IIT JAM Previous Year Question Papers - 2024

IIT Joint Admission Test for Masters

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 Which one of the following is a simple tissue system in plants?

(A) Epidermis

(B) Parenchyma

(C) Phloem

(D) Xylem

Q.2 In DNA replication, the Okazaki fragments are joined by

(A) DNA helicase

(B) DNA ligase

(C) DNA polymerase

(D) DNA primase

Q.3 The most abundant type of RNA in a metabolically active mammalian cell is

(A) mRNA

(B) rRNA

(C) snoRNA

(D) tRNA

Q.4 Which organelle in a eukaryotic cell is the site of electron transport chain?

(A) Endoplasmic reticulum

(B) Golgi apparatus

(C) Mitochondrion

(D) Peroxisome

Q.5 RNA is a polymer of

(A) glycosides

(B) ribonucleosides

(C) ribonucleotides

(D) riboses

Q.6 Which one of the following is present in a bacterial cell?

(A) 28S rRNA

(B) 70S ribosome

(C) Chitinous cell wall

(D) Histones

Q.7 Which color of light excites a natural GFP to emit green fluorescence?

(A) Blue

(B) Green

(C) Infrared

(D) Red

Q.8 Which one of the following hormones promotes fruit ripening?

(A) Abscisic acid

(B) Auxin

(C) Ethylene

(D) Gibberellin

Q.9 Which one of the following has a catalytic RNA?

(A) Ribonuclease H

(B) Ribozyme

(C) RNA polymerase I

(D) RNA polymerase II

Q.10 The number of significant figures in a reported measurement of 0.00361 is

(A) 3

(B) 4

(C) 5

(D) 6

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Match the terminology in **Group I** with the stimulus in **Group II** that generates growth response of plants

Group I	Group II
P. Gravitropism	1. Light
Q. Phototropism	2. Touch
R. Thigmotropism	3. Chemical
S. Chemotropism	4. Gravity

(A) P – 3, Q – 4, R – 2, S – 1

(B) P – 2, Q – 1, R – 3, S – 4

(C) P – 4, Q – 1, R – 2, S – 3

(D) P – 4, Q – 2, R – 1, S – 3

Q.12 The correct hierarchy of taxa in the Linnaean classification of eukaryotes is

(A) kingdom, class, phylum, order, family, genus

(B) kingdom, order, class, phylum, family, genus

(C) kingdom, phylum, order, family, class, genus

(D) kingdom, phylum, class, order, family, genus

Q.13 Which one of the following statements about polyploidy is correct?

(A) Autopolyploids are derived from a single species

(B) Autopolyploids are derived from two different species

(C) Allopolyploids are derived from a single species

(D) Allopolyploids are not fertile when mated with each other

Q.14 Which one of the following hormones is a tyrosine derivative?

(A) Epinephrine

(B) Estradiol

(C) Progesterone

(D) Testosterone

Q.15 Which one of the following immunoglobulins crosses the human placenta?

(A) IgA

(B) IgE

(C) IgG

(D) IgM

Q.16 Determine the correctness or otherwise of the following Assertion [a] and the Reason [r].

Assertion [a]: The resolving power of a transmission electron microscope is higher than that of the light microscope.

Reason [r]: The wavelength of electrons is shorter than that of visible light.

(A) Both [a] and [r] are true and [r] is the correct reason for [a]

(B) Both [a] and [r] are true but [r] is not the correct reason for [a]

(C) Both [a] and [r] are false

(D) [a] is true but [r] is false

Q.17 Match the morphology in **Group I** with the corresponding microorganism in **Group II**

Group I	**Group II**
P. Coccus	1. Treponema
Q. Rod	2. Bacillus
R. Comma	3. Neisseria
S. Spiral	4. Vibrio

(A) P – 3, Q – 2, R – 4, S – 1

(B) P – 4, Q – 1, R – 3, S – 2

(C) P – 2, Q – 4, R – 1, S – 3

(D) P – 1, Q – 2, R – 3, S – 4

Q.18 Which one of the following genetic crosses and their results indicates cytoplasmic inheritance?

(A) Wild-type male × mutant female → 100% progeny are mutant

(B) Wild-type male × mutant female → 25% progeny are wild-type

(C) Mutant male × wild-type female → 50% progeny are mutant

(D) Mutant male × wild-type female → 75% progeny are wild-type

Q.19 Which of the following is **NOT** a characteristic morphological feature of apoptotic cells?

(A) Disassembly of nuclear envelope

(B) DNA fragmentation

(C) Increased cell size

(D) Membrane blebbing

Q.20 Competition between two populations in an ecosystem is

(A) beneficial (+) to both the populations

(B) deleterious (-) to both the populations

(C) beneficial (+) to one population, but deleterious (-) to the other population

(D) beneficial (+) to one population, but no effect (0) on the other population

Q.21 Adenine constitutes 0.16 mole fraction in a given single-stranded DNA. What is the mole fraction of uracil in the resultant RNA, if this entire DNA fragment is transcribed?

(A) 0.16

(B) 0.32

(C) 0.34

(D) 0.68

Q.22 Which one of the following is **NOT** used as a component in subunit vaccines?

(A) Capsular polysaccharide

(B) Inactivated exotoxin

(C) Inactivated virus

(D) Viral glycoprotein

Q.23 Metabolic acidosis is associated with decreased plasma level of

(A) bicarbonate

(B) lactate

(C) oxygen

(D) urea

Q.24 Genes in two species that are derived from the same ancestral gene in their most recent common ancestor are called

(A) analogs

(B) heterologs

(C) orthologs

(D) paralogs

Q.25 An object is placed 15 cm in front of a convex mirror, which has a radius of curvature 30 cm. Which one of the following is true of the image formed?

(A) Real and inverted

(B) Real and upright

(C) Virtual and inverted

(D) Virtual and upright

Q.26 If a variable z shows a standard normal distribution, then the percent probability that

$$0 \leq z^2 \leq 1$$

is _____ (*rounded off to the nearest integer*).

(A) 34

(B) 68

(C) 95

(D) 99

Q.27 In chick embryo, the ectoderm generates

(A) alveolar cells

(B) germ cells

(C) neurons

(D) red blood cells

Q.28 The boiling points of Iodomethane, Dibromomethane, Bromomethane, Chloromethane follow the order

(A) Bromomethane > Dibromomethane > Iodomethane > Chloromethane

(B) Bromomethane > Iodomethane > Chloromethane > Dibromomethane

(C) Dibromomethane > Iodomethane > Bromomethane > Chloromethane

(D) Iodomethane > Bromomethane > Chloromethane > Dibromomethane

Q.29 Chromosome duplication during the cell cycle occurs in

(A) G_1 phase

(B) G_2 phase

(C) M phase

(D) S phase

Q.30 Ionic character of the covalent bonds in the compounds Cl_2, HCl, NaCl, NaF follows the order

(A) $Cl_2 > NaCl > HCl > NaF$

(B) $HCl > Cl_2 > NaF > NaCl$

(C) $HCl > NaCl > NaF > Cl_2$

(D) $NaF > NaCl > HCl > Cl_2$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Which of the following is/are lateral meristems?

(A) Cork cambium

(B) Procambium

(C) Protoderm

(D) Vascular cambium

Q.32 Which of the following statement(s) about Golden Rice is/are correct?

(A) Consumption of it increases vitamin A levels

(B) Consumption of it increases vitamin D levels

(C) Consumption of it increases vitamin K levels

(D) It is a transgenic crop containing β-carotene

Q.33 Which of the following statement(s) about eukaryotic DNA topoisomerase is/are correct?

(A) Topoisomerase I creates transient single-strand breaks

(B) Topoisomerase I creates transient double-strand breaks

(C) Topoisomerase II creates transient single-strand breaks

(D) Topoisomerase II creates transient double-strand breaks

Q.34 Which of the following method(s) is/are used to estimate protein concentration?

(A) Anthrone

(B) Biuret

(C) Bradford

(D) Lowry

Q.35 Which of the following is/are example(s) of a lotic ecosystem?

(A) Lake

(B) Pond

(C) River

(D) Stream

Q.36 Which of the following statement(s) about the effect of genetic drift is/are correct?

(A) It can cause changes in the frequency of alleles at random

(B) It is a mechanism of evolution

(C) It can lead to loss of genetic variation within small populations

(D) It is significant in large populations

Q.37 Which of the following technique(s) can be used to determine the three-dimensional structure of an organic compound?

(A) Mass spectrometry

(B) NMR spectroscopy

(C) UV-visible spectroscopy

(D) X-ray crystallography

Q.38 Which of the following entity(ies) is/are found inside the intact nucleus of eukaryotic cells?

(A) Centrosome

(B) Lysosome

(C) Nucleolus

(D) Nucleosome

Q.39 Which of the following is/are trace element(s)?

(A) Mn

(B) P

(C) S

(D) Zn

Q.40 Which of the following is/are true about Retrovirus?

(A) It contains double-stranded RNA genome

(B) It can cause cancer

(C) It contains reverse transcriptase

(D) It contains double-stranded DNA genome

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 A wooden plant accumulates $10\ mg\ kg^{-1}$ of ^{14}C during its life span. A fossil of this plant was discovered and contains $2.5\ mg\ kg^{-1}$ of ^{14}C. The age of this fossil at the time of discovery is _____ years (*rounded off to the nearest integer*).

(Use 5730 years as half-life of ^{14}C)

Q.42 A cylinder contains $50\ L$ of an ideal gas at a pressure of $50\ atm$. Assuming that the temperature remains unchanged, the volume of the gas at $1\ atm$ is _____ L (*rounded off to the nearest integer*).

Q.43 One molecule of the protein myoglobin contains one atom of iron. A myoglobin sample was found to contain 0.34% iron. The molecular weight of myoglobin is _____ $g\ mol^{-1}$ (*rounded off to the nearest integer*).

(Use $55.9\ g\ mol^{-1}$ as atomic mass of iron)

Q.44 The wavelength of visible light for the green color is 600 nm. The energy of photons of this color is _____ eV (*rounded off to one decimal place*).
(Planck's constant = $6.63 \times 10^{-34}\ Js$, $1\ eV\ = 1.6 \times 10^{-19}\ J$,
speed of light = $3 \times 10^{8}\ ms^{-1}$)

Q.45 A ball dropped from a bridge hits the surface of the water in 3 s. The height of the bridge, ignoring air resistance, is _____ m (*rounded off to one decimal place*).
(Use $g\ =\ 9.8\ ms^{-2}$)

Q.46 For a given square, if the area of its incircle is 100 cm^2, then the area of its circumcircle is _____ cm^2 (*rounded off to the nearest integer*).

Q.47 The number of peaks in the ^{1}H NMR spectrum of methoxymethane (CH_3OCH_3) is _____.

Q.48 The amount of agarose required to prepare $250\ mL$ of 0.8% agarose gel is _____ $grams$ (*rounded off to the nearest integer*).

Q.49 Three genes *x*, *y*, and *z* are located on a chromosome in a linear order. If the recombination frequencies between *x* and *y* is 0.15, and between *y* and *z* is 0.10, then the expected frequency of double crossovers is _____ (*rounded off to three decimal places*).

Q.50 A bacterial cell suspension contains $2 \times 10^5\ cells\ mL^{-1}$. The volume of this suspension required to obtain $1.4 \times 10^6\ cells$ is _____ mL (*rounded off to the nearest integer*).

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 The data provided in the table were obtained from the following reaction, carried out at 273 K.

$$A + B \rightarrow C$$

Initial concentration of $[A]$ $mol\ L^{-1}$	Initial concentration of $[B]$ $mol\ L^{-1}$	Initial rate of formation of $[C]$ $mol\ L^{-1}s^{-1}$
0.2	0.2	0.3
0.4	0.2	0.6
0.4	0.4	2.4

The order of the reaction with respect to A is _____.

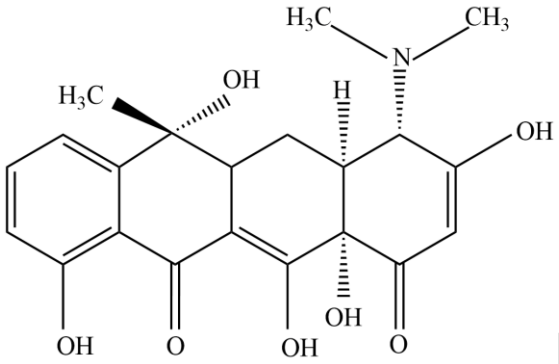
Q.52 Ammonia is synthesized in the Haber process in the following reaction.

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

The temperature above which the reaction becomes spontaneous is _____ K (*rounded off to one decimal place*).

($\Delta H^0 = -92.2\ kJ, \quad \Delta S^0 = -199\ JK^{-1}$)

Q.53 In the given molecule,



the number of chiral centers is _____.

Q.54 Two resistors 2 Ω and 4 Ω are combined in parallel. If this combination is connected to a battery of 16 V, the maximum current that can be drawn from the battery is _____ A (*rounded off to the nearest integer*).

Q.55 A box of mass $20\ kg$ is pulled at constant speed across a floor by a rope. The rope makes an angle of 45^0 with the horizontal. Assuming that friction is negligible, the work done in pulling the box by a distance of $20\ m$ is _____ J (*rounded off to the nearest integer*).
(Use $g = 9.8\ ms^{-2}$)

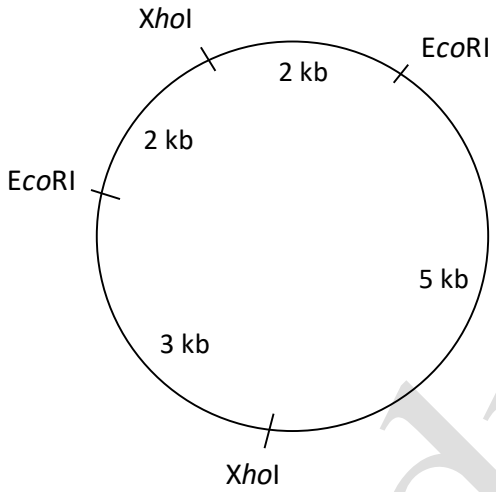
Q.56 Consider an enzyme that follows simple Michaelis-Menten kinetics, and has a K_M of $5\ \mu M$. The initial velocity of the reaction will be 10% of the maximum velocity at a substrate concentration of _____ μM (*rounded off to two decimal places*).

Q.57 The value of $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3}$ is _____(*rounded off to the nearest integer*).

Q.58 A population of 1000 plants are in Hardy-Weinberg equilibrium. Two alleles *R* and *r* determine a particular trait in this population. If the number of plants with *RR* genotype is 640, *Rr* genotype is 320 and *rr* genotype is 40, the frequency of *r* allele (in percentage) in this population is _____ (*rounded off to the nearest integer*).

Q.59 If a fair coin is tossed two times, the probability that the first or the second toss will be heads is _____ (*rounded off to two decimal places*).

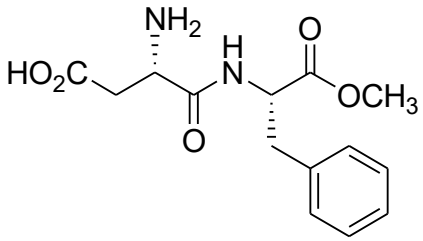
Q.60 The restriction map of a circular plasmid is shown below, along with the indicated distances between the restriction sites.



The plasmid was completely digested with E*co*RI, and X*ho*I. The products were analysed by agarose gel electrophoresis followed by ethidium bromide staining. The number of bands that will be visible in the gel when exposed to UV light is _____.

Section A: Q.1 – Q.10 Carry ONE mark each (Multiple Choice Questions)

Q.1 The following dipeptide derivative is used as an artificial sweetener:



The constituent α-amino acids of this dipeptide are

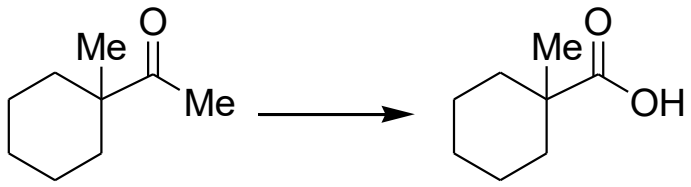
(A) phenylalanine and glutamic acid.

(B) phenylalanine and aspartic acid.

(C) tyrosine and aspartic acid.

(D) tyrosine and glutamic acid.

Q.2 The suitable reagent combination for the following transformation



is

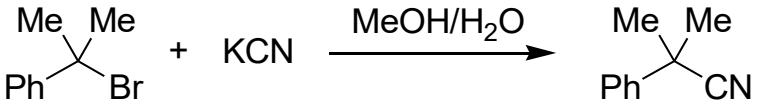
(A) (i) *meta*-chloroperbenzoic acid (*m*-CPBA); (ii) NaOH; (iii) aq. HCl

(B) (i) OsO_4; (ii) aq. HCl

(C) (i) I_2/NaOH; (ii) aq. HCl

(D) (i) dimethyldioxirane (DMDO); (ii) aq. HCl

Q.3 For the reaction



if the concentration of KCN is increased four times, then the rate of the reaction would be

(A) unaffected.

(B) increased by two times.

(C) decreased by four times.

(D) increased by four times.

Q.4 Consider the wavefunction $\psi(x) = N[\exp(ikx) + \exp(-ikx)]$. The complex conjugate $\psi^*(x)$ is

[Given: N is the normalization constant; $i = \sqrt{-1}$]

(A) $N[\exp(-ikx) - \exp(ikx)]$

(B) $N^*[\exp(-ikx) - \exp(ikx)]$

(C) $N^*[\exp(ikx) + \exp(-ikx)]$

(D) $2N[\sin(kx)]$

Q.5 Wavelength of X-rays used in a diffraction experiment is 1.54 Å. X-rays are diffracted from a set of planes with an interplanar spacing of 1.54 Å. Then the angle θ (in degrees) corresponding to the first-order Bragg diffraction is

(A) 30º

(B) 15º

(C) 45º

(D) 90º

Q.6 Identify the reaction for which, at equilibrium, a change in the volume of the closed reaction vessel at a constant temperature will not affect the extent of the reaction.

(A) $\mathrm{CaCO_3}(s) \rightleftharpoons \mathrm{CaO}(s) + \mathrm{CO_2}(g)$

(B) $\mathrm{H_2}(g) + \mathrm{I_2}(g) \rightleftharpoons 2\mathrm{HI}(g)$

(C) $2\mathrm{NO_2}(g) \rightleftharpoons \mathrm{N_2O_4}(g)$

(D) $\mathrm{CO_2}(s) \rightleftharpoons \mathrm{CO_2}(g)$

Q.7 Among $[Ti(H_2O)_6]^{3+}$, $[NiCl_4]^{2-}$, $[CrO_4]^{2-}$, and $[Mn(H_2O)_6]^{2+}$, the complex that exhibits the largest molar absorptivity in the visible region of the electronic absorption spectrum is

(A) $[Ti(H_2O)_6]^{3+}$

(B) $[NiCl_4]^{2-}$

(C) $[CrO_4]^{2-}$

(D) $[Mn(H_2O)_6]^{2+}$

Q.8 $[Co(NH_3)_5(SO_4)]Br$ and $[Co(NH_3)_5Br]SO_4$ are examples of

(A) ionization isomers.

