IIT JAM 2017 Question Paper with Answer Key (All Subjects)

IIT Joint Admission Test for Masters

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SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The antigen binding site of an antibody is present

(A) at the constant region
(B) at the C-terminal
(C) at the variable region
(D) between the constant and the variable region

Q.2 Which of the following is **NOT** involved in eukaryotic translation?

(A) Ribosome (B) Spliceosome (C) mRNA (D) tRNA

Q.3 Which of the following statements is correct?

(A) Gram negative bacteria are colored purple after Gram staining
(B) Gram negative bacteria are commonly more resistant to antibiotics than Gram positive bacteria
(C) Gram negative bacteria cell wall consists of a thick layer of peptidoglycan outside the plasma membrane
(D) Cell wall of Gram negative bacteria does not contain an outer membrane

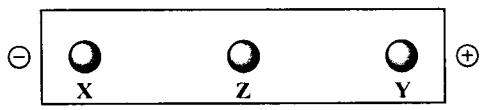
Q.4 The role of enzyme E synthesized by phage ϕX174 during host infection is to

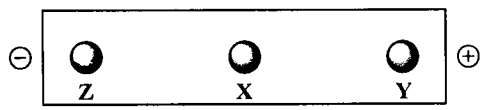
(A) block peptidoglycan synthesis
(B) enhance synthesis of viral +RNA
(C) inhibit lipid metabolism
(D) stimulate dsDNA replication

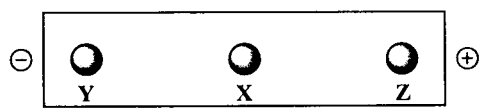
Q.5 Among CH_4, H_2O, NH_3 and PH_3, the molecule having the smallest percent *s* character for the covalent bond (X–H) between the central element (X = C, O, N or P) and hydrogen is

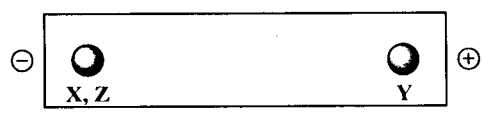
(A) CH_4 (B) H_2O (C) NH_3 (D) PH_3

Q.6 The result of an electrophoretic separation of a mixture of amino acids **X**, **Y** and **Z** at *p*H = 5.0 is represented as (Given the isoelectric points of **X**, **Y**, and **Z** are 9.87, 3.22 and 5.43, respectively)

(A) 

(B) 

(C) 

(D) 

Q.7 $\cos(x + yx) =$

(A) $\cos(x)\cos(yx) - \sin(x)\sin(yx)$ (B) $\cos(x)\cos(yx) + \sin(x)\sin(yx)$
(C) $\cos(x)\sin(yx) - \sin(x)\cos(yx)$ (D) $\cos(x)\sin(yx) + \sin(x)\cos(yx)$

Q.8 If $\begin{bmatrix} x & y \\ p & q \\ u & v \end{bmatrix} R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then the order of R is

(A) 2×3 (B) 3×2 (C) 2×2 (D) 3×3

Q.9 The average energy of a diatomic gaseous molecule at temperature T is $\frac{5}{2} k_B T$ where k_B is Boltzmann's constant. The average energy of this molecule per degree of freedom is

(A) $\frac{1}{2} k_B T$ (B) $\frac{2}{3} k_B T$ (C) $k_B T$ (D) $\frac{3}{2} k_B T$

Q.10 The refractive index of diamond is 2.419. If the speed of light in vacuum is 3×10^8 m s^{-1}, then the speed of light in diamond is

(A) 1.240×10^8 m s^{-1} (B) 1.352×10^8 m s^{-1} (C) 1.521×10^8 m s^{-1} (D) 2.433×10^8 m s^{-1}

Q. 11 – Q. 30 carry two marks each.

Q.11 Which of the following is true of protein synthesis **ONLY** in prokaryotes?

(A) Translation and transcription are coupled
(B) The codon AUG codes for the start signal
(C) The tRNA anticodon can bind to two or more different codons
(D) The functional ribosomes contain two subunits constructed of proteins and RNA

Q.12 Match the entries in Group I with that in Group II

Group I	Group II
P) Phytase	1) paper and pulp processing
Q) Xylanase	2) delignification
R) Laccase	3) gluten complex reduction
S) Bromelain	4) improve mineral availability
	5) phosphorylation

(A) P-4, Q-1, R-3, S-5 (B) P-4, Q-1, R-2, S-3
(C) P-5, Q-4, R-5, S-2 (D) P-5, Q-1, R-2, S-3

Q.13 If an aldol cleavage of glucose-6-phosphate occurs in glycolysis, it will result in

(A) products of equal carbon chain length (B) products of unequal carbon chain length
(C) removal of phosphate group (D) three C_2 compounds

Q.14 The natural geographical distribution of kangaroos is limited to the Australian continent because

(A) abiotic factors determine the distribution
(B) dispersal is limited by accessibility to other continents
(C) kangaroos have not selected habitats in other continents
(D) predators limit the distribution in other continents

Q.15 Which of the following is **NOT** an example of an adaptive defense mechanism against predation?

(A) Bright colors of bird pollinated flower
(B) Insect that resembles a stick
(C) Nicotine in the tobacco plant
(D) Spines on porcupine

Q.16 Match the entries in Group I with that in Group II

Group I	Group II
P) Nucleolus	1) lipid storage
Q) Sphaerosomes	2) breakdown of fatty acids
R) Peroxisomes	3) transport of macromolecules
S) Plasmodesmata	4) RNA synthesis

(A) P-4, Q-3, R-1, S-2
(B) P-4, Q-1, R-2, S-3
(C) P-2, Q-1, R-3, S-4
(D) P-1, Q-3, R-4, S-2

Q.17 The nitrogenase of diazotrophs

(A) contains Cu-S center and uses 12 NADH to reduce one N_2
(B) contains one (4Fe-4S) cluster and uses 8 $FADH_2$ to reduce one N_2
(C) is a complex of Fe-protein and MoFe-protein and uses 16 ATPs to reduce one N_2
(D) is a MoFe protein and uses 4 ATP and 4 $FMNH_2$ to reduce one N_2

Q.18 During eukaryotic cell division, the amount of DNA doubles

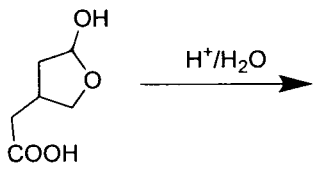
(A) between prophase and anaphase of mitosis
(B) between prophase I and prophase II of meiosis
(C) between the G1 and G2 phases of the cell cycle
(D) during the M phase of the cell cycle

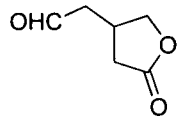
Q.19 The correct sequence of the following events in the human female reproductive cycle is

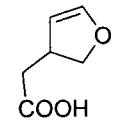
I: Secretion of FSH
II: Growth of corpus luteum
III: Growth of follicle and oogenesis
IV: Ovulation
V: Sudden increase in the levels of LH

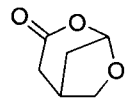
(A) I, II, IV, V, III
(B) II, I, III, IV, V
(C) I, III, V, IV, II
(D) I, V, III, IV, II

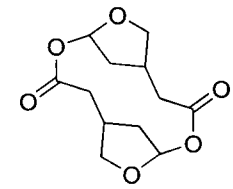
Q.20 The major product formed in the following reaction is



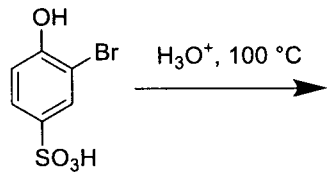
(A) 

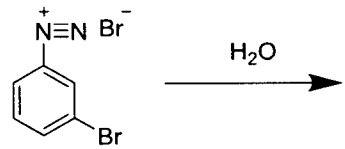
(B) 

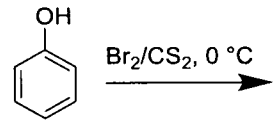
(C) 

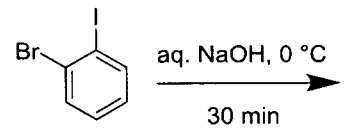
(D) 

Q.21 The reaction that produces *o*-bromophenol as the major product is

(A) 

(B) 

(C) 

(D) 

Q.22 For an autocatalytic second order reaction R → P, the rate law is [where v is rate of the reaction and k is the rate constant]

(A) $v = k\,[R]$ (B) $v = k\,[R]\,[P]$ (C) $v = k\,[R]^2$ (D) $v = k\,[P]^2$

Q.23 In metal-carbonyl complexes, the π-back bonding is

(A) pπ – dπ type
(B) dπ – dπ type
(C) dπ – π* type
(D) dπ – σ* type

Q.24 If $u(x)$ and $v(x)$ are differentiable at $x = 0$, and if $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$ and $v'(0) = 2$, then the value of $\frac{d}{dx}\left(uv + \frac{u}{v}\right)$ at $x = 0$ is

(A) –20 (B) –7 (C) 6 (D) 13

Q.25 Two dice are thrown simultaneously. The probability that the sum of the numbers obtained is divisible by 7 is

(A) $^1/_6$ (B)) $^1/_{36}$ (C) 0 (D)) $^1/_{18}$

Q.26 If one of the diameters of a circle has end points (2, 0) and (4, 0), then the equation of that circle is

(A) $x^2 - 3x + y^2 + 5 = 0$
(B) $x^2 - 4x + y^2 + 6 = 0$
(C) $x^2 - 5x + y^2 + 7 = 0$
(D) $x^2 - 6x + y^2 + 8 = 0$

Q.27 If P = {1, 2, –1, 3}, Q = {0, 4, 1, 3} and R = {1, 6, 7}, then P∩(Q∪R)=

(A) {1, 2} (B) {1, 3} (C) {2, 1} (D) {2, 3}

Q.28 The position of a particle along the y-axis is $y = P\,t^4 + Q$. For the equation to be dimensionally consistent, the dimension of P in terms of length [L] and time [T] is

(A) LT^{-1} (B) LT^{-2} (C) LT^{-3} (D) LT^{-4}

Q.29 Two inductors P and Q having inductance ratio 1:2 are connected in parallel in an electric circuit. The energy stored in the inductors P and Q are in the ratio

(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

Q.30 A body X of mass M moving with velocity v hits a stationary body Y of mass m. If M >> m and X moves with the velocity v', then the velocity of Y after an elastic collision is

(A) $2v$ (B) $v + v'$ (C) $v - v'$ (D) $2v'$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 The cells involved in allergic reactions and containing surface receptors of IgE antibodies and histamine are

(A) Basophils (B) Mast cells (C) Monocytes (D) Neutrophils

Q.32 Which of the following is(are) **INCORRECT** in the regulation of the *trp* operon?

(A) It is an example of a negatively controlled repressible operon
(B) The amino acid Trp inactivates the repressor
(C) The amino acid Trp induces the operon
(D) The repressor binds to the operator in the presence of amino acid Trp

Q.33 Which of the following organs are correctly paired with their function?

(A) Large intestine – Protein digestion
(B) Oral cavity – Starch digestion
(C) Pancreas – Bile production
(D) Small intestine – Fat digestion

Q.34 The $\Delta G^{0\prime}$ for homolactic fermentation converting glucose to lactate is –196 kJ mol^{-1}. If $\Delta G^{0\prime}$ for the formation of ATP is +30.5 kJ mol^{-1}, then

(A) homolactic fermentation is 31% energy efficient
(B) the efficiency of energy conservation is 69%
(C) the energy stored in the form of ATP is 31%
(D) the process results in the loss of 31% of energy

Q.35 Bacterial plasmid genes of non-chromosomal origin are associated with
A) providing resistance against antibacterial agents
B) the degradation of toxic materials
C) the production of certain toxins
D) the transfer of genetic material from one cell to another cell

Q.36 The elements with atomic numbers 19, 37 and 55

(A) form cubic chloride salts with the coordination number of cation being 6
(B) form ionic fluorides with general formula MF
(C) have lowest density of solids in their respective periods
(D) have lowest ionization energy in their respective periods

Q.37 Fehling's solution

(A) contains a copper complex of tartaric acid
(B) forms a brick-red precipitate with glucose
(C) forms a white precipitate with aldehydes
(D) is used as a test reagent for reducing sugars

Q.38 Which of the following point(s) lies(lie) on the plane $2x + 3y + z = 6$?

(A) (0, 0, 6) (B) (0, 2, 0) (C) (1, 1, 1) (D) (3, 0, 0)

Q.39 Kinetic theory of an ideal gas is based upon the following assumption(s)

(A) Gases are made of molecules with negligible volume
(B) The gaseous molecules do not possess kinetic energy
(C) The molecules are in constant random motion
(D) Intermolecular forces of attraction are negligible

Q.40 The electric field and capacitance of a capacitor in the absence of dielectric material are E and C, respectively. When the capacitor is filled with a dielectric material, the electric field and capacitance of the capacitor becomes E' and C', respectively. Which of the following is(are) correct?

(A) $E' > E$ and $C' = C$ (B) $E' < E$ and $C' > C$
(C) $E' = E$ and $C' > C$ (D) $E' > E$ and $C' < C$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Antigen and antibody interaction is shown by the following scheme

$$Ab + Ag \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} Ab\text{-}Ag$$

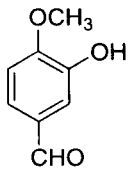
Where Ab is antibody, Ag is antigen and Ab-Ag is antigen-antibody complex. The values of k_1 and k_{-1} are 5×10^{-5} μM^{-1} s^{-1} and 2×10^{-7} s^{-1}, respectively. The dissociation constant for the complex is _____ nM.

Q.42 The population of a bacterial culture increases from one thousand to one billion in five hours. The doubling time of the culture (correct to 1 decimal place) is _____ min.

Q.43 The K_M and v_{max} of lactate dehydrogenase for conversion of pyruvate to lactate are 1 mM and 5 nM s^{-1}, respectively. At 0.25 mM pyruvate, the velocity of the reaction catalyzed by lactate dehydrogenase is _____ nM s^{-1}.

Q.44 A linear DNA contains five restriction sites for EcoRI and three restriction sites for BamHI. The number of fragments that will be generated after digestion with EcoRI is _____.

Q.45 Total number of singlets observed in the ^1H NMR of the following compound is _____.



Q.46 The $[H^+]$ of 0.1 N acetic acid solution is 1.33×10^{-3}. The *p*H of the solution (correct to two decimal places) is _____.

Q.47 The positive root of the equation $x^4 + x^2 - 2 = 0$ is _____ .

Q.48 $\int_0^1 x\,dx + \int_1^2 (2 - x)\,dx =$ _____ .

Q.49 One gram of radioactive nuclei with a half life of 300 days is kept in an open container. The weight of nuclei remaining after 900 days (correct to 1 decimal place) is _____ mg.

Q.50 Two sources P and Q produce electromagnetic waves with wavelengths λ and 2λ, respectively. Source P ejects a photon with a maximum kinetic energy of 4.0 eV from a metal with work function 2.0 eV. The maximum kinetic energy (eV) of a photon ejected by source Q from the same metal is _____ .

Q. 51 – Q. 60 carry two marks each.

Q.51 The standard oxidation potentials for oxidation of NADH and H_2O are + 0.315 V and –0.815 V, respectively. The standard free energy for oxidation of 1 mole of NADH by oxygen under standard conditions (correct to 1 decimal place) is _____ kJ. [Faraday Constant is 96500 C $mole^{-1}$]

Q.52 The K_M and v_{max} of an enzyme are 4 mM and 0.1 nM h^{-1} respectively. In the presence of 1.5 mM inhibitor, the K'_M and v'_{max} of the enzyme are 6 mM and 0.1 nM h^{-1}, respectively. The value of inhibition constant, K_I (correct to 1 decimal place) is _____ mM.

Q.53 The relationship between $log_{10}(MW)$ [where MW = molecular weight in kDa] of a mixture of protein standards and their retention factors (R_f) obtained from native-PAGE is $log_{10}(MW) = -2R_f + 3$. If the measured retention factor for a protein with 180 amino acids is 0.5, then the number of identical monomers in the protein is _____ .

Q.54 In bacteria, a ribosome synthesizes a protein containing 300 amino acids from mRNA in 20 seconds. If the average lifetime of a mRNA is 2 minutes, the number of ribosomes that can translate a single mRNA containing 1350 nucleotides is _____ .

Q.55 In 2 N H_2SO_4, an organic compound shows fluorescence with quantum yield, $\phi_f = 0.42$ and fluorescence rate constant, $k_f = 5.25 \times 10^7\ s^{-1}$. The observed fluorescence life time of it under the same conditions (correct to 1 decimal place) is _____ ns.

Q.56 In acidic solution, permanganate ion is reduced by ferrous ion. The number of electrons involved in the reduction of permanganate ion is _____ .

Q.57 If $\vec{a}$ and $\vec{b}$ are unit vectors and the angle between them is $\frac{\pi}{3}$, then the magnitude of $\vec{a} - \vec{b}$ is _____ .

Q.58 Using the letters in the word TRICK a new word containing five distinct letters is formed such that T appears in the middle. The number of distinct arrangements is _____ .

