COMEDK 2014 Question Paper with Solution

Consortium of Medical, Engineering and Dental Colleges of Karnataka
Under Graduate Entrance Test

SOLVED PAPER – 2014 (COMEDK)

Instructions

- There are 180 questions in all. The number of questions in each section is as given below.

Sections	No. of Questions
Section I : Physics	1-60
Section II : Chemistry	61-120
Section III : Mathematics	121-180

- All the questions are Multiple Choice Questions having four options out of which **ONLY ONE** is correct.
- Candidates will be awarded 1 mark for each correct answer. There will be no negative marking for incorrect answer.
- Time allotted to complete this paper is 3 hrs.

PHYSICS

1. Which of the following statements is not correct regarding conservation laws?

a. A conservation law is a hypothesis based on observations and experiments.
b. Conservation laws do not have a deep connection with symmetries of nature.
c. A conservation law cannot be proved.
d. Conservation of energy, linear momentum, angular momentum are considered to be fundamental laws of Physics.

2. The increase in the height of the tower of height 200 m to get its coverage range tripled is

a. 800 m *b.* 1600 m *c.* 900 m *d.* 700 m

3. The value of g at a height equal to half the radius of the earth from the earth's surface is

a. $\frac{g}{2}$ *b.* $\frac{g}{3}$ *c.* $\frac{4g}{9}$ *d.* $\frac{g}{4}$

4. The maximum and minimum distances of a comet from the sun are 1.4×10^{12} m and 7×10^{10} m. If its velocity nearest to the sun is $6 \times 10^{14}\ ms^{-1}$, what is its velocity in the farthest position?

(Assume the comet to be spherical)

a. 1000 ms^{-1} *b.* 2000 ms^{-1}
c. 3000 ms^{-1} *d.* 4000 ms^{-1}

5. The centre of mass of a system of two bodies of masses M and m, $(M > m)$, separated by a distance d is

a. midway between the bodies
b. closer to the heavier body
c. closer to the lighter body
d. at the centre of the heavier body

6. A pump on the ground floor of a building can pump up water to fill the tank of 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, the power consumed by the pump is ($g = 10\ ms^{-2}$)

a. 4.4 kW *b.* 44 kW
c. 440 kW *d.* 0.44 kW

7. The resistance of a galvanometer is $2.5\,\Omega$ and it requires 50 mA for full scale deflection. The value of shunt resistance required to convert it into an ammeter of range 0 to 5 A is

a. $2.5 \times 10^{-2}\,\Omega$ *b.* $0.25 \times 10^{-2}\,\Omega$
c. $0.025 \times 10^{-2}\,\Omega$ *d.* $0.0025 \times 10^{-2}\,\Omega$

8. Zener diode is used to regulate the voltage supply because, while using a Zener diode

a. voltage remains constant, even when current varies much
b. Both voltage and current remain constant
c. voltage varies much, but current remains constant
d. Both voltage and current vary together

9. In an inelastic collision, which of the following is true?

a. Momentum is conserved but not KE
b. KE is conserved, but not momentum
c. Both momentum and KE are conserved
d. Neither momentum nor KE are conserved

10. If a change in current of 0.01 A in one coil produces a change in magnetic flux of 2×10^{-2} weber in another coil, then the mutual inductance between coils is

a. 0 *b.* 0.5 H *c.* 2 H *d.* 3 H

11. An aeroplane is flying horizontally with a velocity of 360 km h^{-1}. The distance between the tips of the wings of the aeroplane is 50 m. The vertical component of the earth's magnetic field is 4×10^{-4} Wbm^{-2}. The induced emf is

a. 200 V *b.* 20 V *c.* 2V *d.* 0.2 V

12. The inductance in a coil plays the same role as

a. inertia in mechanics
b. energy in mechanics
c. momentum in mechanics
d. force in mechanics

13. The mutual inductance between two coils depends upon

a. the medium between the coils only
b. the separation between the coils only
c. Both the medium and the separation
d. Neither the medium nor the separation

14. The normal magnetic flux passing through a coil changes with time according to the equation $\phi = 6t^2 - 5t + 1$. What is the magnitude of the induced current at $t = 0.253$ s and resistance 10Ω?

a. 1.2 A *b.* 0.8 A
c. 0.6 A *d.* 0.2 A

15. For an electromagnetic wave, which of the following statements is true?

a. Electric field is constant and magnetic field varies.
b. Magnetic field is constant and electric field varies.
c. Both electric and magnetic fields are constant.
d. Both electric and magnetic fields vary.

16. If A is the angle of prism, r angle of refraction, then the condition for minimum deviation is

a. $A = r^2$ *b.* $A = 2r$
c. $A = \frac{r}{2}$ *d.* $A = r$

17. A concave mirror forms an enlarged, erect, virtual image of an object, only when the object is placed

a. at focus
b. between pole and focus
c. at the centre of curvature
d. between focus and centre of curvature

18. A man stands symmetrically between two large plane mirrors fixed to two adjacent walls of a rectangular room. The number of images formed are

a. 4 *b.* 3 *c.* 2 *d.* 6

19. A sinusoidal voltage of peak value 300 V and an angular frequency $\omega = 400$ rads^{-1} is applied to series L-C-R circuit, in which $R = 3\Omega$, $L = 20$ mH and $C = 625\,\mu F$. The peak current in the circuit is

a. $30\sqrt{2}$ A *b.* 60 A
c. 100 A *d.* $60\sqrt{2}$ A

20. A ray of light falls upon a 60° prism $(\mu = \sqrt{2})$ and it suffers minimum deviation. The angle of incidence for this ray must be

a. 30° *b.* 45° *c.* 60° *d.* 75°

21. A source of light of wavelength 5000 Å is placed at one end of a table 2 m long and 5 mm above its flat well polished top. The fringe width of the interference bands seen on a screen located at the end of the table is

a. 2×10^{-5} m *b.* 2×10^{-4} m
c. 2×10^{-3} m *d.* 2×10^{-2} m

22. A screen is placed 2 m away from a narrow slit. It the first minimum lies 5 mm from either side of the central maxima, when plane waves of wavelength 5×10^{-7} m are used, the slit width is

a. 4.8×10^{-4} m *b.* 2×10^{-4} m
c. 5×10^{-4} m *d.* 2.4×10^{-4} m

23. A galaxy moves with respect to the earth, so that sodium line of 589.0 nm is observed at 589.6 nm. The speed of the galaxy is

a. 300 kms^{-1} *b.* 306 kms^{-1}
c. 400 kms^{-1} *d.* 406 kms^{-1}

24. Newton's rings are observed normally in reflected light of wavelength 5000 Å. The diameter of the 10th dark ring is 0.005 m. The radius of curvature of the lens is

a. 1 m *b.* 1.25 m *c.* 2 m *d.* 0.5 m

25. The amount of energy required to separate a hydrogen atom into a proton and an electron is

a. 1.36 eV *b.* 13.6 eV *c.* 0.136 eV *d.* 136 eV

26. The energy equivalent of 1 g of a substance is

a. 4.5×10^{13} J *b.* 9×10^{16} J
c. 9×10^{13} J *d.* 18×10^{13} J

27. Two small drops of mercury, each of radius *r*, coalesce to form a single large drop of radius *R*. The ratio of the total surface energies before and after the change is

a. $1 : 2^{1/3}$ *b.* $2^{1/3} : 1$ *c.* 2 : 1 *d.* 1 : 2

28. Find the energy equivalent of one atomic mass unit in joules and in MeV.

a. 1.66×10^{-10} J, 93.15 MeV
b. 3×10^{-10} J, 9.315 MeV
c. 1.5×10^{-10} J, 931.5 MeV
d. 2.5×10^{-10} J, 931.5 MeV

29. Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 V, what is the output AC signal?

a. 1.5 V *b.* 2 V *c.* 2.5 V *d.* 3.0 V

30. A transistor works as an amplifier when,

a. emitter-base junction is forward biased and collector-base junction is reverse biased
b. emitter-base junction is reverse biased and collector-base junction is forward biased
c. Both junctions are forward biased
d. Both junctions are reverse biased

31. In a common base circuit, collector base voltage changes by 0.6 V and collector current changes by 0.02 mA. Then, the output resistance is

a. $6 \times 10^4\ \Omega$ *b.* $3 \times 10^4\ \Omega$
c. $9 \times 10^4\ \Omega$ *d.* $18 \times 10^4\ \Omega$

32. A repeater in TV transmission is

a. only a receiver
b. only a transmitter
c. a receiver and a transmitter
d. demodulator

33. The waves suitable for transmission of radio signals are

a. infrared
b. waves longer than infrared
c. waves shorter than infrared
d. X-rays

34. A wire of length 2 m is made from 10 cm^3 of copper. A force *F* is applied so that its length increases by 2 mm. Another wire of length 8 m is made from the same volume of copper. If the force *F* is applied to it, its length will increase by

a. 0.8 cm *b.* 1.6 cm
c. 2.4 cm *d.* 3.2 cm

35. A tennis ball of mass *m* strikes a wall with a velocity *v* and retraces the same path. Calculate the change in momentum.

a. $+4mv$ *b.* $-4mv$ *c.* $+2mv$ *d.* $-2mv$

36. A monkey of mass 40 kg climbs up a rope which can stand a maximum tension of 600 N. The rope will break, when the monkey

a. climbs up with an acceleration 6 ms^{-2}
b. climbs down with an acceleration of 4 ms^{-1}
c. climbs up with uniform speed of 5 ms^{-1}
d. slides down the rope freely under gravity

37. A shell of mass 20 g is fired by a gun of mass 100 kg. If the shell leaves the gun with a speed of 80 ms^{-1}, then the speed of recoil of the gun is

a. 3.2 cms^{-1} *b.* 1.6 cms^{-1}
c. 1 cms^{-1} *d.* 2.4 cms^{-1}

38. Two boys are standing at ends *A* and *B* of a ground where $AB = 200$ m. The boy at *B* starts running in a direction perpendicular to *AB* with a speed of 6 ms^{-1}. The boy at *A* starts simultaneously with a velocity of 10 ms^{-1} and catches the other at time, *t* where the time, *t* is

a. 50 s *b.* 20 s
c. 25 s *d.* 12.5 s

39. A car covers the first half distance between two places at 40 kmph and the other half at 60 kmph. The average speed of the car is

a. 48 kmph *b.* 120 kmph
c. 50 kmph *d.* 24 kmph

40. The dimensions of universal gravitational constant is

a. $[M^2L^3T^{-2}]$ *b.* $[M^{-1}L^3T^{-2}]$
c. $[M^{-2}L^3T^{-2}]$ *d.* $[M^2L^{-3}T^{-2}]$

41. Which two of the following quantities are dimensionally equivalent?

(i) Force

(ii) Pressure

(iii) Young's modulus

(iv) Energy

a. (i) and (ii) ***b.*** (i) and (iii)

c. (ii) and (iii) ***d.*** (ii) and (iv)

42. The temperature of an object is 60°C. Its value in Fahrenheit scale is

a. 120°F ***b.*** 130°F ***c.*** 140°F ***d.*** 110°F

43. A pan filled with hot food cools from 94°C to 86°C in 2 min, when the room temperature is 20°C. The time taken for the food to cool from 86°C to 74°C will be

a. 500 s ***b.*** 420 s ***c.*** 200 s ***d.*** 210 s

44. A system containing a ball is oscillating on a frictionless horizontal plane. The position of the mass when its potential energy and its kinetic energy both are equal, is (let A is the amplitude of oscillation)

a. A ***b.*** $A/\sqrt{2}$ ***c.*** $A/2$ ***d.*** $A/\sqrt{3}$

45. The universal gas law $\left(\frac{pV}{T} = \text{constant}\right)$ is applicable to

a. isothermal changes only ***b.*** adiabatic changes only

c. Both (a) and (b) ***d.*** Neither (a) nor (b)

46. On an average, a human heart is found to beat 72 times a minute. Its frequency and period are

a. 1.2 Hz, 0.83 s ***b.*** 2.5 Hz, 1.2 s

c. 2 Hz, 1.2 s ***d.*** 2.5 Hz, 0.83 s

47. A hospital uses an ultrasonic scanner to locate tumors in a tissue. The operating frequency of the scanner is 4.2 MHz. The speed of sound in a tissue is 1.7 km s^{-1}. The wavelength of sound in the tissue is

a. 4×10^{-3} m ***b.*** 8×10^{-3} m

c. 4×10^{-4} m ***d.*** 8×10^{-4} m

48. A policeman on duty detects a drop of 15% in the pitch of the horn of a motor car as it crosses him. If the velocity of sound is 330 ms^{-1}, then calculate the speed of the car.

a. 26.7 ms^{-1} ***b.*** 27.6 ms^{-1}

c. 53.4 ms^{-1} ***d.*** 54.3 ms^{-1}

49. An electric dipole placed in a non-uniform electric field experiences

a. only torque ***b.*** only force

c. Both (a) and (b) ***d.*** Neither (a) nor (b)

50. For a polar molecule, which of the following statements is true?

a. The centre of gravity of electrons and protons coincide.

b. The centre of gravity of electrons and protons do not coincide.

c. The charge distribution is always symmetrical.

d. The dipole moment is always zero.

51. A 10 mF capacitor has been charged to a potential of 100 V. Suddenly if it explodes, the energy given out is

a. 50 J ***b.*** 10^5 J

c. 50 mJ ***d.*** 10^3 J

52. The region between the parallel plates of capacitor is filled with parallel layers of air and paper (of dielectric constant 4). The space between the plates is 1 mm and the thickness of paper is 0.75 mm. The ratio of the voltage across air and paper is

a. $\frac{1}{2}$ ***b.*** $\frac{3}{4}$ ***c.*** $\frac{4}{3}$ ***d.*** $\frac{1}{3}$

53. Two copper wires, one of length 1 m and the other of length 9 m have the same resistance. The diameters are in the ratio

a. 3 : 1 ***b.*** 1 : 3

c. 9 : 1 ***d.*** 1 : 9

54. Which of the following is a correct statement?

a. Heat produced in a conductor varies directly as the current flowing.

b. Heat produced in a conductor varies inversely as the current flowing.

c. Heat produced in a conductor varies directly as the square of the current flowing.

d. Heat produced in a conductor varies inversely as the square of the current flowing.

55. A proton and an α-particle are projected with the same kinetic energy at right angles to a uniform magnetic field. The ratio of the radii of their paths is

a. 2 : 1 ***b.*** 1 : 2 ***c.*** 1 : 1 ***d.*** 2 : 3

56. The magnetic properties of a magnet is lost at its

a. melting point ***b.*** boiling point

c. Curie point ***d.*** triple point

57. Electromagnets are made of soft iron because soft iron has

a. low susceptibility and low retentivity

b. high susceptibility and low retentivity

c. high susceptibility and high retentivity

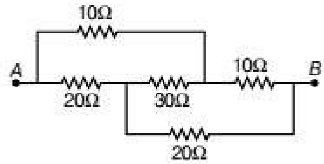
d. low susceptibility and high retentivity

58. Three charges $-q$, $+Q$ and $-q$ are placed at equal distances along a straight line. If the total PE of the system is zero, then the ratio Q/q becomes

a. $\frac{1}{8}$ *b.* $\frac{1}{6}$

c. $\frac{1}{4}$ *d.* $\frac{1}{2}$

59. The equivalent resistance of the circuit between A and B is



a. 30 Ω *b.* 20 Ω *c.* 10 Ω *d.* 8 Ω

60. The TV signals have a bandwidth of 3.7 MHz. The number of TV channels that can be accommodated with a band of 3700 GHz is

a. 10^4 *b.* 10^5 *c.* 10^6 *d.* 10^7

CHEMISTRY

61. Density of 3 M solution of NaCl is 1.25 g/mL. The mass of the solvent in the solution is

a. 1075.4 g *b.* 10.745 g

c. 10.754 g *d.* 1074.5 g

62. An orbital with $n = 3$, $l = 1$ is designated as

a. 1*s* *b.* 3*s* *c.* 3*p* *d.* 3*d*

63. A 150 watt bulb emits light of wavelength 6600 Å and only 8% of the energy is emitted as light. How many photons are emitted by the bulb per second?

a. 4×10^{19} *b.* 3.24×10^{19}

c. 4.23×10^{20} *d.* 3×10^{20}

64. In the following sets of ions, which one is not isoelectronic with the rest of the species?

O^{2-}, F^-, Na, Mg^{2+}, Al^{3+}, Ne

a. Mg^{2+} *b.* Al^{3+}

c. O^{2-} *d.* Na

65. For all gases, at any given pressure, the graph of volume *vs* temperature (in celsius) is a straight line. This graph is called

a. isomer *b.* isochore

c. isobar *d.* isotherm

66. Atomic numbers of vanadium, chromium, nickel and iron are 23, 24, 28 and 26 respectively. Which one of these is expected to have the highest second ionisation enthalpy?

a. Cr *b.* V *c.* Fe *d.* Ni

67. Which of these represents the correct order of their increasing bond order?

a. $C_2^{2-} < He_2^+ < O_2^+ < O_2^-$

b. $He_2^+ < O_2^- < O_2^+ < C_2^{2-}$

c. $O_2^- < O_2^+ < C_2^{2-} < He_2^+$

d. $O_2^+ < C_2^{2-} < O_2^- < He_2^+$

68. Pick up the incorrect statement.

a. Dipole moment of ammonia is due to orbital dipole and resultant dipole in the same direction.

b. In BF_3 bond dipoles are higher but dipole moment is zero.

c. Dipole moment is a vector quantity.

d. O_2 and H_2 show bond dipole due to polarisation.

