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# GATE 2024 Data Science and Artificial Intelligence (DA) Question Paper

Graduate Aptitude Test in Engineering (GATE)

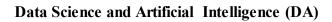
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# **General Aptitude (GA)**

# Q.1 – Q.5 Carry ONE mark Each

Q.1	If '→' denotes increasing order of intensity, then the meaning of the words				
	[sick $\rightarrow$ infirm $\rightarrow$ moribund] is analogous to [silly $\rightarrow$ $\rightarrow$ daft].				
	Which one of the given options is appropriate to fill the blank?				
(A)	frown				
(B)	fawn				
(C)	vein				
(D)	vain				





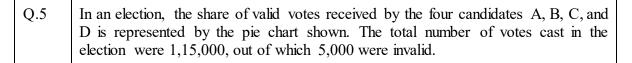
Q.2	The 15 parts of the given figure are to be painted such that no two adjacent parts with shared boundaries (excluding corners) have the same color. The minimum number of colors required is
(A)	4
(B)	3
(C)	5
(D)	6



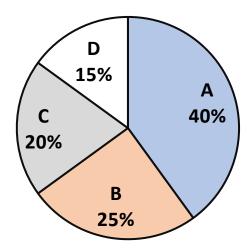


Q.3	How many 4-digit positive integers divisible by 3 can be formed using only the digits {1, 3, 4, 6, 7}, such that no digit appears more than once in a number?
(A)	24
(B)	48
(C)	72
(D)	12
Q.4	The sum of the following infinite series is
	$2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8} + \frac{1}{9} + \frac{1}{16} + \frac{1}{27} + \cdots$
(A)	11/3
(B)	7/2
(C)	13/4
(D)	9/2



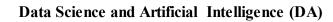


#### Share of valid votes



Based on the data provided, the total number of valid votes received by the candidates B and C is

(A)	45,000
(B)	49,500
(C)	51,750
(D)	54,000



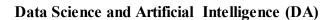


## Q.6 - Q.10 Carry TWO marks Each

Q.6	Thousands of years ago, some people began dairy farming. This coincided with a				
Q.0	number of mutations in a particular gene that resulted in these people developing the ability to digest dairy milk.				
	Based on the given passage, which of the following can be inferred?				
(A)	All human beings can digest dairy milk.				
(B)	No human being can digest dairy milk.				
(C)	Digestion of dairy milk is essential for human beings.				
(D)	In human beings, digestion of dairy milk resulted from a mutated gene.				
Q.7	The probability of a boy or a girl being born is 1/2. For a family having only three children, what is the probability of having two girls and one boy?				
(A)	3/8				
(B)	1/8				
(C)	1/4				
(D)	1/2				



Q.8	Person 1 and Person 2 invest in three mutual funds A, B, and C. The amounts they invest in each of these mutual funds are given in the table.					
			Mutual fund A	Mutual fund B	Mutual fund C	
		Person 1	₹10,000	₹20,000	₹20,000	
		Person 2	₹20,000	₹15,000	₹15,000	
	At the end of one year, the total amount that Person 1 gets is ₹500 more than Person 2. The annual rate of return for the mutual funds B and C is 15% each. What is the annual rate of return for the mutual fund A?					
(A)	7.5%					
(B)	10%					
(C)	15%					
(D)	20%					





Q.9 Three different views of a dice are shown in the figure below.







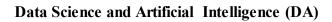
The piece of paper that can be folded to make this dice is

(A) 5 1 4 6 2 3

(B) 5 1 4 2 6 3

(C) 5 1 3 2 4 6

(D) 5 1 4 6 3 2



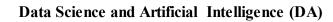


Q.10	Visualize two identical right circular cones such that one is inverted over the other and they share a common circular base. If a cutting plane passes through the vertices of the assembled cones, what shape does the outer boundary of the resulting cross-section make?
(A)	A rhombus
(B)	A triangle
(C)	An ellipse
(D)	A hexagon



# Q.11 – Q.35 Carry ONE mark Each

Q.11	Consider the following statements:  (i) The mean and variance of a Poisson random variable are equal.  (ii) For a standard normal random variable, the mean is zero and the variance is one.  Which ONE of the following options is correct?
(A)	Both (i) and (ii) are true
(B)	(i) is true and (ii) is false
(C)	(ii) is true and (i) is false
(D)	Both (i) and (ii) are false
Q.12	Three fair coins are tossed independently. $T$ is the event that two or more tosses result in heads. $S$ is the event that two or more tosses result in tails.
	What is the probability of the event $T \cap S$ ?
(A)	0
(B)	0.5
(C)	0.25
(D)	1





