IIT JAM 2022 Question Paper (All Subjects)

IIT Joint Admission Test for Masters

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	Which of the following is involved in innate immune response in higher mammals?
(A)	T cell antigen receptor
(B)	B cell antigen receptor
(C)	Toll-like receptor
(D)	Major histocompatibility complex-II molecule
Q.2	Which among the following belongs to the family "Retroviridae"?
(A)	Human Immunodeficiency virus
(B)	Ebola virus
(C)	Dengue virus
(D)	Influenza virus

Q.3	Which of the following is a glycolipid?
(A)	Cerebroside
(B)	Phosphatidylcholine
(C)	Phosphatidylserine
(D)	Cardiolipin
Q.4	Which of the following bacterial component contains “dipicolinic acid”?
(A)	Endospore
(B)	Capsule
(C)	Flagella
(D)	Pili

Q.5	The fossilization process in which mineral rich water penetrates through the pores of decomposed organic matter is known as _____.
(A)	Carbonization
(B)	Chemical fossilization
(C)	Petrifaction
(D)	Microfossilization
Q.6	A random fluctuation in gene frequency is called
(A)	Genetic drift
(B)	Genetic load
(C)	Panmixis
(D)	Genetic shift

Q.7	The number of "Barr Bodies" present in a somatic cell of a woman suffering from Turner syndrome is ____.
(A)	0
(B)	1
(C)	2
(D)	3
Q.8	Which of the following are produced by Mangrove trees to survive in the waterlogged swampy forests?
(A)	Trichomes
(B)	Pneumatophores
(C)	Spermatophores
(D)	Cambia

Q.9	Indeterminate growth in plants is due to the presence of perpetually undifferentiated tissues, called as _____.
(A)	Tracheids
(B)	Meristems
(C)	Parenchyma
(D)	Sclerenchyma
Q.10	The osmotic potential (ψ) of pure water is _____ MPa.
(A)	-1
(B)	0
(C)	0.1
(D)	10

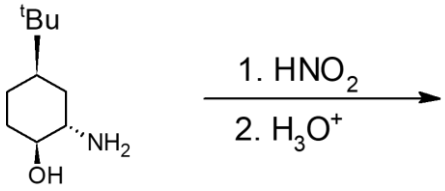
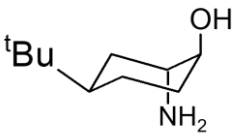
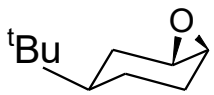
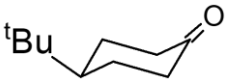
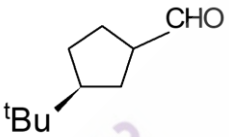
Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Bacteria containing a tuft of flagella that comes out from one pole is called ___.
(A)	Lophotrichous
(B)	Peritrichous
(C)	Monotrichous
(D)	Amphitrichous
Q.12	Which of the following activity is associated with *Klenow* fragment?
(A)	5′-3′ exonuclease activity
(B)	5′-3′ endonuclease activity
(C)	Polymerase activity
(D)	3′-5′ endonuclease activity

Q.13	A frameshift mutation is caused by ____.
(A)	5-Bromouracil
(B)	Acridine
(C)	Glutathione
(D)	Hypoxanthine
Q.14	The zone of a pond system where respiration is more than production is called as _____.
(A)	Limnetic zone
(B)	Littoral zone
(C)	Epilimnion zone
(D)	Benthic zone

Q.15	An organism that causes obstruction of lymphatic system in humans is ____.
(A)	*Borrelia burgdorferi*
(B)	*Brucella abortus*
(C)	*Yersinia pestis*
(D)	*Wuchereria bancrofti*
Q.16	A man having a dominant genetic trait (TT genotype) can taste phenylthiocarbamide (PTC), marries a woman who cannot taste PTC. The PTC tasting ability of their biological son and daughter is
(A)	Son taster; Daughter non-taster
(B)	Daughter taster; Son non-taster
(C)	Both are non-tasters
(D)	Both are tasters

Q.17	Which of the following enzymes is absent in a person suffering from Alkaptonuria?
(A)	Tyrosinase
(B)	Homogentisic acid oxidase
(C)	Catechol dioxygenase
(D)	Phenylalanine hydroxylase
Q.18	The bacterium that can tolerate high concentrations of salt and also ferment mannitol is
(A)	*Staphylococcus aureus*
(B)	*Staphylococcus epidermis*
(C)	*Streptococcus pyogenes*
(D)	*Serratia marcescens*

Q.19	Match the following Group I — Group II P) Streptomycin — 1) Inhibits beta-subunit of RNA polymerase Q) Cycloheximide — 2) Inhibits peptidyl transferase activity of 50S subunit R) Rifamycin — 3) Inhibits peptidyl transferase activity of 60S subunit S) Chloramphenicol — 4) Inhibits binding of formyl methionine tRNA to ribosome
(A)	P-1, Q-3, R-4, S-2
(B)	P-4, Q-3, R-1, S-2
(C)	P-2, Q-3, R-1, S-4
(D)	P-3, Q-4, R-1, S-2

Q.20	The major product formed in the given reaction is 
(A)	
(B)	
(C)	
(D)	

Q.21	DNA gyrase can
(A)	cut single-stranded DNA
(B)	relax supercoiled DNA
(C)	introduce negative supercoiling in DNA
(D)	not utilize ATP
Q.22	The stationary phase of cation-exchange chromatography can be
(A)	DEAE-cellulose
(B)	CM-cellulose
(C)	Sephadex G-50
(D)	Heparin-Sepharose

Q.23	Components of a Transmission Electron Microscope are
(A)	Electron gun, objective lens, positron beam, projector lens
(B)	Neutron beam, projector lens, objective lens, evacuated tube
(C)	Electron beam, projector lens, objective lens, condenser lens
(D)	X-ray beam, projector lens, objective lens, condenser lens
Q.24	In a honey bee population, the workers are infertile but protect the queen from intruders and help in reproduction. This is an example of
(A)	K selection
(B)	Sexual selection
(C)	Kin selection
(D)	Disruptive selection

Q.25	For an enzyme following Michaelis-Menten kinetics, when [S]=K_M then, the velocity v is *([S] is substrate concentration, K_M is Michaelis constant, V_{max} is maximal velocity)*
(A)	[S] × V_{max}
(B)	0.75 × V_{max}
(C)	0.5 × V_{max}
(D)	K_M × V_{max}
Q.26	The net equation for aerobic glycolysis is
(A)	Glucose+2ATP ⟶ 2 lactate+2ADP+2P_i
(B)	Glucose+2ADP+2P_i+2NAD^+ ⟶ 2 pyruvate+2ATP+2NADH+2H_2O+4H^+
(C)	Glucose+2ADP+2P_i ⟶ 2 pyruvate+2ATP+2H_2O
(D)	Glucose+2ADP+2P_i+2NAD^+ ⟶ 2 lactate+2ATP+2NADH+2H_2O+4H^+

Q. 27	In the electron transport chain, flavin mononucleotide (FMN) can adopt _____ as the highest oxidation state and is capable of accepting or donating _____ electrons, respectively.
(A)	2; 2 or 3
(B)	2; 1 or 2
(C)	3; 2 or 3
(D)	3; 1 or 2
Q.28	In bacteria, the σ factor that plays a major role in transcription during the stationary phase is
(A)	σ^{70}
(B)	σ^{54}
(C)	σ^{28}
(D)	σ^{32}

Q.29	A rise in cytosolic calcium ion concentration just after fertilization in a sea urchin egg leads to
(A)	Formation of fertilization envelope
(B)	Acrosomal reaction
(C)	Formation of vegetal pole
(D)	Formation of animal pole
Q.30	In a nephron, _____ follows the ascending limb of the “loop of Henle”.
(A)	Descending limb
(B)	Distal tubule
(C)	Collecting tubule
(D)	Proximal tubule

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Transpirational pull that extends down to the roots in plants can be interrupted by
(A)	Process of cavitation
(B)	Process of gravitation
(C)	Formation of water vapor pockets
(D)	Positive pressure in xylem sap
Q.32	Transfer of plasmids into animal cells can be achieved by
(A)	Electroporation
(B)	Liposome-mediated process
(C)	Calcium chloride treatment
(D)	Sucrose treatment

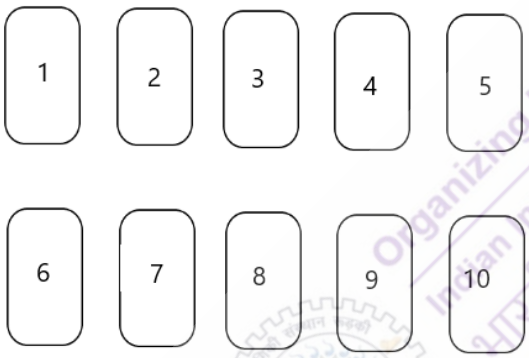
Q.33	Archaeal cell membranes contain lipids that are
(A)	Ether linked
(B)	Ester linked
(C)	Branched alkyl chain
(D)	Linear alkyl chain
Q.34	Which of the following are producers in an ecological system?
(A)	Macrophytes
(B)	Phytoplanktons
(C)	Zooplanktons
(D)	Cyanobacteria

Q.35	Which of the following acts as wound hormones in plants?
(A)	Ethylene
(B)	Cytokinins
(C)	Abscisic acid
(D)	Dextrin
Q.36	The enriched media used to facilitate the growth of fastidious microorganisms are
(A)	Selenite F broth
(B)	Blood agar
(C)	Chocolate agar
(D)	Loeffler's serum

Q.37	Match the bacterial structure to function (i) Cell wall (a) Virulence factor (ii) Glycocalyx (b) Selective permeability (c) Attachment to surfaces (d) Protection from osmotic lysis
(A)	(i)-(b), (ii)-(d)
(B)	(i)-(d), (ii)-(a)
(C)	(i)-(c), (ii)-(b)
(D)	(i)-(d), (ii)-(c)

Q.38	Identify the correct pairs: (i) Thermophile (a) grows optimal at 37 °C (ii) Mesophile (b) grows optimal at low temperature (iii) Psychrophile (c) grows optimal at high saline conditions (iv) Halophile (d) grows optimal at 67 °C
(A)	(i)-(d)
(B)	(ii)-(b)
(C)	(iii)-(a)
(D)	(iv)-(c)
Q.39	A single copy of an allele in sickle-cell heterozygous individuals reduces the frequency and severity of malaria. The reason for this is
(A)	Low oxygen binding capacity of hemoglobin
(B)	Single amino acid substitution in hemoglobin deforms the red blood cells
(C)	Abnormal hemoglobin is toxic for malaria parasite
(D)	Malaria parasite escapes the deformed red blood cells

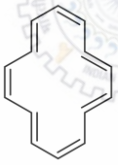
Q.40	The correct statement/s for bimolecular nucleophilic substitution reactions is/are
(A)	It goes through a carbocation formation
(B)	There is an inversion of configuration if the reacting center is chiral
(C)	Reaction is enhanced when carried out in polar solvents
(D)	The reaction intermediate is trigonal bipyramidal

Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	A deck of ten cards is given to you as shown below in the figure. One card is drawn at random from this deck. The probability of selecting a number less than 9 is ____. (*to one decimal place*) 
Q.42	The average of all positive even integers less than or equal to 40 is ___.
Q.43	The smallest positive (non-zero) integer *"n"* for which the expression $\left(\frac{1+i}{1-i}\right)^n = 1$ holds true, is ___.

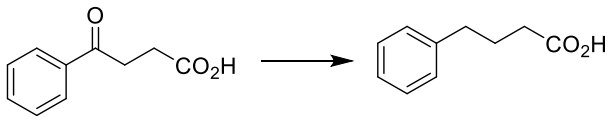
Q.44	Given that A= $(sin\theta\ cos\theta\ tan\theta + sin\theta\ cos\theta\ cot\theta)$, the value of A is
Q.45	An object is placed at the principal focus of a concave lens of focal length 10 cm. The image will be formed at _____cm, between the optical center and the focus of the lens on the same side of the object.
Q.46	What is the maximum number of hydrogen bonds that a water molecule can make in the liquid state?
Q.47	How many pairs of autosomal chromosomes are there in normal humans?

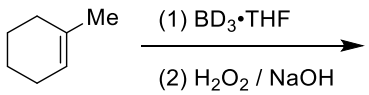
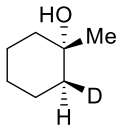
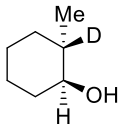
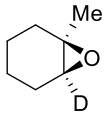
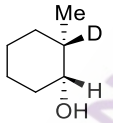
Q.48	Calculate the temperature (in K) at which the resistance of a metal becomes 20% more than its resistance at 300 K. The value of the temperature coefficient of resistance for this metal is 2.0×10^{-4} /K.
Q.49	In the ^{1}H NMR spectrum of ethanol at 400 MHz, the methyl group splits into _____ number of peaks.
Q.50	In a denaturing polyacrylamide gel electrophoresis experiment, pure intact adult human hemoglobin will yield _____(number) bands.

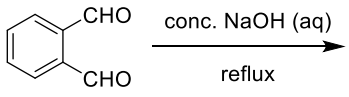
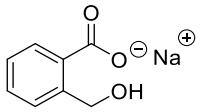
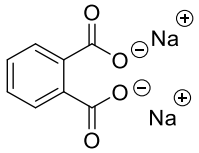
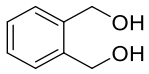
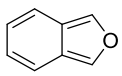
Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	A man throws a ball vertically up in the air with an initial velocity v_1 such that it reaches a height of 12 m with a speed of 12 m/s. If he throws the same ball vertically up with an initial velocity v_2 such that it reaches a maximum height of 12 m. Calculate v_1/v_2. (*up to 2 decimal places*)
Q.52	What is the acceleration due to gravity (m/s^2) on the surface of a planet if its radius is $1/4^{th}$ that of earth and its mass is $1/80^{th}$ that of earth? Assume that the gravity on the surface of the earth is 10 m/s^2.
Q.53	In a randomly mating population, the frequency of 'A' allele is 0.7. What is the frequency of 'Aa' genotype in the next generation according to Hardy-Weinberg's law? (*up to two decimal places*)
Q.54	The potential difference to accelerate an electron was quadrupled. By what factor does the *de Broglie* wavelength of the electron beam change?

Q.55	A 500 nm light is used for imaging in a confocal microscope. What will be the best resolution (in nm) of this microscope?
Q.56	Assuming the molecule shown below is aromatic, the value of "*n*" according to "Hückel's rule" is 
Q.57	In an actively growing population from a single bacterium, 1,048,576 cells are present after 20^{th} generation. How many cells were there in 5^{th} generation?

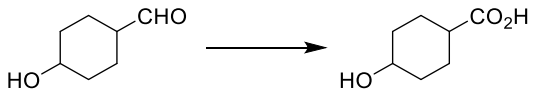
Q.58	A double stranded DNA molecule of total 5000 base pairs long, has a melting temperature of 85 °C. What will be the % AT base pairs in this sample? (*up to one decimal place*).
Q.59	How many GTP molecules are required for the translocation of tRNA from P site to E site during translation elongation process in bacteria?
Q.60	Amongst the molecules given below, the total number of molecules that have at least one sp^2 hybridized atom is _____. C_6H_6, NO_2, BF_3, H_2O_2, SO_2, C_2H_2, *L*-Tryptophan

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	The reagent required for the following transformation  is
(A)	$NaBH_4$
(B)	$LiAlH_4$
(C)	$H_3B{\cdot}THF$
(D)	Zn(Hg)/HCl

Q.2	The major product formed in the following reaction  is
(A)	 (±)
(B)	 (±)
(C)	 (±)
(D)	 (±)

Q.3	The major product formed in the following reaction  is
(A)	
(B)	
(C)	
(D)	

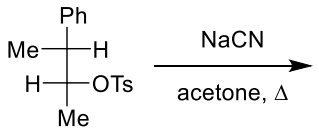
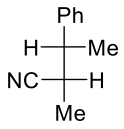
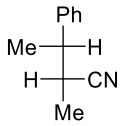
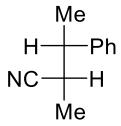
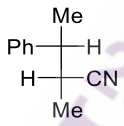
Q.4	The major product formed in the following reaction $K + O_2 \rightarrow$ is
(A)	K_2O
(B)	K_2O_2
(C)	KO_2
(D)	K_2O_3

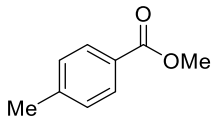
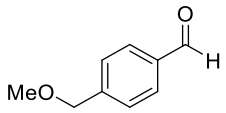
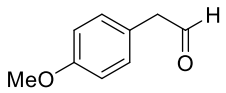
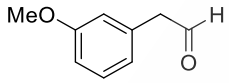
Q.5	Which one of the following options is best suited for effecting the transformation? 
(A)	MnO_2
(B)	DMSO, $(COCl)_2$, Et_3N
(C)	$Al(O\textit{i}\text{-}Pr)_3$
(D)	Ag_2O/NH_4OH

Q.6	The structure of $[XeF_8]^{2-}$ is
(A)	cubic
(B)	hexagonal bipyramid
(C)	square antiprism
(D)	octagonal

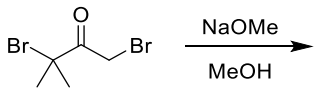
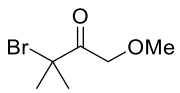
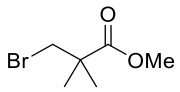
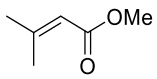
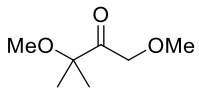
Q.7	Among the following, the compound that forms the strongest hydrogen bond is
(A)	HF
(B)	HCl
(C)	HBr
(D)	HI
Q.8	Among the following, the biomolecule with a direct metal-carbon bond is
(A)	coenzyme B_{12}
(B)	nitrogenase
(C)	chlorophyll
(D)	hemoglobin

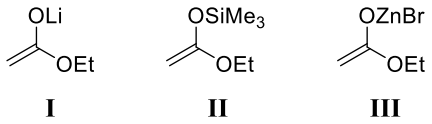
Q.9	For the reaction $H_2PO_2^-(aq) + OH^-(aq) \rightarrow HPO_3^{2-}(aq) + H_2(g)$ the rate expression is $k[H_2PO_2^-][OH^-]^2$. If the concentration of $H_2PO_2^-$ is doubled, the rate is
(A)	tripled
(B)	halved
(C)	doubled
(D)	unchanged
Q.10	The nature of interaction involved at the gas-solid interface in physisorption is
(A)	ionic
(B)	van der Waals
(C)	hydrogen bonding
(D)	covalent

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	The major product formed in the following reaction  is
(A)	
(B)	
(C)	
(D)	

Q.12	An organic compound having molecular formula $C_9H_{10}O_2$ exhibits the following spectral characteristics: ^{1}H NMR: δ 9.72 (t, 1H), 7.1 (d, 2H), 6.7 (d, 2H), 3.8 (s, 3H), 3.6 (d, 2H) IR: ~1720 cm^{-1} The most probable structure of the compound is
(A)	
(B)	
(C)	
(D)	

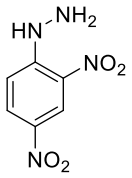
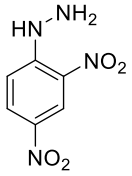
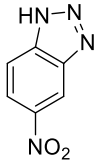
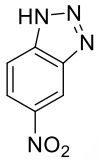
Q.13	The major product formed in the reaction of (2*S*,3*R*)-2-chloro-3-phenylbutane with NaOEt in EtOH is
(A)	(*E*)-2-phenyl-but-2-ene
(B)	(*Z*)-2-phenyl-but-2-ene
(C)	3-phenyl-but-1-ene
(D)	(2*R*,3*R*)-2-ethoxy-3-phenylbutane

Q.14	The major product formed in the following reaction  is
(A)	
(B)	
(C)	
(D)	

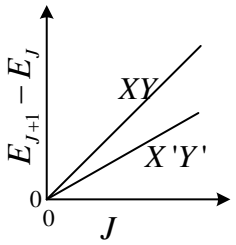
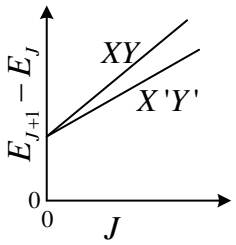
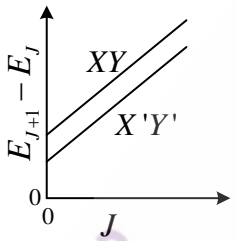
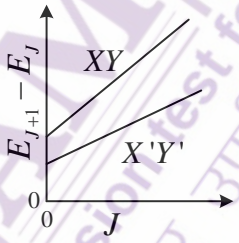
Q.15	The reactivity of the enol derivatives  **I** **II** **III** towards benzaldehyde follows the order
(A)	**I > II > III**
(B)	**III > II > I**
(C)	**II > I > III**
(D)	**I > III > II**
Q.16	All possible lattice types are observed in the
(A)	cubic crystal system
(B)	monoclinic crystal system
(C)	tetragonal crystal system
(D)	orthorhombic crystal system

Q.17	The structure types of $B_{10}H_{10}^{2-}$ and $B_{10}H_{14}$, respectively, are
(A)	*closo* and *nido*
(B)	*nido* and *arachno*
(C)	*nido* and *closo*
(D)	*closo* and *arachno*
Q.18	The ground state and the maximum number of spin-allowed electronic transitions possible in a Co^{2+} tetrahedral complex, respectively, are
(A)	$^4A_2$ and 3
(B)	$^4T_1$ and 2
(C)	$^4A_2$ and 2
(D)	$^4T_1$ and 3

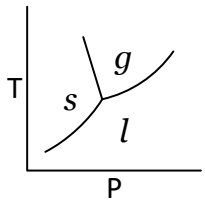
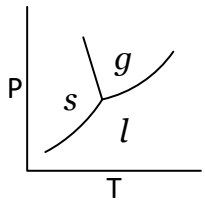
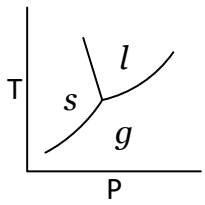
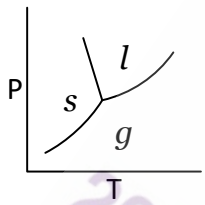
Q.19	The correct statement about the geometries of BH_2^+ and NH_2^+ based on valence shell electron pair repulsion (VSEPR) theory is
(A)	both BH_2^+ and NH_2^+ are trigonal planar
(B)	BH_2^+ is linear and NH_2^+ is trigonal planar
(C)	BH_2^+ is trigonal planar and NH_2^+ is linear
(D)	both BH_2^+ and NH_2^+ are linear
Q.20	The order of increasing CO stretching frequencies in $[Co(CO)_4]^-$, $[Cu(CO)_4]^+$, $[Fe(CO)_4]^{2-}$ and $[Ni(CO)_4]$ is
(A)	$[Cu(CO)_4]^+ < [Ni(CO)_4] < [Co(CO)_4]^- < [Fe(CO)_4]^{2-}$
(B)	$[Fe(CO)_4]^{2-} < [Co(CO)_4]^- < [Ni(CO)_4] < [Cu(CO)_4]^+$
(C)	$[Co(CO)_4]^- < [Fe(CO)_4]^{2-} < [Cu(CO)_4]^+ < [Ni(CO)_4]$
(D)	$[Ni(CO)_4] < [Cu(CO)_4]^+ < [Co(CO)_4]^- < [Fe(CO)_4]^{2-}$

Q.21	The reaction of 2,4-dinitrofluorobenzene with hydrazine produces a yellow orange solid **X** used for the identification of an organic functional group **G**. **X** and **G**, respectively, are
(A)	 and carboxylic acid
(B)	 and aldehyde
(C)	 and aldehyde
(D)	 and carboxylic acid