(B) linkage isomers.

(C) optical isomers.

(D) coordination isomers.

Q.9 The pair of proteins having heme core is

(A) hemoglobin and myoglobin.

(B) hemerythrin and myoglobin.

(C) hemoglobin and hemocyanin.

(D) hemocyanin and hemerythrin.

Q.10 The shape of SCN^- is

(A) linear.

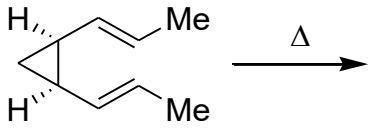
(B) bent.

(C) pyramidal.

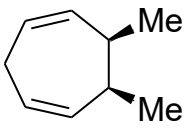
(D) trigonal planar.

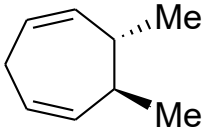
Section A: Q.11 – Q.30 Carry TWO marks each. (Multiple Choice Questions)

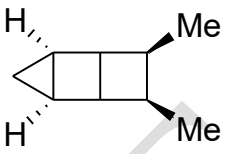
Q.11 The major product formed in the following reaction

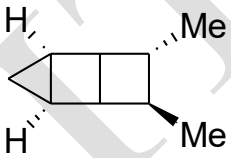


is

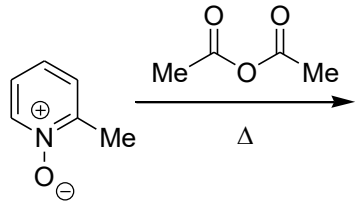
(A) 

(B) 

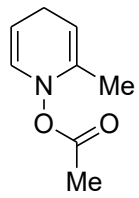
(C) 

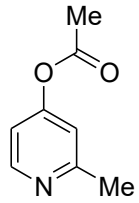
(D) 

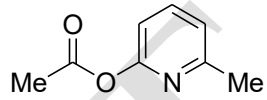
Q.12 The major product formed in the following reaction

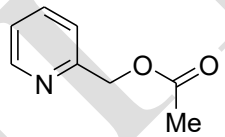


is

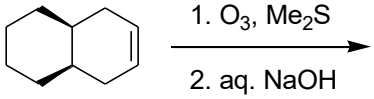
(A) 

(B) 

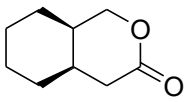
(C) 

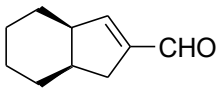
(D) 

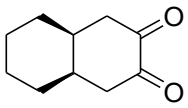
Q.13 The major product formed in the following reaction

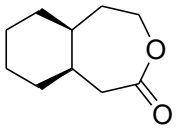


is

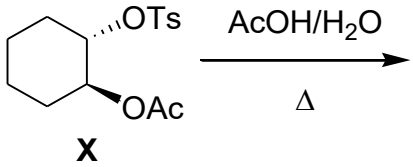
(A) 

(B) 

(C) 

(D) 

Q.14 In the following reaction



X

optically pure ester **X** formed product that did not exhibit optical rotation ($[\alpha]_D = 0$) due to the formation of

(Note: Ts = *para*-toluenesulfonyl; Ac = acetyl)

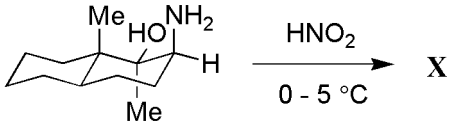
(A) *cis*-1,2-diacetoxycyclohexane.

(B) a racemic mixture of *trans*-1,2-diacetoxycyclohexane.

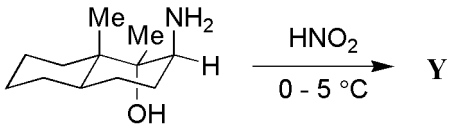
(C) cyclohexene.

(D) cyclohexene oxide.

Q.15 The major products **X** and **Y** in the following reactions

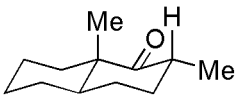
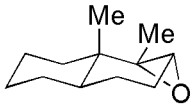


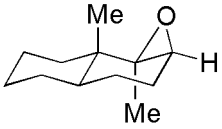
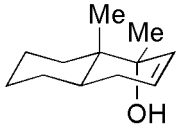
X

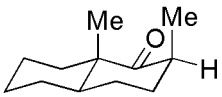
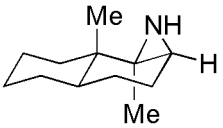


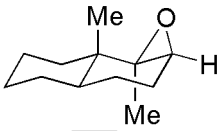
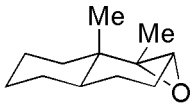
Y

respectively, are

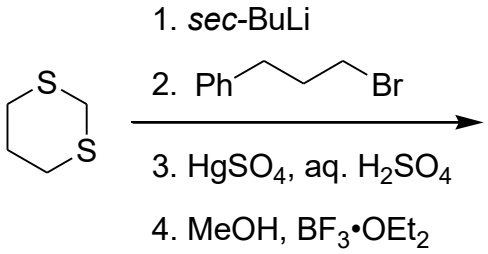
(A)  and 

(B)  and 

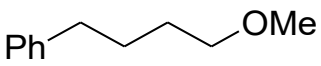
(C)  and 

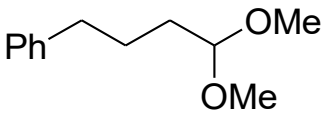
(D)  and 

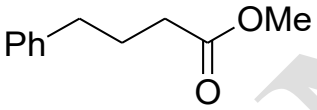
Q.16 The major product formed in the following reaction

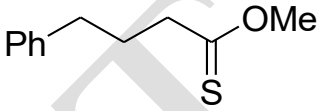


is

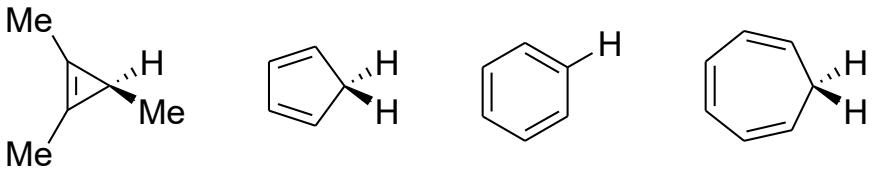
(A) 

(B) 

(C) 

(D) 

Q.17 The acidity of the compounds shown below



I **II** **III** **IV**

follows the order

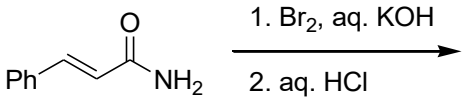
(A) **II** > **I** > **III** > **IV**

(B) **II** > **IV** > **III** > **I**

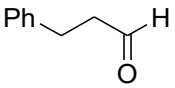
(C) **I** > **II** > **IV** > **III**

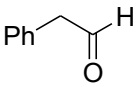
(D) **III** > **IV** > **II** > **I**

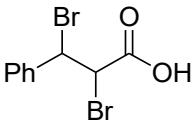
Q.18 The major product formed in the following reaction

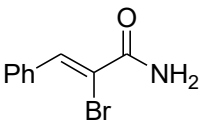


is

(A) 

(B) 

(C) 

(D) 

Q.19 The ratio of osmotic pressures of aqueous solutions of 0.01 M $BaCl_2$ to 0.005 M NaCl is

[Given: Both compounds dissociate completely in water]

(A) 3:1

(B) 1:4

(C) 1:1

(D) 3:2

Q.20 In the cell reaction

$$P^{+}(aq) + Q(s) \rightarrow P(s) + Q^{+}(aq)$$

the EMF of the cell, E_{cell} is zero. The standard EMF of the cell, E^{o}_{cell} is

[Given:
Activities of all solids are unity.
Activity of $P^{+}(aq)$ is 2 M. Activity of $Q^{+}(aq)$ is 1 M.
R = universal gas constant; T = temperature; F = Faraday constant]

(A) $\frac{RT}{F}$

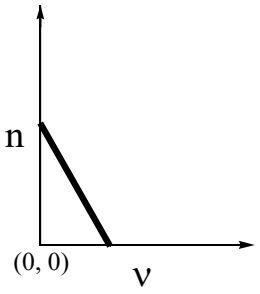
(B) $\frac{RT}{2F}$

(C) $-\frac{RT}{F}\ln(2)$

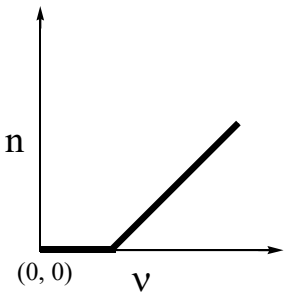
(D) $\frac{RT}{F}\ln(2)$

Q.21 Consider photoelectric effect. The number of incident photons is the same for all frequencies. The plot that best describes the dependence of the number of photoelectrons (n) emitted as a function of the incident light frequency (ν) is

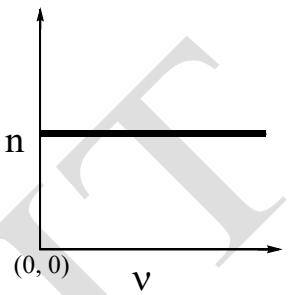
(A)



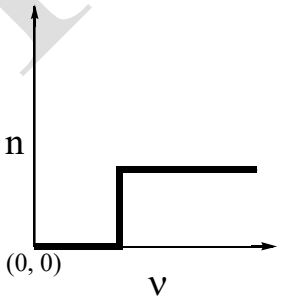
(B)



(C)



(D)



Q.22 If nitrogen and oxygen gases are at the same temperature, the correct statement according to the kinetic theory of gases is

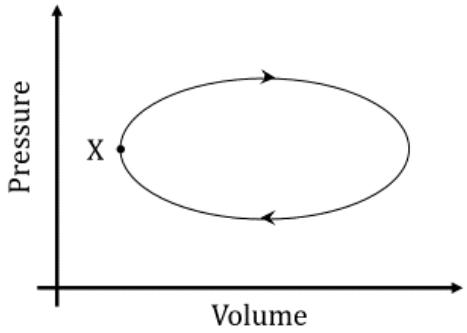
(A) Average kinetic energy of nitrogen and oxygen molecules is inversely proportional to temperature.

(B) For nitrogen and oxygen molecules, the root mean square speed is equal to the most probable speed.

(C) Average speed of nitrogen molecules is less than the average speed of oxygen molecules.

(D) Average kinetic energies of nitrogen and oxygen molecules are equal.

Q.23 A system undergoes one clockwise cycle from point X back to point X as shown in the figure below:



The correct statement about this process is

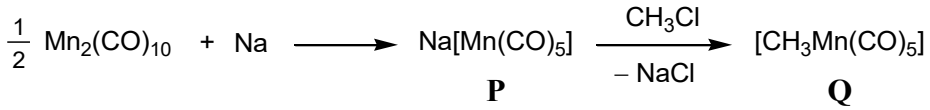
(A) Internal energy of the system decreases at the end of the cycle.

(B) Entropy of the system increases at the end of the cycle.

(C) System performs work on the surroundings during the cycle.

(D) Heat exchanged between system and surroundings is zero during the cycle.

Q.24 For the reaction shown below



the oxidation states of Mn in **P** and **Q**, respectively, are

(A) +1 and +1

(B) –1 and +1

(C) –1 and –1

(D) +1 and –1

Q.25 The number and nature of *d*–*d* transition(s) in the case of Sc^{2+} in an octahedral crystal field, respectively, are

[Ignore spin-orbit coupling and Jahn-Teller distortion.]

(A) 1 and spin allowed.

(B) 3 and spin allowed.

(C) 1 and Laporte allowed.

(D) 3 and Laporte allowed.

Q.26 The *d*-*d* transitions in $[Mn(H_2O)_6]^{2+}$ and $[Ti(H_2O)_4]^{3+}$, respectively, are

[Ignore spin-orbit coupling and Jahn-Teller distortion.]

(A) symmetry allowed and symmetry forbidden.

(B) symmetry forbidden and symmetry allowed.

(C) symmetry allowed and symmetry allowed.

(D) symmetry forbidden and symmetry forbidden.

Q.27 A pair of isosteric compounds is

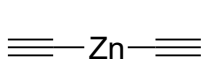
(A) H_2NBH_2 and C_2H_6

(B) $H_3N{\cdot}BH_3$ and C_2H_6

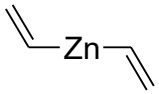
(C) B_2H_6 and C_2H_6

(D) $H_3N{\cdot}BH_3$ and B_2H_6

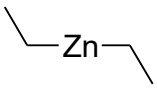
Q.28 Zn–C bond polarity in the compounds below



I



II



III

follows the order

(A) **I** > **II** > **III**

(B) **III** > **II** > **I**

(C) **II** > **III** > **I**

(D) **II** > **I** > **III**

Q.29 B_2 and C_2, respectively, are

(A) paramagnetic and diamagnetic.

(B) diamagnetic and paramagnetic.

(C) paramagnetic and paramagnetic.

(D) diamagnetic and diamagnetic.

Q.30 Mobility of ions

Li^+, Na^+, K^+, Ag^+

in water at 298 K follows the order

(A) $K^+ < Ag^+ < Na^+ < Li^+$

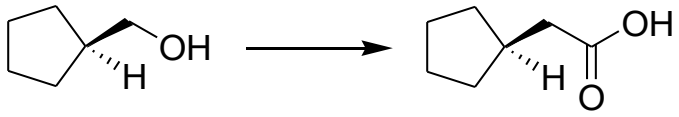
(B) $Li^+ < K^+ < Na^+ < Ag^+$

(C) $Ag^+ < Li^+ < K^+ < Na^+$

(D) $Li^+ < Na^+ < Ag^+ < K^+$

Section B: Q.31 – Q.40 Carry TWO marks each. (Multiple Select Questions)

Q.31 The suitable synthetic route(s) for the following transformation



is/are

(A) (i) *para*-toluenesulfonyl chloride (TsCl), pyridine; (ii) KI; (iii) Mg/Et_2O; (iv) CO_2; (v) aq. HCl

(B) (i) *para*-toluenesulfonyl chloride (TsCl), pyridine; (ii) KCN; (iii) conc. aq. NaOH, reflux; (iv) aq. HCl

(C) (i) CrO_3, H_2SO_4; (ii) $SOCl_2$; (iii) CH_2N_2; (iv) Ag_2O, H_2O

(D) (i) CrO_3, H_2SO_4; (ii) CH_2N_2

Q.32 The compound(s) which on reaction with CH_3MgBr followed by treatment with aqueous NH_4Cl would produce 1-methyl-1-phenylethanol as the major product is/are

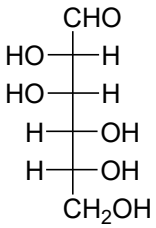
(A) methyl benzoate.

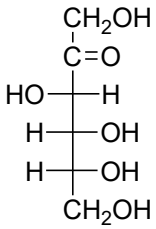
(B) phenyl acetate.

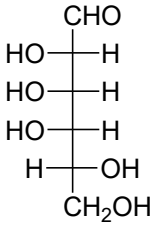
(C) acetaldehyde.

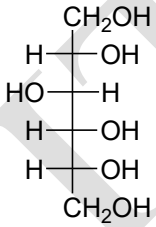
(D) acetophenone.

Q.33 Among the following, the compound(s) which produce the same osazone as that obtained from D-glucose, when reacted with phenylhydrazine, is/are

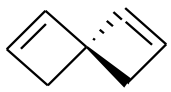
(A) 

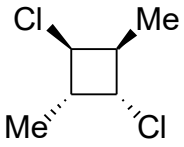
(B) 

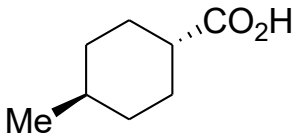
(C) 

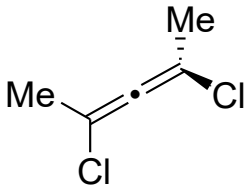
(D) 

Q.34 Among the following, the chiral molecule(s) is/are

(A) 

(B) 

(C) 

(D) 

Q.35 The correct assumption(s) required to derive Langmuir adsorption isotherm is/are

(A) Adsorption is limited to a monolayer on adsorbing surface.

(B) All binding sites on adsorbing surface are identical.

(C) Adsorption of a molecule on a site enhances binding of other molecules on neighboring sites.

(D) Rate of adsorption and rate of desorption are equal at equilibrium.

Q.36 For one mole of an ideal gas, the correct statement(s) is/are

[U = internal energy; V = volume; T = temperature; P = pressure]

(A) $\left(\frac{\partial U}{\partial V}\right)_T = 0$

(B) $\left(\frac{\partial U}{\partial T}\right)_V > 0$

(C) $\left(\frac{\partial P}{\partial T}\right)_V > 0$

(D) $\left(\frac{\partial V}{\partial P}\right)_T > 0$

Q.37 Consider the exothermic chemical reaction $O_2(g) + 2H_2(g) \rightleftharpoons 2H_2O(g)$ at equilibrium in a closed container. The correct statement(s) is/are

(A) At equilibrium, introduction of catalyst increases product formation.

(B) Equilibrium constant decreases with increase in temperature.

(C) The equilibrium constant K_P increases with pressure.

(D) Decrease in volume of reaction vessel increases product formation.

Q.38 Elements and their processes of extraction/purification are given.
The correct pair(s) is/are

(A) Na; Downs process

(B) Ni; Mond process

(C) B; Frasch process

(D) Al; Bayer process

Q.39 The correct statement(s) about the ligand substitution/exchange reaction is/are

(A) The rate is faster in the case of SF_6 than in $[AlF_6]^{3-}$.

(B) The rate is faster in the case of $[Mg(H_2O)_6]^{2+}$ than in $[Sr(H_2O)_6]^{2+}$.

(C) The rate of water exchange is faster in the case of $[Ni(H_2O)_6]^{2+}$ than in $[Co(NH_3)_5(H_2O)]^{3+}$.

(D) The rate is faster in case of $[Cr(H_2O)_6]^{2+}$ than in $[Cr(H_2O)_6]^{3+}$.

Q.40 The stretching frequency of CO in $H_3B{\cdot}CO$ is

(A) greater than the stretching frequency in free CO.

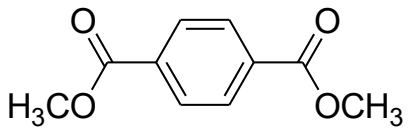
(B) lesser than the stretching frequency in free CO.

(C) lesser than the stretching frequency of CO in $Fe(CO)_5$.

(D) greater than the stretching frequency of CO in $Fe(CO)_5$.

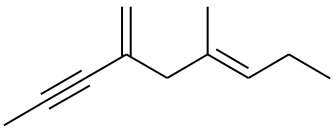
Section C: Q.41 – Q.50 Carry ONE mark each. (Numerical Answer Type)

Q.41 For the following compound



the number of signals expected in the ^{1}H NMR spectrum is _____.

Q.42 Exhaustive hydrogenation of the following compound



under Pd/C generates a saturated hydrocarbon as the product.
The number of stereoisomers possible for this product is _____.

