IIT JAM 2019 Question Paper (All Subjects)

IIT Joint Admission Test for Masters

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The glycosidic linkages in cellulose and amylose are _____, respectively.

(A) α 1-4 and β 1-4
(B) β 1-4 and α 1-4
(C) β 1-4 and α 1-6
(D) α 1-4 and α 1-2

Q.2 A mutation in the operator locus of *lac* operon that confers constitutive expression of β-galactosidase is _____.

(A) *cis* dominant (B) *trans* dominant (C) co-dominant (D) dominant negative

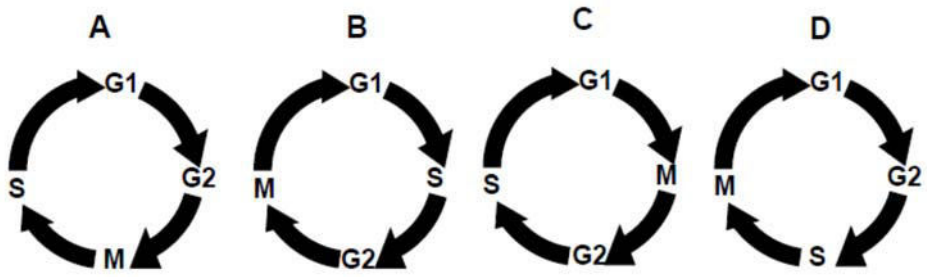
Q.3 Which one of the points $P = \left(\frac{3}{2}, \frac{1}{2}\right)$, $Q = \left(\frac{1}{2}, \frac{3}{2}\right)$, $R = \left(\frac{3}{2}, \frac{11}{2}\right)$ and $S = \left(\frac{11}{2}, \frac{3}{2}\right)$ lies ABOVE the parabola $y = 2x^2$ and INSIDE the circle $x^2 + y^2 = 4$?

(A) P (B) Q (C) R (D) S

Q.4 Let $U = \{1, 2, 3, 4, 5\}$. A subset S is chosen uniformly at random from the non-empty subsets of U. What is the probability that S does NOT have two consecutive elements?

(A) 9/31 (B) 10/31 (C) 11/31 (D) 12/31

Q.5 Which one of the following figures represents the correct sequence of phases in adult eukaryotic cell cycle?



Q.6 At what pH does poly-Glu in an aqueous solution form α-helical structure?

(A) 3 (B) 7 (C) 9 (D) 12

Q.7 The dimensions of coefficient of viscosity are _____.

(A) $ML^{-1}T^{-1}$ (B) $ML^{-1}T^{-2}$ (C) $ML^{-2}T^{-2}$ (D) $ML^{-2}T^{-1}$

Q.8 Match the entries in Group I with the entries in Group II

Group I	**Group II**
(P) Nylon	(i) Isoprene
(Q) Natural rubber	(ii) Hexose
(R) Starch	(iii) Amino acid
(S) Myoglobin	(iv) Adipic acid

(A) P-iv, Q-i, R-ii, S-iii
(B) P-iv, Q-i, R-iii, S-ii
(C) P-iv, Q-iii, R-ii, S-i
(D) P-ii, Q-iv, R-i, S-iii

Q.9 The technique that involves impacting samples with electrons is _____.

(A) NMR spectroscopy
(B) ESI mass spectrometry
(C) IR spectroscopy
(D) UV-vis spectroscopy

Q.10 The orbital angular momentum of hydrogen atom in the ground state is _____.

(A) 0 (B) $\frac{h}{2\pi}$ (C) $\frac{h}{2}$ (D) h

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $a = \frac{\sqrt{5}+1}{2}$ and $b = \frac{\sqrt{5}-1}{2}$. Then, $\lim_{n\to\infty} \frac{a^n+b^n}{a^n-b^n}$

(A) is 1 (B) is ½ (C) is 0 (D) does not exist

Q.12 In how many ways can one write the elements 1, 2, 3, 4 in a sequence x_1, x_2, x_3, x_4 with $x_i \neq i \; \forall i$?

(A) 9 (B) 10 (C) 11 (D) 12

Q.13 Simplify $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$.

(A) $2\sec A$ (B) $2\,\mathrm{cosec}\, A$ (C) $\sec A$ (D) $\mathrm{cosec}\, A$

Q.14 The evolution of eyes in octopus and in human is an example of _____.

(A) divergent evolution
(B) convergent evolution
(C) adaptive radiation
(D) genetic drift

Q.15 Which one of the following modifications occurs both on DNA and protein?

(A) ADP-ribosylation
(B) Methylation
(C) Sumoylation
(D) Ubiquitination

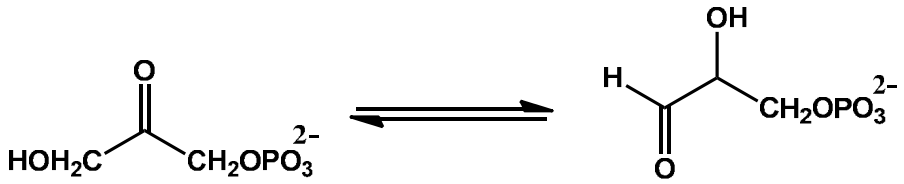
Q.16 Solutions of the following peptides are prepared separately at a concentration of 1 mM. Among these four, which one has the highest A_{280}?

(A) Ser-Val-Trp-Asp-Phe-Gly-Tyr-Trp-Ala
(B) Gln-Leu-Glu-Phe-Thr-Leu-Asp-Gly-Tyr
(C) Met-Gly-Val-Ileu-Asp-Ser-Ala-Trp-His
(D) His-Pro-Gly-Asp-Val-Ileu-Phe-Met-Leu

Q.17 The free energy required to synthesize a mixed anhydride bond of 1,3-bisphosphoglycerate is generated by the oxidation of _____.

(A) an aldehyde to acid
(B) an alcohol to acid
(C) an alcohol to aldehyde
(D) NADH to NAD^+

Q.18 The following reaction is an example of _____.



(A) enolization (B) racemization (C) isomerization (D) epimerization

Q.19 Which one of the following parameters changes upon doubling the enzyme concentration?

(A) K_M (B) V_{max} (C) k_{cat} (D) K_{eq}

Q.20 Which one of the following statements is a correct description of modes of action of taxol and colchicine?

(A) Taxol causes DNA damage and colchicine prevents microtubule formation
(B) Taxol stabilizes microtubules and colchicine inhibits protein synthesis
(C) Taxol destabilizes microtubules and colchicine promotes microtubule formation
(D) Taxol stabilizes microtubules and colchicine prevents microtubule formation

Q.21 In a simple microscope, _____.

(A) a lens with negative power is used
(B) the focal length of the lens is less than the least distance for clear vision
(C) the focal length of the lens is greater than the least distance for clear vision
(D) magnification depends only on the focal length of the lens

Q.22 Which one of the following statements is INCORRECT with respect to bacterial conjugation?

(A) It facilitates transfer of genetic material
(B) It requires flagellum
(C) It can spread antibiotic resistance
(D) It can transfer virulence factors

Q.23 A particle starting from rest is subjected to a constant force. The plot of distance traveled along the direction of the force as a function of time is a/an _____.

(A) straight line (B) circle (C) parabola (D) ellipse

Q.24 Indole acetic acid (IAA) is involved in _____.

(A) gravitropism (B) flowering (C) ripening (D) senescence

Q.25 Which one of the following remains unchanged when light waves enter water from air?

(A) Wavelength (B) Wavenumber (C) Frequency (D) Intensity

Q.26 According to the kinetic theory of gases, the average energy of a diatomic molecule in an ideal gas depends on _____.

(A) mass of each atom and the temperature
(B) mass of each atom and the bond length
(C) mass of each atom, bond length, and temperature
(D) temperature only

Q.27 Match the entries in Group I with entries in Group II

Group I	**Group II**
(P) Bacteria	(i) Malaria
(Q) Virus	(ii) Tuberculosis
(R) Protozoa	(iii) Influenza
(S) Autoantibodies	(iv) Myasthenia gravis

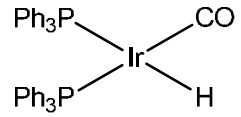
(A) P-ii, Q-i, R-iii, S-iv (B) P-ii, Q-iii, R-i, S-iv
(C) P-iv, Q-iii, R-i, S-ii (D) P-i, Q-iv, R-ii, S-iii

Q.28 pK_a increases in the order _____.

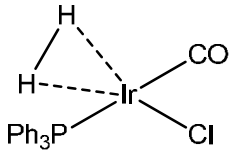
(A) $HN_3 > NH_3OH^+ > N_2H_5^+ > NH_3$
(B) $NH_3OH^+ > N_2H_5^+ > HN_3 > NH_3$
(C) $NH_3 > NH_3OH^+ > N_2H_5^+ > HN_3$
(D) $HN_3 > N_2H_5^+ > NH_3 > NH_3OH^+$

Q.29 H_2 reacts with *trans*-$(Ph_3P)_2Ir(CO)Cl$ to primarily produce _____.

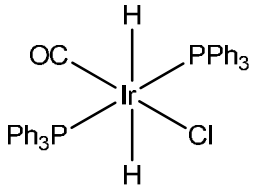
(A)



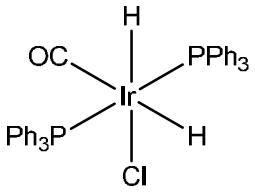
(B)



(C)



(D)



Q.30 Among the following species, the metal center that has the highest number of unpaired electrons is _____.

(A) VCl_4 (B) $Ni(CO)_4$ (C) $[AuCl_4]^-$ (D) $[CdBr_4]^{2-}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Pick the correct statement(s) with respect to the inter-conversion of the topoisomers of a circularly closed double stranded DNA.

(A) Only one strand needs to be cut
(B) Both strands have to be cut
(C) No strand needs to be cut
(D) ATP is required for inter-conversion

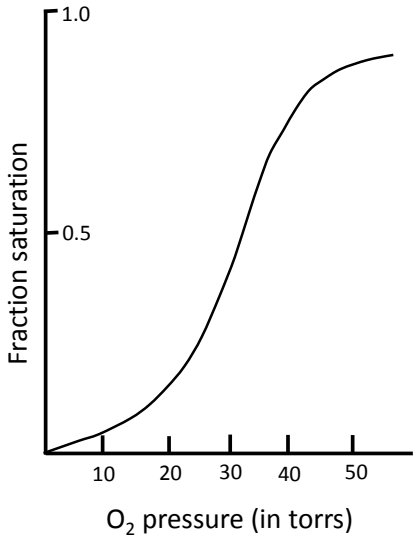
Q.32 Let $U = \{1, 2, \ldots, 15\}$. Let $P \subseteq U$ consist of all prime numbers, $Q \subseteq U$ consist of all even numbers and $R \subseteq U$ consist of all multiples of 3. Let $T = P - Q$. Then, which of the following is/are CORRECT?

(A) $|T| = 5$ and $|T \cup R| = 9$ (B) $|T| = 6$ and $|T \cup R| = 9$
(C) $|T| = 5$ and $|T \cap R| = 1$ (D) $|T| = 6$ and $|T \cap R| = 1$

Q.33 Let $f(x) = (x-1)(x-2)(x-3)(x-4)$ and let $\alpha = f(\frac{3}{2})$, $\beta = f\left(\frac{5}{2}\right)$ and $\gamma = f(\frac{7}{2})$. Which of the following is/are CORRECT?

(A) α and β have the same sign
(B) α and γ have the same sign
(C) β and γ have the same sign
(D) $\alpha\beta$ and $\beta\gamma$ have the same sign

Q.34 The characteristic oxygen binding profile of hemoglobin shown below arises due to the _____.



(A) quaternary structure
(B) subunit dissociation
(C) cooperativity
(D) conformational change

Q.35 The advantage(s) of storing chemical energy in the form of starch and not as free glucose is/are that it_____.

(A) minimizes diffusion
(B) enables compact storage
(C) reduces osmotic pressure
(D) protects against chemical reactivity of aldehyde groups

Q.36 Which of the following cell types can develop from myeloid lineage?

(A) Macrophages (B) T lymphocytes (C) B lymphocytes (D) Erythrocytes

Q.37 Electromagnetic waves _____.

(A) carry energy
(B) carry momentum
(C) are transverse in nature while travelling in vacuum
(D) do not need a material medium to travel

Q.38 Which of the following statement(s) is/are true?

(A) In intrinsic semiconductors, the number of electrons is equal to the number of holes at any temperature
(B) An intrinsic semiconductor changes to an *n*-type semiconductor upon addition of a trivalent element
(C) The shape of the I-V characteristics of a *p-n* diode is a straight line
(D) In the reverse bias condition, the current in a *p-n* diode is due to the minority carriers

Q.39 BF_3 reacts readily with _____.

(A) C_5H_5N (B) $SnCl_2$
(C) SO_3 (D) (C_5H_5N)–$SnCl_2$

Q.40 The reaction of (*R*)-2-bromobutane with CN^- proceeds by _____.

(A) retention of configuration
(B) inversion of configuration
(C) formation of CH_2=$CH(CH_2CH_3)$
(D) formation of (*S*)-2-methylbutanenitrile

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 C_3 plants utilize 18 molecules of ATP to synthesize one molecule of glucose from CO_2. How many molecules of ATP equivalents are used by C_4 plants to synthesize one molecule of glucose from CO_2?

Q.42 A 0.1% (w/v) solution of a protein absorbs 20% of the incident light. What fraction of light is transmitted if the concentration is increased to 0.4%? [Correct to two decimal places]

Q.43 Let XYZ be an equilateral triangle and let P, Q, R be the mid points of YZ, XZ, and XY, respectively.

Let $r = \frac{Area(\Delta\, PQR)}{Area(\Delta\, XYZ)}$.

The value of r is _____.

Q.44 Let N be the set of natural numbers and $f: N \mapsto N$ be defined by

$$f(x) = \begin{cases} x/2, & x \ is \ even \\ 3x+1, & x \ is \ odd \end{cases}$$

Let $f^n(x)$ denote the n-fold composition of $f(x)$. What is the smallest integer $\boldsymbol{n}$ such that $f^n(13) = 1$?

Q.45 Heterozygous female fruit flies with gray body and purple eyes were mated with homozygous males with black body and red eyes. The number of offspring obtained and their phenotypes are shown below:

Number of offspring	**Phenotype**
300	Gray body–purple eyes
347	Black body–red eyes
61	Gray body–red eyes
55	Black body–purple eyes

Calculate the recombination frequency.

Q.46 Proinsulin is an 84 residue polypeptide with six cysteines. How many different disulfide combinations are possible?

Q.47 The refractive index of a liquid relative to air is 1.5. Calculate the ratio of the real depth to the apparent depth when the liquid is taken in a beaker.

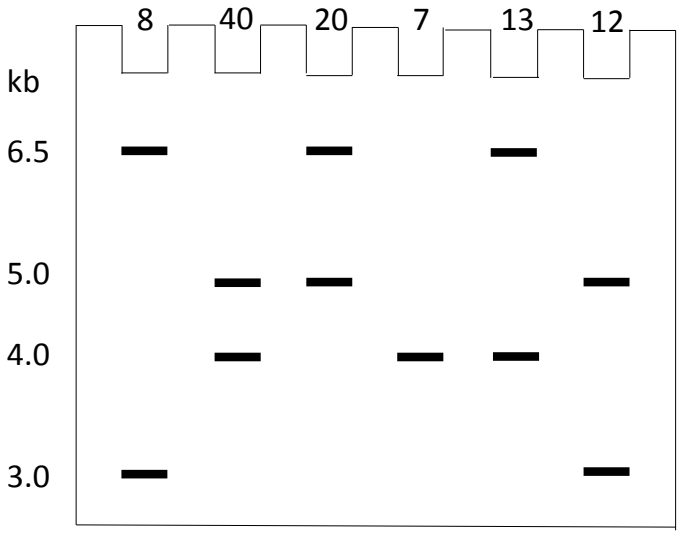
Q.48 A metallic wire of electrical resistance 40 Ω is bent in the form of a square loop. The resistance between any two diagonally opposite corners is _____ Ω.

Q.49 The total number of lone pairs of electrons in NO_2F is _____.

Q.50 The total number of multiplet peaks in the ^1H NMR spectrum of 1,3,5-tri-isopropylbenzene in $CDCl_3$ is _____.

Q. 51 – Q. 60 carry two marks each.

Q.51 A schematic representation of restriction fragment length polymorphism (RFLP) analysis of a sample population is shown below. The number of people exhibiting a given pattern is indicated above the lanes.



Calculate the frequency of 6.5 kb allele. [Correct to two decimal places]

Q.52 The value of $\int_0^{\frac{\pi}{2}} x \sin x \, dx$ is _____.

Q.53 Phosphoglucoisomerase catalyzes the following reaction:

$$\text{Glu-6-P} \rightleftharpoons \text{Fru-6-P}$$

If 0.05% of the original concentration of Glu-6-P remains at equilibrium, then the equilibrium constant of this reaction is_____.

Q.54 In a bacterium, a mutation resulted in an increase of K_S (substrate-specific constant) for ammonium from 50 μM to 5000 μM without affecting μ_{max}. The specific growth rate (μ) of the mutant growing on 0.5 mM ammonium in the medium decreases by a factor of _____.

Q.55 The total number of DNA molecules present after 5 cycles of polymerase chain reaction (PCR) starting with 3 molecules of template DNA is _____.