Q.22	The stability of adducts $H_3B{\cdot}PF_3$, $H_3B{\cdot}NMe_3$, $H_3B{\cdot}CO$, $H_3B{\cdot}OMe_2$ follows the order
(A)	$H_3B{\cdot}OMe_2 < H_3B{\cdot}CO < H_3B{\cdot}PF_3 < H_3B{\cdot}NMe_3$
(B)	$H_3B{\cdot}PF_3 < H_3B{\cdot}CO < H_3B{\cdot}NMe_3 < H_3B{\cdot}OMe_2$
(C)	$H_3B{\cdot}CO < H_3B{\cdot}PF_3 < H_3B{\cdot}NMe_3 < H_3B{\cdot}OMe_2$
(D)	$H_3B{\cdot}PF_3 < H_3B{\cdot}CO < H_3B{\cdot}OMe_2 < H_3B{\cdot}NMe_3$

Q.23	The spacing between successive rotational energy levels of a diatomic molecule *XY* and its heavier isotopic analogue *X'Y'* varies with the rotational quantum number, *J*, as
(A)	
(B)	
(C)	
(D)	

Q.24	The ratio of the 2p $\rightarrow$ 1s transition energy in He^+ to that in the H atom is closest to
(A)	1
(B)	2
(C)	4
(D)	8

Q.25	The phase diagram of water is best represented by
(A)	
(B)	
(C)	
(D)	

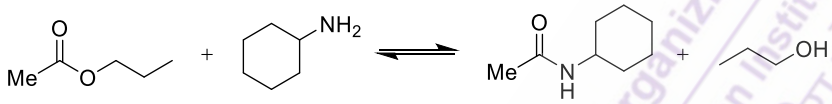
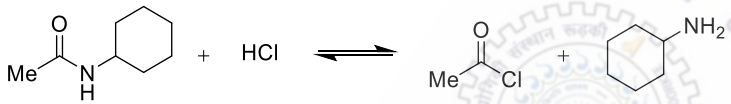
Q.26	Capillary *W* contains water and capillary *M* contains mercury. The contact angles between the capillary wall and the edge of the meniscus at the air-liquid interface in *W* and *M* are θ_W and θ_M , respectively. The contact angles satisfy the conditions
(A)	$\theta_W > 90^\circ$ and $\theta_M > 90^\circ$
(B)	$\theta_W > 90^\circ$ and $\theta_M < 90^\circ$
(C)	$\theta_W < 90^\circ$ and $\theta_M > 90^\circ$
(D)	$\theta_W < 90^\circ$ and $\theta_M < 90^\circ$

Q.27	The Maxwell-Boltzmann distribution $f(v_x)$ of one-dimensional velocities v_x at temperature T is [Given: A is a normalization constant such that $\int_{-\infty}^{\infty} f(v_x)\, dv_x = 1$, and k_B is the Boltzmann constant]
(A)	$A\exp(-mv_x^2 / 2k_BT)$
(B)	$A\exp(-mv_x^2 / k_BT)$
(C)	$Av_x^2 \exp(-mv_x^2 / 2k_BT)$
(D)	$Av_x^2 \exp(-mv_x^2 / k_BT)$

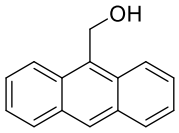
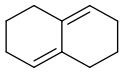
Q.28	The potential for a particle in a one-dimensional box is given as: $V(x) = 0$ for $0 \le x \le L$, and $V(x) = \infty$ elsewhere. The locations of the internal nodes of the eigenfunctions $\psi_n(x)$, $n \ge 2$, are [Given: m is an integer such that $0 < m < n$]
(A)	$x = \frac{m + \frac{1}{2}}{n} L$
(B)	$x = \frac{m}{n} L$
(C)	$x = \frac{m}{n+1} L$
(D)	$x = \frac{m+1}{n+1} L$

Q.29	The number of CO stretching bands in the infrared spectrum of $Fe(CO)_5$ is
(A)	1
(B)	2
(C)	3
(D)	4

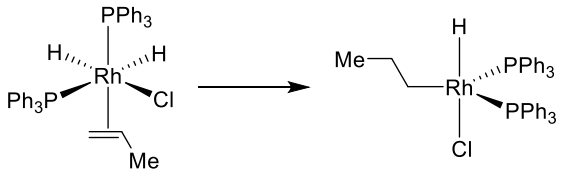
Q.30	The standard Gibbs free energy change for the reaction $H_2O\ (g) \rightarrow H_2\ (g) + \frac{1}{2}O_2\ (g)$ at 2500 K is +118 kJ mol^{-1}. The equilibrium constant for the reaction is [Given: $R = 8.314\ J\ K^{-1}\ mol^{-1}$]
(A)	0.994
(B)	1.006
(C)	3.42×10^{-3}
(D)	292.12

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Among the following, the reaction(s) that favor(s) the formation of the products at 25 °C is/are
(A)	
(B)	
(C)	
(D)	

Q.32	Among the following, the correct statement(s) is/are:
(A)	The first pK_a of malonic acid is lower than the pK_a of acetic acid while its second pK_a is higher than the pK_a of acetic acid.
(B)	The first pK_a of malonic acid is higher than the pK_a of acetic acid while its second pK_a is lower than the pK_a of acetic acid.
(C)	Both the first and the second pK_as of malonic acid are lower than the pK_a of acetic acid.
(D)	Both the first and the second pK_as of malonic acid are higher than the pK_a of acetic acid.

Q.33	The compound(s) that participate(s) in Diels-Alder reaction with maleic anhydride is/are
(A)	
(B)	
(C)	
(D)	

Q.34	Among the following, the suitable route(s) for the conversion of benzaldehyde to acetophenone is/are
(A)	CH_3COCl, anhydrous $AlCl_3$
(B)	(i) $HS(CH_2)_3SH$, $F_3B{\cdot}OEt_2$; (ii) n-BuLi; (iii) MeI; (iv) $HgCl_2$, $CdCO_3$, H_2O
(C)	$NaNH_2$, MeI
(D)	(i) MeMgBr; (ii) aq. acid; (iii) pyridinium chlorochromate (PCC)

Q.35	The reaction  involve(s)
(A)	migratory insertion
(B)	change in electron count of Rh from 18 to 16
(C)	oxidative addition
(D)	change in electron count of Rh from 16 to 18

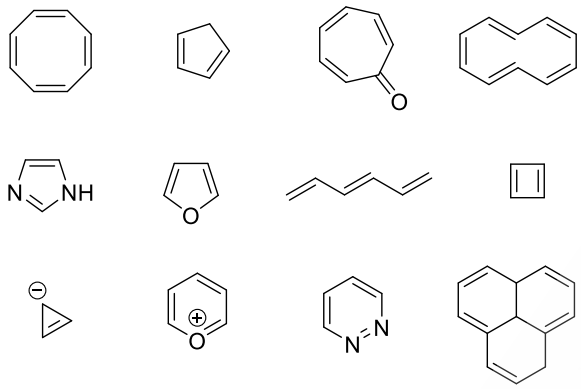
Q.36	The reason(s) for the lower stability of Si_2H_6 compared to C_2H_6 is/are
(A)	silicon is more electronegative than hydrogen
(B)	Si–Si bond is weaker than C–C bond
(C)	Si–H bond is weaker than C–H bond
(D)	the presence of low-lying d-orbitals in silicon

Q.37	For an *N*-atom nonlinear polyatomic gas, the constant volume molar heat capacity $C_{v,m}$ has the expected value of $3(N-1)R$, based on the principle of equipartition of energy. The correct statement(s) about the measured value of $C_{v,m}$ is/are
(A)	The measured $C_{v,m}$ is independent of temperature.
(B)	The measured $C_{v,m}$ is dependent on temperature.
(C)	The measured $C_{v,m}$ is typically lower than the expected value.
(D)	The measured $C_{v,m}$ is typically higher than the expected value.
Q.38	Zinc containing enzyme(s) is/are
(A)	carboxypeptidase
(B)	hydrogenase
(C)	carbonic anhydrase
(D)	urease

Q.39	The conversion of ICl to ICl^+ involve(s)
(A)	the removal of an electron from a π^* molecular orbital of ICl
(B)	an increase in the bond order from 1 in ICl to 1.5 in ICl^+
(C)	the formation of a paramagnetic species
(D)	the removal of an electron from a molecular orbital localized predominantly on Cl
Q.40	The common point defect(s) in a solid is/are
(A)	Wadsley defect
(B)	Schottky defect
(C)	Suzuki defect
(D)	Frenkel defect

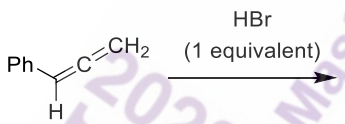
Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 Among the following

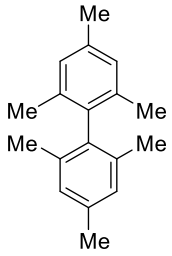
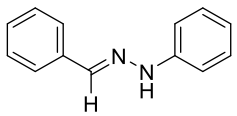


the number of aromatic compounds is _____.

Q.42 The number of stereoisomers possible for the major product formed in the reaction



is _____.

Q.43	The number of signals observed in the ^1H NMR spectrum of the compound  is _____.
Q.44	The reaction of 122 g of benzaldehyde with 108 g of phenylhydrazine gave 157 g of the product  The yield of the product is _____ %. (*round off to the nearest integer*)
Q.45	The B–B bond order in B_2 is _____.
Q.46	The number of unpaired electrons in $[Co(H_2O)_6]^{2+}$ is _____.
Q.47	The number of significant figures in 5.0820×10^2 is _____.

Q.48	The *d* spacing for the first-order X-ray (λ = 1.54 Å) diffraction event of metallic iron (*fcc*) at $2\theta = 20.2°$ is _____ Å. (*round off to three decimal places*)
Q.49	The volume fraction for an element in an *fcc* lattice is _____. (*round off to two decimal places*)
Q.50	A steady current of 1.25 A is passed through an electrochemical cell for 1.5 h using a 12 V battery. The total charge, Q, drawn during this process is _____ Coulombs. (*round off to the nearest integer*)

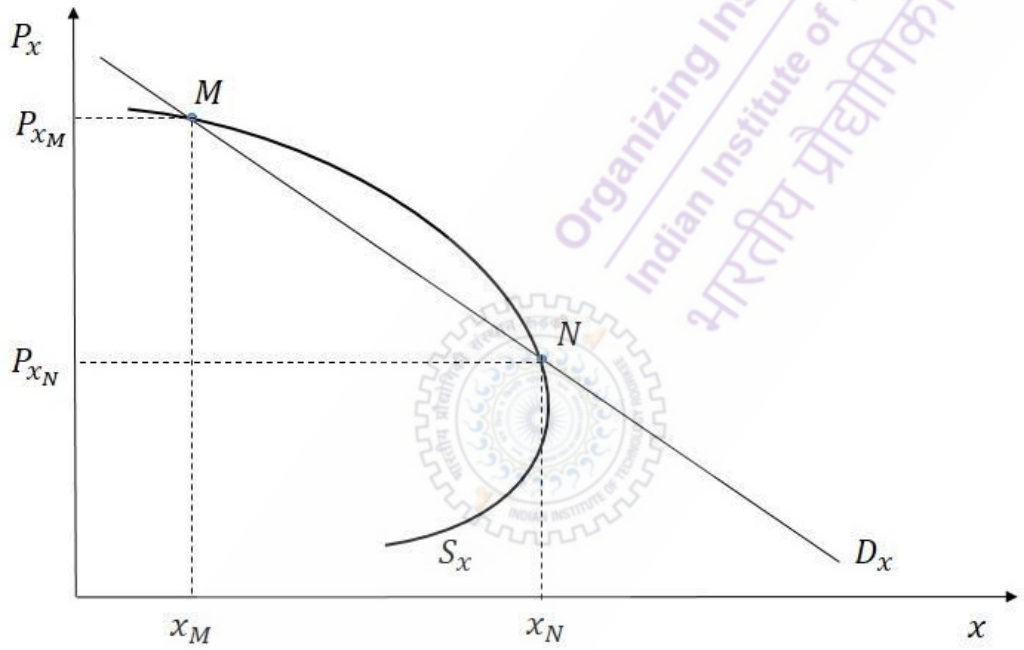
Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	The specific rotation of optically pure (*R*)-1-phenylethylamine is +40 (neat, 20 °C). A synthetic sample of the same compound is shown to contain 4:1 mixture of (*S*)- and (*R*)-enantiomers. The specific rotation of the neat sample at 20 °C is _____. (*round off to the nearest integer*)
Q.52	The number of β particles emitted in the nuclear reaction $^{238}_{92}U \rightarrow {}^{206}_{82}Pb$ is _____.
Q.53	Iron is extracted from its ore via the reaction $Fe_2O_3 + 3\ CO \rightarrow 2\ Fe + 3\ CO_2$ The volume of CO (at STP) required to produce 1 kg of iron is _____ liters. (*round off to the nearest integer*) [Given: Atomic wt. of Fe = 56; assume STP to be 0 °C and 1 atm]

Q.54	Total degeneracy (number of microstates) for a Ti^{3+} ion in spherical symmetry is _____.
Q.55	A galvanic electrochemical cell made of Zn^{2+}/Zn and Cu^{2+}/Cu half-cells produces 1.10 V at 25 °C. The ratio of $[Zn^{2+}]$ to $[Cu^{2+}]$ is maintained at 1.0. The $\Delta G°$ for the reaction when 1.0 mol of Zn gets dissolved is _____ kJ. (*round off to the nearest integer*) [Given: Faraday's constant = 96485 C mol^{-1}]
Q.56	At constant volume, 1.0 kJ of heat is transferred to 2 moles of an ideal gas at 1 atm and 298 K. The final temperature of the ideal gas is _____ K. (*round off to one decimal place*) [Given: R = 8.314 J K^{-1} mol^{-1}]
Q.57	Two close lying bands in a UV spectrum occur at 274 nm and 269 nm. The magnitude of the energy gap between the two bands is _____ cm^{-1}. (*round off to the nearest integer*)

Q.58	The pH of an aqueous buffer prepared using CH_3COOH and $CH_3COO^-Na^+$ is 4.80. The quantity $\frac{[CH_3COO^-] - [CH_3COOH]}{[CH_3COOH]}$ is _____. (*round off to three decimal places*) [Given: pK_a of CH_3COOH in water is 4.75]
Q.59	At constant temperature, 6.40 g of a substance dissolved in 78 g of benzene decreases the vapor pressure of benzene from 0.125 atm to 0.119 atm. The molar mass of the substance is _____ g mol^{-1}. (*round off to one decimal place*) [Given: Mol. wt. of benzene = 78 g mol^{-1}]
Q.60	For a van der Waals gas, the critical temperature is 150 K and the critical pressure is 5×10^6 Pa. The volume occupied by each gas molecule is _____ Å^3. (*round off to two decimal places*) [Given: $R = 8.314$ J mol^{-1} K^{-1}, $N_A = 6.023\times10^{23}$]

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 When the supply curve S_x is backward bending and the demand curve D_x is downward sloping as shown in the figure, there are two equilibria M and N, respectively. Which of the following statements is CORRECT?



(A) Only M is stable equilibrium

(B) Only N is stable equilibrium

(C) Both M and N are stable equilibria

(D) Both M and N are unstable equilibria

Q.2	Which of the following deficits indicates the true current fiscal position of the Indian Economy?
(A)	Revenue Deficit
(B)	Capital Deficit
(C)	Current Account Deficit
(D)	Primary Deficit

Q.3	Which of the following CORRECTLY defines the relationship between the variances of sample means for simple random samples drawn with and without replacement from a normal population?
(A)	$\frac{\sigma^2}{n} > \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$
(B)	$\frac{\sigma^2}{n} \leq \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$
(C)	$\frac{\sigma^2}{n} < \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$
(D)	$\frac{\sigma^2}{n} = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$

Q.4	Suppose that one million unemployed persons in a country are receiving Rs. 6000 per month per person as an unemployment allowance. If the government, instead of paying unemployment allowance, hires all of them at the same amount (Rs. 6000 per month per person) and engages them in digging the pits and filling the same pits. What will be the effect on GDP?
(A)	No effect on GDP
(B)	GDP will rise.
(C)	GDP will fall.
(D)	The effect on GDP will be uncertain.

Q.5	Which amendments to the constitution have provided constitutional status to the rural and urban local bodies in India?
(A)	80^{th} and 81^{st} Amendments
(B)	73^{rd} and 74^{th} Amendments
(C)	92^{nd} and 93^{rd} Amendments
(D)	71^{st} and 72^{nd} Amendments

Q.6	Let W be a subspace of a vector space $\Re^3$. Then, which of the following sets of vectors forms a basis of W?
(A)	$(1, 2, 1)$ and $(1, -2, 5)$
(B)	$(1, 3, 2), (1, -1, 0), (4, -1, 0)$ and $(3, 1, -3)$
(C)	$(1, 1, 1), (1, 2, 3)$ and $(2, -1, 1)$
(D)	$(1, -2, 1), (2, 1, -1)$ and $(7, -4, 1)$

Q.7	From the following, who first examined the close negative relationship between the unemployment rate and the output ratio?
(A)	Alban W. Phillips
(B)	James Tobin
(C)	Arthur M. Okun
(D)	Robert M. Solow
Q.8	In the hypothesis testing, which of the following defines the size of power of the test?
(A)	1 – (Probability of accepting null hypothesis when it is true)
(B)	1 – (Probability of rejecting null hypothesis when it is true)
(C)	1 – (Probability of accepting null hypothesis when it is false)
(D)	1 + (Probability of rejecting null hypothesis when it is not true)

Q.9	Which of the following is NOT a postulate of the Classical Model of full-employment equilibrium?
(A)	Wage-Price flexibility
(B)	Perfect information about the market
(C)	Consumption and saving functions depend on income.
(D)	The price level moves proportionately with the quantity of money.
Q.10	A long-run cost function for a product exhibits economies of scale if
(A)	average cost of production increases when the output increases.
(B)	the production function has decreasing returns to scale.
(C)	average cost of production falls as the output increases.
(D)	average cost of production remains constant as the output increases.

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Let $x^3 + 3y^2 = 4$ for all $x, y \in \Re$, $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2y}{dx^2}$. Then
(A)	$x^2 + y\,y'' + (y')^2 = 0$
(B)	$2x + y'' + 2(y')^2 = 0$
(C)	$x + (y')^2 = 0$
(D)	$x + y\,y'' + (y')^2 = 0$

Q.12 Match **List I** with **List II** and choose the CORRECT option.

List I	**List II**
a. Second Five Year Plan (1956-61)	i. Towards Faster and More Inclusive Growth
b. Fourth Five Year Plan (1969-74)	ii. Removal of Poverty and Attainment of Self-reliance
c. Fifth Five Year Plan (1974-79)	iii. Rapid Industrialization–Heavy and Basic Industries
d. Eleventh Five Year Plan (2007-12)	iv. Family Planning Programmes

(A) (a, ii), (b, i), (c, iv), (d, iii)

(B) (a, iii), (b, iv), (c, i), (d, ii)

(C) (a, iv), (b, iii), (c, ii), (d, i)

(D) (a, iii), (b, iv), (c, ii), (d, i)

Q.13	Let $f:[0,\infty)\to\Re$ be a function defined by $f(x)=\dfrac{x+1}{x+2}$ for all $x\in\Re$. Then f is
(A)	one-one and onto.
(B)	one-one but not onto.
(C)	onto but not one-one.
(D)	neither one-one nor onto.

Q.14	An economy is characterized by the Solow model, with the production function $y = \sqrt{k}$, where y is output per worker and k is capital per worker. The steady-state level of output per worker is $y^{ss} = A^{1/(1-\alpha)} \left(\frac{\gamma}{\delta}\right)^{\alpha/(1-\alpha)}$, where A, γ, δ and α denote productivity, share of output invested (in %), depreciation rate (in %) and capital's share in income (in fraction), respectively. Suppose that $A = 1$, $k = 400$, $\gamma = 50\%$, $\delta = 5\%$ and $\alpha = 1/2$. Then the current output, using the above information, is
(A)	above the steady-state level of output per worker.
(B)	at the steady-state level of output per worker.
(C)	below the steady-state level of output per worker.
(D)	at the Golden Rule level.

Q.15	Which of the following is NOT related to the structural adjustment programmes implemented in India after 1991?
(A)	Deregulation
(B)	Quantitative restrictions on trade
(C)	Fiscal austerity
(D)	Reduction of subsidies

Q.16	Let a second order difference equation be $$y_{n+2} + 4y_n = 4y_{n+1}, \quad n = 2, 3, 4, \ldots, \quad y_0 = 1, \quad y_1 = 4.$$ Then the general solution is
(A)	$(1 + n^2)\, 2^n$
(B)	$(1 + n)\, 2^n$
(C)	$\left(1 + \frac{1}{n}\right) 2^n$
(D)	$(n^2 + n + 1)\, 2^n$

Q.17	Suppose that two random samples of sizes n_1 and n_2 are selected without replacement from two binomial populations with means $\mu_1 = n_1 p_1$, $\mu_2 = n_2 p_2$ and variances $\sigma_1^2 = n_1 p_1 q_1$, $\sigma_2^2 = n_2 p_2 q_2$, respectively. Let the difference of sample proportions $\bar{P}_1$ and $\bar{P}_2$ approximate a normal distribution with mean $(p_1 - p_2)$. Then the standard deviation of the difference of sample proportions $\bar{P}_1$ and $\bar{P}_2$ is
(A)	$\sqrt{\left(\frac{p_1 q_1}{n_1}\right)\left(\frac{N_1 - n_1}{N_1 - 1}\right) + \left(\frac{p_2 q_2}{n_2}\right)\left(\frac{N_2 - n_2}{N_2 - 1}\right)}$
(B)	$\sqrt{\left(\frac{p_1 q_1}{n_1}\right) + \left(\frac{p_2 q_2}{n_2}\right)}$
(C)	$\sqrt{\left(\frac{p_1 q_1 - p_2 q_2}{n_1 + n_2}\right)}$
(D)	$\sqrt{\left(\frac{p_1 q_1}{n_1 + n_2}\right)\left(\frac{N_1 - n_1}{N_1 - 1}\right) + \left(\frac{p_2 q_2}{n_1 + n_2}\right)\left(\frac{N_2 - n_2}{N_2 - 1}\right)}$

Q.18	Which of the following statements is NOT correct in the context of quantity theory of money?
(A)	The quantity of money available determines the price level in the economy.
(B)	The growth rate in the quantity of money available determines the inflation rate in the economy.
(C)	The velocity of money must rise with the increase in the quantity of money in the economy.
(D)	The economy's output is determined by factor supplies and technology, because money is neutral.

Q.19	Let the function $f : \Re^2 \to \Re$ be $f(x, y) = \dfrac{x y^2}{x^3 + 2x^2 y + y^3}$, $f(0, 0) = 0$. Then
(A)	f is differentiable at (0, 0).
(B)	f_x does not exist at (0, 0).
(C)	f_y does not exist at (0, 0).
(D)	f is not continuous at (0, 0).