Q.43 For a zero-order reaction P → Q, the concentration of P becomes half of its initial concentration in 30 minutes after starting the reaction.

The concentration of P becomes zero at _____ minutes.
(*rounded off to the nearest integer*)

Q.44 The magnitude of energy difference between the energy levels $n = 3$ and $n = 2$ of a quantum particle of mass m in a box of length L is $\frac{\mathrm{X}h^2}{8m\mathrm{L}^2}$.

Then X = _____.

(*rounded off to the nearest integer*)

[Given: h is Planck's constant and n denotes the quantum number]

Q.45 The function $\exp(-2(x-1)^2)$ attains a maximum at $x =$ _____.
(*rounded off to the nearest integer*)

Q.46 0.1 M aqueous solution of a weak monobasic acid has pH 2.0. The pK_a of the monobasic acid is _____. (*rounded off to one decimal place*)

Q.47 The enthalpy change for the reaction

$C(g) + \frac{1}{2}O_2(g) \rightarrow CO(g)$ is _____ kJ per mole of $CO(g)$ produced.

(*rounded off to one decimal place*)

[Given:

$C(g) + O_2(g) \rightarrow CO_2(g)$, $\Delta H_{rxn} = -393.5$ kJ per mole of $CO_2(g)$ produced

$CO_2(g) \rightarrow CO(g) + \frac{1}{2}O_2(g)$, $\Delta H_{rxn} = 283.0$ kJ per mole of $CO(g)$ produced]

Q.48 The N–O bond order in $[NO]^-$ is _____.

Q.49 The bond length of CO is 113 pm and its dipole moment ($\vec{\mu}$) is 0.1 D. The charge (in units of electronic charge) on carbon in the CO molecule *including its sign* is _____. (*rounded off to three decimal places*)

[Given: charge of electron = 1.602×10^{-19} C; 1 D = 3.336×10^{-30} C m]

Q.50 The magnetic moment of O_3 molecule is _____ Bohr magneton (B.M.).

(*rounded off to the nearest integer*)

Section C: Q.51 – Q.60 Carry TWO marks each. (Numerical Answer Type)

Q.51 A reaction of 10.50 g of 1,2-diphenylethane-1,2-dione with conc. NaOH followed by aqueous acidic work-up furnished 8.55 g of a carboxylic acid. The yield of the carboxylic acid in this reaction is _____ %.

(*rounded off to the nearest integer*)

Q.52 The specific rotation of an optically pure compound is +75.3 (*c* 1.0 in $CHCl_3$) at 20 °C. A synthetic sample of the same compound showed a specific rotation of +66.3 (*c* 1.0 in $CHCl_3$) at 20 °C. The enantiomeric excess (*ee*) of the synthetic sample is _____ %.
(*rounded off to the nearest integer*)

Q.53 A salt QCl of a certain metal Q is electrolyzed to its elements. 40 g of metal Q is formed at an electrode. The volume of Cl_2 formed at the other electrode at 1 atm pressure and 298 K is _____ litres. (*rounded off to one decimal place*)

[Given: The gas constant $R = 0.082$ L atm mol^{-1} K^{-1}, the molar mass of Q is 40 g mol^{-1} and Cl_2 is assumed to be an ideal gas]

Q.54 If 1 M of a dye in water transmits 50% of incident light at 400 nm, then 2 M of the dye in water transmits _____ % of the incident light at 400 nm.
(*rounded off to the nearest integer*)

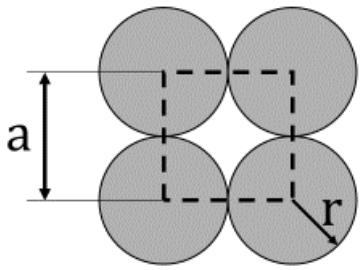
[Given: Both experiments are performed in the same spectrophotometric cell.]

Q.55 A 1.0 L solution is prepared by dissolving 2.0 g of benzoic acid and 4.0 g of sodium benzoate in water. The pH of the resulting solution is _____.
(*rounded off to one decimal place*)

Given:
Molar mass of benzoic acid is $122\ \text{g mol}^{-1}$
Molar mass of sodium benzoate is $144\ \text{g mol}^{-1}$
pK_a of benzoic acid is 4.2

Q.56 The total vapour pressure of an ideal binary liquid mixture of benzene and toluene is 0.3 bar. The vapour pressure of pure benzene is 0.5 bar and that of toluene is 0.2 bar. The mole fraction of benzene in this mixture is _____.
(*rounded off to two decimal places*)

Q.57 The unit cell of a two-dimensional square lattice with lattice parameter a is indicated by the dashed lines as shown below:



The percentage (%) area occupied by the grey circles (of radius r) inside the unit cell is _____. (*rounded off to the nearest integer*)

Q.58 In the oxidation of phosphorus with oxygen, 0.2 mol of P_4 produces _____ g of P_4O_{10}.
(*rounded off to one decimal place*)

[Given: Atomic weight of P = 31; Atomic weight of O = 16]

Q.59 An element E has three isotopes:

$^{28}_{14}$E (abundance 92.21%, atomic mass: 27.977 a.m.u.),

$^{29}_{14}$E (abundance 4.70%, atomic mass: 28.976 a.m.u.), and

$^{30}_{14}$E (abundance 3.09%, atomic mass: 29.974 a.m.u).

The atomic mass of E is _____ a.m.u.

(*rounded off to three decimal places*)

Q.60 The wavelength of the γ-ray emitted in

$^{137m}_{56}\mathrm{Ba} \rightsquigarrow {}^{137}_{56}\mathrm{Ba}$ + γ-ray (0.66 MeV)

is _____ Å. (*rounded off to three decimal places*)

[Given: $h = 6.626 \times 10^{-34}$ J s; $c = 2.998 \times 10^{8}$ m s^{-1}; 1 MeV = 1.602×10^{-13} J]

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	Which one of the following is a non-parametric test?
(A)	χ^2 - test
(B)	t - test
(C)	F - test
(D)	z - test
Q.2	Let x and y be two consumption bundles, assumed to be non-negative and perfectly divisible. Further, the assumptions of completeness, transitivity, reflexivity, non-satiation, continuity, and strict convexity are satisfied. Then, which of the following statements is NOT CORRECT?
(A)	Either $x \gtrsim y$, or $y \gtrsim x$, or both
(B)	$y \succ x$ if y contains more of at least one good and no less of any other
(C)	x is not indifferent to itself
(D)	For x (or y), its better set is strictly convex

Q.3	Consider a production function of the form: $$Y = a \log L + (1-a) \log K, \qquad a \in (0,1), \qquad a \neq 0.5$$ where, Y is output, L is labour, and K is capital. Then, the absolute value of elasticity of substitution is
(A)	1
(B)	a
(C)	$(1-a)$
(D)	∞
Q.4	Consider a closed economy with consumption function $C = 2 + 0.5Y$, where Y is income. The government expenditure is 3 and investment function is $I = 4 - 0.5r$, where r is interest rate. Then, the slope of the IS curve will be
(A)	1
(B)	-0.5
(C)	1.5
(D)	-1

Q.5	Which of the following was announced in the Union Budget 2023-24 to enhance the skills of lakhs of youth in the next 3 years?
(A)	Pradhan Mantri Kaushal Vikas Yojana (PMKVY) 1.0
(B)	Pradhan Mantri Kaushal Vikas Yojana (PMKVY) 2.0
(C)	Pradhan Mantri Kaushal Vikas Yojana (PMKVY) 3.0
(D)	Pradhan Mantri Kaushal Vikas Yojana (PMKVY) 4.0
Q.6	Suppose a random variable X follows an exponential distribution with mean 50. Then, the value of the conditional probability $P(X > 70 \mid X > 60)$ is
(A)	$e^{-\frac{7}{5}}$
(B)	$e^{-\frac{6}{5}}$
(C)	$e^{-\frac{1}{5}}$
(D)	$e^{-\frac{7}{6}}$

Q.7	Which of the following measures was NOT initiated by the Government of India as a part of economic reforms in 1991?
(A)	Announcement of new industrial policy
(B)	Full convertibility of rupee on the capital account
(C)	Removal of Quantitative Restrictions
(D)	Guidelines for investment by Foreign Institutional Investors (FIIs) in the capital market
Q.8	Suppose nominal GDP equals 1,000 units and money supply equals 250 units. Based on the quantity theory of money, the velocity of money equals
(A)	40
(B)	4
(C)	2,50,000
(D)	500

Q.9	Let $S_1 = \{(x,y) \in R^2 : x+y \geq 1,\ x+y \leq 2,\ y \geq x^2,\ x,y \geq 0\}$ and $S_2 = \{(x,y) \in R^2 : x+y \geq 1,\ x+y \leq 2,\ y \leq x^2,\ x,y \geq 0\}$. Then, which of the following is CORRECT ?
(A)	Both S_1 and S_2 are convex sets
(B)	S_1 is a convex set but S_2 is not a convex set
(C)	S_2 is a convex set but S_1 is not a convex set
(D)	Neither S_1 nor S_2 are convex sets
Q. 10	$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ is equal to
(A)	e
(B)	$1/e$
(C)	1
(D)	∞

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Two distinct integers are chosen randomly from 5 consecutive integers. If the random variable X represents the absolute difference between them, then the mean and variance of X are, respectively,
(A)	1 and $\frac{3}{2}$
(B)	2 and 5
(C)	1 and 3
(D)	2 and 1
Q.12	Consider two independent random variables: $X \sim N(5,4)$ and $Y \sim N(3,2)$. If $(2X+3Y) \sim N(\mu, \sigma^2)$, then the values of mean (μ) and variance (σ^2) are
(A)	$\mu = 19$ and $\sigma^2 = 34$
(B)	$\mu = 8$ and $\sigma^2 = 14$
(C)	$\mu = 19$ and $\sigma^2 = 14$
(D)	$\mu = 8$ and $\sigma^2 = 34$

Q. 13	The optimal value of the linear programming problem Maximise $Z = 2x + 3y$ subject to $5x + 4y \leq 20,$ $3x + 5y \leq 15,$ $2x + y \leq 4,$ $x, y \geq 0,$ is
(A)	4
(B)	$\frac{64}{7}$
(C)	9
(D)	$\frac{72}{7}$

Q.14	The solution of the differential equation $$xy\,dx - (x^2 + y^2)\,dy = 0,\ \ y(0) = 1$$ is
(A)	$y = e^{\frac{x}{y}}$
(B)	$y^2 = e^{\frac{x^2}{y^2}}$
(C)	$y^2 = e^{\frac{x}{y}}$
(D)	$y = e^{\frac{x^2}{y^2}}$
Q.15	Which of the following is NOT CORRECT?
(A)	Permanent settlement was introduced by Lord Cornwallis in Bengal in 1793
(B)	The First War of Indian Independence occurred in 1857
(C)	Dadabhai Naoroji prepared the estimate of national income in 1860
(D)	In 1905, Swadeshi Movement was started in India

Q.16

In a two-player game, player 1 can choose either M or N as strategies. Player 2 can choose either X, Y, or Z as strategies. The payoff matrix is as follows:

	X	Y	Z
M	$3, 1$	$0, 0$	$-1, 2$
N	$0, 0$	$1, 3$	$0.5, 1$

Which set of strategy profiles survives iterated elimination of strictly dominated strategies?

(A) (N, Y)

(B) (M, X)

(C) (N, Z)

(D) (M, Z)

Q.17	For a profit maximising monopolist, the ratio of the profit margin to price (also known as the Lerner Index or the relative mark-up) has a relationship with the price-elasticity of demand at the profit maximising price. Then, which of the following statements is CORRECT?
(A)	The larger the elasticity of demand at the profit maximising price, the greater is the relative mark-up
(B)	The power to sustain a price higher than the marginal cost depends only on the profit maximising price
(C)	At the profit maximising price, given costs are greater than zero, the price elasticity of demand is strictly larger than unity
(D)	At the revenue maximising price, the price elasticity of demand is greater than unity

Q.18

To study the effect of X_1 and X_2 on Y, the following regression model is estimated using a large sample:

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

The OLS estimates and standard errors are presented below:

	α	β_1	β_2
Estimates	2.30	0.39	1.80
Standard errors	1.15	0.13	1.00

Given the above information, which of the following is CORRECT?

(A) α is statistically significant at 1% level, β_1 is statistically significant at 5% level, and β_2 is statistically significant at 10% level

(B) α is statistically significant at 1% level, β_1 is statistically significant at 10% level, and β_2 is statistically significant at 5% level

(C) α is statistically significant at 5% level, β_1 is statistically significant at 1% level, and β_2 is statistically significant at 10% level

(D) α is statistically significant at 5% level, β_1 is statistically significant at 10% level, and β_2 is statistically significant at 5% level

Q.19	Suppose high quality and low quality products are sold at the same price to the buyers. The buyers have less information to determine the quality of the product compared to the sellers at the time of purchase. Which of the following problems arises in this situation?
(A)	Moral hazard problem
(B)	Market signaling problem
(C)	Principal-agent problem
(D)	Adverse selection problem
Q.20	Individuals who were either unemployed or out of labour force but had worked for at least 30 days over the reference year were included in the labour force by the NSSO in its labour force surveys. Under which one of the following classifications does the above procedure appear?
(A)	Usual Principal Status
(B)	Usual Principal and Subsidiary Status
(C)	Current Weekly Status
(D)	Current Daily Status

Q.21	Let the production function be given by $$Y_t = A_t K_t^{\alpha} H_t^{\beta} L_t^{1-\alpha-\beta}$$ where, at time t, Y_t is output, A_t is level of Total Factor Productivity, K_t is physical capital, H_t is human capital, and L_t is labour. $\alpha = \frac{1}{5}$ and $\beta = \frac{2}{5}$. If the growth rate of Y_t equals 10 percent, the growth rate of K_t equals 5 percent, the growth rate of H_t equals 5 percent, and the growth rate of L_t equals 10 percent, then the growth rate of A_t is
(A)	2 percent
(B)	3 percent
(C)	5 percent
(D)	10 percent

Q.22	Consider an economy where technology is characterised by the production function: $$Y = 50K^{0.4}L^{0.6}$$ where, Y is output, K is capital, and L is labour. Assuming perfect competition in the product market and in the factor markets, the share of total income paid to labour is equal to
(A)	0.2
(B)	0.3
(C)	0.4
(D)	0.6

Q.23 In a two-player game, player 1 can choose either U or D as strategies. Player 2 can choose either L or R as strategies. Let c be a real number such that $0 < c < 1$. If the payoff matrix is

	L	R
U	$0, 0$	$0, -c$
D	$-c, 0$	$1 - c, 1 - c$

then the number of pure strategy Nash Equilibria in the game equals

(A) 1

(B) 2

(C) 3

(D) 4

Q24	The Rangarajan Panel on 4th June 1993 submitted recommendations related to Balance of Payment (BoP). Which one of the following was NOT a part of the Panel's recommendations?
(A)	Efforts should be made to replace debt flows with equity flows
(B)	The ratio of debt linked to equity should be limited to 1:4
(C)	The minimum targets for foreign reserves should be fixed in such a way that the reserves are generally in a position to accommodate imports of 3 months
(D)	No sovereign guarantee should be extended to private sector

Q.25	According to the "State of Inequality in India Report" from the Institute for Competitiveness, released on 18th May 2022, which of the following statements is CORRECT?
(A)	In India, the percentage of anaemic children under 5 years of age has decreased from 67.1 percent in 2015-16 to 58.6 percent in 2019-21
(B)	The female labour force participation rate in India has increased from 49.8 percent in 2017-18 to 53.5 percent in 2019-20
(C)	Using data from the Periodic Labour Force Survey (PLFS) 2019-20, the report shows that individuals with monthly salary of Rs. 25,000 are among the top 10 percent of total wage earners
(D)	By the end of 2019-20, 95 percent of all schools in India have functional toilets for girls

Q.26	Consider the production function: $$Q(K,L) = \left(2\sqrt{K} + 3\sqrt{L}\right)^2$$ where Q is the output, K is capital, and L is labour. If η_K and η_L denote the output elasticities with respect to capital and labour, respectively, then the value of $(\eta_K + \eta_L)$ is
(A)	2
(B)	1
(C)	4
(D)	0.5
Q.27	Consider a short-run Phillips curve with a constant expected rate of inflation. If the aggregate demand decreases unexpectedly and the labour force remains the same, then what will happen to aggregate price and unemployment rate?
(A)	Aggregate price rises and unemployment rate falls
(B)	Aggregate price falls and unemployment rate rises
(C)	Aggregate price rises and unemployment rate rises
(D)	Aggregate price falls and unemployment rate falls

Q.28	Suppose the price elasticity of demand (e_D) is $-\frac{1}{5}$ and the price elasticity of supply (e_S) is $\frac{2}{5}$. Then, the incidence of a specific (or unit) tax on the firms is equal to
(A)	$\frac{1}{3}$
(B)	$\frac{2}{3}$
(C)	$\frac{1}{2}$
(D)	$\frac{1}{4}$

Q.29	The differential equation satisfied by circles with radius 3 and center lying on the Y-axis is
(A)	$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{9 + x^2}$
(B)	$\left(\frac{dy}{dx}\right)^2 = \frac{9 + y^2}{y^2}$
(C)	$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{9 - x^2}$
(D)	$\left(\frac{dy}{dx}\right)^2 = \frac{9 - y^2}{y^2}$

Q.30	Suppose expected inflation rate (π_t^e) of an individual is formed as: $$\pi_t^e = (1-\theta)\,\bar{\pi} + \theta\,\pi_{t-1}$$ where, $\bar{\pi}$ is constant inflation rate, π_{t-1} is previous year's inflation rate, and $0 \leq \theta \leq 1$ is weight assigned to inflation rate at different points in time. Then, which of the following is NOT CORRECT?
(A)	If $\theta = 0$, then the individual assumes a constant inflation rate
(B)	If $\theta \approx 1$ and $\bar{\pi} < \pi_{t-1}$, then the individual expects this year's inflation rate to be similar to last year
(C)	The original Phillips curve is derived under the assumption of $\theta \approx 1$
(D)	A modified Phillips curve is derived under the assumption of $\theta = 1$

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	In the case of a small open economy with fixed exchange rate regime and imperfect capital mobility, which of the following is/are CORRECT?
(A)	Fiscal contraction will lead to Balance of Payment deficit in the short-run if the slope of LM curve is greater than the slope of Balance of Payment curve
(B)	Fiscal contraction will lead to Balance of Payment deficit in the short-run if the slope of LM curve is less than the slope of Balance of Payment curve
(C)	Monetary expansion leads to Balance of Payment surplus in the short-run irrespective of the slopes of the LM curve and the Balance of Payment curve
(D)	Monetary expansion leads to Balance of Payment deficit in the short-run irrespective of the slopes of the LM curve and the Balance of Payment curve

Q.32	Consider the following three utility functions: $F = (4x_1 + 2x_2), \quad G = \min(4x_1, 2x_2)$ and $H = (\sqrt{x_1} + x_2)$ where, x_1 and x_2 are two goods available at unit prices p_{x_1} and p_{x_2}, respectively. Which of the following is/are CORRECT for the above utility functions?
(A)	The marginal rate of substitution is given by $-1, -2,$ and $-0.5\sqrt{x_1}$ for the utility functions F, G, and H, respectively
(B)	If $p_{x_1} = p_{x_2}$, then the utility maximisation problem with utility function F has a corner solution
(C)	If income is 100 and $p_{x_1} = p_{x_2} = 2$, then in the utility maximisation problem with utility function G, the sum of the optimal values of x_1 and x_2 is 50
(D)	If income is 100, $p_{x_1} = 5$, and $p_{x_2} = 5000$, then in the utility maximisation problem with the utility function H, the optimal value of x_2 is 20

Q.33	The characteristics of pure public good is/are
(A)	rival in consumption
(B)	excludable in consumption
(C)	non-rival in consumption
(D)	non-excludable in consumption

Q.34 Consider a hypothetical economy where only apples and oranges are produced for three years:

	Apples		Oranges	
Year	Quantity (Kg.)	Price (Rs. per Kg.)	Quantity (Kg.)	Price (Rs. per Kg.)
2015	10	180	5	200
2016	15	200	12	300
2017	18	250	15	350

Which of the following is/are CORRECT?

