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## JEE Advanced 2016 Question Paper with Solution

Joint Entrance Examination – Advanced

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# PART I : PHYSICS

## SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -1 In all other cases.

- Q.1 In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength ( $\lambda$ ) of incident light and the corresponding stopping potential ( $V_0$ ) are given below :

$\lambda$ ( $\mu\text{m}$ )	$V_0$ (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that  $c = 3 \times 10^8 \text{ m s}^{-1}$  and  $e = 1.6 \times 10^{-19} \text{ C}$ , Planck's constant (in units of J s) found from such an experiment is

- (A)  $6.0 \times 10^{-34}$     (B)  $6.4 \times 10^{-34}$     (C)  $6.6 \times 10^{-34}$     (D)  $6.8 \times 10^{-34}$

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Space for rough work

### Answers for the above questions

Ans for Q.1: (B)

Q.2 A uniform wooden stick of mass 1.6 kg and length  $l$  rests in an inclined manner on a smooth, vertical wall of height  $h (< l)$  such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of  $30^\circ$  with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio  $h/l$  and the frictional force  $f$  at the bottom of the stick are

( $g = 10 \text{ m s}^{-2}$ )

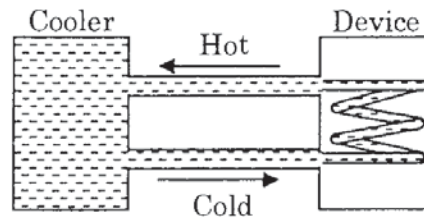
(A)  $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

(B)  $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

(C)  $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3} \text{ N}$

(D)  $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

Q.3 A water cooler of storage capacity 120 litres can cool water at a constant rate of  $P$  watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed  $30^\circ\text{C}$  and the entire stored 120 litres of water is initially cooled to  $10^\circ\text{C}$ . The entire system is thermally insulated. The minimum value of  $P$  (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is  $4.2 \text{ kJ kg}^{-1} \text{ K}^{-1}$  and the density of water is  $1000 \text{ kg m}^{-3}$ )

(A) 1600

(B) 2067

(C) 2533

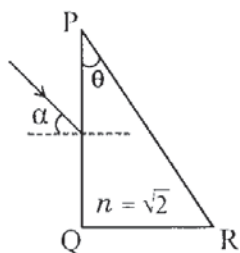
(D) 3933

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**Answers for the above questions**

<b>Ans for Q.2: (D)</b>	<b>Ans for Q.3: (B)</b>
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- Q.4 A parallel beam of light is incident from air at an angle  $\alpha$  on the side PQ of a right angled triangular prism of refractive index  $n = \sqrt{2}$ . Light undergoes total internal reflection in the prism at the face PR when  $\alpha$  has a minimum value of  $45^\circ$ . The angle  $\theta$  of the prism is



(A)  $15^\circ$

(B)  $22.5^\circ$

(C)  $30^\circ$

(D)  $45^\circ$

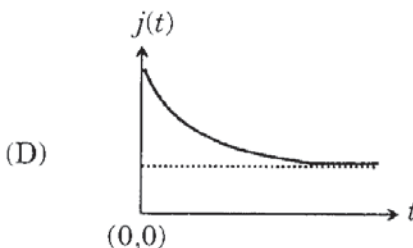
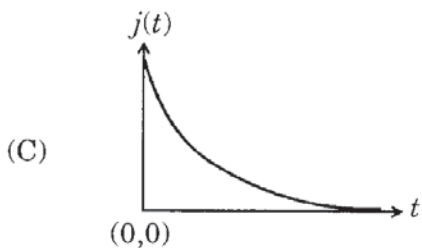
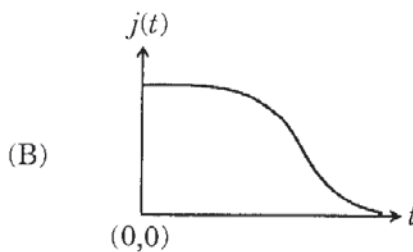
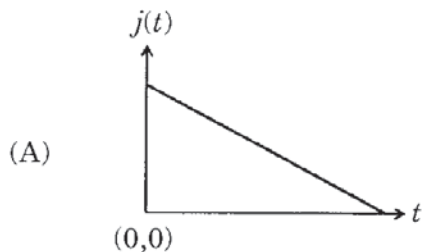
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**Answers for the above questions**

**Ans for Q.4: (A)**

Q.5 An infinite line charge of uniform electric charge density  $\lambda$  lies along the axis of an electrically conducting infinite cylindrical shell of radius  $R$ . At time  $t = 0$ , the space inside the cylinder is filled with a material of permittivity  $\epsilon$  and electrical conductivity  $\sigma$ . The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density  $j(t)$  at any point in the material?




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Space for rough work

**Answers for the above questions**

**Ans for Q.5: (C)**

**SECTION 2 (Maximum Marks: 32)**

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  - Partial Marks* : +1 For darkening a bubble corresponding **to each correct option**, provided **NO** incorrect option is darkened.
  - Zero Marks* : 0 If none of the bubbles is darkened.
  - Negative Marks* : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

- Q.6 Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge  $Ze$  are defined by their principal quantum number  $n$ , where  $n \gg 1$ . Which of the following statement(s) is(are) true?
- (A) Relative change in the radii of two consecutive orbitals does not depend on  $Z$
  - (B) Relative change in the radii of two consecutive orbitals varies as  $1/n$
  - (C) Relative change in the energy of two consecutive orbitals varies as  $1/n^3$
  - (D) Relative change in the angular momenta of two consecutive orbitals varies as  $1/n$

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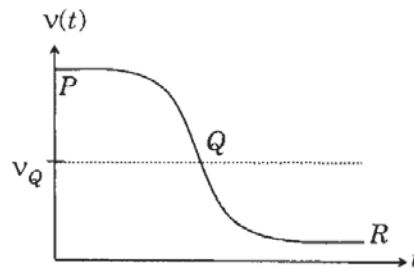
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**Answers for the above questions**

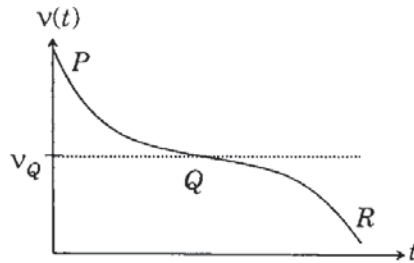
**Ans for Q.6: (A), (B), and (D)**

Q.7 Two loudspeakers  $M$  and  $N$  are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point  $P$ , 1800 m away from the midpoint  $Q$  of the line  $MN$  and moves towards  $Q$  constantly at 60 km/hr along the perpendicular bisector of  $MN$ . It crosses  $Q$  and eventually reaches a point  $R$ , 1800 m away from  $Q$ . Let  $v(t)$  represent the beat frequency measured by a person sitting in the car at time  $t$ . Let  $v_P$ ,  $v_Q$  and  $v_R$  be the beat frequencies measured at locations  $P$ ,  $Q$  and  $R$ , respectively. The speed of sound in air is  $330 \text{ m s}^{-1}$ . Which of the following statement(s) is(are) true regarding the sound heard by the person?

- (A)  $v_P + v_R = 2 v_Q$   
 (B) The rate of change in beat frequency is maximum when the car passes through  $Q$   
 (C) The plot below represents schematically the variation of beat frequency with time



- (D) The plot below represents schematically the variation of beat frequency with time




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Space for rough work

**Answers for the above questions**

**Ans for Q.7: (A), (B), and (C)**

- Q.8 An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?
- (A) The temperature distribution over the filament is uniform
  - (B) The resistance over small sections of the filament decreases with time
  - (C) The filament emits more light at higher band of frequencies before it breaks up
  - (D) The filament consumes less electrical power towards the end of the life of the bulb
- Q.9 A plano-convex lens is made of a material of refractive index  $n$ . When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?
- (A) The refractive index of the lens is 2.5
  - (B) The radius of curvature of the convex surface is 45 cm
  - (C) The faint image is erect and real
  - (D) The focal length of the lens is 20 cm

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.8: (C) and (D)</b>
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<b>Ans for Q.9: (A) and (D)</b>
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Q.10 A length-scale ( $l$ ) depends on the permittivity ( $\epsilon$ ) of a dielectric material, Boltzmann constant ( $k_B$ ), the absolute temperature ( $T$ ), the number per unit volume ( $n$ ) of certain charged particles, and the charge ( $q$ ) carried by each of the particles. Which of the following expression(s) for  $l$  is(are) dimensionally correct?

(A)  $l = \sqrt{\left(\frac{nq^2}{\epsilon k_B T}\right)}$

(B)  $l = \sqrt{\left(\frac{\epsilon k_B T}{nq^2}\right)}$

(C)  $l = \sqrt{\left(\frac{q^2}{\epsilon n^{2/3} k_B T}\right)}$

(D)  $l = \sqrt{\left(\frac{q^2}{\epsilon n^{1/3} k_B T}\right)}$

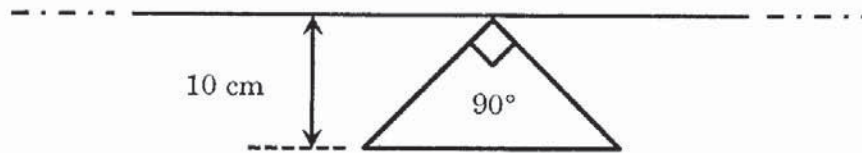
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**Answers for the above questions**

**Ans for Q.10: (B) and (D)**

- Q.11 A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the  $90^\circ$  vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of  $10 \text{ A s}^{-1}$ . Which of the following statement(s) is(are) true?



- (A) The magnitude of induced  $emf$  in the wire is  $\left(\frac{\mu_0}{\pi}\right)$  volt
- (B) If the loop is rotated at a constant angular speed about the wire, an additional  $emf$  of  $\left(\frac{\mu_0}{\pi}\right)$  volt is induced in the wire
- (C) The induced current in the wire is in opposite direction to the current along the hypotenuse
- (D) There is a repulsive force between the wire and the loop

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Space for rough work

**Answers for the above questions**

**Ans for Q.11: (A) and (D)**

Q.12 The position vector  $\vec{r}$  of a particle of mass  $m$  is given by the following equation

$$\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j},$$

where  $\alpha = 10/3 \text{ m s}^{-3}$ ,  $\beta = 5 \text{ m s}^{-2}$  and  $m = 0.1 \text{ kg}$ . At  $t = 1 \text{ s}$ , which of the following statement(s) is(are) true about the particle?