Q.59 An X-ray tube operates at 30 kV. If one electron converts 10% of its energy into a photon at first collision, then the wavelength of the photon (correct to two decimal places) is _____Å.
[$h = 4.14 \times 10^{-15}\ eVs^{-1}$, $c = 3 \times 10^3\ ms^{-1}$ and $e = 1.6 \times 10^{-19}$ C]

Q.60 In a mass spectrometer, a deuteron with kinetic energy 17 MeV enters a uniform magnetic field of 2.4 T with its velocity perpendicular to the field. The deuteron moves in a circular path in the magnetic field. The radius of its path in the magnetic field (correct to two decimal places) is _____ cm. [mass of deuteron is 3.34×10^{-27} kg, 1 MeV = 1.6×10^{-13} J and $e = 1.6 \times 10^{-19}$ C]

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for BT Paper

Paper:**BIOTECHNOLOGY**	Code:**BT**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	**KEY**	**Q. No.**	**KEY**	**Q. No.**	**KEYS**	**Q. No.**	**KEY RANGE**	**Q. No.**	**KEY RANGE**
01	C	**16**	B	**31**	A, B	**41**	4 to 4	**56**	5 to 5
02	B	**17**	C	**32**	B, C	**42**	14.0 to 16.0	**57**	1 to 1
03	B	**18**	C	**33**	B, D	**43**	1 to 1	**58**	24 to 24
04	A	**19**	C	**34**	A, C	**44**	6 to 6	**59**	4.10 to 4.20 or 0.00 to 0.01
05	D	**20**	A	**35**	A, B, C, D	**45**	3 to 3	**60**	34.00 to 36.00
06	A	**21**	A	**36**	B, C, D	**46**	2.86 to 2.89		
07	A	**22**	B	**37**	A, B, D	**47**	1 to 1		
08	A	**23**	C	**38**	A, B, C, D	**48**	1 to 1		
09	A	**24**	C	**39**	A, C, D	**49**	124.0 to 126.0		
10	A	**25**	A	**40**	B or C or B, C	**50**	1 to 1		
11	A	**26**	D			**51**	–219.0 to –217.0		
12	B	**27**	B			**52**	2.9 to 3.1		
13	B	**28**	D			**53**	5 to 5		
14	B	**29**	C			**54**	4.0 to 4.1		
15	A	**30**	B			**55**	7.9 to 8.1		

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The correct order of the boiling points of the compounds is

(A) $CH_4 > SiH_4 > SnH_4 > GeH_4$
(B) $SiH_4 > CH_4 > GeH_4 > SnH_4$
(C) $SnH_4 > GeH_4 > CH_4 > SiH_4$
(D) $SnH_4 > GeH_4 > SiH_4 > CH_4$

Q.2 In the following Latimer diagram, the species that undergoes disproportionation reaction is

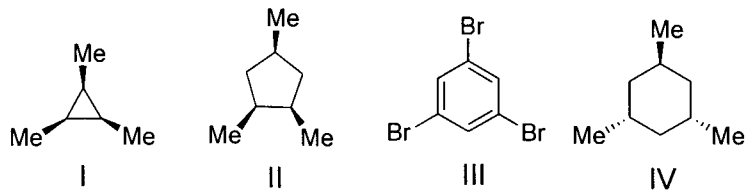
$$MnO_4^- \xrightarrow{+0.56} MnO_4^{2-} \xrightarrow{+0.27} MnO_4^{3-} \xrightarrow{+0.93} MnO_2 \xrightarrow{+0.15} Mn_2O_3 \xrightarrow{-0.25} Mn(OH)_2 \xrightarrow{-1.56} Mn$$

(A) MnO_4^{2-} (B) MnO_4^{3-} (C) Mn_2O_3 (D) $Mn(OH)_2$

Q.3 A yellow precipitate is formed upon addition of aqueous $AgNO_3$ to a solution of

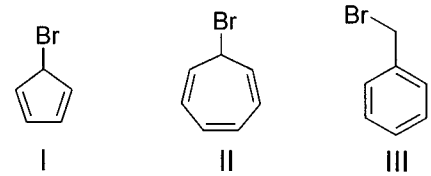
(A) phosphite
(B) pyrophosphate
(C) metaphosphate
(D) orthophosphate

Q.4 The compounds having C_3-axis of symmetry are



(A) I, III and IV (B) I, II and III (C) I and III (D) III and IV

Q.5 The correct order of rate of solvolysis for the following compounds is



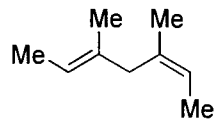
(A) III > II > I (B) II > I > III (C) III > I > II (D) II > III > I

Q.6 In the following sequence of reactions, the overall yield (%) of **O** is

$$L \xrightarrow{92\%\ \text{yield}} M \xrightarrow{78\%\ \text{yield}} N \xrightarrow{85\%\ \text{yield}} O$$

(A) 61 (B) 85 (C) 74 (D) 68

Q.7 Catalytic hydrogenation of the following compound produces saturated hydrocarbon(s). The number of stereoisomer(s) formed is



(A) 1 (B) 2 (C) 3 (D) 4

Q.8 The number of normal modes of vibration in naphthalene is

(A) 55 (B) 54 (C) 48 (D) 49

Q.9 The number of degrees of freedom of liquid water in equilibrium with ice is

(A) 0 (B) 1 (C) 2 (D) 3

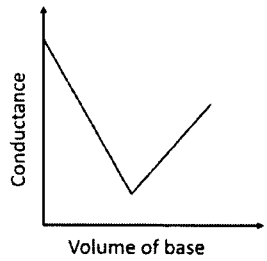
Q.10 A straight line having a slope of $-\Delta U^0/_R$ is obtained in a plot between

(A) lnK_p versus T (B) lnK_C versus T
(C) lnK_p versus 1/T (D) lnK_C versus 1/T

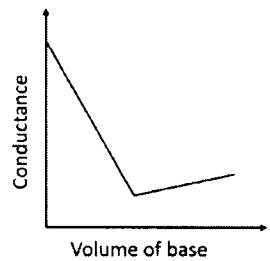
Q. 11 – Q. 30 carry two marks each.

Q.11 In a typical conductometric titration of a strong acid with a weak base, the curve resembles

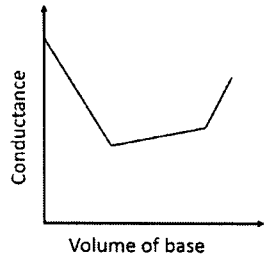
(A)



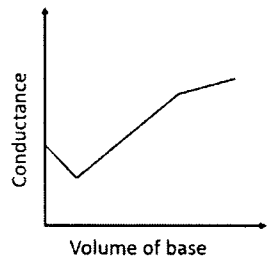
(B)



(C)



(D)



Q.12 The coordination number of Al in crystalline $AlCl_3$ and liquid $AlCl_3$, respectively, is

(A) 4 and 4 (B) 6 and 6 (C) 6 and 4 (D) 3 and 6

Q.13 The homogeneous catalyst used in water-gas shift reaction is

(A) $PdCl_2$ (B) Cr_2O_3
(C) $[RhCl(PPh_3)_3]$ (D) $[RuCl_2(bipyridyl)_2]$

Q.14 Nitrosyl ligand binds to d-metal atoms in linear and bent fashion and behaves, respectively, as

(A) NO^+ and NO^+ (B) NO^+ and NO^- (C) NO^- and NO^- (D) NO^- and NO^+

Q.15 The metal ion (M^{2+}) in the following reaction is

$$M^{2+} + S^{2-} \longrightarrow \text{Black precipitate} \xrightarrow{\text{hot conc. } HNO_3} \text{White precipitate}$$

(A) Mn^{2+} (B) Fe^{2+} (C) Cd^{2+} (D) Cu^{2+}

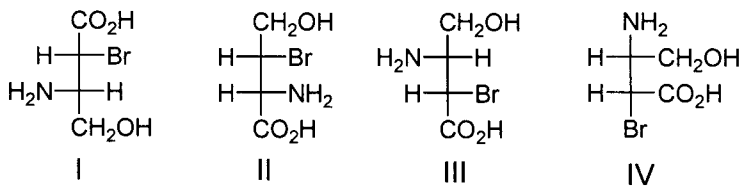
Q.16 The correct order of wavelength of absorption (λ_{max}) of the Cr-complexes is (en = ethylenediamine)

(A) $[CrF_6]^{3-} > [Cr(H_2O)_6]^{3+} > [Cr(en)_3]^{3+} > [Cr(CN)_6]^{3-}$
(B) $[Cr(H_2O)_6]^{3+} > [CrF_6]^{3-} > [Cr(en)_3]^{3+} > [Cr(CN)_6]^{3-}$
(C) $[Cr(CN)_6]^{3-} > [Cr(en)_3]^{3+} > [Cr(H_2O)_6]^{3+} > [CrF_6]^{3-}$
(D) $[Cr(en)_3]^{3+} > [Cr(CN)_6]^{3-} > [Cr(H_2O)_6]^{3+} > [CrF_6]^{3-}$

Q.17 The correct order of enthalpy of hydration for the transition metal ions is

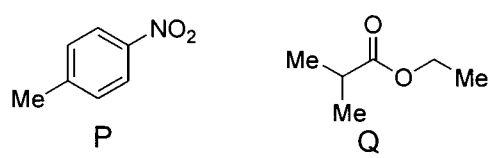
(A) $Cr^{2+} > Mn^{2+} > Co^{2+} > Ni^{2+}$
(B) $Ni^{2+} > Co^{2+} > Mn^{2+} > Cr^{2+}$
(C) $Ni^{2+} > Co^{2+} > Cr^{2+} > Mn^{2+}$
(D) $Cr^{2+} > Mn^{2+} > Ni^{2+} > Co^{2+}$

Q.18 Among the following compounds, the pair of enantiomers is



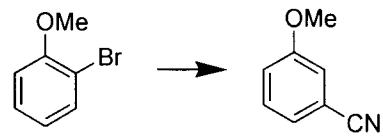
(A) I and IV (B) I and III (C) II and III (D) III and IV

Q.19 The number of proton NMR signals for the compounds **P** and **Q**, respectively, is



(A) 3 and 4 (B) 3 and 5 (C) 4 and 3 (D) 5 and 4

Q.20 The correct set of reagents for the following conversion is



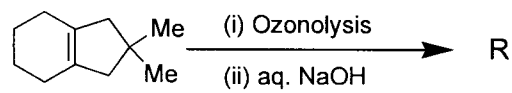
(A) (i) $NaNH_2$/liq.NH_3; (ii) $NaNO_2$/dil. HCl; (iii) CuCN, heat

(B) (i) HNO_3/H_2SO_4; (ii) Zn/HCl; (iii)) $NaNO_2$/dil. HCl; (iv) CuCN, heat

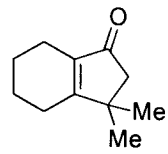
(C) (i) Mg/ether, H_3O^+ ; (ii) $(EtO)_2CO$; (iii) NH_4OH; (iv) PCl_5

(D) (i) Mg/ether, H_3O^+ ; (ii) HNO_3/H_2SO_4; (iii) $NaNO_2$/dil. HCl; (iv) CuCN, heat

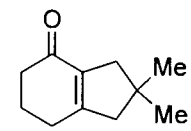
Q.21 The product **R** in the following reaction is



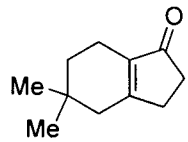
(A)



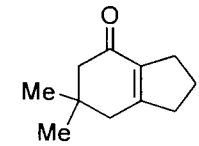
(B)



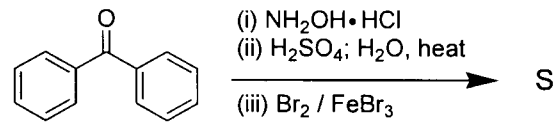
(C)



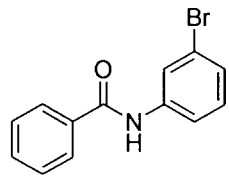
(D)



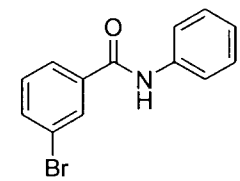
Q.22 The major product **S** of the following reaction is



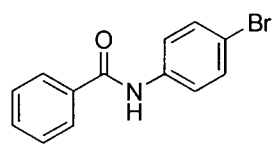
(A)



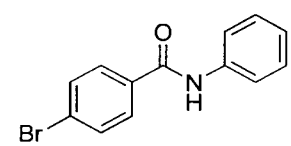
(B)



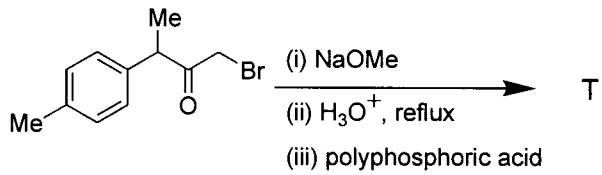
(C)

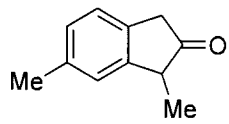


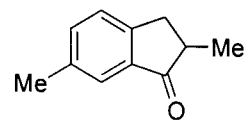
(D)

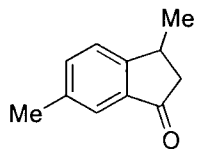


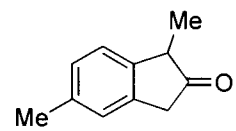
Q.23 In the following reaction, the major product **T** is



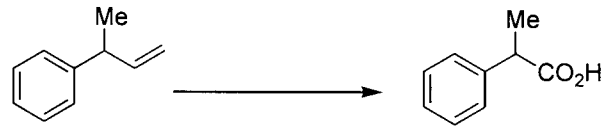
(A)


(B)


(C)


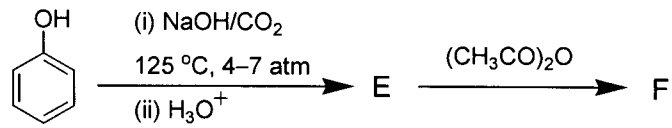
(D)


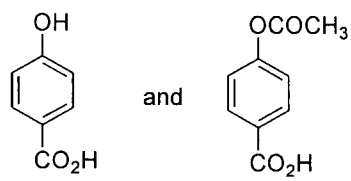
Q.24 The following conversion is carried out using

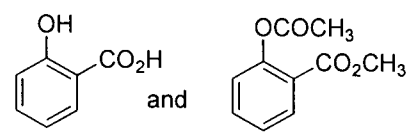


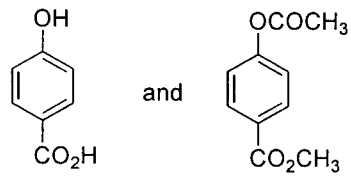
(A) hydroboration-oxidation followed by Jones oxidation
(B) Wacker oxidation followed by haloform reaction
(C) oxymercuration-demercuration followed by Jones oxidation
(D) ozonolysis followed by haloform reaction

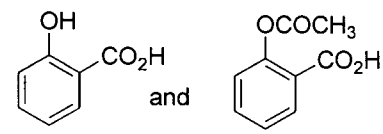
Q.25 In the following reactions, the major products **E** and **F**, respectively, are



(A)


(B)


(C)


(D)


Q.26 $\frac{dy}{dx} = -\frac{y}{x}$ is a differential equation for a/an

(A) circle
(B) ellipse
(C) bell-shaped curve
(D) hyperbola

Q.27 Value of the given determinant is

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$$

(A) –12 (B) 0 (C) 6 (D) 12

Q.28 Ionisation energy of hydrogen atom in ground state is 13.6 eV. The energy released (in eV) for third member of Balmer series is

(A) 13.056 (B) 2.856 (C) 0.967 (D) 0.306

Q.29 For a first order reaction $A(g) \rightarrow 2B(g) + C(g)$, the rate constant in terms of initial pressure (p_0) and pressure at time t (p_t), is given by

(A) $\frac{1}{t} ln \frac{p_0}{p_t - p_0}$ (B) $\frac{1}{t} ln \frac{2p_0}{3p_0 - p_t}$ (C) $\frac{1}{t} ln \frac{3p_0}{p_t - p_0}$ (D) $\frac{1}{t} ln \frac{3p_0}{3p_t - p_0}$

Q.30 For a particle in one-dimensional box of length L with potential energy V(x) = 0 for $L > x > 0$ and V(x) = ∞ for $x \geq L$ and $x \leq 0$, an acceptable wave function consistent with the boundary conditions is (A, B, C and D are constants)

(A) $A cos\left(\frac{n\pi x}{L}\right)$ (B) $B(x + x^2)$ (C) $Cx^3(x - L)$ (D) $\frac{D}{sin\left(\frac{n\pi x}{L}\right)}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 The "heme" containing protein(s) is/are

(A) cytochrome C (B) hemocyanin (C) hemerythrin (D) myoglobin

Q.32 Among the following, the species having see-saw shape is/are

(A) SF_4 (B) XeF_4 (C) ClF_4^+ (D) ClF_4^-

Q.33 The indicator(s) appropriate for the determination of end point in the titration of a weak acid with a strong base is/are

(A) phenolphthalein
(B) thymol blue
(C) bromophenol blue
(D) methyl orange

Q.34 Jahn-Teller distortion is observed in octahedral complexes with d-electron configuration of

(A) d^5- high spin (B) d^5- low spin (C) d^6- high spin (D) d^6- low spin

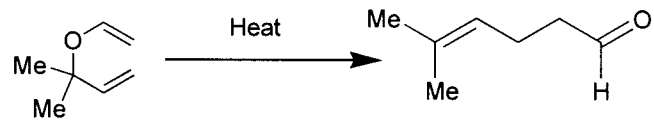
Q.35 Among the following, the correct statement(s) is/are

(A) Guanine is a purine nucleobase
(B) Glycine and proline are achiral amino acids
(C) DNA contains glycosidic bonds and pentose sugars
(D) Sucrose is a non-reducing sugar

Q.36 The **INCORRECT** statement(s) among the following is/are

(A) [4π + 2π] cycloaddition reactions are carried out in presence of light
(B) [2π + 2π] cycloaddition reaction between a keto group and an alkene is photochemically allowed
(C) [4π + 2π] cycloaddition reactions are thermally allowed
(D) Transoid dienes undergo Diels-Alder reactions

Q.37 The following conversion is an example of



(A) oxy-Cope rearrangement (B) sigmatropic rearrangement
(C) Claisen rearrangement (D) pericyclic reaction

Q.38 IR active molecule(s) is/are

(A) CO_2 (B) CS_2
(C) OCS (D) N_2

Q.39 Intensive variable(s) is/are

(A) temperature (B) volume
(C) pressure (D) density

Q.40 Wave nature of electromagnetic radiation is observed in

(A) diffraction (B) interference
(C) photoelectric effect (D) Compton scattering

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

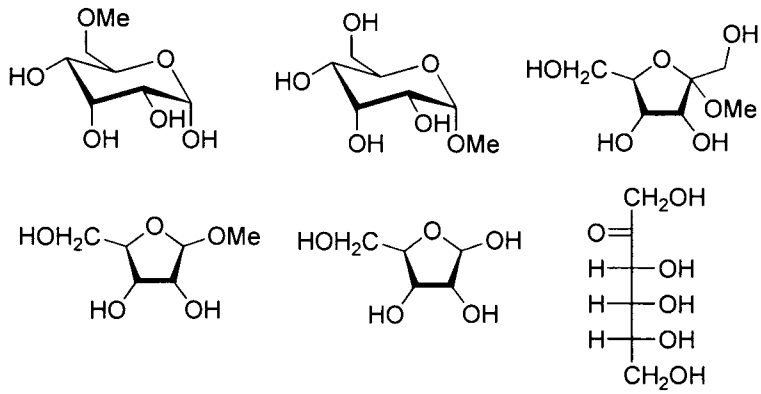
Q. 41 – Q. 50 carry one mark each.

