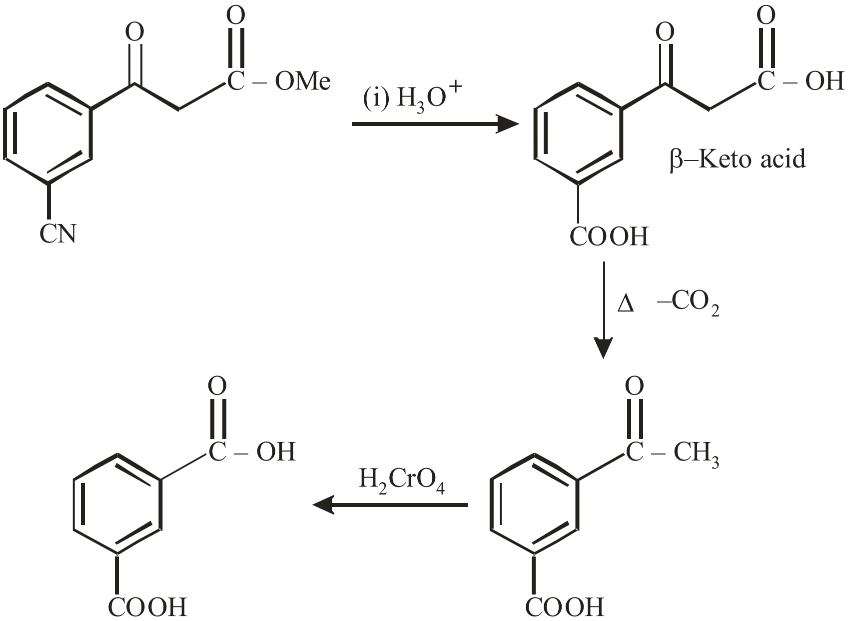
$CH_3-CH_2$
$KMnO_4$, KOH, Δ
$H_3O^+$
HO – C
O


**(P)**



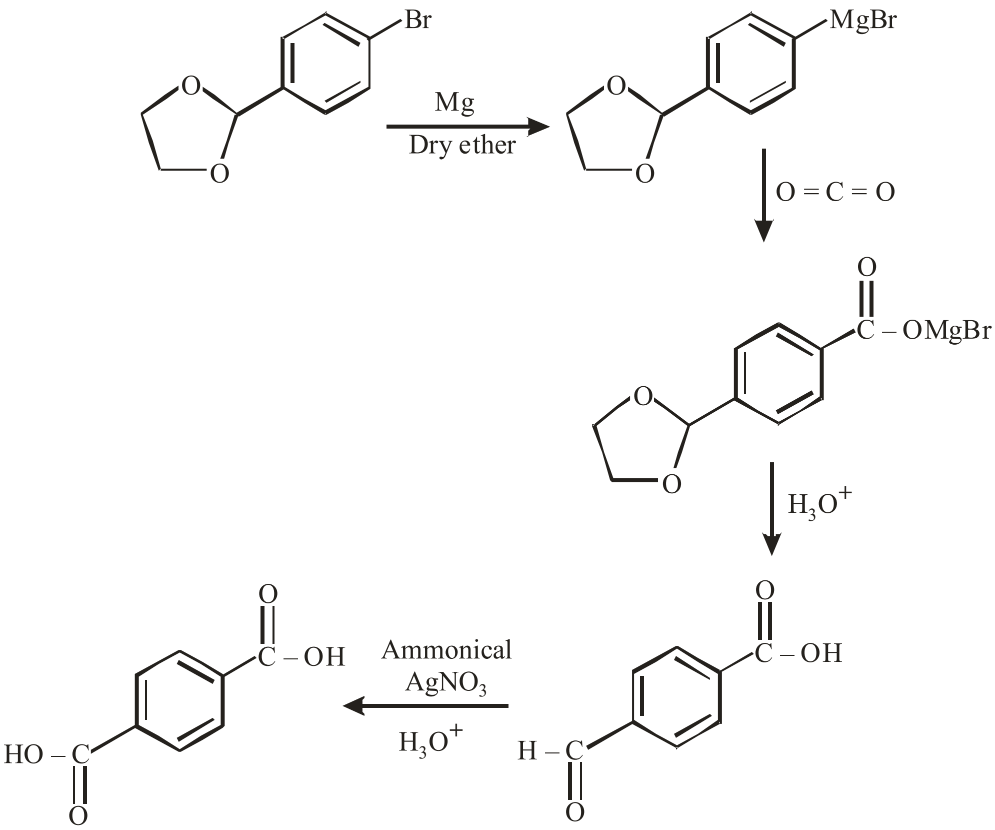
O
MeO – C
O
C – Cl
(i) NaOH, $H_2O$, ($SN^{AE}$)
(ii) $H_3O^+$
O
HO – C
O
C – OH


**(Q)**



O
O
C – OMe
CN
(i) $H_3O^+$
O
O
C – OH
β–Keto acid
COOH
Δ –$CO_2$
O
C – $CH_3$
COOH
$H_2CrO_4$
O
C – OH
COOH


**(R)**



**SECTION-3 : (Maximum Marks : 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +4 If **ONLY** the correct integer is entered;
  *Zero Marks* : 0 In all other cases

---

**8.** $H_2S$ (5 moles) reacts completely with acidified aqueous potassium permanganate solution. In this reaction, the number of moles of water produced is **x**, and the number of moles of electrons involved is **y**. The value of (**x** + **y**) is ____.

**Ans. (18)**

**Sol.** $$2K\overset{+7}{Mn}O_4 + 5H_2S + 3H_2SO_4 \rightarrow K_2SO_4 + 2\overset{+2}{Mn}SO_4 + 5S + 8H_2O$$

x = 8 (moles of $H_2O$ produced)

y = 14 – 4 = 10 (number of electrons involved)

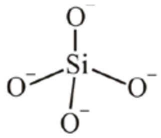
x + y = 10 + 8 = 18

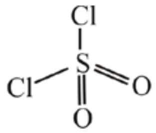
**9.** Among $[I_3]^+$, $[SiO_4]^{4-}$ , $SO_2Cl_2$, $XeF_2$, $SF_4$, $ClF_3$, $Ni(CO)_4$, $XeO_2F_2$, $[PtCl_4]^{2-}$, $XeF_4$, and $SOCl_2$, the total number of species having $sp^3$ hybridised central atom is ______.