69. The mole fraction of dioxygen in a neon-dioxygen mixture is 0.18. If the total pressure of the mixture is 25 bar, the partial pressure of neon in the mixture would be

a. 25.18 bar *b.* 25.82 bar

c. 4.5 bar *d.* 20.5 bar

70. Standard molar enthalpies of formation of $CaCO_3(s)$, $CaO(s)$ and $CO_2(g)$ are -1206.92 kJ mol^{-1}, -635.09 kJ mol^{-1} and -393.51 kJ mol^{-1} respectively. The $\Delta_r H$, for decomposition of $CaCO_3(s)$ is

a. 178.3 kJ mol^{-1} *b.* -178.3 kJ mol^{-1}

c. 1448.5 kJ mol^{-1} *d.* -1448.5 kJ mol^{-1}

71. $\Delta_r G°$ for the conversion of O_2 to ozone,

$\frac{3}{2}O_2(g) \longrightarrow O_3(g)$ at 298 K is

(K_p for this conversion is 1×10^{-29})

a. 16.54 kJ mol^{-1} *b.* 165.4 kJ mol^{-1}

c. 1654 kJ mol^{-1} *d.* 1.654 kJ mol^{-1}

72. In which of the following case, does the reaction go farthest to completion if

a. $K_C = 10^2$ *b.* $K_C = 10$ *c.* $K_C = 10^{-2}$ *d.* $K_C = 1$

73. One mole of PCl_5 is heated in a closed 2 dm^3 vessel. At equilibrium 40% PCl_5 is dissociated. Calculate the equilibrium constant.

a. 0.066 *b.* 0.154 *c.* 0.133 *d.* 0.266

74. The non-existence of PbI_4 and $PbBr_4$ is due to
a. highly oxidising nature of Pb^{4+} ions
b. highly reducing nature of I^- and Br^- ions
c. larger size of Pb^{4+}, Br^- and I^- ions
d. Both (a) and (b)

75. Which of the following species do not show disproportionation reaction?
a. ClO^- *b.* ClO_4^- *c.* ClO_2^- *d.* ClO_3^-

76. Sodium dissolves in liquid NH_3 to give a deep blue solution. This is due to
a. ammoniated Na^+
b. ammoniated Na^-
c. formation of Na^+ / Na^- pair
d. ammoniated electrons

77. Fullerene with formula C_{60} has a structure where every carbon atom is
a. *sp*-hybridised *b.* sp^2-hybridised
c. sp^3-hybridised *d.* not hybridised

78. Maleic acid and fumaric acid are
a. optical isomers
b. geometrical isomers
c. functional isomers
d. positional isomers

79. The IUPAC name of tertiary butyl chloride is
a. 2-chloro-2-methylpropane
b. 3-chlorobutane
c. 4-chlorobutane
d. 1, 2-dichloro-3-methylpropane

80. The compound which on ozonolysis produces a mixture of propanone and ethanal is
a. 2-methyl but-1-ene *b.* 2-pentene
c. 2-pentyne *d.* 2-methyl but-2-ene

81. When 2-bromopentane is heated with alcoholic solution of potassium hydroxide, the major product obtained is
a. pent-1-ene *b.* pent-2-ene
c. pent-1-yne *d.* pent-2-yne

82. Rain water is called acid rain when its pH
a. falls below zero
b. falls below 5.6
c. is above 5.6 but less than 10
d. is above 10

83. Schottky defect in crystals is observed when
a. an ion leaves its normal site and occupies an interstitial site.
b. unequal number of cations and anions are missing from the crystal lattice.
c. equal number of cations and anions are missing from the crystal lattice.
d. there is large difference in size of positive and negative ions.

84. A crystalline solid has A^- ions at the corners and face centres, whereas B^+ ions are at the body centre and edge centres of the unit cell. The simplest formula of the compound will be
a. A_2B *b.* AB_2 *c.* AB_3 *d.* AB

85. The radius of an atom is 300 pm. If it crystallises in a face-centered cubic lattice, the length of the edge of the unit cell is
a. 488.5 pm *b.* 848.5 pm
c. 884.5 pm *d.* 484.5 pm

86. When the concentration is expressed as the number of moles of solute per kilogram of the solvent, it is known as
a. molarity *b.* molality
c. normality *d.* mole fraction

87. van't Hoff factors of equimolal solutions of sodium chloride, barium chloride and glucose in water are
a. 2, 3, 0 respectively *b.* 2, 3, 6 respectively
c. 2, 3, 4 respectively *d.* 2, 3, 1 respectively

88. A one molal solution of sodium chloride in water has the same boiling point as
a. 1 m solution of magnesium sulphate
b. 1 m solution of magnesium chloride
c. 1 m solution of aluminium sulphate
d. 1 m solution of aluminium chloride

89. Vapour pressure of water at 293 K is 17.535 mm Hg. The vapour pressure of water at 293 K containing 25 g of glucose dissolved in 450 g of water is
a. 17.439 mm Hg *b.* 17.535 mm Hg
c. 0.097 mm Hg *d.* 34.973 mm Hg

90. In a electrochemical cell, the reaction will be feasible when
a. $\Delta G = -\text{ve}$, $E = +\text{ve}$
b. $\Delta G = +\text{ve}$, $E = -\text{ve}$
c. $\Delta G = 0$, $E = -\text{ve}$
d. $\Delta G = 0$, $E = 0$

91. The standard emf of the cell $Zn \mid Zn^{2+} \mid\mid Ag^+ \mid Ag$ is 1.56 V. If the standard reduction potential of Ag is 0.8 V, the standard oxidation potential of Zn is
a. − 0.76 V *b.* + 0.76 V
c. − 2.36 V *d.* + 2.36 V

92. The molar conductivities of NaOH, NaCl and $BaCl_2$ at infinite dilution are 2.481×10^{-2} S m^2 mol^{-1}, 1.265×10^{-2}S m^2 mol^{-1} and 2.800×10^{-2}S m^2 mol^{-1} respectively. The molar conductivity of $Ba(OH)_2$ at infinite dilution will be

a. 5.232×10^{-2} S m^2 mol^{-1}
b. 9.654×10^{-2} S m^2 mol^{-1}
c. 4.016×10^{-2} S m^2 mol^{-1}
d. 1.145×10^{-2} S m^2 mol^{-1}

93. The rate of a chemical reaction doubles for every 10°C rise in temperature. If the temperature increases by 60°C, the rate of reaction increases

a. 20 times *b.* 32 times
c. 64 times *d.* 128 times

94. An endothermic reaction, $A \rightarrow B$ has an activation energy as x kJ/mol. If the energy change of the reaction is y kJ, the activation energy of the reverse reaction is

a. $-x$ *b.* $x-y$
c. $x+y$ *d.* $y-x$

95. $aP + bQ \longrightarrow$ products, when $[P]$ is doubled keeping $[Q]$ constant and rate increases 2 times, when $[P]$ is constant and $[Q]$ is doubled and rate increases four times. The overall order is

a. 1 *b.* 2 *c.* 3 *d.* 2.5

96. Semiconductors of very high purity are obtained by

a. liquation
b. vapour phase refining
c. zone refining
d. electrolysis

97. Sodium cyanide is added as a depressant in the froth floatation process when the ore contains a mixture of ZnS and PbS. This is because

a. $Pb(CN)_2$ gets precipitated without any effect on ZnS
b. ZnS forms soluble complex, while PbS forms froth.
c. PbS forms soluble complex, while ZnS forms froth.
d. $Zn(CN)_2$ gets precipitated without any effect on PbS

98. Solid A reacts with strong $NaOH(aq)$ liberating a foul smelling gas B which spontaneously burn in air giving smoky rings. A and B are respectively.

a. p_{red} and PH_3 *b.* p_{White} and PH_3
c. S and H_2S *d.* p_{white} and H_2S

99. $SO_2 + 2H_2S \longrightarrow 3S + 2H_2O$. This equation represents preparation of sulphur sol by

a. hydrolysis *b.* oxidation
c. reduction *d.* double decomposition

100. Which of the following inert gas compounds is not formed?

a. $XeOF_4$ *b.* XeO_3 *c.* XeF_2 *d.* NeF_2

101. Nitrogen shows maximum covalency of 4 whereas, other heavier elements of the group show higher covalency because,

a. it has higher electronegativity
b. it has smaller size
c. it has only 4-orbitals available in the valence shell
d. it prefers to form multiple bonds with atoms

102. The photographic industry relies on the special light sensitive properties of

a. NaI *b.* NaBr *c.* AgCl *d.* AgBr

103. In acidic medium, potassium permanganate oxidises oxalic acid to

a. oxalate *b.* carbon dioxide
c. acetate *d.* acetic acid

104. The complex $[PtCl_2(en)_2]^{2+}$ ion shows

a. structural isomerism only
b. optical isomerism only
c. geometrical and optical isomerism
d. geometrical isomerism only

105. The complex $[Ag(NH_3)_2][Ag(CN)_2]$ has the IUPAC name

a. diamminesilver (I) dicyanosilver (I)
b. diamminesilver (I) dicyanoargentate (I)
c. dicyanosilver (I) diammineargentate (I)
d. diamminesilver (I) dicyanoargentate (I)

106. Molar conductance of a complex of cobalt is zero. Then its structure would be

a. $[Co(NH_3)]_3Cl_3$ *b.* $[Co(NH_3)Cl]Cl_3$
c. $[Co(NH_3)_3Cl_2]Cl$ *d.* $[Co(NH_3)_3Cl_3]$

107. The reaction, $CH_3CH_2I + KOH(aq) \longrightarrow CH_3CH_2OH + KI$ is classified as

a. electrophilic substitution
b. nucleophilic substitution
c. electrophilic addition
d. nucleophilic addition

108. For S_N1 reaction, the order of reactivity of haloalkanes is

a. tertiary halide < secondary halide < primary halide
b. tertiary halide < secondary halide > primary halide
c. tertiary halide > secondary halide > primary halide
d. tertiary halide > secondary halide < primary halide

109. The major product obtained when chlorobenzene is nitrated with HNO_3 + conc.H_2SO_4 is

a. 1-chloro-4-nitrobenzene
b. 1-chloro-2-nitrobenzene
c. 1-chloro-3-nitrobenzene
d. 1-chloro-1-nitrobenzene

110. The unstable intermediate dichlorocarbene [$:CCl_2$] is formed during

a. Kolbe's reaction
b. Friedel-Crafts reaction
c. Williamson synthesis
d. Reimer-Tiemann reaction

111. Phenols are highly acidic compared to alcohols due to

a. the higher molecular mass of phenols
b. the stronger hydrogen bonds in phenols
c. alkoxide ion is a strong conjugate base
d. phenoxide ion is resonance stabilised

112. Arrangement of following compounds based on their boiling points in the increasing order. *n*-butane, 1-butanol, ethoxyethane and 1-propanol will be

a. 1-propanol > *n*-butane > ethoxyethane > 1-butanol
b. *n*-butane < ethoxyethane < 1-propanol < 1-butanol
c. *n*-butane < 1-propanol < ethoxyethane < 1-butanol
d. 1-propanol < *n*-butane < ethoxyethane < 1-butanol

113. Among 2-chloropropanoic acid, 3-chloro-propanoic acid, 2-2-dichloroacetic acid and propanoic acid the K_a values will be in the order,

a. 2, 2-dichloroacetic acid > 2-chloropropanoic acid > 3-chloropropanoic acid > propanoic acid
b. 3-chloropropanoic acid > 2-chloropropanoic acid > 2, 2-dichloroacetic acid > propanoic acid
c. 2, 2-dichloroacetic acid > 3-chloropropanoic acid > 2-chloropropanoic acid > propanoic acid
d. 2, 2-dichloroacetic acid > propanoic acid > 2-chloropropanoic acid > 3-chloropropanoic acid

114. A reaction between a carbonyl compound and a Grignard reagent is termed as

a. nucleophilic addition
b. electrophilic addition
c. S_N1 reaction
d. S_N2 reaction

115. Ethanal is treated with a primary alcohol in presence of HCl gas. The reaction is known as

a. aldol condensation
b. acetal formation
c. cross aldol condensation
d. Cannizzaro reaction

116. $CH_3—CN \xrightarrow{Na/C_2H_5OH} A \xrightarrow{HNO_2} B \xrightarrow[573\text{ K}]{Cu} C$

here *C* is

a. $CH_3—CH_2NHOH$
b. $CH_3—CHO$
c. $CH_3CO—NH_2$
d. $CH_3—COOH$

117. In nucleic acids the nucleotides are linked by

a. ester linkage *b.* amide linkage
c. peptide linkage *d.* glycosidic linkage

118. Amino acids exist as zwitter ions at

a. acidic pH *b.* basic pH
c. neutral pH *d.* isoelectric pH

119. Which one of the following is an addition polymer?

a. Terylene *b.* Nylon-6, 6
c. Neoprene *d.* Teflon

120. Among the following compounds, the only one which is not an artificial sweetening agent is

a. aspartame *b.* sucrose
c. sucralose *d.* saccharin

MATHEMATICS

121. Which of the following is an empty set?

a. $\{x|x$ is a real number and $x^2-1=0\}$

b. $\{x|x$ is a real number and $x^2+3=0\}$

c. $\{x|x$ is a real number and $x^2-9=0\}$

d. $\{x|x$ is a real number and $x^2=x+2\}$

122. The number of proper subsets of a set having $n+1$ elements is

a. 2^{n+1} *b.* $2^{n+1}-1$

c. $2^{n+1}-2$ *d.* 2^{n-2}

123. The domain of the function $y=\sqrt{x-2}+\sqrt{1-x}$ is

a. $x \le 2$ *b.* $x \ge 2$

c. Null set *d.* The set of all numbers

124. If $A=\{1, 2, 3, 4\}$, then which one of the following is reflexive?

a. $\{(1, 1), (2, 3), (3, 3)\}$

b. $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

c. $\{(1, 2), (2, 1) (3, 2), (2, 3)\}$

d. $\{(1, 2), (1, 3), (1, 4)\}$

125. If $f(x)=\frac{2x-3}{3x+4}$, then $f^{-1}\left(\frac{-4}{3}\right)=$

a. Zero *b.* $\frac{3}{4}$

c. $-\frac{2}{3}$ *d.* None of these

126. If $f:[0, \infty) \to [2, \infty)$ is given by $f(x)=x+\frac{1}{x}$, then $f^{-1}(x)$ equals

a. $\frac{x+\sqrt{x^2-4}}{2}$ *b.* $\frac{x}{1+x^2}$

c. $\frac{2x-\sqrt{x^2-4}}{2}$ *d.* $1+\sqrt{x^2-4}$

127. The value of $\frac{\tan 330^\circ \sec 420^\circ \sin 300^\circ}{\tan 135^\circ \sin 210^\circ \sec 315^\circ}$ is equal to

a. $\frac{1}{\sqrt{2}}$ *b.* $\sqrt{2}$ *c.* $\frac{1}{\sqrt{3}}$ *d.* $\sqrt{3}$