(A) Real GDP in the year 2017 (base year = 2016) is Rs. 4,250

(B) Real GDP in the year 2016 (base year = 2015) is Rs. 3,500

(C) Nominal GDP in the year 2015 is Rs. 6,600

(D) Price level, as measured by GDP deflator, increased in 2017 as compared to 2016 (base year = 2016)

Q.35	Let a random variable X has mean μ_x and non-zero variance σ_x^2, and another random variable Y has mean μ_y and non-zero variance σ_y^2. If the correlation coefficient between X and Y is ρ, then which of the following is/are CORRECT?
(A)	$\lvert\rho\rvert \leq 1$
(B)	The regression line of Y on X is $y = \mu_y + \frac{\rho\sigma_x}{\sigma_y}(x - \mu_x)$
(C)	The variance of $X - Y$ is $\sigma_x^2 + \sigma_y^2 - 2\rho\sigma_x\sigma_y$
(D)	$\rho = 0$ implies X and Y are independent random variables

Q.36	Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n > 1$ drawn from a probability distribution having mean μ and non-zero variance σ^2. Then, which of the following is/are CORRECT?
(A)	The sample mean has standard deviation $\frac{\sigma}{\sqrt{n}}$
(B)	The probability distribution of $\frac{\Sigma_{i=1}^{n}(X_i - \mu)}{\sigma\sqrt{n}}$ will tend to follow standard normal distribution as $n \to \infty$
(C)	$\frac{(n-1)S^2}{\sigma^2}$ will follow χ^2 distribution with $(n-1)$ degrees of freedom, where S^2 is the sample variance
(D)	The sample mean is always a consistent estimator of μ

Q.37	Let $M = \begin{pmatrix} \alpha & -6 \\ -1 & 1 \end{pmatrix}$, $\alpha \in R$ be a 2×2 matrix. If the eigenvalues of M are β and 4, then which of the following is/are CORRECT?
(A)	$\alpha + \beta = 1$
(B)	An eigenvector corresponding to β is $[2, 1]^T$
(C)	The rank of the matrix M is 2
(D)	The matrix $M^2 + M$ is invertible
Q. 38	Let $f : R^2 \to R$ be a function defined as $$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$ Then, which of the following is/are CORRECT?
(A)	$\lim_{(x,y) \to (0,0)} f(x,y) = 0$
(B)	$f_x(0,0) = 0$
(C)	$f(x,y)$ is not continuous at $(0,0)$
(D)	Both f_x and f_y do not exist at $(0,0)$

Q.39	Which of the following is/are NOT CORRECT?
(A)	Under the Reserve Bank of India Act, 1938, every scheduled bank has to keep certain minimum cash reserves with the RBI
(B)	CRR is the statutory reserve requirements to be kept by every scheduled bank with the RBI
(C)	A higher SLR increases the capacity of commercial banks to grant loans and advances
(D)	A high SLR can be considered as a tax on the banking system

Q.40	According to the NITI Aayog's "National Multidimensional Poverty Index: A Progress Review 2023", which of the following is/are CORRECT?
(A)	The rural areas in India have experienced fastest decline in percentage of multidimensional poverty from 35.59 percent in 2015-16 to 21.28 percent in 2019-21
(B)	The incidence of poverty in urban areas in India increased from 5.27 percent in 2015-16 to 8.65 percent in 2019-21
(C)	A decline in India's Multidimensional Poverty Index in 2019-21 is due to improvement in all the 12 indicators
(D)	At the national level, there is a decline in the intensity of poverty between 2015-16 and 2019-21

Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	A firm has a production function that is homogenous of degree one given by $Q = 2\sqrt{LK}$, where Q is quantity, L is labour and K is capital. The unit price of L is Rs. 4 and the unit price of K is Rs. 16. Assuming that there is zero fixed cost, the total cost (long run) of producing 10 units of Q is Rs. _____ (*in integer*).
Q.42	Two students A and B are assigned to solve a problem separately. The (conditional) probability that A can solve the problem given that B cannot solve it, is $\frac{1}{5}$. The (conditional) probability that B can solve the problem given that A can solve the problem is $\frac{3}{5}$. The probability that A can solve the problem is $\frac{1}{10}$. Then, the probability that B can solve the problem is _____ (*rounded off to one decimal place*).
Q.43	Suppose the cash reserve ratio is 5 percent in a country. Assume that commercial banks keep zero excess reserve and the cash-to-deposit ratio is 5 percent. To increase the money supply by Rs. 10,500 crores, the central bank of the country should inject Rs. _____ crores (*in integer*).

Q.44	Suppose an Indian company borrowed 300 dollars from a foreign bank at the beginning of the year and repaid it in dollars along with the agreed interest rate of 12 percent per annum. At the time of borrowing, the exchange rate was Rs. 70 per dollar. Assuming zero inflation rate in both the countries, the real cost of borrowing will be zero if the exchange rate is Rs. _____ per dollar at the time of repayment (*rounded off to one decimal place*).
Q.45	There are 32 students in a class. Three courses namely English, Hindi and Mathematics are offered to them. Each student must register for at least one course. If 16 students take English, 8 students take Hindi, 18 students take Mathematics, 4 students take both English and Hindi, 5 students take both Hindi and Mathematics, and 5 students take both English and Mathematics, then the number of students who take Mathematics only is _____ (*in integer*).
Q.46	Let an inverse demand function for a commodity be $p = e^{-\frac{x}{2}}$, where x is the quantity and p is the price. Then, at $p = 0.5$, the consumer surplus is equal to _____ (*rounded off to two decimal places*).

Q.47	The linear system of equations $$x + y = 3,$$ $$x + (k^2 - 8)y = k, \quad k \in R$$ has no solution for $k =$ _____ (*in integer*).
Q.48	A manufacturer producing pens has the following information regarding the cost of production of pens: Output (Q): 1, 2, 3 Total Costs (TC): 4, 13, 32 If the total cost function is of the form $TC(Q) = aQ^2 + bQ + c$ where a, b, and c are constants, then the value of $TC(Q)$ at $Q = 4$ is _____ (*in integer*).

Q.49 Consider the information given in the table below:

Year	Unemployment Rate (in percent)	Number of unemployed (in millions)	Labour Force Participation Rate (in percent)
2010	15	30	70
2020	20	50	80

The percentage change in working-age population from 2010 to 2020 is _____ (*rounded off to two decimal places*).

Q.50 Consider the following information:

Consumption $(C) = 250 + 0.25Y_d$, where Y_d is disposable income

Autonomous Investment $(I_0) = 100$

Government Expenditure $(G_0) = 50$

Income tax rate $(t) = 20$ percent

The equilibrium level of consumption in the economy is _____ (*in integer*).

Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	An individual owns a mobile phone, currently valued at Rs. 40,000. The current wealth of the individual is Rs. 2,00,000 (including the value of the mobile phone). According to reports, there is a 20 percent chance of mobile phone theft and an actuarially fair insurance policy is available to insure the loss of the mobile phone against a theft. The individual's von-Neumann-Morgenstern utility of wealth function is given by $U(W) = \sqrt{W}$, where W is the wealth. Then, the maximum willingness to pay for such an actuarially fair insurance policy is Rs. _____ (*rounded off to nearest integer*).
Q.52	Consider the following AK model where the production function is given by $$Y = AK$$ where Y is output, K is capital, and A is a constant that reflects the level of technology. Suppose there is zero technological progress in the economy and $A = 0.50$. In the economy, the savings rate equals 0.60 and the depreciation rate for the capital stock equals 0.05. The population growth rate equals zero and the size of the labour force is normalised to 1. Based on the AK model, the steady state growth rate of output per capita in the economy equals _____ percent (*in integer*).

<table>
<tr><td></td><td></td></tr>
<tr><td>Q.53</td><td>A regression equation $Y = -2.5 + 2X$ is estimated using the following data:

<table><tr><td>Y</td><td>2</td><td>5</td><td>9</td><td>14</td></tr><tr><td>X</td><td>2</td><td>4</td><td>6</td><td>8</td></tr></table>
The coefficient of determination is _____ (rounded off to two decimal places).</td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td>Q.54</td><td>A consumer’s utility function is given by:

$$u(x_1, x_2) = (2x_1 - 1)^{0.25}(x_2 - 4)^{0.75}$$

If the consumer has a budget of 73 and the unit prices of x_1 and x_2 are given by 2 and 1, respectively, then the value of $(x_1 + x_2)$ is _____ (rounded off to two decimal places).</td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td>Q.55</td><td>An industry has 6 firms in Cournot competition. Each of the 6 firms has zero fixed costs, and a constant marginal cost equal to 20. The product is homogenous and the industry inverse demand function is given by $P = 230 - Q$, where P is the market price and Q is the industry output (sum of outputs of the 6 firms). The market price under Cournot-Nash equilibrium is equal to _____ (in integer).</td></tr>
</table>

Q.56	Let the value of a random sample drawn from a normal distribution with mean 5 and unknown standard deviation σ be $4.8, 4.5, 5.1, 5.2, 5.3, 5.5$. Then, the maximum likelihood estimate of σ^2 is _____ (*rounded off to two decimal places*).
Q.57	An economy produces a consumption good and also has a research sector which produces new ideas. Time is discrete and indexed by $t = 0, 1, 2, ...$ The production function for the consumption good is given by $$Y_t = A_t L_{yt}$$ where, at time t, Y_t is the amount of consumption good produced, A_t is the stock of existing knowledge, and L_{yt} is the amount of labour devoted to production of consumption good. It is known that $A_0 = 1$. The production function for new ideas is given by $$A_{t+1} - A_t = \frac{1}{250} A_t L_{at}$$ where L_{at} is the amount of labour devoted to production of new ideas at time t. Suppose that for all t, $L_{at} = 10$ and $L_{yt} = 90$. Then, the growth rate of the consumption good (Y_t) at $t = 50$ is _____ percent (*in integer*).

Q.58	Consider a closed economy IS-LM model. The goods and the money market equations are respectively given as follows: $$Y = 90 + 0.8Y_d - 100i + G$$ $$M_s = 750 + 0.2Y - 260i$$ where, $Y =$ national income; $Y_d =$ disposable income; $T =$ total tax given by $T = 5 + 0.2Y$; $i =$ interest rate; $G =$ government expenditure $= 300$; $M_s =$ constant money supply $= 950$. The value of T at equilibrium Y is _____ (*rounded off to the nearest integer*).
Q.59	The supply curve is given as $p = 10 + x + 0.1x^2$, where p is the market price and x is the quantity of goods supplied. The change in the producer surplus due to an increase in market price from 30 to 70 is _____ (*rounded off to nearest integer*).

Q.60	There are two goods X and Y and there are two consumers A and B in a pure exchange economy. A and B have Cobb-Douglas utility functions of the form $U_A = 2X^{0.4}Y^{0.6}$ and $U_B = X^{0.3}Y^{0.7}$, respectively. Initially, A is endowed with 50 units of good X and 20 units of good Y. Similarly, B is endowed with 50 units of good X and 20 units of good Y. If the unit price of good Y is normalised to 1, then the equilibrium unit price for good X is _____ (*rounded off to two decimal places*).

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 The plate tectonic setting of Benioff-Wadati zone is

(A) continental rift

(B) subduction zone

(C) passive margin

(D) mid-oceanic ridge

Q.2 Neutron-rich unstable nuclides undergo

(A) β^- (negatron) decay

(B) β^+ (positron) decay

(C) α-decay

(D) electron capture

Q.3 Which one of the following textures is found in alkali olivine basalt?

(A) Rapakivi

(B) Graphic

(C) Blastoporphyritic

(D) Intergranular

Q.4 The mineral assemblage found in a granulite facies metabasalt is

(A) glaucophane + lawsonite + chlorite

(B) orthopyroxene + garnet + plagioclase + clinopyroxene + quartz

(C) actinolite + albite + chlorite + epidote

(D) omphacite + garnet + quartz

Q.5 *Glossopteris* is found in which of the following formations?

(A) Raniganj

(B) Bagra

(C) Lameta

(D) Nimar Sandstone

Q.6 In a sequence of undeformed sedimentary rocks, younger rocks overlie older rocks. This conforms to the principle of

(A) superposition

(B) uniformitarianism

(C) faunal succession

(D) original horizontality

Q.7 Dropstones are found in

(A) Barakar Formation

(B) Talchir Formation

(C) Raniganj Formation

(D) Bijori Formation

Q.8 The sedimentary structure formed by unidirectional current is

(A) trough cross-bedding

(B) oscillation ripple

(C) concretion

(D) hummocky cross-stratification

Q.9 Which of the following is the precursor of petroleum?

(A) Sporinite

(B) Clarain

(C) Kerogen

(D) Vitrain

Q.10 Which of the following is an amorphous variety of SiO_2?

(A) Quartz

(B) Citrine

(C) Agate

(D) Opal

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 The name of an igneous rock having a modal composition of 55% olivine, 40% orthopyroxene and 5% plagioclase, as per the IUGS classification scheme, is

(A) gabbro

(B) troctolite

(C) lherzolite

(D) harzburgite

Q.12 Which of the following is the correct decreasing order of abundance of elements in our solar system?

(A) O > H > Fe > He

(B) O > Fe > H > He

(C) H > O > Fe > He

(D) H > He > O > Fe

Q.13 The suture of a cephalopod having smooth saddles and crenulated lobes is called

(A) orthoceratitic

(B) goniatitic

(C) ceratitic

(D) ammonitic

Q.14 Which of the following is a body fossil?

(A) Coprolite

(B) Footprint

(C) Cast

(D) Stromatolite

Q.15 Match the morphological features in Group I with the corresponding taxa in Group II.

	Group I		**Group II**
P.	Dissepiment	1.	Echinodermata
Q.	Delthyrium	2.	Trilobita
R.	Pygidium	3.	Brachiopoda
S.	Ambulacrum	4.	Anthozoa

(A) P – 4, Q – 3, R – 2, S – 1

(B) P – 4, Q – 3, R – 1, S – 2

(C) P – 2, Q – 3, R – 4, S – 1

(D) P – 3, Q – 4, R – 2, S – 1

Q.16 Match the sedimentary features/structures in Group I with the corresponding processes in Group II.

Group I	**Group II**
P. Stylolite	1. Liquefaction
Q. Pseudonodule	2. Diagenesis
R. Current crescent	3. Organo-sedimentary binding
S. Stromatolite	4. Scouring

(A) P – 2, Q – 1, R – 4, S – 3

(B) P – 2, Q – 3, R – 4, S – 1

(C) P – 3, Q – 1, R – 4, S – 2

(D) P – 4, Q – 1, R – 2, S – 3

Q.17 Match the geomorphic features in Group I with the corresponding environments in Group II.

Group I	**Group II**
P. Dreikanter	1. Glacial
Q. Cirque	2. Beach
R. Natural levee	3. Eolian
S. Berm	4. Fluvial

(A) P – 3, Q – 1, R – 4, S – 2

(B) P – 2, Q – 1, R – 4, S – 3

(C) P – 3, Q – 4, R – 1, S – 2

(D) P – 4, Q – 2, R – 3, S – 1

Q.18 The correct hierarchy of the given stratigraphic units is

(A) Group > Member > Formation > Bed

(B) Eon > Era > Epoch > Period

(C) Group > Formation > Member > Bed

(D) Eon > Era > Series > Systems

Q.19 Match the minerals in Group I with their highest order of interference color in Group II (for 0.03 mm mineral thickness).

Group I	**Group II**
P. Sillimanite	1. First order
Q. Quartz	2. Second order
R. Muscovite	3. Greater than third order
S. Calcite	4. Third order variegated

(A) P – 2, Q – 4, R – 1, S – 3

(B) P – 3, Q – 1, R – 2, S – 4

(C) P – 2, Q – 1, R – 4, S – 3

(D) P – 4, Q – 2, R – 3, S – 1

Q.20 The saturated thickness of an unconfined aquifer is defined by the distance between

(A) the ground surface and the water table

(B) the water table and the underlying confining layer

(C) the water table and the mean sea level

(D) the ground surface and the underlying confining layer

Q.21 Darcy's law quantifies the volume of groundwater flow

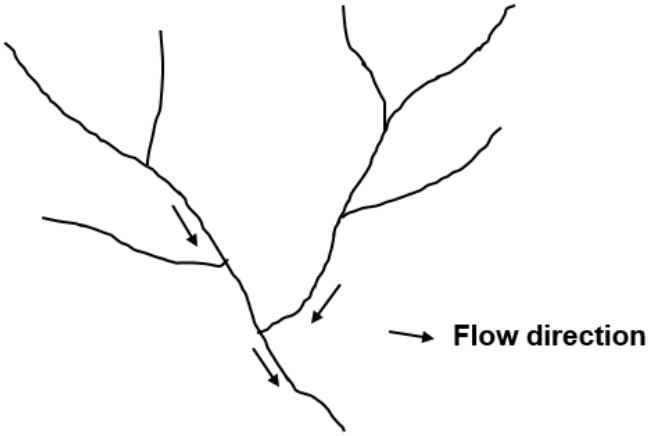
(A) per unit surface area of the aquifer

(B) per unit time

(C) per unit cross-sectional area of the aquifer

(D) per unit cross-sectional area of the aquifer per unit time

Q.22 According to Strahler's stream ordering system, what is the highest order of stream in the given diagram?



(A) 4

(B) 3

(C) 2

(D) 1

Q.23 A tunnel with vertical walls and arched crown is constructed through a set of sedimentary beds. The thickness of individual beds is significantly less than the wall-height of the tunnel. For which of the following conditions, one of the tunnel walls becomes unstable?