Q.13	Consider the matrix $\mathbf{M} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ .  Which <b>ONE</b> of the following statements is <b>TRUE</b> ?		
(A)	The eigenvalues of <b>M</b> are non-negative and real.		
(B)	The eigenvalues of <b>M</b> are complex conjugate pairs.		
(C)	One eigenvalue of $M$ is positive and real, and another eigenvalue of $M$ is zero.		
(D)	One eigenvalue of $M$ is non-negative and real, and another eigenvalue of $M$ is negative and real.		

Q.14	Consider performing depth-first search (DFS) on an undirected and unweighted graph $G$ starting at vertex $s$ . For any vertex $u$ in $G$ , $d[u]$ is the length of the shortest path from $s$ to $u$ . Let $(u, v)$ be an edge in $G$ such that $d[u] < d[v]$ . If the edge $(u, v)$ is explored first in the direction from $u$ to $v$ during the above DFS, then $(u, v)$ becomes a edge.
(A)	tree
(B)	cross
(C)	back
(D)	gray





Q.15	For any twice differentiable function $f: \mathbb{R} \to \mathbb{R}$ , if at some $x^* \in \mathbb{R}$ , $f'(x^*) = 0$ and $f''(x^*) > 0$ , then the function $f$ necessarily has a at $x = x^*$ .  Note: $\mathbb{R}$ denotes the set of real numbers.
(A)	local minimum
(B)	global minimum
(C)	local maximum
(D)	global maximum

Q.16 Match the items in Column 1 with the items in Column 2 in the following table:

Column 1		Column 2
(p) First In First Out (q) Lookup Operation (r) Last In First Out	(i) (ii) (iii)	Stacks Queues Hash Tables

(A) 
$$(p) - (ii), (q) - (iii), (r) - (i)$$

(B) 
$$(p) - (ii), (q) - (i), (r) - (iii)$$

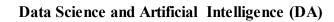
(C) 
$$(p) - (i), (q) - (ii), (r) - (iii)$$

(D) 
$$(p) - (i), (q) - (iii), (r) - (ii)$$



Q.17	Consider the dataset with six datapoints: $\{(x_1, y_1), (x_2, y_2), \dots, (x_6, y_6)\}$ , where $x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ , $x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ , $x_3 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ , $x_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ , $x_5 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ , $x_6 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ and the labels are given by $y_1 = y_2 = y_5 = 1$ , and $y_3 = y_4 = y_6 = -1$ . A hard margin linear support vector machine is trained on the above dataset.  Which <b>ONE</b> of the following sets is a possible set of support vectors?
(A)	$\{x_1, x_2, x_5\}$
(B)	$\{x_3, x_4, x_5\}$
(C)	$\{x_4, x_5\}$
(D)	$\{x_1, x_2, x_3, x_4\}$

Q.18	Match the items in Column 1 with the item	s in Column 2 in the following table:
	Column 1	Column 2
	(p) Principal Component Analysis	(i) Discriminative Model
	(q) Naïve Bayes Classification	(ii) Dimensionality Reduction
	(r) Logistic Regression	(iii) Generative Model
(A)	(p) - (iii), (q) - (i), (r) - (ii)	
(B)	(p) - (ii), (q) - (i), (r) - (iii)	
(C)	(p) - (ii), (q) - (iii), (r) - (i)	
(D)	(p) - (iii), (q) - (ii), (r) - (i)	





Q.19	Euclidean distance based $k$ -means clustering algorithm was run on a dataset of 100 points with $k = 3$ . If the points $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ are both part of cluster 3, then which <b>ONE</b> of the following points is necessarily also part of cluster 3?
(A)	
(B)	$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$
(C)	$\begin{bmatrix} 2 \\ 0 \end{bmatrix}$
(D)	

Q.20	Given a dataset with $K$ binary-valued attributes (where $K > 2$ ) for a two-class classification task, the number of parameters to be estimated for learning a naïve Bayes classifier is
(A)	$2^K + 1$
(B)	2K+1
(C)	$2^{K+1} + 1$
(D)	$K^2 + 1$



