IIT JAM 2018 Question Paper with Answer Key (All Subjects)

IIT Joint Admission Test for Masters

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following protozoan parasites belongs to the phylum *Apicomplexa*?

(A) *Toxoplasma gondii* (B) *Leishmania donovani*
(C) *Entamoeba histolytica* (D) *Trichomonas vaginalis*

Q.2 Which one of the following statements is **CORRECT** for *Mycoplasma*?

(A) Their cells are of definite shape.
(B) They are resistant to lysis by osmotic shock.
(C) Their growth is not inhibited by penicillin.
(D) They are nonpathogenic to human.

Q.3 Which one of the following organelles is enclosed by a single membrane?

(A) Ribosome (B) Mitochondria
(C) Endoplasmic reticulum (D) Centrosome

Q.4 Pyramid of energy in a forest ecosystem is

(A) always inverted. (B) dumb-bell shaped.
(C) spindle shaped. (D) always upright.

Q.5 In the feedback regulation of an enzyme, the end product binds to the

(A) active site of the enzyme. (B) allosteric site of the enzyme.
(C) enzyme-substrate complex. (D) substrate.

Q.6 What is the source of electrons in photosynthesis?

(A) Carbohydrates (B) Water (C) CO_2 (D) NADH

Q.7 The value of $\lim_{n\to\infty} \frac{3n^2+5n+4}{4+2n^2}$ is

(A) 0 (B) 0.75 (C) 1.5 (D) 3

Q.8 Three vectors are as follows:

$\vec{a} = 3\hat{\imath} - 10\hat{\jmath} + 7\hat{k}$

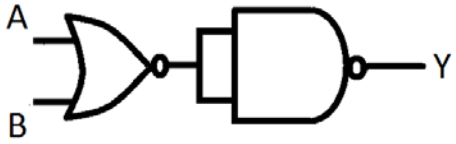
$\vec{b} = -9\hat{\imath} + 6\hat{\jmath} - 47\hat{k}$

$\vec{c} = 11\hat{\imath} - 17\hat{k}$

The value of $(\vec{a} + \vec{b}) \cdot \vec{c}$ is

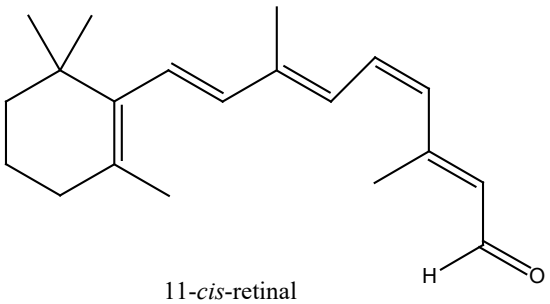
(A) 614 (B) 746 (C) 2 (D) 134

Q.9 The logic operation (OR, AND, NOR or NAND) carried out by following circuit is



(A) AND (B) NOR (C) OR (D) NAND

Q.10 The reaction of 11-*cis*-retinal with the lysine residue of a specific protein forms the light-sensitive pigment in the cells of retina. The light-sensitive pigment is an



11-*cis*-retinal

(A) amide. (B) acid.

(C) anhydride. (D) imine.

Q. 11 – Q. 30 carry two marks each.

Q.11 Viral capsids are made up of morphological subunits called capsomeres. One of the common capsomeres is icosahedral. The icosahedron is a regular polyhedron with

(A) 16 triangular facets and 12 vertices.
(B) 20 triangular facets and 12 vertices.
(C) 16 triangular facets and 16 vertices.
(D) 20 triangular facets and 16 vertices.

Q.12 Which of the following feature(s) should be present in a protein to generate strong immune response (antibody production) in an animal?
I. At least one B-cell epitope
II. At least one T-cell epitope
III. Proteolytic cleavage site(s)

(A) I only (B) II and III (C) I and III (D) I, II and III

Q.13 Match the entries in Group I with that in Group II.

Group I	**Group II**
P) Cholera toxin	1) Endotoxin
Q) Diphtheria toxin	2) Neurotoxin
R) Lipopolysaccharide	3) Enterotoxin
S) Tetanus toxin	4) Cytotoxin

(A) P-1, Q-2, R-3, S-4 (B) P-3, Q-2, R-1, S-4
(C) P-3, Q-4, R-1, S-2 (D) P-4, Q-1, R-2, S-3

Q.14 Proenzyme pepsinogen is secreted from **'P'** of gastric mucosa and converted into active enzyme pepsin on exposure to **'Q'** secreted from '**R'**. Choose the **CORRECT** combination of **P, Q** and **R.**

(A) **P** - chief cells	**Q** - hydrochloric acid	**R** - oxyntic cells
(B) **P** - parietal cells	**Q** - enterokinase	**R** - chief cells
(C) **P** - oxyntic cells	**Q** - hydrochloric acid	**R** - parietal cells
(D) **P** - peptic cells	**Q** - gastrin	**R** - oxyntic cells

Q.15 When bacteria are grown in glucose-depleted media containing high concentration of lactose, expression of *lac* operon genes is activated by

(A) the binding of *lac* repressor in the operator site and cAMP-CAP complex in the CAP site.
(B) the dissociation of bound *lac* repressor from the operator site and binding of cAMP-CAP complex in the CAP site.
(C) the dissociation of bound *lac* repressor only from the operator site.
(D) the dissociation of both bound *lac* repressor from operator site and cAMP-CAP complex from CAP site.

Q.16 Match the hormones in Group I with their functions in Group II

Group I	**Group II**
P) Aldosterone	1) Stimulates the synthesis and secretion of androgens from the testis
Q) Luteinizing hormone (LH)	2) Helps in the re-absorption of Na^+ and water from the kidney
R) Atrial natriuretic factor (ANF)	3) Increases the heart rate and the strength of heart contraction.
S) Epinephrine	4) Causes dilation of blood vessels and reduction of blood pressure

(A) P-2, Q-3, R-4, S-1
(B) P-2, Q-1, R-4, S-3
(C) P-1, Q-2, R-3, S-4
(D) P-3, Q-4, R-2, S-1

Q.17 Match the entries in Group I with that in Group II

Group I	**Group II**
P) Fehling's test	1) α-Amino acid
Q) Ninhydrin reaction	2) Reducing sugar
R) Biuret reaction	3) Sulfhydryl group
S) Nitroprusside reaction	4) Peptide linkage

(A) P-1, Q-2, R-3, S-4
(B) P-3, Q-4, R-1, S-2
(C) P-2, Q-1, R-4, S-3
(D) P-4, Q-1, R-2, S-3

Q.18 Match the entries in Group I with that in Group II

Group I	**Group II**
P) Vitamin B_1	1) Co-enzyme A
Q) Vitamin B_2	2) Flavin mononucleotide
R) Vitamin B_5	3) Pyridoxal phosphate
S) Vitamin B_6	4) Thiamine pyrophosphate

(A) P-4, Q-3, R-2, S-1
(B) P-3, Q-1, R-4, S-2
(C) P-1, Q-2, R-3, S-4
(D) P-4, Q-2, R-1, S-3

Q.19 If $\phi(x) = x^2$ and $\psi(x) = 2^x$, then $\psi(\phi(x))$ is

(A) 2^{x^2} (B) x^2 (C) 2^{2x} (D) x^{2x}

Q.20 The number of three letter words, with or without meaning, which can be formed using letters of the word 'VIRUS' without repetition of letters is

(A) 30 (B) 40 (C) 60 (D) 120

Q.21 What is the solution of $\int x^2 \ln x dx$?
Given C is an arbitrary constant.

(A) $\frac{x^3}{3}\ln x - \frac{x^3}{9} + C$ (B) $\frac{x^3}{3}\ln x + \frac{x^3}{9} + C$
(C) $-\frac{x^3}{9}\ln x + \frac{x^3}{9} + C$ (D) $\frac{x^3}{9}\ln x - \frac{x^3}{3} + C$

Q.22 The area of an equilateral triangle with sides of length α is

(A) $\frac{\sqrt{3}}{4}\alpha^2$ (B) $\frac{\sqrt{3}}{2}\alpha^2$ (C) $\frac{1}{2}\alpha^2$ (D) $\frac{1}{\sqrt{2}}\alpha^2$

Q.23 Nucleus of a radioactive material can undergo beta decay with half life of 4 minutes. Suppose beta decay starts with 4096 nuclei at $t = 0$, the number of nuclei left after 20 minutes would be

(A) 1024 (B) 128 (C) 512 (D) 256

Q.24 Which one of the following shows the **CORRECT** relationship among velocity of light in a medium (v), permittivity of medium (ε) and magnetic permeability of medium (μ)?

(A) $v = \frac{1}{\mu\varepsilon}$ (B) $v = \frac{1}{(\mu\varepsilon)^2}$ (C) $v = \frac{1}{(\mu\varepsilon)^{-2}}$ (D) $v = \frac{1}{\sqrt{\mu\varepsilon}}$

Q.25 A 30 μF capacitor is connected to a 240 V, 50 Hz source. If the frequency of the source is changed from 50 Hz to 200 Hz, the capacitive reactance of the capacitor will

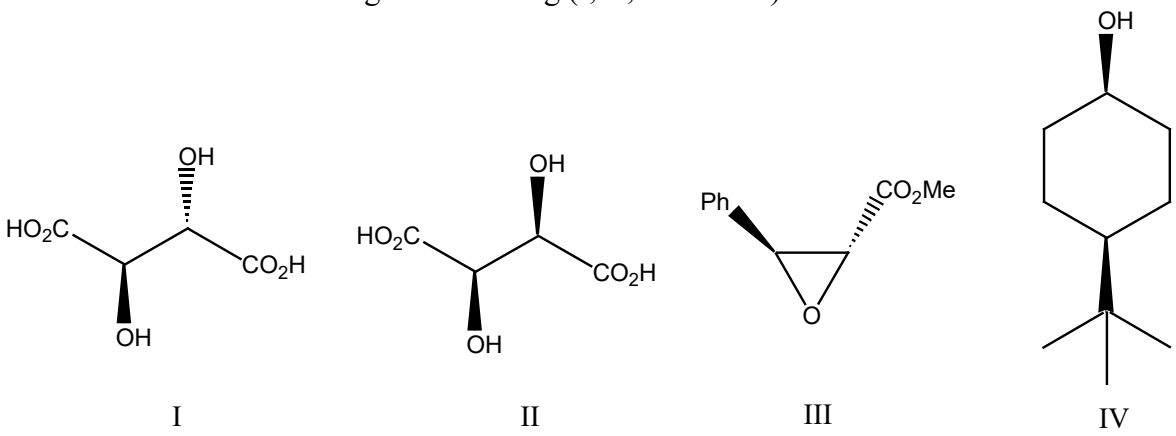
(A) increase by a factor of two.
(B) increase by a factor of four.
(C) decrease by a factor of four.
(D) decrease by a factor of two.

Q.26 Match the entries in Group I (Mechanical system) with analogous quantities in Group II (Electrical system)

Group I	**Group II**
P) Mass	1) Current
Q) Spring constant	2) Voltage
R) Displacement	3) Reciprocal capacitance
S) Velocity	4) Charge
	5) Inductance

(A) P-3, Q-5, R-4, S-1 (B) P-5, Q-3, R-4, S-2
(C) P-3, Q-5, R-4, S-2 (D) P-5, Q-3, R-4, S-1

Q.27 The achiral molecules among the following (I, II, III and IV) are



I II III IV

(A) I and III (B) II and IV (C) III and IV (D) I and IV

Q.28 Match the entries in Group I with those in Group II

Group I	**Group II**
P) Proline	1) Artificial sweetener
Q) Oxytocin	2) Cyclic amino acid
R) Aspartame	3) β-Lactam
S) Penicillin	4) Peptide hormone

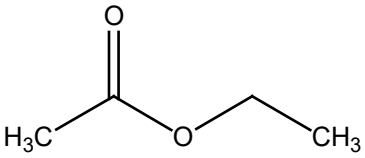
(A) P-2, Q-4, R-1, S-3 (B) P-3, Q-1, R-4, S-2
(C) P-4, Q-3, R-1, S-2 (D) P-2, Q-1, R-4, S-3

Q.29 Which one of the following statements is **CORRECT**?

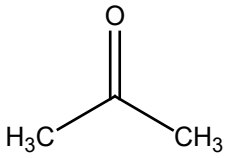
(A) BF_3 is a stronger Lewis acid than BI_3.
(B) CO and CN^- are good π-accepting ligands.
(C) *cis*-Diamminedichloroplatinum (II) has zero dipole moment.
(D) Central atom in BCl_3 is sp^3 hybridized.

Q.30 In the ^{1}H NMR spectrum, which one of the following compounds will show a triplet?

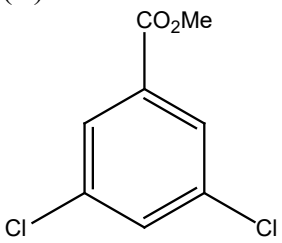
(A)



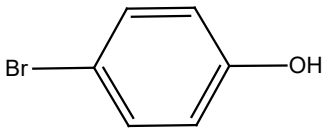
(B)



(C)



(D)



SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Antibody binds to antigen in solution through

(A) ionic interactions.
(B) hydrogen bonds.
(C) van der Waals interactions.
(D) hydrophobic interactions.

Q.32 Plasmid mediated antibiotic resistances in bacteria are acquired by

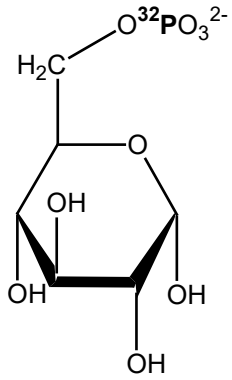
(A) hydrolysis by β-lactamase (penicillin resistance).
(B) expression of aminoglycoside modifying enzyme (kanamycin resistance).
(C) mutation in DNA gyrase (quinolone resistance).
(D) overproduction of dihydrofolate reductase (trimethoprim resistance).

Q.33 Which of the following statements is/are **CORRECT** for G protein–coupled receptor (GPCR) mediated signaling?

(A) GPCRs contain seven membrane spanning regions.
(B) GPCRs are linked to heterotrimeric G protein consisting of α, β and γ subunits.
(C) In the absence of GPCR interacting ligand, α subunit of G protein is bound to GTP and complexed with βγ subunits.
(D) In the presence of GPCR interacting ligand, GTP is displaced from α subunit of G protein by GDP, GDP bound α subunit dissociates from βγ dimer and activates the effector.

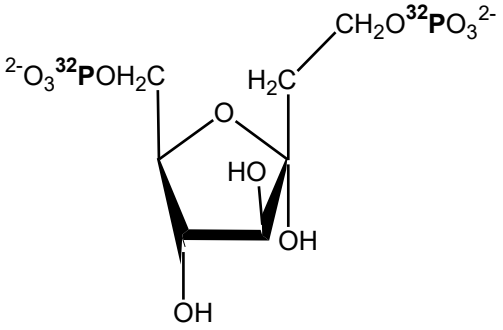
Q.34 Glucose is incubated with enzymes of glycolytic pathway (except pyruvate kinase), gamma ^{32}**P**-ATP and unlabeled inorganic phosphate. Which of the following products is/are formed?

(A)



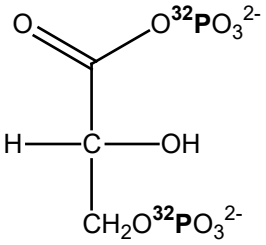
Glucose 6-phosphate

(B)



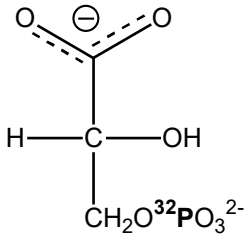
Fructose 1,6-bisphosphate

(C)



1,3-Bisphosphoglycerate

(D)



3-Phosphoglycerate

Q.35 In a double stranded DNA, which of the following ratios is/are always equal to 1? A, T, G and C denote the number of bases.

(A) (A+T)/(G+C) (B) (A+G)/(T+C) (C) A/G (D) (G+T)/(A+C)

Q.36 Consider the equation $x^3 - 1 = 0$. If one of the solutions to this equation is 1, the other solution(s) is/are

(A) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

(B) i

(C) $-i$

(D) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Q.37 Which of the following statements is/are **CORRECT** regarding self-inductance of a long solenoid having cross sectional area (A), length (l) and having n turns per unit length filled with material of relative permeability μ_r?

(A) It depends on the geometry of solenoid.
(B) It does not depend on geometry of solenoid.
(C) It depends on cross sectional area of solenoid.
(D) It depends on relative permeability of the medium.

Q.38 If an optician prescribes a corrective lens of power -2.0 D, the required lens

(A) is a concave lens.
(B) is a convex lens.
(C) has a focal length of +50 cm.
(D) has a focal length of -50 cm.

Q.39 Which of the following statements is/are **CORRECT**?

(A) Absorption occurs at all wavelengths if light passes through a given solution.

(B) The efficiency of a photochemical process is often expressed in terms of quantum yield.

(C) The unit of molar extinction coefficient is litre $mole^{-1}cm$.

(D) The extent of absorption in a dilute solution would be the same if the concentration is doubled and the path-length of light passing through solution is halved.

Q.40 Which of the following pairs of compounds can be distinguished by iodoform test performed in ammonium hydroxide?

(A) CH_3COCH_3 and C_2H_5OH
(B) C_2H_5OH and CH_3OH
(C) CH_3COCH_3 and $C_6H_5COCH_3$
(D) $C_6H_5COCH_3$ and C_2H_5OH

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The total number of genetically different types of gametes that will be produced by a heterozygous plant carrying the genotypes AABbCc is _____.

Q.42 A healthy individual has the cardiac output of 5.5 L and heart rate of 72 beats per minute. The stroke volume of the individual is _____ mL.

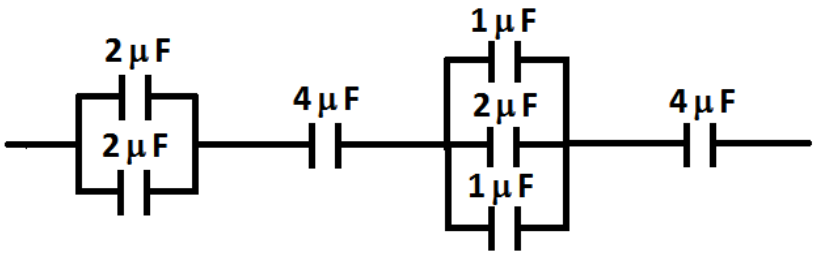
Q.43 Both strands of a DNA molecule are labeled with radioactive thymidine and are allowed to duplicate in an environment containing non-radioactive thymidine. The number of DNA molecules that will contain radioactive thymidine after three duplications is _____.

Q.44 The number of cycles required for complete degradation of Palmitic acid (16 Carbon) by β-oxidation is _____.

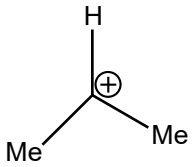
Q.45 The value of $\log_n 4^{-16}$ is -32. The value of n is _____.

Q.46 The determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & -1 & 2 \end{bmatrix}$ is _____.

Q.47 The equivalent capacitance of following assembly of capacitors is _____ μF.

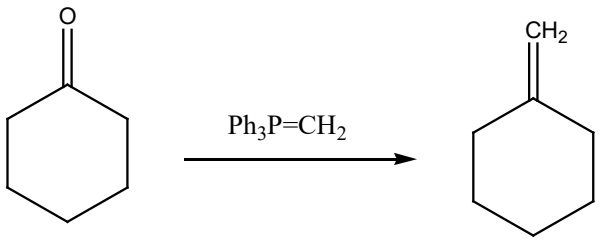


Q.48 The stability of the following carbocation arises from hyperconjugation with _____ number of hydrogen atoms.



Q.49 Oxidation state of Fe in the complex $K_3[Fe(CN)_5NO]$ is (+) _____.

Q.50 The mechanism of the following reaction involves the formation of a _____ membered ring.