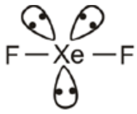
**Ans. (5)**

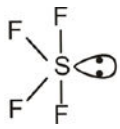
**Sol.**

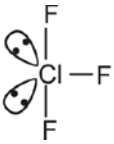
$I_3^+$ :  : $sp^3$

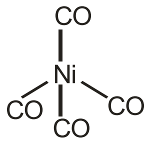
$SiO_4^{4-}$ :  : $sp^3$

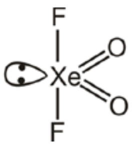
$SO_2Cl_2$ :  : $sp^3$

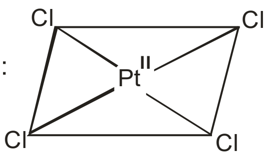
$XeF_2$ :  : $sp^3d$

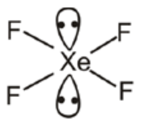
$SF_4$ :  : $sp^3d$

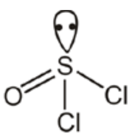
$ClF_3$ :  : $sp^3d$

$[Ni(CO)_4]$ :  : $sp^3$

$XeO_2F_2$ :  : $sp^3d$

$[PtCl_4]^{2-}$ :  : $dsp^2$

$XeF_4$ :  : $sp^3d^2$

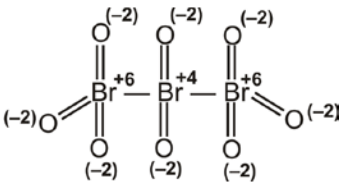
$SOCl_2$ :  : $sp^3$

**10.** Consider the following molecules : $Br_3O_8$, $F_2O$, $H_2S_4O_6$, $H_2S_5O_6$, and $C_3O_2$.

Count the number of atoms existing in their zero oxidation state in each molecule. Their sum is____.

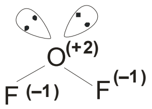
**Ans.** **(6)**

**Sol.** $Br_3O_8$



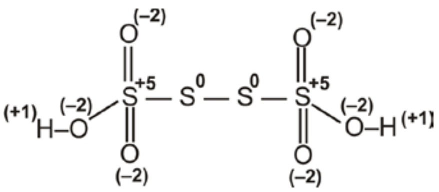
Number of atoms with zero oxidation state = 0

$F_2O$



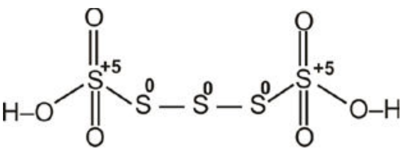
Number of atom with zero oxidation state = 0

$H_2S_4O_6$



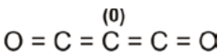
Number of atoms with zero oxidation state = 2

$H_2S_5O_6$



Number of atoms where zero oxidation state = 3

$C_3O_2$



Number of atoms with zero oxidation state = 1

**11.** For $He^+$, a transition takes place from the orbit of radius 105.8 pm to the orbit of radius 26.45 pm. The wavelength (in nm) of the emitted photon during the transition is ___.

[**Use:**

Bohr radius, a = 52.9 pm

Rydberg constant, $R_H = 2.2 \times 10^{-18}$ J

Planck's constant, $h = 6.6 \times 10^{-34}$ J s

Speed of light, $c = 3 \times 10^8$ m $s^{-1}$ ]

**Ans.** **(30)**

**Sol.** For single electron system

$$r = 52.9 \times \frac{n^2}{Z} \text{ pm}$$

Given Z = 2 for $He^+$

$r_2 = 105.8$ pm

So $105.8 = 52.9 \times \frac{n_2^2}{2}$

$n_2 = 2$

$r_1 = 26.45$

So $26.45 = 52.9 \times \frac{n_1^2}{2}$

$n_1 = 1$

So transition is from 2 to 1.

Now $\frac{hc}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

So $\lambda = 30 \times 10^{-9}$ m = 30 nanometer.

Here '$R_H$' is given in terms of energy value.

**12.** 50 mL of 0.2 molal urea solution (density = 1.012 g $mL^{-1}$ at 300 K) is mixed with 250 mL of a solution containing 0.06 g of urea. Both the solutions were prepared in the same solvent. The osmotic pressure (in Torr) of the resulting solution at 300 K is __.

[Use : Molar mass of urea = 60 g $mol^{-1}$; gas constant, R = 62 L Torr $K^{-1}$ $mol^{-1}$ ; Assume, $\Delta_{mix}H = 0$, $\Delta_{mix}V = 0$]

**Ans. (682)**

**Sol.** Weight of 50 ml 0.2 molal urea = V × d = 50 × 1.012 = 50.6 gm

Given 0.2 molal implies

1000 gm solvent has 0.2 moles urea

So weight of solution = 1000 + 0.2 × 60 = 1012 gm.

So wt. of urea in 50.6 gm solution = $\frac{12 \times 50.6}{1012}$ = 0.6 gm

Total urea = 0.6 + 0.06 = 0.66 gm

Total volume = 300 ml

Now, osmotic pressure $\pi = C \times R \times T = \frac{0.66 \times 62 \times 300}{60 \times 0.3}$ = 682 Torr.