(A) The tunnel axis is horizontal and the beds are horizontal

(B) The tunnel axis is parallel to the strike of beds and the beds dip $45^{o} – 60^{o}$

(C) The tunnel axis is parallel to the strike of beds and the beds are vertical

(D) The tunnel axis is perpendicular to the strike of beds and the beds are vertical

Q.24 A plunging fold will NOT show a V-shaped outcrop pattern on a planar ground surface if the plunge of the fold axis is

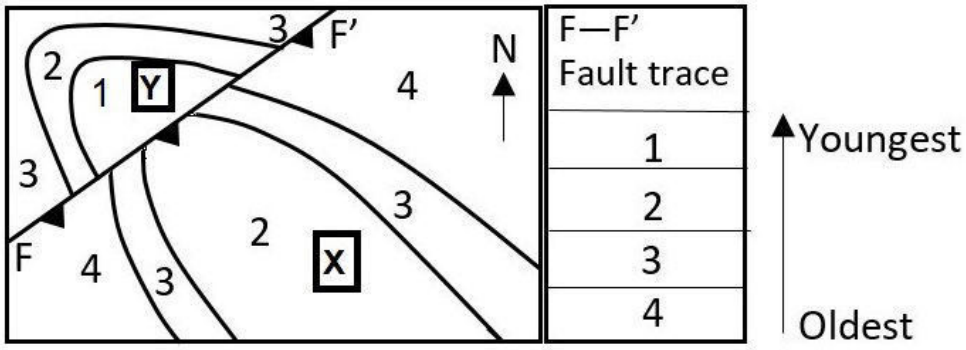
(A) equal to the dip of the ground surface in the same direction

(B) steeper than the dip of the ground surface in the same direction

(C) equal to the dip of the ground surface in the opposite direction

(D) steeper than the dip of the ground surface in the opposite direction

Q.25 The given map shows the outcrop patterns of beds (1 – 4) across a fault plane, F – F', on a flat ground. X and Y refer to the two blocks across F – F'. Which one of the following options is the correct interpretation of the structure depicted on the map?



(A) A south-easterly plunging synform that was subsequently faulted with block X upthrown

(B) A north-westerly plunging antiform that was subsequently faulted with block Y upthrown

(C) A south-easterly plunging antiform that was subsequently faulted with block X downthrown

(D) A north-westerly plunging synform that was subsequently faulted with block Y downthrown

Q.26 The crystal form ‘dome’ contains

(A) two parallel faces related by a 2-fold axis of symmetry

(B) two non-parallel faces related by a 2-fold axis of symmetry

(C) two parallel faces related by a mirror plane

(D) two non-parallel faces related by a mirror plane

Q.27 The symbols [100], {100} and (100) in a crystal represent the sequence

(A) form, line and face

(B) form, face and line

(C) line, face and form

(D) line, form and face

Q.28 Match the stratigraphic units in Group I with the corresponding Archean cratons in Group II.

Group I	**Group II**
P. Bababudan Group	1. Eastern Dharwar
Q. Banded Gneissic Complex-I	2. Western Dharwar
R. Bonai Granite	3. Aravalli
S. Kolar Group	4. Singhbhum

(A) P – 2, Q – 3, R – 4, S – 1

(B) P – 3, Q – 2, R – 1, S – 4

(C) P – 2, Q – 1, R – 4, S – 3

(D) P – 4, Q – 3, R – 2, S – 1

Q.29 Which one of the metamorphic facies sequence in order of increasing metamorphic grade defines thermal metamorphism ?

(A) Sanidinite → pyroxene hornfels → hornblende hornfels → albite-epidote hornfels

(B) Albite-epidote hornfels → hornblende hornfels → sanidinite → pyroxene hornfels

(C) Hornblende hornfels → albite-epidote hornfels → pyroxene hornfels → sanidinite

(D) Albite-epidote hornfels → hornblende hornfels → pyroxene hornfels → sanidinite

Q.30 Nickel ores are NOT associated with

(A) ultramafic igneous rocks

(B) laterites

(C) sea-floor polymetallic nodules

(D) skarns

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Which of the following statements on mantle partial melting are correct?

(A) Shallow melting produces tholeiitic basalts.

(B) Low-degree melting produces alkaline basalts.

(C) Presence of CO_2-rich volatiles favors the formation of tholeiitic basalts.

(D) Presence of H_2O-rich volatiles favors the formation of alkaline basalts.

Q.32 Which of the following fossil groups are from the Siwalik Group?

(A) Proboscidea

(B) Giraffidae

(C) Dinosauria

(D) Equidae

Q.33 The correct stratigraphic successions arranged from the oldest to the youngest are

(A) Uttatur → Trichinopoly → Ariyalur → Niniyur

(B) Chari → Patcham → Umia → Katrol

(C) Chinji → Nagri → Dhok Pathan → Tatrot

(D) Semri → Rewa → Kaimur → Bhander

Q.34 Which of the following combinations are correctly matched?

(A) Photic zone – biogenic carbonate rocks

(B) Delta – progradational coarsening-up succession

(C) Sabkha – shelf storm deposit

(D) Shelf break – submarine fans

Q.35 High drainage density is representative of a terrain with

(A) high relief

(B) arid climate

(C) impermeable surface layer

(D) permeable surface layer

Q.36 Mass-wasting processes are

(A) landslides

(B) lahars

(C) avalanches

(D) sand storms

Q.37 Which ones of the following correspond to the Pyroxene group?

(A) $CaMgSi_2O_6$

(B) $CaAl_2SiO_6$

(C) $Ca_2Si_2O_6$

(D) $NaFeSi_2O_6$

Q.38 Which of the following processes are correctly matched with corresponding deformation structures?

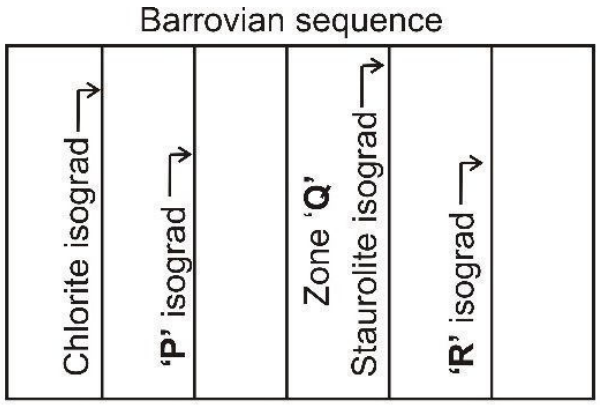
(A) Pressure solution – rock cleavage

(B) Jointing – plumose markings

(C) Layer parallel compression – buckle folds

(D) Cohesion loss – slickensides

Q.39 For the given Barrovian metamorphic sequence, which of the following statements are correct?



A) Grade of metamorphism increases from left to right.

(B) 'P' isograd is the Garnet isograd.

(C) Zone 'Q' is the Garnet zone.

(D) 'R' isograd is the Kyanite isograd.

Q.40 Which ones of the following are formed by brittle deformation?

(A) Cataclasite

(B) Breccia

(C) Mylonite

(D) Gouge

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The value of φ (*phi*) of a sediment grain having a diameter of 0.125 mm is _____. (*In integer*)

Q.42 The vertical separation of a displaced horizontal stratum along a dip-slip reverse fault is 10 m when measured on a section perpendicular to the fault-strike. If the dip of the fault is 30º, the net slip of the fault will be _____ m. (*In integer*)

Q.43 The dips of the normal and overturned limbs of a horizontal-overturned antiform are 30º and 70º, respectively. The interlimb angle of this fold is _____ degrees. (*In integer*)

Q.44 In a mineral with formula $KAl_3Si_3O_{10}(F_{0.5}OH_x)$, the value of 'x' is _____. (*Round off to one decimal place*)

Q.45 The atom percent of Fe in pyrrhotite of composition $Fe_{0.77}S$ is _____. (*Round off to two decimal places*)

Q.46 Consider the univariant metamorphic reaction Albite = Jadeite + Quartz. The minimum number of chemical components required to describe the composition of all the phases is _____. (*In integer*)

Q.47 The mean flow velocity of water in an open channel having an average depth of 0.2 m, and with Froude Number 4, is _____m/s. (*Round off to one decimal place*) (Use g = 9.8 ms^{-2})

Q.48 An aquifer has a cross-sectional area of 10 m^2 and a hydraulic conductivity of 0.25 cm/s. The volume of water that will flow per second through the aquifer for a hydraulic gradient of 0.04 is _____ cm^3. (*Round off to three decimal places*)

Q.49 The geothermal gradient in the continental crust is 0.02 ºC/m. If the surface temperature is 25 ºC, the temperature at a depth of 18 km from the surface is _____ ºC. (*In integer*)

Q.50 The area of a triangular block of a massive orebody is 1500 m^2. If the thickness of the orebody is 5 m, 6 m and 7 m at the three corners of the triangular block, and the ore density is 2.5 tons/m^3, the estimated ore reserve of the block is _____ tons. (*In integer*)

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 Clinopyroxene crystallizing from a basaltic magma has Sm concentration of 24 ppm. If the clinopyroxene-melt partition coefficient for Sm is 1.2, the concentration of Sm in the basaltic magma will be _____ ppm. (*In integer*)

Q.52 The lithostatic pressure at a depth of 36.5 km in the continental crust having an average density of 2800 kg/m^3, is _____GPa. (*Round off to the nearest integer*) (Use g = 9.8 m/s^2)

Q.53 The fraction of $^{24}_{11}Na$ atoms remaining after a decay interval of 5.0 hours will be _____. (*Round off to three decimal places*) (Use $t_{1/2}$ = 15.0 hours)

Q.54 The thickness of a dipping coal bed measured along a vertical drill hole is 15 m. If the dip of the coal bed is 30º, the orthogonal thickness of the coal bed is _____ m. (*Round off to the nearest integer*)

Q.55 The mole fraction of forsterite in olivine with MgO = 29.17 weight %, FeO = 34.65 weight % and SiO_2 = 36.18 weight % is _____. (*Round off to two decimal places*) (Use molecular weight, in g/mol, of MgO = 40.31, FeO = 71.85 and SiO_2 = 60.00)

Q.56 A partially saturated soil sample has a volume of 1200 cc. The volume of water present in the sample is 300 cc. The mass of solid in the sample is 1908 g and the particle density is 2.65 g/cc. The porosity (n) of the soil sample is _____ %. (*In integer*)

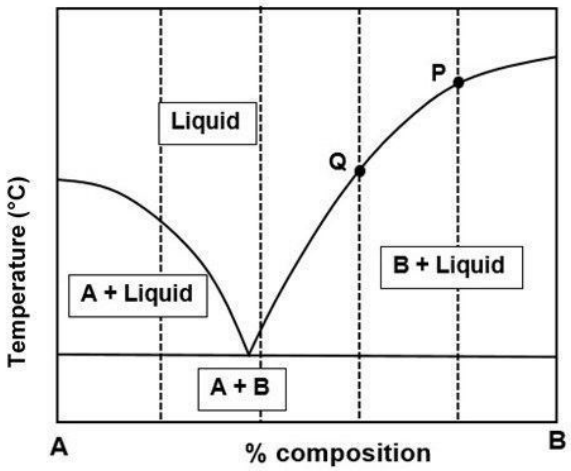
Q.57 A rock element during deformation, experienced a pressure change of 5×10^4 N/m^2, due to which its volume changed from 4 cm^3 to 3.9 cm^3. The bulk modulus of the rock is _____ $\times10^6$ N/m^2. (*In integer*)

Q.58 For an anisotropic crystal of thickness 0.04 mm and refractive indices of 1.636 and 1.486 along the slow and fast directions, respectively, the retardation produced is _____ nm. (*In integer*)

Q.59 An orebody contains pyrite and chalcopyrite in the same molar proportions. The percentage concentration of Cu in the ore will be _____. (*Round off to the nearest integer*)

(Use atomic weight, in g/mol, of Cu = 63.55, Fe = 55.85, S = 32.06)

Q.60 In the given isobaric binary temperature-composition (T-X) phase diagram involving solids A and B, the fraction of melt remaining at point Q for a magma having initial composition P will be _____. (*Round off to one decimal place*)



Special Instructions / Useful Data

$\mathbb{Z}_n = \{\bar{0}, \bar{1}, \ldots, \overline{n-1}\}$ denotes the additive group of integers modulo n

$\mathbb{R} =$ the set of all real numbers

$\mathbb{N} =$ the set of all positive integers

$\mathbb{Z} =$ the set of all integers

$\mathbb{C} =$ the set of all complex numbers

$\mathbb{Q} =$ the set of all rational numbers

$\gcd(r, n) =$ the greatest common divisor of the integers r and n

$S_n =$ the symmetric group of all permutations of $\{1,2, \ldots, n\}$

A_n = the group of all even permutations in S_n

$M_n(\mathbb{C}) =$ the set of all $n \times n$ matrices with entries from $\mathbb{C}$

$M_n(\mathbb{R}) =$ the set of all $n \times n$ matrices with entries from $\mathbb{R}$

$M^T =$ the transpose of the matrix M

$I_n =$ the $n \times n$ identity matrix

$P_n(x) =$ the real vector space of polynomials, in the variable x with real coefficients and having degree at most n, together with the zero polynomial. These polynomials are regarded as functions from $\mathbb{R}$ to $\mathbb{R}$

$\binom{n}{k} =$ the binomial coefficient defined as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$f \circ g =$ the composite function defined by $(f \circ g)(x) = f\big(g(x)\big)$

$A \setminus B$ = the complement of the set B in the set A, that is, $\{x \in A : x \notin B\}$

$\log x =$ the logarithm of x to the base e for a positive number x

$\mathbb{R}^n =$ the n-dimensional Euclidean space

$A \times B$ = the Cartesian product of the sets A and B

$M^{-1} =$ the inverse of an invertible matrix M

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 Let $y_c: \mathbb{R} \to (0, \infty)$ be the solution of the Bernoulli's equation

$$\frac{dy}{dx} - y + y^3 = 0, \qquad y(0) = c > 0.$$

Then, for every $c > 0$, which one of the following is true?

(A) $\lim_{x \to \infty} y_c(x) = 0$

(B) $\lim_{x \to \infty} y_c(x) = 1$

(C) $\lim_{x \to \infty} y_c(x) = e$

(D) $\lim_{x \to \infty} y_c(x)$ does not exist

Q.2 For a twice continuously differentiable function $g: \mathbb{R} \to \mathbb{R}$, define

$$u_g(x,y) = \frac{1}{y}\int_{-y}^{y} g(x+t)dt \quad \text{for } (x,y) \in \mathbb{R}^2, \qquad y > 0.$$

Which one of the following holds for all such g?

(A) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{2}{y}\dfrac{\partial u_g}{\partial y} + \dfrac{\partial^2 u_g}{\partial y^2}$

(B) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{1}{y}\dfrac{\partial u_g}{\partial y} + \dfrac{\partial^2 u_g}{\partial y^2}$

(C) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{2}{y}\dfrac{\partial u_g}{\partial y} - \dfrac{\partial^2 u_g}{\partial y^2}$

(D) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{1}{y}\dfrac{\partial u_g}{\partial y} - \dfrac{\partial^2 u_g}{\partial y^2}$

Q.3 Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = 1 + y \sec x \quad \text{for } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

that satisfies $y(0) = 0$. Then, the value of $y\left(\frac{\pi}{6}\right)$ equals

(A) $\sqrt{3} \log\left(\frac{3}{2}\right)$

(B) $\left(\frac{\sqrt{3}}{2}\right) \log\left(\frac{3}{2}\right)$

(C) $\left(\frac{\sqrt{3}}{2}\right) \log 3$

(D) $\sqrt{3} \log 3$

Q.4 Let $\mathcal{F}$ be the family of curves given by

$$x^2 + 2hxy + y^2 = 1, \quad -1 < h < 1 .$$

Then, the differential equation for the family of orthogonal trajectories to $\mathcal{F}$ is

(A) $(x^2y - y^3 + y)\dfrac{dy}{dx} - (xy^2 - x^3 + x) = 0$

(B) $(x^2y - y^3 + y)\dfrac{dy}{dx} + (xy^2 - x^3 + x) = 0$

(C) $(x^2y + y^3 + y)\dfrac{dy}{dx} - (xy^2 + x^3 + x) = 0$

(D) $(x^2y + y^3 + y)\dfrac{dy}{dx} + (xy^2 + x^3 + x) = 0$

Q.5 Let G be a group of order 39 such that it has exactly one subgroup of order 3 and exactly one subgroup of order 13. Then, which one of the following statements is TRUE?

(A) G is necessarily cyclic

(B) G is abelian but need not be cyclic

(C) G need not be abelian

(D) G has 13 elements of order 13

Q.6 For a positive integer n, let $U(n) = \{\bar{r} \in \mathbb{Z}_n : \gcd(r, n) = 1\}$ be the group under multiplication modulo n. Then, which one of the following statements is TRUE?

(A) $U(5)$ is isomorphic to $U(8)$

(B) $U(10)$ is isomorphic to $U(12)$

(C) $U(8)$ is isomorphic to $U(10)$

(D) $U(8)$ is isomorphic to $U(12)$

Q.7 Which one of the following is TRUE for the symmetric group S_{13}?

(A) S_{13} has an element of order 42

(B) S_{13} has no element of order 35

(C) S_{13} has an element of order 27

(D) S_{13} has no element of order 60

Q.8 Let G be a finite group containing a non-identity element which is conjugate to its inverse. Then, which one of the following is TRUE?

(A) The order of G is necessarily even

(B) The order of G is not necessarily even

(C) G is necessarily cyclic

(D) G is necessarily abelian but need not be cyclic

Q.9 Consider the following statements.

P: If a system of linear equations $Ax = b$ has a unique solution, where A is an $m \times n$ matrix and b is an $m \times 1$ matrix, then $m = n$.

Q: For a subspace W of a nonzero vector space V, whenever $u \in V \setminus W$ and $v \in V \setminus W$, then $u + v \in V \setminus W$.

Which one of the following holds?

(A) Both P and Q are true

(B) P is true but Q is false

(C) P is false but Q is true

(D) Both P and Q are false

Q.10 Let $g: \mathbb{R} \to \mathbb{R}$ be a continuous function. Which one of the following is the solution of the differential equation

$$\frac{d^2y}{dx^2} + y = g(x) \quad \text{for} \quad x \in \mathbb{R},$$

satisfying the conditions $y(0) = 0,\ y'(0) = 1$?

(A) $y(x) = \sin x - \int_0^x \sin(x-t)\, g(t)dt$

(B) $y(x) = \sin x + \int_0^x \sin(x-t)\, g(t)dt$

(C) $y(x) = \sin x - \int_0^x \cos(x-t)\, g(t)dt$

(D) $y(x) = \sin x + \int_0^x \cos(x-t)\, g(t)dt$

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Which one of the following groups has elements of order $1, 2, 3, 4, 5$ but does not have an element of order greater than or equal to 6 ?

(A) The alternating group A_6

(B) The alternating group A_5

(C) S_6

(D) S_5

Q.12 Consider the group $G = \{A \in M_2(\mathbb{R}) : AA^T = I_2\}$ with respect to matrix multiplication. Let

$$Z(G) = \{A \in G : AB = BA, \text{ for all } \quad B \in G\}.$$

Then, the cardinality of $Z(G)$ is

(A) 1

(B) 2

(C) 4

(D) Infinite

Q.13 Let V be a nonzero subspace of the complex vector space $M_7(\mathbb{C})$ such that every nonzero matrix in V is invertible. Then, the dimension of V over $\mathbb{C}$ is

(A) 1

(B) 2

(C) 7

(D) 49

Q.14 For $n \in \mathbb{N}$, let

$$a_n = \frac{1}{(3n+2)(3n+4)} \quad \text{and} \quad b_n = \frac{n^3 + \cos(3^n)}{3^n + n^3}.$$

Then, which one of the following is TRUE?