Q. 51 – Q. 60 carry two marks each.

Q.51 The concentration of a purified enzyme is 10 mg/mL. Ten microlitres of the enzyme solution in a total reaction volume of 1 mL catalyses the formation of 20 nanomoles of product in one minute under optimum conditions. The specific activity of the enzyme is _____ unit/mg.

Q.52 A 100 nucleotide-long single stranded poly-(A) is synthesized from adenosine monophosphate (AMP) at physiological pH.
(Atomic mass of C = 12, H = 1, O = 16, P = 31; at physiological pH, Molecular mass of AMP = 345).

The molecular mass of the resulting poly-(A) at physiological pH is _____.

Q.53 If a colour-blind woman marries a normal man, the chance that their boy child will be colour-blind is _____%.

Q.54 For a 0.1 M aqueous solution of lysine, the pH at which it carries no net charge is _____.

(pK_a values for: α-carboxylic group = 3.1, α-amino group = 8.0, ε-amino group = 10.8)

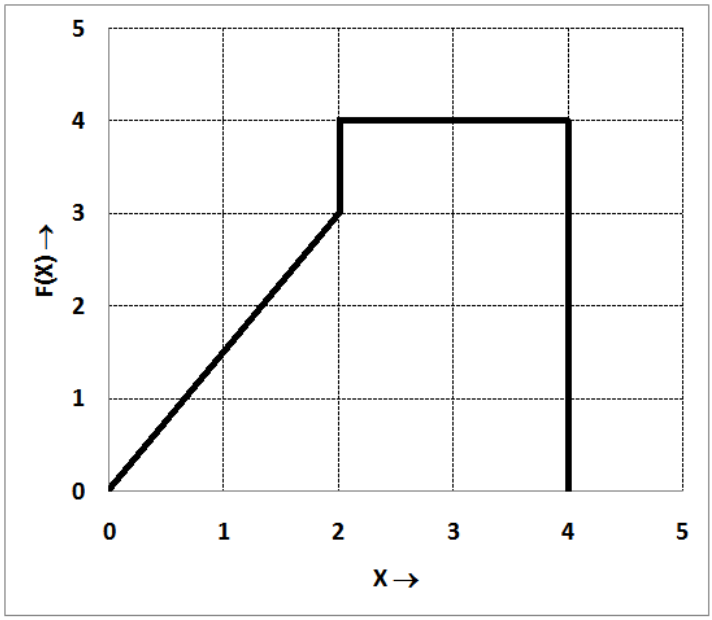
Q.55 For a = _____, the following simultaneous equations have an infinite number of solutions:

$$10x + 13y = 6$$
$$ax + 32.5y = 15$$

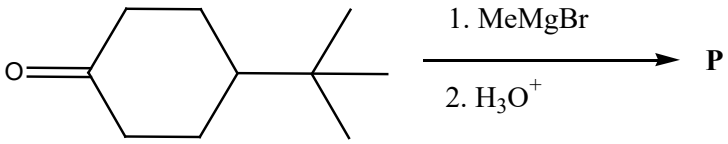
Q.56 If A and B are events such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cup B) = 0.45$, the value of $P(A \cap \bar{B})$ is _____.

Q.57 An ultrasound signal of frequency 50 KHz is sent vertically down into a medium. The signal gets reflected from a depth of 25 mm and returns to source 0.00005 seconds after it is emitted. The wavelength of the ultrasound signal in that medium is _____ cm.

Q.58 The relationship between the applied force F(X) (in Newton) on a body and its displacement X (in metre) is given below. The total amount of work done in moving the body from X = 0 to X = 4 m is _____ Joule.



Q.59 The number of axial C-H bond(s) in the major product (**P**) of the given reaction is _____.



Q.60 A first order reaction is 87.5% complete at the end of 30 minutes. The half-life of the reaction is _____ minute(s).

END OF THE QUESTION PAPER

Paper Code : BT			
Q. No	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	A
2	MCQ	A	C
3	MCQ	A	C
4	MCQ	A	D
5	MCQ	A	B
6	MCQ	A	B
7	MCQ	A	C
8	MCQ	A	A
9	MCQ	A	C
10	MCQ	A	D
11	MCQ	A	B
12	MCQ	A	D
13	MCQ	A	C
14	MCQ	A	A
15	MCQ	A	B
16	MCQ	A	B
17	MCQ	A	C
18	MCQ	A	D
19	MCQ	A	A
20	MCQ	A	C
21	MCQ	A	A
22	MCQ	A	A
23	MCQ	A	B

Paper Code : BT			
Q. No	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	D
25	MCQ	A	C
26	MCQ	A	D
27	MCQ	A	D
28	MCQ	A	A
29	MCQ	A	B
30	MCQ	A	A
31	MSQ	B	A, B, C, D
32	MSQ	B	A, B
33	MSQ	B	A,B
34	MSQ	B	A, B, D
35	MSQ	B	B, D
36	MSQ	B	A, D
37	MSQ	B	A, C, D
38	MSQ	B	A, D
39	MSQ	B	B, D
40	MSQ	B	A, D
41	NAT	C	4.0 to 4.0
42	NAT	C	76 to 77
43	NAT	C	2.0 to 2.0
44	NAT	C	7.0 to 7.0
45	NAT	C	2.0 to 2.0
46	NAT	C	-8.1 to -7.9

Paper Code : BT			
Q. No	**Question Type (QT)**	**Section**	**Key/Range (KY)**
47	NAT	C	0.99 to 1.01
48	NAT	C	6.0 to 6.0
49	NAT	C	2.0 to 2.0
50	NAT	C	4.0 to 4.0
51	NAT	C	0.2 to 0.2
52	NAT	C	32810 to 32825
53	NAT	C	100 to 100
54	NAT	C	9.4 to 9.4
55	NAT	C	25.0 to 25.0
56	NAT	C	0.24 to 0.26
57	NAT	C	1.99 to 2.01
58	NAT	C	10.99 to 11.01
59	NAT	C	5.0 to 5.0
60	NAT	C	9.8 to 10.2

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 On hydrolysis, aluminium carbide produces

(A) CH_4 (B) C_2H_6 (C) C_2H_4 (D) C_2H_2

Q.2 Carbonic anhydrase is an example of

(A) Hydrolysis enzyme (B) Redox enzyme

(C) O_2 transport protein (D) Heme protein

Q.3 The **CORRECT** order of melting points of group 15 trifluorides is

(A) $PF_3 < AsF_3 < SbF_3 < BiF_3$ (B) $BiF_3 < SbF_3 < PF_3 < AsF_3$

(C) $PF_3 < SbF_3 < AsF_3 < BiF_3$ (D) $BiF_3 < AsF_3 < SbF_3 < PF_3$

Q.4 NaF, KF, MgO and CaO are crystalline solids. They have NaCl structure. Their lattice energies vary in the order

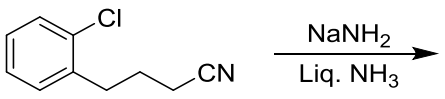
(A) NaF < KF < MgO < CaO

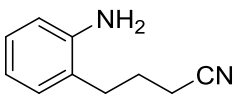
(B) KF < NaF < CaO < MgO

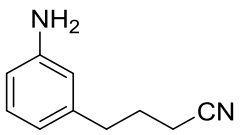
(C) MgO < CaO < NaF < KF

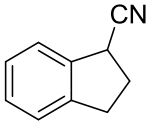
(D) CaO < MgO < KF < NaF

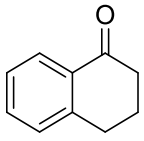
Q.5 The major product formed in the following reaction is



(A) 

(B) 

(C) 

(D) 

Q.6 The compound that contains the most acidic hydrogen is

(A) $H_2C{=}CH_2$ (B) $HC{\equiv}CH$ (C) $H_2C{=}C{=}CH_2$ (D) $H_3C{-}CH_3$

Q.7 The C-2 epimer of D-glucose is

(A) D-Mannose (B) D-Fructose (C) D-Galactose (D) D-Gulose

Q.8 The value of integral $\int_{-2}^{+2} x\, e^{-2x^2} dx$ is

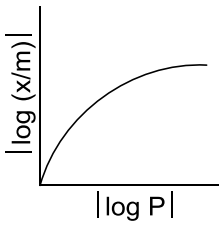
(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

Q.9 The number of crystal systems and the number of Bravais lattices are, respectively,

(A) 14 and 7 (B) 7 and 32 (C) 32 and 14 (D) 7 and 14

Q.10 For adsorption of a gas on a solid surface, the plot that represents Freundlich isotherm is (x = mass of gas, m = mass of adsorbent, P = pressure)

(A)



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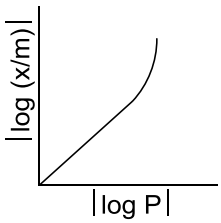
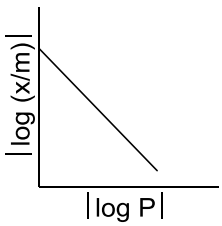
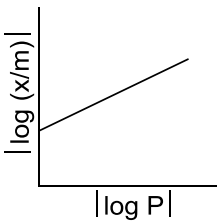


(C)



(D)



Q. 11 – Q. 30 carry two marks each.

Q.11 With respect to periodic properties, the **CORRECT** statement is

(A) Electron affinity order is F > O > Cl

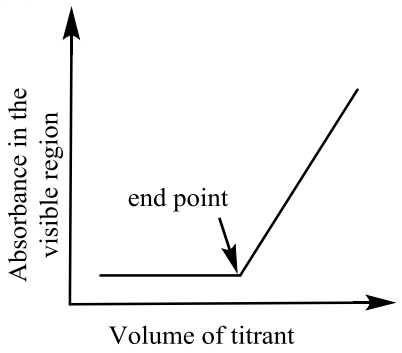
(B) First ionisation energy order is Al > Mg > K

(C) Atomic radius order is N > P > As

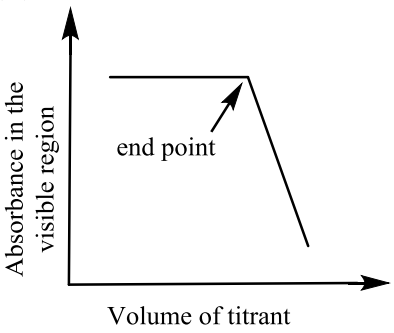
(D) Ionic radius order is $K^+ > Ca^{2+} > Mg^{2+}$

Q.12 Which plot represents a spectrophotometric titration, where the titrant alone absorbs light in the visible region?

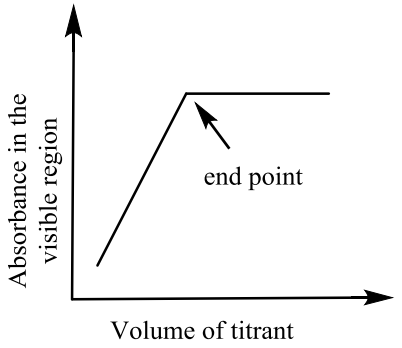
(A)



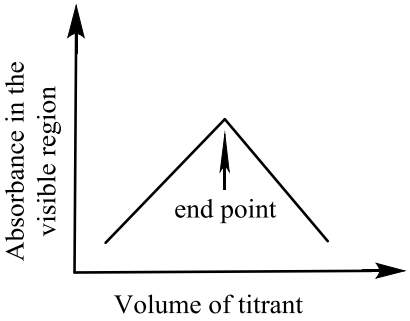
(B)



(C)



(D)



Q.13 Among the following metal carbonyl species, the one with the highest metal-carbon back bonding is

(A) $[Ti(CO)_6]^{2-}$ (B) $[V(CO)_6]^{-}$ (C) $Cr(CO)_6$ (D) $[Mn(CO)_6]^{+}$

Q.14 The **CORRECT** order of Δ_o (the octahedral crystal field splitting of d orbitals) values for the following anionic metal complexes is

(A) $[Ir(CN)_6]^{3-} < [Rh(CN)_6]^{3-} < [RhI_6]^{3-} < [CoI_6]^{3-}$

(B) $[CoI_6]^{3-} < [RhI_6]^{3-} < [Rh(CN)_6]^{3-} < [Ir(CN)_6]^{3-}$

(C) $[CoI_6]^{3-} < [Rh(CN)_6]^{3-} < [RhI_6]^{3-} < [Ir(CN)_6]^{3-}$

(D) $[Ir(CN)_6]^{3-} < [CoI_6]^{3-} < [Rh(CN)_6]^{3-} < [RhI_6]^{3-}$

Q.15 The decay modes of ^{14}C and ^{14}O are

(A) β decay

(B) positron emission

(C) β decay and positron emission, respectively

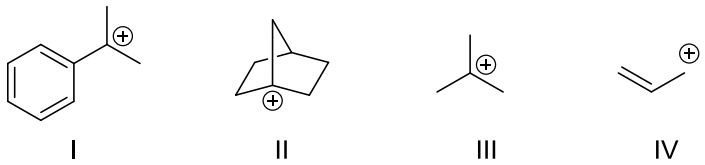
(D) positron emission and β decay, respectively

Q.16 Consider the following four xenon compounds: XeF_2, XeF_4, XeF_6 and XeO_3
The pair of xenon compounds expected to have non-zero dipole moment is

(A) XeF_4 and XeF_6 (B) XeF_2 and XeF_4

(C) XeF_2 and XeO_3 (D) XeF_6 and XeO_3

Q.17 The **CORRECT** order of stability for the following carbocations is

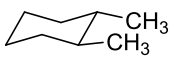


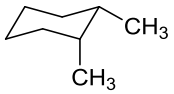
I II III IV

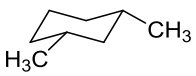
(A) I < III < IV < II (B) III < II < IV < I

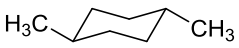
(C) II < IV < III < I (D) IV < III < I < II

Q.18 Among the dimethylcyclohexanes, which one can be obtained in enantiopure form?

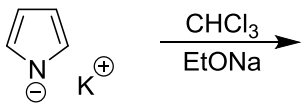
(A) 

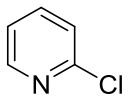
(B) 

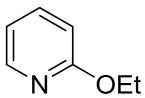
(C) 

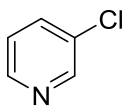
(D) 

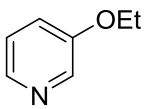
Q.19 The major product formed in the following reaction is



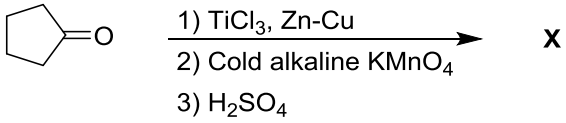
(A) 

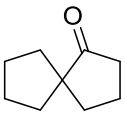
(B) 

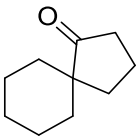
(C) 

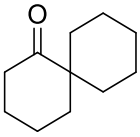
(D) 

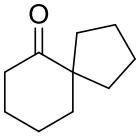
Q.20 The product **X** in the following reaction sequence is



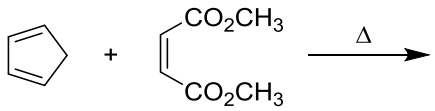
(A) 

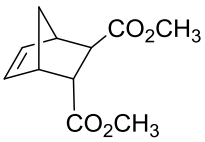
(B) 

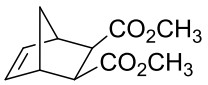
(C) 

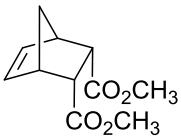
(D) 

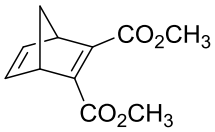
Q.21 The major product formed in the following reaction is



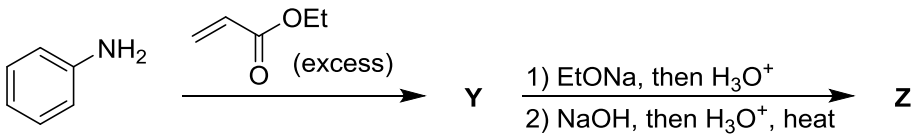
(A) 

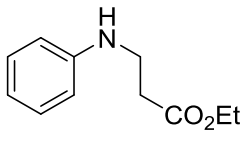
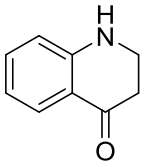
(B) 

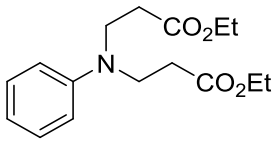
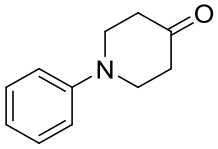
(C) 

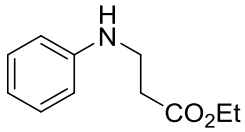
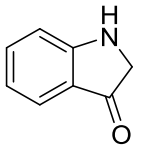
(D) 

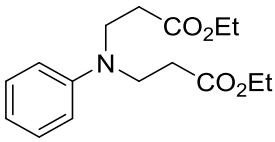
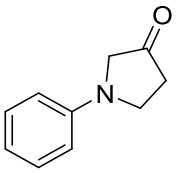
Q.22 The major products **Y** and **Z** in the following reaction sequence are



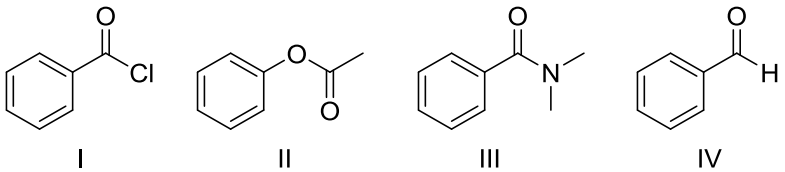
(A) **Y** =  **Z** = 

(B) **Y** =  **Z** = 

(C) **Y** =  **Z** = 

(D) **Y** =  **Z** = 

Q.23 The **CORRECT** order of carbonyl stretching frequencies for the following compounds is



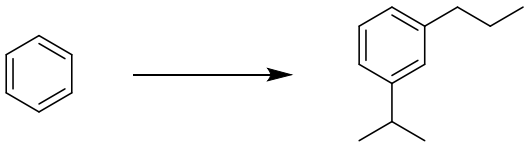
(A) II < I < III < IV

(B) I < III < II < IV

(C) IV < II < III < I

(D) III < IV < II < I

Q.24 The sequence of three steps involved in the following conversion is



(A) (i) Friedel-Crafts alkylation; (ii) Reduction; (iii) Friedel-Crafts acylation

(B) (i) Friedel-Crafts acylation; (ii) Friedel-Crafts alkylation; (iii) Reduction

(C) (i) Friedel-Crafts acylation; (ii) Reduction; (iii) Friedel-Crafts alkylation

(D) (i) Friedel-Crafts alkylation; (ii) Friedel-Crafts acylation; (iii) Reduction

Q.25 The **CORRECT** expression that corresponds to reversible and adiabatic expansion of an ideal gas is

(A) $\Delta U = 0$ (B) $\Delta H = 0$ (C) $\Delta S = 0$ (D) $\Delta G = 0$

Q.26 The electrolyte AB_2 ionises in water as

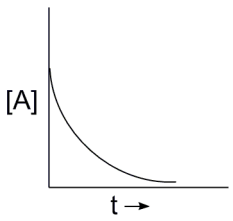
$$AB_2 \rightleftharpoons A^{2+} + 2B^-$$

The mean ionic activity coefficient $(\gamma_{\pm})$ is

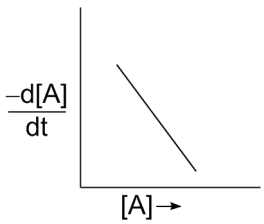
(A) $\boldsymbol{\gamma}_{A^{2+}}^{\frac{1}{2}} \boldsymbol{\gamma}_{B^-}$ (B) $\boldsymbol{\gamma}_{A^{2+}}^{\frac{1}{2}} \boldsymbol{\gamma}_{B^-}^{\frac{2}{3}}$ (C) $\boldsymbol{\gamma}_{A^{2+}}^{\frac{2}{3}} \boldsymbol{\gamma}_{B^-}^{\frac{1}{3}}$ (D) $(\boldsymbol{\gamma}_{A^{2+}} + 2\boldsymbol{\gamma}_{B^-})^{\frac{1}{2}}$

Q.27 The reaction, **A** ⟶ **Products**, follows first-order kinetics. If [A] represents the concentration of reactant at time *t*, the **INCORRECT** variation is shown in

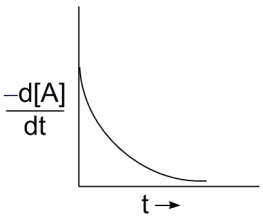
(A)



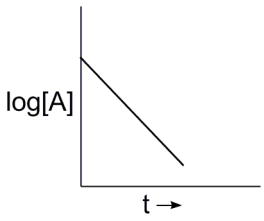
(B)



(C)



(D)



Q.28 The behavior of Cl_2 is closest to ideal gas behavior at

(A) 100 °C and 10.0 atm
(B) 0 ° C and 0.50 atm
(C) 200 °C and 0.50 atm
(D) −100 °C and 10.0 atm

Q.29 A vector $\vec{A} = \vec{\imath} + x\vec{\jmath} + 3\vec{k}$ is rotated through an angle and is also doubled in magnitude resulting in $\vec{B} = 4\vec{\imath} + (4x - 2)\vec{\jmath} + 2\vec{k}$. An acceptable value of x is

(A) 1 (B) 2 (C) 3 (D) $^4/_3$

Q.30 With reference to the variation of molar conductivity (Λ_m) with concentration for a strong electrolyte in an aqueous solution, the **CORRECT** statement is

(A) The asymmetry effect contributes to decrease Λ_m whereas the electrophoretic effect contributes to increase Λ_m
(B) The asymmetry effect contributes to increase Λ_m whereas the electrophoretic effect contributes to decrease Λ_m
(C) Both asymmetry effect and electrophoretic effect contribute to decrease Λ_m
(D) Both asymmetry effect and electrophoretic effect contribute to increase Λ_m

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Which of the following metal(s) is(are) extracted from its(their) sulfide ore(s) by self-reduction/air reduction method?

