JEE Advanced 2018 Question Paper with Solution

Joint Entrance Examination – Advanced

JEE(Advanced) – 2018 TEST PAPER - 1 WITH SOLUTION

(Exam Date: 20-05- 2018)

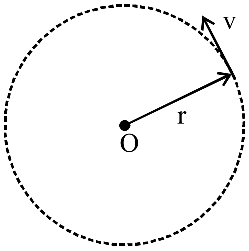
PART-1 : PHYSICS

1. The potential energy of a particle of mass m at a distance r from a fixed point O is given by V (r) = $kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ?

(A) $v=\sqrt{\frac{k}{2m}}R$ (B) $v=\sqrt{\frac{k}{m}}R$ (C) $L=\sqrt{mk}R^2$ (D) $L=\sqrt{\frac{mk}{2}}R^2$

Ans. (B,C)

Sol.



$V=\frac{kr^2}{2}$

$F = -kr$ (towards centre) $\left[F=-\frac{dV}{dr}\right]$

At r = R,

$kR=\frac{mv^2}{R}$ [Centripetal force]

$v=\sqrt{\frac{kR^2}{m}}=\sqrt{\frac{k}{m}}R$

$L=m\sqrt{\frac{k}{m}}R^2$

2. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F}=\left(\alpha t\hat{i}+\beta\hat{j}\right)$ is applied on the body, where α = 1.0 Ns^{-1} and β = 1.0 N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}|=\frac{1}{3}Nm$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v}=\frac{1}{2}\left(\hat{i}+2\hat{j}\right)ms^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}m$

Ans. (A,C)

Sol. $\vec{F} = (\alpha t)\hat{i} + \beta\hat{j}$ [At t = 0, v = 0, $\vec{r} = \vec{0}$]

$\alpha = 1,\ \beta = 1$

$\vec{F} = t\hat{i} + \hat{j}$

$$m\frac{d\vec{v}}{dt} = t\hat{i} + \hat{j}$$

On integrating

$$m\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$$ [m = 1kg]

$$\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$$ [$\vec{r} = \vec{0}$ at t = 0]

On integrating

$$\vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$$

At t = 1 sec, $\vec{\tau} = (\vec{r} \times \vec{F}) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$

$$\vec{\tau} = -\frac{1}{3}\hat{k}$$

$$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$$

At t = 1 $\vec{v} = \left(\frac{1}{2}\hat{i} + \hat{j}\right) = \frac{1}{2}(\hat{i} + 2\hat{j})\,m/sec$

At t = 1 $\vec{s} = \vec{r}_1 - \vec{r}_0$

$= \left[\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right] - [\vec{0}]$

$$\vec{s} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$$

$$|\vec{s}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2} \Rightarrow \frac{\sqrt{10}}{6}m$$

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ.

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases.,

(D) h is proportional to contact angle θ.

Ans. (A,C)

Sol. $\frac{2\sigma}{R} = \rho g h$ [R → Radius of meniscus]

$h = \frac{2\sigma}{R\rho g}$ $R = \frac{r}{\cos\theta}$ [r → radius of capillary; θ → contact angle]

$h = \frac{2\sigma\cos\theta}{r\rho g}$

(A) For given material, θ → constant

$h \propto \frac{1}{r}$

(B) h depend on σ

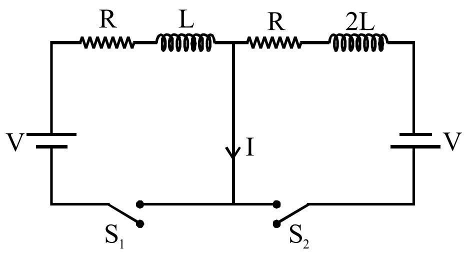
(C) If lift is going up with constant acceleration,

$g_{eff} = (g + a)$

$h = \frac{2\sigma\cos\theta}{r\rho(g+a)}$ It means h decreases

(D) h is proportional to cos θ Not θ

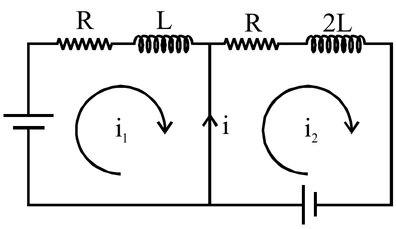
4. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statement(s) is (are) true?



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ell n2$ (D) $\tau = \frac{2L}{R}\ell n2$

Ans. **(B,D)**

Sol.



$i_{max} = (i_2 - i_1)_{max}$

$\Delta i = (i_2 - i_1) = \frac{V}{R}\left[1 - e^{-\left(\frac{R}{2L}\right)t}\right] - \frac{V}{R}\left[1 - e^{\left(-\frac{R}{L}\right)t}\right]$

$\frac{V}{R}\left[e^{-\left(\frac{R}{L}\right)t} - e^{-\left(\frac{R}{2L}\right)t}\right]$

For $(\Delta i)_{max}$ $\frac{d(\Delta i)}{dt} = 0$

$$\frac{V}{R}\left[-\frac{R}{L}e^{-\left(\frac{R}{L}\right)t}-\left(-\frac{R}{2L}\right)e^{-\left(\frac{R}{2L}\right)t}\right]=0$$

$$e^{-\left(\frac{R}{L}\right)t}=\frac{1}{2}e^{-\left(\frac{R}{2L}\right)t}$$

$$e^{-\left(\frac{R}{2L}\right)t}=\frac{1}{2}$$

$$\left(\frac{R}{2L}\right)t=\ell n2$$

$$t=\frac{2L}{R}\ell n2 \rightarrow \text{time when i is maximum.}$$

$$i_{max}=\frac{V}{R}\left[e^{-\frac{R}{L}\left(\frac{2L}{R}\ell n2\right)}-e^{-\left(\frac{R}{2L}\right)\left(\frac{2L}{R}\ell n2\right)}\right]$$

$$|i_{max}|=\frac{V}{R}\left|\left[\frac{1}{4}-\frac{1}{2}\right]\right|=\frac{1}{4}\frac{V}{R}$$

5. Two infinitely long straight wires lie in the xy-plane along the lines x = ±R. The wire located at x = +R carries a constant current I_1 and the wire located at x = –R carries a constant current I_2. A circular loop of radius R is suspended with its centre at (0, 0, $\sqrt{3}R$) and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true ?

(A) If $I_1 = I_2$, then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0)

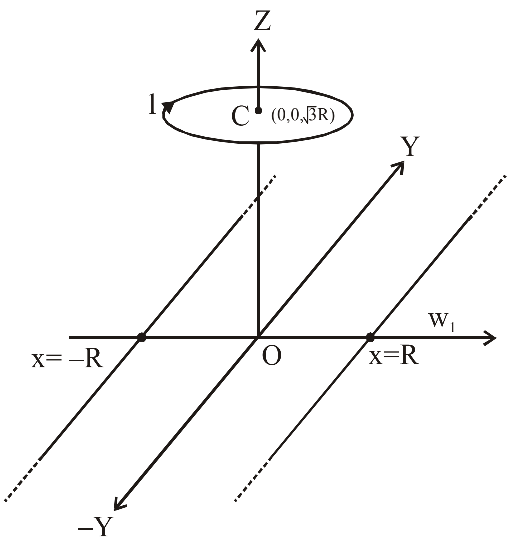
(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

Ans. (A,B,D)

Sol.

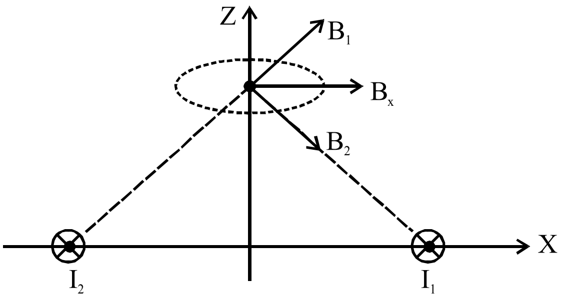


(A) At origin, $\vec{B}=0$ due to two wires if $I_1 = I_2$, hence $\left(\vec{B}_{net}\right)$ at origin is equal to $\vec{B}$ due to ring, which is non-zero.

(B) If $I_1 > 0$ and $I_2 < 0$, $\vec{B}$ at origin due to wires will be along $+\hat{k}$ direction and $\vec{B}$ due to ring is along $-\hat{k}$ direction and hence $\vec{B}$ can be zero at origin.

(C) If $I_1 < 0$ and $I_2 > 0$, $\vec{B}$ at origin due to wires is along $-\hat{k}$ and also along $-\hat{k}$ due to ring, hence $\vec{B}$ cannot be zero.

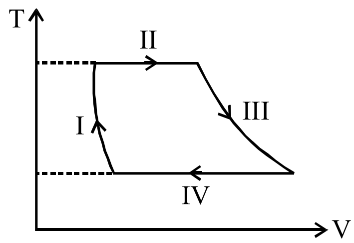
(D)



At centre of ring, $\vec{B}$ due to wires is along x-axis,

hence z-component is only because of ring which $\vec{B}=\frac{\mu_0 i}{2R}\left(-\hat{k}\right)$

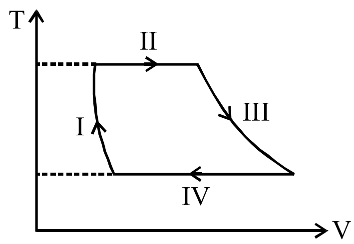
6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (whre V is the volume and T is the temperature). Which of the statements below is (are) true ?



(A) Process I is an isochoric process
(B) In process II, gas absorbs heat
(C) In process IV, gas releases heat
(D) Processes I and II are not isobaric

Ans. (B,C,D)

Sol.



(A) Process-I is not isochoric, V is decreasing.

(B) Process-II is isothermal expansion

$\Delta U = 0,\ W > 0$

$\Delta Q > 0$

(C) Process-IV is isothermal compression,

$\Delta U = 0,\ W < 0$

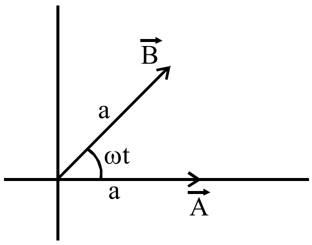
$\Delta Q < 0$

(D) Process-I and III are NOT isobaric because in isobaric process $T \propto V$ hence isobaric T–V graph will be linear.

7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\right)$, whre a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is _____.

Ans. 2.00 sec

Sol.



$$|\vec{A}+\vec{B}| = 2a\cos\frac{\omega t}{2}$$

$$|\vec{A}-\vec{B}| = 2a\sin\frac{\omega t}{2}$$

So $2a\cos\frac{\omega t}{2} = \sqrt{3}\left(2a\sin\frac{\omega t}{2}\right)$

$$\tan\frac{\omega t}{2} = \frac{1}{\sqrt{3}}$$

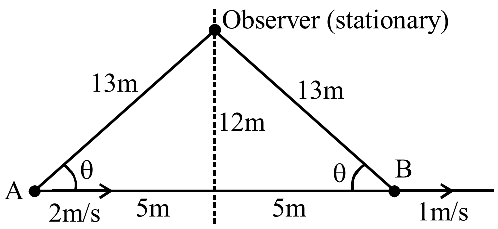
$$\frac{\omega t}{2} = \frac{\pi}{6} \Rightarrow \omega t = \frac{\pi}{3}$$

$\frac{\pi}{6}t = \frac{\pi}{3}$ $\qquad$ t = 2.00 sec

8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is _____

Ans. 5.00 Hz

Sol.



$$\cos\theta = \frac{5}{13}$$

$$f_A = 1430\left[\frac{330}{330-2\cos\theta}\right] = 1430\left[\frac{1}{1-\frac{2\cos\theta}{330}}\right] = 1430\left[1+\frac{2\cos\theta}{330}\right]$$ (By bionomial expansion)

$$f_B = 1430\left[\frac{330}{330+1\cos\theta}\right] = 1430\left[1-\frac{\cos\theta}{330}\right]$$

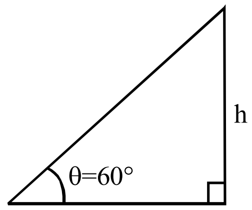
$$\Delta f = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$= 13\left(\frac{5}{13}\right) = 5.00\text{Hz}$$

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}\text{s}$, then the height of the top of the inclined plane, in meters, is _____. Take g = 10 ms^{-2}.

Ans. 0.75m

Sol. $a_c = \dfrac{g\sin\theta}{1+\dfrac{I_C}{MR^2}}$



$$a_{ring} = \frac{g\sin\theta}{2}$$

$$a_{disc} = \frac{2g\sin\theta}{3}$$

$$\frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2 \Rightarrow t_1 = \sqrt{\frac{4h}{g\sin^2\theta}} = \sqrt{\frac{16h}{3g}}$$

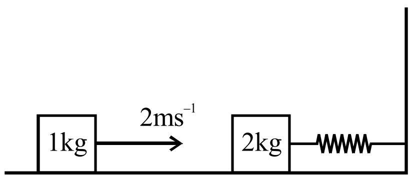
$$\frac{h}{\sin\theta} = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2 \Rightarrow t_2 = \sqrt{\frac{3h}{g\sin^2\theta}} = \sqrt{\frac{4h}{g}}$$

$$\Rightarrow \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\sqrt{h}\left[\frac{4}{\sqrt{3}} - 2\right] = 2-\sqrt{3}$$

$$\sqrt{h} = \frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})} = \frac{\sqrt{3}}{2} \Rightarrow h = \frac{3}{4} = 0.75\text{m}$$

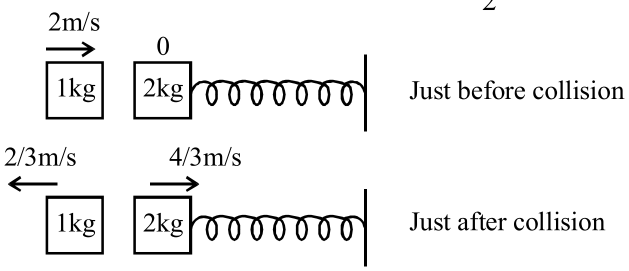
10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _____.



Ans. 2.09 m

Sol. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \sec$

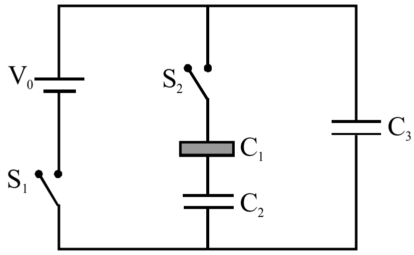
block returns to original position in $\frac{T}{2} = \pi \sec$



$d = \frac{2}{3}(\pi) = \frac{2}{3}(3.14) = 2.0933\text{m}$

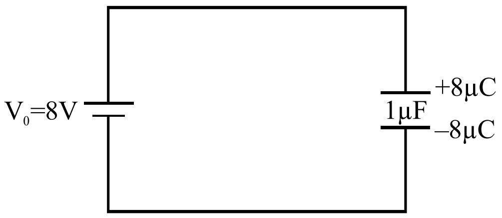
d = 2.09 m

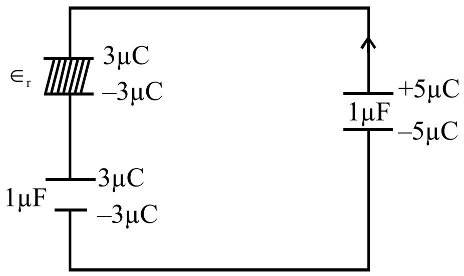
11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of $\in_r$.



Ans. 1.50

Sol.



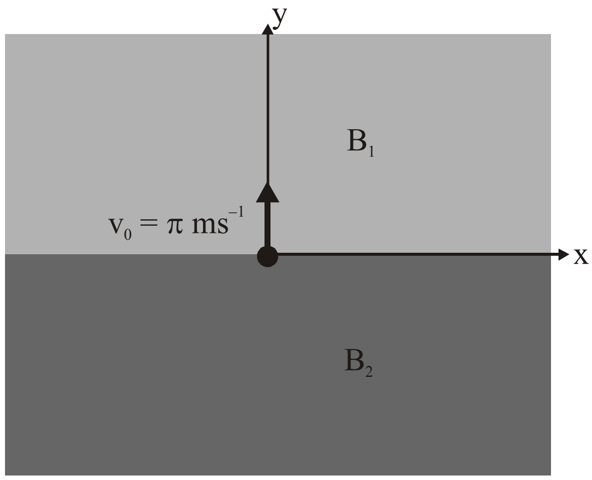


Applying loop rule

$\frac{5}{1} - \frac{3}{\in_r} - \frac{3}{1} = 0 \qquad \frac{3}{\in_r} = 2$

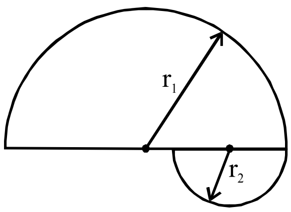
$\in_r = 1.50$

12. In the x-y-plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has a another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi ms^{-1}$ at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is _______.



Ans. 2.00

Sol. (1) Average speed along x-axis



$$\langle v_x \rangle = \frac{\int |\vec{v}_x| dt}{\int dt} = \frac{d_1 + d_2}{t_1 + t_2}$$

(2) We have,

$$r_1 = \frac{mv}{qB_1}, r_2 = \frac{mv}{qB_2}$$

Since $B_1 = \frac{B_2}{4}$

$\therefore$ $r_1 = 4r_2$

Time in $B_1 \Rightarrow \frac{\pi m}{qB_1} = t_1$

Time in $B_2 \Rightarrow \frac{\pi m}{qB_2} = t_2$

Total distance along x-axis $d_1 + d_2 = 2r_1 + 2r_2 = 2\ (r_1 + r_2) = 2\ (5r_2)$

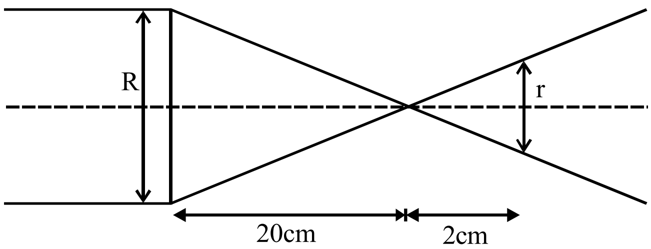
Total time $T = t_1 + t_2 = 5t_2$

$\therefore$ Average speed = $\frac{10r_2}{5t_2} = 2\frac{mv}{qB_2} \times \frac{qB_2}{\pi m} = 2$

13. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is _________.

Ans. 130

Sol.



$$\frac{r}{R}=\frac{2}{20}=\frac{1}{10}$$

$\therefore$ Ratio of area = $\frac{1}{100}$

Let energy incident on lens be E.

$\therefore$ Given $\frac{E}{A}=1.3$

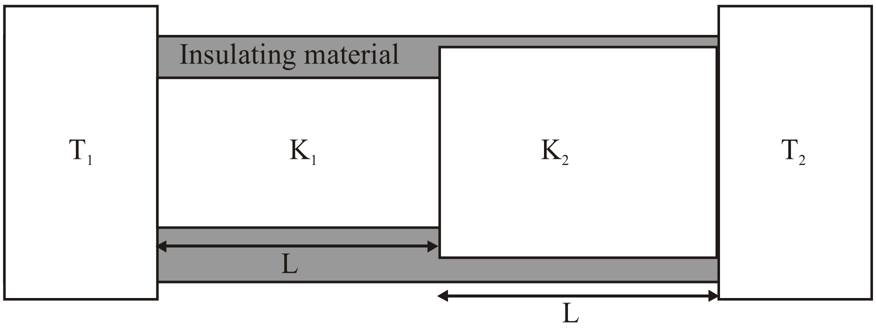
So final, $\frac{E}{a}=??$

E = A × 1.30

Also $\frac{a}{A}=\frac{1}{100}$

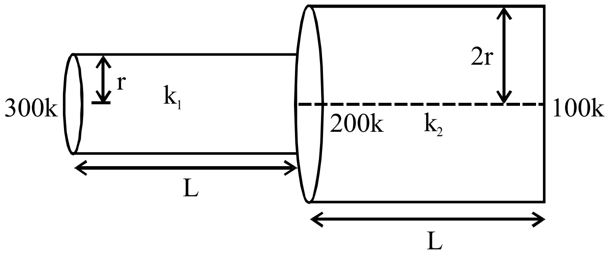
$\therefore$ Average intensity of light at 22 cm = $\frac{E}{a}=\frac{A\times 1.3}{a}=100\times 1.3=130\text{kW}/\text{m}^2$

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = _________.