Q.20	Which of the following measures was announced by the Government of India in the year 1994?
(A)	Full convertibility on capital account
(B)	Full convertibility on current account
(C)	Constitution of the Narasimham Committee on banking sector reforms
(D)	Constitution of the Abid Hussain Committee on trade policies

Q.21	An analyst at the Green Car Co. Ltd. estimated the following demand function for the electric vehicles it sells: $$Q_E = 0.75 - 1.5P_E + 2.5P_F - 0.5P_B + 3.2\,I$$ where Q_E = Number of electric vehicles (in thousand per year), P_E = Unit price of electric vehicle (Rs. in Lakh), P_F = Average unit price of vehicle using fossil fuels (Rs. in Lakh), P_B = Unit price of battery used in electric vehicle (Rs. in Lakh), I = Personal disposable income (Rs. in Lakh). Let P_E = Rs. 6.5 Lakh, P_F = Rs. 4.5 Lakh, P_B = Rs. 0.5 Lakh and I = Rs. 10 Lakh. Then the income elasticity of demand (e_{Q_EI}) and the cross price elasticity of demand ($e_{Q_EP_F}$) satisfy
(A)	$0.98 \leq e_{Q_EI} \leq 0.99$ and $0.33 \leq e_{Q_EP_F} \leq 0.34$
(B)	$0.94 \leq e_{Q_EI} \leq 0.95$ and $0.45 \leq e_{Q_EP_F} \leq 0.46$
(C)	$0.98 \leq e_{Q_EI} \leq 0.99$ and $0.45 \leq e_{Q_EP_F} \leq 0.46$
(D)	$0.94 \leq e_{Q_EI} \leq 0.95$ and $0.33 \leq e_{Q_EP_F} \leq 0.34$

Q.22 Choose the option that represents the original linear programming problem based on the initial simplex tableau given below, where S_i represents slack/surplus variables and A_i represents the artificial variables corresponding to the i^{th} constraint:

C_j			15	25	0	$-M$	$-M$	0
	X_b	b	x	y	S_1	A_1	A_2	S_3
$-M$	A_1	20	7	6	-1	1	0	0
$-M$	A_2	18	3	-2	0	0	1	0
0	S_3	30	8	5	0	0	0	1
	Z_j	$-38M$	$-10M$	$-4M$	M	$-M$	$-M$	0
	$C_j - Z_j$		$15+10M$	$25+4M$	$-M$	0	0	0

(A) Minimize $Z=15x+25y$
subject to $7x+6y \geq 20,\ 3x-2y \leq 18,\ 8x+5y \leq 30\ ;\ x, y \geq 0.$

(B) Maximize $Z=15x+25y$
subject to $7x+6y \geq 20,\ 3x-2y=18,\ 8x+5y \leq 30\ ;\ x, y \geq 0.$

(C) Minimize $Z=15x+25y$
subject to $7x+6y \geq 20,\ 3x-2y=18,\ 8x+5y \geq 30\ ;\ x, y \geq 0.$

(D) Maximize $Z=15x+25y$
subject to $7x+6y=20,\ 3x-2y=18,\ 8x+5y \leq 30\ ;\ x, y \geq 0.$

Q.23	Let a production function be given by $\log Q = \frac{\beta}{\alpha} \log(L^{\alpha} + K^{\alpha})$, where $\alpha \in (-\infty, 1] - \{0\}$ and $\beta > 0$. Then identify the statement that is NOT correct.
(A)	The ratio β/α helps in identification of returns to scale factors.
(B)	For $\beta > 1$, the function exhibits increasing returns to scale, and for $\beta < 1$, it shows decreasing returns to scale.
(C)	The elasticity of substitution is $\frac{1}{1-\alpha}$.
(D)	The elasticity of substitution is $\frac{1}{1-\beta}$.

Q.24	Which of the following statements is NOT correct under the IS-LM (Fixed Price) model?
(A)	The LM curve represents the combinations of income and interest rate, where money market is in equilibrium.
(B)	The IS curve represents the combinations of income and interest rate, where product market (goods and services) is in equilibrium.
(C)	An increase in money supply raises income and reduces interest rate when the IS curve has negative slope and the LM curve has positive slope.
(D)	Monetary policy has a relatively weak effect on income when the interest responsiveness of the demand for money is relatively low.

Q.25	The probability of getting head in a toss of a biased coin is $\frac{2}{3}$. Let the coin be tossed three times independently. Then the probability of getting head in the first two tosses and tail in the final toss is
(A)	$\frac{4}{27}$
(B)	$\frac{1}{8}$
(C)	$\frac{2}{27}$
(D)	$\frac{23}{27}$

Q.26	Consider a pure exchange economy with two goods x and y. Ravi and Suraj are two individuals with utility functions $U_R = \beta \log(xy)$ and $U_S = \left(\frac{x}{y}\right)^{\alpha}$, respectively. The endowments are x_R and y_R for Ravi and x_S and y_S for Suraj such that $x_R + x_S = A$ and $y_R + y_S = B$. Then their contract curve is
(A)	$Ay_R - Bx_R = 0$
(B)	$Ay_R + Bx_R - 2y_R x_R = 0$
(C)	$Ay_R + Bx_R - y_R x_R = 0$
(D)	$Ay_R - Bx_R + 2y_R x_R = 0$

Q.27	Which of the following is NOT correct regarding R-squared (R^2) and Adjusted R-squared $\left(\bar{R}^2\right)$?
(A)	R^2 is a scale invariant statistic.
(B)	$\bar{R}^2$ is always positive.
(C)	R^2 tends to increase if we add an additional explanatory variable.
(D)	$\bar{R}^2 = 1-\left(1-R^2\right)\left(\frac{n-1}{n-k}\right)$, where k is the number of parameters and n is the number of observations.

Q.28	The technical change in the endogenous growth model is endogenized by
(A)	providing incentives to firms to innovate.
(B)	making the saving function dependent on income.
(C)	introducing constraints in capital accumulation.
(D)	assuming a perfectly competitive market structure.

Q.29 Which of the following statements is CORRECT for **Game A** and **Game B**?

Game A: Mary wants to watch a movie and John is interested in watching a football match. Both wish to be together. The payoff matrix is:

		John	
		Movie	*Football*
Mary	*Movie*	(2,1)	(0,0)
	Football	(0,0)	(1,2)

Game B: The Prisoner's dilemma problem is shown below:

		Convict 2	
		Do not confess	*Confess*
Convict 1	*Do not confess*	(-1,-1)	(-9,0)
	Confess	(0,-9)	(-5,-5)

(A) In **Game A**, (Movie, Football) and (Football, Movie) represent Nash equilibrium. In **Game B**, (Do not confess, Do not confess) is the Nash Equilibrium.

(B) In **Game B**, (Confess, Confess) is not a Nash equilibrium but in **Game A**, both (Movie, Football) and (Football, Movie) represent Nash equilibrium.

(C) In **Game B**, the Nash equilibrium is (Do not confess, Do not confess).

(D) In **Game A**, both (Movie, Movie) and (Football, Football) represent Nash equilibrium. In **Game B**, the Nash equilibrium is (Confess, Confess).

Q.30	The short-run production function of a firm is $Q = 200 + 0.2L^2 - 0.0004L^3$. If wage rate equals Rs. 140 and the number of labours (L) is 100, then the Marginal Cost and the Average Variable Cost, respectively, are
(A)	5 and 7.78
(B)	6 and 7.78
(C)	5 and 6.68
(D)	6 and 6.68

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Let $X \sim N\left(\mu_X, \sigma_X^2\right)$ and $Y \sim N\left(\mu_Y, \sigma_Y^2\right)$. Which of the following is/are NOT correct?
(A)	The area $F(X) = \dfrac{1}{\sigma_X \sqrt{2\pi}} \int_{-\infty}^{\mu_X} e^{-\frac{1}{2}\left(\frac{X-\mu_X}{\sigma_X}\right)^2} dx$ is 1.
(B)	The areas under the normal probability curve between the ordinates at $\mu_X \pm 3\sigma_X$ and $\mu_Y \pm 2\sigma_Y$ are 0.9544 and 0.9973, respectively.
(C)	For variable *X*, Quartile Deviation : Mean Absolute Deviation : Standard Deviation $\cong \dfrac{2}{3}\sigma_X : \dfrac{4}{5}\sigma_X : \sigma_X$
(D)	If *X* and *Y* are independent, then $(X - Y) \sim N\left(\mu_X - \mu_Y, \sigma_X^2 + \sigma_Y^2\right)$.

Q.32 Matching **List I** and **List II**, choose the CORRECT option(s).

List I	List II
a. Bombay Plan	i. J. P. Narayan
b. People's Plan	ii. J. R. D. Tata
c. Sarvodaya Plan	iii. M. N. Roy

(A) (a, i), (b, iii)

(B) (a, ii), (b, iii)

(C) (b, iii), (c, i)

(D) (a, ii), (c, iii)

Q.33	Suppose that the regression model is $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \mu_i$, $i = 1, 2, \ldots, n$. Which of the following null hypotheses could be tested using the F-test?
(A)	$\beta_1/\beta_2 = 0$
(B)	$\beta_0 = 0$
(C)	$\beta_1 \beta_2 = 0$
(D)	$\beta_1 = \beta_2 = 0$

Q.34	Let f be defined by $f(x) = \lvert x \rvert + \left\lvert \cos\left(\frac{\pi}{2} - x\right) \right\rvert$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then
(A)	f is continuous on $\left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)$.
(B)	f is differentiable at $x = 0$.
(C)	f is differentiable everywhere except $x = 0$.
(D)	$\lim_{x \to 0} f(x) = 0$.

Q.35	The real exchange rate is given by $e = EP/P^{\star}$, where e is the price of domestic goods in terms of foreign goods, E is the price of domestic currency in terms of foreign currency, P is the domestic price level, $P^{\star}$ is the foreign price level. If the Indian Rupee depreciates vis-à-vis the Japanese Yen, and the Marshall-Lerner condition holds, then
(A)	India's imports will increase.
(B)	India's trade balance will improve.
(C)	foreign demand for Indian goods will increase.
(D)	foreign demand for Indian goods will decrease.

Q.36	The demand function (Q_x^D) and supply function (Q_x^S) are given as: $Q_x^D = f(P_x, I)$ and $Q_x^S = g(P_x, A)$ where I (Income) and A (Advertisement expenses) are the exogenous factors affecting quantity demanded and supplied, respectively. Further, $\frac{\partial f}{\partial P_x} < 0, \frac{\partial g}{\partial P_x} > 0$ but $\frac{\partial f}{\partial I}$ and $\frac{\partial g}{\partial A}$ may have any sign. Considering that there exists an equilibrium ($Q_x^D = Q_x^S = Q$), which of the following is/are CORRECT?
(A)	$e_{P_x A} = \left(\frac{\partial g}{\partial A}\frac{A}{Q}\right) / \left(\frac{\partial f}{\partial P_x}\frac{P_x}{Q} - \frac{\partial g}{\partial P_x}\frac{P_x}{Q}\right)$
(B)	$\frac{dP_x}{dA} = \left(\frac{\partial g}{\partial A}\right) / \left(\frac{\partial f}{\partial P_x} - \frac{\partial g}{\partial P_x}\right)$
(C)	$e_{P_x I} = \left(\frac{\partial g}{\partial I}\frac{I}{Q}\right) / \left(\frac{\partial f}{\partial P_x}\frac{P_x}{Q} - \frac{\partial g}{\partial P_x}\frac{P_x}{Q}\right)$
(D)	The sign of $\frac{dP_x}{dA}$ does not depend on $\frac{\partial g}{\partial A}$.

Q.37	Which of the following statements is/are CORRECT under the Keynesian Cross (Fixed Price) Model?
(A)	The product market and factor market independently determine the full-employment level of output.
(B)	Output is determined in the product market by the aggregate expenditure.
(C)	Money market determines the price level, given the quantity of money and the level of output.
(D)	Employment is determined in the factor market by the output level determined in the product market.

Q.38	Which of the following functions is/are homogeneous?
(A)	$x\cot^{-1}\left(\frac{y}{x}\right)$
(B)	$\sqrt{\frac{x}{y}}+\frac{3x}{y}+7$
(C)	$\frac{x^3+y^3}{3x+4y}$
(D)	$3x^5y+2x^2y^4-3x^3y^4$

Q.39	In the context of Indian agriculture, which of the following statements is/are CORRECT?
(A)	NABARD was established in 1982.
(B)	One of the objectives of setting up of the CACP was to ensure remunerative prices to farmers.
(C)	The APMC Act is related to institutional credit supply in agriculture.
(D)	The National Commission on Agriculture was chaired by V. M. Dandekar.

Q.40	Let a monopolist demand curve be given by $Q = P^e$, where Q is output, P is price, e is the price elasticity of demand $(e < -1)$, and Marginal Cost = Average Cost = α. If P_C and P_M represent the price under perfect competition and monopoly, respectively, then which of the following is/are NOT correct? (CS_M and CS_C represent the consumer surplus under monopoly and perfect competition, respectively.)
(A)	$P_C = \alpha\left(\frac{e}{1+e}\right)$
(B)	$P_M = \alpha\left(\frac{e}{1+e}\right)$
(C)	For $e = -2$, $CS_M = CS_C$.
(D)	For e closer to -1, the ratio CS_M/CS_C increases.

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41

The sum of the eigen values of the square matrix

$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ is _____ (*in integer*).

Q.42

Monthly per capita consumption expenditure (MPCE) of 10 households in a region is given below.

Households	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10
MPCE (in Rs.)	2800	3000	1200	3500	1400	2500	4000	1000	900	1300

Assuming the poverty cutoff (Z) of MPCE to be Rs. 2000, the squared poverty gap ratio is _____ (*round off to 3 decimal places*).

Q.43

Suppose that the full employment level of output of an economy is Rs. 2200 million, expenditure determined level of output is Rs. 2163 million, and the marginal propensity to consume is 0.75. The deflationary gap equals Rs. _____ million (*round off to 2 decimal places*).

Q.44	Let $a,b \in \Re$. If $f(x) = ax + b$ is such that $a + b = 4$ and $f(x + y) = f(x) + f(y) - 2$ for all $x, y \in \Re$, then $\sum_{n=1}^{50} f(n) =$ _____ (*in integer*).
Q.45	The Total Variable Cost (TVC) for a firm is given by $TVC = x^3 - bx^2$. The Total Fixed Cost is 848. The value of b for which the Marginal Cost is minimum at $x = 16$ is _____ (*in integer*).
Q.46	Let the consumption function, tax function, and income identity be given by $C = C_0 + b(Y - T)$, $T = T_0 + tY$, and $Y = C + I_0 + G_0$, respectively, where C_0, I_0, G_0, and T_0 are autonomous consumption, investment, government expenditure, and tax, respectively. If $b = 0.75$ and $t = 0.1$, then an increase in G_0 by Rs. 20 million will increase Y by Rs. _____ million (*round off to 2 decimal places*).

Q.47	Let the system of equations be $\alpha u + w = 0,\ u + \alpha v = 0,\ v + \alpha w = 0$, where $\alpha \in \Re$. Then the system has infinite solutions if $\alpha =$_____ (*in integer*).
Q.48	Assume that the cost function for the i^{th} firm in an industry is given by $$C_i = 0.25q_i^2 + 2q_i + 5, \quad i = 1, 2, \ldots, 150,$$ where C_i and q_i are cost and output for the i^{th} firm, respectively. Let the aggregate inverse demand function be $P = 10 - 0.01Q$, where P is the unit price and Q is the aggregate output. Assuming perfect competition, the equilibrium quantity is _____ (*in integer*).

Q.49

The following table presents the national income related aggregates (at current prices) for the year 2019-20:

National income related aggregates	Rs. Lakh Crores
Net factor income earned abroad	10
Private income	175
GNP at factor cost	210
NNP at factor cost	195
Retained earnings of Nation's private sector	10
Corporate tax	25
Household direct tax	28
Personal income	140
Miscellaneous receipts of government administrative departments	0

The personal disposable income for the year 2019-20 is Rs. _____ Lakh Crores (*in integer*).

Q.50

The following table provides a list of countries selling Big Mac and market exchange rates in January 2019.

Country	**Big Mac** (Price in local currency)	**Market Exchange Rate** (Local currency per USD)
United States	5.58 USD	1.00
Norway	50.00 Kroner	8.53 Kroner/USD
Japan	390.00 Yen	108.44 Yen/USD
Mexico	49.00 Pesos	17.31 Pesos/USD
China	20.90 Yuan	6.85 Yuan/USD
Russia	110.17 Rubles	66.69 Rubles/USD
India	178.00 Rupees	69.69 Rupees/USD

Using the above information, the cheapest price (in USD) of Big Mac is _____ (*round off to 2 decimal places*).

Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	An individual faces an uncertain prospect, where wealth could be Rs. 10 Lakh with probability 0.75 and Rs. 7 Lakh with probability 0.25. Let the utility function be $U(w) = w^3$. Then the individual will buy full insurance by paying a premium of Rs. _____ Lakh (*round off to 2 decimal places*).
Q.52	Suppose that per capita GDP of India and USA are growing at annual average rates of 8.8% and 1.8%, respectively. Further, consider that in 2019-20, per capita GDP of USA was USD 41099 and per capita GDP of India was USD 1570. Assuming that the two countries continue to grow at the above rates, India's per capita GDP will be equal to the per capita GDP of USA in _____ years (*round off to 2 decimal places*).
Q.53	If $\int t \log\left(1+\frac{2}{t}\right) dt = g(t)\left(\frac{t^2}{2}-2\right) + f(t)\frac{t^2}{2} + Kt + C$, where C is an arbitrary constant, then $2K$ is _____ (*in integer*).

Q.54	ACD Bank holds a total deposit of Rs. 256412. To expand the money supply in the economy during the COVID-19 pandemic period, the Reserve Bank of India reduces the cash reserve ratio (CRR) from 4.5% to 3.5%. Due to this policy change, the additional money supply generated by ACD Bank is Rs. _____ (*in integer*).
Q.55	Suppose that the regression model is $Y_{n\times 1} = X_{n\times 3}\beta_{3\times 1} + U_{n\times 1}$ with $\beta_{3\times 1} = \begin{bmatrix}\beta_1 & \beta_2 & \beta_3\end{bmatrix}^T$. A random sample of size $n = 23$ on Y and X is drawn from the normal population. Using the data, if a researcher obtains $(X^TX)^{-1} = \begin{bmatrix}0.3 & 0.5 & 0.8\\ 0.4 & -0.6 & 0.2\\ 0.4 & 0.5 & 0.3\end{bmatrix}$, $X^TY = \begin{bmatrix}0.3 & 0.2 & 0.1\end{bmatrix}^T$ and $e^Te = 0.7$, where e denotes the vector of estimated residuals, then the t-statistic to test the null hypothesis $\beta_3 = 0$ is _____ (*round off to 2 decimal places*).
Q.56	Given the production function $Q = 6\sqrt{L}$ and the supply of labour $L = \sqrt{w}$, where L and w denote the number of labours and wage rate, respectively. If the unit price of the product is Rs. 243, then the profit maximizing value of w is Rs. _____ (*in integer*).

Q.57	Given the following information related to product and money markets, **Product Market** $C = 300 + 0.8(Y - T)$ $T = 200 + 0.2(Y)$ $I_0 = 300; G_0 = 400$ **Money Market** $\frac{M_o}{P} = 0.4Y - 200i$ $M_0 = 900; P = 1$ (Fixed) where Y = Income, C = Consumption, T = Tax, I_0 = Autonomous Investment, G_0 = Autonomous Government Expenditure, M_0 = Nominal Money Demand, P = Price, and i = Interest Rate. The equilibrium level of interest rate (in %) is _____ (*round off to 2 decimal places*).
Q.58	Let the linear programming problem be Maximize $Z = -0.2x_1 + x_2$ subject to $2x_1 + 5x_2 \leq 70$, $x_1 + x_2 \leq 20$, $x_1, x_2 \geq 0$. If $x_1 = a$ and $x_2 = b$ is the optimal solution, then $a + b =$ _____ (*in integer*).

Q.59	Let the production function be $Q = \sqrt{L^2 + K^2}$, the unit price of labour (L) and capital (K) be Rs. 30 and Rs. 40, respectively, and the total cost be Rs. 580. Then the maximum value of Q subject to the cost constraint is _____ (*round off to 2 decimal places*).
Q.60	In a market, two firms F_1 and F_2 are producing homogenous products. The inverse demand function is given by $p = 120 - 0.5(q_1 + q_2)$, where p is the unit price of the product, and q_1 and q_2 are the outputs from F_1 and F_2, respectively. Suppose the cost functions of F_1 and F_2 are $C_1 = 20q_1$ and $C_2 = 10 + 0.5q_2^2$, respectively. Then the total profit earned by both the firms assuming a competitive situation is _____ (*in integer*).

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	Which one of the following is a geochronologic unit?
(A)	System
(B)	Period
(C)	Member
(D)	Formation
Q.2	Which one of the following must have thickness less than 1 cm?
(A)	Lamina
(B)	Bed
(C)	Stratum
(D)	Layer

Q.3	Which one of the following organisms became extinct during the Cretaceous-Tertiary mass extinction event?
(A)	Trilobite
(B)	Ammonite
(C)	Brachiopod
(D)	Echinoderm
Q.4	Match the geomorphic features in **Group I** with the related processes in **Group II**. **Group I** — **Group II** P. Cirque — 1. Fluvial Q. Ventifact — 2. Glacial R. Point bar — 3. Volcanic S. Tephra — 4. Aeolian
(A)	P-2, Q-4, R-1, S-3
(B)	P-2, Q-3, R-1, S-4
(C)	P-4, Q-2, R-3, S-1
(D)	P-1, Q-2, R-3, S-4

Q.5	Which one of the given textural features results from exsolution?
(A)	Ophitic
(B)	Perthitic
(C)	Graphic
(D)	Glomeroporphyritic
Q.6	In the holosymmetric class of the Cubic System, how many more faces does the {110} form have compared to the {111} form?
(A)	2
(B)	4
(C)	6
(D)	8

Q.7	Which one of the following seismic waves involves compression and rarefaction (but not rotation) of the material that it passes through?
(A)	P-waves
(B)	S-waves
(C)	Rayleigh waves
(D)	Love waves
Q.8	Realgar and orpiment are both minerals of arsenic (As) and have the same chemical composition. Which one of the following properties can be used to distinguish between the two minerals in hand specimen?
(A)	Lustre
(B)	Hardness
(C)	Colour
(D)	Fracture

Q.9	Buckle folds result from
(A)	layer parallel shortening
(B)	layer perpendicular slip
(C)	layer parallel shearing
(D)	layer perpendicular shortening
Q.10	Sandstone beds above a magmatic body are domal in shape, while the beds below are horizontal. The magmatic body is a
(A)	Batholith
(B)	Laccolith
(C)	Lopolith
(D)	Sill

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Match the morphological features in **Group I** with the corresponding fossil groups in **Group II**. **Group I** — **Group II** P. Pedicle Foramen — 1. Trilobita Q. Pallial Sinus — 2. Cephalopoda R. Pygidium — 3. Pelecypoda S. Siphuncle — 4. Brachiopoda
(A)	P-4, Q-3, R-1, S-2
(B)	P-4, Q-1, R-2, S-3
(C)	P-3, Q-4, R-1, S-2
(D)	P-2, Q-1, R-4, S-3
Q.12	The Triassic-Jurassic boundary lies within which one of the following stratigraphic units?
(A)	Panchet Formation
(B)	Dharmaram Formation
(C)	Pachmarhi Formation
(D)	Denwa Formation

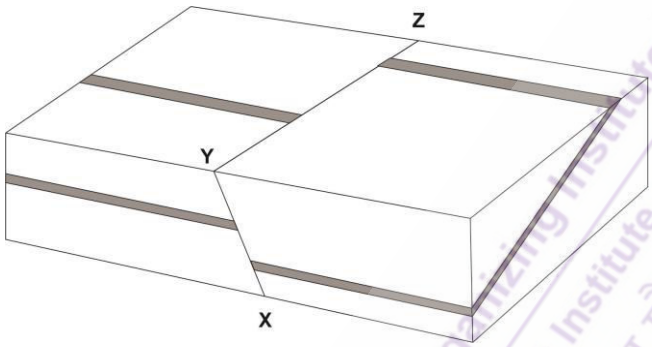
Q.13	Which one is the correct order of stability of the minerals (arranged from the most stable to the least stable) during chemical weathering?
(A)	Muscovite > Amphibole > Quartz > Olivine
(B)	Quartz > Amphibole > Olivine > Muscovite
(C)	Quartz > Muscovite > Amphibole > Olivine
(D)	Muscovite > Olivine > Quartz > Amphibole
Q.14	Match the following sedimentary rocks in **Group I** with their compositions in **Group II**. **Group I** — **Group II** P. Packstone — 1. <15% matrix, > 25% rock fragments Q. Grainstone — 2. >15% matrix, >25% feldspar R. Lithic arenite — 3. Grain supported, contains no mud S. Arkosic wacke — 4. Grain supported, contains mud
(A)	P-4, Q-3, R-1, S-2
(B)	P-3, Q-4, R-2, S-1
(C)	P-3, Q-1, R-4, S-2
(D)	P-2, Q-4, R-1, S-3