(A) Cu (B) Al (C) Au (D) Pb

Q.32 In a saturated calomel electrode, the saturation is with respect to

(A) KCl (B) Hg_2Cl_2 (C) $HgCl_2$ (D) AgCl

Q.33 Consider the following six solid binary oxides: CaO, Al_2O_3, PbO, Cs_2O, SiO_2 and Sb_2O_3. The pair(s) of ionic oxides is(are)

(A) CaO and Al_2O_3 (B) CaO and PbO (C) Cs_2O and Al_2O_3 (D) SiO_2 and Sb_2O_3

Q.34 Choose the **CORRECT** answer(s) with respect to the magnesium-EDTA titration carried out in the pH range 7 – 10.5, using Solochrome black as indicator

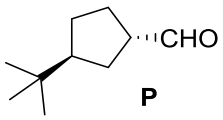
(A) Magnesium–indicator complex is more stable than the magnesium–EDTA complex

(B) At the end point, the colour changes from red to blue

(C) After the end point, the colour of the solution is due to the indicator

(D) pH range of 7 – 10.5 is necessary for observing the specific colour change

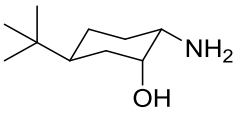
Q.35 On reaction with $NaNO_2$ and HCl, which of the following amino alcohol(s) will yield compound **P**?

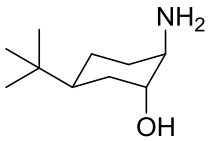


P

(A) 

(B) 

(C) 

(D) 

Q.36 The **CORRECT** statement(s) about carbene is(are)

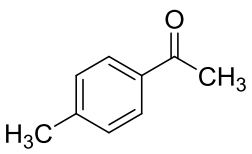
(A) Carbene is a neutral species

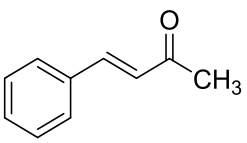
(B) Carbene is an intermediate in the Curtius rearrangement

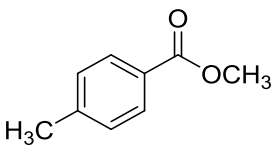
(C) Carbene can insert into both σ and π-bonds

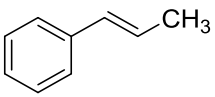
(D) Carbene is generated from amines on reaction with nitrous acid

Q.37 The compound(s) that shows(show) positive haloform test is(are)

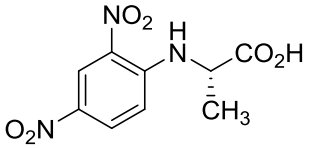
(A) 

(B) 

(C) 

(D) 

Q.38 Tetrapeptide(s) that gives(give) the following product on reaction with Sanger's reagent followed by hydrolysis is(are)



(A) Ala-Gly-Leu-Phe

(B) Asp-Phe-Leu-Pro

(C) Asp-Gly-Tyr-Phe

(D) Ala-Phe-Tyr-Pro

Q.39 Which of the following set(s) of quantum numbers is(are) **NOT** allowed?

(A) $n = 3$, $l = 2$, $m_l = -1$

(B) $n = 4$, $l = 0$, $m_l = -1$

(C) $n = 3$, $l = 3$, $m_l = -3$

(D) $n = 5$, $l = 3$, $m_l = +2$

Q.40 The **CORRECT** expression(s) for isothermal expansion of 1 mol of an ideal gas is(are)

(A) $\Delta A = RT\ ln\frac{V_{initial}}{V_{final}}$

(B) $\Delta G = RT\ ln\frac{V_{initial}}{V_{final}}$

(C) $\Delta H = RT\ ln\frac{V_{final}}{V_{initial}}$

(D) $\Delta S = R\ ln\frac{V_{final}}{V_{initial}}$

SECTION – C

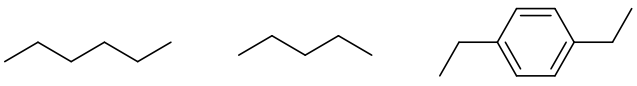
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

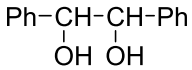
Q.41 The number of possible isomers for [Pt(py)(NH_3)BrCl] is _____. (py is pyridine)

Q.42 The volume of 0.3 M ferrous ammonium sulphate solution required for the completion of redox titration with 20 mL of 0.1 M potassium dichromate solution is _____mL.

Q.43 Among the following hydrocarbon(s), how many of them would give rise to three groups of proton NMR peaks with 2:2:3 integration ratio?



Q.44 The number of stereoisomers possible for the following compound is _____.



Q.45 The number of hydrogen bond(s) present in a guanine-cytosine base pair is _____.

Q.46 The time for 50% completion of a zero order reaction is 30 min. Time for 80% completion of this reaction is _____ min.

Q.47 Consider the reaction $CO(g) + \frac{1}{2} O_2(g) \longrightarrow CO_2(g)$.
The value of ΔU for the reaction at 300 K is –281.8 kJ mol^{-1}. The value of ΔH at same temperature is _____ kJ mol^{-1} (rounded up to the first decimal place).
[R = 8.3 J K^{-1} mol^{-1}]

Q.48 The nuclear spin quantum number (I) of a nucleus is $\frac{3}{2}$. When placed in an external magnetic field, the number of possible spin energy states it can occupy is _____.

Q.49 The value of $\mathbf{C_v}$ for 1 mol of N_2 gas predicted from the principle of equipartition of energy, ignoring vibrational contribution, is _____ J K^{-1} mol^{-1} (rounded up to two decimal places).
[R = 8.3 JK^{-1}mol^{-1}]

Q.50 Assuming ideal gas behavior, the density of O_2 gas at 300 K and 1.0 atm is _____ g L^{-1} (rounded up to two decimal places).
[R = 0.082 L atm mol^{-1} K^{-1}, molar mass of O_2 = 32]

Q. 51 – Q. 60 carry two marks each.

Q.51 How many of the following interhalogen species have 2 lone pairs of electrons on the central atom?

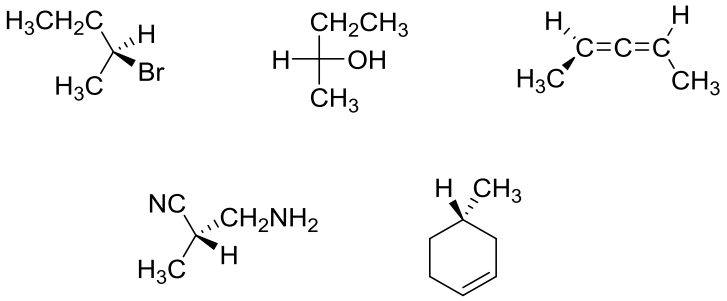
ClF_3, ClF_2^-, ClF_5 and ICl_2^+

Q.52 ^{24}Na decays to one-fourth of its initial amount in 29.8 hours. Its decay constant is_____ hour^{-1} (rounded up to four decimal places).

Q.53 The magnitude of crystal field stabilization energy (CFSE) of octahedral $[Ti(H_2O)_6]^{3+}$ complex is 7680 cm^{-1}. The wavelength at the maximum absorption (λ_{max}) of this complex is _____ nm (rounded up to the nearest integer).

Q.54 Elemental analysis of an organic compound containing C, H and O gives percentage composition: C: 39.9 % and H: 6.7 %. If the molecular weight of the compound is 180, the number of carbon atoms present in the molecule is_____.

Q.55 The number of compounds having *S*-configuration among the following is _____.



Q.56 The *emf* of a standard cadmium cell is 1.02 V at 300 K. The temperature coefficient of the cell is -5.0×10^{-5} V K^{-1}. The value of ΔH° for the cell is _____ kJ mol^{-1} (rounded up to two decimal places).
[1 F = 96500 C mol^{-1}]

Q.57 For the reaction $H_2(g) + \frac{1}{2}\,O_2(g) \longrightarrow H_2O(l)$, the following information is given
T = 300 K
$\Delta\bar{H}^\circ$ = –285 kJ mol^{-1} $\bar{S}^o_{H_2O}(l)$ = 70 J K^{-1} mol^{-1}
$\bar{S}^o_{O_2}(g)$ = 204 J K^{-1} mol^{-1} $\bar{S}^o_{H_2}(g)$ = 130 J K^{-1} mol^{-1}

$\Delta\bar{S}^o_{universe}$ for the reaction is _____ J K^{-1} mol^{-1}.

Q.58 For H_2 molecule, the fundamental vibrational frequency ($\bar{\upsilon}_e$) can be taken as 4400 cm^{-1}. The zero-point energy of the molecule is _____ kJ mol^{-1} (rounded up to two decimal places).
[h = 6.6×10^{-34} J s, c = 3×10^8 m s^{-1}, $N_A = 6\times10^{23}$ mol^{-1}]

Q.59 The solubility of PbI_2 in 0.10 M KI(aq) is _____ $\times10^{-7}$ M (rounded up to two decimal places).
[The solubility product, $K_{sp} = 7.1\times10^{-9}$]

Q.60 The electron of a hydrogen atom is in its n^{th} Bohr orbit having de Broglie wavelength of 13.4 Å. The value of n is _____ (rounded up to the nearest integer).
[Radius of n^{th} Bohr orbit = $0.53n^2$ Å, π = 3.14]

END OF THE QUESTION PAPER

Paper Code : CY			
Q. No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	A
2	MCQ	A	A
3	MCQ	A	A
4	MCQ	A	B
5	MCQ	A	C
6	MCQ	A	B
7	MCQ	A	A
8	MCQ	A	A
9	MCQ	A	D
10	MCQ	A	D
11	MCQ	A	D
12	MCQ	A	A
13	MCQ	A	A
14	MCQ	A	B
15	MCQ	A	C
16	MCQ	A	D
17	MCQ	A	C
18	MCQ	A	A
19	MCQ	A	C
20	MCQ	A	D
21	MCQ	A	C
22	MCQ	A	B
23	MCQ	A	D

Paper Code : CY			
Q. No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	B
25	MCQ	A	C
26	MCQ	A	Marks To All (MTA)
27	MCQ	A	B
28	MCQ	A	C
29	MCQ	A	B
30	MCQ	A	C
31	MSQ	B	A,D
32	MSQ	B	A,B
33	MSQ	B	A,C
34	MSQ	B	B,C,D
35	MSQ	B	B,C
36	MSQ	B	A,C
37	MSQ	B	A,B
38	MSQ	B	A,D
39	MSQ	B	B,C
40	MSQ	B	A,B,D
41	NAT	C	3 to 3
42	NAT	C	40 to 40
43	NAT	C	2 to 2
44	NAT	C	3 to 3
45	NAT	C	3 to 3
46	NAT	C	48 to 48

Paper Code : CY			
Q. No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
47	NAT	C	-286.0 to -282.0
48	NAT	C	4 to 4
49	NAT	C	20.00 to 21.00
50	NAT	C	1.29 to 1.31
51	NAT	C	2 to 2
52	NAT	C	0.0460 to 0.0470
53	NAT	C	520 to 521
54	NAT	C	6 to 6
55	NAT	C	4 to 4
56	NAT	C	-201.00 to -198.00
57	NAT	C	786 to 790
58	NAT	C	25.80 to 26.40
59	NAT	C	7.0 to 7.2
60	NAT	C	4 to 4

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one among the following planets in the Solar system is most similar in size to the Earth?

(A) Mercury (B) Venus (C) Neptune (D) Uranus

Q.2 In which one of the following tectonic settings are the highest mountain chains and thickest crust found?

(A) Island arc (B) Continental arc
(C) Continental collision (D) Transcurrent

Q.3 The second-most abundant oxide in the Earth's crust is

(A) Al_2O_3 (B) SiO_2 (C) CaO (D) Na_2O

Q.4 The type of dentition found in *Trigonia* is

(A) schizodont (B) taxodont (C) pachydont (D) isodont

Q.5 Which one of the following minerals has isolated $(SiO_4)^{4-}$ tetrahedra linked by divalent cations in octahedral coordination?

(A) Muscovite (B) Quartz (C) Beryl (D) Olivine

Q.6 Which one of the following is NOT found in an extensional setting?

(A) Normal faults (B) Horsts (C) Rifts (D) Thrust faults

Q.7 The texture characterized by exsolved lamellae of albite in K-feldspar is known as

(A) myrmekite (B) graphic (C) perthite (D) antiperthite

Q.8 Fissility is best shown by

(A) sandstone (B) siltstone (C) shale (D) limestone

Q.9 Petroleum is NOT commercially produced from

(A) Krishna–Godavari basin (B) Cauvery–Palar basin
(C) Cambay basin (D) Vindhyan basin

Q.10 Among the following, the mineral showing acicular habit is

(A) kyanite (B) tourmaline (C) biotite (D) sillimanite

Q. 11 – Q. 30 carry two marks each.

Q.11 Isostasy involves _____ continental mountain belts.

(A) compensation in (B) creation of (C) destruction of (D) thrusting in

Q.12 Identify the pair from the following list that is NOT correctly matched.

(A) Caldera – stratovolcano
(B) Pillow basalt – subaerial eruption
(C) Ropy lava – pahoehoe flow
(D) Amygdales – filled vesicles

Q.13 Wilson orogenic cycle in continents is initiated by

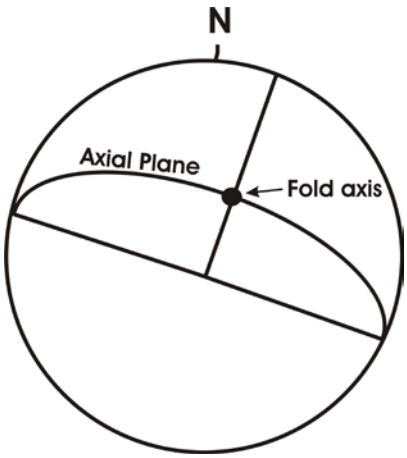
(A) collision (B) rifting (C) drifting (D) subduction

Q.14 Match the processes in Group I with corresponding geomorphic features in Group II.

Group I	**Group II**
P. Dissolution	1. Mushroom rocks
Q. Abrasion	2. Exfoliation domes
R. Deposition	3. Sinkholes
S. Onion skin weathering	4. Moraines

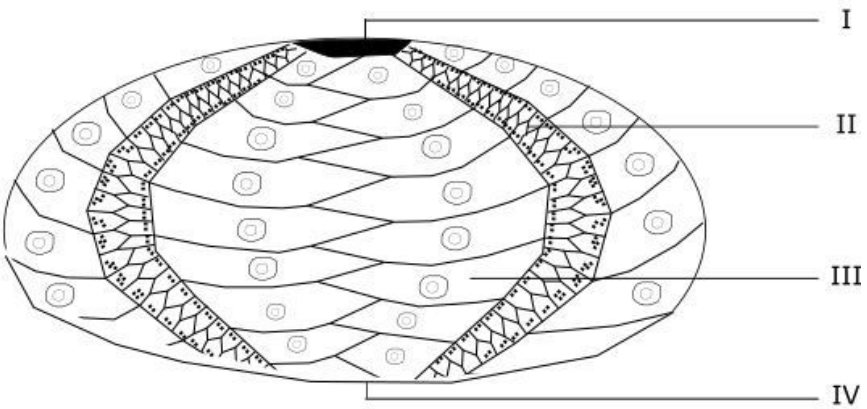
(A)	(B)	(C)	(D)
P-1	P-3	P-3	P-2
Q-3	Q-1	Q-1	Q-1
R-2	R-2	R-4	R-4
S-4	S-4	S-2	S-3

Q.15 The orientations of the fold axis and axial plane in the given figure indicate



(A) reclined fold (B) vertical fold (C) recumbent fold (D) horizontal fold

Q.16 Identify the correct morphological features corresponding to numbers I – IV in the echinoid illustrated below:



(A) I-Periproct	II-Ambulacra	III-Interambulacra	IV-Peristome
(B) I-Periproct	II-Interambulacra	III-Ambulacra	IV-Peristome
(C) I-Peristome	II-Interambulacra	III-Ambulacra	IV-Periproct
(D) I-Peristome	II-Ambulacra	III-Interambulacra	IV-Periproct

Q.17 The correct order of marine benthic habitats with increasing water depths is

(A) abyssal, bathyal, neritic (B) neritic, abyssal, bathyal
(C) neritic, bathyal, abyssal (D) bathyal, abyssal, neritic

Q.18 Which one of the following invertebrates has the most primitive visual system?

(A) Ammonites (B) Brachiopods (C) Gastropods (D) Trilobites

Q.19 The correct chronological sequence (older to younger) of the Precambrian stratigraphic units listed below is

(A) Sargur Group, Chitradurga Group, Alwar Group, Kaimur Group
(B) Chitradurga Group, Sargur Group, Kaimur Group, Alwar Group
(C) Sargur Group, Alwar Group, Chitradurga Group, Kaimur Group
(D) Sargur Group, Chitradurga Group, Kaimur Group, Alwar Group

Q.20 Match the Formations in Group I with corresponding characteristic fossils in Group II.

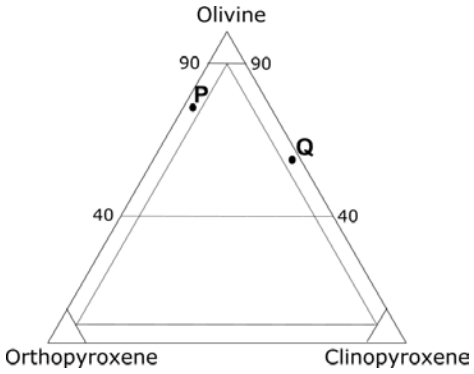
Group I	**Group II**
P. Barakar Formation	1. *Stegodon*
Q. Uttatur Formation	2. *Sauropoda*
R. Dhok Pathan Formation	3. *Belemnites*
S. Lameta Formation	4. *Glossopteris*

(A)	(B)	(C)	(D)
P-3	P-4	P-3	P-4
Q-1	Q-3	Q-4	Q-3
R-4	R-1	R-1	R-2
S-2	S-2	S-2	S-1

Q.21 Which one of the following sedimentary structures is NOT used for determining top and bottom of beds?