128. If $\sin(120-A)=\sin(120-B)$ and $0<A, B<\pi$ then all values of A and B are given by

a. $A+B=\frac{\pi}{3}$ *b.* $A=B$ or $A+B=\frac{\pi}{3}$

c. $A=B$ *d.* $A+B=0$

129. If $\cot^{-1}\frac{1}{5}+\cot^{-1}\frac{1}{3}-\cot^{-1}\frac{4}{7}=\cot^{-1}x$, then x is equal to

a. 0 *b.* 3 *c.* $\frac{1}{\sqrt{3}}$ *d.* ∞

130. If $1+i=(x+iy)(u+iv)$, then $\tan^{-1}\left(\frac{y}{x}\right)+\cot^{-1}\left(\frac{u}{v}\right)$ has the value

a. $n\pi+\frac{\pi}{6}, n \in I$ *b.* $2n\pi+\frac{\pi}{3}, n \in I$

c. $n\pi+\frac{\pi}{4}, n \in I$ *d.* $n\pi-\frac{\pi}{3}, n \in I$

131. For a natural number n, which one is the correct statement?

a. $1^3+2^3+....+n^3=(1+2+.....+n)^2$

b. $1^3+2^3+....+n^3>(1+2+.....+n)^2$

c. $1^3+2^3+....+n^3<(1+2+....+n)^2$

d. $1^3+2^3+....+n^3 \ne (1+2+....+n)^2$

132. If a complex number lies in the III quadrant. Find the quadrant in which its conjugate lies.

a. I quadrant *b.* II quadrant

c. III quadrant *d.* IV quadrant

133. If β satisfies the inequation $x^2-x-6>0$, then a value exists for

a. $\cos^{-1}(\cos\beta)$ *b.* $\sec(\sec^{-1}\beta)$

c. $\tan(\tan^{-1}\beta)$ *d.* $\sin(\sin^{-1}\beta)$

134. If ${}^{56}P_{r+6} : {}^{54}P_{r+3}=30800:1$, then

a. $r=41$ *b.* $r=51$

c. $r=31$ *d.* None of these

135. If the coefficient of x^5 and x^6 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is

a. 51 *b.* 31

c. 41 *d.* None of these

136. The term independent of x in $\left[\sqrt{\left(\frac{x}{3}\right)}+\sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$ is

a. $\frac{5}{4}$ *b.* $\frac{7}{4}$

c. $\frac{9}{4}$ *d.* None of these

137. The sum of the series $\frac{1^2}{1\cdot 2}+\frac{1^2+2^2}{2\cdot 3}+\frac{1^2+2^2+3^2}{3\cdot 4}+\ldots$ upto 20 terms is

a. $\frac{205}{3}$ *b.* $\frac{200}{3}$

c. $\frac{220}{3}$ *d.* $\frac{210}{3}$

138. The angle between lines $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$ is

a. $\frac{\pi}{6}$ *b.* $\frac{3\pi}{4}$

c. $\frac{5\pi}{2}$ *d.* $\frac{\pi}{3}$

139. If a is a parameter then an equation of a family of lines having the sum of the intercepts on axes equal to 7 is

a. $4x+3y=12a$

b. $3x+4y=7a$

c. $7x+ay=a(7-a)$

d. $ay=(7-a)(a-x)$

140. The equation of the ellipse whose centre is at the origin and the X-axis is major axis, which passes through the points $(-3, 1)$ and $(2, -2)$ is

a. $5x^2+3y^2=32$

b. $3x^2+5y^2=32$

c. $5x^2-3y^2=32$

d. $3x^2+5y^2+32=0$

141. The equation of the lines joining the vertex of the parabola $y^2=6x$ to the point on it which have abscissa 24 are

a. $y\pm 2x=0$ *b.* $2y\pm x=0$

c. $x\pm y=0$ *d.* $2x\pm 3y=0$

142. From a point $P(a, b, c)$ perpendicular PA, PB are drawn to yz and zx planes. Find the equation of the plane OAB, where O is the origin.

a. $bcx+cay+abz=0$

b. $bcx+cay-abz=0$

c. $bcx-cay+abz=0$

d. $-bcx+cay+abz=0$

143. Find the distance of a point $(1, 2, 3)$ from the plane $3y+4z+4=0$.

a. 4.4 *b.* 4 *c.* 4.04 *d.* 4.44

144. AB and CD are two line segments, where $A(2, 3, 0)$, $B(6, 9, 0)$, $C(-6, -9, 0)$. P and Q are mid-point of AB and CD, respectively and L is the mid-point of PQ. Find the distance of L from the plane $3x+4z+25=0$

a. 25 *b.* 15 *c.* 5 *d.* 40

145. Determine the plane through the intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$

a. $-51x-15y-50z-173=0$

b. $51x+15y-50z+173=0$

c. $51x-15y+50z-173=0$

d. $51x+50y+15z+173=0$

146. $\lim\limits_{x\to 0}\frac{xa^x-x}{1-\cos x}$ is equal to

a. $\log a$ *b.* $\frac{1}{2}\log a$ *c.* $2\log a$ *d.* $2\log 2$

147. If $y=\tan x$, then $\frac{d^2y}{dx^2}=$

a. $1+y^2$ *b.* $2y(1+y^2)$

c. $y(1+y^2)$ *d.* $2y(1-y^2)$

148. If $y=\tan^{-1}\left(\frac{a-x}{1+ax}\right)$, then $\frac{dy}{dx}=$

a. $\frac{1}{(1+x^2)}$ *b.* $\frac{a}{(1+ax^2)}$

c. $-\frac{1}{(1+x^2)}$ *d.* $\frac{x}{(1+x^2)}$

149. If $f(x)=\begin{cases}[x]+[-x], & x\neq 2\\ K, & x=2\end{cases}$, then $f(x)$ is continuous at $x=2$, provided K is equal to

a. 2 *b.* 1 *c.* -1 *d.* 0

150. The curve $x^2-xy+y^2=27$ has tangents parallel to X-axis at

a. $(-3, -6)$ and $(3, -6)$

b. $(3, 6)$ and $(-3, -6)$

c. $(-3, 6)$ and $(-3, -6)$

d. $(3, -6)$ and $(-3, 6)$

151. The point on the circle $x^2+y^2=2$ at the abscissa and ordinate increase at the same ate is

a. $(-1, -1)$ *b.* $(1, -1)$

c. $(1, 1)$ *d.* $(-1, 4)$

152. A spherical balloon is being inflated at the rate of 35 cm/min. When its radius is 7 cm, its surface area increases at the rate of

a. 10 cm^2/min *b.* 15 cm^2/min

c. 20 cm^2/min *d.* 25 cm^2/min

153. $\int e^x(\cos x - \sin x)\,dx$ is equal to

a. $e^x \cos x + C$ *b.* $e^x \sin x + C$

c. $-e^x \cos x + C$ *d.* $-e^x \sin x + C$

154. The solution of $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is

a. $xy^2 = 2y^5 + C$ *b.* $yx^2 = 2y^5 + C$

c. $x^2y^2 = 2y^5 + C$ *d.* None of these

155. The solution of

$(x+y)^2\left(x\frac{dy}{dx} + y\right) = xy\left(1 + \frac{dy}{dx}\right)$ is

a. $\log(xy) = -\frac{1}{x+y} + C$

b. $\log\left(\frac{x}{y}\right) = -\frac{1}{x+y} + C$

c. $\log(xy) = \frac{1}{x-y} + C$

d. None of the above

156. Find $\sin\theta$, if θ is the angle between the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + \hat{j} + 2\hat{k}$.

a. $\sqrt{\frac{5}{21}}$ *b.* $\frac{5}{\sqrt{21}}$ *c.* $\frac{4}{\sqrt{21}}$ *d.* $\sqrt{\frac{3}{21}}$

157. If **a** and **b** are two vectors of magnitude 2, each inclined at an $\angle 60°$, then angle between **a** and **a** + **b** is

a. 30° *b.* 45° *c.* 60° *d.* 90°

158. If $\begin{bmatrix} -2 & 5 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$, then (x, y) is

a. (1, 2) *b.* (−1, 2) *c.* (1, −2) *d.* (2, 1)

159. If the product of the matrix $B = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

with a matrix A has the inverse $C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$,

then A^{-1} equals

a. $\begin{bmatrix} -3 & -5 & 5 \\ 0 & 9 & 14 \\ 2 & 2 & 9 \end{bmatrix}$ *b.* $\begin{bmatrix} -3 & 5 & 5 \\ 0 & 0 & 9 \\ 2 & 14 & 16 \end{bmatrix}$

c. $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 0 & 2 \\ 2 & 14 & 6 \end{bmatrix}$ *d.* $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$

160. A and B are two matrices such that $AB = B$, $BA = A$, then $A^2 + B^2 =$

a. $2AB$ *b.* $2BA$ *c.* $A + B$ *d.* AB

161. If α, β, γ are the roots of $x^3 + a^2x + b = 0$, then the value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

a. $-a^3$ *b.* $a^3 - 3b$ *c.* a^3 *d.* 0

162. The value of $\begin{vmatrix} 4 & 4 & 4 \\ (a+a^{-1})^2 & (b+b^{-1})^2 & (c+c^{-1})^2 \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix}$ is

a. 0 *b.* $4abc$

c. $4(abc)^{-1}$ *d.* $4[abc + (abc)^{-1}]$

163. Given that, $P(A) = 0.1$, $P(B \mid A) = 0.6$ and $P(B \mid A^C) = 0.3$ what is $P(A \mid B) = ?$

a. $\frac{2}{11}$ *b.* $\frac{4}{11}$ *c.* $\frac{7}{11}$ *d.* $\frac{9}{11}$

164. A card is picked at random from a pack of cards. Given that the picked card is a queen, what is the probability that it is a spade?

a. $\frac{1}{3}$ *b.* $\frac{4}{13}$ *c.* $\frac{1}{4}$ *d.* $\frac{1}{2}$

165. If the occurrence of an event A implies that occurrence of an event B, the $P(A^C \cap B^C)$ is equal to

a. $P(B^C)$ *b.* $P(A^C)\,P(B^C)$

c. $P(A^C)$ *d.* $1 - P(A \cap B)$

166. A certain item is manufactured by 3 factory F_1, F_2 and F_3 with 30% of item made in F_1, 20% in F_2 and 50% in F_3. It is found that 2% of the items produced by F_1. 3% of the items produced by F_2 and 4% of the items produced by F_3 are defective. Suppose that an items selected at random from the stock is found defective. What is the probability that the item came from F_1?

a. $\frac{1}{16}$ *b.* $\frac{1}{8}$ *c.* $\frac{1}{3}$ *d.* $\frac{3}{16}$

167. Ram obtained 60 and 85 in first two unit tests. The minimum marks he should get in the third test to have an average of atleast 55 marks is

a. $x \geq 20$ *b.* $x \leq 20$

c. $x > 20$ *d.* $x < 20$

168. Coefficient of variation of two distributions are 60 and 70, and their standard deviation are 21 and 16 respectively. What are their arithmetic means?

a. 35, 20 *b.* 35, 22.85
c. 30, 22.85 *d.* 30, 20

169. Consider an experiment E in which a box contains 10 identical tickets numbered 1 to 10 and 2 tickets are drawn at random from the box. What is the probability that both the tickets have even number on them?

a. $\frac{4}{9}$ *b.* $\frac{1}{3}$ *c.* $\frac{2}{9}$ *d.* $\frac{1}{9}$

170. Which of the following is not a logical statement?

a. Two non-empty sets have always a non-empty intersection.
b. The real number 'n' is less than 2.
c. Two individuals are always related.
d. None of the above

171. A mathematical model written to construct a maximum area rectangle out of a thread of length 10 cm is given by maximise lb.
Such that $2(l+b)=10$, $l, b>0$, where l and b are the length and breadth of the rectangle.
This is not a linear programming problem because

a. l and b is always positive.
b. first constraints is an equation.
c. the objective function is no maximise.
d. objective function is not linear.

172. Laksmi wants to buy few bangles and ear drops. Each bangle costs ₹ 5 and each ear drop costs ₹ 10. She should buy atleast 6 bangles and atmost 2 ear drops. If she buys x bangles and y ear drops with minimum expenditure, then the formulation for this linear programming is

a. Maximize $5x+10y$ subject to $x \ge 6$,$y \le 2$, x, $y \ge 0$.
b. Minimise $5x+10y$ subject to $x \ge 6$, $y \le 2$, $x, y \ge 0$.
c. Maximise $x+y$ subject to $5x+10y \le 50$, $x, y \ge 0$.
d. Maximise $6x+2y$ subject to $5x+10y \le 50$, x, $y \ge 0$.

173. The function $f(x)=\log(x+\sqrt{x^2+1})$ is

a. an even function.
b. an odd function.
c. a periodic function.
d. neither an even nor an odd function.

174. The number of real roots of the equation $x^4+\sqrt{x^4+20}=22$ is

a. 4 *b.* 2 *c.* 0 *d.* 1

175. If the sum of first n natural numbers is $\frac{1}{78}$ times the sum their cubes, then the value of n is

a. 11 *b.* 12 *c.* 13 *d.* 14

176. If a polygon of n sides 275 diagonals, then n is equal to

a. 25 *b.* 35 *c.* 20 *d.* 15

177. 7^9+9^7 is divisible by

a. 128 *b.* 24 *c.* 64 *d.* 72

178. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2-bc & b^2-ca & c^2-ab \end{vmatrix}$ is equal to

a. 0 *b.* 1
c. abc *d.* $(a-b)(b-c)(c-a)$

179. The number of values of x in $[0, 2\pi]$ satisfying the equation $3\cos 2x-10\cos x+7=0$ is

a. 1 *b.* 2 *c.* 3 *d.* 4

180. $\lim_{x \to -\infty} \frac{2x-1}{\sqrt{x^2+2x+1}}$ is equal to

a. 2 *b.* -2 *c.* 1 *d.* -1

ANSWERS

Physics

1. (b)	**2.** (b)	**3.** (c)	**4.** (*)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (b)
21. (b)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (a)
31. (b)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (a)	**40.** (b)
41. (c)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (c)	**50.** (b)
51. (a)	**52.** (c)	**53.** (b)	**54.** (c)	**55.** (c)	**56.** (c)	**57.** (b)	**58.** (c)	**59.** (*)	**60.** (c)

Chemistry

61. (d)	**62.** (c)	**63.** (a)	**64.** (d)	**65.** (c)	**66.** (a)	**67.** (b)	**68.** (d)	**69.** (d)	**70.** (a)
71. (b)	**72.** (a)	**73.** (c)	**74.** (d)	**75.** (b)	**76.** (d)	**77.** (b)	**78.** (b)	**79.** (a)	**80.** (d)
81. (b)	**82.** (b)	**83.** (c)	**84.** (d)	**85.** (b)	**86.** (b)	**87.** (d)	**88.** (a)	**89.** (a)	**90.** (a)
91. (b)	**92.** (a)	**93.** (c)	**94.** (b)	**95.** (c)	**96.** (c)	**97.** (b)	**98.** (b)	**99.** (b)	**100.** (d)
101. (c)	**102.** (d)	**103.** (b)	**104.** (c)	**105.** (b,d)	**106.** (d)	**107.** (b)	**108.** (c)	**109.** (a)	**110.** (d)
111. (d)	**112.** (b)	**113.** (a)	**114.** (a)	**115.** (b)	**116.** (b)	**117.** (a)	**118.** (d)	**119.** (c, d)	**120.** (b)

Mathematics

121. (b)	**122.** (b)	**123.** (c)	**124.** (b)	**125.** (d)	**126.** (a)	**127.** (b)	**128.** (b)	**129.** (a)	**130.** (c)
131. (a)	**132.** (b)	**133.** (b)	**134.** (a)	**135.** (c)	**136.** (d)	**137.** (c)	**138.** (a)	**139.** (d)	**140.** (b)
141. (b)	**142.** (b)	**143.** (a)	**144.** (c)	**145.** (b)	**146.** (c)	**147.** (b)	**148.** (c)	**149.** (c)	**150.** (b)
151. (b)	**152.** (a)	**153.** (a)	**154.** (a)	**155.** (a)	**156.** (a)	**157.** (a)	**158.** (d)	**159.** (d)	**160.** (c)
161. (d)	**162.** (a)	**163.** (a)	**164.** (c)	**165.** (a)	**166.** (d)	**167.** (a)	**168.** (b)	**169.** (c)	**170.** (d)
171. (d)	**172.** (b)	**173.** (b)	**174.** (b)	**175.** (b)	**176.** (a)	**177.** (c)	**178.** (a)	**179.** (d)	**180.** (b)

Note (*) None of the option is correct.

HINTS & SOLUTIONS

Physics

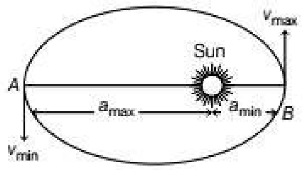
1. *(b)* The laws of conservation have a deep connection with symmetries of nature. This is first theorised by Noether. Noether's first theorem states that any differentiable symmetry of the action of a physical system has a corresponding conservation law. This means that conservation laws are observed because of the symmetries of nature.

2. *(b)* Given, height of the tower, $h_1 = 200$ m

If R be the radius of the earth, then coverage range,
$$d = \sqrt{2Rh} \Rightarrow d \propto \sqrt{h}$$
$$\Rightarrow \quad \frac{d_1}{d_2} = \sqrt{\frac{h_1}{h_2}}$$
Since, $\quad d_2 = 3d_1$
$$\therefore \quad \frac{d_1}{3d_1} = \sqrt{\frac{h_1}{h_2}}$$
$$\Rightarrow \quad \frac{1}{3} = \sqrt{\frac{200}{h_2}}$$
$$\Rightarrow \quad \frac{1}{9} = \frac{200}{h_2}$$
$$\Rightarrow \quad h_2 = 1800 \text{ m}$$
$\therefore$ Increase in the height of the tower
$= 1800 \text{ m} - 200 \text{ m} = 1600$ m

3. *(c)* The value of g at height h from the surface of the earth is given as
$$g_h = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$
$$= \frac{g}{\left(1 + \frac{R_e/2}{R_e}\right)^2} \qquad \left(\because h = \frac{R_e}{2}\right)$$
$$= \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{4g}{9}$$

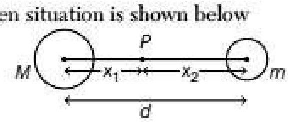
4. *(*)* The given situation is shown below



$$a_{max} = 1.4 \times 10^{12} \text{ m}$$
$$a_{min} = 7 \times 10^{10} \text{ m}$$

The velocity of the comet is maximum, when it is nearest to the sun and minimum when it is farthest from the sun.
$$\therefore \quad v_{max} = 6 \times 10^{14} \text{ m/s}$$
Applying the law of conservation of angular momentum at points A and B, we get
$$mv_{min}a_{max} = mv_{max}a_{min} \qquad (\because L = mvr)$$
$$\Rightarrow \quad v_{min} = \frac{v_{max} \times a_{min}}{a_{max}}$$
$$= \frac{6 \times 10^{14} \times 7 \times 10^{10}}{1.4 \times 10^{12}}$$
$$= 3 \times 10^{13} \text{ ms}^{-1}$$
No option is correct.