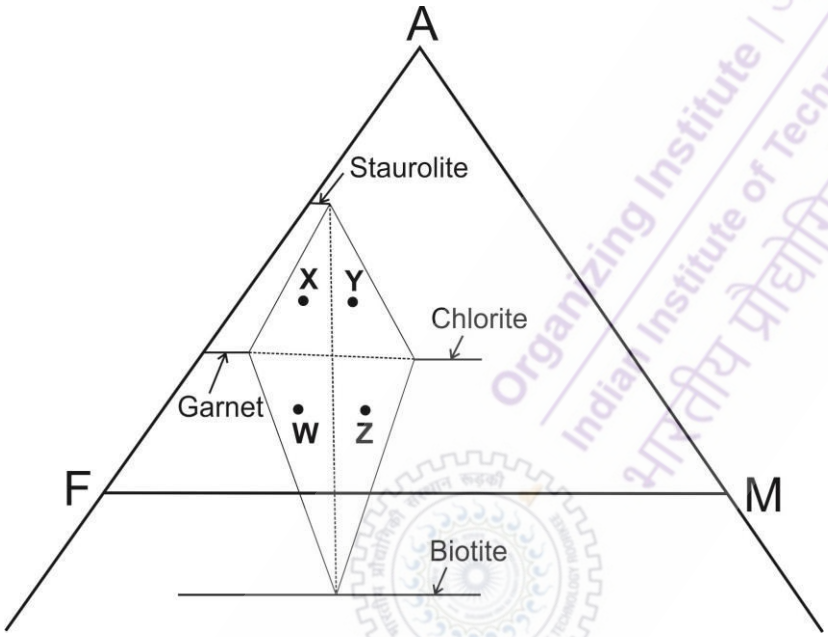
Q.15	Match the parameters in **Group I** with their corresponding dimensions in **Group II**. **Group I** — **Group II** P. Shear modulus — 1. $M^0L^0T^0$ Q. Hydraulic conductivity — 2. $M^1L^{-3}T^0$ R. Volumetric strain — 3. $M^0L^1T^{-1}$ S. Dry density — 4. $M^1L^{-1}T^{-2}$
(A)	P-4, Q-3, R-1, S-2
(B)	P-3, Q-1, R-2, S-4
(C)	P-2, Q-3, R-4, S-1
(D)	P-3, Q-4, R-1, S-2

Q.16	Match the countries in **Group I** with the plate tectonic features in **Group II** that cause seismic activity in them. **Group I** — **Group II** P. Iceland — 1. Subduction Zone Q. Indonesia — 2. Transform Fault R. Nepal — 3. Mid-Oceanic Ridge S. New Zealand — 4. Continental Collision
(A)	P-3, Q-1, R-4, S-2
(B)	P-3, Q-1, R-2, S-4
(C)	P-1, Q-3, R-4, S-2
(D)	P-2, Q-1, R-4, S-3

Q.17	Which one of the magnitude scales given below **DOES NOT** saturate while estimating size of earthquakes?
(A)	Local magnitude scale (M_L)
(B)	Body wave magnitude scale (M_b)
(C)	Surface wave magnitude scale (M_S)
(D)	Moment magnitude scale (M_W)
Q.18	What is the minimum number of forms that an actual crystal must contain in Class 1 (Pedial) of the Triclinic System?
(A)	1
(B)	2
(C)	3
(D)	4

Q.19	The apparent dip of a plane is measured to be 45º towards NE. The true dip of the plane is
(A)	55º towards SSW
(B)	40º towards NNE
(C)	48º towards ENE
(D)	40º towards E
Q.20	A horizontal upright fold will have a
(A)	vertical fold axis and horizontal axial plane
(B)	horizontal fold axis and vertical axial plane
(C)	horizontal fold axis and axial plane with any dip
(D)	plunging fold axis on a vertical axial plane

Q.21	The displacement of the bed shown in the figure below is caused by a single movement along fault XYZ. Of the options given below, which fault-type can explain the observed displacement? 
(A)	Strike-slip
(B)	Reverse
(C)	Normal
(D)	Trace-slip

Q.22	In the AFM diagram below, W, X, Y and Z represent four pelitic assemblages. Which combinations of assemblages contain the same mineral phases in equilibrium in the staurolite zone? 
(A)	(X-Y) & (W-Z)
(B)	(X-W) & (Y-Z)
(C)	(X-Z) & (Y-W)
(D)	(W-X-Y) & (Z)

Q.23	Match the rock types in **Group I** with their characteristic mineral assemblages in **Group II**. **Group I** **Group II** P. Diorite 1. plagioclase – orthopyroxene ± clinopyroxene Q. Tonalite 2. olivine – orthopyroxene – clinopyroxene R. Norite 3. plagioclase – hornblende ± quartz S. Lherzolite 4. quartz – plagioclase ± K-feldspar
(A)	P-4, Q-3, R-2, S-1
(B)	P-2, Q-1, R-3, S-4
(C)	P-3, Q-4, R-1, S-2
(D)	P-1, Q-3, R-4, S-2

Q.24	Match the mineral deposit types in **Group I** with the water types in **Group II** considered dominantly responsible for their origin. **Group I** P. Porphyry copper deposits Q. Mississippi Valley Type deposits R. Roll-front uranium deposits S. Epithermal gold deposits **Group II** 1. Meteoric water 2. Groundwater 3. Magmatic water 4. Connate water
(A)	P-4, Q-3, R-2, S-1
(B)	P-3, Q-4, R-1, S-2
(C)	P-3, Q-4, R-2, S-1
(D)	P-4, Q-1, R-2, S-3

Q.25	Match the minerals in **Group I** with their optical properties in **Group II**. **Group I** — **Group II** P. Sodalite — 1. Mottled extinction Q. Tourmaline — 2. Isotropic R. Calcite — 3. Pleochroic from blue to brown S. Muscovite — 4. Twinkling effect
(A)	P-4, Q-3, R-2, S-1
(B)	P-2, Q-3, R-4, S-1
(C)	P-3, Q-1, R-2, S-4
(D)	P-1, Q-3, R-4, S-2
Q.26	The contact between the Talchir Formation and the underlying Precambrian basement is
(A)	an angular unconformity
(B)	a disconformity
(C)	a paraconformity
(D)	a nonconformity

Q.27	Increased diversity of siphonate bivalves occurred in response to
(A)	the Cambrian explosion in the Paleozoic
(B)	increased temperature in the Cenozoic
(C)	increased predation pressure in the Mesozoic
(D)	increased oxygen level in the Proterozoic
Q.28	An index fossil should have
(A)	large geographic range and small temporal range
(B)	small geographic range and large temporal range
(C)	small geographic range and small temporal range
(D)	large geographic range and large temporal range

Q.29	Match the formations in **Group I** with corresponding stratigraphic periods in **Group II**. **Group I** — **Group II** P. *Syringothyris* Limestone — 1. Permian Q. Karai Shale — 2. Jurassic R. Chari — 3. Carboniferous S. Barren Measures — 4. Cretaceous
(A)	P-1, Q-2, R-3, S-4
(B)	P-2, Q-4, R-1, S-3
(C)	P-3, Q-4, R-2, S-1
(D)	P-4, Q-1, R-2, S-3
Q.30	Which one of the given statements is correct?
(A)	van der Waal's bonding is absent in silicate minerals
(B)	Sulfide minerals form by covalent bonding between metal and sulfur
(C)	Silicate minerals have a significant component of metallic bonding
(D)	Metal-sulfide formation does not involve splitting of d-orbitals

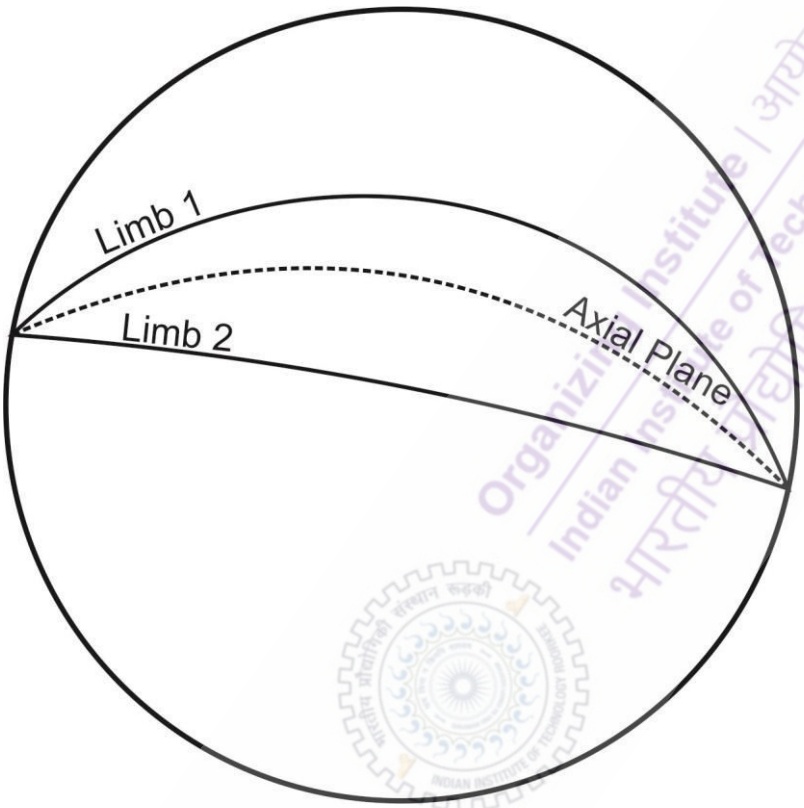
Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Which of the following structures form in marine environment?
(A)	Lateral accretionary surfaces
(B)	Hummocky cross stratification
(C)	Herringbone cross stratification
(D)	Barchanoids
Q.32	Identify the correct stratigraphic successions ordered from oldest to youngest.
(A)	Papaghni Group - Kurnool Group - Nallamalai Group - Chitravati Group
(B)	Semri Group - Kaimur Group - Rewa Group - Bhander Group
(C)	Papaghni Group - Chitravati Group - Nallamalai Group - Kurnool Group
(D)	Semri Group - Rewa Group - Bhander Group - Kaimur Group

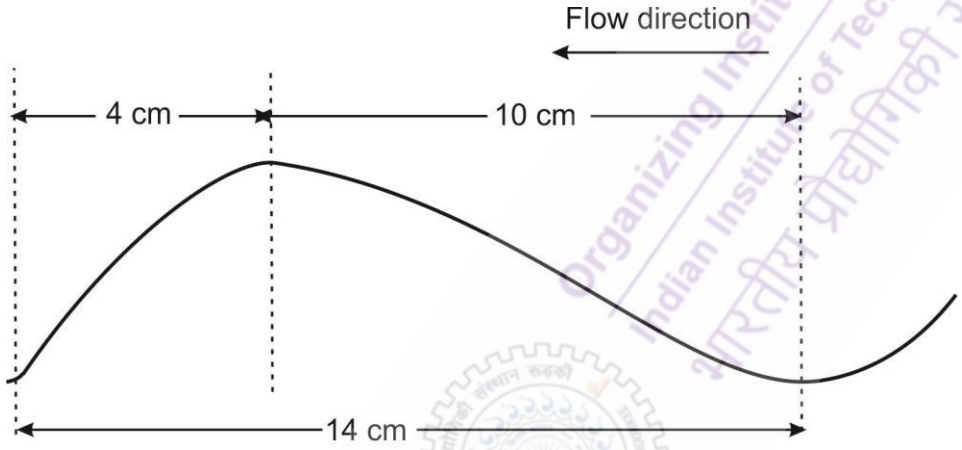
Q.33	Which of the following stratigraphic units contain coal seams?
(A)	Barakar Formation
(B)	Lakadong Formation
(C)	Pachmarhi Formation
(D)	Panchet Formation
Q.34	Which of the following statements are CORRECT?
(A)	*Mytilus* represents byssally attached bivalves
(B)	*Nautilus* is the only living cephalopod genus with a coiled external shell
(C)	The cidaroids are the only echinoid group still living
(D)	Trilobites did not moult

Q.35	Which of the following genera are stem fossils?
(A)	*Dadoxylon*
(B)	*Dicroidium*
(C)	*Vertebraria*
(D)	*Ptilophyllum*
Q.36	Which of the following statements are correct?
(A)	Abutments are the sides of the valley supporting the dam structure
(B)	Spillways can control the release of water from the reservoir
(C)	The toe of the dam is the upstream edge of the base of the dam structure
(D)	Galleries serve as passages through the dam

Q.37	The acceleration due to gravity on the Earth's surface depends on
(A)	latitude
(B)	longitude
(C)	elevation
(D)	topography of the surrounding terrain
Q.38	A metamorphosed basaltic assemblage can include the minerals
(A)	garnet-omphacite
(B)	hornblende-plagioclase
(C)	garnet-staurolite
(D)	glaucophane-lawsonite

Q.39	Which of the following pairs represent correct plutonic – volcanic equivalents?
(A)	Granodiorite – dacite
(B)	Norite – basalt
(C)	Dunite – komatiite
(D)	Nepheline syenite – phonolite

Q.40	Based on the given stereographic projection, the fold can be classified as 
(A)	reclined fold
(B)	vertical fold
(C)	overturned fold
(D)	non-plunging fold

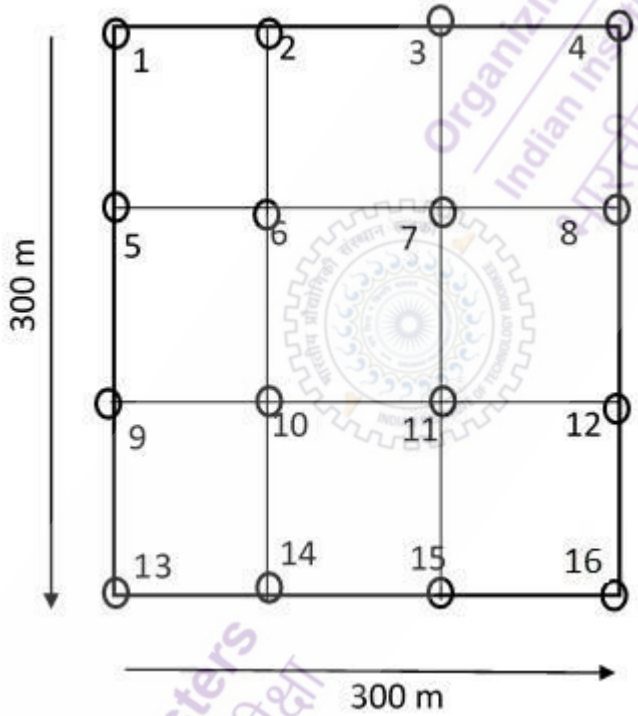
Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	The Ripple Symmetry Index (RSI) for the given hypothetical asymmetric ripple is _____. (*Round off to one decimal place)* 
Q.42	Within a fourth order drainage basin, the total lengths of the 1st, 2nd, 3rd and 4th order streams are 10.5 km, 7.5 km, 5.5 km and 1.5 km, respectively. If the drainage density of the basin is 0.5 km^{-1}, the basin area is _____ km^2. (*In integer*)
Q.43	A soil has a void ratio of 0.5. The total porosity of the soil is _____. (*Round off to two decimal places*)

Q.44	The average unit weight of the uppermost part of the crust is 25000 N/m^3. The vertical stress at a depth of 1 km would be _____ MPa. (*In integer*)
Q.45	The radius of the Earth's circular orbit round the Sun is 149×10^6 km. The Earth takes 365 days to orbit the Sun. The tangential velocity of the Earth is _____ km/hour. ($\pi = 3.14$) (*Round off to one decimal place)*
Q.46	A borehole inclined at 60º to the horizontal pierces a vertical basaltic dyke of uniform thickness. If the length of the basaltic drill core along the core axis is 12 m, the thickness of the dyke is _____ m. (*In integer*)
Q.47	A P-ray arrives at the mantle-core boundary at an angle 25º with respect to the normal. At what angle to the normal does it enter the core? (P-wave velocity in the lower mantle is 13.7 km/s and outer core is 8.1 km/s) (*Round off to two decimal places*)

Q.48	The mass of the Earth is 80 times that of the Moon while the radius of the Earth is four times that of the Moon. The surface gravity of the Earth is _____ times that of the Moon? (*In integer)*
Q.49	A hypothetical rock contains the assemblage kyanite, sillimanite and quartz. The variance (degree of freedom) of the assemblage is _____. (*In integer)*
Q.50	The cut-off grade of copper is 0.45 wt%. A mine has 1 million tonne of waste with a grade of 0.25 wt%. The mine also has stock of high grade ore with a grade of 1.8 wt%. How much of this high grade ore (in million tonne) must be blended with the waste to sell the blended ore at a grade of 0.5 wt%? (*Round off to three decimal places)*

Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	The maximum and minimum principal stresses in a zone of active normal faulting are 28 MPa and 8 MPa, respectively. The fault plane strikes N30ºE and dips 60º towards SE. Considering Anderson's theory of faulting, the normal stress on the fault plane is _____ MPa. (*In integer*)
Q.52	A granite block starts sliding on a slope (inclination of 30º with the horizontal) under the effect of gravity only, along the true direction of inclination of the slope and hits the ground in 4 seconds. Considering zero friction and zero cohesion during sliding, the vertical height of the point (with respect to the ground) from where the block was dislodged is _____ m. ($g = 10\ m/s^2$) (*In integer*)
Q.53	A cylindrical soil sample is encased in an open-ended inclined tube with a diameter of 100 mm. There is a constant supply of water from the upper end of the sample and the outflow from the other end is collected in a beaker. The average amount of water collected is 1000 mm^3 every 10 sec. The average outflow velocity is_____ mm/sec. ($\pi = 3.14$) (*Round off to three decimal places*)

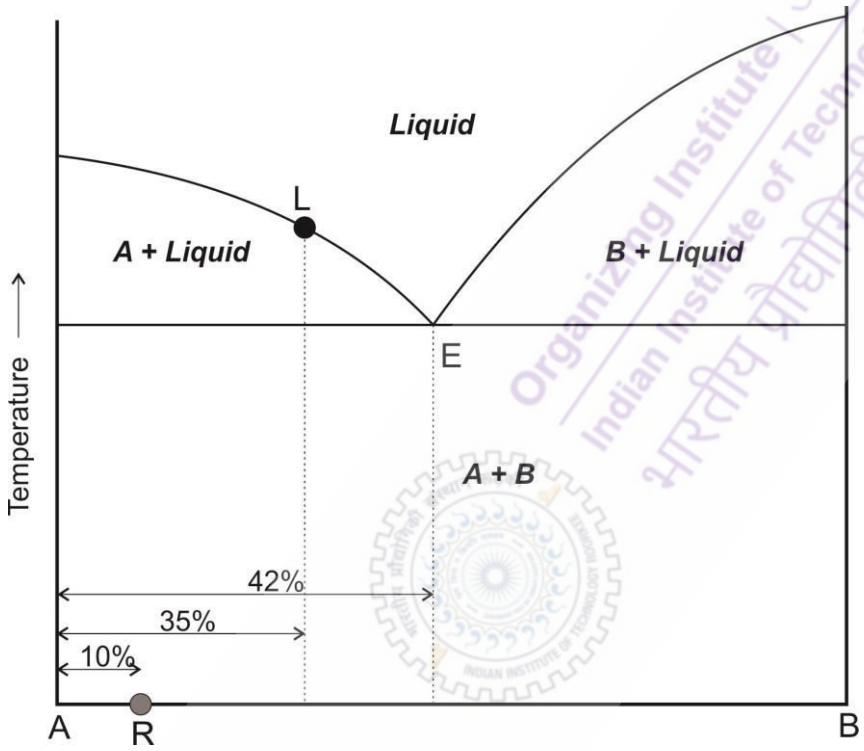
Q.54	Using Airy's hypothesis, calculate the thickness of the root beneath a 4 km high mountain in isostatic equilibrium with a 40 km thick continental crust of density 2800 kg/m^3 and a mantle of density 3300 kg/m^3. Express your answer in km. (*Round off to one decimal place)*
Q.55	Given atomic weights of Cu, Fe and S as 63.55, 55.85 and 32.10, respectively, find out the weight of copper (in gram) metal in an ore (no associated gangue) of 1 kg weight constituting of bornite, chalcopyrite and chalcocite present in weight fractions of 0.4, 0.4 and 0.2, respectively. (*Round off to one decimal place)*

Q.56	An ore body defined by a 300 m × 300 m area is shown in the figure in which the drill hole locations on equally spaced square grid are marked (numbers 1 – 16). The average thickness of the ore body at the 4 interior points is 10.8 m, at the 4 corners is 11.0 m and at the remaining 8 boundary locations is 10.5 m, respectively. The corresponding average grades are 1.5, 1.9 and 1.8 wt%, respectively. Calculate the average grade (in wt%) of the full ore body using the **Included Area Method**. (*Round off to two decimal places).* 
Q.57	The $^{87}Sr/^{86}Sr$ ratio of a 1000 Ma granite was measured as 0.8001. If its $^{87}Rb/^{86}Sr$ ratio is 2.499, what was the Sr isotopic ratio of the source at the time of derivation of the granite? (decay constant of $^{87}Rb = 1.39 \times 10^{-11}$ yr^{-1}) (*Round off to three places of decimals)*

Q.58	The coefficients of permeability of two aquifers – 1 and 2, are 60 m/day and 40 m/day, respectively. Their saturated thicknesses are 30 m and 15 m, respectively. Assuming steady state Darcian flow, the transmissivity of aquifer 1 is _____ times that of aquifer 2. (*In integer*)
Q.59	Assume that ^{218}Po, with a half-life of 138 days, is in secular equilibrium with ^{238}U whose half-life is 4.5×10^9 y. How many grams of ^{218}Po will be present for each gram of ^{238}U in the mineral? Express your answer in **logarithm (to the base 10)**. (*Round off to two decimal places*)

Q.60

The figure below is an isobaric binary temperature-composition (T-X) plot. What amount (in %) of the equilibrium melting of rock R will generate a melt of composition L? *(Round off to one decimal place)*



Special Instructions / Useful Data

Special Instructions / Useful Data	
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R}, i = 1, 2, \ldots, n\},\ n = 2, 3, \ldots$
$\ln x$	Natural logarithm of $x,\ x > 0$
$\det(M)$	Determinant of a square matrix M
I_n	$n \times n$ identity matrix, $n = 2, 3, 4, \ldots$
E^c	Complement of a set E
$P(E)$	Probability of an event E
$P(E \mid F)$	Conditional probability of an event E given the occurrence of the event F
$E(X)$	Expectation of a random variable X
$Var(X)$	Variance of a random variable X
$U(a, b)$	Continuous uniform distribution on the interval $(a, b),\ -\infty < a < b < \infty$
$Exp(\lambda)$	Exponential distribution with the probability density function, for $\lambda > 0$, $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2, $\mu \in \mathbb{R}$, $\sigma > 0$
$\Phi(\cdot)$	The cumulative distribution function of $N(0, 1)$ distributed random variable
χ^2_n	Central chi-square distribution with n degrees of freedom, $n = 1, 2, \ldots$
$F_{m,n}$	Snedecor's central F-distribution with (m, n) degrees of freedom, $m, n = 1, 2, \ldots$
$t_{n,\alpha}$	A constant such that $P(X > t_{n,\alpha}) = \alpha$, where X has central Student's t-distribution with n degrees of freedom, $n = 1, 2, \ldots;\ \alpha \in (0, 1)$
$\Phi(1.645) = 0.95, \quad \Phi(0.355) = 0.6387$ $t_{8,0.0185} = 2.5$	

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	Let $\{a_n\}_{n\geq 1}$ be a sequence of non-zero real numbers. Then which one of the following statements is true?
(A)	If $\left\{\frac{a_{n+1}}{a_n}\right\}_{n\geq 1}$ is a convergent sequence, then $\{a_n\}_{n\geq 1}$ is also a convergent sequence
(B)	If $\{a_n\}_{n\geq 1}$ is a bounded sequence, then $\{a_n\}_{n\geq 1}$ is a convergent sequence
(C)	If $\lvert a_{n+2} - a_{n+1}\rvert \leq \frac{3}{4}\lvert a_{n+1} - a_n\rvert$ for all $n \geq 1$, then $\{a_n\}_{n\geq 1}$ is a Cauchy sequence
(D)	If $\{\lvert a_n\rvert\}_{n\geq 1}$ is a Cauchy sequence, then $\{a_n\}_{n\geq 1}$ is also a Cauchy sequence

