IIT JAM 2021 Question Paper (All Subjects)

IIT Joint Admission Test for Masters

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

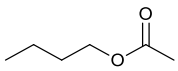
MULTIPLE CHOICE QUESTIONS (MCQ)

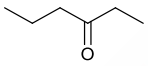
Q. 1 – Q.10 carry one mark each.

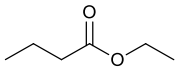
Q.1 An acid contains C, H and O atoms. On combustion analysis, 0.454 g of the acid gives 0.418 g of H_2O and 1.023 g of CO_2. What is the empirical formula of the acid?

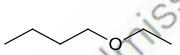
(A) $C_4H_5O_2$ (B) C_3H_6O (C) CH_2O (D) C_5H_8O

Q.2 Ethylbutyrate is responsible for the odor of pineapple. Which one of the following is the structure of ethyl butyrate?

(A) 

(B) 

(C) 

(D) 

Q.3 If the blood groups of mother and father are AB and O, respectively, what are the blood groups possible for their child?

(A) AB or A (B) AB (C) A or B (D) AB, A, B or O

Q.4 Which one of the following features distinguishes between gymnosperms and angiosperms?

(A) Seed formation (B) Vascular tissues
(C) Seed cover (D) Gamete production

Q.5 Ecosystem ecology is the study of

(A) An organism's behavior towards environmental challenges
(B) Factors that affect the interactions of individuals in a population
(C) Interactions among biotic and abiotic components
(D) Factors that affect the interactions among communities in an ecosystem

Q.6 Bacterial strains that do not grow in the absence of a specific nutrient are called

(A) Heterotrophs (B) Chemotrophs (C) Autotrophs (D) Auxotrophs

Q.7 The type of immunological protection provided by plasma therapy is

(A) Natural active (B) Natural passive
(C) Artificial active (D) Artificial passive

Q.8 Which one of the following components of bacterial cell acts as endotoxin?

(A) Peptidoglycan of Gram-positive bacteria (B) Lipopolysaccharide

(C) Porins (D) Peptidoglycan of Gram-negative bacteria

Q.9 The moment of force in terms of fundamental dimensions is

(A) MLT^{-1} (B) MLT^{-2} (C) $ML^{-1}T^{-1}$ (D) $ML^{2}T^{-2}$

Q.10 Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -5 \\ 0 & 1 \end{pmatrix}$. If $AX + 3B = 0$, then the determinant of X is

(A) -18 (B) -6 (C) 6 (D) 18

Q. 11 – Q. 30 carry two marks each.

Q.11 In a genetic cross between plants bearing violet flowers and green seeds (*VVGG*), and white flower and yellow seeds (*vvgg*), the following phenotypic distribution was obtained in the F2 progeny (assume both parents to be pure breeding for both the traits, and self-cross at F1 generation):

i) 2340 plants with violet flowers and green seeds
ii) 47 plants with violet flowers and yellow seeds
iii) 43 plants with white flowers and green seeds
iv) 770 plants with white flowers and yellow seeds

Which one of the following interpretations explains the above phenotypic distribution?

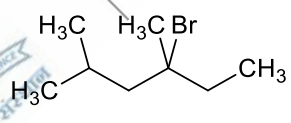
(A) Same genes control both flower and seed colors

(B) Genes for flower and seed colors are genetically interacting

(C) Genes for flower and seed colors are present on the same chromosome

(D) Flower color in this plant species is a polygenic trait

Q.12 IUPAC name of the following molecule is



(A) 3-Bromo-3,5-dimethyl hexane

(B) 4-Bromo-2,4-dimethyl hexane

(C) 3-Bromo-2-isobutyl butane

(D) 4-Bromo-2-methyl-4-ethyl pentane

Q.13 Which one of the following features/properties does glucose acquire through intramolecular hemiacetal formation?

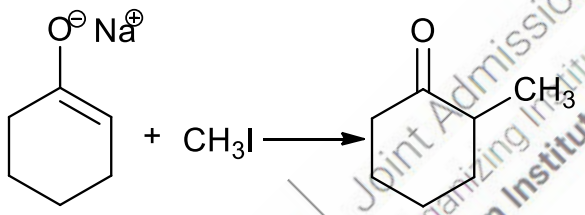
(A) Ability to function as a reducing agent

(B) An additional chiral carbon

(C) Ability to form anhydride linkage with non-carbohydrate moieties such as the inorganic phosphate

(D) Ability to form epimers

Q.14 The following methylation is carried out in various solvents such as benzene, tetrahydrofuran (THF), dimethoxyethane (DME), dimethyl sulfoxide (DMSO) and N,N-dimethylformamide (DMF). Which one of the following is TRUE for the effect of solvent on the reaction rate?



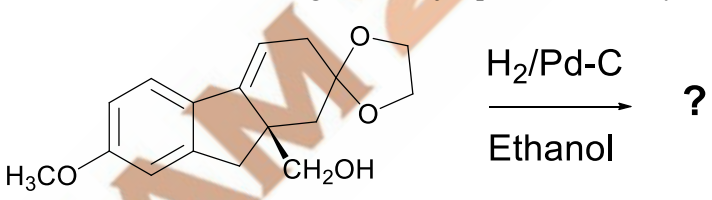
(A) DMSO > DMF > DME > THF > Benzene

(B) Benzene > THF > DME > DMF > DMSO

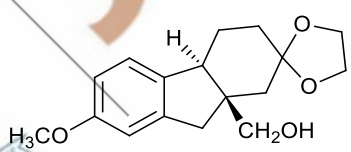
(C) DME > DMSO > DMF > THF > Benzene

(D) THF > Benzene > DME > DMSO > DMF

Q.15 Which one of the following is the major product of the hydrogenation reaction given below?



(A)



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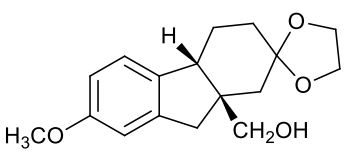

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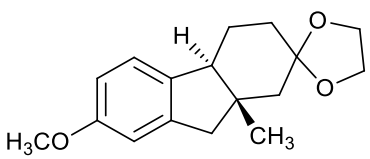

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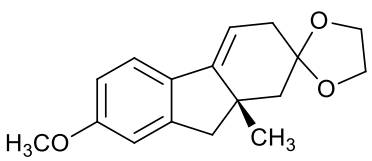
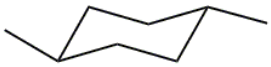
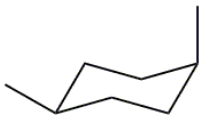
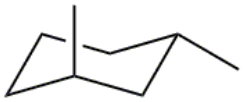
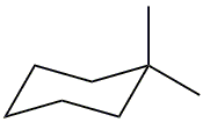


Q.16 Which one of the following isomers is thermodynamically most stable?

(A) 

(B) 

(C) 

(D) 

Q.17 What is the significance of the isomerization of glucose 6-phosphate to fructose 6-phosphate for the progression of glycolysis?

(A) As functional groups, ketones are more reactive than aldehydes

(B) Cleavage of glucose 1,6-bisphosphate will not yield dihydroxy acetone phosphate and glyceraldehyde 3-phosphate

(C) The carbonyl group at carbon-2 (C-2) in fructose facilitates the cleavage of the bond between C-3 and C-4

(D) Phosphorylation of glucose 6-phosphate to glucose 1,6-bisphosphate is irreversible

Q.18 What is the role of bile salts in the mammalian digestive system?

(A) Bile salts convert pepsinogen to pepsin, and thus facilitate protein digestion

(B) Bile salts emulsify fat, and thus aid in fat digestion

(C) Bile salts are excretory products produced by the liver, and do not participate in digestion

(D) Bile salts facilitate digestion of all types of macromolecules in the small intestine

Q.19 Presence of which one of the following in the urine indicates pregnancy in human?

(A) Progesterone

(B) Follicle-stimulating hormone and luteinizing hormone

(C) Estrogen

(D) Human chorionic gonadotropin

Q.20 Which one of the following processes emerged earliest during the course of evolution?

(A) Antigen presentation

(B) Antibody production

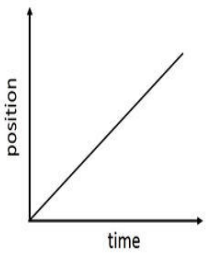
(C) Phagocytosis

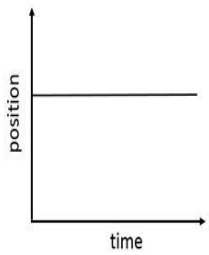
(D) Thymic education

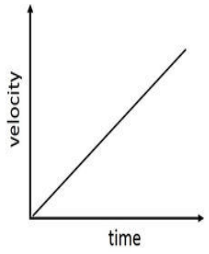
Q.21 Which one of the following microscopic techniques provides a 3-dimensional perspective of live, unstained and transparent specimens obtained from the wild?

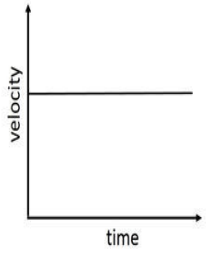
(A) Confocal microscopy
(B) Fluorescence microscopy
(C) Phase contrast microscopy
(D) Differential interference contrast (Nomarski) microscopy

Q.22 Which one of the following represents the motion of an object with a positive acceleration?

(A) 

(B) 

(C) 

(D) 

Q.23 A stationary enemy ship is docked in the sea at a distance of 1.0 km from the coastline. A gun located at the sea level on the coastline can fire projectiles at a velocity of 120 m/s. What is the angle (in degrees) above the horizontal at which the gun must fire to hit the ship? [g = 9.8 m/s^2]

(A) 21.4 (B) 42.9 (C) 23.6 (D) 47.1

Q.24 If $x + \frac{1}{x} = 1$, then the value of $x^6 + \frac{1}{x^6}$ is

(A) -2 (B) -1 (C) 1 (D) 2

Q.25 The value of the integral $\int_0^4 (x - f(x))\ dx$, where $f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 3, & 3 \le x < 4 \\ 4, & 4 \le x < 5 \end{cases}$ is

(A) 2 (B) 1 (C) -1 (D) -2

Q.26 In plants, the ovules are attached to the ovary by

(A) Placenta (B) Synergids (C) Embryo sac (D) Tube cells

Q.27 The lack of linear correlation between the genome sizes and genetic complexities among various species is known as

(A) C-value paradox (B) Genetic diversity (C) G-value paradox (D) Central dogma

Q.28 Match the cell junctions listed in **Group A** with their correct functions listed in **Group B**:

Group A	Group B
(I). Adherens junction	(P). Joins actin bundles in neighboring cells
(II). Desmosome	(Q). Joins intermediate filaments in neighboring cells
(III). Tight junction	(R). Seals neighboring cells
(IV). Gap junction	(S). Allows diffusion of molecules between adjacent cells

(A) I-S; II-P; III-Q; IV-R (B) I-Q, II-R; III-P; IV-S

(C) I-Q; II-R; III-S; IV-P (D) I-P; II-Q; III-R; IV-S

Q.29 In mammals, females have two X chromosomes and males have one X chromosome. Equal expression of X-chromosome genes in both sexes is ensured by

(A) Dosage compensation (B) Histone code

(C) RNA silencing (D) Heterochromatin formation

Q.30 The difference between mitosis and meiosis I is

(A) Sister chromatids separate in mitosis, whereas homologous chromosomes separate in meiosis I

(B) The nuclear membrane is absent during mitotic metaphase, but not in meiotic metaphase

(C) The DNA is double helical in meiosis I but not in mitosis

(D) Unlike in mitotic metaphase, chromosomes do not align at the equatorial plate in meiosis I

SECTION - B

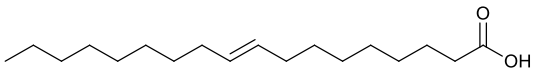
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Infrared (IR) spectroscopy is used for determining certain aspects of the structure of organic compounds. Which of the following statement(s) is/are FALSE?

(A) IR radiation induces electronic transitions
(B) IR peak intensities are related to molecular mass
(C) Most organic functional groups absorb in a characteristic region of the IR spectrum
(D) Each element absorbs at a characteristic wavelength

Q.32 Oleic acid, shown below, is



(A) A saturated fatty acid
(B) An unsaturated fatty acid
(C) Insoluble in water
(D) Soluble in acetone

Q.33 Cyclic AMP (cAMP) acts as a second messenger for which of the following primary signaling molecule(s)?

(A) Retinoic acid
(B) Prostaglandins
(C) Cortisol
(D) Epinephrine

Q.34 Which of the following is/are TRUE about the electron carrier, ubiquinone (coenzyme Q)?

(A) Its ability to accept two electrons, one at a time, enables ubiquinone to function at the junction between a 2-electron donor and a 1-electron acceptor
(B) Being small and hydrophobic, ubiquinone readily shuttles between protein-based electron transfer complexes within the membrane
(C) Its hydrophilic nature and high affinity for protons enable ubiquinone to transport protons readily within the intermembrane space of mitochondria.
(D) Its ability to interact with Heme C of cytochromes enables electron transport in the mitochondrial membrane

Q.35 Which of the following is/are common to both prokaryotic and eukaryotic gene expression?

(A) Coupled transcription and translation
(B) Post-translational modification
(C) Genetic code
(D) Presence of the sequence TATA in the promoter

Q.36 Which of the following molecular genetic technique(s) is/are used in forensic science?

(A) Coimmunoprecipitation
(B) DNA fingerprinting
(C) Restriction fragment length polymorphism
(D) Electrophoretic mobility shift assay

Q.37 Which of the following is/are involved in the initiation of DNA replication?

(A) RhoA
(B) oriC
(C) Sigma factor
(D) DnaA

Q.38 Which of the following pairs is/are analogous structures?

(A) Human hands and bat wings
(B) Butterfly wings and bat wings
(C) Bat wings and bird wings
(D) Dolphin flippers and fish fins

Q.39 A charged particle accelerated by a potential V moves in a circular path with a velocity v in a uniform magnetic field B that is perpendicular to the motion. Which of the following is/are correct if the value of V is increased?

(A) Kinetic energy of the particle increases
(B) Radius of the circular path increases
(C) Time period of the motion increases
(D) Work done by the magnetic field increases

Q.40 A function $f: D \rightarrow \mathbb{R}$ is defined as $f(x) = \frac{x^2+1}{x^2+x+1}$, where $D \subseteq \mathbb{R}$ is the domain. The domain(s) on which the function $f(x)$ is one to one is/are

(A) Natural numbers (B) Integers (C) Rational numbers (D) Irrational numbers

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 1.45 g of sucrose ($C_{12}H_{22}O_{11}$) is dissolved in 30.0 ml of water. Molality (rounded off to 3 decimals) of the resulting solution is _____ m.

Q.42 For a gene present on human chromosome 4, the maximum number of alleles that may be detected by sequencing the genome of 5 males and 10 females is _____.

Q.43 The amount of hydrogen required to reduce 30 g of 2-butene is _____ g (rounded off to 2 decimals).

Q.44 The molar concentration of water in pure water is _____ M (rounded off to 1 decimal).

Q.45 The number of triplet codon(s) for methionine is ___.

Q.46 The number of peptide bonds in a 20-residue linear peptide is ___.

Q.47 When the molecular weight of human immunoglobulin light chain is 24 kDa, the total molecular weight of human IgG is ___ kDa.

Q.48 The maximum number of genotypes possible for gametes formed from a diploid cell of the genotype *AaBBcCDd* is _____.

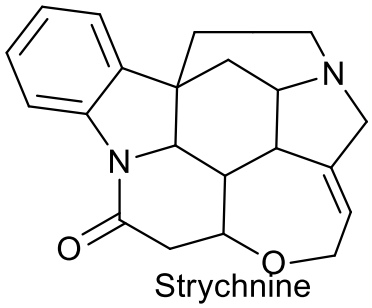
Q.49 The de Broglie wavelength of a proton moving at a speed of 1.0 m/s is _____ Å.
[Planck's constant = 6.626 x 10^{-34} m^2kg/s; m_p = 1.67 x 10^{-27} kg]

Q.50 The distance between the parallel lines $2x + 5y = 7$ and $2x + 5y = 15$ is _____ (rounded off to 2 decimals).

Q. 51 – Q. 60 carry two marks each.

Q.51 At 25°C and pH 7.0, the concentrations of glucose 1-phosphate and glucose 6-phosphate are 2.0 mM and 38 mM, respectively at equilibrium. The standard free energy change for the conversion of glucose 1-phosphate to glucose 6-phosphate is ____ J/mol. [R = 8.315 J mol^{-1} K^{-1}]

Q.52



Strychnine

The number of chiral carbons in strychnine is _____.

Q.53 The number of polypeptide chains in a core nucleosome is ____.

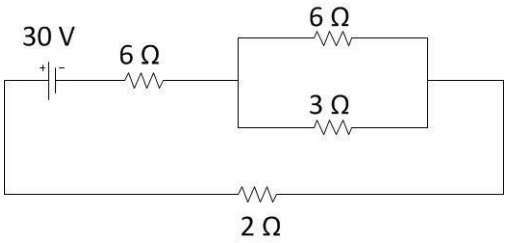
Q.54 While performing a PCR, the student forgot to add one of the two primers. The number of molecules of single-stranded DNA produced after 25 PCR cycles is ____

Q.55 A double helical DNA molecule is composed of 32 mol % of adenosine. The mol % of cytosine in this DNA molecule is _____.

Q.56 In a compound microscope, the magnification power of the objective lens is 100x, and that of the eye piece (ocular lens) is 10x. The magnification power of the microscope is _____ x.

Q.57 In a population at Hardy-Weinberg equilibrium, for *gene-X* only two alleles, namely ***A*** and ***a***, are found. If frequency of allele ***A*** is 0.2 and the frequency of allele ***a*** is 0.8, the frequency of the heterozygote genotype ***Aa*** in that population will be _____ (correct to 2 decimal places).

Q.58 In the circuit shown below, the power dissipated across the 3Ω resistor is _____ W.



Q.59 The equation $\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)=\beta$ has a solution, where β is a natural number. Then β is _____.

Q.60 The velocity of blood in a blood vessel of 2.0 cm radius is 30 cm/s. When the blood vessel bifurcates into 2 smaller vessels of radius 1.0 cm each, the velocity of blood in each of the smaller vessels is _____ cm/s. Assume that the vessel walls are rigid, and blood is incompressible.

END OF THE QUESTION PAPER

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SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The CORRECT order of pK_a for the compounds **I** to **IV** in water at 298 K is

$HCo(CO)_4$	$HCo(CO)_3(PPh_3)$	$HCo(CO)_3(P(OPh)_3)$	$HCo(CO)_2(PPh_3)_2$
I	**II**	**III**	**IV**

(A) **I** > **II** > **III** > **IV**

(B) **IV** > **III** > **II** > **I**

(C) **IV** > **II** > **III** > **I**

(D) **I** > **III** > **II** > **IV**

Q.2 For Na^+, Mg^{2+}, Al^{3+} and F^-, the CORRECT order of ionic radii is

(A) $Al^{3+} > Mg^{2+} > Na^+ > F^-$

(B) $Al^{3+} > Na^+ > Mg^{2+} > F^-$

(C) $F^- > Na^+ > Mg^{2+} > Al^{3+}$

(D) $Na^+ > F^- > Mg^{2+} > Al^{3+}$

Q.3 Spin-only magnetic moments (in BM) of $[NiCl_2(PPh_3)_2]$ and $[Mn(NCS)_6]^{4-}$, respectively, are

(A) 0.00 and 5.92

(B) 2.83 and 1.89

(C) 0.00 and 1.89

(D) 2.83 and 5.92

Q.4 Two sets of quantum numbers with the same number of radial nodes are

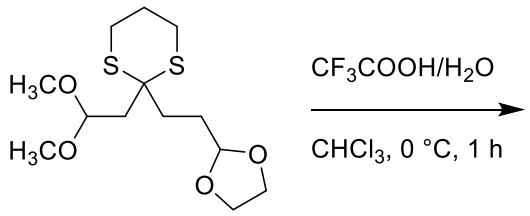
(A) n = 3; l = 0; m_l = 0 and n = 2; l = 0; m_l = 0

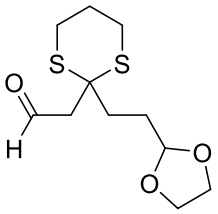
(B) n = 3; l = 1; m_l = 1 and n = 2; l = 1; m_l = 0

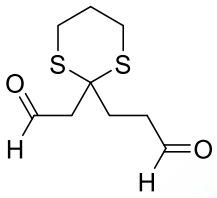
(C) n = 3; l = 2; m_l = 0 and n = 2; l = 1; m_l = 0

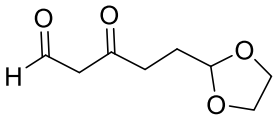
(D) n = 3; l = 1; m_l = –1 and n = 2; l = 1; m_l = 0

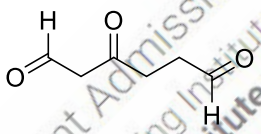
Q.5 The major product formed in the following reaction is



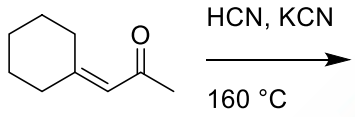
(A) 

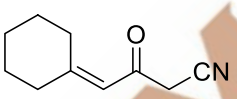
(B) 

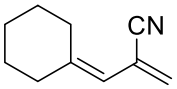
(C) 

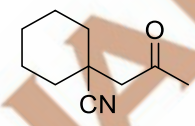
(D) 

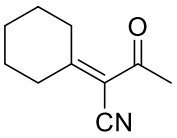
Q.6 The major product formed in the following reaction is



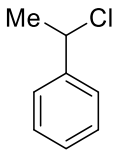
(A) 

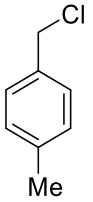
(B) 

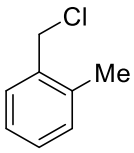
(C) 

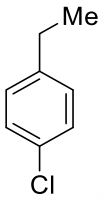
(D) 

Q.7 A compound shows ^{1}H NMR peaks at δ-values (in ppm) 7.31 (2H), 7.21 (2H), 4.5 (2H) and 2.3 (3H). The structure of the compound is

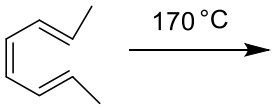
(A) 

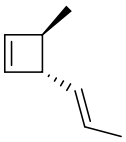
(B) 

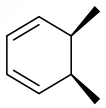
(C) 

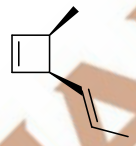
(D) 

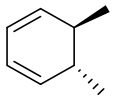
Q.8 The major product formed in the following reaction is



(A) 

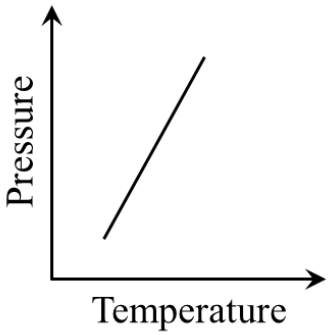
(B) 

(C) 

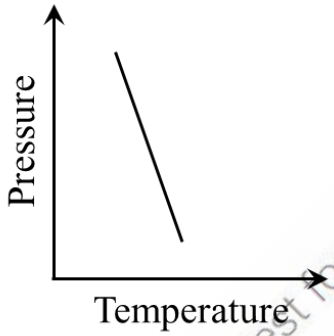
(D) 

Q.9 A pure substance **M** has lesser density in solid state than in liquid state. The ΔS_{fusion} of **M** is +25 J K^{-1} mol^{-1}. The CORRECT representative Pressure-Temperature diagram for the fusion of **M** is

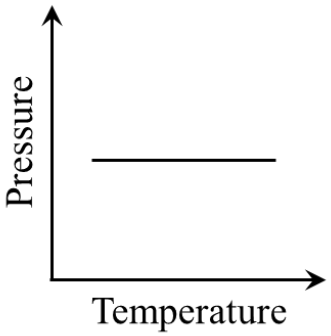
(A)



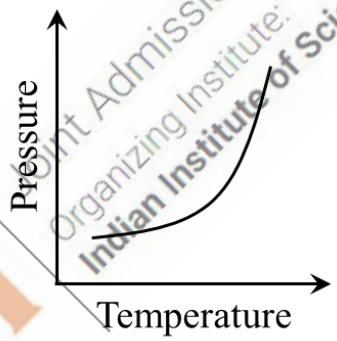
(B)