Ans. 4.00

Sol.



We have in steady state,

$$\left(\frac{200-300}{\frac{L}{k_1\pi r^2}}\right)+\left(\frac{200-100}{\frac{L}{k_2\pi(2r)^2}}\right)=0$$

$$\Rightarrow \frac{k_1\pi r^2\times 100}{L}=\frac{100k_2\pi\times 4r^2}{L}$$

$$\Rightarrow \frac{k_1}{k_2}=4$$

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while [$\in_0$] and [μ_0] stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

15. The relation between [E] and [B] is :-

(A) [E] = [B][L][T] (B) $[E] = [B][L]^{-1}[T]$ (C) $[E] = [B][L][T]^{-1}$ (D) $[E] = [B][L]^{-1}[T]^{-1}$

Ans. (C)

Sol. We have $\frac{E}{C} = B$

$\therefore$ [B] = $\frac{[E]}{[C]} = [E]L^{-1}T^{1}$

$\Rightarrow [E] = [B]\ [L][T^{-1}]$

PARAGRAPH "X"

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(There are two questions based on Paragraph "X", the question given below is one of them)

16. The relation between $[\in_0]$ and $[\mu_0]$ is :-

(A) $[\mu_0] = [\in_0][L]^2[T]^{-2}$

(B) $[\mu_0] = [\in_0][L]^{-2}[T]^2$

(C) $[\mu_0] = [\in_0]^{-1}[L]^2[T]^{-2}$

(D) $[\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$

Ans. (D)

Sol. We have,

$$C = \frac{1}{\sqrt{\mu_0 \in_0}}$$

$$\therefore [C^2] = \left[\frac{1}{\mu_0 \in_0}\right]$$

$$\Rightarrow L^2T^{-2} = \frac{1}{[\mu_0][\in_0]}$$

$\Rightarrow [\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in Δy/y, is $1 \mp (\Delta y / y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\frac{\Delta x}{x} << 1, \frac{\Delta y}{y} << 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on Paragraph "A", the question given below is one of them)

17. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa(Δa/a << 1), then what is the error Δr in determining r ?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$ (C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

Ans. (B)

Sol. $r = \left(\frac{1-a}{1+a}\right)$

$$\frac{\Delta r}{r} = \frac{\Delta(1-a)}{(1-a)} + \frac{\Delta(1+a)}{(1+a)}$$

$$= \frac{\Delta a}{(1-a)} + \frac{\Delta a}{(1+a)}$$

$$= \frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$$

$$\therefore \Delta r = \frac{2\Delta a}{(1-a)(1+a)} \frac{(1-a)}{(1+a)} = \frac{2\Delta a}{(1+a)^2}$$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in Δy/y, is $1 \mp (\Delta y / y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\frac{\Delta x}{x} << 1, \frac{\Delta y}{y} << 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on Paragraph "A", the question given below is one of them)

18. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| << 1$, In (1 + x) = x up to first power in x. The error Δλ, in the determination of the decay constant λ, in s^{-1}, is :-

(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

Ans. **(C)**

Sol. $N = N_0 e^{-\lambda t}$

$\ell nN = \ell nN_0 - \lambda t$

$$\frac{dN}{N} = -d\lambda t$$

Converting to error,

$$\frac{\Delta N}{N} = \Delta\lambda t$$

$$\therefore \Delta\lambda = \frac{40}{2000 \times L} = 0.02$$ (N is number of nuclei left undecayed)

JEE(Advanced) – 2018 TEST PAPER - 1 WITH SOLUTION

(Exam Date: 20-05-2018)

PART-2 : CHEMISTRY

1. The compound(s) which generate(s) N_2 gas upon thermal decomposition below 300°C is (are)

(A) NH_4NO_3 (B) $(NH_4)_2Cr_2O_7$ (C) $Ba(N_3)_2$ (D) Mg_3N_2

Ans. (B,C)

Sol. (A) $NH_4NO_3 \xrightarrow[\text{below } 300°C]{\Delta} N_2O + 2H_2O$

(B) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

(C) $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

(D) Mg_3N_2 (it does not decompose into N_2)

2. The correct statement(s) regarding the binary transition metal carbonyl compounds is (are)

(Atomic numbers : Fe = 26, Ni = 28)

(A) Total number of valence shell electrons at metal centre in $Fe(CO)_5$ or $Ni(CO)_4$ is 16

(B) These are predominantly low spin in nature

(C) Metal - carbon bond strengthens when the oxidation state of the metal is lowered

(D) The carbonyl C–O bond weakens when the oxidation state of the metal is increased

Ans. (B,C)

Sol. (A) $[Fe(CO_5)]$ & $[Ni(CO)_4]$ complexes have 18-electrons in their valence shell.

(B) Carbonyl complexes are predominantly low spin complexes due to strong ligand field.

(C) As electron density increases on metals (with lowering oxidation state on metals), the extent of synergic bonding increases. Hence M–C bond strength increases

(D) While positive charge on metals increases and the extent of synergic bond decreases and hence C–O bond becomes stronger.

3. Based on the compounds of group 15 elements , the correct statement(s) is (are)

(A) Bi_2O_5 is more basic than N_2O_5

(B) NF_3 is more covalent than BiF_3

(C) PH_3 boils at lower temperature than NH_3

(D) The N–N single bond is stronger than the P–P single bond

Ans. (A,B,C)

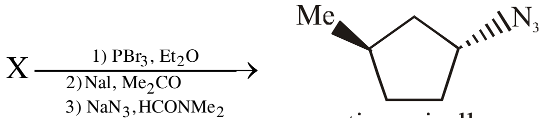
Sol. (A) Bi_2O_5 is metallic oxide but N_2O_5 is non metallic oxide therefore Bi_2O_5 is basic but N_2O_5 is acidic.

(B) In NF_3 , N and F are non metals but BiF_3 , Bi is metal but F is non metal therefore NF_3 is more covalent than BiF_3.

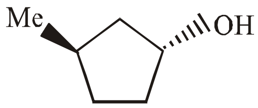
(C) In PH_3 hydrogen bonding is absent but in NH_3 hydrogen bonding is present therefore PH_3 boils at lower temperature than NH_3.

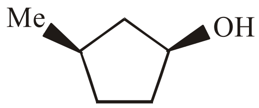
(D) Due to small size in N–N single bond l.p. – l.p. repulsion is more than P–P single bond therefore N–N single bond is weaker than the P–P single bond.

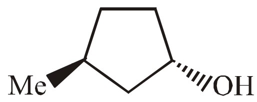
4. In the following reaction sequence, the correct structure(s) of X is (are)

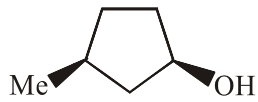


enantiomerically pure

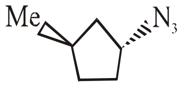
(A) 

(B) 

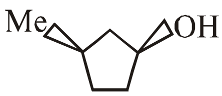
(C) 

(D) 

Ans. (B)

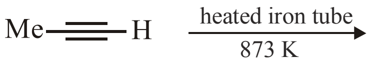
Sol. X $\xrightarrow[(2)NaI, Me_2C=O]{(1)PBr_3Et_2O}$ 

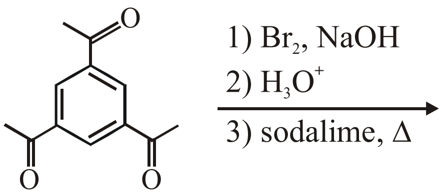
$(3)NaN_3, HCONMe_2$

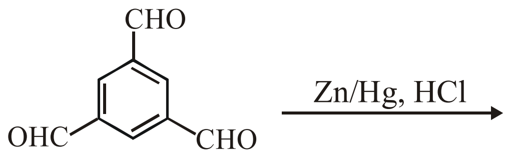
all the three reaction are S_{N^2} so X is 

5. The reaction(s) leading to the formation of 1,3,5-trimethylbenzene is (are)

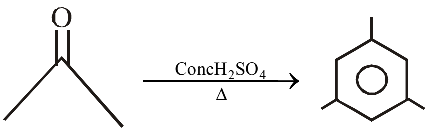
(A) 

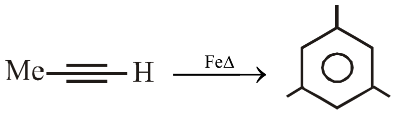
(B) 

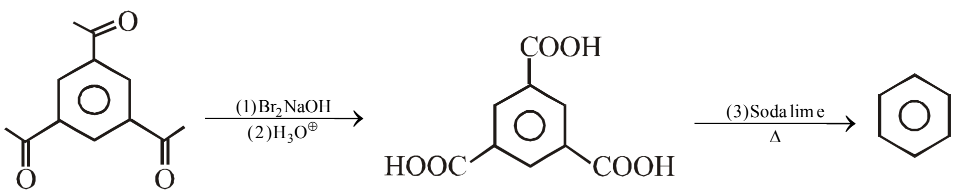
(C) 

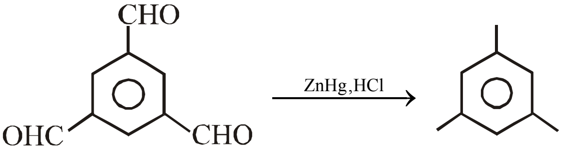
(D) 

Ans. (A,B,D)

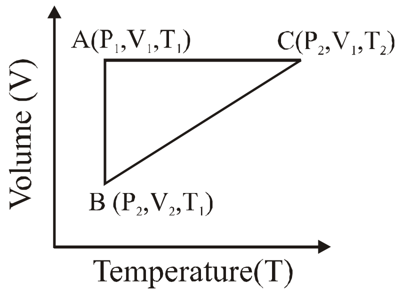
Sol. (A) 

(B) 

(C) 

(D) 

6. A reversible cyclic process for an ideal gas is shown below. Here, P , V and T are pressure , volume and temperature , respectively. The thermodynamic parameters q, w, H and U are heat, work, enthalpy and internal energy, respectively.



The correct option(s) is (are)

(A) $q_{AC} = \Delta U_{BC}$ and $w_{AB} = P_2 (V_2 - V_1)$

(B) $w_{BC} = P_2 (V_2 - V_1)$ and $q_{BC} = \Delta H_{AC}$

(C) $\Delta H_{CA} < \Delta U_{CA}$ and $q_{AC} = \Delta U_{BC}$

(D) $q_{BC} = \Delta H_{AC}$ and $\Delta H_{CA} > \Delta U_{CA}$

Ans. (B,C)

Sol. AC $\rightarrow$ Isochoric

AB $\rightarrow$ Isothermal

BC $\rightarrow$ Isobaric

$q_{AC} = \Delta U_{BC} = nC_V (T_2 - T_1)$

$W_{AB} = nRT_1 \ln\left(\frac{V_2}{V_1}\right)$ A (wrong)

$q_{BC} = \Delta H_{AC} = nC_P (T_2 - T_1)$

$W_{BC} = - P_2(V_1 - V_2)$ B (correct)

$nC_P (T_1 - T_2) < nC_V(T_1 - T_2)$ C (correct)

$\Delta H_{CA} < \Delta U_{CA}$

D (wrong)

7. Among the species given below, the total number of diamagnetic species is____.

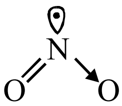
H atom, NO_2 monomer, O_2^- (superoxide), dimeric sulphur in vapour phase,

Mn_3O_4, $(NH_4)_2[FeCl_4]$, $(NH_4)_2[NiCl_4]$, K_2MnO_4, K_2CrO_4

Ans. (1)

Sol.

* H-atom =  $1s^1$ — Paramagnetic

* NO_2 = 

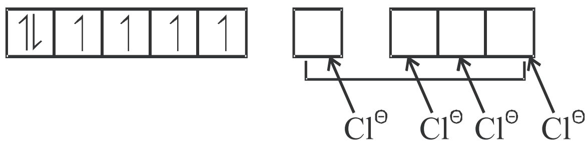
odd electron species — Paramagnetic

* O_2^- (superoxide) = One unpaired electrons in π* M.O. — Paramagnetic

* S_2 (in vapour phase) = same as O_2, two unpaired e^-s are present in π* M.O. — Paramagnetic

* $Mn_3O_4 = 2\overset{+2}{Mn}O.\overset{+4}{Mn}O_2$ — Paramagnetic

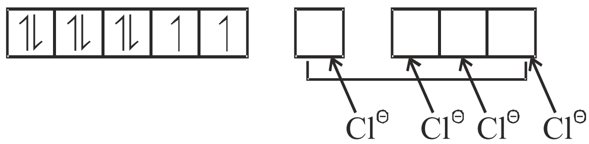
* $(NH_4)_2[FeCl_4]$ = $Fe^{+2} = 3d^6\ 4s^0$



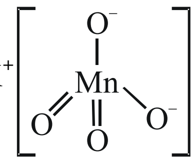
sp^3- hybridisation — Paramagnetic

* $(NH_4)_2\ [NiCl_4]$ = $Ni = 3d^8\ 4s^2$

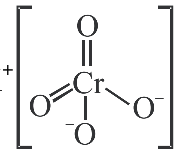
$Ni^{+2} = 3d^8\ 4s^0$



sp^3 - hybridisation — Paramagnetic

* $K_2MnO_4 = 2K^+$ 

, Mn^{+6} = [Ar] $3d^1$ — Paramagnetic

* $K_2CrO_4 = 2K^+$ 

, Cr^{+6} = [Ar] $3d^0$ — Diamagnetic

8. The ammonia prepared by treating ammonium sulphate with calcium hydroxide is completely used by $NiCl_2.6H_2O$ to form a stable coordination compound. Assume that both the reactions are 100% complete. If 1584 g of ammonium sulphate and 952g of $NiCl_2.6H_2O$ are used in the preparation, the combined weight (in grams) of gypsum and the nickel-ammonia coordination compound thus produced is___.

(Atomic weights in g mol^{-1}: H = 1, N = 14, O = 16, S = 32, Cl = 35.5, Ca = 40, Ni = 59)

Ans. (2992)

$$\underset{\substack{1584\,g\\=12\text{ mol}}}{(NH_4)_2SO_4} + Ca(OH)_2 \rightarrow \underset{\substack{\text{gypsum }(M=172)\\12\text{ mol}}}{CaSO_4.2H_2O} + \underset{24\,\text{mole}}{2NH_3}$$

$$\underset{952\,g\,=\,4\text{ mol}}{NiCl_2 \cdot 6H_2O} + \underset{24\,\text{mol}}{6NH_3} \rightarrow \underset{\substack{(M=232)\\4\text{ mol}}}{\left[Ni(NH_3)_6\right]Cl_2} + 6H_2O$$

Total mass = 12 × 172 + 4 × 232 = 2992 g

9. Consider an ionic solid MX with NaCl structure. Construct a new structure (Z) whose unit cell is constructed from the unit cell of MX following the sequential instructions given below. Neglect the charge balance.

(i) Remove all the anions (X) except the central one

(ii) Replace all the face centered cations (M) by anions (X)

(iii) Remove all the corner cations (M)

(iv) Replace the central anion (X) with cation (M)

The value of $\left(\frac{\text{number of anions}}{\text{number of cations}}\right)$ in Z is____.

Ans. (3)

Sol. $X^{\ominus} \Rightarrow$ O.V.

$M^{+} \Rightarrow$ FCC

	M^+	X^-
(i)	4	1
(ii)	4–3	3+1
(iii)	4 – 3 – 1	3+1
(iv)	1	3

$Z = \frac{3}{1} = 3$

10. For the electrochemical cell,

$$Mg(s)|Mg^{2+}(aq, 1M)||Cu^{2+}(aq, 1M) | Cu(s)$$

the standard emf of the cell is 2.70 V at 300 K. When the concentration of Mg^{2+} is changed to x M, the cell potential changes to 2.67 V at 300 K. The value of x is____.

(given, $\frac{F}{R} = 11500\ KV^{-1}$, where F is the Faraday constant and R is the gas constant, ln(10) = 2.30)

Ans. (10)

Sol. $Mg(s) + Cu^{2+}(aq) \longrightarrow Mg^{2+}(aq) + Cu(s)$

$E^\circ_{Cell} = 2.70 \qquad E_{Cell} = 2.67 \qquad Mg^{2+} = x\ M$

$Cu^{2+} = 1\ M$

$$E_{Cell} = E^\circ_{Cell} - \frac{RT}{nF}\ln x$$

$$2.67 = 2.70 - \frac{RT}{2F}\ln x$$

$$-0.03 = -\frac{R \times 300}{2F} \times \ln x$$

$$\ln x = \frac{0.03 \times 2}{300} \times \frac{F}{R}$$

$$= \frac{0.03 \times 2 \times 11500}{300 \times 1}$$

$\ln x = 2.30 = \ln(10)$

$x = 10$

11. A closed tank has two compartments A and B, both filled with oxygen (assumed to be ideal gas). The partition separating the two compartments is fixed and is a perfect heat insulator (Figure 1). If the old partition is replaced by a new partition which can slide and conduct heat but does NOT allow the gas to leak across (Figure 2), the volume (in m^3) of the compartment A after the system attains equilibrium is____.

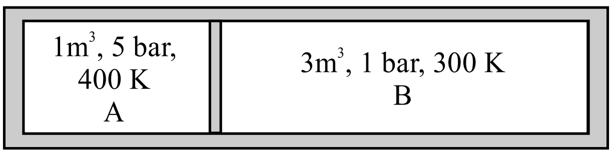


Figure 1

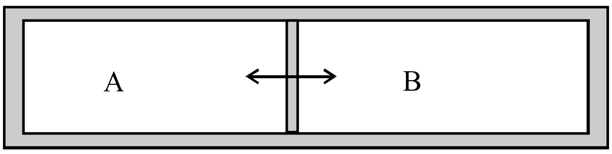


Figure 2

Ans. (2.22)

Sol. $P_1 = 5$ $\quad$ $P_2 = 1$

$v_1 = 1$ $\quad$ $v_2 = 3$

$T_1 = 400$ $\quad$ $T_2 = 300$

$n_1 = \frac{5}{400R}$ $\quad$ $n_2 = \frac{3}{300R}$

Let volume be (v + x) $\quad$ v = (3–x) $\quad$ 15 – 5x = 4 + 4x

$$\frac{P_A}{T_A} = \frac{P_B}{T_B}$$

$$\Rightarrow \frac{n_{b_1} \times R}{v_{b_1}} = \frac{n_{b_2} \times R}{v_{b_2}}$$

$$\Rightarrow \frac{5}{400(4+x)} = \frac{3}{300R(3-x)}$$

$\Rightarrow$ 5(3 – x) = 4 + 4x

$$\Rightarrow x = \frac{11}{9}$$

$$v = 1 + x = 1 + \frac{11}{9} = \left(\frac{20}{9}\right) = 2.22$$

12. Liquids A and B form ideal solution over the entire range of composition. At temperature T, equimolar binary solution of liquids A and B has vapour pressure 45 Torr. At the same temperature, a new solution of A and B having mole fractions x_A and x_B, respectively, has vapour pressure of 22.5 Torr. The value of x_A/x_B in the new solution is____.