Q.56 Two identical, infinite conducting plates are kept parallel to each other and are separated by a distance *d*. The uniform charge densities on the plates are +σ and -σ. The electric field at a point between the two plates is $n\left(\frac{\sigma}{\varepsilon_0}\right)$, where ***n*** is _____.

(ε_0 is the permittivity of free space)

Q.57 The concentration of NaCl (in mM) formed at the stoichiometric equivalence point when 10 mL of 0.1 M HCl solution is titrated with 0.2 M NaOH solution is _____. (as an integer)

Q.58 The standard emf of a cell (in V) involving the reaction, 2 Ag^{+} (aq.) → Ag (s) + Ag^{2+} (aq.) at 298 K is _____. [Correct to two decimal places]

[Given: Ag^{+} (aq.) + e → Ag (s); E^{o} = 0.62 V and Ag^{2+} (aq.) + e → Ag^{+} (aq.); E^{o} = 0.12 V]

Q.59 Let $\vec{a} = 4\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$ and $\vec{b} = 7\hat{\imath} + \hat{\jmath} - 12\hat{k}$. If $\vec{a} \times \vec{b} = \alpha\hat{\imath} + \beta\hat{\jmath} + \gamma\hat{k}$, then the value of $\alpha + \beta + \gamma$ equals_____.

Q.60 An infinitely long solenoid of radius *r* and number of turns per unit length *n* carries a steady current *I*. The ratio of the magnetic fields at a point on the axis of the solenoid to a point *r*/2 from the axis is _____.

END OF THE QUESTION PAPER

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SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 For a reaction of the type A + B $\rightarrow$ Products, the unit of the rate constant is mol L^{-1} s^{-1}. The overall order of the reaction is

(A) 0 (B) 1 (C) 2 (D) 3

Q.2 The thermodynamic criterion for spontaneity of a process in a system under constant volume and temperature and in the absence of any work other than expansion work (if any) is

(A) change in entropy is positive
(B) change in enthalpy is negative
(C) change in Helmholtz free energy is negative
(D) change in Gibbs free energy is negative

Q.3 The number of vibrational mode(s) of a carbon dioxide molecule that can be detected using infrared spectroscopy is

(A) 1 (B) 2 (C) 3 (D) 4

Q.4 For three non-coplanar vectors **a**, **b** and **c**, the expression $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ can be written as

(A) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ (B) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{c})$

(C) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{a} \cdot \mathbf{c})$ (D) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

Q.5 Correct trend in the bond order is

(A) $O_2^+ > O_2^{2-} > O_2^-$
(B) $O_2^- > O_2^+ > O_2^{2-}$
(C) $O_2^{2-} > O_2^- > O_2^+$
(D) $O_2^+ > O_2^- > O_2^{2-}$

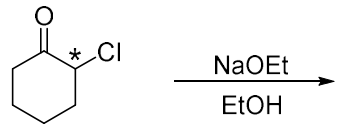
Q.6 The correct option for the metal ion present in the active site of myoglobin, hemocyanin and vitamin B_{12}, respectively, is

(A) iron, iron and zinc
(B) molybdenum, iron and copper
(C) iron, copper and cobalt
(D) molybdenum, copper and cobalt

Q.7 The correct order of wavelength (λ_{max}) of the halide to metal charge-transfer band of $[Co(NH_3)_5Cl]^{2+}$ (I), $[Co(NH_3)_5Br]^{2+}$ (II) and $[Co(NH_3)_5I]^{2+}$ (III), is

(A) III < II < I
(B) I < II < III
(C) II < III < I
(D) I < III < II

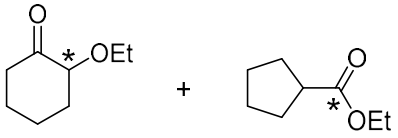
Q.8 The correct option for the major products of the following reaction is



(* represents isotopically labelled carbon atom)

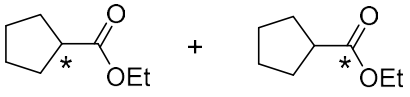
(A)

(B)

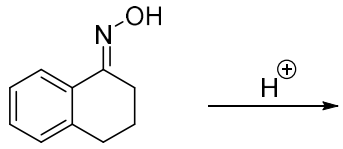


(C)

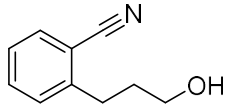
(D)



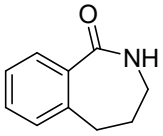
Q.9 The major product formed in the following reaction is



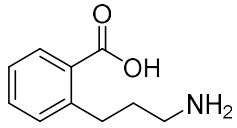
(A)



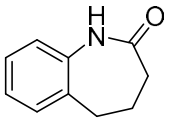
(B)



(C)



(D)



Q.10 The complementary strand for the following single strand of DNA is

5' ← A — T — G — C — T → 3'

(A)
3' ← T — A — C — G — A → 5'

(B)
3' ← A — T — G — C — T → 5'

(C)
5' ← T — A — C — G — A → 3'

(D)
5' ← A — A — C — G — T → 3'

Q. 11 – Q. 30 carry two marks each.

Q.11 The function $f(x) = x\, e^{-x^2}$ has a minimum at

(A) $x = \sqrt{2}$ (B) $x = -\sqrt{2}$ (C) $x = \frac{1}{\sqrt{2}}$ (D) $x = -\frac{1}{\sqrt{2}}$

Q.12 The correct option for the number of bending modes of vibration in each of H_2O, CS_2 and SO_2 molecules, respectively, is

(A) 1, 2 and 2
(B) 2, 2 and 1
(C) 2, 1, and 2
(D) 1, 2 and 1

Q.13 The total number of degrees of freedom of an HBr molecule that is constrained to translate along a straight line but does not have any constraints for its rotation and vibration is

(A) 6 (B) 5 (C) 4 (D) 3

Q.14 According to the kinetic theory of gases, the ratio of the root mean square velocity of molecular oxygen and molecular hydrogen at 300 K is

(A) $1:1$ (B) $1:2\sqrt{2}$ (C) $1:4$ (D) $1:16$

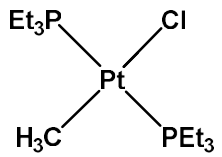
Q.15 The half-life of the chemical reaction, A $\rightarrow$ Product, for initial reactant concentrations of 0.1 and 0.4 mol L^{-1} are 200 and 50 s, respectively. The order of the reaction is

(A) 0 (B) 1 (C) 2 (D) 3

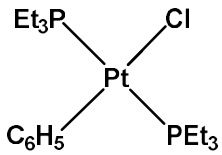
Q.16 The ratio of the nearest neighbor atomic distances in body-centered cubic (bcc) and face-centered cubic (fcc) crystals with the same unit cell edge length is

(A) $\sqrt{\frac{3}{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

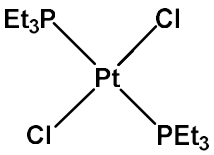
Q.17 The correct trend in the rate of substitution of Cl^- by pyridine in the following complexes is



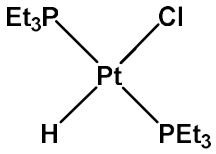
(I)



(II)



(III)



(IV)

(A) III < II < I < IV
(B) II < III < I < IV
(C) I < II < III < IV
(D) III < II < IV < I

Q.18 In qualitative inorganic analysis of metal ions, the ion which precipitates as sulfide in the presence of H_2S in warm dilute HCl is

(A) Cr^{3+} (B) Al^{3+} (C) Co^{2+} (D) Bi^{3+}

Q.19 The correct statement regarding the observed magnetic properties of NO, O_2, B_2, and C_2 in their ground state is

(A) NO, B_2, and C_2 are paramagnetic
(B) B_2, O_2 and NO are paramagnetic
(C) O_2, C_2 and NO are paramagnetic
(D) O_2, B_2 and C_2 are paramagnetic

Q.20 The observed magnetic moments of octahedral Mn^{3+}, Fe^{3+} and Co^{3+} complexes are 4.95, 6.06 and 0.00 BM, respectively. The correct option for the electronic configuration of Mn^{3+}, Fe^{3+} and Co^{3+} metal ions in these complexes, respectively, is

(A) $t_{2g}^4e_g^0$, $t_{2g}^3e_g^2$ and $t_{2g}^4e_g^2$
(B) $t_{2g}^3e_g^1$, $t_{2g}^5e_g^0$ and $t_{2g}^6e_g^0$
(C) $t_{2g}^3e_g^1$, $t_{2g}^3e_g^2$ and $t_{2g}^6e_g^0$
(D) $t_{2g}^3e_g^1$, $t_{2g}^3e_g^2$ and $t_{2g}^4e_g^2$

Q.21 Among the following compounds, the one having the lowest boiling point is

(A) $SnCl_4$ (B) $GeCl_4$ (C) $SiCl_4$ (D) CCl_4

Q.22 The correct option having one complex from each of the following pairs which is more reactive towards the oxidative addition reaction by hydrogen molecule is

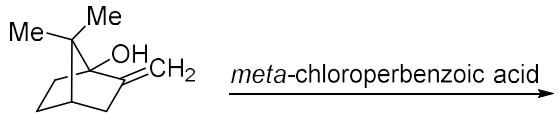
Pair 1: $IrCl(PMe_3)_3$ (I) and $IrCl(CO)(PMe_3)_2$ (II)
Pair 2: $IrCl(CO)(PPh_3)_2$ (III) and $IrCl_3(PPh_3)$ (IV)

(A) (I) and (III) (B) (I) and (IV) (C) (II) and (III) (D) (II) and (IV)

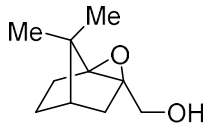
Q.23 Among the following, the correct statement is

(A) The density follows the order, Cs > Rb > Li > Na.
(B) The solubility in water follows the order, $Cs_2CO_3 > K_2CO_3 > Na_2CO_3 > Li_2CO_3$.
(C) The first ionization potential follows the order, Li > K > Na > Cs.
(D) The melting point follows the order, $MgCl_2 > BeCl_2 > CaCl_2 > SrCl_2$.

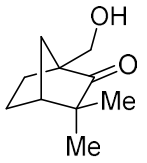
Q.24 The major product of the following reaction is



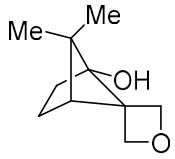
(A)



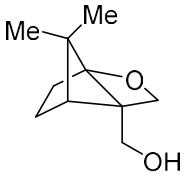
(B)



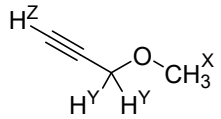
(C)



(D)

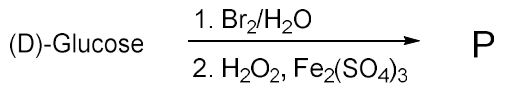


Q.25 In ^{1}H NMR spectrum of the given molecule, the correct order of chemical shifts of the labelled protons (H^X, H^Y, H^Z) is



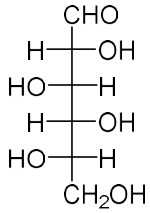
(A) $H^Z > H^X > H^Y$
(B) $H^Z > H^Y > H^X$
(C) $H^X > H^Y > H^Z$
(D) $H^Y > H^X > H^Z$

Q.26 In the following reaction of (D)-Glucose, a product P is formed.

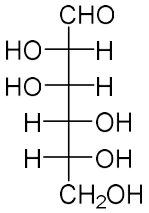


Among the following compounds, the one which will give the same product (P) under identical reaction conditions is

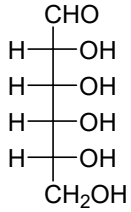
(A)



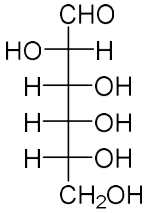
(B)



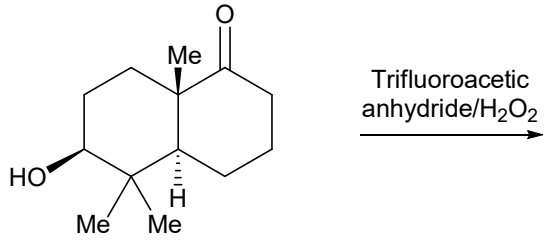
(C)



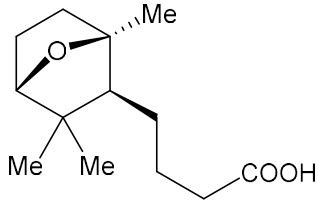
(D)



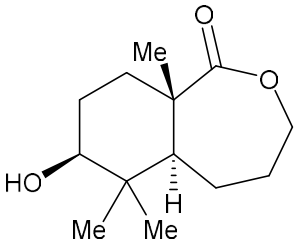
Q.27 The major product of the following reaction is



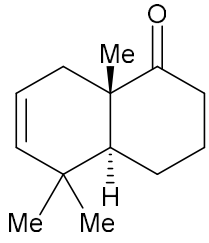
(A)



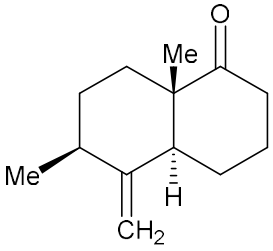
(B)



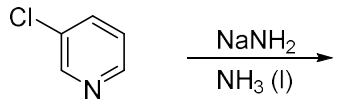
(C)



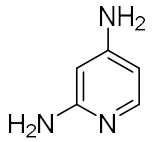
(D)



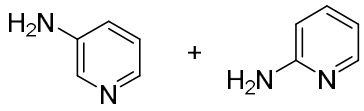
Q.28 The correct option for the product(s) of the following reaction is



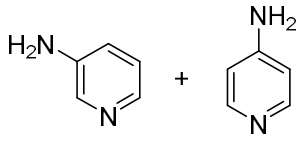
(A)



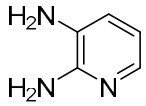
(B)



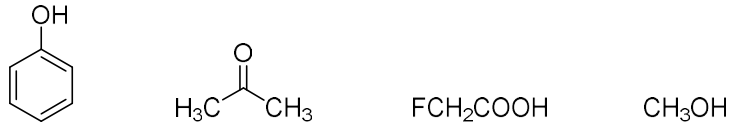
(C)



(D)



Q.29 The increasing order of acidity of the given molecules in aqueous media is



I II III IV

(A) IV < I < II < III

(B) II < I < IV < III

(C) II < IV < I < III

(D) IV < II < I < III

Q.30 The compound formed upon subjecting an aliphatic amine to Lassaigne's test is

(A) $NaNH_2$ (B) $NaNO_2$ (C) NaCN (D) NaN_3

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 The eigenvalue(s) of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is/are

(A) –1 (B) 1 (C) 2 (D) 3

Q.32 The unit of the constant '*a*' in van der Waals equation of state of a real gas can be expressed as

(A) m^6 Pa mol^{-2}
(B) m^6 J mol^{-2}
(C) m^3 Pa mol^{-2}
(D) m^3 J mol^{-2}

Q.33 Among the following, microwave active molecule(s) is/are

(A) *trans*-dichloroethene
(B) 1,2-dinitrobenzene
(C) 3-methylphenol
(D) *para*-aminophenol

Q.34 The true statement(s) regarding the brown ring test carried out in the laboratory for the detection of NO_3^- is/are

(A) Brown ring is due to the formation of the iron nitrosyl complex.
(B) Concentrated nitric acid is used for the test.
(C) The complex formed in the reaction is $[Fe(CN)_5NO]^{2-}$.
(D) The brown colored complex is paramagnetic in nature.

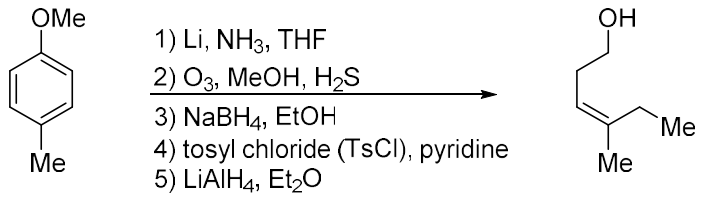
Q.35 The true statement(s) regarding the carbonic anhydrase enzyme is/are

(A) It is involved in peptide bond cleavage.
(B) Redox inactive Zn^{2+} ion is involved in the catalytic activity of this enzyme.
(C) Activated M-OH_2 (M = metal ion) acts as the nucleophile in the enzyme.
(D) The metal ion is coordinated to the side chain of histidine residues.

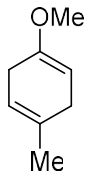
Q.36 The correct statement(s) about NO_2, NO_2^+ and CO_2 is /are

(A) Both NO_2 and CO_2 are paramagnetic.
(B) NO_2 is paramagnetic and NO_2^+ is diamagnetic
(C) Both CO_2 and NO_2^+ have linear geometry.
(D) CO_2 and NO_2^+ are isoelectronic.

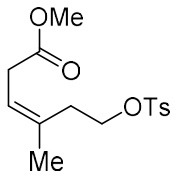
Q.37 The compound(s) formed as intermediate(s) in the following reaction sequence is/are



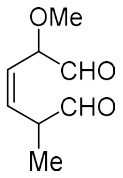
(A)



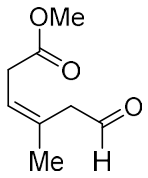
(B)



(C)



(D)



Q.38 The correct statement(s) among the following is/are

(A) Secondary structure of a polypeptide describes the number and type of amino acid residues.
(B) Uracil is a pyrimidine nucleobase.
(C) Natural fatty acids have odd number of carbon atoms.
(D) Reaction of (D)-glucose with $Ca(OH)_2$ gives a product mixture containing (D)-fructose, (D)-mannose, and (D)-glucose.