Q.21	Consider performing uniform hashing on an open address hash table with load factor $\alpha = \frac{n}{m} < 1$ , where $n$ elements are stored in the table with $m$ slots. The expected number of probes in an unsuccessful search is at most $\frac{1}{1-\alpha}$ .  Inserting an element in this hash table requires at most probes, on average.
(A)	$\ln\left(\frac{1}{1-\alpha}\right)$
(B)	$\frac{1}{1-\alpha}$
(C)	$1+\frac{\alpha}{2}$
(D)	$\frac{1}{1+\alpha}$

Q.22 For any binary classification dataset, let  $S_B \in \mathbb{R}^{d \times d}$  and  $S_W \in \mathbb{R}^{d \times d}$  be the between-class and within-class scatter (covariance) matrices, respectively. The Fisher linear discriminant is defined by  $u^* \in \mathbb{R}^d$ , that maximizes

$$J(u) = \frac{u^T S_B u}{u^T S_W u}$$

If  $\lambda = J(u^*)$ ,  $S_W$  is non-singular and  $S_B \neq 0$ , then  $(u^*, \lambda)$  must satisfy which **ONE** of the following equations?

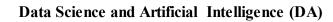
Note:  $\mathbb{R}$  denotes the set of real numbers.

- $(A) S_W^{-1} S_B u^* = \lambda u^*$
- (B)  $S_W u^* = \lambda S_B u^*$
- (C)  $S_B S_W u^* = \lambda u^*$
- (D)  $u^{*T}u^* = \lambda^2$



Q.23	Let $h_1$ and $h_2$ be two admissible heuristics used in $A^*$ search.  Which <b>ONE</b> of the following expressions is always an admissible heuristic?
(A)	$h_1 + h_2$ $h_1 \times h_2$
(C)	$h_1/h_2, (h_2 \neq 0)$
(D)	$ h_1 - h_2 $

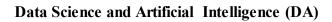
Q.24	Consider five random variables $U, V, W, X$ , and $Y$ whose joint distribution satisfies:
	P(U,V,W,X,Y) = P(U)P(V)P(W U,V)P(X W)P(Y W)
	Which ONE of the following statements is FALSE?
(A)	Y is conditionally independent of $V$ given $W$
(B)	X is conditionally independent of $U$ given $W$
(C)	U and $V$ are conditionally independent given $W$
(D)	Y and $X$ are conditionally independent given $W$





Q.25	Consider the following statement:
	In adversarial search, $\alpha - \beta$ pruning can be applied to game trees of any depth where $\alpha$ is the <u>(m)</u> value choice we have formed so far at any choice point along the path for the MAX player and $\beta$ is the <u>(n)</u> value choice we have formed so far at any choice point along the path for the MIN player.
	Which ONE of the following choices of (m) and (n) makes the above statement valid?
(A)	(m) = highest, (n) = highest
(B)	(m) = lowest, (n) = highest
(C)	(m) = highest, (n) = lowest
(D)	(m) = lowest, (n) = lowest

Q.26	Consider a database that includes the following relations:
	Defender(name, rating, side, goals)
	Forward(name, rating, assists, goals)
	Team(name, club, price)
	Which <b>ONE</b> of the following relational algebra expressions checks that every name occurring in Team appears in either Defender or Forward, where $\phi$ denotes the empty set?
(A)	$\Pi_{name}$ (Team) $\setminus (\Pi_{name}$ (Defender) $\cap \Pi_{name}$ (Forward)) = $\phi$
(B)	$(\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$
(C)	$\Pi_{name}(\text{Team}) \setminus \left(\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})\right) = \phi$
(D)	$(\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$





Q.27	Let the minimum, maximum, mean and standard deviation values for the attribute <i>income</i> of data scientists be ₹46000, ₹170000, ₹96000, and ₹21000, respectively. The z-score normalized <i>income</i> value of ₹106000 is closest to which <b>ONE</b> of the following options?
(A)	0.217
(B)	0.476
(C)	0.623
(D)	2.304

Q.28	Consider the following tree traversals on a full binary tree:
	(i) Preorder (ii) Inorder (iii) Postorder
	Which of the following traversal options is/are sufficient to uniquely reconstruct the full binary tree?
(A)	(i) and (ii)
(B)	(ii) and (iii)
(C)	(i) and (iii)
(D)	(ii) only