Q.41 The number of isomeric structures of di-substituted borazine ($B_3N_3H_4X_2$) is _____

Q.42 The number of S–S bond(s) in tetrathionate ion is _____

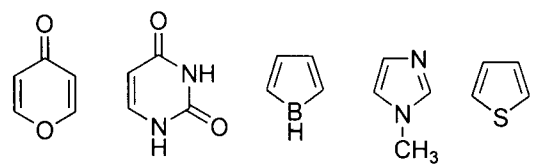
Q.43 The number of unpaired electron(s) in K_2NiF_6 is _____

Q.44 The number of reducing sugars among the following is _____



Q.45 The maximum number of dipeptides that could be obtained by reaction of phenylalanine with leucine is _____

Q.46 Among the following, the number of aromatic compound(s) is _____



Q.47 At an operating frequency of 350 MHz, the shift (in Hz) of resonance from TMS (tetramethylsilane) of a proton with chemical shift of 2 ppm is _____

Q.48 At 298 K and 1 atm, the molar enthalpies of combustion of cyclopropane and propene are –2091 kJ mol^{-1} and –2058 kJ mol^{-1}, respectively. The enthalpy change (in kJ mol^{-1}) for the conversion of one mole of propene to one mole of cyclopropane is _____

Q.49 For a cell reaction, $Pb(s) + Hg_2Cl_2(s) \rightarrow PbCl_2(s) + 2Hg(l)$, $\left(\frac{\partial E^0}{\partial T}\right)_p$ is 1.45×10^{-4} VK^{-1}. The entropy change (in J mol^{-1} K^{-1}) for the reaction is _____

[Given: 1 F = 96500 C mol^{-1}]

Q.50 For a reaction $2A + B \rightarrow C + D$, if rate of consumption of A is 0.1 mol $L^{-1}s^{-1}$, the rate of production of C (in mol $L^{-1}s^{-1}$) is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 The standard reduction potentials of Ce^{4+}/Ce^{3+} and Fe^{3+}/Fe^{2+} are 1.44 and 0.77 V, respectively. The $\log_{10}K$ (K is the equilibrium constant) value for the following reaction is _____
(final answer should be rounded off to two decimal places)

$$Ce^{4+} + Fe^{2+} \rightleftharpoons Ce^{3+} + Fe^{3+}$$

[Given: $RT/F = 0.0257\ V$]

Q.52 A radioactive element undergoes 80% radioactive decay in 300 min. The half-life for this species in minutes is _____

Q.53 Silver crystallizes in a face-centered cubic lattice. The lattice parameter of silver (in picometer) is _____

[Given: Avogadro's number = 6.023×10^{23} mol^{-1}, molar mass of silver = 107.87 g mol^{-1} and density of crystal = 10.5 g cm^{-3}]

Q.54 The amount of bromine (atomic wt.= 80) required (in gram) for the estimation of 42.3 g of phenol (molecular wt. = 94 g mol^{-1}) is _____

Q.55 The total number of pair of enantiomers possible with molecular formula $C_5H_{12}O$ is _____

Q.56 In 200 g of water, 0.01 mole of NaCl and 0.02 mole of sucrose are dissolved. Assuming solution to be ideal, the depression in freezing point of water (in °C) will be _____
(final answer should be rounded off to two decimal places)

[Given: $K_f(H_2O) = 1.86\ \text{K kg mol}^{-1}$]

Q.57 The adsorption of a gas follows the Langmuir isotherm with $K = 1.25\ \text{kPa}^{-1}$ at 25 °C. The pressure (in Pa) at which surface coverage is 0.2 is _____

Q.58 The separation of 123 planes (in nm) in an orthorhombic cell with $a = 0.25$ nm, $b = 0.5$ nm and $c = 0.75$ nm is _____
(final answer should be rounded off to two decimal places)

Q.59 A vessel contains a mixture of H_2 and N_2 gas. The density of this gas mixture is 0.2 g L^{-1} at 300 K and 1 atm. Assuming that both the gases behave ideally, the mole fraction of N_2 (g) in the vessel is _____
(final answer should be rounded off to two decimal places)

[Given: R = 0.082 L atm mol^{-1} K^{-1}, atomic wt. of hydrogen = 1.0 and atomic wt. of nitrogen = 14.0]

Q.60 Consider an isothermal reversible compression of one mole of an ideal gas in which the pressure of the system is increased from 5 atm to 30 atm at 300 K. The entropy change of the surroundings (in J K^{-1}) is _____
(final answer should be rounded off to two decimal places)

[Given: R = 8.314 J mol^{-1} K^{-1}]

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for CY Paper

Paper:**CHEMISTRY**	Code:**CY**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	**KEY**	**Q. No.**	**KEY**	**Q. No.**	**KEYS**	**Q. No.**	**KEY RANGE**	**Q. No.**	**KEY RANGE**
01	D	**16**	A	**31**	A, D	**41**	4 to 4	**56**	0.35 to 0.39
02	B	**17**	C	**32**	A, C	**42**	3 to 3	**57**	200 to 200
03	D	**18**	B	**33**	A, B	**43**	0 to 0	**58**	0.14 to 0.15
04	C	**19**	A	**34**	B, C	**44**	3 to 3	**59**	0.10 to 0.12
05	D	**20**	A	**35**	A, C, D	**45**	2 to 2 or 4 to 4	**60**	14.80 to 15.00
06	A	**21**	D	**36**	A, D	**46**	4 to 4		
07	C	**22**	C	**37**	B, C, D	**47**	700 to 700		
08	C	**23**	C	**38**	A, B, C	**48**	33 to 33		
09	B	**24**	B	**39**	A, C, D	**49**	27.90 to 28.10		
10	D	**25**	D	**40**	A, B	**50**	0.05 to 0.05		
11	B	**26**	D			**51**	11.30 to 11.38		
12	C	**27**	A			**52**	128 to 130		
13	D	**28**	B			**53**	408 to 409		
14	B	**29**	B			**54**	216 to 216		
15	D	**30**	C			**55**	4 to 4		

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following minerals exhibits luminescence when exposed to ultraviolet light?

(A) Cassiterite (B) Wolframite (C) Tantalite (D) Scheelite

Q.2 In which one of the following mass extinction periods trilobites became extinct?

(A) Devonian (B) Permian (C) Triassic (D) Cretaceous

Q.3 En-echelon sigmoidal 'gash' veins indicate

(A) ductile shear zone
(B) brittle-ductile shear zone
(C) brittle shear zone
(D) saddle reef structure

Q.4 Which one of the following primary sedimentary structures is NOT used for palaeocurrent analysis?

(A) Current crescent
(B) Flute marks
(C) Symmetrical wave ripples
(D) Imbrication of pebbles

Q.5 The age of the Patcham Formation is

(A) Permian (B) Triassic (C) Jurassic (D) Cretaceous

Q.6 Rivers that receive water from groundwater seepage are termed as

(A) effluent rivers
(B) consequent rivers
(C) influent rivers
(D) braided rivers

Q.7 Conservative plate boundary is represented by

(A) normal fault
(B) growth fault
(C) transform fault
(D) reverse fault

Q.8 Which one of the following prismatic crystal forms belongs to the hexagonal crystal system?

(A) $\{11\bar{2}0\}$ (B) $\{h0\bar{h}l\}$ (C) $\{0001\}$ (D) $\{hk0\}$

Q.9 The characteristic rock of contact metamorphism is

(A) hornfels (B) blueschist (C) eclogite (D) granulite

Q.10 The volcanic equivalent of nepheline syenite is

(A) rhyolite (B) basanite (C) phonolite (D) andesite

Q. 11 – Q. 30 carry two marks each.

Q.11 Identify the correct match between mineral/ore and its physical property.

(A) Hematite - Yellow streak
(B) Barite - High specific gravity
(C) Psilomelane - Comb structure
(D) Azurite - Distinctive green colour

Q.12 Match the mineral deposits in Group I with their Indian occurrences in Group II.

Group I	Group II
P. Chromite	1. Jhamarkotra, Rajasthan
Q. Magnesite	2. Gudur, Andhra Pradesh
R. Mica	3. Byrapur, Karnataka
S. Phosphorite	4. Chalk Hills, Tamil Nadu

(A)	(B)	(C)	(D)
P-1	P-3	P-3	P-2
Q-4	Q-2	Q-4	Q-1
R-3	R-4	R-2	R-3
S-2	S-1	S-1	S-4

Q.13 A helically coiled ammonite *Turrilites* is differentiated from externally resembling Gastropoda *Turritella* by

(A) apical angle
(B) number of whorls
(C) direction of coiling
(D) chambered shell

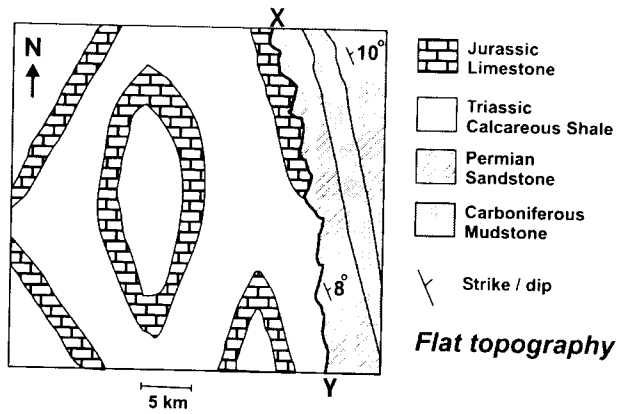
Q.14 The facial suture of trilobites running through the genal angle is known as

(A) proparian (B) marginal (C) gonatoparian (D) opisthoparian

Q.15 Which one of the following statements is correct for Class 1B (Parallel) folds?

(A) Orthogonal thickness at hinge > that at limb
(B) Axial planar thickness at hinge = that at limb
(C) Dip isogons are parallel
(D) Dip isogons are convergent

Q.16 In the given map, the **X-Y** surface has the same orientation as in the Palaeozoic sequence. **X-Y** represents



(A) angular unconformity
(B) non-conformity
(C) normal fault
(D) thrust

Q.17 Match the sedimentary features in Group I with the corresponding sedimentary environments of their formation in Group II.

Group I	Group II
P. Point Bar	1. Tidal
Q. Barchan	2. Meandering fluvial channel
R. Dropstone	3. Aeolian
S. Herring-bone cross stratification	4. Glacial

(A)	(B)	(C)	(D)
P-3	P-2	P-2	P-2
Q-4	Q-3	Q-4	Q-3
R-1	R-4	R-3	R-1
S-2	S-1	S-1	S-4

Q.18 Which one of the following lithostratigraphic units is of Phanerozoic Eon?

(A) Sargur Group
(B) Semri Group
(C) Uttatur Group
(D) Papaghni Group

Q.19 Match the geological processes (Group I) with their examples in Indian stratigraphy (Group II).

Group I	Group II
P. Permo-Carboniferous glaciation	1. Ariyalur Group
Q. Cretaceous marine transgression	2. Siwalik Group
R. Neogene fluvial sedimentation	3. Talchir Formation
S. Cretaceous inter-trappean sedimentation	4. Lameta Formation

(A)	(B)	(C)	(D)
P-3	P-2	P-3	P-4
Q-1	Q-3	Q-2	Q-1
R-2	R-4	R-1	R-2
S-4	S-1	S-4	S-3

Q.20 Check dams are constructed in association with main dam in the

(A) upstream of the main dam to check the siltation of the reservoir
(B) downstream of the main dam to check the siltation of the reservoir
(C) upstream of the main dam to check the seepage from the reservoir
(D) downstream of the main dam to check the seepage from the reservoir

Q.21 A geological formation neither containing nor transmitting water is termed as

(A) aquiclude
(B) aquitard
(C) aquifer
(D) aquifuge

Q.22 Which one of the following sequences of silicate structures indicates an increasing degree of sharing of corners of $(SiO_4)^{4-}$ tetrahedra?

(A) Nesosilicate → Single-chain inosilicate → Phyllosilicate → Tectosilicate
(B) Tectosilicate → Phyllosilicate → Single-chain inosilicate → Nesosilicate
(C) Nesosilicate → Phyllosilicate → Single-chain inosilicate → Tectosilicate
(D) Single-chain inosilicate → Nesosilicate → Phyllosilicate → Tectosilicate

Q.23 Match the igneous bodies in Group I with their ages in Group II.

Group I	Group II
P. Singhbhum granite	1. Neoproterozoic
Q. Malani rhyolite	2. Cretaceous
R. Deccan volcanics	3. Permian
S. Panjal Traps	4. Archaean

(A)	(B)	(C)	(D)
P-3	P-4	P-4	P-3
Q-2	Q-1	Q-3	Q-4
R-4	R-2	R-2	R-2
S-1	S-3	S-1	S-1

Q.24 The tubefeet in echinoids emerge through

(A) interambulacral plates
(B) ambulacral plates
(C) bourrelets
(D) plastron

Q.25 A mineral with a point group symmetry 2/m

(A) has two optic axes
(B) shows inclined extinction in (100) section
(C) shows straight extinction in (010) section
(D) is uniaxial

Q.26 The progressive metamorphic isograd sequence that explains Barrovian metamorphism in pelite is

(A) chlorite → staurolite → biotite → kyanite → sillimanite
(B) chlorite → andalusite → cordierite → sillimanite
(C) chlorite → biotite → garnet → staurolite → kyanite → sillimanite
(D) sillimanite → kyanite → staurolite → garnet → biotite → chlorite

Q.27 In metabasic rocks, plagioclase is not stable in

(A) granulite facies
(B) epidote amphibolite facies
(C) amphibolite facies
(D) eclogite facies

Q.28 A sandstone has < 5% matrix. The recalculated modal compositions of feldspar, quartz and rock fragments are 45%, 35%, 20%, respectively. The sandstone is classified as

(A) feldspathic wacke
(B) quartz wacke
(C) lithic arkose
(D) subfeldsarenite

Q.29 Match the earth layers (Group I) with corresponding approximate thicknesses (Group II).

Group I	Group II
P. Lithosphere	1. 2900 km
Q. Mantle	2. 2250 km
R. Outer Core	3. 1200 km
S. Inner Core	4. 100 km

(A)	(B)	(C)	(D)
P-4	P-4	P-4	P-3
Q-1	Q-1	Q-3	Q-2
R-2	R-3	R-1	R-1
S-3	S-2	S-2	S-4

Q.30 Pressure (1GPa = 10 kbar) and temperature at the centre of the Earth are estimated to be

(A) 360 GPa, 2600 K
(B) 450 GPa, 6000 K
(C) 360 GPa, 6000 K
(D) 450 GPa, 2600 K

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

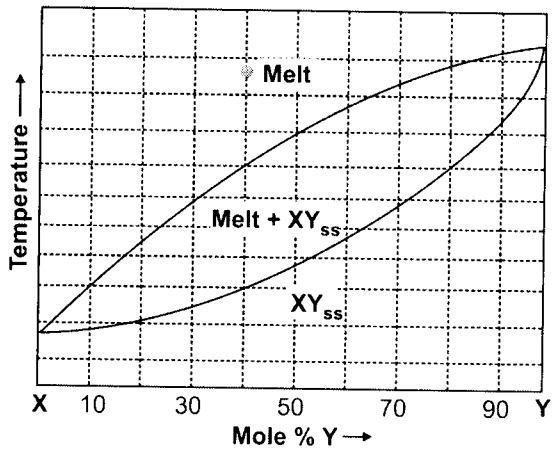
Q.31 Choose the landform(s) resulting from glacial erosion.

(A) Fjords (B) Moraines (C) Drumlins (D) Cirques

Q.32 Choose the correct combination(s) of type of dentition of Bivalvia and the corresponding representative genus.

(A) Taxodont - *Nucula*
(B) Isodont - *Spondylus*
(C) Pachydont - *Hippurites*
(D) Desmodont - *Mya*

Q.33 Shown below is an isobaric binary temperature-composition phase diagram in the system X-Y with complete miscibility between X and Y.



Which of the following statements is/are correct for crystallization of a starting melt of composition $X_{60}Y_{40}$ (the dot in the diagram)?

(A) The first formed crystal has a composition of $X_{20}Y_{80}$.
(B) The final melt composition during equilibrium crystallization is $X_{90}Y_{10}$.
(C) In case of fractional crystallization, the final melt is enriched in X than $X_{90}Y_{10}$.
(D) For fractional crystallization, the final crystal composition is $X_{60}Y_{40}$.

Q.34 Choose the correct combination(s) of textural features of magmatic rocks with corresponding petrological processes from the following.

(A) ophitic texture in dolerite - *peritectic crystallization*
(B) perthite in granite - *slow subsolidus cooling*
(C) spinifex texture in komatiite - *eruption of ultramafic lava*
(D) orthopyroxene rim around olivine in peridotite - *eutectic crystallization*

Q.35 Which of the following statements is/are NOT correct?

(A) (110) lies in zone [001]
(B) (021) lies in zone [100]
(C) (101) lies in zone [010]
(D) (111) lies in zone $[1\bar{1}1]$

Q.36 In an outcrop we find that the bedding planes are vertical and cleavage surfaces are horizontal. Which of the following fold types is/are inferred from this observation?

(A) Upright fold
(B) Recumbent fold
(C) Vertical fold
(D) Neutral fold

Q.37 Which of the following stratigraphic unit(s) is/are coal/lignite bearing?

(A) Barakar Formation
(B) Barail Group
(C) Cuddalore Formation
(D) Ariyalur Formation

Q.38 The Toposheet No(s). immediately adjacent to Toposheet No. 55J/8 is/are

(A) 55K/2 (B) 55J/12 (C) 55J/6 (D) 55K/5

Q.39 Which of the following is/are NOT true for texturally immature sandstone?

(A) Clay content is high
(B) Little or no clay present
(C) Grains are well sorted
(D) Grains are rounded

Q.40 P and S waves originate at earthquake focus and travel through the earth. Which of the following statements for these waves is/are correct?

(A) S-wave shadow zone is 154° wide
(B) P-wave shadow zones are 49° wide
(C) P-wave velocity abruptly increases downward at mantle-core boundary
(D) P-wave velocity abruptly drops downward at mantle-core boundary

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 An object is spotted at S60°E front bearing from the observer. If the position is interchanged, the front bearing value in degree from North (measured clockwise) is ____.

Q.42 The mole% of forsterite component in olivine with chemical formula $Mg_{1.8}Fe_{0.2}SiO_4$ is ____.

Q.43 The Weiss symbol of a crystal face is 4a: 2b: c. The value of h in the corresponding Miller Index (hkl) is ____.

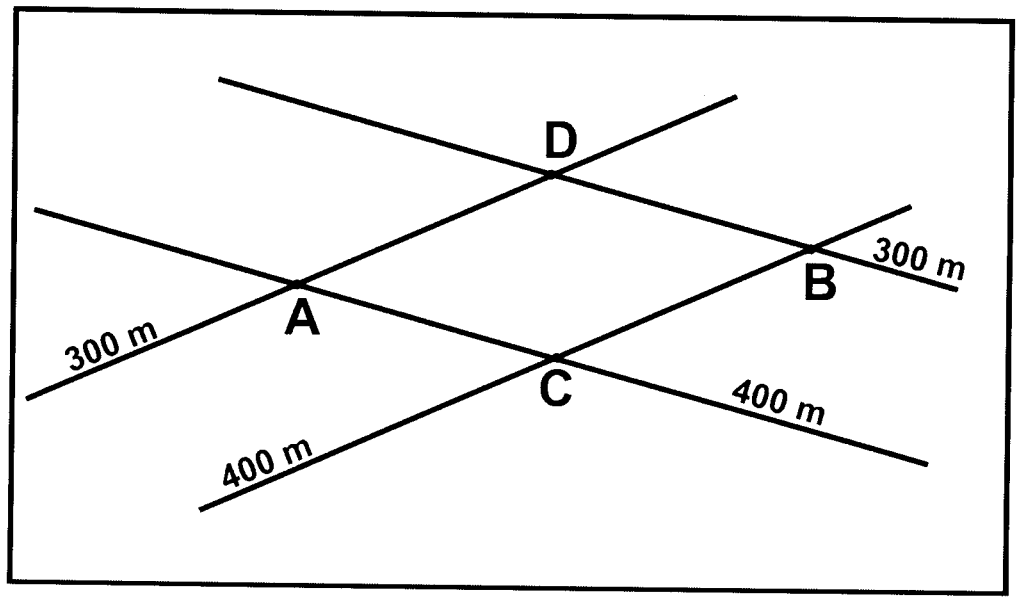
Q.44 In a mineral with chemical formula AT_4O_8, the ionic radii of A and O are 1.12 Å and 1.40 Å, respectively. The co-ordination number of cation A is _____.

Q.45 Aluminium (Al) can occur in both tetrahedral and octahedral co-ordinations in silicates. The amount of octahedral Al in a pyroxene crystal of composition $Mg_{1.4}Fe_{0.4}Al_{0.4}Si_{1.8}O_6$ is ____ (give answer in one decimal place).