(given that the vapour pressure of pure liquid A is 20 Torr at temperature T)

Ans. (19)

Sol. $45 = P_A^o \times \frac{1}{2} + P_B^o \times \frac{1}{2}$

$P_A^o + P_B^o = 90$ ....(1)

given $P_A^o = 20$ torr

$P_B^o = 70$ torr

$\Rightarrow$ 22.5 torr = 20 x_A + 70 (1 – x_A)

= 70 – 50 x_A

$$x_A = \left(\frac{70 - 22.5}{50}\right) = 0.95$$

x_B = 0.05

So $\frac{x_A}{x_B} = \frac{0.95}{0.05} = 19$

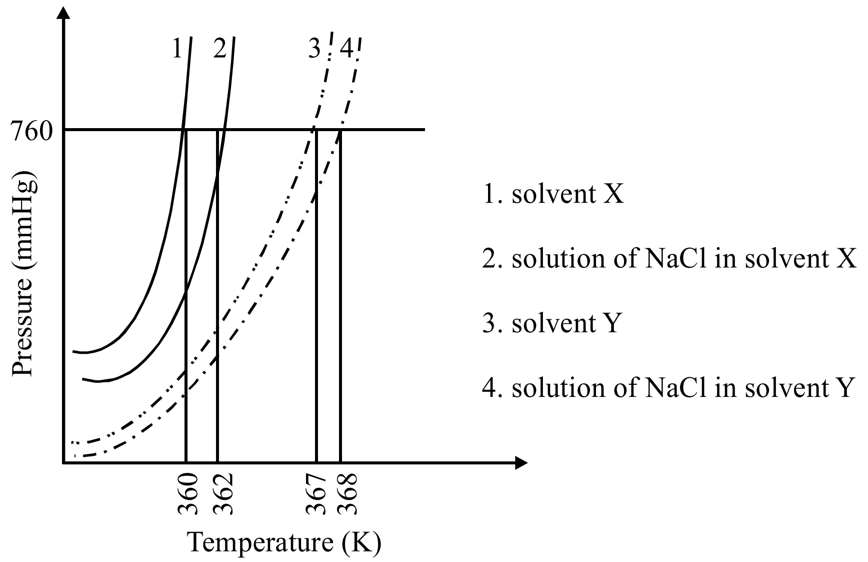
13. The solubility of a salt of weak acid(AB) at pH 3 is Y × 10^{-3} mol L^{-1}. The value of Y is___.

(Given that the value of solubility product of AB (K_{sp}) = 2 × 10^{-10} and the value of ionization constant of HB(K_a) = 1 × 10^{-8})

Ans. (4.47)

Sol. $S=\sqrt{K_{sp}\left(\frac{[H^+]}{K_a}+1\right)}=\sqrt{2\times10^{-10}\left(\frac{10^{-3}}{10^{-8}}+1\right)}\simeq\sqrt{2\times10^{-5}}$ = 4.47 × 10^{-3} M

14. The plot given below shows P–T curves (where P is the pressure and T is the temperature) for two solvents X and Y and isomolal solutions of NaCl in these solvents. NaCl completely dissociates in both the solvents.



On addition of equal number of moles a non-volatile solute S in equal amount (in kg) of these solvents, the elevation of boiling point of solvent X is three times that of solvent Y. Solute S is known to undergo dimerization in these solvents. If the degree of dimerization is 0.7 in solvent Y, the degree of dimerization in solvent X is ___.

Ans. (0.05)

From graph

For solvent X' ΔT_{bx} = 2

$\Delta T_{bx} = m_{NaCl} \times K_{b(x)}$ ......(1)

For solvent 'Y' ΔT_{by} = 1

$\Delta T_{b(y)} = m_{NaCl} \times K_{b(y)}$ ......(2)

Equation (1)/(2)

$\Rightarrow \frac{K_{b(x)}}{K_{b(y)}} = 2$

For solute S

$$\underset{\substack{1 \\ 1-\alpha}}{2(S)} \rightarrow \underset{\alpha/2}{S_2}$$

$i = (1 - \alpha/2)$

$$\Delta T_{b(x)(s)} = \left(1-\frac{\alpha_1}{2}\right)K_{b(x)}$$

$$\Delta T_{b(y)(s)} = \left(1-\frac{\alpha_2}{2}\right)K_{b(y)}$$

Given $\Delta T_{b(x)(s)} = 3\Delta T_{b(y)(s)}$

$$\left(1-\frac{\alpha_1}{2}\right) K_{b(x)} = 3 \times \left(1-\frac{\alpha_2}{2}\right) \times k_{b(y)}$$

$$2\left(1-\frac{\alpha_1}{2}\right) = 3 \left(1-\frac{\alpha_2}{2}\right)$$

$\alpha_2 = 0.7$

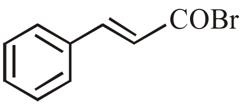
so $\alpha_1 = 0.05$

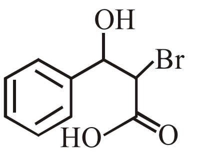
Paragraph "X"

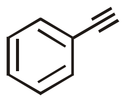
Treatment of benzene with CO/HCl in the presence of anhydrous $AlCl_3$/CuCl followed by reaction with Ac_2O/NaOAc gives compound X as the major product. Compound X upon reaction with Br_2/Na_2CO_3, followed by heating at 473 K with moist KOH furnishes Y as the major product. Reaction of X with H_2/Pd-C, followed by H_3PO_4 treatment gives Z as the major product.

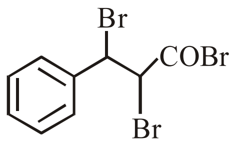
(There are two questions based on PARAGRAPH "X", the question given below is one of them)

15. The compound Y is :-

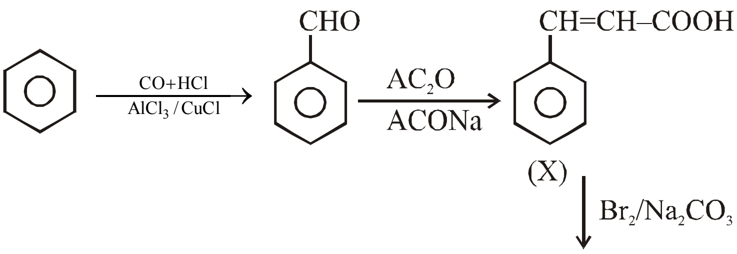
(A) 

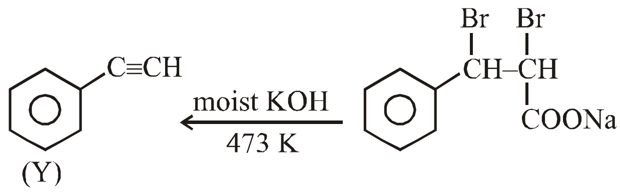
(B) 

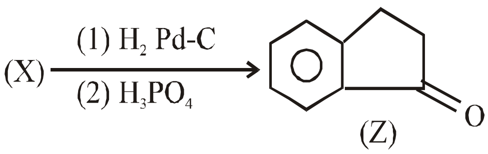
(C) 

(D) 

Ans. (C)





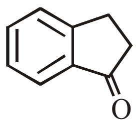
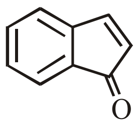
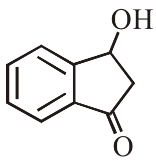
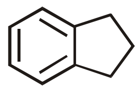


Paragraph "X"

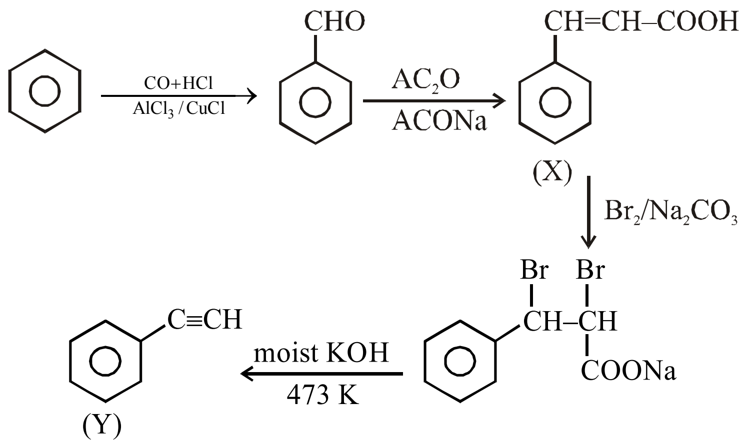
Treatment of benzene with CO/HCl in the presence of anhydrous $AlCl_3$/CuCl followed by reaction with Ac_2O/NaOAc gives compound X as the major product. Compound X upon reaction with Br_2/Na_2CO_3, followed by heating at 473 K with moist KOH furnishes Y as the major product. Reaction of X with H_2/Pd-C, followed by H_3PO_4 treatment gives Z as the major product.

(There are two question based on PARAGARAPH "X", the question given below is one of them)

16. The compound Z is :-

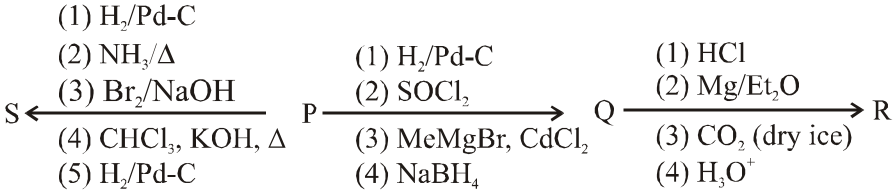
(A)  (B)  (C)  (D) 

Ans. (A)



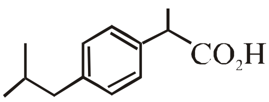
Paragraph "A"

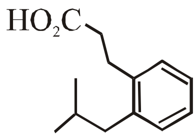
An organic acid P ($C_{11}H_{12}O_2$) can easily be oxidized to a dibasic acid which reacts with ethyleneglycol to produce a polymer dacron. Upon ozonolysis, **P** gives an aliphatic ketone as one of the products. **P** undergoes the following reaction sequences to furnish **R** via **Q.** The compound **P** also undergoes another set of reactions to produce **S.**

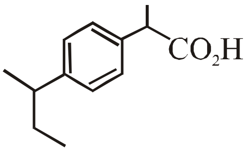


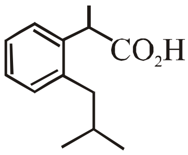
(There are two questions based on PARAGRAPH "A", the question given below is one of them)

17. The compound **R** is

(A) 

(B) 

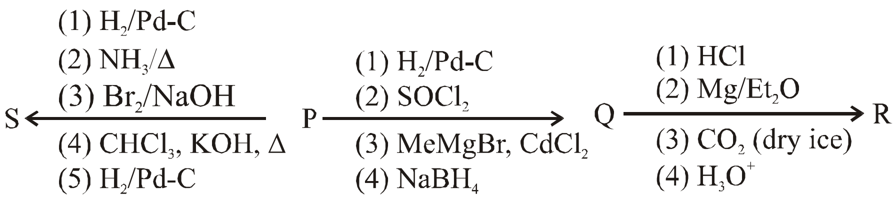
(C) 

(D) 

Ans. (A)

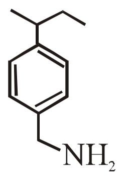
Paragraph "A"

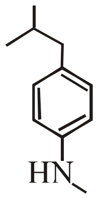
An organic acid P ($C_{11}H_{12}O_2$) can easily be oxidized to a dibasic acid which reacts with ethyleneglycol to produce a polymer dacron. Upon ozonolysis, P gives an aliphatic ketone as one of the products. P undergoes the following reaction sequences to furnish R via Q. The compound P also undergoes another set of reactions to produce S.

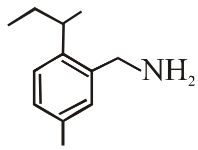


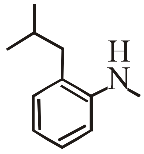
(There are two questions based on PARAGRAPH "A", the question given below is one of them)

18. The compound **S** is

(A) 

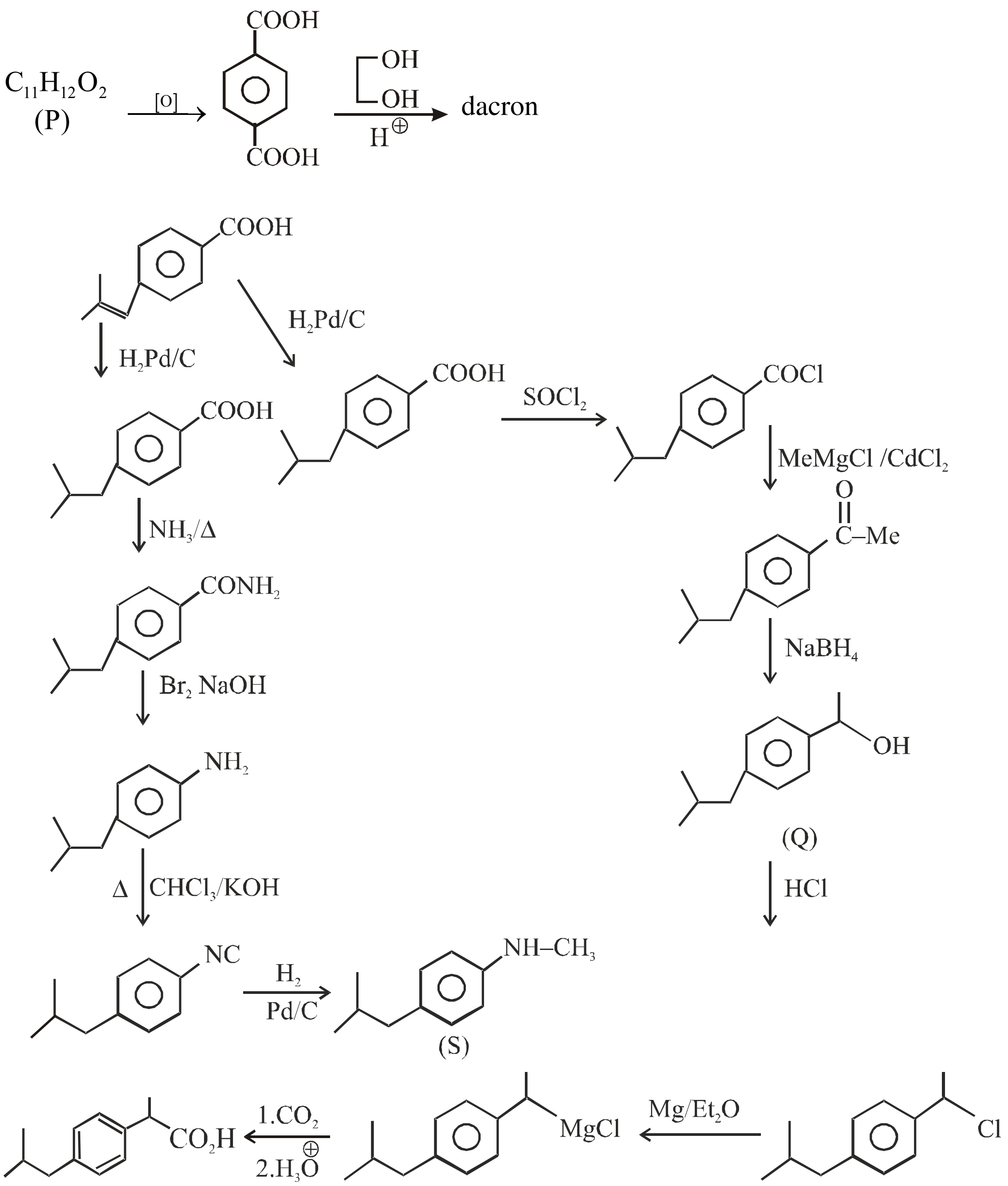
(B) 

(C) 

(D) 

Ans. (B)

Solution 17 & 18.



JEE(Advanced) – 2018 TEST PAPER - 1 WITH SOLUTIONS

(Exam Date: 20-05-2018)

PART-1 : MATHEMATICS

SECTION-1

1. For a non-zero complex number z, let arg(z) denotes the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement(s) is (are) **FALSE** ?

(A) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f : \mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition

$\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line

Ans. (A,B,D)

Sol. (A) $\arg(-1-i) = -\frac{3\pi}{4}$,

(B) $f(t) = \arg(-1+it) = \begin{cases} \pi - \tan^{-1}(t), & t \geq 0 \\ -\pi + \tan^{-1}(t), & t < 0 \end{cases}$

Discontinuous at t = 0.

(C) $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$

$= \arg z_1 - \arg(z_2) + 2n\pi - \arg(z_1) + \arg(z_2) = 2n\pi$.

(D) $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$

$\Rightarrow \quad \dfrac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ is real.

$\Rightarrow$ z, z_1, z_2, z_3 are concyclic.

2. In a triangle PQR, let $\angle PQR = 30°$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE ?

(A) $\angle QPR = 45°$

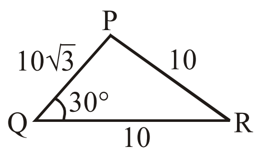
(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120°$

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3}-15$

(D) The area of the circumcircle of the triangle PQR is 100π.

Ans. (B,C,D)

Sol. $\cos 30° = \dfrac{\left(10\sqrt{3}\right)^2 + (10)^2 - (PR)^2}{2\times 10\sqrt{3}\times 10}$



$\Rightarrow$ PR = 10

$\because$ QR = PR $\Rightarrow$ $\angle PQR = \angle QPR$

$\angle QPR = 30°$

(B) area of Δ PQR $= \dfrac{1}{2}\times 10\sqrt{3}\times 10\times \sin 30° = \dfrac{1}{2}\times 10\times 10\sqrt{3}\times \dfrac{1}{2}$

$= 25\sqrt{3}$

$\angle QRP = 180° - (30° + 30°) = 120°$

(C) $r = \dfrac{\Delta}{S} = \dfrac{25\sqrt{3}}{\left(\dfrac{10+10+10\sqrt{3}}{2}\right)} = \dfrac{25\sqrt{3}}{10+5\sqrt{3}}$

$= 5\sqrt{3}.\left(2-\sqrt{3}\right) = 10\sqrt{3}-15$

(D) $R = \dfrac{a}{2\sin A} = \dfrac{10}{2\sin 30°} = 10$

$\therefore$ Area $= \pi R^2 = 100\pi$

3. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE ?

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, –1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_2 is $\frac{2}{\sqrt{3}}$

Ans. (C,D)

Sol. D.C. of line of intersection (a, b, c)

$\Rightarrow$ $2a + b - c = 0$

$a + 2b + c = 0$

$\frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1}$

$\therefore$ D.C. is (1, –1, 1)

(B) $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$

$\Rightarrow$ $\frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$

$\Rightarrow$ lines are parallel.

(C) Acute angle between P_1 and P_2 = $\cos^{-1}\left(\frac{2\times1+1\times2-1\times1}{\sqrt{6}\sqrt{6}}\right)$

$= \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

(D) Plane is given by $(x - 4) - (y - 2) + (z + 2) = 0$

$\Rightarrow$ $x - y + z = 0$

Distance of (2, 1, 1) from plane = $\frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

4. For every twice differentiable function $f : \mathbb{R} \rightarrow [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE ?