5. *(b)* The given situation is shown below



Let P be the location of centre of mass.

If x_1 and x_2 be the distances of the centre of mass P from the two bodies of mass M and $m(M > m)$ respectively, then
$$Mx_1 = mx_2$$
$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{m}{M}$$
Since, $\quad M > m$
$$\therefore \quad \frac{x_1}{x_2} < 1$$
$$\Rightarrow \quad x_1 < x_2$$
Thus, position of centre of mass is closer to the heavier body.

6. *(b)* Volume of water in tank, $V = 30 \text{ m}^3$

Time taken to fill the tank,
$t = 15 \text{ min} = 15 \times 60 = 900$ s

Height of the tank, $h = 40$ m

$\therefore$ Mass of pumped water, m = Volume × Density of water
$$= 30 \times 10^3 = 3 \times 10^4 \text{ kg}$$
Work done by the pump to fill the tank,
$$W = mgh$$
$$= 3 \times 10^4 \times 10 \times 40$$
$$= 1.2 \times 10^7 \text{ J}$$
$\therefore$ Output power of the pump,
$$P_o = \frac{W}{t} = \frac{1.2 \times 10^7}{900} = \frac{4}{3} \times 10^4 \text{ W}$$

As we know, efficiency $= \dfrac{\text{Output power } (P_o)}{\text{Input power } (P_i)}$

$$\Rightarrow \qquad 0.3 = \frac{4/3 \times 10^4}{P_i}$$

$$\Rightarrow \qquad P_i = \frac{4 \times 10^4}{3 \times 0.3} = 4.4 \times 10^4 \text{ W} = 44 \text{ kW}$$

7. *(a)* Given, resistance of galvanometer, $R_g = 2.5\ \Omega$

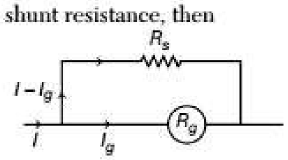
Full scale deflection current,

$$I_g = 50 \text{ mA} = 50 \times 10^{-3} \text{ A}$$
$$= 0.05 \text{ A}$$

Range of galvanometer is (0-5)A

$\therefore \qquad I = 5$ A

If R_s be the shunt resistance, then



$$(I - I_g)R_s = I_g R_g$$

$$\Rightarrow \qquad R_s = \frac{I_g R_g}{I - I_g}$$
$$= \frac{0.05 \times 2.5}{5 - 0.05}$$
$$= 0.025\ \Omega = 2.5 \times 10^{-2}\ \Omega$$

8. *(a)* In a Zener diode, voltage across it remains constant even when current varies much or wide range. This property of the Zener diode is used to regulate the voltage supply.

9. *(a)* In an inelastic collision, momentum is conserved but kinetic energy is not conserved.

10. *(c)* Given, $\Delta I = 0.01$ A

$$\Delta\phi = 2 \times 10^{-2} \text{ Wb}$$

We know that, $\Delta\phi = M\Delta I$

where, M is mutual inductance between the coils.

$$\therefore \qquad M = \frac{\Delta\phi}{\Delta I}$$
$$= \frac{2 \times 10^{-2}}{0.01} = 2 \text{ H}$$

11. *(c)* Given, velocity of aeroplane,

$$v = 360 \text{ kmh}^{-1}$$
$$= 360 \times \frac{5}{18} \text{ ms}^{-1} = 100 \text{ ms}^{-1}$$

Distance between the tips of wings,

$$l = 50 \text{ m}$$

Vertical component of earth's magnetic field, $B_V = 4 \times 10^{-4}$ Wbm^{-2}

$\therefore$ Induced emf, $e = B_V vl$

$$= 4 \times 10^{-4} \times 100 \times 50 = 2 \text{ V}$$

12. *(a)* The inductance in a coil plays the same role as inertia plays in mechanics.

13. *(c)* The mutual inductance between the two coils depends upon the medium and the separation between them both.

14. *(d)* Given, magnetic flux

$$\phi = 6t^2 - 5t + 1, R = 10\ \Omega$$

Induced emf,

$$e = -\frac{d\phi}{dt}$$
$$= -\frac{d}{dt}(6t^2 - 5t + 1)$$
$$e = -12t + 5$$

At $t = 0.253$ s, $e = -12 \times 0.253 + 5$

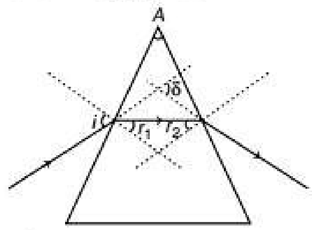
$$= -3.036 + 5 = 1.964 \text{ V}$$

$\therefore$ Induced current, $I = \dfrac{e}{R}$

$$= \frac{1.964}{10} = 0.1964 \text{ A} \simeq 0.2 \text{ A}$$

15. *(d)* In electromagnetic wave, both electric and magnetic fields vary with time.

16. *(b)* For refraction through prism,



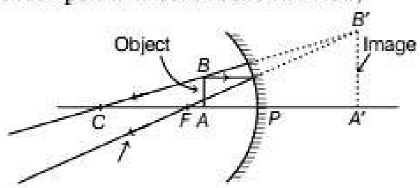
$$A = r_1 + r_2$$

For the case of minimum deviation,

$$r_1 = r_2 = r$$

$$\therefore \qquad A = r + r = 2r$$

17. *(b)* Image formed by a concave mirror is erect, enlarged and virtual only when object is placed between pole and focus as shown below,



18. *(b)* Number of images when an object is placed symmetrically between plane mirrors inclined at an angle θ, is given as

$$n = \frac{360}{\theta} - 1$$

Here, $\qquad \theta = 90°$

$$\therefore \qquad n = \frac{360}{90} - 1 = 4 - 1 = 3$$

19. *(b)* Given, $V_m = 300$ V, $\omega = 400$ rads^{-1}

$$R = 3\Omega, L = 20 \text{ mH} = 20 \times 10^{-3} \text{ H}$$

and $$C = 625\ \mu\text{F} = 625 \times 10^{-6} \text{ F}$$

Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, $$X_L = \omega L = 400 \times 20 \times 10^{-3} = 8\Omega$$

and $$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 625 \times 10^{-6}} = 4\ \Omega$$

$\because$ $$Z = \sqrt{3^2 + (8-4)^2} = \sqrt{25} = 5\Omega$$

$\therefore$ Peak current, $I_m = \dfrac{V_m}{Z} = \dfrac{300}{5} = 60$ A

20. *(b)* Given, angle of prism, $A = 60°$

$$\mu = \sqrt{2}$$

We know that,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2}$$

$$\Rightarrow \sqrt{2} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\frac{60}{2}}$$

$$\Rightarrow \sin\left(\frac{60° + \delta_m}{2}\right) = \sqrt{2}\sin 30°$$

$$\Rightarrow \sin\left(\frac{60° + \delta_m}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{60° + \delta_m}{2}\right) = \sin 45°$$

$$\Rightarrow \frac{60° + \delta_m}{2} = 45°$$

$$\Rightarrow \delta_m = 30°$$

In the case of minimum deviation, angle of incidence,

$$i = \frac{A + \delta_m}{2} = \frac{60° + 30°}{2} = 45°$$

21. *(b)* Given, wavelength of light used,

$$\lambda = 5000\ \text{Å} = 5 \times 10^{-7} \text{ m}$$
$$D = 2 \text{ m}$$
$$d = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

$\therefore$ Fringe width, $\beta = \dfrac{D\lambda}{d}$

$$= \frac{2 \times 5 \times 10^{-7}}{5 \times 10^{-3}} = 2 \times 10^{-4} \text{ m}$$

22. *(b)* Given, distance between source and screen, $D = 2$ m

Distance of first minimum from the central maximum, $y = 5 \text{ mm} = 5 \times 10^{-3}$ m

Wavelength, $\lambda = 5 \times 10^{-7}$ m

We know that,

$$y_n = \frac{n\lambda D}{d}$$

For first minimum, $n = 1$

$\therefore$ $$y_1 = \frac{\lambda D}{d}$$

$$\Rightarrow d = \frac{\lambda D}{y_1} = \frac{5 \times 10^{-7} \times 2}{5 \times 10^{-3}} = 2 \times 10^{-4} \text{ m}$$

23. *(b)* Given, wavelength of sodium line,

$$\lambda = 589.0 \text{ nm} = 5.89 \times 10^{-7} \text{ m}$$

Observed wavelength,

$$\lambda' = 589.6 \text{ nm} = 5.896 \times 10^{-7} \text{m}$$

$\therefore$ Change in wavelength,

$$\Delta\lambda = \lambda' - \lambda$$
$$= 5.896 \times 10^{-7} - 5.89 \times 10^{-7}$$
$$= 0.006 \times 10^{-7} \text{ m} = 6 \times 10^{-10} \text{ m}$$

According to Doppler's shift equation,
Speed of galaxy is given as

$$v = \frac{\Delta\lambda}{\lambda} \times c$$
$$= \frac{6 \times 10^{-10}}{5.89 \times 10^{-7}} \times 3 \times 10^8$$
$$= 3.06 \times 10^5 \text{ m/s}$$
$$= 306 \times 10^3 \text{ m/s} = 306 \text{ km/s}$$

24. *(b)* In Newton's ring experiment, diameter of *n*th dark ring,

$$D_n = 2\sqrt{n\lambda R}$$

$$\Rightarrow R = \frac{D_n^2}{4n\lambda} \quad \ldots \text{(i)}$$

Given, $n = 10$, $D_{10} = 0.005$ m

$$\lambda = 5000\ \text{Å} = 5 \times 10^{-7} \text{ m}$$

$\therefore$ Putting these values in Eq. (i), we get

$$R = \frac{(0.005)^2}{4 \times 10 \times 5 \times 10^{-7}} = 1.25 \text{ m}$$

25. *(b)* The amount of energy required to separate a hydrogen atom into a proton and an electron is equal to its ionisation energy which is equal to 13.6 eV.

26. *(c)* Given, $m = 1 \text{ g} = 10^{-3}$ kg

According to Einstein's mass-energy equation,

$$E = mc^2 = 10^{-3} \times (3 \times 10^8)^2 = 9 \times 10^{13} \text{ J}$$

27. *(b)* As, radius of bigger drop, $R = n^{1/3}\, r = 2^{1/3} r$

$$\Rightarrow R^2 = 2^{2/3}\, r^2 \Rightarrow \frac{r^2}{R^2} \text{ or } 2^{-2/3}$$

$$\frac{\text{Initial surface energy}}{\text{Final surface energy}} = \frac{2(4\pi r^2 T)}{(4\,\pi R^2 T)} = 2\left(\frac{r^2}{R^2}\right)$$
$$= 2 \times 2^{-2/3} = 2^{1/3} \text{ or } 2^{1/3} : 1$$

28. *(c)* Given, mass is equivalent to 1 amu, i.e. $m = 1$ amu

$$= 1.6605 \times 10^{-27} \text{ kg}$$

According to Einstein's mass-energy equivalent equation,

$$E = mc^2$$
$$= 1.6605 \times 10^{-27} \times (3 \times 10^8)^2$$
$$= 1.49 \times 10^{-10} \text{ J}$$
$$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ eV}$$
$$= 931.5 \times 10^6 \text{ eV}$$
$$= 931.5 \text{ MeV}$$

29. *(b)* Voltage gain of first amplifier,

$$A_1 = 10$$

Voltage gain of second amplifier,

$$A_2 = 20$$

Input signal voltage,

$$V_i = 0.01 \text{ V}$$

Total voltage gain, when two amplifiers are connected in series (cascaded), is given as

$$A = A_1 A_2 = 10 \times 20 = 200$$

We know that, voltage gain

$$A = \frac{\text{output voltage } (V_o)}{\text{input voltage } (V_i)}$$

$$\Rightarrow \quad 200 = \frac{V_o}{0.01}$$

$$\Rightarrow \quad V_o = 200 \times 0.01 = 2 \text{ V}$$

30. *(a)* A transistor works as an amplifier, when emitter-base junction is in forward biased and collector-base junction is in reverse biased.

31. *(b)* In common base circuit, change in collector base voltage,

$$V_{CB} = 0.6 \text{ V}$$
$$\Delta I_C = 0.02 \text{ mA} = 2 \times 10^{-5} \text{ A}$$

$\therefore$ Output resistance,

$$R_{out} = \frac{\Delta V_{CB}}{\Delta I_C} = \frac{0.6}{2 \times 10^{-5}} = 3 \times 10^4 \ \Omega$$

32. *(c)* Repeater is used between a receiver and a transmitter to extend transmission of radiowaves in communication system, so that signal can cover longer distances or be received on the other side of an obstruction.

33. *(b)* The waves suitable for transmission of radio signals are radiowaves. Radiowaves are a type of electromagnetic radiation with wavelengths in the electromagnetic spectrum longer than infrared radiation.

34. *(d)* Given, change in length of wire, $l_1 = 2$ mm, $l_2 = ?$, length of wire, $L_1 = 2$ m and length of another wire, $L_2 = 8$ m

Change in length, $l = \frac{FL}{AY} = \frac{FL^2}{(A \cdot L)Y} = \frac{FL^2}{VY}$

$[\because V = A \cdot L]$

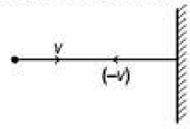
where, Y is Young's modulus.

$\therefore \quad l \propto L^2 \quad$ (as V, Y and F are constants)

$$\frac{l_2}{l_1} = \left(\frac{L_2}{L_1}\right)^2 = \left(\frac{8}{2}\right)^2 = 16$$

$$\Rightarrow \quad l_2 = 16 l_1 = 16 \times 2 \text{ mm}$$
$$= 32 \text{ mm} = 3.2 \text{ cm}$$

35. *(d)* The given situation is shown below



Change in momentum,

$$\Delta p = p_f - p_i = m(-v) - (mv) = -mv - mv = -2mv$$

36. *(a)* Mass of the monkey, $m = 40$ kg

Maximum tension that the rope can bear, $T_{max} = 600$ N

In option (a), acceleration of monkey, $a = 6 \text{ m/s}^2$ (upward)

$\therefore$ By equation of motion,

$$T - mg = ma$$
$$\Rightarrow \quad T = m(g + a)$$
$$= 40(10 + 6) = 640 \text{ N}$$

Since, $T > T_{max}$, hence in this case, rope will break. Therefore, option (a) is correct.

In option (b), acceleration of monkey, $a = 4 \text{ m/s}^2$ (downward)

$\therefore$ By equation of motion,

$$T = m(g - a) = 40(10 - 4) = 240 \text{ N}$$

Since, $T < T_{max}$, then the rope will not break.

In option (c), monkey climbs up with uniform speed, $v = 5$ m/s

$\therefore$ Acceleration, $a = 0 \text{ m/s}^2$

Hence, by equation of motion, $T = mg$

$$= 40 \times 10 = 400 \text{ N}$$

Since, $T < T_{max}$, hence rope will not break.

In option (d), acceleration, $a = g$

$$\therefore \quad T = m(g - a) = m(g - g) = 0 \text{ N}.$$

Since, $T < T_{max}$, the rope will not break.

Therefore, only option (a) is correct.

37. *(b)* Given, mass of shell, $m_s = 20 \text{ g} = 0.02$ kg

Mass of gun, $m_g = 100$ kg

Speed of shell, $v_s = 80$ m/s

Let v_g be the speed of the recoil of gun, then according to law of conservation of linear momentum.

Total initial momentum = Total final momentum

$$\Rightarrow \quad 0 = m_s v_s + m_g v_g$$
$$\Rightarrow \quad 0 = 0.02 \times 80 + 100 v_g$$
$$\Rightarrow \quad 0 = 1.6 + 100 v_g$$

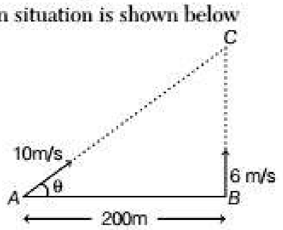
$$\Rightarrow \quad v_g = \frac{-1.6}{100}$$

$$= -1.6 \times 10^{-2} \text{ m/s} = -1.6 \text{ cm/s}$$

Negative sign indicates that gun moves in opposite direction to that of shell.

