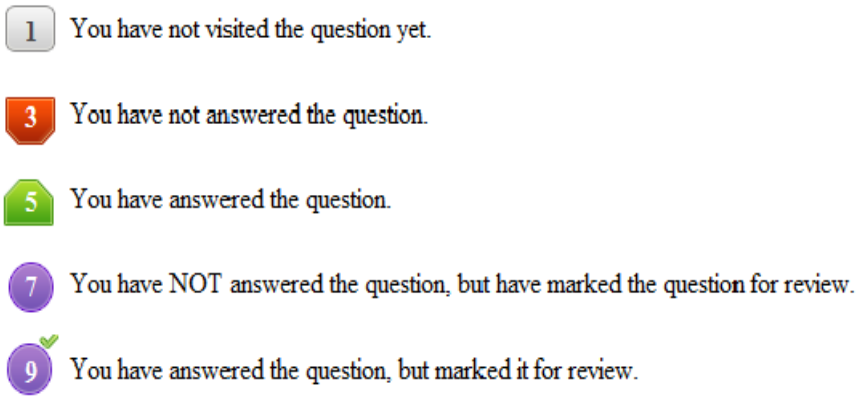
IIT JAM 2015 Question Paper (All Subjects)

IIT Joint Admission Test for Masters

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

BT: Biotechnology

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following most accurately describes the process of natural selection?

(A) Selection of one species over a competing species
(B) Selection of individuals that successfully defend themselves against enemies
(C) Selection of individuals that produce more than the average number of offspring
(D) Selection of individuals that are more attractive to the opposite sex

Q.2 Identify the statement that is **TRUE** of operons.

(A) Fine regulation of the expression of individual genes are made possible by operons
(B) Only genes involved in carbohydrate metabolism are present in operons
(C) Feedback inhibition of the biosynthesis of multiple enzymes by a single small molecule is made possible by operons
(D) In the case of inducible operons, the inducer binds to the operator

Q.3 Signaling pathways usually comprise of several intermediate steps that are arranged in the form of a cascade. What is the primary outcome of such an arrangement?

(A) Specificity of signal transduction
(B) Specificity of the cellular response
(C) Amplification of the cellular response
(D) Fine-tuning of the cellular response

Q.4 Which among the following contain(s) oxygen-rich blood in the human vascular system?
I. Right ventricle
II. Aorta
III. Pulmonary vein

(A) I only (B) I and II only
(C) I, II and III (D) II and III only

Q.5 Choose the option that lists the correct sequence of steps involved in gene therapy.
P. Injection of expression vector into patient
Q. Wild-type gene is inserted into expression vector
R. Wild-type gene is isolated and cloned
S. Wild-type gene is transcribed and translated in the patient

(A) Q, S, P, R (B) Q, P, R, S (C) R, P, Q, S (D) R, Q, P, S

Q.6 Cephalin, a biological surfactant, is

(A) choline phosphoglyceride
(B) ethanolamine phosphoglyceride
(C) glycosphingolipid
(D) sphingolipid

Q.7 The major product(s) produced by gas phase UV irradiation of 2-pentanone is (are)

(A) acetone and ethene
(B) acetic acid and propionic acid
(C) 2-pentanol
(D) cyclopentane

Q.8 If a projectile lifts off from the surface of the Earth with a speed of 11.2 km.s^{-1}, then it can escape from the Earth's gravitational field completely. This is called the escape velocity. If the radius of the Earth were 2 times larger and the mass 8 times larger, then the escape velocity (in km.s^{-1}) would be

(A) 5.6 (B) 11.2 (C) 22.4 (D) 44.8

Q.9 The speed of an electron (v), in the lowest energy orbit in the Bohr model of the Hydrogen atom divided by the speed of light in vacuum (c), is given by (where m is the mass of the electron, M is the mass of the proton, ε_0 is the permittivity of free space, a_0 is the Bohr radius)

(A) $\frac{v}{c} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}$

(B) $\frac{v}{c} = \frac{e^4}{32\pi^2\,\varepsilon_0^2\hbar^2c^2}$

(C) $\frac{v}{c} = \frac{m}{M}$

(D) $\frac{v}{c} = \frac{\hbar}{mca_0}$

Q.10 Let $\mathbb{R}$ be the set of all real numbers. Consider the sets $P = \{x \in \mathbb{R} : (x-1)(x^2+1) = 0\}$, $Q = \{x \in \mathbb{R} : x^2 - 9x + 2 = 0\}$ and $S = \{x \in \mathbb{R} : x = 5y \text{ for some } y \in \mathbb{R}\}$. Then the set $(P \cap S) \cup Q$ contains

(A) exactly two elements (B) exactly three elements
(C) exactly four elements (D) infinitely many elements

Q. 11 – Q. 30 carry two marks each.

Q.11 In a population growing according to the logistic growth model

(A) individuals reproduce according to their physiological capacity
(B) the per capita rate of increase approaches zero as the population nears the carrying capacity
(C) the number of births is always more than the number of deaths
(D) the birth-to-death ratio is NOT influenced by the carrying capacity

Q.12 Which part of the genomic DNA contains the sequence corresponding to the 5' untranslated region (5' UTR)?

(A) Exon (B) Intron
(C) Upstream of the transcription start site (D) Upstream of the promoter

Q.13 Which one of the following is **NOT TRUE** of RNA polymerase II?

(A) It requires a primer to initiate the transcription
(B) It makes an RNA copy of only one strand of a double-stranded DNA at any given time
(C) It does not synthesize rRNA and tRNA
(D) It catalyzes the polymerization of RNA only in the 5'→3' direction

Q.14 Choose the option that shows the correct pairing of the cellular components with their corresponding function.

P. Dynein	i. Movement of organelles
Q. Desmosome	ii. Membrane vesicle
R. Endosome	iii. Beating of flagella
S. Kinesin	iv. Attachment of cells together

(A) P-iv, Q-i, R-ii, S-iii
(B) P-iii, Q-iv, R-ii, S-i
(C) P-iii, Q-i, R-ii, S-iv
(D) P-ii, Q-i, R-iv, S-iii

Q.15 An enzyme shows highest activity in the pH range 2.0 - 3.0. At pH 4.0 and pH 7.0, the enzyme exhibits 50% and 1%, respectively, of its highest activity. Which of the following states of an amino acid residue in the catalytic site is most responsible for its activity profile?

(A) A protonated Asp
(B) A deprotonated Asp
(C) A deprotonated Asn
(D) A protonated Asn

Q.16 The specific productivity (q_p) of cellulase production by *Aspergillus niger* follows a linear relationship with the specific growth rate (μ) and is of the form $q_p = \alpha\mu + \beta$, where α and β are constants. Assuming that the values of α and β are 0.006 and 25, respectively, which type of product formation kinetics is **TRUE**?

(A) Growth-dependent kinetics
(B) Non-growth-dependent kinetics
(C) Both growth- and non-growth-dependent kinetics
(D) Inhibition kinetics

Q.17 Choose the option that shows the correct pairing of the diseases with their corresponding causative organisms.

P. Chagas disease	i. *Trypanosoma gambiense*
Q. Sleeping sickness	ii. *Plasmodium falciparum*
R. Malaria	iii. *Yersinia pestis*
S. Plague	iv. *Trypanosoma cruzi*

(A) P-iv, Q-i, R-iii, S-ii
(B) P-iv, Q-i, R-ii, S-iii
(C) P-i, Q-iv, R-ii, S-iii
(D) P-i, Q-iv, R-iii, S-ii

Q.18 Choose the option that shows the correct pairing of the products with their corresponding microorganisms.

P. Citric acid	i. *Micromonospora purpurea*
Q. Polyhydroxyalkonates	ii. *Zymomonas mobilis*
R. Gentamycin	iii. *Aspergillus niger*
S. Ethanol	iv. *Ralstonia eutropha*

(A) P-ii, Q-iii, R-i, S-iv
(B) P-iv, Q-ii, R-iii, S-i
(C) P-iii, Q-iv, R-i, S-ii
(D) P-iii, Q-i, R-iv, S-ii

Q.19 Determine the correctness or otherwise of the following Assertion [a] and Reason [r].
Assertion [a]. B cells secrete antibodies against a virus while cytotoxic T cells kill virus-infected cells.
Reason [r]. B cells confer active immunity while cytotoxic T cells confer passive immunity.

(A) [a] and [r] are true and [r] is the correct reason for [a]
(B) [a] and [r] are true but [r] is not the correct reason for [a]
(C) [a] is true but [r] is false
(D) [a] false but [r] is true

Q.20 Which one of the following options shows the correct pairing of the enzyme with its corresponding application?

P. Papain	i. Gluten complex reduction
Q. Bromelain	ii. Immuno assay marker enzyme
R. Peroxidase	iii. Maltose syrup preparation
S. β-amylase	iv. Meat tenderizer

(A) P-iv, Q-i, R-ii, S-iii
(B) P-iv, Q-iii, R-i, S-ii
(C) P-iv, Q-i, R-iii, S-ii
(D) P-i, Q-ii, R-iv, S-iii

Q.21 The rate constant for the reaction O (g) + O_3 (g) → $2O_2$ (g) is 8.0×10^{-15} cm^3 $molecule^{-1}$ s^{-1}. The rate constant in dm^3 mol^{-1} s^{-1}, would be

(A) 4.8×10^{-6}
(B) 4.8×10^{6}
(C) 4.8×10^{-9}
(D) 8.0×10^{6}

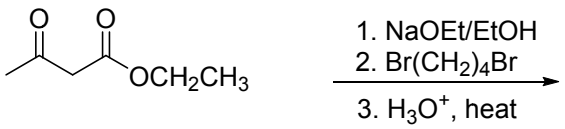
Q.22 The UV spectrum of 2-butanone and the UV spectrum of methyl vinyl ketone (MVK) are independently recorded and compared. Among the various λ_{max}, 185 nm, 219 nm, 277 nm and 324 nm, the absorption at λ_{max}= 324 nm is due to

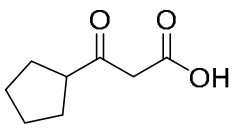
(A) n→π* transition in 2-butanone
(B) n→π* transition in MVK
(C) π→π* transition in 2-butanone
(D) n→σ* transition in 2-butanone

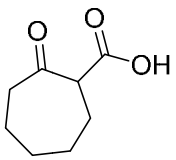
Q.23 The compound *meso* 2,3-dibromobutane is obtained by

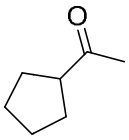
(A) electrophilic addition of HBr to (*E*)-1-bromobut-2-ene
(B) electrophilic addition of Br_2 to (*E*)-2-butene
(C) electrophilic addition of Br_2 to (*Z*)-2-butene
(D) nucleophilic addition of Br_2 to (*Z*)-1-bromobut-2-ene

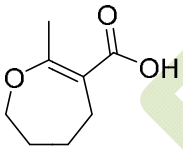
Q.24 The major product in the following reaction is



(A) 

(B) 

(C) 

(D) 

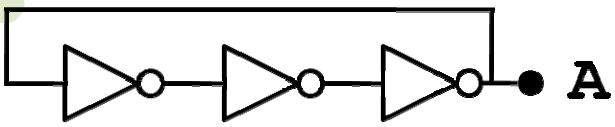
Q.25 An archaeological sample (remains of an animal) containing ^{14}C isotope of Carbon is found to give 10 beta decays per minute per gram of Carbon. It is known that the natural abundance of ^{14}C in organic matter that is in equilibrium with the atmosphere today will give 15 beta decays per minute per gram of Carbon. The half life of ^{14}C is known to be 5730 years. The estimated age of the sample (in years) is

(A) 3010 (B) 3350 (C) 3500 (D) 3800

Q.26 The minimum light intensity that the human eye can perceive is 10^{-10} Wm^{-2}. The area of the opening of our eye (the pupil) is approximately 0.4 cm^{-2}. Consider yellow light with wavelength λ=600 nm. The number of photons incident on the retina per second at the minimum intensity for the eye to respond is

(A) 1.5×10^3 (B) 5×10^3 (C) 8×10^3 (D) 1.2×10^4

Q.27 Three NOT gates are connected in series and the output of the last gate is fed back to the input of the first one as shown in the figure. Each gate has a propagation delay of T_d =1 nano second, which means that the gate requires 1 nano second to change the output after the signal arrives at the input. What is the expected output at point A?



(A) Sine wave with a frequency of 666 MHz (B) Square wave with a frequency of 666 MHz
(C) Random white noise (D) Square wave with a frequency of 333 MHz

Q.28 Let ^nC_r denote the number $\frac{n!}{r!(n-r)!}$. Then for $n = 100$, the sum of the series

$1 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots \cdots + (-1)^r {}^nC_r + \cdots \cdots + (-1)^n {}^nC_n$ is

(A) 0 (B) 1 (C) 2 (D) 1024

Q.29 The lengths of two sides of a triangle are 2 units and 3 units and the angle included by these two sides is 60°. The length of the third side of the triangle will be

(A) $\sqrt{5}$ units (B) $\sqrt{7}$ units (C) 4 units (D) 5 units

Q.30 If A and B are two skew-symmetric matrices, the matrix $AB + BA$ must be

(A) skew-symmetric (B) symmetric
(C) invertible (D) NOT invertible

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1 In a large wild flower population, assume that no new mutations occur and that no natural selection operates. What factor(s) will affect the frequency of a genotype in this population?

(A) Non-random mating
(B) Gene flow
(C) Out-breeding within the population
(D) Invasion of a new pathogen that kills a large number of individuals in the population

Q.2 What is (are) the difference(s) between microtubules and microfilaments?

(A) Microtubules are made up of tubulin and microfilaments are made up of intermediate filaments
(B) Microtubules are important for compression resistance and microfilaments bear tension
(C) Microtubules are important for the functions of cilia and flagella and the microfilaments are important for cytoplasmic streaming
(D) Microtubules – muscle contraction; microfilaments – ciliary movement

Q.3 N and N_0 represent the number of viable cells at time 't' during sterilization and at the start of sterilization (t=0), respectively. Assuming that cell death follows first order kinetics and that k is the death rate constant, which of the following relationship(s) is/are correct?

(A) $N = N_0 e^{kt}$ (B) $-\ln (N/N_0) = k t$ (C) $N = N_0 k t^2$ (D) $N - N_0 = k t$

Q.4 Which of the following statements about antigen-antibody (Ag-Ab) complexes is (are) **TRUE**?

(A) Hydrogen bonds and van der Waals forces participate in Ag-Ab interactions
(B) Ionic bonds and hydrophobic bonds participate in Ag-Ab interactions
(C) The combined strengths of all interactions between a single antigen binding site on an antibody and a single epitope is called avidity
(D) Antibody elicited by one antigen can cross react with an unrelated antigen

Q.5 The superoxide ion, O_2^-, is produced by the reaction $K+O_2 \rightarrow KO_2$. The correct statement(s) pertaining to oxygen and superoxide ion is (are):

(A) oxygen is paramagnetic and has two unpaired electrons
(B) the bond order in oxygen is 2
(C) the bond order in superoxide is 1.5
(D) the superoxide ion is not paramagnetic

Q.6 Among the following compounds, which of these will show two singlets in their ^{1}H-NMR spectrum?

(A) 1,4-Dichlorobenzene (B) 1,2-Dichlorobenzene
(C) Dimethoxymethane (D) Methylacetate

Q.7 Among the following pairs of co-ordination compounds, the pair(s) which represent(s) a case of "ionization isomerism" is (are):

(A) $[Pt(en)_2Cl_2]Br_2$ and $[Pt(en)_2Br_2]Cl_2$

(B) $[Cr(NH_3)_4ClBr]NO_2$ and $[Cr(NH_3)_4ClNO_2]Br$

(C) $[Co(NH_3)_6][Cr(CN)_6]$ and $[Cr(NH_3)_6][Co(CN)_6]$

(D) $[Co(NH_3)_5Br]SO_4$ and $[Co(NH_3)_5\ SO_4]Br$

Q.8 The "strong nuclear force" holds the protons and neutrons (nucleons) together in the nucleus of an atom. It is found that the binding energy per nucleon (for the nucleus of an element) when plotted against the mass number (A) of that element changes very little for $30 < A < 150$. The binding energy is lower for $A << 30$ or $A >> 150$. This leads us to conclude that

(A) the strong nuclear force must oscillate with distance with a periodicity approximately same as the size of a proton or neutron
(B) the fusion of two elements, both with $A << 30$ or fission of an element $A >> 150$ may release energy
(C) the strong nuclear force changes very slowly with distance (i.e. It is long ranged on the scale of the size of nucleus)
(D) the strong nuclear force goes to zero very rapidly with distance (i.e. It is short ranged on the scale of the size of nucleus)

Q.9 Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$. Then

(A) $\vec{a}$ and $\vec{b}$ are parallel to each other
(B) $\vec{a}$ and $\vec{b}$ are perpendicular to each other
(C) $\vec{a}$ is NOT a scalar multiple of $\vec{b}$
(D) $\vec{a} \times \vec{b} = \vec{0}$

Q.10 Let $\mathbb{N}$ be the set of all natural numbers. Consider the relation R on $\mathbb{N}$ given by $R = \{(m,n) : m-n \text{ is divisible by } 2\}$. Then

(A) R is symmetric and transitive
(B) R is symmetric but NOT transitive
(C) R is reflexive but NOT symmetric
(D) R is reflexive and transitive

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

Q.1 The pH of a 0.1 M solution of monosodium succinate ($pKa_1 = 4.19$ and $pKa_2 = 5.57$) is ____

Q.2 The deactivation rate constant of an enzyme is 0.346 h^{-1}. Assuming that the deactivation process follows first order kinetics, the half life of the enzyme in minutes is _____

Q.3 An enzyme preparation containing 10 mg/ml protein shows a specific activity of 50 U/mg. The initial velocity of reaction in a standard 1 ml reaction mixture containing 10 µl of the preparation in $\mu mol.ml^{-1}.min^{-1}$ is _____

Q.4 The number of peaks in the ^{13}C-NMR spectrum of $CDCl_3$ is ____

Q.5 The number of phosphorous-hydrogen bonds in H_3PO_2 is ____

Q.6 A man weighing 70 kg stands on a weighing scale which is placed in an elevator. The elevator is moving up towards its destination floor with a velocity of 1.0 ms^{-1}. As it approaches the destination floor it starts slowing down, such that it comes to rest in 2 seconds. Assuming the acceleration due to gravity, g = 9.8 ms^{-2}, the reading of the weighing scale just *before* the elevator comes to rest is ____

Q.7 A 50-metre tall antenna transmits at 107 MHz (one of the FM radio broadcast frequencies). Calculate the maximum distance from the antenna at which the transmitted signal can be heard. Ignore atmospheric attenuation and give your answer correct to the nearest kilometer only. You are given that the radius of the earth is 6400 km.