Q.29	Let $x$ and $y$ be two propositions. Which of the following statements is a tautology /are tautologies?
(A)	$(\neg x \land y) \Rightarrow (y \Rightarrow x)$
(B)	$(x \land \neg y) \Rightarrow (\neg x \Rightarrow y)$
(C)	$(\neg x \land y) \Rightarrow (\neg x \Rightarrow y)$
(D)	$(x \land \neg y) \Rightarrow (y \Rightarrow x)$

Q.30 Consider sorting the following array of integers in ascending order using an in-place Quicksort algorithm that uses the last element as the pivot.

60 70 80 90 100
-----------------

The minimum number of swaps performed during this Quicksort is \_\_\_\_\_.



Q.31 Consider the following two tables named Raider and Team in a relational database maintained by a Kabaddi league. The attribute *ID* in table Team references the primary key of the Raider table, *ID*.

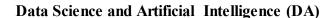
Raider				
ID	Name	Raids	RaidPoints	
1	Arjun	200	250	
2	Ankush	190	219	
3	Sunil	150	200	
4	Reza	150	190	
5	Pratham	175	220	
6	Gopal	193	215	

Team		
City	ID	<b>BidPoints</b>
Jaipur	2	200
Patna	3	195
Hyderabad	5	175
Jaipur	1	250
Patna	4	200
Jaipur	6	200

The SQL query described below is executed on this database:

SELECT \*
FROM Raider, Team
WHERE Raider.ID=Team.ID AND City="Jaipur" AND
RaidPoints > 200;

The number of rows returned by this query is .





### Q.32 The fundamental operations in a double-ended queue D are:

insertFirst(e) - Insert a new element e at the beginning of D.

insertLast(e) - Insert a new element e at the end of D.

removeFirst() - Remove and return the first element of D.

removeLast() - Remove and return the last element of D.

In an empty double-ended queue, the following operations are performed:

insertFirst(10)

insertLast(32)

a ←removeFirst()

insertLast(28)

insertLast(17)

- a ←removeFirst()
- $\mathbf{a} \leftarrow \text{removeLast()}$

The value of **a** is .

# Q.33 Let $f: \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \frac{1}{1 + e^{-x}}$ .

The value of the derivative of f at x where f(x) = 0.4 is \_\_\_\_\_ (rounded off to **two** decimal places).

**Note**:  $\mathbb{R}$  denotes the set of real numbers.



Q.34 The sample average of 50 data points is 40. The updated sample average after including a new data point taking the value of 142 is \_\_\_\_\_.

Consider the  $3 \times 3$  matrix  $\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 4 & 3 & 6 \end{bmatrix}$ .

The determinant of  $(\mathbf{M}^2 + 12\mathbf{M})$  is \_\_\_\_\_.

A fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) is repeatedly thrown independently.

What is the expected number of times the die is thrown until two consecutive throws of even numbers are seen?

(A) 2

(B) 4

(C) 6

(D) 8





Q.37 Let  $f: \mathbb{R} \to \mathbb{R}$  be a function. **Note**:  $\mathbb{R}$  denotes the set of real numbers.

$$f(x) = \begin{cases} -x, & \text{if } x < -2\\ ax^2 + bx + c, & \text{if } x \in [-2, 2]\\ x, & \text{if } x > 2 \end{cases}$$

Which **ONE** of the following choices gives the values of a, b, c that make the function f continuous and differentiable?