(C)



(D)



Q.10 Among the following, the matrices with non-zero determinant are

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 1 & 3 & 0 \\ 4 & 3 & 1 & 4 \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 2 \\ 3 & 4 & 1 & 3 \\ 4 & 1 & 2 & 4 \end{bmatrix}$$

(A) **P**, **Q** and **R**

(B) **P**, **R** and **S**

(C) **P**, **Q** and **S**

(D) **Q**, **R** and **S**

Q. 11 – Q. 30 carry two marks each.

Q.11 Reaction of BCl_3 with NH_4Cl at 140 °C produces compound **P**. Further, **P** reacts with $NaBH_4$ to give a colorless liquid **Q**. The reaction of **Q** with H_2O at 100 °C produces compound **R** and a diatomic gas **S**. Among the following, the CORRECT statement is

(A) **P** is $B_3N_3H_6$

(B) **R** is $[B(OH)NH]_3$

(C) **Q** is $[BClNH]_3$

(D) **S** is Cl_2

Q.12 The complex that does **NOT** obey the 18-electron rule is
(*Given*: Atomic numbers of Ti, Mn, Ta and Ir are 22, 25, 73 and 77, respectively)

(A) $[(\eta^5\text{-}C_5H_5)Ti(CO)_4]^-$

(B) $[Mn(SnPh_3)_2(CO)_4]^-$

(C) $[(\eta^5\text{-}C_5H_5)Ir(CH_2)(PMe_3)]$

(D) $[TaCl_3(PEt_3)_2(CHCMe_3)]$

Q.13 Hybridization of the central atoms in I_3^-, ClF_3 and SF_4, respectively, are

(A) sp^3d, sp^2 and dsp^2

(B) sp, sp^3d and dsp^2

(C) sp^3d, sp^3d and sp^3d

(D) sp, sp^2 and sp^3d

Q.14 Reaction of $[Ni(CN)_4]^{2-}$ with metallic potassium in liquid ammonia at –33 °C yields complex **E**. The geometry and magnetic behavior of **E**, respectively, are

(A) Square planar and diamagnetic

(B) Tetrahedral and diamagnetic

(C) Octahedral and paramagnetic

(D) Square pyramidal and paramagnetic

Q.15 The decreasing order of C=C bond length in the following complexes is

$[Cl_3Pt(CH_2{=}CH_2)]^-$ **I** $[Cl_3Pt(C(CN)_2{=}C(CN)_2)]^-$ **II** $[Cl_3Pt(CF_2{=}CH_2)]^-$ **III** $[Cl_3Pt(CF_2{=}CF_2)]^-$ **IV**

(A) **II** > **III** > **IV** > **I**

(B) **IV** > **II** > **III** > **I**

(C) **II** > **IV** > **III** > **I**

(D) **IV** > **II** > **I** > **III**

Q.16 The CORRECT combination for metalloenzymes given in **Column I** with their catalytic reactions in **Column II** is

Column I	**Column II**
(i) Cytochrome P-450	(**K**) $2H_2O_2 \longrightarrow 2H_2O + O_2$
(ii) Catalase	(**L**) $R\text{-}CH_2OH + O_2 \longrightarrow R\text{-}CHO + H_2O_2$ (R = alkyl or aryl)
(iii) Galactose oxidase	(**M**) $O_2 + 4H^+ + 4e^- \longrightarrow 2H_2O$
(iv) Cytochrome c oxidase	(**N**) $R\text{-}H + O_2 + 2e^- + 2H^+ \longrightarrow R\text{-}OH + H_2O$ (R = alkyl or aryl)

(A) (i)–(**M**); (ii)–(**N**); (iii)–(**K**); (iv)–(**L**)

(B) (i)–(**N**); (ii)–(**L**); (iii)–(**K**); (iv)–(**M**)

(C) (i)–(**N**); (ii)–(**K**); (iii)–(**L**); (iv)–(**M**)

(D) (i)–(**M**); (ii)–(**K**); (iii)–(**L**); (iv)–(**N**)

Q.17 According to the crystal field theory, *d*–*d* transition observed in $[Ti(H_2O)_6]^{3+}$ is

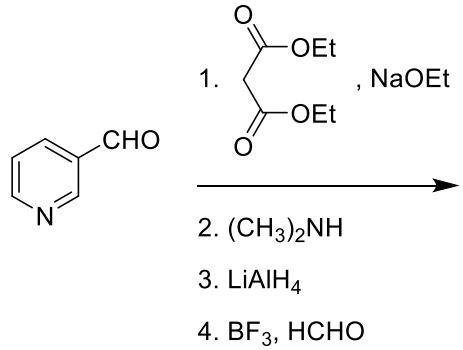
(A) Laporte forbidden and spin forbidden

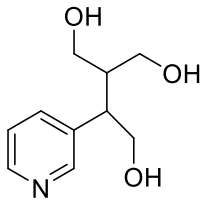
(B) Laporte allowed and spin forbidden

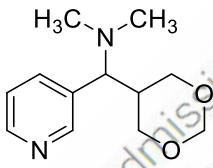
(C) Laporte allowed and spin allowed

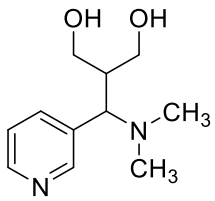
(D) Laporte forbidden and spin allowed

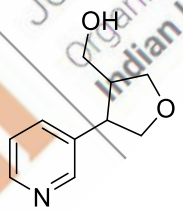
Q.18 The major product formed in the following reaction sequence is



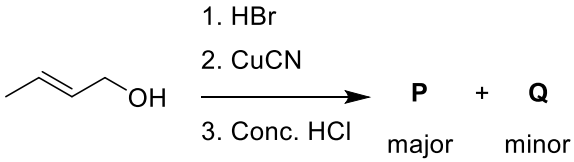
(A) 

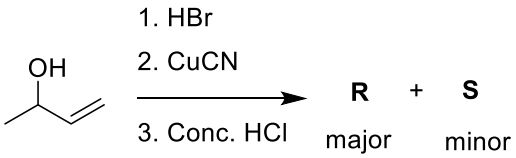
(B) 

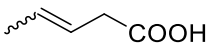
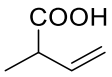
(C) 

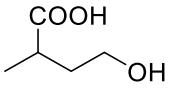
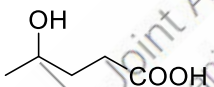
(D) 

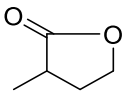
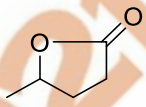
Q.19 The products **P**, **Q**, **R** and **S** formed in the following reactions are

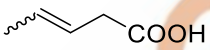
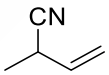




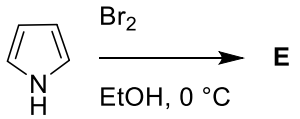
(A) **P** = **R** =  and **Q** = **S** = 

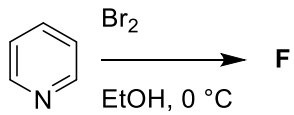
(B) **P** = **S** =  and **Q** = **R** = 

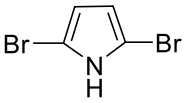
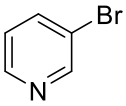
(C) **P** = **S** =  and **Q** = **R** = 

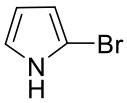
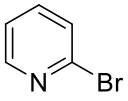
(D) **P** = **R** =  and **Q** = **S** = 

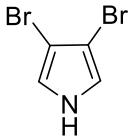
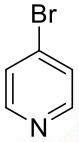
Q.20 The major products **E** and **F** formed in the following reactions are

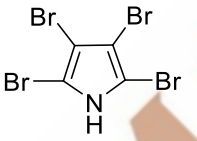
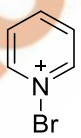




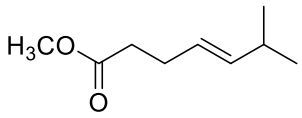
(A) **E** =  and **F** = 

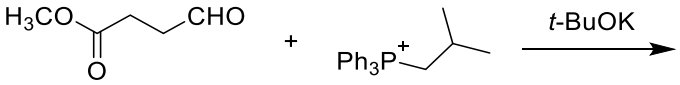
(B) **E** =  and **F** = 

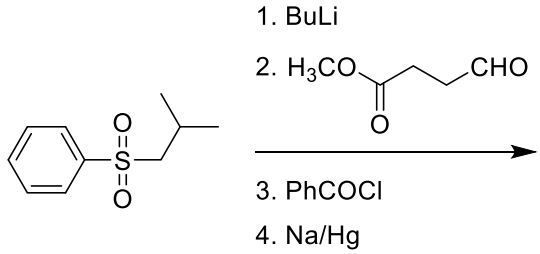
(C) **E** =  and **F** = 

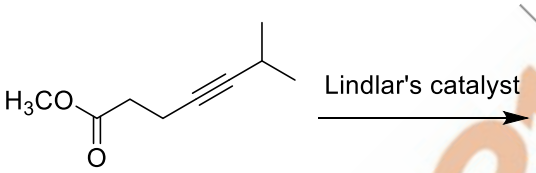
(D) **E** =  and **F** = 

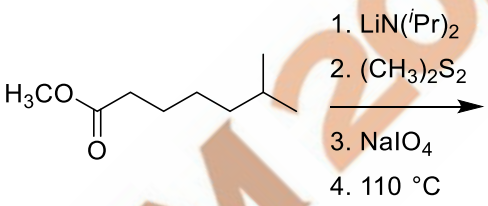
Q.21 The reaction that produces the following as a major product is



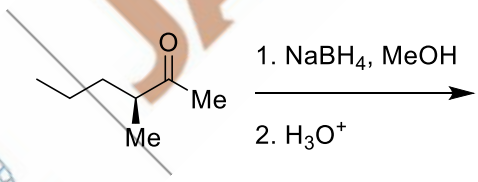
(A) 

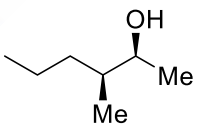
(B) 

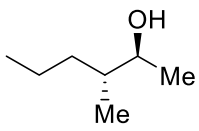
(C) 

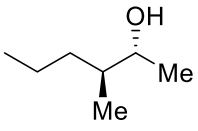
(D) 

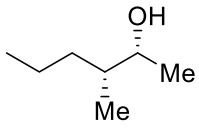
Q.22 The major product formed in the following reaction is



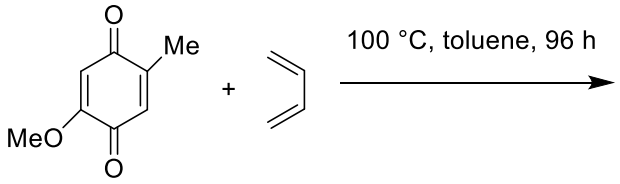
(A) 

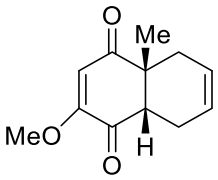
(B) 

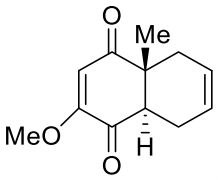
(C) 

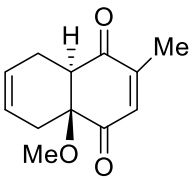
(D) 

Q.23 The major product formed in the following reaction is



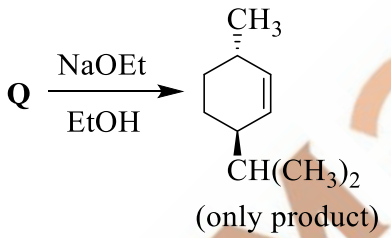
(A) 

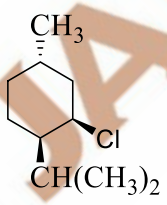
(B) 

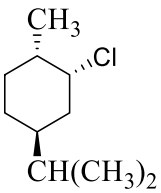
(C) 

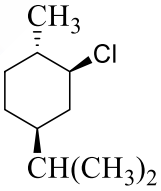
(D) 

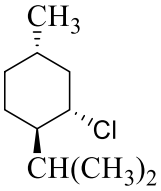
Q.24 In the following reaction, compound **Q** is



(A) 

(B) 

(C) 

(D) 

Q.25 Monochromatic X-rays having energy 2.8×10^{-15} J diffracted (first order) from (200) plane of a cubic crystal at an angle 8.5°. The length of unit cell in Å of the crystal (*rounded off to one decimal place*) is
(*Given*: Planck's constant, $h = 6.626 \times 10^{-34}$ J s; $c = 3.0 \times 10^{8}$ m s^{-1})

(A) 2.4 (B) 3.4 (C) 4.8 (D) 9.8

Q.26 For $\alpha > 0$, the value of the integral $\int_{-\infty}^{+\infty} xe^{-\alpha x^2}\, dx$ is

(A) $\sqrt{\frac{\pi}{\alpha}}$ (B) ∞

(C) 0 (D) 1

Q.27 The volume correction factor for a non-ideal gas in terms of critical pressure (p_c), critical molar volume (V_c), critical temperature (T_c) and gas constant (R) is

(A) $\frac{RT_c}{8p_c}$ (B) $\frac{27R^2{T_c}^2}{64p_c}$ (C) $\frac{8p_cV_c}{3T_c}$ (D) $3p_c{V_c}^2$

Q.28 Half-life $(t_{1/2})$ of a chemical reaction varies with the initial concentration of reactant (A_o) as given below:

A_o (mol L^{-1})	5×10^{-2}	4×10^{-2}	3×10^{-2}
$t_{1/2}$ (s)	360	450	600

The order of the reaction is

(A) 0 (B) 1 (C) 2 (D) 3

Q.29 The CORRECT statement regarding the molecules BF_3 and CH_4 is

(A) Both BF_3 and CH_4 are microwave active

(B) Both BF_3 and CH_4 are infrared active

(C) CH_4 is microwave active and infrared inactive

(D) BF_3 is microwave active and infrared active

Q.30 For the consecutive reaction,

$$X \xrightarrow{k_X} Y \xrightarrow{k_Y} Z$$

C_0 is the initial concentration of X. The concentrations of X, Y and Z at time t are C_X, C_Y and C_Z, respectively. The expression for the concentration of Y at time t is

(A) $\frac{k_X C_0}{k_Y - k_X}\left(e^{-k_X t} - e^{-k_Y t}\right)$

(B) $\frac{k_X C_X}{k_Y - k_X}\left(e^{-k_X t} - e^{-k_Y t}\right)$

(C) $\frac{k_X C_0}{k_Y - k_X}\left(e^{-k_Y t} - e^{-k_X t}\right)$

(D) $\frac{k_X C_X}{k_Y - k_X}\left(e^{-k_Y t} - e^{-k_X t}\right)$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 The CORRECT statement(s) about the species is (are)

(A) $CpMo(CO)_3$ and $CpW(CO)_3$ are isoelectronic (where Cp is cyclopentadienyl)

(B) CH_2^- and NH_2 are isolobal and isoelectronic

(C) BH and CH are isolobal and isoelectronic

(D) CH_3 and $Mn(CO)_5$ are isolobal

Q.32 The complex(es) that show(s) Jahn-Teller distortion is (are)

(A) $[Co(CN)_5(H_2O)]^{3-}$ (B) $[NiF_6]^{2-}$

(C) $[Mn(CNMe)_6]^{2+}$ (D) $[Co(en)_2F_2]^+$

Q.33 The CORRECT statement(s) about sodium nitroprusside is (are)

(A) It is a paramagnetic complex

(B) Nitroprusside ion is formed in the brown ring test for nitrates

(C) It is used for the detection of S^{2-} in aqueous solution

(D) It contains nitrosyl ligand as NO^+

Q.34 The pigment responsible for red color in tomato has one functional group. The CORRECT statement(s) about this functional group is (are)

(A) It decolorizes bromine water

(B) It gives hydrazone derivative on reaction with 2,4-dinitrophenylhydrazine

(C) It gets cleaved on reaction with ozone

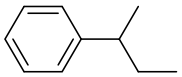
(D) It gives positive silver mirror test

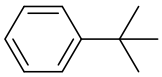
Q.35 Hantzsch pyridine synthesis involves several steps. Some of those are

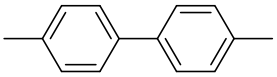
(A) Aldol reaction (B) Darzens reaction

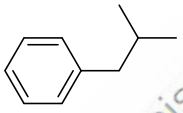
(C) Mannich reaction (D) Michael addition

Q.36 The compound(s), which give(s) benzoic acid on oxidation with $KMnO_4$, is (are)

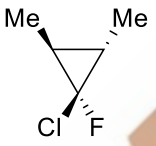
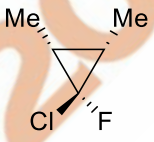
(A) 

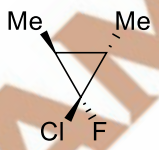
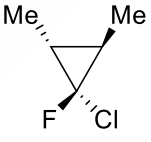
(B) 

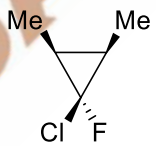
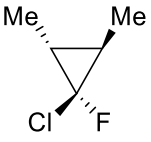
(C) 

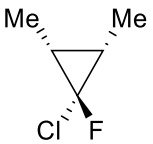
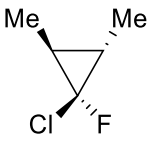
(D) 

Q.37 The products **P** and **Q** formed in the reaction are

(A) **P** =  **Q** = 

(B) **P** =  **Q** = 

(C) **P** =  **Q** = 

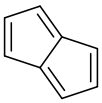
(D) **P** =  **Q** = 

Q.38 The functional group(s) in reducing sugar that tests positive with Tollen's reagent is (are)

(A) Aldehyde (B) Ketone

(C) Hemi-acetal (D) Acetal

Q.39 Among the following, the anti-aromatic compound(s) is (are)

(A)  (B) 

(C)  (D) 

Q.40 The CORRECT Maxwell relation(s) derived from the fundamental equations of thermodynamics is (are)

(A) $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$ (B) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

(C) $\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial p}{\partial S}\right)_V$ (D) $\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The total number of optically active isomers of dichloridobis(glycinato)cobaltate(III) ion is _____.

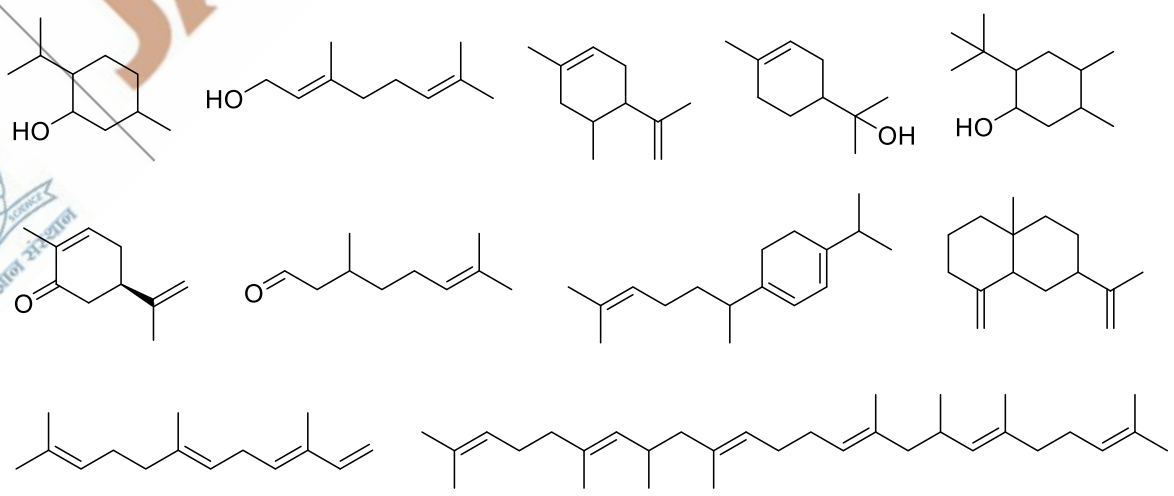
Q.42 The total number of microstates possible for a d^8 electronic configuration is _____.

Q.43 For the following fusion reaction,
$4\ {}^{1}H \longrightarrow {}^{4}He + 2\beta^{+} + 2\upsilon + \gamma$
the *Q*-value (energy of the reaction) in MeV (*rounded off to one decimal place*) is _____.
(*Given*: Mass of ${}^{1}H$ nucleus is 1.007825 *u* and mass of ${}^{4}He$ nucleus is 4.002604 *u*)

Q.44 MgO crystallizes as rock salt structure with unit cell length 2.12 Å. From electrostatic model, the calculated lattice energy in kJ mol^{-1} (*rounded off to the nearest integer*) is _____.
(*Given*: $N_A = 6.022 \times 10^{23}\ mol^{-1}$; Madelung constant = 1.748;
$\varepsilon_0 = 8.854 \times 10^{-12}\ J^{-1}\ C^2\ m^{-1}$; charge of an electron = 1.602×10^{-19} C)

Q.45 Calcium crystallizes in *fcc* lattice of unit cell length 5.56 Å and density 1.4848 g cm^{-3}. The percentage of Schottky defects (*rounded off to one decimal place*) in the crystal is _____.
(*Given*: Atomic mass of Ca is 40 g mol^{-1}; $N_A = 6.022 \times 10^{23}\ mol^{-1}$)

Q.46 Among the following, the total number of terpenes(terpenoids) is _____.



Q.47 A buffer solution is prepared by mixing 0.3 M NH_3 and 0.1 M NH_4NO_3. If K_b of NH_3 is 1.6×10^{-5} at 25 °C, then the pH (*rounded off to one decimal place*) of the buffer solution at 25 °C is _____.

Q.48 The dissociation constant of a weak monoprotic acid is 1.6×10^{-5} and its molar conductance at infinite dilution is 360.5×10^{-4} mho m^2 mol^{-1}. For 0.01 M solution of this acid, the specific conductance is $n \times 10^{-2}$ mho m^{-1}. The value of n (*rounded off to two decimal places*) is _____.

Q.49 Adsorption of a toxic gas on 1.0 g activated charcoal is 0.75 cm^3 both at 2.5 atm, 140 K and at 30.0 atm, 280 K. The isosteric enthalpy for adsorption of the gas in kJ mol^{-1} (*rounded off to two decimal places*) is _____.
(*Given*: $R = 8.314$ J K^{-1} mol^{-1})

Q.50 If the root mean square speed of hydrogen gas at a particular temperature is 1900 m s^{-1}, then the root mean square speed of nitrogen gas at the same temperature, in m s^{-1} (*rounded off to the nearest integer*), is _____.
(*Given*: atomic mass of H is 1 g mol^{-1}; atomic mass of N is 14 g mol^{-1})

Q. 51 – Q. 60 carry two marks each.

Q.51 If the crystal field splitting energy of $[Co(NH_3)_4]^{2+}$ is 5900 cm^{-1}, then the magnitude of its crystal field stabilization energy, in kJ mol^{-1} (*rounded off to one decimal place*), is _____.

Q.52 A salt mixture (1.0 g) contains 25 wt% of $MgSO_4$ and 75 wt% of $\mathbf{M}_2SO_4$. Aqueous solution of this salt mixture on treating with excess $BaCl_2$ solution results in the precipitation of 1.49 g of $BaSO_4$. The atomic mass of **M** in g mol^{-1} (*rounded off to two decimal places*) is _____.
(*Given*: the atomic masses of Mg, S, O, Ba and Cl are 24.31, 32.06, 16.00, 137.33 and 35.45 g mol^{-1}, respectively)

Q.53 The intensity of a monochromatic visible light is reduced by 90% due to absorption on passing through a 5.0 mM solution of a compound. If the path length is 4 cm, then the molar extinction coefficient of the compound in M^{-1} cm^{-1} is _____.