Q.8 The total number of mappings from the set $\{1,2\}$ to the set $\{3,4,5,6,7\}$ is _____

Q.9 The value of the complex number $(1+i)^{150}+(1-i)^{150}$ is _____

Q.10 Let the function $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}\dfrac{\tan x}{x} & \text{if } x\neq 0;\\ k & \text{if } x=0.\end{cases}$$

If f is continuous at $x=0$, then the value of k must be equal to _____

Q. 11 – Q. 20 carry two marks each.

Q.11 Assume that the gene ***UNC*** is essential for the coordinated movement of a nematode and that the wild-type allele ***U*** is dominant over the mutant allele ***u***. Similarly, the wild-type allele ***D*** of another gene ***DPY***, which is responsible for the normal body length, is dominant over the mutant allele ***d***. Assume the ***UNC*** and ***DPY*** are on two different chromosomes. If a female of genotype ***UUDD*** mates with a male of the genotype ***uudd***, the percentage of the F2 progeny that will display uncoordinated movement but will have normal body length is _____

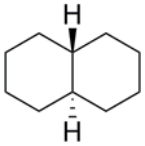
Q.12 The pH of gastric juice in the stomach is 2.0. However the pH inside the cells that line the stomach is 7.0. For transport of protons from inside the cell to the stomach, the free energy change (ΔG) in $kJmol^{-1}$at 37°C is _____
[Assume Universal Gas constant R = 8.314 $Jmol^{-1}K^{-1}$]

Q.13 A solution containing NAD^+ and NADH has an optical density of 0.233 at 340 nm and 1.000 at 260 nm. While this solution absorbs at 260 nm, NADH alone absorbs at 340 nm. All measurements are carried out in a 1-cm cuvette. Given the extinction coefficients (ε) (see the table below), the concentration of the oxidized form of the cofactor in μM is ____

Compound	ε (M^{-1} cm^{-1})	
	260 nm	340 nm
NAD^+	18000	0
NADH	15000	6220

Q.14 *Drosophila melanogaster* is a diploid organism having 8 chromosomes. The number of combinations of chromosomes which are possible in its gametes is ____

Q.15 The number of equatorial hydrogens in the following structure is _____



Q.16 The axis of rotation of the earth makes an angle of 66.5° with the plane containing the Earth's orbit around the Sun (called the plane of the ecliptic). If this angle were 50°, then the area of the Earth's surface from which a "midnight Sun" (24 hour daylight) can be observed would change. The ratio of the new area to the previous area is _____

Q.17 A nuclear power plant generates 1000 Megawatts (MW) of electrical power and used half of its fuel supply in 5 years. The reactor uses ^{235}U with 33% efficiency for the conversion of heat released by nuclear fission to electrical power. Each atom of ^{235}U releases 200 MeV of energy. How many tons of ^{235}U did the reactor start with? (1 ton = 1000 kg; Avogadro number = 6.023×10^{23} mol^{-1})

Q.18 A bat emitting ultrasound at 50 kHz is flying directly towards a solid wall with a speed of 3 ms^{-1}. If the speed of sound in air is 330 ms^{-1}, the frequency of the reflected signal (in kHz) heard by the bat will be _____

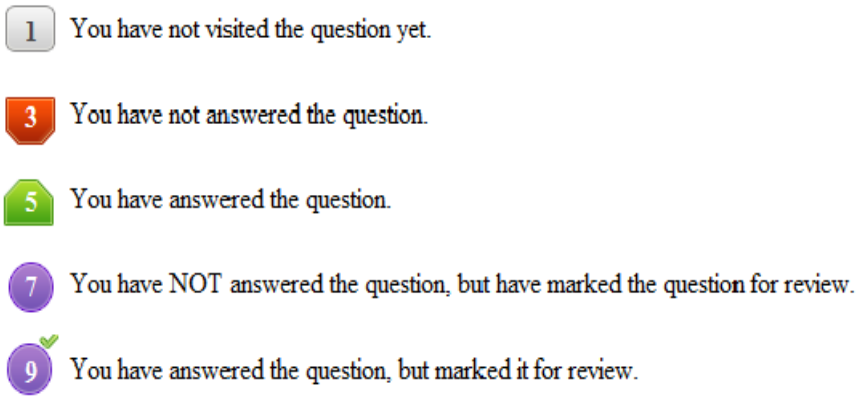
Q.19 A circle is given by the equation $2x^2 + 2y^2 + 8x - 20y + 10 = 0$. The area of a square whose side equals the radius of the circle is _____

Q.20 The value of the integral $\int_0^{\pi} |\cos x| \, dx$ is _____

END OF THE QUESTION PAPER

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

CY: Chemistry

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The first row transition metal complexes having tetrahedral geometry are high-spin due to

(A) $\Delta_t > P$ (B) $\Delta_t < P$ (C) $\Delta_t = P$ (D) $\Delta_t > \Delta_o$

Q.2 The species responsible for the superacidity of SbF_5-HSO_3F system is

(A) HSO_3F (B) SbF_5 (C) HF (D) $H_2SO_3F^+$

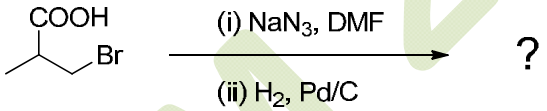
Q.3 A filter paper moistened with cadmium acetate solution turns yellow upon exposure to H_2S. The transition responsible for the yellow colour is

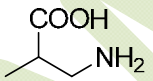
(A) d−d
(B) metal-to-ligand charge transfer
(C) ligand-to-metal charge transfer
(D) σ−σ*

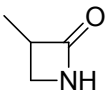
Q.4 Low-spin iron(III) center is present in

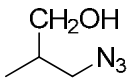
(A) deoxy form of hemoglobin
(B) oxy form of hemoglobin
(C) hemocyanin
(D) carbonic anhydrase

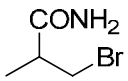
Q.5 The major product formed in the following reaction is



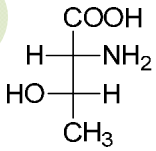
(A) 

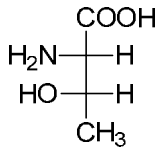
(B) 

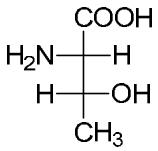
(C) 

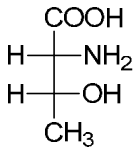
(D) 

Q.6 The structure of (2*S*,3*R*)-2-amino-3-hydroxy butanoic acid is

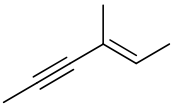
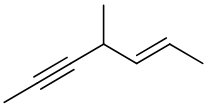
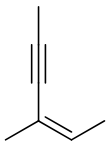
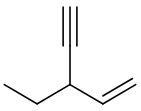
(A) 

(B) 

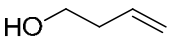
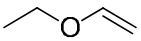
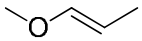
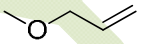
(C) 

(D) 

Q.7 The ene-yne that produces a chiral compound upon treatment with Lindlar's catalyst is

(A)  (B)  (C)  (D) 

Q.8 An organic compound **P** (C_4H_8O) is positive to Bayer's test, but inert to sodium metal. On treatment with conc. HCl, **P** gives CH_3CH_2Cl and CH_3CHO. The structure of **P** is

(A)  (B)  (C)  (D) 

Q.9 Which one of the following is an identity matrix?

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Q.10 The intermolecular van der Waals potential is inversely proportional to r^6. The corresponding force is proportional to

(A) $\frac{1}{r^5}$ (B) $\frac{1}{r^6}$ (C) $\frac{1}{r^7}$ (D) $\frac{1}{r^{12}}$

Q. 11 – Q. 30 carry two marks each.

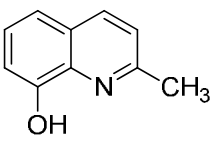
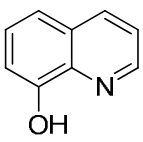
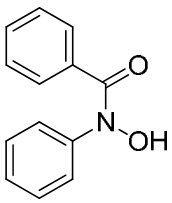
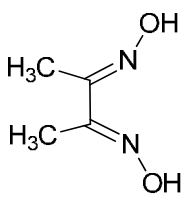
Q.11 The normal spinel among the following mixed metal-oxides is

(A) $CoFe_2O_4$ (B) $NiFe_2O_4$ (C) $CuFe_2O_4$ (D) $ZnFe_2O_4$

Q.12 The ground state term for a free ion with $3d^7$ configuration is

(A) $^4F_{3/2}$ (B) $^4F_{9/2}$ (C) $^4F_{1/2}$ (D) $^4F_{5/2}$

Q.13 The reagent 'oxine' commonly used in analytical chemistry is

(A)  (B)  (C)  (D) 

Q.14 The species having trigonal pyramidal shape is

(A) NO_3^- (B) CO_3^{2-} (C) BrF_3 (D) SO_3^{2-}

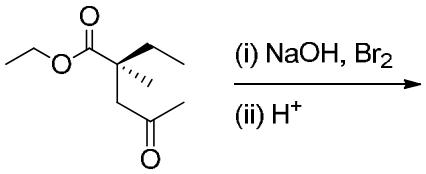
Q.15 The correct statement about ionization potential (IP) is

(A) Non-metallic character of an element decreases as the IP increases
(B) IP decreases down the group in the periodic table
(C) Second IP of Ca is larger than second IP of K
(D) IP decreases on going from left to right in the periodic table

Q.16 The Volhard method is used for the estimation of

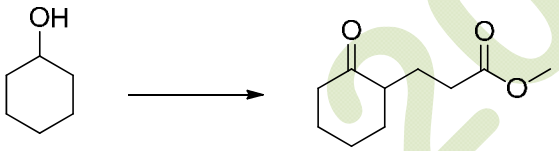
(A) cyanide ion by titration with silver nitrate
(B) silver ion directly
(C) oxygen in water
(D) glucose in blood

Q.17 The set of products formed in the following reaction is



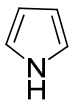
(A) $CHBr_3$ and a racemic acid
(B) $CHBr_3$ and a chiral acid
(C) $CHBr_3$ and a racemic ester
(D) CH_2Br_2 and a chiral ester

Q.18 The correct set of reagents required for the following transformation is

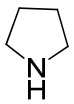


(A) (i) CrO_3; (ii) acrylonitrile; (iii) H_3O^+
(B) (i) O_2; (ii) methyl acrylate
(C) (i) CrO_3; (ii) NaOMe/MeOH, methyl acrylate; (iii) H_3O^+
(D) (i) H_2O; (ii) methyl acrylate

Q.19 The correct order of the p*K*a values for the conjugate acids of heterocyclic compounds given below is

(I) 

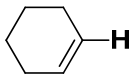
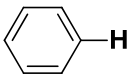
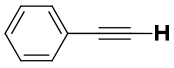
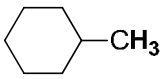
(II) 

(III) 

(IV) 

(A) **II > III > I > IV** (B) **IV > II > III > I** (C) **III > II > IV > I** (D) **III > IV > II > I**

Q.20 The correct order of the ^1H NMR chemical shift values (δ) for the indicated hydrogens (in bold) in the following compounds is

(I)  **(II)**  **(III)**  **(IV)** 

(A) **I > II > III > IV** (B) **II > I > III > IV** (C) **III > II > I > IV** (D) **II > III > IV > I**

Q.21 Which of the following statements are correct for S_NAr reaction?

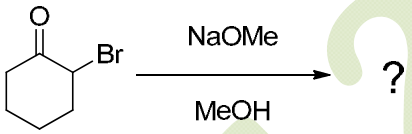
(i) Follows second order kinetics
(ii) $K_H/K_D > 1$
(iii) Involves carbanion-type intermediate
(iv) Involves two transition states

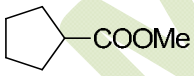
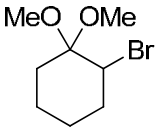
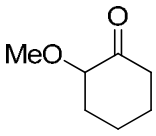
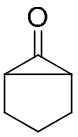
(A) (i) and (ii) only (B) (ii) and (iii) only
(C) (i), (iii) and (iv) only (D) (i) and (iii) only

Q.22 According to the equipartition principle, the predicted high temperature limiting value of the molar heat capacity at constant volume for C_2H_2 is

(A) 5.5R (B) 6.0R (C) 9.0R (D) 9.5R

Q.23 The major product formed in the following reaction is



(A)  (B)  (C)  (D) 

Q.24 At 25 °C, the solubility product (K_{sp}) of CaF_2 in water is 3.2×10^{-11}. The solubility (in mole per kg of water) of the salt at the same temperature (ignore ion pairing) is

(A) 4.0×10^{-6} (B) 3.2×10^{-4} (C) 2.5×10^{-4} (D) 2.0×10^{-4}

Q.25 For an isothermal free expansion of an ideal gas into vacuum, which one of the following set of values is correct?

(A) $\Delta U = 0, q > 0, w < 0$ (B) $\Delta U > 0, q > 0, w = 0$
(C) $\Delta U = 0, q = 0, w = 0$ (D) $\Delta U < 0, q = 0, w < 0$

Q.26 The kinetics of the reaction $2N_2O_5 \rightarrow 4NO_2 + O_2$ in liquid bromine medium was measured independently for three different initial concentrations of N_2O_5: 0.11, 0.07 and 0.05 mol L^{-1}. The half-life of the reaction was found to be 4.5 hours for all these concentrations. The order of the reaction is

(A) 0 (B) 1 (C) 2 (D) 0.5

Q.27 The concentration of K^+ ion inside a biological cell is 20 times higher than outside. The magnitude of potential difference between the two sides is [Given: 2.303 RT/F = 59 mV]

(A) 0 mV (B) 26 mV (C) 77 mV (D) 177 mV

Q.28 The correct order of the fundamental vibrational frequencies of the following diatomic molecules is

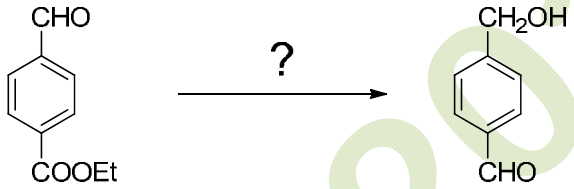
(A) $^1H^{35}Cl > {}^1H^{37}Cl > {}^2D^{35}Cl$

(B) $^2D^{35}Cl > {}^1H^{37}Cl > {}^1H^{35}Cl$

(C) $^1H^{37}Cl > {}^1H^{35}Cl > {}^2D^{35}Cl$

(D) $^1H^{37}Cl > {}^2D^{35}Cl > {}^1H^{35}Cl$

Q.29 Identify the correct reagents required for the following transformation



(A) (i) $NaBH_4$; (ii) H_3O^+

(B) (i) $LiAlH_4$; (ii) H_3O^+

(C) (i) $HOCH_2CH_2OH$, H^+; (ii) $LiAlH_4$; (iii) H_3O^+

(D) (i) $HSCH_2CH_2SH$, H^+; (ii) $LiAlH_4$; (iii) H_3O^+

Q.30 The complex that is expected to show orbital contribution to the overall magnetic moment is

(A) $[Cr(CN)_6]^{3-}$ (B) $[Co(H_2O)_6]^{2+}$ (C) $[Ni(en)_3]^{2+}$ (D) $[Cu(NH_3)_6]^{2+}$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1 The common feature(s) of Rb^+, Kr and Br^- is/are that they

(A) have same number of valence electrons
(B) have same magnitude of effective nuclear charge
(C) have same magnitude of first ionization potential
(D) are isoelectronic species

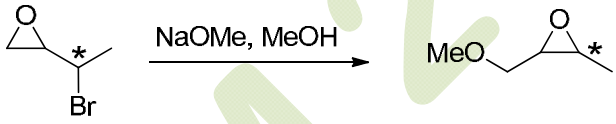
Q.2 The characteristics of the blue solution of sodium in liquid ammonia is/are

(A) diamagnetic
(B) paramagnetic
(C) reducing in nature
(D) conducts electricity

Q.3 Which of the following statement(s) is/are true about the transition metal-alkene complexes?

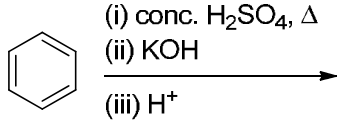
(A) Back-bonding weakens the double bond of the alkene
(B) σ-Bonding and back-bonding synergistically strengthen metal-alkene interaction
(C) Electron-withdrawing substituents on alkene reduce back-bonding
(D) π-Acidic co-ligands on metal strengthen back-bonding

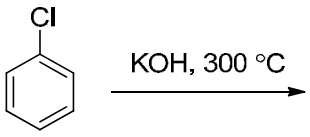
Q.4 Which of the following statement(s) is/are true about the reaction given below?

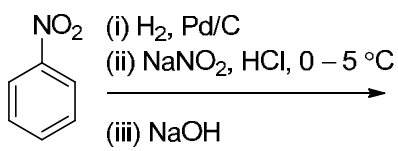


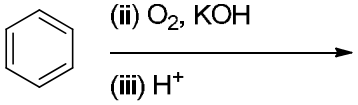
(A) It involves a carbocation intermediate
(B) Rearrangement is due to S_N1 reaction mechanism
(C) It proceeds via a concerted S_N2 pathway
(D) It involves neighbouring group participation

Q.5 The reaction(s) which give(s) phenol is/are

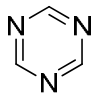
(A) 

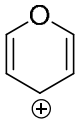
(B) 

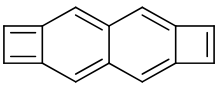
(C) 

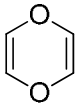
(D) 

Q.6 Which of the following species is/are aromatic in nature?

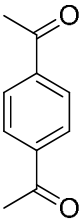
(A) 

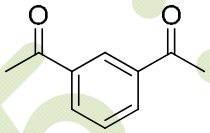
(B) 

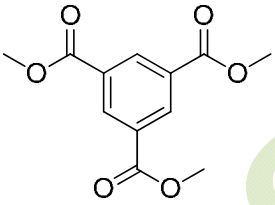
(C) 

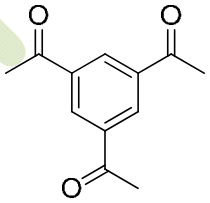
(D) 

Q.7 Which of the following compound(s) show(s) only two signals in ^{1}H NMR and a strong IR band at ~1690 cm^{-1}?

(A) 

(B) 

(C) 

(D) 

Q.8 If $\hat{x} = x \times$ and $\hat{p}_x = \frac{h}{2\pi i}\frac{d}{dx}$, then the value(s) of $\hat{p}_x\hat{x} - \hat{x}\hat{p}_x$ is/are

(A) $\frac{\hbar}{i}$ (B) $-i\hbar$ (C) 0 (D) $\frac{i}{\hbar}$

Q.9 At what angle(s) of incidence, X-rays of wavelength 5.0 Å will produce diffracted beam from the (110) planes in a simple cubic lattice with a = 10 Å?