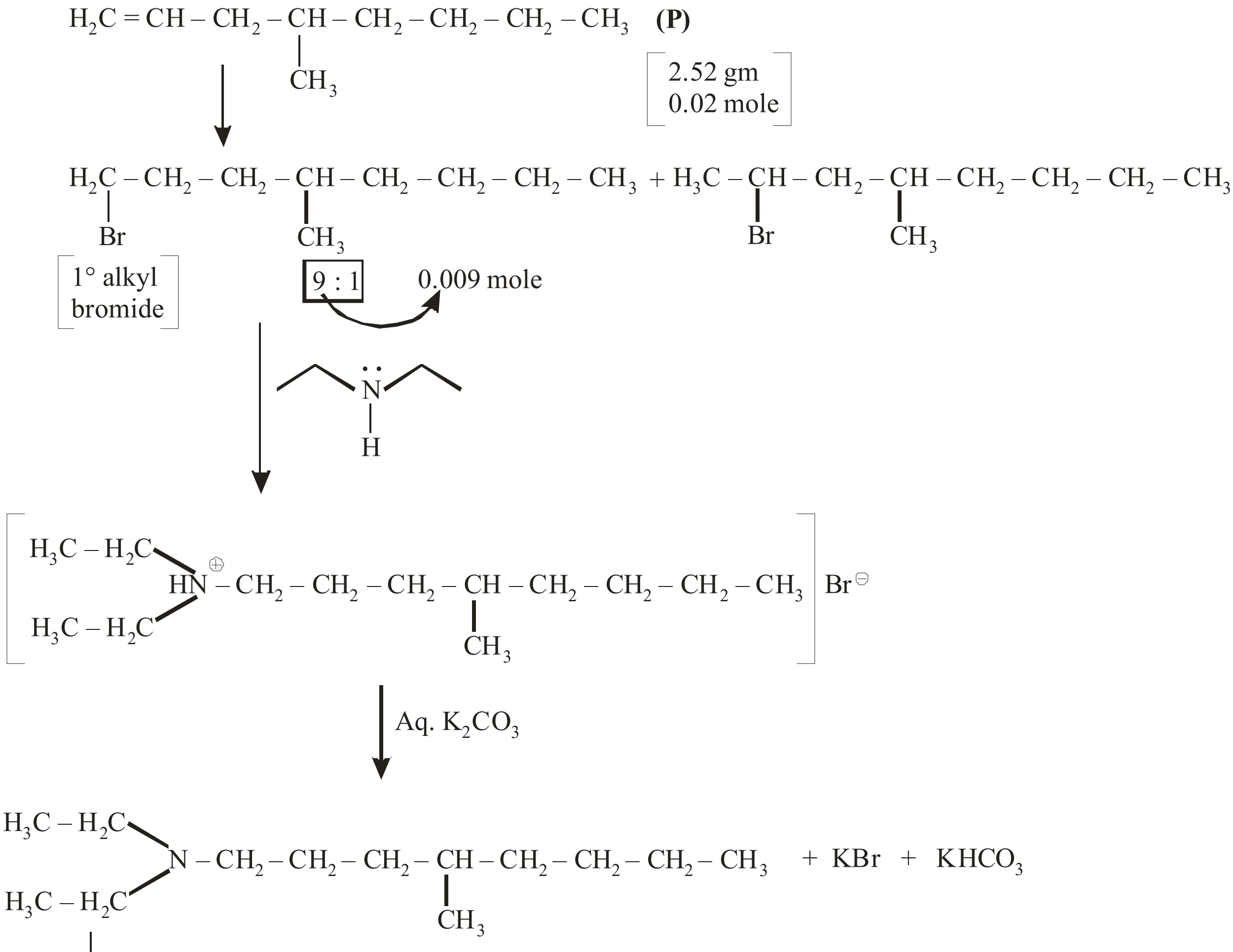
**13.** The reaction of 4-methyloct-ene (**P**, 2.52 g) with HBr in the presence of $(C_6H_5CO)_2O_2$ gives two isomeric bromides in a 9 : 1 ratio, with combined yield of 50%. Of these, the entire amount of the primary alkyl bromide was reacted with an appropriate amount of diethylamine followed by treatment with eq. $K_2CO_3$ to given a non-ionic product **S** in 100% yield.

The mass (in mg) of **S** obtained is __.

[Use molar mass (in g mol$^{-1}$) : H = 1, C = 12, N = 14, Br = 80]

**Ans. (1791)**

**Sol.**




mol. mass = 199 gm

(S) 0.009 mole

= 199 × 0.009 = 1791 mg

## SECTION-4 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +3 If ONLY the correct numerical value is entered in the designated place;
  *Zero Marks* : 0 In all other cases.

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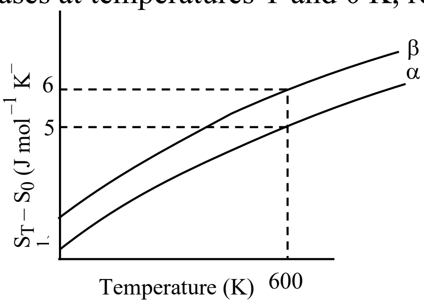
"PARAGRAPH I"

The entropy versus temperature plot for phases α and β at 1 bar pressure is given.
$S_T$ and $S_0$ are entropies of the phases at temperatures T and 0 K, respectively.




The transition temperature for α to β phase change is 600 K and $C_{P,\beta} - C_{P,\alpha} = 1$ J mol$^{-1}$ K$^{-1}$. Assume $(C_{P,\beta} - C_{P,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{P,\alpha}$ and $C_{P,\beta}$ are heat capacities of α and β phases, respectively.

**14.** The value of entropy change, $S_\beta - S_\alpha$ (in J mol$^{-1}$ K$^{-1}$), at 300 K is __.
[Use : ln 2 = 0.69
Given : $S_\beta - S_\alpha = 0$ at 0 K]

**Ans. (0.31)**

**Sol.** At 1 bar

$$\alpha \longrightarrow \beta$$

$$S^o_{\alpha(600)} = S^o_{\alpha(300)} + C_{P(\alpha)}\ell n\frac{600}{300}$$

$$S^o_{\beta(600)} = S^o_{\beta(300)} + C_{P(\beta)}\ell n\frac{600}{300}$$

$$S^o_{\beta(600)} - S^o_{\alpha(600)} = S^o_{\beta(300)} - S^o_{\alpha(300)} + (C_{P(\beta)} - C_{P(\alpha)})\ \ell n 2$$

$$6 - 5 = S^o_{\beta(300)} - S^o_{\alpha(300)} + 1 \times \ell n 2$$

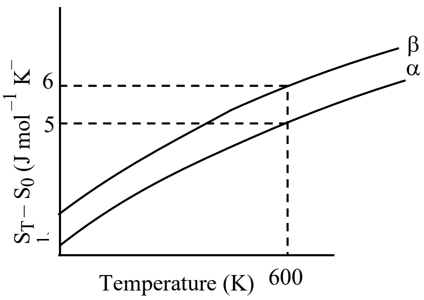
$$1 = S^o_{\beta(300)} - S^o_{\alpha(300)} + 0.69$$

$$\text{So } S^o_{\beta(300)} - S^o_{\alpha(300)} = 0.31$$

"PARAGRAPH I"

The entropy versus temperature plot for phases $\alpha$ and $\beta$ 1 bar pressure is given.