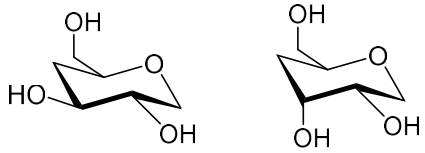
Q.39 The diastereomeric pair(s) among the following option(s) is/are

(A)

(B)

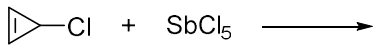
(C)

(D)



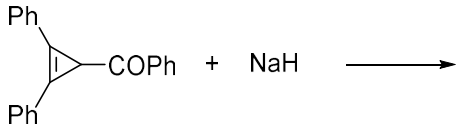
Q.40 The reaction(s) that result(s) in the formation of aromatic species is/are

(A)

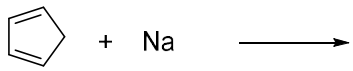


(B)

(C)



(D)



SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The bond order of N_2^+ ion is _____. (Round off to one decimal place)

Q.42 One liter of a buffer solution contains 0.004 mole of acetic acid (pK_a = 4.76) and 0.4 mole of sodium acetate. The pH of the solution is _____. (Round off to two decimal places)

Q.43 The limiting molar conductivity of La^{3+} and Cl^- ions in aqueous medium at 298 K are 209.10×10^{-4} and 76.35×10^{-4} S m^2 mol^{-1}, respectively. The transport number of Cl^- in an infinitely dilute aqueous solution of $LaCl_3$ at 298 K is _____. (Round off to two decimal places)

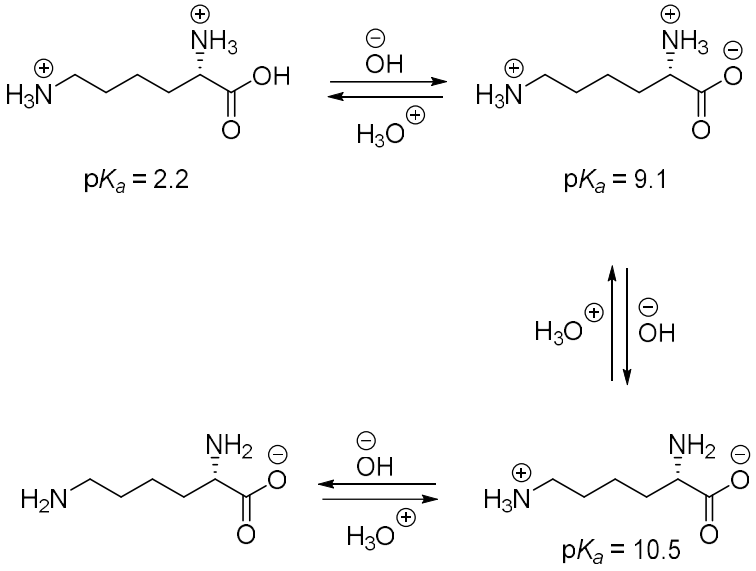
Q.44 The magnetic field strength required to excite an isolated proton to its higher spin state with an electromagnetic radiation of 300 MHz is _____ Tesla (T). (Round off to two decimal places)
[Magnetogyric ratio of proton is 26.75×10^7 rad T^{-1} s^{-1}]

Q.45 The value of n for the complex $[Fe(CO)_4(SiMe_3)]^n$ satisfying the 18-electron rule is _____.

Q.46 In the structure of P_4O_{10}, the number of P-O-P bond(s) is _____.

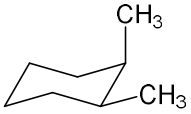
Q.47 Number of vertices in an icosahedral *closo*-borane is _____.

Q.48 Based on the information given below, the isoelectric point (p*I*) of lysine is _____. (Round off to one decimal place)



Q.49 (*R*)-2-methyl-1-butanol has a specific rotation of +13.5°. The specific rotation of 2-methyl-1-butanol containing 40% of the (*S*)-enantiomer is _____ °. (Round off to one decimal place)

Q.50 The number of gauche-butane interaction(s) in the following compound is_____.



Q. 51 – Q. 60 carry two marks each.

Q.51 The ionization energy of hydrogen atom is 13.6 eV and the first ionization energy of sodium atom is 5.1 eV. The effective nuclear charge experienced by the valance electron of sodium atom is _____. (Round off to one decimal place)

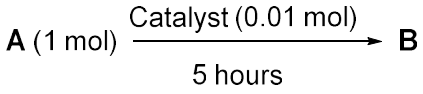
Q.52 One mole of an ideal gas is subjected to an isothermal increase in pressure from 100 kPa to 1000 kPa at 300 K. The change in Gibbs free energy of the system is _____ kJ mol^{-1}. (Round off to one decimal place)
[Given: Gas constant (R) = 8.3 J K^{-1} mol^{-1}]

Q.53 One liter of an aqueous urea solution contains 6 g of urea. The osmotic pressure of the solution at 300 K (assuming an ideal behavior) is _____ kPa. (Round off to one decimal place)
[Given: Molecular weight of urea is 60, gas constant (R) is 8.3 J K^{-1} mol^{-1}]

Q.54 A first order reflection of X-ray from {220} plane of copper crystal is observed at a glancing angle of 22°. The wavelength of the X-ray used is _____ pm. (Round off to one decimal place)
[Given: Copper forms fcc crystal with unit cell edge length of 361 pm.]

Q.55 The collision flux of a monoatomic gas on copper surface is 3.0×10^{18} m^{-2} s^{-1}. Note that copper surface forms a square lattice with lattice constant of 210 pm. If the sticking coefficient of the atom with copper is 1.0, the time taken by the gas to form a complete monolayer on the surface is _____ s. (Round off to one decimal place)

Q.56 The turnover frequency (TOF) for the catalytic reaction,

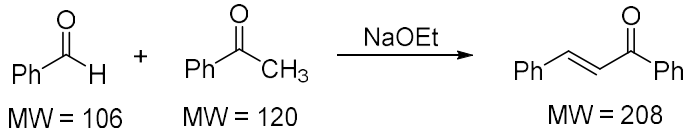


with 90% yield of the product is _____ $hour^{-1}$. (Round off to the nearest integer)

Q.57 A radioactive sample decays to 10% of its initial amount in 4600 minutes. The rate constant of this process is _____ $hour^{-1}$. (Round off to two decimal places)

Q.58 Given that the radius of the first Bohr orbit of hydrogen atom is 53 pm, the radius of its third Bohr orbit is _____ pm. (Round off to the nearest integer)

Q.59 5.3 g of benzaldehyde was reacted with an excess of acetophenone to produce 5.2 g of the enone product as per the reaction shown below. The yield of the reaction is _____ %. (Round off to the nearest integer)



(MW = Molecular weight)

Q.60 Assume that the reaction of MeMgBr with ethylacetate proceeds with 100% conversion to give *tert*-butanol. The volume of 0.2 M solution of MeMgBr required to convert 10 mL of a 0.025 M solution of ethylacetate to *tert*-butanol is _____ mL. (Round off to one decimal place)

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

Q.1 Which of the following is associated with a divergent plate boundary?

(A) Ridge (B) Trench (C) Island arc (D) Accretionary prism

Q.2 Shear waves do not travel through the

(A) upper continental crust
(B) upper mantle
(C) lower mantle
(D) outer core

Q.3 Fossils of burrows and footprints are known as

(A) pseudofossils (B) coprolites (C) body fossils (D) trace fossils

Q.4 Horst and graben structures are typically formed by

(A) normal faulting
(B) strike-slip faulting
(C) reverse faulting
(D) thrust faulting

Q.5 *Dicroidium* is known from the

(A) Pachmarhi Formation
(B) Raniganj Formation
(C) Panchet Formation
(D) Denwa Formation

Q.6 Permian is a/an

(A) Eon (B) Era (C) Epoch (D) Period

Q.7 Polymorphic minerals have

(A) different crystal forms and identical composition
(B) different crystal forms and different compositions
(C) identical crystal form and different compositions
(D) identical crystal form and identical composition

Q.8 Turbidites commonly form in

(A) fluvial environment
(B) deep marine environment
(C) tidal flat environment
(D) beach environment

Q.9 Which of the following is the fundamental constituent of humic coal?

(A) Mineral matter (B) Maceral (C) Lithotype (D) Kerogen

Q.10 Which of the following mineral assemblages characterizes blueschist facies metamorphism of a mafic rock?

(A) Glaucophane + lawsonite
(B) Hornblende + plagioclase ± epidote ± garnet
(C) Omphacite + garnet
(D) Phengite + chlorite + garnet

Q. 11 – Q. 30 carry two marks each.

Q.11 Match the geomorphic features in Group I with corresponding environments in Group II.

Group I	**Group II**
P. Eskers	1. Beach
Q. Natural levee	2. Volcanic
R. Chenier ridge	3. Glacial
S. Plug dome	4. Fluvial

(A)	(B)	(C)	(D)
P-3	P-3	P-2	P-2
Q-4	Q-1	Q-1	Q-4
R-1	R-4	R-4	R-1
S-2	S-2	S-3	S-3

Q.12 Match the bivalves in Group I with their modes of life in Group II

Group I	**Group II**
P. *Gryphaea*	1. Swimming
Q. *Mya*	2. Unattached, free-lying
R. *Mytilus*	3. Burrowing
S. *Pecten*	4. Sessile byssate

(A)	(B)	(C)	(D)
P-4	P-4	P-2	P-2
Q-1	Q-3	Q-3	Q-1
R-2	R-2	R-4	R-4
S-3	S-1	S-1	S-3

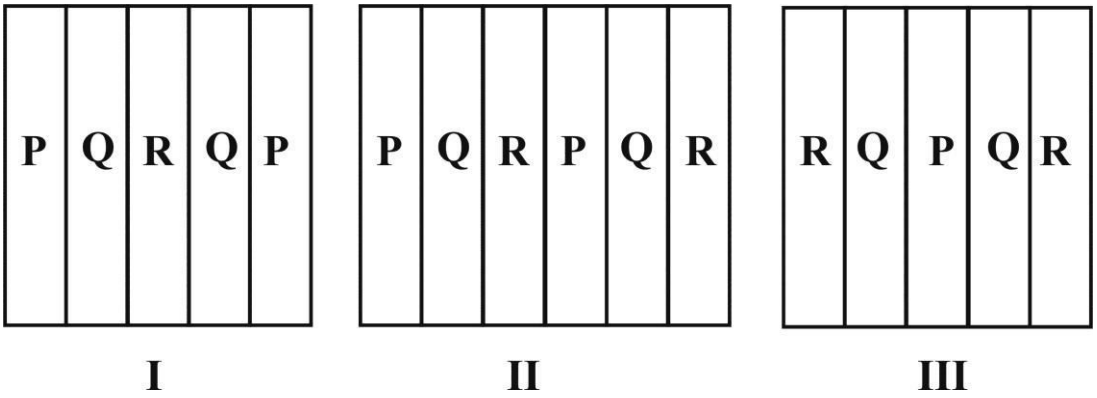
Q.13 Which of the following changes occurred during the evolution of Equidae?

(A) Number of lateral digits or toes increases
(B) Decrease in hypsodonty
(C) Lengthening of skull in front of the orbit
(D) Limb ratios remained constant

Q.14 Which of the following rocks occurs typically in a ductile shear zone?

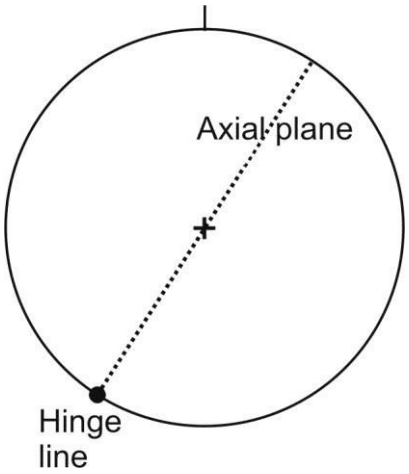
(A) Gouge (B) Breccia (C) Pseudotachylite (D) Mylonite

Q.15 Figures I, II and III are the outcrop patterns of three inclined beds P, Q and R on a flat ground. P is the oldest and R is the youngest amongst these beds. Identify the correct option that explains repetition of beds in the figures.



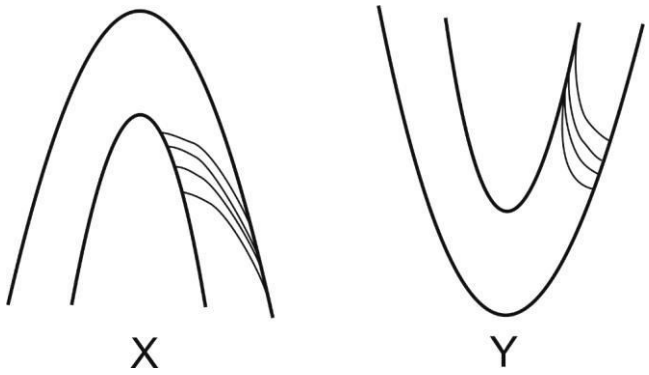
(A) I- anticline; II- syncline; III- fault
(B) I- syncline; II-anticline; III- fault
(C) I- fault; II- anticline; III- syncline
(D) I- syncline; II- fault; III-anticline

Q.16 The figure shows stereographic projections of the axial plane and the hinge line of a fold. Which one of the following folds is represented in the figure?



(A) Upright plunging (B) Upright non-plunging
(C) Reclined (D) Recumbent

Q.17 Figures X and Y show profile sections of folds traced by two cross-bedded sandstone beds. Which one of the following is the correct interpretation?



(A) X- Antiformal anticline; Y- Antiformal syncline
(B) X- Antiformal anticline; Y- Synformal syncline
(C) X- Overturned antiform; Y- Overturned synform
(D) X-Antiformal syncline; Y- Synformal anticline

Q.18 On a geological map of a flat area, the plunge direction of an antiformal anticline is

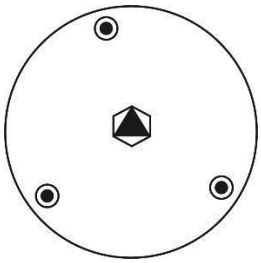
(A) towards the fold closure
(B) away from the fold closure
(C) towards the dip direction of the left limb
(D) towards the dip direction of the right limb

Q.19 Match the Precambrian lithounits in Group I with corresponding cratons in Group II.

Group I	**Group II**
P. Kolhan Group	1. Aravalli
Q. Debari Group	2. Bastar
R. Bababudan Group	3. Singhbhum
S. Sukma Group	4. Dharwar

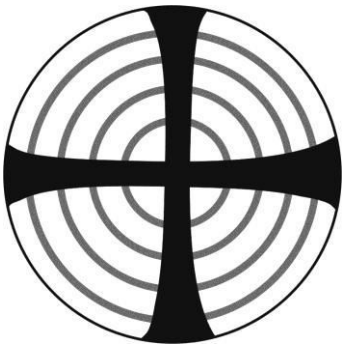
(A)	(B)	(C)	(D)
P-2	P-3	P-3	P-2
Q-4	Q-4	Q-1	Q-1
R-1	R-1	R-4	R-4
S-3	S-2	S-2	S-3

Q.20 The stereographic projection given below represents a



(A) trigonal rhombohedron
(B) hexagonal scalenohedron
(C) trigonal dipyramid
(D) hexagonal dipyramid

Q.21 Which of the following minerals exhibits the optic interference figure given below?



(A) Quartz (B) Diopside (C) Garnet (D) Orthoclase

Q.22 Match the minerals in Group I with corresponding characteristic properties in Group II.

Group-I	**Group-II**
P. Labradorite	1. High specific gravity
Q. Actinolite	2. Play of colors
R. Fluorite	3. Acicular habit
S. Barite	4. Four sets of cleavage

(A)	(B)	(C)	(D)
P-4	P-4	P-2	P-2
Q-1	Q-1	Q-3	Q-3
R-2	R-3	R-4	R-1
S-3	S-2	S-1	S-4

Q.23 Match the metamorphic rocks in Group I with corresponding parent rocks in Group II.

Group-I	**Group-II**
P. Amphibolite	1. Marl
Q. Khondalite	2. Shale
R. Calc-gneiss	3. Sandstone
S. Quartzite	4. Basalt

(A)	(B)	(C)	(D)
P-4	P-4	P-2	P-4
Q-3	Q-2	Q-4	Q-3
R-1	R-1	R-1	R-2
S-2	S-3	S-3	S-1

Q.24 Match the fuels in Group I with corresponding areas of occurrence in Group II.

Group I	**Group II**
P. Uranium	1. Vastan, Gujarat
Q. Lignite	2. Singrauli, Madhya Pradesh
R. Bituminous Coal	3. Digboi, Assam
S. Petroleum	4. Jaduguda, Jharkhand

(A)	(B)	(C)	(D)
P-4	P-4	P-3	P-2
Q-1	Q-1	Q-4	Q-4
R-3	R-2	R-2	R-1
S-2	S-3	S-1	S-3

Q.25 Match the features in Group I with their characteristic rocks / rock suite in Group II.