(A) Mud cracks
(B) Load and flame structures
(C) Sharp-crested wave ripples
(D) Plane lamination

Q.22 Identify the rocks P and Q in the diagram as per the IUGS classification.



(A) P – Websterite, Q – Wehrlite
(B) P – Dunite, Q – Websterite
(C) P – Websterite, Q – Dunite
(D) P – Harzburgite, Q – Wehrlite

Q.23 Which one of the following is produced by a closed-system metamorphic reaction between muscovite and quartz?

(A) orthoclase + sillimanite
(B) orthoclase + biotite
(C) plagioclase + biotite
(D) plagioclase + sillimanite

Q.24 The assemblage staurolite + garnet + biotite + muscovite + quartz in pelites is stable in

(A) greenschist facies
(B) amphibolite facies
(C) granulite facies
(D) pyroxene hornfels facies

Q.25 Conglomerates are commonly deposited in

(A) aeolian dunes
(B) tidal flats
(C) alluvial fans
(D) river flood plains

Q.26 Match the mineral deposits in Group I with corresponding Indian occurrences in Group II.

Group I	**Group II**
P. Iron	1. Mangampet, Andhra Pradesh
Q. Uranium	2. Balaghat, Madhya Pradesh
R. Manganese	3. Narwa Pahar, Jharkhand
S. Baryte	4. Hospet, Karnataka

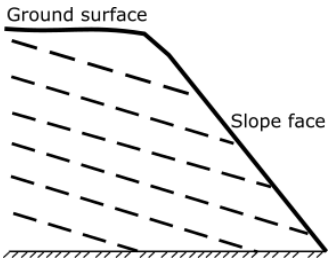
(A)	(B)	(C)	(D)
P-1	P-4	P-3	P-4
Q-3	Q-1	Q-4	Q-3
R-4	R-3	R-2	R-2
S-2	S-2	S-1	S-1

Q.27 Which one of the following processes is responsible for the formation of syngenetic Ni-Cu sulphide ore in gabbro-noritic rocks?

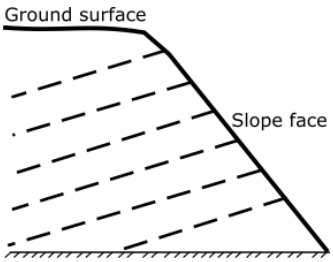
(A) Hydrothermal replacement
(B) Volcanic exhalation
(C) Liquid immiscibility
(D) Contact metamorphism

Q.28 Dashed lines in the figures given below represent joints. Considering only the orientations of the joints and the slope face, which one of the following represents the most stable slope?

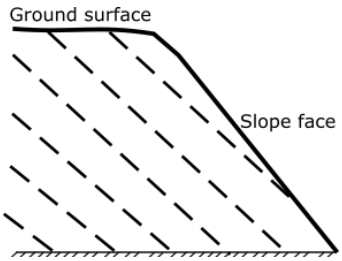
(A)



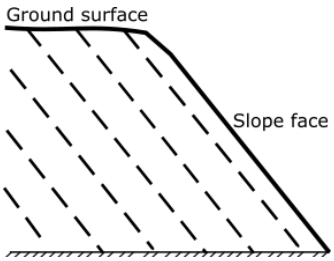
(B)



(C)



(D)



Q.29 Match the morphological features/life processes in Group I with corresponding organisms in Group II.

Group I	**Group II**
P. Water vascular system	1. Cephalopoda
Q. Moulting	2. Echinodermata
R. Jet propulsion locomotion	3. Brachiopoda
S. Lophophore	4. Trilobita

(A)	(B)	(C)	(D)
P-2	P-3	P-2	P-4
Q-3	Q-4	Q-4	Q-3
R-1	R-2	R-1	R-2
S-4	S-1	S-3	S-1

Q.30 Match the plutonic rocks in Group I with corresponding volcanic equivalents in Group II.

Group I	**Group II**
P. Granite	1. Andesite
Q. Syenite	2. Basalt
R. Diorite	3. Rhyolite
S. Gabbro	4. Trachyte

(A)	(B)	(C)	(D)
P-2	P-3	P-4	P-3
Q-3	Q-4	Q-3	Q-4
R-4	R-1	R-1	R-2
S-1	S-2	S-2	S-1

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Which of the following change(s) when a dipping bed with a plunging lineation is rotated about a vertical axis?

(A) Dip amount of bed
(B) Plunge amount of lineation
(C) Plunge direction of lineation
(D) Strike of bed

Q.32 Which of the following indicate(s) the presence of directed stress in a rock?

(A) Porphyritic texture
(B) Schistosity
(C) Gneissosity
(D) Mylonitic texture

Q.33 The correct combination(s) of ranks and corresponding categories of stratigraphic units is/are

(A) Formation – Lithostratigraphy
(B) System – Chronostratigraphy
(C) Period – Chronostratigraphy
(D) Group – Biostratigraphy

Q.34 The correct order(s) of stability of silica polymorphs with increasing pressure is/are

(A) Quartz – Coesite – Stishovite
(B) Quartz – Stishovite – Coesite
(C) Tridymite – Coesite – Stishovite
(D) Tridymite – Stishovite – Coesite

Q.35 Which of the following statement(s) is/are correct for the upper hemisphere stereographic projection of a crystal given below?

(A) Angle between the axes, $\alpha = \beta = \gamma = 90^{o}$
(B) Crystal contains 1 tetrad
(C) Crystal contains 4 diads
(D) Crystal contains 5 mirror planes

Q.36 Which of the following statement(s) is/are correct?

(A) An isotropic mineral remains dark through 360º rotation of stage under crossed polars
(B) Pleochroism is the change of colour of a mineral during rotation under crossed polars
(C) Minerals of the Triclinic system are optically uniaxial
(D) Melatope in an interference figure marks the emergence of an optic axis

Q.37 Hermatypic corals are typically found in

(A) the photic zone
(B) warm and clear water
(C) cool deep water
(D) reefs

Q.38 Choose the characteristic mineral(s) formed in the supergene enriched zone of a sulphide deposit.

(A) Psilomelane (B) Covellite (C) Cassiterite (D) Chalcocite

Q.39 Which of the following is/are true for crystallization of plagioclase phenocrysts from a basic magma forming a layered intrusion?

(A) Cumulus texture at the base
(B) Anorthite-rich early plagioclase at the base
(C) Albite-rich late plagioclase at the top
(D) Quench texture at the base

Q.40 Major mass extinction events occurred in the

(A) end Silurian
(B) end Carboniferous
(C) end Permian
(D) early Devonian

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 When plotted on a map of 1:50000 scale, a 2 km long dyke exposed on a horizontal surface has a length of _____cm (answer in one decimal place).

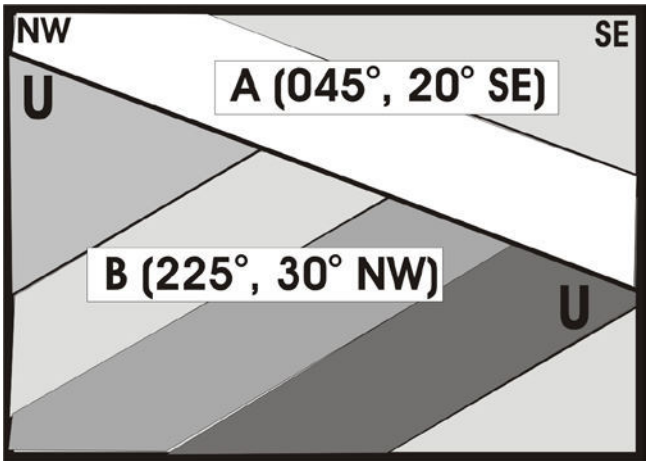
Q.42 The valency of iron in hematite is _____.

Q.43 A crustal rock is at a lithostatic pressure of 3 kbar and a temperature of 275ºC. If the lithostatic pressure increases at a uniform rate of 0.3 kbar/km, and the surface temperature is 25ºC, the geothermal gradient (in ºC/km) is_____ (answer in one decimal place).

Q.44 The absolute difference in the Moh's hardness values of the two silicates among the minerals listed below is _____.

Apatite, Corundum, Gypsum, Talc, Topaz

Q.45 Attitudes of beds in sequences A (younger) and B (older), separated by an unconformity UU, are given in the following sectional view. If UU was horizontal when sequence A was deposited, the dip amount of beds in sequence B at that time was _____ (answer in one decimal place).

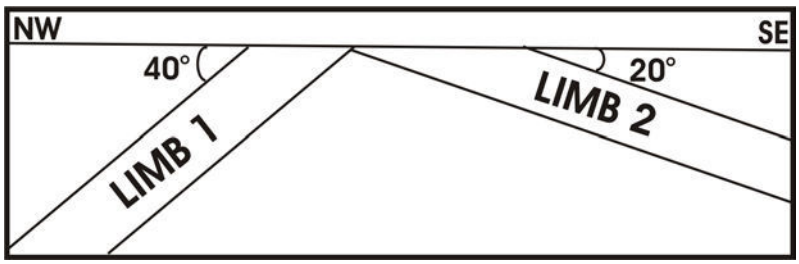


Q.46 The number of alpha (α) particles emitted to produce a daughter isotope of ^{206}Pb from a parent isotope of ^{238}U by radioactive decay is _____.

Q.47 The dip slip on a fault 000º, 30ºE is 10 m. Assuming slip equals separation here, the throw on the fault is _____ m (answer in one decimal place).

Q.48 A continuous 10 m thick sequence of shale was deposited in 10,000 years at uniform rate of sedimentation. The number of samples that must be collected at equal stratigraphic intervals to sample the succession every 500 years is _____.

Q.49 Attitudes of the two limbs of a non-plunging kink fold shown below are 045º, 20ºSE and 045º, 40ºNW. The dip amount (in degrees) of the axial plane of the kink fold is _____ (answer in one decimal place).



Q.50 In the garnet formula $(Fe_{2.5}Mg_{0.3}Ca_x)Al_2Si_3O_{12}$, x represents the number of atoms of Ca. The mole % of grossular in the garnet is _____ (answer in one decimal place).

Q. 51 – Q. 60 carry two marks each.

Q.51 Assuming the Earth to be an ideal sphere, the volume % of the core relative to the total volume of the Earth is _____ (answer in one decimal place).

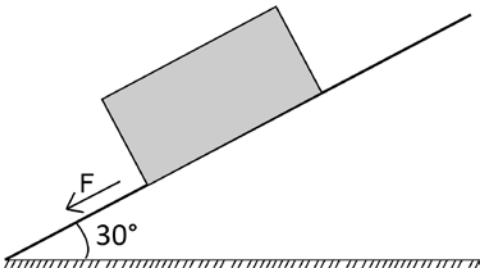
Q.52 Based on 8 oxygen atoms, the number of silicon atoms in a plagioclase of composition $Ab_{20}An_{80}$ is ____ (answer in one decimal place).

Q.53 600 tons of low grade iron ore (40% Fe) are blended with 400 tons of high grade iron ore (65% Fe). The grade of the blended ore is _____ % Fe (answer in one decimal place).

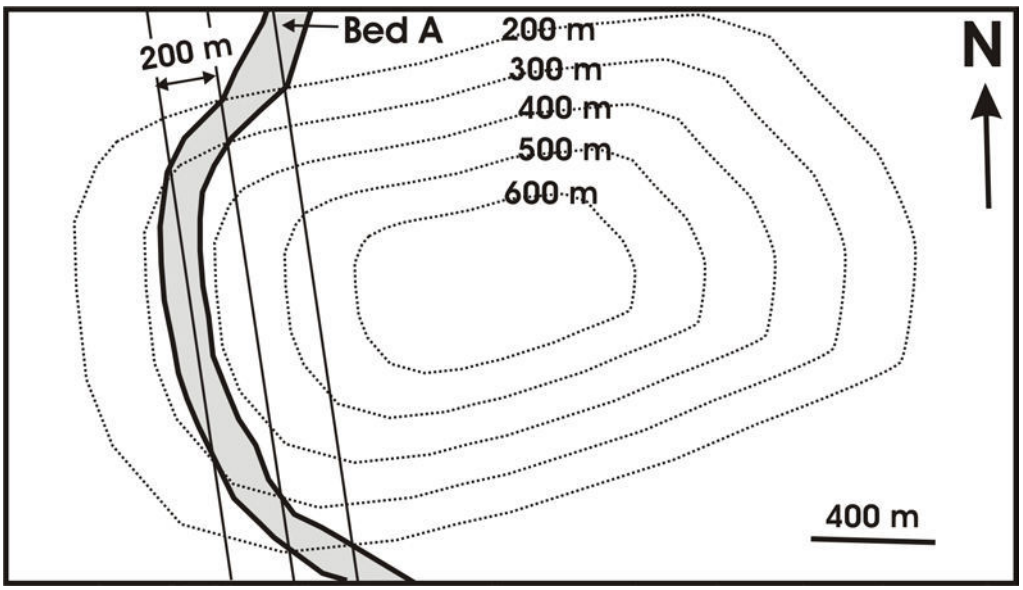
Q.54 The mass of a fully dried rock sample of volume 100 cm^3 is 300 g. The mass of the sample, when fully saturated with water of density 1.00 g/cm^3, is 325 g. Assuming no volume change, the computed porosity of the rock is _____ % (answer in one decimal place).

Q.55 When a dunite comprising pure forsterite undergoes melting, the weight % of MgO in the melt is _____ (answer in one decimal place; given molecular weights of SiO_2 = 60.08; MgO = 40.30).

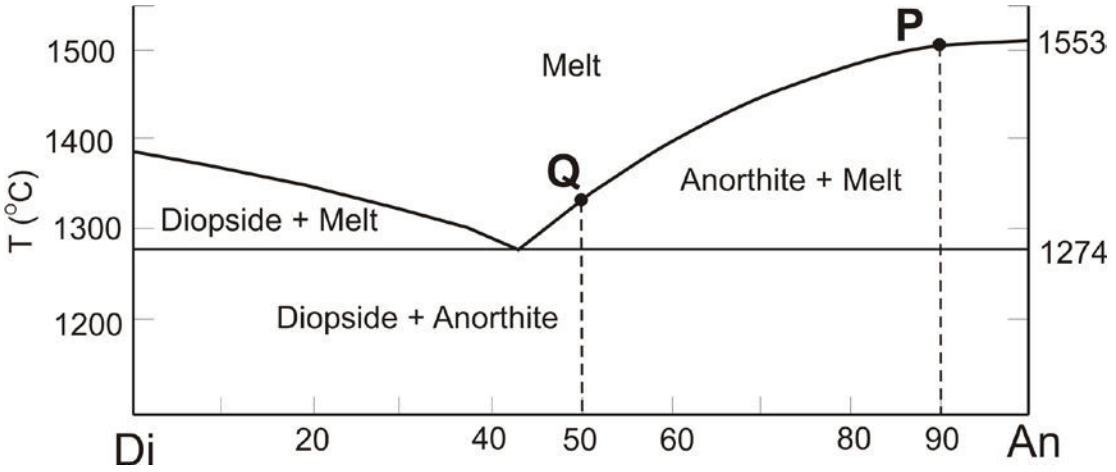
Q.56 A block of rock with a mass of 72 kg slides on a surface inclined at an angle of 30º as shown in the figure. Assuming no cohesion and friction, the force 'F' is _____ Newton (answer in one decimal place; acceleration due to gravity = 9.8 m/s^2).



Q.57 The true thickness of Bed A in the map given below is _____ m (answer in one decimal place).



Q.58 A melt containing 900 moles of anorthite and 100 moles of diopside undergoes crystallization. The number of moles of anorthite that crystallizes as the melt composition moves from P to Q is ____.



Q.59 A confined sandstone aquifer with a uniform cross-sectional area of 7 m^2 and a hydraulic conductivity of 2 m/s, transmits water across a hydraulic gradient of 3.2. Assuming steady state Darcian flow, the volumetric flow rate through the aquifer is _____ m^3/s (answer in one decimal place).

Q.60 A diamondiferous lamproite is ultrapotassic and has a molar K_2O/Na_2O ratio of 11. If the Na_2O content of the rock is 0.62 wt%, the K_2O content is _____ wt% (answer in one decimal place; molecular weight of Na_2O = 61.98, and K_2O = 94.20).

END OF THE QUESTION PAPER

Paper Code : GG			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	B
2	MCQ	A	C
3	MCQ	A	A
4	MCQ	A	A
5	MCQ	A	D
6	MCQ	A	D
7	MCQ	A	C
8	MCQ	A	C
9	MCQ	A	D
10	MCQ	A	D
11	MCQ	A	A
12	MCQ	A	B
13	MCQ	A	B
14	MCQ	A	C
15	MCQ	A	A
16	MCQ	A	A
17	MCQ	A	C
18	MCQ	A	D
19	MCQ	A	A
20	MCQ	A	B
21	MCQ	A	D
22	MCQ	A	D
23	MCQ	A	A

Paper Code : GG			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	B
25	MCQ	A	C
26	MCQ	A	D
27	MCQ	A	C
28	MCQ	A	B
29	MCQ	A	C
30	MCQ	A	B
31	MSQ	B	C,D
32	MSQ	B	B,C,D
33	MSQ	B	A,B
34	MSQ	B	A , C
35	MSQ	B	A,B,C,D
36	MSQ	B	A,D
37	MSQ	B	A,B,D
38	MSQ	B	B,D
39	MSQ	B	A,B,C
40	MSQ	B	C
41	NAT	C	4.0 to 4.0
42	NAT	C	3 to 3
43	NAT	C	25.0 to 25.0
44	NAT	C	7 to 7
45	NAT	C	50.0 to 50.0
46	NAT	C	8 to 8

Paper Code : GG			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
47	NAT	C	5.0 to 5.0
48	NAT	C	20 to 20
49	NAT	C	70.0 to 90.0
50	NAT	C	6.6 to 6.8
51	NAT	C	14.0 to 18.0
52	NAT	C	2.2 to 2.2
53	NAT	C	50.0 to 50.0
54	NAT	C	25.0 to 25.0
55	NAT	C	57.0 to 57.5
56	NAT	C	352.8 to 352.8
57	NAT	C	89.0 to 90.0
58	NAT	C	800 to 800
59	NAT	C	44.8 to 44.8
60	NAT	C	10.3 to 10.5

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Special Instructions/Useful Data	
	All angles are in radian
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1, x_2, \ldots, x_n): x_i \in \mathbb{R}, 1 \leq i \leq n\}$
M^T	Transpose of the matrix M
f'	Derivative of the function f
$P(E)$	Probability of the event E
$E(X)$	Expectation of the random variable X
$Var(X)$	Variance of the random variable X
i.i.d.	Independently and identically distributed
$U(a, b)$	Continuous uniform distribution on $(a, b), -\infty < a < b < \infty$
$\Phi(a)$	$\frac{1}{\sqrt{2\,\pi}} \int_{-\infty}^{a} e^{-x^2/2}\, dx$
$\Gamma(p)$	The gamma function $\Gamma(p) = \int_0^{\infty} e^{-t}\ t^{p-1}\, dt, \quad p > 0$
$n!$	The factorial function $n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 2$ and, for $n \geq 1$,

$$a_{n+1} = \frac{2\,a_n + 1}{a_n + 1}.$$

Then

(A) $1.5 \leq a_n \leq 2$, for all natural number $n \geq 1$

(B) there exists a natural number $n \geq 1$ such that $a_n > 2$

(C) there exists a natural number $n \geq 1$ such that $a_n < 1.5$

(D) there exists a natural number $n \geq 1$ such that $a_n = \frac{1+\sqrt{5}}{2}$