Q.46 The birefringence of a mineral of thickness 30 μm and retardation 0.27 μm is ____ (give answer in three decimal places).

Q.47 Two limbs of a vertical chevron fold strike S70°E and N55°E. The value of the interlimb angle of the fold is ____ (degree).

Q.48 The schematic map given below shows intersecting strike lines of the same lithological contact. In the map, AB and CD are 5 cm and 3.5 cm, respectively. The scale of the map is 1 cm = 100 m. The plunge of the fold axis is ____ degrees (give answer in one decimal place).

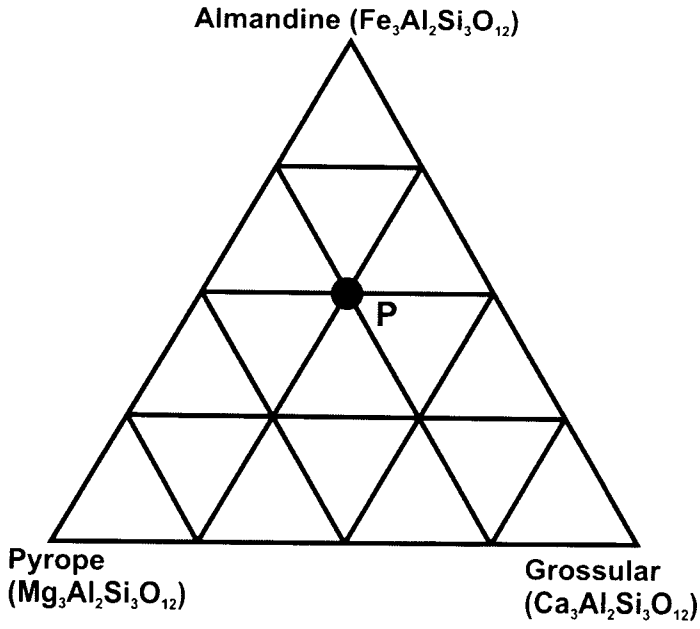


Q.49 The core-rim compositions of a normally zoned plagioclase crystal are as follows:

Core: $Ca_{0.6}\ Na_x\ Al_{1.6}\ Si_{2.4}\ O_8$ Rim: $Ca_{0.4}\ Na_y\ Al_{1.4}\ Si_{2.6}\ O_8$

The amount of increase of Na atom from core to rim per formula unit of plagioclase is _____ (give answer in one decimal place).

Q.50 Considering garnet chemical formula in 12 oxygen basis, the number of Mg cations in a garnet of chemical composition P (as shown in the figure) is ____ (give answer in two decimal places).



Q. 51 – Q. 60 carry two marks each.

Q.51 A fault surface in an outcrop has slickenside lineation whose pitch is 30°. The horizontal slip on the fault is 1.25 m, as determined from displaced vein. The net slip on the fault is ____ meter (give answer in two decimal places).

Q.52 In an outcrop, we find a Belemnite fossil broken into five rectangular pieces (boudins) of equal size. Long dimension of each boudin is 1.35 cm. Gap between adjacent boudins in all cases is 0.25 cm. Note that the long dimensions of boudins are perfectly aligned. The % elongation is ____ (give answer in one decimal place).

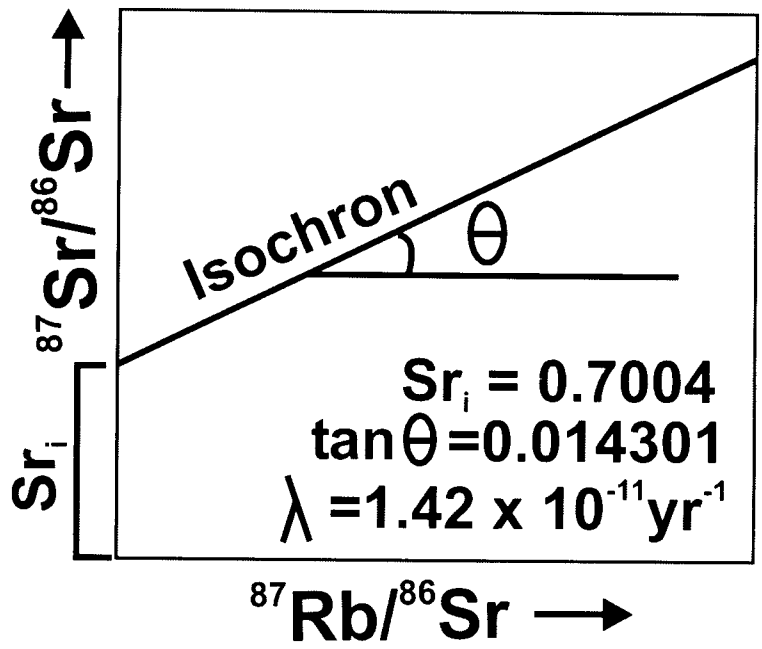
Q.53 A horizontal cylindrical ore body (diameter = 20 m, length = 200 m) has 5% metal content and density of 3500 kg/m^3. The reserve of the ore body is ____ million ton(s) (give answer in two decimal places).

Q.54 A drainage basin of fourth order covers an area of 40 sq. km. Within the basin, total length of 1st order drainage is 12.5 km, 2nd order drainage is 8.8 km, 3rd order drainage is 4.7 km and 4th order drainage is 4.0 km. The drainage density of the basin is ____ km^{-1} (give answer in two decimal places).

Q.55 Age of granitic rocks can be determined using Rb-Sr whole rock radioactive dating method and the following age equation,

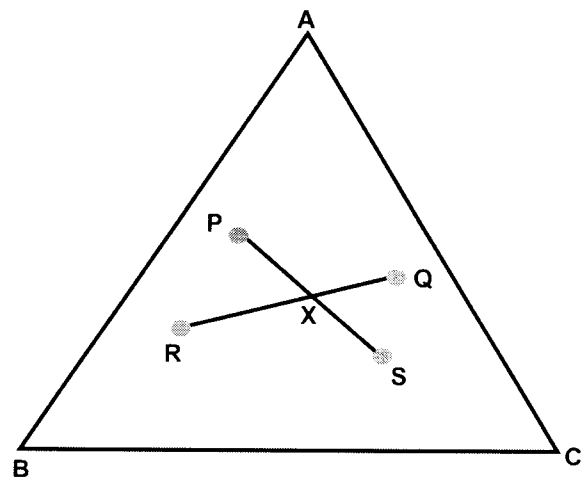
$$(^{87}Sr/^{86}Sr) = (^{87}Sr/^{86}Sr)_i + (^{87}Rb/^{86}Sr)(e^{\lambda t} - 1)$$

For a suite of representative co-magmatic granitic rocks, the Rb-Sr whole rock isochron plot and relevant data are shown in the diagram. The age of granite is calculated at ____Ga (1Ga = 10^9 yrs, give answer in one decimal place).



Q.56 Consider a granulite facies metamorphic rock with peak metamorphic condition at 9 kbar, 850°C. Assume a single layer crust of $\rho = 3000$ kg/m^3 and $g = 10$ m/sec^2 during metamorphism. The depth of burial during peak metamorphism is _____km. 1 Pascal = 1 kg/m/sec^2 and 1 bar = 10^5 Pascals.

Q.57 Consider four minerals P, Q, R and S in a three component chemical system (A-B-C) as shown in the figure. For a crossing tie-line relationship, the variance (degrees of freedom) of the equilibrium mineral assemblage at X is ____.

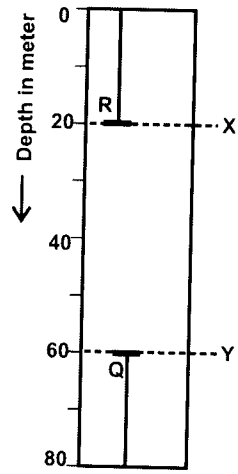


Q.58 The refractive indices of four minerals (P, Q, R, S) are as follows:

P (α = 1.712, β = 1.721, γ = 1.727),
Q (ε = 1.553, ω = 1.544),
R (α = 1.664, β = 1.672, γ = 1.694) and
S (ω = 1.658, ε = 1.486)

The value of maximum birefringence among all the minerals is ____ (give answer in three decimal places).

Q.59 In a sedimentary succession shown in the figure, the last occurrence of the fossil species Q (dated 50 Ma) and the first occurrence of the fossil species R (dated 30 Ma) are recorded at Y and X, respectively. The estimated rate of sedimentation is ___ m/million yrs (assume constant rate of sedimentation).



Q.60 The top surface of a coal seam is exposed at 150 m contour level on a hill top at location **A**. The same surface of the seam is also exposed on a river bed at location **B** at the 50 m contour level. The aerial distance **A-B** is 1 km. The amount of dip of the coal seam along **A-B** is ____ (degree). Give answer in one decimal place.

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for GG Paper

Paper:**GEOLOGY**	Code:**GG**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	**KEY**	**Q. No.**	**KEY**	**Q. No.**	**KEYS**	**Q. No.**	**KEY RANGE**	**Q. No.**	**KEY RANGE**
01	D	**16**	D	**31**	A, D	**41**	300 – 300	**56**	30 – 30
02	B	**17**	B	**32**	A, B, C, D	**42**	90 – 90	**57**	1 – 1
03	B	**18**	C	**33**	A, B, C	**43**	1 – 1	**58**	0.172 – 0.172
04	C	**19**	A	**34**	B, C	**44**	8 – 8	**59**	2 – 2
05	C	**20**	A	**35**	D	**45**	0.2 – 0.2	**60**	5.0 – 6.0
06	A	**21**	D	**36**	B, D	**46**	0.009 - 0.009		
07	C	**22**	A	**37**	A, B, C	**47**	55 – 55		
08	A	**23**	B	**38**	B, D	**48**	15.8 – 16.0		
09	A	**24**	B	**39**	B, C, D	**49**	0.2 – 0.2		
10	C	**25**	A	**40**	A, D	**50**	0.75 – 0.75		
11	B	**26**	C			**51**	1.42 – 1.46		
12	C	**27**	D			**52**	14.7 – 14.9		
13	D	**28**	C			**53**	0.21 – 0.23		
14	C	**29**	A			**54**	0.75 – 0.75		
15	D	**30**	C			**55**	0.9 – 1.1		

Special Instructions/Useful Data	
$\mathbb{N}$	Set of all natural numbers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{R}$	Set of all real numbers
P^T	Transpose of the matrix P
$\mathbb{R}^n$	$\{(x_1, x_2, \ldots, x_n)^T \mid x_i \in \mathbb{R}, i = 1, 2, \ldots, n\}$
g'	Derivative of a real valued function g
g''	Second derivative of a real valued function g
$P(A)$	Probability of an event A
i.i.d.	Independently and identically distributed
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
$F_{m,n}$	F distribution with (m, n) degrees of freedom
t_n	Student's t distribution with n degrees of freedom
χ_n^2	Central Chi-squared distribution with n degrees of freedom
$\Phi(x)$	Cumulative distribution function of $N(0,1)$
A^C	Complement of a set A
$E(X)$	Expectation of a random variable X
$\mathrm{Var}(X)$	Variance of a random variable X
$\mathrm{Cov}(X, Y)$	Covariance between random variables X and Y
$r!$	Factorial of an integer $r > 0$, $0! = 1$
$\Phi(0.25) = 0.5987, \Phi(0.5) = 0.6915, \Phi(0.625) = 0.7341, \Phi(0.71) = 0.7612,$ $\Phi(1) = 0.8413, \Phi(1.125) = 0.8697, \Phi(1.5) = 0.9332, \Phi(1.64) = 0.95,$ $\Phi(2) = 0.9772$	

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The imaginary parts of the eigenvalues of the matrix

$$P = \begin{pmatrix} 3 & 2 & 5 \\ 2 & -3 & 6 \\ 0 & 0 & -3 \end{pmatrix}$$

are

(A) $0, 0, 0$ (B) $2, -2, 0$ (C) $1, -1, 0$ (D) $3, -3, 0$

Q.2 Let $u, v \in \mathbb{R}^4$ be such that $u = (1 \quad 2 \quad 3 \quad 5)^T$ and $v = (5 \quad 3 \quad 2 \quad 1)^T$. Then the equation $uv^T x = v$ has

(A) infinitely many solutions (B) no solution

(C) exactly one solution (D) exactly two solutions

Q.3 Let $u_n = \left(4 - \frac{1}{n}\right)^{\frac{(-1)^n}{n}}$, $n \in \mathbb{N}$ and let $l = \lim_{n\to\infty} u_n$.

Which of the following statements is TRUE?

(A) $l = 0$ and $\sum_{n=1}^{\infty} u_n$ is convergent

(B) $l = \frac{1}{4}$ and $\sum_{n=1}^{\infty} u_n$ is divergent

(C) $l = \frac{1}{4}$ and $\{u_n\}_{n\geq 1}$ is oscillatory

(D) $l = 1$ and $\sum_{n=1}^{\infty} u_n$ is divergent

Q.4 Let $\{a_n\}_{n\geq 1}$ be a sequence defined as follows:

$$a_1 = 1 \text{ and } a_{n+1} = \frac{7a_n + 11}{21}, n \in \mathbb{N}.$$

Which of the following statements is TRUE?

(A) $\{a_n\}_{n\geq 1}$ is an increasing sequence which diverges

(B) $\{a_n\}_{n\geq 1}$ is an increasing sequence with $\lim_{n\to\infty} a_n = \frac{11}{14}$

(C) $\{a_n\}_{n\geq 1}$ is a decreasing sequence which diverges

(D) $\{a_n\}_{n\geq 1}$ is a decreasing sequence with $\lim_{n\to\infty} a_n = \frac{11}{14}$

Q.5 Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ x^3, & \text{if } 0 < x \leq 1 \\ \frac{3}{x^5}, & \text{if } x > 1 \end{cases}.$$

Then $P\left(\frac{1}{2} < X < 2\right)$ equals

(A) $\frac{15}{16}$ (B) $\frac{11}{16}$ (C) $\frac{7}{12}$ (D) $\frac{3}{8}$

Q.6 Let X be a random variable with the moment generating function

$$M_X(t) = \frac{1}{216}(5 + e^t)^3, \quad t \in \mathbb{R}.$$

Then $P(X > 1)$ equals

(A) $\frac{2}{27}$ (B) $\frac{1}{27}$ (C) $\frac{1}{12}$ (D) $\frac{2}{9}$

Q.7 Let X be a discrete random variable with the probability mass function

$$p(x) = k(1 + |x|)^2, \quad x = -2, -1, 0, 1, 2,$$

where k is a real constant. Then $P(X = 0)$ equals

(A) $\frac{1}{9}$ (B) $\frac{2}{27}$ (C) $\frac{1}{27}$ (D) $\frac{1}{81}$

Q.8 Let the random variable X have uniform distribution on the interval $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$. Then $P(\cos X > \sin X)$ is

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Q.9 Let $\{X_n\}_{n \geq 1}$ be a sequence of i.i.d. random variables having common probability density function

$$f(x) = \begin{cases} xe^{-x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, $n = 1,2, \ldots$. Then $\lim_{n \to \infty} P(\bar{X}_n = 2)$ equals

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1

Q.10 Let X_1, X_2, X_3 be a random sample from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}, \quad \theta > 0.$$

Which of the following estimators of θ has the smallest variance for all $\theta > 0$?

(A) $\frac{X_1+3X_2+X_3}{5}$ (B) $\frac{X_1+X_2+2X_3}{4}$

(C) $\frac{X_1+X_2+X_3}{3}$ (D) $\frac{X_1+2X_2+3X_3}{6}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Player P_1 tosses 4 fair coins and player P_2 tosses a fair die independently of P_1. The probability that the number of heads observed is more than the number on the upper face of the die, equals

(A) $\frac{7}{16}$ (B) $\frac{5}{32}$ (C) $\frac{17}{96}$ (D) $\frac{21}{64}$

Q.12 Let X_1 and X_2 be i.i.d. continuous random variables with the probability density function

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}.$$

Using Chebyshev's inequality, the lower bound of $P\left(|X_1 + X_2 - 1| \leq \frac{1}{2}\right)$ is

(A) $\frac{5}{6}$ (B) $\frac{4}{5}$ (C) $\frac{3}{5}$ (D) $\frac{1}{3}$

Q.13 Let X_1, X_2, X_3 be i.i.d. discrete random variables with the probability mass function

$$p(k) = \left(\frac{2}{3}\right)^{k-1}\left(\frac{1}{3}\right), \quad k = 1, 2, 3, \ldots$$

Let $Y = X_1 + X_2 + X_3$. Then $P(Y \geq 5)$ equals

(A) $\frac{1}{9}$ (B) $\frac{8}{9}$ (C) $\frac{2}{27}$ (D) $\frac{25}{27}$

Q.14 Let X and Y be continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} cx(1-x), & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases},$$

where c is a positive real constant. Then $E(X)$ equals

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$ (D) $\frac{1}{3}$

Q.15 Let X and Y be continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases} x+y, & \text{if } 0<x<1, 0<y<1 \\ 0, & \text{otherwise} \end{cases}.$$

Then $P\left(X+Y>\frac{1}{2}\right)$ equals

(A) $\frac{23}{24}$ (B) $\frac{1}{12}$ (C) $\frac{11}{12}$ (D) $\frac{1}{24}$

Q.16 Let $X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_n$ be i.i.d. $N(0,1)$ random variables. Then

$$W=\frac{n\left(\sum_{i=1}^{m} X_i\right)^2}{m\left(\sum_{j=1}^{n} Y_j^2\right)}$$

has

(A) χ^2_{m+n} distribution (B) t_n distribution

(C) $F_{m,n}$ distribution (D) $F_{1,n}$ distribution

Q.17 Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables with the probability mass function

$$f(x)=\begin{cases} \frac{1}{4}, & \text{if } x=4 \\ \frac{3}{4}, & \text{if } x=8 \\ 0, & \text{otherwise} \end{cases}.$$

Let $\bar{X}_n=\frac{1}{n}\sum_{i=1}^{n} X_i$, $n=1,2,\ldots$. If $\lim_{n\to\infty} P(m\leq \bar{X}_n\leq M)=1$, then possible values of m and M are

(A) $m=2.1,\ M=3.1$ (B) $m=3.2,\ M=4.1$

(C) $m=4.2,\ M=5.7$ (D) $m=6.1,\ M=7.1$

Q.18 Let $x_1=1.1,\ x_2=0.5,\ x_3=1.4,\ x_4=1.2$ be the observed values of a random sample of size four from a distribution with the probability density function

$$f(x|\theta)=\begin{cases} e^{\theta-x}, & \text{if } x\geq\theta \\ 0, & \text{otherwise} \end{cases}, \quad \theta\in(-\infty,\infty).$$

Then the maximum likelihood estimate of θ^2 is

(A) 0.5 (B) 0.25 (C) 1.21 (D) 1.44

Q.19 Let $x_1 = 2,\ x_2 = 1,\ x_3 = \sqrt{5},\ x_4 = \sqrt{2}$ be the observed values of a random sample of size four from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{1}{2\theta}, & \text{if } -\theta \le x \le \theta \\ 0, & \text{otherwise} \end{cases}, \quad \theta > 0.$$

Then the method of moments estimate of θ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.20 Let $X_1,\ X_2$ be a random sample from an $N(0,\theta)$ distribution, where $\theta > 0$. Then the value of k, for which the interval $\left(0, \frac{X_1^2+X_2^2}{k}\right)$ is a 95% confidence interval for θ, equals