(A) 6.8° (B) 10.2° (C) 20.7° (D) 45.0°

Q.10 Which of the following thermodynamic relation(s) is/are correct?

(A) $\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial P}{\partial S}\right)_V$ (B) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$

(C) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ (D) $\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

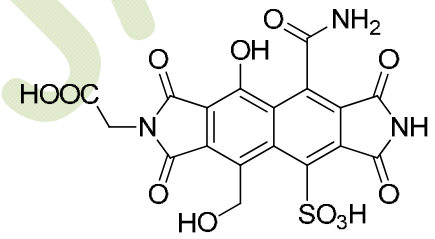
Q.1 Given that the crystal field stabilization energy for $[Co(H_2O)_6]^{2+}$ is 7360 cm^{-1}, the calculated value of Δ_o in kJ mol^{-1} is _____

Q.2 The amount (in grams) of potassium dichromate (MW = 294) present in 75 mL of 0.16 M aqueous solution is _____

Q.3 Given that the expected spin-only magnetic moment for $(Et_4N)_2[NiCl_4]$ is 2.83 μ_B, the total number of unpaired electrons in this complex is _____

Q.4 The pK_a values of lysine are 2.18, 8.95 and 10.79. The isoelectric point of lysine is _____

Q.5 The amount (in grams) of NaOH (MW = 40) required for complete neutralization of one mole of the following compound is _____



Q.6 In the given list, the total number of compounds that form a clear homogenous solution on treatment with cold dilute H_2SO_4 is _____

1-propyne
cyclohexanone
cyclohexane
1-propene
propan-1-amine
propoxypropane
tetrahydropyran
ethyl butanoate
pyridine

Q.7 In the gas phase, the ratio of excluded volume to molecular volume for a spherical molecule is _____

Q.8 The pK_a values of H_3PO_4 are 2.12, 7.21 and 12.67. The pH of a phosphate buffer containing 0.2 M NaH_2PO_4 and 0.1 M Na_2HPO_4 is _____

Q.9 For the reaction, $2SO_2 + O_2 \leftrightharpoons 2SO_3$, the equilibrium constant K_P = 5.0 at 207 °C. If the partial pressures of SO_2, O_2 and SO_3 are 1.0×10^{-3}, 0.20 and 1.0×10^{-4}, respectively, then the Gibbs free energy of the reaction ($\Delta_r G$) in kJ mol^{-1} at 207 °C is _____ [Given: R = 8.314 J K^{-1} mol^{-1}]

Q.10 Two moles of an ideal gas is expanded isothermally and reversibly from 5 to 1 bar at 298 K. The change in the entropy (in J K^{-1}) of the system is _____

Q. 11 – Q. 20 carry two marks each.

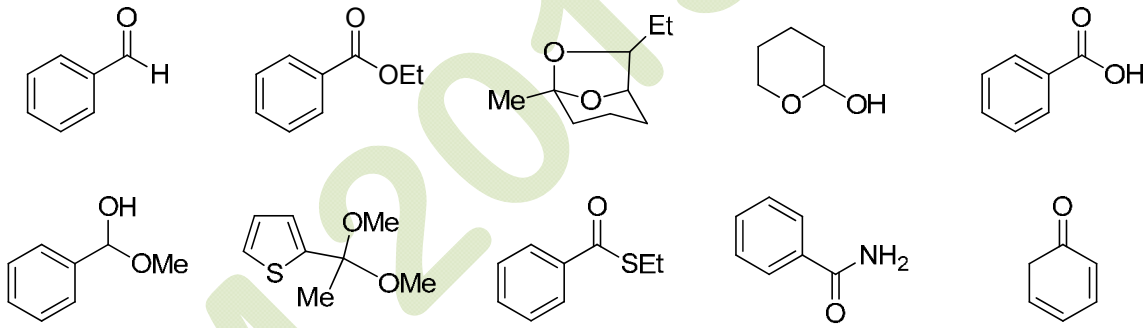
Q.11 The ionic radii of Cs^+ and Cl^- ions are 181 and 167 pm, respectively. The Born exponents for the He, Ne, Ar, Kr and Xe configurations are 5, 7, 9, 10 and 12, respectively. If the value of $\frac{ANe^2}{4\pi\varepsilon_0}$ is 2.45×10^{-4} J m, the lattice energy (in kJ mol^{-1}) of CsCl according to Born-Landé equation is _____

Q.12 A 2.5×10^{-4} M solution of a complex exhibits an absorption maximum at 625 nm with an absorbance of 0.90 when measured in a cuvette with a path length of 1.5 cm. The absorbance of 1.5×10^{-3} M solution of the same complex recorded in a cuvette with a path length of 0.2 cm is _____

Q.13 A wood specimen containing ^{14}C taken from an ancient palace showed 24 counts in 3 minutes per gram of carbon in a detector. However, a fresh wood showed 52 counts in 2 minutes per gram of carbon. Assuming no background signal in the detector and half life of ^{14}C as 5730 years, the age (in years) of the wood specimen is _____

Q.14 The magnetic field (in Tesla) required for flipping a ^{1}H nucleus in an NMR spectrometer operating at 400 MHz is _____ [Given: $\gamma = 2.67 \times 10^{8}\ T^{-1}\ s^{-1}$, $\pi = 3.14$]

Q.15 The total number of compounds (shown below) that form phenylhydrazone derivatives under acidic conditions is _____



Q.16 The number of possible monoalkylated products formed in the Friedel-Crafts reaction of anisole with 2-chloro-3-methylbutane in the presence of anhydrous $AlCl_3$ at 50 °C is _____

Q.17 For a reaction, the rate constant at 25 °C is doubled when the temperature is raised to 45 °C. The activation energy (in kJ mol^{-1}) of the reaction is _____ [Given: ln2 = 0.693]

Q.18 The standard reduction potentials of the Fe^{3+}/Fe^{2+} and Fe^{2+}/Fe couples are 0.77 and −0.44 V, respectively. The standard reduction potential (in V) for the Fe^{3+}/Fe couple is _____

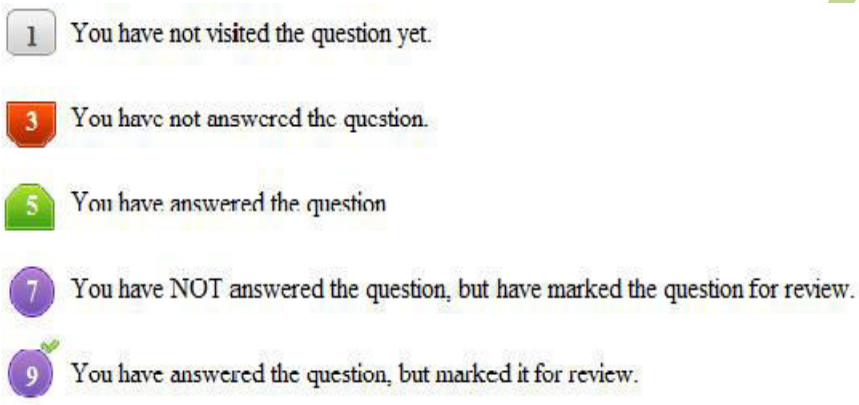
Q.19 When a perfect monolayer of stearic acid is formed at the air-water interface, each molecule of stearic acid (MW = 284, density = 0.94 g cm^{-3}) occupies an area of 20 $Å^2$. The length (in Å) of the molecule is _____

Q.20 In an ideal monoatomic gas, the speed of sound is given by $\sqrt{\frac{5RT}{3M}}$. If the speed of sound in argon at 25 °C is 1245 km h^{-1}, the root mean square velocity in m s^{-1} is _____

END OF THE QUESTION PAPER

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

GG: Geology

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which amongst the following planets has the highest number of known satellites?

(A) Mars (B) Uranus (C) Venus (D) Mercury

Q.2 Identify the mineral which has specific gravity >5, hardness <3 and gives grey streak.

(A) Barite (B) Sphalerite (C) Graphite (D) Galena

Q.3 A unit which is semi-permeable and does not yield any significant quantity of groundwater is called:

(A) Aquiclude (B) Aquifer (C) Aquitard (D) Artesian Aquifer

Q.4 A sandstone with less than 15% matrix content is known as:

(A) Siltstone (B) Mudstone (C) Greywacke (D) Arenite

Q.5 Which of the following mineral deposit is formed exclusively by surface geological processes?

(A) Wollastonite (B) Asbestos (C) Corundum (D) Bauxite

Q.6 Graphic texture is indicative of:

(A) Eutectic crystallization
(B) Crystal settling
(C) Fractional crystallization
(D) Magma mixing

Q.7 Which of the following is NOT a form of the isometric system?

(A) Octahedron (B) Tetrahedron (C) Trapezohedron (D) Rhombohedron

Q.8 Which of the following group of fossils became extinct at the Permo-Triassic boundary?

(A) Graptolite (B) Trilobite (C) Ammonite (D) Ediacaran

Q.9 Arrange the following granitoid bodies in order of their decreasing age.

(P) Erinpura Granite
(Q) Closepet Granite
(R) Singhbhum Granite
(S) Ladakh Granitoids

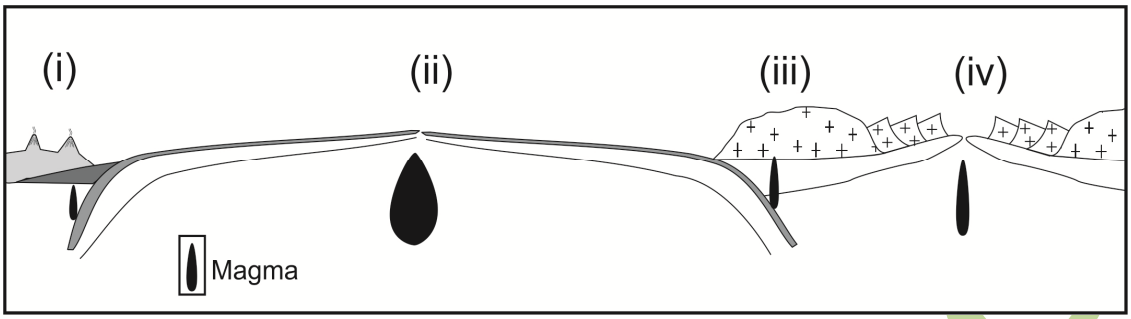
(A) R-P-Q-S
(B) S-Q-R-P
(C) R-Q-P-S
(D) Q-R-S-P

Q.10 Which of the following Brachiopod genera is considered as "living fossil"?

(A) *Lingula* (B) *Productus* (C) *Atrypa* (D) *Spirifer*

Q. 11 – Q. 30 carry two marks each.

Q.11 In the given figure (i), (ii), (iii), and (iv) denote the plate tectonic settings: Mid Oceanic Ridge (M), Island Arc (I), Continental Arc (C) and Rift Zone (R).



Which one of the following options is correctly matched?

(A) (i) – M, (ii) – I, (iii) – C, (iv) – R
(B) (i) – R, (ii) – I, (iii) – M, (iv) – C
(C) (i) – I, (ii) – R, (iii) – M, (iv) – C
(D) (i) – I, (ii) – M, (iii) – C, (iv) – R

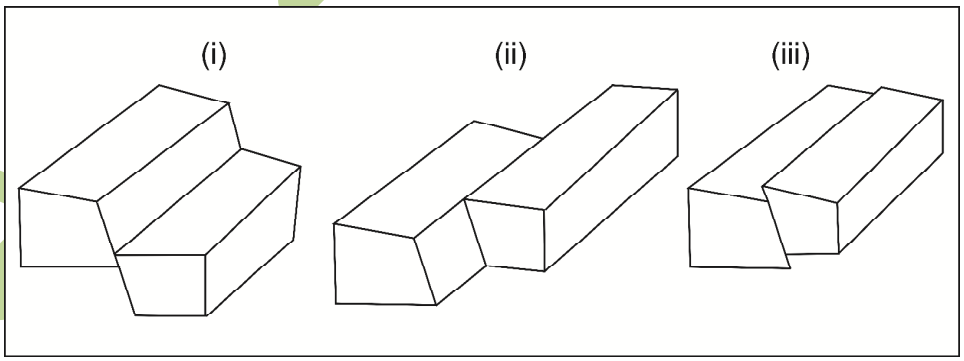
Q.12 In an undulated topography, a set of uniformly inclined beds show 'V'-shaped outcrop pattern with apex pointing downstream in a valley. This indicates:

(A) The dip of the beds is more than the slope of the valley
(B) The dip of the beds is less than the slope of the valley
(C) The beds are vertical
(D) The beds are horizontal

Q.13 Silica-rich viscous magmas often produce:

(A) Stratovolcano (B) Shield volcano (C) Dome (D) Fissure eruption

Q.14 Identify the faults in the diagrams (i), (ii), and (iii)



(A) (i) – Normal, (ii) – Dextral strike slip, (iii) – Reverse
(B) (i) – Normal, (ii) – Sinistral strike slip, (iii) – Reverse
(C) (i) – Reverse, (ii) – Dextral strike slip, (iii) – Normal
(D) (i) – Reverse, (ii) – Sinistral strike slip, (iii) – Normal

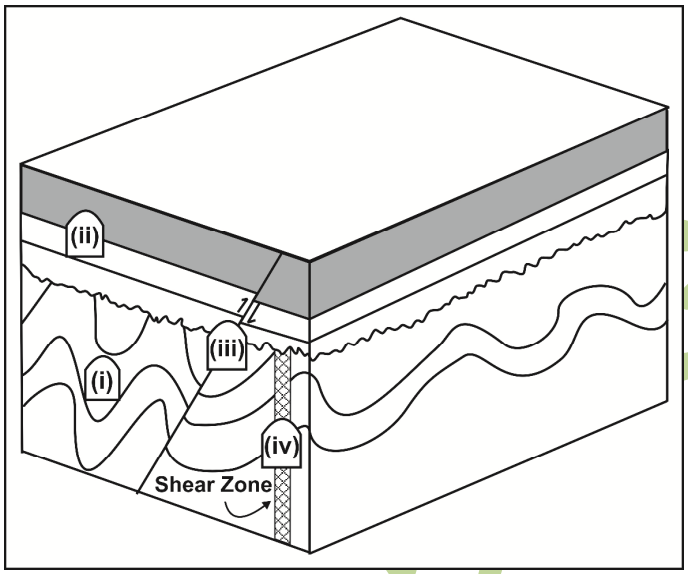
Q.15 A mine excavation made along the strike of a 2 meter thick tabular ore body dipping 30° is called:

(A) Crosscut (B) Raise (C) Drive (D) Shaft

Q.16 An ore body showing 'saddle reef' structure is formed by:

(A) Early magmatic crystallization process
(B) Liquid immiscibility
(C) Hydrothermal process
(D) Metamorphic process

Q.17 In the given block diagram, select the most suitable site amongst (i), (ii), (iii), and (iv) for constructing a tunnel.



(A) (i) (B) (ii) (C) (iii) (D) (iv)

Q.18 Match the metals listed in **Group-I** with their ores in **Group-II**

Group – I	**Group – II**
(i) Tin	(P) Scheelite
(ii) Tungsten	(Q) Pyrolusite
(iii) Copper	(R) Cassiterite
(iv) Manganese	(S) Bornite

(A) (i) – R, (ii) – P, (iii) – S, (iv) – Q
(B) (i) – Q, (ii) – P, (iii) – S, (iv) – R
(C) (i) – R, (ii) – Q, (iii) – P, (iv) – S
(D) (i) – P, (ii) – S, (iii) – R, (iv) – Q

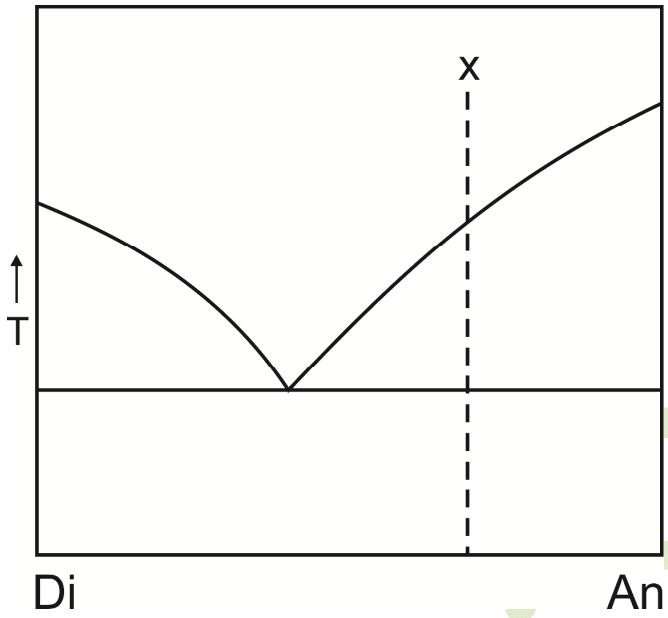
Q.19 Which one of the following minerals is correctly matched with its optical property?

(A) Sillimanite – Isotropic
(B) Orthoclase – High relief
(C) Pyroxene – Nearly orthogonal cleavage
(D) Garnet – Lamellar twinning

Q.20 The mineral coesite is expected to be stable in which of the following metamorphic facies?

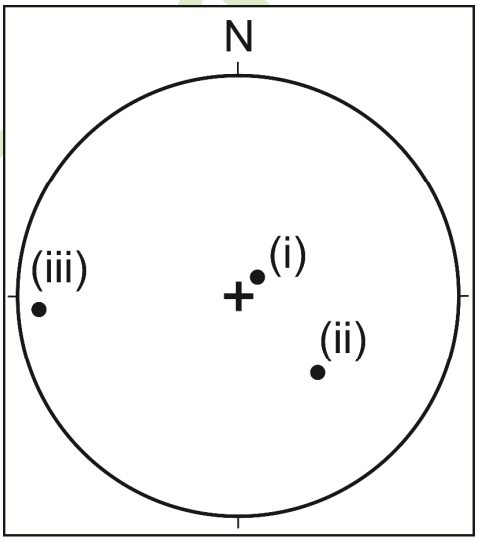
(A) Greenschist (B) Blueschist (C) Eclogite (D) Granulite

Q.21 In the given phase diagram, the sequence of minerals crystallizing from a magma of composition "X" under equilibrium conditions is:



(A) An→ Di (B) An→ An + Di (C) Di→ An (D) Di→ Di + An

Q.22 In the given stereo plot, poles to three bedding planes are plotted as points (i), (ii) and (iii). Find the correct order of increasing dips of the beds.