Q.2 The value of

$$\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^{n^2} e^{-2n}$$

is

(A) e^{-2} (B) e^{-1} (C) e (D) e^{2}

Q.3 Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$be two convergent sequences of real numbers. For $n \geq 1$, define $u_n = \max\{a_n, b_n\}$ and $v_n = \min\{a_n, b_n\}$. Then

(A) neither $\{u_n\}_{n\geq 1}$ nor $\{v_n\}_{n\geq 1}$ converges

(B) $\{u_n\}_{n\geq 1}$ converges but $\{v_n\}_{n\geq 1}$ does not converge

(C) $\{u_n\}_{n\geq 1}$ does not converge but $\{v_n\}_{n\geq 1}$ converges

(D) both $\{u_n\}_{n\geq 1}$ and $\{v_n\}_{n\geq 1}$ converge

Q.4 Let $M = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$. If I is the 2×2 identity matrix and $\mathbf{0}$ is the 2×2 zero matrix, then

(A) $20\,M^2 - 13\,M + 7\,I = \mathbf{0}$

(B) $20\,M^2 - 13\,M - 7\,I = \mathbf{0}$

(C) $20\,M^2 + 13\,M + 7\,I = \mathbf{0}$

(D) $20\,M^2 + 13\,M - 7\,I = \mathbf{0}$

Q.5 Let X be a random variable with the probability density function

$$f(x) = \begin{cases} \frac{\alpha^p}{\Gamma(p)} e^{-\alpha x} x^{p-1}, & x \geq 0, \alpha > 0, p > 0, \\ 0, & \text{otherwise.} \end{cases}$$

If $E(X) = 20$ and $Var(X) = 10$, then (α, p) is

(A) $(2, 20)$ (B) $(2, 40)$ (C) $(4, 20)$ (D) $(4, 40)$

Q.6 Let X be a random variable with the distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{4} + \frac{4x - x^2}{8}, & 0 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$$

Then

$$P(X = 0) + P(X = 1.5) + P(X = 2) + P(X \geq 1)$$

equals

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{7}{8}$ (D) 1

Q.7 Let X_1, X_2 and X_3 be i.i.d. $U(0, 1)$ random variables. Then $E\left(\frac{X_1 + X_2}{X_1 + X_2 + X_3}\right)$ equals

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.8 Let $x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3$ and $x_5 = 0$ be the observed values of a random sample of size 5 from a discrete distribution with the probability mass function

$$f(x; \theta) = P(X = x) = \begin{cases} \frac{\theta}{3}, & x = 0, \\ \frac{2\theta}{3}, & x = 1, \\ \frac{1 - \theta}{2}, & x = 2,3, \end{cases}$$

where $\theta \in [0, 1]$ is the unknown parameter. Then the maximum likelihood estimate of θ is

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{5}{7}$ (D) $\frac{5}{9}$

Q.9 Consider four coins labelled as $1, 2, 3$ and 4. Suppose that the probability of obtaining a 'head' in a single toss of the i^{th} coin is $\frac{i}{4}, i = 1, 2, 3, 4$. A coin is chosen uniformly at random and flipped. Given that the flip resulted in a 'head', the conditional probability that the coin was labelled either 1 or 2 equals

(A) $\frac{1}{10}$ (B) $\frac{2}{10}$ (C) $\frac{3}{10}$ (D) $\frac{4}{10}$

Q.10 Consider the linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i; i = 1, 2, \ldots, n$, where ϵ_i's are i.i.d. standard normal random variables. Given that

$$\frac{1}{n}\sum_{i=1}^{n} x_i = 3.2\,, \qquad \frac{1}{n}\sum_{i=1}^{n} y_i = 4.2, \quad \frac{1}{n}\sum_{j=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = 1.5 \text{ and}$$

$$\frac{1}{n}\sum_{j=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)\left(y_j - \frac{1}{n}\sum_{i=1}^{n} y_i\right) = 1.7,$$

the maximum likelihood estimates of β_0 and β_1, respectively, are

(A) $\frac{17}{15}$ and $\frac{32}{75}$ (B) $\frac{32}{75}$ and $\frac{17}{15}$

(C) $\frac{17}{15}$ and $\frac{43}{75}$ (D) $\frac{43}{75}$ and $\frac{17}{15}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $f: [-1,\ 1] \to \mathbb{R}$ be defined by $f(x) = \frac{x^2 + [\sin \pi x]}{1+|x|}$, where $[y]$ denotes the greatest integer less than or equal to y. Then

(A) f is continuous at $-\frac{1}{2}, 0, 1$

(B) f is discontinuous at $-1, 0, \frac{1}{2}$

(C) f is discontinuous at $-1, -\frac{1}{2}, 0, \frac{1}{2}$

(D) f is continuous everywhere except at 0

Q.12 Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - \frac{\cos x}{2}$ and $g(x) = \frac{x \sin x}{2}$. Then

(A) $f(x) = g(x)$ for more than two values of x

(B) $f(x) \neq g(x)$, for all x in $\mathbb{R}$

(C) $f(x) = g(x)$ for exactly one value of x

(D) $f(x) = g(x)$ for exactly two values of x

Q.13 Consider the domain $D = \{ (x, y) \in \mathbb{R}^2 : x \leq y \}$ and the function $h: D \to \mathbb{R}$ defined by

$$h\big((x, y)\big) = (x-2)^4 + (y-1)^4, (x, y) \in D.$$

Then the minimum value of h on D equals

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

Q.14 Let $M = [X \quad Y \quad Z]$ be an orthogonal matrix with $X, Y, Z \in \mathbb{R}^3$ as its column vectors. Then

$$Q = X X^T + Y Y^T$$

(A) is a skew-symmetric matrix

(B) is the 3×3 identity matrix

(C) satisfies $Q^2 = Q$

(D) satisfies $QZ = Z$

Q.15 Let $f:[0,3] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 0, & 0 \le x < 1, \\ e^{x^2} - e, & 1 \le x < 2 \\ e^{x^2} + 1, & 2 \le x \le 3. \end{cases}$$

Now, define $F:[0,3] \to \mathbb{R}$ by

$$F(0) = 0 \text{ and } F(x) = \int_0^x f(t)dt \text{ , for } 0 < x \le 3.$$

Then

(A) F is differentiable at $x = 1$ and $F'(1) = 0$

(B) F is differentiable at $x = 2$ and $F'(2) = 0$

(C) F is not differentiable at $x = 1$

(D) F is differentiable at $x = 2$ and $F'(2) = 1$

Q.16 If x, y and z are real numbers such that $4x + 2y + z = 31$ and $2x + 4y - z = 19$, then the value of $9x + 7y + z$

(A) cannot be computed from the given information

(B) equals $\frac{281}{3}$

(C) equals $\frac{182}{3}$

(D) equals $\frac{218}{3}$

Q.17 Let $M = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$. If

$$V = \left\{(x, y, 0) \in \mathbb{R}^3 : M\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\} \text{ and } W = \left\{(x, y, z) \in \mathbb{R}^3 : M\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\},$$

then

(A) the dimension of V equals 2

(B) the dimension of W equals 2

(C) the dimension of V equals 1

(D) $V \cap W = \{(0,0,0)\}$

Q.18 Let M be a 3×3 non-zero, skew-symmetric real matrix. If I is the 3×3 identity matrix, then

(A) M is invertible

(B) the matrix $I + M$ is invertible

(C) there exists a non-zero real number α such that $\alpha I + M$ is not invertible

(D) all the eigenvalues of M are real

Q.19 Let X be a random variable with the moment generating function

$$M_X(t) = \frac{6}{\pi^2}\sum_{n\geq 1}\frac{e^{t^2/2n}}{n^2},\ t \in \mathbb{R}.$$

Then $P(X \in \mathbb{Q})$, where $\mathbb{Q}$ is the set of rational numbers, equals

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Q.20 Let X be a discrete random variable with the moment generating function

$$M_X(t) = \frac{(1+3\,e^t)^2(3+e^t)^3}{1024}, t \in \mathbb{R}.$$

Then

(A) $E(X) = \frac{9}{4}$ (B) $Var(X) = \frac{15}{32}$

(C) $P(X \geq 1) = \frac{27}{1024}$ (D) $P(X = 5) = \frac{3}{1024}$

Q.21 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables with X_n having the probability density function as

$$f_n(x) = \begin{cases} \frac{1}{2^{n/2}\ \Gamma(\frac{n}{2})}\ e^{-\frac{x}{2}}\ x^{(\frac{n}{2}-1)}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\lim_{n\to\infty}\left[P\left(X_n > \frac{3}{4}n\right) + P(\,X_n > n + 2\sqrt{2n})\right]$$

equals

(A) $1 + \Phi(2)$ (B) $1 - \Phi(2)$ (C) $\Phi(2)$ (D) $2 - \Phi(2)$

Q.22 Let X be a Poisson random variable with mean $\frac{1}{2}$. Then $E((X+1)!)$ equals

(A) $2\,e^{-\frac{1}{2}}$ (B) $4\,e^{-\frac{1}{2}}$ (C) $4\,e^{-1}$ (D) $2\,e^{-1}$

Q.23 Let X be a standard normal random variable. Then $P(X^3 - 2X^2 - X + 2 > 0)$ equals

(A) $2\Phi(1) - 1$ (B) $1 - \Phi(2)$

(C) $2\Phi(1) - \Phi(2)$ (D) $\Phi(2) - \Phi(1)$

Q.24 Let X and Y have the joint probability density function

$$f(x,y) = \begin{cases} 2, & 0 \le x \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a = E(Y|X = \frac{1}{2})$ and $b = Var(Y|X = \frac{1}{2})$. Then (a, b) is

(A) $\left(\frac{3}{4}, \frac{7}{12}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{48}\right)$

(C) $\left(\frac{1}{4}, \frac{7}{12}\right)$ (D) $\left(\frac{3}{4}, \frac{1}{48}\right)$

Q.25 Let X and Y have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \frac{m+n}{21}, & m = 1,2,3; n = 1,2, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(X = 2|Y = 2)$ equals

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.26 Let X and Y be two independent standard normal random variables. Then the probability density function of $Z = \frac{|X|}{|Y|}$ is

(A) $f(z) = \begin{cases} \frac{\sqrt{1/2}}{\sqrt{\pi}}\, e^{-\frac{z}{2}}\, z^{-\frac{1}{2}}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$

(B) $f(z) = \begin{cases} \frac{2}{\sqrt{2\pi}}\, e^{-z^2/2}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$

(C) $f(z) = \begin{cases} e^{-z}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$

(D) $f(z) = \begin{cases} \frac{2}{\pi}\, \frac{1}{(1+z^2)}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$

Q.27 Let X and Y have the joint probability density function

$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Then the correlation coefficient between X and Y equals

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{2}{\sqrt{3}}$

Q.28 Let $x_1 = -2, x_2 = 1$ and $x_3 = -1$ be the observed values of a random sample of size three from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \frac{1}{2\theta + 1}, & x \in \{-\theta, -\theta + 1, \dots, 0, \dots, \theta\}, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \Theta = \{1, 2, \dots\}$ is the unknown parameter. Then the method of moment estimate of θ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.29 Let X be a random sample from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \frac{1}{\theta}, & x = 1, 2, \dots, \theta, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \Theta = \{20, 40\}$ is the unknown parameter. Consider testing

$$H_0: \theta = 40 \text{ against } H_1: \theta = 20$$

at a level of significance $\alpha = 0.1$. Then the uniformly most powerful test rejects H_0 if and only if

(A) $X \leq 4$ (B) $X > 4$

(C) $X \geq 3$ (D) $X < 3$

Q.30 Let X_1 and X_2 be a random sample of size 2 from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \theta, & x = 0, \\ 1 - \theta, & x = 1, \end{cases}$$

where $\theta \in \Theta = \{0.2, 0.4\}$ is the unknown parameter. For testing $H_0: \theta = 0.2$ against $H_1: \theta = 0.4$, consider a test with the critical region

$$C = \{(x_1, x_2) \in \{0,1\} \times \{0,1\} : x_1 + x_2 < 2\}.$$

Let α and β denote the probability of Type I error and power of the test, respectively.

Then (α, β) is

(A) $(0.36, 0.74)$ (B) $(0.64, 0.36)$

(C) $(0.05, 0.64)$ (D) $(0.36, 0.64)$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$$a_n = \sum_{k=n+1}^{2n} \frac{1}{k}, \quad n \geq 1.$$

Then which of the following statement(s) is (are) true?

(A) $\{a_n\}_{n\geq 1}$ is an increasing sequence

(B) $\{a_n\}_{n\geq 1}$ is bounded below

(C) $\{a_n\}_{n\geq 1}$ is bounded above

(D) $\{a_n\}_{n\geq 1}$ is a convergent sequence

Q.32 Let $\sum_{n\geq 1} a_n$ be a convergent series of positive real numbers. Then which of the following statement(s) is (are) true?

(A) $\sum_{n\geq 1}(a_n)^2$ is always convergent

(B) $\sum_{n\geq 1}\sqrt{a_n}$ is always convergent

(C) $\sum_{n\geq 1}\frac{\sqrt{a_n}}{n}$ is always convergent

(D) $\sum_{n\geq 1}\frac{\sqrt{a_n}}{n^{1/4}}$ is always convergent

Q.33 Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 3$ and, for $n \geq 1$,

$$a_{n+1} = \frac{a_n^2 - 2\,a_n + 4}{2}.$$

Then which of the following statement(s) is (are) true?

(A) $\{a_n\}_{n\geq 1}$ is a monotone sequence

(B) $\{a_n\}_{n\geq 1}$ is a bounded sequence

(C) $\{a_n\}_{n\geq 1}$ does not have finite limit, as $n \to \infty$

(D) $\lim_{n\to\infty} a_n = 2$

Q.34 Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^4(2 + \sin\frac{1}{x}), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then which of the following statement(s) is (are) true?

(A) f attains its minimum at 0

(B) f is monotone

(C) f is differentiable at 0

(D) $f(x) > 2\,x^4 + x^3$, for all $x > 0$

Q.35 Let P be a probability function that assigns the same weight to each of the points of the sample space $\Omega = \{1,2,3,4\}$. Consider the events $E = \{1,2\}, F = \{1,3\}$ and $G = \{3,4\}$. Then which of the following statement(s) is (are) true?

(A) E and F are independent

(B) E and G are independent

(C) F and G are independent

(D) E, F and G are independent

Q.36 Let $X_1, X_2, \ldots, X_n, n \geq 5$, be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statement(s) is (are) true?

(A) A 95% confidence interval of θ has to be of finite length

(B) $(\min\{X_1, X_2, \ldots, X_n\} + \frac{1}{n}\ln(0.05),\ \min\{X_1, X_2, \ldots, X_n\})$ is a 95% confidence interval of θ

(C) A 95% confidence interval of θ can be of length 1

(D) A 95% confidence interval of θ can be of length 2

Q.37 Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0,\theta)$, where $\theta > 0$ is the unknown parameter. Let $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$. Then which of the following is (are) consistent estimator(s) of θ^3?

(A) $8\,X_n^3$

(B) $X_{(n)}^3$

(C) $\left(\frac{2}{n}\sum_{i=5}^{n} X_i\right)^3$

(D) $\frac{n\,X_{(n)}^3+1}{n+1}$

Q.38 Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} c(\theta)\, e^{-(x-\theta)}, & x \geq 2\theta, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statement(s) is (are) true?

(A) The maximum likelihood estimator of θ is $\frac{\min\{X_1, X_2, \ldots, X_n\}}{2}$

(B) $c(\theta) = 1$, for all $\theta \in \mathbb{R}$

(C) The maximum likelihood estimator of θ is $\min\{X_1, X_2, \ldots, X_n\}$

(D) The maximum likelihood estimator of θ does not exist

Q.39 Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \theta^2 x\, e^{-\theta x}, & x > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta > 0$ is the unknown parameter. If $Y = \sum_{i=1}^{n} X_i$, then which of the following statement(s) is (are) true?

(A) Y is a complete sufficient statistic for θ

(B) $\frac{2n}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(C) $\frac{2n-1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(D) $\frac{2n+1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

Q.40 Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta, \theta+1)$, where $\theta \in \mathbb{R}$ is the unknown parameter. Let $U = \max\{X_1, X_2, \ldots, X_n\}$ and $V = \min\{X_1, X_2, \ldots, X_n\}$. Then which of the following statement(s) is (are) true?

(A) U is a consistent estimator of θ

(B) V is a consistent estimator of θ

(C) $2U - V - 2$ is a consistent estimator of θ

(D) $2V - U + 1$ is a consistent estimator of θ

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$$a_n = \frac{1+3+5+\cdots+(2n-1)}{n!}, \quad n \geq 1.$$

Then $\sum_{n\geq 1} a_n$ converges to _____

Q.42 Let

$$S = \left\{(x,y) \in \mathbb{R}^2 : x, y \geq 0, \qquad \sqrt{4-(x-2)^2} \leq y \leq \sqrt{9-(x-3)^2}\right\}.$$

Then the area of S equals _____

Q.43 Let $S = \{(x,y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$. Then the area of S equals _____

Q.44 Let

$$J = \frac{1}{\pi}\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{3}{2}}\,dt\,.$$

Then the value of J equals _____

Q.45 A fair die is rolled three times independently. Given that 6 appeared at least once, the conditional probability that 6 appeared exactly twice equals _____

Q.46 Let X and Y be two positive integer valued random variables with the joint probability mass function

$$P(X=m, Y=n) = \begin{cases} g(m)\,h(n), & m,n \geq 1, \\ 0, & \text{otherwise}, \end{cases}$$

where $g(m) = \left(\frac{1}{2}\right)^{m-1}, m \geq 1$ and $h(n) = \left(\frac{1}{3}\right)^{n}, n \geq 1$. Then $E(X\,Y)$ equals _____

Q.47 Let E, F and G be three events such that

$$P(E \cap F \cap G) = 0.1, P(G|F) = 0.3 \text{ and } P(E|F \cap G) = P(E|F).$$

Then $P(G|E \cap F)$ equals _____

Q.48 Let A_1, A_2 and A_3 be three events such that

$$P(A_i) = \frac{1}{3}, i = 1, 2, 3;\ P(A_i \cap A_j) = \frac{1}{6}, 1 \leq i \neq j \leq 3 \text{ and } P(A_1 \cap A_2 \cap A_3) = \frac{1}{6}.$$

Then the probability that none of the events A_1, A_2, A_3 occur equals _____

Q.49 Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with the probability density function

$$f(x) = \frac{1}{4}\, e^{-|x-4|} + \frac{1}{4} e^{-|x-6|}, x \in \mathbb{R}.$$

Then $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to _____

Q.50 Let $x_1 = 1.1, x_2 = 2.2$ and $x_3 = 3.3$ be the observed values of a random sample of size three from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & x > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in \Theta = \{1, 2, \ldots\}$ is the unknown parameter. Then the maximum likelihood estimate of θ equals _____

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that f' is continuous on $\mathbb{R}$ with $f'(3) = 18$. Define

$$g_n(x) = n\left(f\left(x + \frac{5}{n}\right) - f\left(x - \frac{2}{n}\right)\right).$$

Then $\lim_{n\to\infty} g_n(3)$ equals _____

Q.52 Let $M = \sum_{i=1}^{4} X_i X_i^T$, where

$X_1^T = [1 \quad -1 \quad 1 \quad 0]$, $X_2^T = [1 \quad 1 \quad 0 \quad 1]$, $X_3^T = [1 \quad 3 \quad 1 \quad 0]$ and $X_4^T = [1 \quad 1 \quad 1 \quad 0]$.