- (A) The velocity  $\vec{v}$  is given by  $\vec{v} = (10\hat{i} + 10\hat{j}) \text{ ms}^{-1}$
- (B) The angular momentum  $\vec{L}$  with respect to the origin is given by  $\vec{L} = -(5/3)\hat{k} \text{ N m s}$
- (C) The force  $\vec{F}$  is given by  $\vec{F} = (\hat{i} + 2\hat{j}) \text{ N}$
- (D) The torque  $\vec{\tau}$  with respect to the origin is given by  $\vec{\tau} = -(20/3)\hat{k} \text{ N m}$

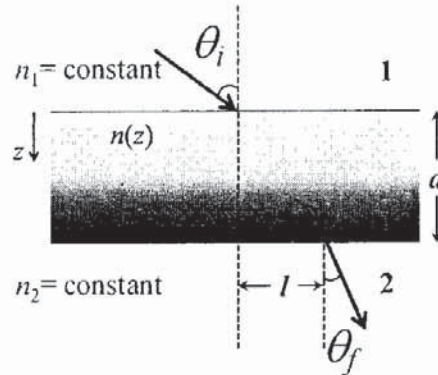
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**Answers for the above questions**

**Ans for Q.12: (A), (B) and (D)**

Q.13 A transparent slab of thickness  $d$  has a refractive index  $n(z)$  that increases with  $z$ . Here  $z$  is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices  $n_1$  and  $n_2 (> n_1)$ , as shown in the figure. A ray of light is incident with angle  $\theta_i$  from medium 1 and emerges in medium 2 with refraction angle  $\theta_f$  with a lateral displacement  $l$ .



Which of the following statement(s) is(are) true?

- (A)  $n_1 \sin \theta_i = n_2 \sin \theta_f$  (B)  $n_1 \sin \theta_i = (n_2 - n_1) \sin \theta_f$   
 (C)  $l$  is independent of  $n_2$  (D)  $l$  is dependent on  $n(z)$

Space for rough work

Answers for the above questions

Ans for Q.13: (A), (C) and (D)

**SECTION 3 (Maximum Marks: 15)**

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct answer is darkened.  
*Zero Marks* : 0 In all other cases.

- Q.14 A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated ( $P$ ) by the metal. The sensor has a scale that displays  $\log_2(P/P_0)$ , where  $P_0$  is a constant. When the metal surface is at a temperature of  $487^\circ\text{C}$ , the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to  $2767^\circ\text{C}$ ?
- Q.15 The isotope  ${}^{12}_5\text{B}$  having a mass 12.014 u undergoes  $\beta$ -decay to  ${}^{12}_6\text{C}$ .  ${}^{12}_6\text{C}$  has an excited state of the nucleus ( ${}^{12}_6\text{C}^*$ ) at 4.041 MeV above its ground state. If  ${}^{12}_5\text{B}$  decays to  ${}^{12}_6\text{C}^*$ , the maximum kinetic energy of the  $\beta$ -particle in units of MeV is  
(1 u = 931.5 MeV/ $c^2$ , where  $c$  is the speed of light in vacuum).

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.14: (9)</b>	<b>Ans for Q.15: (8) OR (9)</b>
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- Q.16 A hydrogen atom in its ground state is irradiated by light of wavelength  $970 \text{ \AA}$ . Taking  $hc/e = 1.237 \times 10^{-6} \text{ eV m}$  and the ground state energy of hydrogen atom as  $-13.6 \text{ eV}$ , the number of lines present in the emission spectrum is
- Q.17 Consider two solid spheres P and Q each of density  $8 \text{ gm cm}^{-3}$  and diameters  $1 \text{ cm}$  and  $0.5 \text{ cm}$ , respectively. Sphere P is dropped into a liquid of density  $0.8 \text{ gm cm}^{-3}$  and viscosity  $\eta = 3$  poiseulles. Sphere Q is dropped into a liquid of density  $1.6 \text{ gm cm}^{-3}$  and viscosity  $\eta = 2$  poiseulles. The ratio of the terminal velocities of P and Q is
- Q.18 Two inductors  $L_1$  (inductance  $1 \text{ mH}$ , internal resistance  $3 \text{ } \Omega$ ) and  $L_2$  (inductance  $2 \text{ mH}$ , internal resistance  $4 \text{ } \Omega$ ), and a resistor  $R$  (resistance  $12 \text{ } \Omega$ ) are all connected in parallel across a  $5 \text{ V}$  battery. The circuit is switched on at time  $t = 0$ . The ratio of the maximum to the minimum current ( $I_{\max} / I_{\min}$ ) drawn from the battery is

**END OF PART I : PHYSICS**

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**Answers for the above questions**

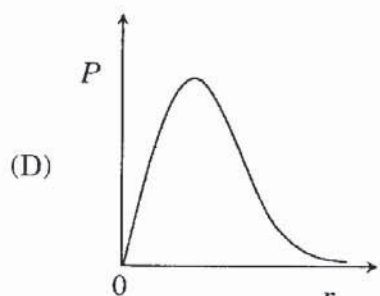
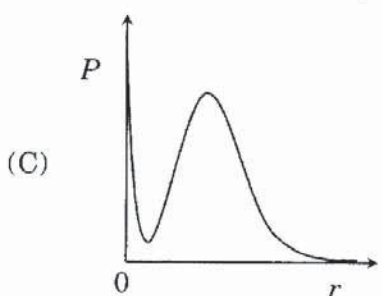
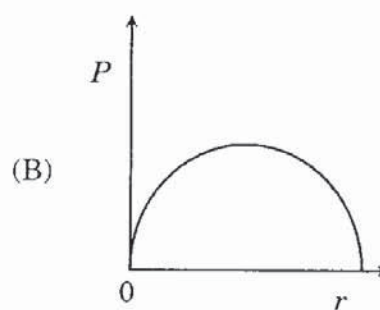
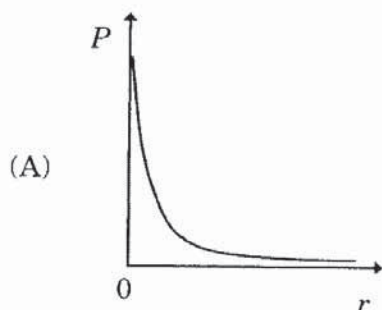
<b>Ans for Q.16: (6)</b>	<b>Ans for Q.17: (3)</b>	<b>Ans for Q.18: (8)</b>
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## PART II : CHEMISTRY

### SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -1 In all other cases.

Q.19  $P$  is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness,  $dr$ , at a distance  $r$  from the nucleus. The volume of this shell is  $4\pi r^2 dr$ . The qualitative sketch of the dependence of  $P$  on  $r$  is



Space for rough work

Answers for the above questions

Ans for Q.19: (D)

- Q.20 One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings ( $\Delta S_{surr}$ ) in  $\text{J K}^{-1}$  is  
(1 L atm = 101.3 J)
- (A) 5.763                      (B) 1.013                      (C) -1.013                      (D) -5.763
- Q.21 The increasing order of atomic radii of the following Group 13 elements is
- (A) Al < Ga < In < Tl                      (B) Ga < Al < In < Tl  
(C) Al < In < Ga < Tl                      (D) Al < Ga < Tl < In
- Q.22 Among  $[\text{Ni}(\text{CO})_4]$ ,  $[\text{NiCl}_4]^{2-}$ ,  $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}$ ,  $\text{Na}_3[\text{CoF}_6]$ ,  $\text{Na}_2\text{O}_2$  and  $\text{CsO}_2$ , the total number of paramagnetic compounds is
- (A) 2                      (B) 3                      (C) 4                      (D) 5
- Q.23 On complete hydrogenation, natural rubber produces
- (A) ethylene-propylene copolymer                      (B) vulcanised rubber  
(C) polypropylene                      (D) polybutylene

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Space for rough work

**Answers for the above questions**

Ans for Q.20: (C)	Ans for Q.21: (B)	Ans for Q.22: (B)	Ans for Q.23: (A)
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**SECTION 2 (Maximum Marks: 32)**

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  - Partial Marks* : +1 For darkening a bubble corresponding to **each correct option**, provided **NO** incorrect option is darkened.
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  - Negative Marks* : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

Q.24 According to the Arrhenius equation,

- (A) a high activation energy usually implies a fast reaction.
- (B) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.
- (C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.
- (D) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

Q.25 A plot of the number of neutrons ( $N$ ) against the number of protons ( $P$ ) of stable nuclei exhibits upward deviation from linearity for atomic number,  $Z > 20$ . For an unstable nucleus having  $N/P$  ratio less than 1, the possible mode(s) of decay is(are)

- (A)  $\beta^-$ -decay ( $\beta$  emission)
- (B) orbital or  $K$ -electron capture
- (C) neutron emission
- (D)  $\beta^+$ -decay (positron emission)

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Space for rough work

**Answers for the above questions**

**Ans for Q.24: (B), (C) and (D)**

**Ans for Q.25: (B) and (D)**

- Q.26 The crystalline form of borax has  
 (A) tetranuclear  $[B_4O_5(OH)_4]^{2-}$  unit  
 (B) all boron atoms in the same plane  
 (C) equal number of  $sp^2$  and  $sp^3$  hybridized boron atoms  
 (D) one terminal hydroxide per boron atom
- Q.27 The compound(s) with TWO lone pairs of electrons on the central atom is(are)  
 (A)  $BrF_5$             (B)  $ClF_3$             (C)  $XeF_4$             (D)  $SF_4$
- Q.28 The reagent(s) that can selectively precipitate  $S^{2-}$  from a mixture of  $S^{2-}$  and  $SO_4^{2-}$  in aqueous solution is(are)  
 (A)  $CuCl_2$             (B)  $BaCl_2$             (C)  $Pb(OOCCH_3)_2$     (D)  $Na_2[Fe(CN)_5NO]$
- Q.29 Positive Tollen's test is observed for
- (A) (B)
- (C) (D)

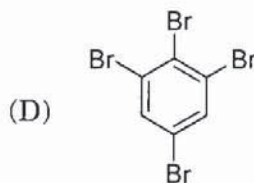
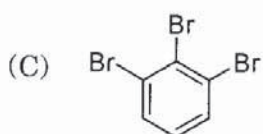
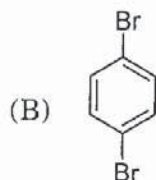
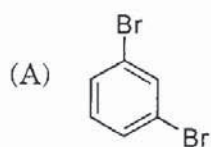
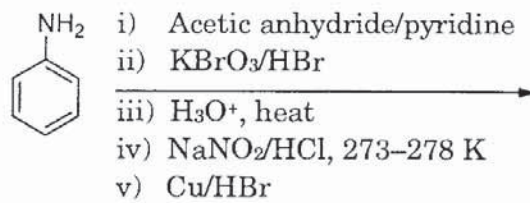
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**Answers for the above questions**

<b>Ans for Q.26: (A), (C) and (D)</b>	<b>Ans for Q.27: (B) and (C)</b>
<b>Ans for Q.28: (A) OR (A) and (C)</b>	<b>Ans for Q.29: (A), (B) and (C)</b>

Q.30 The product(s) of the following reaction sequence is(are)



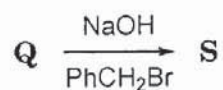
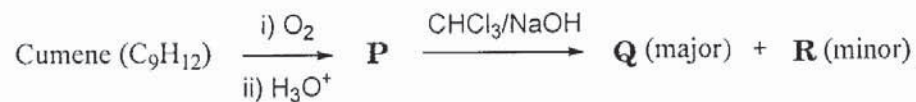
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Space for rough work

**Answers for the above questions**

**Ans for Q.30: (B)**

Q.31 The correct statement(s) about the following reaction sequence is(are)



- (A) **R** is steam volatile
- (B) **Q** gives dark violet coloration with 1% aqueous  $\text{FeCl}_3$  solution
- (C) **S** gives yellow precipitate with 2, 4-dinitrophenylhydrazine
- (D) **S** gives dark violet coloration with 1% aqueous  $\text{FeCl}_3$  solution

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Space for rough work

### Answers for the above questions

Ans for Q.31: (B) and (C)
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**SECTION 3 (Maximum Marks: 15)**

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct answer is darkened.  
*Zero Marks* : 0 In all other cases.