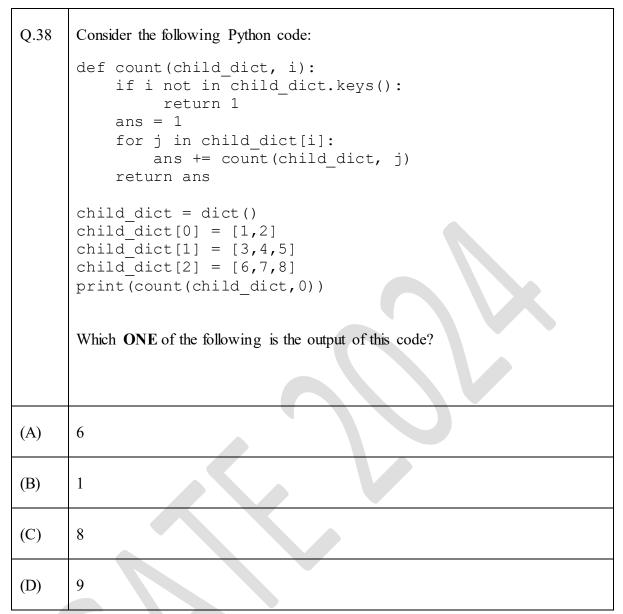
(A) 
$$a = \frac{1}{4}, b = 0, c = 1$$

(B) 
$$a = \frac{1}{2}, b = 0, c = 0$$

(C) 
$$a = 0, b = 0, c = 0$$

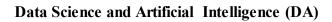
(D) 
$$a = 1, b = 1, c = -4$$







Q.39	Consider the function <b>computeS</b> (X) whose pseudocode is given below:
	$\frac{\texttt{computeS}(X)}{S[1] \leftarrow 1}$ for $i \leftarrow 2$ to $length(X)$ $S[i] \leftarrow 1$ if $X[i-1] \leq X[i]$ $S[i] \leftarrow S[i] + S[i-1]$ end if end for return $S$ Which $\textbf{ONE}$ of the following values is returned by the function $\textbf{computeS}(X)$ for $X = [6, 3, 5, 4, 10]$ ?
(A)	[1, 1, 2, 3, 4]
(B)	[1, 1, 2, 3, 3]
(C)	[1, 1, 2, 1, 2]
(D)	[1, 1, 2, 1, 5]





Q.40	Let $F(n)$ denote the maximum number of comparisons made while searching for an entry in a sorted array of size $n$ using binary search.  Which <b>ONE</b> of the following options is <b>TRUE</b> ?
(A)	$F(n) = F(\lfloor n/2 \rfloor) + 1$
(B)	$F(n) = F(\lfloor n/2 \rfloor) + F(\lceil n/2 \rceil)$
(C)	$F(n) = F(\lfloor n/2 \rfloor)$
(D)	F(n) = F(n-1) + 1

Q.41	Consider the following Python function:
	<pre>def fun(D, s1, s2):     if s1 &lt; s2:         D[s1], D[s2] = D[s2], D[s1]         fun(D, s1+1, s2-1)</pre>
	What does this Python function fun() do? Select the <b>ONE</b> appropriate option below.
(A)	It finds the smallest element in D from index s1 to s2, both inclusive.
(B)	It performs a merge sort in-place on this list D between indices s1 and s2, both inclusive.
(C)	It reverses the list D between indices s1 and s2, both inclusive.
(D)	It swaps the elements in D at indices s1 and s2, and leaves the remaining elements unchanged.

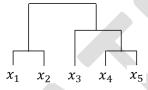


Q.42 Consider the table below, where the  $(i, j)^{th}$  element of the table is the distance between points  $x_i$  and  $x_j$ . Single linkage clustering is performed on data points,  $x_1, x_2, x_3, x_4, x_5$ .

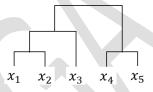
	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
$x_1$	0	1	4	3	6
$x_2$	1	0	3	5	3
$x_3$	4	3	0	2	5
$x_4$	3	5	2	0	1
$x_5$	6	3	5	1	0

Which ONE of the following is the correct representation of the clusters produced?

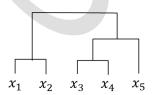
(A)



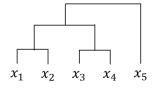
(B)



(C)



(D)





Consider the two neural networks (NNs) shown in Figures 1 and 2, with ReLU activation ( $ReLU(z) = \max\{0, z\}$ ,  $\forall z \in \mathbb{R}$ ).  $\mathbb{R}$  denotes the set of real numbers. The connections and their corresponding weights are shown in the Figures. The biases at every neuron are set to 0. For what values of p, q, r in Figure 2 are the two NNs equivalent, when  $x_1, x_2, x_3$  are positive?