(A) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is divergent

(B) $\sum_{n=1}^{\infty} a_n$ is divergent but $\sum_{n=1}^{\infty} b_n$ is convergent

(C) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent

(D) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

Q.15

Let $a = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \\ 0 \end{bmatrix}$. Consider the following two statements.

P: The matrix $I_4 - aa^T$ is invertible.

Q: The matrix $I_4 - 2aa^T$ is invertible.

Then, which one of the following holds?

(A) P is false but Q is true

(B) P is true but Q is false

(C) Both P and Q are true

(D) Both P and Q are false

Q.16 Let A be a 6×5 matrix with entries in $\mathbb{R}$ and B be a 5×4 matrix with entries in $\mathbb{R}$. Consider the following two statements.

P: For all such nonzero matrices A and B, there is a nonzero matrix Z such that AZB is the 6×4 zero matrix.

Q: For all such nonzero matrices A and B, there is a nonzero matrix Y such that BYA is the 5×5 zero matrix.

Which one of the following holds?

(A) Both P and Q are true

(B) P is true but Q is false

(C) P is false but Q is true

(D) Both P and Q are false

Q.17 Let $P_{11}(x)$ be the real vector space of polynomials, in the variable x with real coefficients and having degree at most 11, together with the zero polynomial. Let

$$E = \{s_0(x), s_1(x), \ldots, s_{11}(x)\}, \qquad F = \{r_0(x), r_1(x), \ldots, r_{11}(x)\}$$

be subsets of $P_{11}(x)$ having 12 elements each and satisfying

$$s_0(3) = s_1(3) = \cdots = s_{11}(3) = 0\,, \qquad r_0(4) = r_1(4) = \cdots = r_{11}(4) = 1\,.$$

Then, which one of the following is TRUE?

(A) Any such E is not necessarily linearly dependent and any such F is not necessarily linearly dependent

(B) Any such E is necessarily linearly dependent but any such F is not necessarily linearly dependent

(C) Any such E is not necessarily linearly dependent but any such F is necessarily linearly dependent

(D) Any such E is necessarily linearly dependent and any such F is necessarily linearly dependent

Q.18 For the differential equation

$$y(8x - 9y)dx + 2x(x - 3y)dy = 0\,,$$

which one of the following statements is TRUE?

(A) The differential equation is not exact and has x^2 as an integrating factor

(B) The differential equation is exact and homogeneous

(C) The differential equation is not exact and does not have x^2 as an integrating factor

(D) The differential equation is not homogeneous and has x^2 as an integrating factor

Q.19 For $x \in \mathbb{R}$, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

For $x, y \in \mathbb{R}$, define

$$\min\{x, y\} = \begin{cases} x & \text{if } x \leq y, \\ y & \text{otherwise.} \end{cases}$$

Let $f: [-2\pi, 2\pi] \to \mathbb{R}$ be defined by

$$f(x) = \sin(\min\{x, x - \lfloor x \rfloor\}) \text{ for } x \in [-2\pi, 2\pi].$$

Consider the set $S = \{x \in [-2\pi, 2\pi]: f \text{ is discontinuous at } x\}$.

Which one of the following statements is TRUE?

(A) S has 13 elements

(B) S has 7 elements

(C) S is an infinite set

(D) S has 6 elements

Q.20 Define the sequences $\{a_n\}_{n=3}^{\infty}$ and $\{b_n\}_{n=3}^{\infty}$ as

$$a_n = (\log n + \log\log n)^{\log n} \quad \text{and} \quad b_n = n^{\left(1+\frac{1}{\log n}\right)}.$$

Which one of the following is TRUE?

(A) $\sum_{n=3}^{\infty} \frac{1}{a_n}$ is convergent but $\sum_{n=3}^{\infty} \frac{1}{b_n}$ is divergent

(B) $\sum_{n=3}^{\infty} \frac{1}{a_n}$ is divergent but $\sum_{n=3}^{\infty} \frac{1}{b_n}$ is convergent

(C) Both $\sum_{n=3}^{\infty} \frac{1}{a_n}$ and $\sum_{n=3}^{\infty} \frac{1}{b_n}$ are divergent

(D) Both $\sum_{n=3}^{\infty} \frac{1}{a_n}$ and $\sum_{n=3}^{\infty} \frac{1}{b_n}$ are convergent

Q.21 For $p, q, r \in \mathbb{R}$, $r \neq 0$ and $n \in \mathbb{N}$, let

$$a_n = p^n\, n^q \left(\frac{n}{n+2}\right)^{n^2} \text{ and } b_n = \frac{n^n}{n!\; r^n}\left(\sqrt{\frac{n+2}{n}}\right).$$

Then, which one of the following statements is TRUE?

(A) If $1 < p < e^2$ and $q > 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent

(B) If $e^2 < p < e^4$ and $q > 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent

(C) If $1 < r < e$, then $\sum_{n=1}^{\infty} b_n$ is convergent

(D) If $\frac{1}{e} < r < 1$, then $\sum_{n=1}^{\infty} b_n$ is convergent

Q.22 Let $P_7(x)$ be the real vector space of polynomials, in the variable x with real coefficients and having degree at most 7, together with the zero polynomial. Let T: $P_7(x) \to P_7(x)$ be the linear transformation defined by

$$T\big(f(x)\big) = f(x) + \frac{df(x)}{dx}\ .$$

Then, which one of the following is TRUE?

(A) T is not a surjective linear transformation

(B) There exists $k \in \mathbb{N}$ such that T^k is the zero linear transformation

(C) 1 and 2 are the eigenvalues of T

(D) There exists $r \in \mathbb{N}$ such that $(T - I)^r$ is the zero linear transformation, where I is the identity map on $P_7(x)$

Q.23 For $\alpha \in \mathbb{R}$, let $y_\alpha(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 2y = \frac{1}{1+x^2} \quad \text{for } x \in \mathbb{R}$$

satisfying $y(0) = \alpha$. Then, which one of the following is TRUE?

(A) $\lim_{x\to\infty} y_\alpha(x) = 0$ for every $\alpha \in \mathbb{R}$

(B) $\lim_{x\to\infty} y_\alpha(x) = 1$ for every $\alpha \in \mathbb{R}$

(C) There exists an $\alpha \in \mathbb{R}$ such that $\lim_{x\to\infty} y_\alpha(x)$ exists but its value is different from 0 and 1

(D) There is an $\alpha \in \mathbb{R}$ for which $\lim_{x\to\infty} y_\alpha(x)$ does not exist

Q.24 Consider the following two statements.

P: There exist functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ such that f is continuous at $x = 1$ and g is discontinuous at $x = 1$ but $g \circ f$ is continuous at $x = 1$.

Q: There exist functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ such that both f and g are discontinuous at $x = 1$ but $g \circ f$ is continuous at $x = 1$.

Which one of the following holds?

(A) Both P and Q are true

(B) Both P and Q are false

(C) P is true but Q is false

(D) P is false but Q is true

Q.25 Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{(x^2+1)^2}{x^4+x^2+1} \quad \text{for } x \in \mathbb{R}.$$

Then, which one of the following is TRUE?

(A) f has exactly two points of local maxima and exactly three points of local minima

(B) f has exactly three points of local maxima and exactly two points of local minima

(C) f has exactly one point of local maximum and exactly two points of local minima

(D) f has exactly two points of local maxima and exactly one point of local minimum

Q.26 Let $f: \mathbb{R} \to \mathbb{R}$ be a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2e^x \text{ for } x \in \mathbb{R}.$$

Consider the following statements.

P: If $f(x) > 0$ for all $x \in \mathbb{R}$, then $f'(x) > 0$ for all $x \in \mathbb{R}$.

Q: If $f'(x) > 0$ for all $x \in \mathbb{R}$, then $f(x) > 0$ for all $x \in \mathbb{R}$.

Then, which one of the following holds?

(A) P is true but Q is false

(B) P is false but Q is true

(C) Both P and Q are true

(D) Both P and Q are false

Q.27 For $a > b > 0$, consider

$$D = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq a^2 \text{ and } x^2 + y^2 \geq b^2\right\}.$$

Then, the surface area of the boundary of the solid D is

(A) $4\pi(a+b)\sqrt{a^2-b^2}$

(B) $4\pi\left(a^2 - b\sqrt{a^2-b^2}\right)$

(C) $4\pi(a-b)\sqrt{a^2-b^2}$

(D) $4\pi\left(a^2 + b\sqrt{a^2-b^2}\right)$

Q.28 For $n \geq 3$, let a regular n-sided polygon P_n be circumscribed by a circle of radius R_n and let r_n be the radius of the circle inscribed in P_n. Then

$$\lim_{n\to\infty}\left(\frac{R_n}{r_n}\right)^{n^2}$$

equals

(A) $e^{(\pi^2)}$

(B) $e^{\left(\frac{\pi^2}{2}\right)}$

(C) $e^{\left(\frac{\pi^2}{3}\right)}$

(D) $e^{(2\pi^2)}$

Q.29 Let L_1 denote the line $y = 3x + 2$ and L_2 denote the line $y = 4x + 3$. Suppose that $f: \mathbb{R} \to \mathbb{R}$ is a four times continuously differentiable function such that the line L_1 intersects the curve $y = f(x)$ at exactly three distinct points and the line L_2 intersects the curve $y = f(x)$ at exactly four distinct points. Then, which one of the following is TRUE?

(A) $\frac{df}{dx}$ does not attain the value 3 on $\mathbb{R}$

(B) $\frac{d^2f}{dx^2}$ vanishes at most once on $\mathbb{R}$

(C) $\frac{d^3f}{dx^3}$ vanishes at least once on $\mathbb{R}$

(D) $\frac{df}{dx}$ does not attain the value $\frac{7}{2}$ on $\mathbb{R}$

Q.30 Define the function $f: \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = 12xy\, e^{-(2x+3y-2)}.$$

If (a, b) is the point of local maximum of f, then $f(a, b)$ equals

(A) 2

(B) 6

(C) 12

(D) 0

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers.

Then, which of the following statements is/are always TRUE?

(A) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n^2$ converges absolutely

(B) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n^3$ converges absolutely

(C) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges

(D) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^3$ converges

Q.32 Which of the following statements is/are TRUE?

(A) $\sum_{n=1}^{\infty} n \log\left(1+\frac{1}{n^3}\right)$ is convergent

(B) $\sum_{n=1}^{\infty} \left(1-\cos\left(\frac{1}{n}\right)\right) \log n$ is convergent

(C) $\sum_{n=1}^{\infty} n^2 \log\left(1+\frac{1}{n^3}\right)$ is convergent

(D) $\sum_{n=1}^{\infty} \left(1-\cos\left(\frac{1}{\sqrt{n}}\right)\right) \log n$ is convergent

Q.33 Which of the following statements is/are TRUE?

(A) The additive group of real numbers is isomorphic to the multiplicative group of positive real numbers

(B) The multiplicative group of nonzero real numbers is isomorphic to the multiplicative group of nonzero complex numbers

(C) The additive group of real numbers is isomorphic to the multiplicative group of nonzero complex numbers

(D) The additive group of real numbers is isomorphic to the additive group of rational numbers

Q.34 Let $f:(1,\infty)\to(0,\infty)$ be a continuous function such that for every $n\in\mathbb{N}$, $f(n)$ is the smallest prime factor of n. Then, which of the following options is/are CORRECT?

(A) $\lim_{x\to\infty} f(x)$ exists

(B) $\lim_{x\to\infty} f(x)$ does not exist

(C) The set of solutions to the equation $f(x)=2024$ is finite

(D) The set of solutions to the equation $f(x)=2024$ is infinite

Q.35 Let

$$S = \{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\},$$

and $f : S \to \mathbb{R}$ be given by

$$f(x, y) = 2x^2 + 3y^2 - \log x - \frac{1}{6}\log y.$$

Then, which of the following statements is/are TRUE?

(A) There is a unique point in S at which $f(x, y)$ attains a local maximum

(B) There is a unique point in S at which $f(x, y)$ attains a local minimum

(C) For each point $(x_0, y_0) \in S$, the set $\{(x, y) \in S : f(x, y) = f(x_0, y_0)\}$ is bounded

(D) For each point $(x_0, y_0) \in S$, the set $\{(x, y) \in S : f(x, y) = f(x_0, y_0)\}$ is unbounded

Q.36 The center $Z(G)$ of a group G is defined as

$$Z(G) = \{x \in G : xg = gx \text{ for all } g \in G\}.$$

Let $|G|$ denote the order of G. Then, which of the following statements is/are TRUE for any group G?

(A) If G is non-abelian and $Z(G)$ contains more than one element, then the center of the quotient group $G/Z(G)$ contains only one element

(B) If $|G| \geq 2$, then there exists a non-trivial homomorphism from $\mathbb{Z}$ to G

(C) If $|G| \geq 2$ and G is non-abelian, then there exists a non-identity isomorphism from G to itself

(D) If $|G| = p^3$, where p is a prime number, then G is necessarily abelian

Q.37 For a matrix M, let Rowspace(M) denote the linear span of the rows of M and Colspace(M) denote the linear span of the columns of M. Which of the following hold(s) for all $A, B, C \in M_{10}(\mathbb{R})$ satisfying $A = BC$?

(A) Rowspace(A) $\subseteq$ Rowspace(B)

(B) Rowspace(A) $\subseteq$ Rowspace(C)

(C) Colspace(A) $\subseteq$ Colspace(B)

(D) Colspace(A) $\subseteq$ Colspace(C)

Q.38 Define $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ as follows

$$f(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m}\ (m!)^2} \text{ and } g(x) = \frac{x}{2} \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m}\ (m+1)!\ m!} \text{ for } x \in \mathbb{R}.$$

Let $x_1, x_2, x_3, x_4 \in \mathbb{R}$ be such that $0 < x_1 < x_2$, $0 < x_3 < x_4$,

$$f(x_1) = f(x_2) = 0, \qquad f(x) \neq 0 \text{ when } x_1 < x < x_2 ,$$

$$g(x_3) = g(x_4) = 0 \quad \text{and } g(x) \neq 0 \text{ when } x_3 < x < x_4.$$

Then, which of the following statements is/are TRUE?

(A) The function f does not vanish anywhere in the interval (x_3, x_4)

(B) The function f vanishes exactly once in the interval (x_3, x_4)

(C) The function g does not vanish anywhere in the interval (x_1, x_2)

(D) The function g vanishes exactly once in the interval (x_1, x_2)

Q.39 For $0 < \alpha < 4$, define the sequence $\{x_n\}_{n=1}^{\infty}$ of real numbers as follows:

$$x_1 = \alpha \text{ and } x_{n+1} + 2 = -x_n(x_n - 4) \text{ for } n \in \mathbb{N}.$$

Which of the following is/are TRUE?

(A) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (0,1)$

(B) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (1,2)$

(C) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (2,3)$

(D) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (3,4)$

Q.40 Consider

$$G = \{m + n\sqrt{2} : m, n \in \mathbb{Z}\}$$

as a subgroup of the additive group $\mathbb{R}$.

Which of the following statements is/are TRUE?

(A) G is a cyclic subgroup of $\mathbb{R}$ under addition

(B) $G \cap I$ is non-empty for every non-empty open interval $I \subseteq \mathbb{R}$

(C) G is a closed subset of $\mathbb{R}$

(D) G is isomorphic to the group $\mathbb{Z} \times \mathbb{Z}$, where the group operation in $\mathbb{Z} \times \mathbb{Z}$ is defined by $(m_1, n_1) + (m_2, n_2) = (m_1 + m_2, n_1 + n_2)$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The area of the region

$$R = \left\{ (x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1 \text{ and } \frac{1}{4} \leq xy \leq \frac{1}{2} \right\}$$

is _____ (*rounded off to two decimal places*).

Q.42 Let $y: \mathbb{R} \to \mathbb{R}$ be the solution to the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 1$$

satisfying $y(0) = 0$ and $y'(0) = 1$.

Then, $\lim_{x \to \infty} y(x)$ equals _____ (*rounded off to two decimal places*).

Q.43 For $\alpha > 0$, let $y_\alpha(x)$ be the solution to the differential equation

$$2\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 0$$

satisfying the conditions

$$y(0) = 1, \quad y'(0) = \alpha.$$

Then, the smallest value of α for which $y_\alpha(x)$ has no critical points in $\mathbb{R}$ equals _____ (*rounded off to the nearest integer*).

Q.44 Consider the 4×4 matrix

$$M = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix}.$$

If $a_{i,j}$ denotes the $(i,j)^{\text{th}}$ entry of M^{-1}, then $a_{4,1}$ equals _____ (*rounded off to two decimal places*).

Q.45 Let $P_{12}(x)$ be the real vector space of polynomials in the variable x with real coefficients and having degree at most 12, together with the zero polynomial. Define

$$V = \left\{ f \in P_{12}(x) : f(-x) = f(x) \text{ for all } x \in \mathbb{R} \text{ and } f(2024) = 0 \right\}.$$

Then, the dimension of V is _____

Q.46 Let

$$S = \left\{ f : \mathbb{R} \to \mathbb{R} : f \text{ is a polynomial and } f\big(f(x)\big) = \big(f(x)\big)^{2024} \text{ for } x \in \mathbb{R} \right\}.$$

Then, the number of elements in S is _____

Q.47 Let $a_1 = 1, b_1 = 2$ and $c_1 = 3$. Consider the convergent sequences

$$\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty} \text{ and } \{c_n\}_{n=1}^{\infty}$$

defined as follows:

$$a_{n+1} = \frac{a_n + b_n}{2}, \qquad b_{n+1} = \frac{b_n + c_n}{2} \text{ and } c_{n+1} = \frac{c_n + a_n}{2} \text{ for } n \geq 1.$$

Then,

$$\sum_{n=1}^{\infty} b_n c_n (a_{n+1} - a_n) + \sum_{n=1}^{\infty} (b_{n+1} c_{n+1} - b_n c_n) a_{n+1}$$

equals _____ (*rounded off to two decimal places*)

Q.48 Let

$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, (x-1)^2 + y^2 \leq 1, z \geq 0\}.$$

Then, the surface area of S equals _____ (*rounded off to two decimal places*).

Q.49 Let $P_7(x)$ be the real vector space of polynomials in x with degree at most 7, together with the zero polynomial. For $r = 1, 2, \ldots, 7,$ define

$$s_r(x) = x(x-1)\cdots\big(x-(r-1)\big) \text{ and } s_0(x) = 1.$$

Consider the fact that $B = \{s_0(x), s_1(x), \ldots, s_7(x)\}$ is a basis of $P_7(x)$.