(A) $-\log_e(0.95)$ (B) $-2\log_e(0.95)$ (C) $-\frac{1}{2}\log_e(0.95)$ (D) 2

Q.21 Let $X_1,\ X_2,\ X_3,\ X_4$ be a random sample from $N(\theta_1, \sigma^2)$ distribution and $Y_1,\ Y_2,\ Y_3,\ Y_4$ be a random sample from $N(\theta_2, \sigma^2)$ distribution, where $\theta_1, \theta_2 \in (-\infty, \infty)$ and $\sigma > 0$. Further suppose that the two random samples are independent. For testing the null hypothesis $H_0: \theta_1 = \theta_2$ against the alternative hypothesis $H_1: \theta_1 > \theta_2$, suppose that a test ψ rejects H_0 if and only if $\sum_{i=1}^4 X_i > \sum_{j=1}^4 Y_j$. The power of the test ψ at $\theta_1 = 1 + \sqrt{2},\ \theta_2 = 1$ and $\sigma^2 = 4$ is

(A) 0.5987 (B) 0.7341 (C) 0.7612 (D) 0.8413

Q.22 Let X be a random variable having a probability density function $f \in \{f_0, f_1\}$, where

$$f_0(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

and

$$f_1(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

For testing the null hypothesis $H_0: f \equiv f_0$ against $H_1: f \equiv f_1$, based on a single observation on X, the power of the most powerful test of size $\alpha = 0.05$ equals

(A) 0.425 (B) 0.525 (C) 0.625 (D) 0.725

Q.23 If

$$\int_{y=0}^{1} \int_{x=y}^{2-\sqrt{1-(y-1)^2}} f(x,y)dx\,dy$$

$$= \int_{x=0}^{1} \int_{y=0}^{\alpha(x)} f(x,y)dy\,dx + \int_{x=1}^{2} \int_{y=0}^{\beta(x)} f(x,y)dy\,dx,$$

then $\alpha(x)$ and $\beta(x)$ are

(A) $\alpha(x) = x,\ \beta(x) = 1 + \sqrt{1-(x-2)^2}$ (B) $\alpha(x) = x,\ \beta(x) = 1 - \sqrt{1-(x-2)^2}$

(C) $\alpha(x) = 1 + \sqrt{1-(x-2)^2},\ \beta(x) = x$ (D) $\alpha(x) = 1 - \sqrt{1-(x-2)^2},\ \beta(x) = x$

Q.24 Let $f: [0,1] \to \mathbb{R}$ be a function defined as

$$f(t) = \begin{cases} t^3\left(1 + \frac{1}{5}\cos(\log_e t^4)\right) & \text{if } t \in (0,1] \\ 0 & \text{if } t = 0 \end{cases}.$$

Let $F: [0,1] \to \mathbb{R}$ be defined as

$$F(x) = \int_0^x f(t)dt\,.$$

Then $F''(0)$ equals

(A) 0 (B) $\frac{3}{5}$ (C) $-\frac{5}{3}$ (D) $\frac{1}{5}$

Q.25 Consider the function

$$f(x,y) = x^3 - y^3 - 3x^2 + 3y^2 + 7\ , x,y \in \mathbb{R}.$$

Then the local minimum (m) and the local maximum (M) of f are given by

(A) $m = 3,\ M = 7$ (B) $m = 4,\ M = 11$

(C) $m = 7,\ M = 11$ (D) $m = 3,\ M = 11$

Q.26 For $c \in \mathbb{R}$, let the sequence $\{u_n\}_{n\geq 1}$ be defined by

$$u_n = \frac{\left(1 + \frac{c}{n}\right)^{n^2}}{\left(3 - \frac{1}{n}\right)^n}\ .$$

Then the values of c for which the series $\sum_{n=1}^{\infty} u_n$ converges are

(A) $\log_e 6 < c < \log_e 9$ (B) $c < \log_e 3$

(C) $\log_e 9 < c < \log_e 12$ (D) $\log_e 3 < c < \log_e 6$

Q.27 If for a suitable $\alpha > 0$,

$$\lim_{x\to 0}\left(\frac{1}{e^{2x} - 1} - \frac{1}{\alpha x}\right)$$

exists and is equal to l $(|\,l\,| < \infty)$, then

(A) $\alpha = 2,\ l = 2$ (B) $\alpha = 2,\ l = -\frac{1}{2}$

(C) $\alpha = \frac{1}{2}, l = -2$ (D) $\alpha = \frac{1}{2},\ l = \frac{1}{2}$

Q.28 Let

$$P = \int_0^1 \frac{dx}{\sqrt{8 - x^2 - x^3}}.$$

Which of the following statements is TRUE?

(A) $\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) < P < \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{2}\right)$ (B) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{2}\right) < P < \sin^{-1}\left(\frac{1}{2}\right)$

(C) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) < P < \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ (D) $\sin^{-1}\left(\frac{1}{2}\right) < P < \frac{\sqrt{3}}{2}\sin^{-1}\left(\frac{1}{2}\right)$

Q.29 Let Q, A, B be matrices of order $n \times n$ with real entries such that Q is orthogonal and A is invertible. Then the eigenvalues of $Q^T A^{-1} B Q$ are always the same as those of

(A) AB (B) $Q^T A^{-1} B$ (C) $A^{-1} B Q^T$ (D) BA^{-1}

Q.30 Let $(x(t), y(t))$, $1 \leq t \leq \pi$, be the curve defined by

$$x(t) = \int_1^t \frac{\cos z}{z^2}\, dz \quad \text{and} \quad y(t) = \int_1^t \frac{\sin z}{z^2}\, dz \ .$$

Let L be the length of the arc of this curve from the origin to the point P on the curve at which the tangent is perpendicular to the x-axis. Then L equals

(A) $\sqrt{2}$ (B) $\frac{\pi}{\sqrt{2}}$ (C) $1 - \frac{2}{\pi}$ (D) $\frac{\pi}{2} + \sqrt{2}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $v \in \mathbb{R}^k$ with $v^T v \neq 0$. Let

$$P = I - 2\frac{vv^T}{v^T v},$$

where I is the $k \times k$ identity matrix. Then which of the following statements is (are) TRUE?

(A) $P^{-1} = I - P$ (B) -1 and 1 are eigenvalues of P

(C) $P^{-1} = P$ (D) $(I + P)v = v$

Q.32 Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be sequences of real numbers such that $\{a_n\}_{n\geq 1}$ is increasing and $\{b_n\}_{n\geq 1}$ is decreasing. Under which of the following conditions, the sequence $\{a_n + b_n\}_{n\geq 1}$ is always convergent?

(A) $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ are bounded sequences

(B) $\{a_n\}_{n\geq 1}$ is bounded above

(C) $\{a_n\}_{n\geq 1}$ is bounded above and $\{b_n\}_{n\geq 1}$ is bounded below

(D) $a_n \to \infty$ and $b_n \to -\infty$

Q.33 Let $f: [0,1] \to [0,1]$ be defined as follows:

$$f(x) = \begin{cases} x, & \text{if } x \in \mathbb{Q} \cap [0,1] \\ x + \frac{2}{3}, & \text{if } x \in \mathbb{Q}^C \cap \left(0, \frac{1}{3}\right) \\ x - \frac{1}{3}, & \text{if } x \in \mathbb{Q}^C \cap \left(\frac{1}{3}, 1\right) \end{cases}.$$

Which of the following statements is (are) TRUE?

(A) f is one-one and onto

(B) f is not one-one but onto

(C) f is continuous on $\mathbb{Q} \cap [0,1]$

(D) f is discontinuous everywhere on $[0,1]$

Q.34 Let $f(x)$ be a nonnegative differentiable function on $[a,b] \subset \mathbb{R}$ such that $f(a) = 0 = f(b)$ and $|f'(x)| \leq 4$. Let L_1 and L_2 be the straight lines given by the equations $y = 4(x-a)$ and $y = -4(x-b)$, respectively. Then which of the following statements is (are) TRUE?

(A) The curve $y = f(x)$ will always lie below the lines L_1 and L_2

(B) The curve $y = f(x)$ will always lie above the lines L_1 and L_2

(C) $\left| \int_a^b f(x)dx \right| < (b-a)^2$

(D) The point of intersection of the lines L_1 and L_2 lie on the curve $y = f(x)$

Q.35 Let E and F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$ and $P(E) + P(F) \geq 1$. Which of the following statements is (are) TRUE?

(A) $P(E^C) \leq P(F)$

(B) $P(E \cup F) < P(E^C \cup F^C)$

(C) $P(E|F^C) \geq P(F^C|E)$

(D) $P(E^C|F) \leq P(F|E^C)$

Q.36 The cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{4}{9}, & \text{if } 0 \le x < 1 \\ \frac{8}{9}, & \text{if } 1 \le x < 2 \\ 1, & \text{if } x \ge 2 \end{cases}.$$

Which of the following statements is (are) TRUE?

(A) The random variable X takes positive probability only at two points

(B) $P(1 \le X \le 2) = \frac{5}{9}$

(C) $E(X) = \frac{2}{3}$

(D) $P(0 < X < 1) = \frac{4}{9}$

Q.37 Let X_1, X_2 be a random sample from a distribution with the probability mass function

$$f(x|\theta) = \begin{cases} 1-\theta, & \text{if } x = 0 \\ \theta, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}, \qquad 0 < \theta < 1.$$

Which of the following is (are) unbiased estimator(s) of θ?

(A) $\frac{X_1+X_2}{2}$ (B) $\frac{X_1^2+X_2}{2}$ (C) $\frac{X_1^2+X_2^2}{2}$ (D) $\frac{X_1+X_2-X_1^2}{2}$

Q.38 Let X_1, X_2, X_3 be a random sample from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}, \qquad \theta > 0.$$

If $\delta(X_1, X_2, X_3)$ is an unbiased estimator of θ, which of the following CANNOT be attained as a value of the variance of δ at $\theta = 1$?

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.5

Q.39 Let $X_1, X_2, \ldots, X_n$ $(n \ge 2)$ be a random sample from a distribution with the probability density function

$$f(x|\theta) = \begin{cases} \frac{x}{\theta^2} e^{-x/\theta}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}, \qquad \theta > 0.$$

Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Which of the following statistics is (are) sufficient but NOT complete?

(A) $\bar{X}$ (B) $\bar{X}^2 + 3$ (C) $(X_1, \sum_{i=2}^{n} X_i)$ (D) $(X_1, \bar{X})$

Q.40 Let X_1, X_2, X_3, X_4 be a random sample from an $N(\theta, 1)$ distribution, where $\theta \in (-\infty, \infty)$. Suppose the null hypothesis $H_0: \theta = 1$ is to be tested against the hypothesis $H_1: \theta < 1$ at $\alpha = 0.05$ level of significance. For what observed values of $\sum_{i=1}^{4} X_i$, the uniformly most powerful test would reject H_0?

(A) -1 (B) 0 (C) 0.5 (D) 0.8

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let the random variable X have uniform distribution on the interval (0, 1) and $Y = -2\log_e X$. Then $E(Y)$ equals _____

Q.42 If $Y = \log_{10} X$ has $N(\mu, \sigma^2)$ distribution with moment generating function $M_Y(t) = e^{5t+2t^2}$, $t \in (-\infty, \infty)$, then $P(X < 1000)$ equals _____

Q.43 Let X_1, X_2, X_3, X_4, X_5 be independent random variables with $X_1 \sim N(200, 8)$, $X_2 \sim N(104, 8)$, $X_3 \sim N(108, 15)$, $X_4 \sim N(120, 15)$ and $X_5 \sim N(210, 15)$. Let $U = \frac{X_1+X_2}{2}$ and $V = \frac{X_3+X_4+X_5}{3}$. Then $P(U > V)$ equals _____

Q.44 Let X and Y be discrete random variables with the joint probability mass function

$$p(x, y) = \frac{1}{25}(x^2 + y^2), \quad \text{if } x = 1,2\,; y = 0,1,2.$$

Then $P(Y = 1 \mid X = 1)$ equals _____

Q.45 Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}.$$

Then $9\text{Cov}(X, Y)$ equals _____

Q.46 Let $X_1, X_2, X_3, Y_1, Y_2, Y_3, Y_4$ be i.i.d. $N(\mu, \sigma^2)$ random variables. Let $\bar{X} = \frac{1}{3}\sum_{i=1}^{3} X_i$ and $\bar{Y} = \frac{1}{4}\sum_{j=1}^{4} Y_j$. If $k\sqrt{\frac{15}{7}} \frac{(\bar{X} - \bar{Y})}{\sqrt{\left\{\sum_{i=1}^{3}(X_i - \bar{X})^2 + \sum_{j=1}^{4}(Y_j - \bar{Y})^2\right\}}}$ has t_ν distribution, then $(\nu - k)$ equals _____

Q.47 Let $f\colon \left[0,\frac{\pi}{2}\right] \to \mathbb{R}$ be defined as

$$f(x) = \alpha x + \beta \sin x\,,$$

where $\alpha, \beta \in \mathbb{R}$. Let f have a local minimum at $x = \frac{\pi}{4}$ with $f\left(\frac{\pi}{4}\right) = \frac{\pi-4}{4\sqrt{2}}$.

Then $8\sqrt{2}\,\alpha + 4\,\beta$ equals _____

Q.48 The area bounded between two parabolas $y = x^2 + 4$ and $y = -x^2 + 6$ is _____

Q.49 For $j = 1,2,\ldots,5$, let P_j be the matrix of order 5×5 obtained by replacing the j^{th} column of the identity matrix of order 5×5 with the column vector $v = (5 \quad 4 \quad 3 \quad 2 \quad 1)^T$. Then the determinant of the matrix product $P_1P_2P_3P_4P_5$ is _____

Q.50 Let

$$u_n = \frac{18n+3}{(3n-1)^2(3n+2)^2}\,, \quad n \in \mathbb{N}.$$

Then $\sum_{n=1}^{\infty} u_n$ equals _____

Q. 51 – Q. 60 carry two marks each.

Q.51 Let a unit vector $v = (v_1 \quad v_2 \quad v_3)^T$ be such that $Av = 0$ where

$$A = \begin{pmatrix} \frac{5}{6} & -\frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{6} & -\frac{1}{3} & \frac{5}{6} \end{pmatrix}.$$

Then the value of $\sqrt{6}\,(|v_1| + |v_2| + |v_3|)$ equals _____

Q.52 Let

$$F(x) = \int_0^x e^t(t^2 - 3t - 5)dt\,, \quad x > 0.$$

Then the number of roots of $F(x) = 0$ in the interval $(0,4)$ is _____

Q.53 A tangent is drawn on the curve $y = \frac{1}{3}\sqrt{x^3}$, $(x > 0)$ at the point $P\left(1, \frac{1}{3}\right)$ which meets the x-axis at Q. Then the length of the closed curve $OQPO$, where O is the origin, is _____

Q.54 The volume of the region

$$R = \{(x, y, z) \in \mathbb{R}^3 : x + y + z \leq 3,\ y^2 \leq 4x, 0 \leq x \leq 1, y \geq 0, z \geq 0\}$$

is _____

Q.55 Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8}, & \text{if } 0 < x < 2 \\ \frac{k}{8}, & \text{if } 2 \leq x \leq 4 \\ \frac{6-x}{8}, & \text{if } 4 < x < 6 \\ 0, & \text{otherwise.} \end{cases},$$

where k is a real constant. Then $P(1 < X < 5)$ equals _____

Q.56 Let X_1, X_2, X_3 be independent random variables with the common probability density function

$$f(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}.$$

Let $Y = \min\{X_1, X_2, X_3\}$, $E(Y) = \mu_y$ and $\text{Var}(Y) = \sigma_y^2$. Then $P(Y > \mu_y + \sigma_y)$ equals _____

Q.57 Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{2}e^{-x}, & \text{if } |y| \leq x,\ x > 0 \\ 0, & \text{otherwise} \end{cases}.$$

Then $E(X \mid Y = -1)$ equals _____

Q.58 Let X and Y be discrete random variables with $P(Y \in \{0,1\}) = 1$,

$$P(X = 0) = \frac{3}{4}, \quad P(X = 1) = \frac{1}{4},$$
$$P(Y = 1 | X = 1) = \frac{3}{4}, \quad P(Y = 0 | X = 0) = \frac{7}{8}.$$

Then $3P(Y = 1) - P(Y = 0)$ equals _____

Q.59 Let $X_1, X_2, \ldots, X_{100}$ be i.i.d. random variables with $E(X_1) = 0$, $E(X_1^2) = \sigma^2$, where $\sigma > 0$. Let $S = \sum_{i=1}^{100} X_i$. If an approximate value of $P(S \leq 30)$ is 0.9332, then σ^2 equals_____

Q.60 Let X be a random variable with the probability density function

$$f(x|r,\lambda) = \frac{\lambda^r}{(r-1)!}\, x^{r-1} e^{-\lambda x}, \quad x > 0, \lambda > 0, r > 0.$$

If $E(X) = 2$ and $\mathrm{Var}(X) = 2$, then $P(X < 1)$ equals _____

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for MS Paper

Paper: **MATHEMATICAL STATISTICS**	Code: **MS**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	KEY	Q. No.	KEY	Q. No.	KEYS	Q. No.	KEY RANGE	Q. No.	KEY RANGE
01	A	16	D	31	B, C	41	(2.0, 2.0)	56	(0.13, 0.14)
02	B	17	D	32	A, C	42	(0.15, 0.16)	57	(2.0, 2.0)
03	D	18	B	33	A, D	43	(0.97, 0.98)	58	(0.12, 0.13)
04	D	19	C	34	A, C	44	(0.25, 0.25)	59	(4.0, 4.0)
05	A	20	B	35	A, C, D	45	(0.15 0.17)	60	(0.25, 0.27)
06	A	21	D	36	B, C	46	(3.0, 3.0)		
07	C	22	B	37	A, B, C	47	(4.0, 4.0)		
08	D	23	B	38	A, B, C	48	(2.6, 2.7)		
09	A	24	A	39	C, D	49	(120.0, 120.0)		
10	C	25	D	40	A, B, C	50	(0.25, 0.25)		
11	C	26	B			51	(4.0, 4.0)		
12	C	27	B			52	(0, 0)		
13	B	28	A			53	(2.1, 2.2)		
14	C	29	D			54	(2.1, 2.3)		
15	A	30	C			55	(0.87, 0.88)		

Notation

$\mathbb{Z}_n$	Set of all residue classes modulo n
$X \setminus Y$	The set of elements from X which are not in Y
$\mathbb{N}$	The set of all natural numbers 1,2,3, ...
$\mathbb{R}$	The set of all real numbers
S_n	Set of all permutations of the set $\{1,2, \ldots, n\}$
$GL_n(\mathbb{R})$	Set of all $n \times n$ invertible matrices with real entries
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes in a three dimensional rectangular coordinate system, respectively
M^T	Transpose of a matrix M

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Consider the function $f(x,y) = 5 - 4\sin x + y^2$ for $0 < x < 2\pi$ and $y \in \mathbb{R}$. The set of critical points of $f(x,y)$ consists of

(A) a point of local maximum and a point of local minimum
(B) a point of local maximum and a saddle point
(C) a point of local maximum, a point of local minimum and a saddle point
(D) a point of local minimum and a saddle point

Q.2 Let $\varphi: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that φ' is strictly increasing with $\varphi'(1) = 0$. Let α and β denote the minimum and maximum values of $\varphi(x)$ on the interval $[2,3]$, respectively. Then which one of the following is TRUE?