(A) (i) – (ii) – (iii) (B) (i) – (iii) – (ii) (C) (ii) – (i) – (iii) (D) (iii) – (i) – (ii)

Q.23 Size of a kettle lake depends upon:

(A) Size of the sediment carried by the glacier
(B) Width of the glacier valley
(C) Height of the valley through which the glacier moves
(D) Size of the buried ice block within moraines

Q.24 Match the sedimentary rocks given in **Group-I** with their characteristics in **Group-II**

Group – I	**Group – II**
(i) Sandstone	(P) Chemical sediment
(ii) Arkose	(Q) Grain size 1/16 – 2 mm
(iii) Limestone	(R) Feldspar-rich
(iv) Shale	(S) Grain size < 1/16 mm

(A) (i) – R, (ii) – P, (iii) – S, (iv) – Q
(B) (i) – Q, (ii) – R, (iii) – P, (iv) – S
(C) (i) – Q, (ii) – R, (iii) – S, (iv) – P
(D) (i) – P, (ii) – S, (iii) – R, (iv) – Q

Q.25 Arrange the following formations of the Gondwana Supergroup in order of their decreasing age:

(i) Raniganj Formation
(ii) Barakar Formation
(iii) Kulti Formation
(iv) Karharbari Formation

(A) (i) – (iii) – (ii) – (iv)
(B) (i) – (ii) – (iii) – (iv)
(C) (iv) – (iii) – (ii) – (i)
(D) (iv) – (ii) – (iii) – (i)

Q.26 Match the Lamellibranchia genera in **Group-I** with their dentition pattern in **Group-II**

Group – I	**Group – II**
(i) *Glycimeris*	(P) Schizodont
(ii) *Trigonia*	(Q) Taxodont
(iii) *Lucina*	(R) Dysodont
(iv) *Mytilus*	(S) Heterodont

(A) (i) – Q, (ii) – P, (iii) – S, (iv) – R
(B) (i) – P, (ii) – Q, (iii) – R, (iv) – S
(C) (i) – Q, (ii) – P, (iii) – R, (iv) – S
(D) (i) – S, (ii) – R, (iii) – Q, (iv) – P

Q.27 Drainage density of a basin is defined as:

(A) Sum of the number of streams of all orders divided by the area of the basin
(B) Total length of streams of all orders divided by the area of the basin
(C) Total number of the highest order streams divided by the total number of the next lower order streams
(D) Length of the largest stream divided by the area of the basin

Q.28 S- wave is terminated at:

(A) Crust—Mantle boundary
(B) Lithosphere—Asthenosphere boundary
(C) Mantle—Core boundary
(D) Inner and Outer core boundary

Q.29 Arrange the elements Fe, O, H, He, Si in decreasing order of their abundance in the solar system.

(A) H > He > O > Si > Fe
(B) He > H > Si > O > Fe
(C) H > He > O > Fe > Si
(D) Si > Fe > H > He > O

Q.30 A dolerite dyke metamorphosed under amphibolite facies condition is expected to have the mineral assemblage:

(A) Chlorite + Actinolite + Albite
(B) Lawsonite + Glaucophane + Epidote
(C) Orthopyroxene + Clinopyroxene + Plagioclase
(D) Hornblende + Plagioclase

SECTION - B

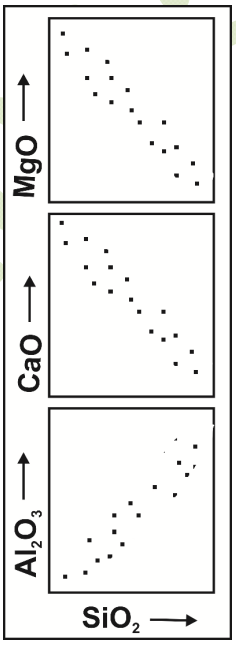
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1 A pyrite-chalcopyrite-bearing copper ore body has undergone supergene enrichment. Choose the correct statement(s).

(A) Limonite is present in the gossan zone
(B) Cuprite is present in the zone of oxidation
(C) Chalcocite is present in the zone of supergene enrichment
(D) Malachite is present in the primary zone

Q.2 The geochemical trends illustrated in the diagrams for a suite of cogenetic and coeval igneous rocks indicate:



(A) Olivine crystallization
(B) Clinopyroxene crystallization
(C) Plagioclase crystallization
(D) Alkali feldspar crystallization

Q.3 Which of the following chronostratigraphic unit(s) is/are correctly matched with the corresponding geochronologic unit(s)?

(A) System—Period
(B) Stage—Era
(C) Series—Epoch
(D) Chronozone—Age

Q.4 Which of the following statements related to depositional environments is/are correct?

(A) Herringbone cross-stratification indicates glacio-fluvial environment
(B) Dune is characterized by the presence of finer-grains at the top and coarser-grains at the bottom
(C) Dropstone is of glacial origin
(D) Bouma sequence indicates turbidite deposit

Q.5 Which of the following fossil genera is/are invertebrate?

(A) *Megalodon*
(B) *Spondylus*
(C) *Stegodon*
(D) *Telemastodon*

Q.6 Which of the following trilobites have two to four thoracic segments and are eyeless?

(A) *Agnostus*
(B) *Olenellus*
(C) *Calymene*
(D) *Microdiscus*

Q.7 Which of the following formation(s) was/were deposited in glacial and/or fluvial environment?

(A) Subathu Formation
(B) Karewa Formation
(C) Indus Formation
(D) Blaini Formation

Q.8 Which of the following statements is/are correct?

(A) Rajpura-Dariba in Rajasthan is a working Pb-Zn deposit
(B) Iron ore is mined in Noamundi, Odisha
(C) The largest copper mine in India is located in Malanjkhand, Madhya Pradesh
(D) Manganese deposit is found in the Mansar Formation, Maharashtra

Q.9 Which of the following stratigraphic units are correctly matched with their ages and geographical localities?

(A) Niniyur Formation – Cretaceous – Cauvery basin
(B) Long Formation – Oligocene – Andaman-Nicobar
(C) Zewan Formation – Permian – Kashmir-Lidar valley
(D) Bhander Group – Early Proterozoic – Vindhyan basin

Q.10 Petroleum producing basins of India belong to:

(A) Cambrian (B) Cretaceous (C) Tertiary (D) Quaternary

SECTION – C

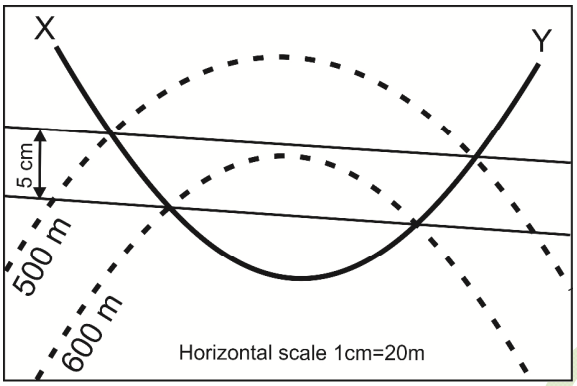
NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

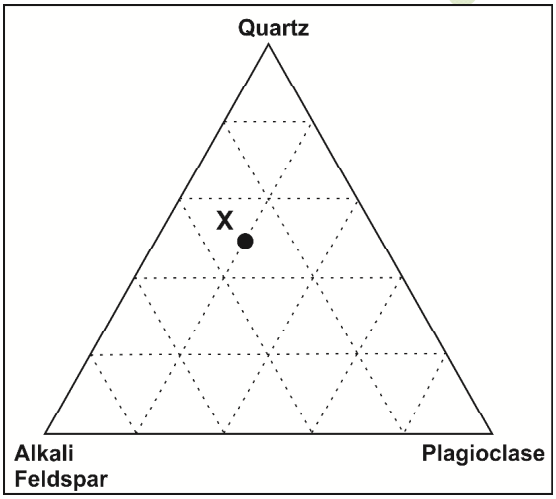
Q.1 On a toposheet of 1:50,000 scale, the distance between upper and lower traces of a bed is 5 cm. The actual distance on ground in kilometers is_____.

Q.2 On a flat topography the outcrop width of a bed is 30m. If the true dip of the bed is 30°, the actual thickness of the bed in meters is_____.

Q.3 In the given map, the true dip (in degree) of the bedding plane X-Y is_____.

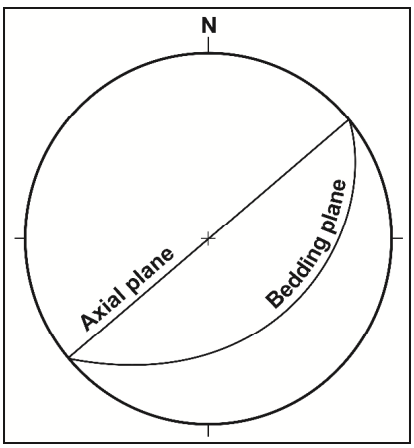


Q.4 In the given diagram, the percentage of Plagioclase in a rock of composition 'X' is_____.



Q.5 The birefringence of a uniaxial mineral having refractive indices of 1.658 and 1.486 is_____.

Q.6 In the given stereoplot, the plunge amount of the fold axis is_____.



Q.7 A crystal face has the following intercepts for the crystallographic axes: $1a_1$, $1a_2$, $\frac{1}{2}a_3$, ∞c. The Miller-Bravais indices for the face is_____.

Q.8 Total number of mirror planes in the $\frac{4}{m}\ \bar{3}\ \frac{2}{m}$ point group is_____.

Q.9 A radioactive isotope has 1024 atoms. How many atoms will remain after 4 half-lives?

Q.10 Waste water discharged from a coal mine has hydrogen ion $[H^+]$ concentration of 10^{-6} moles/liter. The pH of the water is_____.

Q. 11 – Q. 20 carry two marks each.

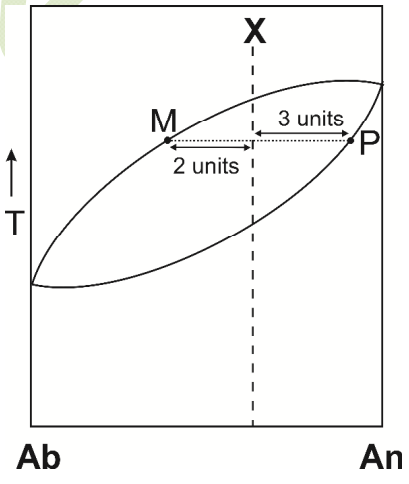
Q.11 An ore body is dipping 30° towards west on a flat topography. At what distance (in meters) should a borehole be placed so that it intersects the ore body perpendicularly at a vertical depth of 70 meters?

Q.12 Using the phase rule, the maximum number of phases in a 3 (three) component geological system having 1 (one) degree of freedom is_____.

Q.13 The lithostatic pressure in Mega Pascal (MPa) at a depth of 10 km in a granite batholith having density 2700 kg/m^3 is_____.

(Acceleration due to gravity = 9.8 m/s^2)

Q.14 In the given phase diagram, the weight percent of melt at point 'M' for a crystallizing magma of bulk composition 'X' is_____.



Q.15 Calculate the average atomic weight (answer to be given up to 3 decimal places) of Rubidium using the given data.

Isotope	Abundance (%)	Atomic weight (a.m.u.)
^{85}Rb	72.17	84.912
^{87}Rb	27.83	86.909

Q.16 In an oblique slip fault having 30° dip, the net slip is 10 meters at an angle of 45° to the strike of the fault plane. The dip slip component of the fault in meters is_____.
(Answer to be given up to 2 decimal places).

Q.17 A 10 cm^3 sandstone block has a mass of 20 grams. If the average density of sand grains is 2.5 g/cm^3, the porosity (in percent) of the sandstone is_____.

Q.18 A mineral has a weight of 4.5 grams in air and 3.2 grams in water. Assuming density of water to be 1g/cm^3, the specific gravity (up to 2 decimal places) of the mineral is_____.

Q.19 The duration of the Proterozoic Eon in Ma is_____.

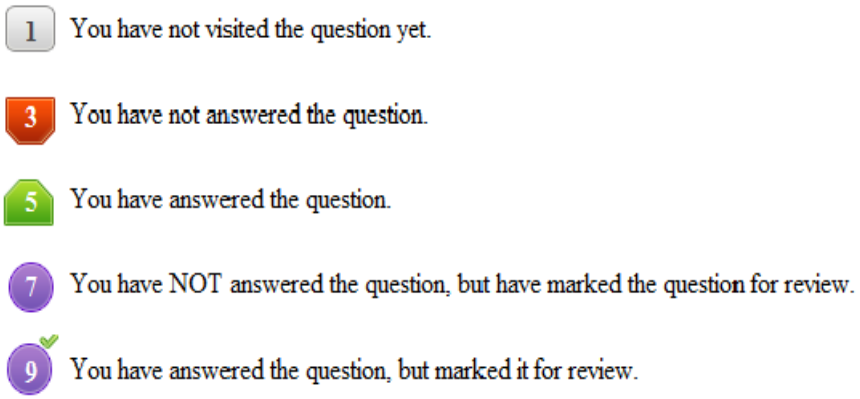
Q.20 From the given equation, calculate how many moles of magnetite will be produced by the reduction of 1 (one) mole of hematite (answer to be given up to 3 decimal places).

$$\mathbf{6Fe_2O_3 \rightarrow 4Fe_3O_4 + O_2}$$

END OF THE QUESTION PAPER

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

MS: Mathematical Statistics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

Special Instructions / Useful Data	
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1, \ldots, x_n) : x_i \in \mathbb{R},\ i = 1, 2, \ldots, n\}$
$E(X)$	Expectation of the random variable X
$P(A)$	Probability of the event A
$\bar{X}_n$	$\frac{1}{n}\sum_{i=1}^{n} X_i$
$U(a,b)$	Continuous uniform distribution on (a,b), $-\infty < a < b < \infty$
$N(\mu, \sigma^2)$	Normal distribution with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$
$Bin(n, p)$	Binomial distribution with n trials and success probability p
$Poisson(\theta)$	Poisson distribution with parameter θ
$Geom(p)$	Geometric distribution with parameter p, whose probability mass function is given by $P(X = x) = p(1-p)^{x-1}$, $x = 1, 2, \ldots$
$Gamma(\alpha, \beta)$	Gamma distribution with parameters α and β, whose probability density function is given by $f(x) = \begin{cases} \frac{x^{\alpha-1}\ e^{-x/\beta}}{\Gamma(\alpha)\ \beta^{\alpha}}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$
$X_n \xrightarrow{P} X$	The sequence of random variables $\{X_n\}$ converges in probability to the random variable X
$X_n \xrightarrow{d} X$	The sequence of random variables $\{X_n\}$ converges in distribution to the random variable X
$\binom{n}{x}$	Binomial coefficient, equal to $\frac{n!}{x!\,(n-x)!}$
$\log(x)$	Natural logarithm of x
I	Identity matrix
$\Phi(x)$	Cumulative distribution function of $N(0,1)$
Special values	$\Phi(1.285) = 0.900$, $\Phi(1.645) = 0.950$, $\Phi(1.96) = 0.975$ $e = 2.718$, $\pi = 3.142$ $\log(2) = 0.693$

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $X_1,\ldots,X_n$ be a random sample from a population with probability density function

$$f_\theta(x)=\begin{cases}\theta e^{-\theta x}, & x>0,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta>0$ is an unknown parameter.

Then, the uniformly minimum variance unbiased estimator for $\frac{1}{\theta}$ is

(A) $\frac{1}{\bar{X}_n}$

(B) $\sum_{i=1}^{n} X_i$

(C) $\bar{X}_n$

(D) $\frac{1}{\sum_{i=1}^{n} X_i}$

Q.2 Let $X_1,\ldots,X_{100}$ be independent and identically distributed $N(0,1)$ random variables.

The correlation between $\sum_{i=1}^{98} X_i$ and $\sum_{i=3}^{100} X_i$ is equal to

(A) 0 (B) $96/98$ (C) $98/100$ (D) 1

Q.3 Consider the problem of testing $H_0:\theta=0$ against $H_1:\theta=1/2$ based on a single observation X from $U(\theta,\theta+1)$ population. The power of the test "Reject H_0 if $X>\frac{2}{3}$" is

(A) $1/6$ (B) $5/6$ (C) $1/3$ (D) $2/3$

Q.4 The probability mass function of a random variable X is given by

$$P(X=x)=k\binom{n}{x},\ x=0,1,\ldots,n,$$

where k is a constant. The moment generating function $M_X(t)$ is

(A) $\dfrac{\left(1+e^t\right)^n}{2^n}$

(B) $\dfrac{2^n}{\left(1+e^t\right)^n}$

(C) $\dfrac{1}{2^n\left(1+e^t\right)^n}$

(D) $2^n\left(1+e^t\right)^n$

Q.5 Suppose A and B are events with $P(A)=0.5$, $P(B)=0.4$ and $P\left(A\cap B^c\right)=0.2$. Then $P\left(B^c \mid A\cup B\right)$ is equal to

(A) $1/2$ (B) $1/3$ (C) $1/4$ (D) 0

Q.6 Let $X_1,\ldots,X_n$ be a random sample from a $Gamma(\alpha,\beta)$ population, where $\beta>0$ is a known constant. The rejection region of the most powerful test for $H_0:\alpha=1$ against $H_1:\alpha=2$ is of the form

(A) $\prod_{i=1}^{n}X_i>K$ (B) $\sum_{i=1}^{n}X_i>K$ (C) $\prod_{i=1}^{n}X_i<K$ (D) $\sum_{i=1}^{n}X_i<K$

Q.7 Which of the following is NOT a linear transformation?

(A) $T:\mathbb{R}^3\to\mathbb{R}^2$ defined by $T(x,y,z)=(x,z)$

(B) $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x,y-1,z)$

(C) $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x,y)=(2x,y-x)$

(D) $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x,y)=(y,x)$

Q.8 If a sequence $\{x_n\}$ is monotone and bounded, then

(A) there exists a subsequence of $\{x_n\}$ that diverges

(B) there may exist a subsequence of $\{x_n\}$ that is not monotone

(C) all subsequences of $\{x_n\}$ converge to the same limit

(D) there exist at least two subsequences of $\{x_n\}$ which converge to distinct limits

Q.9 Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x(x-1)(x-2)$. Then

(A) f is one-one and onto
(B) f is neither one-one nor onto
(C) f is one-one but not onto
(D) f is not one-one but onto

Q.10 Which of the following statements is true for all real numbers x?