$\therefore$ Speed of recoil of gun = 1.6 cm/s

38. *(c)* The given situation is shown below



$$v_A = 10 \text{ m/s}$$

$$v_B = 6 \text{ m/s}$$

The two boys meet at point C after a time t.

Horizontal component of velocity of v_A.

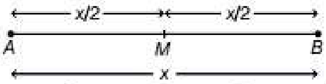
$$v_{AB} = v_A \cos\theta = 10\cos\theta$$

$$= 10\left(\frac{\sqrt{10^2 - 6^2}}{10}\right)$$

$$= 8 \text{ m/s}$$

$$\therefore \quad t = \frac{AB}{v_{AB}} = \frac{200}{8} = 25 \text{ s}$$

39. *(a)* The given situation is shown below



Let the distance between the two places is x km.

$$\therefore \quad AM = MB = \frac{x}{2} \text{ km}$$

Time taken by the car to cover first half (AM) with the speed of 40 km/h is given as

$$t_1 = \frac{AM}{40} = \frac{x/2}{40} = \frac{x}{80} \text{ h}$$

Similarly, time taken by the car to cover second half (MB) with the speed of 60 km/h is given as

$$t_2 = \frac{MB}{60} = \frac{x/2}{60} = \frac{x}{120} \text{ h}$$

$\therefore$ Average speed of car,

$$v_{\text{avg}} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{x}{\frac{x}{80} + \frac{x}{120}} = \frac{x}{\frac{5x}{240}}$$

$$= \frac{240}{5} = 48 \text{ km/h}$$

40. *(b)* According to Newton's law of gravitation,

$$F = \frac{Gm_1m_2}{r^2}$$

where, F = force between two objects of masses m_1 and m_2 and r = distance between m_1 and m_2.

$$\therefore \quad G = \frac{Fr^2}{m_1m_2}$$

$$\Rightarrow \quad [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

41. *(c)* We know that,

Force = Mass $\times$ Acceleration

$$\Rightarrow \quad F = m \times a$$

$$\Rightarrow \quad [F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

$$\text{Pressure}\,(p) = \frac{\text{Force}(F)}{\text{Area}(A)}$$

$$\Rightarrow \quad [p] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

$$\text{Young's modulus, } Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \quad [Y] = \frac{[\text{Stress}]}{[\text{Strain}]} = \frac{[F/A]}{[\text{Strain}]}$$

$$= \frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$$

$$= [ML^{-1}T^{-2}]$$

Energy = Work done = Force $\times$ Displacement

$$\Rightarrow \quad [\text{Energy}] = [\text{Force}] \times [\text{Displacement}]$$

$$= [MLT^{-2}][L] = [ML^2T^{-2}]$$

Hence, we see that dimensions of pressure and Young's modulus is same.

42. *(c)* Temperature of object in °C,

$$T_C = 60°\text{C}$$

Temperature of object in Fahrenheit scale, T_F = ?

We know that,

$$\frac{T_C}{5} = \frac{T_F - 32}{9}$$

$$\Rightarrow \quad \frac{60}{5} = \frac{T_F - 32}{9}$$

$$\Rightarrow \quad 12 = \frac{T_F - 32}{9}$$

$$\Rightarrow \quad T_F = 108 + 32 = 140°\text{F}$$

43. *(d)* According to Newton's law of cooling,

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right) \quad \ldots \text{(i)}$$

where, T_s is the temperature of surrounding.

For the first case, $T_1 = 94°$C, $T_2 = 86°$C

$t = 2$ min $= 120$ s, $T_s = 20°$C

$\therefore$ Putting these values in Eq. (i), we get

$$\frac{94 - 86}{120} = K\left(\frac{94 + 86}{2} - 20\right)$$

$$\Rightarrow \quad K = \frac{8}{120 \times 70} \quad \ldots \text{(ii)}$$

For the second case, $T_1 = 86°C, T_2 = 74°C$

$\therefore$ From Eq. (i), we get

$$\frac{86-74}{t} = K\left(\frac{86+74}{2} - 20\right)$$

$$\Rightarrow \quad \frac{12}{t} = \frac{8}{120 \times 70}(60)$$

$$\Rightarrow \quad t = 210 \text{ s}$$

44. *(b)* Kinetic energy of the system in SHM,

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - x^2)$$

Potential energy of the system in SHM, $\text{PE} = \frac{1}{2}m\omega^2x^2$

According to question, PE = KE

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$\Rightarrow \quad x^2 = A^2 - x^2$$

$$\Rightarrow \quad 2x^2 = A^2 \Rightarrow x = \frac{A}{\sqrt{2}}$$

45. *(c)* The universal gas law, $\frac{pV}{T}$ = constant is applicable to both isothermal and adiabatic changes.

46. *(a)* Frequency of human heart,

$$f = \frac{72}{60}\text{ Hz} = \frac{6}{5}\text{ Hz} = 1.2\text{ Hz}$$

Time period, $T = \frac{1}{f} = \frac{1}{1.2} = 0.83$ s

47. *(c)* Given, frequency, $f = 4.2\text{ MHz} = 4.2 \times 10^6$ Hz

Velocity of sound, $v = 1.7\text{ kms}^{-1} = 1.7 \times 10^3\text{ ms}^{-1}$

The wavelength of sound in the tissue is given as

$$\lambda = \frac{v}{f} \qquad (\because v = f\lambda)$$

$$= \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4 \times 10^{-4}\text{ m}$$

48. *(a)* Given, velocity of sound, $v = 330$ m/s

Let frequency of horn be f and speed of the car be v_c. The frequency of the horn of the car heard by the policeman before it crosses him is given as

$$f' = f\left(\frac{v}{v - v_c}\right) \qquad \ldots \text{(i)}$$

and after it crosses him is given as

$$f'' = f\left(\frac{v}{v + v_c}\right) \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{f''}{f'} = \frac{v - v_c}{v + v_c} = \frac{330 - v_c}{330 + v_c}$$

$$\Rightarrow \quad \frac{f''}{f'} = \frac{330 - v_c}{330 + v_c} \qquad \ldots \text{(iii)}$$

Since, $f'' = f' - 15\%$ of $f' = 0.85\ f'$

$\therefore$ From Eq. (iii), we get

$$\frac{0.85\ f'}{f'} = \frac{330 - v_c}{330 + v_c}$$

$$\Rightarrow \quad 0.85 = \frac{330 - v_c}{330 + v_c} \Rightarrow v_c = 26.7\text{ ms}^{-1}$$

49. *(c)* When an electric dipole is placed in a non-uniform electric field, then it experiences both force and torque. Since, forces on the charges are not linear in non-uniform electric field, so the dipole will also experience a non-zero torque along with a net force.

50. *(b)* In a polar molecule, the centre of gravity of electrons and protons do not coincide.

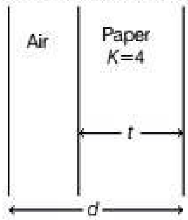
51. *(a)* Given, capacitance, $C = 10\text{ mF} = 10^{-2}$ F

Potential, $V = 100$ V

When capacitor explodes, then its whole stored potential energy is given out in the form of heat and lightning.

$$\therefore \quad E_{\text{out}} = \frac{1}{2}CV^2 = \frac{1}{2} \times 10^{-2} \times (100)^2 = 50\text{J}$$

52. *(c)* The given situation is shown below



$d = 1$ mm

Thickness of paper, $t = 0.75$ mm

According to diagram, it is clear that given capacitor is equivalent to a two capacitors connected in series. In series combination, charge on each capacitor is same.

Hence, $Q_{\text{air}} = Q_{\text{paper}}$

$$\Rightarrow \quad C_{\text{air}}V_{\text{air}} = C_{\text{paper}}V_{\text{paper}}$$

$$\frac{V_{\text{air}}}{V_{\text{paper}}} = \frac{C_{\text{paper}}}{C_{\text{air}}} = \frac{\frac{\varepsilon_0 KA}{t}}{\frac{\varepsilon_0 A}{d - t}}$$

$$= \frac{K(d-t)}{t} = \frac{4(1 - 0.75)}{0.75} = \frac{4}{3}$$

53. *(b)* For first copper wire,

$$l_1 = 1\text{ m}$$

For second copper wire, $l_2 = 9$ m

We have to find, diameter ratio of two wires, i.e. $\frac{d_1}{d_2} = ?$

Since, both copper wires have same resistances.

i.e. $\quad R_1 = R_2$

$$\Rightarrow \quad \rho \cdot \frac{l_1}{A_1} = \rho \cdot \frac{l_2}{A_2} \qquad \left(\because R = \rho . \frac{l}{A}\right)$$

$$\Rightarrow \quad \frac{l_1}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{l_2}{\left(\frac{\pi d_2^2}{4}\right)}$$

$$\Rightarrow \quad \frac{l_1}{d_1^2} = \frac{l_2}{d_2^2}$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3} \text{ or } 1:3$$

54. *(c)* Heat produced in a conductor is given as

$$H = i^2Rt$$

$$\Rightarrow \quad H \propto i^2$$

where, i = current, R = resistance and t = time.

55. *(c)* Radius of circular path of a charged particle in uniform magnetic field, when it enters perpendicular direction of magnetic field.

$$r = \frac{mv}{Bq} \qquad \dots \text{(i)}$$

where, m = mass, v = velocity and q = charge.

We know that, kinetic energy,

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2K}{m}} \qquad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$r = \frac{m}{Bq} \cdot \sqrt{\frac{2K}{m}}$$

$$r = \frac{\sqrt{2Km}}{Bq}$$

For the same value of kinetic energy,

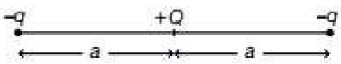
$$r \propto \frac{\sqrt{m}}{q}$$

$$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}} \cdot \frac{q_\alpha}{q_p} = \sqrt{\frac{m_p}{4m_p}} \cdot \left(\frac{2q_p}{q_p}\right) = \frac{1}{1} \text{ or } 1:1$$

56. *(c)* The magnetic properties of a magnet is lost at its Curie point because above this temperature. Magnetic domains to be disrupted permanently.

57. *(b)* Electromagnets are made of soft iron because soft iron has a very high value of susceptibility and low retentitivity.

58. *(c)* The given situation is shown below



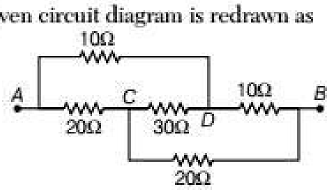
Total potential energy of the system = 0

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left[\frac{(-q)Q}{a} + \frac{(+Q)(-q)}{a} + \frac{(-q)(-q)}{2a}\right] = 0$$

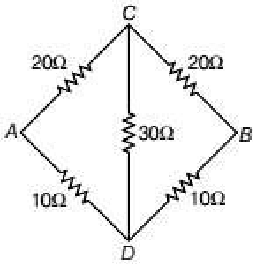
$$\Rightarrow \quad -\frac{qQ}{a} - \frac{qQ}{a} + \frac{q^2}{2a} = 0 \Rightarrow \frac{2qQ}{a} = \frac{q^2}{2a}$$

$$\Rightarrow \quad 2Q = \frac{q}{2} \Rightarrow \frac{Q}{q} = \frac{1}{4}$$

59. *(*)* The given circuit diagram is redrawn as



⇓

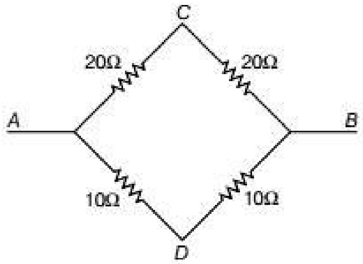


Since,

$$\frac{R_{AC}}{R_{AD}} = \frac{R_{BC}}{R_{DB}}$$

Given circuit is balanced Wheatstone bridge, hence 30 Ω resistance is useless.

Therefore,



$$\therefore \quad R_{AB} = (20 + 20) \| (10 + 10)$$

$$= 40 \| 20 = \frac{40 \times 20}{40 + 20}$$

$$= \frac{800}{60} = \frac{40}{3}\ \Omega = 13.33\ \Omega$$

60. *(c)* Bandwidth of each TV channel = 3.7 MHz

$$= 3.7 \times 10^6 \text{ Hz}$$

Total bandwidth available

$$= 3700 \text{ GHz} = 3.7 \times 10^{12} \text{ Hz}$$

Number of TV channels

$$= \frac{\text{Total bandwidth available}}{\text{Bandwidth of one channel}}$$

$$= \frac{3.7 \times 10^{12}}{3.7 \times 10^6} = 10^6$$

Chemistry

61. *(d)* $d = 1.25$ g/mL, conc. of solution = 3 M

= 3 moles in one litre of the solution.

Molar mass of NaCl = $23 + 35.5 = 58.5$ g mol^{-1}

Volume of solution = 1 L = 1000 mL

Mass of solution = $d \times V$

$= 1.25$ g mL$^{-1} \times 1000$ mL = 1250 g

Mass of solute (NaCl) = $n \times$ molar mass

$= 3 \times 58.5 = 175.5$ g

Mass of solvent = mass of solution − mass of solute

$= 1250 - 175.5 = 1074.5$ g

62. *(c)* For $3p$-orbital, $n = 3$ and $l = 1$.

63. *(a)* Power of the bulb = 150 W = 150 J s^{-1}

As only 8% of the energy is emitted as light so, the total energy emitted per second

$$= \frac{150\ \text{J} \times 8}{100} = 12\ \text{J}$$

Energy of one photon, $E = h\nu = \dfrac{hc}{\lambda}$

$$= \frac{(6.626 \times 10^{-34}\ \text{Js}) \times (3 \times 10^{8}\ \text{ms}^{-1})}{6600 \times 10^{-10}\ \text{m}}$$

$$= 3.0118 \times 10^{-19}\ \text{J}$$

∴ Number of photons emitted per second

$$= \frac{12\ \text{J}}{3.0118 \times 10^{-19}\ \text{J}} = 3.98 \times 10^{19} = 4.0 \times 10^{19}$$

64. *(d)* $O^{2-}(10e^-)$, $F^-(10e^-)$, $Na\,(11e^-)$, $Mg^{2+}(10e^-)$, $Al^{3+}(10e^-)$ and $Ne(10e^-)$

Thus, Na is not isoelectronic with the rest of the species.

65. *(c)* The graph of volume *vs* temperature at constant pressure is called isobar.

66. *(a)* Second ionisation enthalpy of Cr is highest because after the removal of 1st electron, Cr acquires a stable half-filled d^5 configuration thus, removal of 2nd electron is very difficult.

67. *(b)* $He_2^+ = \sigma 1s^2, \sigma^* 1s^1$

Bond order $= \dfrac{1}{2}(N_b - N_a)$

N_b is number of bonding electrons

N_a is number of anti-bonding electrons

$$= \frac{1}{2}(2-1) = \frac{1}{2} = 0.5$$

$C_2^{2-} = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, (\pi 2p_x^2 = \pi 2p_y^2), \sigma 2p_z^2$

$$BO = \frac{1}{2}(10-4) = \frac{6}{2} = 3.0$$

$O_2^+ = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, (\pi 2p_x^2 = \pi 2p_y^2), \pi^* 2p_x^1$

$$BO = \frac{1}{2}(10-5) = \frac{5}{2} = 2.5$$

$O_2^- = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, (\pi 2p_x^2 = \pi 2p_y^2), (\pi^* 2p_x^2 = \pi^* 2p_y^1)$

$$BO = \frac{1}{2}(10-7) = \frac{3}{2} = 1.5$$

Thus, the correct order of increasing bond order is $He_2^+ < O_2^- < O_2^+ < C_2^{2-}$.

68. *(d)* O_2 and H_2 do not show bond dipole as they are homoatomic molecules, hence, they are non-polar.

69. *(d)* Given, $p_{\text{total}} = 25$ bar, $\chi_{O_2} = 0.18$

Using the relation; $\chi_{O_2} + \chi_{Ne} = 1$

$\chi_{Ne} = 1 - 0.18 = 0.82$

$p_{Ne} = \chi_{Ne} \times p_{\text{tota}} = 0.82 \times 25 = 20.5$ bar

70. *(a)* $CaCO_3(s) \longrightarrow CaO\,(s) + CO_2(g)$

$\Delta_r H^\circ = \Delta_f H^\circ(CaO) + \Delta_f H^\circ(CO_2) - \Delta_f H^\circ(CaCO_3)$

$= -635.09 + (-393.51) - (-1206.92)$

$= 178.3$ kJ mol^{-1}.

71. *(b)* $\Delta_r G^\circ = -2.303\ RT \log K_p$

$= -2.303 \times 8.314 \times 298 \times \log(1 \times 10^{-29})$

$= 165469.6$ J mol^{-1} = 165.47 kJ mol^{-1}

72. *(a)* Equilibrium constant, $K_C = \dfrac{[\text{Products}]}{[\text{Reactants}]}$

Higher value of K_C, indicates higher concentration of products which means the reaction goes more towards the completion.