Q.54 The surface tension (γ) of a solution, prepared by mixing 0.02 mol of an organic acid in 1 L of pure water, is represented as

$$\gamma^* - \gamma = A \log(1 + Bc)$$

γ^* is the surface tension of pure water, A = 0.03 N m^{-1}, B = 50 mol^{-1} L and c is concentration in mol L^{-1}. The excess concentration of the organic acid at the surface of the liquid, determined by Gibbs adsorption equation at 300 K is $n \times 10^{-6}$ mol m^{-2}. The value of n (*rounded off to two decimal places*) is _____.
(*Given*: R = 8.314 J K^{-1} mol^{-1})

Q.55 The separation of energy levels in the rotational spectrum of CO is 3.8626 cm^{-1}. The bond length (assume it does not change during rotation) of CO in Å (*rounded off to two decimal places*) is _____.
(*Given*: Planck's constant $h = 6.626 \times 10^{-34}$ J s; $N_A = 6.022 \times 10^{23}$ mol^{-1}; atomic mass of C is 12 g mol^{-1}; atomic mass of O is 16 g mol^{-1}; $c = 3 \times 10^8$ m s^{-1})

Q.56 A dilute solution prepared by dissolving a nonvolatile solute in one liter water shows a depression in freezing point of 0.186 K. This solute neither dissociates nor associates in water. The boiling point of the solution in K (*rounded off to three decimal places*) is _____.
(*Given*: For pure water, boiling point = 373.15 K;
cryoscopic constant = 1.86 K (mol kg^{-1})$^{-1}$; ebullioscopic constant = 0.51 K (mol kg^{-1})$^{-1}$)

Q.57 The thermodynamic data at 298 K for the decomposition reaction of limestone at equilibrium is given below

$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$

Thermodynamic quantity	$CaCO_3(s)$	$CaO(s)$	$CO_2(g)$
μ° (kJ mol^{-1})	–1128.8	–604.0	–394.4
ΔH_f° (kJ mol^{-1})	–1206.9	–635.1	–393.5

The partial pressure of $CO_2(g)$ in atm evolved on heating limestone (*rounded off to two decimal places*) at 1200 K is _____.
(*Given*: R = 8.314 J K^{-1} mol^{-1})

Q.58 The mean ionic activity coefficient of 0.004 molal $CaCl_2$ in water at 298 K (*rounded off to three decimal places*) is _____.
(*Given*: Debye-Hückel constant for an aqueous solution at 298 K is 0.509 kg$^{1/2}$ mol$^{-1/2}$)

Q.59 For the reaction,

$$\mathbf{Q} + \mathbf{R} \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} \mathbf{X} \xrightarrow{k_2} \mathbf{P}$$

$k_1 = 2.5 \times 10^5$ L mol^{-1} s^{-1}, $k_{-1} = 1.0 \times 10^4$ s^{-1} and $k_2 = 10$ s^{-1}. Under steady state approximation, the rate constant for the overall reaction in L mol^{-1} s^{-1} (*rounded off to the nearest integer*) is _____.

Q.60 For the molecule,

$CH_3–CH=CH–CH(OH)–CH=CH–CH=C(CH_3)_2$

the number of all possible stereoisomers is _____.

END OF THE QUESTION PAPER

SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 – Q.10 Carry ONE mark each.

Q.1 When the expected future marginal product of capital increases, then the IS curve

(A) shifts up and to the right

(B) shifts down and to the left

(C) becomes steeper

(D) becomes flatter

Q.2 An unanticipated inflation would cause

(A) redistribution of wealth from lenders to borrowers

(B) redistribution of wealth from borrowers to lenders

(C) gains for both borrowers and lenders

(D) losses for both borrowers and lenders

Q.3 Let $\{x_1, x_2, \cdots, x_n\}$ be the realization of a randomly drawn sample of size n with sample mean $\bar{x}$, and k be a real number other than $\bar{x}$. Let S_1 and S_2 be the sums of squared deviations defined as

$$S_1 = \sum_{i=1}^{n}(x_i - \bar{x})^2 \quad \text{and} \quad S_2 = \sum_{i=1}^{n}(x_i - k)^2.$$

Then,

(A) $S_1 > S_2$

(B) $S_1 > S_2$ only if $\bar{x} < k$

(C) $S_1 < S_2$

(D) $S_1 > S_2$ only if $\bar{x} > k$

Q.4 You have a budget of Rs. 4000 and would like to purchase LPG cylinders from a local seller who charges Rs. 50 per cylinder. The seller has a subsidy scheme by which if you return the empty cylinder purchased from him, you will get a refund of Rs. 20 per cylinder. You cannot borrow money from anyone. The maximum number of cylinders you can purchase is

(A) 131

(B) 132

(C) 133

(D) 134

Q.5 Which one of the following is **NOT** a feature of the New Industrial Policy, 1991?

(A) Abolition of industrial licensing

(B) Privatisation of public industries

(C) Removal of restrictions on foreign trade

(D) Restrictions on foreign technology agreements

Q.6 Which one of the following is a possible reason for underestimation of the official poverty ratio in India?

(A) Changes in the World Bank's definition of poverty

(B) Price indices used in the official poverty estimation may not be adequately capturing the actual increase in the cost of living over the years

(C) Existence of identical poverty lines for all the states and union territories

(D) Existence of identical poverty lines for rural and urban areas

Q.7 Which one of the following committees is **NOT** associated with financial sector reforms in India?

(A) Raghuram Rajan Committee (2013)

(B) Narasimham Committee (1991)

(C) Tarapore Committee (1997)

(D) Urjit Patel Committee (2013)

Q.8 The differential equation

$$(3x^2y + y^3)dx + (x^3 + 3xy^2)dy = 0$$

is

(A) homogenous and exact

(B) neither separable nor exact

(C) exact and not homogenous

(D) homogenous and not exact

Q.9 Which one of the following statements is correct?

(A) If $\langle a_n \rangle$ is a bounded sequence, then it is convergent

(B) If $\langle a_n \rangle$ is a convergent sequence, then it is monotonic

(C) If $\langle a_n \rangle$ is a convergent sequence and converges to zero, then the series $\sum_{n=1}^{\infty} a_n$ is convergent

(D) If a series $\sum_{n=1}^{\infty} a_n$ is convergent, then the sequence $\langle a_n \rangle$ is convergent and converges to zero

Q.10 Let $||.||$ and $\langle .,. \rangle$ denote the standard norm and inner product in $\mathbb{R}^n$, respectively. If $u, v \in \mathbb{R}^3$ such that $||u|| = ||v|| = 2$ and the angle between u and v is $\pi/3$ then

(A) $||u - v|| = 2\sqrt{2}$

(B) $\langle u, v \rangle = 2\sqrt{3}$

(C) $||u - v|| = 2\sqrt{3}$

(D) $||u + v|| = 2\sqrt{3}$

Q. 11 – Q. 30 Carry TWO marks each.

Q.11 A monopoly canteen serves packed meals to two groups of consumers, group X and group Y. The demand for packed meals for X and Y are given by,

$$Q_X = 200 - 4P \text{ and } Q_Y = 300 - P,$$

where P is the uniform price per unit. The unit cost of producing each meal is Rs. 50. The value of P (in Rs.) that maximizes the canteen's profit is

(A) 75

(B) 50

(C) 125

(D) 175

Q.12 Consider a Solow growth model without technological progress. The production function is

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

where Y_t, K_t and N_t are the aggregate output, capital and population at time t, respectively. The population grows at a constant rate of $g_N > 0$, savings rate is constant at $s \in (0,1)$ and capital depreciates at a constant rate of $\delta \geq 0$. Denote per capita capital as

$$k_t = K_t / N_t$$

and define the steady state as a situation where $k_{t+1} = k_t = k^*$ where k^* is a positive constant. Suppose the population growth rate exogenously increases to g_N' . At the new steady state, the aggregate output will grow at a rate

(A) g_N

(B) g_N'

(C) $(1-\alpha)g_N$

(D) $(1-\alpha)g_N'$

Q.13 The utility from wealth (w) for an individual is given by $u(w) = \sqrt{w}$. The individual owns a risky asset that is equally likely to yield either Rs. 400 or Rs. 900. The risk premium of the asset (in Rs.) is

(A) 5

(B) 25

(C) 625

(D) 650

Q.14 Let $\widehat{\alpha_1}$ and $\widehat{\alpha_2}$ be two independent unbiased estimators of the parameter α with standard errors σ_1 and σ_2, respectively, with $\sigma_1 \neq \sigma_2$. The linear combination of $\widehat{\alpha_1}$ and $\widehat{\alpha_2}$ that yields an unbiased estimator of α with the minimum variance is

(A) $\left(\frac{\sigma_1}{\sigma_1+\sigma_2}\right)\widehat{\alpha_1} + \left(\frac{\sigma_2}{\sigma_1+\sigma_2}\right)\widehat{\alpha_2}$

(B) $\left(\frac{\sigma_2}{\sigma_1+\sigma_2}\right)\widehat{\alpha_1} + \left(\frac{\sigma_1}{\sigma_1+\sigma_2}\right)\widehat{\alpha_2}$

(C) $\left(\frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)\widehat{\alpha_1} + \left(\frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)\widehat{\alpha_2}$

(D) $\left(\frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)\widehat{\alpha_1} + \left(\frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)\widehat{\alpha_2}$

Q.15 Let X be a uniformly distributed random variable in $[0, b]$. If the critical region for testing the null hypothesis $H_0: b = 2$ against the alternative hypothesis $H_A: b \neq 2$ is $\{x \leq 0.1 \text{ or } x \geq 1.9\}$, where x is the value of a single draw of the random variable X, then the probability of Type-I error is

(A) 0.2

(B) 0.1

(C) 0.05

(D) 0.01

Q.16 Let X be a uniformly distributed random variable in $[a, b]$. The values of an independently drawn sample of size five from X are given by {1.3, 0.8, 9.5, 20.2, 8.2}. Let $\hat{a}$ and $\hat{b}$ denote the Maximum Likelihood Estimates for the parameters a and b, respectively. Then,

(A) $\hat{a} = 0.8;\ \hat{b} = 20.2$

(B) $\hat{a} = 1.3;\ \hat{b} = 9.5$

(C) $\hat{a} = 1.3;\ \hat{b} = 8.2$

(D) $\hat{a} = 0;\ \hat{b} = 21$

Q.17 There are only two firms in an industry producing a homogenous product and having identical production technology. The cost function of firm i is

$$C_i(q_i) = {q_i}^2, \text{ for } i = 1,2;$$

where q_i is the quantity produced by firm i. The market demand for the product is $p = 100 - q$, where p is the unit price and $q = q_1 + q_2$ is the aggregate quantity. Assuming the firms are price takers, the competitive equilibrium solution of p and q in this market is

(A) $p = 80, q = 20$

(B) $p = 20, q = 80$

(C) $p = \frac{200}{3}, q = \frac{100}{3}$

(D) $p = 50, q = 50$

Q.18 An upstream paper mill dumps effluents in a river. The total benefit and total cost to the mill are $TB = 120Q - Q^2$ and $TC = 20Q$, respectively, where Q is the amount of output it produces. The environmental cost due to the negative externality is $EC = Q^2$. The government wants to impose a production tax of t per unit of output on the mill.

The value of t to achieve the socially optimal level of production is

(A) 6

(B) 25

(C) 50

(D) 70

Q.19 Which one of the following statements is **NOT** correct regarding changes in the occupational structure of the workforce between 1951 and 1991 in India?

(A) Proportion of cultivators has increased

(B) Proportion of agriculture labour has increased

(C) Proportion of those employed in the tertiary sector has increased

(D) Proportion of those employed in the primary sector has decreased

Q.20 Let W be a subspace of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$ spanned by $\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$. Which one of the following vectors lies in W?

(A) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Q.21 Let f, $g : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = xe^{-x} \quad \text{and} \quad g(x) = x|x| .$$

Then, on $\mathbb{R}$,

(A) both f and g are convex

(B) f is convex and g is not convex

(C) f is not quasiconvex and g is quasiconvex

(D) neither f nor g is quasiconvex

Q.22 Let $(x_1^* = 1,\ x_2^* = 0,\ x_3^* = 2)$ be an optimal solution of the linear programming problem

$$\begin{aligned} \text{minimize} \quad & x_1 + 5x_2 + 2x_3 \\ \text{subject to} \quad & x_1 - x_2 \leq 1 \\ & x_1 + x_2 + x_3 \geq 3 \\ & x_1,\ x_2,\ x_3 \geq 0\ . \end{aligned}$$

If $(\lambda_1^*\ ,\ \lambda_2^*)$ is an optimal solution of its dual, then

(A) $2\lambda_1^* = 3\lambda_2^*$

(B) $2\lambda_1^* = \lambda_2^*$

(C) $\lambda_1^* = 2\lambda_2^*$

(D) $\lambda_1^* = \lambda_2^*$

Q.23 Let X and Y be two independent random variables with the cumulative distribution functions

$$F_X(x) = 1 - \left(\frac{3}{4}\right)^x, \qquad x = 1,2,3,\cdots$$

$$G_Y(y) = 1 - \left(\frac{2}{3}\right)^y, \qquad y = 1,2,3,\cdots,$$

respectively. Let $Z = \min\{X,\ Y\}$. Then, the probability $P(Z \geq 6)$ is

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{63}{64}$

(D) $\frac{31}{32}$

Q.24 Let X and Y be two random variables with the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 6xy & \text{if} \quad 0 < y \leq \sqrt{x} \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then, the conditional probability $P(Y \geq \frac{1}{3} \mid X = \frac{2}{3})$ is

(A) $\frac{1}{2}$

(B) $\frac{5}{9}$

(C) $\frac{5}{6}$

(D) $\frac{3}{4}$

Q25 Which one of the following statements is **NOT** correct in the context of economic planning in India?

(A) In the investment strategy for the Second Five Year Plan, a high priority was accorded to the development of heavy capital goods industries over light industries

(B) The sectoral allocation to industry was the highest in the First Five Year Plan

(C) Plan Holiday for three years was declared after the Third Five Year Plan

(D) In each of the first ten Five Year Plan periods, the average incremental capital-output ratio (ICOR) did not exceed 10 %

Q.26 For any two sets $S_1, S_2 \subset \mathbb{R}$, define the set $S_1 - S_2 = \{ x \in S_1,\ x \notin S_2 \}$. Let

$$P = \{ x \in \mathbb{R} : x^2 - 2x - 3 \leq 0 \} \quad \text{and}$$

$$Q = \{ x \in \mathbb{R} : \log_5(1 + x^2) \leq 1 \}.$$

Then,

(A) $P - Q = [2, 3]$

(B) $Q - P = (1, 2]$

(C) $P - Q = [-3, -2)$

(D) $Q - P = [-2, -1)$

Q.27 The workforce participation rate of a country is 60%. This country has a population of 100 million of which 6 million are unemployed. The unemployment rate for this country is

(A) $\frac{2}{11}$

(B) $\frac{1}{11}$

(C) $\frac{3}{50}$

(D) $\frac{1}{10}$

Q.28 According to John Maynard Keynes, which one of the following statements is correct for a closed economy operating at less than the full employment level of output?

(A) Savings determines investment

(B) Investment determines savings

(C) Changes in the money supply have no impact on output

(D) Speculative demand for money is determined by the output level

Q.29 A monopolist is facing a downward sloping linear market demand. His variable cost of production is zero. The profit maximizing price will

(A) lie in the strictly inelastic region of the demand curve

(B) lie in the strictly elastic region of the demand curve

(C) be at the unitary elastic point of the demand curve

(D) be equal to the marginal cost of production

Q.30 X pays Rs. 5 lakhs to a person to transport fake currency worth Rs. 50 lakhs. The Police department pays Rs. 5 lakhs to a detective to investigate the crime. The detective's income is taxed at 10 %. If the above transactions happen in the same year and within the boundary of a country, the contribution of these transactions to GDP (in Rs. lakhs) is

(A) 5.5

(B) 5

(C) 10

(D) 4.5

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 Carry TWO marks each.

Q.31 An amateur singer has just recorded his first music album with a recording company. The demand for his album is given by $Q = 40000 - 800P$, where Q is the number of albums sold and P is the price of each album. Furthermore, per unit cost of producing each album is given by Rs. 8. A profit maximizing recording company has offered the following contract options to the singer

(i) 20% of the revenue from the sales of the album

(ii) Rs. 2 per album sold

(iii) A fixed fee of Rs. 32,000

Which of the following is/are correct?

(A) Contract (i) yields the highest payment to the singer

(B) Contract (ii) yields the highest payment to the singer

(C) Contract (iii) yields the highest payment to the singer

(D) Contract (ii) and (iii) yield the same payment to the singer

Q.32 There are two firms in an oligopolistic industry competing in prices and selling a homogenous product. Total cost of production for firm i is

$$C_i(q_i) = 10q_i, \quad i = 1,2;$$

where q_i is the quantity produced by firm i. Suppose firm i sets price p_i and firm j sets price p_j. The market demand faced by firm i is given by

$$q_i(p_i, p_j) = \begin{cases} 100 - p_i; & \text{if} \quad p_i < p_j \\ 0; & \text{if} \quad p_i > p_j \\ \dfrac{100 - p_i}{2}; & \text{if} \quad p_i = p_j \end{cases}$$

for all $i, j = 1,2$ and $i \neq j$. Price can only take integer values in this market. Nash equilibrium/equilibria is/are given by

(A) $p_1 = 10, \quad p_2 = 10$

(B) $p_1 = 12, \quad p_2 = 12$

(C) $p_1 = 40, \quad p_2 = 40$

(D) $p_1 = 11, \quad p_2 = 11$

Q.33 Which of the following statements is/are correct about the Indian economy during the colonial period?

(A) The average annual growth of per capita income was lower during the period 1920-25 to 1947 than the period 1865 to 1920-25.

(B) The colonial administration generated a large amount of revenue from peasants by raising the land revenue.

(C) The British brought capital from England for the construction of Railways and passed on the burden of interest on it to the Indian tax payers.

(D) Dadabhai Naoroji's estimates of the drain of wealth from India to England included, among other things, the home charges.

Q.34 In the context of Expectations Augmented Phillips Curve (EAPC), which of the following statements is/are correct?

(A) An increase in the natural rate of unemployment shifts EAPC to the left.

(B) An increase in the expected inflation shifts EAPC up and to the right.

(C) If actual unemployment rate equals the natural rate of unemployment, the unanticipated inflation equals zero.

(D) As long as actual unemployment rate exceeds the natural rate of unemployment, the actual inflation rate exceeds the expected inflation.

Q.35 Let f be a function defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ as

$$f(x) = \frac{\cos\left(\frac{\pi}{2} + |x|\right)}{\sin\left(\frac{\pi}{2} - |x|\right)}.$$

Then,

(A) f is not continuous at $x = 0$

(B) f is continuous but not differentiable at $x = 0$

(C) $\lim_{x \to 0^+} \frac{f(x) - f(0)}{x} = -1$

(D) $f'(0) = -1$

Q.36 Let x , $y \in \mathbb{R}$ and the matrix

$$M = \begin{bmatrix} x+y & x-y \\ x-y & x+y \end{bmatrix}.$$

Also, let $adj(M)$ be the adjoint and $\det(M)$ be the determinant of the matrix M.

If $M \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, then

(A) $x + y = -\frac{3}{4}$

(B) $x - y = \frac{3}{4}$

(C) $\det(M) = -1$

(D) $\det\big(adj(M)\big) = 1$

Q.37 The net inflow of foreign currency into a country on current account and capital account combined is negative in a particular year. The country could be following a fixed or a flexible exchange rate regime.

Which of the following scenarios is/are possible for the country's economy in that year?

(A) The country's foreign exchange reserves may increase

(B) The country's exchange rate may appreciate

(C) The country's foreign exchange reserves may decrease

(D) The country's exchange rate may depreciate

Q.38 Let $k \in \mathbb{R}$. Which of the following statements is/are correct for the roots of the quadratic equation

$$x^2 + 2(k+1)x + 9k - 5 = 0$$

(A) If $k \leq 1$, then the roots are real and positive

(B) If $2 \leq k \leq 4$, then the roots are complex

(C) If $4 < k < 6$, then the roots are real and opposite in sign

(D) If $k \geq 6$, then the roots are real and negative

Q.39 If the number of employed workers in a country increases while its population does not change, then the unemployment rate in the country

(A) will always increase

(B) will always decrease

(C) may increase

(D) may decrease

Q.40 There are two sellers, H and L, in a second-hand goods market where product quality varies. The sellers know the quality of their own product but the buyers cannot distinguish the product quality without further information. Sellers' valuation of their own product is based on the quality. H is willing to sell his product with quality Q_H at a price P_H per unit and L is willing to sell the product with quality Q_L at a price P_L per unit such that

$$Q_H > Q_L \quad \text{and} \quad P_H > P_L .$$

This market will suffer from

(A) adverse selection

(B) moral hazard

(C) market failure

(D) excess supply

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 Carry ONE mark each.

Q.41 The amount of money a gambler can win in a casino is determined by three independent rolls of a six-faced fair dice. The gambler wins Rs. 800 if he gets three sixes, Rs. 400 if he gets two sixes, and Rs. 100 in the event of getting only one six. The gambler does not win or lose any money in all other possible outcomes. The probability that a gambler will win at least Rs. 100 is _____. (round off to 2 decimal places)

Q.42 Consider an economy where the full employment output is 1 trillion Rupees and the natural rate of unemployment is 6 %. If actual unemployment rate is 8 %, then according to Okun's law, the absolute gap between the full employment output and actual output (in billion Rupees) will be _____. (in integer)

Q.43 The values of normalized indices for a country are as follows.

Dimension	Value of normalized index
Standard of living	0.4
Education	0.2
Health	0.8

Following the current UNDP methodology, the value of Human Development Index (HDI) for the country is _____. (round off to 1 decimal place)

Q.44 The value of the integral

$$\int_0^9 \frac{x-1}{1+\sqrt{x}}\, dx$$

is _____. (in integer)

Q.45 Consider the first order difference equation

$$x_n = \left(\frac{n+1}{n}\right) x_{n-1}, \quad n = 1, 2, 3, \ldots$$

If $x_0 = 2$, then $x_{100} - x_{50}$ equals _____. (in integer)

Q.46 In a small open economy, the desired domestic savings (S^d) and the desired domestic investment (I^d) are as follows, where r^w is the world real interest rate.

$$S^d = 10 + 100\, r^w$$

$$I^d = 15 - 100\, r^w$$

If $r^w = 3\,\%$, the current account balance in the equilibrium would be _____. (in integer)

Q.47 Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ be two normally distributed random variables, where $\mu_1 = 2$, $\mu_2 = 3$ and $\sigma_1^2 = 4$, $\sigma_2^2 = 9$. The correlation coefficient between them is 0.5. The variance of the random variable $(X_1 + X_2)$ is _____. (in integer)

Q.48 A consumer always spends 50% of his monthly income on food. Introduction of value added tax on food items has led to a 20% increase in food prices while his monthly income remained unchanged. The consumer's price elasticity of demand for food is _____. (in integer)

Q.49 The utility function of a consumer from consumption of x_1 and x_2 is given by

$$u(x_1, x_2) = x_1 + 2\sqrt{x_2}.$$

At the current prices and income, the consumer's optimal consumption bundle is given by $(x_1 = 10, x_2 = 10)$. The consumer's optimal choice of x_2, if his income increases by 50% but prices remain unchanged, is _____. (in integer)

Q.50 The following data relate to a country's GDP in 2012-13 (in local currency).