- Q.32 The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is  $2.0 \text{ g cm}^{-3}$ . The ratio of the molecular weights of the solute and solvent,  $\left(\frac{MW_{\text{solute}}}{MW_{\text{solvent}}}\right)$ , is
- Q.33 The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases  $x$  times. The value of  $x$  is

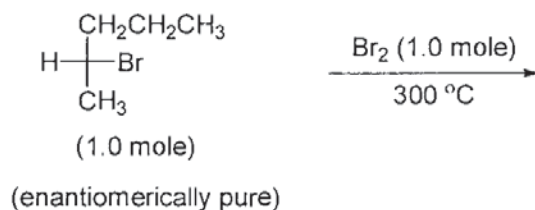
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Space for rough work

**Answers for the above questions**

<b>Ans for Q.32: (9)</b>	<b>Ans for Q.33: (4)</b>
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- Q.34 In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce **X** moles of a sulphur containing product. The magnitude of **X** is
- Q.35 The number of geometric isomers possible for the complex  $[\text{CoL}_2\text{Cl}_2]^-$  ( $\text{L} = \text{H}_2\text{NCH}_2\text{CH}_2\text{O}^-$ ) is
- Q.36 In the following monobromination reaction, the number of possible chiral products is



**END OF PART II : CHEMISTRY**

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.34: (6)</b>	<b>Ans for Q.35: (5)</b>	<b>Ans for Q.36: (5)</b>
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## PART III : MATHEMATICS

### SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -1 In all other cases.

- Q.37 Let  $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$ . Suppose  $\alpha_1$  and  $\beta_1$  are the roots of the equation  $x^2 - 2x \sec \theta + 1 = 0$  and  $\alpha_2$  and  $\beta_2$  are the roots of the equation  $x^2 + 2x \tan \theta - 1 = 0$ . If  $\alpha_1 > \beta_1$  and  $\alpha_2 > \beta_2$ , then  $\alpha_1 + \beta_2$  equals
- (A)  $2(\sec \theta - \tan \theta)$  (B)  $2 \sec \theta$   
(C)  $-2 \tan \theta$  (D) 0
- Q.38 A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is
- (A) 380 (B) 320 (C) 260 (D) 95
- Q.39 Let  $S = \left\{ x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2} \right\}$ . The sum of all distinct solutions of the equation  $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$  in the set  $S$  is equal to
- (A)  $-\frac{7\pi}{9}$  (B)  $-\frac{2\pi}{9}$  (C) 0 (D)  $\frac{5\pi}{9}$

Space for rough work

### Answers for the above questions

Ans for Q.37: (C)

Ans for Q.38: (A)

Ans for Q.39: (C)

Q.40 A computer producing factory has only two plants  $T_1$  and  $T_2$ . Plant  $T_1$  produces 20% and plant  $T_2$  produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

$P$  (computer turns out to be defective given that it is produced in plant  $T_1$ )

$= 10P$  (computer turns out to be defective given that it is produced in plant  $T_2$ ),

where  $P(E)$  denotes the probability of an event  $E$ . A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant  $T_2$  is

- (A)  $\frac{36}{73}$                       (B)  $\frac{47}{79}$                       (C)  $\frac{78}{93}$                       (D)  $\frac{75}{83}$

Q.41 The least value of  $\alpha \in \mathbb{R}$  for which  $4\alpha x^2 + \frac{1}{x} \geq 1$ , for all  $x > 0$ , is

- (A)  $\frac{1}{64}$                       (B)  $\frac{1}{32}$                       (C)  $\frac{1}{27}$                       (D)  $\frac{1}{25}$

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.40: (C)</b>	<b>Ans for Q.41: (C)</b>
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SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  - Partial Marks* : +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened.
  - Zero Marks* : 0 If none of the bubbles is darkened.
  - Negative Marks* : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

Q.42 Consider a pyramid  $OPQRS$  located in the first octant ( $x \geq 0, y \geq 0, z \geq 0$ ) with  $O$  as origin, and  $OP$  and  $OR$  along the  $x$ -axis and the  $y$ -axis, respectively. The base  $OPQR$  of the pyramid is a square with  $OP = 3$ . The point  $S$  is directly above the mid-point  $T$  of diagonal  $OQ$  such that  $TS = 3$ . Then

- (A) the acute angle between  $OQ$  and  $OS$  is  $\frac{\pi}{3}$
- (B) the equation of the plane containing the triangle  $OQS$  is  $x - y = 0$
- (C) the length of the perpendicular from  $P$  to the plane containing the triangle  $OQS$  is  $\frac{3}{\sqrt{2}}$
- (D) the perpendicular distance from  $O$  to the straight line containing  $RS$  is  $\sqrt{\frac{15}{2}}$

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Space for rough work

**Answers for the above questions**

**Ans for Q.42: (B), (C) and (D)**

Q.43 Let  $f: (0, \infty) \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(x) = 2 - \frac{f(x)}{x}$  for all  $x \in (0, \infty)$  and  $f(1) \neq 1$ . Then

(A)  $\lim_{x \rightarrow 0^+} f\left(\frac{1}{x}\right) = 1$

(B)  $\lim_{x \rightarrow 0^+} x f\left(\frac{1}{x}\right) = 2$

(C)  $\lim_{x \rightarrow 0^+} x^2 f'(x) = 0$

(D)  $|f(x)| \leq 2$  for all  $x \in (0, 2)$

Q.44 Let  $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$ , where  $\alpha \in \mathbb{R}$ . Suppose  $Q = [q_{ij}]$  is a matrix such that  $PQ = kI$ , where

$k \in \mathbb{R}$ ,  $k \neq 0$  and  $I$  is the identity matrix of order 3. If  $q_{23} = -\frac{k}{8}$  and  $\det(Q) = \frac{k^2}{2}$ , then

(A)  $\alpha = 0, k = 8$

(B)  $4\alpha - k + 8 = 0$

(C)  $\det(P \operatorname{adj}(Q)) = 2^9$

(D)  $\det(Q \operatorname{adj}(P)) = 2^{13}$

Space for rough work

### Answers for the above questions

**Ans for Q.43: (A)**

**Ans for Q.44: (B) and (C)**

Q.45 In a triangle  $XYZ$ , let  $x, y, z$  be the lengths of sides opposite to the angles  $X, Y, Z$ , respectively, and  $2s = x + y + z$ . If  $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$  and area of incircle of the triangle  $XYZ$  is  $\frac{8\pi}{3}$ , then

- (A) area of the triangle  $XYZ$  is  $6\sqrt{6}$
- (B) the radius of circumcircle of the triangle  $XYZ$  is  $\frac{35}{6}\sqrt{6}$
- (C)  $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$
- (D)  $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

Q.46 A solution curve of the differential equation  $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$ ,  $x > 0$ , passes through the point  $(1, 3)$ . Then the solution curve

- (A) intersects  $y = x + 2$  exactly at one point
- (B) intersects  $y = x + 2$  exactly at two points
- (C) intersects  $y = (x + 2)^2$
- (D) does **NOT** intersect  $y = (x + 3)^2$

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Space for rough work

### Answers for the above questions

<b>Ans for Q.45: (A), (C) and (D)</b>	<b>Ans for Q.46: (A) and (D)</b>
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Q.47 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$  and  $h: \mathbb{R} \rightarrow \mathbb{R}$  be differentiable functions such that  $f(x) = x^3 + 3x + 2$ ,  $g(f(x)) = x$  and  $h(g(g(x))) = x$  for all  $x \in \mathbb{R}$ . Then

- (A)  $g'(2) = \frac{1}{15}$  (B)  $h'(1) = 666$   
 (C)  $h(0) = 16$  (D)  $h(g(3)) = 36$

Q.48 The circle  $C_1: x^2 + y^2 = 3$ , with centre at  $O$ , intersects the parabola  $x^2 = 2y$  at the point  $P$  in the first quadrant. Let the tangent to the circle  $C_1$  at  $P$  touches other two circles  $C_2$  and  $C_3$  at  $R_2$  and  $R_3$ , respectively. Suppose  $C_2$  and  $C_3$  have equal radii  $2\sqrt{3}$  and centres  $Q_2$  and  $Q_3$ , respectively. If  $Q_2$  and  $Q_3$  lie on the  $y$ -axis, then

- (A)  $Q_2Q_3 = 12$   
 (B)  $R_2R_3 = 4\sqrt{6}$   
 (C) area of the triangle  $OR_2R_3$  is  $6\sqrt{2}$   
 (D) area of the triangle  $PQ_2Q_3$  is  $4\sqrt{2}$

Q.49 Let  $RS$  be the diameter of the circle  $x^2 + y^2 = 1$ , where  $S$  is the point  $(1, 0)$ . Let  $P$  be a variable point (other than  $R$  and  $S$ ) on the circle and tangents to the circle at  $S$  and  $P$  meet at the point  $Q$ . The normal to the circle at  $P$  intersects a line drawn through  $Q$  parallel to  $RS$  at point  $E$ . Then the locus of  $E$  passes through the point(s)

- (A)  $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$  (B)  $\left(\frac{1}{4}, \frac{1}{2}\right)$  (C)  $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$  (D)  $\left(\frac{1}{4}, -\frac{1}{2}\right)$

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Space for rough work

**Answers for the above questions**

Ans for Q.47: (B) and (C)	Ans for Q.48: (A), (B) and (C)	Ans for Q.49: (A) and (C)
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**SECTION 3 (Maximum Marks: 15)**

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct answer is darkened.  
*Zero Marks* : 0 In all other cases.