(A) There exist r, s $\in \mathbb{R}$, where r < s, such that f is one-one on the open interval (r, s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$

(C) $\lim_{x\rightarrow\infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

Ans. **(A,B,D)**

Sol. $f(x)$ can't be constant throughout the domain. Hence we can find $x \in (r, s)$ such that $f(x)$ is one-one option (A) is true.

Option (B) : $|f'(x_0)| = \left|\frac{f(0) - f(-4)}{4}\right| \le 1$ (LMVT)

Option (C) : $f(x) = \sin\left(\sqrt{85}x\right)$ satisfies given condition

but $\lim\limits_{x\to\infty} \sin\left(\sqrt{85}\right)$ D.N.E.

$\Rightarrow$ Incorrect

Option (D) : $g(x) = f^2(x) + (f'(x))^2$

$|f'(x_1) \le 1$ (by LMVT)

$|f(x_1)| \le 2$ (given)

$\Rightarrow$ $g(x_1) \le 5$ $\exists x_1 \in (-4, 0)$

Similarly $g(x_2) \le 5$ $\exists x_2 \in (0, 4)$

$g(0) = 85$ $\Rightarrow$ $g(x)$ has maxima in (x_1, x_2) say at α.

$g'(\alpha) = 0$ & $g(\alpha) \ge 85$

$2f'(\alpha)\ (f(\alpha) + f''(\alpha)) = 0$

If $f'(\alpha) = 0$ $\Rightarrow$ $g(\alpha) = f^2(\alpha) \ge 85$ Not possible

$\Rightarrow$ $f(\alpha) + f''(\alpha) = 0$ $\exists \alpha \in (x_1, x_2) \in (-4, 4)$

option (D) correct.

5. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{(f(x) - g(x))})g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE ?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Ans. **(B,C)**

Sol. $f'(x) = e^{(f(x) - g(x))}\ g'(x)\ \forall\ x \in \mathbb{R}$

$\Rightarrow$ $e^{-f(x)}.\ f'(x) - e^{-g(x)}g'(x) = 0$

$\Rightarrow$ $\int\left(e^{-f(x)} f'(x) - e^{-g(x)}.g'(x)\right)dx = C$

$\Rightarrow$ $-e^{-f(x)} + e^{-g(x)} = C$

$\Rightarrow$ $-e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$

$\Rightarrow \quad -\frac{1}{e}+e^{-g(1)}=-e^{-f(2)}+\frac{1}{e}$

$\Rightarrow \quad e^{-f(2)}+e^{-g(1)}=\frac{2}{e}$

$\therefore \quad e^{-f(2)}<\frac{2}{e}$ and $e^{-g(1)}<\frac{2}{e}$

$\Rightarrow \quad -f(2) < \ln 2 - 1$ and $-g(1) < \ln 2 - 1$

$\Rightarrow \quad f(2) > 1 - \ln 2$ and $g(1) > 1 - \ln 2$

6. Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function such that $f(x)=1-2x+\int_0^x e^{x-t} f(t)dt$ for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE ?

(A) The curve $y = f(x)$ passes through the point (1, 2)

(B) The curve $y = f(x)$ passes through the point (2, –1)

(C) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi-2}{4}$

(D) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi-1}{4}$

Ans. (B,C)

Sol. $f(x) = 1-2x+\int_0^x e^{x-t} f(t)dt$

$\Rightarrow \quad e^{-x} f(x) = e^{-x}(1-2x)+\int_0^x e^{-t} f(t)dt$

Differentiate w.r.t. x.

$-e^{-x} f(x) + e^{-x} f'(x) = -e^{-x}(1-2x) + e^{-x}(-2) + e^{-x} f(x)$

$\Rightarrow \quad -f(x) + f'(x) = -(1 - 2x) - 2 + f(x).$

$\Rightarrow \quad f'(x) - 2f(x) = 2x - 3$

Integrating factor = e^{-2x}.

$f(x).e^{-2x} = \int e^{-2x}(2x-3)dx$

$= (2x-3)\int e^{-2x}dx - \int\left((2)\int e^{-2x}dx\right)dx$

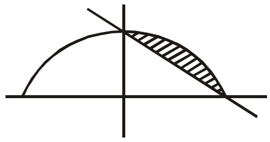
$= \frac{(2x-3)e^{-2x}}{-2} - \frac{e^{-2x}}{2} + c$

$f(x) = \frac{2x-3}{-2} - \frac{1}{2} + ce^{2x}$

$f(0) = \frac{3}{2} - \frac{1}{2} + c = 1 \Rightarrow c = 0$

$\therefore \quad f(x) = 1 - x$

Area $= \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$



SECTION-2

7. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is ——

Ans. **(8)**

Sol. $\log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$

$= \left(\log_2 9\right)^{2\log^2_{\log_2 9}} \times 7^{\frac{1}{2}\log_7 4}$

$= 4 \times 2 = 8$

8. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is ——

Ans. **(625)**

Sol. Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$ Total No. of digits

$= 5 \times 5 \times 5 \times 5 = 625$

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ... , and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ——

Ans. **(3748)**

Sol. X : 1, 6, 11, ..., 10086

Y : 9, 16, 23, ..., 14128

$X \cap Y$: 16, 51, 86, ...

Let $m = n(X \cap Y)$

$\therefore \quad 16 + (m - 1) \times 35 \leq 10086$

$\Rightarrow \quad m \leq 288.71$

$\Rightarrow \quad m = 288$

$\therefore \quad n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 2018 + 2018 - 288 = 3748$

10. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^{i}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^{i} - \sum_{i=1}^{\infty}(-x)^{i}\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ——

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume value in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

Ans. **(2)**

Sol. $\sum_{i=1}^{\infty} x^{i+1} = \frac{x^2}{1-x}$

$\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^{i} = \frac{x}{2-x}$

$\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^{i} = \frac{-x}{2+x}$

$\sum_{i=1}^{\infty}(-x)^{i} = \frac{-x}{1+x}$

To have real solutions

$$\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^{i} = \sum_{i=1}^{\infty}\left(\frac{-x}{2}\right)^{i} - \sum_{i=1}^{\infty}(-x)^{i}$$

$$\frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{-x}{2+x} + \frac{x}{1+x}$$

$x(x^3 + 2x^2 + 5x - 2) = 0$

$\therefore$ $x = 0$ and let $f(x) = x^3 + 2x^2 + 5x - 2$

$$f\left(\frac{1}{2}\right).f\left(-\frac{1}{2}\right) < 0$$

Hence two solutions exist

11. For each positive integer n, let

$$y_n = \frac{1}{n}(n+1)(n+2)...(n+n)^{1/n}$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is ——

Ans. **(1)**

Sol. $y_n = \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).....\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$

$y_n = \prod_{r=1}^{n}\left(1+\frac{r}{n}\right)^{1/n}$

$\log y_n = \frac{1}{n}\sum_{r=1}^{n}\ell n\left(1+\frac{r}{n}\right)$

$\Rightarrow \lim_{n\to\infty}\log y_n = \lim_{x\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ell n\left(1+\frac{r}{n}\right)$

$\Rightarrow \log L = \int_0^1 \ell n(1+x)dx$

$\Rightarrow \log L = \log\frac{4}{e}$

$\Rightarrow L = \frac{4}{e}$

$\Rightarrow [L] = 1$

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b} = 0$. For some x, y $\in \mathbb{R}$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a}\times\vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ——

Ans. **(3)**

Sol. $\vec{c} = x\vec{a} + y\vec{b} + \vec{a}\times\vec{b}$

$\vec{c}.\vec{a} = x$ and x = 2cosα

$\vec{c}.\vec{b} = y$ and y = 2cosα

Also, $|\vec{a}\times\vec{b}| = 1$

$\therefore \quad \vec{c} = 2\cos(\vec{a}+\vec{b}) + \vec{a}\times\vec{b}$

$\vec{c}^2 = 4\cos^2\alpha(\vec{a}+\vec{b})^2 + (\vec{a}\times\vec{b})^2 + 2\cos\alpha(\vec{a}+\vec{b}).(\vec{a}\times\vec{b})$

$4 = 8\cos^2\alpha + 1$

$8\cos^2\alpha = 3$

13. Let a, b, c be three non-zero real numbers such that the equation

$$\sqrt{3}a\cos x + 2b\sin x = c, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is ——

Ans. (0.5)

Sol. $\sqrt{3}\cos x+\frac{2b}{a}\sin x=\frac{c}{a}$

Now, $\sqrt{3}\cos\alpha+\frac{2b}{a}\sin\alpha=\frac{c}{a}$ ..... (1)

$\sqrt{3}\cos\beta+\frac{2b}{a}\sin\beta=\frac{c}{a}$ ..... (2)

$$\sqrt{3}\left[\cos\alpha-\cos\beta\right]+\frac{2b}{a}(\sin\alpha-\sin\beta)=0$$

$$\sqrt{3}\left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]+\frac{2b}{a}\left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]=0$$

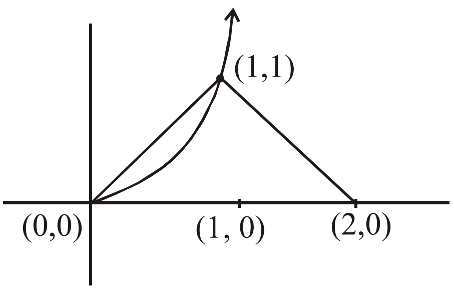
$$-\sqrt{3}+2\sqrt{3}.\frac{b}{a}=0$$

$$\frac{b}{a}=\frac{1}{2}=0.5$$

14. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ——

Ans. (4)

Sol.



$$\text{Area} = \int_0^1 (x-x^n)dx=\frac{3}{10}$$

$$\left[\frac{x^2}{2}-\frac{x^{n+1}}{n+1}\right]_0^1=\frac{3}{10}$$

$\frac{1}{2}-\frac{1}{n+1}=\frac{3}{10}$ $\therefore$ n + 1 = 5

$\Rightarrow$ n = 4

SECTION-3

Paragraph "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(*There are two question based on Paragraph "X", the question given below is one of them*)

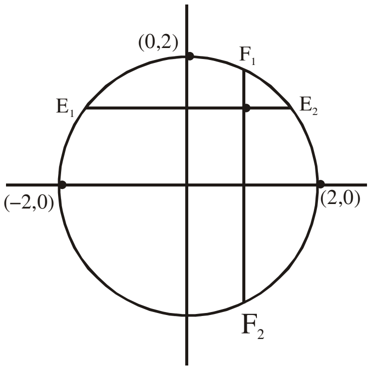
15. Let E_1E_2 and F_1F_2 be the chord of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slop –1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents of S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3 and G_3 lie on the curve

(A) $x + y = 4$ (B) $(x - 4)^2 + (y - 4)^2 = 16$

(C) $(x - 4)(y - 4) = 4$ (D) $xy = 4$

Ans. **(A)**

Sol.



co-ordinates of E_1 and E_2 are obtained by solving $y = 1$ and $x^2 + y^2 = 4$

$\therefore$ $E_1\left(-\sqrt{3},1\right)$ and $E_2\left(\sqrt{3},1\right)$

co-ordinates of F_1 and F_2 are obtained by solving

$x = 1$ and $x^2 + y^2 = 4$

$F_1\left(1,\sqrt{3}\right)$ and $F_2\left(1,-\sqrt{3}\right)$

Tangent at E_1 : $-\sqrt{3}x + y = 4$

Tangent at E_2 : $\sqrt{3}x + y = 4$

$\therefore$ $E_3(0, 4)$

Tangent at F_1 : $x + \sqrt{3}y = 4$

Tangent at F_2 : $x - \sqrt{3}y = 4$

$\therefore$ $F_3(4, 0)$

and similarly $G_3(2, 2)$

(0, 4), (4, 0) and (2, 2) lies on $x + y = 4$

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$

(*There are two questions based on Paragraph "X", the question given below is one of them*)

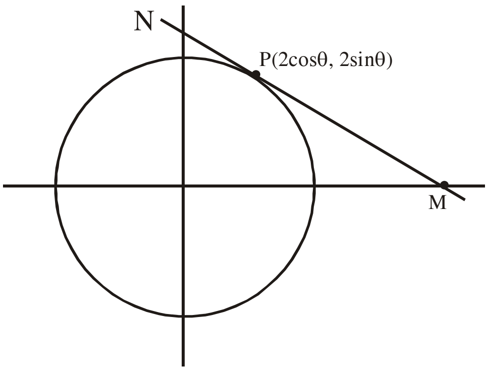
16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve -

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

Ans. (D)

Sol.



Tangent at $P(2\cos\theta, 2\sin\theta)$ is $x\cos\theta + y\sin\theta = 2$

$M(2\sec\theta, 0)$ and $N(0, 2\text{cosec}\theta)$

Let midpoint be (h, k)

$h = \sec\theta,\ \ k = \text{cosec}\theta$

$$\frac{1}{h^2} + \frac{1}{k^2} = 1$$

$$\frac{1}{x^2} + \frac{1}{y^2} = 1$$

PARAGRAPH "A"

There are five students S_1, S_2, S_4 and S_5 in a music class and for them there are five sets R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A". the question given below is one of them*)

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and **NONE** of the remaining students gets the seat previously allotted to him/her is -

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

Ans. (A)

Sol. Required probability = $\frac{4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)}{5!} = \frac{9}{120} = \frac{3}{40}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A", the question given below is one of them*)

18. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is-

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

Ans. **(C)**

Sol. $n(T_1 \cap T_2 \cap T_3 \cap T_4)$ = Total – $n\left(\overline{T}_1 \cup \overline{T}_2 \cup \overline{T}_3 \cup \overline{T}_4\right)$

$= 5! - \left({}^4C_1 4!2! - \left({}^3C_1.3!2! + {}^3C_1 3!2!2!\right) + \left({}^2C_1 2!2! + {}^4C_1.2.2!\right) - 2\right)$

$= 14$

Probability $= \frac{14}{5!} = \frac{7}{60}$

JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION

(Exam Date: 20-05-2018)

PART-1 : PHYSICS

1. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

Ans. (A,B,D)

Sol. $\frac{dk}{dt} = \gamma t$ **as** $k = \frac{1}{2}mv^2$

$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$

$\therefore m\int_0^v vdv = \gamma\int_0^t tdt$

$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$

$v = \sqrt{\frac{\gamma}{m}}t$ .....(i)

$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}}$ = constant

since F = ma

$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m}$ = constant

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true ?

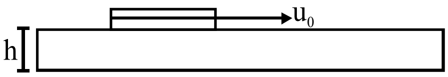
(A) The resistive force of liquid on the plate is inversely proportional to h

(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with u_0.

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

Ans. (A,C,D)

Sol. 

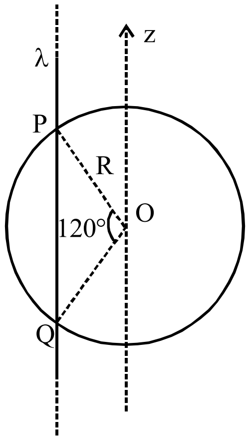
Viscous force is given by F = $-\eta A \frac{dv}{dy}$ since h is very small therefore, magnitude of viscous force is given by

$$F = \eta A \frac{\Delta v}{\Delta y}$$

$\therefore\ F = \frac{\eta A u_0}{h} \Rightarrow F \propto \eta \ \&\ F \propto u_0$; $\qquad F \propto \frac{1}{h}$, $F \propto A$

Since plate is moving with constant velocity, same force must be acting on the floor.

3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true ?



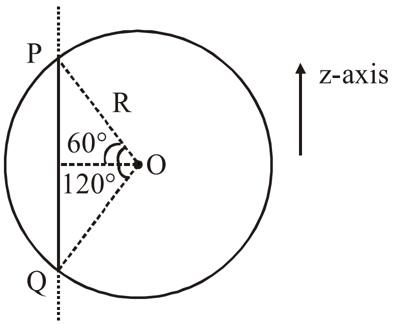
(A) The electric flux through the shell is $\sqrt{3}R\lambda/\varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}R\lambda/\varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

Ans. (A,B)

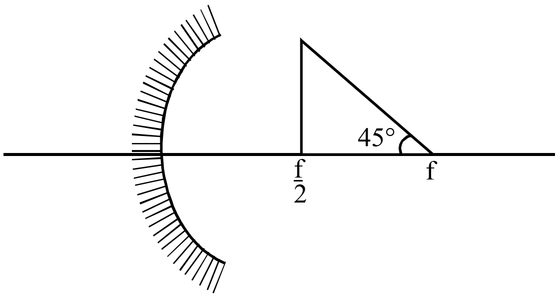
Sol. 

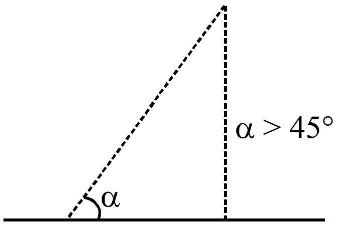
Field due to straight wire is perpendicular to the wire & radially outward. Hence $E_z = 0$

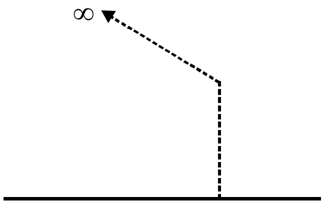
Length, PQ = 2R sin 60 = $\sqrt{3}R$ According to Gauss's law

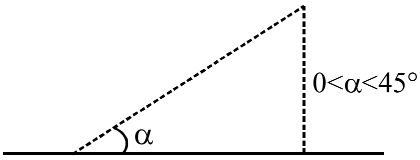
total flux = $\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\epsilon_0} = \frac{\lambda\sqrt{3}R}{\epsilon_0}$

4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire ? (These figures are not to scale.) ?



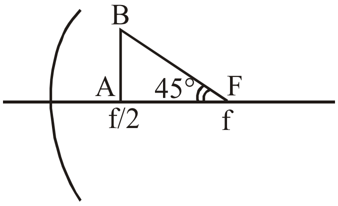
(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol. 

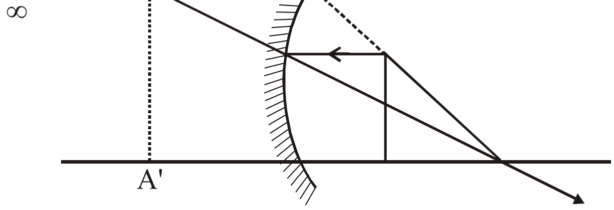
Distance of point A is f/2

Let A' is the image of A from mirror, for this image

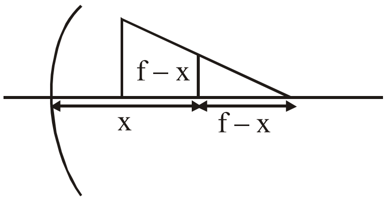
$$\frac{1}{v}+\frac{1}{-f/2}=\frac{1}{-f}$$

$$\frac{1}{v}=\frac{2}{f}-\frac{1}{f}=\frac{1}{f}$$

image of line AB should be perpendicular to the principle axis & image of F will form at infinity, therefor correct image diagram is



OR



$$\frac{f}{f-u} = \frac{h_2}{h_1}$$

$$h_2 = \frac{-f(f-x)}{-f+x}$$

h_2 = f

5. In a radioactive decay chain, $^{232}_{90}Th$ nucleus decays to $^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true ?

(A) N_α = 5 (B) N_α = 6 (C) N_β = 2 (D) N_β = 4

Ans. (A,C)

Sol. $^{232}_{90}Th$ is converting into $^{212}_{82}Pb$

Change in mass number (A) = 20

$\therefore$ no of α particle = $\frac{20}{4} = 5$

Due to 5 α particle, z will change by 10 unit.