Item	Value
GDP	59,816
Private sector investment	17,811
Exports	14,498
Investment expenditure by the government	5,087
Net Factor Income from Abroad	125
Consumption expenditure by the government	6,620
Private sector consumption	35,695

The value of this country's imports (in local currency) in 2012-13 is _____. (in integer)

Q. 51 – Q. 60 Carry TWO mark each.

Q.51 Amar has an endowment of food $F_A = 2$ and water $W_A = 5$. Barun has an endowment of food $F_B = 8$ and water $W_B = 5$. Amar's utility function is given by

$$U_A(f_A, w_A) = f_A^2 \, w_A;$$

where f_A and w_A are his consumption of food and water, respectively. Barun's utility function is given by

$$U_B(f_B, w_B) = \min\{f_B, w_B\};$$

where f_B and w_B are his consumption of food and water, respectively. They exchange food and water at prices p_f and p_w, respectively, to maximize their utilities. In the competitive equilibrium, $\frac{p_f}{p_w}$ equals _____. (in integer)

Q.52 The supply and demand curves of a vaccine are

$$q = 14 + 5p \text{ and } q = 329 - 5p,$$

respectively, where p is price per unit of vaccine and q is quantity of vaccine. The government decides that the maximum price of the vaccine would be Rs. 25 per unit. To avoid any shortage in supply at the ceiling price, the government also decides to subsidize the sellers so that the market clears. Subsidy is given on per unit basis. The total expenditure of the government in providing the subsidy is Rs. _____. (in integer)

Q.53 A firm has two manufacturing plants, 1 and 2 to produce the same product. The total costs of production are given by

$$TC_1 = 500 + 30Q_1 \text{ and } TC_2 = 1500 + 20Q_2$$

in plants 1 and 2, respectively, where Q_1 and Q_2 are the respective quantities. The demand for the product is given by $Q^d = 150 - \frac{P}{3}$, where P is the price per unit. The value of Q_1 that maximizes the profit of the firm is _____. (in integer)

Q.54 Let $y(x) > 0$ be a solution of the differential equation

$$\frac{dy}{dx} + y = y^2 .$$

If $y(\ln 2) = \frac{1}{3}$, where $\ln$ denotes the natural logarithmic function, then $y(\ln 3)$ equals _____. (round off to 2 decimal places)

Q.55 The optimal value of the constrained optimization problem

$$\text{minimize} \quad 2xy$$

$$\text{subject to} \quad 9x^2 + 4y^2 \leq 36$$

is _____. (in integer)

Q.56 For some $\beta > 0$, let the variables x_1 and x_3 be the optimal basic feasible solution of the linear programming problem

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 - x_2 + x_3 = 9 \\ & x_1 + 2\,x_2 - \beta x_3 = 1 \\ & x_1\,, x_2\,, x_3 \geq 0\,. \end{aligned}$$

If the optimal value is 7, then β equals _____. (in integer)

Q.57 Let X_1, X_2, X_3, X_4 be independent random variables following the standard normal distribution. Let Y be defined as, $Y = (X_1 + X_2)^2 + (X_3 + X_4)^2$. Then the variance of Y equals _____. (in integer)

Q.58 The aggregate production function for a country is, $Y = 10N - 0.005N^2$, where N is the quantity of labor input. The aggregate labor supply function is $N = 55 + 5w$, where w is the real wage rate. Assuming perfectly competitive labor and product markets, the equilibrium real wage is _____. (in integer)

Q.59 Individuals in a country start earning and consuming at the age of 18 years, retire at the age of 60 years and die at the age of 90 years, without leaving any debt and bequests. The income of an individual at age t (in years) is given by the expression $100t - t^2$. The price level is constant and the interest rate is zero. According to the life cycle theory of consumption, the average annual consumption of an individual is _____. (in integer)

Q.60 The IS-LM model for a closed economy is given below, where Y is the output, C is the consumption, I is the investment, T is the income tax, M^d is the money demand, P is the price level, r is the real interest rate, π^e is expected inflation rate and G is the government expenditure.

$$C = 200 + 0.8\,(Y - T) - 500\,r$$

$$I = 200 - 500\,r$$

$$T = 20 + 0.25\,Y$$

$$\frac{M^d}{P} = 0.5\,Y - 250\,(r + \pi^e)$$

If $G = 196$, $\pi^e = 0.1$, the nominal money supply equals 9890 and the full employment output equals 1000, the full employment equilibrium price level in the economy is _____. (in integer)

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Among the following rocks, the one with highest metamorphic grade is

(A) chlorite schist

(B) glaucophane schist

(C) phyllite

(D) gneiss

Q.2 The Earth's radius is maximum at which one of the following latitudes?

(A) 0°

(B) 40° N

(C) 60° S

(D) 90°

Q.3 The closest value to the percentage of the Earth's surface covered by the oceans is

(A) 30%

(B) 50%

(C) 70%

(D) 90%

Q.4 Which is the shallowest among the marine environments listed below?

(A) Neritic

(B) Littoral

(C) Abyssal

(D) Bathyal

Q.5 Among the following, the tungsten-bearing mineral is

(A) bornite

(B) cassiterite

(C) scheelite

(D) greenockite

Q.6 The host rock of Pb-Zn deposit at Zawar is

(A) quartzite

(B) phyllite

(C) dolomite

(D) gneiss

Q.7 Bituminous coal deposits in India occur in which one of the following formations?

(A) Barren Measures Formation

(B) Barakar Formation

(C) Naredi Formation

(D) Cuddalore Formation

Q.8 Identify the plant fossil from the following list.

(A) *Glossopteris*

(B) *Fenestella*

(C) *Productus*

(D) *Cidaris*

Q.9 The igneous body with dome or mushroom-like shape is known as a

(A) lopolith

(B) ring dike

(C) sill

(D) laccolith

Q.10 Which one of the following stratigraphic units belongs to the Cretaceous?

(A) Lameta Formation

(B) Talchir Boulder Bed

(C) *Fenestella* Shale

(D) Kasauli Formation

Q. 11 – Q. 30 carry two marks each.

Q.11 Select the youngest volcanic event out of the following.

(A) Rajmahal volcanics

(B) Dalma volcanics

(C) Panjal volcanics

(D) Deccan volcanics

Q.12 Which among the following is the only possible plunge for a lineation located on the foliation plane striking 20° N and dipping 40° southeasterly?

(A) 20° SE

(B) 20° NW

(C) 45° SE

(D) 40° NW

Q.13 Which one of the following tectonic plates has the maximum average velocity?

(A) Eurasian

(B) Pacific

(C) African

(D) North American

Q.14 A limestone contains lime mud and around 25% allochems, which are separated from each other. The name of the rock as per Dunham's classification is

(A) mudstone

(B) wackestone

(C) packstone

(D) grainstone

Q.15 Find the **CORRECT** statement out of the following.

(A) Convolute laminae form by desiccation.

(B) Load cast is an erosional structure.

(C) Prod mark is found at the bottom of a bed.

(D) Wave ripple occurs at the top of a turbidite deposit.

Q.16 Which one of the following crystal forms **DOES NOT** belong to the di-tetragonal pyramidal class?

(A) c-Pedion

(B) Prism of 1st order

(C) Di-tetragonal prism

(D) Tetragonal dipyramid

Q.17 If a coarse-grained igneous rock is composed of >90% of plagioclase and <10% of olivine and pyroxene, then the name of this rock according to the IUGS classification is

(A) anorthosite

(B) olivine gabbro

(C) tonalite

(D) olivine websterite

Q.18 Which one of the following represents the compositional change in plagioclase during the crystallization of mafic magma?

(A) Na/Ca ratio decreases; Al/Si ratio increases

(B) Both Na/Ca and Al/Si ratios increase

(C) Na/Ca ratio increases; Al/Si ratio decreases

(D) Both Na/Ca and Al/Si ratios decrease

Q.19 Choose the **CORRECT** sequence of older to younger formations in the stratigraphy of the Cuddapah Supergroup

(A) Pulivendla-Gulcheru-Vempalle-Tadpatri

(B) Gulcheru-Vempalle-Pulivendla-Tadpatri

(C) Gulcheru-Pulivendla-Tadpatri-Vempalle

(D) Vempalle-Gulcheru-Tadpatri-Pulivendla

Q.20 Match the economic deposits (**Group I**) with their places of occurrence (**Group II**).

Group I	**Group II**
P. Iron ore	1. Bhatan
Q. Base metal	2. Sukinda
R. Chromite	3. Rampura-Agucha
S. Uranium	4. Bellary

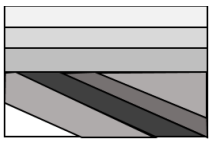
(A) P-4; Q-1; R-2; S-3

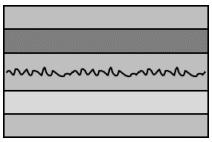
(B) P-2; Q-4; R-3; S-1

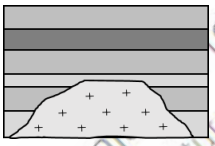
(C) P-2; Q-3; R-1; S-4

(D) P-4; Q-3; R-2; S-1

Q.21 Select the answer that is a **CORRECT** match for the three types of unconformities. (Grey bands = sediments, [+] = igneous rock).

P. Disconformity 1. 

Q. Non-conformity 2. 

R. Angular Unconformity 3. 

(A) P-2; Q-3; R-1

(B) P-2; Q-1; R-3

(C) P-1; Q-3; R-2

(D) P-3; Q-2; R-1

Q.22 Which one of the following statements is **FALSE**?

(A) Ammonites have fluted septa.

(B) Brachiopods have a pedicle.

(C) Echinoids have genal spines.

(D) Trilobites have a pygidium.

Q.23 In the context of phylogeny of horses, the **CORRECT** chronological order from old to young is

(A) *Hyracotherium, Mesohippus, Merychippus, Equus*

(B) *Hyracotherium, Merychippus, Mesohippus, Equus*

(C) *Equus, Merychippus, Mesohippus, Hyracotherium*

(D) *Merychippus, Equus, Mesohippus, Hyracotherium*

Q.24 Choose the **CORRECT** match between items in **Group I** with the items in **Group II**.

Group I	**Group II**
P. Polarity zone	1. Biostratigraphy
Q. Formation	2. Chronostratigraphy
R. Biozone	3. Magnetostratigraphy
S. Epoch	4. Lithostratigraphy

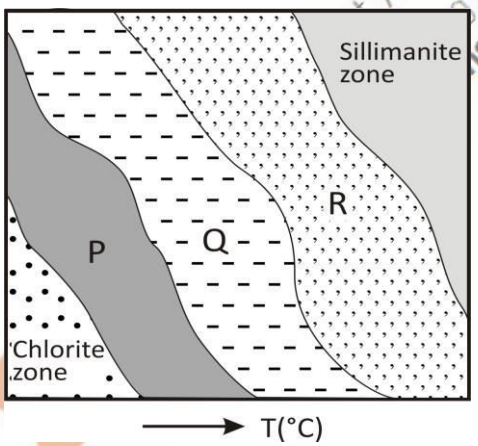
(A) P-1; Q-2; R-3; S-4

(B) P-2; Q-3; R-1; S-4

(C) P-3; Q-4; R-1; S-2

(D) P-3; Q-4; R-2; S-1

Q.25 The following diagram represents the prograde sequence of metamorphic zones that develop during Buchan-type of metamorphism (<4 kbar) of typical pelites. Identify the zones labelled P, Q and R.



(A) P - Biotite zone; Q - Garnet zone; R - Kyanite zone

(B) P - Garnet zone; Q - Kyanite zone; R - Staurolite zone

(C) P - Biotite zone; Q - Cordierite zone; R - Andalusite zone

(D) P - Andalusite zone; Q - Biotite zone; R - Cordierite zone

Q.26 Match the minerals in **Group I** with the corresponding composition in **Group II**.

Group I	**Group II**
P. Leucite	1. $(K,Na)AlSi_3O_8$
Q. Andradite	2. $NaAlSi_2O_6$
R. Sanidine	3. $KAlSi_2O_6$
S. Jadeite	4. $Ca_3Fe_2Si_3O_{12}$

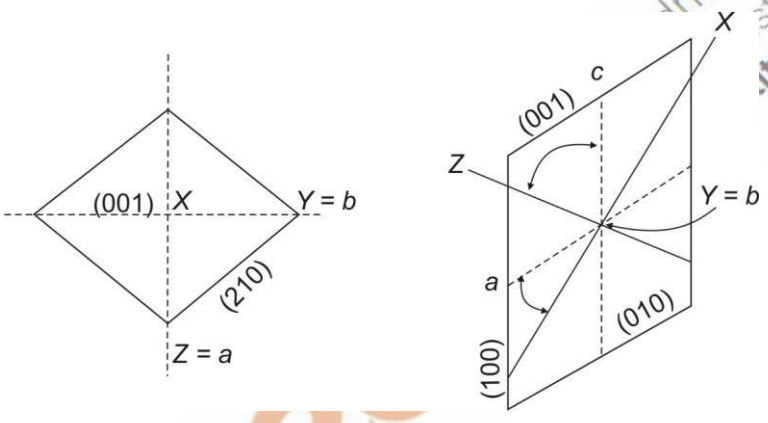
(A) P-3; Q-1; R-4; S-2

(B) P-2; Q-3; R-1; S-4

(C) P-1; Q-4; R-2; S-3

(D) P-3; Q-4; R-1; S-2

Q.27 Choose the **CORRECT** pair of crystal systems that represents the optic orientation shown in the figure. (*X,Y,Z* are the principal optical vibration directions and *a,b,c* are the crystallographic axes).



(A) Orthorhombic - Triclinic

(B) Orthorhombic - Monoclinic

(C) Cubic - Hexagonal

(D) Tetragonal - Monoclinic

Q.28 Match the environment representing physical geological processes in **Group I** with the corresponding geomorphic landform/feature in **Group II**.

Group I	**Group II**
P. Aeolian	1. Drumlin
Q. Glacial	2. Tombolo
R. Fluvial	3. Yardang
S. Coastal	4. Natural levee

(A) P-2; Q-1; R-4; S-3

(B) P-2; Q-4; R-1; S-3

(C) P-3; Q-1; R-4; S-2

(D) P-3; Q-4; R-1; S-2

Q.29 Match the items in **Group I** with the corresponding items in **Group II**.

Group I	**Group II**
P. Chalcocite	1. Supergene enrichment
Q. Bauxite	2. Mechanical accumulation
R. Monazite placers	3. Magmatic crystallization
S. Chromite	4. Residual processes

(A) P-1; Q-4; R-2; S-3

(B) P-2; Q-3; R-4; S-1

(C) P-2; Q-4; R-1; S-3

(D) P-1; Q-3; R-2; S-4

Q.30 Which one of the following statements is **FALSE**?

(A) Perched water table exists within the zone of aeration.

(B) Juvenile water is derived from sediment diagenesis.

(C) Zone of aeration lies above the zone of saturation.

(D) Both aquiclude and aquifuge are impermeable.

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Fossils from which of the following invertebrate classes show pentameral symmetry?

(A) Echinoidea

(B) Anthozoa

(C) Cephalopoda

(D) Trilobita

Q.32 The attitude of the two limbs of a fold was measured as striking 4° N, dipping 85° easterly and striking 30° N, dipping 60° easterly. Which of the following is/are **TRUE** for describing the geometry of the fold?

(A) Synform

(B) Antiform

(C) Overturned

(D) Plunging

Q.33 Which of the following statement(s) is/are **CORRECT** regarding ophitic texture?

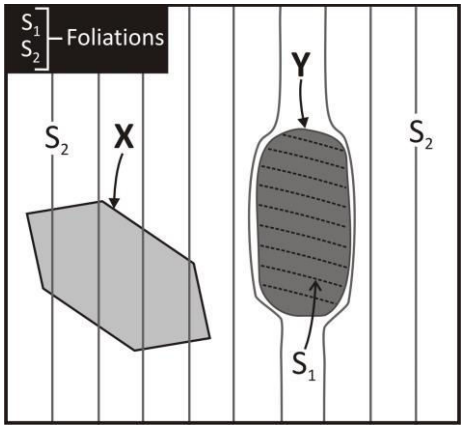
(A) Plagioclase laths are completely enclosed by large pyroxene crystals.

(B) Intergrowth occurs between quartz and alkali-feldspar.

(C) It is a variety of poikilitic texture.

(D) It is a texture observed in peridotite.

Q.34 On the basis of the following schematic diagram, choose the **CORRECT** statement(s).



(A) Minerals **X** and **Y** predate matrix foliation (S_2).

(B) Mineral **X** grew after the development of matrix foliation (S_2).

(C) Mineral **Y** grew during or after the development of internal foliation (S_1).

(D) Minerals **X** and **Y** postdate matrix foliation (S_2).

Q.35 A marine organic-rich, black shale with tiny pyrite crystals shows complete absence of body or trace fossils. Which of the following statement(s) is/are **TRUE**?

(A) The sediments were deposited in low-energy conditions.

(B) The deposition took place in dysoxic to anoxic conditions.

(C) The rate of sedimentation was high.

(D) The environment was stressful for survival of living organisms.

Q.36 Which mineral(s) among the following represent(s) AB_2O_4 composition?

(A) Spinel

(B) Magnetite

(C) Chromite

(D) Ilmenite

Q.37 Which among the following statement(s) is/are **TRUE**?

(A) Attrition is more dominant in aeolian than in glacial environment.

(B) Centrifugal force drives the sediment-laden water outward when the river channel meanders.

(C) U-shaped valley is a common fluvial geomorphic feature.

(D) The downstream water velocity in a river channel increases upward from the channel bed towards the water surface.

Q.38 Which of the following statement(s) regarding hydrocarbon occurrence is/are **CORRECT**?

(A) Gandhar field is in Cambay basin.

(B) Oil and gas occur in Mesozoic reservoir rocks in Bombay High field.

(C) Digboi field is in Assam basin.

(D) Hydrocarbon occurs in limestone reservoir in Ankleshwar field.

Q.39 Following are the statements regarding types of sandstone as per Pettijohn's classification. Which is/are the **CORRECT** statement(s) out of the following?

(A) Arkose contains more than 25% feldspar.

(B) Greywacke contains more than 90% matrix.

(C) Litharenite contains more than 25% lithic fragment.

(D) Quartz arenite contains more than 95% quartz.

Q.40 Choose the **CORRECT** statement(s) out of the following.

(A) Shoreline shifts landward during transgression.

(B) Shoreline shifts seaward during transgression.

(C) Delta deposits preserve the record of transgression.

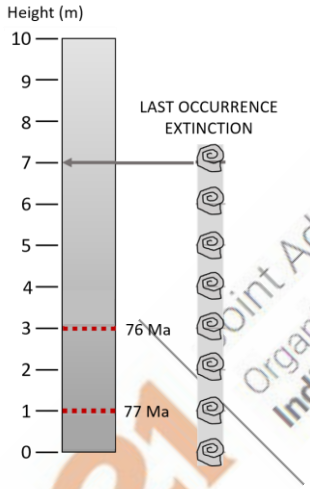
(D) Incised river valley forms because of transgression.

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

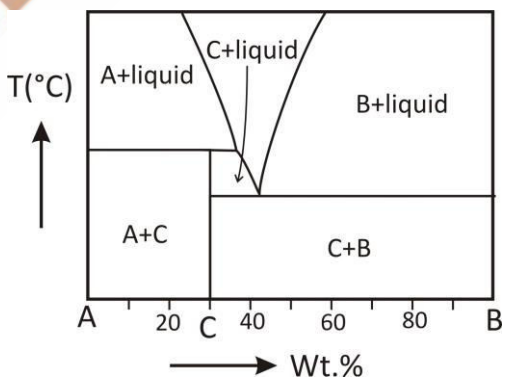
Q. 41 – Q. 50 carry one mark each.

Q.41 The given section with uniform lithology and sedimentation rate records two ash layers dated at 77 Ma and 76 Ma, respectively. An index fossil species present in the lower part of the section becomes extinct at a horizon 7m above the base. The estimated age of the extinction event is _____ Ma. (Answer in integer).



Q.42 A hollow discoid (cylindrical) microfossil has an outer diameter of 20 μm, height 10 μm and wall thickness 1 μm. The internal volume that can be occupied by the organism is _____ μm^3. (use $\pi = 3.14$) (Round off to one decimal place).

Q.43 In the following isobaric temperature-composition diagram, the number of common phases in all the invariant points is _____. (Answer in integer).



Q.44 A muscovite has the following composition in which iron is ferrous. The amount of 'Al' in the tetrahedral site is _____ (per formula unit). (Round off to two decimal places).

Muscovite composition: $KAl_{2.50}Fe_{0.25}Si_{3.25}O_{10}(OH)_2$

Q.45 The density of a 200 g gabbro sample, cut in the form of a cube, is 3125 kg/m^3. The length of the sample is _____ mm. (Answer in integer).

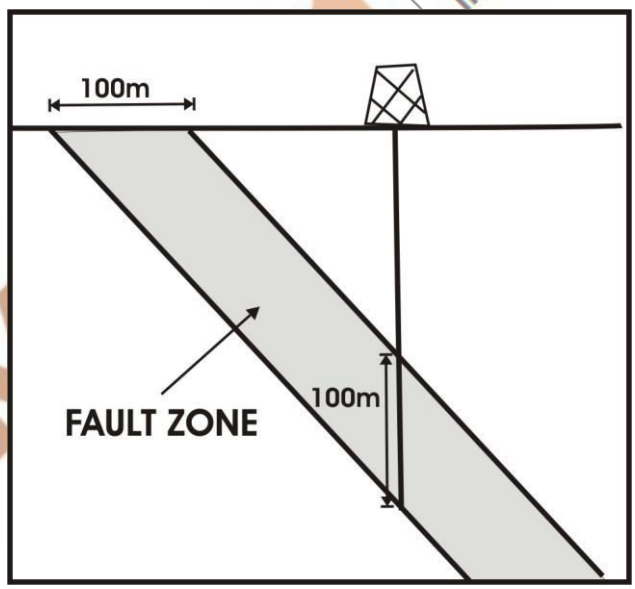
Q.46 A drill run of 3 m was carried out in a coalfield site, where rock core samples were recovered only for a cumulative length of 255 cm. The core loss in percentage is equal to _____. (Answer in integer).

Q.47 During concretionary growth of a spherical grain of radius 2 Å, the rate of change of surface area with respect to change in radius of the grain is _____ $\times 10^{-8}$ cm (use $\pi = 3.14$) (Round off to two decimal places).

Q.48 The weight loss during the conversion of 1 mole of gypsum to anhydrite is _____ % (atomic weights of Ca = 40.0, S = 32.0, O = 16.0, H = 1.0). (Round off to two decimal places).

Q.49 A bed with an attitude 020°, 30° NW is rotated 55° counter-clockwise (looking northerly) along its strike line. The dip of the bed after rotation will be _____° NW. (Answer in integer).

Q.50 The width of the outcrop of a fault zone on a flat surface is 100 m as shown in the figure. A vertical borehole through the fault zone measured its vertical thickness to be 100 m. The true thickness of the fault zone is _____ m. (Round off to two decimal places).



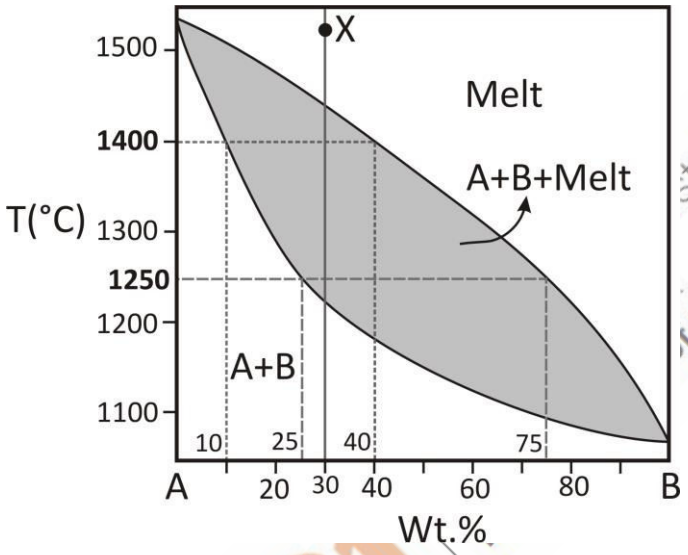
Q. 51 – Q. 60 carry two marks each.

Q.51 In an oblique slip fault with an attitude 000, 30° E, the net slip vector has a length of 20 m and a rake of 30° S on the fault plane. The displacement of a horizontal bed along the fault trace in a plane perpendicular to the strike of the fault is _____ m. (Answer in integer).

Q.52 If the activity of a radioactive mineral falls from 800 counts/s to 500 counts/s in 80 minutes, half-life of the mineral is _____ minutes. (Round off to two decimal places).

Q.53 In a laboratory experiment, water discharge through a porous rock sample in 2 hours was 10 cm^3. The cylindrical rock sample is 10 cm long and has a diameter of 50 mm. If the discharge occurred at a constant head of 300 cm, the coefficient of permeability of the rock sample is _____ $\times 10^{-6}$ cm/s. (Round off to two decimal places).