Q.2	Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by $$f(x) = \begin{cases} \lim\limits_{h \to 0} \dfrac{(x+h)\sin\left(\frac{1}{x}+h\right) - x\sin\frac{1}{x}}{h}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$ Then which one of the following statements is NOT true?
(A)	$f\left(\frac{2}{\pi}\right) = 1$
(B)	$f\left(\frac{1}{\pi}\right) = \frac{1}{\pi}$
(C)	$f\left(-\frac{2}{\pi}\right) = -1$
(D)	f is not continuous at $x = 0$

Q.3	Let $f:\mathbb{R}\to\mathbb{R}$ be the function defined by $$f(x)=\det\begin{pmatrix}1+x & 9 & 9\\ 9 & 1+x & 9\\ 9 & 9 & 1+x\end{pmatrix}.$$ Then the maximum value of f on the interval $[9,10]$ equals
(A)	118
(B)	112
(C)	114
(D)	116

Q.4	Let A and B be two events such that $0<P(A)<1$ and $0<P(B)<1$. Then which one of the following statements is NOT true?
(A)	If $P(A\|B)>P(A)$, then $P(B\|A)>P(B)$
(B)	If $P(A\cup B)=1$, then A and B cannot be independent
(C)	If $P(A\|B)>P(A)$, then $P(A^c\|B)<P(A^c)$
(D)	If $P(A\|B)>P(A)$, then $P(A^c\|B^c)<P(A^c)$

Q.5	If $M(t),\ t \in \mathbb{R}$, is the moment generating function of a random variable, then which one of the following is NOT the moment generating function of any random variable?
(A)	$\frac{5e^{-5t}}{1-4t^2}M(t),\ \lvert t\rvert < \frac{1}{2}$
(B)	$e^{-t}M(t),\ t \in \mathbb{R}$
(C)	$\frac{1+e^t}{2(2-e^t)}M(t),\ t < \ln 2$
(D)	$M(4t),\ t \in \mathbb{R}$

Q.6	Let X be a random variable having binomial distribution with parameters $n\ (>1)$ and $p\ (0<p<1)$. Then $E\left(\frac{1}{1+X}\right)$ equals
(A)	$\frac{1-(1-p)^{n+1}}{(n+1)p}$
(B)	$\frac{1-p^{n+1}}{(n+1)(1-p)}$
(C)	$\frac{(1-p)^{n+1}}{n(1-p)}$
(D)	$\frac{1-p^n}{(n+1)p}$

Q.7	Let (X, Y) be a random vector having the joint probability density function $f(x,y) = \begin{cases} \frac{\sqrt{2}}{\sqrt{\pi}}\ e^{-2x}\ e^{-\frac{(y-x)^2}{2}}, & 0 < x < \infty, -\infty < y < \infty \\ 0, & \text{otherwise.} \end{cases}$ Then $E(Y)$ equals
(A)	$\frac{1}{2}$
(B)	2
(C)	1
(D)	$\frac{1}{4}$

Q.8	Let X_1 and X_2 be two independent and identically distributed discrete random variables having the probability mass function $$f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3, \ldots \\ 0, & \text{otherwise.} \end{cases}$$ Then $P(\min\{X_1, X_2\} \geq 5)$ equals
(A)	$\frac{1}{256}$
(B)	$\frac{1}{512}$
(C)	$\frac{1}{64}$
(D)	$\frac{9}{256}$

Q.9	Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from $Exp\left(\frac{1}{\theta}\right)$ distribution, where $\theta > 0$ is unknown. If $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then which one of the following statements is NOT true?
(A)	$\overline{X}$ is the uniformly minimum variance unbiased estimator of θ
(B)	$\overline{X}^2$ is the uniformly minimum variance unbiased estimator of θ^2
(C)	$\frac{n}{n+1}\overline{X}^2$ is the uniformly minimum variance unbiased estimator of θ^2
(D)	$Var\left(E\left(X_n \mid \overline{X}\right)\right) \leq Var(X_n)$

Q.10	Let $X_1, X_2, \ldots, X_n$ $(n \geq 3)$ be a random sample from a $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$ are both unknown. Then which one of the following is a simple null hypothesis?
(A)	$H_0: \mu < 5,\ \sigma^2 = 3$
(B)	$H_0: \mu = 5,\ \sigma^2 > 3$
(C)	$H_0: \mu = 5,\ \sigma^2 = 3$
(D)	$H_0: \mu = 5$

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	$\lim_{n\to\infty} \frac{6}{n+2}\left\{\left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(2+\frac{n-1}{n}\right)^2\right\}$ equals
(A)	38
(B)	36
(C)	32
(D)	30

Q.12	Let $f: \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $$f(x,y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \cos y, & x \neq 0 \\ 0, & x = 0. \end{cases}$$ Then which one of the following statements is NOT true?
(A)	f is continuous at $(0,0)$
(B)	The partial derivative of f with respect to x is not continuous at $(0,0)$
(C)	The partial derivative of f with respect to y is continuous at $(0,0)$
(D)	f is not differentiable at $(0,0)$

Q.13	Let $f:[1,2] \to \mathbb{R}$ be the function defined by $$f(t) = \int_1^t \sqrt{x^2 e^{x^2} - 1}\ dx.$$ Then the arc length of the graph of f over the interval $[1,2]$ equals
(A)	$e^2 - \sqrt{e}$
(B)	$e - \sqrt{e}$
(C)	$e^2 - e$
(D)	$e^2 - 1$

Q.14	Let $F:[0,2] \to \mathbb{R}$ be the function defined by $$F(x) = \int_{x^2}^{x+2} e^{x\,[t]}\, dt,$$ where $[t]$ denotes the greatest integer less than or equal to t. Then the value of the derivative of F at $x = 1$ equals
(A)	$e^3 + 2e^2 - e$
(B)	$e^3 - e^2 + 2e$
(C)	$e^3 - 2e^2 + e$
(D)	$e^3 + 2e^2 + e$

Q.15	Let the system of equations $$x + ay + z = 1$$ $$2x + 4y + z = -b$$ $$3x + y + 2z = b + 2$$ have infinitely many solutions, where a and b are real constants. Then the value of $2a + 8b$ equals
(A)	-11
(B)	-10
(C)	-13
(D)	-14

Q.16	Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. Then the sum of all the elements of A^{100} equals
(A)	101
(B)	103
(C)	102
(D)	100

Q.17	Suppose that four persons enter a lift on the ground floor of a building. There are seven floors above the ground floor and each person independently chooses her exit floor as one of these seven floors. If each of them chooses the topmost floor with probability $\frac{1}{3}$ and each of the remaining floors with an equal probability, then the probability that no two of them exit at the same floor equals
(A)	$\frac{200}{729}$
(B)	$\frac{220}{729}$
(C)	$\frac{240}{729}$
(D)	$\frac{180}{729}$

Q.18	A year is chosen at random from the set of years $\{2012, 2013, \ldots, 2021\}$. From the chosen year, a month is chosen at random and from the chosen month, a day is chosen at random. Given that the chosen day is the 29^{th} of a month, the conditional probability that the chosen month is February equals
(A)	$\frac{279}{9965}$
(B)	$\frac{289}{9965}$
(C)	$\frac{269}{9965}$
(D)	$\frac{259}{9965}$

Q.19	Suppose that a fair coin is tossed repeatedly and independently. Let X denote the number of tosses required to obtain for the first time a tail that is immediately preceded by a head. Then $E(X)$ and $P(X > 4)$, respectively, are
(A)	4 and $\frac{5}{16}$
(B)	4 and $\frac{11}{16}$
(C)	6 and $\frac{5}{16}$
(D)	6 and $\frac{11}{16}$

Q.20	Let X be a random variable with the moment generating function $$M(t) = \frac{1}{(1-4t)^5}, \quad t < \frac{1}{4}.$$ Then the lower bounds for $P(X < 40)$, using Chebyshev's inequality and Markov's inequality, respectively, are
(A)	$\frac{4}{5}$ and $\frac{1}{2}$
(B)	$\frac{5}{6}$ and $\frac{1}{2}$
(C)	$\frac{4}{5}$ and $\frac{5}{6}$
(D)	$\frac{5}{6}$ and $\frac{5}{6}$

Q.21	In a store, the daily demand for milk (in litres) is a random variable having $Exp(\lambda)$ distribution, where $\lambda > 0$. At the beginning of the day, the store purchases $c\ (> 0)$ litres of milk at a fixed price $b\ (> 0)$ per litre. The milk is then sold to the customers at a fixed price $s\ (> b)$ per litre. At the end of the day, the unsold milk is discarded. Then the value of c that maximizes the expected net profit for the store equals
(A)	$-\frac{1}{\lambda}\ln\left(\frac{b}{s}\right)$
(B)	$-\frac{1}{\lambda}\ln\left(\frac{b}{s+b}\right)$
(C)	$-\frac{1}{\lambda}\ln\left(\frac{s-b}{s}\right)$
(D)	$-\frac{1}{\lambda}\ln\left(\frac{s}{s+b}\right)$

Q.22	Let X_1, X_2 and X_3 be three independent and identically distributed random variables having $U(0,1)$ distribution. Then $E\left[\left(\frac{\ln X_1}{\ln X_1 X_2 X_3}\right)^2\right]$ equals
(A)	$\frac{1}{6}$
(B)	$\frac{1}{3}$
(C)	$\frac{1}{8}$
(D)	$\frac{1}{4}$

Q.23	Let (X, Y) be a random vector having bivariate normal distribution with parameters $E(X) = 0,\ Var(X) = 1,\ E(Y) = -1,\ Var(Y) = 4$ and $\rho(X, Y) = -\frac{1}{2}$, where $\rho(X, Y)$ denotes the correlation coefficient between X and Y. Then $P(X + Y > 1 \mid 2X - Y = 1)$ equals
(A)	$\Phi\left(-\frac{1}{2}\right)$
(B)	$\Phi\left(-\frac{1}{3}\right)$
(C)	$\Phi\left(-\frac{1}{4}\right)$
(D)	$\Phi\left(-\frac{4}{3}\right)$

Q.24	Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having the common probability density function $$f(x)=\begin{cases}\frac{2}{x^3}, & x\geq 1\\ 0, & \text{otherwise.}\end{cases}$$ If $\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} X_i-\theta\right|<\epsilon\right)=1$ for all $\epsilon>0$, then θ equals
(A)	4
(B)	2
(C)	ln 4
(D)	ln 2

Q.25	Let $0.2, 1.2, 1.4, 0.3, 0.9, 0.7$ be the observed values of a random sample of size 6 from a continuous distribution with the probability density function $$f(x) = \begin{cases} 1, & 0 < x \leq \frac{1}{2} \\ \frac{1}{2\theta - 1}, & \frac{1}{2} < x \leq \theta \\ 0, & \text{otherwise,} \end{cases}$$ where $\theta > \frac{1}{2}$ is unknown. Then the maximum likelihood estimate and the method of moments estimate of θ, respectively, are
(A)	$\frac{7}{5}$ and 2
(B)	$\frac{47}{60}$ and $\frac{32}{15}$
(C)	$\frac{7}{5}$ and $\frac{32}{15}$
(D)	$\frac{7}{5}$ and $\frac{47}{60}$

Q.26	For $n = 1, 2, 3, ...$, let the joint moment generating function of (X, Y_n) be $$M_{X,Y_n}(t_1, t_2) = e^{\frac{t_1^2}{2}}(1 - 2t_2)^{-\frac{n}{2}}, \quad t_1 \in \mathbb{R}, t_2 < \frac{1}{2}.$$ If $T_n = \frac{\sqrt{n}\,X}{\sqrt{Y_n}}$, $n \geq 1$, then which one of the following statements is true?
(A)	The minimum value of n for which $Var(T_n)$ is finite is 2
(B)	$E(T_{10}^3) = 10$
(C)	$Var(X + Y_4) = 7$
(D)	$\lim_{n\to\infty} P(\lvert T_n\rvert > 3) = 1 - \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^3 e^{-\frac{t^2}{2}}\, dt$

Q.27	Let $X_{(1)} < X_{(2)} < \cdots < X_{(9)}$ be the order statistics corresponding to a random sample of size 9 from $U(0,1)$ distribution. Then which one of the following statements is NOT true?
(A)	$E\left(\frac{X_{(9)}}{1-X_{(9)}}\right)$ is finite
(B)	$E\left(X_{(5)}\right) = 0.5$
(C)	The median of $X_{(5)}$ is 0.5
(D)	The mode of $X_{(5)}$ is 0.5

Q.28	Let $X_1, X_2, \ldots, X_{16}$ be a random sample from a $N(4\mu, 1)$ distribution and $Y_1, Y_2, \ldots, Y_8$ be a random sample from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is unknown. Assume that the two random samples are independent. If you are looking for a confidence interval for μ based on the statistic $8\overline{X} + \overline{Y}$, where $\overline{X} = \frac{1}{16}\sum_{i=1}^{16} X_i$ and $\overline{Y} = \frac{1}{8}\sum_{i=1}^{8} Y_i$, then which one of the following statements is true?
(A)	There exists a 90% confidence interval for μ of length less than 0.1
(B)	There exists a 90% confidence interval for μ of length greater than 0.3
(C)	$\left[\frac{8\overline{X}+\overline{Y}}{33} - \frac{1.645}{2\sqrt{66}}, \frac{8\overline{X}+\overline{Y}}{33} + \frac{1.645}{2\sqrt{66}}\right]$ is the unique 90% confidence interval for μ
(D)	μ always belongs to its 90% confidence interval

Q.29	Let X_1, X_2, X_3, X_4 be a random sample from a distribution with the probability mass function $$f(x) = \begin{cases} \theta^x(1-\theta)^{1-x}, & x = 0, 1 \\ 0, & \text{otherwise,} \end{cases}$$ where $\theta \in (0, 1)$ is unknown. Let $0 < \alpha \leq 1$. To test the hypothesis $H_0: \theta = \frac{1}{2}$ against $H_1: \theta > \frac{1}{2}$, consider the size α test that rejects H_0 if and only if $\sum_{i=1}^{4} X_i \geq k_\alpha$, for some $k_\alpha \in \{0, 1, 2, 3, 4\}$. Then for which one of the following values of α, the size α test does NOT exist?
(A)	$\frac{1}{16}$
(B)	$\frac{1}{4}$
(C)	$\frac{11}{16}$
(D)	$\frac{5}{16}$

Q.30	Let $X_1,\ X_2,\ X_3,\ X_4$ be a random sample from a Poisson distribution with unknown mean $\lambda > 0$. For testing the hypothesis $$H_0: \lambda = 1 \text{ against } H_1: \lambda = 1.5,$$ let β denote the power of the test that rejects H_0 if and only if $\sum_{i=1}^{4} X_i \geq 5$. Then which one of the following statements is true?
(A)	$\beta > 0.80$
(B)	$0.75 < \beta \leq 0.80$
(C)	$0.70 < \beta \leq 0.75$
(D)	$0.65 < \beta \leq 0.70$

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_n = \frac{1}{3^n}$ for all $n \geq 1$. Then which of the following statements is/are true?
(A)	$\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is a convergent series
(B)	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(a_1 + a_2 + \ldots + a_n)$ is a convergent series
(C)	The radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ is $\frac{1}{3}$
(D)	$\sum_{n=1}^{\infty} a_n \sin\frac{1}{a_n}$ is a convergent series

Q.32	Let $f: \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $$f(x, y) = 8(x^2 - y^2) - x^4 + y^4.$$ Then which of the following statements is/are true?
(A)	f has 9 critical points
(B)	f has a saddle point at $(2, 2)$
(C)	f has a local maximum at $(-2, 0)$
(D)	f has a local minimum at $(0, -2)$

Q.33	If $n \geq 2$, then which of the following statements is/are true?
(A)	If A and B are $n \times n$ real orthogonal matrices such that $\det(A) + \det(B) = 0$, then $A + B$ is a singular matrix
(B)	If A is an $n \times n$ real matrix such that $I_n + A$ is non-singular, then $I_n + (I_n + A)^{-1}(I_n - A)$ is a singular matrix
(C)	If A is an $n \times n$ real skew-symmetric matrix, then $I_n - A^2$ is a non-singular matrix
(D)	If A is an $n \times n$ real orthogonal matrix, then $\det(A - \lambda I_n) \neq 0$ for all $\lambda \in \{x \in \mathbb{R} : x \neq \pm 1\}$

Q.34	Let $\Omega = \{1, 2, 3, \dots\}$ be the sample space of a random experiment and suppose that all subsets of Ω are events. Further, let P be a probability function such that $P(\{i\}) > 0$ for all $i \in \Omega$. Then which of the following statements is/are true?
(A)	For every $\epsilon > 0$, there exists an event A such that $0 < P(A) < \epsilon$
(B)	There exists a sequence of disjoint events $\{A_k\}_{k \geq 1}$ with $P(A_k) \geq 10^{-6}$ for all $k \geq 1$
(C)	There exists $j \in \Omega$ such that $P(\{j\}) \geq P(\{i\})$ for all $i \in \Omega$
(D)	Let $\{A_k\}_{k \geq 1}$ be a sequence of events such that $\sum_{k=1}^{\infty} P(A_k) < \infty$. Then for each $i \in \Omega$ there exists $N \geq 1$ (which may depend on i) such that $i \notin \bigcup_{k=N}^{\infty} A_k$

Q.35	A university bears the yearly medical expenses of each of its employees up to a maximum of Rs. 1000. If the yearly medical expenses of an employee exceed Rs. 1000, then the employee gets the excess amount from an insurance policy up to a maximum of Rs. 500. If the yearly medical expenses of a randomly selected employee has $U(250, 1750)$ distribution and Y denotes the amount the employee gets from the insurance policy, then which of the following statements is/are true?
(A)	$E(Y) = \frac{500}{3}$
(B)	$P(Y > 300) = \frac{3}{10}$
(C)	The median of Y is zero
(D)	The quantile of order 0.6 for Y equals 100

Q.36	Let X and Y be two independent random variables having $N(0,\ \sigma_1^2)$ and $N(0,\ \sigma_2^2)$ distributions, respectively, where $0 < \sigma_1 < \sigma_2$. Then which of the following statements is/are true?
(A)	$X + Y$ and $X - Y$ are independent
(B)	$2X + Y$ and $X - Y$ are independent if $2\sigma_1^2 = \sigma_2^2$
(C)	$X + Y$ and $X - Y$ are identically distributed
(D)	$X + Y$ and $2X - Y$ are independent if $2\sigma_1^2 = \sigma_2^2$

Q.37	Let (X, Y) be a discrete random vector. Then which of the following statements is/are true?
(A)	If X and Y are independent, then X^2 and $\|Y\|$ are also independent.
(B)	If the correlation coefficient between X and Y is 1, then $P(Y = aX + b) = 1$ for some $a, b \in \mathbb{R}$
(C)	If X and Y are independent and $E[(XY)^2] = 0$, then $P(X = 0) = 1$ or $P(Y = 0) = 1$
(D)	If $Var(X) = 0$, then X and Y are independent

Q.38	Let X_1, X_2 and X_3 be three independent and identically distributed random variables having $N(0,1)$ distribution. If $$U = \frac{2X_1^2}{(X_2+X_3)^2} \quad \text{and} \quad V = \frac{2(X_2-X_3)^2}{2X_1^2+(X_2+X_3)^2},$$ then which of the following statements is/are true?
(A)	U has $F_{1,1}$ distribution and V has $F_{1,2}$ distribution
(B)	U has $F_{1,1}$ distribution and V has $F_{2,1}$ distribution
(C)	U and V are independent
(D)	$\frac{1}{2}V(1+U)$ has $F_{2,3}$ distribution

Q.39	Let $X_1,\ X_2,\ X_3,\ X_4$ be a random sample from a continuous distribution with the probability density function $f(x) = \frac{1}{2}\ e^{-\lvert x-\theta\rvert},\ x \in \mathbb{R}$, where $\theta \in \mathbb{R}$ is unknown. Let the corresponding order statistics be denoted by $X_{(1)} < X_{(2)} < X_{(3)} < X_{(4)}$. Then which of the following statements is/are true?
(A)	$\frac{1}{2}(X_{(2)} + X_{(3)})$ is the unique maximum likelihood estimator of θ
(B)	$(X_{(1)},\ X_{(2)},\ X_{(3)},\ X_{(4)})$ is a sufficient statistic for θ
(C)	$\frac{1}{4}(X_{(2)} + X_{(3)})\ (X_{(2)} + X_{(3)} + 2)$ is a maximum likelihood estimator of $\theta(\theta + 1)$
(D)	$(X_1X_2X_3,\ X_1X_2X_4)$ is a complete statistic

Q.40	Let $X_1, X_2, \ldots, X_n$ $(n > 1)$ be a random sample from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is unknown. Let $0 < \alpha < 1$. To test the hypothesis $H_0: \mu = 0$ against $H_1: \mu = \delta$, where $\delta > 0$ is a constant, let β denote the power of the size α test that rejects H_0 if and only if $\frac{1}{n}\sum_{i=1}^{n} X_i > c_\alpha$, for some constant c_α. Then which of the following statements is/are true?
(A)	For a fixed value of δ, β increases as α increases
(B)	For a fixed value of α, β increases as δ increases
(C)	For a fixed value of δ, β decreases as α increases
(D)	For a fixed value of α, β decreases as δ increases

Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_{1+5m} = 2$, $a_{2+5m} = 3, a_{3+5m} = 4, a_{4+5m} = 5, a_{5+5m} = 6,\ m = 0, 1, 2, \ldots$. Then $\limsup_{n\to\infty} a_n + \liminf_{n\to\infty} a_n$ equals _____

Q.42	Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $$20(x-y) \leq f(x) - f(y) \leq 20(x-y) + 2(x-y)^2$$ for all $x, y \in \mathbb{R}$ and $f(0) = 2$. Then $f(101)$ equals _____

Q.43	Let A be a 3×3 real matrix such that $\det(A) = 6$ and $$adj\, A = \begin{pmatrix} 1 & -1 & 2 \\ 5 & 7 & 1 \\ -1 & 1 & 1 \end{pmatrix},$$ where $adj\, A$ denotes the adjoint of A. Then the trace of A equals _____ (round off to 2 decimal places)

Q.44	Let X and Y be two independent and identically distributed random variables having $U(0,1)$ distribution. Then $P(X^2 < Y < X)$ equals _____ (round off to 2 decimal places)

Q.45	Consider a sequence of independent Bernoulli trials, where $\frac{3}{4}$ is the probability of success in each trial. Let X be a random variable defined as follows: If the first trial is a success, then X counts the number of failures before the next success. If the first trial is a failure, then X counts the number of successes before the next failure. Then $2E(X)$ equals _____

Q.46	Let X be a random variable denoting the amount of loss in a business. The moment generating function of X is $$M(t) = \left(\frac{2}{2-t}\right)^2, \quad t < 2.$$ If an insurance policy pays 60% of the loss, then the variance of the amount paid by the insurance policy equals _____ (round off to 2 decimal places)

Q.47	Let (X, Y) be a random vector having the joint moment generating function $$M(t_1, t_2) = \left(\frac{1}{2}\, e^{-t_1} + \frac{1}{2}\, e^{t_1}\right)^2 \left(\frac{1}{2} + \frac{1}{2}\, e^{t_2}\right)^2, \quad (t_1, t_2) \in \mathbb{R}^2.$$ Then $P(\lvert X + Y\rvert = 2)$ equals _____ (round off to 2 decimal places)

Q.48	Let X_1 and X_2 be two independent and identically distributed random variables having χ^2_2 distribution and $W = X_1 + X_2$. Then $P(W > E(W))$ equals _____ (round off to 2 decimal places)

Q.49	Let $2.5, -1.0, 0.5, 1.5$ be the observed values of a random sample of size 4 from a continuous distribution with the probability density function $$f(x) = \frac{1}{8}\, e^{-\lvert x-2\rvert} + \frac{3}{4\sqrt{2\pi}}\, e^{-\frac{1}{2}(x-\theta)^2}, \quad x \in \mathbb{R},$$ where $\theta \in \mathbb{R}$ is unknown. Then the method of moments estimate of θ equals _____ (round off to 2 decimal places)