Figure 1

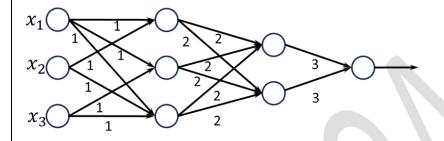
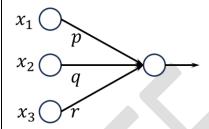
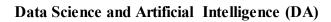


Figure 2



- (A) p = 36, q = 24, r = 24
- (B) p = 24, q = 24, r = 36
- (C) p = 18, q = 36, r = 24
- (D) p = 36, q = 36, r = 36





Q.44	Consider a state space where the start state is number 1. The successor function for the state numbered $n$ returns two states numbered $n+1$ and $n+2$ . Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list.  Which <b>ONE</b> of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?
(A)	BFS expands more states than DFS.
(B)	DFS expands more states than BFS.
(C)	Both BFS and DFS expand equal number of states.
(D)	Both BFS and DFS do not reach the goal state number 6.

Q.45	Consider the following sorting algorithms:
	(i) Bubble sort (ii) Insertion sort (iii) Selection sort
	Which <b>ONE</b> among the following choices of sorting algorithms sorts the numbers in the array [4, 3, 2, 1, 5] in increasing order after <b>exactly two</b> passes over the array?
(A)	(i) only
(B)	(iii) only
(C)	(i) and (iii) only
(D)	(ii) and (iii) only



Q.46	Given the relational schema $R = (U, V, W, X, Y, Z)$ and the set of functional
	dependencies:

$$\{U \to V, U \to W, WX \to Y, WX \to Z, V \to X\}$$

Which of the following functional dependencies can be derived from the above set?

(A) 
$$VW \rightarrow YZ$$

(B) 
$$WX \rightarrow YZ$$

(C) 
$$VW \rightarrow U$$

(D) 
$$VW \rightarrow Y$$



Q.47 Select all choices that are subspaces of  $\mathbb{R}^3$ .

**Note**:  $\mathbb{R}$  denotes the set of real numbers.

(A) 
$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \colon \mathbf{x} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$$

(B) 
$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \colon \mathbf{x} = \alpha^2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \beta^2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$$

(C) 
$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \colon 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0 \right\}$$

(D) 
$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \colon 5x_1 + 2x_3 + 4 = 0 \right\}$$



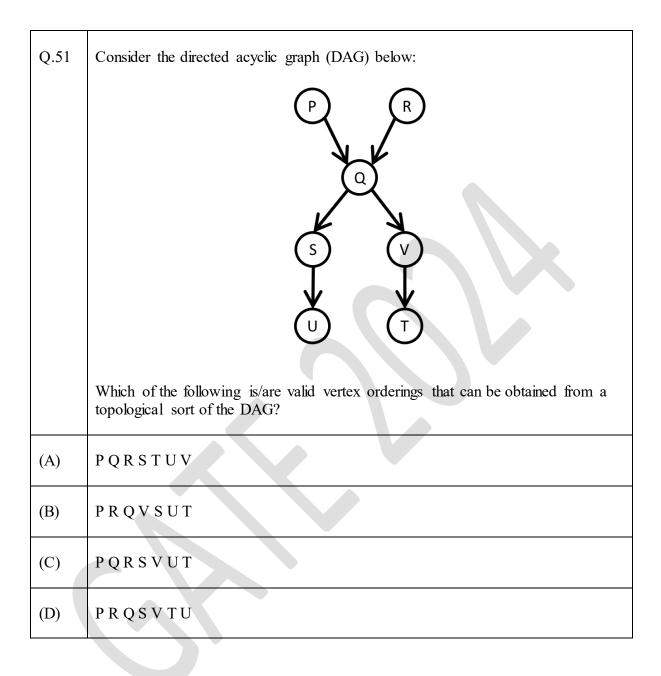
Q.48	Which of the following statements is/are TRUE?  Note: R denotes the set of real numbers.
(A)	There exist $M \in \mathbb{R}^{3\times 3}$ , $p \in \mathbb{R}^3$ , and $q \in \mathbb{R}^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.
(B)	There exist $M \in \mathbb{R}^{3 \times 3}$ , $p \in \mathbb{R}^3$ , and $q \in \mathbb{R}^3$ such that $Mx = p$ has no solutions and $Mx = q$ has infinite solutions.
(C)	There exist $M \in \mathbb{R}^{2\times 3}$ , $p \in \mathbb{R}^2$ , and $q \in \mathbb{R}^2$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.
(D)	There exist $M \in \mathbb{R}^{3 \times 2}$ , $p \in \mathbb{R}^3$ , and $q \in \mathbb{R}^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has no solutions.