Q.50 The total number of distinct  $x \in \mathbb{R}$  for which  $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$  is

Q.51 Let  $m$  be the smallest positive integer such that the coefficient of  $x^2$  in the expansion of  $(1+x)^2 + (1+x)^3 + \dots + (1+x)^{49} + (1+mx)^{50}$  is  $(3n+1) {}^{51}C_3$  for some positive integer  $n$ . Then the value of  $n$  is

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.50: (2)</b>	<b>Ans for Q.51: (5)</b>
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Q.52 The total number of distinct  $x \in [0, 1]$  for which  $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$  is

Q.53 Let  $\alpha, \beta \in \mathbb{R}$  be such that  $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$ . Then  $6(\alpha + \beta)$  equals

Q.54 Let  $z = \frac{-1 + \sqrt{3}i}{2}$ , where  $i = \sqrt{-1}$ , and  $r, s \in \{1, 2, 3\}$ . Let  $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$  and  $I$  be the identity matrix of order 2. Then the total number of ordered pairs  $(r, s)$  for which  $P^2 = -I$  is

**END OF THE QUESTION PAPER**

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.52: (1)</b>	<b>Ans for Q.53: (7)</b>	<b>Ans for Q.54: (1)</b>
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# PART I : PHYSICS

## SECTION 1 (Maximum Marks: 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -1 In all other cases.

- Q.1 The electrostatic energy of  $Z$  protons uniformly distributed throughout a spherical nucleus of radius  $R$  is given by

$$E = \frac{3 Z(Z-1)e^2}{5 4\pi\epsilon_0 R}$$

The measured masses of the neutron,  ${}^1_1\text{H}$ ,  ${}^{15}_7\text{N}$  and  ${}^{15}_8\text{O}$  are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the  ${}^{15}_7\text{N}$  and  ${}^{15}_8\text{O}$  nuclei are same,  $1 \text{ u} = 931.5 \text{ MeV}/c^2$  ( $c$  is the speed of light) and  $e^2/(4\pi\epsilon_0) = 1.44 \text{ MeV fm}$ . Assuming that the difference between the binding energies of  ${}^{15}_7\text{N}$  and  ${}^{15}_8\text{O}$  is purely due to the electrostatic energy, the radius of either of the nuclei is

(1 fm =  $10^{-15}$  m)

- (A) 2.85 fm      (B) 3.03 fm      (C) 3.42 fm      (D) 3.80 fm
- Q.2 An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?
- (A) 64      (B) 90      (C) 108      (D) 120

Space for rough work

### Answers for the above questions

Ans for Q.1: (C)

Ans for Q.2: (C)

- Q.3 A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure  $P_i = 10^5 \text{ Pa}$  and volume  $V_i = 10^{-3} \text{ m}^3$  changes to a final state at  $P_f = (1/32) \times 10^5 \text{ Pa}$  and  $V_f = 8 \times 10^{-3} \text{ m}^3$  in an adiabatic quasi-static process, such that  $P^3V^5 = \text{constant}$ . Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at  $P_i$  followed by an isochoric (isovolumetric) process at volume  $V_f$ . The amount of heat supplied to the system in the two-step process is approximately
- (A) 112 J                      (B) 294 J                      (C) 588 J                      (D) 813 J
- Q.4 The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at  $10^\circ\text{C}$ . Now the end P is maintained at  $10^\circ\text{C}$ , while the end S is heated and maintained at  $400^\circ\text{C}$ . The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is  $1.2 \times 10^{-5} \text{ K}^{-1}$ , the change in length of the wire PQ is
- (A) 0.78 mm                      (B) 0.90 mm                      (C) 1.56 mm                      (D) 2.34 mm

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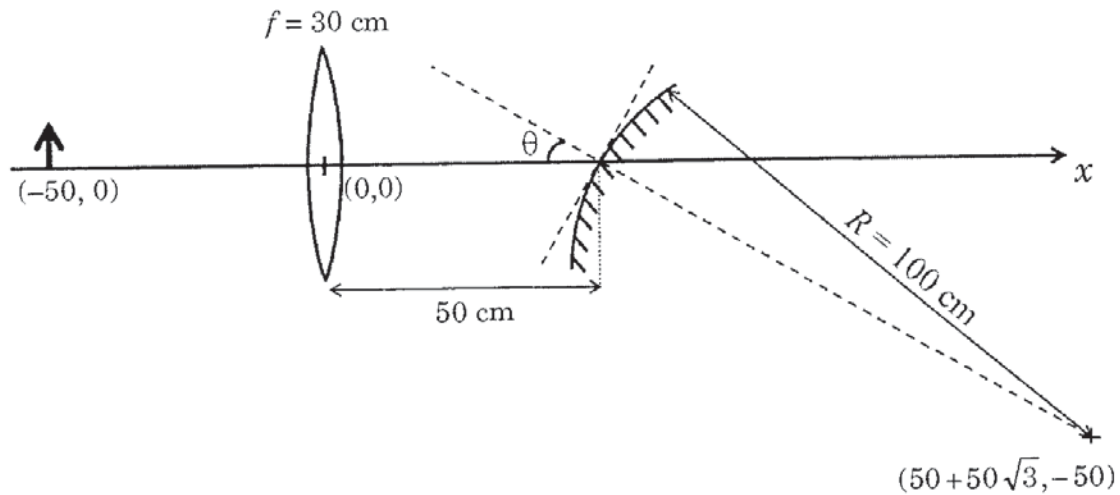
Space for rough work

**Answers for the above questions**

<b>Ans for Q.3: (C)</b>	<b>Ans for Q.4: (A)</b>
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- Q.5 A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle  $\theta = 30^\circ$  to the axis of the lens, as shown in the figure.



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point  $(x, y)$  at which the image is formed are

- (A)  $(0, 0)$       (B)  $(50 - 25\sqrt{3}, 25)$       (C)  $(25, 25\sqrt{3})$       (D)  $(125/3, 25/\sqrt{3})$

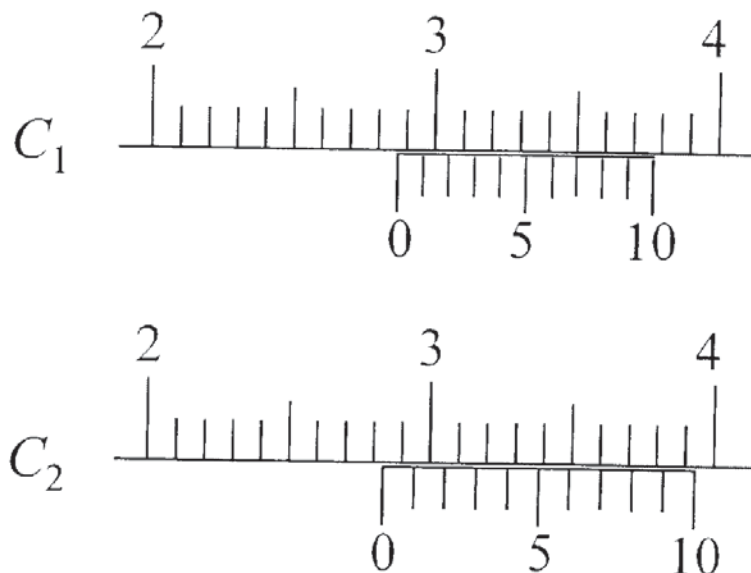
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Space for rough work

**Answers for the above questions**

**Ans for Q.5: (C)**

Q.6 There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers ( $C_1$ ) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper ( $C_2$ ) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers  $C_1$  and  $C_2$ , respectively, are



- (A) 2.85 and 2.82 (B) 2.87 and 2.83 (C) 2.87 and 2.86 (D) 2.87 and 2.87

Space for rough work

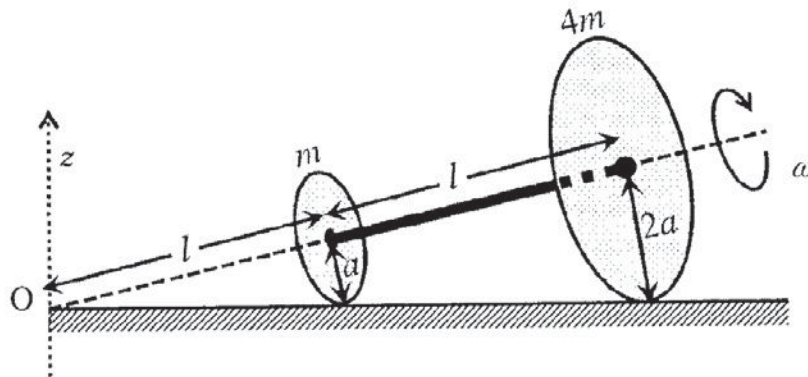
**Answers for the above questions**

**Ans for Q.6: (B)**

**SECTION 2 (Maximum Marks: 32)**

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  - Partial Marks* : +1 For darkening a bubble corresponding to **each correct option**, provided **NO** incorrect option is darkened.
  - Zero Marks* : 0 If none of the bubbles is darkened.
  - Negative Marks* : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

Q.7 Two thin circular discs of mass  $m$  and  $4m$ , having radii of  $a$  and  $2a$ , respectively, are rigidly fixed by a massless, rigid rod of length  $l = \sqrt{24}a$  through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is  $\omega$ . The angular momentum of the entire assembly about the point 'O' is  $\vec{L}$  (see the figure). Which of the following statement(s) is(are) true?