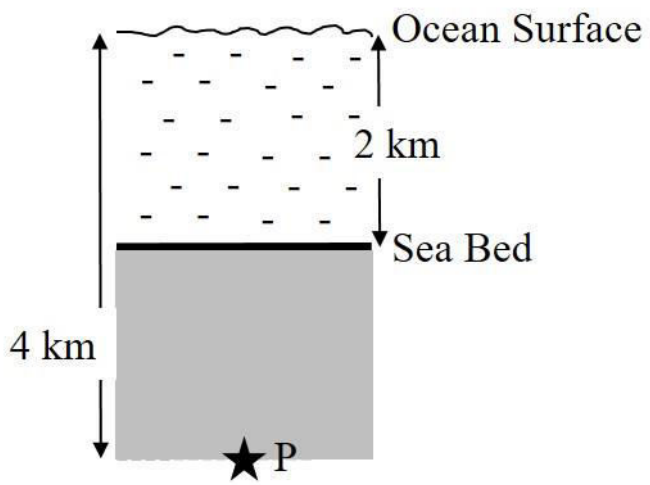
Q.54 The following diagram represents a binary phase diagram for the system A–B at atmospheric pressure. If 'X' is the initial composition of melt, then the amount of melt that converts to solid when the magma cools from 1400 °C to 1250 °C is _____ %. (Round off to two decimal places).



Q.55 The following table shows modal abundance and mineral composition data of a plutonic igneous rock. The amount of SiO_2 in bulk composition of the rock is _____ %. (Round off to two decimal places).

Mineral	Mode (%)	SiO_2 (wt. %)	CaO (wt. %)	MgO (wt. %)
Olivine	45	34	-	66
Clinopyroxene	35	55	25	20
Orthopyroxene	20	58	-	42

Q.56 Refer to the schematic sketch given (not to scale). Assume average saturated density of oceanic crustal rocks = 3200 kg/m^3, density of ocean water = 1000 kg/m^3, and acceleration due to gravity = 10 m/s^2. The overburden pressure at a point (P) located 2 km below seabed and 4 km below the ocean surface is _____ MPa. (Answer in integer).

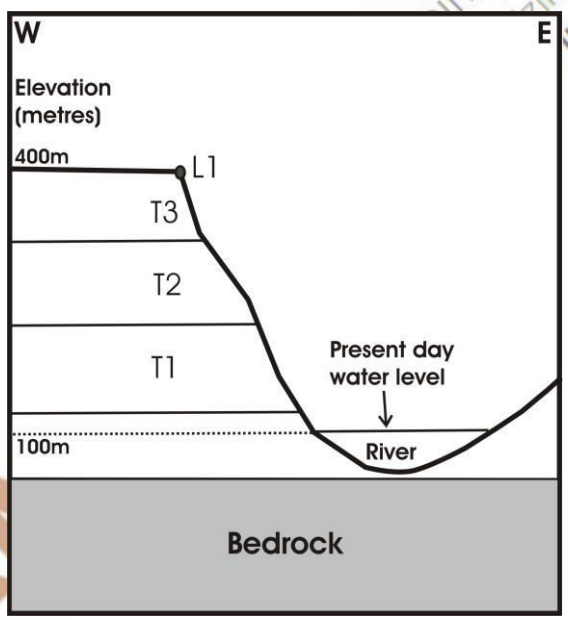


Q.57 If the indices of refraction of a uniaxial section are $\varepsilon = 1.653$ and $\omega = 1.544$, and the retardation between the two rays is 550 nm, then the thickness of the section is _____ μm. (Round off to two decimal places).

Q.58 A crystal has lattice parameters of *a* : 4.26 Å, *b* : 10 Å and *c* : 3.44 Å, respectively. A plane intercepts on the *a*, *b* and *c* axes at 2.13 Å, 10 Å and 1.72 Å, respectively. The Miller Indices for the plane, written as an integer, is _____(Answer in integer).

Q.59 In the uvavorite garnet ($Ca_3^{+2}Cr_2^{+3}Si_3^{+4}O_{12}$), Ca is in cubic coordination, Cr is in octahedral coordination and Si is in tetrahedral coordination. The electrostatic bond strength of the Ca^{2+} central ion is _____. (Round off to two decimal places).

Q.60 In a structurally controlled fluvial setting, an asymmetric flight of river terraces T1, T2, T3 shown in the figure was sampled at location L1. The age of the sample at L1 was 30 ka (kiloyears). Assuming that the terraces were formed entirely due to deformation related uplift, the average uplift rate in the past 30 ka in the region was _____ mm/yr. (Answer in integer).



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2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Special Instructions/ Useful Data	
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R}, i = 1, 2, \ldots, n\}$
$\det(M)$	Determinant of a matrix M
I_n	Identity matrix of order $n \times n, n = 2, 3, \ldots$
g'	First derivative of a real valued function g
g''	Second derivative of a real valued function g
F^c	Complement of an event F
$P(F)$	Probability of an event F
$P(F\|G)$	Conditional probability of an event F given the occurrence of event G
$X \sim f$	The probability density/mass function of the random variable X is f
$E(X)$	Expectation of a random variable X
$\mathrm{Var}(X)$	Variance of a random variable X
$U(a, b)$	Continuous uniform distribution on the interval $(a, b), -\infty < a < b < \infty$
$\mathrm{Poisson}(\theta)$	Poisson distribution with mean $\theta, \theta \in (0, \infty)$
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2, $\mu \in (-\infty, \infty), \sigma^2 \in (0, \infty)$
χ^2_n	Central chi-square distribution with n degrees of freedom, $n = 1,2, \ldots$
$F_{m,n}$	F distribution with (m, n) degrees of freedom, $m, n = 1,2, \ldots$
$\Phi(\cdot)$	Distribution function of $N(0, 1)$
$\|x\|$	Absolute value of x
MLE	Maximum Likelihood Estimator
$n!$	$n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1, \ n = 1, 2, 3, \ldots,$ and $0! = 1$
$\binom{n}{k}$	$\frac{n!}{k!\,(n-k)!}, k = 0,1,2, \ldots, n$ and $n = 1, 2, \ldots$; $\binom{0}{0} = 1$
$\max\{a_1, a_2, \ldots, a_n\}$	Maximum of real numbers $a_1, a_2, \ldots, a_n \ (n \geq 2)$
$\min\{a_1, a_2, \ldots, a_n\}$	Minimum of real numbers $a_1, a_2, \ldots, a_n \ (n \geq 2)$
$\ln x$	Natural logarithm of x

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The value of the limit

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right)^{\frac{1}{n}}$$

is equal to

(A) e (B) $\frac{1}{e}$ (C) $\frac{3}{e}$ (D) $\frac{4}{e}$

Q.2 Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by

$$f(x)=x^7+5x^3+11x+15,\ x\in\mathbb{R}.$$

Then, which of the following statements is **TRUE**?

(A) f is both one-one and onto

(B) f is neither one-one nor onto

(C) f is one-one but NOT onto

(D) f is onto but NOT one-one

Q.3 The value of the limit

$$\lim_{x\to 0}\frac{e^{-3x}-e^{x}+4x}{5(1-\cos x)}$$

is equal to

(A) 1 (B) 0 (C) $\frac{2}{5}$ (D) $\frac{8}{5}$

Q.4 The value of the limit

$$\lim_{n\to\infty}\sum_{k=0}^{n}\binom{2n}{k}\frac{1}{4^n}$$

is equal to

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) $\frac{1}{4}$

Q.5 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables with probability density function

$$f(x)=\begin{cases}1, & \text{if } 0<x<1\\ 0, & \text{otherwise}\end{cases}.$$

Then, the value of the limit

$$\lim_{n\to\infty}P\left(-\frac{1}{n}\sum_{i=1}^{n}\ln X_i\leq 1+\frac{1}{\sqrt{n}}\right)$$

is equal to

(A) $\frac{1}{2}$ (B) $\Phi(1)$ (C) 0 (D) $\Phi(2)$

Q.6 Let X be a $U(0,1)$ random variable and let $Y=X^2$. If ρ is the correlation coefficient between the random variables X and Y, then $48\,\rho^2$ is equal to

(A) 48 (B) 45 (C) 35 (D) 30

Q.7 Let M be a 3×3 real matrix. Let $\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}$ and $\begin{pmatrix}0\\-1\\\alpha\end{pmatrix}$ be the eigenvectors of M corresponding to three distinct eigenvalues of M, where α is a real number. Then, which of the following is **NOT** a possible value of α?

(A) 0 (B) 1 (C) -2 (D) 2

Q.8 If the series $\sum_{n=1}^{\infty} a_n$ converges absolutely, then which of the following series diverges?

(A) $\sum_{n=1}^{\infty} |a_{2n}|$

(B) $\sum_{n=1}^{\infty} \frac{a_n + a_{n+1}}{2}$

(C) $\sum_{n=1}^{\infty} (a_n)^3$

(D) $\sum_{n=2}^{\infty} \left(\frac{1}{(\ln n)^2} + a_n \right)$

Q.9 There are three urns, labeled, Urn 1, Urn 2 and Urn 3. Urn 1 contains 2 white balls and 2 black balls, Urn 2 contains 1 white ball and 3 black balls and Urn 3 contains 3 white balls and 1 black ball. Consider two coins with probability of obtaining head in their single trials as 0.2 and 0.3. The two coins are tossed independently once, and an urn is selected according to the following scheme: Urn 1 is selected if 2 heads are obtained; Urn 3 is selected if 2 tails are obtained; otherwise Urn 2 is selected. A ball is then drawn at random from the selected urn. Then

$$P(\text{Urn 1 is selected} \mid \text{the ball drawn is white})$$

is equal to

(A) $\frac{6}{109}$ (B) $\frac{12}{109}$ (C) $\frac{1}{18}$ (D) $\frac{1}{9}$

Q.10 Let X be a random variable with probability density function

$$f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty.$$

Then, which of the following statements is **FALSE**?

(A) $E(X\,|X|) = 0$

(B) $E(X\,|X|^2) = 0$

(C) $E\left(|X| \sin\left(\frac{X}{|X|}\right)\right) = 0$

(D) $E\left(|X| \sin^2\left(\frac{X}{|X|}\right)\right) = 0$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \dfrac{y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}.$$

Let $f_x(x,y)$ and $f_y(x,y)$ denote the first order partial derivatives of $f(x,y)$ with respect to x and y, respectively, at the point (x,y). Then, which of the following statements is **FALSE**?

(A) $f_x(x,y)$ exists and is bounded at every $(x,y) \in \mathbb{R}^2$

(B) $f_y(x,y)$ exists and is bounded at every $(x,y) \in \mathbb{R}^2$

(C) $f_y(0,0)$ exists and $f_y(x,y)$ is continuous at $(0,0)$

(D) f is NOT differentiable at $(0,0)$

Q.12 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed $N(0,1)$ random variables. Then,

$$\lim_{n\to\infty} P\left(\frac{\sum_{i=1}^{n} X_i^4 - 3n}{\sqrt{32\,n}} \leq \sqrt{6}\right)$$

is equal to

(A) $\frac{1}{2}$ (B) $\Phi(\sqrt{2}\,)$ (C) 0 (D) $\Phi(1)$

Q.13 Consider a sequence of independent Bernoulli trials with probability of success in each trial as $\frac{1}{3}$. The probability that three successes occur before four failures is equal to

(A) $\frac{179}{243}$ (B) $\frac{179}{841}$ (C) $\frac{233}{729}$ (D) $\frac{179}{1215}$

Q.14 Let X and Y be independent $N(0,1)$ random variables and $Z = \frac{|X|}{|Y|}$. Then, which of the following expectations is finite?

(A) $E\left(\frac{1}{\sqrt{Z}}\right)$ (B) $E\left(Z\sqrt{Z}\right)$ (C) $E(Z)$ (D) $E\left(\frac{1}{Z\sqrt{Z}}\right)$

Q.15 Consider three coins having probabilities of obtaining head in a single trial as $\frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$, respectively. A player selects one of these three coins at random (each coin is equally likely to be selected). If the player tosses the selected coin five times independently, then the probability of obtaining two tails in five tosses is equal to

(A) $\frac{85}{384}$ (B) $\frac{255}{384}$

(C) $\frac{125}{384}$ (D) $\frac{64}{384}$

Q.16 Let X be a random variable having the probability density function

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

Define $Y = [X]$, where $[X]$ denotes the largest integer not exceeding X. Then, $E(Y^2)$ is equal to

(A) $\frac{e(e+1)}{e-1}$ (B) $\frac{e+1}{(e-1)^2}$

(C) $\frac{e(e+1)^2}{e-1}$ (D) $\frac{(e+1)^2}{(e-1)^2}$

Q.17 Let X be a continuous random variable having the moment generating function

$$M(t) = \frac{e^t - 1}{t}, \qquad t \neq 0.$$

Let $\alpha = P(48\,X^2 - 40\,X + 3 > 0)$ and $\beta = P((\ln X)^2 + 2\,\ln X - 3 > 0)$.

Then, the value of $\alpha - 2\ln\beta$ is equal to

(A) $\frac{10}{3}$ (B) $\frac{19}{3}$ (C) $\frac{13}{3}$ (D) $\frac{17}{3}$

Q.18 Let $X_1, X_2, \ldots, X_n$ $(n \geq 3)$ be a random sample from Poisson(θ), where $\theta \in (0, \infty)$ is unknown and let

$$T = \sum_{i=1}^{n} X_i.$$

Then, the uniformly minimum variance unbiased estimator of $e^{-2\theta}\theta^3$

(A) is $\frac{T}{n}\left(\frac{T}{n} - 1\right)\left(\frac{T}{n} - 2\right)\left(1 - \frac{2}{n}\right)^{T-3}$

(B) is $\frac{T(T-1)(T-2)(n-2)^{T-3}}{n^T}$

(C) does NOT exist

(D) is $e^{-\frac{2T}{n}}\left(\frac{T}{n}\right)^3$

Q.19 Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from $U(\theta - 5, \theta + 5)$, where $\theta \in (0, \infty)$ is unknown. Let $T = \max\{X_1, X_2, \ldots, X_n\}$ and $U = \min\{X_1, X_2, \ldots, X_n\}$. Then, which of the following statements is **TRUE**?

(A) $\frac{T+U}{2}$ is the unique MLE of θ

(B) $\frac{2}{T+U}$ is an MLE of $\frac{1}{\theta}$

(C) MLE of $\frac{1}{\theta}$ does NOT exist

(D) $U + 8$ is an MLE of θ

Q.20 Let X and Y be random variables having chi-square distributions with 6 and 3 degrees of freedom, respectively. Then, which of the following statements is **TRUE**?

(A) $P(X > 0.7) > P(Y > 0.7)$

(B) $P(X > 0.7) < P(Y > 0.7)$

(C) $P(X > 3) < P(Y > 3)$

(D) $P(X < 6) > P(Y < 6)$

Q.21 Let (X, Y) be a random vector with joint moment generating function

$$M(t_1, t_2) = \frac{1}{(1-(t_1+t_2))(1-t_2)}, \qquad -\infty < t_1 < \infty, -\infty < t_2 < \min\{1, 1-t_1\}.$$

Let $Z = X + Y$. Then, $\mathrm{Var}(Z)$ is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Q.22 Let X be a continuous random variable with distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ a\,x^2, & \text{if } 0 \le x < 2, \\ 1, & \text{if } x \ge 2 \end{cases}$$

for some real constant a. Then, $E(X)$ is equal to

(A) $\frac{4}{3}$ (B) $\frac{1}{4}$ (C) 1 (D) 0

Q.23 Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with probability density function

$$f(x;\theta) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0, \infty)$ is unknown. Let $\alpha \in (0,1)$ be fixed and let β be the power of the most powerful test of size α for testing $H_0: \theta = 1$ against $H_1: \theta = 2$.

Consider the critical region

$$R = \left\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^{n} x_i > \frac{1}{2}\chi^2_{2n}(1-\alpha)\right\},$$

where for any $\gamma \in (0,1)$, $\chi^2_{2n}(\gamma)$ is a fixed point such that $P\left(\chi^2_{2n} > \chi^2_{2n}(\gamma)\right) = \gamma$. Then, the critical region R corresponds to the

(A) most powerful test of size α for testing $H_0: \theta = 1$ against $H_1: \theta = 2$

(B) most powerful test of size $1-\alpha$ for testing $H_0^*: \theta = 2$ against $H_1^*: \theta = 1$

(C) most powerful test of size β for testing $H_0^*: \theta = 2$ against $H_1^*: \theta = 1$

(D) most powerful test of size $1-\beta$ for testing $H_0^*: \theta = 2$ against $H_1^*: \theta = 1$

Q.24 Let

$$S = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{k} \left(\frac{1}{4}\right)^k \text{ and } T = \sum_{k=1}^{\infty} \frac{1}{k} \left(\frac{1}{5}\right)^k.$$

Then, which of the following statements is **TRUE**?

(A) $S - T = 0$ (B) $5S - 4T = 0$

(C) $4S - 5T = 0$ (D) $16S - 25T = 0$

Q.25 Let E_1, E_2, E_3 and E_4 be four events such that

$$P(E_i|E_4) = \frac{2}{3}, i = 1,2,3;\ P\left(E_i \cap E_j^c \middle| E_4\right) = \frac{1}{6}, i,j = 1,2,3; i \neq j \text{ and } P(E_1 \cap E_2 \cap E_3^c|E_4) = \frac{1}{6}$$

Then, $P(E_1 \cup E_2 \cup E_3|E_4)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{7}{12}$

Q.26 Let $a_1 = 5$ and define recursively

$$a_{n+1} = 3^{\frac{1}{4}} (a_n)^{\frac{3}{4}}, \qquad n \geq 1.$$

Then, which of the following statements is **TRUE**?

(A) $\{a_n\}$ is monotone increasing, and $\lim_{n\to\infty} a_n = 3$

(B) $\{a_n\}$ is monotone decreasing, and $\lim_{n\to\infty} a_n = 3$

(C) $\{a_n\}$ is non-monotone, and $\lim_{n\to\infty} a_n = 3$

(D) $\{a_n\}$ is decreasing, and $\lim_{n\to\infty} a_n = 0$

Q.27 Consider the problem of testing $H_0: X \sim f_0$ against $H_1: X \sim f_1$ based on a sample of size 1, where

$$f_0(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \text{ and } f_1(x) = \begin{cases} 2 - 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Then, the probability of Type II error of the most powerful test of size $\alpha = 0.1$ is equal to

(A) 0.81 (B) 0.91 (C) 0.1 (D) 1

Q.28 For $a \in \mathbb{R}$, consider the system of linear equations

$$\begin{aligned} a\,x + a\,y &= a + 2 \\ x + a\,y + (a-1)z &= a - 4 \\ a\,x + a\,y + (a-2)z &= -8 \end{aligned}$$

in the unknowns x, y and z. Then, which of the following statements is **TRUE**?

(A) The given system has a unique solution for $a = 1$

(B) The given system has infinitely many solutions for $a = 2$

(C) The given system has a unique solution for $a = -2$

(D) The given system has infinitely many solutions for $a = -2$

Q.29 Let $\{a_n\}_{n \geq 1}$ be a sequence of real numbers such that $a_n \geq 1$, for all $n \geq 1$. Then, which of the following conditions imply the divergence of $\{a_n\}_{n \geq 1}$?

(A) $\{a_n\}_{n \geq 1}$ is non-increasing

(B) $\sum_{n=1}^{\infty} b_n$ converges, where $b_1 = a_1$ and $b_n = a_{n+1} - a_n$, for all $n > 1$

(C) $\lim_{n \to \infty} \frac{a_{2n+1}}{a_{2n}} = \frac{1}{2}$

(D) $\left\{\sqrt{a_n}\right\}_{n \geq 1}$ converges

Q.30 Let E_1, E_2 and E_3 be three events such that $P(E_1) = \frac{4}{5}, P(E_2) = \frac{1}{2}$ and $P(E_3) = \frac{9}{10}$. Then, which of the following statements is **FALSE**?

(A) $P(E_1 \cup E_2 \cup E_3) \geq \frac{9}{10}$

(B) $P(E_2 \cap E_3) \leq \frac{1}{2}$

(C) $P(E_1 \cap E_2 \cap E_3) \leq \frac{1}{6}$

(D) $P(E_1 \cup E_2) \geq \frac{4}{5}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Consider the linear system $A\underline{x} = \underline{b}$, where A is an $m \times n$ matrix, $\underline{x}$ is an $n \times 1$ vector of unknowns and $\underline{b}$ is an $m \times 1$ vector. Further, suppose there exists an $m \times 1$ vector $\underline{c}$ such that the linear system $A\underline{x} = \underline{c}$ has **NO** solution. Then, which of the following statements is/are necessarily **TRUE**?

(A) If $m \leq n$ and $\underline{d}$ is the first column of A, then the linear system $A\underline{x} = \underline{d}$ has a unique solution

(B) If $m \geq n$, then $\text{Rank}(A) < n$

(C) $\text{Rank}(A) < m$

(D) If $m > n$, then the linear system $A\underline{x} = \underline{0}$ has a solution other than $\underline{x} = \underline{0}$

Q.32 Let A be a 3×3 real matrix such that $A \neq I_3$ and the sum of the entries in each row of A is 1. Then, which of the following statements is/are necessarily **TRUE**?

(A) $A - I_3$ is an invertible matrix

(B) The set $\{\underline{x} \in \mathbb{R}^3 : (A - I_3)\underline{x} = \underline{0}\}$ has at least two elements ($\underline{x}$ is a column vector)

(C) The characteristic polynomial, $p(\lambda)$, of $A + 2A^2 + A^3$ has $(\lambda - 4)$ as a factor

(D) A cannot be an orthogonal matrix

Q.33 Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, 1)$, where $\theta \in (-\infty, \infty)$ is unknown. Consider the problem of testing $H_0: \theta \leq 0$ against $H_1: \theta > 0$. Let $\beta(\theta)$ denote the power function of the likelihood ratio test of size α $(0 < \alpha < 1)$ for testing H_0 against H_1. Then, which of the following statements is/are **TRUE**?

(A) $\beta(\theta) > \beta(0)$, for all $\theta > 0$

(B) $\beta(\theta) < \beta(0)$, for all $\theta > 0$

(C) The critical region of the likelihood test of size α is

$$\left\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n: \sqrt{n}\ \frac{\sum_{i=1}^n x_i}{n} > \tau_{\alpha/2}\right\},$$

where $\tau_{\alpha/2}$ is a fixed point such that $P(Z > \tau_{\alpha/2}) = \frac{\alpha}{2}$, $Z \sim N(0, 1)$.

(D) The critical region of the likelihood test of size α is

$$\left\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n: \sqrt{n}\ \frac{\sum_{i=1}^n x_i}{n} < \tau_{\alpha}\right\},$$

where τ_{α} is a fixed point such that $P(Z > \tau_{\alpha}) = \alpha$, $Z \sim N(0, 1)$.

Q.34 Consider the function

$$f(x, y) = 3x^2 + 4xy + y^2, \qquad (x, y) \in \mathbb{R}^2.$$

If $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$, then which of the following statements is/are **TRUE**?

(A) The maximum value of f on S is $3 + \sqrt{5}$

(B) The minimum value of f on S is $3 - \sqrt{5}$

(C) The maximum value of f on S is $2 + \sqrt{5}$

(D) The minimum value of f on S is $2 - \sqrt{5}$

Q.35 Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Then, which of the following statements is/are necessarily **TRUE**?

(A) f'' is continuous

(B) If $f'(0) = f'(1)$, then $f''(x) = 0$ has a solution in $(0, 1)$

(C) f' is bounded on $[8,\ 10]$

(D) f'' is bounded on $(0,\ 1)$

Q.36 Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be independent and identically distributed random variables with probability density function

$$f(x) = \begin{cases} \frac{1}{x^2}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}.$$

Then, which of the following random variables has/have finite expectation?

(A) X_1 (B) $\frac{1}{X_2}$ (C) $\sqrt{X_1}$ (D) $\min\{X_1, \ldots, X_n\}$

Q.37 A sample of size n is drawn randomly (without replacement) from an urn containing $5n^2$ balls, of which $2n^2$ are red balls and $3n^2$ are black balls. Let X_n denote the number of red balls in the selected sample. If $\ell = \lim_{n\to\infty} \frac{E(X_n)}{n}$ and $m = \lim_{n\to\infty} \frac{\mathrm{Var}(X_n)}{n}$, then which of the following statements is/are **TRUE**?

(A) $\ell + m = \frac{16}{25}$ (B) $\ell - m = \frac{3}{25}$

(C) $\ell m = \frac{14}{125}$ (D) $\frac{\ell}{m} = \frac{5}{3}$

Q.38 Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{2\theta}, & -\theta \leq x \leq \theta \\ 0, & |x| > \theta \end{cases},$$

where $\theta \in (0, \infty)$ is unknown. If $R = \min\{X_1, X_2, \ldots, X_n\}$ and $S = \max\{X_1, X_2, \ldots, X_n\}$, then which of the following statements is/are **TRUE**?