Q.50	Let $X_1, X_2, \ldots, X_{25}$ be a random sample from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is unknown. Consider testing of the hypothesis $H_0: \mu = 5.2$ against $H_1: \mu = 5.6$. The null hypothesis is rejected if and only if $\frac{1}{25}\sum_{i=1}^{25} X_i > k$, for some constant k. If the size of the test is 0.05, then the probability of type-II error equals _____ (round off to 2 decimal places)

Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $f(x, y) = x^2 - 12y$. If M and m be the maximum value and the minimum value, respectively, of the function f on the circle $x^2 + y^2 = 49$, then $\lvert M\rvert + \lvert m\rvert$ equals _____

Q.52	The value of $$\int_0^2 \int_0^{2-x} (x+y)^2 \, e^{\frac{2y}{x+y}} \, dy \, dx$$ equals _____ (round off to 2 decimal places)

Q.53	Let $A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$ and let $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector corresponding to the smallest eigenvalue of A, satisfying $x_1^2 + x_2^2 + x_3^2 = 1$. Then the value of $\lvert x_1 \rvert + \lvert x_2 \rvert + \lvert x_3 \rvert$ equals _____ (round off to 2 decimal places)

Q.54	Five men go to a restaurant together and each of them orders a dish that is different from the dishes ordered by the other members of the group. However, the waiter serves the dishes randomly. Then the probability that exactly one of them gets the dish he ordered equals _____ (round off to 2 decimal places)

Q.55	Let X be a random variable having the probability density function $$f(x) = \begin{cases} ax^2 + b, & 0 \le x \le 3 \\ 0, & \text{otherwise,} \end{cases}$$ where a and b are real constants, and $P(X \ge 2) = \frac{2}{3}$. Then $E(X)$ equals _____ (round off to 2 decimal places)

Q.56	A vaccine, when it is administered to an individual, produces no side effects with probability $\frac{4}{5}$, mild side effects with probability $\frac{2}{15}$ and severe side effects with probability $\frac{1}{15}$. Assume that the development of side effects is independent across individuals. The vaccine was administered to 1000 randomly selected individuals. If X_1 denotes the number of individuals who developed mild side effects and X_2 denotes the number of individuals who developed severe side effects, then the coefficient of variation of $X_1 + X_2$ equals _____ (round off to 2 decimal places)

Q.57	Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having $U(0,1)$ distribution. Let $Y_n = n\min\{X_1, X_2, \ldots, X_n\}$, $n \geq 1$. If Y_n converges to Y in distribution, then the median of Y equals _____ (round off to 2 decimal places)

Q.58	Let $X_{(1)} < X_{(2)} < X_{(3)} < X_{(4)} < X_{(5)}$ be the order statistics based on a random sample of size 5 from a continuous distribution with the probability density function $$f(x) = \begin{cases} \frac{1}{x^2}, & 1 < x < \infty \\ 0, & \text{otherwise.} \end{cases}$$ Then the sum of all possible values of $r \in \{1,2,3,4,5\}$ for which $E(X_{(r)})$ is finite equals _____

Q.59

Consider the linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i, i = 1, 2, \ldots, 6,$ where β_0 and β_1 are unknown parameters and ϵ_i's are independent and identically distributed random variables having $N(0, 1)$ distribution. The data on (x_i, y_i) are given in the following table.

x_i	1.0	2.0	2.5	3.0	3.5	4.5
y_i	2.0	3.0	3.5	4.2	5.0	5.4

If $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the least squares estimates of β_0 and β_1, respectively, based on the above data, then $\widehat{\beta_0} + \widehat{\beta_1}$ equals _____ (round off to 2 decimal places)

Q.60

Let $X_1, X_2, \ldots, X_9$ be a random sample from a $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$ are unknown. Let the observed values of $\overline{X} = \frac{1}{9}\sum_{i=1}^{9} X_i$ and $S^2 = \frac{1}{8}\sum_{i=1}^{9}(X_i - \overline{X})^2$ be 9.8 and 1.44, respectively. If the likelihood ratio test is used to test the hypothesis $H_0: \mu = 8.8$ against $H_1: \mu > 8.8$, then the p-value of the test equals _____ (round off to 3 decimal places)

Notation and Terminology

$\mathbb{N} =$ the set of all positive integers.

$\mathbb{Z} =$ the set of all integers.

$\mathbb{Q} =$ the set of all rational numbers.

$\mathbb{R} =$ the set of all real numbers.

$\mathbb{R}^n =$ the n-dimensional Euclidean space.

$\mathbb{C} =$ the set of all complex numbers.

$M_n(\mathbb{R})$ = the real vector space of all $n \times n$ matrices with entries in $\mathbb{R}$.

$M_n(\mathbb{C})$ = the complex vector space of all $n \times n$ matrices with entries in $\mathbb{C}$.

$\gcd(m, n)$ = the greatest common divisor of the integers m and n.

$M^{\mathsf{T}} =$ the transpose of the matrix M.

$A - B =$ the complement of the set B in the set A, that is, $\{x \in A : x \notin B\}$.

$\ln x =$ the natural logarithm of x (to the base e).

$|x| =$ the absolute value of x.

$y', y'', y''' =$ the first, second and the third derivatives of the function y, respectively.

$S_n =$ the symmetric group consisting of all permutations of $\{1,2, \dots, n\}$.

$\mathbb{Z}_n =$ the additive group of integers modulo n.

$f \circ g$ is the composite function defined by $(f \circ g)(x) = f(g(x))$.

The phrase '**real vector space**' refers to a vector space over $\mathbb{R}$.

Section A: Q.1 – Q.10 carry ONE mark each.	
Q.1	Consider the 2×2 matrix $M = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \in M_2(\mathbb{R})$. If the eighth power of M satisfies $M^8 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$, then the value of x is
(A)	21
(B)	22
(C)	34
(D)	35
Q.2	The rank of the 4×6 matrix $\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ with entries in $\mathbb{R}$, is
(A)	1
(B)	2
(C)	3
(D)	4

Q.3	Let V be the real vector space consisting of all polynomials in one variable with real coefficients and having degree at most 6, together with the zero polynomial. Then which one of the following is true?
(A)	$\{f \in V : f(1/2) \notin \mathbb{Q}\}$ is a subspace of V.
(B)	$\{f \in V : f(1/2) = 1\}$ is a subspace of V.
(C)	$\{f \in V : f(1/2) = f(1)\}$ is a subspace of V.
(D)	$\{f \in V : f'(1/2) = 1\}$ is a subspace of V.
Q.4	Let G be a group of order 2022. Let H and K be subgroups of G of order 337 and 674, respectively. If $H \cup K$ is also a subgroup of G, then which one of the following is FALSE?
(A)	H is a normal subgroup of $H \cup K$.
(B)	The order of $H \cup K$ is 1011.
(C)	The order of $H \cup K$ is 674.
(D)	K is a normal subgroup of $H \cup K$.

Q.5	The radius of convergence of the power series $\sum_{n=1}^{\infty}\left(\frac{n^3}{4^n}\right)x^{5n}$ is
(A)	4
(B)	$\sqrt[5]{4}$
(C)	$\frac{1}{4}$
(D)	$\frac{1}{\sqrt[5]{4}}$

Q.6	Let (x_n) and (y_n) be sequences of real numbers defined by $x_1 = 1,\ \ y_1 = \frac{1}{2},\ \ x_{n+1} = \frac{x_n + y_n}{2},$ and $y_{n+1} = \sqrt{x_n y_n}$ for all $n \in \mathbb{N}$. Then which one of the following is true?
(A)	(x_n) is convergent, but (y_n) is not convergent.
(B)	(x_n) is not convergent, but (y_n) is convergent.
(C)	Both (x_n) and (y_n) are convergent and $\lim_{n\to\infty} x_n > \lim_{n\to\infty} y_n$.
(D)	Both (x_n) and (y_n) are convergent and $\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n$.
Q.7	Suppose $a_n = \frac{3^n + 3}{5^n - 5}$ and $b_n = \frac{1}{(1+n^2)^{\frac{1}{4}}}$ for $n = 2,3,4, \ldots.$ Then which one of the following is true?
(A)	Both $\sum_{n=2}^{\infty} a_n$ and $\sum_{n=2}^{\infty} b_n$ are convergent.
(B)	Both $\sum_{n=2}^{\infty} a_n$ and $\sum_{n=2}^{\infty} b_n$ are divergent.
(C)	$\sum_{n=2}^{\infty} a_n$ is convergent and $\sum_{n=2}^{\infty} b_n$ is divergent.
(D)	$\sum_{n=2}^{\infty} a_n$ is divergent and $\sum_{n=2}^{\infty} b_n$ is convergent.

Q.8	Consider the series $$\sum_{n=1}^{\infty} \frac{1}{n^m\left(1+\frac{1}{n^p}\right)}$$ where m and p are real numbers. Under which of the following conditions does the above series converge?
(A)	$m > 1$.
(B)	$0 < m < 1$ and $p > 1$.
(C)	$0 < m \leq 1$ and $0 \leq p \leq 1$.
(D)	$m = 1$ and $p > 1$.

Q.9	Let c be a positive real number and let $u: \mathbb{R}^2 \to \mathbb{R}$ be defined by $$u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} e^{s^2} ds \quad \text{for} \quad (x,t) \in \mathbb{R}^2.$$ Then which one of the following is true?
(A)	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ on $\mathbb{R}^2$.
(B)	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ on $\mathbb{R}^2$.
(C)	$\frac{\partial u}{\partial t}\frac{\partial u}{\partial x} = 0$ on $\mathbb{R}^2$.
(D)	$\frac{\partial^2 u}{\partial t\, \partial x} = 0$ on $\mathbb{R}^2$.

Q.10	Let $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Consider the functions $$u : \mathbb{R}^2 - \{(0,0)\} \to \mathbb{R} \quad \text{and} \quad v : \mathbb{R}^2 - \{(0,0)\} \to \mathbb{R}$$ given by $$u(x,y) = x - \frac{x}{x^2+y^2} \quad \text{and} \quad v(x,y) = y + \frac{y}{x^2+y^2}.$$ The value of the determinant $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ at the point $(\cos\theta, \sin\theta)$ is equal to
(A)	$4\sin\theta$.
(B)	$4\cos\theta$.
(C)	$4\sin^2\theta$.
(D)	$4\cos^2\theta$.

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Consider the open rectangle $G = \{(s,t) \in \mathbb{R}^2 : \ 0 < s < 1 \text{ and } 0 < t < 1\}$ and the map $T: G \to \mathbb{R}^2$ given by $$T(s,t) = \left(\frac{\pi s(1-t)}{2}, \ \frac{\pi(1-s)}{2}\right) \quad \text{for } (s,t) \in G \ .$$ Then the area of the image $T(G)$ of the map T is equal to
(A)	$\frac{\pi}{4}$
(B)	$\frac{\pi^2}{4}$
(C)	$\frac{\pi^2}{8}$
(D)	1

Q.12	Let T denote the sum of the convergent series $$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots+\frac{(-1)^{n+1}}{n}+\cdots$$ and let S denote the sum of the convergent series $$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+\cdots=\sum_{n=1}^{\infty}a_n,$$ where $$a_{3m-2}=\frac{1}{2m-1},\quad a_{3m-1}=\frac{-1}{4m-2}\quad\text{and}\quad a_{3m}=\frac{-1}{4m}\quad\text{for } m\in\mathbb{N}.$$ Then which one of the following is true?
(A)	$T=S$ and $S\neq 0$.
(B)	$2T=S$ and $S\neq 0$.
(C)	$T=2S$ and $S\neq 0$.
(D)	$T=S=0$.

Q.13	Let $u: \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function such that $u(0) > 0$ and $u'(0) > 0$. Suppose u satisfies $$u''(x) = \frac{u(x)}{1+x^2} \quad \text{for all } x \in \mathbb{R}.$$ Consider the following two statements: I. The function uu' is monotonically increasing on $[0, \infty)$. II. The function u is monotonically increasing on $[0, \infty)$. Then which one of the following is correct?
(A)	Both I and II are false.
(B)	Both I and II are true.
(C)	I is false, but II is true.
(D)	I is true, but II is false.

Q.14	The value of $$\lim_{n\to\infty}\sum_{k=2}^{n}\frac{\sqrt{n+1}-\sqrt{n}}{k(\ln k)^2}$$ is equal to
(A)	∞
(B)	1
(C)	e
(D)	0

Q.15	For $t \in \mathbb{R}$, let $[t]$ denote the greatest integer less than or equal to t. Define functions $h: \mathbb{R}^2 \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ by $h(x,y) = \begin{cases} \frac{-1}{x^2-y} & \text{if } x^2 \neq y, \\ 0 & \text{if } x^2 = y \end{cases}$ and $g(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$ Then which one of the following is FALSE?
(A)	$\lim_{(x,y)\to(\sqrt{2},\pi)} \cos\left(\frac{x^2y}{x^2+1}\right) = \frac{-1}{2}.$
(B)	$\lim_{(x,y)\to(\sqrt{2},2)} e^{h(x,y)} = 0.$
(C)	$\lim_{(x,y)\to(e,e)} \ln\left(x^{y-[y]}\right) = e - 2.$
(D)	$\lim_{(x,y)\to(0,0)} e^{2y} g(x) = 1.$

Q.16	Let $P \in M_4(\mathbb{R})$ be such that P^4 is the zero matrix, but P^3 is a nonzero matrix. Then which one of the following is FALSE?
(A)	For every nonzero vector $v \in \mathbb{R}^4$, the subset $\{v, Pv, P^2v, P^3v\}$ of the real vector space $\mathbb{R}^4$ is linearly independent.
(B)	The rank of P^k is $4-k$ for every $k \in \{1,2,3,4\}$.
(C)	0 is an eigenvalue of P.
(D)	If $Q \in M_4(\mathbb{R})$ is such that Q^4 is the zero matrix, but Q^3 is a nonzero matrix, then there exists a nonsingular matrix $S \in M_4(\mathbb{R})$ such that $S^{-1}QS = P$.

Q.17	For $X, Y \in M_2(\mathbb{R})$, define $(X, Y) = XY - YX$. Let $\mathbf{0} \in M_2(\mathbb{R})$ denote the zero matrix. Consider the two statements: $P:\ \big(X,(Y,Z)\big) + \big(Y,(Z,X)\big) + \big(Z,(X,Y)\big) = \mathbf{0}$ for all $X, Y, Z \in M_2(\mathbb{R})$. $Q:\ \big(X,(Y,Z)\big) = \big((X,Y),Z\big)$ for all $X, Y, Z \in M_2(\mathbb{R})$. Then which one of the following is correct?
(A)	Both P and Q are true.
(B)	P is true, but Q is false.
(C)	P is false, but Q is true.
(D)	Both P and Q are false.

Q.18	Consider the system of linear equations $$\begin{aligned} x+y+t &= 4,\\ 2x-4t &= 7,\\ x+y+z &= 5,\\ x-3y-z-10t &= \lambda, \end{aligned}$$ where x, y, z, t are variables and λ is a constant. Then which one of the following is true?
(A)	If $\lambda = 1$, then the system has a unique solution.
(B)	If $\lambda = 2$, then the system has infinitely many solutions.
(C)	If $\lambda = 1$, then the system has infinitely many solutions.
(D)	If $\lambda = 2$, then the system has a unique solution.
Q.19	Consider the group $(\mathbb{Q}, +)$ and its subgroup $(\mathbb{Z}, +)$. For the quotient group $\mathbb{Q}/\mathbb{Z}$, which one of the following is FALSE?
(A)	$\mathbb{Q}/\mathbb{Z}$ contains a subgroup isomorphic to $(\mathbb{Z}, +)$.
(B)	There is exactly one group homomorphism from $\mathbb{Q}/\mathbb{Z}$ to $(\mathbb{Q}, +)$.
(C)	For all $n \in \mathbb{N}$, there exists $g \in \mathbb{Q}/\mathbb{Z}$ such that the order of g is n.
(D)	$\mathbb{Q}/\mathbb{Z}$ is not a cyclic group.

Q.20	For $P \in M_5(\mathbb{R})$ and $i, j \in \{1,2,\ldots,5\}$, let p_{ij} denote the $(i,j)^{\text{th}}$ entry of P. Let $S = \{P \in M_5(\mathbb{R}) : p_{ij} = p_{rs}$ for $i, j, r, s \in \{1,2,\ldots,5\}$ with $i + r = j + s\}$. Then which one of the following is FALSE?
(A)	S is a subspace of the vector space over $\mathbb{R}$ of all 5×5 symmetric matrices.
(B)	The dimension of S over $\mathbb{R}$ is 5.
(C)	The dimension of S over $\mathbb{R}$ is 11.
(D)	If $P \in S$ and all the entries of P are integers, then 5 divides the sum of all the diagonal entries of P.

Q.21	On the open interval $(-c, c)$, where c is a positive real number, $y(x)$ is an infinitely differentiable solution of the differential equation $$\frac{dy}{dx} = y^2 - 1 + \cos x,$$ with the initial condition $y(0) = 0$. Then which one of the following is correct?
(A)	$y(x)$ has a local maximum at the origin.
(B)	$y(x)$ has a local minimum at the origin.
(C)	$y(x)$ is strictly increasing on the open interval $(-\delta, \delta)$ for some positive real number δ.
(D)	$y(x)$ is strictly decreasing on the open interval $(-\delta, \delta)$ for some positive real number δ.

Q.22	Let $H : \mathbb{R} \to \mathbb{R}$ be the function given by $H(x) = \frac{1}{2}(e^x + e^{-x})$ for $x \in \mathbb{R}$. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $$f(x) = \int_0^{\pi} H(x \sin\theta) d\theta \quad \text{for } x \in \mathbb{R}.$$ Then which one of the following is true?
(A)	$xf''(x) + f'(x) + xf(x) = 0$ for all $x \in \mathbb{R}$.
(B)	$xf''(x) - f'(x) + xf(x) = 0$ for all $x \in \mathbb{R}$.
(C)	$xf''(x) + f'(x) - xf(x) = 0$ for all $x \in \mathbb{R}$.
(D)	$xf''(x) - f'(x) - xf(x) = 0$ for all $x \in \mathbb{R}$.

Q.23	Consider the differential equation $$y'' + ay' + y = \sin x \quad \text{for } x \in \mathbb{R}. \qquad (**)$$ Then which one of the following is true?
(A)	If $a = 0$, then all the solutions of $(**)$ are unbounded over $\mathbb{R}$.
(B)	If $a = 1$, then all the solutions of $(**)$ are unbounded over $(0, \infty)$.
(C)	If $a = 1$, then all the solutions of $(**)$ tend to zero as $x \to \infty$.
(D)	If $a = 2$, then all the solutions of $(**)$ are bounded over $(-\infty, 0)$.
Q.24	For $g \in \mathbb{Z}$, let $\overline{g} \in \mathbb{Z}_{37}$ denote the residue class of g modulo 37. Consider the group $U_{37} = \{\overline{g} \in \mathbb{Z}_{37} : 1 \leq g \leq 37 \text{ with } \gcd(g, 37) = 1\}$ with respect to multiplication modulo 37. Then which one of the following is FALSE?
(A)	The set $\{\overline{g} \in U_{37} : \overline{g} = (\overline{g})^{-1}\}$ contains exactly 2 elements.
(B)	The order of the element $\overline{10}$ in U_{37} is 36.
(C)	There is exactly one group homomorphism from U_{37} to $(\mathbb{Z}, +)$.
(D)	There is exactly one group homomorphism from U_{37} to $(\mathbb{Q}, +)$.

Q.25	For some real number c with $0 < c < 1$, let $\phi: (1-c, 1+c) \to (0, \infty)$ be a differentiable function such that $\phi(1) = 1$ and $y = \phi(x)$ is a solution of the differential equation $$(x^2 + y^2)dx - 4xy\,dy = 0.$$ Then which one of the following is true?
(A)	$\left(3\left(\phi(x)\right)^2 + x^2\right)^2 = 4x.$
(B)	$\left(3\left(\phi(x)\right)^2 - x^2\right)^2 = 4x.$
(C)	$\left(3\left(\phi(x)\right)^2 + x^2\right)^2 = 4\phi(x).$
(D)	$\left(3\left(\phi(x)\right)^2 - x^2\right)^2 = 4\phi(x).$

Q.26	For a 4×4 matrix $M \in M_4(\mathbb{C})$, let $\overline{M}$ denote the matrix obtained from M by replacing each entry of M by its complex conjugate. Consider the real vector space $$H = \{M \in M_4(\mathbb{C}) : \quad M^{\top} = \overline{M}\}$$ where $M^{\top}$ denotes the transpose of M. The dimension of H as a vector space over $\mathbb{R}$ is equal to
(A)	6
(B)	16
(C)	15
(D)	12

Q.27	Let a, b be positive real numbers such that $a < b$. Given that $$\lim_{N\to\infty} \int_0^N e^{-t^2} dt = \frac{\sqrt{\pi}}{2},$$ the value of $$\lim_{N\to\infty} \int_0^N \frac{1}{t^2}\left(e^{-at^2} - e^{-bt^2}\right) dt$$ is equal to
(A)	$\sqrt{\pi}(\sqrt{a} - \sqrt{b})$.
(B)	$\sqrt{\pi}(\sqrt{a} + \sqrt{b})$.
(C)	$-\sqrt{\pi}(\sqrt{a} + \sqrt{b})$.
(D)	$\sqrt{\pi}(\sqrt{b} - \sqrt{a})$.

Q.28	For $-1 \leq x \leq 1$, if $f(x)$ is the sum of the convergent power series $$x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \cdots + \frac{x^n}{n^2} + \cdots$$ then $f\left(\frac{1}{2}\right)$ is equal to
(A)	$\int_0^{\frac{1}{2}} \frac{\ln(1-t)}{t}\, dt.$
(B)	$-\int_0^{\frac{1}{2}} \frac{\ln(1-t)}{t}\, dt.$
(C)	$\int_0^{\frac{1}{2}} \mathrm{t}\ln(1+t)\, dt.$
(D)	$\int_0^{\frac{1}{2}} \mathrm{t}\ln(1-t)\, dt.$

Q.29	For $n \in \mathbb{N}$ and $x \in [1, \infty)$, let $$f_n(x) = \int_0^{\pi} \left(x^2 + (\cos\theta)\sqrt{x^2-1}\right)^n d\theta.$$ Then which one of the following is true?
(A)	$f_n(x)$ is not a polynomial in x if n is odd and $n \geq 3$.
(B)	$f_n(x)$ is not a polynomial in x if n is even and $n \geq 4$.
(C)	$f_n(x)$ is a polynomial in x for all $n \in \mathbb{N}$.
(D)	$f_n(x)$ is not a polynomial in x for any $n \geq 3$.
Q.30	Let P be a 3×3 real matrix having eigenvalues $\lambda_1 = 0, \lambda_2 = 1$ and $\lambda_3 = -1$. Further, $v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $v_2 = \begin{pmatrix}1\\1\\0\end{pmatrix}$ and $v_3 = \begin{pmatrix}1\\0\\1\end{pmatrix}$ are eigenvectors of the matrix P corresponding to the eigenvalues λ_1, λ_2 and λ_3, respectively. Then the entry in the first row and the third column of P is
(A)	0
(B)	1
(C)	-1
(D)	2

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Let $(-c, c)$ be the largest open interval in $\mathbb{R}$ (where c is either a positive real number or $c = \infty$) on which the solution $y(x)$ of the differential equation $\frac{dy}{dx} = x^2 + y^2 + 1$ with initial condition $y(0) = 0$ exists and is unique. Then which of the following is/are true?
(A)	$y(x)$ is an odd function on $(-c, c)$.
(B)	$y(x)$ is an even function on $(-c, c)$.
(C)	$\big(y(x)\big)^2$ has a local minimum at 0.
(D)	$\big(y(x)\big)^2$ has a local maximum at 0.

Q.32	Let S be the set of all continuous functions $f:[-1,1] \to \mathbb{R}$ satisfying the following three conditions: (i) f is infinitely differentiable on the open interval $(-1,1)$, (ii) the Taylor series $$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$ of f at 0 converges to $f(x)$ for each $x \in (-1,1)$, (iii) $f\left(\frac{1}{n}\right) = 0$ for all $n \in \mathbb{N}$. Then which of the following is/are true?
(A)	$f(0) = 0$ for every $f \in S$.
(B)	$f'\left(\frac{1}{2}\right) = 0$ for every $f \in S$.
(C)	There exists $f \in S$ such that $f'\left(\frac{1}{2}\right) \neq 0$.
(D)	There exists $f \in S$ such that $f(x) \neq 0$ for some $x \in [-1,1]$.