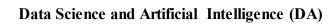


Q.49	Let $\mathbb{R}$ be the set of real numbers, $U$ be a subspace of $\mathbb{R}^3$ and $M \in \mathbb{R}^{3\times 3}$ be the matrix corresponding to the projection on to the subspace $U$ .
	Which of the following statements is/are TRUE?
(A)	If $U$ is a 1-dimensional subspace of $\mathbb{R}^3$ , then the null space of $M$ is a 1-dimensional subspace.
(B)	If $U$ is a 2-dimensional subspace of $\mathbb{R}^3$ , then the null space of $M$ is a 1-dimensional subspace.
(C)	$M^2 = M$
(D)	$M^3 = M$

Q.50	Consider the function $f: \mathbb{R} \to \mathbb{R}$ where $\mathbb{R}$ is the set of all real numbers. $f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2} + 1$ Which of the following statements is/are <b>TRUE</b> ?
(A)	x = 0 is a local maximum of $f$
(B)	x = 3 is a local minimum of $f$
(C)	x = -1 is a local maximum of $f$
(D)	x = 0 is a local minimum of $f$





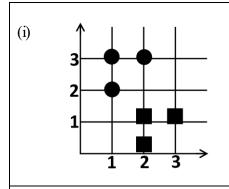


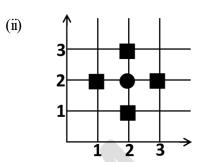


Q.52	Let <i>H</i> , <i>I</i> , <i>L</i> , and <i>N</i> represent height, number of internal nodes, number of leaf nodes, and the total number of nodes respectively in a rooted binary tree.
	Which of the following statements is/are always TRUE?
(A)	$L \le I + 1$
(B)	$H + 1 \le N \le 2^{H+1} - 1$
(C)	$H \le I \le 2^H - 1$
(D)	$H \le L \le 2^{H-1}$

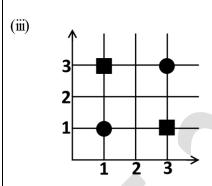


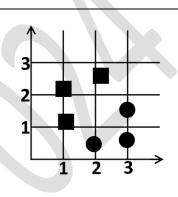
Q.53 Consider the following figures representing datasets consisting of two-dimensional features with two classes denoted by circles and squares.





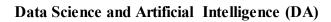
(iv)





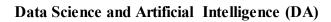
Which of the following is/are TRUE?

- (A) (i) is linearly separable.
- (B) (ii) is linearly separable.
- (C) (iii) is linearly separable.
- (D) (iv) is linearly separable.





Q.54	Let game(ball, rugby) be true if the ball is used in rugby and false otherwise.
	Let shape(ball, round) be true if the ball is round and false otherwise.
	Consider the following logical sentences:
	$s1: \forall ball \neg game(ball, rugby) \Longrightarrow shape(ball, round)$
	$s2: \forall ball \neg shape(ball, round) \Longrightarrow game(ball, rugby)$
	$s3: \forall ball \ game(ball, rugby) \Rightarrow \neg \ shape(ball, round)$
	$s4: \forall ball \ shape(ball, \ round) \Rightarrow \neg \ game(ball, \ rugby)$
	Which of the following choices is/are logical representations of the assertion,
	"All balls are round except balls used in rugby"?
(A)	s1 ∧ s3
(B)	s1 \(\lambda\) s2
(C)	s2 \(\lambda\) s3
(D)	s3 \(\cdot s4\)





Q.55	An OTT company is maintaining a large disk-based relational database of different movies with the following schema:		
	Movie( <u>ID</u> , CustomerRating) Genre( <u>ID</u> , Name) Movie_Genre( <u>MovieID</u> , <u>GenreID</u> )		
	Consider the following SQL query on the relation database above:		
	SELECT * FROM Movie, Genre, Movie_Genre WHERE		
	<pre>Movie.CustomerRating &gt; 3.4 AND Genre.Name = "Comedy" AND Movie_Genre.MovieID = Movie.ID AND Movie_Genre.GenreID = Genre.ID;</pre>		
	This SQL query can be sped up using which of the following indexing options?		
A	B <sup>+</sup> tree on all the attributes.		
В	Hash index on Genre. Name and B <sup>+</sup> tree on the remaining attributes.		
С	Hash index on Movie. CustomerRating and B <sup>+</sup> tree on the remaining attributes.		
C D			
	attributes.		