If

$$x^5 = \sum_{k=0}^{7} \alpha_{5,k}\, s_k(x)\,,$$

where $\alpha_{5,k} \in \mathbb{R}$, then $\alpha_{5,2}$ equals _____ (*rounded off to two decimal places*)

Q.50 Let

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 2 & 0 & 0 & 0 & -4 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix}.$$

If $p(x)$ is the characteristic polynomial of M, then $p(2) - 1$ equals _____

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 For $\alpha \in (-2\pi, 0)$, consider the differential equation

$$x^2 \frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + y = 0 \quad \text{for} \quad x > 0.$$

Let D be the set of all $\alpha \in (-2\pi, 0)$ for which all corresponding real solutions to the above differential equation approach zero as $x \to 0^+$. Then, the number of elements in $D \cap \mathbb{Z}$ equals _____

Q.52 The value of

$$\lim_{t\to\infty} \left(\left(\log\left(t^2 + \frac{1}{t^2} \right) \right)^{-1} \int_1^{\pi t} \frac{\sin^2 5x}{x} dx \right)$$

equals _____ (*rounded off to two decimal places*).

Q.53 Let T be the planar region enclosed by the square with vertices at the points $(0,1), (1,0), (0,-1)$ and $(-1,0)$. Then, the value of

$$\iint_T \Big(\cos(\pi(x-y)) - \cos(\pi(x+y))\Big)^2 \, dx\, dy$$

equals _____ (*rounded off to two decimal places*).

Q.54 Let

$$S = \{(x,y,z) \in \mathbb{R}^3 : \; x^2 + y^2 + z^2 < 1\}.$$

Then, the value of

$$\frac{1}{\pi} \iiint_S \Big((x-2y+z)^2 + \; (2x-y-z)^2 + (x-y+2z)^2 \Big) dxdydz$$

equals _____ (*rounded off to two decimal places*).

Q.55 For $n \in \mathbb{N}$, if

$$a_n = \frac{1}{n^3+1} + \frac{2^2}{n^3+2} + \cdots + \frac{n^2}{n^3+n}$$

then the sequence $\{a_n\}_{n=1}^{\infty}$ converges to _____ (*rounded off to two decimal places*)

Q.56 Consider the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 - 4x^2 + 4x - 6$.

For $c \in \mathbb{R}$, let

$$S(c) = \left\{ x \in \mathbb{R} : f(x) = c \right\}$$

and $|S(c)|$ denote the number of elements in $S(c)$. Then, the value of

$$|S(-7)| + |S(-5)| + |S(3)|$$

equals _____

Q.57 Let $c > 0$ be such that

$$\int_0^c e^{s^2}\, ds = 3$$

Then, the value of

$$\int_0^c \left(\int_x^c e^{x^2+y^2}\, dy \right) dx$$

equals _____(*rounded off to one decimal place*).

Q.58 For $k \in \mathbb{N}$, let $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} = 1$. A function $f: [0,1] \to \mathbb{R}$ is said to be piecewise linear with nodes $t_1, \ldots, t_k$, if for each $j = 1, 2, \ldots, k+1$, there exist $a_j \in \mathbb{R}$, $b_j \in \mathbb{R}$ such that

$$f(t) = a_j + b_j t \text{ for } t_{j-1} < t < t_j .$$

Let V be the real vector space of all real valued continuous piecewise linear functions on $[0,1]$ with nodes $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. Then, the dimension of V equals _____

Q.59 For $n \in \mathbb{N}$, let

$$a_n = \frac{1}{n^{n-1}} \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} \frac{n^k}{k+1}$$

and $\beta = \lim_{n \to \infty} a_n$. Then, the value of $\log \beta$ equals _____ (*rounded off to two decimal places*).

Q.60 Define the function $f: (-1,1) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ by

$$f(x) = \sin^{-1} x$$

Let a_6 denote the coefficient of x^6 in the Taylor series of $\left(f(x)\right)^2$ about $x = 0$. Then, the value of $9a_6$ equals _____ (*rounded off to two decimal places*).

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 The total number of Na and Cl ions per unit cell of the NaCl crystal is:

(A) 2

(B) 4

(C) 8

(D) 16

Q.2 The sum of three binary numbers, 10110.10, 11010.01, and 10101.11, in decimal system is:

(A) 70.75

(B) 70.25

(C) 70.50

(D) 69.50

Q.3 Which of the following matrices is Hermitian as well as unitary?

(A) $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$

Q.4 The divergence of a 3-dimensional vector $\frac{\hat{r}}{r^3}$ ($\hat{r}$ is the unit radial vector) is:

(A) $-\frac{1}{r^4}$

(B) Zero

(C) $\frac{1}{r^3}$

(D) $-\frac{3}{r^4}$

Q.5 The magnitudes of spin magnetic moments of electron, proton and neutron are μ_e, μ_p and μ_n, respectively. Then,

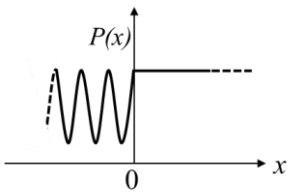
(A) $\mu_e > \mu_p > \mu_n$

(B) $\mu_e = \mu_p > \mu_n$

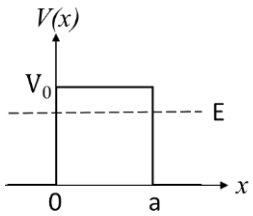
(C) $\mu_e < \mu_p < \mu_n$

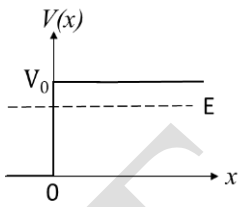
(D) $\mu_e < \mu_p = \mu_n$

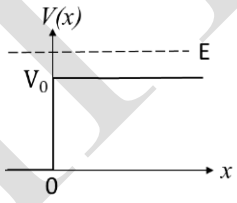
Q.6 A particle moving along the x-axis approaches $x = 0$ from $x = -\infty$ with a total energy E. It is subjected to a potential $V(x)$. For time $t \to \infty$, the probability density $P(x)$ of the particle is schematically shown in the figure.

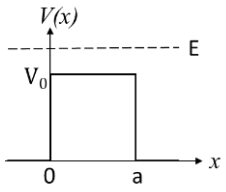


The correct option for the potential $V(x)$ is:

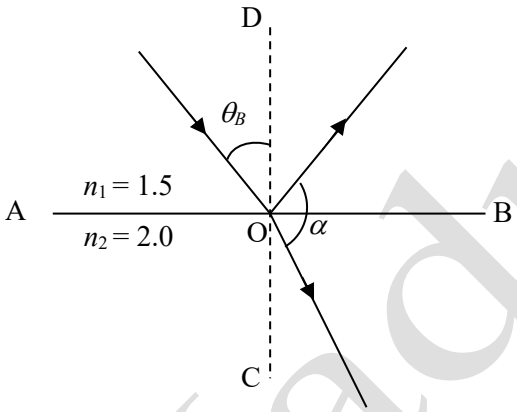
(A)


(B)


(C)


(D)


Q.7 A plane electromagnetic wave is incident on an interface AB separating two media (refractive indices $n_1 = 1.5$ and $n_2 = 2.0$) at Brewster angle θ_B, as schematically shown in the figure. The angle α (in degrees) between the reflected wave and the refracted wave is:



(A) 120

(B) 116

(C) 90

(D) 74

Q.8 If the electric field of an electromagnetic wave is given by,

$$\vec{E} = (4\hat{x} + 3\hat{y})e^{i(\omega t + ax - 600y)},$$

then the value of a is:

(all values are in the SI units)

(A) 450

(B) -450

(C) 800

(D) -800

Q.9 A vector field is expressed in the cylindrical coordinate system (s, ϕ, z) as,

$$\vec{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}.$$

If this field represents an electrostatic field, then the possible values of A and B, respectively, are:

(A) 1 and 0

(B) 0 and 1

(C) -1 and 1

(D) 1 and -1

Q.10 Which of the following types of motion may be represented by the trajectory, $y(x) = ax^2 + bx + c$?

(Here a, b, and c are constants; x, y are the position coordinates)

(A) Projectile motion in a uniform gravitational field

(B) Simple harmonic motion

(C) Uniform circular motion

(D) Motion on an inclined plane in a uniform gravitational field

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 A crystal plane of a lattice intercepts the principal axes $\vec{a}_1, \vec{a}_2,$ and $\vec{a}_3$ at $3a_1$, $4a_2$, and $2a_3$, respectively. The Miller indices of the plane are:

(A) (436)

(B) (342)

(C) (634)

(D) (243)

Q.12 The number of atoms in the *basis* of a primitive cell of hexagonal closed packed structure is:

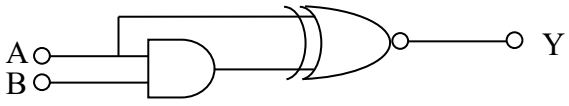
(A) 1

(B) 2

(C) 3

(D) 4

Q.13 Consider the following logic circuit.



The output Y is LOW when:

(A) *A* is HIGH and *B* is LOW

(B) *A* is LOW and *B* is HIGH

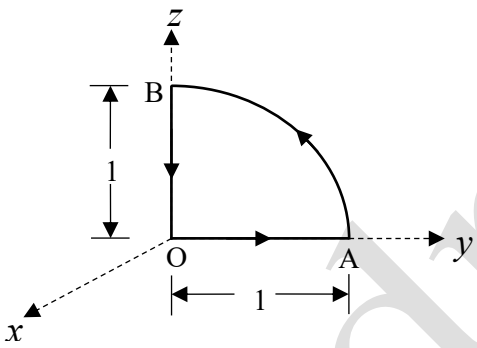
(C) Both *A* and *B* are LOW

(D) Both *A* and *B* are HIGH

Q.14 The value of the line integral for the vector,

$$\vec{v} = 2\hat{x} + yz^2\hat{y} + (3y + z^2)\hat{z}$$

along the closed path OABO (as shown in the figure) is:



(Path AB is the arc of a circle of unit radius)

(A) $\frac{1}{4}(3\pi - 1)$

(B) $3\pi - \frac{1}{4}$

(C) $\frac{3\pi}{4} - 1$

(D) $3\pi - 1$

Q.15 In the *x*-*y* plane, a vector is given by

$$\vec{F}(x,y) = \frac{-y\hat{x} + x\hat{y}}{x^2 + y^2}.$$

The magnitude of the flux of $\vec{\nabla} \times \vec{F}$, through a circular loop of radius 2, centered at the origin, is:

(A) π

(B) 2π

(C) 4π

(D) 0

Q.16 The roots of the polynomial, $f(z) = z^4 - 8z^3 + 27z^2 - 38z + 26$, are $z_1, z_2, z_3, \& z_4$, where z is a complex variable. Which of the following statements is correct?

(A) $\dfrac{z_1 + z_2 + z_3 + z_4}{z_1 z_2 z_3 z_4} = -\dfrac{4}{19}$

(B) $\dfrac{z_1 + z_2 + z_3 + z_4}{z_1 z_2 z_3 z_4} = \dfrac{4}{13}$

(C) $\dfrac{z_1 z_2 z_3 z_4}{z_1 + z_2 + z_3 + z_4} = -\dfrac{26}{27}$

(D) $\dfrac{z_1 z_2 z_3 z_4}{z_1 + z_2 + z_3 + z_4} = \dfrac{13}{19}$

Q.17 The ultraviolet catastrophe in the classical (Rayleigh-Jeans) theory of cavity radiation is attributed to the assumption that

(A) the standing waves of all allowed frequencies in the cavity have the same average energy

(B) the density of the standing waves in the cavity is independent of the shape and size of the cavity

(C) the allowed frequencies of the standing waves inside the cavity have no upper limit

(D) the number of allowed frequencies for the standing waves in a frequency range ν to $(\nu + d\nu)$ is proportional to ν^2

Q.18 Given that the rest mass of electron is $0.511\text{MeV}/c^2$, the speed (in units of c) of an electron with kinetic energy 5.11MeV is closest to:

(A) 0.996

(B) 0.993

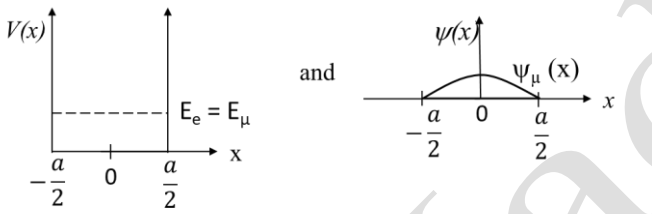
(C) 0.990

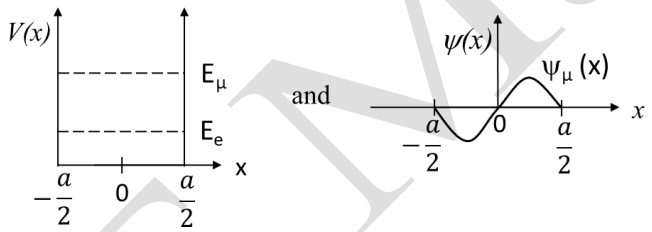
(D) 0.998

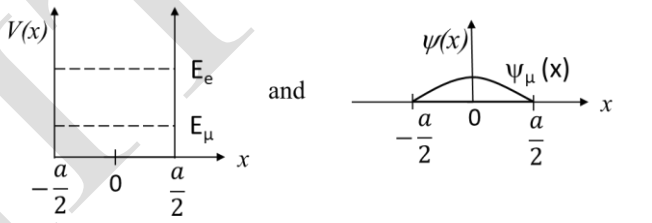
Q.19 A one-dimensional infinite square-well potential is given by:

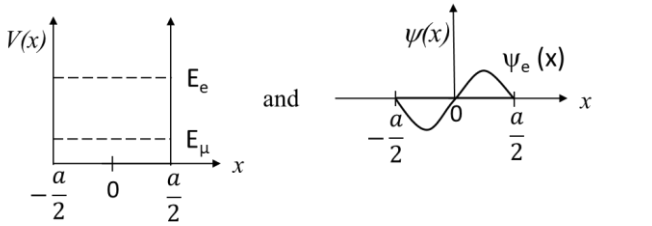
$$V(x) = 0 \quad for -\frac{a}{2} < x < +\frac{a}{2}$$
$$= \infty \quad elsewhere$$

Let $E_e(x)$ and $\psi_e(x)$ be the ground state energy and the corresponding wave function, respectively, if an electron (e) is trapped in that well. Similarly, let $E_\mu(x)$ and $\psi_\mu(x)$ be the corresponding quantities, if a muon (μ) is trapped in the well. Choose the correct option:

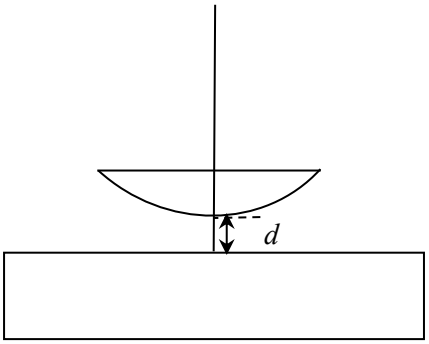
(A) 

(B) 

(C) 

(D) 

Q.20 In a Newton's rings experiment (using light of free space wavelength 580nm), there is an air gap of height d between the glass plate and a plano-convex lens (see figure). The central fringe is observed to be bright.



The least possible value of d (in nm) is:

(A) 145

(B) 290

(C) 580

(D) 72.5

Q.21 Linearly polarized light (free space wavelength λ_0= 600nm) is incident normally on a retarding plate ($n_e - n_o = 0.05$ at λ_0= 600nm). The emergent light is observed to be linearly polarized, irrespective of the angle between the direction of polarization and the optic axis of the plate. The minimum thickness (in μm) of the plate is:

(A) 6

(B) 3

(C) 2

(D) 1

Q.22 A 15.7mW laser beam has a diameter of 4mm. If the amplitude of the associated magnetic field is expressed as $\frac{A}{\sqrt{\varepsilon_0 c^3}}$, the value of A is:

(ε_0 is the free space permittivity and c is the speed of light)

(A) 50

(B) 35.4

(C) 100

(D) 70.8

Q.23 The plane $z = 0$ separates two linear dielectric media with relative permittivities $\varepsilon_{r1} = 4$ and $\varepsilon_{r2} = 3$, respectively. There is no free charge at the interface. If the electric field in the medium 1 is $\vec{E}_1 = 3\hat{x} + 2\hat{y} + 4\hat{z}$, then the displacement vector $\vec{D}_2$ in medium 2 is:

(ε_0 is the permittivity of free space)

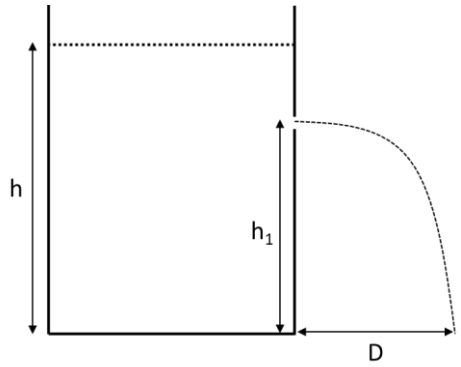
(A) $(3\hat{x} + 4\hat{y} + 6\hat{z})\varepsilon_0$

(B) $(3\hat{x} + 6\hat{y} + 8\hat{z})\varepsilon_0$

(C) $(9\hat{x} + 6\hat{y} + 16\hat{z})\varepsilon_0$

(D) $(4\hat{x} + 2\hat{y} + 3\hat{z})\varepsilon_0$

Q.24 A tank, placed on the ground, is filled with water up to a height h. A small hole is made at a height h_1 such that $h_1 < h$. The water jet emerging from the hole strikes



the ground at a horizontal distance D, as shown schematically in the figure. Which of the following statements is correct?

(g is the acceleration due to gravity)

(A) Velocity at h_1 is $\sqrt{2gh_1}$

(B) $D = 2(h - h_1)$

(C) D will be maximum when $h_1 = \frac{2}{3}h$

(D) The maximum value of D is h

Q.25 An incompressible fluid is flowing through a vertical pipe (height h and cross-sectional area A_o). A thin mesh, having n circular holes of area A_h, is fixed at the bottom end of the pipe. The speed of the fluid entering the top-end of the pipe is v_o. The volume flow rate from an individual hole of the mesh is given by:

(g is the acceleration due to gravity)

(A) $\frac{A_o}{n}\sqrt{v_o^2 + 2gh}$

(B) $\frac{A_o}{n}\sqrt{v_o^2 + gh}$

(C) $n(A_o - A_h)\sqrt{v_o^2 + 2gh}$

(D) $n(A_o - A_h)\sqrt{v_o^2 + gh}_o$

Q.26 A ball is dropped from a height h to the ground. If the coefficient of restitution is e, the time required for the ball to stop bouncing is proportional to:

(A) $\frac{2+e}{1-e}$

(B) $\frac{1+e}{1-e}$

(C) $\frac{1-e}{1+e}$

(D) $\frac{2-e}{1+e}$

Q.27 A cylinder-piston system contains N atoms of an ideal gas. If t_{avg} is the average time between successive collisions of a given atom with other atoms. If the temperature T of the gas is increased isobarically, then t_{avg} is proportional to :

(A) $\sqrt{T}$

(B) $\frac{1}{\sqrt{T}}$

(C) T

(D) $\frac{1}{T}$

Q.28 A gas consists of particles, each having three translational and three rotational degrees of freedom. The ratio of specific heats, C_p/C_v, is:

(C_p and C_v are the specific heats at constant pressure and constant volume, respectively)

(A) 5/3

(B) 7/5

(C) 4/3

(D) 3/2

Q.29 If two traveling waves, given by

$y_1 = A_0 \sin(kx - \omega t)$ and $y_2 = A_0 \sin(\alpha kx - \beta\omega t)$

are superposed, which of the following statements is correct?