(A) $\beta = \varphi(3)$ (B) $\alpha = \varphi(2.5)$ (C) $\beta = \varphi(2.5)$ (D) $\alpha = \varphi(3)$

Q.3 The number of generators of the additive group $\mathbb{Z}_{36}$ is equal to

(A) 6 (B) 12 (C) 18 (D) 36

Q.4
$$\lim_{n\to\infty} \frac{\pi}{n} \sum_{k=1}^{n} \sin\left(\frac{\pi}{2} + \frac{5\pi}{2} \cdot \frac{k}{n}\right) =$$

(A) $\frac{2\pi}{5}$ (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{5\pi}{2}$

Q.5 Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $g(u,v) = f(u^2 - v^2)$, then

$$\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2} =$$

(A) $4(u^2 - v^2)f''(u^2 - v^2)$
(B) $4(u^2 + v^2)f''(u^2 - v^2)$
(C) $2f'(u^2 - v^2) + 4(u^2 - v^2)f''(u^2 - v^2)$
(D) $2(u - v)^2 f''(u^2 - v^2)$

Q.6
$$\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx =$$

(A) $\frac{1+\cos 1}{2}$ (B) $1 - \cos 1$ (C) $1 + \cos 1$ (D) $\frac{1-\cos 1}{2}$

Q.7 Let $f_1(x), f_2(x), g_1(x), g_2(x)$ be differentiable functions on $\mathbb{R}$. Let $F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ be the determinant of the matrix $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$. Then $F'(x)$ is equal to

(A) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$

(B) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(C) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(D) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

Q.8 Let

$$f(x) = \frac{x + |x|(1+x)}{x} \sin\left(\frac{1}{x}\right), \qquad x \neq 0.$$

Write $L = \lim_{x \to 0^-} f(x)$ and $R = \lim_{x \to 0^+} f(x)$. Then which one of the following is TRUE?

(A) L exists but R does not exist

(B) L does not exist but R exists

(C) Both L and R exist

(D) Neither L nor R exists

Q.9 If $\lim_{T \to \infty} \int_0^T e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then

$$\lim_{T \to \infty} \int_0^T x^2 e^{-x^2} dx =$$

(A) $\frac{\sqrt{\pi}}{4}$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $\sqrt{2\pi}$ (D) $2\sqrt{\pi}$

Q.10 If

$$f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ (1-x)(px+q) & \text{if } x \geq 0 \end{cases}$$

satisfies the assumptions of Rolle's theorem in the interval $[-1, 1]$, then the ordered pair (p, q) is

(A) $(2, -1)$ (B) $(-2, -1)$ (C) $(-2, 1)$ (D) $(2, 1)$

Q. 11 – Q. 30 carry two marks each.

Q.11 The flux of the vector field

$$\vec{F} = \left(2\pi x + \frac{2x^2y^2}{\pi}\right)\hat{\imath} + \left(2\pi xy - \frac{4y}{\pi}\right)\hat{\jmath}$$

along the outward normal, across the ellipse $x^2 + 16y^2 = 4$ is equal to

(A) $4\pi^2 - 2$ (B) $2\pi^2 - 4$ (C) $\pi^2 - 2$ (D) 2π

Q.12 Let $\mathcal{M}$ be the set of all invertible 5×5 matrices with entries 0 and 1. For each $M \in \mathcal{M}$, let $n_1(M)$ and $n_0(M)$ denote the number of 1's and 0's in M, respectively. Then

$$\min_{M \in \mathcal{M}} |n_1(M) - n_0(M)| =$$

(A) 1 (B) 3 (C) 5 (D) 15

Q.13 Let $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Then

$$\lim_{n \to \infty} M^n x$$

(A) does not exist (B) is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(C) is $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (D) is $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Q.14 Let $\vec{F} = (3 + 2xy)\hat{\imath} + (x^2 - 3y^2)\hat{\jmath}$ and let L be the curve

$$\vec{r}(t) = e^t \sin t\, \hat{\imath} + e^t \cos t\, \hat{\jmath}, \qquad 0 \le t \le \pi.$$

Then

$$\int_L \vec{F} \cdot d\vec{r} =$$

(A) $e^{-3\pi} + 1$ (B) $e^{-6\pi} + 2$ (C) $e^{6\pi} + 2$ (D) $e^{3\pi} + 1$

Q.15 The line integral of the vector field

$$\vec{F} = zx\,\hat{\imath} + xy\,\hat{\jmath} + yz\,\hat{k}$$

along the boundary of the triangle with vertices (1,0,0), (0,1,0) and (0,0,1), oriented anti-clockwise, when viewed from the point (2,2,2), is

(A) $\frac{-1}{2}$ (B) -2 (C) $\frac{1}{2}$ (D) 2

Q.16 The area of the surface $z = \frac{xy}{3}$ intercepted by the cylinder $x^2 + y^2 \le 16$ lies in the interval

(A) $(20\pi, 22\pi]$ (B) $(22\pi, 24\pi]$ (C) $(24\pi, 26\pi]$ (D) $(26\pi, 28\pi]$

Q.17 For $a > 0, b > 0$, let $\vec{F} = \frac{x\hat{j} - y\,\hat{\imath}}{b^2x^2+a^2y^2}$ be a planar vector field. Let

$$C = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = a^2 + b^2\}$$

be the circle oriented anti-clockwise. Then

$$\oint_C \vec{F} \cdot d\vec{r} =$$

(A) $\frac{2\pi}{ab}$ (B) 2π (C) $2\pi ab$ (D) 0

Q.18 The flux of $\vec{F} = y\,\hat{\imath} - x\hat{j} + z^2\hat{k}$ along the outward normal, across the surface of the solid

$$\left\{(x,y,z) \in \mathbb{R}^3 \,\middle|\, 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le \sqrt{2 - x^2 - y^2}\right\}$$

is equal to

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$ (C) $\frac{8}{3}$ (D) $\frac{4}{3}$

Q.19 Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(2) = 2$ and

$$|f(x) - f(y)| \le 5(|x - y|)^{3/2}$$

for all $x \in \mathbb{R}, y \in \mathbb{R}$. Let $g(x) = x^3 f(x)$. Then $g'(2) =$

(A) 5 (B) $\frac{15}{2}$ (C) 12 (D) 24

Q.20 Let $f: \mathbb{R} \to [0, \infty)$ be a continuous function. Then which one of the following is NOT TRUE?

(A) There exists $x \in \mathbb{R}$ such that $f(x) = \frac{f(0)+f(1)}{2}$

(B) There exists $x \in \mathbb{R}$ such that $f(x) = \sqrt{f(-1)f(1)}$

(C) There exists $x \in \mathbb{R}$ such that $f(x) = \int_{-1}^{1} f(t)dt$

(D) There exists $x \in \mathbb{R}$ such that $f(x) = \int_{0}^{1} f(t)dt$

Q.21 The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{(-3)^{n+2}} \frac{(4x-12)^n}{n^2+1}$$

is

(A) $\frac{10}{4} \leq x < \frac{14}{4}$

(B) $\frac{9}{4} \leq x < \frac{15}{4}$

(C) $\frac{10}{4} \leq x \leq \frac{14}{4}$

(D) $\frac{9}{4} \leq x \leq \frac{15}{4}$

Q.22 Let $\mathcal{P}_3$ denote the real vector space of all polynomials with real coefficients of degree at most 3. Consider the map $T: \mathcal{P}_3 \to \mathcal{P}_3$ given by $T(p(x)) = p''(x) + p(x)$. Then

(A) T is neither one-one nor onto

(B) T is both one-one and onto

(C) T is one-one but not onto

(D) T is onto but not one-one

Q.23 Let $f(x,y) = \frac{x^2y}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Then

(A) $\frac{\partial f}{\partial x}$ and f are bounded

(B) $\frac{\partial f}{\partial x}$ is bounded and f is unbounded

(C) $\frac{\partial f}{\partial x}$ is unbounded and f is bounded

(D) $\frac{\partial f}{\partial x}$ and f are unbounded

Q.24 Let S be an infinite subset of $\mathbb{R}$ such that $S \setminus \{\alpha\}$ is compact for some $\alpha \in S$. Then which one of the following is TRUE?

(A) S is a connected set

(B) S contains no limit points

(C) S is a union of open intervals

(D) Every sequence in S has a subsequence converging to an element in S

Q.25 $$\sum_{n=1}^{\infty} \tan^{-1} \frac{2}{n^2} =$$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

Q.26 Let $0 < a_1 < b_1$. For $n \geq 1$, define

$$a_{n+1} = \sqrt{a_n b_n} \text{ and } b_{n+1} = \frac{a_n + b_n}{2}.$$

Then which one of the following is NOT TRUE?

(A) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal

(B) Both $\{a_n\}$ and $\{b_n\}$ converge and the limits are equal

(C) $\{b_n\}$ is a decreasing sequence

(D) $\{a_n\}$ is an increasing sequence

Q.27

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{9}} + \cdots + \frac{1}{\sqrt{3n}+\sqrt{3n+3}}\right) =$$

(A) $1+\sqrt{3}$ (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{1+\sqrt{3}}$

Q.28 Which one of the following is TRUE?

(A) Every sequence that has a convergent subsequence is a Cauchy sequence

(B) Every sequence that has a convergent subsequence is a bounded sequence

(C) The sequence $\{\sin n\}$ has a convergent subsequence

(D) The sequence $\left\{n \cos\frac{1}{n}\right\}$ has a convergent subsequence

Q.29 A particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x}\sin x$$

is

(A) $\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(B) $-\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(C) $-\frac{e^{2x}}{5}(2\cos x + \sin x)$

(D) $\frac{e^{2x}}{5}(2\cos x - \sin x)$

Q.30 Let $y(x)$ be the solution of the differential equation

$$(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$$

satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(A) $\frac{e}{e-1}$ (B) $\frac{2e}{e-1}$ (C) $\frac{e}{1-e}$ (D) 0

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 For $\alpha, \beta \in \mathbb{R}$, define the map $\varphi_{\alpha,\beta}: \mathbb{R} \to \mathbb{R}$ by $\varphi_{\alpha,\beta}(x) = \alpha x + \beta$. Let

$$G = \{\varphi_{\alpha,\beta} \mid (\alpha,\beta) \in \mathbb{R}^2\}$$

For $f, g \in G$, define $g \circ f \in G$ by $(g \circ f)(x) = g(f(x))$. Then which of the following statements is/are TRUE?

(A) The binary operation $\circ$ is associative

(B) The binary operation $\circ$ is commutative

(C) For every $(\alpha,\beta) \in \mathbb{R}^2$, $\alpha \neq 0$ there exists $(a,b) \in \mathbb{R}^2$ such that $\varphi_{\alpha,\beta} \circ \varphi_{a,b} = \varphi_{1,0}$

(D) $(G, \circ)$ is a group

Q.32 The volume of the solid

$$\left\{(x,y,z) \in \mathbb{R}^3 \;\middle|\; 1 \leq x \leq 2, \quad 0 \leq y \leq \frac{2}{x}, \quad 0 \leq z \leq x\right\}$$

is expressible as

(A) $\int_1^2 \int_0^{2/x} \int_0^x dz\, dy\, dx$

(B) $\int_1^2 \int_0^x \int_0^{2/x} dy\, dz\, dx$

(C) $\int_0^2 \int_1^z \int_0^{2/x} dy\, dx\, dz$

(D) $\int_0^2 \int_{\max\{z,1\}}^2 \int_0^{2/x} dy\, dx\, dz$

Q.33 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function. Then which of the following statements is/are TRUE?

(A) If f is differentiable at (0,0), then all directional derivatives of f exist at (0,0)

(B) If all directional derivatives of f exist at (0,0), then f is differentiable at (0,0)

(C) If all directional derivatives of f exist at (0,0), then f is continuous at (0,0)

(D) If the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous in a disc centered at (0,0), then f is differentiable at (0,0)

Q.34 If X and Y are $n \times n$ matrices with real entries, then which of the following is/are TRUE?

(A) If $P^{-1}XP$ is diagonal for some real invertible matrix P, then there exists a basis for $\mathbb{R}^n$ consisting of eigenvectors of X

(B) If X is diagonal with distinct diagonal entries and $XY = YX$, then Y is also diagonal

(C) If X^2 is diagonal, then X is diagonal

(D) If X is diagonal and $XY = YX$ for all Y, then $X = \lambda I$ for some $\lambda \in \mathbb{R}$

Q.35 Let G be a group of order 20 in which the conjugacy classes have sizes $1, 4, 5, 5, 5$. Then which of the following is/are TRUE?

(A) G contains a normal subgroup of order 5

(B) G contains a non-normal subgroup of order 5

(C) G contains a subgroup of order 10

(D) G contains a normal subgroup of order 4

Q.36 Let $\{x_n\}$ be a real sequence such that $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$. Then which of the following statements is/are TRUE?

(A) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 1

(B) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 2

(C) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to 1

(D) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to -3

Q.37 Let S be the set of all rational numbers in $(0,1)$. Then which of the following statements is / are TRUE?

(A) S is a closed subset of $\mathbb{R}$

(B) S is not a closed subset of $\mathbb{R}$

(C) S is an open subset of $\mathbb{R}$

(D) Every $x \in (0,1) \setminus S$ is a limit point of S

Q.38 Let M be an $n \times n$ matrix with real entries such that $M^3 = I$. Suppose that $Mv \neq v$ for any non-zero vector v. Then which of the following statements is / are TRUE?

(A) M has real eigenvalues

(B) $M + M^{-1}$ has real eigenvalues

(C) n is divisible by 2

(D) n is divisible by 3

Q.39 Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = (y-1)(y-3)$$

satisfying the condition $y(0) = 2$. Then which of the following is/are TRUE?

(A) The function $y(x)$ is not bounded above

(B) The function $y(x)$ is bounded

(C) $\lim_{x \to +\infty} y(x) = 1$

(D) $\lim_{x \to -\infty} y(x) = 3$

Q.40 Let $k, \ell \in \mathbb{R}$ be such that every solution of

$$\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + \ell y = 0$$

satisfies $\lim_{x\to\infty} y(x) = 0$. Then

(A) $3k^2 + \ell < 0$ and $k > 0$

(B) $k^2 + \ell > 0$ and $k < 0$

(C) $k^2 - \ell \leq 0$ and $k > 0$

(D) $k^2 - \ell > 0, k > 0$ and $\ell > 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 If the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 = c_1$, $c_1 > 0$, are given by $y = c_2x^\alpha$, $c_2 \in \mathbb{R}$, then $\alpha =$ _____

Q.42 Let G be a subgroup of $GL_2(\mathbb{R})$ generated by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$. Then the order of G is _____

Q.43 Consider the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 7 & 8 & 6 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 1 & 7 & 6 & 8 & 2 \end{pmatrix}$ in S_8. The number of $\eta \in S_8$ such that $\eta^{-1}\sigma\eta = \tau$ is equal to _____

Q.44 Let P be the point on the surface $z = \sqrt{x^2 + y^2}$ closest to the point $(4,2,0)$. Then the square of the distance between the origin and P is _____

Q.45 $$\left(\int_0^1 x^4(1-x)^5 dx\right)^{-1} = \_\_\_\_\_$$

Q.46 Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Let M be the matrix whose columns are $v_1, v_2, 2v_1 - v_2, v_1 + 2v_2$ in that order. Then the number of linearly independent solutions of the homogeneous system of linear equations $Mx = 0$ is _____

Q.47 $$\frac{1}{2\pi}\left(\frac{\pi^3}{1!\,3}-\frac{\pi^5}{3!\,5}+\frac{\pi^7}{5!\,7}-\cdots+\frac{(-1)^{n-1}\pi^{2n+1}}{(2n-1)!\,(2n+1)}+\cdots\right)=$$ _____

Q.48 Let P be a 7×7 matrix of rank 4 with real entries. Let $\boldsymbol{a}\in\mathbb{R}^7$ be a column vector. Then the rank of $P+\boldsymbol{a}\boldsymbol{a}^T$ is at least _____

Q.49 For $x>0$, let $[x]$ denote the greatest integer less than or equal to x. Then

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\cdots+\left[\frac{10}{x}\right]\right)=$$ _____

Q.50 The number of subgroups of $\mathbb{Z}_7\times\mathbb{Z}_7$ of order 7 is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $y(x)$, $x>0$ be the solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=0$$

satisfying the conditions $y(1)=1$ and $y'(1)=0$. Then the value of $e^2y(e)$ is _____

Q.52 Let T be the smallest positive real number such that the tangent to the helix

$$\cos t\;\hat{\imath}+\sin t\,\hat{\jmath}+\frac{t}{\sqrt{2}}\hat{k}$$

at $t=T$ is orthogonal to the tangent at $t=0$. Then the line integral of $\vec{F}=x\hat{\jmath}-y\,\hat{\imath}$ along the section of the helix from $t=0$ to $t=T$ is _____

Q.53 Let $f(x)=\frac{\sin \pi x}{\pi \sin x}$, $x\in(0,\pi)$, and let $x_0\in(0,\pi)$ be such that $f'(x_0)=0$. Then

$$\left(f(x_0)\right)^2(1+(\pi^2-1)\sin^2 x_0)=$$ _____

Q.54 The maximum order of a permutation σ in the symmetric group S_{10} is _____

Q.55 Let $a_n=\sqrt{n}$, $n\geq 1$, and let $s_n=a_1+a_2+\cdots+a_n$. Then

$$\lim_{n\to\infty}\left(\frac{a_n/s_n}{-\ln(1-a_n/s_n)}\right)=$$ _____

Q.56 For a real number x, define $\lceil x \rceil$ to be the smallest integer greater than or equal to x. Then

$$\int_0^1 \int_0^1 \int_0^1 (\lceil x \rceil + \lceil y \rceil + \lceil z \rceil)\, dx\, dy\, dz = \_\_\_\_\_$$

Q.57 For $x > 1$, let

$$f(x) = \int_1^x \left(\sqrt{\log t} - \frac{1}{2}\log\sqrt{t}\right) dt$$

The number of tangents to the curve $y = f(x)$ parallel to the line $x + y = 0$ is _____

Q.58 Let $\alpha, \beta, \gamma, \delta$ be the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$ _____

Q.59 The radius of convergence of the power series

$$\sum_0^\infty n!\, x^{n^2}$$

is _____

Q.60 If

$$y(x) = \int_{\sqrt{x}}^x \frac{e^t}{t} dt, \quad x > 0$$

then $y'(1) =$ _____

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for MA Paper

Paper: **MATHEMATICS**	Code:**MA**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	KEY	Q. No.	KEY	Q. No.	KEYS	Q. No.	KEY RANGE	Q. No.	KEY RANGE
01	D	16	A	31	A, C	41	1.9 – 2.1	56	2.9 – 3.1
02	A	17	A	32	A, B, D	42	5.9 – 6.1	57	0.9 – 1.1
03	B	18	D	33	A, D	43	-0.01 – +0.01	58	5.9 – 6.1
04	C	19	D	34	A, B, D	44	9.9 – 10.1	59	0.9 – 1.1
05	B	20	C	35	A, C	45	1259.9 – 1260.1	60	1.34 – 1.36
06	D	21	D	36	A, C	46	1.9 – 2.1		
07	B	22	B	37	B, D	47	0.49 – 0.51		
08	A	23	B	38	B, C	48	2.9 – 3.1		
09	A	24	D	39	B, C, D	49	54.9 – 55.1		
10	D	25	C	40	C, D	50	7.9 – 8.1		
11	B	26	A			51	2.9 – 3.1		
12	A	27	C			52	2.0 – 2.2		
13	C	28	C			53	0.9 – 1.1		
14	D	29	C			54	29.9 – 30.1		
15	C	30	B			55	0.9 – 1.1		

Useful information:

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

$\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$

$c = 3.0 \times 10^8\ ms^{-1}$

$e = 1.60 \times 10^{-19}\ C$

$m_e = 9.11 \times 10^{-31}$ kg = 0.51 MeV c^{-2}

$m_p = 1.6727 \times 10^{-27}$ kg = 938.28 MeV c^{-2}

$m_n = 1.6750 \times 10^{-27}$ kg = 939.56 MeV c^{-2}

$G = 6.67 \times 10^{-11}\mathbf{m^3kg^{-1}s^{-2}}$

$h = 6.626 \times 10^{-34}$ Js $= 4.135 \times 10^{-15}$ eV s

$\hbar = 1.054 \times 10^{-34}$ Js $= 6.582 \times 10^{-16}$ eV s

$\sigma = 5.67 \times 10^{-8}\ W\ m^{-2}K^{-4}$

$k_B = 1.38 \times 10^{-23}\ JK^{-1} = 8.62 \times 10^{-5}$ eV K^{-1}

Ground state energy of the Hydrogen atom = -13.6 eV

hc = 1240 eV nm

k_BT at 300 K = 25 meV

Eccentricity: $\varepsilon = \sqrt{1 + \frac{2E\ell^2}{m(GMm)^2}}$

$$C_p = \left(\frac{\partial V}{\partial T}\right)_p \left[\left(\frac{\partial U}{\partial V}\right)_p + p\right]$$

Maxwell's relations:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V,$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p,$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V,$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

Unless otherwise stated, all units are in SI.