Since given change is 8, therefore no. of β particle is 2

6. In an experment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true ?

(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 cm

(C) The wavelength of the sound wave is 66.4 cm

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic

Ans. (A,C or A,B,C)

Sol. Let n_1 harmonic is corresponding to 50.7 cm & n_2 harmonic is corresponding 83.9 cm.

since both one consecutive harmonics.

$\therefore$ their difference = $\frac{\lambda}{2}$

$\therefore$ $\frac{\lambda}{2}$ = (83.9 – 50.7) cm

$$\frac{\lambda}{2} = 33.2 \text{ cm.}$$

$$\lambda = 66.4 \text{ cm}$$

$$\therefore \frac{\lambda}{4} = 16.6 \text{ cm}$$

length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ & 50.7 cm must be closed to an odd multiple of this length as $16.6 \times 3 = 49.8$ cm. therefore 50.7 is 3^{rd} harmonic

If end correction is e, then

$$e + 50.7 = \frac{3\lambda}{4}$$

$$e = 49.8 - 50.7 = -0.9 \text{ cm}$$

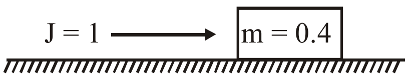
speed of sound, $v = f\lambda$

$$\therefore v = 500 \times 66.4 \text{ cm/sec} = 332.000 \text{ m/s}$$

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is.................. Take $e^{-1} = 0.37$?

Ans. 6.30

Sol.



$$v = v_0 e^{-t/\tau}$$

$$v_0 = \frac{J}{m} = 2.5 \text{ m/s}$$

$$v = v_0 e^{-t/\tau}$$

$$\frac{dx}{dt} = v_0 e^{-t/\tau}$$

$$\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt \qquad \int e^{-x} dx = \frac{e^{-x}}{-1}$$

$$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right]_0^\tau$$

$$x = 2.5\ (-4)\ (e^{-1} - e^{0})$$

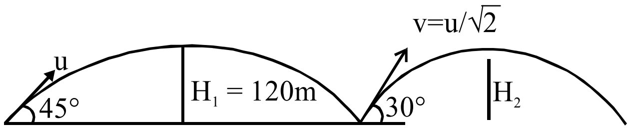
$$x = 25\ (-4)\ (0.37 - 1)$$

$$x = 6.30 \text{ ans.}$$

8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is................

Ans. 30.00

Sol.



$$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \quad ....(i)$$

when half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g} \quad .....(ii)$$

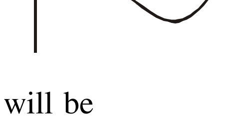
from (i) & (ii)

$H_2 = \frac{H_1}{4}$ = 30 m on 30.00

9. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \,\hat{i}$ where E_0 = 1.0 N C^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is..........................

Ans. 2.00

Sol. n = 10^{-3} kg q = 1C t = 0

$E = E_0 \sin \omega t \equiv$ 

Force on particle will be

$F = qE = qE_0 \sin \omega t$

at v_{max}, a, F = 0 $\qquad qE_0 \sin \omega t = 0$

$F = qE_0 \sin\omega t$

$$\frac{dv}{dt} = q\frac{E_0}{m}\sin\omega t$$

$$\int_0^v dv = \int_0^{\pi/\omega} \frac{qE_0}{m}\sin\omega t\,dt$$

$$v - 0 = \frac{qE_0}{m\omega}[-\cos\omega t]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega}[(-\cos\pi)-(-\cos 0)]$$

$$v = \frac{1\times 1}{10^{-3}10^{3}}\times 2 = 2 \text{ m/s}$$

Ans. 2. 00

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field porduced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is...........

Ans. 5.55

Sol. $n = 50$ turns $\quad A = 2 \times 10^{-4}\ m^2$

$B = 0.02$ T $\quad K = 10^{-4}$

$Q_m = 0.2$ rad $\quad R_g = 50\ \Omega$

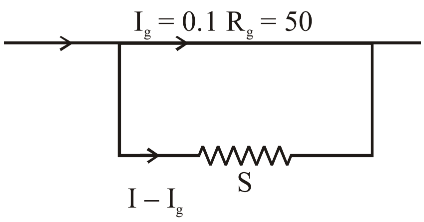
$I_A = 0 - 1.0$ A $\quad \tau = MB = C\theta$, $M = nIA$

$BINA = C\theta$

$0.02 \times 1 \times 50 \times 2 \times 10^{-4} = 10^{-4} \times 0.2\ 10$

$I_g = 0.1$ A

For galvanometer, resistance is to be connected to ammeter in shunt.



$I_g \times R_g = (I - I_g)S$

$0.1 \times 50 = (1 - 0.1)\ S$

$$S = \frac{50}{9} = 5.55$$

11. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2kg, the vernier scale division which coincides with a main scale division is....... Take g = 10 ms^{-2} and π = 3.2.

Ans. 3.00

Sol. d = 0.5 mm $Y = 2 \times 10^{11}$ ℓ = 1m

$$\Delta\ell = \frac{F\ell}{Ay} = \frac{mg\ell}{\frac{\pi d^2}{4}y} = \frac{1.2\times10\times1}{\frac{\pi}{4}\times(5\times10^{-4})^2\times2\times10^{11}}$$

$$\Delta\ell = \frac{1.2\times10}{\frac{3.2}{4}\times25\times10^{-8}\times2\times10^{11}}$$

$$= \frac{12}{0.8\times25\times2\times10^3} = \frac{12}{40\times10^3} = 0.3\text{mm}$$

so 3^{rd} division of vernier scale will coincicle with main scale.

12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is............ .

Ans. 900

Sol. $v_i = v$

$v_F = 8v$

For adiabatic process $\left\{\gamma = \frac{5}{3}\right.$ for monoatomic process

$$T_1V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$$

$$100(v)^{2/3} = T_2(8v)^{2/3}$$

T_2 = 25 k

$$\Delta U = nc_V\Delta T = 1\left(\frac{FR}{2}\right)[100 - 25] = 12 \times 75 = 900 \text{ Joule}$$

13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficinecy is 100% A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is........ Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV = 1.6×10^{-19} J.?

Ans. 24

Sol. Power = nhν   n = number of photons per second

Since KE = 0, hν = ϕ

200 = n[6.25 × 1.6 × 10^{-19} Joule]

$$n = \frac{200}{1.6\times10^{-19}\times6.25}$$

As photon is just above threshold frequency KE_{max} is zero and they are accelrated by potential difference of 500V.

$KE_f = q\Delta V$

$$\frac{P^2}{2m} = q\Delta V \Rightarrow P = \sqrt{2mq\Delta V}$$

Since efficiency is 100%, number of electrons = number of photons per second

As photon is completely absorbed force exerted = nmv

$$= \frac{200}{6.25\times1.6\times10^{-19}}\times\sqrt{2\left(9\times10^{-31}\right)\times1.6\times10^{-19}\times500}$$

$$= \frac{3\times200\times10^{-25}\times\sqrt{1600}}{6.25\times1.6\times10^{-19}} = \frac{2\times40}{6.25\times1.6}\times10^{-4}\times3 = 24$$

14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is............. .

Ans. 3

Sol. $\Delta E_{2-1} = 13.6\times z^2\left[1-\frac{1}{4}\right] = 13.6\times z^2\left[\frac{3}{4}\right]$

$$\Delta E_{3-2} = 13.6\times z^2\left[\frac{1}{4}-\frac{1}{9}\right] = 13.6\times z^2\left[\frac{5}{36}\right]$$

$\Delta E_{2-1} = \Delta E_{3-2} + 74.8$

$$13.6\times z^2\left[\frac{3}{4}\right] = 13.6\times z^2\left[\frac{5}{36}\right] + 74.8$$

$$13.6\times z^2\left[\frac{3}{4}-\frac{5}{36}\right] = 74.8$$

$z^2 = 9$

z = +3 ans

15. The electric field E is measured at a point P(0,0,d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

List–I	**List–II**
P. E is indpendent of d	**1.** A point charge Q at the origin
Q. $E \propto \frac{1}{d}$	2. A small dipole with point charges Q at $(0,0,\ell)$ and $-Q$ at $(0,0,-\ell)$. Take $2\ell << d$
R. $E \propto \frac{1}{d^2}$	3. An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S. $E \propto \frac{1}{d^3}$	4. Two infinite wires carrying uniform linear Charge density parallel to the x - axis. The one along $(y = 0, z = \ell)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -\ell)$ has a charge density $-\lambda$. Take $2\ell << d$
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

(A) P → 5 ; Q → 3, 4 ; R → 1 ; S → 2
(B) P → 5 ; Q → 3, ; R → 1,4 ; S → 2
(C) P → 5 ; Q → 3, ; R → 1,2 ; S → 4
(D) P → 4 ; Q → 2, 3 ; R → 1 ; S → 5

Ans. **(B)**

Sol. (i) $E = \frac{KQ}{d^2} \Rightarrow E \propto \frac{1}{d^2}$

(ii) Dipole

$$E = \frac{2kp}{d^3}\sqrt{1+3\cos^2\theta}$$

$E \propto \frac{1}{d^3}$ for dipole

(iii) For line charge

$$E = \frac{2k\lambda}{d}$$

$$E \propto \frac{1}{d}$$

(iv) $E = \frac{2K\lambda}{d-\ell} - \frac{2K\lambda}{d+\ell}$

$$= 2K\lambda\left[\frac{d+\ell-d+\ell}{d^2-\ell^2}\right]$$

$$E = \frac{2K\lambda(2\ell)}{d^2\left[1-\frac{\ell^2}{d^2}\right]}$$

$$E \propto \frac{1}{d^2}$$

(v) Electric field due to sheet

$$\in = \frac{\sigma}{2\in_0}$$

$\in$ = v is independent of r

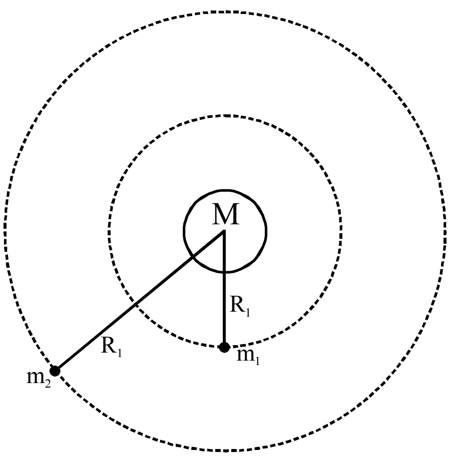
16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

List–I	List–II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

(A) P → 4 ; Q → 2 ; R → 1 ; S → 3
(B) P → 3 ; Q → 2 ; R → 4 ; S → 1
(C) P → 2 ; Q → 3 ; R → 1 ; S → 4
(D) P → 2 ; Q → 3 ; R → 4 ; S → 1

Ans. (B)

Sol.



$$\left.\begin{array}{l}\frac{m_1}{m_2} = 2 \\ \frac{R_1}{R_2} = \frac{1}{4}\end{array}\right\} \text{given}$$

$$\frac{GMm_1}{R_1^2} = \frac{m_1 v_1^2}{R_1}$$

$$v_1^2 = \frac{GM}{R_1}, \quad v_2^2 = \frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{R_2}{R_1} = 4$$

(P) $\frac{v_1}{v_2} = 2$

(Q) $L = mvR$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_2 v_2 R_2} = 2 \times 2 \times \frac{1}{4} = 1$$

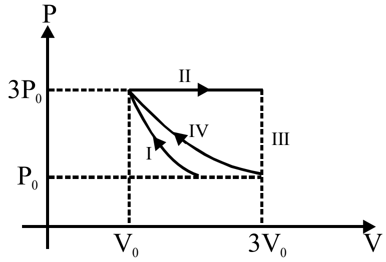
(R) $K = \frac{1}{2} mv^2$

$$\frac{K_1}{K_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = 2 \times (2)^2 = 8$$

(S) $T = 2\pi R/V$

$$\frac{T_1}{T_2} = \frac{R_1}{v_1} \times \frac{v_2}{R_2} = \frac{R_1}{R_2} \times \frac{v_2}{v_1} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II.



List–I	**List–II**
P. In process I	1. Work done by the gas is zero
Q. In process II	2. Temperature of the gas remains unchanged
R. In process III	3. No heat is exchanged between the gas and its surroundings
S. In process IV	4. Work done by the gas is 6 P_0V_0

(A) $P \rightarrow 4$; $Q \rightarrow 3$; $R \rightarrow 1$; $S \rightarrow 2$

(B) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 4$

(C) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$

(D) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 2$; $S \rightarrow 1$

Ans. **(C)**

Sol. Process – I is an adiabatic process

$\Delta Q = \Delta U + W \qquad \Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperatuer of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process – II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process

(Volume remain constant)

$\Delta Q = \Delta U + W$

$W = 0$

$\Delta Q = \Delta U$

Process – IV is an isothermal process

(Temperature remains constant)

$\Delta Q = \Delta U + W$

$\Delta U = 0$

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq B$. In each case, the force acting on the particle is either zero or conservative. In List–II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List–I with those quantities in List–II, which are conserved for that path

	List–I		**List–II**
P.	$\vec{r}(t) = \alpha t\,\hat{i} + \beta t\,\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\,\hat{i} + \beta\sin\omega t\,\hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$	3.	K
S	$\vec{r}(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$	4.	U
		5.	E

(A) P $\rightarrow$ 1,2,3,4,5 ; Q $\rightarrow$ 2,5 ; R $\rightarrow$ 2,3,4,5 ; S $\rightarrow$ 5

(B) P $\rightarrow$ 1,2,3,4,5 ; Q $\rightarrow$ 3,5 ; R $\rightarrow$ 2,3,4,5 ; S $\rightarrow$ 2,5

(C) P $\rightarrow$ 2,3,4 ; Q $\rightarrow$ 5; R $\rightarrow$ 1,2,4 ; S $\rightarrow$ 2,5

(D) P $\rightarrow$ 1,2,3,5 ; Q $\rightarrow$ 2,5 ; R $\rightarrow$ 2,3,4,5 ; S $\rightarrow$ 2,5

Ans. (A)

Sol. (P) $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = \alpha\hat{i} + \beta\hat{j} \{\text{constant}\}$$

$$\vec{a} = \frac{\overrightarrow{dv}}{dt} = 0$$

$\vec{P} = m\vec{v}$ (remain constant)

$$k = \frac{1}{2}mv^2 \{\text{remain constant}\}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{i}\right] = 0$$

$\Rightarrow$ U $\rightarrow$ constant

E = K + U

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0$$

$\vec{L}$ = constant

(Q) $\vec{r} = \alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = -\alpha\omega\sin(\omega t)\hat{i} + \beta\omega\cos(\omega t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\omega^2\cos(\omega t)\hat{i} - \beta\omega^2\sin(\omega t)\hat{j}$$

$$= -\omega^2\left[\alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}\right]$$

$$\vec{a} = -\omega^2\vec{r}$$

$\vec{\tau} = \vec{r} \times \vec{F} = 0$ {$\vec{r}$ and $\vec{F}$ are parallel}

$$\Delta U = -\int \vec{F}.dr = +\int_0^r m\omega^2.r.dr$$

$$\Delta U = m\omega^2\left[\frac{r^2}{2}\right]$$

$U \propto r^2$

$$r = \sqrt{\alpha^2\cos^2(\omega t) + \beta^2\sin^2(\omega t)}$$

r is a function of time (t)

U depends on r hence it will change with time
Total energy remain constant because force is central.

(R) $\vec{r}(t) = \alpha\left(\cos\omega t\hat{i} + \sin(\omega t)\hat{j}\right)$

$\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = \alpha\left[-\omega\sin(\omega t)\hat{i} + \omega\cos(\omega t)\hat{j}\right]$

$|\vec{v}| = \alpha\omega$ (Speed remains constant)

$\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = \alpha\left[-\omega^2\cos(\omega t)\hat{i} - \omega^2\sin(\omega t)\hat{j}\right]$

$= -\alpha\omega^2\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right]$

$\vec{a}(t) = -\omega^2(\vec{r})$

$\vec{\tau} = \vec{F}\times\vec{r} = 0$

$|\vec{r}| = \alpha$ (remain constant)

Force is central in nature and distance from fixed point is constant.
Potential energy remains constant
Kinetic energy is also constant (speed is constant)

(S) $\vec{r} = \alpha t\hat{i} + \dfrac{\beta}{2}t^2\hat{j}$

$\vec{v} = \dfrac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j}$ (speed of particle depends on 't')

$\vec{a} = \dfrac{d\vec{v}}{dt} = \beta\hat{j}$ {constant}

$\vec{F} = m\vec{a}$ {constant}

$\Delta U = -\int \vec{F}.d\vec{r} = -m\int_0^t \beta\hat{j}.\left(\alpha\hat{i} + \beta t\hat{j}\right)dt$

$U = \dfrac{-m\beta^2t^2}{2}$

$k = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\alpha^2 + \beta^2t^2\right)$

$E = k + U = \dfrac{1}{2}m\alpha^2$ [remain constant]

JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION

(Exam Date: 20-05-2018)

PART-1 : CHEMISTRY

1. The correct option(s) regarding the complex $[Co(en)\ (NH_3)_3(H_2O)]^{3+}$:-

(en = $H_2NCH_2CH_2NH_2$) is (are)

(A) It has two geometrical isomers

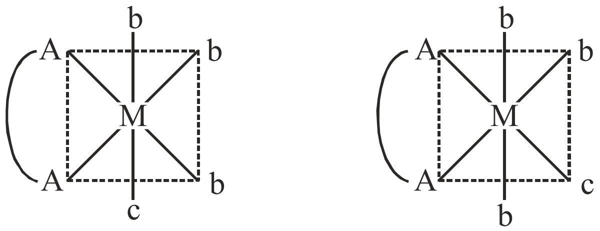
(B) It will have three geometrical isomers if bidentate 'en' is replaced by two cyanide ligands

(C) It is paramagnetic

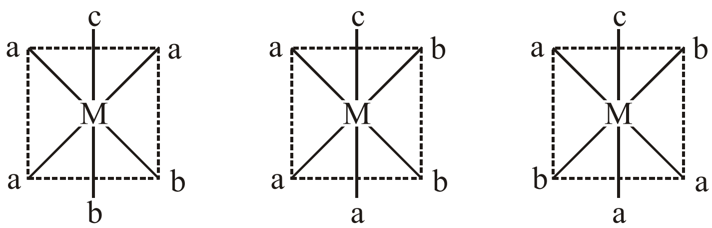
(D) It absorbs light at longer wavelength as compared to $[Co(en)\ (NH_3)_4]^{3+}$

Ans. (A,B,D)

Sol. (A) $[Co(en)(NH_3)_3(H_2O)]^{+3}$ complex is type of $[M(AA)b_3c]$ have two G.I.



(B) If (en) is replaced by two cynide ligand, complex will be type of $[Ma_3b_2c]$ and have 3 G.I.



(C) $[Co(en)(NH_3)_3(H_2O)]^{3+}$ have d^6 configuration (t_{2g}^6) on central metal with SFL therefore it is dimagnetic in nature.

(D) Complex $[Co(en)(NH_3)_3(H_2O)]^{3+}$ have lesser CFSE (Δ_O) value than $[Co(en)(NH_3)_4]^{3+}$ therefore complex $[Co(en)(NH_3)_3(H_2O)]^{+}$ absorbs longer wavelength for d–d transition.

2. The correct option(s) to distinguish nitrate salts of Mn^{2+} and Cu^{2+} taken separately is (are) :-

(A) Mn^{2+} shows the characteristic green colour in the flame test

(B) Only Cu^{2+} shows the formation of precipitate by passing H_2S in acidic medium

(C) Only Mn^{2+} shows the formation of precipitate by passing H_2S in faintly basic medium

(D) Cu^{2+}/Cu has higher reduction potential than Mn^{2+}/Mn (measured under similar conditions)

Ans. (B,D)

Sol. (A) Cu^{+2} and Mn^{+2} both gives green colour in flame test and cannot distinguished.