$S_T$ and $S_0$ are entropies of the phases at temperatures T and 0 K, respectively




The transition temperature for $\alpha$ to $\beta$ phase change is 600 K and $C_{P,\beta} - C_{P,\alpha} = 1 J\ mol^{-1}\ K^{-1}$. Assume $(C_{P,\beta} - C_{P,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{P,\alpha}$ and $C_{P,\beta}$ are heat capacities of $\alpha$ and $\beta$ phases, respectively.

**15.** The value of enthalpy change, $H_\beta - H_\alpha$ (in J mol$^{-1}$), at 300 K is __.

**Ans. (300)**

**Sol.** As the phase transition temperature is 600 K

So at 600 K $\Delta G^\circ_{rxn} = 0$

So $\Delta H^\circ_{reaction\ (600)} = T\ \Delta S^\circ_{reaction\ (600)}$

$\Delta H^\circ_{(600)} = 600 \times 1 = 600$ Joule/mole

So $\Delta H_{600} - \Delta H_{300} = \Delta C_P (T_2 - T_1)$

$\Delta H_{600} - \Delta H_{300} = 1 \times 300$

$\Delta H_{300} = \Delta H_{600} - 300 = 600 - 300 = 300$ Joule/mole.

"PARAGRAPH II"

A trinitro compound, 1, 3,5 tris-(4-nitrophenyl) benzene, on complete reaction with an excess of Sn/HCl gives major product, which on treatment with an excess of $NaNO_2$/HCl at 0°C provides **P** as the product. **P**, upon treatment with excess of $H_2O$ at room temperature, gives the product **Q.** Bromination of **Q** in aqueous medium furnishes the product **R.** The compound **P** upon treatment with an excess of phenol under basic conditions gives the product **S.**

The molar mass difference between compounds **Q** and **R** is 474 $mol^{-1}$ and between compounds **P** and **S** is 172.5 g $mol^{-1}$.

**16.** The number of heteroatoms present in one molecule of **R** is _____.

[Use: Molar mass (in g $mol^{-1}$): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5

Atoms other than C and H are considered as heteroatoms]

**Ans. (9)**

"PARAGRAPH II"

A trinitro compound, 1, 3,5 tris-(4-nitrophenyl) benzene, on complete reaction with an excess of Sn/HCl gives major product, which on treatment with an excess of $NaNO_2$/HCl at 0°C provides **P** as the product. **P,** upon treatment with excess of $H_2O$ at room temperature, gives the product **Q.** Bromination of **Q** in aqueous medium furnishes the product **R.** The compound **P** upon treatment with an excess of phenol under basic conditions gives the product **S.**

The molar mass difference between compounds **Q** and **R** is 474 $mol^{-1}$ and between compounds **P** and **S** is 172.5 g $mol^{-1}$.

**17.** The total number of carbon atoms and heteroatoms present in one molecule of **S** is **_____.**

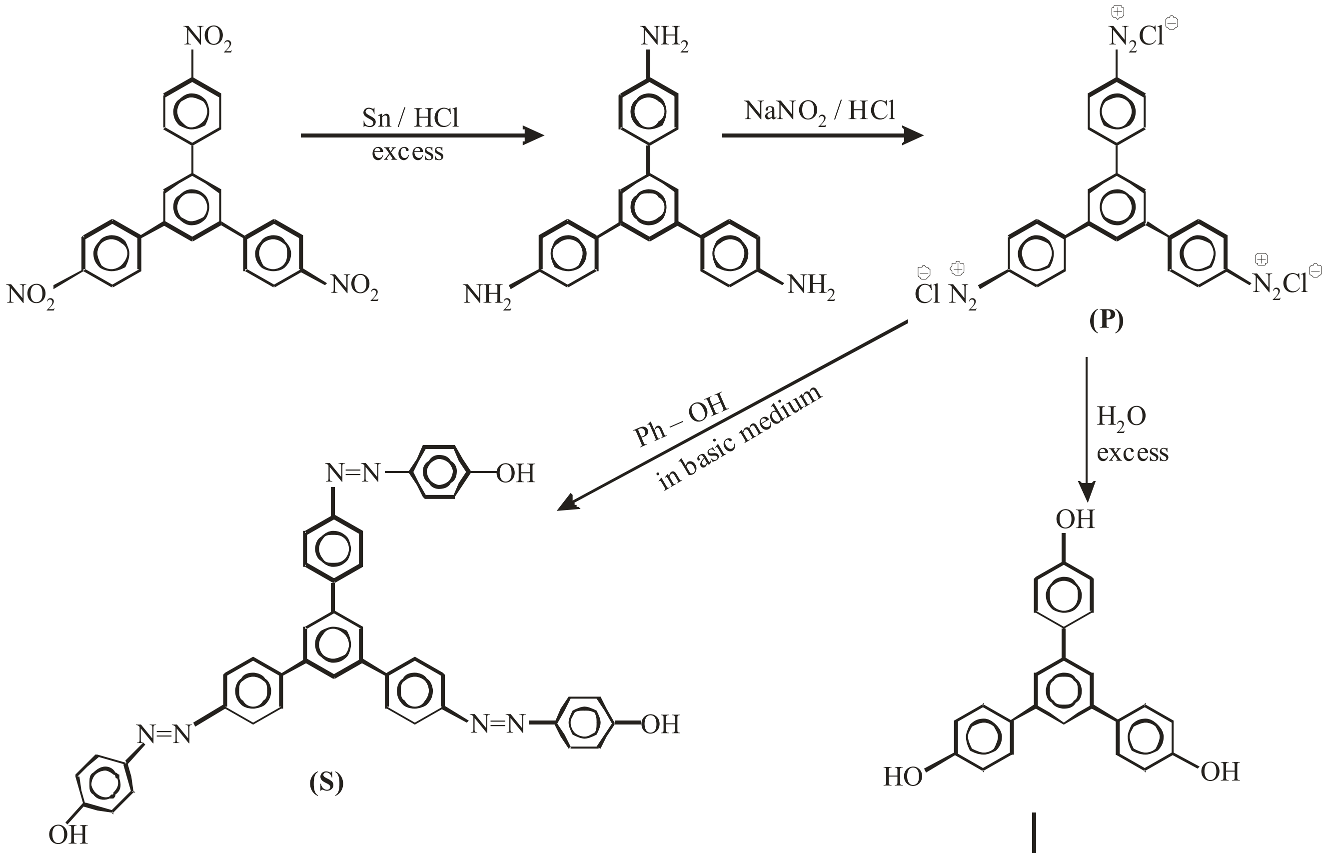
[Use: Molar mass in g $mol^{-1}$]: H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5