Q.33	Define $f:[0,1] \to [0,1]$ by $$f(x) = \begin{cases} 1 & \text{if } x = 0, \\ \frac{1}{n} & \text{if } x = \frac{m}{n} \text{ for some } m, n \in \mathbb{N} \text{ with } m \leq n \text{ and } \gcd(m,n) = 1, \\ 0 & \text{if } x \in [0,1] \text{ is irrational.} \end{cases}$$ and define $g:[0,1] \to [0,1]$ by $$g(x) = \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{if } x \in (0,1]. \end{cases}$$ Then which of the following is/are true?
(A)	f is Riemann integrable on $[0,1]$.
(B)	g is Riemann integrable on $[0,1]$.
(C)	The composite function $f \circ g$ is Riemann integrable on $[0,1]$.
(D)	The composite function $g \circ f$ is Riemann integrable on $[0,1]$.

Q.34	Let S be the set of all functions $f: \mathbb{R} \to \mathbb{R}$ satisfying $$\|f(x) - f(y)\|^2 \leq \|x - y\|^3 \quad \text{for all } x, y \in \mathbb{R}.$$ Then which of the following is/are true?
(A)	Every function in S is differentiable.
(B)	There exists a function $f \in S$ such that f is differentiable, but f is not twice differentiable.
(C)	There exists a function $f \in S$ such that f is twice differentiable, but f is not thrice differentiable.
(D)	Every function in S is infinitely differentiable.

Q.35	A real-valued function $y(x)$ defined on $\mathbb{R}$ is said to be **periodic** if there exists a real number $T > 0$ such that $y(x+T) = y(x)$ for all $x \in \mathbb{R}$. Consider the differential equation $$\frac{d^2y}{dx^2} + 4y = \sin(ax), \quad x \in \mathbb{R}, \qquad (*)$$ where $a \in \mathbb{R}$ is a constant. Then which of the following is/are true?
(A)	All solutions of $(*)$ are periodic for every choice of a.
(B)	All solutions of $(*)$ are periodic for every choice of $a \in \mathbb{R} - \{-2, 2\}$.
(C)	All solutions of $(*)$ are periodic for every choice of $a \in \mathbb{Q} - \{-2, 2\}$.
(D)	If $a \in \mathbb{R} - \mathbb{Q}$, then there is a unique periodic solution of $(*)$.

Q.36	Let M be a positive real number and let $u, v : \mathbb{R}^2 \to \mathbb{R}$ be continuous functions satisfying $\sqrt{u(x,y)^2 + v(x,y)^2} \geq M\sqrt{x^2 + y^2}$ for all $(x,y) \in \mathbb{R}^2$. Let $F: \mathbb{R}^2 \to \mathbb{R}^2$ be given by $F(x,y) = \big(u(x,y), v(x,y)\big)$ for $(x,y) \in \mathbb{R}^2$. Then which of the following is/are true?
(A)	F is injective.
(B)	If K is open in $\mathbb{R}^2$, then $F(K)$ is open in $\mathbb{R}^2$.
(C)	If K is closed in $\mathbb{R}^2$, then $F(K)$ is closed in $\mathbb{R}^2$.
(D)	If E is closed and bounded in $\mathbb{R}^2$, then $F^{-1}(E)$ is closed and bounded in $\mathbb{R}^2$.

Q.37	Let G be a finite group of order at least two and let e denote the identity element of G. Let $\sigma: G \to G$ be a bijective group homomorphism that satisfies the following two conditions: (i) If $\sigma(g) = g$ for some $g \in G$, then $g = e$, (ii) $(\sigma \circ \sigma)(g) = g$ for all $g \in G$. Then which of the following is/are correct?
(A)	For each $g \in G$, there exists $h \in G$ such that $h^{-1}\sigma(h) = g$.
(B)	There exists $x \in G$ such that $x\sigma(x) \neq e$.
(C)	The map σ satisfies $\sigma(x) = x^{-1}$ for every $x \in G$.
(D)	The order of the group G is an odd number.

Q.38	Let (x_n) be a sequence of real numbers. Consider the set $$P = \{n \in \mathbb{N} : x_n > x_m \text{ for all } m \in \mathbb{N} \text{ with } m > n\}.$$ Then which of the following is/are true?
(A)	If P is finite, then (x_n) has a monotonically increasing subsequence.
(B)	If P is finite, then no subsequence of (x_n) is monotonically increasing.
(C)	If P is infinite, then (x_n) has a monotonically decreasing subsequence.
(D)	If P is infinite, then no subsequence of (x_n) is monotonically decreasing.
Q.39	Let V be the real vector space consisting of all polynomials in one variable with real coefficients and having degree at most 5, together with the zero polynomial. Let $T: V \to \mathbb{R}$ be the linear map defined by $T(1) = 1$ and $$T\big(x(x-1)\cdots(x-k+1)\big) = 1 \quad \text{for } 1 \le k \le 5.$$ Then which of the following is/are true?
(A)	$T(x^4) = 15$.
(B)	$T(x^3) = 5$.
(C)	$T(x^4) = 14$.
(D)	$T(x^3) = 3$.

Q.40	Let P be a fixed 3×3 matrix with entries in $\mathbb{R}$. Which of the following maps from $M_3(\mathbb{R})$ to $M_3(\mathbb{R})$ is/are linear?
(A)	$T_1: M_3(\mathbb{R}) \to M_3(\mathbb{R})$ given by $T_1(M) = MP - PM$ for $M \in M_3(\mathbb{R})$.
(B)	$T_2: M_3(\mathbb{R}) \to M_3(\mathbb{R})$ given by $T_2(M) = M^2P - P^2M$ for $M \in M_3(\mathbb{R})$.
(C)	$T_3: M_3(\mathbb{R}) \to M_3(\mathbb{R})$ given by $T_3(M) = MP^2 + P^2M$ for $M \in M_3(\mathbb{R})$.
(D)	$T_4: M_3(\mathbb{R}) \to M_3(\mathbb{R})$ given by $T_4(M) = MP^2 - PM^2$ for $M \in M_3(\mathbb{R})$.
Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	The value of the limit $$\lim_{n\to\infty}\left(\frac{(1^4+2^4+\cdots+n^4)}{n^5}+\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{n+1}}+\frac{1}{\sqrt{n+2}}+\cdots+\frac{1}{\sqrt{4n}}\right)\right)$$ is equal to _____. (*Rounded off to two decimal places*)

Q.42	Consider the function u: $\mathbb{R}^3 \to \mathbb{R}$ given by $$u(x_1, x_2, x_3) = x_1x_2^4x_3^2 - x_1^3x_3^4 - 26x_1^2x_2^2x_3^3.$$ Let $c \in \mathbb{R}$ and $k \in \mathbb{N}$ be such that $$x_1\frac{\partial u}{\partial x_2} + 2x_2\frac{\partial u}{\partial x_3}$$ evaluated at the point (t, t^2, t^3), equals ct^k for every $t \in \mathbb{R}$. Then the value of k is equal to _____.
Q.43	Let $y(x)$ be the solution of the differential equation $$\frac{dy}{dx} + 3x^2y = x^2, \quad \text{for } x \in \mathbb{R},$$ satisfying the initial condition $y(0) = 4$. Then $\lim_{x\to\infty} y(x)$ is equal to _____. (*Rounded off to two decimal places*)
Q.44	The sum of the series $$\sum_{n=1}^{\infty} \frac{1}{(4n-3)(4n+1)}$$ is equal to _____. (*Rounded off to two decimal places*)
Q.45	The number of distinct subgroups of $\mathbb{Z}_{999}$ is _____.

Q.46	The number of elements of order 12 in the symmetric group S_7 is equal to _____.
Q.47	Let $y(x)$ be the solution of the differential equation $$xy^2y' + y^3 = \frac{\sin x}{x} \quad \text{for } x > 0,$$ satisfying $y\left(\frac{\pi}{2}\right) = 0$. Then the value of $y\left(\frac{5\pi}{2}\right)$ is equal to _____. (*Rounded off to two decimal places*)
Q.48	Consider the region $$G = \{(x, y, z) \in \mathbb{R}^3 : 0 < z < x^2 - y^2,\ x^2 + y^2 < 1\}.$$ Then the volume of G is equal to _____. (*Rounded off to two decimal places*)

Q.49	Given that $y(x)$ is a solution of the differential equation $$x^2y'' + xy' - 4y = x^2$$ on the interval $(0, \infty)$ such that $\lim_{x\to 0^+} y(x)$ exists and $y(1) = 1$. The value of $y'(1)$ is equal to _____. (*Rounded off to two decimal places*)
Q.50	Consider the family $\mathcal{F}_1$ of curves lying in the region $$\{(x, y) \in \mathbb{R}^2 : y > 0 \text{ and } 0 < x < \pi\}$$ and given by $$y = \frac{c(1 - \cos x)}{\sin x}, \quad \text{where } c \text{ is a positive real number.}$$ Let $\mathcal{F}_2$ be the family of orthogonal trajectories to $\mathcal{F}_1$. Consider the curve $\mathcal{C}$ belonging to the family $\mathcal{F}_2$ passing through the point $\left(\frac{\pi}{3}, 1\right)$. If a is a real number such that $\left(\frac{\pi}{4}, a\right)$ lies on $\mathcal{C}$, then the value of a^4 is equal to _____. (*Rounded off to two decimal places*)

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51

For $t \in \mathbb{R}$, let $[t]$ denote the greatest integer less than or equal to t.

Let $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 4\}$. Let $f: D \to \mathbb{R}$ and $g: D \to \mathbb{R}$ be defined by $f(0,0) = g(0,0) = 0$ and

$$f(x,y) = [x^2 + y^2]\frac{x^2y^2}{x^4 + y^4}, \qquad g(x,y) = [y^2]\frac{xy}{x^2 + y^2}$$

for $(x, y) \neq (0, 0)$. Let E be the set of points of D at which both f and g are discontinuous. The number of elements in the set E is _____.

Q.52

If G is the region in $\mathbb{R}^2$ given by

$$G = \left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1, \ \frac{x}{\sqrt{3}} < y < \sqrt{3}x, \ x > 0, \ y > 0\right\}$$

then the value of

$$\frac{200}{\pi}\iint_G x^2\,dx\,dy$$

is equal to _____. (*Rounded off to two decimal places*)

Q.53

Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$ and let $A^{\top}$ denote the transpose of A. Let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ be column vectors with entries in $\mathbb{R}$ such that $u_1^2 + u_2^2 = 1$ and $v_1^2 + v_2^2 + v_3^2 = 1$. Suppose

$$Au = \sqrt{2}\, v \quad \text{and} \quad A^{\top} v = \sqrt{2}\, u.$$

Then $|u_1 + 2\sqrt{2}\, v_1|$ is equal to _____. (*Rounded off to two decimal places*)

Q.54

Let $f: [0, \pi] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} (x - \pi)e^{\sin x} & \text{if } 0 \le x \le \dfrac{\pi}{2}, \\ xe^{\sin x} + \dfrac{4}{\pi} & \text{if } \dfrac{\pi}{2} < x \le \pi. \end{cases}$$

Then the value of

$$\int_0^{\pi} f(x)dx$$

is equal to _____. (*Rounded off to two decimal places*)

Q.55

Let r be the radius of convergence of the power series

$$\frac{1}{3}+\frac{x}{5}+\frac{x^2}{3^2}+\frac{x^3}{5^2}+\frac{x^4}{3^3}+\frac{x^5}{5^3}+\frac{x^6}{3^4}+\frac{x^7}{5^4}+\cdots.$$

Then the value of r^2 is equal to _____. (*Rounded off to two decimal places*)

Q.56

Define $f:\mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y)=x^2+2y^2-x \quad \text{for } (x,y)\in\mathbb{R}^2.$$

Let $D=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\le 1\}$ and $E=\left\{(x,y)\in\mathbb{R}^2 : \frac{x^2}{4}+\frac{y^2}{9}\le 1\right\}$.

Consider the sets

$$D_{\max}=\{(a,b)\in D : f \text{ has absolute maximum on } D \text{ at } (a,b)\},$$

$$D_{\min}=\{(a,b)\in D : f \text{ has absolute minimum on } D \text{ at } (a,b)\},$$

$$E_{\max}=\{(c,d)\in E : f \text{ has absolute maximum on } E \text{ at } (c,d)\},$$

$$E_{\min}=\{(c,d)\in E : f \text{ has absolute minimum on } E \text{ at } (c,d)\}.$$

Then the total number of elements in the set

$$D_{\max}\cup D_{\min}\cup E_{\max}\cup E_{\min}$$

is equal to _____.

Q.57	Consider the 4×4 matrix $M = \begin{pmatrix} 11 & 10 & 10 & 10 \\ 10 & 11 & 10 & 10 \\ 10 & 10 & 11 & 10 \\ 10 & 10 & 10 & 11 \end{pmatrix}$. Then the value of the determinant of M is equal to _____.
Q.58	Let σ be the permutation in the symmetric group S_5 given by $\sigma(1) = 2, \quad \sigma(2) = 3, \quad \sigma(3) = 1, \quad \sigma(4) = 5, \quad \sigma(5) = 4.$ Define $N(\sigma) = \{\tau \in S_5 : \sigma \circ \tau = \tau \circ \sigma\}.$ Then the number of elements in $N(\sigma)$ is equal to _____.
Q.59	Let $f: (-1,1) \to \mathbb{R}$ and $g: (-1,1) \to \mathbb{R}$ be thrice continuously differentiable functions such that $f(x) \neq g(x)$ for every nonzero $x \in (-1,1)$. Suppose $f(0) = \ln 2, \quad f'(0) = \pi, \quad f''(0) = \pi^2, \quad \text{and} \quad f'''(0) = \pi^9$ and $g(0) = \ln 2, \quad g'(0) = \pi, \quad g''(0) = \pi^2, \quad \text{and} \quad g'''(0) = \pi^3.$ Then the value of the limit $\lim_{x \to 0} \frac{e^{f(x)} - e^{g(x)}}{f(x) - g(x)}$ is equal to _____. (*Rounded off to two decimal places*)

Q.60	If $f:[0,\infty) \to \mathbb{R}$ and $g:[0,\infty) \to [0,\infty)$ are continuous functions such that $\int_0^{x^3+x^2} f(t)dt = x^2$ and $\int_0^{g(x)} t^2 dt = 9(x+1)^3$ for all $x \in [0,\infty)$, then the value of $$f(2) + g(2) + 16\, f(12)$$ is equal to _____. (*Rounded off to two decimal places*)

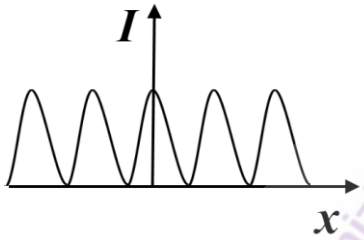
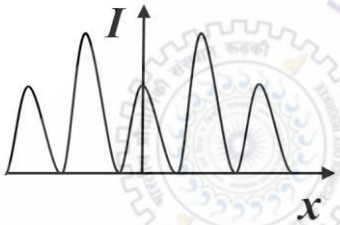
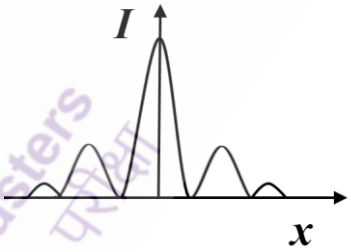
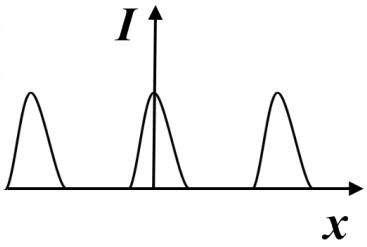
Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	The equation $z^2 + \bar{z}^2 = 4$ in the complex plane (where $\bar{z}$ is the complex conjugate of z) represents
(A)	Ellipse
(B)	Hyperbola
(C)	Circle of radius 2
(D)	Circle of radius 4
Q.2	A rocket (S') moves at a speed $\frac{c}{2}$ *m/s* along the positive x-axis, where c is the speed of light. When it crosses the origin, the clocks attached to the rocket and the one with a stationary observer (S) located at $x = 0$ are both set to zero. If S observes an event at (x, t), the same event occurs in the S' frame at
(A)	$x' = \frac{2}{\sqrt{3}}\left(x - \frac{ct}{2}\right)$ and $t' = \frac{2}{\sqrt{3}}\left(t - \frac{x}{2c}\right)$
(B)	$x' = \frac{2}{\sqrt{3}}\left(x + \frac{ct}{2}\right)$ and $t' = \frac{2}{\sqrt{3}}\left(t - \frac{x}{2c}\right)$
(C)	$x' = \frac{2}{\sqrt{3}}\left(x - \frac{ct}{2}\right)$ and $t' = \frac{2}{\sqrt{3}}\left(t + \frac{x}{2c}\right)$
(D)	$x' = \frac{2}{\sqrt{3}}\left(x + \frac{ct}{2}\right)$ and $t' = \frac{2}{\sqrt{3}}\left(t + \frac{x}{2c}\right)$

Q.3	Consider a classical ideal gas of N molecules in equilibrium at temperature T. Each molecule has two energy levels, $-\epsilon$ and ϵ. The mean energy of the gas is
(A)	0
(B)	$N\epsilon \tanh\left(\frac{\epsilon}{k_B T}\right)$
(C)	$-N\epsilon \tanh\left(\frac{\epsilon}{k_B T}\right)$
(D)	$\frac{\epsilon}{2}$
Q.4	At a temperature T, let β and κ denote the volume expansivity and isothermal compressibility of a gas, respectively. Then $\frac{\beta}{\kappa}$ is equal to
(A)	$\left(\frac{\partial P}{\partial T}\right)_V$
(B)	$\left(\frac{\partial P}{\partial V}\right)_T$
(C)	$\left(\frac{\partial T}{\partial P}\right)_V$
(D)	$\left(\frac{\partial T}{\partial V}\right)_P$

Q.5	The resultant of the binary subtraction $1110101 - 0011110$ is
(A)	1001111
(B)	1010111
(C)	1010011
(D)	1010001
Q.6	Consider a particle trapped in a three-dimensional potential well such that $U(x,y,z) = 0$ for $0 \leq x \leq a,\ 0 \leq y \leq a,\ 0 \leq z \leq a$ and $U(x,y,z) = \infty$ everywhere else. The degeneracy of the 5th excited state is
(A)	1
(B)	3
(C)	6
(D)	9

Q.7	A particle of mass m and angular momentum L moves in space where its potential energy is $U(r) = kr^2 \quad (k > 0)$ and r is the radial coordinate. If the particle moves in a circular orbit, then the radius of the orbit is
(A)	$\left(\frac{L^2}{mk}\right)^{\frac{1}{4}}$
(B)	$\left(\frac{L^2}{2mk}\right)^{\frac{1}{4}}$
(C)	$\left(\frac{2L^2}{mk}\right)^{\frac{1}{4}}$
(D)	$\left(\frac{4L^2}{mk}\right)^{\frac{1}{4}}$

Q.8	Consider a two-dimensional force field $\overrightarrow{F}(x,y) = (5x^2 + ay^2 + bxy)\,\hat{x} + (4x^2 + 4xy + y^2)\,\hat{y}$. If the force field is conservative, then the values of a and b are
(A)	$a = 2$ and $b = 4$
(B)	$a = 2$ and $b = 8$
(C)	$a = 4$ and $b = 2$
(D)	$a = 8$ and $b = 2$
Q.9	Consider an electrostatic field $\vec{E}$ in a region of space. Identify the **INCORRECT** statement.
(A)	The work done in moving a charge in a closed path inside the region is zero
(B)	The curl of $\vec{E}$ is zero
(C)	The field can be expressed as the gradient of a scalar potential
(D)	The potential difference between any two points in the region is always zero

Q.10	Which one of the following figures correctly depicts the intensity distribution for Fraunhofer diffraction due to a single slit? Here, x denotes the distance from the centre of the central fringe and I denotes the intensity.
(A)	
(B)	
(C)	
(D)	

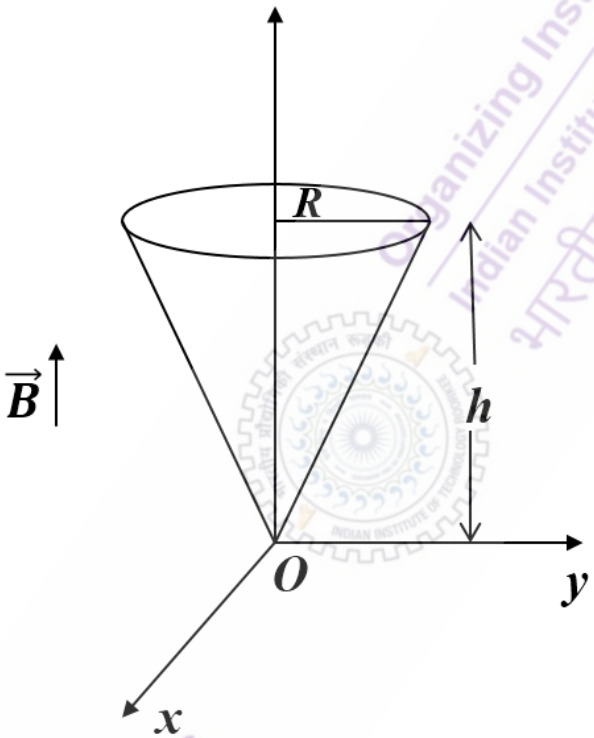
Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	The function $f(x) = e^{\sin x}$ is expanded as a Taylor series in x, around $x = 0$, in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$. The value of $a_0 + a_1 + a_2$ is
(A)	0
(B)	$\frac{3}{2}$
(C)	$\frac{5}{2}$
(D)	5
Q.12	Consider a unit circle C in the xy plane, centered at the origin. The value of the integral $\oint[(\sin x - y)dx - (\sin y - x)dy]$ over the circle C, traversed anticlockwise, is
(A)	0
(B)	2π
(C)	3π
(D)	4π

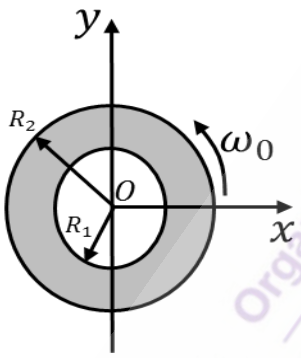
Q.13	The current through a series RL circuit, subjected to a constant *emf* $\mathcal{E}$, obeys $L\frac{di}{dt} + iR = \mathcal{E}$. Let $L = 1\ mH$, $R = 1\ k\Omega$ and $\mathcal{E} = 1\ V$. The initial condition is $i(0) = 0$. At $t = 1\ \mu s$, the current in mA is
(A)	$1 - 2e^{-2}$
(B)	$1 - 2e^{-1}$
(C)	$1 - e^{-1}$
(D)	$2 - 2e^{-1}$
Q.14	An ideal gas in equilibrium at temperature T expands isothermally to twice its initial volume. If ΔS, ΔU and ΔF denote the changes in its entropy, internal energy and Helmholtz free energy respectively, then
(A)	$\Delta S < 0, \Delta U > 0, \Delta F < 0$
(B)	$\Delta S > 0, \Delta U = 0, \Delta F < 0$
(C)	$\Delta S < 0, \Delta U = 0, \Delta F > 0$
(D)	$\Delta S > 0, \Delta U > 0, \Delta F = 0$

Q.15	In a dilute gas, the number of molecules with free path length $\geq x$ is given by $N(x) = N_0 e^{-x/\lambda}$, where N_0 is the total number of molecules and λ is the mean free path. The fraction of molecules with free path lengths between λ and 2λ is
(A)	$\frac{1}{e}$
(B)	$\frac{e}{e-1}$
(C)	$\frac{e^2}{e-1}$
(D)	$\frac{e-1}{e^2}$