(B) Cu^{+2} belongs to group-II of cationic radical will gives ppt. of CuS in acidic medium.

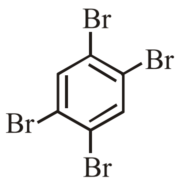
(C) Cu^{+2} and Mn^{+2} both form ppt. in basic medium.

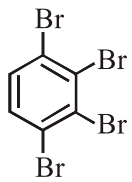
(D) $Cu^{+2}/Cu = +0.34$ V (SRP)

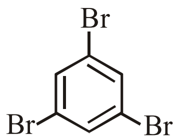
$Mn^{+2}/Mn = -\ 1.18$ V (SRP)

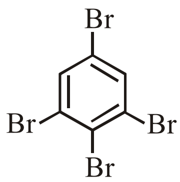
3. Aniline reacts with mixed acid (conc. HNO_3 and conc. H_2SO_4) at 288 K to give P (51%), Q (47%) and R (2%). The major product(s) the following reaction sequence is (are) :-

R $\xrightarrow[\text{3) } H_3O^+ \text{ 4) } NaNO_2,\ HCl/273\text{-}278K \text{ 5) EtOH, } \Delta]{\text{1) } Ac_2O,\ \text{pyridine 2) } Br_2,\ CH_3CO_2H}$ S $\xrightarrow[\text{3) } NaNO_2,\ HCl/273\text{-}278K \text{ 4) } H_3PO_2]{\text{1) Sn/HCl 2) } Br_2/H_2O \text{ (excess)}}$ major product(s)

(A) 

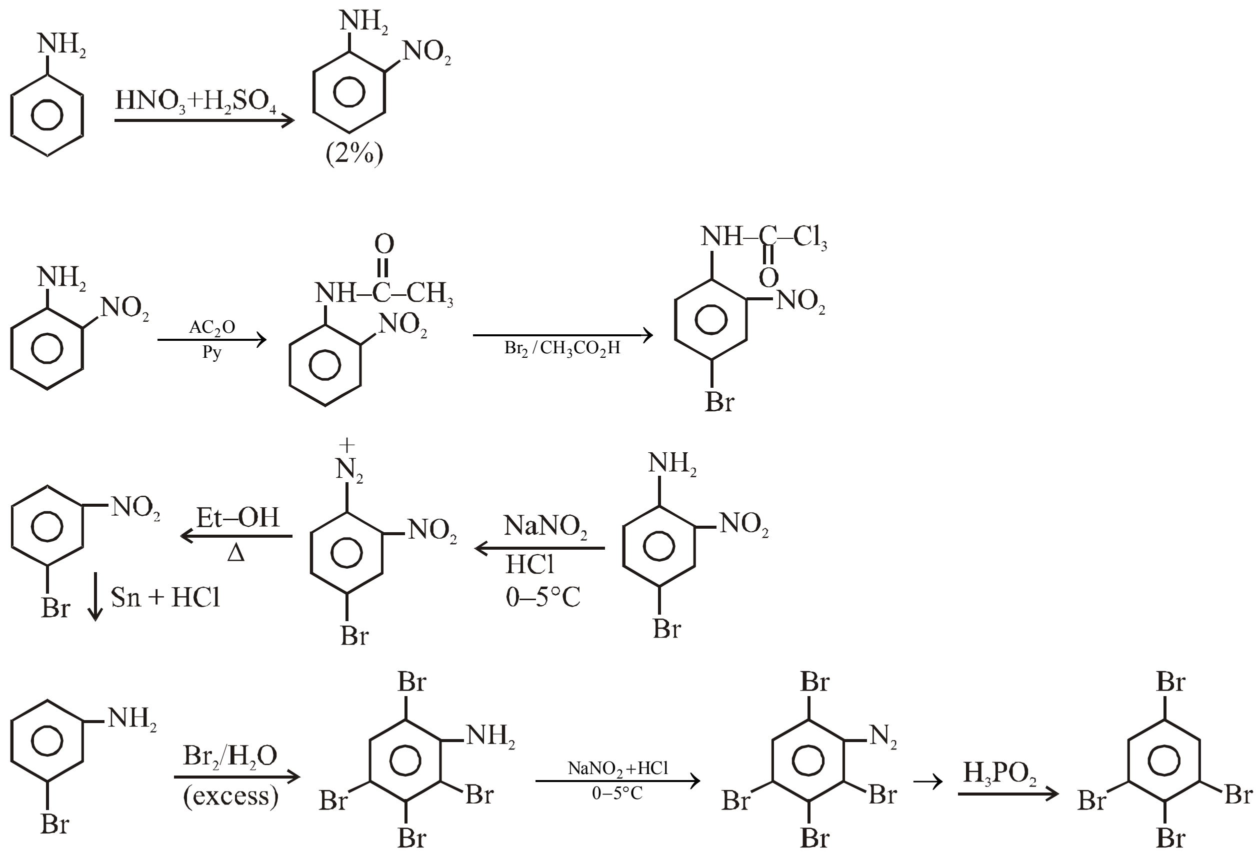
(B) 

(C) 

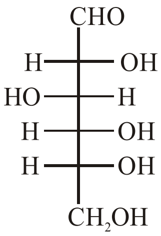
(D) 

Ans. **(D)**

Sol.

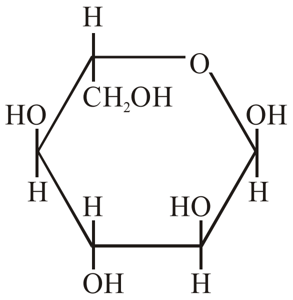


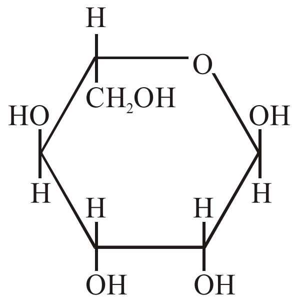
4. The Fischer presentation of D-glucose is given below.

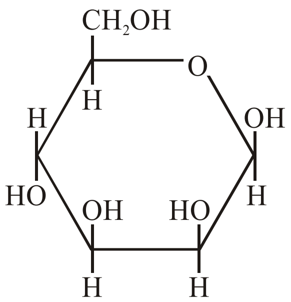


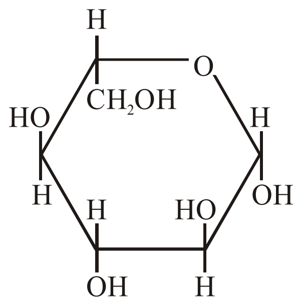
D-glucose

The correct structure(s) of β-L-glucopyranose is (are) :-

(A) 

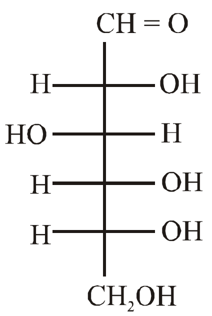
(B) 

(C) 

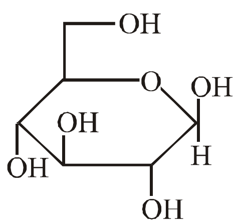
(D) 

Ans. (D)

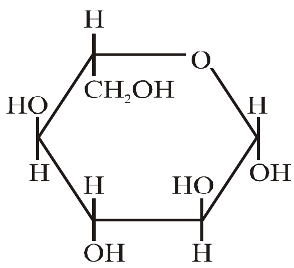
Sol.



D-glucose



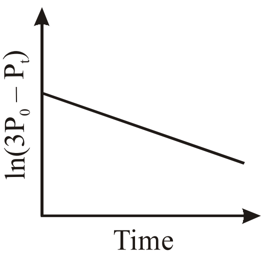
β-D-glucopyranose

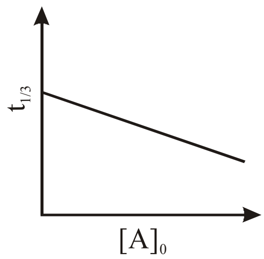


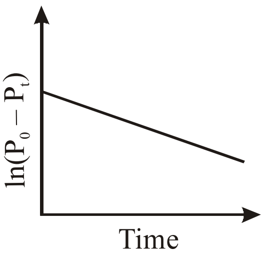
β-L-glucopyranose

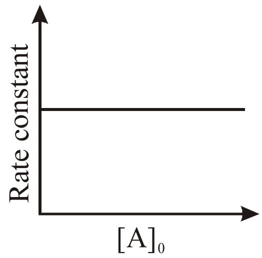
5. For a first order reaction A(g) $\rightarrow$ 2B(g) + C(g) at constant volume and 300 K, the total pressure at the beginning (t = 0) and at time t are P_0 and P_t, respectively. Initially, only A is present with concentration $[A]_0$, and $t_{1/3}$ is the time required for the partial pressure of A to reach $1/3^{rd}$ of its initial value. The correct option(s) is (are) :-

(Assume that all these gases behave as ideal gases)

(A) 

(B) 

(C) 

(D) 

Ans. (A,D)

Sol.

$$\begin{array}{llll} & A \longrightarrow & 2B & + C \\ t = 0 & P_0 & - & - \\ t = t & P_0 - P & 2P & P \end{array}$$

$P_0 + 2P = P_t$

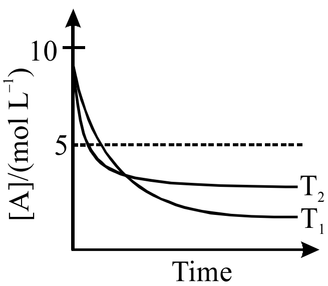
$$K = \frac{1}{t}\ln\frac{P_0}{P_0 - P} = \frac{1}{t}\ln\frac{P_0}{P_0 - \frac{(P_t - P_0)}{2}}$$

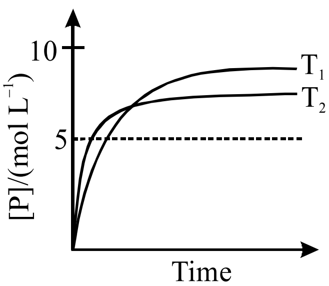
$$K = \frac{1}{t}\ln\frac{2P_0}{3P_0 - P_t} \Rightarrow -Kt + \ln 2P_0 = \ln(3P_0 - P_t)$$

and $t_{1/3} = \frac{1}{K}\ln\frac{P_0}{P_0/3} = \frac{1}{K}\ln 3 = \text{constant}$

Rate constant does not depends on concentration

6. For a reaction, A $\rightleftharpoons$ P, the plots of [A] and [P] with time at temperatures T_1 and T_2 are given below.





If $T_2 > T_1$, the correct statement(s) is (are)

(Assume ΔH^θ and ΔS^θ are independent of temperature and ratio of lnK at T_1 to lnK at T_2 is greater than $\frac{T_2}{T_1}$. Here H, S, G and K are enthalpy, entropy, Gibbs energy and equilibrium constant, respectively.)

(A) $\Delta H^\theta < 0$, $\Delta S^\theta < 0$

(B) $\Delta G^\theta < 0$, $\Delta H^\theta > 0$

(C) $\Delta G^\theta < 0$, $\Delta S^\theta < 0$

(D) $\Delta G^\theta < 0$, $\Delta S^\theta > 0$

Ans. (A,C)

Sol. $A \rightleftharpoons P$

given $T_2 > T_1$

$$\frac{\ln K_1}{\ln K_2} > \frac{T_2}{T_1}$$

$\Rightarrow T_1 \ln k_1 > T_2 \ln k_2$

$\Rightarrow -\Delta G^\circ_1 > -\Delta G^\circ_2$

$\Rightarrow (-\Delta H^\circ + T_1\Delta S^\circ) > (-\Delta H^\circ + T_2\Delta S^\circ)$

$\Rightarrow T_1 \Delta S^\circ > T_2\Delta S^\circ$

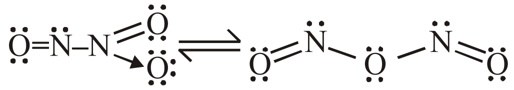
$\Rightarrow \Delta S^\circ < 0$

7. The total number of compounds having at least one bridging oxo group among the molecules given below is______.

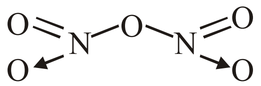
N_2O_3, N_2O_5, P_4O_6, P_4O_7, $H_4P_2O_5$, $H_5P_3O_{10}$, $H_2S_2O_3$, $H_2S_2O_5$

Ans. (5 or 6)

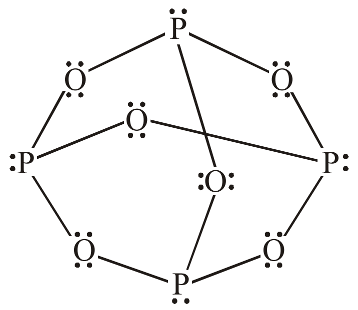
Sol. N_2O_3 =



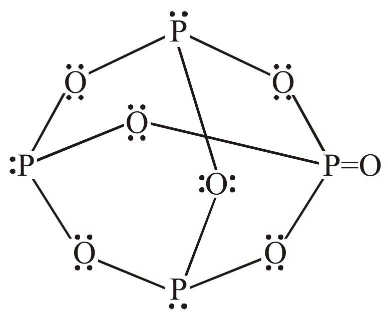
N_2O_5 =

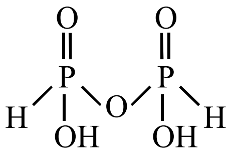


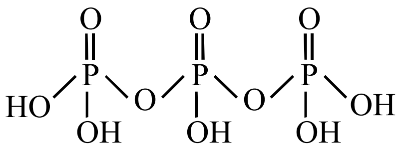
P_4O_6 =

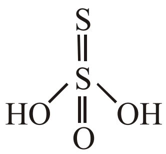


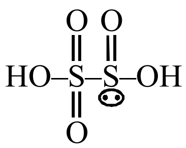
P_4O_7 =



$H_4P_2O_5$ = 

$H_5P_3O_{10}$ = 

$H_2S_2O_3$ = 

$H_2S_2O_5$ = 

8. Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnance such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is ______ .

(Atomic weights in g mol^{-1} : O = 16, S = 32, Pb = 207)

Ans. (6.47)

Sol. $PbS + O_2 \longrightarrow \quad Pb + SO_2$

$$\frac{1000}{32}\text{mol} \qquad \frac{1000}{32}\times 207\text{ gm}$$

mol of Pb = mol of O_2

$$= \frac{1000}{32}\text{mol}$$

$$\therefore \text{ mass of Pb} = \frac{1000}{32}\times 207\text{g}$$

$$= \frac{207}{32}\text{kg} = 6.47 \text{ kg}$$

9. To measure the quantity of $MnCl_2$ dissolved in an aqueous solution, it was completely converted to $KMnO_4$ using the reaction,

$MnCl_2 + K_2S_2O_8 + H_2O \rightarrow KMnO_4 + H_2SO_4 + HCl$ (equation not balanced).

Few drops of concentrated HCl were added to this solution and gently warmed. Further, oxalic acid (225 g) was added in portions till the colour of the permanganate ion disappeard. The quantity of $MnCl_2$ (in mg) present in the initial solution is _____.

(Atomic weights in g mol^{-1} : Mn = 55, Cl = 35.5)

Ans. (126)

Sol. $\underset{\text{a mole}}{MnCl_2} + K_2S_2O_8 + H_2O \rightarrow \underset{\text{a mole}}{KMnO_4} + H_2SO_4 + HCl$

$C_2O_4^{--} + MnO_4^- \xrightarrow{H^+} CO_2$

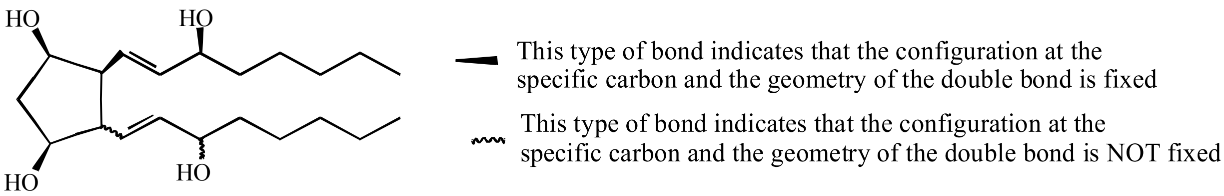
m_{eq} of $C_2O_4^{--}$ = m_{eq} of MnO_4^-

$2 \times 0.225/90 = a \times 5$

$a = 1 \times [55 + 71]$

$= 126$ mg

10. For the given compound X, the total number of optically active stereoisomers is____.

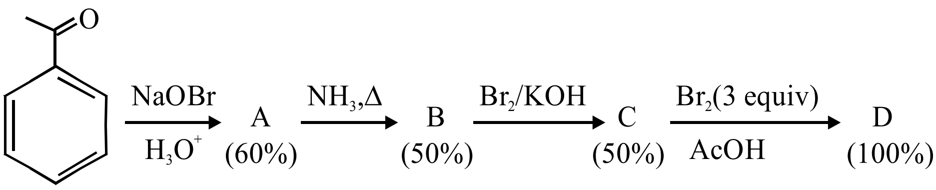


X

Ans. (7)

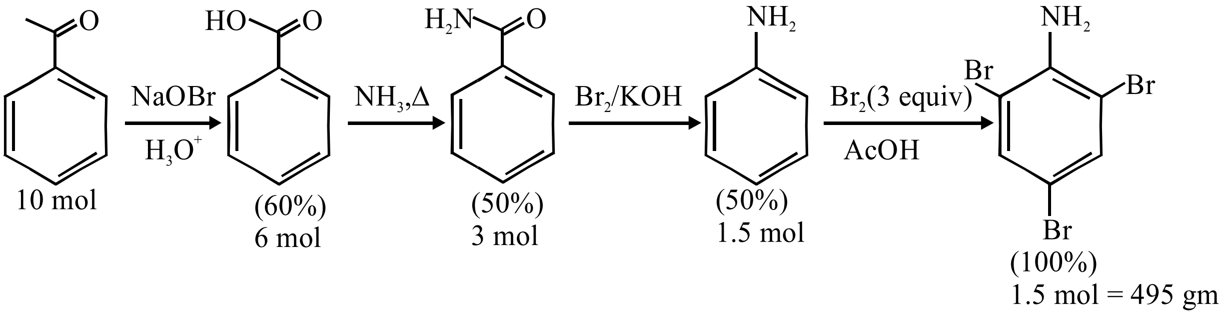
11. In the following reaction sequence, the amount of D (in g) formed from 10 moles of acetophenone is____.

(Atomic weight in g mol^{-1}: H = 1, C = 12, N = 14, O = 16, Br = 80. The yield (%) corresponding to the product in each step is given in the parenthesis)



Ans. (495)

Sol.



12. The surface of copper gets tarnished by the formation of copper oxide. N_2 gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the N_2 gas contains 1 mole % of water vapour as impurity. The water vapour oxidises copper as per the reaction given below :

$2Cu(s) + H_2O(g) \rightarrow Cu_2O(s) + H_2(g)$

p_{H_2} is the minimum partial pressure of H_2 (in bar) needed to prevent the oxidation at 1250 K. The value of $\ln(p_{H_2})$ is ___.

(Given : total pressure = 1 bar, R (universal gas constant) = 8 $JK^{-1}mol^{-1}$, ln(10) = 2.3. Cu(s) and $Cu_2O(s)$ are mutually immiscible.