(A) $e^{-x}\le 1-x$ (B) $e^{-x}\ge 1-x$
(C) $e^{x}\ge 1-x$ (D) $e^{x}\le 1-x$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $X_1,\ldots,X_n$ be a random sample from a $Poisson(\theta)$ population, where $\theta>0$ is unknown. The Cramer-Rao lower bound for the variance of any unbiased estimator of $g(\theta)=\theta e^{-\theta}$ equals

(A) $\frac{1}{n}\theta(1-\theta)^2 e^{-2\theta}$
(B) $\theta(1-\theta)^2 e^{-2\theta}$
(C) $\theta(1-\theta)e^{-\theta}$
(D) $\frac{1}{n}\theta(1-\theta)e^{-\theta}$

Q.12 Let X and Y be two independent random variables such that $X\sim U(0,2)$ and $Y\sim U(1,3)$. Then $P(X<Y)$ equals

(A) $1/2$ (B) $3/4$ (C) $7/8$ (D) 1

Q.13 There are two boxes, each containing two components. Each component is defective with probability 1/4, independent of all other components. The probability that exactly one box contains exactly one defective component equals

(A) $3/8$ (B) $5/8$ (C) $15/32$ (D) $17/32$

Q.14 Consider a normal population with unknown mean μ and variance $\sigma^2=9$. To test $H_0:\mu=0$ against $H_1:\mu\neq 0,$ a random sample of size 100 is taken. Based on this sample, the test of the form $|\bar{X}_n|>K$ rejects the null hypothesis at 5% level of significance. Then, which of the following is a possible 95% confidence interval for μ?

(A) $(-0.488,\ 0.688)$ (B) $(-1.96,\ 1.96)$
(C) $(0.422,\ 1.598)$ (D) $(0.588,\ 1.96)$

Q.15 Let $X_1,\ldots,X_n$ be a random sample from a population with probability density function

$$f_\theta(x)=\begin{cases}(\theta+1)\,x^{\theta}, & 0<x<1,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta>0$ is unknown. The maximum likelihood estimator of θ is

(A) $\sum_{i=1}^{n}\log X_i$

(B) $\dfrac{n-\sum_{i=1}^{n}\log X_i}{\sum_{i=1}^{n}\log X_i}$

(C) $\dfrac{n+\sum_{i=1}^{n}\log X_i}{-\sum_{i=1}^{n}\log X_i}$

(D) $\prod_{i=1}^{n}X_i$

Q.16 Let $X_1,\ldots,X_n$ be a random sample from a population with probability density function

$$f_\theta(x)=\begin{cases}\dfrac{4}{\theta}x^3\,e^{-x^4/\theta}, & x>0,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta>0$ is unknown. Then, a consistent estimator for θ is

(A) $\dfrac{1}{n}\sum_{i=1}^{n}X_i^4$

(B) $\left(\sum_{i=1}^{n}\dfrac{X_i}{n}\right)^4$

(C) $\bar{X}_n$

(D) $\prod_{i=1}^{n}X_i^3$

Q.17 Let the probability density function of a random variable X be given by

$$f(x)=\alpha\,\mathrm{e}^{-x^2-\beta x},\ -\infty<x<\infty.$$

If $E(X)=-\dfrac{1}{2}$, then

(A) $\alpha=\dfrac{1}{\sqrt{\pi}}e^{-1/4}$ and $\beta=1$

(B) $\alpha=\dfrac{1}{\sqrt{\pi}}e^{-1/4}$ and $\beta=-1$

(C) $\alpha=\sqrt{\pi}\,e^{-1/4}$ and $\beta=1$

(D) $\alpha=\sqrt{\pi}\,e^{-1/4}$ and $\beta=-1$

Q.18 Let X be a single observation from a population having an exponential distribution with mean $1/\lambda$. Consider the problem of testing $H_0: \lambda = 2$ against $H_1: \lambda = 4$. For the test with rejection region $X \geq 3$, let $\alpha = P(\text{Type I error})$ and $\beta = P(\text{Type II error})$. Then

(A) $\alpha = e^{-6}$ and $\beta = 1 - e^{-12}$
(B) $\alpha = e^{-12}$ and $\beta = 1 - e^{-6}$
(C) $\alpha = 1 - e^{-12}$ and $\beta = e^{-6}$
(D) $\alpha = e^{-6}$ and $\beta = e^{-12}$

Q.19 Let Y be an exponential random variable with mean $1/\theta$, where $\theta > 0$. The conditional distribution of X given Y has Poisson distribution with mean Y. Then, the variance of X is

(A) $\frac{1}{\theta^2}$ (B) $\frac{\theta+1}{\theta}$ (C) $\frac{\theta^2+1}{\theta^2}$ (D) $\frac{\theta+1}{\theta^2}$

Q.20 2000 cashew nuts are mixed thoroughly in flour. The entire mixture is divided into 1000 equal parts and each part is used to make one biscuit. Assume that no cashews are broken in the process. A biscuit is picked at random. The probability that it contains no cashew nuts is

(A) between 0 and 0.1
(B) between 0.1 and 0.2
(C) between 0.2 and 0.3
(D) between 0.3 and 0.4

Q.21 Suppose $X_1, \ldots, X_n$ are independent random variables and $X_k \sim N(0, k\sigma^2)$, $k = 1, \ldots, n$, where σ^2 is unknown. The maximum likelihood estimator for σ^2 is

(A) $\frac{1}{n}\sum_{k=1}^{n} X_k^2$

(B) $\frac{1}{n}\sum_{k=1}^{n} (X_k - \bar{X}_n)^2$

(C) $\frac{1}{n}\sum_{k=1}^{n} \frac{X_k^2}{k}$

(D) $\frac{\sum_{k=1}^{n} X_k^2}{\sum_{k=1}^{n} k}$

Q.22 Let $X_1, \ldots, X_{10}$ be independent and identically distributed $U(-5,5)$ random variables. Then, the distribution of the random variable $Y = -2\sum_{i=1}^{10} \log(|X_i|/5)$ is

(A) χ^2_{10} (B) $10\,\chi^2_2$ (C) χ^2_{20} (D) $\frac{1}{2}\chi^2_{20}$

Q.23 Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function so that $f(x)f'(x)<0$ for all x. Then, which of the following is necessarily true?

(A) f is an increasing function
(B) f is a decreasing function
(C) $|f|$ is an increasing function
(D) $|f|$ is a decreasing function

Q.24 Let M be the matrix $\begin{pmatrix}2 & 3\\ 1 & -1\end{pmatrix}$. Which of the following matrix equations does M satisfy?

(A) $M^2+3M+5I=0$
(B) $M^3-M^2-5M=0$
(C) $M^3-3M^2+M=0$
(D) $M^2-M+5I=0$

Q.25 If the determinant of an $n\times n$ matrix A is zero, then

(A) $\text{rank}(A)\le n-2$
(B) the trace of A is zero
(C) zero is an eigenvalue of A
(D) $x=0$ is the only solution of $Ax=0$

Q.26 Let $f:(0,\infty)\to\mathbb{R}$ be given by

$$f(x)=\log x - x + 2.$$

Then, the number of roots of f is

(A) 0 (B) 1 (C) 2 (D) 3

Q.27 The number of distinct real values of x for which the matrix

$$\begin{pmatrix}x & 1 & 1\\ 1 & x & 1\\ 1 & 1 & x\end{pmatrix}$$

is singular is

(A) 1 (B) 2 (C) 3 (D) infinite

Q.28 Suppose $f:\mathbb{R}^2 \to \mathbb{R}$ is a continuous function. Define $h:\mathbb{R} \to \mathbb{R}$ by

$$h(x) = \int_0^x \int_0^x f(u,v)\,du\,dv.$$

Then $h'(1)$ is equal to

(A) $2f(1,1)$

(B) $f(1,0) + f(0,1)$

(C) $\int_0^1 f(t,t)\,dt$

(D) $\int_0^1 \big(f(1,t) + f(t,1)\big)\,dt$

Q.29 Let A be a 5×3 real matrix of rank 2. Let $b \in \mathbb{R}^5$ be a non-zero vector that is in the column space of A. Let $S = \{x \in \mathbb{R}^3 : Ax = b\}$. Define the translation of a subspace V of $\mathbb{R}^3$ by $x_0 \in \mathbb{R}^3$ as the set $x_0 + V = \{x_0 + v : v \in V\}$. Then

(A) S is the empty set
(B) S has only one element
(C) S is a translation of a 1 dimensional subspace
(D) S is a translation of a 2 dimensional subspace

Q.30 Let $f:\mathbb{R} \to \mathbb{R}$ be a differentiable function whose derivative is continuous. Then

$$\lim_{n \to \infty} (n+1) \int_0^1 x^n f(x)\,dx$$

(A) is equal to 0

(B) is infinite

(C) is equal to $\int_0^1 f(x)\,dx$

(D) is equal to $f(1)$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1 Suppose $\{a_n\}, \{b_n\}$ are sequences such that $a_n > 0, b_n > 0$ for all $n \geq 1$. Given that $\sum a_n$ converges and $\sum b_n$ diverges, which of the following statements is (are) necessarily FALSE?

(A) $\sum (a_n + b_n)$ converges

(B) $\sum \frac{a_n}{b_n}$ converges

(C) $\sum \frac{b_n}{a_n}$ converges

(D) $\sum a_n b_n$ converges

Q.2 Consider the ordinary differential equation

$$x\frac{dy}{dx} + y = x \text{ for } 0 < x < 1.$$

Which of the following is (are) solution(s) to the above?

(A) $y(x) = \frac{x}{2}$

(B) $y(x) = \frac{x}{2} + \frac{2}{x}$

(C) $y(x) = \frac{x}{2} - \frac{2}{x}$

(D) $y(x) = 0$

Q.3 Let $f : [0,1] \to \mathbb{R}$ be a continuous function such that

$$f(0) = -1, \ f\left(\frac{1}{2}\right) = 1, \ f(1) = -1.$$

Then

(A) f attains the value 0 at least twice in $[0,1]$

(B) f attains the value 0 exactly twice in $[0,1]$

(C) f attains the value 0 exactly once in $[0,1]$

(D) the range of f is $[-1,1]$

Q.4 Let $f : \mathbb{R} \to \mathbb{R}$. Define $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = f(x)\left(f(x) + f(-x)\right).$$

Then

(A) g is even for all f

(B) g is odd for all f

(C) g is even if f is even

(D) g is even if f is odd

Q.5 Which of the following matrices can be the variance-covariance matrix of a random vector $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$?

(A) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

Q.6 Let $X_1, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ population, where $-\infty < \theta < \infty$ is unknown. Which of the following statistics is (are) sufficient for θ?

(A) $\sum_{i=1}^{n} X_i$ (B) $\left(X_1, \sum_{i=2}^{n} X_i \right)$

(C) $\left(X_1, X_2 + X_3, \sum_{i=4}^{n} X_i \right)$ (D) $\left(X_1, X_2, X_3, \sum_{i=4}^{n} X_i \right)$

Q.7 Let $X_1, \ldots, X_n$ be a random sample from a $N(\theta, \theta^2)$ distribution, where $\theta > 0$ is unknown. Let

$$T_1 = \sum_{i=1}^{n} X_i \text{ and } T_2 = \sum_{i=1}^{n} X_i^2.$$

Which of the following statements is (are) correct?

(A) $\frac{T_1^2}{n^2}$ is unbiased for θ^2

(B) $\frac{T_2}{2n}$ is unbiased for θ^2

(C) $\frac{T_1^2}{n^2}$ is consistent for θ^2

(D) $\frac{T_2}{2n}$ is consistent for θ^2

Q.8 Suppose X and Y are independent and identically distributed random variables with finite variance σ^2. Which of the following expressions is (are) equal to σ^2 ?

(A) $E\left(X^2\right) - \left(E\left(Y\right)\right)^2$ (B) $E\left(\left(\frac{X+Y}{2}\right)^2\right) - \left(E\left(\frac{X+Y}{2}\right)\right)^2$

(C) $\frac{1}{2} E\left(\left(X-Y\right)^2\right)$ (D) $\min_{a \in \mathbb{R}} E\left(X-a\right)^2$

Q.9 Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with mean 2 and variance 4. Which of the following statements is (are) true?

(A) $\bar{X}_n \xrightarrow{P} 2$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} 4$

(C) $\sqrt{n}\frac{\left(\bar{X}_n - 2\right)}{2} \xrightarrow{d} N(0,1)$

(D) $E\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}_n\right)^2\right) = 4$ for all $n \geq 2$

Q.10 Let $X_1, \ldots, X_n$ (assume $n \geq 2$) be a random sample from an $N\left(\mu, \sigma^2\right)$ population where $-\infty < \mu < \infty$ and $\sigma^2 > 0$ are unknown. Which of the following statements is (are) true?

(A) The maximum likelihood estimator of μ attains the Cramer-Rao lower bound

(B) The uniformly minimum variance unbiased estimator of μ attains the Cramer-Rao lower bound

(C) The maximum likelihood estimator of σ^2 is an unbiased estimator of σ^2

(D) The relative efficiency of the maximum likelihood estimator of σ^2 with respect to the uniformly minimum variance unbiased estimator of σ^2 is strictly less than 1

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

Q.1 Let X and Y be independent exponentially distributed random variables with means $1/4$ and $1/6$ respectively. Let $Z = \min\{X, Y\}$. Then $E(Z) =$ _____.

Q.2 Let X be a $Geom(0.4)$ random variable. Then $P(X = 5 | X \geq 2) =$ _____.

Q.3 X is a single observation from a $Bin(1, p)$ population, where $p \in [1/5, 4/5]$ is unknown. If the observed value of X is 0, then the maximum likelihood estimator of p is _____.

Q.4 X is a random variable with density $f(x) = \frac{1}{4} e^{-|x|/2}$, $-\infty < x < \infty$. Then $E(|X|) =$ _____.

Q.5 A system comprising of n identical components works if at least one of the components works. Each of the components works with probability 0.8, independent of all other components. The minimum value of n for which the system works with probability at least 0.97 is _____.

Q.6 Let X be a normal random variable with mean 2 and variance 4, and $g(a) = P(a \leq X \leq a+2)$. The value of a that maximizes $g(a)$ is _____.

Q.7 The volume of the solid formed by revolving the curve $y = x$ between $x = 0$ and $x = 1$ about the x-axis is equal to _____.

Q.8 Let $[x]$ be the greatest integer less than or equal to x. Then $\int_{-1}^{2} [x]\,dx =$ _____.

Q.9 The number of real solutions of the equation $x^3 + 3x^2 + 3x + 7 = 0$ is _____.

Q.10 Let $f:\mathbb{R}\to\mathbb{R}$ be a non-constant, three times differentiable function. If $f\left(1+\frac{1}{n}\right)=1$ for all integers n, then $f''(1)=$ _____.

Q. 11 – Q. 20 carry two marks each.

Q.11 Let $Y\sim U(0,1)$. The conditional probability density function of X given Y is

$$f_{X|Y}(x|y)=\begin{cases}\frac{1}{y}, & \text{if } 0<x<y<1,\\ 0, & \text{otherwise.}\end{cases}$$

Then $E(X)=$ _____.

Q.12 The probability density function of a random variable X is given by

$$f(x)=\begin{cases}\frac{1}{4}, & \text{if } |x|\le 1,\\ \frac{1}{4x^2}, & \text{otherwise.}\end{cases}$$

Then $P\left(-\frac{1}{2}\le X\le 2\right)=$ _____.

Q.13 Let $X_1,\ldots,X_n$ be independent and identically distributed random variables with $U(0,1)$ distribution. Then

$\lim_{n\to\infty} P\left(\sum_{i=1}^{n} X_i\le\frac{n}{2}+n^{3/4}\right)=$ _____.

Q.14 Based on 20 observations $(x_1,y_1),\ldots,(x_{20},y_{20})$, the following values are obtained.

$$\sum_{i=1}^{20}x_i=6,\ \sum_{i=1}^{20}x_i^2=14,\ \sum_{i=1}^{20}y_i=9 \text{ and } \sum_{i=1}^{20}x_iy_i=20.$$

For $X=1$, the predicted value of Y based on a least squares fit of a linear regression model of Y on X is _____.

Q.15 The cumulative distribution function of a random variable X is given by

$$F(x)=\begin{cases}0, & \text{if } x<0,\\ \frac{1}{4}+\frac{1}{6}\left(4x-x^2\right), & \text{if } 0\le x<1,\\ 1, & \text{if } x\ge 1.\end{cases}$$

Then $P\left(X=0 \,\middle|\, 0\le X<1\right)=$ _____.

Q.16 The probability density function $f(x)$ of a random variable X is symmetric about 0. Then $\int_{-2}^{2}\int_{-\infty}^{x} f(u)\,du\,dx=$ _____.

Q.17 The length of the curve $y=\sqrt{4-x^2}$ from $x=-\sqrt{2}$ to $x=\sqrt{2}$ is equal to _____.

Q.18 The system of equations

$$\begin{aligned} x+y+2z &= 2\\ 2x+3y-z &= 5\\ 4x+7y+cz &= 6 \end{aligned}$$

does NOT have a solution. Then, the value of c must be equal to _____.

Q.19 Let $y(x)$ be a solution to the differential equation

$$y''-2y'+y=0,\quad y(0)=1 \text{ and } y'(0)=1.$$

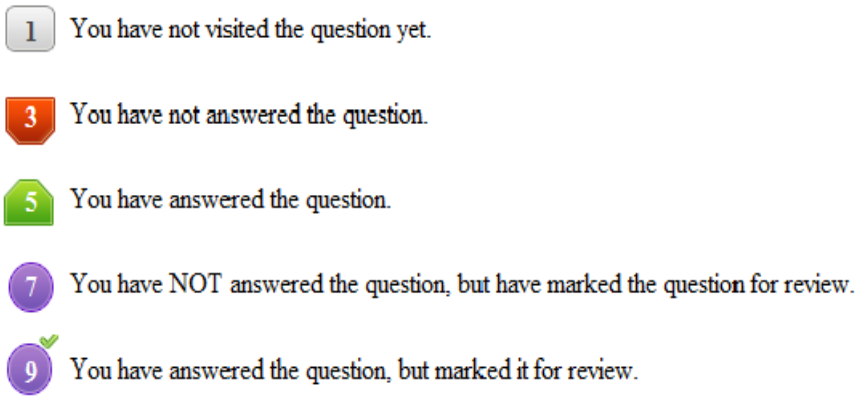
Then $\lim_{x\to-\infty} y(x)=$ _____.

Q.20 The area of the region in the first quadrant enclosed by the curves $y=0$, $y=x$ and $y=\frac{2}{x}-1$ is equal to _____.