Then the rank of M equals _____

Q.53 Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function with $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} f'(x) = 2$. Then

$$\lim_{x\to\infty}\left(1 + \frac{f(x)}{x^2}\right)^x$$

equals _____

Q.54 The value of

$$\int_0^{\frac{\pi}{2}}\left(\int_0^x e^{\sin y}\sin x \; dy\right) dx$$

equals _____

Q.55 Let X be a random variable with the probability density function

$$f(x) = \begin{cases} 4x^k, & 0 < x < 1, \\ x - \dfrac{x^2}{2}, & 1 \le x < 2, \\ 0, & \text{otherwise}, \end{cases}$$

where k is a positive integer. Then $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$ equals _____

Q.56 Let X and Y be two discrete random variables with the joint moment generating function

$$M_{X,Y}(t_1, t_2) = \left(\frac{1}{3}\, e^{t_1} + \frac{2}{3}\right)^2 \left(\frac{2}{3}\, e^{t_2} + \frac{1}{3}\right)^3, t_1, t_2 \in \mathbb{R}.$$

Then $P(2X + 3Y > 1)$ equals _____

Q.57 Let X_1, X_2, X_3 and X_4 be i.i.d. discrete random variables with the probability mass function

$$P(X_1 = n) = \begin{cases} \frac{3^{n-1}}{4^n}, & n = 1,2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(X_1 + X_2 + X_3 + X_4 = 6)$ equals _____

Q.58 Let X be a random variable with the probability mass function

$$P(X = n) = \begin{cases} \frac{1}{10}, & n = 1,2, \cdots, 10, \\ 0, & \text{otherwise.} \end{cases}$$

Then $E(\max\{X, 5\})$ equals _____

Q.59 Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \Theta = \{2,3\}$ is the unknown parameter. For testing

$$H_0: \theta = 2 \text{ against } H_1: \theta = 3,$$

let α and β be the size and power, respectively, of the test that rejects H_0 if and only if $X \geq 3.5$. Then $\alpha + \beta$ equals _____

Q.60 A fair die is rolled four times independently. For $i = 1, 2, 3, 4$, define

$$Y_i = \begin{cases} 1, & \text{if 6 appears in the } i^{th} \text{ throw,} \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(\max\{Y_1, Y_2, Y_3, Y_4\} = 1)$ equals _____

END OF THE QUESTION PAPER

Paper Code : MS			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	A
2	MCQ	A	A
3	MCQ	A	D
4	MCQ	A	B
5	MCQ	A	B
6	MCQ	A	C
7	MCQ	A	C
8	MCQ	A	B
9	MCQ	A	C
10	MCQ	A	D
11	MCQ	A	B
12	MCQ	A	D
13	MCQ	A	C
14	MCQ	A	C
15	MCQ	A	A
16	MCQ	A	D
17	MCQ	A	C
18	MCQ	A	B
19	MCQ	A	A
20	MCQ	A	A
21	MCQ	A	D
22	MCQ	A	B
23	MCQ	A	C

Paper Code : MS			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	D
25	MCQ	A	A
26	MCQ	A	D
27	MCQ	A	C
28	MCQ	A	B
29	MCQ	A	A
30	MCQ	A	D
31	MSQ	B	A,B,C,D
32	MSQ	B	A,C
33	MSQ	B	A,C
34	MSQ	B	A,C
35	MSQ	B	A,C
36	MSQ	B	B,C,D
37	MSQ	B	B,C,D
38	MSQ	B	A
39	MSQ	B	A,C
40	MSQ	B	B,C,D
41	NAT	C	5.40 to 5.50
42	NAT	C	7.80 to 7.90
43	NAT	C	1.90 to 2.10
44	NAT	C	0.35 to 0.40
45	NAT	C	0.16 to 0.17
46	NAT	C	2.50 to 3.50

Paper Code : MS			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
47	NAT	C	0.25 to 0.35
48	NAT	C	0.30 to 0.40
49	NAT	C	4.75 to 5.25
50	NAT	C	1.90 to 2.10
51	NAT	C	125 to 127
52	NAT	C	3.0 to 3.0
53	NAT	C	7.25 to 7.50
54	NAT	C	1.70 to 1.80
55	NAT	C	0.85 to 0.90
56	NAT	C	0.97 to 0.99
57	NAT	C	0.01 to 0.03
58	NAT	1	6.25 to 6.75
59	NAT	C	1.10 to 1.20
60	NAT	C	0.50 to 0.53

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Useful information

$\mathbb{N}$	set of all natural numbers $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	set of all integers $\{0, \pm 1, \pm 2, \dots\}$
$\mathbb{Q}$	set of all rational numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{C}$	set of all complex numbers
$\mathbb{R}^n$	n-dimensional Euclidean space $\{(x_1, x_2, \dots, x_n) \mid x_j \in \mathbb{R}, 1 \leq j \leq n\}$
S_n	group of all permutations of n distinct symbols
$\mathbb{Z}_n$	group of congruence classes of integers modulo n
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$M_{m\times n}(\mathbb{R})$	real vector space of all matrices of order $m \times n$ with entries in $\mathbb{R}$
sup	supremum
inf	infimum

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following is TRUE?

(A) $\mathbb{Z}_n$ is cyclic if and only if n is prime

(B) Every proper subgroup of $\mathbb{Z}_n$ is cyclic

(C) Every proper subgroup of S_4 is cyclic

(D) If every proper subgroup of a group is cyclic, then the group is cyclic

Q.2 Let $a_n = \frac{b_{n+1}}{b_n}$, where $b_1 = 1$, $b_2 = 1$ and $b_{n+2} = b_n + b_{n+1}$, $n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is

(A) $\frac{1-\sqrt{5}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$ (C) $\frac{1+\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{5}}{2}$

Q.3 If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in a vector space over $\mathbb{R}$, then which one of the following sets is also linearly independent?

(A) $\{v_1 + v_2 - v_3,\ 2v_1 + v_2 + 3v_3,\ 5v_1 + 4v_2\}$

(B) $\{v_1 - v_2,\ v_2 - v_3,\ v_3 - v_1\}$

(C) $\{v_1 + v_2 - v_3,\ \ v_2 + v_3 - v_1,\ \ v_3 + v_1 - v_2,\ \ v_1 + v_2 + v_3\}$

(D) $\{v_1 + v_2,\ \ v_2 + 2v_3,\ \ v_3 + 3v_1\}$

Q.4 Let a be a positive real number. If f is a continuous and even function defined on the interval $[-a, a]$, then $\int_{-a}^{a} \frac{f(x)}{1+e^x}\, dx$ is equal to

(A) $\int_0^a f(x)\, dx$ (B) $2\int_0^a \frac{f(x)}{1+e^x}\, dx$

(C) $2\int_0^a f(x)\, dx$ (D) $2a\int_0^a \frac{f(x)}{1+e^x}\, dx$

Q.5 The tangent plane to the surface $z = \sqrt{x^2 + 3y^2}$ at $(1, 1, 2)$ is given by

(A) $x - 3y + z = 0$ (B) $x + 3y - 2z = 0$

(C) $2x + 4y - 3z = 0$ (D) $3x - 7y + 2z = 0$

Q.6 In $\mathbb{R}^3$, the cosine of the acute angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z - x^2 - y^2 + 3 = 0$ at the point $(2, 1, 2)$ is

(A) $\frac{8}{5\sqrt{21}}$ (B) $\frac{10}{5\sqrt{21}}$ (C) $\frac{8}{3\sqrt{21}}$ (D) $\frac{10}{3\sqrt{21}}$

Q.7 Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a scalar field, $\vec{v}: \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field and let $\vec{a} \in \mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then which one of the following is FALSE?

(A) $curl(f\,\vec{v}) = grad(f) \times \vec{v} + f\, curl(\vec{v})$

(B) $div(grad(f)) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f$

(C) $curl(\vec{a} \times \vec{r}) = 2\,|\vec{a}|\,\vec{r}$

(D) $div\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0$, for $\vec{r} \neq \vec{0}$

Q.8 In $\mathbb{R}^2$, the family of trajectories orthogonal to the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$ is given by

(A) $x^{4/3} + y^{4/3} = c^{4/3}$ (B) $x^{4/3} - y^{4/3} = c^{4/3}$

(C) $x^{5/3} - y^{5/3} = c^{5/3}$ (D) $x^{2/3} - y^{2/3} = c^{2/3}$

Q.9 Consider the vector space V over $\mathbb{R}$ of polynomial functions of degree less than or equal to 3 defined on $\mathbb{R}$. Let $T : V \to V$ be defined by $(Tf)(x) = f(x) - xf'(x)$. Then the rank of T is

(A) 1 (B) 2 (C) 3 (D) 4

Q.10 Let $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ for $n \in \mathbb{N}$. Then which one of the following is TRUE for the sequence $\{s_n\}_{n=1}^{\infty}$

(A) $\{s_n\}_{n=1}^{\infty}$ converges in $\mathbb{Q}$

(B) $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence but does not converge in $\mathbb{Q}$

(C) the subsequence $\{s_{k^n}\}_{n=1}^{\infty}$ is convergent in $\mathbb{R}$, only when k is even natural number

(D) $\{s_n\}_{n=1}^{\infty}$ is not a Cauchy sequence

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n} & , \quad if\ n\ is\ odd \\ 1 + \frac{1}{2^n} & , \quad if\ n\ is\ even \end{cases}$, $n \in \mathbb{N}$.

Then which one of the following is TRUE?

(A) $\sup\{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

(B) $\liminf(a_n) = \limsup(a_n) = \frac{3}{2}$

(C) $\sup\{a_n \mid n \in \mathbb{N}\} = 2$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

(D) $\liminf(a_n) = 1$ and $\limsup(a_n) = 3$

Q.12 Let $a, b, c \in \mathbb{R}$. Which of the following values of a, b, c do NOT result in the convergence of the series

$$\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c} \quad ?$$

(A) $|a| < 1,\ b \in \mathbb{R}, c \in \mathbb{R}$ (B) $a = 1,\ b > 1,\ c \in \mathbb{R}$

(C) $a = 1, b \geq 0, c < 1$ (D) $a = -1,\ b \geq 0, c > 0$

Q.13 Let $a_n = n + \frac{1}{n}$, $n \in \mathbb{N}$. Then the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{a_{n+1}}{n!}$ is

(A) $e^{-1} - 1$ (B) e^{-1} (C) $1 - e^{-1}$ (D) $1 + e^{-1}$

Q.14 Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and let $c_n = \sum_{k=0}^{n} a_{n-k} a_k$, where $n \in \mathbb{N} \cup \{0\}$. Then which one of the following is TRUE?

(A) Both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent

(B) $\sum_{n=0}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} c_n$ is not convergent

(C) $\sum_{n=1}^{\infty} c_n$ is convergent but $\sum_{n=0}^{\infty} a_n$ is not convergent

(D) Neither $\sum_{n=0}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} c_n$ is convergent

Q.15 Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable functions such that f is strictly increasing and g is strictly decreasing. Define $p(x) = f(g(x))$ and $q(x) = g(f(x))$, $\forall x \in \mathbb{R}$. Then, for $t > 0$, the sign of $\int_0^t p'(x)\,(\,q'(x) - 3)dx$ is

(A) positive (B) negative (C) dependent on t (D) dependent on f and g

Q.16 For $x \in \mathbb{R}$, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then which one of the following is FALSE?

(A) $\lim_{x\to 0} \frac{f(x)}{x} = 0$

(B) $\lim_{x\to 0} \frac{f(x)}{x^2} = 0$

(C) $\frac{f(x)}{x^2}$ has infinitely many maxima and minima on the interval $(0,1)$

(D) $\frac{f(x)}{x^4}$ is continuous at $x = 0$ but not differentiable at $x = 0$

Q.17 Let $f(x,y) = \begin{cases} \frac{xy}{(x^2+y^2)^\alpha}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Then which one of the following is TRUE for f at the point $(0,0)$?

(A) For $\alpha = 1$, f is continuous but not differentiable

(B) For $\alpha = \frac{1}{2}$, f is continuous and differentiable

(C) For $\alpha = \frac{1}{4}$, f is continuous and differentiable

(D) For $\alpha = \frac{3}{4}$, f is neither continuous nor differentiable

Q.18 Let $a, b \in \mathbb{R}$ and let $f: \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. If $z = e^u f(v)$, where $u = ax + by$ and $v = ax - by$, then which one of the following is TRUE?

(A) $b^2 z_{xx} - a^2 z_{yy} = 4a^2 b^2 e^u f'(v)$ (B) $b^2 z_{xx} - a^2 z_{yy} = -4e^u f'(v)$

(C) $bz_x + az_y = abz$ (D) $bz_x + az_y = -abz$

Q.19 Consider the region D in the yz plane bounded by the line $y = \frac{1}{2}$ and the curve $y^2 + z^2 = 1$, where $y \geq 0$. If the region D is revolved about the z-axis in $\mathbb{R}^3$, then the volume of the resulting solid is

(A) $\frac{\pi}{\sqrt{3}}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\pi\sqrt{3}}{2}$ (D) $\pi\sqrt{3}$

Q.20 If $\vec{F}(x,y) = (3x - 8y)\hat{\imath} + (4y - 6xy)\hat{\jmath}$ for $(x,y) \in \mathbb{R}^2$, then $\oint_C \vec{F} \cdot d\vec{r}$, where C is the boundary of the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ oriented in the anti-clockwise direction, is

(A) $\frac{5}{2}$ (B) 3 (C) 4 (D) 5

Q.21 Let U, V and W be finite dimensional real vector spaces, $T: U \to V$, $S: V \to W$ and $P: W \to U$ be linear transformations. If range (ST) = nullspace (P), nullspace (ST) = range (P) and rank (T) = rank (S), then which one of the following is TRUE?

(A) nullity of T = nullity of S
(B) dimension of $U \neq$ dimension of W
(C) If dimension of $V = 3$, dimension of $U = 4$, then P is not identically zero
(D) If dimension of $V = 4$, dimension of $U = 3$ and T is one-one, then P is identically zero

Q.22 Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx} + y = f(x)$, for $x \geq 0$, $y(0) = 0$, where $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$. Then $y(x) =$

(A) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2(e - 1)e^{-x}$ when $x \geq 1$
(B) $2(1 - e^{-x})$ when $0 \leq x < 1$ and 0 when $x \geq 1$
(C) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2(1 - e^{-1})e^{-x}$ when $x \geq 1$
(D) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2\,e^{1-x}$ when $x \geq 1$

Q.23 An integrating factor of the differential equation $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(x + x\,y^2)\,dy = 0$ is

(A) x^2 (B) $3\log_e x$ (C) x^3 (D) $2\log_e x$

Q.24 A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is

(A) $e^{e^x}e^{-x}$ (B) $e^{e^x}e^{-2x}$ (C) $e^{e^x}e^{2x}$ (D) $e^{e^x}e^{x}$

Q.25 Let G be a group satisfying the property that $f: G \to \mathbb{Z}_{221}$ is a homomorphism implies $f(g) = 0$, $\forall g \in G$. Then a possible group G is

(A) $\mathbb{Z}_{21}$ (B) $\mathbb{Z}_{51}$ (C) $\mathbb{Z}_{91}$ (D) $\mathbb{Z}_{119}$

Q.26 Let H be the quotient group $\mathbb{Q}/\mathbb{Z}$. Consider the following statements.

I. Every cyclic subgroup of H is finite.

II. Every finite cyclic group is isomorphic to a subgroup of H.

Which one of the following holds?

(A) I is TRUE but II is FALSE
(B) II is TRUE but I is FALSE
(C) both I and II are TRUE
(D) neither I nor II is TRUE

Q.27 Let I denote the 4×4 identity matrix. If the roots of the characteristic polynomial of a 4×4 matrix M are $\pm\sqrt{\frac{1\pm\sqrt{5}}{2}}$, then $M^8 =$

(A) $I + M^2$ (B) $2I + M^2$ (C) $2I + 3M^2$ (D) $3I + 2M^2$

Q.28 Consider the group $\mathbb{Z}^2 = \{(a,b) | \, a,b \in \mathbb{Z}\}$ under component-wise addition. Then which of the following is a subgroup of $\mathbb{Z}^2$?

(A) $\{(a,b) \in \mathbb{Z}^2 | \, ab = 0\}$
(B) $\{(a,b) \in \mathbb{Z}^2 | \, 3a + 2b = 15\}$
(C) $\{(a,b) \in \mathbb{Z}^2 | \, 7 \text{ divides } ab\}$
(D) $\{(a,b) \in \mathbb{Z}^2 | \, 2 \text{ divides } a \text{ and } 3 \text{ divides } b\}$

Q.29 Let $f: \mathbb{R} \to \mathbb{R}$ be a function and let J be a bounded open interval in $\mathbb{R}$. Define

$$W(f,J) = \sup \{f(x) \mid x \in J\} - \inf \{f(x) \mid x \in J\} .$$

Which one of the following is FALSE?

(A) $W(f,J_1) \leq W(f,J_2)$ if $J_1 \subset J_2$
(B) If f is a bounded function in J and $J \supset J_1 \supset J_2 \cdots \supset J_n \supset \cdots$ such that the length of the interval J_n tends to 0 as $n \to \infty$, then $\lim_{n\to\infty} W(f,J_n) = 0$
(C) If f is discontinuous at a point $a \in J$, then $W(f,J) \neq 0$
(D) If f is continuous at a point $a \in J$, then for any given $\epsilon > 0$ there exists an interval $I \subset J$ such that $W(f,I) < \epsilon$

Q.30 For $x > \frac{-1}{2}$, let $f_1(x) = \frac{2x}{1+2x}$, $f_2(x) = \log_e(1+2x)$ and $f_3(x) = 2x$. Then which one of the following is TRUE?

(A) $f_3(x) < f_2(x) < f_1(x)$ for $0 < x < \frac{\sqrt{3}}{2}$

(B) $f_1(x) < f_3(x) < f_2(x)$ for $x > 0$

(C) $f_1(x) + f_2(x) < \frac{f_3(x)}{2}$ for $x > \frac{\sqrt{3}}{2}$

(D) $f_2(x) < f_1(x) < f_3(x)$ for $x > 0$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $f: \mathbb{R}\setminus\{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x^3}$. On which of the following interval(s) is f one-one?

(A) $(-\infty, -1)$ (B) $(0, 1)$ (C) $(0, 2)$ (D) $(0, \infty)$

Q.32 The solution(s) of the differential equation $\frac{dy}{dx} = (\sin 2x)\, y^{1/3}$ satisfying $y(0) = 0$ is (are)

(A) $y(x) = 0$

(B) $y(x) = -\sqrt{\frac{8}{27}}\ \sin^3 x$

(C) $y(x) = \sqrt{\frac{8}{27}}\ \sin^3 x$

(D) $y(x) = \sqrt{\frac{8}{27}}\ \cos^3 x$

Q.33 Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$, where

f interchanges α and β but fixes γ and δ,

g interchanges β and γ but fixes α and δ,

h interchanges γ and δ but fixes α and β.

Which of the following permutations interchange(s) α and δ but fix(es) β and γ?

(A) $f \circ g \circ h \circ g \circ f$ (B) $g \circ h \circ f \circ h \circ g$ (C) $g \circ f \circ h \circ f \circ g$ (D) $h \circ g \circ f \circ g \circ h$

Q.34 Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) FALSE?

(A) If P and Q are compact, then $P \cup Q$ is also compact

(B) If P and Q are not connected, then $P \cup Q$ is also not connected

(C) If $P \cup Q$ and P are closed, then Q is closed

(D) If $P \cup Q$ and P are open, then Q is open

Q.35 Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ denote the group of non-zero complex numbers under multiplication. Suppose $Y_n = \{ z \in \mathbb{C} \mid z^n = 1\},\ n \in \mathbb{N}$. Which of the following is (are) subgroup(s) of $\mathbb{C}^*$?

(A) $\bigcup_{n=1}^{100} Y_n$ (B) $\bigcup_{n=1}^{\infty} Y_{2^n}$ (C) $\bigcup_{n=100}^{\infty} Y_n$ (D) $\bigcup_{n=1}^{\infty} Y_n$

Q.36 Suppose $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the following system of linear equations.
$x + y + z = \alpha,\ x + \beta y + z = \gamma,\ x + y + \alpha z = \beta$. If this system has at least one solution, then which of the following statements is (are) TRUE?