Atoms other than C and H are considered as heteroatoms

**Ans. (51)**

**Sol.**

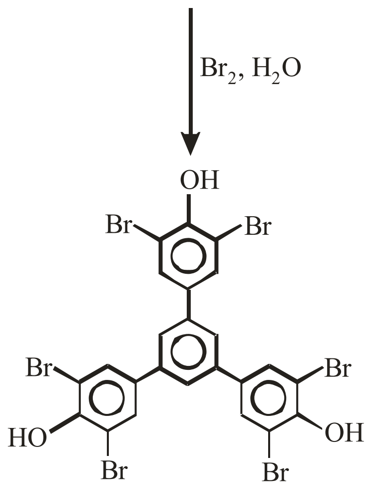
**Common solution for Q.no. 16 and 17**




Number of carbons = 42

Number of heteroatoms = 09

Total = 51




**(R)**

Number of hetero atoms in R is 9

# MATHEMATICS

**SECTION-1 : (Maximum Marks : 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If **ONLY** the correct option is chosen;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : −1 In all other cases.

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**1.** Let $f:[(1,\infty)\to\mathbb{R}$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)dt = xf(x)-\frac{x^3}{3}, x\in[1,\infty)$.

Let $e$ denote the base of the natural logarithm. Then the value of f($e$) is

(A) $\frac{e^2+4}{3}$

(B) $\frac{\log_e 4+e}{3}$

(C) $\frac{4e^2}{3}$

(D) $\frac{e^2-4}{3}$

**Ans. (C)**

**Sol.** Diff. wr.t 'x'

$3f(x) = f(x) + xf'(x) - x^2$

$\frac{dy}{dx}-\left(\frac{2}{x}\right)y = x$

$IF = e^{-2\ell nx} = \frac{1}{x^2}$

$y\left(\frac{1}{x^2}\right)=\int x.\frac{1}{x^2}dx$

$y = x^2\ell nx + cx^2$

$\therefore\ y(1)=\frac{1}{3}\Rightarrow c=\frac{1}{3}$

$y(e)=\frac{4e^2}{3}$

**2.** Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is.

(A) $\frac{1}{3}$ (B) $\frac{5}{21}$ (C) $\frac{4}{21}$ (D) $\frac{2}{7}$

**Ans. (B)**

**Sol.** $P(H)=\frac{1}{3};P(T)=\frac{2}{3}$

Req. prob = P(HH or HTHH or HTHTHH or …….)

+ P(THH or THTHH or THTHTHH or ….)

$$=\frac{\frac{1}{3}.\frac{1}{3}}{1-\frac{2}{3}.\frac{1}{3}}+\frac{\frac{2}{3}.\frac{1}{3}.\frac{1}{3}}{1-\frac{2}{3}.\frac{1}{3}}=\frac{5}{21}$$

**3.** For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\cot^{-1}\left(\frac{9-y^2}{6y}\right)=\frac{2\pi}{3}$ for $0<|y|<3$, is equal to

(A) $2\sqrt{3}-3$ (B) $3-2\sqrt{3}$

(C) $4\sqrt{3}-6$ (D) $6-4\sqrt{3}$

**Ans. (C)**

**Sol.** Case-I : $y \in (-3,0)$

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right)+\pi+\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3}$$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=-\frac{\pi}{3}$$

$y^2-6\sqrt{3}y-9=0 \Rightarrow y=3\sqrt{3}-6$ $(\because y \in (-3,0))$

Case-I : $y \in (0,3)$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right)=\frac{2\pi}{3} \Rightarrow \sqrt{3}y^2+6y-9\sqrt{3}=0$$

$y=\sqrt{3}$ or $y=-3\sqrt{3}$ (rejected)

sum $=\sqrt{3}+3\sqrt{3}-6=4\sqrt{3}-6$

**4.** Let the position vectors of the points P,Q,R and S be $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=3\hat{i}+6\hat{j}+3\hat{k}$, $\vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively. Then which of the following statements is true?