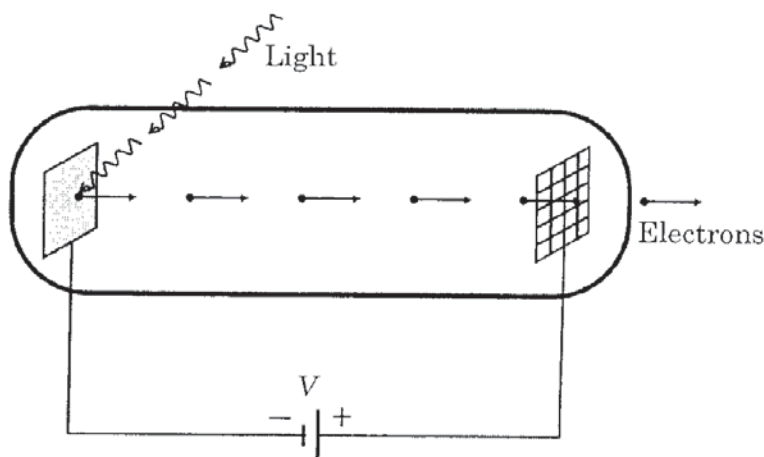
- (A) The center of mass of the assembly rotates about the  $z$ -axis with an angular speed of  $\omega/5$
- (B) The magnitude of angular momentum of center of mass of the assembly about the point O is  $81ma^2\omega$
- (C) The magnitude of angular momentum of the assembly about its center of mass is  $17ma^2\omega/2$
- (D) The magnitude of the  $z$ -component of  $\vec{L}$  is  $55ma^2\omega$

Space for rough work

**Answers for the above questions**

**Ans for Q.7: (A) OR (A) and (C)**

- Q.8 Light of wavelength  $\lambda_{ph}$  falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is  $\phi$  and the anode is a wire mesh of conducting material kept at a distance  $d$  from the cathode. A potential difference  $V$  is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is  $\lambda_e$ , which of the following statement(s) is(are) true?



- (A)  $\lambda_e$  decreases with increase in  $\phi$  and  $\lambda_{ph}$   
(B)  $\lambda_e$  is approximately halved, if  $d$  is doubled  
(C) For large potential difference ( $V \gg \phi/e$ ),  $\lambda_e$  is approximately halved if  $V$  is made four times  
(D)  $\lambda_e$  increases at the same rate as  $\lambda_{ph}$  for  $\lambda_{ph} < hc/\phi$

Space for rough work

### Answers for the above questions

Ans for Q.8: (C)

- Q.9 In an experiment to determine the acceleration due to gravity  $g$ , the formula used for the time period of a periodic motion is  $T = 2\pi \sqrt{\frac{7(R-r)}{5g}}$ . The values of  $R$  and  $r$  are measured to be  $(60 \pm 1)$  mm and  $(10 \pm 1)$  mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true?
- (A) The error in the measurement of  $r$  is 10%
- (B) The error in the measurement of  $T$  is 3.57%
- (C) The error in the measurement of  $T$  is 2%
- (D) The error in the determined value of  $g$  is 11%
- Q.10 Consider two identical galvanometers and two identical resistors with resistance  $R$ . If the internal resistance of the galvanometers  $R_G < R/2$ , which of the following statement(s) about any one of the galvanometers is(are) true?
- (A) The maximum voltage range is obtained when all the components are connected in series
- (B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer
- (C) The maximum current range is obtained when all the components are connected in parallel
- (D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

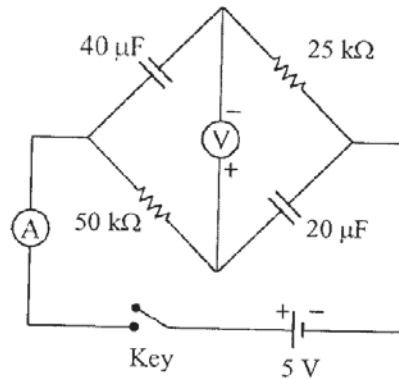
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### Answers for the above questions

<b>Ans for Q.9: (A), (B) and (D)</b>	<b>Ans for Q.10: (B) and (C)</b>
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- Q.11 In the circuit shown below, the key is pressed at time  $t = 0$ . Which of the following statement(s) is(are) true?



- (A) The voltmeter displays  $-5$  V as soon as the key is pressed, and displays  $+5$  V after a long time
- (B) The voltmeter will display  $0$  V at time  $t = \ln 2$  seconds
- (C) The current in the ammeter becomes  $1/e$  of the initial value after 1 second
- (D) The current in the ammeter becomes zero after a long time
- Q.12 A block with mass  $M$  is connected by a massless spring with stiffness constant  $k$  to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude  $A$  about an equilibrium position  $x_0$ . Consider two cases: (i) when the block is at  $x_0$ ; and (ii) when the block is at  $x = x_0 + A$ . In both the cases, a particle with mass  $m (< M)$  is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass  $m$  is placed on the mass  $M$ ?
- (A) The amplitude of oscillation in the first case changes by a factor of  $\sqrt{\frac{M}{m+M}}$ , whereas in the second case it remains unchanged
- (B) The final time period of oscillation in both the cases is same
- (C) The total energy decreases in both the cases
- (D) The instantaneous speed at  $x_0$  of the combined masses decreases in both the cases

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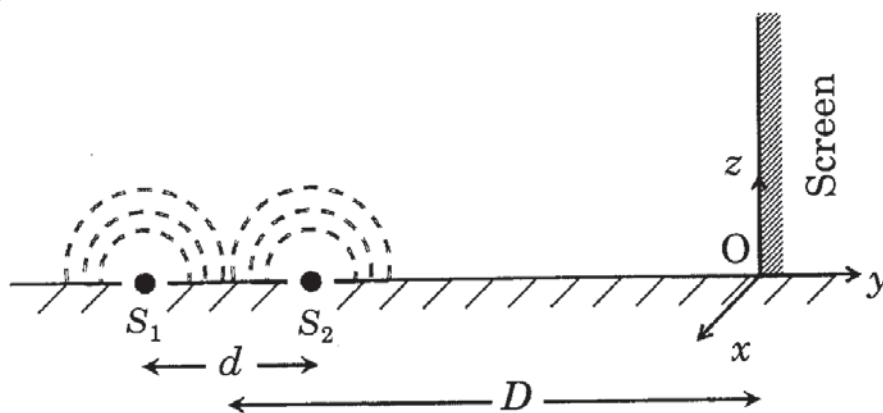
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### Answers for the above questions

**Ans for Q.11: (A), (B), (C) and (D)**

**Ans for Q.12: (A), (B) and (D)**

- Q.13 While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the  $x$ - $y$  plane containing two small holes that act as two coherent point sources ( $S_1, S_2$ ) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the  $x$ - $z$  plane (for  $z > 0$ ) at a distance  $D = 3$  m from the mid-point of  $S_1S_2$ , as shown schematically in the figure. The distance between the sources  $d = 0.6003$  mm. The origin  $O$  is at the intersection of the screen and the line joining  $S_1S_2$ . Which of the following is(are) true of the intensity pattern on the screen?



- (A) Straight bright and dark bands parallel to the  $x$ -axis  
 (B) The region very close to the point  $O$  will be dark  
 (C) Hyperbolic bright and dark bands with foci symmetrically placed about  $O$  in the  $x$ -direction  
 (D) Semi circular bright and dark bands centered at point  $O$

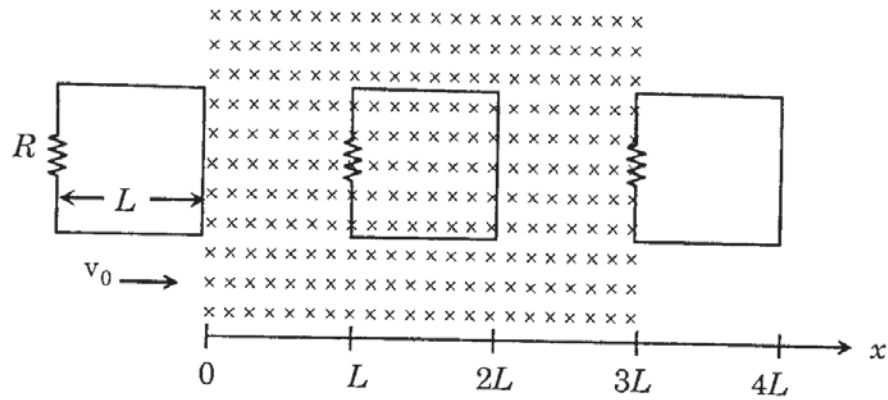
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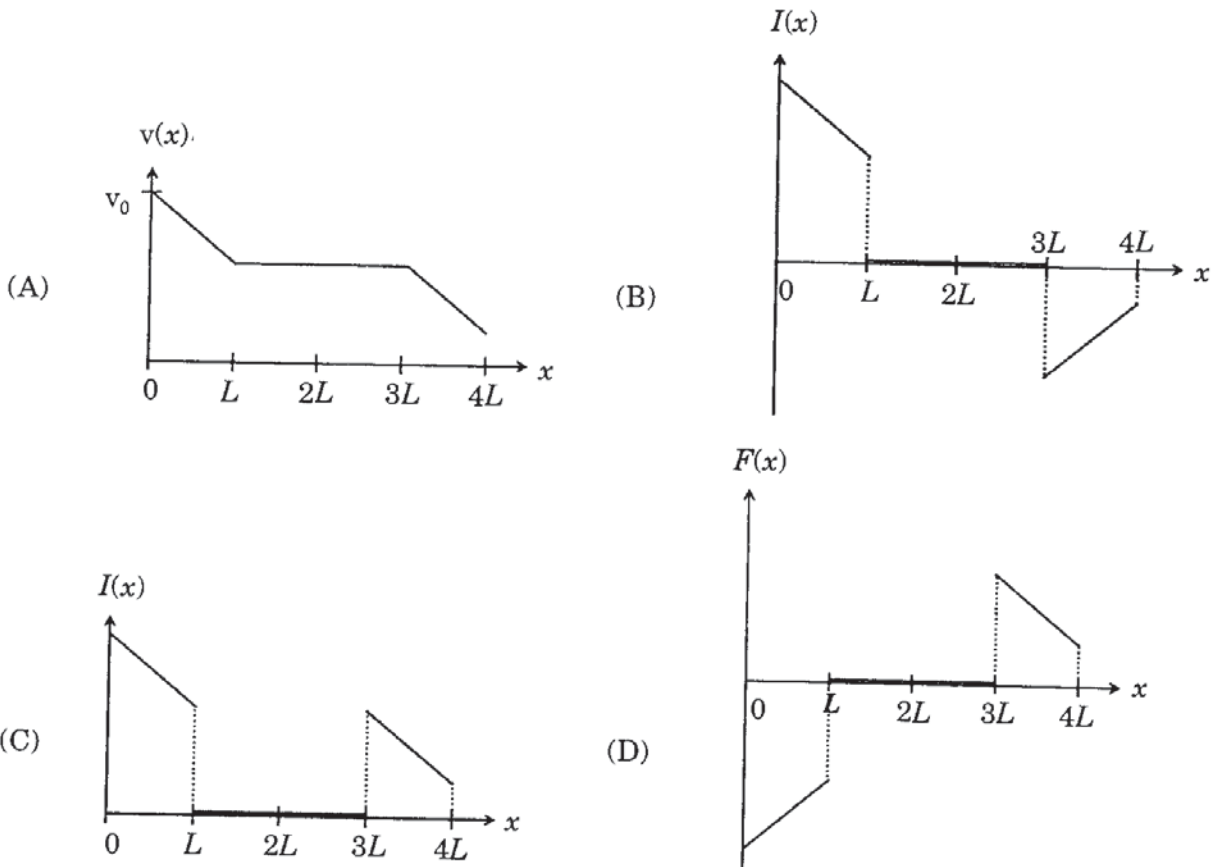
**Answers for the above questions**

**Ans for Q.13: (B) and (D)**

Q.14 A rigid wire loop of square shape having side of length  $L$  and resistance  $R$  is moving along the  $x$ -axis with a constant velocity  $v_0$  in the plane of the paper. At  $t = 0$ , the right edge of the loop enters a region of length  $3L$  where there is a uniform magnetic field  $B_0$  into the plane of the paper, as shown in the figure. For sufficiently large  $v_0$ , the loop eventually crosses the region. Let  $x$  be the location of the right edge of the loop. Let  $v(x)$ ,  $I(x)$  and  $F(x)$  represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of  $x$ . Counter-clockwise current is taken as positive.