SECTION – A

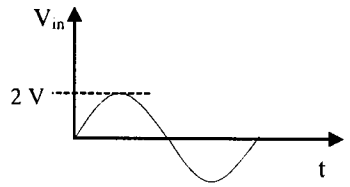
MULTIPLE CHOICE QUESTIONS (MCQ)

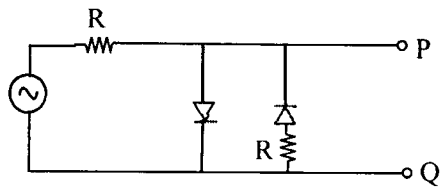
Q. 1 – Q.10 carry one mark each.

Q.1 The dispersion relation for electromagnetic waves travelling in a plasma is given as $\omega^2 = c^2k^2 + \omega_p^2$, where c and ω_p are constants. In this plasma, the group velocity is:

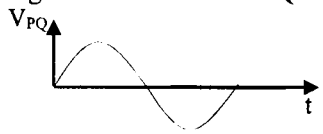
(A) proportional to but not equal to the phase velocity.
(B) inversely proportional to the phase velocity.
(C) equal to the phase velocity.
(D) a constant.

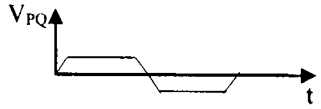
Q.2 Consider the following circuit with two identical Si diodes. The input ac voltage waveform has the peak voltage $V_P = 2V$, as shown.

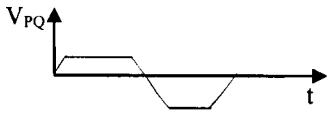


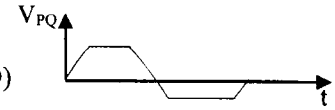


The voltage waveform across PQ will be represented by:

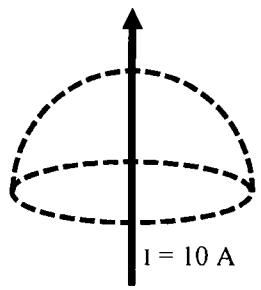
(A) 

(B) 

(C) 

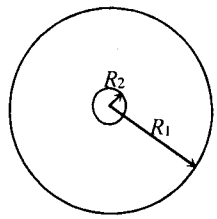
(D) 

Q.3 A current I = 10 A flows in an infinitely long wire along the axis of a hemisphere (see figure). The value of $\int(\vec{\nabla} \times \vec{B}).\overrightarrow{ds}$ over the hemispherical surface as shown in the figure is:



(A) 10 μ_o (B) 5 μ_o (C) 0 (D) 7.5 μ_o

Q.4 Consider two, single turn, co-planar, concentric coils of radii R_1 and R_2 with $R_1 >> R_2$. The mutual inductance between the two coils is proportional to

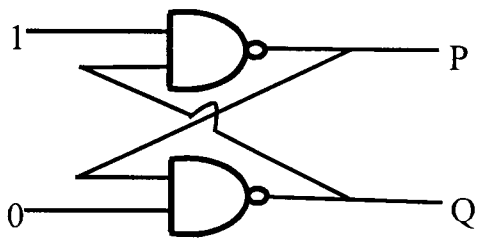


(A) R_1/R_2
(B) R_2/R_1
(C) R_2^2/R_1
(D) R_1^2/R_2

Q.5 If the Boolean function $Z = PQ+PQR+PQRS+PQRST+PQRSTU$, then $\bar{Z}$ is:

(A) $\bar{P}\bar{Q}+\bar{R}(\bar{S}+\overline{T+U})$
(B) $\bar{P}\bar{Q}$
(C) $\bar{P}+\bar{Q}$
(D) $\bar{P}+\bar{Q}+\bar{R}+\bar{S}+\bar{T}+\bar{U}$

Q.6 Shown in the figure is a combination of logic gates. The output values at P and Q are correctly represented by which of the following?



(A) 0 0 (B) 1 1 (C) 0 1 (D) 1 0

Q.7 Which of the following is due to inhomogeneous refractive index of earth's atmosphere?

(A) Red colour of the evening Sun.
(B) Blue colour of the sky.
(C) Oval shape of the evening Sun.
(D) Large apparent size of the evening Sun.

Q.8 For the three matrices given below, which one of the choices is correct?

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

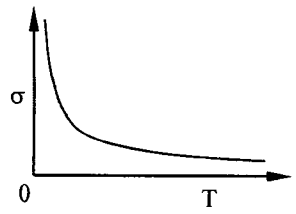
(A) $\sigma_1\sigma_2 = -i\sigma_3$
(B) $\sigma_1\sigma_2 = i\sigma_3$
(C) $\sigma_1\sigma_2 + \sigma_2\sigma_1 = I$
(D) $\sigma_3\sigma_2 = -i\sigma_1$

Q.9 A plane in a cubic lattice makes intercepts of a, $a/2$ and $2a/3$ with the three crystallographic axes, respectively. The Miller indices for this plane are:

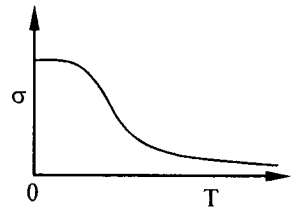
(A) (2 4 3) (B) (3 4 2) (C) (6 3 4) (D) (1 2 3)

Q.10 Which one of the following schematic curves best represents the variation of conductivity σ of a metal with temperature T?

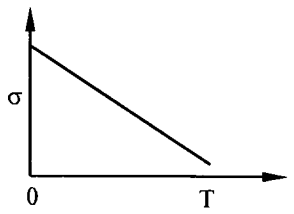
(A)



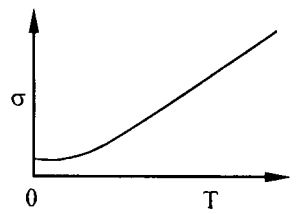
(B)



(C)

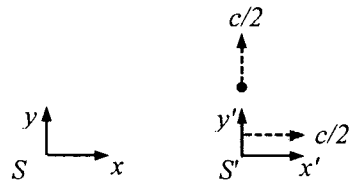


(D)



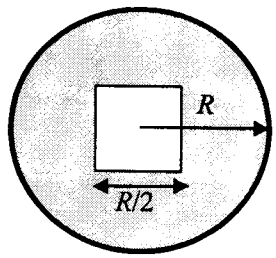
Q. 11 – Q. 30 carry two marks each.

Q.11 Consider an inertial frame S' moving at speed $c/2$ away from another inertial frame S along the common x-x'axis, where c is the speed of light. As observed from S', a particle is moving with speed $c/2$ in the y' direction, as shown in the figure. The speed of the particle as seen from S is:



(A) $c/\sqrt{2}$ (B) $c/2$ (C) $\sqrt{7}c/4$ (D) $\sqrt{3}c/5$

Q.12 Consider a uniform thin circular disk of radius R and mass M. A concentric square of side $R/2$ is cut out from the disk (see figure). What is the moment of inertia of the resultant disk about an axis passing through the centre of the disk and perpendicular to it?



(A) $I = \frac{MR^2}{4}\left[1 - \frac{1}{48\pi}\right]$

(B) $I = \frac{MR^2}{2}\left[1 - \frac{1}{48\pi}\right]$

(C) $I = \frac{MR^2}{4}\left[1 - \frac{1}{24\pi}\right]$

(D) $I = \frac{MR^2}{2}\left[1 - \frac{1}{24\pi}\right]$

Q.13 Consider a system of N particles obeying classical statistics, each of which can have an energy 0 or E. The system is in thermal contact with a reservoir maintained at a temperature T. Let k denote the Boltzmann constant. Which one of the following statements regarding the total energy U and the heat capacity C of the system is correct?

(A) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE}{kT}\frac{e^{E/kT}}{(1+e^{E/kT})^2}$

(B) $U = \frac{NE}{kT}\frac{E}{1+e^{E/kT}}$ and $C = k\frac{NE}{kT}\frac{e^{E/kT}}{(1+e^{-E/kT})^2}$

(C) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE^2}{(kT)^2}\frac{e^{E/kT}}{(1+e^{E/kT})^2}$

(D) $U = \frac{NE}{1+e^{E/kT}}$ and $C = k\frac{NE^2}{(kT)^2}\frac{e^{E/kT}}{(1+e^{E/kT})^2}$

Q.14 The integral of the vector $\vec{A}(\rho, \varphi, z) = \frac{40}{\rho} cos\varphi\ \hat{\rho}$ (standard notation for cylindrical coordinates is used) over the volume of a cylinder of height L and radius R_o is:

(A) $20\pi R_o L\ (\hat{\imath} + \hat{\jmath})$

(B) 0

(C) $40\pi R_o L\ \hat{\jmath}$

(D) $40\pi R_o L\ \hat{\imath}$

Q.15 Consider Rydberg (hydrogen-like) atoms in a highly excited state with n around 300. The wavelength of radiation coming out of these atoms for transitions to the adjacent states lies in the range:

(A) Gamma rays ($\lambda \sim$ pm)
(B) UV ($\lambda \sim$ nm)
(C) Infrared ($\lambda \sim \mu$m)
(D) RF ($\lambda \sim$ m)

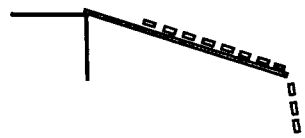
Q.16 A uniform rigid meter-scale is held horizontally with one of its end at the edge of a table and the other supported by hand. Some coins of negligible mass are kept on the meter scale as shown in the figure.

As the hand supporting the scale is removed, the scale starts rotating about its edge on the table and the coins start moving. If a photograph of the rotating scale is taken soon after, it will look closest to:

(A) 

(B) 

(C) 

(D) 

Q.17 Consider two identical, finite, isolated systems of constant heat capacity C at temperatures T_1 and T_2 ($T_1 > T_2$). An engine works between them until their temperatures become equal. Taking into account that the work performed by the engine will be maximum ($= W_{max}$) if the process is reversible (equivalently, the entropy change of the entire system is zero), the value of W_{max} is:

(A) $C(T_1 - T_2)$

(B) $C\,(T_1 - T_2)/2$

(C) $C\left(T_1 + T_2 - \sqrt{T_1 T_2}\right)$

(D) $C\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$

Q.18 A white dwarf star has volume V and contains N electrons so that the density of electrons is $n = \frac{N}{V}$. Taking the temperature of the star to be 0 K, the average energy per electron in the star is $\epsilon_0 = \frac{3\hbar^2}{10m}(3\pi^2 n)^{2/3}$, where m is the mass of the electron. The electronic pressure in the star is:

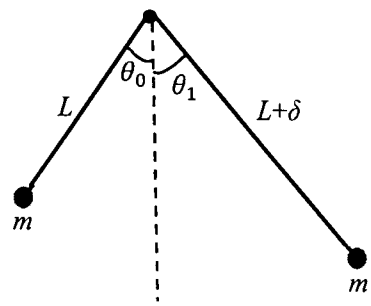
(A) $n\epsilon_0$

(B) $2n\epsilon_0$

(C) $\frac{1}{3}n\epsilon_0$

(D) $\frac{2}{3}n\epsilon_0$

Q.19 A pendulum is made of a massless string of length L and a small bob of negligible size and mass m. It is released making an angle θ_0 (<< 1 rad) from the vertical. When passing through the vertical, the string slips a bit from the pivot so that its length increases by a small amount δ (δ << L) in negligible time. If it swings up to angle θ_1 on the other side before starting to swing back, then to a good approximation which of the following expressions is correct?



(A) $\theta_1 = \theta_0$

(B) $\theta_1 = \theta_0\left(1 - \frac{\delta}{2L}\right)$

(C) $\theta_1 = \theta_0\left(1 - \frac{\delta}{L}\right)$

(D) $\theta_1 = \theta_0\left(1 - \frac{3\delta}{2L}\right)$

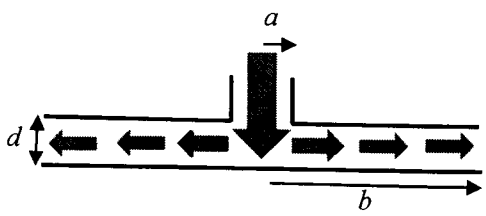
Q.20 To demonstrate Bernoulli's principle, an instructor arranges two circular horizontal plates of radii b each with distance d ($d << b$) between them (see figure). The upper plate has a hole of radius a in the middle. On blowing air at a speed v_0 through the hole so that the flow rate of air is $\pi a^2 v_0$, it is seen that the lower plate does not fall. If the density of air is ρ, the upward force on the lower plate is well approximated by the formula (assume that the region with $r < a$ does not contribute to the upward force and the speed of air at the edges is negligible):

(A) $\frac{\pi\rho v_0^2 a^4}{4d^2} ln\left(\frac{b}{a}\right)$

(B) $\frac{\pi\rho v_0^2 a^2 b^2}{4d^2} ln\left(\frac{b}{a}\right)$

(C) $\frac{\pi\rho v_0^2 d^4}{2ab} ln\left(\frac{b}{a}\right)$

(D) $\frac{2\pi\rho v_0^2 a^4}{d^2} ln\left(\frac{b}{a}\right)$

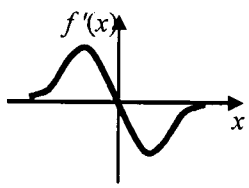


Q.21 KCl has the NaCl type structure which is fcc with two-atom basis, one at (0,0,0) and the other at $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$. Assume that the atomic form factors of K^+ and Cl^- are identical. In an x-ray diffraction experiment on KCl, which of the following (h k l) peaks will be observed?

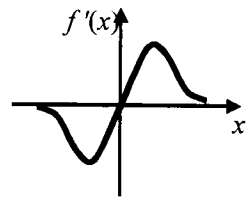
(A) (1 0 0) (B) (1 1 0) (C) (1 1 1) (D) (2 0 0)

Q.22 Which one of the following graphs represents the derivative $f'(x) = \frac{df}{dx}$ of the function $f(x) = \frac{1}{1+x^2}$ most closely (graphs are schematic and not drawn to scale)?

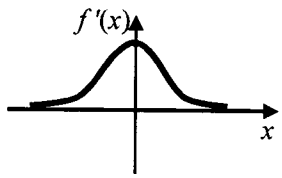
(A)



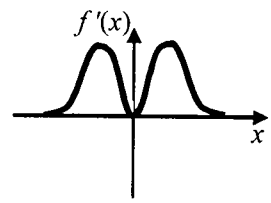
(B)



(C)



(D)



Q.23 In the radiation emitted by a black body, the ratio of the spectral densities at frequencies 2ν and ν will vary with ν as:

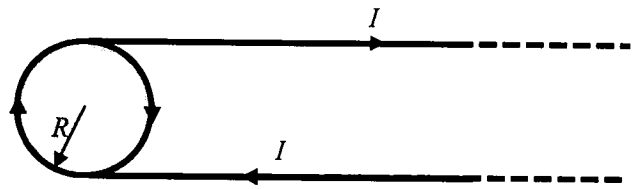
(A) $\left[e^{h\nu/k_BT} - 1\right]^{-1}$

(B) $\left[e^{h\nu/k_BT} + 1\right]^{-1}$

(C) $\left[e^{h\nu/k_BT} - 1\right]$

(D) $\left[e^{h\nu/k_BT} + 1\right]$

Q.24 Consider a thin long insulator coated conducting wire carrying current I. It is now wound once around an insulating thin disc of radius R to bring the wire back on the same side, as shown in the figure. The magnetic field at the centre of the disc is equal to:



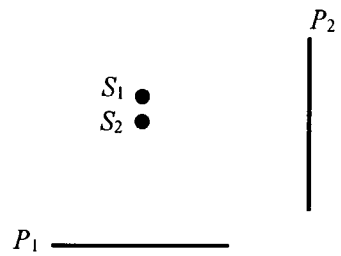
(A) $\frac{\mu_0 I}{2R}$

(B) $\frac{\mu_0 I}{4R}\left[3 + \frac{2}{\pi}\right]$

(C) $\frac{\mu_0 I}{4R}\left[1 + \frac{2}{\pi}\right]$

(D) $\frac{\mu_0 I}{2R}\left[1 + \frac{1}{\pi}\right]$

Q.25 Consider two coherent point sources (S_1 and S_2) separated by a small distance along a vertical line and two screens P_1 and P_2 placed as shown in Figure. Which one of the choices represents the shapes of the interference fringes at the central regions on the screens?



(A) Circular on P_1 and straight lines on P_2.

(B) Circular on P_1 and circular on P_2.

(C) Straight lines on P_1 and straight lines on P_2.

(D) Straight lines on P_1 and circular on P_2.

Q.26 The electric field of an electromagnetic wave is given by $\vec{E} = (2\hat{k} - 3\hat{j}) \times 10^{-3}\sin[10^7(x + 2y + 3z - \beta t)]$. The value of β is (c is the speed of light):

(A) $\sqrt{14}c$
(B) $\sqrt{12}c$
(C) $\sqrt{10}c$
(D) $\sqrt{7}c$

Q.27 Unpolarized light is incident on a combination of a polarizer, a λ/2 plate and a λ/4 plate kept one after the other. What will be the output polarization for the following configurations?
Configuration 1: Axes of the polarizer, the λ/2 plate and the λ/4 plate are all parallel to each other.
Configuration 2: The λ/2 plate is rotated by 45° with respect to configuration 1.
Configuration 3: The λ/4 plate is rotated by 45° with respect to configuration 1.