(A) The points P,Q,R and S are **NOT** coplanar

(B) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b}\times\vec{d}$ is 95

**Ans. (B)**

**Sol.** $P\left(\hat{i}+2\hat{j}-5\hat{k}\right)=P\left(\vec{a}\right)$

$Q\left(3\hat{i}+6\hat{j}+3\hat{k}\right)=Q\left(\vec{b}\right)$

$R\left(\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}\right)=R\left(\vec{c}\right)$

$S\left(2\hat{i}+\hat{j}+\hat{k}\right)=S\left(\vec{d}\right)$

$$\frac{\vec{b}+2\vec{d}}{3}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}$$

$$\frac{5\vec{c}+4\vec{a}}{9}=\frac{21\hat{i}+24\hat{j}+15\hat{k}}{9}$$

$$\Rightarrow\frac{\vec{b}+2\vec{d}}{3}=\frac{5\vec{c}+4\vec{a}}{9}$$

so [B] is correct.

option –D

$$\left|\vec{b}\times\vec{d}\right|^2=\left|\vec{b}\right|\left|\vec{d}\right|^2-\left(\vec{b}.\vec{d}\right)^2$$

$= (9 + 36 + 9)\ (4 + 1 + 1) – (6 + 6 + 3)^2$

$= 54 \times 6 – (15)^2$

$= 324 – 225$

$= 99$

## SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If unanswered; |
| *Negative Marks* | : –2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

---

**5.** Let $M=(a_{ij}), i, j \in \{1, 2, 3\}$, be the $3 \times 3$ matrix such that $a_{ij} = 1$ if $j+1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is (are) true ?

(A) M is invertible

(B) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\} \neq \{\mathbf{0}\}$, where $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the $3 \times 3$ identity matrix

**Ans. (B,C)**

**Sol.** $M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$|M| = -1 + 1 = 0 \Rightarrow$ M is singular so non-invertible

(B) $M\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$

$\left.\begin{array}{l} a_1 + a_2 + a_3 = -a_1 \\ a_1 + a_3 = -a_2 \\ a_2 = -a_3 \end{array}\right\} \Rightarrow a_1 = 0$ and $a_2 + a_3 = 0$ infinite solutions exists [B] is correct.

Option (D)

$M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

$|M - 2I| = 0 \Rightarrow$ [D] is wrong

Option (C) :

$MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x + y + z = 0$

$x + z = 0$

$y = 0$

$\therefore$ Infinite solution

[C] is correct

**6.** Let $f : (0,1) \to \mathbb{R}$ be the function defined as $f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right)$, where [x] denotes the greatest integer less than or equal to x . Then which of the following statements is(are) true?

(A) The function f is discontinuous exactly at one point in (0,1)

(B) There is exactly one point in (0,1) at which the function f is continuous but **NOT** differentiable

(C) The function f is **NOT** differentiable at more than three points in (0,1)

(D) The minimum value of the function f is $-\frac{1}{512}$

**Ans. (A,B)**

**Sol.** $$f(x)=\begin{cases} 0 & ; \quad 0<x<\frac{1}{4} \\ \left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & ; \quad \frac{1}{4}\le x<\frac{1}{2} \\ 2\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & ; \quad \frac{1}{2}\le x<\frac{3}{4} \\ 3\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & ; \quad \frac{3}{4}\le x<1 \end{cases}$$

f(x) is discontinuous at $x=\frac{3}{4}$ only

$$f'(x)=\begin{cases} 0 & ; \quad 0<x<\frac{1}{4} \\ 2\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)+\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{1}{4}<x<\frac{1}{2} \\ 4\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)+2\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{1}{2}<x<\frac{3}{4} \\ 6\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)+3\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{3}{4}<x<1 \end{cases}$$

f(x) is non-differentiable at $x=\frac{1}{2}$ and $\frac{3}{4}$

minimum values of f(x) occur at $x=\frac{5}{12}$ whose value is $-\frac{1}{432}$

**7.** Let S be the set of all twice differentiable functions f from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2f}{dx^2}(x)>0$ for all $x \in (-1,1)$. For $f \in S$, let $X_f$ be the number of points $x \in (-1,1)$ for which f (x) = x. Then which of the following statements is(are) true?

(A) There exists a function $f \in S$ such that $X_f = 0$

(B) For every function $f \in S$, we have $X_f \le 2$

(C) There exists a function $f \in S$ such that $X_f = 2$

(D) There does **NOT** exist any function f in S such that $X_f = 1$

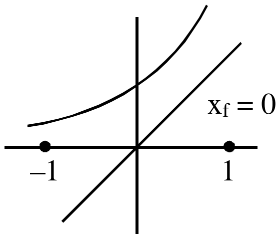
**Ans. (A,B,C)**

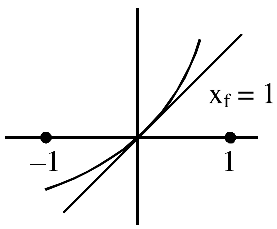
**Sol.** S = Set of all twice differentiable functions $f : R \to R$

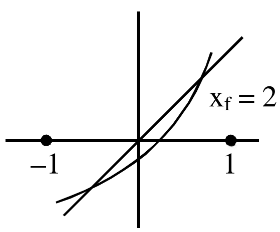
$\frac{d^2f}{dx^2} > 0$ in $(-1, 1)$

Graph 'f' is Concave upward.

Number of solutions of $f(x) = x \to x_f$

(1) 


(2) 


(3) 


$\Rightarrow$ Graph of $y = f(x)$ can intersect graph of $y = x$ at atmost two points $\Rightarrow 0 \le x_f \le 2$

**<u>Aliter</u>**

$\frac{d^2f(x)}{dx^2} > 0$

Let $\phi(x) = f(x) - x$

$\phi''(x) > 0$

$\therefore$ $\phi'(x) = 0$ has atmost 1 root in $x \in (-1, 1)$

$\therefore$ $\phi(x) = 0$ has atmost 2 roots in $x \in (-1, 1)$

$\therefore$ $x_f \le 2$

## SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 If **ONLY** the correct integer is entered;

*Zero Marks* : 0 In all other cases

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**8.** For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}}dt \text{ is}$$

**Ans. (0)**

**Sol.** $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{t-\cos t}}{1+t^{2023}}dt$

$$f'(x) = \frac{e^{x\tan^{-1}x-\cos(x\tan^{-1}x)}}{1+(x\tan^{-1}x)^{2023}} \cdot \left(\frac{x}{1+x^2} + \tan^{-1}x\right)$$