Which of the following schematic plot(s) is(are) correct? (Ignore gravity)



Space for rough work

Answers for the above questions

Ans for Q.14: (A) and (B)



**SECTION 3 (Maximum Marks: 12)**

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 In all other cases.

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### PARAGRAPH 1

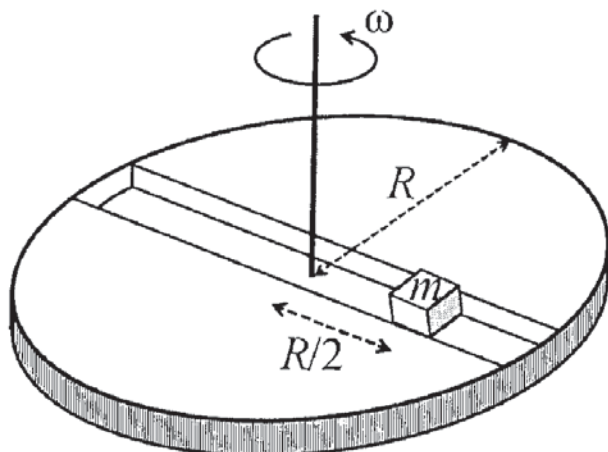
A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity  $\omega$  is an example of a non-inertial frame of reference.

The relationship between the force  $\vec{F}_{\text{rot}}$  experienced by a particle of mass  $m$  moving on the rotating disc and the force  $\vec{F}_{\text{in}}$  experienced by the particle in an inertial frame of reference is

$$\vec{F}_{\text{rot}} = \vec{F}_{\text{in}} + 2m(\vec{v}_{\text{rot}} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where  $\vec{v}_{\text{rot}}$  is the velocity of the particle in the rotating frame of reference and  $\vec{r}$  is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius  $R$  rotating counter-clockwise with a constant angular speed  $\omega$  about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the  $x$ -axis along the slot, the  $y$ -axis perpendicular to the slot and the  $z$ -axis along the rotation axis ( $\vec{\omega} = \omega \hat{k}$ ). A small block of mass  $m$  is gently placed in the slot at  $\vec{r} = (R/2)\hat{i}$  at  $t = 0$  and is constrained to move only along the slot.



Q.15 The distance  $r$  of the block at time  $t$  is

- (A)  $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$  (B)  $\frac{R}{2} \cos \omega t$  (C)  $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$  (D)  $\frac{R}{2} \cos 2\omega t$

Q.16 The net reaction of the disc on the block is

- (A)  $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$  (B)  $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$   
 (C)  $-m\omega^2 R \cos \omega t \hat{j} - mg\hat{k}$  (D)  $m\omega^2 R \sin \omega t \hat{j} - mg\hat{k}$

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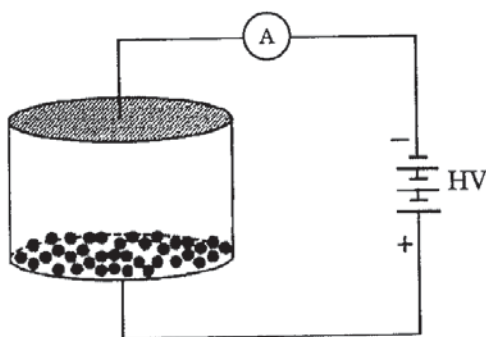
**Answers for the above questions**

**Ans for Q.15: (A)**

**Ans for Q.16: (B)**

## PARAGRAPH 2

Consider an evacuated cylindrical chamber of height  $h$  having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius  $r \ll h$ . Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at  $+V_0$  and the top plate at  $-V_0$ . Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



- Q.17 Which one of the following statements is correct?
- (A) The balls will stick to the top plate and remain there
  - (B) The balls will bounce back to the bottom plate carrying the same charge they went up with
  - (C) The balls will bounce back to the bottom plate carrying the opposite charge they went up with
  - (D) The balls will execute simple harmonic motion between the two plates
- Q.18 The average current in the steady state registered by the ammeter in the circuit will be
- (A) zero
  - (B) proportional to the potential  $V_0$
  - (C) proportional to  $V_0^{1/2}$
  - (D) proportional to  $V_0^2$

**END OF PART I : PHYSICS**

Space for rough work

**Answers for the above questions**

**Ans for Q.17: (C)**

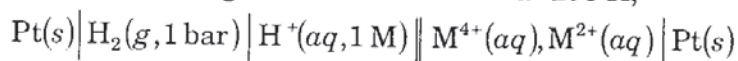
**Ans for Q.18: (D)**

## PART II : CHEMISTRY

### SECTION 1 (Maximum Marks: 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -1 In all other cases.

Q.19 For the following electrochemical cell at 298 K,



$$E_{\text{cell}} = 0.092 \text{ V when } \frac{[\text{M}^{2+}(aq)]}{[\text{M}^{4+}(aq)]} = 10^x .$$

$$\text{Given : } E_{\text{M}^{4+}/\text{M}^{2+}}^0 = 0.151 \text{ V ; } 2.303 \frac{RT}{F} = 0.059 \text{ V}$$

The value of  $x$  is

- (A) -2                      (B) -1                      (C) 1                      (D) 2

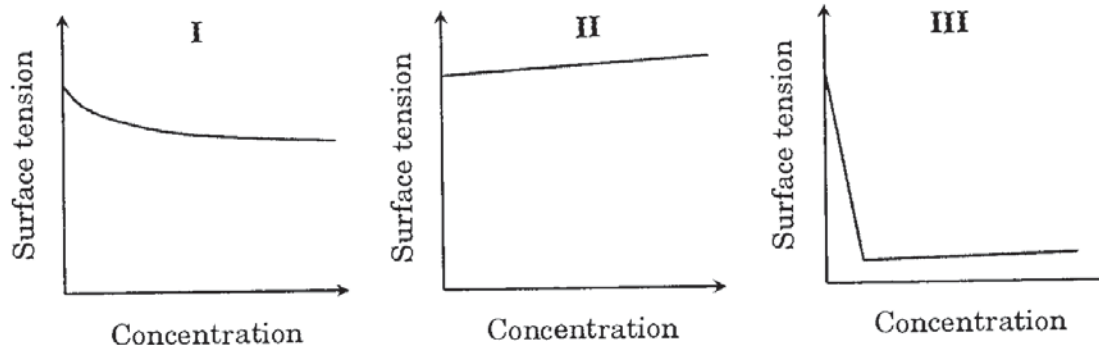
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### Answers for the above questions

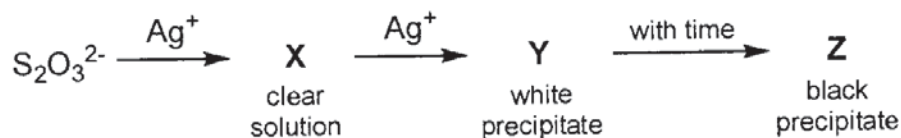
Ans for Q.19: (D)

Q.20 The qualitative sketches I, II and III given below show the variation of surface tension with molar concentration of three different aqueous solutions of KCl, CH<sub>3</sub>OH and CH<sub>3</sub>(CH<sub>2</sub>)<sub>11</sub>OSO<sub>3</sub><sup>-</sup>Na<sup>+</sup> at room temperature. The correct assignment of the sketches is



- (A) I : KCl                      II : CH<sub>3</sub>OH                      III : CH<sub>3</sub>(CH<sub>2</sub>)<sub>11</sub>OSO<sub>3</sub><sup>-</sup>Na<sup>+</sup>  
 (B) I : CH<sub>3</sub>(CH<sub>2</sub>)<sub>11</sub>OSO<sub>3</sub><sup>-</sup>Na<sup>+</sup>    II : CH<sub>3</sub>OH                      III : KCl  
 (C) I : KCl                      II : CH<sub>3</sub>(CH<sub>2</sub>)<sub>11</sub>OSO<sub>3</sub><sup>-</sup>Na<sup>+</sup>    III : CH<sub>3</sub>OH  
 (D) I : CH<sub>3</sub>OH                      II : KCl                      III : CH<sub>3</sub>(CH<sub>2</sub>)<sub>11</sub>OSO<sub>3</sub><sup>-</sup>Na<sup>+</sup>

Q.21 In the following reaction sequence in aqueous solution, the species X, Y and Z, respectively, are



- (A) [Ag(S<sub>2</sub>O<sub>3</sub>)<sub>2</sub>]<sup>3-</sup>, Ag<sub>2</sub>S<sub>2</sub>O<sub>3</sub>, Ag<sub>2</sub>S                      (B) [Ag(S<sub>2</sub>O<sub>3</sub>)<sub>3</sub>]<sup>5-</sup>, Ag<sub>2</sub>SO<sub>3</sub>, Ag<sub>2</sub>S  
 (C) [Ag(SO<sub>3</sub>)<sub>2</sub>]<sup>3-</sup>, Ag<sub>2</sub>S<sub>2</sub>O<sub>3</sub>, Ag                      (D) [Ag(SO<sub>3</sub>)<sub>3</sub>]<sup>3-</sup>, Ag<sub>2</sub>SO<sub>4</sub>, Ag

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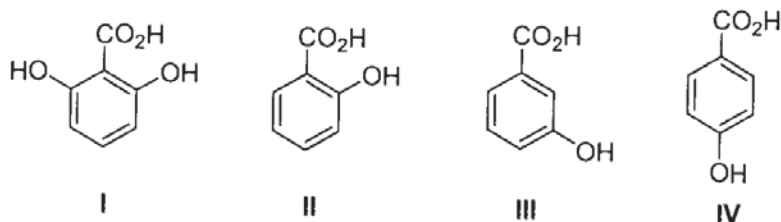
### Answers for the above questions