Group I	**Group II**
P. Continental arc	1. Olivine tholeiite
Q. Mid-oceanic ridge	2. Alkaline rock
R. Collision zones	3. Andesite
S. Continental rift	4. Ophiolite

(A)	(B)	(C)	(D)
P-2	P-1	P-3	P-3
Q-1	Q-3	Q-1	Q-2
R-3	R-4	R-4	R-1
S-4	S-2	S-2	S-4

Q.26 Which of the following statements are correct?

I. In a planar tabular cross-bedding, the upper and lower bounding surfaces are planar and parallel
II. In a planar tabular cross-bedding, the upper and lower bounding surfaces are planar but not parallel
III. In a trough cross-bedding, both the upper and the lower bounding surfaces are curved
IV. In a trough cross-bedding, the upper bounding surface is planar and the lower bounding surface is curved

(A) II and III (B) II, III and IV (C) I, II and III (D) I, III and IV

Q.27 Match the sedimentary structures in Group I with the corresponding processes in Group II.

Group I	**Group II**
P. Stromatolite	1. Desiccation
Q. Flute cast	2. Liquefaction
R. Mud crack	3. Biostratification
S. Dish and pillar	4. Erosion

(A)	(B)	(C)	(D)
P-3	P-3	P-4	P-2
Q-4	Q-4	Q-3	Q-3
R-1	R-2	R-1	R-4
S-2	S-1	S-2	S-1

Q.28 An ore mineral has the following physical properties: (1) metallic lustre, (2) steel-grey color, (3) two sets of octahedral cleavage, (4) high specific gravity, and (5) makes mark on paper. Identify the ore mineral.

(A) sphalerite (B) magnetite (C) galena (D) bornite

Q.29 Which of the following are the characteristics of a typical porphyry copper deposit?

I. Associated with granitic rocks
II. Low-grade and high tonnage of ore
III. Presence of alteration zones
IV. Occurrence in convergent tectonic setting

(A) II only (B) I and IV only (C) I, II, III and IV (D) I, III and IV only

Q.30 Match the metal deposits in Group I with the corresponding processes in Group II.

Group I	**Group II**
P. Aluminum	1. Mechanical concentration
Q. Iron	2. Contact metasomatism
R. Tungsten	3. Residual concentration
S. Thorium	4. Chemical sedimentation

(A)	(B)	(C)	(D)
P-1	P-3	P-3	P-2
Q-3	Q-4	Q-4	Q-3
R-2	R-2	R-1	R-4
S-4	S-1	S-2	S-1

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Which of the following statement(s) is/are correct for a plot of ($^{87}Sr/^{86}Sr$) versus ($^{87}Rb/^{86}Sr$) of a rock which has evolved in a closed system?

(A) The slopes of the lines of evolution for the minerals in the rock are all equal and positive
(B) The slopes of the lines of evolution for the minerals in the rock are all equal and negative
(C) The slope of the isochron is identical in direction to the slope of the lines of evolution for the minerals in the rock
(D) The slope of the isochron is opposite in direction to the slope of the lines of evolution for the minerals in the rock

Q.32 Which of the following statements is/are correct with reference to subsurface water?

(A) The perched water table occurs below the water table
(B) Vadose water occurs in the zone of aeration
(C) For significant groundwater underflow, the medium must be highly permeable
(D) Intersection of the water table with the land surface can result in the formation of a spring

Q.33 Which of the following geomorphic features indicate(s) the presence of a fault?

(A) Triangular facets (B) Sudden topographic jump
(C) Ox-bow lake (D) Point bar

Q.34 Which of the following gastropods has/have siphonal canal?

(A) *Cerithium* (B) *Cypraea* (C) *Fusus* (D) *Murex*

Q.35 Which of the following is/are mode(s) of preservation of fossils?

(A) Small organisms get trapped on viscous resins secreted by trees
(B) Leaves preserved as thin films of carbon
(C) Negative impressions of hard parts of organisms
(D) Mineral growths that branch like ferns

Q.36 Select the pair(s) that is/are correctly matched.

(A) Lathi Formation – Jurassic wood fossils
(B) Pinjor Formation – Cretaceous ammonoid fossils
(C) Panchet Formation – Triassic reptile fossils
(D) Kota Formation – Permian glossopterid fossils

Q.37 Choose the reaction(s) indicating metamorphism of pelites at T> 600 °C and P< 8 kbar.

(A) albite + actinolite $\rightarrow$ hornblende + quartz
(B) muscovite + quartz $\rightarrow$ sillimanite + K-feldspar + H_2O
(C) muscovite + biotite + quartz + H_2O $\rightarrow$ sillimanite + melt
(D) albite $\rightarrow$ jadeite + quartz

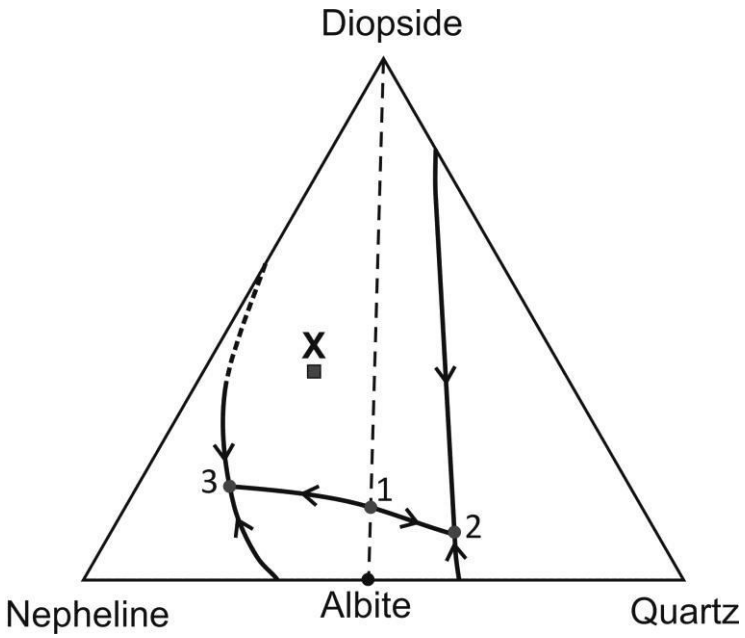
Q.38 Select the correct statement(s).

(A) Laccoliths are convex upward whereas lopoliths are convex downward bodies
(B) Laccoliths can form from any type of magma whereas lopoliths form only from basaltic magma
(C) Laccoliths show layered structures whereas lopoliths lack any layering
(D) Laccoliths are smaller in dimensions than lopoliths

Q.39 Select the correct statement(s).

(A) Phenocrysts are found in igneous rocks whereas porphyroblasts are found in metamorphic rocks
(B) Phenocrysts are indicators of crystallization history whereas porphyroblasts indicate metamorphic history
(C) Porphyroblasts are relict grains, relatively larger than matrix minerals in deformed igneous rocks
(D) Poikiloblastic texture is found in igneous rocks whereas poikilitic texture is found in metamorphic rocks

Q.40 Based on the given schematic ternary phase diagram, choose the correct statement(s).



(A) Point 1 is the thermal minimum in the system.
(B) The diopside – albite join acts as a thermal barrier in the system.
(C) Point 2 and 3 are ternary eutectics.
(D) Melt of composition X will give rise to a rock containing nepheline+albite.

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The intensity of an earthquake of magnitude 8 on the Richter scale is greater than the intensity of an earthquake of magnitude 5 on the same scale by _____ times.

Q.42 Assume: (i) geothermal gradient = 25 °C/km in the crust, (ii) density of the crustal rocks = 3000 kg/m^3, and (iii) acceleration due to gravity = 10 m/s^2. Based on these values, the lithostatic pressure at a point where temperature is 400 °C will be _____ MPa.

Q.43 The radii of A^{+2} and B^- ions are 1.12Å and 1.31Å, respectively. The coordination number of A^{+2} in mineral AB_2 is_____.

Q.44 In a sedimentary rock, the diameters of two grains A and B are 1ϕ and 3ϕ, respectively. The difference in diameters between A and B is _____ mm (rounded off to two decimal places).

Q.45 A light ray passes from mineral A to mineral B having refractive indices of 1.750 and 1.430, respectively. The limiting value of angle of incidence above which the light ray undergoes total internal reflection is _____ degree (rounded off to one decimal place).

Q.46 Throw and heave of a bed offset by a normal fault are 100 m and 200 m, respectively. The dip of the fault plane is _____ degree (rounded off to one decimal place).

Q.47 The difference between Si/O ratios of K-feldspar and olivine is _____ (answer in two decimal places).

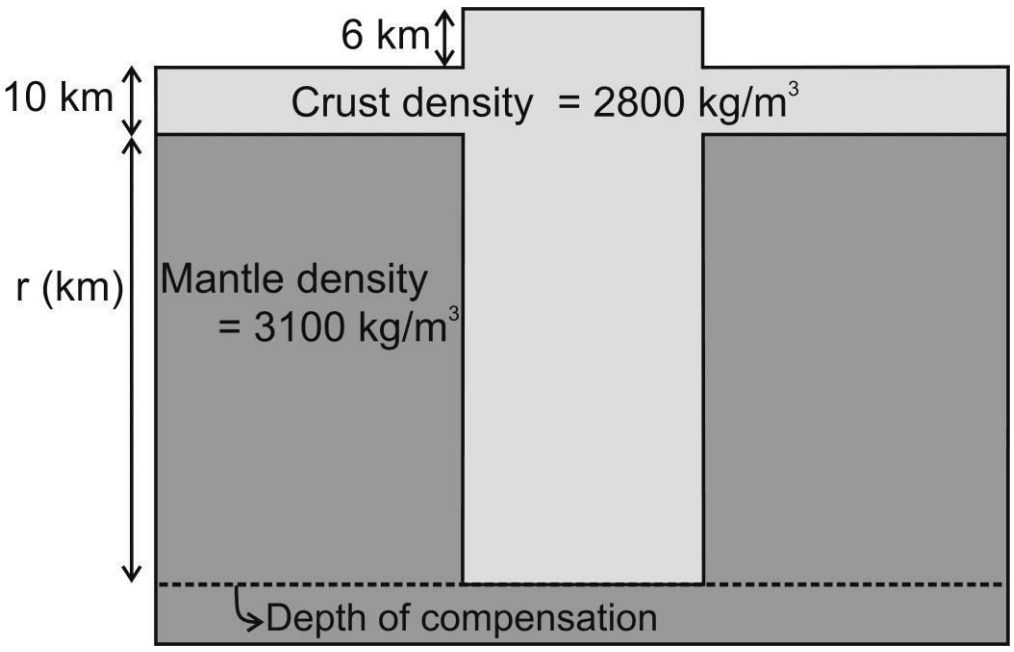
Q.48 If a crystal contains 5 faces and 8 edges, the number of vertices in the crystal is_____.

Q.49 At a temperature of 298.15 Kelvin, the free energy change of a reaction (ΔG^0) is 19.737 kCal/mole. If the universal gas constant (R) = 1.98717 Calorie/degree/mole, the $\log_{10}$ of the equilibrium constant K is _____ (rounded off to two decimal places).

Q.50 A normal fault displaces a sandstone bed such that the dip-slip and the strike-slip components are 3 m and 4 m, respectively. The net-slip of the displacement is_____ m.

Q. 51 – Q. 60 carry two marks each.

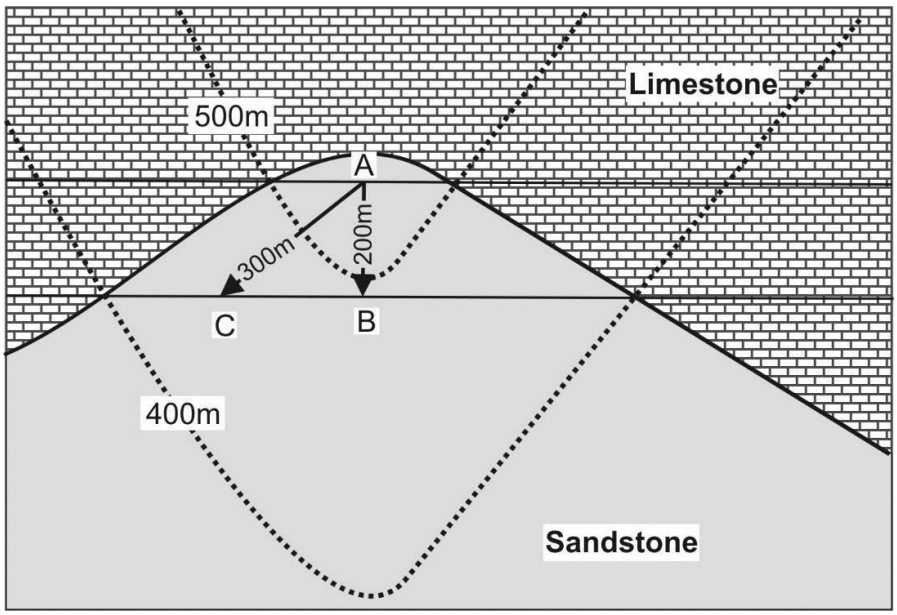
Q.51 In the given diagram, a 6 km high plateau is supported by a crustal root of thickness r. The system is in isostatic equilibrium as per Airy's hypothesis of isostasy. Densities of the crust and the mantle are 2800 kg/m^3 and 3100 kg/m^3, respectively. Assuming the acceleration due to gravity to be same throughout the region, the thickness of the root (r) is _____ km.



Q.52 A 50 kg granite boulder gets dislodged from a cliff of height 20 m and undergoes an absolute vertical free fall. If the acceleration due to gravity is 10 m/s^2, the boulder will hit the ground with a velocity of _____ m/s.

Q.53 Mass and volume of a fully dried soil sample are 500 g and 250 cm^3, respectively. The average density of the particles in the soil sample is 2.5 g/cm^3. The void ratio of the soil sample is _____%.

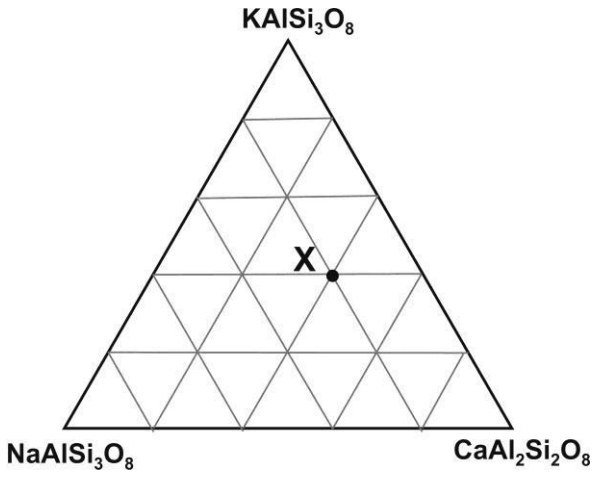
Q.54 The geological map shows the contact between sandstone and limestone. The two dotted curves are the contours of 400 m and 500 m, respectively. The difference between the dip angles of the contact surface along the AB and AC directions is _____ degree (rounded off to two decimal places).



Q.55 A tabular ore body of 9 km^2 area and an average thickness of 9 m has a density of 3000 kg/m^3. The tonnage (in million tonnes) of the ore body is _____.

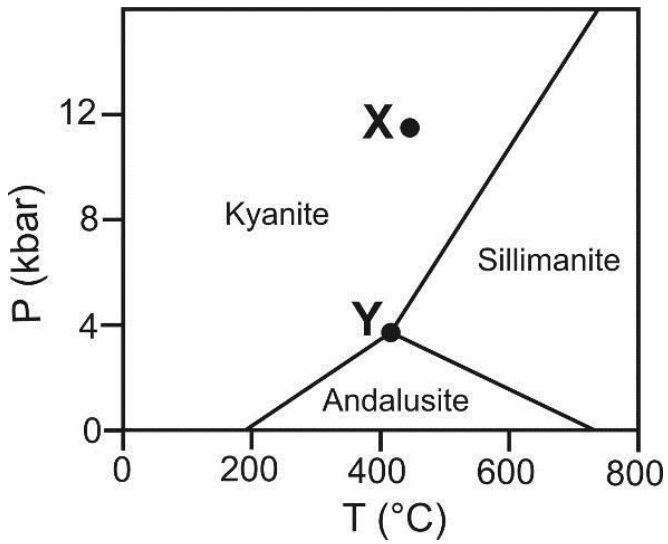
Q.56 Assume that the orbit of the earth is a circle of radius 150×10^6 km. The gravitational constant and the earth's orbital velocity are given as 6.7×10^{-11} Nm^2/kg^2 and 30×10^3 m/s, respectively. The calculated mass of the sun is _____$\times10^{30}$ kg (rounded off to two decimal places).

Q.57 The difference between X_{An} and X_{Ab} in a feldspar represented by X in the given triangular diagram is _____ (answer in one decimal place).



Q.58 Two vertical wells penetrating a confined aquifer are 200 m apart. The water surface elevations in these wells are 35 m and 40 m above a common reference datum. The discharge per unit area through the aquifer is 0.05 m/day. Using Darcy's law, the coefficient of permeability is _____m/day.

Q.59 The given P–T diagram shows the relative stability ranges of andalusite, sillimanite and kyanite. The difference in degrees of freedom at points X and Y is _____.



Q.60 The core and rim compositions of garnet are $(Fe_{0.75}Ca_{0.90}Mn_{1.35})Al_2Si_3O_{12}$ and $(Fe_{0.90}Ca_{1.35}Mn_{0.75})Al_2Si_3O_{12}$, respectively. The difference in mole fractions of spessertine between the core and the rim is _____ (answer in one decimal place).