(A) For $\alpha = \beta = 1$, the resultant wave is a standing wave

(B) For $\alpha = \beta = -1$, the resultant wave is a standing wave

(C) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is $\frac{3}{2}\omega$

(D) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is 3ω

Q.30 Suppose that there is a dispersive medium whose refractive index depends on the wavelength as given by $n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}$. The value of λ at which the group and phase velocities would be the same, is:

(A) $\sqrt{\frac{2b}{a}}$

(B) $\sqrt{\frac{b}{2a}}$

(C) $\sqrt{\frac{3b}{a}}$

(D) $\sqrt{\frac{b}{3a}}$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 A pure Si crystal can be converted to an *n*-type crystal by doping with

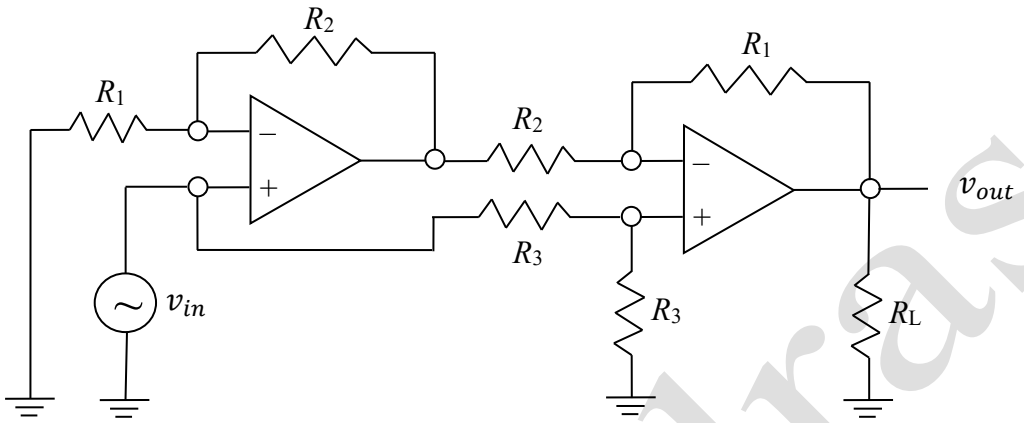
(A) P

(B) As

(C) Sb

(D) In

Q.32 In the following OP-AMP circuit, v_{in} and v_{out} represent the input and output signals, respectively.



Choose the correct statement(s):

(A) v_{out} is out-of-phase with v_{in}

(B) Gain is unity when $R_1 = R_2$

(C) v_{out} is in-phase with v_{in}

(D) v_{out} is zero

Q.33 A spring-mass system (spring constant 80N/m and damping coefficient 40N-s/m), initially at rest, is lying along the y-axis in the horizontal plane. One end of the spring is fixed and the mass (5kg) is attached at its other end. The mass is pulled along the y-axis by 0.5m from its equilibrium position and then released. Choose the correct statement(s).

(Ignore mass of the spring)

(A) Motion will be under damped

(B) Trajectory of the mass will be $y(t) = \frac{1}{2}(1 + t)e^{-4t}$

(C) Motion will be critically damped

(D) Trajectory of the mass will be $y(t) = \frac{1}{2}(1 + 4t)e^{-4t}$

Q.34 Consider two different Compton scattering experiments, in which X-rays and γ-rays of wavelength (λ) 1.024Å and 0.049Å, respectively, are scattered from stationary free electrons. The scattered wavelength (λ') is measured as a function of the scattering angle (θ). If Compton shift is $\Delta\lambda = \lambda' - \lambda$, then which of the following statement(s) is/are true:

($h = 6.63 \times 10^{-34}$J.s, $m_e = 9.11 \times 10^{-31}$kg, $c = 3 \times 10^{8}$m/s)

(A) For γ-rays, $\lambda'_{max} \approx 0.098$Å

(B) For X-rays, $(\Delta\lambda)_{max}$ is observed at $\theta = 180^{\circ}$

(C) For X-rays, $(\Delta\lambda)_{max} \approx 1.049$Å

(D) For γ-rays, at $\theta = 90^{\circ}$, $\lambda' \approx 0.049$Å

Q.35 A particle of mass m, having an energy E and angular momentum L, is in a parabolic trajectory around a planet of mass M. If the distance of the closest approach to the planet is r_m, which of the following statement(s) is(are) true?

(G is the Gravitational constant)

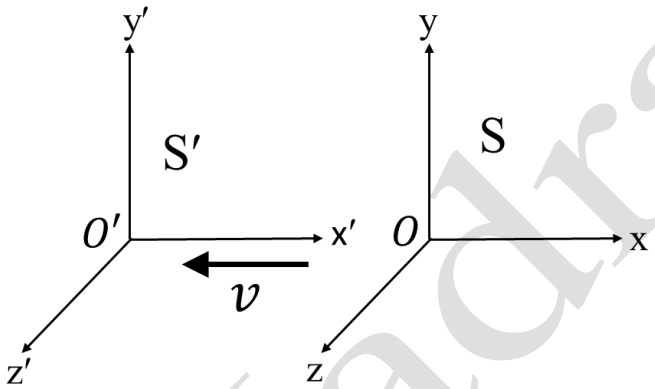
(A) $E > 0$

(B) $E = 0$

(C) $L = \sqrt{2GMm^2r_m}$

(D) $L = \sqrt{2GM^2mr_m}$

Q.36 The inertial frame S' is moving away from the inertial frame S with a speed $v = 0.6c$ along the negative x-direction (see figure). The origins O' and O of the frames coincide at $t = t' = 0$. As observed in the frame S', two events occur simultaneously at two points on the x'-axis with a separation of $\Delta x' = 5$m. If, Δt and Δx are the magnitudes of the time interval and the space interval, respectively, between the events in S, then which of the following statements is(are) correct?



($c = 3 \times 10^8$ m/s)

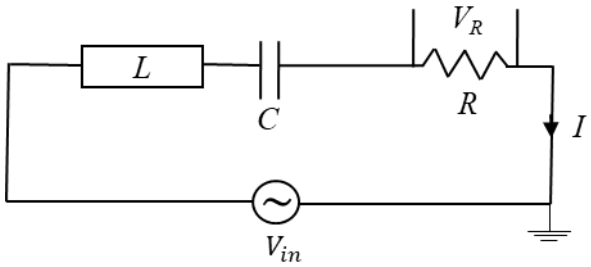
(A) $\Delta t = 12.5$ns

(B) $\Delta t = 4.2$ns

(C) $\Delta x = 10.6$m

(D) $\Delta x = 6.25$m

Q.37 For the LCR AC-circuit (resonance frequency ω_o) shown in the figure below, choose the correct statement(s).



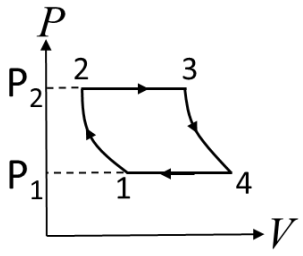
(A) ω_o depends on the values of L, C, and R

(B) At $\omega = \omega_o$, voltage V_R and current I are in-phase

(C) The amplitude of V_R at $\omega = \omega_o/2$ is independent of R

(D) The amplitude of V_R at $\omega = \omega_o$ is independent of L and C

Q.38 The P-V diagram of an engine is shown in the figure below. The temperatures at points 1, 2, 3 and 4 are T_1, T_2, T_3 and T_4, respectively. 1 $\rightarrow$2 and 3$\rightarrow$4 are adiabatic processes, and 2$\rightarrow$3 and 4$\rightarrow$1 are isochoric processes.



Identify the correct statement(s).

[γ is the ratio of specific heats C_p (at constant P) and C_v (at constant V)]

(A) $T_1T_3 = T_2T_4$

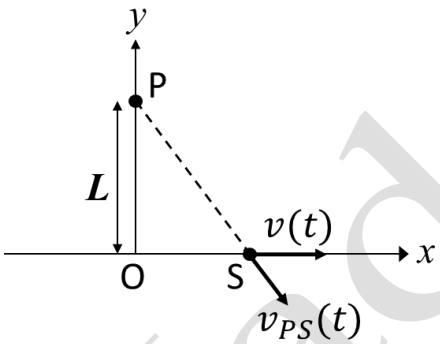
(B) The efficiency of the engine is $1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

(C) The change in entropy for the entire cycle is zero

(D) $T_1T_2 = T_3T_4$

Q.39 A whistle S of sound frequency *f* is oscillating with angular frequency ω along the *x*-axis. Its instantaneous position and the velocity are given by $x(t) = a\sin(\omega t)$ and $v(t) = v_0 \cos(\omega t)$, respectively. An observer P is located on the *y*-axis at a distance *L* from the origin (see figure). Let $v_{PS}(t)$ be the component of $v(t)$ along the line joining the source and the observer. Choose the correct option(s):

(Here a and v_0 are constants)



(A) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2 sin^2 \omega t + L^2}} \sin(2\omega t)$

(B) The observed frequency will be *f* when the source is at $x = 0$ and $x = \pm a$

(C) The observed frequency will be *f* when the source is at position $x = \pm\frac{a}{2}$

(D) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2 + L^2}} \sin(2\omega t)$

Q.40 One mole of an ideal monoatomic gas, initially at temperature T_o is expanded from an initial volume V_o to $2.5V_o$. Which of the following statements is(are) correct?

(R is the ideal gas constant)

(A) When the process is isothermal, the work done is $RT_o \ln 2$

(B) When the process is isothermal, the change in internal energy is zero

(C) When the process is isobaric, the work done is $\frac{3}{2}RT_o$

(D) When the process is isobaric, the change in internal energy is $\frac{9}{2}RT_o$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 Consider a *p*-*n* junction diode which has 10^{23} acceptor-atoms/m^3 in the *p*-side and 10^{22} donor-atoms/m^3 in the *n*-side. If the depletion width in the *p*-side is 0.16μm, then the value of depletion width in the *n*-side will be _____μm. (Rounded off to one decimal place)

Q.42 The co-ordinate system (*x,y,z*) is transformed to the system (*u,v,w*), as given by:

$$u = 2x + 3y - z$$

$$v = x - 4y + z$$

$$w = x + y$$

The Jacobian of the above transformation is _____.

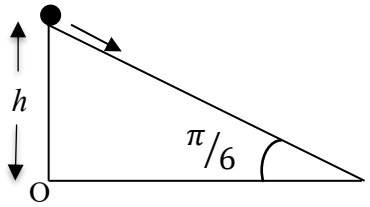
Q.43 Two sides of a triangle OAB are given by:

$$\overrightarrow{OA} = \hat{x} + 2\hat{y} + \hat{z}$$

$$\overrightarrow{OB} = 2\hat{x} - \hat{y} + 3\hat{z}$$

The area of the triangle is _____. (Rounded off to one decimal place)

Q.44 A particle of mass 1kg, initially at rest, starts sliding down from the top of a frictionless inclined plane of angle $\pi/6$ (as schematically shown in the figure). The magnitude of the torque on the particle about the point O after a time 2seconds is _____N-m. (Rounded off to nearest integer)



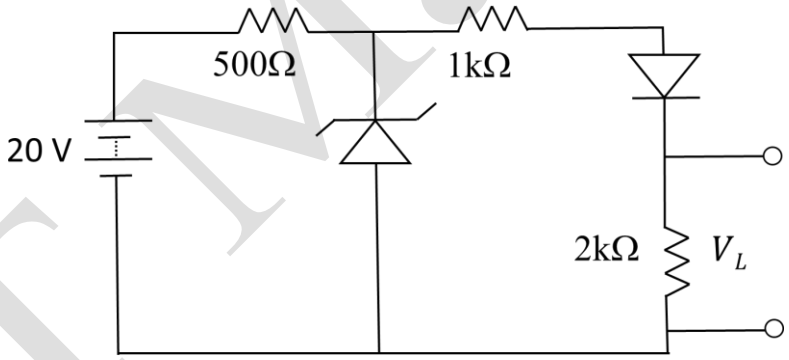
(Take $g = 10\text{m/s}^2$)

Q.45 The moment of inertia of a solid hemisphere (mass M and radius R) about the axis passing through the hemisphere and parallel to its flat surface is $\frac{2}{5}MR^2$. The distance of the axis from the center of mass of the hemisphere (in units of R) is_____. (Rounded off to two decimal places)

Q.46 A collimated light beam of intensity I_0 is incident normally on an air-dielectric (refractive index 2.0) interface. The intensity of the reflected light is _____I_0. (Rounded off to two decimal places)

Q.47 A charge of -9C is placed at the center of a concentric spherical shell made of a linear dielectric material (relative permittivity 9) and having inner and outer radii of 0.1m and 0.2m, respectively. The total charge induced on its inner surface is _____C. (Rounded off to two decimal place)

Q.48 A Zener diode (rating 10V, 2W) and a normal diode (turn-on voltage 0.7V) are connected in a circuit as shown in the figure. The voltage drop V_L across the 2kΩ resistance is _____V. (Rounded off to one decimal place)



Q.49 The Fermi energy of a system is 5.5eV. At 500K, the energy of a level for which the probability of occupancy is 0.2, is _____eV. (Rounded off to two decimal places)

(Boltzmann constant $k_B = 8.62 \times 10^{-5}$ eV/K)

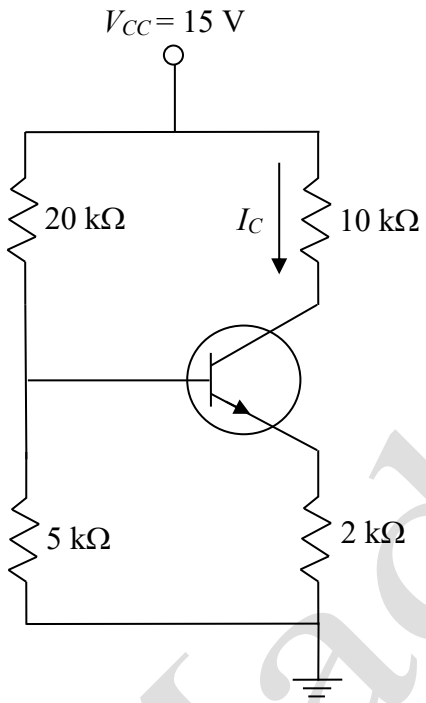
Q.50 One mole of an ideal monoatomic gas is heated in a closed container, first from 273K to 303K, and then from 303K to 373K. The net change in the entropy is _____R. (Rounded off to two decimal places)

(R is the ideal gas constant)

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 For a simple cubic crystal, the smallest inter-planar spacing d that can be determined from its second order of diffraction using monochromatic X-rays of wavelength 1.32Å is _____Å. (Round off to two decimal places)

Q.52 A transistor (β=100, $V_{BE} = 0.7V$) is connected as shown in the circuit below.



The current I_C will be _____ mA. (Rounded off to two decimal places)

Q.53 In the Taylor expansion of function, $F(x) = e^x \sin x$, around $x = 0$, the coefficient of x^5 is _____. (Rounded off to three decimal places)

Q.54 A stationary nitrogen ($^{14}_{7}N$) nucleus is bombarded with α - particle ($^{4}_{2}He$) and the following nuclear reaction takes place:

$$^{4}_{2}He + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

Mass: $4.003u$ $14.003u$ $16.999u$ $1.008u$

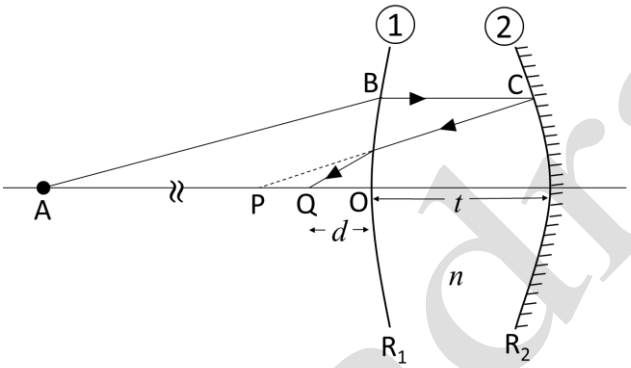
If the kinetic energies of $^{4}_{2}He$ and $^{1}_{1}H$ are 5.314MeV and 4.012MeV, respectively, then the kinetic energy of $^{17}_{8}O$ is _____MeV. (Rounded off to one decimal place)

(Masses are given in units of u = 931.5MeV/c^2)

Q.55 A satellite of mass 10kg, in a circular orbit around a planet, is having a speed v=200m/s. The total energy of the satellite is _____kJ. (Rounded off to nearest integer)

Q.56 When a system of multiple long narrow slits (width 2μm and period 4μm) is illuminated with a laser of wavelength 600nm. There are 40 minima between the two consecutive principal maxima observed in its diffraction pattern. Then maximum resolving power of the system is _____.

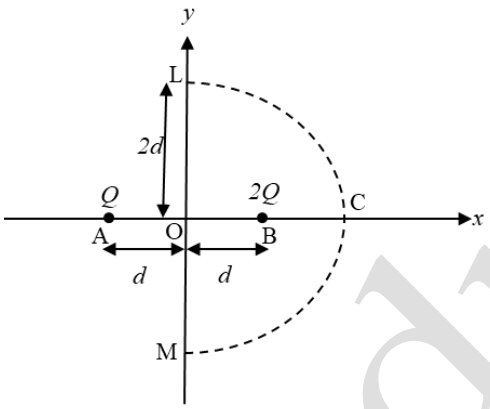
Q.57 Consider a thick biconvex lens (thickness t = 4cm and refractive index n = 1.5) whose magnitudes of the radii of curvature R_1 and R_2, of the first and second surfaces are 30cm and 20cm, respectively. Surface 2 is silvered to act as a mirror. A point object is placed at point A on the axis (OA = 60cm) as shown in the figure. If its image is formed at point Q, the distance d between O and Q is _____cm. (Rounded off to two decimal places)



Q.58 An unstable particle created at a point P moves with a constant speed of 0.998c until it decays at a point Q. If the lifetime of the particle in its rest frame is 632ns, the distance between points P and Q is _____m. (Rounded off to the nearest integer)

($c = 3 \times 10^8$ m/s)

Q.59 Two positive charges Q and $2Q$ are kept at points A and B, separated by a distance $2d$, as shown in the figure. MCL is a semicircle of radius $2d$ centered at the origin O. If Q = 2C and d = 10cm, the value of the line integral $\int_M^L \vec{E}.\overrightarrow{dl}$ (where $\vec{E}$ represents electric field) along the path MCL will be _____V.



Q.60 A time dependent magnetic field inside a long solenoid of radius 0.05m is given by $\vec{B}(t) = B_0 \sin \omega t\, \hat{z}$. If ω=100rad/s and B_0= 0.98Weber/m^2, then the amplitude of the induced electric field at a distance of 0.07m from the axis of the solenoid is _____V/m. (Rounded off to two decimal places)