(A) (R, S) is jointly sufficient for θ

(B) S is an MLE of θ

(C) $\max\{|X_1|, |X_2|, \ldots, |X_n|\}$ is a complete and sufficient statistic for θ

(D) Distribution of $\frac{R}{S}$ does NOT depend on θ

Q.39 Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function

$$f(x;\theta) = \begin{cases} \frac{3x^2}{\theta} e^{-x^3/\theta}, & x > 0 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0, \infty)$ is unknown.

If $T = \sum_{i=1}^{n} X_i^3$, then which of the following statements is/are **TRUE**?

(A) $\frac{n-1}{T}$ is the unique uniformly minimum variance unbiased estimator of $\frac{1}{\theta}$

(B) $\frac{n}{T}$ is the unique uniformly minimum variance unbiased estimator of $\frac{1}{\theta}$

(C) $(n-1)\sum_{i=1}^{n} \frac{1}{X_i^3}$ is the unique uniformly minimum variance unbiased estimator of $\frac{1}{\theta}$

(D) $\frac{n}{T}$ is the MLE of $\frac{1}{\theta}$

Q.40 Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0, \infty)$ is unknown. Then, which of the following statements is/are **TRUE**?

(A) Cramer-Rao lower bound, based on $X_1, X_2, \ldots, X_n$, for the estimand θ^3 is $9\frac{\theta^6}{n}$

(B) Cramer-Rao lower bound, based on $X_1, X_2, \ldots, X_n$, for the estimand θ^3 is $\frac{\theta^2}{n}$

(C) There does NOT exist any unbiased estimator of $\frac{1}{\theta}$ which attains the Cramer-Rao lower bound

(D) There exists an unbiased estimator of $\frac{1}{\theta}$ which attains the Cramer-Rao lower bound

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let α, β and γ be the eigenvalues of $M = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 3 \\ -1 & 2 & 2 \end{bmatrix}$. If $\gamma = 1$ and $\alpha > \beta$, then the value of $2\alpha + 3\beta$ is _____.

Q.42 Let $M = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$ be a 2×2 matrix. If $\alpha = \det(M^4 - 6I_2)$, then the value of α^2 is _____.

Q.43 Let $S = \{(x, y) \in \mathbb{R}^2 : 2 \leq x \leq y \leq 4\}$. Then, the value of the integral

$$\iint_S \frac{1}{4 - x}\, dx\, dy$$

is _____.

Q.44 Let $A = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq y \leq z \leq 1\}$. Let α be the value of the integral

$$\iiint_A x\, y\, z\; dx\, dy\, dz.$$

Then, $384\,\alpha$ is equal to _____.

Q.45 Let f_0 and f_1 be the probability mass functions given by

x	1	2	3	4	5	6
$f_0(x)$	0.1	0.1	0.1	0.1	0.1	0.5
$f_1(x)$	0.1	0.1	0.2	0.2	0.2	0.2

Consider the problem of testing the null hypothesis $H_0: X \sim f_0$ against $H_1: X \sim f_1$ based on a single sample X. If α and β, respectively, denote the size and power of the test with critical region $\{x \in \mathbb{R} : x > 3\}$, then $10(\alpha + \beta)$ is equal to _____.

Q.46 Let $5, 10, 4, 15, 6$ be an observed random sample of size 5 from a distribution with probability density function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta \\ 0, & \text{otherwise} \end{cases}$$

$\theta \in (-\infty,\ 3]$ is unknown. Then, the maximum likelihood estimate of θ based on the observed sample is equal to _____.

Q.47 Let

$$\alpha = \lim_{n\to\infty} \sum_{m=n^2}^{2n^2} \frac{1}{\sqrt{5\,n^4 + n^3 + m}}$$

Then, $10\sqrt{5}\,\alpha$ is equal to _____.

Q.48 Let X be a random variable having the probability density function

$$f(x) = \frac{1}{8\sqrt{2\pi}}\left(2\,e^{-\frac{x^2}{2}} + 3\,e^{-\frac{x^2}{8}}\right), \quad -\infty < x < \infty.$$

Then, $4\,E(X^4)$ is equal to _____.

Q.49 Let X be a random variable with moment generating function

$$M_X(t) = \frac{1}{12} + \frac{1}{6}e^{t} + \frac{1}{3}e^{2t} + \frac{1}{4}e^{-t} + \frac{1}{6}e^{-2t}, t \in \mathbb{R}.$$

Then, $8\,E(X)$ is equal to _____.

Q.50 Let β denote the length of the curve $y = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

Then, the value of $3\sqrt{2}\,(e^{\beta} - 1)$ is equal to _____.

Q.51 – Q. 60 carry two marks each.

Q.51 Let $S \subseteq \mathbb{R}^2$ be the region bounded by the parallelogram with vertices at the points $(1,0), (3,2), (3,5)$ and $(1,3)$. Then, the value of the integral $\iint_S (x + 2y)\, dx\, dy$ is equal to _____.

Q.52 Let $A = \left\{ (x,y) \in \mathbb{R}^2 : x^2 - \frac{1}{2\sqrt{\pi}} < y < x^2 + \frac{1}{2\sqrt{\pi}} \right\}$ and let the joint probability density function of (X,Y) be

$$f(x,y) = \begin{cases} e^{-(x-1)^2}, & (x,y) \in A \\ 0, & \text{otherwise} \end{cases}$$

Then, the covariance between the random variables X and Y is equal to _____.

Q.53 Let X_1 and X_2 be independent $N(0,1)$ random variables. Define

$$\text{sgn}(u) = \begin{cases} -1, & \text{if } u < 0 \\ 0, & \text{if } u = 0 \\ 1, & \text{if } u > 0 \end{cases}$$

Let $Y_1 = X_1 \text{ sgn}(X_2)$ and $Y_2 = X_2 \text{ sgn}(X_1)$. If the correlation coefficient between Y_1 and Y_2 is α, then $\pi\alpha$ is equal to _____.

Q.54 Let

$$a_n = \sum_{k=2}^{n} \binom{n}{k} \frac{2^k (n-2)^{n-k}}{n^n}, \quad n = 2, 3, \ldots.$$

Then, $e^2 \lim_{n\to\infty} (1 - a_n)$ is equal to _____.

Q.55 Let E_1, E_2, E_3 and E_4 be four independent events such that $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{4}$ and $P(E_4) = \frac{1}{5}$. Let p be the probability that at most two events among E_1, E_2, E_3 and E_4 occur. Then, $240\, p$ is equal to _____.

Q.56 Let the random vector (X,Y) have the joint probability mass function

$$f(x,y) = \begin{cases} \binom{10}{x}\binom{5}{y}\left(\frac{1}{4}\right)^{x-y+5}\left(\frac{3}{4}\right)^{y-x+10}, & x = 0,1,\ldots,10;\ y = 0,1,\ldots,5 \\ 0, & \text{otherwise} \end{cases}$$

Let $Z = Y - X + 10$. If $\alpha = E(Z)$ and $\beta = \text{Var}(Z)$, then $8\alpha + 48\beta$ is equal to _____.

Q.57 Let $S = \left\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq \pi,\ \min\{\sin x, \cos x\} \leq y \leq \max\{\sin x, \cos x\}\right\}$.

If α is the area of S, then the value of $2\sqrt{2}\,\alpha$ is equal to _____.

Q.58 The number of real roots of the polynomial

$$f(x) = x^{11} - 13\,x + 5$$

is _____.

Q.59 Let $\alpha = \lim_{n\to\infty} \left(1 + n \sin\frac{3}{n^2}\right)^{2n}$. Then, $\ln \alpha$ is equal to _____.

Q.60 Let $\phi: (-1,\ 1) \to \mathbb{R}$ be defined by

$$\phi(x) = \int_{x^7}^{x^4} \frac{1}{1+t^3}\, dt.$$

If $\alpha = \lim_{x\to 0} \frac{\phi(x)}{e^{2x^4} - 1}$, then $42\,\alpha$ is equal to _____.

END OF THE QUESTION PAPER

$\mathbb{N} = \{1, 2, \ldots\}$.

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.

$\mathbb{Q} =$ the set of rational numbers.

$\mathbb{R} =$ the set of real numbers.

$\mathbb{R}^n =$ the n-dimensional real space with the Euclidean topology.

$\mathbb{C} =$ the set of complex numbers.

$\mathbb{C}^n =$ the n-dimensional complex space with the Euclidean topology.

$M_n(\mathbb{R}), M_n(\mathbb{C}) =$ the vector space of $n \times n$ real or complex matrices, respectively.

$f', \ f'' =$ the first and second derivatives of the function f, respectively.

$f^{(n)} =$ the nth. derivative of the function f.

$\int_C$ stands for the line integral over the curve C.

$I_n =$ the $n \times n$ identity matrix.

$A^{-1} =$ the inverse of an invertible matrix A.

$S_n =$ the permutation group on n symbols.

$\hat{i} = (1, 0, 0), \hat{j} = (0, 1, 0)$ and $\hat{k} = (0, 0, 1)$.

$\ln x =$ the natural logarithm of x (to the base e).

$|X| =$ the number of elements in a finite set X.

$\mathbb{Z}_n =$ the additive group of integers modulo n.

$\arctan(x)$ denotes the unique $\theta \in (-\pi/2, \pi/2)$ such that $\tan\theta = x$.

All vector spaces are over the real or complex field, unless otherwise stated.

SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

Q. 1 Let $0 < \alpha < 1$ be a real number. The number of differentiable functions $y : [0, 1] \to [0, \infty)$, having continuous derivative on $[0, 1]$ and satisfying

$$\begin{aligned} y'(t) &= (y(t))^{\alpha}, \quad t \in [0, 1], \\ y(0) &= 0, \end{aligned}$$

is

(A) exactly one.

(B) exactly two.

(C) finite but more than two.

(D) infinite.

Q. 2 Let $P : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $P(x) > 0$ for all $x \in \mathbb{R}$. Let y be a twice differentiable function on $\mathbb{R}$ satisfying $y''(x) + P(x)y'(x) - y(x) = 0$ for all $x \in \mathbb{R}$. Suppose that there exist two real numbers a, b $(a < b)$ such that $y(a) = y(b) = 0$. Then

(A) $y(x) = 0$ for all $x \in [a, b]$.

(B) $y(x) > 0$ for all $x \in (a, b)$.

(C) $y(x) < 0$ for all $x \in (a, b)$.

(D) $y(x)$ changes sign on (a, b).

Q. 3 Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $f(x) = f(x + 1)$ for all $x \in \mathbb{R}$. Then

(A) f is not necessarily bounded above.

(B) there exists a unique $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$.

(C) there is no $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$.

(D) there exist infinitely many $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$.

Q. 4 Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that for all $x \in \mathbb{R}$,

$$\int_0^1 f(xt)\, dt = 0. \qquad (*)$$

Then

(A) f must be identically 0 on the whole of $\mathbb{R}$.

(B) there is an f satisfying $(*)$ that is identically 0 on $(0, 1)$ but not identically 0 on the whole of $\mathbb{R}$.

(C) there is an f satisfying $(*)$ that takes both positive and negative values.

(D) there is an f satisfying $(*)$ that is 0 at infinitely many points, but is not identically zero.

Q. 5 Let p and t be positive real numbers. Let D_t be the closed disc of radius t centered at $(0, 0)$, i.e., $D_t = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq t^2\}$. Define

$$I(p, t) = \iint_{D_t} \frac{dxdy}{(p^2 + x^2 + y^2)^p}.$$

Then $\lim_{t\to\infty} I(p, t)$ is finite

(A) only if $p > 1$. (B) only if $p = 1$.

(C) only if $p < 1$. (D) for no value of p.

Q. 6 How many elements of the group $\mathbb{Z}_{50}$ have order 10?

(A) 10 (B) 4 (C) 5 (D) 8

Q. 7 For every $n \in \mathbb{N}$, let $f_n : \mathbb{R} \to \mathbb{R}$ be a function. From the given choices, pick the statement that is the negation of

"For every $x \in \mathbb{R}$ and for every real number $\epsilon > 0$, there exists an integer $N > 0$ such that $\sum_{i=1}^{p} |f_{N+i}(x)| < \epsilon$ for every integer $p > 0$."

(A) For every $x \in \mathbb{R}$ and for every real number $\epsilon > 0$, there does not exist any integer $N > 0$ such that $\sum_{i=1}^{p} |f_{N+i}(x)| < \epsilon$ for every integer $p > 0$.

(B) For every $x \in \mathbb{R}$ and for every real number $\epsilon > 0$, there exists an integer $N > 0$ such that $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \epsilon$ for some integer $p > 0$.

(C) There exists $x \in \mathbb{R}$ and there exists a real number $\epsilon > 0$ such that for every integer $N > 0$, there exists an integer $p > 0$ for which the inequality $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \epsilon$ holds.

(D) There exists $x \in \mathbb{R}$ and there exists a real number $\epsilon > 0$ such that for every integer $N > 0$ and for every integer $p > 0$ the inequality $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \epsilon$ holds.

Q. 8 Which one of the following subsets of $\mathbb{R}$ has a non-empty interior?

(A) The set of all irrational numbers in $\mathbb{R}$.

(B) The set $\{a \in \mathbb{R} : \sin(a) = 1\}$.

(C) The set $\{b \in \mathbb{R} : x^2 + bx + 1 = 0$ has distinct roots$\}$.

(D) The set of all rational numbers in $\mathbb{R}$.

Q. 9 For an integer $k \geq 0$, let P_k denote the vector space of all real polynomials in one variable of degree less than or equal to k. Define a linear transformation $T : P_2 \longrightarrow P_3$ by

$$Tf(x) = f''(x) + xf(x).$$

Which one of the following polynomials is not in the range of T?

(A) $x + x^2$ (B) $x^2 + x^3 + 2$ (C) $x + x^3 + 2$ (D) $x + 1$

Q. 10 Let $n > 1$ be an integer. Consider the following two statements for an arbitrary $n \times n$ matrix A with complex entries.

I. If $A^k = I_n$ for some integer $k \geq 1$, then all the eigenvalues of A are k^{th} roots of unity.

II. If, for some integer $k \geq 1$, all the eigenvalues of A are k^{th} roots of unity, then $A^k = I_n$.

Then

(A) both I and II are TRUE.

(B) I is TRUE but II is FALSE.

(C) I is FALSE but II is TRUE.

(D) neither I nor II is TRUE.

Q. 11 – Q. 30 carry two marks each.

Q. 11 Let $M_n(\mathbb{R})$ be the real vector space of all $n \times n$ matrices with real entries, $n \geq 2$. Let $A \in M_n(\mathbb{R})$. Consider the subspace W of $M_n(\mathbb{R})$ spanned by $\{I_n, A, A^2, \ldots\}$. Then the dimension of W over $\mathbb{R}$ is necessarily

(A) ∞. (B) n^2. (C) n. (D) at most n.

Q. 12 Let y be the solution of

$$(1+x)y''(x) + y'(x) - \frac{1}{1+x}y(x) = 0, \quad x \in (-1, \infty),$$
$$y(0) = 1, \; y'(0) = 0.$$

Then

(A) y is bounded on $(0, \infty)$.

(B) y is bounded on $(-1, 0]$.

(C) $y(x) \geq 2$ on $(-1, \infty)$.

(D) y attains its minimum at $x = 0$.

Q. 13 Consider the surface $S = \{(x, y, xy) \in \mathbb{R}^3 : x^2 + y^2 \leq 1\}$. Let $\vec{F} = y\hat{i} + x\hat{j} + \hat{k}$. If $\hat{n}$ is the continuous unit normal field to the surface S with positive z-component, then

$$\iint_S \vec{F} \cdot \hat{n} \, dS$$

equals

(A) $\frac{\pi}{4}$. (B) $\frac{\pi}{2}$. (C) π. (D) 2π.

Q. 14 Consider the following statements.

I. The group $(\mathbb{Q}, +)$ has no proper subgroup of finite index.

II. The group $(\mathbb{C} \setminus \{0\}, \cdot)$ has no proper subgroup of finite index.

Which one of the following statements is true?

(A) Both I and II are TRUE.

(B) I is TRUE but II is FALSE.

(C) II is TRUE but I is FALSE.

(D) Neither I nor II is TRUE.

Q. 15 Let $f : \mathbb{N} \to \mathbb{N}$ be a bijective map such that

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^2} < +\infty.$$

The number of such bijective maps is

(A) exactly one.

(B) zero.

(C) finite but more than one.

(D) infinite.

Q. 16 Define

$$S = \lim_{n\to\infty} \Big(1 - \frac{1}{2^2}\Big)\Big(1 - \frac{1}{3^2}\Big) \cdots \Big(1 - \frac{1}{n^2}\Big).$$

Then

(A) $S = 1/2$. (B) $S = 1/4$. (C) $S = 1$. (D) $S = 3/4$.

Q. 17 Let $f : \mathbb{R} \to \mathbb{R}$ be an infinitely differentiable function such that for all $a, b \in \mathbb{R}$ with $a < b$,

$$\frac{f(b) - f(a)}{b - a} = f'\Big(\frac{a + b}{2}\Big).$$

Then

(A) f must be a polynomial of degree less than or equal to 2.

(B) f must be a polynomial of degree greater than 2.

(C) f is not a polynomial.

(D) f must be a linear polynomial.

Q. 18 Consider the function

$$f(x) = \begin{cases} 1 & \text{if } x \in (\mathbb{R} \setminus \mathbb{Q}) \cup \{0\}, \\ 1 - \frac{1}{p} & \text{if } x = \frac{n}{p},\ n \in \mathbb{Z} \setminus \{0\},\ p \in \mathbb{N} \text{ and } \gcd(n, p) = 1. \end{cases}$$

Then

(A) all $x \in \mathbb{Q} \setminus \{0\}$ are strict local minima for f.

(B) f is continuous at all $x \in \mathbb{Q}$.

(C) f is not continuous at all $x \in \mathbb{R} \setminus \mathbb{Q}$.

(D) f is not continuous at $x = 0$.

Q. 19 Consider the family of curves $x^2 - y^2 = ky$ with parameter $k \in \mathbb{R}$. The equation of the orthogonal trajectory to this family passing through $(1, 1)$ is given by

(A) $x^3 + 3xy^2 = 4$.

(B) $x^2 + 2xy = 3$.

(C) $y^2 + 2x^2y = 3$.

(D) $x^3 + 2xy^2 = 3$.

Q. 20 Which one of the following statements is true?

(A) Exactly half of the elements in any even order subgroup of S_5 must be even permutations.

(B) Any abelian subgroup of S_5 is trivial.

(C) There exists a cyclic subgroup of S_5 of order 6.

(D) There exists a normal subgroup of S_5 of index 7.

Q. 21 Let $f : [0, 1] \to [0, \infty)$ be a continuous function such that

$$\big(f(t)\big)^2 < 1 + 2\int_0^t f(s)\, ds, \text{ for all } t \in [0, 1].$$

Then

(A) $f(t) < 1 + t$ for all $t \in [0, 1]$.

(B) $f(t) > 1 + t$ for all $t \in [0, 1]$.

(C) $f(t) = 1 + t$ for all $t \in [0, 1]$.

(D) $f(t) < 1 + \frac{t}{2}$ for all $t \in [0, 1]$.

Q. 22 Let A be an $n \times n$ invertible matrix and C be an $n \times n$ nilpotent matrix. If $X = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}$ is a $2n \times 2n$ matrix (each X_{ij} being $n \times n$) that commutes with the $2n \times 2n$ matrix $B = \begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}$, then

(A) X_{11} and X_{22} are necessarily zero matrices.

(B) X_{12} and X_{21} are necessarily zero matrices.

(C) X_{11} and X_{21} are necessarily zero matrices.

(D) X_{12} and X_{22} are necessarily zero matrices.

Q. 23 Let $D \subseteq \mathbb{R}^2$ be defined by $D = \mathbb{R}^2 \setminus \{(x, 0) : x \in \mathbb{R}\}$. Consider the function $f : D \to \mathbb{R}$ defined by

$$f(x, y) = x \sin \frac{1}{y}.$$

Then

(A) f is a discontinuous function on D.

(B) f is a continuous function on D and cannot be extended continuously to any point outside D.

(C) f is a continuous function on D and can be extended continuously to $D \cup \{(0, 0)\}$.

(D) f is a continuous function on D and can be extended continuously to the whole of $\mathbb{R}^2$.

Q. 24 Which one of the following statements is true?

(A) $(\mathbb{Z}, +)$ is isomorphic to $(\mathbb{R}, +)$.

(B) $(\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Q}, +)$.

(C) $(\mathbb{Q}/\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Q}/2\mathbb{Z}, +)$.

(D) $(\mathbb{Q}/\mathbb{Z}, +)$ is isomorphic to $(\mathbb{Q}, +)$.

Q. 25 Let y be a twice differentiable function on $\mathbb{R}$ satisfying

$$y''(x) = 2 + e^{-|x|}, \quad x \in \mathbb{R},$$
$$y(0) = -1, \quad y'(0) = 0.$$

Then

(A) $y = 0$ has exactly one root.

(B) $y = 0$ has exactly two roots.

(C) $y = 0$ has more than two roots.

(D) there exists an $x_0 \in \mathbb{R}$ such that $y(x_0) \geq y(x)$ for all $x \in \mathbb{R}$.

Q. 26 Let $f : [0, 1] \to [0, 1]$ be a non-constant continuous function such that $f \circ f = f$. Define

$$E_f = \{x \in [0, 1] : f(x) = x\}.$$

Then

(A) E_f is neither open nor closed.

(B) E_f is an interval.

(C) E_f is empty.

(D) E_f need not be an interval.

Q. 27 Let g be an element of S_7 such that g commutes with the element $(2, 6, 4, 3)$. The number of such g is

(A) 6. (B) 4. (C) 24. (D) 48.

Q. 28 Let G be a finite abelian group of odd order. Consider the following two statements:

I. The map $f : G \to G$ defined by $f(g) = g^2$ is a group isomorphism.

II. The product $\prod_{g \in G} g = e$.

(A) Both I and II are TRUE.

(B) I is TRUE but II is FALSE.

(C) II is TRUE but I is FALSE.

(D) Neither I nor II is TRUE.

Q. 29 Let $n \geq 2$ be an integer. Let $A : \mathbb{C}^n \longrightarrow \mathbb{C}^n$ be the linear transformation defined by

$$A(z_1, z_2, \ldots, z_n) = (z_n, z_1, z_2, \ldots, z_{n-1}).$$

Which one of the following statements is true for every $n \geq 2$?

(A) A is nilpotent.

(B) All eigenvalues of A are of modulus 1.

(C) Every eigenvalue of A is either 0 or 1.

(D) A is singular.

Q. 30 Consider the two series

I. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+(1/n)}}$ and II. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2-n^{1/n}}}$

Which one of the following holds?

(A) Both I and II converge.

(B) Both I and II diverge.

(C) I converges and II diverges.

(D) I diverges and II converges.

SECTION – B
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q. 31 Let $f : \mathbb{R} \to \mathbb{R}$ be a function with the property that for every $y \in \mathbb{R}$, the value of the expression

$$\sup_{x \in \mathbb{R}} [xy - f(x)]$$

is finite. Define $g(y) = \sup_{x \in \mathbb{R}} [xy - f(x)]$ for $y \in \mathbb{R}$. Then

(A) g is even if f is even.

(B) f must satisfy $\lim_{|x| \to \infty} \frac{f(x)}{|x|} = +\infty$.

(C) g is odd if f is even.

(D) f must satisfy $\lim_{|x| \to \infty} \frac{f(x)}{|x|} = -\infty$.

Q. 32 Consider the equation

$$x^{2021} + x^{2020} + \cdots + x - 1 = 0.$$

Then

(A) all real roots are positive.

(B) exactly one real root is positive.

(C) exactly one real root is negative.

(D) no real root is positive.

Q. 33 Let $D = \mathbb{R}^2 \setminus \{(0, 0)\}$. Consider the two functions $u, v : D \to \mathbb{R}$ defined by

$$u(x, y) = x^2 - y^2 \text{ and } v(x, y) = xy.$$

Consider the gradients ∇u and ∇v of the functions u and v, respectively. Then

(A) ∇u and ∇v are parallel at each point (x, y) of D.

(B) ∇u and ∇v are perpendicular at each point (x, y) of D.

(C) ∇u and ∇v do not exist at some points (x, y) of D.