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Special Instructions / Useful Data

$\mathbb{R}$	The set of all real numbers
P^T	Transpose of the matrix P
$\mathbb{R}^n$	$\left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} : x_i \in \mathbb{R}, i = 1, 2, \ldots, n \right\}$
f'	Derivative of the differentiable function f
I_n	$n \times n$ identity matrix
$P(E)$	Probability of the event E
$E(X)$	Expectation of the random variable X
$Var(X)$	Variance of the random variable X
i.i.d.	Independently and identically distributed
$U(a, b)$	Continuous uniform distribution on $(a, b), -\infty < a < b < \infty$
$Exp(\lambda)$	Exponential distribution with probability density function, for $\lambda > 0$, $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
$\Phi(a)$	$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-\frac{u^2}{2}} du$
χ^2_n	Central Chi-squared distribution with n degrees of freedom
$t_{n,\alpha}$	A constant such that $P\left(X > t_{n,\alpha}\right) = \alpha$, where X has Student's t-distribution with n degrees of freedom
$n!$	$n(n-1)\cdots 3 \cdot 2 \cdot 1$ for $n = 1, 2, 3$ ..., and $0! = 1$
$\Phi(1.65) = 0.950, \quad \Phi(1.96) = 0.975$ $t_{4,\,0.05} = 2.132, \qquad t_{4,\,0.10} = 1.533$	

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers. Which one of the following statements is always TRUE?

(A) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{x_n\}_{n\geq 1}$ is monotone

(B) If $\{x_n^2\}_{n\geq 1}$ is a convergent sequence, then the sequence $\{x_n\}_{n\geq 1}$ does not converge

(C) If the sequence $\{|x_{n+1} - x_n|\}_{n\geq 1}$ converges to 0, then the series $\sum_{n=1}^{\infty} x_n$ is convergent

(D) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{e^{x_n}\}_{n\geq 1}$ is also a convergent sequence

Q.2 Consider the function $f(x, y) = x^3 - 3xy^2,\ x, y \in \mathbb{R}$. Which one of the following statements is TRUE?

(A) f has a local minimum at $(0, 0)$

(B) f has a local maximum at $(0, 0)$

(C) f has global maximum at $(0, 0)$

(D) f has a saddle point at $(0, 0)$

Q.3 If $F(x) = \int_{x^3}^{4} \sqrt{4 + t^2}\, dt$, for $x \in \mathbb{R}$, then $F'(1)$ equals

(A) $-3\sqrt{5}$ (B) $-2\sqrt{5}$ (C) $2\sqrt{5}$ (D) $3\sqrt{5}$

Q.4 Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$ and $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}$. Suppose that $\begin{bmatrix}3\\-2\end{bmatrix} = \alpha\begin{bmatrix}1\\2\end{bmatrix} + \beta\begin{bmatrix}2\\1\end{bmatrix}$ and $T\left(\begin{bmatrix}3\\-2\end{bmatrix}\right) = \begin{bmatrix}a\\b\end{bmatrix}$. Then $\alpha + \beta + a + b$ equals

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Q.5 Two biased coins C_1 and C_2 have probabilities of getting heads $\frac{2}{3}$ and $\frac{3}{4}$, respectively, when tossed. If both coins are tossed independently two times each, then the probability of getting exactly two heads out of these four tosses is

(A) $\frac{1}{4}$ (B) $\frac{37}{144}$ (C) $\frac{41}{144}$ (D) $\frac{49}{144}$

Q.6 Let X be a discrete random variable with the probability mass function

$$P(X=n)=\begin{cases}\dfrac{-2c}{n}, & n=-1,-2,\\ d, & n=0,\\ cn, & n=1,2,\\ 0, & \text{otherwise,}\end{cases}$$

where c and d are positive real numbers. If $P(|X|\le 1)=3/4$, then $E(X)$ equals

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q.7 Let X be a Poisson random variable and $P(X=1)+2P(X=0)=12P(X=2)$. Which one of the following statements is TRUE?

(A) $0.40 < P(X=0) \le 0.45$ (B) $0.45 < P(X=0) \le 0.50$

(C) $0.50 < P(X=0) \le 0.55$ (D) $0.55 < P(X=0) \le 0.60$

Q.8 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. discrete random variables with the probability mass function

$$P(X_1=m)=\begin{cases}\dfrac{(\log_e 2)^m}{2(m!)}, & m=0,1,2,\ldots,\\ 0, & \text{otherwise.}\end{cases}$$

If $S_n=X_1+X_2+\cdots+X_n$, then which one of the following sequences of random variables converges to 0 in probability?

(A) $\frac{S_n}{n\log_e 2}$ (B) $\frac{S_n-n\log_e 2}{n}$ (C) $\frac{S_n-\log_e 2}{n}$ (D) $\frac{S_n-n}{\log_e 2}$

Q.9 Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function

$$f(x)=\frac{1}{2\sqrt{2\pi}}\left[e^{-\frac{1}{2}(x-2\mu)^2}+e^{-\frac{1}{2}(x-4\mu)^2}\right],\quad -\infty < x < \infty.$$

If $T=X_1+X_2+\cdots+X_n$, then which one of the following is an unbiased estimator of μ?

(A) $\frac{T}{n}$ (B) $\frac{T}{2n}$ (C) $\frac{T}{3n}$ (D) $\frac{T}{4n}$

Q.10 Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. Instead of observing $X_1, X_2, \ldots, X_n$, we observe $Y_1, Y_2, \ldots, Y_n$, where $Y_i = e^{X_i},\ i = 1, 2, \ldots, n$. To test the hypothesis

$$H_0: \theta = 1 \text{ against } H_1: \theta \neq 1$$

based on the random sample $Y_1, Y_2, \ldots, Y_n$, the rejection region of the likelihood ratio test is of the form, for some $c_1 < c_2$,

(A) $\sum_{i=1}^{n} Y_i \leq c_1$ or $\sum_{i=1}^{n} Y_i \geq c_2$ (B) $c_1 \leq \sum_{i=1}^{n} Y_i \leq c_2$

(C) $c_1 \leq \sum_{i=1}^{n} \log_e Y_i \leq c_2$ (D) $\sum_{i=1}^{n} \log_e Y_i \leq c_1$ or $\sum_{i=1}^{n} \log_e Y_i \geq c_2$

Q. 11 – Q. 30 carry two marks each.

Q.11 $\sum_{n=4}^{\infty} \frac{6}{n^2-4n+3}$ equals

(A) $\frac{5}{2}$ (B) 3 (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

Q.12 $\lim_{n\to\infty} \frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(\pi^n+e^n)^{1/n}\ \log_e n}$ equals

(A) $\frac{1}{\pi}$ (B) $\frac{1}{e}$ (C) $\frac{e}{\pi}$ (D) $\frac{\pi}{e}$

Q.13 A possible value of $b \in \mathbb{R}$ for which the equation $x^4 + bx^3 + 1 = 0$ has no real root is

(A) $\frac{-11}{5}$ (B) $\frac{-3}{2}$ (C) 2 (D) $\frac{5}{2}$

Q.14 Let the Taylor polynomial of degree 20 for $\frac{1}{(1-x)^3}$ at $x = 0$ be $\sum_{n=0}^{20} a_n x^n$. Then a_{15} is

(A) 136 (B) 120 (C) 60 (D) 272

Q.15 The length of the curve $y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3} + 7$ from $x = 1$ to $x = 8$ equals

(A) $\frac{99}{8}$ (B) $\frac{117}{8}$ (C) $\frac{99}{4}$ (D) $\frac{117}{4}$

Q.16 The volume of the solid generated by revolving the region bounded by the parabola $x = 2y^2 + 4$ and the line $x = 6$ about the line $x = 6$ is

(A) $\frac{78\pi}{15}$ (B) $\frac{91\pi}{15}$ (C) $\frac{64\pi}{15}$ (D) $\frac{117\pi}{15}$

Q.17 Let P be a 3×3 non-null real matrix. If there exist a 3×2 real matrix Q and a 2×3 real matrix R such that $P = QR$, then

(A) $P\boldsymbol{x} = \boldsymbol{0}$ has a unique solution, where $\boldsymbol{0} \in \mathbb{R}^3$

(B) there exists $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\boldsymbol{x} = \boldsymbol{b}$ has no solution

(C) there exists a non-zero $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\boldsymbol{x} = \boldsymbol{b}$ has a unique solution

(D) there exists a non-zero $\boldsymbol{b} \in \mathbb{R}^3$ such that $P^T\boldsymbol{x} = \boldsymbol{b}$ has a unique solution

Q.18 If $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & -1 \end{bmatrix}$ and $6P^{-1} = aI_3 + bP - P^2$, then the ordered pair (a, b) is

(A) $(3, 2)$ (B) $(2, 3)$ (C) $(4, 5)$ (D) $(5, 4)$

Q.19 Let E, F and G be any three events with $P(E) = 0.3$, $P(F|E) = 0.2$, $P(G|E) = 0.1$ and $P(F \cap G|E) = 0.05$. Then $P(E - (F \cup G))$ equals

(A) 0.155 (B) 0.175 (C) 0.225 (D) 0.255

Q.20 Let E and F be any two independent events with $0 < P(E) < 1$ and $0 < P(F) < 1$. Which one of the following statements is **NOT** TRUE?

(A) $P(\text{Neither } E \text{ nor } F \text{ occurs}) = (P(E) - 1)(P(F) - 1)$

(B) $P(\text{Exactly one of } E \text{ and } F \text{ occurs}) = P(E) + P(F) - P(E)P(F)$

(C) $P(E \text{ occurs but } F \text{ does not occur}) = P(E) - P(E \cap F)$

(D) $P(E \text{ occurs given that } F \text{ does not occur}) = P(E)$

Q.21 Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{1}{3}\, x^7 e^{-x^2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then the distribution of the random variable $W = 2X^2$ is

(A) χ_2^2 (B) χ_4^2 (C) χ_6^2 (D) χ_8^2

Q.22 Let X be a continuous random variable with the probability density function

$$f(x) = \frac{e^x}{(1+e^x)^2}, \quad -\infty < x < \infty.$$

Then $E(X)$ and $P(X > 1)$, respectively, are

(A) 1 and $(1+e)^{-1}$ (B) 0 and $2(1+e)^{-2}$

(C) 2 and $(2+2e)^{-1}$ (D) 0 and $(1+e)^{-1}$

Q.23 The lifetime (in years) of bulbs is distributed as an $Exp(1)$ random variable. Using Poisson approximation to the binomial distribution, the probability (round off to 2 decimal places) that out of the fifty randomly chosen bulbs at most one fails within one month equals

(A) 0.05 (B) 0.07 (C) 0.09 (D) 0.11

Q.24 Let X follow a beta distribution with parameters $m\ (>0)$ and 2. If $P\left(X \le \frac{1}{2}\right) = \frac{1}{2}$, then $Var(X)$ equals

(A) $\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{25}$ (D) $\frac{1}{40}$

Q.25 Let X_1, X_2 and X_3 be i.i.d. $U(0,1)$ random variables. Then $P(X_1 > X_2 + X_3)$ equals

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q.26 Let X and Y be i.i.d. $U(0,1)$ random variables. Then $E(X|X > Y)$ equals

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.27 Let -1 and 1 be the observed values of a random sample of size two from $N(\theta, \theta)$ distribution. The maximum likelihood estimate of θ is

(A) 0 (B) 2 (C) $\frac{-\sqrt{5}-1}{2}$ (D) $\frac{\sqrt{5}-1}{2}$

Q.28 Let X_1 and X_2 be a random sample from a continuous distribution with the probability density function

$$f(x) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x-\theta}{\theta}}, & x > \theta, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta > 0$. If $X_{(1)} = \min\{X_1, X_2\}$ and $\overline{X} = \frac{(X_1+X_2)}{2}$, then which one of the following statements is TRUE?

(A) $\left(\overline{X}, X_{(1)}\right)$ is sufficient and complete
(B) $\left(\overline{X}, X_{(1)}\right)$ is sufficient but not complete
(C) $\left(\overline{X}, X_{(1)}\right)$ is complete but not sufficient
(D) $\left(\overline{X}, X_{(1)}\right)$ is neither sufficient nor complete

Q.29 Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)$. To test the hypothesis

$H_0: f(x) = \frac{1}{\sqrt{\pi}}\, e^{-x^2}, -\infty < x < \infty$ against $H_1: f(x) = e^{-2|x|}, -\infty < x < \infty$,

the rejection region of the most powerful size α test is of the form, for some $c > 0$,

(A) $\sum_{i=1}^{n}(X_i - 1)^2 \geq c$ (B) $\sum_{i=1}^{n}(X_i - 1)^2 \leq c$

(C) $\sum_{i=1}^{n}(|X_i| - 1)^2 \geq c$ (D) $\sum_{i=1}^{n}(|X_i| - 1)^2 \leq c$

Q.30 Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. To test $H_0: \theta = 0$ against $H_1: \theta = 1$, assume that the critical region is given by $\frac{1}{n}\sum_{i=1}^{n} X_i > \frac{3}{4}$. Then the minimum sample size required so that $P(\text{Type I error}) \leq 0.05$ is

(A) 3 (B) 4 (C) 5 (D) 6

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} x_n$ converges. Which of the following statements is (are) always TRUE?

(A) The series $\sum_{n=1}^{\infty} \sqrt{x_n x_{n+1}}$ converges

(B) $\lim_{n\to\infty} n\, x_n = 0$

(C) The series $\sum_{n=1}^{\infty} \sin^2 x_n$ converges

(D) The series $\sum_{n=1}^{\infty} \frac{\sqrt{x_n}}{1+\sqrt{x_n}}$ converges

Q.32 Let $f: \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and differentiable on $(-\infty, 0) \cup (0, \infty)$. Which of the following statements is (are) always TRUE?

(A) If f is differentiable at 0 and $f'(0) = 0$, then f has a local maximum or a local minimum at 0

(B) If f has a local minimum at 0, then f is differentiable at 0 and $f'(0) = 0$

(C) If $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$, then f has a global maximum at 0

(D) If $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then f has a global maximum at 0

Q.33 Let P be a 2×2 real matrix such that every non-zero vector in $\mathbb{R}^2$ is an eigenvector of P. Suppose that λ_1 and λ_2 denote the eigenvalues of P and $P\begin{bmatrix}\sqrt{2}\\ \sqrt{3}\end{bmatrix} = \begin{bmatrix}2\\ t\end{bmatrix}$ for some $t \in \mathbb{R}$. Which of the following statements is (are) TRUE?

(A) $\lambda_1 \neq \lambda_2$

(B) $\lambda_1 \lambda_2 = 2$

(C) $\sqrt{2}$ is an eigenvalue of P

(D) $\sqrt{3}$ is an eigenvalue of P

Q.34 Let P be an $n \times n$ non-null real skew-symmetric matrix, where n is even. Which of the following statements is (are) always TRUE?

(A) $P\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions, where $\boldsymbol{0} \in \mathbb{R}^n$

(B) $P\boldsymbol{x} = \lambda\boldsymbol{x}$ has a unique solution for every non-zero $\lambda \in \mathbb{R}$

(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^T Q = I_n$

(D) The sum of all the eigenvalues of P is zero

Q.35 Let X be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \dfrac{1+x^2}{10}, & 0 \le x < 1, \\ \dfrac{3+x^2}{10}, & 1 \le x < 2, \\ 1, & x \ge 2. \end{cases}$$

Which of the following statements is (are) TRUE?

(A) $P(1 < X < 2) = \frac{3}{10}$ (B) $P(1 < X \le 2) = \frac{3}{5}$

(C) $P(1 \le X < 2) = \frac{1}{2}$ (D) $P(1 \le X \le 2) = \frac{4}{5}$

Q.36 Let X and Y be i.i.d. $Exp(\lambda)$ random variables. If $Z = \max\{X - Y, 0\}$, then which of the following statements is (are) TRUE?

(A) $P(Z = 0) = \frac{1}{2}$

(B) The cumulative distribution function of Z is $F(z) = \begin{cases} 0, & z < 0, \\ 1 - \frac{1}{2}e^{-\lambda z}, & z \ge 0 \end{cases}$

(C) $P(Z = 0) = 0$

(D) The cumulative distribution function of Z is $F(z) = \begin{cases} 0, & z < 0, \\ 1 - e^{-\lambda z/2}, & z \ge 0 \end{cases}$

Q.37 Let the discrete random variables X and Y have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \dfrac{e^{-2}}{m!\,n!}, & m = 0,1,2,\ldots;\ n = 0,1,2,\ldots, \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following statements is (are) TRUE?

(A) The marginal distribution of X is Poisson with mean 2

(B) The random variables X and Y are independent

(C) The covariance between X and $X + \sqrt{3}\,Y$ is 1

(D) $P(Y = n) = (n+1)P(Y = n+1)$ for $n = 0,1,2,\ldots$

Q.38 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. continuous random variables with the probability density function

$$f(x) = \begin{cases} 2e^{-2\left(x-\frac{1}{2}\right)}, & x \geq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

If $S_n = X_1 + X_2 + \cdots + X_n$ and $\overline{X}_n = S_n/n$, then the distributions of which of the following sequences of random variables converge(s) to a normal distribution with mean 0 and a finite variance?