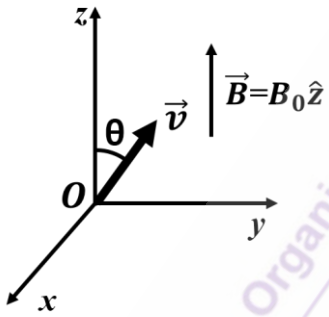
Q.16	Consider a quantum particle trapped in a one-dimensional potential well in the region $[-L/2 < x < L/2]$, with infinitely high barriers at $x = -L/2$ and $x = L/2$. The stationary wave function for the ground state is $\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$. The uncertainties in momentum and position satisfy
(A)	$\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x = 0$
(B)	$\Delta p = \frac{2\pi\hbar}{L}$ and $0 < \Delta x < \frac{L}{2\sqrt{3}}$
(C)	$\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x > \frac{L}{2\sqrt{3}}$
(D)	$\Delta p = 0$ and $\Delta x = \frac{L}{2}$
Q.17	Consider a particle of mass m moving in a plane with a constant radial speed $\dot{r}$ and a constant angular speed $\dot{\theta}$. The acceleration of the particle in (r, θ) coordinates is
(A)	$2r\dot{\theta}^2\hat{r} - \dot{r}\dot{\theta}\hat{\theta}$
(B)	$-r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$
(C)	$\ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta}$
(D)	$\ddot{r}\theta\hat{r} + r\ddot{\theta}\hat{\theta}$

Q.18	A planet of mass m moves in an elliptical orbit. Its maximum and minimum distances from the Sun are R and r, respectively. Let G denote the universal gravitational constant, and M the mass of the Sun. Assuming $M >> m$, the angular momentum of the planet with respect to the center of the Sun is
(A)	$m\sqrt{\frac{2GMRr}{(R+r)}}$
(B)	$m\sqrt{\frac{GMRr}{2(R+r)}}$
(C)	$m\sqrt{\frac{GMRr}{(R+r)}}$
(D)	$2m\sqrt{\frac{2GMRr}{(R+r)}}$

Q.19	Consider a conical region of height h and base radius R with its vertex at the origin. Let the outward normal to its base be along the positive z-axis, as shown in the figure. A uniform magnetic field, $\vec{B} = B_0\hat{z}$ exists everywhere. Then the magnetic flux through the base (Φ_b) and that through the curved surface of the cone (Φ_c) are 
(A)	$\Phi_b = B_0\pi R^2$; $\Phi_c = 0$
(B)	$\Phi_b = -\frac{1}{2}B_0\pi R^2$; $\Phi_c = \frac{1}{2}B_0\pi R^2$
(C)	$\Phi_b = 0$; $\Phi_c = -B_0\pi R^2$
(D)	$\Phi_b = B_0\pi R^2$; $\Phi_c = -B_0\pi R^2$

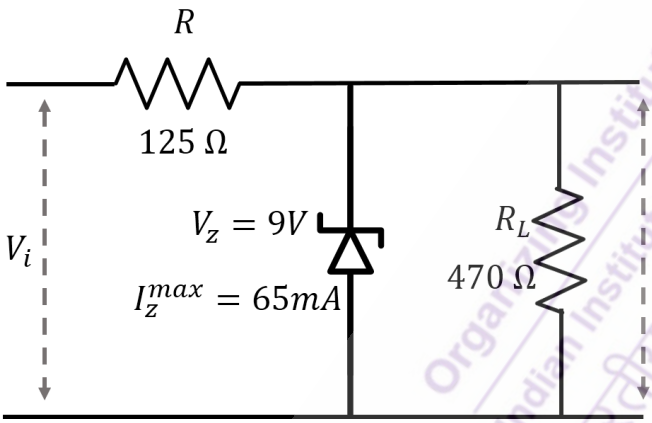
Q.20	Consider a thin annular sheet, lying on the xy-plane, with R_1 and R_2 as its inner and outer radii, respectively. If the sheet carries a uniform surface-charge density σ and spins about the origin O with a constant angular velocity $\vec{\omega} = \omega_0 \hat{z}$ then, the total current flow on the sheet is 
(A)	$\frac{2\pi\sigma\omega_0\left(R_2^{\,3} - R_1^{\,3}\right)}{3}$
(B)	$\sigma\omega_0\left(R_2^{\,3} - R_1^{\,3}\right)$
(C)	$\frac{\pi\sigma\omega_0\left(R_2^{\,3} - R_1^{\,3}\right)}{3}$
(D)	$\frac{2\pi\sigma\omega_0(R_2 - R_1)^3}{3}$

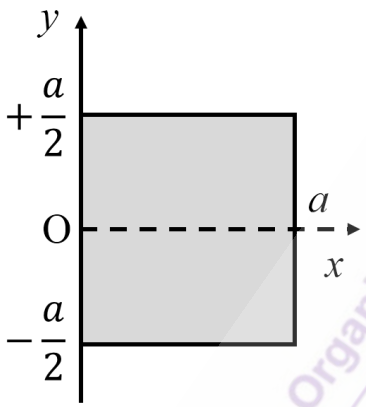
Q.21	A radioactive nucleus has a decay constant λ and its radioactive daughter nucleus has a decay constant 10λ. At time $t = 0$, N_0 is the number of parent nuclei and there are no daughter nuclei present. $N_1(t)$ and $N_2(t)$ are the number of parent and daughter nuclei present at time t, respectively. The ratio $N_2(t)/N_1(t)$ is
(A)	$\frac{1}{9}\left[1 - e^{-9\lambda t}\right]$
(B)	$\frac{1}{10}\left[1 - e^{-10\lambda t}\right]$
(C)	$\left[1 - e^{-10\lambda t}\right]$
(D)	$\left[1 - e^{-9\lambda t}\right]$

Q.22	A uniform magnetic field $\vec{B} = B_0\hat{z}$, where $B_0 > 0$ exists as shown in the figure. A charged particle of mass m and charge q ($q > 0$) is released at the origin, in the yz-plane, with a velocity $\vec{v}$ directed at an angle $\theta = 45°$ with respect to the positive z-axis. Ignoring gravity, which one of the following is **TRUE**. 
(A)	The initial acceleration $\vec{a} = \frac{qvB_0}{\sqrt{2}\,m}\hat{x}$
(B)	The initial acceleration $\vec{a} = \frac{qvB_0}{\sqrt{2}\,m}\hat{y}$
(C)	The particle moves in a circular path
(D)	The particle continues in a straight line with constant speed

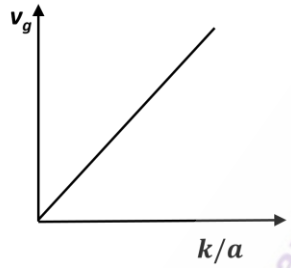
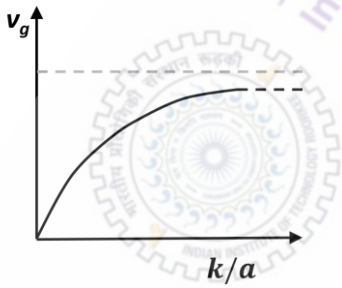
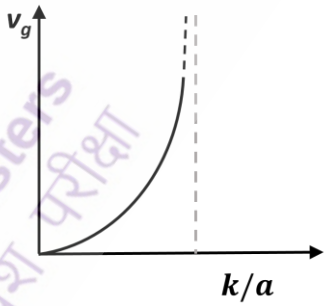
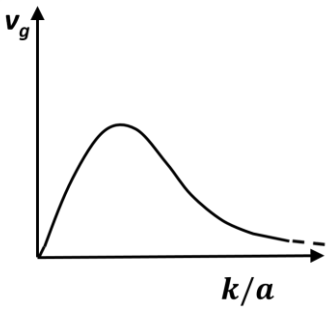
Q.23	For an ideal intrinsic semiconductor, the Fermi energy at 0 K
(A)	lies at the top of the valence band
(B)	lies at the bottom of the conduction band
(C)	lies at the center of the bandgap
(D)	lies midway between center of the bandgap and bottom of the conduction band

Q.24	A circular loop of wire with radius R is centered at the origin of the xy-plane. The magnetic field at a point within the loop is, $\vec{B}(\rho, \phi, z, t) = k\rho^3 t^3 \hat{z}$, where k is a positive constant of appropriate dimensions. Neglecting the effects of any current induced in the loop, the magnitude of the induced *emf* in the loop at time t is
(A)	$\frac{6\pi k t^2 R^5}{5}$
(B)	$\frac{5\pi k t^2 R^5}{6}$
(C)	$\frac{3\pi k t^2 R^5}{2}$
(D)	$\frac{\pi k t^2 R^5}{2}$

Q.25	For the given circuit, $R = 125\ \Omega,\ R_L = 470\ \Omega,\ V_z = 9\ V,$ and I_z^{max}= 65 mA. The minimum and maximum values of the input voltage (V_i^{min} and V_i^{max}) for which the Zener diode will be in the 'ON' state are 
(A)	$V_i^{min} = 9.0\ V$ and $V_i^{max} = 11.4\ V$
(B)	$V_i^{min} = 9.0\ V$ and $V_i^{max} = 19.5\ V$
(C)	$V_i^{min} = 11.4\ V$ and $V_i^{max} = 15.5\ V$
(D)	$V_i^{min} = 11.4\ V$ and $V_i^{max} = 19.5\ V$

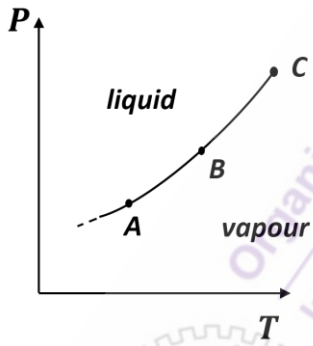
Q.26	A square laminar sheet with side a and mass M, has mass per unit area given by $\sigma(x) = \sigma_0\left[1 - \frac{x}{a}\right]$, (see figure). Moment of inertia of the sheet about y-axis is 
(A)	$\frac{Ma^2}{2}$
(B)	$\frac{Ma^2}{4}$
(C)	$\frac{Ma^2}{6}$
(D)	$\frac{Ma^2}{12}$

Q.27	A particle is subjected to two simple harmonic motions along the x and y axes, described by $x(t) = a\sin(2\omega t + \pi)$ and $y(t) = 2a\sin(\omega t)$. The resultant motion is given by
(A)	$\frac{x^2}{a^2} + \frac{y^2}{4a^2} = 1$
(B)	$x^2 + y^2 = 1$
(C)	$y^2 = x^2\left(1 - \frac{x^2}{4a^2}\right)$
(D)	$x^2 = y^2\left(1 - \frac{y^2}{4a^2}\right)$
Q.28	For a certain thermodynamic system, the internal energy $U = PV$ and P is proportional to T^2. The entropy of the system is proportional to
(A)	UV
(B)	$\sqrt{\frac{U}{V}}$
(C)	$\sqrt{\frac{V}{U}}$
(D)	$\sqrt{UV}$

Q.29	The dispersion relation for certain type of waves is given by $\omega = \sqrt{k^2 + a^2}$, where k is the wave vector and a is a constant. Which one of the following sketches represents v_g, the group velocity?
(A)	
(B)	
(C)	
(D)	

Q.30	Consider a binary number with m digits, where m is an even number. This binary number has alternating 1's and 0's, with digit 1 in the highest place value. The decimal equivalent of this binary number is
(A)	$2^m - 1$
(B)	$\frac{(2^m - 1)}{3}$
(C)	$\frac{(2^{m+1} - 1)}{3}$
(D)	$\frac{2}{3}(2^m - 1)$

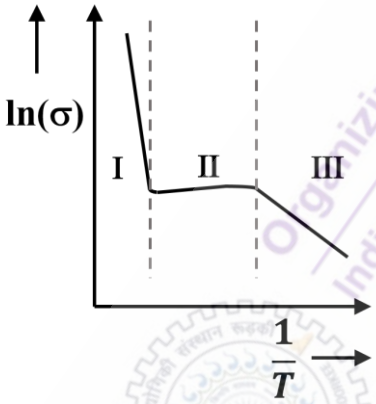
Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Consider the 2×2 matrix $M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$, where $a, b > 0$. Then,
(A)	M is a real symmetric matrix
(B)	One of the eigenvalues of M is greater than b
(C)	One of the eigenvalues of M is negative
(D)	Product of eigenvalues of M is b
Q.32	In the Compton scattering of electrons, by photons incident with wavelength λ,
(A)	$\frac{\Delta\lambda}{\lambda}$ is independent of λ
(B)	$\frac{\Delta\lambda}{\lambda}$ increases with decreasing λ
(C)	there is no change in photon's wavelength for all angles of deflection of the photon
(D)	$\frac{\Delta\lambda}{\lambda}$ increases with increasing angle of deflection of the photon

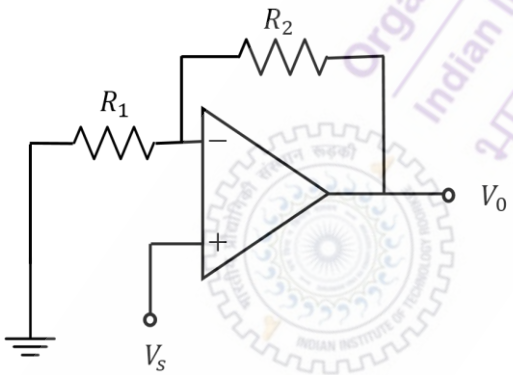
Q.33	The figure shows a section of the phase boundary separating the vapour (1) and liquid (2) states of water in the $P-T$ plane. Here, C is the critical point. μ_1, v_1 and s_1 are the chemical potential, specific volume and specific entropy of the vapour phase respectively, while μ_2, v_2 and s_2 respectively denote the same for the liquid phase. Then 
(A)	$\mu_1 = \mu_2$ along AB
(B)	$v_1 = v_2$ along AB
(C)	$s_1 = s_2$ along AB
(D)	$v_1 = v_2$ at the point C

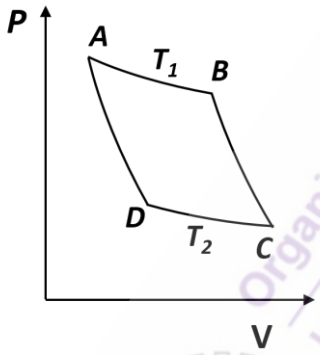
Q.34	A particle is executing simple harmonic motion with time period T. Let x, v and a denote the displacement, velocity and acceleration of the particle, respectively, at time *t.* Then,
(A)	$\frac{aT}{x}$ does not change with time
(B)	$(aT + 2\pi v)$ does not change with time
(C)	x and v are related by an equation of a straight line
(D)	v and a are related by an equation of an ellipse
Q.35	A linearly polarized light beam travels from origin to point A $(1,0,0)$. At the point A, the light is reflected by a mirror towards point B $(1,-1,0)$. A second mirror located at point B then reflects the light towards point C $(1,-1,1)$. Let $\hat{n}(x,\ y,\ z)$ represent the direction of polarization of light at (x,y,z).
(A)	If $\hat{n}(0,\ 0,\ 0) = \hat{y}$, then $\hat{n}(1,\ -1,\ 1) = \hat{x}$
(B)	If $\hat{n}(0,\ 0,\ 0) = \hat{z}$, then $\hat{n}(1,\ -1,\ 1) = \hat{y}$
(C)	If $\hat{n}(0,\ 0,\ 0) = \hat{y}$, then $\hat{n}(1,\ -1,\ 1) = \hat{y}$
(D)	If $\hat{n}(0,\ 0,\ 0) = \hat{z}$, then $\hat{n}(1,\ -1,\ 1) = \hat{x}$

Q.36	Let (r, θ) denote the polar coordinates of a particle moving in a plane. If $\hat{r}$ and $\hat{\theta}$ represent the corresponding unit vectors, then
(A)	$\frac{d\hat{r}}{d\theta} = \hat{\theta}$
(B)	$\frac{d\hat{r}}{dr} = -\hat{\theta}$
(C)	$\frac{d\hat{\theta}}{d\theta} = -\hat{r}$
(D)	$\frac{d\hat{\theta}}{dr} = \hat{r}$
Q.37	The electric field associated with an electromagnetic radiation is given by $E = a(1 + cos\omega_1 t)cos\omega_2 t$. Which of the following frequencies are present in the field?
(A)	ω_1
(B)	$\omega_1 + \omega_2$
(C)	$\lvert\omega_1 - \omega_2\rvert$
(D)	ω_2

Q.38	A string of length L is stretched between two points $x = 0$ and $x = L$ and the endpoints are rigidly clamped. Which of the following can represent the displacement of the string from the equilibrium position?
(A)	$x \cos\left(\frac{\pi x}{L}\right)$
(B)	$x \sin\left(\frac{\pi x}{L}\right)$
(C)	$x\left(\frac{x}{L} - 1\right)$
(D)	$x\left(\frac{x}{L} - 1\right)^2$
Q.39	The Boolean expression $Y = \overline{PQ}\mathrm{R} + \mathrm{Q}\bar{R} + \bar{P}QR + PQR$ simplifies to
(A)	$\bar{P}R + Q$
(B)	$PR + \bar{Q}$
(C)	$P + R$
(D)	$Q + R$

Q.40	For an n-type silicon, an extrinsic semiconductor, the natural logarithm of normalized conductivity (σ) is plotted as a function of inverse temperature. Temperature interval-I corresponds to the intrinsic regime, interval-II corresponds to saturation regime and interval-III corresponds to the freeze-out regime, respectively. Then 
(A)	the magnitude of the slope of the curve in the temperature interval-I is proportional to the bandgap, E_g
(B)	the magnitude of the slope of the curve in the temperature interval-III is proportional to the ionization energy of the donor, E_d
(C)	in the temperature interval-II, the carrier density in the conduction band is equal to the density of donors
(D)	in the temperature interval-III, all the donor levels are ionized

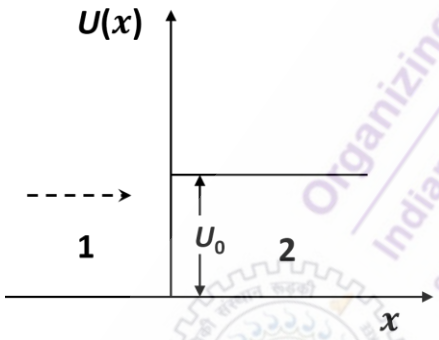
Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	The integral $\iint (x^2 + y^2) dxdy$ over the area of a disk of radius 2 in the xy plane is ___π.
Q.42	For the given operational amplifier circuit $R_1 = 120\ \Omega$, $R_2 = 1.5\ k\Omega$ and $V_s = 0.6\ V$, then the output current I_0 is _____ mA. 

Q.43	For an ideal gas, AB and CD are two isothermals at temperatures T_1 and T_2 ($T_1 > T_2$), respectively. AD and BC represent two adiabatic paths as shown in figure. Let V_A, V_B , V_C and V_D be the volumes of the gas at A, B, C and D respectively. If $\frac{V_C}{V_B} = 2$, then $\frac{V_D}{V_A} =$ _____. 
Q.44	A satellite is revolving around the Earth in a closed orbit. The height of the satellite above Earth's surface at perigee and apogee are 2500 km and 4500 km, respectively. Consider the radius of the Earth to be 6500 km. The eccentricity of the satellite's orbit is ____ (Round off to 1 decimal place).
Q.45	Three masses $m_1 = 1, m_2 = 2$ and $m_3 = 3$ are located on the x-axis such that their center of mass is at $x = 1$. Another mass $m_4 = 4$ is placed at x_0 and the new center of mass is at $x = 3$. The value of x_0 is _____.

Q.46	A normal human eye can distinguish two objects separated by $0.35\,m$ when viewed from a distance of $1.0\,km$. The angular resolution of eye is _____seconds (Round off to the nearest integer).
Q.47	A rod with a proper length of 3 m moves along x-axis, making an angle of 30^0 with respect to the x-axis. If its speed is $\frac{c}{2}\,m/s$, where c is the speed of light, the change in length due to Lorentz contraction is ____m (Round off to 2 decimal places). [Use $c = 3 \times 10^8\,m/s$]
Q.48	Consider the Bohr model of hydrogen atom. The speed of an electron in the second orbit $(n = 2)$ is _____$\times 10^6\,m/s$ (Round off to 2 decimal places). [Use $h = 6.63 \times 10^{-34}\,Js,\ e = 1.6 \times 10^{-19}\,C,\ \epsilon_0 = 8.85 \times 10^{-12}\,C^2m^2/N$]
Q.49	Consider a unit circle C in the xy plane with center at the origin. The line integral of the vector field, $\vec{F}(x,y,z) = -2y\hat{x} - 3z\hat{y} + x\hat{z}$, taken anticlockwise over C is _____π.

Q.50	Consider a p-n junction at $T = 300\ K$. The saturation current density at reverse bias is $-20\ \mu A/cm^2$. For this device, a current density of magnitude $10\ \mu A/cm^2$ is realized with a forward bias voltage, V_F. The same magnitude of current density can also be realized with a reverse bias voltage, V_R. The value of $\lvert V_F/V_R \rvert$ is _____ (Round off to 2 decimal places).

Section C: Q.51 – Q.60 Carry TWO marks each.	
Q.51	Consider the second order ordinary differential equation, $y'' + 4y' + 5y = 0$. If $y(0) = 0$ and $y'(0) = 1$, then the value of $y(\pi/2)$ is _____ (Round off to 3 decimal places).
Q.52	A box contains a mixture of two different ideal monoatomic gases, 1 and 2, in equilibrium at temperature T. Both gases are present in equal proportions. The atomic mass for gas 1 is m, while the same for gas 2 is $2m$. If the *rms* speed of a gas molecule selected at random is $v_{rms} = x\sqrt{\frac{k_B T}{m}}$, then x is _____ (Round off to 2 decimal places).
Q.53	A hot body with constant heat capacity $800\ J/K$ at temperature $925\ K$ is dropped gently into a vessel containing $1\ kg$ of water at temperature $300\ K$ and the combined system is allowed to reach equilibrium. The change in the total entropy ΔS is _____ J/K (Round off to 1 decimal place). [Take the specific heat capacity of water to be $4200\ J/kg\ K$. Neglect any loss of heat to the vessel and air and change in the volume of water.]

Q.54	Consider an electron with mass m and energy E moving along the x-axis towards a finite step potential of height U_0 as shown in the figure. In region **1** ($x < 0$), the momentum of the electron is $p_1 = \sqrt{2mE}$. The reflection coefficient at the barrier is given by $R = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2$, where p_2 is the momentum in region **2**. If, in the limit $E \gg U_0$, $R \approx \frac{U_0^2}{nE^2}$, then the integer n is ___. 
Q.55	A current density for a fluid flow is given by, $$\vec{J}(x, y, z, t) = \frac{8e^t}{(1+x^2+y^2+z^2)}\hat{x}\,.$$ At time $t = 0$, the mass density $\rho(x, y, z, 0) = 1$. Using the equation of continuity, $\rho(1,1,1,1)$ is found to be _____ (Round off to 2 decimal places).
Q.56	The work done in moving a $-5\,\mu C$ charge in an electric field $\vec{E} = \left(8r\,\sin\theta\,\hat{r} + 4r\cos\theta\,\hat{\theta}\right) V/m$, from a point $A(r, \theta) = \left(10, \frac{\pi}{6}\right)$ to a point $B(r, \theta) = \left(10, \frac{\pi}{2}\right)$, is _____ m$J$.

Q.57	A pipe of 1 m length is closed at one end. The air column in the pipe resonates at its fundamental frequency of 400 Hz. The number of nodes in the sound wave formed in the pipe is _____. [Speed of sound = 320 m/s]
Q.58	The critical angle of a crystal is 30°. Its Brewster angle is _____ degrees (Round off to the nearest integer).
Q.59	In an LCR series circuit, a non-inductive resistor of 150 Ω, a coil of 0.2 H inductance and negligible resistance, and a 30 μF capacitor are connected across an ac power source of 220 V, 50 Hz. The power loss across the resistor is ____W (Round off to 2 decimal places).
Q.60	A charge q is uniformly distributed over the volume of a dielectric sphere of radius a. If the dielectric constant ϵ_r = 2, then the ratio of the electrostatic energy stored inside the sphere to that stored outside is_____ (Round off to 1 decimal place).

END OF THE QUESTION PAPER