(D) ∇u and ∇v at each point (x, y) of D span $\mathbb{R}^2$.

Q. 34 Consider the two functions $f(x, y) = x + y$ and $g(x, y) = xy - 16$ defined on $\mathbb{R}^2$. Then

(A) the function f has no global extreme value subject to the condition $g = 0$.

(B) the function f attains global extreme values at $(4, 4)$ and $(-4, -4)$ subject to the condition $g = 0$.

(C) the function g has no global extreme value subject to the condition $f = 0$.

(D) the function g has a global extreme value at $(0, 0)$ subject to the condition $f = 0$.

Q. 35 Let $f : (a, b) \to \mathbb{R}$ be a differentiable function on (a, b). Which of the following statements is/are true?

(A) $f' > 0$ in (a, b) implies that f is increasing in (a, b).

(B) f is increasing in (a, b) implies that $f' > 0$ in (a, b).

(C) If $f'(x_0) > 0$ for some $x_0 \in (a, b)$, then there exists a $\delta > 0$ such that $f(x) > f(x_0)$ for all $x \in (x_0, x_0 + \delta)$.

(D) If $f'(x_0) > 0$ for some $x_0 \in (a, b)$, then f is increasing in a neighbourhood of x_0.

Q. 36 Let G be a finite group of order 28. Assume that G contains a subgroup of order 7. Which of the following statements is/are true?

(A) G contains a unique subgroup of order 7.

(B) G contains a normal subgroup of order 7.

(C) G contains no normal subgroup of order 7.

(D) G contains at least two subgroups of order 7.

Q. 37 Which of the following subsets of $\mathbb{R}$ is/are connected?

(A) The set $\{x \in \mathbb{R} : x \text{ is irrational}\}$.

(B) The set $\{x \in \mathbb{R} : x^3 - 1 \geq 0\}$.

(C) The set $\{x \in \mathbb{R} : x^3 + x + 1 \geq 0\}$.

(D) The set $\{x \in \mathbb{R} : x^3 - 2x + 1 \geq 0\}$.

Q. 38 Consider the four functions from $\mathbb{R}$ to $\mathbb{R}$:

$$f_1(x) = x^4 + 3x^3 + 7x + 1, \quad f_2(x) = x^3 + 3x^2 + 4x, \quad f_3(x) = \arctan(x)$$

and

$$f_4(x) = \begin{cases} x & \text{if } x \notin \mathbb{Z}, \\ 0 & \text{if } x \in \mathbb{Z}. \end{cases}$$

Which of the following subsets of $\mathbb{R}$ are open?

(A) The range of f_1.

(B) The range of f_2.

(C) The range of f_3.

(D) The range of f_4.

Q. 39 Let V be a finite dimensional vector space and $T : V \to V$ be a linear transformation. Let $\mathcal{R}(T)$ denote the range of T and $\mathcal{N}(T)$ denote the null space $\{v \in V : Tv = 0\}$ of T. If $\text{rank}(T) = \text{rank}(T^2)$, then which of the following is/are necessarily true?

(A) $\mathcal{N}(T) = \mathcal{N}(T^2)$.

(B) $\mathcal{R}(T) = \mathcal{R}(T^2)$.

(C) $\mathcal{N}(T) \cap \mathcal{R}(T) = \{0\}$.

(D) $\mathcal{N}(T) = \{0\}$.

Q. 40 Let $m > 1$ and $n > 1$ be integers. Let A be an $m \times n$ matrix such that for some $m \times 1$ matrix b_1, the equation $Ax = b_1$ has infinitely many solutions. Let b_2 denote an $m \times 1$ matrix different from b_1. Then $Ax = b_2$ has

(A) infinitely many solutions for some b_2.

(B) a unique solution for some b_2.

(C) no solution for some b_2.

(D) finitely many solutions for some b_2.

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q. 41 The number of cycles of length 4 in S_6 is _____.

Q. 42 The value of

$$\lim_{n\to\infty}\left(3^n+5^n+7^n\right)^{\frac{1}{n}}$$

is _____.

Q. 43 Let $B=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2\le 1\}$ and define $u(x,y,z)=\sin\left((1-x^2-y^2-z^2)^2\right)$ for $(x,y,z)\in B$. Then the value of

$$\iiint_B\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)dxdydz$$

is _____.

Q. 44 Consider the subset $S=\{(x,y) : x^2+y^2>0\}$ of $\mathbb{R}^2$. Let

$$P(x,y)=\frac{y}{x^2+y^2} \text{ and } Q(x,y)=-\frac{x}{x^2+y^2}$$

for $(x,y)\in S$. If C denotes the unit circle traversed in the counter-clockwise direction, then the value of

$$\frac{1}{\pi}\int_C (Pdx+Qdy)$$

is _____.

Q. 45 Consider the set $A=\{a\in\mathbb{R} : x^2=a(a+1)(a+2) \text{ has a real root }\}$. The number of connected components of A is _____.

Q. 46 Let V be the real vector space of all continuous functions $f:[0,2]\to\mathbb{R}$ such that the restriction of f to the interval $[0,1]$ is a polynomial of degree less than or equal to 2, the restriction of f to the interval $[1,2]$ is a polynomial of degree less than or equal to 3 and $f(0)=0$. Then the dimension of V is equal to _____.

Q. 47 The number of group homomorphisms from the group $\mathbb{Z}_4$ to the group S_3 is _____.

Q. 48 Let $y : \left(\frac{9}{10}, 3\right) \to \mathbb{R}$ be a differentiable function satisfying

$$(x - 2y)\frac{dy}{dx} + (2x + y) = 0, \quad x \in \left(\frac{9}{10}, 3\right), \quad \text{and } y(1) = 1.$$

Then $y(2)$ equals _____.

Q. 49 Let $\vec{F} = (y+1)e^y \cos(x)\hat{i} + (y+2)e^y \sin(x)\hat{j}$ be a vector field in $\mathbb{R}^2$ and C be a continuously differentiable path with the starting point $(0, 1)$ and the end point $(\frac{\pi}{2}, 0)$. Then

$$\int_C \vec{F} \cdot d\vec{r}$$

equals _____.

Q. 50 The value of

$$\frac{\pi}{2} \lim_{n\to\infty} \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{8}\right) \cdots \cos\left(\frac{\pi}{2^{n+1}}\right)$$

is _____.

Q. 51 – Q. 60 carry two marks each.

Q. 51 The number of elements of order two in the group S_4 is equal to _____.

Q. 52 The least possible value of k, accurate up to two decimal places, for which the following problem

$$y''(t) + 2y'(t) + ky(t) = 0, t \in \mathbb{R},$$
$$y(0) = 0, y(1) = 0, y(1/2) = 1,$$

has a solution is _____.

Q. 53 Consider those continuous functions $f : \mathbb{R} \to \mathbb{R}$ that have the property that given any $x \in \mathbb{R}$,

$$f(x) \in \mathbb{Q} \text{ if and only if } f(x+1) \in \mathbb{R} \setminus \mathbb{Q}.$$

The number of such functions is _____.

Q. 54 The largest positive number a such that

$$\int_0^5 f(x)dx + \int_0^3 f^{-1}(x)dx \geq a$$

for every strictly increasing surjective continuous function $f : [0, \infty) \to [0, \infty)$ is _____.

Q. 55 Define the sequence

$$s_n = \begin{cases} \frac{1}{2^n} \sum_{j=0}^{n-2} 2^{2j} & \text{if } n > 0 \text{ is even,} \\ \frac{1}{2^n} \sum_{j=0}^{n-1} 2^{2j} & \text{if } n > 0 \text{ is odd.} \end{cases}$$

Define $\sigma_m = \frac{1}{m} \sum_{n=1}^{m} s_n$. The number of limit points of the sequence $\{\sigma_m\}$ is _____.

Q. 56 The determinant of the matrix

$$\begin{pmatrix} 2021 & 2020 & 2020 & 2020 \\ 2021 & 2021 & 2020 & 2020 \\ 2021 & 2021 & 2021 & 2020 \\ 2021 & 2021 & 2021 & 2021 \end{pmatrix}$$

is _____.

Q. 57 The value of

$$\lim_{n\to\infty} \int_0^1 e^{x^2} \sin(nx)\, dx$$

is _____.

Q. 58 Let S be the surface defined by

$$\{(x, y, z) \in \mathbb{R}^3 : z = 1 - x^2 - y^2,\ z \geq 0\}.$$

Let $\vec{F} = -y\hat{i} + (x-1)\hat{j} + z^2\hat{k}$ and $\hat{n}$ be the continuous unit normal field to the surface S with positive z-component. Then the value of

$$\frac{1}{\pi} \iint_S \left(\nabla \times \vec{F}\right) \cdot \hat{n}\, dS$$

is _____.

Q. 59 Let $A = \begin{pmatrix} 2 & -1 & 3 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{pmatrix}$. Then the largest eigenvalue of A is _____.

Q. 60 Let $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$. Consider the linear map T_A from the real vector space $M_4(\mathbb{R})$ to itself defined by $T_A(X) = AX - XA,$ for all $X \in M_4(\mathbb{R})$. The dimension of the range of T_A is _____.

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The function $e^{\cos x}$ is Taylor expanded about $x = 0$. The coefficient of x^2 is

(A) $-\frac{1}{2}$ (B) $-\frac{e}{2}$ (C) $\frac{e}{2}$ (D) Zero

Q.2 Let M be a 2 ×2 matrix. Its trace is 6 and its determinant has value 8. Its eigenvalues are

(A) 2 and 4 (B) 3 and 3 (C) 2 and 6 (D) -2 and -3

Q.3 A planet is in a highly eccentric orbit about a star. The distance of its closest approach is 300 times smaller than its farthest distance from the star. If the corresponding speeds are v_c and v_f, then $\frac{v_c}{v_f}$ is

(A) $\frac{1}{300}$ (B) $\frac{1}{\sqrt{300}}$ (C) $\sqrt{300}$ (D) 300

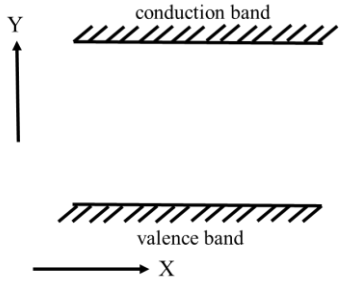
Q.4 An object of density ρ is floating in a liquid with 75% of its volume submerged. The density of the liquid is

(A) $\frac{4}{3}\rho$ (B) $\frac{3}{2}\rho$ (C) $\frac{8}{5}\rho$ (D) 2ρ

Q.5 An experiment with a Michelson interferometer is performed in vacuum using a laser of wavelength 610 nm. One of the beams of the interferometer passes through a small glass cavity 1.3 cm long. After the cavity is completely filled with a medium of refractive index n, 472 dark fringes are counted to move past a reference line. Given that the speed of light is 3×10^8 m/s, the value of n is

(A) 1.01 (B) 1.04 (C) 1.06 (D) 1.10

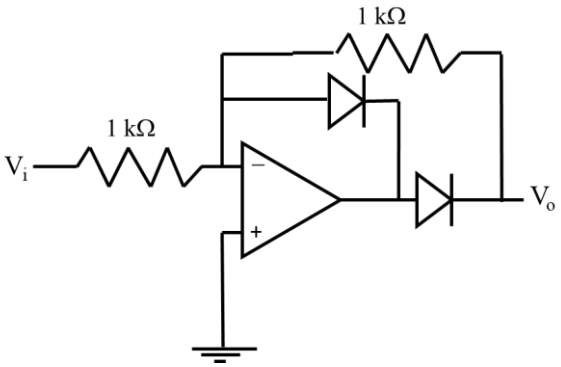
Q.6 For a semiconductor material, the conventional flat band energy diagram is shown in the figure. The variables Y, X, respectively, are



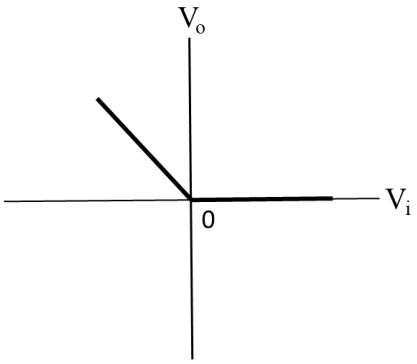
(A) Energy, Momentum (B) Energy, Distance

(C) Distance, Energy (D) Momentum, Energy

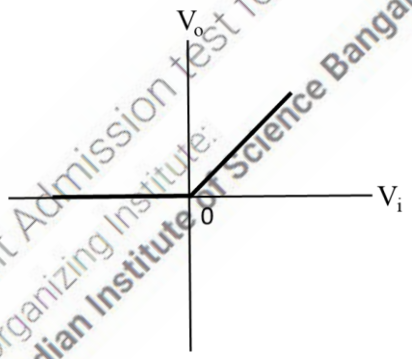
Q.7 For the given circuit, V_D is the threshold voltage of the diode. The graph that best depicts the variation of V_o with V_i is



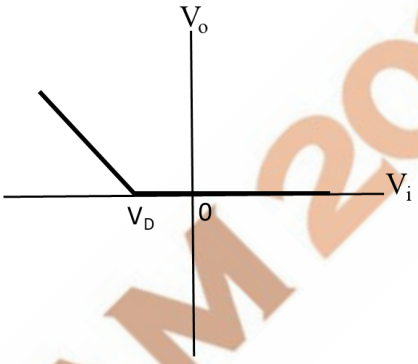
(A)



(B)



(C)



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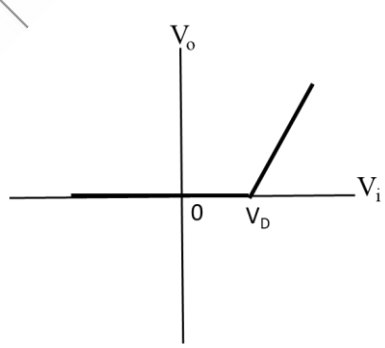


Q.8 Arrange the following telescopes, where D is the telescope diameter and λ is the wavelength, in order of decreasing resolving power:

I. $D = 100$ m, $\lambda = 21$ cm
II. $D = 2$ m, $\lambda = 500$ nm
III. $D = 1$ m, $\lambda = 100$ nm
IV. $D = 2$ m, $\lambda = 10$ mm

(A) III, II, IV, I

(B) II, III, I, IV

(C) IV, III, II, I

(D) III, II, I, IV

Q.9 Metallic lithium has *bcc* crystal structure. Each unit cell is a cube of side a. The number of atoms per unit volume is

(A) $\frac{1}{a^3}$ (B) $\frac{2}{\sqrt{2}a^3}$

(C) $\frac{2}{a^3}$ (D) $\frac{4}{a^3}$

Q.10 The moment of inertia of a solid sphere (radius R and mass M) about the axis which is at a distance of $\frac{R}{2}$ from the center is

(A) $\frac{3}{20}MR^2$ (B) $\frac{1}{2}MR^2$ (C) $\frac{13}{20}MR^2$ (D) $\frac{9}{10}MR^2$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let (x, y) denote the coordinates in a rectangular Cartesian coordinate system C. Let (x', y') denote the coordinates in another coordinate system C', defined by

$$x' = 2x + 3y$$
$$y' = -3x + 4y$$

The area element in C', is

(A) $\frac{1}{17}dx'dy'$ (B) $12dx'dy'$ (C) $dx'dy'$ (D) $x'dx'dy'$

Q.12 Three events, $E_1(ct = 0, x = 0), E_2(ct = 0, x = L)$ and $E_3(ct = 0, x = -L)$ occur, as observed in an inertial frame S. Frame S' is moving with a speed v along the positive x-direction with respect to S. In S', let t_1', t_2', t_3' be the respective times at which E_1, E_2, and E_3 occurred. Then,

(A) $t_2' < t_1' < t_3'$

(B) $t_1' = t_2' = t_3'$

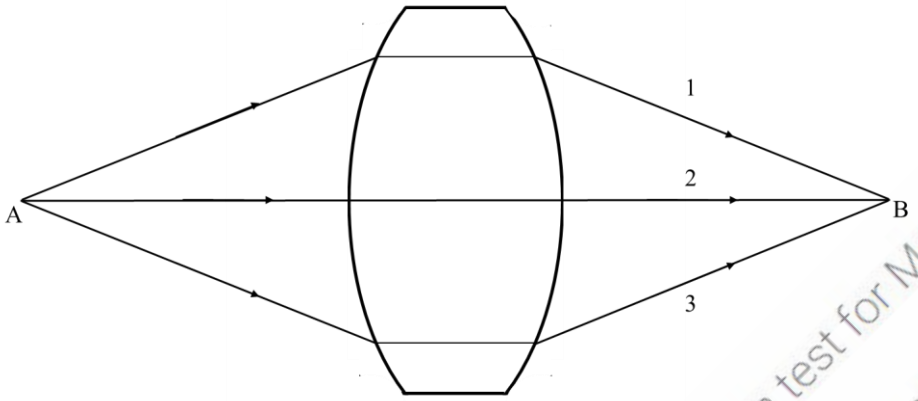
(C) $t_3' < t_1' < t_2'$

(D) $t_3' < t_2' < t_1'$

Q.13 The solution $y(x)$ of the differential equation $y\frac{dy}{dx} + 3x = 0,\ y(1) = 0$, is described by

(A) an ellipse (B) a circle (C) a parabola (D) a straight line

Q.14 In the figure below, point A is the object and point B is the image formed by the lens. Let l_1, l_2 and l_3 denote the optical path lengths of the three rays 1, 2 and 3, respectively. Identify the correct statement.



(A) $l_1 = l_2 = l_3$

(B) $l_1 > l_2 < l_3$

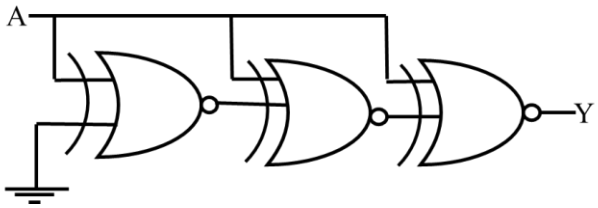
(C) $l_1 = l_3 < l_2$

(D) $l_1 = l_3 > l_2$

Q.15 A particle, initially at the origin in an inertial frame S, has a constant velocity $V\hat{\imath}$. Frame S' is rotating about the z-axis with angular velocity ω (anticlockwise). The coordinate axes of S' coincide with those of S at $t = 0$. The velocity of the particle (V_x', V_y') in the S' frame, at $t = \frac{\pi}{2\,\omega}$ is

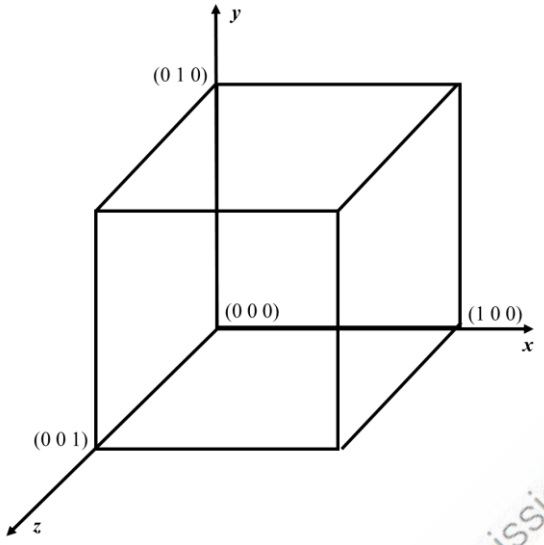
(A) $\left(-\frac{V\pi}{2}, -V\right)$ (B) $(-V, -V)$ (C) $\left(\frac{V\pi}{2}, -V\right)$ (D) $\left(\frac{3V\pi}{2}, -V\right)$

Q.16 For the given circuit, the output Y is



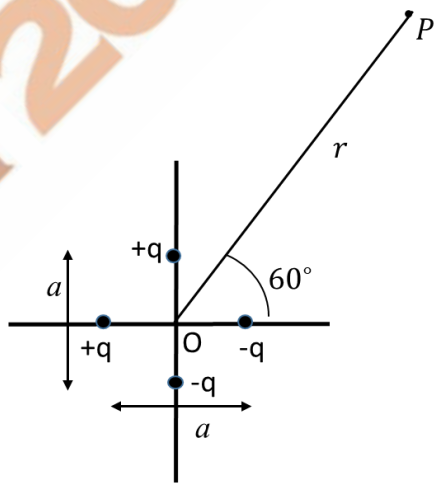
(A) 0 (B) 1 (C) A (D) $\overline{A}$

Q.17 The total charge contained within the cube (see figure), in which the electric field is given by $\vec{E} = K(4x^2\,\hat{\imath} + 3y\,\hat{\jmath})$, where ϵ_0 is the permittivity of free space, is



(A) $7K\epsilon_0$ (B) $5K\epsilon_0$ (C) $3K\epsilon_0$ (D) Zero

Q.18 Four charges are placed very close to each other, as shown. The separation between the two charges on the y-axis is a. The separation between the two charges on the x-axis is also a. The leading order (non-vanishing) form of the electrostatic potential, at point P, at a distance r from the origin $(r \gg a)$, is



(A) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{2r^2}\left(\sqrt{3}-1\right)$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{2qa}{r^2}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(\sqrt{5}-1\right)$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{qa}{r^2}\left(1-\sqrt{3}\right)$

Q.19 At $t = 0$, N_0 number of a radioactive nuclei A start decaying into B with a decay constant λ_a. The daughter nuclei B decay into nuclei C with a decay constant λ_b. Then, the number of nuclei B at small time t (to the leading order) is

(A) $\lambda_a N_0 t$ (B) $(\lambda_a - \lambda_b)N_0 t$ (C) $(\lambda_a + \lambda_b)N_0 t$ (D) $\lambda_b N_0 t$

Q.20 The electric field of an electromagnetic wave has the form $\vec{E} = E_0\, cos(\omega t - kz)\hat{\imath}$. At $t=0$, a test particle of charge q is at $z = 0$, and has velocity $\vec{v} = 0.5c\hat{k}$, where c is the speed of light. The total instantaneous force on the particle is

(A) $\frac{qE_0}{2}\hat{\imath}$ (B) $\frac{qE_0}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$ (C) $\frac{qE_0}{2}(\hat{\imath} - \hat{k})$ (D) Zero

Q.21 The *rms* velocity of molecules of oxygen gas is given by v at some temperature T. The molecules of another gas have the same *rms* velocity at temperature $\frac{T}{16}$. The second gas is

(A) Hydrogen (B) Helium (C) Nitrogen (D) Neon

Q.22 A system undergoes a thermodynamic transformation from state S_1 to state S_2 via two different paths *1* and *2*. The heat absorbed and work done along path *1* are 50 J and 30 J, respectively. If the heat absorbed along path *2* is 30 J, the work done along path *2* is

(A) Zero (B) 10 J (C) 20 J (D) 30 J

Q.23 The condition for maxima in the interference of two waves $A\, e^{\,i\left(\frac{k_0}{2}(\sqrt{3}\,x+y)-\omega t\right)}$ and $A\, e^{\,i\left(\frac{k_0}{\sqrt{2}}(x+y)-\omega t\right)}$ is given in terms of the wavelength λ and m, an integer, by

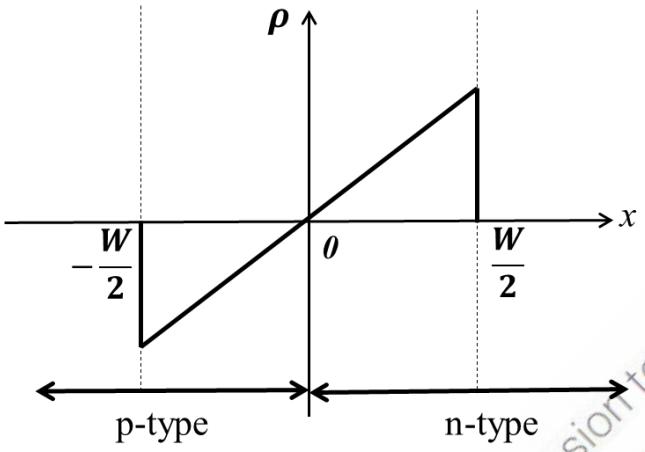
(A) $\left(\sqrt{3}-\sqrt{2}\right)x + \left(1-\sqrt{2}\right)y = 2m\lambda$