(A) $\frac{S_n - n}{\sqrt{n}}$ (B) $\frac{S_n}{\sqrt{n}}$ (C) $\sqrt{n}\left(\overline{X}_n - 1\right)$ (D) $\frac{\sqrt{n}\left(\overline{X}_n - 1\right)}{2}$

Q.39 Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U(\theta,\ 0)$ distribution, where $\theta < 0$. If $T_n = \min\{X_1, X_2, \ldots, X_n\}$, then which of the following sequences of estimators is (are) consistent for θ?

(A) T_n (B) $T_n - 1$ (C) $T_n + \frac{1}{n}$ (D) $T_n - 1 - \frac{1}{n^2}$

Q.40 Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function, for $\lambda > 0$,

$$f(x) = \begin{cases} 2\lambda x e^{-\lambda x^2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

To test the hypothesis $H_0: \lambda = \frac{1}{2}$ against $H_1: \lambda = \frac{3}{4}$ at the level α $(0 < \alpha < 1)$, which of the following statements is (are) TRUE?

(A) The most powerful test exists for each value of α

(B) The most powerful test does not exist for some values of α

(C) If the most powerful test exists, it is of the form: Reject H_0 if $X_1^2 + X_2^2 + \cdots + X_n^2 \leq c$ for some $c > 0$

(D) If the most powerful test exists, it is of the form: Reject H_0 if $X_1^2 + X_2^2 + \cdots + X_n^2 \geq c$ for some $c > 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 $\lim_{n\to\infty} \frac{\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{n+n}}{n\sqrt{n}}$ (round off to 2 decimal places) equals _____

Q.42 Let $f:[0,2] \to \mathbb{R}$ be such that $|f(x)-f(y)| \le |x-y|^{4/3}$ for all $x, y \in [0,2]$. If $\int_0^2 f(x)dx = \frac{2}{3}$, then $\sum_{k=1}^{2019} f\left(\frac{1}{k}\right)$ equals _____

Q.43 The value (round off to 2 decimal places) of the double integral

$$\int_0^9 \int_{\sqrt{x}}^3 \frac{1}{1+y^3}\,dydx$$

equals _____

Q.44 If $\begin{bmatrix} \frac{\sqrt{5}}{3} & -\frac{2}{3} & c \\ \frac{2}{3} & \frac{\sqrt{5}}{3} & d \\ a & b & 1 \end{bmatrix}$ is a real orthogonal matrix, then $a^2+b^2+c^2+d^2$ equals _____

Q.45 Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals _____

Q.46 Let X be a random variable with the moment generating function

$$M_X(t) = \left(\frac{e^{\frac{t}{2}} + e^{-\frac{t}{2}}}{2}\right)^2, \qquad -\infty < t < \infty.$$

Using Chebyshev's inequality, the upper bound for $P\left(|X| > \sqrt{\frac{2}{3}}\right)$ equals _____

Q.47 In a production line of a factory, each packet contains four items. Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and approves the packet for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected packets at least two are approved for shipment equals _____

Q.48 Let X be the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again X number of times independently, and let Y be the number of heads obtained in these X number of tosses. Then $E(X+2Y)$ equals _____

Q.49 Let $0, 1, 0, 0, 1$ be the observed values of a random sample of size five from a discrete distribution with the probability mass function $P(X=1) = 1 - P(X=0) = 1 - e^{-\lambda}$, where $\lambda > 0$. The method of moments estimate (round off to 2 decimal places) of λ equals _____

Q.50 Let X_1, X_2, X_3 be a random sample from $N(\mu_1, \sigma^2)$ distribution and Y_1, Y_2, Y_3 be a random sample from $N(\mu_2, \sigma^2)$ distribution. Also, assume that (X_1, X_2, X_3) and (Y_1, Y_2, Y_3) are independent. Let the observed values of $\sum_{i=1}^{3}\left[X_i - \frac{1}{3}(X_1+X_2+X_3)\right]^2$ and $\sum_{i=1}^{3}\left[Y_i - \frac{1}{3}(Y_1+Y_2+Y_3)\right]^2$ be 1 and 5, respectively. The length (round off to 2 decimal places) of the shortest 90% confidence interval of $\mu_1 - \mu_2$ equals _____

Q. 51 – Q. 60 carry two marks each.

Q.51 $\lim_{n\to\infty}\left[n - \frac{n}{e}\left(1+\frac{1}{n}\right)^n\right]$ equals _____

Q.52 For any real number y, let $[y]$ be the greatest integer less than or equal to y and let $\{y\} = y - [y]$. For $n = 1, 2, \ldots$, and for $x \in \mathbb{R}$, let

$$f_{2n}(x) = \begin{cases} \left[\frac{\sin x}{x}\right], & x \neq 0, \\ 1, & x = 0, \end{cases} \quad \text{and} \quad f_{2n-1}(x) = \begin{cases} \left\{\frac{\sin x}{x}\right\}, & x \neq 0, \\ 1, & x = 0. \end{cases}$$

Then $\lim_{x\to 0}\sum_{k=1}^{100} f_k(x)$ equals _____

Q.53 The volume (round off to 2 decimal places) of the region in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 2$ and $y + z = 4$ equals _____

Q.54 If $ad - bc = 2$ and $ps - qr = 1$, then the determinant of $\begin{bmatrix} a & b & 0 & 0 \\ 3 & 10 & 2p & q \\ c & d & 0 & 0 \\ 2 & 7 & 2r & s \end{bmatrix}$ equals _____

Q.55 In an ethnic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease. But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places), that a randomly selected adult male from this population does not have heart disease given that the test indicates high cholesterol level, equals _____

Q.56 Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} ax^2, & 0 < x < 1, \\ bx^{-4}, & x \geq 1, \\ 0, & \text{otherwise}, \end{cases}$$

where a and b are positive real numbers. If $E(X) = 1$, then $E(X^2)$ equals _____

Q.57 Let X and Y be jointly distributed continuous random variables, where Y is positive valued with $E(Y^2) = 6$. If the conditional distribution of X given $Y = y$ is $U(1 - y, 1 + y)$, then $Var(X)$ equals _____

Q.58 Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. $N(0, 1)$ random variables. If $T = X_1^2 + X_2^2 + \cdots + X_{10}^2$, then $\mathrm{E}\left(\frac{1}{T}\right)$ equals _____

Q.59 Let X_1, X_2, X_3 be a random sample from a continuous distribution with the probability density function

$$f(x) = \begin{cases} e^{-(x-\mu)}, & x > \mu, \\ 0, & \text{otherwise}. \end{cases}$$

Let $X_{(1)} = \min\{X_1, X_2, X_3\}$ and $c > 0$ be a real number. Then $\left(X_{(1)} - c, X_{(1)}\right)$ is a 97% confidence interval for μ, if c (round off to 2-decimal places) equals _____

Q.60 Let X_1, X_2, X_3, X_4 be a random sample from a discrete distribution with the probability mass function $P(X = 0) = 1 - P(X = 1) = 1 - p$, for $0 < p < 1$. To test the hypothesis

$$H_0: p = \frac{3}{4} \quad \text{against} \quad H_1: p = \frac{4}{5},$$

consider the test:

$$\text{Reject } H_0 \text{ if } X_1 + X_2 + X_3 + X_4 > 3.$$

Let the size and power of the test be denoted by α and γ, respectively. Then $\alpha + \gamma$ (round off to 2 decimal places) equals _____

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Notation

$\mathbb{N}$	set of all natural numbers $1, 2, 3, \cdots$
$\mathbb{R}$	set of all real numbers
$M_{m\times n}(\mathbb{R})$	real vector space of all matrices of size $m \times n$ with entries in $\mathbb{R}$
$\emptyset$	empty set
$X \setminus Y$	set of all elements from the set X which are not in the set Y
$\mathbb{Z}_n$	group of all congruence classes of integers modulo n
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system, respectively
S_n	group of all permutations of the set $\{1, 2, 3, \cdots, n\}$
ln	logarithm to the base e
log	logarithm to the base 10
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$\det(M)$	determinant of a square matrix M

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by

$$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m} \text{ and } b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m} .$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(A) Both $\{a_n\}$ and $\{b_n\}$ are divergent
(B) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent
(C) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent
(D) Both $\{a_n\}$ and $\{b_n\}$ are convergent

Q.2 Let $T \in M_{m\times n}(\mathbb{R})$. Let V be the subspace of $M_{n\times p}(\mathbb{R})$ defined by

$$V = \{X \in M_{n\times p}(\mathbb{R}) : TX = 0\}.$$

Then the dimension of V is

(A) $pn -$ rank(T)
(B) $mn - p$ rank(T)
(C) $p(m -$ rank$(T))$
(D) $p(n -$ rank$(T))$

Q.3 Let $g: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f: \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x, y, z) = g(x^2 + y^2 - 2z^2).$$

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

(A) $4(x^2 + y^2 - 4z^2)\, g''(x^2 + y^2 - 2z^2)$
(B) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2)$
(C) $4(x^2 + y^2 - 2z^2)\, g''(x^2 + y^2 - 2z^2)$
(D) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

Q.4 Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$ for all $n \geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is 4, then the power series $\sum_{n=0}^{\infty} b_n x^n$

(A) converges for all x with $|x| < 2$
(B) converges for all x with $|x| > 2$
(C) does not converge for any x with $|x| > 2$
(D) does not converge for any x with $|x| < 2$

Q.5 Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(A) $\mathbb{Q}_+ \subseteq S$
(B) $S \subseteq \mathbb{Q}_+$
(C) $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \emptyset$
(D) $S \cap \mathbb{Q}_+ \neq \emptyset$

Q.6 If $x^h y^k$ is an integrating factor of the differential equation

$$y(1+xy)\,dx + x(1-xy)\,dy = 0,$$

then the ordered pair (h,k) is equal to

(A) $(-2,-2)$ (B) $(-2,-1)$ (C) $(-1,-2)$ (D) $(-1,-1)$

Q.7 If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to

(A) 1 (B) 4 (C) 5 (D) 9

Q.8 The equation of the tangent plane to the surface $x^2 z + \sqrt{8 - x^2 - y^4} = 6$ at the point $(2,0,1)$ is

(A) $2x + z = 5$ (B) $3x + 4z = 10$
(C) $3x - z = 10$ (D) $7x - 4z = 10$

Q.9 The value of the integral

$$\int_{y=0}^{1} \int_{x=0}^{1-y^2} y \sin\left(\pi(1-x)^2\right) dx\, dy$$

is

(A) $\frac{1}{2\pi}$ (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

Q.10 The area of the surface generated by rotating the curve $x = y^3$, $0 \leq y \leq 1$, about the y-axis, is

(A) $\frac{\pi}{27}10^{3/2}$ (B) $\frac{4\pi}{3}(10^{3/2} - 1)$ (C) $\frac{\pi}{27}(10^{3/2} - 1)$ (D) $\frac{4\pi}{3}10^{3/2}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let H and K be subgroups of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H \cap K$ is

(A) 3 (B) 4 (C) 6 (D) 12

Q.12 Let P be a 4×4 matrix with entries from the set of rational numbers. If $\sqrt{2} + i$, with $i = \sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4×4 identity matrix, then

(A) $P^4 = 4P^2 + 9I$ (B) $P^4 = 4P^2 - 9I$ (C) $P^4 = 2P^2 - 9I$ (D) $P^4 = 2P^2 + 9I$

Q.13 The set $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$, as a subset of $\mathbb{R}$, is

(A) connected and compact
(B) connected but not compact
(C) not connected but compact
(D) neither connected nor compact

Q.14 The set $\left\{\frac{1}{m}+\frac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}$, as a subset of $\mathbb{R}$, is

(A) compact and open
(B) compact but not open
(C) not compact but open
(D) neither compact nor open

Q.15 For $-1 < x < 1$, the sum of the power series $1 + \sum_{n=2}^{\infty} (-1)^{n-1} n^2 x^{n-1}$ is

(A) $\frac{1-x}{(1+x)^3}$
(B) $\frac{1+x^2}{(1+x)^4}$
(C) $\frac{1-x}{(1+x)^2}$
(D) $\frac{1+x^2}{(1+x)^3}$

Q.16 Let $f(x) = (\ln x)^2$, $x > 0$. Then

(A) $\lim_{x\to\infty} \frac{f(x)}{x}$ does not exist
(B) $\lim_{x\to\infty} f'(x) = 2$
(C) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big) = 0$
(D) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big)$ does not exist

Q.17 Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$. Then $f(1)$ lies in the interval

(A) $(0,\ e^{-1})$ (B) $\left(e^{-1}, \sqrt{e}\right)$ (C) $(\sqrt{e},\ e)$ (D) (e, ∞)

Q.18 For which one of the following values of k, the equation

$$2x^3 + 3x^2 - 12x - k = 0$$

has three distinct real roots?

(A) 16 (B) 20 (C) 26 (D) 31

Q.19 Which one of the following series is divergent?

(A) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$
(B) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$
(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$
(D) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

Q.20 Let S be the family of orthogonal trajectories of the family of curves

$$2x^2 + y^2 = k, \text{ for } k \in \mathbb{R} \text{ and } k > 0.$$

If $\mathcal{C} \in S$ and $\mathcal{C}$ passes through the point $(1, 2)$, then $\mathcal{C}$ also passes through

(A) $(4, -\sqrt{2})$ (B) $(2, -4)$ (C) $(2, 2\sqrt{2})$ (D) $(4, 2\sqrt{2})$

Q.21 Let x, $x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

(A) $e + 1$ (B) $2e + 3$ (C) $3e + 2$ (D) $3e + 1$

Q.22 The function

$$f(x, y) = x^3 + 2xy + y^3$$

has a saddle point at

(A) $(0, 0)$ (B) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$ (C) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ (D) $(-1, -1)$

Q.23 The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

(A) $\frac{\pi}{2}(17^{3/2} - 1)$ (B) $\pi(17^{3/2} - 1)$ (C) $\frac{\pi}{6}(17^{3/2} - 1)$ (D) $\frac{\pi}{3}(17^{3/2} - 1)$

Q.24 Let $\mathcal{C}$ be the circle $(x - 1)^2 + y^2 = 1$, oriented counter clockwise. Then the value of the line integral

$$\oint_{\mathcal{C}} -\frac{4}{3}xy^3\,dx + x^4\,dy$$

is

(A) 6π (B) 8π (C) 12π (D) 14π

Q.25 Let $\vec{F}(x, y, z) = 2y\,\hat{\imath} + x^2\,\hat{\jmath} + xy\,\hat{k}$ and let $\mathcal{C}$ be the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$. Then the value of

$$\left| \oint_{\mathcal{C}} \vec{F} \cdot d\vec{r} \right|$$

is

(A) π (B) $\frac{3\pi}{2}$ (C) 2π (D) 3π

Q.26 The tangent line to the curve of intersection of the surface $x^2 + y^2 - z = 0$ and the plane $x + z = 3$ at the point $(1, 1, 2)$ passes through

(A) $(-1, -2, 4)$ (B) $(-1, 4, 4)$ (C) $(3, 4, 4)$ (D) $(-1, 4, 0)$

Q.27 The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Q.28 Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty} a_n$ converges if the series

(A) $\sum_{n=1}^{\infty} a_n^2$ converges
(B) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges
(C) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges
(D) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

Q.29 For $\beta \in \mathbb{R}$, define

$$f(x, y) = \begin{cases} \dfrac{x^2 |x|^\beta y}{x^4 + y^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then, at $(0, 0)$, the function f is

(A) continuous for $\beta = 0$
(B) continuous for $\beta > 0$
(C) not differentiable for any β
(D) continuous for $\beta < 0$

Q.30 Let $\{a_n\}$ be a sequence of positive real numbers such that

$$a_1 = 1, \ a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0 \text{ for all } n \geq 1.$$

Then the sum of the series $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ lies in the interval

(A) $(1, 2]$ (B) $(2, 3]$ (C) $(3, 4]$ (D) $(4, 5]$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_8$
II. There is NO surjective (onto) homomorphism from $\mathbb{Z}_8$ to G

Then

(A) I is true
(B) I is false
(C) II is true
(D) II is false

Q.32 Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by

$$f(x) = yxy^{-1},\ g(x) = x^{-1} \text{ and } h = g \circ g.$$

Then

(A) g and h are homomorphisms and f is not a homomorphism
(B) h is a homomorphism and g is not a homomorphism
(C) f is a homomorphism and g is not a homomorphism
(D) f, g and h are homomorphisms

Q.33 Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S\big(T(v)\big) = 0$ for all $v \in V$. Then

(A) rank(T) $\geq$ nullity(S)
(B) rank(S) $\geq$ nullity(T)
(C) rank(T) $\leq$ nullity(S)
(D) rank(S) $\leq$ nullity(T)

Q.34 Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let g be a differentiable scalar function. Then

(A) $\nabla \cdot \left(\vec{F} \times \vec{G}\right) = \vec{G} \cdot \nabla \times \vec{F} - \vec{F} \cdot \nabla \times \vec{G}$
(B) $\nabla \cdot \left(\vec{F} \times \vec{G}\right) = \vec{G} \cdot \nabla \times \vec{F} + \vec{F} \cdot \nabla \times \vec{G}$
(C) $\nabla \cdot \left(g\vec{F}\right) = g\nabla \cdot \vec{F} - \nabla \mathrm{g} \cdot \vec{F}$
(D) $\nabla \cdot \left(g\vec{F}\right) = g\nabla \cdot \vec{F} + \nabla \mathrm{g} \cdot \vec{F}$

Q.35 Consider the intervals $S = (0, 2]$ and $T = [1, 3)$. Let S° and T° be the sets of interior points of S and T, respectively. Then the set of interior points of $S \setminus T$ is equal to

(A) $S \setminus T^\circ$ (B) $S \setminus T$ (C) $S^\circ \setminus T^\circ$ (D) $S^\circ \setminus T$

Q.36 Let $\{a_n\}$ be the sequence given by

$$a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right\}, \quad n \geq 1.$$

Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$?