73. *(c)*

	$PCl_5 \rightleftharpoons$	PCl_3 +	Cl_2
Initial conc.	$\frac{1}{2} = 0.5$	0	0
Equili. conc.	$\frac{1-0.4}{2} = \frac{0.6}{2}$	$\frac{0.4}{2}$	$\frac{0.4}{2}$

$$K_C = \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{\frac{0.4}{2} \times \frac{0.4}{2}}{\frac{0.6}{2}} = \frac{(0.4)^2}{2 \times 0.6} = 0.133$$

74. *(d)* The non-existence of PbI_4 and $PbBr_4$ is probably due to the strong oxidising power of Pb^{4+} ions and strong reducing power of I^- and Br^- ions.

75. *(b)* ClO_4^- does not show disproportionate because in this oxoanion chlorine is present in its highest oxidation state (+7). The disproportionation reactions for the other three oxoanions of chlorine are as follows :

$$3\overset{+1}{Cl}O^- \longrightarrow 2\overset{-1}{Cl^-} + \overset{+5}{Cl}O_3^-$$

$$6\overset{+3}{Cl}O_2^- \xrightarrow{h\nu} 4\overset{+5}{Cl}O_3^- + 2\overset{-1}{Cl^-}$$

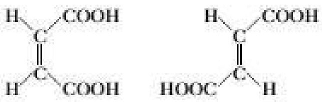
$$4\overset{+5}{Cl}O_3^- \longrightarrow \overset{-1}{Cl^-} + 3\overset{+7}{Cl}O_4^-$$

76. *(d)* $Na + (x + y)NH_3 \longrightarrow [Na(NH_3)_x]^+ + [e(NH_3)_y]^-$.

The deep blue colour of the solution is due to the ammoniated electron which absorbs energy in the visible region of light and thus, imparts blue colour to the solution.

77. *(b)* In fullerene with formula C_{60}, all the carbon atoms are equal and they undergo sp^2 hybridisation.

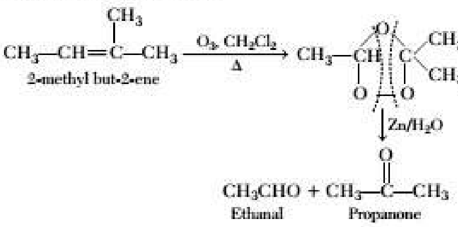
78. *(b)* Maleic acid and fumaric acid both are geometrical isomers.



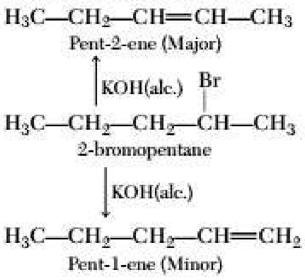
Maleic acid *cis*-butenedioic acid — Fumaric acid *trans*-butenedioic acid

79. *(a)* IUPAC name of tertiary butyl chloride is 2-chloro-2-methylpropane.

80. *(d)* 2-methyl but-2-ene on ozonolysis produce mixture of ethanal and propanone.



81. *(b)* According to Saytzeff rule, in dehydrohalogenation reactions, the preferred product is that alkene which has the greater number of alkyl groups attached to the doubly bonded carbon atoms. Thus, 2-bromopentane gives pent-2-ene as the major product.

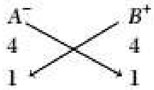


82. *(b)* Rain water is called acid rain when its pH falls below 5.6.

83. *(c)* The number of missing cations and anions are equal in Schottky defect to maintain electrical neutrality.

84. *(d)* Number of A^- ions $= \underset{\text{(At corners)}}{\frac{1}{8} \times 8} + \underset{\text{(At face centres)}}{\frac{1}{2} \times 6} = 4$

Number of B^+ ions $= \underset{\text{(At body centre)}}{1} + \underset{\text{(At edge centres)}}{12 \times \frac{1}{4}} = 4$



The simplest formula of the compound is *AB*.

85. *(b)* For fcc, $a = 2\sqrt{2}r$

$$= 2 \times 1.414 \times 300 \text{ pm} = 848.5 \text{ pm}$$

86. *(b)* When the concentration is expressed as the number of moles of solute per kilogram of solvent, it is known as molality.

87. *(d)* $NaCl(aq) \longrightarrow Na^+(aq) + Cl^-(aq)$

van't Hoff factor, $i = 2$

$BaCl_2(aq) \longrightarrow Ba^{2+}(aq) + 2\,Cl^-(aq)$, $i = 3$

Glucose does not undergo dissociation or association hence, $i = 1$

88. *(a)* $\Delta T_b = iK_b m$

Two solutions having same molality and same van't Hoff factors will have same elevation in boiling points (ΔT_b) and thus, have same boiling points (T_b).

For NaCl, $i = 2$; $MgSO_4$, $i = 2$; $MgCl_2$, $i = 3$;

$Al_2(SO_4)_3$, $i = 5$ and $AlCl_3$, $i = 4$.

Thus, one molal solution of sodium chloride in water has the same boiling point as one molal solution of $MgSO_4$.

89. *(a)* $\dfrac{p^\circ - p_s}{p^\circ} = \dfrac{w_2 \times M_1}{w_1 \times M_2}$

$$\frac{17.535 - p_s}{17.535} = \frac{25 \times 18}{450 \times 180} = 5.55 \times 10^{-3}$$

$$17.535 - p_s = 5.55 \times 10^{-3} \times 17.535$$

$$p_s = 17.535(1 - 0.00555)$$

$$p_s = 17.535 \times 0.99445$$

$$p_s = 17.437 \text{ mm Hg.}$$

90. *(a)* In a electrochemical cell, the reaction will be feasible when, ΔG is – ve which is possible only when E is +ve as $\Delta G = -nFE$.

91. *(b)* For the cell, $Zn \mid Zn^{2+} \parallel Ag^+ \mid Ag$

$$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$$

$$1.56\text{ V} = E^\circ_{Ag^+/Ag} - E^\circ_{Zn^{2+}/Zn}$$

$$1.56\text{ V} = 0.8\text{ V} - E^\circ_{Zn^{2+}/Zn}$$

$$E^\circ_{Zn^{2+}/Zn} = -0.76\text{ V}$$

$$\therefore \quad E^\circ_{Zn/Zn^{2+}} = -E^\circ_{Zn^{2+}/Zn} = -(-0.76) = +0.76\text{ V}$$

92. *(a)* $\Lambda^\circ_m[Ba(OH)_2]$

$= \Lambda^\circ_m(BaCl_2) + 2\Lambda^\circ_m(NaOH) - 2\Lambda^\circ_m(NaCl)$

$= (2.8 + 2 \times 2.481 - 2 \times 1.265) \times 10^{-2}\text{ S m}^2\text{mol}^{-1}$

$= (2.8 + 4.962 - 2.53) \times 10^{-2}\text{S m}^2\text{ mol}^{-1}$

$= 5.232 \times 10^{-2}\text{S m}^2\text{mol}^{-1}$

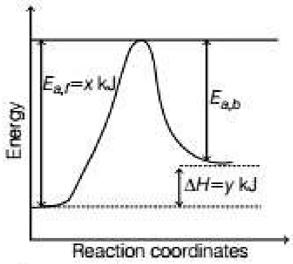
93. *(c)* For every 10°C rise in temperature, the rate of a chemical reaction doubles thus, for 60°C rise in temperature the rate of reaction increases by 2^n.

where, $n = \frac{\Delta T}{10} = \frac{60}{10} = 6$

$\therefore \quad r_1 = 2^n r = 2^6 r \Rightarrow r_1 = 64r$

So, when the temperature increases by 60°C, then the rate of reaction increases by 64 times.

94. *(b)* Consider the following graph,



From the above graph

$$x = y + E_{a,b}$$
$$E_{a,b} = x - y \text{ kJ}$$

95. *(c)* For a reaction,

$$aP + bQ \rightarrow \text{Products}$$

$$r = k[P]^a[Q]^b \quad \ldots \text{(i)}$$

$$2r = k[2P]^a[Q]^b \quad \ldots \text{(ii)}$$

$$4r = k[P]^a[2Q]^b \quad \ldots \text{(iii)}$$

On dividing Eqs. (ii) by (i), we get

$$2 = 2^a \Rightarrow a = 1$$

On dividing Eqs. (iii) by (i), we get

$$4 = 2^b \Rightarrow b = 2$$

$\therefore$ Overall order $= a + b = 1 + 2 = 3$.

96. *(c)* Semiconductor of very high purity are obtained by zone refining. Zone refining method is very useful for producing semiconductors and other metals of very high purity, e.g., Ge, Si, B, Ga and In.

97. *(b)* NaCN forms a soluble complex with ZnS thus, it selectively prevents ZnS from coming to the froth but allows PbS to come with the froth.

$$4NaCN + ZnS \longrightarrow \underset{\text{[Soluble complex]}}{Na_2[Zn(CN)_4]} + Na_2S$$

98. *(b)* $\underset{\substack{\text{White}\\\text{phosphorus}\\(A)}}{P_4(s)} + 3NaOH(aq) + 3H_2O(l) \longrightarrow \underset{\substack{\text{Phosphine}\\(B)}}{PH_3(g)} + 3NaH_2PO_2(aq)$

Phosphine has unpleasant odour like that of garlic or rotten fish. It burns in air to give clouds of P_4O_{10} which act as smoke screens.

99. *(b)* The reaction,

$SO_2 + 2H_2S \xrightarrow{\text{Oxidation}} 3S\text{ (sol.)} + 2H_2O$, represent the preparation of sulphur sol by oxidation.

100. *(d)* NeF_2 does not form inert gas compound due to high ionisation enthalpy because of small size.

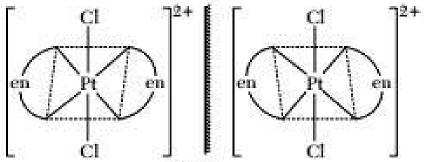
101. *(c)* Covalency of nitrogen is restricted to four due to absence of *d*-orbitals in its valence shell. Only four orbitals (one 2*s* and three 2*p*-orbitals) are available in its valence shell.

102. *(d)* The photographic industry relies on the special light sensitive properties of AgBr.

103. *(b)* In acidic medium, oxalic acid is oxidised to CO_2 (carbon dioxide) by $KMnO_4$.

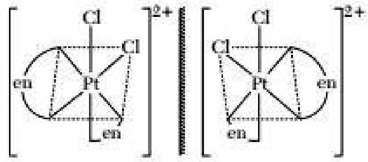
$$5(COOH)_2 + 2KMnO_4 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2MnSO_4 + 10\,CO_2 + 8H_2O$$

104. *(c)* $[PtCl_2(en)_2]^{2+}$ complex forms geometrical isomers (*cis* and *trans*). *Trans*-isomer does not show optical isomerism, since it is symmetrical while *cis*-isomer shows optical isomerism as it is unsymmetrical.



Mirror

trans-$[PtCl_2(en)_2]^{2+}$ isomer—optically inactive
(Superimposable mirror images)



Mirror

cis-$[PtCl_2(en)_2]^{2+}$ isomer—optically inactive
(Non-superimposable mirror images)

105. *(b, d)* Both options are same. The IUPAC name of complex $[Ag(NH_3)_2]^+[Ag(CN)_2]^-$ is diamminesilver (I) dicyanoargentate (I).

106. *(d)* Molar conductance of $[Co(NH_3)_3Cl_3]$ is zero as it does not ionise in solution.

107. *(b)* Alkyl halides are hydrolysed to corresponding alcohols by boiling with aqueous alkali solution (NaOH or KOH).

$$CH_3CH_2I + KOH \longrightarrow CH_3CH_2OH + KI$$

This is nucleophilic substitution reaction in which the attacking nucleophile is OH^-.

108. *(c)* S_N1 reaction is two steps reaction in which carbocation is formed as an intermediate in step I (rate determining step). Greater the stability of carbocation, greater will be its ease of formation from alkylhalide and faster will be the rate of reaction.

Stability of carbocation follows the order :

Tertiary > Secondary > Primary

due to decreasing +*I*-effect.

Thus, the order of reactivity of haloalkanes towards S_N1 reaction is

tertiary halide > secondary halide > primary halide.

109. *(a)* The given reaction takes place as follows :

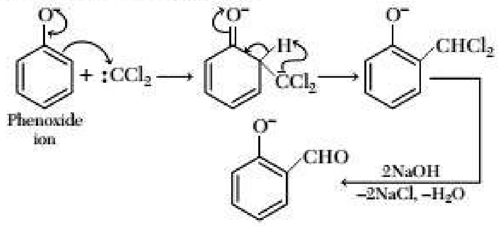
Chlorine is a *o*, *p*-directing and deactivating group. Chlorine is deactivating because of its –*I*-effect. As inductive effect is distance dependent so, electron density is lower at *ortho* position than *para* position. Thus, the nitration occurs at *para* position.

110. *(d)* The unstable intermediate [$:CCl_2$] is formed in Reimer-Tiemann reaction which is an electrophilic substitution reaction and occurs through the following steps.

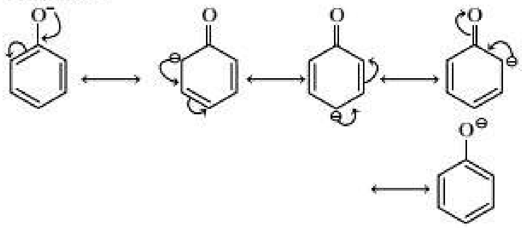
(i) **Generation of electrophile**

$$HO^- + CHCl_3 \rightleftharpoons H_2O + CCl_3^- \longrightarrow \underset{\text{Dichlorocarbene (Electrophile)}}{:CCl_2} + Cl^-$$

(ii) **Electrophilic substitution**



111. *(d)* Phenols are highly acidic compared to alcohols due to formation of phenoxide ion which is stabilised by resonance.



112. *(b)* Boiling point increases with increase in molecular mass so, 1-butanol has higher boiling point than 1-propanol.

Unlike alcohols, ethers do not form hydrogen bonds thus, they have lower boiling points than the corresponding alcohols.

Due to weak dipole-dipole interactions, the boiling points of lower ethers are only slightly higher than those of the *n*-alkanes having comparable molecular masses.

Thus, the increasing order of boiling points is

n-butane < ethoxyethane < 1-propanol < 1-butanol.

113. *(a)* Acidic strength $\propto k_a$ values

Due to – *I*-effect of electron withdrawing —Cl group, chloropropanoic acid is stronger acid than propanoic acid. Further, greater the number of electron withdrawing substitutents, greater would be the acidic strength.

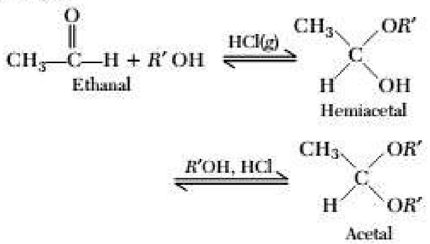
Inductive effect decreases rapidly with distance and so, is the acidic strength. Hence, the correct order of acidic strength (or k_a values) will be

$$\underset{\text{2,2-dichloroacetic acid}}{Cl_2CHCOOH} > \underset{\text{2-chloropropanoic acid}}{ClCH(CH_3)COOH} >$$

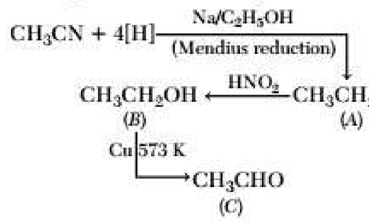
$$\underset{\text{3-chloropropanoic acid}}{ClCH_2CH_2COOH} > \underset{\text{Propanoic acid}}{CH_3CH_2COOH}$$

114. *(a)* Addition of Grignard reagents to carbonyl compounds is an example of nucleophilic addition reactions.

115. *(b)* The given reaction is known as acetal formation reaction.



116. *(b)* The complete reaction takes place as follows :



117. *(a)* In nucleic acids the nucleotides are joined together by phosphodiester linkage between 5′ and 3′ carbon atoms of the pentose sugar.

118. *(d)* Amino acids exists as Zwitter ions at isoelectric pH.

119. *(c, d)* Neoprene and teflon are formed by addition polymerisation, while terylene and nylon-6,6 are formed by condensation polymerisation.

120. *(b)* Sucrose is natural sweetener while aspartame, sucralose and saccharin are artificial sweeteners.

Mathematics

121. *(b)* In option (b), $x^2 + 3 = 0$

$\Rightarrow \quad x = \sqrt{-3} \in C$

$\Rightarrow$ x is not a real number.

$\Rightarrow$ It is an empty set.

122. *(b)* If a set having n elements then its number of subsets $= 2^n$

$\therefore$ Numbers of proper susbets of a set having $(n+1)$ elements $= 2^{n+1} - 1$.