END OF THE QUESTION PAPER

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

MA: Mathematics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

Notation

$\mathbb{N}$ - The set of natural numbers = $\{1,2,3, \ldots\}$

$\mathbb{Z}$ - The set of integers

$\mathbb{Q}$ - The set of rational numbers

$\mathbb{R}$ - The set of real numbers

$\mathbb{C}$ - The set of complex numbers

S_n - The group of permutations of n distinct symbols

$\mathbb{Z}_n$ - The group of integers modulo n

$M_n(\mathbb{R})$ - The vector space of $n \times n$ real matrices

$\hat{\imath}, \hat{\jmath}, \hat{k}$ - Standard mutually orthogonal unit vectors

i - Imaginary number $\sqrt{-1}$

$\bar{a}$ - Complex conjugate of a

$\bar{A}$ - Complex conjugate of matrix A

A^T - Transpose of matrix A

$\emptyset$ - Empty set

sup - supremum

inf - infimum

y' - Derivative of y

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Suppose N is a normal subgroup of a group G. Which one of the following is true?

(A) If G is an infinite group then G/N is an infinite group
(B) If G is a nonabelian group then G/N is a nonabelian group
(C) If G is a cyclic group then G/N is an abelian group
(D) If G is an abelian group then G/N is a cyclic group

Q.2 Let $y(x) = u(x)\sin x + v(x)\cos x$ be a solution of the differential equation $y'' + y = \sec x$. Then $u(x)$ is

(A) $\ln|\cos x| + C$ (B) $-x + C$
(C) $x + C$ (D) $\ln|\sec x| + C$

Q.3 Let a, b, c, d be distinct non-zero real numbers with $a + b = c + d$. Then an eigenvalue of the matrix $\begin{bmatrix} a & b & 1 \\ c & d & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is

(A) $a + c$ (B) $a + b$ (C) $a - b$ (D) $b - d$

Q.4 Let S be a nonempty subset of $\mathbb{R}$. If S is a finite union of disjoint bounded intervals, then which one of the following is true?

(A) If S is not compact, then $\sup S \notin S$ and $\inf S \notin S$
(B) Even if $\sup S \in S$ and $\inf S \in S$, S need not be compact
(C) If $\sup S \in S$ and $\inf S \in S$, then S is compact
(D) Even if S is compact, it is not necessary that $\sup S \in S$ and $\inf S \in S$

Q.5 Let $\{x_n\}$ be a convergent sequence of real numbers. If $x_1 > \pi + \sqrt{2}$ and $x_{n+1} = \pi + \sqrt{x_n - \pi}$ for $n \geq 1$, then which one of the following is the limit of this sequence?

(A) $\pi + 1$ (B) $\pi + \sqrt{2}$ (C) π (D) $\pi + \sqrt{\pi}$

Q.6 The volume of the portion of the solid cylinder $x^2 + y^2 \leq 2$ bounded above by the surface $z = x^2 + y^2$ and bounded below by the xy-plane is

(A) π (B) 2π (C) 3π (D) 4π

Q.7 Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function with $f(0)=0$. If for all $x\in\mathbb{R}$, $1<f'(x)<2$, then which one of the following statements is true on $(0,\infty)$?

(A) f is unbounded
(B) f is increasing and bounded
(C) f has at least one zero
(D) f is periodic

Q.8 If an integral curve of the differential equation $(y-x)\frac{dy}{dx}=1$ passes through $(0,0)$ and $(\alpha,1)$, then α is equal to

(A) $2-e^{-1}$ (B) $1-e^{-1}$ (C) e^{-1} (D) $1+e$

Q.9 An integrating factor of the differential equation

$$\frac{dy}{dx}=\frac{2xy^2+y}{x-2y^3}$$

is

(A) $\frac{1}{y}$ (B) $\frac{1}{y^2}$ (C) y (D) y^2

Q.10 Let A be a nonempty subset of $\mathbb{R}$. Let $I(A)$ denote the set of interior points of A. Then $I(A)$ can be

(A) empty
(B) singleton
(C) a finite set containing more than one element
(D) countable but not finite

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S_3 be the group of permutations of three distinct symbols. The direct sum $S_3\oplus S_3$ has an element of order

(A) 4 (B) 6 (C) 9 (D) 18

Q.12 The orthogonal trajectories of the family of curves $y=C_1x^3$ are

(A) $2x^2+3y^2=C_2$
(B) $3x^2+y^2=C_2$
(C) $3x^2+2y^2=C_2$
(D) $x^2+3y^2=C_2$

Q.13 Let G be a nonabelian group. Let $\alpha\in G$ have order 4 and let $\beta\in G$ have order 3. Then the order of the element $\alpha\beta$ in G

(A) is 6
(B) is 12
(C) is of the form $12k$ for $k\geq 2$
(D) need not be finite

Q.14 Let S be the bounded surface of the cylinder $x^2 + y^2 = 1$ cut by the planes $z = 0$ and $z = 1 + x$. Then the value of the surface integral $\iint_S 3z^2 d\sigma$ is equal to

(A) $\int_0^{2\pi} (1 + \cos\theta)^3 d\theta$ (B) $\int_0^{2\pi} \sin\theta \cos\theta\, (1 + \cos\theta)^2 d\theta$

(C) $\int_0^{2\pi} (1 + 2\cos\theta)^3 d\theta$ (D) $\int_0^{2\pi} \sin\theta \cos\theta\, (1 + 2\cos\theta)^2 d\theta$

Q.15 Suppose that the dependent variables z and w are functions of the independent variables x and y, defined by the equations $f(x, y, z, w) = 0$ and $g(x, y, z, w) = 0$, where $f_z g_w - f_w g_z = 1$. Which one of the following is correct?

(A) $z_x = f_w g_x - f_x g_w$ (B) $z_x = f_x g_w - f_w g_x$
(C) $z_x = f_z g_x - f_x g_z$ (D) $z_x = f_z g_w - f_z g_x$

Q.16 Let $A = \begin{bmatrix} 0 & 1-i \\ -1-i & i \end{bmatrix}$ and $B = A^T \overline{A}$. Then

(A) an eigenvalue of B is purely imaginary
(B) an eigenvalue of A is zero
(C) all eigenvalues of B are real
(D) A has a non-zero real eigenvalue

Q.17 The limit

$$\lim_{x \to 0+} \frac{1}{\sin^2 x} \int_{\frac{x}{2}}^{x} \sin^{-1} t\, dt$$

is equal to

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$

Q.18 Let $P_2(\mathbb{R})$ be the vector space of polynomials in x of degree at most 2 with real coefficients. Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. If a linear transformation $T: P_2(\mathbb{R}) \to M_2(\mathbb{R})$ is defined as

$$T(f) = \begin{bmatrix} f(0) - f(2) & 0 \\ 0 & f(1) \end{bmatrix}$$

then

(A) T is one-one but not onto
(B) T is onto but not one-one
(C) Range (T) = span $\left\{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\right\}$
(D) Null (T) = span $\{x^2 - 2x,\ 1 - x\}$

Q.19

Let $B_1 = \{(1,2),\ (2,-1)\}$ and $B_2 = \{(1,0),\ (0,1)\}$ be ordered bases of $\mathbb{R}^2$. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $[T]_{B_1,B_2}$, the matrix of T with respect to B_1 and B_2, is $\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$, then $T(5,5)$ is equal to

(A) $(-9,\ 8)$ (B) $(9,\ 8)$ (C) $(-15,\ -2)$ (D) $(15,\ 2)$

Q.20

Let $S = \cap_{n=1}^{\infty}\left(\left[0,\ \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n},\ 1\right]\right)$. Which one of the following statements is **FALSE**?

(A) There exist sequences $\{a_n\}$ and $\{b_n\}$ in [0,1] such that $S = [0,1] \setminus \cup_{n=1}^{\infty}(a_n,\ b_n)$
(B) $[0,1] \setminus S$ is an open set
(C) If A is an infinite subset of S, then A has a limit point
(D) There exists an infinite subset of S having no limit points

Q.21 Let $f: \mathbb{R} \to \mathbb{R}$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in $[0,1]$, then the sequence $\{f(a_n)\}$ is

(A) increasing (B) bounded
(C) convergent (D) not necessarily bounded

Q.22

Which one of the following statements is true for the series $\sum_{n=1}^{\infty}(-1)^n \frac{(2n)!}{n^{2n}}$?

(A) The series converges conditionally but not absolutely
(B) The series converges absolutely
(C) The sequence of partial sums of the series is bounded but not convergent
(D) The sequence of partial sums of the series is unbounded

Q.23

The sequence $\left\{\cos\left(\frac{1}{2}\tan^{-1}\left(-\frac{n}{2}\right)^n\right)\right\}$ is

(A) monotone and convergent
(B) monotone but not convergent
(C) convergent but not monotone
(D) neither monotone nor convergent

Q.24

If $y(t)$ is a solution of the differential equation $y'' + 4y = 2e^t$, then

$$\lim_{t\to\infty} e^{-t}\, y(t)$$

is equal to

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{2}{7}$ (D) $\frac{2}{9}$

Q.25

For what real values of x and y, does the integral $\int_x^y (6 - t - t^2)dt$ attain its maximum?

(A) $x = -3,\ y = 2$ (B) $x = 2,\ y = 3$
(C) $x = -2,\ y = 2$ (D) $x = -3,\ y = 4$

Q.26 The area of the planar region bounded by the curves $x = 6y^2 - 2$ and $x = 2y^2$ is

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{2\sqrt{2}}{3}$ (C) $\frac{4\sqrt{2}}{3}$ (D) $\sqrt{2}$

Q.27

For $n \geq 2$, let $f_n: \mathbb{R} \to \mathbb{R}$ be given by $f_n(x) = x^n \sin x$. Then at $x = 0$, f_n has a

(A) local maximum if n is even

(B) local maximum if n is odd

(C) local minimum if n is even

(D) local minimum if n is odd

Q.28

For $m, n \in \mathbb{N}$, define $f_{m,n}(x) = \begin{cases} x^m \sin\left(\frac{1}{x^n}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$

Then at $x = 0$, $f_{m,n}$ is

(A) differentiable for each pair m, n with $m > n$

(B) differentiable for each pair m, n with $m < n$

(C) not differentiable for each pair m, n with $m > n$

(D) not differentiable for each pair m, n with $m < n$

Q.29

Let G and H be nonempty subsets of $\mathbb{R}$, where G is connected and $G \cup H$ is not connected. Which one of the following statements is true for all such G and H ?

(A) If $G \cap H = \emptyset$, then H is connected

(B) If $G \cap H = \emptyset$, then H is not connected

(C) If $G \cap H \neq \emptyset$, then H is connected

(D) If $G \cap H \neq \emptyset$, then H is not connected

Q.30

Let $f : \{ (x, y) \in \mathbb{R}^2 : x > 0,\ y > 0 \} \to \mathbb{R}$ be given by

$$f(x, y) = x^{\frac{1}{3}} y^{\frac{-4}{3}} \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{\sqrt{x^2 + y^2}}$$

Then the value of

$$g(x, y) = \frac{x f_x(x,y) + y f_y(x,y)}{f(x,y)}$$

(A) changes with x but not with y

(B) changes with y but not with x

(C) changes with x and also with y

(D) neither changes with x nor with y

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1

Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \int_{-5}^{x} (t-1)^3 dt$.
In which of the following interval(s), f takes the value 1?

(A) $[-6,\ 0]$ (B) $[-2,\ 4]$ (C) $[2,\ 8]$ (D) $[6,\ 12]$

Q.2

Which of the following statements is (are) true?

(A) $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$

(B) $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_9$

(C) $\mathbb{Z}_4 \oplus \mathbb{Z}_6$ is isomorphic to $\mathbb{Z}_{24}$

(D) $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{30}$

Q.3

Which of the following conditions implies (imply) the convergence of a sequence $\{x_n\}$ of real numbers?

(A) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $|x_{n+1} - x_n| < \varepsilon$

(B) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $\frac{1}{(n+1)^2}|x_{n+1} - x_n| < \varepsilon$

(C) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $(n+1)^2|x_{n+1} - x_n| < \varepsilon$

(D) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all m, n with $m > n \geq n_0$, $|x_m - x_n| < \varepsilon$

Q.4

Let $\vec{F}$ be a vector field given by $\vec{F}(x, y, z) = -y\,\hat{\imath} + 2\,x\,y\,\hat{\jmath} + z^3\hat{k}$, for $(x, y, z) \in \mathbb{R}^3$. If C is the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $y + z = 2$, then which of the following is (are) equal to $\left|\int_C \vec{F} \cdot d\vec{r}\right|$?

(A) $\int_0^{2\pi}\int_0^1 (1 + 2r\sin\theta) r\, dr d\theta$

(B) $\int_0^{2\pi}\left(\frac{1}{2} + \frac{2}{3}\sin\theta\right) d\theta$

(C) $\int_0^{2\pi}\int_0^1 (1 + 2r\sin\theta)\; dr d\theta$

(D) $\int_0^{2\pi}(1 + \sin\theta)\, d\theta$

Q.5

Let V be the set of 2×2 matrices $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with complex entries such that $a_{11} + a_{22} = 0$. Let W be the set of matrices in V with $a_{12} + \overline{a_{21}} = 0$. Then, under usual matrix addition and scalar multiplication, which of the following is (are) true?

(A) V is a vector space over $\mathbb{C}$
(B) W is a vector space over $\mathbb{C}$
(C) V is a vector space over $\mathbb{R}$
(D) W is a vector space over $\mathbb{R}$

Q.6

The initial value problem

$$y' = \sqrt{y},\; y(0) = \alpha,\; \alpha \geq 0$$

has

(A) at least two solutions if $\alpha = 0$

(B) no solution if $\alpha > 0$

(C) at least one solution if $\alpha > 0$

(D) a unique solution if $\alpha = 0$

Q.7 Which of the following statements is (are) true on the interval $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x < \cos(\sin x)$

(B) $\tan x < x$

(C) $\sqrt{1+x} < 1 + \frac{x}{2} - \frac{x^2}{8}$

(D) $\frac{1-x^2}{2} < \ln(2+x)$

Q.8 Let $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

At $(0,0)$,

(A) f is not continuous

(B) f is continuous, and both f_x and f_y exist

(C) f is differentiable

(D) f_x and f_y exist but f is not differentiable

Q.9

Let $f, g : [0,1] \to [0,1]$ be functions. Let $R(f)$ and $R(g)$ be the ranges of f and g, respectively. Which of the following statements is (are) true?

(A) If $f(x) \leq g(x)$ for all $x \in [0,1]$, then sup $R(f) \leq$ inf $R(g)$

(B) If $f(x) \leq g(x)$ for some $x \in [0,1]$, then inf $R(f) \leq$ sup $R(g)$

(C) If $f(x) \leq g(y)$ for some $x, y \in [0,1]$, then inf $R(f) \leq$ sup $R(g)$

(D) If $f(x) \leq g(y)$ for all $x, y \in [0,1]$, then sup $R(f) \leq$ inf $R(g)$

Q.10

Let $f: (-1,1) \longrightarrow \mathbb{R}$ be the function defined by

$$f(x) = x^2 e^{1/(1-x^2)}$$

Then

(A) f is decreasing in $(-1,0)$

(B) f is increasing in $(0,1)$

(C) $f(x) = 1$ has two solutions in $(-1,1)$

(D) $f(x) = 1$ has no solutions in $(-1,1)$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

Q.1

Let C be the straight line segment from $P(0,\ \pi)$ to $Q\left(4,\ \frac{\pi}{2}\right)$, in the xy-plane. Then the value of $\int_C e^x(\cos y\, dx - \sin y\, dy)$ is _____

Q.2

Let S be the portion of the surface $z = \sqrt{16 - x^2}$ bounded by the planes $x = 0,\ x = 2,\ y = 0,$ and $y = 3$. The surface area of S, correct upto three decimal places, is _____

Q.3

The number of distinct normal subgroups of S_3 is _____

Q.4

Let $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \left(1 + \frac{x}{y}\right)^2, & y \neq 0 \\ 0, & y = 0 \end{cases}$$

If the directional derivative of f at $(0,0)$ exists along the direction $\cos\alpha\ \hat{\imath} + \sin\alpha\ \hat{\jmath}$, where $\sin\alpha \neq 0$, then the value of $\cot\alpha$ is _____

Q.5

Let $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y,z) = \sin x + 2\, e^{\frac{y}{2}} + z^2$$

The maximum rate of change of f at $\left(\frac{\pi}{4},\ 0,\ 1\right)$, correct upto three decimal places, is _____

Q.6

If the power series

$$\sum_{n=0}^{\infty} \frac{n!}{n^n} x^{2n}$$

converges for $|x| < c$ and diverges for $|x| > c$, then the value of c, correct upto three decimal places, is _____

Q.7

If $5^{2015} \equiv n \pmod{11}$ and $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then n is equal to _____

Q.8

If the set $\left\{\begin{bmatrix} x & -x \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ x & x \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\right\}$ is linearly dependent in the vector space of all 2×2 matrices with real entries, then x is equal to _____

Q.9

Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^6 - 1, & x \in \mathbb{Q} \\ 1 - x^6, & x \notin \mathbb{Q} \end{cases}$$

The number of points at which f is continuous, is _____

Q.10

Let $f: (0,1) \to \mathbb{R}$ be a continuously differentiable function such that f' has finitely many zeros in $(0,1)$ and f' changes sign at exactly two of these points. Then for any $y \in \mathbb{R}$, the maximum number of solutions to $f(x) = y$ in $(0,1)$ is _____

Q. 11 – Q. 20 carry two marks each.

Q.11 Let R be the planar region bounded by the lines $x = 0$, $y = 0$ and the curve $x^2 + y^2 = 4$, in the first quadrant. Let C be the boundary of R, oriented counter-clockwise. Then the value of

$$\oint_C x(1-y)\,dx + (x^2 - y^2)\,dy$$

is _____

Q.12 Suppose G is a cyclic group and $\sigma, \tau \in G$ are such that order(σ) = 12 and order(τ) = 21. Then the order of the smallest group containing σ and τ is _____

Q.13 The limit

$$\lim_{n\to\infty} \sum_{k=2}^{n} \frac{1}{k^3 - k}$$

is equal to _____

Q.14 Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. Let V be a subspace of $M_2(\mathbb{R})$ defined by

$$V = \left\{ A \in M_2(\mathbb{R}) : A \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} A \right\}$$

Then the dimension of V is _____

Q.15

Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{y}{\sin y}, & y \neq 0 \\ 1, & y = 0 \end{cases}$$

Then the integral

$$\frac{1}{\pi^2}\int_{x=0}^{1}\int_{y=\sin^{-1}x}^{\frac{\pi}{2}} f(x,y)\,dy\,dx$$

correct upto three decimal places, is _____

Q.16

The coefficient of $\left(x-\frac{\pi}{4}\right)^3$ in the Taylor series expansion of the function

$$f(x) = 3\sin x \cos\left(x+\frac{\pi}{4}\right), \qquad x \in \mathbb{R}$$

about the point $\frac{\pi}{4}$, correct upto three decimal places, is _____

Q.17

If $\int_0^x \left(e^{-t^2}+\cos t\right)\,dt$ has the power series expansion $\sum_{n=1}^{\infty} a_n x^n$, then a_5, correct upto three decimal places, is equal to _____

Q.18

Let ℓ be the length of the portion of the curve $x = x(y)$ between the lines $y = 1$ and $y = 3$, where $x(y)$ satisfies

$$\frac{dx}{dy} = \frac{\sqrt{1+y^2+y^4}}{y}, \qquad x(1) = 0$$

The value of ℓ, correct upto three decimal places, is _____

Q.19 The limit

$$\lim_{x \to 0+} \frac{9}{x}\left(\frac{1}{\tan^{-1}x} - \frac{1}{x}\right)$$

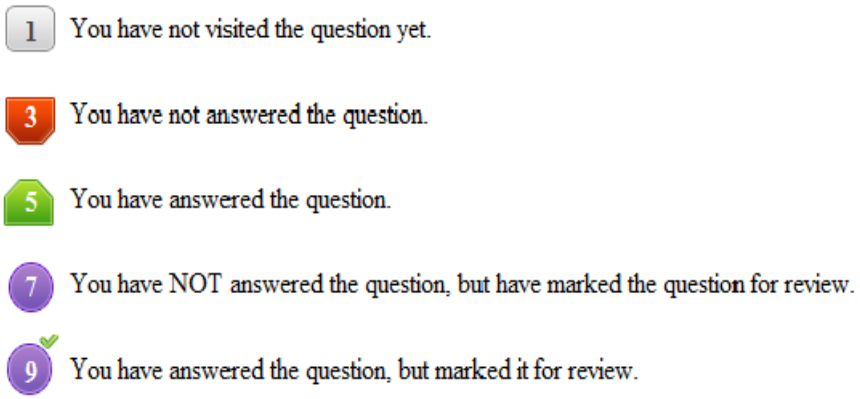
is equal to _____

Q.20 Let P and Q be two real matrices of size 4×6 and 5×4, respectively. If $\text{rank}(Q) = 4$ and $\text{rank}(QP) = 2$, then $\text{rank}(P)$ is equal to _____

END OF THE QUESTION PAPER

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

PH: Physics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

Useful Data

Speed of light in vacuum (c): 3×10^{8} m s^{-1}

Planck's constant (h): 6.62×10^{-34} J s

hbar times speed of light in vacuum ($\hbar c$): 200×10^{-15} MeV m

Universal gas constant (R): 8.3 J K^{-1} $mole^{-1}$

Mass of the electron (m_e): 9.1×10^{-31} Kg

Charge of the electron (e): -1.6×10^{-19} C

Acceleration due to gravity (g): 9.8 m s^{-2}

Free space permittivity (ε_0): 8.85×10^{-12} N m^2 C^{-2}

In multiple choice questions having numerical answers, choose the option nearest to the correct answer.