(A) Both the subsequences are convergent
(B) Only one of the subsequences is convergent
(C) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$
(D) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

Q.37 Let

$$f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x| \text{ and } g(x) = x^2 \text{ for } x \in \mathbb{R}.$$

If $h(x) = f\big(g(x)\big)$, then

(A) h is not differentiable at $x = 0$
(B) $h'\big(\sqrt{\pi}\big) = 0$
(C) $h''(x) = 0$ has a solution in $(-\pi, \pi)$
(D) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

Q.38 Let $f: \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by

$$f(x) = (\sin x)^{\pi} - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE?

(A) f is an increasing function
(B) f is a decreasing function
(C) $f(x) > 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$
(D) $f(x) < 0$ for some $x \in \left(0, \frac{\pi}{2}\right)$

Q.39 Let

$$f(x,y) = \begin{cases} \dfrac{|x|}{|x| + |y|}\sqrt{x^4 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Then at $(0,0)$,

(A) f is continuous
(B) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$ does not exist
(C) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y} = 0$
(D) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

Q.40 Let $\{a_n\}$ be the sequence of real numbers such that

$$a_1 = 1 \text{ and } a_{n+1} = a_n + a_n^2 \text{ for all } n \geq 1.$$

Then

(A) $a_4 = a_1(1 + a_1)(1 + a_2)(1 + a_3)$
(B) $\lim\limits_{n\to\infty} \frac{1}{a_n} = 0$
(C) $\lim\limits_{n\to\infty} \frac{1}{a_n} = 1$
(D) $\lim\limits_{n\to\infty} a_n = 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let x be the 100-cycle $(1 \;\; 2 \;\; 3 \cdots 100)$ and let y be the transposition $(49 \;\; 50)$ in the permutation group S_{100}. Then the order of xy is _____

Q.42 Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$$W_1 = \{(x_1, x_2, \ldots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 4\},$$

$$W_2 = \{(x_1, x_2, \ldots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 5\}.$$

Then the dimension of $W_1 \cap W_2$ is _____

Q.43 Consider the following system of three linear equations in four unknowns x_1, x_2, x_3 and x_4

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4, \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 5, \\ x_1 + 3x_2 + 5x_3 + kx_4 &= 5. \end{aligned}$$

If the system has no solutions, then $k =$ _____

Q.44 Let $\vec{F}(x, y) = -y\,\hat{\imath} + x\,\hat{\jmath}$ and let $\mathcal{C}$ be the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

oriented counter clockwise. Then the value of $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ (round off to 2 decimal places) is_____

Q.45 The coefficient of $\left(x - \frac{\pi}{2}\right)$ in the Taylor series expansion of the function

$$f(x) = \begin{cases} \dfrac{4(1 - \sin x)}{2x - \pi}, & x \neq \dfrac{\pi}{2} \\ 0, & x = \dfrac{\pi}{2} \end{cases}$$

about $x = \frac{\pi}{2}$, is _____

Q.46 Let $f:[0,1] \to \mathbb{R}$ be given by

$$f(x) = \frac{\left(1+x^{\frac{1}{3}}\right)^3 + \left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}.$$

Then

$$\max\{f(x): x \in [0,1]\} - \min\{f(x): x \in [0,1]\}$$

is _____

Q.47 If

$$g(x) = \int_{x(x-2)}^{4x-5} f(t)\,dt, \text{ where } f(x) = \sqrt{1+3x^4} \text{ for } x \in \mathbb{R}$$

then $g'(1) =$ _____

Q.48 Let

$$f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2-y^2}, & x^2-y^2 \neq 0 \\ 0, & x^2-y^2 = 0. \end{cases}$$

Then the directional derivative of f at $(0,0)$ in the direction of $\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ is _____

Q.49 The value of the integral

$$\int_{-1}^{1}\int_{-1}^{1} |x+y|\,dx\,dy$$

(round off to 2 decimal places) is _____

Q.50 The volume of the solid bounded by the surfaces $x = 1-y^2$ and $x = y^2-1$, and the planes $z = 0$ and $z = 2$ (round off to 2 decimal places) is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis (round off to 2 decimal places) is _____

Q.52 The greatest lower bound of the set

$$\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\},$$

(round off to 2 decimal places) is _____

Q.53 Let $G = \{n \in \mathbb{N} : n \leq 55,\ \gcd(n, 55) = 1\}$ be the group under multiplication modulo 55. Let $x \in G$ be such that $x^2 = 26$ and $x > 30$. Then x is equal to_____

Q.54 The number of critical points of the function

$$f(x, y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$$

is _____

Q.55 The number of elements in the set $\{x \in S_3 : x^4 = e\}$, where e is the identity element of the permutation group S_3, is _____

Q.56 If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$, $y, z \in \mathbb{R}$, is an eigenvector corresponding to a real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$ then $z - y$ is equal to_____

Q.57 Let M and N be any two 4×4 matrices with integer entries satisfying

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then the maximum value of $\det(M) + \det(N)$ is _____

Q.58 Let M be a 3×3 matrix with real entries such that $M^2 = M + 2I$, where I denotes the 3×3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma = -4$, then $\alpha + \beta + \gamma$ is equal to_____

Q.59 Let $y(x) = xv(x)$ be a solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = 0.$$

If $v(0) = 0$ and $v(1) = 1$, then $v(-2)$ is equal to_____

Q.60 If $y(x)$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0,\ y(0) = 2,\ \frac{dy}{dx}(0) = 0,$$

then $y(\ln 2)$ is (round off to 2 decimal places) equal to _____

END OF THE QUESTION PAPER

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

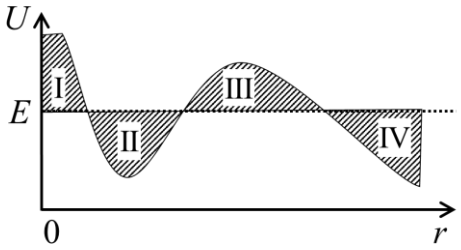
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The function $f(x) = \dfrac{8x}{x^2 + 9}$ is continuous everywhere except at

(A) $x = 0$ (B) $x = \pm 9$ (C) $x = \pm 9i$ (D) $x = \pm 3i$

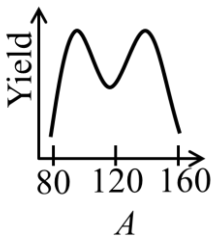
Q.2 A classical particle has total energy E. The plot of potential energy (U) as a function of distance (r) from the centre of force located at $r = 0$ is shown in the figure. Which of the regions are forbidden for the particle?



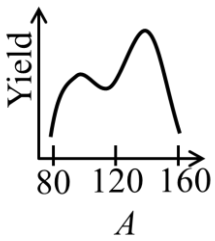
(A) I and II (B) II and IV (C) I and IV (D) I and III

Q.3 In the thermal neutron induced fission of ^{235}U, the distribution of relative number of the observed fission fragments (Yield) versus mass number (A) is given by

(A)



(B)



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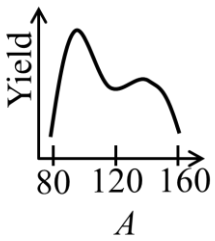

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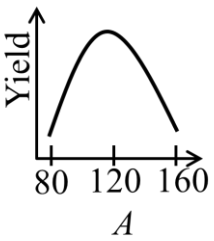
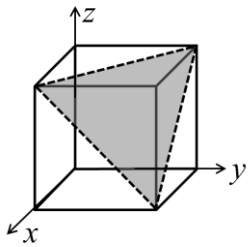
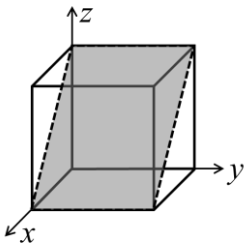


Q.4 Which one of the following crystallographic planes represent (101) Miller indices of a cubic unit cell?

(A)



(B)



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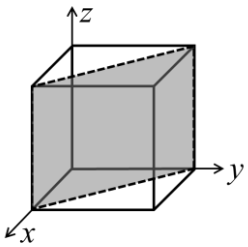
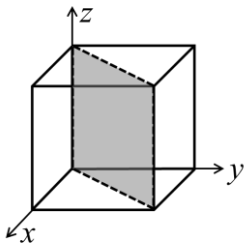


(D)



Q.5 The Fermi-Dirac distribution function $[n(\varepsilon)]$ is
(k_B is the Boltzmann constant, T is the temperature and ε_F is the Fermi energy)

(A) $n(\varepsilon) = \dfrac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_BT}} - 1}$ (B) $n(\varepsilon) = \dfrac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_BT}} - 1}$ (C) $n(\varepsilon) = \dfrac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_BT}} + 1}$ (D) $n(\varepsilon) = \dfrac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_BT}} + 1}$

Q.6 If $\phi(x,y,z)$ is a scalar function which satisfies the Laplace equation, then the gradient of ϕ is

(A) Solenoidal and irrotational
(B) Solenoidal but not irrotational
(C) Irrotational but not solenoidal
(D) Neither solenoidal nor irrotational

Q.7 In a heat engine based on the Carnot cycle, heat is added to the working substance at constant

(A) Entropy (B) Pressure (C) Temperature (D) Volume

Q.8 Isothermal compressibility is given by

(A) $\dfrac{1}{V}\left(\dfrac{\partial V}{\partial P}\right)_T$ (B) $\dfrac{1}{P}\left(\dfrac{\partial P}{\partial V}\right)_T$ (C) $-\dfrac{1}{V}\left(\dfrac{\partial V}{\partial P}\right)_T$ (D) $-\dfrac{1}{P}\left(\dfrac{\partial P}{\partial V}\right)_T$

Q.9 For using a transistor as an amplifier, choose the correct option regarding the resistances of base-emitter (R_{BE}) and base-collector (R_{BC}) junctions

(A) Both R_{BE} and R_{BC} are very low
(B) Very low R_{BE} and very high R_{BC}
(C) Very high R_{BE} and very low R_{BC}
(D) Both R_{BE} and R_{BC} are very high

Q.10 A unit vector perpendicular to the plane containing $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$ is

(A) $\dfrac{1}{\sqrt{26}}(-\hat{i} + 3\hat{j} - 4\hat{k})$
(B) $\dfrac{1}{\sqrt{19}}(-\hat{i} + 3\hat{j} - 3\hat{k})$
(C) $\dfrac{1}{\sqrt{35}}(-\hat{i} + 5\hat{j} - 3\hat{k})$
(D) $\dfrac{1}{\sqrt{35}}(-\hat{i} - 5\hat{j} - 3\hat{k})$

Q. 11 – Q. 30 carry two marks each.

Q.11 A thin lens of refractive index 3/2 is kept inside a liquid of refractive index 4/3. If the focal length of the lens in air is 10 cm, then its focal length inside the liquid is

(A) 10 cm (B) 30 cm (C) 40 cm (D) 50 cm

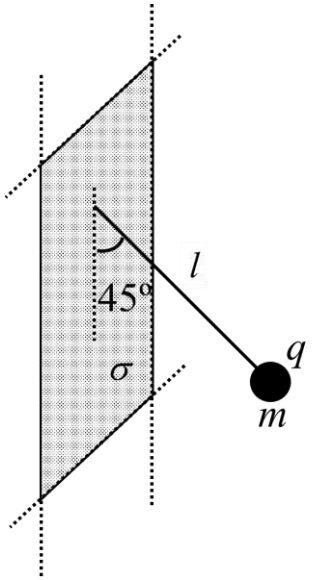
Q.12 The eigenvalues of $\begin{pmatrix} 3 & i & 0 \\ -i & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ are

(A) 2, 4 and 6 (B) $2i$, $4i$ and 6 (C) $2i$, 4 and 8 (D) 0, 4 and 8

Q.13 For a quantum particle confined inside a cubic box of side L, the ground state energy is given by E_0. The energy of the first excited state is

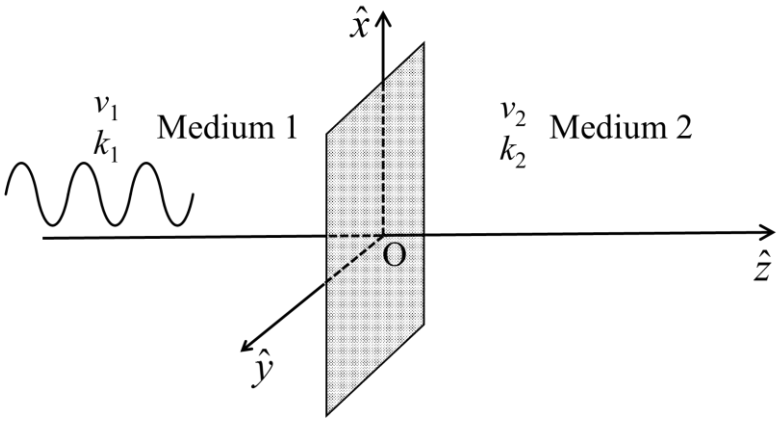
(A) $2E_0$ (B) $\sqrt{2}\,E_0$ (C) $3E_0$ (D) $6E_0$

Q.14 A small spherical ball having charge q and mass m, is tied to a thin massless non-conducting string of length l. The other end of the string is fixed to an infinitely extended thin non-conducting sheet with uniform surface charge density σ. Under equilibrium, the string makes an angle 45° with the sheet as shown in the figure. Then σ is given by (g is the acceleration due to gravity and ε_0 is the permittivity of free space)



(A) $\frac{mg\varepsilon_0}{q}$ (B) $\sqrt{2}\frac{mg\varepsilon_0}{q}$ (C) $2\frac{mg\varepsilon_0}{q}$ (D) $\frac{mg\varepsilon_0}{q\sqrt{2}}$

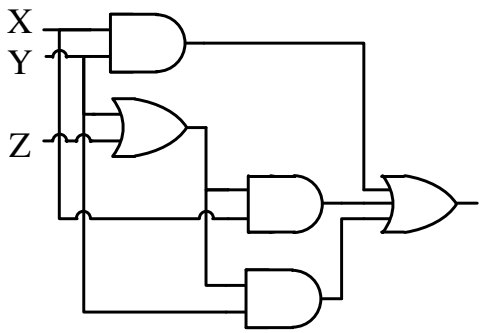
Q.15 Consider the normal incidence of a plane electromagnetic wave with electric field given by $\vec{E} = E_0 \exp\left[i(k_1 z - \omega t)\right]\hat{x}$ over an interface at $z = 0$ separating two media [wave velocities v_1 and v_2 ($v_2 > v_1$) and wave vectors k_1 and k_2, respectively] as shown in figure. The magnetic field vector of the reflected wave is
(ω is the angular frequency)



(A) $\frac{E_0}{v_1}\exp\left[i(k_1 z - \omega t)\right]\hat{y}$ (B) $\frac{E_0}{v_1}\exp\left[i(-k_1 z - \omega t)\right]\hat{y}$

(C) $\frac{-E_0}{v_1}\exp\left[i(-k_1 z - \omega t)\right]\hat{y}$ (D) $\frac{-E_0}{v_1}\exp\left[i(k_1 z - \omega t)\right]\hat{y}$

Q.16 The output of following logic circuit can be simplified to



(A) X + YZ (B) Y + XZ (C) XYZ (D) X + Y + Z

Q.17 A red star having radius r_R at a temperature T_R and a white star having radius r_W at a temperature T_W, radiate the same total power. If these stars radiate as perfect black bodies, then

(A) $r_R > r_W$ and $T_R > T_W$ (B) $r_R < r_W$ and $T_R > T_W$

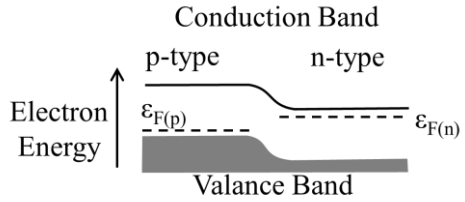
(C) $r_R > r_W$ and $T_R < T_W$ (D) $r_R < r_W$ and $T_R < T_W$

Q.18 The mass per unit length of a rod (length 2 m) varies as $\rho = 3x$ kg/m. The moment of inertia (in kg m^2) of the rod about a perpendicular-axis passing through the tip of the rod (at $x = 0$) is

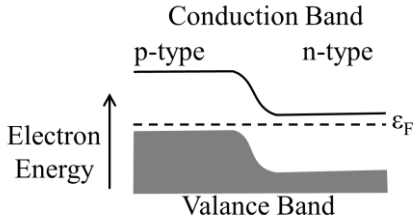
(A) 10 (B) 12 (C) 14 (D) 16

Q.19 For a forward biased p-n junction diode, which one of the following energy-band diagrams is correct?
(ε_F is the Fermi energy)

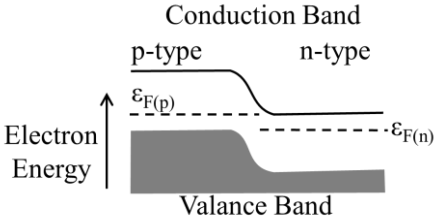
(A)