At 1250 K : $2Cu(s) + 1/2O_2(g) \rightarrow Cu_2O(s)$; $\Delta G^\theta = -78,000$ J mol^{-1}

$H_2(g) + 1/2O_2(g) \rightarrow H_2O(g)$; $\Delta G^\theta = -1,78,000$ J mol^{-1}; G is the Gibbs energy)

Ans. (–14.6)

Sol. $2Cu(s) + \frac{1}{4}O_2(g) \rightarrow 1Cu_2O(s)$ $\quad \Delta G^\circ = -78$ kJ

$[H_2(g) + \frac{1}{2}O_2 \rightarrow H_2O(g)$ $\quad \Delta G^\circ = -178$ kJ$] \times (-1)$

Hence, $2Cu(s) + H_2O(g) \rightarrow Cu_2O + H_2(g)$ $\quad \Delta G^\circ = +100$ kJ

$\Delta G = \Delta G^\circ + RT \ln Q$

$$0 = +100 + \frac{8}{1000} \times 1250 \ln \frac{p_{H_2}}{p_{H_2O}}$$

$$-\frac{100 \times 1000}{8} = 1250 \ln \frac{p_{H_2}}{\left(\frac{1}{100} \times 1\right)}$$

$\ln p_{H_2} = -14.6$

13. Consider the following reversible reaction,

$$A(g) + B(g) \rightleftharpoons AB(g).$$

The activition energy of the backward reaction exceeds that of the forward reaction by 2RT (in J mol^{-1}). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of ΔG^θ (in J mol^{-1}) for the reaction at 300 K is____.

(Given ; ln (2) = 0.7, RT = 2500 J mol^{-1} at 300 K and G is the Gibbs energy)

Ans. (8500)

Sol. $A_{(g)} + B_{(g)} \rightleftharpoons AB_{(g)}$

$E_{ab} - E_{af} = 2RT \quad \Rightarrow \Delta H = -2RT \quad$ and $\frac{A_f}{A_b} = 4$

$$K_{eq} = \left(\frac{K_f}{K_b}\right) = \frac{A_f e^{-E_{af}/RT}}{A_b e^{-E_{ab}/RT}} = 4(e^2)$$

$\Delta G^\circ = -RT \ln K = -2500 \times \ln(4 \times e^2) = -8500$ J/mol

$\therefore$ Absolute value of $\Delta G^\circ = 8500$ J/mol

14. Consider an electrochemical cell: $A(s) \mid A^{n+}$ (aq, 2M) $\parallel B^{2n+}$ (aq, 1M) $\mid B(s)$. The value of ΔH^θ for the cell reaction is twice that of ΔG^θ at 300 K. If the emf of the cell is zero, the ΔS^θ (in JK^{-1} mol^{-1}) of the cell reaction per mole of B formed at 300 K is___.

(Given : ln (2) = 0.7, R (universal gas constant) = 8.3 J K^{-1} mol^{-1}. H, S and G are enthalpy, entropy and Gibbs energy, respectively.)

Ans. (–11.62)

Sol. A(s) | A^{+n} (aq, 2M) || B^{+2n} (aq, 1M) | B(s)

$\Delta H^\circ = 2\Delta G^\circ_0$ $\qquad$ $E_{cell} = 0$

Cell Rx $\quad A \rightarrow A^{+n} + ne^- \;] \times 2$

$B^{+2n} + 2n\, e^- \rightarrow B(s)$

$$2A(s) + \underset{1M}{B^{+2n}}(aq) \rightarrow 2\underset{2M}{A^{+n}}(aq) + B(s)$$

$$\Delta G = \Delta G^\circ + RT \ln \frac{[A^{+n}]^2}{[B^{+2n}]}$$

$$\Delta G^\circ = -RT \ln \frac{[A^{+n}]^2}{[B^{+2n}]} = -RT.\ln \frac{2^2}{1} = -RT.\ln 4$$

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

$$\Delta G^\circ = 2\Delta G^\circ - T\Delta S^\circ$$

$$\Delta S^\circ = \frac{\Delta G^\circ}{T} = -\frac{RT \ln 4}{T}$$

$$= -8.3 \times 2 \times 0.7 = -11.62 \text{ J/K.mol}$$

15. Match each set of hybrid orbitals from LIST-I with complex (es) given in LIST-II.

LIST-I	**LIST-II**
P. dsp^2	1. $[FeF_6]^{4-}$
Q. sp^3	2. $[Ti(H_2O)_3Cl_3]$
R. sp^3d^2	3. $[Cr(NH_3)_6]^{3+}$
S. d^2sp^3	4. $[FeCl_4]^{2-}$
	5. $Ni(CO)_4$
	6. $[Ni(CN)_4]^{2-}$

The correct option is

(A) P $\rightarrow$ 5; Q $\rightarrow$ 4,6; R $\rightarrow$ 2,3; S $\rightarrow$ 1

(B) P $\rightarrow$ 5,6; Q $\rightarrow$ 4; R $\rightarrow$ 3; S $\rightarrow$ 1,2

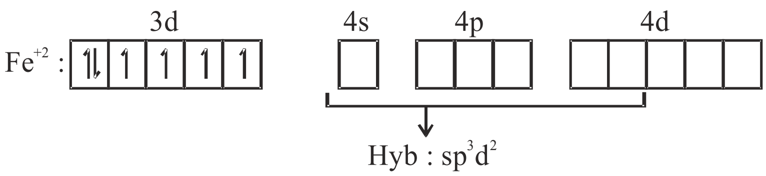
(C) P $\rightarrow$ 6; Q $\rightarrow$ 4,5; R $\rightarrow$ 1; S $\rightarrow$ 2,3

(D) P $\rightarrow$ 4,6; Q $\rightarrow$ 5,6; R $\rightarrow$ 1,2; S $\rightarrow$ 3

Ans. (C)

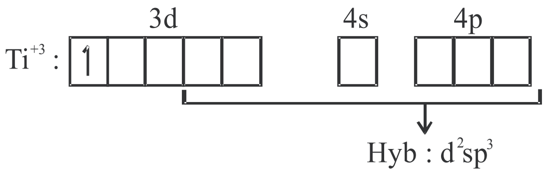
Sol. [1] $[FeF_6]^{4-}$

$Fe^{2+} = [Ar]3d^6$,



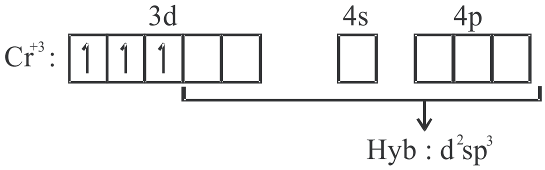
[2] $[Ti(H_2O)_3Cl_3]$

$Ti^{+3} = [Ar]3d^1$,



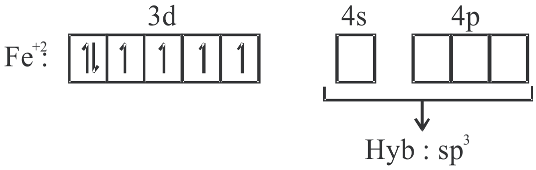
[3] $[Cr(NH_3)_6]^{3+}$

$Cr^{+3} = [Ar]3d^3$,



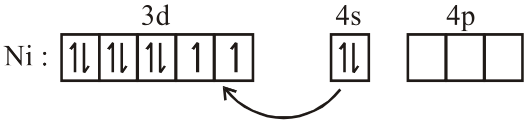
[4] $[FeCl_4]^{2-}$

$Fe^{2+} = [Ar]3d^6$,

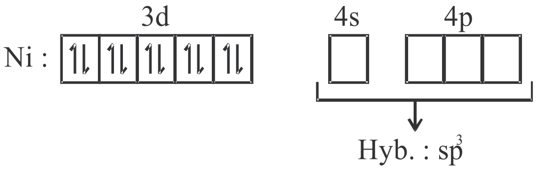


[5] $[Ni(CO)_4]$

Ni : $3d^8$ $4s^2$



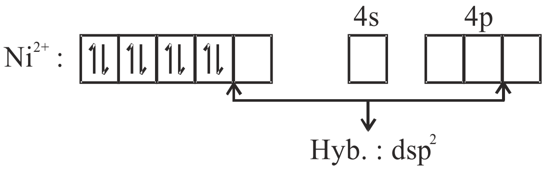
Back pairing of electrons due to presence of strong field ligand



[6] $[Ni(CN)_4]^{2-}$

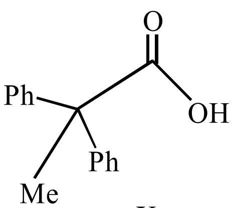
Ni^{2+} : $3d^8$

Electron pairing take place due to presence of S.F.L.



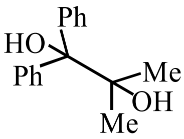
16. The desired product X can be prepared by reacting the major product of the reactions in LIST-I with one or more appropriate reagents in LIST-II.

(given, order of migratory aptitude: aryl > alkyl > hydrogen)

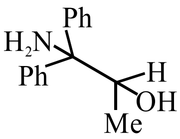


X

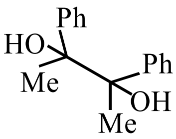
LIST-I | **LIST-II**

P.  $+ H_2SO_4$

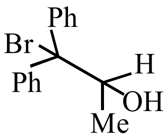
1. I_2, NaOH

Q.  $+ HNO_2$

2. $[Ag(NH_3)_2]OH$

R.  $+ H_2SO_4$

3. Fehling solution

S.  $+ AgNO_3$

4. HCHO, NaOH

5. NaOBr

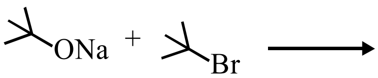
The correct option is

(A) P → 1; Q → 2,3; R → 1,4; S → 2,4
(B) P → 1,5; Q → 3,4; R → 4,5; S → 3
(C) P → 1,5; Q → 3,4; R → 5; S → 2,4
(D) P → 1,5; Q → 2,3; R → 1,5; S → 2,3

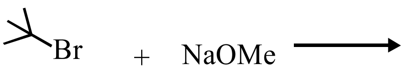
Ans. (D)

17. LIST-I contains reactions and LIST-II contains major products.

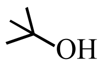
LIST-I | **LIST-II**

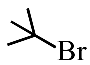
P. 

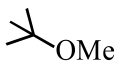
Q. 

R. 

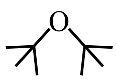
S. 

1. 

2. 

3. 

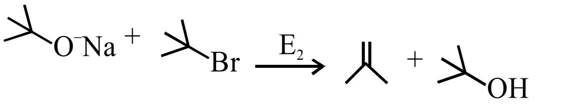
4. 

5. 

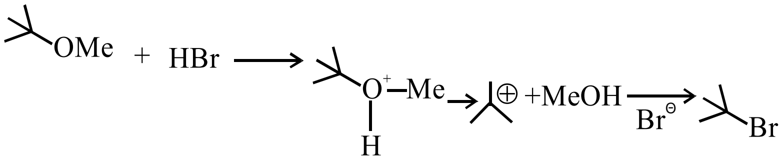
Match each reaction in LIST-I with one or more product in LIST-II and choose the correct option.

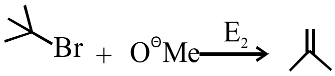
(A) P → 1,5; Q → 2; R → 3; S → 4
(B) P → 1,4; Q → 2; R → 4; S → 3
(C) P → 1,4; Q → 1,2; R → 3,4; S → 4
(D) P → 4,5; Q → 4; R → 4; S → 3,4

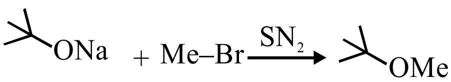
Ans. (B)

Sol. P. 

(Elimination product)

Q. 

R. 

S. 

18. Dilution process of different aqueous solutions; with water, are given in LIST-I. The effects of dilution of the solutions on $[H^+]$ are given in LIST-II.

(Note : Degree of dissociation (α) of weak acid and weak base is << 1; degree of hydrolysis of salt <<1; $[H^+]$ represents the concentration of H^+ ions)

LIST-I	**List-II**
P. (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	1. the vale of $[H^+]$ does not change on dilution
Q. (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	2. the value of $[H^+]$ change to half of its initial value on dilution
R. (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	3. the value of $[H^+]$ changes to two times of its initial value on dilution
S. 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	4. the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
	5. the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

Match each process given in LIST-I with one or more effect(s) in LIST-II. The correct option is

(A) $P \to 4$; $Q \to 2$; $R \to 3$; $S \to 1$
(B) $P \to 4$; $Q \to 3$; $R \to 2$; $S \to 3$
(C) $P \to 1$; $Q \to 4$; $R \to 5$; $S \to 3$
(D) $P \to 1$; $Q \to 5$; $R \to 4$; $S \to 1$

Ans. (D)

Sol. P. $\underset{0.1M,\,20ml}{CH_3COOH} + \underset{0.1M,10ml}{NaOH} \rightarrow CH_3COONa + H_2O$

pH = pKa $\Rightarrow$ $[H^+]$ will not change on dilution

correct match : P-1

Q. $\underset{0.1M,\,20ml}{CH_3COOH} + \underset{0.1M,20ml}{NaOH} \rightarrow CH_3COONa + H_2O$

— — 0.05 M

$$[OH^-] = \sqrt{K_H C} = \sqrt{\left(\frac{k_w}{k_a}C\right)}$$

$$[H^+]_1 = \sqrt{\frac{k_w k_a}{C}}$$

$$\frac{[H^+]_2}{[H^+]_1} = \sqrt{\frac{C_1}{C_2}} = \sqrt{\frac{0.05}{0.025}} = \sqrt{2}$$

correct match : Q-5

R. $\underset{0.1M,20ml}{NH_4OH} + \underset{0.1M,20ml}{HCl} \rightarrow \underset{0.05M}{NH_4Cl}$

$$[H^+] = \sqrt{K_H C}$$

$$\frac{[H^+]_2}{[H^+]_1} = \sqrt{\frac{C_2}{C_1}} = \frac{1}{\sqrt{2}}$$

correct match : R-4

S. Because of dilution solubility does not change so $[H^+]$ = constant

JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION

(Exam Date: 20-05-2018)

PART-1 : MATHEMATICS

SECTION 1

1. For any positive integer n, define $f_n : (0, \infty) \to \mathbb{R}$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty).$$

(Here, the inverse trigonometric function $\tan^{-1}x$ assume values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Then, which of the following statement(s) is (are) TRUE ?

(A) $\sum_{j=1}^{5} \tan^2\left(f_j(0)\right) = 55$

(B) $\sum_{j=1}^{10} \left(1+f_j'(0)\right)\sec^2\left(f_j(0)\right) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan\left(f_n(x)\right) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2\left(f_n(x)\right) = 1$

Ans. (D)

Sol. $f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right)$

$$f_n(x) = \sum_{j=1}^{n} [\tan^{-1}(x+j) - \tan^{-1}(x+j-1)]$$

$f_n(x) = \tan^{-1}(x + n) - \tan^{-1}x$

$\therefore \tan(f_n(x)) = \tan[\tan^{-1}(x + n) - \tan^{-1}x]$

$$\tan(f_n(x)) = \frac{(x+n)-x}{1+x(x+n)}$$

$$\tan(f_n(x)) = \frac{n}{1+x^2+nx}$$

$\therefore \sec^2(f_n(x)) = 1 + \tan^2(f_n(x))$

$$\sec^2(f_n(x)) = 1+\left(\frac{n}{1+x^2+nx}\right)^2$$

$$\lim_{x\to\infty}\sec^2(f_n(x)) = \lim_{x\to\infty}1+\left(\frac{n}{1+x^2+nx}\right)^2 = 1$$

2. Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangents to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE ?

(A) The point (–2, 7) lies in E_1

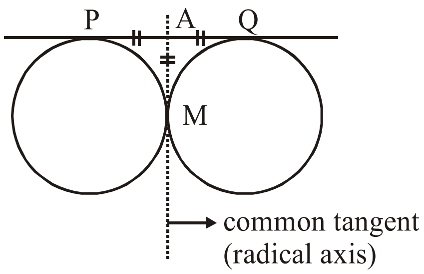
(B) The point $\left(\frac{4}{5},\frac{7}{5}\right)$ does **NOT** lie in E_2

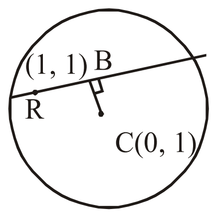
(C) The point $\left(\frac{1}{2},1\right)$ lies in E_2

(D) The point $\left(0,\frac{3}{2}\right)$ does **NOT** lie in E_1

Ans. (D)

Sol.





AP = AQ = AM

Locus of M is a circle having PQ as its diameter

Hence, $E_1 : (x - 2)(x + 2) + (y - 7)(y + 5) = 0$ and $x \neq \pm 2$

Locus of B (midpoint)

is a circle having RC as its diameter

$E_2 : x(x - 1) + (y - 1)^2 = 0$

Now, after checking the options, we get (D)

3. Let S be the of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution of each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

Ans. **(A,D)**

Sol. We find D = 0 & since no pair of planes are parallel, so there are infinite number of solutions.

Let $\alpha P_1 + \lambda P_2 = P_3$

$\Rightarrow \quad P_1 + 7P_2 = 13P_3$

$\Rightarrow \quad b_1 + 7b_2 = 13b_3$

(A) $D \neq 0 \Rightarrow$ unique solution for any b_1, b_2, b_3

(B) $D = 0$ but $P_1 + 7P_2 \neq 13P_3$

(C) As planes are parallel and there exist infinite ordered triplet for which they will be non coincident although satisfying $b_1 + 7b_2 = 13b_3$.

$\therefore$ rejected.

(D) $D \neq 0$

4. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0, 0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then the which of the following statement(s) is (are) TRUE ?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

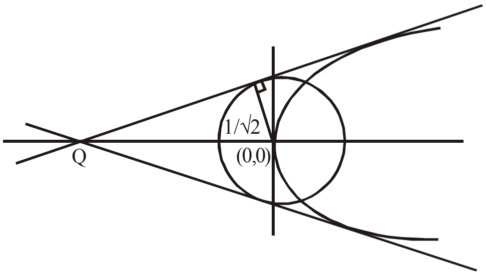
(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

Ans. (A,C)

Sol.



Let equation of common tangent is $y = mx + \frac{1}{m}$

$$\therefore \quad \left|\frac{0+0+\frac{1}{m}}{\sqrt{1+m^2}}\right| = \frac{1}{\sqrt{2}} \Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow m = \pm 1$$

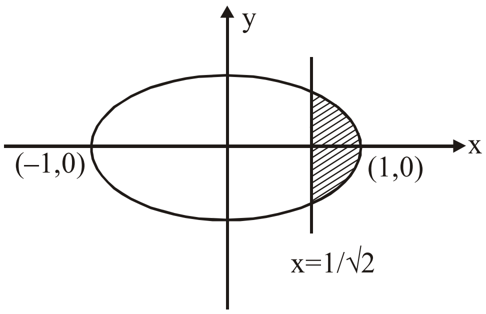
Equation of common tangents are $y = x + 1$ and $y = -x - 1$

point Q is $(-1, 0)$

$\therefore$ Equation of ellipse is $\frac{x^2}{1} + \frac{y^2}{1/2} = 1$

(A) $e = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$ and $LR = \frac{2b^2}{a} = 1$

(C)



$$\text{Area } \; 2.\int_{1/\sqrt{2}}^{1} \frac{1}{\sqrt{2}}.\sqrt{1-x^2}\,dx = \sqrt{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1}$$

$$= \sqrt{2}\left[\frac{\pi}{4} - \left(\frac{1}{4} + \frac{\pi}{8}\right)\right] = \sqrt{2}\left(\frac{\pi}{8} - \frac{1}{4}\right) = \frac{\pi - 2}{4\sqrt{2}}$$

correct answer are (A) and (D)

5. Let s, t, r be the non-zero complex numbers and L be the set of solutions $z = x + iy \left(x, y \in \mathbb{R}, i = \sqrt{-1}\right)$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE ?