123. *(c)* Given, function is

$y = \sqrt{x-2} + \sqrt{1-x}$

Since, $x - 2 \geq 0$ and $1 - x \geq 0$

$\Rightarrow \quad x \geq 2 \quad \ldots \text{(i)}$

$\Rightarrow \quad x \leq 1 \quad \ldots \text{(ii)}$

$\therefore$ From Eqs. (i) and (ii), $x = \phi$

124. *(b)* Given, $A = [1, 2, 3, 4]$

Let R be a reflexive relation on A, then for each $x \in A$, $(a, a) \in R$

$\therefore$ Option (b) is true.

125. *(d)* $f(x) = \dfrac{2x-3}{3x+4}$

Let $f(x) = y = \dfrac{2x-3}{3x+4}$

On cross multiplication, we get

$3xy + 4y = 2x - 3$

$\Rightarrow x(3y - 2) = -3 - 4y$

$\Rightarrow \quad x = \dfrac{-3-4y}{3y-2} \Rightarrow x = f^{-1}(y) = \dfrac{-3-4y}{3y-2}$

Put $y = -\dfrac{4}{3}$, we get

$$f^{-1}\left(-\frac{4}{3}\right) = \frac{-3 - 4 \times \left(-\frac{4}{3}\right)}{3\left(-\frac{4}{3}\right) - 2}$$

$$= \frac{-3 + \frac{16}{3}}{-4-2} = \frac{7}{3 \times (-6)} = -\frac{7}{18}$$

126. *(a)* Let $f(x) = y = x + \dfrac{1}{x}$

or $xy = x^2 + 1$ or $x^2 - xy + 1 = 0$

Since, $x \in [0, \infty)$

$\therefore \quad D \geq 0 \Rightarrow y^2 - 4 \geq 0 \Rightarrow y \in [2, \infty)$

$\therefore \quad x = \dfrac{y \pm \sqrt{y^2-4}}{2}$

i.e. $\quad x = \dfrac{y - \sqrt{y^2-4}}{2}$ or $x = \dfrac{y - \sqrt{y^2-4}}{2}$

$\Rightarrow \quad f^{-1}(y) = \dfrac{y + \sqrt{y^2-4}}{2}$

or $\quad f^{-1}(y) = \dfrac{y - \sqrt{y^2-4}}{2}$

Replace by x,

$f^{-1}(x) = \dfrac{x + \sqrt{x^2-4}}{2}$

or $\quad f^{-1}(x) = \dfrac{x - \sqrt{x^2-4}}{2}$

127. *(b)* We have, $\dfrac{\tan 330° \sec 420° \sin 300°}{\tan 135° \sin 210° \sec 315°}$

$$= \frac{\tan(360-30)° \sec(360+60)° \sin(360-60)°}{\tan(180-45)° \sin(180+30)° \sec(360-45)°}$$

$$= \frac{-\tan 30° \times \sec 60° \times (-\sin 60°)}{(-\tan 45°)(-\sin 30°)\sec 45°}$$

$[\because \tan(2\pi - \theta) = -\tan\theta, \sec(2\pi \pm \theta) = \sec\theta,$
$\sin(2\pi - \theta) = -\sin\theta, \ \sin(\pi + \theta) = -\sin\theta,$
$\tan(\pi - \theta) = -\tan\theta]$

$$= \frac{1 \times 2 \times \sqrt{3} \times 2}{\sqrt{3} \times 2 \times 1 \times \sqrt{2}} = \sqrt{2}$$

128. *(b)* Given, equation is

$\sin(120 - A) = \sin(120 - B)$

Since, sine is positive in II quadrant.

$\therefore$ Either $120 - A = 120 - B$

$\Rightarrow \quad A = B$

or $\quad 120 - A = 180 - (120 - B)$

$\Rightarrow \quad 120 - A = 60 + B \Rightarrow A + B = \dfrac{\pi}{3}$

129. *(a)* Given, equation is

$\cot^{-1}\dfrac{1}{5} + \cot^{-1}\dfrac{1}{3} - \cot^{-1}\dfrac{4}{7} = \cot^{-1} x$

$\Rightarrow \tan^{-1} 5 + \left(\tan^{-1} 3 - \tan^{-1}\dfrac{7}{4}\right) = \tan^{-1}\dfrac{1}{x}$

$\Rightarrow \tan^{-1} 5 + \tan^{-1}\left(\dfrac{3 - \frac{7}{4}}{1 + 3 \times \frac{7}{4}}\right) = \tan^{-1}\dfrac{1}{x}$

$\Rightarrow \tan^{-1} 5 + \tan^{-1}\left(\dfrac{1}{5}\right) = \tan^{-1}\dfrac{1}{x}$

$\Rightarrow \tan^{-1} 5 + \cot^{-1} 5 = \tan^{-1}\dfrac{1}{x}$

$\Rightarrow \tan^{-1}\dfrac{1}{x} = \dfrac{\pi}{2} \quad \left[\because \tan^{-1}\theta + \cot^{-1}\theta = \dfrac{\pi}{2}\right]$

$\Rightarrow \dfrac{1}{x} = \tan\dfrac{\pi}{2} = \infty \Rightarrow x = 0$

130. *(c)* Given, equation is

$$1 + i = (x + iy)(u + iv)$$

$$\Rightarrow 1 + i = (xu - yv) + i(xv + yu)$$

Comparing real and imaginary parts, we get

$$xu - yv = 1 \quad \ldots \text{(i)}$$

$$xv + yu = 1 \quad \ldots \text{(ii)}$$

Multiply Eq. (i) by u and Eq. (ii) by v and then adding, we get

$$x(u^2 + v^2) = u + v \Rightarrow x = \frac{u + v}{u^2 + v^2}$$

From Eq. (i)

$$y = \frac{xu - 1}{v} = \frac{u - v}{u^2 + v^2}$$

(Substituting the value of x)

Now, $\tan^{-1}(y/x) + \cot^{-1}(u/v)$

$$\tan^{-1}\left(\frac{u - v}{u + v}\right) + \cot^{-1}(u/v)$$

$$= \tan^{-1}\left(\frac{1 - \frac{v}{u}}{1 + \frac{v}{u}}\right) + \cot^{-1}(u/v)$$

$$= \tan^{-1}(1) - \tan^{-1}\left(\frac{v}{u}\right) + \tan^{-1}\left(\frac{v}{u}\right)$$

$$= n\pi + \pi/4, n \in I$$

131. *(a)* $1^3 + 2^3 + \ldots + n^3$

$$= \left(\frac{n(n+1)}{2}\right)^2 = (1 + 2 + \ldots + n)^2$$

132. *(b)* Let $z = a + ib$

Then, in third quadrant $a < 0, b < 0$.

Its conjugate $\bar{z} = \overline{a + ib} = a - ib = a + (-ib)$

$= a + ik$, where $k = -b$

$\Rightarrow \quad a < 0, k > 0$

$\Rightarrow \quad \bar{z}$ lies in II quadrant.

133. *(b)* Since, β satisfy the inequation

$$x^2 - x - 6 > 0$$

$$\Rightarrow \quad \beta^2 - \beta - 6 > 0$$

$$\Rightarrow \quad (\beta - 3)(\beta + 2) > 0$$

$$\Rightarrow \quad \beta > 3, \beta < -2$$

134. *(a)* Given, ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$$\Rightarrow \quad \frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\Rightarrow \quad \frac{56! \times (51 - r)!}{(50 - r)! \times 54!} = \frac{30800}{1}$$

$$\Rightarrow 56 \times 55 \times (51 - r) = 30800$$

$$\Rightarrow \quad 51 - r = 10 \Rightarrow r = 41$$

135. *(c)* Given, expansion is $\left(2 + \frac{x}{3}\right)^n$

Let t_{r+1} be general term.

Then, $t_{r+1} = {}^nC_r 2^{n-r}\left(\frac{x}{3}\right)^r = {}^nC_r 2^{n-r} \cdot 3^{-r} x^r$

Since, coefficient of x^5 and x^6 are equal.

$$\therefore \quad {}^nC_6 2^{n-6} 3^{-6} = {}^nC_5 2^{n-5} 3^{-5}$$

$$\Rightarrow \quad \frac{{}^nC_6}{{}^nC_5} = 2 \times 3 \Rightarrow \frac{n! \times 5! \times (n-5)!}{(n-6)! \times 6! \times n!} = 6$$

$$\Rightarrow \quad \frac{n-5}{6} = 6 \Rightarrow n - 5 = 36 \Rightarrow n = 41$$

136. *(d)* Given, expansion is $\left[\sqrt{\left(\frac{x}{3}\right)} + \sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$

Let t_{r+1} be general term, then

$$t_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\sqrt{\frac{3}{2x^2}}\right)^r$$

$$= {}^{10}C_r \frac{x^{\frac{10-r}{2}}}{3^{\frac{10-r}{2}}} \cdot \frac{3^{r/2}}{2^{r/2} \cdot x^r} = {}^{10}C_r x^{5 - \frac{r}{2} - r} \cdot \frac{3^{r-5}}{2^{r/2}}$$

For the term independent of x.

Puts $5 - \frac{r}{2} - r = 0$

$$\Rightarrow \quad 5 - \frac{3}{2}r = 0 \Rightarrow \frac{3}{2}r = 5 \Rightarrow r = \frac{10}{3}$$

Fractional value of r is not possible. So, no term is independent of x.

137. *(c)* Let

$$S = \frac{1^2}{1 \cdot 2} + \frac{1^2 + 2^2}{2 \cdot 3} + \frac{1^2 + 2^2 + 3^2}{3 \cdot 4} + \ldots$$

upto 20 terms

Let t_n be nth terms of series.

Then, $t_n = \frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n \cdot (n+1)}$

$$= \frac{\Sigma n^2}{n(n+1)} = \frac{n(n+1)(2n+1)}{6 \cdot n(n+1)} = \frac{2n+1}{6}$$

Taking summation on both sides

$$\sum_{n=1}^{20} t_n = \frac{2}{6}\sum_{n=1}^{20} n + \frac{1}{6}\sum_{n=1}^{20} 1$$

$$= \frac{1}{3} \times \left(\frac{20(20+1)}{2}\right) + \frac{1}{6} \times 20$$

$$= \frac{1}{3}\left(\frac{20 \times 21}{2}\right) + \left(\frac{10}{3}\right) = \frac{210}{3} + \frac{10}{3} = \frac{220}{3}$$

138. *(a)* Given, equations of lines are

$$\sqrt{3}x + y = 1 \quad \ldots \text{(i)}$$

and $\quad x + \sqrt{3}y = 1 \quad \ldots \text{(ii)}$

Let m_1 and m_2 be slopes of Eqs. (i) and (ii), then

$$m_1 = -\sqrt{3}, m_2 = -\frac{1}{\sqrt{3}}$$

Let θ be angle between them, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\cdot\frac{1}{\sqrt{3}}}\right| = \left|\frac{-3+1}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

139. *(d)* Since, equation of family of lines is $\frac{x}{a} + \frac{y}{b} = 1$ ... (i)

Sum of intercepts $= a + b = 7$ (given)

$\Rightarrow \quad b = 7 - a$

Substitute $b = 7 - a$ in Eq. (i), we get

$$\frac{x}{a} + \frac{y}{7-a} = 1$$

$\Rightarrow (7-a)x + ay = a(7-a) \Rightarrow ay = (7-a)(a-x)$

140. *(b)* Let equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since it passes through the points $(-3, 1)$ and $(2, -2)$

$\therefore \quad \frac{(-3)^2}{a^2} + \frac{(1)^2}{b^2} = 1$ and $\frac{(2)^2}{a^2} + \frac{(-2)^2}{b^2} = 1$

$\Rightarrow \quad \frac{9}{a^2} + \frac{1}{b^2} = 1$ ... (i)

$\Rightarrow \quad \frac{4}{a^2} + \frac{4}{b^2} = 1$ ...(ii)

Multiply by 4 in Eq. (i) and subtracting Eq. (i) from Eq. (ii),

$$3a^2 = 32 \Rightarrow a^2 = \frac{32}{3}$$

Substituting in Eq. (i), gives

$$\frac{9 \times 3}{32} + \frac{1}{b^2} = 1$$

$\Rightarrow \quad \frac{1}{b^2} = 1 - \frac{27}{32} = \frac{5}{32} \Rightarrow b^2 = \frac{32}{5}$

$\therefore$ Required equation is

$$\frac{3x^2}{32} + \frac{5y^2}{32} = 1 \text{ or } 3x^2 + 5y^2 = 32$$

141. *(b)* Given, equation of parabola is $y^2 = 6x$

Substitute $x = 24$

$y^2 = 6 \times 24$

$y = \pm\sqrt{6 \times 24} = \pm\sqrt{6 \times 6 \times 4} = \pm(6 \times 2) = \pm 12$

$\therefore$ Point on the parabola is (24, 12), (24, − 12) vertex of given parabola is (0, 0).

$\therefore$ Equation of lines passing through (0, 0) and (24, 12), (24, − 12) is

$$y = \frac{12}{24}x, y = \frac{-12}{24}x$$

$\Rightarrow \quad 2y - x = 0, 2y + x = 0$

$\Rightarrow \quad 2y \pm x = 0$ are required equations of lines.

142. *(b)* $P(a, b, c)$ and PA and PB are perpendicular to YZ and ZX planes. Hence, coordinate of A and B are $(0, b, c)$ and $(a, 0, c)$ respectively.

Equation of plane passing through (0, 0, 0), $(0, b, c)$ and $(a, 0, c)$ is $\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$

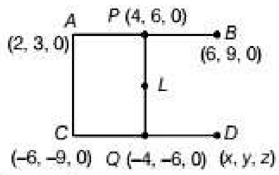
$\Rightarrow \quad x(bc - 0) - y(0 - ac) + z(0 - ab) = 0$

$\Rightarrow \quad bcx + acy - abz = 0$

143. *(a)* Perpendicular distance of point (1, 2, 3) from plane $3y + 4z + 4 = 0$

$$= \left|\frac{3(2) + 4(3) + 4}{\sqrt{(3)^2 + (4)^2}}\right| = \left|\frac{6+16}{5}\right| = \left|\frac{22}{5}\right| = 4.4$$

144. *(c)*



Let coordinate of D is (x, y, z).

Using mid-point formula,

$$Q = \left(\frac{x-6}{2}, \frac{y-9}{2}, \frac{z+0}{2}\right)$$

Also, $\quad P = \left(\frac{2+6}{2}, \frac{3+9}{2}, \frac{0+0}{2}\right) = (4, 6, 0)$

Since, $AC \parallel PQ$

$\therefore$ D.R'.s of line AC = D.R' s of line PQ

$\Rightarrow (-8, -12, 0) = \left(\frac{x-14}{2}, \frac{y-21}{2}, \frac{z}{2}\right)$

$\Rightarrow x = -2, y = -3, z = 0$

$\Rightarrow D(-2, -3, 0) \Rightarrow Q(-4, -6, 0)$

If L is mid-point of PQ, then

$$L\left(\frac{4-4}{2}, \frac{6-6}{2}, 0\right) = (0, 0, 0)$$

$\therefore$ Perpendicular distance of L (0, 0, 0) from the plane $3x + 4z + 25 = 0$ is

$$\left|\frac{3(0) + 4(0) + 25}{\sqrt{(3)^2 + (4)^2}}\right| = \left|\frac{25}{\sqrt{25}}\right| = \sqrt{25} = 5$$

145. *(b)* Equation of plane through the intersection of planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is

$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0$

or $(1 + 2k)x + (2 + k)y + (3 - k)z + (5k - 4) = 0$ ... (i)

D.R.' s of normal of plane (i) are

$$= \langle (1 + 2k), (2 + k), (3 - k) \rangle$$

Given, $5x + 3y + 6z + 8 = 0$ ... (ii)

D.R.'s of plane (ii) are $< 5, 3, 6 >$.
Since Eq. (i) is Perpendicular to the plane (ii),

$\therefore\ 5(1 + 2k) + (2 + k)3 + 6(3 - k) = 0$

$\Rightarrow\ 5 + 10k + 6 + 3k + 18 - 6k = 0$

$\Rightarrow\ 7k + 29 = 0 \Rightarrow k = \frac{-29}{7}$

$\therefore$ Required equation of plane is

$$(x - 2y + 3z - 4) + \left(-\frac{29}{7}\right)(2x + y - z + 5) = 0$$

$\Rightarrow 7x + 14y + 21z - 28 - 58x - 29y + 29z - 145 = 0$

$\Rightarrow -51x - 15y + 50z - 173 = 0$

$\Rightarrow 51x + 15y - 50z + 173 = 0$

146. *(c)* $\lim_{x \to 0} \frac{xa^x - x}{1 - \cos x} = \lim_{x \to 0} \frac{x(a^x - 1)}{2\sin^2 \frac{x}{2}}$

$[\because \cos 2\theta = 1 - 2\sin^2 \theta]$

$$= \lim_{x \to 0} \frac{a^x - 1}{\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2} \times \frac{2}{x}$$

$$= \frac{\lim_{x \to 0} 2\left(\frac{a^x - 1}{x}\right)}{\left[\lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}}\right]^2} = 2\log a$$

147. *(b)* Given, $y = \tan x$

Differentiating w.r.t. x both sides,

$$\frac{dy}{dx} = \sec^2 x$$

Taking again derivative w.r.t. x,

$$\frac{d^2y}{dx^2} = 2\sec x \cdot \sec x \tan x$$

$$= 2\sec^2 x \tan x = 2\tan x(1 + \tan^2 x)$$

$$= 2y(1 + y^2)$$

148. *(c)* Given, $y = \tan^{-1}\left(\frac{a - x}{1 + ax}\right)$

$\Rightarrow\ y = \tan^{-1} a - \tan^{-1} x$

Taking derivative w.r.t. x on both sides, we get

$$\frac{dy}{dx} = -\frac{1}{1 + x^2}$$

149. *(c)* $f(x) = \begin{cases} [x] + [-x], & x \neq 2 \\ K, & x = 2 \end{cases}$

Since, $f(x)$ is continuous at $x = 2$

$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

Now, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} [x] + [-x]$

$= 1 + (-2) = -1$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} [x] + [-x] = 2 - 3 = -1$

$\Rightarrow\ K = -1$

150. *(b)* Given, equation of curve is

$$x^2 - xy + y^2 = 27 \quad \ldots \text{(i)}$$

Taking derivative w.r.t. x on both sides

$$2x - \frac{xdy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(2y - x) = y - 2x \Rightarrow \frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

Since, curve has tangent parallel to X-axis.