Q.57 Let X be a random variable exponentially distributed with parameter  $\lambda > 0$ . The probability density function of X is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \ge 0\\ 0, & otherwise \end{cases}$$

If 5E(X) = Var(X), where E(X) and Var(X) indicate the expectation and variance of X, respectively, the value of  $\lambda$  is \_\_\_\_\_ (rounded off to **one** decimal place).

Q.58 Consider two events T and S. Let  $\overline{T}$  denote the complement of the event T. The probability associated with different events are given as follows:

$$P(\overline{T}) = 0.6$$
,  $P(S|T) = 0.3$ ,  $P(S|T) = 0.6$ 

Then, P(T|S) is \_\_\_\_\_ (rounded off to **two** decimal places).

Q.59 Consider a joint probability density function of two random variables X and Y

$$f_{X,Y}(x,y) = \begin{cases} 2xy, & 0 < x < 2, & 0 < y < x \\ 0, & \text{otherwise} \end{cases}$$

Then, E[Y|X = 1.5] is \_\_\_\_\_.

Q.60 Evaluate the following limit:

$$\lim_{x \to 0} \frac{\ln((x^2 + 1)\cos x)}{x^2} = \underline{\hspace{1cm}}.$$



Q.61 Let 
$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$
, and let  $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$  be the singular values of the matrix

 $M = uu^T$  (where  $u^T$  is the transpose of u). The value of  $\sum_{i=1}^5 \sigma_i$  is \_\_\_\_\_.

Q.62 Details of ten international cricket games between two teams "Green" and "Blue" are given in Table C. This table consists of matches played on different pitches, across formats along with their winners. The attribute *Pitch* can take one of two values: spin-friendly (represented as *S*) or pace-friendly (represented as *F*). The attribute *Format* can take one of two values: one-day match (represented as *O*) or test match (represented as *T*).

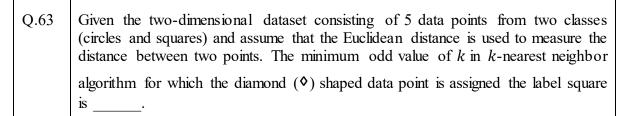
A cricket organization would like to use the information given in Table C to develop a decision-tree model to predict outcomes of future games between these two teams.

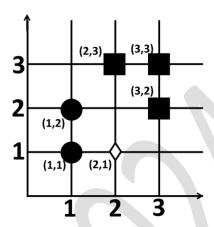
To develop such a model, the computed InformationGain(C, *Pitch*) with respect to the Target is \_\_\_\_\_ (rounded off to **two** decimal places).

Table C

Match	Pitch	Format	Winner (Target)
Number			
1	S	T	Green
2	S	T	Blue
3	F	0	Blue
4	S	0	Blue
5	F	T	Green
6	F	0	Blue
7	S	0	Green
8	F	T	Blue
9	F	0	Blue
10	S	0	Green

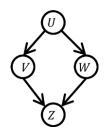








Q.64 Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:



	$P(\cdot)$
U = 0	0.5
U=1	0.5

	$P(V=0 \cdot)$	$P(V=1 \cdot)$
U = 0	0.5	0.5
U = 1	0.5	0.5
	$P(W=0 \cdot)$	$P(W=1 \cdot)$
U = 0	1	0
U=1	0	1

		$P(Z=0 \cdot)$	$P(Z=1 \cdot)$
V = 0	W = 0	0.5	0.5
V = 0	W = 1	1	0
V = 1	W = 0	1	0
V = 1	W = 1	0.5	0.5

The value of P(U = 1, V = 1, W = 1, Z = 1) =\_\_\_\_\_ (rounded off to **three** decimal places).

Q.65 Two fair coins are tossed independently. X is a random variable that takes a value of 1 if both tosses are heads and 0 otherwise. Y is a random variable that takes a value of 1 if at least one of the tosses is heads and 0 otherwise.

The value of the covariance of X and Y is \_\_\_\_\_ (rounded off to **three** decimal places).