Ans for Q.20: (D)

Ans for Q.21: (A)

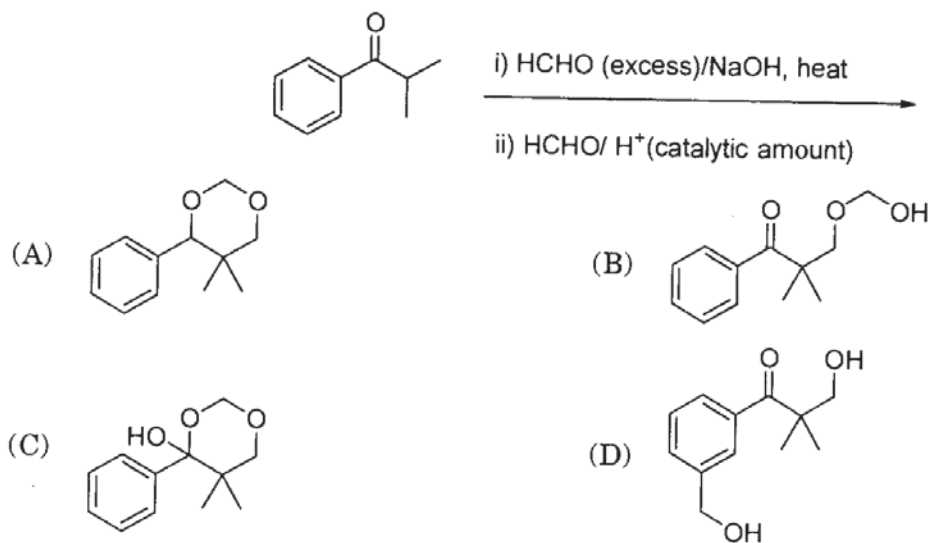
- Q.22 The geometries of the ammonia complexes of  $\text{Ni}^{2+}$ ,  $\text{Pt}^{2+}$  and  $\text{Zn}^{2+}$ , respectively, are
- (A) octahedral, square planar and tetrahedral  
 (B) square planar, octahedral and tetrahedral  
 (C) tetrahedral, square planar and octahedral  
 (D) octahedral, tetrahedral and square planar

- Q.23 The correct order of acidity for the following compounds is



- (A) I > II > III > IV  
 (B) III > I > II > IV  
 (C) III > IV > II > I  
 (D) I > III > IV > II

- Q.24 The major product of the following reaction sequence is



Space for rough work

### Answers for the above questions

**Ans for Q.22: (A)**

**Ans for Q.23: (A)**

**Ans for Q.24: (A)**

**SECTION 2 (Maximum Marks: 32)**

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.  
*Partial Marks* : +1 For darkening a bubble corresponding to **each correct option**, provided **NO** incorrect option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

Q.25 According to Molecular Orbital Theory,

- (A)  $C_2^{2-}$  is expected to be diamagnetic
- (B)  $O_2^{2+}$  is expected to have a longer bond length than  $O_2$
- (C)  $N_2^+$  and  $N_2^-$  have the same bond order
- (D)  $He_2^+$  has the same energy as two isolated He atoms

Q.26 Mixture(s) showing positive deviation from Raoult's law at 35 °C is(are)

- (A) carbon tetrachloride + methanol
- (B) carbon disulphide + acetone
- (C) benzene + toluene
- (D) phenol + aniline

Q.27 The **CORRECT** statement(s) for cubic close packed (*ccp*) three dimensional structure is(are)

- (A) The number of the nearest neighbours of an atom present in the topmost layer is 12
- (B) The efficiency of atom packing is 74%
- (C) The number of octahedral and tetrahedral voids per atom are 1 and 2, respectively
- (D) The unit cell edge length is  $2\sqrt{2}$  times the radius of the atom

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Space for rough work

**Answers for the above questions**

Ans for Q.25: (A) and (C)

Ans for Q.26: (A) and (B)

Ans for Q.27: (B), (C) and (D)

- Q.28 Extraction of copper from copper pyrite ( $\text{CuFeS}_2$ ) involves
- (A) crushing followed by concentration of the ore by froth-flotation
  - (B) removal of iron as slag
  - (C) self-reduction step to produce 'blister copper' following evolution of  $\text{SO}_2$
  - (D) refining of 'blister copper' by carbon reduction
- Q.29 The nitrogen containing compound produced in the reaction of  $\text{HNO}_3$  with  $\text{P}_4\text{O}_{10}$
- (A) can also be prepared by reaction of  $\text{P}_4$  and  $\text{HNO}_3$
  - (B) is diamagnetic
  - (C) contains one N-N bond
  - (D) reacts with Na metal producing a brown gas
- Q.30 For 'invert sugar', the correct statement(s) is(are)  
(Given: specific rotations of (+)-sucrose, (+)-maltose, L-(-)-glucose and L-(+)-fructose in aqueous solution are  $+66^\circ$ ,  $+140^\circ$ ,  $-52^\circ$  and  $+92^\circ$ , respectively)
- (A) 'invert sugar' is prepared by acid catalyzed hydrolysis of maltose
  - (B) 'invert sugar' is an equimolar mixture of D-(+)-glucose and D-(-)-fructose
  - (C) specific rotation of 'invert sugar' is  $-20^\circ$
  - (D) on reaction with  $\text{Br}_2$  water, 'invert sugar' forms saccharic acid as one of the products

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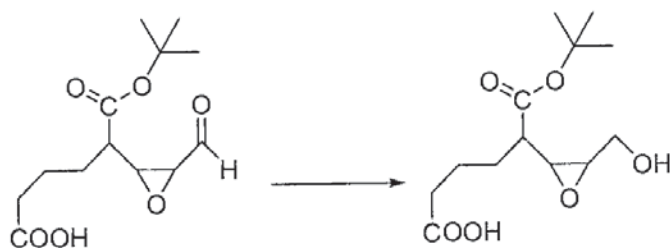
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**Answers for the above questions**

<b>Ans for Q.28: (A), (B) and (C)</b>	<b>Ans for Q.29: (B) and (D)</b>	<b>Ans for Q.30: (B) and (C)</b>
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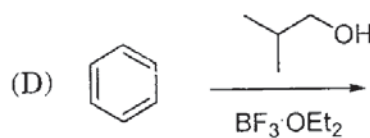
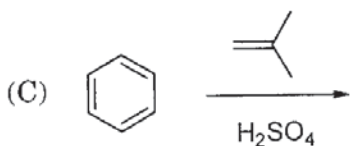
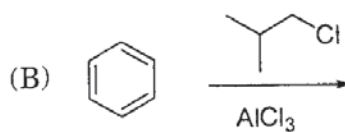
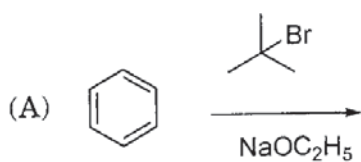


Q.31 Reagent(s) which can be used to bring about the following transformation is(are)



- (A)  $\text{LiAlH}_4$  in  $(\text{C}_2\text{H}_5)_2\text{O}$       (B)  $\text{BH}_3$  in THF  
 (C)  $\text{NaBH}_4$  in  $\text{C}_2\text{H}_5\text{OH}$       (D) Raney Ni/ $\text{H}_2$  in THF

Q.32 Among the following, reaction(s) which gives(give) *tert*-butyl benzene as the major product is(are)



Space for rough work

**Answers for the above questions**

**Ans for Q.31: (C) and (D)**

**Ans for Q.32: (B), (C) and (D)**

**SECTION 3 (Maximum Marks: 12)**

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 In all other cases.

**PARAGRAPH 1**

Thermal decomposition of gaseous  $X_2$  to gaseous X at 298 K takes place according to the following equation :



The standard reaction Gibbs energy,  $\Delta_r G^\circ$ , of this reaction is positive. At the start of the reaction, there is one mole of  $X_2$  and no X. As the reaction proceeds, the number of moles of X formed is given by  $\beta$ . Thus,  $\beta_{equilibrium}$  is the number of moles of X formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally. (Given :  $R = 0.083 \text{ L bar K}^{-1} \text{ mol}^{-1}$ )

Q.33 The equilibrium constant  $K_p$  for this reaction at 298 K, in terms of  $\beta_{equilibrium}$ , is

- (A)  $\frac{8\beta_{equilibrium}^2}{2 - \beta_{equilibrium}}$  (B)  $\frac{8\beta_{equilibrium}^2}{4 - \beta_{equilibrium}^2}$  (C)  $\frac{4\beta_{equilibrium}^2}{2 - \beta_{equilibrium}}$  (D)  $\frac{4\beta_{equilibrium}^2}{4 - \beta_{equilibrium}^2}$

Q.34 The **INCORRECT** statement among the following, for this reaction, is

- (A) Decrease in the total pressure will result in formation of more moles of gaseous X  
 (B) At the start of the reaction, dissociation of gaseous  $X_2$  takes place spontaneously  
 (C)  $\beta_{equilibrium} = 0.7$   
 (D)  $K_C < 1$

Space for rough work

**Answers for the above questions**

**Ans for Q.33: (B)**

**Ans for Q.34: (C)**

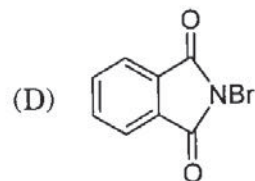
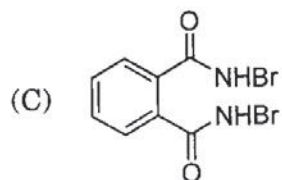
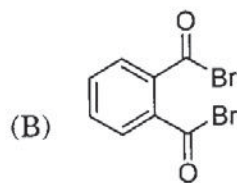
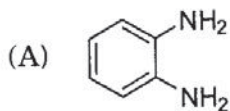
PARAGRAPH 2

Treatment of compound **O** with  $\text{KMnO}_4/\text{H}^+$  gave **P**, which on heating with ammonia gave **Q**. The compound **Q** on treatment with  $\text{Br}_2/\text{NaOH}$  produced **R**. On strong heating, **Q** gave **S**, which on further treatment with ethyl 2-bromopropanoate in the presence of  $\text{KOH}$  followed by acidification, gave a compound **T**.



(O)

Q.35 The compound **R** is



Q.36 The compound **T** is

(A) glycine

(B) alanine

(C) valine

(D) serine

END OF PART II : CHEMISTRY

Space for rough work

Answers for the above questions

Ans for Q.35: (A)

Ans for Q.36: (B)

## PART III : MATHEMATICS

### SECTION 1 (Maximum Marks: 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 If none of the bubbles is darkened.  
*Negative Marks* : -1 In all other cases.