(A) If $\alpha = 1$ then $\gamma = 1$
(B) If $\beta = 1$ then $\gamma = \alpha$
(C) If $\beta \neq 1$ then $\alpha = 1$
(D) If $\gamma = 1$ then $\alpha = 1$

Q.37 Let $m, n \in \mathbb{N},\ m < n,\ P \in M_{n\times m}(\mathbb{R}),\ Q \in M_{m\times n}(\mathbb{R})$. Then which of the following is (are) NOT possible?

(A) $rank\ (PQ) = n$
(B) $rank\ (QP) = m$
(C) $rank\ (PQ) = m$
(D) $rank\ (QP) = \left\lceil \frac{m+n}{2} \right\rceil$, the smallest integer larger than or equal to $\frac{m+n}{2}$

Q.38 If $\vec{F}(x, y, z) = (2x + 3yz)\hat{\imath} + (3xz + 2y)\hat{\jmath} + (3xy + 2z)\hat{k}$ for $(x, y, z) \in \mathbb{R}^3$, then which among the following is (are) TRUE?

(A) $\nabla \times \vec{F} = \vec{0}$
(B) $\oint_C \vec{F} \cdot d\vec{r} = 0$ along any simple closed curve C
(C) There exists a scalar function $\phi: \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \cdot \vec{F} = \phi_{xx} + \phi_{yy} + \phi_{zz}$
(D) $\nabla \cdot \vec{F} = 0$

Q.39 Which of the following subsets of $\mathbb{R}$ is (are) connected?

(A) $\{ x \in \mathbb{R} \mid x^2 + x > 4\}$
(B) $\{ x \in \mathbb{R} \mid x^2 + x < 4\}$
(C) $\{ x \in \mathbb{R} \mid |x| < |x - 4|\}$
(D) $\{ x \in \mathbb{R} \mid |x| > |x - 4|\}$

Q.40 Let S be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) TRUE?

(A) S contains an interval

(B) There is a sequence in S which does not converge to 2018

(C) There is an element $y \in S$, $y \neq 2018$ such that y is also an interior point of S

(D) There is a point $z \in S$, such that $|z - 2018| = 0.002018$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The order of the element $(1\ 2\ 3)\ (2\ 4\ 5)\ (4\ 5\ 6)$ in the group S_6 is _____

Q.42 Let $\phi(x, y, z) = 3y^2 + 3yz$ for $(x, y, z) \in \mathbb{R}^3$. Then the absolute value of the directional derivative of ϕ in the direction of the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-2}$, at the point $(1, -2, 1)$ is _____

Q.43 Let $f(x) = \sum_{n=0}^{\infty}(-1)^n x(x-1)^n$ for $0 < x < 2$. Then the value of $f\left(\frac{\pi}{4}\right)$ is _____

Q.44 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2 y\,(x-y)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Then $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ at the point $(0,0)$ is _____

Q.45 Let $f(x,y) = \sqrt{x^3 y}\ \sin\left(\frac{\pi}{2}\, e^{\left(\frac{y}{x}-1\right)}\right) + xy\ \cos\left(\frac{\pi}{3}\, e^{\left(\frac{x}{y}-1\right)}\right)$ for $(x,y) \in \mathbb{R}^2$, $x > 0$, $y > 0$.

Then $f_x(1,1) + f_y(1,1) =$ _____

Q.46 Let $f: [0, \infty) \to [0, \infty)$ be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$. If $f(x) = \int_0^x \sqrt{f(t)}\, dt$, then $f(6) =$ _____

Q.47 Let $a_n = \frac{(1+(-1)^n)}{2^n} + \frac{(1+(-1)^{n-1})}{3^n}$. Then the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ about $x = 0$ is _____

Q.48 Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is _____

Q.49 Let W_1 be the real vector space of all 5×2 matrices such that the sum of the entries in each row is zero. Let W_2 be the real vector space of all 5×2 matrices such that the sum of the entries in each column is zero. Then the dimension of the space $W_1 \cap W_2$ is _____

Q.50 The coefficient of x^4 in the power series expansion of $e^{\sin x}$ about $x = 0$ is _____ (correct up to three decimal places).

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $a_k = (-1)^{k-1}$, $s_n = a_1 + a_2 + \cdots + a_n$ and $\sigma_n = (s_1 + s_2 + \cdots + s_n)/n$, where $k, n \in \mathbb{N}$. Then $\lim_{n\to\infty} \sigma_n$ is _____ (correct up to one decimal place).

Q.52 Let $f: \mathbb{R} \to \mathbb{R}$ be such that f'' is continuous on $\mathbb{R}$ and $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -1$. Then $\lim_{x\to\infty} \left(f\left(\sqrt{\frac{2}{x}}\right)\right)^x$ is _____ (correct up to three decimal places).

Q.53 Suppose x, y, z are positive real numbers such that $x + 2y + 3z = 1$. If M is the maximum value of xyz^2, then the value of $\frac{1}{M}$ is _____

Q.54 If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces

$$x = -1, \quad x = 1, \quad y = -1, \quad y = 1, \quad z = 2, \quad y^2 + z^2 = 2$$

is $\alpha - \pi$, then $\alpha =$ _____

Q.55 If $\alpha = \int_{\pi/6}^{\pi/3} \frac{\sin t + \cos t}{\sqrt{\sin 2t}}\, dt$, then the value of $\left(2 \sin \frac{\alpha}{2} + 1\right)^2$ is _____

Q.56 The value of the integral

$$\int_0^1 \int_x^1 y^4 e^{xy^2}\, dy\, dx$$

is _____ (correct up to three decimal places).

Q.57 Suppose $Q \in M_{3\times3}(\mathbb{R})$ is a matrix of rank 2. Let $T: M_{3\times3}(\mathbb{R}) \to M_{3\times3}(\mathbb{R})$ be the linear transformation defined by $T(P) = QP$. Then the rank of T is _____

Q.58 The area of the parametrized surface

$$S = \{((2 + \cos u)\cos v,\ (2 + \cos u)\sin v,\ \sin u) \in \mathbb{R}^3 \mid 0 \le u \le \tfrac{\pi}{2},\ 0 \le v \le \tfrac{\pi}{2}\}$$

is _____ (correct up to two decimal places).

Q.59 If $x(t)$ is the solution to the differential equation $\frac{dx}{dt} = x^2 t^3 + xt$, for $t > 0$, satisfying $x(0) = 1$, then the value of $x(\sqrt{2})$ is _____ (correct up to two decimal places).

Q.60 If $y(x) = v(x)\sec x$ is the solution of $y'' - (2\tan x)\, y' + 5y = 0$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, satisfying $y(0) = 0$ and $y'(0) = \sqrt{6}$, then $v\left(\frac{\pi}{6\sqrt{6}}\right)$ is _____ (correct up to two decimal places).

END OF THE QUESTION PAPER

Paper Code : MA			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	B
2	MCQ	A	D
3	MCQ	A	D
4	MCQ	A	A
5	MCQ	A	B
6	MCQ	A	C
7	MCQ	A	C
8	MCQ	A	B
9	MCQ	A	C
10	MCQ	A	B
11	MCQ	A	A
12	MCQ	A	C
13	MCQ	A	D
14	MCQ	A	B
15	MCQ	A	A
16	MCQ	A	D
17	MCQ	A	C
18	MCQ	A	A
19	MCQ	A	C
20	MCQ	A	B
21	MCQ	A	C
22	MCQ	A	A
23	MCQ	A	C

Paper Code : MA			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	B
25	MCQ	A	A
26	MCQ	A	C
27	MCQ	A	C
28	MCQ	A	D
29	MCQ	A	B
30	MCQ	A	C
31	MSQ	B	B
32	MSQ	B	A,B,C
33	MSQ	B	A,D
34	MSQ	B	B,C,D
35	MSQ	B	B,C,D
36	MSQ	B	A,B
37	MSQ	B	A,D
38	MSQ	B	A,B,C
39	MSQ	B	B,C,D
40	MSQ	B	A,B,C
41	NAT	C	4 to 4
42	NAT	C	6.5 to 7.5
43	NAT	C	1 to 1
44	NAT	C	1 to 1
45	NAT	C	3 to 3
46	NAT	C	9 to 9

Paper Code : MA			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
47	NAT	C	2 to 2
48	NAT	C	0 to 0
49	NAT	C	4 to 4
50	NAT	C	-0.130 to -0.120
51	NAT	C	0.4 to 0.6
52	NAT	C	0.350 to 0.380
53	NAT	C	1140 to 1160
54	NAT	C	5.99 to 6.01
55	NAT	C	2.9 to 3.1
56	NAT	C	0.230 to 0.250
57	NAT	C	6 to 6
58	NAT	C	6.30 to 6.70
59	NAT	C	-2.80 to -2.70
60	NAT	C	0.5 to 0.5

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $f(x,y) = x^3 - 2y^3$. The curve along which $\nabla^2 f = 0$ is

(A) $x = \sqrt{2}$ y
(B) $x = 2y$
(C) $x = \sqrt{6}y$
(D) $x = -y/2$

Q.2 A curve is given by $\vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$. The unit vector of the tangent to the curve at t =1 is

(A) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$
(B) $\frac{\hat{\imath}+\hat{\jmath}+2\hat{k}}{\sqrt{6}}$
(C) $\frac{\hat{\imath}+2\hat{\jmath}+2\hat{k}}{3}$
(D) $\frac{\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$

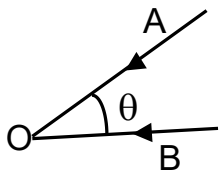
Q.3 There are three planets in circular orbits around a star at distances a, $4a$ and $9a$, respectively. At time $t = t_0$, the star and the three planets are in a straight line. The period of revolution of the closest planet is T. How long after t_0 will they again be in the same straight line?

(A) $8T$ (B) $27T$ (C) $216T$ (D) $512T$

Q.4 A current I is flowing through the sides of an equilateral triangle of side a. The magnitude of the magnetic field at the centroid of the triangle is

(A) $\frac{9\mu_0 I}{2\pi a}$
(B) $\frac{\mu_0 I}{\pi a}$
(C) $\frac{3\mu_0 I}{2\pi a}$
(D) $\frac{3\mu_0 I}{\pi a}$

Q.5 Two vehicles A and B are approaching an observer O at rest with equal speed as shown in the figure. Both vehicles have identical sirens blowing at a frequency f_s . The observer hears these sirens at frequency f_A and f_B, respectively from the two vehicles. Which one of the following is correct?



(A) $f_A = f_B < f_s$
(B) $f_A = f_B > f_s$
(C) $f_A > f_B > f_s$
(D) $f_A < f_B < f_s$

Q.6 Three infinite plane sheets carrying uniform charge densities $-\sigma, 2\sigma, 3\sigma$ are placed parallel to the x-z plane at $y = a, 3a, 4a$, respectively. The electric field at the point $(0, 2a, 0)$ is

(A) $\frac{4\sigma}{\varepsilon_0}\hat{\jmath}$
(B) $-\frac{3\sigma}{\varepsilon_0}\hat{\jmath}$
(C) $-\frac{2\sigma}{\varepsilon_0}\hat{\jmath}$
(D) $\frac{\sigma}{\varepsilon_0}\hat{\jmath}$

Q.7 Two boxes A and B contain an equal number of molecules of the same gas. If the volumes are V_A and V_B, and λ_A and λ_B denote respective mean free paths, then

(A) $\lambda_A = \lambda_B$

(B) $\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$

(C) $\frac{\lambda_A}{V_A^{1/3}} = \frac{\lambda_B}{V_B^{1/3}}$

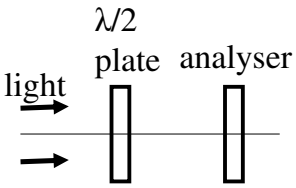
(D) $\lambda_A V_A = \lambda_B V_B$

Q.8 Let T_g and T_e be the kinetic energies of the electron in the ground and the third excited states of a hydrogen atom, respectively. According to the Bohr model, the ratio T_g/T_e is

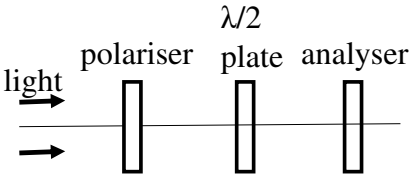
(A) 3 (B) 4 (C) 9 (D) 16

Q.9 Which one of the following arrangements of optical components can be used to distinguish between an unpolarised light and a circularly polarised light?

(A)



(B)



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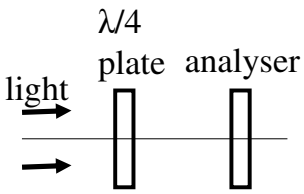
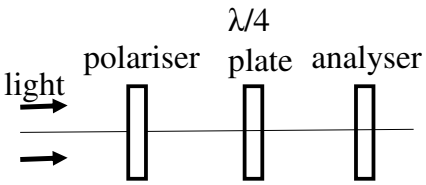
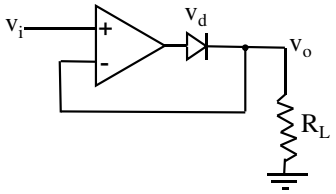


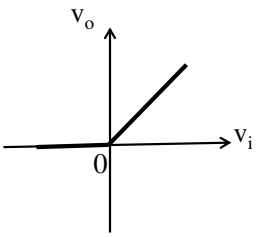
(D)



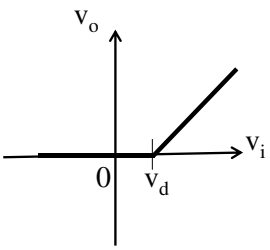
Q.10 Which one of the following graphs shows the correct variation of v_o with v_i? Here, v_d is the voltage drop across the diode and the Op-Amp is assumed to be ideal.



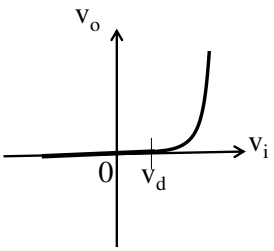
(A)



(B)



(C)



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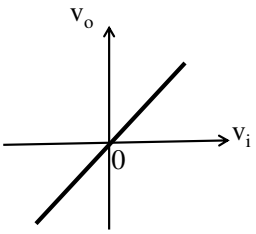
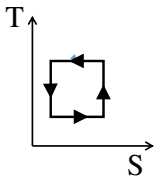


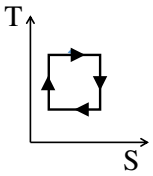
Q. 11 – Q. 30 carry two marks each.

Q.11 Which one of the figures correctly represents the T-S diagram of a Carnot engine?

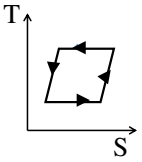
(A)



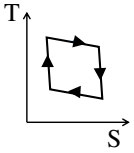
(B)



(C)



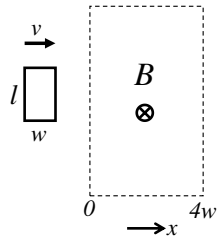
(D)



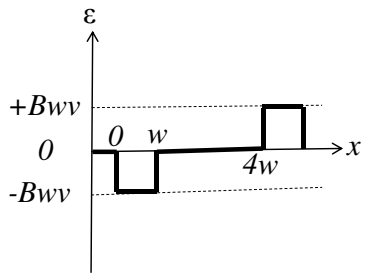
Q.12 The plane of polarisation of a plane polarized light rotates by 60° after passing through a wave plate. The pass-axis of the wave plate is at an angle α with respect to the plane of polarisation of the incident light. The wave plate and α are

(A) $\lambda/4$, 60° (B) $\lambda/2$, 30° (C) $\lambda/2$, 120° (D) $\lambda/4$, 30°

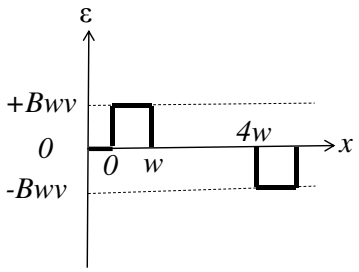
Q.13 A rectangular loop of dimensions l and w moves with a constant speed of v through a region containing a uniform magnetic field B directed into the paper and extending a distance of $4w$. Which of the following figures correctly represents the variation of emf (ε) with the position (x) of the front end of the loop?



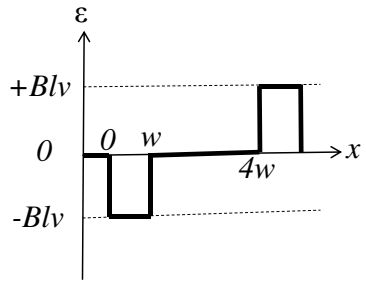
(A)



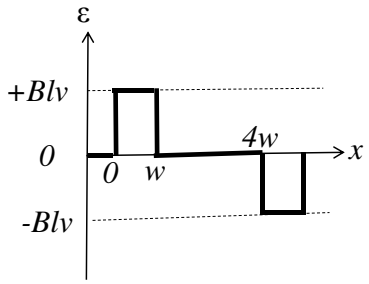
(B)



(C)



(D)



Q.14 The equation of state for one mole of a non-ideal gas is given by $PV = A\left(1 + \frac{B}{V}\right)$, where the coefficients A and B are temperature dependent. If the volume changes from V_1 to V_2 in an isothermal process, the work done by the gas is

(A) $AB\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$

(B) $AB \ln\left(\frac{V_2}{V_1}\right)$

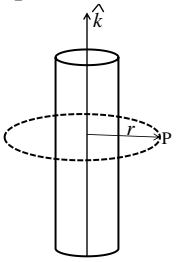
(C) $A \ln\left(\frac{V_2}{V_1}\right) + AB\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$

(D) $A \ln\left(\frac{V_2 - V_1}{V_1}\right) + B$

Q.15 An ideal gas consists of three dimensional polyatomic molecules. The temperature is such that only one vibrational mode is excited. If R denotes the gas constant, then the specific heat at constant volume of one mole of the gas at this temperature is

(A) $3R$ (B) $\frac{7}{2}R$ (C) $4R$ (D) $\frac{9}{2}R$

Q.16 A long solenoid is carrying a time dependent current such that the magnetic field inside has the form $\vec{B}(t) = B_0 t^2 \hat{k}$, where $\hat{k}$ is along the axis of the solenoid. The displacement current at the point P on a circle of radius *r* in a plane perpendicular to the axis



(A) is inversely proportional to *r* and radially outward.
(B) is inversely proportional to *r* and tangential.
(C) increases linearly with time and is tangential.
(D) is inversely proportional to r^2 and tangential.

Q.17 Consider an ensemble of thermodynamic systems, each of which is characterized by the same number of particles, pressure and temperature. The thermodynamic function describing the ensemble is

(A) Enthalpy
(B) Helmholtz free energy
(C) Gibbs free energy
(D) Entropy

Q.18 Given a spherically symmetric charge density $\rho(r) = \begin{cases} kr^2 , & r < R \\ 0 , & r > R \end{cases}$ (*k* being a constant), the electric field for $r < R$ is (take the total charge as *Q*)

(A) $\frac{Qr^3}{4\pi\varepsilon_0 R^5}\hat{r}$
(B) $\frac{3Qr^2}{4\pi\varepsilon_0 R^4}\hat{r}$
(C) $\frac{5Qr^3}{8\pi\varepsilon_0 R^5}\hat{r}$
(D) $\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

Q.19 An infinitely long solenoid, with its axis along $\hat{k}$, carries a current *I*. In addition there is a uniform line charge density λ along the axis. If $\vec{S}$ is the energy flux, in cylindrical coordinates$(\hat{\rho}, \hat{\phi}, \hat{k})$, then

(A) $\vec{S}$ is along $\hat{\rho}$
(B) $\vec{S}$ is along $\hat{k}$
(C) $\vec{S}$ has non zero components along $\hat{\rho}$ and $\hat{k}$
(D) $\vec{S}$ is along $\hat{\rho} \times \hat{k}$

Q.20 Consider two waves $y_1 = a\,cos(\omega t - kz)$ and $y_2 = a\,cos[(\omega + \Delta\omega)t - (k + \Delta k)z]$. The group velocity of the superposed wave will be ($\Delta\omega \ll \omega$ and $\Delta k \ll k$)

(A) $\frac{(\omega-\Delta\omega)}{(k-\Delta k)}$
(B) $\frac{(2\omega+\Delta\omega)}{(2k+\Delta k)}$
(C) $\frac{\Delta\omega}{\Delta k}$
(D) $\frac{(\omega+\Delta\omega)}{(k+\Delta k)}$

Q.21 Consider a convex lens of focal length *f*. A point object moves towards the lens along its axis between 2*f* and *f*. If the speed of the object is V_O, then its image would move with speed V_I. Which of the following is correct?