(A) Linear for configuration 1, linear for configuration 2, circular for configuration 3.
(B) Linear for configuration 1, circular for configuration 2, circular for configuration 3.
(C) Circular for configuration 1, circular for configuration 2, circular for configuration 3.
(D) Circular for configuration 1, linear for configuration 2, circular for configuration 3.

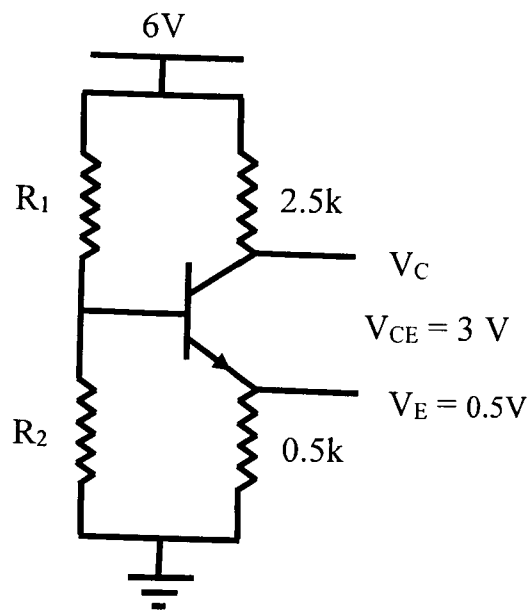
Q.28 For the Fourier series of the following function of period 2π

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

the ratio (to the nearest integer) of the Fourier coefficients of the first and the third harmonic is:

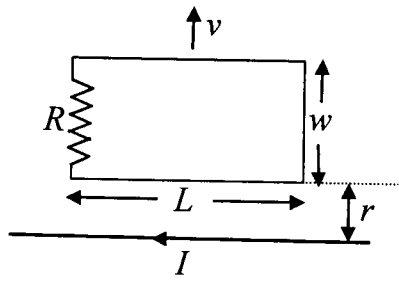
(A) 1 (B) 2 (C) 3 (D) 6

Q.29 An n-p-n transistor is connected in a circuit as shown in the figure. If $I_C = 1$ mA, $\beta = 50$, $V_{BE} = 0.7$ V, and the current through R_2 is 10 I_B, where I_B is the base current. Then the ratio R_1/R_2 is:



(A) 0.375 (B) 0.25 (C) 0.5 (D) 0.275

Q.30 A rectangular loop of dimension L and width w moves with a constant velocity v away from an infinitely long straight wire carrying a current I in the plane of the loop as shown in the figure below. Let R be the resistance of the loop. What is the current in the loop at the instant the near-side is at a distance r from the wire?



(A) $\frac{\mu_0 IL}{2\pi R}\frac{wv}{r[r+2w]}$

(B) $\frac{\mu_0 IL}{2\pi R}\frac{wv}{r[2r+w]}$

(C) $\frac{\mu_0 IL}{2\pi R}\frac{wv}{r[r+w]}$

(D) $\frac{\mu_0 IL}{2\pi R}\frac{wv}{2r[r+w]}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 A photon of frequency ν strikes an electron of mass m initially at rest. After scattering at an angle ϕ, the photon loses half of its energy. If the electron recoils at an angle θ, which of the following is (are) true?

(A) $\cos\phi = \left(1 - \frac{mc^2}{h\nu}\right)$

(B) $\sin\theta = \left(1 - \frac{mc^2}{h\nu}\right)$

(C) The ratio of the magnitudes of momenta of the recoiled electron and scattered photon is $\frac{\sin\phi}{\sin\theta}$.

(D) Change in photon wavelength is $\frac{h}{mc}(1 - 2\cos\phi)$.

Q.32 For an atomic nucleus with atomic number Z and mass number A, which of the following is (are) correct?

(A) Nuclear matter and nuclear charge are distributed identically in the nuclear volume.
(B) Nuclei with $Z > 83$ and $A > 209$ emit α-radiation.
(C) The surface contribution to the binding energy is proportional to $A^{2/3}$.
(D) β-decay occurs when the proton to neutron ratio is large, but not when it is small.

Q.33 Consider a one-dimensional harmonic oscillator of angular frequency ω. If 5 identical particles occupy the energy levels of this oscillator at zero temperature, which of the following statement(s) about their ground state energy E_0 is (are) correct?

(A) If the particles are electrons, $E_0 = \frac{13}{2}\hbar\omega$.
(B) If the particles are protons, $E_0 = \frac{25}{2}\hbar\omega$.
(C) If the particles are spin-less fermions, $E_0 = \frac{25}{2}\hbar\omega$.
(D) If the particles are bosons, $E_0 = \frac{5}{2}\hbar\omega$.

Q.34 For a point dipole of dipole moment $\vec{p} = p\hat{z}$ located at the origin, which of the following is (are) correct?

(A) The electric field at (0, b, 0) is zero.
(B) The work done in moving a charge q from (0, b, 0) to (0, 0, b) is $\frac{qp}{4\pi\epsilon_0 b^2}$.
(C) The electrostatic potential at (b, 0, 0) is zero.
(D) If a charge q is kept at (0, 0, b) it will exert a force of magnitude $\frac{qp}{4\pi\epsilon_0 b^3}$ on the dipole.

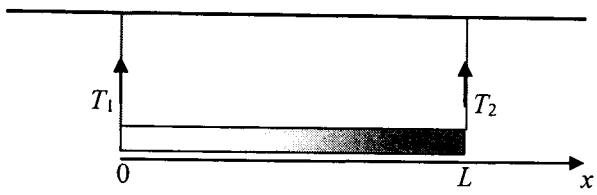
Q.35 A dielectric sphere of radius R has constant polarization $\vec{P} = P_0\hat{z}$ so that the field inside the sphere is $\vec{E}_{in} = -\frac{P_0}{3\epsilon_0}\hat{z}$. Then, which of the following is (are) correct?

(A) The bound surface charge density is $P_o\cos\theta$.
(B) The electric field at a distance r on the z-axis varies as $\frac{1}{r^2}$ for $r >> R$.
(C) The electric potential at a distance $2R$ on the z-axis is $\frac{P_0 R}{12\epsilon_0}$.
(D) The electric field outside is equivalent to that of a dipole at the origin.

Q.36 Consider a circular parallel plate capacitor of radius R with separation d between the plates ($d << R$). The plates are placed symmetrically about the origin. If a sinusoidal voltage $V = V_0 \sin\omega t$ is applied between the plates, which of the following statement(s) is (are) true?

(A) The maximum value of the Poynting vector at $r = R$ is $\frac{V_0^2\epsilon_0\omega R}{4d^2}$.
(B) The average energy per cycle flowing out of the capacitor is zero.
(C) The magnetic field inside the capacitor is constant.
(D) The magnetic field lines inside the capacitor are circular with the curl being independent of r.

Q.37 The linear mass density of a rod of length L varies from one end to the other as $\lambda_o\left(1+\frac{x^2}{L^2}\right)$, where x is the distance from one end with tensions T_1 and T_2 in them (see figure), and λ_o is a constant. The rod is suspended from a ceiling by two massless strings. Then, which of the following statement(s) is (are) correct?



(A) The mass of the rod is $\frac{2\lambda_o L}{3}$.

(B) The center of gravity of the rod is located at $x = \frac{9L}{16}$.

(C) The tension T_1 in the left string is $\frac{7\lambda_o Lg}{12}$.

(D) The tension T_2 in the right string is $\frac{3\lambda_o Lg}{2}$.

Q.38 An object of mass m with non-zero angular momentum ℓ is moving under the influence of gravitational force of a much larger mass (ignore drag). Which of the following statement(s) is (are) correct?

(A) If the total energy of the system is negative, then the orbit is always circular.
(B) The motion of m always occurs in a two-dimensional plane.
(C) If the total energy of the system is 0, then the orbit is a parabola.
(D) If the area of the particle's bound orbit is S, then its time period is $2mS/\ell$.

Q.39 A particle of mass m fixed in space is observed from a frame rotating about its z-axis with angular speed ω. The particle is in the frame's xy plane at a distance R from its origin. If the Coriolis and centrifugal forces on the particle are $\vec{F}_{COR}$ and $\vec{F}_{CFG}$, respectively, then (all the symbols have their standard meaning and refer to the rotating frame),

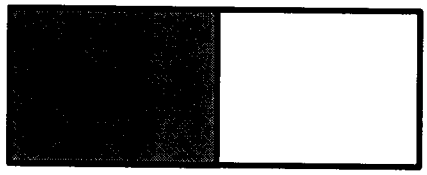
(A) $\vec{F}_{COR} + \vec{F}_{CFG} = 0$

(B) $\vec{F}_{COR} + \vec{F}_{CFG} = -m\omega^2 R\,\hat{r}$

(C) $\vec{F}_{COR} = -2m\omega^2 R\,\hat{r}$

(D) $\vec{F}_{CFG} = -m\omega^2 R\,\hat{r}$

Q.40 An isolated box is divided into two equal compartments by a partition (see figure). One compartment contains a van der Waals gas while the other compartment is empty. The partition between the two compartments is now removed. After the gas has filled the entire box and equilibrium has been achieved, which of the following statement(s) is (are) correct?



(A) Internal energy of the gas has not changed.
(B) Internal energy of the gas has decreased.
(C) Temperature of the gas has increased.
(D) Temperature of the gas has decreased.

SECTION – C

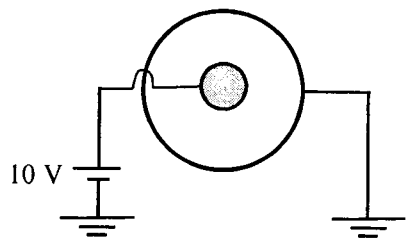
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 An intrinsic semiconductor of band gap 1.25 eV has an electron concentration 10^{10} cm^{-3} at 300 K. Assume that its band gap is independent of temperature and that the electron concentration depends only exponentially on the temperature. If the electron concentration at 200 K is $Y \times 10^{N}$ cm^{-3} ($1 < Y < 10$; N = integer), then the value of N is:

Q.42 A particle of unit mass is moving in a one-dimensional potential $V(x) = x^2 - x^4$. The minimum mechanical energy (in the same units as $V(x)$) above which the motion of the particle cannot be bounded for any given initial condition is:
(Specify your answer to two digits after the decimal point.)

Q.43 In a coaxial cable, the radius of the inner conductor is 2 mm and that of the outer one is 5 mm. The inner conductor is at a potential of 10 V, while the outer conductor is grounded. The value of the potential at a distance of 3.5 mm from the axis is:
(Specify your answer in volts to two digits after the decimal point.)



Q.44 Sand falls on a conveyor belt at the rate of 1.5 kg/s. If the belt is moving with a constant speed of 7 m/s, the power needed to keep the conveyer belt running is:
(Specify your answer in Watts to two digits after the decimal point.)

Q.45 A particle of mass m is placed in a three-dimensional cubic box of side a. What is the degeneracy of its energy level with energy $14\left(\frac{\hbar^2\pi^2}{2ma^2}\right)$?
(Express your answer as an integer.)

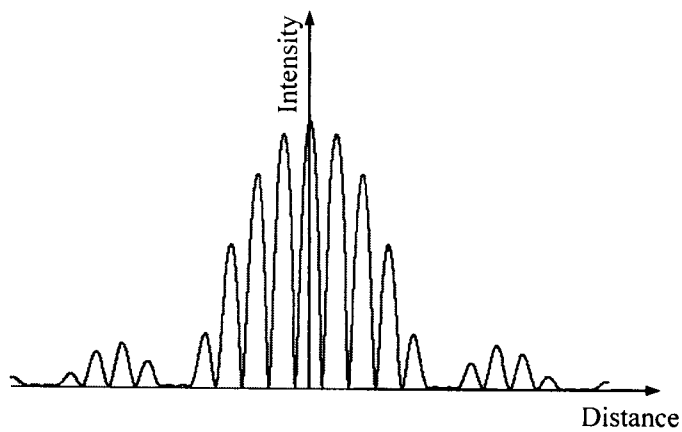
Q.46 The wave number of an electromagnetic wave incident on a metal surface is $(20\pi + 750\, i)$ m^{-1} inside the metal, where $i = \sqrt{-1}$. The skin depth of the wave in the metal is:
(Specify your answer in mm to two digits after the decimal point.)

Q.47 Consider a Carnot engine operating between temperatures of 600 K and 400 K. The engine performs 1000 J of work per cycle. The heat (in Joules) extracted per cycle from the high temperature reservoir is:
(Specify your answer to two digits after the decimal point.)

Q.48 Unpolarized light of intensity I_0 passes through a polarizer P_1. The light coming out of the polarizer falls on a quarter-wave plate with its optical axis at 45° with respect to the polarization axis of P_1 and then passes through another polarizer P_2 with its polarization axis perpendicular to that of P_1. The intensity of the light coming out of P_2 is I. The ratio I_0/I is:
(Specify your answer to two digits after the decimal point.)

Q.49 An anti-reflection film coating of thickness 0.1 μm is to be deposited on a glass plate for normal incidence of light of wavelength 0.5 μm. What should be the refractive index of the film?
(Specify your answer to two digits after the decimal point.)

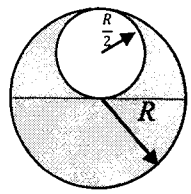
Q.50 Intensity versus distance curve for a double slit diffraction experiment is shown in the figure below. If the width of each of the slits is 0.7 μm, what is the separation between the two slits in micrometers?
(Specify your answer to two digits after the decimal point.)



Q. 51 – Q. 60 carry two marks each.

Q.51 The volume integral of the function $f(r, \theta, \varphi) = r^2 cos\theta$ over the region ($0 \leq r \leq 2$, $0 \leq \theta \leq \pi/3$ and $0 \leq \varphi \leq 2\pi$) is:
(Specify your answer to two digits after the decimal point.)

Q.52 A sphere of radius R has a uniform charge density ρ. A sphere of smaller radius $\frac{R}{2}$ is cut out from the original sphere, as shown in the figure below. The center of the cut out sphere lies at $z = \frac{R}{2}$. After the smaller sphere has been cut out, the magnitude of the electric field at $z = -\frac{R}{2}$ is $\frac{\rho R}{n\epsilon_0}$. The value of the integer n is:



Q.53 In planar polar co-ordinates, an object's position at time t is given as $(r, \theta) = (e^t \text{ m}, \sqrt{8}\, t \text{ rad})$. The magnitude of its acceleration in m/s^2 at $t = 0$ (to the nearest integer) is:

Q.54 Consider two particles moving along the x-axis. In terms of their coordinates x_1 and x_2, their velocities are given as $\frac{dx_1}{dt} = x_2 - x_1$ and $\frac{dx_2}{dt} = x_1 - x_2$, respectively. When they start moving from their initial locations of $x_1(0) = 1$ and $x_2(0) = -1$, the time dependence of both x_1 and x_2 contains a term of the form e^{at}, where a is a constant. The value of a (an integer) is:

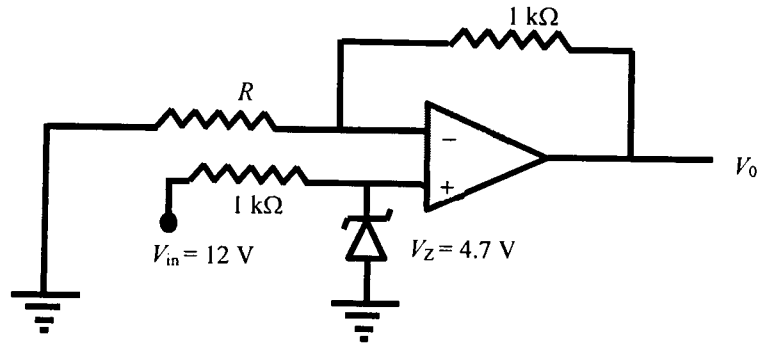
Q.55 For a proton to capture an electron to form a neutron and a neutrino (assumed massless), the electron must have some minimum energy. For such an electron the de Broglie wavelength in picometers is:
(Specify your answer to two digits after the decimal point.)

Q.56 Starting with the equation $TdS = dU + p\,dV$ and using the appropriate Maxwell's relation along with the expression for heat capacity C_p (see useful information), the derivative $\left(\frac{\partial p}{\partial T}\right)_S$ for a substance can be expressed in terms of its specific heat c_p, density ρ, coefficient of volume expansion β and temperature T. For ice, $c_p = 2010$ J/kg-K, $\rho = 10^3$ kg/m^3 and $\beta = 1.6 \times 10^{-4}$/°K. If the value of $\left(\frac{\partial p}{\partial T}\right)_S$ at 270 K is $N \times 10^7$ Pa/K, then the value of N is:
(Specify your answer to two digits after the decimal point.)

Q.57 In an electron microscope, electrons are accelerated through a potential difference of 200 kV. What is the best possible resolution of the microscope?
(Specify your answer in picometers to two digits after the decimal point.)

Q.58 Consider the differential equation $y'' + 2y' + y = 0$. If $y(0) = 0$ and $y'(0) = 1$, then the value of $y(2)$ is:
(Specify your answer to two digits after the decimal point.)

Q.59 An OPAMP is connected in a circuit with a Zener diode as shown in the figure. The value of resistance R in kΩ for obtaining a regulated output $V_0 = 9$ V is:
(Specify your answer to two digits after the decimal point.)



Q.60 At $t = 0$, a particle of mass m having velocity v_0 starts moving through a liquid kept in a horizontal tube and experiences a drag force ($F_d = -k\frac{dx}{dt}$). It covers a distance L before coming to rest. If the times taken to cover the distances $L/2$ and $L/4$ are t_2 and t_4, respectively, then the ratio t_2/t_4 (ignoring gravity) is:
(Specify your answer to two digits after the decimal point.)

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for PH Paper

Paper:**PHYSICS**	Code:**PH**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	**KEY**	**Q. No.**	**KEY**	**Q. No.**	**KEYS**	**Q. No.**	**KEY RANGE**	**Q. No.**	**KEY RANGE**
01	B	**16**	B	**31**	A, C	**41**	4 to 4	**56**	4.60 to 4.70
02	C	**17**	D	**32**	B, C or A, B, C	**42**	0.24 to 0.26	**57**	2.30 to 3.20
03	A	**18**	D	**33**	A, C, D	**43**	3.79 to 3.99	**58**	0.25 to 0.29
04	C	**19**	D	**34**	B, C	**44**	73.01 to 73.99	**59**	1.05 to 1.15
05	C	**20**	A	**35**	A, C, D	**45**	6 to 6	**60**	2.35 to 2.50
06	C	**21**	D	**36**	A, B, D	**46**	1.30 to 1.36		
07	C	**22**	A	**37**	B, C	**47**	2995.00 to 3005.00		
08	A	**23**	B	**38**	B, C, D	**48**	4.00 to 4.00		
09	A	**24**	B	**39**	B, C	**49**	1.25 to 1.25		
10	B	**25**	A	**40**	A, D	**50**	3.48 to 3.52		
11	C	**26**	A			**51**	15.00 to 15.15		
12	B	**27**	A			**52**	8 to 8		
13	C or D	**28**	C			**53**	9 to 9		
14	D	**29**	Marks to all			**54**	-2 to -2		
15	D	**30**	C			**55**	1.00 to 1.10		