$\therefore$ Slope of tangent $= 0$

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{y - 2x}{2y - x} = 0$$

$$\Rightarrow y = 2x \quad \ldots \text{(ii)}$$

Now, solving Eqs. (i) and (ii), we get

$$x^2 - 2x^2 + 4x^2 = 27 \Rightarrow 3x^2 = 27 \Rightarrow x = \pm 3$$

For $x = 3$, $y = 6$ and $x = -3$, $y = -6$

$\therefore$ Points are $(3, 6)$ and $(-3, -6)$.

151. *(b)* Given, equation of circle is

$$x^2 + y^2 = 2 \quad \ldots \text{(i)}$$

Taking derivative w.r.t. 't' on both sides.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

If abscissa and ordinate increase at the same rate, we have

$$\frac{dx}{dt} = \frac{dy}{dt}$$

$$x\frac{dx}{dt} + y\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt}(x + y) = 0$$

Since, $\frac{dx}{dt} \neq 0$

$$\Rightarrow x + y = 0 \Rightarrow x = -y \quad \ldots \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$x^2 + (-x)^2 = 2 \Rightarrow x = \pm 1$$

For $x = 1$, $y = -1$ and $x = -1$, $y = 1$

Required point are $(1, -1)$ and $(-1, 1)$.

152. *(a)* Given, $\frac{dV}{dt} = 35$

where, V is volume of spherical balloon.

Also, $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 35 \Rightarrow \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = 35$$

$$\Rightarrow \frac{dr}{dt} = \frac{35 \times 3}{4\pi \times 3r^2}$$

Let S be surface area of sphere, then $S = 4\pi r^2$

Taking derivative w.r.t. r

$$\frac{dS}{dt} = 8\pi \times r\frac{dr}{dt} = 8\pi \times r \times \frac{35 \times 3}{4\pi \times 3r^2}$$

Substitute, $r = 7$

$$\frac{dS}{dt} = \frac{2 \times 35 \times 3}{3 \times 7} = 10 \text{ cm}^2/\text{min}$$

153. *(a)* Let $I = \int e^x(\cos x - \sin x)\,dx$

$$= \int e^x \cos x\,dx - \int e^x \sin x\,dx$$

$$= \int e^x \cos x\,dx - [e^x(-\cos x) - \int e^x(-\cos x)\,dx]$$

$$= \int e^x \cos x\,dx + e^x \cos x - \int e^x \cos x\,dx$$

$$= e^x \cos x + C$$

154. *(a)* Given, differential equation is

$$(2x - 10y^3)\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{2x - 10y^3}$$

$$\Rightarrow \frac{dx}{dy} = \frac{2x - 10y^3}{-y} = \frac{-2x}{y} + 10y^2$$

$$\Rightarrow \frac{dx}{dy} + \frac{2}{y}x = 10y^2$$

Compare with linear differential equation

$$\frac{dx}{dy} + Px = Q \Rightarrow \text{IF} = e^{\int \frac{2}{y}dy} = e^{\log y^2} = y^2$$

$\therefore$ Required solution is $x.\,y^2 = \int 10y^2 \cdot y^2 dy + C$

i.e. $xy^2 = 2y^5 + C$

155. *(a)* Given, differential equation is

$$\Rightarrow (xy)^{-1}\left(x\frac{dy}{dx} + y\right) = (x+y)^{-2}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \int (xy)^{-1}\left(x\frac{dy}{dx} + y\right)dx = \int (x+y)^{-2}\left(1 + \frac{dy}{dx}\right)dx \quad \ldots\text{(i)}$$

Using integral,

$$\int (f(x))^n f'(x)dx = \frac{(f(x))^{n+1}}{n+1}$$

and $$\int \frac{f'(x)}{f(x)}dx = \log(f(x)) + C$$

From Eq. (i) $$\log(xy) = \frac{(x+y)^{-1}}{-1} + C$$

$$\Rightarrow \log(xy) = \frac{-1}{x+y} + C$$

156. *(a)* Let $\mathbf{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b} = \hat{i} + \hat{j} + 2\hat{k}$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(2-2) - \hat{j}(6-2) + \hat{k}(3-1) = -4\hat{j} + 2\hat{k}$$

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(-4)^2 + (2)^2} = \sqrt{16+4} = \sqrt{20}$$

Since, $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$,

If θ is angle between a and b.

$$\sqrt{20} = \sqrt{9+1+4}\,\sqrt{1+1+4}\,\sin\theta$$

$$\Rightarrow \sin\theta = \frac{\sqrt{20}}{\sqrt{6}\,\sqrt{14}}$$

$$\Rightarrow \sin\theta = \sqrt{\frac{20}{6 \times 14}} = \sqrt{\frac{5}{21}}$$

157. *(a)* Let θ be angle between $\mathbf{a}$ and $\mathbf{b}$ then $\theta = 60°$ (given)

Since, $|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a} \cdot \mathbf{b}$

$$= 4 + 4 + (2 \times 2 \times 2 \times \cos 60°)$$

$$= 8 + 8\cos 60° = 8 + 4 = 12$$

$$\Rightarrow |\mathbf{a} + \mathbf{b}| = \sqrt{12} = 2\sqrt{3}$$

Now, $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = |\mathbf{a}||\mathbf{a} + \mathbf{b}|\cos x$

where x is angle between a and $\mathbf{a} + \mathbf{b}$.

$$\Rightarrow \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} = 4\sqrt{3}\cos x$$

$$\Rightarrow 4 + 2 \times 2\cos 60° = 4\sqrt{3}\cos x$$

$$\Rightarrow 6 = 4\sqrt{3}\cos x$$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow x = 30°$$

158. *(d)* $$\begin{bmatrix} -2 & 5 \\ 3 & -1 \end{bmatrix}_{2 \times 2} \begin{bmatrix} x \\ y \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 3 \\ -1 \end{bmatrix}_{2 \times 1}$$

$$\Rightarrow \begin{bmatrix} -2x + 5y \\ 3x - y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$\Rightarrow -2x + 5y = 1,\ 3x - y = 5$$

or $$y = 3x - 5 \quad \ldots\text{(i)}$$

$$\Rightarrow -2x + 5(3x - 5) = 1$$

$$\Rightarrow -2x + 15x - 25 = 1$$

$$\Rightarrow 13x = 26 \Rightarrow x = 2$$

Substituting $x = 2$ in Eq. (i), we get

$$y = 6 - 5 = 1$$

Hence, $(x, y) = (2, 1)$

159. *(d)* $(BA)^{-1} = C$ (given)

or $A^{-1}B^{-1} = C$

$$\text{or } A^{-1}\begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$$

Multiply by B on both sides, we get

$$A^{-1}(B^{-1}B) = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\text{or} \quad A^{-1} = \begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}_{3 \times 3}$$

160. (c) Given, $AB = B$

Multiply by A on both sides

$$ABA = BA \quad \dots (i)$$

Also, $BA = A$

Multiply by B on both sides

$$BAB = AB \quad \dots (ii)$$

Adding Eqs. (i) and (ii), we get

$$ABA + BAB = BA + AB$$

$$\Rightarrow A(BA) + B(AB) = BA + AB$$

$$\Rightarrow AA + BB = A + B$$

$$\Rightarrow A^2 + B^2 = A + B$$

161. (d) Given, equation is $x^3 + a^2x + b = 0$

Since, α, β, γ are its roots.

$\therefore$ Sum of roots $= \alpha + \beta + \gamma = 0 \quad \dots (i)$

Now, $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

Using operation $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} (\alpha+\beta+\gamma) & \beta & \gamma \\ (\alpha+\beta+\gamma) & \gamma & \alpha \\ (\alpha+\beta+\gamma) & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0 \quad \text{[Using Eq. (i)]}$$

162. (a) Given determinant is

$$\begin{vmatrix} 4 & 4 & 4 \\ (a+a^{-1})^2 & (b+b^{-1})^2 & (c+c^{-1})^2 \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix}$$

Using operation $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 4 & 4 & 4 \\ 4aa^{-1} & 4bb^{-1} & 4cc^{-1} \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix} = 0$$

[$\because R_1$ and R_2 are identical rows]

163. (a) Given, $P(B \mid A) = 0.6$

$$\Rightarrow \frac{P(B \cap A)}{P(A)} = 0.6$$

$$\Rightarrow P(B \cap A) = 0.6 \times 0.1 = 0.06$$

Also, $P(B \mid A^C) = 0.3$

$$\Rightarrow \frac{P(B \cap A^C)}{P(A^C)} = 0.3$$

$$\Rightarrow \frac{P(B) - P(B \cap A)}{1 - P(A)} = 0.3$$

$$\Rightarrow \frac{P(B) - 0.06}{0.9} = 0.3$$

$$\Rightarrow P(B) = 0.33$$

Now, $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.06}{0.33} = \dfrac{2}{11}$

164. (c) Let A be the event that a card is queen and B be the event that it is a spade.

Now, $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{1/52}{4/52} = \dfrac{1}{4}$

165. (a) $P(A^C \cap B^C) = P((A \cup B)^C) = 1 - P(A \cup B)$

Since, probability of occurrence of an event A implies the occurrence of event B,

$\therefore A \subset B \Rightarrow A \cup B = B$

$\Rightarrow P(A^C \cap B^C) = 1 - P(B) = P(B^C)$

166. (d) Let events T_1, T_2, T_3 be the following

T_1 : The item is manufactured by factory F_1.

T_2 : The item is manufactured by factory F_2.

T_3 : : The item is manufactured by factory F_3.

Clearly, T_1, T_2, T_3 are mutually exclusive and exhaustive events.

$P(T_1) = 30\% = 0.3$, $P(T_2) = 20\% = 0.2$

$P(T_3) = 50\% = 05$

Let E be the event that item is defective.

Now, $P(E|T_1) = 2\% = 0.02$

$P(E|T_2) = 3\% = 0.03$

$P(E|T_3) = 4\% = 0.04$

Hence, by Bayes' theorem, we have

$$P(T_1 | E) = \frac{P(T_1)\, P(E|T_1)}{P(T_1)P(E|T_1) + P(T_2)(P(E|T_2) + P(T_3)P(E|T_3)}$$

$$= \frac{0.3 \times 0.02}{0.3 \times 0.02 + 0.2 \times 0.03 + 05 \times 0.04}$$

$$= \frac{0.006}{0.006 + 0.006 + 0.20} = \frac{0.006}{0.032} = \frac{6}{32} = \frac{3}{16}$$

167. (a) To get an average of atleast 55 marks,

We have,

$$\frac{60 + 85 + x}{3} \geq 55$$

$$\Rightarrow 145 + x \geq 165$$

$$\Rightarrow x \geq 165 - 145$$

$$\Rightarrow x \geq 20$$

168. (b) Coefficient of variation $= \dfrac{\sigma}{\bar{x}} \times 100$

For first distribution, $60 = \dfrac{21}{\bar{x}} \times 100$

$$\bar{x} = \frac{21 \times 100}{60} = 35$$

For second distribution, $70 = \dfrac{16}{\bar{x}} \times 100$

$$\bar{x} = \frac{16 \times 100}{70} = 22.85$$

Hence, required means are 35, 22.85.

169. *(c)* Required probability $= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$

170. *(d)* Two non-empty sets have always non-empty intersection $\rightarrow$ logical

The real number n is less 2 $\rightarrow$ logical

Two individuals are always related $\rightarrow$ logical

171. *(d)* Objective function is not linear.

172. *(b)* Minimise $5x + 10y$

Subject to $x \geq 6$

$y \leq 2$

$x, y \geq 0$

173. *(b)* Given,

$$f(x) = \log(x + \sqrt{x^2 + 1})$$

$$\therefore \quad f(x) + f(-x) = \log(x + \sqrt{x^2 + 1}) + \log(-x + \sqrt{x^2 + 1}) = \log(1) = 0$$

$$\Rightarrow \quad f(-x) = -f(x)$$

Hence, $f(x)$ is an odd function.

174. *(b)* Given, $x^4 + \sqrt{x^4 + 20} = 22$

Add both sides 20, we get

$$x^4 + 20 + \sqrt{x^4 + 20} = 22 + 20$$

Let $\sqrt{x^4 + 20} = y$

$$\therefore \quad y^2 + y - 42 = 0$$

$$\Rightarrow \quad (y - 6)(y + 7) = 0$$

$$\Rightarrow \quad y = 6 \qquad [\because y \neq -7]$$

$$\Rightarrow \sqrt{x^4 + 20} = 6 \Rightarrow x^4 + 20 = 36$$

$$\Rightarrow \quad x^4 = 16 \Rightarrow x = \pm 2$$

Hence, the number of real roots of the equation is 2.

175. *(b)* Since, $\Sigma n = \frac{1}{78} \Sigma n^3$

$$\Rightarrow \quad \frac{n(n+1)}{2} = \frac{1}{78} \times \frac{n^2(n+1)^2}{4}$$

$$\Rightarrow \quad n^2 + n - 156 = 0$$

$$\Rightarrow \quad (n + 13)(n - 12) = 0$$

$$\Rightarrow \quad n = 12 \qquad [\because n \neq -13]$$

176. *(a)* A polygon of n sides has number of diagonals

$$= \frac{n(n-3)}{2} = 275 \qquad \text{[given]}$$

$$\Rightarrow \quad n^2 - 3n - 550 = 0$$

$$\Rightarrow \quad (n - 25)(n + 22) = 0$$

$$\therefore \quad n = 25 \qquad [\because n \neq -22]$$

177. *(c)* Now, $7^9 = (8 - 1)^9 = -1(1 - 8)^9$

$$= -1 + {}^9C_1 8 - {}^9C_2 8^2 + \ldots + {}^9C_9 8^9$$

and $9^7 = (1 + 8)^7 = 1 + {}^7C_1 8 + {}^7C_2 8^2 + {}^7C_3 8^3 + \ldots + {}^7C_7 8^7$

$$\therefore \quad 7^9 + 9^7 = 8({}^9C_1 + {}^7C_1) + 8^2({}^7C_2 - {}^9C_2) + \ldots$$

$$= 8(9 + 7) + 8^2(21 - 36) + \ldots$$

$$= 64 \times 2 + 64(-15) + \ldots$$

Hence, it is divisible by 64.

178. *(a)* Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 - bc & b^2 - ca & c^2 - ab \end{vmatrix}$

On applying $C_3 \rightarrow C_3 - C_2$ and $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ a & b - a & c - b \\ a^2 - bc & \begin{matrix}(b^2 - a^2) \\ + c(b - a)\end{matrix} & \begin{matrix}(c^2 - b^2) \\ + a(c - b)\end{matrix} \end{vmatrix}$$

$$= 1[(b - a)(c - b)(c + b + a) - (c - b)(b - a)(b + a + c)] = 0$$

179. *(d)* Given, $3\cos 2x - 10\cos x + 7 = 0$

$$\Rightarrow \quad 6\cos^2 x - 10\cos x + 4 = 0 \qquad [\because \cos 2x = 2\cos^2 x - 1]$$

$$\Rightarrow 2(3\cos x - 2)(\cos x - 1) = 0$$

$$\Rightarrow \quad \cos x = 1 \text{ or } \cos x = \frac{2}{3}$$

Hence, $\cos x$ is positive in Ist and IVth quadrants.

Hence, the total number of solutions is 4.

180. *(b)* $\lim_{x \to -\infty} \frac{2x - 1}{\sqrt{x^2 + 2x + 1}}$

$$= \lim_{y \to \infty} \frac{-2 - \frac{1}{y}}{\sqrt{1 - \frac{2}{y} + \frac{1}{y^2}}} \qquad [\text{put } x = -y, x \to -\infty]$$

$$= -\frac{2}{1} = -2$$