(A) $V_I = V_O$; the image moves away from the lens.
(B) $V_I = -V_O$; the image moves towards the lens.
(C) $V_I > V_O$; the image moves away from the lens.
(D) $V_I < V_O$; the image moves away from the lens.

Q.22 A disc of radius R_1 having uniform surface density has a concentric hole of radius $R_2 < R_1$. If its mass is *M*, the principal moments of inertia are

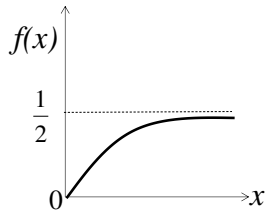
(A) $\frac{M(R_1^2-R_2^2)}{2}, \frac{M(R_1^2-R_2^2)}{4}, \frac{M(R_1^2-R_2^2)}{4}$
(B) $\frac{M(R_1^2+R_2^2)}{2}, \frac{M(R_1^2+R_2^2)}{4}, \frac{M(R_1^2+R_2^2)}{4}$
(C) $\frac{M(R_1^2+R_2^2)}{2}, \frac{M(R_1^2+R_2^2)}{4}, \frac{M(R_1^2+R_2^2)}{8}$
(D) $\frac{M(R_1^2-R_2^2)}{2}, \frac{M(R_1^2-R_2^2)}{4}, \frac{M(R_1^2-R_2^2)}{8}$

Q.23 The function $f(x) = \begin{cases} x, -\pi < x < 0 \\ -x, 0 < x < \pi \end{cases}$
is expanded as a Fourier series of the form $a_0 + \sum_{n=1}^{\infty} a_n cos(nx) + \sum_{n=1}^{\infty} b_n sin(nx)$. Which of the following is true?

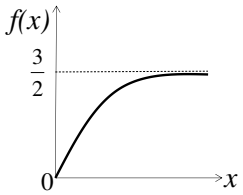
(A) $a_0 \neq 0, b_n = 0$
(B) $a_0 \neq 0, b_n \neq 0$
(C) $a_0 = 0, b_n = 0$
(D) $a_0 = 0, b_n \neq 0$

Q.24 Which one of the following curves correctly represents (schematically) the solution for the equation $\frac{df}{dx} + 2f = 3\,; f(0) = 0$?

(A)



(B)



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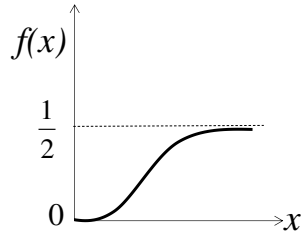

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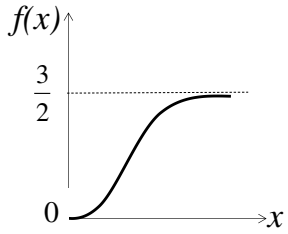


Q.25 The mean momentum $\bar{p}$ of a nucleon in a nucleus of mass number *A* and atomic number *Z* depends on *A*, *Z* as

(A) $\bar{p} \propto A^{\frac{1}{3}}$ (B) $\bar{p} \propto Z^{\frac{1}{3}}$ (C) $\bar{p} \propto A^{-\frac{1}{3}}$ (D) $\bar{p} \propto (AZ)^{-\frac{2}{3}}$

Q.26 The Boolean expression $(\overline{AB})(\bar{A}+B)(A+\bar{B})$ can be simplified to

(A) $A+B$ (B) $\bar{A}B$ (C) $\overline{A+B}$ (D) AB

Q.27 Consider the transformation to a new set of coordinates (ξ,η) from rectangular Cartesian coordinates (x, y), where $\xi = 2x + 3y$ and $\eta = 3x - 2y$. In the (ξ,η) coordinate system, the area element $dxdy$ is

(A) $\frac{1}{13}d\xi d\eta$ (B) $\frac{2}{13}d\xi d\eta$ (C) $5d\xi d\eta$ (D) $\frac{3}{5}d\xi d\eta$

Q.28 A particle of mass m is in a one dimensional potential $V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$.

At some instant its wave function is given by $\psi(x) = \frac{1}{\sqrt{3}}\psi_1(x) + i\sqrt{\frac{2}{3}}\psi_2(x)$, where $\psi_1(x)$and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

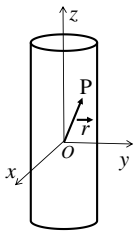
(A) $\langle x\rangle = \frac{L}{2}$; $\langle E\rangle = \frac{\hbar^2}{2m}\frac{3\pi^2}{L^2}$ (B) $\langle x\rangle = \frac{2L}{3}$; $\langle E\rangle = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}$

(C) $\langle x\rangle = \frac{L}{2}$; $\langle E\rangle = \frac{\hbar^2}{2m}\frac{8\pi^2}{L^2}$ (D) $\langle x\rangle = \frac{2L}{3}$; $\langle E\rangle = \frac{\hbar^2}{2m}\frac{4\pi^2}{3L^2}$

Q.29 A raindrop falls under gravity and captures water molecules from atmosphere. Its mass changes at the rate $\lambda m(t)$, where λ is a positive constant and $m(t)$ is the instantaneous mass. Assume that acceleration due to gravity is constant and water molecules are at rest with respect to earth before capture. Which of the following statements is correct?

(A) The speed of the raindrop increases linearly with time.
(B) The speed of the raindrop increases exponentially with time.
(C) The speed of the raindrop approaches a constant value when $\lambda t >> 1$.
(D) The speed of the raindrop approaches a constant value when $\lambda t << 1$.

Q.30 A particle P of mass m is constrained to move on the surface of a cylinder under a force $-k\vec{r}$ as shown in figure (k is the positive constant). Which of the following statements is correct? (Neglect friction.)



(A) Total energy of the particle is *not* conserved.
(B) The motion along z direction is simple harmonic.
(C) Angular momentum of the particle about O increases with time.
(D) Linear momentum of the particle is conserved.

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

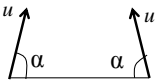
Q.31 Let matrix $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$. If det ($M$) = 0, then

(A) M is symmetric.
(B) M is invertible.
(C) One eigenvalue is 13.
(D) Its eigenvectors are orthogonal.

Q.32 Let $f(x) = 3x^6 - 2x^2 - 8$. Which of the following statements is (are) true?

(A) The sum of all its roots is zero.
(B) The product of its roots is $-\frac{8}{3}$.
(C) The sum of all its roots is $\frac{2}{3}$.
(D) Complex roots are conjugates of each other.

Q.33 Two projectiles of identical mass are projected from the ground with same initial angle (α) with respect to earth surface and same initial velocity (u) in the same plane. They collide at the highest point of their trajectories and stick to each other. Which of the following statements is (are) correct?



(A) The momentum of the combined object immediately after the collision is zero.
(B) Kinetic energy is conserved in the collision.
(C) The combined object moves vertically downward.
(D) The combined object moves in a parabolic path.

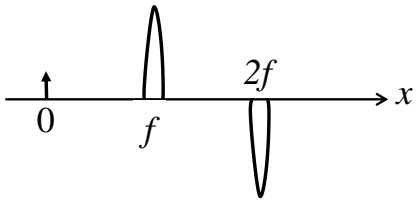
Q.34 Two beams of light in the visible range (400 nm - 700 nm) interfere with each other at a point. The optical path difference between them is 5000 nm. Which of the following wavelengths will interfere constructively at the given point?

(A) 416.67 nm (B) 555.55 nm
(C) 625 nm (D) 666.66 nm

Q.35 Which of the following relations is (are) true for thermodynamic variables?

(A) $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$
(B) $TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$
(C) $dF = -SdT + PdV$
(D) $dG = -SdT + VdP$

Q.36 Consider a convex lens of focal length *f*. The lens is cut along a diameter in two parts. The two lens parts and an object are kept as shown in the figure. The images are formed at following distances from the object:



(A) 2*f* (B) 3*f* (C) 4*f* (D) ∞

Q.37 Let the electric field in some region *R* be given by $\vec{E} = e^{-y^2}\hat{\imath} + e^{-x^2}\hat{\jmath}$. From this we may conclude that

(A) *R* has a non-uniform charge distribution.
(B) *R* has no charge distribution.
(C) *R* has a time dependent magnetic field.
(D) The energy flux in *R* is zero everywhere.

Q.38 In presence of a magnetic field $B\hat{\jmath}$ and an electric field $(-E)\hat{k}$, a particle moves undeflected. Which of the following statements is (are) correct?

(A) The particle has positive charge, velocity = $-\frac{E}{B}\hat{\imath}$
(B) The particle has positive charge, velocity = $\frac{E}{B}\hat{\imath}$
(C) The particle has negative charge, velocity = $-\frac{E}{B}\hat{\imath}$
(D) The particle has negative charge, velocity = $\frac{E}{B}\hat{\imath}$

Q.39 In a *pn* junction, dopant concentration on the *p*- side is higher than that on the *n*-side. Which of the following statements is (are) correct, when the junction is unbiased?

(A) The width of the depletion layer is larger on the *n*-side.
(B) At thermal equilibrium the Fermi energy is higher on the *p*- side.
(C) In the depletion region, number of negative charges per unit area on the *p*- side is equal to number of positive charges per unit area on the *n*- side.
(D) The value of the built-in potential barrier depends on the dopant concentration.

Q.40 Which of the combinations of crystal structure and their coordination number is (are) correct?

(A) body centered cubic – 8 (B) face centered cubic – 6
(C) diamond – 4 (D) hexagonal closed packed – 12

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The coefficient of x^3 in the Taylor expansion of sin(sin x) around $x = 0$ is _____.
(Specify your answer upto two digits after the decimal point.)

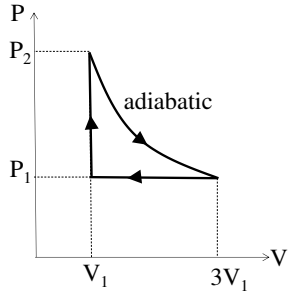
Q.42 A particle of mass m is moving along the positive x direction under a potential $V(x) = \frac{1}{2}kx^2 + \frac{\lambda}{2x^2}$ (k and λ are positive constants). If the particle is slightly displaced from its equilibrium position, it oscillates with an angular frequency (ω) _____.
(Specify your answer in units of $\sqrt{\frac{k}{m}}$ as an integer.)

Q.43 A planet has average density same as that of the earth but it has only $1/8$ of the mass of the earth. If the acceleration due to gravity at the surface is g_p and g_e for the planet and earth, respectively, then $\frac{g_p}{g_e}$ = _____.
(Specify your answer upto one digit after the decimal point.)

Q.44 In a grating with grating constant $d = a + b$, where a is the slit width and b is the separation between the slits, the diffraction pattern has the fourth order missing. The value of $\frac{b}{a}$ is _____.
(Specify your answer as an integer.)

Q.45 Consider an electromagnetic plane wave $\vec{E} = E_0(\hat{\imath} + b\hat{\jmath})\, cos\left[\frac{2\pi}{\lambda}\{ct - (x - \sqrt{3}y)\}\right]$, where λ is the wavelength, c is the speed of light and b is a constant. The value of b is _____.
(Specify your answer upto two digits after the decimal point.)

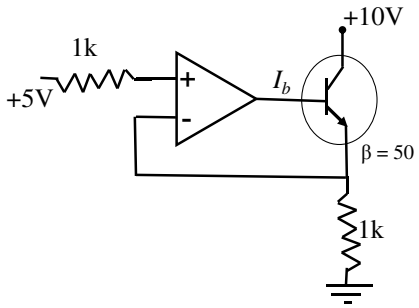
Q.46 Consider a monoatomic ideal gas operating in a closed cycle as shown in the P-V diagram given below. The ratio $\frac{P_1}{P_2}$ is _____.
(Specify your answer upto two digits after the decimal point.)



Q.47 Consider the first order phase transition of the sublimation of zinc. Assume the vapor to be an ideal gas and the molar volume of solid to be negligible. Experimentally, it is found that $log_{10}(P) = -\frac{C_1}{T} + C_2$ where P is the vapor pressure in Pascal, T is in K, $C_1 = 6790$ K and $C_2 = 9$. The latent heat of sublimation of zinc from the Clausius - Clapeyron equation is _____ kJ/mole. (R = 8.314 J/mole.K)
(Specify your answer in kJ/mole upto one digit after the decimal point.)

Q.48 A system of 8 non-interacting electrons is confined by a three dimensional potential $V(r) = \frac{1}{2}m\omega^2 r^2$. The ground state energy of the system in units of $\hbar\omega$ is _____.
(Specify your answer as an integer.)

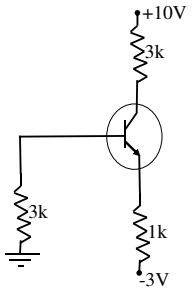
Q.49 For the given circuit, value of the base current (I_b) of the *npn* transistor will be _____ mA. (β is the current gain and assume Op-Amp as ideal.)
(Specify your answer in mA upto two digits after the decimal point.)



Q.50 The lattice constant of unit cell of NaCl crystal is 0.563 nm. X-rays of wavelength 0.141 nm are diffracted by this crystal. The angle at which the first order maximum occurs is _____degrees.
(Specify your answer in degrees upto two digits after the decimal point.)

Q. 51 – Q. 60 carry two marks each.

Q.51 For the following circuit, the collector voltage with respect to ground will be _____V. (Emitter diode voltage is 0.7 V and β_{DC} of the transistor is large.)
(Specify your answer in volts upto one digit after the decimal point.)



Q.52 A body of mass 1 kg is moving under a central force in an elliptic orbit with semi major axis 1000 m and semi minor axis 100 m. The orbital angular momentum of the body is 100 kg m^2 s^{-1}. The time period of motion of the body is _____ hours.
(Specify your answer in hours upto two digits after the decimal point.)

Q.53 The moon moves around the earth in a circular orbit with a period of 27 days. The radius of the earth (R) is 6.4×10^6 m and the acceleration due to gravity on the earth surface is 9.8 ms^{-2}. If D is the distance of the moon from the center of the earth, the value of D/R will be _____.
(Specify your answer upto one digit after the decimal point.)

Q.54 A syringe is used to exert 1.5 atmospheric pressure to release water horizontally. The speed of water immediately after ejection is _____. (take 1 atmospheric pressure = 10^5 Pascal, density of water = 10^3 kg m^{-3})
(Specify your answer in ms^{-1} as an integer.)

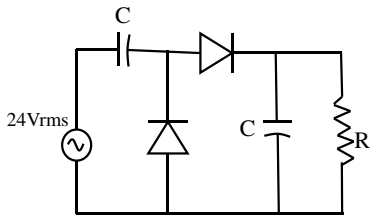
Q.55 Consider a slit of width 18 μm which is being illuminated simultaneously with light of orange color (wavelength 600 nm) and of blue color (wavelength 450 nm). The diffraction pattern is observed on a screen kept at a distance in front of the slit. The smallest angle at which only the orange color is observed is θ_1, and the smallest angle at which only the blue color is observed is θ_2. The angular difference $\theta_2 - \theta_1$ (in degrees) is _____.
(Specify your answers upto two digits after the decimal point.)

Q.56 A particle of mass *m* is moving in a circular orbit given by $x = R\cos(\omega t)$; $y = R\sin(\omega t)$, as observed in an inertial frame S_1. Another inertial frame S_2 moves with uniform velocity $\vec{v} = \omega R\hat{\imath}$ with respect to S_1. S_1 and S_2 are related by Galilean transformation, such that the origins coincide at $t = 0$. The magnitude of the angular momentum of the particle at $t = \frac{2\pi}{\omega}$, as observed in S_2 about its origin, is expressed as $(mR^2\omega)x$. Then *x* is _____.
(Specify your answer upto two digits after the decimal point.)

Q.57 Rod R_1 has a rest length 1m and rod R_2 has a rest length of 2m. R_1 and R_2 are moving with respect to the laboratory frame with velocities $+v\hat{\imath}$ and $-v\hat{\imath}$, respectively. If R_2 has a length of 1m in the rest frame of R_1, $\frac{v}{c}$ is given by _____.
(Specify your answer upto two digits after the decimal point.)

Q.58 Two events E_1 and E_2 take place in an inertial frame *S* with respective time-space coordinates (in SI units): $E_1(t_1 = 0, \overrightarrow{r_1} = 0)$ and $E_2(t_2 = 0, x_2 = 10^8, y_2 = 0, z_2 = 0)$. Another inertial frame S' is moving with respect to S with a velocity $\vec{v} = 0.8\,c\,\hat{\imath}$. The time difference $(t_2' - t_1')$as observed in S' is ____s. ($c = 3\times10^8$ ms^{-1})
(Specify your answer in seconds upto two digits after the decimal point.)

Q.59 In the following circuit, the time constant *RC* is much greater than the period of the input signal. Assume diode as ideal and resistance R to be large. The dc output voltage across resistance *R* will be _____V.
(Specify your answer in volts upto one digit after the decimal point.)



Q.60 For a metal, the electron density is 6.4×10^{28} m^{-3}. The Fermi energy is _____eV.
(h = 6.626×10^{-34} J s , $m_e = 9.11 \times 10^{-31}$ kg, 1eV = 1.6×10^{-19} J)
(Specify your answer in electron volts (eV) upto one digit after the decimal point.)

END OF THE QUESTION PAPER

Paper Code: PH			
Q.No	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	B
2	MCQ	A	D
3	MCQ	A	C
4	MCQ	A	A
5	MCQ	A	B
6	MCQ	A	B
7	MCQ	A	B
8	MCQ	A	D
9	MCQ	A	C
10	MCQ	A	A
11	MCQ	A	B
12	MCQ	A	B
13	MCQ	A	C
14	MCQ	A	C
15	MCQ	A	C
16	MCQ	A	B
17	MCQ	A	C
18	MCQ	A	A
19	MCQ	A	D
20	MCQ	A	C
21	MCQ	A	C
22	MCQ	A	B
23	MCQ	A	A

Paper Code: PH			
Q.No	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	B
25	MCQ	A	C
26	MCQ	A	C
27	MCQ	A	A
28	MCQ	A	A
29	MCQ	A	C
30	MCQ	A	B
31	MSQ	B	A,C,D
32	MSQ	B	A,B,D
33	MSQ	B	A,C
34	MSQ	B	A,B,C
35	MSQ	B	A,B,D
36	MSQ	B	B,C,D or C,D
37	MSQ	B	B,C
38	MSQ	B	B,D
39	MSQ	B	A,C,D
40	MSQ	B	A,C,D
41	NAT	C	-0.35 to -0.30
42	NAT	C	2 to 2
43	NAT	C	0.4 to 0.6
44	NAT	C	3 to 3
45	NAT	C	0.55 to 0.59
46	NAT	C	0.14 to 0.18

Paper Code: PH			
Q.No	**Question Type (QT)**	**Section**	**Key/Range (KY)**
47	NAT	C	129.0 to 131.0 or 55.0 to 58.0
48	NAT	C	18 to 18
49	NAT	C	0.09 to 0.11
50	NAT	C	12.00 to 15.00
51	NAT	C	2.9 to 3.5
52	NAT	C	1.70 to 1.80
53	NAT	C	59.0 to 60.0
54	NAT	C	10 to 10
55	NAT	C	0.45 to 0.55
56	NAT	C	5.25 to 5.30 or -5.30 to -5.25
57	NAT	C	0.50 to 0.60
58	NAT	C	-0.46 to -0.42
59	NAT	C	66.0 to 69.0
60	NAT	C	5.6 to 6.0