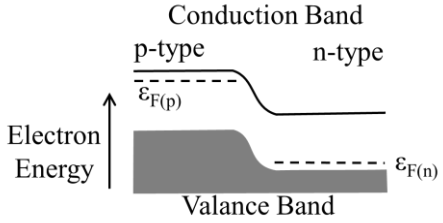
(B)



(C)



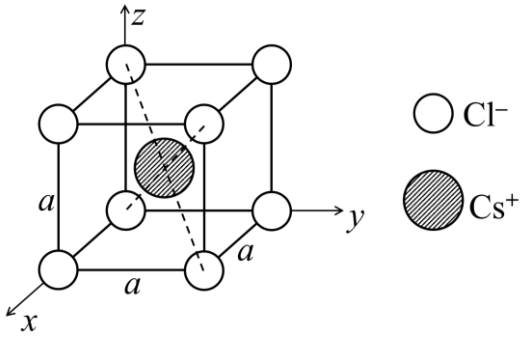
(D)



Q.20 The amount of work done to increase the speed of an electron from $c/3$ to $2c/3$ is ($c = 3\times10^8$ m/s and rest mass of electron is 0.511 MeV)

(A) 56.50 keV (B) 143.58 keV (C) 168.20 keV (D) 511.00 keV

Q.21 The location of Cs^+ and Cl^- ions inside the unit cell of CsCl crystal is shown in the figure. The Bravais lattice of CsCl is



(A) simple cubic
(B) body centered orthorhombic
(C) face centered cubic
(D) base centered orthorhombic

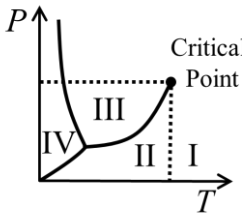
Q.22 A γ-ray photon emitted from a ^{137}Cs source collides with an electron at rest. If the Compton shift of the photon is 3.25×10^{-13} m, then the scattering angle is closest to
(Planck's constant $h = 6.626 \times 10^{-34}$ J s, electron mass $m_e = 9.109 \times 10^{-31}$ kg and velocity of light in free space $c = 3\times10^8$ m/s)

(A) 45° (B) 60° (C) 30° (D) 90°

Q.23 During free expansion of an ideal gas under adiabatic condition, the internal energy of the gas

(A) Decreases
(B) Initially decreases and then increases
(C) Increases
(D) Remains constant

Q.24 In the given phase diagram for a pure substance, regions I, II, III, IV, respectively represent



(A) Vapor, Gas, Solid, Liquid
(B) Gas, Vapor, Liquid, Solid
(C) Gas, Liquid, Vapor, Solid
(D) Vapor, Gas, Liquid, Solid

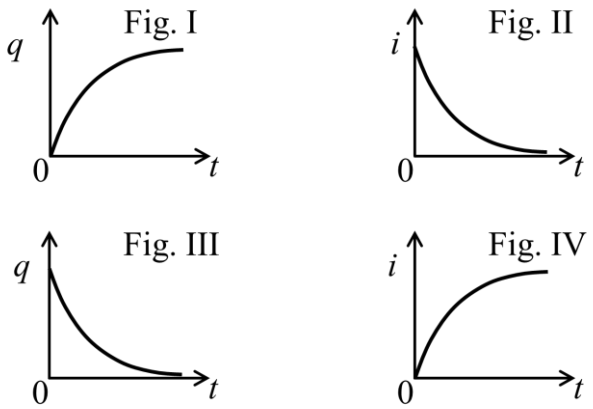
Q.25 Light of wavelength λ (in free space) propagates through a dispersive medium with refractive index $n(\lambda)=1.5 + 0.6\lambda$. The group velocity of a wave travelling inside this medium in units of 10^8 m/s is

(A) 1.5 (B) 2.0 (C) 3.0 (D) 4.0

Q.26 The maximum number of intensity minima that can be observed in the Fraunhofer diffraction pattern of a single slit (width 10 μm) illuminated by a laser beam (wavelength 0.630 μm) will be

(A) 4 (B) 7 (C) 12 (D) 15

Q.27 During the charging of a capacitor C in a series RC circuit, the typical variations in the magnitude of the charge $q(t)$ deposited on one of the capacitor plates, and the current $i(t)$ in the circuit, respectively are best represented by



(A) Fig. I and Fig. II
(B) Fig. I and Fig. IV
(C) Fig. III and Fig. II
(D) Fig. III and Fig. IV

Q.28 Which one of the following is an impossible magnetic field $\vec{B}$?

(A) $\vec{B} = 3x^2z^2\hat{x} - 2xz^3\hat{z}$

(B) $\vec{B} = -2xy\,\hat{x} + yz^2\hat{y} + \left(2yz - \frac{z^3}{3}\right)\hat{z}$

(C) $\vec{B} = \left(xz + 4y\right)\hat{x} - yx^3\hat{y} + \left(x^3z - \frac{z^2}{2}\right)\hat{z}$

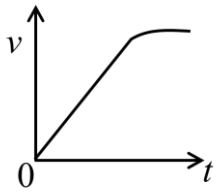
(D) $\vec{B} = -6xz\,\hat{x} + 3yz^2\hat{y}$

Q.29 If the motion of a particle is described by $x = 5\cos(8\pi t)$, $y = 5\sin(8\pi t)$ and $z = 5t$, then the trajectory of the particle is

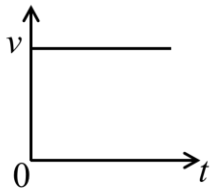
(A) Circular (B) Elliptical (C) Helical (D) Spiral

Q.30 A ball of mass m is falling freely under gravity through a viscous medium in which the drag force is proportional to the instantaneous velocity v of the ball. Neglecting the buoyancy force of the medium, which one of the following figures best describes the variation of v as a function of time t?

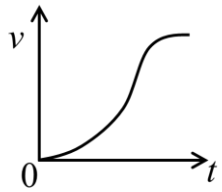
(A)



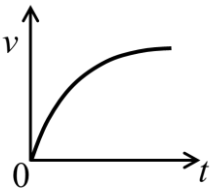
(B)



(C)



(D)



SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 The relation between the nuclear radius (R) and the mass number (A), given by $R = 1.2\,A^{1/3}$ fm, implies that

(A) The central density of nuclei is independent of A
(B) The volume energy per nucleon is a constant
(C) The attractive part of the nuclear force has a long range
(D) The nuclear force is charge dependent

Q.32 Consider an object moving with a velocity $\vec{v}$ in a frame which rotates with a constant angular velocity $\vec{\omega}$. The Coriolis force experienced by the object is

(A) along $\vec{v}$
(B) along $\vec{\omega}$
(C) perpendicular to both $\vec{v}$ and $\vec{\omega}$
(D) always directed towards the axis of rotation

Q.33 The gradient of a scalar field $S(x,y,z)$ has the following characteristic(s).

(A) Line integral of a gradient is path-independent
(B) Closed line integral of a gradient is zero
(C) Gradient of S is a measure of the maximum rate of change in the field S
(D) Gradient of S is a scalar quantity

Q.34 A thermodynamic system is described by the P, V, T coordinates. Choose the valid expression(s) for the system.

(A) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_V$

(B) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial P}{\partial T}\right)_V$

(C) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial P}\right)_T$

(D) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = \left(\frac{\partial V}{\partial P}\right)_T$

Q.35 Which of the following statement(s) is/are true?

(A) Newton's laws of motion and Maxwell's equations are both invariant under Lorentz transformations.
(B) Newton's laws of motion and Maxwell's equations are both invariant under Galilean transformations.
(C) Newton's laws of motion are invariant under Galilean transformations and Maxwell's equations are invariant under Lorentz transformations.
(D) Newton's laws of motion are invariant under Lorentz transformations and Maxwell's equations are invariant under Galilean transformations.

Q.36 For an underdamped harmonic oscillator with velocity $v(t)$,

(A) Rate of energy dissipation varies linearly with $v(t)$
(B) Rate of energy dissipation varies as square of $v(t)$
(C) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$
(D) For weak damping, the amplitude decays exponentially to zero

Q.37 Out of the following statements, choose the correct option(s) about a perfect conductor.

(A) The conductor has an equipotential surface
(B) Net charge, if any, resides only on the surface of conductor
(C) Electric field cannot exist inside the conductor
(D) Just outside the conductor, the electric field is always perpendicular to its surface

Q.38 In the X-ray diffraction pattern recorded for a simple cubic solid (lattice parameter a = 1 Å) using X rays of wavelength 1 Å, the first order diffraction peak(s) would appear for the

(A) (100) planes
(B) (112) planes
(C) (210) planes
(D) (220) planes

Q.39 Consider a classical particle subjected to an attractive inverse-square force field. The total energy of the particle is E and the eccentricity is ε. The particle will follow a parabolic orbit if

(A) $E > 0$ and $\varepsilon = 1$
(B) $E < 0$ and $\varepsilon < 1$
(C) $E = 0$ and $\varepsilon = 1$
(D) $E < 0$ and $\varepsilon = 1$

Q.40 An atomic nucleus X with half-life T_X decays to a nucleus Y, which has half-life T_Y. The condition(s) for secular equilibrium is(are)

(A) $T_X \simeq T_Y$ (B) $T_X < T_Y$ (C) $T_X \ll T_Y$ (D) $T_X \gg T_Y$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 In a typical human body, the amount of radioactive ^{40}K is 3.24×10^{-5} percent of its mass. The activity due to ^{40}K in a human body of mass 70 kg is _____ kBq.
(Round off to 2 decimal places)
(Half-life of $^{40}K = 3.942 \times 10^{16}$ s, Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1})

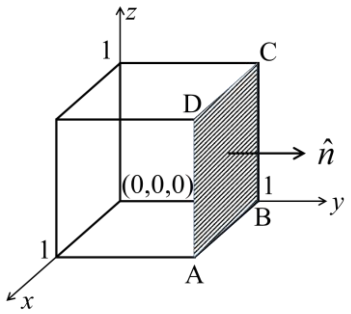
Q.42 Sodium (Na) exhibits body-centered-cubic (BCC) crystal structure with atomic radius 0.186 nm. The lattice parameter of Na unit cell is _____ nm.
(Round off to 2 decimal places)

Q.43 Light of wavelength 680 nm is incident normally on a diffraction grating having 4000 lines/cm. The diffraction angle (in degrees) corresponding to the third-order maximum is _____.
(Round off to 2 decimal places)

Q.44 Two gases having molecular diameters D_1 and D_2, and mean free paths λ_1 and λ_2, respectively, are trapped separately in identical containers.
If $D_2 = 2D_1$, then λ_1/λ_2 = _____.
(Assume there is no change in other thermodynamic parameters)

Q.45 An object of 2 cm height is placed at a distance of 30 cm in front of a concave mirror with radius of curvature 40 cm. The height of the image is _____ cm.

Q.46 The flux of the function $\vec{F} = (y^2)\hat{x} + (3xy - z^2)\hat{y} + (4yz)\hat{z}$ passing through the surface ABCD along $\hat{n}$ is _____.
(Round off to 2 decimal places)

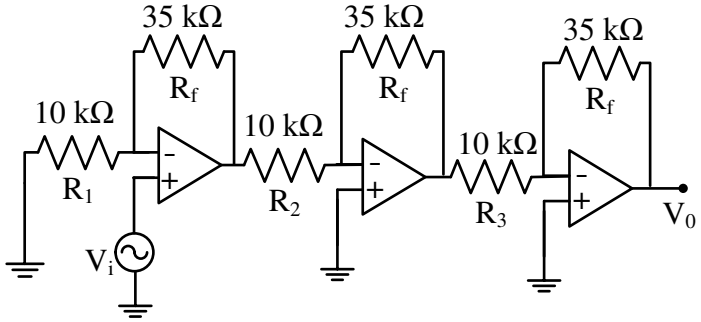


Q.47 The electrostatic energy (in units of $\frac{1}{4\pi\varepsilon_0}$ J) of a uniformly charged spherical shell of total charge 5 C and radius 4 m is _____.
(Round off to 3 decimal places)

Q.48 An infinitely long very thin straight wire carries uniform line charge density $8\pi \times 10^{-2}$ C/m. The magnitude of electric displacement vector at a point located 20 mm away from the axis of the wire is _____ C/m^2.

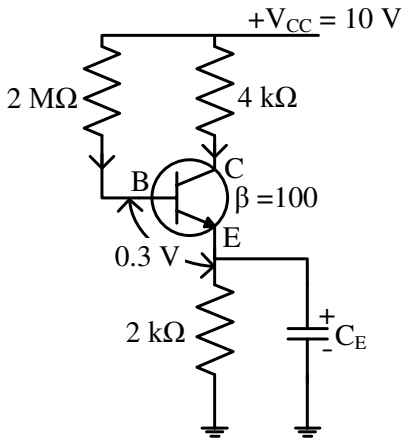
Q.49 The 7th bright fringe in the Young's double slit experiment using a light of wavelength 550 nm shifts to the central maxima after covering the two slits with two sheets of different refractive indices n_1 and n_2 but having same thickness 6 μm. The value of $|n_1 - n_2|$ is _____.
(Round off to 2 decimal places)

Q.50 For the input voltage V_i = (200 mV) sin (400t), the amplitude of the output voltage (V_0) of the given OPAMP circuit is _____ V.
(Round off to 2 decimal places)



Q. 51 – Q. 60 carry two marks each.

Q.51 The value of emitter current in the given circuit is _____ μA.
(Round off to 1 decimal place)



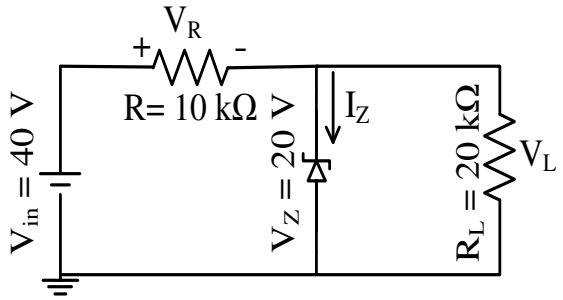
Q.52 The value of $\left|\int_0^{3+i} (\bar{z})^2 dz\right|^2$, along the line $3y = x$, where $z = x + iy$ is _____.
(Round off to 1 decimal place)

Q.53 If the wavelength of $K\alpha_2$ X-ray line of an element is 1.544Å, then the atomic number (Z) of the element is _____.
(Rydberg constant R = $1.097\times10^7 m^{-1}$ and velocity of light $c = 3\times10^8$ m/s)

Q.54 A proton is confined within a nucleus of size 10^{-13} cm. The uncertainty in its velocity is _____ $\times 10^8$ m/s.
(Round off to 2 decimal places)
(Planck's constant $h = 6.626 \times 10^{-34}$ J s and proton mass $m_p = 1.672 \times 10^{-27}$ kg)

Q.55 Given the wave function of a particle $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} x\right)$ for $0 < x < L$ and 0 elsewhere, the probability of finding the particle between $x = 0$ and $x = L/2$ is _____.
(Round off to 1 decimal place)

Q.56 The Zener current I_Z for the given circuit is _____ mA.

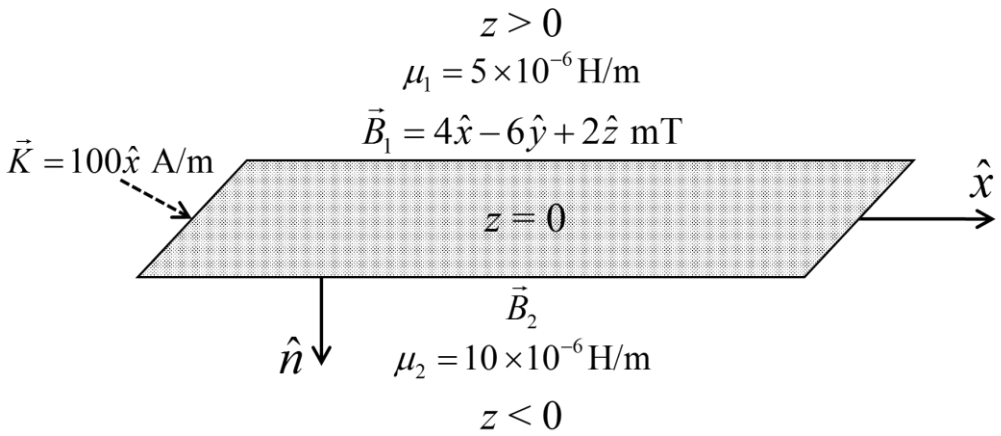


Q.57 If the diameter of the Earth is increased by 4% without changing the mass, then the length of the day is _____hours.
(Take the length of the day before the increment as 24 hours. Assume the Earth to be a sphere with uniform density.)
(Round off to 2 decimal places)

Q.58 A di-atomic gas undergoes adiabatic expansion against the piston of a cylinder. As a result, the temperature of the gas drops from 1150 K to 400 K. The number of moles of the gas required to obtain 2300 J of work from the expansion is _____.
(The gas constant $R = 8.314$ J mol^{-1}K^{-1}.)
(Round off to 2 decimal places)

Q.59 The decimal equivalent of the binary number 110.101 is _____.

Q.60 A surface current $\vec{K} = 100\,\hat{x}$ A/m flows on the surface $z = 0$, which separates two media with magnetic permeabilities μ_1 and μ_2 as shown in the figure. If the magnetic field in the region 1 is $\vec{B}_1 = 4\hat{x} - 6\hat{y} + 2\hat{z}$ mT, then the magnitude of the normal component of $\vec{B}_2$ will be _____ mT.



END OF THE QUESTION PAPER