SECTION – A

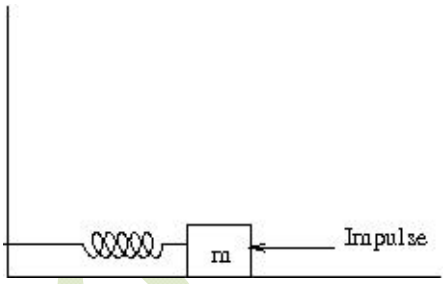
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 A system consists of N number of particles, $N \gg 1$. Each particle can have only one of the two energies E_1 or $E_1 + \varepsilon\,(\varepsilon > 0)$. If the system is in equilibrium at a temperature T, the average number of particles with energy E_1 is

(A) $\frac{N}{2}$
(B) $\frac{N}{e^{\varepsilon/kT}+1}$
(C) $\frac{N}{e^{-\varepsilon/kT}+1}$
(D) $Ne^{-\varepsilon/kT}$

Q.2 A mass m, lying on a horizontal, frictionless surface, is connected to one end of a spring. The other end of the spring is connected to a wall, as shown in the figure. At $t = 0$, the mass is given an impulse.



The time dependence of the displacement and the velocity of the mass (in terms of non-zero constants A and B) are given by

(A) $x(t) = A\ \sin\omega t,\ v(t) = B\ \cos\omega t$
(B) $x(t) = A\ \sin\omega t,\ v(t) = B\ \sin\omega t$
(C) $x(t) = A\ \cos\omega t,\ v(t) = B \sin\omega t$
(D) $x(t) = A\ \cos\omega t,\ v(t) = B\ \cos\omega t$

Q.3 A particle with energy E is incident on a potential given by

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x \geq 0 \end{cases}.$$

The wave function of the particle for $E < V_0$, in the region $x > 0$ (in terms of positive constants A, B and k) is

(A) $Ae^{kx} + Be^{-kx}$
(B) Ae^{-kx}
(C) $Ae^{ikx} + Be^{-ikx}$
(D) Zero

Q.4 The electric field of a light wave is given by $\vec{E} = E_0[\hat{\imath}\ \sin(\omega t - kz) + \hat{\jmath}\sin(\omega t - kz - \frac{\pi}{4})]$. The polarization state of the wave is

(A) Left handed circular
(B) Right handed circular
(C) Left handed elliptical
(D) Right handed elliptical

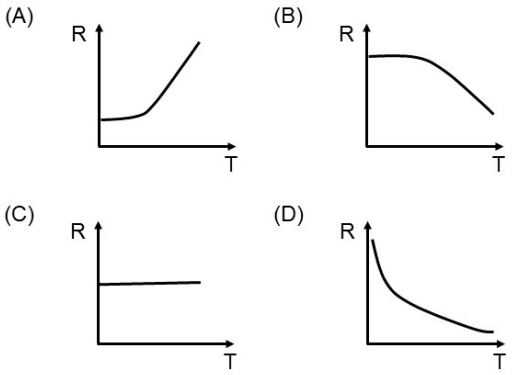
Q.5 Consider the coordinate transformation $x' = \frac{x+y}{\sqrt{2}}, y' = \frac{x-y}{\sqrt{2}}$. The relation between the area elements $dx'dy'$ and $dxdy$ is given by $dx'dy' = Jdxdy$. The value of J is

(A) 2 (B) 1 (C) -1 (D) -2

Q.6 The trace of a 2×2 matrix is 4 and its determinant is 8. If one of the eigenvalues is $2(1+i)$, the other eigenvalue is

(A) $2(1-i)$ (B) $2(1+i)$ (C) $(1+2i)$ (D) $(1-2i)$

Q.7 Temperature dependence of resistivity of a metal can be described by



Q.8 A proton from outer space is moving towards earth with velocity $0.99c$ as measured in earth's frame. A spaceship, traveling parallel to the proton, measures proton's velocity to be $0.97c$. The approximate velocity of the spaceship, in the earth's frame, is

(A) $0.2c$ (B) $0.3c$
(C) $0.4c$ (D) $0.5c$

Q.9 A charge q is at the center of two concentric spheres. The outward electric flux through the inner sphere is ϕ while that through the outer sphere is 2ϕ. The amount of charge contained in the region between the two spheres is

(A) $2q$ (B) q (C) $-q$ (D) $-2q$

Q.10 At room temperature, the speed of sound in air is 340 m/sec. An organ pipe with both ends open has a length L= 29 cm. An extra hole is created at the position L/2. The lowest frequency of sound produced is

(A) 293 Hz (B) 586 Hz (C) 1172 Hz (D) 2344 Hz

Q. 11 – Q. 30 carry two marks each.

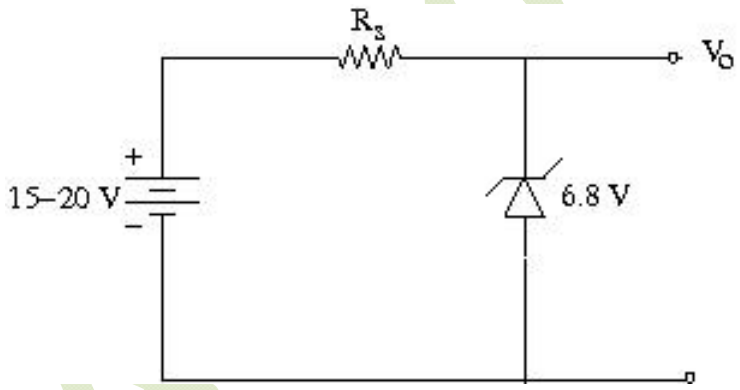
Q.11 A system comprises of three electrons. There are three single particle energy levels accessible to each of these electrons. The number of possible configurations for this system is

(A) 1 (B) 3 (C) 6 (D) 7

Q.12 A rigid and thermally isolated tank is divided into two compartments of equal volume V, separated by a thin membrane. One compartment contains one mole of an ideal gas A and the other compartment contains one mole of a different ideal gas B. The two gases are in thermal equilibrium at a temperature T. If the membrane ruptures, the two gases mix. Assume that the gases are chemically inert. The change in the total entropy of the gases on mixing is

(A) 0 (B) R ln 2 (C) $\frac{3}{2}R$ ln 2 (D) $2R$ ln 2

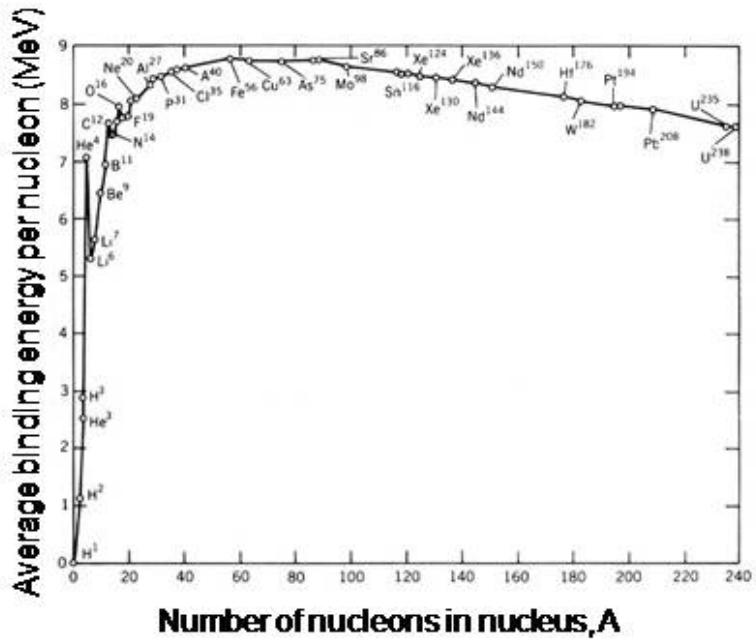
Q.13 A Zener regulator has an input voltage in the range 15V-20V and a load current in the range of 5 mA-20 mA. If the Zener voltage is 6.8V, the value of the series resistor should be



(A) 390 Ω (B) 420 Ω

(C) 440 Ω (D) 460 Ω

Q.14 The variation of binding energy per nucleon with respect to the mass number of nuclei is shown in the figure.



Consider the following reactions:

(1) $^{238}_{92}\mathrm{U} \rightarrow {}^{206}_{82}\mathrm{Pb} + 10p + 22n$
(2) $^{238}_{92}\mathrm{U} \rightarrow {}^{206}_{82}\mathrm{Pb} + 8^{4}_{2}He + 6e^{-}$
Which one of the following statements is true for the given decay modes of $^{238}_{92}\mathrm{U}$?

(A) Both (1) and (2) are allowed
(B) Both (1) and (2) are forbidden
(C) (1) is forbidden and (2) is allowed
(D) (1) is allowed and (2) is forbidden

Q.15 A rigid triangular molecule consists of three non-collinear atoms joined by rigid rods. The constant pressure molar specific heat (C_p) of an ideal gas consisting of such molecules is

(A) $6R$
(B) $5R$
(C) $4R$
(D) $3R$

Q.16 A satellite moves around the earth in a circular orbit of radius R centered at the earth. A second satellite moves in an elliptic orbit of major axis 8R, with the earth at one of the foci. If the former takes 1 day to complete a revolution, the latter would take

(A) 21.6 days (B) 8 days (C) 3 hours (D) 1.1 hour

Q.17 A positively charged particle, with a charge q, enters a region in which there is a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$, both directed parallel to the positive y- axis. At $t = 0$, the particle is at the origin and has a speed v_0 directed along the positive x- axis. The orbit of the particle, projected on the x-z plane, is a circle. Let T be the time taken to complete one revolution of this circle. The y-coordinate of the particle at $t = T$ is given by

(A) $\frac{\pi^2 mE}{2qB^2}$ (B) $\frac{2\pi^2 mE}{qB^2}$

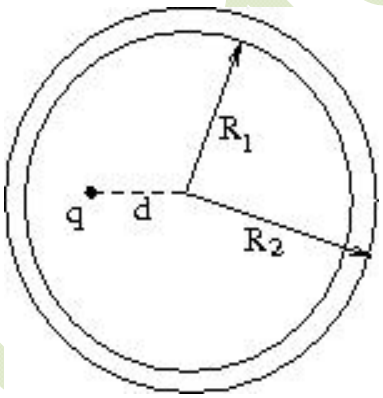
(C) $\frac{\pi^2 mE}{qB^2} + \frac{v_0 \pi m}{qB}$ (D) $\frac{2\pi m v_0}{qB}$

Q.18 Vibrations of diatomic molecules can be represented as those of harmonic oscillators. Two halogen molecules X_2 and Y_2 have fundamental vibrational frequencies $\nu_X = 16.7 \times 10^{12}$ Hz and $\nu_Y = 26.8 \times 10^{12}$ Hz, respectively. The respective force constants are $K_X = 325$ N/m and $K_Y = 446$ N/m. The atomic masses of F, Cl and Br are 19.0, 35.5 and 79.9 atomic mass unit respectively. The halogen molecules X_2 and Y_2 are

(A) $X_2 = F_2$ and $Y_2 = Cl_2$ (B) $X_2 = Cl_2$ and $Y_2 = F_2$

(C) $X_2 = Br_2$ and $Y_2 = F_2$ (D) $X_2 = F_2$ and $Y_2 = Br_2$

Q.19 A hollow, conducting spherical shell of inner radius R_1 and outer radius R_2 encloses a charge q inside, which is located at a distance d $(< R_1)$ from the centre of the spheres. The potential at the centre of the shell is



(A) Zero (B) $\frac{1}{4\pi\epsilon_0}\frac{q}{d}$

(C) $\frac{1}{4\pi\epsilon_0}\left(\frac{q}{d} - \frac{q}{R_1}\right)$ (D) $\frac{1}{4\pi\epsilon_0}\left(\frac{q}{d} - \frac{q}{R_1} + \frac{q}{R_2}\right)$

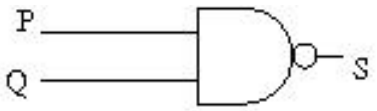
Q.20 Doppler effect can be used to measure the speed of blood through vessels. Sound of frequency 1.0522 MHz is sent through the vessels along the direction of blood flow. The reflected sound generates a beat signal of frequency 100 Hz. The speed of sound in blood is 1545 m/sec. The speed of blood through the vessel, in m/sec, is

(A) 14.68 (B) 1.468 (C) 0.1468 (D) 0.01468

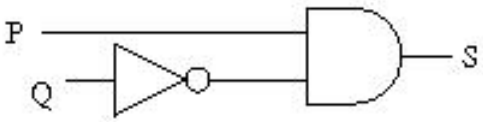
Q.21 Which of the following circuits represent the Boolean expression

$$S = \overline{\overline{P + QR} + \overline{Q}P}$$

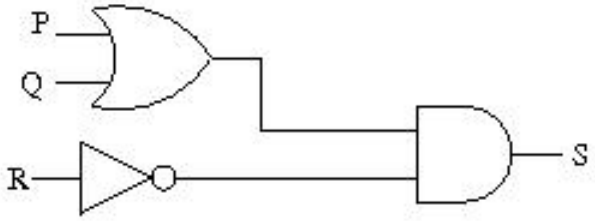
(A)



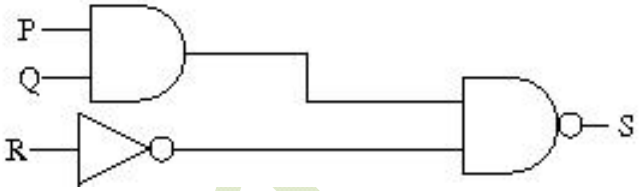
(B)



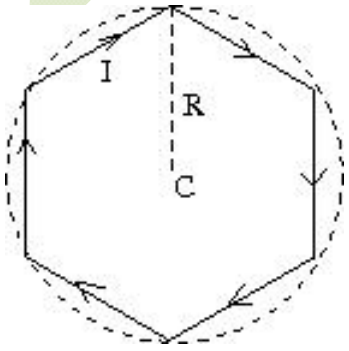
(C)



(D)



Q.22 A conducting wire is in the shape of a regular hexagon, which is inscribed inside an imaginary circle of radius R, as shown. A current I flows through the wire. The magnitude of the magnetic field at the center of the circle is



(A) $\frac{\sqrt{3}\mu_0 I}{2\pi R}$

(B) $\frac{\mu_0 I}{2\sqrt{3}\pi R}$

(C) $\frac{\sqrt{3}\mu_0 I}{\pi R}$

(D) $\frac{3\mu_0 I}{2\pi R}$

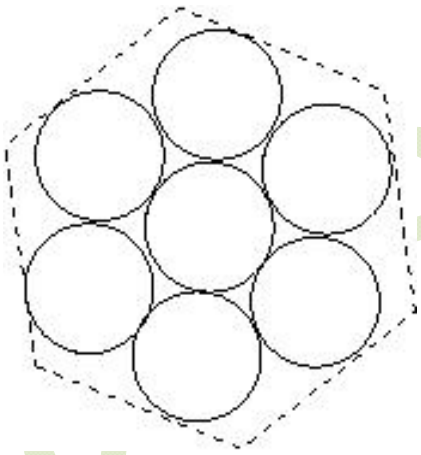
Q.23 An observer is located on a horizontal, circular turntable which rotates about a vertical axis passing through its center, with a uniform angular speed of 2 rad/sec. A mass of 10 grams is sliding without friction on the turntable. At an instant when the mass is at a distance of 8 cm from the axis, it is observed to move towards the center with a speed of 6 cm/sec. The net force on the mass, as seen by the observer at that instant, is

(A) 0.0024 N (B) 0.0032 N
(C) 0.004 N (D) 0.006 N

Q.24 Miller indices of a plane in cubic structure that contains all the directions [100], [011] and [111] are

(A) (011) (B) (101) (C) (100) (D) (110)

Q.25 Seven uniform disks, each of mass m and radius r, are inscribed inside a regular hexagon, as shown. The moment of inertia of this system of seven disks, about an axis passing through the central disk and perpendicular to the plane of the disks, is