For $x < 0,\ \tan^{-1}x \in \left(-\frac{\pi}{2}, 0\right)$

For $x \geq 0,\ \tan^{-1}x \in \left[0, \frac{\pi}{2}\right)$

$\Rightarrow x\tan^{-1}x \geq 0\ \forall\ x \in R$

$$\text{And } \frac{x}{1+x^2} + \tan^{-1}x = \begin{cases} >0 & \text{For } x>0 \\ <0 & \text{For } x<0 \\ 0 & \text{For } x=0 \end{cases}$$

f'

− +

point of minima

Hence minimum value is $f(0) = \int_0^0 = 0$

**9.** For $x \in \mathbb{R}$, let y(x) be a solution of the differential equation

$$(x^2 - 5)\frac{dy}{dx} - 2xy = -2x(x^2 - 5)^2 \text{ such that } y(2) = 7.$$

Then the maximum value of the function y(x) is

**Ans. (16)**

**Sol.** $\dfrac{dy}{dx} - \dfrac{2x}{x^2 - 5}y = -2x(x^2 - 5)$

$$IF = e^{-\int \frac{2x}{x^2-5}dx} = \frac{1}{(x^2 - 5)}$$

$$y.\frac{1}{x^2 - 5} = \int -2x.dx + c$$

$$\Rightarrow \frac{y}{x^2 - 5} = -x^2 + c$$

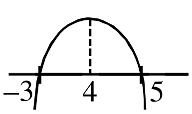
$x = 2, y = 7$

$$\frac{7}{-1} = -4 + c \;\Rightarrow c = -3$$

$y = -(x^2 - 5)(x^2 + 3)$

put $x^2 = t > 0$

$y = -(t - 5)(t + 3)$




$y_{max} = 16$ when $x^2 = 1$

$y_{max} = 16$

**10.** Let X be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to

**Ans. (31)**

**Sol.** No. of elements in X which are multiple of 5

$$\left.\begin{array}{l}
\underbrace{_\ _\ _\ _}_{1,2,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \dfrac{\underline{|4}}{\underline{|3}} = 4 \\[2ex]
\underbrace{_\ _\ _\ _}_{1,4,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \dfrac{\underline{|4}}{\underline{|2}} = 12 \\[2ex]
\underbrace{_\ _\ _\ _}_{4,2,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \dfrac{\underline{|4}}{\underline{|3}} = 4 \\[2ex]
\underbrace{_\ _\ _\ _}_{2,2,4,4}\ \underbrace{0}_{\text{fixed}} \rightarrow \dfrac{\underline{|4}}{\underline{|2}\,\underline{|2}} = 6 \\[2ex]
\underbrace{_\ _\ _\ _}_{1,2,4,4}\ \underbrace{0}_{\text{fixed}} \rightarrow \dfrac{\underline{|4}}{\underline{|2}} = 12
\end{array}\right\}\text{Total} = 38$$

Among these 38 elements, let us calculate when element is not divisible by 20

$$\left.\begin{array}{l}
\underbrace{_\ _\ _}_{2,2,2}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \dfrac{\underline{|3}}{\underline{|3}} = 1 \\[2ex]
\underbrace{_\ _\ _}_{2,2,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \dfrac{\underline{|3}}{\underline{|2}} = 3 \\[2ex]
\underbrace{_\ _\ _}_{2,4,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \dfrac{\underline{|3}}{\underline{|2}} = 3
\end{array}\right\}\text{Total} = 7$$

$\therefore\ p = \dfrac{38-7}{38}\quad \therefore\ 38p = 31$

**11.** Let $A_1$, $A_2$, $A_3$, ........, $A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let $PA_i$ denote the distance between the points P and $A_i$ for i = 1,2,....,8. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \cdot \cdots\cdots PA_8$, is

**Ans. (512)**

**Sol.** $z^8 - 2^8 = (z-2)(z-\alpha)(z-\alpha^2)\ldots(z-\alpha^7)$

Put $z = 2e^{i\theta}$

$2^8(e^{i8\theta} - 1) = (2e^{i\theta} - 2)(2e^{i\theta} - \alpha)\ldots..(2e^{i\theta} - \alpha^7)$

Take mod

$2^8|e^{i8\theta} - 1| = PA_1\, PA_2 \ldots PA_8$

$2^8|2\sin 4\theta| = PA_1\, PA_2 \ldots PA_8$

$(PA_1 \cdot PA_2 \ldots PA_8)_{max} = 512$

**12.** Let $R=\left\{\begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a,b,c,d \in \{0,3,5,7,11,13,17,19\}\right\}$. Then the number of invertible matrices in R is

**Ans. (3780)**

**Sol.** Let us calculate when $|R| = 0$

Case-I $ad = bc = 0$

Now $ad = 0$

$\Rightarrow$ Total – (When none of a & d is 0)

$= 8^2 - 1 = 15$ ways

Similarly $bc = 0 \Rightarrow 15$ ways

$\therefore 15 \times 15 = 225$ ways of $ad = bc = 0$

Case-II $ad = bc \neq 0$

either $a = d = b = c$ OR $a \neq d,\ b \neq d$ but $ad = bc$

${}^7C_1 = 7$ ways ${}^7C_2 \times 2 \times 2 = 84$ ways

Total 91 ways

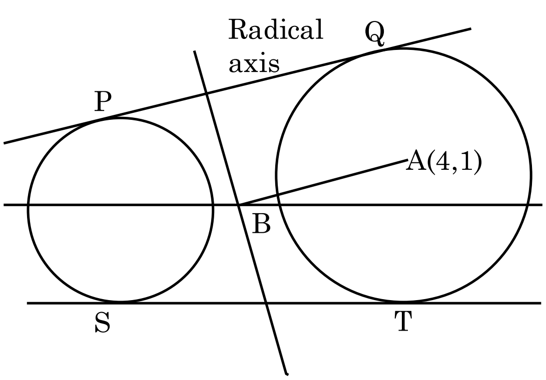
$\therefore |R| = 0$ in $225 + 91 = 316$ ways