(B) $\left(\sqrt{3}+\sqrt{2}\right)x + \left(1-\sqrt{2}\right)y = 2m\lambda$

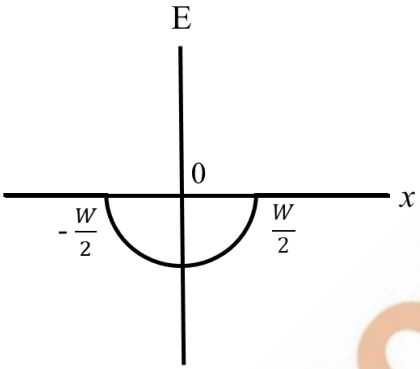
(C) $\left(\sqrt{3}-\sqrt{2}\right)x - \left(1-\sqrt{2}\right)y = m\lambda$

(D) $\left(\sqrt{3}-\sqrt{2}\right)x + \left(1-\sqrt{2}\right)y = (2m+1)\lambda$

Q.24 A semiconductor *pn* junction at thermal equilibrium has the space charge density $\rho(x)$ profile as shown in the figure. The figure that best depicts the variation of the electric field E with x is (W denotes the width of the depletion layer)



(A)



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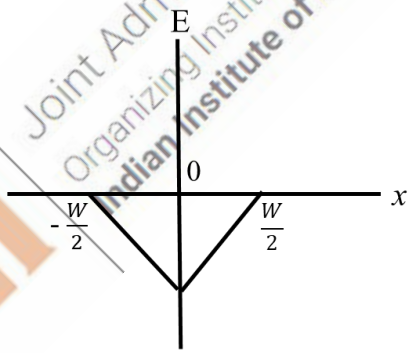
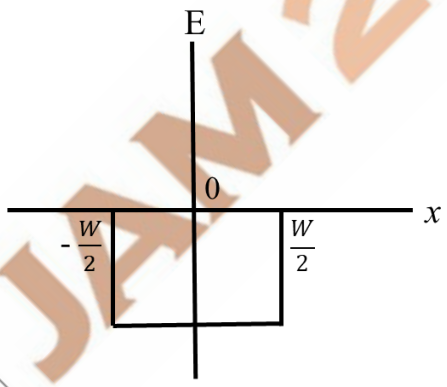


(C)



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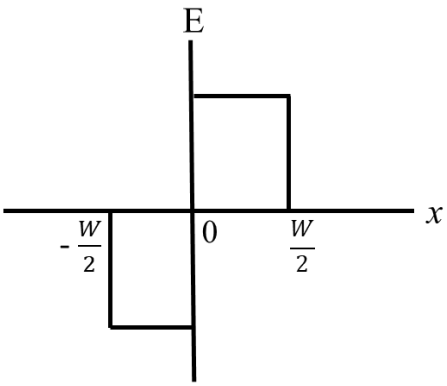
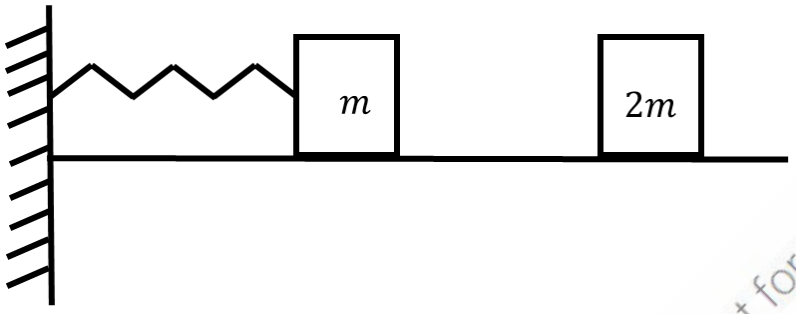


Q.25 A mass m is connected to a massless spring of spring constant k, which is fixed to a wall. Another mass $2m$, having kinetic energy E, collides collinearly with the mass m completely inelastically (see figure). The entire set up is placed on a frictionless floor. The maximum compression of the spring is



(A) $\sqrt{\frac{4}{3}\frac{E}{k}}$ (B) $\sqrt{\frac{E}{3k}}$ (C) $\sqrt{\frac{E}{5k}}$ (D) $\sqrt{\frac{E}{7k}}$

Q.26 A linearly polarized light falls on a quarter wave plate and the emerging light is found to be elliptically polarized. The angle between the fast axis of the quarter wave plate and the plane of polarization of the incident light, can be

(A) 30° (B) 45° (C) 90° (D) 180°

Q.27 The expression for the magnetic field that induces the electric field $\vec{E} = K\left(yz\hat{\imath} + 3z\hat{\jmath} + 4y\hat{k}\right)\cos(\omega t)$ is

(A) $-\frac{K}{\omega}(\hat{\imath} + y\hat{\jmath} - z\hat{k})\sin(\omega t)$

(B) $-\frac{K}{\omega}(\hat{\imath} + y\hat{\jmath} + z\hat{k})\sin(\omega t)$

(C) $-\frac{K}{\omega}(\hat{\imath} - y\hat{\jmath} + z\hat{k})\sin(\omega t)$

(D) $-\frac{K}{\omega}(\hat{\imath} + y\hat{\jmath} + z\hat{k})\sin(\omega t)$

Q.28 In the Fourier series expansion of two functions $f_1(t) = 4t^2 + 3$ and $f_2(t) = 6t^3 + 7t$ in the interval $-\frac{T}{2}\ to\ +\frac{T}{2}$, the Fourier coefficients a_n and b_n (a_n and b_n are coefficients of $\cos(n\omega t)$ and $\sin(n\omega t)$, respectively) satisfy

(A) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n = 0$ for $f_2(t)$

(B) $a_n \neq 0$ and $b_n = 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

(C) $a_n \neq 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$

(D) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n \neq 0$ for $f_2(t)$

Q.29 A thin circular disc lying in the xy-plane has a surface mass density σ, given by

$$\sigma(r) = \begin{cases} \sigma_0\left(1 - \frac{r^2}{R^2}\right) & if\ r \leq R \\ 0 & if\ r > R \end{cases}$$

where r is the distance from its center. Its moment of inertia about the z-axis, passing through its center is

(A) $\frac{\sigma_0 R^4}{4}$ (B) $\frac{\pi\sigma_0 R^4}{6}$ (C) $\sigma_0 R^4$ (D) $2\,\pi\sigma_0 R^4$

Q.30 The radial component of acceleration in plane polar coordinates is given by

(A) $\frac{d^2r}{dt^2}$

(B) $\frac{d^2r}{dt^2} - r\left(\frac{d\,\theta}{dt}\right)^2$

(C) $\frac{d^2r}{dt^2} + r\left(\frac{d\,\theta}{dt}\right)^2$

(D) $2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\,\theta}{dt^2}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 A gaseous system, enclosed in an adiabatic container, is in equilibrium at pressure P_1 and volume V_1. Work is done on the system in a quasi-static manner due to which the pressure and volume change to P_2 and V_2, respectively, in the final equilibrium state. At every instant, the pressure and volume obey the condition $PV^{\gamma} = C$, where $\gamma = \frac{C_P}{C_V}$ and C is a constant. If the work done is zero, then identify the correct statement(s).

(A) $P_2V_2 = P_1V_1$

(B) $P_2V_2 = \gamma P_1V_1$

(C) $P_2V_2 = (\gamma + 1)P_1V_1$

(D) $P_2V_2 = (\gamma - 1)P_1V_1$

Q.32 An isolated ideal gas is kept at a pressure P_1 and volume V_1. The gas undergoes free expansion and attains a pressure P_2 and volume V_2. Identify the correct statement(s). $\left(\gamma = \frac{C_P}{C_V}\right)$

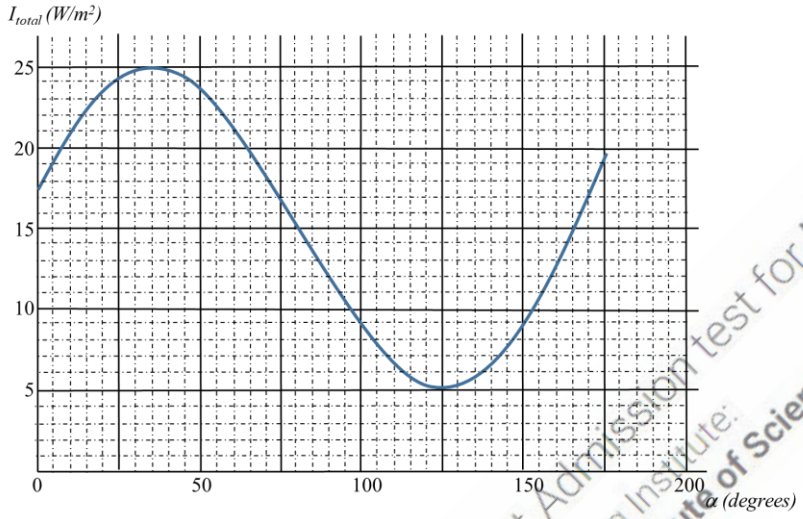
(A) This is an adiabatic process

(B) $P_1V_1 = P_2V_2$

(C) $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

(D) This is an isobaric process

Q.33 A beam of light traveling horizontally consists of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization is oriented at an angle θ with respect to the vertical. The figure shows the total intensity I_{total} after the light passes through a polarizer as a function of the angle α, that the axis of the polarizer makes with respect to the vertical. Identify the correct statement(s).



(A) $\theta = 125°$

(B) $I_p = 5$ W/m^2

(C) $I_0 = 17.5$ W/m^2

(D) $I_0 = 10$ W/m^2; $I_p = 20$ W/m^2

Q.34 Consider the following differential equation that describes the oscillations of a physical system:

$$\alpha \frac{d^2y}{dt^2} + \beta \frac{dy}{dt} + \gamma y = 0$$

If α and β are held fixed, and γ is increased, then,

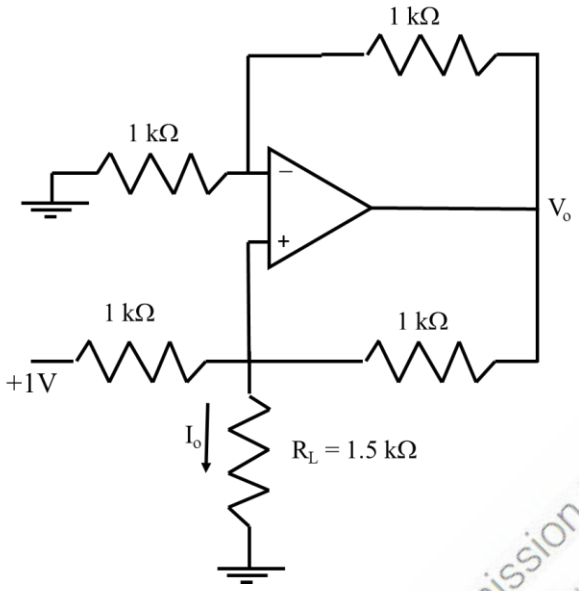
(A) the frequency of oscillations increases

(B) the oscillations decay faster

(C) the frequency of oscillations decreases

(D) the oscillations decay slower

Q.35 For the given circuit, identify the correct statement(s).



(A) $I_o = 1$ mA

(B) $V_o = 3$ V

(C) If R_L is doubled, I_o will change to 0.5 mA

(D) If R_L is doubled, V_o will change to 6 V

Q.36 A Carnot engine operates between two temperatures, $T_L = 100$ K and $T_H = 150$ K. Each cycle of the engine lasts for 0.5 seconds during which the power delivered is 500 J/second. Let Q_H be the corresponding heat absorbed by the engine and Q_L be the heat lost. Identify the correct statement(s).

(A) $Q_H = 750$ J

(B) $\frac{Q_H}{Q_L} \leq \frac{2}{3}$

(C) The change in entropy of the engine and the hot bath in a cycle is 5 J/K

(D) The change in entropy of the engine in 0.5 seconds is zero

Q.37 A time independent conservative force $\vec{F}$ has the form, $\vec{F} = 3y\hat{\imath} + f(x,y)\hat{\jmath}$. Its magnitude at $x = y = 0$ is 8. The allowed form(s) of $f(x,y)$ is(are)

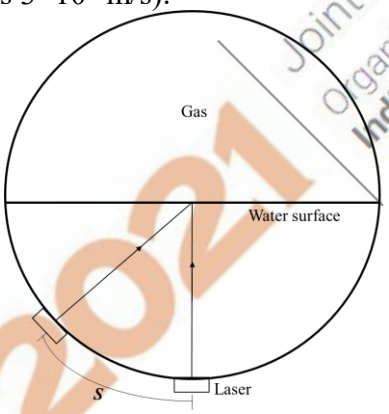
(A) $3x + 8$

(B) $2x + 8(y-1)^2$

(C) $3x + 8e^{-y^2}$

(D) $2x + 8\cos y$

Q.38 The figure shows the cross-section of a hollow cylindrical tank, 2.2 m in diameter, which is half filled with water (refractive index of 1.33). The space above the water is filled with a gas of unknown refractive index. A small laser moves along the bottom surface and aims a light beam towards the center (see figure). When the laser moves a distance of S = 1.09 m or beyond from the lowest point in the water, no light enters the gas. Identify the correct statement(s) (speed of light is 3×10^8 m/s).



(A) The refractive index of the gas is 1.05

(B) The time taken for the light beam to travel from the laser to the rim of the tank when $S <$ 1.09 m is 8.9 ns

(C) The time taken for the light beam to travel from the laser to the rim of the tank when $S >$ 1.09 m is 9.7 ns

(D) The critical angle for the water-gas interface is 56.77°

Q.39 Identify the correct statement(s) regarding nuclei

(A) The uncertainty in the momentum of a proton in a nucleus is roughly 10^5 times the uncertainty in the momentum of the electron in the ground state of Hydrogen atom

(B) The volume of a nucleus grows linearly with the number of nucleons in it

(C) The energy of γ rays due to de-excitation of a nucleus can be of the order of MeV

(D) ^{56}Fe is the most stable nucleus

Q.40 A particle of mass m is in an infinite square well potential of length L. It is in a superposed state of the first two energy eigenstates, as given by $\psi(x) = \frac{1}{\sqrt{3}}\psi_{n=1}(x) + \sqrt{\frac{2}{3}}\psi_{n=2}(x)$. Identify the correct statement(s). h is Planck's constant.

(A) $\langle p \rangle = 0$

(B) $\Delta p = \frac{\sqrt{3}h}{2L}$

(C) $\langle E \rangle = \frac{3h^2}{8mL^2}$

(D) $\Delta x = 0$

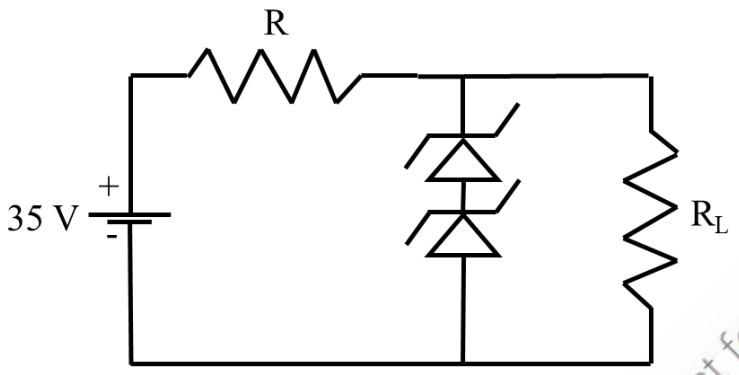
SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 One of the roots of the equation, $z^6 - 3z^4 - 16 = 0$ is given by $z_1 = 2$. The value of the product of the other five roots is _____.

Q.42 The following Zener diode voltage regulator circuit is used to obtain 20 V regulated output at load resistance R_L from a 35 V dc power supply. Zener diodes are rated at 5W and 10V. The value of the resistance R is _____ Ω.



Q.43 A small conducting square loop of side l is placed inside a concentric large conducting square loop of side L ($L \gg l$). The value of mutual inductance of the system is expressed as $\frac{n\mu_0 l^2}{\pi L}$. The value of n is _____ (Round off to two decimal places).

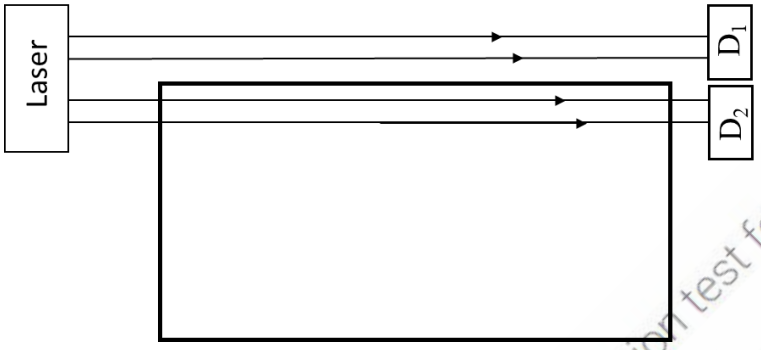
Q.44 Consider N_1 number of ideal gas particles enclosed in a volume V_1. If the volume is changed to V_2 and the number of particles is reduced by half, the mean free path becomes four times of its initial value. The ratio $\frac{V_1}{V_2}$ is _____ (Round off to one decimal place).

Q.45 A particle is moving with a velocity $0.8c\hat{j}$ (c is the speed of light) in an inertial frame S_1. Frame S_2 is moving with a velocity $0.8c\hat{\imath}$ with respect to S_1. Let E_1 and E_2 be the respective energies of the particle in the two frames. Then, $\frac{E_2}{E_1}$ is _____ (Round off to two decimal places).

Q.46 At some temperature T, two metals A and B, have Fermi energies ϵ_A and ϵ_B, respectively. The free electron density of A is 64 times that of B. The ratio $\frac{\epsilon_A}{\epsilon_B}$ is _____.

Q.47 A crystal has monoclinic structure, with lattice parameters, a = 5.14 Å, b = 5.20 Å, c = 5.30 Å and angle β = 99°. It undergoes a phase transition to tetragonal structure with lattice parameters, a = 5.09 Å and c = 5.27 Å. The fractional change in the volume $\left|\frac{\Delta V}{V}\right|$ of the crystal due to this transition is _____ (Round off to two decimal places).

Q.48 A laser beam shines along a block of transparent material of length 2.5 m. Part of the beam goes to the detector D_1 while the other part travels through the block and then hits the detector D_2. The time delay between the arrivals of the two light beams is inferred to be 6.25 ns. The speed of light $c = 3\times10^8$ m/s. The refractive index of the block is _____ (Round off to two decimal places).

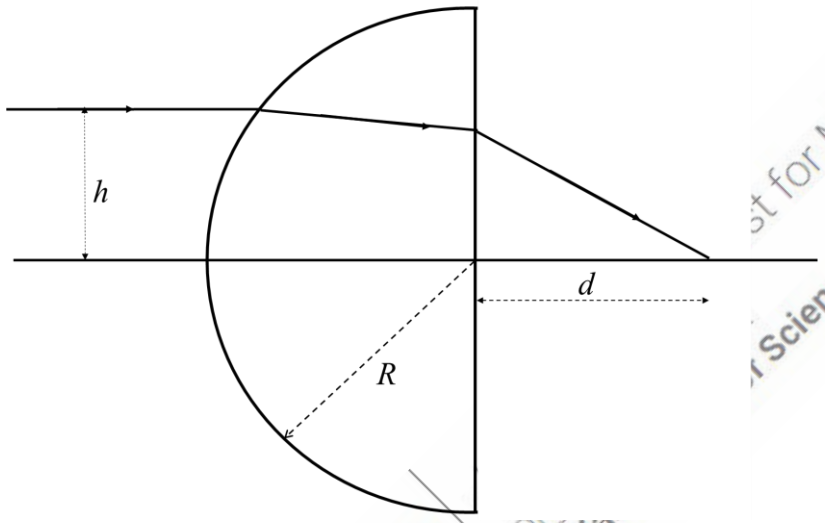


Q.49 An ideal blackbody at temperature T, emits radiation of energy density u. The corresponding value for a material at temperature $\frac{T}{2}$ is $\frac{u}{256}$. Its emissivity is _____ (Round off to three decimal places).

Q.50 A particle with positive charge 10^{-3} C and mass 0.2 kg is thrown upwards from the ground at an angle 45° with the horizontal with a speed of 5 m/s. The projectile moves through a horizontal electric field of 10 V/m, which is in the same direction as the horizontal component of the initial velocity of the particle. The acceleration due to gravity is $10\,\frac{\mathrm{m}}{\mathrm{s}^2}$. The range is _____ m. (Round off to three decimal places).

Q. 51 – Q. 60 carry two marks each.

Q.51 Consider a hemispherical glass lens (refractive index is 1.5) having radius of curvature $R = 12$ cm for the curved surface. An incoming ray, parallel to the optical axis, is incident on the curved surface at a height $h = 1$ cm above the optical axis, as shown in the figure. The distance d (from the flat surface of the lens) at which the ray crosses the optical axis is _____ cm (Round off to two decimal places).

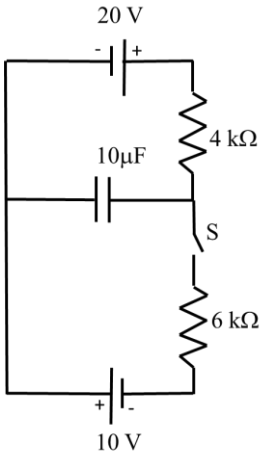


Q.52 Twenty non-interacting spin ½ particles are trapped in a three-dimensional simple harmonic oscillator potential of frequency ω. The ground state energy of the system, in units of $\hbar\omega$, is _____.

Q.53 A thin film of alcohol is spread over a surface. When light from a tunable source is incident normally, the intensity of reflected light at the detector is maximum for $\lambda = 640$ nm and minimum for $\lambda = 512$ nm. Taking the refractive index of alcohol to be 1.36 for both the given wavelengths, the minimum thickness of the film would be _____ nm (Round off to two decimal places).

Q.54 For the Boolean expression $Y = A\,B\,C + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C}$, the number of combinations for which the output $Y = 1$ is _____.

Q.55 An *RC* circuit is connected to two dc power supplies, as shown in the figure. With switch *S* open, the capacitor is fully charged. *S* is then closed at time $t = 0$. The voltage across the capacitor at $t = 2.4$ milliseconds is _____ V (Round off to one decimal place).

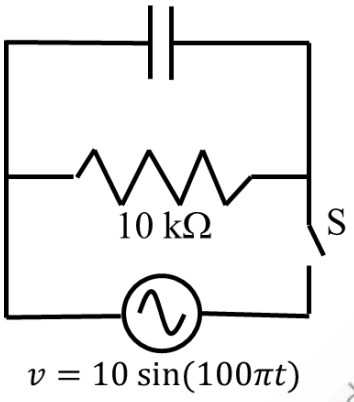


Q.56 A current I is uniformly distributed across a long straight nonmagnetic wire ($\mu_r = 1$) of circular cross-section with radius a. Two points P and Q are at distances $\frac{a}{3}$ and $9a$, respectively, from the axis of the wire. The ratio of the magnetic fields at points P and Q is _____.

Q.57 A particle A of mass m is moving with a velocity $v\hat{\imath}$, and collides elastically with a particle B, of mass $2m$. B is initially at rest. After collision, A moves with a velocity $v_A\hat{\jmath}$. If v_B is the final speed of B, then $v_A^2 = kv_B^2$. The value of k is _____.

Q.58 In an X-ray diffraction experiment with Cu crystals having lattice parameter 3.61 Å, X-rays of wavelength of 0.090 nm are incident on the family of planes {1 1 0}. The highest order present in the diffraction pattern is _____.

Q.59 A parallel plate capacitor having plate area of 50 cm^2 and separation of 0.1 mm is completely filled with a dielectric (dielectric constant $K = 10$). The capacitor is connected to a 10 kΩ resistance and an alternating voltage $v = 10\sin(100\pi t)$, as shown in the figure. The switch S is initially open and then closed at $t = 0$. The ratio of the displacement current in the capacitor, to the current in the resistance, at time $t = \frac{2}{\pi}$ seconds is _____ (Round off to three decimal places).



Q.60 The wavelength of characteristic K_α X-ray photons from Mo (atomic number 42) is _____ Å. (Round off to one decimal place).
(speed of light is 3×10^8 m/s; Rydberg constant $R = 1.09 \times 10^7$/m)

END OF THE QUESTION PAPER