(A) $\frac{7}{2}mr^2$ (B) $7mr^2$ (C) $\frac{13}{2}mr^2$ (D) $\frac{55}{2}mr^2$

Q.26 A nucleus has a size of 10^{-15} m. Consider an electron bound within a nucleus. The estimated energy of this electron is of the order of

(A) 1 MeV (B) 10^2 MeV
(C) 10^4 MeV (D) 10^6 MeV

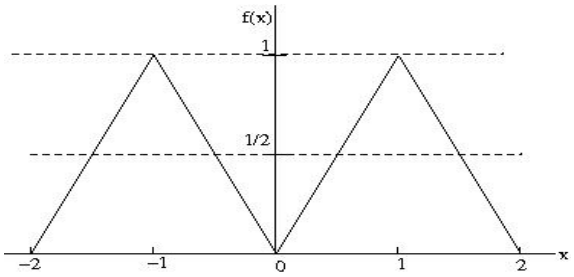
Q.27 Consider a vector field $\vec{F} = y\hat{\imath} + xz^3\hat{\jmath} - zy\hat{k}$. Let C be the circle $x^2 + y^2 = 4$ on the plane $z = 2$, oriented counter-clockwise. The value of the contour integral $\oint_C \vec{F} \cdot d\vec{r}$ is

(A) 28π (B) 4π (C) -4π (D) -28π

Q.28 Consider the equation $\frac{dy}{dx} = \frac{y^2}{x}$ with the boundary condition $y(1) = 1$. Ou t of the following, the range of x in which y is real and finite, is

(A) $-\infty \leq x \leq -3$ (B) $-3 \leq x \leq 0$
(C) $0 \leq x \leq 3$ (D) $3 \leq x \leq \infty$

Q.29 The Fourier series for an arbitrary periodic function with period $2L$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$. For the particular periodic function shown in the figure, the value of a_0 is



(A) 0 (B) 0.5 (C) 1 (D) 2

Q.30 The phase of the complex number $(1 + i)i$ in the polar representation is

(A) $\pi/4$
(B) $\pi/2$
(C) $3\pi/4$
(D) $5\pi/4$

SECTION - B

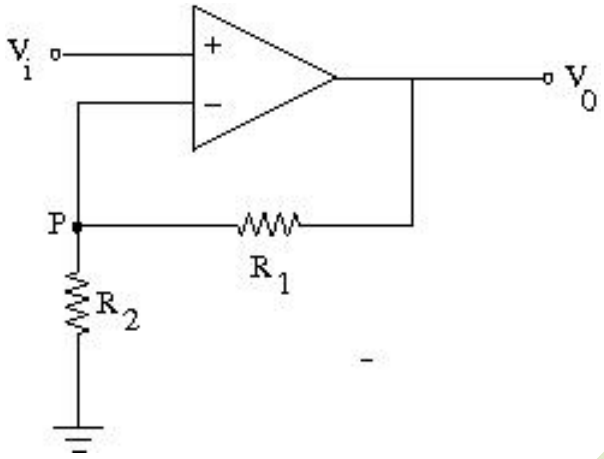
MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1 For an electromagnetic wave traveling in free space, the electric field is given by $\vec{E} = 100\cos(10^8 t + kx)\hat{j}$ V/m. Which of the following statements are true?

(A) The wavelength of the wave in meter is 6π
(B) The corresponding magnetic field is directed along the positive z direction
(C) The Poynting vector is directed along the positive z direction
(D) The wave is linearly polarized

Q.2 In an ideal Op-Amp circuit shown below, R_1=3kΩ, R_2=1 kΩ and $V_i = 0.5 \sin \omega t$ (in Volt). Which of the following statements are true?

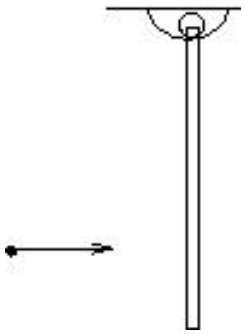


(A) The current through R_1= The current through R_2
(B) The potential at P is $V_0 \frac{R_2}{R_1}$
(C) The amplitude of V_0 is 2V
(D) The output voltage V_0 is in phase with V_i

Q.3 A particle of mass m is moving in x-y plane. At any given time t, its position vector is given by $\vec{r}(t) = A\cos\omega t\,\hat{\imath} + B\sin\omega t\,\hat{\jmath}$, where A, B and ω are constants with $A \neq B$. Which of the following statements are true?

(A) Orbit of the particle is an ellipse
(B) Speed of the particle is constant
(C) At any given time t, the particle experiences a force towards origin
(D) The angular momentum of the particle is $m\omega AB\hat{k}$

Q.4 A rod is hanging vertically from a pivot. A particle, traveling in horizontal direction, collides with the rod as shown in the figure. For the rod-particle system, consider the linear momentum and the angular momentum about the pivot. Which of the following statements are **NOT** true?



(A) Both linear momentum and angular momentum are conserved
(B) Linear momentum is conserved but angular momentum is not
(C) Linear momentum is not conserved but angular momentum is conserved
(D) Neither linear momentum nor angular momentum are conserved

Q.5 A particle is moving in a two-dimensional potential well

$$V(x,y) = 0, \quad 0 \le x \le L, 0 \le y \le 2L$$
$$= \infty, \quad \text{elsewhere.}$$

Which of the following statements about the ground state energy E_1 and ground state eigenfunction φ_0 are true?

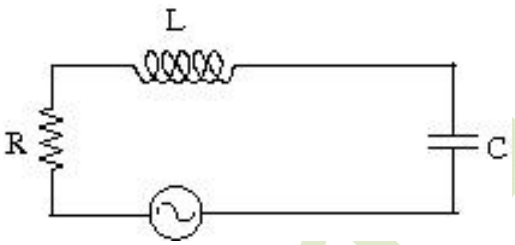
(A) $E_1 = \frac{\hbar^2\pi^2}{mL^2}$

(B) $E_1 = \frac{5\hbar^2\pi^2}{8mL^2}$

(C) $\varphi_0 = \frac{\sqrt{2}}{L}\sin\frac{\pi x}{L}\sin\frac{\pi y}{2L}$

(D) $\varphi_0 = \frac{\sqrt{2}}{L}\cos\frac{\pi x}{L}\cos\frac{\pi y}{2L}$

Q.6 Consider the circuit, consisting of an AC function generator $V(t) = V_0 \sin 2\pi\nu t$ with $V_0 = 5V$, an inductor $L = 8.0\ mH$, resistor $R = 5\ \Omega$ and a capacitor $C = 100\ \mu F$. Which of the following statements are true if we vary the frequency?

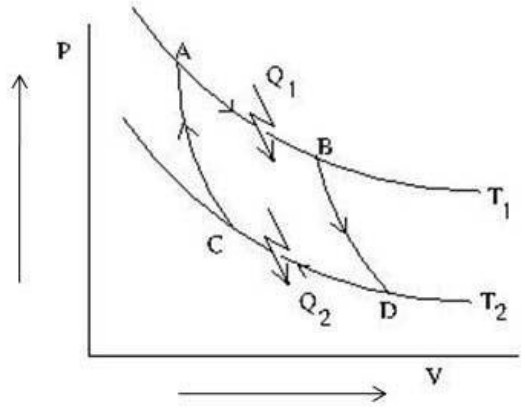


(A) The current in the circuit would be maximum at $\nu = 178$ Hz
(B) The capacitive reactance increases with frequency
(C) At resonance, the impedance of the circuit is equal to the resistance in the circuit
(D) At resonance, the current in the circuit is out of phase with the source voltage

Q.7 Muons are elementary particles produced in the upper atmosphere. They have a life time of 2.2 μs. Consider muons which are traveling vertically towards the earth's surface at a speed of $0.998c$. For an observer on earth, the height of the atmosphere above the surface of the earth is 10.4 km. Which of the following statements are true?

(A) The muons can never reach earth's surface
(B) The apparent thickness of earth's atmosphere in muon's frame of reference is 0.96 km
(C) The lifetime of muons in earth's frame of reference is 34.8 μs
(D) Muons traveling at a speed greater than $0.998c$ reach the earth's surface

Q.8 As shown in the P-V diagram, AB and CD are two isotherms at temperatures T_1 and T_2, respectively ($T_1 > T_2$). AC and BD are two reversible adiabats. In this Carnot cycle, which of the following statements are true?



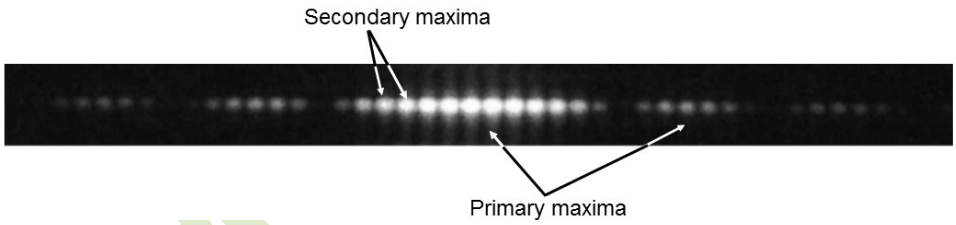
(A) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

(B) The entropy of the source decreases

(C) The entropy of the system increases

(D) Work done by the system $W = Q_1 - Q_2$

Q.9 The following figure shows a double slit Fraunhofer diffraction pattern produced by two slits, each of width a, separated by a distance b, $a < b$. Which of the following statements are correct?



(A) Reducing a increases the separation between consecutive primary maxima
(B) Reducing a increases the separation between consecutive secondary maxima
(C) Reducing b increases the separation between consecutive primary maxima
(D) Reducing b increases the separation between consecutive secondary maxima

Q.10 A unit cube made of a dielectric material has a polarization $\vec{P} = 3\hat{\imath} + 4\hat{\jmath}$ units. The edges of the cube are parallel to the Cartesian axes. Which of the following statements are true?

(A) The cube carries a volume bound charge of magnitude 5 units
(B) There is a charge of magnitude 3 units on both the surfaces parallel to the y-z plane
(C) There is a charge of magnitude 4 units on both the surfaces parallel to the x-z plane
(D) There is a net non-zero induced charge on the cube

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

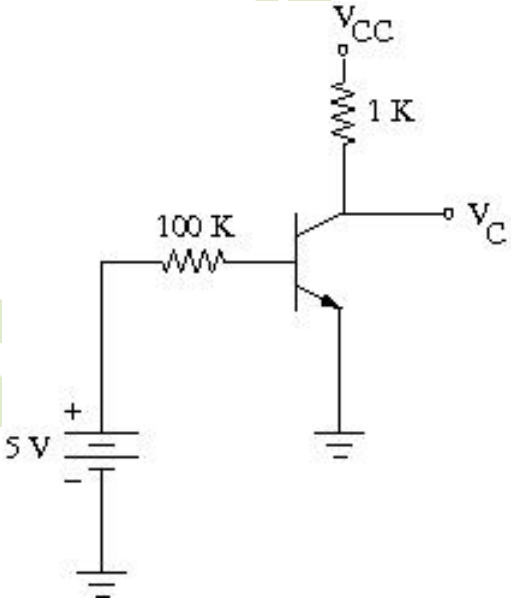
Q. 1 – Q. 10 carry one mark each.

Q.1 The power radiated by sun is 3.8×10^{26} W and its radius is 7×10^5 km. The magnitude of the Poynting vector (in W/cm^2) at the surface of the sun is _____.

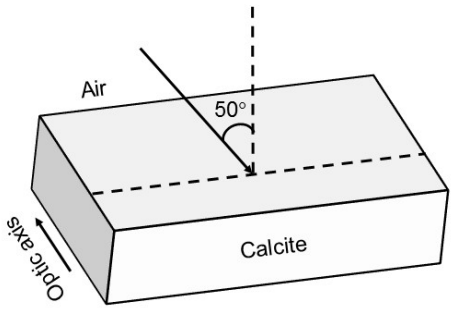
Q.2 A particle is in a state which is a superposition of the ground state φ_0 and the first excited state φ_1 of a one-dimensional quantum harmonic oscillator. The state is given by $\Phi = \frac{1}{\sqrt{5}}\varphi_0 + \frac{2}{\sqrt{5}}\varphi_1$. The expectation value of the energy of the particle in this state (in units of $\hbar\omega$, ω being the frequency of the oscillator) is _____.

Q.3 In an experiment on charging of an initially uncharged capacitor, an RC circuit is made with the resistance $R = 10$ kΩ and the capacitor C = 1000 μF along with a voltage source of 6V. The magnitude of the displacement current through the capacitor (in μA), 5 seconds after the charging has started, is _____.

Q.4 In the given circuit, V_{CC} = 10V and β = 100 for the n-p-n transistor. The collector voltage V_C (in Volts) is _____.



Q.5 Unpolarized light is incident on a calcite plate at an angle of incidence 50°, as shown in the figure. Take $n_0 = 1.6584$ and $n_e = 1.4864$ for calcite. The angular separation (in degrees) between the two emerging rays within the plate is _____.



Q.6 In the hydrogen atom spectrum, the ratio of the longest wavelength in the Lyman series (final state n = 1) to that in the Balmer series (final state n = 2) is _____.

Q.7 A rod is moving with a speed of $0.8c$ in a direction at 60° to its own length. The percentage contraction in the length of the rod is _____.

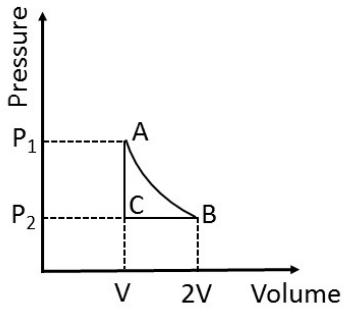
Q.8 X-rays of wavelength 0.24 nm are Compton scattered and the scattered beam is observed at an angle of 60^o relative to the incident beam. The Compton wavelength of the electron is 0.00243 nm. The kinetic energy of scattered electrons in eV is _____.

Q.9 A diode at room temperature ($kT = 0.025$ eV) with a current of 1μA has a forward bias voltage $V_F = 0.4$ V. For $V_F = 0.5$ V, the value of the diode current (in μA) is _____.

Q.10 GaAs has a diamond structure. The number of Ga-As bonds per atom which have to be broken to fracture the crystal in the (001) plane is _____.

Q. 11 – Q. 20 carry two marks each.

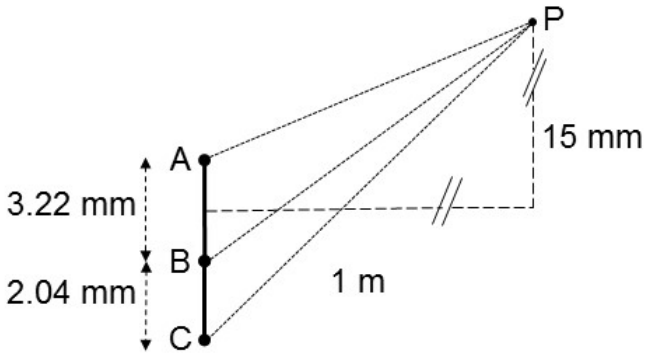
Q.11 In the thermodynamic cycle shown in the figure, one mole of a monatomic ideal gas is taken through a cycle. AB is a reversible isothermal expansion at a temperature of 800 K in which the volume of the gas is doubled. BC is an isobaric contraction to the original volume in which the temperature is reduced to 300 K. CA is a constant volume process in which the pressure and temperature return to their initial values. The net amount of heat (in Joules) absorbed by the gas in one complete cycle is _____.



Q.12 In a region of space, a time dependent magnetic field $B(t) = 0.4\,t$ Tesla points vertically upwards. Consider a horizontal, circular loop of radius 2 cm in this region. The magnitude of the electric field (in mV/m) induced in the loop is _____.

Q.13 A plane electromagnetic wave of frequency 5×10^{14} Hz and amplitude 10^3 V/m, traveling in a homogeneous dielectric medium of dielectric constant 1.69, is incident normally at the interface with a second dielectric medium of dielectric constant 2.25. The ratio of the amplitude of the transmitted wave to that of the incident wave is _____.

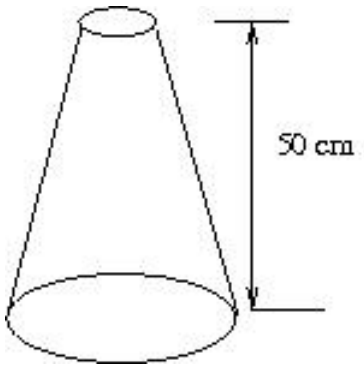
Q.14 For the arrangement given in the following figure, the coherent light sources A, B and C have individual intensities of 2 mW/m^2, 2 mW/m^2 and 5 mW/m^2, respectively at point P. The wavelength of each of the sources is 600 nm. The resultant intensity at point P (in mW/m^2) is _____.



Q.15 One gram of ice at 0^{o}C is melted and heated to water at 39^{o}C. Assume that the specific heat remains constant over the entire process. The latent heat of fusion of ice is 80 Calories/gm. The entropy change in the process (in Calories per degree) is _____.

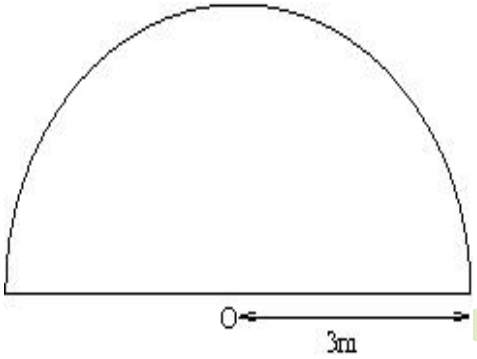
Q.16 A uniform disk of mass m and radius R rolls, without slipping, down a fixed plane inclined at an angle 30° to the horizontal. The linear acceleration of the disk (in m/sec^2) is _____.

Q.17 A nozzle is in the shape of a truncated cone, as shown in the figure. The area at the wide end is 25 cm^2 and the narrow end has an area of 1 cm^2. Water enters the wider end at a rate of 500 gm/sec. The height of the nozzle is 50 cm and it is kept vertical with the wider end at the bottom. The magnitude of the pressure difference in kPa (1 kPa = 10^3 N/m^2) between the two ends of the nozzle is _____.



Q.18 A block of mass 2 kg is at rest on a horizontal table. The coefficient of friction between the block and the table is 0.1. A horizontal force 3 N is applied to the block. The speed of the block (in m/s) after it has moved a distance 10 m is _____.

Q.19 A homogeneous semi-circular plate of radius R = 3 m is shown in the figure. The distance of the center of mass of the plate (in meter) from the point O is _____.



Q.20 Consider a 20μm diameter p-n junction fabricated in silicon. The donor density is 10^{16} per cm^3. The charge developed on the n-side is 1.6×10^{-13} C. Then the width (in μm) of the depletion region on the n-side of the p-n junction is _____.

END OF THE QUESTION PAPER