Q.37 Let  $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$  and  $I$  be the identity matrix of order 3. If  $Q = [q_{ij}]$  is a matrix such that

$$P^{50} - Q = I, \text{ then } \frac{q_{31} + q_{32}}{q_{21}} \text{ equals}$$

- (A) 52                      (B) 103                      (C) 201                      (D) 205

Q.38 Let  $b_i > 1$  for  $i = 1, 2, \dots, 101$ . Suppose  $\log_e b_1, \log_e b_2, \dots, \log_e b_{101}$  are in Arithmetic Progression (A.P.) with the common difference  $\log_e 2$ . Suppose  $a_1, a_2, \dots, a_{101}$  are in A.P. such that  $a_1 = b_1$  and  $a_{51} = b_{51}$ . If  $t = b_1 + b_2 + \dots + b_{51}$  and  $s = a_1 + a_2 + \dots + a_{51}$ , then

- (A)  $s > t$  and  $a_{101} > b_{101}$                       (B)  $s > t$  and  $a_{101} < b_{101}$   
 (C)  $s < t$  and  $a_{101} > b_{101}$                       (D)  $s < t$  and  $a_{101} < b_{101}$

Q.39 The value of  $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$  is equal to

- (A)  $3 - \sqrt{3}$                       (B)  $2(3 - \sqrt{3})$                       (C)  $2(\sqrt{3} - 1)$                       (D)  $2(2 + \sqrt{3})$

Space for rough work

### Answers for the above questions

**Ans for Q.37: (B)**

**Ans for Q.38: (B)**

**Ans for Q.39: (C)**

Q.40 The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$  is equal to

- (A)  $\frac{\pi^2}{4} - 2$       (B)  $\frac{\pi^2}{4} + 2$       (C)  $\pi^2 - e^{\frac{\pi}{2}}$       (D)  $\pi^2 + e^{\frac{\pi}{2}}$

Q.41 Area of the region  $\{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\}$  is equal to

- (A)  $\frac{1}{6}$       (B)  $\frac{4}{3}$       (C)  $\frac{3}{2}$       (D)  $\frac{5}{3}$

Q.42 Let  $P$  be the image of the point  $(3, 1, 7)$  with respect to the plane  $x - y + z = 3$ . Then the equation of the plane passing through  $P$  and containing the straight line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$  is

- (A)  $x + y - 3z = 0$       (B)  $3x + z = 0$   
(C)  $x - 4y + 7z = 0$       (D)  $2x - y = 0$

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**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.40: (A)</b>	<b>Ans for Q.41: (C)</b>	<b>Ans for Q.42: (C)</b>
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**SECTION 2 (Maximum Marks: 32)**

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
 

*Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

*Partial Marks* : +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened.

*Zero Marks* : 0 If none of the bubbles is darkened.

*Negative Marks* : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

Q.43 Let  $f(x) = \lim_{n \rightarrow \infty} \left( \frac{n^n (x+n)(x+\frac{n}{2}) \cdots (x+\frac{n}{n})}{n! (x^2+n^2)(x^2+\frac{n^2}{4}) \cdots (x^2+\frac{n^2}{n^2})} \right)^{\frac{x}{n}}$ , for all  $x > 0$ . Then

- (A)  $f\left(\frac{1}{2}\right) \geq f(1)$                       (B)  $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$
- (C)  $f'(2) \leq 0$                               (D)  $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

- Q.44 Let  $a, b \in \mathbb{R}$  and  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$ . Then  $f$  is
- (A) differentiable at  $x = 0$  if  $a = 0$  and  $b = 1$
- (B) differentiable at  $x = 1$  if  $a = 1$  and  $b = 0$
- (C) **NOT** differentiable at  $x = 0$  if  $a = 1$  and  $b = 0$
- (D) **NOT** differentiable at  $x = 1$  if  $a = 1$  and  $b = 1$

Space for rough work

**Answers for the above questions**

**Ans for Q.43: (B) and (C)**

**Ans for Q.44: (A) and (B)**

Q.45 Let  $f: \mathbb{R} \rightarrow (0, \infty)$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be twice differentiable functions such that  $f''$  and  $g''$  are continuous functions on  $\mathbb{R}$ . Suppose  $f'(2) = g(2) = 0$ ,  $f''(2) \neq 0$  and  $g'(2) \neq 0$ .  
If  $\lim_{x \rightarrow 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$ , then

- (A)  $f$  has a local minimum at  $x = 2$
- (B)  $f$  has a local maximum at  $x = 2$
- (C)  $f''(2) > f(2)$
- (D)  $f(x) - f''(x) = 0$  for at least one  $x \in \mathbb{R}$

Q.46 Let  $f: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$  and  $g: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$  be functions defined by  $f(x) = [x^2 - 3]$  and  $g(x) = |x|f(x) + |4x - 7|f(x)$ , where  $[y]$  denotes the greatest integer less than or equal to  $y$  for  $y \in \mathbb{R}$ . Then

- (A)  $f$  is discontinuous exactly at three points in  $\left[-\frac{1}{2}, 2\right]$
- (B)  $f$  is discontinuous exactly at four points in  $\left[-\frac{1}{2}, 2\right]$
- (C)  $g$  is **NOT** differentiable exactly at four points in  $\left(-\frac{1}{2}, 2\right)$
- (D)  $g$  is **NOT** differentiable exactly at five points in  $\left(-\frac{1}{2}, 2\right)$

Space for rough work

### Answers for the above questions

Ans for Q.45: (A) and (D)

Ans for Q.46: (B) and (C)

Q.47 Let  $a, b \in \mathbb{R}$  and  $a^2 + b^2 \neq 0$ . Suppose  $S = \left\{ z \in \mathbb{C} : z = \frac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0 \right\}$ , where  $i = \sqrt{-1}$ .

If  $z = x + iy$  and  $z \in S$ , then  $(x, y)$  lies on

- (A) the circle with radius  $\frac{1}{2a}$  and centre  $\left(\frac{1}{2a}, 0\right)$  for  $a > 0, b \neq 0$
- (B) the circle with radius  $-\frac{1}{2a}$  and centre  $\left(-\frac{1}{2a}, 0\right)$  for  $a < 0, b \neq 0$
- (C) the  $x$ -axis for  $a \neq 0, b = 0$
- (D) the  $y$ -axis for  $a = 0, b \neq 0$

Q.48 Let  $P$  be the point on the parabola  $y^2 = 4x$  which is at the shortest distance from the center  $S$  of the circle  $x^2 + y^2 - 4x - 16y + 64 = 0$ . Let  $Q$  be the point on the circle dividing the line segment  $SP$  internally. Then

- (A)  $SP = 2\sqrt{5}$
- (B)  $SQ : QP = (\sqrt{5} + 1) : 2$
- (C) the  $x$ -intercept of the normal to the parabola at  $P$  is 6
- (D) the slope of the tangent to the circle at  $Q$  is  $\frac{1}{2}$

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Space for rough work

### Answers for the above questions

Ans for Q.47: (A), (C) and (D)	Ans for Q.48: (A), (C) and (D)
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Q.49 Let  $a, \lambda, \mu \in \mathbb{R}$ . Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(s) is(are) correct?

- (A) If  $a = -3$ , then the system has infinitely many solutions for all values of  $\lambda$  and  $\mu$
- (B) If  $a \neq -3$ , then the system has a unique solution for all values of  $\lambda$  and  $\mu$
- (C) If  $\lambda + \mu = 0$ , then the system has infinitely many solutions for  $a = -3$
- (D) If  $\lambda + \mu \neq 0$ , then the system has no solution for  $a = -3$

Q.50 Let  $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$  be a unit vector in  $\mathbb{R}^3$  and  $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$ . Given that there exists

a vector  $\vec{v}$  in  $\mathbb{R}^3$  such that  $|\hat{u} \times \vec{v}| = 1$  and  $\hat{w} \cdot (\hat{u} \times \vec{v}) = 1$ . Which of the following statement(s) is(are) correct?

- (A) There is exactly one choice for such  $\vec{v}$
- (B) There are infinitely many choices for such  $\vec{v}$
- (C) If  $\hat{u}$  lies in the  $xy$ -plane then  $|u_1| = |u_2|$
- (D) If  $\hat{u}$  lies in the  $xz$ -plane then  $2|u_1| = |u_3|$

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Space for rough work

### Answers for the above questions

<b>Ans for Q.49: (B), (C) and (D)</b>	<b>Ans for Q.50: (B) and (C)</b>
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**SECTION 3 (Maximum Marks: 12)**

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:  
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened.  
*Zero Marks* : 0 In all other cases.

**PARAGRAPH 1**

Football teams  $T_1$  and  $T_2$  have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of  $T_1$  winning, drawing and losing a game against  $T_2$  are  $\frac{1}{2}$ ,  $\frac{1}{6}$  and  $\frac{1}{3}$ , respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let  $X$  and  $Y$  denote the total points scored by teams  $T_1$  and  $T_2$ , respectively, after two games.

Q.51  $P(X > Y)$  is

- (A)  $\frac{1}{4}$                       (B)  $\frac{5}{12}$                       (C)  $\frac{1}{2}$                       (D)  $\frac{7}{12}$

Q.52  $P(X = Y)$  is

- (A)  $\frac{11}{36}$                       (B)  $\frac{1}{3}$                       (C)  $\frac{13}{36}$                       (D)  $\frac{1}{2}$

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Space for rough work

**Answers for the above questions**

**Ans for Q.51: (B)**

**Ans for Q.52: (C)**

**PARAGRAPH 2**

Let  $F_1(x_1, 0)$  and  $F_2(x_2, 0)$ , for  $x_1 < 0$  and  $x_2 > 0$ , be the foci of the ellipse  $\frac{x^2}{9} + \frac{y^2}{8} = 1$ . Suppose a parabola having vertex at the origin and focus at  $F_2$  intersects the ellipse at point  $M$  in the first quadrant and at point  $N$  in the fourth quadrant.

Q.53 The orthocentre of the triangle  $F_1MN$  is

- (A)  $\left(-\frac{9}{10}, 0\right)$       (B)  $\left(\frac{2}{3}, 0\right)$       (C)  $\left(\frac{9}{10}, 0\right)$       (D)  $\left(\frac{2}{3}, \sqrt{6}\right)$

Q.54 If the tangents to the ellipse at  $M$  and  $N$  meet at  $R$  and the normal to the parabola at  $M$  meets the  $x$ -axis at  $Q$ , then the ratio of area of the triangle  $MQR$  to area of the quadrilateral  $MF_1NF_2$  is

- (A) 3 : 4      (B) 4 : 5      (C) 5 : 8      (D) 2 : 3

**END OF THE QUESTION PAPER**

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.53: (A)</b>
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<b>Ans for Q.54: (C)</b>
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