$|R| \neq 0$ in $8^4 - 316 = 3780$

**13.** Let $C_1$ be the circle of radius 1 with center at the origin. Let $C_2$ be the circle of radius r with center at the point $A = (4,1)$, where $1 < r < 3$. Two distinct common tangents PQ and ST of $C_1$ and $C_2$ are drawn. The tangent PQ touches $C_1$ at P and $C_2$ at Q. The tangent ST touches $C_1$ at S and $C_2$ at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If $AB = \sqrt{5}$, then the value of $r^2$ is

**Ans. (2)**

**Sol.**



Let $C_2$ $(x-4)^2+(y-1)^2=r^2$

radical axis $8x+2y-17=1-r^2$

$8x+2y=18-r^2$

$B\left(\frac{18-r^2}{8},0\right)$ $A(4,1)$

$AB=\sqrt{5}$

$$\sqrt{\left(\frac{18-r^2}{8}-4\right)^2+1}=\sqrt{5}$$

$r^2=2$

$\Rightarrow n=\sin\alpha+\cos\alpha$

**SECTION-4 : (Maximum Marks : 12)**

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If ONLY the correct numerical value is entered in the designated place;

*Zero Marks* : 0 In all other cases.

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**PARAGRAPH "I"**

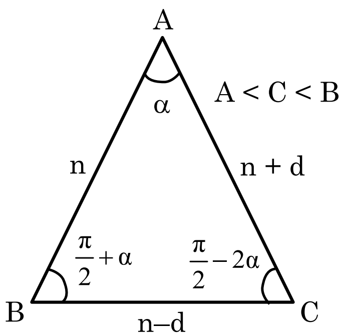
Consider on obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

**(There are two questions based on PARAGRAPH "I", the question given below is one of them)**

**14.** Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

**Ans. (1008.00)**

**Sol.**




$n - d = 2\sin\alpha \qquad \ldots(1)$

$n + d = 2\sin\left(\frac{\pi}{2}+\alpha\right)$

$\Rightarrow n + d = 2\cos\alpha \qquad \ldots.(2)$

$n = 2\sin\left(\frac{\pi}{2}-2\alpha\right)$

$\Rightarrow n = 2\cos 2\alpha \qquad \ldots(3)$

$\Rightarrow 2\cos2\alpha = \sin\alpha + \cos\alpha$

$\Rightarrow 2(\cos\alpha - \sin\alpha) = 1$

$\Rightarrow \sin 2\alpha = \frac{3}{4}$

Then, $a = \frac{1}{2}.n.(n+d).\sin\alpha = \frac{1}{2}.2\cos 2\alpha.2\cos\alpha.\sin\alpha$

$= \sin 2\alpha.\cos 2\alpha$

$= \frac{3}{4}\times\frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{16}$

$(64a)^2 = \left(64\times\frac{3\sqrt{7}}{16}\right)^2 = 16\times9\times7 = 1008$

**PARAGRAPH "I"**

Consider on obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

**(There are two questions based on PARAGRAPH "I", the question given below is one of them)**

**15.** Then the inradius of the triangle ABC is

**Ans. (0.25)**

**Sol.** From above equation in Ques. 14

$$r = \frac{\Delta}{s} = \frac{1}{2}\frac{n(n+d)\sin\alpha}{\left(\frac{3n}{2}\right)}$$

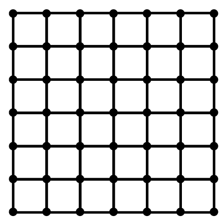
$$= \frac{(n+d).\sin\alpha}{3}$$

$$= \frac{2\cos\alpha.\sin\alpha}{3} \quad \text{(from (2))}$$

$$r = \frac{\sin 2\alpha}{3} = \frac{1}{4}$$

**PARAGRAPH "II"**

Consider the $6 \times 6$ square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that $A_i$ and $A_j$ are friends if they are adjacent along a row or along a column. Assume that each point $A_i$ has an equal chance of being chosen.

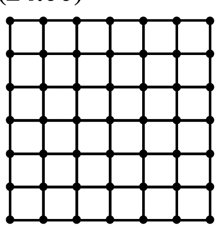


**(There are two questions based on PARAGRAPH "II", the question given below is one of them)**

**16.** Let $p_i$ be the probability that a randomly chosen point has $i$ many friends, $i = 0,1,2,3,4$. Let $X$ be a random variable such that for $i = 0,1,2,3,4$, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is

**Ans. (24.00)**

**Sol.**



$P_i$ = Probability that randomly selected points has friends

$P_0 = 0$ (0 friends)

$P_1 = 0$ (exactly 1 friends)

$$P_2 = \frac{{}^{4}C_1}{{}^{49}C_1} = \frac{4}{9} \text{ (exactly 2 friends)}$$

$$P_3 = \frac{{}^{20}C_1}{{}^{49}C_1} = \frac{20}{49} \text{ (exactly 3 friends)}$$

$$P_4 = \frac{{}^{25}C_1}{{}^{49}C_1} = \frac{25}{49} \text{ (exactly 4 friends)}$$

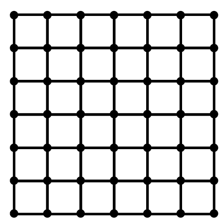
| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(x)$ | 0 | 0 | $\frac{4}{49}$ | $\frac{20}{49}$ | $\frac{25}{49}$ |

Mean = E(x) = $\sum x_i P_i = 0 + 0 + \frac{8}{49} + \frac{60}{49} + \frac{100}{49} = \frac{168}{49}$

$$7(E(x)) = \frac{168}{49} \times 7 = 24$$

**PARAGRAPH "II"**

Consider the 6 × 6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that $A_i$ and $A_j$ are friends if they are adjacent along a row or along a column. Assume that each point $A_i$ has an equal chance of being chosen.



**(There are two questions based on PARAGRAPH "II", the question given below is one of them)**

**17.** Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots, A_{49}$. Let $p$ be the probability that they are friends. Then the value of $7p$ is

**Ans. (0.50)**

**Sol.** Total number of ways of selecting 2 persons = ${}^{49}C_2$

Number of ways in which 2 friends are selected = 6 × 7 × 2 = 84

$$7P = \frac{84 \times 2}{49 \times 48} \times 7 = \frac{1}{2}$$