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

Ans. (A,C,D)

Sol. Given

$sz + t\bar{z} + r = 0$ (1)

$\bar{z} = x - iy$ (Conjugate of z)

Taking conjugate throughout $\bar{s}\bar{z} + \bar{t}z + \bar{r} = 0$ (2)

Adding (1) and (2)

$$\left(s + \bar{t}\right)z + \left(\bar{s} + t\right)\bar{z} + \left(r + \bar{r}\right) = 0$$

And Subtracting (1) and (2)

$$\left(s - \bar{t}\right)z + \left(t - \bar{s}\right)\bar{z} + \left(r - \bar{r}\right) = 0$$

For unique solution

$$\frac{t + \bar{s}}{t - s} \neq \frac{s + \bar{t}}{s - \bar{t}}$$

On further simplification $\Rightarrow |t| \neq |s|$

Hence option A proved.

If the lines coincide, then

$$\frac{t + \bar{s}}{t - \bar{s}} = \frac{\bar{t} + s}{s - t} = \frac{r + \bar{r}}{r - \bar{r}}$$

On comparing

$$\frac{t + \bar{s}}{t - \bar{s}} = \frac{r + \bar{r}}{r - \bar{r}}$$

and simplification, we get $\Rightarrow |s| = |t|$

The lines can be parallel or coincidental.

Since, no concrete outcome.

Hence, option B is not correct.

Clearly L is either a single or represents a line and $|z-1+i|=5$ represents a circle.

$\therefore$ Intersection of L and $\{|z-1+i|=5\}$ is ATMOST 2.

Hence, option C is correct.

Let $s=\alpha_1+i\beta_1; t=\alpha_2+i\beta_2$ *and* $r=\alpha_3+i\beta_3$

Then $sz+t\bar{z}+r=0$

$$\Rightarrow(\alpha_1+\alpha_2)x+(\beta_2-\beta_1)y+\alpha_3=0$$
$$\text{and}\quad(\beta_1+\beta_2)x+(\alpha_1-\alpha_2)y+\beta_3=0$$

If L has more than 1 element then it implies L will have $\propto$ elements.

As L represents linear equation in x and y.

Hence, option D is correct.

6. Let $f:(0,\pi)\to\mathbb{R}$ be a twice differentiable function such that

$$\lim_{t\to x}\frac{f(x)\sin t-f(t)\sin x}{t-x}=\sin^2 x \text{ for all } x\in(0,\pi).$$

If $f\left(\frac{\pi}{6}\right)=-\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

(A) $f\left(\frac{\pi}{4}\right)=\frac{\pi}{4\sqrt{2}}$

(B) $f(x)<\frac{x^4}{6}-x^2$ for all $x\in(0,\pi)$

(C) There exists $\alpha\in(0,\pi)$ such that $f'(\alpha)=0$

(D) $f''\left(\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)=0$

Ans. (B,C,D)

Sol. $\lim_{t\to x}\frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$

by using L'Hopital

$\lim_{t\to x}\frac{f(x)\cos t - f'(t)\sin x}{1} = \sin^2 x$

$\Rightarrow$ f(x)cosx –f'(x)sinx = sin^{2}x

$\Rightarrow \ -\left(\frac{f'(x)\sin x - f(x)\cos x}{\sin^2 x}\right) = 1$

$\Rightarrow \ -d\left(\frac{f(x)}{\sin x}\right) = 1$

$\Rightarrow \ \frac{f(x)}{\sin x}$=-x+c

Put x = $\frac{\pi}{6}$ & $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$

$\therefore$ c = 0 $\Rightarrow$ f(x) = –xsinx

(A) $f\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}\frac{1}{\sqrt{2}}$

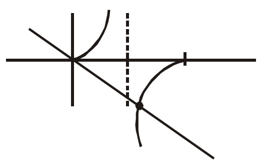
(B) f(x) = –x sinx

as sinx > $x - \frac{x^3}{6}$, –x sinx < – x^2 + $\frac{x^4}{6}$

$\therefore$ f(x) < – x^2 + $\frac{x^4}{6}$ $\forall$ x $\in$ (0, π)

(C) f'(x) = –sinx – x cosx

f'(x) = 0 $\Rightarrow$ tanx = –x $\Rightarrow$ there exist $\alpha \in (0, \pi)$ for which f'(α) = 0



(D) f"(x) = –2cosx + xsinx

$f''\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, $f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

$f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION 2

7. The value of the integral

$$\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$$

is ______ .

Ans. **(2)**

Sol. $\displaystyle\int_0^{\frac{1}{2}} \frac{\left(1+\sqrt{3}\right)dx}{\left[(1+x)^2(1-x)^6\right]^{1/4}}$

$$\int_0^{\frac{1}{2}} \frac{\left(1+\sqrt{3}\right)dx}{(1+x)^2\left[\frac{(1-x)^6}{(1+x)^6}\right]^{1/4}}$$

Put $\frac{1-x}{1+x} = t \Rightarrow \frac{-2dx}{(1+x)^2} = dt$

$$I = \int_1^{1/3} \frac{\left(1+\sqrt{3}\right)dt}{-2t^{6/4}} = \frac{-(1+\sqrt{3})}{2} \times \left|\frac{-2}{\sqrt{t}}\right|_1^{1/3} = \left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right) = 2$$

8. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is ______ .

Ans. **(4)**

Sol. $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \underbrace{(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)}_{x} - \underbrace{(a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)}_{y}$

Now if $x \le 3$ and $y \ge -3$

the Δ can be maximum 6

But it is not possible

as $x = 3 \Rightarrow$ each term of $x = 1$

and $y = 3 \Rightarrow$ each term of $y = -1$

$\Rightarrow \prod_{i=1}^{3} a_i b_i c_i = 1$ and $\prod_{i=1}^{3} a_i b_i c_i = -1$

which is contradiction

so now next possibility is 4

which is obtained as $\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 1(1+1) - 1(-1-1) + 1(1-1) = 4$

9. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____ .

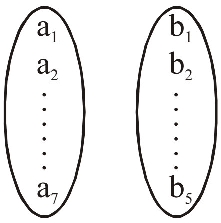
Ans. (119)

Sol. $n(X) = 5$

$n(Y) = 7$

$\alpha \rightarrow$ Number of one-one function = ${}^7C_5 \times 5!$

$\beta \rightarrow$ Number of onto function Y to X



1, 1, 1, 1, 3 1, 1, 1, 2, 2

$$\frac{7!}{3!4!} \times 5! + \frac{7!}{(2!)^3 3!} \times 5! = \left({}^7C_3 + 3.{}^7C_3\right)5! = 4 \times {}^7C_3 \times 5!$$

$$\frac{\beta - \alpha}{5!} = 4 \times {}^7C_3 - {}^7C_5 = 4 \times 35 - 21 = 119$$

10. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = (2 + 5y)(5y - 2),$$

then the value of $\lim_{x \to -\infty} f(x)$ is _____ .

Ans. (0.4)

Sol. $\frac{dy}{dx} = 25y^2 - 4$

So, $\frac{dy}{25y^2 - 4} = dx$

Integrating, $\frac{1}{25} \times \frac{1}{2 \times \frac{2}{5}} \ln \left| \frac{y - \frac{2}{5}}{y + \frac{2}{5}} \right| = x + c$

$\Rightarrow \quad \ln\left|\frac{5y-2}{5y+2}\right| = 20(x+c)$

Now, c = 0 as f(0) = 0

Hence $\quad \left|\frac{5y-2}{5y+2}\right| = e^{(20x)}$

$\underset{x\to-\infty}{\text{let}}\left|\frac{5f(x)-2}{5f(x)+2}\right| = \underset{x\to-\infty}{\text{let}}\ e^{(20x)}$

Now, RHS = 0 $\quad \Rightarrow \quad \underset{x\to-\infty}{\text{let}}(5f(x)-2) = 0$

$\Rightarrow \quad \underset{x\to-\infty}{\text{let}}\ f(x) = \frac{2}{5}$

11. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 1$ and satisfying the equation

$$f(x + y) = f(x)f'(y) + f'(x)f(y) \text{ for all } x, y \in \mathbb{R}.$$

Then, then value of $\log_e(f(4))$ is ______ .

Ans. **(2)**

Sol. P(x, y) : $f(x + y) = f(x)f'(y) + f'(x) f(y)\ \forall\ x, y \in R$

P(0, 0) : $f(0) = f(0)f'(0) + f'(0) f(0)$

$\Rightarrow \quad 1 = 2f'(0)$

$\Rightarrow \quad f'(0) = \frac{1}{2}$

P(x, 0) : $f(x) = f(x).\ f'(0) + f'(x).f(0)$

$\Rightarrow \quad f(x) = \frac{1}{2}f(x) + f'(x)$

$\Rightarrow \quad f'(x) = \frac{1}{2}f(x)$

$\Rightarrow \quad f(x) = e^{\frac{1}{2}x}$

$\Rightarrow \quad \ln(f(4)) = 2$

12. Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is ______ .

Ans. **(8)**

Sol. Let $P(\alpha, \beta, \gamma)$

$Q(0, 0, \gamma)$ &

$R(\alpha, \beta, -\gamma)$

Now, $\overline{PQ} \parallel \hat{i}+\hat{j} \Rightarrow \left(\alpha\hat{i}+\beta\hat{j}\right) \parallel \left(\hat{i}+\hat{j}\right)$

$\Rightarrow \quad \alpha = \beta$

Also, mid point of PQ lies on the plane $\Rightarrow \dfrac{\alpha}{2}+\dfrac{\beta}{2}=3 \Rightarrow \alpha + \beta = 6 \Rightarrow \alpha = 3$

Now, distance of point P from X-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow \quad \beta^2+\gamma^2=25 \ \Rightarrow \gamma^2=16$

as $\beta = \alpha = 3$

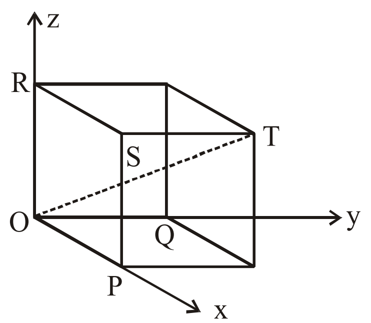
as $\gamma = 4$

Hence, $PR = 2\gamma = 8$

13. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{SP}$, $\vec{q}=\overrightarrow{SQ}$, $\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is _____ .

Ans. **(0.5)**

Sol.



$\vec{p}=\overrightarrow{SP}=\left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}-\hat{j}-\hat{k}\right)$

$\vec{q} = \overrightarrow{SQ} = \left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right) = \frac{1}{2}\left(-\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{r} = \overrightarrow{SR} = \left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}\left(-\hat{i} - \hat{j} + \hat{k}\right)$

$\vec{t} = \overrightarrow{ST} = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}\left(\hat{i} + \hat{j} + \hat{k}\right)$

$$\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right| = \frac{1}{4}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ -1 & 1 & -1 \end{vmatrix} \times \frac{1}{4}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{16}\left|\left(2\hat{i} + 2\hat{j}\right)\times\left(-2\hat{i} + 2\hat{j}\right)\right| = \left|\frac{\hat{k}}{2}\right| = \frac{1}{2}$$

14. Let $X = \left({}^{10}C_1\right)^2 + 2\left({}^{10}C_2\right)^2 + 3\left({}^{10}C_3\right)^2 + ... + 10\left({}^{10}C_{10}\right)^2$, where ${}^{10}C_r$, $r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is _____ .

Ans. (646)

Sol. $X = \sum_{r=0}^{n} r.\left({}^{n}C_r\right)^2; n = 10$

$X = n.\sum_{r=0}^{n} {}^{n}C_r.{}^{n-1}C_{r-1}$

$X = n.\sum_{r=1}^{n} {}^{n}C_{n-r}.{}^{n-1}C_{r-1}$

$X = n.{}^{2n-1}C_{n-1};\ n = 10$

$X = 10.{}^{19}C_9$

$\frac{X}{1430} = \frac{1}{143}.{}^{19}C_9$

$= 646$

SECTION 3

15. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$

and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}$.

$\left(\text{Here, the inverse trigonometric function } \sin^{-1}x \text{ assumes values in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].\right)$

Let $f : E_1 \to \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \to \mathbb{R}$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

LIST-I	LIST-II
P. The range of f is	**1.** $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q. The range of g contains	**2.** $(0, 1)$
R. The domain of f contains	**3.** $\left[-\frac{1}{2}, \frac{1}{2}\right]$
S. The domain of g is	**4.** $(-\infty, 0) \cup (0, \infty)$
	5. $\left(-\infty, \frac{e}{e-1}\right]$
	6. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) **P → 4; Q → 2; R → 1; S → 1**

(B) **P → 3; Q → 3; R → 6; S → 5**

(C) **P → 4; Q → 2; R → 1; S → 6**

(D) **P → 4; Q → 3; R → 6; S → 5**

Ans. (A)

Sol. $E_1 : \dfrac{x}{x-1} > 0$

+   –   +
0   1

$\Rightarrow \quad E_1 : x \in (-\infty, 0) \cup (1, \infty)$

$E_2 : -1 \le \ell n\left(\dfrac{x}{x+1}\right) \le 1$

$\dfrac{1}{e} \le \dfrac{x}{x-1} \le e$

Now $\dfrac{x}{x-1} - \dfrac{1}{e} \ge 0$

$\Rightarrow \quad \dfrac{(e-1)x+1}{e(x-1)} \ge 0$

+   –   +
–1/(e–1)   1

$\Rightarrow \quad x \in \left(-\infty, \dfrac{1}{1-e}\right] \cup (1, \infty)$

also $\dfrac{x}{x-1} - e \le 0$

$\dfrac{(e-1)x-e}{x-1} \ge 0$

+   –   +
1   e/(e–1)

$\Rightarrow \quad x \in (-\infty, 1) \cup \left[\dfrac{e}{e-1}, \infty\right]$

So $\quad E_2 : \left(-\infty, \dfrac{1}{1-e}\right) \cup \left[\dfrac{e}{e-1}, \infty\right]$

as Range of $\dfrac{x}{x-1}$ is $R^+ - \{1\}$

$\Rightarrow$ Range of f is $R - \{0\}$ or $(-\infty, 0) \cup (0, \infty)$

Range of g is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right] \setminus \{0\}$ or $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

Now $P \to 4$, $Q \to 2$, $R \to 1$, $S \to 1$

Hence A is correct

16. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 body and 2 girls.

(ii) Let α_2 be the total number of ways in which the committe can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committe can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the commitee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I	**LIST-II**
P. The value of α_1 is	**1.** 136
Q. The value of α_2 is	**2.** 189
R. The value of α_3 is	**3.** 192
S. The value of α_4 is	**4.** 200
	5. 381
	6. 461

The correct option is :-

(A) **$P \to 4$; $Q \to 6$, $R \to 2$; $S \to 1$**

(B) **$P \to 1$; $Q \to 4$; $R \to 2$; $S \to 3$**

(C) **$P \to 4$; $Q \to 6$, $R \to 5$; $S \to 2$**

(D) **$P \to 4$; $Q \to 2$; $R \to 3$; $S \to 1$**

Ans. (C)

Sol. (1) $\alpha_1 = \binom{6}{3}\binom{5}{2} = 200$

So $P \to 4$

(2) $\alpha_2 = \binom{6}{1}\binom{5}{1} + \binom{6}{2}\binom{5}{2} + \binom{6}{3}\binom{5}{3} + \binom{6}{4}\binom{5}{4} + \binom{6}{5}\binom{5}{5}$

$= \binom{11}{5} - 1$

= 46!

So Q → 6

(3) $\alpha_3 = \binom{5}{2}\binom{6}{3} + \binom{5}{3}\binom{6}{2} + \binom{5}{4}\binom{6}{1} + \binom{5}{5}\binom{6}{0}$

$= \binom{11}{5} - \binom{5}{0}\binom{6}{5} - \binom{5}{1}\binom{6}{4}$

= 381

So R → 5

(4) $\alpha_2 = \binom{5}{2}\binom{6}{2} - \binom{4}{1}\binom{5}{1} + \binom{5}{3}\binom{6}{1} - \binom{4}{2}\binom{1}{1} + \binom{5}{4} = 189$

So S → 2

17. Let H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a > b > 0, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

LIST-I	LIST-II
P. The length of the conjugate axis of H is	**1.** 8
Q. The eccentricity of H is	**2.** $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	**3.** $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	**4.** 4

The correct option is :

(A) **P → 4; Q → 2, R → 1; S → 3**

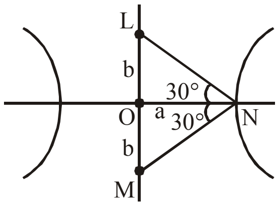
(B) **P → 4; Q → 3; R → 1; S → 2**

(C) **P → 4; Q → 1, R → 3; S → 2**

(D) **P → 3; Q → 4; R → 2; S → 1**

Ans. (B)

Sol.



$$\tan 30° = \frac{b}{a}$$

$\Rightarrow \quad a = b\sqrt{3}$

Now area of $\Delta LMN = \frac{1}{2}.2b.b\sqrt{3}$

$4\sqrt{3} = \sqrt{3}b^2$

$\Rightarrow \quad b = 2 \quad \& \quad a = 2\sqrt{3}$

$\Rightarrow \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{2}{\sqrt{3}}$

P. Length of conjugate axis = 2b = 4

So P $\rightarrow$ 4

Q. Eccentricity $e = \frac{2}{\sqrt{3}}$

So Q $\rightarrow$ 3

R. Distance between foci = 2ae

$= 2\left(2\sqrt{3}\right)\left(\frac{2}{\sqrt{3}}\right) = 8$

So R $\rightarrow$ 1

S. Length of latus rectum $= \frac{2b^2}{a} = \frac{2(2)^2}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

So S $\rightarrow$ 2

18. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \to \mathbb{R}$ and $f_4 : \mathbb{R} \to \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x + 2))]$, where for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

List-I	**List-II**
P. the function f_1 is	**1.** **NOT** continuous at $x = 0$
Q. The function f_2 is	**2.** continuous at $x = 0$ and **NOT** differentiable at $x = 0$
R. The function f_3 is	**3.** differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
S. The function f_4 is	**4.** differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is :

(A) **P → 2; Q → 3, R → 1; S → 4**

(B) **P → 4; Q → 1; R → 2; S → 3**

(C) **P → 4; Q → 2, R → 1; S → 3**

(D) **P → 2; Q → 1; R → 4; S → 3**

Ans. (D)

Sol. (i) $f(x) = \sin\sqrt{1-e^{-x^2}}$

$$f_1'(x) = \cos\sqrt{1-e^{-x^2}}\,.\frac{1}{2\sqrt{1-e^{-x^2}}}\left(0-e^{-x^2}.(-2x)\right)$$

at x = 0 $f_1'(x)$ does not exist

So. P $\rightarrow$ 2

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$\lim_{x\to 0^+} \frac{\sin x}{x}\frac{x}{\tan^{-1} x} = 1$$

$\Rightarrow$ $f_2(x)$ does not continuous at x = 0

So Q $\rightarrow$ 1

(iii) $f_3(x) = \left[\sin \ell n(x+2)\right] = 0$

$1 < x + 2 < e^{\pi/2}$

$\Rightarrow$ $0 < \ell n(x + 2) < \dfrac{\pi}{2}$

$\Rightarrow$ $0 < \sin(\ell n(x + 2) < 1$

$\Rightarrow$ $f_3(x) = 0$

So R $\rightarrow$ 4

(iv) $f_4(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

So S $\rightarrow$ 3